ENGR 213: EXAM #2, 5/13/20, Sean Lai

Honor statement:

I have neither given nor received unauthorized assistance on this Midterm.

signed: Sean Lai

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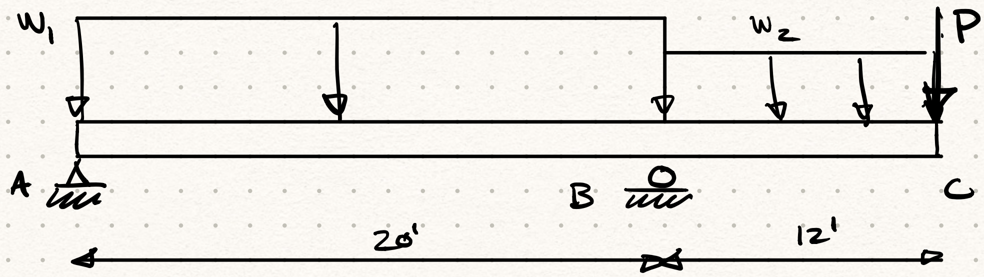
(1)

G:



$w_1 = 500\,lb/ft$, $w_2 = 300\,lb/ft$, $P = 300\,lb$

a) $A_1 = w_1 \cdot 20 = 10000\ lbs$

$A_2 = w_2 \cdot 12 = 3600\ lbs$

$\Sigma F_y = 0 \longrightarrow A_y + B_y - A_1 - B_1 - P = 0$

$\Sigma M_A = 0 \longrightarrow -A_1(10) + B_y(20) - A_2(26) - P(32) = 0$

$\rightarrow B_y = 10160\ lbs$

$A_y = 3740\ lbs$

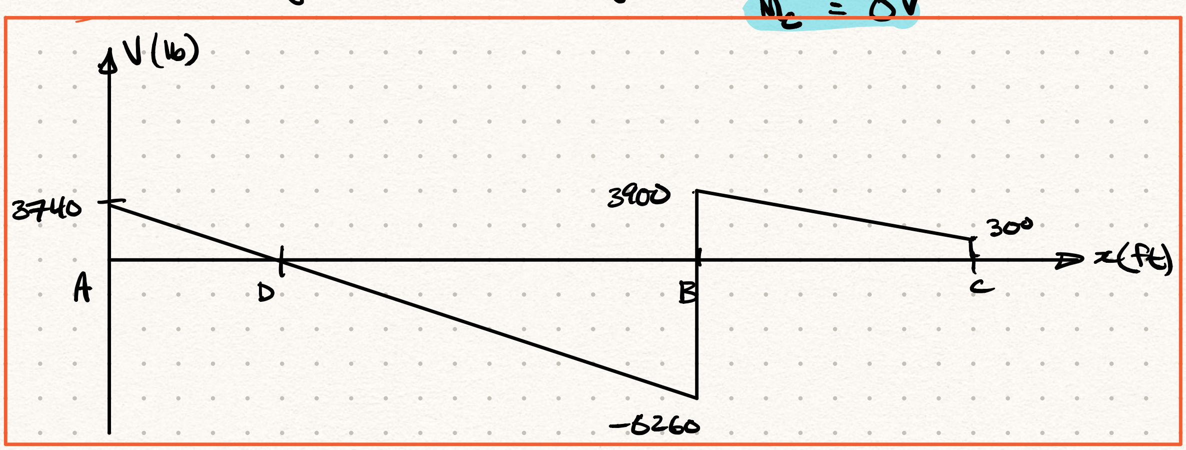
from left, $M_A = 0$

$M_B - A_y \cdot 20 + A_1 \cdot 10 = 0$

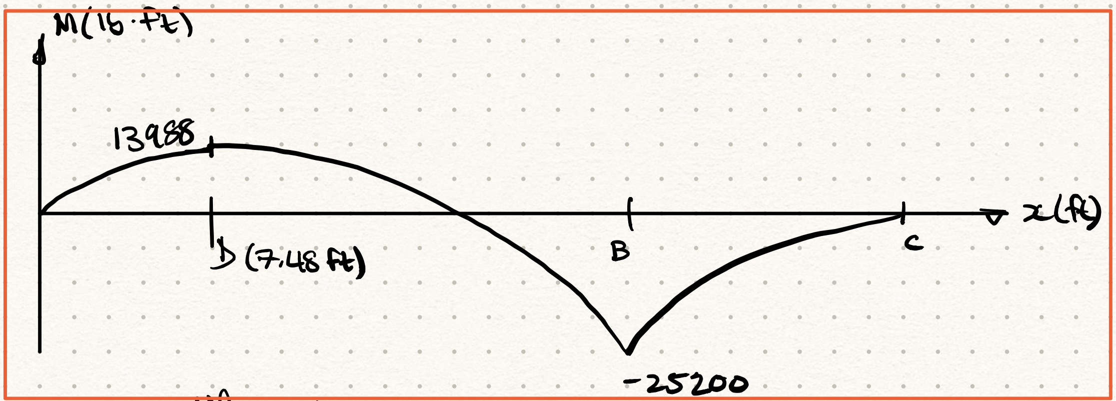
$M_B = -25200\ lb\,ft$

$M_C - A_y \cdot 32 + A_1 \cdot 22 - B_y \cdot 12 + A_2(6) = 0$

$M_C = 0$ ✓



1a) continued $\longrightarrow$



$$\frac{dV}{dx} = -w, \quad \frac{dM}{dx} = V$$

A $\rightarrow$ D

$$\int dV = \int w_1\, dx \rightarrow V - V_0 = -w_1 x$$
$$V = -w_1 x + V_0$$

$$M - M_0 = \int (-w_1 x + V_0)\, dx$$
$$M = \frac{-w_1 x^2}{2} + V_0 x + \cancel{M_0}^{\,0}$$

Max M at $V = 0$

