

Q1. 21 January Shift 1

Let $y = y(x)$ be the solution curve of the differential equation $(1 + x^2)dy + (y - \tan^{-1} x)dx = 0, y(0) = 1$. Then the value of $y(1)$ is :

- (1) $\frac{4}{e^{\pi/4}} + \frac{\pi}{2} - 1$ (2) $\frac{2}{e^{\pi/4}} + \frac{\pi}{4} - 1$
(3) $\frac{2}{e^{\pi/4}} - \frac{\pi}{4} - 1$ (4) $\frac{4}{e^{\pi/4}} - \frac{\pi}{2} - 1$

Q2. 21 January Shift 2

Let $y = y(x)$ be the solution of the differential equation $\sec x \frac{dy}{dx} - 2y = 2 + 3 \sin x, x \in (-\frac{\pi}{2}, \frac{\pi}{2}), y(0) = \frac{7}{4}$. Then $y(\frac{\pi}{6})$ is equal to :

- (1) $-\frac{5}{4}$ (2) $-\frac{5}{2}$ (3) $-3\sqrt{3} - 7$ (4) $-3\sqrt{2} - 7$

Q3. 22 January Shift 1

Let the solution curve of the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx, x > 0, y(1) = 0$, be $y = y(x)$. Then $y(3)$ is equal to

- (1) 1 (2) 4 (3) 2 (4) 6

Q4. 22 January Shift 1

Let $f : [1, \infty) \rightarrow \mathbb{R}$ be a differentiable function. If $6 \int_1^x f(t)dt = 3xf(x) + x^3 - 4$ for all $x \geq 1$, then the value of $f(2) - f(3)$ is

- (1) -4 (2) 3 (3) 4 (4) -3

Q5. 22 January Shift 2

If $y = y(x)$ satisfies the differential equation $16(\sqrt{x + 9\sqrt{x}})(4 + \sqrt{9 + \sqrt{x}}) \cos y dy = (1 + 2 \sin y)dx, x > 0$ and $y(256) = \frac{\pi}{2}, y(49) = \alpha$, then $2 \sin \alpha$ is equal to :

- (1) $2(\sqrt{2} - 1)$ (2) $3(\sqrt{2} - 1)$ (3) $2\sqrt{2} - 1$ (4) $\sqrt{2} - 1$

Q6. 23 January Shift 1

Let $y = y(x)$ be the solution of the differential equation $x^4 dy + (4x^3 y + 2 \sin x)dx = 0, x > 0, y(\frac{\pi}{2}) = 0$. Then $\pi^4 y(\frac{\pi}{3})$ is equal to :

- (1) 72 (2) 92 (3) 64 (4) 81

Q7. 23 January Shift 1

Let f be a twice differentiable non-negative function such that $(f(x))^2 = 25 + \int_0^x ((f(t))^2 + (f'(t))^2) dt$. Then the mean of $f(\log_e(1)), f(\log_e(2)), \dots, f(\log_e(625))$ is equal to ____.

Q8. 23 January Shift 2

If the solution curve $y = f(x)$ of the differential equation

$$(x^2 - 4)y' - 2xy + 2x(4 - x^2)^2 = 0, x > 2,$$

passes through the point $(3, 15)$, then the local maximum value of f is ____.

Q9. 24 January Shift 2

Let $y = y(x)$ be a differentiable function in the interval $(0, \infty)$ such that $y(1) = 2$, and $\lim_{t \rightarrow x} \left(\frac{t^2 y(x) - x^2 y(t)}{x - t} \right) = 3$ for each $x > 0$. Then $2y(2)$ is equal to

- (1) 23 (2) 27 (3) 12 (4) 18

Q10. 28 January Shift 1

Let $y = y(x)$ be the solution of the differential equation $x \frac{dy}{dx} - \sin 2y = x^3 (2 - x^3) \cos^2 y$, $x \neq 0$. If $y(2) = 0$, then $\tan(y(1))$ is equal to

- (1) $-\frac{3}{4}$ (2) $\frac{3}{4}$ (3) $-\frac{7}{4}$ (4) $\frac{7}{4}$

Q11. 28 January Shift 2

Let $y = y(x)$ be the solution of the differential equation $x \frac{dy}{dx} - y = x^2 \cot x$, $x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then $6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$ is equal to :

- (1) 3π (2) π (3) $-\pi$ (4) -3π

ANSWER KEYS

1. (2) 2. (2) 3. (2) 4. (2) 5. (3) 6. (3) 7. 1565 8. 16
9. (1) 10. (4) 11. (3)