

Instructor's and Solutions Manual

to accompany

Mechanics of Materials

Fourth Edition

Volume I, Chapters 1-6

Ferdinand P. Beer

Late of Lehigh University

E. Russell Johnston, Jr.

University of Connecticut

John T. DeWolf

University of Connecticut

Prepared by

Dean P. Updike

Lehigh University

PROPRIETARY AND CONFIDENTIAL

This Manual is the proprietary property of The McGraw-Hill Companies, Inc. ("McGraw-Hill") and protected by copyright and other state and federal laws. By opening and using this Manual the user agrees to the following restrictions, and if the recipient does not agree to these restrictions, the Manual should be promptly returned unopened to McGraw-Hill: This Manual is being provided only to authorized professors and instructors for use in preparing for the classes using the affiliated textbook. No other use or distribution of this Manual is permitted. This Manual may not be sold and may not be distributed to or used by any student or other third party. No part of this Manual may be reproduced, displayed or distributed in any form or by any means, electronic or otherwise, without the prior written permission of the McGraw-Hill.



Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St. Louis
Bangkok Bogotá Caracas Kuala Lumpur Lisbon London Madrid Mexico City
Milan Montreal New Delhi Santiago Seoul Singapore Sydney Taipei Toronto

TABLE OF CONTENTS

To the Instructor	v
Description of Material Contained in <i>Mechanics of Materials, 4th edition</i>	vii
Table I : List of Topics Covered in <i>Mechanics of Materials, 4th edition</i>	xvi
Table II: Classification and Description of Problems.....	xviii
Table III: Sample Assignment Schedule (50% of Problems in SI Units and 50% in U.S. Customary Units)	xxxvi
Table IV: Sample Assignment Schedule (75% of Problems in SI Units and 25% in U.S. Customary Units)	xxxvii
Problem Solutions	xxxix

text for the various loading conditions encountered. Thus, students are presented at an early stage with a method of solution that combines the analysis of deformations with the conventional analysis of forces used in statics, and will have become thoroughly familiar with it by the end of the course. The concept of plastic deformation is also introduced in Chap. 2, where it is applied to the analysis of members under axial loading, while problems involving the plastic deformation of circular shafts and of prismatic beams are considered in optional sections of Chaps. 3 and 4, respectively. On the other hand, while the concept of stress at a point is introduced in Chap. 1, the discussion of the transformation of stresses is delayed until Chap. 7, after students have discovered for themselves the need for special techniques, such as Mohr's circle. In this edition, shear and bending-moment diagrams are introduced

at the beginning of Chap. 5 and applied immediately to the design of beams in that chapter and in the next.

A brief description, chapter by chapter, of the topics included in the text will be found in the following pages. It is hoped that this material will help instructors organize their courses to best fit the needs of their students.

The authors of *Mechanics of Materials*, 4/e, wish to thank Professor Dean P. Updike of the Department of Mechanical Engineering and Mechanics at Lehigh University for having written the problem solutions contained in this Manual.

*E. Russell Johnston, Jr.
John T. DeWolf*

TO THE INSTRUCTOR:

As indicated in its preface, *Mechanics of Materials* is designed for the first course in mechanics of materials - or strength of materials - offered to engineering students in the sophomore or junior year. However, because of the large number of optional sections that have been included and the maturity of approach that has been achieved, this text can also be used to teach a more advanced course.

The text has been divided into units, each corresponding to a well-defined topic and consisting of one or several theory sections followed by sample problems and a large number of problems to be assigned. In order to accommodate courses of varying emphases, considerably more material has been included than can be covered effectively in a single three-credit-hour course. To assist the instructors in making up a schedule of assignments that best fits their classes, the various topics presented in the text have been listed in Table I and both a minimum and a maximum number of periods to be spent on each topic have been suggested. Topics have been divided into three categories: core topics that will probably be covered in every course; additional topics that can be selected to complement this core to form courses of various emphases; and finally topics that can be used with more advanced students.

The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty, with problems requiring special attention indicated by asterisks. The instructor's attention is called to the fact that problems have been arranged in groups of six or more, all problems of the same group being closely related. This means that the instructor will easily find additional problems to amplify a particular point that has been brought up in the

discussion of a problem assigned for homework. Since half of the problems are stated in SI units and half in U.S. customary units, it also means that the instructor has the choice of assigning problems using SI units and problems using U.S. customary units in whatever proportion is found to be most desirable for a given class. To assist in the preparation of homework assignments Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. It should also be noted that answers to all problems with a number set in roman type are given at the end of the text, while problems with a number set in italic are not.

In Table III six alternative lists of possible assignments have been suggested. Four of these lists consist of problems whose answers are given at the end of the text, and two of problems whose answers are not. Half of the problems in each list are stated in SI units and half in U.S. customary units. For those instructors who wish to emphasize the use of SI units, four additional lists of problems have been given in Table IV, in which 75% of the problems use SI units. Since the lists in Tables III and IV cover the entire text, instructors will want to select those groups of sections that are best suited to the course they wish to teach. For a typical one-semester course consisting of 42 class meetings and including four quizzes, as many as 38 of the 46 available groups can be selected.

Since the approach used in this text differs in a number of respects from the approach used in other books, the instructor is advised to read the preface to *Mechanics of Materials*, in which the authors have outlined their general philosophy. Attention is particularly called to the fact that statically indeterminate problems are first discussed in Chapter 2 and are considered throughout the

text for the various loading conditions encountered. Thus, students are presented at an early stage with a method of solution that combines the analysis of deformations with the conventional analysis of forces used in statics, and will have become thoroughly familiar with it by the end of the course. The concept of plastic deformation is also introduced in Chap. 2, where it is applied to the analysis of members under axial loading, while problems involving the plastic deformation of circular shafts and of prismatic beams are considered in optional sections of Chaps. 3 and 4, respectively. On the other hand, while the concept of stress at a point is introduced in Chap. 1, the discussion of the transformation of stresses is delayed until Chap. 7, after students have discovered for themselves the need for special techniques, such as Mohr's circle. In this edition, shear and bending-moment diagrams are introduced

at the beginning of Chap. 5 and applied immediately to the design of beams in that chapter and in the next.

A brief description, chapter by chapter, of the topics included in the text will be found in the following pages. It is hoped that this material will help instructors organize their courses to best fit the needs of their students.

The authors of *Mechanics of Materials*, 4/e, wish to thank Professor Dean P. Updike of the Department of Mechanical Engineering and Mechanics at Lehigh University for having written the problem solutions contained in this Manual.

*E. Russell Johnston, Jr.
John T. DeWolf*

DESCRIPTION OF THE MATERIAL CONTAINED IN "MECHANICS OF MATERIALS", 4th edition

Chapter 1 Introduction – Concept of Stress

The main purpose of this chapter is to introduce the concept of stress. After a short review of Statics in Sec. 1.2 emphasizing the use of free-body diagrams, Secs. 1.3 through 1.7 discuss *normal stresses* under an axial loading, *shearing stresses* – with applications to pins and bolts in single and double shear – and *bearing stresses*. Section 1.4 introduces the concepts of analysis and design, while Sec. 1.5 emphasizes the fact that stresses are inherently statically indeterminate and that, at this point, normal stresses under an axial loading can only be assumed to be uniformly distributed. Moreover, such an assumption requires that the axial loading be centric.

Section 1.8 is devoted to the application of these concepts to the analysis of a *simple structure*. Section 1.9 describes how students should approach the solution of a problem in mechanics of materials, and Sec. 1.10 discusses the *numerical accuracy* to be expected in such a solution. Problems included in the first lesson also serve as a review of the methods of analysis of trusses, frames, and mechanisms learned in statics.

Section 1.11 discusses the determination of normal and shearing stresses *on oblique planes* under an axial loading, while Sec. 1.12 introduces the components of stress under general loading conditions. This section emphasizes the fact that the components of the shearing stresses exerted on perpendicular planes, such as τ_{xy} and τ_{yx} , must be equal. It also introduces the students to the concept of transformation of stress. However, the study of the computational techniques associated with the transformation of stress at a point is delayed

until Chap. 7, after students have discovered for themselves the need for such techniques.

Section 1.13 is devoted to design considerations. It introduces the concepts of *ultimate load*, *ultimate stress*, and *factor of safety*. It also discusses the reasons for the use of factors of safety in engineering practice. The section ends with an optional presentation of an alternative method of design, *Load and Resistance Factor Design*.

Chapter 2 Stress and Strain – Axial Loading

This chapter is devoted to the analysis and design of members under a centric axial loading. Sections 2.1 and 2.2 introduce the concept of *normal strain*, while Sec. 2.3 describes the general properties of the stress-strain diagrams of ductile and brittle materials and defines the *yield strength*, *ultimate strength*, and *breaking strength* of a material. Section 2.4, which is optional, defines *true stress* and *true strain*. Section 2.5 introduces *Hooke's law*, the *modulus of elasticity*, and the *proportional limit* of a material. It defines as *isotropic* those materials whose mechanical properties are independent of the direction considered and as *anisotropic* those whose mechanical properties depend upon that direction. Among the latter are *fiber-reinforced composite materials*, which are described in this section.

Section 2.6 discusses the elastic and the plastic behavior of a material and defines its *elastic limit*, while Sec. 2.7 is devoted to fatigue and the behavior of materials under repeated loadings. The first lesson of Chap. 2 ends with Sec. 2.8, which shows how Hooke's law can be used to determine the deformation of a rod of

uniform or variable cross section under one or several loads, and introduces the concept of *relative displacement*. Section 2.9 discusses *statically indeterminate problems* involving members under an axial load. As indicated in the preface of the text and in the introduction to this manual, the authors believe it is important to introduce the students at an early stage to the concept of statical indeterminacy and to show them how the analysis of deformations can be used in the solution of problems that cannot be solved by the methods of statics alone. It will also help them realize that stresses, being statically indeterminate, can be computed only by considering the corresponding distribution of strains. Section 2.10 discusses the *thermal expansion* of rods and shows how to determine stresses in statically indeterminate members subjected to temperature changes.

Section 2.11 introduces the concept of *lateral strain* for an isotropic material and defines *Poisson's ratio*. Section 2.12 discusses the *multiaxial loading* of a structural element and derives the generalized Hooke's law for such a loading. Since this derivation is based on the application of the *principle of superposition*, this principle is also introduced in Sec. 2.12, and the conditions under which it can be used are clearly stated. Section 2.13 is optional. It discusses the change in volume of a material under a multiaxial loading and defines the *dilatation* and the *bulk modulus* or *modulus of compression* of a given material.

Section 2.14 introduces the concept of *shearing strain*. It should be noted that the authors define the shearing strain as the change in the angle formed by the faces of the element of material considered, and not as the angle through which one of these faces rotates. Hooke's law for shearing stress and strain and the *modulus of rigidity* are also introduced in this section, as well as the *generalized Hooke's law* for a homogeneous, isotropic material under

the most general stress conditions. Section 2.15 points out that strains, just as stresses, depend upon the orientation of the planes considered. It also establishes the fact that the constants E , v , and G are not independent from each other and derives Eq. (2.43), which expresses the relation among these three constants. Section 2.16, which is optional, extends the stress-strain relationships to *fiber-reinforced composite materials*. The relations obtained are expressed by Eqs. (2.45) and (2.47) and involve three different values of the modulus of elasticity and six different values of Poisson's ratio.

Section 2.17 discusses the *distribution of the normal stresses* under a centric axial loading and shows that this distribution depends upon the manner in which the loads are applied. However, except in the immediate vicinity of the points of application of the loads, the distribution of stresses can be assumed uniform. This result verifies Saint-Venant's principle. Section 2.18 discusses *stress concentrations* near circular holes and fillets in flat bars under axial loading.

Section 2.19 is devoted to the *plastic deformation* of members under centric axial loads and introduces the concept of an *elastoplastic material*. As stated in the preface of the text, the authors believe that students should be exposed to the concept of plastic deformation in the first course in mechanics of materials, if only to let them realize the limitations of the assumption of a linear stress-strain relation in engineering applications. By introducing this concept early in the course in connection with axial loading, rather than later with torsion or bending, one makes it easier for the students to understand and accept it. For the same reason, *residual stresses* are discussed in Sec. 2.20 in connection with axial loading. However, since some instructors may not want to include the concept of residual stresses in an elementary course, this section is optional and can be omitted.

without any prejudice to the understanding of the rest of the text.

Chapter 3

Torsion

Section 3.1 introduces this type of loading, while Sec. 3.2 establishes the relation that must be satisfied, on the basis of statics, by the shearing stresses in a given section of a shaft subjected to a torque. This condition, however, does not suffice to determine the stresses, and one must analyze the deformations which occur in the shaft. This is done in Sec. 3.3, where it is proved that the distribution of shearing strains in a circular shaft is linear. It should be noted that the discussion presented in Sec. 3.3 is based solely on the assumption of rigid end plates, rather than on arbitrary and gratuitous assumptions regarding the deformations of a shaft. The results obtained in this and the following sections clearly depend upon the validity of this assumption, but can be extended to other loading conditions through the application of Saint-Venant's principle.

Section 3.4 is devoted to the analysis of the shearing stresses in the elastic range and presents the derivation of the *elastic torsion formulas* for circular shafts. The section ends with remarks on the transformation of stresses in torsion and the comparison between the failures of ductile and brittle materials in torsion.

The formula for the *angle of twist* of a shaft in the elastic range is derived in Sec. 3.5. This section also contains various applications involving the twisting of single and gear-connected shafts. Section 3.6 deals with the solution of problems involving *statically indeterminate shafts*.

Section 3.7 is devoted to the *design of transmission shafts* and begins with the determination of the torque required to transmit a given power at a given speed, both in SI and

U.S. customary units. Note that the effect of bending on the design of transmission shafts will be discussed in Sec. 8.3, which is optional. Section 3.8 discusses *stress concentrations* at fillets in circular shafts.

Sections 3.9 through 3.11 deal with the *plastic deformations* and *residual stresses* in circular shafts and are optional. Since a similar presentation of the plastic deformations and residual stresses of members in pure bending is given in Chap. 4, the instructor may decide to include only one of these presentations in the course. Section 3.9 describes the general method for the determination of the torque corresponding to a given maximum shearing stress in a shaft made of a material with a *nonlinear stress-strain diagram*, while Secs. 3.10 and 3.11 deal, respectively, with the deformations and the residual stresses in shafts made of an *elastoplastic material*. Sections 3.12 and 3.13 are also optional. They are devoted, respectively, to the torsion of solid members and thin-walled hollow shafts of *noncircular section*.

Chapter 4

Pure Bending

Section 4.1 defines this type of loading and shows how the results obtained in the following sections can be applied to the analysis of other types of loading as well, namely, eccentric axial loadings and transverse loadings. Section 4.2 establishes the relation that must be satisfied, on the basis of statics, by the *normal stresses* in a given section of a member subjected to pure bending. This condition, however, does not suffice to determine the stresses, and one must analyze the *deformations* that occur in the member. This is done in Sec. 4.3, where it is proved that the distribution of normal stresses in a symmetric member in pure bending is linear. It should be noted that no assumption is made in this discussion regarding the deformations of the member, except that the couples should be applied

in such a way that the ends of the member remain plane. Whether this can actually be accomplished is discussed at the end of Sec. 4.5.

Section 4.4 is devoted to the analysis of the normal stresses in the elastic range and presents the derivation of the *elastic flexure formulas*. It also defines the elastic section modulus and ends with the derivation of the formula for the *curvature* of an elastic beam. Section 4.5 discusses the *anticlastic curvature* of members in pure bending and also states the loading conditions required for the ends of the member to remain plane.

Section 4.6 discusses the determination of stresses in members made of several materials and defines the *transformed section* of such members. It also shows how the transformed section can be used to determine the radius of curvature of the member. The section ends with a discussion of the stresses in *reinforced-concrete beams*. Section 4.7 deals with the stress concentrations at fillets and grooves in flat bars under pure bending.

Sections 4.8 through 4.11 are optional. They discuss the plastic deformations and residual stresses in members subjected to pure bending in much the same way that these were discussed in Secs. 3.9 through 3.11 in the case of members in torsion. Section 4.8 describes the general method for the determination of the bending moment corresponding to a given maximum normal stress in a member possessing two planes of symmetry and made of a material with a nonlinear stress-strain diagram. Section 4.9 deals with members made of an *elastoplastic material* and derives formulas relating the thickness of the elastic core and the radius of curvature with the applied bending moment in the case of members with a rectangular cross section. It also defines the *shape factor* and the *plastic section modulus* of members with a nonrectangular section. Section 4.10 deals with

the determination of the plastic moment of members made of an elastoplastic material and possessing a single plane of symmetry, while Sec. 4.11 discusses *residual stresses*.

Section 4.12 shows how the stresses due to a *two-dimensional eccentric axial loading* can be obtained by replacing the given eccentric load by a centric load and a couple, and superposing the corresponding stresses. Attention is called to the fact that the neutral axis *does not pass* through the centroid of the section.

Section 4.13 deals with the *unsymmetric bending* of elastic members. It is first shown that the neutral axis of a cross section will coincide with the axis of the bending couple if, and only if, the axis of the couple is directed along one of the principal centroidal axes of the cross section. It is then shown that stresses due to unsymmetric bending can always be determined by resolving the given bending couple into two component couples directed along the principal axes of the section and superposing the corresponding stresses.

This method of analysis is extended in Sec. 4.14 to the determination of the stresses due to an *eccentric axial loading in three-dimensional space*. The eccentric load is replaced by an equivalent system consisting of a centric load and two bending couples, and the corresponding stresses are superposed.

Section 4.15 is optional; it deals with the bending of *curved members*.

Chapter 5 Analysis and Design of Beams for Bending

In Sec. 5.1 beams are defined as slender prismatic members subjected to transverse loads and are classified according to the way in which they are supported. It is shown that the internal forces in any given cross section are equivalent to a shear force V and a bending couple M . The

bending couple M creates *normal stresses* in the section, while the shear force V creates *shearing stresses*. The former is determined in this chapter, using the flexure formula (5.2), while the latter will be discussed in Chap. 6.

Since the dominant criterion in the design of beams for strength is usually the bending stresses in the beam, the determination of the maximum value of the bending moment in the beam is the most important factor to be considered. To facilitate the determination of the bending moment in any given section of the beam, *shear and bending-moment diagrams* will be drawn, using free-body diagrams of various portions of the beam (Sec. 5.2).

An alternative method, based on relations among load, shear, and bending moment, is presented in Sec. 5.3. To maintain the interest of the students, most of the problems to be assigned are focused on the engineering applications of these methods and call for the determination, not only of the shear and bending moment, but also of the *normal stresses in the beam*.

Section 5.4 is devoted to the *design of prismatic beams* based on the allowable normal stress for the material used. Sample Problems and problems to be assigned include wooden beams of rectangular cross section, as well as rolled-steel W and S beams. An optional paragraph on page 334 describes the application of *Load and Resistance Factor Design* to beams under transverse loading.

Section 5.5 introduces the concept of *singularity functions* and shows how these functions can provide an alternative and effective method for the determination of the shear and bending moment at any point of a beam under the most general loading condition. While this section is optional, it should be included in the lesson schedule if singularity functions are to be used later for the determination of the slope and deflection of a beam (Sec. 9.6). It is pointed out

on page 348 that singularity functions are particularly well suited to the *use of computers*, and several optional problems requiring the use of a computer (Probs. 5.129 through 5.136) have been included in this assignment.

Section 5.6, which is optional, is devoted to *nonprismatic beams*, such as forged or cast beams designed to be of constant strength, and rolled-steel beams reinforced with cover plates.

Chapter 6

Stresses in Beams and Thin-Walled Members Due to Shear

It is shown in Sec. 6.1 that a transverse load creates *shearing stresses* as well as normal stresses in a beam. Considering first the horizontal face of a beam element, it is shown in Sec. 6.2 that the horizontal shear per unit length q , or *shear flow*, is equal to VQ/I . This result is applied in Example 6.01 to the determination of the shear force in the nails connecting three planks forming a wooden beam, as well as in Probs. 6.1 through 6.8.

In Sec. 6.3 the *average shearing stress* τ_{ave} exerted on the horizontal face of the beam element is obtained by dividing the shear flow q by the width t of the beam:

$$\tau_{ave} = \frac{VQ}{It} \quad (6.6)$$

Note that since the shearing stresses τ_{xy} and τ_{yx} exerted at a given point are equal, the expression obtained also represents the average shearing stress exerted at a given height on a vertical section of the beam. This formula is used to determine shearing stresses in a beam made of glued planks in Sample Prob. 6.1 and to design a timber beam in Sample Prob. 6.2. Problems 6.11 through 6.14 and 6.21 through 6.24 call for the determination of shearing stresses in various types of beams.

In Examples 6.02 and 6.03 the designs obtained on the basis of normal stresses, respectively, for a timber beam in Sample Prob. 5.7 and for a rolled-steel beam in Sample Prob. 5.8 are checked and found to be acceptable from the point of view of shearing stresses. Problems 6.15 through 6.18 call for a similar verification of beam designs obtained in Chap. 5.

Section 6.5 is optional and discusses the distribution of stresses in a narrow rectangular beam.

In Sec. 6.6 the expression $q = VQ/I$ obtained on Sec. 6.2 for the shear flow on the horizontal face of a beam element is shown to remain valid for the curved surface of a beam element of arbitrary shape. It is then applied in Example 6.04 and in Probs. 6.29 through 6.33 to the determination of the shearing forces and shearing stresses in nailed and glued vertical surfaces.

Section 6.7 deals with the determination of shearing stresses in *thin-walled members* and shows that Eq. (6.6) can be applied to the determination of the average shearing stress in a section of arbitrary orientation.

Section 6.8, which is optional, describes the formation of *plastic zones* in beams subjected to transverse loads.

Section 6.9, which is also optional, deals with the *unsymmetric loading* of thin-walled members, the determination of the *shear center*, and the computation of the shearing stresses caused by a shearing force exerted at the shear center.

Chapter 7 Transformations of Stress and Strain

After a short introduction (Sec. 7.1), formulas for the *transformation of plane stress* under a rotation of axes are derived in Sec. 7.2,

while the principal planes of stress, principal stresses, and maximum shearing stress are determined in Sec. 7.3.

Section 7.4 is devoted to the use of *Mohr's circle*. It should be noted that the convention used in the text provides for a rotation on Mohr's circle in the *same sense* as the corresponding rotation of the element; in other words, this convention is the same as that used in statics for the transformation of moments and products of inertia. Attention is called to the statement at the top of page 439 of the text and the accompanying Fig. 7.20.

Section 7.5 discusses the general (*three-dimensional*) state of stress and establishes the fact that three principal axes of stress and three principal stresses exist. Section 7.6 shows how three different Mohr's circles can be used to represent the transformations of stress associated with rotations of the element about the principal axes. The results obtained are used to show that in a state of plane stress, the maximum shearing stress does not necessarily occur in the plane of stress.

Sections 7.7 and 7.8 are optional. Section 7.7 presents the two criteria most commonly used to predict whether a ductile material will yield under a given state of plane stress, while Sec. 7.8 discusses the two criteria used to predict the *fracture of brittle materials*.

Section 7.9 deals with stresses in *thin-walled pressure vessels*; it is limited to the analysis of cylindrical and spherical pressure vessels.

The second part of the chapter (Sects. 7.10 through 7.13) deals with *transformations of strain* and is optional. Section 7.10 presents the derivation of the formulas for the transformation of strain under a rotation of axes. It should be noted that this derivation is based on the consideration of

an oblique triangle (Fig. 7.62) and the use of the law of cosines, and that the determination of the shearing strain is facilitated by the use of Eq. (7.43), which relates it to the normal strain along the coordinate axes and their bisector.

Section 7.11 introduces *Mohr's circle for plane strain*, and Sec. 7.12 discusses the three-dimensional analysis of strain and its application to the determination of the maximum shearing strain in states of plane strain and of plane stress. Section 7.13 deals with the use of strain rosettes for the determination of states of plane strain.

Chapter 8 Principal Stresses under a Given Loading

This chapter is devoted to the determination of the *principal stresses* and *maximum shearing stress* in beams, transmission shafts subjected to transverse loads as well as to torques, and bodies of arbitrary shape under combined loadings.

In Sec. 8.1 it is shown that, while only normal stresses occur on a square element with horizontal and vertical faces located at the surface of a beam, shearing stresses will occur if the element is rotated through 45° (Fig. 8.1). The reverse situation is observed for an element with horizontal and vertical faces subjected only to shearing stresses (Fig. 8.2). The analysis of beams, therefore, should include the determination of the principal stresses and maximum shearing stress at various points.

This is done in Sec. 8.3 for cantilever beams of various rectangular sections subjected to a single concentrated load at their free end. It is found that the principal stress σ_{\max} does not exceed the maximum normal stress σ_m determined by the method of Chap. 5 except very close to the load. While this result holds for most beams of nonrectangular section, it may not be valid for rolled-steel W and S beams, and the

analysis and design of such beams should include the determination of the principal stress σ_{\max} at the junction of the web with the flanges of the beam. (See Sample Probs. 8.1 and 8.2, and Probs. 8.1 through 8.14).

Section 8.3 is devoted to the analysis and design of *transmission shafts* using gears or sprocket wheels to transmit power to and from the shaft. These shafts are subjected to transverse loads as well as to torques. The design of such shafts is the subject of Sample Prob. 8.3 and Probs. 8.15 through 8.30.

The determination of the stresses at a given point K of a body due to a *combined loading* is the subject of Sec. 8.4. First, the loading is reduced to an equivalent system of forces and couples in a section of the body containing K . Next, the normal and shearing stresses are determined at K . Finally, using one of the methods of transformation of stresses presented in Chap. 7, the principal planes, principal stresses, and maximum shearing stress may be determined at K . This procedure is illustrated in Example 8.01 and Sample Probs. 8.4 and 8.5.

Chapter 9 Deflection of Beams

The relation derived in Chap. 4 between the curvature of a beam and the bending moment is recalled in Sec. 9.2 and used to predict the variation of the curvature along the beam. In Sec. 9.3, the equation of the *elastic curve* for a beam is obtained through two successive integrations, after the bending moment has been expressed as a function of the coordinate x . Examples 9.01 and 9.02 show how the boundary conditions can be used to determine the two constants of integration in the cases of a cantilever beam and of a simply supported beam. Example 9.03 indicates how to proceed when the bending moment must be represented by two different functions of x .

Section 9.4 is optional; it shows in the case of a beam supporting a distributed load, how the equation of the elastic curve can be obtained directly from the function representing the load distribution through the use of four successive integrations.

Section 9.5 is devoted to the analysis of *statically indeterminate beams* and to the determination of the reactions at their supports. It is suggested that a minimum of two lessons be spent on Secs. 9.2 through 9.5 if neither the use of singularity functions (Sec. 9.6) nor the moment-area method (Secs. 9.9 through 9.14) are to be covered in the course.

Section 9.6 is devoted to the use of *singularity functions* for the determination of beam deflections and slopes. It is optional and assumes that Sec. 5.5 has been covered previously. It is recommended that both Secs. 5.5 and 9.6 be included in the course, since singularity functions provide the students with an effective and versatile method for the determination of deflections and slopes under the most diverse loading conditions. In addition, and as indicated earlier, singularity functions are well suited to the use of computers. Two problems requiring the use of computers (Probs. 9.49 and 9.50) may be included in this assignment.

Section 9.7 discusses the *method of superposition* for the determination of beam deflections and slopes. It shows how the expressions given in Appendix D for various simple loadings can be used to obtain the deflection and slope of a beam supporting a more complex loading. In Sec. 9.8, the method of superposition is applied to the determination of the reactions at the supports of *statically indeterminate beams*.

Sections 9.9 through 9.14 are optional. They deal with the application of the moment-area methods to the determination of the deflection of beams and may be omitted in

courses that place a greater emphasis on analytical methods and make use of singularity functions. It should be noted, however, that these methods provide a very practical means for the determination of the deflection and slope of beams of *variable cross section*.

The two moment-area theorems are derived in Sec. 9.9 and are immediately applied in Sec. 9.10 to the computation of the slope and deflection of *cantilever beams* and *beams with symmetric loadings* (simply supported or overhanging beams). Section 9.11 shows how to draw a *bending-moment diagram by parts*. This approach greatly facilitates the determination of moment areas in all but the simplest loading situations.

Section 9.12 deals with simply supported and overhanging beams with *unsymmetric loadings*. The analysis of such beams hinges on the use of a reference tangent drawn through one of the supports after the tangential deviation of the second support has been computed from the bending-moment diagram. Section 9.13 describes how to locate the point of *maximum deflection* and how to compute that deflection.

Section 9.14 deals with the analysis of *statically indeterminate beams* and the determination of the reactions at their supports.

Chapter 10 Columns

Section 10.2 introduces the concept of *stability of a structure*. The example considered in this section consists of a block supported by two spring-connected rigid rods. It is shown that the position of equilibrium in which both rods are aligned is stable if this position is the only possible position of equilibrium of the system. The same criterion is applied to an elastic pin-ended column in Sec. 10.3 in order to derive *Euler's formula*. Section 10.4 shows how Euler's

formula for pin-ended columns can be used to determine the critical load of columns with other end conditions.

Section 10.5 is optional; it deals with the eccentric loading of a column and gives the derivation of the *secant formula*.

Section 10.6 discusses the *design of columns under a centric load* and presents the empirical formulas developed by various engineering associations for the design of steel columns, aluminum columns, and wood columns. Page 643 is devoted to an optional discussion of the application of *Load and Resistance Factor Design* to steel columns.

Section 10.7 discusses the *design of columns under an eccentric load* and presents two of the most frequently used methods: the *allowable-stress method* and the *interaction method*.

Chapter 11 Energy Methods

Section 11.2 introduces the concept of *strain energy* by considering the work required to stretch a rod of uniform cross section. This work, which is equal to the area under the load-deformation curve, represents the strain energy of the rod. The *strain-energy density* is defined in Sec. 11.3, as well as the *modulus of toughness* and the *modulus of resilience* of a given material. The formula for the elastic strain energy associated with normal stresses is derived in Sec. 11.4, as well as the expressions for the strain energy corresponding to an axial loading and to pure bending. The formula for the strain energy associated with shearing stresses is derived in Sec. 11.5, as well as the expressions corresponding to torsion and transverse loading.

Section 11.6, which is optional, covers the strain energy for a general state of stress and derives an expression for the *distortion energy per unit volume*, both in the general case of three-dimensional stress and in the particular case of plane stress.

Section 11.7 discusses *impact loadings* and Sec. 11.8 the design of a structure for an impact load. To facilitate the solution of impact-loading problems, it is shown in Sec. 11.9 that the strain energy of a structure subjected to a single concentrated load \mathbf{P} can be obtained by equating the strain energy to the work of \mathbf{P} . (Appendix D is used to express the deflection in terms of P). As shown in Sec. 11.10, the reverse procedure can be used to determine the deflection of a structure at the point of application of a single load \mathbf{P} or a single couple \mathbf{M} ; the strain energy of the structure is computed from one of the formulas derived in Secs. 11.4 and 11.5, and the work of \mathbf{P} or \mathbf{M} is equated to the expression obtained for the strain energy.

Sections 11.11 through 11.14 are optional. In Sec. 11.11 an expression for the strain energy of a structure subjected to several loads is obtained by computing the work of the loads as they are successively applied. Reversing the order in which the loads are applied, one proves *Maxwell's reciprocal theorem*. The expression obtained for the strain energy of the structure is used in Sec. 11.12 to prove *Castiglano's theorem*. Section 11.13 is devoted to the application of Castiglano's theorem to the determination of the deflection and slope of a beam and to the deflection of a point in a truss. Finally, Sec. 11.14 deals with the application of Castiglano's theorem to the determination of the reactions at the supports of statically indeterminate structures such as beams and trusses.

