ENGR 213 HW #7, Due: 5/20/20, Sean Lai

CH 7: 31, 40, 43, 47, 71, 102, 108

For Mohr's:

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

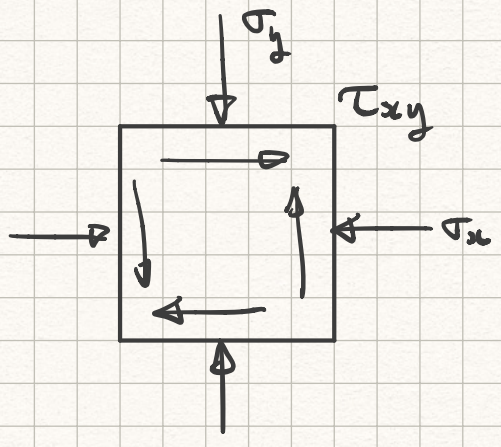
$$\boxed{\sigma_{max,min} = \sigma_{ave} \pm R}$$

$$\tan 2\theta_s = -\frac{1}{\tan 2\theta_p} \rightarrow |\theta_p - \theta_s| = 45^\circ$$

$$\boxed{\tau_{max} = R}$$

7.31:

Given:



$\sigma_x = -60$ MPa
$\sigma_y = -40$ MPa
$\tau_{xy} = 35$ MPa

Find:
a) Principle planes
b) Principle stresses
a') Orientation of τ_{max} plane
b') τ_{max}
c') corresponding σ

} using Mohr's circle

Solution:

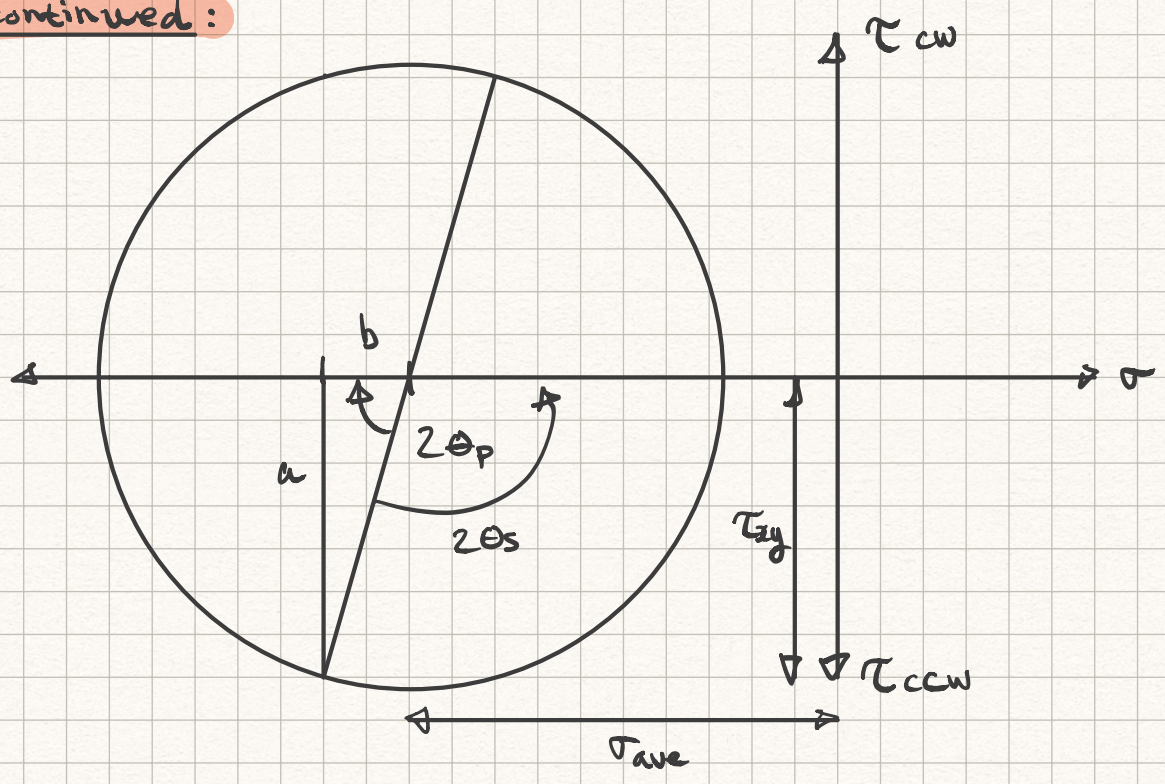
Find σ_{ave}:

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$$

$$a = \tau_{xy} = 35 \text{ MPa}$$
$$b = \frac{1}{2}(\sigma_x - \sigma_y) = 10 \text{ MPa}$$
$$R = \sqrt{a^2 + b^2} = 36.4 \text{ MPa}$$

CONTINUED →

7.31 continued :



a) $\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{a}{b}\right)$

$\theta_{p_1} = -37°$ cw → neg. or $\theta_{p_2} = \theta_{p_1} + 90° = +53°$

b)

$\sigma_{max} = \sigma_{ave} - R$ $\quad$ $\sigma_{min} = \sigma_{ave} + R$

$\sigma_{max} = -86.4\ MPa$ $\quad$ $\sigma_{min} = -13.6\ MPa$

a') $\tan 2\theta_s = -\frac{1}{\tan 2\theta_p}$

$\rightarrow \theta_s = -\frac{1}{2}\tan^{-1}\left(\frac{1}{\tan 2\theta_p}\right)$

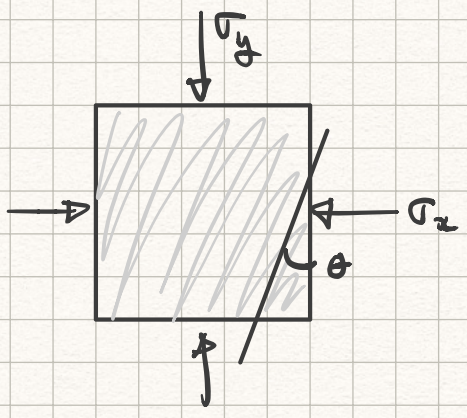
$\theta_{s_1} = +8°$ or $\theta_{s_2} = \theta_{s_1} + 90° = 98°$

b') $\tau_{max} = R = 36.4\ MPa$

c') σ @ $\tau_{max} = \sigma_{ave} = -50\ MPa$

7.40:

Given:



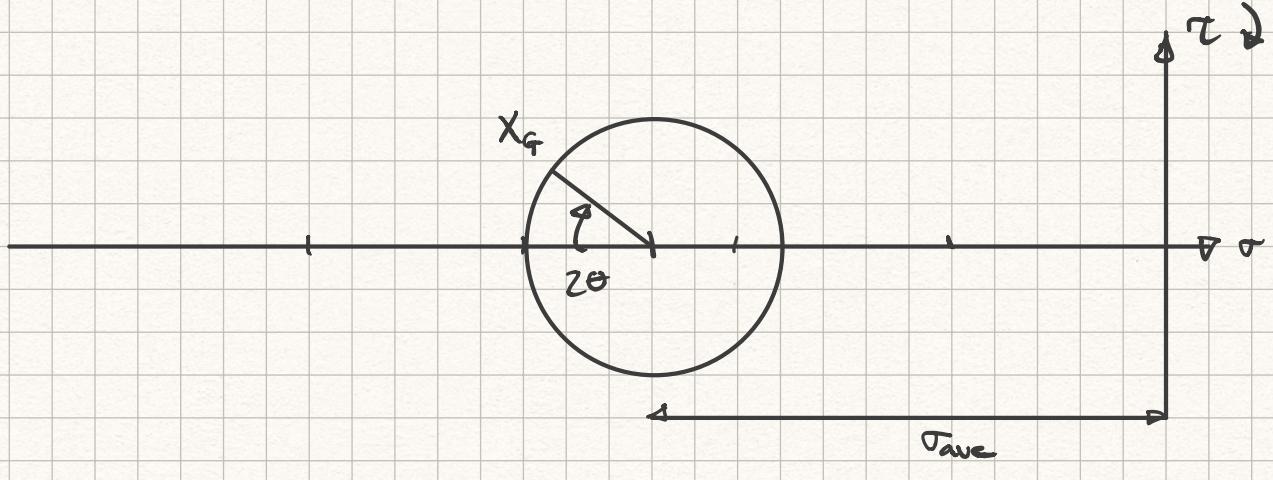
$\sigma_x = -3\,MPa$
$\sigma_y = -1.8\,MPa$
$\theta = 15°$

Find: a) τ_G, $\parallel$ wood grain
b) σ_G, $\perp$ wood grain

Solution:

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2.4\,MPa$$

X: $(\sigma_x, \tau_{xy}) = (-3, 0)$
Y: $(\sigma_y, \tau_{xy}) = (-1.8, 0)$

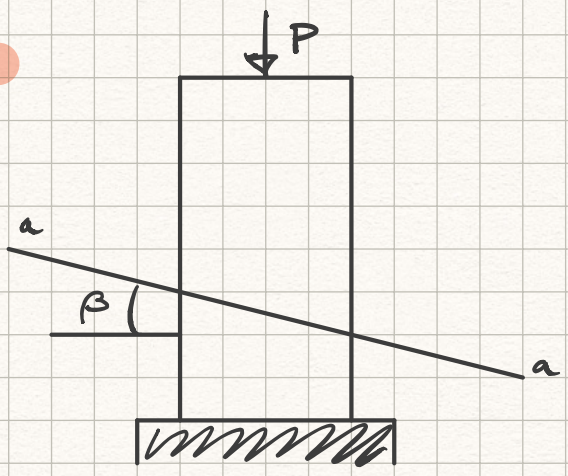


$\rightarrow R = 0.6$, $\tau_{max} = 0.6$

$\tau_G = R \sin 2\theta \longrightarrow \tau_G = 0.3\,MPa$
$\sigma_G = \sigma_{ave} - R\cos 2\theta \longrightarrow \sigma_G = -2.92\,MPa$

7.13:

Given:

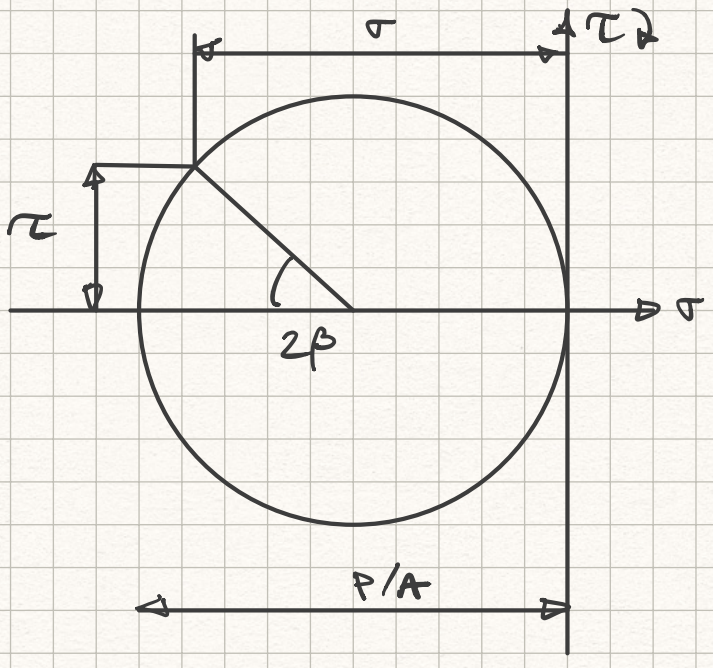


$\sigma = -15$ ksi
$\tau = 5$ ksi
} @ a-a

Find: a) β
b) σ_{max} for post

Solution:

from figure, $\sigma_x = 0$ → X: $(\sigma_x, \tau_{xy}) = (0, 0)$
Y: $(\sigma_y, \tau_{xy}) = (-P/A, 0)$



$$\tau = R \sin 2\beta$$
$$\sigma = R + R\cos 2\beta$$
$$R = \frac{\tau}{\sin 2\beta}$$
$$\sigma = \frac{\tau}{\sin 2\beta} + \frac{\tau}{\sin 2\beta} \cdot \cos 2\beta$$
$$\frac{\sigma}{\tau} = \csc 2\beta + \cot 2\beta$$

... ugh

CONTINUED →

$$R^2 = \tau^2 + (\sigma - R)^2$$
$$R^2 = \tau^2 + \sigma^2 - 2\sigma R + R^2$$
$$2\sigma R = \tau^2 + \sigma^2$$
$$R = \frac{\tau^2}{2\sigma} + \frac{\sigma}{2}$$

$$R = 8.333 \text{ ksi}$$

$$\tau = R \sin 2\beta \longrightarrow \beta = \frac{1}{2}\sin^{-1}\left(\frac{\tau}{R}\right)$$

