Counterfactuals

Current Formal Models of Counterfactuals and Causation

Dean McHugh

Institute of Logic, Language and Computation
University of Amsterdam

NASSLLI 2025, University of Washington, Seattle

Plan

1. Challenges
   - Challenges to the intuitive idea of similarity
   - Strengthening with a possibility
   - Reciprocity
   - Static counterexamples to Reciprocity
   - Substitution
2. Causal Sufficiency
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency
3. An aboutness approach to counterfactuals
   - Invalidating substitution
   - Invalidating conjunctive sufficiency

Plan

1. Challenges
   - Challenges to the intuitive idea of similarity
   - Strengthening with a possibility
   - Reciprocity
   - Static counterexamples to Reciprocity
   - Substitution
2. Causal Sufficiency
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency
3. An aboutness approach to counterfactuals
   - Invalidating substitution
   - Invalidating conjunctive sufficiency

Plan

Fine's Nixon example

Lewis (1973): the similarity order represents overall similarity, measuring similarity across the world as a whole.

Fine's Nixon example

Lewis (1973): the similarity order represents overall similarity, measuring similarity across the world as a whole.

> *The counterfactual 'If Nixon had pressed the button there would have been a nuclear holocaust' is true or can be imagined to be so. Now suppose that there never will be a nuclear holocaust. Then that counterfactual is, on Lewis' analysis, very likely false. For given any world in which antecedent and consequent are both true it will be easy to imagine a closer world in which the antecedent is true but the consequent false. For we need only imagine a change that prevents the holocaust but that does not require such a great divergence from reality.*

(Fine 1975, p. 452)

Fine's Nixon example

Lewis (1973): the similarity order represents overall similarity, measuring similarity across the world as a whole.

> *The counterfactual 'If Nixon had pressed the button there would have been a nuclear holocaust' is true or can be imagined to be so. Now suppose that there never will be a nuclear holocaust. Then that counterfactual is, on Lewis' analysis, very likely false. For given any world in which antecedent and consequent are both true it will be easy to imagine a closer world in which the antecedent is true but the consequent false. For we need only imagine a change that prevents the holocaust but that does not require such a great divergence from reality.*
>
> (Fine 1975, p. 452)

Some respond with a time-dependent notion of similarity (Bennett 1984, 2003, Arregui 2005, Ippolito 2013, Khoo 2017)

Lewis (1979) tries to derive the asymmetry of time from the asymmetry of counterfactual dependence: later affairs depend counterfactually on earlier ones, not vice versa

Elga (2001) argues that this proposal fails from the perspective of statistical mechanics

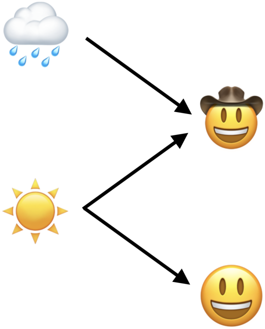
Tichý's (1976) puzzle

In bad weather, Jones always takes his hat.

In fine weather, there's a 50:50 chance Jones takes his hat.

Actual context: The weather is bad and Jones is wearing his hat.

(1) If the weather had been fine, Jones would have been wearing his hat.



Plan

Strengthening with a Possibility (aka rational monotonicity):

$$\frac{A > C \qquad A \mathbin{\Diamond\!\!\rightarrow} B}{(A \wedge B) > C}$$

Strengthening with a Possibility (aka rational monotonicity):

$$\frac{A > C \qquad A \mathbin{\Diamond\!\!\rightarrow} B}{(A \wedge B) > C}$$

This is valid on both Stalnaker's semantics and Lewis' (1973) sphere semantics for counterfactuals.

Strengthening with a Possibility (aka rational monotonicity):

$$\frac{A > C \qquad A \mathbin{\Diamond\!\!\rightarrow} B}{(A \wedge B) > C}$$

This is valid on both Stalnaker's semantics and Lewis' (1973) sphere semantics for counterfactuals.

On Lewis's ordering semantics,
it corresponds to the order being almost connected.

$\leq_w$ is *almost connected* just in case for all worlds w, x, y, z,

$$\text{if } x <_w z \text{ then } x <_w y \text{ or } y <_w z$$

where $<_w$ is the strict version of $\leq_w$:
$x <_w y$ just in case $x \leq_w y$ but not $y \leq_w x$

Counterexample to strengthening with a possibility

A counterexample was given by Ginsberg (1986, p. 50).
Here is one from Boylan and Schultheis (2017, 2021):

> *Alice, Billy, and Carol are playing a simple game of dice. Anyone who gets an odd number wins $10; anyone who gets even loses $10. The die rolls are, of course, independent. What Alice rolls has no effect on what Billy rolls and vice versa. Likewise for Alice and Carol as well as for Billy and Carol. Each player throws their dice. Alice gets odd; Billy gets even; Carol gets odd.*

(2) a. If Alice and Billy had thrown the same type of number, at least one person would still have won $10.

b. If Alice and Billy had thrown the same type of number, Alice, Billy, and Carol could have all thrown the same type of number.

c. If Alice, Billy, and Carol had all thrown the same type of number, at least one person would still have won $10.

