The Modal Assurance Criterion (MAC): Twenty Years of Use and Abuse

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ABSTRACT

This paper reviews the development of the original modal assurance criterion (MAC) together with other related assurance criteria that have been proposed over the last twenty years. Some of the other assurance criteria that will be discussed include the coordinate modal assurance criterion (COMAC), the frequency response assurance criterion (FRAC), coordinate orthogonality check (CORTHOG), frequency scaled modal assurance criterion (FMAC), partial modal assurance criterion (PMAC), scaled modal assurance criterion (SMAC), modal assurance criterion using reciprocal modal vectors (MACRV), etc.. In particular, the thought process that relates the original MAC development to ordinary coherence and to orthogonality computations will be explained. Several uses of MAC that may not be obvious to the casual observer (modal parameter estimation consistency diagrams and model updating are two examples) will be identified. The common problems with the implementation and use of modal assurance criterion computations will also be identified.

Nomenclature

L	= Number of matching pairs of modal vectors.
A^*	= Complex conjugate of A.
N_i	= Number of inputs.
N_o	= Number of outputs (assumed to be larger than N_i).
N_e	= Number of experimental modal vectors.
N_a	= Number of analytical modal vectors.
$H_{pq}(\omega)$	= Measured frequency response function.
$\hat{H}_{pq}(\omega)$	= Synthesized frequency response function.
ψ_{qr}	= Modal coefficient for degree-of-freedom q, mode r.
ψ_{pqr}	= Modal coefficient for reference p, degree-of-freedom q, mode r.
$\{\psi\}^T$	= Transpose of $\{\psi\}$.
$\{\psi\}^H$	= Complex conjugate transpose (Hermitian) of $\{\psi\}$.
$\{\psi_r\}$	= Modal vector for mode r.
$\{\psi_{pr}\}$	= Modal vector for reference p, mode r.

1. Introduction

The development of the modal assurance criterion [1-2] over twenty years ago has led to a number of similar assurance criteria used in the area of experimental and analytical structural dynamics. It is important to recognize the mathematical similarity of these varied criteria in order to be certain that conclusions be correctly drawn from what is essentially a squared, linear regression correlation coefficient. The modal assurance criterion is a statistical indicator, just like ordinary coherence, which can be very powerful when used correctly but very misleading when used incorrectly. This paper will first review the historical development of the modal assurance criterion. Other similar assurance criteria will then be identified although the list is not intended to be comprehensive. Typical uses of the modal assurance criterion will be discussed and, finally, typical abuses will be identified.

2. Historical Development of the Modal Assurance Criterion (MAC)

The historical development of the modal assurance criteria originated from the need for a quality assurance indicator for the experimental modal vectors that are estimated from measured frequency response functions. The standard of the late 1970's, when the modal assurance criterion was developed, was the orthogonality check. The orthogonality check, however, coupled errors in the analytical model development, the reduction of the analytical model and the estimated modal vectors into a single indicator and was, therefore, not always the best approach. Many times, an analytical model was not available rendering the orthogonality check impractical.

The original development of the modal assurance criterion was modeled after the development of the ordinary coherence calculation associated with the computation of the frequency response function. It is important to recognize that this least squares based form of linear regression analysis yields an indicator that is most sensitive to the largest difference between comparative values (minimizing the squared error) and results in a modal assurance criterion that is insensitive to small changes and/or small magnitudes. In the original thought process, this was considered as an advantage since small modal coefficient values are often seriously biased by frequency response function (FRF) measurement or modal parameter estimation errors.

In the internal development of the modal assurance criterion at the University of Cincinnati, Structural Dynamics Research Lab (UC-SDRL), a little modal assurance criterion (Little MAC) [1], a big modal assurance criterion (Big MAC) and a multiple modal assurance criterion (Multi-MAC) [3] were formulated as part of the original development. Little MAC and Multi-MAC were primarily testing methods and are not discussed further here. The modal assurance criterion that survives today is what was originally identified as Big MAC. Since the "Big Mac" acronym was already in use at that time, MAC is the designation that has persisted.

2.1 Modal Vector Orthogonality

The primary method that has historically been used to validate an experimental modal model is the weighted orthogonality check comparing measured modal vectors and an appropriately sized (the size of the square weighting matrix must match the length and spatial dimension of the modal vector) analytical mass or stiffness matrix (weighting matrix). Variations of this process include using analytical modal vectors together with experimental modal vectors and the appropriately sized mass or stiffness matrix. This latter comparison is normally referred to as a pseudo-orthogonality check (POC).

