Conditional modality with alternatives:
Free choice, independence, and substitution

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Abstract

This paper brings together two ideas. First, that statements under a modal are interpreted as conditional antecedents, what we call 'conditional modality'. *Possibly A* states that if *A* were true, there would be some case where the relevant ideals are met. Dually, *necessarily A* states that if *A* were false, there would be no case where the relevant ideals are met. Second, conditional antecedents are interpreted via sets of alternatives, with some items—such as disjunction and *any*—introducing multiple alternatives. Combining them returns, in a uniform and automatic way, a solution to three challenges facing the standard theory of modality: free choice inferences, independence inferences, and counterexamples to substitution of logical equivalents.

Keywords: Modality, Conditionals, Alternatives, Free choice, Independence inferences, Substitution

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1 Introduction

According to the standard theory of modality, for any sentence *A*, *possibly A* means that *A* is true at some world in the relevant domain, and *necessarily A* means that *A* is true at every such world. The relevant domain depends on the theory in question; it may, for example, be the set of accessible worlds, as in Kripke semantics, or the set of accessible worlds coming closest to the relevant ideals, as in Kratzer's (1981) theory.

This theory faces several well-known challenges. We focus on three facing the analysis of possibility in particular. First, it fails to account for free choice (Ross 1941,

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von Wright 1968, Kamp 1973). If you may have ice cream or cake, then intuitively, you may have ice cream and you may have cake. It moreover fails to account for wide-scope free choice (Zimmermann 2000).

Second, the standard theory fails to account for what Booth (2022) calls *independence inferences*. Booth observes that from disjunctions under possibility modals, we often infer that each disjunct is possible independently of the other. Menéndez-Benito (2010) observes analogous inferences for free choice indefinites such as *any*.

Third, the standard theory predicts that two sentences true in all the same worlds have the same modal status: both are possible, or neither are; both are necessary, or neither are. Call this *substitution of logical equivalents* (*substitution*, for short). As we will see, there are intuitive counterexamples to substitution.

This article develops an analysis of modality that uniformly accounts for free choice, independence inferences, and counterexamples to substitution. It is unique in placing conditionals at its heart. As we see in greater detail below, something like this was previously suggested for deontic logic by Kanger (1957) and Anderson (1956, 1958), for metaphysical modality by Williamson (2007), and is reminiscent of how Japanese, Korean, and Burmese often express modality (Akatsuka 1992, Wymann 1996, Kaufmann 2017a, Chung 2019). The idea is this.[1]

(1) a. *Possibly A* means: if *A* were true, there would be some case where the relevant ideals are met.

b. *Necessarily A* means: if *A* were false, there would be no case where the relevant ideals are met.

The relevant ideals are given by the modal flavour in question: epistemic, deontic, and so on. To illustrate, asking 'May I open the window?' on this approach amounts to something like asking 'Would it be OK if I opened the window?' Since conditionals play a central role in this theory, we call it *the conditional theory of modality*. In symbols, we may express it like so.

$$\begin{aligned} \Diamond A &\equiv A \mathbin{\Diamond\!\!\rightarrow} \mathit{good} \\ \Box A &\equiv \neg A \mathbin{\Box\!\!\rightarrow} \neg \mathit{good} \end{aligned}$$

[1]We have formulated the conditional in the past with subjunctive mood. This typically implies that the antecedent is false, though we regard this as an implicature (see Ippolito 2003, Leahy 2011, 2018) rather than part of the conditional's literal meaning. Following Stalnaker (1975), we take the subjunctive to be the more neutral form, imposing fewer requirements than the indicative (for instance, that the antecedent be compatible with the common ground, something we do not assume for the conditionals to come). For this reason, we opt for the subjunctive form in what follows. We are grateful to [Suppressed for blind review] for discussion of this point.

where $\Diamond$ denotes possibility, $\Box$ necessity, '$A \Diamond\!\!\rightarrow C$' says that C holds at some world that results from supposing A, '$A \Box\!\!\rightarrow C$' says that C holds at every such world, and *good* states that the relevant ideals are met.[2]

When we evaluate modal statements on the conditional theory, the A in *possibly A* and *necessarily A* appears inside a conditional antecedent.[3] This shift in perspective paves the way to applying developments in the theory of conditionals to modals. One important recent development is the use of *alternatives* (Alonso-Ovalle 2006, 2009, Ciardelli 2016, Santorio 2018, Willer 2018, Khoo 2022). On this approach, conditional antecedents are interpreted via sets of propositions, called their 'alternatives'. For instance, a disjunctive conditional antecedent *if A or B* may have as alternatives the individual disjuncts A and B.

The goal of this paper is to show how applying the theory of alternatives to modals via the conditional theory provides a uniform solution to the three challenges above facing the standard theory of modality.

2 The standard theory

We assume a standard first-order modal language (denoted $\mathscr{L}$), and use uppercase italic letters $A, B, C, \ldots$ for sentences of this language.

Kripke semantics is arguably the most well-known semantics of modals. A Kripke model $M = (W, R, V)$ consists of a non-empty set of possible worlds W, a binary accessibility relation R over worlds, and a valuation V assigning to each atomic sentence and world a truth value 0 or 1. For any world w we let $R[w] = \{w' \in W \mid wRw'\}$ denote the set of worlds accessible from w. We assume a standard semantics for the connectives. A negated sentence $\neg A$ is true just in case A is not true, a disjunction $A \vee B$ is true just in case at least one disjunct is true, and a conjunction $A \wedge B$ is true just in case both conjuncts are true, and so on.

According to Kripke semantics, a statement holds necessarily just in case it holds at all accessible worlds, and holds possibly just in case it holds at some accessible world.[4]

(2) **Kripke Semantics.** Let $M = (W, R, V)$ be a Kripke model.

a. $[\![\Box A]\!]^w = 1$ iff $[\![A]\!]^{w'} = 1$ for all w' in $R[w]$.

b. $[\![\Diamond A]\!]^w = 1$ iff $[\![A]\!]^{w'} = 1$ for some w' in $R[w]$.

[2]Section 5.1 gives a formal analysis of what it means for the relevant ideals to be met.

[3]We use the term 'conditional antecedent' broadly here, covering not only overt *if*-clauses, but also sentences that appear as the left argument of $\Diamond\!\!\rightarrow$ and $\Box\!\!\rightarrow$.

[4]Strictly speaking, the interpretation function $[\![\cdot]\!]^w$ should also be relative to the model, $[\![\cdot]\!]^{M,w}$ Since it is clear from context which model is in question, we leave out the superscript.

Kratzer (1981b) has proposed enriching Kripke semantics by assigning an order to each world. A Kratzer model is given by $M = (W, R, \leq, V)$ where (W, R, V) is a Kripke model and $\leq$ is a function assigning to each world w a reflexive and transitive order $\leq_w$ over worlds.[5] Given a set of worlds S, a world w' is among the best worlds in S, relative to $\leq_w$, just in case no world in S is strictly better than it according to $\leq_w$. This set of worlds we denote by $\textsc{best}_w(S) := \{w' \in S : \neg\exists w'' \in S, w'' <_w w'\}$.[6]

For simplicity, we adopt the limit assumption for each order $\leq_w$. For any world w, every non-empty set of worlds S contains some best world with respect to w: $\textsc{best}_w(S)$ is empty only if S is (for further discussion see Kaufmann 2017b). This allows us to state Kratzer's semantics as follows.

(3) **Kratzer Semantics.** Let $M = (W, R, \leq, V)$ be a Kratzer model.

a. $[\![\Box A]\!]^w = 1$ iff $[\![A]\!]^{w'} = 1$ for all w' in $\textsc{best}_w(R[w])$.

b. $[\![\Diamond A]\!]^w = 1$ iff $[\![A]\!]^{w'} = 1$ for some w' in $\textsc{best}_w(R[w])$.

When the order is trivial (for instance, every world is as good as every other world, $w' \leq_w w''$ for all worlds, or only as good as itself, $w' \leq_w w''$ only if $w' = w''$), Kratzer semantics reduces to Kripke semantics: $\textsc{best}_w(S)$ is simply S for any set S. Kripke semantics is therefore a special case of Kratzer's semantics. For this reason, in what follows we focus exclusively on Kratzer's semantics, knowing that we may recover Kripke semantics by plugging in a trivial order. Given the widespread popularity of Kratzer's semantics in philosophy and linguistics, we refer to it as *the standard theory*.

3 Three challenges for a theory of possibility

3.1 Free choice

On the standard theory, $\Diamond A$ entails $\Diamond(A \vee B)$. Ross (1941) observed that this seems incorrect for imperatives. Later, von Wright (1968) and Kamp (1973) observed this for permission: 'If we are told that we may do this thing or that thing, we normally understand this to mean that we may do the one thing *but also* the other thing' (von Wright 1968:4). The same goes for epistemic modals (Zimmermann 2000). 'John

[5] Kratzer derives the accessibility relation for each world w from a set of propositions $f(w)$, which she calls the *modal base* at w, where world w' is accessible from world w (wRw') just in case every proposition in $f(w)$ is true at w'. Kratzer derives the order for each world from a set of propositions $g(w)$, called the *ordering source* at w, where $w' \leq_{g(w)} w''$ just in case every proposition in $g(w)$ that is true at w'' is true at w'. This additional structure will not affect our claims in this paper; for simplicity, we state the account in terms of accessibility relations and orders directly.

[6] We define the strict order $<_w$ as usual: $w' <_w w''$ just in case $w' \leq_w w''$ but not $w'' \leq_w w'$.

might be in Paris or Berlin' intuitively implies that he might be in Paris and that he might be in Berlin. This is the first challenge facing the standard theory.

A related problem, noted by Zimmermann (2000), is that the same inferences often arise even when the disjunction takes wide scope. 'You may go to Brixton or you may go to Victoria' intuitively implies 'You may go to Brixton' and 'You may go to Victoria'. Likewise, 'He might be in Brixton or he might be in Victoria' intuitively implies 'He might be in Brixton' and 'He might be in Victoria'.[7] This is also not predicted by the standard theory, according to which $A \vee B$ does not in general entail $A \wedge B$. Similar remarks apply to *any*. Vendler observes that if someone says, 'Take any one of them' they 'grant you the unrestricted liberty of individual choice' (Vendler 1967:79–80), an effect he calls *freedom of choice*.

3.2 Independence

Most accounts of free choice are tailored to capture the inference from $\Diamond(A \vee B)$ to $\Diamond A \wedge \Diamond B$. Booth (2022) argues that this is not quite right. Here is an illustration.[8] Alice and Bob are siblings in an orphanage. The orphanage has a rule that siblings must not be separated. It is permitted to adopt both Alice and Bob, and to adopt neither, but forbidden to adopt one without the other. The adoption agent informs prospective parents, 'You may adopt Alice or Bob'. Is what the agent said correct?

