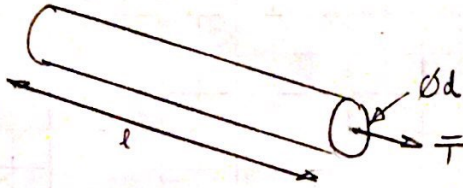


CH3 #'s : 4, 11, 20, 40, 42, 45, 71, 76, 77, 87, 89

3.4

Given:



$$\begin{aligned} d &= 3 \text{ in} \\ l &= 4 \text{ ft} \\ T &= 40 \text{ kip}\cdot\text{in} \end{aligned}$$

Find: a) Find maximum shearing stress τ for solid shaft
 b) " " " " " " hollow shaft with lin ID.

Solution:

$$T = \frac{J\tau}{\rho} \quad \text{or} \quad \tau = \frac{T\rho}{J}$$

$$\begin{aligned} \text{a) } \rho &= \frac{d}{2}, \quad J = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 \\ \rightarrow \tau &= \frac{40 \text{ kip}\cdot\text{in} \cdot 1.5 \text{ in}}{\frac{\pi}{2} (1.5 \text{ in})^4} \end{aligned}$$

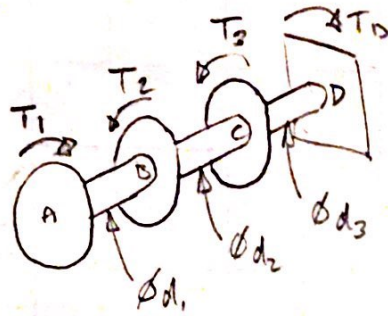
$$\tau = 7.55 \text{ kip/in}^2 \text{ or ksi}$$

$$\begin{aligned} \text{b) } \tau &= \frac{T\rho}{J}, \quad J = \frac{\pi}{2} (p_o^4 - p_i^4) \\ \rightarrow \tau &= \frac{T p_o \cdot 2}{\pi (p_o^4 - p_i^4)} \end{aligned}$$

$$\tau = 7.64 \text{ ksi}$$

3.11:

Given:



$$\begin{aligned} T_1 &= 48 \text{ Nm} \\ T_2 &= 144 \text{ Nm} \\ T_3 &= 60 \text{ Nm} \end{aligned}$$

$$\begin{aligned} d_1 &= 15 \text{ mm} \\ d_2 &= 18 \text{ mm} \\ d_3 &= 21 \text{ mm} \end{aligned}$$

Find: a) shaft w/ highest stress
b) magnitude of that stress

Solution:

$$\sum M = 0 \rightarrow T_1 + T_2 + T_3 - T_D = 0$$

$$T_D = -156 \text{ Nm}$$

AB:



$$\tau_{AB} = \frac{T_1 \cdot c}{J}, \quad J = \frac{\pi}{2} c^4, \quad c = \frac{d_1}{2}$$

$$\tau_{AB} = \frac{T_1 \cdot 16}{\pi d_1^3}$$

$$\tau_{AB} = 72.4 \text{ MPa}$$

BC: opposing torques, can't be lowest stress

CD:

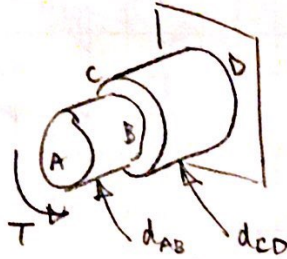
$$\tau_{CD} = \frac{T_D \cdot c}{J}, \quad J = \frac{\pi}{2} c^4, \quad c = \frac{d_3}{2}$$

$$\tau_{CD} = \frac{T_D \cdot 16}{\pi d_3^3}$$

$$\tau_{CD} = 85.8 \text{ MPa} \quad | \quad \leftarrow \text{highest stress a), b)}$$

3.20:

Given:



$$\begin{aligned}d_{AB} &= 60 \text{ mm} \\d_{CD} &= 90 \text{ mm}, \quad t = 6 \text{ mm} \\ \tau_{\max} &= 75 \text{ MPa}\end{aligned}$$

Find: Largest T that can be applied

Solution:

$$\text{AB: } \tau_{AB} = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4, \quad c = \frac{d_{AB}}{2}$$

$$\begin{aligned}\rightarrow T &= \tau_{AB} \frac{\pi}{2} c^3 \\ T &= 3.18 \text{ kNm}\end{aligned}$$

$$\begin{aligned}\text{CD: } T &= \tau_{AB} \frac{J}{c}, \quad J = \frac{\pi}{2} (c_2^4 - c_1^4) \\ T &= \tau_{AB} \cdot \frac{\pi}{2c_2} (c_2^4 - c_1^4)\end{aligned}$$