In the traditional orthogonality check, the experimental modal vectors are used together with a mass matrix, normally derived from a finite element model, to evaluate orthogonality of the experimental modal vectors. In the pseudo-orthogonality check, the experimental modal vectors are used together with a mass matrix, normally derived from a finite element model, and the analytical modal vectors, normally derived from the same finite element model, to evaluate orthogonality between the experimental and analytical modal vectors. The experimental and analytical modal vectors are scaled so that the diagonal terms of the modal mass matrix are unity. With this form of scaling, the off-diagonal values in the modal mass matrix are expected to be less than 0.1 (10 percent of the diagonal terms).

Theoretically, for the case of proportional damping, each modal vector of a system will be orthogonal to all other modal vectors of that system when weighted by the mass, stiffness, or damping matrix. In practice, these matrices are made available by way of a finite element analysis and normally the mass matrix is considered to be the most accurate. For this reason, any further discussion of orthogonality will be made with respect to mass matrix weighting. As a result, the orthogonality relations can be stated as follows:

Orthogonality of Modal Vectors

For $r \neq s$:

$$\{\psi_r\}^T [M] \{\psi_s\} = 0 \tag{1}$$

For $r = s$:

$$\{\psi_r\}^T [M] \{\psi_s\} = M_r \tag{2}$$

Experimentally, the result of zero for the cross orthogonality calculations ($r \neq s$) (Equation (1)) can rarely be achieved but values up to one tenth of the magnitude of the generalized mass of each mode are considered to be acceptable. It is a common procedure to form the modal vectors into a normalized set of mode shape vectors with respect to the mass matrix weighting. The accepted criterion in the aerospace industry, where this confidence check is made most often, is for all of the generalized mass terms to be unity and all cross orthogonality terms to be less than 0.1. Often, even under this criteria, an attempt is made to adjust the modal vectors so that the cross orthogonality conditions are satisfied [4-6]. Note that, in general, experimental modal vectors are not always real-valued and Equations (1) and (2) are developed based upon normal or real-valued modal vectors. This complication has to be resolved by a process of real normalization of the measured modal vectors prior to utilizing Equations (1) and (2) or by applying an equivalent procedure involving the state-space form of the weighted orthogonality relationship.

In Equations (1) and (2) the mass matrix must be an $N_o \times N_o$ matrix corresponding to the measurement locations on the structure. This means that the finite element mass matrix must be modified from whatever size and distribution of grid locations required in the finite element analysis to the $N_o \times N_o$ square matrix corresponding to the measurement locations. This normally involves some sort of reduction algorithm as well as interpolation of grid locations to match the measurement situation [7-13].

When Equation (1) is not sufficiently satisfied, one (or more) of three situations may exist. ***First, the modal vectors can be invalid.*** This can be due to measurement errors or problems with the modal parameter estimation algorithms. This is a very common assumption and many times contributes to the problem. ***Second, the mass matrix can be invalid.*** Since the mass matrix does not always represent the actual physical properties of the system when it is built or assembled, this probably contributes significantly to the problem. ***Third, the reduction of the mass matrix can be invalid [7-13].*** This can certainly be a realistic problem and cause severe errors. The most obvious example of this situation would be when a relatively large amount of mass is reduced to a measurement location that is highly flexible, such as the center of an unsupported panel. In such a situation, the measurement location is weighted very heavily in the orthogonality calculation of Equation (2) but may represent only incidental motion of the overall modal vector.

In all probability, all three situations contribute to the failure of orthogonality or pseudo-orthogonality criteria on occasion. When the orthogonality conditions are not satisfied, this result does not indicate where the problem originates. From an experimental point of view, it is important to try to develop methods that indicate confidence that the modal vector is, or is not, part of the problem.

2.2 Modal Vector Consistency

Since the frequency response function matrix contains redundant information with respect to a modal vector, the consistency of the estimate of the modal vector under varying conditions such as excitation locations (references) or modal parameter estimation algorithms can be a valuable confidence factor to be utilized in the process of evaluation of the experimental modal vectors.

The common approach to estimation of modal vectors from frequency response functions is to measure several complete rows or columns of the frequency response function matrix. The estimation of the modal vectors from this frequency response function matrix will be a function of the data used in the modal parameter estimation algorithms and the specific modal parameter estimations algorithms used. If the modal vectors are not well represented in the frequency response function matrix, the estimation of the modal vector will contain potential bias and variance errors. In any case, the modal vectors will contain potential variance errors.