Intuitively, it is not. As Booth observes, $\Diamond(A \vee B)$ seems to imply $\Diamond(A \wedge \neg B)$ and $\Diamond(B \wedge \neg A)$, expressing the possibility of each disjunct without the other. Booth calls these *independence inferences*. These are not predicted by the standard theory of modality, nor by most accounts of free choice (such as Fox 2007a, Goldstein 2019, or Aloni 2023). This is because there is a deontically ideal world where you adopt Alice, and one where you adopt Bob; namely, one where you adopt both. Without further refinement,

[7]Some accounts only predict narrow-scope free choice (Aloni 2007, Willer 2018, Bar-Lev and Fox 2020). On their behalf, Simons (2005) and Meyer and Sauerland (2016) argue that cases of wide-scope free choice, despite appearances, in fact involve narrow-scope. We prefer the null hypothesis here that what you see is what you get: these are genuine cases of wide-scope disjunction. There are reasons to be sceptical of the reduction of wide-scope to narrow-scope free choice. First, Alonso-Ovalle (2006) observes that wide-scope free choice may arise even when the two modals are distinct, as in 'You may email us or you can reach the Business License office at 949 644-3141'. It is unclear how *may* and *can* here are both instances of the same modal. Second, following a generalisation by Larson (1985), we can use *either* to force disjunction to take wide-scope. Cremers et al. (2017) found experimental evidence of wide-scope free choice, even with *either*. They found, for example, that 'Either Mary can have a pizza or she can have a hamburger' has a free choice reading, implying that Mary can have a pizza and that she can have a hamburger. There is, however, more to be said than we can here (see, for example, Bar-Lev 2018:§2.3.1). We aim to provide an account of wide-scope free choice for those who take it to be a genuine phenomenon.

[8]This case is not due to Booth, but we believe it illustrates the point in a rather clear way.

then, these predict the agent's utterance to be perfectly fine.

Menéndez-Benito (2010:36) makes parallel observations for free choice *any* and Spanish *cualquiera*. She offers the Canasta example. At a certain point while playing Canasta, the player has two options: (i) take all of the cards in the discard pile, (ii) take none of them. Consider (4).

(4) You may take any of the cards in the discard pile.

This is intuitively incorrect, but the standard theory, as well as standard accounts of free choice, predict it to be true. We assume that *any* expresses existential quantification.[9] Then parallel to the inference from $\Diamond(A \vee B)$ to $\Diamond A \wedge \Diamond B$ is the inference from $\Diamond \exists x Ax$ to $\forall x \Diamond Ax$. In the Canasta example, we may express this inference on the standard theory as 'For any combination of cards x in the discard pile, there is a deontically ideal world in which you take x'. This is true, since the world where the player takes every card in the discard pile is deontically ideal.

To solve the problem, Menéndez-Benito appeals to exhaustive readings. (4) communicates that for each collection of cards in the discard pile, it is permitted to take those *and only those* cards.[10] This correctly predicts (4) to sound incorrect in the Canasta case. One might apply an analogous strategy to independence inferences from disjunction, arguing that the disjunction in 'You may adopt Alice or Bob' is read exclusively, as 'You may adopt Alice but not Bob, or Bob but not Alice', which explains why it too sounds wrong in the adoption case.

This runs into a problem observed by Szabolcsi (2019), adapting an example of Elbourne's (2002). Consider (5).

(5) Any bishop may meet a bishop.

This is a perfectly consistent thing to say. Interpreted exhaustively, incorporating the free choice inference, it says that for every bishop b, it is permitted that b meets a bishop *and only b meets a bishop*. But this is contradictory, since meeting is symmetric: if bishop b meets a bishop c, c also meets a bishop; namely, b.[11] The point can be repeated

[9]For arguments that *any* is existential, see Horn (1972) and Ladusaw (1979). Kadmon and Landman (1993), Krifka (1995), Lahiri (1998), and Chierchia (2006) account for its behaviour assuming an existential semantics. Though this assumption is controversial: for an alternative view on which *any* is universal, see Carlson (1981) and Dayal (2004).

[10]Farkas (2008) offers another proposal that also assumes that the alternatives of universal free choice items (such as *any*) denote mutually exclusive propositions. This is also part of Dayal's (2013) Viability Constraint.

[11] This assumes, following Szabolcsi (2019), that the free choice interpretation applies to *any bishop* but not to *a bishop*. Applying the free choice interpretation to both may result in different readings, depending

with any symmetric predicate whatsoever: ‘Anyone is allowed to make eye contact with someone’, ‘Any pawn is allowed to be one square away from another pawn’, and so on.

A plausible reply is that the exhaustive interpretation does not arise when it results in a contradictory meaning, as in (5). Alsop (2024) proposes that *any* is optionally exhaustified, with pragmatics determining when it is. To test this, we can reformulate the above examples to be true without an exhaustive interpretation, but contradictory with it. If the exhaustive reading is pragmatically determined, we expect the exhaustive interpretation not to arise and the sentences to be fine. Are they? Consider (6).

(6) You must take all or none of the cards in the discard pile and may take any of the cards in the discard pile.

The second conjunct is intuitively incorrect in the Canasta context. Plausibly, the first conjunct exerts a strong pressure not to exhaustify *any* in the second conjunct, since the exhaustive reading renders the utterance as a whole contradictory. But without exhaustivity, the standard theory predicts (6) to be perfectly fine, for the same reason as (4).

Secondly, we may combine Menéndez-Benito and Szabolcsi’s examples by putting symmetric predicates in a Canasta-style, all-or-nothing context. Suppose there is a group of bishops who may meet all together or not at all (to prevent schisms, let’s say). In this context (5) still sounds incorrect. However, with exhaustivity, it is contradictory, while without exhaustivity, the standard theory predicts it to be true.[12]

There is also an argument against accounting for independence inferences from disjunction using exclusive readings. It is one which, to my knowledge, has not been observed before.[13] Suppose an immigration law states, ‘In this document, “Ireland” refers to the island consisting of the Republic of Ireland and Northern Ireland, and “the United Kingdom” refers to the nation consisting of England, Scotland, Wales, and Northern Ireland’. The law later states:

on how exhaustivity interacts with multiple alternatives. We can get around this by reformulating the example. Consider ‘Any king may meet royalty’ or ‘Any clergy member may meet clergy’. These are intuitively consistent, but incorporating the free choice and exhaustive interpretation predicts them to be contradictory; saying, for example, that for every king *K*, there is a deontically ideal world where *K and only K* meets royalty.

[12] Granted, we might also interpret ‘a bishop’ exhaustively here. In response, following note 11, suppose there is a group of clergy who may meet all together or not at all. ‘Any clergy member may meet clergy’ intuitively also sounds incorrect in this context.

[13] Booth (2022:note 21) also argues against appealing to an exclusive interpretation. In addition, most accounts of free choice of which we are aware (including Fox 2007a, Goldstein 2019, and Aloni 2023) assume that the embedded disjunction is read inclusively.

(7) If you hold an E2 visa, you may travel to Ireland or the United Kingdom.

Intuitively, this permits those with an E2 visa to travel to Northern Ireland.[14] But on an exclusive reading, it would state that you may visit Ireland but not the United Kingdom, or the United Kingdom but not Ireland. This clearly does not permit one to visit Northern Ireland. It is silent on whether one may go there.

Summing up, disjunction in possibility statements sometimes seems to be read exclusively ('You may adopt Alice or Bob') and sometimes not ('You may visit Ireland or the United Kingdom'). Likewise, *any* in possibility statements sometimes seems to be read exhaustively ('You may take any cards from the discard pile') and sometimes not ('Any bishop may meet a bishop'). We therefore face the challenge of explaining why exclusive/exhaustive inferences inside possibility statements seem to sometimes arise and sometimes not. Call this the *exhaustivity problem*.

3.3 Substitution

The third challenge facing the standard theory is that it predicts two sentences true in all the same worlds to have the same modal status: both are possible or neither are, both are necessary or neither are.

There are intuitive counterexamples to substitution. Suppose a concert hall limits the number of tickets each customer may buy to a maximum of five. Is Alice allowed to buy more than four tickets? Is she allowed to buy five or more tickets?

Intuitively, she is allowed to buy more than four tickets, since she may buy five, but it is wrong to say that she may buy five or more, since she may not buy more than five. But the embedded statements are logically equivalent: Alice buys more than four tickets if and only if she buys five or more. In general, whenever we are counting discrete things—such as concert tickets—*more than n* and $n+1$ *or more* give rise to the same truth conditions. They come apart, however, under possibility modals.[15]

[14] We do not rule out that this may also have an exclusive reading. However, intuitively the most salient reading is one where it permits one to visit Northern Ireland. While this judgement would benefit from experimental testing, what is nonetheless clear is that the judgement that 'You may adopt Alice or Bob' is unacceptable in the adoption case is far more robust than the judgement that 'You may visit Ireland or the United Kingdom' does not permit one to visit Northern Ireland. The discrepancy here makes it unlikely that appeal to an exclusive reading of disjunction accounts for the adoption case (even if, admittedly, an exclusive reading may play some role).

[15] This example is based on one by Fox (2007b:103), who judges it acceptable to say, 'John is allowed to smoke more than six cigarettes. More specifically, he is allowed to smoke seven but not more than seven'. We slightly modified the example since *smoking cigarettes* is arguably not discrete—one can smoke half a cigarette. If John smoked six and a half cigarettes, then he smoked more than six cigarettes but did not smoke seven or more: the expressions are no longer equivalent. But one can't buy four and a half tickets.

For a second counterexample, consider the following case from Fine (2014a). Eve finds herself in Alternative Eden, which contains one forbidden apple a_0 and infinitely many apples which Eve is permitted to eat, $a_1, a_2, \ldots$. Compare:

(8) a. Eve is permitted to eat infinitely many of the apples $a_1, a_2, \ldots$.
b. Eve is permitted to eat infinitely many of the apples $a_0, a_1, a_2, \ldots$.

Fine observes that the first is intuitively correct in this scenario, while the second is not. But Eve eats infinitely many of $a_1, a_2, \ldots$ if and only if she eats infinitely many of $a_0, a_1, \ldots$. For a difference of one cannot separate the finite from the infinite.[16]

The lesson Fine draws from his case is for a statement to be permitted, it must be permitted for *every way* in which it might obtain. We might call this principle *universal realisability* for permission, or say that permission is *fine-grained*.[17] Fine represents the notion using truthmaker semantics. We say that a part of a world, or state, *exactly verifies* a sentence just in case the state's obtaining is wholly relevant to sentence's truth (Fine 2017). A sentence's exact verifiers are, loosely put, the 'ways' for it to obtain. Crucially for substitution, two statements may be true in all the same worlds but differ in their exact verifiers. For instance, a state of Eve eating a_0 along with infinitely many of $a_1, a_2, \ldots$ exactly verifies that she eats infinitely many of $a_0, a_1, \ldots$, but not that she eats infinitely many of $a_1, a_2, \ldots$, since it contains a part—eating a_0—irrelevant to the latter sentence's truth.