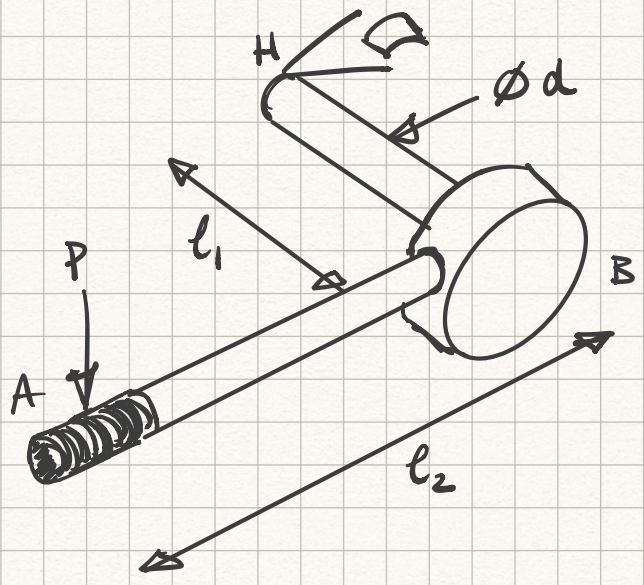
a) $\beta = 18.43^\circ$

b) $\sigma_{max} = 2R$

$$\sigma_{max} = -16.667 \text{ ksi}$$

7.47:

Given:



P = 24 lb
d = 0.75 in
ℓ_1 = 6 in
ℓ_2 = 10 in

Find: a) principal σ's
b) τ_{max}

Solution:

$$\overline{T}_B = +P\ell_2 \quad (ccw)$$

$$\tau = \frac{T\rho}{J} \quad \text{or} \quad \tau = \frac{\overline{T}\rho}{J}, \quad \rho = \frac{d}{2}$$

$$J = \frac{\pi}{2}\rho^4$$

$$\rightarrow \tau = \frac{P\ell_2 \cdot \frac{d}{2}}{\frac{\pi}{2}\left(\frac{d}{2}\right)^3}$$

$$= \frac{16 P\ell_2}{\pi d^3}$$

$$\tau = 2897.32 \text{ psi}$$

$$M_H = P\ell_1 \quad (cw)$$

$$\sigma = \frac{Mc}{I}, \quad c = \frac{d}{2}, \quad I = \frac{\pi}{4}c^4$$

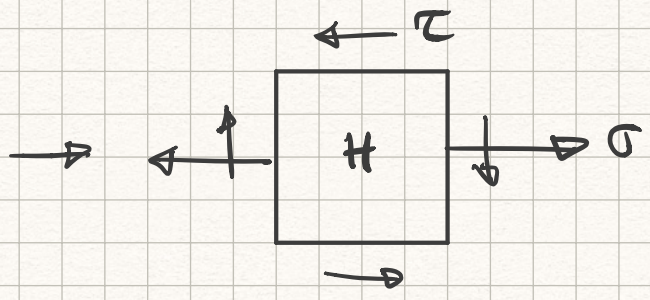
$$\sigma = \frac{P\ell_1 c}{\frac{\pi}{4}c^4}$$

CONTINUED →

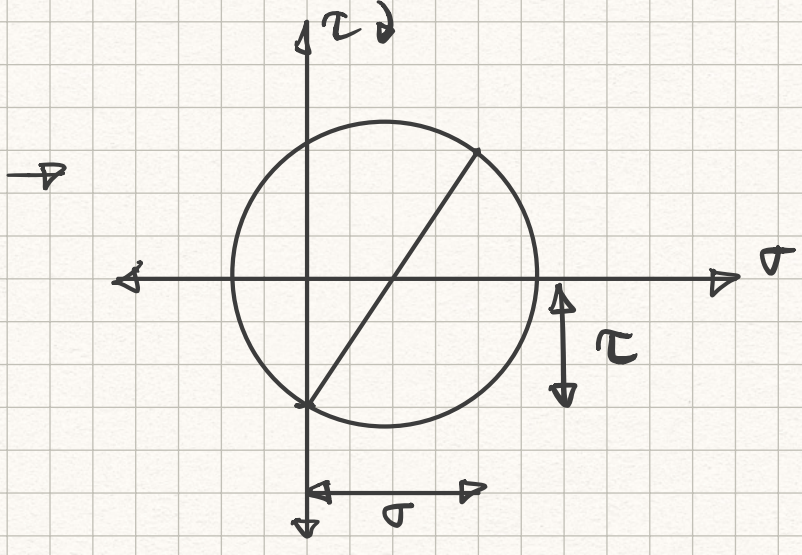
7.47 continued:

$$\sigma = \frac{4P\ell_1}{\pi\left(\frac{d}{2}\right)^3}$$

$\sigma = 3476.79$ psi



X: (σ, τ)
Y: $(0, -\tau)$



$\left(\frac{\sigma}{2}\right)^2 + \tau^2 = R^2 \longrightarrow$ $R = 3378.8$ psi

$$\sigma_{max} = R + \frac{\sigma}{2}$$

a)

