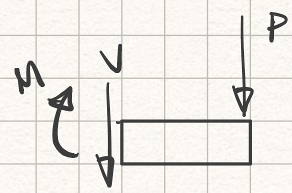
ENGR 213 HW#8, Due 6/1/20, Sean Lai

CH 9: 1, 4, 9, 10, 24, 65, 73, 75, 79

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}, \quad \frac{d^4y}{dx^4} = \frac{w(x)}{EI}, \quad \frac{dV}{dx} = -w \quad \frac{dM}{dx} = V$$

9.1

Given:



Find:
a) eq of elastic curve for AB
b) deflection at free end, B.
c) slope at free end

Solution:

$$M(0) = +PL \rightarrow dM = +P\,dx$$
$$M = +Px + C, \quad M=0 \text{ @ } x=L$$
$$M = +Px - PL \quad \Rightarrow C = -PL$$
$$M = -P(L-x)$$

$$\rightarrow \frac{d^2y}{dx^2} = \frac{-P(L-x)}{EI}$$
$$\frac{dy}{dx} = \frac{-P}{EI}\left[Lx - \frac{x^2}{2}\right] + C_1, \quad \text{@ } x=0,\ C_1 = 0$$
$$y = \frac{-P}{EI}\left[L\frac{x^2}{2} - \frac{x^3}{6}\right] + C_2, \quad \text{@ } x=0,\ C_2 = 0$$

a) $\rightarrow \boxed{y = \frac{-Px^2}{6EI}\left[3L - x\right]}$

CONTINUED →

9.1 Continued:

b) $$y(L) = \frac{-P(L)^2}{6EI}(3L-L)$$

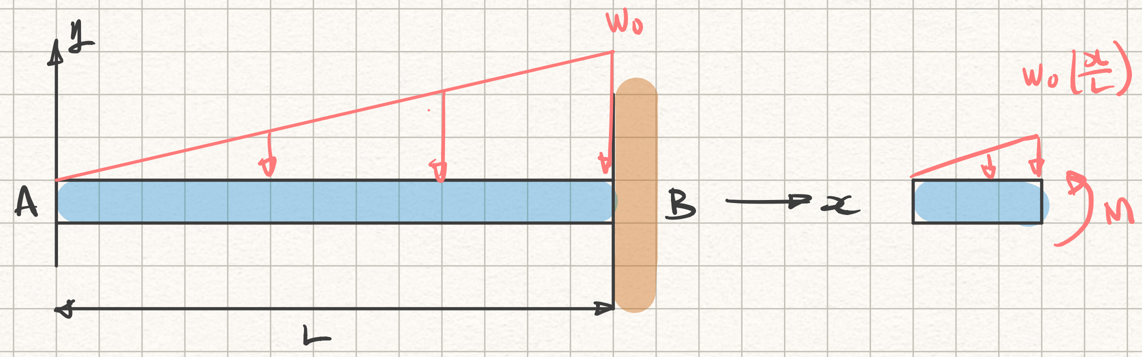
$$\boxed{y(L) = -\frac{PL^3}{3EI}}$$

c) $$\frac{dy}{dx}(x=L) = \frac{-P}{EI}\left[L(L) - \frac{L^2}{2}\right]$$

$$\boxed{\left.\frac{dy}{dx}\right|_{x=L} = \frac{-PL^2}{2EI}}$$

9.4:

Given:



Find: a) eq of elastic curve for AB
b) deflection at free end, A.
c) slope at free end A

Solution:

$$dV = -w\,dx \longrightarrow V = -\frac{w_0}{L}\frac{x^2}{2}$$

$$dM = V\,dx \longrightarrow M = \frac{-w_0}{6L}x^3 + c$$

$$\text{at } x=0,\ M=0 \longrightarrow c=0$$

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

$$\frac{dy}{dx} = -\frac{w_0}{6LEI}\frac{x^4}{4} + c_1$$

at $x=L$, $y'=0$ $\rightarrow$ $c_1 = \frac{+w_0}{24LEI}\cdot L^4$

$$c_1 = \frac{w_0L^3}{24EI}$$

$$y = -\frac{w_0}{24LEI}\int x^4\,dx + \frac{w_0L^3}{24EI}\int dx$$

$$y = \frac{-w_0}{120LEI}x^5 + \frac{w_0L^3}{24EI}x + c_2$$

at $x=L$, $y=0$ $\rightarrow$ $0 = \frac{-w_0}{120EI}L^4 + \frac{w_0L^4}{24EI} + c_2 = 0$

$$c_2 = -\frac{w_0L^4}{30EI}$$

a) $\rightarrow$ $\boxed{y = \frac{-w_0}{120LEI}x^5 + \frac{w_0L^3}{24EI}x - \frac{w_0L^4}{30EI}}$