Fine (2014a) proposes that a statement is permitted just in case each of its verifiers are permitted. This gets the Alternative Eden case right: Eve is permitted to eat infinitely many of the apples $a_1, a_2, \ldots$, but not permitted to eat infinitely many of the apples $a_0, a_1, \ldots$, since one way for her to do the latter is to eat the forbidden fruit a_0 along with infinitely many others.

Here are two problems for universal realisability for permission. First, it struggles with the contrast between 'Alice may buy more than four tickets' and 'Alice may buy five or more'. Consider the embedded statements:

(9) a. Alice buys more than four tickets.
b. Alice buys five or more tickets.

[16]If one thinks that the lists $a_0, a_1, \ldots$ may be playing a role, note that we can reformulate the example without them. Suppose that the forbidden apple a_0 is red and that the permitted apples $a_1, a_2, \ldots$ are green. Then 'Eve may eat infinitely many of the green apples' is intuitively correct, while 'Eve may eat infinitely many of the apples' is not.

[17]The term 'universal realisability' comes from Fine (2012), who proposes Universal Realizability of the Antecedent: a conditional is true just in case it is true for every way in which its antecedent might be true.

Which states are wholly relevant to their truth? Plausibly, for both, these are the state of Alice buying five tickets, the state of her buying six tickets, and so on; in general, the state of her buying n tickets, for $n = 5, 6, \ldots$. Thus the sentences have the same exact verifiers. They are, in the words of Fine (2014b:556), *exactly equivalent*. Given this, truthmaker semantics cannot distinguish them.

Second, there are cases where permission seems to be *coarse*: something is permitted even though there are impermissible ways for it to obtain (something predicted by the standard theory). Suppose Alice and Bob are colleagues in Quebec. They know both English and French. Bill 96 requires them to speak French at work, but they defy the law by talking to each other at work in English. Does the law allow them to talk to each other at work? Intuitively, it does. Here permission is coarse: not every way for them to do so is permitted—English is forbidden. In terms of exact verification, the state of Alice and Bob talking to each other at work in English is, plausibly, wholly relevant to truth of 'Alice and Bob talk to each other at work'.

For a second example, suppose there are two switches, A and B. Each switch can be either up or down. The rules specify that A must be up and that B may be in any position. Are the positions of the switches allowed to agree? Intuitively, they are. For they are allowed to both be up. But not every way for them to be in the same position is permitted. The rules prohibit them from both being down.

We face a dilemma. If permission is indeed coarse, we should expect 'Alice may buy five or more tickets' and 'Eve may eat infinitely many of the apples $a_0, a_2, \ldots$' to both be true. This is because one way for Alice to buy five or more tickets is to buy exactly five, and one way for Eve to eat infinitely many of $a_0, a_2, \ldots$ is to eat infinitely many of $a_1, a_2, \ldots$ without a_0, both of which are permitted. So some examples suggest that permission is fine, others that it is coarse. But what determines when it is one and when it is the other? Call this the *grain problem*. It is a problem that, to our knowledge, has not been considered before.

4 Conditional–Possibility parallels

We have seen three challenges facing the standard theory of modality: free choice, independence inferences, and counterexamples to substitution. While this behaviour may seem puzzling, there is another environment where the same behaviour is entirely familiar: conditional antecedents. The goal of this section is to show that when we compare how possibility modals and conditional antecedents behave with respect to the challenges above, we find that they behave in exactly the same way. These parallels will set the stage for our conditional theory of modality to come.

4.1 Free choice

First, just as $\Diamond(A \vee B)$ implies $\Diamond A$ and $\Diamond B$, *if* $A \vee B$, C implies *if* A, C and *if* B, C. For example, from 'if you had coffee or tea, you'd be happy' we have a strong tendency to infer both 'if you had coffee, you'd be happy' and 'if you had tea, you'd be happy'. This is the much-discussed rule of simplification of disjunctive antecedents (SDA).[18] Similarly, just as 'you may post the letter' does not imply 'you may post the letter or burn down the post office', with conditionals, 'if you posted the letter, things would be fine' does not imply 'if you posted the letter or burned down the post office, things would be fine'.

Alonso-Ovalle (2006:18–19) observes that *might* counterfactuals also have simplification readings, a point which applies to possibility modals in general. 'If the die landed 1 or 2, you could win' has a prominent reading on which it implies both 'if the die landed 1, you could win' and 'if the die landed 2, you could win'.[19] The remaining observations in this section also apply with a possibility modal in the consequent; for brevity, in the rest of this section we formulate our examples using *will* and *would*.

Similarly, just as $\Diamond A \vee \Diamond B$ often implies $\Diamond A$ and $\Diamond B$, disjunctions of whole conditionals often imply each conditional, as observed by Wiggins and Edgington (1997), Geurts (2005), Khoo (2021a), and McHugh (2025a).[20] 'If you went to Brixton, things would be fine, or if you went to Victoria, things would be fine' is naturally read as implying each conditional. In addition, both possibility modals and conditionals have an ignorance reading—brought out by the continuation, '... but I don't know which'—on which the disjunction behaves classically: true just in case at least one of the conditionals is true.

4.2 Independence

Second, just as with a disjunctive possibility statement, $\Diamond(A \vee B)$, where we often infer that each disjunct is permitted independently of the other, with a conditional 'if A or B,

[18] For other proposals that aim to reduce free choice inferences to SDA, see Hilpinen (1982), Asher and Bonevac (2005), van Rooij (2006), Klinedinst (2007), Barker (2010), Franke (2011), and Willer (2018). These proposals are limited to standard free choice inferences, and do not consider the other challenges to the standard theory that we aim to account for.

[19] Granted, these have an SDA invalidating reading, brought out by: 'if the die landed on 1 or 2, it could land on 2, in which case you would lose'. We discuss counterexamples to SDA in Section 6.

[20] In particular, McHugh offers an account of why $\Box A \vee \Box B$ does not imply $\Box A \wedge \Box B$, an inference surprisingly predicted by a number of theories of wide-scope free choice, such as Zimmermann (2000), Geurts (2005), and Aloni (2023). We may also use this account to also block the unattested inference from $\Box A \vee \Box B$ to $\Box A \wedge \Box B$ on the present theory.

C', we often suppose each disjunct independently of the other. Take the adoption case, where the rules require that Alice and Bob not be separated. Consider:

(10) If you adopted Alice or Bob, the rules would be met.

This is intuitively false. Indeed, it sounds equivalent to the conjunction of 'if you adopted Alice but not Bob, the rules would be met' and 'if you adopted Bob but not Alice, the rules would be met'.

When we interpret a conditional with a disjunctive antecedent, do we typically consider the case where both disjuncts hold? There is experimental evidence that we do not. Consider the following scenario due to Ciardelli, Zhang, and Champollion (2018). Two switches, A and B, are connected to a light. Each switch can be up or down. The light is on just in case both switches are in the same position. Currently, both switches are up. Consider:

(11) If switch A or switch B was down, the light would be off.

Ciardelli, Zhang, and Champollion tested this sentence experimentally, finding that a majority of participants judged it to be true, matching intuition.[21] It seems that when we interpret a disjunctive antecedent conditional, we suppose each disjunct independently (supposing A but not B, or B but not A), parallel to Booth's (2022) observation that for $\Diamond(A \vee B)$ to be true, each disjunct must be possible independently.

This suggests that 'if A or B, C' is often intuitively equivalent to 'if A, C, and if B, C'.[22] From this we may derive independence inferences for conditionals from the assumption that the disjuncts are conditionally independent, in the sense that supposing one leaves the other unchanged.[23] For then, in a scenario where A and B are currently both false, when we suppose A, B remains false, and when we suppose B, A remains false. We only consider cases where exactly one of the disjuncts is true. This sounds remarkably like exhaustivity, but instead of deriving it via separate mechanism (such as scalar implicature or exhaustification), it follows automatically from the truth conditions of counterfactuals under the assumptions that the disjuncts are conditionally independent and currently both false.

When the independence assumption fails, we observe that the disjuncts are not supposed in isolation. To illustrate, imagine Alice and Bob are a couple, planning to visit

[21] For an argument that this is not explained by an exclusive interpretation of disjunction, see Ciardelli, Zhang, and Champollion (2018:§2.6.2).

[22] Though see Section 6 for some counterexamples to this equivalence.

[23] Arregui (2011) has previously argued that facts about dependence play a role in the interpretation of both counterfactuals and deontic modals, though not to account for independence inferences.

either Paris or Berlin together. Consider:

(12) If Alice or Bob were in Paris, the other would be in Paris too.

This is intuitively true. It is clearly not equivalent to (13).

(13) If Alice but not Bob were in Paris, or Bob but not Alice were in Paris, the other would be in Paris too.

The two disjuncts are conditionally *dependent*. In contrast, if Alice and Bob are strangers, travelling independently, (12) is no longer true. When we imagine one moving, we leave the other where they are, rendering the conditional antecedent equivalent to 'if only Alice or only Bob were in Paris'. Here exhaustivity emerges as a mirage of independence.

While the exclusivity problem for permission statements is puzzling, the same behaviour in conditional antecedents has a simple explanation. In the adoption case, we naturally assume that adopting one child is independent of adopting the other, in the sense that when we imagine adopting one child, we imagine the other staying in their current, unadopted state. This explains why 'if you adopted Alice, the rules would be met' and 'if you adopted Bob, the rules would be met' have a prominent false reading. 'If you adopted Alice' takes us to worlds where we adopt Alice and not Bob, in which case the rules are broken; similarly for 'if you adopted Bob'.[24]

In this respect, the adoption case is just like the switches case from Ciardelli, Zhang, and Champollion, where we assume that the two switches are independent. When we imagine one switch in a different position, we imagine the other staying where it currently is. In contrast, when we imagine one visiting the island of Ireland or the UK, we do not necessarily imagine them still remaining outside the other country. For one way to visit the island of Ireland or the UK is to visit Northern Ireland. The disjuncts are not independent. Accordingly, given 'If you visited Ireland, you'd be allowed in, and if you visited the UK, you'd be allowed in', we naturally infer that if you went to Northern Ireland, you'd be allowed in.

