Assume that $\tilde{y}$ is $n \times 1$ and X is $n \times (p+1)$ we can augment X as

$$X^* = \begin{pmatrix} X \\ \\ wI \end{pmatrix}$$

where λI is $(p+1) \times (p+1)$ making X^* a $(n+p+1) \times (p+1)$ matrix, for

$\lambda > 0$. We can augment $\tilde{y}$ as $\tilde{y}^* = \begin{pmatrix} \tilde{y} \\ 0 \end{pmatrix}$ where

$\tilde{y}^*$ is $(n+p+1) \times 1$ vector.

We can write the least squares problem now as:

$$\| \vec{y}^* - X^* \beta \|_2^2$$

$$= \left\| \begin{pmatrix} \vec{y} - X\beta \\ W\beta \end{pmatrix} \right\|_2^2$$

$$= \| \vec{y} - X\beta \|_2^2 + W^2 \| \beta \|_2^2$$

If we write the Lasso this

becomes:

$$\|\vec{y}^{*} - X^{*}\beta\|_2^2 + t\|\beta\|_1$$

$$= \|\vec{y} - X\beta\|_2^2 + w^2\|\beta\|_2^2 + t\|\beta\|_1$$

If $w^2 = \lambda\alpha$

$w = \sqrt{\lambda\alpha}$

and $t = \lambda(1-\alpha)$ we recover the elastic-net using X^{*} and $\vec{y}^{*}$. $\square$