CONTINUED →

9.4 Continued:

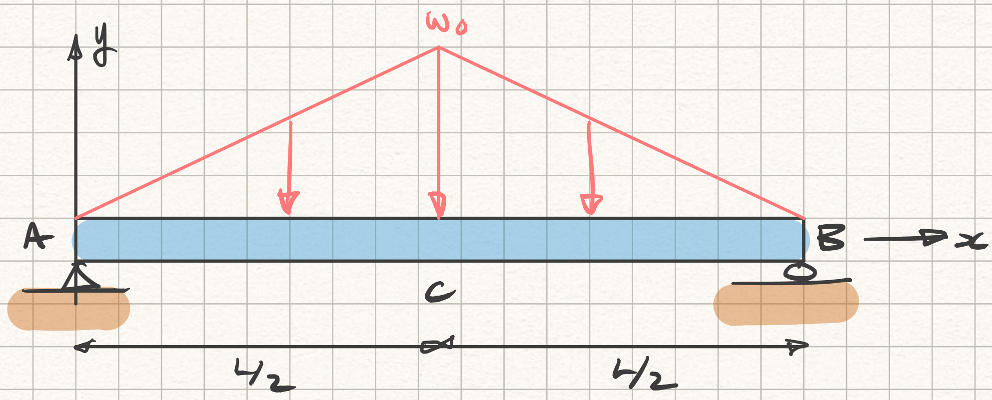
b) $$\boxed{y(0) = -\frac{w_0 L^4}{30EI}}$$

c) $$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{w_0}{6LEI}\frac{(0)^4}{4} + \frac{w_0 L^3}{24EI}$$

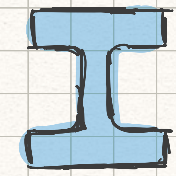
$$\rightarrow \boxed{\left.\frac{dy}{dx}\right|_{x=0} = \frac{w_0 L^3}{24EI}}$$

9.9:

Given:



W10 X 33



$L = 12\,ft = 144\,in$, $w_0 = 3\,kips/ft. = 0.25\,kips/in$, $E = 29 \cdot 10^6\,psi$, $I = 171\,in^4$

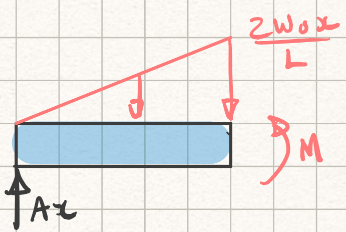
Find: a) slope at A
b) deflection at C

Solution: let force at C.O.M be P, where $P = \frac{w_0 L}{2}$

then:

$$A_y = B_y = \frac{w_0 L}{4} \quad \text{by symmetry.}$$

for $0 \le x \le \frac{L}{2}$:



$$dV = -w\,dx$$

$$V = -\frac{w_0}{L}x^2 + c_V \quad , \quad @\ x=0,\ V = \frac{w_0 L}{4} \ \rightarrow\ c_V = \frac{w_0 L}{4}$$

$$\therefore \ V = -\frac{w_0}{L}\left(x^2 - \frac{L^2}{4}\right)$$

now, $dM = V\,dx$

$$M = -\frac{w_0}{L}\left[\frac{x^3}{3} - \frac{L^2 x}{4}\right] + \cancel{c_M}^{\,0} \quad @\ x=0,\ M=0 \ \rightarrow\ c_M = 0$$

$$\frac{dy}{dx} = \frac{1}{EI}\left[-\frac{w_0}{L}\left[\frac{x^4}{12} - \frac{L^2x^2}{8}\right] + c'\right]$$

