C. M. GARELLI, B. QUASSIATI
and M. VIGONE
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On the Energy Determination of the Heavy Primaries.

C. M. GARELLI, B. QUASSIATI and M. VIGONE

Istituto di Fisica dell'Università - Torino
Istituto Nazionale di Fisica Nucleare - Sezione di Torino

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Summary. — The formula suggested by PETERS for the energy determination of the heavy primaries, requires the knowledge of the mean energy of evaporation of the α-particles. The present work establishes that this energy depends slightly from the size of the evaporating nucleus.

1. – Introduction.

During a preceding work of our group, in which the heavy primaries of the cosmic radiation have been studied, we noticed a peculiar feature of the angular distribution of the α-particles emitted in the splittings of the heavy primaries (see: R. CESTER, A. DEBENEDETTI, C. M. GARELLI, B. QUASSIATI, L. TALLONE and M. VIGONE: *Nuovo Cimento*, **7**, 371 (1958), Sect. **15**). The angle of the α-particles in the laboratory system is related to the energy of the primary. The relation normally used is the one given by PETERS [1]:

$$\sqrt{\overline{\theta_\alpha^2}} = \left(\frac{\overline{E_\alpha^*} M}{3p_0^2}\right)^{\frac{1}{2}},$$

where $\overline{E_\alpha^*}$ is the mean kinetic energy of the α-particles in the rest system, M is the proton mass and p_0 is the momentum per nucleon of the incident particle. Following the information on the energy of the evaporation α-part-

(1) B. PETERS: *Progress in Cosmic Rays*, **1**, 193 (1952).

icles given by LE COUTEUR and others [2], PETERS used the value $E^*_\alpha = 12$ MeV and gave the expression

$$\sqrt{\overline{\theta^2_\alpha}} = \frac{0.06}{\varepsilon_0},$$

where θ_α is measured in radians and ε_0 is the total energy per nucleon of the incident particle in GeV.

The feature found by our group is that, in the laboratory system, the mean angle of the α-particles with the direction of the primary increases when the charge of the incident nucleus increases. The same effect has been previously found by FOWLER *et al.* [3].

The observed increase of the mean angle of the α-particles of splitting can be explained with three different hypotheses;

a) the values 0.06 for the coefficient of Peters' formula is correct for any charge of the incident nucleus and then the energy spectrum of the heavy primaries of the cosmic radiation depends on the charge of the primary;

b) the mean energy of emission of the α-particles in the rest system is not always equal to 12 MeV, but depends on the size of the nucleus from which the α-particles evaporated;

c) the mean energy of emission of the α-particles in the rest system is not always equal to 12 MeV, but depends on the type of collision in which the α-particles are emitted.

In order to decide between these hypotheses, we studied the energy of the α-particles evaporated in the stars produced by the heavy primaries. The analysis of these type of collisions (nucleus-nucleus collisions) enables us to get illuminating information; in fact:

— if hypothesis *a*) is correct we must find that the energy of emission of the α-particles of evaporation is the same for any kind of target and incident nucleus;

— if hypothesis *b*) is correct we must find a variation of the energy of emission of the α-particles of evaporation with the size of the target nucleus;

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(2) J. B. HARDING: *Phil. Mag.*, **40**, 530 (1949); K. J. LE COUTEUR: *Proc. Phys. Soc.*, A **63**, 259 (1950); D. H. PERKINS: *Proc. Roy. Soc.*, A **203**, 399 (1950); N. PAGE: *Proc. Phys. Soc.*, A **63**, 250 (1950); G. BERNARDINI, G. CORTINI and A. MANFREDINI: *Phys. Rev.*, **79**, 952 (1950); J. B. HARDING: *Phys. Mag.*, **42**, 63 (1951).

(3) P. H. FOWLER, R. R. HILLIER and C. J. WADDINGTON: *Phil. Mag.*, **2**, 293 (1957).

— if hypothesis *c*) is correct we must find a variation of the energy of emission of the α-particles of evaporation both from the type of the target nucleus as from the type of the incident nucleus.

In the last two cases it is necessary to apply a correction factor to Peters' formula in order to obtain a more accurate value of the energy of the primary.

2. – Experimental procedure and results.