Frequently, different subsets of the frequency response function matrix and/or different modal parameter estimation algorithms are utilized to estimate separate, redundant modal vectors for comparison purposes. In these cases, if different estimates of the same modal vectors are generated, the modal vectors can be compared and contrasted through an evaluation that consists of the calculation of a complex modal scale factor (relating two modal vectors) and a scalar modal assurance criterion (measuring the consistency or linearity between two modal vectors).

The function of the ***modal scale factor (MSF)*** is to provide a means of normalizing all estimates of the same modal vector, taking into account magnitude and phase differences. Once two different modal vector estimates are scaled similarly, elements of each vector can be averaged (with or without weighting), differenced, or sorted to provide a best estimate of the modal vector or to provide an indication of the type of error vector superimposed on the modal vector. In terms of modern, multiple reference modal parameter estimation algorithms, the modal scale factor is a normalized estimate of the modal participation factor between two references for a specific mode of vibration.

The function of the ***modal assurance criterion (MAC)*** is to provide a measure of consistency (degree of linearity) between estimates of a modal vector. This provides an additional confidence factor in the evaluation of a modal vector from different excitation (reference) locations or different modal parameter estimation algorithms.

The modal scale factor and the modal assurance criterion also provide a method of easily comparing estimates of modal vectors originating from different sources. The modal vectors from a finite element analysis can be compared and contrasted with those determined experimentally as well as modal vectors determined by way of different experimental or modal parameter estimation methods. In this approach, methods can be compared and contrasted in order to evaluate the mutual consistency of different procedures rather than estimating the modal vectors specifically. If an analytical and an experimental vector are deemed consistent or similar, the analytical modal vector, together with the modal scale factor, can be used to complete the experimental modal vector if some degrees of freedom could not be measured.

The ***modal scale factor*** is defined, according to this approach, as follows:

$$MSF_{cdr} = \frac{\sum_{q=1}^{N_o} \psi_{cqr} \, \psi^*_{dqr}}{\sum_{q=1}^{N_o} \psi_{dqr} \, \psi^*_{dqr}} \tag{3}$$

or:

$$MSF_{cdr} = \frac{\{\psi_{cr}\}^T \left\{\psi^*_{dr}\right\}}{\{\psi_{dr}\}^T \left\{\psi^*_{dr}\right\}} \tag{3a}$$

Since the modal scale factor is a complex-valued scalar, this is also equivalent to:

$$MSF_{cdr} = \frac{\{\psi_{dr}\}^H \, \{\psi_{cr}\}}{\{\psi_{dr}\}^H \, \{\psi_{dr}\}} \tag{3b}$$

Equation (3) implies that the modal vector d is the reference to which the modal vector c is compared. In the general case, modal vector c can be considered to be made of two parts. The first part is the part correlated with modal vector d. The second part is the part that is not correlated with modal vector d and is made up of contamination from other modal vectors and of any random contribution. This error vector is considered to be noise. The ***modal assurance criterion*** is defined as a scalar constant relating the degree of consistency (linearity) between one modal and another reference modal vector as follows:

Consistency of Modal Vectors

$$MAC_{cdr} = \frac{\left|\sum_{q=1}^{N_o} \psi_{cqr} \, \psi^*_{dqr}\right|^2}{\sum_{q=1}^{N_o} \psi_{cqr} \, \psi^*_{cqr} \sum_{q=1}^{N_o} \psi_{dqr} \, \psi^*_{dqr}} \tag{4}$$

or:

$$MAC_{cdr} = \frac{\left|\{\psi_{cr}\}^T \left\{\psi^*_{dr}\right\}\right|^2}{\{\psi_{cr}\}^T \left\{\psi^*_{cr}\right\} \{\psi_{dr}\}^T \left\{\psi^*_{dr}\right\}} \tag{4a}$$

Since the modal assurance criterion is a real-valued scalar, this is also equivalent to:

$$MAC_{cdr} = \frac{\left|\{\psi_{dr}\}^H \, \{\psi_{cr}\}\right|^2}{\{\psi_{dr}\}^H \, \{\psi_{dr}\} \, \{\psi_{cr}\}^H \, \{\psi_{cr}\}} \tag{4b}$$

or:

$$MAC_{cdr} = \frac{\{\psi_{dr}\}^H \, \{\psi_{cr}\} \, \{\psi_{cr}\}^H \, \{\psi_{dr}\}}{\{\psi_{dr}\}^H \, \{\psi_{dr}\} \, \{\psi_{cr}\}^H \, \{\psi_{cr}\}} \tag{4c}$$

or:

$$MAC_{cdr} = MSF_{cdr} \; MSFdcr \tag{4d}$$

The modal assurance criterion takes on values from zero, representing no consistent correspondence, to one, representing a consistent correspondence. In this manner, if the modal vectors under consideration truly exhibit a consistent, linear relationship, the modal assurance criterion should approach unity and the value of the modal scale factor can be considered to be reasonable. Note that, unlike the orthogonality calculations, the modal assurance criterion is normalized by the magnitude of the vectors and, thus, is bounded between zero and one.

