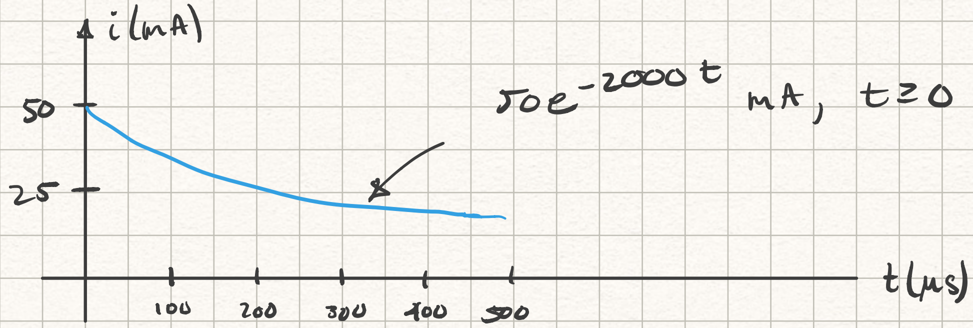
ENGR 221 HW #7, Due 6/5/20, Sean Lai

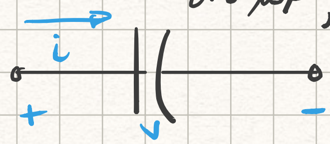
CH 6: 19, 23, 25, 28, 31, 39

6.19

Given:



0.5 μF, −20V



Find: a) E in μJ is stored at $t = 500\,\mu s$

b) " " " " " " $t = \infty$

Solution:

$$i = C\frac{dv}{dt}, \quad p = vi = Cv\frac{dv}{dt}, \quad w = \frac{1}{2}Cv^2$$

$$v(t) = \frac{1}{C}\int_0^t i\,d\tau + v(0), \quad v(0) = -20V$$

$$= \frac{1}{0.5e-6}\int_0^{500e-6} 50e-3\, e^{-2e3}\, d\tau - 20$$

$$= \frac{1}{0.5e-6}\left[\frac{50e-3}{-2e3}\left[e^{-2e3 \cdot 500e-6} - 1\right]\right] - 20$$

$$= -5e1\left[e^{-1} - 1\right] - 20$$

$v(500\mu s) = 11.606\,V$

$$w = \frac{1}{2}Cv^2$$
$$= \frac{1}{2} \cdot 0.5e-6 \cdot (11.606)^2$$

a) $\boxed{w = 33.67\,\mu J}$

b) $v(\infty) = 30$

$$w = \frac{1}{2} \cdot 0.5e-6\,(30)^2 \rightarrow \boxed{w = 225\,\mu J}$$

6.23:

Given: Using realistic inductor values in Appendix H. No energy stored

Find: Series/parallel combinations to yield equivalent inductances below. Minimize inductor count.

a) 8 mH
b) 45 μH
c) 180 μH

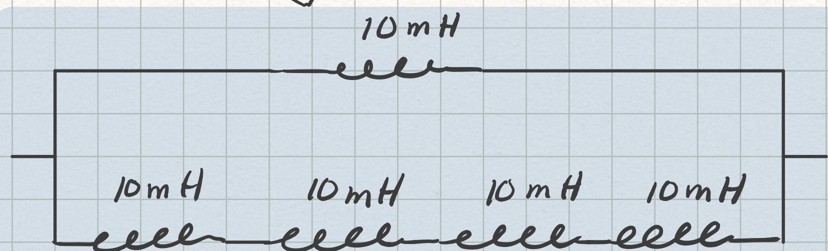
Solution: inductors add like resistors.

Available inductors: 10 μH, 100 μH, 1 mH, 10 mH

a) $L_{eq} = \left[\frac{1}{L_1} + \frac{1}{L_2} + \cdots\right]^{-1} + L_\alpha + L_\beta + \cdots$

$$\frac{1}{8} = \frac{5}{40} = \left[\frac{4}{40} + \frac{1}{40}\right]$$

$$= \left[\underbrace{\frac{1}{10} + \frac{1}{40}}\right]$$



b) $45 = 10 + 10 + 10 + 10 + 5$ 6 inductors...

