Markovian Noise Modelling and Parameter Extraction Framework for Quantum Devices

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QSUN
Quantum Research Group of Stellenbosch University

SA QuTI South African Quantum Technology Initiative

NITheCS
National Institute for Theoretical and Computational Sciences

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Motivation

- There have been various claims in recent literature about the behaviour of IBMQ devices.
- IBM claims their devices fit within the ideal Markovian model.
- Recent publications have claimed that they are non-Markovian, and propose ways to mitigate this.
- Hardware parameters provided by IBMQ are calibrated infrequently and on different sets of data using simple curve-fitting methods, rather than full Quantum Process Tomography (QPT).
- There is no data provided on the behaviour of neighbouring qubits and how they influence each other.

IBMQ Qubits

- The IBMQ devices use superconducting transmon qubits in various topologies which are connected through coupling channels necessary for multi-qubit operations such as entanglement.
- These systems are modelled by a simple Duffing oscillator Hamiltonian,

$$\mathcal{H} = \frac{\omega_0}{2}\sigma_0^z + \frac{\omega_1}{2}\sigma_1^z + J_{0,1}\left(\sigma_0^+\sigma_1^- + \sigma_0^-\sigma_1^+\right),$$

for a two-qubit example where the Pauli matrix index indicates a tensor product index,

$$\hat{O}_i = \underbrace{\mathbb{I}}_{0} \otimes \underbrace{\mathbb{I}}_{1} \otimes \cdots \otimes \underbrace{\hat{O}}_{i} \otimes \cdots \otimes \underbrace{\mathbb{I}}_{N-1}.$$

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General Parameters

- The Duffing oscillator model could be too simple of a model, so a more generalised Hamiltonian is worth investigating, for example,

$$\mathcal{H} = \frac{1}{2}\left(\vec{\omega}_0 \cdot \vec{\sigma}_0\right) + \frac{1}{2}\left(\vec{\omega}_1 \cdot \vec{\sigma}_1\right) + \vec{\sigma}_0 \mathbf{J} \vec{\sigma}_1,$$

$$\vec{\sigma}_0 \mathbf{J} \vec{\sigma}_1 = \begin{pmatrix} \sigma_0^x & \sigma_0^y & \sigma_0^z \end{pmatrix} \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix} \begin{pmatrix} \sigma_1^x \\ \sigma_1^y \\ \sigma_1^z \end{pmatrix}.$$

Calibration Data

- IBMQ only provides a limited set of hardware parameters, such as qubit frequency, relaxation and decoherence times, and coupling strengths.
- An important parameter which could be extracted from the dynamics is the qubit temperature from average photon emission,

$$\langle n \rangle = \frac{1}{e^{\hbar\omega/k_B T} - 1}.$$

Calibration data Last calibrated: about 1 hour ago

Map view | Graph view | Table view

Qubit	T1 (us)	T2 (us)	Frequency (GHz)	Anharmonicity (GHz)	Readout assignment error	Prob meas0
Q0	132.31	326.66	4.833	-0.34168	3.650e-2	0.0454
Q1	150.02	115.94	4.624	-0.32772	3.220e-2	0.0248
Q2	115.25	71.9	4.82	-0.34078	1.860e-2	0.0256
Q3	128.9	111.47	4.742	-0.34318	2.310e-2	0.0166
Q4	143.17	136.24	4.816	-0.34099	2.640e-2	0.033

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Open Quantum Systems

- To model the dynamics of these qubits and extract useful parameters about the hardware, the framework of open quantum systems provides a natural test-bed.
- The IBMQ devices are claimed to evolve as Markovian, or “memoryless," systems which are more simple and stable than non-Markovian ones.
- This is, however, just a claim by the backend and worth more investigation.
- To investigate, the hardware parameters can be used in a Markovian model and compared to experimental results.
- The Markovian model of choice is a GKSL Master Equation,

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i\left[\mathcal{H},\rho\right] + \sum_{i=0}^{N-1}\Bigg[\gamma_i\left(\langle n_i\rangle + 1\right)\left(\sigma_i^-\rho\sigma_i^+ - \frac{1}{2}\left\{\sigma_i^+\sigma_i^-,\rho\right\}\right) + \gamma_i\langle n_i\rangle\left(\sigma_i^+\rho\sigma_i^- - \frac{1}{2}\left\{\sigma_i^-\sigma_i^+,\rho\right\}\right)\Bigg].$$