TABLE I: LIST OF TOPICS COVERED IN *MECHANICS OF MATERIALS, 4th edition*

		Suggested Number of Periods		
Sections	Topics	Core Topics	Additional Topics	Advanced Topics
Chapter 1: Introduction – Concept of Stress				
1.1-10	Stress Under Axial Loading	1 - 2		
1.11-13	Components of Stress; Factor of Safety	1		
Chapter 2: Stress and Strain – Axial Loading				
2.1-8	Stress-Strain Diagrams; Deformations Under Axial Loading	1 - 2		
2.9-10	Statically Indeterminate Problems	1		
2.11-12	Poisson's Ratio; Generalized Hooke's Law	1		
*2.13	Dilatation; Bulk Modulus		0.25 - 0.5	
2.14-15	Shearing Strain	0.5		
*2.16	Stress-Strain Relationships for Fiber-Reinforced Composite Materials		0.5 - 1	
2.17-19	Stress Concentrations; Plastic Deformations	0.5 - 1		
*2.20	Residual Stresses		0.5	
Chapter 3: Torsion				
3.1-4	Stresses in Elastic Range	1		
3.5-6	Angle of Twist; Statically Indeterminate Shafts	1 - 2		
3.7-8	Design of Transmission Shafts; Stress Concentrations	1		
*3.9-11	Plastic Deformations; Residual Stresses		1 - 2	
*3.12-13	Noncircular Members; Thin-Walled Hollow Shafts			1 - 2
Chapter 4: Pure Bending				
4.1-5	Stresses in Elastic Range	1 - 2		
4.6-7	Members Made of Several Materials; Stress Concentrations	1 - 2		
*4.8-11	Plastic Deformations; Residual Stresses		1 - 2	
4.12	Eccentric Axial Loading	1 - 2		
4.13-14	Unsymmetric Bending; General Eccentric Axial Loading	1 - 2		
*4.15	Bending of Curved Members			1 - 2
Chapter 5: Analysis and Design of Beams for Bending				
5.1-2	Shear and Bending-Moment Diagrams	1 - 1.5		
5.3	Using Relations among w , V , and M	1 - 1.5		
5.4	Design of Prismatic Beams in Bending	1 - 2		
*5.5	Use of Singularity Functions to Determine V and M		1 - 2	
*5.6	Nonprismatic Beams		1 - 2	
Chapter 6: Shearing Stresses in Beams and Thin-Walled Members				
6.1-4	Shearing Stresses in Beams	1 - 2		
*6.5	Shearing Stresses in Narrow Rectangular Beam		0.25	
6.6-7	Shearing Stresses in Thin-Walled Members	1 - 2		
*6.8	Plastic Deformations		0.25	
*6.9	Unsymmetric Loading; Shear Center			1 - 2

TABLE I: LIST OF TOPICS COVERED IN *MECHANICS OF MATERIALS, 4th edition*
(CONTINUED)

Sections	Topics	Suggested Number of Periods		
		Core Topics	Additional Topics	Advanced Topics
Chapter 7: Transformation of Stress and Strain				
7.1-3	Transformation of Plane Stress	1 - 2		
7.4	Mohr's Circle for Plane Stress	1 - 2		
7.5-6	Three-Dimensional Analysis of Stress	0.5 - 1		
*7.7-8	Yield and Fracture Criteria		0.5 - 1	
7.9	Thin-Walled Pressure Vessels	0.5 - 1		
*7.10-12	Analysis of Strain; Mohr's Circle		1 - 1.5	
*7.13	Strain Rosette			0.5
Chapter 8: Principal Stresses under a Given Loading				
*8.1-2	Principal Stresses in a Beam		0.5 - 1	
*8.3	Design of Transmission Shafts		0.5 - 1	
*8.4	Stresses under Combined Loadings		1 - 3	
Chapter 9: Deflection of Beams				
9.1-3	Equation of Elastic Curve	0.5 - 1		
*9.4	Direct Determination of Elastic Curve from Load Distribution		0.5	
9.5	Statically Indeterminate Beams	0.5 - 1		
*9.6	Use of Singularity Functions		1 - 2	
9.7-8	Method of Superposition	1 - 2		
Application of Moment-Area Theorems to:				
*9.9-11	Cantilever Beams and Beams with Symmetric Loadings		1 - 2	
*9.12-13	Beams with Unsymmetric Loadings; Maximum Deflection		1 - 1.5	
*9.14	Statically Indeterminate Beams		0.5	
Chapter 10: Columns				
10.1-4	Euler's Column Formula	1 - 2		
*10.5	Eccentric Loading; Secant Formula		1	
10.6	Design of Columns under a Centric Load	1 - 2		
10.7	Design of Columns under an Eccentric Load	1 - 2		
Chapter 11: Energy Methods				
11.1-5	Strain Energy	1 - 2		
*11.6	Strain Energy for General State of Stress	0.5		
11.7-9	Impact Loading	0.5 - 1		
11.10	Deflections by Work-Energy Method	0.5 - 1		
*11.11-13	Castigliano's Theorem	1 - 2		
*11.14	Statically Indeterminate Structures	1 - 2		
Total Number of Periods		24 - 41½	21 - 38½	3 - 6

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
Mechanics of Materials 4th Edition

SI Units	U.S. Units	Problem description
<u>CHAPTER 1: INTRODUCTION - CONCEPT OF STRESS</u>		
1.1,2	1.3,4	Normal stress under axial loading: in bars
1.5, <u>6</u>		
1.7,9	1.8,10	in pin-connected structures
1.13,14	1.11,12	in trusses and mechanisms
1.15,18	1.16,17	Shearing stress
1.19,20	1.21,22	Bearing stress between flat surfaces
1.23,25	1.24,28	Shearing and bearing stresses at pin-connected joints
1.26,27		
1.29,30	1.31,32	Stresses on an oblique plane
1.33,34	1.35,36	Factor of safety: in tension
1.37,40	1.38,39	in shear
1.41,42		
1.43,44	1.45,46	in structures involving links and pins
1.47,48		
1.53,54	1.49,50	*1.58 Load and Resistance Factor Design
1.55,56	1.51,52	
*1.58	*1.57	
1.59,61	1.60,62	Review problems
1.64,66	1.63,65	
1.67, <u>68</u>	1.69,70	
1.C2,C4,C6	1.C1,C3, C5	Computer problems
<u>CHAPTER 2: STRESS AND STRAIN - AXIAL LOADING</u>		
2.2,4	2.1,3	Stresses and deformations in statically determinate structures: simple rods and wires
2.6,8	2.5,7	
2.9,10	2.12,14	multiple-criteria problems
2.11,13		
2.15,16	2.19,20	composite rods and members
2.17, <u>18</u>		
2.23,24	2.21,22	members of trusses and simple frames
2.25,26	2.27,28	
<u>2.29</u>	2.30	computation of deformations by integration
<u>2.31</u>	2.32	true strain

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
2.35,36		2.33,34	Statically indeterminate structures (constant temperature): with members undergoing equal deformations
2.37,38			composite rods with both ends restrained
2.39,40			
2.41,42			with members undergoing unequal deformations
2.43,44			
2.45,46			Statically indeterminate structures (with temperature changes): with members undergoing equal deformations
2.48,49		2.47,50	composite rods with both ends restrained
2.53,54		2.51,52	rods with gaps
2.55		2.56,57	with unequal deformations
2.58,59			
2.60			
2.61,62		2.63,65	Poisson's ratio and generalized Hooke's law: uniaxial loading
2.64,66			
2.67,68		2.69,70	biaxial loading
2.71,72		<u>2.73,74</u>	derivation of formulas
2.75,76		2.77,78	Hooke's law for shearing stress and strain
2.79,80		2.81,82	
*2.83,85		*2.84,86	dilatation
*2.87,*88			Problems involving cylindrical coordinates
*2.83,*86		*2.84,*85	Dilatation
*2.89,*90			Theory problems
*2.91,92			Problems involving composites
2.93,94		2.95,96	Stress concentrations in flat bars
2.99,100		2.97,98	Plastic deformations under axial loading:
2.103,104		2.101,102	simple problems
2.105,106			
2.107,108		2.109,110	more complex problems
2.111,112			
2.114,*115		2.113	problems involving temperature change
2.116,117			problems involving a structure
*2.118,120		*2.119	problems involving residual stresses
*2.120,121		*2.115,116	
2.124,128		2.125,126	Review problems
2.131,132		2.127,129	
2.133,135		2.130,134	
2.C1,C3,C6		2.C2,C4,C5	Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*		Problem description
SI Units	U.S. Units	
<u>CHAPTER 3: TORSION</u>		
		Shearing stresses:
3.3,4	3.1,2	in simple shafts
3.5,6	3.7,8	
3.11,12	3.9,10	in shafts subjected to several torques
3.13,14		
3.15,16	3.19,20	in composite shafts
3.17,18		
3.21,22	3.23,24	in gear-connected shafts
3.25,26	3.27,28	
3.29	3.30	special problems
		Angle of twist:
3.31,34	3.32,33	in simple shafts
3.35,36		in shafts subjected to several torques
3.37	3.38	in composite shafts
3.39,40	<u>3.41</u> ,42	in gear-connected shafts
	3.43,44	
3.46	3.45,47	Design of shafts based on allowable stress and allowable angle of twist
<u>3.49</u>	3.48	
	3.50,51	
		Statically indeterminate shafts:
3.54,55	3.52,53	with inner core and outer shell of different materials
3.56,57	3.58,59	
<u>3.62,63</u>	<u>3.60,61</u>	Special problems
		Design of shafts:
3.65,66	3.64,67	easy problems with solid shafts
3.69,71	3.68,70	hollow shafts
3.72,73		
	3.74,75	including angle of twist
	3.76,77	gear-connected shafts
3.78,79	3.80,81	multiple-criteria problems
3.82,83		
3.86,87	3.84,85	Stress concentrations in stepped shafts
3.90	3.88,89	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*		Mechanics of Materials 4 th Edition
SI Units	U.S. Units	Problem description
3.92,93	3.91,94	Plastic deformations of shafts: Shafts made of elastoplastic material: stresses in solid shafts
3.95		
3.96,97	<u>3.99</u> ,100	angle of twist for solid shafts
3.98		
3.101,102	<u>3.103</u> ,104	hollow and tapered shafts derivation
<u>3.105</u>		
3.106,107	3.108,109	Shafts made of a material with a nonlinear stress-strain diagram
3.110,114	3.111,112	Residual stresses and permanent angle of twist in shafts made of an
<u>3.115</u> ,118	<u>3.113</u> ,116	elastoplastic material
	3.117	
<hr/>		
3.119,120	3.121,122	Bars with rectangular cross section: easy problems
3.123,124	3.125,126	
3.129	<u>3.127</u> ,128	comparing circular and rectangular shafts application to structural shapes
3.131,132	<u>3.130</u> ,133	
<hr/>		
3.135,137	3.134,136	Thin-walled hollow shafts: determine shearing stresses
3.138, <u>141</u>	3.139,140	
3.142	<u>3.143</u>	
<u>3.144</u>	<u>3.145</u>	Special problems
<hr/>		
3.148,150	<u>3.146</u> ,147	Review problems
3.151,152	<u>3.149</u> ,153	
3.156, <u>157</u>	<u>3.154</u> , <u>155</u>	
<hr/>		
3.C1,C2,C3	3.C4,C5,C6	Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number. Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
CHAPTER 4: PURE BENDING			
4.3,4		4.1,2	Normal stresses: in beams with horizontal plane of symmetry
4.5,6			
4.9,10		4.7,8	in unsymmetrical beams (first locate centroid)
4.11,12		4.13,14	Resultant force on portion of cross section
4.15,16		4.17,18	Beams with different allowable stresses in tension and compression
4.19,20			
4.22,26		4.21,23	Maximum stress and radius of curvature
		4.24,25	
4.27		4.28	Maximization of beam strength
4.31		4.29,30	Anticlastic curvature
4.32			Special problem on theory
4.33,34		4.37,38	Stresses in composite beams: symmetric beams of two materials
4.35,36			
4.39,40		4.41,42	unsymmetric beams of two materials
4.43,44		4.45,46	Radius of curvature in composite beams
4.47,48		4.49,50	Stresses in reinforced concrete beams
		4.51	
4.52,54		4.53	in beams with balanced design
		4.55,56	Beams of three materials
4.57,58			Composite beams with circular cross section
4.59		*4.60	Beams with different moduli of elasticity in tension and compression
4.63,64		4.61,62	Stress concentrations in flat bars in pure bending
4.65,66			
4.69,70		4.67,68	Plastic deformation in pure bending (elastoplastic material): plastic zone in rectangular beams
		4.71,72	including radius of curvature
4.73,74		4.75,76	plastic zone and radius of curvature for symmetric beams
4.77,78		4.79,80	plastic moment and shape factor of symmetric beams
4.81,82		4.83,84	plastic moment of unsymmetric beams
4.86		4.85	plastic moment of thick-walled pipe
			Residual stresses in symmetric beams:
4.87,88		4.89,90	after M_p has been applied and removed
4.91,93		4.92,94	after a given plastic zone has been developed
			Special problems:
4.95			residual radius of curvature
4.96		4.97,98	Plastic deformation of beams with a nonlinear stress-strain diagram

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
4.102,103	4.100,101		Eccentric loading in plane of symmetry of member: find stress in symmetric section
4.104	<u>4.99</u> ,105		
4.106,107	<u>4.109</u> ,110		design of symmetric section
4.108	4.111		
4.113,114	<u>4.112</u> ,115		find stress in unsymmetric section
4.117,118	4.116,119		
4.121,122	4.120,123		computation of loads from strain measurements
4.124,125			
4.127,131	4.126, <u>128</u>		Unsymmetric bending with one or two planes of symmetry: bending moment at an angle with horizontal
	4.129, <u>130</u>		
4.132,134	4.133,136		section at an angle with horizontal
*4.139	*4.137, <u>138</u>		Bending of unsymmetric section (principal axes must be determined) General eccentric bending:
4.140, <u>141</u>	4.142,143		symmetric beam; find stresses
4.144,145			symmetric beam; find allowable load or dimension
4.146,147	4.148, <u>149</u>		Bending of unsymmetric beams; determine largest bending moment
4.151	4.150		
<u>4.152</u> ,153	4.154, <u>155</u>		Special problems and problems on theory
4.156,157	4.158, <u>159</u>		Curved beams: with rectangular cross section
4.160,164	4.161, <u>162</u>		in pure bending under eccentric loading
	4.163		
4.165,166	4.167,168		with unsymmetric cross section
4.171,172	4.169,170		with circular cross section
4.175,176	4.173, <u>174</u>		with trapezoidal cross section
	4.177,178		with triangular cross section
<u>4.180</u> ,181	4.182,*183		special problems and derivations of formulas

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
4.186, <u>188</u>		4.184, <u>185</u>	Review problems
4.189, <u>193</u>		4.187, <u>190</u>	
4.194, <u>195</u>		4.191,192	

4.C1,C3		4.C2,C4	Computer problems
<u>4.C5,C7</u>		4.C6	

CHAPTER 5: ANALYSIS AND DESIGN OF BEAMS FOR BENDING

<u>5.1,2</u>	<u>5.3,4</u>	Using the free-body diagram of a portion of a beam: draw V and M diagrams (easy problems)
<u>5.5,6</u>		
5.7,8	5.9,10	draw V and M diagrams and determine maximum values of $ V $ and $ M $
5.11,13	5.12,14	
5.16,18	5.15,17	find maximum normal stress in given beam section
5.19,20		
5.22,23	5.21,25	draw V and M diagrams and find maximum normal stress in beam
5.24,26		
5.27,29	5.28,31	determine given parameter to minimize normal stress in beam
5.30,32	5.33	
<u>5.34,35</u>	<u>5.36,37</u>	Using relations among w , V and M whenever appropriate: draw V and M diagrams (easy problems)
<u>5.38,39</u>		
<u>5.40,41</u>	5.42,43	draw V and M diagrams and determine maximum values of $ V $ and $ M $
5.44,45		
5.47,49	5.46,48	find maximum normal stress in a given beam section
<u>5.50,51</u>	<u>5.52,53</u>	write equations for V and M and find maximum value of $ M $,
5.54,55	5.56,57	draw V and M diagrams and find maximum normal stress in beam
5.60,61	5.58,59	
*5.66,*63	*5.64	Special problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*		Problem description
SI Units	U.S. Units	
5.65,69	5.66,67	Design of timber beams
5.70	5.68	
5.73,74	5.71,72	Design of steel beams, W shapes
5.75,76	5.77,78	Design of steel beams, S shapes
5.79,80	5.81,82	Design of steel beams, miscellaneous shapes
5.84	5.83	Design of beams resting on ground
5.87,88	5.85,86	Find allowable load for beam of unsymmetric cross section with allowable stresses in tension and compression
5.90,91	5.89	Design of beam systems
5.92,93		
*5.96,*97	*5.94,*95	Design of beams using LRFD
<u>5.98,99</u>	<u>5.100,101</u>	Using singularity functions write equations for $V(x)$ and $M(x)$ and find M at given point in beam
<u>5.102,103</u>	<u>5.104,105</u>	
5.107,109	5.106,108	find $ M _{\max}$ in beam
<u>5.110,111</u>		find σ_{\max} in beam
5.112,113		find $ M _{\max}$ and σ_{\max} in beam
5.116,117	5.114,115	design beam, knowing allowable stress
5.118,119	5.120,121	Using a computer and step functions, calculate V and M along the beam
5.122,123	5.124,125	calculate V and M along the beam , and determine σ_{\max} in the beam
		Nonuniform beams
		Beams of constant strength:
5.128,129	5.126,127	beams of uniform width and variable depth
5.130,131	.	
5.132,133	5.134,135	built-up timber beams
<u>5.136</u>	<u>5.137</u>	beams of circular cross section
<u>5.138</u>	5.139	beams of uniform depth and variable width
5.140,141	5.142,143	Rolled-steel beams with cover plates
5.144,145	5.146,147	
5.148,149	5.150,151	Tapered beams
5.152,155	5.153,153	Review problems
<u>5.157,159</u>	5.156,158	
5.161	5.160,162	
	5.163	
5.C1,C2	<u>5.C3,C4</u>	Computer problems
5.C6	5.C5	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>CHAPTER 6: SHEARING STRESSES IN BEAMS AND THIN-WALLED MEMBERS</u>			
6.3,4		6.1,2	Shearing forces in nails and bolts, using horizontal cuts
6.5,6		6.7,8	
6.10,11		6.9,12	Shearing stresses in beams
6.13,14			
6.16,18		6.15,17	Checking earlier designs of beams for shearing stresses
6.19,20			
6.21,23		6.22,24	Beams with unsymmetric sections
6.25,26		6.27,28	Beams with various geometric sections
6.29,31		6.30,32	Shearing forces and shearing stresses on arbitrary cuts due to vertical shear
6.33,34			
6.35,36		6.37,38	Shearing stresses in extruded beams
6.39,40		6.41,42	
6.45,46		6.43,44	Shearing stresses in bolts
6.48,49		6.47,50	Shearing stresses and shear flow in thin-walled members
6.52,53		6.55,54	Special problems
6.55,56		6.57,59	Shearing stresses in composite beams
6.58		6.60	Plastic behavior
6.61,62		6.63,64	Shear center in thin-walled beams with horizontal and vertical portions
6.67,68		6.65,66	Shear center and shearing stresses in extruded beams
6.70,71		6.69,72	Shear center in thin-walled beams with oblique portions
6.73,74			Shear center in thin-walled beams with circular portions
6.77,78		6.75,76	Problems involving location of shear center
6.79		6.80	Special problems
*6.81,*82		*6.83,*84	Shearing stresses in semicircular shapes, angle shapes and Z shapes
*6.87,*88		*6.85,*86	
6.89,91		6.90,92	Review problems
6.94,95		6.93,96	
6.97,100		6.98,99	
6.C1		6.C2	Computer problems
6.C3,C4		6.C5,C6	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
CHAPTER 7: TRANSFORMATION OF STRESS AND STRAIN			
7.1,4		7.2,3	Find stresses on oblique plane from equilibrium of wedge
7.7,8		7.5,6	Find principal planes and stresses
7.11,12		7.9,10	Find planes of maximum shearing stress and corresponding stresses
7.13,14		7.15,16	Find stresses on a given plane
7.17,20		7.18,19	Stresses on oblique planes - simple applications
7.21,22			
7.25,26		7.23,24	Find principal stresses and/or maximum shearing stress in loaded shaft
7.28,30		7.27,29	Special problems involving determination of a stress to satisfy a given requirement
7.31,32		7.33,34	Using Mohr's circle, determine: principal planes and stresses, and maximum shearing stress
7.35,36		7.37,38	stresses on oblique plane - simple applications
7.39,42		7.40,41	
7.43,44			principal stresses and/or maximum shearing stress in loaded shaft
7.47,48		7.45,46	solve special problems involving determination of a stress to satisfy a given requirement
7.50,53		7.49,51	find principal planes and stresses resulting from superposition of two states of stress
7.54, <u>56</u>		7.55, <u>57</u>	find range of values of a parameter for which a certain stress will not be
7.60,61		7.63	
7.62			Derivation of a formula involving Mohr's circle
7.64		7.65	
7.66,67			In-plane and out-of-plane maximum shearing stress
7.68,69			Maximum shearing stress in a three dimensional state of stress
		7.70,71	
		7.72,73	Maximum shearing stress (more advanced problems)
7.74,76		7.75,77	
7.78,79			Determine whether material will yield under given state of stress. If no yield occurs find the factor of safety
*7.80			Find load for which yield will occur
7.83,84		7.81,82	Will rupture occur under a given state of stress?
7.87,88		7.85,86	Find stress or load for which rupture will occur
7.89,90		7.91,92	
7.93,95		7.94,97	
7.96			

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
7.98,99		7.100,101	Spherical pressure vessels (easy problems)
7.102, <u>103</u>			
7.105,107		7.104, <u>106</u>	Cylindrical pressure vessels (easy problems)
		7.108,109	
7.110,111		7.112, <u>113</u>	Stresses in welds in cylindrical pressure vessels
7. <u>116</u> ,117		7.114,115	
7.118, <u>119</u>		7.120,121	Pressure vessels subjected to external loadings
7.122,123		7.124,125	Shrunk fit rings
7.126,127		7.128, <u>129</u>	Find state of strain associated with given rotation using formulas of Sec. 7.10
7. <u>130</u> ,131		7.132,133	using Mohr's circle
			Find principal strains and maximum shearing strain (in plane and out of plane):
7. <u>134</u> , <u>135</u>		7.136,137	for a state of plane stress
7. <u>138</u> , <u>139</u>		7.140,141	for a state of plane strain
7.142,144		7.143,145	Problems involving strain rosettes
7. <u>146</u>		7.147	
7.148		7.149	Problems involving use of Mohr's circle and Hooke's law
7.152,153		7.150, <u>151</u>	Applications of the use of strain rosettes
		7.154, <u>155</u>	
7.158, <u>161</u>		7.156, <u>157</u>	Review problems
7.164, <u>165</u>		7.159,160	
7.166		7.162, <u>163</u>	
		7.167	
7.C1,C2		7.C3,C4	Computer problems
7.C5,C6		7.C7,C8	
CHAPTER 8: PRINCIPAL STRESSES UNDER GIVEN LOADING CONDITIONS			
			Principal stresses in rolled-steel beams:
8.1,2		8.3,4	find σ_m in beam and σ_{max} at junction of flanges and web
8.7,8		8.5,6	design beam, taking σ_m , τ_m , and σ_{max} into account
8.9,10		8.11,12	check earlier design for σ_{max} at junction of flanges and web
		8.13,14	
			Design of transmission shafts:
8.15,16		8.19,20	loading represented by forces and couples
8. <u>17</u> , <u>18</u>		8. <u>21</u>	
8.22			
8.27,28		8.23,24	loading represented by input and output power
8.29, <u>30</u>		8.25,26	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

SI Units	U.S. Units	Problem description
		For beam of rectangular cross section under axial, bending and transverse loading (easy problems), find: normal and shearing stresses
8.31,32 8.34,35	8.33,36	
		For beams of circular cross section under axial, bending, and transverse loading, find: normal and shearing stresses principal stresses and maximum shearing stress
8.39,40 8.41,42	8.37,38 8.43,44	
		For beams of rectangular cross section under axial, bending and transverse loading, find: normal and shearing stresses principal stresses and maximum shearing stress
8.45,46 8.49,50	8.47,48 8.51,52	
		For steel beams and structural tubes under axial, bending and transverse loading, find: normal and shearing stresses principal stresses, principal planes, and maximum shearing stress
8.53,54 8.56,57 8.59	8.55 8.58 8.60	Special problems
*8.63	*8.61,62 *8.64	Problems involving torsion of rectangular sections or structural shapes
8.66, <u>67</u> 8.68,70 8.75,76	8.65,69 8.71,72 8. <u>73</u> ,74	Review problems
8.C1,C2 8.C3,C6	8.C4,C5 8.C7	Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>CHAPTER 9: DEFLECTION OF BEAMS</u>			
<u>9.1,3</u>	<u>9.2,4</u>		Using the integration method, determine the equation of the elastic curve and the deflection and/or slope at specific points for: cantilever beams
<u>9.5,6</u>			
<u>9.7</u>	9.8		overhanging beams simply supported beams
9.9	9.10		symmetrical loading
9.11	9.12		unsymmetrical loading
<u>9.14,15</u>	<u>9.13</u>		beams and loadings requiring the use of 2 equations and 4 constants of integration
<u>9.16</u>			
<u>9.17</u>	<u>9.18</u>		direct determination of the elastic curve from an analytic function of $w(x)$ For a statically indeterminate beam (first degree), determine: reaction at the roller support
<u>9.19,20</u>	<u>9.21,22</u>		
9.24	9.23		
<u>9.25,26</u>	<u>9.27,28</u>		reaction at the roller support and draw the M diagram (use of 2 equations and 4 constants of integration required)
<u>9.29,30</u>	<u>9.31,32</u>		reaction at the roller support and the deflection at a given point (use of 2 equations and 4 constants of integration required)
<u>9.33</u>	<u>9.34</u>		For a statically indeterminate beam (second degree), determine the reaction at one end and draw the M diagram
<hr/>			
<u>9.35,36</u>	<u>9.37,38</u>		Using singularity functions, determine the equation of the elastic curve and the deflection and/or slope at specified points: easy problems
<u>9.39,40</u>			problems involving overhanging beams
<u>9.41,42</u>	<u>9.43,44</u>		problems with distributed loads
9.45,48	9.46,47		problems with numerical data
<u>9.49,50</u>	<u>9.51,52</u>		For a statically indeterminate beam (first degree), determine the reaction at the roller support and the deflection at a specified point.
9.53,56	9.54,55		
<u>9.57</u>	<u>9.58</u>		For a statically indeterminate beam (second degree), determine the reaction at the roller support and the deflection at a specified point.
9.59,62	9.60,61		determine the maximum deflection
9.63,64			Problems involving the reduction of a load to a force-couple system.

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

SI Units	U.S. Units	Problem description
		Using method of superposition, determine the deflection and slope at specified points in:
<u>9.65,66</u>	<u>9.67,68</u>	cantilever beams
<u>9.71,72</u>	<u>9.69,70</u>	simply supported beams
<u>9.73,74</u>	<u>9.75,76</u>	cantilever beams (with numerical data)
<u>9.77,78</u>		simply supported beams (with numerical data)
<u>9.79,80</u>	<u>9.81,82</u>	statically indeterminate beams (first degree)
<u>9.83</u>	<u>9.84</u>	statically indeterminate beams (second degree)
<u>9.85,88</u>	<u>9.86,87</u>	combined beams, determinate (with numerical data)
<u>9.89,92</u>	<u>9.90,91</u>	statically indeterminate beams (with numerical data)
<u>9.93</u>	<u>9.94</u>	Combined bending and torsion of rods
		Using the moment-area method, determine the slope and/or deflection at specified points in
		cantilever beams
<u>9.95,96</u>	<u>9.97,98</u>	with simple loadings
<u>9.99,100</u>		with more complex loads
<u>9.101,104</u>	<u>9.102,103</u>	with numerical data
<u>9.106,108</u>	<u>9.105,107</u>	with variable EI
<u>9.109,110</u>	<u>9.113,114</u>	simply supported beams
<u>9.111,112</u>		with symmetric loadings
<u>9.115,116</u>		with variable EI
<u>9.117,118</u>	<u>9.121,122</u>	with numerical data
<u>9.119,120</u>		
<u>*9.123</u>	<u>*9.124</u>	special problems
		Simply supported and overhanging beams with unsymmetric loadings
<u>9.126,127</u>	<u>9.125,128</u>	simply supported beams (easy problems)
<u>9.129,131</u>	<u>9.130,132</u>	simply supported beams with numerical data
<u>9.133</u>	<u>9.134</u>	overhanging beams
<u>9.135,137</u>	<u>9.136,138</u>	overhanging beams (with numerical data)
<u>9.139</u>	<u>9.140</u>	simply supported beams with variable EI
		Find maximum deflection for:
<u>9.141</u>	<u>9.142</u>	simply supported beams
<u>9.143</u>	<u>9.144</u>	with numerical data
<u>9.145</u>	<u>9.146</u>	overhanging beams
		Statically indeterminate beams (first degree)
<u>9.149,150</u>	<u>9.147,148</u>	single span
	<u>9.151,152</u>	two span beams, find all reactions
<u>9.153</u>	<u>9.154</u>	single span beams (with numerical data)
<u>9.155</u>	<u>9.156</u>	simply supported beams with additional elastic support at midspan

* Problems that do not involve any specific system of units have been indicated by underlining their number.

Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
9.157, <u>158</u>		<u>9.159,160</u>	Review problems
9. <u>161</u> ,163		9.162,165	
9. <u>164</u> ,168		9.166,167	

9.C1,C3	9.C2,C5	Computer problems
9.C4,C8	9.C6,C7	

CHAPTER 10: COLUMNS

10. <u>1</u> ,2	10. <u>3</u> ,4	Stability of rigid-rod-and-spring systems: easy problems, single spring
10. <u>5</u> ,6	10.7,8	systems with two or more springs
10.9	10.10	Application of Euler's formula to the critical loading of pin-ended columns: short struts
10.12, <u>13</u>	<u>10.11</u>	comparison of critical loads for various cross sections
10.14		
10. <u>17</u> ,18	10.15,16	Allowable loading for pin-ended columns: rolled-steel shapes
10.19,20		multiple-member structures
10.23,24	10.21,22	columns with various end conditions
10.27,28	10.25,26	
10.30, <u>31</u>	10.29,33	Application of the secant formula to the eccentric loading of columns: find σ_{max} and either deflection or e for a given load
10.32,34		
10.37,38	10.35,36	find σ_{max} and load for a given deflection and eccentricity
	10.39,40	
10.41,42		temperature induced loading
10.45, <u>46</u>	10.43,44	find P_{all} for given e , σ_{max} and F.S. (using Fig. 10.24) design problems
10.49,50	10.47,48	find column length
10.51,52		find factor of safety of column
10.53,54	10.55,56	find cross section of column

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
10.58,60		10.57,59	Analysis of columns under centric load: columns with simple cross section: steel columns
10.62		10.61	aluminum columns
10.64		10.63	timber columns
10.65		10.66	columns with built-up cross sections: timber column
10.67,68		10.69	aluminum column
10.70			steel columns
10.72,73		10.71	Design of columns under a centric load: aluminum columns
10.74,76		10.75	timber columns
10.77,80		10.78,79	steel columns
10.83,84		10.81,82	Application of LRFD formulas: analysis of columns under a centric loading design of columns under a centric loading
*10.85		*10.86	
*10.87		*10.88	
10.89,90			Analysis of columns under an eccentric load: steel columns, find allowable load
10.91,92		10.93,94	aluminum columns, find allowable load timber columns, find allowable load
10.95,96			Design of columns under an eccentric load: find maximum allowable length: aluminum column
10.97,98		10.99,100	timber columns
		10.101,102	steel columns
10.103,104			design cross section: timber columns
10.105,106		10.107,108	steel columns, rectangular cross section
		10.109,110	aluminum columns, rectangular or circular cross sections
10.111,112		10.113,114	steel column, wide-flange shape
10.115,116			steel tube
<u>10.117,121</u>		<u>10.118,119</u>	Review problems
<u>10.123,126</u>		<u>10.120,122</u>	
<u>10.127,128</u>		<u>10.124,125</u>	
10.C1,C3,C6		10.C2,C4,C5	Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
CHAPTER 11: ENERGY METHODS			
11.2,3		11.1,4	Modulus of resilience
			Modulus of resilience and modulus of toughness:
11.5		11.6	from stress-strain diagram
11.8		11.7	from load-deflection diagram of a tensile test
11.10		11.9	Strain energy under axial loads:
11.11,12			strain energy under a given load
11.13		11.14	maximum allowable strain energy
11.15		11.16	factor of safety
11.17,19		11.18,20	strain energy by integration of approximate methods
		11.21,22	strain energy of trusses
11.23,24		11.25,26	with numerical data
11.27		11.28	Strain energy in bending:
11.29,30		11.31,32	derivation of formulas
11.33		11.34	with numerical data
11.35			Strain energy in torsion:
			by integration
		11.36,37	Maximum-distortion-energy criterion for 3-dimensional state of stress
		11.38,39	
11.40		11.41	Special problems
Impact loading:			
11.42,43		11.44,45	of rods (horizontal impact)
11.46,47			of rods (vertical impact)
11.49,50		11.48	of beams (horizontal impact)
11.52,53		11.51	of beams (vertical impact)
11.55,56		11.54	problems on theory
Use of work-energy method to determine:			
			deflection or slope of:
11.57,58		11.59,60	prismatic beams
11.63,64		11.61,62	nonprismatic beams
11.65,66		11.67,69	angle of twist of shafts
11.68			
11.70			angle of twist of a thin-walled hollow shaft
11.76		11.74,75	deflection of a joint of a truss

* Problems that do not involve any specific system of units have been indicated by underlining their number.
Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>11.78,80</u>		<u>11.77,79</u>	Work of several loads applied to a beam
<u>11.81</u>		<u>11.82</u>	<u>Determinate structures.</u> Use Castiglano's theorem to determine: deflection and/or slope of beams
<u>11.83,84</u>		<u>11.85,86</u>	
<u>11.87,88</u>		<u>11.91,92</u>	
<u>11.89,90</u>			
11.95,98		11.93,94 11.96,97	deflection and/or slope of beams (with numerical data)
11.101,102		11.99,100	deflection of a given joint in a truss
11.103,104			
<u>11.105,106</u>		<u>11.107,108</u>	deflection and/or slope at a given point in a bent or curved rod
<u>11.109,110</u>			<u>Indeterminate structures (first degree).</u> Use Castiglano's theorem to determine: reaction at roller support and draw bending-moment diagram
<u>11.111,112</u>		<u>11.113,114</u> <u>11.115,116</u>	
11.117,118		<u>11.119,120</u>	force in member(s) of a truss
<u>11.121,122</u>			
11.123,124		11.125,126	Review problems
11.127,131		11.128,129	
11.133,134		11.130,132	
11.C3,C4,C5		11.C1,C2,C6	Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
Answers are not given to problems with a number set in italic type.

TABLE III: SAMPLE ASSIGNMENT SCHEDULE FOR A COURSE IN MECHANICS OF MATERIALS *Mechanics of Materials, 4th Edition*

The six lists of suggested assignments contained in this table cover the entire text. The instructor can select those assignments that are covered in the course.

50% OF THE PROBLEMS IN EACH OF THESE LISTS USE SI UNITS AND 50% U.S. CUSTOMARY UNITS

| < ----- For these lists answers to all of the problems are given in the back of the book ----- | < ----- For Lists 5 and 6 no answers are given ----- |

Group	Sections	Topics	List 1	List 2	List 3	List 4	List 5	List 6
1	1.1	Introduction						
2	1.2-10	Stresses Under Axial Loading	1.4, 7, 21, 23	1.3, 14, 16, 27	1.2, 18, 28	1.1, 10, 20, 24	1.6, 12, 17, 25	1.5, 11, 22, 25
3	1.11-13	Components of Stress; Factor of Safety	1.30, 38, 43, 49	1.29, 39, 44, 51	1.31, 42, 45, 53	1.32, 41, 46, 54	1.36, 40, 47, 52	1.35, 37, 48, 50
4	2.1-8	Stress-Strain Diagram	2.2, 14, 15, 21	2.4, 12, 16, 22	2.1, 16, 19, 24	2.3, 9, 20, 25	2.5, 13, 18, 27	2.7, 11, 17, 28
5	2.9-10	Statically Indeterminate Problems	2.34, 40, 50, 55	2.33, 39, 47, 60	2.35, 45, 49, 56	2.36, 46, 48, 57	2.37, 44, 51, 59	2.38, 43, 52, 58
6	2.11-16	Generalized Hooke's Law; Poisson's Ratio	2.65, 68, 78, 87	2.63, 67, 77, 88	2.62, 69, 75, 84	2.61, 70, 76, 86	2.66, 72, 81, 85	2.64, 71, 82, 85
7	2.17-20	Stress Concentration, Plastic Behavior	2.94, 101, 111, 119	2.93, 102, 112, 113	2.95, 106, 109, 117	2.96, 105, 116, 116	2.97, 104, 107, 122	2.98, 103, 108, 123
8	3.1-4	Stresses in Tension	3.3, 10, 15, 27	3.5, 9, 17, 23	3.11, 13, 25	3.7, 11, 20, 21	3.2, 12, 18, 28	3.8, 12, 16, 24
9	3.5-6	Angle of Twist; Indeterminate Shafts	3.52, 39, 45, 57	3.33, 37, 47, 56	3.31, 44, 46, 53	3.36, 38, 49, 52	3.35, 42, 50, 59	3.34, 43, 51, 58
10	3.7-8	Transmission Shafts; Stress Concentrations	3.65, 77, 78, 85	3.66, 76, 83, 84	3.64, 71, 80, 87	3.67, 72, 81, 86	3.68, 75, 82, 89	3.70, 74, 79, 88
11	3.9-11	Plastic Deformations; Residual Stresses	3.91, 98, 104, 114	3.94, 97, 104, 110	3.92, 100, 101, 112	3.93, 99, 102, 111	3.95, 103, 106, 115	3.96, 105, 107, 118
12	3.12-13	Nonsigular and Thin-Walled members	3.119, 129, 134, 142	3.120, 126, 136, 141	3.121, 127, 135, 145	3.122, 125, 137, 140	3.123, 133, 139, 144	3.124, 130, 139, 143
13	4.1-5	Stresses and Deformations in the Elastic Range	4.6, 14, 15, 23	4.3, 13, 16, 21	4.1, 10, 17, 26	4.2, 9, 20, 22	4.7, 12, 18, 25	4.8, 11, 19, 24
14	4.6-7	Members Made of Several Materials	4.37, 40, 49, 63	4.38, 39, 59, 66	4.33, 42, 47, 62	4.34, 41, 54, 61	4.35, 46, 51, 64	4.36, 45, 53, 63
15	4.8-11	Plastic Deformations; Residual Stresses	4.69, 76, 82, 90	4.72, 75, 86, 89	4.67, 77, 83, 88	4.71, 77, 84, 87	4.68, 78, 81, 94	4.72, 78, 85, 92
16	4.12	Eccentric Axial Loading in a Plane of Symmetry	4.100, 137, 119, 124	4.101, 106, 116, 121	4.99, 109, 113, 123	4.103, 105, 117, 120	4.102, 111, 112, 123	4.104, 110, 115, 122
17	4.13-14	Unsymmetric Bending	4.127, 136, 140, 150	4.136, 133, 144, 148	4.126, 135, 142, 151	4.129, 134, 143, 146	4.128, 132, 141, 149	4.130, 132, 145, 149
18	4.15	Bending of Curved Members	4.158, 164, 169, 176	4.139, 164, 170, 175	4.156, 167, 171, 173	4.160, 162, 171, 178	4.156, 163, 174, 179	4.157, 168, 177, 179
19	5.1-2	Shear and Bending-Moment Diagrams	5.71, 17, 22, 28	5.8, 15, 25, 28	5.9, 18, 22, 29	5.10, 20, 25, 27	5.12, 16, 23, 33	5.14, 19, 24, 31
20	5.3	Using Relations Among w, V and M	5.42, 49, 56, 62	5.43, 47, 59, 62	5.40, 48, 54, 63	5.45, 46, 60, 63	5.41, 53, 57, 64	5.42, 51, 58, 64
21	5.4	Design of Prismatic Beams in Bending	5.65, 72, 79, 89	5.70, 71, 80, 89	5.68, 74, 81, 91	5.68, 73, 82, 90	5.66, 78, 85, 93	5.67, 77, 86, 92
22	5.5	Use of Singularity Functions to determine V and M	5.98, 108, 116, 124	5.105, 106, 111, 121	5.100, 109, 114, 123	5.101, 107, 115, 118	5.102, 111, 112, 125	5.103, 110, 117, 120
23	5.6	Nonprismatic Beams	5.126, 138, 147, 148	5.127, 132, 142, 148	5.131, 139, 141, 151	5.128, 134, 145, 150	5.129, 137, 146, 149	5.130, 135, 143, 149
24	6.1-5	Shearing Stresses in a Beam	6.4, 9, 19, 22	6.3, 12, 20, 24	6.2, 13, 15, 21	6.1, 14, 17, 23	6.8, 10, 16, 27	6.7, 11, 18, 25
25	6.6-8	Shearing Stresses in Thin-Walled Members	6.3, 30, 36, 44, 54	6.32, 35, 43, 53	6.29, 38, 46, 52	6.34, 37, 45, 51	6.31, 42, 50, 58	6.33, 41, 47, 56
26	6.9	Shear Center	6.61, 69, 77, 83	6.62, 72, 78, 84	816, 62, 70, 75, 81	6.64, 71, 76, 82	6.66, 73, 79, 85	6.65, 74, 80, 86
27	7.1-3	Transformation of Plane Stress	7.1, 10, 17, 24	7.4, 9, 22, 23	7.2, 13, 18, 26	7.3, 11, 19, 25	7.5, 14, 20, 29	7.6, 12, 21, 27
28	7.4	Mohr's Circle for Plane Stress	7.33, 44, 45, 60	7.34, 39, 46, 54	7.31, 38, 47, 56	7.32, 37, 48, 55	7.36, 42, 49, 61	7.36, 42, 55, 61
29	7.5-8	Three-Dimensional Stress Analysis; Fracture Criteria	7.66, 77, 83, 97	7.69, 75, 87, 91	7.72, 79, 81, 95	7.71, 74, 92, 99	7.70, 76, 84, 92	7.73, 78, 88, 94
30	7.9	Stresses in Thin-Walled Pressure Vessels	7.100, 107, 112, 123	7.101, 105, 115, 118	7.98, 109, 110, 124	7.102, 104, 116, 120	7.99, 106, 113, 122	7.103, 108, 114, 119
31	7.10-13	Analysis and Measurement of Strain	7.127, 140, 146, 154	7.131, 136, 145, 150	7.129, 138, 143, 153	7.133, 137, 145, 152	7.128, 139, 142, 155	7.132, 135, 147, 151
32	8.1-3	Principal Stresses in a Beam; Design of Shatsis	8.4, 12, 16, 24	8.3, 11, 15, 23	8.2, 10, 20, 28	8.1, 9, 19, 27	8.6, 14, 18, 26	8.5, 15, 17, 25
33	8.4	Stresses Under Combined Loadings	8.36, 40, 52, 54	8.35, 39, 51, 53	8.32, 38, 50, 58	8.31, 37, 49, 55	8.34, 44, 48, 57	8.35, 44, 42, 56
34	9.1-5	Deflection of Beams by Integration	9.1, 10, 24, 28	9.2, 12, 22, 27	9.3, 9, 23, 26	9.4, 11, 21, 25	9.5, 16, 20, 32	9.7, 14, 19, 31
35	9.6	Use of Singularity Functions	9.35, 48, 52, 62	9.36, 45, 51, 59	9.37, 47, 50, 61	9.38, 46, 49, 60	9.39, 44, 54, 64	9.40, 43, 55, 63
36	9.7-8	Method of Superposition; Statically Indet. Beams	9.68, 76, 82, 87	9.67, 73, 81, 86	9.66, 74, 80, 88	9.65, 73, 79, 85	9.70, 78, 84, 91	9.69, 77, 83, 96
37	9.9-10	Moment-Area Method	9.95, 104, 109, 119	9.96, 101, 110, 120	9.97, 103, 113, 117	9.98, 102, 114, 118	9.99, 105, 115, 121	9.100, 107, 116, 122
38	9.12-14	Moment-Area Method; Max Deflection; Indet. Beams	9.125, 138, 142, 148	9.129, 136, 144, 154	9.126, 134, 144, 153	9.130, 133, 146, 147	9.128, 137, 141, 152	9.132, 135, 145, 151
39	10.1-4	Columns; Euler's Formula	10.1, 10, 20, 22	10.2, 11, 19, 21	10.3, 9, 16, 24	10.4, 12, 15, 23	10.8, 13, 18, 26	10.7, 14, 17, 25
40	10.5	Eccentric Loading; Secant Formula	10.33, 38, 48, 54	10.29, 37, 47, 53	10.34, 36, 50, 56	10.32, 35, 49, 55	10.31, 40, 44, 52	10.32, 39, 43, 51
41	10.6	Design of Columns Under a Centric Load	10.58, 69, 76, 81	10.60, 65, 74, 79	10.57, 65, 71, 77	10.63, 67, 75, 80	10.59, 70, 73, 78	10.61, 68, 72, 82
42	10.7	Design of Columns Under an Eccentric Load	10.94, 96, 108, 116	10.93, 95, 107, 115	10.90, 100, 104, 114	10.89, 99, 103, 113	10.92, 102, 110, 112	10.91, 101, 109, 111
43	10.1-6	Elastic Strain Energy	11.2, 14, 18, 23	11.3, 9, 20, 32	11.1, 10, 21, 30	11.7, 13, 22, 24	11.6, 12, 12, 31	11.4, 11, 11, 26
44	10.7-10	Design for Impact Loading; Deflections	11.44, 52, 59, 76	11.45, 49, 62, 71	11.42, 48, 57, 75	11.46, 48, 58, 74	11.47, 51, 60, 73	11.43, 51, 61, 73
45	10.11-13	Castigliano's Theorem	11.81, 90, 93, 101	11.82, 83, 96, 104	11.77, 92, 94, 100	11.79, 85, 98, 99	11.78, 86, 94, 102	11.80, 91, 97, 105
46	10.14	Statistically Indeterminate Structures	11.107, 112, 117	11.108, 114, 119	11.105, 114, 117	11.106, 116, 118	11.110, 115, 120	11.109, 113, 120

The four lists of suggested assignments contained in this table cover the entire text. The instructor can select those assignments that are covered in the course.

