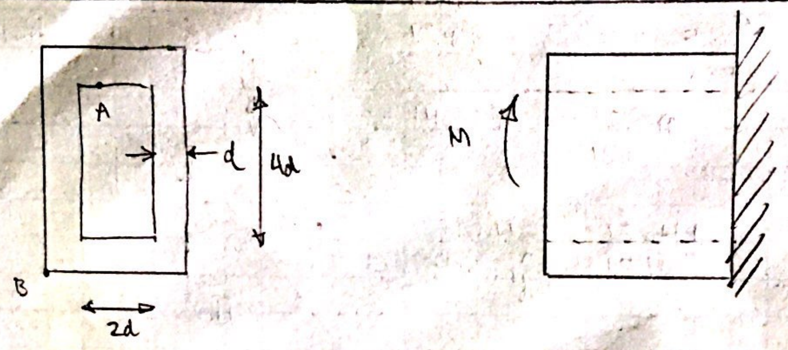
ENGR HW#4 | Due 4/29/20 | Sean Lai

CH 4, #s : 1, 11, 15, 23, 31, 34, 41, 49, 61, 63, 100, 106

4.1:

Given:



$d = 20mm$
$M = 15 kN \cdot m$

Find: a) σ_A
b) σ_b

Solution: $I = \frac{1}{12}bh^3$

$$\Sigma I = \frac{1}{12}\left(4d\cdot(6d)^3 - 2d\cdot(4d)^3\right)$$
$$= \frac{1}{12}(864 - 128)d^4$$
$$= 61.3\,(20mm)^4$$
$$I_{tot} = 9.81\cdot 10^{-6}\ m^4$$

a) $\sigma_A = \frac{-Mc}{I}$, $c = 2d = 40mm = 0.04m$

$$= \frac{-15kN\cdot m\cdot 0.04m}{9.81\cdot 10^{-6} m^4}$$

$\boxed{\sigma_A = -61.2\ MPa}$

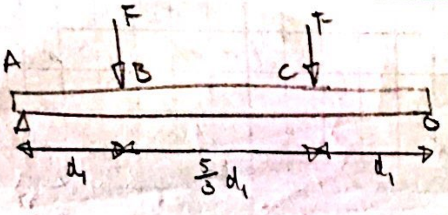
b) $\sigma_B = \frac{+Mc}{I}$, $c = 3d = 0.06m$

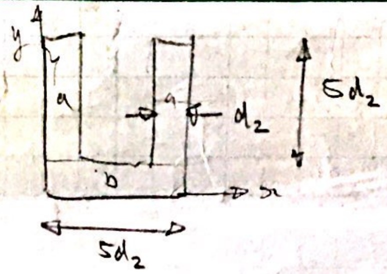
$$= \frac{15kN\cdot m\cdot 0.06m}{9.81\cdot 10^{-6} m^4}$$

$\boxed{\sigma_B = 91.7\ MPa}$

4.11

Given:





$d_1 = 150\ mm$
$d_2 = 10\ mm$
$F = 10\ kN$

Find: maximum compressive + tensile stresses

Solution:

$$M_{section} = F \cdot d_1 = 1.5\ kN \cdot m$$

$$\bar{Y} = \frac{5d_2^2 \cdot \frac{d_2}{2} + 2 \cdot 5d_2^2 \cdot 3.5d_2}{15d_2^2}$$

$$\bar{Y} = \frac{\frac{5}{2} + 35}{15} d_2$$

$$\bar{Y} = 25mm$$

$$I = 2I_a + I_b, \quad I_x = \frac{1}{12}bh^3$$
$$\bar{I}_a = \frac{1}{12} d_2 \cdot (5d_2)^3 = 10.42\, d_2^4$$
$$\bar{I}_b = \frac{1}{12} 5d_2 (d_2)^3 = 0.417\, d_2^4$$

