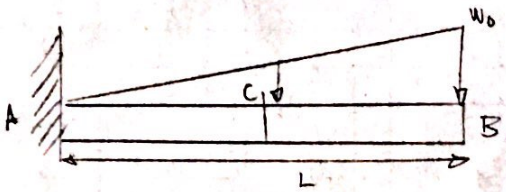
ENGR 213 HW #5 | Due 5/6/20 | Sean Lai

CH5, #'s 3, 8, 9, 11, 21, 36, 52, 56, 65, 71, 91

5.3:

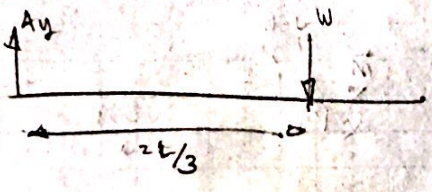
Given:



Find: a) draw shear and bending moment diagrams
b) determine eq's of shear and bending moment curves.

Solution:

FBD:

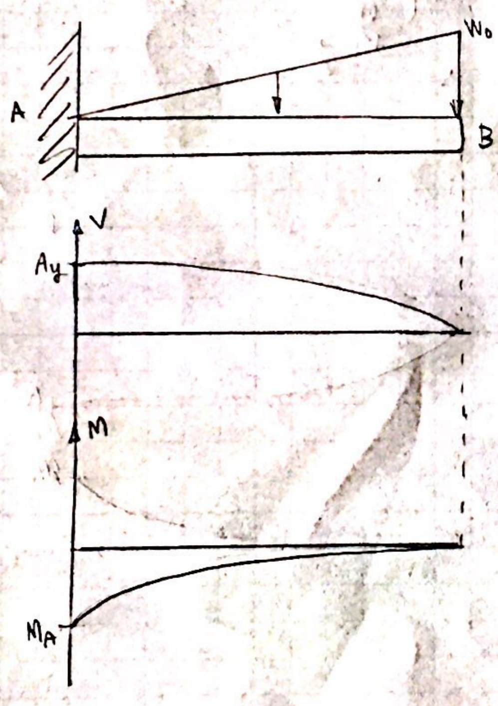


$W = \frac{w_0 L}{2}$

$\Sigma M_A: \; M_A - \frac{2L}{3}\left(\frac{w_0 L}{2}\right) = 0$

$\rightarrow \; M_A = \frac{w_0 L^2}{3}, \quad A_y = W$

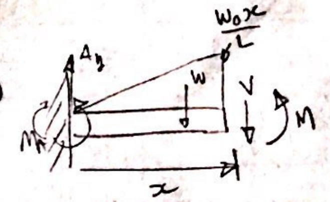
a)



$A_y = \frac{w_0 L}{2}$

$M_A = \frac{w_0 L^2}{3}$

b)



$W = \frac{w_0 x^2}{L}, \quad A_y = \frac{w_0 L}{2} \rightarrow \frac{w_0 L}{2} - \frac{w_0 x^2}{L} = V$

$V = w_0\left(\frac{L}{2} - \frac{x^2}{L}\right)$

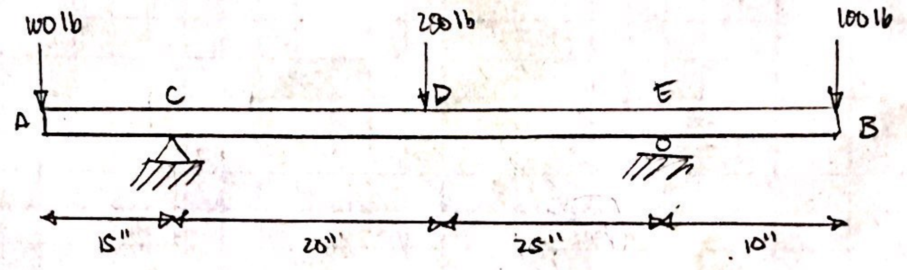
$\Sigma M_x = + \frac{w_0 x^2}{2L} \cdot \frac{x}{3} - \frac{w_0 L}{2} x + M_A + M = 0$

$\rightarrow M = +\frac{w_0 L x}{2} + \frac{w_0 L^2}{3} - \frac{w_0 x^3}{6L}$

$$\boxed{\begin{aligned} V &= w_0\left(\frac{L}{2} - \frac{x^2}{L}\right) \\ M &= -\frac{w_0 L x}{2} - \frac{w_0 L^2}{3} - \frac{w_0 x^3}{6L} \end{aligned}}$$