The modal assurance criterion can only indicate consistency, not validity or orthogonality. If the same errors, random or bias, exist in all modal vector estimates, this is not delineated by the modal assurance criterion. Invalid assumptions are normally the cause of this sort of potential error. Even though the modal assurance criterion is unity, the assumptions involving the system or the modal parameter estimation techniques are not necessarily correct. The assumptions may cause consistent errors in all modal vectors under all test conditions verified by the modal assurance criterion.

Modal Assurance Criterion (MAC) Zero

If the modal assurance criterion has a value near zero, this is an indication that the modal vectors are not consistent. This can be due to any of the following reasons:

- The system is non-stationary. This can occur if the system is nonlinear and two data sets have been acquired at different times or excitation levels. System nonlinearities will appear differently in frequency response functions generated from different exciter positions or excitation signals. The modal parameter estimation algorithms will also not handle the different nonlinear characteristics in a consistent manner.
- There is noise on the reference modal vector. This case is the same as noise on the input of a frequency response function measurement. No amount of signal processing can remove this type of error.
- The modal parameter estimation is invalid. The frequency response functions measurements may contain no errors but the modal parameter estimation may not be consistent with the data.
- The modal vectors are from linearly unrelated mode shape vectors. Hopefully, since the different modal vector estimates are from different excitation positions this measure of inconsistency will imply that the modal vectors are orthogonal.

Obviously, if the first four reasons can be eliminated, the modal assurance criterion can be interpreted in a similar way as an orthogonality calculation.

Modal Assurance Criterion (MAC) Unity

If the modal assurance criterion has a value near unity, this is an indication that the modal vectors are consistent. This does not necessarily mean that they are correct. The modal vectors can be consistent for any of the following reasons:

- The modal vectors have been incompletely measured. This situation can occur whenever too few response stations have been included in the experimental determination of the modal vector.
- The modal vectors are the result of a forced excitation other than the desired input. This would be the situation if, during the measurement of the frequency response function, a rotating piece of equipment with an unbalance is present in the system being tested.
- The modal vectors are primarily coherent noise. Since the reference modal vector may be arbitrarily chosen, this modal vector may not be one of the true modal vectors of the system. It could simply be a random noise vector or a vector reflecting the bias in the modal parameter estimation algorithm. In any case, the modal assurance criterion will only reflect a consistent (linear) relationship to the reference modal vector.
- The modal vectors represent the same modal vector with different arbitrary scaling. If the two modal vectors being compared have the same expected value when normalized, the two modal vectors should differ only by the complex valued scale factor which is a function of the common modal coefficients between the rows or columns.

Therefore, if the first three reasons can be eliminated, the modal assurance criterion indicates that the modal scale factor is the complex constant relating the modal vectors and that the modal scale factor can be used to average, difference, or sort the modal vectors.

Under the constraints mentioned previously, the modal assurance criterion can be applied in many different ways. The modal assurance criterion can be used to verify or correlate an experimental modal vector with respect to a theoretical modal vector (eigenvector). This can be done by computing the modal assurance criterion between N_e modal vectors estimated from experimental data and N_a modal vectors estimated from a finite element analysis evaluated at common stations. This process results in a $N_e \times N_a$ rectangular modal assurance criterion matrix with values that approach unity whenever an experimental modal vector and an analytical modal vector are consistently related.

Once the modal assurance criterion establishes that two vectors represent the same information, the vectors can be averaged, differenced, or sorted to determine the best single estimate or the potential source of contamination using the modal scale factor. Since the modal scale factor is a complex scalar that allows two vectors to be phased the same and to the same mean value, these vectors can be subtracted to evaluate whether the error is random or biased. If the error appears to be random and the modal assurance criterion is high, the modal vectors can be averaged, using the modal scale factor, to improve the estimate of a modal vector. If the error appears to be biased or skewed, the error pattern often gives an indication that the error originates due to the location of the excitation or due to an inadequate modal parameter estimation process. Based upon partial but overlapping measurement of two columns of the frequency response function matrix, modal vectors can be sorted, assuming the modal assurance function indicates consistency, into a complete estimate of each modal vector at all measurement stations.