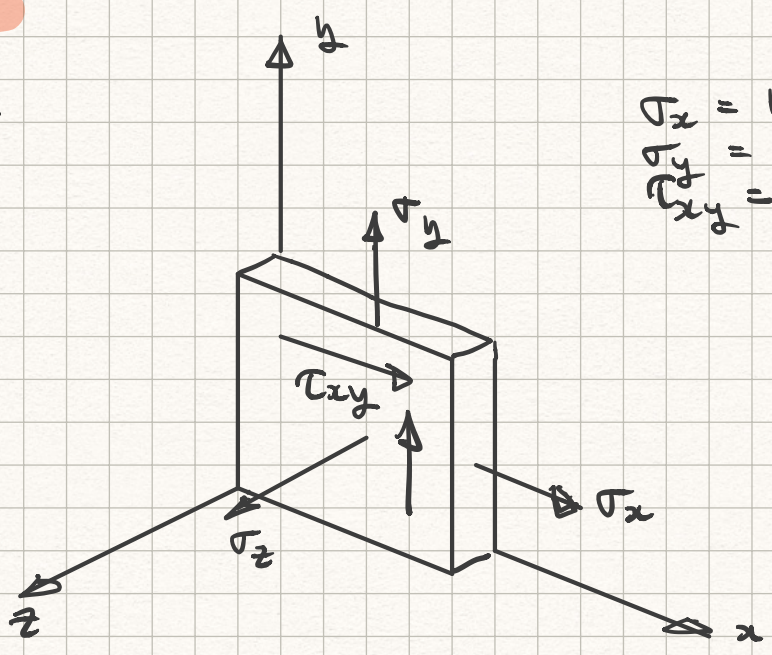
$\sigma_{max} = 5117$ psi

$\sigma_{min} = -1640$ psi

b) $\tau_{max} = R = 3378.8$ psi

7.71:

Given:



$\sigma_x = 170$ MPa
$\sigma_y = 100$ MPa
$\tau_{xy} = 84$ MPa

Find: τ_{max} when...

a) $\sigma_z = 0$

b) $\sigma_z = +60$ MPa

c) $\sigma_z = -60$ MPa

Solution:

$\tau_{xy\ avg} = 135$

X: $(\sigma_x, -\tau)$
Y: $(\sigma_y, +\tau)$

$a = \tau_{xy} = 84$

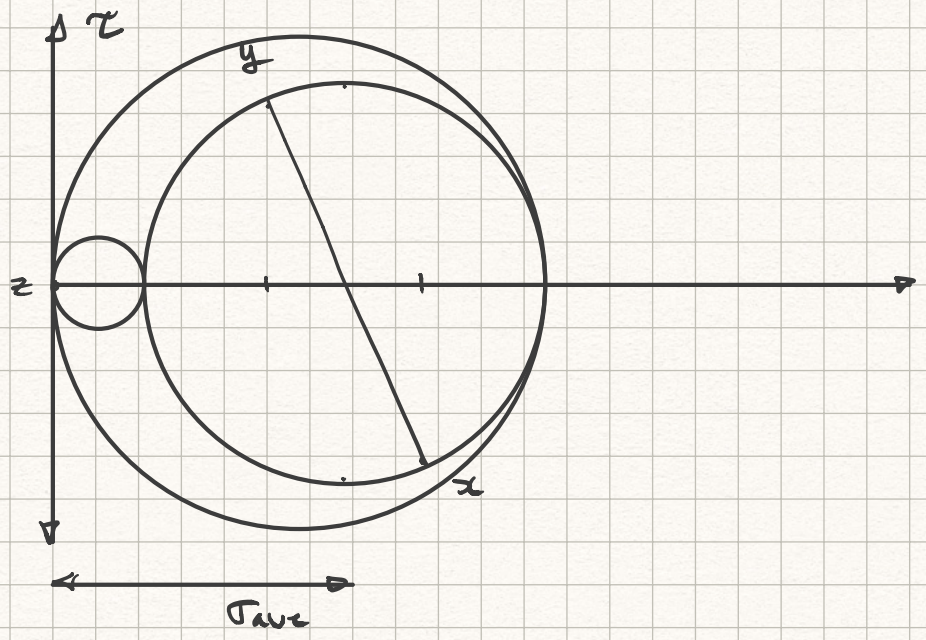
$b = \frac{1}{2}(\sigma_x - \sigma_y) = 35$

