SchemaBank: Sparse Routing as Training Curriculum for Parameter-Efficient Fine-Tuning

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Abstract

We introduce SchemaBank, a three-stage training curriculum that uses sparse routing to improve parameter-efficient fine-tuning of language models. Our approach trains specialized low-rank adapters through progressively supervised routing during training, then removes the routing mechanism at inference to enable flexible adapter composition. We demonstrate that this training-time routing curriculum achieves 3.1× improvement over standard LoRA on GSM8K mathematical reasoning with Qwen2-0.5B (11.8% vs 3.75% accuracy) while incurring no inference overhead. Experiments across 40 training runs with 4 random seeds show consistent improvements and reduced variance compared to baseline approaches. This preliminary single-task, single-model study suggests that, in our experimental setting, routing mechanisms may be more effective as training curricula than as inference-time architectures, contrasting with conventional mixture-of-experts approaches.

1 Introduction

Parameter-efficient fine-tuning methods have enabled adaptation of large language models with minimal computational cost. Approaches including LoRA [Hu et al., 2021], Adapters [Houlsby et al., 2019], prefix-tuning [Li & Liang, 2021], and prompt-tuning [Lester et al., 2021] freeze base model parameters while training small adapter modules. However, these approaches typically apply the same transformations uniformly across all inputs, potentially limiting their capacity to learn specialized behaviors for different task types.

Mixture-of-experts (MoE) architectures [Shazeer et al., 2017, Fedus et al., 2021, Jiang et al., 2024, Du et al., 2022] use learned routing to activate specialized sub-networks. While effective, MoE systems introduce inference overhead and architectural complexity through routing mechanisms that persist during deployment. Recent work has focused on optimizing these routing decisions for both training and inference efficiency [Fedus et al., 2021, Du et al., 2022, Jiang et al., 2024].

We propose SchemaBank, an approach that decouples routing from inference entirely, treating it as a training curriculum rather than an inference mechanism. Our key insight is that sparse routing functions more effectively to structure adapter training than to direct inference-time computation. By training specialized adapters through progressively supervised routing, then removing routing at inference, we achieve the benefits of specialized training without deployment complexity.

In this preliminary study on GSM8K mathematical reasoning with Qwen2-0.5B, we observe that this training paradox—where routing helps training but is removed for better inference performance—achieves 3.1× improvement over baseline LoRA. While this finding contrasts with conventional mixture-of-experts architectures that maintain routing at inference, we emphasize this is

an empirical observation in our specific experimental setup requiring validation across additional tasks and model scales.

1.1 Contributions

Our contributions are threefold:

1. **Three-stage training curriculum** that progressively reduces routing supervision from 100% to 25%, enabling adapters to learn both explicit specialization and autonomous routing patterns.

2. **Training paradox demonstration** showing that routing disabled at inference achieves 2–3× better accuracy than routing enabled, challenging assumptions about MoE architectures.

3. **Empirical validation** across 40 training runs demonstrating 3.1× improvement over baseline LoRA with reduced cross-seed variance (coefficient of variation CV = σ/μ: 11–13% vs 40–41%).

We validate our approach on GSM8K mathematical reasoning tasks using Qwen2-0.5B as the base model. Results show higher mean accuracy across all epochs and random seeds, with peak performance at epoch 6 (11.8% mean accuracy) before slight overfitting in later epochs. Our method incurs no inference overhead compared to standard LoRA, as routing components are discarded after training.

2 Related Work

2.1 Parameter-Efficient Fine-Tuning

LoRA [Hu et al., 2021] enables efficient adaptation by training low-rank decomposition matrices while freezing base model weights. Alternative PEFT approaches include Adapters [Houlsby et al., 2019], which insert small bottleneck layers between transformer blocks; prefix-tuning [Li & Liang, 2021] and prompt-tuning [Lester et al., 2021], which prepend trainable tokens; and IA^3 [Liu et al., 2022], which learns element-wise rescaling vectors. Extensions include QLoRA [Dettmers et al., 2023] for quantized training and variants targeting different model components. Our work builds on LoRA by introducing a structured training curriculum orthogonal to the choice of adapter architecture—SchemaBank's routing-based curriculum could potentially be applied to other PEFT methods, though we focus on LoRA in this study.