$$I_a = \bar{I}_a + A_a d^2 \qquad d = 3.5d_2 - 25mm = 10mm, \quad A_a = 5d_2^2$$
$$I_a = 10.42\, d_2^4 + 5d_2^4$$
$$I_a = 15.42\, d_2^4$$

$$I_b = \bar{I}_b + A_b d^2 \qquad d = 0.5d_2 - 2.5d_2 = -2d_2, \quad A_b = 5d_2^2$$
$$= 0.417 d_2^4 + 5d_2^2 \cdot (-2d_2)^2$$
$$I_b = 20.417\, d_2^4$$

$$I = 51.257\, d_2^4 = 5.126 \cdot 10^{-7}\ m^4$$

$\sigma_{max,c}$ at top, $\sigma_{max,t}$ at bottom.

$$\rightarrow \sigma_c = \frac{-M \cdot c_c}{I}, \quad c_c = |6d_2 - 2.5d_2|$$

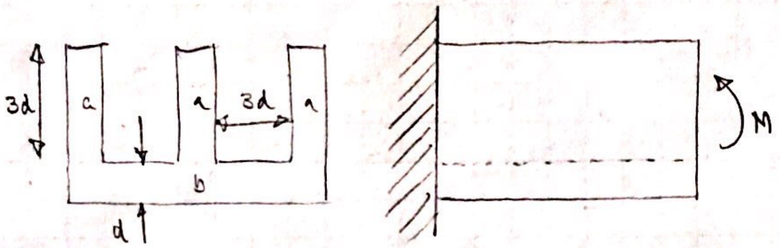
$$\boxed{\sigma_c = -102.4\ MPa}$$

$$\sigma_t = \frac{+M \cdot c_t}{I} \qquad c_t = |0 - 2.5d_2|$$

$$\boxed{\sigma_t = +73.16\ MPa}$$

4.15:

Given:



$d = 0.5\text{in}$
$\sigma_t = 12\text{ksi}$
$\sigma_c = 16\text{ksi}$

Find: Largest M

Solution:

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$\bar{Y} = \frac{\frac{d}{2} \cdot 9d^2 + 3 \cdot 2.5d \cdot 3d^2}{9d^2 + 9d^2}$$

$$\bar{Y} = 1.5d = 0.75\text{in}$$

$$I = 3I_a + I_b \quad , \quad I = \bar{I} + Ad^2$$

a:
$$\bar{I}_a = \tfrac{1}{12} d(3d)^3$$
$$\bar{I}_a = 2.25d^4$$
$$I_a = \bar{I}_a + 3d^2 \cdot \left(d + \tfrac{3}{2}d - \bar{Y}\right)^2 \quad , \quad \bar{Y} = \tfrac{3}{2}d$$
$$= 2.25d^4 + 3d^2 (d)^2$$
$$\underline{I_a = 5.25d^4}$$

$$\bar{I}_b = \tfrac{1}{12} \cdot 9d \cdot d^3$$
$$\bar{I}_b = \tfrac{3}{4}d^4$$
$$I_b = \bar{I}_b + 9d^2 \left(\tfrac{1}{2}d - \bar{Y}\right)^2$$
$$I_b = \tfrac{3}{4}d^4 + 9d^2$$
$$I_b = 9.75d^4$$

$$\rightarrow I = 3I_a + I_b$$
$$I = 25.5d^4$$
$$I = 1.594\text{ in}^4$$

$$\sigma = \frac{Mc}{I} \rightarrow M = \frac{\sigma I}{c}$$

$$M_t = \frac{\sigma_t I}{c} \qquad c = \left|\bar{Y} - \tfrac{d}{2}\right| = d = 0.5\text{in}$$

$$= \frac{12\text{ksi} \cdot 1.594\text{in}^4}{0.5\text{in}}$$

$$M_t = 38.26\text{ kip}\cdot\text{in}$$

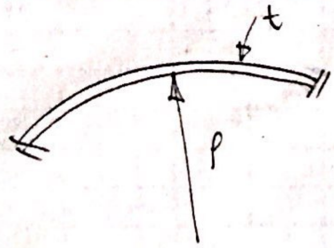
$$M_c = \frac{\sigma_c I}{c} \quad , \quad c = |\bar{Y} - 4d| = 2.5d = 1.25\text{in}$$

$$= \frac{16\text{ksi} \cdot 1.594\text{in}^4}{1.25\text{in}}$$

$M_c = 20.40$ kip·in $\rightarrow$ M_c smaller $\rightarrow$ $\boxed{20.40 \text{ kip}\cdot\text{in}}$

