Disjunctions of Modals and Conditionals

A Solution to Zimmerman's Problem

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Institute for Logic, Language and Computation

University of Amsterdam

Plan

1. Data
   - Disjunctions of universal modals
   - Motivation from theories of wide-scope free choice
   - Disjunctions of conditionals
2. Analysis
   - Ingredients
   - Predictions
3. Deriving the equivalence of $\Box A \vee \Box B$ and $\Box(A \vee B)$
4. Remaining Issues
   - Negating the whole or a part of the first disjunct
   - A single modal?
5. Comparison with Geurts (2005)
6. Conclusion

(1) a. You must do clean your room or you must walk the dog.
b. They keys must be in the drawer or John must have taken them.

Discussed by Geurts (2005)

(2) a. If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.
b. If Alice had come to the party, Charlie would have come. Or if Alice had come, Darius would have come.

Discussed by Woods (1997), Geurts (2005), and Khoo (2021)

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(53a) You must do clean your room or you must walk the dog.

(53a) You must do clean your room or you must walk the dog.

- It is not true that you must clean your room: you may walk the dog and leave your room as it is.
- It is not true that you must walk the dog: you may clean your room and keep the dog home.

(53a) You must do clean your room or you must walk the dog.

- It is not true that you must clean your room: you may walk the dog and leave your room as it is.
- It is not true that you must walk the dog: you may clean your room and keep the dog home.

(53b) They keys must be in the drawer or John must have taken them.

- It is not true that they must be in the drawer: John might have taken them).
- It is not true that John must have taken them: they might be in the drawer.

(53a) You must do clean your room or you must walk the dog.

- It is not true that you must clean your room: you may walk the dog and leave your room as it is.
- It is not true that you must walk the dog: you may clean your room and keep the dog home.

(53b) They keys must be in the drawer or John must have taken them.

- It is not true that they must be in the drawer: John might have taken them).
- It is not true that John must have taken them: they might be in the drawer.

Puzzle 1

Classical logic: $\Box A \vee \Box B$ is incompatible with $\neg\Box A \wedge \neg\Box B$.

Question regarding disjunction and necessity in modal logic

Asked 9 days ago Modified 8 days ago Viewed 136 times

2

I have a question regarding disjunction and necessity.

Is the following theorem provable in any system of modal logic, or is it not generally true?

$\vdash A \lor B \to \Box A \lor \Box B$

I was thinking about using the truth functional definition of OR to answer the question myself. I think it's true, but I don't know how to prove it.

logic modal-logic

Share Improve this question Follow

asked May 27 at 12:35
lee pappas
887 ●1 ●8

2 It is trivially false, take $B = \neg A$. – Conifold May 27 at 12:43

Necessity and possibility are nothing but a half-baked attempt at something probability theory does better. ("Necessary" = probability of 1, "possible" = nonzero probability) – causative May 27 at 18:21

The question is, is the following a theorem of any modal logic?

$\vdash A \lor B \rightarrow \Box A \lor \Box B$

To me it seems to be a consequence of the truth functional definition of OR. If 'A or B' is true then A must be true or B must be true.

Thus, any modal logic

in which does not have

$\vdash A \lor B \rightarrow \Box A \lor \Box B$

is perforce inconsistent.

So I followed my instinct, and used the truth functional definition of OR to answer the question.

David Lippman, *Math in Society*:

(3) A conditional statement [$p \to q$] and its contrapositive [$\neg q \to \neg p$] are logically equivalent. The converse [$q \to p$] and inverse [$\neg p \to \neg q$] of a conditional statement are logically equivalent.
In other words, the original statement and the contrapositive must agree with each other; they must both be true, or they must both be false. Similarly, the converse and the inverse must agree with each other; they must both be true, or they must both be false.
Be aware that symbolic logic cannot represent the English language perfectly.

The Wikipedia page for *Kill Your Darlings* states:

(4) Ginsberg is faced with possible expulsion from Columbia. Either he must be expelled or he must embrace establishment values. He chooses the former.

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Motivation from theories of wide-scope free choice

(Zimmermann 2000, Geurts 2005, Aloni 2023)

(5) He might be in Brixton or he might be in Victoria.
$\rightsquigarrow$ He might be in Brixton.
$\rightsquigarrow$ He might be in Victoria.

(6) You may go to Brixton or you may go to Victoria.
$\rightsquigarrow$ You may go to Brixton.
$\rightsquigarrow$ You may go to Victoria.

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A welcome prediction

Zimmermann (2000), Geurts (2005), and Aloni (2023) predict:
when the speaker is an authority on the modal facts,
$\Diamond A \vee \Diamond B$ implies $\Diamond A \wedge \Diamond B$.

