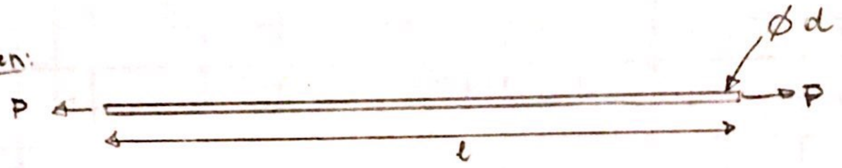
ENGR 213 HW # 2 | Due 4/15/20 | Sean Lai

CH2: 2, 9, 17, 21, 33, 35, 43, 51, 58, 61, 79, 98, 100

2.2:

Given:




$d = 0.25$ in
$\ell = 4.8$ ft $= 57.6$ in
$E = 29 \cdot 10^6$ psi
$P = 750$ lb

Find: a) elongation of wire
b) normal stress $\sigma$

Solution:

$E = \frac{\sigma}{\varepsilon}$ , $\sigma = \frac{P}{A}$ , $A = \frac{\pi}{4} d^2 = \frac{\pi}{64}$ in

$\sigma = \frac{750 \text{ lb}}{\frac{\pi}{64} \text{ in}}$

$\boxed{\sigma = 15{,}279 \text{ psi} \quad b)}$

$\varepsilon = \frac{\sigma}{E}$
$= \frac{15279 \text{ psi}}{29 \cdot 10^6 \text{ psi}}$

$\varepsilon = 5.27 \cdot 10^{-4}$

let e be elongation: note: will use $\delta$ from now on

$e = \varepsilon \cdot \ell$
$\boxed{e = 0.0303 \text{ in} \quad a)}$

Scanned with CamScanner

2.9:

Given: steel rod → $l = 4m$
$\delta \leq 3mm = 0.003m$
$\sigma_m \leq 150\,MPa$
$P = 10\,kN$
$E = 200\,GPa$

Find: required diameter: $d$

Solution:

① stress limit: $\sigma_m = \frac{P}{A}$

$$A = \frac{P}{\sigma_m} \quad , \quad A = \frac{\pi}{4} d^2$$

$$\rightarrow d_\sigma = \sqrt{\frac{P}{\sigma_m} \cdot \frac{4}{\pi}} \qquad ①$$

$$d_\sigma = 0.0092m = 9.2mm$$

② strain limit: $\delta = \varepsilon \cdot l$

$$\varepsilon = \frac{\delta}{l}$$

$$E = \frac{\sigma}{\varepsilon}$$

$$\sigma = E\varepsilon$$

from ①: $d = \sqrt{\frac{P}{\sigma} \cdot \frac{4}{\pi}} \quad , \quad \sigma = E\frac{\delta}{l}$

$$d = \sqrt{\frac{Pl}{E\delta} \cdot \frac{4}{\pi}}$$

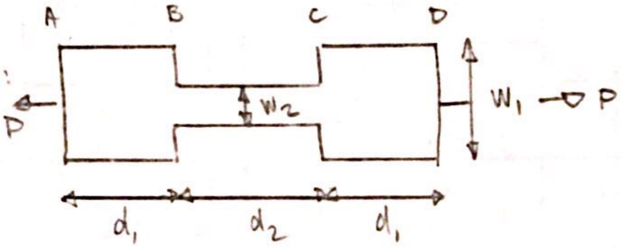
$$d = 0.0092m = \boxed{9.2\,mm}$$

oh, they're the same!

Scanned with CamScanner

2.17:

Given:




$w_1 = 1\,in$, $w_2 = 0.4\,in$
$d_1 = 1.6\,in$, $d_2 = 2\,in$
$t = 0.25\,in$
$P = 350\,lb$
$E = 0.45 \cdot 10^6\,psi$

Find: a) total deformation of specimen
b) deformation of BC

Solution:

a) $$\delta = \int_0^L \frac{P\,dx}{AE}$$

$$= \underset{①}{\int_A^B \frac{P\,dx}{A_{AB}E}} + \underset{②}{\int_B^C \frac{P\,dx}{A_{BC}E}} + \underset{③}{\int_C^D \frac{P\,dx}{A_{CD}E}}$$

note, ① and ③ are the same

$A_{AB} = w_1 t$, $A_{BC} = w_2 t$

$$\rightarrow \delta = 2\int_A^B \frac{P\,dx}{w_1 t E} + \int_B^C \frac{P\,dx}{w_2 t E}$$

