Truthmaker Semantics for Causal Sufficiency

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Truthmaker Semantics and Modal Logic
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Plan

1. Analysing sufficiency
2. Reciprocity
3. Sufficiency via aboutness

The need for sufficiency

(1) a. Ali has an Irish passport because he was born in Ireland.
    b. Ali has an Irish passport because he was born in Europe.

The need for sufficiency

(1) a. Ali has an Irish passport because he was born in Ireland.
b. Ali has an Irish passport because he was born in Europe.

(2) a. Being born in Ireland caused Ali to get an Irish passport.
b. Being born in Europe caused Ali to get an Irish passport.

The need for sufficiency

(3) a. Sue was allowed into the bar because she's over 21.
b. Sue was allowed into the bar because she's over 16.

(4) a. The fact that Sue is over 21 caused the bouncer to let her in.
b. The fact that Sue is over 16 caused the bouncer to let her in.

The need for sufficiency

(5) *The radio spontaneously starts playing music.*

A: Why did the radio turn on?
B: I have no idea. I didn't touch it.
A: I see it's plugged in, and it needs to be plugged in to turn on.
B: Right, but I still have no idea why it started playing.

The need for sufficiency with reasons

Sami and Jan are fun on their own, but always fight when together. A heard that they are both attending a party and therefore decids to skip it.

(6) a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.
b. I'm skipping the party for one reason: because Sami and Jan are going.

(7) a. The reasons why I'm skipping the party are that Sami is going and that Jan is going.
b. The reason why I'm skipping the party is that Sami and Jan are going.

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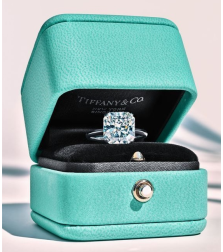
b. The reason why I'm skipping the party is that Sami and Jan are going.

My intuitive judgement: the (a)-sentences are odd, the (b)-sentences are fine.

The need for sufficiency with reasons

Tiffany & Co. have trademarked a specific pastel shade of blue, called Tiffany Blue.

Suppose Costco chooses to sell its rings in Tiffany Blue boxes. Tiffany sues for trademark infringement.



(8) The/A reason Tiffany sued Costco was that Costco's boxes are blue.

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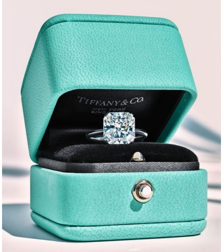

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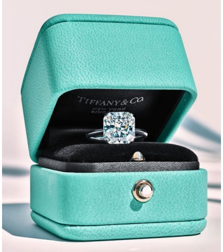

(8) The/A reason Tiffany sued Costco was that Costco's boxes are blue.

(9) The/A reason Tiffany sued Costco was that Costco's boxes are Tiffany Blue.

(10)
  a. The boxes being blue motivated Tiffany to sue Costco.
  b. The boxes being Tiffany Blue motivated Tiffany to sue Costco.

The need for sufficiency with reasons

Sami and Jan are each miserable people. Even one of them going to a party is enough to make it a dull event.

(11) a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.
b. I'm skipping the party for one reason: because Sami and Jan are going.

(12) a. The reasons I'm skipping the party are that Sami is going and that Jan is going.
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My intuitive judgement: the (a)- and (b)-sentences are both fine.

The sufficiency requirement

- *E because C* $\Rightarrow$ *C* is sufficient for *E*.
- *C cause E* $\Rightarrow$ *C* is sufficient for *E*.

What does it mean for *C* to be sufficient for *E*?

Sufficiency is not logical entailment

(13) a. My laptop turned on because I pushed the power button.
b. Pushing the power button caused the laptop to turn on.

$\not\Rightarrow$ In every logically possible world where I push the power button, the laptop turns on.

These are assertable even though there is a logically possible world where the laptop's battery is empty.

C sufficient for *E* just in case *if C would E* is true.

Motivation: Counterfactual dependence approaches to causation (Collins, Hall, and Paul 2004)

E.g. Lewis (1973): *C* causally influences *E* just in case the following two counterfactuals are true

- "if *C* were true, *E* would be true"
- "if *C* were false, *E* would be false"

We often paraphrase causal claims with counterfactuals

- The *but for* test for causation in law (Hart and Honoré 1959): "but for the cause, the effect would not have occurred"

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Modal Uniformity

If we express sufficiency with conditionals, we can appeal to a common modality for both the positive condition ("if the cause occurred...") and the negative condition ("if the cause had not occurred...")

C sufficient for *E* just in case *if C would E* is true

Problem

Many existing semantics of conditionals validate conjunctive sufficiency, predicting that *C* and *E* together entail *if C would E*.

