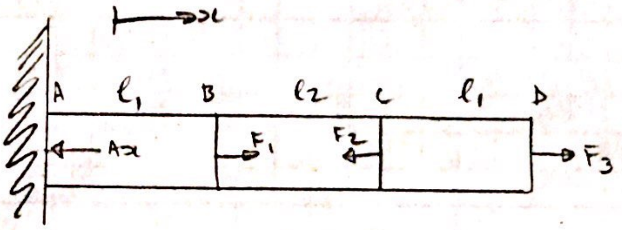
ENGR 213 EXAM #1 | Wed 4/22/20 | Sean Lai

Honor Statement:

I have neither given nor received unauthorized assistance on this Midterm.

signed: [signature] Sean Lai

Q1:



$A = 900\,mm^2$
$F_1 = 45\,kN$
$F_2 = 90\,kN$
$F_3 = 70\,kN$
$\alpha = 11.7 \cdot 10^{-6}\,/°C$
$E = 200\,GPa$

$\left.\begin{array}{l} l_1 = 800\,mm \\ l_2 = 1200\,mm \end{array}\right\}$ @ 28°C

$\Sigma F_x: -A_x + F_1 - F_2 + F_3 = 0$

$A_x = -90\,kN + 45\,kN + 70\,kN$

a) $A_x = \underline{25\,kN}$

AB: $P_{AB} = -A_x$

$\rightarrow \sigma_{AB} = \frac{P}{A} = \frac{-25kN}{900mm^2} = \boxed{27.8\text{ MPa tension (AB)}}$

BC: $P_{BC} = -A_x + F_1$

$\rightarrow \sigma_{BC} = \frac{-A_x + F_1}{A} = \frac{20kN}{900mm^2} \boxed{= 22.2\text{ MPa compression (BC)}}$

CD: $P_{CD} = -A_x + F_1 - F_2$

$\rightarrow \sigma_{CD} = \frac{-70kN}{900mm^2} = \boxed{77.8\text{ MPa tension (CD)}}$

b) $\delta = \frac{Pl}{AE}$

$\delta_{tot} = \delta_{AB} + \delta_{BC} + \delta_{CD}$, tension +, comp. −

$= \frac{1}{E}\left(\sigma_{AB} l_1 + \sigma_{BC} l_2 + \sigma_{CD} l_1\right)$

$= \frac{1}{E}\left[(22.8 + 77.8\text{ MPa})(800\text{ mm}) - (22.2\text{ MPa})(1200\text{mm})\right]$

$\delta_{tot} = 2.92 \cdot 10^{-4}\text{ m}$

$\rightarrow \boxed{\delta_{tot b} = 0.292\text{ mm}}$

c) $\delta_{temp} = \alpha \Delta T \cdot L$, $\Delta T = 140°C - 28°C$, $L = 2l_1 + l_2$

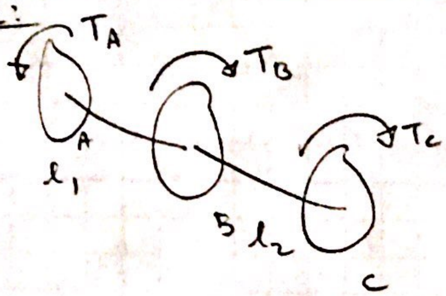
$\delta_{temp} = 11.7 \cdot 10^{-6} \cdot 112 \cdot 2.8\text{ m}$

$\delta_{temp} = 3.67 \cdot 10^{-3}\text{ m} = 3.67\text{mm}$

$\rightarrow \delta_{tot} = \delta_{tot b} + \delta_{temp} = \boxed{3.961\text{ mm}}$

Scanned with CamScanner

Q2:



$T_A = 1600\,Nm$ $\quad l_1 = 500\,mm$
$T_B = 900\,Nm$ $\quad l_2 = 900\,mm$
$T_C = 700\,Nm$
$G = 77\,GPa$ $\quad \emptyset d = 30\,mm$

$$\phi = \frac{Tl}{JG}, \quad J = \frac{\pi}{2}c^4, \quad c = 15\,mm$$

$$\phi_{B/A} = \frac{T_B\, l_1}{\frac{\pi}{2}c^4 \cdot G}$$

$$\boxed{\phi_{B/A} = 0.00735\,rad = 4.21^\circ}$$

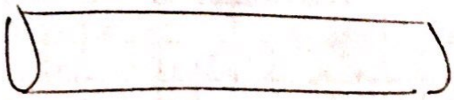
$$\phi_{C/A} = \phi_{B/A} + \phi_{C/B}$$

$$\phi_{C/B} = \frac{T_C\, l_2 \cdot 2}{\pi c^4 \cdot G}$$

$$\phi_{C/B} = 0.103\,rad = 5.90^\circ$$

$$\rightarrow \boxed{\phi_{C/A} = 5.90^\circ + 4.21^\circ = 10.11^\circ}$$

Q3:



$P = 120\ \text{hp}$, $\omega = 1200\ \text{rpm}$

$\tau_{max} = 7000\,\text{ksi}$

$P = T \cdot \omega$

$T = \frac{P}{\omega}$, $\therefore$ 1 hp = 6600 lb·in/s , 1 rpm = $\frac{2\pi}{60}$ rad/s

$$\rightarrow T = \frac{120\ \text{hp} \cdot 6600\ \text{lb}\cdot\text{in/s/hp}}{1200\ \text{rpm} \cdot \frac{2\pi}{60}\ \frac{\text{rad}}{\text{s}} \Big/ \frac{\text{rev}}{\text{min}}}$$

