Comparative study of model correlation methods with application to model order reduction

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Abstract

Model correlation methods are utilized to qualitatively compare a reference model with a modified model. Such methods are commonly used in structural dynamics for model validation and model updating. This article focuses on model correlation methods with the application to model order reduction. Although only used once after reduction, the correlation can be even more expensive compared to the whole reduction process. This issue gains importance when dealing with large models or utilizing correlation methods within optimization processes like model updating where iterative calculations are required. The study presented aims for efficient and expressive correlation methods by pursuing two objectives.

Plenty of different correlation methods can be found in current literature. Within the scope of this study, 60 different methods were implemented in MATLAB. The first objective is to pick out the most preferable methods concerning information quality and computation time. Since every correlation method proves a certain characteristic, a few encouraging methods are selected, which yield by combination a valid frequency range for the further utilization of the reduced model.

When dealing with reduced order models, the dimension disagreement between the eigenvectors of the full and reduced order model has to be taken into account. The article demonstrates how this problem is solved by means of expansion and contraction. The dimension adaption is the key issue when dealing with large models, since the size of the eigenvectors mainly affects calculation time. The second objective lies in finding an appropriate size of the eigenvectors without significantly changing the correlation result. For this purpose, two novel methods are presented, where the eigenvectors are contracted by DoF-selection or adapted to an intermediate dimension by using sensor placement methods.

Besides, the preparation of the data sets is a crucial issue for the correlation success. Therefore the topics of automatic mode pairing, handling of multiple eigenvalues and detection of rigid body modes are also discussed. Finally, the presented methods are demonstrated by means of two example models: The symmetric model of a wheel set axle and the comparatively large and geometrically complex model of a gear box. A final selection of preferred methods concerning efficiency and information quality is presented.

1 Introduction

Model correlation methods are utilized to qualitatively compare a reference model with a modified model and are particularly applied in structural dynamics for model validation and model updating. The data sets for comparison are usually generated by means of analytical FE-modelling and via experimental modal analysis. Alternatively two analytical data sets may be compared, e.g. either for mesh convergence investigations or for model order reduction purposes, where in the latter case the full and the reduced order model are compared with each other. Depending on whether the data is extracted via model or measurement, the data set consists of natural frequencies, mode shapes, transfer functions or system matrices.

Numerous correlation methods can be found in current literature, whereby every correlation method proves a certain characteristic. Depending on the correlation method, the calculation effort can be vast and even exceeds the reduction effort. Therefore, the objective is the investigation of promising methods available and then selecting a few encouraging correlation methods concerning information quality and computation time. The latter is important when dealing with large models. The outcome of this study can be used whenever fast and reliable correlation methods are required, for instance for optimization purposes, e.g. model updating [1] and robust design [2], where the focus lies on iterative processes.

This article focuses the application of correlation methods to model order reduction. For this purpose, distinct specifics must be considered. On the one hand, modern model order reduction techniques are often based upon the approximation of the input-output behaviour. In control theory, where the input-output behaviour is of interest, usually the correlation based upon the frequency response via the infinity-norm is considered [3]. Especially, for an automated model order reduction with an iterative selection of expansion points, the error between original and reduced order system has to be ascertained in a numerically effective manner – this is often done by the H_2-norm of the frequency response [4], [5]. On the other hand, modal reduction methods exist over decades. Their approximation quality is usually assessed by modal correlation methods based on natural frequencies and mode shapes [6], [7], which need much less computational effort. Regarding both aspects the question arises, whether modal correlation methods are able to indicate the input-output behaviour of a mechanical system instead of using expensive frequency-response-based techniques?

Another specific of model order reduction lies in the dimension disagreement between the eigenvectors of the full and reduced order model. This problem is usually overcome by using expansion and contraction methods, e.g. [8]. The issue of dimension adaption plays a key role when dealing with large models, since the size of the eigenvectors mainly affects calculation effort. Finding an appropriate size of the eigenvectors without significantly changing the correlation results represents the second objective of this article. Therefore, two novel methods are presented, where the eigenvectors are contracted by DoF-selection or adapted to an intermediate dimension by choosing a proper DoF-subset according to existing sensor placement methods.

In Section 2 the basics concerning analytical modelling, model order reduction and frequency response are presented. In Section 3 numerous correlation methods are categorized and specified. Since the preparation of the data sets is a crucial issue for the correlation success, automatic mode pairing, handling of multiple eigenvalues and detection of rigid body modes are treated in Section 4. Section 5 deals with the dimension adaption of the eigenvectors due to model order reduction. Finally, by using two different example models a preferred approach for the effective and reliable correlation for large models is derived in Section 6.

2 Fundamentals

The numeric modal analysis and the model order reduction constitute the basics for correlating the dynamic behaviour of the original and reduced order model. Fundamentals and required notations are introduced.

2.1 Numerical modal analysis

The correlation methods presented are all based on validation models gained by means of the linear-elastic Finite Element Method (FEM). For the FEM-formulation the elastic continuum is discretized via Finite Elements (FE). Therefore, a Ritz-approach for the element shape functions and a weak formulation of the equilibrium is used, which leads to a linear system of N ordinary differential equations of second order

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{D}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f} \tag{1}$$

with displacement vector $\mathbf{x} \in \mathbb{R}^N$, external load vector $\mathbf{f} \in \mathbb{R}^N$ and the system matrices for mass, damping and stiffness $\{\mathbf{M}, \mathbf{D}, \mathbf{K}\} \in \mathbb{R}^{N\times N}$ [9]. Gyroscopic terms are not considered and the damping assumption is based on a modal approach. The second order state space formulation of equation (1) can be gained by defining an input $\mathbf{u}(t)$ and output vector $\mathbf{y}(t)$. The mapping of the N equations onto the i inputs and o outputs is done by the input $\mathbf{B} \in \mathbb{R}^{N \times i}$ and output matrix $\mathbf{C} \in \mathbb{R}^{o \times N}$:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{D}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{B}\mathbf{u}(t), \quad \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \quad . \tag{2}$$

Following, the numerical modal analysis is outlined, by extracting natural frequencies and eigenvalues which are required for correlation purposes. Since only modal damping is considered, the homogenous solution of the undamped linear system (1) is used which leads to the general eigenvalue problem

$$\left(\mathbf{K} - \omega_{0,i}^2 \cdot \mathbf{M}\right)\boldsymbol{\phi}_i = \mathbf{0}, \quad i = 1, \dots, p \tag{3}$$

with the i-th eigenvalue $\lambda_i = \omega_{0,i}^2$ and the corresponding eigenvector $\boldsymbol{\phi}_i$. For mechanical models without kinematic boundary conditions, null eigenvalues are included which indicate rigid body motion. Due to computational reasons, not all possible N modes are extracted, but only $p \ll N$ representative ones. Selected eigenvalues and eigenmodes are contained in the spectral matrix $\boldsymbol{\Lambda} \in \mathbb{R}^{p \times p}$ and the modal matrix $\boldsymbol{\Phi} \in \mathbb{R}^{N \times p}$:

$$\boldsymbol{\Lambda} = \operatorname{diag}\left(\lambda_1, \cdots, \lambda_i, \cdots, \lambda_p\right), \quad \boldsymbol{\Phi} = \left(\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_i, \cdots, \boldsymbol{\phi}_p\right) \quad . \tag{4}$$