4.23:

Given:



$t = 0.30\,in$
$\rho = 2.5ft = 30\,in$
$E = 29 \cdot 10^6\ psi$

Find: a) σ_{max}
b) M

Solution:

$$\sigma_{max} = \frac{Ec}{\rho}, \quad c = \frac{t}{2}$$

$$= \frac{29 \cdot 10^6\,psi \cdot 0.30in}{2 \cdot 30in}$$

a) $\boxed{\sigma_{max} = 1.45 \cdot 10^5\ psi}$

b) $\sigma = \frac{Mc}{I} \rightarrow M = \frac{\sigma I}{c}$

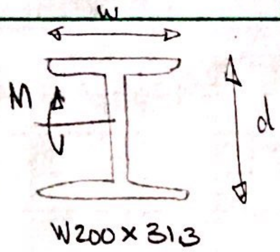
$I = \frac{1}{4}\pi c^4$

$M = \frac{\sigma \cdot \pi c^3}{4} \qquad c = \frac{t}{2}$

$\boxed{M = 384\ lb \cdot in.}$

4.31:

Given:



$A = 3970\ mm^2$
$d = 210\ mm$
$I_x = 31.3 \cdot 10^6\ mm^4$
$M = 45\ kN \cdot m$
$w = 134\ mm$

$E = 200\ GPa$
$\nu = 0.29$
$I_y = 4.07 \cdot 10^6\ mm^4$

Find: a) ρ radius of curvature
b) ρ' " " " transverse cross-section

Solution:

$\sigma_{max} = \frac{Ec}{\rho} \rightarrow \rho = \frac{Ec}{\sigma_{max}}$, $\sigma = \frac{Mc}{I}$

$\rightarrow \rho = \frac{EI}{M}$, $I = I_x$

a) $\boxed{\rho = 139.1\ m}$

b) $\epsilon' = \nu\epsilon$, $\epsilon \propto \frac{1}{\rho}$

$\rightarrow \frac{1}{\rho'} = \nu \cdot \frac{1}{\rho}$

$\rho' = \frac{\rho}{\nu}$

$\boxed{\rho' = 480\ m}$

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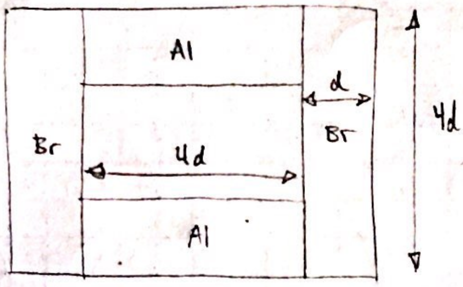
4.34:

Given:



$E_A = 70\ GPa$ $\quad E_B = 105\ GPa$

$\sigma_A = 100\ MPa$ $\quad \sigma_B = 160\ MPa$

$d = 8mm$

Find: M_{max} about horizontal

Solution: Turn $Al \rightarrow Brass$

let $n = \frac{E_A}{E_B} = \frac{2}{3} \rightarrow A_{Al \rightarrow Br} = A_{Al} \cdot n = 4d \frac{E_A}{E_B}$