$\rightarrow R = \sqrt{35^2 + 84^2}$

$R = 91$

CONTINUED →

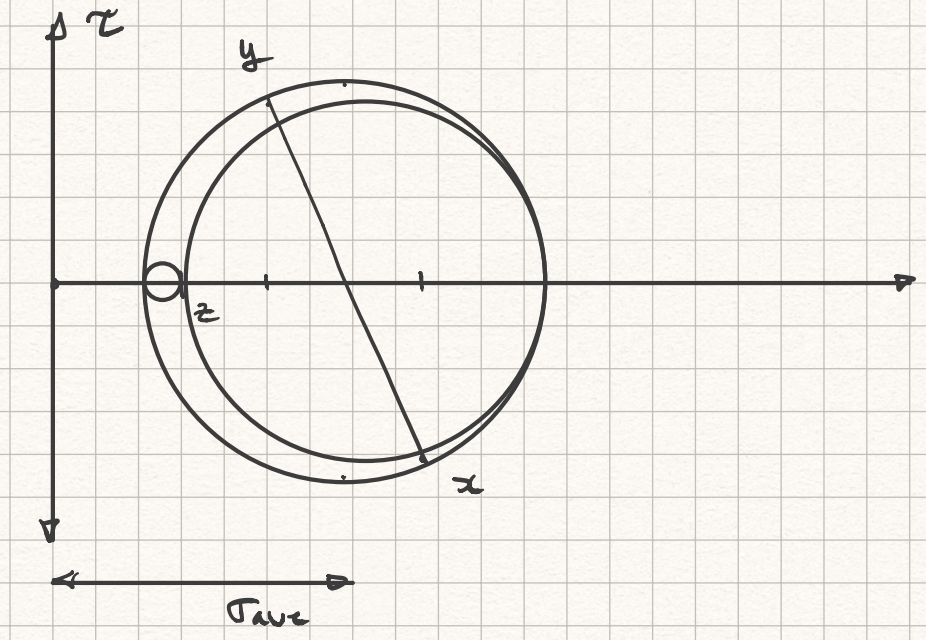
a)



$$\tau_{max} = \frac{\sigma_{ave} + R}{2}$$

$$\boxed{\tau_{max} = 113 \text{ MPa}}$$

b)

