



Theory and Practice of Finite Element Methods

Triangulation, DoFHandler, FiniteElement

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Aims for this module

- Gain familiarity with three core classes
 - **Triangulation**
 - **DoFHandler**
 - **FiniteElement**
- Create and interrogate meshes
- Create and interrogate sparsity patterns



Reference material

- Main page
<https://dealii.org/current/doxygen/deal.II/index.html>
- Tutorials
 - Step-1
https://dealii.org/current/doxygen/deal.II/step_1.html
 - Step-49
https://dealii.org/current/doxygen/deal.II/step_49.html
 - Step-2
https://dealii.org/current/doxygen/deal.II/step_2.html

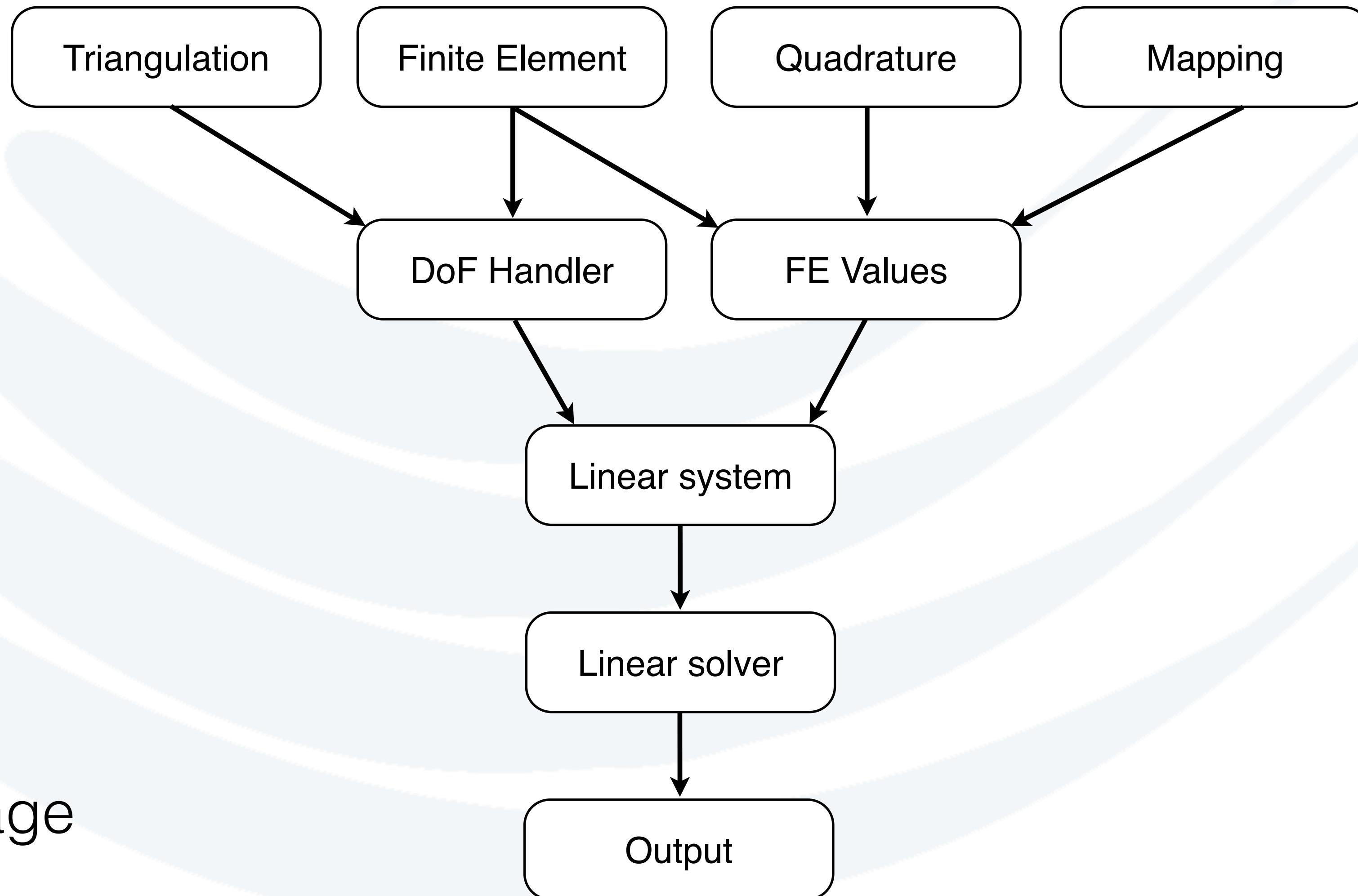


First and **BIGGEST** tip

- Program **defensively**
 - Program and test in **debug** mode
 - Additional compiler warnings
 - Add assertions
 - Perform studies in **release** mode



