A Conditional Theory of Permission and Obligation

Dean McHugh & Arsen Anisimov

Fifteenth International Tbilisi Symposium on Logic, Language and Computation

Thursday 11 September, 2025
Kutaisi, Georgia

The tamada should make another toast.

You are allowed to eat the leftover khinkali.

The standard theory

Hanson's Kripke semantics for deontic logic (1965)

v is accessible from w

just in case

"v is permitted with respect to w"

SEMANTICS FOR DEONTIC LOGIC

WILLIAM H. HANSON

1. *Introduction*

In this paper I consider some deontic logics that are counterparts, in an obvious sense, of the alethic modal logics M, the Brouwersche system, S4, and S5. In section 2 these logics are specified and some of their relations to other modal logics, both alethic and deontic, are indicated. In sections 3 and 4 I adapt the semantical techniques of Kripke [5] for alethic modal logic to deontic logic. Specifically, section 3 contains a model-theoretic definition of validity and section 4 a corresponding method of semantic tableaux for each deontic logic under consideration. In section 5 each deontic logic is proved consistent and complete with respect to the valid formulas of its corresponding model theory, and the method of semantic tableaux is shown to constitute a decision procedure for each. Modalities are discussed in section 6. It is shown that each deontic logic has the same number of nonequivalent modalities as its corresponding alethic logic.

Hanson (1965):

“*A* is obligatory at *w*”

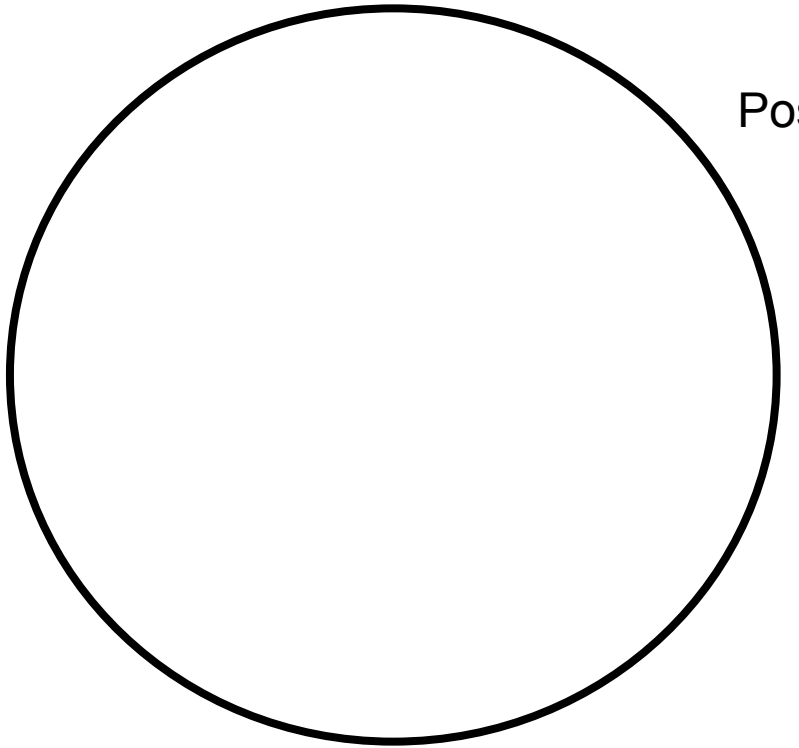
means

A is true at every permitted world from the perspective of *w*

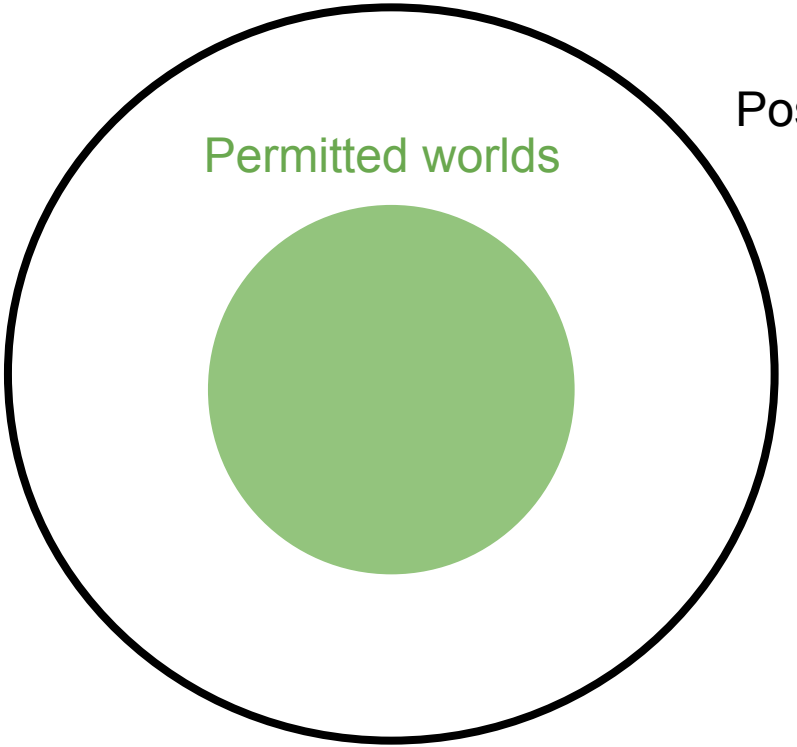
“*A* is obligatory at *w*”

means

A is true at every permitted world from the perspective of *w*

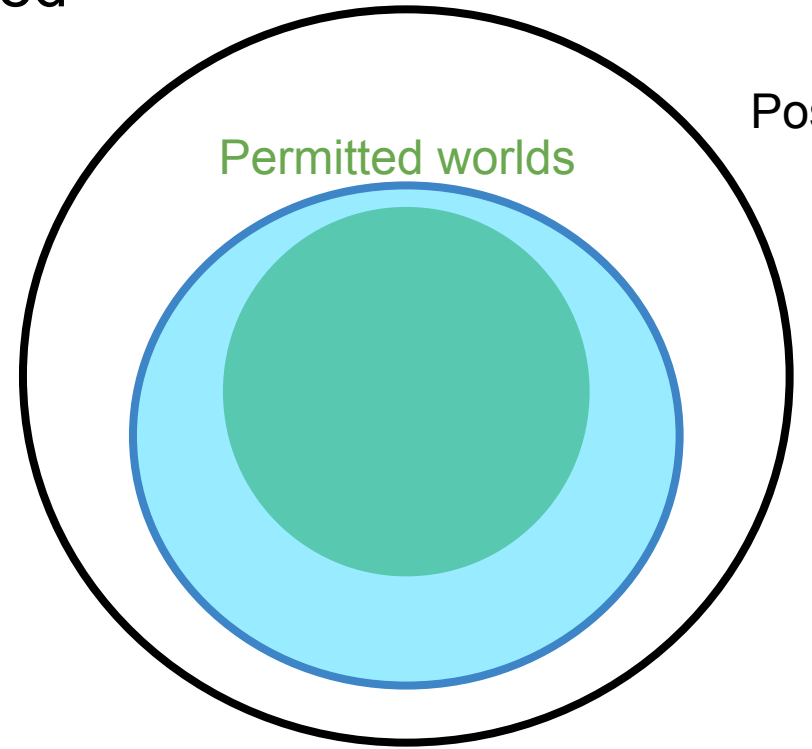


Possible worlds

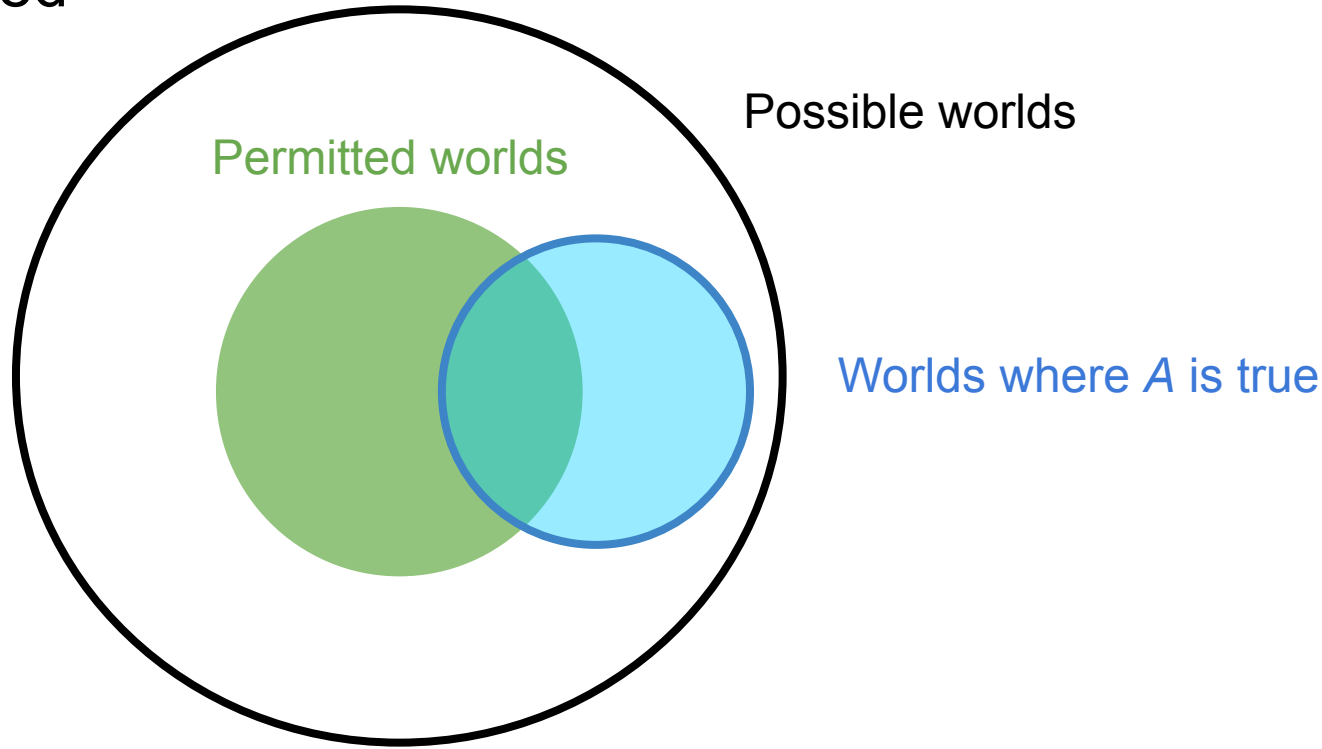


Possible worlds

A is obligated



A is permitted



A challenge

Holly M. Smith,
‘David Lewis’s Semantics of Deontic Logic’ (1977)

Two trains approach an intersection in a dense forest at full speed.

