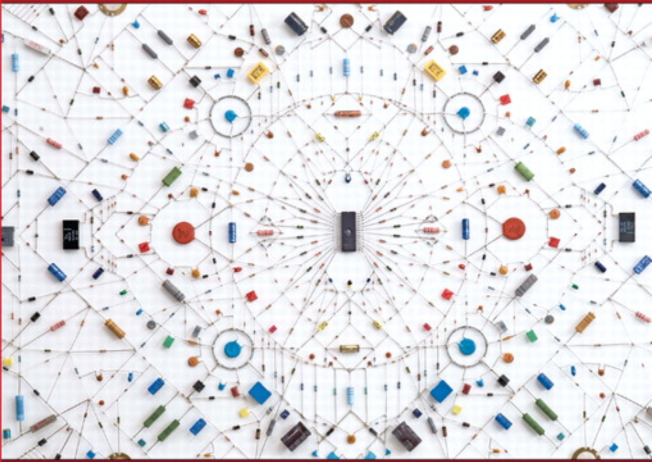
INSTRUCTOR SOLUTIONS MANUAL for

NILSSON • RIEDEL



ELECTRIC CIRCUITS

10th Edition

1 Circuit Variables

Assessment Problems

AP 1.1 Use a product of ratios to convert two-thirds the speed of light from meters per second to miles per second:

$$\left(\frac{2}{3}\right)\frac{3\times 10^8\ \text{m}}{1\ \text{s}}\cdot\frac{100\ \text{cm}}{1\ \text{m}}\cdot\frac{1\ \text{in}}{2.54\ \text{cm}}\cdot\frac{1\ \text{ft}}{12\ \text{in}}\cdot\frac{1\ \text{mile}}{5280\ \text{feet}}=\frac{124{,}274.24\ \text{miles}}{1\ \text{s}}$$

Now set up a proportion to determine how long it takes this signal to travel 1100 miles:

$$\frac{124{,}274.24\ \text{miles}}{1\ \text{s}}=\frac{1100\ \text{miles}}{x\ \text{s}}$$

Therefore,

$$x=\frac{1100}{124{,}274.24}=0.00885=8.85\times 10^{-3}\ \text{s}=8.85\ \text{ms}$$

AP 1.2 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

$$\$100\ \text{billion}\ =\$100\times 10^9$$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1\ \text{year}}{365.25\ \text{days}}\cdot\frac{1\ \text{day}}{24\ \text{hours}}\cdot\frac{1\ \text{hour}}{60\ \text{mins}}\cdot\frac{1\ \text{min}}{60\ \text{secs}}\cdot\frac{1\ \text{sec}}{1000\ \text{ms}}=\frac{1\ \text{year}}{31.5576\times 10^9\ \text{ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100\times 10^9}{1\ \text{year}}\cdot\frac{1\ \text{year}}{31.5576\times 10^9\ \text{ms}}=\frac{100}{31.5576}=\$3.17/\text{ms}$$

AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or $i = \frac{dq}{dt}$ In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. (1.2) to find an expression for charge in terms of current:

$$q(t) = \int_0^t i(x)\,dx$$

We are given the expression for current, i, which can be substituted into the above expression. To find the total charge, we let $t \to \infty$ in the integral. Thus we have

$$q_{\text{total}} = \int_0^\infty 20e^{-5000x}\,dx = \frac{20}{-5000}e^{-5000x}\Big|_0^\infty = \frac{20}{-5000}(e^{-\infty} - e^0)$$

$$= \frac{20}{-5000}(0-1) = \frac{20}{5000} = 0.004 \text{ C} = 4000\,\mu\text{C}$$

AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. (1.2):

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\right]$$

$$= \frac{d}{dt}\left(\frac{1}{\alpha^2}\right) - \frac{d}{dt}\left(\frac{t}{\alpha}e^{-\alpha t}\right) - \frac{d}{dt}\left(\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= 0 - \left(\frac{1}{\alpha}e^{-\alpha t} - \alpha\frac{t}{\alpha}e^{-\alpha t}\right) - \left(-\alpha\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha}\right)e^{-\alpha t}$$

$$= te^{-\alpha t}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t:

$$\frac{di}{dt} = \frac{d}{dt}(te^{-\alpha t}) = e^{-\alpha t} + t(-\alpha)e^{\alpha t} = (1-\alpha t)e^{-\alpha t} = 0$$

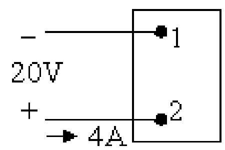
Since $e^{-\alpha t}$ never equals 0 for a finite value of t, the expression equals 0 only when $(1-\alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t, the current is

$$i = \frac{1}{\alpha}e^{-\alpha/\alpha} = \frac{1}{\alpha}e^{-1}$$

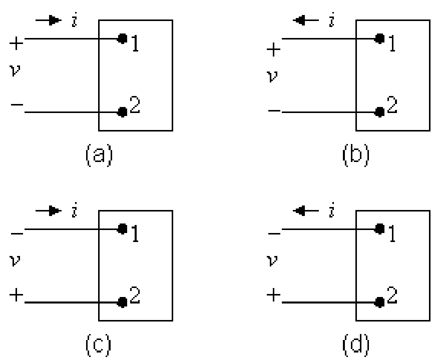
Remember in the problem statement, $\alpha = 0.03679$. Using this value for α,

$$i = \frac{1}{0.03679}e^{-1} \cong 10 \text{ A}$$

AP 1.5 Start by drawing a picture of the circuit described in the problem statement:



Also sketch the four figures from Fig. 1.6:



[**a**] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

$$\text{(a) } v = -20\,\text{V}, \qquad i = -4\,\text{A}; \quad \text{(b) } v = -20\,\text{V}, \qquad i = 4\,\text{A}$$

$$\text{(c) } v = 20\,\text{V}, \qquad i = -4\,\text{A}; \qquad \text{(d) } v = 20\,\text{V}, \qquad i = 4\,\text{A}$$

[**b**] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80\,\text{W}$. Since the power is greater than 0, the box is absorbing power.

[**c**] From the calculation in part (b), the box is absorbing 80 W.

AP 1.6 [**a**] Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. To find the time at which the power is maximum, find the first derivative of the power with respect to time, set the resulting expression equal to zero, and solve for time:

$$p = (80{,}000te^{-500t})(15te^{-500t}) = 120 \times 10^4 t^2 e^{-1000t}$$

$$\frac{dp}{dt} = 240 \times 10^4 te^{-1000t} - 120 \times 10^7 t^2 e^{-1000t} = 0$$

Therefore,

$240 \times 10^4 - 120 \times 10^7 t = 0$

Solving,

$$t = \frac{240 \times 10^4}{120 \times 10^7} = 2 \times 10^{-3} = 2 \text{ ms}$$

[b] The maximum power occurs at 2 ms, so find the value of the power at 2 ms:

$$p(0.002) = 120 \times 10^4 (0.002)^2 e^{-2} = 649.6 \text{ mW}$$

[c] From Eq. (1.3), we know that power is the time rate of change of energy, or $p = dw/dt$. If we know the power, we can find the energy by integrating Eq. (1.3). To find the total energy, the upper limit of the integral is infinity:

$$\begin{aligned} w_{\text{total}} &= \int_0^\infty 120 \times 10^4 x^2 e^{-1000x}\,dx \\ &= \frac{120 \times 10^4}{(-1000)^3} e^{-1000x}[(-1000)^2 x^2 - 2(-1000)x + 2)\Big|_0^\infty \\ &= 0 - \frac{120 \times 10^4}{(-1000)^3} e^0 (0 - 0 + 2) = 2.4 \text{ mJ} \end{aligned}$$

AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440 \text{ MW}$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

Chapter Problems

P 1.1 $\dfrac{(260 \times 10^6)(540)}{10^9} = 104.4$ gigawatt-hours

P 1.2 $\dfrac{(480)(320)\text{ pixels}}{1\text{ frame}} \cdot \dfrac{2\text{ bytes}}{1\text{ pixel}} \cdot \dfrac{30\text{ frames}}{1\text{ sec}} = 9.216 \times 10^6\text{ bytes/sec}$

$(9.216 \times 10^6\text{ bytes/sec})(x\text{ secs}) = 32 \times 2^{30}\text{ bytes}$

$x = \dfrac{32 \times 2^{30}}{9.216 \times 10^6} = 3728\text{ sec} = 62\text{ min} \approx 1\text{ hour of video}$

P 1.3 **[a]** $\dfrac{20{,}000\text{ photos}}{(11)(15)(1)\text{ mm}^3} = \dfrac{x\text{ photos}}{1\text{ mm}^3}$

$x = \dfrac{(20{,}000)(1)}{(11)(15)(1)} = 121\text{ photos}$

[b] $\dfrac{16 \times 2^{30}\text{ bytes}}{(11)(15)(1)\text{ mm}^3} = \dfrac{x\text{ bytes}}{(0.2)^3\text{ mm}^3}$

$x = \dfrac{(16 \times 2^{30})(0.008)}{(11)(15)(1)} = 832{,}963\text{ bytes}$

P 1.4 $(4\text{ cond.}) \cdot (845\text{ mi}) \cdot \dfrac{5280\text{ ft}}{1\text{ mi}} \cdot \dfrac{2526\text{ lb}}{1000\text{ ft}} \cdot \dfrac{1\text{ kg}}{2.2\text{ lb}} = 20.5 \times 10^6\text{ kg}$

P 1.5 Volume = area × thickness

Convert values to millimeters, noting that $10\text{ m}^2 = 10^6\text{ mm}^2$

$10^6 = (10 \times 10^6)(\text{thickness})$

$\Rightarrow\ \text{thickness} = \dfrac{10^6}{10 \times 10^6} = 0.10\text{ mm}$

P 1.6 **[a]** We can set up a ratio to determine how long it takes the bamboo to grow $10\,\mu$m First, recall that $1\text{ mm} = 10^3\mu$m. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250\text{ mm}}{1\text{ day}} \cdot \frac{1\text{ day}}{24\text{ hours}} \cdot \frac{1\text{ hour}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456}\text{ mm/s}$$

Use a ratio to determine the time it takes for the bamboo to grow $10\,\mu$m:

$$\frac{10/3456 \times 10^{-3}\text{ m}}{1\text{ s}} = \frac{10 \times 10^{-6}\text{ m}}{x\text{ s}} \qquad \text{so} \qquad x = \frac{10 \times 10^{-6}}{10/3456 \times 10^{-3}} = 3.456\text{ s}$$

[b] $\dfrac{1 \text{ cell length}}{3.456 \text{ s}} \cdot \dfrac{3600 \text{ s}}{1 \text{ hr}} \cdot \dfrac{(24)(7) \text{ hr}}{1 \text{ week}} = 175{,}000$ cell lengths/week

P 1.7 [a] First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 0.125e^{-2500t}$$

Therefore, $dq = 0.125e^{-2500t}\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 0.125\int_0^t e^{-2500y}\,dy$$

We solve the integral and make the substitutions for the limits of the integral:

$$q(t) - q(0) = 0.125\frac{e^{-2500y}}{-2500}\bigg|_0^t = 50 \times 10^{-6}(1 - e^{-2500t})$$

But $q(0) = 0$ by hypothesis, so

$$q(t) = 50(1 - e^{-2500t})\,\mu\text{C}$$

[b] As $t \to \infty$, $q_T = 50\,\mu$C.

[c] $q(0.5 \times 10^{-3}) = (50 \times 10^{-6})(1 - e^{(-2500)(0.0005)}) = 35.675\,\mu$C.

P 1.8 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 20\cos 5000t$$

Therefore, $dq = 20\cos 5000t\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 20\int_0^t \cos 5000y\,dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 20\frac{\sin 5000y}{5000}\bigg|_0^t = \frac{20}{5000}\sin 5000t - \frac{20}{5000}\sin 5000(0) = \frac{20}{5000}\sin 5000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 4 \times 10^{-3}\sin 5000t\,\text{C} = 4\sin 5000t\,\text{mC}$

P 1.9 [a] First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 40te^{-500t}$$

Therefore, $dq = 40te^{-500t}\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 40\int_0^t ye^{-500y}\,dy$$

We solve the integral and make the substitutions for the limits of the integral:

$$q(t) - q(0) = 40\frac{e^{-500y}}{(-500)^2}(-500y-1)\Bigg|_0^t = 160\times10^{-6}e^{-500t}(-500t-1) + 160\times10^{-6}$$

$$= 160\times10^{-6}(1 - 500te^{-500t} - e^{-500t})$$

But $q(0) = 0$ by hypothesis, so

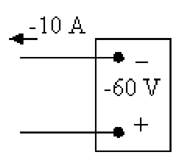
$$q(t) = 160(1 - 500te^{-500t} - e^{-500t})\,\mu\text{C}$$

[b] $q(0.001) = (160)[1 - 500(0.001)e^{-500(0.001)} - e^{-500(0.001)} = 14.4\,\mu\text{C}.$

P 1.10 $n = \dfrac{35\times10^{-6}\text{ C/s}}{1.6022\times10^{-19}\text{ C/elec}} = 2.18\times10^{14}\text{ elec/s}$

P 1.11 $w = qV = (1.6022\times10^{-19})(6) = 9.61\times10^{-19} = 0.961\text{ aJ}$

P 1.12 [a]



$p = vi = (40)(-10) = -400$ W

Power is being delivered by the box.

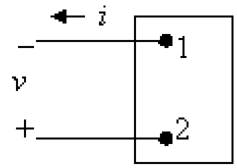
[b] Entering

[c] Gaining

P 1.13 [a] $p = vi = (-60)(-10) = 600$ W, so power is being absorbed by the box.

[b] Entering

[c] Losing



P 1.14 Assume we are standing at box A looking toward box B. Use the passive sign convention to get $p = vi$, since the current i is flowing into the + terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = (30)(6) = 180$ W   180 W from A to B

[b] $p = (-20)(-8) = 160$ W   160 W from A to B

[c] $p = (-60)(4) = -240$ W   240 W from B to A

[d] $p = (40)(-9) = -360$ W   360 W from B to A

P 1.15 **[a]** In Car A, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the + terminal of the battery of Car A). Therefore using the passive sign convention,
$p = vi = (30)(12) = 360$ W.
Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.

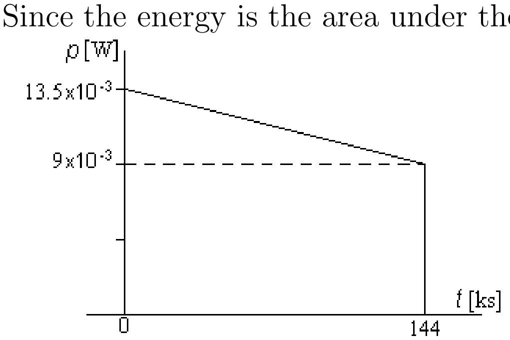
[b] $w(t) = \int_0^t p\,dx; \qquad 1\text{ min} = 60\text{ s}$

$$w(60) = \int_0^{60} 360\,dx$$

$$w = 360(60 - 0) = 360(60) = 21{,}600 \text{ J} = 21.6 \text{ kJ}$$

P 1.16 $p = vi; \qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 40 hr = 144,000 s = 144 ks

$$p(0) = (1.5)(9 \times 10^{-3}) = 13.5 \times 10^{-3} \text{ W}$$

$$p(144 \text{ ks}) = (1)(9 \times 10^{-3}) = 9 \times 10^{-3} \text{ W}$$

$$w = (9 \times 10^{-3})(144 \times 10^3) + \frac{1}{2}(13.5 \times 10^{-3} - 9 \times 10^{-3})(144 \times 10^3) = 1620 \text{ J}$$

P 1.17 $p = (12)(100 \times 10^{-3}) = 1.2$ W; $\quad 4 \text{ hr} \cdot \dfrac{3600 \text{ s}}{1 \text{ hr}} = 14{,}400 \text{ s}$

$$w(t) = \int_0^t p\,dt \qquad w(14{,}400) = \int_0^{14{,}400} 1.2\,dt = 1.2(14{,}400) = 17.28 \text{ kJ}$$

P 1.18 **[a]** $p = vi = (15e^{-250t})(0.04e^{-250t}) = 0.6e^{-500t}$ W

$$p(0.01) = 0.6e^{-500(0.01)} = 0.6e^{-5} = 0.00404 = 4.04 \text{ mW}$$

[b] $w_{\text{total}} = \displaystyle\int_0^\infty p(x)\,dx = \int_0^\infty 0.6e^{-500x}\,dx = \frac{0.6}{-500}e^{-500x}\bigg|_0^\infty$

$$= -0.0012(e^{-\infty} - e^0) = 0.0012 = 1.2 \text{ mJ}$$

P 1.19 **[a]** $p = vi = (0.05e^{-1000t})(75 - 75e^{-1000t}) = (3.75e^{-1000t} - 3.75e^{-2000t})$ W

$$\frac{dp}{dt} = -3750e^{-1000t} + 7500e^{-2000t} = 0 \qquad \text{so} \qquad 2e^{-2000t} = e^{-1000t}$$

$2 = e^{1000t}$ so $\ln 2 = 1000t$ thus p is maximum at $t = 693.15\,\mu\text{s}$

$$p_{\text{max}} = p(693.15\,\mu\text{s}) = 937.5 \text{ mW}$$

[b] $w = \displaystyle\int_0^\infty [3.75e^{-1000t} - 3.75e^{-2000t}]\,dt = \left[\frac{3.75}{-1000}e^{-1000t} - \frac{3.75}{-2000}e^{-2000t}\bigg|_0^\infty\right]$

$$= \frac{3.75}{1000} - \frac{3.75}{2000} = 1.875 \text{ mJ}$$

P 1.20 **[a]** $p = vi = 0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}$

$p(625\,\mu\text{s}) = 42.2$ mW

[b]
$$\begin{aligned} w(t) &= \int_0^t (0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}) \\ &= 140.625 - 78.125e^{-3200t} + 250e^{-2000t} - 312.5e^{-800t}\mu\text{J} \\ w(625\,\mu\text{s}) &= 12.14\,\mu\text{J} \end{aligned}$$

[c] $w_{\text{total}} = 140.625\,\mu$J

P 1.21 **[a]**

$$\begin{aligned} p &= vi \\ &= [(1500t+1)e^{-750t}](0.04e^{-750t}) \\ &= (60t+0.04)e^{-1500t} \\ \frac{dp}{dt} &= 60e^{-1500t} - 1500e^{-1500t}(60t+0.04) \\ &= -90{,}000te^{-1500t} \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $t = 0$
so p_{max} occurs at $t = 0$.

[b]

$$\begin{aligned} p_{\text{max}} &= [(60)(0) + 0.04]e^0 = 0.04 \\ &= 40 \text{ mW} \end{aligned}$$

[c]

$$\begin{aligned} w &= \int_0^t p dx \\ w &= \int_0^t 60xe^{-1500x}\,dx + \int_0^t 0.04e^{-1500x}\,dx \\ &= \frac{60e^{-1500x}}{(-1500)^2}(-1500x-1)\bigg|_0^t + 0.04\frac{e^{-1500x}}{-1500}\bigg|_0^t \end{aligned}$$

When $t = \infty$ all the upper limits evaluate to zero, hence
$w = \frac{60}{225 \times 10^4} + \frac{0.04}{1500} = 53.33\,\mu\text{J}.$

P 1.22 **[a]**

$$\begin{aligned} p &= vi \\ &= [(3200t+3.2)e^{-1000t}][(160t+0.16)e^{-1000t}] \\ &= e^{-2000t}[512{,}000t^2 + 1024t + 0.512] \\ \frac{dp}{dt} &= e^{-2000t}[1{,}024{,}000t + 1024] - 2000e^{-2000t}[512{,}000t^2 + 1024t + 0.512] \\ &= -e^{-2000t}[1024 \times 10^6 t^2 + 1{,}024{,}000t] \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $t = 0$
so p_{max} occurs at $t = 0$.

[b]

$$\begin{aligned} p_{\text{max}} &= e^{-0}[0 + 0 + 0.512] \\ &= 512 \text{ mW} \end{aligned}$$

[c]

$$\begin{aligned} w &= \int_0^t p dx \\ w &= \int_0^t 512{,}000x^2e^{-2000x}\,dx + \int_0^t 1024xe^{-2000x}\,dx + \int_0^t 0.512e^{-2000x}\,dx \\ &= \frac{512{,}000e^{-2000x}}{-8 \times 10^9}[4 \times 10^6x^2 + 4000x + 2]\bigg|_0^t + \\ &\quad \frac{1024e^{-2000x}}{4 \times 10^6}(-2000x-1)\bigg|_0^t + \frac{0.512e^{-2000x}}{-2000}\bigg|_0^t \end{aligned}$$

When $t \to \infty$ all the upper limits evaluate to zero, hence

$$w = \frac{(512{,}000)(2)}{8 \times 10^9} + \frac{1024}{4 \times 10^6} + \frac{0.512}{2000}$$

$$w = 128 \times 10^{-6} + 256 \times 10^{-6} + 256 \times 10^{-6} = 640\,\mu\text{J}.$$

P 1.23 **[a]** We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t. The calculations are shown below:

$$\begin{aligned}
p &= 0 \quad t < 0, \qquad p = 0 \quad t > 40\text{ s} \\
p &= vi = t(1 - 0.025t)(4 - 0.2t) = 4t - 0.3t^2 + 0.005t^3\text{ W} \qquad 0 \le t \le 40\text{ s} \\
\frac{dp}{dt} &= 4 - 0.6t + 0.015t^2 = 0.015(t^2 - 40t + 266.67) \\
\frac{dp}{dt} &= 0 \qquad \text{when } t^2 - 40t + 266.67 = 0 \\
t_1 &= 8.453\text{ s}; \qquad t_2 = 31.547\text{ s} \\
& \quad \text{(using the polynomial solver on your calculator)} \\
p(t_1) &= 4(8.453) - 0.3(8.453)^2 + 0.005(8.453)^3 = 15.396\text{ W} \\
p(t_2) &= 4(31.547) - 0.3(31.547)^2 + 0.005(31.547)^3 = -15.396\text{ W}
\end{aligned}$$

Therefore, maximum power is being delivered at $t = 8.453$ s.

[b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\text{max}} = 15.396$ W (delivered)

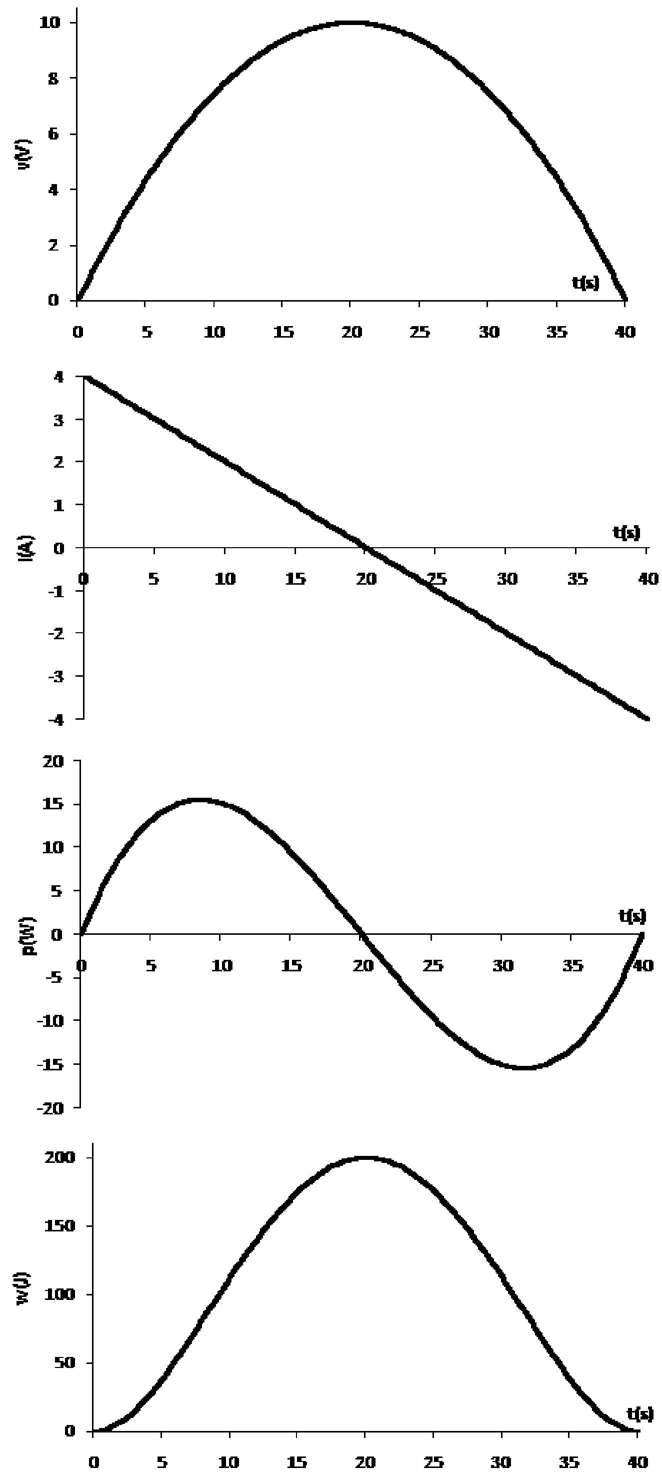
[c] As we saw in part (a), the other "maximum" power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 31.547$ s.

[d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\text{max}} = 15.396$ W (extracted)

[e] $w = \int_0^t p\,dx = \int_0^t (4x - 0.3x^2 + 0.005x^3)\,dx = 2t^2 - 0.1t^3 + 0.00125t^4$

$$\begin{aligned}
w(0) &= 0\text{ J} & w(30) &= 112.5\text{ J} \\
w(10) &= 112.5\text{ J} & w(40) &= 0\text{ J} \\
w(20) &= 200\text{ J}
\end{aligned}$$

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:



P 1.24 **[a]** $v(10 \text{ ms}) = 400e^{-1}\sin 2 = 133.8 \text{ V}$

$i(10 \text{ ms}) = 5e^{-1}\sin 2 = 1.67 \text{ A}$

$p(10 \text{ ms}) = vi = 223.80 \text{ W}$

$$\begin{aligned}
\text{[b]}\quad p &= vi = 2000e^{-200t}\sin^2 200t \\
&= 2000e^{-200t}\left[\frac{1}{2} - \frac{1}{2}\cos 400t\right] \\
&= 1000e^{-200t} - 1000e^{-200t}\cos 400t \\
w &= \int_0^\infty 1000e^{-200t}\,dt - \int_0^\infty 1000e^{-200t}\cos 400t\,dt \\
&= 1000\left.\frac{e^{-200t}}{-200}\right|_0^\infty \\
&\quad -1000\left.\left\{\frac{e^{-200t}}{(200)^2 + (400)^2}[-200\cos 400t + 400\sin 400t]\right\}\right|_0^\infty \\
&= 5 - 1000\left[\frac{200}{4\times 10^4 + 16\times 10^4}\right] = 5 - 1 \\
w &= 4\ \text{J}
\end{aligned}$$

P 1.25 [a] $p = vi = 2000\cos(800\pi t)\sin(800\pi t) = 1000\sin(1600\pi t)$ W

Therefore, $p_{\max} = 1000$ W

[b] $p_{\max}(\text{extracting}) = 1000$ W

$$\begin{aligned}
\text{[c]}\quad p_{\text{avg}} &= \frac{1}{2.5\times 10^{-3}}\int_0^{2.5\times 10^{-3}} 1000\sin(1600\pi t)\,dt \\
&= 4\times 10^5\left[\frac{-\cos 1600\pi t}{1600\pi}\right]_0^{2.5\times 10^{-3}} = \frac{250}{\pi}[1 - \cos 4\pi] = 0
\end{aligned}$$

[d]

$$\begin{aligned}
p_{\text{avg}} &= \frac{1}{15.625\times 10^{-3}}\int_0^{15.625\times 10^{-3}} 1000\sin(1600\pi t)\,dt \\
&= 64\times 10^3\left[\frac{-\cos 1600\pi t}{1600\pi}\right]_0^{15.625\times 10^{-3}} = \frac{40}{\pi}[1 - \cos 25\pi] = 25.46\ \text{W}
\end{aligned}$$

P 1.26

$$\begin{aligned}
\text{[a]}\quad q &= \text{area under } i \text{ vs. } t \text{ plot} \\
&= \tfrac{1}{2}(8)(12{,}000) + (16)(12{,}000) + \tfrac{1}{2}(16)(4000) \\
&= 48{,}000 + 192{,}000 + 32{,}000 = 272{,}000\ \text{C}
\end{aligned}$$

$$\begin{aligned}
\text{[b]}\quad w &= \int p\,dt = \int vi\,dt \\
v &= 250\times 10^{-6}t + 8 \qquad 0 \le t \le 16\ \text{ks}
\end{aligned}$$

$0 \le t \le 12{,}000s$:

$$\begin{aligned}
i &= 24 - 666.67\times 10^{-6}t \\
p &= 192 + 666.67\times 10^{-6}t - 166.67\times 10^{-9}t^2 \\
w_1 &= \int_0^{12{,}000} (192 + 666.67\times 10^{-6}t - 166.67\times 10^{-9}t^2)\,dt \\
&= (2304 + 48 - 96)10^3 = 2256\ \text{kJ}
\end{aligned}$$

12,000 s $\leq t \leq$ 16,000 s:

$$\begin{aligned}
i &= 64 - 4 \times 10^{-3}t \\
p &= 512 - 16 \times 10^{-3}t - 10^{-6}t^2 \\
w_2 &= \int_{12,000}^{16,000} (512 - 16 \times 10^{-3}t - 10^{-6}t^2)\,dt \\
&= (2048 - 896 - 789.33)10^3 = 362.667 \text{ kJ} \\
w_T &= w_1 + w_2 = 2256 + 362.667 = 2618.667 \text{ kJ}
\end{aligned}$$

P 1.27 **[a]** 0 s $\leq t <$ 10 ms:

$v = 8$ V; $\quad i = 25t$ A; $\quad p = 200t$ W

10 ms $< t \leq$ 30 ms:

$v = -8$ V; $\quad i = 0.5 - 25t$ A; $\quad p = 200t - 4$ W

30 ms $\leq t <$ 40 ms:

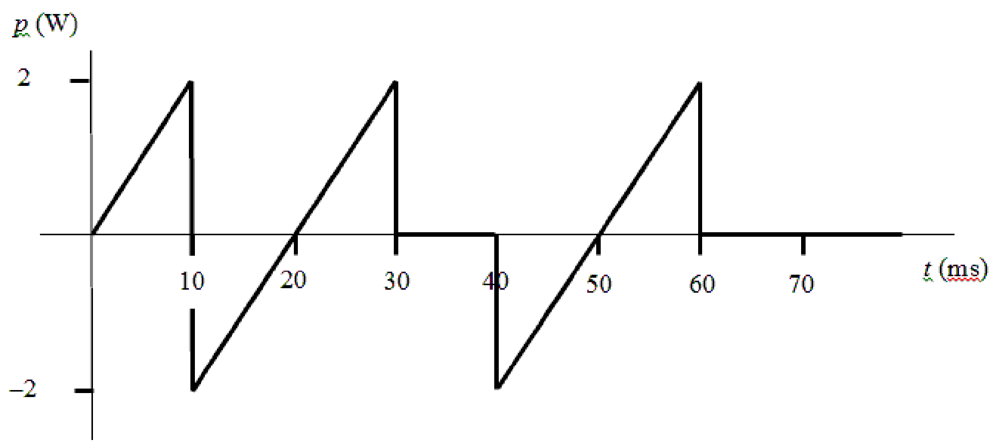
$v = 0$ V; $\quad i = -250$ mA; $\quad p = 0$ W

40 ms $< t \leq$ 60 ms:

$v = 8$ V; $\quad i = 25t - 1.25$ A; $\quad p = 200t - 10$ W

$t >$ 60 ms:

$v = 0$ V; $\quad i = 250$ mA; $\quad p = 0$ W



[b] Calculate the area under the curve from zero up to the desired time:

$$\begin{aligned}
w(0.01) &= \tfrac{1}{2}(2)(0.01) = 10 \text{ mJ} \\
w(0.03) &= w(0.01) - \tfrac{1}{2}(2)(0.01) + \tfrac{1}{2}(2)(0.01) = 10 \text{ mJ} \\
w(0.08) &= w(0.03) - \tfrac{1}{2}(2)(0.01) + \tfrac{1}{2}(2)(0.01) = 10 \text{ mJ}
\end{aligned}$$

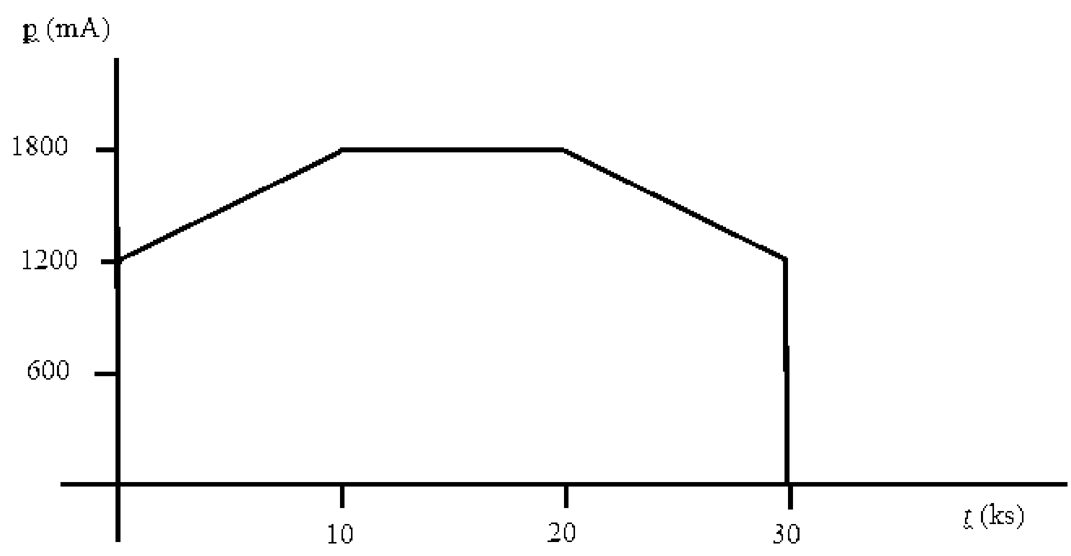
P 1.28 **[a]**

[b]

$$\begin{aligned}
i(t) &= 10 + 0.5 \times 10^{-3}t \text{ mA}, & 0 \leq t \leq 10 \text{ ks} \\
i(t) &= 15 \text{ mA}, & 10 \text{ ks} \leq t \leq 20 \text{ ks} \\
i(t) &= 25 - 0.5 \times 10^{-3}t \text{ mA}, & 20 \text{ ks} \leq t \leq 30 \text{ ks} \\
i(t) &= 0, & t > 30 \text{ ks}
\end{aligned}$$

$p = vi = 120i$ so

$$\begin{aligned}
p(t) &= 1200 + 0.06t \text{ mW}, & 0 \leq t \leq 10 \text{ ks} \\
p(t) &= 1800 \text{ mW}, & 10 \text{ ks} \leq t \leq 20 \text{ ks} \\
p(t) &= 3000 - 0.06t \text{ mW}, & 20 \text{ ks} \leq t \leq 30 \text{ ks} \\
p(t) &= 0, & t > 30 \text{ ks}
\end{aligned}$$



[c] To find the energy, calculate the area under the plot of the power:

$$w(10\text{ ks}) = \frac{1}{2}(0.6)(10{,}000) + (1.2)(10{,}000) = 15\text{ kJ}$$

$$w(20\text{ ks}) = w(10\text{ ks}) + (1.8)(10{,}000) = 33\text{ kJ}$$

$$w(10\text{ ks}) = w(20\text{ ks}) + \frac{1}{2}(0.6)(10{,}000) + (1.2)(10{,}000) = 48\text{ kJ}$$

P 1.29 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i, as shown below:

$$\begin{aligned}
p_\text{a} &= -v_\text{a}i_\text{a} = -(40)(-4\times10^{-3}) = 160\text{ mW}\\
p_\text{b} &= v_\text{b}i_\text{b} = (-24)(-4\times10^{-3}) = 96\text{ mW}\\
p_\text{c} &= -v_\text{c}i_\text{c} = -(-16)(4\times10^{-3}) = 64\text{ mW}\\
p_\text{d} &= -v_\text{d}i_\text{d} = -(-80)(-1.5\times10^{-3}) = -120\text{ mW}\\
p_\text{e} &= v_\text{e}i_\text{e} = (40)(2.5\times10^{-3}) = 100\text{ mW}\\
p_\text{f} &= v_\text{f}i_\text{f} = (120)(-2.5\times10^{-3}) = -300\text{ mW}
\end{aligned}$$

Remember that if the power is positive, the circuit element is absorbing power, whereas is the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:
$\sum P_\text{dev} = 120 + 300 = 420$ mW;
$\sum P_\text{abs} = 160 + 96 + 64 + 100 = 420$ mW
Thus, the power balances and the total power absorbed in the circuit is 420 mW.

P 1.30

$$\begin{aligned}
p_\text{a} &= -v_\text{a}i_\text{a} = -(-3000)(-250\times10^{-6}) = -0.75\text{ W}\\
p_\text{b} &= -v_\text{b}i_\text{b} = -(4000)(-400\times10^{-6}) = 1.6\text{ W}\\
p_\text{c} &= -v_\text{c}i_\text{c} = -(1000)(400\times10^{-6}) = -0.4\text{ W}\\
p_\text{d} &= v_\text{d}i_\text{d} = (1000)(150\times10^{-6}) = 0.15\text{ W}\\
p_\text{e} &= v_\text{e}i_\text{e} = (-4000)(200\times10^{-6}) = -0.8\text{ W}\\
p_\text{f} &= v_\text{f}i_\text{f} = (4000)(50\times10^{-6}) = 0.2\text{ W}
\end{aligned}$$

Therefore,

$$\sum P_\text{abs} = 1.6 + 0.15 + 0.2 = 1.95\text{ W}$$

$$\sum P_\text{del} = 0.75 + 0.4 + 0.8 = 1.95\text{ W} = \sum P_\text{abs}$$

Thus, the interconnection does satisfy the power check.

P 1.31 **[a]** From the diagram and the table we have

$$\begin{aligned}
p_{\rm a} &= -v_{\rm a}i_{\rm a} = -(46.16)(6) = -276.96 \text{ W} \\
p_{\rm b} &= v_{\rm b}i_{\rm b} = (14.16)(4.72) = 66.8352 \text{ W} \\
p_{\rm c} &= v_{\rm c}i_{\rm c} = (-32)(-6.4) = 204.8 \text{ W} \\
p_{\rm d} &= -v_{\rm d}i_{\rm d} = -(22)(1.28) = -28.16 \text{ W} \\
p_{\rm e} &= v_{\rm e}i_{\rm e} = (-33.6)(-1.68) = 56.448 \text{ W} \\
p_{\rm f} &= v_{\rm f}i_{\rm f} = (66)(0.4) = 26.4 \text{ W} \\
p_{\rm g} &= v_{\rm g}i_{\rm g} = (2.56)(1.28) = 3.2768 \text{ W} \\
p_{\rm h} &= -v_{\rm h}i_{\rm h} = -(-0.4)(0.4) = 0.16 \text{ W} \\
\sum P_{\rm del} &= 276.96 + 28.16 = 305.12 \text{ W} \\
\sum P_{\rm abs} &= 66.8352 + 204.8 + 56.448 + 26.4 + 3.2768 + 0.16 = 357.92 \text{ W}
\end{aligned}$$

Therefore, $\sum P_{\rm del} \neq \sum P_{\rm abs}$ and the subordinate engineer is correct.

[b] The difference between the power delivered to the circuit and the power absorbed by the circuit is

$$-305.12 + 357.92 = 52.8 \text{ W}$$

One-half of this difference is 26.4 W, so it is likely that $p_{\rm f}$ is in error. Either the voltage or the current probably has the wrong sign. (In Chapter 2, we will discover that using KCL at the node connecting components f and h, the current $i_{\rm f}$ should be -0.4 A, not 0.4 A!) If the sign of $p_{\rm f}$ is changed from negative to positive, we can recalculate the power delivered and the power absorbed as follows:

$$\begin{aligned}
\sum P_{\rm del} &= 276.96 + 28.16 + 26.4 = 331.52 \text{ W} \\
\sum P_{\rm abs} &= 66.8352 + 204.8 + 56.448 + 3.2768 + 0.16 = 331.52 \text{ W}
\end{aligned}$$

Now the power delivered equals the power absorbed and the power balances for the circuit.

P 1.32 **[a]** Remember that if the circuit element is absorbing power, the power is positive, whereas if the circuit element is supplying power, the power is negative. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal: $\sum P_{\rm sup} = 600 + 50 + 600 + 1250 = 2500$ W;
$\sum P_{\rm abs} = 400 + 100 + 2000 = 2500$ W
Thus, the power balances.

[b] The current can be calculated using $i = p/v$ or $i = -p/v$, with proper application of the passive sign convention:

$$
\begin{aligned}
i_{\rm a} &= -p_{\rm a}/v_{\rm a} = -(-600)/(400) = 1.5 \text{ A} \\
i_{\rm b} &= p_{\rm b}/v_{\rm b} = (-50)/(-100) = 0.5 \text{ A} \\
i_{\rm c} &= p_{\rm c}/v_{\rm c} = (400)/(200) = 2.0 \text{ A} \\
i_{\rm d} &= p_{\rm d}/v_{\rm d} = (-600)/(300) = -2.0 \text{ A} \\
i_{\rm e} &= p_{\rm e}/v_{\rm e} = (100)/(-200) = -0.5 \text{ A} \\
i_{\rm f} &= -p_{\rm f}/v_{\rm f} = -(2000)/(500) = -4.0 \text{ A} \\
i_{\rm g} &= p_{\rm g}/v_{\rm g} = (-1250)/(-500) = 2.5 \text{ A}
\end{aligned}
$$

P 1.33 **[a]** If the power balances, the sum of the power values should be zero:

$$p_{\rm total} = 0.175 + 0.375 + 0.150 - 0.320 + 0.160 + 0.120 - 0.660 = 0$$

Thus, the power balances.

[b] When the power is positive, the element is absorbing power. Since elements a, b, c, e, and f have positive power, these elements are absorbing power.

[c] The voltage can be calculated using $v = p/i$ or $v = -p/i$, with proper application of the passive sign convention:

$$
\begin{aligned}
v_{\rm a} &= p_{\rm a}/i_{\rm a} = (0.175)/(0.025) = 7 \text{ V} \\
v_{\rm b} &= p_{\rm b}/i_{\rm b} = (0.375)/(0.075) = 5 \text{ V} \\
v_{\rm c} &= -p_{\rm c}/i_{\rm c} = -(0.150)/(-0.05) = 3 \text{ V} \\
v_{\rm d} &= p_{\rm d}/i_{\rm d} = (-0.320)/(0.04) = -8 \text{ V} \\
v_{\rm e} &= -p_{\rm e}/i_{\rm e} = -(0.160)/(0.02) = -8 \text{ V} \\
v_{\rm f} &= p_{\rm f}/i_{\rm f} = (0.120)/(-0.03) = -4 \text{ V} \\
v_{\rm g} &= p_{\rm g}/i_{\rm g} = (-0.66)/(0.055) = -12 \text{ V}
\end{aligned}
$$

P 1.34

$$
\begin{aligned}
p_{\rm a} &= v_{\rm a}i_{\rm a} = (120)(-10) = -1200 \text{ W} \\
p_{\rm b} &= -v_{\rm b}i_{\rm b} = -(120)(9) = -1080 \text{ W} \\
p_{\rm c} &= v_{\rm c}i_{\rm c} = (10)(10) = 100 \text{ W} \\
p_{\rm d} &= -v_{\rm d}i_{\rm d} = -(10)(-1) = 10 \text{ W} \\
p_{\rm e} &= v_{\rm e}i_{\rm e} = (-10)(-9) = 90 \text{ W} \\
p_{\rm f} &= -v_{\rm f}i_{\rm f} = -(-100)(5) = 500 \text{ W} \\
p_{\rm g} &= v_{\rm g}i_{\rm g} = (120)(4) = 480 \text{ W} \\
p_{\rm h} &= v_{\rm h}i_{\rm h} = (-220)(-5) = 1100 \text{ W}
\end{aligned}
$$

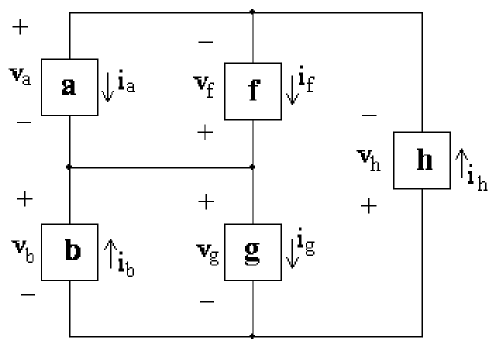
$$\sum P_{\rm del} = 1200 + 1080 = 2280 \text{ W}$$

$\sum P_{\rm abs} = 100 + 10 + 90 + 500 + 480 + 1100 = 2280$ W

Therefore, $\sum P_{\rm del} = \sum P_{\rm abs} = 2280$ W

Thus, the interconnection now satisfies the power check.

P 1.35 **[a]** The revised circuit model is shown below:



[b] The expression for the total power in this circuit is

$$v_{\rm a}i_{\rm a} - v_{\rm b}i_{\rm b} - v_{\rm f}i_{\rm f} + v_{\rm g}i_{\rm g} + v_{\rm h}i_{\rm h}$$

$$= (120)(-10) - (120)(10) - (-120)(3) + 120i_{\rm g} + (-240)(-7) = 0$$

Therefore,

$$120i_{\rm g} = 1200 + 1200 - 360 - 1680 = 360$$

so

$$i_{\rm g} = \frac{360}{120} = 3 \text{ A}$$

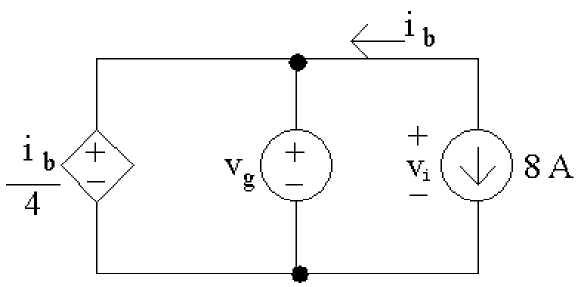
Thus, if the power in the modified circuit is balanced the current in component g is 3 A.

2

Circuit Elements

Assessment Problems

AP 2.1



[a] Note that the current $i_{\rm b}$ is in the same circuit branch as the 8 A current source; however, $i_{\rm b}$ is defined in the opposite direction of the current source. Therefore,

$$i_{\rm b} = -8\,{\rm A}$$

Next, note that the dependent voltage source and the independent voltage source are in parallel with the same polarity. Therefore, their voltages are equal, and

$$v_{\rm g} = \frac{i_{\rm b}}{4} = \frac{-8}{4} = -2\,{\rm V}$$

[b] To find the power associated with the 8 A source, we need to find the voltage drop across the source, v_i. Note that the two independent sources are in parallel, and that the voltages $v_{\rm g}$ and v_1 have the same polarities, so these voltages are equal:

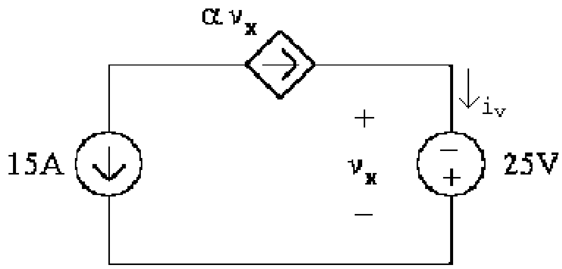
$$v_i = v_g = -2\,{\rm V}$$

Using the passive sign convention,

$$p_s = (8\,{\rm A})(v_i) = (8\,{\rm A})(-2\,{\rm V}) = -16\,{\rm W}$$

Thus the current source generated 16 W of power.

AP 2.2



[a] Note from the circuit that $v_x = -25$ V. To find α note that the two current sources are in the same branch of the circuit but their currents flow in opposite directions. Therefore

$$\alpha v_x = -15\,\text{A}$$

Solve the above equation for α and substitute for v_x,

$$\alpha = \frac{-15\,\text{A}}{v_x} = \frac{-15\,\text{A}}{-25\,\text{V}} = 0.6\ \text{A/V}$$

[b] To find the power associated with the voltage source we need to know the current, i_v. Note that this current is in the same branch of the circuit as the dependent current source and these two currents flow in the same direction. Therefore, the current i_v is the same as the current of the dependent source:

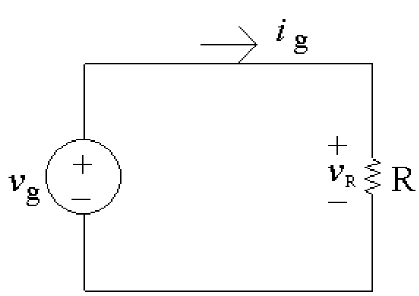
$$i_v = \alpha v_x = (0.6)(-25) = -15\,\text{A}$$

Using the passive sign convention,

$$p_s = -(i_v)(25\,\text{V}) = -(-15\,\text{A})(25\,\text{V}) = 375\,\text{W}.$$

Thus the voltage source dissipates 375 W.

AP 2.3



[a] The resistor and the voltage source are in parallel and the resistor voltage and the voltage source have the same polarities. Therefore these two voltages are the same:

$$v_R = v_g = 1\,\text{kV}$$

Note from the circuit that the current through the resistor is $i_g = 5$ mA. Use Ohm's law to calculate the value of the resistor:

$$R = \frac{v_R}{i_g} = \frac{1\,\text{kV}}{5\,\text{mA}} = 200\,\text{k}\Omega$$

Using the passive sign convention to calculate the power in the resistor,

$$p_R = (v_R)(i_g) = (1\,\text{kV})(5\,\text{mA}) = 5\,\text{W}$$

The resistor is dissipating 5 W of power.

[b] Note from part (a) the $v_R = v_g$ and $i_R = i_g$. The power delivered by the source is thus

$$p_{\text{source}} = -v_g i_g \qquad \text{so} \qquad v_g = -\frac{p_{\text{source}}}{i_g} = -\frac{-3\,\text{W}}{75\,\text{mA}} = 40\,\text{V}$$

Since we now have the value of both the voltage and the current for the resistor, we can use Ohm's law to calculate the resistor value:

$$R = \frac{v_g}{i_g} = \frac{40\,\text{V}}{75\,\text{mA}} = 533.33\,\Omega$$

The power absorbed by the resistor must equal the power generated by the source. Thus,

$$p_R = -p_{\text{source}} = -(-3\,\text{W}) = 3\,\text{W}$$

[c] Again, note the $i_R = i_g$. The power dissipated by the resistor can be determined from the resistor's current:

$$p_R = R(i_R)^2 = R(i_g)^2$$

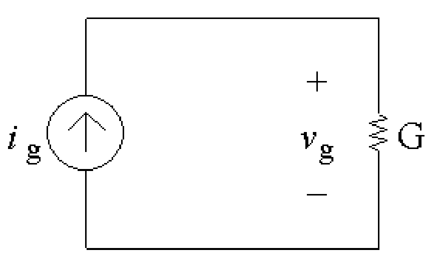
Solving for i_g,

$$i_g^2 = \frac{p_r}{R} = \frac{480\,\text{mW}}{300\,\Omega} = 0.0016 \qquad \text{so} \qquad i_g = \sqrt{0.0016} = 0.04\,\text{A} = 40\,\text{mA}$$

Then, since $v_R = v_g$

$$v_R = Ri_R = Ri_g = (300\,\Omega)(40\,\text{mA}) = 12\,\text{V} \qquad \text{so} \qquad v_g = 12\,\text{V}$$

AP 2.4



[a] Note from the circuit that the current through the conductance G is i_g, flowing from top to bottom, because the current source and the

conductance are in the same branch of the circuit so must have the same current. The voltage drop across the current source is v_g, positive at the top, because the current source and the conductance are also in parallel so must have the same voltage. From a version of Ohm's law,

$$v_g = \frac{i_g}{G} = \frac{0.5\,\text{A}}{50\,\text{mS}} = 10\,\text{V}$$

Now that we know the voltage drop across the current source, we can find the power delivered by this source:

$$p_{\text{source}} = -v_g i_g = -(10)(0.5) = -5\,\text{W}$$

Thus the current source delivers 5 W to the circuit.

[b] We can find the value of the conductance using the power, and the value of the current using Ohm's law and the conductance value:

$$p_g = Gv_g^2 \qquad \text{so} \qquad G = \frac{p_g}{v_g^2} = \frac{9}{15^2} = 0.04\,\text{S} = 40\,\text{mS}$$

$$i_g = Gv_g = (40\,\text{mS})(15\,\text{V}) = 0.6\,\text{A}$$

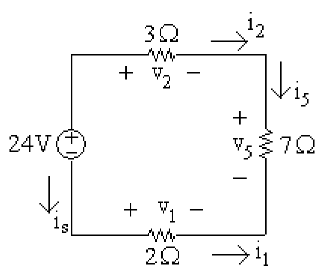
[c] We can find the voltage from the power and the conductance, and then use the voltage value in Ohm's law to find the current:

$$p_g = Gv_g^2 \qquad \text{so} \qquad v_g^2 = \frac{p_g}{G} = \frac{8\,\text{W}}{200\,\mu\text{S}} = 40{,}000$$

Thus $\qquad v_g = \sqrt{40{,}000} = 200\,\text{V}$

$$i_g = Gv_g = (200\,\mu\text{S})(200\,\text{V}) = 0.04\,\text{A} = 40\,\text{mA}$$

AP 2.5 **[a]** Redraw the circuit with all of the voltages and currents labeled for every circuit element.



Write a KVL equation clockwise around the circuit, starting below the voltage source:

$$-24\,\text{V} + v_2 + v_5 - v_1 = 0$$

Next, use Ohm's law to calculate the three unknown voltages from the three currents:

$$v_2 = 3i_2; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1$$

A KCL equation at the upper right node gives $i_2 = i_5$; a KCL equation at the bottom right node gives $i_5 = -i_1$; a KCL equation at the upper left node gives $i_s = -i_2$. Now replace the currents i_1 and i_2 in the Ohm's law equations with i_5:

$v_2 = 3i_2 = 3i_5; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1 = -2i_5$

Now substitute these expressions for the three voltages into the first equation:

$24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12i_5$

Therefore $i_5 = 24/12 = 2$ A

[b] $v_1 = -2i_5 = -2(2) = -4\,\text{V}$

[c] $v_2 = 3i_5 = 3(2) = 6\,\text{V}$

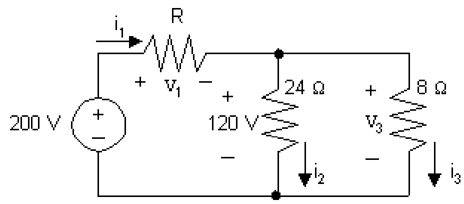
[d] $v_5 = 7i_5 = 7(2) = 14\,\text{V}$

[e] A KCL equation at the lower left node gives $i_s = i_1$. Since $i_1 = -i_5$, $i_s = -2\,\text{A}$. We can now compute the power associated with the voltage source:

$p_{24} = (24)i_s = (24)(-2) = -48\,\text{W}$

Therefore 24 V source is delivering 48 W.

AP 2.6 Redraw the circuit labeling all voltages and currents:



We can find the value of the unknown resistor if we can find the value of its voltage and its current. To start, write a KVL equation clockwise around the right loop, starting below the $24\,\Omega$ resistor:

$-120\,\text{V} + v_3 = 0$

Use Ohm's law to calculate the voltage across the $8\,\Omega$ resistor in terms of its current:

$v_3 = 8i_3$

Substitute the expression for v_3 into the first equation:

$-120\,\text{V} + 8i_3 = 0 \qquad \text{so} \qquad i_3 = \dfrac{120}{8} = 15\,\text{A}$

Also use Ohm's law to calculate the value of the current through the $24\,\Omega$ resistor:

$$i_2 = \frac{120\,\text{V}}{24\,\Omega} = 5\,\text{A}$$

Now write a KCL equation at the top middle node, summing the currents leaving:

$$-i_1 + i_2 + i_3 = 0 \qquad \text{so} \qquad i_1 = i_2 + i_3 = 5 + 15 = 20\,\text{A}$$

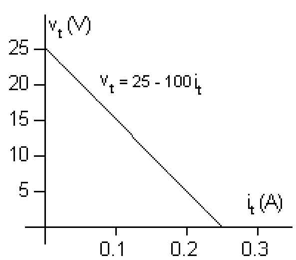
Write a KVL equation clockwise around the left loop, starting below the voltage source:

$$-200\,\text{V} + v_1 + 120\,\text{V} = 0 \qquad \text{so} \qquad v_1 = 200 - 120 = 80\,\text{V}$$

Now that we know the values of both the voltage and the current for the unknown resistor, we can use Ohm's law to calculate the resistance:

$$\text{R} = \frac{v_1}{i_1} = \frac{80}{20} = 4\,\Omega$$

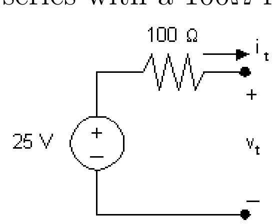
AP 2.7 **[a]** Plotting a graph of v_t versus i_t gives



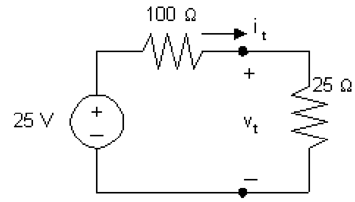
Note that when $i_t = 0$, $v_t = 25$ V; therefore the voltage source must be 25 V. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω resistor, as shown below:



[b] Draw the circuit model from part (a) and attach a $25\,\Omega$ resistor:



To find the power delivered to the $25\,\Omega$ resistor we must calculate the current through the $25\,\Omega$ resistor. Do this by first using KCL to recognize that the current in each of the components is i_t, flowing in a clockwise direction. Write a KVL equation in the clockwise direction, starting below the voltage source, and using Ohm's law to express the voltage drop across the resistors in the direction of the current i_t flowing through the resistors:

$$-25\,\mathrm{V} + 100i_t + 25i_t = 0 \qquad \text{so} \qquad 125i_t = 25 \qquad \text{so} \qquad i_t = \frac{25}{125} = 0.2\,\mathrm{A}$$

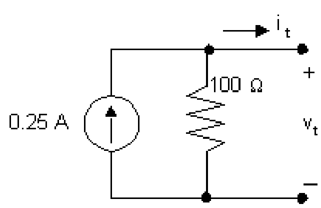
Thus, the power delivered to the $25\,\Omega$ resistor is

$$p_{25} = (25)i_t^2 = (25)(0.2)^2 = 1\,\mathrm{W}.$$

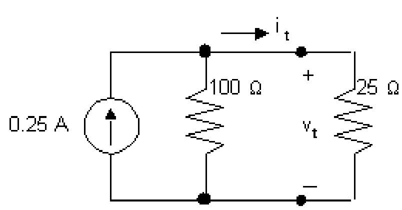
AP 2.8 **[a]** From the graph in Assessment Problem 2.7(a), we see that when $v_t = 0$, $i_t = 0.25\,\mathrm{A}$. Therefore the current source must be 0.25 A. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 0.25 A current source in parallel with a 100Ω resistor, as shown below:



[b] Draw the circuit model from part (a) and attach a $25\,\Omega$ resistor:



Note that by writing a KVL equation around the right loop we see that the voltage drop across both resistors is v_t. Write a KCL equation at the top center node, summing the currents leaving the node. Use Ohm's law to specify the currents through the resistors in terms of the voltage drop across the resistors and the value of the resistors.

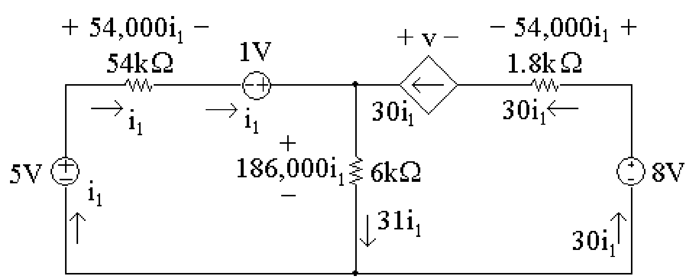
$$-0.25 + \frac{v_t}{100} + \frac{v_t}{25} = 0, \qquad \text{so} \qquad 5v_t = 25, \qquad \text{thus} \qquad v_t = 5\,\text{V}$$

$$p_{25} = \frac{v_t^2}{25} = 1\,\text{W}.$$

AP 2.9 First note that we know the current through all elements in the circuit except the 6 kΩ resistor (the current in the three elements to the left of the 6 kΩ resistor is i_1; the current in the three elements to the right of the 6 kΩ resistor is $30i_1$). To find the current in the 6 kΩ resistor, write a KCL equation at the top node:

$$i_1 + 30i_1 = i_{6\text{k}} = 31i_1$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_1. The results are shown in the figure below:



[a] To find i_1, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 5V source:

$$-5\,\text{V} + 54{,}000i_1 - 1\,\text{V} + 186{,}000i_1 = 0$$

Solving for i_1

$$54{,}000i_1 + 186{,}000i_1 = 6\,\text{V} \qquad \text{so} \qquad 240{,}000i_1 = 6\,\text{V}$$

Thus,

$$i_1 = \frac{6}{240{,}000} = 25\,\mu\text{A}$$

[b] Now that we have the value of i_1, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$+v - 54{,}000i_1 + 8\,\text{V} - 186{,}000i_1 = 0$$

Thus,

$v = 240{,}000 i_1 - 8\,\mathrm{V} = 240{,}000(25 \times 10^{-6}) - 8\,\mathrm{V} = 6\,\mathrm{V} - 8\,\mathrm{V} = -2\,\mathrm{V}$

We now know the values of voltage and current for every circuit element. Let's construct a power table:

Element	**Current (μA)**	**Voltage (V)**	**Power Equation**	**Power (μW)**
5 V	25	5	$p = -vi$	-125
54 kΩ	25	1.35	$p = Ri^2$	33.75
1 V	25	1	$p = -vi$	-25
6 kΩ	775	4.65	$p = Ri^2$	3603.75
Dep. source	750	-2	$p = -vi$	1500
1.8 kΩ	750	1.35	$p = Ri^2$	1012.5
8 V	750	8	$p = -vi$	-6000

[c] The total power generated in the circuit is the sum of the negative power values in the power table:

$-125\,\mu\mathrm{W} + -25\,\mu\mathrm{W} + -6000\,\mu\mathrm{W} = -6150\,\mu\mathrm{W}$

Thus, the total power generated in the circuit is $6150\,\mu$W.

[d] The total power absorbed in the circuit is the sum of the positive power values in the power table:

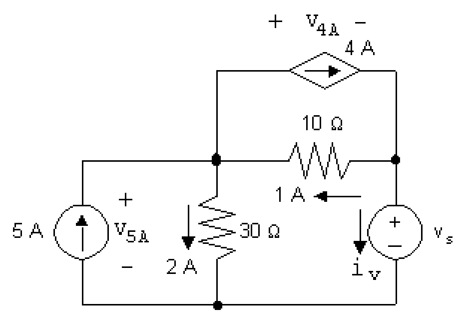
$33.75\,\mu\mathrm{W} + 3603.75\,\mu\mathrm{W} + 1500\,\mu\mathrm{W} + 1012.5\,\mu\mathrm{W} = 6150\,\mu\mathrm{W}$

Thus, the total power absorbed in the circuit is $6150\,\mu$W.

AP 2.10 Given that $i_\phi = 2\,\mathrm{A}$, we know the current in the dependent source is $2i_\phi = 4\,\mathrm{A}$. We can write a KCL equation at the left node to find the current in the 10 Ω resistor. Summing the currents leaving the node,

$$-5\,\mathrm{A} + 2\,\mathrm{A} + 4\,\mathrm{A} + i_{10\Omega} = 0 \qquad \text{so} \qquad i_{10\Omega} = 5\,\mathrm{A} - 2\,\mathrm{A} - 4\,\mathrm{A} = -1\,\mathrm{A}$$

Thus, the current in the 10 Ω resistor is 1 A, flowing right to left, as seen in the circuit below.



[a] To find v_s, write a KVL equation, summing the voltages counter-clockwise around the lower right loop. Start below the voltage source.

$$-v_s + (1\,\text{A})(10\,\Omega) + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_s = 10\,\text{V} + 60\,\text{V} = 70\,\text{V}$$

[b] The current in the voltage source can be found by writing a KCL equation at the right-hand node. Sum the currents leaving the node

$$-4\,\text{A} + 1\,\text{A} + i_v = 0 \qquad \text{so} \qquad i_v = 4\,\text{A} - 1\,\text{A} = 3\,\text{A}$$

The current in the voltage source is 3 A, flowing top to bottom. The power associated with this source is

$$p = vi = (70\,\text{V})(3\,\text{A}) = 210\,\text{W}$$

Thus, 210 W are absorbed by the voltage source.

[c] The voltage drop across the independent current source can be found by writing a KVL equation around the left loop in a clockwise direction:

$$-v_{5A} + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_{5A} = 60\,\text{V}$$

The power associated with this source is

$$p = -v_{5A}i = -(60\,\text{V})(5\,\text{A}) = -300\,\text{W}$$

This source thus delivers 300 W of power to the circuit.

[d] The voltage across the controlled current source can be found by writing a KVL equation around the upper right loop in a clockwise direction:

$$+v_{4A} + (10\,\Omega)(1\,\text{A}) = 0 \qquad \text{so} \qquad v_{4A} = -10\,\text{V}$$

The power associated with this source is

$$p = v_{4A}i = (-10\,\text{V})(4\,\text{A}) = -40\,\text{W}$$

This source thus delivers 40 W of power to the circuit.

[e] The total power dissipated by the resistors is given by

$$(i_{30\Omega})^2(30\,\Omega) + (i_{10\Omega})^2(10\,\Omega) = (2)^2(30\,\Omega) + (1)^2(10\,\Omega) = 120 + 10 = 130\,\text{W}$$

Problems

P 2.1 [**a**] Yes, independent voltage sources can carry the 5 A current required by the connection; independent current source can support any voltage required by the connection, in this case 5 V, positive at the bottom.

[**b**] 20 V source: absorbing

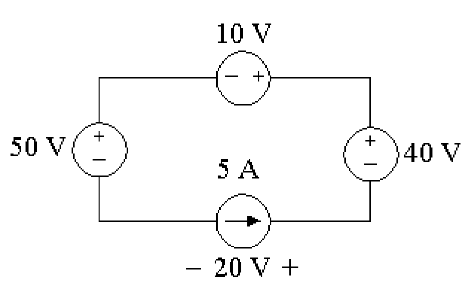
15 V source: developing (delivering)

5 A source: developing (delivering)

[**c**]
$$P_{20\text{V}} = (20)(5) = 100\text{ W} \quad \text{(abs)}$$
$$P_{15\text{V}} = -(15)(5) = -75\text{ W} \quad \text{(dev/del)}$$
$$P_{5\text{A}} = -(5)(5) = -25\text{ W} \quad \text{(dev/del)}$$
$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 100\text{ W}$$

[**d**] The interconnection is valid, but in this circuit the voltage drop across the 5 A current source is 35 V, positive at the top; 20 V source is developing (delivering), the 15 V source is developing (delivering), and the 5 A source is absorbing:
$$P_{20\text{V}} = -(20)(5) = -100\text{ W} \quad \text{(dev/del)}$$
$$P_{15\text{V}} = -(15)(5) = -75\text{ W} \quad \text{(dev/del)}$$
$$P_{5\text{A}} = (35)(5) = 175\text{ W} \quad \text{(abs)}$$
$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 175\text{ W}$$

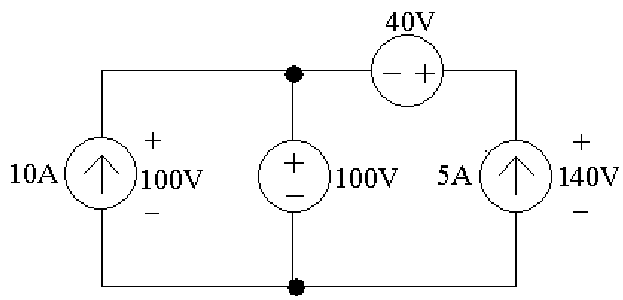
P 2.2 The interconnect is valid since the voltage sources can all carry 5 A of current supplied by the current source, and the current source can carry the voltage drop required by the interconnection. Note that the branch containing the 10 V, 40 V, and 5 A sources must have the same voltage drop as the branch containing the 50 V source, so the 5 A current source must have a voltage drop of 20 V, positive at the right. The voltages and currents are summarize in the circuit below:



$$
\begin{aligned}
P_{50\text{V}} &= (50)(5) = 250 \text{ W} \quad \text{(abs)} \\
P_{10\text{V}} &= (10)(5) = 50 \text{ W} \quad \text{(abs)} \\
P_{40\text{V}} &= -(40)(5) = -200 \text{ W} \quad \text{(dev)} \\
P_{5\text{A}} &= -(20)(5) = -100 \text{ W} \quad \text{(dev)}
\end{aligned}
$$

$\sum P_{\text{dev}} = 300$ W

P 2.3 The interconnection is valid. The 10 A current source has a voltage drop of 100 V, positive at the top, because the 100 V source supplies its voltage drop across a pair of terminals shared by the 10 A current source. The right hand branch of the circuit must also have a voltage drop of 100 V from the left terminal of the 40 V source to the bottom terminal of the 5 A current source, because this branch shares the same terminals as the 100 V source. This means that the voltage drop across the 5 A current source is 140 V, positive at the top. Also, the two voltage sources can carry the current required of the interconnection. This is summarized in the figure below:



From the values of voltage and current in the figure, the power supplied by the current sources is calculated as follows:

$$
\begin{aligned}
P_{10\text{A}} &= -(100)(10) = -1000 \text{ W} \quad \text{(1000 W supplied)} \\
P_{5\text{A}} &= -(140)(5) = -700 \text{ W} \quad \text{(700 W supplied)}
\end{aligned}
$$

$\sum P_{\text{dev}} = 1700$ W

P 2.4 The interconnection is not valid. Note that the 3 A and 4 A sources are both connected in the same branch of the circuit. A valid interconnection would require these two current sources to supply the same current in the same direction, which they do not.

P 2.5 The interconnection is valid, since the voltage sources can carry the currents supplied by the 2 A and 3 A current sources, and the current sources can carry whatever voltage drop from the top node to the bottom node is required by the interconnection. In particular, note the the voltage drop between the top and bottom nodes in the right hand branch must be the same as the voltage drop between the top and bottom nodes in the left hand branch. In particular, this means that

$$-v_1 + 8 \text{ V} = 12 \text{ V} + v_2$$

Hence any combination of v_1 and v_2 such that $v_1 + v_2 = -4\,\text{V}$ is a valid solution.

P 2.6 [**a**] Because both current sources are in the same branch of the circuit, their values must be the same. Therefore,

$$\frac{v_1}{50} = 0.4 \qquad \rightarrow \qquad v_1 = 0.4(50) = 20 \;\; \text{V}$$

[**b**] $p = v_1(0.4) = (20)(0.4) = 8$ W (absorbed)

P 2.7 [**a**] The voltage drop from the top node to the bottom node in this circuit must be the same for every path from the top to the bottom. Therefore, the voltages of the two voltage sources are equal:

$$-\alpha i_\Delta = 6$$

Also, the current i_Δ is in the same branch as the 15 mA current source, but in the opposite direction, so

$$i_\Delta = -0.015$$

Substituting,

$$-\alpha(-0.015) = 6 \qquad \rightarrow \qquad \alpha = \frac{6}{0.015} = 400$$

The interconnection is valid if $\alpha = 400$ V/A.

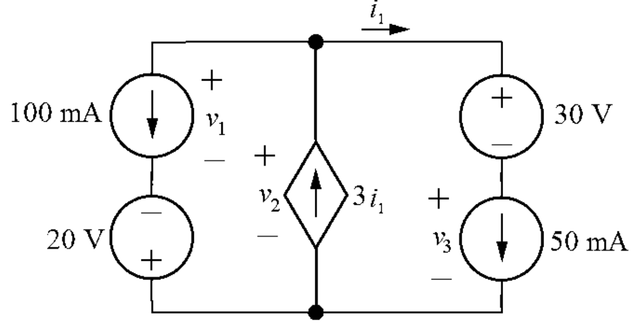
[**b**] The voltage across the current source must equal the voltage across the 6 V source, since both are connected between the top and bottom nodes. Using the passive sign convention,

$$p = vi = (6)(0.015) = 0.09 = 90 \text{ mW}$$

[**c**] Since the power is positive, the current source is absorbing power.

P 2.8 [**a**] Yes, each of the voltage sources can carry the current required by the interconnection, and each of the current sources can carry the voltage drop required by the interconnection. (Note that $i_1 = 50$ mA.)

[**b**] No, because the voltage drop between the top terminal and the bottom terminal cannot be determined. For example, define v_1, v_2, and v_3 as shown:



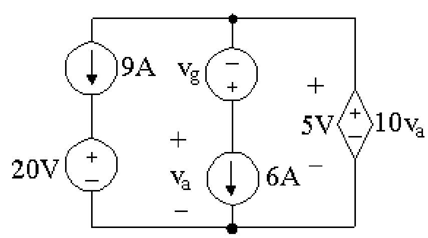
The voltage drop across the left branch, the center branch, and the right branch must be the same, since these branches are connected at the same two terminals. This requires that

$v_1 - 20 = v_2 = v_3 + 30$

But this equation has three unknown voltages, so the individual voltages cannot be determined, and thus the power of the sources cannot be determined.

P 2.9 The interconnection is invalid. In the middle branch, the value of the current i_x must be 50 mA, since the 50 mA current source supplies current in this branch in the same direction as the current i_x. Therefore, the voltage supplied by the dependent voltage source in the right hand branch is $1800(0.05) = 90$ V. This gives a voltage drop from the top terminal to the bottom terminal in the right hand branch of $90 + 60 = 150$ V. But the voltage drop between these same terminals in the left hand branch is 30 V, due to the voltage source in that branch. Therefore, the interconnection is invalid.

P 2.10



First, $10v_a = 5$ V, so $v_a = 0.5$ V. Then recognize that each of the three branches is connected between the same two nodes, so each of these branches must have the same voltage drop. The voltage drop across the middle branch is 5 V, and since $v_a = 0.5$ V, $v_g = 0.5 - 5 = -4.5$ V. Also, the voltage drop across the left branch is 5 V, so $20 + v_{9A} = 5$ V, and $v_{9A} = -15$ V, where v_{9A} is positive at the top. Note that the current through the 20 V source must be 9 A, flowing from top to bottom, and the current through the v_g is 6 A flowing from top to bottom. Let's find the power associated with the left and middle branches:

$p_{9A} = (9)(-15) = -135\,\text{W}$
$p_{20V} = (9)(20) = 180\,\text{W}$
$p_{v_g} = -(6)(-4.5) = 27\,\text{W}$
$p_{6A} = (6)(0.5) = 3\,\text{W}$

Since there is only one component left, we can find the total power:

$p_{\text{total}} = -135 + 180 + 27 + 3 + p_{\text{ds}} = 75 + p_{\text{ds}} = 0$

so p_{ds} must equal -75 W.

Therefore,

$$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 210\,\text{W}$$

P 2.11 [**a**] Using the passive sign convention and Ohm's law,

$$v = Ri = (3000)(0.015) = 45 \text{ V}$$

[**b**] $P_{\rm R} = \dfrac{v^2}{R} = \dfrac{45^2}{3000} = 0.675 = 675 \text{ mW}$

[**c**] Using the passive sign convention with the current direction reversed,

$$v = -Ri = -(3000)(0.015) = -45 \text{ V}$$

$$P_{\rm R} = \frac{v^2}{R} = \frac{-45^2}{3000} = 0.675 = 675 \text{ mW}$$

P 2.12 [**a**] Using the passive sign convention and Ohm's law,

$$i = -\frac{v}{R} = -\frac{40}{2500} = -0.016 = -16 \text{ mA}$$

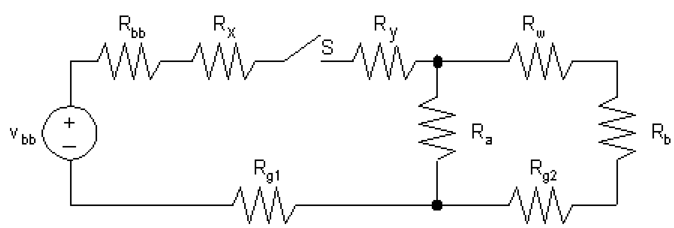
[**b**] $P_{\rm R} = Ri^2 = (2500)(-0.016)^2 = 0.64 = 640 \text{ mW}$

[**c**] Using the passive sign convention with the voltage polarity reversed,

$$i = \frac{v}{R} = \frac{40}{2500} = 0.016 = 16 \text{ mA}$$

$$P_{\rm R} = Ri^2 = (2500)(0.016)^2 = 0.64 = 640 \text{ mW}$$

P 2.13 [**a**]



[**b**]

V_{bb}	=	no-load voltage of battery
R_{bb}	=	internal resistance of battery
R_x	=	resistance of wire between battery and switch
R_y	=	resistance of wire between switch and lamp A
$R_{\rm a}$	=	resistance of lamp A
$R_{\rm b}$	=	resistance of lamp B
R_w	=	resistance of wire between lamp A and lamp B
R_{g1}	=	resistance of frame between battery and lamp A
R_{g2}	=	resistance of frame between lamp A and lamp B
S	=	switch

P 2.14 Since we know the device is a resistor, we can use Ohm's law to calculate the resistance. From Fig. P2.14(a),

$$v = Ri \qquad \text{so} \qquad R = \frac{v}{i}$$

Using the values in the table of Fig. P2.14(b),

$$R = \frac{-7200}{-6} = \frac{-3600}{-3} = \frac{3600}{3} = \frac{7200}{6} = \frac{10{,}800}{9} = 1.2\,\text{k}\Omega$$

Note that this value is found in Appendix H.

P 2.15 Since we know the device is a resistor, we can use the power equation. From Fig. P2.15(a),

$$p = vi = \frac{v^2}{R} \qquad \text{so} \qquad R = \frac{v^2}{p}$$

Using the values in the table of Fig. P2.13(b)

$$R = \frac{(-8)^2}{640 \times 10^{-3}} = \frac{(-4)^2}{160 \times 10^{-3}} = \frac{(4)^2}{160 \times 10^{-3}} = \frac{(8)^2}{640 \times 10^{-3}}$$

$$= \frac{(12)^2}{1440 \times 10^{-3}} = \frac{(16)^2}{2560 \times 10^{-3}} = 100\,\Omega$$

Note that this value is found in Appendix H.

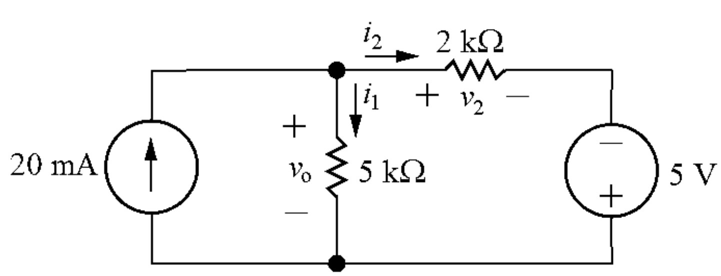
P 2.16 The resistor value is the ratio of the power to the square of the current: $R = \frac{p}{i^2}$. Using the values for power and current in Fig. P2.16(b),

$$\frac{8.25 \times 10^{-3}}{(0.5 \times 10^{-3})^2} = \frac{33 \times 10^{-3}}{(1 \times 10^{-3})^2} = \frac{74.25 \times 10^{-3}}{(1.5 \times 10^{-3})^2} = \frac{132 \times 10^{-3}}{(2 \times 10^{-3})^2}$$

$$= \frac{206.25 \times 10^{-3}}{(2.5 \times 10^{-3})^2} = \frac{297 \times 10^{-3}}{(3 \times 10^{-3})^2} = 33\text{ k}\Omega$$

Note that this is a value from Appendix H.

P 2.17 Label the unknown resistor currents and voltages:



[**a**] KCL at the top node: $0.02 = i_1 + i_2$

KVL around the right loop: $-v_o + v_2 - 5 = 0$

Use Ohm's law to write the resistor voltages in the previous equation in terms of the resistor currents:

$$-5000i_1 + 2000i_2 - 5 = 0 \qquad \rightarrow \qquad -5000i_1 + 2000i_2 = 5$$

Multiply the KCL equation by -2000 and add it to the KVL equation to eliminate i_2:

$$-2000(i_1 + i_2) + (-5000i_1 + 2000i_2) = -2000(0.02) + 5 \quad \rightarrow \quad -7000i_1 = -35$$

Solving,

$$i_1 = \frac{-35}{-7000} = 0.005 = 5 \text{ mA}$$

Therefore,

$$v_o = Ri_1 = (5000)(0.005) = 25 \text{ V}$$

[**b**] $p_{20\text{mA}} = -(0.02)v_o = -(0.02)(25) = -0.5$ W

$$i_2 = 0.02 - i_1 = 0.02 - 0.005 = 0.015 \text{ A}$$

$$p_{5\text{V}} = -(5)i_2 = -(5)(0.015) = -0.075 \text{ W}$$

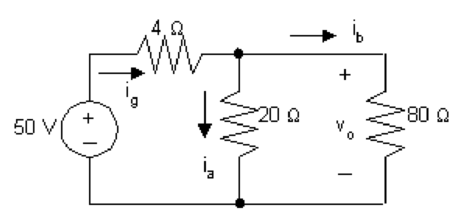
$$p_{5\text{k}} = 5000i_1^2 = 5000(0.005)^2 = 0.125 \text{ W}$$

$$p_{2\text{k}} = 2000i_2^2 = 2000(0.015)^2 = 0.45 \text{ W}$$

$$p_{\text{total}} = p_{20\text{mA}} + p_{5\text{V}} + p_{5\text{k}} + p_{2\text{k}} = -0.5 - 0.075 + 0.125 + 0.45 = 0$$

Thus the power in the circuit balances.

P 2.18 [**a**]



$$\begin{aligned} 20i_\text{a} &= 80i_\text{b} \qquad i_g = i_\text{a} + i_\text{b} = 5i_\text{b} \\ i_\text{a} &= 4i_\text{b} \\ 50 &= 4i_g + 80i_\text{b} = 20i_\text{b} + 80i_\text{b} = 100i_\text{b} \\ i_\text{b} &= 0.5 \text{ A, therefore, } i_\text{a} = 2 \text{ A} \quad \text{and} \quad i_g = 2.5 \text{ A} \end{aligned}$$

[**b**] $i_\text{b} = 0.5$ A

[**c**] $v_o = 80i_\text{b} = 40$ V

[d] $p_{4\Omega} = i_g^2(4) = 6.25(4) = 25$ W

$p_{20\Omega} = i_a^2(20) = (4)(20) = 80$ W

$p_{80\Omega} = i_b^2(80) = 0.25(80) = 20$ W

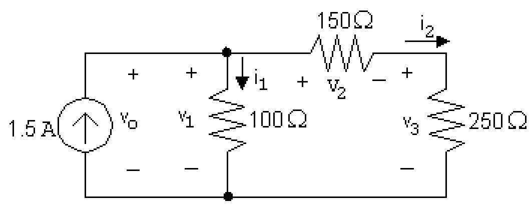
[e] p_{50V} (delivered) $= 50i_g = 125$ W

Check:

$\sum P_{\text{dis}} = 25 + 80 + 20 = 125\,\text{W}$

$\sum P_{\text{del}} = 125\,\text{W}$

P 2.19



[a] Write a KCL equation at the top node:

$-1.5 + i_1 + i_2 = 0$ so $i_1 + i_2 = 1.5$

Write a KVL equation around the right loop:

$-v_1 + v_2 + v_3 = 0$

From Ohm's law,

$v_1 = 100i_1, \quad v_2 = 150i_2, \quad v_3 = 250i_2$

Substituting,

$-100i_1 + 150i_2 + 250i_2 = 0$ so $-100i_1 + 400i_2 = 0$

Solving the two equations for i_1 and i_2 simultaneously,

$i_1 = 1.2\,\text{A}$ and $i_2 = 0.3\,\text{A}$

[b] Write a KVL equation clockwise around the left loop:

$-v_o + v_1 = 0$ but $v_1 = 100i_1 = 100(1.2) = 120\,\text{V}$

So $v_o = v_1 = 120\,\text{V}$

[c] Calculate power using $p = vi$ for the source and $p = Ri^2$ for the resistors:

$p_{\text{source}} = -v_o(1.5) = -(120)(1.5) = -180\,\text{W}$

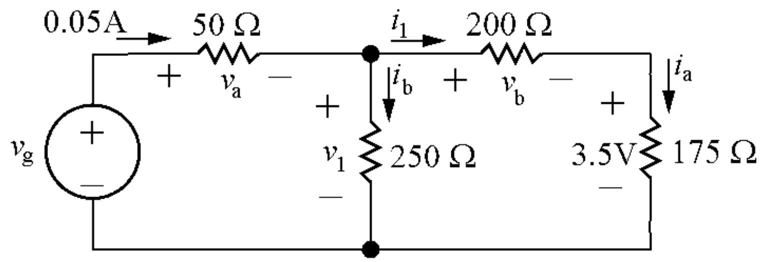
$p_{100\Omega} = 1.2^2(100) = 144\,\text{W}$

$p_{150\Omega} = 0.3^2(150) = 13.5\,\text{W}$

$p_{250\Omega} = 0.3^2(250) = 22.5\,\text{W}$

$\sum P_{\text{dev}} = 180\,\text{W} \qquad \sum P_{\text{abs}} = 144 + 13.5 + 22.5 = 180\,\text{W}$

P 2.20 Label the unknown resistor voltages and currents:



[a] $i_a = \dfrac{3.5}{175} = 0.02\,\text{A}$ (Ohm's law)

$i_1 = i_a = 0.02\,\text{A}$ (KCL)

[b] $v_b = 200i_1 = 200(0.02) = 4\,\text{V}$ (Ohm's law)

$-v_1 + v_b + 3.5 = 0$ so $v_1 = 3.5 + v_b = 3.5 + 4 = 7.5\,\text{V}$ (KVL)

[c] $v_a = 0.05(50) = 2.5\,\text{V}$ (Ohm's law)

$-v_g + v_a + v_1 = 0$ so $v_g = v_a + v_1 = 2.5 + 7.5 = 10\,\text{V}$ (KVL)

[d] $p_g = v_g(0.05) = 10(0.05) = 0.5\,\text{W}$

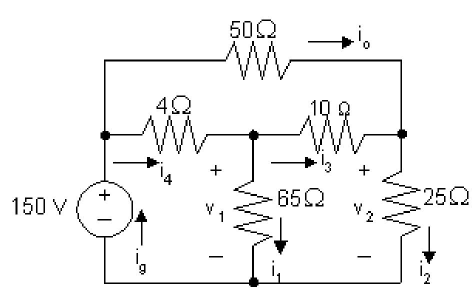
P 2.21 [a] Use KVL for the right loop to calculate the voltage drop across the right-hand branch v_o. This is also the voltage drop across the middle branch, so once v_o is known, use Ohm's law to calculate i_o:

$$\begin{aligned} v_o &= 1000i_a + 4000i_a + 3000i_a = 8000i_a = 8000(0.002) = 16\text{ V} \\ 16 &= 2000i_o \\ i_o &= \frac{16}{2000} = 8\text{ mA} \end{aligned}$$

[b] KCL at the top node: $i_g = i_a + i_o = 0.002 + 0.008 = 0.010\text{ A} = 10\text{ mA}$.

[c] The voltage drop across the source is v_0, seen by writing a KVL equation for the left loop. Thus,
$p_g = -v_o i_g = -(16)(0.01) = -0.160\text{ W} = -160\text{ mW}$.
Thus the source delivers 160 mW.

P 2.22 [a]



$v_2 = 150 - 50(1) = 100\text{V}$

$$i_2 = \frac{v_2}{25} = 4\text{A}$$

$$i_3 + 1 = i_2, \qquad i_3 = 4 - 1 = 3\text{A}$$

$$v_1 = 10i_3 + 25i_2 = 10(3) + 25(4) = 130\text{V}$$

$$i_1 = \frac{v_1}{65} = \frac{130}{65} = 2\text{A}$$

Note also that

$$i_4 = i_1 + i_3 = 2 + 3 = 5\,\text{A}$$

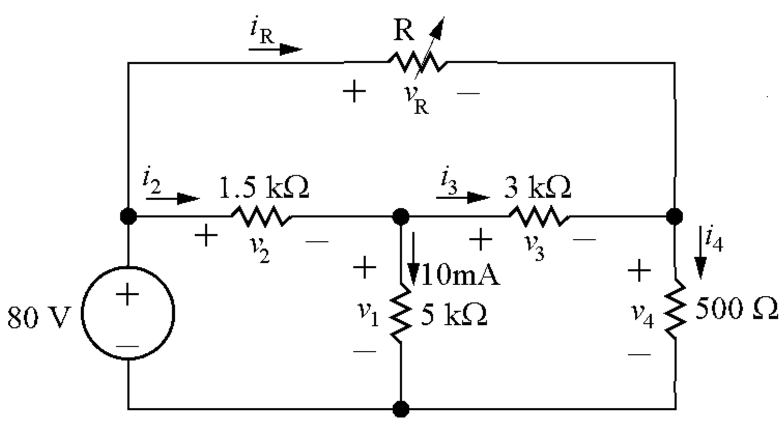
$$i_g = i_4 + i_o = 5 + 1 = 6\,\text{A}$$

[b]
$$\begin{aligned} p_{4\Omega} &= 5^2(4) = 100\text{ W} \\ p_{50\Omega} &= 1^2(50) = 50\text{ W} \\ p_{65\Omega} &= 2^2(65) = 260\text{ W} \\ p_{10\Omega} &= 3^2(10) = 90\text{ W} \\ p_{25\Omega} &= 4^2(25) = 400\text{ W} \end{aligned}$$

[c] $\sum P_{\text{dis}} = 100 + 50 + 260 + 90 + 400 = 900\,\text{W}$

$$P_{\text{dev}} = 150 i_g = 150(6) = 900\,\text{W}$$

P 2.23 Label all unknown resistor voltages and currents:

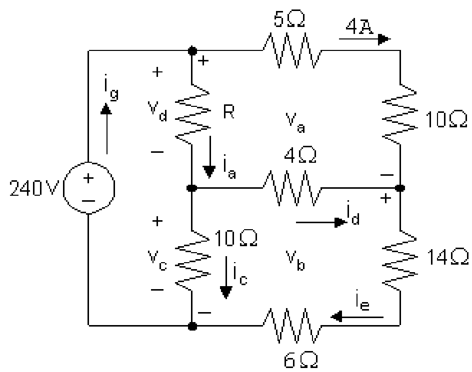


Ohms' law for 5 kΩ resistor: $v_1 = (0.01)(5000) = 50$ V
KVL for lower left loop: $-80 + v_2 + 50 = 0 \quad \rightarrow \quad v_2 = 80 - 50 = 30$ V
Ohm's law for 1.5 kΩ resistor: $i_2 = v_2/1500 = 30/1500 = 20$ mA
KCL at center node:
$i_2 = i_3 + 0.01 \quad \rightarrow \quad i_3 = i_2 - 0.01 = 0.02 - 0.01 = 0.01 = 10$ mA
Ohm's law for 3 kΩ resistor $v_3 = 3000 i_3 = 3000(0.01) = 30$ V
KVL for lower right loop:
$-v_1 + v_3 + v_4 = 0 \quad \rightarrow \quad v_4 = v_1 - v_3 = 50 - 30 = 20$ V

Ohm's law for $500\,\Omega$ resistor: $\quad i_4 = v_4/500 = 20/500 = 0.04 = 40$ mA
KCL for right node:
$i_3 + i_{\rm R} = i_4 \quad \rightarrow \quad i_{\rm R} = i_4 - i_3 = 0.04 - 0.01 = 0.03 = 30$ mA
KVL for outer loop:
$-80 + v_{\rm R} + v_4 = 0 \quad \rightarrow \quad v_{\rm R} = 80 - v_4 = 80 - 20 = 60$ V
Therefore,

$$R = \frac{v_{\rm R}}{i_{\rm R}} = \frac{60}{0.03} = 2000 = 2\ \mathrm{k}\Omega$$

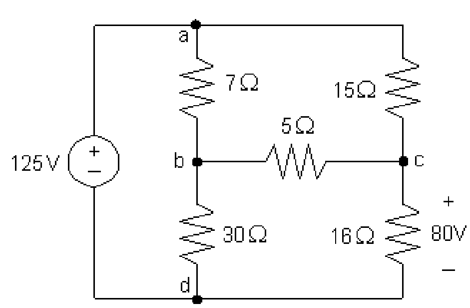
P 2.24 [**a**]



$v_a = (5 + 10)(4) = 60\,\mathrm{V}$
$-240 + v_a + v_b = 0 \qquad$ so $\qquad v_b = 240 - v_a = 240 - 60 = 180\,\mathrm{V}$
$i_e = v_b/(14 + 6) = 180/20 = 9\,\mathrm{A}$
$i_d = i_e - 4 = 9 - 4 = 5\,\mathrm{A}$
$v_c = 4i_d + v_b = 4(5) + 180 = 200\,\mathrm{V}$
$i_c = v_c/10 = 200/10 = 20\,\mathrm{A}$
$v_d = 240 - v_c = 240 - 200 = 40\,\mathrm{V}$
$i_a = i_d + i_c = 5 + 20 = 25\,\mathrm{A}$
$R = v_d/i_a = 40/25 = 1.6\,\Omega$

[**b**] $i_g = i_a + 4 = 25 + 4 = 29$ A
p_g (supplied) $= (240)(29) = 6960$ W

P 2.25 [**a**]



$i_{\rm cd} = 80/16 = 5\,\mathrm{A}$

$v_{\rm ac} = 125 - 80 = 45 \qquad$ so $\qquad i_{\rm ac} = 45/15 = 3\,\mathrm{A}$

$i_{ac} + i_{bc} = i_{cd}$ so $i_{bc} = 5 - 3 = 2\,\text{A}$

$v_{ab} = 15i_{ac} - 5i_{bc} = 15(3) - 5(2) = 35\,\text{V}$ so $i_{ab} = 35/7 = 5\,\text{A}$

$i_{bd} = i_{ab} - i_{bc} = 5 - 2 = 3\,\text{A}$

Calculate the power dissipated by the resistors using the equation $p_R = Ri_R^2$:

$p_{7\Omega} = (7)(5)^2 = 175\,\text{W}$ $p_{30\Omega} = (30)(3)^2 = 270\,\text{W}$

$p_{15\Omega} = (15)(3)^2 = 135\,\text{W}$ $p_{16\Omega} = (16)(5)^2 = 400\,\text{W}$

$p_{5\Omega} = (5)(2)^2 = 20\,\text{W}$

[b] Calculate the current through the voltage source:

$i_{ad} = -i_{ab} - i_{ac} = -5 - 3 = -8\,\text{A}$

Now that we have both the voltage and the current for the source, we can calculate the power supplied by the source:

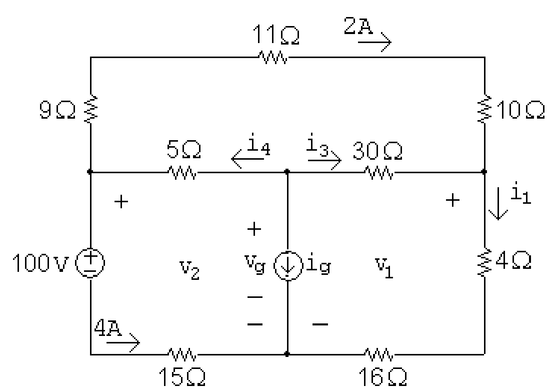
$p_g = 125(-8) = -1000\,\text{W}$ thus p_g (supplied) $= 1000\,\text{W}$

[c] $\sum P_{\text{dis}} = 175 + 270 + 135 + 400 + 20 = 1000\,\text{W}$

Therefore,

$\sum P_{\text{supp}} = \sum P_{\text{dis}}$

P 2.26 **[a]**



$v_2 = 100 + 4(15) = 160\,\text{V};$ $v_1 = 160 - (9 + 11 + 10)(2) = 100\,\text{V}$

$i_1 = \dfrac{v_1}{4 + 16} = \dfrac{100}{20} = 5\,\text{A};$ $i_3 = i_1 - 2 = 5 - 2 = 3\,\text{A}$

$v_g = v_1 + 30i_3 = 100 + 30(3) = 190\,\text{V}$

$i_4 = 2 + 4 = 6\,\text{A}$

$i_g = -i_4 - i_3 = -6 - 3 = -9\,\text{A}$

[b] Calculate power using the formula $p = Ri^2$:

$p_{9\,\Omega} = (9)(2)^2 = 36\,\text{W};$ $\quad p_{11\,\Omega} = (11)(2)^2 = 44\,\text{W}$

$p_{10\,\Omega} = (10)(2)^2 = 40\,\text{W};$ $\quad p_{5\,\Omega} = (5)(6)^2 = 180\,\text{W}$

$p_{30\,\Omega} = (30)(3)^2 = 270\,\text{W};$ $\quad p_{4\,\Omega} = (4)(5)^2 = 100\,\text{W}$

$p_{16\,\Omega} = (16)(5)^2 = 400\,\text{W};$ $\quad p_{15\,\Omega} = (15)(4)^2 = 240\,\text{W}$

[c] $v_g = 190\,\text{V}$

[d] Sum the power dissipated by the resistors:

$$\sum p_{\text{diss}} = 36 + 44 + 40 + 180 + 270 + 100 + 400 + 240 = 1310\,\text{W}$$

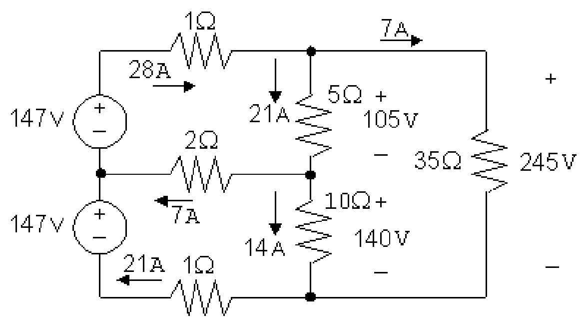
The power associated with the sources is

$$p_{\text{volt-source}} = (100)(4) = 400\,\text{W}$$

$$p_{\text{curr-source}} = v_g i_g = (190)(-9) = -1710\,\text{W}$$

Thus the total power dissipated is $1310 + 400 = 1710$ W and the total power developed is 1710 W, so the power balances.

P 2.27 [a] Start by calculating the voltage drops due to the currents i_1 and i_2. Then use KVL to calculate the voltage drop across and $35\,\Omega$ resistor, and Ohm's law to find the current in the $35\,\Omega$ resistor. Finally, KCL at each of the middle three nodes yields the currents in the two sources and the current in the middle $2\,\Omega$ resistor. These calculations are summarized in the figure below:



$$p_{147(\text{top})} = -(147)(28) = -4116\ \text{W}$$

$$p_{147(\text{bottom})} = -(147)(21) = -3087\ \text{W}$$

Therefore the top source supplies 4116 W of power and the bottom source supplies 3087 W of power.

[b]

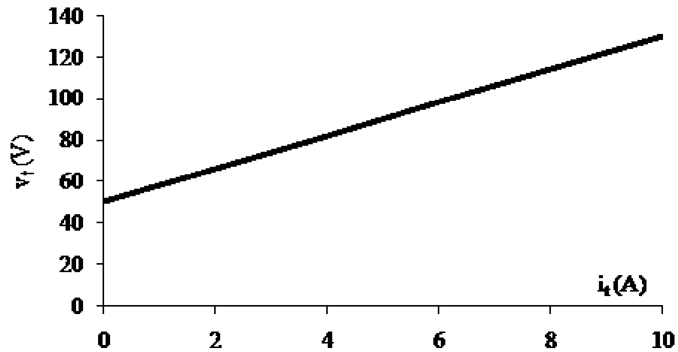
$$\sum P_{\text{dis}} = (28)^2(1) + (7)^2(2) + (21)^2(1) + (21)^2(5) + (14)^2(10) + (7)^2(35)$$

$$= 784 + 98 + 441 + 2205 + 1960 + 1715 = 7203\ \text{W}$$

$$\sum P_{\text{sup}} = 4116 + 3087 = 7203\ \text{W}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{sup}} = 7203$ W

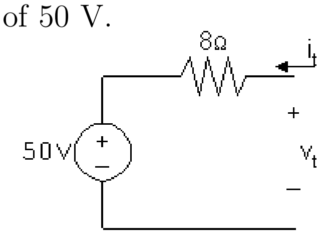
P 2.28 [a] Plot the $v - i$ characteristic



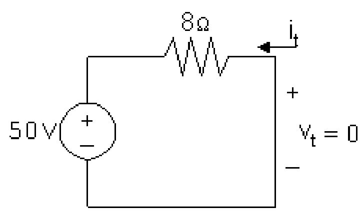
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(130 - 50)}{(10 - 0)} = 8\,\Omega$$

When $i_t = 0$, $v_t = 50$ V; therefore the ideal voltage source has a voltage of 50 V.



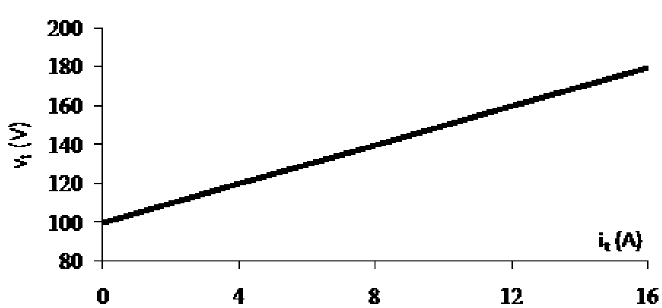
[b]



When $v_t = 0$, $\qquad i_t = \dfrac{-50}{8} = -6.25\text{A}$

Note that this result can also be obtained by extrapolating the $v - i$ characteristic to $v_t = 0$.