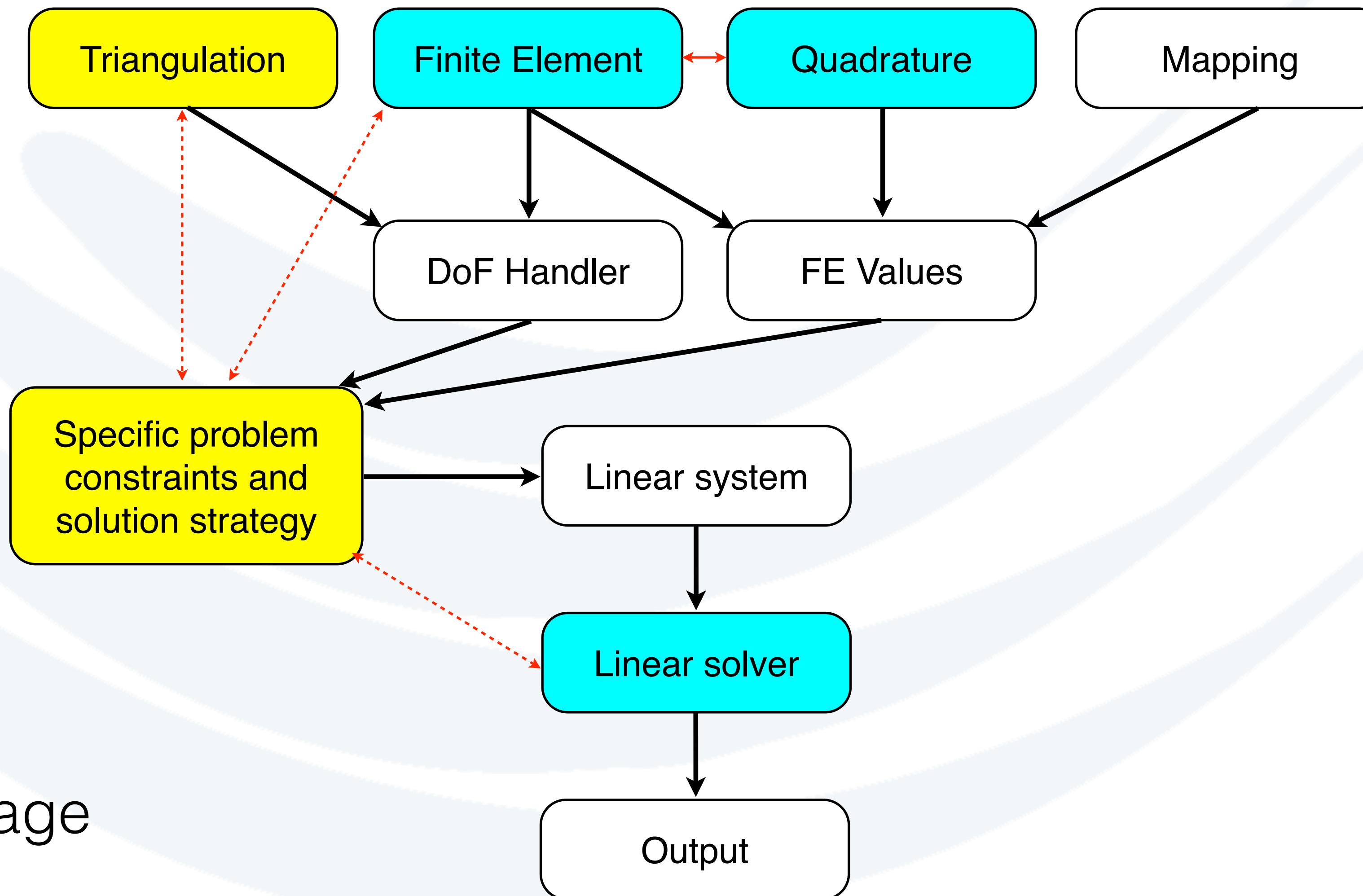
Structure of a prototypical FE problem



Main page

<https://dealii.org/current/doxygen/deal.II/index.html>

Structure of a prototypical FE problem

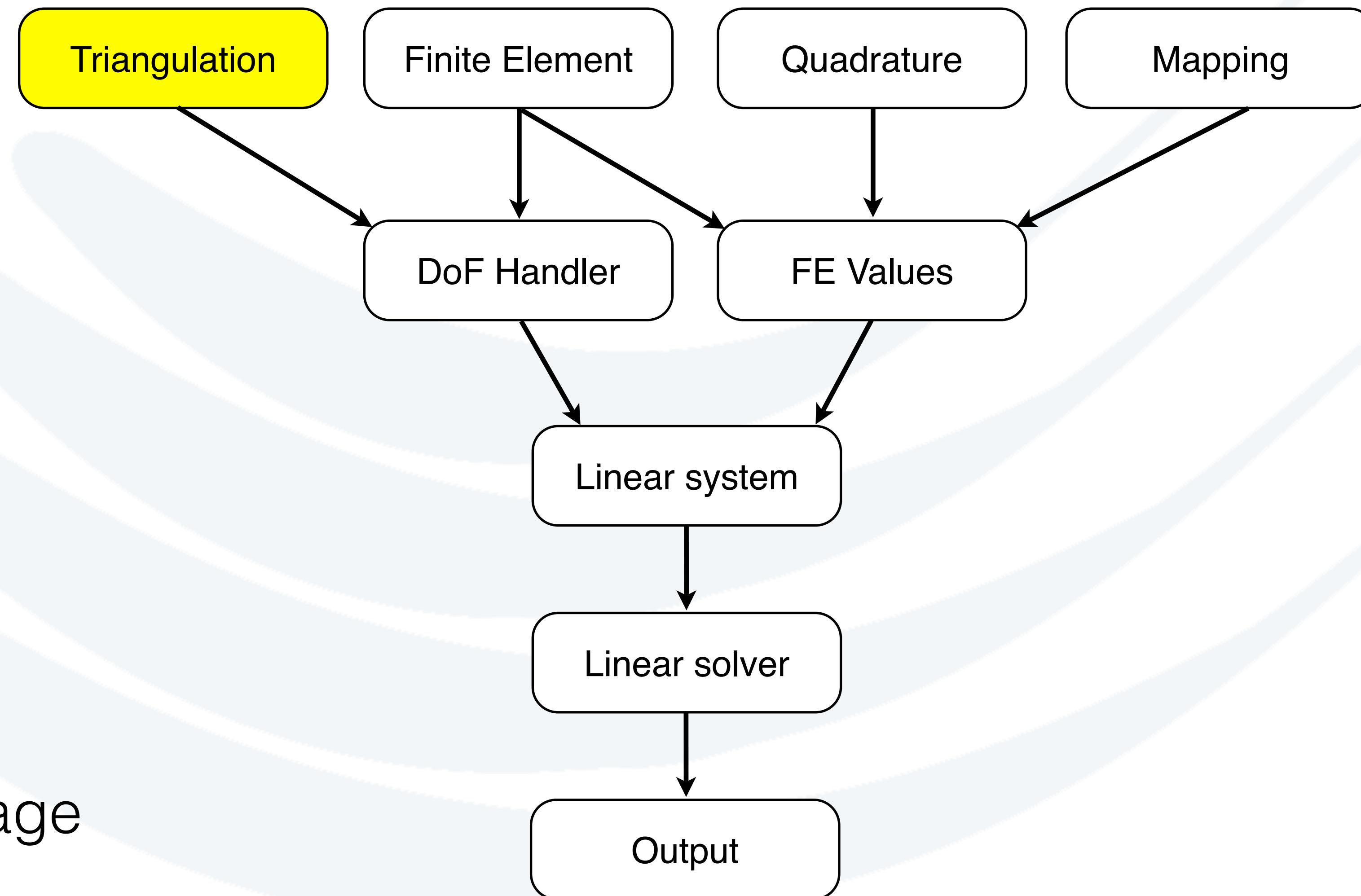


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Structure of a prototypical FE problem



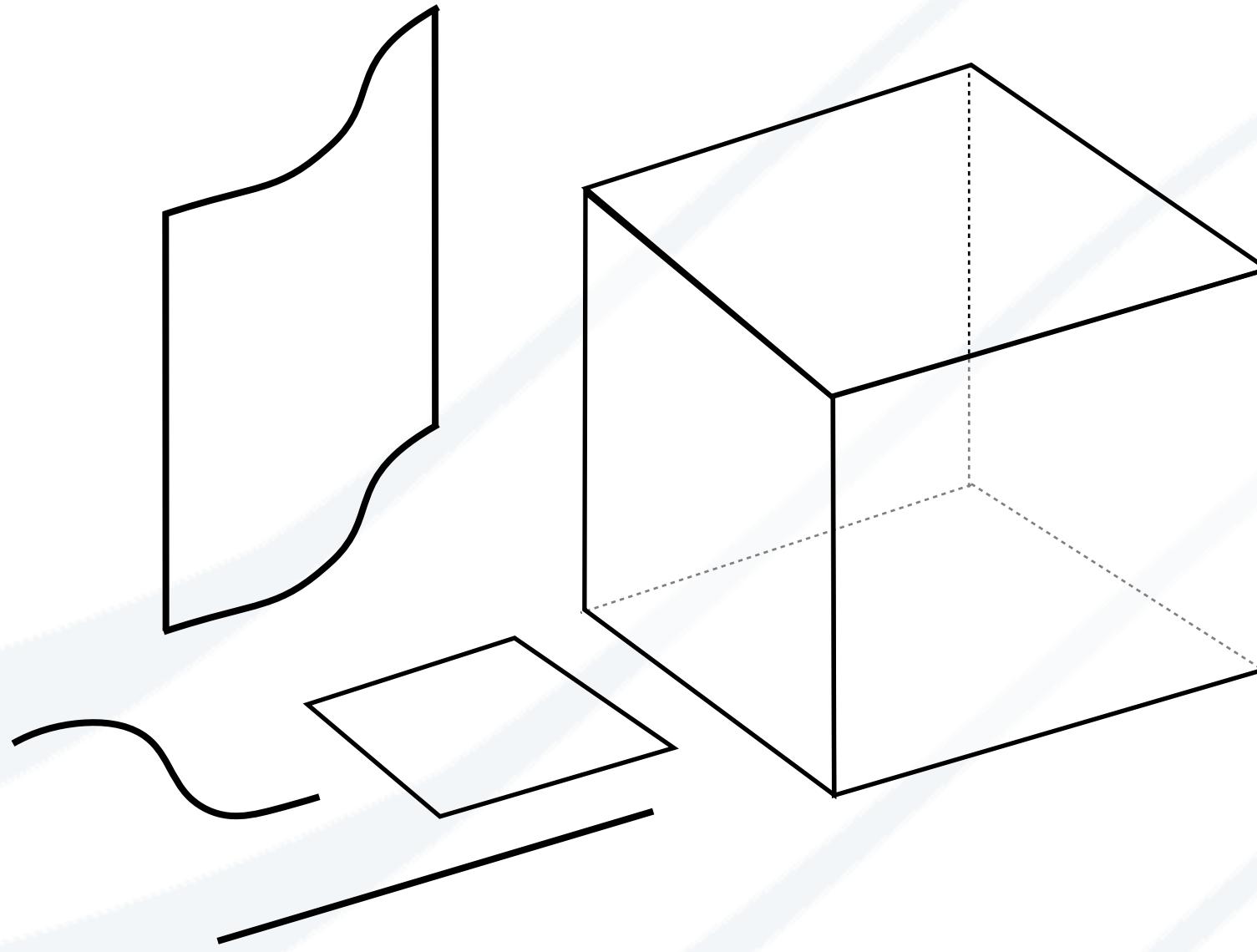
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Interaction with geometry: the Triangulation class