(7) He must be in Brixton or he must be in Victoria.
$\not\rightsquigarrow$ He must be in Brixton.
$\not\rightsquigarrow$ He must be in Victoria.

(8) He must go to Brixton or he must go to Victoria.
$\not\rightsquigarrow$ He must go to Brixton.
$\not\rightsquigarrow$ He must go to Victoria.

An unwelcome prediction

Zimmermann (2000), Geurts (2005), and Aloni (2023) also predict: when the speaker is an authority on the modal facts, $\Box A \vee \Box B$ implies $\Box A \wedge \Box B$.

Geurts (2005): this is "clearly wrong".

One may explicitly affirm authority/indisputability without *Must A or must B* meaning *Must A and must B*

(9) I am your parent, so I make the rules. And I'm telling you that you have to do your homework or you have to walk the dog.
$\not\rightsquigarrow$ You have to do your homework and you have to walk the dog.

$\Box A \vee \Box B$ winds up meaning something like $\Box(A \vee B)$

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(10) a. Everyone is in the kitchen or the garden.

b. $\neq$ Everyone is in the kitchen or everyone is in the garden.

$\Box A \vee \Box B$ winds up meaning something like $\Box(A \vee B)$

(10) a. Everyone is in the kitchen or the garden.
b. $\neq$ Everyone is in the kitchen or everyone is in the garden.

(11) a. You must clean the kitchen or you must walk the dog.
b. $\neq$ In every normatively best world you clean the kitchen, or in every normatively best world you walk the dog.

Plan

(54a) If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.

(54a) If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.

$\rightsquigarrow$ If Alice had come to the party, Charlie would have come.

$\rightsquigarrow$ If Bob had come to the party, Charlie would have come.

(54a) If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.
$\rightsquigarrow$ If Alice had come to the party, Charlie would have come.
$\rightsquigarrow$ If Bob had come to the party, Charlie would have come.

Conjunctive reading

(54a) If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.
$\rightsquigarrow$ If Alice had come to the party, Charlie would have come.
$\rightsquigarrow$ If Bob had come to the party, Charlie would have come.

Conjunctive reading

(54b) If Alice had come to the party, Charlie would have come. Or if Alice had come, Darius would have come.

(54a) If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.

$\rightsquigarrow$ If Alice had come to the party, Charlie would have come.

$\rightsquigarrow$ If Bob had come to the party, Charlie would have come.

Conjunctive reading

(54b) If Alice had come to the party, Charlie would have come. Or if Alice had come, Darius would have come.

$\not\rightsquigarrow$ If Alice had come to the party, Charlie would have come.

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(Exclusive) disjunctive reading

Puzzle 2

Why do we by default interpret *If A, C or if B, C* conjunctively but *If A, C or if A, D* disjunctively?

(12) If you take the morning train, you'll arrive at 12:00,
or if you take the afternoon train, you'll arrive at 16:00.

Prominent conjunctive reading

(12) If you take the morning train, you'll arrive at 12:00,
or if you take the afternoon train, you'll arrive at 16:00.

Prominent conjunctive reading

Context: Alice's best friend is Bob. Charlie's best friend is Deepta. The hearer has a crush on Bob and Deepta.

(13) If you invite Alice, Bob will come,
or if you invite Charlie, Deepta will come.

Prominent conjunctive reading

(14) PRESENT (GENERIC)
If Jia takes the tofu she enjoys her meal,
or if she takes the dumplings she enjoys her meal.

(15) PRESENT + WILL (EPISODIC)
If Jia takes the tofu she will enjoy her meal,
or if she takes the dumplings she will enjoy her meal.

(16) PAST SIMPLE
If Jia took the tofu she would enjoy her meal,
or if she took the dumplings she would enjoy her meal.

(17) PAST PERFECT
If Jia has taken the tofu she would have enjoyed her meal,
or if she had taken the dumplings she would have enjoyed her meal.

(18) CONDITIONAL CONJUNCTION
Take the tofu and you'll enjoy your meal,
or take the dumplings and you'll enjoy your meal.

(19) And if a soul sin … if he do not utter it, then he shall bear his iniquity.
Or if a soul touch any unclean thing … he also shall be unclean, and guilty.
Or if he touch the uncleanness of man … when he knoweth of it, then he shall be guilty.
(*Leviticus* 5:1–3, King James Version, 1611).

A disjunction word links the clauses of Leviticus 5 in: Chinese (*huò*), the original Hebrew (*o*), Hungarian (*vagy*), Icelandic (*eða*), Māori (*rānei*), Urdu (*yâ*), Somali (*ama*), Welsh (*neu*) and Yoruba (*tàbí*).

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Our account has three ingredients:

1. **Disjunction's dynamic effect**

   In $A \vee B$, B's local context entails $\neg A$,
   and perhaps also symmetrically, A's local context entails $\neg B$.