**75% OF THE PROBLEMS IN EACH OF THESE LISTS USE SI UNITS AND 25% U.S. CUSTOMARY UNITS
ANSWERS TO ALL OF THESE PROBLEMS ARE GIVEN IN THE BACK OF THE BOOK**

Group	Sections	Topics	List 1a	List 2a	List 3a	List 4a
1	1.1	Introduction	1.4, 14, 15, 27, 1.30, 39, 43, 56	1.3, 7, 19, 23 1.29, 38, 44, 55	1.2, 9, 18, 24 1.34, 41, 45, 54	1.113, 20, 28 1.33, 42, 46, 53
2	1.2-10	Stresses Under Axial Loading	2.2, 12, 15, 23	2.4, 14, 16, 26	2.6, 9, 19, 25	2.8, 10, 20, 24
3	1.11-13	Components of Stress; Factor of Safety	2.34, 39, 53, 60	2.33, 40, 54, 55	2.35, 41, 49, 57	2.36, 42, 48, 56
4	2.1-8	Stress-Strain Diagram	2.65, 67, 79, 88	2.63, 68, 80, 87	2.62, 74, 75, 86	2.61, 73, 76, 84
5	2.9-10	Statically Indeterminate Problems	2.94, 102, 111, 115	2.93, 101, 112, 114	2.99, 105, 109, 116	2.100, 106, 110, 117
6	2.11-16	Generalized Hooke's Law; Poisson's Ratio	3.3, 9, 15, 22	3.5, 10, 17, 26	3.4, 11, 19, 21	3.6, 15, 20, 25
7	2.17-20	Stress Concentration; Plastic Behavior	3.32, 37, 54, 56	3.33, 39, 55, 57	3.31, 40, 46, 52	3.36, 41, 49, 53
8	3.1-4	Stresses in Torsion	3.65, 76, 78, 90	3.66, 77, 83, 90	3.69, 72, 80, 86	3.73, 71, 81, 87
9	3.5-6	Angle of Twist; Indeterminate Shafts	3.91, 97, 108, 110	3.94, 109, 114	3.92, 101, 102, 111	3.93, 102, 101, 112
10	3.7-8	Transmission Shafts; Stress Concentrations	3.119, 126, 138, 141	3.120, 129, 138, 142	3.121, 131, 135, 140	3.122, 132, 137, 145
11	3.9-11	Plastic Deformations; Residual Stresses	4.6, 13, 15, 27	4.3, 14, 16, 28	4.4, 9, 17, 22	4.5, 10, 20, 26
12	3.12-13	Noncircular and Thin-Walled members	4.37, 39, 48, 65	4.38, 40, 52, 66	4.33, 43, 47, 61	4.34, 44, 54, 62
13	4.1-5	Stresses and Deformations in the Elastic Range	4.69, 75, 82, 91	4.73, 76, 86, 93	4.70, 77, 83, 87	4.74, 77, 84, 88
14	4.6-7	Members Made of Several Materials	4.100, 106, 118, 121	4.101, 107, 114, 124	4.99, 108, 113, 120	4.102, 108, 117, 123
15	4.8-11	Plastic Deformations; Residual Stresses	4.127, 133, 140, 147	4.131, 136, 144, 147	4.127, 134, 142, 146	4.131, 135, 143, 151
16	4.12	Eccentric Axial Loading in a Plane of Symmetry	4.158, 164, 172, 175	4.159, 164, 172, 176	4.156, 165, 171, 178	4.160, 166, 171, 175
17	4.13-14	Unsymmetric Bending	5.7, 15, 22, 30	5.8, 17, 26, 32	5.11, 20, 21, 27	5.13, 18, 25, 29
18	4.15	Bending of Curved Members	5.42, 47, 55, 62	5.43, 49, 61, 62	5.40, 50, 54, 63	5.45, 52, 56, 65
19	5.1-2	Shear and Bending-Moment Diagrams	5.65, 71, 79, 87	5.70, 72, 80, 88	5.75, 73, 81, 90	5.76, 74, 82, 91
20	5.3	Using Relations Among w, V, and M	5.98, 106, 116, 122	5.105, 108, 113, 119	5.104, 107, 114, 118	5.99, 109, 115, 123
21	5.4	Design of Prismatic Beams in Bending	5.126, 132, 140, 148	5.127, 138, 144, 148	5.131, 133, 141, 150	5.128, 136, 145, 151
22	5.5	Use of Singularity Functions to determine V and M	6.4, 12, 19, 26	6.3, 9, 20, 28	6.6, 14, 15, 23	6.5, 13, 17, 21
23	5.6	Nonprismatic Beams	6.30, 35, 49, 53	6.32, 36, 48, 54	6.29, 39, 46, 51	6.34, 40, 45, 52
24	6.1-5	Shearing Stresses in a Beam	6.61, 72, 77, 88	6.62, 69, 78, 87	6.68, 71, 75, 82	6.67, 70, 76, 81
25	6.6-8	Shearing Stresses in Thin-Walled Members	7.1, 9, 17, 28	7.4, 10, 22, 30	7.7, 11, 18, 25	7.8, 13, 19, 26
26	6.9	Shear center	7.35, 39, 50, 54	7.34, 44, 52, 60	7.31, 41, 47, 55	7.32, 41, 48, 56
27	7.1-3	Transformation of Plane Stress	7.66, 75, 83, 93	7.69, 77, 87, 90	7.68, 74, 81, 89	7.67, 79, 82, 95
28	7.4	Mohr's Circle for Plane Stress	7.100, 105, 111, 118	7.101, 107, 117, 123	7.98, 105, 110, 120	7.102, 107, 116, 124
29	7.5-8	Three-Dimensional Stress Analysis; Fracture Criteria	7.127, 136, 146, 152	7.131, 140, 144, 153	7.126, 137, 143, 152	7.130, 138, 145, 153
30	7.9	Stresses in Thin-Walled Pressure Vessels	8.4, 11, 16, 29	8.31, 12, 15, 30	8.8, 9, 20, 27	8.7, 10, 19, 28
31	7.10-13	Analysis and Measurement of Strain	8.36, 39, 46, 53	8.33, 40, 45, 54	8.32, 41, 50, 55	8.31, 42, 49, 58
32	8.1-3	Principal Stresses in a Beam; Design of Shafts	9.1, 12, 24, 29	9.2, 19, 22, 30	9.6, 11, 23, 25	9.8, 9, 21, 26
33	8.4	Stresses Under Combined Loadings	9.35, 45, 53, 59	9.36, 48, 56, 62	9.37, 41, 50, 60	9.38, 42, 49, 61
34	9.1-5	Deflection of Beams by Integration	9.68, 75, 82, 89	9.67, 76, 81, 92	9.72, 73, 80, 85	9.71, 74, 79, 88
35	9.6	Use of Singularity Functions	10.1, 11, 20, 27	10.2, 10, 19, 28	10.5, 12, 16, 19	10.6, 9, 15, 24
36	9.7-8	Method of Superposition, Statically Indet. Beams	10.33, 37, 46, 53	10.29, 38, 45, 54	10.34, 41, 50, 55	10.32, 42, 49, 56
37	9.9-10	Moment-Area Method	10.58, 66, 76, 84	10.60, 69, 74, 83	10.62, 67, 71, 80	10.64, 65, 75, 77
38	9.12-14	Moment-Area Method; Max Deflection; Indet. Beams	10.94, 95, 106, 115	10.93, 96, 105, 116	10.90, 97, 104, 113	9.131, 134, 146, 153
39	10.1-4	Columns; Euler's Formula	11.2, 9, 18, 25	11.3, 14, 20, 29	11.5, 13, 24, 24	11.8, 10, 30, 36
40	10.5	Eccentric Loading; Secant Formula	11.44, 49, 63, 71	11.45, 52, 64, 76	11.42, 53, 57, 74	11.46, 50, 58, 75
41	10.6	Design of Columns Under a Centric Load	11.81, 83, 95, 104	11.82, 90, 98, 101	11.77, 84, 95, 99	11.79, 89, 98, 100
42	10.7	Design of Columns Under an Eccentric Load	11.107, 111, 121	11.108, 121, 122	11.105, 112, 117	11.106, 114, 118
43	11.1-6	Elastic Strain Energy				
44	11.7-10	Design for Impact Loading; Deflections				
45	11.11-13	Castigliano's Theorem				
46	11.14	Statically Indeterminate Structures				

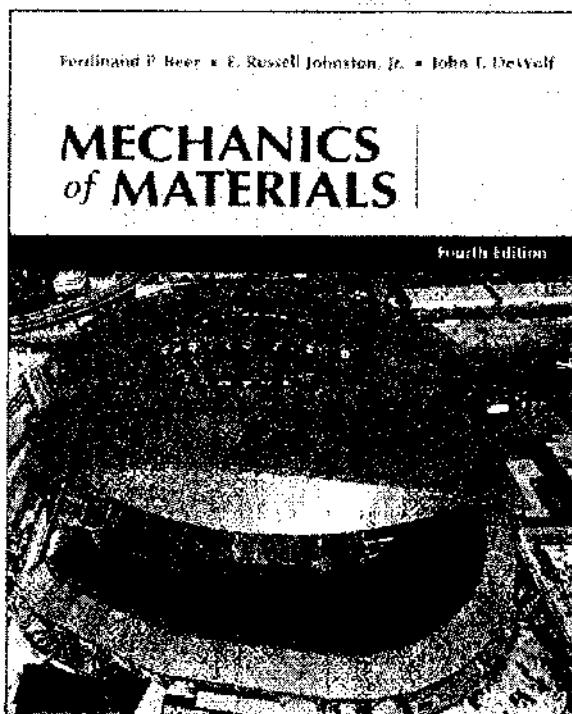
PROPRIETARY AND CONFIDENTIAL

This Manual is the proprietary property of The McGraw-Hill Companies, Inc. (McGraw-Hill) and protected by copyright and other state and federal laws. By opening and using this Manual the user agrees to the following restrictions, and if the recipient does not agree to these restrictions, the Manual should be promptly returned unopened to McGraw-Hill: This Manual is being provided only to authorized professors and instructors for use in preparing for the classes using the affiliated textbook. No other use or distribution of this Manual is permitted. This Manual may not be sold and may not be distributed to or used by any student or other third party. No part of this Manual may be reproduced, displayed or distributed in any form or by any means, electronic or otherwise, without the prior written permission of the McGraw-Hill.

Instructor's and Solutions Manual

Volume II, Chapters 7-11

to accompany



Prepared by
Dean P. Updike

Instructor's and Solutions Manual

to accompany

Mechanics of Materials

Fourth Edition

Volume II, Chapters 7-11

Ferdinand P. Beer

Late of Lehigh University

E. Russell Johnston, Jr.

University of Connecticut

John T. DeWolf

University of Connecticut

Prepared by

Dean P. Updike

Lehigh University

PROPRIETARY AND CONFIDENTIAL

This Manual is the proprietary property of The McGraw-Hill Companies, Inc. ("McGraw-Hill") and protected by copyright and other state and federal laws. By opening and using this Manual the user agrees to the following restrictions, and if the recipient does not agree to these restrictions, the Manual should be promptly returned unopened to McGraw-Hill: This Manual is being provided only to authorized professors and instructors for use in preparing for the classes using the affiliated textbook. No other use or distribution of this Manual is permitted. This Manual may not be sold and may not be distributed to or used by any student or other third party. No part of this Manual may be reproduced, displayed or distributed in any form or by any means, electronic or otherwise, without the prior written permission of the McGraw-Hill.



Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St. Louis
Bangkok Bogotá Caracas Kuala Lumpur Lisbon London Madrid Mexico City
Milan Montreal New Delhi Santiago Seoul Singapore Sydney Taipei Toronto

The McGraw-Hill Companies

Instructor's and Solutions Manual, Volume II to accompany
MECHANICS OF MATERIALS, FOURTH EDITION
Ferdinand P. Beer, E. Russell Johnston, Jr., and John T. DeWolf

Published by McGraw-Hill Higher Education, an imprint of the The McGraw-Hill Companies,
Inc., 1221 Avenue of the Americas, New York, NY 10020. Copyright © 2006, 2002, 1992, 1981
by The McGraw-Hill Companies, Inc. All rights reserved.

The contents, or parts thereof, may be reproduced in print form solely for classroom use with
MECHANICS OF MATERIALS, FOURTH EDITION, provided such reproductions bear copyright
notice, but may not be reproduced in any other form or for any other purpose without the prior
written consent of The McGraw-Hill Companies, Inc., including, but not limited to, network or
other electronic storage or transmission, or broadcast for distance learning.

This book is printed on acid-free paper.

2 3 4 5 6 7 8 9 0 BKM/BKM 0 9 8 7

ISBN-13: 978-0-07-298095-0

ISBN-10: 0-07-298095-8

www.mhhe.com

TABLE OF CONTENTS

To the Instructor	v
Description of Material Contained in <i>Mechanics of Materials, 4th edition</i>	vii
Table I : List of Topics Covered in <i>Mechanics of Materials, 4th edition</i>	xvi
Table II: Classification and Description of Problems.....	xviii
Table III: Sample Assignment Schedule (50% of Problems in SI Units and 50% in U.S. Customary Units)	xxxvi
Table IV: Sample Assignment Schedule (75% of Problems in SI Units and 25% in U.S. Customary Units)	xxxvii
Problem Solutions	xxxix

TO THE INSTRUCTOR:

As indicated in its preface, *Mechanics of Materials* is designed for the first course in mechanics of materials - or strength of materials - offered to engineering students in the sophomore or junior year. However, because of the large number of optional sections that have been included and the maturity of approach that has been achieved, this text can also be used to teach a more advanced course.

The text has been divided into units, each corresponding to a well-defined topic and consisting of one or several theory sections followed by sample problems and a large number of problems to be assigned. In order to accommodate courses of varying emphases, considerably more material has been included than can be covered effectively in a single three-credit-hour course. To assist the instructors in making up a schedule of assignments that best fits their classes, the various topics presented in the text have been listed in Table I and both a minimum and a maximum number of periods to be spent on each topic have been suggested. Topics have been divided into three categories: core topics that will probably be covered in every course; additional topics that can be selected to complement this core to form courses of various emphases; and finally topics that can be used with more advanced students.

The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty, with problems requiring special attention indicated by asterisks. The instructor's attention is called to the fact that problems have been arranged in groups of six or more, all problems of the same group being closely related. This means that the instructor will easily find additional problems to amplify a particular point that has been brought up in the

discussion of a problem assigned for homework. Since half of the problems are stated in SI units and half in U.S. customary units, it also means that the instructor has the choice of assigning problems using SI units and problems using U.S. customary units in whatever proportion is found to be most desirable for a given class. To assist in the preparation of homework assignments Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. It should also be noted that answers to all problems with a number set in roman type are given at the end of the text, while problems with a number set in italic are not.

In Table III six alternative lists of possible assignments have been suggested. Four of these lists consist of problems whose answers are given at the end of the text, and two of problems whose answers are not. Half of the problems in each list are stated in SI units and half in U.S. customary units. For those instructors who wish to emphasize the use of SI units, four additional lists of problems have been given in Table IV, in which 75% of the problems use SI units. Since the lists in Tables III and IV cover the entire text, instructors will want to select those groups of sections that are best suited to the course they wish to teach. For a typical one-semester course consisting of 42 class meetings and including four quizzes, as many as 38 of the 46 available groups can be selected.

Since the approach used in this text differs in a number of respects from the approach used in other books, the instructor is advised to read the preface to *Mechanics of Materials*, in which the authors have outlined their general philosophy. Attention is particularly called to the fact that statically indeterminate problems are first discussed in Chapter 2 and are considered throughout the

text for the various loading conditions encountered. Thus, students are presented at an early stage with a method of solution that combines the analysis of deformations with the conventional analysis of forces used in statics, and will have become thoroughly familiar with it by the end of the course. The concept of plastic deformation is also introduced in Chap. 2, where it is applied to the analysis of members under axial loading, while problems involving the plastic deformation of circular shafts and of prismatic beams are considered in optional sections of Chaps. 3 and 4, respectively. On the other hand, while the concept of stress at a point is introduced in Chap. 1, the discussion of the transformation of stresses is delayed until Chap. 7, after students have discovered for themselves the need for special techniques, such as Mohr's circle. In this edition, shear and bending-moment diagrams are introduced

at the beginning of Chap. 5 and applied immediately to the design of beams in that chapter and in the next.

A brief description, chapter by chapter, of the topics included in the text will be found in the following pages. It is hoped that this material will help instructors organize their courses to best fit the needs of their students.

The authors of *Mechanics of Materials*, 4/e, wish to thank Professor Dean P. Updike of the Department of Mechanical Engineering and Mechanics at Lehigh University for having written the problem solutions contained in this Manual.

*E. Russell Johnston, Jr.
John T. DeWolf*

DESCRIPTION OF THE MATERIAL CONTAINED IN "MECHANICS OF MATERIALS", 4th edition

Chapter 1 **Introduction – Concept of Stress**

The main purpose of this chapter is to introduce the concept of stress. After a short review of Statics in Sec. 1.2 emphasizing the use of free-body diagrams, Secs. 1.3 through 1.7 discuss *normal stresses* under an axial loading, *shearing stresses* – with applications to pins and bolts in single and double shear – and *bearing stresses*. Section 1.4 introduces the concepts of analysis and design, while Sec. 1.5 emphasizes the fact that stresses are inherently statically indeterminate and that, at this point, normal stresses under an axial loading can only be assumed to be uniformly distributed. Moreover, such an assumption requires that the axial loading be centric.

Section 1.8 is devoted to the application of these concepts to the analysis of a *simple structure*. Section 1.9 describes how students should approach the solution of a problem in mechanics of materials, and Sec. 1.10 discusses the *numerical accuracy* to be expected in such a solution. Problems included in the first lesson also serve as a review of the methods of analysis of trusses, frames, and mechanisms learned in statics.

Section 1.11 discusses the determination of normal and shearing stresses on *oblique planes* under an axial loading, while Sec. 1.12 introduces the components of stress under general loading conditions. This section emphasizes the fact that the components of the shearing stresses exerted on perpendicular planes, such as τ_{xy} and τ_{yx} , must be equal. It also introduces the students to the concept of transformation of stress. However, the study of the computational techniques associated with the transformation of stress at a point is delayed

until Chap. 7, after students have discovered for themselves the need for such techniques.

Section 1.13 is devoted to design considerations. It introduces the concepts of *ultimate load*, *ultimate stress*, and *factor of safety*. It also discusses the reasons for the use of factors of safety in engineering practice. The section ends with an optional presentation of an alternative method of design, *Load and Resistance Factor Design*.

Chapter 2 **Stress and Strain – Axial Loading**

This chapter is devoted to the analysis and design of members under a centric axial loading. Sections 2.1 and 2.2 introduce the concept of *normal strain*, while Sec. 2.3 describes the general properties of the stress-strain diagrams of ductile and brittle materials and defines the *yield strength*, *ultimate strength*, and *breaking strength* of a material. Section 2.4, which is optional, defines *true stress* and *true strain*. Section 2.5 introduces *Hooke's law*, the *modulus of elasticity*, and the *proportional limit* of a material. It defines as *isotropic* those materials whose mechanical properties are independent of the direction considered and as *anisotropic* those whose mechanical properties depend upon that direction. Among the latter are *fiber-reinforced composite materials*, which are described in this section.

Section 2.6 discusses the elastic and the plastic behavior of a material and defines its *elastic limit*, while Sec. 2.7 is devoted to fatigue and the behavior of materials under repeated loadings. The first lesson of Chap. 2 ends with Sec. 2.8, which shows how Hooke's law can be used to determine the deformation of a rod of

uniform or variable cross section under one or several loads, and introduces the concept of *relative displacement*. Section 2.9 discusses *statically indeterminate problems* involving members under an axial load. As indicated in the preface of the text and in the introduction to this manual, the authors believe it is important to introduce the students at an early stage to the concept of statical indeterminacy and to show them how the analysis of deformations can be used in the solution of problems that cannot be solved by the methods of statics alone. It will also help them realize that stresses, being statically indeterminate, can be computed only by considering the corresponding distribution of strains. Section 2.10 discusses the *thermal expansion* of rods and shows how to determine stresses in statically indeterminate members subjected to temperature changes.

Section 2.11 introduces the concept of *lateral strain* for an isotropic material and defines *Poisson's ratio*. Section 2.12 discusses the *multiaxial loading* of a structural element and derives the generalized Hooke's law for such a loading. Since this derivation is based on the application of the *principle of superposition*, this principle is also introduced in Sec. 2.12, and the conditions under which it can be used are clearly stated. Section 2.13 is optional. It discusses the change in volume of a material under a multiaxial loading and defines the *dilatation* and the *bulk modulus* or *modulus of compression* of a given material.

Section 2.14 introduces the concept of *shearing strain*. It should be noted that the authors define the shearing strain as the change in the angle formed by the faces of the element of material considered, and not as the angle through which one of these faces rotates. Hooke's law for shearing stress and strain and the *modulus of rigidity* are also introduced in this section, as well as the *generalized Hooke's law* for a homogeneous, isotropic material under

the most general stress conditions. Section 2.15 points out that strains, just as stresses, depend upon the orientation of the planes considered. It also establishes the fact that the constants E , v , and G are not independent from each other and derives Eq. (2.43), which expresses the relation among these three constants. Section 2.16, which is optional, extends the stress-strain relationships to *fiber-reinforced composite materials*. The relations obtained are expressed by Eqs. (2.45) and (2.47) and involve three different values of the modulus of elasticity and six different values of Poisson's ratio.

Section 2.17 discusses the *distribution of the normal stresses* under a centric axial loading and shows that this distribution depends upon the manner in which the loads are applied. However, except in the immediate vicinity of the points of application of the loads, the distribution of stresses can be assumed uniform. This result verifies Saint-Venant's principle. Section 2.18 discusses *stress concentrations* near circular holes and fillets in flat bars under axial loading.

Section 2.19 is devoted to the *plastic deformation* of members under centric axial loads and introduces the concept of an *elastoplastic material*. As stated in the preface of the text, the authors believe that students should be exposed to the concept of plastic deformation in the first course in mechanics of materials, if only to let them realize the limitations of the assumption of a linear stress-strain relation in engineering applications. By introducing this concept early in the course in connection with axial loading, rather than later with torsion or bending, one makes it easier for the students to understand and accept it. For the same reason, *residual stresses* are discussed in Sec. 2.20 in connection with axial loading. However, since some instructors may not want to include the concept of residual stresses in an elementary course, this section is optional and can be omitted.

without any prejudice to the understanding of the rest of the text.

Chapter 3 Torsion

Section 3.1 introduces this type of loading, while Sec. 3.2 establishes the relation that must be satisfied, on the basis of statics, by the shearing stresses in a given section of a shaft subjected to a torque. This condition, however, does not suffice to determine the stresses, and one must analyze the deformations which occur in the shaft. This is done in Sec. 3.3, where it is proved that the distribution of shearing strains in a circular shaft is linear. It should be noted that the discussion presented in Sec. 3.3 is based solely on the assumption of rigid end plates, rather than on arbitrary and gratuitous assumptions regarding the deformations of a shaft. The results obtained in this and the following sections clearly depend upon the validity of this assumption, but can be extended to other loading conditions through the application of Saint-Venant's principle.

Section 3.4 is devoted to the analysis of the shearing stresses in the elastic range and presents the derivation of the *elastic torsion formulas* for circular shafts. The section ends with remarks on the transformation of stresses in torsion and the comparison between the failures of ductile and brittle materials in torsion.

The formula for the *angle of twist* of a shaft in the elastic range is derived in Sec. 3.5. This section also contains various applications involving the twisting of single and gear-connected shafts. Section 3.6 deals with the solution of problems involving *statically indeterminate shafts*.

Section 3.7 is devoted to the *design of transmission shafts* and begins with the determination of the torque required to transmit a given power at a given speed, both in SI and

U.S. customary units. Note that the effect of bending on the design of transmission shafts will be discussed in Sec. 8.3, which is optional. Section 3.8 discusses *stress concentrations* at fillets in circular shafts.

Sections 3.9 through 3.11 deal with the *plastic deformations* and *residual stresses* in circular shafts and are optional. Since a similar presentation of the plastic deformations and residual stresses of members in pure bending is given in Chap. 4, the instructor may decide to include only one of these presentations in the course. Section 3.9 describes the general method for the determination of the torque corresponding to a given maximum shearing stress in a shaft made of a material with a *nonlinear stress-strain diagram*, while Secs. 3.10 and 3.11 deal, respectively, with the deformations and the residual stresses in shafts made of an *elastoplastic material*. Sections 3.12 and 3.13 are also optional. They are devoted, respectively, to the torsion of solid members and thin-walled hollow shafts of *noncircular section*.

Chapter 4 Pure Bending

Section 4.1 defines this type of loading and shows how the results obtained in the following sections can be applied to the analysis of other types of loading as well, namely, eccentric axial loadings and transverse loadings. Section 4.2 establishes the relation that must be satisfied, on the basis of statics, by the *normal stresses* in a given section of a member subjected to pure bending. This condition, however, does not suffice to determine the stresses, and one must analyze the *deformations* that occur in the member. This is done in Sec. 4.3, where it is proved that the distribution of normal stresses in a symmetric member in pure bending is linear. It should be noted that no assumption is made in this discussion regarding the deformations of the member, except that the couples should be applied

in such a way that the ends of the member remain plane. Whether this can actually be accomplished is discussed at the end of Sec. 4.5.

Section 4.4 is devoted to the analysis of the normal stresses in the elastic range and presents the derivation of the *elastic flexure formulas*. It also defines the elastic section modulus and ends with the derivation of the formula for the *curvature* of an elastic beam. Section 4.5 discusses the *anticlastic curvature* of members in pure bending and also states the loading conditions required for the ends of the member to remain plane.

Section 4.6 discusses the determination of stresses in members made of several materials and defines the *transformed section* of such members. It also shows how the transformed section can be used to determine the radius of curvature of the member. The section ends with a discussion of the stresses in *reinforced-concrete beams*. Section 4.7 deals with the stress concentrations at fillets and grooves in flat bars under pure bending.

Sections 4.8 through 4.11 are optional. They discuss the plastic deformations and residual stresses in members subjected to pure bending in much the same way that these were discussed in Secs. 3.9 through 3.11 in the case of members in torsion. Section 4.8 describes the general method for the determination of the bending moment corresponding to a given maximum normal stress in a member possessing two planes of symmetry and made of a material with a nonlinear stress-strain diagram. Section 4.9 deals with members made of an *elastoplastic material* and derives formulas relating the thickness of the elastic core and the radius of curvature with the applied bending moment in the case of members with a rectangular cross section. It also defines the *shape factor* and the *plastic section modulus* of members with a nonrectangular section. Section 4.10 deals with

the determination of the plastic moment of members made of an elastoplastic material and possessing a single plane of symmetry, while Sec. 4.11 discusses *residual stresses*.

Section 4.12 shows how the stresses due to a *two-dimensional eccentric axial loading* can be obtained by replacing the given eccentric load by a centric load and a couple, and superposing the corresponding stresses. Attention is called to the fact that the neutral axis *does not pass* through the centroid of the section.

Section 4.13 deals with the *unsymmetric bending* of elastic members. It is first shown that the neutral axis of a cross section will coincide with the axis of the bending couple if, and only if, the axis of the couple is directed along one of the principal centroidal axes of the cross section. It is then shown that stresses due to unsymmetric bending can always be determined by resolving the given bending couple into two component couples directed along the principal axes of the section and superposing the corresponding stresses.

This method of analysis is extended in Sec. 4.14 to the determination of the stresses due to an *eccentric axial loading in three-dimensional space*. The eccentric load is replaced by an equivalent system consisting of a centric load and two bending couples, and the corresponding stresses are superposed.

Section 4.15 is optional; it deals with the bending of *curved members*.

Chapter 5

Analysis and Design of Beams for Bending

In Sec. 5.1 beams are defined as slender prismatic members subjected to transverse loads and are classified according to the way in which they are supported. It is shown that the internal forces in any given cross section are equivalent to a shear force V and a bending couple M . The

bending couple M creates *normal stresses* in the section, while the shear force V creates *shearing stresses*. The former is determined in this chapter, using the flexure formula (5.2), while the latter will be discussed in Chap. 6.

Since the dominant criterion in the design of beams for strength is usually the bending stresses in the beam, the determination of the maximum value of the bending moment in the beam is the most important factor to be considered. To facilitate the determination of the bending moment in any given section of the beam, *shear and bending-moment diagrams* will be drawn, using free-body diagrams of various portions of the beam (Sec. 5.2).

An alternative method, based on relations among load, shear, and bending moment, is presented in Sec. 5.3. To maintain the interest of the students, most of the problems to be assigned are focused on the engineering applications of these methods and call for the determination, not only of the shear and bending moment, but also of the *normal stresses in the beam*.

Section 5.4 is devoted to the *design of prismatic beams* based on the allowable normal stress for the material used. Sample Problems and problems to be assigned include wooden beams of rectangular cross section, as well as rolled-steel W and S beams. An optional paragraph on page 334 describes the application of *Load and Resistance Factor Design* to beams under transverse loading.

Section 5.5 introduces the concept of *singularity functions* and shows how these functions can provide an alternative and effective method for the determination of the shear and bending moment at any point of a beam under the most general loading condition. While this section is optional, it should be included in the lesson schedule if singularity functions are to be used later for the determination of the slope and deflection of a beam (Sec. 9.6). It is pointed out

on page 348 that singularity functions are particularly well suited to the *use of computers*, and several optional problems requiring the use of a computer (Probs. 5.129 through 5.136) have been included in this assignment.

Section 5.6, which is optional, is devoted to *nonprismatic beams*, such as forged or cast beams designed to be of constant strength, and rolled-steel beams reinforced with cover plates.

Chapter 6

Stresses in Beams and Thin-Walled Members Due to Shear

It is shown in Sec. 6.1 that a transverse load creates *shearing stresses* as well as normal stresses in a beam. Considering first the horizontal face of a beam element, it is shown in Sec. 6.2 that the horizontal shear per unit length q , or *shear flow*, is equal to VQ/I . This result is applied in Example 6.01 to the determination of the shear force in the nails connecting three planks forming a wooden beam, as well as in Probs. 6.1 through 6.8.

In Sec. 6.3 the *average shearing stress* τ_{ave} exerted on the horizontal face of the beam element is obtained by dividing the shear flow q by the width t of the beam:

$$\tau_{ave} = \frac{VQ}{It} \quad (6.6)$$

Note that since the shearing stresses τ_{xy} and τ_{yx} exerted at a given point are equal, the expression obtained also represents the average shearing stress exerted at a given height on a vertical section of the beam. This formula is used to determine shearing stresses in a beam made of glued planks in Sample Prob. 6.1 and to design a timber beam in Sample Prob. 6.2. Problems 6.11 through 6.14 and 6.21 through 6.24 call for the determination of shearing stresses in various types of beams.

In Examples 6.02 and 6.03 the designs obtained on the basis of normal stresses, respectively, for a timber beam in Sample Prob. 5.7 and for a rolled-steel beam in Sample Prob. 5.8 are checked and found to be acceptable from the point of view of shearing stresses. Problems 6.15 through 6.18 call for a similar verification of beam designs obtained in Chap. 5.

Section 6.5 is optional and discusses the distribution of stresses in a narrow rectangular beam.

In Sec. 6.6 the expression $q = VQ/I$ obtained on Sec. 6.2 for the shear flow on the horizontal face of a beam element is shown to remain valid for the curved surface of a beam element of arbitrary shape. It is then applied in Example 6.04 and in Probs. 6.29 through 6.33 to the determination of the shearing forces and shearing stresses in nailed and glued vertical surfaces.

Section 6.7 deals with the determination of shearing stresses in *thin-walled members* and shows that Eq. (6.6) can be applied to the determination of the average shearing stress in a section of arbitrary orientation.

Section 6.8, which is optional, describes the formation of *plastic zones* in beams subjected to transverse loads.