This explains why disjunction in conditional antecedents sometimes appears to be exclusive and sometimes not. In fact we need not appeal to exclusivity at all. The seeming exclusive inference arises entirely from independence. When the disjuncts are independent—as in the switches and adoption cases—the disjunction acts as if it is exclusive, while when they are dependent—as in the Ireland/UK or Alice and Bob

[24][Redacted for blind review] helpfully points out that, with some explicit contextual support, these may sound true. An example is: 'If you adopted Alice, the rules would be met, provided you also adopt Bob'. It's worth pointing out that same goes for permission: 'You may adopt Alice, provided you also adopt Bob' is true. In this case too, conditionals and permission statements behave in the same way.

travelling cases—they do not. Whether independence holds, in turn, is determined by what scenarios we consider when we interpret the conditional antecedent, something that every theory of conditionals seeks to provide.

Similar to disjunction, we may trace an exhaustive interpretation of *any* in conditional antecedents to facts about independence. Recall the Canasta case: one may take all or none of the cards in the discard pile. When we imagine one taking some cards from the pile, we imagine them taking *only* those cards. As before, this follows from independence. Whether one takes some cards from the pile is independent of whether one takes the others. This follows without appealing to exhaustivity at all.

To examine a case where independence fails, consider a conditional analogue of Szabolcsi's example.

(14) If any bishop met a bishop, they would bless each other.

When we interpret this, for each bishop *b*, we do not suppose that only bishop *b* meets a bishop. This, after all, is impossible. We also imagine the other bishop meeting *b*, due to the symmetry of meeting. Here 'bishop *b* meets a bishop' and 'bishop *c* meets a bishop' are not independent, for the two bishops might be meeting one another. When we suppose that bishop *b* meets a bishop, the sentence 'bishop *c* meets a bishop' does not necessarily keep its current truth value. It may go from false to true. To account for this, we need not say that an exhaustive interpretation has disappeared. The lack of independence alone suffices.

4.3 Substitution

Third, just like permission statements, conditional antecedents violate substitution. There are sentences *A* and *B* that are true in all the same worlds, while 'if *A*, *C*' and 'if *B*, *C*' are not. Fine (2014a:328) observes this for Alternative Eden.[25] Recall that there is one forbidden apple a_0 and infinitely many safe apples $a_1, a_2, \dots$. Compare:

(15) a. If Eve were to eat infinitely many of the apples $a_1, a_2, \dots$, she would be fine.

b. If Eve were to eat infinitely many of the apples $a_0, a_1, \dots$, she would be fine.

The first is intuitively true but the second is not. Their antecedents, however, are true in all the same worlds.

[25] Goodsell (2022) makes the same point using a scenario and conditionals with essentially the same structure as Fine's.

Also like permission statements, conditionals exhibit their own version of the grain problem. Some cases suggest that, for a conditional to be true, it must be true for every way in which its antecedent might be true—a principle Fine (2012) calls *universal realisability of the antecedent*. To illustrate with Nute's (1980) example:

(16) If we had had good weather this summer or the sun had grown cold, we would have had a bumper crop.

Intuitively, for this to be true, it must be true when we suppose either disjunct. Likewise, (15b) intuitively implies that for every infinite collection of apples whatsoever, if Eve ate every apple from the collection, she would be fine.

Other cases point the other way: a conditional can be true even though it is false for some ways in which its antecedent might be true. Embry (2014) makes the point with the following examples. If it had snowed yesterday, I would have gone skiing (...but not if it had snowed 100 feet, in that case I would have stayed home). If Sue were to take some of these pills, she would get better (...but not if she were to take 25 of them, which would just make her sicker). If the passenger hadn't been sitting up front during the collision, they would have survived (...but not if they had instead been between the two colliding cars). Not every way for it to snow yesterday, for Sue to take some of these pills, or for the driver to not be in the driver's seat will do.[26] Just as with permission statements, we face the task of articulating when conditional antecedents are coarse and when they are fine.

Summing up this section, we have seen three parallels between possibility statements and conditionals. For each of above challenges to the standard theory of possibility, we find that conditionals behave in the same way. We believe that these are not three coincidences, but three instances of a single underlying phenomenon, three instances of a systematic connection between modals and conditionals. The goal of the rest of this paper is to provide this connection. Once this is in place, a solution to the three challenges facing the standard theory will fall out in an automatic and uniform way.

[26]One might reply on behalf of universal realisability that some of these ways for things to occur are outlandish, or contextually irrelevant, so we ignore them. (In a similar vein, Fine (2012:232) argues that when we judge 'If Spain had fought with the Axis or the Allies, she would have fought with the Axis' true, we regard Spain fighting alongside the Allies as counterfactually impossible.) One objection to this reply is that it draws an unprincipled distinction between those cases that are excluded by the semantics of counterfactuals proper (such as the match being wet in Goodman's (1947) 'If the match were struck, it would light'), and those that are excluded by contextual irrelevance (such as it snowing 100 feet). For further discussion of this reply, see McHugh (2022:202) and note 45 below.

5 Conditional modality

There is a way of thinking about modality that has been proposed in scattered places over the years, with conditionals at its heart. This section first discusses the history of this approach, and expresses it in terms of conditional selection functions. We then generalise the approach to all modal flavours whatsoever.

5.1 The conditional theory of deontic modals

In Leibniz's *Elements of Natural Law*, he defines what is permitted as 'what is possible for a good person to do' and what is obligatory as 'what is necessary for a good person to do'.[27] Hilpinen (2017:159–60) suggests paraphrasing Leibniz's thought here using conditionals: one is permitted to do something just in case *if they do it*, possibly, they are a good person, and one is obligated to do something just in case *if they do not do it*, necessarily, they are not a good person.

In the 1950s, Kanger (1957) proposed analysing *ought* A as $N(Q \supset A)$, where 'N is the notion of analytic necessity', Q 'a constant stating what morality prescribes', and $\supset$ the material conditional.[28] This is equivalent to $N(\neg A \supset \neg Q)$, stating that every world where A is false is one where what morality prescribes has not been met. Around the same time, Anderson (1958:103) proposed that 'to say that p is obligatory is to say that failure of p leads to a state-of-affairs $\mathscr{P}$ which is "bad" '.[29] He later wrote that 'it is *analytic* of the notion of obligation that if an obligation is not fulfilled, then something has gone wrong' (Anderson 1967:346–47). 'You must not murder', for example, means that if you murder, necessarily, the rules have been broken. That sounds exactly right.[30]

This is remarkably close to how a number of languages typically express permission and obligation, such as Japanese, Korean, and Burmese.[31] Examples (17) and (18), due to Akatsuka (1992), show this for Japanese.

(17) *Permission:*

Tabe-temo ii.

[27] Leibniz to Arnauld, 1671; *Sämtliche Schriften und Briefe* II i 173–174; *Philosophical Papers and Letters*, 150.

[28] See von Wright (1968) for a similar proposal.

[29] The negation of $\mathscr{P}$, stating that all obligations have been met, has also been called 'the good thing' by Lokhorst (2006) and Barker (2010), and 'OK' by Asher and Bonevac (2005).

[30] For further work of the connection between conditionals and deontic modals, see von Wright (1968) and Arregui (2011).

[31] Kaufmann (2017a) calls these *conditional evaluative constructions*. For discussion, see Akatsuka (1992), Wymann (1996), Nauze (2008), Knoob (2008), Narrog (2009), Kaufmann (2017a), and Chung (2019).

eat even if good

Literally: "It is good even if you eat" = "You may eat"

(18) *Obligation:*

Tabenakere-ba ikenai/ dame da.

eat Neg if can go Neg no good is

Literally: "It is not good if you don't eat" = "You must eat"

Beyond deontic modality, Williamson (2007:157, 297) also considers analysing metaphysical possibility and necessity in terms of counterfactuals. Let $\Box\!\!\rightarrow$ denote the counterfactual conditional, and let $A \Diamond\!\!\rightarrow C$ abbreviate $\neg(A \Box\!\!\rightarrow \neg C)$. Williamson suggests that a statement is metaphysically necessary just in case were it false, a contradiction would obtain, with possibility the dual of necessity. That is, where $\top$ is a tautology and $\bot$ a contradiction, Williamson proposes:[32]

$$\begin{aligned} \Diamond A &\equiv A \Diamond\!\!\rightarrow \top \\ \Box A &\equiv \neg A \Box\!\!\rightarrow \bot \end{aligned}$$

Williamson's analysis of necessity falls out as a special case of the pattern observed for Japanese above when we interpret the good as the truth. (For metaphysical modality this seems apt: what is 'good', metaphysically speaking, is just what is true.)

These observations all point toward the general analysis of possibility and necessity in (1), repeated below.

(1) a. *Possibly* A means: if A were true, there would be some case where the relevant ideals are met.

b. *Necessarily* A means: if A were false, there would be no case where the relevant ideals are met.

5.2 Conditional modality via selection functions

We can formalise these ideas in terms of conditional selection functions.[33] A model is given by $M = (W, f, \leq, V)$ where W, $\leq$, and V are as before, and f is a conditional selection function, taking a sentence and a world (and perhaps other parameters not represented here) and returning a set of worlds. Intuitively, $f(A, w)$ contains the worlds we consider when we suppose A true at w; those that result from supposing 'if A were true'

[32] Note that these equivalences follow automatically if we analyse $\Diamond$ and $\Box$ via Kripke semantics and define that $A \Diamond\!\!\rightarrow C$ (respectively, $A \Box\!\!\rightarrow C$) is true at a world w just in case C is true at some (every) accessible world from w where A is true.

[33] The presentation in this section closely follows that of McHugh (2025b).

at w. Selection functions allow us to represent a wide range of theories of conditionals. For Stalnaker (1968), $f(A,w)$ will contain the unique selected A-world at w; for Lewis (1973), under the limit assumption, it will be the set of closest A-worlds to w; for interventionist semantics of conditionals (Galles and Pearl 1998, Halpern 2000, Hiddleston 2005, Briggs 2012, a.o.), $f(A,w)$ will contain the models that result from intervening to make A true at context w.

We may express the semantics of conditionals in terms of selection functions. Let $A \diamondrightarrow C$ be true at a world w just in case C is true at some world in $f(A,w)$, and $A \boxright C$ be true at w just in case C is true at every world in $f(A,w)$.[34]

It remains to say which worlds are good: what it means for the relevant ideals to be met. For completeness we adopt an analysis here, though for the purposes of solving the three challenges above facing the standard theory, nothing hinges on this. Our solution will apply regardless of what analysis one adopts. We follow McHugh (2025b) in adopting a comparative interpretation of which worlds are good. When we evaluate whether the relevant ideals are met, given a sentence A, we consider what would happen if A were true and what would happen if it were false. The ideals are met at a world w', with respect to A and w, just in case w' is among the best worlds that result from supposing A true or supposing it false w. That is, a world is $good_{A,w}$ just in case it is in $\textsc{best}_w(f(A,w) \cup f(\neg A,w))$. To avoid notational clutter, when the sentence and world under consideration are clear from context, we often omit the subscripts and write $good_{A,w}$ simply as *good*.