The selection function approach (Stalnaker 1968)

> *Consider a possible world in which A is true, and which otherwise differs minimally from the actual world. 'If A, then B' is true (false) just in case B is true (false) in that possible world.*

Let W be the set of possible worlds, and $f : \wp(W) \times W \to W$ a function from propositions to worlds.

Proposal: $A > B$ is true at world w just in case B is true at $f(A, w)$.

Constraints on the selection function:

1. A is true at $f(A, w)$.
2. $f(A, w)$ is the absurd world λ (the world where every proposition is true) only if there is no possible world with respect to w in which A is true.
3. If A is true in w then $f(A, w) = w$.
4. If A is true in $f(B, w)$ and B is true in $f(A, w)$, then $f(A, w) = f(B, w)$.

Condition 3 ensures that $A \wedge C$ entails $A > C$.

The ordering approach (Lewis 1973)

Let W be a set. For any $w \in W$ let $\leq_w$ be a reflexive and transitive binary relation over W. For any sentences A and C and $w \in W$ define that a conditional $A > C$ is true at w (denoted $w \models A > C$) as follows:

$$w \models A > C \quad \text{iff} \quad \forall x \models A \; \exists y \models A \; \big(y \leq_w x \wedge \forall z \leq_w y \; (z \models A \rightarrow C)\big),$$

where $A \rightarrow C$ is the material conditional (that is, equivalent to $\neg A \vee C$).

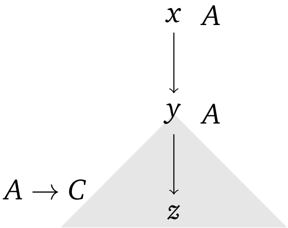


Figure: Illustrating the truth conditions of $A > C$.

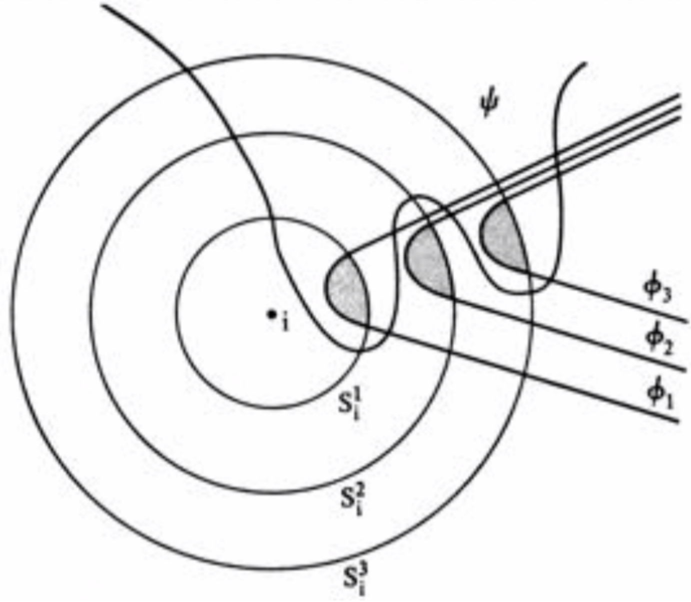


Figure: Lewis (1973) assumes strong centering: every world is more similar to itself than any other world is to it: $w <_w v$ for all w and v distinct from w.

Reciprocity implies conjunctive sufficiency

Lewis and Stalnaker validate reciprocity:

$$\frac{A > B \qquad B > A \qquad B > C}{A > C}\ \text{Reciprocity}$$

$$\frac{A \wedge B}{A > C}\ \text{Conjunctive Sufficiency}$$

Walters and Williams (2013) show that, under mild assumptions, reciprocity also ensures that $A \wedge C$ implies $A > C$.

Consider any true A, C, and any B that is irrelevant to A and C, in the sense that $(B \vee \neg B) > A$ and $(B \vee \neg B) > C$ hold.

$$\frac{A > (B \vee \neg B) \qquad (B \vee \neg B) > A \qquad (B \vee \neg B) > C}{A > C}\ \text{Reciprocity}$$

Given the existence of such a B, Reciprocity tells us that $A \wedge C$ implies $A > C$.

Reciprocity implies conjunctive sufficiency

$$\frac{A > B \qquad \neg A > B}{(A \vee \neg A) > B}\ \text{Disjunction}$$

$$\frac{}{A > \top}\ \text{Tautology}$$

A: Ali was born in Europe.
B: A pebble on a planet billions of light years away weighs 5g.
C: Ali has an Irish passport.