One is travelling North to South, the other East to West.

If both continue they will crash into one another.

Network Configuration	Deontic status
Train A goes, train B goes	Forbidden
Train A goes, train B stops	Permitted
Train A stops, train B goes	Permitted
Train A stops, train B stops	Permitted

Am I permitted to continue through the intersection?

Am I permitted to stop?

Am I permitted to continue through the intersection?

Am I permitted to stop?

Am I permitted to continue through the intersection?

Am I permitted to stop?

Am I permitted to continue through the intersection?

Am I permitted to stop?

Network Configuration	**Deontic status**
Train A goes, train B goes	Forbidden
Train A goes, train B stops	Permitted
Train A stops, train B goes	Permitted
Train A stops, train B stops	Permitted

Am I permitted to continue through the intersection?

Yes.

Am I permitted to stop?

Yes.

Am I permitted to continue through the intersection?

Yes.

Am I permitted to stop?

Yes.

Network Configuration	**Deontic status**
Train A goes, train B goes	Forbidden
Train A goes, train B stops	Permitted
Train A stops, train B goes	Permitted
Train A stops, train B stops	Permitted

You are permitted to continue through the intersection.

You are permitted to stop.

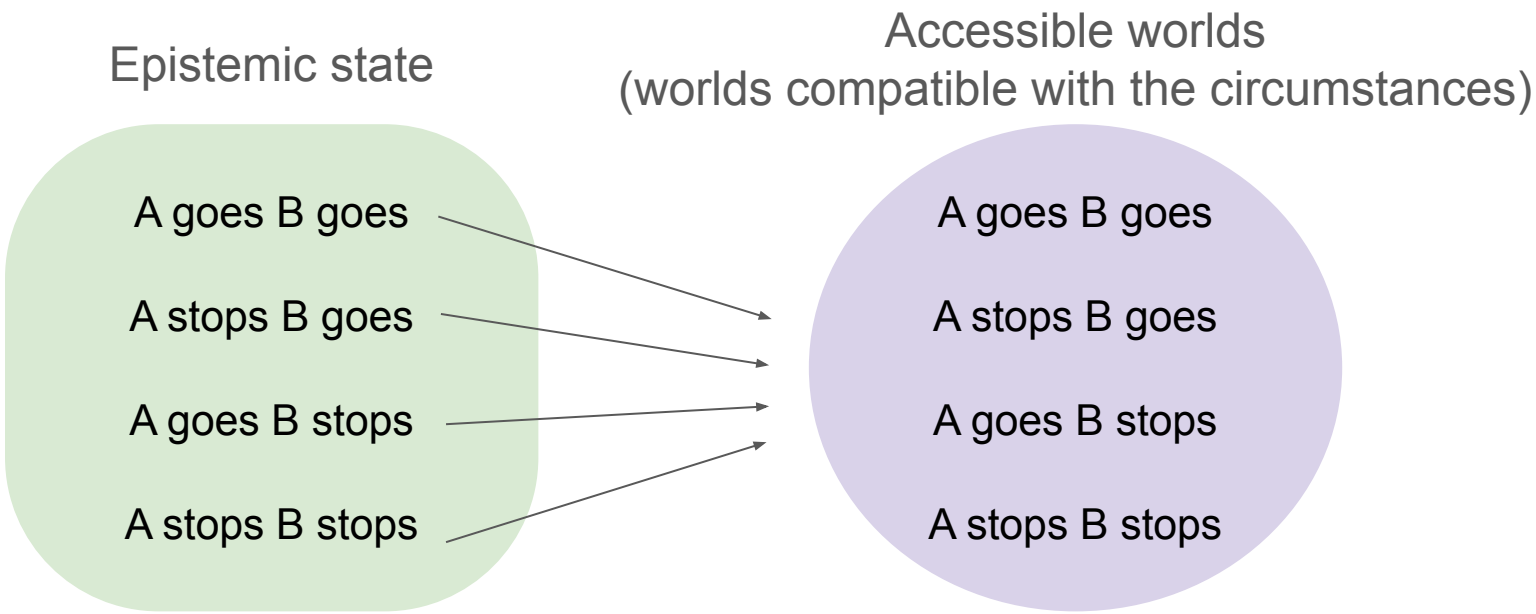
Epistemic state

A goes B goes

A stops B goes

A goes B stops

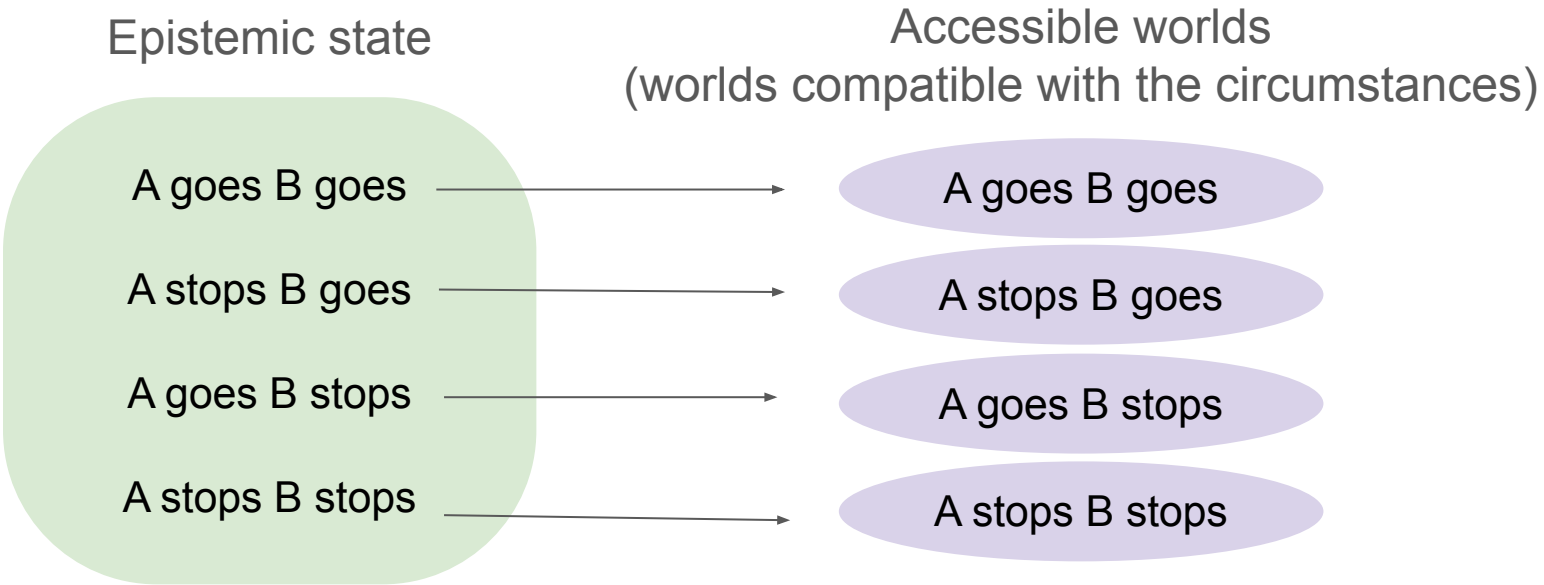
A stops B stops


Epistemic state
A goes B goes
A stops B goes
A goes B stops
A stops B stops
Accessible worlds
(worlds compatible with the circumstances)
A goes B goes
A stops B goes
A goes B stops
A stops B stops

