

Bake-off Kernels in Index Notation

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$$v_{abc}^e = \sum_{i=1}^{nm0} \phi_{ia} \sum_{j=1}^{nm1} \phi_{jb} \sum_{k=1}^{nm2} u_{ijk}^e \phi_{kc}$$

$$w_{abc}^e = v_{abc}^e J_{abc}^e$$

$$u_{ijk}^e = \sum_{a=1}^{nq0} \phi_{ia} \sum_{b=1}^{nq1} \phi_{jb} \sum_{c=1}^{nq2} \phi_{kc} w_{abc}^e$$

Bake-off Kernel 5 - Phase 1

$$qr_{ijk}^e = \sum_{n=1}^{nm0} u_{njk}^e D_{in}$$

$$qs_{ijk}^e = \sum_{n=1}^{nm1} u_{ink}^e D_{jn}$$

$$qt_{ijk}^e = \sum_{n=1}^{nm2} u_{ijn}^e D_{kn}$$

Bake-off Kernel 5 - Phase 2

$$rqr_{ijk}^e = G_{1,ijk}^e qr_{ijk}^e + G_{2,ijk}^e qs_{ijk}^e + G_{3,ijk}^e qt_{ijk}^e$$

$$rqs_{ijk}^e = G_{2,ijk}^e qr_{ijk}^e + G_{4,ijk}^e qs_{ijk}^e + G_{5,ijk}^e qt_{ijk}^e$$

$$rqt_{ijk}^e = G_{3,ijk}^e qr_{ijk}^e + G_{5,ijk}^e qs_{ijk}^e + G_{6,ijk}^e qt_{ijk}^e$$

$$\underline{u}_{ijk}^e = \sum_{n=1}^{nm0} D_{ni} rqr_{njk}^e + \sum_{n=1}^{nm1} D_{nj} rqs_{ink}^e + \sum_{n=1}^{nm2} D_{nk} rqt_{ijn}^e$$

NOTES This kernel is a combination of BK1 and BK5:

- 1** Backward sweep from BK1 is applied.
- 2** Phases from BK5 are performed.
- 3** Forward sweep from BK1 is applied.

Bake-off Kernel 3 - Phase 1

$$v_{abc}^e = \sum_{i=1}^{nm0} \phi_{ia} \sum_{j=1}^{nm1} \phi_{jb} \sum_{k=1}^{nm2} u_{ijk}^e \phi_{kc}$$

Bake-off Kernel 3 - Phase 2

$$qr_{abc}^e = \sum_{n=1}^{nq0} v_{nbc}^e D_{an}$$

$$qs_{abc}^e = \sum_{n=1}^{nq1} v_{anc}^e D_{bn}$$

$$qt_{abc}^e = \sum_{n=1}^{nq2} v_{abn}^e D_{cn}$$

Bake-off Kernel 3 - Phase 3

$$rqr_{abc}^e = G_{1,abc}^e qr_{abc}^e + G_{2,abc}^e qs_{abc}^e + G_{3,abc}^e qt_{abc}^e$$

$$rqs_{abc}^e = G_{2,abc}^e qr_{abc}^e + G_{4,abc}^e qs_{abc}^e + G_{5,abc}^e qt_{abc}^e$$

$$rqt_{abc}^e = G_{3,abc}^e qr_{abc}^e + G_{5,abc}^e qs_{abc}^e + G_{6,abc}^e qt_{abc}^e$$

$$w_{abc}^e = \sum_{n=1}^{nq0} D_{nar} rqr_{nbc}^e + \sum_{n=1}^{nq1} D_{nbr} rqs_{anc}^e + \sum_{n=1}^{nq2} D_{ncr} rqt_{abn}^e$$

$$\underline{u}_{ijk}^e = \sum_{a=1}^{nq0} \phi_{ia} \sum_{b=1}^{nq1} \phi_{jb} \sum_{c=1}^{nq2} \phi_{kc} w_{abc}^e$$

References

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