235 stars produced in the emulsion by the heavy primaries ($Z \geqslant 3$) have been taken into account and the evaporation tracks of each star have been analysed. In order to distinguish the tracks of the α-particles from the ones of protons, deuterons and tritons, the most suitable measurement on tracks at the end of their range is the gap-count. A calibration on protons and α-tracks of different dip angle with the emulsion surface has been made and it has been established that the separation is possible until the dip angle is not greater than 45°. Table I collects the results of the calibration for the last 50 μm of track.

TABLE I.

dip angle	number of gaps on the last 50 μm of track	
	protons	α-particles
0° ÷ 5°	13	3
5° ÷ 22°	9	2.5
22° ÷ 35°	7	1.5
35° ÷ 45°	5	0.6

The experimental work has been done in the following way: we selected the evaporation tracks whose dip angle was $\leqslant 45°$ and on the whole range of these tracks we made the gap-count; of each track that, according to this measurement, appeared to be a track of an α-particle, we determined accurately the range and the angle with the direction of the primary. As the primaries of the stars studied in the present work have a very small dip angle with respect to the emulsion surface (the range per plate of the primaries was always greater than 3 mm), the evaporation tracks that have been analysed are more or less the ones whose vertical angle (*) with respect to the direction of the primary is $\leqslant 45°$.

(*) By « vertical angle » between two tracks, we indicate the angle between these two tracks when we project them on one plane perpendicular to the emulsion surface.

1661

The angular distribution of the α-particles, calculated taking into account the experimental conditions above described, results to be isotropic when the primary of the star is an L nucleus ($3 \leqslant Z \leqslant 5$), while it presents a forward excess when the primary is a M nucleus ($6 \leqslant Z \leqslant 10$) or a H nucleus ($Z > 10$). The anysotropy is due to the momentum transferred to the target nucleus.

In Table II we report the mean kinetic energy $\overline{E_\alpha}$ of the α-particles in the laboratory system and the angle with the direction of the primary, $\varphi_{\frac{1}{2}}$, in which half of the tracks are contained. The energy has been calculated from the range-energy relation; the errors quoted are the statistical ones.

TABLE II.

primary	L	M	H
$\overline{E_\alpha}$	18.0 ± 1.5	20.8 ± 1.4	22.4 ± 3.3
$\varphi_{\frac{1}{2}}$	$90° \pm 7°$	$79° \pm 5°$	$66° \pm 7°$

The mean energy of the α-particles in the rest system, $\overline{E_\alpha^*}$, can be evaluated making use of the formula:

$$\beta_\alpha^* = \operatorname{tg} \varphi_{\frac{1}{2}} \sqrt{\frac{\gamma_\alpha^2 - 1}{1 + \gamma_\alpha^2 \operatorname{tg}^2 \varphi_{\frac{1}{2}}}}, \tag{1}$$

where β_α^* is the velocity of the α-particle in the rest system in velocity of light units and γ_α is the total energy of the α-particle in the laboratory system in protonic mass units. The formula (1) is valid under the assumption that the distribution of the α-particles is isotropic in the rest system of the evaporating nucleus. The results of this calculation are given in the first row of Table III.

We checked the results obtained with another method of determining the $\overline{E_\alpha^*}$ that does not depend directly from the values of $\overline{E_\alpha}$ and $\varphi_{\frac{1}{2}}$. In fact, the momentum $\boldsymbol{P}_\alpha$ of each α-particle in the laboratory system is the composition of the momentum $\boldsymbol{P}_\alpha^*$ of the α-particle in the rest system with the momentum $\boldsymbol{T}$ transferred to the target nucleus:

$$\boldsymbol{P}_\alpha = \boldsymbol{P}_\alpha^* + \boldsymbol{T}. \tag{2}$$

The projection of this relation on the direction of the incident nucleus gives: $P_\alpha \cos\varphi = P_\alpha^* \cos\varphi^* + T$, where φ is the angle of the α-particle with the direction of the primary in the laboratory system and φ^* is the analogous angle in the rest system. Assuming that the distribution of the α-particles is isotropic in the rest system, we obtain:

$$\frac{\sum P_\alpha \cos\varphi}{n_\alpha} = T\,. \tag{3}$$

Using the formulae (2) and (3) we calculate the mean value of the energy of the α-particle in the rest system. The results obtained with this method are given in the second row of Table III and are in perfect agreement with the ones obtained with the first method.