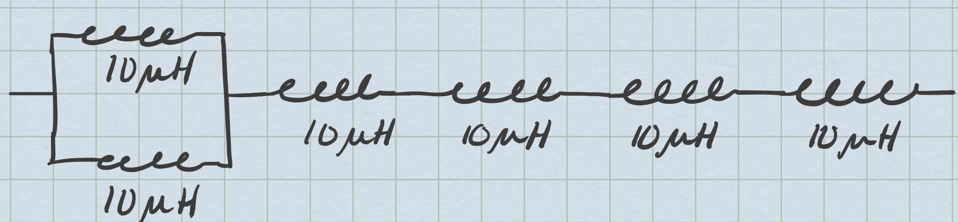
$5 = 10 \| 10$ $2 \times L$

$40 \rightarrow \frac{1}{40} = \frac{5}{200} = \left[\frac{2}{100} + \frac{1}{200}\right]$

$= 100 \| 100 \| 200$ $4 \times L$

still 6

the simple one it is!



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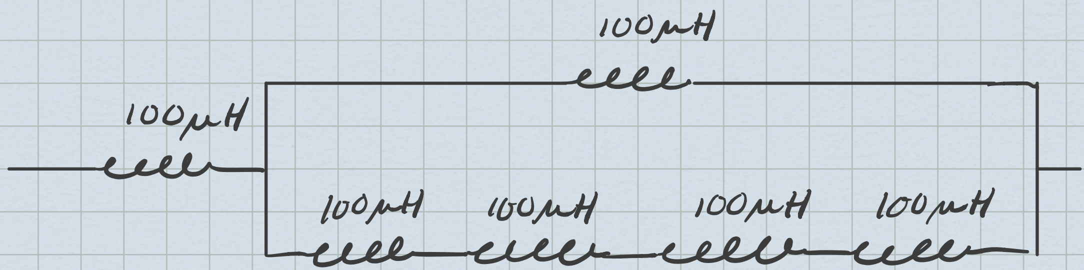
6.23 Continued:

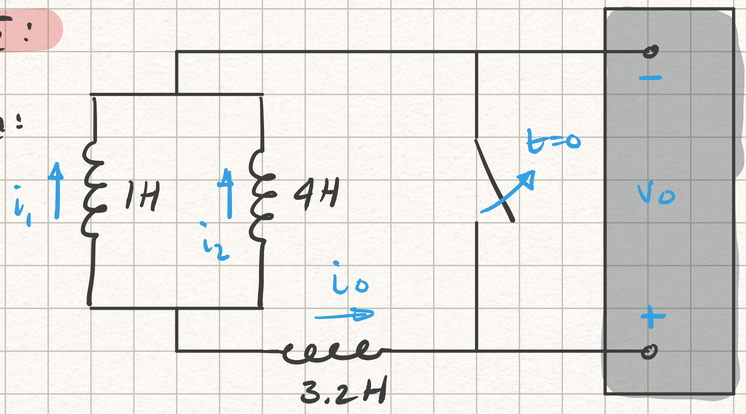
c) $180 = \underset{1}{100} + \underbrace{80}$

$\hookrightarrow \frac{1}{80} = \frac{5}{400} = \left[\underbrace{\frac{1}{100} + \frac{1}{400}}_{5}\right]$



6.25:

Given:



for $t > 0$:

$V_o = 2000 e^{-100t}$ V

$i_1(0) = -6A$

$i_2(0) = 1A$

Find:

a) $i_o(0)$
b) $i_o(t)$, $t \geq 0$
c) $i_1(t)$, $t \geq 0$
d) $i_2(t)$, $t \geq 0$
e) initial energy stored in L's
f) total energy delived to black box
g) energy trapped in the ideal L's

Solution:

$$L_{eq} = 3.2H + 1 \| 4$$

$$L_{eq} = 4.0H$$

a) $i_o(0) = -i_1 - i_2$

$= 6A - 1A$

$$\boxed{i_o(0) = 5A}$$

b)