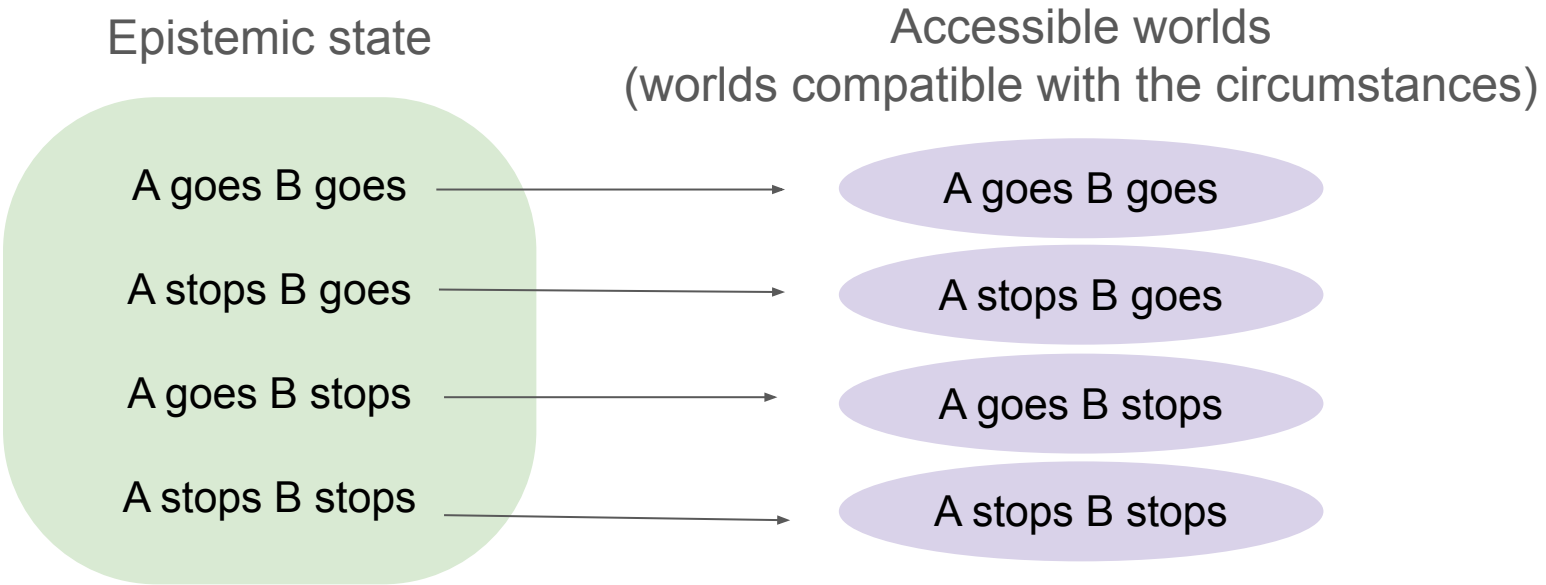
the kind of facts we take into account for circumstantial modality are a rather slippery matter

— Angelika Kratzer (1981),
‘The notional category of modality’

Why if the circumstances fix what each train will in fact do?



Why if the circumstances fix what each train will in fact do?



Upshot: Deontic reasoning is trivial
Something is permitted or obligated just in case it actually occurs

Conditional modality

May I open the window?

May I open the window?

Is there at least one allowed case in which I open the window?

May I open the window?

Would it be OK if I opened the window?

Leibniz:

what is permitted is “what is possible for a good person to do”

what is obligatory is “what is necessary for a good person to do”

Hilpinen’s paraphrase:

something is permitted just in case **if** someone does it, possibly, they are a good person

something is obligatory just in case **if** they don't do it, necessarily, they are not a good person

Leibniz, G.W. (1670). Elementa iuris naturalis [Elements of natural law]. *Philosophical papers and letters*.
Hilpinen, Risto (2017). Deontic logic. *The Blackwell guide to philosophical logic*.

Stig Kanger (1957)

“ought *A*” expresses $N(\neg A \supset \neg Q)$

- “*N* is the notion of analytic necessity”
- $\supset$ is the material conditional ($A \supset B$ is equivalent to $\neg A \vee B$)
- *Q* is “a constant stating what morality prescribes”

Kanger, Stig. New Foundations for Ethical Theory. *Deontic Logic: Introductory and Systematic Readings*.

Alan Ross Anderson (1958)

“to say that *p* is obligatory is to say that failure of *p* **leads to** a state-of-affairs … which is ‘bad’ ”

Anderson, Alan Ross (1958). A reduction of deontic logic to alethic modal logic. *Mind*.

Frank Jackson (1984)

“It ought to be that *A* out of {*A*, *A*1, …} iff what **would** be the case **were *A* true** is better than what **would** be the case **were *Ai*-true**, for all *i* ”

Jackson, Frank (1984), On the semantics and logic of obligation. *Mind*.

Tim Williamson (2007)

A is metaphysically necessary:

If *A* were false, in every case, a contradiction would obtain

A is metaphysically possible:

If *A* were true, in some case, no contradiction would obtain

Williamson, Timothy (2007). *The Philosophy of Philosophy*.

Permission:

Tabe-temo ii.

eat even if good

Literally: “It is good even if you eat.” = “You may eat.”

Obligation:

Tabenakere-ba ikenai/ dame da.

eat Neg if can go Neg no good is

Literally: “It is not good if you don’t eat” = “You must eat.”

— Norito Akatsuka (1992), ‘Japanese modals are conditionals’

Let *f* be a conditional selection function

- It takes a statement and a world and returns a set of worlds
- *f*(*A*, *w*) contains the worlds that result from supposing “if *A*…” at *w*

Let f be a conditional selection function

- It takes a statement and a world and returns a set of worlds
- $f(A, w)$ contains the worlds that result from supposing “if A…” at w

$$A \Diamond\!\!\rightarrow C \text{ is true at } w \quad \text{iff} \quad C \text{ is true at some world in } f(A, w)$$

$$A \Box\!\!\rightarrow C \text{ is true at } w \quad \text{iff} \quad C \text{ is true at every world in } f(A, w)$$

Let *f* be a conditional selection function

- It takes a statement and a world and returns a set of worlds
- *f*(*A*, *w*) contains the worlds that result from supposing “if *A*…” at *w*

$$A \Diamond\!\!\rightarrow C \text{ is true at } w \quad \text{iff} \quad C \text{ is true at some world in } f(A, w)$$

$$A \Box\!\!\rightarrow C \text{ is true at } w \quad \text{iff} \quad C \text{ is true at every world in } f(A, w)$$

Let *good* state that the relevant ideals are met.