Section 6.9, which is also optional, deals with the *unsymmetric loading* of thin-walled members, the determination of the *shear center*, and the computation of the shearing stresses caused by a shearing force exerted at the shear center.

Chapter 7 Transformations of Stress and Strain

After a short introduction (Sec. 7.1), formulas for the *transformation of plane stress* under a rotation of axes are derived in Sec. 7.2,

while the principal planes of stress, principal stresses, and maximum shearing stress are determined in Sec. 7.3.

Section 7.4 is devoted to the use of *Mohr's circle*. It should be noted that the convention used in the text provides for a rotation on Mohr's circle in the *same sense* as the corresponding rotation of the element; in other words, this convention is the same as that used in statics for the transformation of moments and products of inertia. Attention is called to the statement at the top of page 439 of the text and the accompanying Fig. 7.20.

Section 7.5 discusses the general (*three-dimensional*) state of stress and establishes the fact that three principal axes of stress and three principal stresses exist. Section 7.6 shows how three different Mohr's circles can be used to represent the transformations of stress associated with rotations of the element about the principal axes. The results obtained are used to show that in a state of plane stress, the maximum shearing stress does not necessarily occur in the plane of stress.

Sections 7.7 and 7.8 are optional. Section 7.7 presents the two criteria most commonly used to predict *whether a ductile material will yield* under a given state of plane stress, while Sec. 7.8 discusses the two criteria used to predict the *fracture of brittle materials*.

Section 7.9 deals with stresses in *thin-walled pressure vessels*; it is limited to the analysis of cylindrical and spherical pressure vessels.

The second part of the chapter (Secs. 7.10 through 7.13) deals with *transformations of strain* and is optional. Section 7.10 presents the derivation of the formulas for the transformation of strain under a rotation of axes. It should be noted that this derivation is based on the consideration of

an oblique triangle (Fig. 7.62) and the use of the law of cosines, and that the determination of the shearing strain is facilitated by the use of Eq. (7.43), which relates it to the normal strain along the coordinate axes and their bisector.

Section 7.11 introduces *Mohr's circle for plane strain*, and Sec. 7.12 discusses the three-dimensional analysis of strain and its application to the determination of the maximum shearing strain in states of plane strain and of plane stress. Section 7.13 deals with the use of strain rosettes for the determination of states of plane strain.

Chapter 8 Principal Stresses under a Given Loading

This chapter is devoted to the determination of the *principal stresses* and *maximum shearing stress* in beams, transmission shafts subjected to transverse loads as well as to torques, and bodies of arbitrary shape under combined loadings.

In Sec. 8.1 it is shown that, while only normal stresses occur on a square element with horizontal and vertical faces located at the surface of a beam, shearing stresses will occur if the element is rotated through 45° (Fig. 8.1). The reverse situation is observed for an element with horizontal and vertical faces subjected only to shearing stresses (Fig. 8.2). The analysis of beams, therefore, should include the determination of the principal stresses and maximum shearing stress at various points.

This is done in Sec. 8.3 for cantilever beams of various rectangular sections subjected to a single concentrated load at their free end. It is found that the principal stress σ_{\max} does not exceed the maximum normal stress σ_m determined by the method of Chap. 5 except very close to the load. While this result holds for most beams of nonrectangular section, it may not be valid for rolled-steel W and S beams, and the

analysis and design of such beams should include the determination of the principal stress σ_{\max} at the junction of the web with the flanges of the beam. (See Sample Probs. 8.1 and 8.2, and Probs. 8.1 through 8.14).

Section 8.3 is devoted to the analysis and design of *transmission shafts* using gears or sprocket wheels to transmit power to and from the shaft. These shafts are subjected to transverse loads as well as to torques. The design of such shafts is the subject of Sample Prob. 8.3 and Probs. 8.15 through 8.30.

The determination of the stresses at a given point K of a body due to a *combined loading* is the subject of Sec. 8.4. First, the loading is reduced to an equivalent system of forces and couples in a section of the body containing K . Next, the normal and shearing stresses are determined at K . Finally, using one of the methods of transformation of stresses presented in Chap. 7, the principal planes, principal stresses, and maximum shearing stress may be determined at K . This procedure is illustrated in Example 8.01 and Sample Probs. 8.4 and 8.5.

Chapter 9 Deflection of Beams

The relation derived in Chap. 4 between the curvature of a beam and the bending moment is recalled in Sec. 9.2 and used to predict the variation of the curvature along the beam. In Sec. 9.3, the equation of the *elastic curve* for a beam is obtained through two successive integrations, after the bending moment has been expressed as a function of the coordinate x . Examples 9.01 and 9.02 show how the boundary conditions can be used to determine the two constants of integration in the cases of a cantilever beam and of a simply supported beam. Example 9.03 indicates how to proceed when the bending moment must be represented by two different functions of x .

Section 9.4 is optional; it shows in the case of a beam supporting a distributed load, how the equation of the elastic curve can be obtained directly from the function representing the load distribution through the use of four successive integrations.

Section 9.5 is devoted to the analysis of *statically indeterminate beams* and to the determination of the reactions at their supports. It is suggested that a minimum of two lessons be spent on Secs. 9.2 through 9.5 if neither the use of singularity functions (Sec. 9.6) nor the moment-area method (Secs. 9.9 through 9.14) are to be covered in the course.

Section 9.6 is devoted to the use of *singularity functions* for the determination of beam deflections and slopes. It is optional and assumes that Sec. 5.5 has been covered previously. It is recommended that both Secs. 5.5 and 9.6 be included in the course, since singularity functions provide the students with an effective and versatile method for the determination of deflections and slopes under the most diverse loading conditions. In addition, and as indicated earlier, singularity functions are well suited to the use of computers. Two problems requiring the use of computers (Probs. 9.49 and 9.50) may be included in this assignment.

Section 9.7 discusses the *method of superposition* for the determination of beam deflections and slopes. It shows how the expressions given in Appendix D for various simple loadings can be used to obtain the deflection and slope of a beam supporting a more complex loading. In Sec. 9.8, the method of superposition is applied to the determination of the reactions at the supports of *statically indeterminate beams*.

Sections 9.9 through 9.14 are optional. They deal with the application of the moment-area methods to the determination of the deflection of beams and may be omitted in

courses that place a greater emphasis on analytical methods and make use of singularity functions. It should be noted, however, that these methods provide a very practical means for the determination of the deflection and slope of beams of *variable cross section*.

The two moment-area theorems are derived in Sec. 9.9 and are immediately applied in Sec. 9.10 to the computation of the slope and deflection of *cantilever beams* and *beams with symmetric loadings* (simply supported or overhanging beams). Section 9.11 shows how to draw a *bending-moment diagram by parts*. This approach greatly facilitates the determination of moment areas in all but the simplest loading situations.

Section 9.12 deals with simply supported and overhanging beams with *unsymmetric loadings*. The analysis of such beams hinges on the use of a reference tangent drawn through one of the supports after the tangential deviation of the second support has been computed from the bending-moment diagram. Section 9.13 describes how to locate the point of *maximum deflection* and how to compute that deflection.

Section 9.14 deals with the analysis of *statically indeterminate beams* and the determination of the reactions at their supports.

Chapter 10 Columns

Section 10.2 introduces the concept of *stability of a structure*. The example considered in this section consists of a block supported by two spring-connected rigid rods. It is shown that the position of equilibrium in which both rods are aligned is stable if this position is the only possible position of equilibrium of the system. The same criterion is applied to an elastic pin-ended column in Sec. 10.3 in order to derive *Euler's formula*. Section 10.4 shows how Euler's

formula for pin-ended columns can be used to determine the critical load of columns with other end conditions.

Section 10.5 is optional; it deals with the eccentric loading of a column and gives the derivation of the *secant formula*.

Section 10.6 discusses the *design of columns under a centric load* and presents the empirical formulas developed by various engineering associations for the design of steel columns, aluminum columns, and wood columns. Page 643 is devoted to an optional discussion of the application of *Load and Resistance Factor Design* to steel columns.

Section 10.7 discusses the *design of columns under an eccentric load* and presents two of the most frequently used methods: the *allowable-stress method* and the *interaction method*.

Chapter 11 Energy Methods

Section 11.2 introduces the concept of *strain energy* by considering the work required to stretch a rod of uniform cross section. This work, which is equal to the area under the load-deformation curve, represents the strain energy of the rod. The *strain-energy density* is defined in Sec. 11.3, as well as the *modulus of toughness* and the *modulus of resilience* of a given material. The formula for the elastic strain energy associated with normal stresses is derived in Sec. 11.4, as well as the expressions for the strain energy corresponding to an axial loading and to pure bending. The formula for the strain energy associated with shearing stresses is derived in Sec. 11.5, as well as the expressions corresponding to torsion and transverse loading.

Section 11.6, which is optional, covers the strain energy for a general state of stress and derives an expression for the *distortion energy per unit volume*, both in the general case of three-dimensional stress and in the particular case of plane stress.

Section 11.7 discusses *impact loadings* and Sec. 11.8 the design of a structure for an impact load. To facilitate the solution of impact-loading problems, it is shown in Sec. 11.9 that the strain energy of a structure subjected to a single concentrated load \mathbf{P} can be obtained by equating the strain energy to the work of \mathbf{P} . (Appendix D is used to express the deflection in terms of P). As shown in Sec. 11.10, the reverse procedure can be used to determine the deflection of a structure at the point of application of a single load \mathbf{P} or a single couple \mathbf{M} ; the strain energy of the structure is computed from one of the formulas derived in Secs. 11.4 and 11.5, and the work of \mathbf{P} or \mathbf{M} is equated to the expression obtained for the strain energy.

Sections 11.11 through 11.14 are optional. In Sec. 11.11 an expression for the strain energy of a *structure subjected to several loads* is obtained by computing the work of the loads as they are successively applied. Reversing the order in which the loads are applied, one proves *Maxwell's reciprocal theorem*. The expression obtained for the strain energy of the structure is used in Sec. 11.12 to prove *Castigliano's theorem*. Section 11.13 is devoted to the application of Castigliano's theorem to the determination of the deflection and slope of a beam and to the deflection of a point in a truss. Finally, Sec. 11.14 deals with the application of Castigliano's theorem to the determination of the reactions at the supports of statically indeterminate structures such as beams and trusses.

TABLE I: LIST OF TOPICS COVERED IN *MECHANICS OF MATERIALS, 4th edition*

Sections	Topics	Suggested Number of Periods		
		Core Topics	Additional Topics	Advanced Topics
Chapter 1: Introduction – Concept of Stress				
1.1-10	Stress Under Axial Loading	1 - 2		
1.11-13	Components of Stress; Factor of Safety	1		
Chapter 2: Stress and Strain – Axial Loading				
2.1-8	Stress-Strain Diagrams; Deformations Under Axial Loading	1 - 2		
2.9-10	Statically Indeterminate Problems	1		
2.11-12	Poisson's Ratio; Generalized Hooke's Law	1		
*2.13	Dilatation; Bulk Modulus		0.25 - 0.5	
2.14-15	Shearing Strain	0.5		
*2.16	Stress-Strain Relationships for Fiber-Reinforced Composite Materials		0.5 - 1	
2.17-19	Stress Concentrations; Plastic Deformations	0.5 - 1		
*2.20	Residual Stresses		0.5	
Chapter 3: Torsion				
3.1-4	Stresses in Elastic Range	1		
3.5-6	Angle of Twist; Statically Indeterminate Shafts	1 - 2		
3.7-8	Design of Transmission Shafts; Stress Concentrations	1		
*3.9-11	Plastic Deformations; Residual Stresses		1 - 2	
*3.12-13	Noncircular Members; Thin-Walled Hollow Shafts			1 - 2
Chapter 4: Pure Bending				
4.1-5	Stresses in Elastic Range	1 - 2		
4.6-7	Members Made of Several Materials; Stress Concentrations	1 - 2		
*4.8-11	Plastic Deformations; Residual Stresses		1 - 2	
4.12	Eccentric Axial Loading	1 - 2		
4.13-14	Unsymmetric Bending; General Eccentric Axial Loading	1 - 2		
*4.15	Bending of Curved Members			1 - 2
Chapter 5: Analysis and Design of Beams for Bending				
5.1-2	Shear and Bending-Moment Diagrams	1 - 1.5		
5.3	Using Relations among w , V , and M	1 - 1.5		
5.4	Design of Prismatic Beams in Bending	1 - 2		
*5.5	Use of Singularity Functions to Determine V and M		1 - 2	
*5.6	Nonprismatic Beams			1 - 2
Chapter 6: Shearing Stresses in Beams and Thin-Walled Members				
6.1-4	Shearing Stresses in Beams	1 - 2		
*6.5	Shearing Stresses in Narrow Rectangular Beam		0.25	
6.6-7	Shearing Stresses in Thin-Walled Members	1 - 2		
*6.8	Plastic Deformations		0.25	
*6.9	Unsymmetric Loading; Shear Center			1 - 2

TABLE I: LIST OF TOPICS COVERED IN *MECHANICS OF MATERIALS, 4th edition*
(CONTINUED)

Sections	Topics	Suggested Number of Periods		
		Core Topics	Additional Topics	Advanced Topics
Chapter 7: Transformation of Stress and Strain				
7.1-3	Transformation of Plane Stress	1 - 2		
7.4	Mohr's Circle for Plane Stress	1 - 2		
7.5-6	Three-Dimensional Analysis of Stress	0.5 - 1		
*7.7-8	Yield and Fracture Criteria		0.5 - 1	
7.9	Thin-Walled Pressure Vessels	0.5 - 1		
*7.10-12	Analysis of Strain; Mohr's Circle		1 - 1.5	
*7.13	Strain Rosette		0.5	
Chapter 8: Principal Stresses under a Given Loading				
*8.1-2	Principal Stresses in a Beam		0.5 - 1	
*8.3	Design of Transmission Shafts		0.5 - 1	
*8.4	Stresses under Combined Loadings		1 - 3	
Chapter 9: Deflection of Beams				
9.1-3	Equation of Elastic Curve	0.5 - 1		
*9.4	Direct Determination of Elastic Curve from Load Distribution		0.5	
9.5	Statically Indeterminate Beams	0.5 - 1		
*9.6	Use of Singularity Functions		1 - 2	
9.7-8	Method of Superposition	1 - 2		
Application of Moment-Area Theorems to:				
*9.9-11	Cantilever Beams and Beams with Symmetric Loadings		1 - 2	
*9.12-13	Beams with Unsymmetric Loadings; Maximum Deflection		1 - 1.5	
*9.14	Statically Indeterminate Beams		0.5	
Chapter 10: Columns				
10.1-4	Euler's Column Formula	1 - 2		
*10.5	Eccentric Loading; Secant Formula		1	
10.6	Design of Columns under a Centric Load		1 - 2	
10.7	Design of Columns under an Eccentric Load		1 - 2	
Chapter 11: Energy Methods				
11.1-5	Strain Energy		1 - 2	
*11.6	Strain Energy for General State of Stress		0.5	
11.7-9	Impact Loading		0.5 - 1	
11.10	Deflections by Work-Energy Method		0.5 - 1	
*11.11-13	Castigliano's Theorem		1 - 2	
*11.14	Statically Indeterminate Structures		1 - 2	
Total Number of Periods		24 - 41½	21 - 38½	3 - 6

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
Mechanics of Materials 4th Edition

SI Units	U.S. Units	Problem description
<u>CHAPTER 1: INTRODUCTION - CONCEPT OF STRESS</u>		
1.1,2 1.5,6 1.7,9 1.13,14 1.15,18 1.19,20 1.23,25 1.26,27	1.3,4 1.8,10 1.11,12 1.16,17 1.21,22 1.24,28	Normal stress under axial loading: in bars in pin-connected structures in trusses and mechanisms Shearing stress Bearing stress between flat surfaces Shearing and bearing stresses at pin-connected joints
1.29,30 1.33,34 1.37,40 1.41,42 1.43,44 1.47,48 1.53,54 1.55,56 *1.58	1.31,32 1.35,36 1.38,39 1.45,46 1.49,50 1.51,52 *1.57	Stresses on an oblique plane Factor of safety: in tension in shear in structures involving links and pins Load and Resistance Factor Design
1.59,61 1.64,66 1.67, <u>68</u>	1.60,62 1.63,65 1.69,70	Review problems
1.C2,C4,C6	1.C1,C3, C5	Computer problems
<u>CHAPTER 2: STRESS AND STRAIN - AXIAL LOADING</u>		
2.2,4 2.6,8 2.9,10 2.11, <u>13</u> 2.15,16 2.17, <u>18</u> 2.23,24 2.25,26 2.29 2.31	2.1,3 2.5,7 2.12,14 2.19,20 2.21,22 2.27,28 2.30 2.32	Stresses and deformations in statically determinate structures: simple rods and wires multiple-criteria problems composite rods and members members of trusses and simple frames computation of deformations by integration true strain

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*		Problem description
SI Units	U.S. Units	
2.35,36	2.33,34	Statically indeterminate structures (constant temperature): with members undergoing equal deformations
2.37,38		composite rods with both ends restrained
2.39,40		
2.41,42		with members undergoing unequal deformations
2.43,44		
2.45,46		Statically indeterminate structures (with temperature changes): with members undergoing equal deformations
2.48,49	2.47,50	composite rods with both ends restrained
2.53,54	2.51,52	rods with gaps
2.55	2.56,57	with unequal deformations
2.58,59		
2.60		
2.61,62	2.63,65	Poisson's ratio and generalized Hooke's law: uniaxial loading
<u>2.64,66</u>		
2.67,68	2.69,70	biaxial loading
<u>2.71,72</u>	<u>2.73,74</u>	derivation of formulas
2.75,76	2.77,78	Hooke's law for shearing stress and strain
2.79,80	2.81,82	
*2.83,85	*2.84,86	dilatation
*2.87,*88		Problems involving cylindrical coordinates
*2.83,*86	*2.84,*85	Dilatation
*2.89,*90		Theory problems
*2.91,92		Problems involving composites
2.93,94	2.95,96	Stress concentrations in flat bars
2.99,100	2.97,98	
2.103,104	2.101,102	Plastic deformations under axial loading: simple problems
2.105,106		
2.107,108	2.109,110	more complex problems
2.111,112		
2.114,*115	2.113	problems involving temperature change
2.116,117		problems involving a structure
*2.118,120	*2.119	problems involving residual stresses
*2.120,121	*2.115,116	
2.124,128	2.125,126	Review problems
2.131,132	2.127,129	
2.133,135	2.130,134	
2.C1,C3,C6	2.C2,C4,C5	Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>CHAPTER 3: TORSION</u>			
			Shearing stresses:
3.3,4		3.1,2	in simple shafts
3.5,6		3.7,8	
3.11, <i>12</i>		3.9,10	in shafts subjected to several torques
3.13, <i>14</i>			
3.15, <i>16</i>		3.19,20	in composite shafts
3.17, <i>18</i>			
3.21,22		3.23,24	in gear-connected shafts
3.25,26		3.27,28	
3.29		3.30	special problems
			Angle of twist:
3.31, <i>34</i>		3.32,33	in simple shafts
3.35,36			in shafts subjected to several torques
3.37		3.38	in composite shafts
3.39,40		<u>3.41</u> ,42	in gear-connected shafts
		3.43,44	
3.46		3.45,47	Design of shafts based on allowable stress and allowable angle of twist
<u>3.49</u>		3.48	
		3.50, <u>51</u>	
			Statically indeterminate shafts:
3.54,55		3.52,53	with inner core and outer shell of different materials
3.56,57		3.58,59	
3.62, <u>63</u>		<u>3.60</u> ,61	Special problems
			Design of shafts:
3.65,66		3.64,67	easy problems with solid shafts
3.69,71		3.68,70	hollow shafts
3.72, <u>73</u>			
		3.74,75	including angle of twist
		3.76,77	gear-connected shafts
3.78,79		3.80,81	multiple-criteria problems
3.82,83			
3.86,87		3.84,85	Stress concentrations in stepped shafts
3.90		3.88,89	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
3.92,93 3.95 3.96,97 3.98 3.101,102 3.105 3.106,107 3.110,114 3.115,118	3.91,94 <u>3.99</u> ,100 <u>3.103</u> ,104 <u>3.108</u> ,109 3.111,112 <u>3.113</u> , <u>116</u> 3.117		Plastic deformations of shafts: Shafts made of elastoplastic material: stresses in solid shafts angle of twist for solid shafts hollow and tapered shafts derivation Shafts made of a material with a nonlinear stress-strain diagram Residual stresses and permanent angle of twist in shafts made of an elastoplastic material
3.119,120 3.121,122			Bars with rectangular cross section: easy problems
3.131,132	<u>3.130</u> , <u>133</u>		comparing circular and rectangular shafts application to structural shapes
3.135,137 3.138,141 3.142 3.144	3.134,136 3.139,140 <u>3.143</u> <u>3.145</u>		Thin-walled hollow shafts: determine shearing stresses Special problems
3.148,150 3.151,152 3.156,157	3.146,147 3.149,153 <u>3.154</u> , <u>155</u>		Review problems
3.C1,C2,C3	3.C4,C5,C6		Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>CHAPTER 4: PURE BENDING</u>			
4.3,4	4.1,2		Normal stresses: in beams with horizontal plane of symmetry
4.5,6			
4.9,10	4.7,8		in unsymmetrical beams (first locate centroid)
4.11,12	4.13,14		Resultant force on portion of cross section
4.15,16	4.17,18		Beams with different allowable stresses in tension and compression
4.19,20			
4.22,26	4.21,23		Maximum stress and radius of curvature
	4.24,25		
4.27	4.28		Maximization of beam strength
4.31	4.29,30		Anticlastic curvature
4.32			Special problem on theory
<hr/>			
4.33,34	4.37,38		Stresses in composite beams: symmetric beams of two materials
4.35,36			
4.39,40	4.41,42		unsymmetric beams of two materials
4.43,44	4.45,46		Radius of curvature in composite beams
4.47,48	4.49,50		Stresses in reinforced concrete beams
	4.51		
4.52,54	4.53		in beams with balanced design
	4.55,56		Beams of three materials
4.57,58			Composite beams with circular cross section
4.59	*4.60		Beams with different moduli of elasticity in tension and compression
4.63,64	4.61,62		Stress concentrations in flat bars in pure bending
4.65,66			
<hr/>			
4.69,70	4.67,68		Plastic deformation in pure bending (elastoplastic material): plastic zone in rectangular beams
	4.71,72		including radius of curvature
4.73,74	4.75,76		plastic zone and radius of curvature for symmetric beams
4.77,78	4.79,80		plastic moment and shape factor of symmetric beams
4.81,82	4.83,84		plastic moment of unsymmetric beams
4.86	4.85		plastic moment of thick-walled pipe
			Residual stresses in symmetric beams:
4.87,88	4.89,90		after M_p has been applied and removed
4.91,93	4.92,94		after a given plastic zone has been developed
			Special problems:
4.95			residual radius of curvature
4.96	4.97,98		Plastic deformation of beams with a nonlinear stress-strain diagram

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
4.102,103	4.100,101		Eccentric loading in plane of symmetry of member: find stress in symmetric section
4.104	<u>4.99</u> ,105		
4.106,107	<u>4.109</u> , <u>110</u>		design of symmetric section
4.108	<u>4.111</u>		
4.113,114	<u>4.112</u> , <u>115</u>		find stress in unsymmetric section
4.117,118	4.116,119		
4.121, <u>122</u>	4.120,123		computation of loads from strain measurements
4.124, <u>125</u>			
4.127,131	4.126, <u>128</u>		Unsymmetric bending with one or two planes of symmetry: bending moment at an angle with horizontal
	4.129, <u>130</u>		
4.132,134	4.133,136		section at an angle with horizontal
*4.139	*4.137, <u>138</u>		Bending of unsymmetric section (principal axes must be determined) General eccentric bending:
4.140, <u>141</u>	4.142,143		symmetric beam; find stresses
4.144, <u>145</u>			symmetric beam; find allowable load or dimension
4.146, <u>147</u>	4.148, <u>149</u>		Bending of unsymmetric beams; determine largest bending moment
4.151	4.150		
<u>4.152</u> ,153	<u>4.154</u> ,155		Special problems and problems on theory
4.156,157	4.158,159		Curved beams: with rectangular cross section
4.160,164	4.161,162		in pure bending under eccentric loading
	4.163		
4.165,166	4.167,168		with unsymmetric cross section
4.171,172	4.169,170		with circular cross section
4.175,176	4.173, <u>174</u>		with trapezoidal cross section
	4.177,178		with triangular cross section
<u>4.180</u> ,181	4.182,*183		special problems and derivations of formulas

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
4.186, <u>188</u>	4.184, <u>185</u>		Review problems
4.189, <u>193</u>	4.187, <u>190</u>		
4. <u>194</u> ,195	4.191,192		
4.C1,C3	4.C2,C4		Computer problems
<u>4.C5,C7</u>	4.C6		

CHAPTER 5: ANALYSIS AND DESIGN OF BEAMS FOR BENDING

<u>5.1,2</u>	<u>5.3,4</u>	Using the free-body diagram of a portion of a beam: draw V and M diagrams (easy problems)
<u>5.5,6</u>		
5.7,8	5.9,10	draw V and M diagrams and determine maximum values of $ V $ and $ M $
5.11,13	5.12,14	
5.16,18	5.15,17	find maximum normal stress in given beam section
5.19,20		
5.22,23	5.21,25	draw V and M diagrams and find maximum normal stress in beam
5.24,26		
5.27,29	5.28,31	determine given parameter to minimize normal stress in beam
5.30,32	5.33	
<u>5.34,35</u>	<u>5.36,37</u>	Using relations among w , V and M whenever appropriate: draw V and M diagrams (easy problems)
<u>5.38,39</u>		
5.40,41	5.42,43	draw V and M diagrams and determine maximum values of $ V $ and $ M $
5.44,45		
5.47,49	5.46,48	find maximum normal stress in a given beam section
5.50,51	5.52,53	write equations for V and M and find maximum value of $ M $
5.54,55	5.56,57	draw V and M diagrams and find maximum normal stress in beam
5.60,61	5.58,59	
*5.66,*63	*5.64	Special problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*		Problem description
SI Units	U.S. Units	
5.65,69	<u>5.66,67</u>	Design of timber beams
5.70	5.68	
5.73,74	5.71,72	Design of steel beams, W shapes
5.75,76	5.77,78	Design of steel beams, S shapes
5.79,80	5.81,82	Design of steel beams, miscellaneous shapes
5.84	5.83	Design of beams resting on ground
5.87,88	5.85,86	Find allowable load for beam of unsymmetric cross section with allowable stresses in tension and compression
5.90,91	5.89	Design of beam systems
5.92,93		
*5.96,*97	*5.94,*95	Design of beams using LRFD
5.98,99	<u>5.100,101</u>	Using singularity functions write equations for $V(x)$ and $M(x)$ and find M at given point in beam
<u>5.102,103</u>	<u>5.104,105</u>	
5.107,109	5.106,108	find $ M _{\max}$ in beam
5.110,111		find σ_{\max} in beam
5.112,113		find $ M _{\max}$ and σ_{\max} in beam
5.116,117	5.114,115	design beam, knowing allowable stress
5.118,119	5.120,121	Using a computer and step functions, calculate V and M along the beam
5.122,123	5.124,125	calculate V and M along the beam , and determine σ_{\max} in the beam
		Nonuniform beams
		Beams of constant strength:
5.128,129	5.126,127	beams of uniform width and variable depth
5.130,131		
5.132,133	5.134,135	built-up timber beams
<u>5.136</u>	<u>5.137</u>	beams of circular cross section
<u>5.138</u>	5.139	beams of uniform depth and variable width
5.140,141	5.142,143	Rolled-steel beams with cover plates
5.144,145	5.146,147	
5.148,149	5.150,151	Tapered beams
5.152,155	5.153,153	Review problems
<u>5.157,159</u>	5.156,158	
5.161	5.160,162	
	5.163	
5.C1,C2	<u>5.C3,C4</u>	Computer problems
5.C6	5.C5	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>CHAPTER 6: SHEARING STRESSES IN BEAMS AND THIN-WALLED MEMBERS</u>			
6.3,4		<u>6.1,2</u>	Shearing forces in nails and bolts, using horizontal cuts
6.5,6		<u>6.7,8</u>	
6.10,11		<u>6.9,12</u>	Shearing stresses in beams
6.13,14			
6.16,18		<u>6.15,17</u>	Checking earlier designs of beams for shearing stresses
6.19,20			
6.21,23		<u>6.22,24</u>	Beams with unsymmetric sections
<u>6.25,26</u>		<u>6.27,28</u>	Beams with various geometric sections
6.29,31		<u>6.30,32</u>	Shearing forces and shearing stresses on arbitrary cuts due to vertical shear
6.33,34			
6.35,36		<u>6.37,38</u>	Shearing stresses in extruded beams
6.39,40		<u>6.41,42</u>	
6.45,46		<u>6.43,44</u>	Shearing stresses in bolts
6.48,49		<u>6.47,50</u>	Shearing stresses and shear flow in thin-walled members
<u>6.52,53</u>		<u>6.55,54</u>	Special problems
<u>6.55,56</u>		<u>6.57,59</u>	Shearing stresses in composite beams
<u>6.58</u>		<u>6.60</u>	Plastic behavior
<u>6.61,62</u>		<u>6.63,64</u>	Shear center in thin-walled beams with horizontal and vertical portions
6.67,68		<u>6.65,66</u>	Shear center and shearing stresses in extruded beams
6.70,71		<u>6.69,72</u>	Shear center in thin-walled beams with oblique portions
<u>6.73,74</u>			Shear center in thin-walled beams with circular portions
6.77,78		<u>6.75,76</u>	Problems involving location of shear center
<u>6.79</u>		<u>6.80</u>	Special problems
<u>*6.81,*82</u>		<u>*6.83,*84</u>	Shearing stresses in semicircular shapes, angle shapes and Z shapes
<u>*6.87,*88</u>		<u>*6.85,*86</u>	
6.89,91		<u>6.90,92</u>	Review problems
6.94,95		<u>6.93,96</u>	
6.97,100		<u>6.98,99</u>	
6.C1		<u>6.C2</u>	Computer problems
<u>6.C3,C4</u>		<u>6.C5,C6</u>	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>CHAPTER 7: TRANSFORMATION OF STRESS AND STRAIN</u>			
7.1,4		7.2,3	Find stresses on oblique plane from equilibrium of wedge
7.7,8		7.5,6	Find principal planes and stresses
7.11,12		7.9,10	Find planes of maximum shearing stress and corresponding stresses
7.13,14		7.15,16	Find stresses on a given plane
7.17,20		7.18,19	Stresses on oblique planes - simple applications
7.21,22			
7.25,26		7.23,24	Find principal stresses and/or maximum shearing stress in loaded shaft
7.28,30		7.27,29	Special problems involving determination of a stress to satisfy a given requirement
Using Mohr's circle, determine:			
7.31,32		7.33,34	principal planes and stresses, and maximum shearing stress
7.35,36		7.37,38	
7.39,42		7.40,41	stresses on oblique plane - simple applications
7.43,44			
7.47,48		7.45,46	principal stresses and/or maximum shearing stress in loaded shaft
7.50,53		7.49,51	solve special problems involving determination of a stress to satisfy a given requirement
7.54, <u>56</u>		7.55, <u>57</u>	find principal planes and stresses resulting from superposition of two states of stress
		7.58,59	
7.60,61		7.63	find range of values of a parameter for which a certain stress will not be
7.62			
7.64		7.65	Derivation of a formula involving Mohr's circle
7.66,67			In-plane and out-of-plane maximum shearing stress
7.68,69			
		7.70,71	Maximum shearing stress in a three dimensional state of stress
		7.72,73	
7.74,76		7.75,77	Maximum shearing stress (more advanced problems)
7.78,79			
*7.80			
7.83,84		7.81,82	Determine whether material will yield under given state of stress. If no yield occurs find the factor of safety
7.87,88		7.85,86	Find load for which yield will occur
7.89,90		7.91,92	Will rupture occur under a given state of stress?
7.93,95		7.94,97	Find stress or load for which rupture will occur
7.96			

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
7.98,99		7.100,101	Spherical pressure vessels (easy problems)
<u>7.102,103</u>			
7.105,107		7.104, <u>106</u>	Cylindrical pressure vessels (easy problems)
		7.108,109	
7.110,111		7.112, <u>113</u>	Stresses in welds in cylindrical pressure vessels
<u>7.116,117</u>		7.114,115	
7.118, <u>119</u>		7.120,121	Pressure vessels subjected to external loadings
<u>7.122,123</u>		7.124,125	Shrunk fit rings
<u>7.126,127</u>		<u>7.128,129</u>	Find state of strain associated with given rotation using formulas of Sec. 7.10
<u>7.130,131</u>		7.132,133	using Mohr's circle Find principal strains and maximum shearing strain (in plane and out of plane):
<u>7.134,135</u>		<u>7.136,137</u>	for a state of plane stress
<u>7.138,139</u>		<u>7.140,141</u>	for a state of plane strain
7.142,144		7.143,145	Problems involving strain rosettes
<u>7.146</u>		<u>7.147</u>	
7.148		7.149	Problems involving use of Mohr's circle and Hooke's law
7.152,153		7.150, <u>151</u>	Applications of the use of strain rosettes
		7.154, <u>155</u>	
7.158, <u>161</u>		7.156,157	Review problems
7.164, <u>165</u>		7.159,160	
7.166		7.162, <u>163</u>	
		7.167	
7.C1,C2		7.C3,C4	Computer problems
7.C5,C6		7.C7,C8	
<u>CHAPTER 8: PRINCIPAL STRESSES UNDER GIVEN LOADING CONDITIONS</u>			
Principal stresses in rolled-steel beams:			
8.1,2		8.3,4	find σ_m in beam and σ_{max} at junction of flanges and web
8.7,8		8.5,6	design beam, taking σ_m , r_m , and σ_{max} into account
8.9,10		8.11,12	check earlier design for σ_{max} at junction of flanges and web
		8.13, <u>14</u>	
Design of transmission shafts:			
8.15,16		8.19,20	loading represented by forces and couples
8.17, <u>18</u>		8.21	
8.22			
8.27,28		8.23,24	loading represented by input and output power
8.29,30		8.25,26	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
8.31,32		8.33,36	For beam of rectangular cross section under axial, bending and transverse loading (easy problems), find: normal and shearing stresses
8.34,35			For beams of circular cross section under axial, bending, and transverse loading, find: normal and shearing stresses principal stresses and maximum shearing stress
8.39,40		8.37,38	For beams of rectangular cross section under axial, bending and transverse loading, find: normal and shearing stresses
8.41,42		8.43,44	principal stresses and maximum shearing stress
8.45,46		8.47,48	For steel beams and structural tubes under axial, bending and transverse loading, find: normal and shearing stresses
8.49,50		8.51,52	principal stresses, principal planes, and maximum shearing stress
8.53,54		8.55	Special problems
8.56,57		8.58	*8.63 *8.61,62 Problems involving torsion of rectangular sections or structural shapes
8.59		8.60	*8.64
8.66,67		8.65,69	Review problems
8.68,70		8.71,72	
8.75,76		8. <u>73</u> ,74	
8.C1,C2		8.C4,C5	Computer problems
8.C3,C6		8.C7	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>CHAPTER 9: DEFLECTION OF BEAMS</u>			
<u>9.1,3</u>	<u>9.2,4</u>		Using the integration method, determine the equation of the elastic curve and the deflection and/or slope at specific points for: cantilever beams
<u>9.5,6</u>			
<u>9.7</u>	<u>9.8</u>		overhanging beams
			simply supported beams
<u>9.9</u>	<u>9.10</u>		symmetrical loading
<u>9.11</u>	<u>9.12</u>		unsymmetrical loading
<u>9.14,15</u>	<u>9.13</u>		beams and loadings requiring the use of 2 equations and 4 constants of integration
<u>9.16</u>			
<u>9.17</u>	<u>9.18</u>		direct determination of the elastic curve from an analytic function of $w(x)$ For a statically indeterminate beam (first degree), determine: reaction at the roller support
<u>9.19,20</u>	<u>9.21,22</u>		
<u>9.24</u>	<u>9.23</u>		
<u>9.25,26</u>	<u>9.27,28</u>		reaction at the roller support and draw the M diagram (use of 2 equations and 4 constants of integration required)
<u>9.29,30</u>	<u>9.31,32</u>		reaction at the roller support and the deflection at a given point (use of 2 equations and 4 constants of integration required)
<u>9.33</u>	<u>9.34</u>		For a statically indeterminate beam (second degree), determine the reaction at one end and draw the M diagram
Using singularity functions, determine the equation of the elastic curve and the deflection and/or slope at specified points:			
<u>9.35,36</u>	<u>9.37,38</u>		easy problems
<u>9.39,40</u>			problems involving overhanging beams
<u>9.41,42</u>	<u>9.43,44</u>		problems with distributed loads
<u>9.45,48</u>	<u>9.46,47</u>		problems with numerical data
<u>9.49,50</u>	<u>9.51,52</u>		For a statically indeterminate beam (first degree), determine the reaction at the roller support and the deflection at a specified point.
<u>9.53,56</u>	<u>9.54,55</u>		
<u>9.57</u>	<u>9.58</u>		For a statically indeterminate beam (second degree), determine the reaction at the roller support and the deflection at a specified point.
<u>9.59,62</u>	<u>9.60,61</u>		determine the maximum deflection
<u>9.63,64</u>			Problems involving the reduction of a load to a force-couple system.