For subsequent purposes a modal transformation $\mathbf{x} = \boldsymbol{\Phi}\mathbf{q}$ is necessary, whereby the displacement vector $\mathbf{x}$ is represented via modal coordinates $\mathbf{q}$ by using the modal matrix $\boldsymbol{\Phi}$ for transformation. Application onto the equation of motion (1) leads to a decoupled linear system of equations for the modal damping assumption:

$$\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi}\ddot{\mathbf{q}} + \boldsymbol{\Phi}^T\mathbf{D}\boldsymbol{\Phi}\dot{\mathbf{q}} + \boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi}\mathbf{q} = \boldsymbol{\Phi}^T\mathbf{f} \quad \Rightarrow \quad \widetilde{\mathbf{M}}\ddot{\mathbf{q}} + \widetilde{\mathbf{D}}\dot{\mathbf{q}} + \widetilde{\mathbf{K}}\mathbf{q} = \widetilde{\mathbf{f}} \quad . \tag{5}$$

The diagonalized system matrices contain the modal coefficients and can be rewritten in vector format:

$$\widetilde{\mathbf{m}} = \operatorname{diag}(\widetilde{\mathbf{M}}), \quad \widetilde{\mathbf{d}} = \operatorname{diag}(\widetilde{\mathbf{D}}), \quad \widetilde{\mathbf{k}} = \operatorname{diag}(\widetilde{\mathbf{K}}) \quad . \tag{6}$$

Since the modal matrix is not unique, the normalization of the eigenvectors plays an important role. Normalizing the eigenvectors to modal mass $\widetilde{m}_i$ by $\boldsymbol{\phi}_{N,i} = \boldsymbol{\phi}_i \cdot (\widetilde{m}_i)^{-1/2}$ leads to the orthogonality properties:

$$\widetilde{\mathbf{M}} = \mathbf{E}, \quad \widetilde{\mathbf{K}} = \boldsymbol{\Lambda} \quad \text{with} \quad \boldsymbol{\omega}_0^2 = \operatorname{diag}(\boldsymbol{\Lambda}) \quad . \tag{7}$$

Eigenvectors are distinguished into displacement eigenvectors $\boldsymbol{\phi}_i$ and force eigenvectors $\boldsymbol{\phi}_{F,j} = \mathbf{K} \cdot \boldsymbol{\phi}_j$. Substitution into the eigenvalue problem (3) gives the following relationship for $i \neq j$:

$$\boldsymbol{\phi}_i^T \cdot \boldsymbol{\phi}_{F,j} = \boldsymbol{\phi}_i^T \cdot \mathbf{K} \cdot \boldsymbol{\phi}_j = \omega_{0,i}^2 \cdot \boldsymbol{\phi}_i^T \cdot \mathbf{M} \cdot \boldsymbol{\phi}_j = 0 \quad . \tag{8}$$

Equation (8) is an energetic interpretation and states that a force eigenvector cannot do work on any other displacement eigenvector [10]. Consequently, eigenvectors are orthogonal with respect to the mass matrix.

2.2 Principle of model order reduction

The purpose of a linear model order reduction is to acquire a projection matrix $\mathbf{V} \in \mathbb{R}^{N \times n}$ with $n \ll N$ to approximate the displacement vector $\mathbf{x} \in \mathbb{R}^N$ of equation (1) in such a way, that the dynamic behaviour of the original FE-model is described as exact as possible [11]. By projecting equation (1) onto a subspace of a smaller dimension with basis $\mathbf{V}$, the equation of motion of the reduced order model is derived

$$\overline{\mathbf{M}}\ddot{\bar{\mathbf{x}}} + \overline{\mathbf{D}}\dot{\bar{\mathbf{x}}} + \overline{\mathbf{K}}\bar{\mathbf{x}} = \bar{\mathbf{f}} \tag{9}$$

with $\bar{\mathbf{x}} \in \mathbb{R}^n$ representing the displacement vector of the reduced order system by $\mathbf{x} \approx \mathbf{V} \cdot \bar{\mathbf{x}}$. The system matrices and the load vector as well as the input-output matrices of the reduced order system follow:

$$\begin{aligned} \overline{\mathbf{M}} = (\mathbf{V}^T \cdot \mathbf{M} \cdot \mathbf{V}) \in \mathbb{R}^{n \times n}, \quad \overline{\mathbf{D}} = (\mathbf{V}^T \cdot \mathbf{D} \cdot \mathbf{V}) \in \mathbb{R}^{n \times n}, \quad \overline{\mathbf{K}} = (\mathbf{V}^T \cdot \mathbf{K} \cdot \mathbf{V}) \in \mathbb{R}^{n \times n}, \\ \bar{\mathbf{f}} = (\mathbf{V}^T \cdot \mathbf{f}) \in \mathbb{R}^n, \quad \overline{\mathbf{B}} = (\mathbf{V}^T \cdot \mathbf{B}) \in \mathbb{R}^{n \times i}, \quad \overline{\mathbf{C}} = (\mathbf{C} \cdot \mathbf{V}) \in \mathbb{R}^{o \times n} \quad . \end{aligned} \tag{10}$$

The aim of the model order reduction is to firstly define an appropriate reduced displacement vector $\bar{\mathbf{x}}$ and afterwards a sufficient mapping matrix $\mathbf{V}$ to approximate the dynamic behaviour of the original model within a predefined frequency range. In current literature, plenty reduction methods can be looked up to gain the matrix $\mathbf{V}$, a short extract of methods is given following.

Reduction methods that produce a reduced displacement vector with physically interpretable coordinates are the Guyan [12], the CMS (Component Mode Synthesis) [13] and the iterated IRS (Improved Reduction Sequence) reduction [6]. Methods that project onto the modal subspace are the modal reduction mentioned in equation (5) with $\mathbf{V} = \mathbf{\Phi} \in \mathbb{R}^{N \times p}$ and the SEREP (System Equivalent Reduction Expansion Process) reduction [14], [8]. Both groups mainly approximate the natural frequencies and the respective mode shapes only. Modern reduction methods, e.g. based on KRYLOV subspaces (KSM) and on Balanced Truncation (BT), gain reduced displacement vectors in a general subspace with coordinates that do not belong to the physical subspace anymore. Several promising methods can be found in [4], [15], [16], [5]. The latter group approximates the frequency response function within a certain frequency range.

Several reduction methods require a partition into master m and slave s coordinates with $m \cup s = N$ and $m \cap s = \varnothing$. Details can be found in [14], [16]. This partitioning is needed for the contraction of the eigenvectors by DoF-selection in Section 5.

2.3 Frequency response

The frequency response is the relationship between the force excitation at DoF l and the displacment response at DoF k for a frequency sampling point Ω. By transforming equation (1) into the frequency domain via the fourier transformation the relation $\mathcal{F}(u_k) = H_{kl}(\Omega) \cdot \mathcal{F}(f_l)$ follows. Considering all possible combinations of l and k, the frequency response matrix $\mathbf{H}(\Omega) \in \mathbb{C}^{N \times N}$ within a predefined frequency range can be gained:

$$\mathbf{H}(\Omega) = \left(-\Omega^2\mathbf{M} + \mathrm{j}\Omega\mathbf{D} + \mathbf{K}\right)^{-1} \quad . \tag{11}$$

The inversion is needed for each sampling point Ω and in the case of $\Omega = \omega_0$ only possible when considering damping. Since the application of equation (11) is not feasible for large systems with $N \gg 10^4$, a modal description $\widetilde{\mathbf{H}}(\Omega)$ is reasonable according to formulation (5):

$$\widetilde{\mathbf{H}}(\Omega) = \mathbf{\Phi} \cdot \left(-\Omega^2\mathbf{M} + \mathrm{j}\Omega\mathbf{D} + \mathbf{K}\right)^{-1} \cdot \mathbf{\Phi}^T \quad \text{with} \quad \mathbf{\Phi} \in \mathbb{R}^{N \times N} \quad . \tag{12}$$

