

ENGR30003

Numerical Programming for Engineers

Semester 2, 2017

Workshop on Root Finding

Exercise 1

For function $f(x) = x^2 - 2x - 3 = 0$,

- (a) Find the two roots for $f(x)$ analytically.
- (b) Show that $f(x)$ can be rearranged into following forms

$$x = g_1(x) = \sqrt{2x + 3}$$

$$x = g_2(x) = \frac{3}{x-2}$$

$$x = g_3(x) = \frac{(x^2-3)}{2}$$

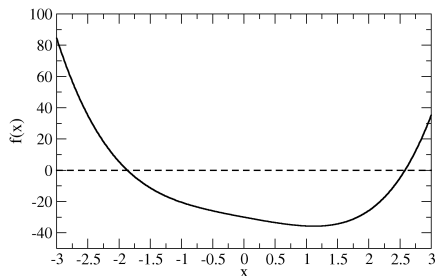
- (c) With these three expressions for $g_i(x)$, use the fixed point iteration method to find the approximate root for $f(x)$. Start with the initial guess of $x_0 = 4$. Compare your solutions to the answer you get in part (a).
- (d) Sketch all the $g_i(x)$ and show what is happening graphically.

Exercise 2 I

Consider equation

$$f(x) = x^4 + 30 \exp(-x/4) - 60.$$

Plotting the function over interval $-3 \leq x \leq 3$



$f(x)$ has two roots, one at $x_{r_1} < 0$ and one at $x_{r_2} > 0$.
Write down estimated values of roots.

Exercise 2 II

- Write a C code that uses the bisection method to estimate the roots of $f(x)$.
 - 1 Using initial guesses $x_l = 2$ and $x_u = 3$, run your program until the error ε drops below 10^{-4} . How many iterations does it take? When the error drops below 10^{-4} , what is the value of x'_r ?
 - 2 Using initial guesses $x_l = -2.5$ and $x_u = -1$, run your program until the error ε drops below 10^{-4} . How many iterations does it take? When the error drops below 10^{-4} , what is the value of x'_r ?

Exercise 2 III

- Write a C code that uses the Newton–Raphson method to estimate the roots of $f(x)$.
 - 1 Start with initial guess $x_0 = -2.5$. Can you find x_r using the Newton–Raphson method? Sketch $f(x)$ and show what is happening graphically.
 - 2 Start with initial guess $x_0 = 3$. Can you find x_r using the Newton–Raphson method? Sketch $f(x)$ and show what is happening graphically.
 - 3 Start with initial guess $x_0 = 1.1229$. Can you find x_r using the Newton–Raphson method? Sketch $f(x)$ and show what is happening graphically.

Exercise 2 IV

- From results above, compare the Newton–Raphson results with those from the Bisection method and discuss advantages and disadvantages of either method.