$\rightarrow$ M_{max} @ $x = \frac{V_0}{w_1} = 7.48$ ft.

$$M(7.48\text{ft}) = \frac{-w_1}{2}(7.48)^2 + 3740(7.48)$$

note: max in A$\rightarrow$D region $\quad M_{max} = 13987.6$ lb·ft

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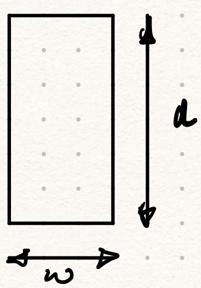
b)



$w = 3.125''$

$\sigma_{all} = 2000$ psi

$\sigma = \frac{Mc}{I}$, $c = \frac{d}{2}$, $M_{max} = |M_B| = 25200$ lb·ft $= 302400$ lb·in

$I = \frac{1}{12} w d^3$

$$\rightarrow \sigma = \frac{M\left(\frac{d}{2}\right)}{\frac{1}{12} w d^3} = \frac{6M}{wd^2}$$

$$d = \sqrt{\frac{6M}{w\,\sigma_{all}}} = \boxed{17.04\text{in}}$$

c)

$$S = \frac{M_{max}}{\sigma_{all}} \quad , \quad M_{max} = 25200 \text{ lb ft} = 302400 \text{ lb in} \quad , \quad \sigma_{all} = 24000 \text{ psi}$$

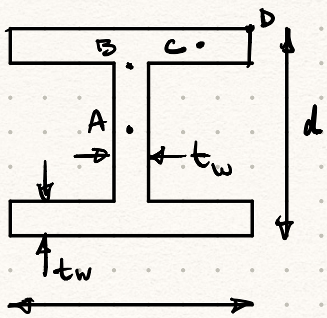
$\rightarrow \quad S > 12.6$

most economical has lowest area with $S > 12.6$

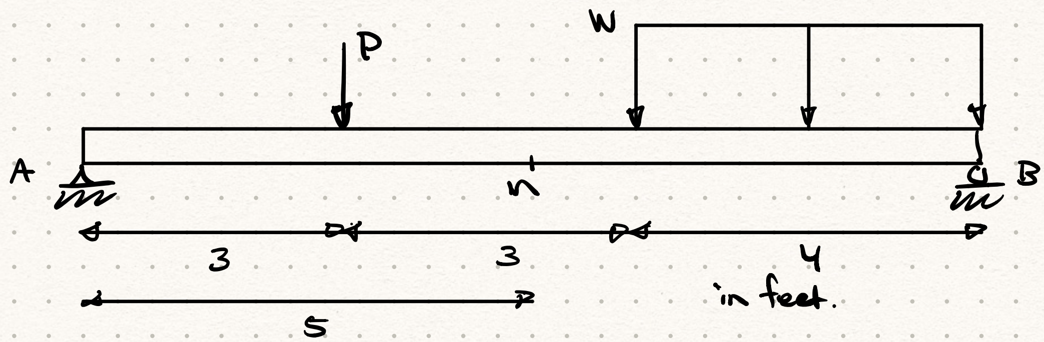
$\rightarrow \boxed{W10 \times 15}$

②

<u>G:</u> W4x13 beam



$A = 3.83 \text{ in}^2$
$d = 4.16 \text{ in}$
$t_f = 0.345 \text{ in}$
$t_w = 0.280 \text{ in}$
$I_{xx} = 11.3 \text{ in}^4$