TABLE III.

primary	M	H
$\overline{E^*_\alpha}$	20.0 ± 1.0	18.0 ± 2.0
$\overline{E^*_\alpha}$	20.4 ± 1.3	20.0 ± 2.0

If we compare the figures of Table III with the value of the mean energy of the α-particles obtained for the L primary, $\overline{E_\alpha} = \overline{E^*_\alpha} = 18.0 \pm 1.5$ (see Table II), we can conclude that the mean energy of the α-particles evaporated from the target nucleus does not depend on the charge of the incident nucleus.

With the same methods above described, we calculate the mean energy of the α-particles evaporated from the nuclei of C, N, O of the emulsion and that of the α-particles evaporated from the nuclei of Ag, Br of the emulsion. For this purpose, we divided the studied stars in stars with a total number of grey and black prongs $\leqslant 7$ ($N_h \leqslant 7$) and stars with a total number of grey and black prongs > 7 ($N_h > 7$). The stars with $N_h \leqslant 7$ are due to collisions with the light elements of the emulsion and to peripheral collisions with the heavy elements. From the emulsion composition and from the evaluation of the cross-section in emulsion, the percentage of collisions with a C, N, O target is estimated to be 26%. The stars from which the α-particles are emitted, are stars with $N_h \leqslant 7$ in 36% of the cases. Consequently 28% of the stars with $N_h \leqslant 7$ represent peripheral collisions with heavy nuclei.

The results, uncorrected for peripheral collisions' contamination, are:

$\overline{E^*_\alpha} = (13.2 \pm 1.2)$ MeV for α-particles evaporated from C, N, O nuclei.

$\overline{E^*_\alpha} = (21.6 \pm 1.2)$ MeV for α-particles evaporated from Ag, Br nuclei.

If we correct these results for peripheral collisions contamination, we obtain

$\overline{E^*_\alpha} = (10.0 \pm 1.2)$ MeV for α-particles evaporated from C, N, O nuclei

$\overline{E^*_\alpha} = (21.6 \pm 1.2)$ MeV for α-particles evaporated from Ag, Br nuclei

These figures represent the lower limits for the energy of the α-particles, because they have been calculated including in the mean also the evaporation tracks with a range $\leqslant 10\ \mu m$, for which the experimental method used to distinguish the α-particle from the protons, does not give reliable results. The higher limits, obtained excluding from the calculation of the mean energy every track with a range $\leqslant 10\ \mu m$, are the following:

$\overline{E^*_\alpha} = (10.0 \pm 1.2)$ MeV for α-particles evaporated from C, N, O nuclei.

$\overline{E^*_\alpha} = (23.2 \pm 1.3)$ MeV for α-particles evaporated from Ag, Br nuclei.

Also these last figures have been corrected for the percentage of peripheral collisions.

In Table IV are summarized the lower and higher limits of the mean energy of the α-particles evaporated from nuclei of different size.

TABLE IV.

target nucleus	C, N, O	Ag, Br
$\overline{E^*_\alpha}$ (lower limit)	10.0 ± 1.2 (MeV)	21.6 ± 1.2 (MeV)
$\overline{E^*_\alpha}$ (higher limit)	10.0 ± 1.2 (MeV)	23.2 ± 1.3 (MeV)

In the work of our group already quoted has been given also the variation of the mean angle of the α-particles of splitting with the number of nucleon pairs involved in the collision. This variation could give an indication that the effect does not depend only on the size of the nucleus, but also on the type of collision. In the present work we find an analogous variation of the energy of the α-particles of evaporation with the number of nucleon pairs involved in the collision. The results are given in Table V.

The number of nucleon pairs depends both on the size of the incident nucleus as on the size of the target nucleus. From the results now obtained (see Table III and Table IV) we think that the variation of the mean energy of the α-particles shown in Table V is a consequence of the variation due to the size of the nucleus, i.e., if there is a dependence of the energy of evaporation on the type of collision, this dependence is certainly much smaller than the dependence on the size of the nucleus. This fact is in agreement with the slight dependence of the mean energy of the evaporation products on the excitation energy [4].

TABLE V.

N_{np}	$\overline{E^*_\alpha}$ (MeV)
0-1-2	11.6 ± 2.0
3-4-5	16.8 ± 2.0
6-7-8-9-10-11	21.2 ± 2.0
11	22.0 ± 1.6

N_{np} indicates the number of nucleon pairs involved in the collision.