By considering only p modes $\boldsymbol{\phi}_i$ with natural frequency $\omega_{0,i}$ and modal mass $\widetilde{m}_i$, equation (12) can be interpreted as summation over p modes:

$$\widetilde{\mathbf{H}}(\Omega) \approx \sum_{i=1}^{p} \frac{\boldsymbol{\phi}_i\, \boldsymbol{\phi}_i^T}{-\Omega^2\, \widetilde{m}_i + \mathrm{j}\, \Omega\, \widetilde{d}_i + \widetilde{k}_i} = \sum_{i=1}^{p} \frac{1}{\widetilde{m}_i} \cdot \frac{\boldsymbol{\phi}_i\, \boldsymbol{\phi}_i^T}{-\Omega^2 + 2\mathrm{j}\, D_i\, \omega_{0,i}\, \Omega + \omega_{0,i}^2} \quad . \tag{13}$$

By normalizing the eigenvectors to modal mass, $\widetilde{\mathbf{m}}$ becomes a unit vector. If no modal damping is explicitly considered, the damping can be applied by LEHR's damping ratio D_i. The common value of $D = 0.001$ is used. Rigid body modes are usually excluded. As known, $p \ll N$ modes are sufficient for the correlation, since the addends decay by increasing frequency. Depending on the model and frequency range of interest, values between 100 and 1000 are usually adequate. The individual frequency response function between DoF k and l reads:

$$\widetilde{H}_{kl}(\Omega) \approx \sum_{i=1}^{p} \frac{1}{\widetilde{m}_i} \cdot \frac{\phi_{ki}\, \phi_{li}}{-\Omega^2 + 2\mathrm{j}\, D_i\, \omega_{0,i}\, \Omega + \omega_{0,i}^2} \quad . \tag{14}$$

Based on equation (2) the frequency response function $\mathbf{H}(\Omega)$ can be considered between $i = o = a$ inputs and outputs (IO) only. This can be accomplished in inverse and modal from:

$$^{IO}\mathbf{H}(\Omega) = \mathbf{C} \cdot \left(-\Omega^2\mathbf{M} + i\Omega\mathbf{D} + \mathbf{K}\right)^{-1} \cdot \mathbf{B}, \quad ^{IO}\widetilde{\mathbf{H}}(\Omega) \approx \sum_{i=1}^{p} \frac{1}{\widetilde{m}_i} \cdot \frac{\mathbf{C} \cdot \boldsymbol{\phi}_i\, \boldsymbol{\phi}_i^T \cdot \mathbf{B}}{-\Omega^2 + 2\mathrm{j}\, D_i\, \omega_{0,i}\, \Omega + \omega_{0,i}^2} \quad . \tag{15}$$

The equations (11) through (15) can be applied for the original as well as for the reduced order model. For an DoF-based comparison, an dimension adaption according to Section 5 has to be regarded.

3 Correlation Methods

For comparing the dynamic behaviour of two models, the results of analytical or experimental analysis are required. In either case, two data sets exist which will be denoted with r for *reference model* and v for *validation model*.

There is an enormous variety of correlation methods, several are also provided by commercial software [17]. But an objective and purely quantitative assessment is difficult according to ZANG, since no concise guidelines exist for an objective and quantitative assessment [18, p.139f]. And according to WU an ultimative criterion does not exist, since every method proves a certain characteristic [19, p.19]. Consequently, a combination of methods is mandatory and only a few criteria have to be considered. The works of DONDERS [2] and WIJKER [20] give a definite selection of criteria as well as recommendations about the limit values. Based on experience, numerous references usually lead to non-consistent recommendations.

3.1 Categorization

For categorizing correlation criteria, EWINS introduced correlation levels concerning *modal* quantities and *response* quantities [7, p.447f]. PASCUAL divided into *local* and *global* correlation criteria as well as into *modal domain* and *frequency domain* [21]. The term *local* means the comparison of single coordinates and *global* means the comparison of the whole system. KOUTSOVASILIS divided into *eigenfrequency related criteria* and *eigenvector related criteria* [16], which can be extended by *response related criteria* in [11]. Criteria based on system matrices or modal matrices complete the listing. Based on the existing classifications, an integrated definition is introduced:

1. eigenvalue-based criteria (Table 1, 3 criteria)
2. eigenvector-based criteria (Table 2, Table 3 and Table 4, 17 MAC-criteria + 15 other criteria)
3. frequency-response-based criteria (Table 5 and Table 6, 18 criteria)
4. system-matrix-based criteria (Table 7, 8 criteria)

The amount of information as well as the calculation effort increase in the given order. Besides, other criteria are available, like the comparison of mass properties or the results of a static or transient analysis [19], which are not part of this article. If all numerical correlation methods fail, the visualization of mode shapes can be helpful. The following four surveys (Table 1 through Table 7) give a detailed overview of correlation methods corresponding to the categories given above. The criteria are divided into global (G) and local DoF-based (L) methods. Although being very comprehensive, the presented methods raise no claim to completeness.

3.2 Eigenvalue-based methods

For quantifying the differences in the natural frequencies, eigenvalue-based correlation criteria can be used. Table 1 gives an overview over three available criteria.

Criteria	**Abbr.**	**DoF**	**Inputs**	**Outputs**	**Remarks**	**Ref.**
Normalized Relative Frequency Difference	NRFD	G	$\boldsymbol{\omega}_r, \boldsymbol{\omega}_v$	vector	relative difference between corresponding natural frequencies	[1]
Natural Frequency Difference	NFD	G	$\boldsymbol{\omega}_r, \boldsymbol{\omega}_v$	matrix	relative difference between all natural frequencies	[7]
Natural Frequency Correlation Coefficient	NFCC	G	$\boldsymbol{\omega}_r, \boldsymbol{\omega}_v$	value	standard deviation of corresponding natural frequencies	[7]

Table 1: eigenvalue-based correlation methods

A previously mode shape determination and mode pairing is mandatory. The natural frequencies of the first p modes are compared, neglecting rigid body modes. Required capabilities can be found in Subsection 4.1 and Subsection 4.3. The most common method is a comparison of natural frequencies by the Normalized Relative Frequency Difference (NRFD) [1], which gives the percental error between each corresponding natural frequency of the reference $f_{r,i}$ and the validation model $f_{v,i}$ and yields values between 0 and 1:

$$\mathrm{NRFD}_i = \frac{|f_{r,i} - f_{v,i}|}{f_{r,i}} = \left|1 - \frac{f_{v,i}}{f_{r,i}}\right|, \quad i = 1 \dots p \quad . \tag{16}$$

3.3 Eigenvector-based methods

Due to the fact that a pure frequency comparison is in most cases insufficient, mode shape vectors are regarded, which can be achieved by using eigenvector-based correlation methods. Since the dimensions of the original model and of the reduced model usually do not match, a previous adaptation of the eigenvector dimension has to be ensured, see Section 5. The following eigenvector-based criteria assume dimension-adapted eigenvectors $\boldsymbol{\phi}_{r,i}$ and $\boldsymbol{\phi}_{v,i}$ as well as dimension-adapted system matrices.

The scope of eigenvector-based criteria is widespread. The best known method is the Modal Assurance Criterion (MAC), of which 16 modifications are gathered in Table 2 and Table 3. These cover weighted formulations, more sensitive MACs, MACs for complex modes and local methods for a DoF-based comparison, e.g. the Coordinate MAC (COMAC). ALLEMANG presents a lively overview of MAC-variants in [22].