$$\Diamond A \quad \equiv \quad A \Diamond\!\!\rightarrow good$$

$$\Box A \quad \equiv \quad \neg A \Box\!\!\rightarrow \neg good$$

Train A is permitted to go

Train B is permitted to go

If train A went, …

If train B went, …

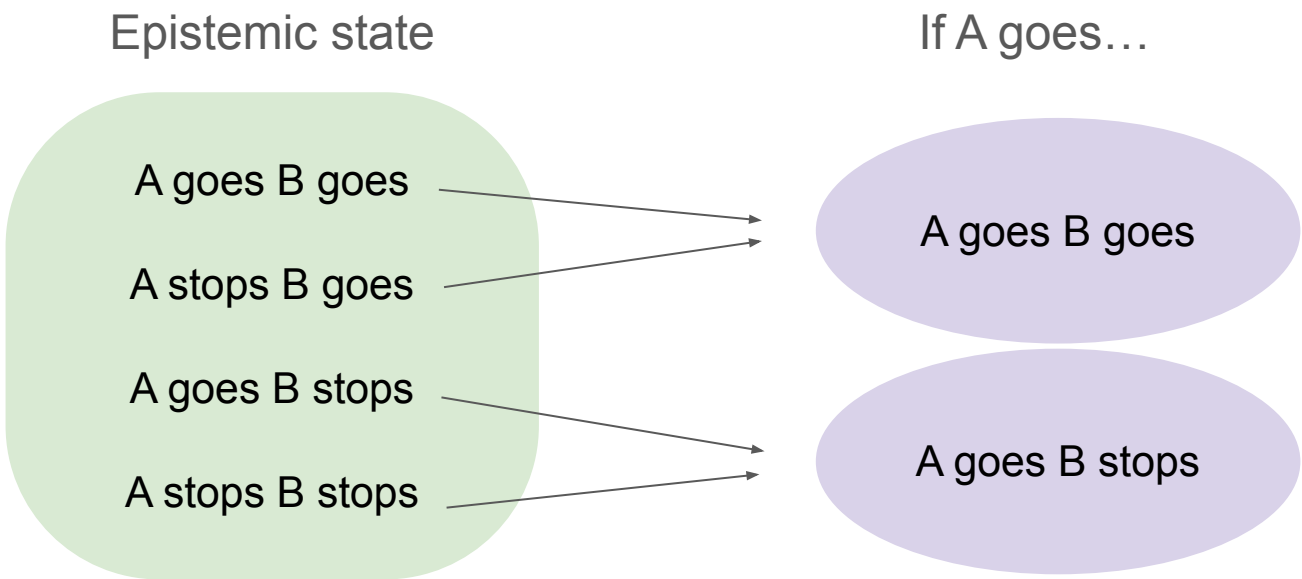
Epistemic state

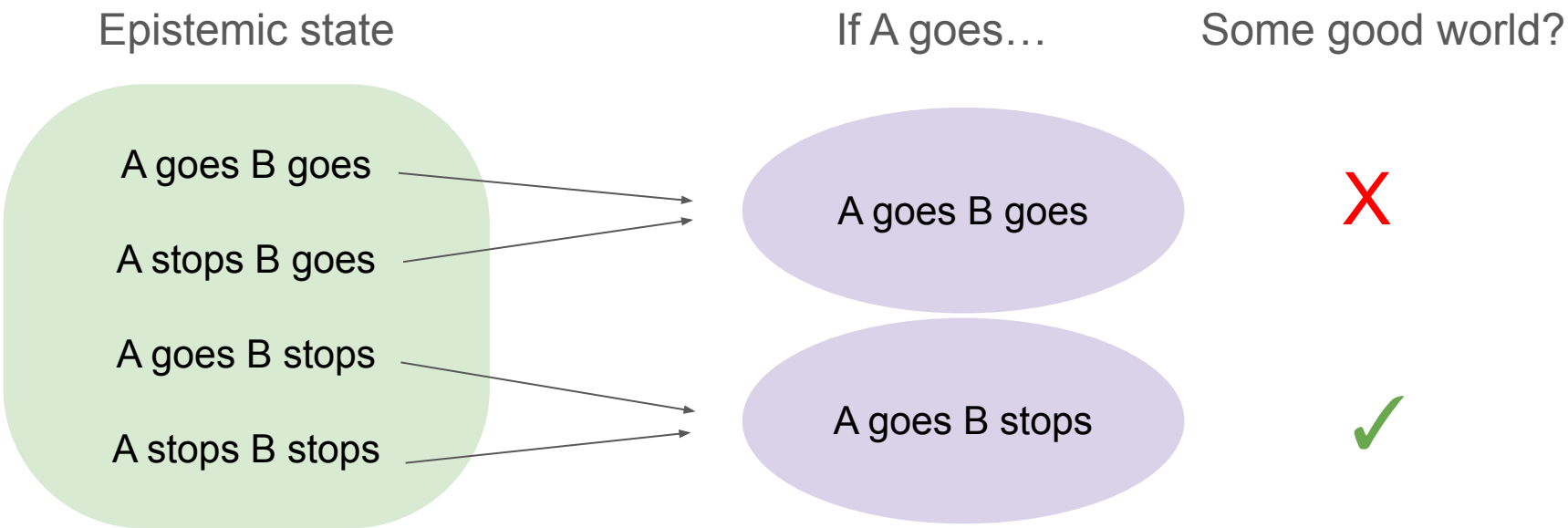
A goes B goes

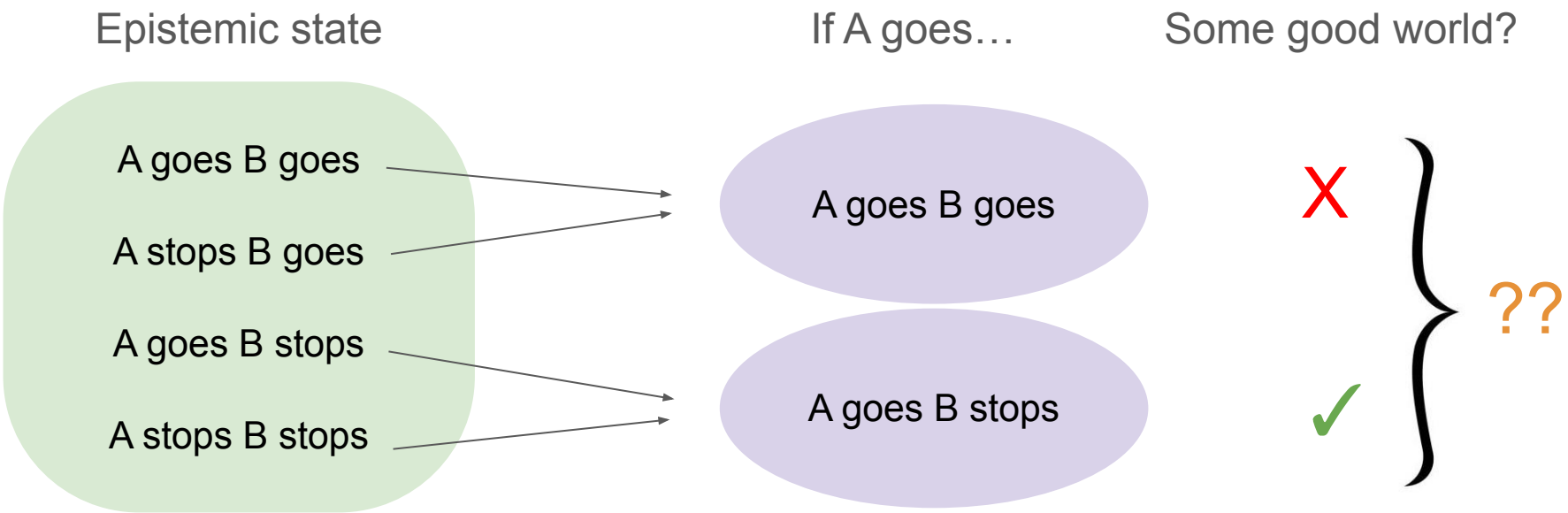
A stops B goes

A goes B stops

A stops B stops


Epistemic state
If A goes…
A goes B goes
A stops B goes
A goes B stops
A stops B stops
A goes B goes
A goes B stops


Epistemic state
If A goes…
Some good world?
A goes B goes
A stops B goes
A goes B stops
A stops B stops
A goes B goes
A goes B stops
X
✓


Epistemic state
If A goes…
Some good world?
A goes B goes
A stops B goes
A goes B stops
A stops B stops
A goes B goes
A goes B stops
X
✓
??

Prediction: “Train A is permitted to go” is unassertable

Its truth depends on what train B does

Vice versa for train B


NATIONAL ASSEMBLY
OF QUÉBEC
FORTY-SECOND LEGISLATURE
FIRST SESSION
Bill 96
An Act respecting French, the official and common language of Québec

Alice and Bob are colleagues in Quebec

They know both French and English

Bill 96 requires them to speak French at work

They choose to continue to speak English at work

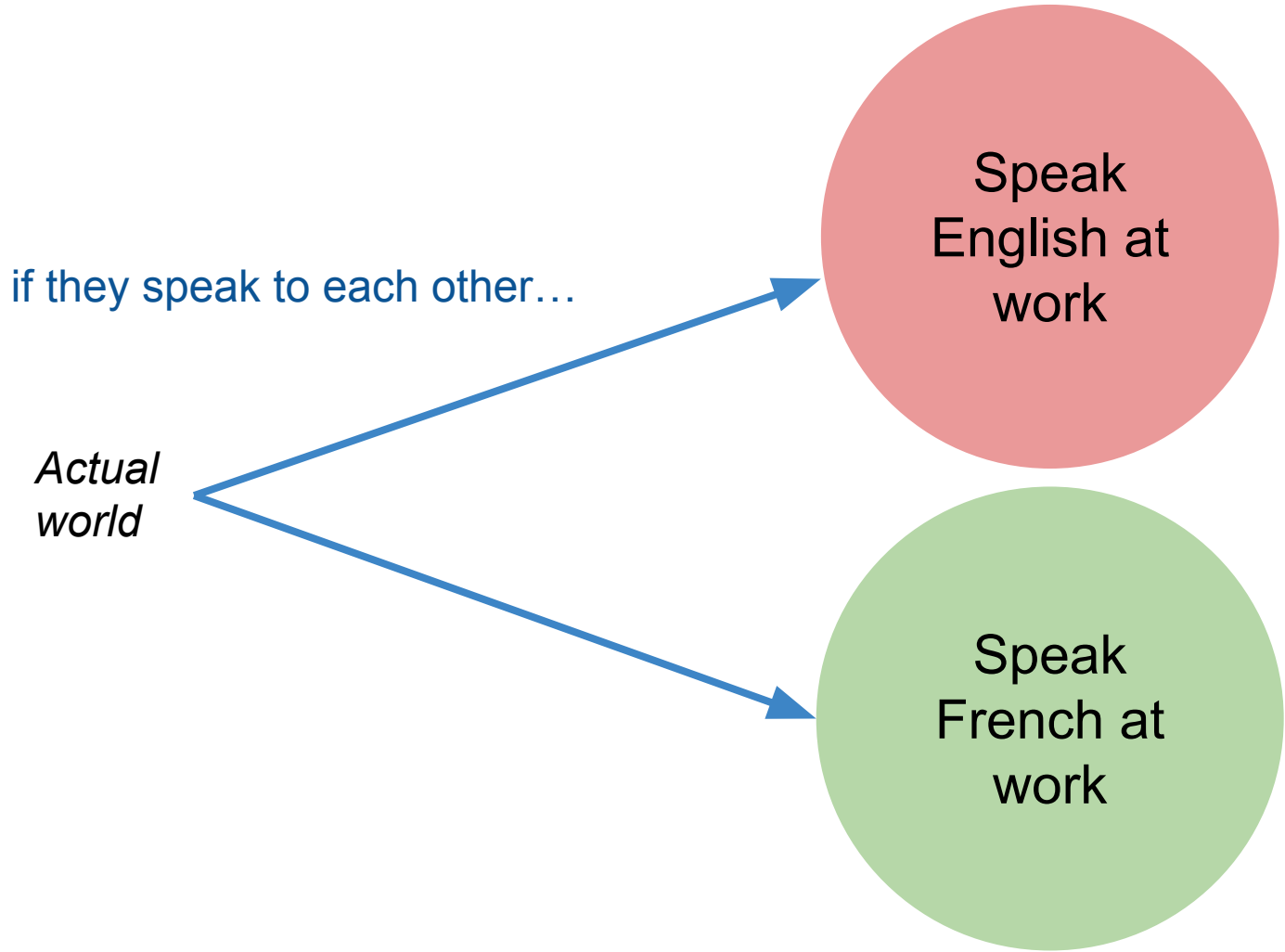
Alice and Bob are colleagues in Quebec

They know both French and English

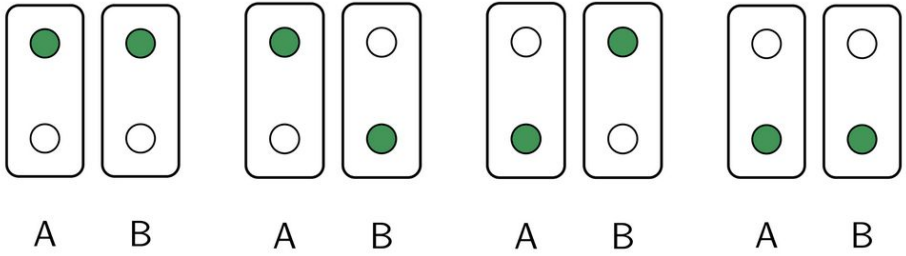
Bill 96 requires them to speak French at work