at $x = \frac{L}{2}$, $y' = 0$ $\rightarrow$

$$0 = \frac{1}{EI}\left[-\frac{w_0}{L}\left[\frac{L^4}{192} - \frac{L^4}{32}\right] + c'\right]$$

$$c' = \frac{1}{EI} \cdot -\frac{5}{192} w_0 L^3$$

$$\therefore \ \frac{dy}{dx} = \frac{1}{EI}\left[-\frac{w_0}{L}\left[\frac{x^4}{12} - \frac{L^2x^2}{8} + \frac{5}{192}L^4\right]\right]$$

at $x = 0$

$$\frac{dy}{dx} = c' = \frac{-5}{192}\frac{w_0 L^3}{EI}$$

a) $$\boxed{\frac{dy}{dx} = -3.92 \cdot 10^{-3} \quad \text{or} \quad \theta_A = -3.92 \cdot 10^{-3}}$$

since $\tan\theta \sim \theta$ for $|\theta| \ll 1$

CONTINUED →

9.9 Continued:

b) $$\frac{dy}{dx} = \frac{1}{EI}\left[-\frac{w_0}{L}\left[\frac{x^4}{12} - \frac{L^2x^2}{8} + \frac{5}{192}L^4\right]\right]$$

$$y = \frac{1}{EI}\left[-\frac{w_0}{L}\left[\frac{x^5}{60} - \frac{L^2x^3}{24} + \frac{5}{192}L^4x\right]\right]$$

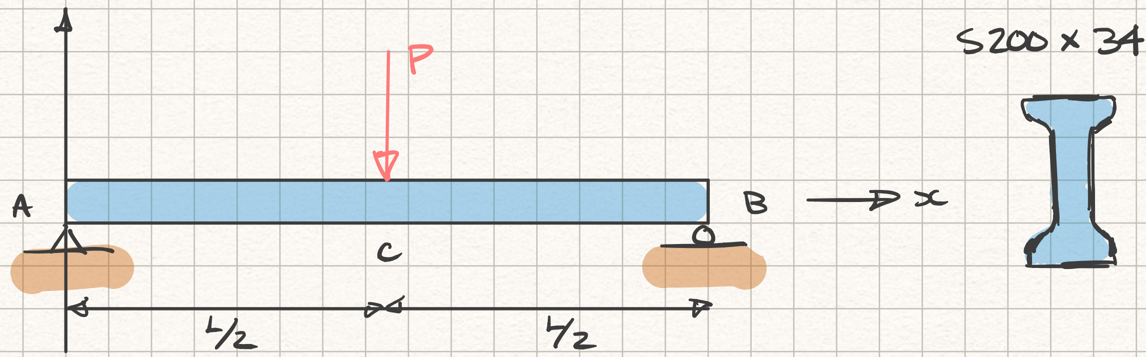
at $x = \frac{L}{2}$ $$y = \frac{-w_0}{EIL}\left[\frac{L^5}{1920} - \frac{L^5}{192} + \frac{5}{384}L^5\right]$$

$$y = \frac{-1}{120}\,\frac{w_0L^4}{EI}$$

$$\boxed{y = 0.1806 \text{ in}}$$

9.10 :

Given :



$P = 60\,kN$ $\quad L = 2m$ $\quad E = 200\,GPa$ $\quad I = 26.9 \cdot 10^6\ mm^4 = 2.69 \cdot 10^{-5}\ m^4$

Find : a) slope at A
b) deflection at C

Solution : $A_y = B_y = \frac{P}{2}$

$V = \frac{P}{2}$

for $0 \le x \le \frac{L}{2}$ $\quad M = \frac{P}{2}x + \cancel{C_M}^{\,0}$ $\quad$ at $x=0$, $M=0 \rightarrow C_M = 0$

$$\frac{dy}{dx} = \frac{1}{EI}\left[\frac{P}{4}x^2 + c'\right] \quad \text{at } x = \frac{L}{2},\ y' = 0$$

$$0 = \frac{1}{EI}\left[\frac{P}{16}L^2 + c'\right]$$

$$c' = -\frac{L^2P}{16EI}$$

$$\therefore \frac{dy}{dx} = \frac{1}{EI}\left[\frac{P}{4}\left(x^2 - \frac{L^2}{4}\right)\right]$$