Criteria	**Abbr.**	**DoF**	**Inputs**	**Outputs**	**Remarks**	**Ref.**
Modal Assurance Criterion	MAC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	matrix	assessment of directions between all eigenvectors	[23]
Autocorrelated MAC	AutoMAC, AutoNCO	G	$\mathbf{\Phi}_r$ or $\mathbf{\Phi}_v$	matrix	MAC of model with itself, detection of spatial aliasing	[24]
Normalized Cross Orthogonality	NCO / modMAC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}_r$ or $\mathbf{M}_v$	matrix	mass-weighted MAC, orthogonality check	[7] [16]
Weighted MAC	WMAC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{K}_r$ or $\mathbf{K}_v$	matrix	stiffness-weighted MAC, orthogonality check	[7]
SEREP Cross Orthogonality Check	SCO	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	matrix	MAC weighted with reduced mass matrix	[7]
Scaled MAC	SMAC_1	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	matrix	MAC with trans. and rot. DoF weighted separately	[22]
Frequency-weighted MAC	MACF	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\boldsymbol{\omega}_r, \boldsymbol{\omega}_v$	matrix	MAC weigthed with NFD-matrix	[22]
Frequency-scaled MAC	FMAC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\boldsymbol{\omega}_r, \boldsymbol{\omega}_v$	2D-plot	MAC over frequency	[7]
Spatial MAC	SMAC_2	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	2 different matrices	product of SMSF-matices	[25]
Energy-based MAC	EMAC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{K}_r$ or $\mathbf{K}_v$	matrix	weigthed MAC with modal strain energy	[24]
Modal Comparison Criteria Matrix	MCCM	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}_r$ or $\mathbf{M}_v$	matrix	similar to NCO	[26]
MAC Square Root	MACSQ	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	matrix	amplification of off-diagonal of MAC	[22]
Linearized MAC	LMAC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	matrix	amplification of off-diagonal of MAC	[24]

Table 2: eigenvector-based correlation methods (MAC-variants) – part 1

Criteria	Abbr.	DoF	Inputs	Outputs	Remarks	Ref.
Inverse MAC	IMAC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	matrix	amplification of off-diagonal of MAC	[27]
Complex MAC	MACX	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	matrix	used for non-mono-phased eigenvectors	[28]
Pole-weighted Complex MAC	MACXP	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	matrix	used for eigenvectors with only few entries	[28]
Coordinate MAC	COMAC	L	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	vector	correlation of single DoFs acc. to MAC	[29]
Enhanced Coordinate MAC	ECOMAC	L	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	vector	correction of systematic errors, normalization of modal matrix required	[29]

Table 3: eigenvector-based correlation methods (MAC-variants) – part 2

Besides the MAC-methods, other mode-shape-based methods exist; mainly modal scale factors, mode indications, e.g. Mode Indicator Functions (MIF), and orthogonality checks, e.g. Pseudo Orthogonality Check (POC), as well as complex checks. Table 4 gives a detailed overview over 15 criteria.

Criteria	Abbr.	DoF	Inputs	Outputs	Remarks	Ref.
Modal Scale Factor	MSF	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	2 different matrices	normalization of modal matrix required, method of least squares	[23]
Spatial Modal Scale Factor	SMSF	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	2 different matrices	pseudo-inverse, method of least squares	[25]
Standard Orthogonality Check	ORTHOG	G	$\mathbf{\Phi}_v$, $\mathbf{M}_r$	matrix	orthogonality check	[7]
Pseudo Orthogonality Check	POC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}_r$	matrix	orthogonality check, square of NCO	[29]
Minimum-Maximum-Method	MIN, MAX	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	2 different matrices	more sensitive than MAC	[30]
Mass Normalized Vector Difference	MNVD	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}$	matrix	magnitude and direction of eigenvectors	[26]
Stiffness Normalized Vector Difference	SNVD	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{K}$	matrix	magnitude and direction of eigenvectors	[26]
Modal Phase Colinearity	MPC	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	1 vector per matrix	identification of non-mono-phased eigenvectors	[28]
Mode Indicator Function	MIF	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	1 vector per matrix	distinction between real and complex eigenvectors	[31]
Mean Phase Deviation	MPD	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	2 different vectors	distinction between real and complex eigenvectors	[31]
Normalized Modal Difference	NMD	L	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	vector	absolute deviation of single eigenvector coordinates	[32]
Coordinate Orthogonality Check	CORTHOG	L	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	vector	absolute deviation of single eigenvector coordinates	[33]
Coordinate Modal Error Function	COMEF	L	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$	vector	correction of systematic errors, normalization of modal matrix required	[34]

Table 4: eigenvector-based correlation methods (other methods)

Following, a few methods are explained in detail. The Modal Assurance Criterion (MAC) introduced in [23] compares the directions of two eigenvectors based on the inner product and yields values between 0 and 1:

$$\mathrm{MAC}_{ij} = \frac{(\boldsymbol{\phi}_{r,i}^H \cdot \boldsymbol{\phi}_{v,j})^2}{(\boldsymbol{\phi}_{r,i}^H \cdot \boldsymbol{\phi}_{r,i}) \cdot (\boldsymbol{\phi}_{v,j}^H \cdot \boldsymbol{\phi}_{v,j})}, \quad i,j = 1 \dots p \quad . \tag{17}$$

For purely real eigenvectors the conjugated-complex transposed $(\dots)^H$ can be replaced by the simple transposed $(\dots)^T$. The ideal MAC-matrix is a unit matrix for corresponding eigenvectors in ascending order. For a good correlation values on the main diagonal greater 0.9 are expected, non-correlated vectors on the main diagonal are less 0.6 [28]. For a geometrically orthogonal correlation the off-diagonal terms shall be less 0.1. The utilization of vectorization techniques in MATLAB allows a fast and efficient calculation.

The Normalized Cross Orthogonality Check (NCO) is a weighted formulation of the MAC (17), with the weighting matrix $\mathbf{W}$ being the mass matrix $\mathbf{M}$ [7]. Depending on the adapted model dimension, the mass matrix can belong to the reference model $\mathbf{M}_r$ or to the validation model $\mathbf{M}_v$. The NCO embodies an orthogonality check according to (8) since an energetic comparison is made:

$$\mathrm{NCO}_{ij} = \frac{\left(\boldsymbol{\phi}_{r,i}^H \cdot \mathbf{W} \cdot \boldsymbol{\phi}_{v,j}\right)^2}{\left(\boldsymbol{\phi}_{r,i}^H \cdot \mathbf{W} \cdot \boldsymbol{\phi}_{r,i}\right) \cdot \left(\boldsymbol{\phi}_{v,j}^H \cdot \mathbf{W} \cdot \boldsymbol{\phi}_{v,j}\right)}, \quad \mathbf{W} \equiv \mathbf{M}, \quad i,j = 1 \dots p \quad . \tag{18}$$

Compared to the MAC, the off-diagonal terms are reduced and expected to be much less 0.1. The main diagonal is mostly unaffected. Due to additional vector-matrix multiplications the NCO needs more computational effort. Other weighted MAC variants even take more calculation time when using inverse operations, the pseudo-inverse or square-roots. Although the NCO already includes an orthogonality check, other criteria proving the orthogonality are available, e.g. the Standard Orthogonality Check (ORTHOG) [7] or the Pseudo Orthogonality Check (POC) [29].