* Problems that do not involve any specific system of units have been indicated by underlining their number.
Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*		Problem description
SI Units	U.S. Units	
		Using method of superposition, determine the deflection and slope at specified points in:
9.65,66	<u>9.67,68</u>	cantilever beams
9.71,72	<u>9.69,70</u>	simply supported beams
9.73,74	<u>9.75,76</u>	cantilever beams (with numerical data)
9.77,78		simply supported beams (with numerical data)
<u>9.79,80</u>	<u>9.81,82</u>	statically indeterminate beams (first degree)
<u>9.83</u>	<u>9.84</u>	statically indeterminate beams (second degree)
9.85,88	<u>9.86,87</u>	combined beams, determinate (with numerical data)
9.89,92	<u>9.90,91</u>	statically indeterminate beams (with numerical data)
9.93	9.94	Combined bending and torsion of rods
		Using the moment-area method, determine the slope and/or deflection at specified points in
		cantilever beams
<u>9.95,96</u>	<u>9.97,98</u>	with simple loadings
<u>9.99,100</u>		with more complex loads
9.101,104	9.102,103	with numerical data
<u>9.106,108</u>	<u>9.105,107</u>	with variable EI
		simply supported beams
<u>9.109,110</u>	<u>9.113,114</u>	with symmetric loadings
<u>9.111,112</u>		
<u>9.115,116</u>		with variable EI
9.117,118	9.121,122	with numerical data
9.119,120		
*9.123	<u>9.124</u>	special problems
		Simply supported and overhanging beams with unsymmetric loadings
9.126,127	<u>9.125,128</u>	simply supported beams (easy problems)
9.129,131	9.130,132	simply supported beams with numerical data
<u>9.133</u>	<u>9.134</u>	overhanging beams
<u>9.135,137</u>	9.136,138	overhanging beams (with numerical data)
<u>9.139</u>	<u>9.140</u>	simply supported beams with variable EI
		Find maximum deflection for:
<u>9.141</u>	<u>9.142</u>	simply supported beams
9.143	9.144	with numerical data
		overhanging beams
9.145	9.146	Statically indeterminate beams (first degree)
<u>9.149,150</u>	<u>9.147,148</u>	single span
	<u>9.151,152</u>	two span beams, find all reactions
9.153	9.154	single span beams (with numerical data)
9.155	9.156	simply supported beams with additional elastic support at midspan

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*		Problem description
SI Units	U.S. Units	
9.157, <u>158</u>	<u>9.159</u> ,160	Review problems
9. <u>161</u> ,163	9.162,165	
9. <u>164</u> , <u>168</u>	9.166,167	
9.C1,C3 9.C4,C8	9.C2,C5 9.C6,C7	Computer problems

CHAPTER 10: COLUMNS

10.1,2	<u>10.3</u> ,4	Stability of rigid-rod-and-spring systems: easy problems, single spring
<u>10.5</u> ,6	10.7,8	systems with two or more springs
10.9	10.10	Application of Euler's formula to the critical loading of pin-ended columns: short struts
10.12, <u>13</u>	<u>10.11</u>	comparison of critical loads for various cross sections
10.14		
10.17, <u>18</u>	10.15,16	Allowable loading for pin-ended columns: rolled-steel shapes
10.19,20		multiple-member structures
10.23,24	10.21,22	columns with various end conditions
10.27,28	10.25,26	
10.30, <u>31</u>	10.29,33	Application of the secant formula to the eccentric loading of columns: find σ_{max} and either deflection or e for a given load
10.32, <u>34</u>		
10.37, <u>38</u>	10.35,36 <u>10.39</u> ,40	find σ_{max} and load for a given deflection and eccentricity
10.41,42		temperature induced loading
10.45, <u>46</u>	10.43, <u>44</u>	find P_{all} for given e , σ_{max} and F.S. (using Fig. 10.24) design problems
10.49, <u>50</u>	10.47,48	find column length
10.51, <u>52</u>		find factor of safety of column
10.53, <u>54</u>	10.55,56	find cross section of column

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)

Mechanics of Materials 4th Edition

SI Units	U.S. Units	Problem description
		Analysis of columns under centric load: columns with simple cross section: steel columns aluminum columns timber columns
10.58,60	10.57, <i>59</i>	columns with built-up cross sections: timber column aluminum column steel columns
10.62	10.6 <i>1</i>	
10.64	10.63	
10.65	10.66	
10.67, <i>68</i>	10.69	
10.70		Design of columns under a centric load: aluminum columns timber columns steel columns
10.72, <i>73</i>	10.71	
10.74, <i>76</i>	10.75	
10.77, <i>80</i>	10.78, <i>79</i>	
10.83, <i>84</i>	10.81, <i>82</i>	
*10.85	*10.86	Application of LRFD formulas: analysis of columns under a centric loading
*10.87	*10.88	design of columns under a centric loading
10.89, <i>90</i>		Analysis of columns under an eccentric load: steel columns, find allowable load
10.91, <i>92</i>	10.93,94	aluminum columns, find allowable load timber columns, find allowable load
10.95, <i>96</i>		Design of columns under an eccentric load: find maximum allowable length: aluminum column
10.97, <i>98</i>	10.99,100	timber columns steel columns
	10.101, <i>102</i>	
10.103, <i>104</i>		design cross section: timber columns
10.105, <i>106</i>	10.107, <i>108</i>	steel columns, rectangular cross section
	10.109, <i>110</i>	aluminum columns, rectangular or circular cross sections
10.111, <i>112</i>	10.113, <i>114</i>	steel column, wide-flange shape
10.115, <i>116</i>		steel tube
<u>10.117,<i>121</i></u>	<u>10.118,<i>119</i></u>	Review problems
<u>10.123,<i>126</i></u>	<u>10.120,<i>122</i></u>	
<u>10.127,<i>128</i></u>	<u>10.124,<i>125</i></u>	
10.C1,C3,C6	10.C2,C4,C5	Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
CHAPTER 11: ENERGY METHODS			
11.2,3		11.1,4	Modulus of resilience Modulus of resilience and modulus of toughness:
11.5		11.6	from stress-strain diagram
11.8		11.7	from load-deflection diagram of a tensile test
11.10		11.9	Strain energy under axial loads: strain energy under a given load
11.11,12			maximum allowable strain energy
11.13		11.14	factor of safety
11.15		11.16	strain energy by integration of approximate methods
11.17,19		11.18,20	strain energy of trusses with numerical data
11.23,24		11.25,26	Strain energy in bending: derivation of formulas
11.27		11.28	with numerical data
11.29,30		11.31,32	Strain energy in torsion: by integration
11.33		11.34	Maximum-distortion-energy criterion for 3-dimensional state of stress
11.35		11.36,37	
		11.38,39	
11.40		11.41	Special problems
11.42,43		11.44,45	Impact loading: of rods (horizontal impact)
11.46,47			of rods (vertical impact)
11.49,50		11.48	of beams (horizontal impact)
11.52,53		11.51	of beams (vertical impact)
11.55,56		11.54	problems on theory
11.57,58		11.59,60	Use of work-energy method to determine: deflection or slope of: prismatic beams
11.63,64		11.61,62	nonprismatic beams
11.65,66		11.67,69	angle of twist of shafts
11.68			angle of twist of a thin-walled hollow shaft
11.70			deflection of a joint of a truss
11.76		11.74,75	

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS (CONTINUED)
Mechanics of Materials 4th Edition

Problem Number*	SI Units	U.S. Units	Problem description
<u>11.78,80</u>		<u>11.77,79</u>	Work of several loads applied to a beam
<u>11.81</u>		<u>11.82</u>	Determinate structures. Use Castigiano's theorem to determine: deflection and/or slope of beams
<u>11.83,84</u>		<u>11.85,86</u>	
<u>11.87,88</u>		<u>11.91,92</u>	
<u>11.89,90</u>			
11.95,98	11.93,94	11.96,97	deflection and/or slope of beams (with numerical data)
11.101, <i>102</i>	11.99,100		deflection of a given joint in a truss
11.103,104			
<u>11.105,106</u>		<u>11.107,108</u>	deflection and/or slope at a given point in a bent or curved rod
<u>11.109,110</u>			
<u>11.111,112</u>		<u>11.113,114</u>	<u>Indeterminate structures</u> (first degree). Use Castigiano's theorem to determine: reaction at roller support and draw bending-moment diagram
		<u>11.115,116</u>	
11.117,118		<u>11.119,120</u>	force in member(s) of a truss
<u>11.121,122</u>			
11.123,124	11.125, <i>126</i>		Review problems
11.127, <i>131</i>	11.128,129		
11.133,134	11.130,132		
11.C3,C4,C5	11.C1,C2,C6		Computer problems

* Problems that do not involve any specific system of units have been indicated by underlining their number.
 Answers are not given to problems with a number set in italic type.

The six lists of suggested assignments contained in this table cover the entire text. The instructor can select those assignments that are covered in the course.

50% OF THE PROBLEMS IN EACH OF THESE LISTS USE SI UNITS AND 50% U.S. CUSTOMARY UNITS

| <----For these lists answers to all of the problems are given in the back of the book-----> | -----> For Lists 5 and 6 no answers are given-->

Group	Sections	Topics	List 1	List 2	List 3	List 4	List 5	List 6
1	1.1	Introduction	1.4.7,21,23	1.3,14,16,27	1.2,8,18,28	1.1,10,20,24	1.6,12,17,26	1.5,11,22,25
2	1.2-10	Stresses Under Axial Loading	1.30,38,43,49	1.29,39,44,51	1.31,42,45,53	1.32,41,46,54	1.35,40,47,52	1.35,37,48,50
3	1.11-13	Components of Stress; Factor of Safety	2.2,4,15,21	2.4,12,16,22	2.1,10,19,24	2.3,9,20,25	2.5,13,18,27	2.7,11,17,28
4	2.1-8	Stress-Strain Diagram	2.34,40,50,55	2.33,39,47,60	2.35,45,49,56	2.36,46,48,57	2.37,44,51,59	2.38,43,52,58
5	2.9-10	Statically Indeterminate Problems	2.65,68,78,87	2.63,67,77,88	2.62,69,75,84	2.61,70,76,86	2.66,72,81,83	2.64,71,82,85
6	2.11-16	Generalized Hooke's Law; Poisson's Ratio	2.94,101,111,119	2.93,102,112,113	2.95,106,109,117	2.96,105,110,116	2.97,104,107,122	2.98,103,108,123
7	2.17-20	Stress Concentration; Plastic Behavior	3.3,10,15,27	3.5,9,17,23	3.1,13,25	3.7,11,20,21	3.2,14,18,28	3.8,12,16,24
8	3.1-4	Stresses in Torsion	3.32,39,45,57	3.33,37,47,56	3.31,44,46,53	3.36,38,49,52	3.35,42,50,59	3.34,43,51,58
9	3.5-6	Angle of Twist; Indeterminate Shafts	3.65,77,78,85	3.66,76,83,84	3.64,71,80,87	3.67,72,81,86	3.68,75,82,89	3.70,74,79,88
10	3.7-8	Transmission Shafts; Stress Concentrations	3.91,98,104,114	3.94,97,104,110	3.92,100,101,112	3.93,99,102,111	3.95,103,106,115	3.96,103,107,118
11	3.9-11	Plastic Deformations; Residual Stresses	3.119,129,134,142	3.120,126,136,141	3.121,127,135,145	3.122,125,137,140	3.123,133,149,144	3.124,130,139,143
12	3.12-13	Noncircular and Thin-Walled members	4.6,14,15,23	4.3,13,16,21	4.1,10,17,26	4.2,9,20,22	4.7,12,18,25	4.8,11,19,24
13	4.1-5	Stresses and Deformations in the Elastic Range	4.37,40,49,65	4.38,39,50,66	4.53,42,47,62	4.34,41,54,61	4.35,46,51,64	4.36,45,53,63
14	4.6-7	Members Made of Several Materials	4.69,76,82,90	4.73,75,86,89	4.67,77,83,88	4.71,77,84,87	4.68,78,81,94	4.72,78,85,92
15	4.8-11	Plastic Deformations; Residual Stresses	4.100,107,119,124	4.101,106,116,121	4.99,109,113,123	4.103,105,111,120	4.102,111,112,125	4.104,110,115,122
16	4.12	Eccentric Axial Loading in a Plane of Symmetry	4.127,136,140,150	4.131,133,144,148	4.126,135,142,151	4.129,134,143,146	4.128,132,141,149	4.130,132,145,149
17	4.13-14	Unsymmetric Bending	4.158,164,169,176	4.159,164,170,175	4.156,167,171,173	4.160,162,171,178	4.156,163,174,179	4.157,168,177,179
18	4.15	Bending of Curved Members	5.7,17,22,28	5.8,15,26,28	5.9,18,21,29	5.10,20,25,27	5.12,16,23,33	5.14,19,24,31
19	5.1-2	Shear and Bending-Moment Diagrams	5.42,49,56,62	5.43,47,59,62	5.40,48,54,63	5.45,46,60,63	5.41,53,57,64	5.44,51,58,64
20	5.3	Using Relations Among w, V, and M	5.65,72,79,89	5.76,71,80,89	5.68,74,81,91	5.68,73,82,90	5.66,78,85,93	5.67,77,86,92
21	5.4	Design of Prismatic Beams in Bending	5.98,108,116,124	5.105,106,113,121	5.100,109,114,123	5.101,107,111,118	5.102,111,112,125	5.103,110,117,120
22	5.5	Use of Singularity Functions to determine V and M	5.126,138,147,148	5.127,132,142,148	5.131,139,141,151	5.128,134,145,150	5.129,137,146,149	5.130,135,143,149
23	5.6	Nonprismatic Beams	6.4,9,19,22	6.3,12,20,24	6.2,13,15,22	6.1,14,17,23	6.8,10,16,27	6.7,11,18,25
24	6.1-5	Shearing Stresses in a Beam	6.3,30,36,44,54	6.32,32,35,43,53	6.29,38,46,52	6.34,37,45,51	6.31,42,50,58	6.33,41,47,56
25	6.6-8	Shearing Stresses in Thin-Walled Members	6.61,69,77,83	6.62,72,78,84	6.63,63,70,75,81	6.64,71,76,82	6.65,71,79,85	6.65,74,80,86
26	6.9	Shear Center	7.1,10,17,24	7.4,9,22,23	7.2,13,18,26	7.3,11,19,25	7.5,12,20,29	7.6,12,21,27
27	7.1-3	Transformation of Plane Stress	7.33,44,45,60	7.34,39,46,54	7.31,38,47,56	7.32,37,48,55	7.36,42,49,61	7.36,42,55,61
28	7.4	Mohr's Circle for Plane Stress	7.66,77,83,97	7.69,78,87,91	7.72,79,81,95	7.75,74,92,89	7.76,84,94	7.73,78,94
29	7.5-8	Three-Dimensional Stress Analysis; Fracture Criteria	7.100,107,112,123	7.101,105,115,118	7.98,109,110,124	7.102,104,116,120	7.99,106,113,122	7.103,108,114,119
30	7.9	Stresses in Thin-Walled Pressure Vessels	7.127,140,146,154	7.131,136,144,150	7.129,138,143,153	7.133,137,145,152	7.128,139,142,155	7.132,135,147,151
31	7.10-13	Analysis and Measurement of Strain	8.4,12,16,24	8.1,11,15,23	8.2,10,20,28	8.1,9,19,27	8.6,14,18,26	8.5,13,17,25
32	8.1-3	Principal Stresses in a Beam; Design of Shafts	8.36,40,52,54	8.33,38,50,58	8.31,37,49,55	8.34,44,54,57	8.35,43,47,56	8.37,44,54,57
33	8.4	Stresses Under Combined Loadings	9.1,10,24,28	9.2,12,22,27	9.3,9,23,26	9.4,11,21,25	9.5,16,20,32	9.7,14,19,31
34	9.1-5	Deflection of Beams by Integration	9.35,48,52,62	9.36,45,51,59	9.37,47,50,61	9.38,46,49,66	9.39,44,54,64	9.40,43,55,63
35	9.6	Use of Singularity Functions	9.68,76,82,87	9.67,75,81,86	9.66,74,80,88	9.65,73,79,85	9.70,78,84,91	9.69,77,83,90
36	9.7-8	Method of Superposition; Statically Indet. Beams	9.95,104,119,119	9.96,101,110,120	9.97,103,113,117	9.98,102,114,118	9.99,105,115,121	9.100,107,116,122
37	9.9-10	Moment-Area Method; Max Deflection; Indet. Beams	9.125,138,142,148	9.126,136,144,154	9.130,134,144,153	9.130,133,146,147	9.128,137,141,152	9.132,135,145,151
38	9.12-14	Moment-Area Method; Max Deflection; Indet. Beams	10.1,10,20,22	10.2,11,19,21	10.3,9,16,24	10.4,12,15,23	10.8,13,18,26	10.7,14,17,25
39	10.1-4	Columns; Euler's Formula	10.33,38,48,54	10.29,37,47,53	10.34,36,50,56	10.32,35,49,55	10.31,40,44,52	10.30,39,43,51
40	10.5	Eccentric Loading; Secant Formula	10.58,69,76,81	10.60,66,74,79	10.57,65,71,77	10.63,67,75,80	10.59,70,73,76	10.61,68,72,82
41	10.6	Design of Columns Under a Centric Load	10.94,96,108,116	10.93,95,107,115	10.90,100,104,114	10.89,99,103,113	10.92,102,110,112	10.91,104,109,111
42	10.7	Design of Columns Under an Eccentric Load	11.2,14,18,23	11.3,9,20,32	11.1,10,21,30	11.7,13,22,24	11.6,12,12,31	11.4,11,11,26
43	11.1-6	Elastic Strain Energy	11.4,52,59,76	11.45,49,62,71	11.42,48,57,75	11.46,48,58,74	11.47,51,60,73	11.43,51,61,73
44	11.7-10	Design for Impact Loading; Deflections	11.8,90,93,101	11.82,83,96,104	11.77,92,94,100	11.79,85,98,99	11.78,86,94,102	11.80,91,97,103
45	11.11-13	Castigliano's Theorem	11.105,114,117	11.108,111,119	11.106,116,118	11.116,115,120	11.109,113,120	11.110,113,120
46	11.14	Statically Indeterminate Structures						

TABLE IV: SAMPLE ASSIGNMENT SCHEDULE FOR A COURSE IN MECHANICS OF MATERIALS Mechanics of Materials 4th Edition

The four lists of suggested assignments contained in this table cover the entire text. The instructor can select those assignments that are covered in the course.

75% OF THE PROBLEMS IN EACH OF THESE LISTS USE SI UNITS AND 25% U.S. CUSTOMARY UNITS

ANSWERS TO ALL OF THESE PROBLEMS ARE GIVEN IN THE BACK OF THE BOOK

Group	Sections	Topics	List 1a	List 2a	List 3a	List 4a
1	1.1	Introduction				
2	1.2-10	Stresses Under Axial Loading	1.4; 4,15,27;	1,3,7,19,23	1,2,9,18,24	1,113,20,28
3	1.11-13	Components of Stress; Factor of Safety	1,30,39,43,56	1,29,38,44,55	1,34,41,45,54	1,33,42,46,53
4	2.1-8	Stress-Strain Diagram	2,2,12,15,23	2,4,14,16,26	2,6,9,19,25	2,8,10,20,24
5	2.9-10	Statically Indeterminate Problems	2,34,39,53,60	2,33,40,54,55	2,35,41,49,57	2,36,42,48,56
6	2.11-16	Generalized Hooke's Law; Poisson's Ratio	2,65,67,79,88	2,63,68,80,87	2,62,74,75,86	2,61,73,76,84
7	2.17-20	Stress Concentration; Plastic Behavior	2,94,102,111,115	2,93,101,112,114	2,99,105,109,116	2,100,106,110,117
8	3.1-4	Stresses in Torsion	3,3,9,15,22	3,5,10,17,26	3,4,11,19,21	3,6,13,20,25
9	3.5-6	Angle of Twist; Indeterminate Shafts	3,22,37,54,56	3,33,39,55,57	3,31,40,46,52	3,36,41,49,53
10	3.7-8	Transmission Shafts: Stress Concentrations	3,65,76,78,90	3,66,77,83,90	3,69,72,80,86	3,73,77,81,87
11	3.9-11	Plastic Deformations; Residual Stresses	3,91,97,108,110	3,94,98,109,114	3,92,101,102,111	3,93,102,101,112
12	3.12-13	Noncircular and Thin-Walled members	3,119,126,138,141	3,120,129,138,142	3,121,131,135,140	3,122,132,137,145
13	4.1-5	Stresses and Deformations in the Elastic Range	4,6,13,15,27	4,3,14,16,28	4,4,9,17,22	4,5,10,20,26
14	4.6-7	Members Made of Several Materials	4,37,39,48,65	4,38,40,52,66	4,33,43,47,61	4,34,44,46,62
15	4.8-11	Plastic Deformations; Residual Stresses	4,69,75,82,91	4,73,76,86,93	4,70,77,83,87	4,74,77,84,88
16	4.12	Eccentric Axial Loading in a Plane of Symmetry	4,100,106,118,121	4,101,107,114,124	4,98,108,113,120	4,103,108,117,123
17	4.13-14	Unsymmetric Bending	4,127,133,140,147	4,131,136,144,147	4,127,134,142,146	4,131,135,143,151
18	4.15	Bending of Curved Members	4,158,164,172,175	4,159,164,172,176	4,156,165,171,178	4,160,166,171,179
19	5.1-2	Shear and Bending-Moment Diagrams	5,7,15,22,30	5,8,17,26,32	5,11,20,21,27	5,13,18,23,29
20	5.3	Using Relations Among w , V , and M	5,42,47,55,62	5,43,49,61,62	5,40,50,54,63	5,45,52,56,63
21	5.4	Design of Prismatic Beams in Bending	5,65,71,79,87	5,70,72,80,88	5,75,73,81,90	5,76,74,82,91
22	5.5	Use of Singularity Functions to determine V and M	5,98,106,116,122	5,105,108,113,119	5,104,107,114,118	5,99,109,115,123
23	5.6	Nonprismatic Beams	5,126,132,140,148	5,127,138,144,148	5,131,133,141,150	5,128,136,145,151
24	6.1-5	Shearing Stresses in a Beam	6,4,12,19,26	6,3,9,20,28	6,6,14,15,23	6,5,13,17,21
25	6.6-8	Shearing Stresses in Thin-Walled Members	6,30,35,49,53	6,32,36,48,54	6,29,39,46,51	6,34,40,45,52
26	6.9	Shear center	6,61,72,77,88	6,62,69,78,87	6,68,71,75,82	6,67,70,76,81
27	7.1-3	Transformation of Plane Stress	7,1,9,17,28	7,4,10,22,30	7,7,11,18,25	7,8,13,19,26
28	7.4	Mohr's Circle for Plane Stress	7,33,39,50,54	7,34,44,52,60	7,31,41,47,55	7,32,41,48,56
29	7.5-8	Three-Dimensional Stress Analysis; Fracture Criteria	7,66,75,83,93	7,69,77,87,90	7,68,74,83,89	7,67,79,82,95
30	7.9	Stresses in Thin-Walled Pressure Vessels	7,100,105,111,118	7,101,107,117,123	7,98,105,110,120	7,102,107,116,124
31	7.10-13	Analysis and Measurement of Strain	7,127,136,146,152	7,131,140,144,153	7,126,137,143,152	7,130,138,145,153
32	8.1-3	Principal Stresses in a Beam; Design of Shafts	8,4,11,16,29	8,3,12,15,30	8,8,9,20,27	8,7,10,19,28
33	8.4	Stresses Under Combined Loadings	8,36,39,46,53	8,33,40,45,54	8,32,41,45,55	8,31,42,49,58
34	9.1-5	Deflection of Beams by Integration	9,1,12,24,29	9,2,19,22,30	9,6,11,23,25	9,8,9,21,26
35	9.6	Use of Singularity Functions	9,35,38,53,59	9,36,48,56,62	9,37,41,50,60	9,38,42,49,61
36	9.7-8	Method of Superposition; Statically Indet. Beams	9,66,75,82,89	9,67,76,81,92	9,72,73,80,85	9,71,74,79,88
37	9.9-10	Moment-Area Method	9,95,101,111,120	9,96,104,112,119	9,97,108,113,118	9,98,105,114,117
38	9.12-14	Moment-Area Method; Max Deflection; Indet. Beams	9,125,136,142,149	9,129,138,143,150	9,127,133,144,147	9,131,134,146,153
39	10.1-4	Columns: Euler's Formula	10,1,11,20,27	10,2,10,19,28	10,5,12,16,19	10,6,9,15,24
40	10.5	Eccentric Loading; Secant Formula	10,33,37,46,53	10,29,38,45,54	10,34,41,50,55	10,32,42,49,56
41	10.6	Design of Columns Under a Centric Load	10,58,66,76,84	10,60,69,74,83	10,62,67,71,80	10,64,65,75,77
42	10.7	Design of Columns Under an Eccentric Load	10,94,95,106,115	10,93,96,105,116	10,90,97,104,113	10,89,98,103,114
43	11.1-6	Elastic Strain Energy	11,2,9,18,25	11,3,14,20,29	11,5,15,24,24	11,8,10,30,30
44	11.7-10	Design for Impact Loading; Deflections	11,44,49,63,71	11,45,52,64,74	11,42,53,57,74	11,46,50,56,75
45	11.11-13	Castigliano's Theorem	11,81,83,95,104	11,82,83,98,101	11,77,84,95,99	11,79,89,98,100
46	11.14	Statically Indeterminate Structures	11,107,111,121	11,108,112,122	11,105,112,117	11,106,114,118

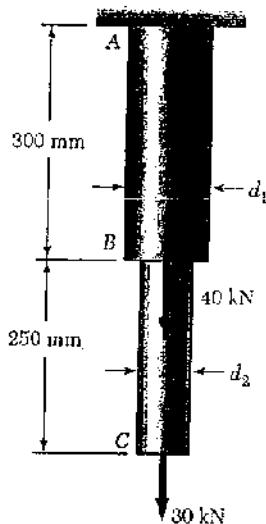
Problem Solutions

by
Dean Updike

Chapter 1

Problem 1.1

1.1 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that $d_1 = 50 \text{ mm}$ and $d_2 = 30 \text{ mm}$, find average normal stress at the midsection of (a) rod AB , (b) rod BC .



(a) Rod AB

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{70 \times 10^3}{1.9635 \times 10^{-3}} = 35.7 \times 10^6 \text{ Pa}$$

$$\sigma_{AB} = 35.7 \text{ MPa}$$

(b) Rod BC

$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

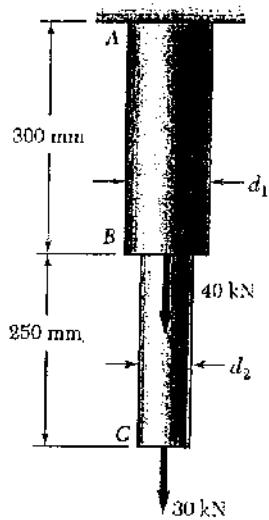
$$A = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{30 \times 10^3}{706.86 \times 10^{-6}} = 42.4 \times 10^6 \text{ Pa}$$

$$\sigma_{BC} = 42.4 \text{ MPa}$$

Problem 1.2

1.2 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 140 MPa in either rod, determine the smallest allowable values of d_1 and d_2 .



Rod AB.

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4}d_1^2} = \frac{4P}{\pi d_1^2}$$

$$d_1 = \sqrt{\frac{4P}{\pi \sigma_{AB}}} = \sqrt{\frac{(4)(70 \times 10^3)}{\pi(140 \times 10^6)}} = 25.2 \times 10^{-3} \text{ m}$$

$$d_1 = 25.2 \text{ mm}$$

Rod BC

$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4}d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi \sigma_{BC}}} = \sqrt{\frac{(4)(30 \times 10^3)}{\pi(140 \times 10^6)}} = 16.52 \times 10^{-3} \text{ m}$$

$$d_2 = 16.52 \text{ mm}$$

Problem 1.3

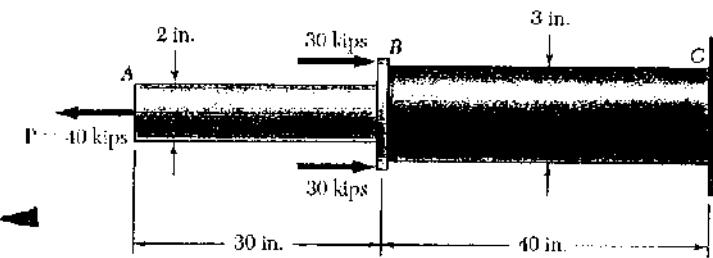
1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the average normal stress at the midsection of (a) rod *AB*, (b) rod *BC*.

(a) Rod *AB*.

$$P = 40 \text{ kips} \quad (\text{tension})$$

$$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi (2)^2}{4} = 3.1416 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{40}{3.1416} = 12.73 \text{ ksi}$$



(b) Rod *BC*.

$$F = 40 - (2)(30) = -20 \text{ kips. i.e. 20 kips compression.}$$

$$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi (3)^2}{4} = 7.0686 \text{ in}^2$$

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-20}{7.0686} = -2.83 \text{ ksi}$$

Problem 1.4

1.4 In Prob. 1.3, determine the magnitude of the force *P* for which the tensile stress in rod *AB* has the same magnitude as the compressive stress in rod *BC*.

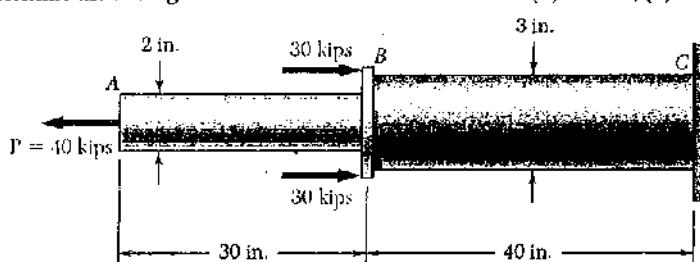
1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the average normal stress at the midsection of (a) rod *AB*, (b) rod *BC*.

$$A_{AB} = \frac{\pi (2)^2}{4} = 3.1416 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{3.1416} \\ = 0.31831 P$$

$$A_{BC} = \frac{\pi (3)^2}{4} = 7.0686 \text{ in}^2$$

$$\sigma_{BC} = \frac{(2)(30) - P}{A_{AB}} \\ = \frac{60 - P}{7.0686} = 8.4883 - 0.14147 P$$

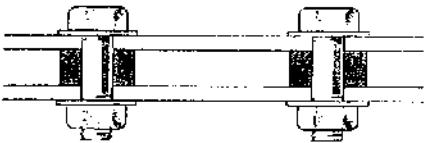


Equating σ_{AB} to σ_{BC}

$$0.31831 P = 8.4883 - 0.14147 P$$

$$P = 18.46 \text{ kips}$$

Problem 1.5



1.5 Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time the spacer pushes that plate upward with a compressive force P_s . In order to maintain equilibrium,

$$P_b = P_s$$

For the bolt $\sigma_b = \frac{P_b}{A_b} = \frac{4P_b}{\pi d_b^2}$ or $P_b = \frac{\pi}{4} \sigma_b d_b^2$

For the spacer $\sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \frac{\pi}{4} \sigma_s (d_s^2 - d_b^2)$

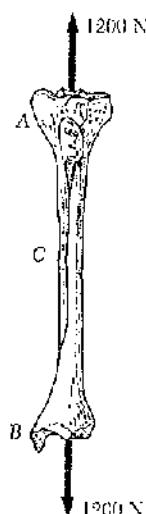
Equating P_b and P_s

$$\frac{\pi}{4} \sigma_b d_b^2 = \frac{\pi}{4} \sigma_s (d_s^2 - d_b^2)$$

$$d_s = \sqrt{1 + \frac{\sigma_b}{\sigma_s}} d_b \quad \sqrt{1 + \frac{200}{130}} \quad (16) \qquad d_s = 25.5 \text{ mm} \quad \blacktriangleleft$$

Problem 1.6

1.6 A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.



$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

$$\text{Geometry: } A = \frac{\pi}{4} (d_1^2 - d_2^2)$$

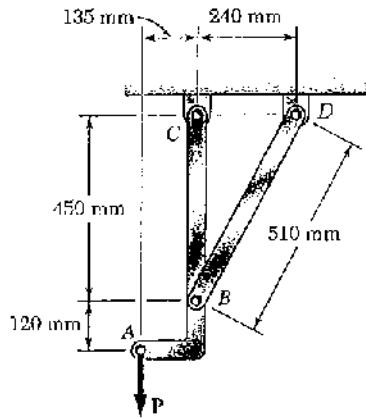
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)} \\ = 222.9 \times 10^{-6} \text{ m}^2$$

$$d_2 = 14.93 \times 10^{-3} \text{ m} \quad d_2 = 14.93 \text{ mm} \quad \blacktriangleleft$$

Problem 1.7

1.7 Knowing that the central portion of the link BD has a uniform cross-sectional area of 800 mm^2 , determine the magnitude of the load P for which the normal stress in that portion of BD is 50 MPa .

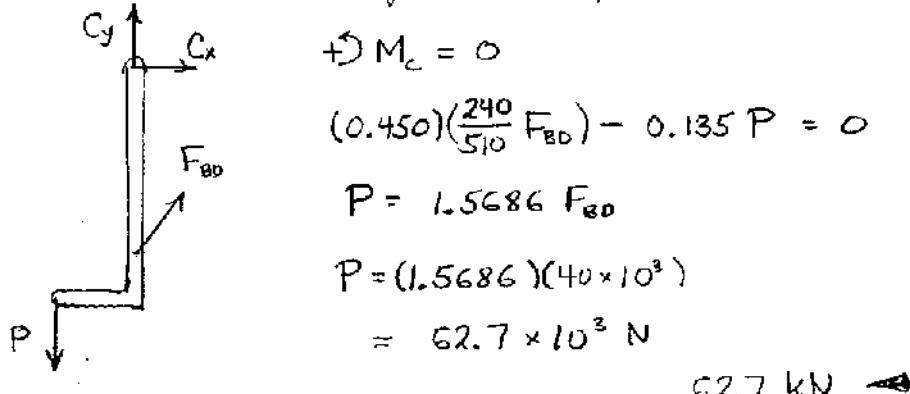


$$\sigma_{BD} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$$

$$A_{BD} = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \sigma_{BD} A_{BD} = (50 \times 10^6)(800 \times 10^{-6}) = 40 \times 10^3 \text{ N}$$

Draw free body diagram of body ABD.



$$\therefore M_C = 0$$

$$(0.450)\left(\frac{240}{510} F_{BD}\right) - 0.135 P = 0$$

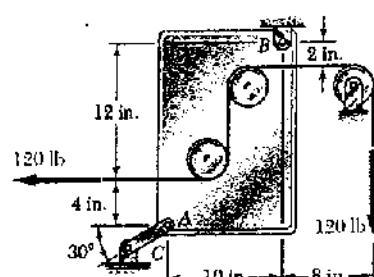
$$P = 1.5686 F_{BD}$$

$$P = (1.5686)(40 \times 10^3) \\ = 62.7 \times 10^3 \text{ N}$$

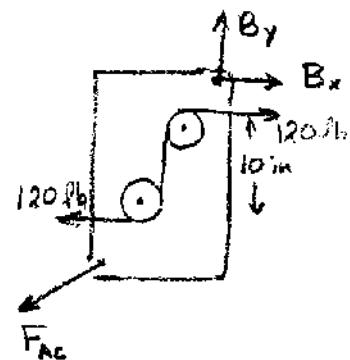
62.7 kN

Problem 1.8

1.8 Link AC has a uniform rectangular cross section $\frac{1}{8}$ in. thick and 1 in. wide. Determine the normal stress in the central portion of the link.