5.3 Conditional modality generalised via accessibility relations

We have expressed the conditional approach to modality in terms of selection functions. However, the standard approach to modality is not framed in terms of selection functions, but in terms of accessibility relations. There is, nonetheless, a natural correspondence between the two, one that allows us to express the conditional theory in terms of accessibility relations. This will allow us to represent diverse modal flavours within the conditional theory by plugging in different accessibility relations for different modal flavours in the usual way (epistemic accessibility, deontic accessibility, and so on).

To draw the correspondence, we need to make one small but significant change to

[34]For simplicity, we express conditionals in terms of a connective, though nothing hinges on this. As Kratzer (1986) has persuasively argued, conditionals in natural language do not express a connective, but rather a restriction of a modal's domain, what is known as the *restrictor view*. Our formalisation of the conditional may easily be adapted to the restrictor view by taking $f(A,w)$ to be the highest-ranked worlds (according to the ordering source) in the modal base that is updated to include A.

the standard theory of modality. On the standard theory, accessibility is taken to depend on a variety of factors, such as, most importantly, the conversational context (see Kratzer 1981b). But it is not taken to depend on the statement whose modal status is at issue—the A in $\Diamond A$ and $\Box A$—what is often called the modal's *prejacent*. Conditional selection functions, on the other hand, do depend on the conditional antecedent. To express selection functions in terms of accessibility relations, we therefore need accessibility to be sensitive to the statement at issue, a phenomenon McHugh (2025b) calls *condition dependence*.

We may reformulate the conditional theory of deontic modals in terms of condition-dependent accessibility relations as follows. Given a sentence A, let us say that a world w' is *A-accessible* from a world w (denoted $wR_A w'$) just in case w' is among the worlds we consider when we suppose 'if A were true' or 'if A were false' at w.

$$wR_A w' \quad \text{if and only if} \quad w' \in f(A,w) \cup f(\neg A,w) \qquad (f \textit{ to } R)$$

This gives us two ways to analyse the conditional. $A \Diamond\!\!\rightarrow C$ (respectively, $A \Box\!\!\rightarrow C$) is true at w just in case...

in terms of selection functions: ...C is true at some (every) world in $f(A,w)$.

in terms of accessibility relations: ...C is true at some (every) A-accessible world from w where A is true.

These two ways of defining the conditional are equivalent, assuming the above analysis of A-accessibility (*f to R*), and naturally, that the selection function satisfies success: A is true at every selected A-world. Given this analysis in terms of accessibility relations, it follows that a world is *good*, with respect to a sentence A and world w, just in case it is among the best A-accessible worlds from w. We therefore have two equivalent ways to express the conditional theory: in terms of selection functions or accessibility relations.[35]

This correspondence between selection functions and accessibility relations opens the door to a general, conditional theory of modality. This is because we already have a general theory of modality in terms of accessibility relations. For deontic modals, in line with the conditional theory above, we may take the A-accessible worlds to be those that result from supposing 'if A' or 'if $\neg A$' (for an argument in favour of this

[35] We may, if desired, also go the other way, defining the conditional selection function in terms of accessibility. Take the selected A-worlds to be the A-accessible worlds where A is true (call this principle *R to r*). Then *f to R* and *R to f* will be equivalent under certain conditions. If the selection function satisfies success (A is true at every selected A-world), *f to R* implies *R to f*. While if each sentence has same accessibility relation as its negation ($R_A = R_{\neg A}$), *R to f* implies *f to R*.

over the standard semantics of deontic logic, see McHugh 2025b). Alternatively, if we prefer the standard theory of deontic modals, we may take the A-accessible worlds to be deontically accessible according to the standard deontic accessibility relation.

For other modal flavours we may, if we like, plug in the usual accessibility relation. With epistemic modals, say, we may give a standard treatment *à la* Hintikka (1962), whereby one world w' is epistemically accessible from a world w just in case w' is compatible with the relevant agent's evidence at w. In this case A-accessibility will be the same as B-accessibility for any sentences A and B: A-accessibility will reduce to plain accessibility, and the condition dependence of the accessibility relation will play no role.

Putting all this together, we formulate the conditional theory of modals in terms of accessibility relations as follows.

The Conditional Theory of Modality. Given a language (set of sentences) $\mathscr{L}$, a model of the conditional theory is given by $M = (W, R, \leq, V)$, where

> W is a non-empty set of worlds;
> $R : \mathscr{L} \to W \times W$ assigns to each sentence A an accessibility relation R_A;
> $\leq : W \to W \times W$ assigns to each world an order $\leq_w$ over worlds;
> $V : W \times \mathscr{L} \to \{0, 1\}$ is a valuation, assigning to each world and sentence a truth value.

We say $A \Diamond\!\!\rightarrow C$ (respectively, $A \Box\!\!\rightarrow C$) is true at w just in case C is true at some (every) A-accessible world from w where A is true: for some (every) $w' \in W$ such that wR_Aw' and $V(A, w') = 1$, $V(C, w') = 1$. A world w' is *good*, with respect to A and w, just in case it is among the best A-accessible worlds from w: wR_Aw' and for no w'' with wR_Aw'' is $w'' <_w w'$. The truth-conditions of modal statements are given by:

$$\llbracket \Diamond A \rrbracket^w = 1 \quad \text{iff} \quad A \Diamond\!\!\rightarrow \; good_{A,w} \text{ is true at } w;$$
$$\llbracket \Box A \rrbracket^w = 1 \quad \text{iff} \quad \neg A \Box\!\!\rightarrow \neg good_{A,w} \text{ is true at } w.$$

How far have we strayed from the standard theory of modality? As McHugh (2025a) shows, not very. We can express the conditional theory in a way that is almost identical to standard theory. The only difference is to replace the standard accessibility relation R with the above, condition-dependent accessibility relation R_A.

Fact 1. $\Diamond A$ (respectively, $\Box A$) is true at w according to the conditional theory just in case A is true at some (every) world in $\text{BEST}_w(R_A[w])$.

Thus the standard theory falls out as a special case of the conditional theory; namely, when we take A-accessibility, for any sentence A whatsoever, to be the familiar accessibility relation R from the standard theory ($R_A = R$ for every sentence A).

Nonetheless, the standard and conditional theories take markedly different perspectives on modal statements. Uniquely, on the conditional theory, the modal prejacent A appears inside a conditional antecedent: it represents modal statements as having a conditional meaning. This shift in perspective opens the door to solving the three challenges above in a way unavailable to the standard theory. This is because it allows us to take our favourite account of conditional antecedents and automatically apply it to permission statements. In what follows, we show this for one account in particular: alternatives.

6 Alternatives in conditional antecedents

One rule that has received a great deal of attention in the literature on conditionals is simplification of disjunctive antecedents (SDA): the inference from *if* $A \vee B$, C to *if* A, C and *if* B, C. Broadly, there are three approaches to SDA available: claim it is valid, invalid, or that disjunctive antecedent conditionals are ambiguous between a reading that validates SDA and one that does not.[36] Here we adopt an ambiguity approach.[37] Following Pollock (1984:145), we call the SDA-validating reading the 'distributive' reading and the SDA-invalidating reading the 'literal' reading.

A general way to express this is to say that disjunctions are associated with a set of sentences, known as an *alternative set*. The possible alternative sets for a disjunction are the set consisting of each disjunct, and the singleton set containing the whole disjunction.[38] The first generates the distributive reading, the second the literal reading. We let *if* $\{A, B\}, C$ denote the distributive reading and *if* $\{A \vee B\}, C$ the literal reading. The ambiguity approach says that *if A or B, C* is ambiguous between these two readings.

Disjunction is not the only alternative-generating item. In general, theories that appeal to alternatives assume that a small class of logical expressions generate alter-

[36] Among those who claim SDA is valid are Nute (1975), Ellis, Jackson, and Pargetter (1977), Fine (2012), Starr (2014), and Willer (2018). Among those who claim it is invalid—at least on some readings—are Nute (1980b, 1984), Bennett (2003), van Rooij (2006), and Lassiter (2018). An ambiguity approach is endorsed by Hilpinen (1982), Loewer (1976), McKay and Inwagen (1977), Warmbrōd (1981), Pollock (1984), Alonso-Ovalle (2009), Santorio (2018), and Khoo (2021b).

[37] This is by no means the only approach available, though since our primary focus is on modality, rather than conditionals proper, we limit our attention to one approach (the most plausible one we believe to be currently available). One who prefers an alternative approach to SDA may plug their favourite account into the conditional theory here.

[38] See Alonso-Ovalle (2006, 2009) and Ciardelli (2016: especially note 11).

natives, including disjunction, existential quantification, and questions (Hamblin 1973, Kratzer and Shimoyama 2002, Alonso-Ovalle 2006, Aloni 2007, Liao 2011, Rawlins 2013, Ciardelli, Groenendijk, and Roelofsen 2018, Hengeveld, Iatridou, and Roelofsen 2023). For instance, the antecedent 'if any bishop meets a bishop' has a distributive reading, *if* $\{x \textit{ meets a bishop} \mid x \textit{ is a bishop}\}$, and a literal reading, *if* $\{\exists x(x \textit{ is a bishop} \wedge x \textit{ meets a bishop})\}$. In general, where D is the relevant domain, *if any x Ax, C* has a distributive reading *if* $\{Ax \mid x \in D\}, C$, and a literal reading *if* $\{\exists x \in D\, Ax\}, C$.

We propose that a conditional with an alternative-generating item in its antecedent has whatever status it has when the item is replaced by its alternatives.[39] This formulation presupposes that each alternative results in the same status, called a homogeneity presupposition.[40] More precisely, we can formulate this idea as follows. For any set of sentences Alt and sentences A and S, let $[\![S]\!]^{Alt \mapsto A}$ be the result of replacing every occurrence of Alt in S with A. Finally, we let *if A, C* stand for whichever conditional is in question, such as $A \mathrel{\Box\!\!\rightarrow} C$ or $A \mathrel{\Diamond\!\!\rightarrow} C$.[41]

For any set of sentences Alt and sentence C, *if Alt, C*

(i) presupposes $[\![\textit{if } Alt, C]\!]^{Alt \mapsto A} = [\![\textit{if } Alt, C]\!]^{Alt \mapsto B}$ for all $A, B \in Alt$,

(ii) and asserts $[\![\textit{if } Alt, C]\!]^{Alt \mapsto A} = 1$ for all $A \in Alt$.

Alternative-free theories of conditionals fall out as a special case; namely, when there is a single alternative: *if* $\{A\}, C$ is equivalent to *if* A, C.

6.1 Evidence for the ambiguity account

Support for the homogeneity condition in (i) comes from downward-entailing environments (environments that reverse the direction of entailment, such as negation) and probability judgements. Santorio (2018:524) observes that 'None of my friends would have fun at the party if Alice or Bob went' intuitively implies both 'None of my friends would have fun at the party if Alice went' and 'None of my friends would have fun at the party if Bob went'.[42] Condition (ii) alone does not predict this, but adding to it the

[39]Simons (2005:278), for example, proposes that 'when a modal (or other operator) takes a non-singleton set as argument, the operator in some sense interacts with each member of the argument set'.