3.4 Frequency-response-based methods

Both above mentioned categories exhibit difficulties concerning mode identification and mode pairing, especially when dealing with multiple eigenvalues. This can be overcome by using frequency-response-based methods, which are sophisticated and standard in control theory. But the evaluation of each single frequency response function (FRF) is expensive, see Subsection 2.3. All methods presented are based on (13) and (15) for a predefined frequency range. Since each individual response function (IRF) has to be evaluated at each sampling point Ω, correlation criteria exist which downsize the matrix $\mathbf{H}$ and only return a well-interpretable vector or even scalar value by condensation or averaging. Most-known criteria that provide a frequency-based vector containing values between 0 and 1 are the Frequency Response Assurance Criterion (FRAC) and the Frequency Domain Assurance Criterion (FDAC) as well as the relative error of the infinity-norm (H_∞). The 18 criteria presented below are divided into global (Table 5) and local (Table 6) methods.

The calculation of the frequency response in modal formulation according to equation (13) and (15) is still very expensive. The effort can be reduced by only considering the main diagonal of $\mathbf{H}$ or by using adaptive sampling points Ω instead of equidistant sampling points [35]. Further speed-ups can be accomplished by avoiding loops in MATLAB and using vectorization instead. Another way to further improve performance is the utilization of a reduced sized system, which will be discussed in Section 5.

A common criteria that yields condensated results is the relative error of the infinity-norm H_∞ which gives the frequency dependent error $\boldsymbol{\epsilon}_\infty(\Omega)$ [36]:

$$\boldsymbol{\epsilon}_\infty(\Omega) = \frac{\|\mathbf{H}_r(\Omega) - \mathbf{H}_v(\Omega)\|_\infty}{\|\mathbf{H}_r(\Omega)\|_\infty}, \quad \|\mathbf{H}(\Omega)\|_\infty = \sup\left(\sigma_{max}(\mathbf{H}(\Omega))\right) \quad . \tag{19}$$

Criteria	Abbr.	DoF	Inputs	Outputs	Remarks	Ref.
FRF with Infinity and Frobenius error norms	FRF, H_∞, H_F	G	$\mathbf{H}_r$, $\mathbf{H}_v$	vector each	two normalized FRFs over frequency or relative error	[7] [36]
FRF with H2 error norm	H_2	G	$\mathbf{H}_r$, $\mathbf{H}_v$	value	two normalized FRFs over frequency or relative error	[4]
Frequency Domain Assurance Criterion / Global Shape Criterion	FDAC / GSC	G	$\mathbf{H}_r$, $\mathbf{H}_v$	vector	MAC in frequency domain	[21] [18] [1]
Modified FDAC / Global Amplitude Criterion	modFDAC / GAC	G	$\mathbf{H}_r$, $\mathbf{H}_v$	vector	FDAC based on amplitude	[18] [1]
Improved FDAC	IFDAC	G	$\mathbf{H}_r$, $\mathbf{H}_v$	vector	FDAC based on sign	[21]
Response Vector Assurance Criterion	RVAC	G	$\mathbf{H}_r$, $\mathbf{H}_v$	vector	FDAC with only one column of FRF-matrix	[37]
Global Frequency Response Correlation	GFRC	G	$\mathbf{H}_r$, $\mathbf{H}_v$	vector	similar to FDAC with root scaling	[38]
FRF Root Mean Square	FRFRMS	G	$\mathbf{H}_r$, $\mathbf{H}_v$	vector	relative error of magnification factor of FRFs	[39]
Modal FRF Assurance Criterion	MFAC	G	$\mathbf{H}_r$, $\mathbf{H}_v$	matrix	mixed formulation of MAC and FDAC	[40]
Frequency-band Correlation Factor	-	G	$\mathbf{H}_r$, $\mathbf{H}_v$	matrix	symmetry of FRF-matrix	[37]

Table 5: frequency-response-based correlation methods (global)

Criteria	Abbr.	DoF	Inputs	Outputs	Remarks	Ref.
Individual Response Function	IRF	L	$H_{r,kl}$, $H_{v,kl}$	vector	two single FRFs over frequency or relative error	[7]
Frequency Response Assurance Criterion	FRAC	L	$\mathbf{H}_r$, $\mathbf{H}_v$	matrix	COMAC in frequency domain	[40]
Frequency Amplitude Assurance Criterion	FAAC	L	$\mathbf{H}_r$, $\mathbf{H}_v$	matrix	FRAC based on amplitude	[18]
FRF Curvature Method	-	L	$\mathbf{H}_r$, $\mathbf{H}_v$	vector	curvature of FRF (second deviation)	[41]
Gap Smoothing Method	GSM	L	$\mathbf{H}_r$, $\mathbf{H}_v$	matrix	curvature of FRF (second deviation)	[42]
Residual imag FRF Method	-	L	$\mathbf{H}_r$, $\mathbf{H}_v$	matrix	imaginary parts of FRF	[42]

Table 6: frequency-response-based correlation methods (local)

3.5 System-matrix-based methods

Besides comparing modal data and frequency response functions, system or modal matrices can be directly correlated. System-matrix-based methods are outlined and gathered in Table 7. The correlation requires equal-sized matrices. Comparison based on a smaller sized dimension is profitable for saving calculation time, see Section 5.

Criteria	Abbr.	DoF	Inputs	Outputs	Remarks	Ref.
Modal Matching Array	MMA	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}_r$, $\mathbf{M}_v$, $\mathbf{K}_r$, $\mathbf{K}_v$	2 different matrices	comparison of modal system matrices	[43]
Modal Mass Difference	MMD	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}_r$ or $\mathbf{M}_v$	matrix	relative error of modal mass matrix	[26]
Modal Stiffness Difference	MSD	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{K}_r$ or $\mathbf{K}_v$	matrix	relative error of modal stiffness matrix	[26]
Average Accelerance Difference	AAD	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}_r$ or $\mathbf{M}_v$	vector	amplitude difference in FRF, reconstructable from MMD	[26]
Average Flexibility Difference	AFD	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{K}_r$ or $\mathbf{K}_v$	vector	amplitude difference in FRF, reconstructable from MSD	[26]
Effective Modal Mass	EMM_1	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}_r$, $\mathbf{M}_v$, $\mathbf{K}_r$, $\mathbf{K}_v$	value	relative error of sum of modal masses	[20]
Error Matrix Method	EMM_2	G	$\mathbf{\Phi}_r$, $\mathbf{\Phi}_v$, $\mathbf{M}_r$, $\mathbf{K}_r$	2 different matrices	comparison of expanded system matrices	[19] [7]
Error Vector	-	L	$\mathbf{\Phi}_v$, $\mathbf{M}_r$, $\mathbf{K}_r$	1 vector per mode	residuum of eigenvalue problem	[44]

Table 7: system-matrix-based correlation methods

4 Model preparation

As addressed before, the preparation of the data sets is a crucial issue for the correlation success. The preparation covers an automatic mode pairing, the handling of multiple eigenvalues, e.g. for models with symmetry, and the detection of rigid body modes.

4.1 Mode pairing

The assignment between eigenvectors and natural frequencies by rearrangement is called mode pairing. This is mandatory for using eigenvalue-based criteria and important for eigenvector-based criteria for a meaningful correlation. The mode pairing algorithm is based on the NCO [31] and sorts the eigenvectors to be validated in such a way that the maxima of the NCO are moved to the main diagonal. According to GROVERS only NCO-values above 70% are to be considered. Since this line cannot be drawn clearly, an automatic mode pairing algorithm is applied, where the optimal MAC-limit is chosen iteratively, starting with a fairly low value. The algorithm stops as soon as a definite assignment of modes is possible, meaning that only one possible rearrangement exists. Since the NCO-matrix is only calculated once, the automatic algorithm is not time consuming. Multiple eigenvalues cannot be treated with this procedure.