Use the plate together with two pulleys as a free body. Note that the cable tension causes a clockwise couple to act on the body.



$$\therefore \sum M_B = 0$$

$$-(12 + 4)(F_{AC} \cos 30^\circ) + (10)(F_{AC} \sin 30^\circ) - 1200 = 0$$

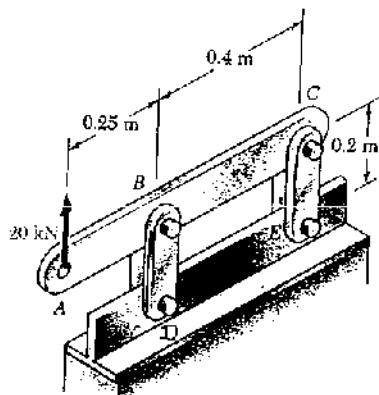
$$F_{AC} = -\frac{1200}{16 \cos 30^\circ - 10 \sin 30^\circ} = -135.50 \text{ lb.}$$

$$\text{Area of link } AC: \quad A = 1 \text{ in.} \times \frac{1}{8} \text{ in.} = 0.125 \text{ in.}^2$$

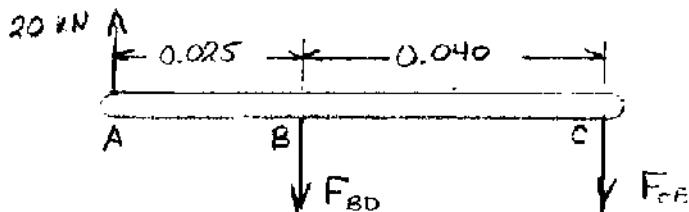
$$\text{Stress in link } AC: \quad \sigma_{AC} = \frac{F_{AC}}{A} = -\frac{135.50}{0.125} = -1084 \text{ psi} = 1.084 \text{ ksi}$$

Problem 1.9

1.9 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points B and D, (b) points C and E.



Use bar ABC as a free body.



$$\sum M_c = 0 \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N} \quad \text{Link BD is in tension}$$

$$\sum M_B = 0 \quad -(0.040)F_{CE} - (0.025)(20 \times 10^3) = 0$$

$$F_{CE} = -12.5 \times 10^3 \text{ N} \quad \text{Link CE is in compression}$$

Net area of one link for tension = $(0.008)(0.036 - 0.016)$

= $160 \times 10^{-6} \text{ m}^2$. For two parallel links $A_{\text{net}} = 320 \times 10^{-6} \text{ m}^2$

Tensile stress in link BD

$$(a) \quad \sigma_{BD} = \frac{F_{BD}}{A_{\text{net}}} = \frac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.56 \times 10^6 \text{ or } 101.6 \text{ MPa} \quad \blacksquare$$

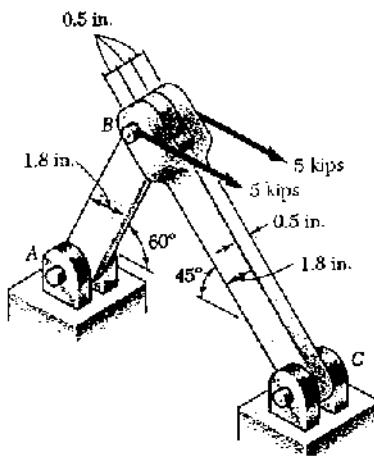
Area for one link in compression = $(0.008)(0.036)$

= $288 \times 10^{-6} \text{ m}^2$. For two parallel links $A = 576 \times 10^{-6} \text{ m}^2$

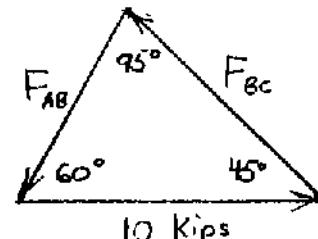
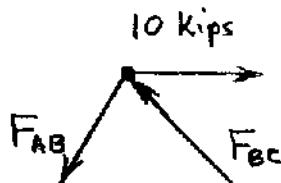
$$(b) \quad \sigma_{CE} = \frac{F_{CE}}{A} = \frac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.70 \times 10^6 \text{ or } -21.7 \text{ MPa} \quad \blacksquare$$

Problem 1.10

1.10 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.



Use joint *B* as free body.



Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ}$$

$$F_{AB} = 7.3205 \text{ kips} \quad F_{BC} = 8.9658 \text{ kips.}$$

Link *AB* is a tension member

$$\text{Minimum section at pin } A_{\text{net}} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$$

$$(a) \text{ Stress in AB} \quad \sigma_{AB} = \frac{F_{AB}}{A_{\text{net}}} = \frac{7.3205}{0.5} = 14.64 \text{ ksi}$$

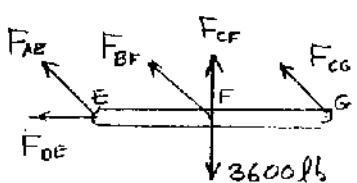
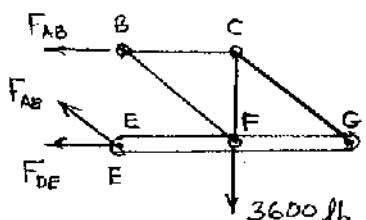
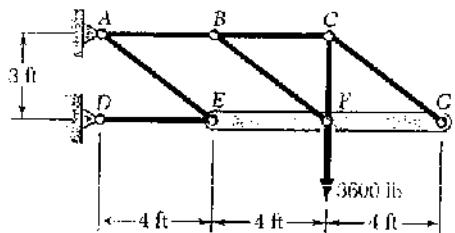
Link *BC* is a compression member

$$\text{Cross sectional area is } A = (1.8)(0.5) = 0.9 \text{ in}^2$$

$$(b) \text{ Stress in BC} \quad \sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-8.9658}{0.9} = -9.96 \text{ ksi}$$

Problem 1.11

1.11 The rigid bar EFG is supported by the truss system shown. Knowing that the member CG is a solid circular rod of 0.75-in. diameter, determine the normal stress in CG.



Using portion EFGCB as a free body

$$\uparrow \sum F_y = 0: \quad \frac{3}{5} F_{AE} - 3600 = 0$$

$$F_{AE} = 6000 \text{ lb.}$$

Using beam EFG as a free body

$$\rightarrow M_F = 0: \quad -(4) \frac{3}{5} F_{AE} + (4) \left(\frac{3}{5} F_{CG} \right) = 0$$

$$F_{CG} = F_{AE} = 6000 \text{ lb.}$$

Cross sectional area of member CG

$$A_{CG} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.75)^2 = 0.44179 \text{ in}^2$$

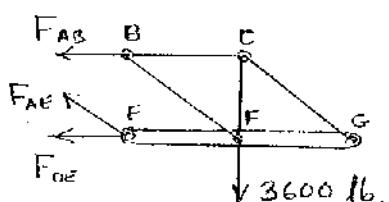
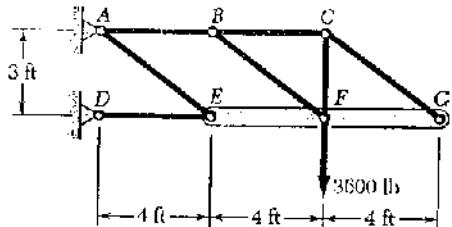
Normal stress in CG.

$$\sigma_{CG} = \frac{F_{CG}}{A_{CG}} = \frac{6000}{0.44179} = 13580 \text{ psi}$$

$$13.58 \text{ ksi}$$

Problem 1.12

1.12 The rigid bar EFG is supported by the truss system shown. Determine the cross-sectional area of member AE for which the normal stress in the member is 15 ksi.



Using portion EFGCB as a free body

$$\uparrow \sum F_y = 0: \quad \frac{3}{5} F_{AE} - 3600 = 0$$

$$F_{AE} = 6000 \text{ lb.} = 6.00 \text{ kips}$$

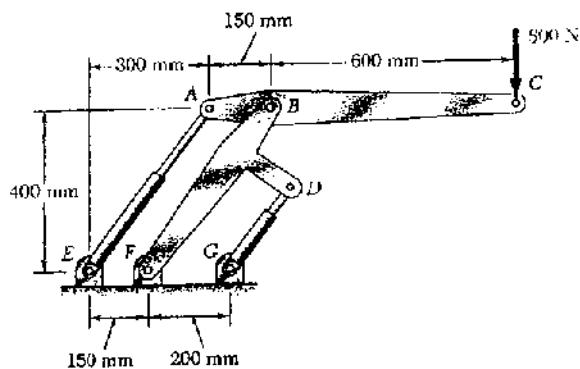
Stress in member AE $\sigma_{AE} = 15 \text{ ksi.}$

$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}}$$

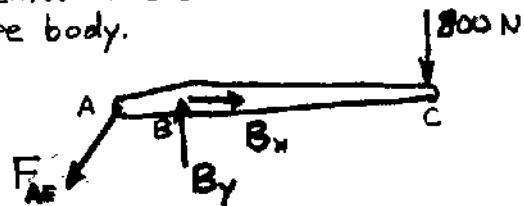
$$A_{AE} = \frac{F_{AE}}{\sigma_{AE}} = \frac{6.00}{15} = 0.400 \text{ in}^2$$

Problem 1.13

1.13 Two hydraulic cylinders are used to control the position of the robotic arm ABC . Knowing that the control rods attached at A and D each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (a) member AE , (b) member DG .



Use member ABC
as free body.

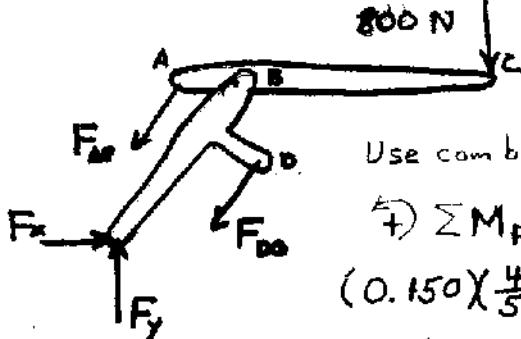


$$\textcircled{+} \sum M_B = 0 \quad (0.150) \frac{4}{5} F_{AE} - (0.600)(800) = 0 \quad F_{AE} = 4 \times 10^3 \text{ N}$$

Area of rod in member AE is $A = \frac{\pi d^2}{4} = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.16 \times 10^{-6} \text{ m}^2$

$$\text{Stress in rod } AE: \quad \sigma_{AE} = \frac{F_{AE}}{A} = \frac{4 \times 10^3}{314.16 \times 10^{-6}} = 12.73 \times 10^6 \text{ Pa}$$

$$(a) \quad \sigma_{AE} = 12.73 \text{ MPa}$$



Use combined members ABC and BFD as free body.

$$\textcircled{+} \sum M_F = 0$$

$$(0.150) \left(\frac{4}{5} F_{AE} \right) - (0.200) \left(\frac{4}{5} F_{DG} \right)$$

$$- (1.050 - 0.350)(800) = 0 \quad F_{DG} = -1500 \text{ N}$$

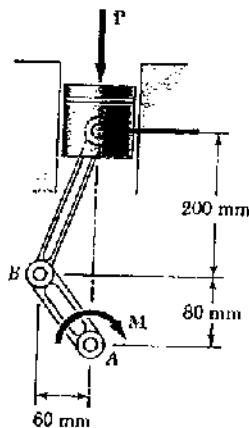
Area in rod DG is $A = \frac{\pi d^2}{4} = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.16 \times 10^{-6} \text{ m}^2$

$$\text{Stress in rod } DG: \quad \sigma_{DG} = \frac{F_{DG}}{A} = \frac{-1500}{3.1416 \times 10^{-6}} = -4.77 \times 10^6 \text{ Pa}$$

$$(b) \quad \sigma_{DG} = -4.77 \text{ MPa}$$

Problem 1.14

1.14 A couple M of magnitude 1500 N·m is applied to the crank of an engine. For the position shown, determine (a) the force P required to hold the engine system in equilibrium, (b) the average normal stress in the connecting rod BC , which has a 450-mm² uniform cross section.

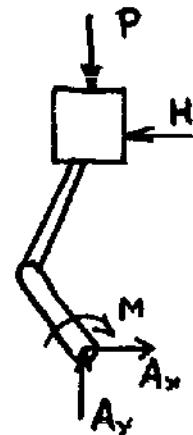


Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y .

$$\textcircled{C} \sum M_A = 0$$

$$(0.280 \text{ m})H - 1500 \text{ N}\cdot\text{m} = 0$$

$$H = 5.3571 \times 10^3 \text{ N}$$

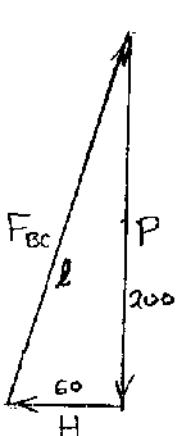


Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BC} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$$

$$\frac{P}{H} = \frac{200}{60} \therefore P = 17.86 \times 10^3 \text{ N}$$

$$P = 17.86 \text{ kN} \quad \blacktriangleleft$$



(a)

$$\frac{F_{BC}}{H} = \frac{208.81}{60} \therefore F_{BC} = 18.643 \times 10^3 \text{ N}$$

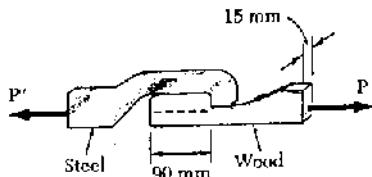
Rod BC is a compression member. Its area is $450 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$

$$\text{Stress } \sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^5 \text{ Pa}$$

$$(b) \quad \sigma_{BC} = -41.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 1.15

1.15 When the force P reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.



Area being sheared

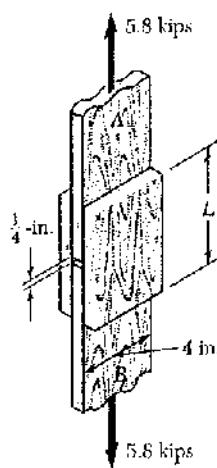
$$A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$$

Force $P = 8 \times 10^3 \text{ N}$

Shearing stress $\tau = \frac{P}{A} = \frac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^3 \text{ Pa} = 5.93 \text{ MPa}$ \blacktriangleleft

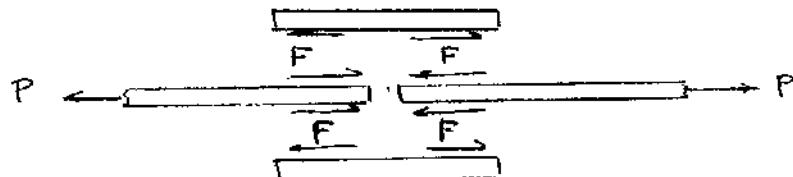
Problem 1.16

1.16 The wooden members A and B are to be joined by plywood splice plates which will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be $\frac{1}{4}$ in., determine the smallest allowable length L if the average shearing stress in the glue is not to exceed 120 psi.



There are four separate areas that are glued. Each of these areas transmits one half the the 5.8 kip force. Thus

$$F = \frac{1}{2}P = \frac{1}{2}(5.8) = 2.9 \text{ kips} = 2900 \text{ lb.}$$



Let l = length of one glued area and w = 4 in. be its width.

For each glued area $A = lw$

Average shearing stress $\tau = \frac{F}{A} = \frac{F}{lw}$

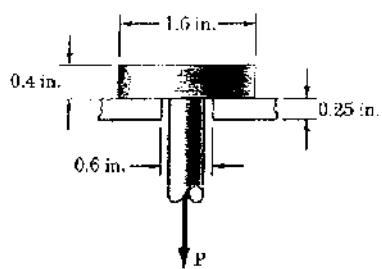
The allowable shearing stress is $\tau = 120 \text{ psi}$

Solving for l $l = \frac{F}{\tau w} = \frac{2900}{(120)(4)} = 6.0417 \text{ in.}$

Total length L $L = l + (\text{gap}) + l = 6.0417 + \frac{1}{4} + 6.0417$

$$= 12.33 \text{ in.} \quad \blacktriangleleft$$

Problem 1.17



1.17 A load P is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load P that may be applied to the rod.

$$\text{For steel } A_1 = \pi d t = \pi (0.6)(0.4) \\ = 0.7540 \text{ in}^2$$

$$\tau_1 = \frac{P}{A_1} \therefore P = A_1 \tau_1 = (0.7540)(18) \\ = 13.57 \text{ kips}$$

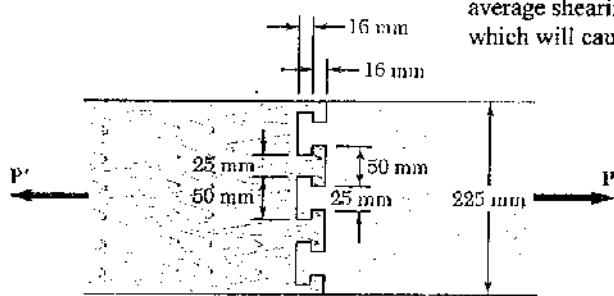
$$\text{For aluminum } A_2 = \pi d t = \pi (1.6)(0.25) = 1.2566 \text{ in}^2$$

$$\tau_2 = \frac{P}{A_2} \therefore P = A_2 \tau_2 = (1.2566)(10) = 12.57 \text{ kips}$$

Limiting value of P is the smaller value $\therefore P = 12.57 \text{ kips}$

Problem 1.18

1.18 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.



Six areas must be sheared off when the joint fails. Each of these areas has dimensions 16 mm \times 12 mm, its area being

$$A = (16)(12) = 192 \text{ mm}^2 = 192 \times 10^{-6} \text{ m}^2$$

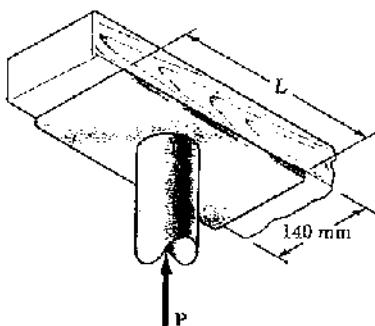
At failure the force F carried by each of areas is

$$F = 2A = (8 \times 10^6)(192 \times 10^{-6}) = 1536 \text{ N} = 1.536 \text{ kN}$$

Since there are six failure areas $P = 6F = (6)(1.536) = 9.22 \text{ kN}$

Problem 1.19

1.19 The axial force in the column supporting the timber beam shown is $P = 75 \text{ kN}$. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa.



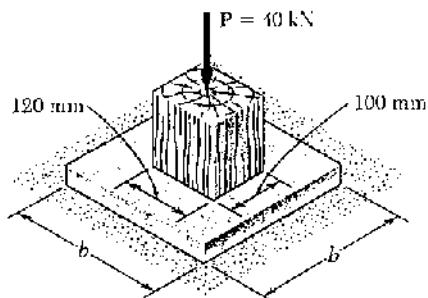
SOLUTION

$$\sigma_b = \frac{P}{A} = \frac{P}{Lw}$$

$$\text{Solving for } L: L = \frac{P}{\sigma_b w} = \frac{75 \times 10^3}{(3.0 \times 10^6)(0.140)} \\ 178.6 \cdot 10^{-3} \text{ m}$$

$$L = 178.6 \text{ mm}$$

Problem 1.20



1.20 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (a) the maximum bearing stress on the concrete footing, (b) the size of the footing for which the average bearing stress in the soil is 145 kPa.

(a) Bearing stress on concrete footing.

$$P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$$

$$A = (100)(120) = 12 \times 10^3 \text{ mm}^2 = 12 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.33 \times 10^6 \text{ Pa}$$

3.33 MPa

(b) Footing area.

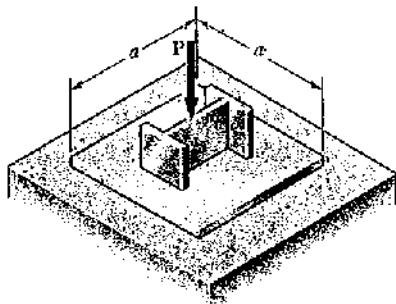
$$P = 40 \times 10^3 \text{ N} \quad \sigma = 145 \text{ kPa} = 145 \times 10^3 \text{ Pa}$$

$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$$

Since the area is square $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m} \quad b = 525 \text{ mm}$$

Problem 1.21



1.21 An axial load P is supported by a short W8 × 40 column of cross-sectional area $A = 11.7 \text{ in}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side a of the plate that will provide the most economical and safe design.

$$\text{For the column} \quad \sigma = \frac{P}{A}$$

$$\text{or} \quad P = \sigma A = (30)(11.7) = 351 \text{ kips}$$

$$\text{For the } a \times a \text{ plate,} \quad \sigma = 3.0 \text{ ksi}$$

$$A = \frac{P}{\sigma} = \frac{351}{3.0} = 117 \text{ in}^2$$

$$\text{Since the plate is square} \quad A = a^2$$

$$a = \sqrt{A} = \sqrt{117} = 10.82 \text{ in.}$$

Problem 1.22



1.22 Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is $\frac{1}{2}$ in. and the inner diameter of each washer is $\frac{5}{8}$ in., which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter d_o of the washers, knowing that the average normal stress in the bolts is 5 ksi and that the bearing stress between the washers and the planks must not exceed 1.2 ksi.

$$\text{Bolt: } A_{\text{bolt}} = \frac{\pi}{4} d_b^2 = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$\sigma_b = \frac{P}{A} \therefore \text{Tensile force in bolt: } P = \sigma_b A = (5)(0.19635) = 0.98175 \text{ kips}$$

Washer: inside diameter = $d_i = \frac{5}{8}$ in., outside diameter = d_o .

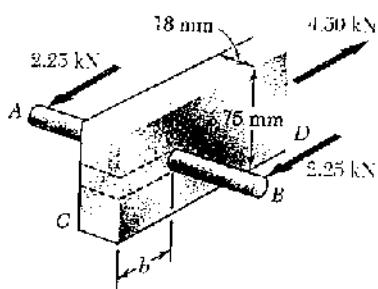
$$\text{Bearing area } A_w = \frac{\pi}{4} (d_o^2 - d_i^2) \text{ and } A_w = \frac{P}{\sigma_b}$$

$$\text{Equating } \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{P}{\sigma_b}$$

$$d_o^2 = d_i^2 + \frac{4P}{\pi\sigma_b} = \left(\frac{5}{8}\right)^2 + \frac{(4)(0.98175)}{\pi(12.5)} = 1.4323 \text{ in}^2$$

$$d_o = 1.197 \text{ in.}$$

Problem 1.23



1.23 A 0.12-mm-diameter steel rod AB is fitted to a round hole near end C of the wooden member CD . For the loading shown, determine (a) the maximum average normal stress in the wood, (b) the distance b for which the average shearing stress is 620 kPa on the surfaces indicated by the dashed lines, (c) the average bearing stress on the wood.

(a) Maximum average normal stress in the wood.

$$A_{\text{net}} = (75 - 12)(18) = 1.134 \times 10^3 \text{ mm}^2 = 1.134 \times 10^{-3} \text{ m}^2$$

$$P = 4.50 \text{ kN} = 4.50 \times 10^3 \text{ N}$$

$$\sigma = \frac{P}{A_{\text{net}}} = \frac{4.50 \times 10^3}{1.134 \times 10^{-3}} = 3.97 \times 10^6 \text{ Pa} \quad 3.97 \text{ MPa}$$

$$(b) \tau = \frac{P}{A} = \frac{P}{2bt}$$

$$b = \frac{P}{2t\tau} = \frac{4.50 \times 10^3}{(2)(18 \times 10^{-3})(620 \times 10^3)} = 202 \times 10^{-3} \text{ m}$$

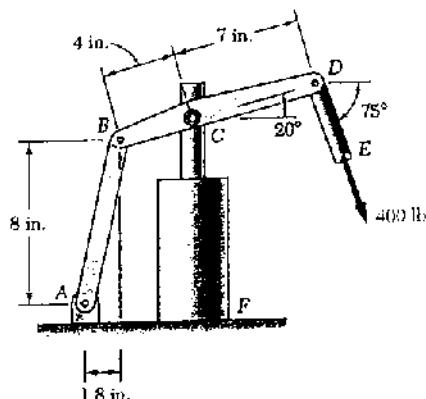
$$b = 202 \text{ mm}$$

$$(c) \sigma_b = \frac{P}{dt} = \frac{4.50 \times 10^3}{(12 \times 10^{-3})(18 \times 10^{-3})} = 20.8 \times 10^6 \text{ Pa}$$

$$20.8 \text{ MPa}$$

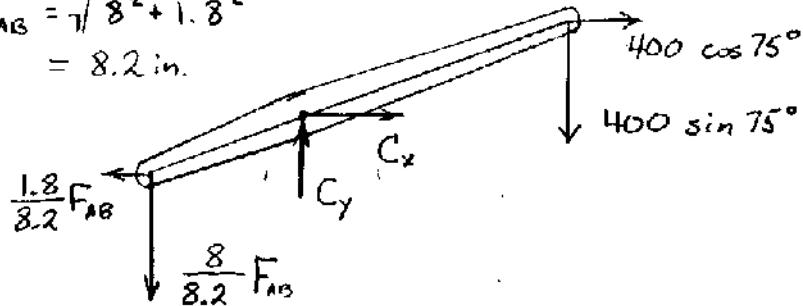
Problem 1.24

1.24 The hydraulic cylinder CF , which partially controls the position of rod DE , has been locked in the position shown. Member BD is $\frac{5}{8}$ in. thick and is connected to the vertical rod by a $\frac{3}{8}$ -in.-diameter bolt. Determine (a) the average shearing stress in the bolt, (b) the bearing stress at C in member BD .



Use member BCD as a free body, and note that AB is a two force member.

$$l_{AB} = \sqrt{8^2 + 1.8^2} \\ = 8.2 \text{ in.}$$



$$\begin{aligned} \text{Sum of moments about C: } & (4 \cos 20^\circ) \left(\frac{8}{8.2} F_{AB} \right) - (4 \sin 20^\circ) \left(\frac{1.8}{8.2} F_{AB} \right) \\ & - (7 \cos 20^\circ)(400 \sin 75^\circ) - (7 \sin 20^\circ)(400 \cos 75^\circ) = 0 \\ 3.36678 F_{AB} - 2789.35 & = 0 \quad \therefore F_{AB} = 828.49 \text{ lb.} \end{aligned}$$

$$\text{Sum of forces in x-direction: } -\frac{1.8}{8.2} F_{AB} + C_x + 400 \cos 75^\circ = 0$$

$$C_x = \frac{(1.8)(828.49)}{8.2} - 400 \cos 75^\circ = 78.34 \text{ lb.}$$

$$\text{Sum of forces in y-direction: } -\frac{8}{8.2} F_{AB} + C_y - 400 \sin 75^\circ = 0$$

$$C_y = \frac{(8)(828.49)}{8.2} + 400 \sin 75^\circ = 1194.65 \text{ lb.}$$

$$C = \sqrt{C_x^2 + C_y^2} = 1197.2 \text{ lb.}$$

(a) Shearing stress in the bolt: $P = 1197.2 \text{ lb.}$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{3}{8} \right)^2 = 0.11045 \text{ in.}^2$$

$$\tau = \frac{P}{A} = \frac{1197.2}{0.11045} = 10.84 \times 10^3 \text{ psi} = 10.84 \text{ ksi}$$

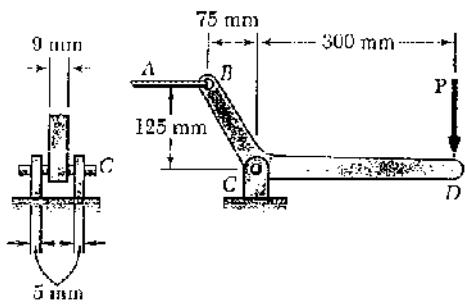
(b) Bearing stress at C in member BCD: $P = 1197.2 \text{ lb.}$

$$A_b = d t = \left(\frac{3}{8} \right) \left(\frac{5}{8} \right) = 0.234375 \text{ in.}^2$$

$$\sigma_b = \frac{P}{A_b} = \frac{1197.2}{0.234375} = 5.11 \times 10^3 \text{ psi} = 5.11 \text{ ksi}$$

Problem 1.25

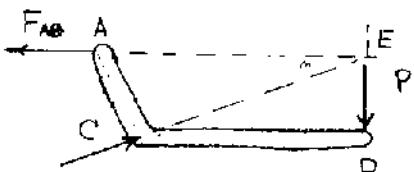
1.25 A 6-mm-diameter pin is used at connection C of the pedal shown. Knowing that $P = 500 \text{ N}$, determine (a) the average shearing stress in the pin, (b) the nominal bearing stress in the pedal at C, (c) the nominal bearing stress in each support bracket at C.



Draw free body diagram of ACD.

Since ACD is a 3-force member,
the reaction at C

is directed toward point E, the intersection
of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325 \text{ mm}$.

$$+\uparrow \sum F_y = 0: \frac{125}{325} C - P = 0 \quad C = 2.6P = (2.6)(500) = 1300 \text{ N}$$

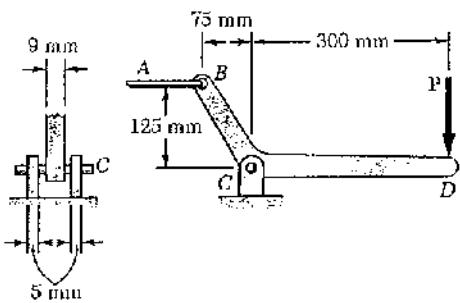
$$(a) \tau_{\text{pin}} = \frac{\frac{1}{2}C}{A_{\text{pin}}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \frac{2C}{\pi d^2} = \frac{(2)(1300)}{\pi(6 \times 10^{-3})^2} = 23.0 \times 10^6 \text{ Pa} \quad 23.0 \text{ MPa}$$

$$(b) \sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1300}{(6 \times 10^{-3})(9 \times 10^{-3})} = 24.1 \times 10^6 \text{ Pa} \quad 24.1 \text{ MPa}$$

$$(c) \sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1300}{(2)(6 \times 10^{-3})(5 \times 10^{-3})} = 21.7 \times 10^6 \text{ Pa} \quad 21.7 \text{ MPa}$$

Problem 1.26

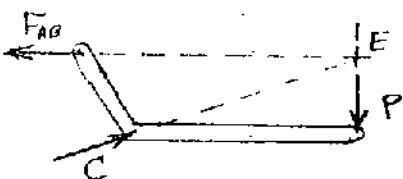
1.26 Knowing that a force P of magnitude 750 N is applied to the pedal shown, determine (a) the diameter of the pin at C for which the average shearing stress in the pin is 40 MPa, (b) the corresponding bearing stress in the pedal at C, (c) the corresponding bearing stress in each support bracket at C.



Draw free body diagram of ACD.

Since ACD is a 3-force member,
the reaction at C

is directed toward point E, the intersection
of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325 \text{ mm}$

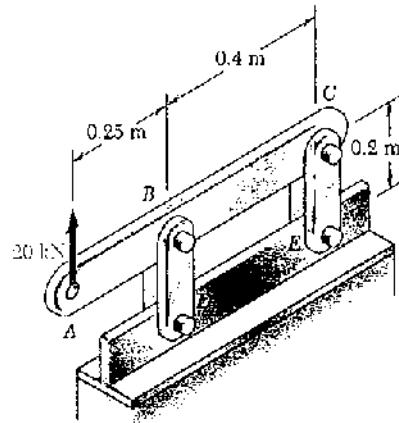
$$+\uparrow \sum F_y = 0: \frac{125}{325} C - P = 0 \quad C = 2.6P = (2.6)(750) = 1950 \text{ N}$$

$$(a) \tau_{\text{pin}} = \frac{\frac{1}{2}C}{A_{\text{pin}}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \frac{2C}{\pi d^2} = \frac{(2)(1950)}{\pi(40 \times 10^6)} = 5.57 \times 10^{-3} \text{ m} \quad 5.57 \text{ mm}$$

$$(b) \sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1950}{(5.57 \times 10^{-3})(9 \times 10^{-3})} = 38.9 \times 10^6 \text{ Pa} \quad 38.9 \text{ MPa}$$

$$(c) \sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1950}{(2)(5.57 \times 10^{-3})(5 \times 10^{-3})} = 35.0 \times 10^6 \text{ Pa} \quad 35.0 \text{ MPa}$$

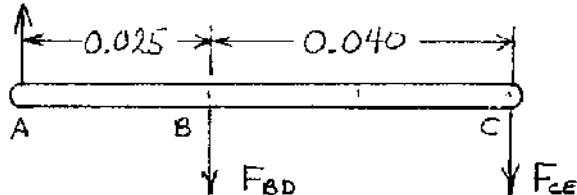
Problem 1.27



1.27 For the assembly and loading of Prob. 1.9, determine (a) the average shearing stress in the pin at *B*, (b) the average bearing stress at *B* in member *BD*, (c) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10×50 -mm uniform rectangular cross section.

1.9 Each of the four vertical links has an 8×36 -mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points *B* and *D*, (b) points *C* and *E*.

Use bar *ABC* as a free body



$$\sum M_c = 0 \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) Shear pin at *B* $\tau = \frac{F_{BD}}{2A}$ for double shear

where $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \quad 80.8 \text{ MPa}$$

(b) Bearing: link *BD* $A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

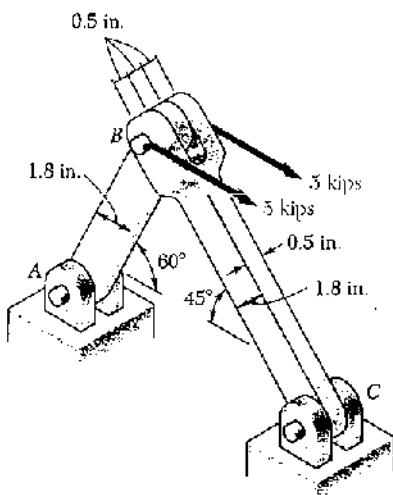
$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \quad 127.0 \text{ MPa}$$

(c) Bearing in *ABC* at *B*

$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \quad 203 \text{ MPa}$$

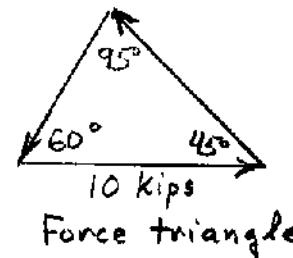
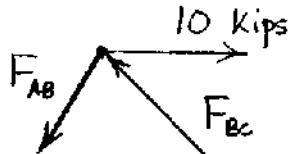
Problem 1.28



1.28 For the assembly and loading of Prob. 1.10, determine (a) the average shearing stress in the pin at C, (b) the average bearing stress at C in member BC, (c) the average bearing stress at B in member BC.

1.10 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (a) in link AB, (b) in link BC.