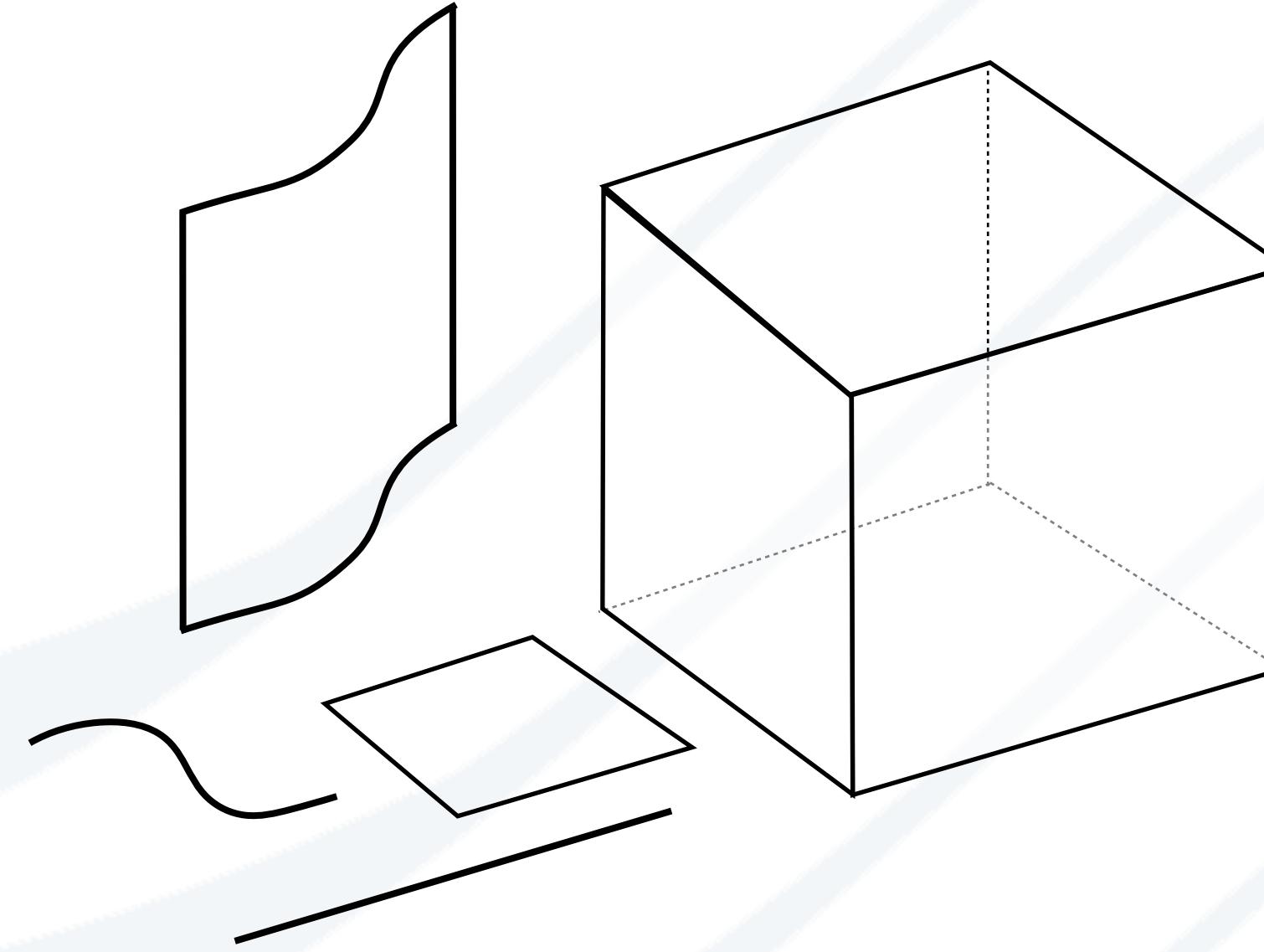
- Describes problem geometry
 - Support for lines, quad, hex elements
 - Conceptually even higher order!
 - Structured/unstructured meshes
 - Co-dimension 1 or 2 case
- Grid creation
 - Built-in basic grid generation and manipulation tools
 - Can read in grids