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   - SYMMETRIC: $(\neg B \rightarrow A) \vee (\neg B \rightarrow A)$

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   and perhaps also symmetrically, A's local context entails $\neg B$.

   - ASYMMETRIC: $A \vee (\neg A \to B)$
   - SYMMETRIC: $(\neg B \to A) \vee (\neg B \to A)$

2. **Wide scope free choice**

   $A \vee B$ can be strengthened to $A \wedge B$

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2. **Wide scope free choice**

   $A \vee B$ can be strengthened to $A \wedge B$

3. **Bounded modality**

   Modal bases are restricted to local contexts (Mandelkern 2019)

Disjunction's dynamic effect

Familiar from dynamic semantics (Heim 1982, Veltman 1996, Chierchia 1995, Beaver 2001), the wider literature on presupposition projection (Schlenker 2008, 2009, Chemla 2009), and what Klinedinst and Rothschild (2012) call 'non-truth-tabular' disjunction.

Evidence from presupposition filtering

- Presuppositions must be satisfied in their local context
  (Heim 1983, Beaver 2001, Schlenker 2009, among many others)
- The local context for B in $A \vee B$ is $\neg A$
- If B presupposes P, provided $\neg A$ entails P,
  the global context need not entail P.

Filtering in plain disjunctions

Karttunen (1973):

(20) a. Either all of Jack's letters have been held up or he has not written any.
b. Either Jack has not written any letters or all of them have been held up.

Partee (in Roberts 1987):

(21) a. There's no bathroom in this house or it's in a funny place.
b. The bathroom is in a funny place or there's no bathroom here.

Filtering in disjunctions of modals

(22) a. Either there's no bathroom in this house or it must be in a funny place.

b. Either the bathroom is in a funny place or there must be no bathroom here.

(23) a. There must be no bathroom in this house or it must be in a funny place.

b. The bathroom must be in a funny place or there must be no bathroom here.

Picking up material below the modal

Meyer (2016) cites the following example, from Heim and Kratzer (1998)

(24) Pronouns must be generated with an index or else they will be uninterpretatable.

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Without *else*:

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Without *else*:

(25) Pronouns must be generated with an index or they will be uninterpretatable.

Futher examples from Meyer:

(26) a. We are not allowed to go or we risk an incident
b. I'm afraid John could have missed the bus or he would be here by now.

Presupposition filtering with disjunction

Given *If A, C, or if B, D*, we can interpret the second conditional assuming that *A*, the antecedent of the first, is false.

(27) a. If there's no bathroom in this house, I'll go home, or if it's in a funny place, I'll ask the host for directions.

b. If the bathroom is in a funny place, I'll ask the host for directions, or if there's no bathroom in this house, I'll go home.

Alternatively, given *If A, C, or if B, D*, we can interpret the second conditional assuming that *C*, the consequent of the first, is false.

(28) a. Our new office will have no bathroom if we hire Radical Architects. Or if we hire them they'll put it in a strange place.

b. Our new office bathroom will be in a funny place if we hire Radical Architects. Or if we hire them it will have no bathroom.

(29) ASYMMETRIC DYNAMIC EFFECT + CONJUNCTIVE INTERPRETATION

a. You must do clean your room and if you do not clean your room, you must walk the dog.

b. They keys must be in the drawer and if they are not in the drawer, John must have taken them.

(30) SYMMETRIC DYNAMIC EFFECT + CONJUNCTIVE INTERPRETATION

a. If you do not walk the dog, you must do clean your room, and if you do not clean your room, you must walk the dog.

b. If John has not taken the keys, they must be in the drawer, and if they are not in the drawer, John must have taken them.

Bounded modality

Let R be a binary accessibility relation over worlds.
For any world w let $R[w] = w' : wRw'$ be the set of worlds accessible from w.

(31) **Kripke.**
$[\![\Box A]\!]^{c,w}$ is true just in case $R[w] \subseteq [\![A]\!]^c$.

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$[\![\Box A]\!]^{c,w}$ is true just in case $R[w] \subseteq [\![A]\!]^{c}$.

(32) **Bounded Kripke.**
$[\![\Box A]\!]^{c,w}$ is true just in case $R[w] \cap c \subseteq [\![A]\!]^{c}$.