$$= \frac{P}{tE}\left(\frac{2d_1}{w_1} + \frac{d_2}{w_2}\right)$$

$$\boxed{\delta_t = 0.0255\,in}$$

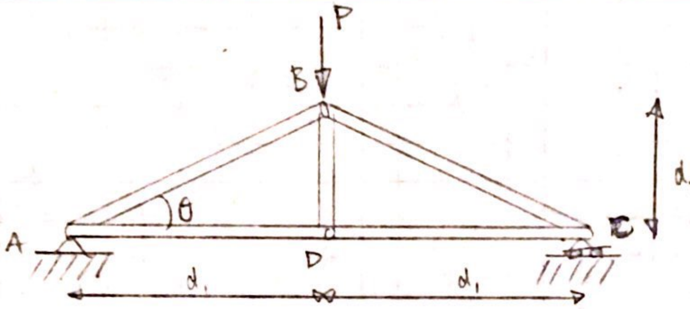
b) $$\delta_{BC} = \frac{P}{tE}\frac{d_2}{w_2}$$

$$\rightarrow \boxed{\delta_{BC} = 0.0156\,in}$$

Scanned with CamScanner

2.21 :

Given:




$P = 228\ kN$

$d_1 = 4m \quad d_2 = 2.5m$

$E = 200 GPa$

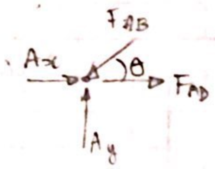
$A_{AB} = 2400\ mm^2$

$A_{AD} = 1800\ mm^2$

Find: $\delta_{AB}$ , $\delta_{AD}$

Solution: note BD is zero force member due to no loading at D.

at A:




$A_y , C_y = \frac{P}{2}$

$\theta = \tan^{-1}\left(\frac{d_2}{d_1}\right) = 32°$

$\rightarrow A_y = F_{AB} \sin\theta \qquad \Sigma F_y$

$F_{AB} = \frac{A_y}{\sin\theta}$

$F_{AB} = 215.1\ kN$

$F_{AB} \cos\theta = F_{AD} \qquad \Sigma F_x$

$F_{AD} = 182.4\ kN$

$\delta = \frac{Pl}{AE}$ , $l_{AB} = \sqrt{d_1^2 + d_2^2}$

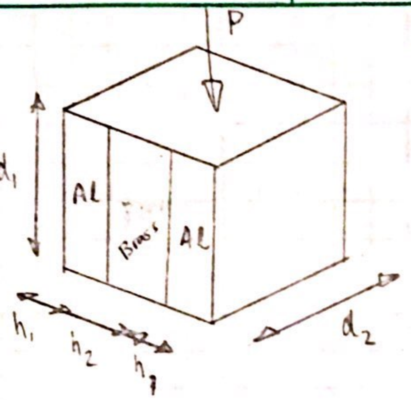
$l_{AB} = 4.72m$ , $l_{AD} = 4m$

$$\boxed{\begin{aligned} \delta_{AB} &= \frac{F_{AB} l_{AB}}{A_{AB} E} = -0.00212\ m = -2.12\ mm \\ \delta_{AD} &= \frac{F_{AD} l_{AD}}{A_{AD} E} = +0.00202\ m = +2.03\ mm \end{aligned}}$$

Scanned with CamScanner

2.33.

Given:




$E_{Al} = 70$ GPa
$E_B = 105$ GPa

$d_1 = 300$ mm $\quad d_2 = 60$ mm
$h_1 = 10$ mm $\quad h_2 = 40$ mm

$P = 450$ kN

Find: $\sigma_B$, $\sigma_{Al}$

Solution:

$$\delta_{A1} = \frac{P_A \ell}{A_A E_A} \qquad \ell = d_1 , \; A_A = 2 \cdot d_2 h_1$$

$$\delta_B = \frac{P_B \ell}{A_B E_B} \qquad \ell = d_1 , \; A_B = d_2 h_2$$

and $\delta_A = \delta_B$

$$\rightarrow \frac{P_B \ell}{A_B E_B} = \frac{P_A \ell}{A_A E_A} , \text{ and } P_A + P_B = P$$

system of eq:

| $P_A$ | $P_B$ | k |
|---|---|---|
| $+\frac{1}{A_A E_A}$ | $-\frac{1}{A_B E_B}$ | 0 |
| 1 | 1 | P |