Interaction with geometry: the Triangulation class

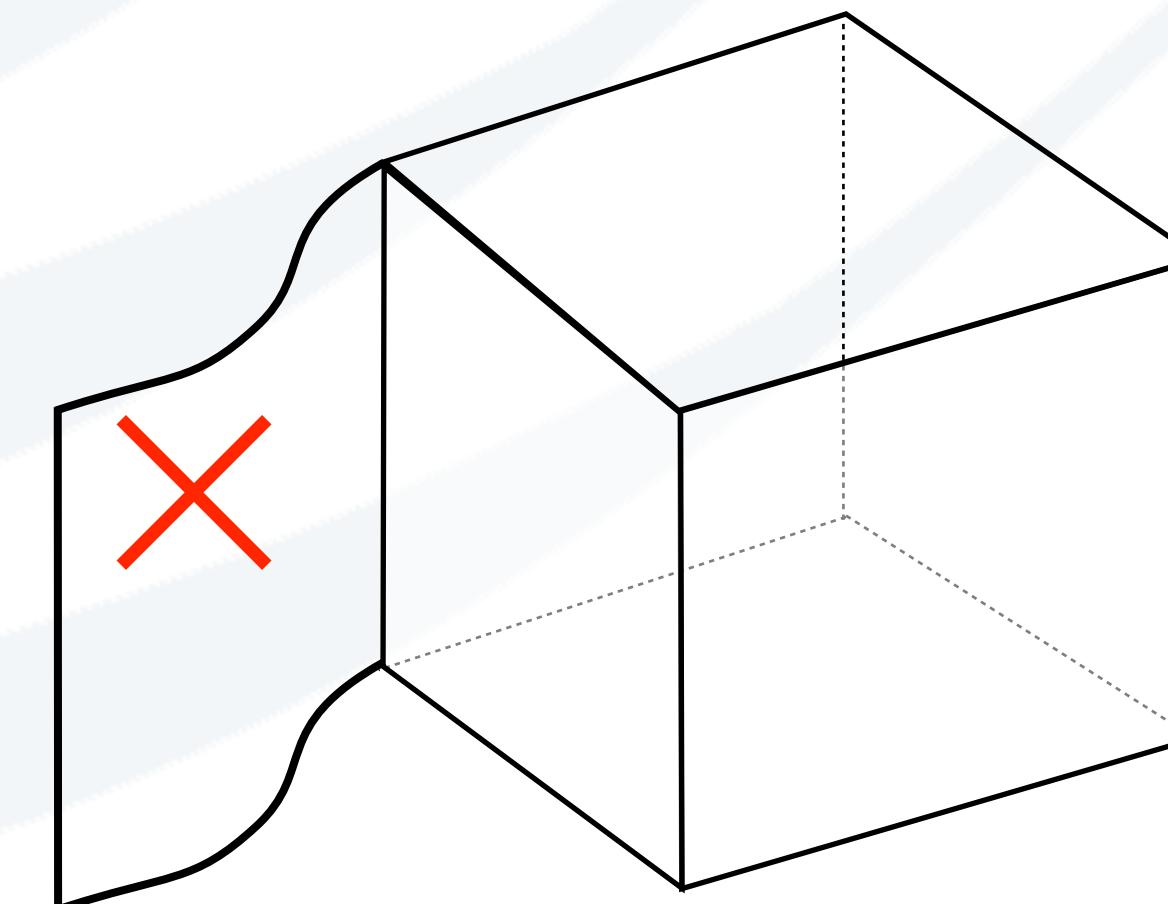
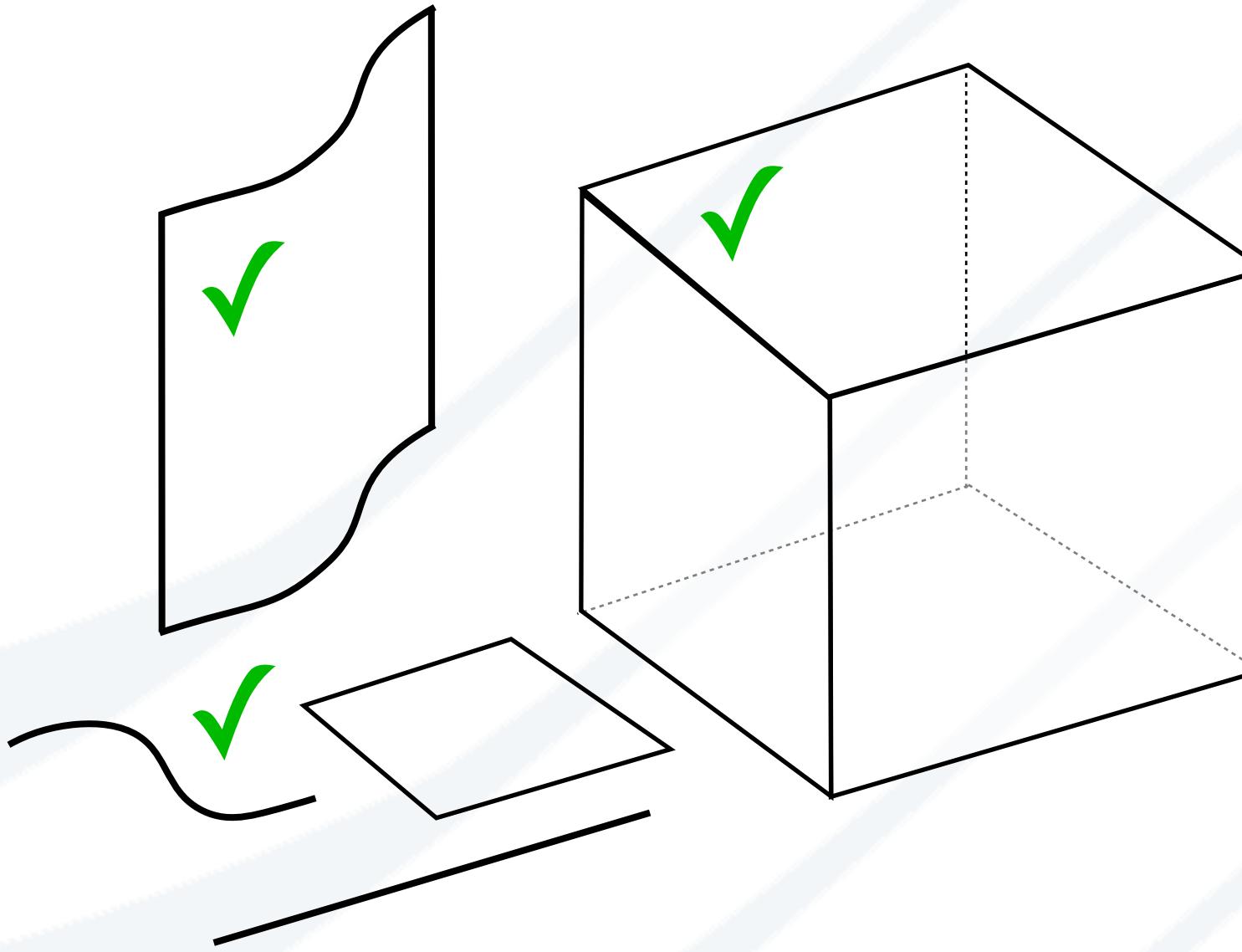
- Assign helper ID's
 - Materials
 - Boundaries
 - Manifolds
- Allows storage of custom data-structure attached to each cell/face
- Cells know about neighbour cells
 - Useful for DG methods





Interaction with geometry: the Triangulation class

- Can enforce topologies
 - Manifolds on boundary
 - Internal manifolds
- Disadvantage
 - Cannot mix triangulation types
 - e.g. Volumetric body with extended manifold surface





