Lenders require compensation with interest for the use of their money because money loaned out is:

- Not available to the lender to purchase desired goods today
- Cannot be put to work by the lender to earn a rate of return in another use
    - The opportunity cost of deploying money in one activity is the lost opportunity to deploy it in the second most attractive alternative activity

The time value of money means that the value of a dollar partly depends on **when** it is received

- A $ today is more valuable than a $ next year--and upfront costs sting more (e.g. college tuition)
- **[Inflation** (an overall increase in the price level for all goods) also erodes the value of money over time but we will defer discussion of inflation for another time]

This may be the most important information contained in this entire class:

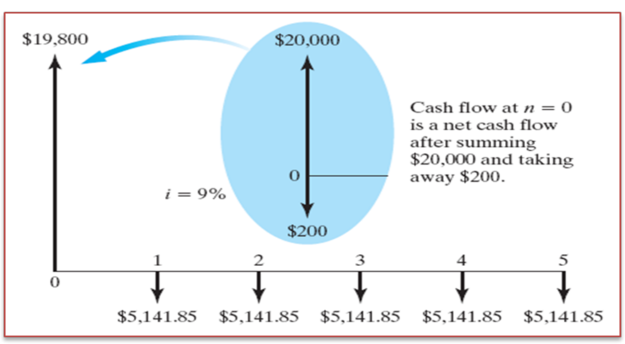
- There is a way to compare the value of money at different points in time
- We can reduce a **sequence of benefits and costs** to a **single point in time**

IMPORTANT: Engineering economics problems always use the **end-of-period convention:**

- All cash flow transactions are placed at the end of an **interest period** which is:
    - the time period over which interest is calculated (monthly, quarterly, or yearly)
        - **n** (small n) = an interest period and **N** (big N) = total number of interest periods
- Example: Suppose Fred put $1,000 in the bank in January 2017 at 5% interest with an interest period of a year. The end-of-period convention means we record the deposit at the end of 2017, and he is not credited any interest in 2017 (Mainly because of this, banks have *interest periods* of a month or quarter, even though they quote their rates as *annual percentages*).

Solving simple time value of money problems

- A **cash-flow diagram**—a graphical representation of cash transactions through time helps conceptualize time value of money problems. We return to them frequently in this class.
- Example: Cornelia goes to a bank and takes out a loan for $20,000 at 9% interest over 5 years, requiring a $200 loan origination fee upfront and annual payments of $5,141.85 over 5 years. This information is summarized in the following cash flow diagram:



Notes:

- A **cash inflow** is represented by ↑ with a length that reflects the size of the inflow
- A **cash outflow** is represented by ↓ with a length that reflects the size of the outflow
- If there is more than one cash flow in a time period, **net cash flow** is shown
- The present is time 0, N = 5, n = 1 year, and **i** (small i) = the interest rate = 9%

There are two possible ways to calculate interest:

1. **Simple Interest**
   Interest calculated on original investment amount only, i.e. *simple interest is not earned on prior reinvested interest payments—it does not compound*.
   - Less common than compound interest in the "real" world

Example: Suppose $100 (P) is invested at an interest rate (i) = 5% for 5 years (N). Let **I** be the $ amount of interest earned and F be the future value of the deposit. Find I and F:

$I = (iP)N$ Answer: $25 [0.05 x $100 x 5]

$F = P + I$
$F = P + (iP)N$
$F = P(1 + iN)$ Answer: $125 [$100 + $25]

2. **Compound Interest**
   - Calculated on total amount (P + I) accumulated at the end of previous interest period.
   - **Always assume compound interest applies**--unless you are explicitly told otherwise
   - Using the example above, we can illustrate the power of compounding:

Period	Amount at beginning of interest period	Interest earned for period	Amount at end of interest period
1	$100	$100 (0.05) = $5	$105
2	$105	$105 (0.05) = $5.25	$110.25
3	$110.25	$110.25(0.05) = $5.51	$115.76
4	$115.76	$115.76(0.05) = $5.79	$121.55
5	$121.55	$121.55(0.05) = $6.08	$127.63

   - Comparing simple interest to compound interest, $2.63 in extra interest may seem small, but compounding is **powerful when N or i are large**

The formula for compound interest is:

$$F = P(1 + i)^N$$

Because:

Period	Amount at beginning of interest period	Interest earned for period	Amount at end of interest period
1	P	iP	$P + iP = P(1 + i)$
2	$P(1+ i)$	$iP(1+ i)$	$P(1+ i) + i[P(1+ i)]$ $= P(1 + i)(1 + i)$ $=P(1 + i)^2$
3	$P(1 + i)^2$	$iP(1+i)^2$	$P(1 + i)^2 + iP(1+i)^2$ $F = P(1 + i)^3$

And so on, I stopped after 3 periods, but you get the idea. In this case F is the value of the cash at the end of year 3 (N = 3)