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GKSL Master Equation

- This equation,

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i\left[\mathcal{H},\rho\right] + \sum_{i=0}^{N-1}\Bigg[\gamma_i\left(\langle n_i\rangle + 1\right)\left(\sigma_i^-\rho\sigma_i^+ - \frac{1}{2}\left\{\sigma_i^+\sigma_i^-,\rho\right\}\right) + \gamma_i\langle n_i\rangle\left(\sigma_i^+\rho\sigma_i^- - \frac{1}{2}\left\{\sigma_i^-\sigma_i^+,\rho\right\}\right)\Bigg].$$

  describes the evolution of a density matrix in a dissipative environment.
- The unitary evolution, $-i\left[\mathcal{H},\rho\right]$ occurs in isolated systems.
- The decay of the quantum states is contributed to by spontaneous emission, described by the average photon number $\langle n_i\rangle$.
- The dissipation rate, γ_i, from any decay mechanism, is what controls the speed of decay to the complete ground state.

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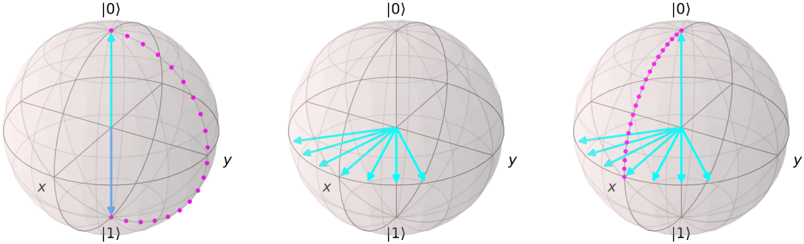
Relaxation and Decoherence

- The typical benchmarks for the decay of quantum states are the relaxation and decoherence times.
- The relaxation time, T_1, is the characteristic time taken for a qubit to de-excite to the ground state.
- The decoherence time, T_2, is the characteristic time taken for a qubit in a transverse state, such as $|+\rangle$, to depahse into a steady state through coupling to longitudinal noise.



(a) Relaxation (b) Pure dephasing (c) Transverse relaxation

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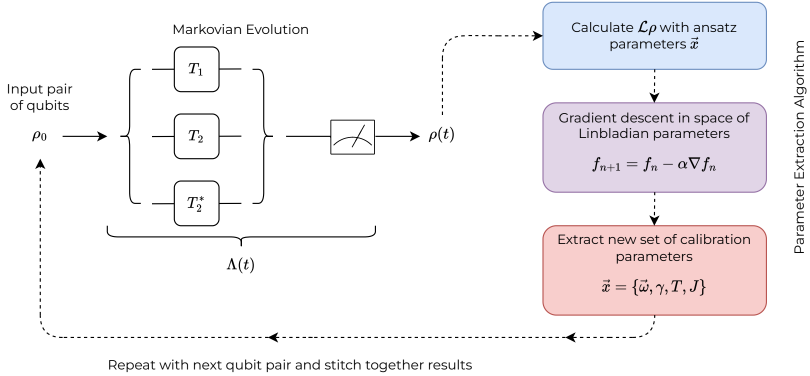
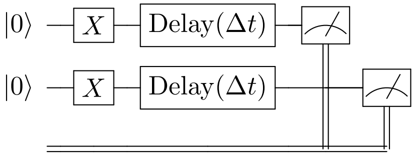
Framework


Markovian Evolution
T_1
Input pair of qubits
ρ_0
T_2
T_2^*
$\Lambda(t)$
$\rho(t)$
Calculate $\mathcal{L}\rho$ with ansatz parameters $\vec{x}$
Gradient descent in space of Linbladian parameters
$f_{n+1} = f_n - \alpha \nabla f_n$
Extract new set of calibration parameters
$\vec{x} = \{\vec{\omega}, \gamma, T, J\}$
Parameter Extraction Algorithm
Repeat with next qubit pair and stitch together results