Intuitively, the following are true: $B > A$, $\neg B > A$, $B > C$, $\neg B > C$.
E.g. "if the pebble didn't weight 5g, Ali would still have been born in Europe."

$$\frac{A > (B \vee \neg B) \qquad (B \vee \neg B) > A \qquad (B \vee \neg B) > C}{A > C}\ \text{Reciprocity}$$

Kratzer's semantics of *would*-conditionals

There is an intuitive and appealing way of thinking about the truthconditions for counterfactuals. It is an analysis that, in my heart of hearts, I have always believed to be correct...

A "would"-counterfactual is true in a world w iff every way of adding propositions that are true in w to the antecedent while preserving consistency reaches a point where the resulting set of propositions logically implies the consequent.

— Kratzer (2012, p. 127)

Sufficiency in Structural Causal Models

Other theories (such as Nadathur and Lauer 2020, Halpern 2016, and Beckers 2021) model causal sufficiency using structural causal models (Pearl 2000).

Problem: it is generally required that variables in a structural causal model be logically independent (see Hitchcock (2004, p. 145), Hitchcock (2007, p. 502), Woodward (2003, p. 128); Woodward (2016, p. 1053), McDonald (2025))

But the causes above exhibit a logical relationship: being born in Ireland entails being born in Europe, and being over 21 entails being over 10.

In light of the logical independence constraint, it is hard to see how one may find a structural causal model that simultaneously contains variables for, say, "Ali was born in Ireland" and "Ali was born in Europe."

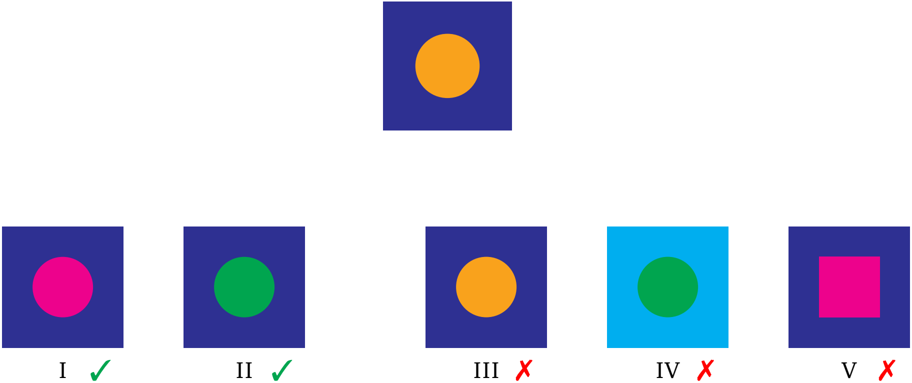
Proposed Solution: Aboutness

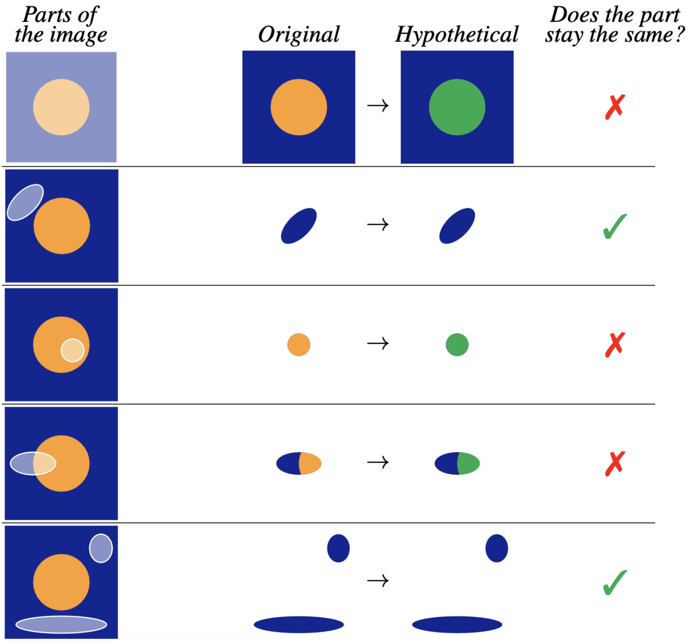
"Sue is over 16" is about Sue's age

When we interpret "Sue was allowed into the bar because she's over 16" we vary her age.

When we restrict to worlds where she is over 16, we find that in some of them, she is not allowed into the bar.

C is sufficient for E at w iff
allowing C to vary at w, all resulting C-cases are E-cases


I ✓
II ✓
III ✗
IV ✗
V ✗


Parts of the image
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Does the part stay the same?