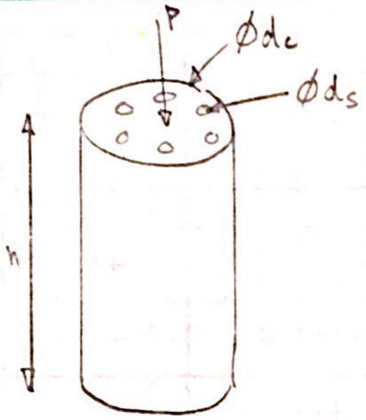
$\Rightarrow P_A = 113$ kN
$P_B = 337$ kN

$$\boxed{\begin{aligned} \rightarrow \sigma_A &= \frac{P_A}{A_A} = -94 \text{ MPa} \\ \sigma_B &= \frac{P_B}{A_B} = -140 \text{ MPa} \end{aligned}}$$

Scanned with CamScanner

2.35:

Given:




$d_c = 18\,in$
$d_s = 1.125\,in$
$h = 54\,in$
$E_c = 4.2 \cdot 10^6$ psi
$E_s = 29 \cdot 10^6$ psi
$P = 350$ kip

Find: $\sigma_s$, $\sigma_c$

Solution: $A_s = 6 \cdot \frac{\pi}{4} d_s^2 = 5.964\,in^2$

$A_c = \frac{\pi}{4} d_c^2 - A_s = 248.5\,in^2$

$\delta_s = \frac{P_s h}{A_s E_s}$ $\quad \delta_c = \frac{P_c h}{A_c E_c}$, $\delta_s = \delta_c$ $\quad P_s + P_c = P$

from book, $P_s = \frac{A_s E_s P}{A_s E_s + A_c E_c}$

$P_s = 49.75$ kip

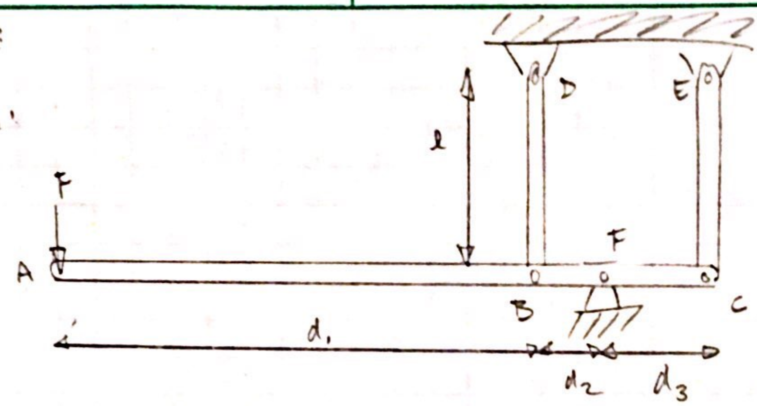
$\rightarrow P_c = 300.25$ kip

$$\boxed{\begin{aligned} \sigma_s &= \frac{P_s}{A_s} = -8.342\text{ ksi} \\ \sigma_c &= \frac{P_c}{A_c} = -1.208\text{ ksi} \end{aligned}}$$

Scanned with CamScanner

2.43:

Given:




$F = 2\,kN$
$l = 225\,mm$
$d_1 = 550\,mm$
$d_2 = 75\,mm$
$d_3 = 100\,mm$
$A_{DB} = A_{EC} = 200\,mm^2$
$E = 105\,GPa$

Find: deflection at A, given rigid ABC

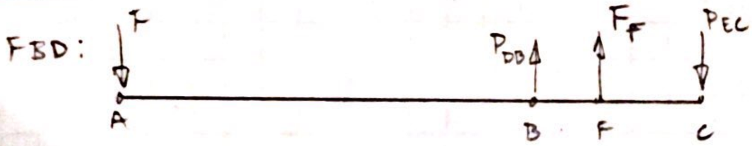
Solution: by similar triangles,

$$\delta_{BD} = \frac{d_2}{d_3}\delta_{EC} = 0.75\,\delta_{EC}$$

what we're looking for $\rightarrow$ $$\delta_A = \frac{(d_1+d_2)}{d_3}\delta_{EC} = 6.25\,\delta_{EC}$$

determine forces:

FBD:




$$\Sigma F_y: \; P_{DB} + F_F - P_{EC} - F = 0 \quad ①$$

$$\Sigma M_C: \; +F(d_1+d_2+d_3) - P_{DB}(d_2+d_3) - F_F(d_3) = 0 \quad ②$$

$$\delta_{BD} = \frac{P_{DB}\,l}{AE}, \quad \delta_{EC} = \frac{P_{EC}\,l}{AE}$$

since $\delta_{BD} = 0.75\,\delta_{EC} \longrightarrow P_{DB} = 0.75\,P_{EC}$ ③

system of eq:

$$\begin{array}{ccc|c} P_{DB} & F_F & P_{EC} & k \\ \hline 1 & 1 & -1 & F \\ +(d_2+d_3) & +d_3 & 0 & F(d_1+d_2+d_3) \\ 1 & 0 & -0.75 & 0 \end{array} \rightarrow \begin{array}{l} P_{DB} = 6\,kN \\ F_F = 4\,kN \\ P_{EC} = 8\,kN \end{array}$$

$$\delta_{EC} = \frac{P_{EC}\,l}{AE} = 8.57 \cdot 10^{-5}\,m$$

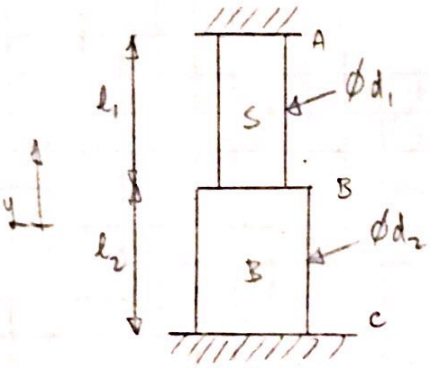
$$\delta_A = \delta_{EC}\frac{(d_1+d_2)}{d_3} = 5.35 \cdot 10^{-4}\,m$$

$$\rightarrow \boxed{\delta_A = 0.535\,mm}$$

Scanned with CamScanner

2.51:

Given:




$l_1 = 250\,mm$, $l_2 = 300\,mm$
$d_1 = 30\,mm$, $d_2 = 50\,mm$

$E_S = 200\,GPa$ $E_B = 105\,GPa$
$\alpha_S = 11.7 \cdot 10^{-6}\,/^\circ C$ $\alpha_B = 20.9 \cdot 10^{-6}\,/^\circ C$

Find: $P_c$ compressive force when T temperature rises 50°C

Solution: $\delta_{TB} + \delta_{TS} = \delta_{PB} + \delta_{PS}$

$\delta_{TB} = \alpha_B \Delta T l_2$ $\delta_{TS} = \alpha_S \Delta T l_1$

$\delta_{PB} = \dfrac{P l_2}{A_B E_B}$ $\delta_{PS} = \dfrac{P l_1}{A_S E_S}$

$\rightarrow P = \dfrac{\delta_{PB} A_B E_B}{l_2}$ $P = \dfrac{\delta_{PS} A_S E_S}{l_1}$

$\rightarrow \delta_{PB} = \delta_{PS} \cdot \dfrac{l_2}{l_1} \cdot \dfrac{A_S E_S}{A_B E_B}$

$\rightarrow \delta_{TB} + \delta_{TS} = \delta_{PS}\left[1 + \dfrac{l_2}{l_1}\dfrac{A_S E_S}{A_B E_B}\right]$

$\delta_{TB} = \alpha_B \Delta T l_2$
$= 0.3135\,mm$

$A_B = \frac{\pi}{4} d_2^2$
$= 1.96 \cdot 10^{-3}\,m$

$\delta_{TS} = \alpha_S \Delta T l_1$
$= 0.1463\,mm$

$A_S = \frac{\pi}{4} d_1^2$
$= 7.07 \cdot 10^{-4}\,m$

$\rightarrow 0.4598\,mm = 1.82\,\delta_{PS}$

$\delta_{PS} = 0.252\,mm$

$\delta_{PS} + \delta_{PB} = 0.4598\,m$

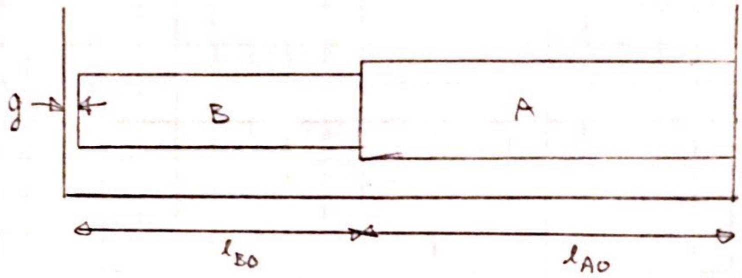
$\rightarrow \delta_{PB} = 1.97\,mm$

$$\boxed{P = \frac{\delta_{PS} A_S E_S}{l_1} = 142.5\,kN}$$

Scanned with CamScanner

2.58 :