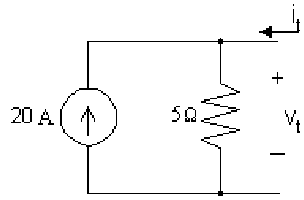
P 2.29 [a] Plot the v—i characteristic:



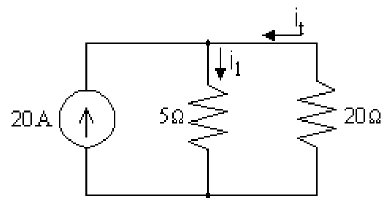
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(180 - 100)}{(16 - 0)} = 5\,\Omega$$

When $i_t = 0$, $v_t = 100$ V; therefore the ideal current source must have a current of $100/5 = 20$ A



[**b**] We attach a $20\,\Omega$ resistor to the device model developed in part (a):



Write a KCL equation at the top node:

$20 + i_t = i_1$

Write a KVL equation for the right loop, in the direction of the two currents, using Ohm's law:

$5i_1 + 20i_t = 0$

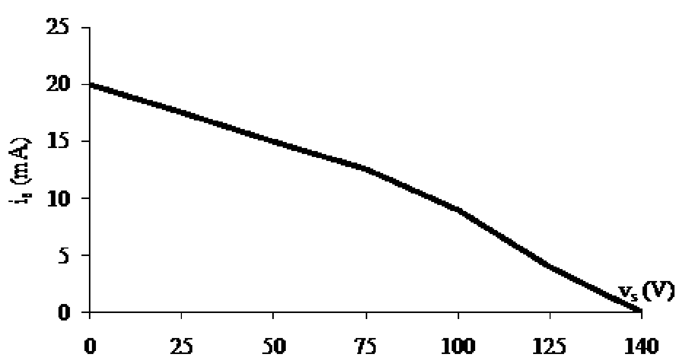
Combining the two equations and solving,

$5(20 + i_t) + 20i_t = 0$ so $25i_t = -100;$ thus $i_t = -4\,\text{A}$

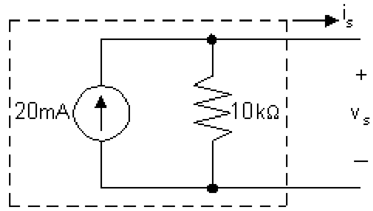
Now calculate the power dissipated by the resistor:

$p_{20\,\Omega} = 20i_t^2 = 20(-4)^2 = 320\,\text{W}$

P 2.30 [**a**]

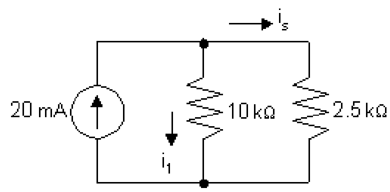


[b] $\Delta v = 25\text{V}; \quad \Delta i = 2.5 \text{ mA}; \quad R = \dfrac{\Delta v}{\Delta i} = 10 \text{ k}\Omega$



[c] $10{,}000 i_1 = 2500 i_s, \qquad i_1 = 0.25 i_s$

$0.02 = i_1 + i_s = 1.25 i_s, \qquad i_s = 16 \text{ mA}$

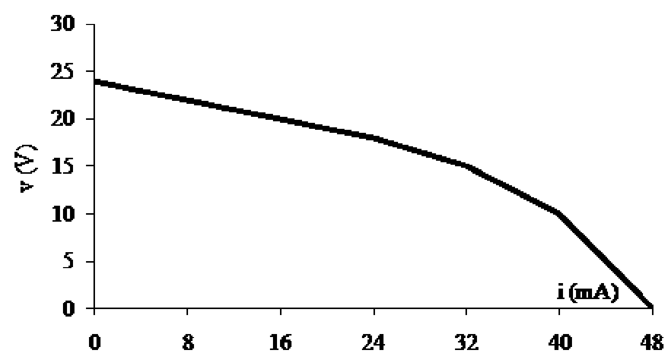


[d] $v_s(\text{open circuit}) = (20 \times 10^{-3})(10 \times 10^3) = 200$ V

[e] The open circuit voltage can be found in the table of values (or from the plot) as the value of the voltage v_s when the current $i_s = 0$. Thus, $v_s(\text{open circuit}) = 140$ V (from the table)

[f] Linear model cannot predict the nonlinear behavior of the practical current source.

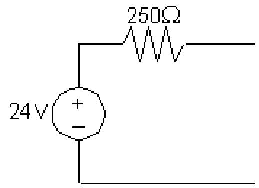
P 2.31 [a] Begin by constructing a plot of voltage versus current:



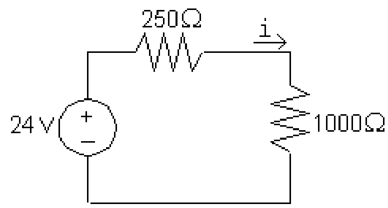
[b] Since the plot is linear for $0 \leq i_s \leq 24$ mA amd since $R = \Delta v/\Delta i$, we can calculate R from the plotted values as follows:

$$R = \frac{\Delta v}{\Delta i} = \frac{24 - 18}{0.024 - 0} = \frac{6}{0.024} = 250\,\Omega$$

We can determine the value of the ideal voltage source by considering the value of v_s when $i_s = 0$. When there is no current, there is no voltage drop across the resistor, so all of the voltage drop at the output is due to the voltage source. Thus the value of the voltage source must be 24 V. The model, valid for $0 \leq i_s \leq 24$ mA, is shown below:



[c] The circuit is shown below:

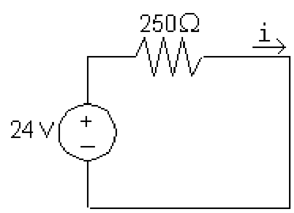


Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-24\,\text{V} + 250i + 1000i = 0 \qquad \text{so} \qquad 1250i = 24\,\text{V}$$

$$\text{Thus,} \qquad i = \frac{24\,\text{V}}{1250\,\Omega} = 19.2\,\text{mA}$$

[d] The circuit is shown below:



Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-24\,\text{V} + 250i = 0 \qquad \text{so} \qquad 250i = 24\,\text{V}$$

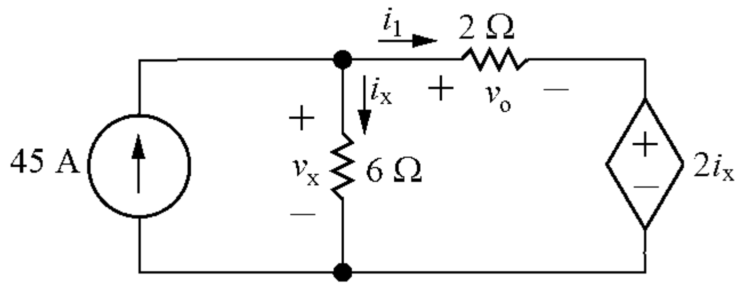
$$\text{Thus,} \qquad i = \frac{24\,\text{V}}{250\,\Omega} = 96\,\text{mA}$$

[e] The short circuit current can be found in the table of values (or from the plot) as the value of the current i_s when the voltage $v_s = 0$. Thus,

$$i_{sc} = 48\,\text{mA} \qquad \text{(from table)}$$

[f] The plot of voltage versus current constructed in part (a) is not linear (it is piecewise linear, but not linear for all values of i_s). Since the proposed circuit model is a linear model, it cannot be used to predict the nonlinear behavior exhibited by the plotted data.

P 2.32 Label unknown voltage and current:



$$-v_x + v_o + 2i_x = 0 \qquad \text{(KVL)}$$

$$v_x = 6i_x \qquad \text{(Ohm's law)}$$

Therefore

$$-6i_x + v_o + 2i_x = 0 \quad \text{so} \quad v_o = 4i_x$$

Thus

$$i_x = \frac{v_o}{4}$$

Also,

$$i_1 = \frac{v_o}{2} \qquad \text{(Ohm's law)}$$

$$45 = i_x + i_1 \qquad \text{(KCL)}$$

Substituting for the currents i_x and i_1:

$$45 = \frac{v_o}{4} + \frac{v_o}{2} = \frac{3v_o}{4}$$

Thus

$$v_o = 45\left(\frac{4}{3}\right) = 60\,\text{V}$$

The only two circuit elements that could supply power are the two sources, so calculate the power for each source:

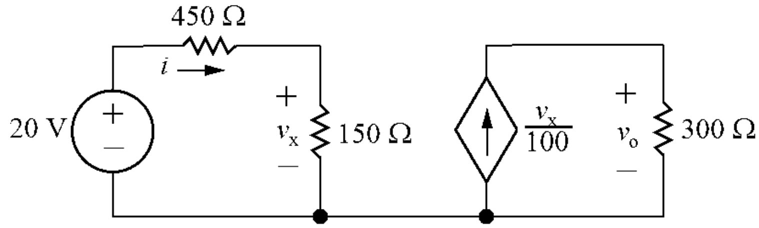
$$v_x = 6i_x = 6\frac{v_o}{4} = 6(60/4) = 90\,\text{V}$$

$$p_{45\text{V}} = -45v_x = -45(90) = -4050\,\text{W}$$

$$p_{\text{d.s.}} = (2i_x)i_1 = 2(v_o/4)(v_o/2) = 2(60/4)(60/2) = 900\,\text{W}$$

Only the independent voltage source is supplying power, so the total power supplied is 4050 W.

P 2.33 Label unknown current:



$$-20 + 450i + 150i = 0 \qquad \text{(KVL and Ohm's law)}$$

$$\text{so} \qquad 600i = 20 \quad \to \quad i = 33.33\,\text{mA}$$

$$v_x = 150i = 150(0.0333) = 5\,\text{V} \qquad \text{(Ohm's law)}$$

$$v_o = 300\left(\frac{v_x}{100}\right) = 300(5/100) = 15\,\text{V} \qquad \text{(Ohm's law)}$$

Calculate the power for all components:

$$p_{20\text{V}} = -20i = -20(0.0333) = -0.667\,\text{W}$$

$$p_{\text{d.s.}} = -v_o\left(\frac{v_x}{100}\right) = -(15)(5/100) = -0.75\,\text{W}$$

$$p_{450} = 450i^2 = 450(0.033)^2 = 0.5\,\text{W}$$

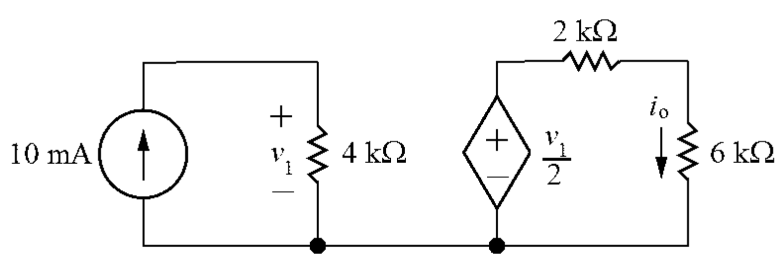
$$p_{150} = 150i^2 = 150(0.033)^2 = 0.1667\,\text{W}$$

$$p_{300} = \frac{v_o^2}{300} = \frac{15^2}{300} = 0.75\,\text{W}$$

Thus the total power absorbed is

$$p_{\text{abs}} = 0.5 + 0.1667 + 0.75 = 1.4167\,\text{W}$$

P 2.34 The circuit:



$$v_1 = (4000)(0.01) = 40\,\text{V} \qquad \text{(Ohm's law)}$$

$$\frac{v_1}{2} = 2000i_\text{o} + 6000i_\text{o} = 8000i_\text{o} \qquad \text{(KVL)}$$

Thus,

$$i_\text{o} = \frac{v_1/2}{8000} = \frac{40/2}{8000} = 0.0025 = 2.5\,\text{mA}$$

Calculate the power for all components:

$$p_{10\text{mA}} = -(0.01)v_1 = -(0.01)(40) = -0.4\,\text{W}$$

$$p_{\text{d.s.}} = -(v_1/2)i_\text{o} = -(40/2)(2.5 \times 10^{-3}) = -0.05\,\text{W}$$

$$p_{4\text{k}} = \frac{v_1^2}{4000} = \frac{40^2}{4000} = 0.4\,\text{W}$$

$$p_{2\text{k}} = 2000i_\text{o}^2 = 2000(2.5 \times 10^{-3})^2 = 0.0125\,\text{W}$$

$$p_{6\text{k}} = 6000i_\text{o}^2 = 6000(2.5 \times 10^{-3})^2 = 0.0375\,\text{W}$$

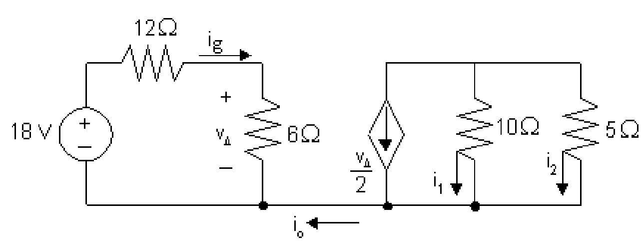
Therefore,

$$p_{\text{total}} = -0.4 - 0.05 + 0.4 + 0.0125 + 0.0375 = 0$$

Thus the power in the circuit balances.

P 2.35 **[a]** $i_o = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



$18 = (12 + 6)i_g \qquad i_g = 1$ A

$v_\Delta = 6i_g = 6\text{V} \qquad v_\Delta/2 = 3$ A

$10i_1 = 5i_2,$ so $i_1 + 2i_1 = -3$ A; therefore, $i_1 = -1$ A

[c] $i_2 = 2i_1 = -2$ A.

P 2.36 **[a]** $-50 - 20i_\sigma + 18i_\Delta = 0$

$-18i_\Delta + 5i_\sigma + 40i_\sigma = 0 \quad$ so $\quad 18i_\Delta = 45i_\sigma$

Therefore, $\quad -50 - 20i_\sigma + 45i_\sigma = 0, \quad$ so $\quad i_\sigma = 2\,\text{A}$

$18i_\Delta = 45i_\sigma = 90;$ so $i_\Delta = 5\,\text{A}$

$v_o = 40i_\sigma = 80\,\text{V}$

[b] i_g = current out of the positive terminal of the 50 V source
v_d = voltage drop across the $8i_\Delta$ source

$i_g = i_\Delta + i_\sigma + 8i_\Delta = 9i_\Delta + i_\sigma = 47\,\text{A}$

$v_d = 80 - 20 = 60\,\text{V}$

$$\begin{aligned}
\sum P_\text{gen} &= 50i_g + 20i_\sigma i_g = 50(47) + 20(2)(47) = 4230\text{ W} \\
\sum P_\text{diss} &= 18i_\Delta^2 + 5i_\sigma(i_g - i_\Delta) + 40i_\sigma^2 + 8i_\Delta v_d + 8i_\Delta(20) \\
&= (18)(25) + 10(47 - 5) + 4(40) + 40(60) + 40(20) \\
&= 4230\text{ W; Therefore,} \\
\sum P_\text{gen} &= \sum P_\text{diss} = 4230\text{ W}
\end{aligned}$$

P 2.37 $40i_2 + \dfrac{5}{40} + \dfrac{5}{10} = 0; \quad i_2 = -15.625\text{ mA}$

$v_1 = 80i_2 = -1.25\text{ V}$

$25i_1 + \dfrac{(-1.25)}{20} + (-0.015625) = 0; \quad i_1 = 3.125\text{ mA}$

$v_g = 60i_1 + 260i_1 = 320i_1$

Therefore, $v_g = 1$ V.

P 2.38 $i_E - i_B - i_C = 0$

$i_C = \beta i_B \qquad$ therefore $i_E = (1 + \beta)i_B$

$i_2 = -i_B + i_1$

$V_o + i_E R_E - (i_1 - i_B)R_2 = 0$

$-i_1 R_1 + V_{CC} - (i_1 - i_B)R_2 = 0 \quad$ or $\quad i_1 = \dfrac{V_{CC} + i_B R_2}{R_1 + R_2}$

$$V_o + i_E R_E + i_B R_2 - \frac{V_{CC} + i_B R_2}{R_1 + R_2} R_2 = 0$$

Now replace i_E by $(1+\beta)i_B$ and solve for i_B. Thus

$$i_B = \frac{[V_{CC}R_2/(R_1 + R_2)] - V_o}{(1+\beta)R_E + R_1 R_2/(R_1 + R_2)}$$

P 2.39 Here is Equation 2.25:

$$i_{\rm B} = \frac{(V_{\rm CC}R_2)/(R_1 + R_2) - V_0}{(R_1 R_2)/(R_1 + R_2) + (1+\beta)R_{\rm E}}$$

$$\frac{V_{CC}R_2}{R_1 + R_2} = \frac{(10)(60{,}000)}{100{,}000} = 6\text{V}$$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{(40{,}000)(60{,}000)}{100{,}000} = 24\ \text{k}\Omega$$

$$i_B = \frac{6 - 0.6}{24{,}000 + 50(120)} = \frac{5.4}{30{,}000} = 0.18\ \text{mA}$$

$$i_C = \beta i_B = (49)(0.18) = 8.82\ \text{mA}$$

$$i_E = i_C + i_B = 8.82 + 0.18 = 9\ \text{mA}$$

$$v_{3d} = (0.009)(120) = 1.08\text{V}$$

$$v_{bd} = V_o + v_{3d} = 1.68\text{V}$$

$$i_2 = \frac{v_{bd}}{R_2} = \frac{1.68}{60{,}000} = 28\,\mu\text{A}$$

$$i_1 = i_2 + i_B = 28 + 180 = 208\,\mu\text{A}$$

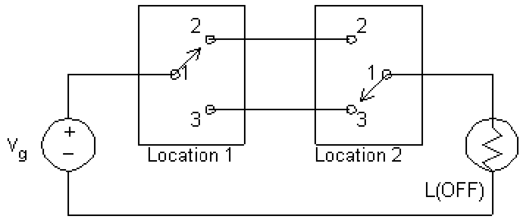
$$v_{\rm ab} = 40{,}000(208 \times 10^{-6}) = 8.32\,\text{V}$$

$$i_{CC} = i_C + i_1 = 8.82 + 0.208 = 9.028\ \text{mA}$$

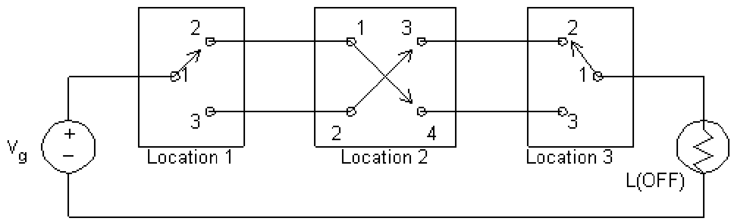
$$v_{13} + (8.82 \times 10^{-3})(750) + 1.08 = 10\ \text{V}$$

$$v_{13} = 2.305\,\text{V}$$

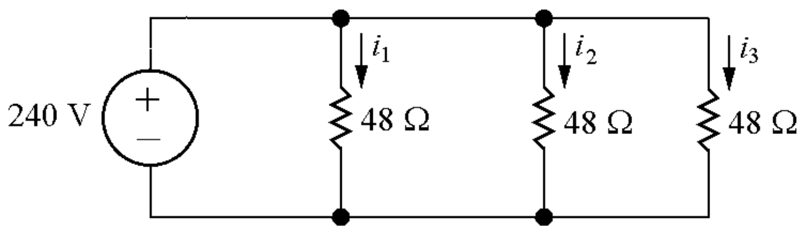
P 2.40 [**a**]



[**b**]



P 2.41 Each radiator is modeled as a $48\,\Omega$ resistor:



Write a KVL equation for each of the three loops:

$$-240 + 48i_1 = 0 \quad \rightarrow \quad i_1 = \frac{240}{48} = 5\,\mathrm{A}$$

$$-48i_1 + 48i_2 = 0 \quad \rightarrow \quad i_2 = i_1 = 5\,\mathrm{A}$$

$$-48i_2 + 48i_3 = 0 \quad \rightarrow \quad i_3 = i_2 = 5\,\mathrm{A}$$

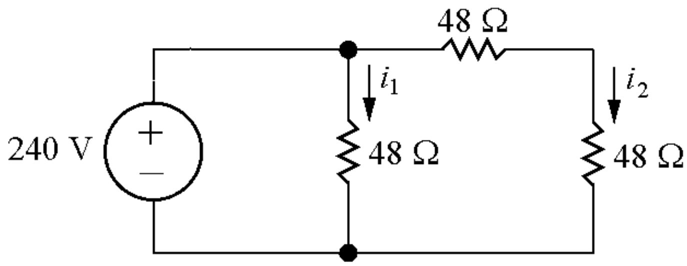
Therefore, the current through each radiator is 5 A and the power for each radiator is

$$p_{\mathrm{rad}} = Ri^2 = 48(5)^2 = 1200\,\mathrm{W}$$

There are three radiators, so the total power for this heating system is

$$p_{\mathrm{total}} = 3p_{\mathrm{rad}} = 3(1200) = 3600\,\mathrm{W}$$

P 2.42 Each radiator is modeled as a $48\,\Omega$ resistor:



Write a KVL equation for the left and right loops:

$$-240 + 48i_1 = 0 \quad \rightarrow \quad i_1 = \frac{240}{48} = 5\,\text{A}$$

$$-48i_1 + 48i_2 + 48i_2 = 0 \quad \rightarrow \quad i_2 = \frac{i_1}{2} = \frac{5}{2} = 2.5\,\text{A}$$

The power for the center radiator is

$$p_{\text{cen}} = 48i_1^2 = 48(5)^2 = 1200\,\text{W}$$

The power for each of the radiators on the right is

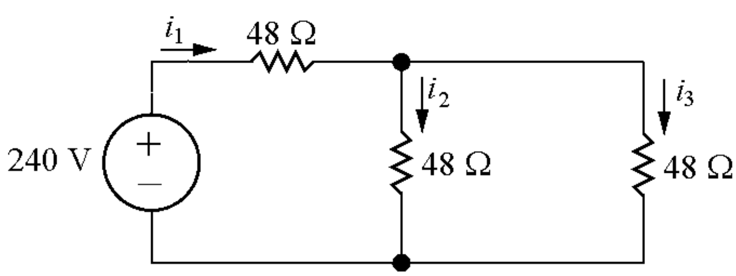
$$p_{\text{right}} = 48i_2^2 = 48(2.5)^2 = 300\,\text{W}$$

Thus the total power for this heating system is

$$p_{\text{total}} = p_{\text{cen}} + 2p_{\text{right}} = 1200 + 2(300) = 1800\,\text{W}$$

The center radiator produces 1200 W, just like the three radiators in Problem 2.41. But the other two radiators produce only 300 W each, which is 1/4th of the power of the radiators in Problem 2.41. The total power of this configuration is 1/2 of the total power in Fig. 2.41.

P 2.43 Each radiator is modeled as a $48\,\Omega$ resistor:



Write a KVL equation for the left and right loops:

$$-240 + 48i_1 + 48i_2 = 0$$

$$-48i_2 + 48i_3 = 0 \quad \rightarrow \quad i_2 = i_3$$

Write a KCL equation at the top node:

$$i_1 = i_2 + i_3 \quad \rightarrow \quad i_1 = i_2 + i_2 = 2i_2$$

Substituting into the first KVL equation gives

$$-240 + 48(2i_2) + 48i_2 = 0 \quad \rightarrow \quad i_2 = \frac{240}{3(48)} = 1.67\,\text{A}$$

Solve for the currents i_1 and i_3:

$$i_3 = i_2 = 1.67\,\text{A}; \qquad i_1 = 2i_2 = 2(1.67) = 3.33\,\text{A}$$

Calculate the power for each radiator using the current for each radiator:

$$p_{\text{left}} = 48i_1^2 = 48(3.33)^2 = 533.33\,\text{W}$$

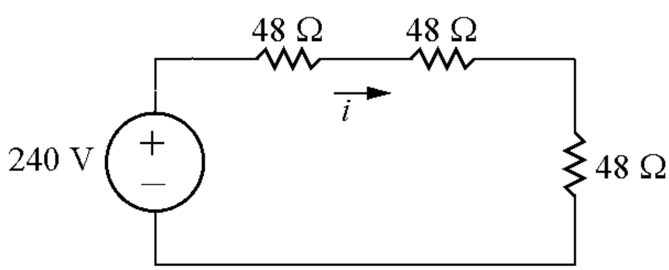
$$p_{\text{middle}} = p_{\text{right}} = 48i_2^2 = 48(1.67)^2 = 133.33\,\text{W}$$

Thus the total power for this heating system is

$$p_{\text{total}} = p_{\text{left}} + p_{\text{middle}} + p_{\text{right}} = 533.33 + 133.33 + 133.33 = 800\,\text{W}$$

All radiators in this configuration have much less power than their counterparts in Fig. 2.41. The total power for this configuration is only 22.2% of the total power for the heating system in Fig. 2.41.

P 2.44 Each radiator is modeled as a $48\,\Omega$ resistor:



Write a KVL equation for this loop:

$$-240 + 48i + 48i + 48i = 0 \quad \rightarrow \quad i = \frac{240}{3(48)} = 1.67\,\text{A}$$

Calculate the power for each radiator:

$$p_{\text{rad}} = 48i^2 = 48(1.67)^2 = 133.33\,\text{W}$$

Calculate the total power for this heating system:

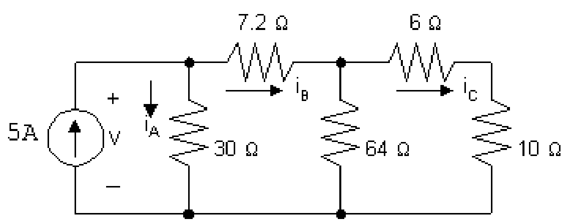
$$p_{\text{total}} = 3p_{\text{rad}} = 3(133.33) = 400\,\text{W}$$

Each radiator has much less power than the radiators in Fig. 2.41, and the total power of this configuration is just 1/9th of the total power in Fig. 2.41.

3 Simple Resistive Circuits

Assessment Problems

AP 3.1



Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the $6\,\Omega$ resistor and the $10\,\Omega$ resistor in series:

$$6\,\Omega + 10\,\Omega = 16\,\Omega$$

Now combine this $16\,\Omega$ resistor in parallel with the $64\,\Omega$ resistor:

$$16\,\Omega \| 64\,\Omega = \frac{(16)(64)}{16+64} = \frac{1024}{80} = 12.8\,\Omega$$

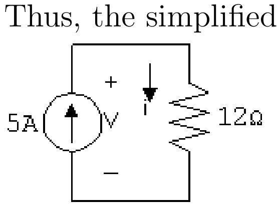
This equivalent $12.8\,\Omega$ resistor is in series with the $7.2\,\Omega$ resistor:

$$12.8\,\Omega + 7.2\,\Omega = 20\,\Omega$$

Finally, this equivalent $20\,\Omega$ resistor is in parallel with the $30\,\Omega$ resistor:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = \frac{600}{50} = 12\,\Omega$$

Thus, the simplified circuit is as shown:



[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the $12\,\Omega$ equivalent resistor:

$$v = (12\,\Omega)(5 \text{ A}) = 60 \text{ V}$$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula $p = -vi$ to find the power associated with the source:

$$p = -(60 \text{ V})(5 \text{ A}) = -300 \text{ W}$$

Thus, the source delivers 300 W of power to the circuit.

[c] We now can return to the original circuit, shown in the first figure. In this circuit, $v = 60$ V, as calculated in part (a). This is also the voltage drop across the $30\,\Omega$ resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i_A = \frac{60 \text{ V}}{30\,\Omega} = 2 \text{ A}$$

Now write a KCL equation at the upper left node to find the current i_B:

$$-5 \text{ A} + i_A + i_B = 0 \qquad \text{so} \qquad i_B = 5 \text{ A} - i_A = 5 \text{ A} - 2 \text{ A} = 3 \text{ A}$$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$-v + 7.2i_B + 6i_C + 10i_C = 0$$

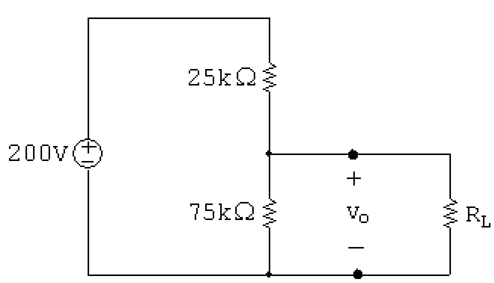
So $\qquad 16i_C = v - 7.2i_B = 60 \text{ V} - (7.2)(3) = 38.4 \text{ V}$

Thus $\qquad i_C = \dfrac{38.4}{16} = 2.4 \text{ A}$

Now that we have the current through the $10\,\Omega$ resistor we can use the formula $p = Ri^2$ to find the power:

$$p_{10\,\Omega} = (10)(2.4)^2 = 57.6 \text{ W}$$

AP 3.2



[a] We can use voltage division to calculate the voltage v_o across the 75 kΩ resistor:

$$v_o(\text{no load}) = \frac{75{,}000}{75{,}000 + 25{,}000}(200\ \text{V}) = 150\ \text{V}$$

[b] When we have a load resistance of 150 kΩ then the voltage v_o is across the parallel combination of the 75 kΩ resistor and the 150 kΩ resistor. First, calculate the equivalent resistance of the parallel combination:

$$75\ \text{k}\Omega \| 150\ \text{k}\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50{,}000\,\Omega = 50\ \text{k}\Omega$$

Now use voltage division to find v_o across this equivalent resistance:

$$v_o = \frac{50{,}000}{50{,}000 + 25{,}000}(200\ \text{V}) = 133.3\ \text{V}$$

[c] If the load terminals are short-circuited, the 75 kΩ resistor is effectively removed from the circuit, leaving only the voltage source and the 25 kΩ resistor. We can calculate the current in the resistor using Ohm's law:

$$i = \frac{200\ \text{V}}{25\ \text{k}\Omega} = 8\ \text{mA}$$

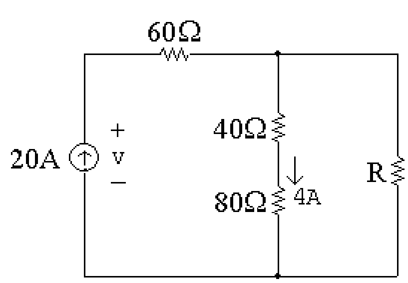
Now we can use the formula $p = Ri^2$ to find the power dissipated in the 25 kΩ resistor:

$$p_{25k} = (25{,}000)(0.008)^2 = 1.6\ \text{W}$$

[d] The power dissipated in the 75 kΩ resistor will be maximum at no load since v_o is maximum. In part (a) we determined that the no-load voltage is 150 V, so be can use the formula $p = v^2/R$ to calculate the power:

$$p_{75k}(\text{max}) = \frac{(150)^2}{75{,}000} = 0.3\ \text{W}$$

AP 3.3



[a] We will write a current division equation for the current throught the 80Ω resistor and use this equation to solve for R:

$$i_{80\Omega} = \frac{R}{R + 40\,\Omega + 80\,\Omega}(20\ \text{A}) = 4\ \text{A} \qquad \text{so} \qquad 20R = 4(R + 120)$$

$$\text{Thus} \qquad 16R = 480 \qquad \text{and} \qquad R = \frac{480}{16} = 30\,\Omega$$

[b] With $R = 30\,\Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i_R = \frac{40+80}{40+80+30}(20\text{ A}) = 16\text{ A} \qquad \text{so} \qquad p_R = (30)(16)^2 = 7680\text{ W}$$

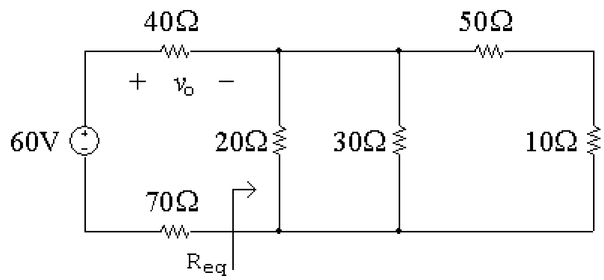
[c] Write a KVL equation around the outer loop to solve for the voltage v, and then use the formula $p = -vi$ to calculate the power delivered by the current source:

$$-v + (60\,\Omega)(20\text{ A}) + (30\,\Omega)(16\text{ A}) = 0 \qquad \text{so} \qquad v = 1200 + 480 = 1680\text{ V}$$

Thus, $\quad p_{\text{source}} = -(1680\text{ V})(20\text{ A}) = -33{,}600\text{ W}$

Thus, the current source generates 33,600 W of power.

AP 3.4



[a] First we need to determine the equivalent resistance to the right of the $40\,\Omega$ and $70\,\Omega$ resistors:

$$R_{\text{eq}} = 20\,\Omega \| 30\,\Omega \| (50\,\Omega + 10\,\Omega) \qquad \text{so} \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{20\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{60\,\Omega} = \frac{1}{10\,\Omega}$$

Thus, $\quad R_{\text{eq}} = 10\,\Omega$

Now we can use voltage division to find the voltage v_o:

$$v_o = \frac{40}{40+10+70}(60\text{ V}) = 20\text{ V}$$

[b] The current through the $40\,\Omega$ resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v_o}{40} = \frac{20\text{ V}}{40\,\Omega} = 0.5\text{ A}$$

This current flows from left to right through the $40\,\Omega$ resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the $20\,\Omega$ resistor and the $50\,\Omega$ and $10\,\Omega$ resistors:

$$20\,\Omega \| (50\,\Omega + 10\,\Omega) = \frac{(20)(60)}{20+60} = 15\,\Omega$$

Now we use current division to find the current in the $30\,\Omega$ branch:

$$i_{30\Omega} = \frac{15}{15+30}(0.5\text{ A}) = 0.16667\text{ A} = 166.67\text{ mA}$$

[**c**] We can find the power dissipated by the $50\,\Omega$ resistor if we can find the current in this resistor. We can use current division to find this current from the current in the $40\,\Omega$ resistor, but first we need to calculate the equivalent resistance of the $20\,\Omega$ branch and the $30\,\Omega$ branch:

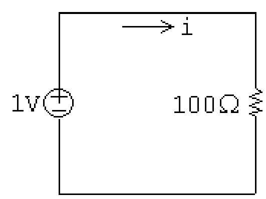
$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = 12\,\Omega$$

Current division gives:

$$i_{50\Omega} = \frac{12}{12+50+10}(0.5\ \text{A}) = 0.08333\ \text{A}$$

Thus, $\quad p_{50\Omega} = (50)(0.08333)^2 = 0.34722\ \text{W} = 347.22\ \text{mW}$

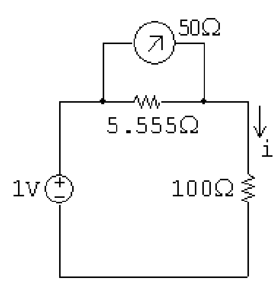
AP 3.5 [**a**]



We can find the current i using Ohm's law:

$$i = \frac{1\ \text{V}}{100\,\Omega} = 0.01\ \text{A} = 10\ \text{mA}$$

[**b**]

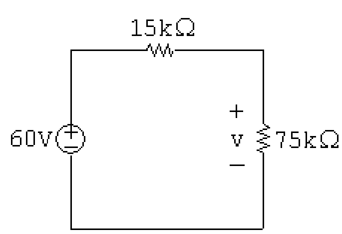


$$R_m = 50\,\Omega \| 5.555\,\Omega = 5\,\Omega$$

We can use the meter resistance to find the current using Ohm's law:

$$i_{\text{meas}} = \frac{1\ \text{V}}{100\,\Omega + 5\,\Omega} = 0.009524 = 9.524\ \ \text{mA}$$

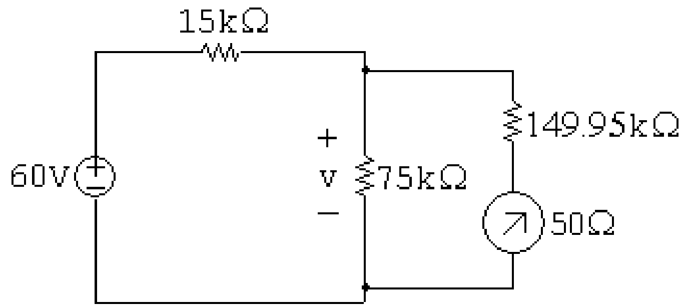
AP 3.6 [**a**]



Use voltage division to find the voltage v:

$$v = \frac{75{,}000}{75{,}000 + 15{,}000}(60\ \text{V}) = 50\ \text{V}$$

[**b**]



The meter resistance is a series combination of resistances:

$$R_m = 149{,}950 + 50 = 150{,}000\,\Omega$$

We can use voltage division to find v, but first we must calculate the equivalent resistance of the parallel combination of the 75 kΩ resistor and the voltmeter:

$$75{,}000\,\Omega \| 150{,}000\,\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50\ \text{k}\Omega$$

Thus, $$v_{\text{meas}} = \frac{50{,}000}{50{,}000 + 15{,}000}(60\ \text{V}) = 46.15\ \text{V}$$

AP 3.7 [**a**] Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$100R_x = (1000)(150) \qquad \text{so} \qquad R_x = \frac{(1000)(150)}{100} = 1500\,\Omega = 1.5\ \text{k}\Omega$$

[**b**] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination R_1 and R_3 and the branch with the series combination of R_2 and R_x. We can find the current in the latter two branches using Ohm's law:

$$i_{R_1,R_3} = \frac{5\ \text{V}}{100\,\Omega + 150\,\Omega} = 20\ \text{mA}; \qquad i_{R_2,R_x} = \frac{5\ \text{V}}{1000 + 1500} = 2\ \text{mA}$$

We can calculate the power dissipated by each resistor using the formula $p = Ri^2$:

$$p_{100\Omega} = (100\,\Omega)(0.02\ \text{A})^2 = 40\ \text{mW}$$

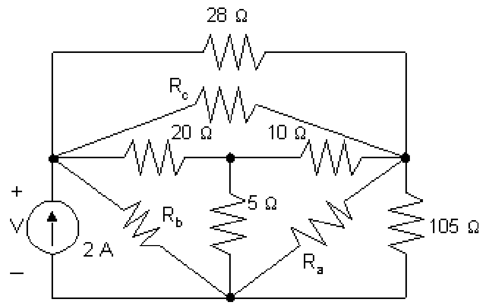
$$p_{150\Omega} = (150\,\Omega)(0.02\ \text{A})^2 = 60\ \text{mW}$$

$$p_{1000\Omega} = (1000\,\Omega)(0.002\ \text{A})^2 = 4\ \text{mW}$$

$$p_{1500\Omega} = (1500\,\Omega)(0.002\ \text{A})^2 = 6\ \text{mW}$$

Since none of the power dissipation values exceeds 250 mW, the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

AP 3.8 Convert the three Y-connected resistors, $20\,\Omega$, $10\,\Omega$, and $5\,\Omega$ to three Δ-connected resistors R_a, R_b, and R_c. To assist you the figure below has both the Y-connected resistors and the Δ-connected resistors

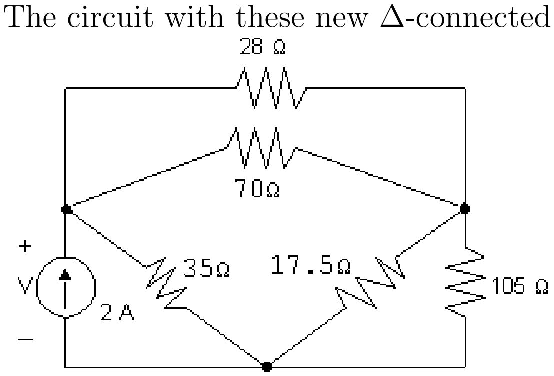


$$R_\text{a} = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\,\Omega$$

$$R_\text{b} = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\,\Omega$$

$$R_\text{c} = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\,\Omega$$

The circuit with these new Δ-connected resistors is shown below:



From this circuit we see that the $70\,\Omega$ resistor is parallel to the $28\,\Omega$ resistor:

$$70\,\Omega \| 28\,\Omega = \frac{(70)(28)}{70 + 28} = 20\,\Omega$$

Also, the $17.5\,\Omega$ resistor is parallel to the $105\,\Omega$ resistor:

$$17.5\,\Omega \| 105\,\Omega = \frac{(17.5)(105)}{17.5 + 105} = 15\,\Omega$$

Once the parallel combinations are made, we can see that the equivalent $20\,\Omega$ resistor is in series with the equivalent $15\,\Omega$ resistor, giving an equivalent resistance of $20\,\Omega + 15\,\Omega = 35\,\Omega$. Finally, this equivalent $35\,\Omega$ resistor is in parallel with the other $35\,\Omega$ resistor:

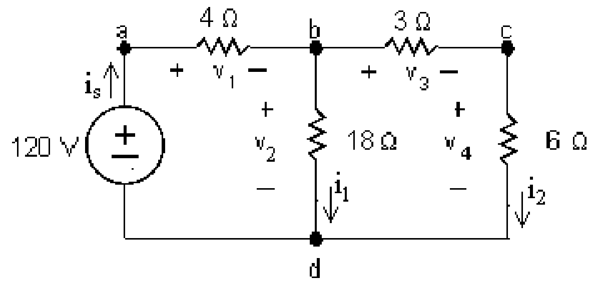
$$35\,\Omega \| 35\,\Omega = \frac{(35)(35)}{35+35} = 17.5\,\Omega$$

Thus, the resistance seen by the 2 A source is $17.5\,\Omega$, and the voltage can be calculated using Ohm's law:

$$v = (17.5\,\Omega)(2\text{ A}) = 35\text{ V}$$

Problems

P 3.1 **[a]** From Ex. 3-1: $i_1 = 4$ A, $\quad i_2 = 8$ A, $\quad i_s = 12$ A
at node b: $\quad -12 + 4 + 8 = 0,$ $\quad$ at node d: $\quad 12 - 4 - 8 = 0$



[b]
$$\begin{aligned} v_1 &= 4i_s = 48 \text{ V} \qquad v_3 = 3i_2 = 24 \text{ V} \\ v_2 &= 18i_1 = 72 \text{ V} \qquad v_4 = 6i_2 = 48 \text{ V} \end{aligned}$$

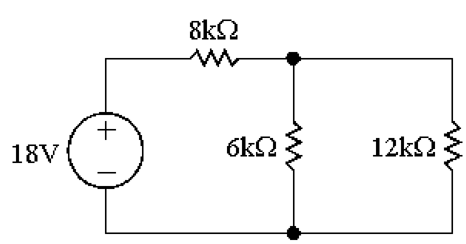
loop abda: $\quad -120 + 48 + 72 = 0,$
loop bcdb: $\quad -72 + 24 + 48 = 0,$
loop abcda: $\quad -120 + 48 + 24 + 48 = 0$

P 3.2 **[a]**
$$\begin{aligned} p_{4\Omega} &= i_s^2 4 = (12)^2 4 = 576 \text{ W} \qquad p_{18\Omega} = (4)^2 18 = 288 \text{ W} \\ p_{3\Omega} &= (8)^2 3 = 192 \text{ W} \qquad p_{6\Omega} = (8)^2 6 = 384 \text{ W} \end{aligned}$$

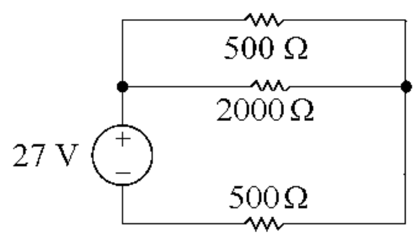
[b] $p_{120\text{V}}(\text{delivered}) = 120 i_s = 120(12) = 1440$ W

[c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440$ W

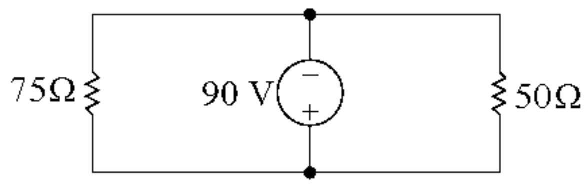
P 3.3 **[a]** The 5 kΩ and 7 kΩ resistors are in series. The simplified circuit is shown below:



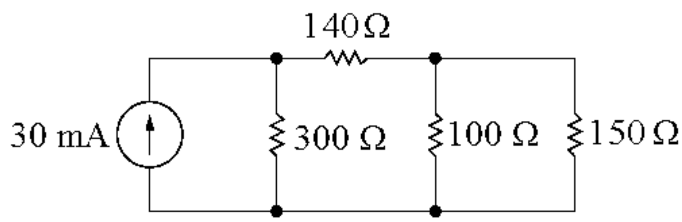
[b] The 800 Ω and 1200 Ω resistors are in series, as are the 300 Ω and 200 Ω resistors. The simplified circuit is shown below:



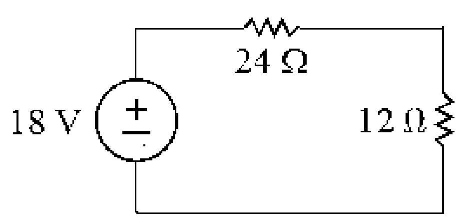
[c] The 35 Ω, 15 Ω, and 25 Ω resistors are in series. as are the 10 Ω and 40 Ω resistors. The simplified circuit is shown below:



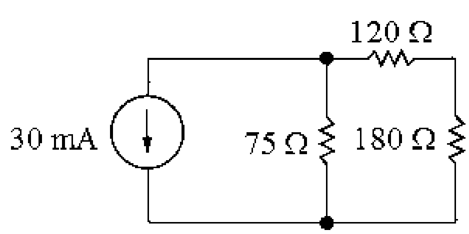
[d] The 50 Ω and 90 Ω resistors are in series, as are the 80 Ω and 70 Ω resistors. The simplified circuit is shown below:



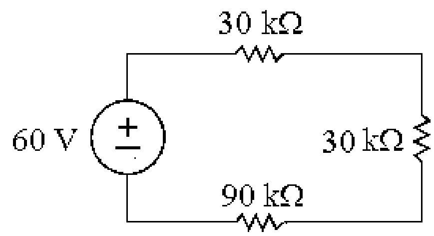
P 3.4 [a] The 36 Ω and 18 Ω resistors are in parallel. The simplified circuit is shown below:



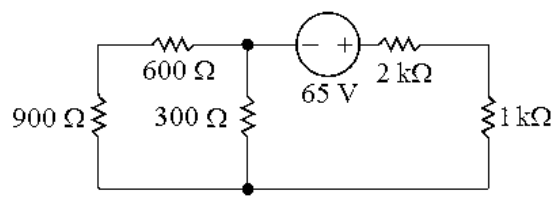
[b] The 200 Ω and 120 Ω resistors are in parallel, as are the 210 Ω and 280 Ω resistors. The simplified circuit is shown below:



[c] The 100 kΩ, 150 kΩ, and 60 kΩ resistors are in parallel, as are the 75 kΩ and 50 kΩ resistors. The simplified circuit is shown below:



[**d**] The 750 Ω and 500 Ω resistors are in parallel, as are the 1.5 kΩ and 3 kΩ resistors. The simplified circuit is shown below:



P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[**a**] Circuit in Fig. P3.3(a):

$$R_{\rm eq} = [(7000 + 5000)\|6000] + 8000 = 12{,}000\|6000 + 8000$$

$$= 4000 + 8000 = 12\ \text{k}\Omega$$

Circuit in Fig. P3.3(b):

$$R_{\rm eq} = [500\|(800 + 1200)] + 300 + 200 = (500\|2000) + 300 + 200$$

$$= 400 + 300 + 200 = 900\,\Omega$$

Circuit in Fig. P3.3(c):

$$R_{\rm eq} = (35 + 15 + 25)\|(10 + 40) = 75\|50 = 30\,\Omega$$

Circuit in Fig. P3.3(d):

$$R_{\rm eq} = ([(70 + 80)\|100] + 50 + 90)\|300 = [(150\|100) + 50 + 90]\|300$$

$$= (60 + 50 + 90)\|300 = 200\|300 = 120\,\Omega$$

[**b**] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.3(a):

$$P = \frac{V_s^2}{R_{\rm eq}} = \frac{18^2}{12{,}000} = 0.027 = 27\ \text{mW}$$

For the circuit in Fig. P3.3(b):

$$P = \frac{V_s^2}{R_{\rm eq}} = \frac{27^2}{900} = 0.81 = 810 \text{ mW}$$

For the circuit in Fig. P3.3(c):

$$P = \frac{V_s^2}{R_{\rm eq}} = \frac{90^2}{30} = 270 \text{ W}$$

For the circuit in Fig. P3.3(d):

$$P = I_s^2(R_{\rm eq}) = (0.03)^2(120) = 0.108 = 108 \text{ mW}$$

P 3.6 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] Circuit in Fig. P3.4(a):

$$R_{\rm eq} = (36\|18) + 24 = 12 + 24 = 36\,\Omega$$

Circuit in Fig. P3.4(b):

$$R_{\rm eq} = 200\|120\|[(210\|280) + 180] = 200\|120\|(120 + 180) = 200\|120\|300 = 60\,\Omega$$

Circuit in Fig. P3.4(c):

$$R_{\rm eq} = (75 \text{ k}\|50 \text{ k}) + (100 \text{ k}\|150 \text{ k}\|60 \text{ k}) + 90 \text{ k} = 30 \text{ k} + 30 \text{ k} + 90 \text{ k} = 150 \text{ k}\Omega$$

Circuit in Fig. P3.4(d):

$$R_{\rm eq} = [(600 + 900)\|750\|500] + (1500\|3000) + 2000 = (1500\|750\|500) + 1000 + 2000$$
$$= 250 + 1000 + 2000 = 3250 = 3.25 \text{ k}\Omega$$

[b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.4(a):

$$P = \frac{V_s^2}{R_{\rm eq}} = \frac{18^2}{36} = 9 \text{ W}$$

For the circuit in Fig. P3.4(b):

$$P = I_s^2(R_{\rm eq}) = (0.03)^2(60) = 0.054 = 54 \text{ mW}$$

For the circuit in Fig. P3.4(c):

$$P = \frac{V_s^2}{R_{\rm eq}} = \frac{60^2}{150{,}000} = 0.024 = 24 \text{ mW}$$

For the circuit in Fig. P3.4(d):

$$P = \frac{V_s^2}{R_{\rm eq}} = \frac{65^2}{3250} = 1.3 \text{ W}$$

P 3.7 **[a]** Circuit in Fig. P3.7(a):

$$R_{\text{eq}} = ([(15\|60) + (30\|45) + 20]\|50) + 25 + 10 = [(12 + 18 + 20)\|50] + 25 + 10$$
$$= (50\|50) + 25 + 10 = 25 + 25 + 10 = 60\,\Omega$$

Circuit in Fig. P3.7(b) – begin by simplifying the $75\,\Omega$ resistor and all resistors to its right:

$$[(18 + 12)\|60 + 30]\|75 = (30\|60 + 30)\|75 = (20 + 30)\|75 = 50\|75 = 30\,\Omega$$

Now simplify the remainder of the circuit:

$$R_{\text{eq}} = ([(30 + 20)\|50] + (20\|60))\|40 = [(50\|50) + 15]\|40 = (25 + 15)\|40$$
$$= 40\|40 = 20\,\Omega$$

Circuit in Fig. P3.7(c) – begin by simplifying the left and right sides of the circuit:

$$R_{\text{left}} = [(1800 + 1200)\|2000] + 300 = (3000\|2000) + 300 = 1200 + 300 = 1500\,\Omega$$

$$R_{\text{right}} = [(500 + 2500)\|1000] + 750 = (3000\|1000) + 750 = 750 + 750 = 1500\,\Omega$$

Now find the equivalent resistance seen by the source:

$$R_{\text{eq}} = (R_{\text{left}}\|R_{\text{right}}) + 250 + 3000 = (1500\|1500) + 250 + 3000$$
$$= 750 + 250 + 3000 = 4000 = 4\ \text{k}\Omega$$

Circuit in Fig. P3.7(d):

$$R_{\text{eq}} = ([(750 + 250)\|1000] + 100)\|([(150 + 600)\|500] + 300)$$
$$= [(1000\|1000) + 100]\|[(750\|500) + 300] = (500 + 100)\|(300 + 300)$$
$$= 600\|600 = 300\,\Omega$$

[b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.7(a):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{30^2}{60} = 15\ \text{W}$$

For the circuit in Fig. P3.7(b):

$$P = I_s^2(R_{\text{eq}}) = (0.08)^2(20) = 0.128 = 128\ \text{mW}$$

For the circuit in Fig. P3.7(c):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{20^2}{4000} = 0.1 = 100\ \text{mW}$$

For the circuit in Fig. P3.7(d):

$$P = I_s^2(R_{\text{eq}}) = (0.05)^2(300) = 0.75 = 750\ \text{mW}$$

P 3.8 **[a]** $R_{\text{ab}} = 24 + (90\|60) + 12 = 24 + 36 + 12 = 72\,\Omega$

[b] $R_{\text{ab}} = [(4\text{ k} + 6\text{ k} + 2\text{ k})\|8\text{ k}] + 5.2\text{ k} = (12\text{ k}\|8\text{ k}) + 5.2\text{ k} = 4.8\text{ k} + 5.2\text{ k} = 10\ \text{k}\Omega$

[c] $R_{\text{ab}} = 1200\|720\|(320 + 480) = 1200\|720\|800 = 288\,\Omega$

P 3.9 Write an expression for the resistors in series and parallel from the right side of the circuit to the left. Then simplify the resulting expression from left to right to find the equivalent resistance.

[a] $R_{\text{ab}} = [(26 + 10)\|18 + 6]\|36 = (36\|18 + 6)\|36 = (12 + 6)\|36 = 18\|36 = 12\,\Omega$

[b] $R_{\text{ab}} = [(12 + 18)\|10\|15\|20 + 16]\|30 + 4 + 14 = (30\|10\|15\|20 + 16)\|30 + 4 + 14$

$$= (4 + 16)\|30 + 4 + 14 = 20\|30 + 4 + 14 = 12 + 4 + 14 = 30\,\Omega$$

[c] $R_{\text{ab}} = (500\|1500\|750 + 250)\|2000 + 1000 = (250 + 250)\|2000 + 1000$

$$= 500\|2000 + 1000 = 400 + 1000 = 1400\,\Omega$$

[d] Note that the wire on the far right of the circuit effectively removes the $60\,\Omega$ resistor!

$$\begin{aligned}
R_{\text{ab}} &= [([(30 + 18)\|16 + 28]\|40 + 20)\|24 + 25 + 10]\|50 \\
&= ([(48\|16 + 28)\|40 + 20]\|24 + 25 + 10)\|50 \\
&= ([(12 + 28)\|40 + 20]\|24 + 25 + 10)\|50 = [(40\|40 + 20)\|24 + 25 + 10]\|50 \\
&= [(20 + 20)\|24 + 25 + 10]\|50 = (40\|24 + 25 + 10)\|50 = (15 + 25 + 10)\|50 \\
&= 50\|50 = 25\,\Omega
\end{aligned}$$

P 3.10 **[a]** $R + R = 2R$

[b] $R + R + R + \cdots + R = nR$

[c] $R + R = 2R = 3000 \quad \text{so} \quad R = 1500 = 1.5\ \text{k}\Omega$
This is a resistor from Appendix H.

[d] $nR = 4000; \qquad \text{so if } n = 4, \quad R = 1\ \text{k}\Omega$
This is a resistor from Appendix H.

P 3.11 **[a]** $R_{\text{eq}} = R\|R = \dfrac{R^2}{2R} = \dfrac{R}{2}$

[b]
$$\begin{aligned}
R_{\text{eq}} &= R\|R\|R\|\cdots\|R \qquad (n\ R\text{'s}) \\
&= R\|\frac{R}{n-1} \\
&= \frac{R^2/(n-1)}{R + R/(n-1)} = \frac{R^2}{nR} = \frac{R}{n}
\end{aligned}$$

[c] $\dfrac{R}{2} = 5000 \quad \text{so} \quad R = 10\ \text{k}\Omega$
This is a resistor from Appendix H.

[d] $\frac{R}{n} = 4000 \quad \text{so} \quad R = 4000n$

If $n = 3 \qquad r = 4000(3) = 12\ \text{k}\Omega$

This is a resistor from Appendix H. So put three 12k resistors in parallel to get 4kΩ.

P 3.12 [a] $v_o = \frac{160(3300)}{(4700 + 3300)} = 66$ V

[b] $i = 160/8000 = 20$ mA

$$P_{R_1} = (400 \times 10^{-6})(4.7 \times 10^3) = 1.88 \text{ W}$$

$$P_{R_2} = (400 \times 10^{-6})(3.3 \times 10^3) = 1.32 \text{ W}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the voltage requirement, first pick R_1 to meet the 0.5 W specification

$$i_{R_1} = \frac{160 - 66}{R_1}, \qquad \text{Therefore,} \left(\frac{94}{R_1}\right)^2 R_1 \leq 0.5$$

Thus, $R_1 \geq \frac{94^2}{0.5} \quad \text{or} \quad R_1 \geq 17{,}672\,\Omega$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 17{,}672}(160) = 66$$

Thus, $R_2 = 12{,}408\,\Omega$

P 3.13 $4 = \frac{20R_2}{R_2 + 40} \quad \text{so} \quad R_2 = 10\,\Omega$

$$3 = \frac{20R_\text{e}}{40 + R_\text{e}} \quad \text{so} \quad R_\text{e} = \frac{120}{17}\,\Omega$$

Thus, $\frac{120}{17} = \frac{10R_\text{L}}{10 + R_\text{L}} \quad \text{so} \quad R_\text{L} = 24\,\Omega$

P 3.14 [a] $v_o = \frac{40R_2}{R_1 + R_2} = 8 \quad \text{so} \quad R_1 = 4R_2$

Let $R_\text{e} = R_2 \| R_\text{L} = \frac{R_2R_\text{L}}{R_2 + R_\text{L}}$

$$v_o = \frac{40R_\text{e}}{R_1 + R_\text{e}} = 7.5 \quad \text{so} \quad R_1 = 4.33R_\text{e}$$

Then, $4R_2 = 4.33R_\text{e} = \frac{4.33(3600R_2)}{3600 + R_2}$

Thus, $R_2 = 300\,\Omega \quad \text{and} \quad R_1 = 4(300) = 1200\,\Omega$

[b] The resistor that must dissipate the most power is R_1, as it has the largest resistance and carries the same current as the parallel combination of R_2 and the load resistor. The power dissipated in R_1 will be maximum when the voltage across R_1 is maximum. This will occur when the voltage divider has a resistive load. Thus,

$$v_{R_1} = 40 - 7.5 = 32.5 \text{ V}$$

$$p_{R_1} = \frac{32.5^2}{1200} = 880.2 \text{ m W}$$

Thus the minimum power rating for all resistors should be 1 W.

P 3.15 Refer to the solution to Problem 3.16. The voltage divider will reach the maximum power it can safely dissipate when the power dissipated in R_1 equals 1 W. Thus,

$$\frac{v_{R_1}^2}{1200} = 1 \quad \text{so} \quad v_{R_1} = 34.64 \text{ V}$$

$$v_o = 40 - 34.64 = 5.36 \text{ V}$$

So, $\dfrac{40R_e}{1200 + R_e} = 5.36$ and $R_e = 185.68\,\Omega$

Thus, $\dfrac{(300)R_L}{300 + R_L} = 185.68$ and $R_L = 487.26\,\Omega$

The minimum value for R_L from Appendix H is $560\,\Omega$.

P 3.16 $R_{eq} = 10\|[6 + 5\|(8 + 12)] = 10\|(6 + 5\|20) = 10\|(6 + 4) = 5\,\Omega$

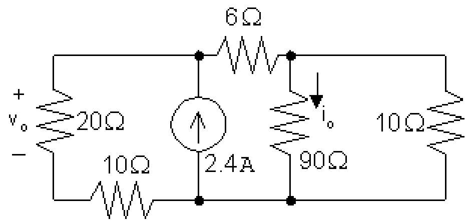
$$v_{10A} = v_{10\Omega} = (10 \text{ A})(5\,\Omega) = 50 \text{ V}$$

Using voltage division:

$$v_{5\Omega} = \frac{5\|(8 + 12)}{6 + 5\|(8 + 12)}(50) = \frac{4}{6 + 4}(50) = 20 \text{ V}$$

Thus, $p_{5\Omega} = \dfrac{v_{5\Omega}^2}{5} = \dfrac{20^2}{5} = 80 \text{ W}$

P 3.17 [a]



$$R_{\text{eq}} = (10 + 20)\|[12 + (90\|10)] = 30\|15 = 10\,\Omega$$

$$v_{2.4\text{A}} = 10(2.4) = 24 \text{ V}$$

$$v_o = v_{20\Omega} = \frac{20}{10 + 20}(24) = 16 \text{ V}$$

$$v_{90\Omega} = \frac{90\|10}{6 + (90\|10)}(24) = \frac{9}{15}(24) = 14.4 \text{ V}$$

$$i_o = \frac{14.4}{90} = 0.16 \text{ A}$$

[b] $p_{6\Omega} = \dfrac{(v_{2.4\text{A}} - v_{90\Omega})^2}{6} = \dfrac{(24 - 14.4)^2}{6} = 15.36 \text{ W}$

[c] $p_{2.4\text{A}} = -(2.4)(24) = -57.6 \text{ W}$

Thus the power developed by the current source is 57.6 W.

P 3.18 Begin by using KCL at the top node to relate the branch currents to the current supplied by the source. Then use the relationships among the branch currents to express every term in the KCL equation using just i_2:

$$0.05 = i_1 + i_2 + i_3 + i_4 = 0.6i_2 + i_2 + 2i_2 + 4i_1 = 0.6i_2 + i_2 + 2i_2 + 4(0.6i_2) = 6i_2$$

Therefore,

$$i_2 = 0.05/6 = 0.00833 = 8.33 \text{ mA}$$

Find the remaining currents using the value of i_2:

$$i_1 = 0.6i_2 = 0.6(0.00833) = 0.005 = 5 \text{ mA}$$

$$i_3 = 2i_2 = 2(0.00833) = 0.01667 = 16.67 \text{ mA}$$

$$i_4 = 4i_1 = 4(0.005) = 0.02 = 20 \text{ mA}$$

Since the resistors are in parallel, the same voltage, 25 V, appears across each of them. We know the current and the voltage for every resistor so we can use Ohm's law to calculate the values of the resistors:

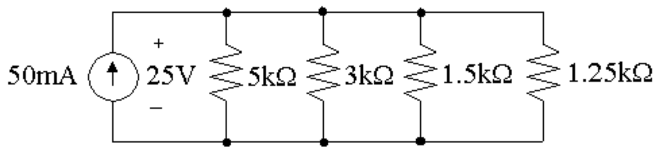
$$R_1 = 25/i_1 = 25/0.005 = 5000 = 5 \text{ k}\Omega$$

$R_2 = 25/i_2 = 25/0.00833 = 3000 = 3\ \text{k}\Omega$

$R_3 = 25/i_3 = 25/0.01667 = 1500 = 1.5\ \text{k}\Omega$

$R_4 = 25/i_4 = 25/0.02 = 1250 = 1.25\ \text{k}\Omega$

The resulting circuit is shown below:



P 3.19 $\dfrac{(24)^2}{R_1 + R_2 + R_3} = 80,$ Therefore, $R_1 + R_2 + R_3 = 7.2\,\Omega$

$$\frac{(R_1 + R_2)24}{(R_1 + R_2 + R_3)} = 12$$

Therefore, $2(R_1 + R_2) = R_1 + R_2 + R_3$

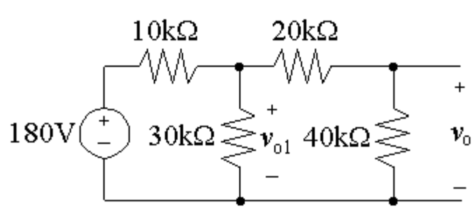
Thus, $R_1 + R_2 = R_3$; $2R_3 = 7.2$; $R_3 = 3.6\,\Omega$

$$\frac{R_2(24)}{R_1 + R_2 + R_3} = 5$$

$4.8R_2 = R_1 + R_2 + 3.6 = 7.2$

Thus, $R_2 = 1.5\,\Omega$; $R_1 = 7.2 - R_2 - R_3 = 2.1\,\Omega$

P 3.20 **[a]**



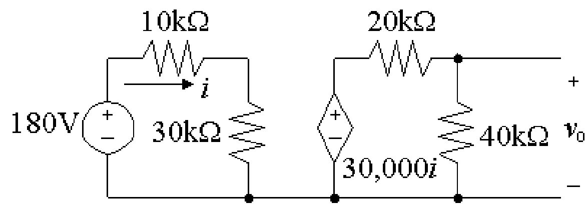
$20\ \text{k}\Omega + 40\ \text{k}\Omega = 60\ \text{k}\Omega$

$30\ \text{k}\Omega \| 60\ \text{k}\Omega = 20\ \text{k}\Omega$

$$v_{o1} = \frac{20{,}000}{(10{,}000 + 20{,}000)}(180) = 120\ \text{V}$$

$$v_o = \frac{40{,}000}{60{,}000}(v_{o1}) = 80\ \text{V}$$

[b]



$$i = \frac{180}{40{,}000} = 4.5 \text{ mA}$$

$$30{,}000i = 135 \text{ V}$$

$$v_o = \frac{40{,}000}{60{,}000}(135) = 90 \text{ V}$$

[c] It removes the loading effect of the second voltage divider on the first voltage divider. Observe that the open circuit voltage of the first divider is

$$v'_{o1} = \frac{30{,}000}{40{,}000}(180) = 135 \text{ V}$$

Now note this is the input voltage to the second voltage divider when the current-controlled voltage source is used.

P 3.21 [a] At no load: $v_o = kv_s = \dfrac{R_2}{R_1 + R_2}v_s.$

At full load: $v_o = \alpha v_s = \dfrac{R_{\text{e}}}{R_1 + R_{\text{e}}}v_s,$ where $R_{\text{e}} = \dfrac{R_oR_2}{R_o + R_2}$

$$\begin{aligned} \text{Therefore} \quad k &= \frac{R_2}{R_1 + R_2} \quad \text{and} \quad R_1 = \frac{(1-k)}{k}R_2 \\ \alpha &= \frac{R_{\text{e}}}{R_1 + R_{\text{e}}} \quad \text{and} \quad R_1 = \frac{(1-\alpha)}{\alpha}R_{\text{e}} \end{aligned}$$

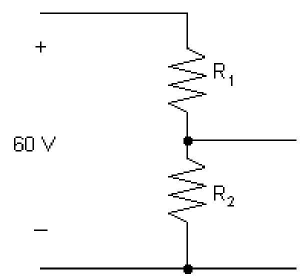
Thus $\left(\dfrac{1-\alpha}{\alpha}\right)\left[\dfrac{R_2R_o}{R_o + R_2}\right] = \dfrac{(1-k)}{k}R_2$

Solving for R_2 yields $R_2 = \dfrac{(k-\alpha)}{\alpha(1-k)}R_o$

Also, $R_1 = \dfrac{(1-k)}{k}R_2 \quad \therefore \quad R_1 = \dfrac{(k-\alpha)}{\alpha k}R_o$

[b]
$$\begin{aligned} R_1 &= \left(\frac{0.05}{0.68}\right)R_o = 2.5 \text{ k}\Omega \\ R_2 &= \left(\frac{0.05}{0.12}\right)R_o = 14.167 \text{ k}\Omega \end{aligned}$$

[c]



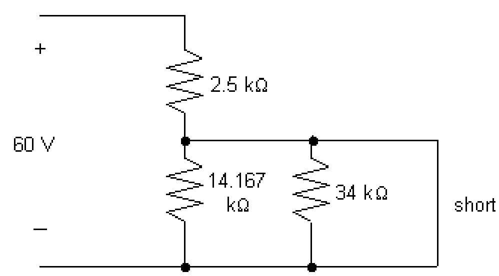
Maximum dissipation in R_2 occurs at no load, therefore,

$$P_{R_2(\text{max})} = \frac{[(60)(0.85)]^2}{14{,}167} = 183.6 \text{ mW}$$

Maximum dissipation in R_1 occurs at full load.

$$P_{R_1(\text{max})} = \frac{[60 - 0.80(60)]^2}{2500} = 57.60 \text{ mW}$$

[d]



$$P_{R_1} = \frac{(60)^2}{2500} = 1.44 \text{ W} = 1440 \text{ mW}$$

$$P_{R_2} = \frac{(0)^2}{14{,}167} = 0 \text{ W}$$

P 3.22 [a] Let v_o be the voltage across the parallel branches, positive at the upper terminal, then

$$i_g = v_oG_1 + v_oG_2 + \cdots + v_oG_N = v_o(G_1 + G_2 + \cdots + G_N)$$

It follows that $$v_o = \frac{i_g}{(G_1 + G_2 + \cdots + G_N)}$$

The current in the k^{th} branch is $i_k = v_oG_k$; Thus,

$$i_k = \frac{i_gG_k}{[G_1 + G_2 + \cdots + G_N]}$$

[b] $$i_5 = \frac{40(0.2)}{2 + 0.2 + 0.125 + 0.1 + 0.05 + 0.025} = 3.2 \text{ A}$$

P 3.23 [a] The equivalent resistance of the 6 kΩ resistor and the resistors to its right is

$$6 \text{ k}\|(5 \text{ k} + 7 \text{ k}) = 6 \text{ k}\|12 \text{ k} = 4 \text{ k}\Omega$$

Using voltage division,

$$v_{6\text{k}} = \frac{4000}{8000 + 4000}(18) = 6 \text{ V}$$

[b] $$v_{5\text{k}} = \frac{5000}{5000 + 7000}(6) = 2.5 \text{ V}$$

P 3.24 **[a]** The equivalent resistance of the 100 Ω resistor and the resistors to its right is

$$100\|(80 + 70) = 100\|150 = 60\,\Omega$$

Using current division,

$$i_{50} = \frac{(50 + 90 + 60)\|300}{50 + 90 + 60}(0.03) = \frac{120}{200}(0.03) = 0.018 = 18 \text{ mA}$$

[b] $$v_{70} = \frac{(80 + 70)\|100}{80 + 70}(0.018) = \frac{60}{150}(0.018) = 0.0072 = 7.2 \text{ mA}$$

P 3.25 **[a]** The equivalent resistance of the circuit to the right of, and including, the 50 Ω resistor is

$$[(60\|15) + (45\|30) + 20]\|50 = 25\,\Omega$$

Thus by voltage division,

$$v_{25} = \frac{25}{25 + 25 + 10}(30) = 12.5 \text{ V}$$

[b] The current in the 25 Ω resistor can be found from its voltage using Ohm's law:

$$i_{25} = \frac{12.5}{25} = 0.5 \text{ A}$$

[c] The current in the 25 Ω resistor divides between two branches – one containing 50 Ω and one containing $(45\|30) + (15\|60) + 20 = 50\,\Omega$. Using current division,

$$i_{50} = \frac{50\|50}{50}(i_{25}) = \frac{25}{50}(0.5) = 0.25 \text{ A}$$

[d] The voltage drop across the 50 Ω resistor can be found using Ohm's law:

$$v_{50} = 50 i_{50} = 50(0.25) = 12.5 \text{ V}$$

[e] The voltage v_{50} divides across the equivalent resistance $(45\|30)\,\Omega$, the equivalent resistance $(15\|60)\,\Omega$, and the 20 Ω resistor. Using voltage division,

$$v_{60} = v_{15\|60} = \frac{15\|60}{(15\|60) + (30\|45) + 20}(12.5) = \frac{12}{12 + 18 + 20}(12.5) = 3 \text{ V}$$

P 3.26 **[a]** The equivalent resistance to the right of the 36 Ω resistor is

$$6 + [18\|(26 + 10)] = 18\,\Omega$$

By current division,

$$i_{36} = \frac{36\|18}{36}(0.45) = 0.15 = 150 \text{ mA}$$

[b] Using Ohm's law,

$$v_{36} = 36i_{36} = 36(0.15) = 5.4 \text{ V}$$

[c] Before using voltage division, find the equivalent resistance of the 18 Ω resistor and the resistors to its right:

$$18\|(26 + 10) = 12\,\Omega$$

Now use voltage division:

$$v_{18} = \frac{12}{12 + 6}(5.4) = 3.6 \text{ V}$$

[d] $v_{10} = \dfrac{10}{10 + 26}(3.6) = 1 \text{ V}$

P 3.27 **[a]** Begin by finding the equivalent resistance of the 30 Ω resistor and all resistors to its right:

$$([(12 + 18)\|10\|15\|20] + 16)\|30 = 12\,\Omega$$

Now use voltage division to find the voltage across the 4 Ω resistor:

$$v_4 = \frac{4}{4 + 12 + 14}(6) = 0.8 \text{ V}$$

[b] Use Ohm's law to find the current in the 4 Ω resistor:

$$i_4 = v_4/4 = 0.8/4 = 0.2 \text{ A}$$

[c] Begin by finding the equivalent resistance of all resistors to the right of the 30 Ω resistor:

$$[(12 + 18)\|10\|15\|20] + 16 = 20\,\Omega$$

Now use current division:

$$i_{16} = \frac{30\|20}{20}(0.2) = 0.12 = 120 \text{ mA}$$

[d] Note that the current in the 16 Ω resistor divides among four branches – $20\,\Omega, 15\,\Omega, 10\,\Omega,$ and $(12 + 18)\,\Omega$:

$$i_{10} = \frac{20\|15\|10\|(12 + 18)}{10}(0.12) = 0.048 = 48 \text{ mA}$$

[e] Use Ohm's law to find the voltage across the 10 Ω resistor:

$$v_{10} = 10i_{10} = 10(0.048) = 0.48 \text{ V}$$

[f] $v_{18} = \dfrac{18}{12+18}(0.48) = 0.288 = 288\text{ mV}$

P 3.28 [a] $v_{6\text{k}} = \dfrac{6}{6+2}(18) = 13.5\text{ V}$

$$v_{3\text{k}} = \frac{3}{3+9}(18) = 4.5\text{ V}$$

$$v_x = v_{6\text{k}} - v_{3\text{k}} = 13.5 - 4.5 = 9\text{ V}$$

[b] $v_{6\text{k}} = \dfrac{6}{8}(V_s) = 0.75V_s$

$$v_{3\text{k}} = \frac{3}{12}(V_s) = 0.25V_s$$

$$v_x = (0.75V_s) - (0.25V_s) = 0.5V_s$$

P 3.29 Use current division to find the current in the branch containing the 10 k and 15 k resistors, from bottom to top

$$i_{10\text{k}+15\text{k}} = \frac{(10\text{ k} + 15\text{ k})\|(3\text{ k} + 12\text{ k})}{10\text{ k} + 15\text{ k}}(18) = 6.75\text{ mA}$$

Use Ohm's law to find the voltage drop across the 15 k resistor, positive at the top:

$$v_{15\text{k}} = -(6.75\text{ m})(15\text{ k}) = -101.25\text{ V}$$

Find the current in the branch containing the 3 k and 12 k resistors, from bottom to top

$$i_{10\text{k}+15\text{k}} = \frac{(10\text{ k} + 15\text{ k})\|(3\text{ k} + 12\text{ k})}{3\text{ k} + 12\text{ k}}(18) = 11.25\text{ mA}$$

Use Ohm's law to find the voltage drop across the 12 k resistor, positive at the top:

$$v_{12\text{k}} = -(12\text{ k})(11.25\text{ m}) = -135\text{ V}$$

$$v_o = v_{15\text{k}} - v_{12\text{k}} = -101.25 - (-135) = 33.75\text{ V}$$

P 3.30 The equivalent resistance of the circuit to the right of the 90 Ω resistor is

$$R_{\text{eq}} = [(150\|75) + 40]\|(30 + 60) = 90\|90 = 45\,\Omega$$

Use voltage division to find the voltage drop between the top and bottom nodes:

$$v_{\text{Req}} = \frac{45}{45+90}(3) = 1 \text{ V}$$

Use voltage division again to find v_1 from v_{Req}:

$$v_1 = \frac{150\|75}{150\|75+40}(1) = \frac{50}{90}(1) = \frac{5}{9} \text{ V}$$

Use voltage division one more time to find v_2 from v_{Req}:

$$v_2 = \frac{30}{30+60}(1) = \frac{1}{3} \text{ V}$$

P 3.31 Find the equivalent resistance of all the resistors except the $2\,\Omega$:

$$5\,\Omega\|20\,\Omega = 4\,\Omega; \qquad 4\,\Omega + 6\,\Omega = 10\,\Omega; \qquad 10\|(15+12+13) = 8\,\Omega = R_{\text{eq}}$$

Use Ohm's law to find the current i_g:

$$i_g = \frac{125}{2+R_{\text{eq}}} = \frac{125}{2+8} = 12.5 \text{ A}$$

Use current division to find the current in the $6\,\Omega$ resistor:

$$i_{6\Omega} = \frac{8}{6+4}(12.5) = 10 \text{ A}$$

Use current division again to find i_o:

$$i_o = \frac{5\|20}{20} i_{6\Omega} = \frac{5\|20}{20}(10) = 2 \text{ A}$$

P 3.32 Use current division to find the current in the $8\,\Omega$ resistor. Begin by finding the equivalent resistance of the $8\,\Omega$ resistor and all resistors to its right:

$$R_{\text{eq}} = ([(20\|80)+4]\|30) + 8 = 20\,\Omega$$

$$i_8 = \frac{60\|R_{\text{eq}}}{R_{\text{eq}}}(0.25) = \frac{60\|20}{20}(0.25) = 0.1875 = 187.5 \text{ mA}$$

Use current division to find i_1 from i_8:

$$i_1 = \frac{30\|[4+(80\|20)]}{30}(i_8) = \frac{30\|20}{30}(0.1875) = 0.075 = 75 \text{ mA}$$

Use current division to find $i_{4\Omega}$ from i_8:

$$i_{4\Omega} = \frac{30\|[4 + (80\|20)]}{4 + (80\|20)}(i_8) = \frac{30\|20}{20}(0.1875) = 0.1125 = 112.5 \text{ mA}$$

Finally, use current division to find i_2 from $i_{4\Omega}$:

$$i_2 = \frac{80\|20}{20}(i_{4\Omega}) = \frac{80\|20}{20}(0.1125) = 0.09 = 90 \text{ mA}$$

P 3.33 The current in the shunt resistor at full-scale deflection is $i_A = i_{\text{fullscale}} - 3 \times 10^{-3}$ A. The voltage across R_A at full-scale deflection is always 150 mV; therefore,

$$R_A = \frac{150 \times 10^{-3}}{i_{\text{fullscale}} - 3 \times 10^{-3}} = \frac{150}{1000 i_{\text{fullscale}} - 3}$$

[a] $R_A = \dfrac{150}{5000 - 3} = 30.018 \text{ m}\Omega$

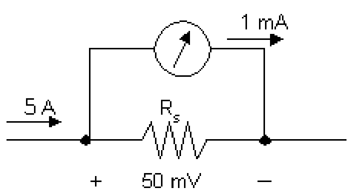
[b] Let R_m be the equivalent ammeter resistance:

$$R_m = \frac{0.15}{5} = 0.03 = 30 \text{ m}\Omega$$

[c] $R_A = \dfrac{150}{100 - 3} = 1.546\,\Omega$

[d] $R_m = \dfrac{0.15}{0.1} = 1.5\,\Omega$

P 3.34



Original meter: $R_e = \dfrac{50 \times 10^{-3}}{5} = 0.01\,\Omega$

Modified meter: $R_e = \dfrac{(0.02)(0.01)}{0.03} = 0.00667\,\Omega$

$\therefore \quad (I_{fs})(0.00667) = 50 \times 10^{-3}$

$\therefore \quad I_{fs} = 7.5$ A

P 3.35 At full scale the voltage across the shunt resistor will be 200 mV; therefore the power dissipated will be

$$P_A = \frac{(200 \times 10^{-3})^2}{R_A}$$

Therefore $R_A \geq \dfrac{(200 \times 10^{-3})^2}{1.0} = 40\ \mathrm{m}\Omega$

Otherwise the power dissipated in R_A will exceed its power rating of 1 W
When $R_A = 40\ \mathrm{m}\Omega$, the shunt current will be

$$i_A = \frac{200 \times 10^{-3}}{40 \times 10^{-3}} = 5\ \mathrm{A}$$

The measured current will be $i_{\mathrm{meas}} = 5 + 0.002 = 5.002\ \mathrm{A}$
$\therefore$ Full-scale reading for practical purposes is 5 A.

P 3.36 [a] The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by

$$R_m = \frac{100\,\mathrm{mV}}{2\,\mathrm{mA}} = 50\,\Omega.$$

We can calculate the current through the real meter using current division:

$$i_m = \frac{(25/12)}{50 + (25/12)}(i_{\mathrm{meas}}) = \frac{25}{625}(i_{\mathrm{meas}}) = \frac{1}{25} i_{\mathrm{meas}}$$

[b] At full scale, $i_{\mathrm{meas}} = 5$ A and $i_{\mathrm{m}} = 2$ mA so $5 - 0.002 = 4998$ mA flows throught the resistor R_A:

$$R_A = \frac{100\ \mathrm{mV}}{4998\ \mathrm{mA}} = \frac{100}{4998}\,\Omega$$

$$i_m = \frac{(100/4998)}{50 + (100/4998)}(i_{\mathrm{meas}}) = \frac{1}{2500}(i_{\mathrm{meas}})$$

[c] Yes

P 3.37 For all full-scale readings the total resistance is

$$R_V + R_{\mathrm{movement}} = \frac{\text{full-scale reading}}{10^{-3}}$$

We can calculate the resistance of the movement as follows:

$$R_{\mathrm{movement}} = \frac{20\ \mathrm{mV}}{1\ \mathrm{mA}} = 20\,\Omega$$

Therefore, $\quad R_V = 1000\ (\text{full-scale reading}) - 20$

[**a**] $R_V = 1000(50) - 20 = 49{,}980\,\Omega$

[**b**] $R_V = 1000(5) - 20 = 4980\,\Omega$

[**c**] $R_V = 1000(0.25) - 20 = 230\,\Omega$

[**d**] $R_V = 1000(0.025) - 20 = 5\,\Omega$

P 3.38 [**a**] $v_{\text{meas}} = (50 \times 10^{-3})[15\|45\|(4980 + 20)] = 0.5612$ V

[**b**] $v_{\text{true}} = (50 \times 10^{-3})(15\|45) = 0.5625$ V

$$\%\text{ error } = \left(\frac{0.5612}{0.5625} - 1\right) \times 100 = -0.224\%$$

P 3.39 The measured value is $60\|20.1 = 15.05618\,\Omega$.

$$i_g = \frac{50}{(15.05618 + 10)} = 1.995526 \text{ A}; \qquad i_{\text{meas}} = \frac{60}{80.1}(1.996) = 1.494768 \text{ A}$$

The true value is $60\|20 = 15\,\Omega$.

$$i_g = \frac{50}{(15 + 10)} = 2 \text{ A}; \qquad i_{\text{true}} = \frac{60}{80}(2) = 1.5 \text{ A}$$

$$\%\text{error } = \left[\frac{1.494768}{1.5} - 1\right] \times 100 = -0.34878\% \approx -0.35\%$$

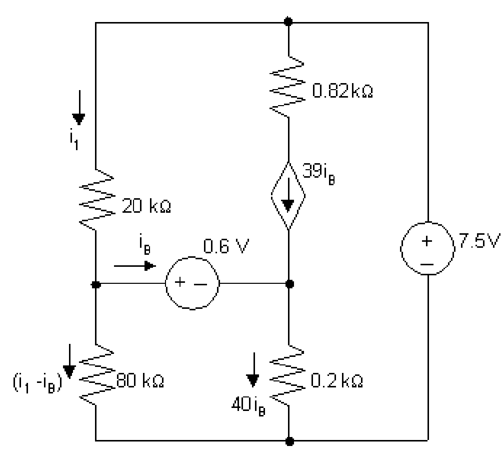
P 3.40 Begin by using current division to find the actual value of the current i_o:

$$i_{\text{true}} = \frac{15}{15 + 45}(50 \text{ mA}) = 12.5 \text{ mA}$$

$$i_{\text{meas}} = \frac{15}{15 + 45 + 0.1}(50 \text{ mA}) = 12.4792 \text{ mA}$$

$$\%\text{ error } = \left[\frac{12.4792}{12.5} - 1\right] 100 = -0.166389\% \approx -0.17\%$$

P 3.41 [**a**]



$$20 \times 10^3 i_1 + 80 \times 10^3 (i_1 - i_{\text{B}}) = 7.5$$

$$80 \times 10^3(i_1 - i_B) = 0.6 + 40i_B(0.2 \times 10^3)$$

$$\therefore \quad 100i_1 - 80i_B = 7.5 \times 10^{-3}$$

$$80i_1 - 88i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 225\,\mu A$

[b] With the insertion of the ammeter the equations become

$$100i_1 - 80i_B = 7.5 \times 10^{-3} \quad \text{(no change)}$$

$$80 \times 10^3(i_1 - i_B) = 10^3 i_B + 0.6 + 40i_B(200)$$

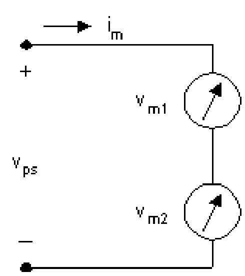
$$80i_1 - 89i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 216\,\mu A$

[c] $\%\ \text{error} = \left(\frac{216}{225} - 1\right) 100 = -4\%$

P 3.42 **[a]** Since the unknown voltage is greater than either voltmeter's maximum reading, the only possible way to use the voltmeters would be to connect them in series.

[b]



$$R_{m1} = (300)(900) = 270\ \text{k}\Omega; \qquad R_{m2} = (150)(1200) = 180\ \text{k}\Omega$$

$$\therefore \quad R_{m1} + R_{m2} = 450\ \text{k}\Omega$$

$$i_{1\ \max} = \frac{300}{270} \times 10^{-3} = 1.11\ \text{mA}; \qquad i_{2\ \max} = \frac{150}{180} \times 10^{-3} = 0.833\ \text{mA}$$

$$\therefore \quad i_{\max} = 0.833\ \text{mA since meters are in series}$$

$$v_{\max} = (0.833 \times 10^{-3})(270 + 180)10^3 = 375\ \text{V}$$

Thus the meters can be used to measure the voltage.

[c] $i_m = \frac{320}{450 \times 10^3} = 0.711\ \text{mA}$

$$v_{m1} = (0.711)(270) = 192\ \text{V}; \qquad v_{m2} = (0.711)(180) = 128\ \text{V}$$

P 3.43 The current in the series-connected voltmeters is

$$i_m = \frac{205.2}{270{,}000} = \frac{136.8}{180{,}000} = 0.76 \text{ mA}$$

$$v_{50\ \text{k}\Omega} = (0.76 \times 10^{-3})(50{,}000) = 38 \text{ V}$$

$$V_{\text{power supply}} = 205.2 + 136.8 + 38 = 380 \text{ V}$$

P 3.44 $R_{\text{meter}} = R_m + R_{\text{movement}} = \dfrac{500 \text{ V}}{1 \text{ mA}} = 1000 \text{ k}\Omega$

$$v_{\text{meas}} = (50 \text{ k}\Omega \| 250 \text{ k}\Omega \| 1000 \text{ k}\Omega)(10 \text{ mA}) = (40 \text{ k}\Omega)(10 \text{ mA}) = 400 \text{ V}$$

$$v_{\text{true}} = (50 \text{ k}\Omega \| 250 \text{ k}\Omega)(10 \text{ mA}) = (41.67 \text{ k}\Omega)(10 \text{ mA}) = 416.67 \text{ V}$$

$$\% \text{ error } = \left(\frac{400}{416.67} - 1\right) 100 = -4\%$$

P 3.45 **[a]** $v_{\text{meter}} = 180$ V

[b] $R_{\text{meter}} = (100)(200) = 20 \text{ k}\Omega$

$$20\|70 = 15.555556 \text{ k}\Omega$$

$$v_{\text{meter}} = \frac{180}{35.555556} \times 15.555556 = 78.75 \text{ V}$$

[c] $20\|20 = 10 \text{ k}\Omega$

$$v_{\text{meter}} = \frac{180}{80}(10) = 22.5 \text{ V}$$

[d] $v_{\text{meter a}} = 180$ V

$$v_{\text{meter b}} + v_{\text{meter c}} = 101.26 \text{ V}$$

No, because of the loading effect.

P 3.46 **[a]**

$$R_1 = (100/2)10^3 = 50 \text{ k}\Omega$$

$$R_2 = (10/2)10^3 = 5 \text{ k}\Omega$$

$$R_3 = (1/2)10^3 = 500\,\Omega$$

[b] Let $i_{\rm a}$ = actual current in the movement

$i_{\rm d}$ = design current in the movement

$$\text{Then \% error} = \left(\frac{i_{\rm a}}{i_{\rm d}} - 1\right)100$$

For the 100 V scale:

$$i_{\rm a} = \frac{100}{50{,}000 + 25} = \frac{100}{50{,}025}, \qquad i_{\rm d} = \frac{100}{50{,}000}$$

$$\frac{i_{\rm a}}{i_{\rm d}} = \frac{50{,}000}{50{,}025} = 0.9995 \qquad \text{\% error} = (0.9995 - 1)100 = -0.05\%$$

For the 10 V scale:

$$\frac{i_{\rm a}}{i_{\rm d}} = \frac{5000}{5025} = 0.995 \qquad \text{\% error} = (0.995 - 1.0)100 = -0.4975\%$$

For the 1 V scale:

$$\frac{i_{\rm a}}{i_{\rm d}} = \frac{500}{525} = 0.9524 \qquad \text{\% error} = (0.9524 - 1.0)100 = -4.76\%$$

P 3.47 From the problem statement we have

$$50 = \frac{V_s(10)}{10 + R_s} \qquad (1) \quad V_s \text{ in mV; } R_s \text{ in M}\Omega$$

$$48.75 = \frac{V_s(6)}{6 + R_s} \qquad (2)$$

[a] From Eq (1) $\quad 10 + R_s = 0.2V_s$

$\therefore \quad R_s = 0.2V_s - 10$

Substituting into Eq (2) yields

$$48.75 = \frac{6V_s}{0.2V_s - 4} \qquad \text{or} \qquad V_s = 52 \text{ mV}$$

[b] From Eq (1)

$$50 = \frac{520}{10 + R_s} \qquad \text{or} \qquad 50R_s = 20$$

So $R_s = 400$ kΩ

P 3.48 **[a]** $R_{\rm movement} = 50\,\Omega$

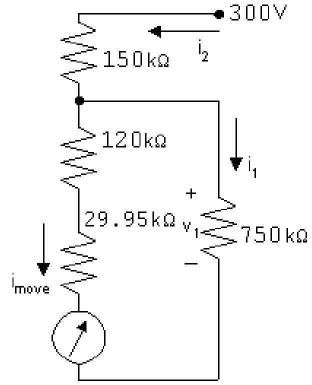
$$R_1 + R_{\rm movement} = \frac{30}{1 \times 10^{-3}} = 30 \text{ k}\Omega \qquad \therefore \quad R_1 = 29{,}950\,\Omega$$

$$R_2 + R_1 + R_{\rm movement} = \frac{150}{1 \times 10^{-3}} = 150 \text{ k}\Omega \qquad \therefore \quad R_2 = 120 \text{ k}\Omega$$

$$R_3 + R_2 + R_1 + R_{\rm movement} = \frac{300}{1 \times 10^{-3}} = 300 \text{ k}\Omega$$

$\therefore \quad R_3 = 150$ kΩ

[b]



$$v_1 = (0.96\text{ m})(150\text{ k}) = 144\text{ V}$$

$$i_{\text{move}} = \frac{144}{120 + 29.95 + 0.05} = 0.96\text{ mA}$$

$$i_1 = \frac{144}{750\text{ k}} = 0.192\text{ mA}$$

$$i_2 = i_{\text{move}} + i_1 = 0.96\text{ m} + 0.192\text{ m} = 1.152\text{ mA}$$

$$v_{\text{meas}} = v_x = 144 + 150 i_2 = 316.8\text{ V}$$

[c] $v_1 = 150$ V; $\qquad i_2 = 1\text{ m} + 0.20\text{ m} = 1.20\text{ mA}$

$$i_1 = 150/750{,}000 = 0.20\text{ mA}$$

$$\therefore \quad v_{\text{meas}} = v_x = 150 + (150\text{ k})(1.20\text{ m}) = 330\text{ V}$$

P 3.49 **[a]** $R_{\text{meter}} = 300\text{ k}\Omega + 600\text{ k}\Omega \| 200\text{ k}\Omega = 450\text{ k}\Omega$

$$450 \| 360 = 200\text{ k}\Omega$$

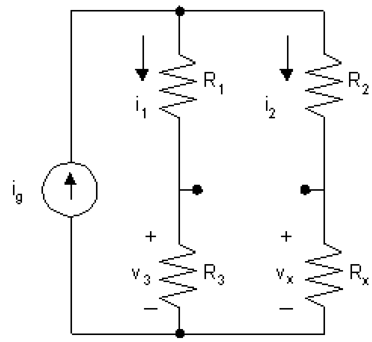
$$V_{\text{meter}} = \frac{200}{240}(600) = 500\text{ V}$$

[b] What is the percent error in the measured voltage?

$$\text{True value } = \frac{360}{400}(600) = 540\text{ V}$$

$$\%\text{ error } = \left(\frac{500}{540} - 1\right) 100 = -7.41\%$$

P 3.50 Since the bridge is balanced, we can remove the detector without disturbing the voltages and currents in the circuit.



It follows that

$$i_1 = \frac{i_g(R_2 + R_x)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_2 + R_x)}{\sum R}$$

$$i_2 = \frac{i_g(R_1 + R_3)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_1 + R_3)}{\sum R}$$

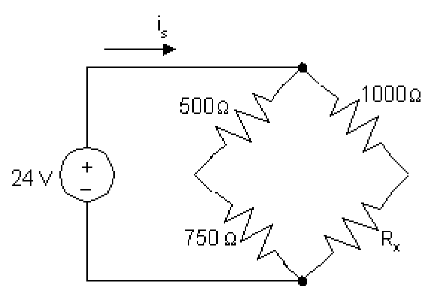
$$v_3 = R_3 i_1 = v_x = i_2 R_x$$

$$\therefore \quad \frac{R_3 i_g(R_2 + R_x)}{\sum R} = \frac{R_x i_g(R_1 + R_3)}{\sum R}$$

$$\therefore \quad R_3(R_2 + R_x) = R_x(R_1 + R_3)$$

From which $R_x = \dfrac{R_2 R_3}{R_1}$

P 3.51 **[a]**



The condition for a balanced bridge is that the product of the opposite resistors must be equal:

$$(500)(R_x) = (1000)(750) \qquad \text{so} \qquad R_x = \frac{(1000)(750)}{500} = 1500\,\Omega$$

[b] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 24 V:

$$i_s = \frac{24\text{ V}}{500\,\Omega + 750\,\Omega} + \frac{24\text{ V}}{1000\,\Omega + 1500\,\Omega} = 28.8\text{ mA}$$

[c] We can use Ohm's law to find the current in each branch:

$$i_{\text{left}} = \frac{24}{500 + 750} = 19.2\text{ mA}$$

$$i_{\text{right}} = \frac{24}{1000 + 1500} = 9.6\text{ mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated by each resistor:

$$p_{500} = (500)(0.0192)^2 = 184.32\text{ mW} \qquad p_{750} = (750)(0.0192)^2 = 276.18\text{ mW}$$

$$p_{1000} = (1000)(0.0096)^2 = 92.16\text{ mW} \qquad p_{1500} = (1500)(0.0096)^2 = 138.24\text{ mW}$$

Thus, the $750\,\Omega$ resistor absorbs the most power; it absorbs 276.48 mW of power.

[d] From the analysis in part (c), the $1000\,\Omega$ resistor absorbs the least power; it absorbs 92.16 mW of power.

P 3.52 Note the bridge structure is balanced, that is $15 \times 5 = 3 \times 25$, hence there is no current in the 5 kΩ resistor. It follows that the equivalent resistance of the circuit is

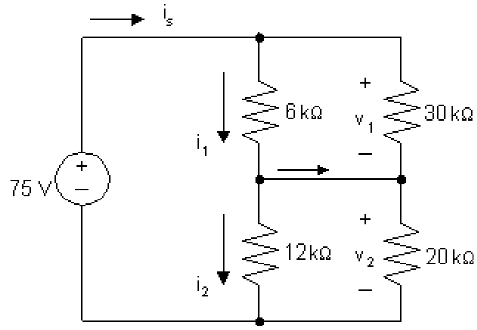
$$R_{\text{eq}} = 750 + (15{,}000 + 3000)\|(25{,}000 + 5000) = 750 + 11{,}250 = 12\ \text{k}\Omega$$

The source current is $192/12{,}000 = 16$ mA.
The current down through the branch containing the 15 kΩ and 3 kΩ resistors is

$$i_{3\text{k}} = \frac{11{,}250}{18{,}000}(0.016) = 10\text{ mA}$$

$$\therefore\ \ p_{3\text{k}} = 3000(0.01)^2 = 0.3\ \text{W}$$

P 3.53 Redraw the circuit, replacing the detector branch with a short circuit.



$6\ \text{k}\Omega \| 30\ \text{k}\Omega = 5\ \text{k}\Omega$

$12\ \text{k}\Omega \| 20\ \text{k}\Omega = 7.5\ \text{k}\Omega$

$$i_s = \frac{75}{12{,}500} = 6\ \text{mA}$$

$$v_1 = 0.006(5000) = 30\ \text{V}$$

$$v_2 = 0.006(7500) = 45\ \text{V}$$

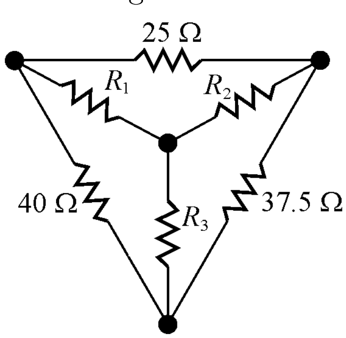
$$i_1 = \frac{30}{6000} = 5\ \text{mA}$$

$$i_2 = \frac{45}{12{,}000} = 3.75\ \text{mA}$$

$$i_{\text{d}} = i_1 - i_2 = 1.25\ \text{mA}$$

P 3.54 In order that all four decades (1, 10, 100, 1000) that are used to set R_3 contribute to the balance of the bridge, the ratio R_2/R_1 should be set to 0.001.