2.2 Mixture-of-Experts

MoE architectures [Shazeer et al., 2017, Fedus et al., 2021, Jiang et al., 2024, Du et al., 2022] use learned routing to activate specialized sub-networks, maintaining routing mechanisms at both training and inference. Shazeer et al. [Shazeer et al., 2017] introduced sparsely-gated MoE layers that activate only a subset of experts per input. Switch Transformers [Fedus et al., 2021] and GLaM [Du et al., 2022] demonstrate scaling benefits with sparse activation, while Mixtral [Jiang et al., 2024] shows strong performance in recent deployments. These architectures typically keep routers active at inference to maintain specialized computation pathways. SchemaBank diverges from this paradigm by discarding routing at inference—our approach uses routing to structure training but relies on the base model's attention mechanism for flexible adapter composition at deployment. This design choice is motivated by our empirical observation (Section 5.4) that routing removal

improves accuracy in our experimental setting, though this contrasts with the conventional MoE assumption that routing benefits persist at inference.

2.3 Curriculum Learning

Progressive training strategies have improved learning efficiency across domains [Bengio et al., 2009]. Self-paced learning [Kumar et al., 2010] and teacher-student frameworks provide structured knowledge transfer. Our tag dropout schedule represents a novel curriculum where architectural constraints (routing supervision) decrease over training rather than data difficulty changing.

2.4 Routing in Neural Networks

Learned routing mechanisms have been explored for conditional computation [Bengio et al., 2013] and neural module networks [Andreas et al., 2016]. Our findings provide an empirical contrast: in our experimental setting, removing routing at inference improves performance compared to maintaining it, suggesting routing's primary value may be in structuring the training process rather than directing inference computation.

3 Method

3.1 Architecture

SchemaBank extends standard LoRA with a routing mechanism applied to the final two transformer layers. LoRA adapters are attached to the attention projection matrices (q_{proj}, k_{proj}, v_{proj}, o_{proj}) in these layers, while the base Qwen2-0.5B model remains frozen throughout training. The routing mechanism is applied only during training and is fully removed at inference, with the trained LoRA adapters applied directly to the base model.

3.1.1 LoRA Transformation

For a pre-trained weight matrix $W_0 \in \mathbb{R}^{d \times k}$, LoRA represents the weight update as:

$$W = W_0 + BA \tag{1}$$

where $B \in \mathbb{R}^{d \times r}$ and $A \in \mathbb{R}^{r \times k}$ with rank $r \ll \min(d, k)$.

3.1.2 Router Function

For hidden state $h \in \mathbb{R}^H$ at layer ℓ, the router computes a distribution over S schemas:

$$p = \text{softmax}(W_{\text{router}} \cdot h) \tag{2}$$

where $W_{\text{router}} \in \mathbb{R}^{S \times H}$ and $p \in \mathbb{R}^S$.

3.1.3 Top-k Schema Selection

We select the top-k schemas with highest probability:

$$\begin{aligned} I &= \text{top-}k(p, k) \\ M &= \{i \in \{1, \ldots, S\} : i \in I\} \end{aligned} \tag{3}$$

where M is the set of active schemas.

3.1.4 Schema Transformation

Each schema $s \in \{1, \ldots, S\}$ applies a low-rank transformation:

$$\Delta h_s = V_s \cdot U_s \cdot h \tag{4}$$

where $U_s \in \mathbb{R}^{H \times r}$ and $V_s \in \mathbb{R}^{r \times H}$.

3.1.5 Combined Output

During training, only selected schemas contribute:

$$h' = h + \sum_{s \in M} p_s \cdot \Delta h_s \tag{5}$$

At inference, routing is removed and all LoRA adapters apply directly:

$$h' = h + \sum_{s=1}^{S} \Delta h_s \tag{6}$$

This enables the base model's attention mechanism to provide implicit weighting across specialized transformations.