A perhaps clearer variant of the counterexample

There are three switches, A, B, and C, connected to a light. Each switch can be up or down. The light is on just in case A and B are in the same position and C is down. Currently, A is down, B is up, and C is down.

(3) If A and B were in the same position, the light would be on.

(4) If A and B were in the same position, A, B and C might be in the same position.

(5) If A, B and C were in the same position, the light would be on.

Plan

Reciprocity (a.k.a. CSO)

$$\frac{A > B \qquad B > A \qquad B > C}{A > C}\ \text{Reciprocity}$$

Lewis's semantics validates Reciprocity

Proof.

For any world w and sentence A, let $w \models A$ denote that A is true at w. Pick any world w and suppose $A > B$, $B > A$ and $B > C$ are true at w. To show that $A > C$ is true at w, pick any $x \models A$. We have to show that there is a $y \models A$ such that $y \leq_w x$ and for all $z \leq_w y$, $z \models A \rightarrow C$, where $\rightarrow$ is the material conditional.

Since $w \models A > B$ and $x \models A$, there is a $v \models A$ such that $v \leq_w x$ and (i) for all $v' \leq_w v$, $v' \models A \rightarrow B$. Since $\leq_w$ is reflexive, $v \leq_w v$, so $v \models A \rightarrow B$. Thus $v \models B$.

Since $w \models B > A$ and $v \models B$, there is a $u \models B$ such that $u \leq_w v$ and (ii) for all $u' \leq_w u$, $u' \models B \rightarrow A$. Since $w \models B > C$ and $u \models B$, there is a $y \models B$ such that $y \leq_w u$ and (iii) for all $z \leq_w y$, $z \models B \rightarrow C$. Since $y \leq_w u$, by (ii), $y \models B \rightarrow A$. Then as $y \models B$, $y \models A$. And as $y \leq_w u \leq_w v \leq_w x$, by transitivity of $\leq_w$, $y \leq_w x$.

We show that $z \models A \rightarrow C$ for all $z \leq_w y$. Pick any $z \leq_w y$. Then $z \leq_w y \leq_w u \leq_w v$, so by transitivity of $\leq_w$, $z \leq_w v$. Then by (i), $z \models A \rightarrow B$. And since $z \leq_w y$, by (iii), $z \models B \rightarrow C$. Hence $z \models A \rightarrow C$. □

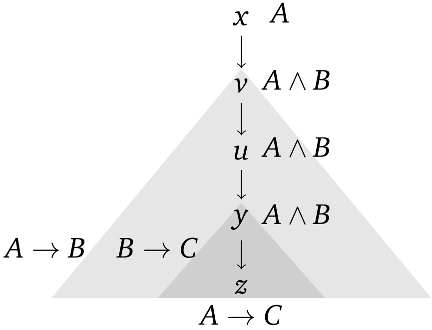


Figure: Illustrating the proof that Reciprocity is valid on Lewis's semantics.

Bacon's counterexample (Bacon 2013)

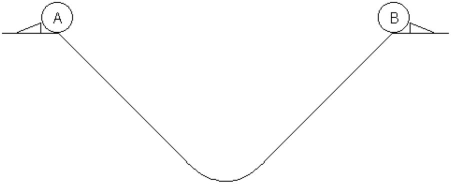


Figure: Bacon's counterexample to reciprocity.

(6) a. If A fell, B would fall.
b. If B fell, A would fall.
c. If A fell, the light would turn green.
d. If B fell, the light would turn red.

Bacon's counterexample (Bacon 2013)

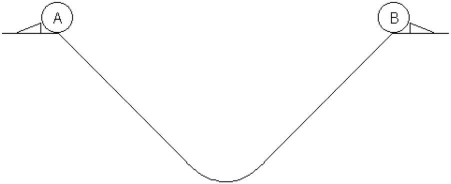


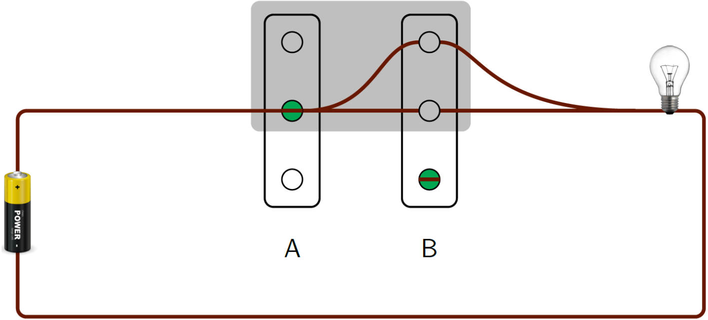
Figure: Bacon's counterexample to reciprocity.

(6) a. If A fell, B would fall.
b. If B fell, A would fall.
c. If A fell, the light would turn green.
d. If B fell, the light would turn red.

(7) a. If A had fallen, B would have fallen.
b. If B had fallen, A would have fallen.
c. If A had fallen, the light would have turned green.
d. If B had fallen, the light would have turned red.