They choose to continue to speak English at work

Does Bill 96 allow Alice and Bob to talk to each other at work?


if they speak to each other…
Actual world
Speak English at work
Speak French at work

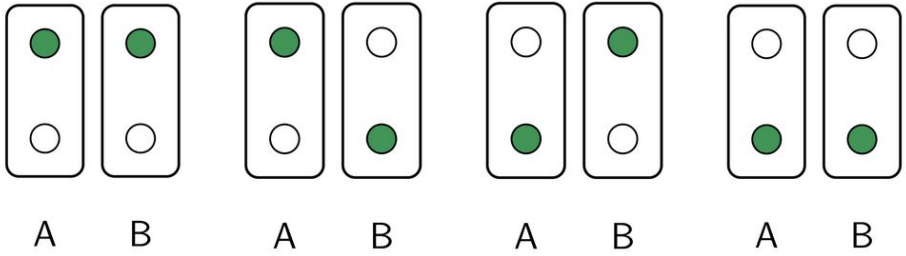
Our experiment



A power plant contains a control panel to manage the flow of electricity. The control panel contains two switches, A and B. Each switch is either up or down. The picture above displays the four possible configurations of the switches, where a green circle indicates the position of each switch. In the power plant, there are rules governing the position of the switches. The rules are given in the table below.

Configuration of the switches	**Status**
A is up, B is up	Allowed
A is up, B is down	Allowed
A is down, B is up	Forbidden
A is down, B is down	Allowed

Show a sentence



A power plant contains a control panel to manage the flow of electricity. The control panel contains two switches, A and B. Each switch is either up or down. The picture above displays the four possible configurations of the switches, where a green circle indicates the position of each switch. In the power plant, there are rules governing the position of the switches. The rules are given in the table below.

Configuration of the switches	**Status**
A is up, B is up	Allowed
A is up, B is down	Allowed
A is down, B is up	Forbidden
A is down, B is down	Allowed

The switches are allowed to be down at the same time.

This sentence is ...

○ True ○ Indeterminate ○ False

The contrast between two conditions:
dependent and independent

Dependent condition, the train scenario

Two trains are approaching an intersection. One is travelling north to south, the other east to west. Each train driver will either apply the brakes or continue going. If both continue, they will crash into each other. The rules governing the train network are given in the table below.

Dependent condition, the train scenario

Two trains are approaching an intersection. One is travelling north to south, the other east to west. Each train driver will either apply the brakes or continue going. If both continue, they will crash into each other. The rules governing the train network are given in the table below.

Possibilities for the train network	**Status**
Train A goes, train B goes	Forbidden
Train A goes, train B stops	Allowed
Train A stops, train B goes	Allowed
Train A stops, train B stops	Allowed

Dependent condition, the train scenario

Possibilities for the train network	Status
Train A goes, train B goes	Forbidden
Train A goes, train B stops	Allowed
Train A stops, train B goes	Allowed
Train A stops, train B stops	Allowed

Train A is allowed to go through the intersection.

— Is it true, false, or indeterminate?

Dependent condition, the train scenario

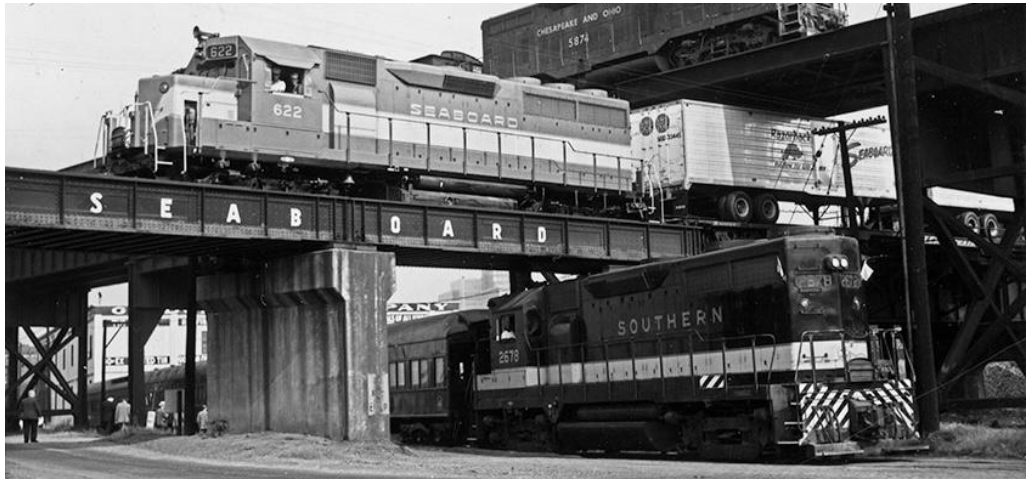
Possibilities for the train network	Status
Train A goes, train B goes	Forbidden
Train A goes, train B stops	Allowed
Train A stops, train B goes	Allowed
Train A stops, train B stops	Allowed

Train A is allowed to go through the intersection.

— Well, I don’t know, it depends.

Independent condition, the train scenario

Two trains are travelling on the train network. There is a lower track and an upper track. One is travelling north to south, the other east to west. Each train driver will either apply the brakes or continue going. There is no danger of the trains crashing into one another. The rules governing the train network are given in the table below.



Independent condition, the train scenario

Configuration of the network	Status
Train A goes, train B goes	Allowed
Train A goes, train B stops	Allowed
Train A stops, train B goes	Allowed
Train A stops, train B stops	Allowed

Train A is allowed to go through the intersection.

— Is it true, false, or indeterminate?

Independent condition, the train scenario

Configuration of the network	Status
Train A goes, train B goes	Allowed
Train A goes, train B stops	Allowed
Train A stops, train B goes	Allowed
Train A stops, train B stops	Allowed

Train A is allowed to go through the intersection.

— True.

Predictions

Dependent condition

Indeterminate

The sentence can be *true* and can be *false* under different conditions.

percentage of true responses → **0**

Independent condition

True

There are no factors prohibiting the test sentence to be *true*.

percentage of true responses → **100**

Participants

109 participants (119 before the exclusion) recruited with Prolific.

Native English speakers.

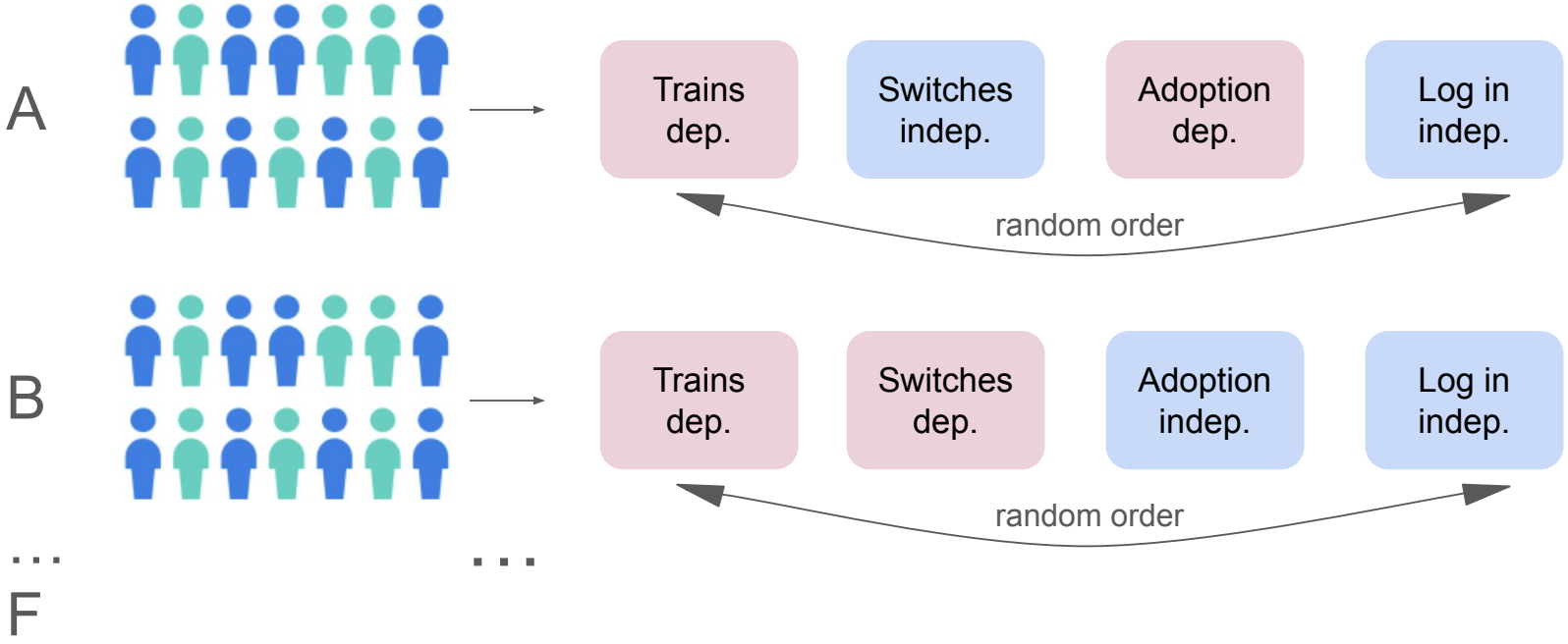
If people answered *Indeterminate*, to any of the questions, they were asked:

- I strongly feel that there is no right answer.
- I just don't know; I am not sure what the answer is.