$T = 6.3 \cdot 10^3$ lb·in or 6.3 kip·in

$\tau_{max} = \frac{Tc}{J}$, $J = \frac{\pi}{2} c^4$

$$\tau_{max} = \frac{Tc \cdot 2}{\pi c^4}$$

$$c^3 = \frac{2T}{\pi \tau_{max}}$$

$$c = \left(\frac{2T}{\pi \cdot \tau_{max}}\right)^{1/3}$$

$$c = \left(\frac{2 \cdot 6.3\ \text{kip in}}{\pi \cdot 7000\ \text{kip/in}^2}\right)^{1/3}$$

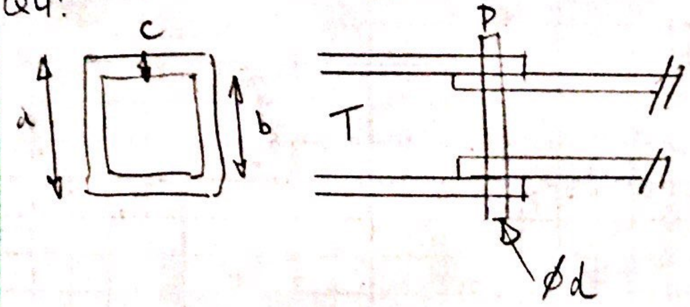
$c = 8.31 \cdot 10^{-2}$ in — seems small... but τ_{max} of 7000 ksi seems large.

$d = 2c = 0.166$ in

$\boxed{\text{shaft must have a a diameter of } \underline{0.166\ \text{in}} \text{ to transmit 120 hp @ 1200 rpm for a } \tau_{max} = 7000\ \text{ksi}}$

Scanned with CamScanner

Q4:



$a = 2.5\,in$
$b = 2.0\,in$
$c = 0.25\,in$

$\sigma_P = 40\,ksi$ $\quad \tau_P = 21\,ksi$
$\tau_T = 30\,ksi$

$d = 0.625\,in$

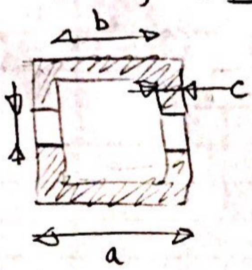
a) $\tau_{max} = \frac{P}{A}$, $A = \pi\left(\frac{d}{2}\right)^2$

2 shearing points:

$P = \tau_T \cdot 2A$
$P = 18.41$ kips

F.O.S of 5 → $\boxed{P_{all} = \frac{P}{5} : 3.68 \text{ kips}}$

b) $P = 7.5$ kips (note: this is outside of F.O.S. of 5)
Assuming axial load is tension, ie. ~~not~~ compression when braking down a hill.



$A = a^2 - b^2 - 2dc$

$A = 1.938\,in^2$

$\sigma = \frac{P}{A}$

$$\boxed{\sigma = \frac{7.5 \text{ kips}}{1.938\,in^2} = 3.871 \text{ ksi}}$$

c) $P = 7.5$ kips , $\sigma_{ball} = 40\,ksi$

$\sigma_B = \frac{P}{A}$, $A = 2 \cdot dc = 0.3125\,in^2$

$\sigma_B = \frac{7.5 \text{ kips}}{0.3125\,in^2} = 24\,ksi$

$F.O.S. = \frac{\sigma_{Ball}}{\sigma_B} = \frac{40}{24}\,ksi$

$\boxed{F.O.S = 1.67}$

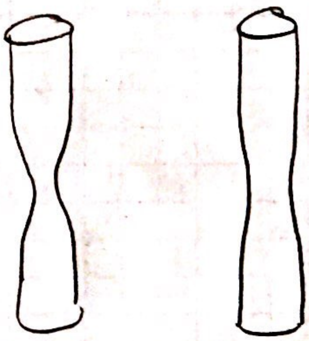
Scanned with CamScanner

Q5:

$$\nu = \frac{\text{lateral } \epsilon}{\text{axial } \epsilon}$$

so larger ν means more lateral ϵ for each quantity of axial ϵ
→ greater thinning / necking

$\nu_A = 0.21$, $\nu_B = 0.33$



B A

Scanned with CamScanner

Q6.:

$$\tau = \frac{Tc}{J} \quad , \quad J = \frac{\pi}{2} c^4$$

$$\tau_A = \frac{2T}{\pi c^3} \quad , \quad c = \frac{1}{2} r$$

$$\tau_A = \frac{16T}{\pi r^3} \quad (1)$$

$$\tau_B = \frac{2T}{\pi c^3} \quad , \quad c = r$$

$$\tau_B = \frac{2T}{\pi r^3} \quad (2)$$

$$\tau_c = \frac{2T}{\pi} \cdot \frac{c_o}{c_o^4 - c_I^4} \quad , \quad c_o = \frac{2}{3} r \quad , \quad c_I = \frac{1}{2} r$$

$$\tau_c = \frac{2T}{\pi} \cdot \frac{2/3\, r}{16/81\, r^4 - \frac{1}{16} r^4}$$

$$\tau_c = \frac{2T}{\pi} \cdot \frac{4.93}{r^3}$$

$$\tau_c \approx \frac{10T}{\pi r^3} \quad (3)$$

$$\boxed{\tau_A > \tau_c > \tau_B}$$

Scanned with CamScanner