P 3.55 Use the figure below to transform the Δ to an equivalent Y:

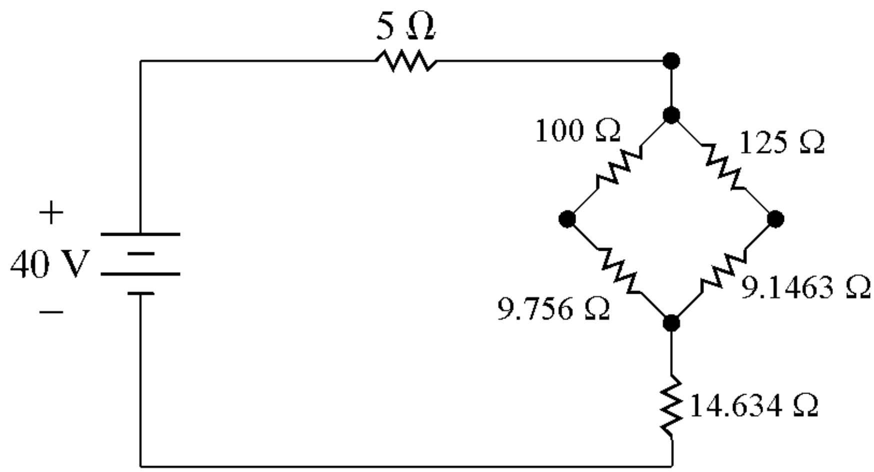


$$R_1 = \frac{(40)(25)}{40 + 25 + 37.5} = 9.756\,\Omega$$

$$R_2 = \frac{(25)(37.5)}{40 + 25 + 37.5} = 9.1463\,\Omega$$

$$R_3 = \frac{(40)(37.5)}{40 + 25 + 37.5} = 14.634\,\Omega$$

Replace the Δ with its equivalent Y in the circuit to get the figure below:



Find the equivalent resistance to the right of the $5\,\Omega$ resistor:

$(100 + 9.756)\|(125 + 9.1463) + 14.634 = 75\,\Omega$

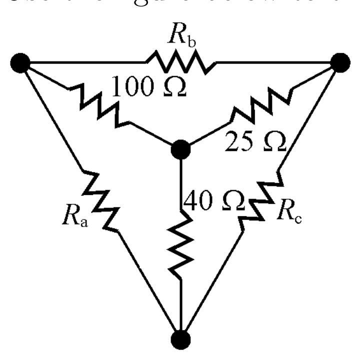
The equivalent resistance seen by the source is thus $5 + 75 = 80\,\Omega$. Use Ohm's law to find the current provided by the source:

$$i_s = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_s = -(40)(0.5) = -20 \text{ W}$$

P 3.56 Use the figure below to transform the Y to an equivalent Δ:

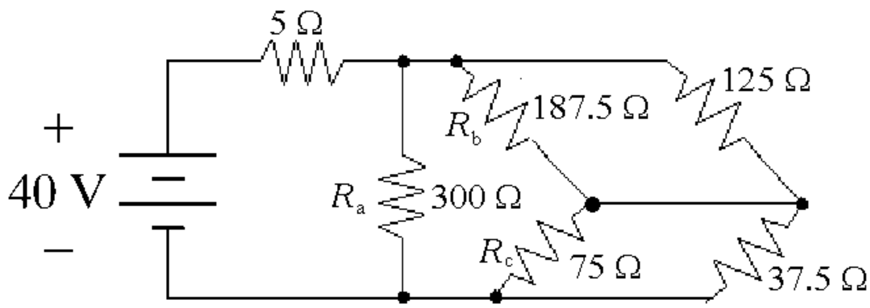


$$R_a = \frac{(25)(100) + (25)(40) + (40)(100)}{25} = \frac{7500}{25} = 300\,\Omega$$

$$R_b = \frac{(25)(100) + (25)(40) + (40)(100)}{40} = \frac{7500}{40} = 187.5\,\Omega$$

$$R_c = \frac{(25)(100) + (25)(40) + (40)(100)}{100} = \frac{7500}{100} = 75\,\Omega$$

Replace the Y with its equivalent Δ in the circuit to get the figure below:



Find the equivalent resistance to the right of the 5 Ω resistor:

$300\|[(125\|187.5) + (37.5\|75)] = 75\,\Omega$

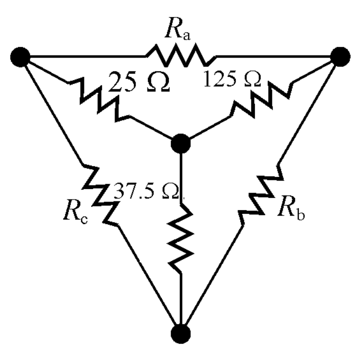
The equivalent resistance seen by the source is thus $5 + 75 = 80\,\Omega$. Use Ohm's law to find the current provided by the source:

$$i_s = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_s = -(40)(0.5) = -20 \text{ W}$$

P 3.57 Use the figure below to transform the Y to an equivalent Δ:

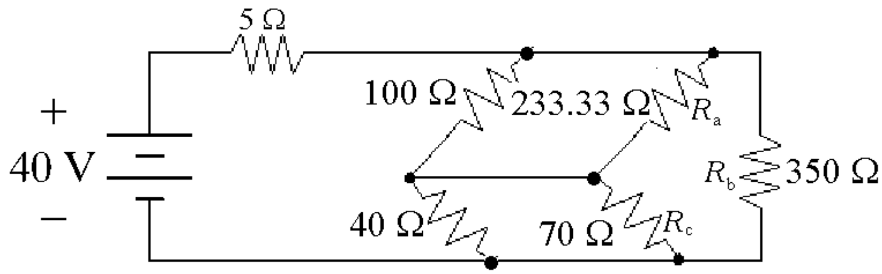


$$R_a = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{37.5} = \frac{8750}{37.5} = 233.33\,\Omega$$

$$R_b = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{25} = \frac{8750}{25} = 350\,\Omega$$

$$R_c = \frac{(25)(125) + (25)(37.5) + (37.5)(125)}{125} = \frac{8750}{125} = 70\,\Omega$$

Replace the Y with its equivalent Δ in the circuit to get the figure below:



Find the equivalent resistance to the right of the $5\,\Omega$ resistor:

$350\|[(100\|233.33) + (40\|70)] = 75\,\Omega$

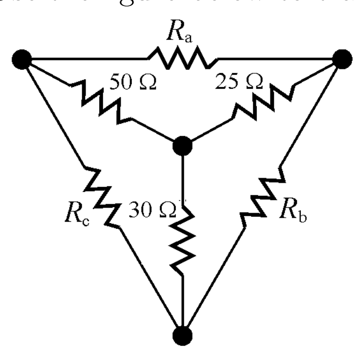
The equivalent resistance seen by the source is thus $5 + 75 = 80\,\Omega$. Use Ohm's law to find the current provided by the source:

$$i_s = \frac{40}{80} = 0.5 \text{ A}$$

Thus, the power associated with the source is

$$P_s = -(40)(0.5) = -20 \text{ W}$$

P 3.58 **[a]** Use the figure below to transform the Y to an equivalent Δ:

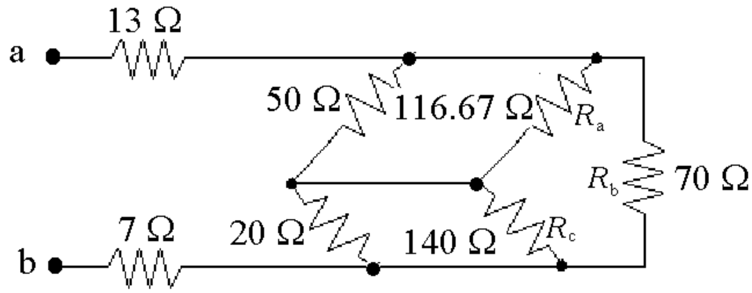


$$R_a = \frac{(25)(30) + (25)(50) + (30)(50)}{30} = \frac{3500}{30} = 116.67\,\Omega$$

$$R_b = \frac{(25)(30) + (25)(50) + (30)(50)}{50} = \frac{3500}{50} = 70\,\Omega$$

$$R_c = \frac{(25)(30) + (25)(50) + (30)(50)}{25} = \frac{3500}{25} = 140\,\Omega$$

Replace the Y with its equivalent Δ in the circuit to get the figure below:



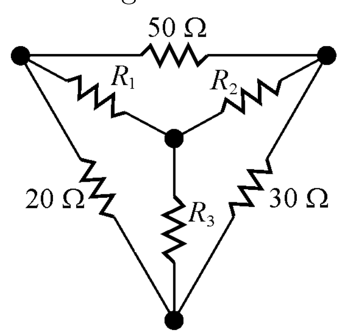
Find the equivalent resistance to the right of the 13 Ω and 7 Ω resistors:

$70\|[(50\|116.67) + (20\|140)] = 30\,\Omega$

Thus, the equivalent resistance seen from the terminals a-b is:

$R_{ab} = 13 + 30 + 7 = 50\,\Omega$

[b] Use the figure below to transform the Δ to an equivalent Y:

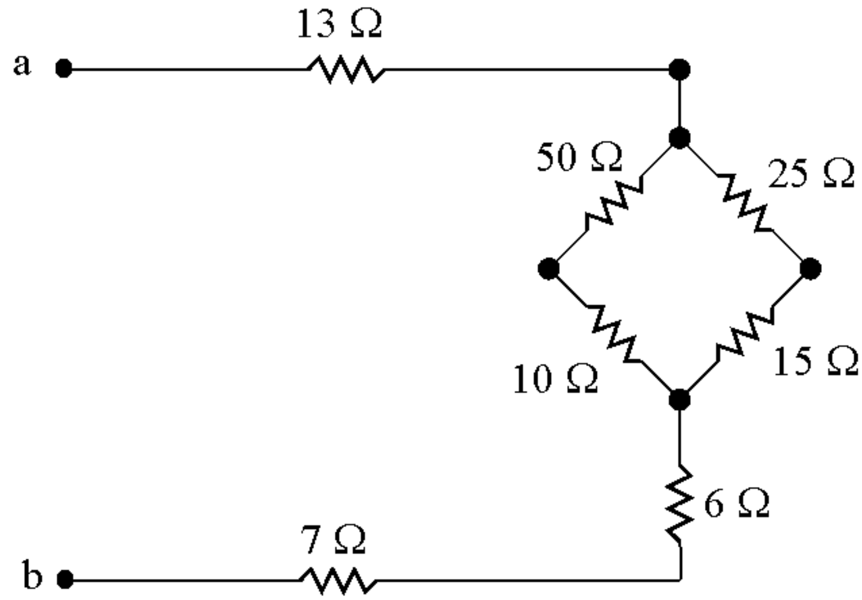


$$R_1 = \frac{(50)(20)}{50 + 20 + 30} = 10\,\Omega$$

$$R_2 = \frac{(50)(30)}{50 + 20 + 30} = 15\,\Omega$$

$$R_3 = \frac{(20)(30)}{50 + 20 + 30} = 6\,\Omega$$

Replace the Δ with its equivalent Y in the circuit to get the figure below:



Find the equivalent resistance to the right of the $13\,\Omega$ and $7\,\Omega$ resistors:

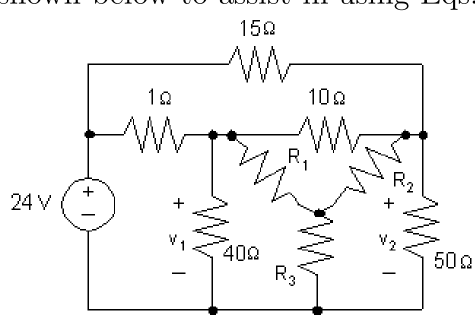
$(50 + 10)\|(25 + 15) + 6 = 30\,\Omega$

Thus, the equivalent resistance seen from the terminals a-b is:

$R_{\text{ab}} = 13 + 30 + 7 = 50\,\Omega$

[c] Convert the delta connection R_1—R_2—R_3 to its equivalent wye.
Convert the wye connection R_1—R_3—R_4 to its equivalent delta.

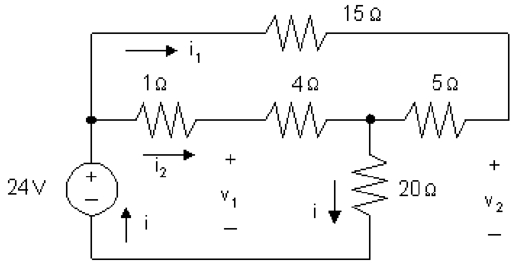
P 3.59 Begin by transforming the Δ-connected resistors $(10\,\Omega, 40\,\Omega, 50\,\Omega)$ to Y-connected resistors. Both the Y-connected and Δ-connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:



Now use Eqs. 3.44 – 3.46 to calculate the values of the Y-connected resistors:

$$R_1 = \frac{(40)(10)}{10 + 40 + 50} = 4\,\Omega; \quad R_2 = \frac{(10)(50)}{10 + 40 + 50} = 5\,\Omega; \quad R_3 = \frac{(40)(50)}{10 + 40 + 50} = 20\,\Omega$$

The transformed circuit is shown below:



The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{\text{eq}} = (15+5)\|(1+4) + 20 = 20\|5 + 20 = 4 + 20 = 24\,\Omega$$

Therefore, the current i in the 24 V source is given by

$$i = \frac{24\text{ V}}{24\,\Omega} = 1\text{ A}$$

Use current division to calculate the currents i_1 and i_2. Note that the current i_1 flows in the branch containing the $15\,\Omega$ and $5\,\Omega$ series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the $1\,\Omega$ and $4\,\Omega$ resistors:

$$i_1 = \frac{4}{15+5}(i) = \frac{4}{20}(1\text{ A}) = 0.2\text{ A}, \qquad \text{and} \qquad i_2 = 1\text{ A} - 0.2\text{ A} = 0.8\text{ A}$$

Now use KVL and Ohm's law to calculate v_1. Note that v_1 is the sum of the voltage drop across the $4\,\Omega$ resistor, $4i_2$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_1 = 4i_2 + 20i = 4(0.8\text{ A}) + 20(1\text{ A}) = 3.2 + 20 = 23.2\text{ V}$$

Finally, use KVL and Ohm's law to calculate v_2. Note that v_2 is the sum of the voltage drop across the $5\,\Omega$ resistor, $5i_1$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_2 = 5i_1 + 20i = 5(0.2\text{ A}) + 20(1\text{ A}) = 1 + 20 = 21\text{ V}$$

P 3.60 **[a]** Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5\,\Omega$$

$$R_2 = \frac{(50)(100)}{200} = 25\,\Omega$$

$$R_3 = \frac{(100)(50)}{200} = 25\,\Omega$$

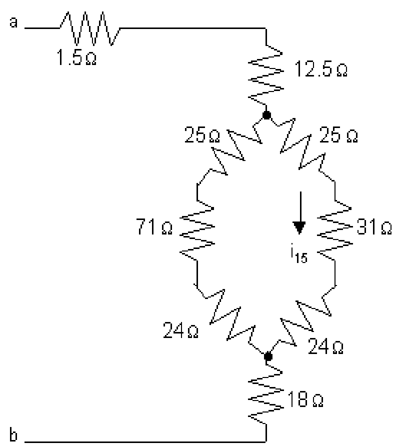
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\,\Omega$$

$$R_5 = \frac{(60)(60)}{200} = 18\,\Omega$$

$$R_6 = \frac{(80)(60)}{200} = 24\,\Omega$$

Now redraw the circuit using the wye equivalents.



$$R_{\rm ab} = 1.5 + 12.5 + \frac{(120)(80)}{200} + 18 = 14 + 48 + 18 = 80\,\Omega$$

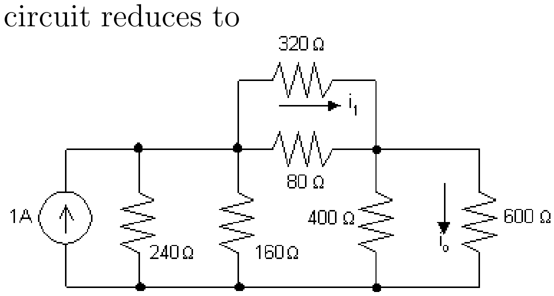
[b] When $v_{\rm ab} = 400$ V

$$i_g = \frac{400}{80} = 5\text{ A}$$

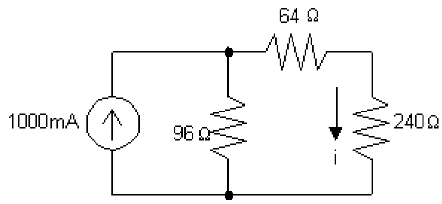
$$i_{31} = \frac{48}{80}(5) = 3\text{ A}$$

$$p_{31\Omega} = (31)(3)^2 = 279\text{ W}$$

P 3.61 [a] After the $20\,\Omega$—$100\,\Omega$—$50\,\Omega$ wye is replaced by its equivalent delta, the circuit reduces to



Now the circuit can be reduced to

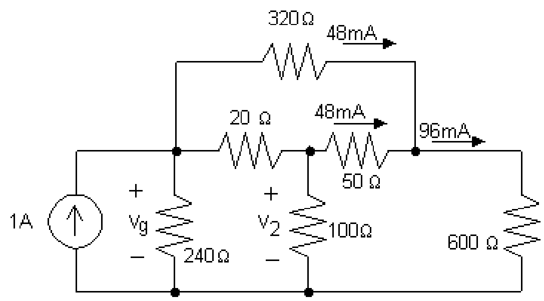


$$i = \frac{96}{400}(1000) = 240 \text{ mA}$$

$$i_o = \frac{400}{1000}(240) = 96 \text{ mA}$$

[b] $i_1 = \frac{80}{400}(240) = 48 \text{ mA}$

[c] Now that i_o and i_1 are known return to the original circuit



$$v_2 = (50)(0.048) + (600)(0.096) = 60 \text{ V}$$

$$i_2 = \frac{v_2}{100} = \frac{60}{100} = 600 \text{ mA}$$

[d] $v_g = v_2 + 20(0.6 + 0.048) = 60 + 12.96 = 72.96$ V

$$p_g = -(v_g)(1) = -72.96 \text{ W}$$

Thus the current source delivers 72.96 W.

P 3.62 $8 + 12 = 20\,\Omega$

$20\|60 = 15\,\Omega$

$15 + 20 = 35\,\Omega$

$35\|140 = 28\,\Omega$

$28 + 22 = 50\,\Omega$

$50\|75 = 30\,\Omega$

$30 + 10 = 40\,\Omega$

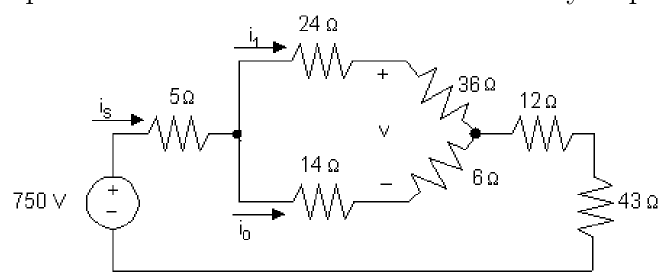
$i_g = 240/40 = 6$ A

$i_o = (6)(50)/125 = 2.4$ A

$i_{140\Omega} = (6 - 2.4)(35)/175 = 0.72$ A

$p_{140\Omega} = (0.72)^2(140) = 72.576$ W

P 3.63 **[a]** Replace the 60—120—20 Ω delta with a wye equivalent to get



$$i_s = \frac{750}{5 + (24+36)\|(14+6) + 12 + 43} = \frac{750}{75} = 10 \text{ A}$$

$$i_1 = \frac{(24+36)\|(14+6)}{24+36}(10) = \frac{15}{60}(10) = 2.5 \text{ A}$$

[b] $i_o = 10 - 2.5 = 7.5$ A

$$v = 36i_1 - 6i_o = 36(2.5) - 6(7.5) = 45 \text{ V}$$

[c] $i_2 = i_o + \dfrac{v}{60} = 7.5 + \dfrac{45}{60} = 8.25$ A

[d] $P_{\text{supplied}} = (750)(10) = 7500$ W

P 3.64
$$\begin{aligned} G_\text{a} &= \frac{1}{R_\text{a}} = \frac{R_1}{R_1R_2 + R_2R_3 + R_3R_1} \\ &= \frac{1/G_1}{(1/G_1)(1/G_2) + (1/G_2)(1/G_3) + (1/G_3)(1/G_1)} \\ &= \frac{(1/G_1)(G_1G_2G_3)}{G_1 + G_2 + G_3} = \frac{G_2G_3}{G_1 + G_2 + G_3} \end{aligned}$$

Similar manipulations generate the expressions for G_b and G_c.

P 3.65 **[a]** Subtracting Eq. 3.42 from Eq. 3.43 gives

$R_1 - R_2 = (R_\text{c}R_\text{b} - R_\text{c}R_\text{a})/(R_\text{a} + R_\text{b} + R_\text{c})$.

Adding this expression to Eq. 3.41 and solving for R_1 gives

$R_1 = R_\text{c}R_\text{b}/(R_\text{a} + R_\text{b} + R_\text{c})$.

To find R_2, subtract Eq. 3.43 from Eq. 3.41 and add this result to Eq. 3.42. To find R_3, subtract Eq. 3.41 from Eq. 3.42 and add this result to Eq. 3.43.

[b] Using the hint, Eq. 3.43 becomes

$$R_1 + R_3 = \frac{R_\text{b}[(R_2/R_3)R_\text{b} + (R_2/R_1)R_\text{b}]}{(R_2/R_1)R_\text{b} + R_\text{b} + (R_2/R_3)R_\text{b}} = \frac{R_\text{b}(R_1 + R_3)R_2}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Solving for R_b gives $R_\text{b} = (R_1R_2 + R_2R_3 + R_3R_1)/R_2$. To find R_a: First use Eqs. 3.44–3.46 to obtain the ratios $(R_1/R_3) = (R_\text{c}/R_\text{a})$ or $R_\text{c} = (R_1/R_3)R_\text{a}$ and $(R_1/R_2) = (R_\text{b}/R_\text{a})$ or $R_\text{b} = (R_1/R_2)R_\text{a}$. Now use these relationships to eliminate R_b and R_c from Eq. 3.42. To find R_c, use Eqs. 3.44–3.46 to obtain the ratios $R_\text{b} = (R_3/R_2)R_\text{c}$ and $R_\text{a} = (R_3/R_1)R_\text{c}$. Now use the relationships to eliminate R_b and R_a from Eq. 3.41.

P 3.66 **[a]** $R_\text{ab} = 2R_1 + \dfrac{R_2(2R_1 + R_\text{L})}{2R_1 + R_2 + R_\text{L}} = R_\text{L}$

Therefore $\quad 2R_1 - R_\text{L} + \dfrac{R_2(2R_1 + R_\text{L})}{2R_1 + R_2 + R_\text{L}} = 0$

Thus $\quad R_\text{L}^2 = 4R_1^2 + 4R_1R_2 = 4R_1(R_1 + R_2)$

When $R_\text{ab} = R_\text{L}$, the current into terminal a of the attenuator will be v_i/R_L.
Using current division, the current in the R_L branch will be

$$\frac{v_i}{R_\text{L}} \cdot \frac{R_2}{2R_1 + R_2 + R_\text{L}}$$

Therefore $\quad v_o = \dfrac{v_i}{R_\text{L}} \cdot \dfrac{R_2}{2R_1 + R_2 + R_\text{L}} R_\text{L}$

and $\quad \dfrac{v_o}{v_i} = \dfrac{R_2}{2R_1 + R_2 + R_\text{L}}$

[b] $(300)^2 = 4(R_1 + R_2)R_1$

$$22{,}500 = R_1^2 + R_1R_2$$

$$\frac{v_o}{v_i} = 0.5 = \frac{R_2}{2R_1 + R_2 + 300}$$

$$\therefore \quad R_1 + 0.5R_2 + 150 = R_2$$

$$0.5R_2 = R_1 + 150$$

$$R_2 = 2R_1 + 300$$

$$\therefore \quad 22{,}500 = R_1^2 + R_1(2R_1 + 300) = 3R_1^2 + 300R_1$$

$$\therefore \quad R_1^2 + 100R_1 - 7500 = 0$$

Solving,

$R_1 = 50\,\Omega$

$R_2 = 2(50) + 300 = 400\,\Omega$

[c] From Appendix H, choose $R_1 = 47\,\Omega$ and $R_2 = 390\,\Omega$. For these values, $R_{\rm ab} \neq R_{\rm L}$, so the equations given in part (a) cannot be used. Instead

$$R_{\rm ab} = 2R_1 + [R_2\|(2R_1 + R_{\rm L})] = 2(47) + 390\|(2(47) + 300)$$

$$= 94 + 390\|394 = 290\,\Omega$$

$$\%\text{ error} = \left(\frac{290}{300} - 1\right)100 = -3.33\%$$

Now calculate the ratio of the output voltage to the input voltage. Begin by finding the current through the top left R_1 resistor, called $i_{\rm a}$:

$$i_{\rm a} = \frac{v_i}{R_{\rm ab}}$$

Now use current division to find the current through the $R_{\rm L}$ resistor, called $i_{\rm L}$:

$$i_{\rm L} = \frac{R_2}{R_2 + 2R_1 + R_{\rm L}}i_{\rm a}$$

Therefore, the output voltage, v_o, is equal to $R_{\rm L}i_{\rm L}$:

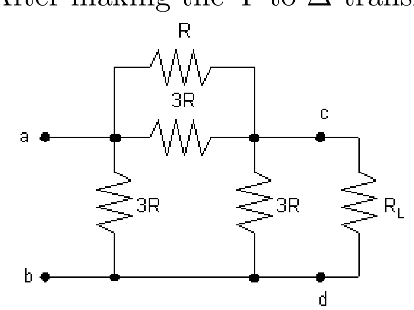
$$v_o = \frac{R_2R_{\rm L}v_i}{R_{\rm ab}(R_2 + 2R_1 + R_{\rm L})}$$

Thus,

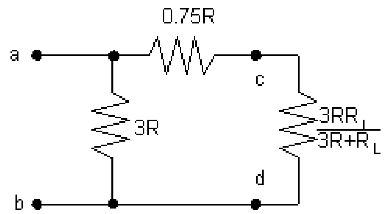
$$\frac{v_o}{v_i} = \frac{R_2R_{\rm L}}{R_{\rm ab}(R_2 + 2R_1 + R_{\rm L})} = \frac{390(300)}{290(390 + 2(47) + 300)} = 0.5146$$

$$\%\text{ error} = \left(\frac{0.5146}{0.5} - 1\right)100 = 2.92\%$$

P 3.67 **[a]** After making the Y-to-Δ transformation, the circuit reduces to



Combining the parallel resistors reduces the circuit to



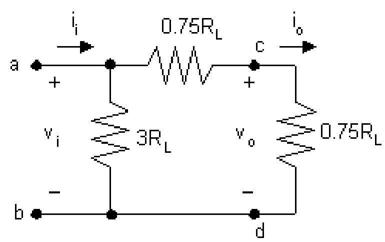
Now note: $\quad 0.75R + \dfrac{3RR_\mathrm{L}}{3R+R_\mathrm{L}} = \dfrac{2.25R^2 + 3.75RR_\mathrm{L}}{3R+R_\mathrm{L}}$

Therefore $\quad R_\mathrm{ab} = \dfrac{3R\left(\dfrac{2.25R^2+3.75RR_\mathrm{L}}{3R+R_\mathrm{L}}\right)}{3R + \left(\dfrac{2.25R^2+3.75RR_\mathrm{L}}{3R+R_\mathrm{L}}\right)} = \dfrac{3R(3R+5R_\mathrm{L})}{15R+9R_\mathrm{L}}$

If $R = R_\mathrm{L}$, we have $\quad R_\mathrm{ab} = \dfrac{3R_\mathrm{L}(8R_\mathrm{L})}{24R_\mathrm{L}} = R_\mathrm{L}$

Therefore $\quad R_\mathrm{ab} = R_\mathrm{L}$

[b] When $R = R_\mathrm{L}$, the circuit reduces to



$i_o = \dfrac{i_i(3R_\mathrm{L})}{4.5R_\mathrm{L}} = \dfrac{1}{1.5}i_i = \dfrac{1}{1.5}\dfrac{v_i}{R_\mathrm{L}}, \qquad v_o = 0.75R_\mathrm{L}i_o = \dfrac{1}{2}v_i,$

Therefore $\quad \dfrac{v_o}{v_i} = 0.5$

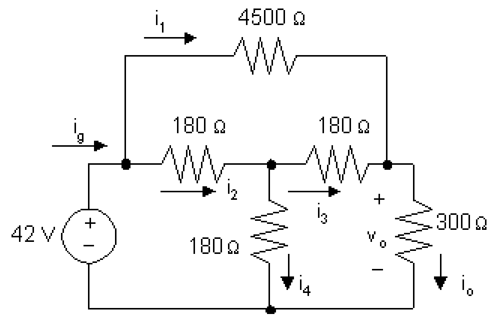
P 3.68 **[a]** $3.5(3R - R_\mathrm{L}) = 3R + R_\mathrm{L}$

$10.5R - 1050 = 3R + 300$

$7.5R = 1350, \qquad R = 180\,\Omega$

$R_2 = \dfrac{2(180)(300)^2}{3(180)^2 - (300)^2} = 4500\,\Omega$

[**b**]



$$v_o = \frac{v_i}{3.5} = \frac{42}{3.5} = 12 \text{ V}$$

$$i_o = \frac{12}{300} = 40 \text{ mA}$$

$$i_1 = \frac{42 - 12}{4500} = \frac{30}{4500} = 6.67 \text{ mA}$$

$$i_g = \frac{42}{300} = 140 \text{ mA}$$

$$i_2 = 140 - 6.67 = 133.33 \text{ mA}$$

$$i_3 = 40 - 6.67 = 33.33 \text{ mA}$$

$$i_4 = 133.33 - 33.33 = 100 \text{ mA}$$

$$p_{4500 \text{ top}} = (6.67 \times 10^{-3})^2(4500) = 0.2 \text{ W}$$

$$p_{180 \text{ left}} = (133.33 \times 10^{-3})^2(180) = 3.2 \text{ W}$$

$$p_{180 \text{ right}} = (33.33 \times 10^{-3})^2(180) = 0.2 \text{ W}$$

$$p_{180 \text{ vertical}} = (100 \times 10^{-3})^2(180) = 0.48 \text{ W}$$

$$p_{300 \text{ load}} = (40 \times 10^{-3})^2(300) = 0.48 \text{ W}$$

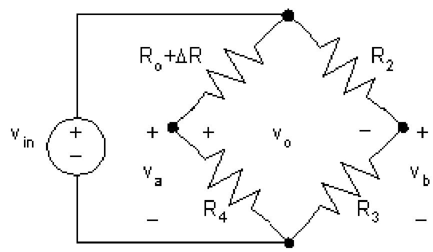
The 180 Ω resistor carrying i_2

[**c**] $p_{180 \text{ left}} = 3.2$ W

[**d**] Two resistors dissipate minimum power – the 4500 Ω resistor and the 180 Ω resistor carrying i_3.

[**e**] They both dissipate 0.2 W.

P 3.69 [**a**]



$$v_a = \frac{v_{in}R_4}{R_o + R_4 + \Delta R}$$

$$v_b = \frac{R_3}{R_2 + R_3}v_{in}$$

$$v_o = v_a - v_b = \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_3}{R_2 + R_3}v_{in}$$

When the bridge is balanced,

$$\frac{R_4}{R_o + R_4}v_{in} = \frac{R_3}{R_2 + R_3}v_{in}$$

$$\therefore \quad \frac{R_4}{R_o + R_4} = \frac{R_3}{R_2 + R_3}$$

$$\begin{aligned} \text{Thus,} \qquad v_o &= \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_4 v_{in}}{R_o + R_4} \\ &= R_4 v_{in}\left[\frac{1}{R_o + R_4 + \Delta R} - \frac{1}{R_o + R_4}\right] \\ &= \frac{R_4 v_{in}(-\Delta R)}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &\approx \frac{-(\Delta R)R_4 v_{in}}{(R_o + R_4)^2}, \qquad \text{since } \Delta R << R_4 \end{aligned}$$

[**b**] $\Delta R = 0.03R_o$

$$R_o = \frac{R_2 R_4}{R_3} = \frac{(1000)(5000)}{500} = 10{,}000\,\Omega$$

$$\Delta R = (0.03)(10^4) = 300\,\Omega$$

$$\therefore \quad v_o \approx \frac{-300(5000)(6)}{(15{,}000)^2} = -40 \text{ mV}$$

[**c**]
$$\begin{aligned} v_o &= \frac{-(\Delta R)R_4 v_{in}}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &= \frac{-300(5000)(6)}{(15{,}300)(15{,}000)} \\ &= -39.2157 \text{ mV} \end{aligned}$$

P 3.70 [a] approx value $= \dfrac{-(\Delta R)R_4 v_{\text{in}}}{(R_o + R_4)^2}$

$$\text{true value} = \frac{-(\Delta R)R_4 v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{(R_o + R_4 + \Delta R)}{(R_o + R_4)}$$

$$\therefore \quad \%\ \text{error} = \left[\frac{R_o + R_4}{R_o + R_4 + \Delta R} - 1\right] \times 100 = \frac{-\Delta R}{R_o + R_4} \times 100$$

Note that in the above expression, we take the ratio of the true value to the approximate value because both values are negative.

But $R_o = \dfrac{R_2 R_4}{R_3}$

$$\therefore \quad \%\ \text{error} = \frac{-R_3 \Delta R}{R_4(R_2 + R_3)}$$

[b] $\%\ \text{error} = \dfrac{-(500)(300)}{(5000)(1500)} \times 100 = -2\%$

P 3.71 $\dfrac{\Delta R(R_3)(100)}{(R_2 + R_3)R_4} = 0.5$

$$\frac{\Delta R(500)(100)}{(1500)(5000)} = 0.5$$

$$\therefore \quad \Delta R = 75\,\Omega$$

$$\%\ \text{change} = \frac{75}{10{,}000} \times 100 = 0.75\%$$

P 3.72 [a] Using the equation for voltage division,

$$V_y = \frac{\beta R_y}{\beta R_y + (1-\beta)R_y} V_S = \frac{\beta R_y}{R_y} V_S = \beta V_S$$

[b] Since β represents the touch point with respect to the bottom of the screen, $(1-\beta)$ represents the location of the touch point with respect to the top of the screen. Therefore, the y-coordinate of the pixel corresponding to the touch point is

$$y = (1-\beta)p_y$$

Remember that the value of y is capped at $(p_y - 1)$.

P 3.73 **[a]** Use the equations developed in the Practical Perspective and in Problem 3.72:

$$V_x = \alpha V_S \qquad \text{so} \qquad \alpha = \frac{V_x}{V_S} = \frac{1}{5} = 0.2$$

$$V_y = \beta V_S \qquad \text{so} \qquad \beta = \frac{V_y}{V_S} = \frac{3.75}{5} = 0.75$$

[b] Use the equations developed in the Practical Perspective and in Problem 3.72:

$$x = (1-\alpha)p_x = (1-0.2)(480) = 384$$

$$y = (1-\beta)p_y = (1-0.75)(800) = 200$$

Therefore, the touch occurred in the upper right corner of the screen.

P 3.74 Use the equations developed in the Practical Perspective and in Problem 3.72:

$$x = (1-\alpha)p_x \qquad \text{so} \qquad \alpha = 1 - \frac{x}{p_x} = 1 - \frac{480}{640} = 0.25$$

$$V_x = \alpha V_S = (0.25)(8) = 2 \text{ V}$$

$$y = (1-\beta)p_y \qquad \text{so} \qquad \beta = 1 - \frac{y}{p_y} = 1 - \frac{192}{1024} = 0.8125$$

$$V_y = \beta V_S = (0.8125)(8) = 6.5 \text{ V}$$

P 3.75 From the results of Problem 3.74, the voltages corresponding to the touch point $(480, 192)$ are

$$V_{x1} = 2 \text{ V}; \qquad V_{y1} = 6.5 \text{ V}$$

Now calculate the voltages corresponding to the touch point $(240, 384)$:

$$x = (1-\alpha)p_x \qquad \text{so} \qquad \alpha = 1 - \frac{x}{p_x} = 1 - \frac{240}{640} = 0.625$$

$$V_{x2} = \alpha V_S = (0.625)(8) = 5 \text{ V}$$

$$y = (1-\beta)p_y \qquad \text{so} \qquad \beta = 1 - \frac{y}{p_y} = 1 - \frac{384}{1024} = 0.625$$

$$V_{y2} = \beta V_S = (0.625)(8) = 5 \text{ V}$$

When the screen is touched at two points simultaneously, only the smaller of the two voltages in the x direction is sensed. The same is true in the y direction. Therefore, the voltages actually sensed are

$$V_x = 2 \text{ V}; \qquad V_y = 5 \text{ V}$$

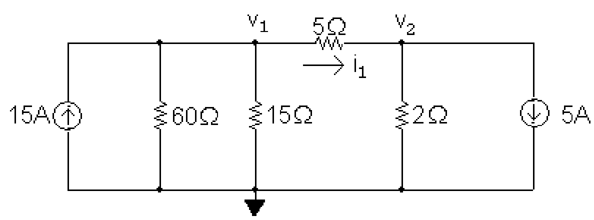
These two voltages identify the touch point as $(480, 384)$, which does not correspond to either of the points actually touched! Therefore, the resistive touch screen is appropriate only for single point touches.

4

Techniques of Circuit Analysis

Assessment Problems

AP 4.1 [**a**] Redraw the circuit, labeling the reference node and the two node voltages:



The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) = 15$$

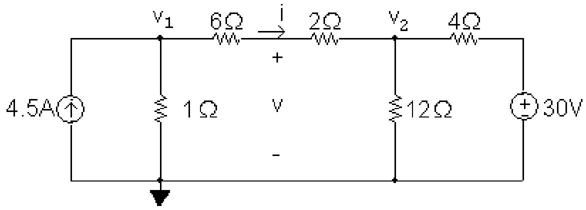
$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{2} + \frac{1}{5}\right) = -5$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[**b**] $p_{15\text{A}} = -(15\,\text{A})v_1 = -(15\,\text{A})(60\,\text{V}) = -900$ W $= 900$ W(delivered)

[**c**] $p_{5\text{A}} = (5\,\text{A})v_2 = (5\,\text{A})(10\,\text{V}) = 50$ W$= -50$ W(delivered)

AP 4.2 Redraw the circuit, choosing the node voltages and reference node as shown:



The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} = 0$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1\left(1 + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 4.5$$

$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Solving, $v_1 = 6$ V $\qquad v_2 = 18\,\text{V}$

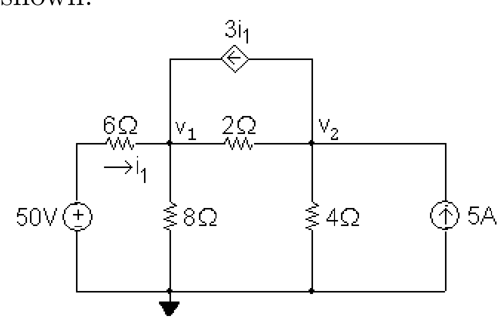
To find the voltage v, first find the current i through the series-connected $6\,\Omega$ and $2\,\Omega$ resistors:

$$i = \frac{v_1 - v_2}{6+2} = \frac{6-18}{8} = -1.5\,\text{A}$$

Using a KVL equation, calculate v:

$$v = 2i + v_2 = 2(-1.5) + 18 = 15\,\text{V}$$

AP 4.3 **[a]** Redraw the circuit, choosing the node voltages and reference node as shown:



The node voltage equations are:

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

The dependent source requires the following constraint equation:

$$i_1 = \frac{50 - v_1}{6}$$

Place these equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{6}+\frac{1}{8}+\frac{1}{2}\right) &+ v_2\left(-\frac{1}{2}\right) &+ i_1(-3) &= \frac{50}{6}\\
v_1\left(-\frac{1}{2}\right) &+ v_2\left(\frac{1}{4}+\frac{1}{2}\right) &+ i_1(3) &= 5\\
v_1\left(\frac{1}{6}\right) &+ v_2(0) &+ i_1(1) &= \frac{50}{6}
\end{aligned}$$

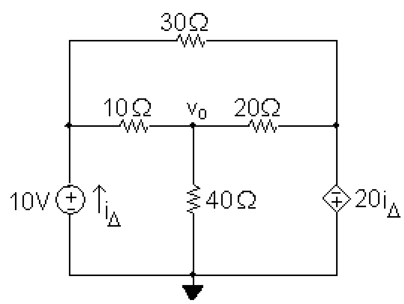
Solving, $v_1 = 32$ V; $v_2 = 16$ V; $i_1 = 3$ A

Using these values to calculate the power associated with each source:

$$\begin{aligned}
p_{50\text{V}} = -50i_1 &= -150 \text{ W}\\
p_{5\text{A}} = -5(v_2) &= -80 \text{ W}\\
p_{3i_1} = 3i_1(v_2 - v_1) &= -144 \text{ W}
\end{aligned}$$

[**b**] All three sources are delivering power to the circuit because the power computed in (a) for each of the sources is negative.

AP 4.4 Redraw the circuit and label the reference node and the node at which the node voltage equation will be written:



The node voltage equation is

$$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0$$

The constraint equation required by the dependent source is

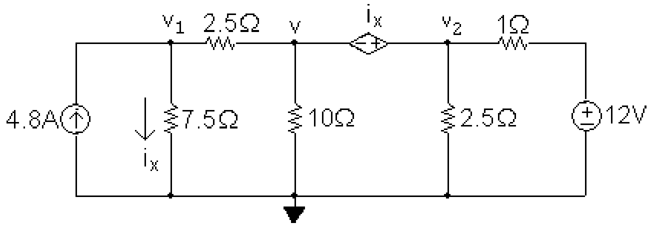
$$i_\Delta = i_{10\,\Omega} + i_{30\,\Omega} = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

Place these equations in standard form:

$$\begin{aligned}
v_o\left(\frac{1}{40}+\frac{1}{10}+\frac{1}{20}\right) &+ i_\Delta(1) &= 1\\
v_o\left(\frac{1}{10}\right) &+ i_\Delta\left(1 - \frac{20}{30}\right) &= 1 + \frac{10}{30}
\end{aligned}$$

Solving, $i_\Delta = -3.2$ A and $v_o = 24$ V

AP 4.5 Redraw the circuit identifying the three node voltages and the reference node:



Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5} + \frac{v_1 - v}{2.5} - 4.8 = 0$$

The supernode equation is

$$\frac{v - v_1}{2.5} + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0$$

The constraint equation due to the dependent source is

$$i_x = \frac{v_1}{7.5}$$

The constraint equation due to the supernode is

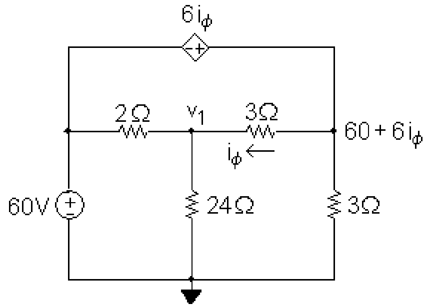
$$v + i_x = v_2$$

Place this set of equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{7.5} + \frac{1}{2.5}\right) &+ v\left(-\frac{1}{2.5}\right) &+ v_2(0) &+ i_x(0) &= 4.8 \\
v_1\left(-\frac{1}{2.5}\right) &+ v\left(\frac{1}{2.5} + \frac{1}{10}\right) &+ v_2\left(\frac{1}{2.5} + 1\right) &+ i_x(0) &= 12 \\
v_1\left(-\frac{1}{7.5}\right) &+ v(0) &+ v_2(0) &+ i_x(1) &= 0 \\
v_1(0) &+ v(1) &+ v_2(-1) &+ i_x(1) &= 0
\end{aligned}$$

Solving this set of equations gives $v_1 = 15$ V, $v_2 = 10$ V, $i_x = 2$ A, and $v = 8$ V.

AP 4.6 Redraw the circuit identifying the reference node and the two unknown node voltages. Note that the right-most node voltage is the sum of the 60 V source and the dependent source voltage.



The node voltage equation at v_1 is

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0$$

The constraint equation due to the dependent source is

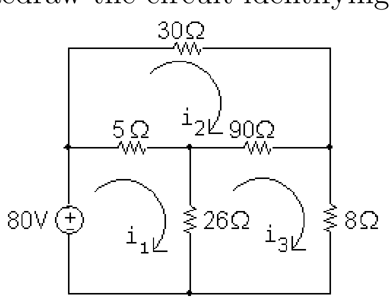
$$i_\phi = \frac{60 + 6i_\phi - v_1}{3}$$

Place these two equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{2} + \frac{1}{24} + \frac{1}{3}\right) &+ i_\phi(-2) &= 30 + 20 \\ v_1\left(\frac{1}{3}\right) &+ i_\phi(1 - 2) &= 20 \end{aligned}$$

Solving, $\quad i_\phi = -4$ A $\quad$ and $\quad v_1 = 48$ V

AP 4.7 **[a]** Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -80 + 5(i_1 - i_2) + 26(i_1 - i_3) &= 0 \\ 30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) &= 0 \\ 8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} 31i_1 - 5i_2 - 26i_3 &= 80 \\ -5i_1 + 125i_2 - 90i_3 &= 0 \\ -26i_1 - 90i_2 + 124i_3 &= 0 \end{aligned}$$

Solving,

$i_1 = 5\,\text{A}; \qquad i_2 = 2\,\text{A}; \qquad i_3 = 2.5\,\text{A}$

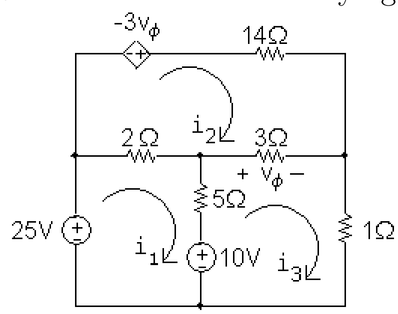
$p_{80V} = -(80)i_1 = -(80)(5) = -400\,\text{W}$

Therefore the 80 V source is delivering 400 W to the circuit.

[b] $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the $8\,\Omega$ resistor dissipates 50 W.

AP 4.8 **[a]** $b = 8, \qquad n = 6, \qquad b - n + 1 = 3$

[b] Redraw the circuit identifying the three mesh currents:



The three mesh-current equations are

$$\begin{aligned} -25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 &= 0 \\ -(-3v_\phi) + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) &= 0 \\ 1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$v_\phi = 3(i_3 - i_2)$

Place these four equations in standard form:

$$\begin{aligned} 7i_1 - 2i_2 - 5i_3 + 0v_\phi &= 15 \\ -2i_1 + 19i_2 - 3i_3 + 3v_\phi &= 0 \\ -5i_1 - 3i_2 + 9i_3 + 0v_\phi &= 10 \\ 0i_1 + 3i_2 - 3i_3 + 1v_\phi &= 0 \end{aligned}$$

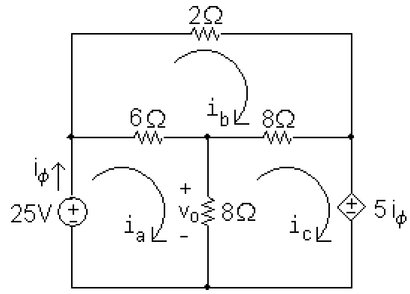
Solving

$i_1 = 4\,\text{A}; \qquad i_2 = -1\,\text{A}; \qquad i_3 = 3\,\text{A}; \qquad v_\phi = 12\,\text{V}$

$p_{\text{ds}} = -(-3v_\phi)i_2 = 3(12)(-1) = -36\,\text{W}$

Thus, the dependent source is delivering 36 W, or absorbing −36 W.

AP 4.9 Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -25 + 6(i_a - i_b) + 8(i_a - i_c) &= 0 \\ 2i_b + 8(i_b - i_c) + 6(i_b - i_a) &= 0 \\ 5i_\phi + 8(i_c - i_a) + 8(i_c - i_b) &= 0 \end{aligned}$$

The dependent source constraint equation is $i_\phi = i_a$. We can substitute this simple expression for i_ϕ into the third mesh equation and place the equations in standard form:

$$\begin{aligned} 14i_a - 6i_b - 8i_c &= 25 \\ -6i_a + 16i_b - 8i_c &= 0 \\ -3i_a - 8i_b + 16i_c &= 0 \end{aligned}$$

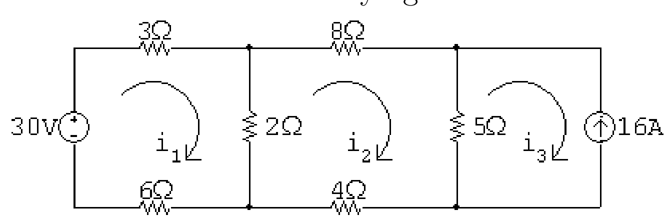
Solving,

$i_a = 4\,\text{A}$; $\qquad i_b = 2.5\,\text{A}$; $\qquad i_c = 2\,\text{A}$

Thus,

$v_o = 8(i_a - i_c) = 8(4 - 2) = 16\,\text{V}$

AP 4.10 Redraw the circuit identifying the mesh currents:



Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16$ A. The remaining two mesh equations are

$$\begin{aligned} -30 + 3i_1 + 2(i_1 - i_2) + 6i_1 &= 0 \\ 8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) &= 0 \end{aligned}$$

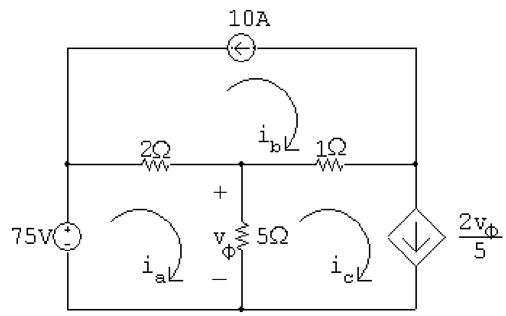
Place these equations in standard form:

$$11i_1 - 2i_2 = 30$$

$$-2i_1 + 19i_2 = -80$$

Solving: $i_1 = 2\,\text{A}$, $i_2 = -4\,\text{A}$, $i_3 = -16\,\text{A}$
The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6\,\text{A}$ $\therefore$ $p_{2\,\Omega} = (6)^2(2) = 72\,\text{W}$
Thus, the $2\,\Omega$ resistors dissipates 72 W.

AP 4.11 Redraw the circuit and identify the mesh currents:



There are current sources on the perimeters of both the i_b mesh and the i_c mesh, so we know that

$$i_\text{b} = -10\,\text{A}; \qquad i_\text{c} = \frac{2v_\phi}{5}$$

The remaining mesh current equation is

$$-75 + 2(i_\text{a} + 10) + 5(i_\text{a} - 0.4v_\phi) = 0$$

The dependent source requires the following constraint equation:

$$v_\phi = 5(i_\text{a} - i_\text{c}) = 5(i_\text{a} - 0.4v_\phi)$$

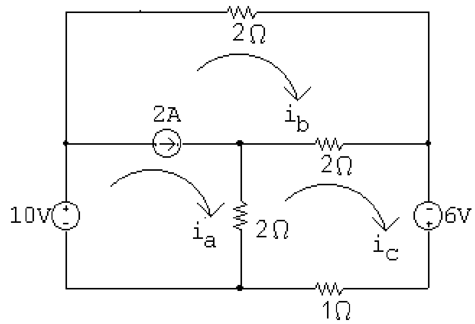
Place the mesh current equation and the dependent source equation is standard form:

$$7i_\text{a} - 2v_\phi = 55$$

$$5i_\text{a} - 3v_\phi = 0$$

Solving: $i_\text{a} = 15\,\text{A}$; $i_\text{b} = -10\,\text{A}$; $i_\text{c} = 10\,\text{A}$; $v_\phi = 25\,\text{V}$
Thus, $i_\text{a} = 15$ A.

AP 4.12 Redraw the circuit and identify the mesh currents:



The 2 A current source is shared by the meshes i_a and i_b. Thus we combine these meshes to form a supermesh and write the following equation:

$$-10 + 2i_b + 2(i_b - i_c) + 2(i_a - i_c) = 0$$

The other mesh current equation is

$$-6 + 1i_c + 2(i_c - i_a) + 2(i_c - i_b) = 0$$

The supermesh constraint equation is

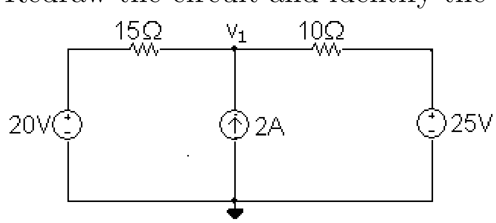
$$i_a - i_b = 2$$

Place these three equations in standard form:

$$\begin{aligned} 2i_a + 4i_b - 4i_c &= 10 \\ -2i_a - 2i_b + 5i_c &= 6 \\ i_a - i_b + 0i_c &= 2 \end{aligned}$$

Solving, $i_a = 7\,\text{A}; \quad i_b = 5\,\text{A}; \quad i_c = 6\,\text{A}$
Thus, $p_{1\,\Omega} = i_c^2(1) = (6)^2(1) = 36\,\text{W}$

AP 4.13 Redraw the circuit and identify the reference node and the node voltage v_1:



The node voltage equation is

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$$

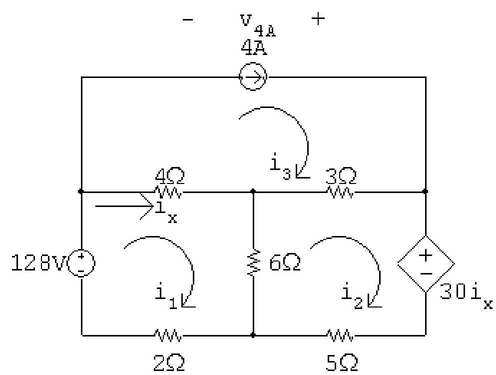
Rearranging and solving,

$$v_1\left(\frac{1}{15}+\frac{1}{10}\right)=2+\frac{20}{15}+\frac{25}{10} \qquad \therefore \quad v_1=35\,\text{V}$$

$$p_{2A}=-35(2)=-70\,\text{W}$$

Thus the 2 A current source delivers 70 W.

AP 4.14 Redraw the circuit and identify the mesh currents:



There is a current source on the perimeter of the i_3 mesh, so $i_3 = 4$ A. The other two mesh current equations are

$$\begin{aligned} -128+4(i_1-4)+6(i_1-i_2)+2i_1 &= 0 \\ 30i_x+5i_2+6(i_2-i_1)+3(i_2-4) &= 0 \end{aligned}$$

The constraint equation due to the dependent source is

$$i_x=i_1-i_3=i_1-4$$

Substitute the constraint equation into the second mesh equation and place the resulting two mesh equations in standard form:

$$\begin{aligned} 12i_1-6i_2 &= 144 \\ 24i_1+14i_2 &= 132 \end{aligned}$$

Solving,

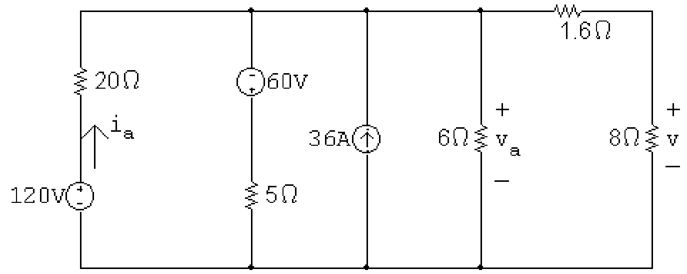
$$i_1=9\,\text{A}; \qquad i_2=-6\,\text{A}; \qquad i_3=4\,\text{A}; \qquad i_x=9-4=5\,\text{A}$$

$$\therefore \quad v_{4\text{A}}=3(i_3-i_2)-4i_x=10\,\text{V}$$

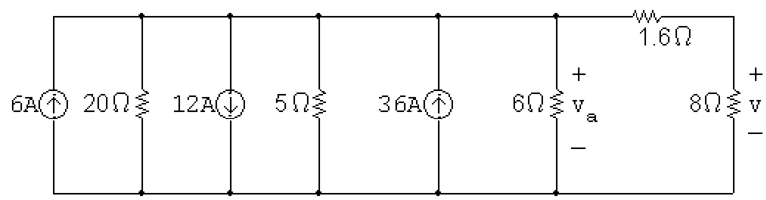
$$p_{4A}=-v_{4\text{A}}(4)=-(10)(4)=-40\,\text{W}$$

Thus, the 2 A current source delivers 40 W.

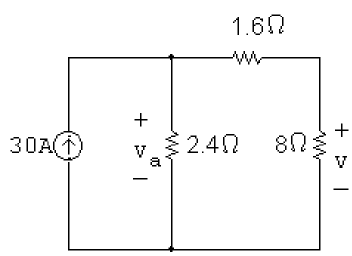
AP 4.15 [**a**] Redraw the circuit with a helpful voltage and current labeled:

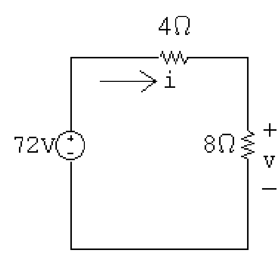


Transform the 120 V source in series with the 20 Ω resistor into a 6 A source in parallel with the 20 Ω resistor. Also transform the −60 V source in series with the 5 Ω resistor into a −12 A source in parallel with the 5 Ω resistor. The result is the following circuit:



Combine the three current sources into a single current source, using KCL, and combine the 20 Ω, 5 Ω, and 6 Ω resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 Ω resistor into a 72 V source in series with the 2.4 Ω resistor. Combine the 2.4 Ω resistor in series with the 1.6 Ω resisor to get a very simple circuit that still maintains the voltage v. The resulting circuit is on the right.





Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48\,\text{V}$$

[**b**] Calculate i in the circuit on the right using Ohm's law:

$$i = \frac{v}{8} = \frac{48}{8} = 6\,\text{A}$$

Now use i to calculate v_a in the circuit on the left:

$$v_\text{a} = 6(1.6 + 8) = 57.6\,\text{V}$$

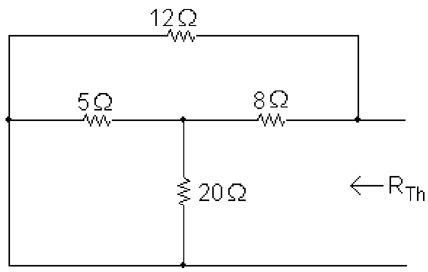
Returning back to the original circuit, note that the voltage v_a is also the voltage drop across the series combination of the 120 V source and 20 Ω resistor. Use this fact to calculate the current in the 120 V source, i_a:

$$i_a = \frac{120 - v_a}{20} = \frac{120 - 57.6}{20} = 3.12\,\text{A}$$

$$p_{120V} = -(120)i_a = -(120)(3.12) = -374.40\,\text{W}$$

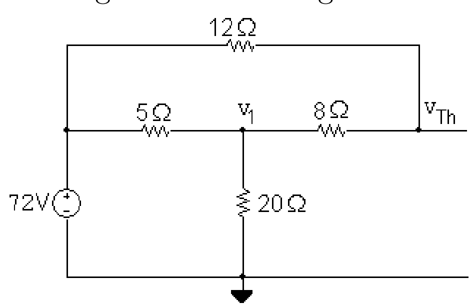
Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find R_{Th}, replace the 72 V source with a short circuit:



Note that the $5\,\Omega$ and $20\,\Omega$ resistors are in parallel, with an equivalent resistance of $5\|20 = 4\,\Omega$. The equivalent $4\,\Omega$ resistance is in series with the $8\,\Omega$ resistor for an equivalent resistance of $4 + 8 = 12\,\Omega$. Finally, the $12\,\Omega$ equivalent resistance is in parallel with the $12\,\Omega$ resistor, so $R_{Th} = 12\|12 = 6\,\Omega$.

Use node voltage analysis to find v_{Th}. Begin by redrawing the circuit and labeling the node voltages:



The node voltage equations are

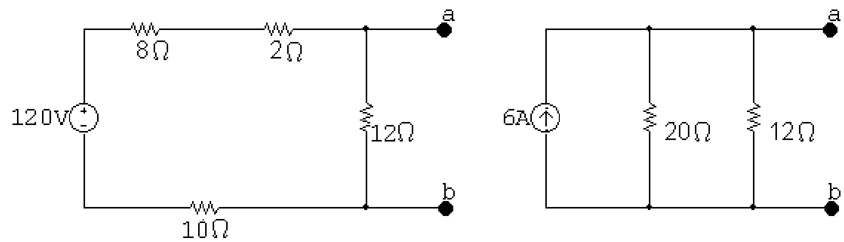
$$\begin{aligned} \frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{Th}}{8} &= 0 \\ \frac{v_{Th} - v_1}{8} + \frac{v_{Th} - 72}{12} &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{5} + \frac{1}{20} + \frac{1}{8}\right) &+ v_{Th}\left(-\frac{1}{8}\right) &= \frac{72}{5} \\ v_1\left(-\frac{1}{8}\right) &+ v_{Th}\left(\frac{1}{8} + \frac{1}{12}\right) &= 6 \end{aligned}$$

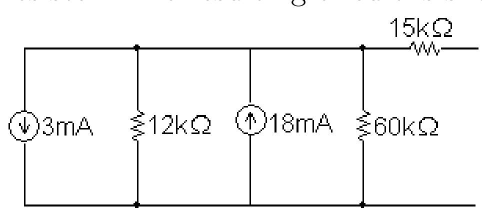
Solving, $v_1 = 60$ V and $v_{Th} = 64.8$ V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a $6\,\Omega$ resistor.

AP 4.17 We begin by performing a source transformation, turning the parallel combination of the 15 A source and $8\,\Omega$ resistor into a series combination of a 120 V source and an $8\,\Omega$ resistor, as shown in the figure on the left. Next, combine the $2\,\Omega$, $8\,\Omega$ and $10\,\Omega$ resistors in series to give an equivalent $20\,\Omega$ resistance. Then transform the series combination of the 120 V source and the $20\,\Omega$ equivalent resistance into a parallel combination of a 6 A source and a $20\,\Omega$ resistor, as shown in the figure on the right.

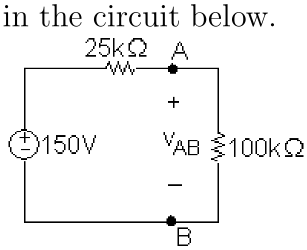


Finally, combine the $20\,\Omega$ and $12\,\Omega$ parallel resistors to give $R_{\rm N} = 20\|12 = 7.5\,\Omega$. Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a $7.5\,\Omega$ resistor.

AP 4.18 Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the -36 V source and 12 kΩ resistor into a parallel combination of a -3 mA source and 12 kΩ resistor. The resulting circuit is shown below:



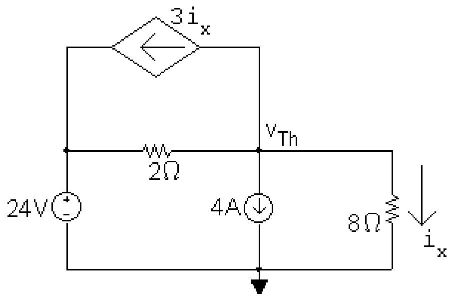
Now combine the two parallel current sources and the two parallel resistors to give a $-3 + 18 = 15$ mA source in parallel with a 12 k$\|$60 k= 10 kΩ resistor. Then transform the 15 mA source in parallel with the 10 kΩ resistor into a 150 V source in series with a 10 kΩ resistor, and combine this 10 kΩ resistor in series with the 15 kΩ resistor. The Thévenin equivalent is thus a 150 V source in series with a 25 kΩ resistor, as seen to the left of the terminals A,B in the circuit below.



Now attach the voltmeter, modeled as a 100 kΩ resistor, to the Thévenin equivalent and use voltage division to calculate the meter reading $v_{\rm AB}$:

$$v_{\rm AB} = \frac{100{,}000}{125{,}000}(150) = 120\,{\rm V}$$

AP 4.19 Begin by calculating the open circuit voltage, which is also $v_{\rm Th}$, from the circuit below:



Summing the currents away from the node labeled $v_{\rm Th}$ We have

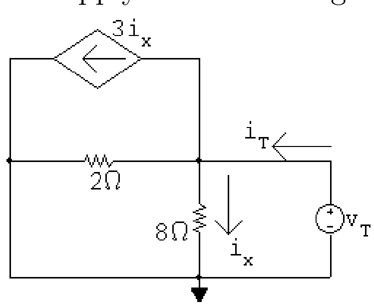
$$\frac{v_{\rm Th}}{8} + 4 + 3i_x + \frac{v_{\rm Th} - 24}{2} = 0$$

Also, using Ohm's law for the $8\,\Omega$ resistor,

$$i_x = \frac{v_{\rm Th}}{8}$$

Substituting the second equation into the first and solving for $v_{\rm Th}$ yields $v_{\rm Th} = 8\,\mathrm{V}$.

Now calculate $R_{\rm Th}$. To do this, we use the test source method. Replace the voltage source with a short circuit, the current source with an open circuit, and apply the test voltage $v_{\rm T}$, as shown in the circuit below:



Write a KCL equation at the middle node:

$$i_{\rm T} = i_x + 3i_x + v_{\rm T}/2 = 4i_x + v_{\rm T}/2$$

Use Ohm's law to determine i_x as a function of $v_{\rm T}$:

$$i_x = v_{\rm T}/8$$

Substitute the second equation into the first equation:

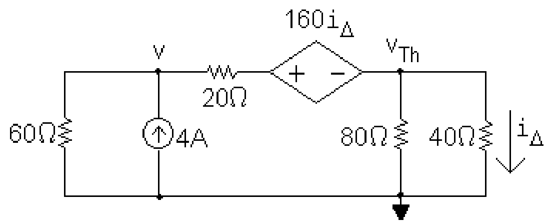
$$i_{\rm T} = 4(v_{\rm T}/8) + v_{\rm T}/2 = v_{\rm T}$$

Thus,

$R_{\mathrm{Th}} = v_{\mathrm{T}}/i_{\mathrm{T}} = 1\,\Omega$

The Thévenin equivalent is an 8 V source in series with a $1\,\Omega$ resistor.

AP 4.20 Begin by calculating the open circuit voltage, which is also v_{Th}, using the node voltage method in the circuit below:



The node voltage equations are

$$\begin{aligned}\frac{v}{60} + \frac{v - (v_{\mathrm{Th}} + 160 i_\Delta)}{20} - 4 &= 0,\\ \frac{v_{\mathrm{Th}}}{40} + \frac{v_{\mathrm{Th}}}{80} + \frac{v_{\mathrm{Th}} + 160 i_\Delta - v}{20} &= 0\end{aligned}$$

The dependent source constraint equation is

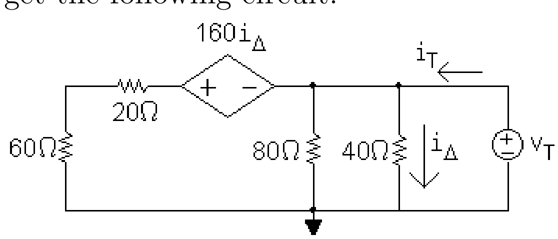
$$i_\Delta = \frac{v_{\mathrm{Th}}}{40}$$

Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

$$\begin{aligned} v\left(\frac{1}{60} + \frac{1}{20}\right) \quad &+ \quad v_{\mathrm{Th}}\left(-\frac{5}{20}\right) &= 4\\ v\left(-\frac{1}{20}\right) \quad &+ \quad v_{\mathrm{Th}}\left(\frac{1}{40} + \frac{1}{80} + \frac{5}{20}\right) &= 0\end{aligned}$$

Solving, $v = 172.5$ V and $v_{\mathrm{Th}} = 30$ V.

Now use the test source method to calculate the test current and thus R_{Th}. Replace the current source with a short circuit and apply the test source to get the following circuit:



Write a KCL equation at the rightmost node:

$$i_{\mathrm{T}} = \frac{v_{\mathrm{T}}}{80} + \frac{v_{\mathrm{T}}}{40} + \frac{v_{\mathrm{T}} + 160 i_\Delta}{80}$$

The dependent source constraint equation is

$$i_\Delta = \frac{v_\mathrm{T}}{40}$$

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

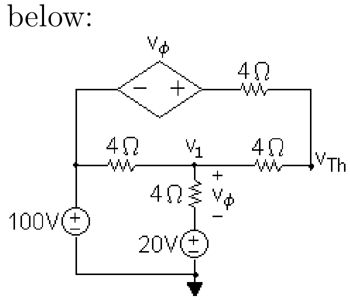
$$i_\mathrm{T} = \frac{v_\mathrm{T}}{10}$$

Therefore,

$$R_\mathrm{Th} = \frac{v_\mathrm{T}}{i_\mathrm{T}} = 10\,\Omega$$

Thus, the Thévenin equivalent is a 30 V source in series with a $10\,\Omega$ resistor.

AP 4.21 First find the Thévenin equivalent circuit. To find v_Th, create an open circuit between nodes a and b and use the node voltage method with the circuit below:



The node voltage equations are:

$$\begin{aligned}
\frac{v_\mathrm{Th} - (100 + v_\phi)}{4} + \frac{v_\mathrm{Th} - v_1}{4} &= 0 \\
\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_\mathrm{Th}}{4} &= 0
\end{aligned}$$

The dependent source constraint equation is

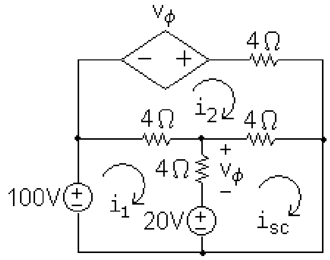
$$v_\phi = v_1 - 20$$

Place these three equations in standard form:

$$\begin{aligned}
v_\mathrm{Th}\left(\frac{1}{4} + \frac{1}{4}\right) &+ v_1\left(-\frac{1}{4}\right) &+ v_\phi\left(-\frac{1}{4}\right) &= 25 \\
v_\mathrm{Th}\left(-\frac{1}{4}\right) &+ v_1\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) &+ v_\phi\,(0) &= 30 \\
v_\mathrm{Th}\,(0) &+ v_1\,(1) &+ v_\phi\,(-1) &= 20
\end{aligned}$$

Solving, $v_\mathrm{Th} = 120$ V, $v_1 = 80$ V, and $v_\phi = 60$ V.

Now create a short circuit between nodes a and b and use the mesh current method with the circuit below:



The mesh current equations are

$$\begin{aligned}
-100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\
-v_\phi + 4i_2 + 4(i_2 - i_{\rm sc}) + 4(i_2 - i_1) &= 0 \\
-20 - v_\phi + 4(i_{\rm sc} - i_2) &= 0
\end{aligned}$$

The dependent source constraint equation is

$v_\phi = 4(i_1 - i_{\rm sc})$

Place these four equations in standard form:

$$\begin{aligned}
4i_1 - 4i_2 + 0i_{\rm sc} + v_\phi &= 80 \\
-4i_1 + 12i_2 - 4i_{\rm sc} - v_\phi &= 0 \\
0i_1 - 4i_2 + 4i_{\rm sc} - v_\phi &= 20 \\
4i_1 + 0i_2 - 4i_{\rm sc} - v_\phi &= 0
\end{aligned}$$

Solving, $i_1 = 45$ A, $i_2 = 30$ A, $i_{\rm sc} = 40$ A, and $v_\phi = 20$ V. Thus,

$$R_{\rm Th} = \frac{v_{\rm Th}}{i_{\rm sc}} = \frac{120}{40} = 3\,\Omega$$

[**a**] For maximum power transfer, $R = R_{\rm Th} = 3\,\Omega$

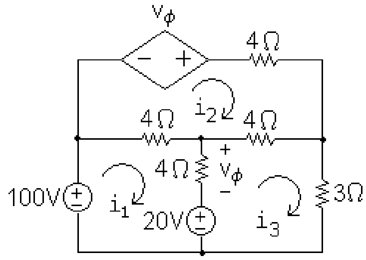
[**b**] The Thévenin voltage, $v_{\rm Th} = 120$ V, splits equally between the Thévenin resistance and the load resistance, so

$$v_{\rm load} = \frac{120}{2} = 60\,\text{V}$$

Therefore,

$$p_{\rm max} = \frac{v_{\rm load}^2}{R_{\rm load}} = \frac{60^2}{3} = 1200\,\text{W}$$

AP 4.22 Sustituting the value $R = 3\,\Omega$ into the circuit and identifying three mesh currents we have the circuit below:



The mesh current equations are:

$$\begin{aligned} -100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\ -v_\phi + 4i_2 + 4(i_2 - i_3) + 4(i_2 - i_1) &= 0 \\ -20 - v_\phi + 4(i_3 - i_2) + 3i_3 &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_3)$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_3 + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_3 - v_\phi &= 0 \\ 0i_1 - 4i_2 + 7i_3 - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_3 - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 30$ A, $i_2 = 20$ A, $i_3 = 20$ A, and $v_\phi = 40$ V.

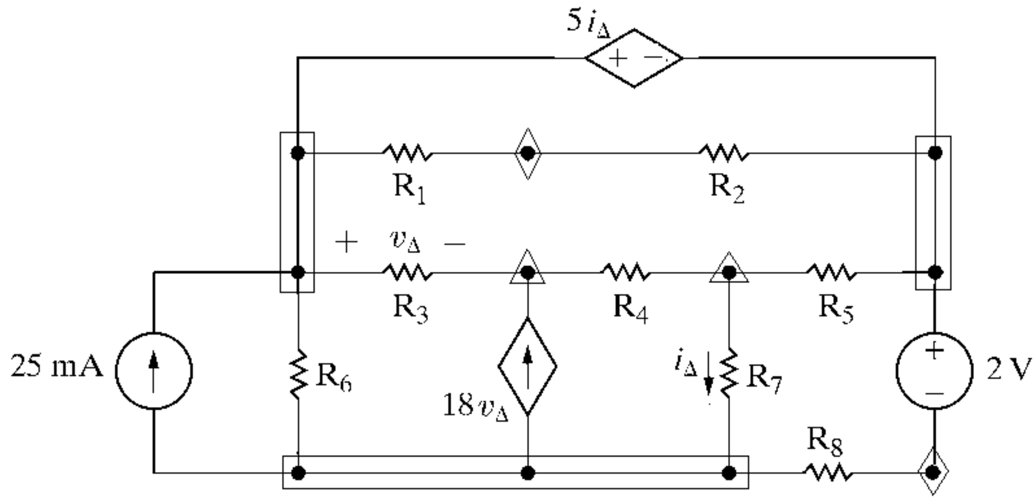
[a] $p_{100\text{V}} = -(100)i_1 = -(100)(30) = -3000$ W. Thus, the 100 V source is delivering 3000 W.

[b] $p_{\text{depsource}} = -v_\phi i_2 = -(40)(20) = -800$ W. Thus, the dependent source is delivering 800 W.

[c] From Assessment Problem 4.21(b), the power delivered to the load resistor is 1200 W, so the load power is $(1200/3800)100 = 31.58\%$ of the combined power generated by the 100 V source and the dependent source.

Problems

P 4.1



[**a**] 12 branches, 8 branches with resistors, 2 branches with independent sources, 2 branches with dependent sources.

[**b**] The current is unknown in every branch except the one containing the 25 mA current source, so the current is unknown in 11 branches.

[**c**] 10 essential branches – $R_1 - R_2$ forms an essential branch as does $R_8 - 2$ V. The remaining eight branches are essential branches that contain a single element.

[**d**] The current is known only in the essential branch containing the current source, and is unknown in the remaining 9 essential branches

[**e**] From the figure there are 7 nodes – three identified by rectangular boxes, two identified by triangles, and two identified by diamonds.

[**f**] There are 5 essential nodes, three identified with rectangular boxes and two identified with triangles.

[**g**] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

P 4.2 [**a**] From Problem 4.1(d) there are 9 essential branches where the current is unknown, so we need 9 simultaneous equations to describe the circuit.

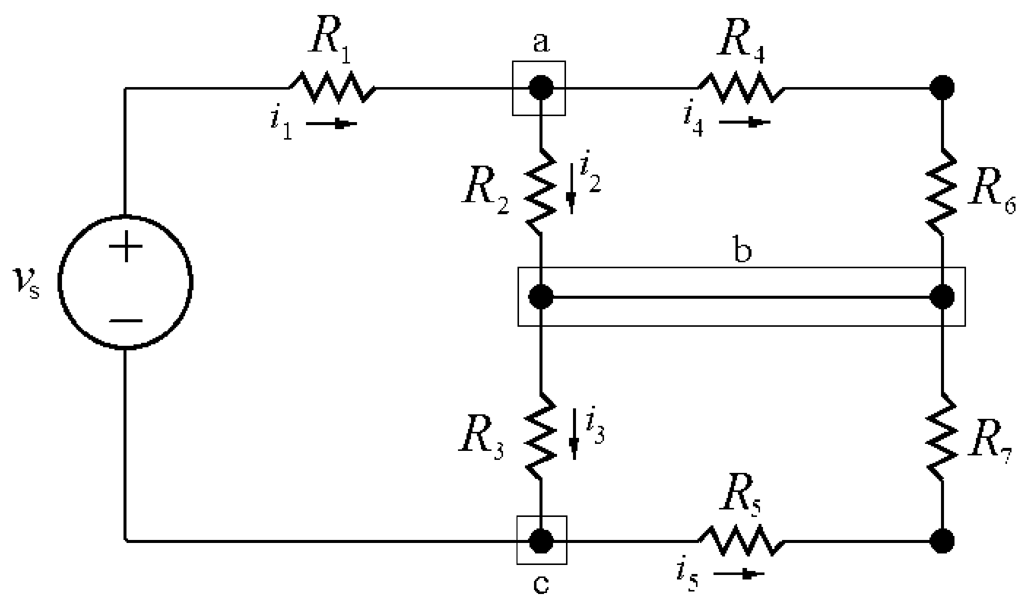
[**b**] From Problem 4.1(f), there are 5 essential nodes, so we can apply KCL at $(5 - 1) = 4$ of these essential nodes. There would also be a dependent source constraint equation.

[**c**] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

[d] We must avoid using the bottom left-most mesh, since it contains a current source, and we have no way of determining the voltage drop across a current source. The two meshes on the bottom that share the dependent source must be handled in a special way.

P 4.3 **[a]** There are eight circuit components, seven resistors and the voltage source. Therefore there are **eight** unknown currents. However, the voltage source and the R_1 resistor are in series, so have the same current. The R_4 and R_6 resistors are also in series, so have the same current. The R_5 and R_7 resistors are in series, so have the same current. Therefore, we only need 5 equations to find the 5 distinct currents in this circuit.

[b]



There are three essential nodes in this circuit, identified by the boxes. At two of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the third node would be dependent on the first two. Therefore there are **two** independent KCL equations.

[c] Sum the currents at any two of the three essential nodes a, b, and c. Using nodes a and c we get

$$-i_1 + i_2 + i_4 = 0$$

$$i_1 - i_3 + i_5 = 0$$

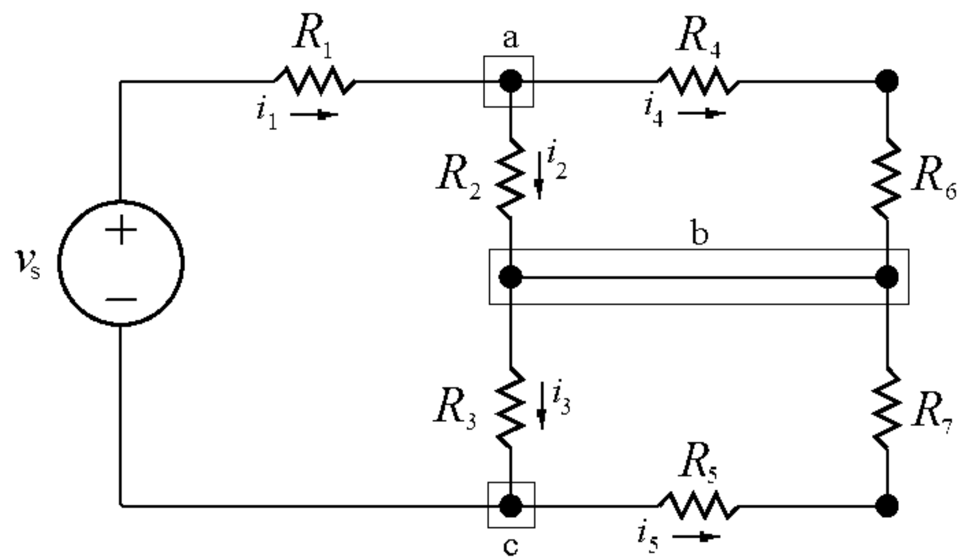
[d] There are three meshes in this circuit: one on the left with the components v_s, R_1, R_2 and R_3; one on the top right with components R_2, R_4, and R_6; and one on the bottom right with components R_3, R_5, and R_7. We can write KVL equations for all three meshes, giving a total of **three** independent KVL equations.

[e] $-v_s + R_1 i_1 + R_2 i_2 + R_3 i_3 = 0$

$R_4 i_4 + R_6 i_4 - R_2 i_2 = 0$

$R_3 i_3 + R_5 i_5 + R_7 i_5 = 0$

P 4.4



[a] At node a: $\quad - i_1 + i_2 + i_4 = 0$

At node b: $\quad - i_2 + i_3 - i_4 - i_5 = 0$

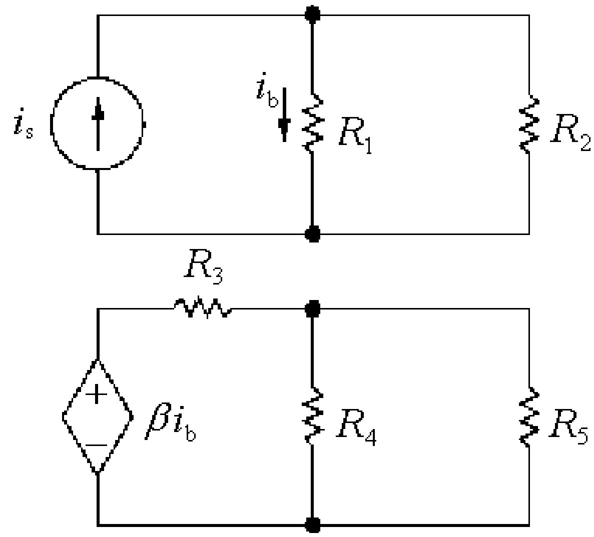
At node c: $\quad i_1 - i_3 + i_5 = 0$

[b] There are many possible solutions. For example, adding the equations at nodes a and c gives the equation at node b:

$(-i_1 + i_2 + i_4) + (i_1 - i_3 + i_5) = 0 \qquad \text{so} \qquad i_2 - i_3 + i_4 + i_5 = 0$

This is the equation at node b with both sides multiplied by -1.

P 4.5



[a] As can be seen from the figure, the circuit has 2 separate parts.

[b] There are 5 nodes – the four black dots and the node betweem the voltage source and the resistor R_3.

[c] There are 7 branches, each containing one of the seven circuit components.

[d] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

P 4.6 Use the lower terminal of the 25 Ω resistor as the reference node.

$$\frac{v_o - 24}{20 + 80} + \frac{v_o}{25} + 0.04 = 0$$

Solving, $\qquad v_o = 4\,\text{V}$

P 4.7 **[a]** From the solution to Problem 4.6 we know $v_o = 4$ V, therefore

$$p_{40\text{mA}} = 0.04 v_o = 0.16\,\text{W}$$

$$\therefore \; p_{40\text{mA}} \text{ (developed) } = -160 \text{ mW}$$

[b] The current into the negative terminal of the 24 V source is

$$i_g = \frac{24 - 4}{20 + 80} = 0.2\,\text{A}$$

$$p_{24\text{V}} = -24(0.2) = -4.8\,\text{W}$$

$$\therefore \; p_{24\text{V}} \text{ (developed) } = 4800 \text{ mW}$$

[c] $p_{20\Omega} = (0.2)^2(20) = 800$ mW

$p_{80\Omega} = (0.2)^2(80) = 3200$ mW

$p_{25\Omega} = (4)^2/25 = 640$ mW

$\sum p_{\rm dev} = 4800$ mW

$\sum p_{\rm dis} = 160 + 800 + 3200 + 640 = 4800$ mW

P 4.8 [a] $\dfrac{v_0 - 24}{20+80} + \dfrac{v_o}{25} + 0.04 = 0; \qquad v_o = 4\,\text{V}$

[b] Let v_x = voltage drop across 40 mA source

$v_x = v_o - (50)(0.04) = 2\,\text{V}$

$p_{40\text{mA}} = (2)(0.04) = 80$ mW so $p_{40\text{mA}}$ (developed) $= -80$ mW

[c] Let i_g = be the current into the positive terminal of the 24 V source

$i_g = (4 - 24)/100 = -0.2\,\text{A}$

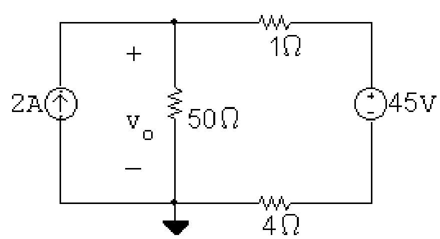
$p_{24\text{V}} = (-0.2)(24) = -4800$ mW so $p_{24\text{V}}$ (developed) $= 4800$ mW

[d] $\sum p_{\rm dis} = (0.2)^2(20) + (0.2)^2(80) + (4)^2/25 + (0.04)^2(50) + 0.08$

$= 4800$ mW

[e] v_o is independent of any finite resistance connected in series with the 40 mA current source

P 4.9



$$-2 + \frac{v_o}{50} + \frac{v_o - 45}{1+4} = 0$$

$v_o = 50\,\text{V}$

$p_{2\text{A}} = -(50)(2) = -100\,\text{W}$ (delivering)

The 2 A source extracts −100 W from the circuit, because it delivers 100 W to the circuit.

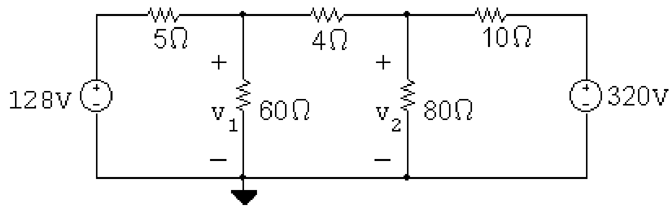
P 4.10 **[a]** $\dfrac{v_o - v_1}{R} + \dfrac{v_o - v_2}{R} + \dfrac{v_o - v_3}{R} + \cdots + \dfrac{v_o - v_n}{R} = 0$

$$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$$

$$\therefore \quad v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n}\sum_{k=1}^{n} v_k$$

[b] $v_o = \dfrac{1}{3}(100 + 80 - 60) = 40\,\text{V}$

P 4.11 **[a]**



$$\frac{v_1 - 128}{5} + \frac{v_1}{60} + \frac{v_1 - v_2}{4} = 0$$
$$\frac{v_2 - v_1}{4} + \frac{v_2}{80} + \frac{v_2 - 320}{10} = 0$$

In standard form,

$$v_1\left(\frac{1}{5} + \frac{1}{60} + \frac{1}{4}\right) + v_2\left(-\frac{1}{4}\right) = \frac{128}{5}$$
$$v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{1}{4} + \frac{1}{80} + \frac{1}{10}\right) = \frac{320}{10}$$

Solving, $v_1 = 162\,\text{V}$; $\quad v_2 = 200\,\text{V}$

$$i_\text{a} = \frac{128 - 162}{5} = -6.8\,\text{A}$$

$$i_\text{b} = \frac{162}{60} = 2.7\,\text{A}$$

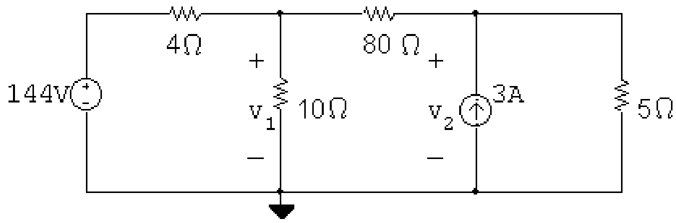
$$i_\text{c} = \frac{162 - 200}{4} = -9.5\,\text{A}$$

$$i_\text{d} = \frac{200}{80} = 2.5\,\text{A}$$

$$i_\text{e} = \frac{200 - 320}{10} = -12\,\text{A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4$ W (abs)
$p_{320V} = (320)(-12) = -3840$ W (dev)
Therefore, the total power developed is 3840 W.

P 4.12



$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \qquad \text{so} \qquad 29v_1 - v_2 = 2880$$

$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \qquad \text{so} \qquad -v_1 + 17v_2 = 240$$

Solving, $v_1 = 100\,\text{V}$; $\qquad v_2 = 20\,\text{V}$

P 4.13 $-6 + \dfrac{v_1}{40} + \dfrac{v_1 - v_2}{8} = 0$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$

Solving, $v_1 = 120$ V; $\qquad v_2 = 96\,\text{V}$

CHECK:

$$p_{40\Omega} = \frac{(120)^2}{40} = 360\,\text{W}$$

$$p_{8\Omega} = \frac{(120 - 96)^2}{8} = 72\,\text{W}$$

$$p_{80\Omega} = \frac{(96)^2}{80} = 115.2\,\text{W}$$

$$p_{120\Omega} = \frac{(96)^2}{120} = 76.8\,\text{W}$$

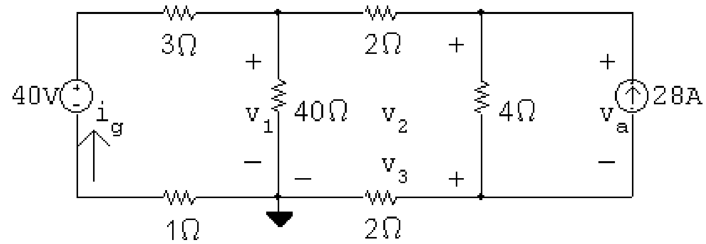
$$p_{6\text{A}} = -(6)(120) = -720\,\text{W}$$

$$p_{1\text{A}} = (1)(96) = 96\,\text{W}$$

$$\sum p_{\text{abs}} = 360 + 72 + 115.2 + 76.8 + 96 = 720\,\text{W}$$

$$\sum p_{\text{dev}} = 720\,\text{W} \quad \text{(CHECKS)}$$

P 4.14 [**a**]



$$\frac{v_1}{40} + \frac{v_1 - 40}{4} + \frac{v_1 - v_2}{2} = 0 \qquad \text{so} \qquad 31v_1 - 20v_2 + 0v_3 = 400$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{4} - 28 = 0 \qquad \text{so} \qquad -2v_1 + 3v_2 - v_3 = 112$$

$$\frac{v_3}{2} + \frac{v_3 - v_2}{4} + 28 = 0 \qquad \text{so} \qquad 0v_1 - v_2 + 3v_3 = -112$$

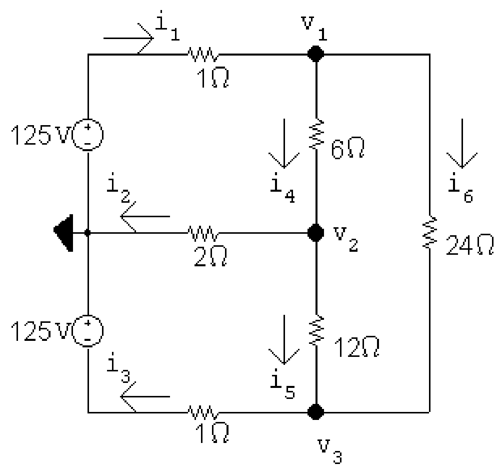
Solving, $v_1 = 60\,\text{V}$; $v_2 = 73\,\text{V}$; $v_3 = -13\,\text{V}$,

[**b**] $i_g = \dfrac{40 - 60}{4} = -5\,\text{A}$

$p_g = (40)(-5) = -200\,\text{W}$

Thus the 40 V source delivers 200 W of power.

P 4.15 [**a**]



$$\frac{v_1 - 125}{1} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} = 0$$

$$\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{12} = 0$$

$$\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} = 0$$

In standard form:

$$
\begin{aligned}
v_1\left(\frac{1}{1}+\frac{1}{6}+\frac{1}{24}\right)+v_2\left(-\frac{1}{6}\right)+v_3\left(-\frac{1}{24}\right) &= 125\\
v_1\left(-\frac{1}{6}\right)+v_2\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{12}\right)+v_3\left(-\frac{1}{12}\right) &= 0\\
v_1\left(-\frac{1}{24}\right)+v_2\left(-\frac{1}{12}\right)+v_3\left(\frac{1}{1}+\frac{1}{12}+\frac{1}{24}\right) &= -125
\end{aligned}
$$

Solving, $v_1 = 101.24\,\text{V};\quad v_2 = 10.66\,\text{V};\quad v_3 = -106.57\,\text{V}$

Thus, $i_1 = \dfrac{125 - v_1}{1} = 23.76\ \text{A} \qquad i_4 = \dfrac{v_1 - v_2}{6} = 15.10\,\text{A}$

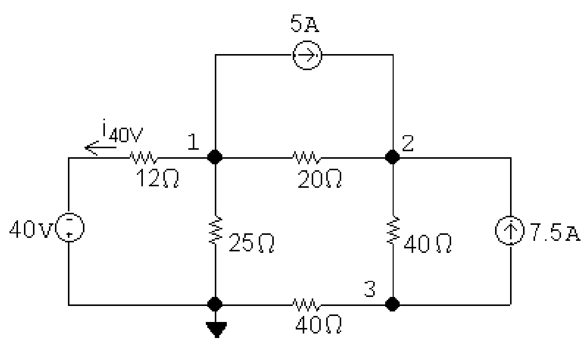
$i_2 = \dfrac{v_2}{2} = 5.33\ \text{A} \qquad i_5 = \dfrac{v_2 - v_3}{12} = 9.77\,\text{A}$

$i_3 = \dfrac{v_3 + 125}{1} = 18.43\ \text{A} \qquad i_6 = \dfrac{v_1 - v_3}{24} = 8.66\,\text{A}$

[b] $\sum P_{\text{dev}} = 125 i_1 + 125 i_3 = 5273.09\,\text{W}$

$\sum P_{\text{dis}} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09\,\text{W}$

P 4.16



$$\frac{v_1 + 40}{12} + \frac{v_1}{25} + \frac{v_1 - v_2}{20} + 5 = 0$$

$$\left[\frac{v_2 - v_1}{20}\right] - 5 + \frac{v_2 - v_1}{40} + -7.5 = 0$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 = 0$$

Solving, $v_1 = -10\,\text{V};\quad v_2 = 132\,\text{V};\quad v_3 = -84\,\text{V};\quad i_{40\text{V}} = \dfrac{-10 + 40}{12} = 2.5\,\text{A}$

$p_{5\text{A}} = 5(v_1 - v_2) = 5(-10 - 132) = -710\,\text{W} \quad \text{(del)}$

$$p_{7.5\text{A}} = (-84 - 132)(7.5) = -1620\,\text{W} \quad \text{(del)}$$

$$p_{40\text{V}} = -(40)(2.5) = -100\,\text{W} \quad \text{(del)}$$

$$p_{12\Omega} = (2.5)^2(12) = 75\,\text{W}$$

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{10^2}{25} = 4\,\text{W}$$

$$p_{20\Omega} = \frac{(v_1 - v_2)^2}{20} = \frac{142^2}{20} = 1008.2\,\text{W}$$

$$p_{40\Omega}(\text{lower}) = \frac{(v_3)^2}{40} = \frac{84^2}{40} = 176.4\,\text{W}$$

$$p_{40\Omega}(\text{right}) = \frac{(v_2 - v_3)^2}{40} = \frac{216^2}{40} = 1166.4\,\text{W}$$

$$\sum p_{\text{diss}} = 75 + 4 + 1008.2 + 176.4 + 1166.4 = 2430\,\text{W}$$

$$\sum p_{\text{dev}} = 710 + 1620 + 100 = 2430\,\text{W} \qquad \text{(CHECKS)}$$

The total power dissipated in the circuit is 2430 W.

P 4.17 $-3 + \dfrac{v_o}{200} + \dfrac{v_o + 5i_\Delta}{10} + \dfrac{v_o - 80}{20} = 0; \quad i_\Delta = \dfrac{v_o - 80}{20}$

[a] Solving, $v_o = 50$ V

[b] $i_{\text{ds}} = \dfrac{v_o + 5i_\Delta}{10}$

$$i_\Delta = (50 - 80)/20 = -1.5\,\text{A}$$

$$\therefore \quad i_{\text{ds}} = 4.25\,\text{A}; \quad 5i_\Delta = -7.5\,\text{V} : \quad p_{\text{ds}} = (-5i_\Delta)(i_{\text{ds}}) = 31.875\,\text{W}$$

[c] $p_{3\text{A}} = -3v_o = -3(50) = -150\,\text{W} \quad \text{(del)}$

$$p_{80\text{V}} = 80i_\Delta = 80(-1.5) = -120\,\text{W} \quad \text{(del)}$$

$$\sum p_{\text{del}} = 150 + 120 = 270\,\text{W}$$

CHECK:

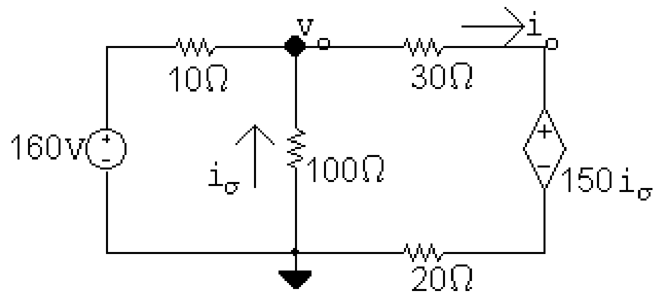
$$p_{200\Omega} = 2500/200 = 12.5\,\text{W}$$

$$p_{20\Omega} = (80 - 50)^2/20 = 900/20 = 45\,\text{W}$$

$$p_{10\Omega} = (4.25)^2(10) = 180.625\,\text{W}$$

$$\sum p_{\text{diss}} = 31.875 + 180.625 + 12.5 + 45 = 270\,\text{W}$$

P 4.18



$$\frac{v_o - 160}{10} + \frac{v_o}{100} + \frac{v_o - 150i_\sigma}{50} = 0; \quad i_\sigma = -\frac{v_o}{100}$$

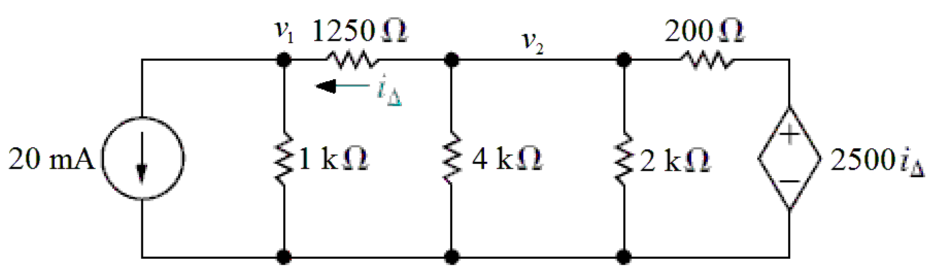
Solving, $v_o = 100\,\text{V}$; $\quad i_\sigma = -1\,\text{A}$

$$i_o = \frac{100 - (150)(-1)}{50} = 5\,\text{A}$$

$$p_{150i_\sigma} = 150 i_\sigma i_o = -750\,\text{W}$$

$\therefore$ The dependent voltage source delivers 750 W to the circuit.

P 4.19



[a] $0.02 + \dfrac{v_1}{1000} + \dfrac{v_1 - v_2}{1250} = 0$

$$\frac{v_2 - v_1}{1250} + \frac{v_2}{4000} + \frac{v_2}{2000} + \frac{v_2 - 2500i_\Delta}{200} = 0$$

$$i_\Delta = \frac{v_2 - v_1}{1250}$$

Solving,

$v_1 = 60\,\text{V}$; $\quad v_2 = 160\,\text{V}$; $\quad i_\Delta = 80$ mA

$$P_{20\text{mA}} = (0.02)v_1 = (0.02)(60) = 1.2\,\text{W (absorbed)}$$

$$i_{\text{ds}} = \frac{v_2 - 2500i_\Delta}{200} = \frac{160 - (2500)(0.08)}{200} = -0.2\,\text{A}$$

$$P_{\text{ds}} = (2500i_\Delta)i_{\text{ds}} = 2500(0.08)(-0.2) = -40\,\text{W (40 W developed)}$$

$$P_{\text{developed}} = 40\,\text{W}$$

[b] $P_{1\text{k}} = \dfrac{v_1^2}{1000} = \dfrac{60^2}{1000} = 3.6\,\text{W}$

$$P_{1250} = 1250 i_\Delta^2 = 1250(0.08)^2 = 8\,\text{W}$$

$$P_{4\text{k}} = \frac{v_2^2}{4000} = \frac{160^2}{4000} = 6.4\,\text{W}$$

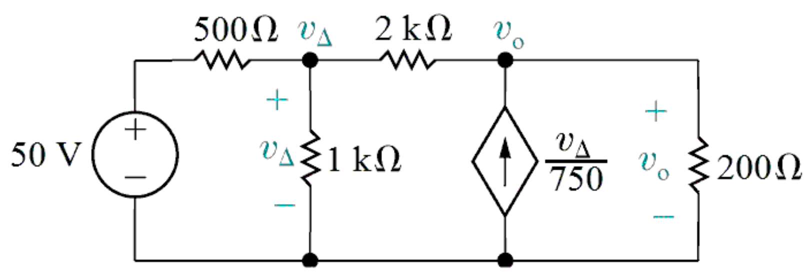
$$P_{2\text{k}} = \frac{v_2^2}{2000} = \frac{160^2}{2000} = 12.8\,\text{W}$$

$$P_{200} = 200 i_{\text{ds}}^2 = 200(-0.2)^2 = 8\,\text{W}$$

$$P_{\text{absorbed}} = P_{20\text{mA}} + P_{1\text{k}} + P_{1250} + P_{4\text{k}} + P_{2\text{k}} + P_{200}$$

$$= 1.2 + 3.6 + 8 + 6.4 + 12.8 + 8 = 40\,\text{W (check)}$$

P 4.20



[a] $\dfrac{v_\Delta - 50}{500} + \dfrac{v_\Delta}{1000} + \dfrac{v_\Delta - v_o}{2000} = 0$

$$\frac{v_o - v_\Delta}{2000} - \frac{v_\Delta}{750} + \frac{v_o}{200} = 0$$

Solving,

$v_\Delta = 30\,\text{V}; \qquad v_o = 10\,\text{V}$

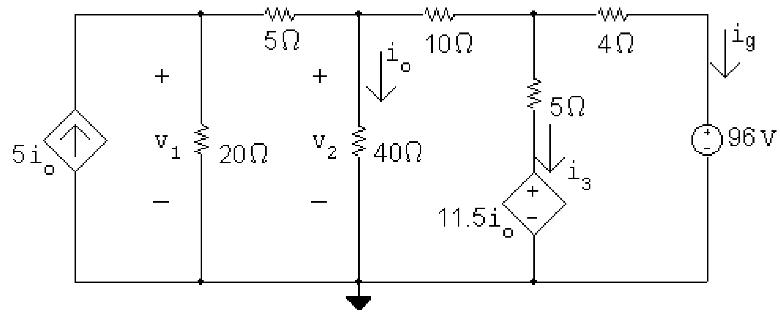
[b] $i_{50\text{V}} = \dfrac{v_\Delta - 50}{500} = \dfrac{30 - 50}{500} = -0.04\,\text{A}$

$P_{50\text{V}} = 50 i_{50\text{V}} = 50(-0.04) = -2\,\text{W}$ (2 W supplied)

$P_{\text{ds}} = -v_o\left(\dfrac{v_\Delta}{750}\right) = -(10)(30/750) = -0.4\,\text{W}$ (0.4 W supplied)

$P_{\text{total}} = 2 + 0.4 = 2.4\,\text{W}$ supplied

P 4.21 **[a]**



$$i_o = \frac{v_2}{40}$$

$$-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0 \qquad \text{so} \qquad 10v_1 - 13v_2 + 0v_3 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} \qquad \text{so} \qquad -8v_1 + 13v_2 - 4v_3 = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i_o}{5} + \frac{v_3 - 96}{4} = 0 \qquad \text{so} \qquad 0v_1 - 63v_2 + 220v_3 = 9600$$

Solving, $v_1 = 156\,\text{V}; \quad v_2 = 120\,\text{V}; \quad v_3 = 78\,\text{V}$

[b] $i_o = \dfrac{v_2}{40} = \dfrac{120}{40} = 3\,\text{A}$

$$i_3 = \frac{v_3 - 11.5i_o}{5} = \frac{78 - 11.5(3)}{5} = 8.7\,\text{A}$$

$$i_g = \frac{78 - 96}{4} = -4.5\,\text{A}$$

$$p_{5i_o} = -5i_ov_1 = -5(3)(156) = -2340\,\text{W(dev)}$$

$$p_{11.5i_o} = 11.5i_oi_3 = 11.5(3)(8.7) = 300.15\,\text{W(abs)}$$

$$p_{96V} = 96(-4.5) = -432\,\text{W(dev)}$$

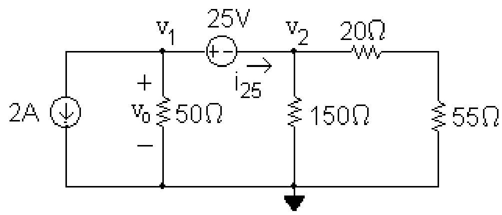
$$\sum p_{\text{dev}} = 2340 + 432 = 2772\,\text{W}$$

CHECK

$$\sum p_{\text{dis}} = \frac{156^2}{20} + \frac{(156-120)^2}{5} + \frac{120^2}{40} + \frac{(120-78)^2}{50}$$
$$+(8.7)^2(5) + (4.5)^2(4) + 300.15 = 2772\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{dis}} = 2772\,\text{W}$$

P 4.22 [**a**]



This circuit has a supernode includes the nodes v_1, v_2 and the 25 V source. The supernode equation is

$$2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{75} = 0$$

The supernode constraint equation is

$$v_1 - v_2 = 25$$

Place these two equations in standard form:

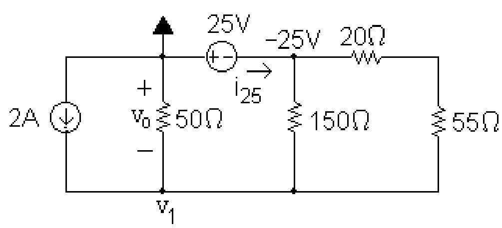
$$\begin{array}{lcccl} v_1\left(\dfrac{1}{50}\right) & + & v_2\left(\dfrac{1}{150} + \dfrac{1}{75}\right) & = & -2 \\ v_1(1) & + & v_2(-1) & = & 25 \end{array}$$

Solving, $v_1 = -37.5$ V and $v_2 = -62.5$ V, so $v_o = v_1 = -37.5$ V.

$$p_{2\text{A}} = (2)v_o = (2)(-37.5) = -75\,\text{W}$$

The 2 A source delivers 75 W.

[**b**]



This circuit now has only one non-reference essential node where the voltage is not known – note that it is not a supernode. The KCL equation at v_1 is

$$-2 + \frac{v_1}{50} + \frac{v_1 + 25}{150} + \frac{v_1 + 25}{75} = 0$$

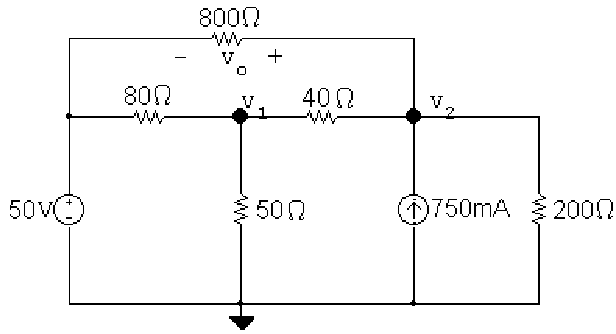
Solving, $v_1 = 37.5$ V so $v_0 = -v_1 = -37.5$ V.

$$p_{2\text{A}} = (2)v_o = (2)(-37.5) = -75\,\text{W}$$

The 2 A source delivers 75 W.