S.8:

Given:



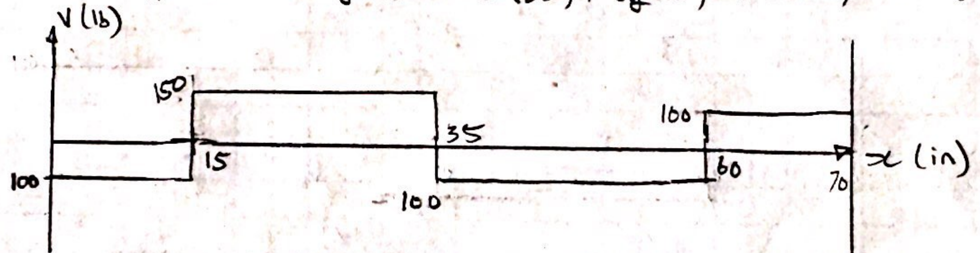
Find: Draw shear and bending moment diagrams and find
a) maximum shear
b) maximum bending moment

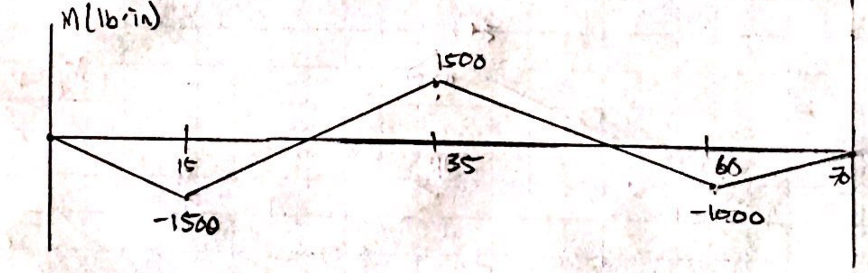
Solution:

$$\Sigma F_y = 0 \rightarrow C_y + E_y - 450 = 0$$

$$\Sigma M_A = 0 \rightarrow C_y(15) - 250(35) + E_y(60) - 100(70)$$

$$\left.\right\} \begin{array}{l} C_y = 250\text{ lb} \\ E_y = 200\text{ lb} \end{array}$$





$$M_{15} + 100(15) = 0 \rightarrow M_{15} = -1500$$
$$M_D + 100(35) - 250(20) = 0 \rightarrow M_D = 1500$$
$$M_E + 100(60) - 250(45) + 250(25) = 0 \rightarrow M_E = 1000$$

a) Maximum shear: $\boxed{|V_{max}| = 150\text{ lb}}$

b) Maximum bending moment: $\boxed{|M_{max}| = 1500\text{ lb in}}$

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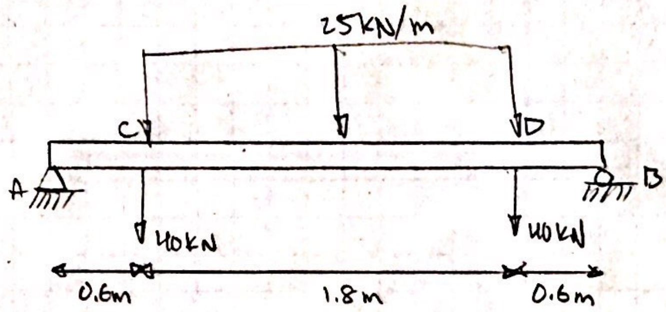
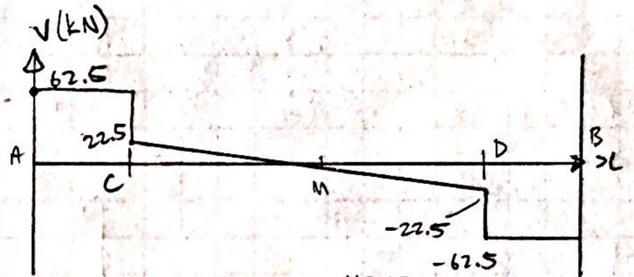
5.9:

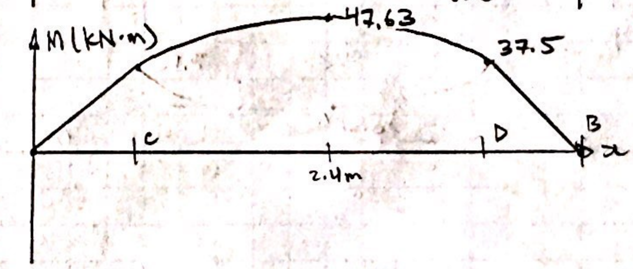
Given:



Find: draw shear + bending moment diagrams and find maximum absolute
a) shear
b) bending moment

Solution:





$\Sigma F_y \rightarrow A_y + B_y - 80kN - 25kN/m(1.8m) = 0$ ①

$\Sigma M_A \rightarrow -40(0.6) - 25 \cdot 1.8 \cdot 1.5 - 40(2.4) + B_y(3) = 0$ ②

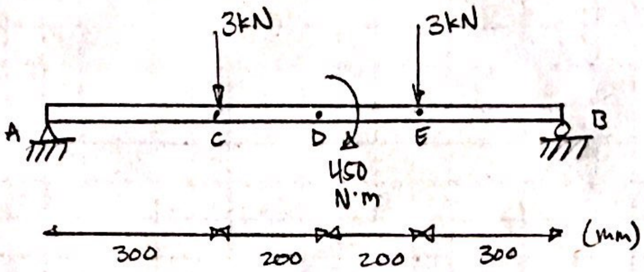
from ②, $B_y = 62.5$ kN
from ①, $A_y = 62.5$ kN

$M_C - A_y(0.6) = 0 \rightarrow M_A = 37.5$ kN·m
$M_M - A_y(1.5) + 40(0.9) + 25(0.9)(0.45) = 47.63$ kN·m
$M_D - A_y(2.4) + 40(1.8) + 25(1.8)(0.9) = 37.5$ kN·m

a) $V_{max} = 62.5$ kN
b) $M_{max} = 47.63$ kN·m

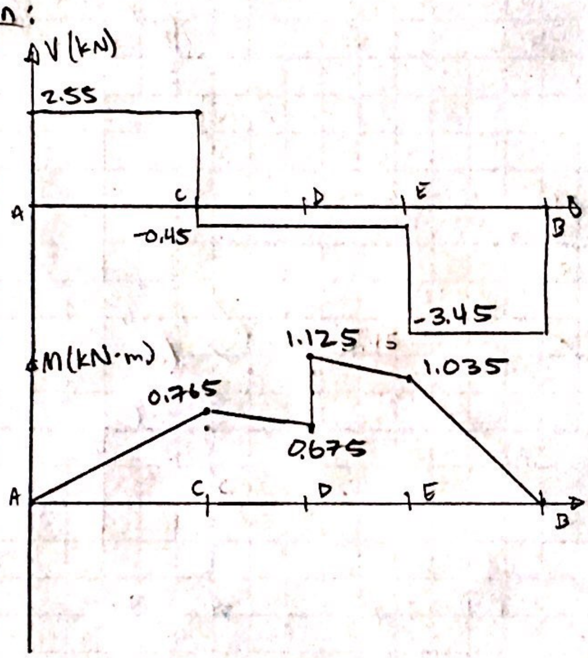
5.11:

Given:



Find: Draw diagrams and find maximum absolute
a) shear
b) bending moment

Solution:



$\Sigma F_y: \; A_y + B_y - 6\,kN = 0$

$\Sigma M_A: \; -3(0.3) - 3(0.7) + B_y(1) - 0.45 = 0$

$B_y = 3.45\,kN$

$\rightarrow A_y = 2.55\,kN$

$M_C - A_y(0.3) = 0 \rightarrow M_C = 0.765$

$M_{DL} - A_y(0.5) + 3(0.2) = 0 \rightarrow M_{D_L} = 0.675$

$M_{DR} - A_y(0.5) + 3(0.2) - 0.45 = 0 \rightarrow M_{D_R} = 1.125$

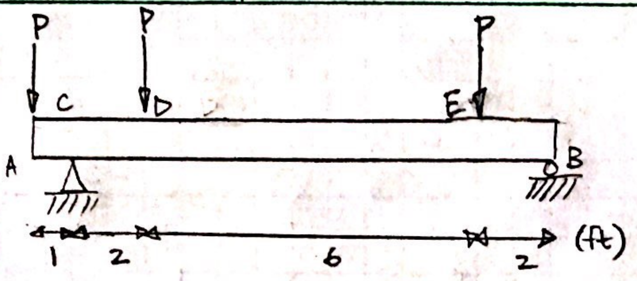
$M_E - A_y(0.7) + 3(0.4) - 0.45 = 0 \rightarrow M_E = 1.035$