4.2 Treatment of multiple eigenvalues

The eigenvectors of models with multiple eigenvalues are not unique. This is especially the case for models with rotational symmetry and causes problems when using eigenvector-based criteria like the MAC. Permutations and shifts usually occur and cause a poor agreement on the main diagonal, see Figure 5 in Section 6. Natural frequencies that are numerically identical or close together can lead to permutations between original model and reduced order model. In case of two symmetry planes orthogonal mode shapes occur which can cause shifts. Depending on the reduction technique, even non-orthogonal mode shapes can arise.

Both described problems can be overcome by using frequency-response-based criteria or by performing a linear combination ahead, whereby the eigenvectors of the reference model $\boldsymbol{\phi}_r$ can be expressed in terms of the eigenvectors of the validation model $\boldsymbol{\phi}_v$ [10]:

$$\boldsymbol{\phi}_{r,i} = \sum_{i=1}^{p} a_i \cdot \boldsymbol{\phi}_{v,i} = \boldsymbol{\Phi}_v \cdot \mathbf{a} \quad \Rightarrow \quad \mathbf{a} = \boldsymbol{\Phi}_v^{-1} \cdot \boldsymbol{\phi}_{r,i} \quad \Rightarrow \quad \widehat{\boldsymbol{\phi}}_{v,i} = \boldsymbol{\Phi}_v \cdot \mathbf{a} \quad . \tag{20}$$

The solution of the linear system (20) for each eigenvector $\boldsymbol{\phi}_{r,i}$ gives the respective coefficients a_i. The corrected eigenvector $\widehat{\boldsymbol{\phi}}_{v,i}$ is generated accordingly. This approach can overcome small permutations as well as shifts. It is only physically adequate to combine eigenvectors that have adjacent natural frequencies. Therefore, eigenvectors are considered that are within a range of 1% of the natural frequencies. The mode pairing algorithm mentioned in Subsection 4.1 has to be performed previously to correct large permutations with clearly differing natural frequencies. An alternative approach can be found in [45], which treats shifts by solving an optimization problem and maximizing the MAC-value.

4.3 Rigid body modes

Almost all correlation criteria require the exclusion of rigid body modes. Therefore, a reliable and automatic detection of rigid body modes for reference and validation model is mandatory. But numerically not all null eigenvalues equal zero and not always 6 rigid body modes exist depending on reduction technique and kinematic boundary conditions. As a result, other adequate methods are required.

The observation of natural frequencies f_i which are sorted in ascending order can be a simple way to determine rigid body modes, where either an absolute ϵ_a or a relative limit ϵ_r can be stated:

$$f_i < \epsilon_a, \quad f_{i+1} - f_i < \epsilon_r \quad . \tag{21}$$

The limits in (21) are hard to choose and highly model dependent. Alternatively, rigid body frequencies are identified by assuming that a significant step in frequency can be detected between the last rigid body mode and the first non-rigid body mode.

Besides natural frequencies, eigenvectors can serve as an identification measure. By using force eigenvectors $\boldsymbol{\phi}_{F,i}$ according to equation (8) the following limit ϵ_F can be stated, where from experience 10^{-4} seems adequate for structural mechanics:

$$||\mathbf{K} \cdot \boldsymbol{\phi}_{F,i}|| < \epsilon_F \approx 10^{-4} \quad . \tag{22}$$

If no stiffness matrix is available, all eigenvectors can be compared against artificially generated rigid body modes. In most cases, rigid body modes are seldom purely translations or rotations, but combined motions. This requires a linear combination of rigid body modes, which leads to a very reliable and useful approach. After generating six artificial rigid body modes $\boldsymbol{\phi}_{RB,i}$, the potential rigid body modes of the model $\boldsymbol{\phi}_i$ must be a linear combination of those six. The approach is equivalent to procedure (20) in Subsection 4.2. After the determination of the coefficients a_i, the eigenvectors $\boldsymbol{\phi}_i$ and $\boldsymbol{\phi}_{RB,i}$ are compared via the NCO. By experience, values on the main diagonal indicating rigid body motion are above 99.9%. According to [20], the artificial rigid body modes can either be generated by using a rotation matrix $\mathbf{A}$ or a stiffness matrix which is partitioned into free f and constrained DoF c. The latter method requires the inversion of nearly the complete stiffness matrix $\mathbf{K}_{ff}$, which is not effective for large systems.

4.4 Spatial aliasing

An adverse definition of master-DoF during a model order reduction can lead to different mode shapes that have identical deflections at the master-DoF – concluding not all modes are orthogonal. This is called spatial aliasing and can be examined by auto-correlation techniques, e.g. the AutoMAC, where a model is compared with itself. An AutoMAC-matrix without off-diagonal terms indicates independent mode shapes. Off-diagonal terms in the AutoMAC-matrix will also cause off diagonal-terms in the MAC-matrix. Consequently, more or other master-DoF have to be chosen for correlation purposes, see Subsection 5.4.

5 Dimension adaptations

When dealing with reduced order models dimension disagreements of eigenvectors are unavoidable, since the eigenvectors of the reference model $\boldsymbol{\phi}_r$ are of length N and the eigenvectors of the validation model $\boldsymbol{\phi}_v$ are of length n ($N \gg n$). All eigenvector-based correlation methods require equal-sized eigenvectors, see Subsection 3.3. Methods using system matrices also expect equal-sized matrices. Most of all other correlation methods can deal with non-equally-sized eigenvectors, but the length of the eigenvectors mainly affects the calculation time, e.g. for frequency-response-based correlation methods. The eigenvector-resizing can be realized by means of expansion, contraction, DoF-selection or adaption to any intermediate dimension. The problem of dimension adaptation is well known for the comparison of experimental and analytical models [19], but will gain especial importance for the purpose of model order reduction.

5.1 Expansion

The modal matrix of the reduced order model $\boldsymbol{\Phi}_v$ can be expanded to the reference dimension by using the projection matrix $\mathbf{V}$:

$$\boldsymbol{\Psi}_v = \mathbf{V} \cdot \boldsymbol{\Phi}_v \quad . \tag{23}$$

Figure 1 (left) illustrates the procedure. For the application of (23) the existence of a projection matrix $\mathbf{V}$ is mandatory. Not all commercial FE-programs provide the projection matrix. Alternatively, compensatory projection methods can be used to gain a projection matrix $\mathbf{V}$, e.g. by SEREP-reduction [14], [7] or Modal Coordinate Method [19]. Depending on the model size, the expansion itself as well as the subsequent correlation can be very expensive.

5.2 Contraction

Alternatively, the reference model of size N can be contracted to the dimension of the reduced order model n, whereby only the master-DoFs are compared, see Figure 1 (right). This inverse operation requires the inverse of the projection matrix $\mathbf{V}^{-1}$, which can be done by using the pseudo-inverse:

$$\boldsymbol{\Theta}_r = \mathbf{V}^{-1} \cdot \boldsymbol{\Phi}_r = \mathbf{V}^{+} \cdot \boldsymbol{\Phi}_r \quad . \tag{24}$$

Like in Subsection 5.1, the projection matrix $\mathbf{V}$ is required and can alternatively be generated by using the mentioned methods. Since the constitution of the inverse mainly depends on the master-partition and reduction method used, an inversion is not always possible.