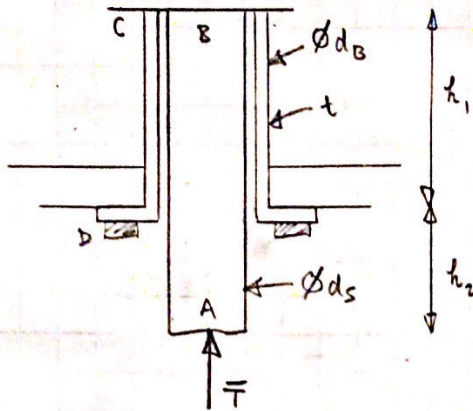
$$\begin{aligned}c_2 &= \frac{d_{CD}}{2}, \quad c_1 = \frac{d_{CD} - 2t}{2} \\ c_2 &= 45 \text{ mm}, \quad c_1 = 39 \text{ mm}\end{aligned}$$

$$T = 4.68 \text{ kNm}$$

\Rightarrow largest allowable $T = 3.18 \text{ kNm}$

3.40 :

Given:



$$\begin{aligned} h_1 &= 8 \text{ in} & h_2 &= 4 \text{ in} \\ d_s &= 1.5 \text{ in} & d_B &= 3 \text{ in} & t &= 0.25 \text{ in} \\ G_s &= 11.2 \cdot 10^6 \text{ psi} & G_B &= 5.6 \cdot 10^6 \text{ psi} \\ \tau_{as} &= 12 \text{ ksi} & \tau_{ab} &= 7 \text{ ksi} \end{aligned}$$

Find: largest angle through which A can be rotated

Solution:

recall: $\phi = \frac{TL}{JG}$

steel rod: $\phi_s = \frac{T \cdot (h_1 + h_2)}{J \cdot G}$, $J = \frac{\pi}{2} \cdot \left(\frac{d_s}{2}\right)^4$

$$\phi_s = \frac{32 T (h_1 + h_2)}{\pi d_s^4 \cdot G}$$

$$\tau_s = \frac{Tc}{J} \rightarrow T = \frac{J \tau_s}{c}$$

$$T = \frac{\pi}{2} c^3 \tau_s$$

$$T_s = \frac{\pi}{16} d_s^3 \tau_s = 7.95 \text{ kip in}$$

for brass,

$$T_b = \frac{J \tau_b}{c}, \quad J = \frac{\pi}{2} (c_o^4 - c_i^4)$$

$$T_b = \frac{\pi}{2} \frac{(c_o^4 - c_i^4)}{c_o} \tau_b, \quad c_o = \frac{d_B}{2}, \quad c_i = \frac{d_B}{2} - t$$

$$T_b = 19.2 \text{ kip in}$$

$\rightarrow T_s = 7.95 \text{ kip}$ highest allowable Torque

$$\phi_s = \frac{32 T_s (h_1 + h_2)}{\pi d_s^4 \cdot G_s} = 0.0171 \text{ rad} = 0.982^\circ$$

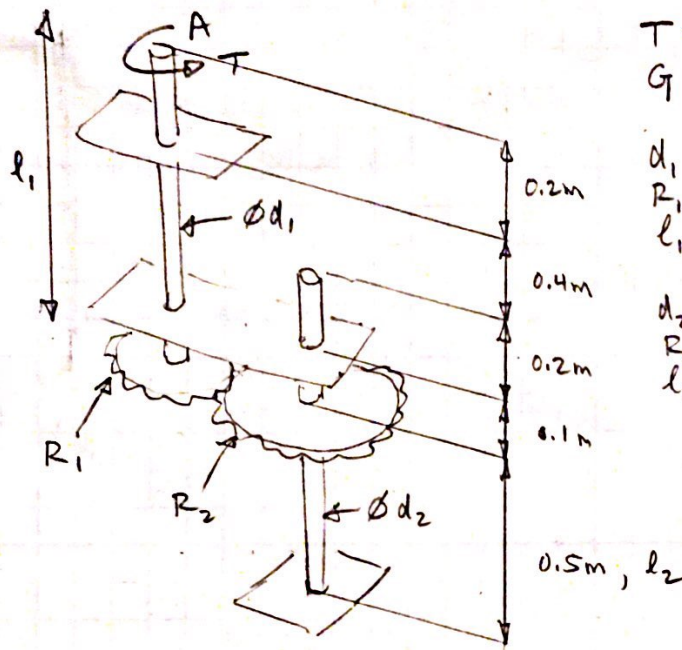
$$\phi_B = \frac{T_s h_1}{J_B G_B}, \quad J_B = \frac{\pi}{2} (c_o^4 - c_i^4)$$