Indeterminate answers which was followed by *I just don’t know* were excluded.

Mixed design

Each participant saw 4 of the scenarios: 2 dependent and 2 independent.
The order of the scenarios was random.



Methods

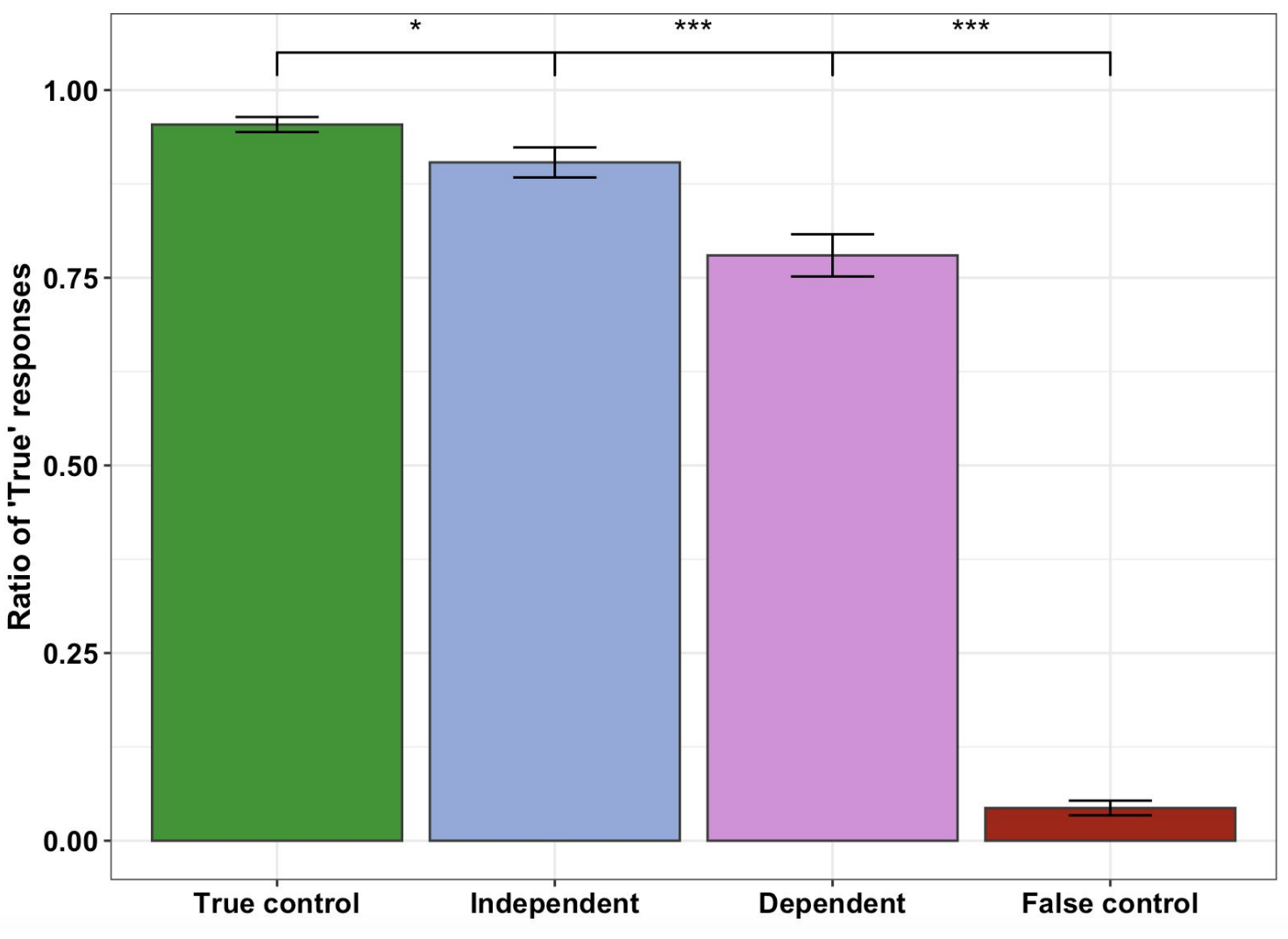
Binomial Generalized Linear Mixed-Effects Model

Independent variable: condition.
Dependent variable: binary response (1 — True, 0 — Indeterminate / False).

Random factors: scenario, participant .

Multiple Comparisons of Means: Tukey Contrasts

R packages: lme4, multcomp.


*
***
***
1.00
0.75
0.50
0.25
0.00
Ratio of 'True' responses
True control
Independent
Dependent
False control

The last test item at the end of each block

7 questions per a scenario (1 test item, controls, filters)
+
1 item, always at the end of the scenario block

There is at least one allowed possibility in which {A happens}.

There is at least one allowed possibility in which train A goes through the intersection.
The expected answer: *True*.

The last test item at the end of each block

There is at least one allowed possibility in which {A happens}.

Dependent condition: 94.4% of *True*

Independent condition: 85.6% of *True*

The last test item at the end of each block

There is at least one allowed possibility in which {A happens}.

Dependent condition: 94.4% of *True* No implicature violation (exactly 1 poss.)

Independent condition: 85.6% of *True* Implicature violation

Possibilities for the train network	**Status**
Train A goes, train B goes	Forbidden
Train A goes, train B stops	Allowed
Train A stops, train B goes	Allowed
Train A stops, train B stops	Allowed

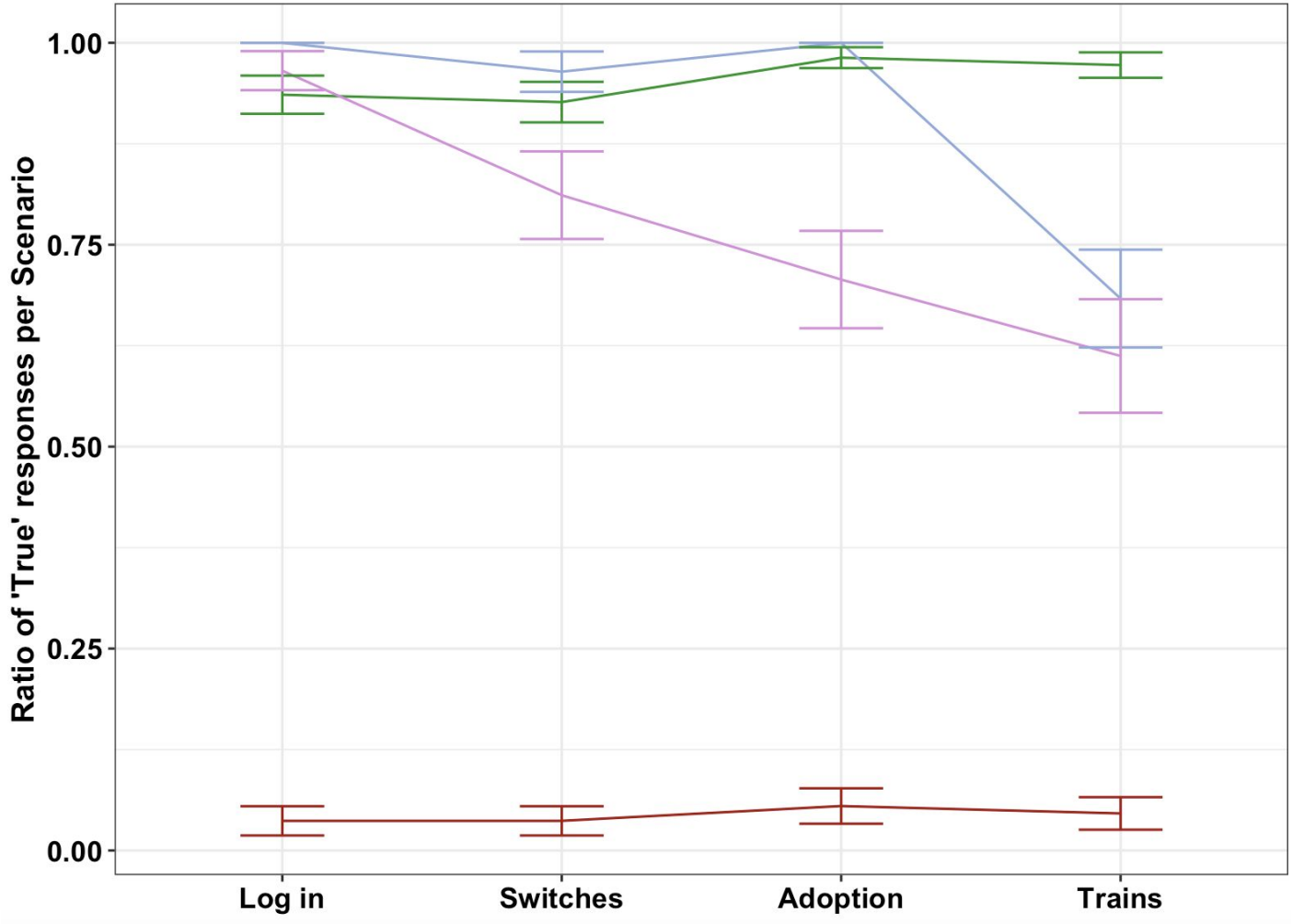
The last test item at the end of each block

There is at least one allowed possibility in which {A happens}.