$P = 4 \text{ kip}, \quad w = 1 \text{ kip/ft}$

$\Sigma F_y = 0 \rightarrow A_y + B_y - P - w(4) = 0$

$\Sigma M_A = 0 \rightarrow -P(3) - 4 \cdot w \cdot 8 + B_y(10) = 0$

$B_y = 4.4 \text{ kip}$
$A_y = 3.6 \text{ kip}$

<u>at M:</u>

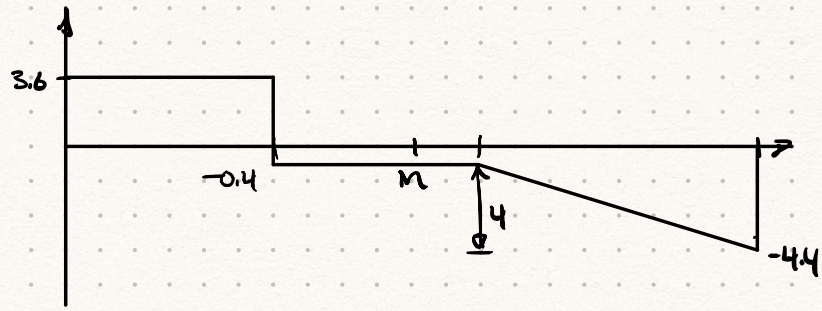
$V_m = A_y - P$
$V_m = -0.4 \text{ kip}$

$M_m + -A_y(5) + P(2) = 0$

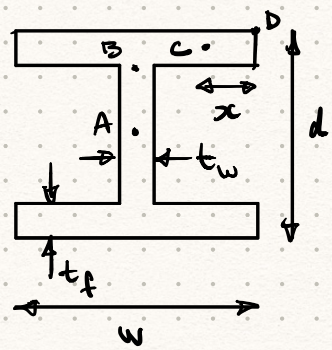
$M_m = +10 \text{ kip ft}$

CONTINUED

CHECK W/ DIAGRAMS



W4x13 beam



$A = 3.83 in^2$
$d = 4.16 in$
$t_f = 0.345 in$
$t_w = 0.280 in$
$I_{xx} = 11.3 in^4$
$x = 1 in$
$w = 4.06 in$

$V_m = -0.4$ kip

$M_m = +10$ kip ft

$$\tau = \frac{VQ}{It}$$

intermediate values:

webbing depth: $d_w = d - 2t_f \rightarrow d_w = 3.47 in$

a) $$Q_A = \Sigma \bar{y} A = \underbrace{\left(\frac{d_w}{4}\right)}_{\bar{y}}\underbrace{\left(\frac{d_w}{2} \cdot t_w\right)}_{A} + \underbrace{\left(\frac{d_w}{2} + \frac{t_f}{2}\right)}_{\bar{y}}\underbrace{\left(w \cdot t_f\right)}_{A}$$

$$Q_A = 3.09 in^3$$

$t_A = t_w \rightarrow$ horizontal shear plane through web

$$\rightarrow \tau_A = \frac{VQ_A}{It_A} = \frac{0.4 kip \cdot 3.09 in^3}{11.3 in^4 \cdot 0.280 in}$$

$$\boxed{\tau_A = 0.391 \text{ ksi} \quad \text{or} \quad 391 \text{ psi}}$$

b) $Q_B = \Sigma \bar{y} A = \underbrace{\left(\frac{d_w}{2} + \frac{t_f}{2}\right)}_{\bar{y}} \underbrace{(w \cdot t_f)}_{A}$

$Q_B = 2.67 \text{ in}^3$

$t_B = t_w \rightarrow$ horizontal shear plane through web

$\rightarrow \tau_B = \frac{VQ_B}{It_B} = \frac{0.4 \text{ kip} \cdot 2.67 \text{ in}^3}{11.3 \text{ in}^4 \cdot 0.280 \text{ in}}$

$\boxed{\tau_B = 0.338 \text{ ksi or } 338 \text{ psi}}$

c) $Q_c = \underbrace{\left(\frac{d_w}{2} + \frac{t_f}{2}\right)}_{\bar{y}} \underbrace{(x \cdot t_f)}_{A}$

$Q_c = 0.658 \text{ in}^3$

$t_c = t_f \rightarrow$ vertical shear plane through flange

$\rightarrow \tau_c = \frac{VQ_c}{It_c} = \frac{0.4 \text{ kip} \cdot 0.658 \text{ in}^3}{11.3 \text{ in}^4 \cdot 0.345 \text{ in}}$

$\boxed{\tau_c = 0.0675 \text{ ksi or } 67.5 \text{ psi}}$

d) $Q_d = 0$ since $A = 0$

$\rightarrow \boxed{\tau_D = 0}$

e) $\boxed{\tau_{max} \text{ occurs at } A, \text{ horizontal centroidal axis.}}$