[**c**] The choice of a reference node in part (b) resulted in one simple KCL equation, while the choice of a reference node in part (a) resulted in a supernode KCL equation and a second supernode constraint equation. Both methods give the same result but the choice of reference node in part (b) yielded fewer equations to solve, so is the preferred method.

P 4.23



The two node voltage equations are:

$$\frac{v_1 - 50}{80} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} = 0$$

$$\frac{v_2 - v_1}{40} - 0.75 + \frac{v_2}{200} + \frac{v_2 - 50}{800} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{80} + \frac{1}{50} + \frac{1}{40}\right) + v_2\left(-\frac{1}{40}\right) = \frac{50}{80}$$

$$v_1\left(-\frac{1}{40}\right) + v_2\left(\frac{1}{40} + \frac{1}{200} + \frac{1}{800}\right) = 0.75 + \frac{50}{800}$$

Solving, $v_1 = 34\,\text{V}$; $\quad v_2 = 53.2\,\text{V}$.

Thus, $v_o = v_2 - 50 = 53.2 - 50 = 3.2\,\text{V}$.

POWER CHECK:

$$i_g = (50 - 34)/80 + (50 - 53.2)/800 = 196\,\text{mA}$$

$$p_{50\text{V}} = -(50)(0.196) = -9.8\,\text{W}$$

$$p_{80\Omega} = (50 - 34)^2/80 = 3.2\,\text{W}$$

$$p_{800\Omega} = (50 - 53.2)^2/800 = 12.8\,\text{mW}$$

$$p_{40\Omega} = (53.2 - 34)^2/40 = 9.216\,\text{W}$$

$$p_{50\Omega} = 34^2/50 = 23.12\,\text{W}$$

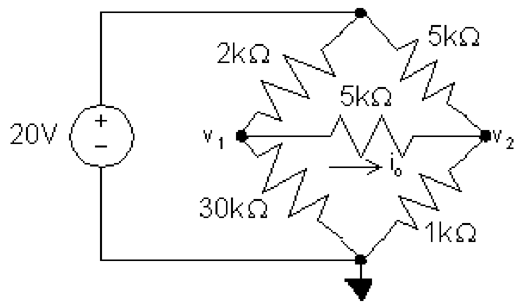
$$p_{200\Omega} = 53.2^2/200 = 14.1512\,\text{W}$$

$$p_{0.75\text{A}} = -(53.2)(0.75) = -39.9\,\text{W}$$

$$\sum p_{\text{abs}} = 3.2 + .0128 + 9.216 + 23.12 + 14.1512 = 49.7\,\text{W}$$

$$\sum p_{\text{del}} = 9.8 + 39.9 = 49.7 \text{ (check)}$$

P 4.24



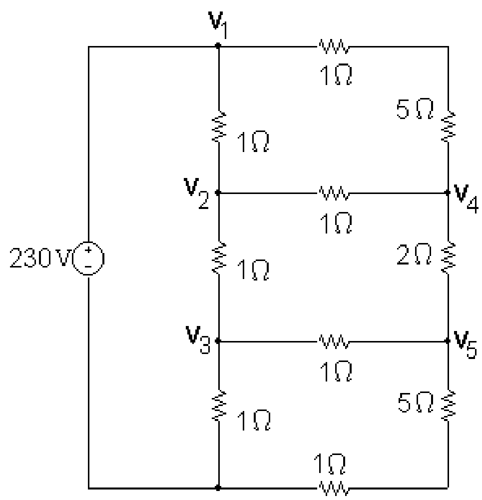
$$\frac{v_1}{30{,}000} + \frac{v_1 - v_2}{5000} + \frac{v_1 - 20}{2000} = 0 \qquad \text{so} \qquad 22v_1 - 6v_2 = 300$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 20}{5000} = 0 \qquad \text{so} \qquad -v_1 + 7v_2 = 20$$

Solving, $v_1 = 15\,\text{V}$; $\qquad v_2 = 5\,\text{V}$

Thus, $i_o = \dfrac{v_1 - v_2}{5000} = 2\ \text{mA}$

P 4.25 **[a]**



$$\frac{v_2 - 230}{1} + \frac{v_2 - v_4}{1} + \frac{v_2 - v_3}{1} = 0 \qquad \text{so} \qquad 3v_2 - 1v_3 - 1v_4 + 0v_5 = 230$$

$$\frac{v_3 - v_2}{1} + \frac{v_3}{1} + \frac{v_3 - v_5}{1} = 0 \qquad \text{so} \qquad -1v_2 + 3v_3 + 0v_4 - 1v_5 = 0$$

$$\frac{v_4 - v_2}{1} + \frac{v_4 - 230}{6} + \frac{v_4 - v_5}{2} = 0 \qquad \text{so} \qquad -12v_2 + 0v_3 + 20v_4 - 6v_5 = 460$$

$$\frac{v_5 - v_3}{1} + \frac{v_5}{6} + \frac{v_5 - v_4}{2} = 0 \qquad \text{so} \qquad 0v_2 - 12v_3 - 6v_4 + 20v_5 = 0$$

Solving, $v_2 = 150\,\text{V}$; $\quad v_3 = 80\,\text{V}$; $\quad v_4 = 140\,\text{V}$; $\quad v_5 = 90\,\text{V}$

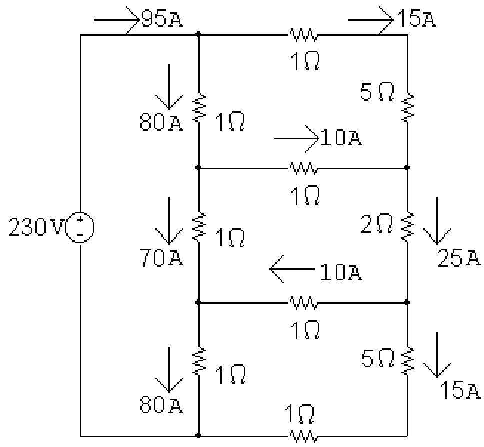
$$i_{2\Omega} = \frac{v_4 - v_5}{2} = \frac{140 - 90}{2} = 25\,\text{A}$$

$$p_{2\Omega} = (25)^2(2) = 1250\,\text{W}$$

[b]

$$i_{230\text{V}} = \frac{v_1 - v_2}{1} + \frac{v_1 - v_4}{6}$$
$$= \frac{230 - 150}{1} + \frac{230 - 140}{6} = 80 + 15 = 95 \text{ A}$$
$$p_{230\text{V}} = (230)(95) = 21{,}850\,\text{W}$$

Check:



$$\sum P_{\text{dis}} = (80)^2(1) + (70)^2(1) + (80)^2(1) + (15)^2(6) + (10)^2(1)$$
$$+(10)^2(1) + (25)^2(2) + (15)^2(6) = 21{,}850\,\text{W}$$

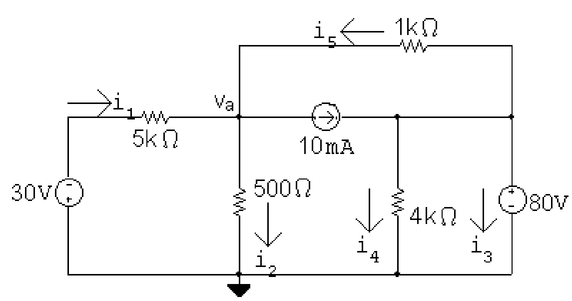
P 4.26 Place $5v_\Delta$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_\Delta - 15}{10} + \frac{v_\Delta}{2} + \frac{v_\Delta - 5v_\Delta}{20} + \frac{v_\Delta - 5v_\Delta}{40} = 0$$

$$12v_\Delta = 60; \qquad v_\Delta = 5\,\text{V}$$

$$v_o = v_\Delta - 5v_\Delta = -4(5) = -20\,\text{V}$$

P 4.27 **[a]**



There is only one node voltage equation:

$$\frac{v_\text{a} + 30}{5000} + \frac{v_\text{a}}{500} + \frac{v_\text{a} - 80}{1000} + 0.01 = 0$$

Solving,

$v_a + 30 + 10v_a + 5v_a - 400 + 50 = 0 \qquad \text{so} \qquad 16v_a = 320$

$\therefore \qquad v_a = 20\,\text{V}$

Calculate the currents:

$$\begin{aligned} i_1 &= (-30-20)/5000 = -10 \text{ mA} \\ i_2 &= 20/500 = 40 \text{ mA} \\ i_4 &= 80/4000 = 20 \text{ mA} \\ i_5 &= (80-20)/1000 = 60 \text{ mA} \end{aligned}$$

$i_3 + i_4 + i_5 - 10 \text{ mA} = 0 \quad \text{so} \quad i_3 = 0.01 - 0.02 - 0.06 = -0.07 = -70 \text{ mA}$

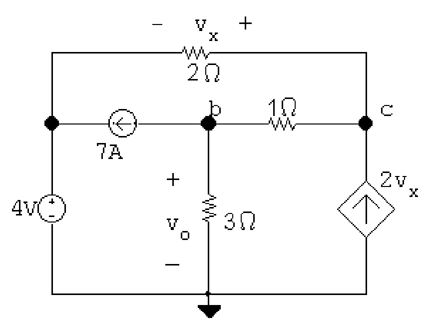
[b]

$$\begin{aligned} p_{30\text{V}} &= (30)(-0.01) = -0.3 \text{ W} \\ p_{10\text{mA}} &= (20-80)(0.01) = -0.6 \text{ W} \\ p_{80\text{V}} &= (80)(-0.07) = -5.6 \text{ W} \\ p_{5\text{k}} &= (-0.01)^2(5000) = 0.5 \text{ W} \\ p_{500\Omega} &= (0.04)^2(500) = 0.8 \text{ W} \\ p_{1\text{k}} &= (80-20)^2/(1000) = 3.6 \text{ W} \\ p_{4\text{k}} &= (80)^2/(4000) = 1.6 \text{ W} \end{aligned}$$

$\sum p_{\text{abs}} = 0.5 + 0.8 + 3.6 + 1.6 = 6.5\,\text{W}$

$\sum p_{\text{del}} = 0.3 + 0.6 + 5.6 = 6.5 \text{ W (checks!)}$

P 4.28



The two node voltage equations are:

$$\begin{aligned} 7 + \frac{v_b}{3} + \frac{v_b - v_c}{1} &= 0 \\ -2v_x + \frac{v_c - v_b}{1} + \frac{v_c - 4}{2} &= 0 \end{aligned}$$

The constraint equation for the dependent source is:

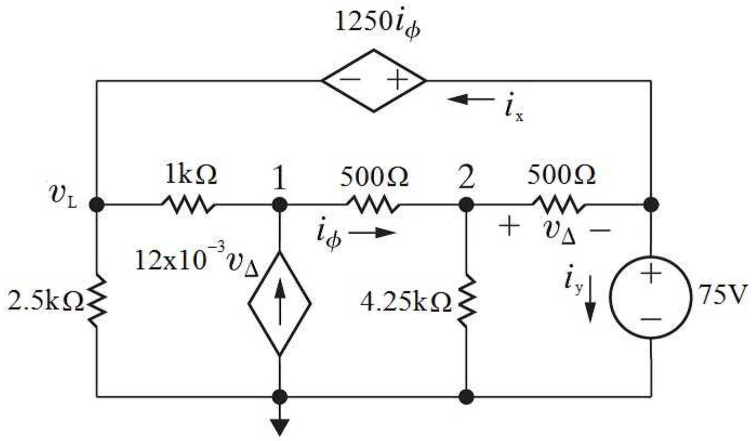
$v_x = v_c - 4$

Place these equations in standard form:

$$
\begin{array}{lclclcl}
v_{\rm b}\left(\frac{1}{3}+1\right) & + & v_{\rm c}(-1) & + & v_x(0) & = & -7 \\
v_{\rm b}(-1) & + & v_{\rm c}\left(1+\frac{1}{2}\right) & + & v_x(-2) & = & \frac{4}{2} \\
v_{\rm b}(0) & + & v_{\rm c}(1) & + & v_x(-1) & = & 4
\end{array}
$$

Solving, $v_{\rm c} = 9\,\text{V}, v_x = 5\,\text{V},$ and $v_o = v_{\rm b} = 1.5\,\text{V}$

P 4.29



[**a**] The left-most node voltage is $75 - 1250i_\phi$. The right-most node voltage is 75 V. Write KCL equations at the essential nodes labeled 1 and 2.

[**b**] From the values given,

$$i_\phi = \frac{v_1 - v_2}{500} = \frac{105 - 85}{500} = 0.04\,\text{A}$$

$$v_\Delta = v_2 - 75 = 85 - 75 = 10 \text{ V}$$

$$v_{\rm L} = 75 - (1250)(0.04) = 25 \text{ V}$$

$$i_{\rm x} = \frac{v_{\rm L} - v_1}{1000} + \frac{v_{\rm L}}{2500} = \frac{25 - 105}{1000} + \frac{25}{2500} = -0.07\,\text{A}$$

$$i_{\rm y} = \frac{v_2 - 75}{500} - i_{\rm x} = \frac{85 - 75}{500} + 0.07 = 0.09\,\text{A}$$

Calculate the total power:

$$P_{\rm dstop} = 1250i_\phi(i_{\rm x}) = 1250(0.04)(-0.07) = -3.5\,\text{W}$$

$$P_{\rm dsbot} = -v_1(12 \times 10^{-3}v_\Delta) = -(105)(12 \times 10^{-3})(10) = -12.6\,\text{W}$$

$$P_{75\text{V}} = 75i_{\rm y} = 75(0.09) = 6.75\,\text{W}$$

$$P_{1\text{k}} = \frac{(v_{\text{L}} - v_1)^2}{1000} = \frac{(25-105)^2}{1000} = 6.4\,\text{W}$$

$$P_{2.5\text{k}} = \frac{v_{\text{L}}^2}{2500} = \frac{25^2}{2500} = 0.25\,\text{W}$$

$$P_{500\text{mid}} = 500 i_\phi^2 = 500(0.04)^2 = 0.8\,\text{W}$$

$$P_{500\text{right}} = \frac{v_\Delta^2}{500} = \frac{10^2}{500} = 0.2\,\text{W}$$

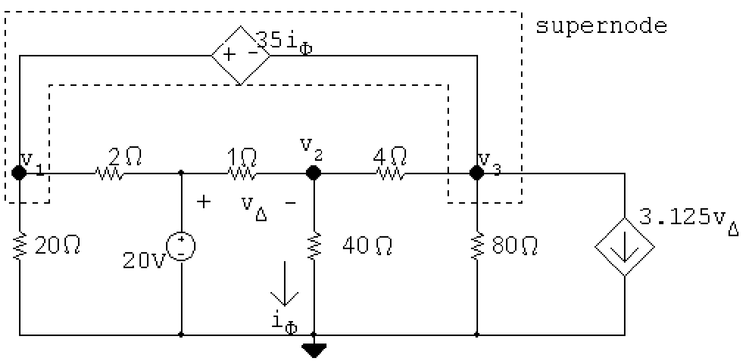
$$P_{4.25\text{k}} = \frac{v_2^2}{4250} = \frac{85^2}{4250} = 1.7\,\text{W}$$

[c] $P_{\text{supplied}} = 3.5 + 12.6 = 16.1\,\text{W}$

$$P_{\text{absorbed}} = 6.75 + 6.4 + 0.25 + 0.8 + 0.2 + 1.7 = 16.1\,\text{W} = P_{\text{supplied}}$$

Therefore the analyst is correct.

P 4.30



Node equations:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125 v_\Delta = 0$$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$$v_1 - 35 i_\phi = v_3$$

$$i_\phi = v_2/40$$

Solving, $v_1 = -20.25\,\text{V}; \quad v_2 = 10\,\text{V}; \quad v_3 = -29\,\text{V}$

Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125\,\text{A}$$

$$p_g \text{ (delivered) } = 20(30.125) = 602.5\,\text{W}$$

P 4.31 From Eq. 4.16, $i_B = v_c/(1 + \beta)R_E$

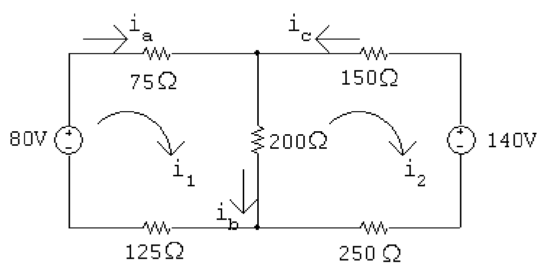
From Eq. 4.17, $i_B = (v_b - V_o)/(1 + \beta)R_E$

From Eq. 4.19,

$$i_B = \frac{1}{(1+\beta)R_E}\left[\frac{V_{CC}(1+\beta)R_E R_2 + V_o R_1 R_2}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} - V_o\right]$$

$$= \frac{V_{CC}R_2 - V_o(R_1+R_2)}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} = \frac{[V_{CC}R_2/(R_1+R_2)] - V_o}{[R_1R_2/(R_1+R_2)] + (1+\beta)R_E}$$

P 4.32 [**a**]



$80 = 400i_1 - 200i_2$

$-140 = -200i_1 + 600i_2$

Solving, $i_1 = 0.1\,\text{A}$; $\quad i_2 = -0.2\,\text{A}$

$i_{\text{a}} = i_1 = 0.1\,\text{A}$; $\quad i_{\text{b}} = i_1 - i_2 = 0.3\,\text{A}$; $\quad i_{\text{c}} = -i_2 = 0.2\,\text{A}$

[**b**] If the polarity of the 140 V source is reversed, we have

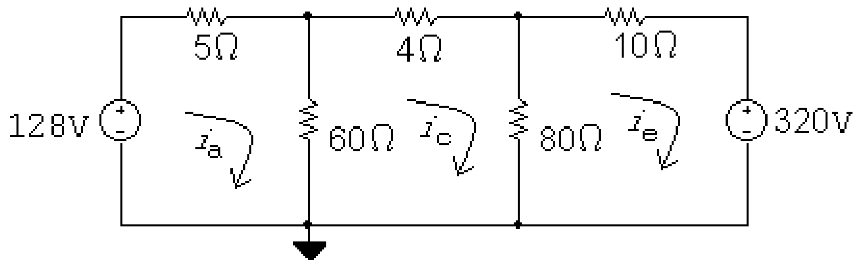
$80 = 400i_1 - 200i_2$

$140 = -200i_1 + 600i_2$

$i_1 = 0.38\,\text{A}$ and $i_2 = 0.36\,\text{A}$

$i_{\text{a}} = i_1 = 0.38\,\text{A}$; $\quad i_{\text{b}} = i_1 - i_2 = 0.02\,\text{A}$; $\quad i_{\text{c}} = -i_2 = -0.36\,\text{A}$

P 4.33 **[a]**



The three mesh current equations are:

$$-128 + 5i_\text{a} + 60(i_\text{a} - i_\text{c}) = 0$$

$$4i_\text{c} + 80(i_\text{c} - i_\text{e}) + 60(i_\text{c} - i_\text{a}) = 0$$

$$320 + 80(i_\text{e} - i_\text{c}) + 10i_\text{e} = 0$$

Place these equations in standard form:

$$i_\text{a}(5 + 60) + i_\text{c}(-60) + i_\text{e}(0) = 128$$

$$i_\text{a}(-60) + i_\text{c}(4 + 80 + 60) + i_\text{e}(-80) = 0$$

$$i_\text{a}(0) + i_\text{c}(-80) + i_\text{e}(80 + 10) = -320$$

Solving, $i_\text{a} = -6.8$ A; $\quad i_\text{c} = -9.5$ A; $\quad i_\text{e} = -12$ A

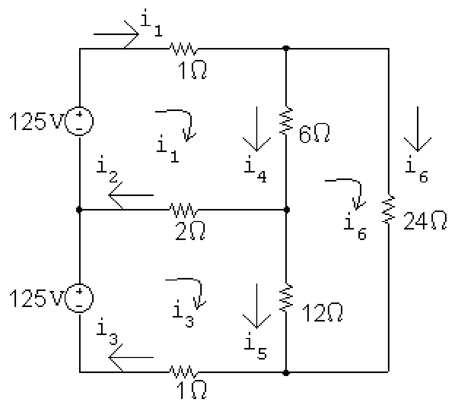
Now calculate the remaining branch currents:

$$i_\text{b} = i_\text{a} - i_\text{c} = 2.7\,\text{A}$$

$$i_\text{d} = i_\text{v} - i_\text{e} = 2.5\,\text{A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4$ W (abs)
$p_{320V} = (320)(-12) = -3840$ W (dev)
Therefore, the total power developed is 3840 W.

P 4.34 [a]



The three mesh current equations are:

$$-125 + 1i_1 + 6(i_1 - i_6) + 2(i_1 - i_3) = 0$$

$$24i_6 + 12(i_6 - i_3) + 6(i_6 - i_1) = 0$$

$$-125 + 2(i_3 - i_1) + 12(i_3 - i_6) + 1i_3 = 0$$

Place these equations in standard form:

$$i_1(1 + 6 + 2) + i_3(-2) + i_6(-6) = 125$$

$$i_1(-6) + i_3(-12) + i_6(24 + 12 + 6) = 0$$

$$i_1(-2) + i_3(2 + 12 + 1) + i_6(-12) = 125$$

Solving, $i_1 = 23.76$ A; $i_3 = 18.43$ A; $i_6 = 8.66$ A
Now calculate the remaining branch currents:

$$i_2 = i_1 - i_3 = 5.33\,\text{A}$$

$$i_4 = i_1 - i_6 = 15.10\,\text{A}$$

$$i_5 = i_3 - i_6 = 9.77\,\text{A}$$

[b] $p_{\text{sources}} = p_{\text{top}} + p_{\text{bottom}} = -(125)(23.76) - (125)(18.43)$

$$= -2969.58 - 2303.51 = -5273\,\text{W}$$

Thus, the power developed in the circuit is 5273 W.
Now calculate the power absorbed by the resistors:

$$p_{1\text{top}} = (23.76)^2(1) = 564.39\,\text{W}$$

$$p_2 = (5.33)^2(2) = 56.79\,\text{W}$$

$$p_{1\text{bot}} = (18.43)^2(1) = 339.59\,\text{W}$$

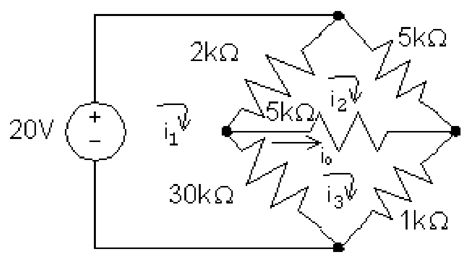
$p_6 = (15.10)^2(6) = 1367.64\,\text{W}$

$p_{12} = (9.77)^2(12) = 1145.22\,\text{W}$

$p_{24} = (8.66)^2(24) = 1799.47\,\text{W}$

The power absorbed by the resistors is
$564.39 + 56.79 + 339.59 + 1367.64 + 1145.22 + 1799.47 = 5273$ W so the power balances.

P 4.35



The three mesh current equations are:

$$-20 + 2000(i_1 - i_2) + 30{,}000(i_1 - i_3) = 0$$

$$5000i_2 + 5000(i_2 - i_3) + 2000(i_2 - i_1) = 0$$

$$1000i_3 + 30{,}000(i_3 - i_1) + 5000(i_3 - i_2) = 0$$

Place these equations in standard form:

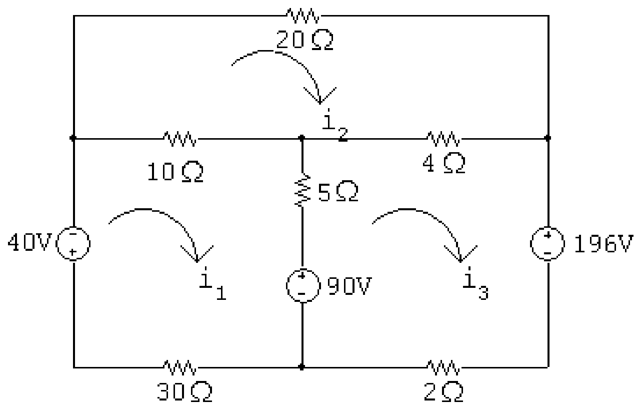
$$i_1(32{,}000) + i_2(-2000) + i_3(-30{,}000) = 20$$

$$i_1(-2000) + i_2(12{,}000) + i_3(-5000) = 0$$

$$i_1(-30{,}000) + i_2(-5000) + i_3(36{,}000) = 0$$

Solving, $i_1 = 5.5$ mA; $\quad i_2 = 3$ mA; $\quad i_3 = 5$ mA
Thus, $i_o = i_3 - i_2 = 2$ mA.

P 4.36 [**a**]



$$40 + 10(i_1 - i_2) + 5(i_1 - i_3) + 90 + 30i_1 = 0$$

$$20i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

$$196 + 2i_3 - 90 + 5(i_3 - i_1) + 4(i_3 - i_2) = 0$$

Solving, $i_1 = -5\,\text{A}; \quad i_2 = -3\,\text{A}; \quad i_3 = -13\,\text{A}$

$$p_{40} = 40i_1 = -200\,\text{W (del)}$$

$$p_{90} = 90(i_1 - i_3) = 720\,\text{W (abs)}$$

$$p_{196} = 196i_3 = -2548\,\text{W (del)}$$

$$\therefore \quad \sum p_{\text{dev}} = 2748\,\text{W}$$

[**b**] $p_{20\Omega} = (-3)^2(20) = 180\,\text{W}$

$$p_{10\Omega} = (2)^2(10) = 40\,\text{W}$$

$$p_{4\Omega} = (10)^2(4) = 400\,\text{W}$$

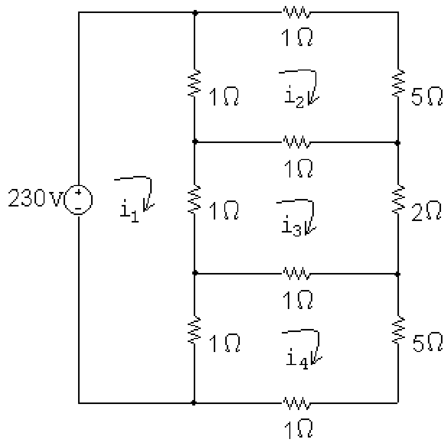
$$p_{5\Omega} = (8)^2(5) = 320\,\text{W}$$

$$p_{30\Omega} = (-5)^2(30) = 750\,\text{W}$$

$$p_{2\Omega} = (-13)^2(2) = 338\,\text{W}$$

$$\therefore \quad \sum p_{\text{abs}} = 720 + 180 + 40 + 400 + 320 + 750 + 338 = 2748\,\text{W}$$

P 4.37 **[a]**



The four mesh current equations are:

$$-230 + 1(i_1 - i_2) + 1(i_1 - i_3) + 1(i_1 - i_4) = 0$$

$$6i_2 + 1(i_2 - i_3) + 1(i_2 - i_1) = 0$$

$$2i_3 + 1(i_3 - i_4) + 1(i_3 - i_1) + 1(i_3 - i_2) = 0$$

$$6i_4 + 1(i_4 - i_1) + 1(i_4 - i_3) = 0$$

Place these equations in standard form:

$$i_1(3) + i_2(-1) + i_3(-1) + i_4(-1) = 230$$

$$i_1(-1) + i_2(8) + i_3(-1) + i_4(0) = 0$$

$$i_1(-1) + i_2(-1) + i_3(5) + i_4(-1) = 0$$

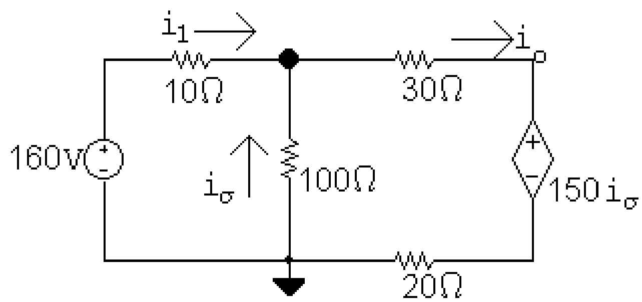
$$i_1(-1) + i_2(0) + i_3(-1) + i_4(8) = 0$$

Solving, $i_1 = 95$ A; $i_2 = 15$ A; $i_3 = 25$ A; $i_4 = 15$ A
The power absorbed by the $5\,\Omega$ resistor is

$$p_5 = i_3^2(2) = (25)^2(2) = 1250\,\text{W}$$

[b] $p_{230} = -(230)i_1 = -(230)(95) = -21{,}850\,\text{W}$

P 4.38



$$-160 + 10i_1 + 100(i_1 - i_o) = 0$$

$$30i_o + 150i_\sigma + 20i_o + 100i_\sigma = 0$$

$$i_\sigma = i_o - i_1$$

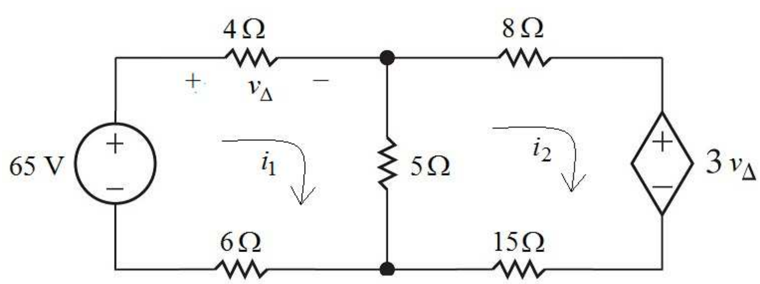
Solving,

$i_1 = 6\,\text{A}; \qquad i_o = 5\,\text{A}; \qquad i_\sigma = -1\,\text{A}$

$$P_{\text{ds}} = (150i_\sigma)i_o = 150(-1)(5) = -750\,\text{W}$$

Thus, 750 W is delivered by the dependent source.

P 4.39



$$-65 + 4i_1 + 5(i_1 - i_2) + 6i_1 = 0$$

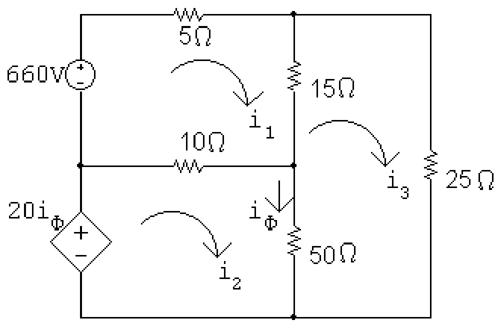
$$8i_2 + 3v_\Delta + 15i_2 + 5(i_2 - i_1) = 0$$

$$v_\Delta = 4i_1$$

Solving, $i_1 = 4\,\text{A} \qquad i_2 = -1\,\text{A} \qquad v_\Delta = 16\,\text{V}$

$$p_{15\Omega} = (-1)^2(15) = 15\,\text{W}$$

P 4.40



$$660 = 30i_1 - 10i_2 - 15i_3$$

$$20i_\phi = -10i_1 + 60i_2 - 50i_3$$

$$0 = -15i_1 - 50i_2 + 90i_3$$

$$i_\phi = i_2 - i_3$$

Solving, $i_1 = 42$ A; $\qquad i_2 = 27$ A; $\qquad i_3 = 22$ A; $\qquad i_\phi = 5$ A

$$20i_\phi = 100\,\text{V}$$

$$p_{20i_\phi} = -100i_2 = -100(27) = -2700\,\text{W}$$

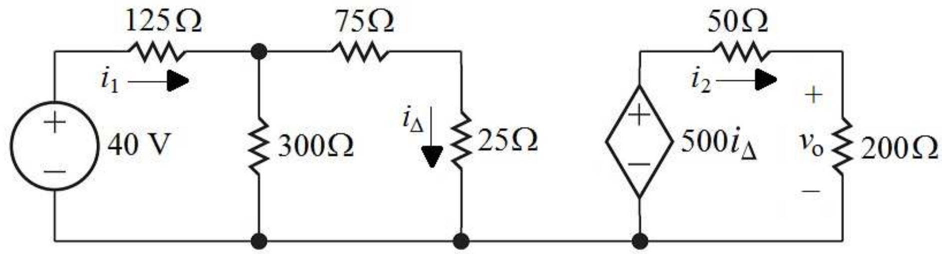
$$\therefore \quad p_{20i_\phi}\ \text{(developed)} = 2700\,\text{W}$$

CHECK:

$$p_{660\text{V}} = -660(42) = -27{,}720\,\text{W (dev)}$$

$$\begin{aligned}
\therefore \quad \sum P_{\text{dev}} &= 27{,}720 + 2700 = 30{,}420\,\text{W} \\
\sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + \\
&\quad (15)^2(10) \\
&= 30{,}420\,\text{W}
\end{aligned}$$

P 4.41 [**a**]



$$40 = 125i_1 + 300(i_1 - i_\Delta)$$

$$0 = 75i_\Delta + 25i_\Delta + 300(i_\Delta - i_1)$$

$$0 = 50i_2 + 200i_2 - 500i_\Delta$$

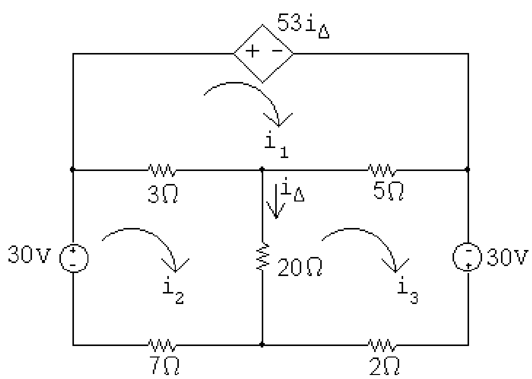
Solving, $i_1 = 0.2$ A; $\quad i_2 = 0.3\,\text{A}$; $\quad i_\Delta = 0.15\,\text{A}$

$$v_o = 200i_2 = 200(0.3) = 60\,\text{V}$$

[**b**] $p_{\text{ds}} = -(500i_\Delta)i_2 = -500(0.15)(0.3) = -22.5\,\text{W}$

$\therefore \quad p_{\text{ds}}$ (delivered) $= 22.5\,\text{W}$

P 4.42



Mesh equations:

$$53i_\Delta + 8i_1 - 3i_2 - 5i_3 = 0$$

$$0i_\Delta - 3i_1 + 30i_2 - 20i_3 = 30$$

$$0i_\Delta - 5i_1 - 20i_2 + 27i_3 = 30$$

Constraint equations:

$$i_\Delta = i_2 - i_3$$

Solving, $i_1 = 110$ A; $i_2 = 52$ A; $i_3 = 60$ A; $i_\Delta = -8$ A

$p_{\text{depsource}} = 53i_\Delta i_1 = (53)(-8)(110) = -46{,}640\,\text{W}$

Therefore, the dependent source is developing 46,640 W.
CHECK:

$p_{30\text{V}} = -30i_2 = -1560\,\text{W}$ (left source)

$p_{30\text{V}} = -30i_3 = -1800\,\text{W}$ (right source)

$\sum p_{\text{dev}} = 46{,}640 + 1560 + 1800 = 50\,\text{kW}$

$p_{3\Omega} = (110-52)^2(3) = 10{,}092\,\text{W}$

$p_{5\Omega} = (110-60)^2(5) = 12{,}500\,\text{W}$

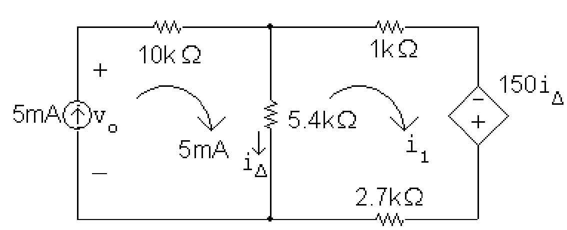
$p_{20\Omega} = (-8)^2(20) = 1280\,\text{W}$

$p_{7\Omega} = (52)^2(7) = 18{,}928\,\text{W}$

$p_{2\Omega} = (60)^2(2) = 7200\,\text{W}$

$\sum p_{\text{diss}} = 10{,}092 + 12{,}500 + 1280 + 18{,}928 + 7200 = 50$ kW

P 4.43 **[a]**

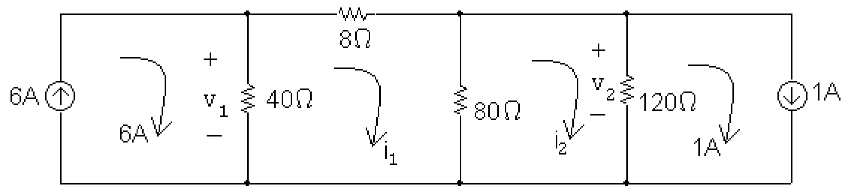


The mesh current equation for the right mesh is:
$5400(i_1 - 0.005) + 3700i_1 - 150(0.005 - i_1) = 0$
Solving, $9250i_1 = 27.75$ $\therefore$ $i_1 = 3$ mA
Then, $i_\Delta = 5 - 3 = 2$ mA

[b] $v_o = (0.005)(10{,}000) + (5400)(0.002) = 60.8\,\text{V}$
$p_{5\text{mA}} = -(60.8)(0.005) = -304\,\text{mW}$
Thus, the 5 mA source delivers 304 mW

[c] $p_{\text{dep source}} = -150i_\Delta i_1 = (-150)(0.002)(0.003) = -0.9\,\text{mW}$
The dependent source delivers 0.9 mW.

P 4.44



Mesh equations:

$128i_1 - 80i_2 = 240$

$-80i_1 + 200i_2 = 120$

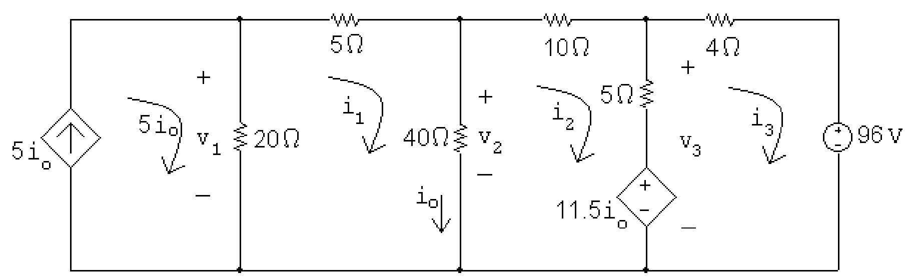
Solving,

$i_1 = 3\,\text{A}; \qquad i_2 = 1.8\,\text{A}$

Therefore,

$v_1 = 40(6 - 3) = 120\,\text{V}; \qquad v_2 = 120(1.8 - 1) = 96\,\text{V}$

P 4.45 **[a]**



Mesh equations:

$65i_1 - 40i_2 + 0i_3 - 100i_o = 0$

$-40i_1 + 55i_2 - 5i_3 + 11.5i_o = 0$

$0i_1 - 5i_2 + 9i_3 - 11.5i_o = 0$

$-1i_1 + 1i_2 + 0i_3 + 1i_o = 0$

Solving,

$i_1 = 7.2\,\text{A}; \qquad i_2 = 4.2\,\text{A}; \qquad i_3 = -4.5\,\text{A}; \qquad i_o = 3\,\text{A}$

Therefore,

$v_1 = 20[5(3) - 7.2] = 156\,\text{V}; \qquad v_2 = 40(7.2 - 4.2) = 120\,\text{V}$

$v_3 = 5(4.2 + 4.5) + 11.5(3) = 78\,\text{V}$

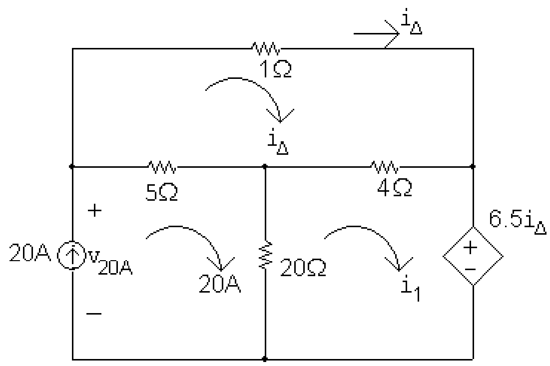
[b] $p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340\,\text{W}$

$p_{11.5i_o} = 11.5i_o(i_2 - i_3) = 11.5(3)(4.2 + 4.5) = 300.15\,\text{W}$

$p_{96\text{V}} = 96i_3 = 96(-4.5) = -432\,\text{W}$

Thus, the total power dissipated in the circuit, which equals the total power developed in the circuit is $2340 + 432 = 2772$ W.

P 4.46



Mesh equations:

$10i_\Delta - 4i_1 = 0$

$-4i_\Delta + 24i_1 + 6.5i_\Delta = 400$

Solving, $i_1 = 15$ A; $\quad i_\Delta = 16$ A

$v_{20\text{A}} = 1i_\Delta + 6.5i_\Delta = 7.5(16) = 120\,\text{V}$

$p_{20\text{A}} = -20v_{20\text{A}} = -(20)(120) = -2400\,\text{W}$ (del)

$p_{6.5i_\Delta} = 6.5i_\Delta i_1 = (6.5)(16)(15) = 1560\,\text{W}$ (abs)

Therefore, the independent source is developing 2400 W, all other elements are absorbing power, and the total power developed is thus 2400 W.
CHECK:

$p_{1\Omega} = (16)^2(1) = 256\,\text{W}$

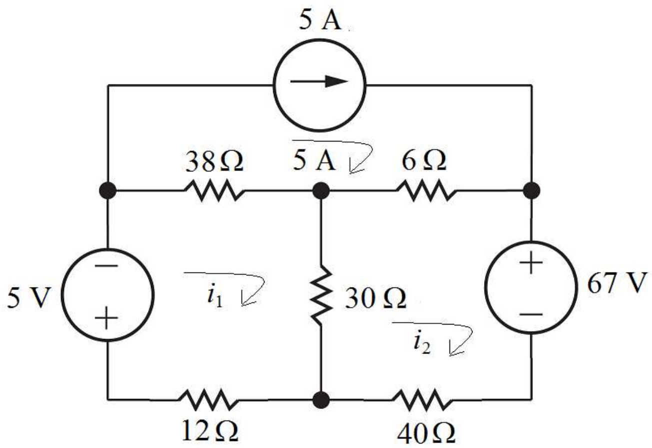
$p_{5\Omega} = (20 - 16)^2(5) = 80\,\text{W}$

$p_{4\Omega} = (1)^2(4) = 4\,\text{W}$

$p_{20\Omega} = (20 - 15)^2(20) = 500\,\text{W}$

$\sum p_{\text{abs}} = 1560 + 256 + 80 + 4 + 500 = 2400\,\text{W}$ (CHECKS)

P 4.47



$$5 + 38(i_1 - 5) + 30(i_1 - i_2) + 12i_1 = 0$$

$$67 + 40i_2 + 30(i_2 - i_1) + 6(i_2 - 5) = 0$$

Solving, $i_1 = 2.5\,\text{A}; \quad i_2 = 0.5\,\text{A}$

[a] $v_{5\text{A}} = 38(2.5 - 5) + 6(0.5 - 5)$

$= -122\,\text{V}$

$p_{5\text{A}} = 5v_{5\text{A}} = 5(-122) = -610\,\text{W}$

Therefore, the 5 A source delivers 610 W.

[b] $p_{5\text{V}} = 5(2.5) = 12.5\,\text{W}$

$p_{67\text{V}} = 67(0.5) = 33.5\,\text{W}$

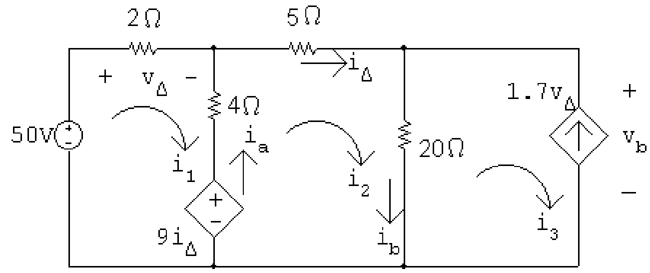
Therefore, only the current source delivers power and the total power delivered is 610 W.

[c] $\sum p_{\text{resistors}} = (2.5)^2(38) + (4.5)^2(6) + (2)^2(30) + (2.5)^2(12) + (0.5)^2(40)$

$= 564\,\text{W}$

$\sum p_{\text{abs}} = 564 + 12.5 + 33.5 = 610\,\text{W} = \sum p_{\text{del}}$ (CHECKS)

P 4.48 [**a**]



Mesh equations:

$$-50 + 6i_1 - 4i_2 + 9i_\Delta = 0$$

$$-9i_\Delta - 4i_1 + 29i_2 - 20i_3 = 0$$

Constraint equations:

$$i_\Delta = i_2; \qquad i_3 = -1.7v_\Delta; \qquad v_\Delta = 2i_1$$

Solving, $i_1 = -5$ A; $\quad i_2 = 16$ A; $\quad i_3 = 17$ A; $\quad v_\Delta = -10$ V

$$9i_\Delta = 9(16) = 144\,\text{V}$$

$$i_\text{a} = i_2 - i_1 = 21\,\text{A}$$

$$i_\text{b} = i_2 - i_3 = -1\,\text{A}$$

$$v_\text{b} = 20i_\text{b} = -20\,\text{V}$$

$$p_{50\text{V}} = -50i_1 = 250\,\text{W (absorbing)}$$

$$p_{9i_\Delta} = -i_\text{a}(9i_\Delta) = -(21)(144) = -3024\,\text{W (delivering)}$$

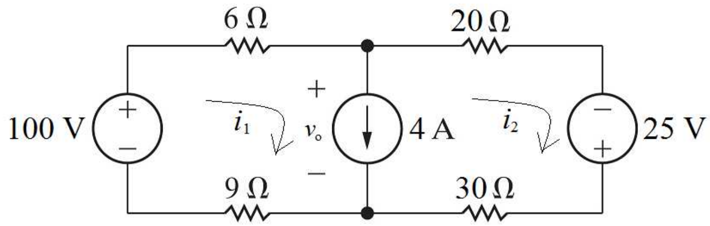
$$p_{1.7\text{V}} = -1.7v_\Delta v_\text{b} = i_3 v_\text{b} = (17)(-20) = -340\,\text{W (delivering)}$$

[**b**] $\sum P_\text{dev} = 3024 + 340 = 3364\,\text{W}$

$$\sum P_\text{dis} = 250 + (-5)^2(2) + (21)^2(4) + (16)^2(5) + (-1)^2(20)$$

$$= 3364\,\text{W}$$

P 4.49



$$-100 + 6i_1 + 20i_2 - 25 + 30i_2 + 9i_1 = 0; \quad i_1 - i_2 = 4$$

Solving, $i_1 = 5\,\text{A}; \quad i_2 = 1\,\text{A}$

$p_{100\text{V}} = -100i_1 = -500\,\text{W}$ (delivered)

$p_{6\,\Omega} = (5)^2(6) = 150\,\text{W}$

$p_{9\,\Omega} = (5)^2(9) = 225\,\text{W}$

$p_{20\,\Omega} = (1)^2(20) = 20\,\text{W}$

$p_{30\,\Omega} = (1)^2(30) = 30\,\text{W}$

$v_o = 20(1) - 25 + 30(1) = 25\,\text{V}$

$p_{4\text{A}} = 4v_o = 100\,\text{W}$

$p_{25\text{V}} = -25i_2 = -25\,\text{W}$ (delivered)

$\sum p_{\text{dev}} = 150 + 225 + 20 + 30 + 100 = 525\,\text{W}$

$\sum p_{\text{diss}} = -500 - 25 = -525\,\text{W}$

Thus the total power dissipated is 525 W.

P 4.50 **[a]** Summing around the supermesh used in the solution to Problem 4.49 gives

$$-67.5 + 6i_1 + 20i_2 - 25 + 30i_2 + 9i_1 = 0; \qquad i_1 - i_2 = 4$$

$\therefore \quad i_1 = 4.5\,\text{A}; \qquad i_2 = 0.5\,\text{A}$

$p_{67.5\text{V}} = -67.5(4.5) = -303.75\,\text{W}$ (del)

$v_o = 20(0.5) - 25 + 30(0.5) = 0\,\text{V}$

$p_{4\text{A}} = 4v_o = 0\,\text{W}$

$p_{25\text{V}} = -25i_2 = -12.5\,\text{W}$ (del)

$\sum p_{\text{diss}} = (4.5)^2(6 + 9) + (0.5)^2(20 + 30) = 316.25\,\text{W}$

$\sum p_{\text{dev}} = 303.75 + 0 + 12.5 = 316.25\,\text{W} = \sum p_{\text{diss}}$

[b] With 4 A current source replaced with a short circuit

$15i_1 = 67.5; \qquad 50i_2 = 25$

Solving,

$i_1 = 4.5\,\text{A}, \qquad i_2 = 0.5\,\text{A}$

$\therefore \quad \sum P_{\text{sources}} = -(67.5)(4.5) - (25)(0.5) = -316.25\,\text{W}$

[c] A 4 A source with zero terminal voltage is equivalent to a short circuit carrying 4 A.

[d] With the new value of the right-hand source, we want $v_o = 0$ but the current in the middle branch must still equal 4 A.
KVL left:

$-100 + 6i_1 + 0 + 9i_1 = 0 \qquad \text{so} \qquad i_1 = 6.667\,\text{A}$

$i_1 - i_2 = 4 \qquad \text{so} \qquad i_2 = i_1 - 4 = 2.667\,\text{A}$

KVL right:

$20i_2 - V_2 + 30i_2 + 0 = 0 \qquad \text{so} \qquad V_2 = 50i_2 = 133.333\,\text{V}$

To check these results, sum around the supermesh with the value of the source on the right as 133.333 V,

$-100 + 6i_1 + 20i_2 - 133.333 + 30i_2 + 9i_1 = 0; \qquad i_1 - i_2 = 4$

Solving,

$i_1 = 6.667\,\text{A}; \qquad i_2 = 2.667\,\text{A}$

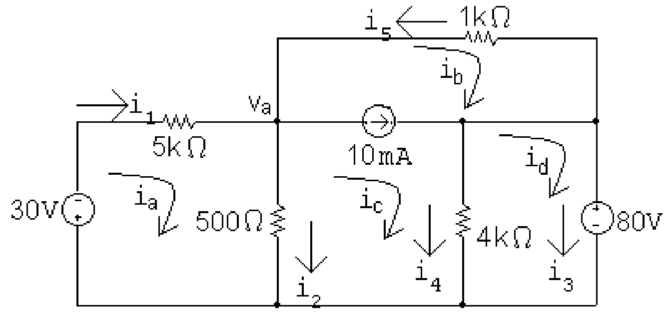
Therefore,

$v_o = 20i_2 - 133.333 + 30i_2 = 0$

Thus,

$P_{4\text{A}} = 4v_o = 0$ (checks)

P 4.51 **[a]**



Supermesh equations:

$1000i_b + 4000(i_c - i_d) + 500(i_c - i_a) = 0$

$i_c - i_b = 0.01$

Two remaining mesh equations:

$5500i_a - 500i_c = -30$

$4000i_d - 4000i_c = -80$

In standard form,

$-500i_a + 1000i_b + 4500i_c - 4000i_d = 0$

$0i_a - 1i_b + 1i_c + 0i_d = 0.01$

$5500i_a + 0i_b - 500i_c + 0i_d = -30$

$0i_a + 0i_b - 4000i_c + 4000i_d = -80$

Solving:

$i_a = -10\,\text{mA};\qquad i_b = -60\,\text{mA};\qquad i_c = -50\,\text{mA};\qquad i_d = -70\,\text{mA}$

Then,

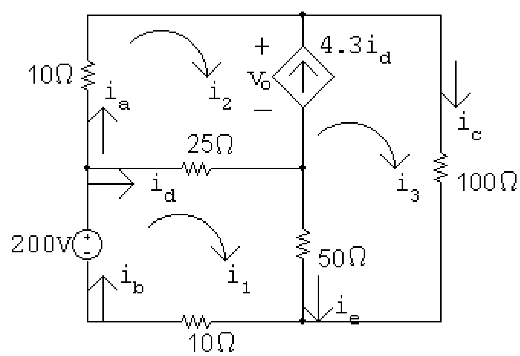
$i_1 = i_a = -10\,\text{mA};\qquad i_2 = i_a - i_c = 40\,\text{mA};\qquad i_3 = i_d = -70\,\text{mA}$

[b] $p_{\text{sources}} = 30(-0.01) + [1000(-0.06)](0.01) + 80(-0.07) = -6.5\,\text{W}$

$p_{\text{resistors}} = 1000(0.06)^2 + 5000(0.01)^2 + 500(0.04)^2$

$+4000(-0.05 + 0.07)^2 = 6.5\,\text{W}$

P 4.52 [**a**]



$$200 = 85i_1 - 25i_2 - 50i_3$$

$$0 = -75i_1 + 35i_2 + 150i_3 \qquad \text{(supermesh)}$$

$$i_3 - i_2 = 4.3(i_1 - i_2)$$

Solving, $i_1 = 4.6$ A; $\qquad i_2 = 5.7$ A; $\qquad i_3 = 0.97$ A

$$i_\text{a} = i_2 = 5.7\,\text{A}; \qquad i_\text{b} = i_1 = 4.6\,\text{A}$$

$$i_\text{c} = i_3 = 0.97\,\text{A}; \qquad i_\text{d} = i_1 - i_2 = -1.1\,\text{A}$$

$$i_\text{e} = i_1 - i_3 = 3.63\,\text{A}$$

[**b**] $10i_2 + v_o + 25(i_2 - i_1) = 0$

$$\therefore \quad v_o = -57 - 27.5 = -84.5\,\text{V}$$

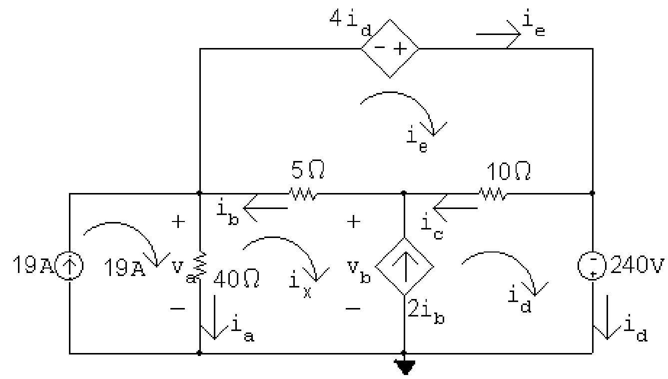
$$p_{4.3i_\text{d}} = -v_o(4.3i_\text{d}) = -(-84.5)(4.3)(-1.1) = -399.685\,\text{W (dev)}$$

$$p_{200\text{V}} = -200(4.6) = -920\,\text{W (dev)}$$

$$\begin{aligned}\sum P_\text{dev} &= 1319.685\,\text{W}\\ \sum P_\text{dis} &= (5.7)^2 10 + (1.1)^2(25) + (0.97)^2 100 + (4.6)^2(10) +\\ &\quad (3.63)^2(50)\\ &= 1319.685\,\text{W}\end{aligned}$$

$$\therefore \quad \sum P_\text{dev} = \sum P_\text{dis} = 1319.685\,\text{W}$$

P 4.53 [**a**]



$$-4i_d + 10(i_e - i_d) + 5(i_e - i_x) = 0$$

$$5(i_x - i_e) + 10(i_d - i_e) - 240 + 40(i_x - 19) = 0$$

$$i_d - i_x = 2i_b = 2(i_e - i_x)$$

Solving, $i_d = 10$ A; $i_e = 18$ A; $i_x = 26$ A

$i_a = 19 - i_x = -7\,\text{A}$; $i_b = i_e - i_x = -8\,\text{A}$; $i_c = i_e - i_d = 8\,\text{A}$;

[**b**] $v_\text{a} = 40i_\text{a} = -280\,\text{V}$; $v_\text{b} = 5i_b + 40i_a = -320\,\text{V}$

$$p_{19\text{A}} = -19v_a = 5320\,\text{W}$$

$$p_{4i_d} = -4i_d i_e = -720\,\text{W}$$

$$p_{2i_a} = -2i_b v_b = -5120\,\text{W}$$

$$p_{240\text{V}} = -240i_d = -2400\,\text{W}$$

$$p_{40\Omega} = (7)^2(40) = 1960\,\text{W} =$$

$$p_{5\Omega} = (8)^2(5) = 320\,\text{W}$$

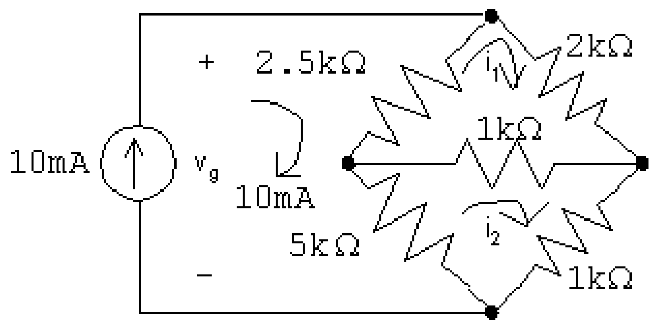
$$p_{10\Omega} = (8)^2(10) = 640\,\text{W}$$

$$\sum P_\text{gen} = 720 + 5120 + 2400 = 8240\,\text{W}$$

$$\sum P_\text{diss} = 5320 + 1960 + 320 + 640 = 8240\,\text{W}$$

P 4.54 [**a**] There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2500(i_1 - 0.01) + 2000i_1 + 1000(i_1 - i_2) = 0$$

$$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000i_2 = 0$$

Place the equations in standard form:

$$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$$

$$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$$

Solving, $i_1 = 6$ mA; $\qquad i_2 = 8$ mA

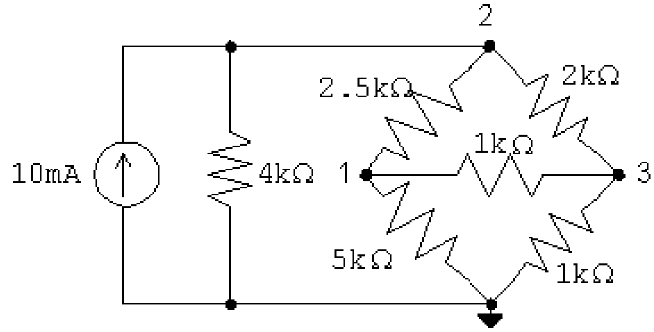
Find the power in the $1\,\mathrm{k}\Omega$ resistor:
$i_{1\mathrm{k}} = i_1 - i_2 = -2\,\mathrm{mA}$
$p_{1\mathrm{k}} = (-0.002)^2(1000) = 4$ mW

[c] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

[d] $v_g = 2000i_1 + 1000i_2 = 12 + 8 = 20\,\mathrm{V}$
$p_{10\mathrm{mA}} = -(20)(0.01) = -200\,\mathrm{mW}$
Thus the 10 mA source develops 200 mW.

P 4.55 **[a]** There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[b]



The node voltage equations are:

$$\begin{aligned}
\frac{v_1}{5000} + \frac{v_1 - v_2}{2500} + \frac{v_1 - v_3}{1000} &= 0 \\
-0.01 + \frac{v_2}{4000} + \frac{v_2 - v_1}{2500} + \frac{v_2 - v_3}{2000} &= 0 \\
\frac{v_3 - v_1}{1000} + \frac{v_3 - v_2}{2000} + \frac{v_3}{1000} &= 0
\end{aligned}$$

Put the equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{5000} + \frac{1}{2500} + \frac{1}{1000}\right) + v_2\left(-\frac{1}{2500}\right) + v_3\left(-\frac{1}{1000}\right) &= 0 \\
v_1\left(-\frac{1}{2500}\right) + v_2\left(\frac{1}{4000} + \frac{1}{2500} + \frac{1}{2000}\right) + v_3\left(-\frac{1}{2000}\right) &= 0.01 \\
v_1\left(-\frac{1}{1000}\right) + v_2\left(-\frac{1}{2000}\right) + v_3\left(\frac{1}{2000} + \frac{1}{1000} + \frac{1}{1000}\right) &= 0
\end{aligned}$$

Solving, $\quad v_1 = 6.67\,\text{V}; \quad v_2 = 13.33\,\text{V}; \quad v_3 = 5.33\,\text{V}$

$p_{10\text{m}} = -(13.33)(0.01) = -133.33\,\text{mW}$

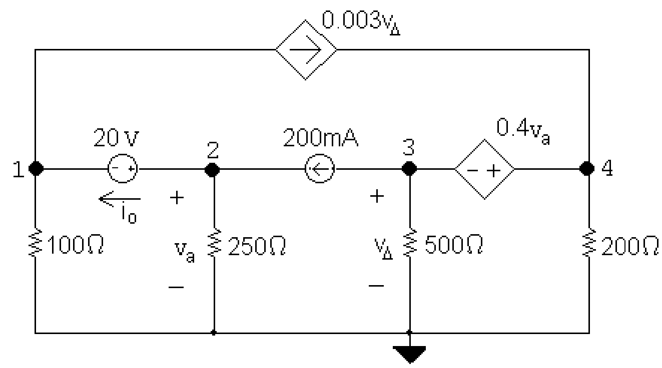
Therefore, the 10 mA source is developing 133.33 mW

P 4.56 **[a]** The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + 0.003v_\Delta + \frac{v_2}{250} - 0.2 = 0$$

$$0.2 + \frac{v_3}{100} + \frac{v_4}{200} - 0.003v_\Delta = 0$$

Constraints:

$$v_2 = v_a; \qquad v_3 = v_\Delta; \qquad v_4 - v_3 = 0.4v_a; \qquad v_2 - v_1 = 20$$

Solving, $v_1 = 24\,\text{V}$; $\quad v_2 = 44\,\text{V}$; $\quad v_3 = -72\,\text{V}$; $\quad v_4 = -54\,\text{V}$.

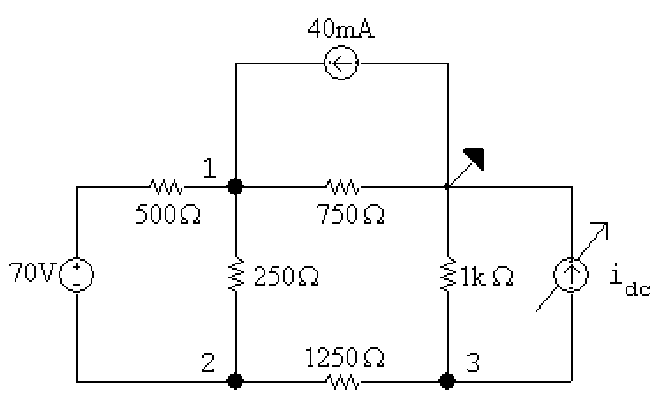
$$i_o = 0.2 - \frac{v_2}{250} = 24\,\text{mA}$$

$$p_{20\text{V}} = 20(0.024) = 480\,\text{mW}$$

Thus, the 20 V source absorbs 480 mW.

P 4.57 **[a]** The mesh-current method does not directly involve the voltage drop across the 40 mA source. Instead, use the node-voltage method and choose the reference node so that a node voltage is identical to the voltage across the 40 mA source.

[b]



Since the 40 mA source is developing 0 W, v_1 must be 0 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2; thus:

$$\frac{0-(70+v_2)}{500} + \frac{0-v_2}{250} + \frac{0}{750} - 0.04 = 0$$

$$\therefore \quad v_2 = -30\,\text{V}$$

Now that we know v_2 we sum the currents away from node 2 to find v_3; thus:

$$\frac{v_2+70-0}{500} + \frac{v_2-0}{250} + \frac{v_2-v_3}{1250} = 0$$

$$\therefore \quad v_3 = -80\,\text{V}$$

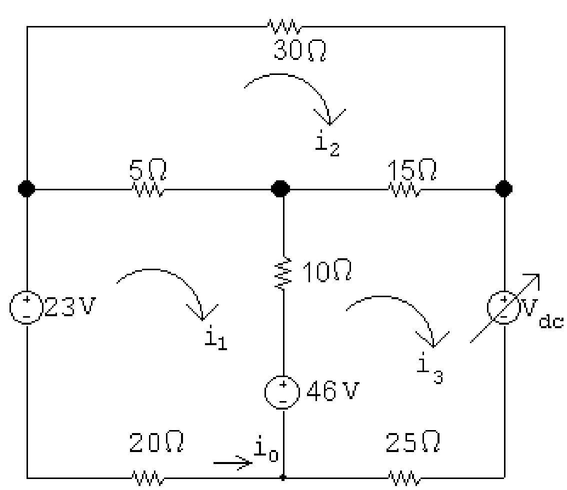
Now that we know v_3 we sum the currents away from node 3 to find i_{dc}; thus:

$$\frac{v_3}{1000} + \frac{v_3-v_2}{1250} + i_{\text{dc}} = 0$$

$$\therefore \quad i_{\text{dc}} = 0.12 = 120 \text{ mA}$$

P 4.58 **[a]** If the mesh-current method is used, then the value of the lower left mesh current is $i_o = 0$. This shortcut will simplify the set of KVL equations. The node-voltage method has no equivalent simplifying shortcut, so the mesh-current method is preferred.

[b]



Write the mesh current equations. Note that if $i_o = 0$, then $i_1 = 0$:

$$\begin{aligned}
-23 + 5(-i_2) + 10(-i_3) + 46 &= 0 \\
30i_2 + 15(i_2 - i_3) + 5i_2 &= 0 \\
V_{\text{dc}} + 25i_3 - 46 + 10i_3 + 15(i_3 - i_2) &= 0
\end{aligned}$$

Place the equations in standard form:

$$\begin{aligned}
i_2(-5) + i_3(-10) + V_{\rm dc}(0) &= -23\\
i_2(30+15+5) + i_3(-15) + V_{\rm dc}(0) &= 0\\
i_2(-15) + i_3(25+10+15) + V_{\rm dc}(1) &= 46
\end{aligned}$$

Solving, $i_2 = 0.6\,\text{A}$; $i_3 = 2\,\text{A}$; $V_{\rm dc} = -45\,\text{V}$

Thus, the value of $V_{\rm dc}$ required to make $i_o = 0$ is -45 V.

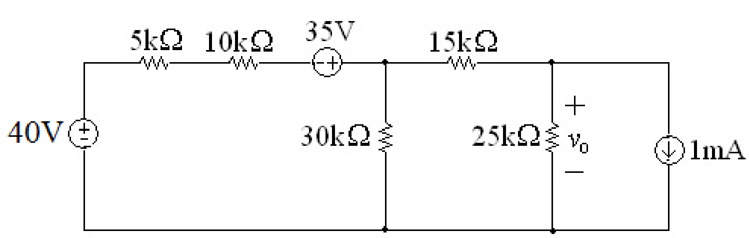
[c] Calculate the power:

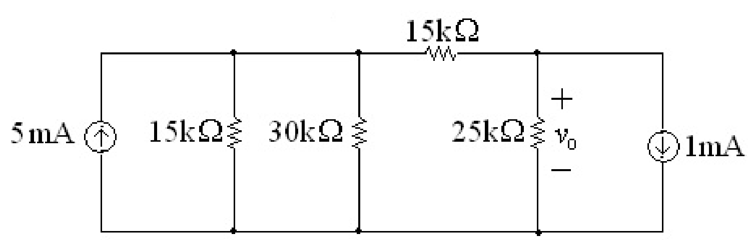
$$\begin{aligned}
p_{23\rm V} &= -(23)(0) = 0\,\text{W}\\
p_{46\rm V} &= -(46)(2) = -92\,\text{W}\\
p_{\rm Vdc} &= (-45)(2) = -90\,\text{W}\\
p_{30\Omega} &= (30)(0.6)^2 = 10.8\,\text{W}\\
p_{5\Omega} &= (5)(0.6)^2 = 1.8\,\text{W}\\
p_{15\Omega} &= (15)(2-0.6)^2 = 29.4\,\text{W}\\
p_{10\Omega} &= (10)(2)^2 = 40\,\text{W}\\
p_{20\Omega} &= (20)(0)^2 = 0\,\text{W}\\
p_{25\Omega} &= (25)(2)^2 = 100\,\text{W}
\end{aligned}$$

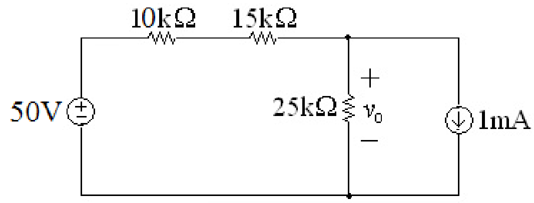
$$\sum p_{\rm dev} = 92 + 90 = 182\,\text{W}$$

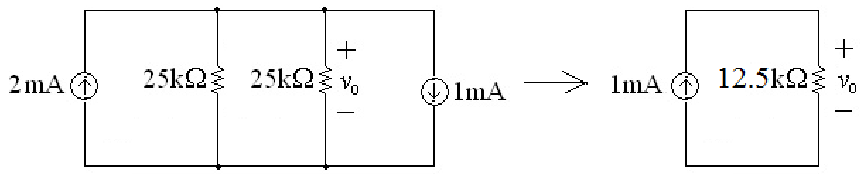
$$\sum p_{\rm dis} = 10.8 + 1.8 + 29.4 + 40 + 0 + 100 = 182\,\text{W(checks)}$$

P 4.59 **[a]**



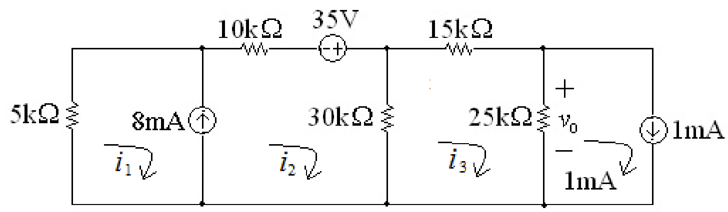






$v_o = (12{,}500)(0.001) = 12.5\,\text{V}$

[b]



$5000i_1 + 40{,}000i_2 - 30{,}000i_3 = 35$

$i_2 - i_1 = 0.008$

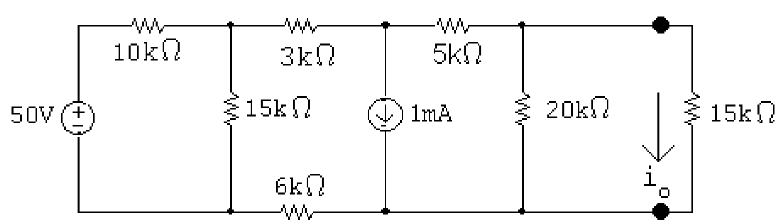
$-30{,}000i_2 + 70{,}000i_3 = 25$

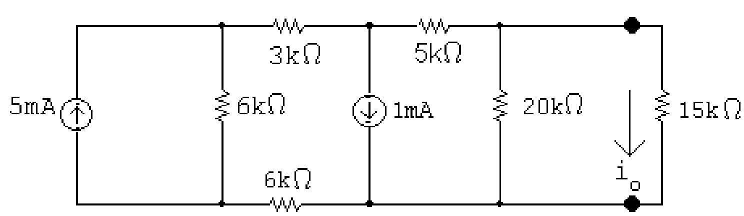
Solving,

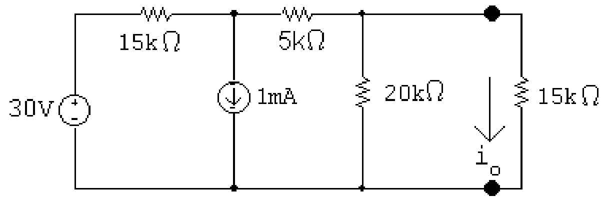
$i_1 = -5.33\,\text{mA}; \qquad i_2 = 2.667\,\text{mA}; \qquad i_3 = 1.5\,\text{mA}$

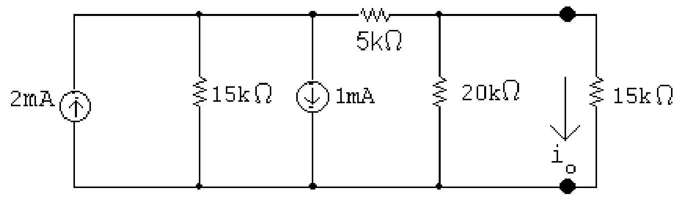
$v_o = (25{,}000)(i_3 - 0.001) = (25{,}000)(0.0005) = 12.5\,\text{V}$

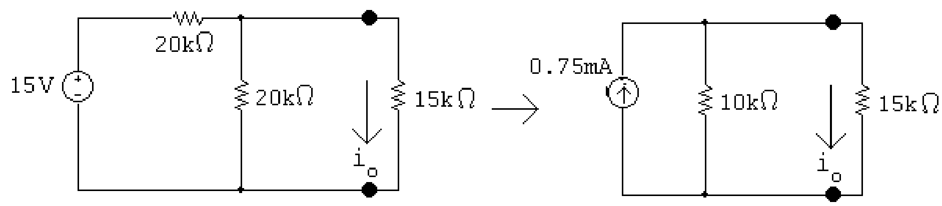
P 4.60 [a]

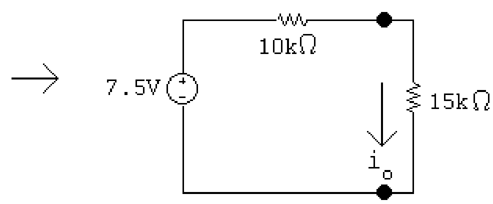






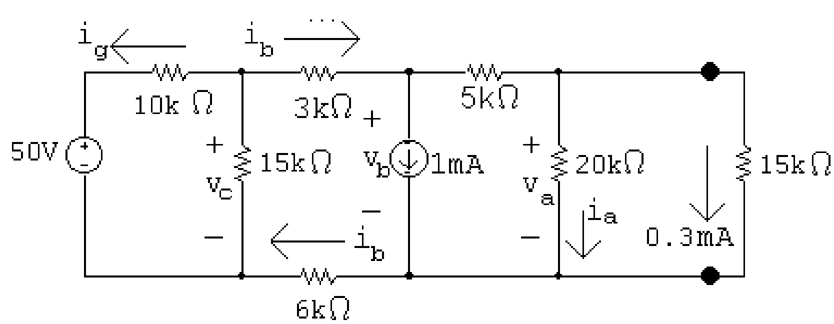






$$i_o = \frac{7.5}{25{,}000} = 0.3\,\text{mA}$$

[**b**]



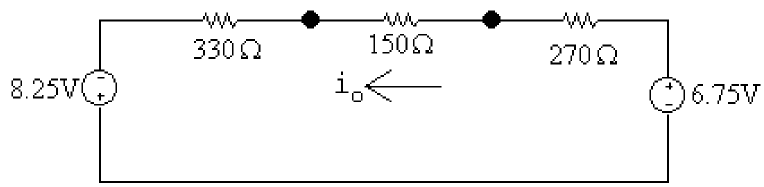
$$\begin{aligned}
v_\text{a} &= (15{,}000)(0.0003) = 4.5\,\text{V}\\
i_\text{a} &= \frac{v_\text{a}}{20{,}000} = 225\,\mu\text{A}\\
i_\text{b} &= 1 + 0.225 + 0.3 = 1.525\,\text{mA}\\
v_\text{b} &= 5000(0.525 \times 10^{-3}) + 4.5 = 7.125\,\text{V}\\
v_\text{c} &= 3000(1.525 \times 10^{-3}) + 7.125 + 6000(1.525 \times 10^{-3}) = 20.85\,\text{V}
\end{aligned}$$

$$
\begin{aligned}
i_g &= \frac{20.85 - 50}{10{,}000} = -2.915\,\text{mA} \\
p_{50\text{V}} &= (50)(-2.915 \times 10^{-3}) = -145.75\,\text{mW}
\end{aligned}
$$

Check:

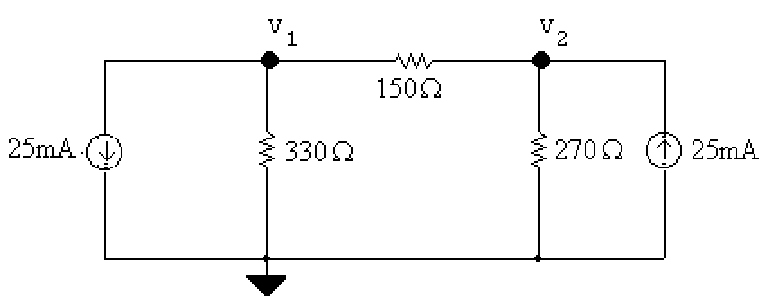
$$
\begin{aligned}
p_{1\text{mA}} &= (7.125)(10^{-3}) = 7.125\,\text{mW} \\
\sum P_{\text{dev}} &= 145.75\,\text{mW} \\
\sum P_{\text{dis}} &= (10{,}000)(2.915 \times 10^{-3})^2 + (20.85)^2/15{,}000 + (9000)(1.525 \times 10^{-3})^2 \\
&\quad + (5000)(0.525 \times 10^{-3})^2 + (20{,}000)(0.225 \times 10^{-3})^2 \\
&\quad + (15{,}000)(0.3 \times 10^{-3})^2 + 7.125 \times 10^{-3} \\
&= 145.75\,\text{mW}
\end{aligned}
$$

P 4.61 [**a**] Apply source transformations to both current sources to get



$$
i_o = \frac{(6.75 + 8.25)}{330 + 150 + 270} = 20\,\text{mA}
$$

[**b**]



The node voltage equations:

$$
\begin{aligned}
0.025 + \frac{v_1}{330} + \frac{v_1 - v_2}{150} &= 0 \\
\frac{v_2}{270} + \frac{v_2 - v_1}{150} - 0.025 &= 0
\end{aligned}
$$

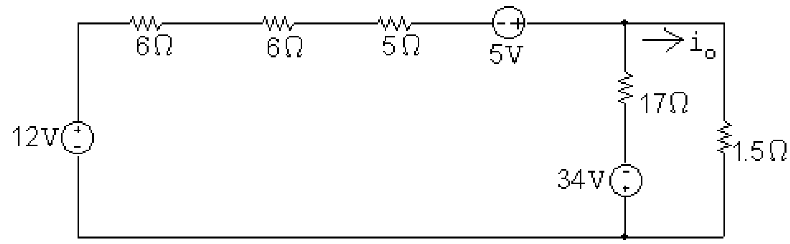
Place these equations in standard form:

$$
\begin{aligned}
v_1\left(\frac{1}{330} + \frac{1}{150}\right) + v_2\left(-\frac{1}{150}\right) &= -0.025 \\
v_1\left(-\frac{1}{150}\right) + v_2\left(\frac{1}{270} + \frac{1}{150}\right) &= 0.025
\end{aligned}
$$

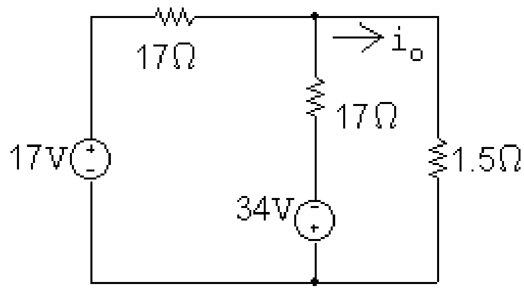
Solving, $v_1 = -1.65$ V; $v_2 = 1.35\,\text{V}$

$$\therefore \quad i_o = \frac{v_2 - v_1}{150} = 20\,\text{mA}$$

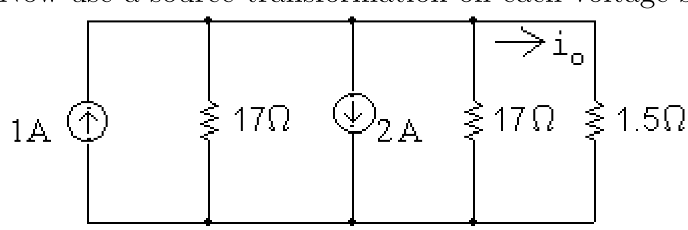
P 4.62 **[a]** Applying a source transformation to each current source yields



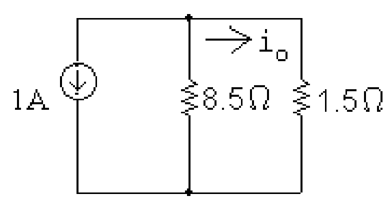
Now combine the 12 V and 5 V sources into a single voltage source and the 6 Ω, 6 Ω and 5 Ω resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

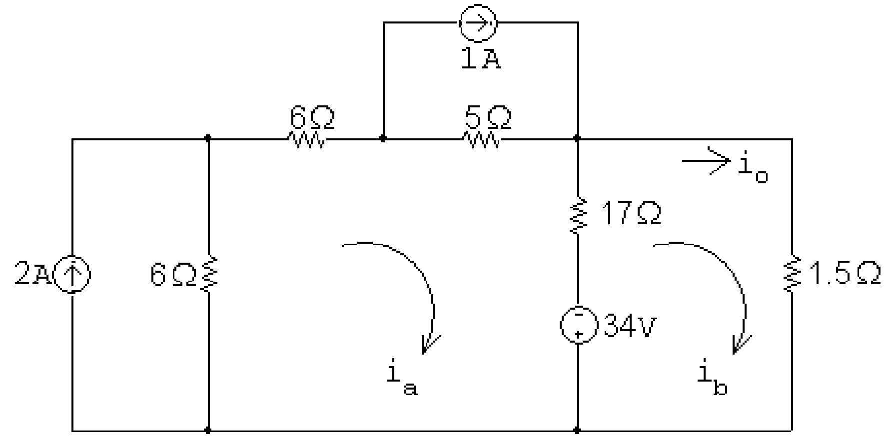


which can be reduced to



$$\therefore \quad i_o = -\frac{8.5}{10}(1) = -0.85\,\text{A}$$

[**b**]

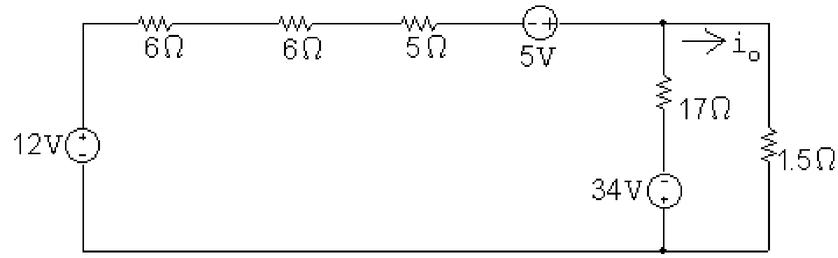


$$34i_a - 17i_b = 12 + 5 + 34 = 51$$

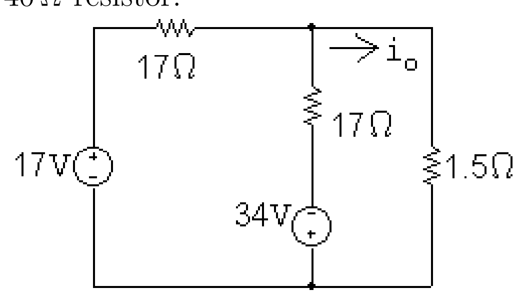
$$-17i_a + 18.5i_b = -34$$

Solving, $i_b = -0.85$ A $= i_o$

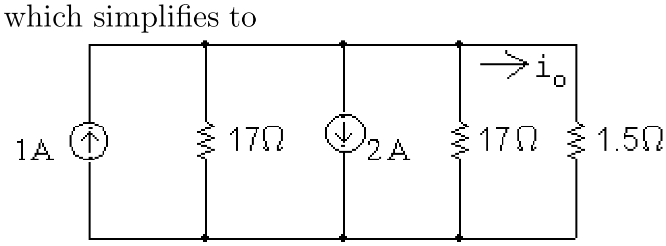
P 4.63 [**a**] First remove the $16\,\Omega$ and $260\,\Omega$ resistors:



Next use a source transformation to convert the 1 A current source and $40\,\Omega$ resistor:

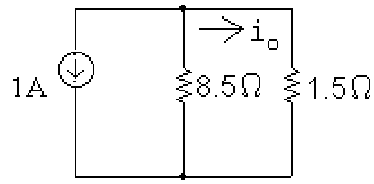


which simplifies to



$$\therefore \quad v_o = \frac{250}{300}(480) = 400\,\text{V}$$

[b] Return to the original circuit with $v_o = 400$ V:



$$i_g = \frac{520}{260} + 1.6 = 3.6\,\text{A}$$

$$p_{520\text{V}} = -(520)(3.6) = -1872\,\text{W}$$

Therefore, the 520 V source is developing 1872 W.

[c] $v_1 = -520 + 1.6(4 + 250 + 6) = -104\,\text{V}$

$$v_g = v_1 - 1(16) = -104 - 16 = -120\,\text{V}$$

$$p_{1\text{A}} = (1)(-120) = -120\,\text{W}$$

Therefore the 1 A source is developing 120 W.

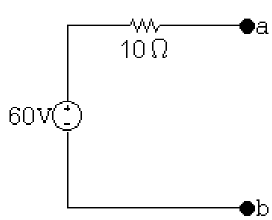
[d] $\sum p_{\text{dev}} = 1872 + 120 = 1992\,\text{W}$

$$\sum p_{\text{diss}} = (1)^2(16) + \frac{(104)^2}{40} + \frac{(520)^2}{260} + (1.6)^2(260) = 1992\,\text{W}$$

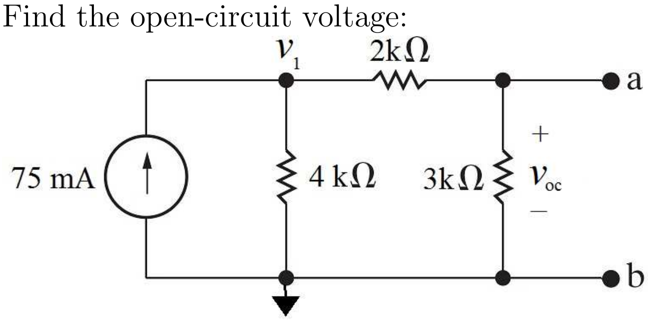
$$\therefore \quad \sum p_{\text{diss}} = \sum p_{\text{dev}}$$

P 4.64 $v_{\text{Th}} = \dfrac{30}{40}(80) = 60\,\text{V}$

$$R_{\text{Th}} = 2.5 + \frac{(30)(10)}{40} = 10\,\Omega$$



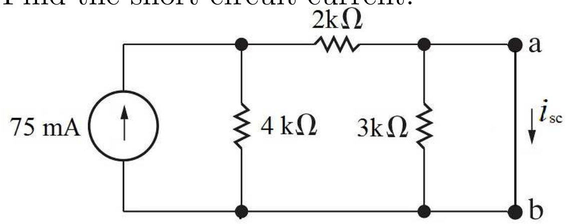
P 4.65 Find the open-circuit voltage:



$$-0.075 + \frac{v_1}{4000} + \frac{v_1}{5000} = 0$$

$$v_1 = 166.67\,\text{V}; \qquad \text{so} \qquad v_{oc} = \frac{3000}{5000}v_1 = 100\,\text{V}$$

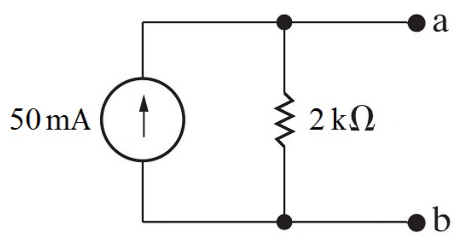
Find the short-circuit current:



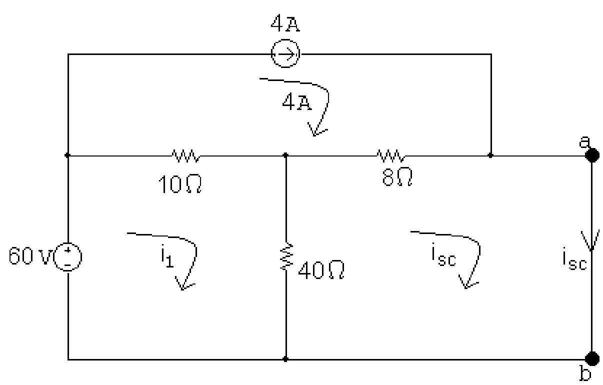
$$i_{sc} = \frac{4000\|2000}{2000}(0.075) = 50\,\text{mA}$$

Thus,

$$I_N = i_{sc} = 50\,\text{mA}; \qquad R_N = \frac{v_{oc}}{i_{sc}} = \frac{100}{0.05} = 2\,\text{k}\Omega$$



P 4.66

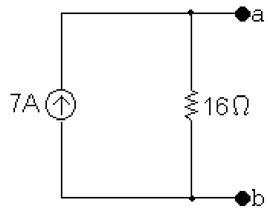


$$50i_1 - 40i_{sc} = 60 + 40$$

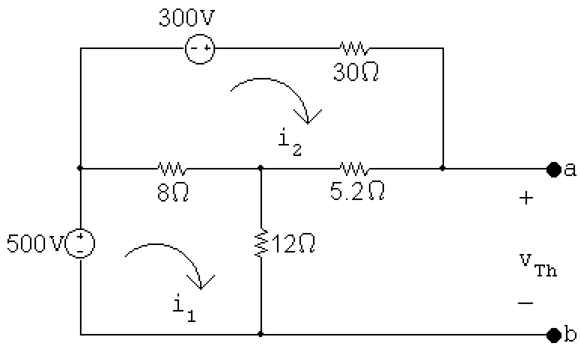
$$-40i_1 + 48i_{scs} = 32$$

$$\text{Solving,} \qquad i_{sc} = 7\,\text{A}$$

$$R_{\text{Th}} = 8 + \frac{(10)(40)}{50} = 16\,\Omega$$



P 4.67 After making a source transformation the circuit becomes



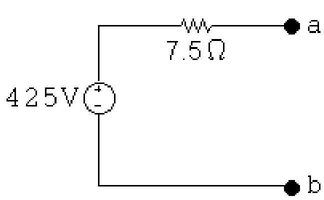
$$500 = 20i_1 - 8i_2$$

$$300 = -8i_1 + 43.2i_2$$

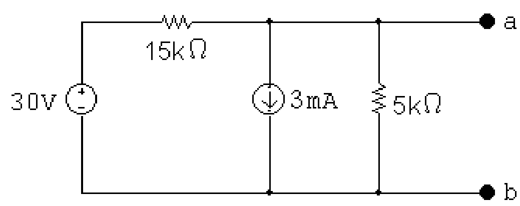
$$\therefore \quad i_1 = 30\,\text{A and } i_2 = 12.5\,\text{A}$$

$$v_{\text{Th}} = 12i_1 + 5.2i_2 = 425\,\text{V}$$

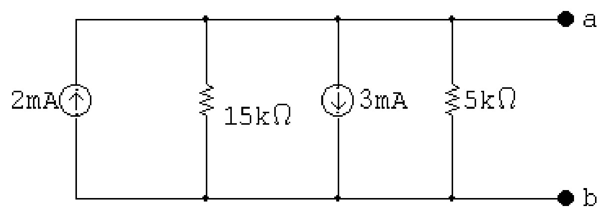
$$R_{\text{Th}} = (8\|12 + 5.2)\|30 = 7.5\,\Omega$$



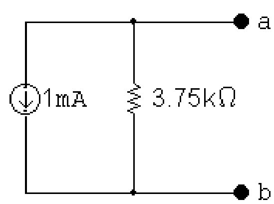
P 4.68 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



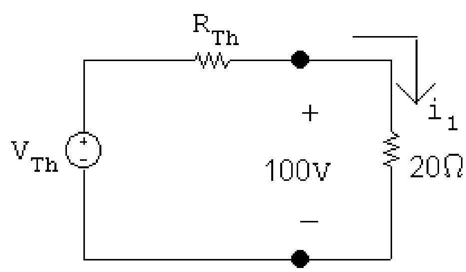
or



Therefore the Norton equivalent is

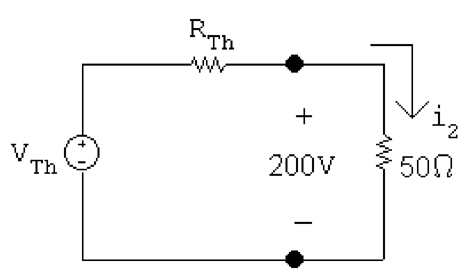


P 4.69



$i_1 = 100/20 = 5\,\text{A}$

$100 = v_{\text{Th}} - 5R_{\text{Th}}, \qquad v_{\text{Th}} = 100 + 5R_{\text{Th}}$

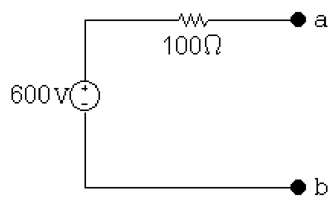


$i_2 = 200/50 = 4\,\text{A}$

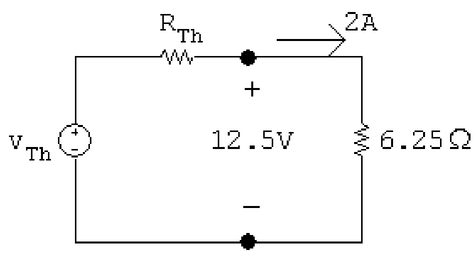
$200 = v_{\text{Th}} - 4R_{\text{Th}}, \qquad v_{\text{Th}} = 200 + 4R_{\text{Th}}$

$\therefore\quad 100 + 5R_{\text{Th}} = 200 + 4R_{\text{Th}} \qquad \text{so} \qquad R_{\text{Th}} = 100\Omega$

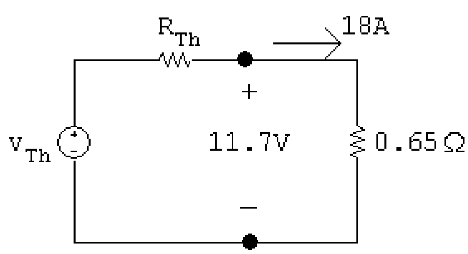
$v_{\text{Th}} = 100 + 500 = 600\,\text{V}$



P 4.70



$12.5 = v_{\text{Th}} - 2R_{\text{Th}}$



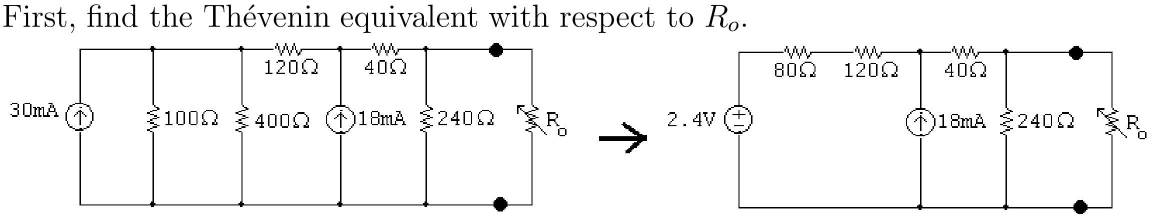
$11.7 = v_{\text{Th}} - 18R_{\text{Th}}$

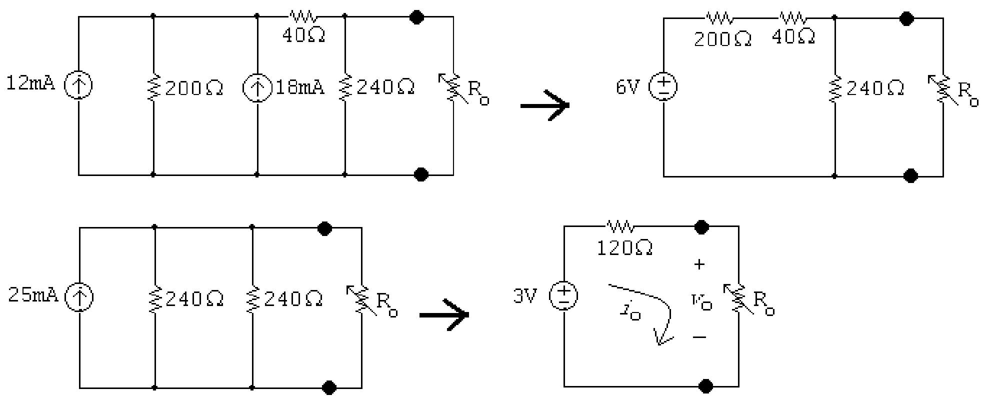
Solving the above equations for V_{Th} and R_{Th} yields

$v_{\text{Th}} = 12.6\,\text{V}, \qquad R_{\text{Th}} = 50\,\text{m}\Omega$

$\therefore\quad I_N = 252\,\text{A}, \qquad R_N = 50\,\text{m}\Omega$

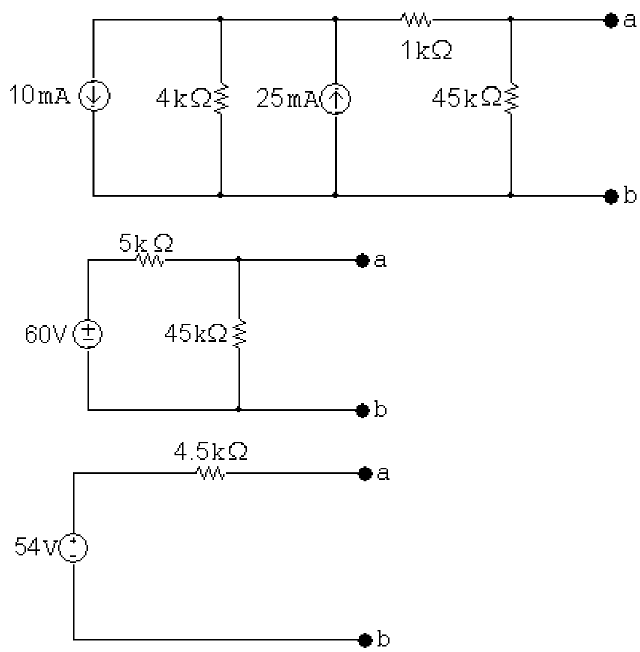
P 4.71 First, find the Thévenin equivalent with respect to R_o.



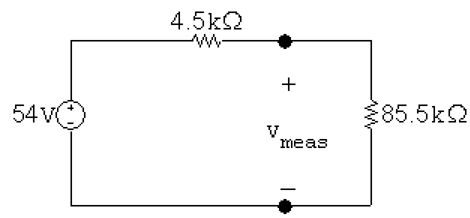


$R_o(\Omega)$	i_o(mA)	v_o(V)
100	13.64	1.364
120	12.5	1.5
150	11.11	1.667
180	10	1.8

P 4.72 [a] First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.



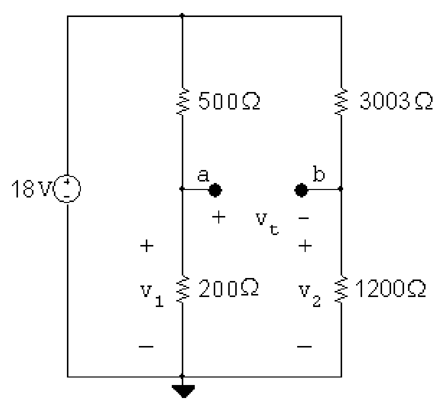
$\therefore \quad v_{\text{Th}} = 54\,\text{V} \qquad R_{\text{Th}} = 4.5\,\text{k}\Omega$



$$v_{\text{meas}} = \frac{54}{90}(85.5) = 51.3\,\text{V}$$

[b] $$\%\text{error} = \left(\frac{51.3 - 54}{54}\right) \times 100 = -5\%$$

P 4.73

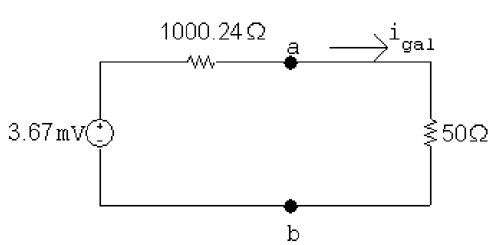


$$v_1 = \frac{200}{700}(18) = 5.143\,\text{V}$$

$$v_2 = \frac{1200}{4203}(18) = 5.139\,\text{V}$$

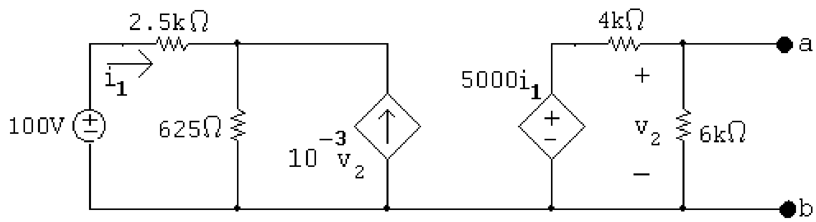
$$v_{\text{Th}} = v_1 - v_2 = 5.143 - 5.139 = 3.67\,\text{mV}$$

$$R_{\text{Th}} = \frac{(500)(200)}{700} + \frac{(3003)(1200)}{4203} = 1000.24\,\Omega$$



$$i_{\text{gal}} = \frac{3.67 \times 10^{-3}}{1050.24} = 3.5\,\mu\text{A}$$

P 4.74



OPEN CIRCUIT

$$100 = 2500i_1 + 625(i_1 + 10^{-3}v_2)$$

$$v_2 = \frac{6000}{10{,}000}(5000i_1)$$

Solving,

$$i_1 = 0.02\,\text{A}; \qquad v_2 = v_{\text{oc}} = 60\,\text{V}$$

SHORT CIRCUIT

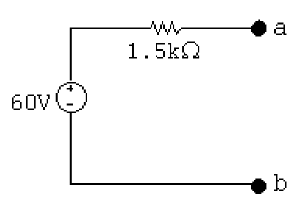
$$v_2 = 0; \qquad i_{\text{sc}} = \frac{5000}{4000}i_1$$

$$i_1 = \frac{100}{2500 + 625} = 0.032\,\text{A}$$

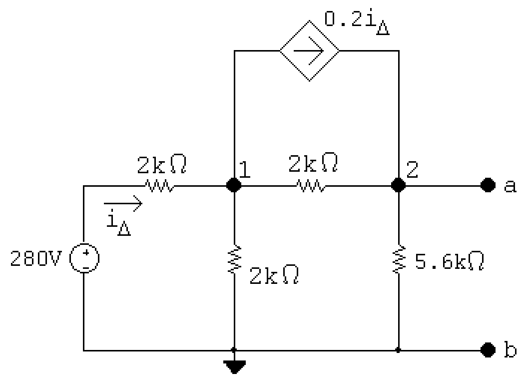
Thus,

$$i_{\text{sc}} = \frac{5}{4}i_1 = 0.04\,\text{A}$$

$$R_{\text{Th}} = \frac{60}{0.04} = 1.5\,\text{k}\Omega$$



P 4.75



The node voltage equations and dependant source equation are:

$$\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta = 0$$

$$\frac{v_2 - v_1}{2000} + \frac{v_2}{5600} - 0.2i_\Delta = 0$$

$$i_\Delta = \frac{280 - v_1}{2000}$$

In standard form:

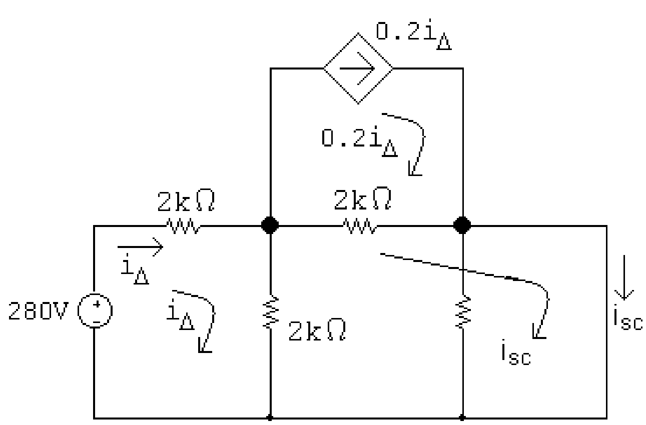
$$v_1\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000}\right) + v_2\left(-\frac{1}{2000}\right) + i_\Delta(0.2) = \frac{280}{2000}$$

$$v_1\left(-\frac{1}{2000}\right) + v_2\left(\frac{1}{2000} + \frac{1}{5600}\right) + i_\Delta(-0.2) = 0$$

$$v_1\left(\frac{1}{2000}\right) + v_2(0) + i_\Delta(1) = \frac{280}{2000}$$

Solving, $v_1 = 120\,\text{V}$; $v_2 = 112\,\text{V}$; $i_\Delta = 0.08\,\text{A}$

$V_{\text{Th}} = v_2 = 112\,\text{V}$



The mesh current equations are:

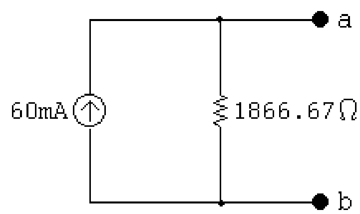
$$-280 + 2000i_\Delta + 2000(i_\Delta - i_{\rm sc}) = 0$$
$$2000(i_{\rm sc} - 0.2i_\Delta) + 2000(i_{\rm sc} - i_\Delta) = 0$$

Put these equations in standard form:

$$i_\Delta(4000) + i_{\rm sc}(-2000) = 280$$
$$i_\Delta(-2400) + i_{\rm sc}(4000) = 0$$

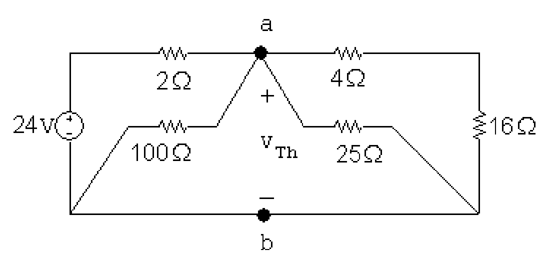
Solving, $i_\Delta = 0.1\,\text{A}$; $\qquad i_{\rm sc} = 0.06\,\text{A}$

$R_{\rm Th} = \dfrac{112}{0.06} = 1866.67\,\Omega$



P 4.76 [a] Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the $4.8\,\Omega$ resistor.