Quantum Circuits

Relaxation Time

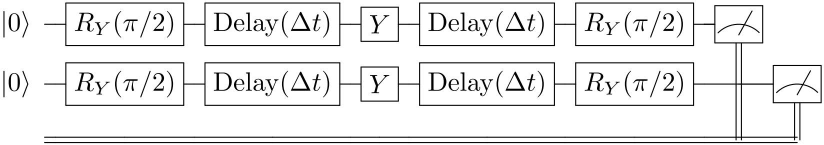
- To implement these quantum noise/decay benchmarks, they need to be translated to gate-based quantum circuits.
- The relaxation time circuit is the initialisation of the $|0\rangle^n$ state, an excitation through an X gate, and a variable delay before measurement,



Quantum Circuits

Decoherence Time

- The decoherence time can have two forms based on the desired dynamics, T_2 which is the standard decoherence time, or T_2^* which is a slight variation to the sequence of operations but leads to different results.
- The standard T_2 sequence takes the initial $|0\rangle^n$ state to the xy-plane where it undergoes decoherence, after which it is flipped using a π-rotation, left to decohere again, and taken back to the ground state.



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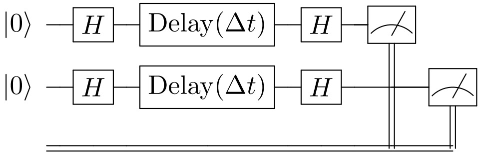
Quantum Circuits

Decoherence Time

- The T_2^* sequence varies this to excite the initial state $|0\rangle^n$ to the xy-plane through a Hadamard operation, then left to decohere before being taken back to the ground state through a Hadamard operation.



Quantum Circuits

- These can be altered to only use one qubit, or make use of multiple qubits to observe the changes in dynamics.
- With multiple qubits, these sequences can be varied further to mix T_1 and T_2 experiments.
- However, with larger quantum circuits, the analytical and numerical modelling becomes exponentially more difficult, making it better to stick to a maximum of 3 qubits.

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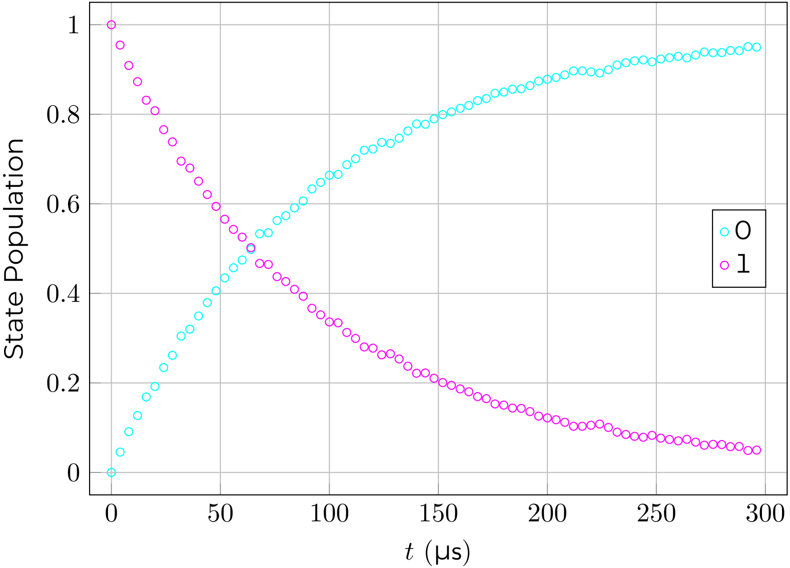
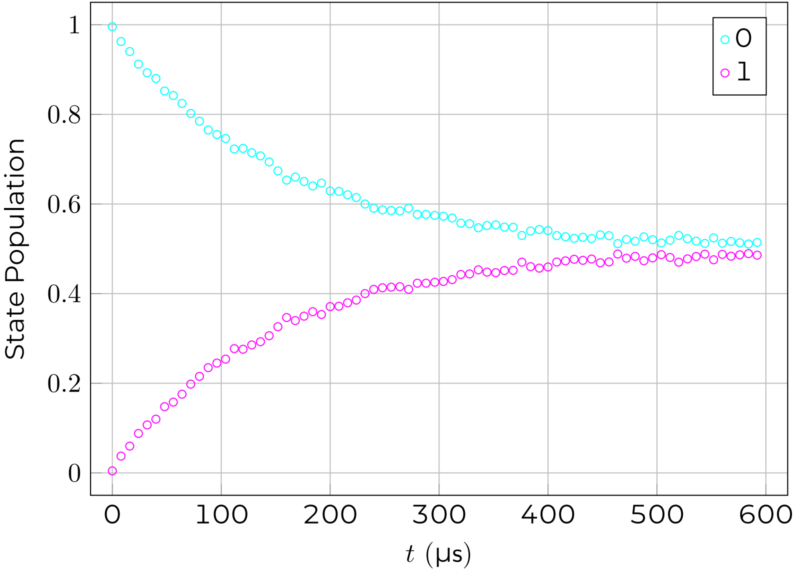
Experimental Data