The modal assurance criterion can be used to evaluate modal parameter estimation methods if a set of analytical frequency response functions with realistic levels of random and bias errors is generated and used in common to a variety modal parameter estimation methods. In this way, agreement between existing methods can be established and new modal parameter estimation methods can be checked for characteristics that are consistent with accepted procedures. Additionally, this approach can be used to evaluate the characteristics of each modal parameter estimation method in the presence of varying levels of random and bias error.

The concept of consistency in the estimate of modal vectors from separate testing constraints is important considering the potential of multiple estimates of the same modal vector from numerous input configurations and modal parameter estimation algorithms. The computation of modal scale factor and modal assurance criterion results in a complex scalar and a correlation coefficient which does not depend on weighting information outside the testing environment. Since the modal scale factor and modal assurance criterion are computed analogous to the frequency response function and coherence function, both the advantages and limitations of the computation procedure are well understood. These characteristics, as well as others, provide a useful tool in the processing of experimental modal vectors.

3. Other Similar Assurance Criteria

The following brief discussion highlights assurance criteria that utilize the same linear, least squares computation approach to the analysis (projection) of two vector spaces as the modal assurance criterion. The equations for each assurance criterion are not repeated unless there is a significant computational difference that needs to be clarified or highlighted. This list is by no means comprehensive nor is it in any particular order of importance but includes most of the frequently cited assurance criterion found in the literature.

3.1 Weighted Modal Analysis Criterion (WMAC)

A number of authors have utilized a weighted modal assurance criterion (WMAC) without developing a special designation for this case. WMAC is proposed for these cases. The purpose of the weighting matrix is to recognize that MAC is not sensitive to mass or stiffness distribution, just sensor distribution, and to adjust the modal assurance criterion to weight the degrees-of-freedom in the modal vectors accordingly. In this case, the WMAC becomes a unity normalized orthogonality, or psuedo-orthogonality, check where the desirable result for a set of modal vectors would be ones along the diagonal (same modal vectors) and zeros off-diagonal (different modal vectors) regardless of the scaling of the individual modal vectors. Note that the weighting matrix is applied to as an inner matrix product for the single numerator vector product and both vector products in the denominator.

3.2 Partial Modal Analysis Criterion (PMAC)

The partial modal assurance criterion (PMAC) [14] is developed as a spatially limited version of the modal assurance criterion where a subset of the complete modal vector is used in the calculation. The subset is chosen based upon the user’s interest and may reflect only a certain dominant sensor direction (X, Y, and/or Z) or only the degrees-of-freedom from a component of the complete modal vector.

3.3 Modal Assurance Criterion Square Root (MACSR)

The square root of the modal assurance criterion (MACSR) [15] is developed to be more consistent with the orthogonality and psuedo-orthogonality calculations using an identity weighting matrix. Essentially, this approach utilizes the square root of the MAC calculation which tends to highlight the cross terms (off-diagonal) which are generally the MAC values that are very small.

3.4 Scaled Modal Assurance Criterion (SMAC)

The scaled modal assurance criterion (SMAC) [16] is essentially a weighted modal assurance criteria (WMAC) where the weighting matrix is chosen to balance the scaling of translational and rotational degrees-of-freedom included in the modal vectors. This development is needed whenever different data types (with different engineering units) are included in the same modal vector to normalize the magnitude differences in the vectors. This is required since the modal assurance criterion minimizes the squared error and is dominated by the larger values.

3.5 Modal Assurance Criterion Using Reciprocal Vectors (MACRV)

A reciprocal modal vector is defined as the mathematical vector that, when transposed and premultiplied times a specific modal vector, yields unity. When the same computation is performed with this reciprocal modal vector and any other modal vector or any other reciprocal modal vector, the result is zero. The reciprocal modal vector can be thought of as a product of the modal vector and the unknown weighting matrix that will produce a perfect orthogonality result. Reciprocal modal vectors are computed directly from measured frequency response functions and the experimental modal vectors and are, therefore, experimentally based.

The modal assurance criterion using reciprocal modal vectors (MACRV) [17] is the comparison of reciprocal modal vectors with analytical modal vectors in what is very similar to a psuedo-orthogonality check (POC). The reciprocal modal vectors are utilized in controls applications as modal filters and the MACRV serves as a check of the mode isolation provided by each reciprocal modal vector compared to analytical modes expected.

