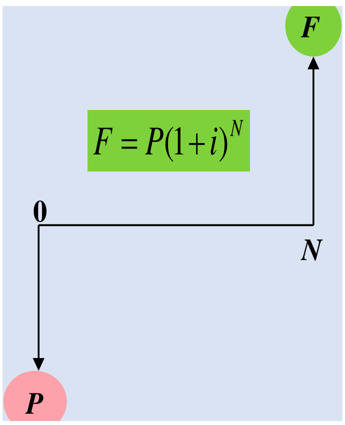
If an investor is indifferent between a sum of money today and a different sum of money in the future, the two sums are said to be **economically equivalent**.

- Example: If Logan would have to toss a coin to choose between $500 on 01/01/2017 or $550 on 01/01/2018 then $500 on 01/01/17 is economically equivalent to $550 on 01/01/2018.

Equivalence calculations using a single sum of money
Compounding (see E6) is used to find what a current sum would be worth at a future date—i.e. what is the *economically equivalent* amount 5 years from now?



Question: Suppose Heeral deposits $2,042 in a bank which pays an interest rate of 8%. How much money will she have in 5 years?
IMPORTANT: interest rates must be converted to *decimal form* for ALL problems. Method: Divide the stated i by 100 (8/100 = 0.08)
Answer:

$$F = P(1 + i)^N$$
$$F = \$2{,}042(1 + 0.08)^5$$
$$F = \$3{,}000$$

We say that $2,042 today is *equivalent* to $3,000 in 5 years at 8% interest

If i = 5%, are these two sums ($2,042 and $3,000) still equivalent?
- No! A lower i means $2,042 will be on a slower growth path
  - (At i = 5%, $2,042 is equivalent to $2,606 in 5 years)

Techniques for solving equivalence problems (which get harder as the term progresses)
- A calculator—fine for simple problems/more accurate than factor tables

Factor Tables (see sample factor table): Developed to make equivalence problems quicker to solve.
- Engineering economics textbook include them, usually as an appendix
- Available from NCEES: https://ncees.org/engineering/fe/ (pg. 127 of Reference Handbook)
- Available on D2L under "Course Content," Module, "FACTOR TABLES."
- Easiest method if available, and essential to learn for FE exam candidates!

APPENDIX C: COMPOUND INTEREST TABLES 611

8% Compound Interest Factors 8%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find F Given P F/P	Present Worth Factor Find P Given F P/F	Sinking Fund Factor Find A Given F A/F	Capital Recovery Factor Find A Given P A/P	Compound Amount Factor Find F Given A F/A	Present Worth Factor Find P Given A P/A	Gradient Uniform Series Find A Given G A/G	Gradient Present Worth Find P Given G P/G	n
1	1.080	.9259	1.0000	1.0800	1.000	0.926	0	0	1
2	1.166	.8573	.4808	.5608	2.080	1.783	0.481	0.857	2
3	1.260	.7938	.3080	.3880	3.246	2.577	0.949	2.445	3
4	1.360	.7350	.2219	.3019	4.506	3.312	1.404	4.650	4
5	1.469	.6806	.1705	.2505	5.867	3.993	1.846	7.372	5

Using factor tables to solve Heeral's problem:
- Find the table for i = 8% (notice 8% at the top of the sample table.)
- Find the column labeled **F/P**. Read this as "Find F, given P".
- Find the row, N = 5 → The factor F/P = 1.469

Solve for F: F = P(F/P,i,N) = $2,042(1.469) = **$3,000**

(Factors in Factor Tables are rounded, so answers will have small discrepancies with calculator answers. Don't worry about this, economic equivalence answers are not very precise, partly to avoid *spurious accuracy*. (The pretense of accuracy when accuracy is impossible is as bad as inaccuracy when accuracy is possible and necessary.)

Discounting tells us what a future sum would be worth *today* (i.e. Find P given F)

We know: $F = P(1 + i)^N$
Rearrange to express P as a function of F: $P = F(1 + i)^{-N}$

(Not sure why the exponent has a negative sign? See Khan Academy video: https://www.khanacademy.org/math/pre-algebra/pre-algebra-exponents-radicals/pre-algebra-negative-exponents/v/negative-exponents)

Let's reimagine Heeral's problem: Suppose Heeral wants to have $3,000 saved in 5 years and the bank is offering an interest rate of 8%. How much money will she need to deposit today to achieve her goal?

- Using the expression for P as a function of F (above), and a calculator-- **$2,042**
- Look at the **Sample Factor Table** and use column: **P/F** which means, "Find P given F".
  - For i = 8% (0.08 in decimal form) and N = 5, the factor for P/F = .6806
  - Solve for P: P = F(P/F, i, N) = $3,000(0.6806) = $2,041.80, or **$2,042** with rounding.

Interesting questions that can be answered using discounting:

- Will the **discounted** present value of the *excess of my expected salary over that of a high school graduate* justify my upfront college costs?
- What is the economic impact of a project with massive upfront costs, and benefits that play out over decades-- like a new Columbia River crossing?

Are these sums also equivalent in the intervening years?
The **future worth** of the cash flow is $2,042 and the **present worth** is $3,000. How about Year 3?

Let $\mathbf{V_n}$ = an equivalent sum of money at the end of a specified period, N, that considers the time value of money. (Note: $P = V_0$ and $F = V_N$).

1. What is $2,042 worth when N = 3? Find V_3:

   $V_3 = P(F/P, 8\%, 3)$

   Look up (F/P,i,N) for i = 8% and N = 3 in sample factor table. (F/P,8%,3) = 1.260.

   V_3 = $2,042(1.260) = **$2,572.92**

2. What is $3,000 worth 2 years prior? (N = 2 because Year 3 is 2 years prior to Year 5)

   $V_3 = F(P/F,i,N)$

   Look up (P/F,i,N) for i = 8% and N = 2 in sample factor table. (P/F, 8%, 2) = 0.8573.

   V_3 = $3,000(0.8573) = **$2,571.90**

(Rounding again, but close enough)

Therefore, cash flows that are equivalent at F and P are also equivalent at *any point* between F and P.