1 Qubit T_1 Time



Experimental Data

1 Qubit T_2 Time



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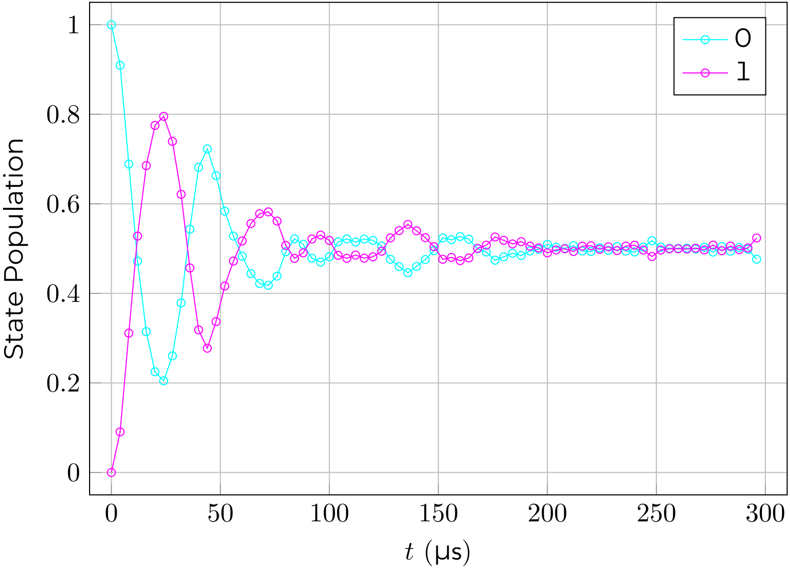
Experimental Data

1 Qubit T_2^* Time



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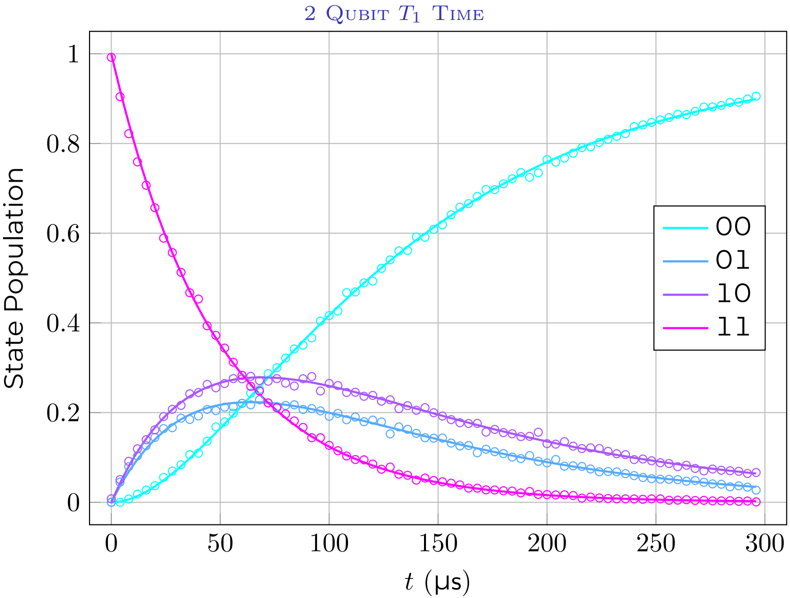
Numerical Solutions