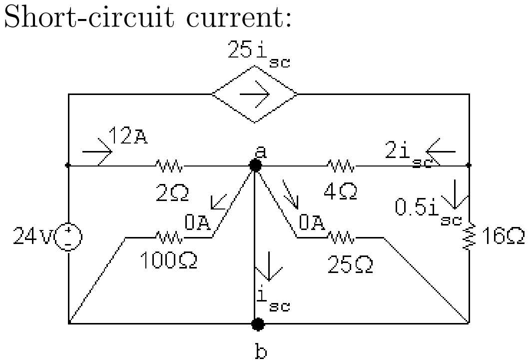
Thévenin voltage: note i_ϕ is zero.



$$\frac{v_{\rm Th}}{100} + \frac{v_{\rm Th}}{25} + \frac{v_{\rm Th}}{20} + \frac{v_{\rm Th} - 16}{2} = 0$$

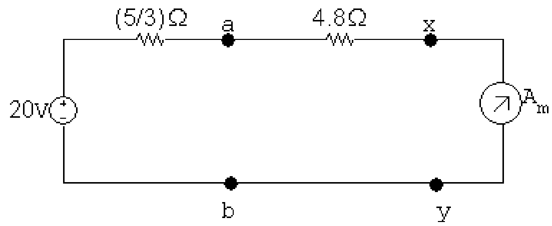
Solving, $v_{\rm Th} = 20\,\text{V}$.

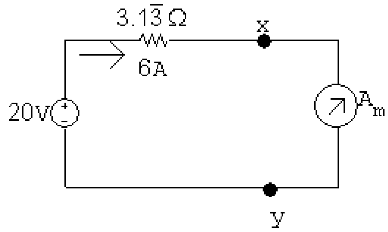
Short-circuit current:



$i_{\rm sc} = 12 + 2i_{\rm sc}, \qquad \therefore\ \ i_{\rm sc} = -12\,\text{A}$

$$R_{\rm Th} = \frac{20}{-12} = -(5/3)\,\Omega$$

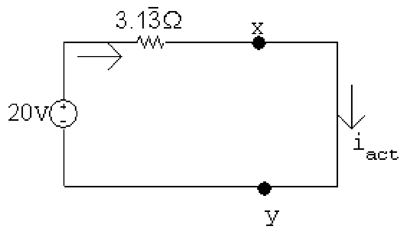




$$R_{\text{total}} = \frac{20}{6} = 3.33\,\Omega$$

$$R_{\text{meter}} = 3.33 - 3.13 = 0.2\,\Omega$$

[b] Actual current:



$$i_{\text{actual}} = \frac{20}{3.13} = 6.38\,\text{A}$$

$$\%\text{ error } = \frac{6 - 6.38}{6.38} \times 100 = -6\%$$

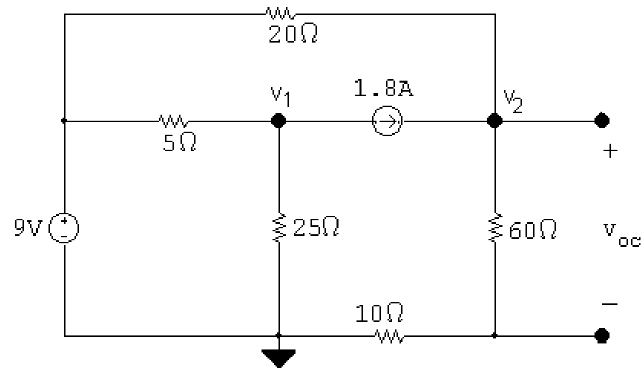
P 4.77 **[a]** Replace the voltage source with a short circuit and find the equivalent resistance from the terminals a,b:

$$R_{\text{Th}} = 10\|30 + 2.5 = 10\,\Omega$$

[b] Replace the current source with an open circuit and the voltage source with a short circuit. Find the equivalent resistance from the terminals a,b:

$$R_{\text{Th}} = 10\|40 + 8 = 16\,\Omega$$

P 4.78 **[a]** Open circuit:

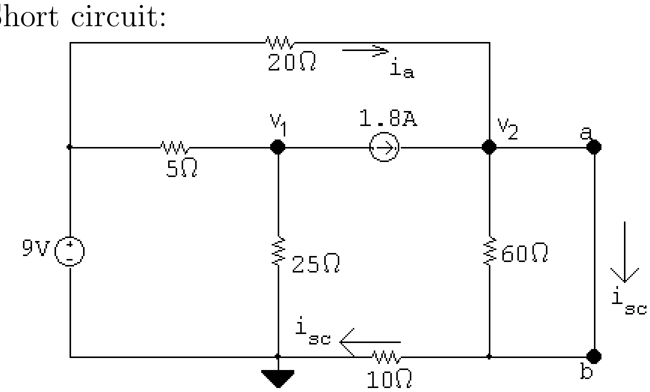


$$\frac{v_2 - 9}{20} + \frac{v_2}{70} - 1.8 = 0$$

$$v_2 = 35\,\text{V}$$

$$v_{\text{Th}} = \frac{60}{70}v_2 = 30\,\text{V}$$

Short circuit:



$$\frac{v_2 - 9}{20} + \frac{v_2}{10} - 1.8 = 0$$

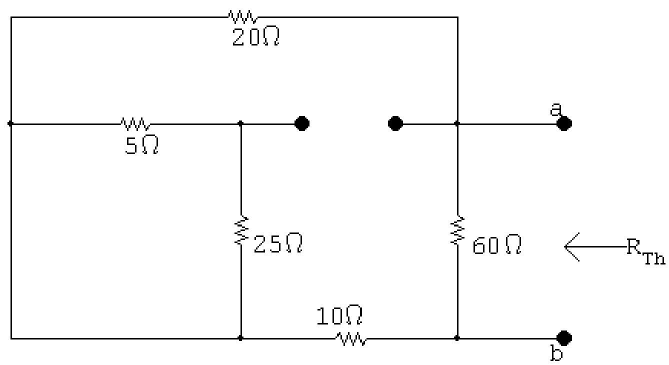
$$\therefore \quad v_2 = 15\,\text{V}$$

$$i_{\text{a}} = \frac{9 - 15}{20} = -0.3\,\text{A}$$

$$i_{\text{sc}} = 1.8 - 0.3 = 1.5\,\text{A}$$

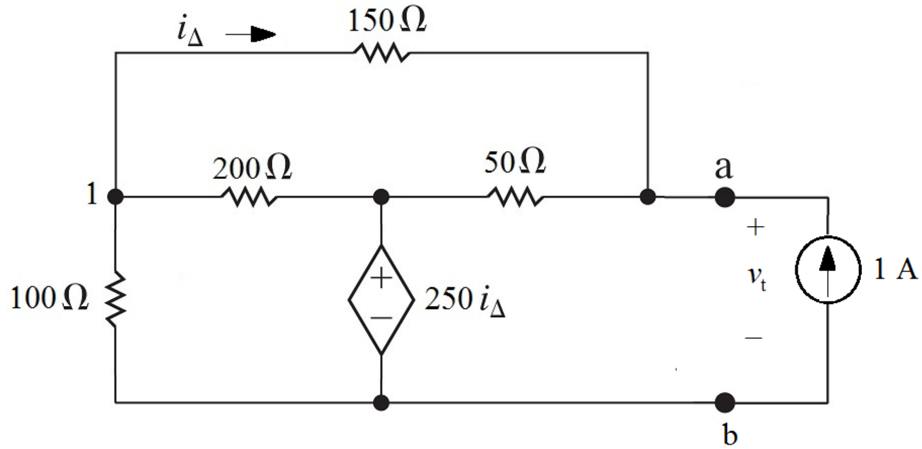
$$R_{\text{Th}} = \frac{30}{1.5} = 20\,\Omega$$

[b]



$R_{\mathrm{Th}} = (20 + 10\|60 = 20\,\Omega$ (CHECKS)

P 4.79 $V_{\mathrm{Th}} = 0$, since circuit contains no independent sources.



$$\frac{v_1}{100} + \frac{v_1 - 250i_\Delta}{200} + \frac{v_1 - v_t}{150} = 0$$

$$\frac{v_t - v_1}{150} + \frac{v_t - 250i_\Delta}{50} - 1 = 0$$

$$i_\Delta = \frac{v_t - v_1}{150}$$

In standard form:

$$v_1\left(\frac{1}{100} + \frac{1}{200} + \frac{1}{150}\right) + v_t\left(-\frac{1}{150}\right) + i_\Delta\left(-\frac{250}{200}\right) = 0$$

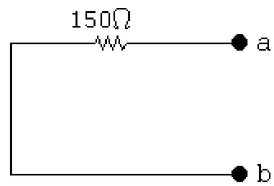
$$v_1\left(-\frac{1}{150}\right) + v_t\left(\frac{1}{150} + \frac{1}{50}\right) + i_\Delta\left(-\frac{250}{50}\right) = 1$$

$$v_1\left(-\frac{1}{150}\right) + v_t\left(\frac{1}{150}\right) + i_\Delta(-1) = 0$$

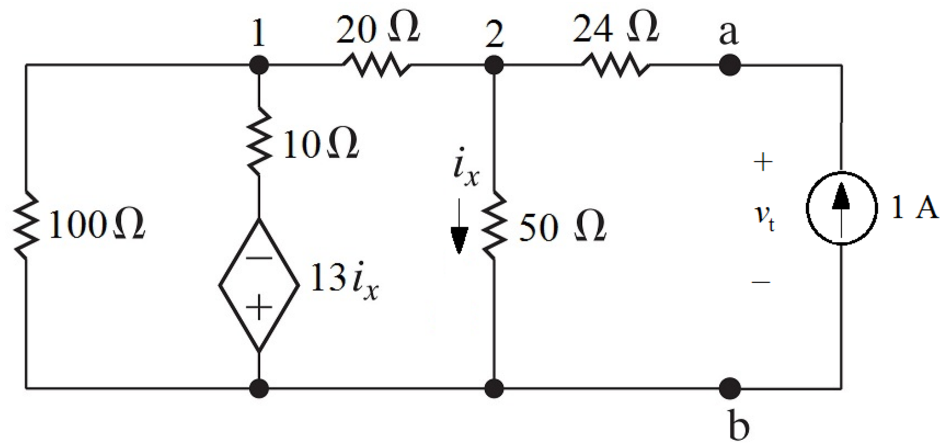
Solving,

$v_1 = 75\,\text{V}; \qquad v_\text{t} = 150\,\text{V}; \qquad i_\Delta = 0.5\,\text{A}$

$$\therefore \quad R_\text{Th} = \frac{v_\text{t}}{1\,\text{A}} = 150\,\Omega$$



P 4.80 Since there is no independent source, $V_\text{Th} = 0$. Now apply a test source at the terminals a,b to find the Thévenin equivalent resistance:



$$\frac{v_1}{100} + \frac{v_1 + 13i_x}{10} + \frac{v_1 - v_2}{20} = 0$$

$$\frac{v_2 - v_1}{20} + \frac{v_2}{50} - 1 = 0$$

$$i_x = \frac{v_2}{50}$$

Solving,

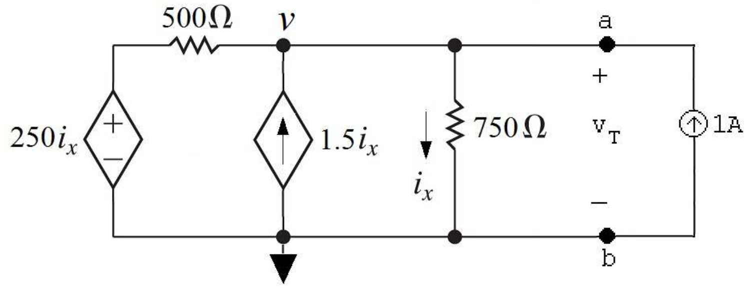
$v_1 = 2.4\,\text{V}; \qquad v_2 = 16\,\text{V}; \qquad i_x = 0.32\,\text{A}$

$v_\text{t} - 24(1) = v_2; \qquad \text{so} \qquad v_\text{t} = 16 + 24 = 40\,\text{V}$

$$R_\text{Th} = \frac{v_\text{t}}{1\,\text{A}} = 40\,\Omega$$

The Thévenin equivalent is simply a $40\,\Omega$ resistor.

P 4.81 $V_{\mathrm{Th}} = 0$ since there are no independent sources in the circuit. Thus we need only find R_{Th}.



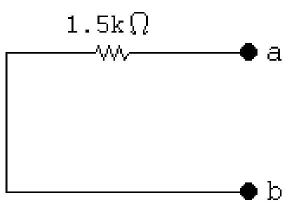
$$\frac{v - 250i_x}{500} - 1.5i_x + \frac{v}{750} - 1 = 0$$

$$i_x = \frac{v}{750}$$

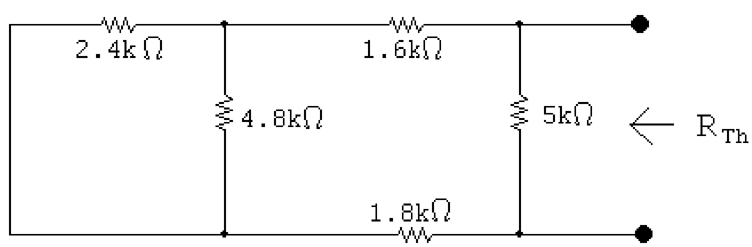
Solving,

$$v = 1500\,\mathrm{V}; \qquad i_x = 2\,\mathrm{A}$$

$$R_{\mathrm{Th}} = \frac{v}{1\,\mathrm{A}} = 1500 = 1.5\,\mathrm{k}\Omega$$



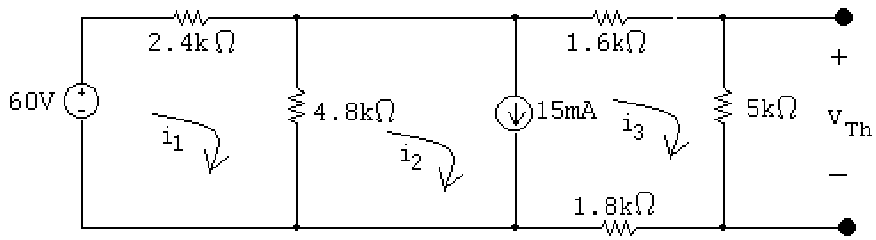
P 4.82 **[a]**



$$R_{\mathrm{Th}} = 5000\|(1600 + 2400\|4800 + 1800) = 2.5\,\mathrm{k}\Omega$$

$$R_o = R_{\mathrm{Th}} = 2.5\,\mathrm{k}\Omega$$

[**b**]



$$7200i_1 - 4800i_2 = 60$$

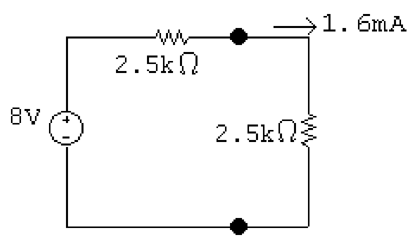
$$-4800i_1 + 4800i_2 + 8400i_3 = 0$$

$$i_2 - i_3 = 0.015$$

Solving,

$i_1 = 19.4\,\text{mA}; \qquad i_2 = 16.6\,\text{mA}; \qquad i_3 = 1.6\,\text{mA}$

$$v_{\text{oc}} = 5000i_3 = 8\,\text{V}$$



$$p_{\max} = (1.6 \times 10^{-3})^2(2500) = 6.4\,\text{mW}$$

[**c**] The resistor closest to $2.5\,\text{k}\Omega$ from Appendix H has a value of $2.7\,\text{k}\Omega$. Use voltage division to find the voltage drop across this load resistor, and use the voltage to find the power delivered to it:

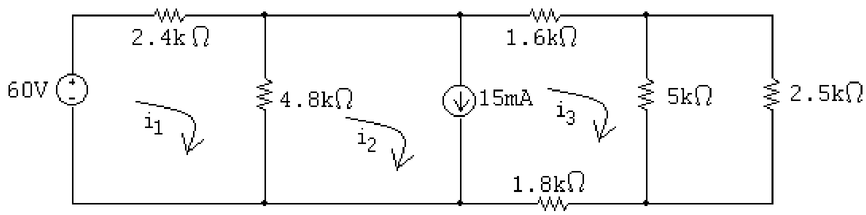
$$v_{2.7\text{k}} = \frac{2700}{2700 + 2500}(8) = 4.154\,\text{V}$$

$$p_{2.7\text{k}} = \frac{(4.154)^2}{2700} = 6.391\,\text{mW}$$

The percent error between the maximum power and the power delivered to the best resistor from Appendix H is

$$\%\ \text{error} = \left(\frac{6.391}{6.4} - 1\right)(100) = -0.1\%$$

P 4.83 Write KVL equations for the left mesh and the supermesh, place them in standard form, and solve:



At i_1: $\quad -60 + 2400i_1 + 4800(i_1 - i_2) = 0$

Supermesh: $\quad 4800(i_2 - i_1) + 1600i_3 + (5000\|2500)i_3 + 1800i_3 = 0$

Constraint: $\quad i_2 - i_3 = 0.015 = 0$

Standard form:

$i_1(7200) + i_2(-4800) + i_3(0) = 60$

$i_1(-4800) + i_2(4800) + i_3(5066.67) = 0$

$i_1(0) + i_2(1) + i_3(-1) = 0.015$

Calculator solution:

$i_1 = 19.933\,\text{mA};\qquad i_2 = 17.4\,\text{mA};\qquad i_3 = 2.4\,\text{mA}$

Calculate voltage across the current source:

$v_{15\text{mA}} = 4800(i_1 - i_2) = 12.16\,\text{V}$

Calculate power delivered by the sources:

$p_{15\text{mA}} = (0.015)(12.16) = 182.4\text{ mW (abs)}$

$p_{60\text{V}} = -60i_1 = -60(0.019933) = -1.196\text{ W (del)}$

$p_{\text{delivered}} = 1.196\,\text{W}$

Calculate power absorbed by the $2.5\,\text{k}\Omega$ resistor and the percentage power:

$$i_{2.5\text{k}} = \frac{5000\|2500}{2500} i_3 = 1.6\,\text{mA}$$

$p_{2.5\text{k}} = (0.0016)^2(2500) = 6.4\text{ mW}$

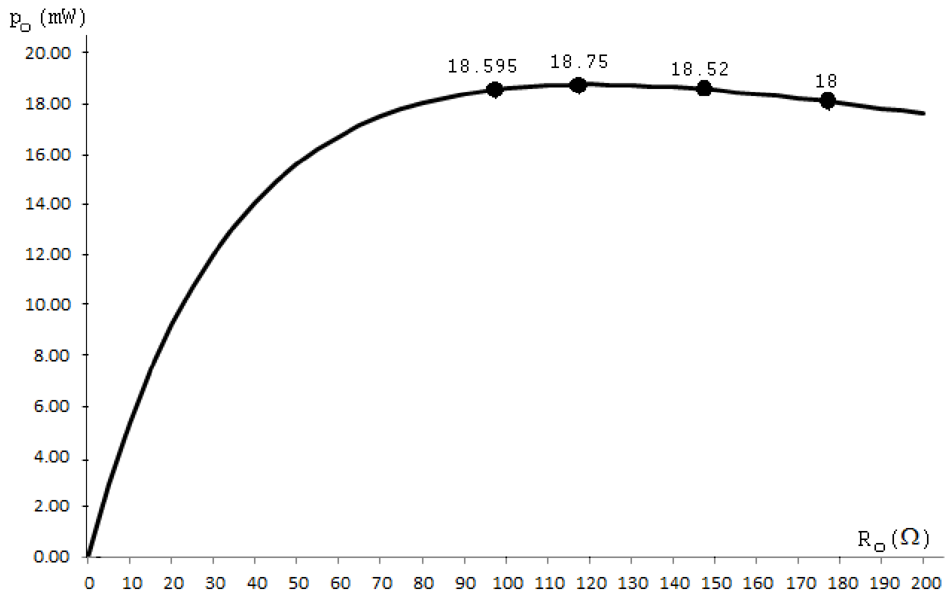
% delivered to R_o: $\quad \dfrac{0.0064}{1.196}(100) = 0.535\%$

P 4.84 [a] From the solution to Problem 4.71 we have

$R_o(\Omega)$	p_o(mW)
100	18.595
120	18.75
150	18.52
180	18

The 120 Ω resistor dissipates the most power, because its value is equal to the Thévenin equivalent resistance of the circuit.

[b]



[c] $R_o = 120\,\Omega$, $\quad p_o = 18.75$ mW, which is the maximum power that can be delivered to a load resistor.

P 4.85 [a] Since $0 \leq R_o \leq \infty$ maximum power will be delivered to the 6 Ω resistor when $R_o = 0$.

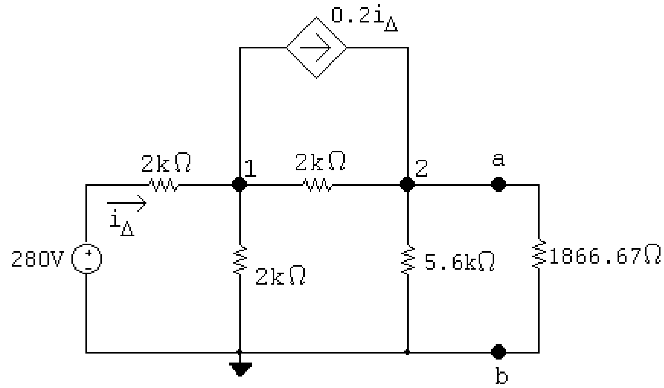
[b] $P = \dfrac{30^2}{6} = 150\,\text{W}$

P 4.86 [a] From the solution of Problem 4.75 we have $R_{\text{Th}} = 1866.67\,\Omega$ and $V_{\text{Th}} = 112$ V. Therefore

$$R_o = R_{\text{Th}} = 1866.67\,\Omega$$

[b] $p = \dfrac{(56)^2}{1866.67} = 1.68\,\text{W}$

[c]



The node voltage equations are:

$$\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta = 0$$

$$\frac{v_2 - v_1}{2000} + \frac{v_2}{5600} + \frac{v_2}{1866.67} - 0.2i_\Delta = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{280 - v_1}{2000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000}\right) + v_2\left(-\frac{1}{2000}\right) + i_\Delta(0.2) = \frac{280}{2000}$$

$$v_1\left(-\frac{1}{2000}\right) + v_2\left(\frac{1}{2000} + \frac{1}{5600} + \frac{1}{1866.67}\right) + i_\Delta(-0.2) = 0$$

$$v_1(1) + v_2(0) + i_\Delta(2000) = 280$$

Solving, $v_1 = 100\,\text{V}$; $v_2 = 56\,\text{V}$; $i_\Delta = 90$ mA

Calculate the power:

$p_{280\text{V}} = -(280)(0.09) = -25.2$ W

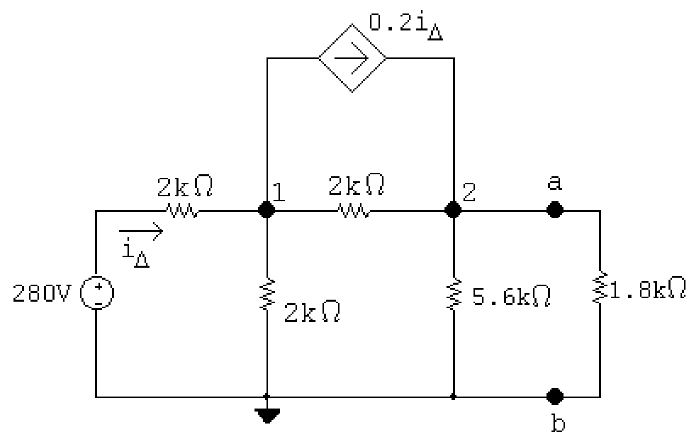
$p_{\text{dep source}} = (v_1 - v_2)(0.2i_\Delta) = 0.792$ W

$\sum p_{\text{dev}} = 25.2\,\text{W}$

$$\% \text{ delivered } = \frac{1.68}{25.2} \times 100 = 6.67\%$$

[d] The 1.8 kΩ resistor in Appendix H is closest to the Thévenin equivalent resistance.

[e] Substitute the 1.8 kΩ resistor into the original circuit and calculate the power developed by the sources in this circuit:



The node voltage equations are:

$$\frac{v_1 - 280}{2000} + \frac{v_1}{2000} + \frac{v_1 - v_2}{2000} + 0.2i_\Delta \quad = \quad 0$$

$$\frac{v_2 - v_1}{2000} + \frac{v_2}{5600} + \frac{v_2}{1800} - 0.2i_\Delta \quad = \quad 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{280 - v_1}{2000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000}\right) + v_2\left(-\frac{1}{2000}\right) + i_\Delta(0.2) = \frac{280}{2000}$$

$$v_1\left(-\frac{1}{2000}\right) + v_2\left(\frac{1}{2000} + \frac{1}{5600} + \frac{1}{1800}\right) + i_\Delta(-0.2) = 0$$

$$v_1(1) + v_2(0) + i_\Delta(2000) = 280$$

Solving, $\quad v_1 = 99.64\,\text{V}; \qquad v_2 = 54.98\,\text{V}; \qquad i_\Delta = 90.18$ mA

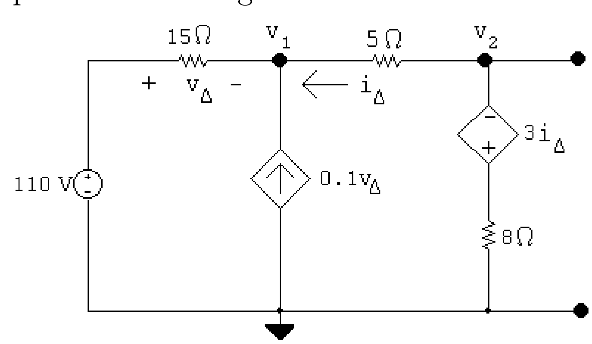
Calculate the power:

$p_{280\text{V}} = -(280)(0.09018) = -25.25$ W

$\sum p_{\text{dev}} = 25.25\,\text{mW}$

$$p_L = (54.98)^2/1800 = 1.68\,\text{W} \ \% \text{ delivered } = \frac{1.68}{25.25} \times 100 = 6.65\%$$

P 4.87 **[a]** Open circuit voltage



Node voltage equations:

$$\frac{v_1 - 110}{15} - 0.1v_\Delta + \frac{v_1 - v_2}{5} = 0$$

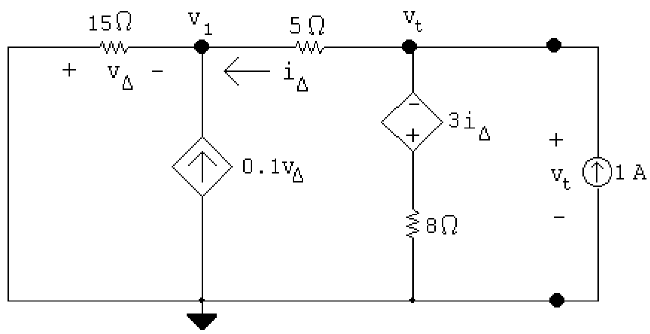
$$\frac{v_2 - v_1}{5} + \frac{v_2 + 3i_\Delta}{8} = 0$$

Constraint equations:

$$i_\Delta = \frac{v_2 - v_1}{5}; \qquad v_\Delta = 110 - v_1$$

Solving, $v_2 = 55\text{ V} = v_{\text{Th}}$

Thévenin resistance using a test source:



$$\frac{v_1}{15} - 0.1v_\Delta + \frac{v_1 - v_t}{5} = 0$$

$$\frac{v_t - v_1}{5} + \frac{v_t + 3i_\Delta}{8} - 1 = 0$$

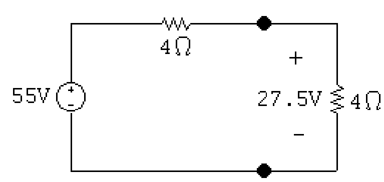
$$i_\Delta = \frac{v_t - v_1}{5}; \qquad v_\Delta = -v_1$$

Solving, $v_t = 4$ V.

$$R_{\text{Th}} = \frac{v_t}{1} = 4\,\Omega$$

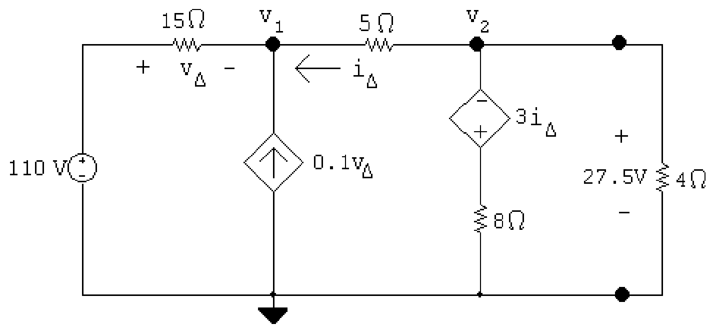
$$\therefore \quad R_o = R_{\text{Th}} = 4\,\Omega$$

[b]



$$p_{\text{max}} = \frac{(27.5)^2}{4} = 189.0625\,\text{W}$$

[**c**]



$$\frac{v_1 - 110}{15} - 0.1v_\Delta + \frac{v_1 - 27.5}{5} = 0$$

$$v_\Delta = 110 - v_1$$

Solving, $v_1 = 65$ V,

$$i_{110\text{V}} = \frac{65 - 110}{15} = -3\,\text{A}$$

$$p_{110\text{V}} = 110(-3) = -330\,\text{W}$$

$$i_\Delta = \frac{27.5 - 65}{5} = -7.5\,\text{A}$$

$$i_{\text{CCVS}} = \frac{27.5 + 3i_\Delta}{8} = 0.625\,\text{A}$$

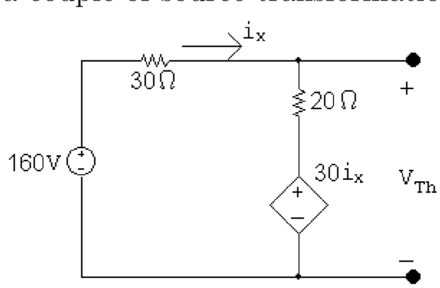
$$p_{\text{CCVS}} = -[3(-7.5)](0.625) = 14.0625\,\text{W}$$

$$p_{\text{VCCS}} = -[0.1(45)](65) = -292.5\,\text{W}$$

$$\sum p_{\text{dev}} = 330 + 292.5 = 622.5\,\text{W}$$

$$\%\text{ delivered } = \frac{189.0625}{622.5} \times 100 = 30.37\%$$

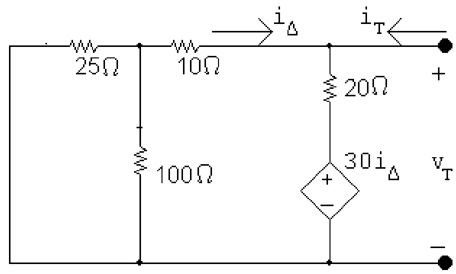
P 4.88 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2\,\text{A}$$

$$V_{\mathrm{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100\,\mathrm{V}$$

Using the test-source method to find the Thévenin resistance gives

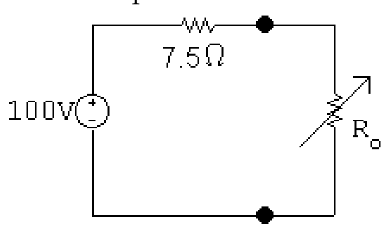


$$i_{\mathrm{T}} = \frac{v_{\mathrm{T}}}{30} + \frac{v_{\mathrm{T}} - 30(-v_{\mathrm{T}}/30)}{20}$$

$$\frac{i_{\mathrm{T}}}{v_{\mathrm{T}}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\mathrm{Th}} = \frac{v_{\mathrm{T}}}{i_{\mathrm{T}}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25} R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

$$R_o^2 - 25R_o + 56.25 = 0$$

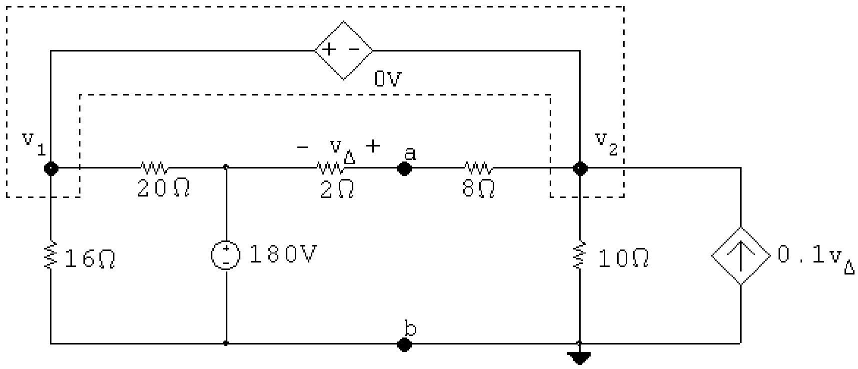
$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

$$R_o = 2.5\,\Omega$$

P 4.89 **[a]** First find the Thévenin equivalent with respect to R_o.

Open circuit voltage: $i_\phi = 0$; $184\phi = 0$



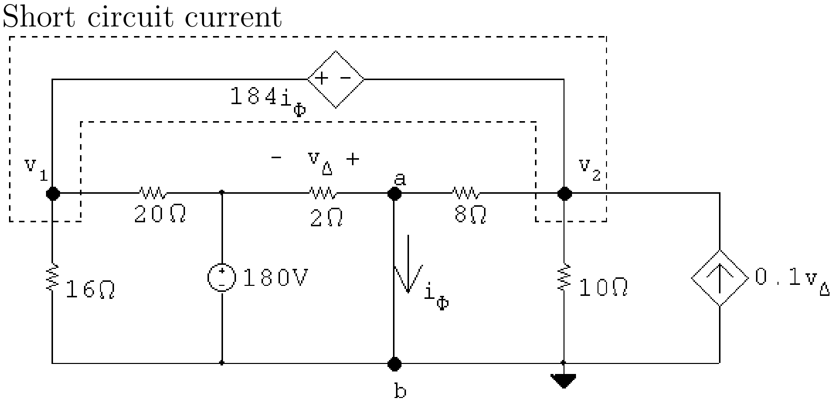
$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_1 - 180}{10} + \frac{v_1}{10} - 0.1v_\Delta = 0$$

$$v_\Delta = \frac{v_1 - 180}{10}(2) = 0.2v_1 - 36$$

$$v_1 = 80\,\text{V}; \qquad v_\Delta = -20\,\text{V}$$

$$V_{\text{Th}} = 180 + v_\Delta = 180 - 20 = 160\,\text{V}$$

Short circuit current



$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_2}{8} + \frac{v_2}{10} - 0.1(-180) = 0$$

$$v_2 + 184i_\phi = v_1$$

$$i_\phi = \frac{180}{2} + \frac{v_2}{8} = 90 + 0.125v_2$$

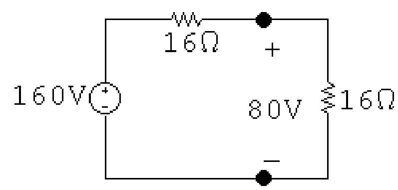
$$v_2 = -640\,\text{V}; \qquad v_1 = 1200\,\text{V}$$

$$i_\phi = i_{\text{sc}} = 10\,\text{A}$$

$$R_{\text{Th}} = V_{\text{Th}}/i_{\text{sc}} = 160/10 = 16\,\Omega$$

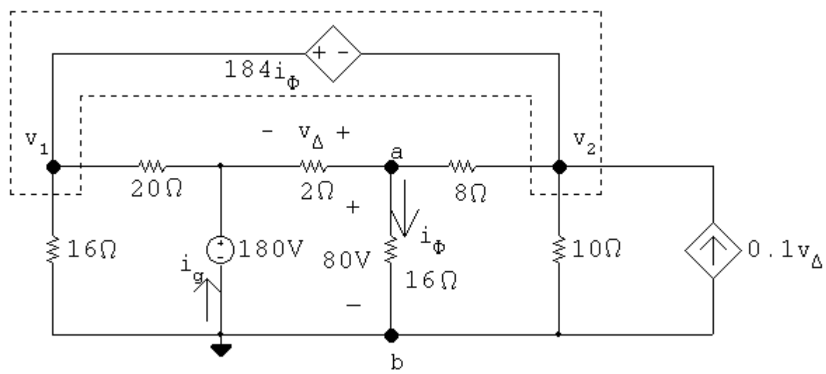
$$\therefore \quad R_o = 16\,\Omega$$

[b]



$$p_{\max} = (80)^2/16 = 400\,\text{W}$$

[c]



$$\frac{v_1}{16} + \frac{v_1 - 180}{20} + \frac{v_2 - 80}{8} + \frac{v_2}{10} - 0.1(80 - 180) = 0$$

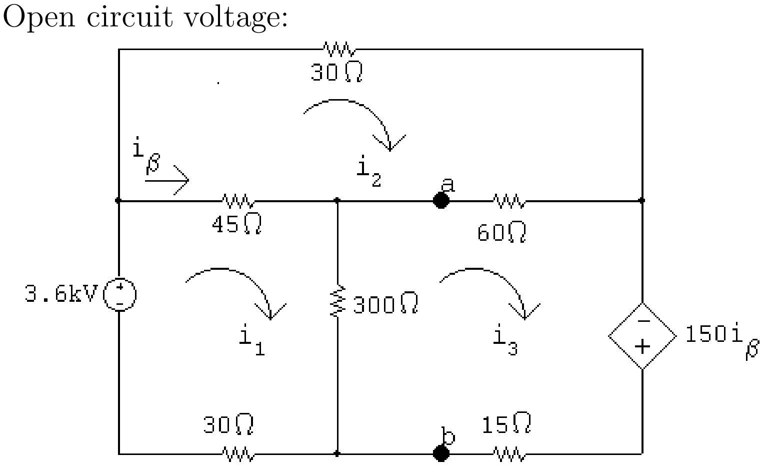
$$v_2 + 184 i_\phi = v_1; \qquad i_\phi = 80/16 = 5\,\text{A}$$

Therefore, $v_1 = 640\,\text{V}$ and $v_2 = -280$ V; thus,

$$i_g = \frac{180 - 80}{2} + \frac{180 - 640}{20} = 27\,\text{A}$$

$$p_{180\text{V}}\ (\text{dev}) = (180)(27) = 4860\,\text{W}$$

P 4.90 [a] Find the Thévenin equivalent with respect to the terminals of R_{L}.
Open circuit voltage:



The mesh current equations are:

$$\begin{aligned}
-3600 + 45(i_1 - i_2) + 300(i_1 - i_3) + 30i_1 &= 0\\
30i_2 + 60(i_2 - i_3) + 45(i_2 - i_1) &= 0\\
-150i_\beta + 15i_3 + 300(i_3 - i_1) + 60(i_3 - i_2) &= 0
\end{aligned}$$

The dependent source constraint equation is:
$i_\beta = i_1 - i_2$

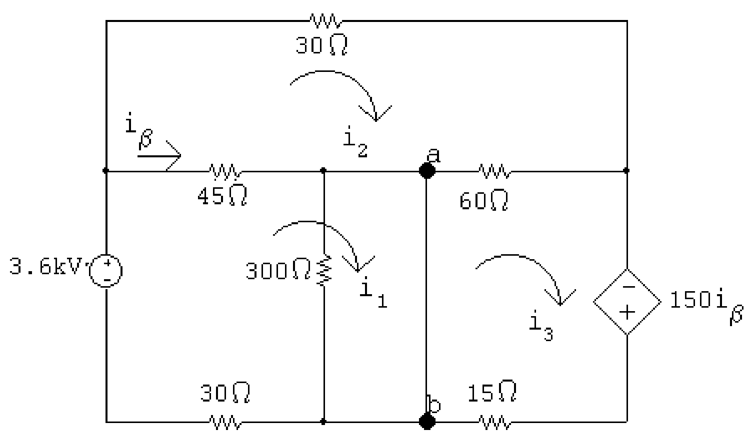
Place these equations in standard form:

$$\begin{aligned}
i_1(45 + 300 + 30) + i_2(-45) + i_3(-300) + i_\beta(0) &= 3600\\
i_1(-45) + i_2(30 + 60 + 45) + i_3(-60) + i_\beta(0) &= 0\\
i_1(-300) + i_2(-60) + i_3(15 + 300 + 60) + i_\beta(-150) &= 0\\
i_1(1) + i_2(-1) + i_3(0) + i_\beta(-1) &= 0
\end{aligned}$$

Solving, $\quad i_1 = 99.6\,\mathrm{A}; \quad i_2 = 78\,\mathrm{A}; \quad i_3 = 100.8\,\mathrm{A}; \quad i_\beta = 21.6\,\mathrm{A}$

$V_{\mathrm{Th}} = 300(i_1 - i_3) = -360\,\mathrm{V}$

Short-circuit current:



The mesh current equations are:

$$\begin{aligned}
-3600 + 45(i_1 - i_2) + 30i_1 &= 0\\
30i_2 + 60(i_2 - i_3) + 45(i_2 - i_1) &= 0\\
-150i_\beta + 15i_3 + 60(i_3 - i_2) &= 0
\end{aligned}$$

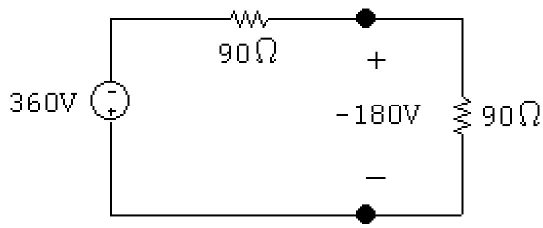
The dependent source constraint equation is:
$i_\beta = i_1 - i_2$

Place these equations in standard form:

$$\begin{aligned}
i_1(45 + 30) + i_2(-45) + i_3(0) + i_\beta(0) &= 3600\\
i_1(-45) + i_2(30 + 60 + 45) + i_3(-60) + i_\beta(0) &= 0\\
i_1(0) + i_2(-60) + i_3(60 + 15) + i_\beta(-150) &= 0\\
i_1(1) + i_2(-1) + i_3(0) + i_\beta(-1) &= 0
\end{aligned}$$

Solving, $\quad i_1 = 92\,\mathrm{A}; \quad i_2 = 73.33\,\mathrm{A}; \quad i_3 = 96\,\mathrm{A}; \quad i_\beta = 18.67\,\mathrm{A}$

$$i_{\text{sc}} = i_1 - i_3 = -4\,\text{A}; \qquad R_{\text{Th}} = \frac{V_{\text{Th}}}{i_{\text{sc}}} = \frac{-360}{-4} = 90\,\Omega$$

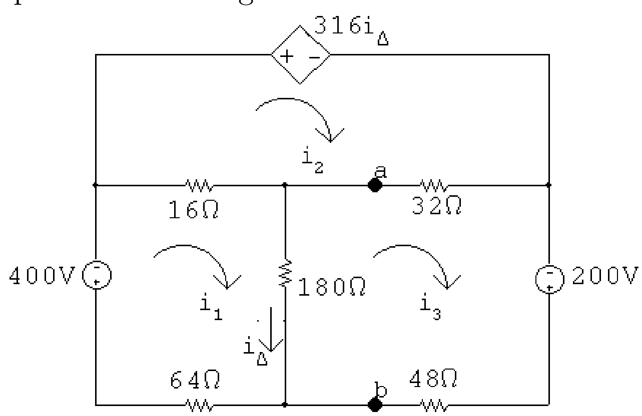


$$R_{\text{L}} = R_{\text{Th}} = 90\,\Omega$$

[b] $p_{\max} = \dfrac{180^2}{90} = 360\,\text{W}$

P 4.91 **[a]** We begin by finding the Thévenin equivalent with respect to the terminals of R_o.

Open circuit voltage



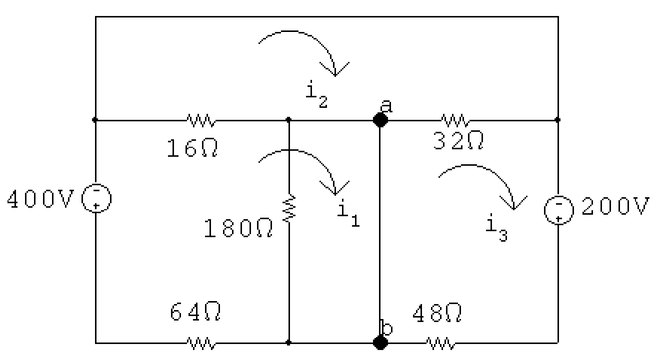
The mesh current equations are:

$$\begin{aligned} 260i_1 - 16i_2 - 180i_3 &= -400 \\ -16i_1 + 48i_2 - 32i_3 + 316(i_1 - i_3) &= 0 \\ -180i_1 - 32i_2 + 260i_3 &= 200 \end{aligned}$$

Solving, $i_1 = 3$ A; $\quad i_2 = 17.5\,\text{A}$; $\quad i_3 = 5\,\text{A}$; $\quad i_\Delta = i_1 - i_3 = -2\,\text{A}$

Also, $\quad V_{\text{Th}} = v_{\text{oc}} = 180 i_\Delta = -360\,\text{V}$

Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $316i_\Delta$ is also zero.

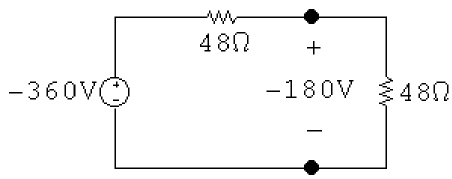
The mesh currents are:

$$\begin{aligned} 80i_1 - 16i_2 + 0i_3 &= -400 \\ -16i_1 + 48i_2 - 32i_3 &= 0 \\ 0i_1 - 32i_2 + 80i_3 &= 200 \end{aligned}$$

Solving, $\quad i_1 = -5\,\text{A}; \quad i_2 = 0\,\text{A}; \quad i_3 = 2.5\,\text{A}$

Then, $\quad i_{sc} = i_1 - i_3 = -7.5\,\text{A}$

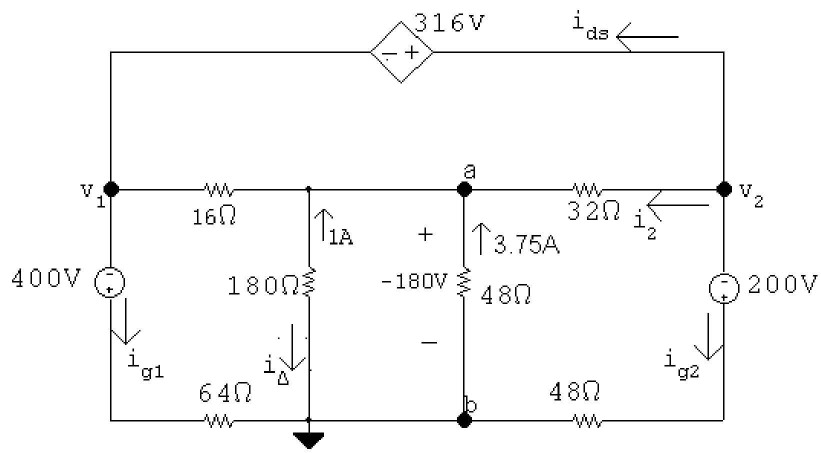
$$R_{Th} = \frac{-360}{-7.5} = 48\,\Omega$$



For maximum power transfer $\quad R_o = R_{Th} = 48\,\Omega$

[**b**] $p_{max} = \dfrac{180^2}{48} = 675\,\text{W}$

[**c**] The problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is −1 A, and hence $316i_\Delta$ is −316 V.



Using the node voltage method to find v_1 and v_2 yields

$$\frac{v_1 + 400}{64} + \frac{v_1 + 180}{16} + \frac{v_2 + 180}{32} + \frac{v_2 + 200}{48} = 0$$

$$v_2 - v_1 = 316$$

Solving, $v_1 = -336$ V; $\qquad v_2 = -20\,\text{V}$.

It follows that

$$i_{g_1} = \frac{-336+400}{64} = 1\,\text{A}$$

$$i_{g_2} = \frac{-20+200}{48} = 3.75\,\text{A}$$

$$i_2 = \frac{-20+180}{32} = 5\,\text{A}$$

$$i_{\text{ds}} = -5 - 3.75 = -8.75\,\text{A}$$

$$p_{400\text{V}} = -400 i_{g_1} = -400\,\text{W}$$

$$p_{200\text{V}} = -200 i_{g_2} = -750\,\text{W}$$

$$p_{\text{ds}} = 316 i_{\text{ds}} = -2765\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = 400 + 750 + 2765 = 3915\,\text{W}$$

$$\therefore \quad \%\ \text{delivered} = \frac{675}{3915}(100) = 17.24\%$$

$\therefore$ 17.24% of developed power is delivered to load

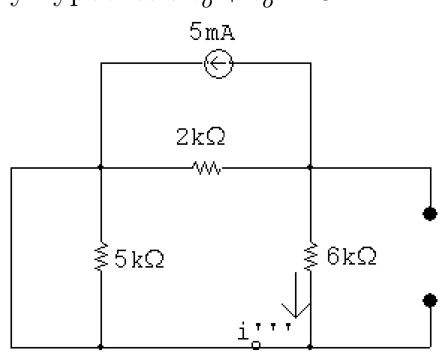
[d] The resistor from Appendix H that is closest to the Thévenin resistance is 47 Ω. To calculate the power delivered to a 47 Ω load resistor, calculate the current using the Thévenin circuit and use it to find the power delivered to the load resistor:

$$i_{47} = \frac{-360}{47+48} = 3.7895\,\text{A}$$

$$p_{47} = 47(3.7895)^2 = 674.9\,\text{W}$$

Thus, using a 47 Ω resistor selected from Appendix H will cause 674.9 W of power to be delivered to the load, compared to the maximum power of 675 W that will be delivered if a 48 Ω resistor is used.

P 4.92 **[a]** By hypothesis $i_o' + i_o'' = 3$ mA.



$$i_o''' = -5\frac{(2)}{(8)} = -1.25\,\text{mA}; \qquad \therefore \quad i_o = 3.5 - 1.25 = 2.25\,\text{mA}$$

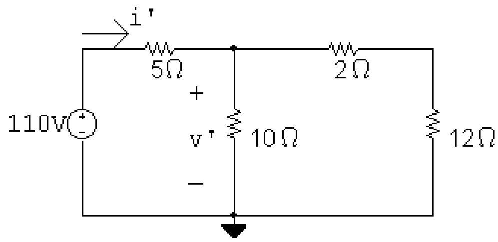
[**b**] With all three sources in the circuit write a single node voltage equation.

$$\frac{v_b}{6} + \frac{v_b - 8}{2} + 5 - 10 = 0$$

$$\therefore \quad v_b = 13.5\,\text{V}$$

$$i_o = \frac{v_b}{6} = 2.25\,\text{mA}$$

P 4.93 [**a**] 110 V source acting alone:

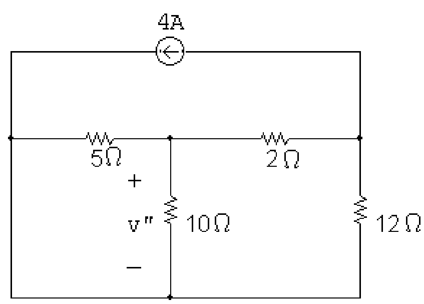


$$R_e = \frac{10(14)}{24} = \frac{35}{6}\,\Omega$$

$$i' = \frac{110}{5 + 35/6} = \frac{132}{13}\,\text{A}$$

$$v' = \left(\frac{35}{6}\right)\left(\frac{132}{13}\right) = \frac{770}{13}\,\text{V} = 59.231\,\text{V}$$

4 A source acting alone:

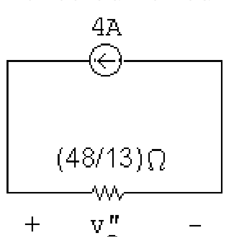


$$5\,\Omega \| 10\,\Omega = 50/15 = 10/3\,\Omega$$

$$10/3 + 2 = 16/3\,\Omega$$

$$16/3 \| 12 = 48/13\,\Omega$$

Hence our circuit reduces to:



It follows that

$$v_a'' = 4(48/13) = (192/13)\,\text{V}$$

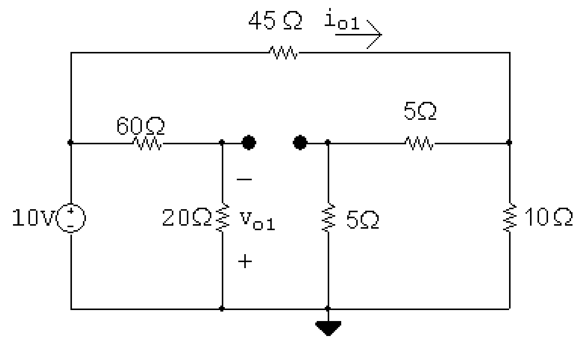
and

$$v'' = \frac{-v_a''}{(16/3)}(10/3) = -\frac{5}{8}v_a'' = -(120/13)\,\text{V} = -9.231\,\text{V}$$

$$\therefore \qquad v = v' + v'' = \frac{770}{13} - \frac{120}{13} = 50\,\text{V}$$

[b] $p = \dfrac{v^2}{10} = 250\,\text{W}$

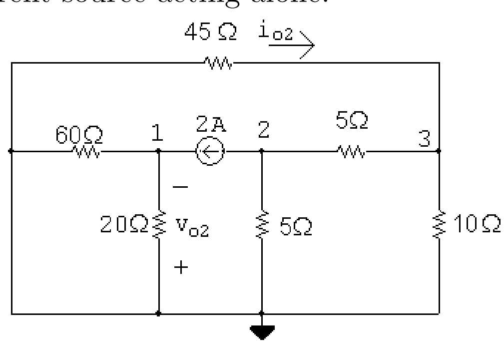
P 4.94 Voltage source acting alone:



$$i_{o1} = \frac{10}{45 + (5+5)\|10} = \frac{10}{45+5} = 0.2\,\text{A}$$

$$v_{o1} = \frac{20}{20+60}(-10) = -2.5\,\text{V}$$

Current source acting alone:



$$\frac{v_2}{5} + 2 + \frac{v_2 - v_3}{5} = 0$$

$$\frac{v_3}{10} + \frac{v_3 - v_2}{5} + \frac{v_3}{45} = 0$$

Solving, $v_2 = -7.25\text{ V} = v_{o2}$; $\qquad v_3 = -4.5\,\text{V}$

$$i_{o2} = -\frac{v_3}{45} = -0.1\,\text{A}$$

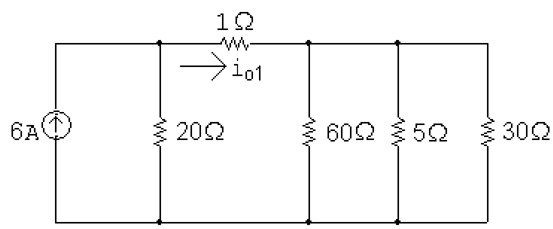
$$i_{20} = \frac{60\|20}{20}(2) = 1.5\,\text{A}$$

$$v_{o2} = -20i_{20} = -20(1.5) = -30\,\text{V}$$

$$\therefore \quad v_o = v_{o1} + v_{o2} = -2.5 - 30 = -32.5\,\text{V}$$

$$i_o = i_{o1} + i_{o2} = 0.2 + 0.1 = 0.3\,\text{A}$$

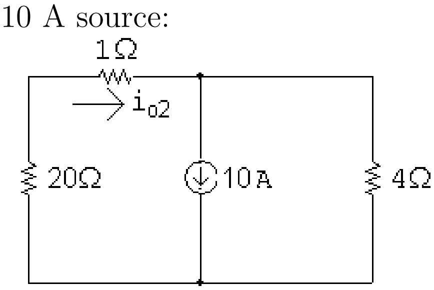
P 4.95 6 A source:



$$30\,\Omega \| 5\,\Omega \| 60\,\Omega = 4\,\Omega$$

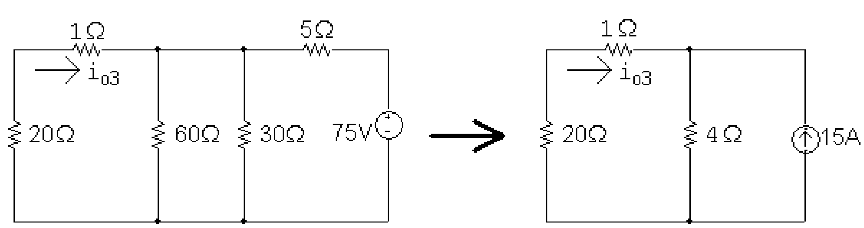
$$\therefore \quad i_{o1} = \frac{20}{20+5}(6) = 4.8\,\text{A}$$

10 A source:



$$i_{o2} = \frac{4}{25}(10) = 1.6\,\text{A}$$

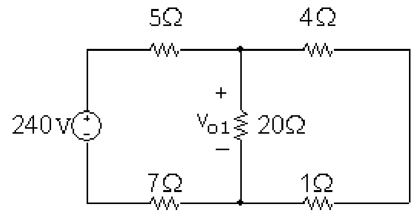
75 V source:



$$i_{o3} = -\frac{4}{25}(15) = -2.4\,\text{A}$$

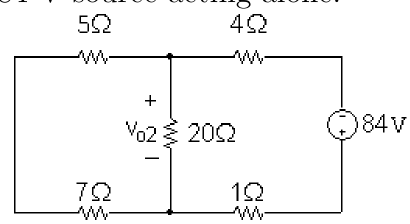
$$i_o = i_{o1} + i_{o2} + i_{o3} = 4.8 + 1.6 - 2.4 = 4\,\text{A}$$

P 4.96 240 V source acting alone:



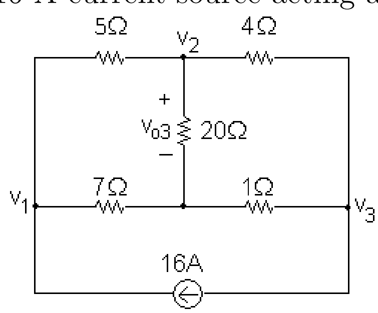
$$v_{o1} = \frac{20\|5}{5 + 7 + 20\|5}(240) = 60\,\text{V}$$

84 V source acting alone:



$$v_{o2} = \frac{20\|12}{1 + 4 + 20\|12}(-84) = -50.4\,\text{V}$$

16 A current source acting alone:



$$\frac{v_1 - v_2}{5} + \frac{v_1}{7} - 16 = 0$$

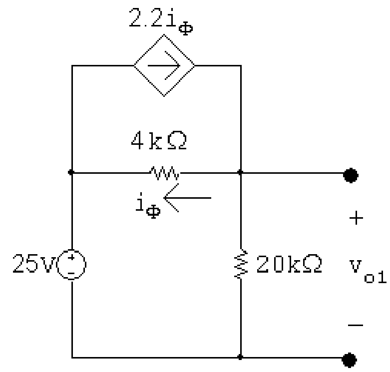
$$\frac{v_2 - v_1}{5} + \frac{v_2}{20} + \frac{v_2 - v_3}{4} = 0$$

$$\frac{v_3 - v_2}{4} + \frac{v_3}{1} + 16 = 0$$

Solving, $v_2 = 18.4\,\text{V} = v_{o3}$. Therefore,

$$v_o = v_{o1} + v_{o2} + v_{o3} = 60 - 50.4 + 18.4 = 28\,\text{V}$$

P 4.97 Voltage source acting alone:

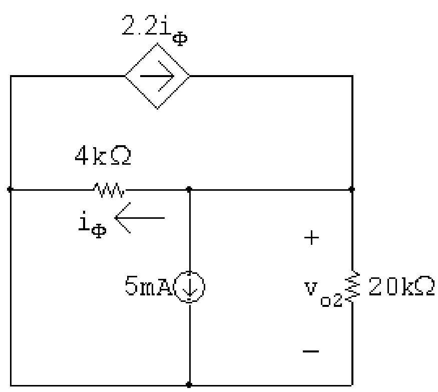


$$\frac{v_{o1} - 25}{4000} + \frac{v_{o1}}{20{,}000} - 2.2\left(\frac{v_{o1} - 25}{4000}\right) = 0$$

Simplifying $\qquad 5v_{o1} - 125 + v_{o1} - 11v_{o1} + 275 = 0$

$\therefore \quad v_{o1} = 30\,\text{V}$

Current source acting alone:



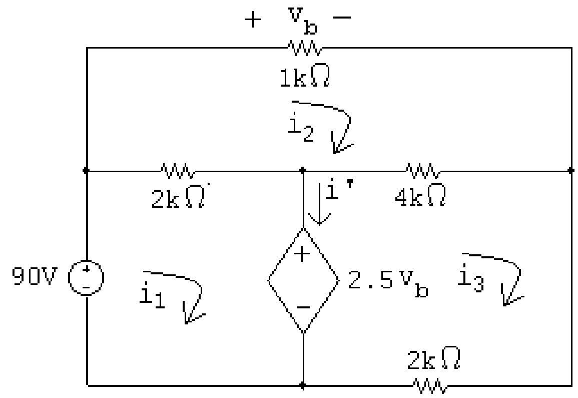
$$\frac{v_{o2}}{4000} + \frac{v_{o2}}{20{,}000} + 0.005 - 2.2\left(\frac{v_{o2}}{4000}\right) = 0$$

Simplifying $\qquad 5v_{o2} + v_{o2} + 100 - 11v_{o2} = 0$

$\therefore \quad v_{o2} = 20\,\text{V}$

$v_o = v_{o1} + v_{o2} = 30 + 20 = 50\,\text{V}$

P 4.98 90-V source acting alone:



$$2000(i_1 - i_2) + 2.5v_b = 90$$

$$-2000i_1 + 7000i_2 - 4000i_3 = 0$$

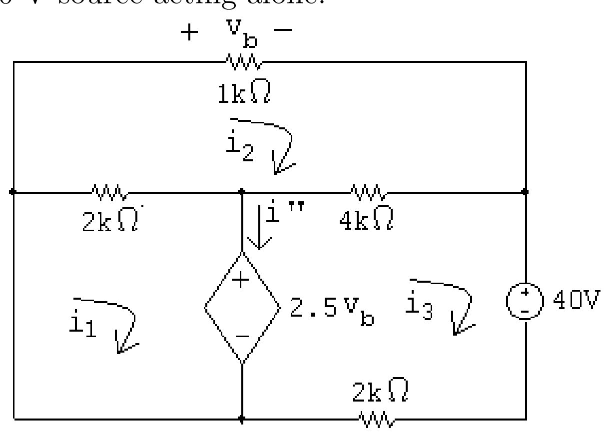
$$-4000i_2 + 6000i_3 - 2.5v_b = 0$$

$$v_b = 1000i_2$$

Solving,

$i_1 = 37.895\,\text{mA};\qquad i_3 = 30.789\,\text{mA};\qquad i' = i_1 - i_3 = 7.105\,\text{mA}$

40-V source acting alone:



$$2000(i_1 - i_2) + 2.5v_b = 0$$

$$-2000i_1 + 7000i_2 - 4000i_3 = 0$$

$$-4000i_2 + 6000i_3 - 2.5v_b = -40$$

$v_b = 1000i_2$

Solving,

$i_1 = 2.105\,\text{mA}; \qquad i_3 = -15.789\,\text{mA}; \qquad i'' = i_1 - i_3 = 17.895\,\text{mA}$

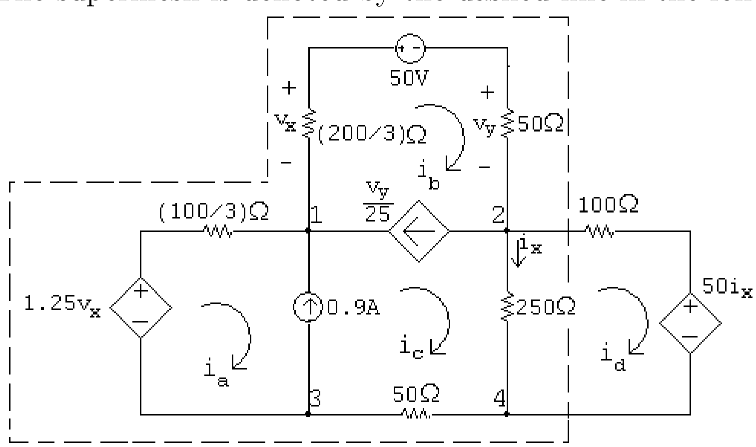
Hence, $\quad i = i' + i'' = 7.105 + 17.895 = 25\,\text{mA}$

P 4.99 **[a]** In studying the circuit in Fig. P4.99 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node-voltage approach will require solving three node voltage equations along with equations involving v_x, v_y, and i_x.

The mesh-current approach will require writing one mesh equation and one supermesh equation plus five constraint equations involving the five sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 50 V source, we will retain the mesh current i_{b} and eliminate the mesh currents i_{a}, i_{c} and i_{d}.

The supermesh is denoted by the dashed line in the following figure.



[b] Summing the voltages around the supermesh yields

$$-1.25v_x + (100/3)i_a + (200/3)i_b + 50 + 50i_b + 250(i_c - i_d) + 50i_c = 0$$

The remaining mesh equation is

$$50i_x + 350i_d - 250i_c = 0$$

The constraint equations are

$$\frac{v_y}{25} = i_b - i_c; \qquad 0.9 = i_c - i_a; \qquad v_x = -(200/3)i_b$$

$$v_y = 50i_b; \qquad i_x = i_c - i_d$$

Solving,

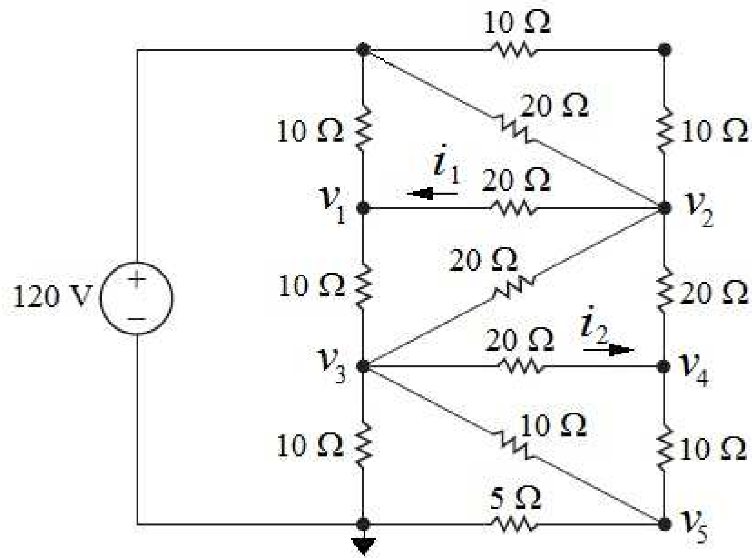
$i_a = -0.3\,\text{A}; \qquad i_b = -0.6\,\text{A}; \qquad i_c = 0.6\,\text{A}; \qquad i_d = 0.4\,\text{A}$

Finally,

$p_{50\text{V}} = 50 i_b = -30\,\text{W}$

The 50 V source delivers 30 W of power.

P 4.100



At v_1: $$\frac{v_1 - 120}{10} + \frac{v_1 - v_2}{20} + \frac{v_1 - v_3}{10} = 0$$

At v_2: $$\frac{v_2 - 120}{20} + \frac{v_2 - 120}{20} + \frac{v_2 - v_1}{20} + \frac{v_1 - v_3}{20} + \frac{v_2 - v_4}{20} = 0$$

At v_3: $$\frac{v_3 - v_1}{10} + \frac{v_3 - v_2}{20} + \frac{v_3 - v_4}{20} + \frac{v_3}{10} + \frac{v_3 - v_5}{10} = 0$$

At v_4: $$\frac{v_4 - v_2}{20} + \frac{v_4 - v_3}{20} + \frac{v_4 - v_5}{10} = 0$$

At v_5: $$\frac{v_5 - v_4}{10} + \frac{v_5 - v_3}{10} + \frac{v_5}{5} = 0$$

A calculator solution yields

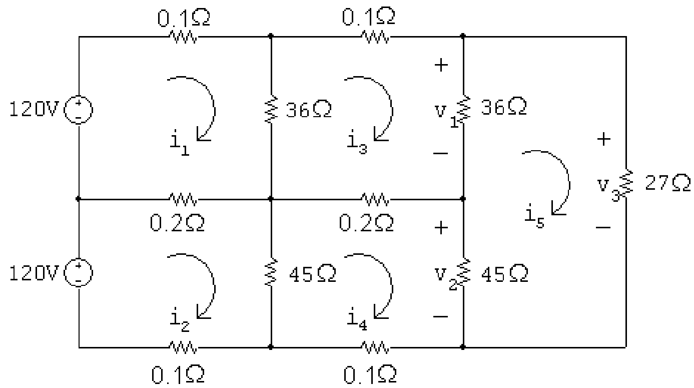
$v_1 = 80$ A; $\qquad v_2 = 80$ A;

$v_3 = 40$ A; $\qquad v_4 = 40$ A;

$v_5 = 20$ A.

$$\therefore \quad i_1 = \frac{v_2 - v_1}{20} = 0\,\text{A}; \qquad i_1 = \frac{v_3 - v_4}{20} = 0\,\text{A}$$

P 4.101

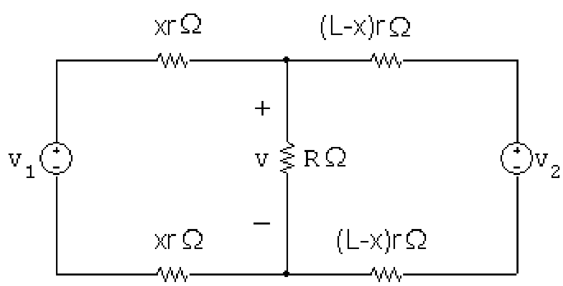


The mesh equations are:

$$\begin{aligned}
i_1(36.3) + i_2(-0.2) + i_3(-36) + i_4(0) + i_5(0) &= 120\\
i_1(-0.2) + i_2(45.3) + i_3(0) + i_4(-45) + i_5(0) &= 120\\
i_1(-36) + i_2(0) + i_3(72.3) + i_4(-0.2) + i_5(-36) &= 0\\
i_1(0) + i_2(-45) + i_3(-0.2) + i_4(90.3) + i_5(-45) &= 0\\
i_1(0) + i_2(0) + i_3(-36) + i_4(-45) + i_5(108) &= 0
\end{aligned}$$

Solving,
$i_1 = 15.226\,\text{A}; \quad i_2 = 13.953\,\text{A}; \quad i_3 = 11.942\,\text{A}; \quad i_4 = 11.314\,\text{A}; \quad i_5 = 8.695\,\text{A}$
Find the requested voltages:
$v_1 = 36(i_3 - i_5) = 118.6\,\text{V}$
$v_2 = 45(i_4 - i_5) = 117.8\,\text{V}$
$v_3 = 27i_5 = 234.8\,\text{V}$

P 4.102 **[a]**



$$\frac{v - v_1}{2xr} + \frac{v}{R} + \frac{v - v_2}{2r(L-x)} = 0$$

$$v\left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(L-x)}\right] = \frac{v_1}{2xr} + \frac{v_2}{2r(L-x)}$$

$$v = \frac{v_1 RL + xR(v_2 - v_1)}{RL + 2rLx - 2rx^2}$$

[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx} = \frac{(RL + 2rLx - 2rx^2)R(v_2 - v_1) - [v_1 RL + xR(v_2 - v_1)]2r(L - 2x)}{D^2}$$

$$\frac{dv}{dx} = 0 \quad \text{when numerator is zero.}$$

The numerator simplifies to

$$x^2 + \frac{2Lv_1}{(v_2 - v_1)}x + \frac{RL(v_2 - v_1) - 2rv_1L^2}{2r(v_2 - v_1)} = 0$$

Solving for the roots of the quadratic yields

$$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1v_2 - \frac{R}{2rL}(v_2 - v_1)^2}\right\}$$

[c] $$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1v_2 - \frac{R}{2rL}(v_1 - v_2)^2}\right\}$$

$v_2 = 1200\,\text{V}, \qquad v_1 = 1000\,\text{V}, \qquad L = 16$ km

$r = 5 \times 10^{-5}\,\Omega/m; \qquad R = 3.9\,\Omega$

$$\frac{L}{v_2 - v_1} = \frac{16{,}000}{1200 - 1000} = 80; \qquad v_1v_2 = 1.2 \times 10^6$$

$$\frac{R}{2rL}(v_1 - v_2)^2 = \frac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$$

$$x = 80\{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\}$$

$$= 80\{-1000 \pm 1050\} = 80(50) = 4000 \text{ m}$$

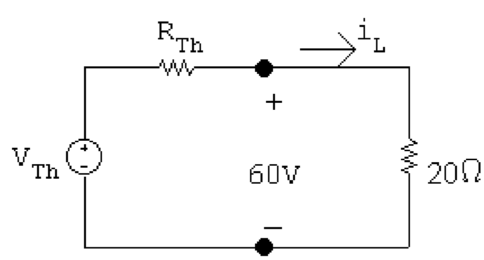
[d]

$$v_{\min} = \frac{v_1RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2}$$

$$= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)}$$

$$= 975\,\text{V}$$

P 4.103 **[a]**



$$v_{oc} = V_{\text{Th}} = 75\,\text{V}; \qquad i_L = \frac{60}{20} = 3\,\text{A}; \qquad i_L = \frac{75 - 60}{R_{\text{Th}}} = \frac{15}{R_{\text{Th}}}$$

Therefore $\quad R_{\text{Th}} = \dfrac{15}{3} = 5\,\Omega$

[b] $i_L = \dfrac{v_o}{R_L} = \dfrac{V_{\text{Th}} - v_o}{R_{\text{Th}}}$

Therefore $\quad R_{\text{Th}} = \dfrac{V_{\text{Th}} - v_o}{v_o/R_L} = \left(\dfrac{V_{\text{Th}}}{v_o} - 1\right) R_L$

P 4.104 $\dfrac{dv_1}{dI_{g1}} = \dfrac{-R_1[R_2(R_3 + R_4) + R_3R_4]}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$

$$\frac{dv_1}{dI_{g2}} = \frac{R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g1}} + \frac{-R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g2}} = \frac{R_3R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

P 4.105 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g1}} = \frac{-25[5(125) + 3750]}{30(125) + 3750} = -\frac{175}{12}\ \text{V/A} = -14.5833\ \text{V/A}$$

and

$$\frac{dv_2}{dI_{g1}} = \frac{-(25)(50)(75)}{30(125) + 3750} = -12.5\ \text{V/A}$$

By hypothesis, $\Delta I_{g1} = 11 - 12 = -1\,\text{A}$

$$\therefore \quad \Delta v_1 = (-\frac{175}{12})(-1) = \frac{175}{12} = 14.583\,\text{V}$$

Thus, $v_1 = 25 + 14.583 = 39.583\,\text{V}$
Also,

$$\Delta v_2 = (-12.5)(-1) = 12.5\,\text{V}$$

Thus, $v_2 = 90 + 12.5 = 102.5\,\text{V}$
The PSpice solution is

$$v_1 = 39.583\,\text{V}$$

and

$$v_2 = 102.5\,\text{V}$$

These values are in agreement with our predicted values.

P 4.106 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g2}} = \frac{(25)(50)(75)}{30(125)+3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g2}} = \frac{(50)(75)(30)}{30(125)+3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_{g2} = 17 - 16 = 1\,\text{A}$

$\therefore \quad \Delta v_1 = (12.5)(1) = 12.5\,\text{V}$

Thus, $v_1 = 25 + 12.5 = 37.5\,\text{V}$
Also,

$\Delta v_2 = (15)(1) = 15\,\text{V}$

Thus, $v_2 = 90 + 15 = 105\,\text{V}$
The PSpice solution is

$v_1 = 37.5\,\text{V}$

and

$v_2 = 105\,\text{V}$

These values are in agreement with our predicted values.

P 4.107 From the solutions to Problems 4.104 — 4.106 we have

$$\frac{dv_1}{dI_{g1}} = -\frac{175}{12} \text{ V/A}; \qquad \frac{dv_1}{dI_{g2}} = 12.5 \text{ V/A}$$

$$\frac{dv_2}{dI_{g1}} = -12.5 \text{ V/A}; \qquad \frac{dv_2}{dI_{g2}} = 15 \text{ V/A}$$

By hypothesis,

$\Delta I_{g1} = 11 - 12 = -1\,\text{A}$

$\Delta I_{g2} = 17 - 16 = 1\,\text{A}$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833\,\text{V}$$

$\Delta v_2 = 12.5 + 15 = 27.5\,\text{V}$

Hence

$v_1 = 25 + 27.0833 = 52.0833\,\text{V}$

$v_2 = 90 + 27.5 = 117.5\,\text{V}$

The PSpice solution is

$v_1 = 52.0830\,\text{V}$

and

$v_2 = 117.5\,\text{V}$

These values are in agreement with our predicted values.

P 4.108 By hypothesis,

$\Delta R_1 = 27.5 - 25 = 2.5\,\Omega$

$\Delta R_2 = 4.5 - 5 = -0.5\,\Omega$

$\Delta R_3 = 55 - 50 = 5\,\Omega$

$\Delta R_4 = 67.5 - 75 = -7.5\,\Omega$

So

$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168\,\text{V}$

$\therefore \quad v_1 = 25 + 4.9168 = 29.9168\,\text{V}$

$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1\,\text{V}$

$\therefore \quad v_2 = 90 - 1.1 = 88.9\,\text{V}$

The PSpice solution is

$v_1 = 29.6710\,\text{V}$

and

$v_2 = 88.5260\,\text{V}$

Note our predicted values are within a fraction of a volt of the actual values.

5

The Operational Amplifier

Assessment Problems

AP 5.1 [**a**] This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \qquad \text{so} \qquad v_o = -5v_s$$

v_s(V)	0.4	2.0	3.5	−0.6	−1.6	−2.4
v_o(V)	−2.0	−10.0	−15.0	3.0	8.0	10.0

Two of the values, 3.5 V and −2.4 V, cause the op amp to saturate.

[**b**] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3\ \text{V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \qquad v_s = -2\ \text{V}$$

Therefore $\quad -2 \leq v_s \leq 3\ \text{V}$

AP 5.2 From Assessment Problem 5.1

$$v_o = (-R_f/R_i)v_s = (-R_x/16{,}000)v_s = (-R_x/16{,}000)(-0.640)$$

$$= 0.64R_x/16{,}000 = 4 \times 10^{-5}R_x$$

Use the negative power supply value to determine one limit on the value of R_x:

$$4 \times 10^{-5}R_x = -15 \qquad \text{so} \qquad R_x = -15/4 \times 10^{-5} = -375\,\text{k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x:

$$4 \times 10^{-5} R_x = 10 \qquad \text{so} \qquad R_x = 10/4 \times 10^{-5} = 250\,\text{k}\Omega$$

Therefore,

$$0 \leq R_x \leq 250\,\text{k}\Omega$$

AP 5.3 **[a]** This is an inverting summing amplifier so

$$v_o = (-R_f/R_\text{a})v_\text{a} + (-R_f/R_\text{b})v_\text{b} = -(250/5)v_\text{a} - (250/25)v_\text{b} = -50v_\text{a} - 10v_\text{b}$$

Substituting the values for v_a and v_b:

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_\text{a} - 10(0.25) = -50v_\text{a} - 2.5 = -10 \text{ V}$$

$$\text{Therefore} \quad 50v_\text{a} = 7.5, \quad \text{so} \quad v_\text{a} = 0.15 \text{ V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_\text{b} = -5 - 10v_\text{b} = -10 \text{ V};$$

$$\text{Therefore} \quad 10v_\text{b} = 5, \quad \text{so} \quad v_\text{b} = 0.5 \text{ V}$$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.
Repeat part (a):

$$v_o = -50v_\text{a} + 10v_\text{b} = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v_o = -50v_\text{a} + 2.5 = -10 \text{ V}; \qquad 50v_\text{a} = 12.5, \quad v_\text{a} = 0.25 \text{ V}$$

Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_\text{b} = 15 \text{ V}; \qquad 10v_\text{b} = 20; \qquad v_\text{b} = 2.0 \text{ V}$$

AP 5.4 **[a]** Write a node voltage equation at v_n; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63{,}000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$$14v_n + v_n - v_o = 0 \qquad \text{so} \qquad v_o = 15v_n$$

Now use voltage division to calculate v_p. We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R_x resistor:

$$v_p = \frac{R_x}{15{,}000 + R_x}(0.400)$$

Now substitute the value $R_x = 60$ kΩ:

$$v_p = \frac{60{,}000}{15{,}000 + 60{,}000}(0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_0

$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8 \text{ V}$$

[b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15\left(\frac{0.4R_x}{15{,}000 + R_x}\right) = 5$$

$$15(0.4R_x) = 5(15{,}000 + R_x) \quad \text{so} \quad R_x = 75\,\text{k}\Omega$$

AP 5.5 **[a]** Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_\text{b} - \frac{50}{10}v_\text{a}$$

Simplify this expression and subsitute in the value for v_b:

$$v_o = 5(v_\text{b} - v_\text{a}) = 20 - 5v_\text{a}$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_\text{a} = 10 \text{ V} \quad \text{so} \quad v_\text{a} = 2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_\text{a} = -10 \text{ V} \quad \text{so} \quad v_\text{a} = 6 \text{ V}$$

Therefore $\quad 2 \leq v_\text{a} \leq 6$ V

[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)} v_{\rm b} - 5v_{\rm a} = 4v_{\rm b} - 5v_{\rm a}$$

Now substitute the value for $v_{\rm b}$:

$$v_o = 4(4) - 5v_{\rm a} = 16 - 5v_{\rm a}$$

Set this expression for v_o to the positive power supply value:

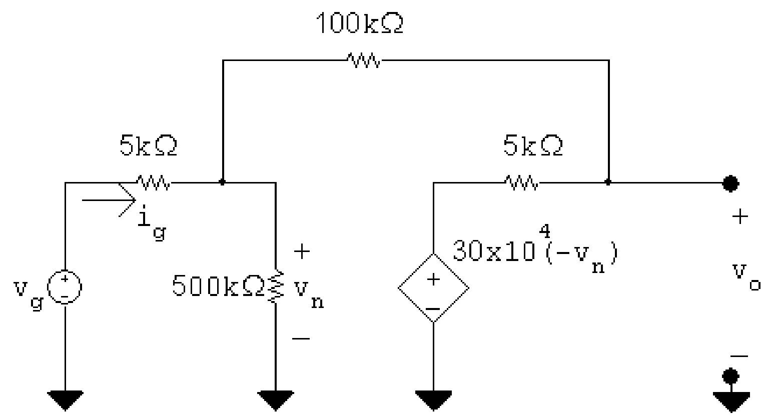
$$16 - 5v_{\rm a} = 10 \text{ V} \quad \text{so} \quad v_{\rm a} = 1.2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_{\rm a} = -10 \text{ V} \quad \text{so} \quad v_{\rm a} = 5.2 \text{ V}$$

Therefore $\quad 1.2 \le v_{\rm a} \le 5.2 \text{ V}$

AP 5.6 **[a]** Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v_n}{500{,}000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_0 = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300{,}000(-v_n)}{5000} + \frac{v_o - v_n}{100{,}000} = 0$$

Multiply through by 100,000 and simplify:

$$20v_o + 6 \times 10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6 \times 10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o:

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6{,}000{,}445.2$$

$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985 v_g; \qquad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n:

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v_g$$

$$v_g = 1 \text{ V}, \qquad v_n = 69.995\,\mu \text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35\,\Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

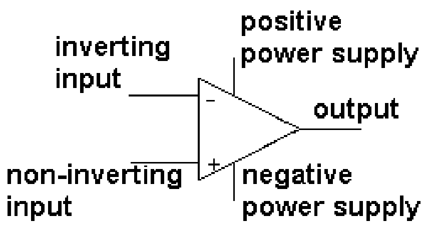
$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$
Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000\,\Omega$$

Problems

P 5.1 [a] The five terminals of the op amp are identified as follows:



[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.

[d] Write a node voltage equation at v_n:

$$\frac{v_n - 2}{4000} + \frac{v_n - v_o}{12{,}000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-2}{4000} - \frac{v_o}{12{,}000} = 0 \quad \text{so} \quad v_o = -6 \text{ V}$$

P 5.2 [a] Let the value of the voltage source be v_s:

$$\frac{v_n - v_s}{4000} + \frac{v_n - v_o}{12{,}000} = 0$$

But $v_n = v_p = 0$. Therefore,

$$v_o = -\frac{12{,}000}{4000}v_s = -3v_s$$

When $v_s = -6$ V, $v_o = -3(-6) = 18$ V; saturates at $v_o = 15$ V.
When $v_s = -3.5$ V, $v_o = -3(-3.5) = 10.5$ V.
When $v_s = -1.25$ V, $v_o = -3(-1.25) = 3.75$ V.
When $v_s = 2.4$ V, $v_o = -3(2.4) = -7.2$ V.
When $v_s = 4.5$ V, $v_o = -3(4.5) = -13.5$ V.
When $v_s = 5.4$ V, $v_o = -3(5.4) = -16.2$ V; saturates at $v_o = -15$ V.

[b] $$-3v_s = 15 \qquad \text{so} \qquad v_s = \frac{15}{-3} = -5 \text{ V}$$

$$-3v_s = -15 \qquad \text{so} \qquad v_s = \frac{-15}{-3} = 5 \text{ V}$$

The range of source voltages that avoids saturation is $-5 \text{ V} \le v_s \le 5$ V.

P 5.3 $$v_p = \frac{5000}{5000 + 10{,}000}(6) = 2\ \text{V} = v_n$$

$$\frac{v_n + 5}{3000} + \frac{v_n - v_o}{6000} = 0$$

$$2(2+5) + (2 - v_o) = 0$$

$$v_o = 16\ \text{V}$$

$$i_\text{L} = \frac{v_o}{8000} = \frac{16}{8000} = 2000 \times 10^{-6}$$

$$i_\text{L} = 2\ \text{mA}$$

P 5.4 $$\frac{v_\text{b} - v_\text{a}}{5000} + \frac{v_\text{b} - v_o}{40{,}000} = 0, \qquad \text{therefore} \quad v_o = 9v_\text{b} - 8v_\text{a}$$

[a] $v_\text{a} = 1.5$ V, $v_\text{b} = 0$ V, $v_o = -12$ V

[b] $v_\text{a} = -0.5$ V, $v_\text{b} = 0$ V, $v_o = 4$ V

[c] $v_\text{a} = 1$ V, $v_\text{b} = 2.5$ V, $v_o = 14.5$ V

[d] $v_\text{a} = 2.5$ V, $v_\text{b} = 1$ V, $v_o = -11$ V

[e] $v_\text{a} = 2.5$ V, $v_\text{b} = 0$ V, $v_o = -20$ V (saturates at -16 V)

[f] If $v_\text{b} = 2$ V, $v_o = 18 - 8v_\text{a} = \pm 16$

$$\therefore \quad 0.25\ \text{V} \leq v_\text{a} \leq 4.25\ \text{V}$$

P 5.5 $$v_o = -(0.5 \times 10^{-3})(10{,}000) = -5\ \text{V}$$

$$\therefore \quad i_o = \frac{v_o}{5000} = \frac{-5}{5000} = -1\,\text{mA}$$

P 5.6 **[a]** $$i_\text{a} = \frac{240 \times 10^{-3}}{8000} = 30\,\mu\text{A}$$

$$v_1 = -40 \times 10^3 i_2 = -3\,\text{V}$$

[b] $$\frac{0 - v_\text{a}}{60{,}000} = i_\text{a} \quad \text{so} \quad v_\text{a} = -60{,}000 i_\text{a} = -1.8\ \text{V}$$

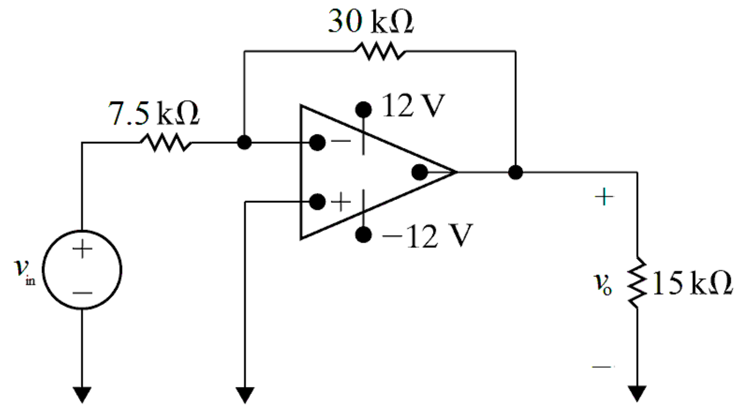
[c] $$\frac{v_\text{a}}{60{,}000} + \frac{v_\text{a}}{40{,}000} + \frac{v_\text{a} - v_\text{o}}{30{,}000} = 0$$

$$\therefore \quad v_\text{o} = 2.25 v_\text{a} = -4.05\ \text{V}$$

[d] $$i_\text{o} = \frac{-v_\text{o}}{20{,}000} + \frac{v_\text{a} - v_\text{o}}{30{,}000} = 277.5\,\mu\text{A}$$

P 5.7 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the 2.2 MΩ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V. Therefore the voltmeter reads 7.7 V.

P 5.8 [a] $\dfrac{30{,}000}{R_{\rm in}} = 4 \qquad \text{so} \qquad R_{\rm in} = \dfrac{30{,}000}{4} = 7500 = 7.5\,\text{k}\Omega$



[b] $-4v_{\rm in} = 12 \qquad \text{so} \qquad v_{\rm in} = \dfrac{12}{-4} = -3\ \text{V}$

$-4v_{\rm in} = -12 \qquad \text{so} \qquad v_{\rm in} = \dfrac{12}{4} = 3\ \text{V}$

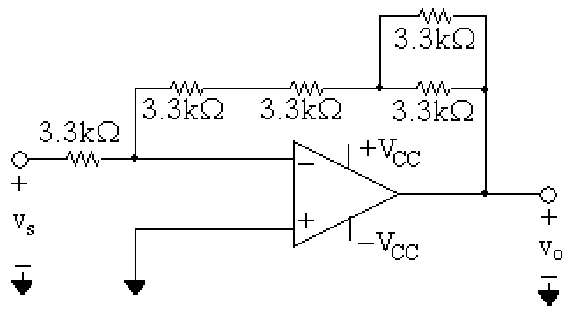
$\therefore \qquad -3\ \text{V} \leq v_{\rm in} \leq 3\ \text{V}$

[c] $-\dfrac{R_{\rm f}}{7500}(2) = -12 \qquad \text{so} \qquad R_{\rm f} = 45\,\text{k}\Omega$

$$\left|\frac{v_{\rm o}}{v_{\rm in}}\right| = \frac{R_{\rm f}}{R_{\rm in}} = \frac{45{,}000}{7500} = 6$$

The amplifier has a gain of 6.

P 5.9 [a] The gain of an inverting amplifier is the negative of the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 2.5, the feedback resistor must be 2.5 times as large as the input resistor. There are many possible designs that use a resistor value chosen from Appendix H. We present one here that uses 3.3 kΩ resistors. Use a single 3.3 kΩ resistor as the input resistor. Then construct a network of 3.3 kΩ resistors whose equivalent resistance is 2.5(3.3) = 8.25 kΩ by connecting two resistors in parallel and connecting the parallel resistors in series with two other resistors. The resulting circuit is shown here:



[b] To amplify signals in the range −2 V to 3 V without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain.

$$-2.5(-2) = 5 \text{ V} \qquad \text{and} \qquad -2.5(3) = -7.5 \text{ V}$$

Thus, the power supplies should have values of −7.5 V and 5 V.

P 5.10 [a] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{2000} + \frac{0 - v_\Delta}{50{,}000} = 0$$

$$-25v_g - v_\Delta = 0, \qquad \therefore \quad v_\Delta = -25(40 \times 10^{-3}) = -1 \text{ V}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{50{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 2v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 2 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore \quad v_o = -1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

$$\text{When} \quad \alpha = 0.2, \qquad v_o = -1(1 + 1.6 + 4) = -6.6 \text{ V}$$

$$\text{When} \quad \alpha = 1, \qquad v_o = -1(1 + 0 + 0) = -1 \text{ V}$$

$$\therefore \quad -6.6 \text{ V} \le v_o \le -1 \text{ V}$$

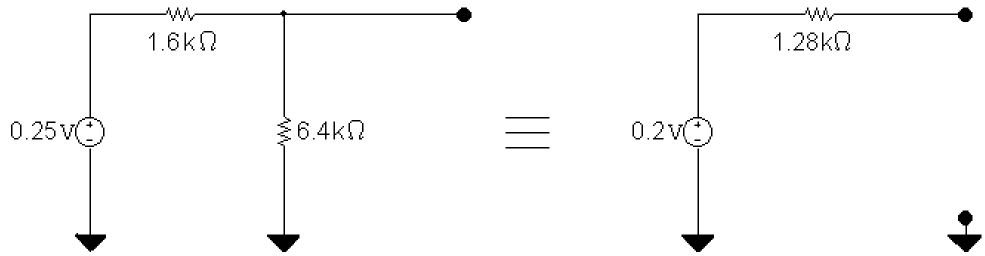
[b] $$-1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right] = -7$$

$$\alpha + 2\alpha(1-\alpha) + (1-\alpha) = 7\alpha$$

$$\alpha + 2\alpha - 2\alpha^2 + 1 - \alpha = 7\alpha$$

$$\therefore \quad 2\alpha^2 + 5\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.186$$

P 5.11 [a] Replace the combination of v_g, 1.6 kΩ, and the 6.4 kΩ resistors with its Thévenin equivalent.



Then $v_o = \dfrac{-[12 + \sigma 50]}{1.28}(0.20)$

At saturation $v_o = -5$ V; therefore

$-\left(\dfrac{12 + \sigma 50}{1.28}\right)(0.2) = -5, \quad \text{or} \quad \sigma = 0.4$

Thus for $0 \leq \sigma \leq 0.40$ the operational amplifier will not saturate.

[b] When $\sigma = 0.272, \qquad v_o = \dfrac{-(12 + 13.6)}{1.28}(0.20) = -4$ V

Also $\dfrac{v_o}{10} + \dfrac{v_o}{25.6} + i_o = 0$

$\therefore \quad i_o = -\dfrac{v_o}{10} - \dfrac{v_o}{25.6} = \dfrac{4}{10} + \dfrac{4}{25.6}\,\text{mA} = 556.25\,\mu\text{A}$

P 5.12 [a] This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\dfrac{220}{44}v_\text{a} - \dfrac{220}{27.5}v_\text{b} - \dfrac{220}{80}v_\text{c} = -5 - 12 + 11 = -6$ V

[c] $v_o = -6 - 8v_\text{b} = \pm 10$

$\therefore \quad v_\text{b} = -0.5$ V when $v_o = 10$ V;

$v_\text{b} = 2$ V when $v_o = -10$ V

$\therefore \quad -0.5\text{ V} \leq v_\text{b} \leq 2\text{ V}$

P 5.13 $v_o = -\left[\dfrac{R_\text{f}}{4000}(0.2) + \dfrac{R_\text{f}}{5000}(0.15) + \dfrac{R_\text{f}}{20{,}000}(0.4)\right]$

$-6 = -0.1 \times 10^{-3}R_\text{f}; \qquad R_\text{f} = 60\,\text{k}\Omega; \qquad \therefore \quad 0 \leq R_\text{f} \leq 60\,\text{k}\Omega$

P 5.14 [a] Write a KCL equation at the inverting input to the op amp:

$$\frac{v_\text{d} - v_\text{a}}{60{,}000} + \frac{v_\text{d} - v_\text{b}}{20{,}000} + \frac{v_\text{d} - v_\text{c}}{36{,}000} + \frac{v_\text{d}}{270{,}000} + \frac{v_\text{d} - v_o}{180{,}000} = 0$$

Multiply through by 180,000, plug in the values of input voltages, and rearrange to solve for v_o:

$$v_o = 180{,}000\left(\frac{3}{60{,}000} + \frac{-3}{20{,}000} + \frac{1}{36{,}000} + \frac{6}{270{,}000} + \frac{6}{180{,}000}\right)$$

$$= -3 \text{ V}$$

[**b**] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{6-3}{60{,}000} + \frac{6-9}{20{,}000} + \frac{6-v_c}{36{,}000} + \frac{6}{270{,}000} + \frac{6-v_o}{180{,}000} = 0$$

Simplify and solve for v_o:

$$9 - 27 + 30 - 5v_c + 4 + 6 - v_o = 0 \quad \text{so} \quad v_o = 22 - 5v_c$$

Set v_o to the positive power supply voltage and solve for v_c:

$$22 - 5v_c = 10 \qquad \therefore \qquad v_c = 2.4 \text{ V}$$

Set v_o to the negative power supply voltage and solve for v_c:

$$22 - 5v_c = -10 \qquad \therefore \qquad v_c = 6.4 \text{ V}$$

Therefore,

$$2.4 \text{ V} \le v_c \le 6.4 \text{ V}$$

P 5.15 [**a**] $$\frac{6-3}{60{,}000} + \frac{6-9}{20{,}000} + \frac{6-5}{36{,}000} + \frac{6}{270{,}000} + \frac{6-v_o}{R_f} = 0$$

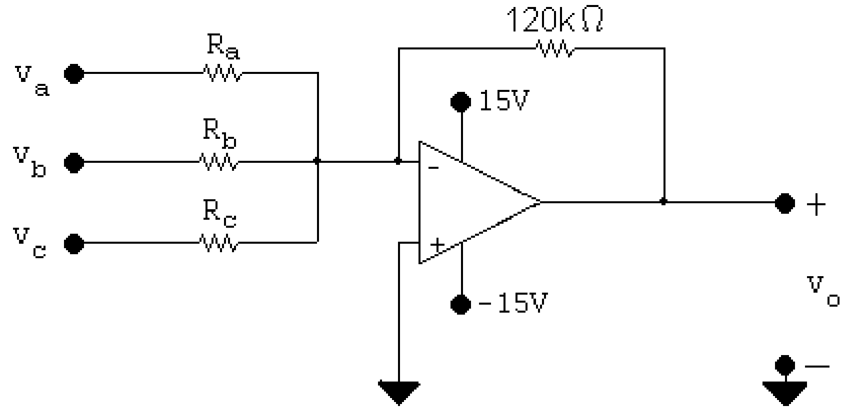
$$\frac{6-v_o}{R_f} = -5\times10^{-5} \quad \text{so} \quad R_f = \frac{6-v_o}{-5\times10^{-5}}$$

For $v_o = 10$ V, $R_f = 80\,\text{k}\Omega$

For $v_o = -10$ V, $R_f < 0$ so this solution is not possible.

