Q1. 21 January Shift 2

If the line $\alpha x + 4y = \sqrt{7}$, where $\alpha \in \mathbf{R}$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in the first quadrant, then one of the focal distances of P is :

(1) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$ (2) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{5}}$ (3) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{5}}$ (4) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{11}}$

Q2. 22 January Shift 2

Let S and S′ be the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $\mathrm{P}(\alpha, \beta)$ be a point on the ellipse in the first quadrant. If $(\mathrm{SP})^2 + (\mathrm{S'P})^2 - \mathrm{SP} \cdot \mathrm{S'P} = 37$, then $\alpha^2 + \beta^2$ is equal to :

(1) 15 (2) 11 (3) 17 (4) 13

Q3. 23 January Shift 1

Let the line $y - x = 1$ intersect the ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ at the points A and B. Then the angle made by the line segment AB at the center of the ellipse is :

(1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$ (2) $\frac{\pi}{2} + 2\tan^{-1}\left(\frac{1}{4}\right)$
(3) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (4) $\pi - \tan^{-1}\left(\frac{1}{4}\right)$

Q4. 24 January Shift 1

Let each of the two ellipses $\mathrm{E}_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ and $\mathrm{E}_2 : \frac{x^2}{\mathrm{A}^2} + \frac{y^2}{\mathrm{B}^2} = 1, (\mathrm{A} < \mathrm{B})$ have eccentricity $\frac{4}{5}$. Let the lengths of the latus recta of E_1 and E_2 be l_1 and l_2, respectively, such that $2l_1^2 = 9l_2$. If the distance between the foci of E_1 is 8, then the distance between the foci of E_2 is

(1) $\frac{32}{5}$ (2) $\frac{8}{5}$ (3) $\frac{16}{5}$ (4) $\frac{96}{5}$

Q5. 24 January Shift 2

Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$, be 30. If its eccentricity is the maximum value of the function $f(t) = -\frac{3}{4} + 2t - t^2$, then $(a^2 + b^2)$ is equal to

(1) 276 (2) 256 (3) 516 (4) 496

Q6. 24 January Shift 2

Let (h, k) lie on the circle $\mathrm{C} : x^2 + y^2 = 4$ and the point $(2h + 1, 3k + 2)$ lie on an ellipse with eccentricity e. Then the value of $\frac{5}{e^2}$ is equal to ____.

Q7. 28 January Shift 2

An ellipse has its center at $(1, -2)$, one focus at $(3, -2)$ and one vertex at $(5, -2)$. Then the length of its latus rectum is :

(1) 6 (2) $4\sqrt{3}$ (3) $\frac{16}{\sqrt{3}}$ (4) $6\sqrt{3}$

Q8. 28 January Shift 2

Let the ellipse $E : \frac{x^2}{144} + \frac{y^2}{169} = 1$ and the hyperbola $H : \frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1$ have the same foci. If e and L respectively denote the eccentricity and the length of the latus rectum of H, then the value of $24(e + L)$ is :

(1) 148 (2) 126 (3) 67 (4) 296

ANSWER KEYS

1. (1) **2.** (4) **3.** (2) **4.** (1) **5.** (4) **6.** 9 **7.** (1) **8.** (4)