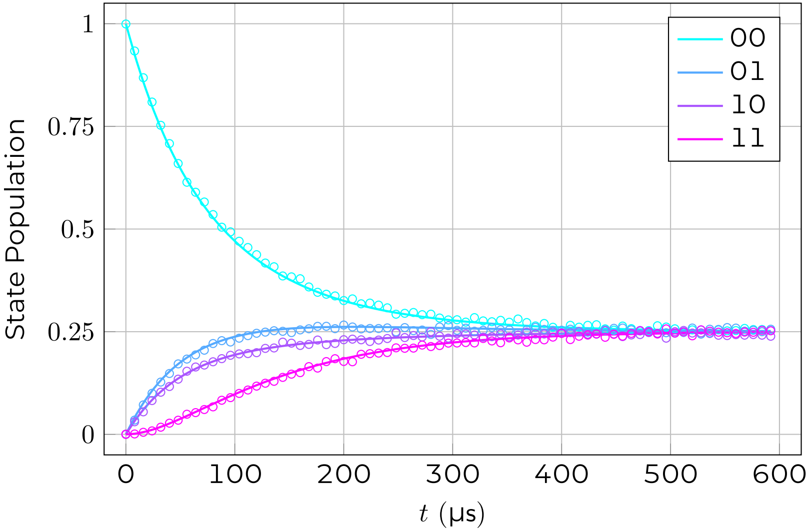
- The IBMQ SDK, *Qiskit*, provides hardware parameters such as the qubit frequency, measured T_1 and T_2 times, and coupling strengths which can be used in calculations related to the quantum device.
- These parameters can be used as initial conditions in the numerical integration of the GKSL Master Equation.
- In this numerical integration, the master equation was solved in the context of the desired quantum channel to match the quantum circuits shown before.
- The numerical integration solutions could then be used in an optimisation algorithm to find parameters which fit the experimental data better than the claimed parameters.

Optimised Results



Optimised Results

2 Qubit T_2 Time



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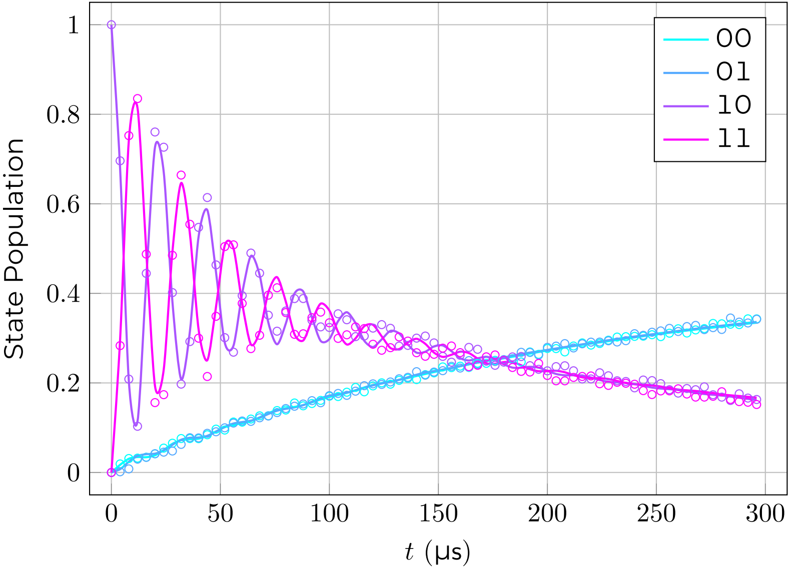
Optimised Results

2 Qubit T_2^* & T_1 Times



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Parameter Extraction

- After the optimisation algorithm found a best fit to all of the experimental data, the parameters of the master equation could be extracted and compared to the parameters which were claimed.
- The findings were that the frequencies were accurate in most cases, the inter-qubit coupling was more complex than the Duffing oscillator form, and the relaxation and decoherence times were slightly varied.
- This proved to be a better and more self-consistent way of measuring these hardware parameters in one experiment rather than several different ones.
- This method also allowed for the calculation of qubit temperatures based on qubit frequency and photon emission, which was consistent with the order of magnitude claimed by the backend, $\sim 10\,\mathrm{mK}$.

