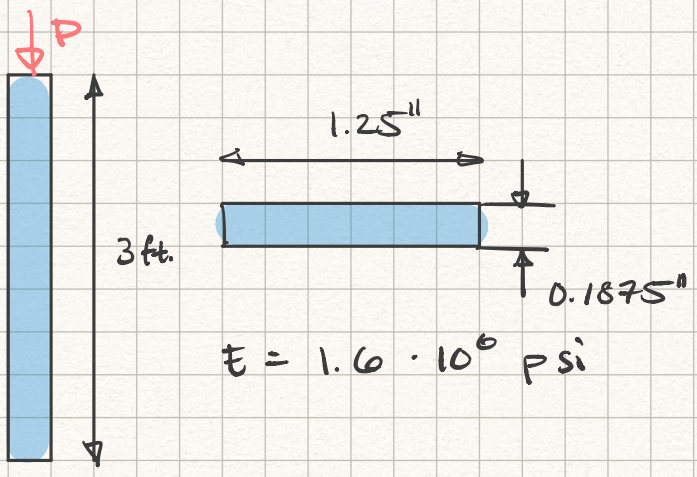
ENGR 213 HW#9, Due 6/1/20, Sean Lai

CH10: 10, 19, 27, 62

10.10

Given:



$E = 1.6 \cdot 10^6$ psi

Find: critical load

Solution:

$$P_{cr} = \frac{\pi^2 E I}{L^2}$$

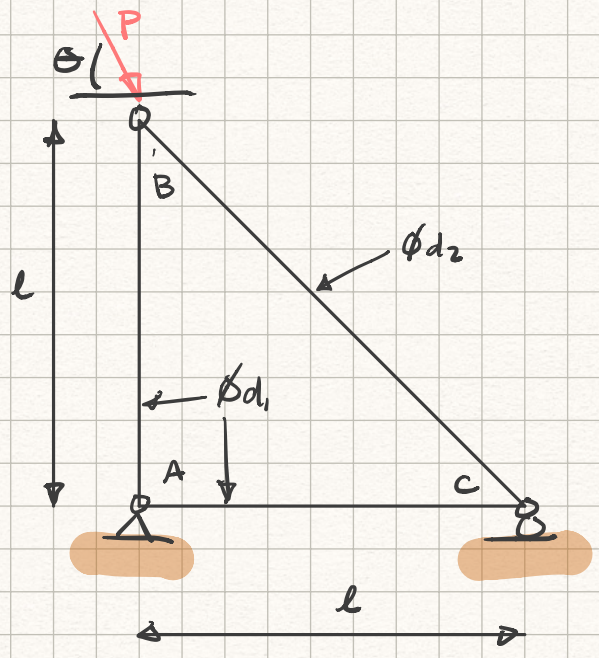
$$I = \frac{1}{12} b h^3$$

$$P_{cr} = \frac{\pi^2 \cdot 1.6E6 \cdot \frac{1}{12} \cdot 1.25 (0.1875)^3}{36^2}$$

$$\boxed{P_{cr} = 8.37 \text{ lb}}$$

10.19:

Given:



$\theta = 70°$

$d_1 = 18\,mm = 0.018\,m$

$d_2 = 22\,mm = 0.022\,m$

$\ell = 1.2\,m$

$P = 5.2\,kN$

$E = 200\,GPa$

Find: Factor of Safety

Solution:

at B: $\Sigma F_x: P_x = BC_x$

$$\rightarrow BC_x = P\cos\theta$$

$$BC\cos 45 = P\cos\theta$$

$$BC = P\frac{\cos\theta}{\cos 45}$$

$$BC = 2.515\,kN$$

$$\Sigma F_y: -P_y + BA + BC_y = 0$$

$$\rightarrow BA = P_y - BC\sin 45$$

$$BA = 3.108\,N$$

$$I_{cyl} = \frac{\pi}{4}r^4$$

$$P_{CR,BC} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^3}{4}\cdot\frac{200\,GPa\cdot(0.011\,m)^4}{(1.2\sqrt{2}\,m)^2}$$