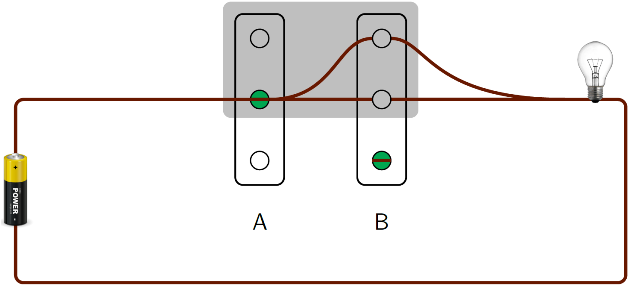
Plan

New experimental evidence against Reciprocity



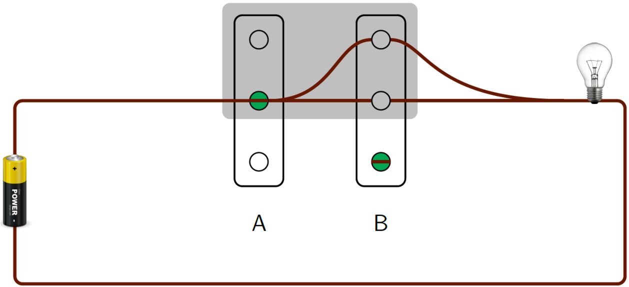
- There are two switches, A and B, are connected to a light.
- Each switch can be up, in the middle, or down.
- The light is on just in case A is in the middle and B is up or in the middle.
- A is in the middle and B is down, so the light is off.

Premise 1



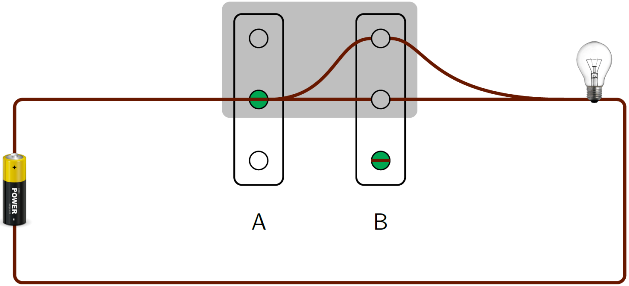
If switch B were in the shaded area,
both switches would be in the shaded area.

Premise 2



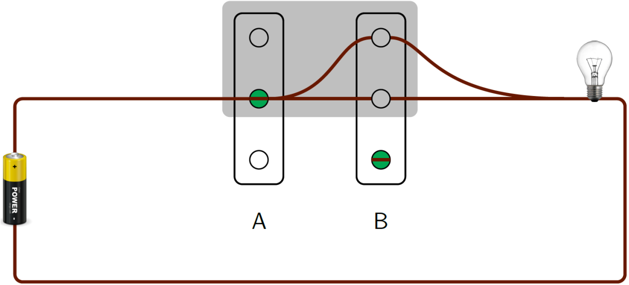
If both switches were in the shaded area,
switch B would be in the shaded area.

Premise 3



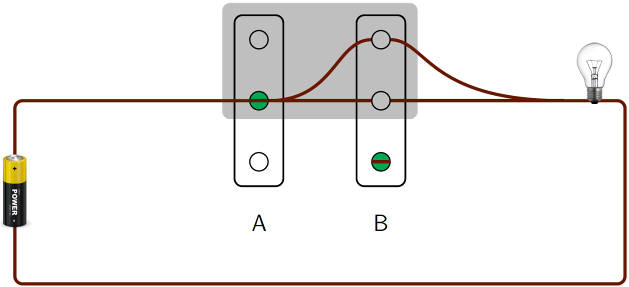
If switch B were in the shaded area,
the light would be on.

Conclusion



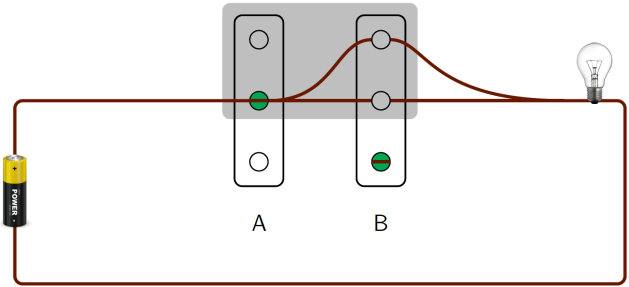
If both switches were in the shaded area,
the light would be on.

True control

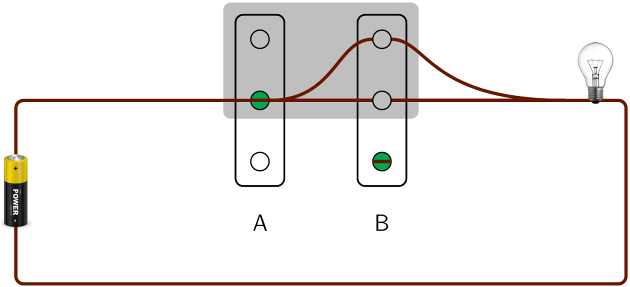


If both switches were in the shaded area,
switch B would be in the shaded area.

False control



If both switches were outside the shaded area, the light would be on.



T	If both switches were in the middle, the light would be on.	
P1	If switch B were in the shaded area, both switches would be in the shaded area.	$B > both$
P2	If both switches were in the shaded area, switch B would be in the shaded area.	$both > B$
P3	If switch B were in the shaded area, the light would be on.	$B > on$
C	If both switches were in the shaded area, the light would be on.	$both > on$
F	If both switches were outside the shaded area, the light would be on.	