$$\phi_B = \frac{7.95 \text{ kip} \cdot \text{in} \cdot 8 \text{ in}}{\frac{\pi}{2} (1.5 \text{ in}^4 - 1.25 \text{ in}^4) \cdot 5.6 \cdot 10^6 \text{ psi}} = 0.00276 \text{ rad} = 0.158^\circ$$

$$\boxed{\phi_{\text{tot}} = \phi_s + \phi_B = 1.14^\circ}$$

3.42

Given:



$$T = 200 \text{ N}\cdot\text{m}$$

$$G = 77.2 \text{ GPa}$$

$$d_1 = 30 \text{ mm} \rightarrow r_1 = 15 \text{ mm}$$

$$R_1 = 60 \text{ mm}$$

$$l_1 = 0.9 \text{ m}$$

$$d_2 = 30 \text{ mm} \rightarrow r_2 = 15 \text{ mm}$$

$$R_2 = 90 \text{ mm}$$

$$l_2 = 0.5 \text{ m}$$

Find: angle through which A rotates

Solution: $T_1 = F_g R_1$, $T_2 = F_g R_2$

$$\frac{T_1}{R_1} = \frac{T_2}{R_2} \rightarrow T_2 = T_1 \cdot \frac{R_2}{R_1}$$

$$\phi_1 = \frac{T_1 l_1}{J \cdot G} , J = \frac{\pi}{2} r^4$$

$$\phi_1 = \frac{T_1 l_1 \cdot 2}{\pi r_1^4 \cdot G}$$

$$\phi_1 = 0.0293 \text{ rad} = 1.68^\circ$$

$$\phi_2 = \frac{T_2 l_2 \cdot 2}{\pi r_2^4 \cdot G}$$

$$\phi_2 = 0.0244 \text{ rad} = 1.40^\circ$$

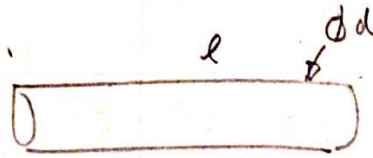
$$\phi_{2/1} = \phi_2 \cdot \frac{R_2}{R_1} = 2.1^\circ$$

$$\rightarrow \phi_{\text{tot}} = \phi_1 + \phi_{2/1} = 3.78^\circ$$

3.45:

Given:

$$\begin{aligned}l &= 1.2 \text{ m} \\ \phi_{all} &= 4^\circ \\ T &= 750 \text{ N}\cdot\text{m} \\ \tau &= 90 \text{ MPa} \\ G &= 77.2 \text{ GPa}\end{aligned}$$



Find: required diameter d

Solution:

$$\phi = \frac{TL}{JG}, \quad J = \frac{\pi}{2} r^4$$

$$\phi = \frac{TL \cdot 2}{\pi r^4 G}$$

$$\rightarrow r = \left(\frac{2TL}{\pi \phi G} \right)^{\frac{1}{4}}, \quad \phi \text{ in rad} \rightarrow \phi_{rad} = 0.0698$$

$$r = 0.01805 \text{ m}$$

$$\rightarrow d = 2r = 36.1 \text{ mm by } G$$

$$\tau = \frac{Tc}{J}$$

$$\tau = \frac{Tc}{\frac{\pi}{2} c^4}$$

$$\rightarrow c^3 = \frac{2T}{\pi \tau}$$

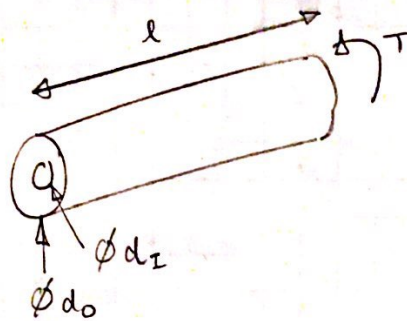
$$c = \left(\frac{2T}{\pi \tau} \right)^{\frac{1}{3}} = 0.0174 \text{ m}$$

$$\rightarrow d = 2c = 34.9 \text{ mm by } \tau$$

$$\text{by } G \text{ larger} \rightarrow \boxed{d = 36.1 \text{ mm}}$$

3.71:

Given:



$$\tau_a = 50 \text{ MPa}$$

$$G = 77.2 \text{ GPa}$$

$$\omega = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

$$d_o = 60 \text{ mm}$$

$$d_i = 25 \text{ mm}$$

$$l = 5 \text{ m}$$

Find:

- maximum power that can be transmitted
- corresponding ϕ

Solution:

$$a) \quad \tau = \frac{Tc}{J} \rightarrow T = \frac{\tau J}{c}, \quad J = \frac{\pi}{2}(c_o^4 - c_i^4), \quad c_o = \frac{d_o}{2}$$

$$T = \frac{\tau_a \cdot \pi (c_o^4 - c_i^4)}{2 \cdot c_o}$$

$$T = 2057 \text{ N}\cdot\text{m}$$

$$P = T\omega$$

$$P = 2057 \text{ N}\cdot\text{m} \cdot 8\pi \text{ rad/s}$$

$$P = 51.7 \text{ kW}$$

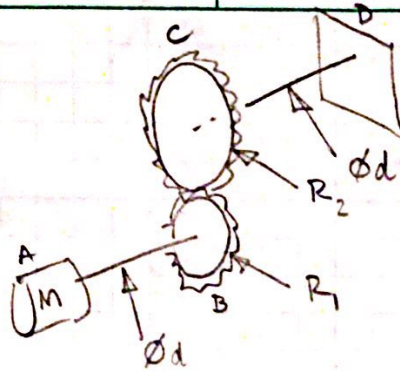
$$b) \quad \phi = \frac{TL}{JG}, \quad J = \frac{\pi}{2}(c_o^4 - c_i^4)$$

$$\phi = \frac{2TL}{\pi(c_o^4 - c_i^4)G}$$

$$\phi = 0.108 \text{ rad} = 6.19^\circ$$

3.76:

Given:



$$P = 16 \text{ hp}$$

$$\omega = 1260 \text{ rpm} = 42\pi \text{ rad/s}$$

$$d = 1 \text{ in}$$

$$R_1 = 3 \text{ in}, R_2 = 5 \text{ in}$$

Find: a) τ_{\max} in AB
b) τ_{\max} in CD

Solution:

Note: $1 \text{ hp} = 550 \text{ lb ft/s} = 6600 \text{ lb in/s}$

$$P = T\omega \rightarrow T = \frac{P}{\omega}$$

$$T = \frac{16 \text{ hp} \cdot 6600 \text{ lb in/s/hp}}{42\pi \text{ rad/s}}$$

$$T_m = 800 \text{ lb in torque from motor}$$

$$\begin{aligned} \text{a) } \tau_{AB} &= \frac{T_m c}{J} \\ &= \frac{2T_m}{\pi c^3}, \quad c = \frac{d}{2} \end{aligned}$$

$$\tau_{AB} = 4.08 \text{ ksi}$$

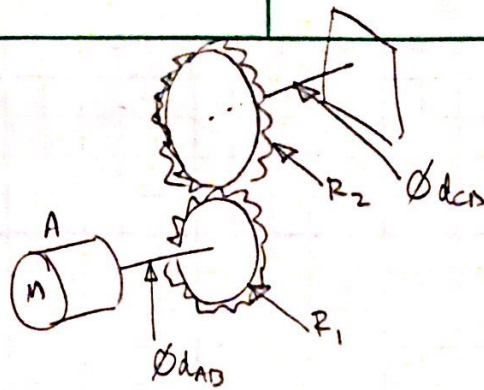
$$\text{b) } \frac{T_m}{R_1} = \frac{T_{CD}}{R_2} \rightarrow T_{CD} = \frac{R_2}{R_1} T_m$$

$$\begin{aligned} \tau_{CD} &= \frac{2T_{CD}}{\pi c^3} \\ &= \frac{2R_2 T_m}{\pi R_1 c^3} = \frac{R_2}{R_1} \tau_{AB} \end{aligned}$$

$$\rightarrow \tau_{CD} = 6.8 \text{ ksi}$$

3.77:

Given:



$$P = 16 \text{ hp}$$

$$\omega = 1260 \text{ rpm}$$

$$\tau_a = 8 \text{ ksi}$$

$$R_1 = 3 \text{ in} \quad R_2 = 5 \text{ in}$$

Find: a) d_{AB}
b) d_{CB}

Solution: $1 \text{ hp} = 6600 \text{ lb in/s}$

a) $P_A = \frac{T_{AB}}{\omega}$, $\omega = 1260 \text{ rpm} = 42\pi \text{ rad/s}$

$$T_{AB} = \frac{16 \text{ hp} \cdot 6600 \text{ lb in/s/hp}}{42\pi \text{ rad/s}}$$

$$T_{AB} = 800 \text{ lb in}$$

$$\tau = \frac{T}{J}, \quad J = \frac{\pi}{2} c^4$$

$$\rightarrow \tau_{AB} = \frac{T_{AB} \cdot 2}{\pi c^3}$$

$$c^3 = \frac{2 T_{AB}}{\pi \tau_{AB}} \rightarrow c = \left[\frac{2 T_{AB}}{\pi \tau_{AB}} \right]^{1/3}$$

$$c_{AB} = 0.40 \text{ in}$$

$$\rightarrow \boxed{d_{AB} = 0.80 \text{ in}}$$

b) $\frac{T_{CB}}{R_2} = \frac{T_{AB}}{R_1}$

$$T_{CB} = \frac{R_2}{R_1} T_{AB}$$

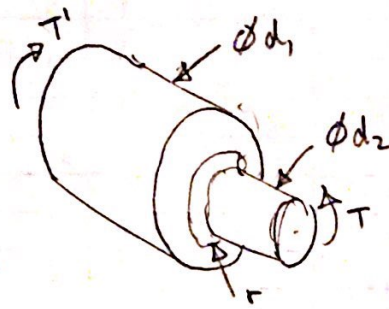
$$\rightarrow c_{CB} = \left[\frac{R_2}{R_1} \cdot \frac{2 T_{AB}}{\pi \tau_{AB}} \right]^{1/3}$$

$$c_{CB} = 0.473 \text{ in}$$

$$\rightarrow \boxed{d_{CB} = 0.947 \text{ in}}$$

3.87:

Given:



$$\omega = 50 \text{ Hz}$$

$$r = 8 \text{ mm}$$

$$d_1 = 60 \text{ mm} \quad d_2 = 30 \text{ mm}$$

$$\tau = 45 \text{ MPa}$$

Find: P_{\max}

Solution:

from p. 188, K for $\frac{D}{d} = 0.267$, $\frac{D}{d} = 2 \rightarrow K \sim 1.17$

$$\tau_{\max} = K \frac{Tc}{J}, \quad J = \frac{\pi}{2} c^4, \quad c = \frac{d_2}{2}$$

$$\tau_{\max} = K \frac{2T}{\pi c^3}$$

$$\rightarrow T = \frac{\tau_{\max}}{K} \cdot \frac{\pi c^3}{2}$$

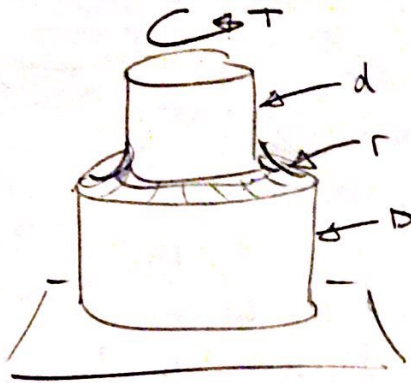
$$T = 204 \text{ N}\cdot\text{m}$$

$$P = T\omega, \quad \omega = 50 \text{ Hz} = 100\pi \text{ rad/s}$$

$$\begin{array}{l} P = 20.4\pi \text{ kW} \\ \boxed{P = 64.1 \text{ kW}} \end{array}$$

3.89:

Given:



$$D = 1 \text{ in}$$
$$r = \frac{1}{2}(D - d)$$
$$T = 200 \text{ lb in}$$

- Find: a) τ_{\max} for $d = 0.8 \text{ in}$
b) τ_{\max} for $d = 0.9 \text{ in}$

Solution:

a) $\tau_{\max} = K \frac{Tc}{J}$

K for $D=1$, $d=0.8$, $r=0.1 \rightarrow K \sim 1.31$

$\tau_{\max} = K \frac{T \cdot \frac{D}{2}}{\frac{\pi D^4}{32}}$, $c = D/2$

$\tau_{\max} \approx 2606 \text{ psi for } d = 0.8 \text{ in}$

b) K for $D=1$, $d=0.9$, $r=0.05 \rightarrow K \sim 1.42$

$\tau_{\max} = K \frac{T}{\pi c^3}$, $c = \frac{D}{2} = 0.45$

$\tau_{\max} \approx 1984 \text{ psi for } d = 0.9 \text{ in}$