Given :




$g = 0.02 in$
$l_{BO} = 14 in$
$l_{AO} = 18 in$
$T_0 = 75°F$

Bronze: $A = 2.4 in^2$
$E = 15 \cdot 10^6$ psi
$\alpha_B = 12 \cdot 10^{-6}/°F$

Aluminum: $A = 2.8 in^2$
$E = 10.6 \cdot 10^6$ psi
$\alpha_A = 12.9 \cdot 10^{-6}/°F$

Find : a) temperature T when $\sigma_A = -11$ ksi
b) $l_A$ at T

Solution : first solve for δ's to close gap, then δ's under stress

stage ⓪ $\delta_{TB} + \delta_{TA} = g$ , $\delta_T = \alpha \Delta T L$

$\alpha_B \Delta T_g l_{BO} + \alpha_A \Delta T_g l_{AO} = g$

$\rightarrow \Delta T_g = \dfrac{g}{\alpha_B l_{BO} + \alpha_A l_{AO}}$

$\Delta T_g = 50°C$ to close gap

stage ①
now, with gap closed:
$\delta_{TB} + \delta_{TA} = \delta_{PB} + \delta_{PA}$ ①

recall, $\delta_T = \alpha \Delta T l$ , $\delta_P = \dfrac{Pl}{EA}$

$l_{B1} = l_{BO} + \delta_{TBO}$
$= l_{BO} + \alpha_B \Delta T_g l_{BO}$
$l_{B1} = 14.01 in$

$l_{A1} = l_{AO}(1 + \alpha_A \Delta T_g)$
$l_{A1} = 18.01 in$

from ① $\Delta T_c(\alpha_B l_{B1} + \alpha_A l_{A1}) = P\left(\dfrac{l_{B1}}{E_B A_B} + \dfrac{l_{A1}}{E_A A_A}\right)$

we know $\sigma_A = -11$ ksi

$\rightarrow \sigma_A = \dfrac{P}{A_A} \rightarrow P = \sigma_A A_A$
$P = 30.8$ ksi

$\rightarrow \Delta T_c = P\left(\dfrac{l_{B1}}{E_B A_B} + \dfrac{l_{A1}}{E_A A_A}\right) \cdot \left(\alpha_B l_{B1} + \alpha_A l_{A1}\right)^{-1}$

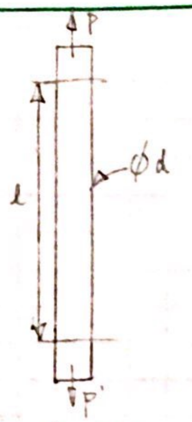
$\Delta T_c = 76.6°F$

$\rightarrow T_f = T_0 + T_g + T_c$
$\boxed{T_f = 201.6 °F}$

$\boxed{l_{A1} = 18.01 in}$

2.61:

Given:




$\ell = 5.0\text{ in}$
$d = 0.625\text{ in}$
$P = 800\text{ lb}$
$\delta_\ell = 0.45\text{ in}$
$\delta_d = -0.025\text{ in}$

Find: $E$, $G$, $\nu$

Solution:

$$\delta_\ell = \frac{P\ell}{EA} \longrightarrow E = \frac{P\ell}{\delta_\ell A}$$

$$\boxed{E = 28973\text{ psi}}$$

$$\nu = -\frac{\varepsilon_d}{\varepsilon_\ell} \qquad \varepsilon_d = \frac{\delta_d}{d}, \quad \varepsilon_\ell = \frac{\delta_\ell}{\ell}$$

$$\boxed{\nu = 0.444}$$

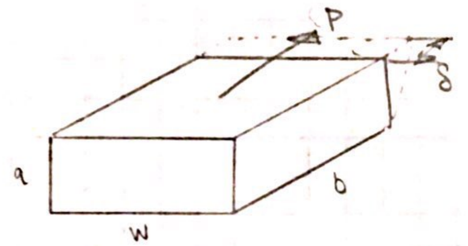
$$G = \frac{E}{2(1+\nu)}$$

$$\boxed{G = 10032\text{ psi}}$$

Scanned with CamScanner

2.79:

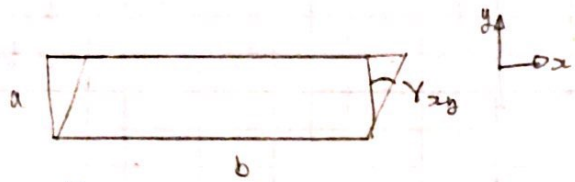
Given:




$w = 8 in$
$P = 5 kip$
$\delta = 0.375 in$ max
$\tau = 60 psi$ max
$G = 130 psi$

Find: a) smallest allowable b
b) smallest required thickness a

Solution:




a) $\tau = \frac{P}{A}$ , $A = bw$

$\rightarrow \tau = \frac{P}{bw}$

$b = \frac{P}{\tau w}$

$\boxed{b = 10.4 in}$

b) $\tau = G\gamma$ , $\gamma = \frac{\delta}{a}$ $\rightarrow$ $a = \frac{\delta}{\gamma}$

$\gamma = \frac{\tau}{G}$

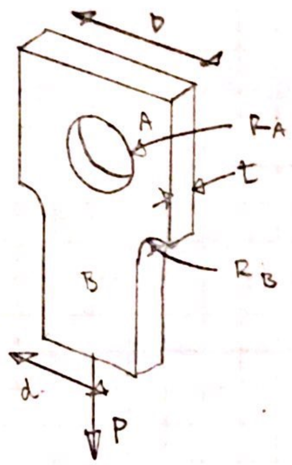
$\rightarrow a = \frac{\delta G}{\tau}$

$\boxed{a = 0.8125 in}$

Scanned with CamScanner

2.98:

Given:




$D = 88\,mm$
$d = 64\,mm$
$R_A = 20\,mm$
$R_B = 15\,mm$
$P = 100\,kN$

Find: minimum t for allowable $\sigma = 125\,MPa$

Hole: $\sigma_{max} = K\sigma_{ave}$

$$\frac{2r}{D} = \frac{2R_A}{D} = 0.455 \rightarrow K_h \approx 2.2$$

$$\sigma_{ave} = \frac{P}{A} \quad A = (D - 2R_A)(t)$$

$$\sigma_{max} = K\frac{P}{(D-2R_A)t}$$

$$\rightarrow t = K\frac{P}{(D-2R_A)\sigma_{max}}$$

$$t_{min} = 0.0367\,m = 36.7\,mm$$

Fillet: $\frac{r}{d} \rightarrow \frac{R_B}{d} = 0.234$, $\frac{D}{d} = 1.375$ $\Big\}$ $K \approx 1.6$

$$\sigma_{ave} = \frac{P}{dt}$$

$$\sigma_{max} = K\frac{P}{dt}$$

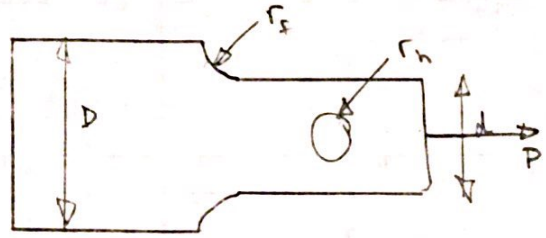
$$\rightarrow t = \frac{KP}{d\sigma_{max}} = 0.02\,m = 20\,mm$$

largest of two minimums $= \boxed{36.7\,mm}$

Scanned with CamScanner

2.100 :

Given:




$r_f = 0.5 \text{ in}$
$r_h = 0.5 \text{ in}$
$D = 6.5 \text{ in}$
$d = 5 \text{ in}$
$t = 0.75 \text{ in}$
$\sigma_{all} = 20 \text{ ksi}$

Find: maximum allowable P

Solution:

fillet: $\frac{D}{d} = 1.3 \quad \frac{r_f}{d} = 0.1$

$\rightarrow K_f \approx 2.05$

hole: $\frac{2r}{d} = 0.2$

$\rightarrow K_h \approx 2.5$

$\sigma_{avef} = \frac{P_f}{A_f} \quad A_f = dt$

$\sigma_{ave_f} = \frac{\sigma_{max}}{K_f}$

$P_f = A_f \frac{\sigma_{max}}{K_f} = 3.66 \cdot 10^4 \text{ lb}$

$\sigma_{aveh} = \frac{P_h}{A_h} \quad A_h = (d - 2r_h)t$

$P_h = A_h \frac{\sigma_{max}}{K_h}$

lowest of two is max allowable P

$\rightarrow \boxed{P_{max} = 2.4 \cdot 10^4 \text{ lb}}$

Scanned with CamScanner