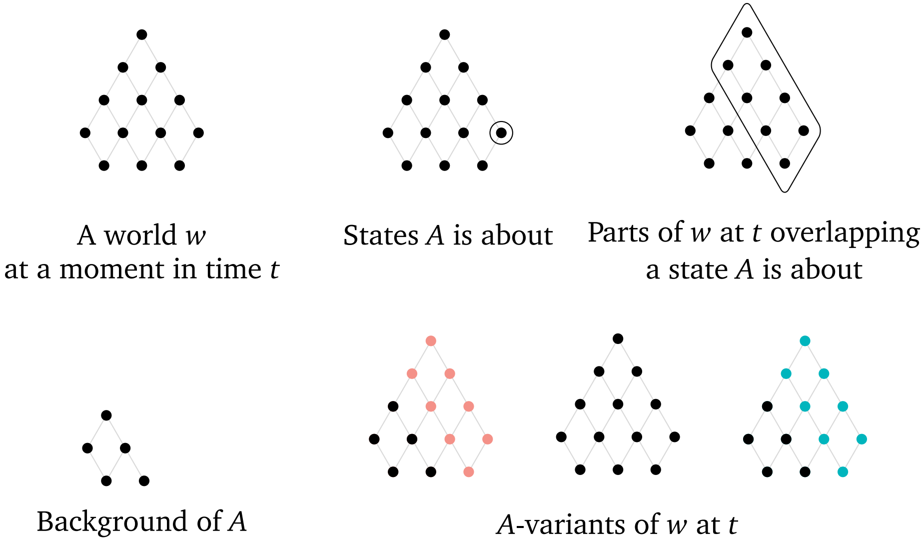
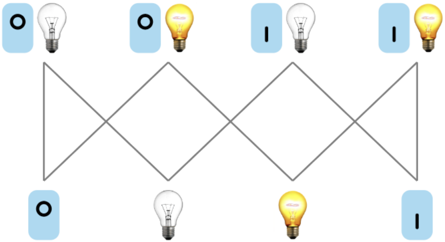
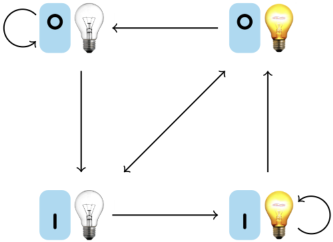


Figure: Steps to construct the A-variants of a world at a moment in time.



(a) Mereological structure.



(b) Nomic possibilities.

Figure: Light switch example. Nomically possible worlds correspond to directed paths in (b).

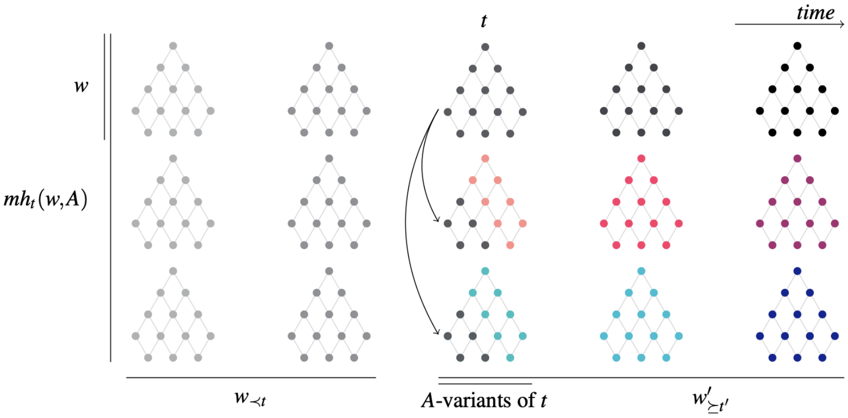


Figure: Constructing the modal horizon.

Definition (Nomic aboutness model)

Where S is a set and $\leq$ a binary relation on S, define

$$\begin{aligned}
Sit &:= S \times I, \text{ where } I \text{ is an arbitrary label set,}\\
M &:= \{t_i \in Sit : t \leq u \textit{ implies } t = u \textit{ for all } u \in S\},\\
W &:= \{(M', \preceq) : M' \subseteq M, \preceq \text{ is a linear order}\}.
\end{aligned}$$

Definition (The modal horizon)

For any sentence A, moment $t \in M$ and world $w \in W$, define

$$mh_t(w, A) := \{w_{\prec t} \frown w'_{\succeq t'} : t' \textit{ is an } A\textit{-variant of } t, t' \in w' \text{ and } w' \in P\}.$$

Fine (2012) offers the following theory of counterfactuals in terms of exact verification and a transition relation between states.

> *If A, would C is true at a world w just in case for every exact verifier t of A and possible outcome u of t at w, u contains an exact verifier of C.*

This straightforwardly accounts for the contrasts we have observed, predicting the relevant counterfactuals to be false ("If Alice were born in Europe, she would have an Irish passport" and "If bob were over 10, the bouncer would have let him in")