at $x = 0$: $\quad \frac{dy}{dx} = c'$

$$\therefore \left.\frac{dy}{dx}\right|_{x=0} = -\frac{L^2P}{16EI}$$

a) $$\boxed{\left.\frac{dy}{dx}\right|_{x=0} = 2.788 \cdot 10^{-3}}$$

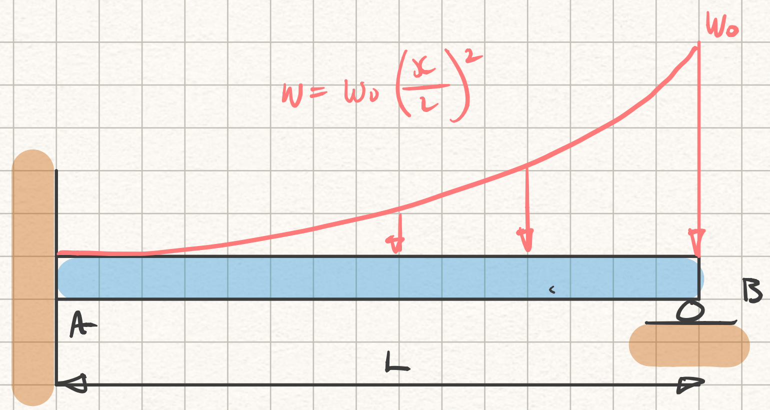
b) $$y = \frac{P}{4EI}\left[\frac{x^3}{3} - \frac{L^2x}{4}\right] + c \quad , \quad y=0 \text{ at } x=0 \rightarrow c=0$$

at $x = \frac{L}{2}$ $\quad y\left(\frac{L}{2}\right) = \frac{P}{4EI}\left[\frac{L^3}{24} - \frac{L^3}{8}\right]$

$$\boxed{y\left(\frac{L}{2}\right) = -1.859 \cdot 10^{-3}\ m}$$

9.24 :

Given :



$w_0 = 15\,kN/m$

$L = 3m$

Find : Reaction at B.

Solution :

$\Sigma F_x = 0$

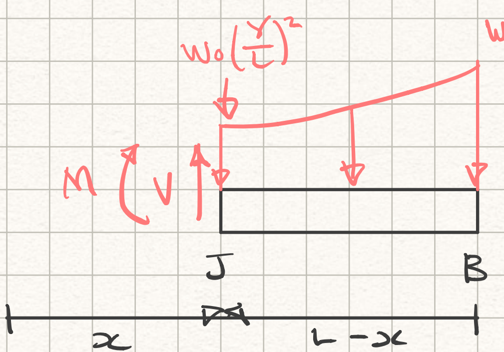
$\Sigma F_y \; : \; A_y + B_y - W_t = 0$

$W_t = \int_0^L \frac{w_0}{L^2} x^2$

$W_t = \frac{w_0 L}{3}$

$\rightarrow \; \boxed{A_y + B_y - \frac{w_0 L}{3} = 0}$

$\Sigma M_A \; : \; B_y L - \frac{w_0 L}{3} x + M_A$



OH BOY, THIS TOOK A WHILE TO UNDERSTAND.

at J :

$$0 = M + B_y L + \int_x^L \underbrace{(\gamma - x)}_{\bar{r}} \underbrace{\left[\frac{w_0}{L^2}\gamma^2\right] d\gamma}_{dA}$$

ΣM due to distributed force.

$$\therefore \quad M = -B_y(L-x) + \frac{w_0}{L^2} \cdot \frac{\gamma^4}{4}\Big|_x^L - \frac{w_0}{L^2} x \frac{\gamma^3}{3}\Big|_x^L$$

$$= -B_y(L-x) + \frac{w_0 L^2}{4} - \frac{w_0 x^4}{4L^2} - \frac{w_0 L x}{3} + \frac{w_0 x^4}{3L^2}$$

$$M = -B_y(L-x) + \frac{1}{12}\frac{w_0}{L^2}x^4 - \frac{w_0 L x}{3} + \frac{w_0 L^2}{4}$$

to check :

