Q1. 21 January Shift 1

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be a twice differentiable function such that the quadratic equation $f(x)\mathrm{m}^2 - 2f'(x)\mathrm{m} + f''(x) = 0$ in m, has two equal roots for every $x \in \mathbf{R}$. If $f(0) = 1, f'(0) = 2$, and (α, β) is the largest interval in which the function $f(\log_e x - x)$ is increasing, then $\alpha + \beta$ is equal to ____.

Q2. 21 January Shift 2

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$ and $f'(a-1) = 0$, where a is a real number. Let $g(x) = f\left(\tan^2 x - 2\tan x + a\right),\ 0 < x < \frac{\pi}{2}$.

Consider the following two statements:

(I) g is increasing in $\left(0, \frac{\pi}{4}\right)$

(II) g is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then,

(1) Neither (I) nor (II) is True
(2) Only (I) is True
(3) Both (I) and (II) are True
(4) Only (II) is True

Q3. 22 January Shift 1

Let $f(x) = x^{2025} - x^{2000}, x \in [0,1]$ and the minimum value of the function $f(x)$ in the interval $[0,1]$ be $(80)^{80}(n)^{-81}$. Then n is equal to

(1) -40
(2) -41
(3) -80
(4) -81

Q4. 24 January Shift 1

Let $(2\alpha, \alpha)$ be the largest interval in which the function $f(t) = \frac{|t+1|}{t^2}, t < 0$, is strictly decreasing. Then the local maximum value of the function $g(x) = 2\log_e(x-2) + \alpha x^2 + 4x - \alpha, x > 2$, is ____

Q5. 24 January Shift 2

Consider the following three statements for the function $f: (0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = |\log_e x| - |x - 1|$:

(I) f is differentiable at all $x > 0$.

(II) f is increasing in $(0, 1)$.

(III) f is decreasing in $(1, \infty)$.

Then.

(1) All (I), (II) and (III) are TRUE.
(2) Only (II) and (III) are TRUE.
(3) Only (I) is TRUE.
(4) Only (I) and (III) are TRUE.

Q6. 28 January Shift 2

Let f be a differentiable function satisfying $f(x)=1-2x+\int_0^x \mathrm{e}^{(x-t)} f(t)\mathrm{dt}, x \in \mathbf{R}$ and let $\mathrm{g}(x)=\int_0^x (f(\mathrm{t})+2)^{15}(\mathrm{t}-4)^6(\mathrm{t}+12)^{17}\mathrm{dt}, x \in \mathbf{R}$. If p and q are respectively the points of local minima and local maxima of g, then the value of $|\mathrm{p}+\mathrm{q}|$ is equal to ____.

ANSWER KEYS

1. 1	**2.** (1)	**3.** (4)	**4.** 4	**5.** (4)	**6.** 9