Bounded modality

(33) **Kratzer.**
Must A is true at a world *w* just in case *A* is true at every world in the modal base at *w* ranked highest according to the ordering source at *w*.

$$[\![\Box A]\!]^{w,f,g,c} = 1 \quad \text{iff} \quad \max_{g(w)} \bigcap f(w) \subseteq [\![A]\!]^{f,g,c}$$

Bounded modality

(33) **Kratzer.**
Must A is true at a world w just in case A is true at every world in the modal base at w ranked highest according to the ordering source at w.

$$[\![\Box A]\!]^{w,f,g,c} = 1 \quad \text{iff} \quad \max_{g(w)} \bigcap f(w) \subseteq [\![A]\!]^{f,g,c}$$

(34) **Bounded Kratzer.**
Must A is true at a world w just in case A is true at every world in the modal base at w *compatible with the local context* that is ranked highest according to the ordering source at w.

$$[\![\Box A]\!]^{w,f,g,c} = 1 \quad \text{iff} \quad \max_{g(w)}(c \cap \bigcap f(w)) \subseteq [\![A]\!]^{f,g,c}$$

Where does bounded modality come from?

Where does bounded modality come from?

Idea: Think of the local context as itself a kind of presupposition.
Heim (1992): presuppositions restrict modal bases.

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Asymmetric readings

(35) a. You have to pay the bill, or you have to go to jail.
b. ??You have to go to jail, or you have to pay the bill.

(similar to the *or else* effect from Schwager 2006)

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(35) a. You have to pay the bill, or you have to go to jail.
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(36) a. You have to pay the bill. If you do not pay the bill, you have to go to jail.
b. ?? You have to go to jail. If you do not go to jail, you have to pay the bill.

Asymmetric readings

(35) a. You have to pay the bill, or you have to go to jail.
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b. ?? You have to go to jail. If you do not go to jail, you have to pay the bill.

- Following Meyer (2016), we adopt Kratzer's (1981) analysis of modality
- $\Box A$ is true at a world w just in case A is true at all worlds in the modal base that come closest to the ideal according to w.

Asymmetric readings

(35) a. You have to pay the bill, or you have to go to jail.
b. ??You have to go to jail, or you have to pay the bill.

(similar to the *or else* effect from Schwager 2006)

(36) a. You have to pay the bill. If you do not pay the bill, you have to go to jail.
b. ?? You have to go to jail. If you do not go to jail, you have to pay the bill.

- Following Meyer (2016), we adopt Kratzer's (1981) analysis of modality
- $\Box A$ is true at a world w just in case A is true at all worlds in the modal base that come closest to the ideal according to w.
- (36a): in all of the normatively best worlds, you pay the bill, and in all of the normatively best worlds where you don't, you go to jail.

Asymmetric readings

(37) "If you were born on or after September 2, 1971 and you are [...] age 9 through 16, you must successfully complete hunter education, OR you must be accompanied."

https://tpwd.texas.gov/education/hunter-education/faq

(38) "To be eligible for Medicaid, you must be a U.S. citizen. Or, you must be within one of the qualified categories of non-citizens."

https://www.illinoislegalaid.org/legal-information/am-i-eligible-medicaid

Symmetric readings

(39) *For Ali to win the game, the die must land on multiple of three.*

a. Ali has to roll a three or he has to roll a six.

b. Ali has to roll a six or he has to roll a three.

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(39) *For Ali to win the game, the die must land on multiple of three.*

a. Ali has to roll a three or he has to roll a six.
b. Ali has to roll a six or he has to roll a three.

(40)
a. If Ali doesn't roll a six, he has to roll a three, and if he doesn't roll a three, he has to roll a six.
b. If Ali doesn't roll a three, he has to roll a six, and if he doesn't roll a six, he has to roll a three.

Symmetric readings

(39) *For Ali to win the game, the die must land on multiple of three.*

a. Ali has to roll a three or he has to roll a six.
b. Ali has to roll a six or he has to roll a three.

(40) a. If Ali doesn't roll a six, he has to roll a three, and if he doesn't roll a three, he has to roll a six.
b. If Ali doesn't roll a three, he has to roll a six, and if he doesn't roll a six, he has to roll a three.

(41) a. ??Ali has to roll a three, and if he doesn't roll a three, he has to roll a six.
b. ??Ali has to roll a six, and if he doesn't roll a six, he has to roll a three.

(54) a. If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.

b. If Alice had come to the party, Charlie would have come. Or if Alice had come, Darius would have come.

(54a) If Alice had come to the party, Charlie would have come.
Or if Bob had come, Charlie would have come.

(54a) If Alice had come to the party, Charlie would have come.
Or if Bob had come, Charlie would have come.

Negating the consequent is not an option:

(42) If Alice had come to the party, Charlie would have come.
Or if Bob but not Charlie had come, Charlie would have come.

(54a) If Alice had come to the party, Charlie would have come.
Or if Bob had come, Charlie would have come.

Negating the consequent is not an option:

(42) If Alice had come to the party, Charlie would have come.
Or if Bob but not Charlie had come, Charlie would have come.