Experimental design

- 80 native English speakers, recruited via Prolific
- Three sentence-picture verification tasks
- Following Romoli, Santorio, and Wittenberg (2022), for each sentence we asked whether it is true, false, or indeterminate.
  - If indeterminate: follow up whether they strongly feel that there is no correct answer or just do not know
  - We excluded the latter responses from the analysis.
- Participants understood the scenarios well
  - Mean accuracy of 89% on the filler items
  - Excluded from the statistical analysis two participants whose error rates on the fillers were above 30%.

Experiment available at
`https://www.tklochowicz.com/experiment_reciprocity`

Results

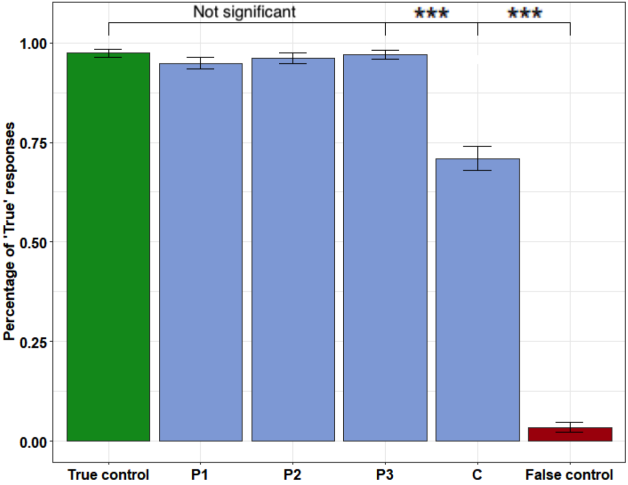


Figure: Percentage of 'True' responses. Error bars denote Standard Errors.

Plan

1. Challenges
   - Challenges to the intuitive idea of similarity
   - Strengthening with a possibility
   - Reciprocity
   - Static counterexamples to Reciprocity
   - Substitution
2. Causal Sufficiency
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency
3. An aboutness approach to counterfactuals
   - Invalidating substitution
   - Invalidating conjunctive sufficiency

Substitution

$$\frac{A \models B \qquad B \models A}{(A > C) \leftrightarrow (B > C)}\ \text{Substitution}$$

If A and B are logically equivalent, they are substitutable salva veritate in conditional antecedents.

Substitution

$$\frac{A \models B \qquad B \models A}{(A > C) \leftrightarrow (B > C)} \text{ Substitution}$$

If A and B are logically equivalent, they are substitutable salva veritate in conditional antecedents.

$$\frac{A \models B}{A > B} \text{ Entailment}$$

Given Entailment, Reciprocity implies Substitution.

$$\frac{\dfrac{A \models B}{A > B} \qquad \dfrac{B \models A}{B > A}}{(A > C) \leftrightarrow (B > C)} \text{ Reciprocity}$$

Infinitely many: a counterexample to Substitution

From Fine (2014, p. 328): There is one poison apple and infinitely many safe apples.

$\cdots \infty$

Infinitely many: a counterexample to Substitution

From Fine (2014, p. 328): There is one poison apple and infinitely many safe apples.

$\cdots \infty$

(8) If Eve ate infinitely many of the green apples,
she would be fine.

Infinitely many: a counterexample to Substitution

From Fine (2014, p. 328): There is one poison apple and infinitely many safe apples.

$\cdots \infty$

(8) If Eve ate infinitely many of the green apples,
she would be fine.

(9) If Eve ate infinitely many of the apples,
she would be fine.

Infinitely many: a counterexample to Substitution

From Fine (2014, p. 328): There is one poison apple and infinitely many safe apples.

$\cdots \infty$

(8) If Eve ate infinitely many of the green apples,
she would be fine.

(9) If Eve ate infinitely many of the apples,
she would be fine.

The antecedents are logically equivalent.

Eve eats infinitely many of the green apples just in case she eats infinitely many of the apples.

Infinitely many: a counterexample to substitution

From Goodsell (2022):
There is a 1 Euro coin and infinitely many pennies, all facing tails.
The light is green just in case the 1 Euro coin is facing tails.

(10) If infinitely many of the coins were facing heads,
the light would still be green.

(11) If infinitely many of the pennies were facing heads,
the light would still be green.

Plan

1. Challenges
   - Challenges to the intuitive idea of similarity
   - Strengthening with a possibility
   - Reciprocity
   - Static counterexamples to Reciprocity
   - Substitution
2. Causal Sufficiency
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency
3. An aboutness approach to counterfactuals
   - Invalidating substitution
   - Invalidating conjunctive sufficiency

Plan

The need for sufficiency

(12) a. Sue was allowed into the bar because she's over 21.
b. Sue was allowed into the bar because she's over 16.

(13) a. The fact that Sue is over 21 caused the bouncer to let her in.
b. The fact that Sue is over 16 caused the bouncer to let her in.

The need for sufficiency

(14) *The radio spontaneously starts playing music.*

A: Why did the radio turn on?

B: I have no idea. I didn't touch it.

A: I see it's plugged in, and it needs to be plugged in to turn on.