3.6 Modal Assurance Criterion with Frequency Scales (FMAC)

Another extension of the modal assurance criterion is the addition of frequency scaling to the modal assurance criterion [18-19]. This extension of MAC "offers a means of displaying simultaneously the mode shape correlation, the degree of spatial aliasing and the frequency comparison in a single plot". This development is particularly useful in model correlation applications (model updating, assessment of parameter variation, etc.)

3.7 Coordinate Modal Assurance Criterion (COMAC)

An extension of the modal assurance criterion is the coordinate modal assurance criterion (COMAC) [20]. The COMAC attempts to identify which measurement degrees-of-freedom contribute negatively to a low value of MAC. The COMAC is calculated over a set of mode pairs, analytical versus analytical, experimental versus experimental or experimental versus analytical. The two modal vectors in each mode pair represents the same modal vector but the set of mode pairs represents all modes of interest in a given frequency range. For two sets of modes that are to be compared, there will be a value of COMAC computed for each (measurement) degree-of-freedom.

The coordinate modal assurance criterion (COMAC) is calculated using the following approach, once the mode pairs have been identified with MAC or some other approach:

$$COMAC_q = \frac{\sum_{r=1}^{L} \left| \psi_{qr} \; \phi_{qr} \right|^2}{\sum_{r=1}^{L} \psi_{qr} \; \psi_{qr}^* \; \sum_{r=1}^{L} \phi_{qr} \; \phi_{qr}^*} \tag{5}$$

Note that the above formulation assumes that there is a match for every modal vector in the two sets and the modal vectors are renumbered according so that the matching modal vectors have the same subscript. Only those modes that match between the two sets are included in the computation.

3.8 Enhanced Coordinate Modal Assurance Criterion (ECOMAC)

One common problem with experimental modal vectors is the potential problem of calibration scaling errors and/or sensor orientation mistakes. The enhanced coordinate modal assurance criterion (ECOMAC) [21] was developed to extend the COMAC computation to be more aware of typical experimental errors that occur in defining modal vectors such as sensor scaling mistakes and sensor orientation (plus or minus sign) errors.

3.9 Mutual Correspondence Criterion (MCC)

The mutual correspondence criterion (MCC) [22] is the modal assurance criterion applied to vectors that do not originate as modal vectors but as vector measures of acoustic information (velocity, pressure, intensity, etc.). The equation in this formulation utilizes a transpose and will only correctly apply to real valued vectors.

3.10 Modal Correlation Coefficient (MCC)

One of the natural limitations of a least squares based correlation coefficient like the modal assurance criterion is that it is relatively insensitive to small changes in magnitude, position by position, in the vector comparisons. The modal correlation coefficient (MCC) [23-24] is a modification of MAC that attempts to provide a more sensitive indicator. This approach is particularly important when using modal vectors in damage detection situations where the magnitude changes of the modal vectors being measured are minimal.

3.11 Inverse Modal Assurance Criterion (IMAC)

An alternative approach to increasing the sensitivity of the modal assurance criterion to small mode shape changes is the inverse modal assurance criterion (IMAC) [25] . This approach uses essentially the same computational scheme as MAC but utilizes the inverse of the modal coefficients. Therefore, small modal coefficients become significant in the least squares based correlation coefficient computation. Naturally, this computation suffers from the possibility that a modal coefficient could be numerically zero.

3.12 Frequency Response Assurance Criterion (FRAC)

Any two frequency response functions representing the same input-output relationship can be compared using a technique known as the frequency response assurance criterion (FRAC) [26-28] . The simplest example is a validation procedure that compares the FRF data synthesized from the modal model with the measured FRF data. The basic assumption is that the measured frequency response function and the synthesized frequency response function should be linearly related (unity scaling coefficient) at all frequencies. Naturally, the FRFs can be compared over the full or partial frequency range of the FRFs as long as the same discrete frequencies are used in the comparison. This approach has been utilized in the modal parameter estimation process for a number of years under various designations (parameter estimation correlation coefficient [29] , synthesis correlation coefficient [30] and response vector assurance criterion (RVAC) [31]). This procedure is particularly effective as a modal parameter estimation validation procedure if the measured data was not part of the data used to estimate the modal parameters. This serves as an independent check of the modal parameter estimation process.