[40]For an earlier proposal that conditional antecedents trigger a homogeneity presupposition, see von Fintel (1997).

[41]The use of substitution in the definition accounts for cases where the alternatives affect material outside the conditional antecedent, as in 'If there were a mouse or squirrel in the kitchen, I'd run away from it,' where the referent of *it* depends on the alternative in question. When a conditional antecedent contains multiple alternative-generating items (as in 'if anyone has any questions'), we assume that the alternatives are computed pointwise (see Hamblin 1973, Kratzer and Shimoyama 2002).

[42]One may reply that Santorio's example is parsed as [None of my friends would have fun at the party] [if Alice or Bob went], with the negation scoping only over the consequent. Parsed in this way, the example

homogeneity condition in (i) does.

For probability judgements, consider the following case, from Santorio (2021:633).

> Alice threw a six-sided die. Before her throw, Bob bet that the outcome would be a 3 or a 4. We don't know the outcome, but we hear two reports. Mary says that the die landed on 2, 3, or 4; Sue says that it landed on 3, 4, or 5. We have no idea whether the reports are accurate.

Consider:

(19) If Mary's report is accurate or Sue's report is accurate, probably Bob won.

Santorio observes that this has a true reading here.[43] Now, it is true on the distributive reading but false on the literal reading. On the distributive reading, it is equivalent to 'if Mary's report is accurate, probably Bob won, and if Sue's report is accurate, probably Bob won', which is true.[44] On the literal reading, it is equivalent to 'If the die landed between 2 and 5, probably Bob won'. This is false since in that case he would only have a half chance of winning, while *probably* requires more than half chance.

Other examples bring out the literal reading. Consider:

(20) If the die landed 2, 3, or 4, probably Bob won.

This also has a true reading here. Now the pattern is reversed: the literal reading is true but the distributive reading is false, since it is false that if the die landed 2, probably Bob won—in that case he certainly lost.

When a disjunctive antecedent conditional appears unembedded, we have a clear preference for the distributive reading over the literal one. The literal reading appears to only be available when one of the alternatives entails the consequent. To illustrate, consider the following pair.[45]

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does not provide evidence for (i). This objection can be met by forcing the negation to take wide-scope. We can do this, for example, by placing a bound pronoun in the antecedent (see McHugh 2023:145), as in 'No girl would be happy if she failed algebra or geography', or with Santorio's example, 'I doubt that, if Alice or Bob went, the party would be fun' (Santorio 2018:524).

[43] As Santorio (2021) observes, this would be unexpected if SDA were an implicature—proposed by Bennett (2003), Klinedinst (2007), van Rooij (2010), Franke (2011), and Bar-Lev and Fox (2020)—since implicatures imply their literal meanings.

[44] We assume a standard semantics of *probably* here: *if A, probably C* is true just in case the selected A-worlds where C is true are more probable than the selected A-worlds where C is false: $Pr(f(A,w) \cap |C|)$ is greater than $Pr(f(A,w) \cap |\neg C|)$; equivalently, $Pr(f(A,w) \cap |C|) > .5$ (see Yalcin 2010:928), where $|C|$ is the set of worlds where C is true.

[45] The first sentence comes from McKay and Inwagen (1977). The contrast was first observed by Nute (1980a). Note that it casts doubt on explanations of SDA in terms of pragmatic shifts in context. Fine

(21) a. If Spain had fought for the Allies or Axis, they would have fought for the Axis.
b. If Spain had fought for the Allies or Axis, Hitler would have been pleased.

The first is intuitively much more acceptable than the second. Both are false on their distributive readings, and true (we may suppose) on their literal readings. The fact that the first is much more acceptable than the second therefore shows that the literal reading is much more accessible in the first than in the second.[46]

We see the same contrast with other alternative-generating items, such as *anyone*.

(22) a. If anyone had eaten my sandwich, Bob would have.
b. If anyone had eaten my sandwich, I would have gotten annoyed.

The first does not require the conditional to hold for each alternative to *anyone*. It does not imply, say, that if Charlie had eaten my sandwich, Bob would have. But the second does require it to hold for each alternative, implying that if Charlie had eaten my sandwich, I would have gotten annoyed.

This theory also allows us to account for Ciardelli, Zhang, and Champollion's experimental results, assuming that disjunctions raise alternatives but negated conjunctions do not (something Ciardelli et al. themselves endorse).[47] 'If switch A or switch B was down, the light would be off' expresses $\{A\ down, B\ down\} \Box\!\!\rightarrow off$, which asserts $(A\ down \Box\!\!\rightarrow off) \wedge (B\ down \Box\!\!\rightarrow off)$ and is intuitively true. 'If switch A and switch B were not both up, the light would be off' expresses $\{\neg(A\ up \wedge B\ up)\} \Box\!\!\rightarrow off$, with a single alternative in the antecedent. Assuming that this antecedent raises the possibility of both switches being down, in which case the light is on, accounts for Ciardelli, Zhang, and Champollion's experimental results.[48]

(2012:232), for example, proposes 'a principle of "Suppositional Accommodation", according to which we always attempt to interpret a counterfactual in such a way that its antecedent *A* represents a genuine counterfactual possibility'. This rule only concerns whether the antecedent is counterfactually possible, and therefore does not predict changes in the *consequent* to affect SDA's validity, contrary to what we observe.

[46] That being said, the literal reading appears to be available given enough explicit contextual support. Nute (1980b:163) notes that the following sounds acceptable: 'if Spain had fought for the Allies or Axis, they would have fought for the Axis. So if Spain had fought for the Allies or Axis, Hitler would have been pleased'. The point is not decisive: one might reply that the sentence is interpreted enthymematically, as in, say, 'so if Spain had fought for the Allies or Axis, [they would have fought for the Axis, in which case] Hitler would have been pleased'. This fits the pattern above, where one of the alternatives entails the consequent.

[47] The assumption that disjunction raises alternatives but negations do not is made by Kratzer and Shimoyama (2002), Alonso-Ovalle (2006), and Ciardelli, Groenendijk, and Roelofsen (2013).

[48] Some theories of conditionals predicting that $\neg(A \wedge B) \Box\!\!\rightarrow C$ implies $(\neg A \wedge \neg B) \Box\!\!\rightarrow C$ include Fine

6.2 Wide-scope disjunction

The theory of alternatives proposed here does not account for why disjunctions of whole conditionals are often interpreted conjunctively; for example, why 'If you go to Brixton, things will be fine, or if you go to Victoria, things will be fine' intuitively implies each conditional. It is currently unclear what mechanism predicts this (for discussion see Wiggins and Edgington 1997, Geurts 2005, Khoo 2021a, and McHugh 2025a). That being said, our point from Section 4.1 still stands, that disjunctions of permissions and disjunctions of conditionals exhibit parallel behaviour here, suggesting that whatever account we find for the conditional data may be applied to account for wide-scope free choice as well.

7 Solving the three challenges

The conditional theory allows us to take the above account of conditional antecedents and derive, wholly automatically, correct predictions for modal statements.

7.1 Free choice

On the conditional theory, $\Diamond A$ is equivalent to $A \Diamond\!\!\rightarrow \mathit{good}$, so $\Diamond(A \vee B)$ is equivalent, on its distributive reading, to $\{A,B\} \Diamond\!\!\rightarrow \mathit{good}$.[49] This asserts $A \Diamond\!\!\rightarrow \mathit{good}$ and $B \Diamond\!\!\rightarrow \mathit{good}$; that is, $\Diamond A$ and $\Diamond B$. Thus we derive free choice, following a number of authors who have sought to reduce free choice inferences to SDA (such as Hilpinen 1982, Asher and Bonevac 2005, van Rooij 2006, Klinedinst 2007, Barker 2010, Franke 2011, and Willer 2018).[50]

In addition, $\Diamond(A \vee B)$ presupposes that $A \Diamond\!\!\rightarrow \mathit{good}$ and $B \Diamond\!\!\rightarrow \mathit{good}$ have the same

(2012), Ciardelli, Zhang, and Champollion (2018), Schulz (2018), and McHugh (2023).

[49]Strictly speaking, these should be $\mathit{good}_{w,A}$, $\mathit{good}_{w,\{A,B\}}$, respectively. We omit the subscripts for simplicity.

[50]What about the literal reading? In Section 6 we saw that when a disjunctive antecedent conditional appears unembedded, we only opt for the literal reading when the consequent entails one of the alternatives. This situation is easy to construct for conditionals but hard—if not impossible—to construct for permission statements. To produce an analogue of McKay and Inwagen's Allies–Axis example for $(A \vee B) \Diamond\!\!\rightarrow \mathit{good}_{A\vee B}$, we would need $\mathit{good}_{A\vee B}$ to entail A or B. A world is a $\mathit{good}_{A\vee B}$-world just in case it is a selected $A \vee B$-world or a selected $\neg(A \vee B)$-world, and no selected $A \vee B$-world or selected $\neg(A \vee B)$-world is better than it. Assuming A and B are not tautologies, this does not logically entail that A is true, nor that B is true, for whenever there is some selected $\neg(A \vee B)$-world, we may take an order which makes this world deontically ideal, but neither A nor B are true there. This shows that, provided there is some selected $\neg(A \vee B)$-world, $\mathit{good}_{A\vee B}$ entails neither A nor B, meaning we cannot construct examples for permission analogous to McKay and Inwagen's example for conditionals.

truth value: $\Diamond A$ and $\Diamond B$ are either both true or both false.[51] This captures dual prohibition. 'You may not have ice cream or cake' implies both 'You may not have ice cream' and 'You may not have cake'. This is for the same reason that 'I doubt things will be fine if you have ice cream or cake' implies both 'I doubt that things will be fine if you have ice cream' and 'I doubt that things will be fine if you have cake': the presupposition of homogeneous alternatives. $\neg(\{A,B\} \Diamond\!\!\rightarrow \mathit{good})$ asserts $\neg((A \Diamond\!\!\rightarrow \mathit{good}) \wedge (B \Diamond\!\!\rightarrow \mathit{good}))$ and presupposes that each conditional has the same truth value. Since they are not both true, they must both be false: $\neg(A \Diamond\!\!\rightarrow \mathit{good}) \wedge \neg(B \Diamond\!\!\rightarrow \mathit{good}))$; that is, $\neg\Diamond A \wedge \neg\Diamond B$.