Negating the antecedent is perfectly possible:

(43)
a. If Alice had come to the party, Charlie would have come. And if Bob but not Alice had come, Charlie would have come.
b. If Alice but not Bob had come to the party, Charlie would have come. And if Bob but not Alice had come, Charlie would have come.

(54b) If Alice had come to the party, Charlie would have come.
Or if Alice had come, Darius would have come.

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Negating the antecedent is not an option:

(44) If Alice had come to the party, Charlie would have come.
Or if Alice but not Alice had come, Charlie would have come.

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Negating the antecedent is perfectly possible:

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And if Alice but not Charlie had come, Darius would have come.

b. If Alice but not Darius had come to the party, Charlie would have come.
And if Alice but not Charlie had come, Darius would have come.

Plan

$$\Box A \vee \Box_{\neg A} B$$

$$
\begin{aligned}
& \Box A \vee \Box_{\neg A} B \\
\equiv \quad & \Box A \vee \Box(\neg A \to B)
\end{aligned}
$$

$$
\begin{aligned}
& \Box A \vee \Box_{\neg A} B \\
\equiv \quad & \Box A \vee \Box(\neg A \to B) \\
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\end{aligned}
$$

$$
\begin{aligned}
& \Box A \vee \Box_{\neg A} B \\
\equiv \quad & \Box A \vee \Box(\neg A \rightarrow B) \\
\equiv \quad & \Box A \vee \Box(A \vee B) \\
\equiv \quad & \Box(A \vee B)
\end{aligned}
$$

$$\Box_{\neg B} A \vee \Box_{\neg A} B$$

$$
\begin{aligned}
& \Box_{\neg B} A \vee \Box_{\neg A} B \\
\equiv \quad & \Box(\neg B \rightarrow A) \vee \Box(\neg A \rightarrow B)
\end{aligned}
$$

$$
\begin{aligned}
& \Box_{\neg B} A \vee \Box_{\neg A} B \\
\equiv \quad & \Box(\neg B \rightarrow A) \vee \Box(\neg A \rightarrow B) \\
\equiv \quad & \Box(B \vee A) \vee \Box(A \vee B)
\end{aligned}
$$

$$
\begin{aligned}
& \Box_{\neg B} A \vee \Box_{\neg A} B \\
\equiv \quad & \Box(\neg B \rightarrow A) \vee \Box(\neg A \rightarrow B) \\
\equiv \quad & \Box(B \vee A) \vee \Box(A \vee B) \\
\equiv \quad & \Box(A \vee B)
\end{aligned}
$$

(34) **A Localised semantics of universal modals**

Must A is true at a world *w* just in case *A* is true at every world in the modal base at *w compatible with the local context* that is ranked highest according to the ordering source at *w*.

$$[\![\Box A]\!]^{w,f,g,c} = 1 \quad \text{iff} \quad \max_{g(w)}(c \cap \bigcap f(w)) \subseteq [\![A]\!]^{f,g,c}$$

Strengthening with a Possibility.

If $(\max_P X) \cap Y$ *is nonempty, then* $\max_P(X \cap Y) = (\max_P X) \cap Y$.

Proposition

Assume bounded modality, that is, the entry in (34), and Strengthening with a Possibility. Then at any world where $\neg\Box A$ and $\neg\Box B$ are true, the following are equivalent (all true or all false).

1. $\Box(A \vee B)$
2. $\Box A \vee \Box_{\neg A}(B)$
3. $\Box_{\neg B} A \vee \Box_{\neg A} B$
4. $\Box_{\neg B} A \wedge \Box_{\neg A} B$

Proof

Pick any parameters w, f, g, c. To simplify notation, for any sentence A let $|A| = [\![A]\!]^{f,g,c}$ be the set of worlds where A is true, D, the modal domain, be the closest worlds in the modal base restricted to the global context, $D_{\neg A}$ the modal domain restricted by $\neg A$, and $D_{\neg B}$ the modal domain restricted by $\neg B$.

$$D = \max_{g(w)}((\bigcap f(w)) \cap c)$$

$$D_{\neg A} = \max_{g(w)}((\bigcap f(w)) \cap c \cap [\![\neg A]\!]^{f,g,c})$$

$$D_{\neg B} = \max_{g(w)}((\bigcap f(w)) \cap c \cap [\![\neg B]\!]^{f,g,c})$$

Proof (continued)

Since $\neg\Box A$ and $\neg\Box B$ are true at w, $D \cap [\![\neg A]\!]^{f,g,c}$ and $D \cap [\![\neg B]\!]^{f,g,c}$ are nonempty. Then by Strengthening with a Possibility, $D_{\neg A} = D \cap |\neg A|$ and $D_{\neg B} = D \cap |\neg B|$. Note that for any sets X, Y and Z, $X \cap \overline{Y} \subseteq Z$ just in case $X \subseteq Y \cup Z$. Then $\Box_{\neg A} B$ is equivalent to $\Box(A \vee B)$.