Interaction with geometry: the Triangulation class

- Demonstration: Step-1, step-49

https://www.dealii.org/current/doxygen/deal.II/step_1.html

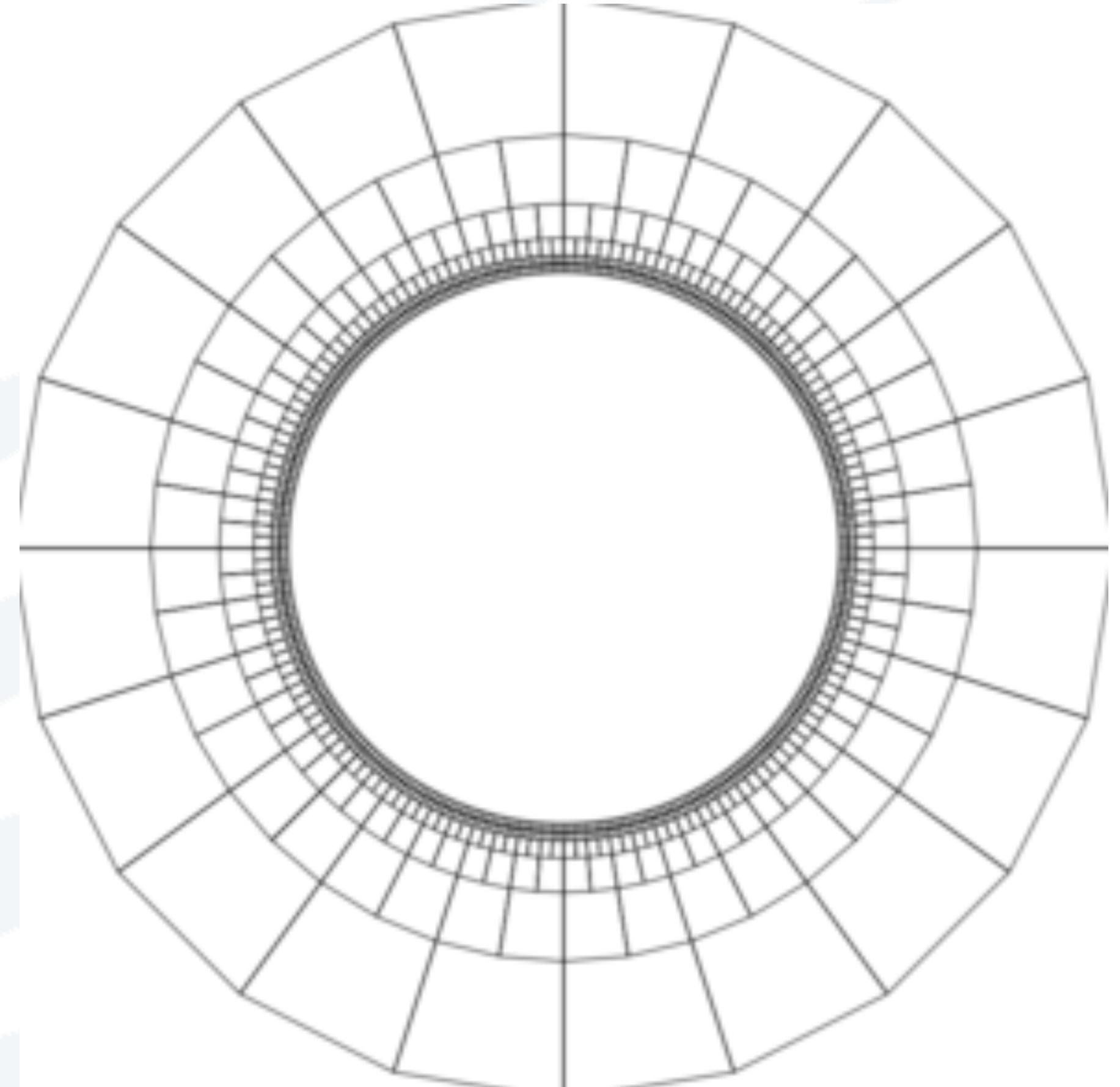
https://www.dealii.org/current/doxygen/deal.II/step_49.html

<http://www.math.colostate.edu/~bangerth/videos.676.5.html>

<http://www.math.colostate.edu/~bangerth/videos.676.6.html>

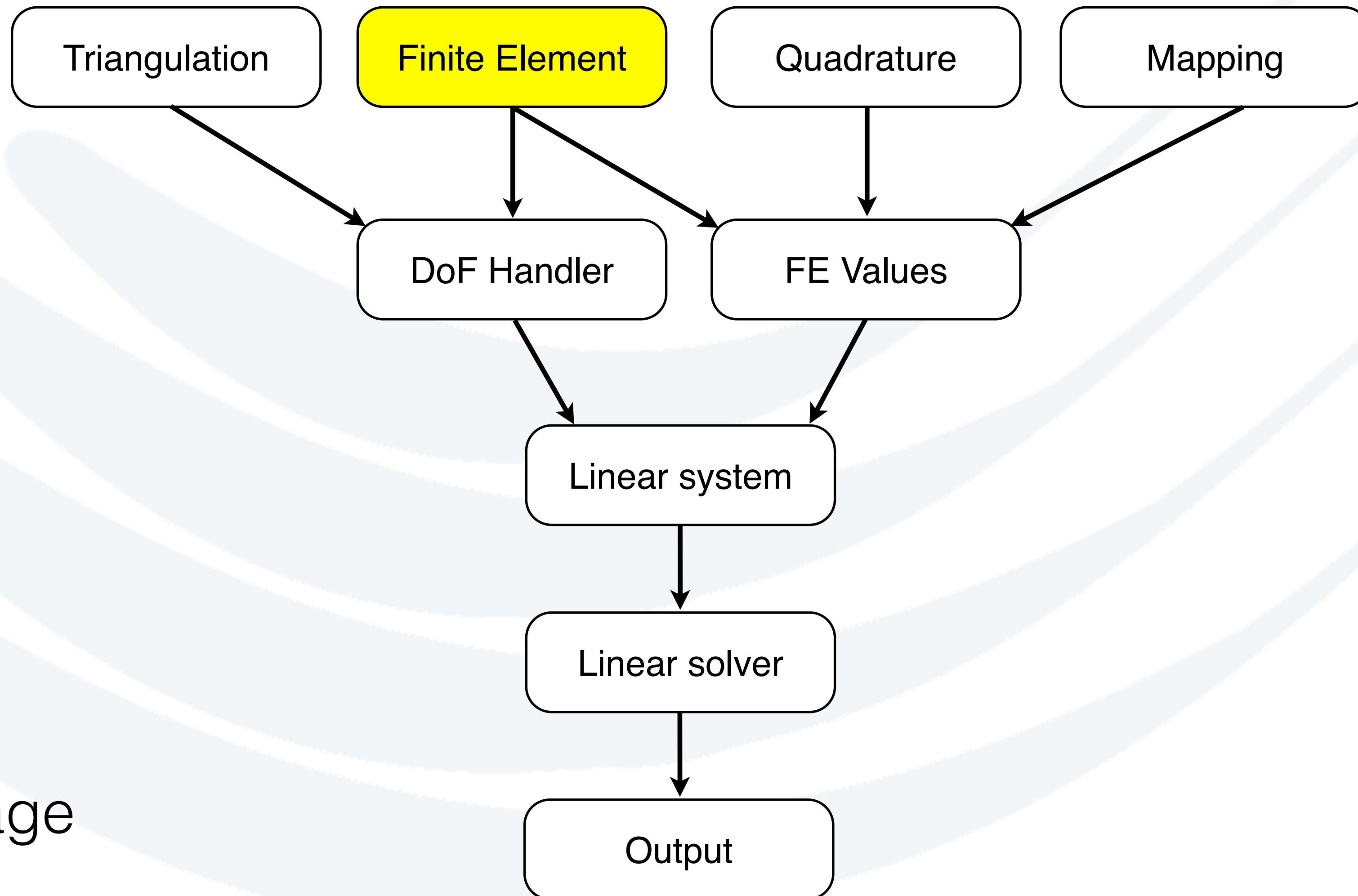
- Key points

- deal.II headers
- Creating a triangulation
- Boundary topology
- Traversing a triangulation
- Querying geometric information
- Manipulating a triangulation
- Aspects of grid refinement
- Visualising a triangulation





