Group Assignment 1

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Problem

Given an array of small integers $a[1, \ldots, n]$ (that contains a least one positive integer), compute

$$\max_{i \leq j} \sum_{k=i}^{j} a[k]$$

Algorithm 1. ***Enumeration****: Loop over each pair of indices $i \leq j$ and compute the sum $\sum_{k=i}^{j} a[k]$*

Psuedocode

```
MaxSubArray(a[1, ..., n])
  max_sum = 0
  sum = 0
  for i = 1, ..., n
    for j = i, ..., n
      for k = i, ..., j
        sum = sum + a[k]
      if sum > max_sum
        max_sum = sum
  return max_sum
```

Run-time analysis

Number of operations

$$\sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=i}^{j} a[k]$$

Asymptotic bounds

$$(\mathcal{O}(n^2) \times (\mathcal{O}(n)\text{time to compute each sum})) = \mathcal{O}(n^3)$$

Algorithm 2. ***Better Enumeration***: *Notice that in the previous algorithm, the same sum is computed many times. In particular, notice that* $\sum_{k=i}^{j} a[k]$ *can be computed from* $\sum_{k=i}^{j-1} a[k]$ *in* $\mathcal{O}(1)$ *time, rather than starting from scratch. Write a new version of the first algorithm that takes advantage of this observation.*

Psuedocode

```
MaxSubArray(a[1, ..., n])
  max_sum = 0
  for i = 1, ..., n
    sum = 0
    for j = i, ..., n
      sum = sum + a[j]
      if sum > max_sum
        max_sum = sum
  return max_sum
```

Run-time analysis

Number of operations

$$\sum_{i=1}^{n} \sum_{j=i}^{n} a[j]$$

Asymptotic bounds

$$((\mathcal{O}(n)\text{i-iterations}) \times (\mathcal{O}(n)\text{j-iterations}) \times (\mathcal{O}(n)\text{time to update sum})) = \mathcal{O}(n^2)$$

Algorithm 3. ***Dynamic Programming***: *Your dynamic programming algorithm should be based on the following idea:*

- *The maximum subarray either uses the last element in the input array, or it doesn't.*

Describe the solution to the maximum subarray problem recursively and mathematically based on the above idea.

Recursive Formula

$$MaxSubArray(a[1,\ldots,n]) = \begin{cases} MaxSubArray\left(a[1,\ldots,\frac{n}{2}]\right) \\ MaxSubArray\left(a[\frac{n}{2},\ldots,n]\right) \\ MaxSuffix\left(a[1,\ldots,\frac{n}{2}]\right) + MaxPrefix\left(a[\frac{n}{2},\ldots,n]\right) \end{cases}$$

```
MaxSuffix(a[low, ..., mid])
  max = 0
  sum = 0
  for i = mid, ..., low
    sum = sum + a[i]
    if sum > max
      max = sum
  return max

MaxSuffix(a[mid, ..., high])
  max = 0
  sum = 0
  for i = mid, ..., high
    sum = sum + a[i]
    if sum > max
      max = sum
  return max
```

Psuedocode

```
MaxSubArray(a[1, ..., n])
  max = 0
  sum = 0
  for i = 1, ..., n
    sum = sum + a[i]
    if sum < 0
      sum = 0
    if sum > max
      max = sum
  return max
```

Run-time analysis

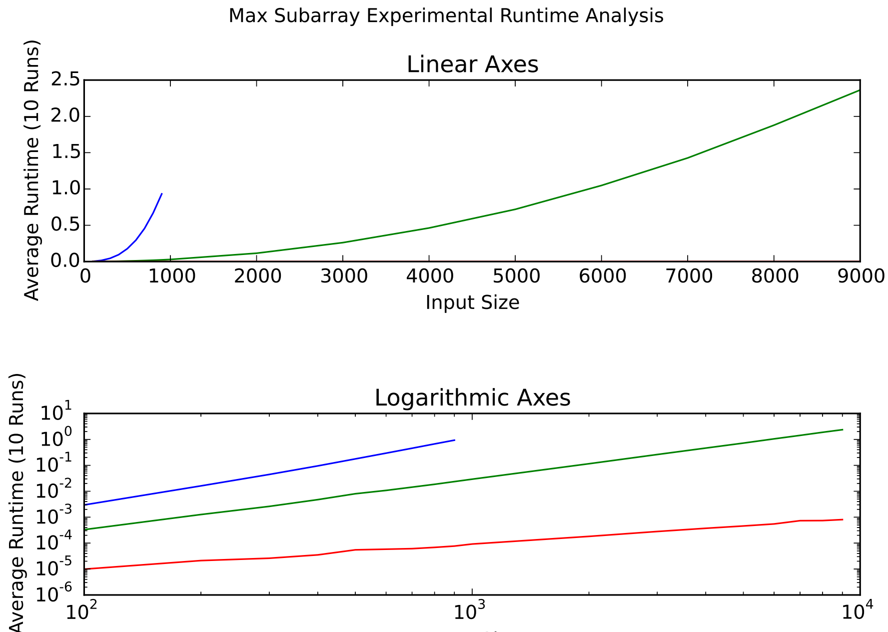
Number of operations

$$\sum_{i=1}^{n} a[i]$$

Asymptotic bounds

$$(\mathcal{O}(n)\text{i-iterations}) \times (\mathcal{O}(c)\text{to compute sum}) = \mathcal{O}(n)$$

1 Graphs



Slope comparision

During our experiments we obtained the following slopes for our algorithms:

- Algorithm 1: 0.3800
- Algorithm 2: 0.5048
- Algorithm 3: 1.0050

This would give us the following runtime complexity for each algorithm:

- Algorithm 1: $\theta(n^{0.3800})$
- Algorithm 2: $\theta(n^{0.5048})$
- Algorithm 3: $\theta(n^{1.0050})$

For our last algorithm, the experimental runtime analysis is accurate to what our theoretical running time. This accuracy can be attributed to the fact that the algorithms linear runtime and the ease of implementation compared to the other algorithms.
The enumeration and better enumeration algorithms, (1 and 2) both had lower runtime complexities when measured than what we would expect. For algorithm 1 we expected a runtime complexity of $\theta(n^3)$ but instead ended up with 0.3800 which could be caused by our experimental data being so small. The second algorithm we expected to run in $\theta(n^2)$ time, but it actually ended up taking $\theta(n^{\frac{1}{2}})$ which suggests we may have made an error when measuring our data, or the algorithm is wrong. However, the algorithm looks fundamentally correct, so I would attribute the error to the experimental data, and inconsistencies caused by computer caching, and such.