Group Assignment 1

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Problem

Given an array of small integers $a[1, \ldots, n]$ (that contains a least one positive integer), compute

$$\max_{i \leq j} \sum_{k=i}^{j} a[k]$$

Algorithm 1. ***Enumeration****: Loop over each pair of indices $i \leq j$ and compute the sum $\sum_{k=i}^{j} a[k]$*

Psuedocode

```
MaxSubArray(a[1, ..., n])
  max_sum = 0
  sum = 0
  for i = 1, ..., n
    for j = i, ..., n
      for k = i, ..., j
        sum = sum + a[k]
      if sum > max_sum
        max_sum = sum
  return max_sum
```

Run-time analysis

Number of operations

$$\sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=i}^{j} a[k]$$

Asymptotic bounds

$$(\mathcal{O}(n^2) \times (\mathcal{O}(n)\text{time to compute each sum})) = \mathcal{O}(n^3)$$

Algorithm 2. ***Better Enumeration****: Notice that in the previous algorithm, the same sum is computed many times. In particular, notice that $\sum_{k=i}^{j} a[k]$ can be computed from $\sum_{k=i}^{j-1} a[k]$ in $\mathcal{O}(1)$ time, rather than starting from scratch. Write a new version of the first algorithm that takes advantage of this observation.*

Psuedocode

```
MaxSubArray(a[1, ..., n])
  max_sum = 0
  for i = 1, ..., n
    sum = 0
    for j = i, ..., n
      sum = sum + a[j]
      if sum > max_sum
        max_sum = sum
  return max_sum
```

Run-time analysis

Number of operations

$$\sum_{i=1}^{n} \sum_{j=i}^{n} a[j]$$

Asymptotic bounds

$$((\mathcal{O}(n)\text{i-iterations}) \times (\mathcal{O}(n)\text{j-iterations}) \times (\mathcal{O}(n)\text{time to update sum})) = \mathcal{O}(n^2)$$

Algorithm 3. ***Dynamic Programming****: Your dynamic programming algorithm should be based on the following idea:*

- *The maximum subarray either uses the last element in the input array, or it doesn't.*

Describe the solution to the maximum subarray problem recursively and mathematically based on the above idea.

Recursive Formula

$$MaxSubArray(a[1,\ldots,n]) = \begin{cases} MaxSubArray\left(a[1,\ldots,\frac{n}{2}]\right) \\ MaxSubArray\left(a[\frac{n}{2},\ldots,n]\right) \\ MaxSuffix\left(a[1,\ldots,\frac{n}{2}]\right) + MaxPrefix\left(a[\frac{n}{2},\ldots,n]\right) \end{cases}$$

```
MaxSuffix(a[low, ..., mid])
  max = 0
  sum = 0
  for i = mid, ..., low
    sum = sum + a[i]
    if sum > max
      max = sum
  return max

MaxPrefix(a[mid, ..., high])
  max = 0
  sum = 0
  for i = mid, ..., high
    sum = sum + a[i]
    if sum > max
      max = sum
  return max
```

Psuedocode

```
MaxSubArray(a[1, ..., n])
  max = 0
  sum = 0
  for i = 1, ..., n
    sum = sum + a[i]
    if sum < 0
      sum = 0
    if sum > max
      max = sum
  return max
```

Run-time analysis

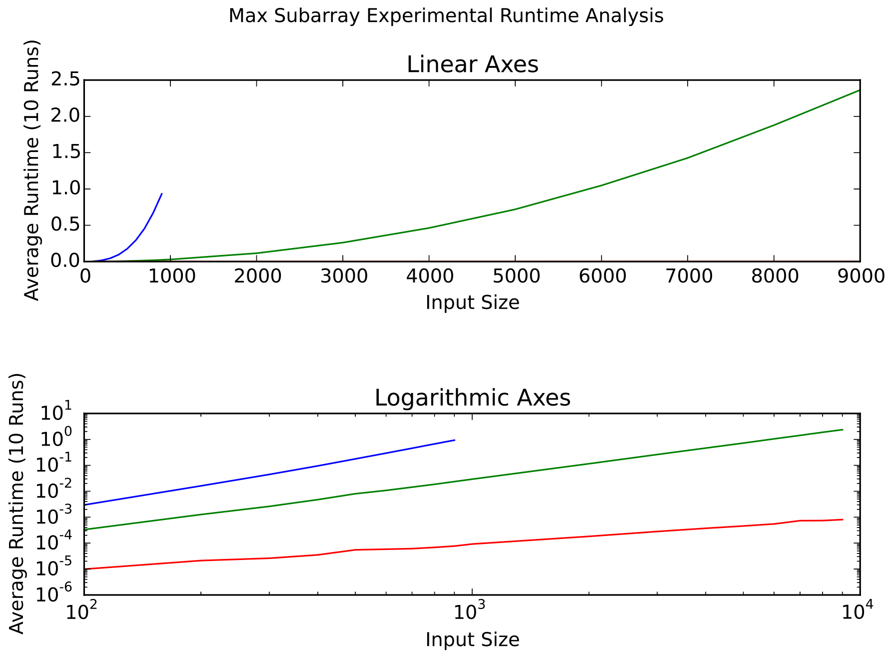
Number of operations

$$\sum_{i=1}^{n} a[i]$$

Asymptotic bounds

$$(\mathcal{O}(n)\text{i-iterations}) \times (\mathcal{O}(c)\text{to compute sum}) = \mathcal{O}(n)$$

1 Graphs



Slope comparision

During our experiments we obtained the following slopes for our algorithms:

- Algorithm 1: 2.6283
- Algorithm 2: 1.9803
- Algorithm 3: 0.9920

This would give us the following runtime complexity for each algorithm:

- Algorithm 1: $\theta(n^{2.6283})$
- Algorithm 2: $\theta(n^{1.9803})$
- Algorithm 3: $\theta(n^{0.9920})$

These values are extremely close to what we would have expected. The only algorithm which had a value that was not completely accurate was algorithm 1. This can be partially attributed to the fact that algorithm 1 was the only algorithm which was not tested with sets greater than 900 elements.