B: Right, but I still have no idea why it started playing.

The need for sufficiency with reasons

Sami and Jan are fun on their own, but always fight when together. A heard that they are both attending a party and therefore decids to skip it.

(15) a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.
b. I'm skipping the party for one reason: because Sami and Jan are going.

(16) a. The reasons why I'm skipping the party are that Sami is going and that Jan is going.
b. The reason why I'm skipping the party is that Sami and Jan are going.

The need for sufficiency with reasons

Sami and Jan are fun on their own, but always fight when together. A heard that they are both attending a party and therefore decids to skip it.

(15) a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.
b. I'm skipping the party for one reason: because Sami and Jan are going.

(16) a. The reasons why I'm skipping the party are that Sami is going and that Jan is going.
b. The reason why I'm skipping the party is that Sami and Jan are going.

My intuitive judgement: the (a)-sentences are odd, the (b)-sentences are fine.

The need for sufficiency with reasons

Sami and Jan are each miserable people. Even one of them going to a party is enough to make it a dull event.

(17) a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.

b. I'm skipping the party for one reason: because Sami and Jan are going.

(18) a. The reasons I'm skipping the party are that Sami is going and that Jan is going.

b. The reason I'm skipping the party is that Sami and Jan are going.

The need for sufficiency with reasons

Sami and Jan are each miserable people. Even one of them going to a party is enough to make it a dull event.

(17)
a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.
b. I'm skipping the party for one reason: because Sami and Jan are going.

(18)
a. The reasons I'm skipping the party are that Sami is going and that Jan is going.
b. The reason I'm skipping the party is that Sami and Jan are going.

My intuitive judgement: the (a)- and (b)-sentences are both fine.

The sufficiency requirement

- *E because C* $\Rightarrow$ C is sufficient for E.
- *C cause E* $\Rightarrow$ C is sufficient for E.

What does it mean for C to be sufficient for E?

Sufficiency is not logical entailment

(19) a. My laptop turned on because I pushed the power button.
b. Pushing the power button caused the laptop to turn on.

$\not\Rightarrow$ In every logically possible world where I push the power button, the laptop turns on.

These are assertable even though there is a logically possible world where the laptop's battery is empty.

Is C sufficient for E just in case *if C would E* is true?

Problem

Many existing semantics of conditionals validate conjunctive sufficiency, predicting that C and E together entail *if C would E*.

Plan

1. Challenges
   - Challenges to the intuitive idea of similarity
   - Strengthening with a possibility
   - Reciprocity
   - Static counterexamples to Reciprocity
   - Substitution
2. Causal Sufficiency
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency
3. An aboutness approach to counterfactuals
   - Invalidating substitution
   - Invalidating conjunctive sufficiency

Reciprocity implies conjunctive sufficiency

$$\frac{A > B \qquad B > A \qquad B > C}{A > C}\ \text{Reciprocity}$$

$$\frac{A \wedge B}{A > C}\ \text{Conjunctive Sufficiency}$$

Walters and Williams (2013) show that, under mild assumptions, reciprocity also ensures that $A \wedge C$ implies $A > C$.

Consider any true A, C, and any B that is irrelevant to A and C, in the sense that $(B \vee \neg B) > A$ and $(B \vee \neg B) > C$ hold.

$$\frac{A > (B \vee \neg B) \qquad (B \vee \neg B) > A \qquad (B \vee \neg B) > C}{A > C}\ \text{Reciprocity}$$

Given the existence of such a B, Reciprocity tells us that $A \wedge C$ implies $A > C$.

Plan

1. Challenges
   - Challenges to the intuitive idea of similarity
   - Strengthening with a possibility
   - Reciprocity
   - Static counterexamples to Reciprocity
   - Substitution
2. Causal Sufficiency
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency
3. An aboutness approach to counterfactuals
   - Invalidating substitution
   - Invalidating conjunctive sufficiency

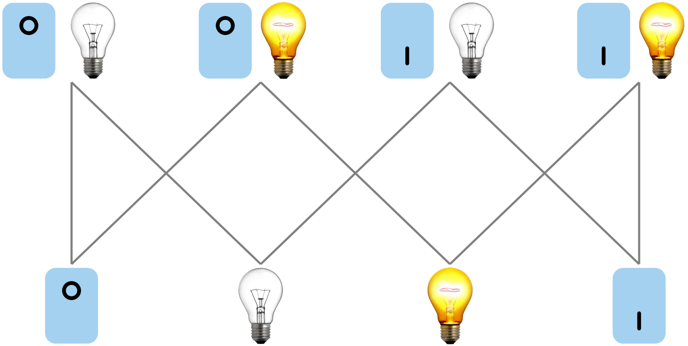


Figure: A state space of the switch and light.