$I_{rect} = \frac{1}{12} bh^3$

$\rightarrow I = \frac{1}{12}\left[(2d + 4nd)(4d)^3 - (4nd)(2d)^3\right]$

$= \frac{1}{12}\left[128d^4 + 256nd^4 - 32nd^4\right]$

$= 23.11\, d^4$

$I = 9.47 \cdot 10^{-8}\ m^4$

$\sigma = \frac{Mc}{I} \rightarrow M = \frac{\sigma I}{c}$

$M = \frac{160MPa \cdot 23.11 d^4}{2d}$

$M_b = 947\ N \cdot m$ brass

Turn Brass $\rightarrow Al$

let $n = \frac{E_B}{E_A} = 1.5$

$I = \frac{1}{12}\left[(2dn + 4d)(4d)^3 - (4d)(2d)^3\right]$

$= \frac{d^4}{12}\left[128n + 256 - 32\right]$

$= 34.67 d^4$

$M = \frac{\sigma I}{c}$

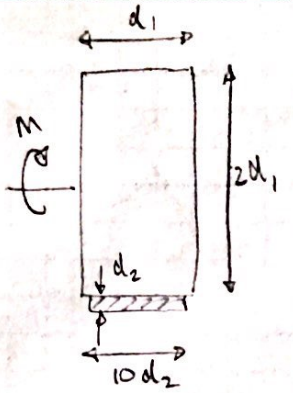
$M = \frac{100\ MPa \cdot 34.67 d^4 m^4}{2d\ m}$

$M_A = 887\ N \cdot m$

M_A smaller $\rightarrow$ $\boxed{887\ N \cdot m}$ largest ~~smallest~~ allowable

4.41 :

Given :



$d_1 = 6in$
$d_2 = 0.5in$
$M = 450$ kip·in

$E_w = 1.8 \cdot 10^6$ psi
$E_s = 29 \cdot 10^6$ psi

Find : a) $\sigma_{max,w}$
b) $\sigma_{max,s}$

Solution : steel → wood

let $n = \frac{E_s}{E_w}$, $A_{s \to w} = n \cdot A_s$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$\bar{Y} = \frac{(d_2 + d_1)(2d_1^2) + (0.5 d_2)(10 d_2^2 \cdot n)}{2d_1^2 + 10 d_2^2 \cdot n}$$

$$\bar{Y} = \frac{478.1 \text{ in}^3}{112.3 \text{ in}^2} = 4.26 \text{ in}$$

$I = I_s + I_w$, $I_s = \bar{I}_s + Ad^2$

$$\bar{I}_s = \frac{1}{12}(10 n d_2)(d_2)^3$$
$$\bar{I}_s = \frac{5n}{6} d_2^4$$

$$\bar{I}_w = \frac{1}{12}(d_1)(2d_1)^3$$
$$\bar{I}_w = \frac{2}{3} d_1^4$$

$$I_s = \bar{I}_s + (10 n d_2^2)(\bar{Y} - \frac{d_2}{2})^2$$
$$I_s = 648.5 \text{ in}^4$$

$$I_w = \bar{I}_w + (2d_1^2)(\bar{Y} - (d_1 + d_2))^2$$
$$I_w = 1225 \text{ in}^4$$

$\rightarrow I = 1874 \text{ in}^4$

a) $\sigma_{max,w} = \frac{-Mc}{I}$, $c = 2d_1 + d_2 - \bar{Y}$

$\boxed{\sigma_{max,w} = -1.979 \text{ ksi}}$

b) $\sigma_{max,s} = \frac{+Mc \cdot n}{I}$, $c = \left|\frac{d_2}{2} - \bar{Y}\right|$ $n = \frac{29}{1.8}$

$\boxed{\sigma_{max,s} = 16.48 \text{ ksi}}$

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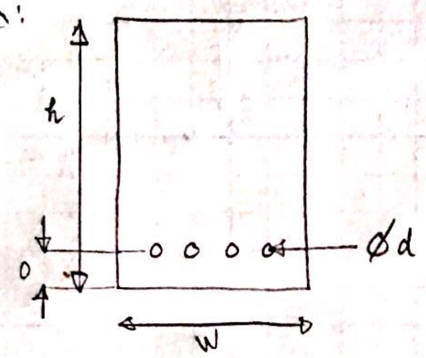
4.49:

Given:



$d = 25$ mm
$o = 60$ mm
$h = 540$ mm
$w = 300$ mm

$M = 175$ kN·m
$E_c = 25$ GPa
$E_s = 200$ GPa

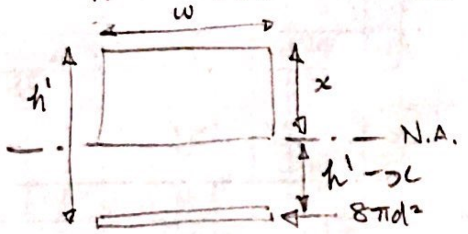
let $h' = h - o = 480$ mm

Find: a) σ_s stress in steel
b) $\sigma_{c,max}$ maximum stress in concrete

Solution:

Split concrete into compressed portion + transformed steel cross section.

$A_s = 4 \cdot \left(\frac{d}{2}\right)^2 \pi = \pi d^2 \longrightarrow nA_s = 8\pi d^2$
$n = E_s / E_c = 8.0$



from figure:

$wx\left(\frac{x}{2}\right) - 8\pi d^2(h' - x) = 0$

$\frac{w}{2}x^2 + 8\pi d^2 x - 8\pi d^2 h' = 0$

solver...

$x = 177.9$ mm

$\rightarrow I = \frac{1}{3}wx^3 + 8\pi d^2(h' - x)^2$
$I = 2.00 \cdot 10^9$ mm^4 $= 2.00 \cdot 10^{-3}$ m^4

a) $\sigma_s = n\frac{Mc}{I}$, $c = h' - x$

$\boxed{\sigma_s = +211 \text{ MPa}}$

b) $\sigma_{max,c} = \frac{Mc}{I}$, $c = x$

$\boxed{\sigma_c = -15.5 \text{ MPa}}$

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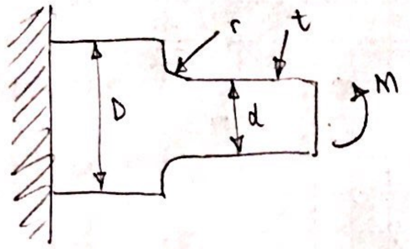
4.61 :

Given:

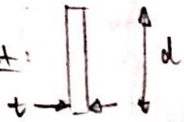


$t = 8mm$
$M = 250 \, N \cdot m$
$D = 80mm$
$d = 40mm$

Find: a) σ for $r = 4mm$
b) σ for $r = 8mm$

Solution:

Front:



$I = \frac{1}{12} t d^3$, $\underbrace{\frac{D}{d} = 2.00 \, , \, \frac{r}{d} = 0.1}_{K \approx 1.88}$

$\rightarrow \sigma = K \frac{Mc}{I}$, $c = \frac{d}{2}$

$\sigma = K \cdot \frac{M \frac{d}{2}}{\frac{1}{12} t d^3}$

a) $\sigma = 6K \frac{M}{td^2} = \boxed{220 \, MPa}$

b) $\frac{D}{d} = 2.00$, $\frac{r}{d} = 0.2 \longrightarrow K \approx 1.5$

$\sigma = 6K \frac{M}{td^2} = \boxed{176 \, MPa}$

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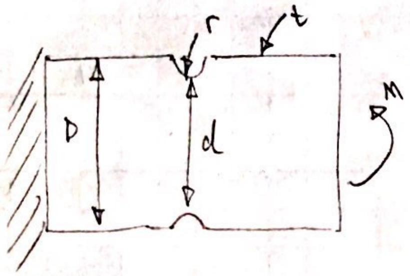
4.63:

Given:



$D = 4.5\text{ in}$
$t = 0.75\text{ in}$
$\sigma_{all} = 8\text{ ksi}$

Find: a) M_{max} for $r = \frac{3}{8}\text{ in}$
b) " " $r = 3/4\text{ in}$

Solution:

a) $d = D - 2r = 3.75\text{ in}$

$\frac{D}{d} = 1.2 \quad \frac{r}{d} = 0.1 \longrightarrow K \approx 2.08$

$I = \frac{1}{12}t(d)^3$

$\sigma = K\frac{Mc}{I} \longrightarrow M = \frac{\sigma I}{Kc} \qquad c = \frac{d}{2}$

$$M = \frac{8\text{ksi} \cdot \frac{1}{12}td^{3\,2}}{2.08\ \frac{d}{2}}$$

$\boxed{M = 6.76\text{ kip}\cdot\text{in}}$

b) $d = D - 2r = 3.0\text{ in}$

$\frac{D}{d} = 1.5 \quad \frac{r}{d} = 0.25 \longrightarrow K = 1.61$

$M = \frac{\sigma I}{Kc}$

$M = \frac{\sigma t d^2}{6K}$

$\boxed{M = 5.59\text{ kip}\cdot\text{in}}$

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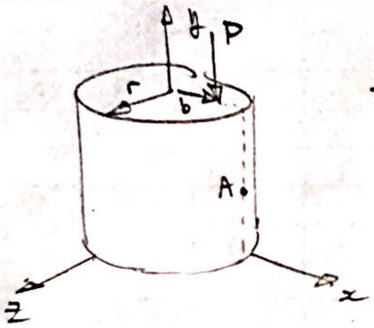
4.100:

Given:



$r = 3 in$
$P = 6 kip$

Find: Stress at A when b is:
a) $b = 0$
b) $b = 1.5 in$
c) $b = 3.0 in$

Solution:

$$\sigma = \frac{-P}{A} - \frac{Mc}{I}$$, both neg. due to compression

where $A = \pi r^2$, $M = Pb$, $c = r - b$, $I = \frac{\pi}{4} r^4$

a) $\sigma_A = \frac{-P}{\pi r^2} - 0$

$$\boxed{\sigma_A = -0.212 \text{ ksi}}$$

b) $\sigma_B = \sigma_A - \frac{Pb(r)}{\frac{\pi}{4} r^4}$ $b = r/2$

$\sigma_B = \sigma_A - \frac{4Pb}{\pi r^3}$

$$\boxed{\sigma_B = -0.636 \text{ ksi}}$$

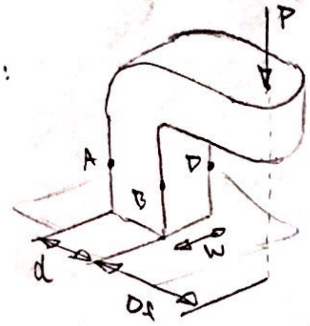
c) $\sigma_c = \sigma_A - \frac{4Pb}{\pi r^3}$ $b = r$

$= \sigma_A - \frac{4P}{\pi r^2}$

$$\boxed{\sigma_c = -1.061 \text{ ksi}}$$

4.106:

Given:



$d = 18\,mm$

$o_f = 40\,mm$

$w = 24\,mm$

$\sigma_A = 80\,MPa$ in ABD

Find: largest P

Solution: most stress will occur at B or D due to stress from moment and force acting in same dir.

$$\sigma = \frac{-P}{A} - \frac{Mc}{I}$$

$$A = dw, \quad M = P(d/2 + o_f), \quad c = \frac{d}{2}, \quad I = \frac{1}{12}wd^3$$

$$\rightarrow \sigma = \frac{-P}{A} - \frac{P(\frac{d}{2}+o_f)\frac{d}{2}}{\frac{1}{12}wd^3}$$

$$\sigma = -P\left(\frac{1}{A} + \frac{(\frac{d}{2}+o_f)\frac{d}{2}}{\frac{1}{12}wd^3}\right)$$

$$\sigma = -P\left(\frac{1}{dw} + \frac{1}{dw}\left(\frac{6(\frac{d}{2}+o_f)}{d}\right)\right)$$

$$\rightarrow P = \sigma dw\left(\frac{1}{1+\frac{6}{d}(\frac{d}{2}+o_f)}\right)$$

$$\boxed{P = 1994N}$$