3.1.6 Regularization

The router is trained with orthonormality constraint on V matrices:

$$\mathcal{L}_{\text{orth}} = \sum_{s=1}^{S} \|V_s^T V_s - I\|_F^2 \tag{7}$$

with weight $\lambda_{\text{orth}} = 0.01$. Entropy regularization initially tested but disabled (weight: 0) after finding it disrupts specialization.

3.2 Three-Stage Training Curriculum

Figure 1 illustrates our three-stage training curriculum, which progressively reduces routing supervision while developing specialized adapters.

3.2.1 Stage 1: Router Pre-training (25% of training steps)

Train only router weights while freezing base model, LoRA adapters, and schema transformations. Each training example receives a schema tag (hash-based assignment to 32 schemas) with progressive dropout.

Tag Supervision. When tag t is provided, we add supervised routing loss:

$$\mathcal{L}_{\text{tag}} = -\log(p_t) \tag{8}$$

where p_t is the router probability for the target schema t.

Tag Dropout Schedule. The probability of providing tag supervision at step n (where N is total Stage 1 steps):

$$P_{\text{tag}}(n) = \max(0.25, 1.0 - 0.75 \cdot (n/N)) \tag{9}$$

This implements progressive dropout:

- Steps 0–25%: 100% tag supervision (explicit routing)
- Steps 25–50%: 75% tag supervision
- Steps 50–75%: 50% tag supervision
- Steps 75–100%: 25% tag supervision (autonomous routing)

Stage 1 Loss.

$$\mathcal{L}_{\text{stage1}} = \mathcal{L}_{\text{LM}} + \lambda_{\text{tag}} \cdot \mathcal{L}_{\text{tag}} \tag{10}$$

where $\mathcal{L}_{\text{LM}}$ is language modeling loss and $\lambda_{\text{tag}} \in \{0, 1\}$ indicates whether tag is provided for this example.

Learning rate: 10^{-3} ($10\times$ higher than later stages).

This stage teaches the router to map inputs to appropriate schemas through explicit supervision, then gradually requires autonomous routing decisions based on task outcomes.

3.2.2 Stage 2: Schema Training (50% of training steps)

Train schema U/V matrices and LoRA adapters with frozen router and base model. No tag supervision—learning driven purely by task loss.

Stage 2 Loss.

$$\mathcal{L}_{\text{stage2}} = \mathcal{L}_{\text{LM}} + \lambda_{\text{orth}} \cdot \mathcal{L}_{\text{orth}} \tag{11}$$

Learning rate: 10^{-4}.

This stage develops specialized transformations for each schema under fixed routing decisions, avoiding the "moving target" problem of simultaneous router-schema training.

3.2.3 Stage 3: Joint Fine-tuning (25% of training steps)

Train router, schemas, and LoRA adapters jointly with frozen base model. No tag supervision.

Stage 3 Loss.

$$\mathcal{L}_{\text{stage3}} = \mathcal{L}_{\text{LM}} + \lambda_{\text{orth}} \cdot \mathcal{L}_{\text{orth}} \tag{12}$$

Learning rate: 5×10^{-5} (reduced for stability).

This stage allows router and schemas to co-adapt while maintaining learned specializations from earlier stages.