$M_B - A_y(1) + 3(0.7+0.3) - 0.45 = 0 \rightarrow M_B = 0$ ✓

$\boxed{\begin{array}{l} a)\; |V_{max}| = 3.45\,kN \\ b)\; |M_{max}| = 1.125\,kN \cdot m \end{array}}$

5.21:

Given:



$P = 25\text{ kips}$

S12×35

Find: Draw diagrams and determine σ_{max} due to bending

Solution:

$\Sigma F_y: \; C_y + B_y - 3P = 0$

$\Sigma M_C: \; P - 2P - 8P + 10B_y = 0$

$B_y = 22.5\text{ kips}$

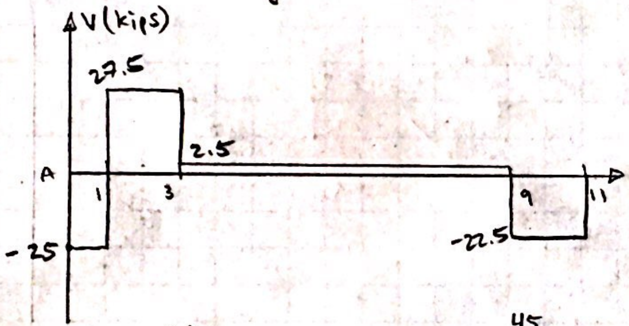
$\rightarrow \; C_y = 52.5\text{ kips}$

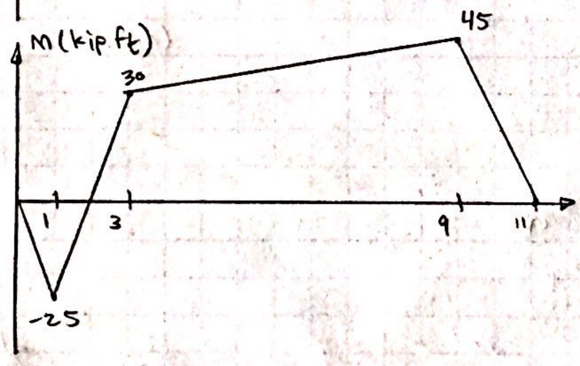
$M_C + 1P = 0 \quad \longrightarrow \quad M_C = -P = -25$

$M_D + 3P - 2C_y = 0 \quad\quad M_D = 30$

$M_E + 9P - 8C_y + 6P = 0 \quad \longrightarrow \quad M_E = 45$

$M_B + 11P - 10C_y + 8P + 2P = 0 \quad \longrightarrow \quad M_B = 0$ ✓





$M_{max} = 45\text{ kip ft.}$

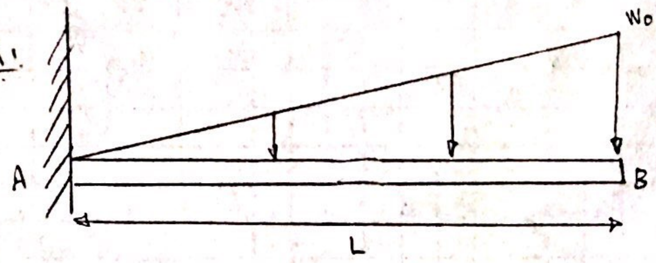
$\sigma_{max} = \dfrac{Mc}{I}$, $c = 6\text{ in}$, $I = 228\text{ in}^4$, $M_{max} = 540\text{ kip in}$

$\sigma_{max} = \dfrac{540\text{ kip in} \cdot 6\text{ in}}{228\text{ in}^4}$

$\boxed{\sigma_{max} = 14.2\text{ ksi}}$

5.36:

Given:

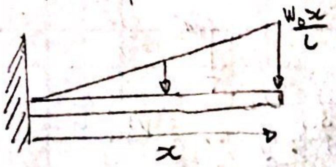


Find: a) Draw shear and bending moment diagrams using derivatives/areas

Solution: $\frac{dV}{dx} = -w$, $\frac{dM}{dx} = V$

ΣF_y: $A_y = \frac{w_o L}{2}$

ΣM_A: $M_A = -\frac{w_o L}{2}\left(\frac{2L}{3}\right) = -\frac{w_o L^2}{3}$



$\rightarrow \int dV = -\int \frac{w_o x}{L} dx$

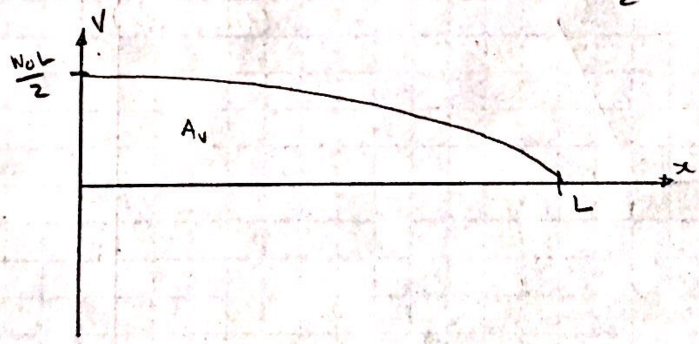
$V - V_A = -\frac{w_o x^2}{2L}$

$\rightarrow V = \frac{w_o L}{2} - \frac{w_o x^2}{2L}$ ①

$\int dM = \int \frac{w_o L}{2} - \frac{w_o x^2}{2L}$

$M - M_A = \frac{w_o L}{2} x - \frac{w_o x^3}{6L}$

$\rightarrow M = \frac{w_o L}{2} x - \frac{w_o x^3}{6L} + -\frac{w_o L^2}{3}$

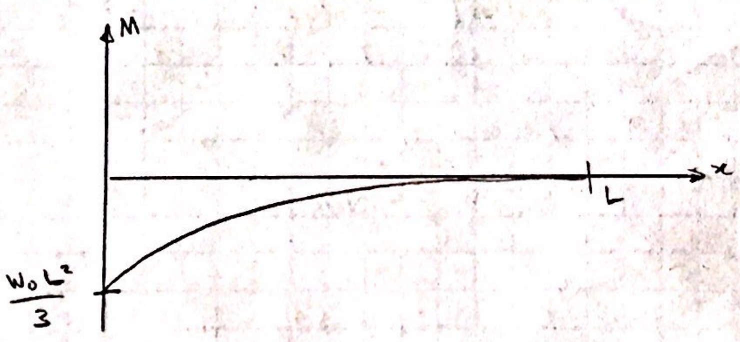


$A_v = \int_0^L \frac{w_o L}{2} - \frac{w_o x^2}{2L} dx$

$A_v = \frac{w_o L}{2} x - \frac{w_o}{6L} x^3 \Big|_0^L$

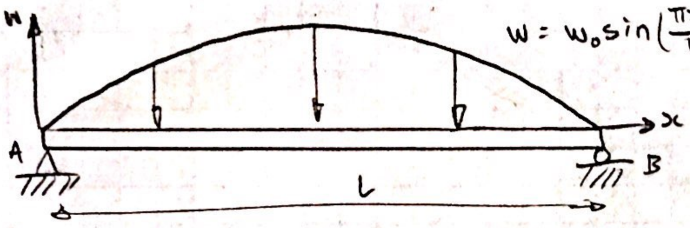
$A_v = \frac{w_o L^2}{2} - \frac{w_o}{6} L^2$

$A_v = \frac{w_o L^2}{3} = -M_A$ ✓



5.52:

Given:



Find: a) Eq's of shear and bending moment
b) $|M_{max}|$

Solution:

a) $\frac{dV}{dx} = -w$, $\frac{dM}{dx} = V$

note, V, M = 0 at points A, B

$\int dV = \int -w_0 \sin\left(\frac{\pi x}{L}\right) dx$ $\quad u = \frac{\pi}{L}x \quad du = \frac{\pi}{L} dx$

$V = \int -w_0 \sin u \frac{L}{\pi} du$

$$\boxed{V = +w_0 \frac{L}{\pi} \cos\left(\frac{\pi}{L}x\right)}$$

$\int dM = \int w_0 \frac{L}{\pi} \cos\left(\frac{\pi}{L}x\right) dx$

$$\boxed{M = w_0 \frac{L^2}{\pi^2} \sin\left(\frac{\pi}{L}x\right)}$$

b) set $\frac{dM}{dx} = 0 \rightarrow w_0 \frac{L}{\pi} \cos\left(\frac{\pi}{L}x\right) = 0$