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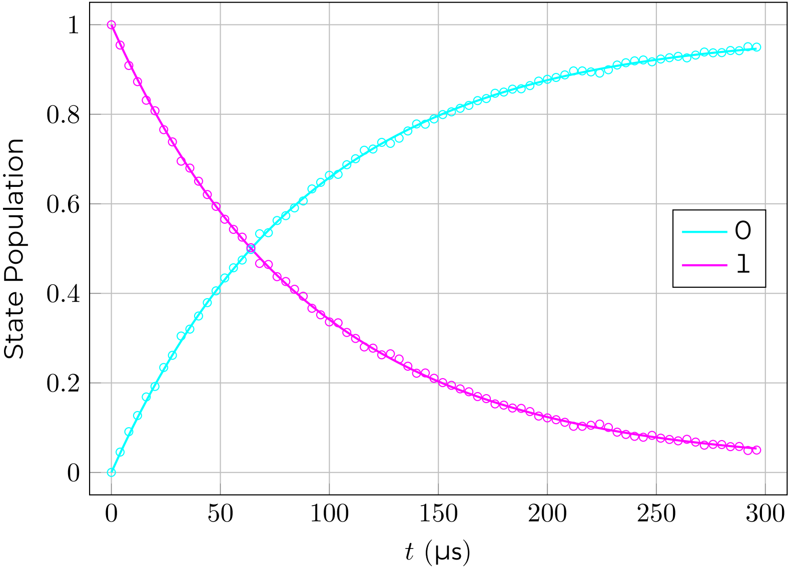
Consistency

- The results were found to be consistent not only between pairings of qubits, but also between systems of various sizes.
- Across pairings of two qubits, the results of overlapping qubits were matching accurately within margin of error between experimental runs.
- The results from the 1-qubit systems matched the 2-qubit and 3-qubit systems.
- This means that the method of optimisation and extraction combines and scales linearly.
- This method can be performed on several sets of 2 and 3 qubits then combined for a complete description of the system, rather than the usual methods of full quantum state/process tomography.