Use joint B as free body



Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \quad F_{BC} = 8.9658 \text{ kips}$$

(a) Shearing stress in pin at C $\tau = \frac{F_{BC}}{2A_p}$

$$A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \quad 8.92 \text{ ksi}$$

(b) Bearing stress at C in member BC $\sigma_b = \frac{F_{BC}}{A}$

$$A = t d = (0.5)(0.8) = 0.4 \text{ in}^2$$

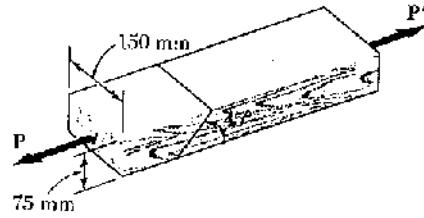
$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \quad 22.4 \text{ ksi}$$

(c) Bearing stress at B in member BC $\sigma_b = \frac{F_{BC}}{A}$

$$A = 2t d = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \quad 11.21 \text{ ksi}$$

Problem 1.29



1.29 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11 \text{ kN}$, determine the normal and shearing stresses in the glued splice.

$$\theta = 90^\circ - 45^\circ = 45^\circ$$

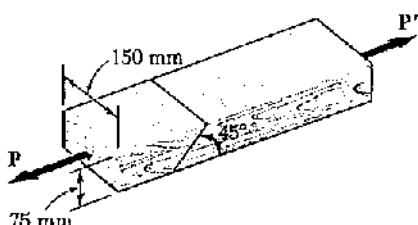
$$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P \cos^2 \theta}{A_o} = \frac{(11 \times 10^3) \cos^2 45^\circ}{11.25 \times 10^{-3}} \\ = 489 \times 10^3 \text{ Pa} \quad \sigma = 489 \text{ kPa}$$

$$\tau = \frac{P \sin 2\theta}{2 A_o} = \frac{(11 \times 10^3) (\sin 90^\circ)}{(2)(11.25 \times 10^{-3})} \\ = 4.89 \times 10^3 \text{ Pa} \quad \tau = 489 \text{ kPa}$$

Problem 1.30



1.30 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (a) the largest load P that can be safely applied, (b) the corresponding tensile stress in the splice.

$$\theta = 90^\circ - 45^\circ = 45^\circ$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

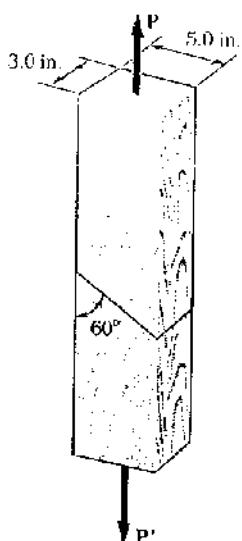
$$\tau = 620 \text{ kPa} = 620 \times 10^3 \text{ Pa}$$

$$\tau = \frac{P \sin 2\theta}{2 A_o}$$

$$(a) \quad P = \frac{2 A_o \tau}{\sin 2\theta} = \frac{(2)(11.25 \times 10^{-3})(620 \times 10^3)}{\sin 90^\circ} \\ = 13.95 \times 10^3 \text{ N} \quad 13.95 \text{ kN}$$

$$(b) \quad \sigma = \frac{P \cos^2 \theta}{A_o} = \frac{(13.95 \times 10^3)(\cos 45^\circ)^2}{11.25 \times 10^{-3}} \\ = 620 \times 10^3 \text{ Pa} \quad 620 \text{ kPa}$$

Problem 1.31



1.31 The 1.4 kip load P is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$$P = 1400 \text{ lb}$$

$$\theta = 90^\circ - 60^\circ = 30^\circ$$

$$A_o = (5.0)(3.0) = 15 \text{ in}^2$$

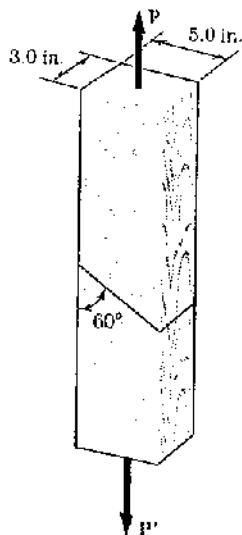
$$\sigma = \frac{P \cos^2 \theta}{A_o} = \frac{(1400)(\cos 30^\circ)^2}{15}$$

$$\sigma = 70.0 \text{ psi}$$

$$\tau = \frac{P \sin 2\theta}{2A_o} = \frac{(1400) \sin 60^\circ}{(2)(15)}$$

$$\tau = 40.4 \text{ psi}$$

Problem 1.32



1.32 Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (a) the largest load P that can be safely supported, (b) the corresponding stress in the splice.

$$A_o = (5.0)(3.0) = 15 \text{ in}^2$$

$$\theta = 90^\circ - 60^\circ = 30^\circ$$

$$\sigma = \frac{P \cos^2 \theta}{A_o}$$

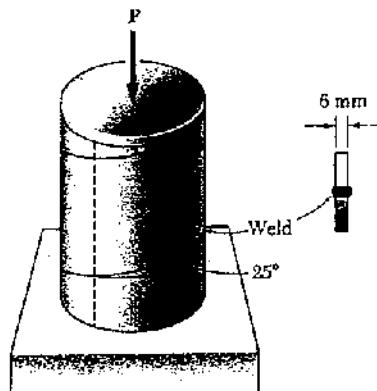
$$(a) P = \frac{\sigma A_o}{\cos^2 \theta} = \frac{(75)(15)}{\cos^2 30^\circ} = 1500 \text{ lb}$$

$$P = 1.500 \text{ kips}$$

$$(b) \tau = \frac{P \sin 2\theta}{2A_o} = \frac{(1500) \sin 60^\circ}{(2)(15)}$$

$$\tau = 43.3 \text{ psi}$$

Problem 1.33



1.33 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that a 250-kN axial force P is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

$$d_o = 0.300 \text{ m} \quad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$$

$$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$$

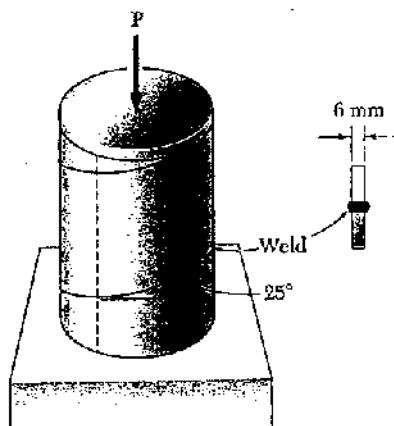
$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2) \\ = 5.54 \times 10^{-3} \text{ m}^2$$

$$\theta = 25^\circ$$

$$\sigma = \frac{P}{A_o} \cos^2 \theta = \frac{-250 \times 10^3 \cos^2 25^\circ}{5.54 \times 10^{-3}} \\ = -37.1 \times 10^6 \quad \sigma = -37.1 \text{ MPa} \quad \blacksquare$$

$$\tau = \frac{P}{2A_o} \sin 2\theta = \frac{-250 \times 10^3 \sin 50^\circ}{(2)(5.54 \times 10^{-3})} \\ = -17.28 \times 10^6 \quad \tau = 17.28 \text{ MPa} \quad \blacksquare$$

Problem 1.34



1.34 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 50 \text{ MPa}$ and $\tau = 30 \text{ MPa}$, determine the magnitude P of the largest axial force that can be applied to the pipe.

$$d_o = 0.300 \text{ m} \quad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$$

$$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$$

$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2) \\ = 5.54 \times 10^{-3} \text{ m}^2$$

$$\theta = 25^\circ$$

$$\text{Based on } |\sigma| = 50 \text{ MPa: } \sigma = \frac{P}{A_o} \cos^2 \theta$$

$$P = \frac{A_o \sigma}{\cos^2 \theta} = \frac{(5.54 \times 10^{-3})(50 \times 10^6)}{\cos^2 25^\circ} = 337 \times 10^3$$

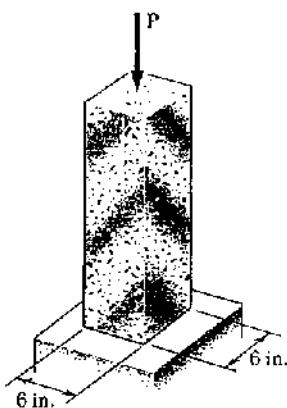
$$\text{Based on } |\tau| = 30 \text{ MPa} \quad \tau = \frac{P}{2A_o} \sin 2\theta$$

$$P = \frac{2A_o \tau}{\sin 2\theta} = \frac{(2)(5.54 \times 10^{-3})(30 \times 10^6)}{\sin 50^\circ} = 434 \times 10^3$$

Smaller value is the allowable value of P & $P = 337 \text{ kN} \quad \blacksquare$

Problem 1.35

1.35 A 240-kip load P is applied to the granite block shown. Determine the resulting maximum value of (a) the normal stress, (b) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.



$$A_o = (6)(6) = 36 \text{ in}^2$$

$$\sigma = \frac{P}{A_o} \cos^2 \theta = \frac{-240}{36} \cos^2 \theta = -6.67 \cos^2 \theta$$

(a) max tensile stress = 0 at $\theta = 90^\circ$

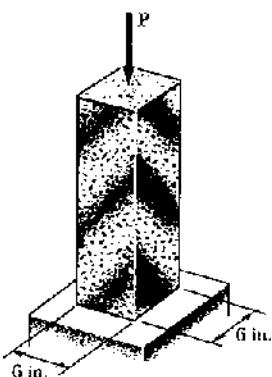
max. compressive stress = 6.67 ksi
at $\theta = 0^\circ$

$$(b) \tau_{\max} = \frac{P}{2A_o} = \frac{240}{(2)(36)} = 3.33 \text{ ksi}$$

at $\theta = 45^\circ$

Problem 1.36

1.36 A centric load P is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 2.5 ksi, determine (a) the magnitude of P , (b) the orientation of the surface on which the maximum shearing stress occurs, (c) the normal stress exerted on the surface, (d) the maximum value of the normal stress in the block.



$$A_o = (6)(6) = 36 \text{ in}^2 \quad \tau_{\max} = 2.5 \text{ ksi}$$

$\theta = 45^\circ$ for plane of τ_{\max}

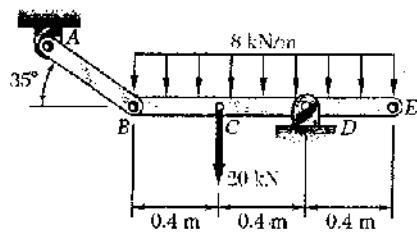
$$(a) \tau_{\max} = \frac{|P|}{2A_o} \therefore |P| = 2A_o \tau_{\max} = (2)(36)(2.5) = 180 \text{ kips}$$

$$(b) \sin 2\theta = 1 \quad 2\theta = 90^\circ \quad \theta = 45^\circ$$

$$(c) \sigma_{45} = \frac{P}{A_o} \cos^2 45^\circ = \frac{P}{2A_o} = \frac{-180}{(2)(36)} = -2.5 \text{ ksi}$$

$$(d) \sigma_{\max} = \frac{P}{A_o} = \frac{-180}{36} = -5 \text{ ksi}$$

Problem 1.37



1.37 Link AB is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for AB for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at A and B.

$$P = (1.2)(8) = 9.6 \text{ kN}$$

$$\rightarrow \sum M_D = 0$$

$$-(0.8)(F_{AB} \sin 35^\circ) + (0.2)(9.6) + (0.4)(20) = 0$$

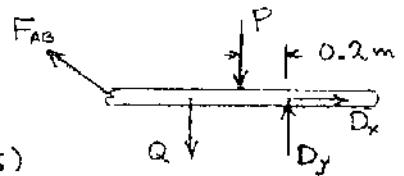
$$F_{AB} = 21.619 \text{ kN} = 21.619 \times 10^3 \text{ N}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{ut}}{F.S.}$$

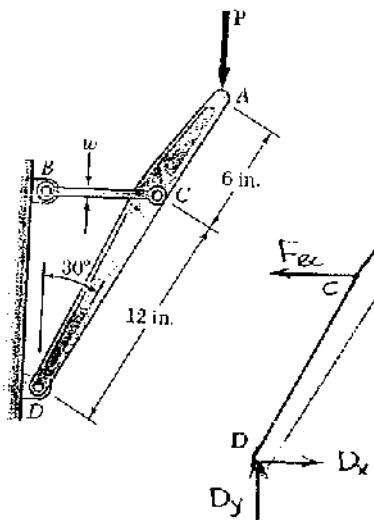
$$A_{AB} = \frac{(F.S.) F_{AB}}{\sigma_{ut}} = \frac{(3.50)(21.619 \times 10^3)}{450 \times 10^6}$$

$$= 168.1 \times 10^{-6} \text{ m}^2$$

$$A_{AB} = 168.1 \text{ mm}^2$$



Problem 1.38



1.38 The horizontal link BC is $\frac{1}{4}$ in. thick, has a width $w = 1.25$ in., and is made of a steel with a 65-ksi ultimate strength in tension. What is the factor of safety if the structure shown is designed to support a load of $P = 10$ kips?

$$+\circlearrowleft \sum M_c = 0$$

$$(12 \cos 30^\circ) F_{AB} - (18 \sin 30^\circ)(10) = 0$$

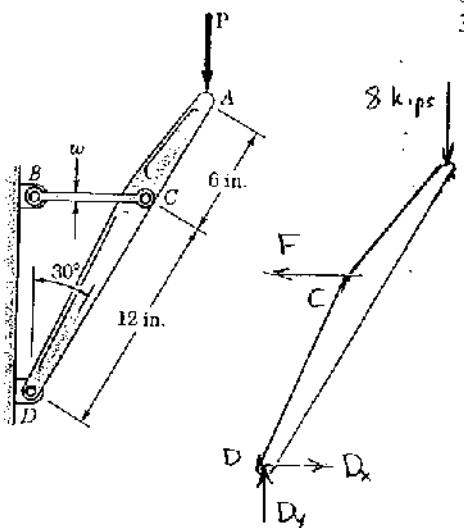
$$F_{BC} = 8.6603 \text{ kips}$$

$$A_{BC} = \left(\frac{1}{4}\right)(1.25) = 0.3125 \text{ in}^2$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{65\text{ksi}}{\text{F.S.}}$$

$$\text{F.S.} = \frac{A_{BC} \sigma_{ult}}{F_{BC}} = \frac{(0.3125)(65)}{8.6603} = 2.35$$

Problem 1.39



1.39 The horizontal link BC is $\frac{1}{4}$ in. thick and is made of a steel with a 65-ksi ultimate strength in tension. What should be the width w of the link if the structure shown is to be designed to support a load $P = 8$ kips with a factor of safety equal to 3?

$$+\circlearrowleft \sum M_c = 0$$

$$(12 \cos 30^\circ) F_{AB} - (18 \sin 30^\circ)(8) = 0$$

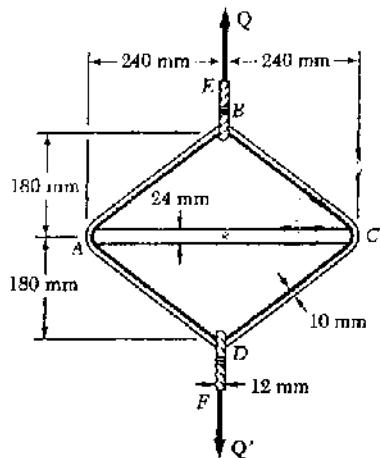
$$F_{BC} = 6.9282 \text{ kips}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{F_{AB}}{tw} = \frac{\sigma_{ult}}{\text{F.S.}}$$

$$w = \frac{(F.S.) F_{BC}}{t \sigma_{ult}} = \frac{(3)(6.9282)}{\left(\frac{1}{4}\right)(65)}$$

$$w = 1.279 \text{ in.}$$

Problem 1.40

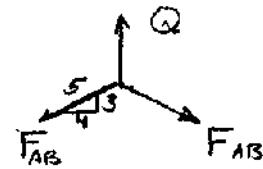


L40 A steel loop $ABCD$ of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod AC . Cables BE and DF , each of 12-mm diameter, are used to apply the load Q . Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load Q that can be applied if an overall factor of safety of 3 is desired.

Using joint B as a free body
and considering symmetry

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

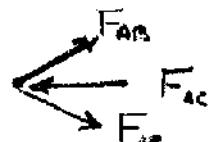
$$Q = \frac{6}{5} F_{AB}$$



Using joint A as a free body
and considering symmetry

$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{6}{5} Q - F_{AC} = 0 \quad \therefore Q = \frac{3}{4} F_{AC}$$



Based on strength of cable BE

$$Q_u = \bar{\sigma}_u A = \bar{\sigma}_u \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop

$$\begin{aligned} Q_u &= \frac{6}{5} F_{AB,u} = \frac{6}{5} \bar{\sigma}_u A = \frac{6}{5} \bar{\sigma}_u \frac{\pi}{4} d^2 \\ &= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N} \end{aligned}$$

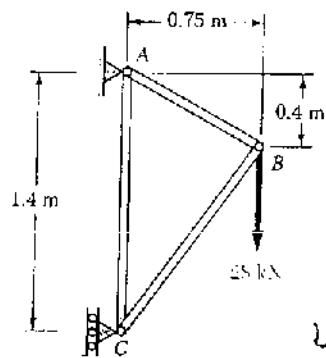
Based on strength of rod AC

$$\begin{aligned} Q_u &= \frac{3}{4} F_{AC,u} = \frac{3}{4} \bar{\sigma}_u A = \frac{3}{4} \bar{\sigma}_u \frac{\pi}{4} d^2 \\ &= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N} \end{aligned}$$

Actual ultimate load Q_u is the smallest $\therefore Q_u = 45.24 \times 10^3 \text{ N}$

$$\begin{aligned} \text{Allowable load } Q &= \frac{Q_u}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3 \text{ N} \\ &= 15.08 \text{ kN} \end{aligned}$$

Problem 1.41



1.41 Members AB and BC of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for both bars, determine the required cross-sectional area of (a) bar AB, (b) bar AC.

Length of member AB

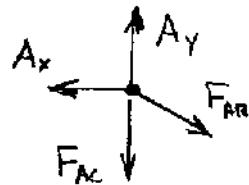
$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

Use entire truss as a free body

$$\therefore \sum M_c = 0 \quad 1.4 A_x - (0.75)(28) = 0 \quad A_x = 15 \text{ kN}$$

$$+\uparrow \sum F_y = 0 \quad A_y - 28 = 0 \quad A_y = 28 \text{ kN}$$

Use joint A as free body



$$\therefore \sum F_x = 0 \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \sum F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_u = 120 \times 10^3 \text{ N}$

For the material $G_u = \frac{P_u}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For member AB $\text{F.S.} = \frac{P_u}{F_{AB}} = \frac{G_u A_{AB}}{F_{AB}}$

$$A_{AB} = \frac{(\text{F.S.}) F_{AB}}{G_u} = \frac{(3.2)(17 \times 10^3)}{300 \times 10^6} = 181.33 \times 10^{-6} \text{ m}^2$$

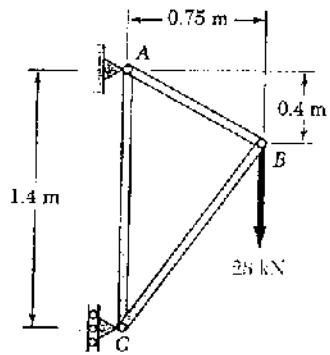
$$A_{AB} = 181.3 \text{ mm}^2$$

(b) For member AC $\text{F.S.} = \frac{P_u}{F_{AC}} = \frac{G_u A_{AC}}{F_{AC}}$

$$A_{AC} = \frac{(\text{F.S.}) F_{AC}}{G_u} = \frac{(3.2)(20 \times 10^3)}{300 \times 10^6} = 213.33 \times 10^{-6} \text{ m}^2$$

$$A_{AC} = 213 \text{ mm}^2$$

Problem 1.42



1.42 Members AB and BC of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar AB has a cross-sectional area of 225 mm^2 , determine (a) the factor of safety for bar AB , (b) the cross-sectional area of bar AC if it is to have the same factor of safety as bar AB .

Length of member AB

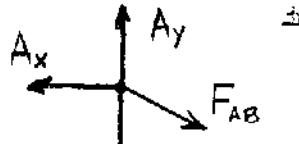
$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

Use entire truss as a free body

$$\sum M_c = 0 \quad 1.4 A_x - (0.75)(28) = 0 \quad A_x = 15 \text{ kN}$$

$$\sum F_y = 0 \quad A_y - 28 = 0 \quad A_y = 28 \text{ kN}$$

Use joint A as free body



$$\sum F_x = 0 \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$\sum F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

$$\text{For the test bar} \quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_u = 120 \times 10^3 \text{ N}$$

$$\text{For the material} \quad \sigma_u = \frac{P_u}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$$

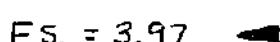
$$(a) \text{ For bar } AB \quad \text{F.S.} = \frac{F_u}{F_{AB}} = \frac{\sigma_u A_{AB}}{F_{AB}} = \frac{(300 \times 10^6)(225 \times 10^{-6})}{17 \times 10^3}$$

$$\text{F.S.} = 3.97$$

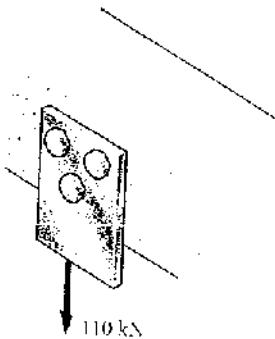
$$(b) \text{ For bar } AC \quad \text{F.S.} = \frac{F_u}{F_{AC}} = \frac{\sigma_u A_{AC}}{F_{AC}}$$

$$A_{AC} = \frac{(\text{F.S.}) F_{AC}}{\sigma_u} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{ m}^2$$

$$A_{AC} = 265 \text{ mm}^2$$



Problem 1.43



1.43 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110 kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.

$$\text{For each bolt } P = \frac{110}{3} = 36.667 \text{ kN}$$

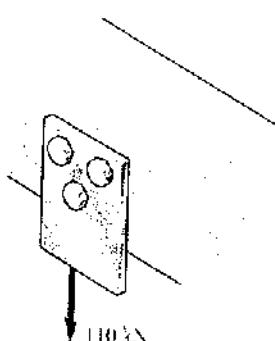
$$\text{Required } P_u = (F.S.)P = (3.35)(36.667) = 122.83 \text{ kN}$$

$$\tau_u = \frac{P_u}{A} = \frac{P_u}{\frac{\pi}{4}d^2} = \frac{4P_u}{\pi d^2}$$

$$d = \sqrt{\frac{4P_u}{\pi \tau_u}} = \sqrt{\frac{(4)(122.83 \times 10^3)}{\pi (360 \times 10^6)}} = 20.8 \times 10^{-3} \text{ m}$$

$$d = 20.8 \text{ mm}$$

Problem 1.44



1.44 Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.

$$\text{For each bolt } A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2 \\ = 254.47 \times 10^{-6} \text{ m}^2$$

$$P_u = A \tau_u = (254.47 \times 10^{-6})(360 \times 10^6) \\ = 91.609 \times 10^3 \text{ N}$$

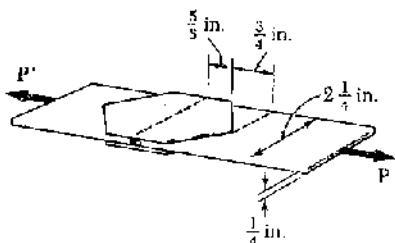
$$\text{For the three bolts } P_u = (3)(91.609 \times 10^3) \\ = 274.83 \times 10^3 \text{ N}$$

Factor of safety,

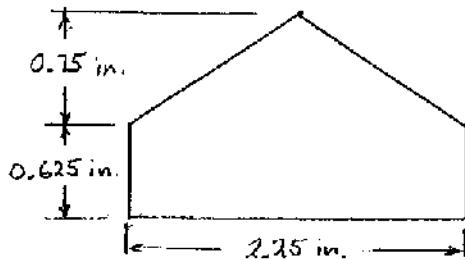
$$\text{F.S.} = \frac{P_u}{P} = \frac{274 \times 10^3}{110 \times 10^3} = 2.50$$

Problem 1.45

1.45 Two plates, each $\frac{1}{8}$ in. thick, are used to splice a plastic strip as shown. Knowing that the ultimate shearing stress of the bonding between the surfaces is 130 psi, determine the factor of safety with respect to shear when $P = 32.5$ lb.



Bond area: (See figure)



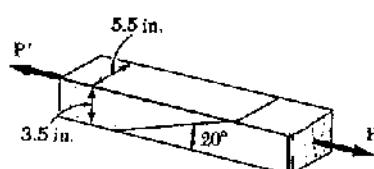
$$A = \frac{1}{2}(2.25)(0.75) + (2.25)(0.625) = 2.25 \text{ in}^2$$

$$P_u = 2 A \tau_u = (2)(2.25)(130) = 585 \text{ lb.}$$

$$\text{F.S.} = \frac{P_u}{P} = \frac{585}{32.5} = 1.800$$

Problem 1.46

1.46 Two wooden members of 3.5 x 5.5-in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 75 psi, determine the largest axial load P that can be safely applied.



$$A_o = (3.5)(5.5) = 19.25 \text{ in}^2$$

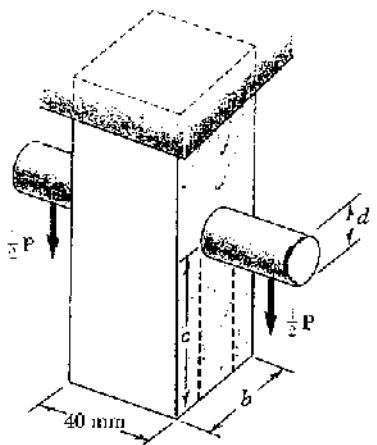
$$\theta = 90^\circ - 20^\circ = 70^\circ$$

$$\gamma = \frac{P}{A_o} \sin \theta \cos \theta = \frac{P}{2A_o} \sin 2\theta$$

$$P = \frac{2A\gamma}{\sin 2\theta} = \frac{(2)(19.25)(75)}{\sin 140^\circ} = 4492 \text{ lb} = 4.49 \text{ kips}$$

Problem 1.47

1.47 A load P is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load P if an overall factor of safety of 3.2 is desired.



Based on double shear in pin

$$P_u = 2A\tau_u = 2 \frac{\pi}{4}d^2\tau_u \\ = \frac{\pi}{4}(2)(0.012)^2(145 \times 10^6) = 32.80 \times 10^3 \text{ N}$$

Based on tension in wood

$$P_u = A\sigma_u = w(b-d)\sigma_u \\ = (0.040)(0.040 - 0.012)(60 \times 10^6) \\ = 67.2 \times 10^3 \text{ N}$$

Based on double shear in the wood

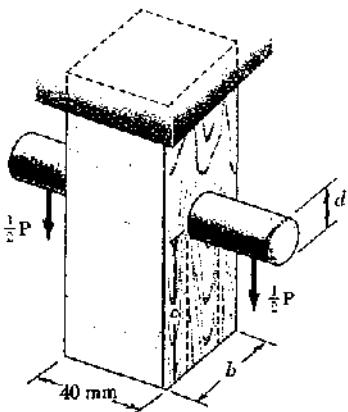
$$P_u = 2A\tau_u = 2w c \tau_u = (2)(0.040)(0.055)(7.5 \times 10^6) \\ = 33.0 \times 10^3 \text{ N}$$

Use smallest $P_u = 32.8 \times 10^3 \text{ N}$

Allowable $P = \frac{P_u}{F.S.} = \frac{32.8 \times 10^3}{3.2} = 10.25 \times 10^3 \text{ N}$

10.25 kN

Problem 1.48



1.48 For the support of Prob. 1.47, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20 \text{ kN}$, determine (a) the factor of safety for the pin, (b) the required values of b and c if the factor of safety for the wooden members is the same as that found in part a for the pin.

1.47 A load P is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40 \text{ mm}$, $c = 55 \text{ mm}$, and $d = 12 \text{ mm}$, determine the load P if an overall factor of safety of 3.2 is desired.

$$\text{Pin: } P = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

$$(a) \text{ Pin: } A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$\text{Double shear } \gamma = \frac{P}{2A} \quad \gamma_u = \frac{P_u}{2A}$$

$$P_u = 2A\gamma_u = (2)(201.06 \times 10^{-6})(145 \times 10^6)$$

$$= 58.336 \times 10^3 \text{ N}$$

$$\text{F.S.} = \frac{P_u}{P} = \frac{58.336 \times 10^3}{20 \times 10^3} = 2.92$$

(b) Tension in wood $P_u = 58.336 \times 10^3 \text{ N}$ for same F.S.

$$\sigma_u = \frac{P_u}{A} = \frac{P_u}{W(b-d)} \quad \text{where } W = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_u}{W\sigma_u} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^3 \text{ m}$$

$$b = 40.3 \text{ mm}$$

Shear in wood $P_u = 58.336 \times 10^3 \text{ N}$ for same F.S.

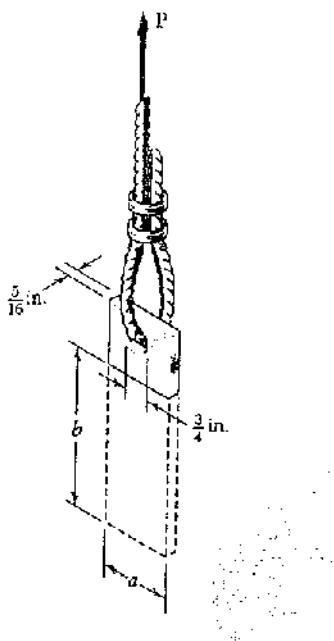
Double shear; each area is $A = WC$

$$\gamma_u = \frac{P_u}{2A} = \frac{P_u}{2WC}$$

$$c = \frac{P_u}{2W\gamma_u} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3} \text{ m}$$

$$c = 97.2 \text{ mm}$$

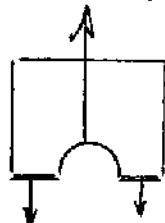
Problem 1.49



1.49 A steel plate $\frac{5}{16}$ in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (a) the required width a of the plate, (b) the minimum depth b to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)

2.5 Kips

Based on tension in plate



$$A = (a - d)t$$

$$P_u = \sigma_u A$$

$$F.S. = \frac{P_u}{P} = \frac{\sigma_u(a - d)t}{P}$$

Solving for a

$$a = d + \frac{(F.S.)P}{\sigma_u t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)(\frac{5}{16})}$$

$$(a) \quad a = 1.550 \text{ in.}$$

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b \quad \tau_u = 0.300 \text{ ksi}$$

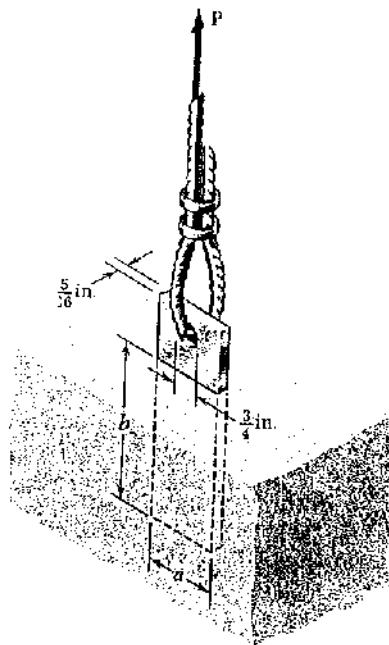
$$P_u = \tau_u A = 2\tau_u(a + t)b \quad F.S. = \frac{P_u}{P}$$

$$\text{Solving for } b \quad b = \frac{(F.S.)P}{2(a + t)\tau_u} = \frac{(3.6)(2.5)}{(2)(1.550 + \frac{5}{16})(0.300)}$$

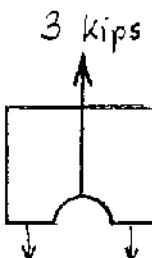
$$(b) \quad b = 8.05 \text{ in.}$$

Problem 1.50

1.50 Determine the factor of safety for the cable anchor in Prob. 1.49 when $P = 3$ kips, knowing that $a = 2$ in. and $b = 7.5$ in.



1.49 A steel plate $\frac{5}{16}$ in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (a) the required width a of the plate, (b) the minimum depth b to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)



Based on tension in plate

$$\begin{aligned} A &= (a - d)t \\ &= (2 - \frac{3}{4})(\frac{5}{16}) = 0.3906 \text{ in}^2 \\ P_u &= \sigma_u A \\ &= (36)(0.3906) = 14.06 \text{ kips} \\ F.S. &= \frac{P_u}{P} = \frac{14.06}{3} = 4.69 \end{aligned}$$

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b = 2(2 + \frac{5}{16})(7.5)$$

$$A = 34.69 \text{ in}^2 \quad \gamma_u = 0.300 \text{ ksi}$$

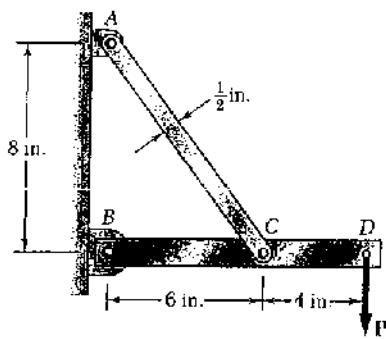
$$P_u = \gamma_u A = (0.300)(34.69) = 10.41 \text{ kips}$$

$$F.S. = \frac{P_u}{P} = \frac{10.41}{3} = 3.47$$

Actual factor of safety is the smaller value

F.S. = 3.47

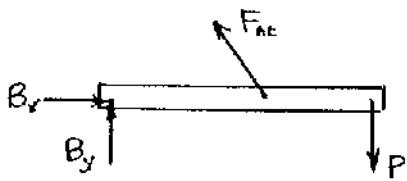
Problem 1.51



1.51 Link AC is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4} \times \frac{1}{2}$ -in. uniform rectangular cross section. It is connected to a support at A and to member BCD at C by $\frac{3}{8}$ -in.-diameter pins, while member BCD is connected to its support at B by a $\frac{5}{16}$ -in.-diameter pin; all of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load P that can be applied at D . Note that link AC is not reinforced around the pin holes.

Use free body BCD .

$$+\sum M_B = 0 : (6)(\frac{8}{10} F_{AC}) - 10 P = 0 \\ P = 0.48 F_{AC} \quad (1)$$



$$\pm \sum F_x = 0 : B_x - \frac{6}{10} F_{AC} = 0$$

$$B_x = \frac{6}{10} F_{AC} = 1.25 P \rightarrow$$

$$+\sum M_C = 0 : -6B_y - 4P = 0$$

$$B_y = -\frac{2}{3}P \quad \text{i.e. } B_y = \frac{2}{3}P \downarrow$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + (\frac{2}{3})^2} P = 1.41667 P \quad P = 0.70588 B \quad (2)$$

Shear in pins at A and C .

$$F_{AC} = \tau A_{pin} = \frac{\tau_0}{F.S.} \frac{\pi}{4} d^2 = \left(\frac{25}{3.25}\right) \left(\frac{\pi}{4}\right) \left(\frac{5}{16}\right)^2 = 0.84959 \text{ kips}$$

Tension on net section at A and C .

$$F_{AC} = 6 A_{net} = \frac{6\tau_0}{F.S.} A_{net} = \left(\frac{65}{3.25}\right) \left(\frac{1}{4}\right) \left(\frac{1}{2} - \frac{3}{8}\right) = 0.625 \text{ kips}$$

Smaller value of F_{AC} is 0.625 kips.

$$\text{From (1)} \quad P = (0.48)(0.625) = 0.300 \text{ kips}$$

Shear in pin at B .

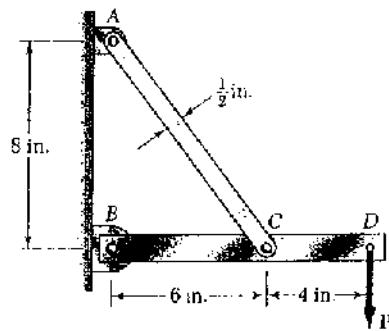
$$B = \tau A_{pin} = \frac{\tau_0}{F.S.} \frac{\pi}{4} d^2 = \left(\frac{25}{3.25}\right) \left(\frac{\pi}{4}\right) \left(\frac{5}{16}\right)^2 = 0.58999 \text{ kips}$$

$$\text{From (2)} \quad P = (0.70588)(0.58999) = 0.416 \text{ kips}$$

Allowable value of P is the smaller value. $P = 0.300 \text{ kips}$

or $P = 300 \text{ lb.}$

Problem 1.52



1.52 Solve Prob. 1.51, assuming that the structure has been redesigned to use $\frac{5}{16}$ -in.-diameter pins at A and C as well as at B and that no other change has been made.

1.51 Link AC is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4} \times \frac{1}{2}$ in. uniform rectangular cross section. It is connected to a support at A and to member BCD at C by $\frac{3}{8}$ -in.-diameter pins, while member BCD is connected to its support at B by a $\frac{5}{16}$ -in.-diameter pin; all of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load P that may be applied at D. Note that link AC is not reinforced around the pin holes.

Use free body BCD

$$+\circlearrowleft \sum M_B = 0 : (6)(\frac{3}{8} F_{AC}) - 10P = 0$$

$$P = 0.48 F_{AC} \quad (1)$$



$$+\uparrow \sum F_y = 0 : B_y - \frac{6}{10} F_{AC} = 0$$

$$B_y = \frac{6}{10} F_{AC} = 1.25 P \rightarrow$$

$$+\circlearrowleft \sum M_C = 0 : -6B_y - 4P = 0 \quad B_y = -\frac{2}{3}P \text{ i.e. } B_y = \frac{2}{3}P \downarrow$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + (\frac{2}{3})^2} P = 1.41667 P \quad P = 0.70583 B \quad (2)$$

Shear in pins at A and C.

$$F_{AC} = \tau A_{pin} = \frac{\tau_u}{F.S.} \frac{\pi}{4} d^2 = (\frac{25}{3.25})(\frac{\pi}{4})(\frac{5}{16})^2 = 0.58999 \text{ kips}$$

Tension on net section at A and C.

$$F_{AC} = \sigma A_{net} = \frac{\sigma_u}{F.S.} A_{net} = (\frac{65}{3.25})(\frac{1}{4})(\frac{1}{2} - \frac{5}{16}) = 0.9375 \text{ kips.}$$

Smaller value of F_{AC} is 0.58999 kips

$$\text{From (1)} \quad P = (0.48)(0.58999) = 0.283 \text{ kips}$$

Shear in pin at B.

$$B = \tau A_{pin} = \frac{\tau_u}{F.S.} \frac{\pi}{4} d^2 = (\frac{25}{3.25})(\frac{\pi}{4})(\frac{5}{16})^2 = 0.58999 \text{ kips}$$

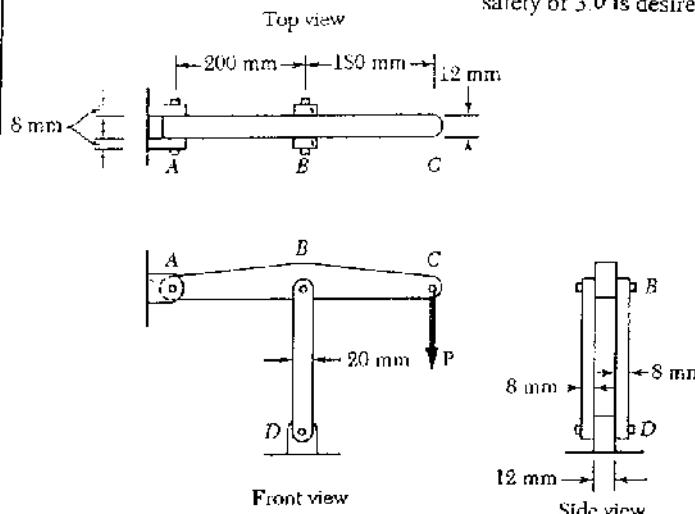
$$\text{From (2)} \quad P = (0.70583)(0.58999) = 0.416 \text{ kips}$$

Allowable value of P is the smaller value. $P = 0.283 \text{ kips}$

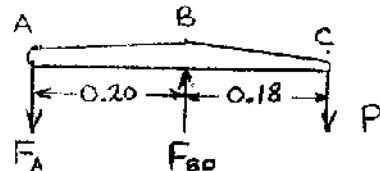
$$P = 283 \text{ lb}$$

Problem 1.53

1.53 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load P if an overall factor of safety of 3.0 is desired.