Literal readings of disjunction in permissions. In Section 6 we saw that the literal reading of disjunction in conditional antecedents is generally unavailable, but becomes available under probability operators. For instance, given that Bob bet that the die landed 3 or 4, when we interpret 'What is the probability that if the die landed 2, 3, or 4, Bob won?' we can easily access the literal reading on which the answer is two thirds. In light of the conditional theory, we would expect disjunctive permissions to behave analogously. Do they? Consider:

> It's time for dessert. Alice's parents play a game to decide what dessert she is allowed to have. They roll a die. If it lands 1 or 2, she is allowed fruit (and only fruit). If it lands 3 or 4, she is allowed ice cream (and only ice cream). If it lands 5 or 6, she is allowed cake (and only cake). The die has just been cast, but we don't know how it landed. What is the probability that Alice is allowed to have ice cream or cake?

We can easily access a reading on which the answer is two thirds. After all, there is a two in three chance that the die lands between 3 and 6. This is the literal reading of 'Alice is allowed to have ice cream or cake': $\{\mathit{ice\ cream} \vee \mathit{cake}\} \Diamond\!\!\rightarrow \mathit{good}$. On the distributive reading, $\{\mathit{ice\ cream}, \mathit{cake}\} \Diamond\!\!\rightarrow \mathit{good}$, 'Alice is allowed to have ice cream or cake' asserts that Alice is allowed to have ice cream *and* that she is allowed to have cake. But there is zero chance of that happening. No matter how the die lands, she is allowed only one dessert. The fact that 'The probability that Alice is allowed to have ice cream or cake is two thirds' has a true reading in the this context shows that disjunction in permission statements has a literal reading under probability operators—exactly parallel to how disjunction behaves in conditional antecedents.

[51] This is none other than Goldstein's (2019) homogeneity presupposition for free choice. Section 8.2 compares the conditional theory with Goldstein's.

7.2 Independence

The theory of alternatives above predicts that $\{A,B\} \Diamond\!\!\rightarrow C$ asserts $(A \Diamond\!\!\rightarrow C) \wedge (B \Diamond\!\!\rightarrow C)$. As discussed in Section 4.2, this allows us to account for the independence inferences when the disjuncts are independent, in the sense that when we suppose one true, we leave the other as it is.

For example, in the adoption case, when we imagine adopting Alice (A), we imagine leaving Bob in his current, unadopted state ($\neg B$). That is, at every A-accessible world where A is true, B is false: $A \Box\!\!\rightarrow \neg B$. In these worlds the normative ideals have been violated. We therefore predict 'You may adopt Alice' to be false in the adoption case, similarly for 'You may adopt Bob', and hence predict 'You may adopt Alice or Bob' to also be false, as desired. It is worth emphasising that this prediction does not require any additional mechanism to handle independence inferences, but followed automatically from disjuncts' independence.

In the Canasta case, we represent 'You may take any collection of cards in the discard pile' as

$$\{\textit{you take } x \mid x \textit{ is a collection of cards in the discard pile}\} \Diamond\!\!\rightarrow \textit{good}.$$

Given the independence of taking each card, when we suppose that one takes a collection of cards, we imagine the other cards left where they are. In a world where the one takes only some of the cards, the rules have been broken.[52] So we correctly predict 'you may take any collection of cards in the discard pile' to be false in the Canasta case.

7.3 Substitution

Our proposal also invalidates substitution. Take the contrast between 'Alice may buy more than four tickets' and 'Alice may buy five or more tickets'. The difference comes down to a difference in alternatives: *more than n* does not raise alternatives, while the disjunction *n or more* does.

(23) a. Alice may buy more than four tickets.
b. $\{\textit{Alice buys more than four tickets}\} \Diamond\!\!\rightarrow \textit{good}$

(24) a. Alice may buy five or more tickets.
b. $\{\textit{Alice buys five tickets}, \textit{Alice buys more than five tickets}\} \Diamond\!\!\rightarrow \textit{good}$

[52] More precisely, in terms of our analysis of *good* from Section 5.3, we may show this as follows. Let Tc denote that you take collection c, which is less than the full pile. Then no Tc-accessible world where Tc is true is good, since there is a better Tc-accessible world where Tc is false; namely, one where you take no cards.

The first is true, under the natural assumption that when we suppose that Alice buys more than four tickets, we consider possibilities where she buys five, six, seven, and so on. At least one of these possibilities is good (namely, one where she buys exactly five), so the sentence is true. But the second is false: it asserts both *Alice buys five* $\Diamond\!\!\rightarrow$ *good* and *Alice buys more than five* $\Diamond\!\!\rightarrow$ *good*. The latter is false, since if she buys more than five, the rules have been broken.

We may also account for Alternative Eden using alternatives. This requires a bit more work, since the sentence 'Eve eats infinitely many apples' seems at first sight to lack any of the typical alternative-generating items, such as disjunction, existential quantification, or question words. We can, nonetheless, express its meaning in terms of them: Eve eats infinitely many apples just in case she eats *some* infinite collection of apples (where we understand that Eve eats a collection of apples just in case she eats every apple in the collection).[53]

This is a familiar move. It is sometimes argued that an expression which does not overtly contain an alternative-generating item nonetheless generates alternatives. An example is *at least*.[54] Like *infinitely many*, *at least* can be expressed in terms of an alternative-generating item: *at least n* is equivalent to *n or more*. Indeed, taking *at least* to raise alternatives *exactly n* and *more than n* solves a problem raised by McDermott (2007). The problem concerns theories of conditionals validating conjunctive sufficiency: the rule that *A and C* implies *if A, would C*.[55] Suppose a coin is to be tossed twice. Before it is tossed Alice bet that it will land heads both times. In fact it does, so she wins. Consider:

(25) If at least one head had come up, Alice would have won.

As McDermott observes, this sounds incorrect. This appears to be a counterexample to conjunctive sufficiency. If, however, *at least n* has alternatives *exactly n* and *more than n*, we predict its default, distributive reading to be:

$$\{\textit{exactly one head comes up}, \textit{more than one head comes up}\} \;\Box\!\!\rightarrow\; \textit{Alice wins}.$$

This asserts that if exactly one head had come up, Alice would have won, which is false. Assuming that *at least n* has an alternative *exactly n* correctly predicts (25) to be

[53] McHugh (2023:155) has previously given an analysis of 'infinitely many' in terms of existential quantification.

[54] Among those arguing that *at least n* generates alternatives *n* and *more than n* are Büring (2008), Coppock and Brochhagen (2013), Blok (2019), and Ciardelli, Groenendijk, and Roelofsen (2018).

[55] Examples include Stalnaker (1968), Lewis (1973), and Kratzer (1981a). Walters and Williams (2013) show how to derive conjunctive sufficiency from minimal assumptions. Cruz et al. (2015) and Politzer and Baratgin (2016) present experimental evidence supporting conjunctive sufficiency.

incorrect—crucially, regardless of whether the underlying theory of conditionals validates conjunctive sufficiency.[56]

So we have reason to think that an item can generate alternatives despite not wearing any of the typical alternative-generating meanings on its sleeve. In this spirit, we propose to represent 'Eve may eat infinitely many of $a_0, a_1, \ldots$' as

$$\{\textit{Eve eats } x \mid x \textit{ is an infinite collection from } a_0, a_1, \ldots\} \mathrel{\Diamond\!\!\rightarrow} \textit{good}.$$

Some of these collections contain the forbidden apple a_0. If Eve eats such a collection, the rules will be broken, so we predict the sentence to be false, as desired. In contrast, we predict 'Eve may eat infinitely many of the apples $a_1, a_2, \ldots$' to be true, since Eve may eat any collection of these apples.

It is worth mentioning that not every expression that can be expressed in terms of disjunction or existential quantification raises alternatives. In event semantics (Davidson 1967, Parsons 1990), for example, declarative statements express the existence of events. 'It snowed yesterday' says that a snowing event took place yesterday: $\exists e(\textit{actual}(e) \wedge \textit{snow}(e) \wedge \textit{runtime}(e) \subseteq \textit{yesterday})$. Now recall Embry's (2014) example from Section 4.3, 'If it had snowed yesterday, I would have gone skiing'. We assign it the following literal and distributive readings, respectively.

$$\{\exists e(\textit{actual}(e) \wedge \textit{snow}(e) \wedge \textit{runtime}(e) \subseteq \textit{yesterday})\} \mathrel{\Box\!\!\rightarrow} \textit{ski}$$
$$\{\textit{actual}(e) \mid \textit{snow}(e) \wedge \textit{runtime}(e) \subseteq \textit{yesterday}\} \mathrel{\Box\!\!\rightarrow} \textit{ski}$$

The first does not require the conditional to hold for every such skiing event. Which skiing events we consider is decided by the conditional selection function in the usual way. But the second does assert that for any snowing event yesterday whatsoever, had it occurred, I would have gone skiing. This runs into Embry's problem from Section 4.3. For in one such skiing event, it snows 100 feet yesterday, in which case I would not have gone skiing. To make the right predictions in this case, we must say that 'it snowed yesterday' does not raise alternatives, even though we may express its meaning in terms of existential quantification. Having a meaning that we can express using disjunction or existential quantification is not enough for an item to generate alternatives. Its disjunctive/existential meaning must also be in some sense 'visible' to the alternative-generating mechanism.

[56] We also predict *at least* to have a literal reading. This seems correct. To bring it out, suppose one coin is fair but that the other is biased to always land tails. 'If at least one coin head had come up, exactly one head would have come up' has a true reading here—the literal reading.

7.4 The grain problem

Our account also provides a solution to the grain problem. Whether permission is fine or coarse is determined by the presence or absence of alternatives. For a permission statement to hold, it must hold for each of its alternatives—ensuring a form of fineness—while within each alternative, permission statements have an existential meaning—ensuring a form of coarseness.

To illustrate coarseness, recall Alice and Bob, who know both French and English but are required to speak French at work. Intuitively they are permitted to talk to each other at work, even though not every way for them to do so is permitted. While *or*, *any*, and *infinitely many* raise alternatives, *talk to each other at work* does not. Thus 'Alice and Bob may talk to each other at work' expresses $\{$*Alice and Bob talk to each other at work*$\}$ $\Diamond\!\!\rightarrow$ *good*, with a single alternative. If they talked to each other, they might speak English and they might speak French. In the latter cases, the rules are met. Since $\Diamond\!\!\rightarrow$ only requires the consequent to hold at *some* of the selected worlds, we correctly predict the sentence to be true.

Similar remarks apply to the switches case. Assuming that 'the switches agree' does not raise alternatives, 'the switches are allowed to agree' expresses $\{$*the switches agree*$\}$ $\Diamond\!\!\rightarrow$ *good*. Were the switches in the same position, they could both be up and they could both be down. When both are up the rules are met, so we correctly predict 'the switches are allowed to agree' to be true.