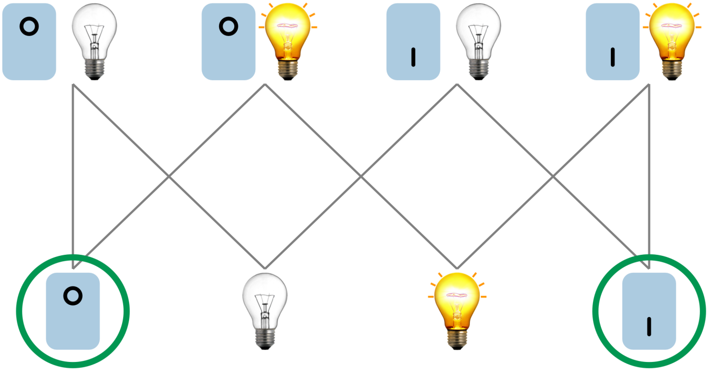


Figure: The states that “the switch is up” is about.

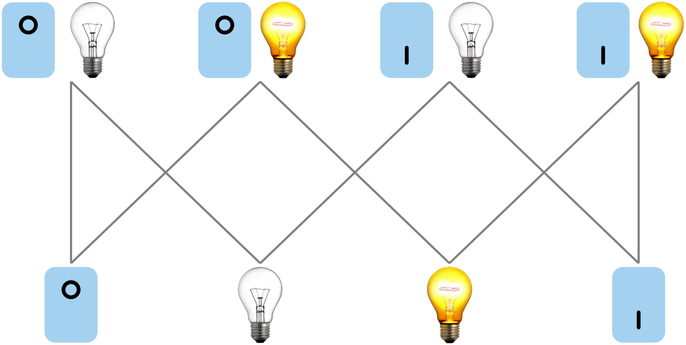
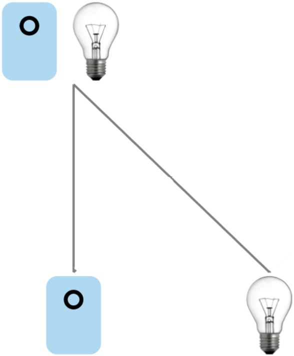
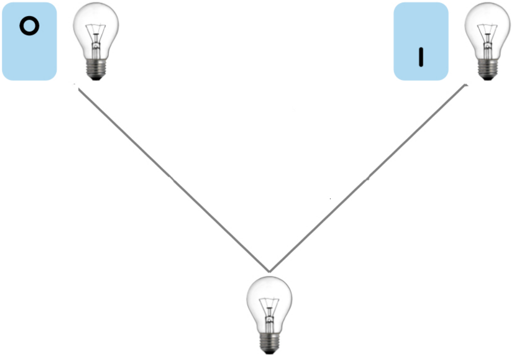


Figure: A state space of the switch and light.




O
I

Some sequences of states are lawful (or, nomically possible) and others are not.

Figure: A lawful sequence.

Some sequences of states are lawful (or, nomically possible) and others are not.

Figure: A lawful sequence.

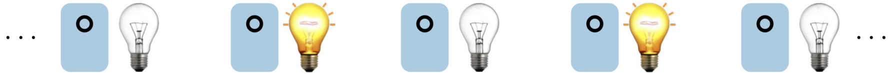


Figure: An unlawful sequence.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
   - This is the *intervention time* t.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
   - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
   - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
   - This gives us a set of time slices, called the *A-variants of w at t*.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
   - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
   - This gives us a set of time slices, called the *A-variants of* w *at* t.
3. Play the laws forward.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
   - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
   - This gives us a set of time slices, called the *A-variants of w at t*.
3. Play the laws forward.
   - Find the lawful futures of the A-variants of w at t.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
   - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
   - This gives us a set of time slices, called the *A-variants of w at t*.
3. Play the laws forward.
   - Find the lawful futures of the A-variants of w at t.
4. Stick on the actual past.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
   - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
   - This gives us a set of time slices, called the *A-variants of w at t*.
3. Play the laws forward.
   - Find the lawful futures of the A-variants of w at t.
4. Stick on the actual past.
   - This gives us the *modal horizon of A at w*.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
    - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
    - This gives us a set of time slices, called the *A-variants of w at t*.
3. Play the laws forward.
    - Find the lawful futures of the A-variants of w at t.
4. Stick on the actual past.
    - This gives us the *modal horizon of A at w*.
5. Restrict to those worlds where the antecedent is true.

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
    - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
    - This gives us a set of time slices, called the *A-variants of w at t*.
3. Play the laws forward.
    - Find the lawful futures of the A-variants of w at t.
4. Stick on the actual past.
    - This gives us the *modal horizon of A at w*.
5. Restrict to those worlds where the antecedent is true.
6. Check whether the consequent is true at the resulting world(s).

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
    - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
    - This gives us a set of time slices, called the *A-variants of w at t*.
3. Play the laws forward.
    - Find the lawful futures of the A-variants of w at t.
4. Stick on the actual past.
    - This gives us the *modal horizon of A at w*.
5. Restrict to those worlds where the antecedent is true.
6. Check whether the consequent is true at the resulting world(s).

A semantics of conditionals (McHugh 2022, 2023)

1. Pick a time at which to imagine the change.
   - This is the *intervention time* t.
2. Vary the part of the world the antecedent A is about at intervention time.
   - This gives us a set of time slices, called the *A-variants of w at t*.
3. Play the laws forward.
   - Find the lawful futures of the A-variants of w at t.
4. Stick on the actual past.
   - This gives us the *modal horizon of A at w*.
5. Restrict to those worlds where the antecedent is true.
6. Check whether the consequent is true at the resulting world(s).