$\rightarrow \frac{\pi}{L}x = \frac{\pi}{2}$

$x = \frac{L}{2}$

$\rightarrow M_{max} = w_0 \frac{L^2}{\pi^2} \cancel{\sin\left(\frac{\pi}{L} \cdot \frac{L}{2}\right)}^{1}$

$$\boxed{M_{max} = w_0 \frac{L^2}{\pi^2}}$$

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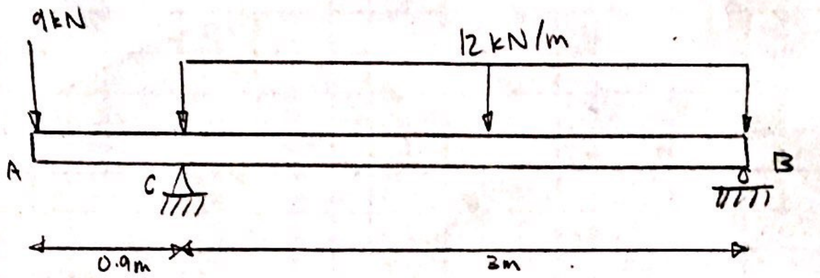
56:

Given:



Find: Draw diagrams and find maximum σ due to bending

Solution:

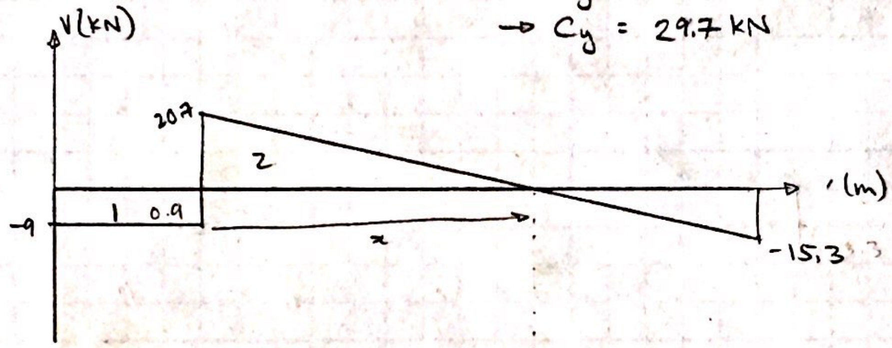
$$\Sigma F_y: \ -9 + C_y - 12(3) + B_y = 0$$

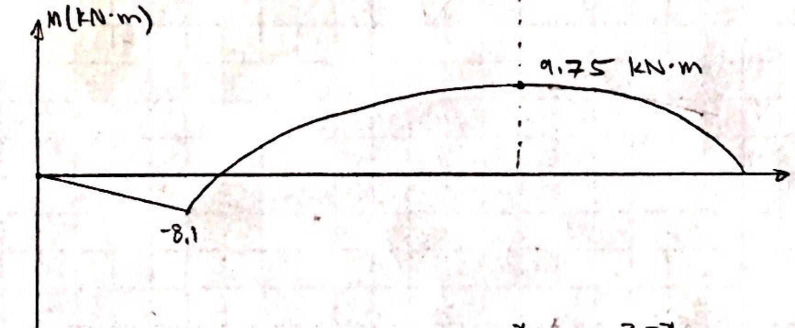
$$C_y + B_y = 45\,kN$$

$$\Sigma M_C: \ +9(0.9) - 12(3)(1.5) + B_y(3) = 0$$

$$B_y = 15.3\,kN$$

$$\rightarrow C_y = 29.7\,kN$$





solve for x in V diagram:

$$\frac{x}{20.7} = \frac{3-x}{15.3}$$

$$x = (3-x)\left(\frac{20.7}{15.3}\right)$$

$$x\left(1+\frac{20.7}{15.3}\right) = 3\left(\frac{20.7}{15.3}\right)$$

$$\rightarrow x = 1.725\,m$$

$$M_x = A_1 + A_2$$

$$= -9(0.9) + \tfrac{1}{2}\cdot 20.7 \cdot 1.725$$

$$= 9.75\,kN\cdot m$$

$$\sigma_{max} = \frac{Mc}{I}, \quad c = \frac{203\,mm}{2}, \quad I = 16.5\cdot 10^{6}\,mm^4$$

$$c = 0.1015\,m \qquad I = 1.65\cdot 10^{-5}\,m^4$$

$$\sigma_{max} = \frac{9.75\,kN\,m \cdot 0.1015\,m}{1.65\cdot 10^{-5}\,m^4}$$

$$\boxed{\sigma_{max} = 60\,MPa}$$

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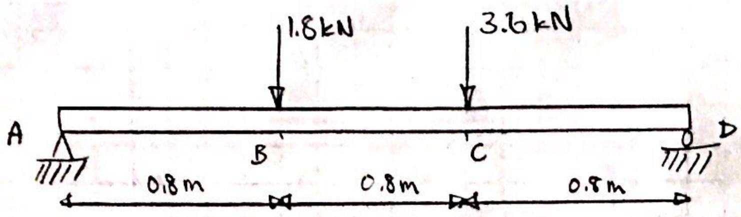
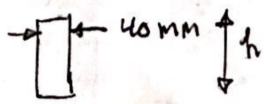
5.65:

Given:





Find: Design a cross section knowing $\sigma_{all} = 12$ MPa (find h)

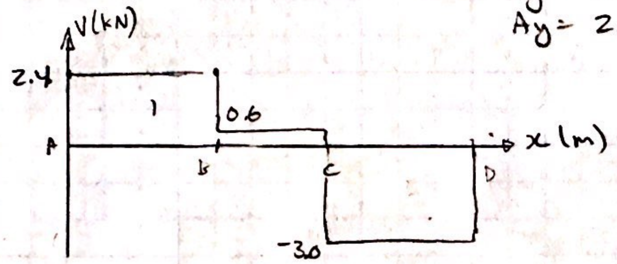
Solution:

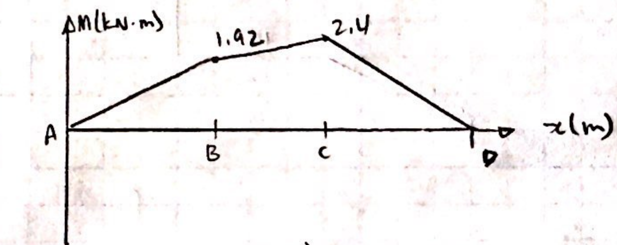
ΣF_y: $A_y + D_y = 5.4$ kN

ΣM_A: $-1.8(0.8) - 3.6(1.6) + D_y(2.4) = 0$

$\rightarrow$ $D_y = 3.0$ kN

$A_y = 2.4$ kN

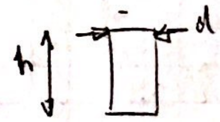




$M_B = A_1 = 2.4(0.8) = 1.92$ kN·m

$M_C = A_1 + 0.6(0.8) = 2.41$ kN·m

$\rightarrow M_{max} = 2.4$ kN·m



$d = 40$ mm $\quad c = \frac{h}{2}$

$I = \frac{1}{12} d h^3$

$\sigma = \frac{Mc}{I}$

$\sigma_{max} = \frac{M_{max} \cdot \frac{h}{2}}{\frac{1}{12} d h^3} = \frac{M_{max} \cdot 6}{d h^2}$

$h^2 = \frac{6 M_{max}}{d \sigma}$

$h = \sqrt{\frac{6 M_{max}}{d \sigma}}$

$h = 0.173$ m $= 173$ m

$\rightarrow$ $\boxed{h > 173 \text{ mm for } \sigma_{all} = 12 \text{ MPa}}$

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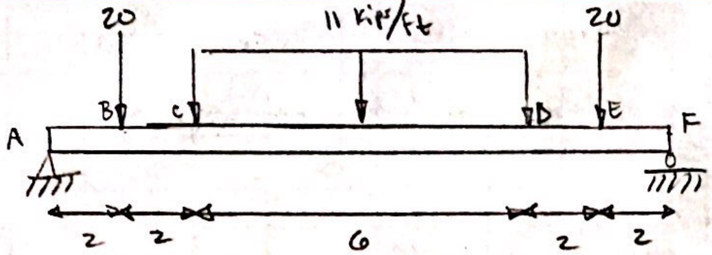
5.71:

Given:



Forces in **kips**
lengths in **ft**

$\sigma_{all} = 24\text{ ksi}$

Find: Most economical W-flange

Solution:

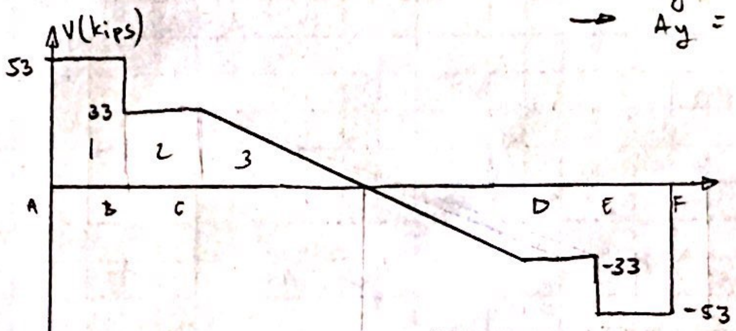
$\Sigma F_y: \quad A_y + F_y - 40 - 11(6) = 0$