$$V = \frac{dM}{dx} = +\frac{1}{3}\frac{w_0}{L^2}x^3 - \frac{w_0 L}{3} + B_y$$

$$A_{JB} = \int_x^L \frac{w_0}{L^2}\gamma^2 d\gamma = \frac{w_0}{3L^2}\left[L^3 - x^3\right] = \frac{w_0 L}{3} - \frac{w_0}{3L^2}x^3$$

$$V = -A_{JB} + B_y \checkmark$$

CONTINUED →

9.24 Continued:

$$M = -B_y(L-x) + \frac{1}{12}\frac{w_o}{L^2}x^4 - \frac{w_o L x}{3} + \frac{w_o L^2}{4}$$

$$EI\frac{dy}{dx} = -B_y L x + \frac{B_y x^2}{2} + \frac{1}{60}\frac{w_o}{L^2}x^5 - \frac{1}{6}w_o L x^2 + \frac{w_o L^2}{4}x + c'$$

at $x=0$, $\frac{dy}{dx}=0$ $\longrightarrow$ $c' = 0$

$$EI\,y = B_y\left[-\frac{Lx^2}{2} + \frac{x^3}{6}\right] + \frac{w_o}{L^2}\left[\frac{1}{360}x^6 - \frac{1}{18}L^3x^3 + \frac{1}{8}L^4x^2\right] + c$$

at $x=0$, $y=0$ $\longrightarrow$ $c = 0$

at $x = L$:

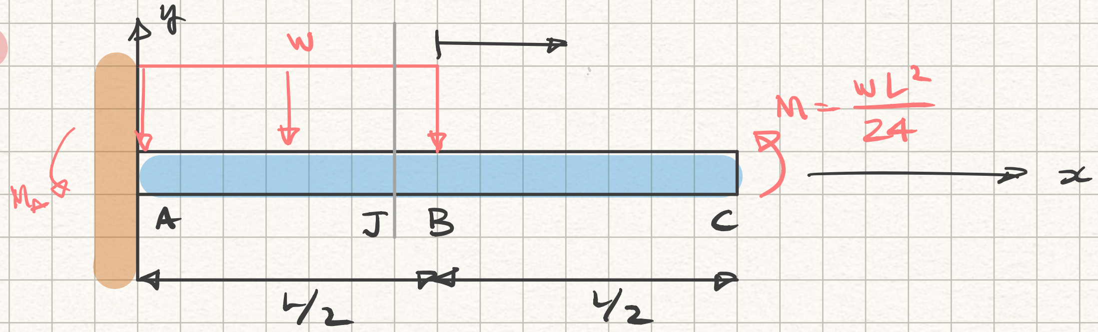
$$0 = B_y\left[-\frac{1}{2} + \frac{1}{6}\right]L^3 + \frac{w_o}{L^2}\left[\frac{1}{360} - \frac{1}{18} + \frac{1}{8}\right]L^6$$

$$B_y = \frac{13}{60}w_o L$$

$$\boxed{B_y = 9.750\text{ kN}}$$

9.65:

Given:



Find: slope and deflection at free end.

Solution:

$$A_y = \frac{wL}{2}$$

ΣM_A: $$0 = M_A - \frac{wL}{2}\left(\frac{L}{4}\right) + \frac{wL^2}{24}$$

$$M_A = \frac{1}{12} wL^2$$

SUPERPOSITION: $$M_{Aw} = \frac{wL}{2}\left(\frac{L}{4}\right) = \frac{wL^2}{8}$$

for w

at J:

$$0 = M + M_{Aw} + wx\left(\frac{x}{2}\right) - A_y x$$

$$M = -\frac{wL^2}{8} - \frac{wx^2}{2} + \frac{wLx}{2}$$

$$EI\theta = w\left[-\frac{x^3}{6} + \frac{Lx^2}{4} - \frac{L^2x}{8}\right]$$

$$EIy = w\left[-\frac{x^4}{24} + \frac{Lx^3}{12} - \frac{L^2x^2}{16}\right]$$

at $x = \frac{L}{2}$:

$$\theta_{wB} = \frac{w}{EI}\left[-\frac{1}{48} + \frac{1}{16} - \frac{1}{16}\right]L^3$$