Analysis of aboutness: a sentence is about the parts of the world that exactly determine its truth value (McHugh 2023, p. 108).

- **The foreground:** the set of states A is about.
- **The background:** the set of states that do not overlap a state in the foreground.

- **The foreground:** the set of states A is about.
- **The background:** the set of states that do not overlap a state in the foreground.

Ceteris paribus

- The background is the *ceteris*, the ‘all else’ in ‘all else being equal’
- *Paribus* means having the *ceteris* as part

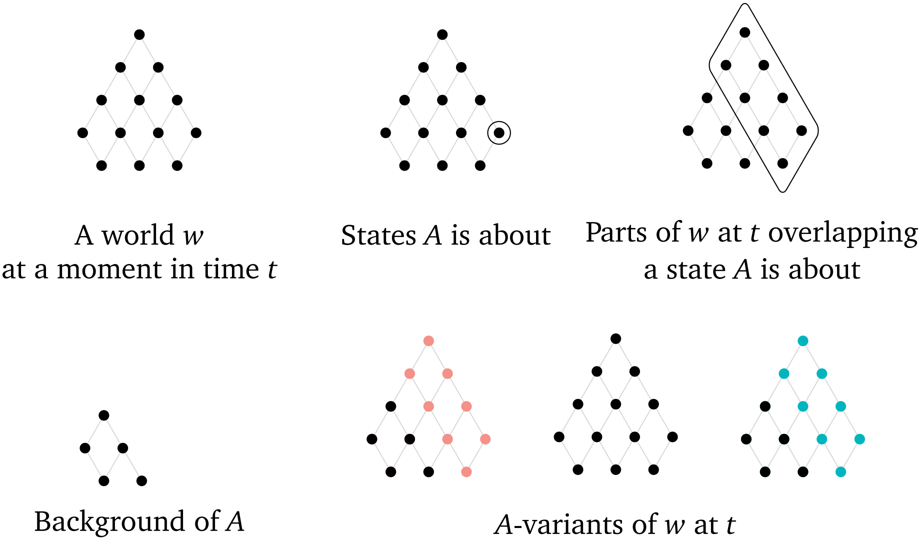


Figure: Steps to construct the A-variants of a world at a moment in time.

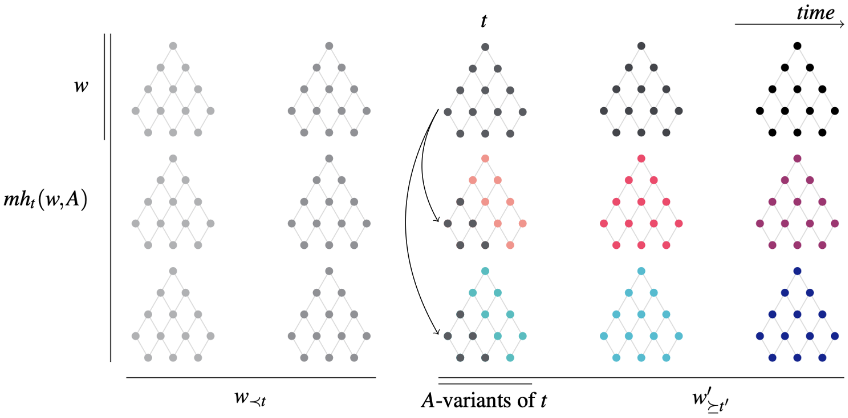


Figure: Constructing the modal horizon.

Plan

1. Challenges
   - Challenges to the intuitive idea of similarity
   - Strengthening with a possibility
   - Reciprocity
   - Static counterexamples to Reciprocity
   - Substitution
2. Causal Sufficiency
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency
3. An aboutness approach to counterfactuals
   - Invalidating substitution
   - Invalidating conjunctive sufficiency

Infinitely many: a counterexample to Substitution

From Fine (2014, p. 328): There is one poison apple and infinitely many safe apples.

$\cdots \infty$

(20) If Eve ate infinitely many of the green apples,
she would be fine.

(21) If Eve ate infinitely many of the apples,
she would be fine.

Infinitely many: a counterexample to Substitution

From Fine (2014, p. 328): There is one poison apple and infinitely many safe apples.

$\cdots \infty$

(20) If Eve ate infinitely many of the green apples,
she would be fine.

(21) If Eve ate infinitely many of the apples,
she would be fine.

"Eve eats infinitely many of the green apples" is about Eve and the green apples.

"Eve eats infinitely many of the apples" is about Eve and the apples.

Plan

1. Challenges
   - Challenges to the intuitive idea of similarity
   - Strengthening with a possibility
   - Reciprocity
   - Static counterexamples to Reciprocity
   - Substitution
2. Causal Sufficiency
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency
3. An aboutness approach to counterfactuals
   - Invalidating substitution
   - Invalidating conjunctive sufficiency

The need for sufficiency

Suppose Sue is 30.

(22) a. Sue was allowed into the bar because she's over 21.
b. Sue was allowed into the bar because she's over 16.

(23) a. The fact that Sue is over 21 caused the bouncer to let her in.
b. The fact that Sue is over 16 caused the bouncer to let her in.

The need for sufficiency

Suppose Sue is 30.