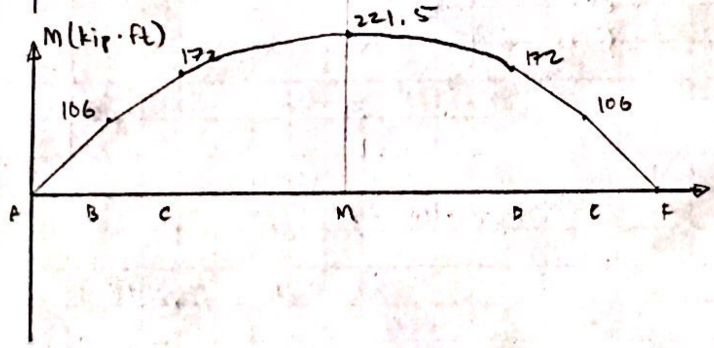
$A_y + F_y = 106\text{ kips}$

$\Sigma M_A: \quad -20(2) - 11(6)(7) - 20(12) + F_y(14) = 0$

$F_y = 53\text{ kips}$

$\rightarrow \quad A_y = 53\text{ kips}$





$M_B - 53(2) = 0 \quad \rightarrow \quad M_B = 106$

$M_C - 53(4) + 20(2) = 0 \quad \rightarrow \quad M_C = 172$

$M_M = A_1 + A_2 + A_3 \quad \rightarrow \quad M_M = 221.5$

$M_{max} = 221.5\text{ kip ft} = 2658\text{ kip}\cdot\text{in}$

$\sigma_{all} = \frac{M_{max}c}{I} \rightarrow I = \frac{M_{max}c}{\sigma_{all}}, \quad S_{min} = \frac{M_{max}}{\sigma_{all}}$

$S_{min} = 110.8\text{ in}^3$

lowest A w/ $S_{min} \rightarrow \boxed{W21\times62}$

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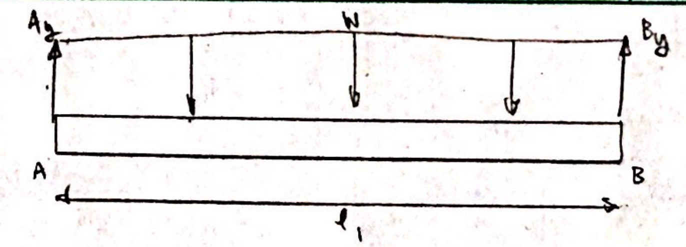
5.91:

Given:

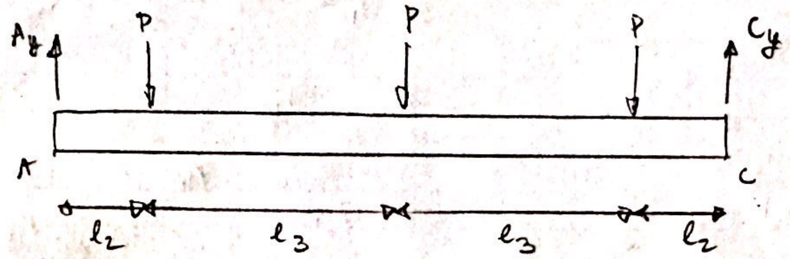
S:



$W = 64$ kip
$l = 12$ ft

$\sigma_{all} = 24$ ksi

W:



$P = 32$ kip.
$l_2 = 4$ft
$l_3 = 8$ft.

Find: a) most economical S beam
b) most economical W beam

Solution:

a) ΣF_y: $A_y + B_y - W = 0$

$\rightarrow \left.\begin{array}{l} A_y = 32 \text{ kip} \\ B_y = 32 \text{ kip} \end{array}\right\}$ by symmetry

$M_m = -A_y \cdot \frac{l_1}{2} + \frac{W}{2}\left(\frac{l_1}{4}\right)$

$= 96$ kip$\cdot$ft $\rightarrow M_{max,s}$

$S > \frac{M_{max,s}}{\sigma_{all}} = \frac{1152 \text{ kip in}}{24 \text{ ksi}}$

$\rightarrow S > 48 \rightarrow \boxed{S15 \times 42.9}$

b) ΣF_y: $A_y + C_y = 3P$

$\rightarrow A_y = C_y = 1.5P$

M_m: $-A_y(12) + P(8) = M_m$

$M_m = 320$ kip$\cdot$ft

$S > \frac{3840 \text{ kip in}}{24 \text{ ksi}}$

$S > 160 \rightarrow \boxed{W27 \times 84}$