Structure of a prototypical FE problem



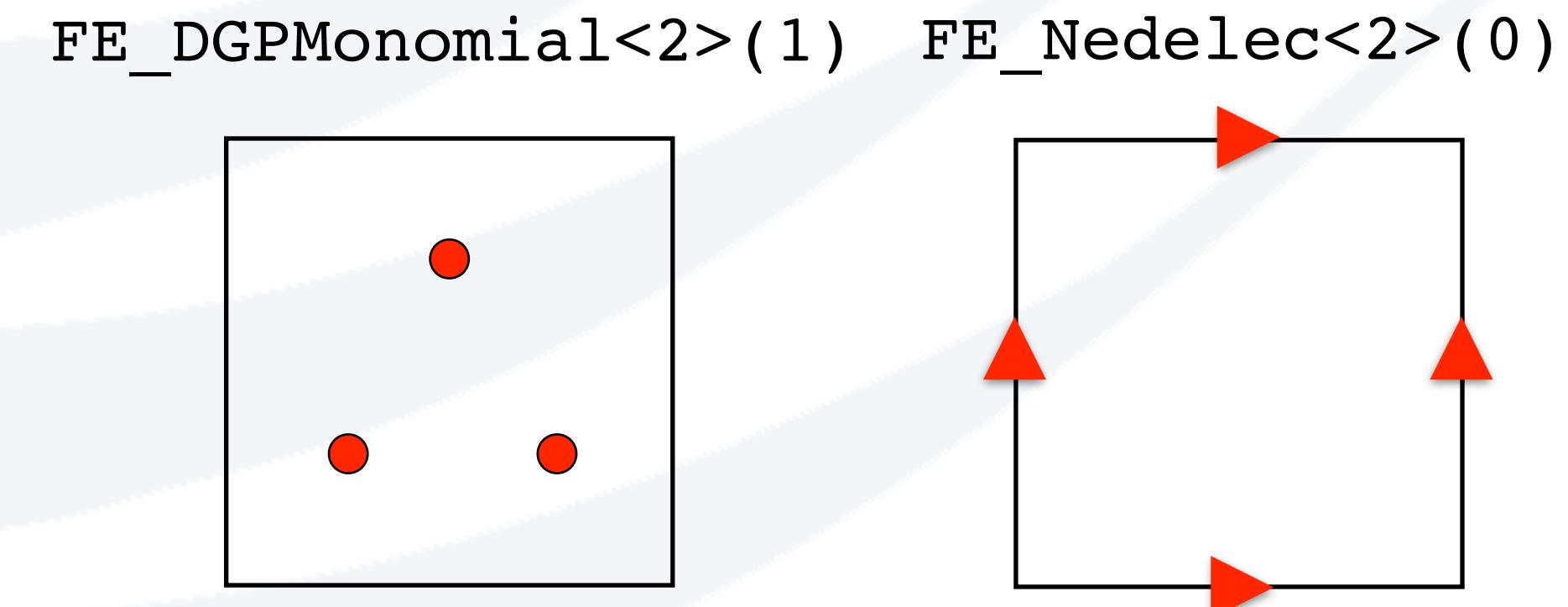
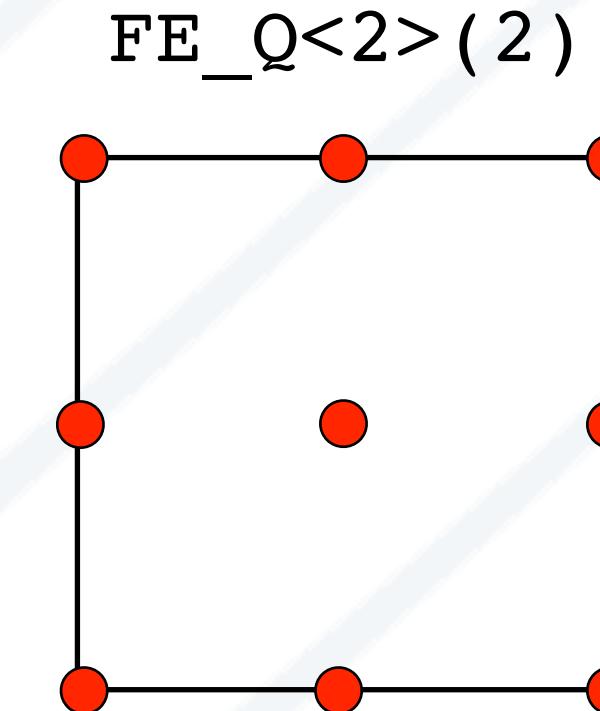
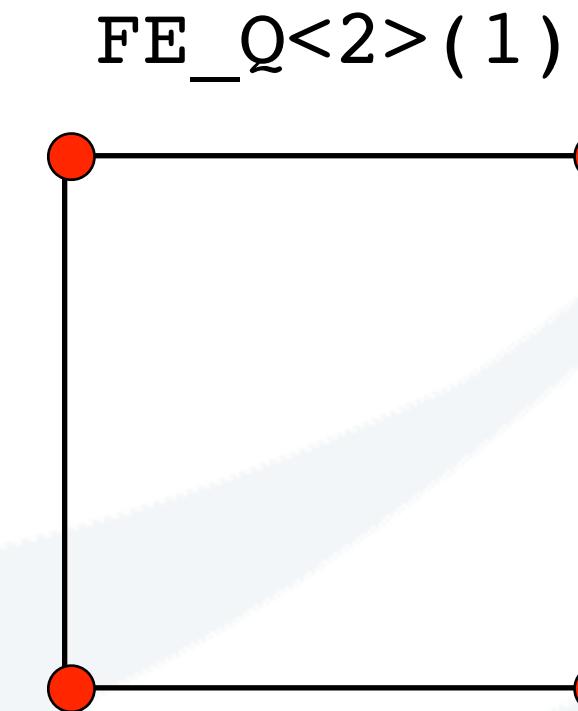
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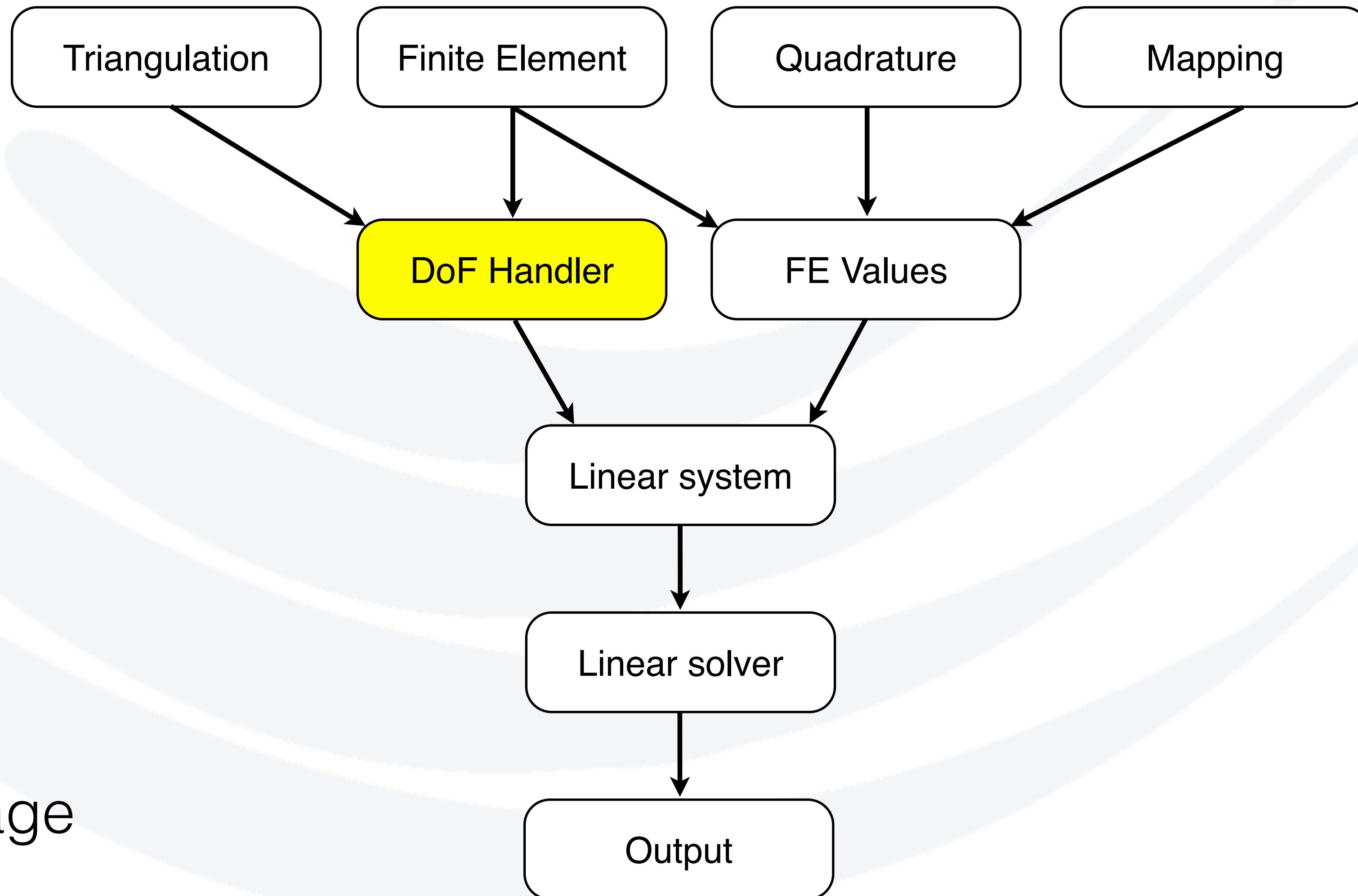
Assigning degrees-of-freedom: the FiniteElement classes

