RUHR-UNIVERSITÄT BOCHUM

Bake-off Kernels in Index Notation

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Bake-off Kernel 1

$$v^e_{abc} = \sum_{i=0}^{nm0} \phi_{ia} \sum_{j=0}^{nm1} \phi_{jb} \sum_{k=0}^{nm2} u^e_{ijk} \phi_{kc}$$

$$w^e_{abc} = v^e_{abc} J_{abc}$$

$$\underline{u}^e_{ijk} = \sum_{a=0}^{nq0} \phi_{ia} \sum_{b=0}^{nq1} \phi_{jb} \sum_{c=0}^{nq2} \phi_{kc} w^e_{abc}$$

Bake-off Kernel 5 - Phase 1

$$qr^e_{ijk} = \sum_{n=0}^{nm0} u^e_{njk} D_{in}$$

$$qs^e_{ijk} = \sum_{n=0}^{nm1} u^e_{ink} D_{jn}$$

$$qt^e_{ijk} = \sum_{n=0}^{nm2} u^e_{ijn} D_{kn}$$

Bake-off Kernel 5 - Phase 2

$$rqr^e_{ijk} = G^e_{1,ijk} qr^e_{ijk} + G^e_{2,ijk} qs^e_{ijk} + G^e_{3,ijk} qt^e_{ijk}$$

$$rqs^e_{ijk} = G^e_{2,ijk} qr^e_{ijk} + G^e_{4,ijk} qs^e_{ijk} + G^e_{5,ijk} qt^e_{ijk}$$

$$rqt^e_{ijk} = G^e_{3,ijk} qr^e_{ijk} + G^e_{5,ijk} qs^e_{ijk} + G^e_{6,ijk} qt^e_{ijk}$$

Bake-off Kernel 5 - Phase 3

$$\underline{u}^e_{ijk} = \sum_{n=0}^{nm0} D_{ni}\, rqr^e_{njk} + \sum_{n=0}^{nm1} D_{nj}\, rqs^e_{ink} + \sum_{n=0}^{nm2} D_{nk}\, rqt^e_{ijn}$$

Bake-off Kernel 3

NOTES This kernel is a combination of BK1 and BK5:

1. Backward sweep from BK1 is applied.
2. Phases from BK5 are performed.
3. Forward sweep from BK1 is applied.

Bake-off Kernel 3 - Phase 1

$$v^e_{abc} = \sum_{i=0}^{nm0} \phi_{ia} \sum_{j=0}^{nm1} \phi_{jb} \sum_{k=0}^{nm2} u^e_{ijk} \phi_{kc}$$

Bake-off Kernel 3 - Phase 2

$$qr^e_{abc} = \sum_{n=0}^{nq0} v^e_{nbc} D_{an}$$

$$qs^e_{abc} = \sum_{n=0}^{nq1} v^e_{anc} D_{bn}$$

$$qt^e_{abc} = \sum_{n=0}^{nq2} v^e_{abn} D_{cn}$$

Bake-off Kernel 3 - Phase 3

$$rqr^e_{abc} = G^e_{1,abc} qr^e_{abc} + G^e_{2,abc} qs^e_{abc} + G^e_{3,abc} qt^e_{abc}$$

$$rqs^e_{abc} = G^e_{2,abc} qr^e_{abc} + G^e_{4,abc} qs^e_{abc} + G^e_{5,abc} qt^e_{abc}$$

$$rqt^e_{abc} = G^e_{3,abc} qr^e_{abc} + G^e_{5,abc} qs^e_{abc} + G^e_{6,abc} qt^e_{abc}$$

Bake-off Kernel 3 - Phase 4

$$w^e_{abc} = \sum_{n=0}^{nq0} D_{ni}\, rqr^e_{nbc} + \sum_{n=0}^{nq1} D_{nj}\, rqs^e_{anc} + \sum_{n=0}^{nq2} D_{nk}\, rqt^e_{abn}$$

Bake-off Kernel 3 - Phase 5

$$\underline{u}^e_{ijk} = \sum_{a=0}^{nq0} \phi_{ia} \sum_{b=0}^{nq1} \phi_{jb} \sum_{c=0}^{nq2} \phi_{kc} w^e_{abc}$$

References

T. Kolev, P. Fischer, M. Min, J. Dongarra, J. Brown, V. Dobrev, T. Warburton, S. Tomov, M. S. Shephard, A. Abdelfattah, et al., *Efficient exascale discretizations: High-order finite element methods*, The International Journal of High Performance Computing Applications, 35 (2021), pp. 527–552.

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