(22) a. Sue was allowed into the bar because she's over 21.
b. Sue was allowed into the bar because she's over 16.

(23) a. The fact that Sue is over 21 caused the bouncer to let her in.
b. The fact that Sue is over 16 caused the bouncer to let her in.

- "Sue is over 21" and "Sue is over 16" are about Sue's age.

The need for sufficiency

Suppose Sue is 30.

(22) a. Sue was allowed into the bar because she's over 21.
     b. Sue was allowed into the bar because she's over 16.

(23) a. The fact that Sue is over 21 caused the bouncer to let her in.
     b. The fact that Sue is over 16 caused the bouncer to let her in.

- "Sue is over 21" and "Sue is over 16" are about Sue's age.
- We vary Sue's age.

The need for sufficiency

Suppose Sue is 30.

(22) a. Sue was allowed into the bar because she's over 21.
 b. Sue was allowed into the bar because she's over 16.

(23) a. The fact that Sue is over 21 caused the bouncer to let her in.
 b. The fact that Sue is over 16 caused the bouncer to let her in.

- "Sue is over 21" and "Sue is over 16" are about Sue's age.
- We vary Sue's age.
- Restrict to those where she is over 21 and 16, respectively.

The need for sufficiency

Suppose Sue is 30.

(22) a. Sue was allowed into the bar because she's over 21.
b. Sue was allowed into the bar because she's over 16.

(23) a. The fact that Sue is over 21 caused the bouncer to let her in.
b. The fact that Sue is over 16 caused the bouncer to let her in.

- "Sue is over 21" and "Sue is over 16" are about Sue's age.
- We vary Sue's age.
- Restrict to those where she is over 21 and 16, respectively.
- This gives us the diverse ways for Sue to be over 16 (17, 18, ...) beyond merely the actual way in which she is over 16 (being her actual age)

References I

Arregui, Ana (2005). On the accessibility of possible worlds: The role of tense and aspect. PhD thesis. University of Massachusetts Amherst. URL: `https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.456.7358&rep=rep1&type=pdf`.

Bacon, Andrew (2013). In defence of a naıve conditional epistemology. *Manuscript*.

Bennett, Jonathan (1984). Counterfactuals and temporal direction. *The Philosophical Review* 93.1, pp. 57–91. DOI: 10.2307/2184413.

— (2003). *A philosophical guide to conditionals*. Oxford University Press.

References II

Boylan, David and Ginger Schultheis (2017). Strengthening principles and counterfactual semantics. *Proceedings of the 21st Amsterdam Colloquium*. Ed. by Floris Roelofsen Alexandre Cremers Thom van Gessel, pp. 155–164. URL: https://semanticsarchive.net/Archive/jZiM2FhZ/AC2017-Proceedings.pdf.

— (2021). How strong is a counterfactual? *The Journal of Philosophy* 118.7, pp. 373–404. DOI: 10.5840/jphil2021118728.

Elga, Adam (2001). Statistical mechanics and the asymmetry of counterfactual dependence. *Philosophy of Science* 68.S3, S313–S324.

Fine, Kit (1975). Critical Notice (review of Lewis 1973). *Mind* 84.1, pp. 451–458. DOI: 10.1093/mind/LXXXIV.1.451.

— (2014). Permission and possible worlds. *dialectica* 68 (3), pp. 317–336. DOI: 10.1111/1746-8361.12068.

References III

- Ginsberg, Matthew L. (1986). Counterfactuals. *Artificial Intelligence* 30, pp. 35–79. DOI: 10.1016/0004-3702(86)90067-6.
- Goodsell, Zachary (2022). Tossing Morgenbesser's Coin. *Analysis* 82.2, pp. 214–221.
- Ippolito, Michela (2013). *Subjunctive Conditionals: A Linguistic Analysis*. MIT Press.
- Khoo, Justin (2017). Backtracking counterfactuals revisited. *Mind* 126.503, pp. 841–910. DOI: 10.1093/mind/fzw005.
- Lewis, David (1973). *Counterfactuals*. Wiley-Blackwell.
- — (1979). Counterfactual dependence and time's arrow. *Noûs*, pp. 455–476. DOI: 10.2307/2215339.
- McHugh, Dean (2022). Aboutness and Modality. *Proceedings of the 23rd Amsterdam Colloquium*, pp. 194–206. DOI: 10.21942/uva.21739718.

References IV

- McHugh, Dean (2023). Causation and Modality: Models and Meanings. PhD thesis. University of Amsterdam. URL: https://eprints.illc.uva.nl/id/eprint/2243.
- Romoli, Jacopo, Paolo Santorio, and Eva Wittenberg (2022). Alternatives in Counterfactuals: What Is Right and What Is Not. *Journal of Semantics* 39.2, pp. 213–260. DOI: 10.1093/jos/ffab023.
- Tichý, Pavel (1976). A counterexample to the Stalnaker-Lewis analysis of counterfactuals. *Philosophical Studies* 29.4, pp. 271–273. DOI: 10.1007/BF00411887.
- Walters, Lee and Robert G. Williams (2013). An argument for conjunction conditionalization. *The Review of Symbolic Logic* 6.4, pp. 573–588. DOI: 10.1017/S1755020313000191.