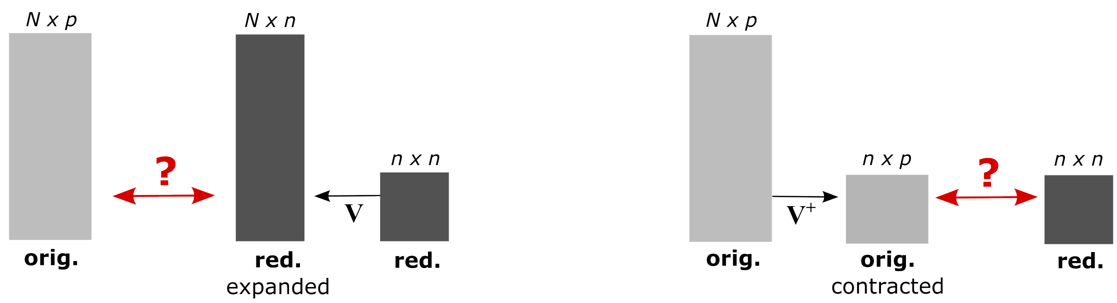

Figure 1: dimension adaption by expansion (left) and contraction (right) of the reduced order model

5.3 Selection

If the projection matrix $\mathbf{V}$ is not known, the master-DoFs can be directly selected without any expansion or contraction, see Figure 2 (left). This method assumes a physical interpretation of master-DoFs, otherwise a transformation into physical subspace is required. The selection approach can easily be applied for Guyan- and CMS-reduction, due to the fact that the master-partition of the expanded model equals the reduced order model. Since most commercial FE-programs only provide the Guyan- and CMS-reduction and do not supply the projection matrix, a selection is favourable. Furthermore, a selection of DoFs is not expensive at all.

Generally, the reduced order model should be expanded according to Subsection 5.1. The following DoF-selection is illustrated in Figure 2 (right). This enhancement covers any reduction method and raises more accurate data sets.

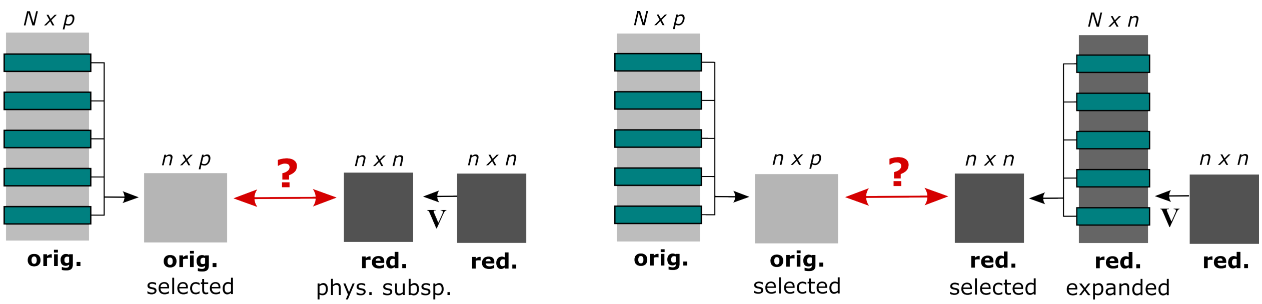

Figure 2: dimension adaption by DoF-selection without (left) and with (right) expansion

5.4 Intermediate dimension

Eventually, the model dimension can be adapted to an intermediate dimension m with $N \gg m > n$. The main advantage lies in the fact that the models to be compared are still fairly small, but having a meaningful number of locations to compare. This is important, since e.g. the MAC is a statistical measure [22], which requires a minimum amount of information. Furthermore, the master-DoFs are not necessarily the important DoFs to avoid spatial aliasing, see Subsection 4.4. The creation of an intermediate dimension can be done by using sensor placement methods, e.g. the Effective Independence (EfI) [46] or the MoGeSeC-Algorithm [47]. Although being quite effective, additional calculation time is required. This is usually compensated by using the fairly small intermediate dimension.

After the determination of an intermediate dimension, the respective equal-sized eigenvectors can be gained by expansion and contraction, see Figure 3 (left), which requires two projection matrices $\mathbf{V}_1$ and $\mathbf{V}_2$. This special case works well for 2-step-reduction techniques. If no projection matrices exist, an expansion of the reduced order model is useful. Due to the mentioned drawbacks of expansion and contraction, the selection approach from Subsection 5.3 can also be applied to the intermediate dimension, see Figure 3 (right).

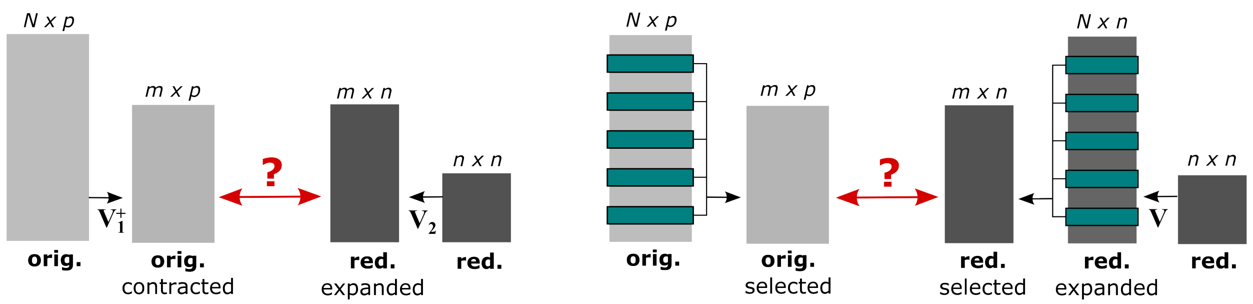

Figure 3: intermediate dimension without (left) and with (right) expansion

6 Results

The presented methods and approaches are demonstrated and discussed by means of two representative validation examples. The first model constitutes a wheel set axle with rotational symmetry, Figure 4 (left). It is meshed by 67639 hexahedron elements with quadratic shape functions and consists of 7 master nodes each with 6 DoF, which are all located on the symmetry axis and are connected to the mesh via rigid connections. The total number of DoF equals 204492 and is reduced to 52 DoF by KSM and CMS including 10 CB-modes. The frequency range of interest is 0 to 4000 Hz (24 modes). The other model of a gear box is fairly larger and geometrically more complex, Figure 4 (right). The tetrahedron mesh consists of 53058 elements. The 9 master nodes have 6 DoF each and are connected to the mesh via multi-point-constraints (MPC). This makes a total number of 310335 DoF which is reduced to 64 DoF by KSM and CMS including 10 CB-modes. The frequency range of interest is 0 to 2000 Hz (60 modes). Both models come along with a minimum number of master-DoF, since only the necessary interfaces for the bearings are provided. Due to the fact that no additional DoFs are defined, the models are suitable for the approaches mentioned in Section 5. The calculations were performed on a 64-bit Win7 system with Intel-Core-i5-2430M CPU (2x 2.4 GHz) and 6 GB RAM. For the comparability of the methods presented, the additional time for data handling is not considered, since the loading and saving makes up the major effort.

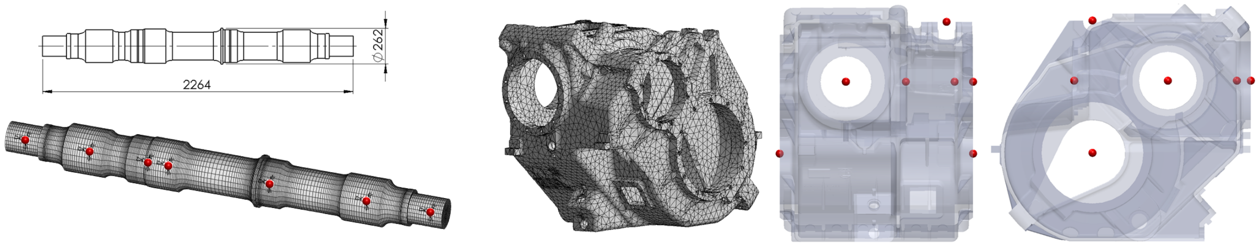

Figure 4: wheel set axle 204492 DoF (left) and gear box 310335 DoF (right) with master nodes (red)

6.1 Selection of correlation methods

Based on the gear box model, a small variety of correlation criteria is presented in Table 8. Depending on the limits given, a number of correlated modes as well as a valid frequency range can be determined. All criteria are based on an expansion of the reduced order model (23) and include a model preparation by automatic mode pairing and linear combination. Due to the vast calculation time, the frequency-response-based criteria are generated based on an intermedite dimension of size 100 for 4050 adaptively chosen sampling points. The time for the calculation of the correlation criteria is listed excluding the effort for model preparation and dimension adaptation. Generally, it is recommended to always use a combination of eigenvalue- and eigenvector-based methods (NRFD, NCO). The frequency-response-based methods give only little additional information and thus are ancillary (ΔH_∞).

