The lasso problem is given by

$$\min_{\beta} \|Y - X\beta\|_2^2 + \lambda \|\beta\|_1 .$$

If we assume the lasso coeff for X_j is $\hat{\beta}_j = a$ and we augment with $X_j^* = X_j$ then the solution set becomes:

$$\hat{\beta}^{lasso} = \min_{\beta} \sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{l=1}^{p} x_{il} \beta_l \right)^2$$

$$\sum_{l=1}^{p} |\beta_l| \leq t$$

$$\Rightarrow \quad \hat{\beta}^{lasso'} = \min_{\beta} \sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{l \neq j}^{p} x_{il} \beta_l - x_{ij} \beta_j^* \right)^2$$

where $\sum_{l=1}^{p} |\beta_l| + |\beta_j^*| \leq t$.

Further, we know $\bar{\beta}_j = \beta_j + \beta_j^*$

for some $j \in \{1, .., p\}$ and reason

that given an optimal solution

to the original Lasso, we can

set $|\bar{\beta}_j| - |\beta_j| - |\beta_j^*| = 0$ when

$\beta_j = a/2 = \beta_j^*$ then

$|a| - |a/2| - |a/2| = 0$ and

so the collinearity for the β_j^{th} term gets absorbed by dividing it by 2 $\square$