$$\Box_{\neg A} B \equiv D_{\neg A} \subseteq |B| \equiv D \cap |\neg A| \subseteq |B| \equiv D \subseteq |A| \cup |B| \equiv \Box(A \vee B)$$

Similarly, $\Box_{\neg B} A$ is equivalent to $\Box(A \vee B)$. Since $\Box A$ entails $\Box(A \vee B)$, $\Box A \vee \Box(A \vee B)$ is equivalent to $\Box(A \vee B)$. Hence $\Box(A \vee B)$ is equivalent to $\Box A \vee \Box_{\neg A} B$.

$$\begin{array}{rcl@{\quad}c@{\quad}l}
\Box A \vee \Box_{\neg A} B & \equiv & \Box A \vee \Box(A \vee B) & \equiv & \Box(A \vee B) \\
\Box_{\neg B} A \vee \Box_{\neg A} B & \equiv & \Box(A \vee B) \vee \Box(A \vee B) & \equiv & \Box(A \vee B) \\
\Box_{\neg B} A \wedge \Box_{\neg A} B & \equiv & \Box(A \vee B) \wedge \Box(A \vee B) & \equiv & \Box(A \vee B)
\end{array}$$

Plan

Plan

1. Data
   - Disjunctions of universal modals
   - Motivation from theories of wide-scope free choice
   - Disjunctions of conditionals
2. Analysis
   - Ingredients
   - Predictions
3. Deriving the equivalence of $\Box A \vee \Box B$ and $\Box(A \vee B)$
4. Remaining Issues
   - Negating the whole or a part of the first disjunct
   - A single modal?
5. Comparison with Geurts (2005)
6. Conclusion

(46) Pronouns must be generated with an index or else they will be uninterpretatable. (from Heim and Kratzer 1998)

(46) Pronouns must be generated with an index or else they will be uninterpretatable. (from Heim and Kratzer 1998)

(47) Mary believes that Bill will come, and Sue knows that Bill will come.

(48) Mary believes that Bill will come, or Sue knows that he won't.

Plan

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The role of *either*

Could disjunction take narrow scope?

The role of *either*

Could disjunction take narrow scope?

(4) Ginsberg is faced with possible expulsion from Columbia. Either he must be expelled or he must embrace establishment values. He chooses the former.

Larson (1985): *either* forces the disjunction to take wide scope.

Mixed modals: $\Diamond A \vee \Box B$

(49) If you are in a lane posted with an EXIT ONLY, you may change lanes, or you must exit the highway if you stay in this lane.[1]

[1] https://web.archive.org/web/20241125064615/https://www.dot.state.pa.us/public/dvspubsforms/bdl/bdl%20manuals/manuals/pa%20drivers%20manual%20by%20chapter/english/pub%2095.pdf.

Mixed modals: $\Diamond A \vee \Box B$

(49) If you are in a lane posted with an EXIT ONLY, you may change lanes, or you must exit the highway if you stay in this lane.[1]

(50) If you are in a lane posted with an EXIT ONLY, you may change lanes, and [if you do not change lanes] you must exit the highway if you stay in this lane.

[1] `https://web.archive.org/web/20241125064615/https://www.dot.state.pa.us/public/dvspubsforms/bdl/bdl%20manuals/manuals/pa%20drivers%20manual%20by%20chapter/english/pub%2095.pdf`.

Plan

By default, A and A' are bound to C, i.e. $A = A' = C$. ... The logical form of [Must B or must B'] is $A\Box B \wedge A'\Box B'$, and it is interpreted against an epistemic background C. ... if $A = A' = C$, the sentence entails that It must be here and there, which is inconsistent with the fact that, as a rule, a chicken [the 'it' in Geurts' examples] cannot be in more than one place at a time. More generally, Disjointness and Non-triviality cannot be satisfied together if either $A = C$ and $A' \subseteq C$ or $A' = C$ and $A \subseteq C$. Therefore, A and A' are allowed to cover only part of C, i.e. $A \subseteq C$ and $A' \subseteq C$.

(Geurts 2005, pp. 396–97)

(51) You must clean your room or you must walk the dog.

Even for the laziest among us, it is not a logical contradiction to do both.

A simple argument against this strategy: Disjointness is incorrect.

Degano, Ramotowska, Marty, Aloni, Breheny, Romoli, and Sudo (2023) found experimental evidence that a disjunction can be accepted even when one of the disjuncts is known to be true.

(52) *Context: it is certain that the mystery box contains a yellow ball.*
The mystery box contains a yellow ball or a blue ball.