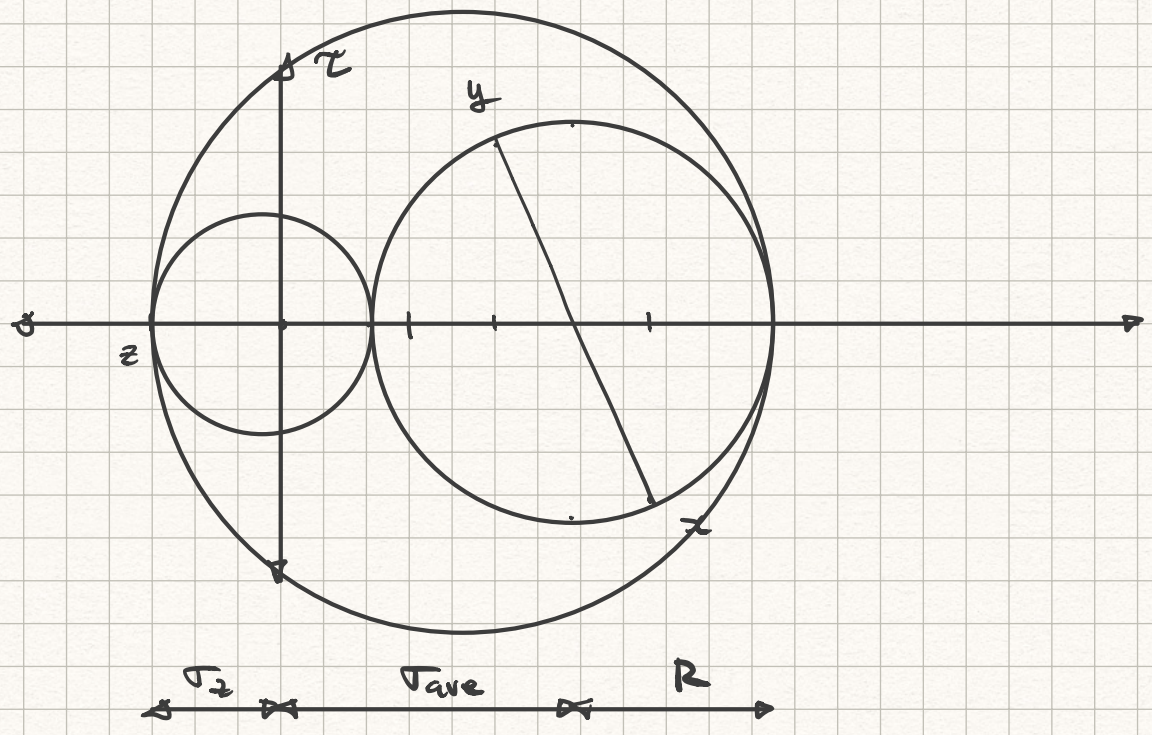


$$\tau_{max} = R$$

$$\boxed{\tau_{max} = 91}$$

CONTINUED →

c)

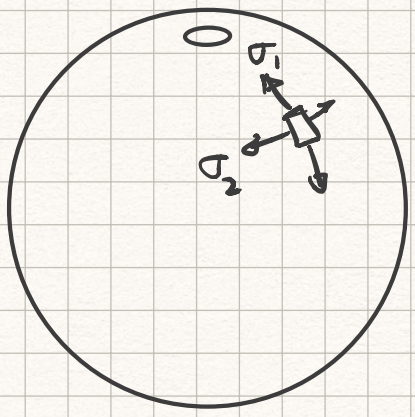


$$\therefore \tau_{max} = \frac{1}{2}(\sigma_z + \sigma_{ave} + R)$$

$$\boxed{\tau_{max} = 143\ MPa}$$

7.102:

Given:



$\varnothing$ $D = 20\,ft. = 240\,in$

$t = 0.4375\,in$

$P = 75\,psi$

$\rightarrow r = 120 - t$

$r = 119.563$

Find: σ_{max}, τ_{max}

Solution:

Eq's 7.36, 7.37: $\sigma = \frac{Pr}{2t}$, $\tau = \frac{\sigma}{2}$

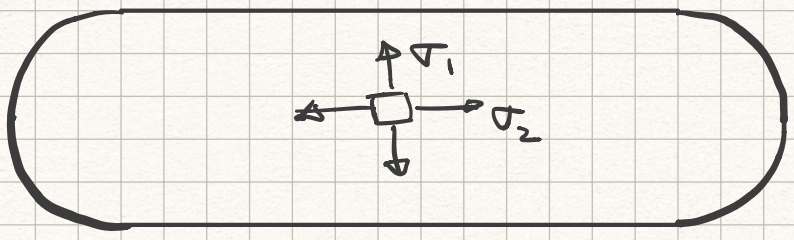
$$\therefore \sigma_{max} = \frac{75\,psi \cdot 119.563}{2 \cdot 0.4375\,in}$$

$$\boxed{\sigma_{max} = 10.25\,ksi}$$

$$\boxed{\tau_{max} = 5.12\,ksi}$$

7.108 !

Given:



$t = 3\ mm$

$O.D. = 320\ mm$

$\rightarrow r = 157\ mm$

LP gas @

1.5 MPa, 38°C

Find: σ_{max}, τ_{max}

Solution:

Eq's 7.30, 7.31: $\underbrace{\sigma_1 = \frac{pr}{t}}_{\text{hoop}}$, $\underbrace{\sigma_2 = \frac{pr}{2t}}_{\text{longitudinal}}$

7.34: $\tau_{max} = \sigma_2$

$$\therefore \sigma_1 = \sigma_{max} = \frac{1.5\,MPa \cdot 0.157}{0.003\,m}$$

$$\boxed{\sigma_{max} = 78.5\ MPa}$$

$$\tau_{max} = \frac{1}{2}\sigma_{max}$$

$$\boxed{\tau_{max} = 39.25\ MPa}$$