CS325 Project 2 Report

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Correctness of Claim 3

Theorem 1. *If $\{z_1, \ldots, z_t\}$ and $\{z'_1, \ldots, z'_s\}$ are two visible sets of lines (each ordered by increasing slope), then the visible subset of $\{z_1, \ldots, z_t\} \cup \{z'_1, \ldots, z'_s\}$ is $\{z_1, \ldots, z_i\} \cup \{z'_j, \ldots, z'_s\}$ for some $i \geq 1$ and $j \leq s$.*

Proof. Let $\{z_1, \ldots, z_t\} \in Z$. Let $\{z'_1, \ldots, z'_s\} \in Z'$.

Let both Z and Z' contain only visible lines in their respective sets. The slopes of all lines in both Z and Z' are ordered by (strictly) increasing slope such that the slope of z_t is strictly less than the slope of z'_1. If both Z and Z' are joined together ($Z \cup Z'$) then there will be subsets of Z and Z' comprised of the visible lines found in $Z \cup Z'$, known as V and V' respectively. These visible subsets are joined as $V \cup V'$.

Because the the slope of $Z[0]$ (or z_1) and the slope of $Z'[s]$ (or z'_s) are steeper than the the remaining slopes in their respective sets (because of the slope ordering mentioned above), these two lines will always be visible and are immediately added to V and V' respectively.

To find the remaining visible subset of $Z \cup Z'$, we first find the visibility of the line $Z[1]$ (as it has the next steepest slope in Z) by using the line comparison method proved in Project 1. If $Z[1]$ is found to not be visible, all of the other lines in Z after $Z[1]$ are not visible either because any line in the set after $Z[1]$ has a slope that is less steep than $Z[1]$ and will thus be found to not be visible if compared to Z' on their own. If $Z[1]$ is visible, it is added to V.

Once a line in Z has been found to be visible or not visible, the line in Z' with the next-steepest slope to $Z'[s]$, $Z'[s-1]$, is determined to be visible or not visible by the method proved in Project 1. If $Z'[s-1]$ is found to not be visible then the remaining lines in Z' are also not visible because their slopes are less steep than $Z'[s-1]$ and they would be found to be not visible if compared to lines in Z on their own. If $Z'[s-1]$ is found to be visible, it is added to V'.

The alternating line comparisons between Z and Z' continue until there are either no lines remaining to compare in Z or no lines remaining to compare in Z'. Any lines left in the remaining set are now known to be visible because there are no lines left in the empty set that could make them not visible. These remaining lines are then added to their respective subset, and the visible subset $V \cup V'$ is complete. □

Run-time Analysis

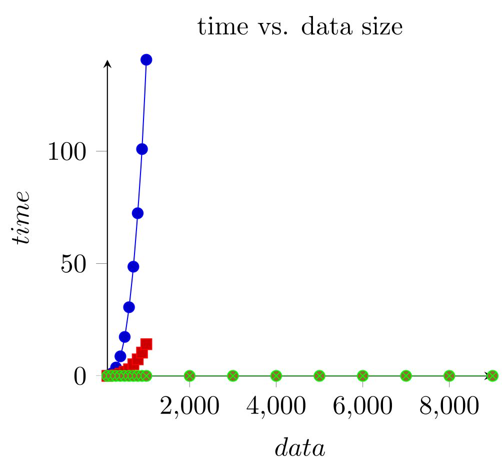
ALGORITHM 4: DIVIDE AND CONQUER

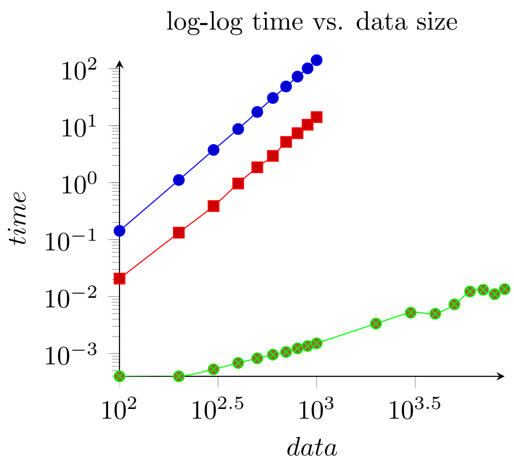
n =number of lines
$j = 0$
$i = j + 1$
$k = i + 1$
lines(i, j, k) visible = true
for $k < n$
  y_{jk} = y-intersect(lines(j, k))
  y_i = [slope(line(i)) $*$ (y-0-intersect(line(j)) - y-0-intersect(line(k)))]
    + [y-intersect(line$(i), 0)$ $*$ slope(line$(k)-$ slope(line(j)))]
  if $y_{jk} > y_i$
    line(i) visible = false
  $j = j + 1$
  $i = i + 1$
  $k = k + 1$
return

Experimental Correctness

Solutions to the instances in the file provided have been submitted to TEACH.

Experimental Analysis





Extrapolation and Interpolation

1. *The biggest instances that could be solved within one hour were not able to be determined.*

2. (a) Algorithm 1 (blue) slope is $\frac{\log(48.599125/8.711271)}{\log(700/400)} = 3.072$

   (b) Algorithm 2 (red) slope is $\frac{\log(5.17444/.965603}{700/400} = 2.99$

   (c) Algorithm 3 (green) slope is $\frac{\log(.00106699999999/.000685000000001}{700/400} = .792$

   It appears that Algorithms 1 and 2 are similar enough to each other to be θ to each other, but Algorithm 3 increases at such a lower rate that it is closer to O for both 1 and 2.