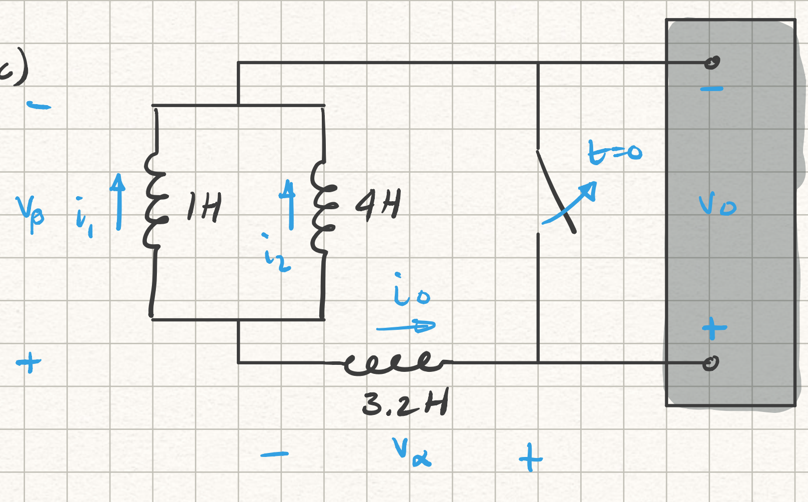
$$i_o(t) = \frac{1}{L_{eq}} \int_0^t -2e3\, e^{-100\tau}\, d\tau + 5$$

note, voltage across L_{eq} is neg. w/ respect to i_o

$$= \frac{-2000}{4 \cdot -100}\left[e^{-100t} - 1\right] + 5$$

$$\boxed{i_o(t) = +5e^{-100t} \text{ A}}$$

c)



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6.25 Continued :

$$v_0 = v_\alpha + v_\beta$$

$$v_\beta = v_0 - v_\alpha$$

now, $v_\alpha = L \cdot -\frac{di_0(t)}{dt}$

$$= 3.2H \cdot -5 \cdot -100e^{-100t}$$

$$= 1600\, e^{-100t}$$

$$v_\beta = 2000e^{-100t} - 1600e^{-100t}$$

$$v_\beta = 400\, e^{-100t}$$

$$i_1(t) = \frac{1}{L_1}\int_0^t 400e^{-100\tau}\, d\tau + -6$$

$$i_1(t) = -4(e^{-100t} - 1) - 6$$

c) $\boxed{i_1(t) = -4e^{-100t} - 2\text{ A}}$

d) $i_2(t) = \frac{1}{L_2}\int_0^t v_\beta\, d\tau + 1$

$$i_2(t) = -1(e^{-100t} - 1) + 1$$

$$\boxed{i_2(t) = -e^{-100t} + 2\text{ A}}$$

e) $w = \frac{1}{2}Li^2$

$$w = w_0 + w_1 + w_2$$

$$w = \frac{1}{2}\left[3.2(5)^2 + 1(6)^2 + 4(1)^2\right]$$

$$\boxed{w(0) = 60\text{ J}}$$

f) $w_{BB} = \frac{1}{2}L_{eq}\, i_{eq}^2$, $L_{eq} = 4H$, $i_{eq} = i_0(0) = 5A$

$$w_{BB} = \frac{1}{2} \cdot 4H \cdot 5A^2$$

$$\boxed{w_{BB} = 50\text{ J}}$$

g) $w_{trapped} = w(0) - w_{BB} \longrightarrow \boxed{w_{trapped} = 10\text{ J}}$

6.28:

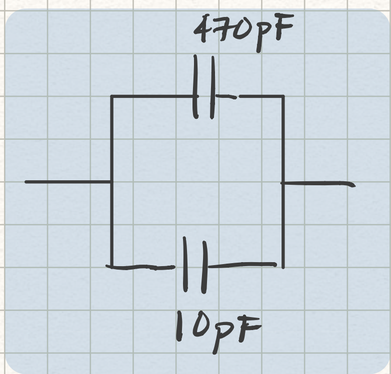
Given: Using realistic caps in Appendix H

