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Bake-off Kernels in Index Notation

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$$v_{abc}^e = \sum_{i=0}^{nm0} \phi_{ia} \sum_{j=0}^{nm1} \phi_{jb} \sum_{k=0}^{nm2} u_{ijk}^e \phi_{kc}$$

$$w_{abc}^e = v_{abc}^e J_{abc}^e$$

$$\underline{u}_{ijk}^e = \sum_{a=0}^{nq0} \phi_{ia} \sum_{b=0}^{nq1} \phi_{jb} \sum_{c=0}^{nq2} \phi_{kc} w_{abc}^e$$

$$qr_{ijk}^e = \sum_{n=0}^{nm0} u_{njk}^e D_{in}$$

$$qs_{ijk}^e = \sum_{n=0}^{nm1} u_{ink}^e D_{jn}$$

$$qt_{ijk}^e = \sum_{n=0}^{nm2} u_{ijn}^e D_{kn}$$

Bake-off Kernel 5 – Phase 2

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$$rqr_{ijk}^e = G_{1,ijk}^e qr_{ijk}^e + G_{2,ijk}^e qs_{ijk}^e + G_{3,ijk}^e qt_{ijk}^e$$

$$rqs_{ijk}^e = G_{2,ijk}^e qr_{ijk}^e + G_{4,ijk}^e qs_{ijk}^e + G_{5,ijk}^e qt_{ijk}^e$$

$$rqt_{ijk}^e = G_{3,ijk}^e qr_{ijk}^e + G_{5,ijk}^e qs_{ijk}^e + G_{6,ijk}^e qt_{ijk}^e$$

Bake-off Kernel 5 – Phase 3

$$\underline{u}^e_{ijk} = \sum_{n=0}^{nm0} D_{ni} \, rqr^e_{njk} + \sum_{n=0}^{nm1} D_{nj} \, rqs^e_{ink} + \sum_{n=0}^{nm2} D_{nk} \, rqt^e_{ijn}$$

NOTES This kernel is a combination of BK1 and BK5:

1. Backward sweep from BK1 is applied.
2. Phases from BK5 are performed.
3. Forward sweep from BK1 is applied.

$$v^e_{abc} = \sum_{i=0}^{nm0} \phi_{ia} \sum_{j=0}^{nm1} \phi_{jb} \sum_{k=0}^{nm2} u^e_{ijk} \phi_{kc}$$

Bake-off Kernel 3 – Phase 2

$$qr_{abc}^e = \sum_{n=0}^{nq0} v_{nbc}^e D_{an}$$

$$qs_{abc}^e = \sum_{n=0}^{nq1} v_{anc}^e D_{bn}$$

$$qt_{abc}^e = \sum_{n=0}^{nq2} v_{abn}^e D_{cn}$$

Bake-off Kernel 3 – Phase 3

$$rqr^e_{abc} = G^e_{1,abc} qr^e_{abc} + G^e_{2,abc} qs^e_{abc} + G^e_{3,abc} qt^e_{abc}$$

$$rqs^e_{abc} = G^e_{2,abc} qr^e_{abc} + G^e_{4,abc} qs^e_{abc} + G^e_{5,abc} qt^e_{abc}$$

$$rqt^e_{abc} = G^e_{3,abc} qr^e_{abc} + G^e_{5,abc} qs^e_{abc} + G^e_{6,abc} qt^e_{abc}$$

$$w_{abc}^e = \sum_{n=0}^{nq0} D_{na} rq r_{nbc}^e + \sum_{n=0}^{nq1} D_{nb} rq s_{anc}^e + \sum_{n=0}^{nq2} D_{nc} rq t_{abn}^e$$

Bake-off Kernel 3 – Phase 5

$$\underline{u}^e_{ijk} = \sum_{a=0}^{nq0} \phi_{ia} \sum_{b=0}^{nq1} \phi_{jb} \sum_{c=0}^{nq2} \phi_{kc} w^e_{abc}$$

▤ T. Kolev, P. Fischer, M. Min, J. Dongarra, J. Brown, V. Dobrev, T. Warburton, S. Tomov, M. S. Shephard, A. Abdelfattah, et al., *Efficient exascale discretizations: High-order finite element methods*, The International Journal of High Performance Computing Applications, 35 (2021), pp. 527–552.

▤ K. Świrydowicz, N. Chalmers, A. Karakus, and T. Warburton, *Acceleration of tensor-product operations for high-order finite element methods*, The International Journal of High Performance Computing Applications, 33 (2019), pp. 735–757.