Dependent condition: 94.4% of *True* No implicature violation (exactly 1 poss.)

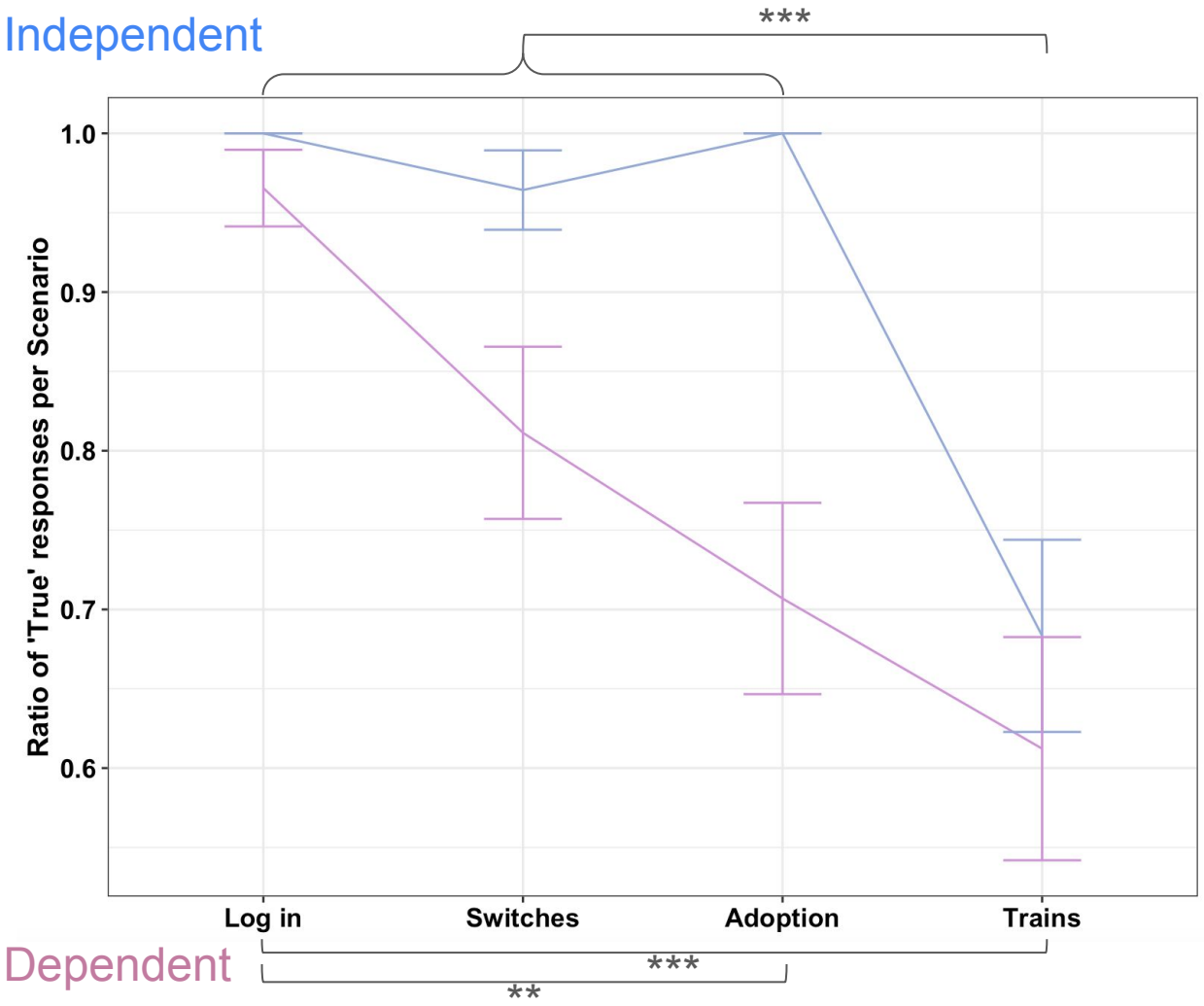
Independent condition: 85.6% of *True* Implicature violation

Configuration of the network	**Status**
Train A goes, train B goes	Allowed
Train A goes, train B stops	Allowed
Train A stops, train B goes	Allowed
Train A stops, train B stops	Allowed



True control

False control



Differences between the scenarios

P-values

*** < 0.0001

** < 0.001

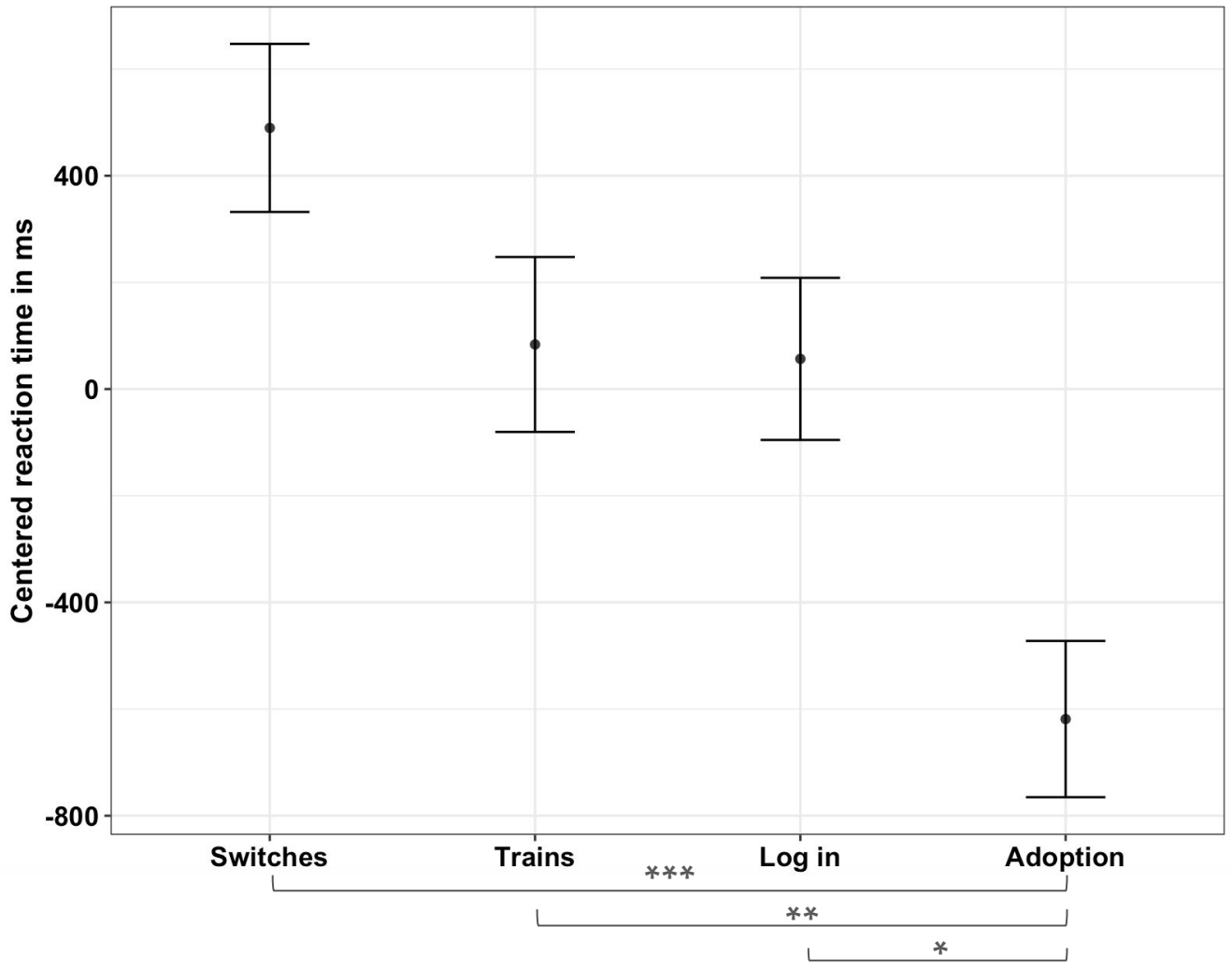
Billy and Suzy can decide freely what to do.