[**b**] $$i_o = -(i_f + i_{16\text{k}}) = -\left[\frac{10-6}{80{,}000} + \frac{10}{16{,}000}\right] = -675\,\mu\text{A}$$

P 5.16 [a]



$$\frac{120{,}000}{R_\mathrm{a}} = 8 \qquad \text{so} \qquad R_\mathrm{a} = \frac{120{,}000}{8} = 15\,\mathrm{k}\Omega$$

$$\frac{120{,}000}{R_\mathrm{b}} = 5 \qquad \text{so} \qquad R_\mathrm{b} = \frac{120{,}000}{5} = 24\,\mathrm{k}\Omega$$

$$\frac{120{,}000}{R_\mathrm{c}} = 12 \qquad \text{so} \qquad R_\mathrm{c} = \frac{120{,}000}{12} = 10\,\mathrm{k}\Omega$$

[b] $-[8(2) + 5v_\mathrm{b} + 12(-1)] = -4 - 5v_\mathrm{b}$

$$-4 - 5v_\mathrm{b} = -15 \qquad \text{so} \qquad 5v_\mathrm{b} = 11 \qquad \text{thus} \qquad v_\mathrm{b} = \frac{11}{5} = 2.2\text{ V}$$

$$-4 - 5v_\mathrm{b} = 15 \qquad \text{so} \qquad -5v_\mathrm{b} = 19 \qquad \text{thus} \qquad v_\mathrm{b} = \frac{-19}{5} = -3.8\text{ V}$$

Thus,

$-3.8 \text{ V} \leq v_b \leq 2.2 \text{ V}$

P 5.17 We want the following expression for the output voltage:

$v_o = -(8v_a + 4v_b + 10v_c + 6v_d)$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage. Pick a feedback resistor with divisors of 8, 4, 10, and 6 – say $120\,\mathrm{k}\Omega$:

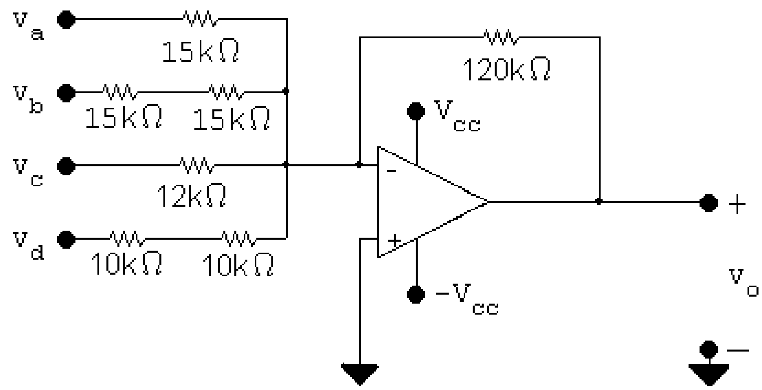
$$v_o = -\left[\frac{120\mathrm{k}}{R_\mathrm{a}}v_\mathrm{a} + \frac{120\mathrm{k}}{R_\mathrm{b}}v_\mathrm{b} + \frac{120\mathrm{k}}{R_\mathrm{c}}v_\mathrm{c} + \frac{120\mathrm{k}}{R_\mathrm{d}}v_\mathrm{d}\right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore \quad R_\mathrm{a} = 120{,}000/8 = 15\,\mathrm{k}\Omega \qquad R_\mathrm{c} = 120{,}000/10 = 12\,\mathrm{k}\Omega$$

$$R_\mathrm{b} = 120{,}000/4 = 30\,\mathrm{k}\Omega \qquad R_\mathrm{d} = 120{,}000/6 = 20\,\mathrm{k}\Omega$$

Now create the 5 resistor values needed from the realistic resistor values in Appendix H. Note that $R_{\rm f} = 120\,\mathrm{k}\Omega$, $R_{\rm a} = 15\,\mathrm{k}\Omega$, and $R_{\rm c} = 12\,\mathrm{k}\Omega$ are already values from Appendix H. Create $R_{\rm b} = 30\,\mathrm{k}\Omega$ by combining two $15\,\mathrm{k}\Omega$ resistors in series. Create $R_{\rm d} = 20\,\mathrm{k}\Omega$ by combining two $10\,\mathrm{k}\Omega$ resistors in series. Of course there are many other acceptable possibilities. The final circuit is shown here:



P 5.18 **[a]** The circuit shown is a non-inverting amplifier.

[b] We assume the op amp to be ideal, so $v_n = v_p = 2$ V. Write a KCL equation at v_n:

$$\frac{2}{25{,}000} + \frac{2 - v_o}{150{,}000} = 0$$

Solving,

$v_o = 14$ V.

P 5.19 **[a]** This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v_p:

$$v_p = \frac{8000}{8000 + 32{,}000} v_s = \frac{v_s}{5}$$

Write a KCL equation at $v_n = v_p = v_s/5$:

$$\frac{v_s/5}{7000} + \frac{v_s/5 - v_o}{56{,}000} = 0$$

Solving,

$$v_o = 8v_s/5 + v_s/5 = 1.8v_s$$

[c] $1.8v_s = 12 \qquad \text{so} \qquad v_s = 6.67\ \text{V}$

$1.8v_s = -15 \qquad \text{so} \qquad v_s = -8.33\ \text{V}$

Thus, $-8.33\ \text{V} \le v_s \le 6.67\ \text{V}$.

P 5.20 **[a]** $v_p = v_n = \dfrac{68}{80}v_g = 0.85v_g$

$$\therefore \quad \frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{63{,}000} = 0;$$

$$\therefore \quad v_o = 2.635v_g = 2.635(4), \qquad v_o = 10.54 \text{ V}$$

[b] $v_o = 2.635v_g = \pm 12$

$$v_g = \pm 4.55 \text{ V}, \qquad -4.55 \le v_g \le 4.55 \text{ V}$$

[c] $\dfrac{0.85v_g}{30{,}000} + \dfrac{0.85v_g - v_o}{R_\text{f}} = 0$

$$\left(\frac{0.85R_\text{f}}{30{,}000} + 0.85\right)v_g = v_o = \pm 12$$

$$\therefore \quad 1.7R_\text{f} + 51 = \pm 360; \qquad 1.7R_\text{f} = 360 - 51; \qquad R_\text{f} = 181.76\,\text{k}\Omega$$

P 5.21 **[a]** From Eq. 5.18,

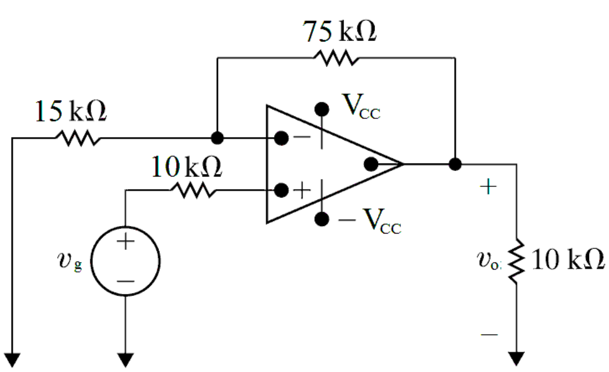
$$v_\text{o} = \frac{R_\text{s} + R_\text{f}}{R_\text{s}}v_\text{g} \qquad \text{so} \qquad \frac{v_\text{o}}{v_\text{g}} = 1 + \frac{R_\text{f}}{R_\text{s}} = 6$$

So,

$$\frac{R_\text{f}}{R_\text{s}} = 5$$

Thus,

$$R_\text{s} = \frac{R_\text{f}}{5} = \frac{75{,}000}{5} = 15\,\text{k}\Omega$$



[b] $v_\text{o} = 6v_\text{g}$

When $v_\text{g} = -2.5$ V, $v_\text{o} = 6(-2.5) = -15$ V.

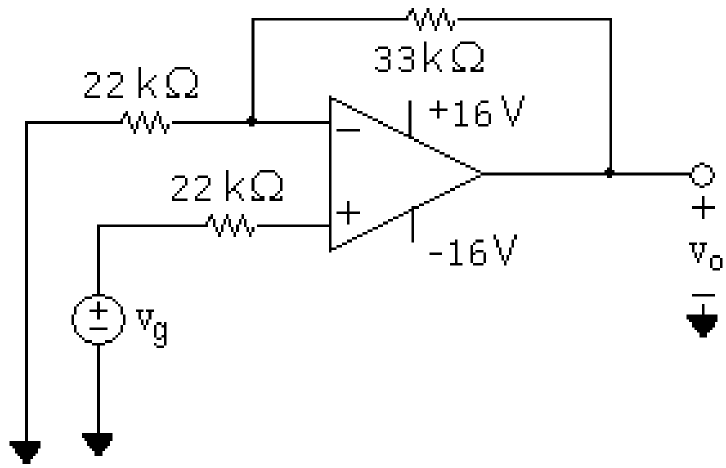
When $v_\text{g} = 1.5$ V, $v_\text{o} = 6(1.5) = 9$ V.

The power supplies can be set at 9 V and -15 V.

P 5.22 **[a]** From the equation for the non-inverting amplifier,

$$\frac{R_s + R_f}{R_s} = 2.5 \qquad \text{so} \qquad R_s + R_f = 2.5R_s \qquad \text{and therefore} \qquad R_f = 1.5R_s$$

Choose $R_s = 22\,\text{k}\Omega$, which is a component in Appendix H. Then $R_f = (1.5)(22) = 33\,\text{k}\Omega$, which is also a resistor value in Appendix H. The resulting non-inverting amplifier circuit is shown here:



[b] $v_o = 2.5v_g = 16 \qquad \text{so} \qquad v_g = 6.4$ V

$v_o = 2.5v_g = -16 \qquad \text{so} \qquad v_g = -6.4$ V

Therefore,

$-6.4 \text{ V} \leq v_g \leq 6.4 \text{ V}$

P 5.23 **[a]** This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v_p and solve for v_p in terms of v_s:

$$\frac{v_p - 5}{16{,}000} + \frac{v_p - v_s}{24{,}000} = 0$$

$$3v_p - 15 + 2v_p - 2v_s = 0 \qquad \text{so} \qquad v_p = 2v_s/5 + 3$$

Now write a KCL equation at v_n and solve for v_o:

$$\frac{v_n}{24{,}000} + \frac{v_n - v_o}{96{,}000} = 0 \qquad \text{so} \qquad v_o = 5v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$$v_o = 5(2v_s/5 + 3) = 2v_s + 15$$

[c] $2v_s + 15 = 10 \qquad \text{so} \qquad v_s = -2.5$ V

$2v_s + 15 = -10 \qquad \text{so} \qquad v_s = -12.5$ V

Thus, $-12.5 \text{ V} \leq v_s \leq -2.5$ V.

P 5.24 **[a]** $\frac{v_p - v_\mathrm{a}}{R_\mathrm{a}} + \frac{v_p - v_\mathrm{b}}{R_\mathrm{b}} + \frac{v_p - v_\mathrm{c}}{R_\mathrm{c}} = 0$

$$\therefore \quad v_p = \frac{R_\mathrm{b}R_\mathrm{c}}{D}v_\mathrm{a} + \frac{R_\mathrm{a}R_\mathrm{c}}{D}v_\mathrm{b} + \frac{R_\mathrm{a}R_\mathrm{b}}{D}v_\mathrm{c}$$

where $D = R_\mathrm{b}R_\mathrm{c} + R_\mathrm{a}R_\mathrm{c} + R_\mathrm{a}R_\mathrm{b}$

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{100{,}000} = 0$$

$$\left(\frac{100{,}000}{20{,}000} + 1\right) v_n = 6v_n = v_o$$

$$\therefore \quad v_o = \frac{6R_\mathrm{b}R_\mathrm{c}}{D}v_\mathrm{a} + \frac{6R_\mathrm{a}R_\mathrm{c}}{D}v_\mathrm{b} + \frac{6R_\mathrm{a}R_\mathrm{b}}{D}v_\mathrm{c}$$

By hypothesis,

$$\frac{6R_\mathrm{b}R_\mathrm{c}}{D} = 1; \qquad \frac{6R_\mathrm{a}R_\mathrm{c}}{D} = 2; \qquad \frac{6R_\mathrm{a}R_\mathrm{b}}{D} = 3$$

Then

$$\frac{6R_\mathrm{a}R_\mathrm{b}/D}{6R_\mathrm{a}R_\mathrm{c}/D} = \frac{3}{2} \qquad \text{so} \qquad R_\mathrm{b} = 1.5R_\mathrm{c}$$

But from the circuit

$$R_\mathrm{b} = 15\,\mathrm{k}\Omega \qquad \text{so} \qquad R_\mathrm{c} = 10\,\mathrm{k}\Omega$$

Similarly,

$$\frac{6R_\mathrm{b}R_\mathrm{c}/D}{6R_\mathrm{a}R_\mathrm{b}/D} = \frac{1}{3} \qquad \text{so} \qquad 3R_\mathrm{c} = R_\mathrm{a}$$

Thus,

$$R_\mathrm{a} = 30\,\mathrm{k}\Omega$$

[b] $v_o = 1(0.7) + 2(0.4) + 3(1.1) = 4.8\ \mathrm{V}$

$$v_n = v_o/6 = 0.8\ \mathrm{V} = v_p$$

$$i_\mathrm{a} = \frac{v_\mathrm{a} - v_p}{30{,}000} = \frac{0.7 - 0.8}{30{,}000} = -3.33\,\mu\mathrm{A}$$

$$i_\mathrm{b} = \frac{v_\mathrm{b} - v_p}{15{,}000} = \frac{0.4 - 0.8}{15{,}000} = -26.67\,\mu\mathrm{A}$$

$$i_\mathrm{c} = \frac{v_\mathrm{c} - v_p}{10{,}000} = \frac{1.1 - 0.8}{10{,}000} = 30\,\mu\mathrm{A}$$

Check:

$$i_\mathrm{a} + i_\mathrm{b} + i_\mathrm{c} = 0? \qquad -3.33 - 26.67 + 30 = 0 \text{ (checks)}$$

P 5.25 **[a]** Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_\mathrm{a}}{R_\mathrm{a}} + \frac{0 - v_o'}{R_\mathrm{b}} + i_n = 0, \qquad i_n = 0$$

Therefore

$$\frac{v_o'}{R_\mathrm{b}} = -\frac{v_\mathrm{a}}{R_\mathrm{a}}, \qquad v_o' = -\frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{a}$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_\mathrm{b} R_\mathrm{d}}{R_\mathrm{c} + R_\mathrm{d}}$$

$$\frac{v_n}{R_\mathrm{a}} + \frac{v_n - v_o''}{R_\mathrm{b}} + i_n = 0, \qquad i_n = 0$$

$$\left(\frac{1}{R_\mathrm{a}} + \frac{1}{R_\mathrm{b}}\right)\left(\frac{R_\mathrm{d}}{R_\mathrm{c} + R_\mathrm{d}}\right) v_\mathrm{b} - \frac{v_o''}{R_\mathrm{b}} = 0$$

$$v_o'' = \left(\frac{R_\mathrm{b}}{R_\mathrm{a}} + 1\right)\left(\frac{R_\mathrm{d}}{R_\mathrm{c} + R_\mathrm{d}}\right) v_\mathrm{b} = \frac{R_\mathrm{d}}{R_\mathrm{a}}\left(\frac{R_\mathrm{a} + R_\mathrm{b}}{R_\mathrm{c} + R_\mathrm{d}}\right) v_\mathrm{b}$$

$$v_o = v_o' + v_o'' = \frac{R_\mathrm{d}}{R_\mathrm{a}}\left(\frac{R_\mathrm{a} + R_\mathrm{b}}{R_\mathrm{c} + R_\mathrm{d}}\right) v_\mathrm{b} - \frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{a}$$

[b] $\frac{R_\mathrm{d}}{R_\mathrm{a}}\left(\frac{R_\mathrm{a} + R_\mathrm{b}}{R_\mathrm{c} + R_\mathrm{d}}\right) = \frac{R_\mathrm{b}}{R_\mathrm{a}},$ therefore $R_\mathrm{d}(R_\mathrm{a} + R_\mathrm{b}) = R_\mathrm{b}(R_\mathrm{c} + R_\mathrm{d})$

$R_\mathrm{d} R_\mathrm{a} = R_\mathrm{b} R_\mathrm{c},$ therefore $\frac{R_\mathrm{a}}{R_\mathrm{b}} = \frac{R_\mathrm{c}}{R_\mathrm{d}}$

When $\frac{R_\mathrm{d}}{R_\mathrm{a}}\left(\frac{R_\mathrm{a} + R_\mathrm{b}}{R_\mathrm{c} + R_\mathrm{d}}\right) = \frac{R_\mathrm{b}}{R_\mathrm{a}}$

Eq. (5.22) reduces to $v_o = \frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{b} - \frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{a} = \frac{R_\mathrm{b}}{R_\mathrm{a}}(v_\mathrm{b} - v_\mathrm{a}).$

P 5.26 **[a]** This is a difference amplifier circuit.

[b] Use Eq. 5.22 with $R_\mathrm{a} = 5\,\mathrm{k\Omega}$, $R_\mathrm{b} = 20\,\mathrm{k\Omega}$, $R_\mathrm{c} = 8\,\mathrm{k\Omega}$, $R_\mathrm{d} = 2\,\mathrm{k\Omega}$, and $v_\mathrm{b} = 5$ V:

$$v_\mathrm{o} = \frac{R_\mathrm{d}(R_\mathrm{a} + R_\mathrm{b})}{R_\mathrm{a}(R_\mathrm{c} + R_\mathrm{d})} v_\mathrm{b} - \frac{R_\mathrm{b}}{R_\mathrm{a}} v_\mathrm{a} = \frac{2(5 + 20)}{5(8 + 2)}(5) - \frac{20}{5} v_\mathrm{a} = 5 - 4v_\mathrm{a}$$

[c] $\frac{2000(5000 + R_\mathrm{f})}{5000(8000 + 2000)}(5) - \frac{R_\mathrm{f}}{5000}(2) = \frac{5000 + R_\mathrm{f}}{5000} - \frac{2R_\mathrm{f}}{5000} = 1 - \frac{R_\mathrm{f}}{5000}$

$1 - \frac{R_\mathrm{f}}{5000} = 10$ so $R_\mathrm{f} < 0$ which is not a possible solution.

$1 - \frac{R_\mathrm{f}}{5000} = -10$ so $R_\mathrm{f} = 5000(11) = 55\,\mathrm{k\Omega}$

P 5.27 **[a]** $v_o = \dfrac{R_{\rm d}(R_{\rm a}+R_{\rm b})}{R_{\rm a}(R_{\rm c}+R_{\rm d})}v_{\rm b} - \dfrac{R_{\rm b}}{R_{\rm a}}v_{\rm a} = \dfrac{120(24+75)}{24(130+120)}(5) - \dfrac{75}{24}(8)$

$$v_o = 9.9 - 25 = -15.1 \text{ V}$$

[b] $\dfrac{v_1 - 8}{24{,}000} + \dfrac{v_1 + 15.1}{75{,}000} = 0 \qquad \text{so} \qquad v_1 = 2.4\ \text{V}$

$$i_{\rm a} = \frac{8-2.4}{24{,}000} = 233\,\mu\text{ A}$$

$$R_{\rm ina} = \frac{v_{\rm a}}{i_{\rm a}} = \frac{8}{233\times 10^{-6}} = 34.3\,\text{k}\Omega$$

[c] $R_{\rm in\,b} = R_{\rm c} + R_{\rm d} = 250\,\text{k}\Omega$

P 5.28 $v_p = \dfrac{1500}{9000}(-18) = -3\ \text{V} = v_n$

$$\frac{-3+18}{1600} + \frac{-3-v_o}{R_{\rm f}} = 0$$

$$\therefore\quad v_o = 0.009375R_{\rm f} - 3$$

$$v_o = 9\ \text{V};\qquad R_{\rm f} = 1280\,\Omega$$

$$v_o = -9\ \text{V};\qquad R_{\rm f} = -640\,\Omega$$

But $\quad R_{\rm f} \geq 0, \qquad \therefore\quad R_{\rm f} = 1.28\,\text{k}\Omega$

P 5.29 $v_o = \dfrac{R_{\rm d}(R_{\rm a}+R_{\rm b})}{R_{\rm a}(R_{\rm c}+R_{\rm d})}v_{\rm b} - \dfrac{R_{\rm b}}{R_{\rm a}}v_{\rm a}$

By hypothesis: $\quad R_{\rm b}/R_{\rm a} = 4; \quad R_{\rm c} + R_{\rm d} = 470\,\text{k}\Omega; \quad \dfrac{R_{\rm d}(R_{\rm a}+R_{\rm b})}{R_{\rm a}(R_{\rm c}+R_{\rm d})} = 3$

$$\therefore\quad \frac{R_{\rm d}}{R_{\rm a}}\frac{(R_{\rm a}+4R_{\rm a})}{470{,}000} = 3 \quad \text{so} \quad R_{\rm d} = 282\,\text{k}\Omega; \quad R_{\rm c} = 188\,\text{k}\Omega$$

Create $R_{\rm d} = 282\,\text{k}\Omega$ by combining a $270\,\text{k}\Omega$ resistor and a $12\,\text{k}\Omega$ resistor in series. Create $R_{\rm c} = 188\,\text{k}\Omega$ by combining a $120\,\text{k}\Omega$ resistor and a $68\,\text{k}\Omega$ resistor in series. Also, when $v_o = 0$ we have

$$\frac{v_n - v_{\rm a}}{R_{\rm a}} + \frac{v_n}{R_{\rm b}} = 0$$

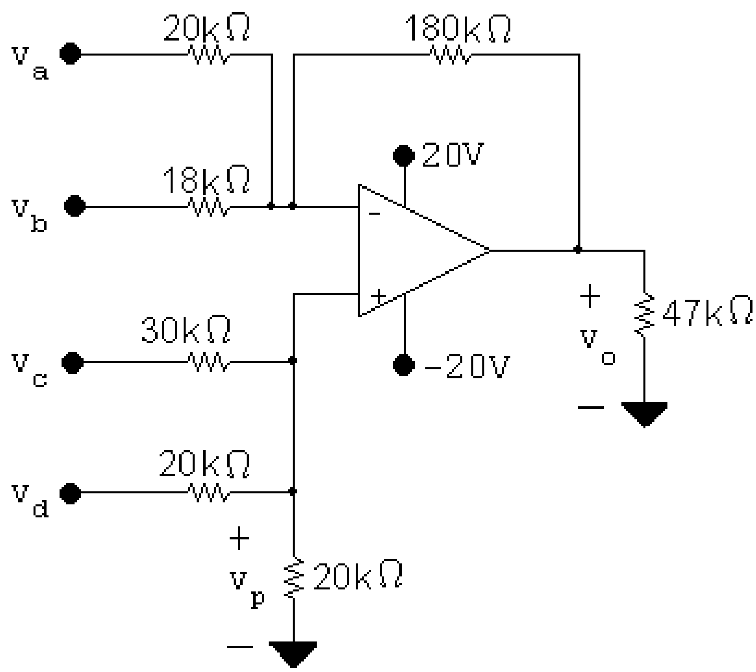
$$\therefore\quad v_n\left(1 + \frac{R_{\rm a}}{R_{\rm b}}\right) = v_{\rm a};\qquad v_n = 0.8v_{\rm a}$$

$$i_{\rm a} = \frac{v_{\rm a} - 0.8v_{\rm a}}{R_{\rm a}} = 0.2\frac{v_{\rm a}}{R_{\rm a}}; \qquad R_{\rm in} = \frac{v_{\rm a}}{i_{\rm a}} = 5R_{\rm a} = 22\,\text{k}\Omega$$

$$\therefore \quad R_{\rm a} = 4.4\,\text{k}\Omega; \qquad R_{\rm b} = 17.6\,\text{k}\Omega$$

Create $R_{\rm a} = 4.4\,\text{k}\Omega$ by combining two $2.2\,\text{k}\Omega$ resistors in series. Create $R_{\rm b} = 17.6\,\text{k}\Omega$ by combining a $12\,\text{k}\Omega$ resistor and a $5.6\,\text{k}\Omega$ resistor in series.

P 5.30 **[a]**



$$\frac{v_p}{20{,}000} + \frac{v_p - v_{\rm c}}{30{,}000} + \frac{v_p - v_{\rm d}}{20{,}000} = 0$$

$$\therefore \quad 8v_p = 2v_{\rm c} + 3v_{\rm d} = 8v_n$$

$$\frac{v_n - v_{\rm a}}{20{,}000} + \frac{v_n - v_{\rm b}}{18{,}000} + \frac{v_n - v_o}{180{,}000} = 0$$

$$\begin{aligned} \therefore \quad v_o &= 20v_n - 9v_{\rm a} - 10v_{\rm b} \\ &= 20[(1/4)v_{\rm c} + (3/8)v_{\rm d}] - 9v_{\rm a} - 10v_{\rm b} \\ &= 20(0.75 + 1.5) - 9(1) - 10(2) = 16\ \text{V} \end{aligned}$$

[b] $v_o = 5v_{\rm c} + 30 - 9 - 20 = 5v_{\rm c} + 1$

$$\pm 20 = 5v_{\rm c} + 1$$

$$\therefore \quad v_{\rm b} = -4.2\ \text{V} \qquad \text{and} \qquad v_{\rm b} = 3.8\ \text{V}$$

$$\therefore \quad -4.2\ \text{V} \leq v_{\rm b} \leq 3.8\ \text{V}$$

P 5.31 $v_p = R_b i_b = v_n$

$$\frac{R_b i_b}{2000} + \frac{R_b i_b - v_o}{R_f} - i_a = 0$$

$$\therefore \qquad R_b i_b \left(\frac{1}{2000} + \frac{1}{R_f}\right) - i_a = \frac{v_o}{R_f}$$

$$\therefore \qquad R_b i_b \left(1 + \frac{R_f}{2000}\right) - R_f i_a = v_o$$

By hypopthesis, $v_o = 8000(i_b - i_a)$. Therefore,

$R_f = 8\,\text{k}\Omega$ (use $3.3\,\text{k}\Omega$ and $4.7\,\text{k}\Omega$ resistors in series)

$$R_b\left(1 + \frac{8000}{2000}\right) = 8000 \qquad \text{so} \qquad R_b = 1.6\,\text{k}\Omega$$

To construct the $1.6\,\text{k}\Omega$ resistor, combine $270\,\Omega$, $330\,\Omega$, and $1\,\text{k}\Omega$ resistors in series.

P 5.32 **[a]** $v_p = \dfrac{\alpha R_g}{\alpha R_g + (R_g - \alpha R_g)} v_g$

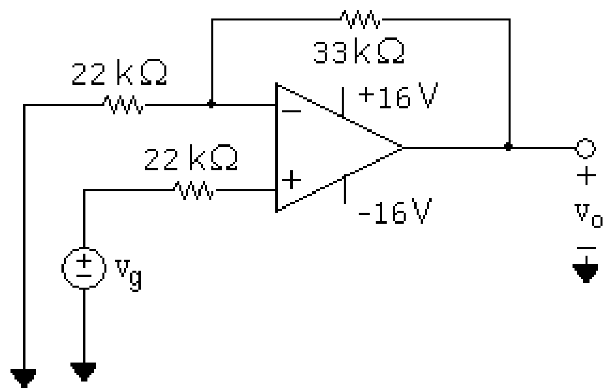
$v_n = v_p = \alpha v_g$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_f} = 0$$

$$(v_n - v_g)\frac{R_f}{R_1} + v_n - v_o = 0$$

$$\begin{aligned} v_o &= \left(1 + \frac{R_f}{R_1}\right)\alpha v_g - \frac{R_f}{R_1} v_g \\ &= (\alpha v_g - v_g)4 + \alpha v_g \\ &= [(\alpha - 1)4 + \alpha] v_g \\ &= (5\alpha - 4) v_g \\ &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	−8 V	0.4	−4 V	0.8	0 V
0.1	−7 V	0.5	−3 V	0.9	1 V
0.2	−6 V	0.6	−2 V	1.0	2 V
0.3	−5 V	0.7	−1 V		



[**b**] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g \alpha + -\left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope } = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad \text{intercept } = -\left(\frac{R_f}{R_1}\right) v_g$$

[**c**] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \qquad \frac{R_f}{R_1} = 2$$

P 5.33 [**a**] $A_{\rm dm} = \dfrac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[**b**] $A_{\rm cm} = \dfrac{(1)(24) - 25(1)}{1(25)} = -0.04$

[**c**] $\text{CMRR} = \left|\dfrac{24.98}{0.04}\right| = 624.50$

P 5.34 $A_{\rm cm} = \dfrac{(3000)(6000) - (6000)R_x}{3000(6000 + R_x)}$

$$A_{\rm dm} = \frac{6000(3000 + 6000) + 6000(6000 + R_x)}{2(3000)(6000 + R_x)}$$

$$\frac{A_{\rm dm}}{A_{\rm cm}} = \frac{R_x + 15{,}000}{2(3000 - R_x)}$$

$$\therefore \quad \frac{R_x + 15{,}000}{2(3000 - R_x)} = \pm 1500 \quad \text{for the limits on the value of } R_x$$

If we use $+1500 \quad R_x = 2994\,\Omega$

If we use $-1500 \quad R_x = 3006\,\Omega$

$2994\,\Omega \le R_x \le 3006\,\Omega$

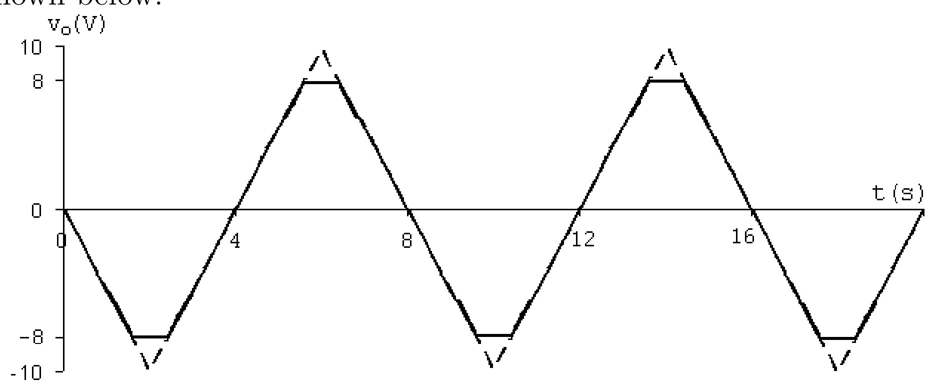
P 5.35 It follows directly from the circuit that $v_o = -(75/15)v_g = -5v_g$
From the plot of v_g we have $\quad v_g = 0, \quad t < 0$

$$\begin{aligned}
v_g &= t && 0 \le t \le 2\\
v_g &= 4 - t && 2 \le t \le 6\\
v_g &= t - 8 && 6 \le t \le 10\\
v_g &= 12 - t && 10 \le t \le 14\\
v_g &= t - 16 && 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

Therefore

$$\begin{aligned}
v_o &= -5t && 0 \le t \le 2\\
v_o &= 5t - 20 && 2 \le t \le 6\\
v_o &= 40 - 5t && 6 \le t \le 10\\
v_o &= 5t - 60 && 10 \le t \le 14\\
v_o &= 80 - 5t && 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 9, the output is clipped at ± 9. The plot is shown below.



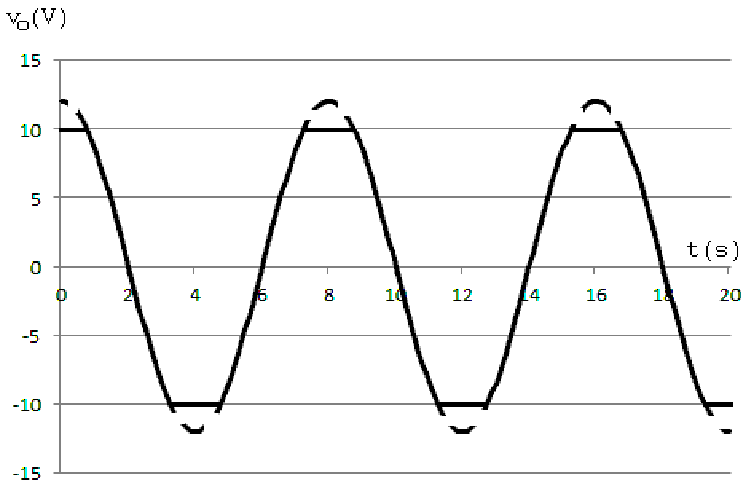
P 5.36 $v_p = \dfrac{5.4}{7.2}v_g = 0.75v_g = 3\cos(\pi/4)t\ \text{V}$

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{60{,}000} = 0$$

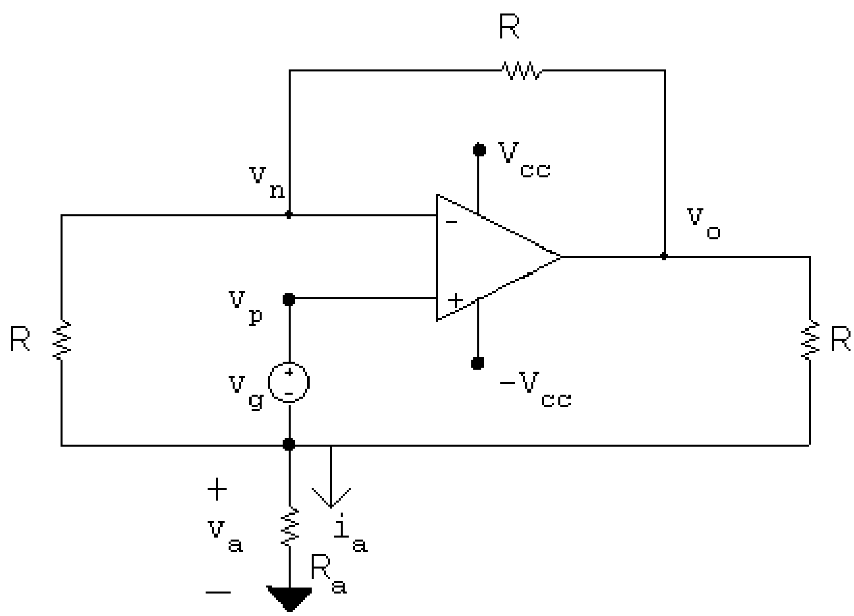
$4v_n = v_o; \qquad v_n = v_p$

$$\therefore \quad v_o = 12\cos(\pi/4)t \ \text{V} \qquad 0 \le t \le \infty$$

but saturation occurs at $v_o = \pm 10$ V



P 5.37 [a]



$$\frac{v_n - v_\text{a}}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_\text{a} = v_o$$

$$\frac{v_\text{a}}{R_\text{a}} + \frac{v_\text{a} - v_n}{R} + \frac{v_\text{a} - v_o}{R} = 0$$

$$v_\text{a}\left[\frac{1}{R_\text{a}} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_\text{a}\left(2 + \frac{R}{R_\text{a}}\right) - v_n = v_o$$

$$v_n = v_p = v_\mathrm{a} + v_g$$

$$\therefore \quad 2v_n - v_\mathrm{a} = 2v_\mathrm{a} + 2v_g - v_\mathrm{a} = v_\mathrm{a} + 2v_g$$

$$\therefore \quad v_\mathrm{a} - v_o = -2v_g \qquad (1)$$

$$2v_\mathrm{a} + v_\mathrm{a}\left(\frac{R}{R_\mathrm{a}}\right) - v_\mathrm{a} - v_g = v_o$$

$$\therefore \quad v_\mathrm{a}\left(1 + \frac{R}{R_\mathrm{a}}\right) - v_o = v_g \qquad (2)$$

Now combining equations (1) and (2) yields

$$-v_\mathrm{a}\frac{R}{R_\mathrm{a}} = -3v_g$$

$$\text{or} \quad v_\mathrm{a} = 3v_g\frac{R_\mathrm{a}}{R}$$

$$\text{Hence} \quad i_\mathrm{a} = \frac{v_\mathrm{a}}{R_\mathrm{a}} = \frac{3v_g}{R} \qquad \text{Q.E.D.}$$

[b] At saturation $v_o = \pm\,\mathrm{V_{cc}}$

$$\therefore \quad v_\mathrm{a} = \pm\,\mathrm{V_{cc}} - 2v_g \qquad (3)$$

and

$$\therefore \quad v_\mathrm{a}\left(1 + \frac{R}{R_\mathrm{a}}\right) = \pm\,\mathrm{V_{cc}} + v_g \qquad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_\mathrm{a}} = \frac{\pm\,\mathrm{V_{cc}} + v_g}{\pm\,\mathrm{V_{cc}} - 2v_g}$$

$$\therefore \quad \frac{R}{R_\mathrm{a}} = \frac{\pm\,\mathrm{V_{cc}} + v_g}{\pm\,\mathrm{V_{cc}} - 2v_g} - 1 = \frac{3v_g}{\pm\,\mathrm{V_{cc}} - 2v_g}$$

$$\text{or} \quad R_\mathrm{a} = \frac{(\pm\,\mathrm{V_{cc}} - 2v_g)}{3v_g}R \qquad \text{Q.E.D.}$$

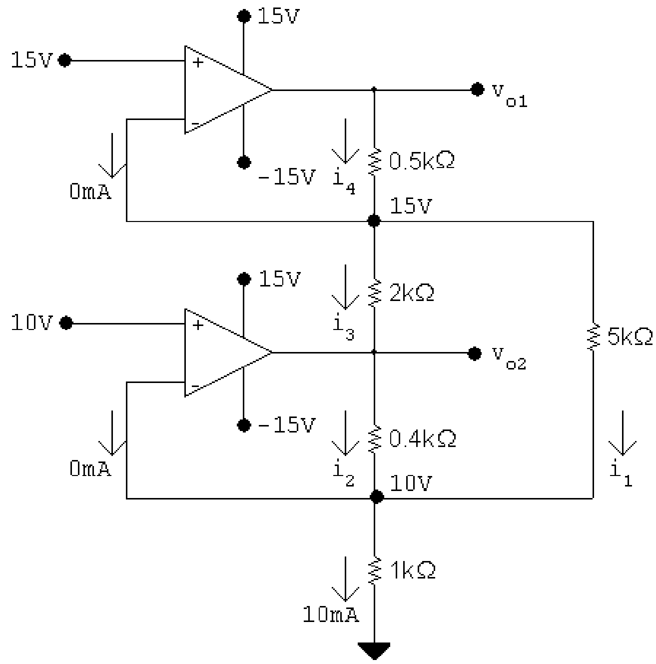
P 5.38 **[a]** $v_p = v_s, \qquad v_n = \dfrac{R_1 v_o}{R_1 + R_2}, \qquad v_n = v_p$

$$\text{Therefore} \quad v_o = \left(\frac{R_1 + R_2}{R_1}\right)v_s = \left(1 + \frac{R_2}{R_1}\right)v_s$$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 5.39



$$i_1 = \frac{15 - 10}{5000} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 10\,\text{mA}; \qquad i_2 = 9\,\text{mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6 \text{ V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7\,\text{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\text{mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85 \text{ V}$$

P 5.40 **[a]** $p_{16\,\text{k}\Omega} = \dfrac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\text{W}$

[b] $v_{16\,\text{k}\Omega} = \left(\dfrac{16}{64}\right)(320) = 80\,\text{mV}$

$$p_{16\,\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4\,\mu\text{W}$$

[c] $\dfrac{p_\text{a}}{p_\text{b}} = \dfrac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification.*
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 5.41 **[a]** Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7\ \text{V}; \qquad v_{o2} = \frac{-220}{33}(-0.15) = 1.0\ \text{V}$$

$$i_{\text{a}} = \frac{v_{o2} - v_{o1}}{1000} = 5.7\,\text{mA}$$

[b] $i_{\text{a}} = 0$ when $v_{o1} = v_{o2}$ so from (a) $v_{o2} = 1$ V

Thus

$$\frac{-47}{10}(v_{\text{L}}) = 1$$

$$v_{\text{L}} = -\frac{10}{47} = -212.77\ \text{mV}$$

P 5.42 **[a]** Assume the op-amp is operating within its linear range, then

$$i_L = \frac{8}{4000} = 2\,\text{mA}$$

For $R_L = 4\,\text{k}\Omega$ $\quad v_o = (4+4)(2) = 16\ \text{V}$

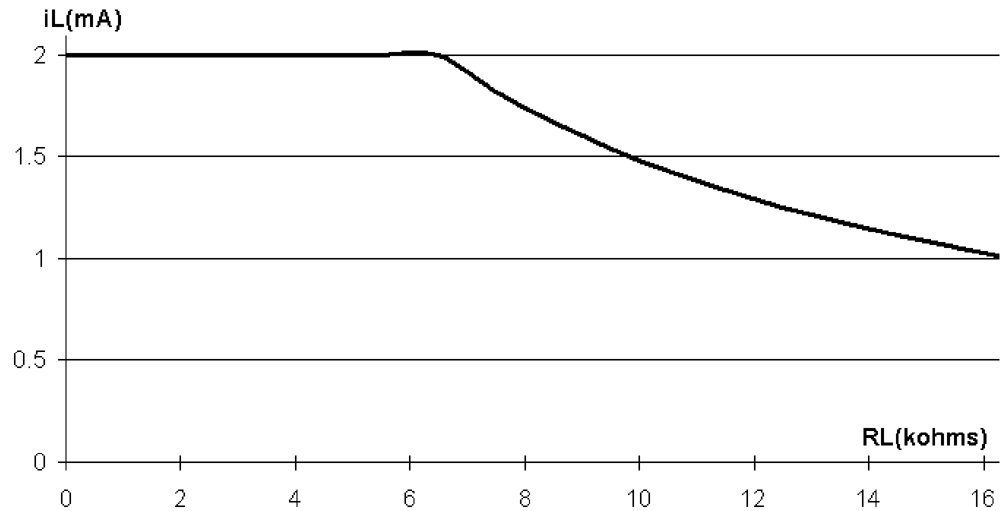
Now since $v_o < 20$ V our assumption of linear operation is correct, therefore

$$i_L = 2\,\text{mA}$$

[b] $20 = 2(4 + R_L); \qquad R_L = 6\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 6\,\text{k}\Omega$. Therefore when $R_L = 6\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 20/(4000 + 16{,}000) = 1\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 50 kΩ resistor is negligible, since the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i_p = i_n = (8-4)/(500 \times 10^3) = 8\,\mu\text{A}$. But $8\,\mu\text{A} \ll 1\,\text{mA}$, hence our assumption is reasonable.

[d]



P 5.43 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R+\Delta R} = v_n\left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f}\right) - \frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R+\Delta R} = \frac{v_{\text{ref}}\left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f}\right)}{(R-\Delta R)\left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f}\right)} - \frac{v_o}{R_f}$$

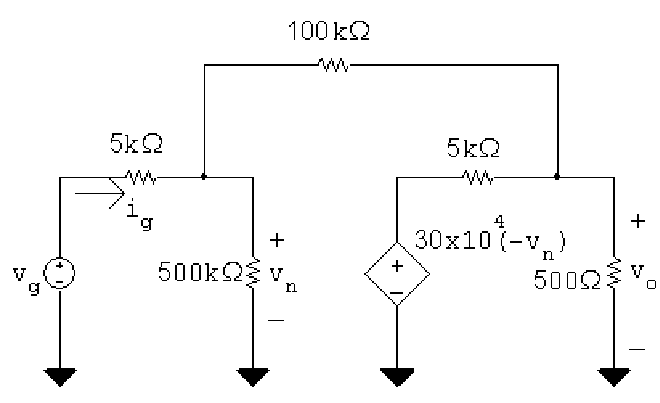
Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}}\left(\frac{1}{R-\Delta R} - \frac{1}{R+\Delta R}\right)$$

Thus,

$$v_o = v_{\text{ref}}\left(\frac{2\Delta R}{R^2 - (\Delta R)^2}\right)R_f$$

P 5.44 [a] Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} + \frac{v_n}{500{,}000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100{,}000} + \frac{v_o}{500} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6 \times 10^6 - 1)v_n + 221v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$v_o = -19.9844$ V; $\qquad v_n = 736.1\,\mu$V

Thus the voltage gain is $v_o/v_g = -19.9844$.

[b] From the solution in part (a), $v_n = 736.1\,\mu$V.

[c] $i_g = \dfrac{v_g - v_n}{5000} = \dfrac{v_g - 736.1 \times 10^{-6} v_g}{5000}$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 736.1 \times 10^{-6}} = 5003.68\,\Omega$$

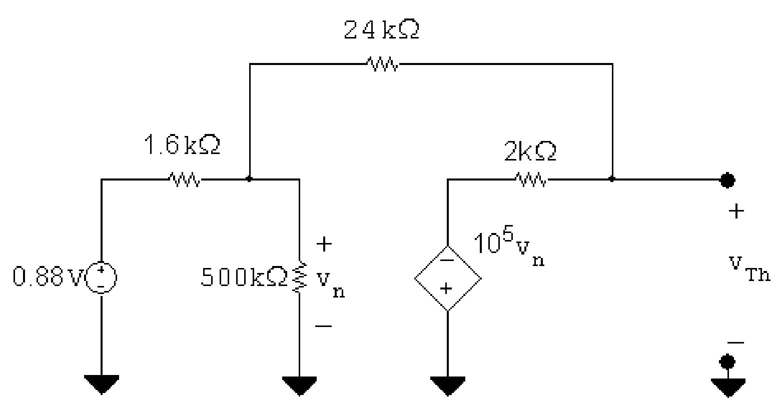
[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite. Therefore,

$v_n = v_p = 0$ V; $\qquad R_g = 5000\,\Omega$

P 5.45 **[a]**

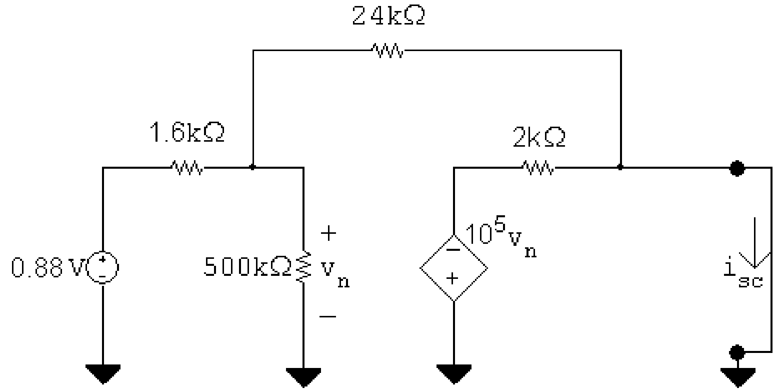


$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n - v_{\rm Th}}{24{,}000} = 0$$

$$\frac{v_{\mathrm{Th}} + 10^5 v_n}{2000} + \frac{v_{\mathrm{Th}} - v_n}{24{,}000} = 0$$

Solving, $v_{\mathrm{Th}} = -13.198$ V

Short-circuit current calculation:

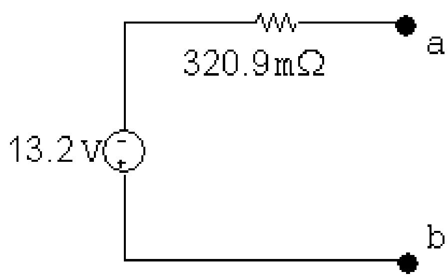


$$\frac{v_n}{500{,}000} + \frac{v_n - 0.88}{1600} + \frac{v_n - 0}{24{,}000} = 0$$

$$\therefore \quad v_n = 0.8225 \text{ V}$$

$$i_{\mathrm{sc}} = \frac{v_n}{24{,}000} - \frac{10^5}{2000} v_n = -41.13 \text{ A}$$

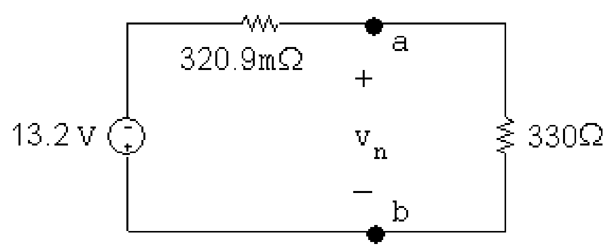
$$R_{\mathrm{Th}} = \frac{v_{\mathrm{Th}}}{i_{\mathrm{sc}}} = 320.9\,\mathrm{m}\Omega$$



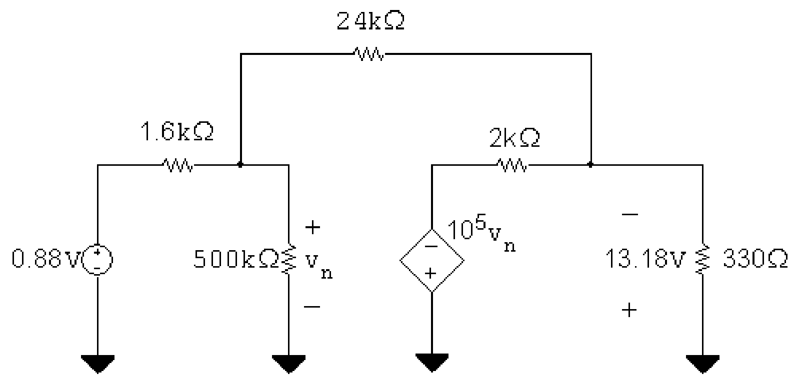
[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{\mathrm{Th}} = 320.9\,\mathrm{m}\Omega$$

[c]



$$v_o = \left(\frac{330}{330.3209}\right)(-13.2) = -13.18 \text{ V}$$



$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n + 13.18}{24{,}000} = 0$$

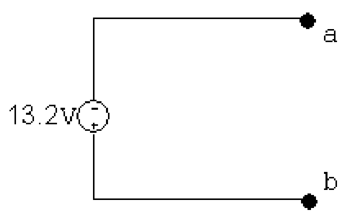
$$\therefore \quad v_n = 942\,\mu\text{V}$$

$$i_g = \frac{0.88 - 942 \times 10^{-6}}{1600} = 549.41\,\mu\text{A}$$

$$R_g = \frac{0.88}{i_g} = 1601.71\,\Omega$$

P 5.46 **[a]** $v_{\text{Th}} = -\frac{24{,}000}{1600}(0.88) = -13.2\ \text{V}$

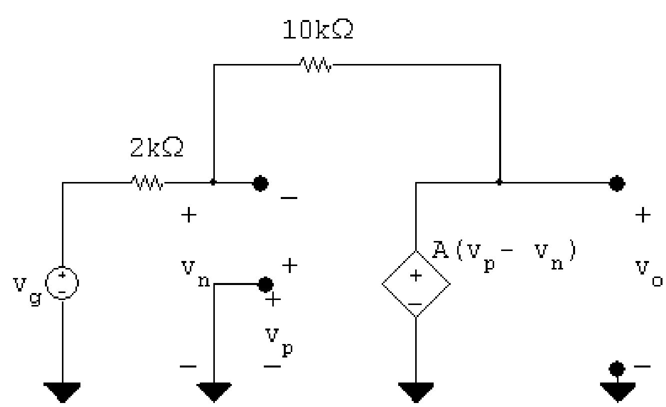
$R_{\text{Th}} = 0,$ since op-amp is ideal



[b] $R_o = R_{\text{Th}} = 0\,\Omega$

[c] $R_g = 1.6\,\text{k}\Omega \quad \text{since} \quad v_n = 0$

P 5.47 **[a]**



$$\frac{v_n - v_g}{2000} + \frac{v_n - v_o}{10{,}000} = 0$$

$$\therefore \quad v_o = 6v_n - 5v_g$$

$$\text{Also} \quad v_o = A(v_p - v_n) = -Av_n$$

$$\therefore \quad v_n = \frac{-v_o}{A}$$

$$\therefore \quad v_o\left(1 + \frac{6}{A}\right) = -5v_g$$

$$v_o = \frac{-5A}{(6 + A)}v_g$$

[b] $v_o = \dfrac{-5(150)(1)}{156} = -4.81 \text{ V}$

[c] $v_o = \dfrac{-5}{1 + (6/A)}(1) = -5 \text{ V}$

[d] $\dfrac{-5A}{A+6}(1) = -0.99(5) \qquad \text{so} \qquad -5A = -4.95(A+6)$

$$\therefore \quad -0.05A = -29.7 \qquad \text{so} \qquad A = 594$$

P 5.48 **[a]** $\dfrac{v_n}{16{,}000} + \dfrac{v_n - v_g}{800{,}000} + \dfrac{v_n - v_o}{200{,}000} = 0 \qquad \text{or} \qquad 55v_n - 4v_o = v_g \qquad \text{Eq (1)}$

$$\frac{v_o}{20{,}000} + \frac{v_o - v_n}{200{,}000} + \frac{v_o - 50{,}000(v_p - v_n)}{8000} = 0$$

$$36v_o - v_n - 125 \times 10^4(v_p - v_n) = 0$$

$$v_p = v_g + \frac{(v_n - v_g)(240)}{800} = (0.7)v_g + (0.3)v_n$$

$$36v_o - v_n - 125 \times 10^4[(0.7)v_g - (0.7)v_n] = 0$$

$$36v_o + 874{,}999v_n = 875{,}000v_g \qquad \text{Eq (2)}$$

Let $v_g = 1$ V and solve Eqs. (1) and (2) simultaneously:

$$v_n = 999.446\,\text{mV} \qquad \text{and} \qquad v_o = 13.49 \text{ V}$$

$$\therefore \qquad \frac{v_o}{v_g} = 13.49$$

[b] From part (a), $v_n = 999.446$ mV.

$$v_p = (0.7)(1000) + (0.3)(999.446) = 999.834\,\text{mV}$$

[c] $v_p - v_n = 387.78\,\mu\text{V}$

[d] $i_g = \dfrac{(1000 - 999.83)10^{-3}}{24 \times 10^3} = 692.47\,\text{pA}$

[**e**] $\dfrac{v_g}{16{,}000} + \dfrac{v_g - v_o}{200{,}000} = 0, \qquad \text{since } v_n = v_p = v_g$

$\therefore \quad v_o = 13.5v_g, \qquad \dfrac{v_o}{v_g} = 13.5$

$v_n = v_p = 1 \text{ V}; \qquad v_p - v_n = 0 \text{ V}; \qquad i_g = 0 \text{ A}$

P 5.49 [**a**] Use Eq. 5.61 to solve for R_f; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2\,\text{k}\Omega$$

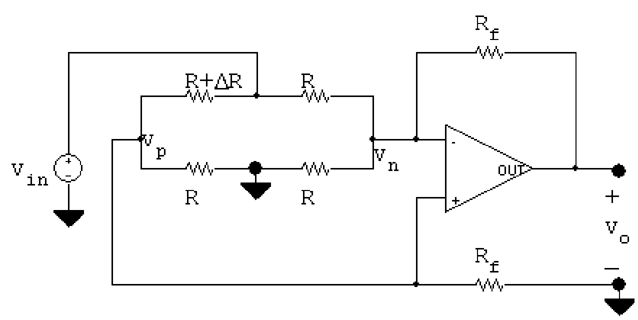
[**b**] Now solve for Δ given $v_o = 50$ mV:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12\ \text{m}\Omega$$

P 5.50 [**a**]



Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\text{in}}}{R_1} = 0$$

$$\therefore \quad v_p\left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1}\right] = \frac{v_{\text{in}}}{R_1}$$

$$\therefore \quad v_p = \frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\text{in}}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_f}\right] - \frac{v_o}{R_f} = \frac{v_{\text{in}}}{R}$$

$$\therefore \quad v_n\left[\frac{R+2R_f}{RR_f}\right] - \frac{v_{\text{in}}}{R} = \frac{v_o}{R_f}$$

$$\therefore \quad \frac{v_o}{R_f} = \left[\frac{R+2R_f}{RR_f}\right]\left[\frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R}\right] - \frac{v_{\text{in}}}{R}$$

$$\therefore \quad \frac{v_o}{R_f} = \left[\frac{R+2R_f}{RR_1 + R_f R_1 + R_f R} - \frac{1}{R}\right] v_{\text{in}}$$

$$\therefore \quad v_o = \frac{[R^2 + 2RR_f - R_1(R+R_f) - RR_f]R_f}{R[R_1(R+R_f) + RR_f]} v_{\text{in}}$$

Now substitute $R_1 = R + \Delta R$ and get

$$v_o = \frac{-\Delta R(R+R_f)R_f v_{\text{in}}}{R[(R+\Delta R)(R+R_f) + RR_f]}$$

If $\Delta R \ll R$

$$v_o \approx \frac{(R+R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R+2R_f)}$$

[b] $$v_o \approx \frac{47\times 10^4(48\times 10^4)(-95)15}{10^8(95\times 10^4)} \approx -3.384 \text{ V}$$

[c] $$v_o = \frac{-95(48\times 10^4)(47\times 10^4)15}{10^4[(1.0095)10^4(48\times 10^4) + 47\times 10^8]} = -3.368 \text{ V}$$

P 5.51 **[a]** $$v_o \approx \frac{(R+R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R+2R_f)}$$

$$v_o = \frac{(R+R_f)(-\Delta R)R_f v_{\text{in}}}{R[(R+\Delta R)(R+R_f) + RR_f]}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R+\Delta R)(R+R_f) + RR_f]}{R^2(R+2R_f)}$$

$$\therefore \quad \text{Error } = \frac{R[(R+\Delta R)(R+R_f) + RR_f] - R^2(R+2R_f)}{R^2(R+2R_f)}$$

$$= \frac{\Delta R}{R}\frac{(R+R_f)}{(R+2R_f)}$$

$$\therefore \quad \%\text{ error } = \frac{\Delta R(R+R_f)}{R(R+2R_f)} \times 100$$

[b] $$\%\text{ error} = \frac{95(48\times 10^4)\times 100}{10^4(95\times 10^4)} = 0.48\%$$

P 5.52 $$1 = \frac{\Delta R(48 \times 10^4)}{10^4(95 \times 10^4)} \times 100$$

$$\therefore \quad \Delta R = \frac{9500}{48} = 197.91667\,\Omega$$

$$\therefore \quad \%\text{ change in } R = \frac{197.19667}{10^4} \times 100 \approx 1.98\%$$

P 5.53 **[a]** It follows directly from the solution to Problem 5.50 that

$$v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f v_{\text{in}}}{R[R_1(R + R_f) + RR_f]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v_o = \frac{(R + R_f)R_f(\Delta R)v_{\text{in}}}{R[(R - \Delta R)(R + R_f) + RR_f]}$$

Now let $\Delta R \ll R$ and get

$$v_o \approx \frac{(R + R_f)R_f \Delta R v_{\text{in}}}{R^2(R + 2R_f)}$$

[b] It follows directly from the solution to Problem 5.50 that

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R - \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \quad \text{Error} = \frac{(R - \Delta R)(R + R_f) + RR_f - R(R + 2R_f)}{R(R + 2R_f)}$$

$$= \frac{-\Delta R(R + R_f)}{R(R + 2R_f)}$$

$$\therefore \quad \%\text{ error} = \frac{-\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

[c] $R - \Delta R = 9810\,\Omega \qquad \therefore \quad \Delta R = 10{,}000 - 9810 = 190\,\Omega$

$$\therefore \quad v_o \approx \frac{(48 \times 10^4)(47 \times 10^4)(190)(15)}{10^8(95 \times 10^4)} \approx 6.768 \text{ V}$$

[d] $$\%\text{ error} = \frac{-190(48 \times 10^4)(100)}{10^4(95 \times 10^4)} = -0.96\%$$

6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 **[a]** $i_g = 8e^{-300t} - 8e^{-1200t}\,\mathrm{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\mathrm{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\mathrm{V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54\,\mathrm{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\mathrm{W}$

[d] $\dfrac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.44766, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\mathrm{s}$$

$$x = 11.0523, \qquad t = \frac{\ln 11.05}{900} = 2.67\,\mathrm{ms}$$

p is maximum at $t = 411.05\,\mu\mathrm{s}$

[e] $p_{\max} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\mathrm{W}$

[f] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\mathrm{ms}$.

[g] $i_{\max} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\mathrm{A}$

$$w_{\max} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\mathrm{mJ}$$

AP 6.2 **[a]** $i = C\frac{dv}{dt} = 24 \times 10^{-6}\frac{d}{dt}[e^{-15{,}000t}\sin 30{,}000t]$

$$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15{,}000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$$p = vi = -649.23\,\text{mW}$$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 **[a]** $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$$= \frac{1}{0.6 \times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$$

[c] $w_{(\text{max})} = \left(\frac{1}{2}\right)Cv_{\text{max}}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 **[a]** $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$$

$$i_1 + i_2 = i$$

AP 6.5 $v_1 = 0.5 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$$v_2 = 0.125 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$$

$$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$$

$$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$$

AP 6.6 **[a]** Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2 + i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i_g + 8\frac{di_g}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2 + i_g)}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di_g}{dt}$$

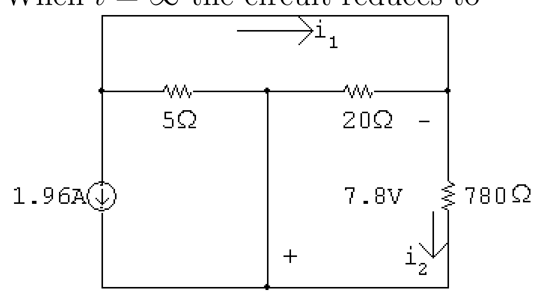
[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to



$$\therefore \quad i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

Also, $\displaystyle\frac{di_g}{dt} = 7.84e^{-4t}$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

$$8\frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8\frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t}$$
$$+0.20 + 19.80e^{-4t} - 20e^{-5t} \stackrel{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5t}$$
$$+(185.60 - 290 + 31.68 + 19.80)e^{-4t} \stackrel{?}{=} -(9.8 + 52.92e^{-4t})$$

$$-9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \stackrel{?}{=} -9.8 - 52.92e^{-4t}$$

$$-9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16\frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16\frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t}$$
$$-8 - 792e^{-4t} + 800e^{-5t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5t}$$
$$+(371.20 + 232 + 63.36 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$-125.44e^{-4t} = -125.44e^{-4t} \quad \text{(OK)}$$

Problems

P 6.1 **[a]** $v = L\dfrac{di}{dt}$

$$= (150 \times 10^{-6})(25)[e^{-500t} - 500te^{-500t}] = 3.75e^{-500t}(1 - 500t)\,\text{mV}$$

[b] $i(5\,\text{ms}) = 25(0.005)(e^{-2.5}) = 10.26\,\text{mA}$

$$v(5\,\text{ms}) = 0.00375(e^{-2.5})(1 - 2.5) = -461.73\,\mu\text{V}$$

$$p(5\,\text{ms}) = vi = (10.26 \times 10^{-3})(-461.73 \times 10^{-6}) = -4.74\,\mu\text{W}$$

[c] delivering $4.74\,\mu$W

[d] $i(5\,\text{ms}) = 10.26\,\text{mA}$ (from part [b])

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(150 \times 10^{-6})(0.01026)^2 = 7.9\,\text{nJ}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1 - 500t = 0 \quad \text{or} \quad t = 2\,\text{ms}$$

$$i_{\text{max}} = 25(0.002)e^{-1} = 18.39\,\text{mA}$$

$$w_{\text{max}} = \frac{1}{2}(150 \times 10^{-6})(0.01839)^2 = 25.38\,\text{nJ}$$

P 6.2 **[a]**

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 4t\,\text{A} && 0 \le t \le 25\,\text{ms} \\ i &= 0.2 - 4t\,\text{A} && 25 \le t \le 50\,\text{ms} \\ i &= 0 && 50\,\text{ms} < t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 500 \times 10^{-3}(4) = 2\,\text{V} \qquad 0 \le t \le 25\,\text{ms}$

$$v = 500 \times 10^{-3}(-4) = -2\,\text{V} \qquad 25 \le t \le 50\,\text{ms}$$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 2\,\text{V} && 0 < t < 25\,\text{ms} \\ v &= -2\,\text{V} && 25 < t < 50\,\text{ms} \\ v &= 0 && 50\,\text{ms} < t \end{aligned}$$

$$p = vi$$

$$
\begin{aligned}
p &= 0 && t < 0 \\
p &= (4t)(2) = 8t\,\mathrm{W} && 0 < t < 25\,\mathrm{ms} \\
p &= (0.2 - 4t)(-2) = 8t - 0.4\,\mathrm{W} && 25 < t < 50\,\mathrm{ms} \\
p &= 0 && 50\,\mathrm{ms} < t
\end{aligned}
$$

$$
\begin{aligned}
w &= 0 && t < 0 \\
w &= \int_0^t (8x)\,dx = 8\frac{x^2}{2}\bigg|_0^t = 4t^2\,\mathrm{J} && 0 < t < 25\,\mathrm{ms} \\
w &= \int_{0.025}^t (8x - 0.4)\,dx + 2.5\times 10^{-3} && \\
&= 4x^2 - 0.4x\bigg|_{0.025}^t + 2.5\times 10^{-3} && \\
&= 4t^2 - 0.4t + 10\times 10^{-3}\,\mathrm{J} && 25 < t < 50\,\mathrm{ms} \\
w &= 0 && 10\,\mathrm{ms} < t
\end{aligned}
$$

P 6.3 **[a]** $i(0) = A_1 + A_2 = 0.12$

$$\frac{di}{dt} = -500A_1e^{-500t} - 2000A_2e^{-2000t}$$

$$v = -25A_1e^{-500t} - 100A_2e^{-2000t}\,\mathrm{V}$$

$$v(0) = -25A_1 - 100A_2 = 3$$

Solving, $A_1 = 0.2$ and $A_2 = -0.08$

Thus,

$$i = 200e^{-500t} - 80e^{-2000t}\,\mathrm{mA} \qquad t \geq 0$$

$$v = -5e^{-500t} + 8e^{-2000t}\,\mathrm{V}, \qquad t \geq 0$$

[b] $i = 0$ when $200e^{-500t} = 80e^{-2000t}$

Therefore

$e^{1500t} = 0.4$ so $t = -610.86\,\mu\mathrm{s}$ which is not possible!

$v = 0$ when $5e^{-500t} = 8e^{-2000t}$

Therefore

$e^{1500t} = 1.6$ so $t = 313.34\,\mu\mathrm{s}$

Thus the power is zero at $t = 313.34\,\mu\mathrm{s}$.

P 6.4 **[a]** From Problem 6.3 we have

$$i = A_1e^{-500t} + A_2e^{-2000t}\,\text{A}$$

$$v = -25A_1e^{-500t} - 100A_2e^{-2000t}\,\text{V}$$

$$i(0) = A_1 + A_2 = 0.12$$

$$v(0) = -25A_1 - 100A_2 = -18$$

Solving, $\quad A_1 = -0.08; \quad A_2 = 0.2$

Thus,

$$i = -80e^{-500t} + 200e^{-2000t}\,\text{mA} \quad t \geq 0$$

$$v = 2e^{-500t} - 20e^{-2000t}\,\text{V} \quad t \geq 0$$

[b] $i = 0 \quad \text{when} \quad 80e^{-500t} = 200e^{-2000t}$

$$\therefore \quad e^{1500t} = 2.5 \quad \text{so} \quad t = 610.86\,\mu\text{s}$$

Thus,

$$i > 0 \quad \text{for} \quad 0 \leq t < 610.86\,\mu\text{s} \qquad \text{and} \qquad i < 0 \quad \text{for} \quad 610.86\,\mu\text{s} < t < \infty$$

$$v = 0 \quad \text{when} \quad 2e^{-500t} = 20e^{-2000t}$$

$$\therefore \quad e^{1500t} = 10 \quad \text{so} \quad t = 1535.06\,\mu\text{s}$$

Thus,

$$v < 0 \quad \text{for} \quad 0 \leq t < 1535.06\,\mu\text{s} \qquad \text{and} \qquad v > 0 \quad \text{for} \quad 1535.06\,\mu\text{s} < t < \infty$$

Therefore,

$$p < 0 \quad \text{for} \quad 0 \leq t < 610.86\,\mu\text{s} \qquad \text{and} \qquad 1535.06\,\mu\text{s} < t < \infty$$

(inductor delivers energy)

$$p > 0 \quad \text{for} \quad 610.86\,\mu\text{s} < t < 1535.06\,\mu\text{s} \quad \text{(inductor stores energy)}$$

[c] The energy stored at $t = 0$ is

$$w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(0.05)(0.12)^2 = 360\,\mu\text{J}$$

$$p = vi = -0.16e^{-1000t} + 2e^{-2500t} - 4e^{-4000t}\,\text{W}$$

For $t > 0$:

$$w = \int_0^\infty -0.16e^{-1000t}\,dt + \int_0^\infty 2e^{-2500t}\,dt - \int_0^\infty 4e^{-4000t}\,dt$$

$$= \frac{-0.16e^{-1000t}}{-1000}\Big|_0^\infty + \frac{2e^{-2500t}}{-2500}\Big|_0^\infty - \frac{4e^{-4000t}}{-4000}\Big|_0^\infty$$

$$= (-1.6 + 8 - 10) \times 10^{-4}$$

$$= -360\,\mu\text{J}$$

Thus, the energy stored equals the energy extracted.

P 6.5 $i = (B_1 \cos 200t + B_2 \sin 200t)e^{-50t}$

$$i(0) = B_1 = 75\,\text{mA}$$

$$\frac{di}{dt} = (B_1 \cos 200t + B_2 \sin 200t)(-50e^{-50t}) + e^{-50t}(-200B_1 \sin 200t + 200B_2 \cos 200t)$$

$$= [(200B_2 - 50B_1)\cos 200t - (200B_1 + 50B_2)\sin 200t]e^{-50t}$$

$$v = 0.2\frac{di}{dt} = [(40B_2 - 10B_1)\cos 200t - (40B_1 + 10B_2)\sin 200t]e^{-50t}$$

$$v(0) = 4.25 = 40B_2 - 10B_1 = 40B_2 - 0.75 \quad \therefore \quad B_2 = 125\,\text{mA}$$

Thus,

$$i = (75\cos 200t + 125\sin 200t)e^{-50t}\,\text{mA}, \qquad t \geq 0$$

$$v = (4.25\cos 200t - 4.25\sin 200t)e^{-50t}\,\text{V}, \qquad t \geq 0$$

$$i(0.025) = -28.25\,\text{mA}; \qquad v(0.025) = 1.513\,\text{V}$$

$$p(0.025) = (-28.25)(1.513) = -42.7\,\text{mW delivering}$$

P 6.6 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$\text{W} = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\text{J}$$

This is energy stored in the inductor at $t = \infty$.

P 6.7 **[a]** $0 \leq t \leq 50\,\text{ms}$:

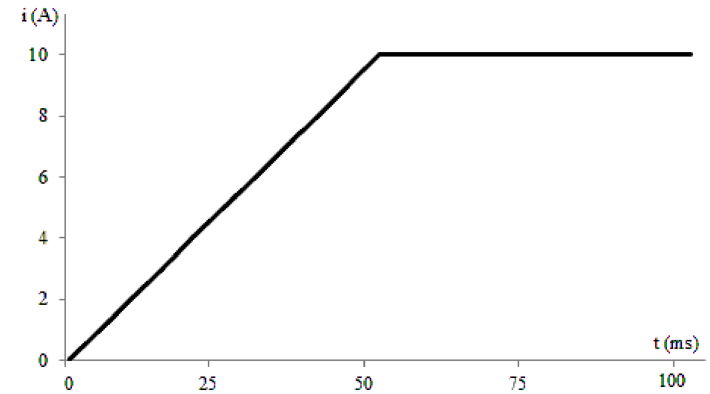
$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{750}\int_0^t 0.15\,dx + 0$$

$$= 200x\,\Big|_0^t = 200t\,\text{A}$$

$$i(0.05) = 200(0.05) = 10\,\text{A}$$

$$t \geq 50\,\text{ms}: \qquad i = \frac{10^6}{750}\int_{50\times10^{-3}}^t (0)\,dx + 10 = 10\,\text{A}$$

[b] $i = 200t\,\text{A}, \quad 0 \le t \le 50\,\text{ms}; \qquad\qquad i = 10\,\text{A}, \quad t \ge 50\,\text{ms}$



P 6.8 $\quad 0 \le t \le 100\,\text{ms}:$

$$i_L = \frac{10^3}{50}\int_0^t 2e^{-100x}\,dx + 0.1 = 40\frac{e^{-100x}}{-100}\Big|_0^t + 0.1$$

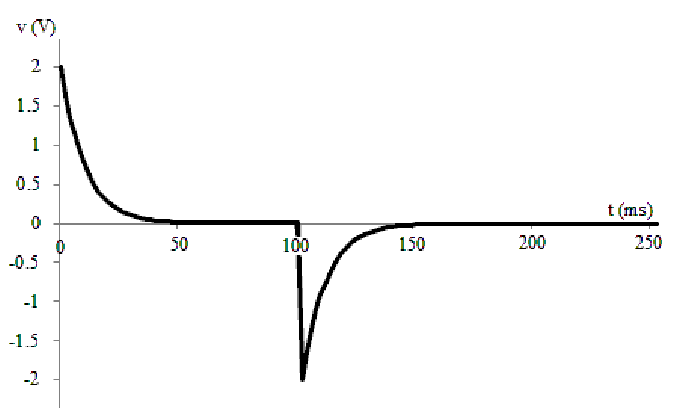
$$= -0.4e^{-100t} + 0.5\,\text{A}$$

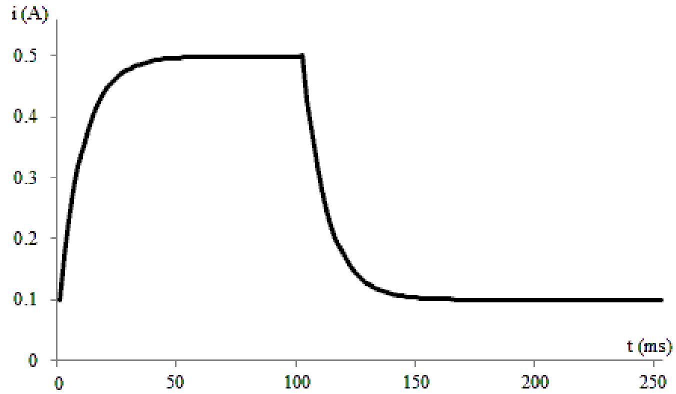
$$i_L(0.1) = -0.4e^{-10} + 0.5 = 0.5\,\text{A}$$

$t \ge 100\,\text{ms}:$

$$i_L = \frac{10^3}{50}\int_{0.1}^t -2e^{-100(x-0.1)}\,dx + 0.5 = -40\frac{e^{-100(x-0.1)}}{-100}\Big|_{0.1}^t + 0.5$$

$$= 0.4e^{-100(t-0.1)} + 0.1\,\text{A}$$





P 6.9 **[a]** $0 \le t \le 25\,\text{ms}:$

$$v = 800t$$

$$i = \frac{1}{10}\int_0^t 800x\,dx + 0 = 80\frac{x^2}{2}\Big|_0^t$$

$$i = 40t^2\ \text{A}$$

$25\,\text{ms} \le t \le 75\,\text{ms}:$

$$v = 20$$

$$i(0.025) = 25\,\text{mA}$$

$$\begin{aligned}\therefore\ i &= \frac{1}{10}\int_{0.025}^t 20\,dx + 0.025\\ &= 2x\Big|_{0.025}^t + 0.025\\ &= 2t - 0.025\ \text{A}\end{aligned}$$

$75\,\text{ms} \le t \le 125\,\text{ms}:$

$$v = 80 - 800t\ \text{V}$$

$$i(0.075) = 2(0.075) - 0.025 = 0.125\ \text{A}$$

$$\begin{aligned}i &= \frac{1}{10}\int_{0.075}^t (80 - 800x)\,dx + 0.125\\ &= \left(8x - \frac{80x^2}{2}\right)\Big|_{0.075}^t + 0.125\\ &= 8t - 40t^2 - 0.25\ \text{A}\end{aligned}$$

$125\,\text{ms} \le t \le 150\,\text{ms}:$

$$v = 800t - 120$$

$$i(0.125) = 8(0.125) - 40(0.125)^2 - 0.25 = 0.125\,\text{A}$$

$$\begin{aligned} i &= \frac{1}{10}\int_{0.125}^{t}(800x - 120)\,dx + 0.125 \\ &= \left(\frac{80x^2}{2} - 12x\right)\Big|_{0.125}^{t} + 0.125 \\ &= 40t^2 - 12t + 1\,\text{A} \end{aligned}$$

$t \geq 150\,\text{ms}:$

$$v = 0$$

$$i(0.150) = 40(0.15)^2 - 12(0.15) + 1 = 0.1\,\text{A}$$

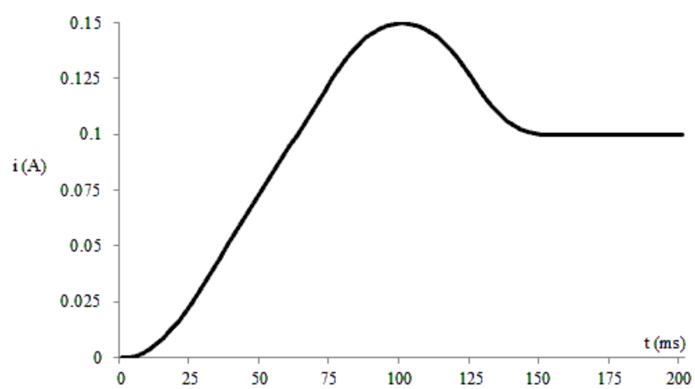
$$\begin{aligned} i &= \frac{1}{10}\int_{0.15}^{t} 0\,dx + 0.1 \\ &= 0.1\,\text{A} \end{aligned}$$

[b] $v = 0 \quad \text{at} \quad t = 100\,\text{ms and } t = 150\,\text{ms}$

$$i(0.1) = 8(0.1) - 40(0.1)^2 - 0.25 = 0.15\,\text{A}$$

$$i(0.15) = 0.1\,\text{A}$$

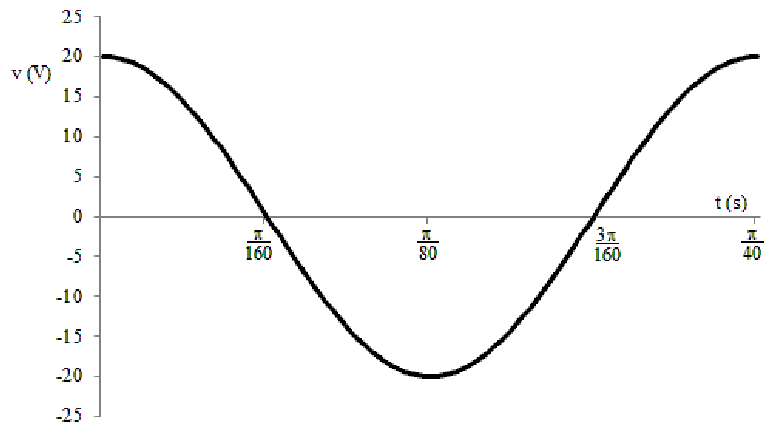
[c]

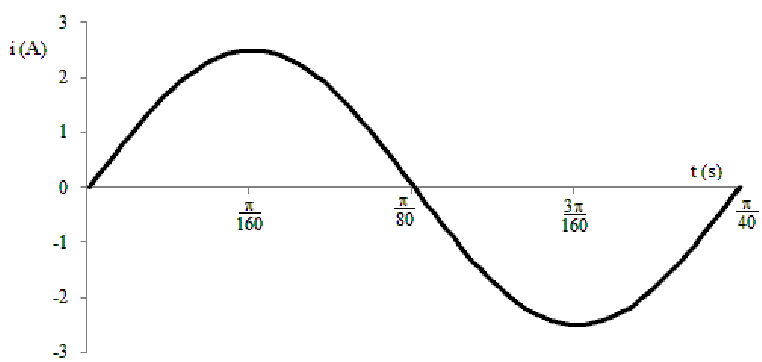


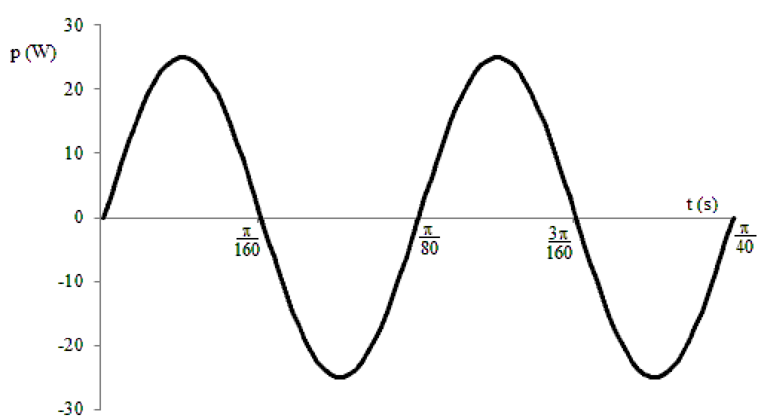
P 6.10 **[a]**

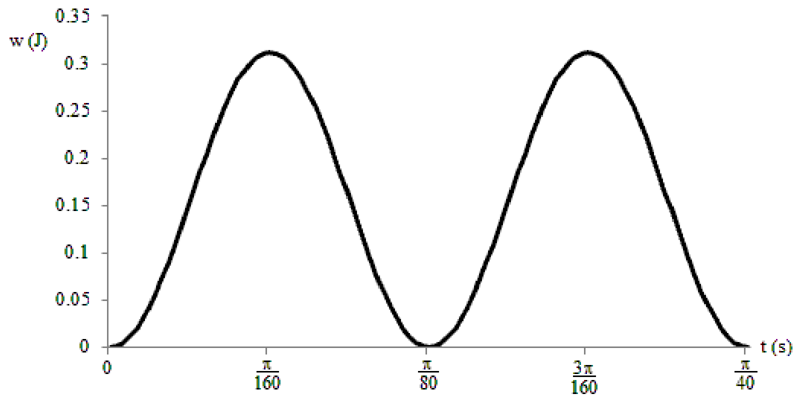
$$\begin{aligned} i &= \frac{1}{0.1}\int_{0}^{t} 20\cos 80x\,dx \\ &= 200\frac{\sin 80x}{80}\Big|_{0}^{t} \\ &= 2.5\sin 80t\,\text{A} \end{aligned}$$

[b]
$$
\begin{aligned}
p &= vi = (20\cos 80t)(2.5\sin 80t) \\
&= 50\cos 80t \sin 80t \\
p &= 25\sin 160t\,\mathrm{W} \\
w &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(0.1)(2.5\sin 80t)^2 \\
&= 312.5\sin^2 80t\,\mathrm{mJ} \\
w &= (156.25 - 156.25\cos 160t)\,\mathrm{mJ}
\end{aligned}
$$









[c] Absorbing power: Delivering power:

$0 \le t \le \pi/160\,\text{s}$ $\qquad$ $\pi/160 \le t \le \pi/80\,\text{s}$

$\pi/80 \le t \le 3\pi/160\,\text{s}$ $\qquad$ $3\pi/160 \le t \le \pi/40\,\text{s}$

P 6.11 [a] $v = L\dfrac{di}{dt}$

$$v = -25\times10^{-3}\frac{d}{dt}[10\cos 400t + 5\sin 400t]e^{-200t}$$

$$= -25\times10^{-3}(-200e^{-200t}[10\cos 400t + 5\sin 400t]$$

$$+e^{-200t}[-4000\sin 400t + 2000\cos 400t])$$

$$v = -25\times10^{-3}e^{-200t}(-1000\sin 400t - 4000\sin 400t)$$

$$= -25\times10^{-3}e^{-200t}(-5000\sin 400t)$$

$$= 125e^{-200t}\sin 400t\,\text{V}$$

$$\frac{dv}{dt} = 125(e^{-200t}(400)\cos 400t - 200e^{-200t}\sin 400t)$$

$$= 25{,}000e^{-200t}(2\cos 400t - \sin 400t)\ \text{V/s}$$

$$\frac{dv}{dt} = 0 \qquad \text{when} \qquad 2\cos 400t = \sin 400t$$

$$\therefore\quad \tan 400t = 2, \qquad 400t = 1.11; \qquad t = 2.77\,\text{ms}$$

[b] $v(2.77\text{ ms}) = 125e^{-0.55}\sin 1.11 = 64.27\,\text{V}$

P 6.12 For $0 \le t \le 1.6\,\text{s}$:

$$i_L = \frac{1}{5}\int_0^t 3\times10^{-3}\,dx + 0 = 0.6\times10^{-3}t$$

$$i_L(1.6\,\text{s}) = (0.6\times10^{-3})(1.6) = 0.96\,\text{mA}$$

$R_m = (20)(1000) = 20\,\mathrm{k}\Omega$

$v_m(1.6\,\mathrm{s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2\,\mathrm{V}$

P 6.13 **[a]** $i = C\dfrac{dv}{dt} = (5 \times 10^{-6})[500t(-2500)e^{-2500t} + 500e^{-2500t}]$

$= 2.5 \times 10^{-3}e^{-2500t}(1 - 2500t)\,\mathrm{A}$

[b] $v(100\,\mu) = 500(100 \times 10^{-6})e^{-0.25} = 38.94\,\mathrm{mV}$

$i(100\,\mu) = (2.5 \times 10^{-3})e^{-0.25}(1 - 0.25) = 1.46\,\mathrm{mA}$

$p(100\,\mu) = vi = (38.94 \times 10^{-3})(1.46 \times 10^{-3}) = 56.86\,\mu\mathrm{W}$

[c] $p > 0$, so the capacitor is absorbing power.

[d] $v(100\,\mu) = 38.94\,\mathrm{mV}$

$w = \frac{1}{2}Cv^2 = \frac{1}{2}(5 \times 10^{-6})(38.94 \times 10^{-3})^2 = 3.79\,\mathrm{nJ}$

[e] The energy is maximum when the voltage is maximum:

$\dfrac{dv}{dt} = 0$ when $(1 - 2500t) = 0$ or $t = 0.4\,\mathrm{ms}$

$v_{\mathrm{max}} = 500(0.4 \times 10^{-3})^2e^{-1} = 73.58\,\mathrm{mV}$

$p_{\mathrm{max}} = \frac{1}{2}Cv_{\mathrm{max}}^2 = 13.53\,\mathrm{nJ}$

P 6.14 **[a]**

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 10t\,\mathrm{A} && 0 \le t \le 2\,\mathrm{s} \\ v &= 40 - 10t\,\mathrm{A} && 2 \le t \le 6\,\mathrm{s} \\ v &= 10t - 80\,\mathrm{A} && 6 \le t \le 8\,\mathrm{s} \\ v &= 0 && 8\,\mathrm{s} < t \end{aligned}$$

[b] $i = C\dfrac{dv}{dt}$

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 2\,\mathrm{mA} && 0 < t < 2\,\mathrm{s} \\ i &= -2\,\mathrm{mA} && 2 < t < 6\,\mathrm{s} \\ i &= 2\,\mathrm{mA} && 6 < t < 8\,\mathrm{s} \\ i &= 0 && 8\,\mathrm{s} < t \end{aligned}$$

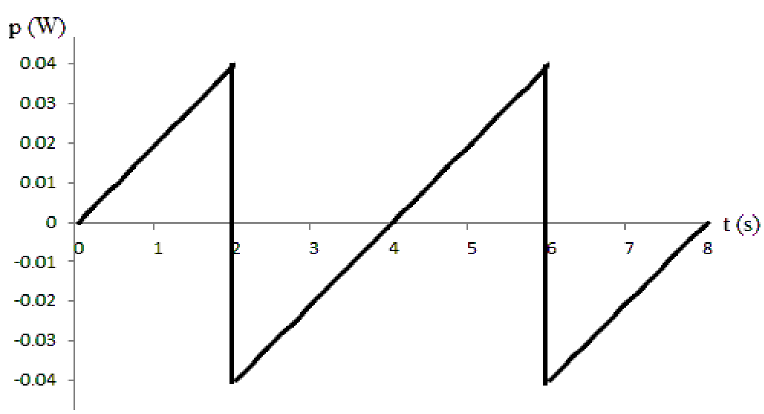
$p = vi$

$$
\begin{aligned}
p &= 0 && t < 0 \\
p &= (10t)(0.002) = 0.02t\,\text{W} && 0 < t < 2\,\text{s} \\
p &= (40 - 10t)(-0.002) = 0.02t - 0.08\,\text{W} && 2 < t < 6\,\text{s} \\
p &= (10t - 80)(0.002) = 0.02t - 0.16\,\text{W} && 6 < t < 8\,\text{s} \\
p &= 0 && 8\,\text{s} < t
\end{aligned}
$$

$w = \int p\,dx$

$$
\begin{aligned}
w &= 0 && t < 0 \\
w &= \int_0^t (0.02x)\,dx = 0.01x^2 \Big|_0^t = 0.01t^2\,\text{J} && 0 < t < 2\,\text{s} \\
w &= \int_2^t (0.02x - 0.08)\,dx + 0.04 \\
&= (0.01x^2 - 0.08x) \Big|_2^t + 0.04 \\
&= 0.01t^2 - 0.08t + 0.16\,\text{J} && 2 < t < 6\,\text{s} \\
w &= \int_6^t (0.02x - 0.16)\,dx + \ 0.04 \\
&= (0.01x^2 - 0.16x) \Big|_6^t + 0.04 \\
&= 0.01t^2 - 0.16t + 0.64\,\text{J} && 6 < t < 8\,\text{s} \\
w &= 0 && 8\,\text{s} < t
\end{aligned}
$$

[c]



From the plot of power above, it is clear that power is being absorbed for $0 < t < 2$ s and for $4\text{ s} < t < 6$ s, because $p > 0$. Likewise, power is being delivered for $2\text{ s} < t < 4$ s and $6\text{ s} < t < 8$ s, because $p < 0$.

P 6.15 [a] $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(5 \times 10^{-6})(60)^2 = 9\,\text{mJ}$

[b] $v = (A_1 + A_2 t)e^{-1500t}$

$$v(0) = A_1 = 60\,\text{V}$$

$$\begin{aligned}\frac{dv}{dt} &= -1500e^{-1500t}(A_1 + A_2 t) + e^{-1500t}(A_2)\\ &= (-1500A_2 t - 1500A_1 + A_2)e^{-1500t}\end{aligned}$$

$$\frac{dv(0)}{dt} = A_2 - 1500A_1$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore \quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{100 \times 10^{-3}}{5 \times 10^{-6}} = 20 \times 10^3$$

$$\therefore \quad 20 \times 10^3 = A_2 - 1500(60)$$

Thus, $A_2 = 20 \times 10^3 + 90 \times 10^3 = 110 \times 10^3\ \frac{\text{V}}{\text{s}}$

[c] $v = (60 + 110 \times 10^3 t)e^{-1500t}$

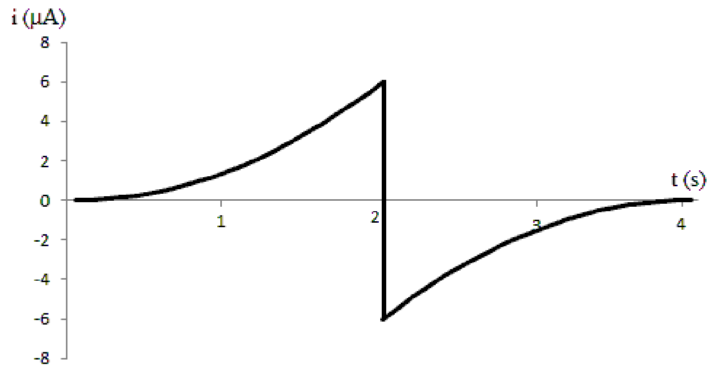
$$i = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(60 + 110 \times 10^3 t)e^{-1500t}$$

$$\begin{aligned} i &= (5 \times 10^{-6})[110{,}000e^{-1500t} - 1500(60 + 110,000t)e^{-1500t}]\\ &= (0.1 - 825t)e^{-1500t}\,\text{A}, \qquad t \geq 0\end{aligned}$$

P 6.16 $i_C = C(dv/dt)$

$0 < t < 2\,\text{s}:$ $\quad i_C = 100 \times 10^{-9}(15)t^2 = 1.5 \times 10^{-6}t^2\,\text{A}$

$2 < t < 4\,\text{s}:$ $\quad i_C = 100 \times 10^{-9}(-15)(t-4)^2 = -1.5 \times 10^{-6}(t-4)^2\,\text{A}$



P 6.17 **[a]** $i = C\dfrac{dv}{dt} = 0, \quad t < 0$

[b] $$i = C\frac{dv}{dt} = 120 \times 10^{-6}\frac{d}{dt}[30 + 5e^{-500t}(6\cos 2000t + \sin 2000t)]$$

$$= 120 \times 10^{-6}[5(-500)e^{-500t}(6\cos 2000t + \sin 2000t)$$

$$+5(2000)e^{-500t}(-6\sin 2000t + \cos 2000t)]$$

$$= -0.6e^{-500t}[\cos 2000t + 12.5\sin 2000t]\,\text{A}, \quad t \geq 0$$

[c] no, $\quad v(0^-) = 60$ V

$v(0^+) = 30 + 5(6) = 60$ V

[d] yes, $\quad i(0^-) = 0$ A

$i(0^+) = -0.6$ A

[e] $v(\infty) = 30\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(120 \times 10^{-6})(30)^2 = 54\,\text{mJ}$$

P 6.18 **[a]**
$$\begin{aligned}
v(20\,\mu\text{s}) &= 12.5 \times 10^9(20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)}\\
v(20\,\mu\text{s}) &= 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10\\
&= 5\,\text{V (start of second interval)}\\
v(40\,\mu\text{s}) &= 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10\\
&= 10\,\text{V (end of second interval)}
\end{aligned}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = (1.25)(50\,\text{m}) = 62.5\,\text{mW}$ (checks)

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

$p(30\,\mu\text{s}) = vi = (8.75)(0.05) = 62.5\,\text{mW}$ (checks)

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$

P 6.19 **[a]**

$$\begin{aligned} v &= \frac{1}{0.5 \times 10^{-6}} \int_0^{500\times10^{-6}} 50 \times 10^{-3} e^{-2000t}\, dt - 20 \\ &= 100 \times 10^3 \frac{e^{-2000t}}{-2000} \Big|_0^{500\times10^{-6}} -20 \\ &= 50(1-e^{-1}) - 20 = 11.61\,\text{V} \\ w &= \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7\,\mu\text{J} \end{aligned}$$

[b] $v(\infty) = 50 - 20 = 30\text{V}$

$$w(\infty) = \frac{1}{2}(0.5 \times 10^{-6})(30)^2 = 225\,\mu\text{J}$$

P 6.20 **[a]** $i = \dfrac{5}{2 \times 10^{-3}}t = 2500t \qquad 0 \le t \le 2\,\text{ms}$

$$i = \frac{-10}{4 \times 10^{-3}}t + 10 = 10 - 2500t \qquad 2 \le t \le 6\,\text{ms}$$

$$i = \frac{10}{4 \times 10^{-3}}t - 20 = 2500t - 20 \qquad 6 \le t \le 10\,\text{ms}$$

$$i = \frac{-5}{2 \times 10^{-3}}t + 30 = 30 - 2500t \qquad 10 \le t \le 12\,\text{ms}$$

$$\begin{aligned} q &= \int_0^{0.002} 2500t\, dt + \int_{0.002}^{0.006} (10 - 2500t)\, dt \\ &= \frac{2500t^2}{2}\Big|_0^{0.002} + \left(10t - \frac{2500t^2}{2}\right)\Big|_{0.002}^{0.006} \\ &= 0.005 - 0 + (0.06 - 0.045) - (0.02 - 0.005) \\ &= 5\,\text{mC} \end{aligned}$$

[b]

$$v = 0.5 \times 10^6 \int_0^{0.002} 2500x\, dx + 0.5 \times 10^6 \int_{0.002}^{0.006} (10 - 2500x)\, dx$$

$$+\,0.5 \times 10^6 \int_{0.006}^{0.01} (2500x - 20)\, dx$$

$$= 0.5 \times 10^6 \left[\frac{2500x^2}{2}\Big|_0^{0.002} + 10x\Big|_{0.002}^{0.006} - \frac{2500x^2}{2}\Big|_{0.002}^{0.006} + \frac{2500x^2}{2}\Big|_{0.006}^{0.01} - 20x\Big|_{0.006}^{0.01}\right]$$

$$= 0.5 \times 10^6[(0.005 - 0) + (0.06 - 0.02) - (0.045 - 0.005)$$

$$+ (0.125 - 0.045) - (0.2 - 0.12)]$$

$$= 2500\,\text{V}$$

$$v(10\,\text{ms}) = 2500\,\text{V}$$

[c] $v(12\,\text{ms}) = v(10\,\text{ms}) + 0.5 \times 10^6 \int_{0.01}^{0.012} (30 - 2500x)\,dx$

$$= 2500 + 0.5 \times 10^6 \left(30x - \frac{2500x^2}{2}\right) \Big|_{0.01}^{0.012}$$

$$= 2500 + 0.5 \times 10^6 (0.36 - 0.18 - 0.3 + 0.125)$$

$$= 2500 + 2500 = 5000\,\text{V}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(2 \times 10^{-6})(5000)^2 = 25\,\text{J}$$

P 6.21 **[a]** $0 \le t \le 10\,\mu\text{s}$

$$C = 0.1\,\mu\text{F} \qquad \frac{1}{C} = 10 \times 10^6$$

$$v = 10 \times 10^6 \int_0^t -0.05\,dx + 15$$

$$v = -50 \times 10^4 t + 15\,\text{V} \qquad 0 \le t \le 10\,\mu\text{s}$$

$$v(10\,\mu\text{s}) = -5 + 15 = 10\,\text{V}$$

[b] $10\,\mu\text{s} \le t \le 20\,\mu\text{s}$

$$v = 10 \times 10^6 \int_{10\times10^{-6}}^t 0.1\,dx + 10 = 10^6 t - 10 + 10$$

$$v = 10^6 t\,\text{V} \qquad 10 \le t \le 20\,\mu\text{s}$$

$$v(20\,\mu\text{s}) = 10^6(20 \times 10^{-6}) = 20\,\text{V}$$

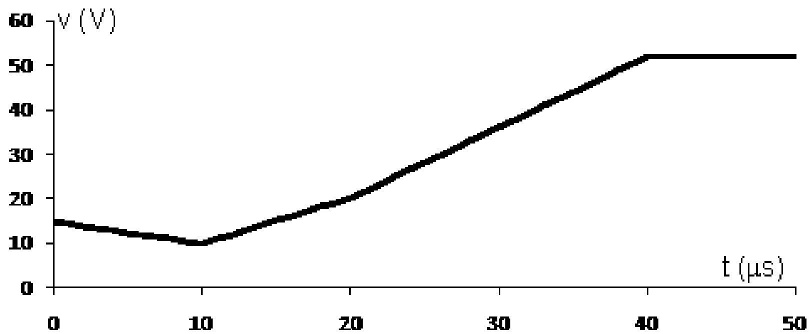
[c] $20\,\mu\text{s} \le t \le 40\,\mu\text{s}$

$$v = 10 \times 10^6 \int_{20\times10^{-6}}^t 1.6\,dx + 20 = 1.6 \times 10^6 t - 32 + 20$$

$$v = 1.6 \times 10^6 t - 12\,\text{V}, \qquad 20\,\mu\text{s} \le t \le 40\,\mu\text{s}$$

[d] $40\,\mu\text{s} \le t < \infty$

$$v(40\,\mu\text{s}) = 64 - 12 = 52\,\text{V} \qquad 40\,\mu\text{s} \le t < \infty$$



P 6.22 [a] $15\|30 = 10\,\text{mH}$

$10 + 10 = 20\,\text{mH}$

$20\|20 = 10\,\text{mH}$

$12\|24 = 8\,\text{mH}$

$10 + 8 = 18\,\text{mH}$

$18\|9 = 6\,\text{mH}$

$L_{\text{ab}} = 6 + 8 = 14\,\text{mH}$

[b] $12 + 18 = 30\,\mu\text{H}$

$30\|20 = 12\,\mu\text{H}$

$12 + 38 = 50\,\mu\text{H}$

$30\|75\|50 = 15\,\mu\text{H}$

$15 + 15 = 30\,\mu\text{H}$

$30\|60 = 20\,\mu\text{H}$

$L_{\text{ab}} = 20 + 25 = 45\,\mu\text{H}$

P 6.23 [a] Combine two 10 mH inductors in parallel to get a 5 mH equivalent inductor. Then combine this parallel pair in series with three 1 mH inductors:

$$10\,\text{m}\|10\,\text{m} + 1\,\text{m} + 1\,\text{m} + 1\,\text{m} = 8\,\text{mH}$$

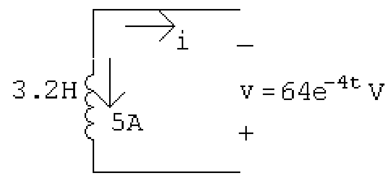
[b] Combine two $10\,\mu$H inductors in parallel to get a $5\,\mu$H inductor. Then combine this parallel pair in series with four more $10\,\mu$H inductors:

$$10\,\mu\|10\,\mu + 10\,\mu + 10\,\mu + 10\,\mu + 10\,\mu = 45\,\mu\text{H}$$

[c] Combine two 100 μH inductors in parallel to get a 50 μH inductor. Then combine this parallel pair with a 100 μH inductor and three 10 μH inductors in series:

$$100\,\mu \| 100\,\mu + 100\,\mu + 10\,\mu + 10\,\mu + 10\,\mu = 180\,\mu\text{H}$$

P 6.24 [a]



$$3.2\frac{di}{dt} = 64e^{-4t} \qquad \text{so} \qquad \frac{di}{dt} = 20e^{-4t}$$

$$\begin{aligned} i(t) &= 20\int_0^t e^{-4x}\,dx - 5 \\ &= 20\frac{e^{-4x}}{-4}\Big|_0^t - 5 \\ i(t) &= -5e^{-4t}\,\text{A} \end{aligned}$$

[b] $4\dfrac{di_1}{dt} = 64e^{-4t}$

$$\begin{aligned} i_1(t) &= 16\int_0^t e^{-4x}\,dx - 10 \\ &= 16\frac{e^{-4x}}{-4}\Big|_0^t - 10 \\ i_1(t) &= -4e^{-4t} - 6\,\text{A} \end{aligned}$$

[c] $16\dfrac{di_2}{dt} = 64e^{-4t} \qquad \text{so} \qquad \dfrac{di_2}{dt} = 4e^{-4t}$

$$\begin{aligned} i_2(t) &= 4\int_0^t e^{-4x}\,dx + 5 \\ &= 4\frac{e^{-4x}}{-4}\Big|_0^t + 5 \\ i_2(t) &= -e^{-4t} + 6\,\text{A} \end{aligned}$$

[d] $p = -vi = (-64e^{-4t})(-5e^{-4t}) = 320e^{-8t}\,\text{W}$

$$
\begin{aligned}
w &= \int_0^\infty p\,dt = \int_0^\infty 320e^{-8t}\,dt \\
&= 320\frac{e^{-8t}}{-8}\Big|_0^\infty \\
&= 40\,\text{J}
\end{aligned}
$$

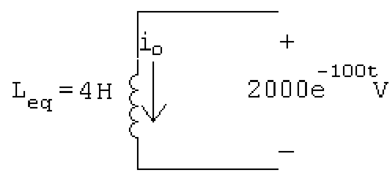
[e] $w = \frac{1}{2}(4)(-10)^2 + \frac{1}{2}(16)(5)^2 = 400\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 400 - 40 = 360\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(4)(-6)^2 + \frac{1}{2}(16)(6)^2 = 360\,\text{J}$ checks

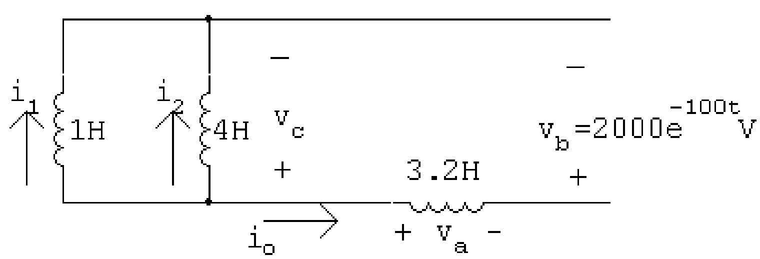
P 6.25 **[a]** $i_o(0) = -i_1(0) - i_2(0) = 6 - 1 = 5\,\text{A}$

[b]



$$
\begin{aligned}
i_o &= -\frac{1}{4}\int_0^t 2000e^{-100x}\,dx + 5 = -500\,\frac{e^{-100x}}{-100}\Big|_0^t + 5 \\
&= 5(e^{-100t} - 1) + 5 = 5e^{-100t}\,\text{A}, \qquad t \geq 0
\end{aligned}
$$

[c]



$$
\begin{aligned}
v_a &= 3.2(-500e^{-100t}) = -1600e^{-100t}\,\text{V} \\
v_c &= v_a + v_b = -1600e^{-100t} + 2000e^{-100t} \\
&= 400e^{-100t}\,\text{V} \\
i_1 &= \frac{1}{1}\int_0^t 400e^{-100x}\,dx - 6 \\
&= -4e^{-100t} + 4 - 6 \\
i_1 &= -4e^{-100t} - 2\,\text{A} \qquad t \geq 0
\end{aligned}
$$

[**d**] $$i_2 = \frac{1}{4}\int_0^t 400e^{-100x}\,dx + 1$$

$$= -e^{-100t} + 2\,\text{A}, \qquad t \geq 0$$

[**e**] $w(0) = \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(1)^2 + \frac{1}{2}(3.2)(5)^2 = 60\,\text{J}$

[**f**] $w_{\text{del}} = \frac{1}{2}(4)(5)^2 = 50\,\text{J}$

[**g**] $w_{\text{trapped}} = 60 - 50 = 10\,\text{J}$

or $\quad w_{\text{trapped}} = \frac{1}{2}(1)(2)^2 + \frac{1}{2}(4)(2)^2 + 10\,\text{J (check)}$

P 6.26 $v_{\text{b}} = 2000e^{-100t}\,\text{V}$

$i_o = 5e^{-100t}\,\text{A}$

$p = 10{,}000e^{-200t}\,\text{W}$

$$w = \int_0^t 10^4 e^{-200x}\,dx = 10{,}000\frac{e^{-200x}}{-200}\Big|_0^t = 50(1 - e^{-200t})\,\text{W}$$

$w_{\text{total}} = 50\,\text{J}$

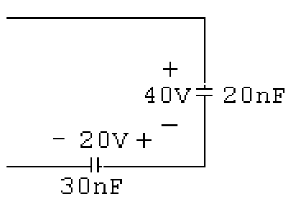
$80\% w_{\text{total}} = 40\,\text{J}$

Thus,

$50 - 50e^{-200t} = 40; \qquad e^{200t} = 5; \qquad \therefore \quad t = 8.05\,\text{ms}$

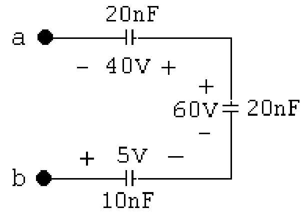
P 6.27 [**a**] $\frac{1}{C_1} = \frac{1}{48} + \frac{1}{24} = \frac{1}{16}; \qquad C_1 = 16\,\text{nF}$

$C_2 = 4 + 16 = 20\,\text{nF}$



$\frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} = \frac{1}{12}; \qquad C_3 = 12\,\text{nF}$

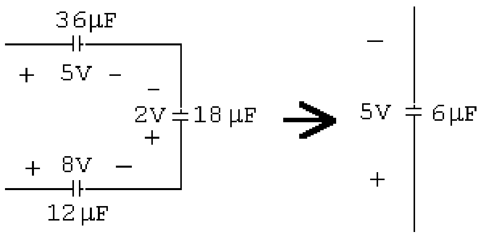
$C_4 = 12 + 8 = 20\,\text{nF}$

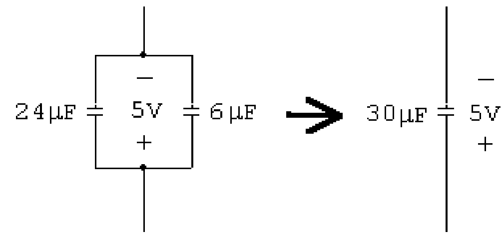


$$\frac{1}{C_5} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{1}{5}; \qquad C_5 = 5\,\text{nF}$$

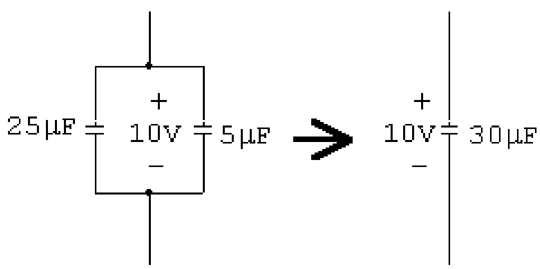
Equivalent capacitance is 5 nF with an initial voltage drop of +15 V.