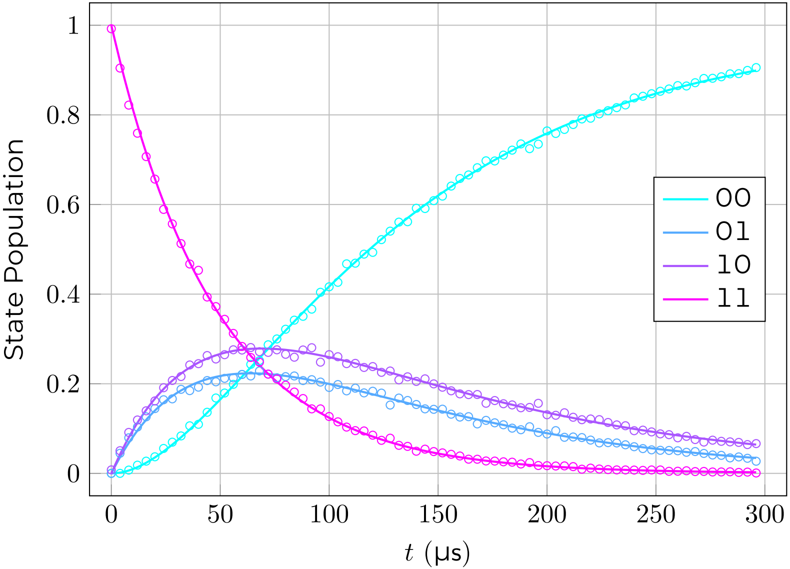
Consistency

1 Qubit



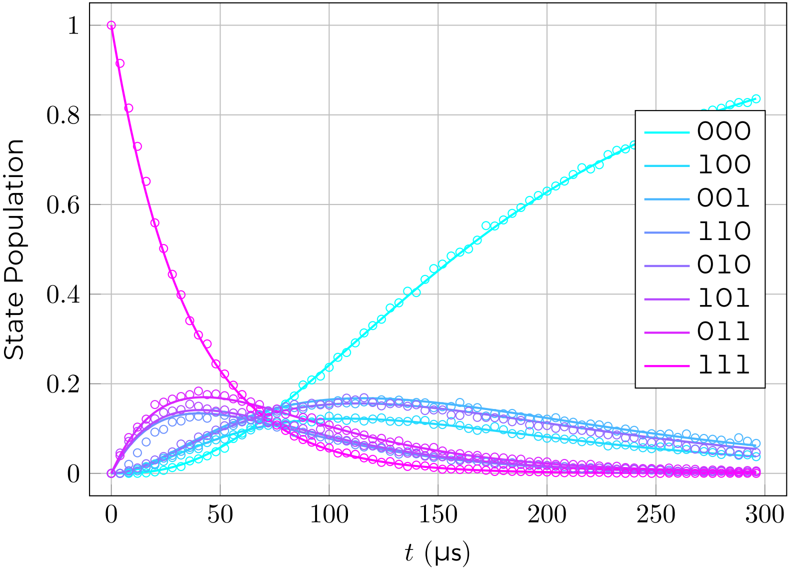
Consistency

2 Qubits



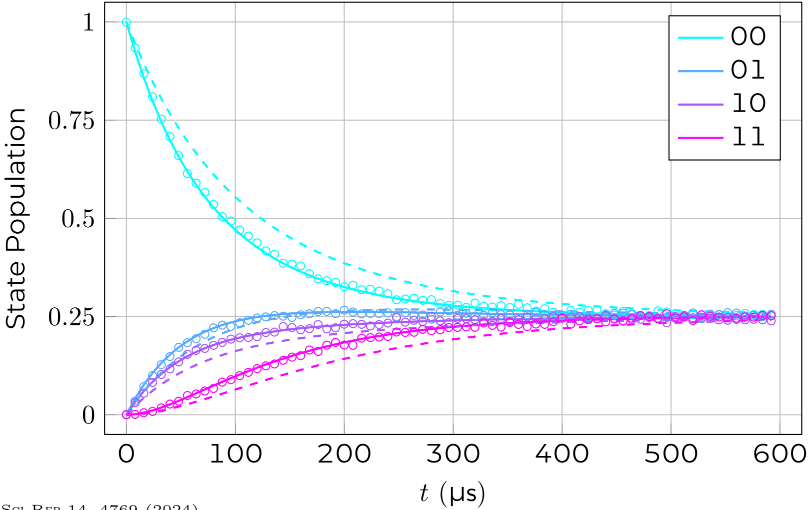
Consistency

3 Qubits



Predictive Ability

- The model is of limited use if it cannot be used on circuits other than those which are used in optimisation.



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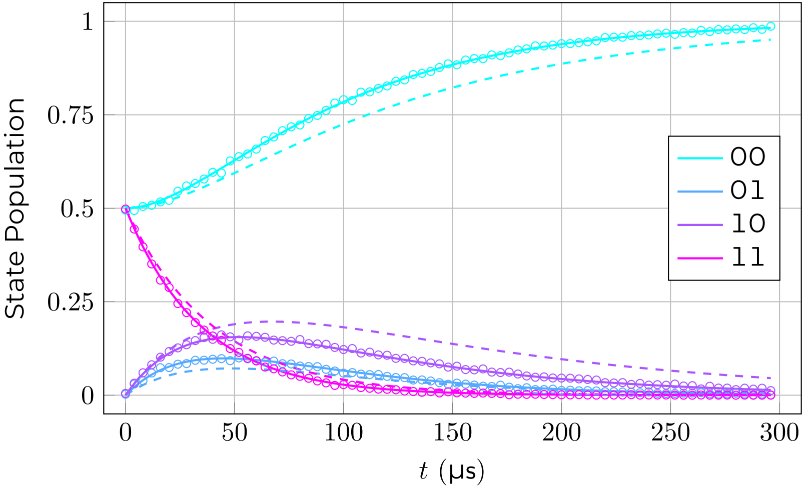
Predictive Ability

- We can use T_2 optimised parameters to obtain better results for any circuit, here shown for the Bell state, $|\Phi^+\rangle$, left to decay.



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QUESTIONS?

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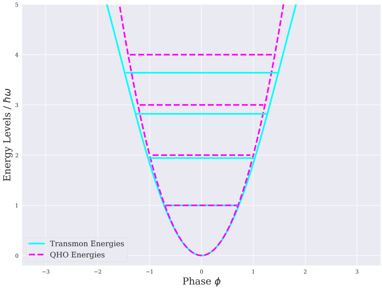
Superconducting Transmon Qubits

- Using superconducting circuits leads to Cooper pairs of electrons, the presence or absence of which form the quantum states $|0\rangle$ and $|1\rangle$, respectively.
- The *transmon* architecture introduces a difference in energy level spacing, to avoid accidentally driving the wrong qubit channel.
- This forms an artificial two-level system which behave as coupled qubits.



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Numerical Solutions

- The Adam optimiser has an adaptive learning rate for more efficient optimisation, and is typically used in artificial neural networks.