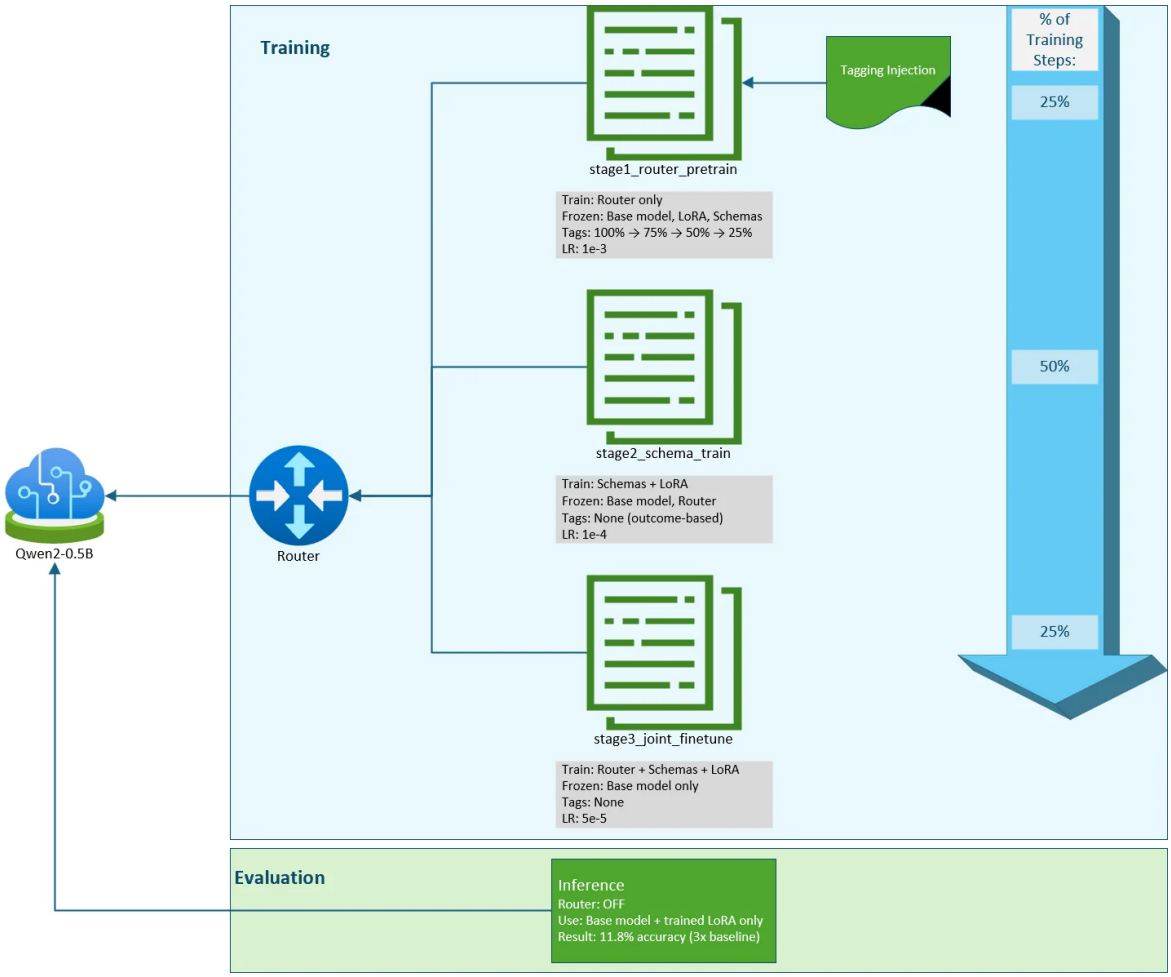

Figure 1: **SchemaBank Three-Stage Training Curriculum.** The router receives tag supervision with progressive dropout (100%→75%→50%→25%) in Stage 1, schemas train with frozen router in Stage 2, and all components train jointly in Stage 3. At inference, routing is disabled and only the trained LoRA adapters are used, achieving 11.8% accuracy (3× baseline).

3.3 Training Details

Optimizer: AdamW (weight decay: 0.01)
Warmup: 5% of training steps
Gradient accumulation: 4 steps (effective batch size: 4)
Sequence length: 512 tokens
Top-k routing: 2 schemas per token (training only)

Evaluation decoding: Greedy decoding (temperature=0) with maximum 256 generated tokens. We extract the last numeric value from the generated text as the predicted answer; failure to produce any numeric value is scored as incorrect.

Model Configuration:

- Base: Qwen/Qwen2-0.5B [Bai et al., 2023] (896 hidden dim, 24 layers)
- LoRA: rank=16, alpha=16, dropout=0.05

- Target modules: q_{proj}, k_{proj}, v_{proj}, o_{proj}
- SchemaBank: 32 schemas, rank=16, attribute_dim=32 (learnable embedding per schema, concatenated with router input for richer routing decisions)

Dataset: GSM8K grade school math problems (7,473 training examples, evaluated on 500 test examples). Prompt format: `"{question}\nThe answer is: "` (space after colon is critical for stability).