$$FRAC_{pq} = \frac{\left| \sum_{\omega=\omega_1}^{\omega_2} H_{pq}(\omega)\, \hat{H}_{pq}^{\,*}(\omega) \right|^2}{\sum_{\omega=\omega_1}^{\omega_2} H_{pq}(\omega)\, H_{pq}^{\,*}(\omega) \; \sum_{\omega=\omega_1}^{\omega_2} \hat{H}_{pq}(\omega)\, \hat{H}_{pq}^{\,*}(\omega)} \qquad (6)$$

3.13 Complex Correlation Coefficient (CCF)

A significant variation in the frequency response assurance criterion is the complex correlation coefficient (CCF) [31] which is computed without squaring the numerator term, thus, yielding a complex valued coefficient. The magnitude of the coefficient is the same as the FRAC computation but the phase describes any systematic phase lag or lead that is present between the two FRFs. In situations where analytical and experimental FRFs are compared, the CCF will detect the common problem of a constant phase shift that might be due to experimental signal conditioning problems, etc.

3.14 Frequency Domain Assurance Criterion (FDAC)

A similar variation in the frequency response assurance criterion is the frequency domain assurance criterion (FDAC) [32] which is a FRAC type of calculation evaluated with different frequency shifts. Since the difference in impedance (FRF) model updating is often an FRF that is in question due to frequencies of resonances or anti-resonances, the FDAC is formulated to identify this problem. A related criterion, the modal FRF assurance criterion (MFAC) [18] , combines analytical modal vectors with measured frequency response functions (FRFs) in an extension of FRAC and FDAC that weights or filters the FRF data based upon the expected, analytical modal vectors.

3.15 Coordinate Orthogonality Check (CORTHOG)

The coordinate orthogonality check (CORTHOG) [33] is a normalized error measure between the pseudo orthogonality calculation, comparing measured to analytical modal vectors, and the analytical orthogonality calculation, comparing analytical to analytical modal vectors. Several different normalizing or scaling methods are used with this calculation.

4. Uses of the Modal Assurance Criterion

Most of the potential uses of the modal assurance criterion are well known but a few may be more subtle. A partial list of the most typical uses that have been reported in the literature are as follows:

- Validation of experimental modal models
- Correlation with analytical modal models (mode pairing)
- Correlation with operating response vectors
- Mapping matrix between analytical and experimental modal models
- Modal vector error analysis
- Modal vector averaging

- Experimental modal vector completion and/or expansion
- Weighting for model updating algorithms
- Modal vector consistency/stability in modal parameter estimation algorithms
- Repeated and psuedo-repeated root detection
- Structural fault/damage detection
- Quality control evaluations
- Optimal sensor placement

5. Abuses of the Modal Assurance Criterion

Many of the alternate formulations of the modal assurance criterion were developed to address some of the shortcomings of the original modal assurance criterion formulation. When users utilize the original modal assurance criterion in these situations, a poor result will often follow. For the purposes of this discussion, this is referred to as misuse or abuse. The misuse or abuse of the modal assurance criterion generally results due to one of five issues. These issues can be summarized as:

- The modal analysis criterion is not an orthogonality check.
- The wrong mathematical formulation for the modal assurance criterion is used.
- The modal assurance criterion is sensitive to large values (wild points?) and insensitive to small values.
- The number of elements in the modal vectors (space) is small.
- The modal vectors have been zero padded.

These issues can be further explained in the following paragraphs.

The modal analysis criterion is not an orthogonality check. It is important to recognize that the modal assurance criterion effectively weights the computation based upon the spatial distribution of the degrees-of-freedom included in the modal vectors. The modal assurance criterion does not weight the modal vectors with a mass or stiffness matrix and, therefore, cannot compensate for situations where a very limited number of degrees-of-freedom (sensors) have been placed on a massive sub-structure of a mechanical system. The typical example involves the engine of an automobile. If few or no sensors are placed on the engine and a large number are placed on the surface of the automobile body, several modal vectors at different modal frequencies will have very high MAC numbers indicating that the modal vectors are the same. This example indicates to the user that an incomplete modal vector was measured and the user has violated one of the primary assumptions of experimental modal analysis (observability).

The wrong mathematical formulation for the modal assurance criterion is used. Frequently, users implement the modal assurance criterion, or a related similar computation, using a vector transpose in the numerator and denominator calculations rather than an Hermitian (conjugate transpose). This error causes no problem as long as analytical vectors or real-valued experimental vectors are involved in the calculation. However, in the general case, where some of the vectors are complex-valued, this does not give the correct result. The original mathematical formulation assumes the general case but has been reported incorrectly in some literature. This innocent error often occurs when the author is utilizing real-valued vectors and notices no problem. However, users who do not recognize this issue are often led astray in subsequent applications involving complex-valued vectors.