Mean acceptance of 94.7%

Without disjointness, there is nothing in Geurts' account to remove the default that $A = A' = C$.

Plan

(53) a. You must do clean your room or you must walk the dog.

b. They keys must be in the drawer or John must have taken them.

Discussed by Geurts (2005)

(54) a. If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.

b. If Alice had come to the party, Charlie would have come. Or if Alice had come, Darius would have come.

Our account has three ingredients:

1. **Disjunction's dynamic effect**

   In $A \vee B$, B's local context entails $\neg A$,
   and perhaps also symmetrically, A's local context entails $\neg B$.

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   - ASYMMETRIC: $A \vee (\neg A \to B)$

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1. **Disjunction's dynamic effect**

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   - SYMMETRIC: $(\neg B \rightarrow A) \vee (\neg B \rightarrow A)$

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   $A \vee B$ can be strengthened to $A \wedge B$

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2. **Wide scope free choice**

   $A \vee B$ can be strengthened to $A \wedge B$

3. **Bounded modality**

   Modal bases are restricted to local contexts (Mandelkern 2019)

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Wide scope free choice (Zimmermann 2000)

(55) If Alice had come to the party, Charlie would have come.
Or if Alice had come, Darius would have come.

Wide scope free choice (Zimmermann 2000)

(55) If Alice had come to the party, Charlie would have come.
Or if Alice had come, Darius would have come.

(56) Detectives may go by bus or they may go by boat.
$\rightsquigarrow$ Detectives may go by bus and may go by boat.

(57) Mr. X might be in Victoria or he might be in Brixton.
$\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.

$$\Diamond A \vee \Diamond B \quad \rightsquigarrow \quad \Diamond A \wedge \Diamond B$$

Bilateral State-Based Modal Logic (BSML; Aloni 2023)

Models

Let $M = (W, R, V)$ be a *model*, where

- W is a set of worlds
- $R \subseteq W \times W$ is a binary relation over W
- $V : W \times Prop \to \{0, 1\}$ is a valuation

and let a set of worlds s be a *state*.

Semantic clauses

$$
\begin{array}{lll}
M, s \models p & \text{iff} & \forall w \in s, V(w, p) = 1 \\
M, s \models A \wedge B & \text{iff} & M, s \models A \text{ and } M, s \models B \\
M, s \models A \vee B & \text{iff} & \exists t, t', s = t \cup t', M, t \models A \text{ and } M, t' \models B \\
M, s \models \Diamond A & \text{iff} & \forall w \in s : M, R[w] \models A \\
M, s \models \text{NE} & \text{iff} & s \neq \emptyset
\end{array}
$$

Ingredient 1: Non-empty states

(56) Detectives may go by bus or they may go by boat.

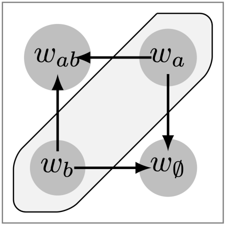
(57) Mr. X might be in Victoria or he might be in Brixton.

$$(\Diamond A \wedge \textsc{ne}) \vee (\Diamond B \wedge \textsc{ne})$$

Ingredient 2: Indisputability

Definition (Indisputability)

R is indisputable in (M, s) iff for all $w, v \in s : R[w] = R[v]$.



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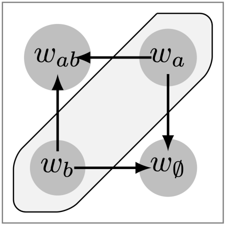

Assuming s represents the information state of the relevant speaker, an indisputable R means that the speaker is fully informed about R, so, for example, if R represents a deontic accessibility relation, indisputability means that the speaker is fully informed about (or has full authority on) what propositions are obligatory or allowed.

(Aloni 2023)

Deriving wide scope free choice (Aloni 2023)

$M, s \models (\Diamond A \wedge \text{NE}) \vee (\Diamond B \wedge \text{NE})$
iff for some nonempty states t, t' with $s = t \cup t'$:

$$t \models \Diamond A \qquad \text{and} \qquad t' \models \Diamond B$$

Deriving wide scope free choice (Aloni 2023)

$M, s \models (\Diamond A \wedge \text{NE}) \vee (\Diamond B \wedge \text{NE})$
iff for some nonempty states t, t' with $s = t \cup t'$:

$$\begin{array}{lcl} t \models \Diamond A & \text{and} & t' \models \Diamond B \\ \forall w \in t : R[w] \models A & \text{and} & \forall w' \in t' : R[w'] \models B \end{array}$$

Fact

If R is indisputable, $(\Diamond A \wedge \text{NE}) \vee (\Diamond B \wedge \text{NE}) \models_{BMSL} \Diamond A \wedge \Diamond B$.

Proof.