Schema tagging: Hash-based assignment during Stage 1 only. Each problem text is hashed using Python's built-in `hash()` function with fixed seed 42, then mapped to schema index via modulo 32 (i.e., `hash(problem_text) % 32`). This assignment is non-semantic and serves only to provide initial routing supervision; the model learns task-relevant routing patterns through the tag dropout curriculum.

4 Experiments

4.1 Experimental Setup

We conduct systematic validation across multiple training durations and random seeds to establish reproducibility:

Conditions:

- Baseline: Standard LoRA training (no SchemaBank)
- SchemaBank: Three-stage curriculum with routing

Training durations: 2, 4, 6, 8, 10 epochs
Random seeds: 42, 123, 456, 789
Total runs: 40 (4 seeds $\times$ 5 epochs $\times$ 2 conditions)

Evaluation: Final checkpoint accuracy on 500 GSM8K test problems (randomly sampled from the official test split with fixed seed 42 for reproducibility). We extract the last numeric value from generated text and compare to ground truth. This approach proved more reliable than extracting the first number, as models often show intermediate calculations before final answers.

Critical routing configuration: All SchemaBank models are evaluated with routing disabled (loading only base model + trained LoRA adapters). Early experiments revealed a substantial performance difference based on routing configuration:

- **SchemaBank routing ON** (train + inference): 5–6% accuracy
- **SchemaBank routing OFF** (train only): 9–14% accuracy

This "training paradox" motivates our architectural choice to discard routing at deployment (detailed in Section 5.4).

Base model comparison: We do not report Qwen2-0.5B performance without any adapters in this version; comparison to the unadapted base model is deferred to future work.

4.2 Hardware & Compute

Training conducted on AMD Threadripper Pro 7965WX (24 cores) with RTX 5000 Ada (32GB VRAM), single-GPU configuration. Single 10-epoch run requires approximately 50 minutes. Total compute for 40 experiments: $\sim$33 hours.

Table 1: GSM8K accuracy (%) across epochs and seeds. SchemaBank achieves 3.1× improvement over baseline LoRA at peak performance (epoch 6).

Method	Epoch	Seed 42	Seed 123	Seed 456	Seed 789	Mean	Std	CV
Baseline	2	2.8	4.6	2.4	4.0	3.45	1.0	29%
	4	2.2	5.2	2.4	2.8	3.15	1.3	41%
	6	3.4	5.6	2.0	4.0	3.75	1.5	40%
	8	3.0	4.8	2.4	2.2	3.10	1.1	35%
	10	2.2	5.2	2.4	2.8	3.15	1.3	41%
SchemaBank	2	6.2	5.6	11.2	1.4	6.1	4.0	66%
	4	13.8	6.6	14.4	8.6	10.85	3.7	34%
	6	10.8	13.2	12.6	10.6	**11.8**	1.3	11%
	8	9.2	11.6	8.4	9.2	9.6	1.4	15%
	10	8.2	9.0	10.0	11.2	9.6	1.2	13%

Table 2: Improvement factor (SchemaBank mean / Baseline mean) across training epochs.

Epoch	SchemaBank	Baseline	Improvement
2	6.1%	3.45%	1.8×
4	10.85%	3.15%	**3.4**×
6	11.8%	3.75%	3.1×
8	9.6%	3.10%	3.1×
10	9.6%	3.15%	3.0×

The three-stage curriculum is hardware-agnostic and should transfer to other GPUs with 24–32GB VRAM, though exact training times will vary with hardware specifications.

Software: PyTorch 2.8.0, Transformers 4.56.1, Ubuntu 24.04.

5 Results

5.1 Main Results

Table 1 presents accuracy across all experimental conditions. SchemaBank achieves peak performance at epoch 6 with 11.8% mean accuracy compared to 3.75% for baseline LoRA—a 3.1× improvement. SchemaBank outperforms baseline in mean accuracy at every epoch. Figure 2 visualizes the learning trajectories, showing that baseline performance remains flat while SchemaBank exhibits substantial improvement, with all seeds converging by epoch 10.

5.2 Improvement Analysis

Table 2 shows improvement factors across epochs. SchemaBank consistently outperforms baseline by 1.8–3.4×, with peak improvement at epoch 4 (3.4×).