Find: Series/parallel combinations to yield capacitances below. Minimize capacitors.

a) 480 pF
b) 600 nF
c) 120 μF

Solution:

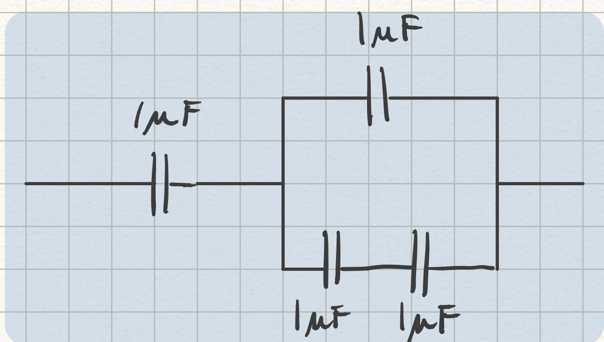
a) 10 pF + 470 pF



b) $$\frac{1}{600} = \frac{5}{3000} = \underbrace{\frac{1}{1000}}_{1\mu F} + \frac{2}{3000} = \frac{1}{1000} + \frac{1}{1500}$$

600nF

$$= \frac{1}{1000} + \frac{1}{\underbrace{1000}_{1\mu} + \underbrace{500}_{1\mu // 1\mu}}$$



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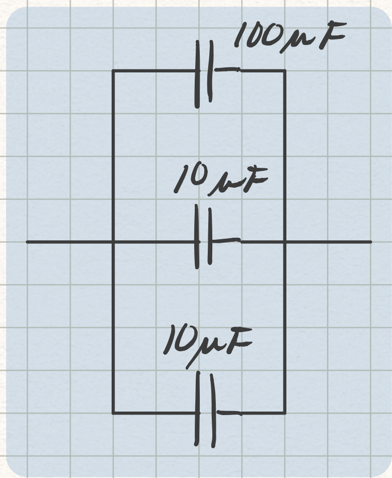
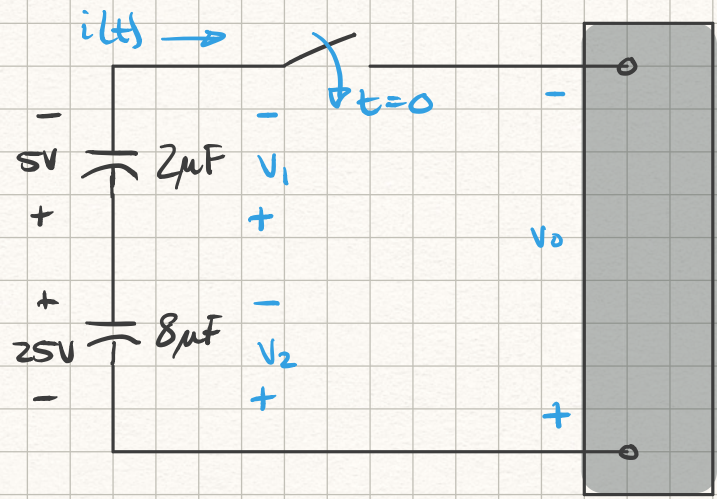
6.28 Continued:

c) $120\mu F = 100 + 10 + 10$



6.31:

Given:



$$i(t) = 800e^{-25t}\ \mu A$$

Find:
a) use C_{eq} to find $v_0(t)$ for $t \geq 0$
b) $v_1(t)$, $t \geq 0$
c) $v_2(t)$, $t \geq 0$
d) energy delivered from $0 \leq t < \infty$
e) energy initially stored in caps
f) energy trapped in ideal caps
g) show that v_1 and v_2 agree with part f)