- By non-emptyness, there are $w \in t$ and $w' \in t'$.

$\square$

Deriving wide scope free choice (Aloni 2023)

$M, s \models (\Diamond A \wedge \text{NE}) \vee (\Diamond B \wedge \text{NE})$
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Fact

If R is indisputable, $(\Diamond A \wedge \text{NE}) \vee (\Diamond B \wedge \text{NE}) \models_{BMSL} \Diamond A \wedge \Diamond B$.

Proof.

- By non-emptyness, there are $w \in t$ and $w' \in t'$.
- Since $t \models \Diamond A$, $R[w] \models A$ and since $t' \models \Diamond B$, $R[w'] \models B$.

$\square$

Deriving wide scope free choice (Aloni 2023)

$M, s \models (\Diamond A \wedge \mathrm{NE}) \vee (\Diamond B \wedge \mathrm{NE})$
iff for some nonempty states t, t' with $s = t \cup t'$:

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Fact

If R is indisputable, $(\Diamond A \wedge \mathrm{NE}) \vee (\Diamond B \wedge \mathrm{NE}) \models_{BMSL} \Diamond A \wedge \Diamond B$.

Proof.

- By non-emptyness, there are $w \in t$ and $w' \in t'$.
- Since $t \models \Diamond A$, $R[w] \models A$ and since $t' \models \Diamond B$, $R[w'] \models B$.
- Pick any $v \in s$. By indisputability, $R[v] = R[w] = R[w']$.

$\square$

Deriving wide scope free choice (Aloni 2023)

$M, s \models (\Diamond A \wedge \text{NE}) \vee (\Diamond B \wedge \text{NE})$
iff for some nonempty states t, t' with $s = t \cup t'$:

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Fact

If R is indisputable, $(\Diamond A \wedge \text{NE}) \vee (\Diamond B \wedge \text{NE}) \models_{BMSL} \Diamond A \wedge \Diamond B$.

Proof.

- By non-emptyness, there are $w \in t$ and $w' \in t'$.
- Since $t \models \Diamond A$, $R[w] \models A$ and since $t' \models \Diamond B$, $R[w'] \models B$.
- Pick any $v \in s$. By indisputability, $R[v] = R[w] = R[w']$.
- So $R[v] \models A$ and $R[v] \models B$. Hence $M, s \models \Diamond A \wedge \Diamond B$.

$\square$

(58) I know exactly under what conditions Charlie will go to the party. If Alice goes, Charlie will go, or if Bob goes, Charlie will go.

(58) I know exactly under what conditions Charlie will go to the party. If Alice goes, Charlie will go, or if Bob goes, Charlie will go.

(59) I know exactly who will go to the party if Alice goes. If Alice goes, Bob will go, or if Alice goes, Charlie will go.

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Indisputability predicts: conjunctive reading ✓

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Problem

Not clear how BSML currently has the resources to derive this contrast.

The implicature approach to free choice

Kratzer and Shimoyama (2002) and Fox (2007). See also **meyer2015generalizedfreechoice**, Bar-Lev and Margulis (2014), Bowler (2014), and Singh et al. (2016).

Conjunctive alternative available	$\Rightarrow$	exclusive inference
Conjunctive alternative unavailable	$\Rightarrow$	conjunctive inference

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Assumption: $A \vee B$ has as alternatives $A \wedge \neg B$ and $B \wedge \neg A$.

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EXCLUSIVE INFERENCE

- $A \wedge B$ is an alternative. $\rightsquigarrow \neg(A \wedge B)$

CONJUNCTIVE INFERENCE

- $A \wedge B$ is not an alternative.

The implicature approach to free choice

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- $\neg(A \wedge \neg B) \quad \wedge \quad \neg(B \wedge \neg A)$

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- $A \wedge B$ is an alternative. $\rightsquigarrow \neg(A \wedge B)$

CONJUNCTIVE INFERENCE

- $A \wedge B$ is not an alternative.
- $\neg(A \wedge \neg B) \quad \wedge \quad \neg(B \wedge \neg A)$
- $A \rightarrow B \quad \wedge \quad B \rightarrow A$

The implicature approach to free choice

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EXCLUSIVE INFERENCE

- $A \wedge B$ is an alternative. $\rightsquigarrow \neg(A \wedge B)$

CONJUNCTIVE INFERENCE

- $A \wedge B$ is not an alternative.
- $\neg(A \wedge \neg B) \quad \wedge \quad \neg(B \wedge \neg A)$
- $A \rightarrow B \quad \wedge \quad B \rightarrow A$
- $(A \vee B) \wedge (A \leftrightarrow B)$ $\rightsquigarrow A \wedge B$

Strategy on the implicature approach

Account for when, and why, the conjunctive alternative to *If A, C or if B, C* is available.