3. – Conclusions.

From the results given in the preceding section, we can conclude that hypothesis *b*) (see introduction) seems to be the correct one. In fact, the results collected in Tables III and IV indicate that, in the laboratory system, the angle of the α-particles of splitting does not depend on the size of the target nucleus, but depends on the charge of the incident nucleus.

According to the experimental data of the present work, the coefficient of Peters' formula must be taken as:

0.06 for the splitting of M nuclei,

0.08 for the splitting of H nuclei.

From the ratio of this coefficients: $0.08/0.06 = 1.33$, we would expect to find that the mean angle of the α-particles emitted in the splitting of the H nuclei is greater than the mean angle of α-particles emitted in the splittings of the M nuclei, by a factor 1.33. Actually this factor, calculated from the data of our group (given in the work already quoted), is: $(2.11 \pm 0.19)°/(1.35 \pm 0.10)° =$

(4) J. B. HARDING, S. LATTIMORE and D. H. PERKINS: *Proc. Roy. Soc.*, A **196**, 325 (1949); D. H. PERKINS: *Phil. Mag.*, **41**, 138 (1950).

$= 1.56 \pm 0.30$. The discrepancy is not large, but we think it useful to discuss a little more about this point.

The values of the mean angle of the α-particles ($\sqrt{\overline{\theta_\alpha^2}} = (2.11 \pm 0.19)°$ in the case of H primaries and $\sqrt{\overline{\theta_\alpha^2}} = (1.35 \pm 0.10)°$ in the case of M primaries) have been calculated as the root mean square of the angles of all the α-particles emitted in the studied events. Looking at Peters' formula, we see that the quantity that is proportional to the coefficient is the root mean square angle of the α-particles emitted in each single event. If we calculate (from the quoted data of our group) the mean of the root mean square angles of each event, we find:

$$\overline{\sqrt{\overline{\theta_\alpha^2}}} = (1.60 \pm 0.22)° \quad \text{in the case of H primaries},$$

$$\overline{\sqrt{\overline{\theta_\alpha^2}}} = (1.16 \pm 0.11)° \quad \text{in the case of M primaries}.$$

The ratio between these two angles is: 1.38 ± 0.36.

The difference between the two methods of calculating a mean value of the angle of the α-particles is due to two facts: first, in one method we make an arithmetic mean and in the other method we make a root mean square mean; second, in one method each event has the same statistical weight, whereas in the other method the events have a statistical weight increasing with the increase of the number of α-particles emitted in the splitting. The arithmetic mean is always lower than the root mean square one; the difference in the statistical weights has an influence on the final result only if there is a variation of the angle of the α-particles that depends on the number of α-particles emitted in the splitting. This variation has been found by FOWLER *et al.* [3] and it is an important point to be considered if we want to understand the very high values of the evaporation energy calculated for the α-particles by these authors.

At the present point of our knowledge it is not possible to say which is the physical meaning of the variation of the angle of the α-particles with the number of α-particles emitted from the same nucleus and which is the phenomenon that produces it. It would be useful to know the energy of evaporation of the α-particles in the rest system when one α-particle only and when more α-particles are evaporated. Unfortunately we cannot use the experimental results of the present work for this purpose, because, having analysed only the evaporation tracks that make a vertical angle $\leqslant 45°$ with the direction of the primary, we do not know the number of α-particles evaporated in each star.

The result of the present work clears up the question about the validity of Peters' formula. Actually the value 0.06 of the coefficient, choosen by PETERS, is the best one for the medium elements, while for the heavy ones a value of 0.08 seems more correct. This change in the value of the coefficient takes into account the variation of the mean energy of evaporation of the α-particles with the size of the nucleus that was already pointed out by PERKINS (5). The mean energies of evaporation found by us are only slightly higher than the ones given by PERKINS, but not so high as the ones suggested by FOWLER *et al.* (3).

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(5) D. H. PERKINS: *Phil. Mag.*, **40**, 601 (1949).

RIASSUNTO

La formula proposta da PETERS per la determinazione dell'energia dei primari pesanti, richiede la conoscenza dell'energia media delle particelle α di evaporazione Il presente lavoro prova che questa energia dipende leggermente dalla dimensione del nucleo che evapora.

1667