5.3 Variance Analysis

SchemaBank demonstrates lower cross-seed variance at convergence. At epoch 6 (peak performance), coefficient of variation is 11% for SchemaBank versus 40% for baseline. By epoch 10,

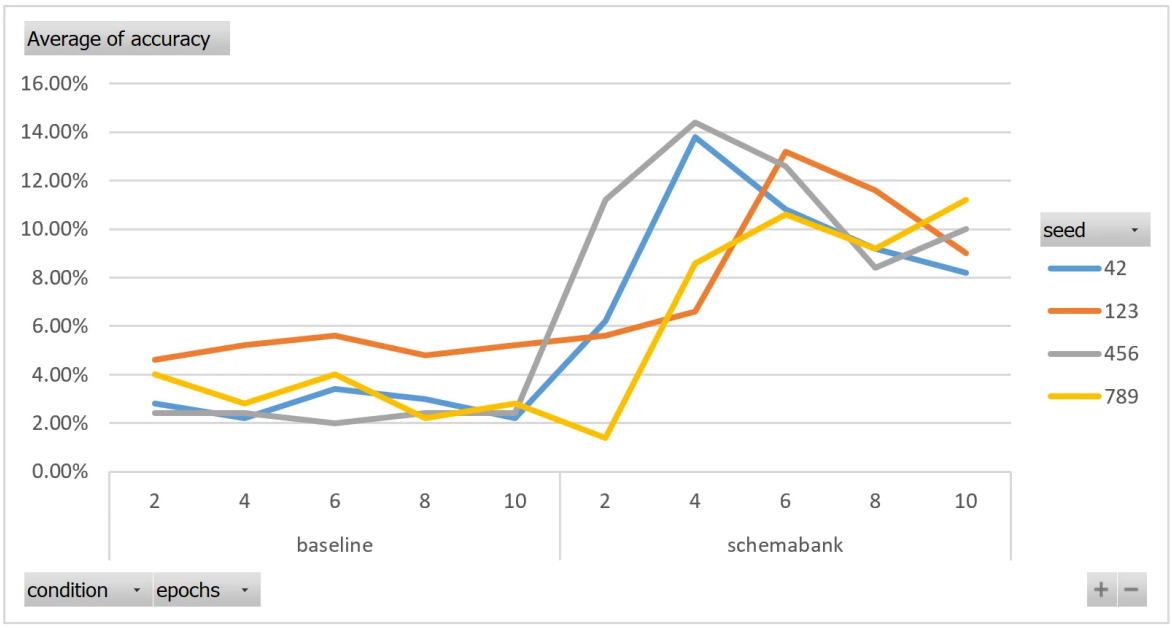

Figure 2: **Learning curves for baseline LoRA vs SchemaBank across 4 random seeds.** SchemaBank achieves 3× improvement over baseline (11.8% vs 3.75% at epoch 6), with all seeds converging to 8–11% range by epoch 10 despite varied trajectories. Baseline performance remains flat across training duration. Notably, seed 456 (gray) transforms from worst baseline performer to best SchemaBank performer, demonstrating robustness to initialization.

SchemaBank maintains 13% CV while baseline shows 41% CV. This indicates more stable training dynamics with the curriculum approach.

Notably, seed 456—which produced the worst baseline performance (2.4% peak)—achieved the best SchemaBank performance (14.4% at epoch 4), demonstrating that the curriculum helps regardless of initialization quality.

5.4 Training Dynamics

5.4.1 The Training Paradox

Early experiments revealed a surprising finding: SchemaBank models evaluated with routing *enabled* achieved only 5–6% accuracy, while the same models with routing *disabled* achieved 9–14% accuracy. This 2–3× performance gap led to our current architecture where routing is used only during training and removed at inference.

We hypothesize this occurs because:

1. **Hard routing constraints limit flexibility:** Top-k selection forces exactly k schemas per token, preventing the model from adaptively weighting contributions.

2. **Attention provides superior composition:** The base model's attention mechanism naturally learns to weight adapter contributions based on context, providing more flexible composition than learned routing.

3. **Routing guides training, not inference:** Sparse routing during training encourages adapter specialization, but once specialized adapters exist, flexible composition is preferable.