$$P_{CR,BC} = 7.88\,kN$$

$$FOS_{BC} = \frac{P_{CR}}{P}$$

$$= \frac{7.88\,kN}{2.515\,kN}$$

$$\rightarrow F.O.S._{BC} = 3.13$$

CONTINUED →

10.19 Continued:

$$P_{CR,BA} = \frac{\pi^3}{4} \cdot \frac{200\ GPa\ (0.009)^4}{(1.2)^2}$$

$$P_{CR,BA} = 7.06\ kN$$

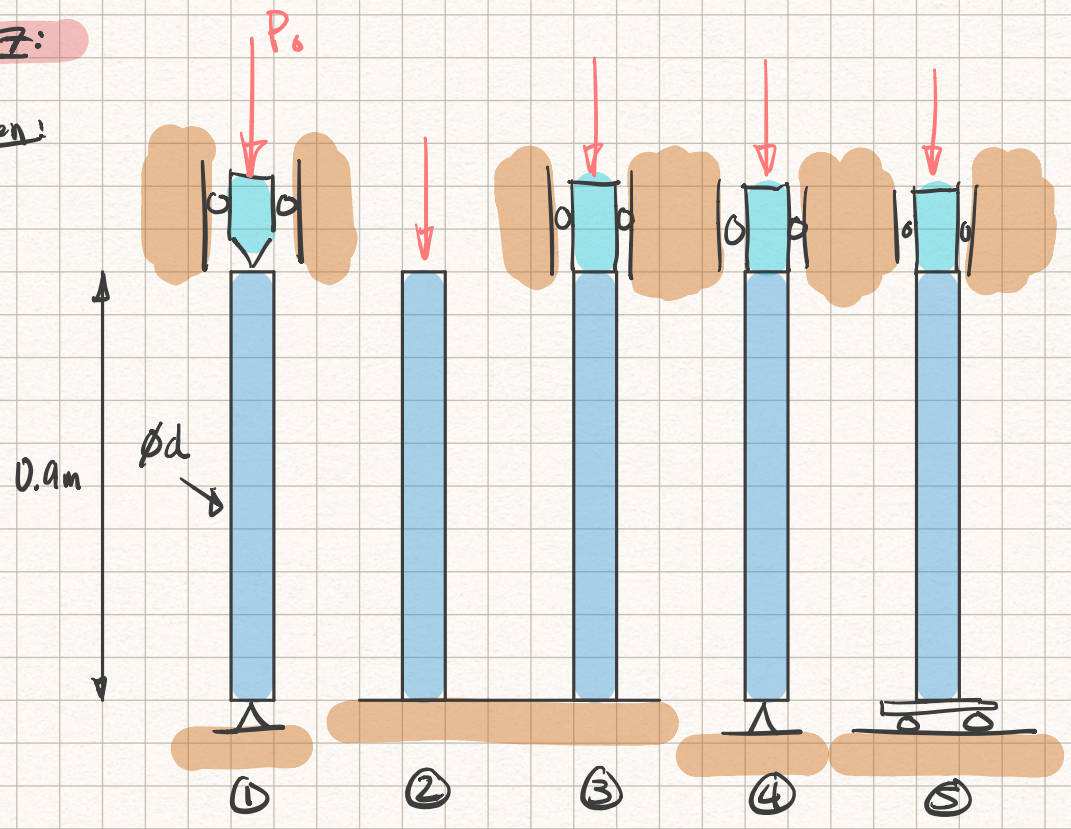
$$FOS_{BA} = \frac{P_{CR}}{P}$$

$$FOS_{BA} = \frac{7.06}{3.108} \longrightarrow FOS_{BA} = 2.27$$

$$F.O.S._{system} = 2.27$$

10.27:

Given:



$d = 20mm$
$P_o = 7.5kN$
$E = 200GPa$

Find: a) F.O.S. in ①

b) ϕd for ② to ⑤ for same FOS as in ①

Solution:

a) $I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$

$P_{cr} = \frac{\pi^3}{64} \cdot \frac{200GPa \cdot 0.02m^4}{0.9m^2}$

$P_{cr} = 19.14\ kN$

$FOS = \frac{P_{cr}}{P}$ → $\boxed{FOS_1 = 2.55}$

b) $P_{cr} = \frac{\pi^3}{64} \cdot \frac{200GPa \cdot d^4}{L^2}$, $L_o = 0.9m$

→ $d = \left[L^2 P_{cr} \cdot \frac{64}{\pi^3} \cdot \frac{1}{200GPa}\right]^{-4}$

at ②, $L = 2L_o$

→ $d_2 = 0.0283m$, $\boxed{d_2 = 28.3mm}$

CONTINUED →

10.27 Continued:

at ③, $L = \frac{L_0}{2}$

$\rightarrow d_3 = 0.0141\,m$, $\boxed{d_3 = 14.14\,mm}$

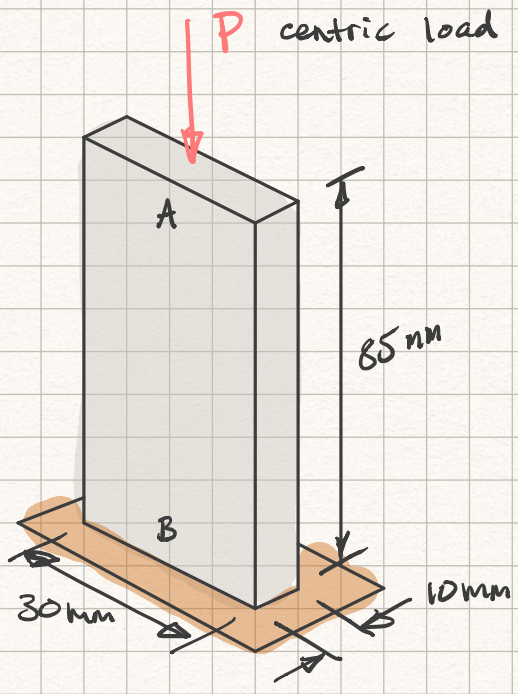
at ④, $L = 0.7L_0$

$\rightarrow d_4 = 0.0167m$, $\boxed{d_4 = 16.7mm}$

at ⑤, $L = 2L_0/2 = L_0$

$\rightarrow d_5 = d_0$ $\boxed{d_5 = 20mm}$

10.62:



Given:

Find: allowable $\bar{P}$ for

a) 6061-T6 Al
b) 2014-T6 Al

Solution:

a) $E = 70\ GPa$, $\sigma_y = 240\ MPa$

$I = \frac{1}{12} 30 \cdot 10^3\ mm^4$

$I = 2.5 \cdot 10^{-9}\ m^4$

$L_e = 2L_0$ (one fixed end)

$L_e = 0.17\ m$

$A = 3 \cdot 10^{-4}\ m^2$

$r = \sqrt{\frac{I}{A}}$

$\rightarrow r = 0.00289\ m$

$\therefore \frac{L_e}{r} = 58.9 < 66$

$\rightarrow \sigma_{all} = [140 - 0.874(L/r)]\ MPa$

CONTINUED →

10.62 Continued:

$$\sigma_{all} = [140 - 0.874(58.9)] = 88.52 \text{ MPa}$$

for $\sigma = \frac{P}{A}$, $P = A\sigma$

$$P_{all} = A\sigma_{all}$$

$$P_{all} = 3 \cdot 10^{-4} \text{ m}^2 \cdot 88.52 \text{ MPa}$$

$$\boxed{P_{all} = 26.56 \text{ kN}}$$

b) $E = 75$ GPa, $\sigma_y = 400$ MPa

from a), $I = 2.5 \cdot 10^{-9} \text{ m}^4$

$A = 3 \cdot 10^{-4} \text{ m}^2$

$$\frac{Le}{r} = 58.9 \geq 55$$

$$\sigma_{all} = \frac{382 \cdot 10^3 \text{ MPa}}{(Le/r)^2}$$

$$\sigma_{all} = 110.1 \text{ MPa}$$

$$P_{all} = A \cdot \sigma_{all}$$

$$\boxed{P_{all} = 33.03 \text{ kN}}$$