Criteria	**Limits**	**CMS (54+10)** [time]	**KSM (64)** [time]
NRFD_i: modes / max. freq.	$\leq 3\%$	4 / 684 Hz [0.01 s]	31 / 1347 Hz [0.01 s]
MAC_{ii}: modes / max. freq. / min. value	$\geq 95\%$	4 / 684 Hz / 98.6%	30 / 1328 Hz / 96.4%
MAC_{ij}: off terms / max. value	$\leq 10\%$	0 / 0.5% [0.3 s]	0 / 0.7% [0.3 s]
NCO_{ii}: modes / max. freq. / min. value	$\geq 95\%$	4 / 684 Hz / 98.5%	30 / 1328 Hz / 96.5%
NCO_{ij}: off terms / max. value	$\leq 1\%$	0 / 0.1% [2.9 s]	0 / 0.0% [2.9 s]
ΔH_∞: max. freq.	$\leq 10\%$	587 Hz [16.9 s]	1209 Hz [22.7 s]
FDAC: max. freq.	$\geq 90\%$	702 Hz [36.2 s]	1453 Hz [43.4 s]

Table 8: extract of correlation results for the reduced gear box with CMS54+10 and KSM64

6.2 Influence of model preparation

The mode pairing algorithm in Subsection 4.1 and the linear combination in Subsection 4.2 are presented at the example of the wheel set axle for the CMS42+10 reduced model, see Figure 5. The mode pairing with 70% NCO-border recombines 7 modes and takes 1.8 s, the automatic algorithm recombines 10 modes and takes 5.0 s with a final NCO-border of 39%. The mode shifts are only corrected by the linear combination (20) which requires additional 1.4 s. Due to robustness, automatic mode pairing in conjunction with a linear combination is always preferred. The detection of rigid body modes by frequency steps (21) requires 0.05 s and 0.27 s by stiffness matrix (22). Both procedures are able to reliably find six rigid body modes.

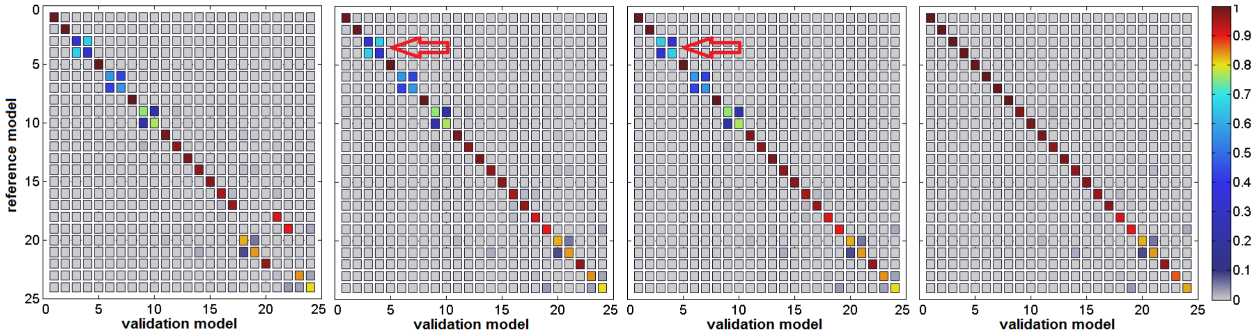

Figure 5: preparation results of wheel set axle via NCO: no preparation (1), only mode pairing (2), only automatic mode pairing (3), automatic mode pairing with linear combination (4)

6.3 Impact of dimension adaptation

The modal adaptation is demonstrated at the wheel set axle for the CMS42+10 reduced model, see Figure 6. The standard approach by expansion (23) takes 1.0 s and produces two modal matrices of size 204492x24 (33.8 MB). The contraction approach (24) slightly overrates the correlation result. After 1.3 s, the handy modal matrices of size 52x24 (8 KB) are gained. But the bottleneck is the solvability of (24), see Subsection 5.2. The dimension adaption by DoF-selection with expansion according to Figure 2 is also very fast (1.0 s) and gives equally sized matrices. But significant off-diagonal terms occur indicating non-orthogonal modes. The correlation at an intermediate dimension with expansion is performed according to Figure 3. Extra time of 15.8 s is needed for the selection of 48 additional DoF by the EFI-algorithm. The procedure leads to small matrices of size 100x24 (20 KB) and mainly orthogonal modes. The correlation at an intermediate dimension is conservative and gives a lower bound for the NOC generated by expansion. It overcomes the drawbacks of the other three approaches and enables a very fast calculation of the criteria listed in Table 8.

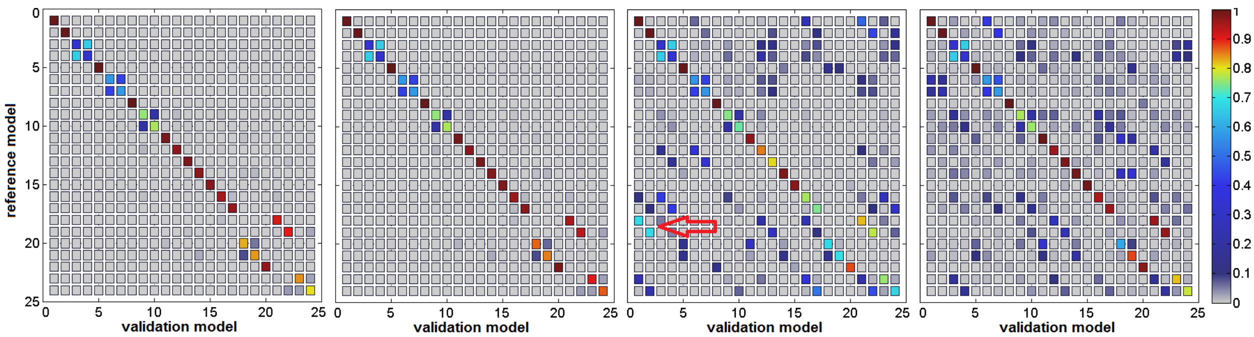

Figure 6: dimension adaptation results of wheel set axle via NCO: expansion (1), contraction (2), selection with expansion (3), intermediate dimension of size 100 by EFI with expansion (4)

7 Conclusion and outlook

The effective and reliable correlation for reduced large models was discussed. Due to an enormous amount of correlation methods available, a few preferable methods were combined, which are able to correctly indicate the input-output behaviour of the mechanical system. Additionally, different approaches for an adaption of the model dimensions were presented. Because of the fast calculation and sufficient results, the correlation at an intermediate dimension is favourable. An efficient utilization of modal correlation methods always requires a preparation of the data sets by mode pairing and treatment of multiple eigenvalues. The study was performed on two representative example models. However, a general assessment can be realized by generating system matrices with artificial error instead of using reduced order models, see [48].

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