$$\theta_{wB} = -\frac{1}{48}\frac{wL^3}{EI} = \theta_{wC}$$

$$y_{wB} = \frac{w}{EI} \cdot L^4\left[\frac{-1}{384} + \frac{1}{96} - \frac{1}{64}\right]$$

$$y_{wB} = \frac{wL^4}{EI} \; \frac{-1}{128}$$

$$y_{wC} = y_{wB} + \left(\frac{L}{2}\right)\theta_{wB}$$

$$y_{wC} = -\frac{7}{384}\frac{wL^4}{EI}$$

CONTINUED →

9.65 Continued:

for M_c :
$$0 = M + M_c$$
$$M = \frac{wL^2}{24}$$
$$EI\theta = \frac{wL^2}{24}x + C_c \quad , \quad \theta = 0 \text{ at } x = 0 \rightarrow C_c = 0$$
$$EIy = \frac{wL^2x^2}{48}$$

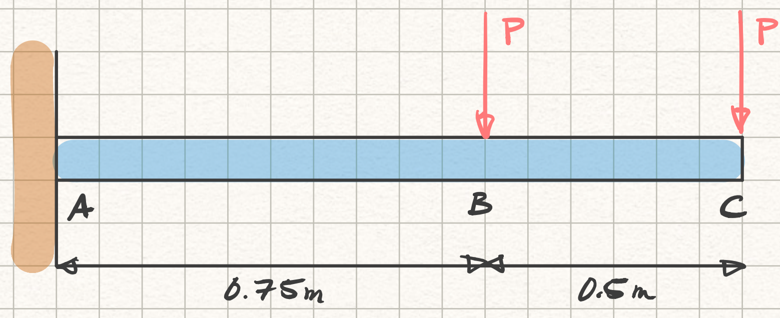
at $x = L$:
$$EI\,\theta_{mc} = \frac{wL^3}{24}$$
$$EI\,y_{mc} = \frac{wL^4}{48}$$

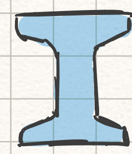
$\rightarrow$ $\theta_c = \theta_{mc} + \theta_{wc}$ $\therefore$ $\boxed{\theta_c = \frac{1}{48}\frac{wL^3}{EI}}$

$y_c = y_{mc} + y_{wc}$ $\therefore$ $\boxed{y_c = \frac{1}{384}\frac{wL^4}{EI}}$

9.73:

Given:





$P = 3kN$, $E = 200 GPa$, $I = 2.52 \cdot 10^6 \ mm^4$
$= 2.52 \cdot 10^{-6} \ m^4$

Find: slope and deflection at C.

Solution:

B: $A_y = P$
$M_A = 0.75P$

$0 = M_B + M_A - A_y x$
$M_B = Px - 0.75P$

$EI\,\theta_B = \frac{P}{2}x^2 - 0.75Px$

$EI\,y_B = \frac{P}{6}x^3 - 0.375Px^2$

at B: $EI\,\theta_B = -0.2813P = \theta_c$
$(x=0.75)$

$y_B = -0.1406P$

$y_c = y_B + \theta_B(0.5)$

$EI\,y_c = -0.2813P$

C: $A_y = P$
$M_A = 1.25P$

$0 = M_c + M_A - A_y x$
$M_c = Px - 1.25P$

$EI\,\theta_c = \frac{P}{2}x^2 - 1.25Px$

$EI\,y_c = \frac{P}{6}x^3 - 0.625Px^2$

CONTINUED →

9.73 Continued :

at A: $EI\theta_c = -0.7813P$
$(x=1.25)$

$EIy_c = -0.6510P$

$\rightarrow \quad EI\theta_T = \theta_B + \theta_c$

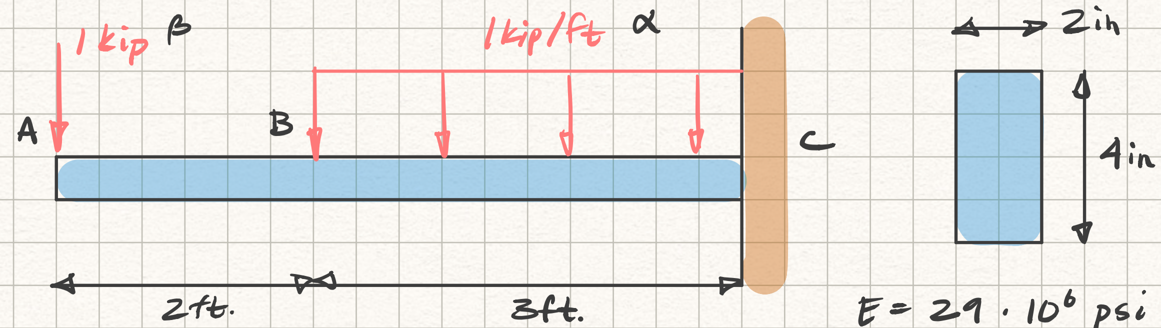
$EIy_T = y_B + y_c$

$\theta_T = -1.0626P/EI \quad \therefore \quad \boxed{\theta_T = 6.325 \cdot 10^{-3}}$

$y_T = -0.9323P/EI \quad \therefore \quad \boxed{y_T = 5.549 \cdot 10^{-3} \ m}$

9.75:

Given:



$E = 29 \cdot 10^6$ psi
$I = 32/3 \text{ in}^4$

Find: Slope and deflection at A

Solution:

α:

$$C_y = 3 \text{ kip}$$

$$M_c = -3 \text{ kip}(1.5 \text{ ft})$$
$$M_c = -4.5 \text{ kip ft}$$

$$0 = M_\alpha + M_c - 1x\left(\frac{x}{2}\right) + C_y x$$

$$M_\alpha = \frac{x^2}{2} - 3x + 4.5$$

$$EI\theta_\alpha = \frac{x^3}{6} - \frac{3}{2}x^2 + 4.5x$$

$$EI y_\alpha = \frac{x^4}{24} - \frac{x^3}{2} + 2.25x^2$$

at B:
($x = 3$ft)

$$\theta_{\alpha B} = \frac{1}{EI} \cdot 4.5 = \theta_{\alpha A}$$

$$y_\alpha = \frac{1}{EI}\left[3.375 - 13.5 + 20.25\right]$$

$$y_{\alpha B} = \frac{1}{EI}\left[10.125\right]$$

$$y_{\alpha A} = y_{\alpha B} + \theta_{\alpha B}(2) \rightarrow y_{\alpha A} = 19.25 \cdot \frac{1}{EI}$$

β:

$$C_y = 1 \text{ kip}$$

$$M_c = -5 \text{ kip ft}$$

$$0 = M_\beta + M_c + C_y x$$

$$M_\beta = 5 - x$$

$$EI\theta_\beta = 5x - \frac{x^2}{2}$$

$$EI y_\beta = \frac{5}{2}x^2 - \frac{x^3}{6}$$

CONTINUED →

9.75 Continued:

at A:
($x = 5ft$)

$$\theta_{\beta A} = \frac{1}{EI}[12.5]$$

$$y_{\beta A} = \frac{1}{EI}[41.67]$$

$$\theta_T = \theta_\alpha + \theta_\beta \qquad y_T = y_\alpha + y_\beta$$

$$\theta_T = \frac{1}{EI}[17 \text{ kip ft}^2] \qquad y_T = \frac{1}{EI}[60.92 \text{ kip ft}^3]$$

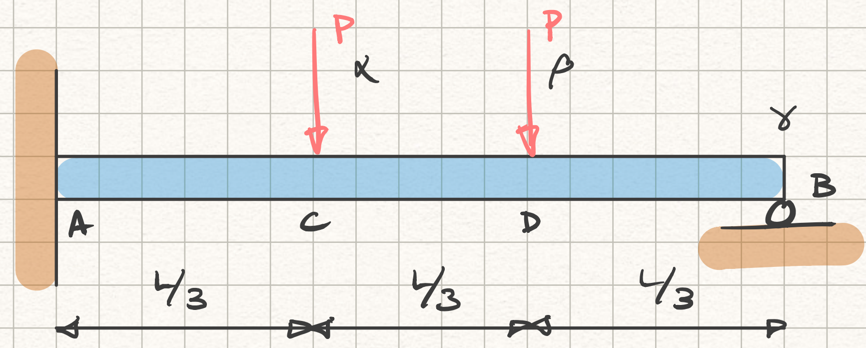
$\therefore$

$$\boxed{\begin{aligned} \theta_T &= 7.91 \cdot 10^{-3} \\ y_T &= -0.3403 \text{ in} \end{aligned}}$$

neg for deflection down

9.79:

Given:



Find: a) Reaction at A
b) Reaction at B

Solution:

α:

$$A_y = P$$

$$M_A = \frac{PL}{3}$$

$$0 = M_\alpha + M_A - A_y x$$

$$M_\alpha = P x - \frac{PL}{3}$$

$$EI\,\theta_\alpha = \frac{P}{2}x^2 - \frac{PL}{3}x$$

$$EI\,y_\alpha = \frac{P}{6}x^3 - \frac{PL}{6}x^2$$

at C: $\left(x = \frac{L}{3}\right)$

$$\theta_\alpha = \frac{1}{EI}\left[\frac{P}{18}L^2 - \frac{P}{9}L^2\right]$$

$$\theta_{\alpha c} = \frac{PL^2}{EI} \cdot -\frac{1}{18} = \theta_{\alpha B}$$

$$y_\alpha = \frac{1}{EI}\left[\frac{P}{162}L^3 - \frac{P}{54}L\right]$$

$$y_{\alpha c} = \frac{PL^3}{EI}\left[-\frac{1}{81}\right]$$

$$y_{\alpha B} = y_{\alpha c} + \theta_{\alpha c}\left(\frac{2L}{3}\right) \longrightarrow y_{\alpha B} = \frac{-4}{81}\frac{PL^3}{EI}$$

β:

$$A_y = P$$

$$M_A = \frac{2PL}{3}$$

$$M_\beta = Px - \frac{2PL}{3}$$

$$EI\,\theta_\beta = \frac{P}{2}x^2 - \frac{2PL}{3}x$$

$$EI\,y_\beta = \frac{P}{6}x^3 - \frac{PL}{3}x^2$$

CONTINUED →

9.79 Continued:

at D:
$(x = \frac{2L}{3})$

$$\theta_{\beta D} = \frac{PL^2}{EI}\left[-\frac{2}{9}\right] = \theta_{\beta B}$$

$$y_{\beta D} = \frac{PL^3}{EI}\left[-\frac{8}{81}\right]$$

$$y_{\beta B} = y_{\beta D} + \theta_{\beta D}\left(\frac{L}{3}\right) \longrightarrow y_{\beta B} = \frac{-14}{81}\frac{PL^3}{EI}$$

$$y_T = y_{\beta B} + y_{\alpha B}$$

$$y_T = -\frac{2}{9}\frac{PL^3}{EI}$$

γ:

$$A_y = -B_y$$

$$M_A = -B_y L$$

$$0 = M_\gamma + M_A - A_y x$$

$$M_\gamma = B_y L - B_y x$$

$$EI\theta_\gamma = B_y L x - B_y \frac{x^2}{2} = B_y\left[Lx - \frac{x^2}{2}\right]$$

$$EI y_\gamma = B_y\left[\frac{Lx^2}{2} - \frac{x^3}{6}\right]$$

at L:

$$B_y = \frac{EI\, y_\gamma}{\frac{Lx^2}{2} - \frac{x^3}{6}} \quad , \quad y_\gamma = -y_T$$

$$B_y = \frac{\cancel{EI} \cdot \frac{P\cancel{L^3}}{\cancel{EI}} \cdot \frac{2}{9}}{\frac{\cancel{L^3}}{2} - \frac{\cancel{L^3}}{6}}$$

b) $\boxed{B_y = \frac{2}{3}P}$

a)
$$A_{yT} = A_{y\alpha} + A_{y\beta} + A_{y\gamma}$$
$$= P + P - \frac{2}{3}P$$

$\therefore \boxed{A_y = \frac{4P}{3}}$

$$M_{AT} = M_{A\alpha} + M_{A\beta} + M_{A\gamma}$$
$$= \frac{PL}{3} + \frac{2PL}{\cancel{3}} - \frac{2}{\cancel{3}}PL$$

$\therefore \boxed{M_A = \frac{PL}{3}}$