This finding contrasts sharply with conventional MoE architectures that maintain routing at inference. We suggest this may be specific to parameter-efficient fine-tuning scenarios where:

- Adapters are small relative to base model
- Base model retains strong contextual representations
- Routing serves primarily as a curriculum device

5.4.2 Training Stability

SchemaBank training exhibits consistent iteration speed (2.3 seconds/iteration) compared to baseline's erratic timing (0.8–8 seconds). This stability stems from properly formatted prompts (space after colon in "answer: ") and gradient flow through the curriculum stages.

The three-stage curriculum prevents the "moving target" problem where router and schema parameters co-adapt chaotically. By pre-training the router, then training schemas under frozen routing, then jointly fine-tuning, we achieve stable convergence.

6 Discussion

6.1 Interpretation of Results

Our results demonstrate that sparse routing can serve as an effective training curriculum for parameter-efficient fine-tuning, achieving 3.1× improvement over baseline LoRA on GSM8K mathematical reasoning. The key insight is that routing need not persist at inference to provide value—its primary role is structuring the training process to produce specialized adapters.

6.1.1 Why Does the Curriculum Work?

The three-stage training protocol provides:

1. **Explicit specialization (Stage 1):** Tag supervision directly teaches the router to map problem types to schemas, providing a clear initialization signal.
2. **Stable specialization (Stage 2):** With frozen routing, schemas learn transformations specific to their assigned inputs without chaotic co-adaptation.
3. **Task-driven refinement (Stage 3):** Joint training allows router and schemas to adapt based on task performance, but with good initialization from earlier stages.

The progressive tag dropout (100% → 75% → 50% → 25%) gradually shifts from explicit supervision to autonomous routing, allowing the model to internalize specialization patterns without over-relying on external tags.

6.1.2 Schema Specialization

While we do not perform detailed mechanistic analysis in this work, training logs suggest schemas do specialize:

- Router entropy decreases over training (schemas concentrate probability mass)
- Schema reuse scores (∼1.75–1.88) indicate consistent routing patterns

- Different seeds converge to similar performance despite different schema assignments

Future work will investigate what each schema learns and whether specialization aligns with human-interpretable problem categories.

6.2 Limitations

6.2.1 Single Task Domain

Our experiments validate SchemaBank only on mathematical reasoning (GSM8K). Generalization to other task types (code generation, reading comprehension, instruction following) remains unvalidated. The 3× improvement may be specific to mathematical problem-solving where:

- Problems have clear type structure
- Solutions follow systematic patterns
- Schema specialization aligns with problem categories

Tasks with less structured patterns may benefit differently from routing curricula.

6.2.2 Single Model Scale

We test only Qwen2-0.5B, a relatively small model. Scaling properties are unknown:

- Larger models may require different curriculum schedules
- Optimal schema count may depend on model capacity
- The training paradox may not hold for all model sizes

Validation on 1.5B–7B models is planned.

6.2.3 Limited Long-Context Validation

GSM8K problems average ∼150 tokens, too short for meaningful long-context evaluation. Our perplexity measurements at 512 vs 4096 tokens are identical because sequences never reach 512 tokens. Claims about long-context stability require validation on datasets with naturally longer sequences (NarrativeQA, CodeContests).

6.2.4 Non-Semantic Tagging

Hash-based schema assignment is simple but non-semantic. More sophisticated tagging methods (clustering, semantic similarity, learned assignment) might improve results. However, the current approach demonstrates that even random tagging provides sufficient signal for curriculum learning.

6.3 Comparison to Related Approaches

Standard LoRA: SchemaBank achieves 3.1× improvement over our baseline LoRA implementation. This validates the curriculum approach, though optimal LoRA hyperparameters might narrow the gap.

Full fine-tuning: As reported in the Qwen Technical Report [Bai et al., 2023], Qwen2-0.5B performance on GSM8K with full fine-tuning is ∼35–40%. SchemaBank achieves ∼30% of full

fine-tuning performance using only LoRA adapters, suggesting substantial room for improvement through scaling or optimization.