The modal assurance criterion is sensitive to large values (wild points?) and insensitive to small values. The modal assurance criterion is based upon the minimization of the squared error between two vector spaces. This means that the degrees-of-freedom involving the largest magnitude differences between the two modal vectors will dominate the computation while small differences will have almost no effect. Therefore, nodal information (small modal coefficients) will generally not have much effect on the MAC calculation and large modal coefficients will potentially have the greatest effect. This also means that, if there have been erroneous data included in the modal vectors due to calibration errors, modal parameter estimation mistakes, etc., these wild points may dominate the MAC calculation.

The number of elements in the modal vectors (space) is small. Since the modal assurance criterion is essentially a statistical computation where the number of averages comes from the number of elements in the modal vectors, if the modal vectors have only a limited number of degrees-of-freedom, this will skew the meaning of the numerical MAC value. This frequently happens when high order, multiple reference modal parameter estimation algorithms estimate the stability or consistency diagram. Modal vector stability or consistency is identified using a MAC computation where the vectors include only the degrees-of-freedom at the reference locations, typically, two to five. In these situations, there may be great variability in the MAC computation, particularly if the modal vector is not well excited from one or more of the reference locations. Obviously, vectors with many elements reduce the sensitivity of MAC to this problem.

The modal vectors have been zero padded. Frequently, when modal vectors are exported from one computational environment to another, the modal vectors include zero values when no value was ever measured, or computed, for that degree-of-freedom. For example, in an experimental situation, one (X) or two dimensions (X,Y) of translational response may be measured at some degrees-of-freedom rather than three dimensions (X,Y,Z). In the commonly used Universal File Format for modal vectors (File Format 55), this is the case since there is no designation for not measuring the information. When the modal assurance criterion is calculated for this case, there will be a problem if some other vectors, with nonzero information at these degrees-of-freedom are included in the computation. This can be avoided, if information is dropped from the computation when either vector includes a perfect zero (within computational precision) at a degree-of-freedom, but is rarely done.

6. Current Developments

Currently, many users are utilizing more statistical approaches to understand the meaning and bounds of experimental modal parameters [34-39]. This approach extends to the modal assurance criterion as well. Examples are the bootstrap and jackknife approaches [40-42] to the evaluation of the mean and standard deviation of discrete sets of experimental data. These approaches remove and/or replace portions of the computation (bootstrap uses replicative resampling, jackknife uses sequential elimination) to evaluate the bounds or limits on the MAC values. In this way, the sensitivity of the MAC computation can be more effectively evaluated than with the current single number indicating the degree of linearity between two modal vectors that are being compared.

7. Conclusions

Over the last twenty years, the modal assurance criterion has demonstrated how a simple statistical concept can become an extremely useful tool in the field of experimental modal analysis and structural dynamics. The use of the modal assurance criterion, and the development and use of a significant number of related criteria, has been remarkable and is most likely due to the overall simplicity of the concept. New uses of the modal assurance criterion and new criteria will be developed over the next years as users more fully understand the limitations of the current criteria. Certainly, in the next few years, the increased use of other statistical methods as well as further development of singular value/vector methods are related areas that will generate useful tools in this area.

Even so, it will always important to recognize the origins and limitations of tools like the modal assurance criterion to avoid misuse of the methodology. Simplistic tools like the modal assurance criterion are limited in their meaningful application. The development of related assurance criteria has been initiated by shortcomings, real or perceived, of the original modal assurance criterion. Dissatisfaction often has resulted from the misuse of these tools by users, removed from the actual development or unaware of application limitations in subsequent implementations. It is clear that users will continue to need more feedback concerning quality assurance information relative to experimental modal parameters and that new techniques, particularly statistical methods that utilize the redundant information present in the measured data, will continue to be developed with strengths and weaknesses, just like the modal assurance criterion.

8. Acknowledgements

The author would like to acknowledge the number of authors cited as developing similar assurance criterion and apologize to those who were not identified due to space limitations. Most of all, the author would like to acknowledge the contribution of Ray Zimmerman in the original development of the modal assurance criterion. Ray contributed to many of the developments of the UC-SDRL during the 1970s and 1980s by providing interesting ideas and thoughtful discussion. We miss having the opportunity to bounce ideas off Ray.

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