Solution:

$$C_{eq} = C_1 \| C_2$$

$$= \left(\frac{1}{2\mu} + \frac{1}{8\mu}\right)^{-1}$$

$$C_{eq} = 1.6\ \mu F$$

$$v_0(t) = \frac{1}{C}\int_0^t i\, d\tau + v_0(0) \quad , \quad \text{from figure, } v_0(0) = -25 + 5 = -20V$$

$$v_0(t) = \frac{800e\text{-}6}{1.6e\text{-}6} \cdot -\frac{1}{25} \cdot \left[e^{-25t} - 1\right] - 20$$

$$v_0(t) = -20e^{-25t} + 20 - 20$$

a) $$\boxed{v_0(t) = -20e^{-25t}\ V}$$

b) $$v_1(t) = \frac{1}{C_1}\int_0^t i(\tau)\, d\tau + 5$$

$$v_1(t) = \frac{800}{2} \cdot -\frac{1}{25}\left[e^{-25t} - 1\right] + 5$$

$$\boxed{v_1(t) = -16e^{-25t} + 21\ V}$$

CONTINUED →

6.31 Continued :

c)

$$v_2(t) = \frac{1}{C_2}\int_0^t i(\tau)\,d\tau + -25$$

$$v_2(t) = \frac{800}{8} \cdot -\frac{1}{25}\left[e^{-25t} - 1\right] - 25$$

$$v_2(t) = -4e^{-25t} + 4 - 25$$

$$\boxed{v_2(t) = -4e^{-25t} - 21 \text{ V}}$$

d)

$$w = \frac{1}{2}Cv^2 \quad , \quad C = C_{eq} \quad , \quad v = v_0(0) = -20$$

$$w = \frac{1}{2}(1.6\mu)(20)^2$$

$$w_d = 320\mu J$$

or check :

$$p = -vi$$

$$p = -(-20e^{-25t})(800 \cdot 10^{-6} e^{-25t})$$

$$p = +1.6 \cdot 10^{-2} e^{-50t}$$

$$dw = p\,dt$$

$$w = \int_0^\infty 1.6 \cdot 10^{-2} e^{-50t}\,dt$$

$$w = -3.2 \cdot 10^{-4}[0 - 1]$$

$$\boxed{w_d = 320\mu J} \checkmark$$

e)

$$w_i = w_1 + w_2$$

$$= \frac{1}{2}\left[C_1 v_1(0)^2 + C_2 v_2(0)^2\right]$$

$$= \frac{1}{2}\left[2\mu(5)^2 + 8\mu(-25)^2\right]$$

$$\boxed{w_i = 2525\mu J}$$

f)

$$w_{trapped} = w_i - w_d$$

$$\boxed{w_{trapped} = 2205\mu J}$$

CONTINUED →

6.31 Continued:

g) $v_1(t) = -16e^{-25t} + 21\ V$

$v_2(t) = -4e^{-25t} - 21\ V$

as $t \to \infty$, $v_1(t) = 21V$
$v_2(t) = -21V$

$w_{trapped} = w_{1\infty} + w_{2\infty}$

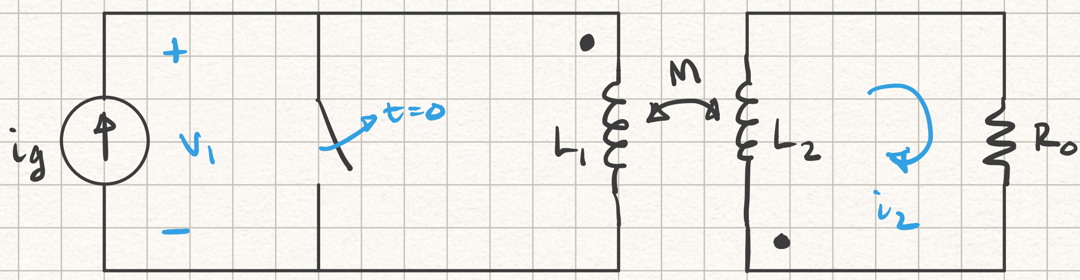
$= \frac{1}{2}[C_1(v_1)^2 + C_2(v_2)^2]$

$= \frac{1}{2}(21)^2[C_1 + C_2]$

$\boxed{w_{trapped} = 2205\ \mu J}$ ✓

6.39:

Given:



No initial energy stored

Find: a) Derive differential eq. for i_2 if:

$L_1 = 5H$, $L_2 = 0.2H$
$M = 0.5H$, $R_o = 10\Omega$

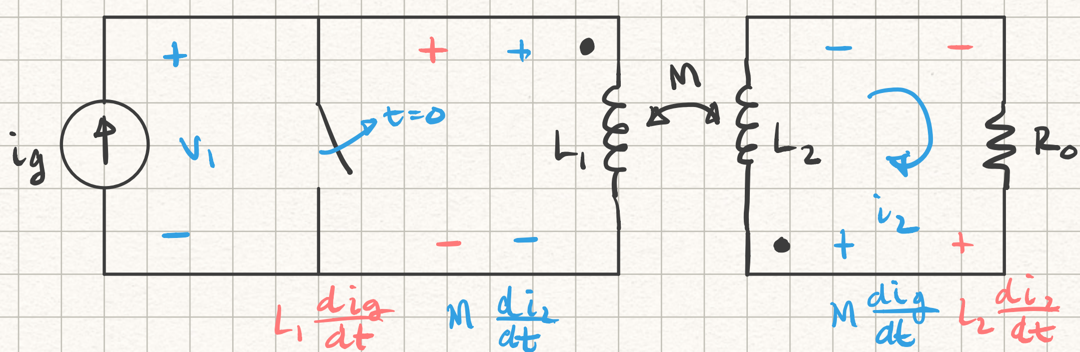
b) Show that for $i_g = e^{-10t} - 10$ A, $t \geq 0$, eq. from a) is satisfied when $i_2 = 625e^{-10t} - 250e^{-50t}$ mA

c) Find an expression for v_1

d) What is the initial value of v_1? Does this make sense in terms of known circuit behavior?

Solution:

a)



mesh 1: $-v_1 + L_1 \frac{di_g}{dt} + M\frac{di_2}{dt} = 0$

mesh 2: $i_2 R_o + L_2 \frac{di_2}{dt} + M\frac{di_g}{dt} = 0$

from mesh 2:
$$\frac{di_2}{dt} = \frac{1}{L_2}\left[-i_2 R_o - M\frac{di_g}{dt}\right]$$

$$\frac{di_2}{dt} = 5\left[-10 i_2 - 0.5\frac{di_g}{dt}\right]$$

$$\boxed{\frac{di_2}{dt} = -50 i_2 - 2.5\frac{di_g}{dt}}$$

CONTINUED →

6.39 Continued:

b) let $i_g = e^{-10t} - 10\,A$, $i_2 = 625e^{-10t} - 250e^{-50t}\,mA$

$$\frac{d}{dt}(i_2) = -50 i_2 - 2.5 \frac{d}{dt} i_g$$

$$\frac{d}{dt} i_2 = -6.25e^{-10t} + 12.5e^{-50t}\ A$$

$$\frac{d}{dt} i_g = -10e^{-10t}\ A$$

$$\rightarrow \ -6.25e^{-10t} + 12.5e^{-50t} = -50\left(0.625e^{-10t} - 0.25e^{-50t}\right) - 2.5\left(-10e^{-10t}\right)$$

$$-6.25e^{-10t} + 12.5e^{-50t} = -31.25e^{-10t} + 25e^{-10t} + 12.5e^{-50t}$$

$$\boxed{\begin{gathered}-6.25e^{-10t} + 12.5e^{-50t} = -6.25e^{-10t} + 12.5e^{-50t} \\ 0 = 0 \ \checkmark\end{gathered}}$$

c) $v_1 = L_1 \frac{di_g}{dt} + M \frac{di_2}{dt}$

using values from b)

$$V_1 = 5\left(-10e^{-10t}\right) + 0.5\left(-6.25e^{-10t} + 12.5e^{-50t}\right)$$

$$V_1 = -50e^{-10t} - 3.125e^{-10t} + 6.25e^{-50t}$$

$$\boxed{V_1 = -53.125e^{-10t} + 6.25e^{-50t}\ V}$$

d) $\boxed{V_1(0) = -46.875\,V}$ ← plugging $t = 0$ into expression above.

The voltage should be negative when the switch is first flipped as L_1 will induce a voltage against the source current i_g. Our calculated V_1 is negative, so this makes sense.