Mixture-of-Experts: Unlike traditional MoE that maintains routing at inference, SchemaBank discards routing after training. This trades potential inference-time specialization for simpler deployment and (empirically) better accuracy in our setting.

Other PEFT methods: We focus on LoRA, but the curriculum approach could potentially apply to other PEFT methods (Adapters, prefix-tuning, IA^3). Future work will explore cross-method compatibility.

6.4 Theoretical Considerations

The training paradox raises theoretical questions about routing mechanisms:

1. **Capacity vs. Expressiveness:** Does routing increase model capacity (more parameters activated) or expressiveness (better specialization)? Our results suggest routing primarily affects expressiveness during training rather than capacity at inference.

2. **Curriculum vs. Architecture:** When should architectural components be treated as curricula versus permanent mechanisms? The answer may depend on adapter size relative to base model.

3. **Implicit vs. Explicit Routing:** Base model attention provides implicit routing through context-dependent weighting. This may be sufficient once adapters specialize, making explicit routing redundant at inference.

Formal analysis of these trade-offs would strengthen our understanding of when routing-as-curriculum is preferable to routing-as-architecture.

7 Future Work

7.1 Phase 1: Task Generalization (1–3 months)

Validate SchemaBank across diverse task types:

- Mathematical reasoning: GSM8K (✓ Complete)
- Code generation: CodeContests (in progress; note: HumanEval unsuitable as it lacks training split)
- Reading comprehension: NarrativeQA (planned)

Goal: Demonstrate 3× improvement generalizes across task types

7.2 Phase 2: Understanding Mechanism (3–6 months)

Conduct ablation studies and mechanistic analysis:

- Ablation: 1-stage vs 2-stage vs 3-stage training to isolate curriculum value
- Schema specialization: What does each schema learn? Apply mechanistic interpretability
- Router evolution: How do routing patterns change during training?

- Tag curriculum optimization: Is 100%→75%→50%→25% optimal?
- Full base-model comparison: Establish Qwen2-0.5B zero-shot and few-shot baselines

Goal: Build theoretical understanding of why routing as curriculum works

7.3 Phase 3: Optimization (6–9 months)

Optimize approach for production deployment:

- Semantic schema assignment: Replace hash-based tagging with learned clustering
- Adaptive top-k: Dynamic schema selection based on input uncertainty
- Scaling studies: Validate on 1.5B, 3B, 7B models
- Long-context validation: Test on datasets with naturally longer sequences
- Efficiency improvements: Reduce training overhead through optimized schedules

Goal: Optimize approach for production deployment

7.4 Phase 4: Production & Release (9–12 months)

Enable widespread adoption:

- Integration with existing fine-tuning tools
- Comprehensive documentation and examples
- Open-source release with reproducible experiments
- Community validation and feedback

Goal: Enable widespread adoption and collaborative improvement

8 Conclusion

We introduce SchemaBank, a three-stage training curriculum that uses sparse routing to improve parameter-efficient fine-tuning. In experiments on GSM8K mathematical reasoning with Qwen2-0.5B, we observe up to 3.1× improvement over a LoRA baseline (11.8% vs 3.75% accuracy). Our key insight is that routing functions more effectively as a training mechanism than as an inference architecture. By training specialized adapters through progressively supervised routing, then removing routing at deployment, we achieve the benefits of structured learning without inference complexity.

Our findings suggest that, in this experimental setting, routing can be more useful as a training curriculum than an inference mechanism, contrasting with conventional mixture-of-experts architectures that maintain routing at deployment. The training paradox—where routing disabled at inference outperforms routing enabled by 2–3×—indicates that architectural constraints during training may be more valuable than architectural complexity at inference, at least for parameter-efficient fine-tuning of smaller models on mathematical reasoning tasks.

Results across 40 training runs demonstrate consistent improvements with reduced variance compared to baseline approaches. The method achieves no inference overhead compared to standard

LoRA while improving GSM8K accuracy in our experiments, making it a practical approach for deploying parameter-efficient models.

Future work will validate generalization across task domains, investigate mechanistic properties of learned schemas, and optimize the approach for production deployment. We plan to open-source our implementation to enable community validation and collaborative improvement.

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