- Built in Finite Elements
 - Continuous
 - Piecewise Lagrange polynomials
 - Discontinuous
 - Monomials
 - Legendre polynomials
 - Vector-valued
 - Nedelec (H_{curl} , C/Dc)
 - Raviart-Thomas (H_{div} , C/Dc)
 - A few more...
 - Can develop finite elements from scratch
 - Specialisation for FE's derived by polynomial expansions
 - Enhanced/bubble elements





Structure of a prototypical FE problem



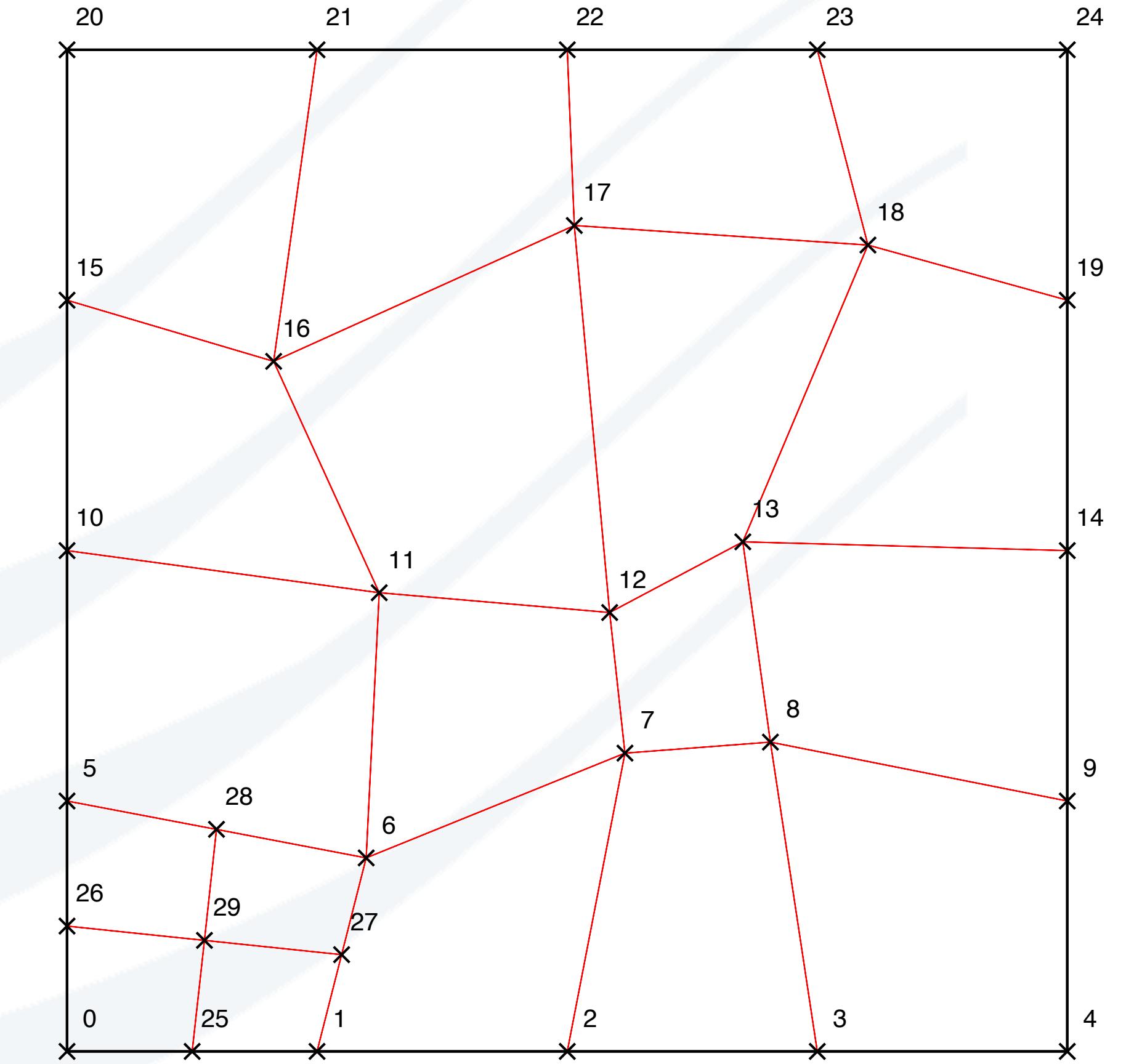
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Assigning degrees-of-freedom: the DoFHandler class