[b] $\frac{1}{36} + \frac{1}{18} + \frac{1}{12} = \frac{1}{6} \quad \therefore \quad C_{\text{eq}} = 6\,\mu\text{F} \qquad\qquad 24 + 6 = 30\,\mu\text{F}$

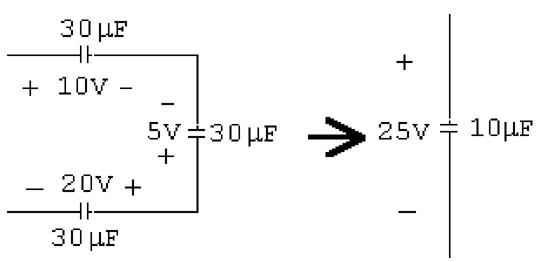




$25 + 5 = 30\,\mu\text{F}$



$$\frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{3}{30} \quad \therefore \quad C_{\text{eq}} = 10\,\mu\text{F}$$



Equivalent capacitance is $10\,\mu$F with an initial voltage drop of +25 V.

P 6.28 [a] Combine a 470 pF capacitor and a 10 pF capacitor in parallel to get a 480 pF capacitor:

(470 p) in parallel with (10 p) = 470 p + 10 p = 480 pF

[**b**] Create a 1200 nF capacitor as follows:

$(1\,\mu)$ in parallel with $(0.1\,\mu)$ in parallel with $(0.1\,\mu)$

$$= 1000\,\text{n} + 100\,\text{n} + 100\,\text{n} = 1200\,\text{nF}$$

Create a second 1200 nF capacitor using the same three resistors. Place these two 1200 nF in series:

$$(1200\,\text{n}) \text{ in series with } (1200\,\text{n}) = \frac{(1200\,\text{n})(1200\,\text{n})}{1200\,\text{n} + 1200\,\text{n}} = 600\,\text{nF}$$

[**a**] Combine two 220 μF capacitors in series to get a 110 μF capacitor. Then combine the series pair in parallel with a 10 μF capacitor to get 120 μF:

$[(220\,\mu)$ in series with $(220\,\mu)]$ in parallel with $(10\,\mu)$

$$= \frac{(220\,\mu)(220\,\mu)}{220\,\mu + 220\,\mu} + 10\,\mu = 120\,\mu\text{F}$$

P 6.29 From Figure 6.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i\,dx + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

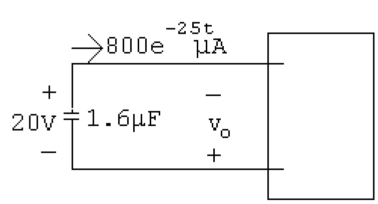
Therefore $\dfrac{1}{C_{\text{eq}}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots\right], \qquad v_{\text{eq}}(0) = v_1(0) + v_2(0) + \cdots$

P 6.30 From Fig. 6.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{\text{eq}} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

P 6.31 [**a**]



$$
\begin{aligned}
v_o &= \frac{10^6}{1.6}\int_0^t 800\times 10^{-6}e^{-25x}\,dx - 20\\
&= 500\frac{e^{-25x}}{-25}\Big|_0^t -20\\
&= -20e^{-25t}\,\text{V},\qquad t\geq 0
\end{aligned}
$$

[b]
$$
\begin{aligned}
v_1 &= \frac{10^6}{2}(800\times 10^{-6})\frac{e^{-25x}}{-25}\Big|_0^t +5\\
&= -16e^{-25t}+21\,\text{V},\qquad t\geq 0
\end{aligned}
$$

[c]
$$
\begin{aligned}
v_2 &= \frac{10^6}{8}(800\times 10^{-6})\frac{e^{-25x}}{-25}\Big|_0^t -25\\
&= -4e^{-25t}-21\,\text{V},\qquad t\geq 0
\end{aligned}
$$

[d]
$$
\begin{aligned}
p &= -vi = -(-20e^{-25t})(800\times 10^{-6})e^{-25t}\\
&= 16\times 10^{-3}e^{-50t}\\
w &= \int_0^\infty 16\times 10^{-3}e^{-50t}\,dt\\
&= 16\times 10^{-3}\frac{e^{-50t}}{-50}\Big|_0^\infty\\
&= -0.32\times 10^{-3}(0-1) = 320\,\mu\text{J}
\end{aligned}
$$

[e]
$$
\begin{aligned}
w &= \tfrac{1}{2}(2\times 10^{-6})(5)^2+\tfrac{1}{2}(8\times 10^{-6})(25)^2\\
&= 2525\,\mu\text{J}
\end{aligned}
$$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 2525-320 = 2205\,\mu\text{J}$

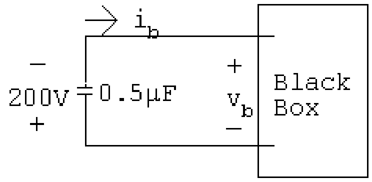
[g]
$$
\begin{aligned}
w_{\text{trapped}} &= \tfrac{1}{2}(2\times 10^{-6})(21)^2+\tfrac{1}{2}(8\times 10^{-6})(-21)^2\\
&= 2205\,\mu\text{J}
\end{aligned}
$$

P 6.32 $\dfrac{1}{C_e} = \dfrac{1}{1}+\dfrac{1}{5}+\dfrac{1}{1.25} = \dfrac{10}{5} = 2$

$\therefore\quad C_2 = 0.5\,\mu\text{F}$

$v_{\text{b}} = 20-250+30 = -200\,\text{V}$

[**a**]



$$v_{\rm b} = -\frac{10^6}{0.5}\int_0^t -5\times 10^{-3}e^{-50x}\,dx - 200$$

$$= 10{,}000\frac{e^{-50x}}{-50}\Big|_0^t -200$$

$$= -200e^{-50t}\,\mathrm{V}$$

[**b**] $$v_{\rm a} = -\frac{10^6}{0.5}\int_0^t -5\times 10^{-3}e^{-50x}\,dx - 20$$

$$= 20(e^{-50t}-1) - 20$$

$$= 20e^{-50t} - 40\,\mathrm{V}$$

[**c**] $$v_{\rm c} = \frac{10^6}{1.25}\int_0^t -5\times 10^{-3}e^{-50x}\,dx - 30$$

$$= 80(e^{-50t}-1) - 30$$

$$= 80e^{-50t} - 110\,\mathrm{V}$$

[**d**] $$v_{\rm d} = 10^6\int_0^t -5\times 10^{-3}e^{-50x}\,dx + 250$$

$$= 100(e^{-50t}-1) + 250$$

$$= 100e^{-50t} + 150\,\mathrm{V}$$

CHECK: $v_{\rm b} = -v_{\rm c} - v_{\rm d} - v_{\rm a}$

$$= -200e^{-50t}\,\mathrm{V} \quad \text{(checks)}$$

[**e**] $$i_1 = 0.2\times 10^{-6}\frac{d}{dt}[100e^{-50t}+150]$$

$$= 0.2\times 10^{-6}(-5000e^{-50t}) = -e^{-50t}\,\mathrm{mA}$$

[**f**] $$i_2 = 0.8\times 10^{-6}\frac{d}{dt}[100e^{-50t}+150] = -4e^{-50t}\,\mathrm{mA}$$

CHECK: $i_{\rm b} = i_1 + i_2 = -5e^{-50t}\,\mathrm{mA}$ (OK)

P 6.33 **[a]**
$$\begin{aligned} w(0) &= \tfrac{1}{2}(0.2\times10^{-6})(250)^2 + \tfrac{1}{2}(0.8\times10^{-6})(250)^2 + \tfrac{1}{2}(5\times10^{-6})(20)^2 \\ &+ \tfrac{1}{2}(1.25\times10^{-6})(30)^2 \\ &= 32{,}812.5\,\mu\text{J} \end{aligned}$$

[b]
$$\begin{aligned} w(\infty) &= \tfrac{1}{2}(5\times10^{-6})(40)^2 + \tfrac{1}{2}(1.25\times10^{-6})(110)^2 + \tfrac{1}{2}(0.2\times10^{-6})(150)^2 \\ &+ \tfrac{1}{2}(0.8\times10^{-6})(150)^2 \\ &= 22{,}812.5\,\mu\text{J} \end{aligned}$$

[c] $w = \dfrac{1}{2}(0.5\times10^{-6})(200)^2 = 10{,}000\,\mu\text{J}$

CHECK: $32{,}812.5 - 22{,}812.5 = 10{,}000\,\mu\text{J}$

[d] $\%\text{ delivered} = \dfrac{10{,}000}{32{,}812.5}\times100 = 30.48\%$

[e]
$$\begin{aligned} w &= \int_0^t(-0.005e^{-50x})(-200e^{-50x})\,dx = \int_0^t e^{-100x}\,dx \\ &= 10(1-e^{-100t})\,\text{mJ} \end{aligned}$$

$\therefore\ 10^{-2}(1-e^{-100t}) = 7.5\times10^{-3}; \qquad e^{-100t} = 0.25$

Thus, $t = \dfrac{\ln 4}{100} = 13.86\,\text{ms}.$

P 6.34
$$\begin{aligned} v_c &= -\frac{1}{10\times10^{-6}}\left(\int_0^t 0.2e^{-800x}\,dx - \int_0^t 0.04e^{-200x}\,dx\right) + 5 \\ &= 25(e^{-800t}-1) - 20(e^{-200t}-1) + 5 \\ &= 25e^{-800t} - 20e^{-200t}\ \text{V} \\ v_L &= 150\times10^{-3}\frac{di_o}{dt} \\ &= 150\times10^{-3}(-160e^{-800t} + 8e^{-200t}) \\ &= -24e^{-800t} + 1.2e^{-200t}\ \text{V} \\ v_o &= v_c - v_L \\ &= (25e^{-800t} - 20e^{-200t}) - (-24e^{-800t} + 1.2e^{-200t}) \\ &= 49e^{-800t} - 21.2e^{-200t}\ \text{V},\ t>0 \end{aligned}$$

P 6.35
$$\begin{aligned} \frac{di_o}{dt} &= (2)\{e^{-5000t}[-1000\sin1000t + 5000\cos1000t] \\ &\quad +(-5000e^{-5000t})[\cos1000t + 5\sin1000t]\} \\ &= e^{-5000t}\{-52{,}000\sin1000t\}\ \text{V} \end{aligned}$$

$$\frac{di_o}{dt}(0^+) = (1)[\sin(0)] = 0$$

$$\therefore \qquad 50 \times 10^{-3}\frac{di_o}{dt}(0^+) = 0 \qquad \text{so} \qquad v_2(0^+) = 0$$

$$v_1(0^+) = 25i_o(0^+) + v_2(0^+) = 25(2) + 0 = 50\,\text{V}$$

P 6.36 **[a]** Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8\frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8\frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$

$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \stackrel{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$

$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \stackrel{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad \text{(OK)}$$

$$8\frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16\frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16\frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$

$$+80 - 4160e^{-5t} + 4080e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$

$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad \text{(OK)}$$

P 6.37 **[a]** Yes, using KVL around the lower right loop

$$v_o = v_{20\Omega} + v_{60\Omega} = 20(i_2 - i_1) + 60i_2$$

[b]
$$\begin{aligned} v_o &= 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 20(-3 - 116e^{-5t} + 119e^{-4t}) + 60 - 3120e^{-5t} + 3060e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t}\ \text{V} \end{aligned}$$

[c]
$$\begin{aligned} v_o &= L_2\frac{d}{dt}(i_g - i_2) + M\frac{di_1}{dt} \\ &= 16\frac{d}{dt}(15 + 36e^{-5t} - 51e^{-4t}) + 8\frac{d}{dt}(4 + 64e^{-5t} - 68e^{-4t}) \\ &= -2880e^{-5t} + 3264e^{-4t} - 2560e^{-5t} + 2176e^{-4t} \\ v_o &= -5440e^{-5t} + 5440e^{-4t}\ \text{V} \end{aligned}$$

P 6.38 **[a]**
$$\begin{aligned} v_g &= 5(i_g - i_1) + 20(i_2 - i_1) + 60i_2 \\ &= 5(16 - 16e^{-5t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 20(1 - 52e^{-5t} + 51e^{-4t} - 4 - 64e^{-5t} + 68e^{-4t}) + \\ &\quad 60(1 - 52e^{-5t} + 51e^{-4t}) \\ &= 60 + 5780e^{-4t} - 5840e^{-5t}\ \text{V} \end{aligned}$$

[b] $v_g(0) = 60 + 5780 - 5840 = 0\,\text{V}$

[c]
$$\begin{aligned} p_{\text{dev}} &= v_g i_g \\ &= 960 + 92{,}480e^{-4t} - 94{,}400e^{-5t} - 92{,}480e^{-9t} + \\ &\quad 93{,}440e^{-10t}\text{W} \end{aligned}$$

[d] $p_{\text{dev}}(\infty) = 960\,\text{W}$

[e] $i_1(\infty) = 4\,\text{A}; \quad i_2(\infty) = 1\,\text{A}; \quad i_g(\infty) = 16\,\text{A};$

$$p_{5\Omega} = (16-4)^2(5) = 720\,\text{W}$$

$$p_{20\Omega} = 3^2(20) = 180\,\text{W}$$

$$p_{60\Omega} = 1^2(60) = 60\,\text{W}$$

$$\sum p_{\text{abs}} = 720 + 180 + 60 = 960\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 960\,\text{W}$$

P 6.39 **[a]** $0.5\dfrac{di_g}{dt} + 0.2\dfrac{di_2}{dt} + 10i_2 = 0$

$$0.2\frac{di_2}{dt} + 10i_2 = -0.5\frac{di_g}{dt}$$

[b] $i_2 = 625e^{-10t} - 250e^{-50t}\,\text{mA}$

$$\frac{di_2}{dt} = -6.25e^{-10t} + 12.5e^{-50t}\,\text{A/s}$$

$$i_g = e^{-10t} - 10\,\text{A}$$

$$\frac{di_g}{dt} = -10e^{-10t}\,\text{A/s}$$

$$0.2\frac{di_2}{dt} + 10i_2 = 5e^{-10t} \qquad \text{and} \qquad -0.5\frac{di_g}{dt} = 5e^{-10t}$$

[c]
$$\begin{aligned} v_1 &= 5\frac{di_g}{dt} + 0.5\frac{di_2}{dt} \\ &= 5(-10e^{-10t}) + 0.5(-6.25e^{-10t} + 12.5e^{-50t}) \\ &= -53.125e^{-10t} + 6.25e^{-50t}\,\text{V}, \qquad t > 0 \end{aligned}$$

[d] $v_1(0) = -53.125 + 6.25 = -46.875\,\text{V}; \qquad$ Also

$$\begin{aligned} v_1(0) &= 5\frac{di_g}{dt}(0) + 0.5\frac{di_2}{dt}(0) \\ &= 5(-10) + 0.5(-6.25 + 12.5) = -46.875\,\text{V} \end{aligned}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.

P 6.40 **[a]** $v_{\rm ab} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$

It follows that $L_{\rm ab} = (L_1 + L_2 + 2M)$

[b] $v_{\rm ab} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$

Therefore $L_{\rm ab} = (L_1 + L_2 - 2M)$

P 6.41 **[a]** $v_{\rm ab} = L_1\frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$

$$0 = L_1\frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{\rm ab} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{\rm ab}$$

from which we have

$$v_{\rm ab} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore \quad L_{\rm ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{\rm ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

P 6.42 When the switch is opened the induced voltage is negative at the dotted terminal. Since the voltmeter kicks downscale, the induced voltage across the voltmeter must be negative at its positive terminal. Therefore, the voltage is negative at the positive terminal of the voltmeter.

Thus, the upper terminal of the unmarked coil has the same instantaneous polarity as the dotted terminal. Therefore, place a dot on the upper terminal of the unmarked coil.

P 6.43 **[a]** Dot terminal 1; the flux is up in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

[b] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[c] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[d] Dot terminal 1; the flux is down in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

P 6.44 **[a]** $\frac{1}{k^2} = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11}N_1i_1 + \mathcal{P}_{21}N_1i_1 = N_1i_1(\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2i_2(\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2i_2}\right)$$

Therefore

$$k^2 = \frac{(\phi_{12}/N_2i_2)(\phi_{21}/N_1i_1)}{(\phi_1/N_1i_1)(\phi_2/N_2i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.45 **[a]** $k = \frac{M}{\sqrt{L_1L_2}} = \frac{22.8}{\sqrt{576}} = 0.95$

[b] $M_{\max} = \sqrt{576} = 24\,\text{mH}$

[c] $\dfrac{L_1}{L_2} = \dfrac{N_1^2\mathcal{P}_1}{N_2^2\mathcal{P}_2} = \left(\dfrac{N_1}{N_2}\right)^2$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{60}{9.6} = 6.25$$

$$\frac{N_1}{N_2} = \sqrt{6.25} = 2.5$$

P 6.46 **[a]** $L_2 = \left(\dfrac{M^2}{k^2 L_1}\right) = \dfrac{(0.09)^2}{(0.75)^2(0.288)} = 50\,\text{mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{288}{50}} = 2.4$$

[b] $\mathcal{P}_1 = \dfrac{L_1}{N_1^2} = \dfrac{0.288}{(1200)^2} = 0.2 \times 10^{-6}\ \text{Wb/A}$

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.05}{(500)^2} = 0.2 \times 10^{-6}\ \text{Wb/A}$$

P 6.47 **[a]** $W = (0.5)L_1 i_1^2 + (0.5)L_2 i_2^2 + M i_1 i_2$

$$M = 0.85\sqrt{(18)(32)} = 20.4\,\text{mH}$$

$$W = [9(36) + 16(81) + 20.4(54)] = 2721.6\,\text{mJ}$$

[b] $W = [324 + 1296 + 1101.6] = 2721.6\,\text{mJ}$

[c] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

[d] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

P 6.48 **[a]** $M = 1.0\sqrt{(18)(32)} = 24\,\text{mH}, \qquad i_1 = 6\,\text{A}$

Therefore $16 i_2^2 + 144 i_2 + 324 = 0, \qquad i_2^2 + 9 i_2 + 20.25 = 0$

Therefore $i_2 = -\left(\dfrac{9}{2}\right) \pm \sqrt{\left(\dfrac{9}{2}\right)^2 - 20.25} = -4.5 \pm \sqrt{0}$

Therefore $i_2 = -4.5\,\text{A}$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

P 6.49 **[a]** $L_1 = N_1^2 \mathcal{P}_1; \qquad \mathcal{P}_1 = \dfrac{72 \times 10^{-3}}{6.25 \times 10^4} = 1152 \text{ nWb/A}$

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.2; \qquad \mathcal{P}_{21} = 2\mathcal{P}_{11}$$

$$\therefore \quad 1152 \times 10^{-9} = \mathcal{P}_{11} + \mathcal{P}_{21} = 3\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = 192 \text{ nWb/A}; \qquad \mathcal{P}_{21} = 960 \text{ nWb/A}$$

$$M = k\sqrt{L_1 L_2} = (2/3)\sqrt{(0.072)(0.0405)} = 36\,\text{mH}$$

$$N_2 = \frac{M}{N_1 \mathcal{P}_{21}} = \frac{36 \times 10^{-3}}{(250)(960 \times 10^{-9})} = 150 \text{ turns}$$

[b] $\mathcal{P}_2 = \dfrac{L_2}{N_2^2} = \dfrac{40.5 \times 10^{-3}}{(150)^2} = 1800 \text{ nWb/A}$

[c] $\mathcal{P}_{11} = 192$ nWb/A [see part (a)]

[d] $\dfrac{\phi_{22}}{\phi_{12}} = \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2 - \mathcal{P}_{12}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2}{\mathcal{P}_{12}} - 1$

$$\mathcal{P}_{21} = \mathcal{P}_{21} = 960 \text{ nWb/A}; \qquad \mathcal{P}_2 = 1800 \text{ nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{1800}{960} - 1 = 0.875$$

P 6.50 $\mathcal{P}_1 = \dfrac{L_1}{N_1^2} = 2 \text{ nWb/A}; \quad \mathcal{P}_2 = \dfrac{L_2}{N_2^2} = 2 \text{ nWb/A}; \quad M = k\sqrt{L_1 L_2} = 180\,\mu\text{H}$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1 N_2} = 1.2 \text{ nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 0.8 \text{ nWb/A}$$

P 6.51 When the touchscreen in the mutual-capacitance design is touched at the point x, y, the touch capacitance C_t is present in series with the mutual capacitance at the touch point, C_{mxy}. Remember that capacitances combine in series the way that resistances combine in parallel. The resulting mutual capacitance is

$$C'_{mxy} = \frac{C_{mxy} C_t}{C_{mxy} + C_t}$$

P 6.52 **[a]** The self-capacitance and the touch capacitance are effectively connected in parallel. Therefore, the capacitance at the x-grid electrode closest to the touch point with respect to ground is

$$C_x = C_p + C_t = 30\,\text{pF} + 15\,\text{pF} = 45\,\text{pF}.$$

The same capacitance exists at the y-grid electrode closest to the touch point with respect to ground.

[b] The mutual-capacitance and the touch capacitance are effectively connected in series. Therefore, the mutual capacitance between the x-grid and y-grid electrodes closest to the touch point is

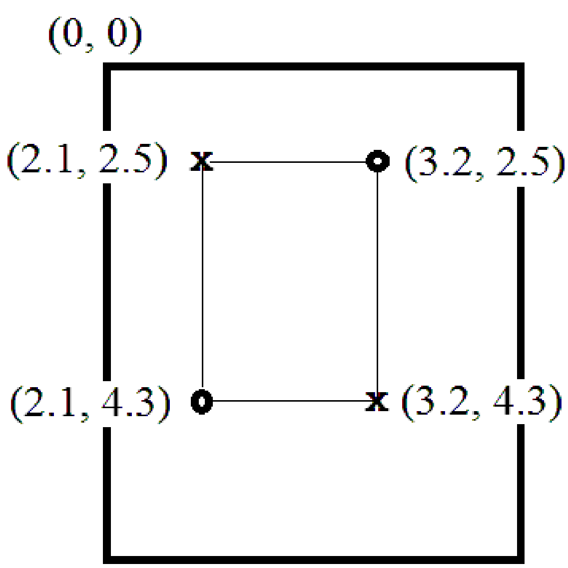
$$C'_{mxy} = \frac{C_{mxy}C_t}{C_{mxy} + C_t} = \frac{(30)(15)}{30 + 15} = 10\,\text{pF}.$$

[c] In the self-capacitance design, touching the screen increases the capacitance being measured at the point of touch. For example, in part (a) the measured capacitance before the touch is 30 pF and after the touch is 45 pF. In the mutual-capacitance design, touching the screen decreases the capacitance being measured at the point of touch. For example, in part (b) the measured capacitance before the touch is 30 pF and after the touch is 10 pF.

P 6.53 [a] The four touch points identified are the two actual touch points and two ghost touch points. Their coordinates, in inches from the upper left corner of the screen, are

$(2.1, 4.3)$; $(3.2, 2.5)$; $(2.1, 2.5)$; and $(3.2, 4.3)$

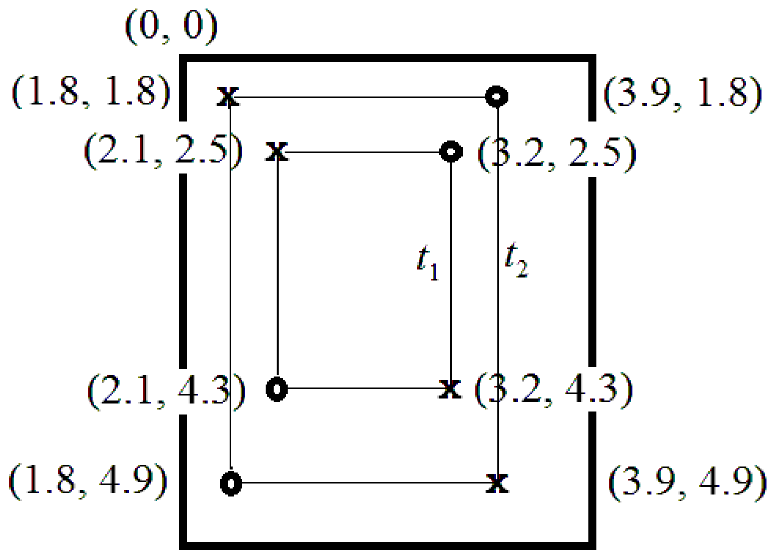
These four coordinates identify a rectangle within the screen, shown below.



[b] The touch points identified at time t_1 are those listed in part (a). The touch points recognized at time t_2 are

$(1.8, 4.9)$; $(3.9, 1.8)$; $(1.8, 1.8)$; and $(3.9, 4.9)$

The first two coordinates are the actual touch points and the last two coordinates are the associated ghost points. Again, the four coordinates identify a rectangle at time t_2, as shown here:

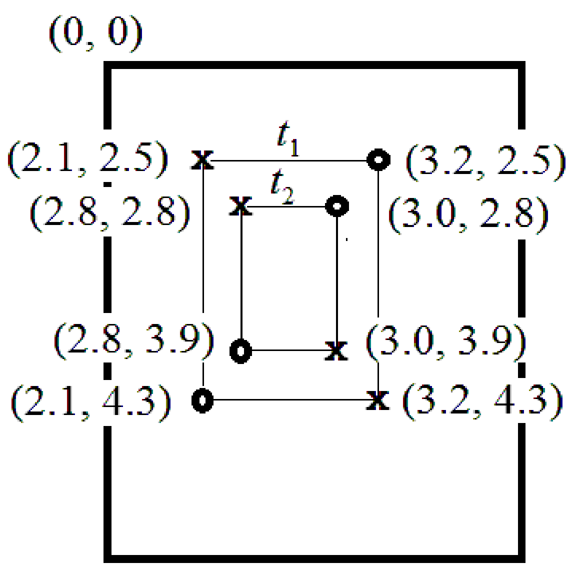


Note that the rectangle at time t_2 is larger than the rectangle at time t_1, so the software would recognize the two fingers are moving toward the edges of the screen. This pinch gesture thus specifies a zoom-in for the screen.

[**c**] The touch points identified at time t_1 are those listed in part (a). The touch points recognized at time t_2 are

$(2.8, 3.9)$; $(3.0, 2.8)$; $(2.8, 2.8)$; and $(3.0, 3.9)$

The first two coordinates are the actual touch points and the last two coordinates are the associated ghost points. Again, the four coordinates identify a rectangle at time t_2, as shown here:



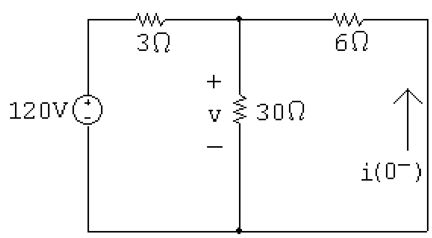
Here, the rectangle at time t_2 is smaller than the rectangle at time t_1, so the software would recognize the two fingers are moving toward the middle of the screen. This pinch gesture thus specifies a zoom-out for the screen.

7

Response of First-Order RL and RC Circuits

Assessment Problems

AP 7.1 **[a]** The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.



First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:

$30\|6 = 5\,\Omega$

Use voltage division to find the voltage drop across the parallel resistors:

$$v = \frac{5}{5+3}(120) = 75\,\text{V}$$

Now find the current using Ohm's law:

$$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\,\text{A}$$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8\times10^{-3})(12.5)^2 = 625\,\text{mJ}$

[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,

$$\tau = \frac{L}{R} = \frac{8\times10^{-3}}{2} = 4\,\text{ms}$$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

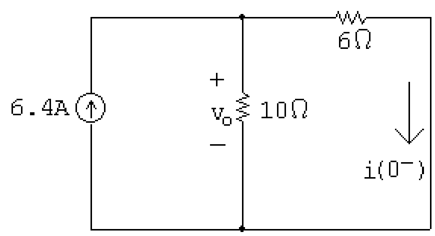
[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$

So $\quad w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8)\times10^{-3}(3.58)^2 = 51.3\,\text{mJ}$

$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$

$\%\ \text{dissipated} = \left(\dfrac{573.7}{625}\right)100 = 91.8\%$

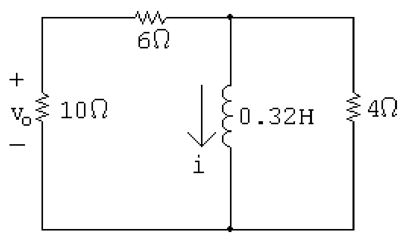
AP 7.2 [a] First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,

$i(0^-) = \dfrac{10}{10+6}(6.4) = 4\,\text{A}$

Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$$R_{\text{eq}} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \frac{L}{R_{\text{eq}}} = \frac{0.32}{3.2} = 0.1\,\text{s}$$

Use the initial inductor current and the time constant to find the current in the inductor:

$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$

Use current division to find the current in the $10\,\Omega$ resistor:

$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$

Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:

$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$

[b] The initial energy stored in the inductor is

$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$

Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:

$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

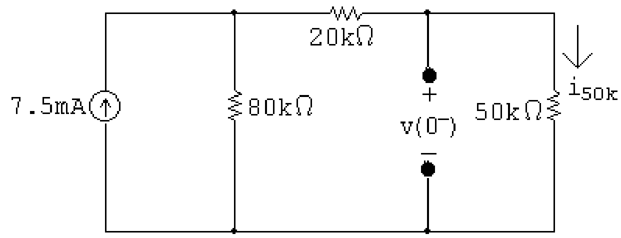
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\ \text{dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 [a] The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\text{k}\Omega$ resistor. First use current division to find the current through the $50\,\text{k}\Omega$ resistor:

$$i_{50\text{k}} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3}(7.5 \times 10^{-3}) = 4\,\text{mA}$$

Use Ohm's law to find the voltage drop:

$v(0^-) = (50 \times 10^3)i_{50\text{k}} = (50 \times 10^3)(0.004) = 200\,\text{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\text{k}\Omega$ resistor remains connected to the capacitor. Thus,

$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V}, \quad t \geq 0$

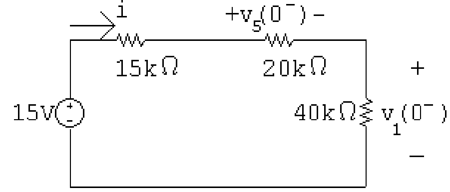
[d] $w(0) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\text{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

AP 7.4 [a] This circuit is actually two RC circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$i = \dfrac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu$F – $20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu$F – $40\,\text{k}\Omega$ subcircuit:

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$

Therefore,

$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$

$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$

Finally,

$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$

$w_1(60\,\text{ms}) = \frac{1}{2}Cv_1^2(60\,\text{ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$

$w_5(60\,\text{ms}) = \frac{1}{2}Cv_5^2(60\,\text{ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$

$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$

Find the initial energy from the initial voltage:

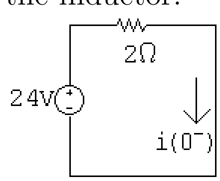
$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$

$\%\text{ dissipated} = (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$

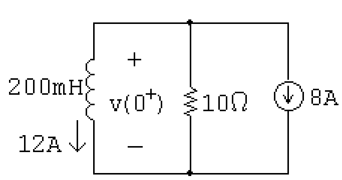
AP 7.5 [a] Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$

Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.

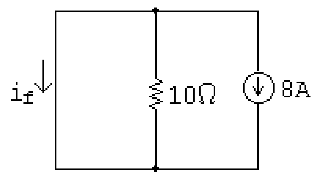


$v(0^+) = -10(8+12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,

$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:



Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,

$i_f = -8\,\text{A}$

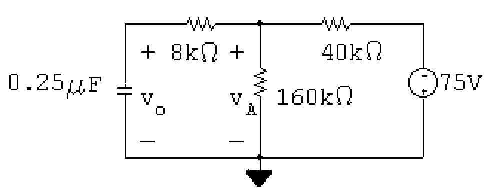
Now,

$$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$$
$$= -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 [a]



From Example 7.6,

$v_o(t) = -60 + 90e^{-100t}\,\text{V}$

Write a KCL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

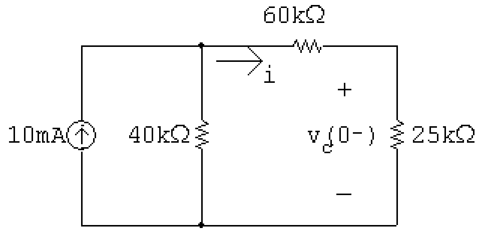
$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

[b] $t \geq 0^{+}$, since there is no requirement that the voltage be continuous in a resistor.

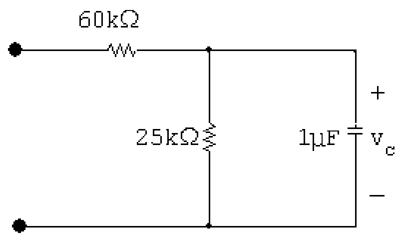
AP 7.7 **[a]** Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\,\text{mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\,\text{V} \quad \text{so} \quad v_c(0^+) = 80\,\text{V}$$

Now use the next circuit, valid for $0 \leq t \leq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $\quad 0 \leq t \leq 100\,\text{ms}$:

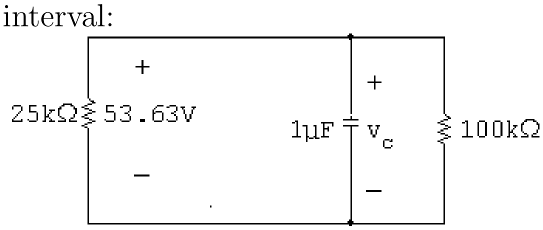
$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V} \quad 0 \leq t \leq 10\,\text{ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \geq 10\,\text{ms}$, using the capacitor voltage from the previous interval:

$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$

Now use the next circuit, valid for $t \geq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $\quad t \geq 10\,\text{ms}$:

$$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$$

Therefore $\quad v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \geq 0.01\,\text{s}$

[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\,\mathrm{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000}dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000}dt = 2.91\,\mathrm{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

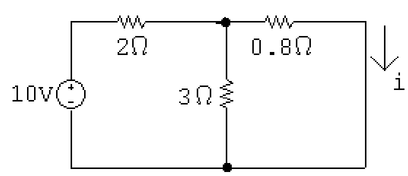
$$w_{100\,\mathrm{k\Omega}} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000}dt = 0.29\,\mathrm{mJ}$$

We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the 25 kΩ resistor and the 100 kΩ resistor.

Check: $w_{\mathrm{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\mathrm{mJ}$

$$w_{\mathrm{diss}} = 2.91 + 0.29 = 3.2\,\mathrm{mJ}$$

AP 7.8 [a] Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.
For $0 \le t \le 1\,\mathrm{s}$,



The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\mathrm{A}$$

The final current in the inductor, which is equal to the current in the 0.8 Ω resistor is

$$I_{\mathrm{F}} = \frac{3}{3 + 0.8}(3.8) = 3\,\mathrm{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$[(2\|3) + 0.8]\|3\|6 = 1\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{1} = 2\,\mathrm{s}$$

Therefore,

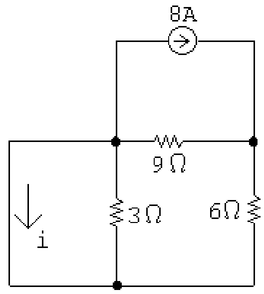
$$i = i_{\mathrm{F}} + [i(0^+) - i_{\mathrm{F}}]e^{-t/\tau} = 3 - 3e^{-0.5t}\,\mathrm{A}, \quad 0 \le t \le 1\,\mathrm{s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$i(1) = 3 - 3e^{-0.5} = 1.18\,\text{A}$

.

[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9+6}(-8) = -4.8\,\text{A}$$

The equivalent resistance seen by the inductor is used to calculate the time constant:

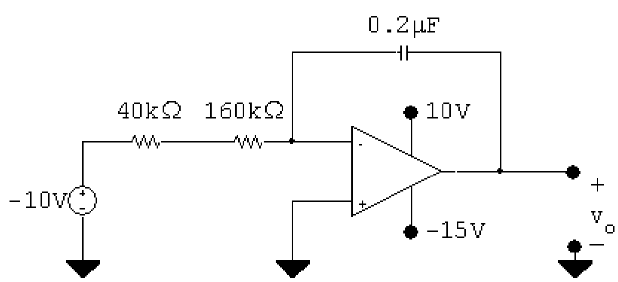
$$3\|(9+6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\text{s}$$

Therefore,

$$i = i_\text{F} + [i(1^+) - i_\text{F}]e^{-(t-1)/\tau}$$

$$= -4.8 + 5.98e^{-1.25(t-1)}\,\text{A}, \quad t \geq 1\,\text{s}$$

AP 7.9 $0 \leq t \leq 32\,\text{ms}$:

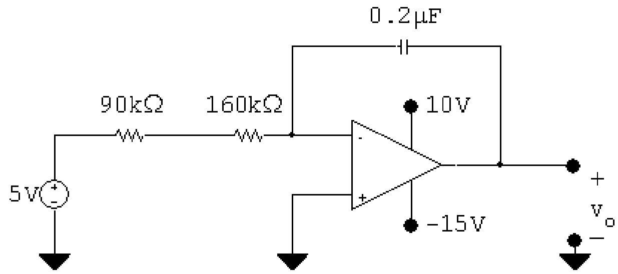


$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\text{V}$$

$t \geq 32\,\text{ms}$:



$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} +8 = -\frac{1}{RC_f}5(t - 32\times10^{-3}) + 8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t - 32\times10^{-3}) + 8 = -100t + 11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \quad \therefore \quad t = 262\,\text{ms}$$

AP 7.10 [**a**] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0+2)e^{-t/\tau}$$

$$\tau = (160\times10^3)(10\times10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$$

[**b**] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1+2)e^{-625t} = -2 + 3e^{-625t}\,\text{V}$$

The analysis for v_o is the same as in part (a):

$$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$$

Problems

P 7.1 **[a]** $i_o(0) = \dfrac{20}{16 + 12 + 4 + 8} = \dfrac{20}{40} = 0.5\,\text{A}$

$i_o(\infty) = 0\,\text{A}$

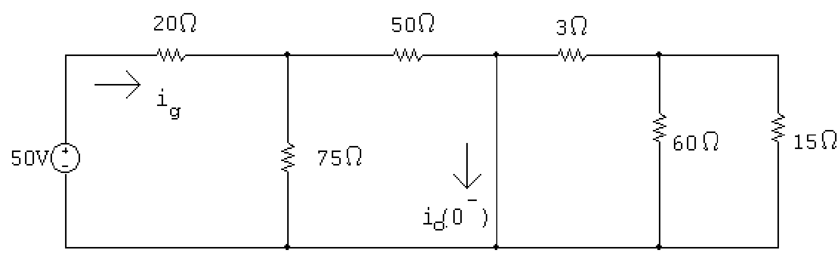
[b] $i_o = 0.5e^{-t/\tau}; \qquad \tau = \dfrac{L}{R} = \dfrac{80 \times 10^{-3}}{12 + 8} = 4\,\text{ms}$

$i_o = 0.5e^{-250t}\,\text{A}, \quad t \geq 0$

[c] $0.5e^{-250t} = 0.1$

$e^{250t} = 5 \qquad \therefore \quad t = 6.44\,\text{ms}$

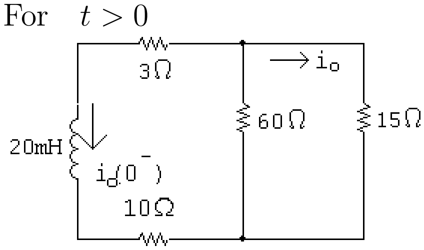
P 7.2 **[a]** For $t < 0$



$$i_g = \frac{50}{20 + (75\|50)} = \frac{50}{50} = 1\,\text{A}$$

$$i_o(0^-) = \frac{50}{75 + 50}(1) = 0.4\,\text{A} = i_o(0^+)$$

For $t > 0$



$$i_o(t) = i_o(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.02}{3 + 60\|15} = 1.33\,\text{ms}; \qquad \frac{1}{\tau} = 750$$

$$i_o(t) = 0.4e^{-750t}\,\text{A}, \qquad t \geq 0$$

[b] $v_\text{L} = L\dfrac{di_o}{dt} = 0.02(-750)(0.4e^{-750t}) = -6e^{-750t}\,\text{V}$

$$v_o = \frac{60\|15}{3 + 60\|15}v_\text{L} = \frac{12}{15}(-6e^{-750t}) = -4.8e^{-750t}\,\text{V} \qquad t \geq 0^+$$

P 7.3 [a] $i(0) = \dfrac{60}{120} = 0.5\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{0.32}{160} = 2\,\text{ms}$

[c] $i = 0.5e^{-500t}\,\text{A}, \qquad t \geq 0$

$$v_1 = L\frac{d}{dt}(0.5e^{-500t}) = -80e^{-500t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = -70i = -35e^{-500t}\,\text{V} \qquad t \geq 0^+$$

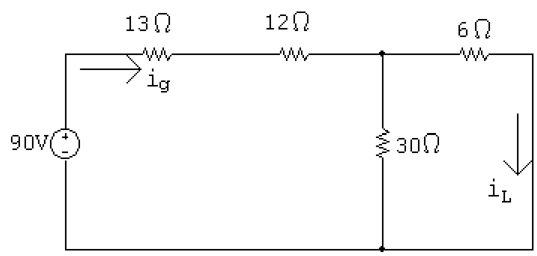
[d] $w(0) = \dfrac{1}{2}(0.32)(0.5)^2 = 40\,\text{mJ}$

$$w_{90\Omega} = \int_0^t 90(0.25e^{-1000x})\,dx = 22.5\frac{e^{-1000x}}{-1000}\Big|_0^t = 22.5(1 - e^{-1000t})\,\text{mJ}$$

$$w_{90\Omega}(1\,\text{ms}) = 0.0225(1 - e^{-1}) = 14.22\,\text{mJ}$$

$$\%\ \text{dissipated} \ = \frac{14.22}{40}(100) = 35.6\%$$

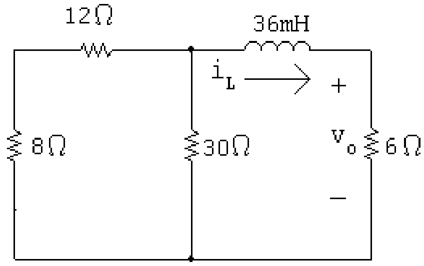
P 7.4 $t < 0$:



$$i_g = \frac{90}{13 + 12 + 6\|30} = 3\,\text{A}$$

$$i_L(0^-) = \frac{30}{36}(3) = 2.5\,\text{A}$$

$t > 0$:



$$R_e = 6 + 30\|(8 + 12) = 6 + 12 = 18\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{36 \times 10^{-3}}{18} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$\therefore \quad i_L = 2.5e^{-500t}\,\text{A}$$

$$v_o = 6i_o = 15e^{-500t}\,\text{V}, \quad t \geq 0^+$$

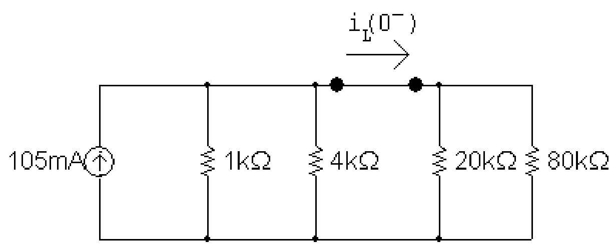
P 7.5 $$p_{6\Omega} = \frac{v_o^2}{6} = \frac{(15)^2}{6}e^{-1000t} = 37.5e^{-1000t}\,\text{W}$$

$$w_{6\Omega} = \int_0^\infty 37.5e^{-1000t}\,dt = 37.5\frac{e^{-1000t}}{-1000}\Big|_0^\infty = 37.5\,\text{mJ}$$

$$w(0) = \frac{1}{2}(36 \times 10^{-3})(2.5)^2 = 112.5\,\text{mJ}$$

$$\%\ \text{diss} = \frac{37.5}{112.5}(100) = 33.33\%$$

P 7.6 **[a]** $t < 0$

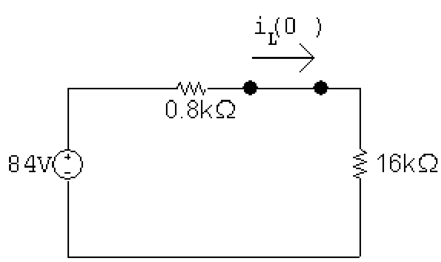


Simplify this circuit by creating a Thévenin equivalant to the left of the inductor and an equivalent resistance to the right of the inductor:

$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$

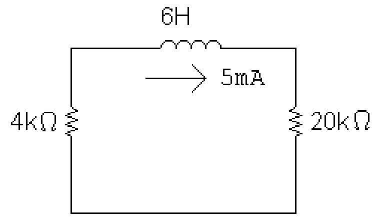
$20\,\text{k}\Omega \| 80\,\text{k}\Omega = 16\,\text{k}\Omega$

$(105 \times 10^{-3})(0.8 \times 10^3) = 84\,\text{V}$



$$i_L(0^-) = \frac{84}{16{,}800} = 5\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{4\text{k}} = 25 \times 10^{-6} e^{-8000t}(4000) = 0.10e^{-8000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\text{J}$$

$$0.10w(0) = 7.5\,\mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t})\,\mu\text{J}$

$$w_{\text{diss}}(114.54\,\mu\text{s}) = 45\,\mu\text{J}$$

$$\% = (45/75)(100) = 60\%$$

P 7.7 **[a]** $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore \quad e^{10^{-3}/\tau} = 2$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{10^{-3}}{\ln 2}$$

$$\therefore \quad L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43\,\text{mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10(1/10)(30 \times 10^{-3}) = -30\,\text{mV}$

$$v_o(t) = -0.03e^{-t/\tau}\,\text{V}$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5}e^{-2t/\tau}$$

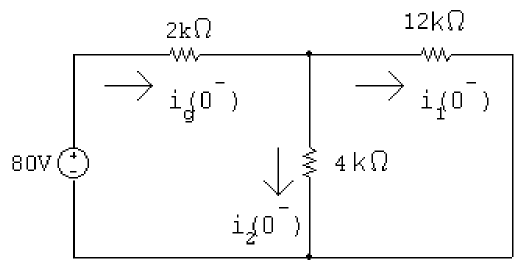
$$w_{10\Omega} = \int_0^{10^{-3}} 9 \times 10^{-5} e^{-2t/\tau}\,dt = 4.5\tau \times 10^{-5}(1 - e^{-2\times 10^{-3}/\tau})$$

$$\tau = \frac{1}{1000 \ln 2} \qquad \therefore \qquad w_{10\Omega} = 48.69\,\text{nJ}$$

$$w_L(0) = \frac{1}{2}Li_L^2(0) = \frac{1}{2}(14.43 \times 10^{-3})(3 \times 10^{-3})^2 = 64.92\,\text{nJ}$$

$$\%\text{ diss in 1 ms} = \frac{48.69}{64.92} \times 100 = 75\%$$

P 7.8 **[a]** $t < 0$



$$4\,\text{k}\Omega \| 12\,\text{k}\Omega = 3\,\text{k}\Omega$$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{80}{(2000 + 3000)} = 16\,\text{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{3000}{12{,}000}(0.016) = 4\,\text{mA}$$

$$i_2(0^-) = \frac{3000}{4000}(0.016) = 12\,\text{mA}$$

[b] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 4\,\text{mA}$$

$$i_2(0^+) = -i_1(0^+) = -4\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{0.64 \times 10^{-3}}{16 \times 10^3} = 4 \times 10^{-5}\,\text{s}; \qquad \dfrac{1}{\tau} = 25{,}000$

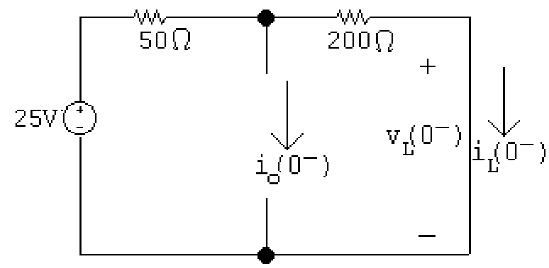
$$i_1(t) = i_1(0^+)e^{-t/\tau} = 4e^{-25{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -4e^{-25{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 12 mA and $i_2(0^+) = -4$ mA.

P 7.9 [a] For $t = 0^-$ the circuit is:

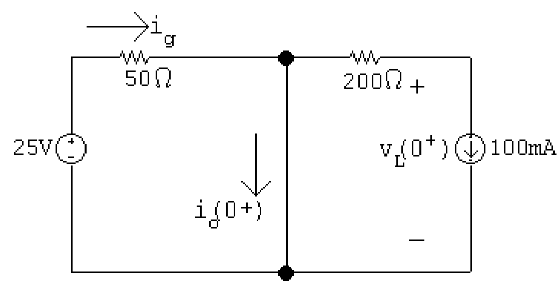


$$i_o(0^-) = 0 \quad \text{since the switch is open}$$

$$i_L(0^-) = \frac{25}{250} = 0.1 = 100\,\text{mA}$$

$$v_L(0^-) = 0 \quad \text{since the inductor behaves like a short circuit}$$

[b] For $t = 0^+$ the circuit is:



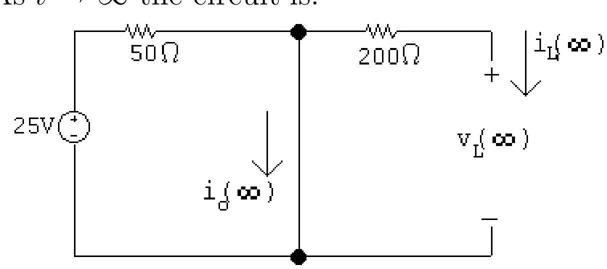
$$i_L(0^+) = i_L(0^-) = 100\,\text{mA}$$

$$i_g = \frac{25}{50} = 0.5 = 500\,\text{mA}$$

$$i_o(0^+) = i_g - i_L(0^+) = 500 - 100 = 400\,\text{mA}$$

$$200 i_L(0^+) + v_L(0^+) = 0 \qquad \therefore \qquad v_L(0^+) = -200 i_L(0^+) = -20\,\text{V}$$

[c] As $t \to \infty$ the circuit is:



$$i_L(\infty) = 0; \qquad v_L(\infty) = 0$$

$$i_o(\infty) = \frac{25}{50} = 500\,\text{mA}$$

[d] $\tau = \dfrac{L}{R} = \dfrac{0.05}{200} = 0.25\,\text{ms}$

$$i_{\text{L}}(t) = 0 + (0.1 - 0)e^{-4000t} = 0.1e^{-4000t}\,\text{A}$$

[e] $i_o(t) = i_{\text{g}} - i_{\text{L}} = 0.5 - 0.1e^{-4000t}\,\text{A}$

[f] $v_{\text{L}}(t) = L\dfrac{di_{\text{L}}}{dt} = 0.05(-4000)(0.1)e^{-4000t} = -20e^{-4000t}\,\text{V}$

P 7.10 $w(0) = \dfrac{1}{2}(10 \times 10^{-3})(5)^2 = 125\,\text{mJ}$

$$0.9w(0) = 112.5\,\text{mJ}$$

$$w(t) = \frac{1}{2}(10 \times 10^{-3})i(t)^2, \qquad i(t) = 5e^{-t/\tau}\,\text{A}$$

$$\therefore\quad w(t) = 0.005(25e^{-2t/\tau}) = 125e^{-2t/\tau})\,\text{mJ}$$

$$w(10\,\mu\text{s}) = 125e^{-20\times10^{-6}/\tau}\,\text{mJ}$$

$$\therefore\quad 125e^{-20\times10^{-6}/\tau} = 112.5 \qquad \text{so} \qquad e^{20\times10^{-6}/\tau} = \frac{10}{9}$$

$$\tau = \frac{20 \times 10^{-6}}{\ln(10/9)} = \frac{L}{R}$$

$$R = \frac{10 \times 10^{-3}\ln(10/9)}{20 \times 10^{-6}} = 52.68\,\Omega$$

P 7.11 **[a]** $w(0) = \dfrac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2 Re^{-2t/\tau}\,dt = I_g^2 R\frac{e^{-2t/\tau}}{(-2/\tau)}\Bigg|_0^{t_o}$$
$$= \frac{1}{2}I_g^2 R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2 L(1 - e^{-2t_o/\tau})$$

$$w_{\text{diss}} = \sigma w(0)$$

$$\therefore\quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \sigma\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\ \ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(10\times 10^{-3})\ln[1/0.9]}{20\times 10^{-6}}$

$$R = 52.68\,\Omega$$

P 7.12 **[a]** $R = \dfrac{v}{i} = 25\,\Omega$

[b] $\tau = \dfrac{1}{10} = 100\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 0.1$

$$L = (0.1)(25) = 2.5\,\text{H}$$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(2.5)(6.4)^2 = 51.2\,\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^t 1024e^{-20x}\,dx = 1024\frac{e^{-20x}}{-20}\Bigg|_0^t = 51.2(1-e^{-20t})\,\text{J}$

$$\%\,\text{dissipated} = \frac{51.2(1-e^{-20t})}{51.2}(100) = 100(1-e^{-20t})$$

$$\therefore\quad 100(1-e^{-20t}) = 60 \qquad \text{so} \qquad e^{-20t} = 0.4$$

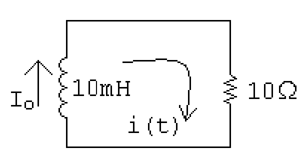
$$\text{Therefore}\quad t = \frac{1}{20}\ln 2.5 = 45.81\,\text{ms}$$

P 7.13 **[a]** Note that there are several different possible solutions to this problem, and the answer to part (c) depends on the value of inductance chosen.

$$R = \frac{L}{\tau}$$

Choose a 10 mH inductor from Appendix H. Then,

$R = \dfrac{0.01}{0.001} = 10\,\Omega$ which is a resistor value from Appendix H.



[b] $i(t) = I_o e^{-t/\tau} = 10e^{-1000t}\,\text{mA}, \qquad t \ge 0$

[c] $w(0) = \frac{1}{2}LI_o^2 = \frac{1}{2}(0.01)(0.01)^2 = 0.5\,\mu\text{J}$

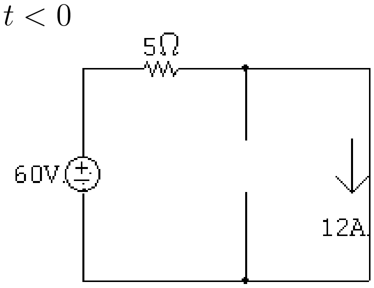
$$w(t) = \frac{1}{2}(0.01)(0.01e^{-1000t})^2 = 0.5 \times 10^{-6}e^{-2000t}$$

So $\quad 0.5 \times 10^{-6}e^{-2000t} = \frac{1}{2}w(0) = 0.25 \times 10^{-6}$

$e^{-2000t} = 0.5 \qquad \text{then} \qquad e^{2000t} = 2$

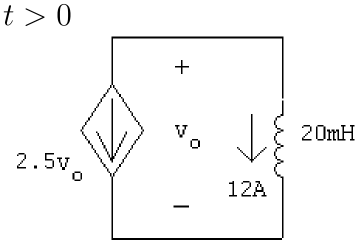
$\therefore \quad t = \frac{\ln 2}{2000} = 346.57\,\mu\text{s} \qquad$ (for a 10 mH inductor)

P 7.14 $\quad t < 0$

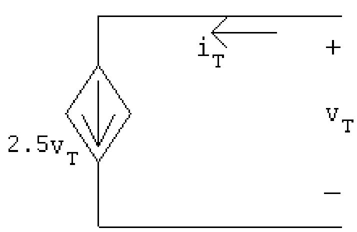


$i_L(0^-) = i_L(0^+) = 12\,\text{A}$

$t > 0$

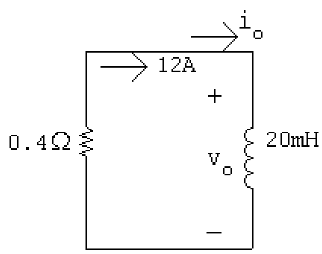


Find Thévenin resistance seen by inductor:



$i_T = 2.5v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{2.5} = 0.4\,\Omega$

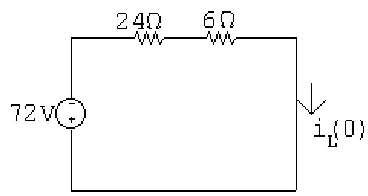
$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{0.4} = 50\,\text{ms}; \qquad 1/\tau = 20$



$$i_o = 12e^{-20t}\,\text{A}, \qquad t \geq 0$$

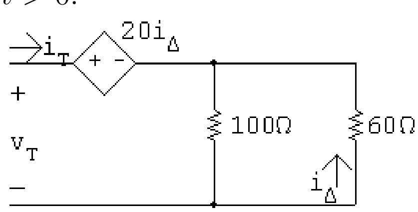
$$v_o = L\frac{di_o}{dt} = (20 \times 10^{-3})(-240e^{-20t}) = -4.8e^{-20t}\,\text{V}, \quad t \geq 0^+$$

P 7.15 **[a]** $t < 0$:



$$i_L(0) = -\frac{72}{24+6} = -2.4\,\text{A}$$

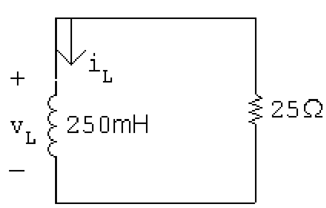
$t > 0$:



$$i_\Delta = -\frac{100}{160}i_T = -\frac{5}{8}i_T$$

$$v_T = 20i_\Delta + i_T\frac{(100)(60)}{160} = -12.5i_T + 37.5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = -12.5 + 37.5 = 25\,\Omega$$



$$\tau = \frac{L}{R} = \frac{250 \times 10^{-3}}{25} \qquad \frac{1}{\tau} = 100$$

$$i_L = -2.4e^{-100t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = 250 \times 10^{-3}(240e^{-100t}) = 60e^{-100t}\,\text{V}, \quad t \geq 0^+$

[c] $i_\Delta = 0.625 i_L = -1.5e^{-100t}\,\text{A} \qquad t \geq 0^+$

P 7.16 $w(0) = \frac{1}{2}(250 \times 10^{-3})(-2.4)^2 = 720\,\text{mJ}$

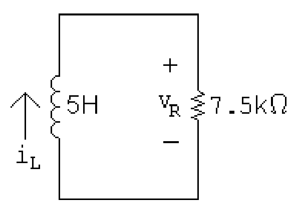
$$p_{60\Omega} = 60(-1.5e^{-100t})^2 = 135e^{-200t}\,\text{W}$$

$$w_{60\Omega} = \int_0^\infty 135e^{-200t}\,dt = 135\frac{e^{-200t}}{-200}\Big|_0^\infty = 675\,\text{mJ}$$

$$\%\text{ dissipated } = \frac{675}{720}(100) = 93.75\%$$

P 7.17 [a] $t > 0$:

$$L_{\text{eq}} = 1.25 + \frac{60}{16} = 5\,\text{H}$$



$$i_L(t) = i_L(0)e^{-t/\tau}\,\text{mA}; \qquad i_L(0) = 2\,\text{A}; \qquad \frac{1}{\tau} = \frac{R}{L} = \frac{7500}{5} = 1500$$

$$i_L(t) = 2e^{-1500t}\,\text{A}, \qquad t \geq 0$$

$$v_R(t) = Ri_L(t) = (7500)(2e^{-1500t}) = 15{,}000e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

$$v_o = -3.75\frac{di_L}{dt} = 11{,}250e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

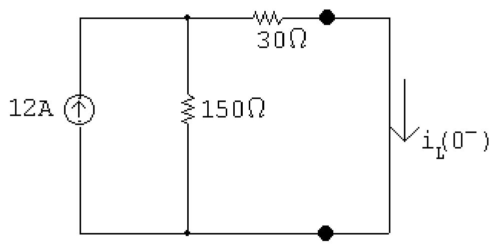
[b] $i_o = \frac{-1}{6}\int_0^t 11{,}250e^{-1500x}\,dx + 0 = 1.25e^{-1500t} - 1.25\,\text{A}$

P 7.18 [a] From the solution to Problem 7.17,

$$w(0) = \frac{1}{2}L_{\text{eq}}[i_L(0)]^2 = \frac{1}{2}(5)(2)^2 = 10\,\text{J}$$

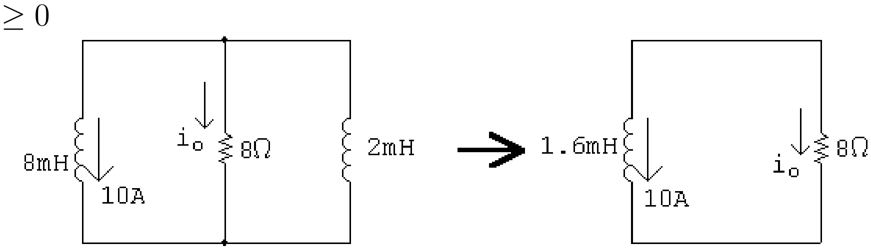
[b] $w_{\text{trapped}} = \frac{1}{2}(10)(1.25)^2 + \frac{1}{2}(6)(1.25)^2 = 12.5\,\text{J}$

P 7.19 **[a]** $t < 0$



$$i_L(0^-) = \frac{150}{180}(12) = 10\,\text{A}$$

$t \geq 0$



$$\tau = \frac{1.6 \times 10^{-3}}{8} = 200 \times 10^{-6}; \qquad 1/\tau = 5000$$

$$i_o = -10e^{-5000t}\,\text{A} \quad t \geq 0$$

[b] $w_{\text{del}} = \dfrac{1}{2}(1.6 \times 10^{-3})(10)^2 = 80\,\text{mJ}$

[c] $0.95 w_{\text{del}} = 76\,\text{mJ}$

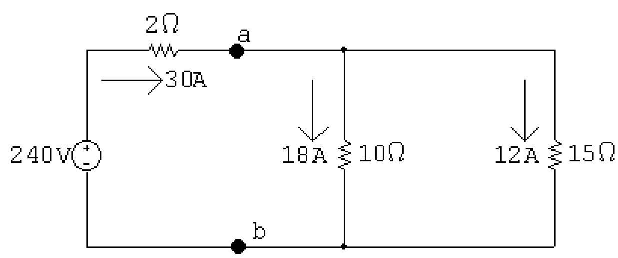
$$\therefore \quad 76 \times 10^{-3} = \int_0^{t_o} 8(100e^{-10{,}000t})\,dt$$

$$\therefore \quad 76 \times 10^{-3} = -80 \times 10^{-3} e^{-10{,}000t} \Big|_0^{t_o} = 80 \times 10^{-3}(1 - e^{-10{,}000t_o})$$

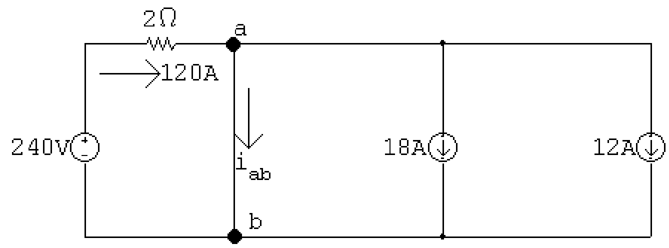
$$\therefore \quad e^{-10{,}000t_o} = 0.05 \quad \text{so} \quad t_o = 299.57\,\mu\text{s}$$

$$\therefore \quad \frac{t_o}{\tau} = \frac{299.57 \times 10^{-6}}{200 \times 10^{-6}} = 1.498 \quad \text{so} \quad t_o \approx 1.498\tau$$

P 7.20 **[a]** $t < 0$:

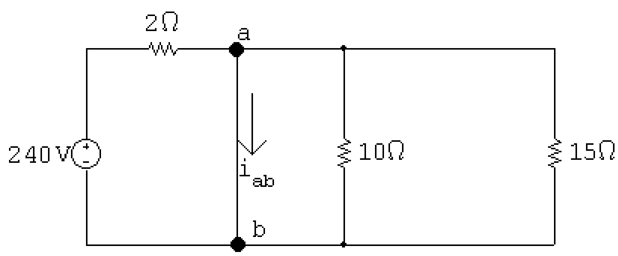


$t = 0^+$:

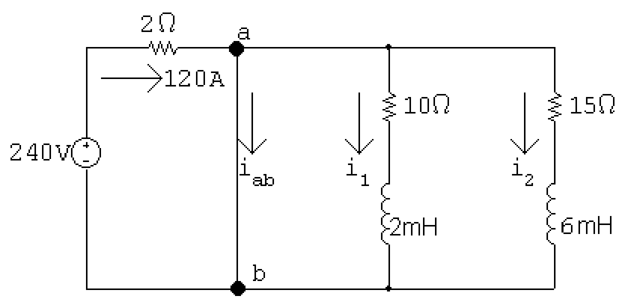


$$120 = i_{ab} + 18 + 12, \qquad i_{ab} = 90\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:



$$i_{ab} = 240/2 = 120\,\text{A}, \quad t = \infty$$



[c] $i_1(0) = 18, \qquad \tau_1 = \dfrac{2 \times 10^{-3}}{10} = 0.2\,\text{ms}$

$$i_2(0) = 12, \qquad \tau_2 = \frac{6 \times 10^{-3}}{15} = 0.4\,\text{ms}$$

$$i_1(t) = 18e^{-5000t}\,\text{A}, \quad t \geq 0$$

$$i_2(t) = 12e^{-2500t}\,\text{A}, \quad t \geq 0$$

$$i_{ab} = 120 - 18e^{-5000t} - 12e^{-2500t}\,\text{A}, \quad t \geq 0$$

$$120 - 18e^{-5000t} - 12e^{-2500t} = 114$$

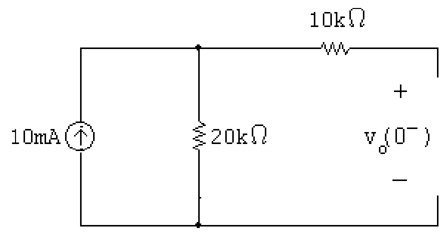
$$6 = 18e^{-5000t} + 12e^{-2500t}$$

Let $\quad x = e^{-2500t} \qquad$ so $\qquad 6 = 18x^2 + 12x$

Solving $\quad x = \dfrac{1}{3} = e^{-2500t}$

$$\therefore \quad e^{2500t} = 3 \qquad \text{and} \qquad t = \frac{\ln 3}{2500} = 439.44\,\mu\text{s}$$

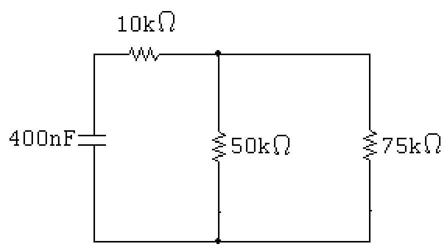
P 7.21 **[a]** For $t < 0$:



$$v(0) = 20{,}000(0.01) = 200\,\text{V}$$

[b] $w(0) = \frac{1}{2}Cv(0)^2 = \frac{1}{2}(400 \times 10^{-9})(200)^2 = 8\,\text{mJ}$

[c] For $t > 0$:

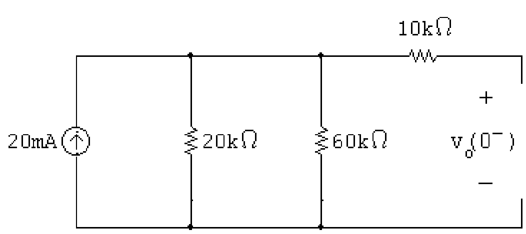


$$R_{\text{eq}} = 10{,}000 + 50{,}000\|75{,}000 = 40\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (40{,}000)(400 \times 10^{-9}) = 16\,\text{ms}$$

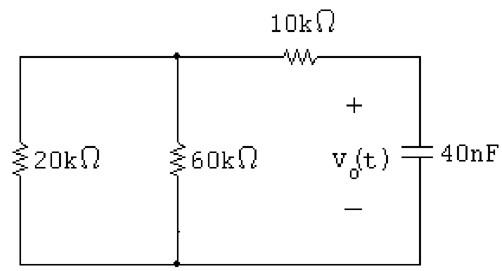
[d] $v(t) = v(0)e^{-t/\tau} = 200e^{-62.5t}\,\text{V} \qquad t \geq 0$

P 7.22 For $t < 0$:



$$V_o = (20{,}000\|60{,}000)(20 \times 10^{-3}) = 300\,\text{V}$$

For $t \geq 0$:

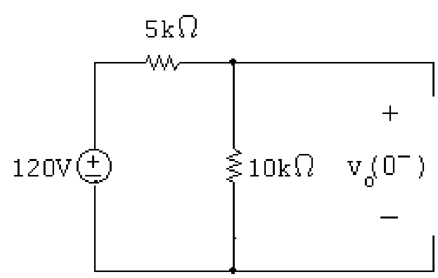


$$R_{\text{eq}} = 10{,}000 + (20{,}000 \| 60{,}000) = 25\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (25{,}000)(40 \times 10^{-9}) = 1\,\text{ms}$$

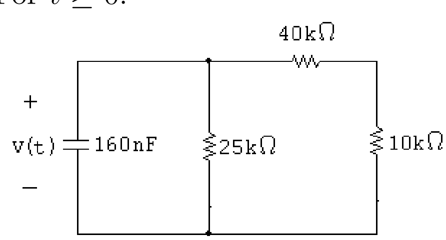
$$v(t) = V_o e^{-t/\tau} = 300e^{-1000t}\,\text{V} \qquad t \geq 0$$

P 7.23 **[a]** For $t < 0$:



$$V_o = \frac{10{,}000}{15{,}000}(120) = 80\,\text{V}$$

For $t \geq 0$:

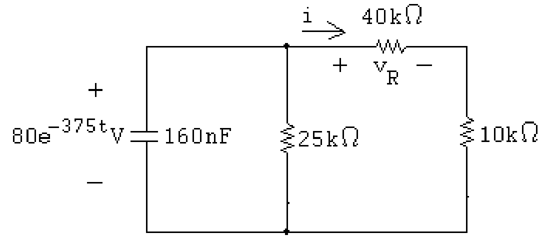


$$R_{\text{eq}} = 25{,}000 \| (40{,}000 + 10{,}000) = 16.67\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (16{,}666/67)(160 \times 10^{-9}) = 2.67\,\text{ms}$$

$$v(t) = V_o e^{-t/\tau} = 80e^{-375t}\,\text{V} \qquad t \geq 0$$

[b] For $t \geq 0$:



$$v_R(t) = \frac{40{,}000}{50{,}000}(80e^{-375t}) = 64e^{-375t}\,\text{V}$$

$$i(t) = \frac{v_R}{40{,}000} = 1.6e^{-375t}\,\text{mA}, \quad t \geq 0^+$$

P 7.24 Using the results of Problem 7.23:

$$w(0) = \frac{1}{2}CV_o^2 = \frac{1}{2}(160 \times 10^{-9})(80)^2 = 512\,\mu\text{J}$$

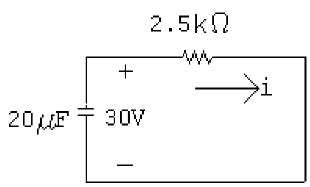
$$p_{40\text{k}} = Ri^2 = (40{,}000)(1.6 \times 10^{-3}e^{-375t})^2 = 0.1024e^{-750t}$$

$$w_{40\text{k}} = \int_0^\infty p_{40k}\,dt = \int_0^\infty 0.1024e^{-750t}\,dt = \frac{0.1024e^{-750t}}{-750}\Bigg|_0^\infty = 136.53\,\mu\text{J}$$

$$\text{percent} = \frac{136.53}{512}(100) = 26.67\%$$

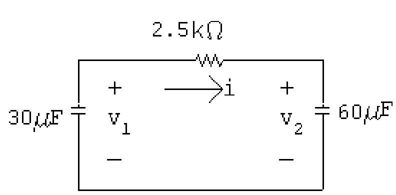
P 7.25 **[a]** $v_1(0^-) = v_1(0^+) = (0.006)(5000) = 30\,\text{V}$ $\qquad v_2(0^+) = 0$

$$C_{\text{eq}} = (30)(40)/90 = 20\,\mu\text{F}$$



$$\tau = (2.5 \times 10^3)(20 \times 10^{-6}) = 50\text{ms}; \qquad \frac{1}{\tau} = 20$$

$$i = \frac{30}{2500}e^{-20t} = 12e^{-20t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-1}{30^{-6}}\int_0^t 12 \times 10^{-3}e^{-20x}\,dx + 30 = 20e^{-20t} + 10\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{60 \times 10^{-6}} \int_0^t 12 \times 10^{-3} e^{-20x}\, dx + 0 = -10e^{-20t} + 10\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(30 \times 10^{-6})(30)^2 = 13.5\,\text{mJ}$

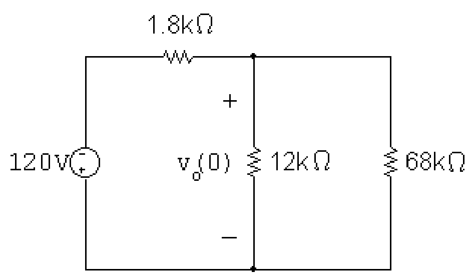
[c] $w_{\text{trapped}} = \frac{1}{2}(30 \times 10^{-6})(10)^2 + \frac{1}{2}(60 \times 10^{-6})(10)^2 = 4.5\,\text{mJ}.$

The energy dissipated by the 2.5 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors:

$$w_{\text{diss}} = \frac{1}{2}(20 \times 10^{-6})(30)^2 = 9\,\text{mJ}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 4.5 + 9 = 13.5\,\text{mJ}; \qquad w(0) = 13.5\,\text{mJ}.$

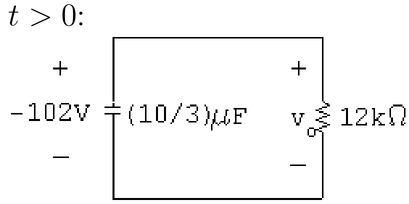
P 7.26 [a] $t < 0$:



$$R_{\text{eq}} = 12\,\text{k}\|8\,\text{k} = 10.2\,\text{k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:



$$\tau = [(10/3) \times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3} e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12\times10^{-3}} 867 \times 10^{-3} e^{-50t}\, dt$$
$$= 17.34 \times 10^{-3}(1 - e^{-50(12\times10^{-3})}) = 7824\,\mu\text{J}$$

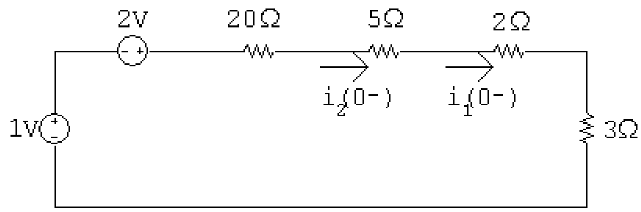
[**b**] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$0.75w(0) = 13\,\text{mJ}$

$$\int_0^{t_o} 867 \times 10^{-3} e^{-50x}\,dx = 13 \times 10^{-3}$$

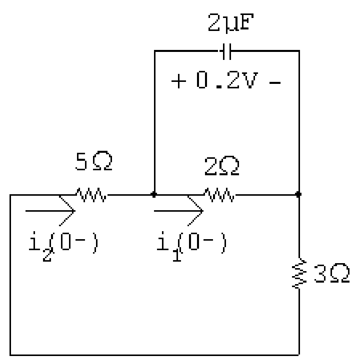
$\therefore\quad 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,\text{ms}$

P 7.27 [**a**] $t < 0$:



$$i_1(0^-) = i_2(0^-) = \frac{3}{30} = 100\,\text{mA}$$

[**b**] $t > 0$:



$$i_1(0^+) = \frac{0.2}{2} = 100\,\text{mA}$$

$$i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$$

[**c**] Capacitor voltage cannot change instantaneously, therefore,

$i_1(0^-) = i_1(0^+) = 100\,\text{mA}$

[**d**] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$i_2(0^-) = 100\,\text{mA} \quad \text{and} \quad i_2(0^+) = 25\,\text{mA}$

[**e**] $v_c = 0.2e^{-t/\tau}\,\text{V}, \qquad t \geq 0$

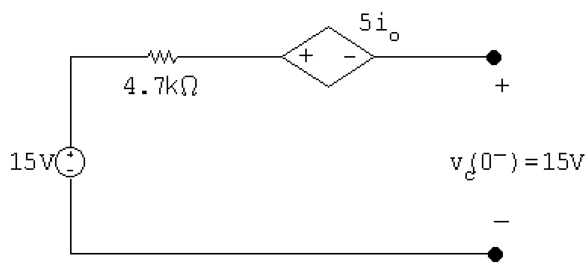
$\tau = R_eC = 1.6(2 \times 10^{-6}) = 3.2\,\mu\text{s}; \qquad \frac{1}{\tau} = 312{,}500$

$$v_c = 0.2e^{-312{,}000t}\,\text{V}, \qquad t \geq 0$$

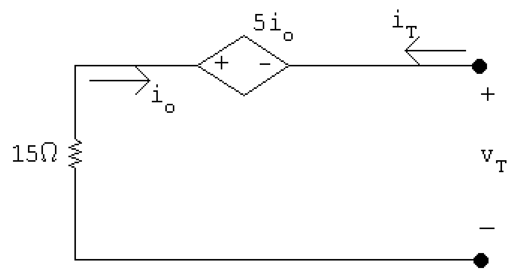
$$i_1 = \frac{v_c}{2} = 0.1e^{-312{,}000t}\,\text{A}, \qquad t \geq 0$$

[f] $i_2 = \dfrac{-v_c}{8} = -25e^{-312{,}000t}\,\text{mA}, \qquad t \geq 0^+$

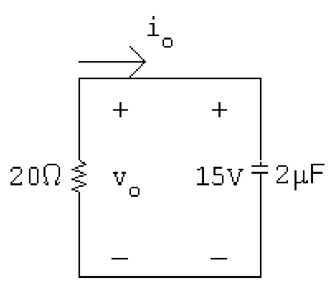
P 7.28 $t < 0$



$t > 0$



$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \quad \therefore \quad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\Omega$$



$$\tau = RC = 40\,\mu\text{s}; \qquad \frac{1}{\tau} = 25{,}000$$

$$v_o = 15e^{-25{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.29 **[a]** $R = \dfrac{v}{i} = 8\,\text{k}\Omega$

[b] $\frac{1}{\tau} = \frac{1}{RC} = 500; \qquad C = \frac{1}{(500)(8000)} = 0.25\,\mu\text{F}$

[c] $\tau = \frac{1}{500} = 2\,\text{ms}$

[d] $w(0) = \frac{1}{2}(0.25 \times 10^{-6})(72)^2 = 648\,\mu\text{J}$

[e] $w_{\text{diss}} = \int_0^{t_o} \frac{(72)^2 e^{-1000t}}{(800)} dt$

$$= 0.648 \frac{e^{-1000t}}{-1000}\Big|_0^{t_o} = 648(1 - e^{-1000t_o})\,\mu\text{J}$$

$$\%\text{diss} = 100(1 - e^{-1000t_o}) = 68 \qquad \text{so} \qquad e^{1000t_o} = 3.125$$

$$\therefore \quad t = \frac{\ln 3.125}{1000} = 1139\,\mu\text{s}$$

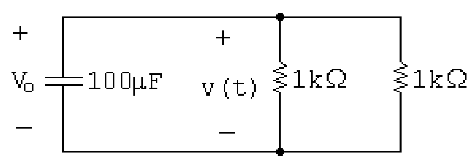
P 7.30 [a] Note that there are many different possible correct solutions to this problem.

$R = \frac{\tau}{C}$

Choose a $100\,\mu$F capacitor from Appendix H. Then,

$$R = \frac{0.05}{100 \times 10^{-6}} = 500\,\Omega$$

Construct a $500\,\Omega$ resistor by combining two $1\,\text{k}\Omega$ resistors in parallel:

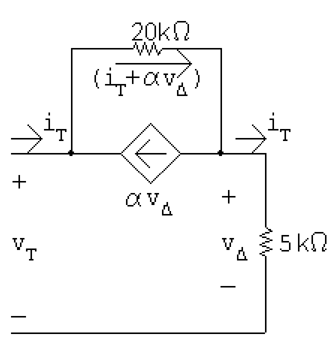


[b] $v(t) = V_o e^{-t/\tau} = 50e^{-20t}\,\text{V}, \qquad t \geq 0$

[c] $50e^{-20t} = 10 \qquad \text{so} \qquad e^{20t} = 5$

$$\therefore \quad t = \frac{\ln 5}{20} = 80.47\,\text{ms}$$

P 7.31 [a]



$$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$$

$$v_\Delta = 5 \times 10^3 i_T$$

$$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha(5 \times 10^3 i_T)$$

$$R_{\text{Th}} = 25{,}000 + 100 \times 10^6 \alpha$$

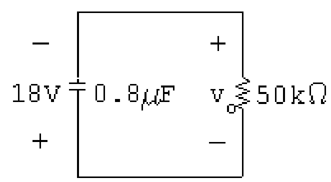
$$\tau = R_{\text{Th}}C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$$

$$R_{\text{Th}} = 50\,\text{k}\Omega = 25{,}000 + 100 \times 10^6 \alpha$$

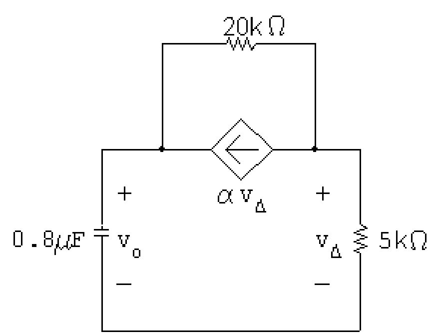
$$\alpha = \frac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4}\,\text{A/V}$$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = -18e^{-25t}\,\text{V}, \quad t \geq 0$$

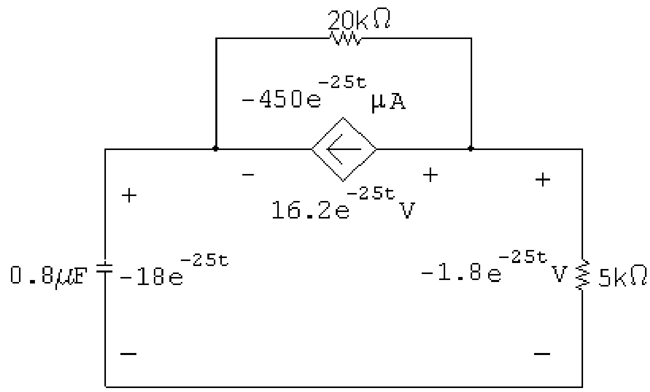


$$\frac{v_\Delta}{5000} + \frac{v_\Delta - v_o}{20{,}000} + 2.5 \times 10^{-4} v_\Delta = 0$$

$$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$$

$$\therefore \quad v_\Delta = \frac{v_o}{10} = -1.8e^{-25t}\,\text{V}, \quad t \geq 0^+$$

P 7.32 **[a]**



$$p_{ds} = (16.2e^{-25t})(-450 \times 10^{-6}e^{-25t}) = -7290 \times 10^{-6}e^{-50t}\,\mathrm{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -145.8\,\mu\mathrm{J}.$$

$\therefore$ dependent source is delivering $145.8\,\mu$J.

[b] $w_{5\mathrm{k}} = \int_0^\infty (5000)(0.36 \times 10^{-3}e^{-25t})^2\,dt = 648 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 12.96\,\mu\mathrm{J}$

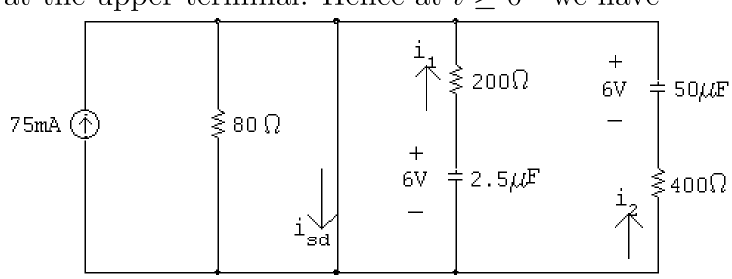
$$w_{20\mathrm{k}} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20{,}000}\,dt = 13{,}122 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 262.44\,\mu\mathrm{J}$$

$$w_c(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.6\,\mu\mathrm{J}$$

$$\sum w_{\mathrm{diss}} = 12.96 + 262.44 = 275.4\,\mu\mathrm{J}$$

$$\sum w_{\mathrm{dev}} = 145.8 + 129.6 = 275.4\,\mu\mathrm{J}.$$

P 7.33 **[a]** At $t = 0^-$ the voltage on each capacitor will be 6 V (0.075×80), positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore \quad i_{sd}(0^+) = 0.075 + \frac{6}{200} + \frac{6}{400} = 120\,\mathrm{mA}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore \quad i_{sd}(\infty) = 75\,\mathrm{mA}$$

[b] $i_{sd}(t) = 0.075 + i_1(t) + i_2(t)$

$$\tau_1 = 200(25 \times 10^{-6}) = 5\,\text{ms}$$

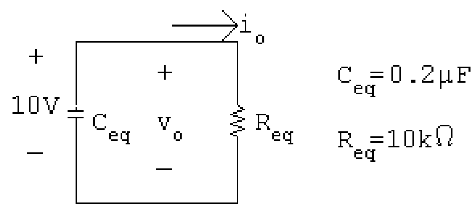
$$\tau_2 = 400(50 \times 10^{-6}) = 20\,\text{ms}$$

$$\therefore\quad i_1(t) = 30e^{-200t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_2(t) = 15e^{-50t}\,\text{mA}, \qquad t \geq 0$$

$$\therefore\quad i_{sd} = 75 + 30e^{-200t} + 15e^{-50t}\,\text{mA}, \qquad t \geq 0^+$$

P 7.34 [a] The equivalent circuit for $t > 0$:



$$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{24\text{k}\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$p_{24\text{k}\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$$

$$w_{24\text{k}\Omega} = \int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0-1) = 3.84\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25 \times 10^{-6})(40)^2 + \frac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45\,\text{mJ}$$

$$\%\ \text{diss}\,(24\,\text{k}\Omega) = \frac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$$

[b] $p_{400\Omega} = 400(1 \times 10^{-3}e^{-500t})^2 = 0.4 \times 10^{-3}e^{-1000t}$

$$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$$

$$\%\ \text{diss}\,(400\,\Omega) = \frac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$$

$$i_{16k\Omega} = e^{-500t}\left(\frac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

$$p_{16k\Omega} = (0.6 \times 10^{-3}e^{-500t})^2(16{,}000) = 5.76 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{16k\Omega} = \int_0^\infty 5.76 \times 10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$$

$$\%\ \text{diss}\,(16\,\text{k}\Omega) = 0.4\%$$

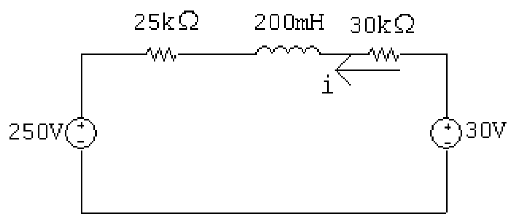
[**c**] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44\,\text{mJ}$$

$$\%\text{ trapped} = \frac{1.44}{1.45} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

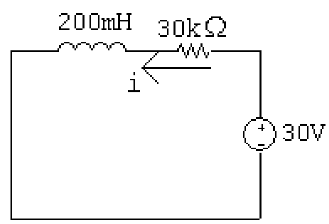
P 7.35 [**a**] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 200 mH inductor. We get



$$i(0^-) = \frac{30 - 250}{25\,\text{k} + 30\,\text{k}} = -4\,\text{mA}$$

$$i(0^-) = i(0^+) = -4\,\text{mA}$$

[**b**] For $t > 0$, the circuit reduces to



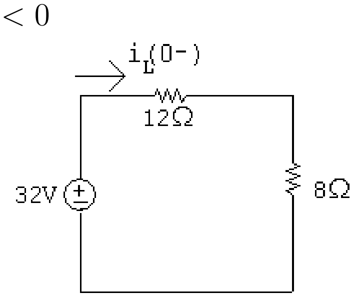
Therefore $i(\infty) = 30/30{,}000 = 1\,\text{mA}$

[**c**] $\tau = \dfrac{L}{R} = \dfrac{200 \times 10^{-3}}{30{,}000} = 6.67\,\mu\text{s}$

[**d**] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

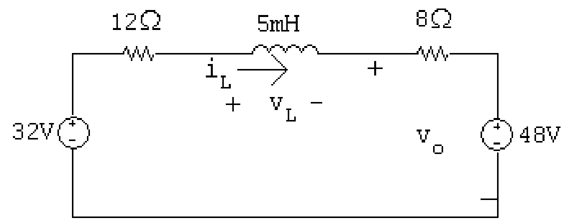
$$= 0.001 + [-0.004 - 0.001]e^{-150{,}000t} = 1 - 5e^{-150{,}000t}\,\text{mA}, \qquad t \geq 0$$

P 7.36 [**a**] $t < 0$



$$i_L(0^-) = \frac{32}{20} = 1.6\,\text{A}$$

$t > 0$



$$i_L(\infty) = \frac{32 - 48}{12 + 8} = -0.8\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{12 + 8} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -0.8 + (1.6 + 0.8)e^{-4000t} = -0.8 + 2.4e^{-4000t}\,\text{A}, \qquad t \geq 0$$

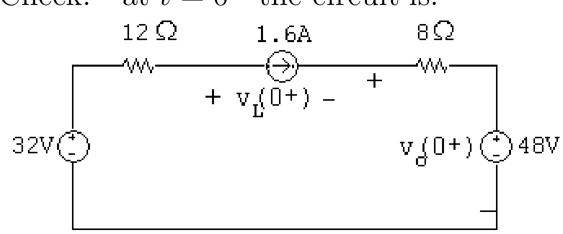
$$v_o = 8i_L + 48 = 8(-0.8 + 2.4e^{-4000t}) + 48 = 41.6 + 19.2e^{-4000t}\,\text{V}, \qquad t \geq 0$$

[b] $v_L = L\dfrac{di_L}{dt} = 5 \times 10^{-3}(-4000)[2.4e^{-4000t}] = -48e^{-4000t}\,\text{V}, \qquad t \geq 0^+$

$v_L(0^+) = -48\,\text{V}$

From part (a) $\quad v_o(0^+) = 0\,\text{V}$

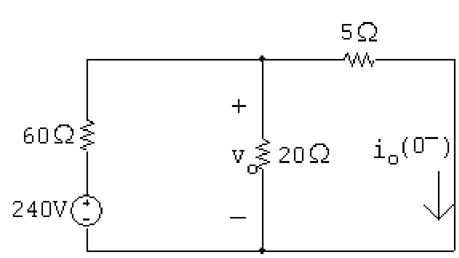
Check: at $t = 0^+$ the circuit is:



$v_o(0^+) = 48 + (8\,\Omega)(1.6\,\text{A}) = 60.8\,\text{V}; \qquad v_\text{L}(0^+) + v_o(0^+) = 12(-1.6) + 32$

$\therefore \qquad v_\text{L}(0^+) = -19.2 + 32 - 60.8 = -48\,\text{V}$

P 7.37 **[a]** $t < 0$



KVL equation at the top node:

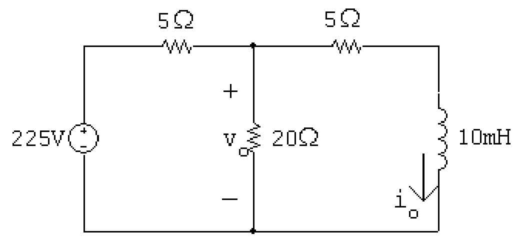
$$\frac{v_o - 240}{60} + \frac{v_o}{20} + \frac{v_o}{5} = 0$$

Multiply by 60 and solve:

$$240 = (3 + 1 + 12)v_o; \qquad v_o = 15\,\text{V}$$

$$\therefore \quad i_o(0^-) = \frac{v_o}{5} = 15/5 = 3\,\text{A}$$

$t > 0$



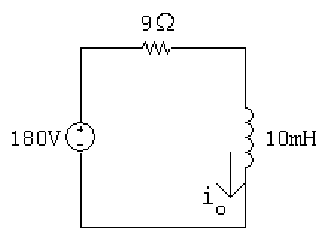
Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{20}{20+5}(225) = 180\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\text{Th}} = 5 + 20\|5 = 5 + 4 = 9\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}}{9} = 1.11\,\text{ms}; \qquad \frac{1}{\tau} = 900$$

$$i_o(\infty) = \frac{190}{9} = 20\,\text{A}$$

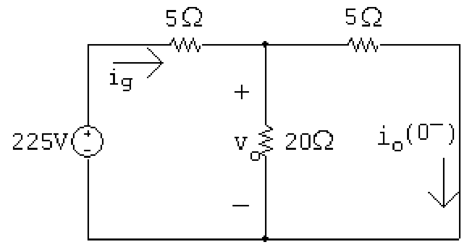
$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 20 + (3 - 20)e^{-900t} = 20 - 17e^{-900t}\,\text{A}, \qquad t \geq 0$$

[b]

$$\begin{aligned} v_o &= 5i_o + L\frac{di_o}{dt} \\ &= 5(20 - 17e^{-900t}) + 0.01(-900)(17e^{-900t}) \\ &= 100 - 85e^{-900t} + 153e^{-900t} \\ v_o &= 100 + 68e^{-900t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

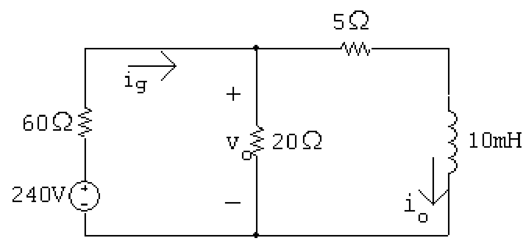
P 7.38 **[a]** $t < 0$



$$i_g = \frac{225}{5 + 20\|5} = \frac{225}{9} = 25\,\text{A}$$

$$\therefore \quad i_o(0^-) = \frac{20\|5}{5}(25) = 20\,\text{A}$$

$t > 0$



$$i_g(\infty) = \frac{240}{60 + 20\|5} = \frac{240}{64} = 3.75\,\text{A}$$

$$i_o(\infty) = \frac{20\|5}{5} i_g(\infty) = 3\,\text{A}$$

$$R_{\text{eq}} = 5 + 20\|60 = 3 + 15 = 20\,\Omega$$

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{10 \times 10^{-3}}{20} = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$$

$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 3 + (20 - 3)e^{-2000t} = 3 + 17e^{-2000t}\,\text{A}, \qquad t \geq 0$$

[b]
$$\begin{aligned} v_o &= 5i_o + L\frac{di_o}{dt} \\ &= 5(3 + 17e^{-2000t}) + 0.01(-2000)(17e^{-2000t}) \\ &= 15 + 65e^{-2000t} - 340e^{-2000t} \\ v_o &= 15 - 255e^{-2000t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

P 7.39 **[a]** From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-(R/L)t}$$

$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 4; \qquad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_oR = -80; \qquad \frac{R}{L} = 40$$

$$\therefore \quad I_o = 4 + \frac{V_s}{R} = 8\,\mathrm{A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \qquad R = 20\,\Omega$$

$$V_s = 80\,\mathrm{V}; \qquad L = \frac{R}{40} = 0.5\,\mathrm{H}$$

[b] $i = 4 + 4e^{-40t}; \qquad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore \quad 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

Let $x = e^{-40t}$:

$x^2 + 2x - 1.25 = 0$; Solving, $x = 0.5$; $x = -2.5$

But $x \geq 0$ for all t. Thus,

$$e^{-40t} = 0.5; \qquad e^{40t} = 2; \qquad t = 25\ln 2 = 17.33\,\mathrm{ms}$$

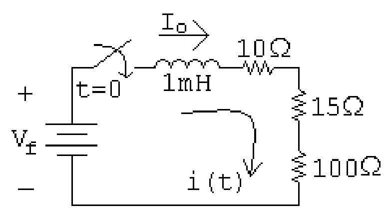
P 7.40 **[a]** Note that there are many different possible solutions to this problem.

$$R = \frac{L}{\tau}$$

Choose a 1 mH inductor from Appendix H. Then,

$$R = \frac{0.001}{8 \times 10^{-6}} = 125\,\Omega$$

Construct the resistance needed by combining $100\,\Omega$, $10\,\Omega$, and $15\,\Omega$ resistors in series:



[b] $i(t) = I_f + (I_o - I_f)e^{-t/\tau}$

$$I_o = 0\,\text{A}; \qquad I_f = \frac{V_f}{R} = \frac{25}{125} = 200\,\text{mA}$$

$$\therefore \quad i(t) = 200 + (0 - 200)e^{-125{,}000t}\,\text{mA} = 200 - 200e^{-125{,}000t}\,\text{mA}, \qquad t \geq 0$$

[c] $i(t) = 0.2 - 0.2e^{-125{,}000t} = (0.75)(0.2) = 0.15$

$$e^{-125{,}000t} = 0.25 \qquad \text{so} \qquad e^{125{,}000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{125{,}000} = 11.09\,\mu\text{s}$$

P 7.41 [a] $v_o(0^+) = -I_g R_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_g R_2 e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to$ zero

[c] $v_{sw} = R_2 i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g \frac{R_1}{R_1 + R_2}$$

Therefore $\quad i_o(t) = \frac{I_g R_1}{R_1+R_2} + \left[I_g - \frac{I_g R_1}{R_1+R_2}\right] e^{-[(R_1+R_2)/L]t}$

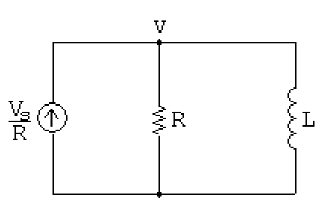
$$i_o(t) = \frac{R_1 I_g}{(R_1+R_2)} + \frac{R_2 I_g}{(R_1+R_2)} e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{sw} = \frac{R_1 I_g}{(1+R_1/R_2)} + \frac{R_2 I_g}{(1+R_1/R_2)} e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[d] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.42 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [d] of Problem 7.41), causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.43 [a]



$$-\frac{V_s}{R} + \frac{v}{R} + \frac{1}{L}\int_0^t v\,dt + I_o = 0$$

Differentiating both sides,

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$\therefore \qquad \frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $$\frac{dv}{dt} = -\frac{R}{L}v$$

$$\frac{dv}{dt}dt = -\frac{R}{L}v\,dt \qquad \text{so} \qquad dv = -\frac{R}{L}v\,dt$$