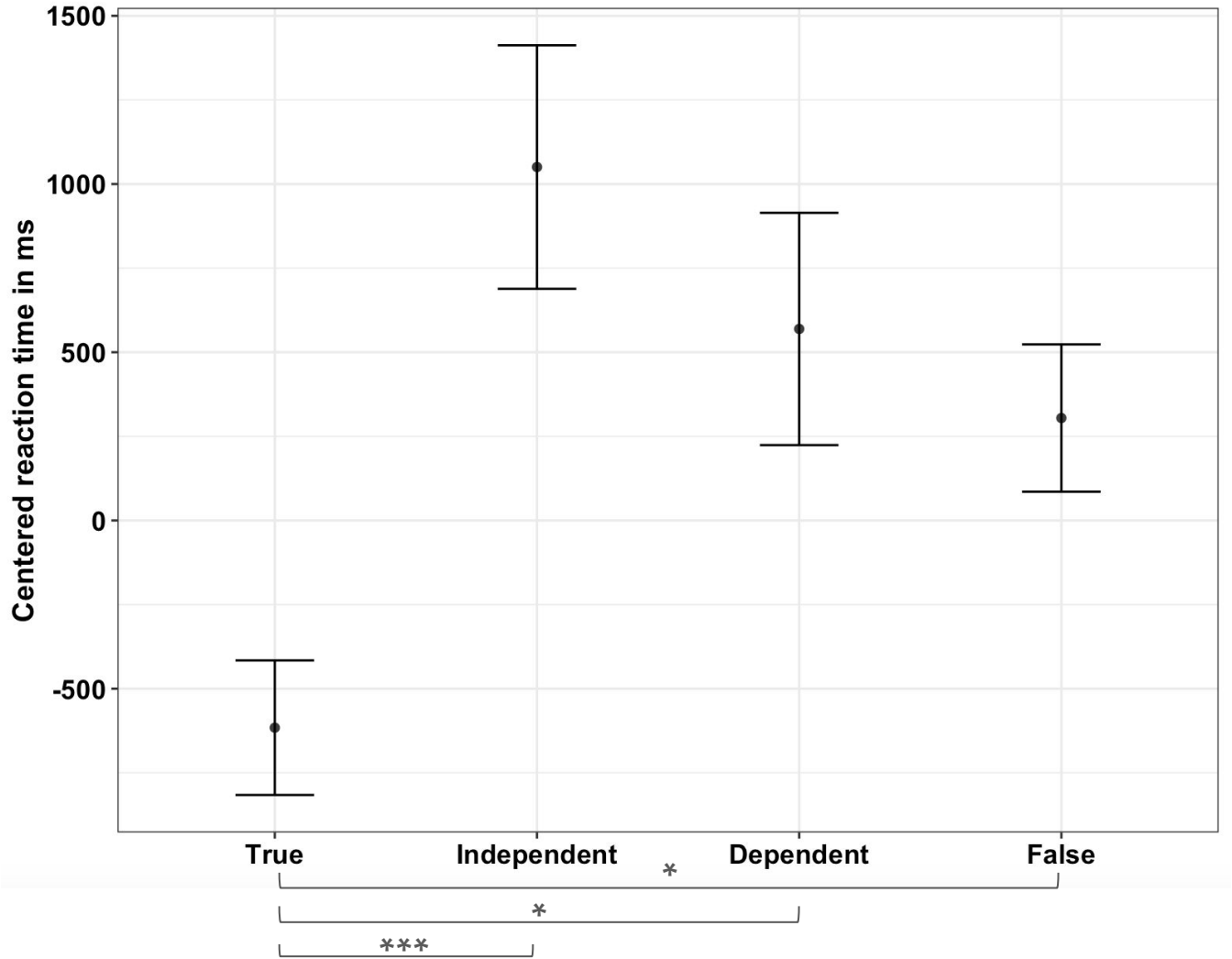
Billy and Suzy are prohibited from logging in simultaneously because the parents will be charged extra costs by their internet provider.

Possibilities for the computer network	**Status**
Both Billy and Suzy are playing online games	Allowed
Billy is playing online games and Suzy is not	Allowed
Suzy is playing online games and Billy is not	Allowed
Neither Billy nor Suzy are playing online games	Allowed

Possibilities for the computer network	**Status**
Both Billy and Suzy are playing online games	Forbidden
Billy is playing online games and Suzy is not	Allowed
Suzy is playing online games and Billy is not	Allowed
Neither Billy nor Suzy are playing online games	Allowed

Billy is allowed to log into the online game network.


Centered reaction time in ms
400
0
-400
-800
Switches
Trains
Log in
Adoption
***
**
*


1500
1000
500
0
-500
Centered reaction time in ms
True
Independent
Dependent
False
*
*
***

Discussion

Despite the contrast between dependent (78%) and the independent (90.3%) conditions, the acceptance rate of the dependent condition was high.

Probably, there were confounding factors:

- “Maybe there are not only two trains on the network”.
- Actors who can communicate (children who want to log in) vs. who cannot (train drivers).

Summary

- The standard theory of permission predicts “Train A is permitted to go” and “Train B is permitted to go” to both be **true**
- The conditional theory predicts them to be **indeterminate**
- We ran an experiment to test this using four scenarios, comparing dependent and independent conditions
- Overall, our test items “X is allowed” were judged significantly lower in the dependent condition than the independent condition
- We observed significant variability between scenarios

Thank you for listening!

Journal of Semantics, 28, 2011: 171–210
doi:10.1093/jos/ffq017
Advance Access publication October 25, 2010

Counterfactual-Style Revisions in the Semantics of Deontic Modals

ANA ARREGUI
University of Ottawa

You’re a guest at your best friend’s wedding. Before the dinner, your friend tells you:

“My brothers-in-law are very shy. I’d like to make sure someone friendly has dinner with them. Could you sit with them?”

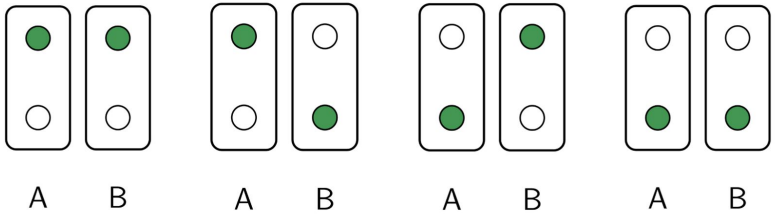
You arrive and see two brothers are sitting at one table, and other (Jim) sitting alone. You sit with the two.

Your friend comes over and says,

“You should be sitting with Jim.”

Your friend comes over and says,

“You should be sitting with John Jack, and Jim.”



A power plant contains a control panel to manage the flow of electricity. The control panel contains two switches, A and B. Each switch is either up or down. The picture above displays the four possible configurations of the switches, where a green circle indicates the position of each switch.
In the power plant, there are rules governing the position of the switches. The rules are given in the table below.

Configuration of the switches	**Status**
A is up, B is up	Allowed
A is up, B is down	Allowed
A is down, B is up	Forbidden
A is down, B is down	Allowed

Configuration of the switches	**Status**
A is up, B is up	Allowed
A is up, B is down	Forbidden
A is down, B is up	Forbidden
A is down, B is down	Allowed

Switch A is allowed to be up.

Alice and Bob are two children in an orphanage. A couple are considering adopting. They are not sure whether they will adopt any children, but visit the orphanage to inquire. The possibilities for adoption are given in the table below.

Adoption possibilities for the couple	**Status**
Adopt both Alice and Bob	Allowed
Adopt Alice and do not adopt Bob	Allowed
Adopt Bob and do not adopt Alice	Allowed
Adopt neither Alice nor Bob	Allowed

Alice and Bob are two children in an orphanage. They are siblings. The orphanage where Alice and Bob are staying has a rule that siblings must not be separated. Potential adoptive parents can only adopt both siblings together.

Adoption possibilities for the couple	**Status**
Adopt both Alice and Bob	Allowed
Adopt Alice and do not adopt Bob	Forbidden
Adopt Bob and do not adopt Alice	Forbidden
Adopt neither Alice nor Bob	Allowed

The couple are allowed to adopt Alice.

Billy and Suzy are two children at home. Billy has his computer turned on downstairs, and Suzy has her computer turned on upstairs. Each child can log into an online network to play games or do offline activities, such as drawing digital images or reading ebooks. The parents are happy for each child to choose whether they want to log into the online network to play games or to do offline activities.

Billy and Suzy are two children at home. Billy has his computer turned on downstairs, and Suzy has her computer turned on upstairs. Each child can log into an online network to play games or do offline activities, such as drawing digital images or reading ebooks. The parents have a rule that Billy and Suzy are strictly forbidden from both logging into the online game network at the same time, since if they do, the network will exceed the number of permitted devices and the parents will be charged extra costs by their internet provider.

Possibilities for the computer network	**Status**
Both Billy and Suzy are playing online games	Allowed
Billy is playing online games and Suzy is not	Allowed
Suzy is playing online games and Billy is not	Allowed
Neither Billy nor Suzy are playing online games	Allowed

Possibilities for the computer network	**Status**
Both Billy and Suzy are playing online games	Forbidden
Billy is playing online games and Suzy is not	Allowed
Suzy is playing online games and Billy is not	Allowed
Neither Billy nor Suzy are playing online games	Allowed

Billy is allowed to log into the online game network.

An imperfection with the data collection: distribution among the groups

A	6
B	2
C	1
D	3
E	4
F	**46**

A	**53**
B	2
C	1
D	3
E	4
F	**46**

	switches	trains	adopt	log in
A	i	i	d	d
B	i	d	i	d
C	i	d	d	i
D	d	i	i	d
E	d	i	d	i
F	d	d	i	i

The reasons why Dean and Arsen are cuties

Reasons

Because we’re sweet

Because we’re handsome

Because we’re kind

Because we help each other

Because we feel 100% comfortable with each other

Because when I’m with you I’m home