7.5 Necessity

So far we have focused exclusively on possibility. Let us briefly discuss our predictions for necessity. What is especially interesting is that obligations and permissions differ in their relationship with alternatives. As discussed in Section 6.1, following Kratzer and Shimoyama (2002) and Ciardelli, Groenendijk, and Roelofsen (2013), we assume that negations do not raise alternatives. On the conditional theory, when we evaluate whether something is necessary, we ask what would happen were it *false*: A is necessary just in case if it were false, the relevant ideals would not be met, $\Box A \equiv \neg A \;\Box\!\!\rightarrow \neg good$.

This predicts that necessity, unlike possibility, has only a literal reading. We therefore predict necessity to be coarse: something can be necessary even though some ways for it to obtain are impossible.[57] This seems right. You must pay your taxes (but not

[57] Cariani (2013:534) observes this for *ought*: 'You ought to give food to your pets, but not every way of giving food to the pets is something you are permitted to do. You may be under more specific requirements. For example, you ought to give them *non-poisonous* food. Presumably this is not the end of it: you ought to give them non-poisonous food *in decent quantities*, and so on'.

in whatever currency you like), you have to submit your assignment (but not after the deadline), for a flag to be the official flag of the France, it must be red, white, and blue (but only in the official shades). For disjunction, we predict $\Box(A \vee B)$ to be equivalent to $\{\neg(A \vee B)\} \mathrel{\Box\!\!\rightarrow} \neg good$. Thus 'you must clean the kitchen or walk the dog' says that if you fail to do both, the rules will be broken—the correct result.

Stepping back, Table 1 summarises, for each item discussed in this section, whether or not we have assumed that it generates multiple alternatives.

Multiple alternatives	**Single alternative**
Disjunction	Negation
At least n	*More than n*
Any	*Talk to each other, agree*
Infinitely many	*It snowed*

Table 1: Which items generate multiple alternatives?

8 Comparisons

8.1 Conditional modality and alternatives in isolation

Our account combines conditional modality with alternatives. Their combination is essential in generating the right results for the data we have considered. To show this, let us see some of the problems that result from adopting only one of these components in isolation.

Without alternatives, the conditional theory fails to predict free choice. It predicts 'You may post the letter or burn down the post office', represented as $(P \vee B) \mathrel{\Diamond\!\!\rightarrow} good$, to be true. This holds under the natural assumption that, if one posts the letter or burns down the post office, among the possibilities we consider is one where they post the letter and do not burn down the post office—a good world. For similar reasons, the conditional theory without alternatives undesirably predicts the following sentences to be true in their respective contexts: 'You may take any of the cards in the discard pile', 'Alice may buy five or more tickets', and 'Eve may eat infinitely many of the apples'.

What if we instead keep alternatives, but replace the conditional theory of modality with the standard theory? Here are two problems with that approach. First, we lose our account of independence inferences. The standard theory plus alternatives predicts 'You may adopt Alice or Bob' and 'You may take any of the cards in the discard pile' to be true in their respective contexts. Further, as we saw in Section 3.2, appeals to exhaustivity run into the exhaustivity problem.

Second, the standard theory plus alternatives struggles with obligation. $\Box(A \vee B)$ does not imply $\Box A \wedge \Box B$. For instance, 'You must clean the kitchen or walk the dog' does not require you to do both. But a naive application of alternatives to the standard theory would predict that disjunction should raise alternatives under permission and obligation alike. On our implementation of alternatives, $\Box(A \vee B)$ would assert—on its default, distributive reading—the result of replacing the disjunction with each disjunct: $\Box A$ and $\Box B$, the wrong result.

8.2 Goldstein's homogeneity account

Goldstein (2019) has proposed a theory of free choice which shares some features with the present approach. In particular, according to both accounts, $\Diamond(A \vee B)$ presupposes that $\Diamond A$ and $\Diamond B$ have the same truth value. We nonetheless see four advantages of the present account over Goldstein's.

First, instead of stipulating the homogeneity effect, as Goldstein does, we derive it from the general behaviour of alternatives for which we have independent evidence. As we saw in Section 6.1, disjunction in conditional antecedents carries the same presupposition.

Second, the homogeneity account, as it stands, does not capture independence inferences, wrongly predicting 'You may adopt Alice or Bob' to be true in the adoption case and 'You may take any cards in the discard pile' to be true in the Canasta case.

Third, since Goldstein assumes that the homogeneity presupposition applies in general, he does not account for the literal reading of permission statements under probability operators observed in Section 7.1.

Finally, while Goldstein does not discuss necessity, when we try to extend his homogeneity idea beyond possibility in the obvious way, we run into trouble. Goldstein proposes that disjunctive permission statements presuppose that both disjuncts have the same status: either both are permitted or neither are. Goldstein's account does not appear to provide any reason why permission should be special in this regard. We would similarly expect a disjunctive *obligation* to presuppose that both disjuncts are obligated or neither are. To test this, suppose Alice must follow the commands of each of her parents (unless they contradict one another, in which case there is a family meeting). One parent tells Alice, 'You must clean the kitchen or walk the dog'. The other tells her, 'You must clean the kitchen'. There is no conflict here. Clearly, Alice can satisfy the commands of both parents at once by cleaning the kitchen. But given a homogeneity presupposition for obligation, the first would presuppose that she must clean the kitchen if and only if she must walk the dog. Since she must clean the kitchen, it would follow that she must also walk the dog—the wrong result.

8.3 Alternatives from negation

We have assumed that negations do not raise alternatives. In contrast, Schulz (2018) and Romoli, Santorio, and Wittenberg (2022) propose that they do. For example, both approaches represent a negated conjunction $\neg(A \wedge B)$ as having three alternatives: $A \wedge \neg B$, $\neg A \wedge B$, and $\neg A \wedge \neg B$. Paired with the present theory, this assigns $\neg\Box(A \wedge B)$ a remarkably strong meaning, predicting it to imply that each alternative is permitted: $\Diamond(A \wedge \neg B)$, $\Diamond(\neg A \wedge B)$, and $\Diamond(\neg A \wedge \neg B)$.[58]

This seems too strong. Consider:

(26) You don't have to read both papers on the reading list, but you do have to read at least one of them. $\neg\Box(A \wedge B) \wedge \Box(A \vee B)$

This is perfectly consistent. But if 'you don't have to read both papers', $\neg\Box(A \wedge B)$, implied 'you are permitted to read neither of them', $\Diamond(\neg A \wedge \neg B)$, we would expect (26) to be as bad as 'You may have ice cream or cake, but you may not have cake'. After all, their unacceptability would be generated via the same mechanism: inferences from alternatives. Similarly, one may assert:

(27) Out of papers A and B, you have to read A, but you don't have to read both. $\Box A \wedge \neg\Box(A \wedge B)$

In this context, neglecting to read paper A is forbidden, $\neg\Diamond\neg A$, as is reading only paper B, $\neg\Diamond(\neg A \wedge B)$.

Epistemic modals behave similarly. The following are consistent.

(28)
a. Alice might not have read both papers, but she read at least one of them.
b. Out of papers A and B, Alice must have read A, but she might not have read both.

Certainly, they are far better than 'Alice might have read paper A or paper B, but she didn't read paper A'.

In contrast, assuming that $\neg\Box(A \wedge B)$ does not raise alternatives, we predict $\neg\Box(A \wedge B)$ to be equivalent to $\neg(\{\neg(A \wedge B)\} \mathrel{\Box\!\!\rightarrow} \neg good)$, which simplifies to $\{\neg(A \wedge B)\} \mathrel{\Diamond\!\!\rightarrow} good$.

[58] This is because $\neg\Box(A \wedge B)$ expresses $\neg(\neg(A \wedge B) \mathrel{\Box\!\!\rightarrow} \neg good)$, which is equivalent to $\neg(A \wedge B) \mathrel{\Diamond\!\!\rightarrow} good$. If negated conjunction has the three alternatives above, on its distributive reading this expresses $\{A \wedge \neg B, \neg A \wedge B, \neg A \wedge \neg B\} \mathrel{\Diamond\!\!\rightarrow} good$, which implies $(A \wedge \neg B) \mathrel{\Diamond\!\!\rightarrow} good$, $(\neg A \wedge B) \mathrel{\Diamond\!\!\rightarrow} good$, and $(\neg A \wedge \neg B) \mathrel{\Diamond\!\!\rightarrow} good$; that is, $\Diamond(A \wedge \neg B)$, $\Diamond(\neg A \wedge B)$, and $\Diamond(\neg A \wedge \neg B)$.

This is true if supposing $\neg(A \wedge B)$ returns at least one good world. In general, this could be a world where only A is true, a world where only B is true, or a world where both are false (provided there are such worlds among the selected $\neg(A \wedge B)$-worlds): we merely require the existence of some such world. This correctly predicts $\neg\Box(A \wedge B)$ to be in general compatible with $\Box(A \vee B)$, $\Box A$, and $\Box(A \wedge \neg B)$.

9 Conclusion

To recap, let's remind ourselves of our assumptions.

1. **Conditional Modality.** Modalised statements are interpreted as conditional antecedents.

    - $\Diamond A$ says that if A were true, there would be some case where the relevant ideals are met: $\Diamond A \equiv A \mathbin{\Diamond\!\!\rightarrow} \textit{good}$.
    - $\Box A$ says that if A were false, there would be no case where the relevant ideals are met: $\Box A \equiv \neg A \mathbin{\Box\!\!\rightarrow} \neg \textit{good}$.

2. **Alternatives.**

    - Some items have multiple alternatives while others do not. For example:
        - Disjunction, *any*, and *infinitely many* have multiple alternatives.
        - Negations, *more than n*, *talk to each other*, and *agree* do not.
    - Items that generate multiple alternatives have a distributive reading and a literal reading.
        - The distributive reading is the default reading, with the literal reading available in certain environments, such as under probability operators and when the consequent entails one of the alternatives.
        - A conditional has whatever status that results from replacing the antecedent with its alternatives.

These assumptions all have independent motivation. Conditional modality has been proposed by Kanger and Anderson for deontic modals, Williamson for metaphysical modals, and represents how Japanese, Korean, and Burmese typically express permission and obligation. The theory of alternatives has been proposed to account for the behaviour of conditional antecedents.

Combining conditional modality returns, in a wholly automatic way, solutions to a wide range of problems facing the standard theory of modality. We receive an account of free choice inferences, independence inferences (one crucially not relying on exhaustivity), and counterexamples to substitution, thereby solving the grain problem. The account moreover correctly predicts the disappearance of free choice under probability operators, and overcomes the limitations facing Goldstein's homogeneity account of free choice.

This price of all this is that we rethink some of our fundamental assumptions about what modal statements express. Adding a modal to a statement does not say that the statement is true in such-and-such worlds. Modals instead use the statement as a conditional antecedent. They are sensitive not only to the statement's truth conditions, but to the worlds we consider when we suppose the statement true or false. When a modal looks at a statement, it sees a vehicle to transport us through logical space—a springboard to other worlds.

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