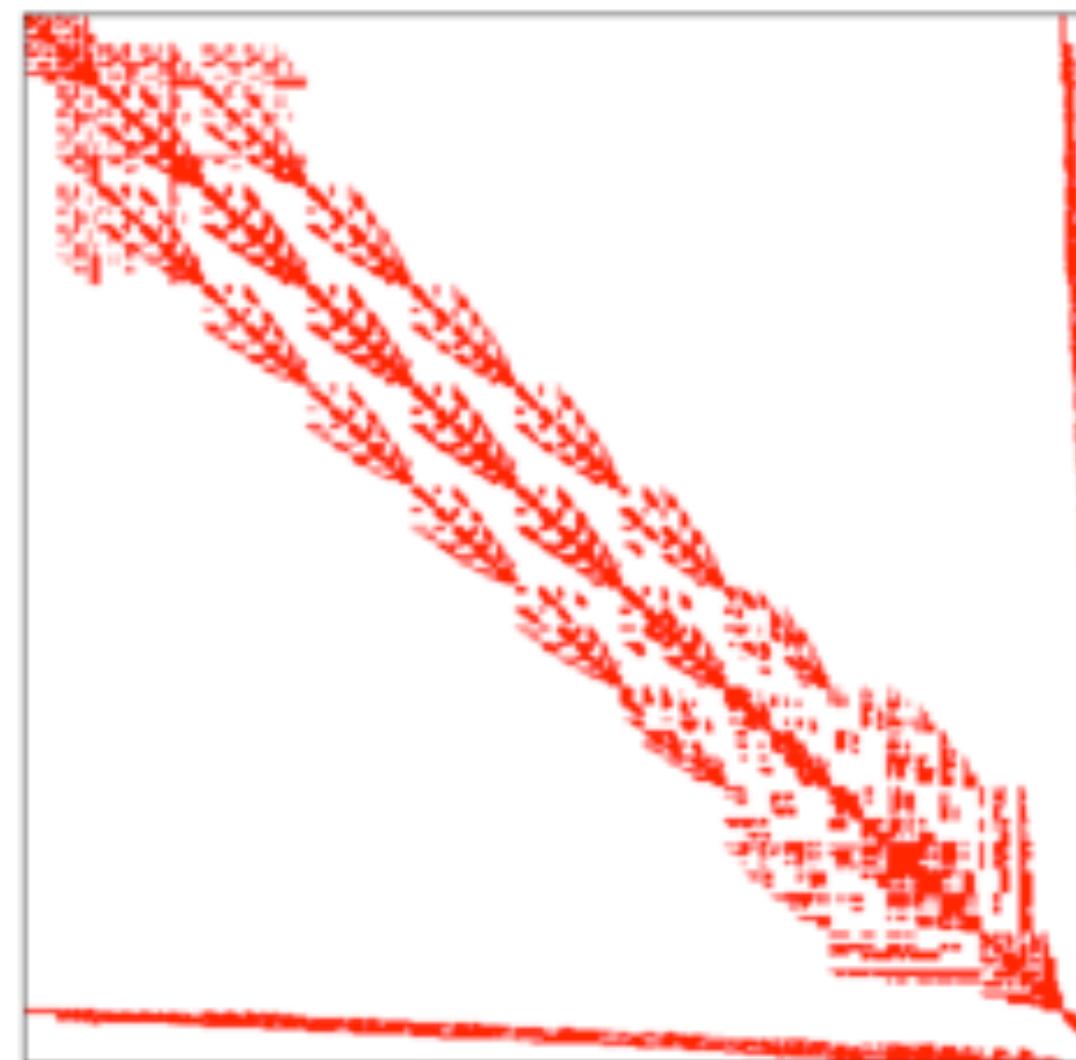
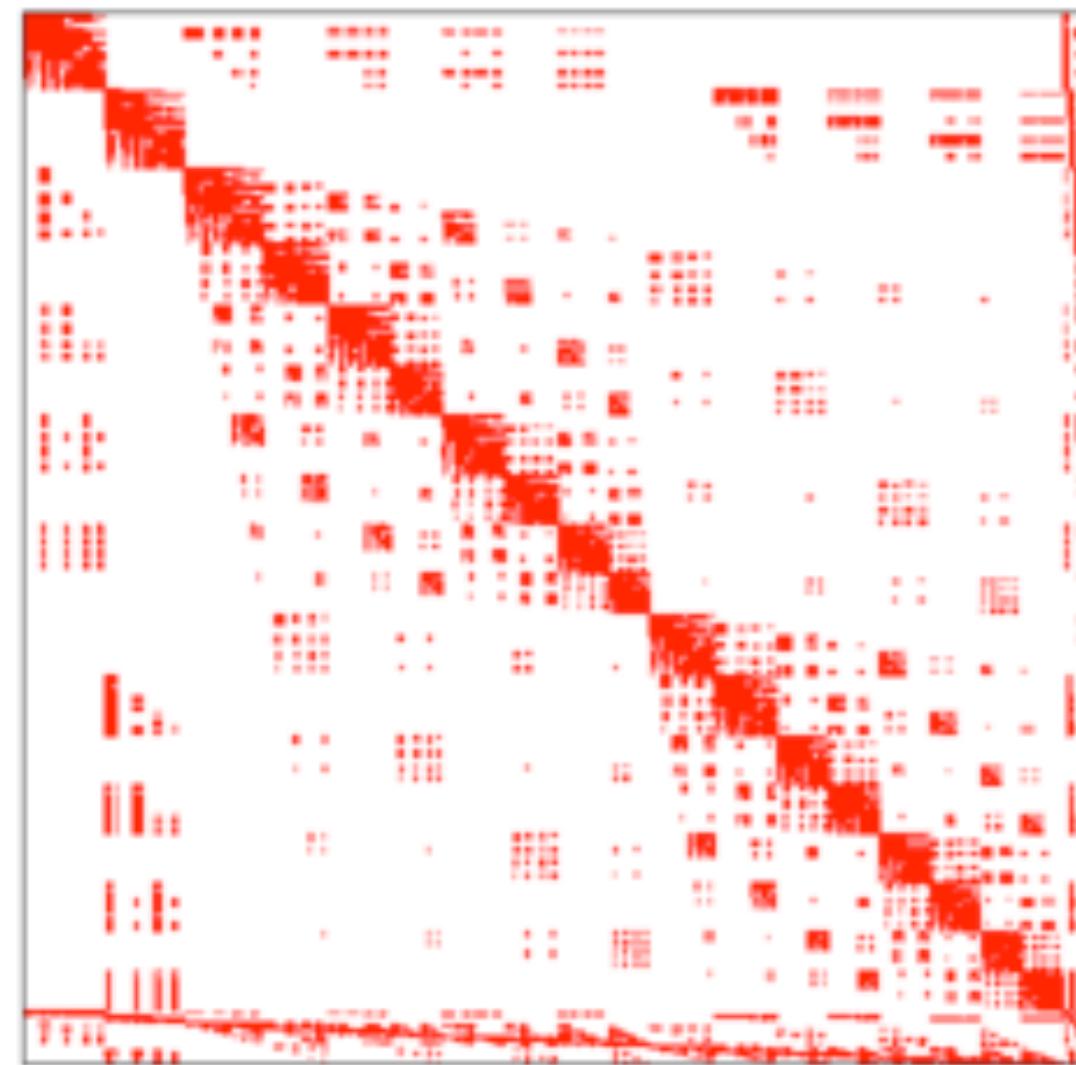
- DoFHandler assigns DoF's to grid
 - Important: separate from Triangulation!
 - Unified way to access DoF's, regardless of FE used
 - e.g. Discontinuous elements: support points not necessarily at vertices
 - Fast access and grid traversal
 - STL-type cell iterators
 - Access to faces, edges through these
 - Disadvantage
 - All cells must have same types of finite-elements*





Assigning degrees-of-freedom: the DoFRenumbering namespace

- Renumbering schemes
 - Cuthill McKee
 - King
 - Downwind
- Reduce bandwidth
- Collect like-components
- Induce block-structure
- Directional (fluid flow)
- MPI subdomain





Assigning degrees-of-freedom: the FiniteElement and DoFHandler classes

- Demonstration: Step-2
https://www.dealii.org/current/doxygen/deal.II/step_2.html
<http://www.math.colostate.edu/~bangerth/videos.676.9.html>
- Key points
 - Choosing a Finite Element
 - Distributing degrees-of-freedom on a mesh
 - Renumbering degrees-of-freedom
 - Visualising sparsity patterns