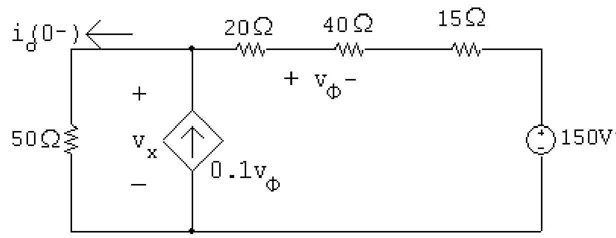
$$\frac{dv}{v} = -\frac{R}{L}dt$$

$$\int_{V_o}^{v(t)} \frac{dx}{x} = -\frac{R}{L}\int_0^t dy$$

$$\ln\frac{v(t)}{V_o} = -\frac{R}{L}t$$

$$\therefore \qquad v(t) = V_o e^{-(R/L)t} = (V_s - RI_o)e^{-(R/L)t}$$

P 7.44 For $t < 0$



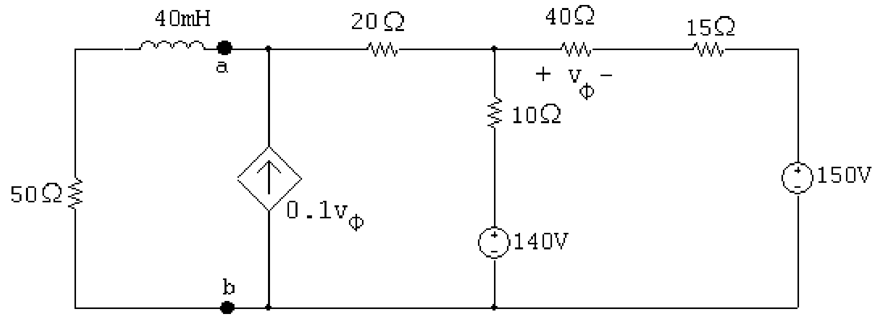
$$\frac{v_x}{50} - 0.1v_\phi + \frac{v_x - 150}{75} = 0$$

$$v_\phi = \frac{40}{75}(v_x - 150)$$

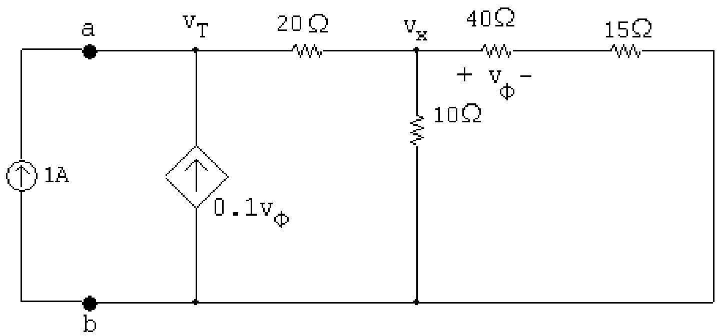
Solving,

$$v_x = 300\,\text{V}; \qquad i_o(0^-) = \frac{v_x}{50} = 6\,\text{A}$$

$t > 0$



Find Thévenin equivalent with respect to a, b. Use a test source to find the Thévenin equivalent resistance:



$$-1 - 0.1v_\phi + \frac{v_\mathrm{T} - v_x}{20} = 0$$

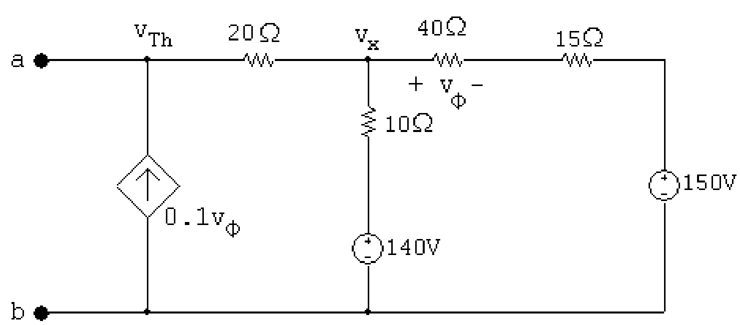
$$\frac{v_x - v_\mathrm{T}}{20} + \frac{v_x}{10} + \frac{v_x}{55} = 0$$

$$v_\phi = \frac{40}{55}v_x$$

Solving,

$$v_\mathrm{T} = 74\,\mathrm{V} \qquad \text{so} \qquad R_\mathrm{Th} = \frac{v_\mathrm{T}}{1\,\mathrm{A}} = 74\,\Omega$$

Find the open circuit voltage with respect to a, b:



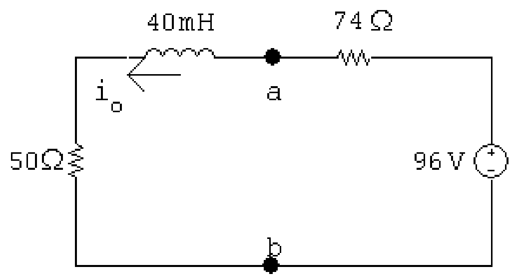
$$-0.1v_\phi + \frac{v_\mathrm{Th} - v_x}{20} = 0$$

$$\frac{v_x - v_{\text{Th}}}{20} + \frac{v_x - 140}{10} + \frac{v_x - 150}{55} = 0$$

$$v_\phi = \frac{40}{55}(v_x - 150)$$

Solving,

$$v_{\text{Th}} = 96\,\text{V}$$

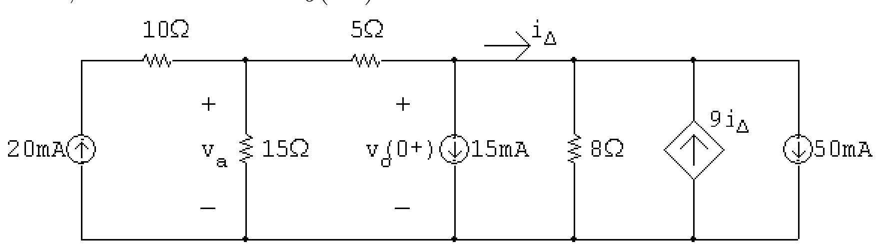


$$i_o(\infty) = 96/124 = 0.774\,\text{A}$$

$$\tau = \frac{40 \times 10^{-3}}{124} = 0.3226\,\text{ms}; \qquad 1/\tau = 3100$$

$$i_o = 0.774 + (6 - 0.774)e^{-3100t} = 0.774 + 5.226e^{-3100t}\,\text{A}, \qquad t \geq 0$$

P 7.45 $t > 0$; calculate $v_o(0^+)$



$$\frac{v_{\text{a}}}{15} + \frac{v_{\text{a}} - v_o(0^+)}{5} = 20 \times 10^{-3}$$

$$\therefore \quad v_{\text{a}} = 0.75 v_o(0^+) + 75 \times 10^{-3}$$

$$15 \times 10^{-3} + \frac{v_o(0^+) - v_{\text{a}}}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3} = 0$$

$$13v_o(0^+) - 8v_{\text{a}} - 360i_\Delta = -2600 \times 10^{-3}$$

$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3}$$

$$\therefore \quad i_\Delta = \frac{v_o(0^+)}{80} + 5 \times 10^{-3}$$

$$\therefore \quad 360 i_\Delta = 4.5 v_o(0^+) + 1800 \times 10^{-3}$$

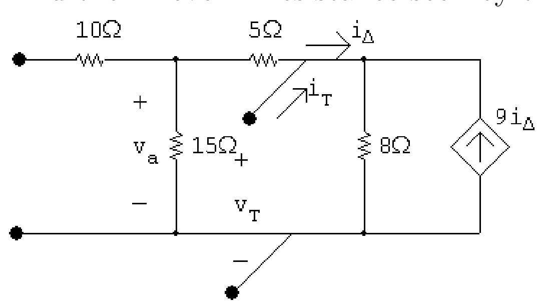
$$8v_{\mathrm{a}} = 6v_o(0^+) + 600 \times 10^{-3}$$

$$\therefore \quad 13v_o(0^+) - 6v_o(0^+) - 600 \times 10^{-3} - 4.5v_o(0^+) -$$
$$1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5v_o(0^+) = -200 \times 10^{-3}; \qquad v_o(0^+) = -80\,\mathrm{mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \qquad \therefore \quad 10i_\Delta = \frac{v_T}{8}; \qquad i_\Delta = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\,\mathrm{S}$$

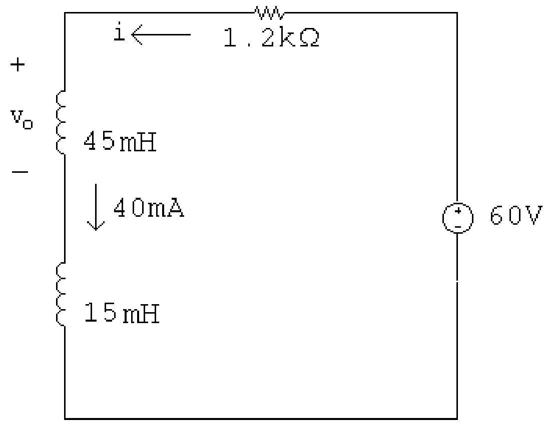
$$\therefore \quad R_{\mathrm{Th}} = 16\Omega$$

$$\tau = \frac{4 \times 10^{-3}}{16} = 0.25\,\mathrm{ms}; \qquad 1/\tau = 4000$$

$$\therefore \quad v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t}\,\mathrm{mV}, \qquad t \geq 0^+$$

P 7.46 For $t < 0$, $\quad i_{45\text{mH}}(0) = 80\,\text{V}/2000\,\Omega = 40\,\text{mA}$

For $t > 0$, after making a Thévenin equivalent of the circuit to the right of the inductors we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-t/\tau}$$

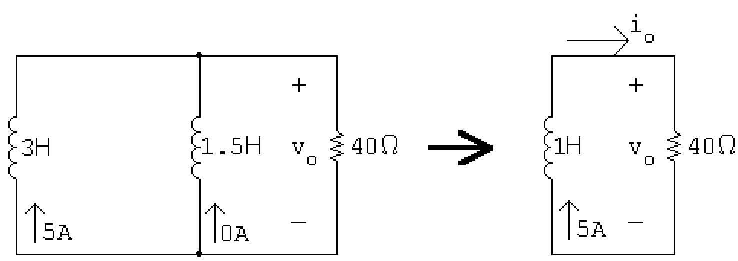
$$\frac{1}{\tau} = \frac{R}{L} = \frac{1200}{60 \times 10^{-3}} = 20{,}000$$

$$I_o = 40\,\text{mA}; \qquad I_f = \frac{V_s}{R} = \frac{60}{1200} = 50\,\text{mA}$$

$$i = 0.05 + (0.04 - 0.05)e^{-20{,}000t} = 50 - 10e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = 0.045\frac{di}{dt} = 0.045(-0.01)(-20{,}000e^{-20{,}000t}) = 9e^{-20{,}000t}\,\text{V}, \qquad t \geq 0^+$$

P 7.47 $t > 0$



$$\tau = \frac{1}{40}$$

$$i_o = 5e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 40i_o = 200e^{-40t}\,\text{V}, \qquad t > 0^+$$

$$200e^{-40t} = 100; \qquad e^{40t} = 2$$

$$\therefore \quad t = \frac{1}{40}\ln 2 = 17.33\,\text{ms}$$

P 7.48 **[a]** $w_{\text{diss}} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(1)(5)^2 = 12.5\,\text{J}$

[b] $i_{3H} = \frac{1}{3}\int_0^t (200)e^{-40x}\,dx - 5$

$$= 1.67(1 - e^{-40t}) - 5 = -1.67e^{-40t} - 3.33\,\text{A}$$

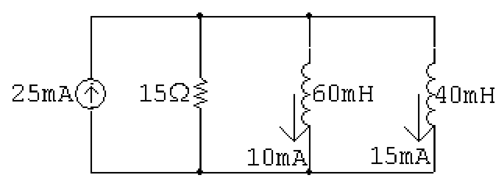
$$i_{1.5H} = \frac{1}{1.5}\int_0^t (200)e^{-40x}\,dx + 0$$

$$= -3.33e^{-40t} + 3.33\,\text{A}$$

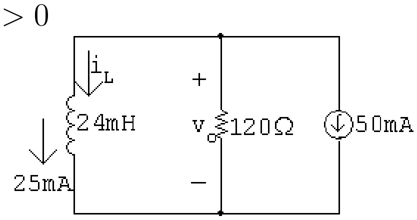
$$w_{\text{trapped}} = \frac{1}{2}(4.5)(3.33)^2 = 25\,\text{J}$$

[c] $w(0) = \frac{1}{2}(3)(5)^2 = 37.5\,\text{J}$

P 7.49 **[a]** $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 25\,\text{mA}; \qquad \tau = \frac{24 \times 10^{-3}}{120} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$i_L(\infty) = -50\,\text{mA}$$

$$i_L = -50 + (25 + 50)e^{-5000t} = -50 + 75e^{-5000t}\,\text{mA}, \qquad t \ge 0$$

$$v_o = -120[75 \times 10^{-3}e^{-5000t}] = -9e^{-5000t}\,\text{V}, \qquad t \ge 0^+$$

[b] $i_1 = \frac{1}{60 \times 10^{-3}}\int_0^t -9e^{-5000x}\,dx + 10 \times 10^{-3} = (30e^{-5000t} - 20)\,\text{mA}, \qquad t \ge 0$

[c] $i_2 = \frac{1}{40 \times 10^{-3}}\int_0^t -9e^{-5000x}\,dx + 15 \times 10^{-3} = (45e^{-5000t} - 30)\,\text{mA}, \qquad t \ge 0$

P 7.50 **[a]** Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $v = I_gR_ge^{-t/\tau}$; $\qquad \tau = L_e/R_g$
Thus

$$i_1 = \frac{1}{L_1}\int_0^t I_gR_ge^{-x/\tau}\,dx = \frac{I_gR_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\Bigg|_0^t = \frac{I_gL_e}{L_1}(1 - e^{-t/\tau})$$

$$i_1 = \frac{I_gL_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_gL_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \dfrac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \dfrac{L_1}{L_1 + L_2}I_g$

P 7.51 **[a]** $v_c(0^+) = -120\,\text{V}$

[b] Use voltage division to find the final value of voltage:

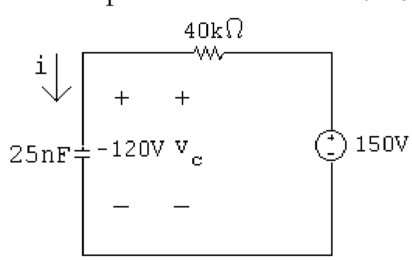
$$v_c(\infty) = \frac{150{,}000}{200{,}000}(200) = 150\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = 150\,\text{V}, \qquad R_{\text{Th}} = 2500 + 150\,\text{k}\|50\,\text{k} = 40\,\text{k}\Omega,$$

Therefore $\tau = R_{\text{eq}}C = (40{,}000)(25 \times 10^{-9}) = 1\,\text{ms}$

The simplified circuit for $t > 0$ is:



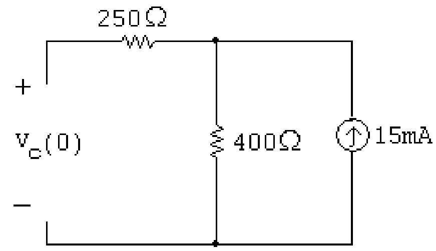
[d] $i(0^+) = \dfrac{150 - (-120)}{40{,}000} = 6.75\,\text{mA}$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= 150 + (-120 - 150)e^{-t/\tau} = 150 - 270e^{-1000t}\,\text{V}, \qquad t \geq 0$$

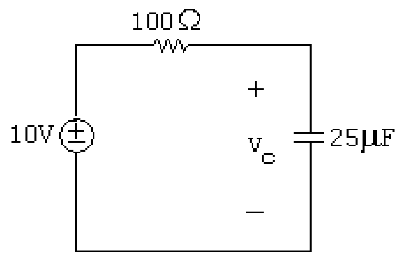
[f] $i = C\dfrac{dv_c}{dt} = (25 \times 10^{-9})(-1000)(-270e^{-1000t}) = 6.75e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

P 7.52 [a] for $t < 0$:



$v_c(0) = 400(0.015) = 6\,\text{V}$

For $t \geq 0$:

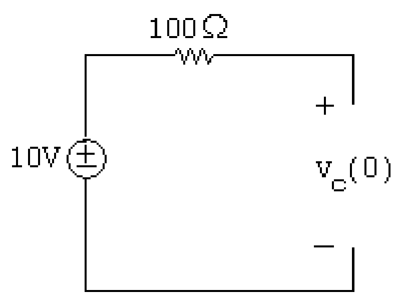


$v_c(\infty) = 10\,\text{V}$

$R_{\text{eq}} = 20\,\Omega \qquad \text{so} \qquad \tau = R_{\text{eq}}C = 250(25 \times 10^{-6}) = 6.25\,\text{ms}$

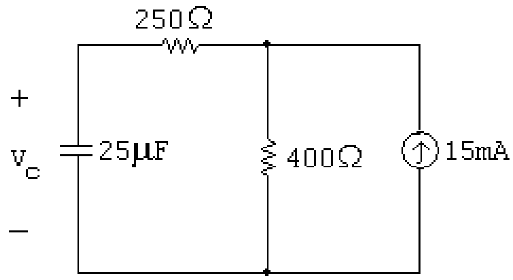
$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = 10 + (6 - 10)e^{-160t} = 10 - 4e^{-160t}\,\text{V}$

[b] For $t < 0$:



$v_c(0) = 10\,\text{V}$

For $t \geq 0$:



$v_c(\infty) = 400(0.015) = 6\,\text{V}$

$R_{\text{eq}} = 100 + 400 = 500\,\Omega \qquad \text{so} \qquad \tau = R_{\text{eq}}C = 500(25 \times 10^{-6}) = 12.5\,\text{ms}$

$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = 6 + (10-6)e^{-80t} = 6 + 4e^{-80t}\,\text{V}$

P 7.53 [a] Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\text{k}}{9\,\text{k} + 3\,\text{k}}(120) = 90\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -60\,\text{V}, \qquad R_{\text{Th}} = 10\,\text{k} + 40\,\text{k} = 50\,\text{k}\Omega$$

$$\tau = R_{\text{Th}}C = 1\,\text{ms} \;= 1000\,\mu\text{s}$$

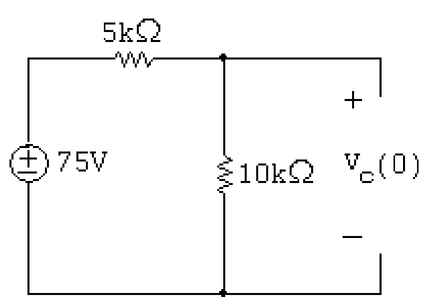
[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\text{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

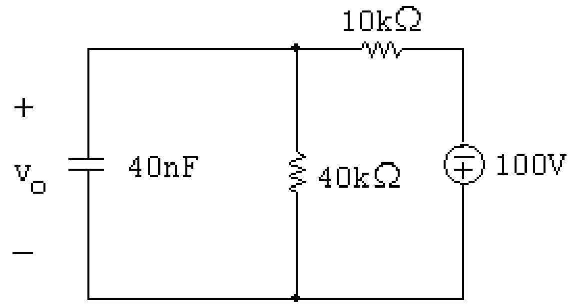
$$\text{Therefore} \quad t = \frac{\ln(150/60)}{1000} = 916.3\,\mu\text{s}$$

P 7.54 [a] For $t < 0$:



$$v_o(0) = \frac{10{,}000}{15{,}000}(75) = 50\,\text{V}$$

For $t \geq 0$:



$$v_o(\infty) = \frac{40{,}000}{50{,}000}(-100) = -80\,\mathrm{V}$$

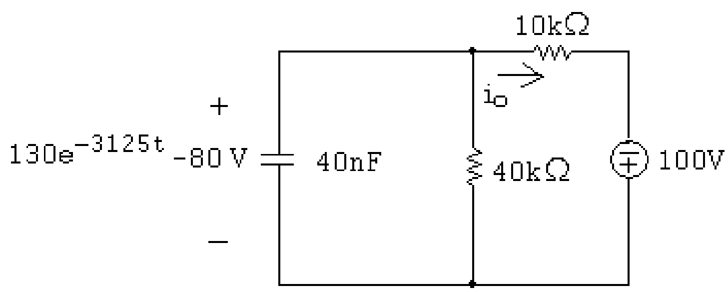
$$R_{\text{eq}} = 40\,\mathrm{k}\|10\,\mathrm{k} = 8\,\mathrm{k\Omega}$$

$$\tau = R_{\text{eq}}C = (8000)(40 \times 10^{-9}) = 0.32\,\mathrm{ms}$$

$$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/\tau} = -80 + (50 + 80)e^{-3125t}$$

$$= -80 + 130e^{-3125t}\,\mathrm{V}$$

[b] For $t \geq 0$:

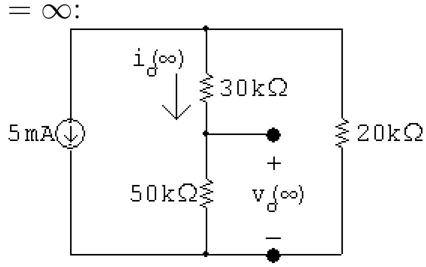


$$i_o = \frac{130e^{-3125t} - 80 + 100}{10{,}000} = 13e^{-3125t} + 2\,\mathrm{mA}$$

P 7.55 $t < 0$:

$$i_o(0^-) = \frac{20}{100}(10 \times 10^{-3}) = 2\,\mathrm{mA}; \qquad v_o(0^-) = (2 \times 10^{-3})(50{,}000) = 100\,\mathrm{V}$$

$t = \infty$:

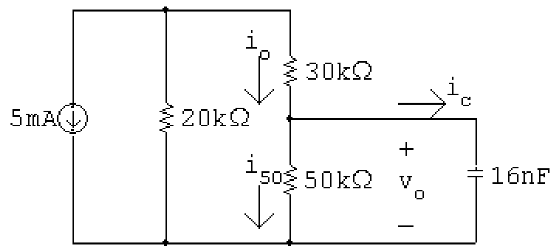


$$i_o(\infty) = -5 \times 10^{-3}\left(\frac{20}{100}\right) = -1\,\mathrm{mA}; \qquad v_o(\infty) = i_o(\infty)(50{,}000) = -50\,\mathrm{V}$$

$R_{\text{Th}} = 50\,\text{k}\Omega \| 50\,\text{k}\Omega = 25\,\text{k}\Omega; \qquad C = 16\,\text{nF}$

$\tau = (25{,}000)(16 \times 10^{-9}) = 0.4\,\text{ms}; \qquad \frac{1}{\tau} = 2500$

$\therefore \quad v_o(t) = -50 + 150e^{-2500t}\,\text{V}, \qquad t \geq 0$

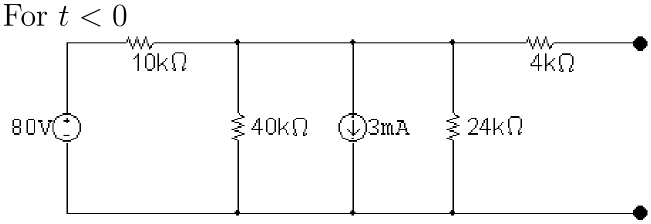


$i_c = C\frac{dv_o}{dt} = -6e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$i_{50k} = \frac{v_o}{50{,}000} = -1 + 3e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$i_o = i_c + i_{50k} = -(1 + 3e^{-2500t})\,\text{mA}, \qquad t \geq 0^+$

P 7.56 For $t < 0$



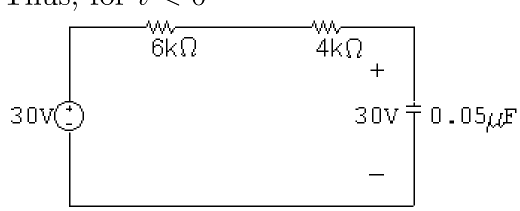
Simplify the circuit:

$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$

$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$

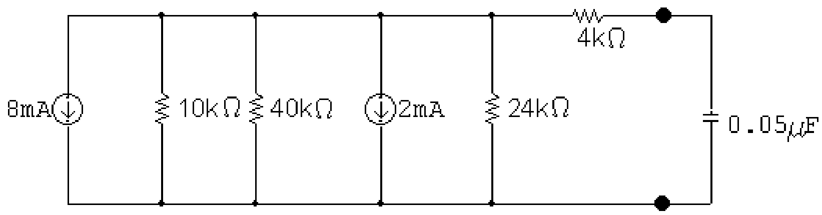
$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$

Thus, for $t < 0$



$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$

$t > 0$



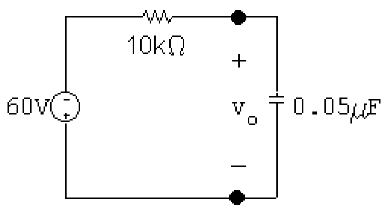
Simplify the circuit:

$8\,\text{mA} + 2\,\text{mA} = 10\,\text{mA}$

$10\,\text{k}\|40\,\text{k}\|24\,\text{k} = 6\,\text{k}\Omega$

$(10\,\text{mA})(6\,\text{k}\Omega) = 60\,\text{V}$

Thus, for $t > 0$



$$v_o(\infty) = -10 \times 10^{-3}(6 \times 10^3) = -60\,\text{V}$$

$$\tau = RC = (10\,\text{k})(0.05\,\mu) = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t}$$

$$= -60 + 90e^{-2000t}\,\text{V} \qquad t \geq 0$$

P 7.57 Use voltage division to find the initial voltage:

$$v_o(0) = \frac{60}{40 + 60}(50) = 30\,\text{V}$$

Use Ohm's law to find the final value of voltage:

$$v_o(\infty) = (-5\,\text{mA})(20\,\text{k}\Omega) = -100\,\text{V}$$

$$\tau = RC = (20 \times 10^3)(250 \times 10^{-9}) = 5\,\text{ms}; \qquad \frac{1}{\tau} = 200$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau}$$

$$= -100 + (30 + 100)e^{-200t} = -100 + 130e^{-200t}\,\text{V}, \qquad t \geq 0$$

P 7.58 **[a]** $v = I_sR + (V_o - I_sR)e^{-t/RC} \qquad i = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$

$\therefore \quad I_sR = 40, \qquad V_o - I_sR = -24$

$\therefore \quad V_o = 16\,\text{V}$

$I_s - \frac{V_o}{R} = 3 \times 10^{-3}; \qquad I_s - \frac{16}{R} = 3 \times 10^{-3}; \qquad R = \frac{40}{I_s}$

$\therefore \quad I_s - 0.4I_s = 3 \times 10^{-3}; \qquad I_s = 5\,\text{mA}$

$R = \frac{40}{5} \times 10^3 = 8\,\text{k}\Omega$

$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20 \times 10^3} = 50\,\text{nF}; \quad \tau = RC = \frac{1}{2500} = 400\,\mu\text{s}$

[b] $v(\infty) = 40\,\text{V}$

$w(\infty) = \frac{1}{2}(50 \times 10^{-9})(1600) = 40\,\mu\text{J}$

$0.81w(\infty) = 32.4\,\mu\text{J}$

$v^2(t_o) = \frac{32.4 \times 10^{-6}}{25 \times 10^{-9}} = 1296; \qquad v(t_o) = 36\,\text{V}$

$40 - 24e^{-2500t_o} = 36; \qquad e^{2500t_o} = 6; \qquad \therefore \quad t_o = 716.70\,\mu\text{s}$

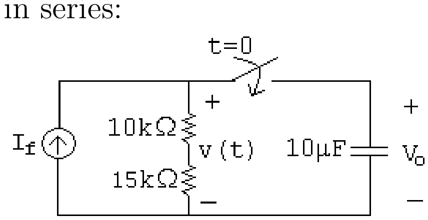
P 7.59 **[a]** Note that there are many different possible solutions to this problem.

$R = \frac{\tau}{C}$

Choose a $10\,\mu$H capacitor from Appendix H. Then,

$R = \frac{0.25}{10 \times 10^{-6}} = 25\,\text{k}\Omega$

Construct the resistance needed by combining $10\,\text{k}\Omega$ and $15\,\text{k}\Omega$ resistors in series:



[b] $v(t) = V_f + (V_o - V_f)e^{-t/\tau}$

$V_o = 100\,\text{V}; \qquad V_f = (I_f)(R) = (1 \times 10^{-3})(25 \times 10^3) = 25\,\text{V}$

$\therefore \quad v(t) = 25 + (100 - 25)e^{-4t}\,\text{V} = 25 + 75e^{-4t}\,\text{V}, \qquad t \geq 0$

[**c**] $v(t) = 25 + 75e^{-4t} = 50 \qquad \text{so} \qquad e^{-4t} = \frac{1}{3}$

$$\therefore \quad t = \frac{\ln 3}{4} = 274.65\,\text{ms}$$

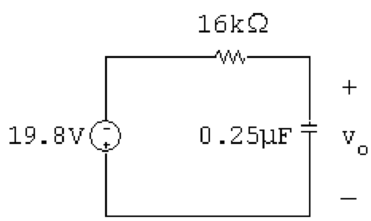
P 7.60 For $t > 0$

$$V_{\text{Th}} = (-25)(16{,}000)i_{\text{b}} = -400 \times 10^3 i_{\text{b}}$$

$$i_{\text{b}} = \frac{33{,}000}{80{,}000}(120 \times 10^{-6}) = 49.5\,\mu\text{A}$$

$$V_{\text{Th}} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8\,\text{V}$$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$



$$v_o(\infty) = -19.8\,\text{V}; \qquad v_o(0^+) = 0$$

$$\tau = (16,000)(0.25 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_o = -19.8 + 19.8e^{-250t}\,\text{V}, \qquad t \geq 0$$

$$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2\,\text{J}$$

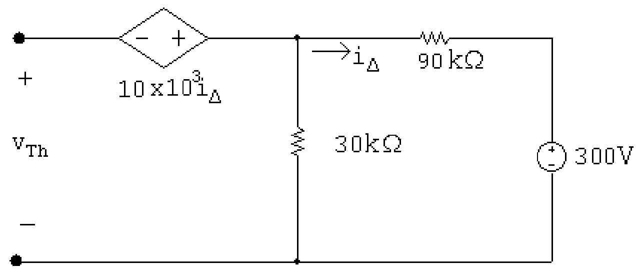
$$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$$

$$1 - e^{-250t} = 0.6$$

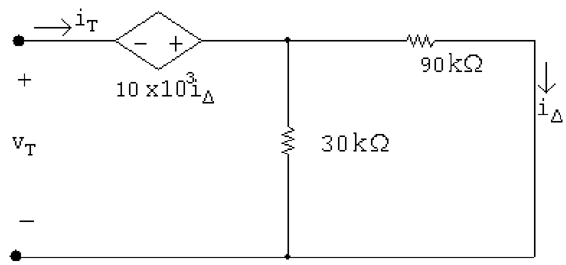
$$e^{-250t} = 0.4 \quad \therefore \quad t = 3.67\,\text{ms}$$

P 7.61 For $t < 0$, $v_o(0) = 90\,\text{V}$

$t > 0$:



$$v_{\text{Th}} = -10 \times 10^3 i_\Delta + (30/120)(300) = -10 \times 10^3 \left(\frac{-300}{120 \times 10^3}\right) + 75 = 100\,\text{V}$$

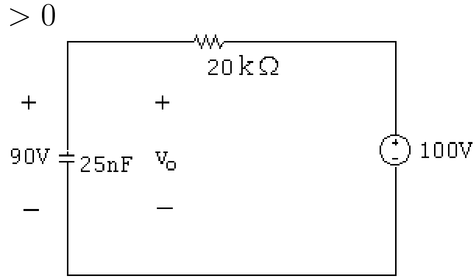


$$v_T = -10 \times 10^3 i_\Delta + 22.5 \times 10^3 i_T = -10 \times 10^3 (30/120) i_T + 22.5 \times 10^3 i_T$$

$$= 20 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\text{k}\Omega$$

$t > 0$



$$v_o = 100 + (90 - 100)e^{-t/\tau}$$

$$\tau = RC = (20 \times 10^3)(25 \times 10^{-9}) = 500 \times 10^{-6}; \qquad \frac{1}{\tau} = 2000$$

$$v_o = 100 - 10e^{-2000t}\,\text{V}, \quad t \geq 0$$

P 7.62 From Problem 7.61,

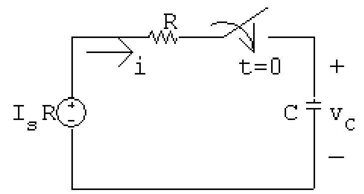
$$v_o(0) = 100\,\text{V}; \qquad v_o(\infty) = 90\,\text{V}$$

$$R_{\text{Th}} = 40\,\text{k}\Omega$$

$$\tau = (40)(25 \times 10^{-6}) = 10^{-3}; \qquad \frac{1}{\tau} = 1000$$

$$v = 90 + (100 - 90)e^{-1000t} = 90 + 10e^{-1000t}\,\text{V}, \quad t \geq 0$$

P 7.63 **[a]**



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\dfrac{di}{dt} = -\dfrac{i}{RC}; \qquad \dfrac{di}{i} = -\dfrac{dt}{RC}$

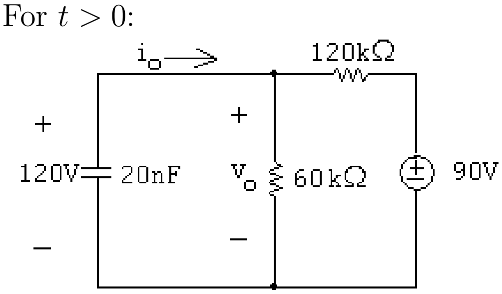
$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.64 **[a]** For $t > 0$:



$$v(\infty) = \frac{60}{180}(90) = 30\,\text{V}$$

$$R_{eq} = 60\,\text{k}\|120\,\text{k} = 40\,\text{k}\Omega$$

$$\tau = R_{eq}C = (40\times 10^3)(20\times 10^{-9}) = 0.8\,\text{ms}; \qquad \frac{1}{\tau} = 1250$$

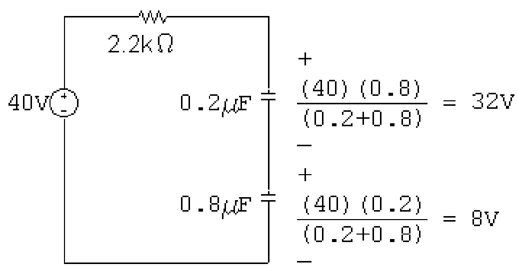
$$v_o = 30 + (120-30)e^{-1250t} = 30 + 90e^{-1250t}\,\text{V}, \qquad t \geq 0^+$$

[b] $$i_o = \frac{v_o}{60{,}000}\frac{v_o}{-}90120{,}000 = \frac{30+90e^{-1250t}}{60{,}000} + \frac{30+90e^{-1250t}-90}{120{,}000}$$

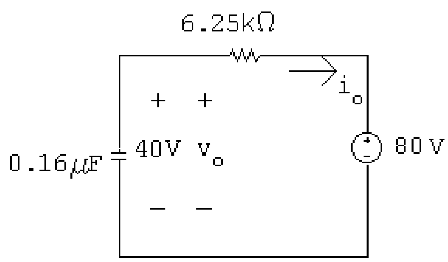
$$= 2.25e^{-1250t}\,\text{mA}$$

$$v_1 = \frac{1}{60\times 10^{-9}} \times 2.25\times 10^{-3}\int_0^t e^{-1250x}\,dx = -30e^{-1250t} + 30\,\text{V}, \quad t \geq 0$$

P 7.65 **[a]** $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16\times 10^{-6})(6.25\times 10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $$i_o = -C\frac{dv_o}{dt} = -0.16\times 10^{-6}[40{,}000e^{-1000t}]$$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $$v_1 = \frac{-1}{0.2\times 10^{-6}}\int_0^t -6.4\times 10^{-3}e^{-1000x}\,dx + 32$$

$$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[d] $v_2 = \dfrac{-1}{0.8 \times 10^{-6}} \displaystyle\int_0^t -6.4 \times 10^{-3} e^{-1000x}\, dx + 8$

$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$

[e] $w_{\text{trapped}} = \dfrac{1}{2}(0.2 \times 10^{-6})(64)^2 + \dfrac{1}{2}(0.8 \times 10^{-6})(16)^2 = 512\,\mu\text{J}.$

P 7.66 **[a]** Let i be the current in the clockwise direction around the circuit. Then

$$V_g = iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx$$

$$= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $\quad i = \dfrac{V_g}{R_g}e^{-t/R_gC_e} = \dfrac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\left.\frac{e^{-x/\tau}}{-1/\tau}\right|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

$$v_2(t) = \frac{V_gC_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

[b] $v_1(\infty) = \dfrac{C_2}{C_1 + C_2}V_g; \qquad v_2(\infty) = \dfrac{C_1}{C_1 + C_2}V_g$

P 7.67 **[a]** $L_{\text{eq}} = \dfrac{(3)(15)}{3 + 15} = 2.5\,\text{H}$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore \quad i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$\therefore \quad \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40 \text{ Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40 \text{ Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40 \text{ Wb-turns}$$

$\therefore \quad i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.68 [a] From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.125 - 0.0625}{0.75 + 0.5} = 50\,\text{mH}$$

$$\tau = \frac{L}{R} = \frac{1}{5000}; \qquad \frac{1}{\tau} = 5000$$

$$\therefore \quad i_o(t) = 40 - 40e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[b] $v_o = 10 - 250i_o = 10 - 250(0.04 + 0.04e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 0.5\dfrac{di_1}{dt} - 0.25\dfrac{di_2}{dt} = 10e^{-5000t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 200e^{-5000t} \text{ A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 200e^{-5000t} - \frac{di_1}{dt}$$

$$\therefore \quad 10e^{-5000t} = 0.5\frac{di_1}{dt} - 50e^{-5000t} + 0.25\frac{di_1}{dt}$$

$$\therefore \quad 0.75\frac{di_1}{dt} = 60e^{-5000t}; \qquad di_1 = 80e^{-5000t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 80e^{-5000y}\,dy$$

$$i_1 = \frac{80}{-5000}e^{-5000y}\Big|_0^t = 16 - 16e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 40 - 40e^{-5000t} - 16 + 16e^{-5000t}$

$$= 24 - 24e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = (0.05)(200)e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 0.5\frac{di_1}{dt} - 0.25\frac{di_2}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 0.25\frac{di_2}{dt} - 0.25\frac{di_1}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 10\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

$i_o(\infty) = 40\,\text{mA}; \qquad i_o(\infty)L_{\text{eq}} = (0.04)(0.05) = 2$ mWb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (16\,\text{m})(500) + (24\,\text{m})(-250) = 2$ mWb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (24\,\text{m})(250) + (16\,\text{m})(-250) = 2$ mWb-turns (ok)

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.69 **[a]** $L_{\text{eq}} = 0.02 + 0.04 + 2(0.015) = 0.09 = 90\,\text{mH}$

$$\tau = \frac{L}{R} = \frac{0.09}{4500} = 20\,\mu\text{s}; \qquad \frac{1}{\tau} = 50{,}000$$

$$i = 20 - 20e^{-50{,}000t}\,\text{mA}, \quad t \geq 0$$

[b] $v_1(t) = 0.02\frac{di}{dt} + 0.015\frac{di}{dt} = 0.035\frac{di}{dt} = 0.035(1000e^{-50{,}000t}) = 35e^{-50{,}000t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 0.04\frac{di}{dt} + 0.015\frac{di}{dt} = 0.055\frac{di}{dt} = 0.055(1000e^{-50{,}000t}) = 55e^{-50{,}000t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 0.02 - 0.02 = 0$, which agrees with initial conditions.

$$90 = 4500i + v_1 + v_2 = 4500(0.02 - 0.02e^{-50{,}000t}) + 35e^{-50{,}000t} + 55e^{-50{,}000t} = 90\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.70 **[a]** $L_{\text{eq}} = 0.02 + 0.04 - 2(0.015) = 0.03 = 30\,\text{mH}$

$$\tau = \frac{L}{R} = \frac{0.03}{4500} = 6.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 150{,}000$$

$$i = 0.02 - 0.02e^{-150{,}000t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 0.02\dfrac{di}{dt} - 0.015\dfrac{di}{dt} = 0.005\dfrac{di}{dt} = 0.005(3000e^{-150{,}000t})$

$$= 15e^{-150{,}000t}\,\text{V}, \quad t \geq 0^+$$

[c] $v_2(t) = 0.04\dfrac{di}{dt} - 0.015\dfrac{di}{dt} = 0.025\dfrac{di}{dt} = 0.025(3000e^{-150{,}000t})$

$$= 75e^{-150{,}000t}\,\text{V}, \quad t \geq 0^+$$

[d] $i(0) = 0$, which agrees with initial conditions.

$$90 = 4500i_1 + v_1 + v_2 = 4500(0.02 - 0.02e^{-150{,}000t}) + 15e^{-150{,}000t}$$
$$+ 75e^{-150{,}000t} = 90\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.71 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{50 - 25}{15 + 10} = 1\,\text{H}$$

$$\tau = \frac{L}{R} = \frac{1}{20}; \qquad \frac{1}{\tau} = 20$$

$$\therefore \quad i_o(t) = 4 - 4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o = 80 - 20i_o = 80 - 80 + 80e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 5\dfrac{di_1}{dt} - 5\dfrac{di_2}{dt} = 80e^{-20t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 80e^{-20t}\ \text{A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 80e^{-20t} - \frac{di_1}{dt}$$

$$\therefore \quad 80e^{-20t} = 5\frac{di_1}{dt} - 400e^{-20t} + 5\frac{di_1}{dt}$$

$$\therefore \quad 10\frac{di_1}{dt} = 480e^{-20t}; \qquad di_1 = 48e^{-20t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 48e^{-20y}\,dy$$

$$i_1 = \frac{48}{-20}e^{-20y}\Big|_0^t = 2.4 - 2.4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 4 - 4e^{-20t} - 2.4 + 2.4e^{-20t}$

$$= 1.6 - 1.6e^{-20t}\,\text{A}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = 1(80)e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 10\frac{di_2}{dt} - 5\frac{di_1}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 80\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

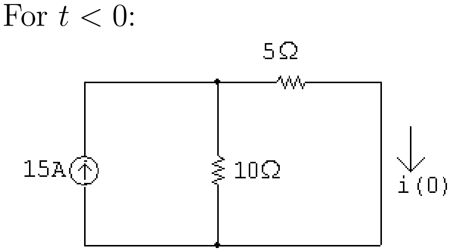
$i_o(\infty) = 4\,\text{A}; \qquad i_o(\infty)L_{\text{eq}} = (4)(1) = 4$ Wb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (2.4)(5) + (1.6)(-5) = 4$ Wb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (1.6)(10) + (2.4)(-5) = 4$ Wb-turns (ok)

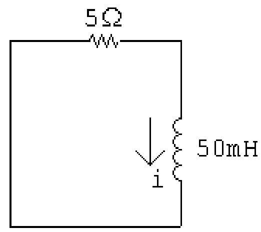
Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.72 For $t < 0$:



$$i(0) = \frac{10}{15}(15) = 10\,\text{A}$$

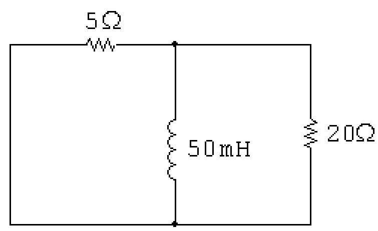
$0 \le t \le 10\,\text{ms}$:



$$i = 10e^{-100t}\,\text{A}$$

$$i(10\,\text{ms}) = 10e^{-1} = 3.68\,\text{A}$$

$10\,\text{ms} \le t \le 20\,\text{ms}$:



$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$$i = 3.68e^{-80(t-0.01)}\,\text{A}$$

$20\,\text{ms} \le t < \infty$:

$$i(20\,\text{ms}) = 3.68e^{-80(0.02-0.01)} = 1.65\,\text{A}$$

$$i = 1.65e^{-100(t-0.02)}\,\text{A}$$

$$v_o = L\frac{di}{dt}; \qquad L = 50\,\text{mH}$$

$$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$$

$$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$$

$$= -8.26e^{-100(t-0.02)}\,\text{V}, \qquad t > 20^{+}\,\text{ms}$$

$$v_o(25\,\text{ms}) = -8.26e^{-100(0.025-0.02)} = -5.013\,\text{V}$$

P 7.73 From the solution to Problem 7.72, the initial energy is

$$w(0) = \frac{1}{2}(50\,\text{mH})(10\,\text{A})^2 = 2.5\,\text{J}$$

$$0.04w(0) = 0.1\,\text{J}$$

$$\therefore \quad \frac{1}{2}(50 \times 10^{-3})i_L^2 = 0.1 \quad \text{so} \quad i_L = 2\,\text{A}$$

Again, from the solution to Problem 7.73, t must be between 10 ms and 20 ms since

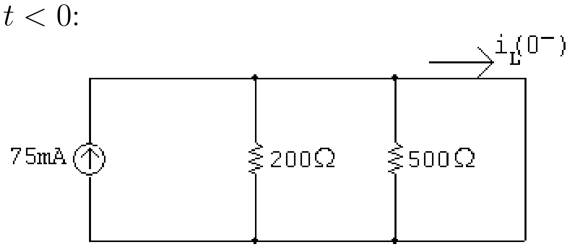
$$i(10\,\text{ms}) = 3.68\,\text{A} \quad \text{and} \quad i(20\,\text{ms}) = 1.65\,\text{A}$$

For $10\,\text{ms} \le t \le 20\,\text{ms}$:

$$i = 3.68e^{-80(t-0.01)} = 2$$

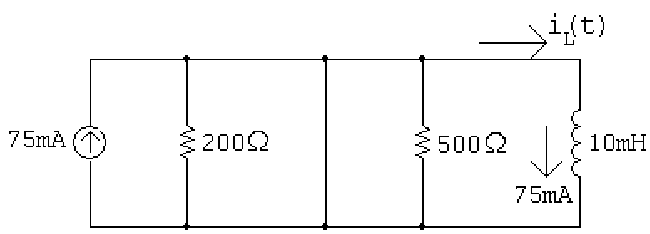
$$e^{80(t-0.01)} = \frac{3.68}{2} \quad \text{so} \quad t - 0.01 = 0.0076 \quad \therefore \quad t = 17.6\,\text{ms}$$

P 7.74 $t < 0$:



$$i_L(0^-) = 75\,\text{mA} = i_L(0^+)$$

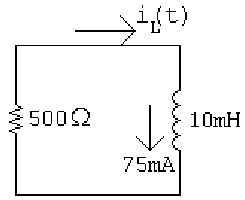
$0 \le t \le 25\,\text{ms}$:



$$\tau = 0.01/0 = \infty$$

$$i_L(t) = 0.075e^{-t/\infty} = 0.075e^{-0} = 75\,\text{mA}$$

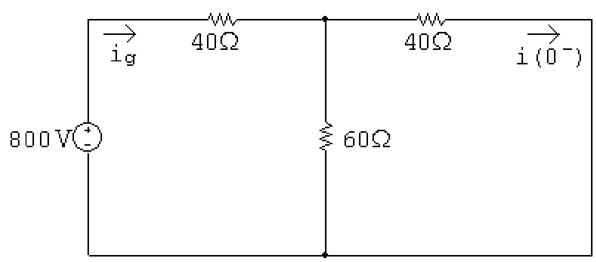
$25\,\text{ms} \le t$:



$$\tau = \frac{0.01}{500} = 20\,\mu\text{s}; \qquad 1/\tau = 50{,}000$$

$$i_L(t) = 75e^{-50{,}000(t-0.025)}\,\text{mA}, \quad t \ge 25\,\text{ms}$$

P 7.75 **[a]** $t < 0$:



Using Ohm's law,

$$i_g = \frac{800}{40 + 60\|40} = 12.5\,\text{A}$$

Using current division,

$$i(0^-) = \frac{60}{60+40}(12.5) = 7.5\,\text{A} = i(0^+)$$

[b] $0 \le t \le 1\,\text{ms}$:

$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

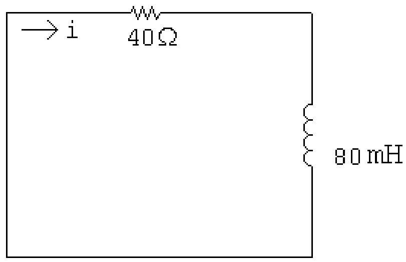
$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120\|60}{80 \times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200\times10^{-6})} = 7.5e^{-0.2} = 6.14\,\text{A}$$

[c] $i(1\,\text{ms}) = 7.5e^{-1} = 2.7591\,\text{A}$

$1\,\text{ms} \le t$:



$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1\,\text{ms})e^{-(t-1\,ms)/\tau} = 2.7591e^{-500(t-0.001)}\,\text{A}$$

$$i(6\text{ms}) = 2.7591e^{-500(0.005)} = 2.7591e^{-2.5} = 226.48\,\text{mA}$$

[d] $0 \le t \le 1\,\text{ms}$:

$$i = 7.5e^{-1000t}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5e^{-1000t}) = -600e^{-1000t}\,\text{V}$$

$$v(1^{-}\text{ms}) = -600e^{-1} = -220.73\,\text{V}$$

[e] $1\,\text{ms} \le t \le \infty$:

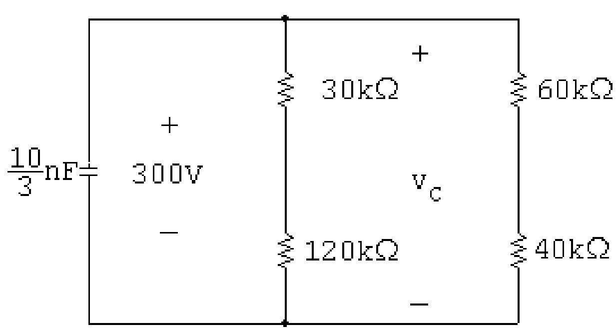
$$i = 2.759e^{-500(t-0.001)}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-500)(2.759e^{-500(t-0.001)})$$

$$= -110.4e^{-500(t-0.001)}\,\text{V}$$

$$v(1^{+}\text{ms}) = -110.4\,\text{V}$$

P 7.76 $0 \le t \le 200\,\mu\text{s}$;

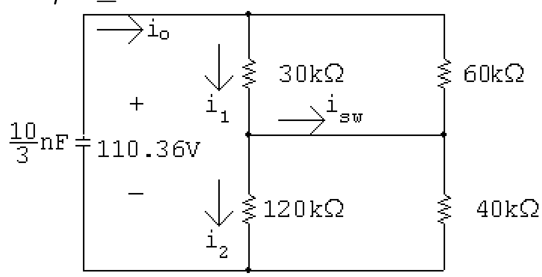


$$R_e = 150\|100 = 60\,\text{k}\Omega; \qquad \tau = \left(\frac{10}{3} \times 10^{-9}\right)(60{,}000) = 200\,\mu\text{s}$$

$v_c = 300e^{-5000t}\,\text{V}$

$v_c(200\,\mu\text{s}) = 300e^{-1} = 110.36\,\text{V}$

$200\,\mu\text{s} \le t < \infty$:



$$R_e = 30\|60 + 120\|40 = 20 + 30 = 50\,\text{k}\Omega$$

$$\tau = \left(\frac{10}{3} \times 10^{-9}\right)(50{,}000) = 166.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 6000$$

$$v_c = 110.36e^{-6000(t\,-\,200\,\mu s)}\,\text{V}$$

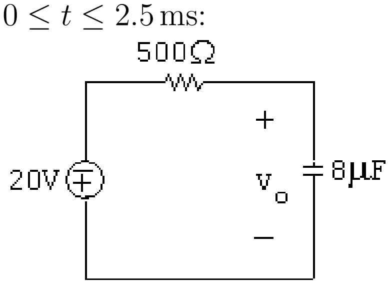
$$v_c(300\,\mu\text{s}) = 110.36e^{-6000(100\,\mu\text{S})} = 60.57\,\text{V}$$

$$i_o(300\,\mu\text{s}) = \frac{60.57}{50{,}000} = 1.21\,\text{mA}$$

$$i_1 = \frac{60}{90}i_o = \frac{2}{3}i_o; \qquad i_2 = \frac{40}{160}i_o = \frac{1}{4}i_o$$

$$i_{\text{sw}} = i_1 - i_2 = \frac{2}{3}i_o - \frac{1}{4}i_o = \frac{5}{12}i_o = \frac{5}{12}(1.21 \times 10^{-3}) = 0.50\,\text{mA}$$

P 7.77 $\quad 0 \le t \le 2.5\,\text{ms}$:

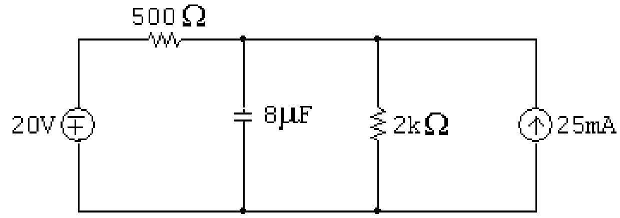


$$\tau = RC = (500)(8 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$$

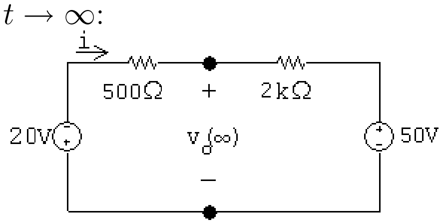
$$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = -20\,\text{V}$$

$v_o = -20 + 20e^{-250t}\,\text{V} \qquad 0 \le t \le 2.5\,\text{ms}$

$2.5\,\text{ms} \le t$:



$t \to \infty$:



$$i = \frac{-70\,\text{V}}{2.5\,\text{k}\Omega} = -28\,\text{mA}$$

$$v_o(\infty) = (-28 \times 10^{-3})(2000) + 50 = -6\,\text{V}$$

$$v_o(0.0025) = -20 + 20e^{-0.625} = -9.29\,\text{V}$$

$$v_o = -6 + (-9.29 + 6)e^{-(t-0.0025)/\tau}$$

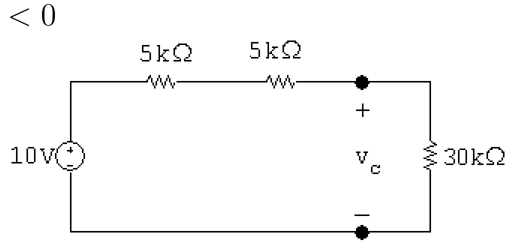
$$R_{\text{Th}} = 2000\|500 = 400\,\Omega$$

$$\tau = (400)(8 \times 10^{-6}) = 3.2\,\text{ms}; \qquad 1/\tau = 312.5$$

$$v_o = -6 - 3.29e^{-312.5(t-0.0025)} \qquad 2.5\,\text{ms} \le t$$

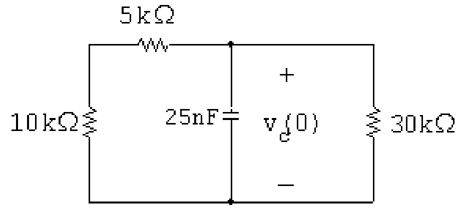
P 7.78 Note that for $t > 0$, $v_o = (10/15)v_c$, where v_c is the voltage across the 25 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{30}{40}(10) = 7.5\,\text{V}$$

$0 \leq t \leq 0.2\,\text{ms}$:



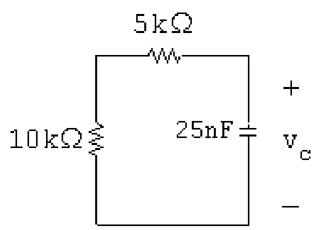
$$\tau = R_e C, \qquad R_e = 15{,}000 \| 30{,}000 = 10\,\text{k}\Omega$$

$$\tau = (10 \times 10^3)(25 \times 10^{-9}) = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$$

$$v_\text{c} = 7.5e^{-4000t}\,\text{V}, \qquad t \geq 0$$

$$v_\text{c}(0.2\,\text{ms}) = 7.5e^{-0.8} = 3.37\,\text{V}$$

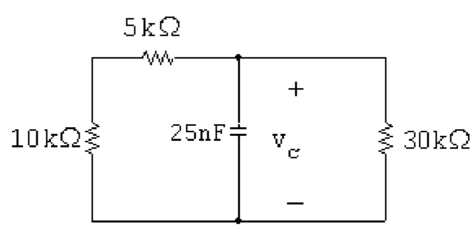
$0.2\,\text{ms} \leq t \leq 0.8\,\text{ms}$:



$$\tau = (15 \times 10^3)(2.5 \times 10^{-9}) = 375\,\mu\text{s}, \qquad \frac{1}{\tau} = 2666.67$$

$$v_\text{c} = 3.37e^{-2666.67(t - 200 \times 10^{-6})}\,\text{V}$$

$0.8\,\text{ms} \leq t <$:



$$\tau = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$$

$$v_\text{c}(0.8\,\text{ms}) = 3.37e^{-2666.67(800-200)10^{-6}} = 3.37e^{-1.6} = 0.68\,\text{V}$$

$$v_\text{c} = 0.68e^{-4000(t - 0.8 \times 10^{-3})}\,\text{V}$$

$$v_\text{c}(1\,\text{ms}) = 0.68e^{-4000(1-0.8)10^{-3}} = 0.68e^{-0.8} = 0.306\,\text{V}$$

$$v_o = (10/15)(0.306) = 0.204\,\text{V}$$

P 7.79 $w(0) = \frac{1}{2}(25 \times 10^{-9})(7.5)^2 = 703.125\,\text{nJ}$

$0 \le t \le 200\,\mu\text{s}$:

$$v_c = 7.5e^{-4000t}; \qquad v_c^2 = 56.25e^{-8000t}$$

$$p_{30\text{k}} = 1.875e^{-8000t}\,\text{mW}$$

$$w_{30\text{k}} = \int_0^{200\times10^{-6}} 1.875 \times 10^{-3} e^{-8000t}\,dt$$
$$= 1.875 \times 10^{-3} \frac{e^{-8000t}}{-8000}\Bigg|_0^{200\times10^{-6}}$$
$$= -234.375 \times 10^{-9}(e^{-1.6} - 1) = 187.1\,\text{nJ}$$

$0.8\,\text{ms} \le t$:

$$v_c = 0.68e^{-4000(t-0.8\times10^{-3})}\,\text{V}; \qquad v_c^2 = 0.46e^{-8000(t-0.8\times10^{-3})}$$

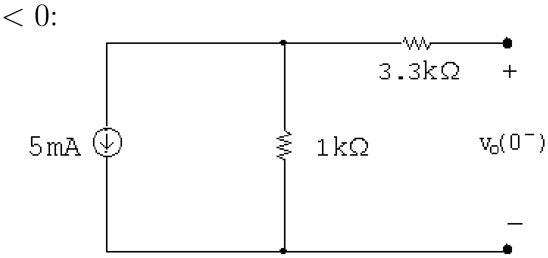
$$p_{30\text{k}} = 15.33e^{-8000(t-0.8\times10^{-3})}\,\mu\text{W}$$

$$w_{30\text{k}} = \int_{0.8\times10^{-3}}^{\infty} 15.33 \times 10^{-6} e^{-8000(t-0.8\times10^{-3})}\,dt$$
$$= 15.33 \times 10^{-6} \frac{e^{-8000(t-0.8\times10^{-3})}}{-8000}\Bigg|_{0.8\times10^{-3}}^{\infty}$$
$$= -1.9 \times 10^{-9}(0 - 1) = 1.9\,\text{nJ}$$

$$w_{30\text{k}} = 187.1 + 1.9 = 189\,\text{nJ}$$

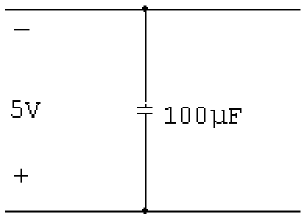
$$\% = \frac{189}{703.125}(100) = 26.88\%$$

P 7.80 $t < 0$:



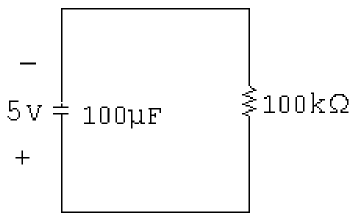
$$v_c(0^-) = -(5)(1000) \times 10^{-3} = -5\,\text{V} = v_c(0^+)$$

$0 \le t \le 5\,\text{s}$:



$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = -5e^{-0} = -5\,\text{V}$

$5\,\text{s} \le t < \infty$:



$\tau = (100)(0.1) = 10\,\text{s}; \qquad 1/\tau = 0.1; \qquad v_o = -5e^{-0.1(t-5)}\,\text{V}$

Summary:

$v_o = -5\,\text{V}, \qquad 0 \le t \le 5\,\text{s}$

$v_o = -5e^{-0.1(t-5)}\,\text{V}, \qquad 5\,\text{s} \le t < \infty$

P 7.81 **[a]** $i_o(0) = 0; \qquad i_o(\infty) = 50\,\text{mA}$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{3000}{75} \times 10^3 = 40{,}000$$

$$i_o = (50 - 50e^{-40{,}000t})\,\text{mA}, \qquad 0 \le t \le 25\,\mu\text{s}$$

$$v_o = 0.075\frac{di_o}{dt} = 150e^{-40{,}000t}\,\text{V}, \qquad 0 \le t \le 25\,\mu s$$

$25\,\mu\text{s} \le t$:

$$i_o(25\mu\text{s}) = 50 - 50e^{-1} = 31.6\,\text{mA}; \qquad i_o(\infty) = 0$$

$$i_o = 31.6e^{-40{,}000(t-25\times10^{-6})}\,\text{mA}$$

$$v_o = 0.075\frac{di_o}{dt} = -94.82e^{-40{,}000(t-25\mu s)}$$

$$\begin{aligned} \therefore\quad t < 0: &\qquad v_o = 0 \\ 0 \le t \le 25\,\mu\text{s}: &\qquad v_o = 150e^{-40{,}000t}\,\text{V} \\ 25\,\mu\text{s} \le t: &\qquad v_o = -94.82e^{-40{,}000(t-25\mu s)}\,\text{V} \end{aligned}$$

[b] $v_o(25^- \mu s) = 150e^{-1} = 55.18\,\text{V}$

$v_o(25^+ \mu s) = -94.82\,\text{V}$

[c] $i_o(25^- \mu s) = i_o(25^+ \mu s) = 31.6\,\text{mA}$

P 7.82 **[a]** $0 \leq t \leq 2.5\,\text{ms}$

$v_o(0^+) = 80\,\text{V}; \qquad v_o(\infty) = 0$

$$\tau = \frac{L}{R} = 2\,\text{ms}; \qquad 1/\tau = 500$$

$v_o(t) = 80e^{-500t}\,\text{V}, \qquad 0^+ \leq t \leq 2.5^-\,\text{ms}$

$v_o(2.5^-\,\text{ms}) = 80e^{-1.25} = 22.92\,\text{V}$

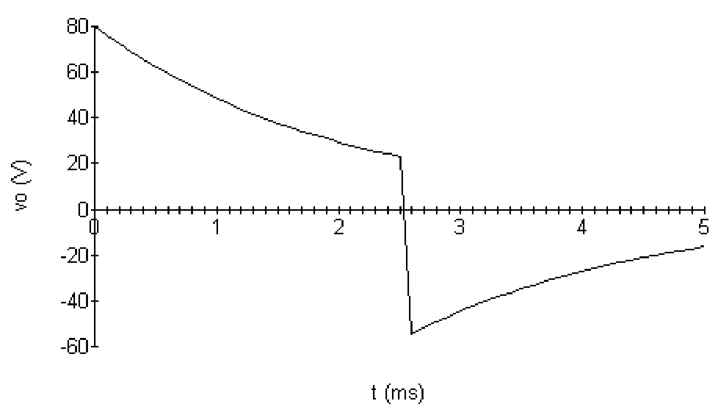
$$i_o(2.5^-\,\text{ms}) = \frac{(80 - 22.92)}{20} = 2.85\,\text{A}$$

$v_o(2.5^+\,\text{ms}) = -20(2.85) = -57.08\,\text{V}$

$v_o(\infty) = 0; \qquad \tau = 2\,\text{ms}; \qquad 1/\tau = 500$

$v_o = -57.08e^{-500(t-0.0025)}\,\text{V} \qquad t \geq 2.5^+\,\text{ms}$

[b]



[c] $v_o(5\,\text{ms}) = -16.35\,\text{V}$

$$i_o = \frac{+16.35}{20} = 817.68\,\text{mA}$$

P 7.83 **[a]** $t < 0; \qquad v_o = 0$

$0 \leq t \leq 25\,\mu\text{s}$:

$\tau = (4000)(50 \times 10^{-9}) = 0.2\,\text{ms}; \qquad 1/\tau = 5000$

$v_o = 10 - 10e^{-5000t}\,\text{V}, \qquad 0 \leq t \leq 25\,\mu\text{s}$

$v_o(25\,\mu\text{s}) = 10(1 - e^{-0.125}) = 1.175\,\text{V}$

$25\,\mu\text{s} \leq t \leq 50\,\mu\text{s}$:

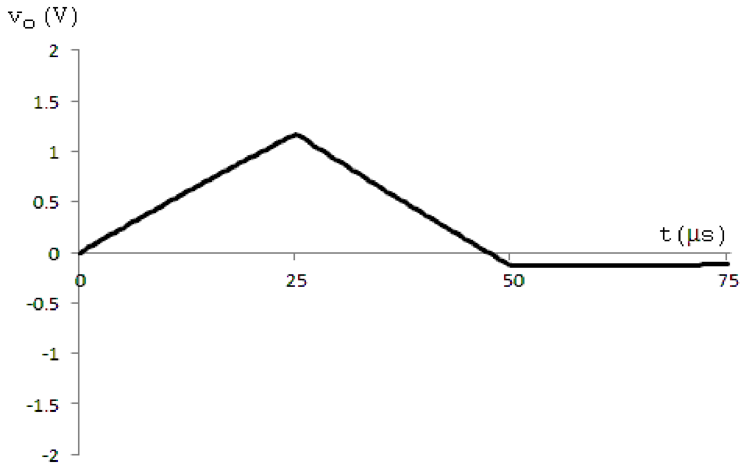
$v_o = -10 + 11.175e^{-5000(t-25\times10^{-6})}\,\text{V}, \quad 25\,\mu\text{s} \leq t \leq 50\,\mu\text{s}$

$v_o(50\,\mu\text{s}) = -10 + 11.175e^{-0.125} = -0.138\,\text{V}$

$t \geq 50\,\mu\text{s}$:

$v_o = -0.138e^{-5000(t-50\times10^{-6})}\,\text{V}, \quad t \geq 50\,\mu\text{s}$

[b]



[c] $t \leq 0: \qquad v_o = 0$

$0 \leq t \leq 25\,\mu\text{s}$:

$\tau = (800)(50 \times 10^{-9}) = 40\,\mu\text{s} \quad 1/\tau = 25{,}000$

$v_o = 10 - 10e^{-25{,}000t}\,\text{V}, \qquad 0 \leq t \leq 25\,\mu\text{s}$

$v_o(25\,\mu\text{s}) = 10 - 10e^{-0.625} = 4.65\,\text{V}$

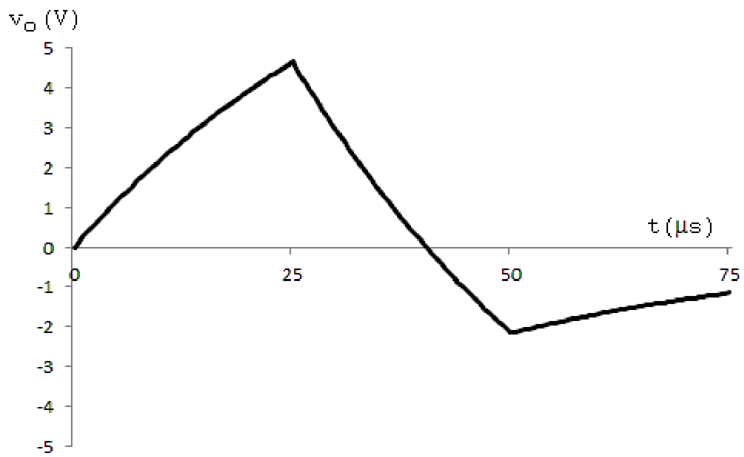
$25\,\mu\text{s} \leq t \leq 50\,\mu\text{s}$:

$v_o = -10 + 14.65e^{-25{,}000(t-25\times10^{-6})}\,\text{V}, \qquad 25\,\mu\text{s} \leq t \leq 50\,\mu\text{s}$

$v_o(50\,\mu\text{s}) = -10 + 14.65e^{-0.625} = -2.16\,\text{V}$

$t \geq 50\,\mu\text{s}$:

$v_o = -2.16e^{-25{,}000(t-50\times10^{-6})}\,\text{V}, \quad t \geq 50\,\mu\text{s}$



P 7.84 **[a]** $0 \leq t \leq 1\,\text{ms}$:

$$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$$

$$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$$

$$v_c = 50 - 50e^{-250t}$$

$$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \leq t \leq 1\,\text{ms}$$

$1\,\text{ms} \leq t < \infty$:

$$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$$

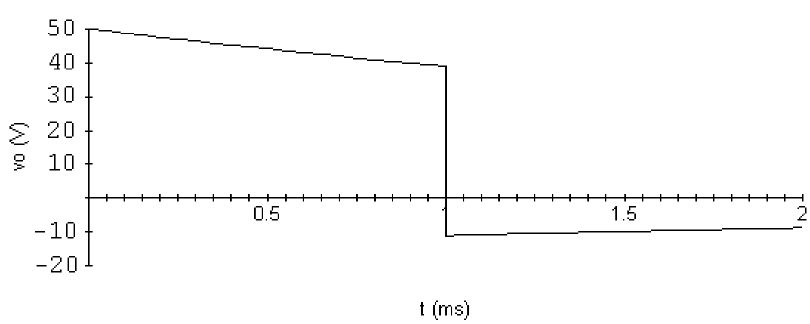
$$v_c(\infty) = 0\,\text{V}$$

$$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$$

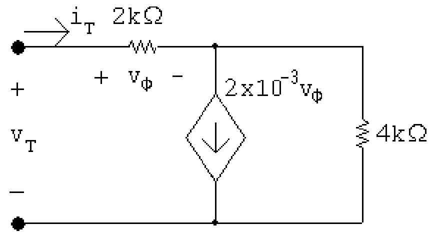
$$v_c = 11.06e^{-250(t-0.001)}\,\text{V}$$

$$v_o = -v_c = -11.06e^{-250(t-0.001)}\,\text{V}, \qquad t \geq 1\,\text{ms}$$

[b]



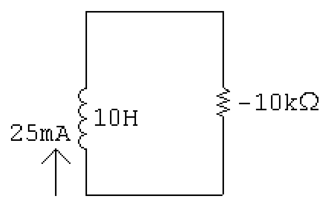
P 7.85



$$v_T = 2000i_T + 4000(i_T - 2 \times 10^{-3}v_\phi) = 6000i_T - 8v_\phi$$

$$= 6000i_T - 8(2000i_T)$$

$$\frac{v_T}{i_T} = -10{,}000$$

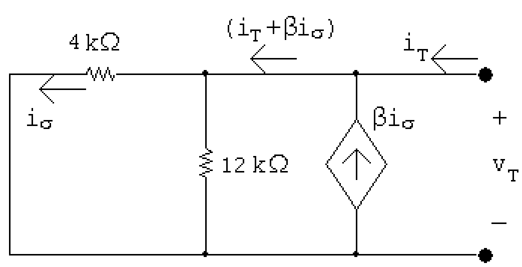


$$\tau = \frac{10}{-10{,}000} = -1\,\text{ms}; \qquad 1/\tau = -1000$$

$$i = 25e^{1000t}\,\text{mA}$$

$$\therefore \quad 25e^{1000t} \times 10^{-3} = 5; \qquad t = \frac{\ln 200}{1000} = 5.3\,\text{ms}$$

P 7.86 **[a]**



Using Ohm's law,

$$v_T = 4000i_\sigma$$

Using current division,

$$i_\sigma = \frac{12{,}000}{12{,}000 + 4000}(i_T + \beta i_\sigma) = 0.75i_T + 0.75\beta i_\sigma$$

Solve for i_σ:

$$i_\sigma(1 - 0.75\beta) = 0.75 i_T$$

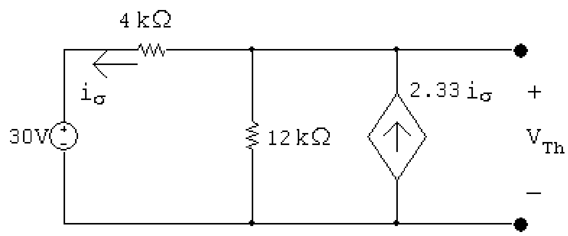
$$i_\sigma = \frac{0.75 i_T}{1 - 0.75\beta}; \qquad v_T = 4000 i_\sigma = \frac{3000 i_T}{(1 - 0.75\beta)}$$

Find β such that $R_{\text{Th}} = -4\,\text{k}\Omega$:

$$R_{\text{Th}} = \frac{v_T}{i_T} = \frac{3000}{1 - 0.75\beta} = -4000$$

$$1 - 0.75\beta = -0.75 \qquad \therefore \quad \beta = 2.33$$

[b] Find V_{Th};



Write a KCL equation at the top node:

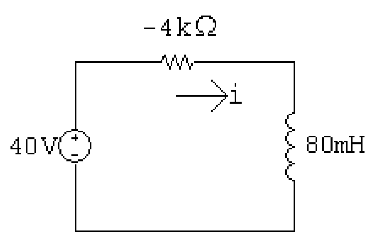
$$\frac{V_{\text{Th}} - 30}{4000} + \frac{V_{\text{Th}}}{12{,}000} - 2.33 i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{\text{Th}} - 30)}{4000} = 0$$

Solving,

$$V_{\text{Th}} = 40\,\text{V}$$



Write a KVL equation around the loop:

$$40 = -4000i + 0.08\frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 500 + 50{,}000i = 25{,}000(i + 0.01)$$

Separate the variables and integrate to find i;

$$\frac{di}{i + 0.01} = 50{,}000\,dt$$

$$\int_0^i \frac{dx}{x+0.01} = \int_0^t 50{,}000\,dx$$

$$\therefore \quad i = -10 + 10e^{50{,}000t}\,\text{mA}$$

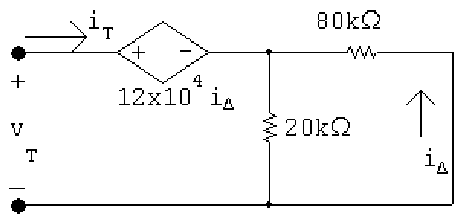
$$\frac{di}{dt} = (10\times 10^{-3})(50{,}000)e^{50{,}000t} = 500e^{50{,}000t}$$

Solve for the arc time:

$$v = 0.08\frac{di}{dt} = 40e^{50{,}000t} = 30{,}000; \qquad e^{50{,}000t} = 750$$

$$\therefore \quad t = \frac{\ln 750}{50{,}000} = 132.4\,\mu\text{s}$$

P 7.87 $t > 0$:



$$v_T = 12\times 10^4 i_\Delta + 16\times 10^3 i_T$$

$$i_\Delta = -\frac{20}{100}i_T = -0.2i_T$$

$$\therefore \quad v_T = -24\times 10^3 i_T + 16\times 10^3 i_T$$

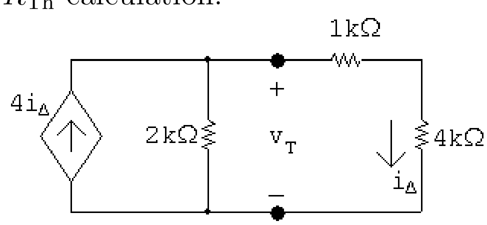
$$R_{\text{Th}} = \frac{v_T}{i_T} = -8\,\text{k}\Omega$$

$$\tau = RC = (-8\times 10^3)(2.5\times 10^{-6}) = -0.02 \quad 1/\tau = -50$$

$$v_c = 20e^{50t}\,\text{V}; \qquad 20e^{50t} = 20{,}000$$

$$50t = \ln 1000 \quad \therefore \quad t = 138.16\,\text{ms}$$

P 7.88 Find the Thévenin equivalent with respect to the terminals of the capacitor. R_{Th} calculation:

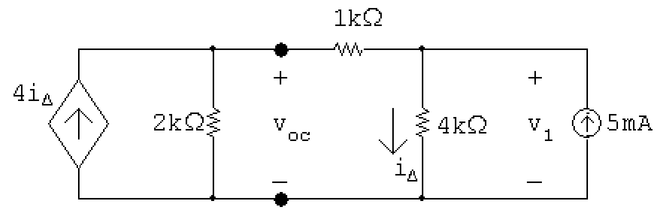


$$i_T = \frac{v_T}{2000} + \frac{v_T}{5000} - 4\frac{v_T}{5000}$$

$$\therefore \quad \frac{i_T}{v_T} = \frac{5+2-8}{10{,}000} = -\frac{1}{10{,}000}$$

$$\frac{v_T}{i_T} = -\frac{10{,}000}{1} = -10\,\text{k}\Omega$$

Open circuit voltage calculation:



The node voltage equations:

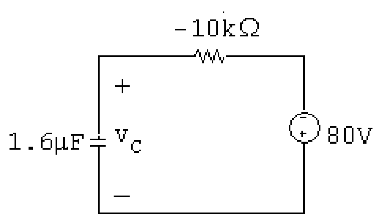
$$\frac{v_{\text{oc}}}{2000} + \frac{v_{\text{oc}} - v_1}{1000} - 4i_\Delta = 0$$

$$\frac{v_1 - v_{\text{oc}}}{1000} + \frac{v_1}{4000} - 5 \times 10^{-3} = 0$$

The constraint equation:

$$i_\Delta = \frac{v_1}{4000}$$

Solving, $v_{\text{oc}} = -80\,\text{V}, \quad v_1 = -60\,\text{V}$



$$v_c(0) = 0; \qquad v_c(\infty) = -80\,\text{V}$$

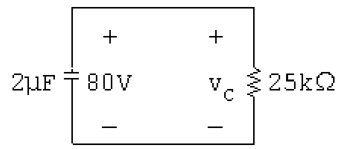
$$\tau = RC = (-10{,}000)(1.6 \times 10^{-6}) = -16\,\text{ms}; \qquad \frac{1}{\tau} = -62.5$$

$$v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = -80 + 80e^{62.5t} = 14{,}400$$

Solve for the time of the maximum voltage rating:

$$e^{62.5t} = 181; \qquad 62.5t = \ln 181; \qquad t = 83.09\,\text{ms}$$

P 7.89 [**a**]



$$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$$

$$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$$

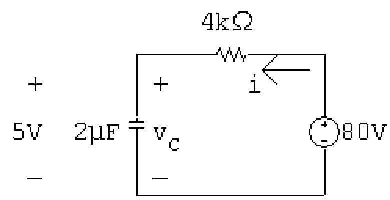
$$v_c = 80e^{-20t}\,\text{V}$$

$$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \frac{\ln 16}{20} = 138.63\,\text{ms}$$

[**b**] $0^+ \leq t \leq 138.63^-$ ms:

$$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$$

$t \geq 138.63^+$ ms:



$$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$$

$$v_c(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad t \geq 138.63\,\text{ms}$$

$$\begin{aligned} i &= 2 \times 10^{-6}(9375)e^{-125(t-0.13863)} \\ &= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad t \geq 138.63^+\,\text{ms} \end{aligned}$$

[**c**] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \qquad \Delta t = \frac{\ln 6.25}{125} \cong 14.66\,\text{ms}$$

P 7.90 $v_o = -\dfrac{1}{R(0.5 \times 10^{-6})}\displaystyle\int_0^t 4\,dx + 0 = \dfrac{-4t}{R(0.5 \times 10^{-6})}$

$$\frac{-4(15 \times 10^{-3})}{R(0.5 \times 10^{-6})} = -10$$

$$\therefore \qquad R = \frac{-4(15 \times 10^{-3})}{-10(0.5 \times 10^{-6})} = 12\,\text{k}\Omega$$

P 7.91 $v_o = \dfrac{-4t}{R(0.5\times 10^{-6})} + 6 = \dfrac{-4(40\times 10^{-3})}{R(0.5\times 10^{-6})} + 6 = -10$

$$\therefore \qquad R = \frac{-4(40\times 10^{-3})}{-16(0.5\times 10^{-6})} = 20\,\text{k}\Omega$$

P 7.92 $RC = (80\times 10^3)(250\times 10^{-9}) = 20\,\text{ms}; \qquad \dfrac{1}{RC} = 50$

[a] $t < 0: \qquad v_o = 0$

[b] $0 \le t \le 2\,\text{s}:$

$$v_o = -50\int_0^t 0.075\,dx = -3.75t\,\text{V}$$

[c] $2\,\text{s} \le t \le 4\,\text{s};$

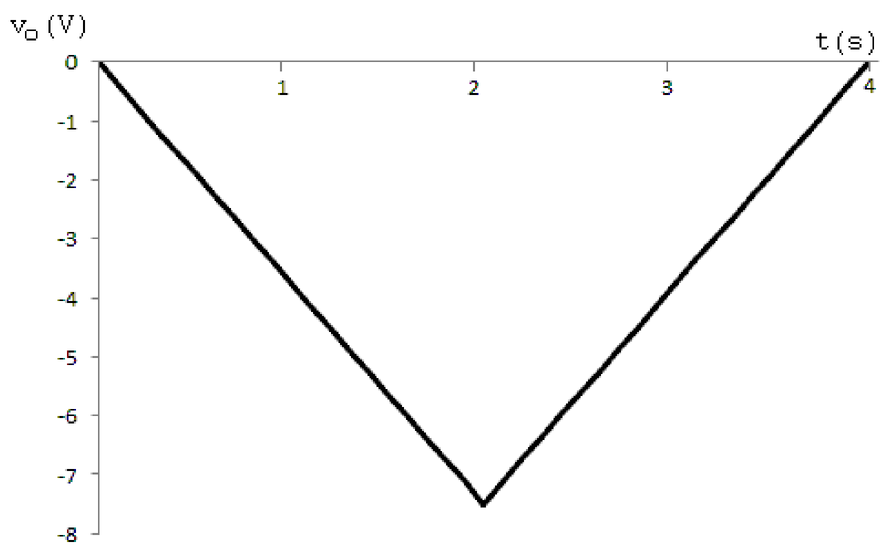
$$v_o(2) = -3.75(2) = -7.5\,\text{V}$$

$$v_o(t) = -50\int_2^t -0.075\,dx - 7.5 = 3.75(t-2) - 7.5 = 3.75t - 15\,\text{V}$$

[d] $t \ge 4\,\text{s}:$

$$v_o(4) = 15 - 15 = 0\,\text{V}$$

$$v_o(t) = 0\,\text{V}$$



P 7.93 Write a KCL equation at the inverting input to the op amp, where the voltage is 0:

$$\frac{0 - v_g}{R_i} + \frac{0 - v_o}{R_f} + C_f\frac{d}{dt}(0 - v_o) = 0$$

$$\therefore \qquad \frac{dv_o}{dt} + \frac{1}{R_fC_f}v_o = -\frac{v_g}{R_iC_f}$$

Note that this first-order differential equation is in the same form as Eq. 7.50 if $I_s = -v_g/R_i$. Therefore, its solution is the same as Eq. 7.51:

$$v_o = \frac{-v_gR_f}{R_i} + \left(V_o - \frac{-v_gR_f}{R_i}\right)e^{-t/R_fC_f}$$

[a] $v_o = 0, \qquad t < 0$

[b] $R_fC_f = (4 \times 10^6)(250 \times 10^{-9}) = 1; \qquad \frac{1}{R_fC_f} = 1$

$$\frac{-v_gR_f}{R_i} = \frac{-(0.075)(4 \times 10^6)}{80{,}000} = -3.75$$

$$V_o = v_o(0) = 0$$

$$\therefore \qquad v_o = -3.75 + (0 + 3.75)e^{-t} = -3.75(1 - e^{-t})\,\text{V}, \qquad 0 \le t \le 2\,\text{s}$$

[c] $\frac{1}{R_fC_f} = 1$

$$\frac{-v_gR_f}{R_i} = \frac{-(-0.075)(4 \times 10^6)}{80{,}000} = 3.75$$

$$V_o = v_o(2) = -3.75(1 - e^{-2}) = -3.24\,\text{V}$$

$$\therefore \qquad v_o = 3.75 + [-3.24 - 3.75]e^{-(t-2)}$$

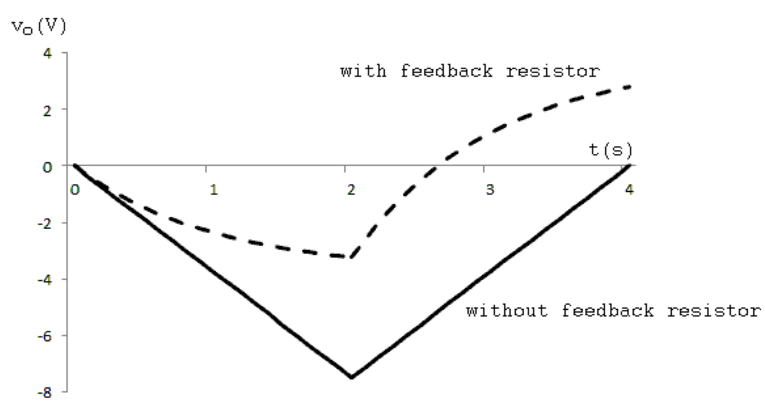
$$= 3.75 - 6.99e^{-(t-2)}\,\text{V}, \quad 2\,\text{s} \le t \le 4\,\text{s}$$

[d] $\frac{1}{R_fC_f} = 1$

$$\frac{-v_gR_f}{R_i} = 0$$

$$V_o = v_o(4) = 3.75 - 6.99e^{-2} = 2.8\,\text{V}$$

$$v_o = 0 + (2.8 - 0)e^{-(t-4)} = 2.8e^{-(t-4)}\,\text{V}, \qquad 4\,\text{s} \le t$$



P 7.94 [a] $\dfrac{C dv_p}{dt} + \dfrac{v_p - v_b}{R} = 0;$ therefore $\dfrac{dv_p}{dt} + \dfrac{1}{RC}v_p = \dfrac{v_b}{RC}$

$$\frac{v_n - v_a}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

therefore $\dfrac{dv_o}{dt} = \dfrac{dv_n}{dt} + \dfrac{v_n}{RC} - \dfrac{v_a}{RC}$

But $v_n = v_p$

Therefore $\dfrac{dv_n}{dt} + \dfrac{v_n}{RC} = \dfrac{dv_p}{dt} + \dfrac{v_p}{RC} = \dfrac{v_b}{RC}$

Therefore $\dfrac{dv_o}{dt} = \dfrac{1}{RC}(v_b - v_a);$ $v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_b - v_a)\,dy$

[b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

[c] $v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_b - v_a)\,dx$

$$RC = (50 \times 10^3)(10 \times 10^{-9}) = 0.5\,\text{ms}$$

$$v_b - v_a = -25\,\text{mV}$$

$$v_o = \frac{1}{0.0005}\int_0^t -25 \times 10^{-3} dx = -50t$$

$$-50t_{sat} = -6; \qquad t_{sat} = 120\,\text{ms}$$

P 7.95 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36\,\text{V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36 - 14}{80{,}000} + 2.5 \times 10^{-6}\frac{d}{dt}(-36 - v_o) = 0$$

$$\therefore \quad 2.5 \times 10^{-6}\frac{dv_o}{dt} = \frac{-50}{80{,}000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \quad \therefore \quad dv_o = -250dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250\int_0^t dy \quad \therefore \quad v_o(t) - v_o(0) = -250t$$

$$v_o(0) = -36 + 56 = 20\,\text{V}$$

$$v_o(t) = -250t + 20$$

Find the time when the voltage reaches 0:

$$0 = -250t + 20 \qquad \therefore \qquad t = \frac{20}{250} = 80\,\text{ms}$$

P 7.96 [a] $RC = (1000)(800 \times 10^{-12}) = 800 \times 10^{-9}; \qquad \frac{1}{RC} = 1{,}250{,}000$

$0 \leq t \leq 1\,\mu\text{s}$:

$$v_g = 2 \times 10^6 t$$

$$v_o = -1.25 \times 10^6 \int_0^t 2 \times 10^6 x\, dx + 0$$

$$= -2.5 \times 10^{12} \frac{x^2}{2}\Big|_0^t = -125 \times 10^{10} t^2\,\text{V}, \quad 0 \leq t \leq 1\,\mu\text{s}$$

$$v_o(1\,\mu\text{s}) = -125 \times 10^{10}(1 \times 10^{-6})^2 = -1.25\,\text{V}$$

$1\,\mu\text{s} \leq t \leq 3\,\mu\text{s}$:

$$v_g = 4 - 2 \times 10^6 t$$

$$v_o = -125 \times 10^4 \int_{1\times10^{-6}}^t (4 - 2 \times 10^6 x)\, dx - 1.25$$

$$= -125 \times 10^4 \left[4x \Big|_{1\times10^{-6}}^t - 2 \times 10^6 \frac{x^2}{2} \Big|_{1\times10^{-6}}^t \right] - 1.25$$

$$= -5 \times 10^6 t + 5 + 125 \times 10^{10} t^2 - 1.25 - 1.25$$

$$= 125 \times 10^{10} t^2 - 5 \times 10^6 t + 2.5\,\text{V}, \quad 1\,\mu\text{s} \leq t \leq 3\,\mu\text{s}$$

$$v_o(3\,\mu\text{s}) = 125 \times 10^{10}(3 \times 10^{-6})^2 - 5 \times 10^6(3 \times 10^{-6}) + 2.5$$

$$= -1.25$$

$3\,\mu\text{s} \leq t \leq 4\,\mu\text{s}$:

$$v_g = -8 + 2 \times 10^6 t$$

$$v_o = -125 \times 10^4 \int_{3\times10^{-6}}^t (-8 + 2 \times 10^6 x)\, dx - 1.25$$

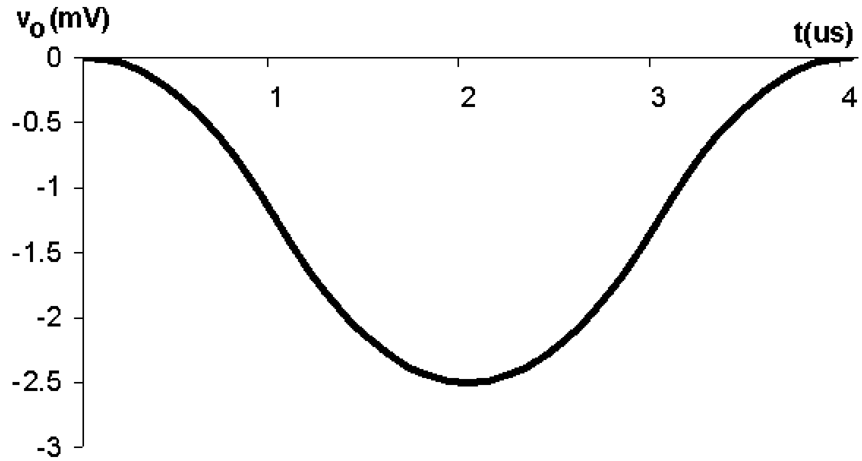
$$= -125 \times 10^4 \left[-8x \Big|_{3\times10^{-6}}^t + 2 \times 10^6 \frac{x^2}{2} \Big|_{3\times10^{-6}}^t \right] - 1.25$$

$$= 10^7 t - 30 - 125 \times 10^{10} t^2 + 11.25 - 1.25$$

$$= -125 \times 10^{10} t^2 + 10^7 t - 20\,\text{V}, \quad 3\,\mu\text{s} \leq t \leq 4\,\mu\text{s}$$

$$v_o(4\,\mu\text{s}) = -125 \times 10^{10}(4 \times 10^{-6})^2 + 10^7(4 \times 10^{-6}) - 20 = 0$$

[b]



[c] The output voltage will also repeat. This follows from the observation that at $t = 4\,\mu$s the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\,\mu$s as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.97 [a] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{b2} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{ce2} = 0$, therefore $i_{b1} = 0$. When $i_{b1} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

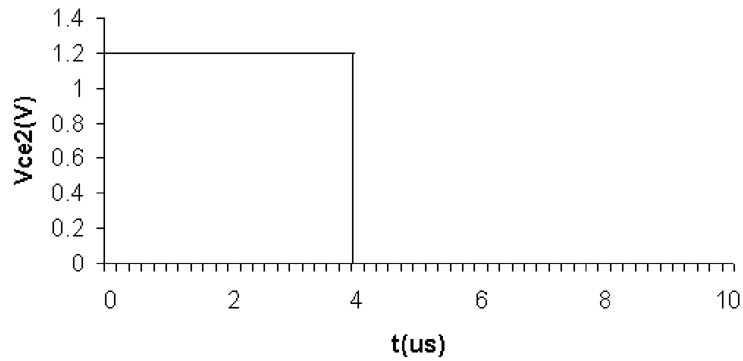
[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_L)$ and i_{b1} jumps to $V_{CC}/(R_1 + R_L)$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{be2} = 0$. The equation for v_{be2} is $v_{be2} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{be2} = 0$ when $t = RC\ \ln 2$, therefore T_2 stays OFF for $RC\ \ln 2$ seconds.

P 7.98 [a] For $t < 0$, $v_{ce2} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{ce2} = \left(\frac{V_{CC}}{R_1 + R_L}\right) R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

T_2 remains open for $(23{,}083)(250) \times 10^{-12}\ \ln 2 \cong 4\,\mu$s.

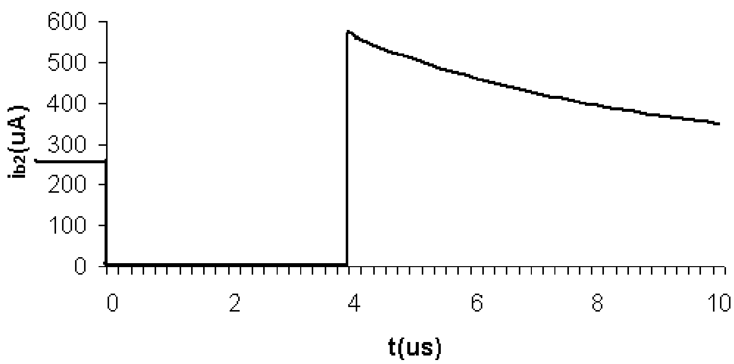


[b] $i_{b2} = \dfrac{V_{CC}}{R} = 259.93\,\mu\text{A}, \qquad -5 \le t \le 0\,\mu\text{s}$

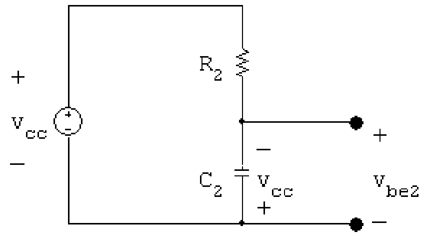
$i_{b2} = 0, \qquad 0 < t < RC\,\ln 2$

$$i_{b2} = \frac{V_{CC}}{R} + \frac{V_{CC}}{R_L} e^{-(t-RC\,\ln 2)/R_LC}$$

$$= 259.93 + 300e^{-0.2\times10^6(t-4\times10^{-6})}\,\mu\text{A}, \qquad RC\,\ln 2 < t$$

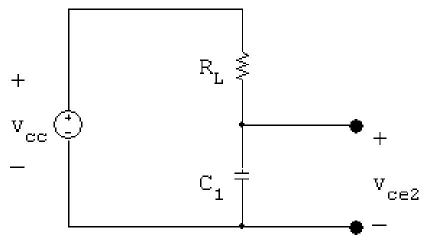


P 7.99 **[a]** While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{be2} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, v_{be2} is zero, T_2 turns ON. This makes $v_{be1} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{\rm be2} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[**b**] While T_2 is OFF and T_1 is ON, the output voltage $v_{\rm ce2}$ is the same as the voltage across C_1, thus



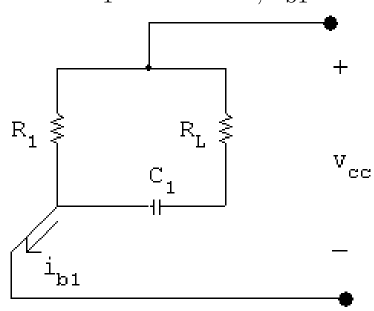
It follows that $v_{\rm ce2} = V_{CC} - V_{CC}e^{-t/R_{\rm L}C_1}$.

[**c**] T_2 will be OFF until $v_{\rm be2}$ reaches zero. As soon as $v_{\rm be2}$ is zero, $i_{\rm b2}$ will become positive and turn T_2 ON. $v_{\rm be2} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2\ \ln 2$.

[**d**] When $t = R_2C_2\ \ln 2$, we have

$$v_{\rm ce2} = V_{CC} - V_{CC}e^{-[(R_2C_2\ \ln 2)/(R_{\rm L}C_1)]} = V_{CC} - V_{CC}e^{-10\ \ln 2} \cong V_{CC}$$

[**e**] Before T_1 turns ON, $i_{\rm b1}$ is zero. At the instant T_1 turns ON, we have

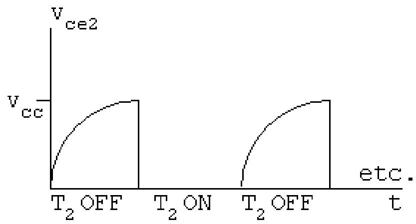


$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\rm L}}e^{-t/R_{\rm L}C_1}$$

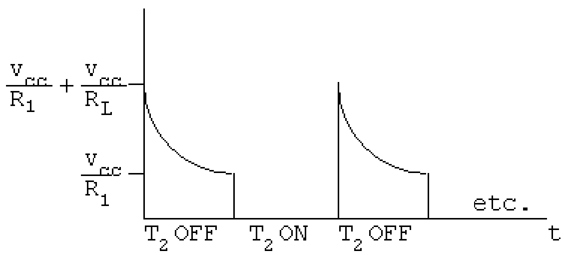
[**f**] At the instant T_2 turns back ON, $t = R_2C_2\ \ln 2$; therefore

$$i_{\rm b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\rm L}}e^{-10\ \ln 2} \cong \frac{V_{CC}}{R_1}$$

[g]



[h]



P 7.100 **[a]** $t_{\mathrm{OFF2}} = R_2C_2 \ln 2 = 18 \times 10^3(2 \times 10^{-9}) \ln 2 \cong 25\,\mu\mathrm{s}$

[b] $t_{\mathrm{ON2}} = R_1C_1 \ln 2 \cong 25\,\mu\mathrm{s}$

[c] $t_{\mathrm{OFF1}} = R_1C_1 \ln 2 \cong 25\,\mu\mathrm{s}$

[d] $t_{\mathrm{ON1}} = R_2C_2 \ln 2 \cong 25\,\mu\mathrm{s}$

[e] $i_{\mathrm{b1}} = \dfrac{9}{3} + \dfrac{9}{18} = 3.5\,\mathrm{mA}$

[f] $i_{\mathrm{b1}} = \dfrac{9}{18} + \dfrac{9}{3}e^{-6\ln 2} \cong 0.5469\,\mathrm{mA}$

[g] $v_{\mathrm{ce2}} = 9 - 9e^{-6\ln 2} \cong 8.86\,\mathrm{V}$

P 7.101 **[a]** $t_{\mathrm{OFF2}} = R_2C_2 \ln 2 = (18 \times 10^3)(2.8 \times 10^{-9}) \ln 2 \cong 35\,\mu\mathrm{s}$

[b] $t_{\mathrm{ON2}} = R_1C_1 \ln 2 \cong 37.4\,\mu\mathrm{s}$

[c] $t_{\mathrm{OFF1}} = R_1C_1 \ln 2 \cong 37.4\,\mu\mathrm{s}$

[d] $t_{\mathrm{ON1}} = R_2C_2 \ln 2 = 35\,\mu\mathrm{s}$

[e] $i_{\mathrm{b1}} = 3.5\,\mathrm{mA}$

[f] $i_{\mathrm{b1}} = \dfrac{9}{18} + 3e^{-5.6\ln 2} \cong 0.562\,\mathrm{mA}$

[g] $v_{\mathrm{ce2}} = 9 - 9e^{-5.6\ln 2} \cong 8.81\,\mathrm{V}$

Note in this circuit T_2 is OFF $35\,\mu$s and ON $37.4\,\mu$s of every cycle, whereas T_1 is ON $35\,\mu$s and OFF $37.4\,\mu$s every cycle.

P 7.102 If $R_1 = R_2 = 50R_{\mathrm{L}} = 100\,\mathrm{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\mathrm{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\mathrm{pF}$$

If $R_1 = R_2 = 6R_L = 12\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \le C_1 \le 5.77\,\text{nF}$ and $519.37\,\text{pF} \le C_2 \le 4.33\,\text{nF}$

P 7.103 [**a**] $0 \le t \le 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right) e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore \quad e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\text{H}$$

[**b**] $0 \le t \le t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right) e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore \quad 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31 \ln 1.25}{60} \cong 0.10\,\text{s}$$

P 7.104 From the Practical Perspective,

$$v_C(t) = 0.75V_S = V_S(1 - e^{-t/RC})$$

$$0.25 = e^{-t/RC} \qquad \text{so} \qquad t = -RC \ln 0.25$$

In the above equation, t is the number of seconds it takes to charge the capacitor to $0.75V_S$, so it is a period. We want to calculate the heart rate, which is a frequency in beats per minute, so $H = 60/t$. Thus,

$$H = \frac{60}{-RC \ln 0.25}$$

P 7.105 In this problem, $V_{max} = 0.6V_S$, so the equation for heart rate in beats per minute is

$$H = \frac{60}{-RC \ln 0.4}$$

Given $R = 150\,\text{k}\Omega$ and $C = 6\,\mu\text{F}$,

$$H = \frac{60}{-(150{,}000)(6 \times 10^{-6}) \ln 0.4} = 72.76$$

Therefore, the heart rate is about 73 beats per minute.

P 7.106 From the Practical Perspective,

$$v_C(t) = V_{max} = V_S(1 - e^{-t/RC})$$

Solve this equation for the resistance R:

$$\frac{V_{max}}{V_S} = 1 - e^{-t/RC} \qquad \text{so} \qquad e^{-t/RC} = 1 - \frac{V_{max}}{V_S}$$

Then, $$\frac{-t}{RC} = \ln\left(1 - \frac{V_{max}}{V_S}\right)$$

$$\therefore \qquad R = \frac{-t}{C\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

In the above equation, t is the time it takes to charge the capacitor to a voltage of V_{max}. But t and the heart rate H are related as follows:

$$H = \frac{60}{t}$$

Therefore,

$$R = \frac{-60}{HC\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

P 7.107 From Problem 7.106,

$$R = \frac{-60}{HC\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

Note that from the problem statement,

$$\frac{V_{max}}{V_S} = 0.68$$

Therefore,

$$R = \frac{-60}{(70)(2.5 \times 10^{-6})\ln(1 - 0.68)} = 301\,\text{k}\Omega$$