Statics: Use ABC as free body.



$$\sum M_A = 0 \quad 0.20 F_A - 0.18 P = 0$$

$$P = \frac{10}{9} F_A$$

$$\sum M_A = 0 \quad 0.20 F_{BD} - 0.38 P = 0$$

~~$$P = \frac{10}{19} F_{BD}$$~~

Based on double shear in pin A

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.008)^2 = 50.266 \times 10^{-6} \text{ m}^2$$

$$F_A = \frac{2 \tau_u A}{F.S.} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3 \text{ N}$$

$$P = \frac{10}{9} F_A = 3.72 \times 10^3 \text{ N}$$

Based on double shear in pins at B and D

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.012)^2 = 113.10 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2 \tau_u A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3 \text{ N}$$

Based on compression in links BD

$$\text{For one link } A = (0.020)(0.008) = 160 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2 \sigma_u A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3 \text{ N}$$

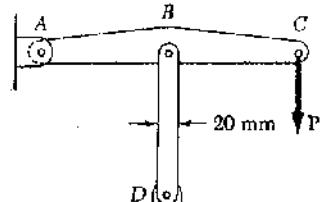
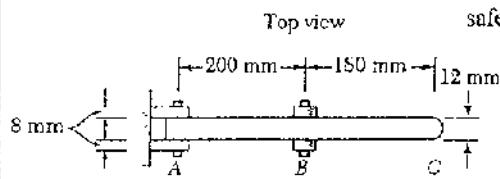
Allowable value of P is smallest : $P = 3.72 \times 10^3 \text{ N}$

3.72 KN

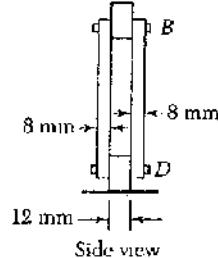
Problem 1.54

1.54 In an alternative design for the structure of Prob. 1.53, a pin of 10-mm-diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load P if an overall factor of safety of 3.0 is desired.

1.53 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load P if an overall factor of safety of 3.0 is desired.

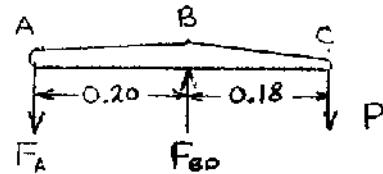


Front view



Side view

Statics : Use ABC as free body.



$$\sum M_A = 0 \quad 0.20 F_A - 0.18 P = 0$$

$$P = \frac{10}{9} F_A$$

$$\sum M_A = 0 \quad 0.20 F_{BD} - 0.38 P = 0$$

$$P = \frac{10}{9} F_{BD}$$

Based on double shear in pin A

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.010)^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$F_A = \frac{2 \tau u A}{F.S.} = \frac{(2)(100 \times 10^6)(78.54 \times 10^{-6})}{3.0} = 5.236 \times 10^3 \text{ N}$$

$$P = \frac{10}{9} F_A = 5.82 \times 10^3 \text{ N}$$

Based on double shear in pins at B and D

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.012)^2 = 113.10 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2 \tau u A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3 \text{ N}$$

$$P = \frac{10}{9} F_{BD} = 8.37 \times 10^3 \text{ N}$$

Based on compression in links BD

$$\text{For one link } A = (0.020)(0.008) = 160 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2 \sigma u A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3 \text{ N}$$

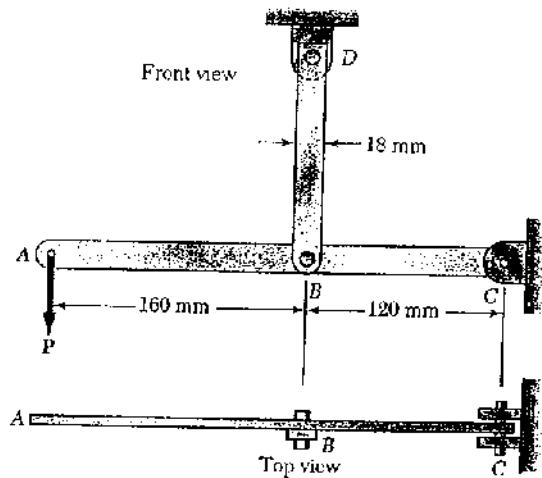
$$P = \frac{10}{9} F_{BD} = 29.7 \times 10^3 \text{ N}$$

Allowable value of P is smallest : $P = 3.97 \times 10^3 \text{ N}$

3.97 kN

Problem 1.55

1.55 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3 is desired, determine the largest load P that can be applied at A. Note that link BD is not reinforced around the pin holes.



Use free body ABC.

$$+\circlearrowleft \sum M_C = 0:$$

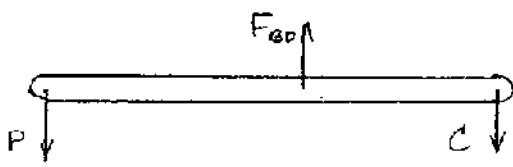
$$0.280 P - 0.120 F_{BD} = 0$$

$$P = \frac{3}{7} F_{BD} \quad (1)$$

$$+\circlearrowleft \sum M_B = 0:$$

$$0.160 P - 0.120 C = 0$$

$$P = \frac{3}{4} C \quad (2)$$



Tension on net section of link BD.

$$\begin{aligned} F_{BD} &= \sigma A_{net} = \frac{\sigma_u}{F.S.} A_{net} \\ &= \frac{(400 \times 10^6)}{3} (6 \times 10^{-3}) (18 - 10) (10^{-3}) \\ &= 6.40 \times 10^3 N \end{aligned}$$

Shear in pins at B and D.

$$F_{BD} = 2 \tau A_{pin} = \frac{\tau_u}{F.S.} \frac{\pi}{4} d^2 = \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (10 \times 10^{-3})^2 = 3.9270 \times 10^3 N$$

Smaller value of F_{BD} is $3.9270 \times 10^3 N$.

$$\text{From (1)} \quad P = \left(\frac{3}{7} \right) (3.9270 \times 10^3) = 1.683 \times 10^3 N$$

Shear in pin at C

$$C = 2 \tau A_{pin} = 2 \frac{\tau_u}{F.S.} \frac{\pi}{4} d^2 = (2) \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (6 \times 10^{-3})^2 = 2.8274 \times 10^3 N$$

$$\text{From (2)} \quad P = \left(\frac{3}{4} \right) (2.8274 \times 10^3) = 2.12 \times 10^3 N$$

Smaller value of P is allowable value.

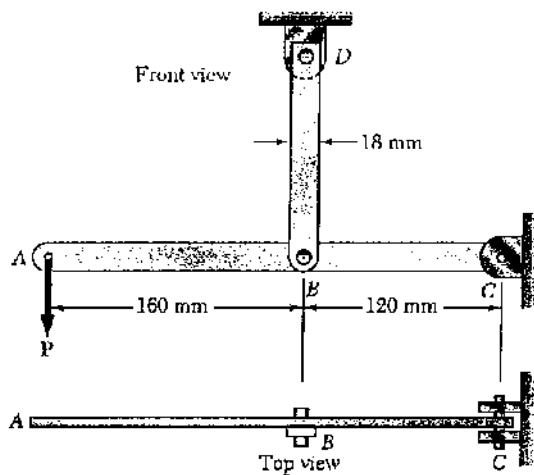
$$P = 1.683 \times 10^3 N$$

$$P = 1.683 kN$$

Problem 1.56

1.56 Solve Prob. 1.55, assuming that the structure has been redesigned to use 12-mm-diameter pins at *B* and *D* and no other change has been made.

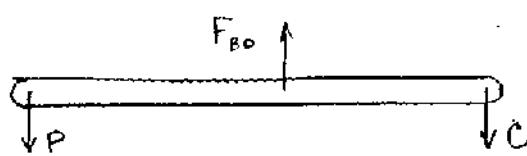
1.55 In the steel structure shown, a 6-mm-diameter pin is used at *C* and 10-mm-diameter pins are used at *B* and *D*. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link *BD*. Knowing that a factor of safety of 3 is desired, determine the largest load *P* that may be applied at *A*. Note that link *BD* is not reinforced around the pin holes.



Use free body ABC.

$$+\odot \sum M_C = 0 : \\ 0.280 P - 0.120 F_{BD} = 0 \\ P = \frac{3}{7} F_{BD} \quad (1)$$

$$+\odot \sum M_B = 0 : \\ 0.160 P - 0.120 C = 0 \\ P = \frac{3}{4} C \quad (2)$$



Tension on net section of link *BD*.

$$F_{BD} = \sigma A_{net} = \frac{\sigma_u}{F.S.} A_{net} \\ = \left(\frac{400 \times 10^6}{3} \right) \left(\sigma_u \times 10^3 \right) (18 - 12) (10^{-3}) \\ = 4.80 \times 10^3 \text{ N}$$

Shear in pins at *B* and *D*.

$$F_{BD} = 2A_{pin} = \frac{\sigma_u}{F.S.} \frac{\pi d^2}{4} = \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (12 \times 10^{-3})^2 = 5.6549 \times 10^3 \text{ N}$$

Smaller value of F_{BD} is $4.80 \times 10^3 \text{ N}$.

$$\text{From (1)} \quad P = \left(\frac{3}{7} \right) (4.80 \times 10^3) = 2.06 \times 10^3 \text{ N}$$

Shear in pin at *C*

$$C = 2A_{pin} = 2 \frac{\sigma_u}{F.S.} \frac{\pi d^2}{4} = 2 \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (6 \times 10^{-3})^2 = 2.8274 \times 10^3 \text{ N}$$

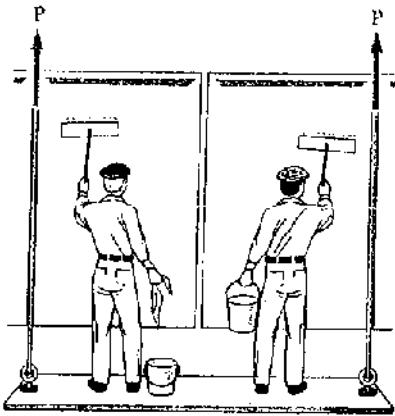
$$\text{From (2)} \quad P = \left(\frac{3}{4} \right) (2.8274 \times 10^3) = 2.12 \times 10^3 \text{ N}$$

Smaller value of *P* is the allowable value.

$$P = 2.06 \times 10^3 \text{ N}$$

$$P = 2.06 \text{ kN} \quad \blacksquare$$

Problem 1.57



*1.57 The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weights 160 lb and each of the window washers is assumed to weight 195 lb with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (a) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_d = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (b) What is the conventional factor of safety for the selected cables?

$$\begin{aligned}\gamma_d P_d + \gamma_L P_L &= \phi P_u \\ P_u &= \frac{\gamma_d P_d + \gamma_L P_L}{\phi} \\ &= \frac{(1.2)(\frac{1}{2} \times 160) + (1.5)(\frac{3}{4} \times 2 \times 195)}{0.85} \\ &= 629 \text{ lb.}\end{aligned}$$

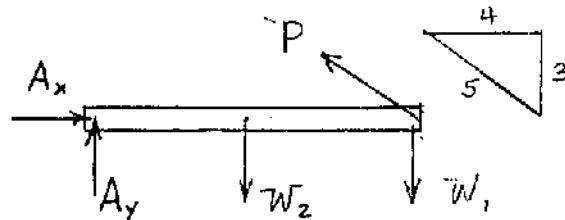
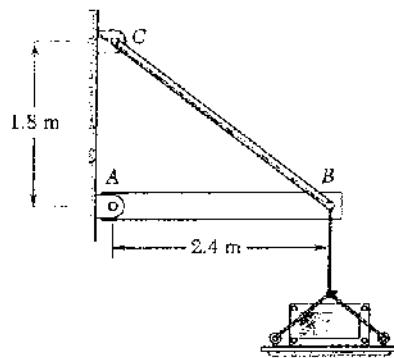
Conventional factor of safety

$$P = P_d + P_L = \frac{1}{2} \times 80 + 0.75 \times 2 \times 195 = 372.5 \text{ lb}$$

$$\text{F.S.} = \frac{P_u}{P} = \frac{629}{372.5} = 1.689$$

Problem 1.58

*1.58 A 40-kg platform is attached to the end *B* of a 50-kg wooden beam *AB*, which is supported as shown by a pin at *A* and by a slender steel rod *BC* with a 12-kN ultimate load. (a) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_d = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (b) What is the corresponding conventional factor of safety for rod *BC*?



$$\sum M_A = 0 \quad (2.4) \frac{3}{5}P - 2.4 W_1 - 1.2 W_2 \therefore P = \frac{5}{3}W_1 + \frac{5}{6}W_2$$

$$\text{For dead loading } W_1 = (40)(9.81) = 392.4 \text{ N}$$

$$W_2 = (50)(9.81) = 490.5 \text{ N}$$

$$P_d = \left(\frac{5}{3}\right)(392.4) + \left(\frac{5}{6}\right)(490.5) = 1.0628 \times 10^3 \text{ N}$$

$$\text{For live loading } W_1 = mg \quad W_2 = 0$$

$$P_L = \frac{5}{3}mg \quad \text{from which } m = \frac{3}{5} \frac{P_L}{g}$$

Design criterion

$$\gamma_d P_d + \gamma_L P_L = \phi P_u$$

$$P_L = \frac{\phi P_u - \gamma_d P_d}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^3)}{1.6}$$

$$= 5.920 \times 10^3 \text{ N}$$

$$\text{Allowable load } m = \frac{3}{5} \frac{5.92 \times 10^3}{9.81} = 362 \text{ kg}$$

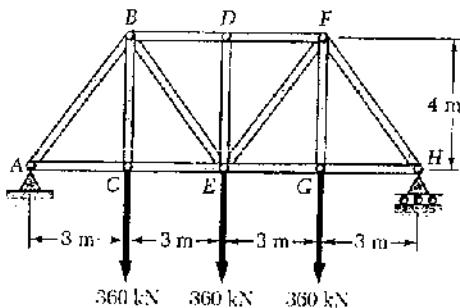
Conventional factor safety

$$P = P_d + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3 \text{ N}$$

$$\text{F.S.} = \frac{P_u}{P} = \frac{12 \times 10^3}{6.983 \times 10^3} = 1.718$$

Problem 1.59

1.59 For the Pratt bridge truss and loading shown, determine the average stress in member BE , knowing that the cross-sectional area of that member is 3750 mm^2 .

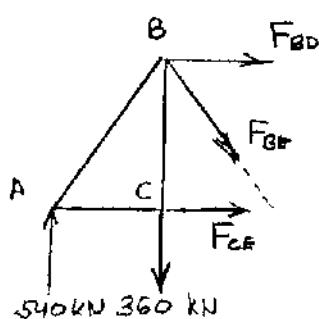


Use entire truss as free body

$$+\circlearrowleft \sum M_H = 0:$$

$$(3)(360) + (6)(360) + (9)(360) - 12 A_y = 0$$

$$A_y = 540 \text{ kN}$$



Use portion of truss to the left of a section cutting members BD , BE , and CE .

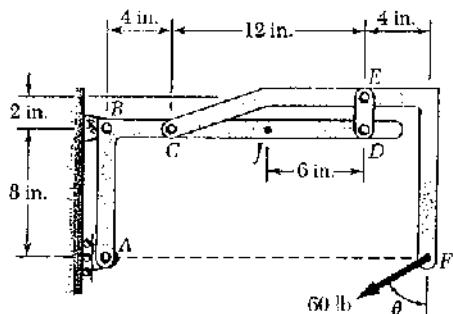
$$+\uparrow \sum F_y = 0$$

$$540 - 360 - \frac{14}{5} F_{BE} = 0 \quad F_{BE} = 22.5 \text{ kN}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{22.5 \times 10^3}{3750 \times 10^{-6}} = 60 \times 10^6 \text{ Pa} \quad 60.0 \text{ MPa}$$

Problem 1.60

1.60 Knowing that link DE is 1 in. wide and $\frac{1}{8}$ in. thick, determine the normal stress in the central portion of that link when (a) $\theta = 0$, (b) $\theta = 90^\circ$.



Use member CDF as a free body

$$+\circlearrowleft \sum M_c = 0$$

$$-12 F_{DE} - (8)(60 \sin \theta) - (16)(60 \cos \theta) = 0$$

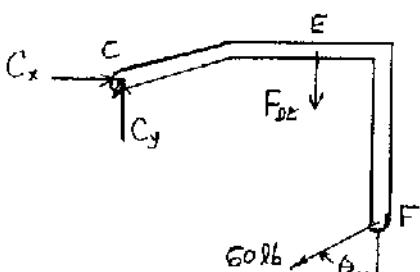
$$F_{DE} = -40 \sin \theta - 80 \cos \theta \quad \text{lb}$$

$$A_{DE} = (1)(\frac{1}{8}) = 0.125 \text{ in}^2$$

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

$$(a) \theta = 0: F_{DE} = -80 \text{ lb.}$$

$$\sigma_{DE} = \frac{-80}{0.125} = -640 \text{ psi}$$

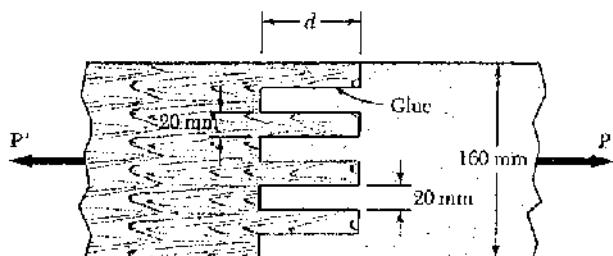


$$(b) \theta = 90^\circ: F_{DE} = -40 \text{ lb.}$$

$$\sigma_{DE} = \frac{-40}{0.125} = -320 \text{ psi}$$

Problem 1.61

1.61 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6 \text{ kN}$.



Seven surfaces carry the total load $P = 7.6 \text{ kN} = 7.6 \times 10^3 \text{ N}$

Let $t = 22 \text{ mm}$

Each glue area is $A = dt$

$$\tau = \frac{P}{7A} \quad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3} \text{ m}^2$$

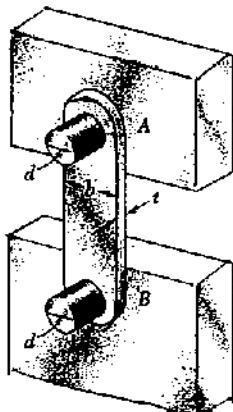
$$= 1.32404 \times 10^3 \text{ mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2$$

$d = 60.2 \text{ mm}$

Problem 1.62

1.62 Link AB , of width $b = 2 \text{ in.}$ and thickness $t = \frac{1}{4} \text{ in.}$, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -20 ksi , and that the average shearing stress in each of the two pins is 12 ksi , determine (a) the diameter d of the pins, (b) the average bearing stress in the link.



Rod AB is in compression.

$$A = bt \text{ where } b = 2 \text{ in. and } t = \frac{1}{4} \text{ in}$$

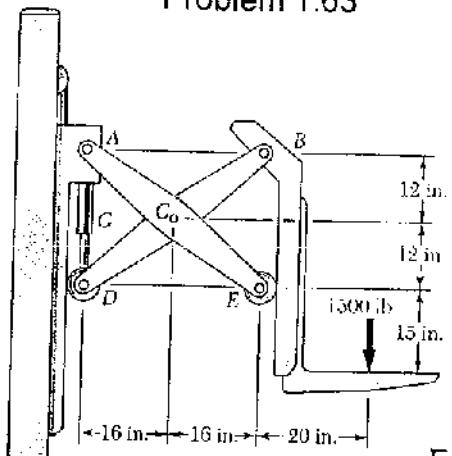
$$P = -\sigma A = -(-20)(2)(\frac{1}{4}) = 10 \text{ kips}$$

$$\text{Pin: } \tau_p = \frac{P}{A_p} \text{ and } A_p = \frac{\pi}{4} d^2$$

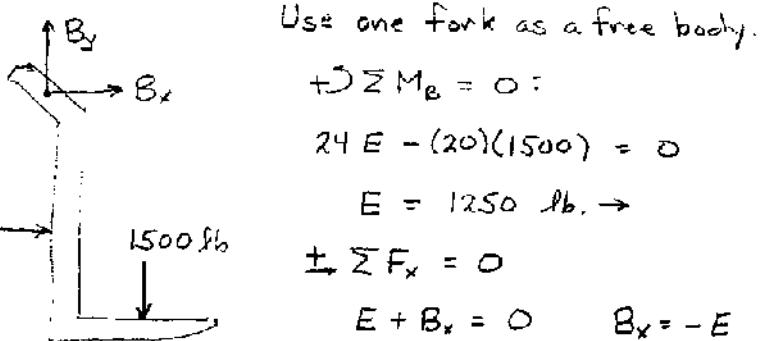
$$(a) d = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{4P}{\pi Z_p}} = \sqrt{\frac{(4)(10)}{\pi(12)}} = 1.030 \text{ in.}$$

$$(b) \sigma_b = \frac{P}{bt} = \frac{10}{(1.030)(0.25)} = 38.8 \text{ ksi}$$

Problem 1.63



1.63 Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 1500 lb. Knowing that the thickness of member BD is $\frac{1}{8}$ in., determine (a) the average shearing stress in the $\frac{1}{2}$ -in.-diameter pin at B , (b) the bearing stress at B in member BD .



Use one fork as a free body.

$$+\uparrow \sum M_E = 0:$$

$$24E - (20)(1500) = 0$$

$$E = 1250 \text{ lb.} \rightarrow$$

$$\pm \sum F_x = 0$$

$$E + B_x = 0 \quad B_x = -E$$

$$B_x = 1250 \text{ lb.} \leftarrow$$

$$+\uparrow \sum F_y = 0: \quad B_y - 1500 = 0 \quad B_y = 1500 \text{ lb}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1250^2 + 1500^2} = 1952.56 \text{ lb.}$$

(a) Shearing stress in pin at B .

$$A_{pin} = \frac{\pi}{4} d_{pin}^2 = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

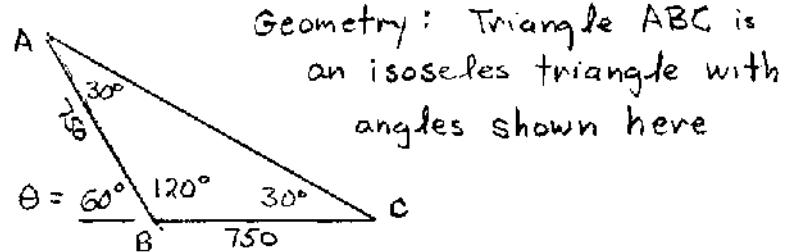
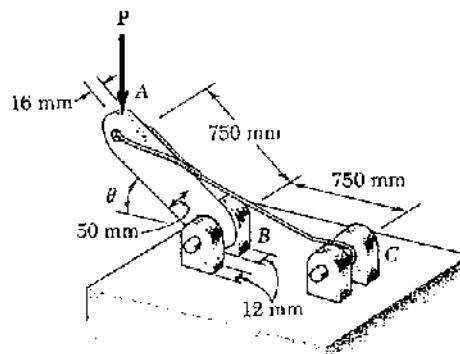
$$\tau = \frac{B}{A_{pin}} = \frac{1952.56}{0.19635} = 9.94 \times 10^3 \text{ psi} \quad 9.94 \text{ ksi} \quad \blacktriangleleft$$

(b) Bearing stress at B .

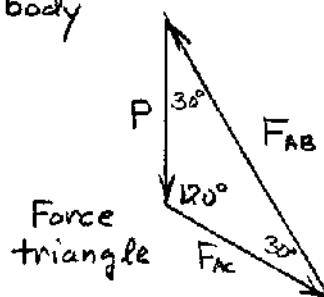
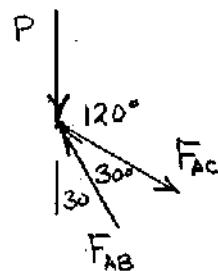
$$\sigma = \frac{B}{dt} = \frac{1952.56}{\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)} = 6.25 \times 10^3 \text{ psi} \quad 6.25 \text{ ksi} \quad \blacktriangleleft$$

Problem 1.64

1.64 Determine the largest load P that can be applied at A when $\theta = 60^\circ$, knowing that the average shearing stress in the 10-mm-diameter pin at B must not exceed 120 MPa and that the average bearing stress in member AB and in the bracket at B must not exceed 90 MPa.



Use joint A as free body



law of sines applied to force triangle

$$\frac{P}{\sin 30^\circ} = \frac{F_{AB}}{\sin 120^\circ} = \frac{F_{AC}}{\sin 30^\circ}$$

$$P = \frac{F_{AB} \sin 30^\circ}{\sin 120^\circ} = 0.57735 F_{AB}$$

$$P = \frac{F_{AC} \sin 30^\circ}{\sin 30^\circ} = F_{AC}$$

IF shearing stress in pin at B is critical

$$A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.010)^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = 2A_p \tau = (2)(78.54 \times 10^{-6})(120 \times 10^6) = 18.850 \times 10^3 \text{ N}$$

IF bearing stress in member AB at bracket at A is critical

$$A_b = t d = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = A_b \sigma_b = (160 \times 10^{-6})(90 \times 10^6) = 14.40 \times 10^3 \text{ N}$$

IF bearing stress in the bracket at B is critical

$$A_b = 2t d = (2)(0.012)(0.010) = 240 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = A_b \sigma_b = (240 \times 10^{-6})(90 \times 10^6) = 21.6 \times 10^3 \text{ N}$$

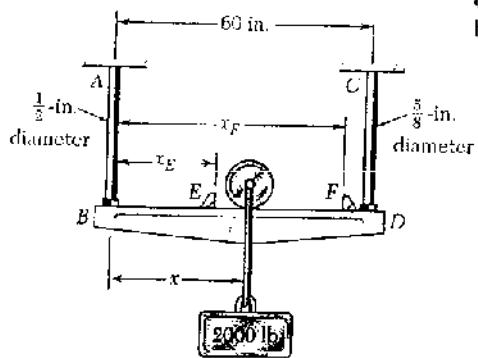
Allowable F_{AB} is the smallest, i.e. $14.40 \times 10^3 \text{ N}$

Then, from Statics $P_{allow} = (0.57735)(14.40 \times 10^3)$

$$= 8.31 \times 10^3 \text{ N}$$

$$8.31 \text{ kN}$$

Problem 1.65

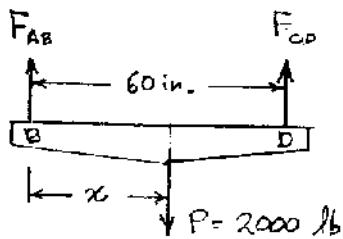


1.65 The 2000-lb load may be moved along the beam BD to any position between stops at E and F . Knowing that $\sigma_{all} = 6$ ksi for the steel used in rods AB and CD , determine where the stops should be placed if the permitted motion of the load is to be as large as possible.

Permitted member forces:

$$AB: (F_{AB})_{max} = \sigma_{all} A_{AB} = (6)(\frac{\pi}{4})(\frac{1}{2})^2 \\ = 1.17810 \text{ kips}$$

$$CD: (F_{CD})_{max} = \sigma_{all} A_{CD} = (6)(\frac{\pi}{4})(\frac{5}{8})^2 \\ = 1.84078 \text{ kips}$$



Use member BEFD as a free body.

$$P = 2000 \text{ lb} = 2.000 \text{ kips}$$

$$+\rightarrow \sum M_D = 0$$

$$-(60)F_{AB} + (60 - x_E)P = 0$$

$$60 - x_E = \frac{60 F_{AB}}{P} = \frac{(60)(1.17810)}{2.000} \\ = 35.343$$

$$x_E = 24.7 \text{ in.}$$

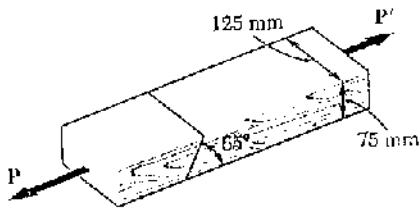
$$+\rightarrow \sum M_B = 0$$

$$60 F_{CD} - x_F P = 0$$

$$x_F = \frac{60 F_{CD}}{P} = \frac{(60)(1.84078)}{2.000}$$

$$x_F = 55.2 \text{ in.}$$

Problem 1.66



1.66 Two wooden members of 75×125 -mm uniform rectangular cross section are joined by the simple glued joint shown. Knowing that $P = 3.6$ kN and that the ultimate strength of the glue is 1.1 MPa in tension and 1.4 MPa in shear, determine the factor of safety.

$$\theta = 90^\circ - 65^\circ = 25^\circ$$

$$P = 3.6 \text{ kN} = 3.6 \times 10^3 \text{ N}$$

$$A_o = (75)(125) = 9.375 \times 10^3 \text{ mm}^2 = 9.375 \times 10^{-3} \text{ m}^2$$

Tension on glue area.

$$\sigma = \frac{P \cos^2 \theta}{A_o} = \frac{(3.6 \times 10^3)(\cos 25^\circ)^2}{9.375 \times 10^{-3}}$$

$$= 315.415 \times 10^3 \text{ Pa}$$

$$\text{F.S.} = \frac{\sigma_u}{\sigma} = \frac{1.1 \times 10^6}{315.415 \times 10^3} = 3.49$$

Shear on glue area.

$$\tau = \frac{P \sin \theta}{2A_o} = \frac{(3.6 \times 10^3)(\sin 25^\circ)}{2(9.375 \times 10^{-3})}$$

$$= 147.081 \times 10^3 \text{ Pa}$$

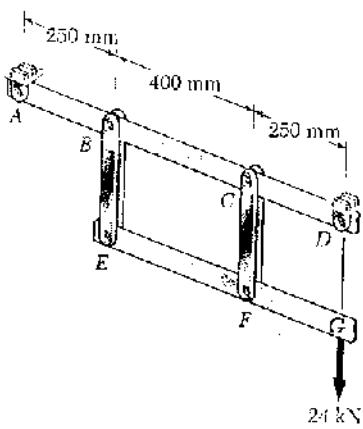
$$\text{F.S.} = \frac{\tau_u}{\tau} = \frac{1.4 \times 10^6}{147.081 \times 10^3} = 9.52$$

The smaller factor of safety governs

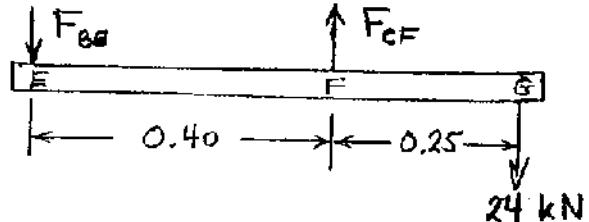
$$\text{F.S.} = 3.49$$

Problem 1.67

1.67 Each of the two vertical links CF connecting the two horizontal members AD and EG has a 10×40 -mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at C and F has a 20 -mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.



Use member EFG as free body.



$$\textcircled{D} \sum M_E = 0$$

$$0.40 F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \times 10^3 \text{ N}$$

Based on tension in links CF

$$A = (b - d)t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6} \text{ m}^2 \text{ (one link)}$$

$$F_u = 25_u A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3 \text{ N}$$

Based on double shear in pins

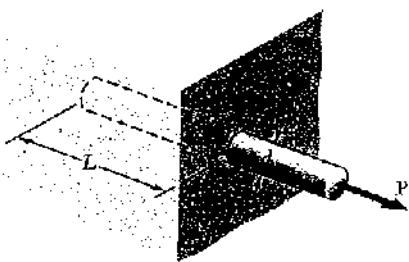
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$F_u = 2 Z_u A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3 \text{ N}$$

Actual F_u is smaller value, i.e. $F_u = 94.248 \times 10^3 \text{ N}$

$$\text{Factor of safety} \quad F.S. = \frac{F_u}{F_{CF}} = \frac{94.248 \times 10^3}{39 \times 10^3} = 2.42$$

Problem 1.68



1.68 A force P is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

$$\text{For shear, } A = \pi d L$$

$$P = \tau_{all} A = \tau_{all} \pi d L$$

$$\text{For tension, } A = \frac{\pi}{4} d^2$$

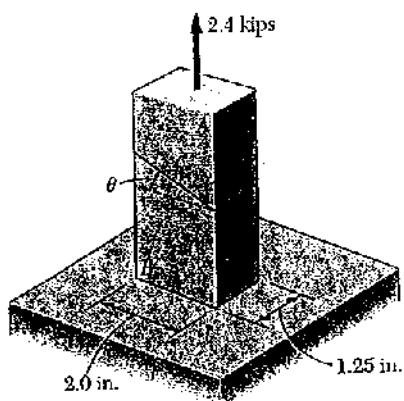
$$P = \sigma_{all} A = \sigma_{all} \left(\frac{\pi}{4} d^2 \right)$$

$$\text{Equating } \tau_{all} \pi d L = \sigma_{all} \frac{\pi}{4} d^2$$

Solving for L

$$L_{min} = \sigma_{all} d / 4 \tau_{all}$$

Problem 1.69



1.69 The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine (a) the value of θ for which the factor of safety of the member is maximum, (b) the corresponding value of the factor of safety. (Hint: Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

$$A_o = (2.0)(1.25) = 2.50 \text{ in}^2$$

At the optimum angle $(F.S.)_\sigma = (F.S.)_\tau$

$$\text{Normal stress: } \sigma = \frac{P}{A_o} \cos^2 \theta \therefore P_{\sigma, \sigma} = \frac{\sigma_u A_o}{\cos^2 \theta}$$

$$(F.S.)_\sigma = \frac{P_{\sigma, \sigma}}{P} = \frac{\sigma_u A_o}{P \cos^2 \theta}$$

$$\text{Shearing stress: } \tau = \frac{P}{A_o} \sin \theta \cos \theta \therefore P_{\tau, \tau} = \frac{\tau_u A_o}{\sin \theta \cos \theta}$$

$$(F.S.)_\tau = \frac{P_{\tau, \tau}}{P} = \frac{\tau_u A_o}{P \sin \theta \cos \theta}$$

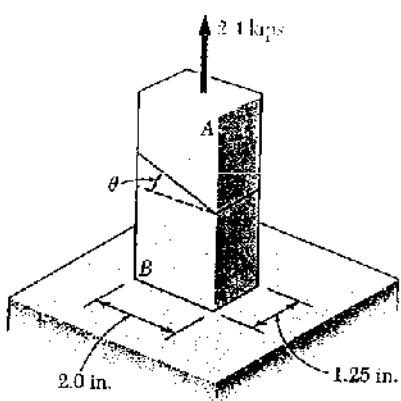
$$\text{Equating: } \frac{\sigma_u A_o}{P \cos^2 \theta} = \frac{\tau_u A_o}{P \sin \theta \cos \theta}$$

$$\text{Solving: } \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{\tau_u}{\sigma_u} = \frac{1.3}{2.5} = 0.520 \quad (\text{a}) \theta_{opt} = 27.5^\circ$$

$$(\text{b}) \quad P_u = \frac{\sigma_u A_o}{\cos^2 \theta} = \frac{(12.5)(2.50)}{\cos^2 27.5^\circ} = 7.94 \text{ kips}$$

$$F.S. = \frac{P_u}{P} = \frac{7.94}{2.4} = 3.31$$

Problem 1.70



1.70 The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

$$A_o = (2.0)(1.25) = 2.50 \text{ in}^2$$

$$P = 2.4 \text{ kips} \quad P_u = (F.S)P = 7.2 \text{ kips}$$

Based on tensile stress

$$\sigma_u = \frac{P_u}{A_o} \cos^2 \theta$$

$$\cos^2 \theta = \frac{\sigma_u A_o}{P_u} = \frac{(2.5)(2.50)}{7.2} = 0.86805$$

$$\cos \theta = 0.93169$$

$$\theta = 21.3^\circ$$

$$\theta \geq 21.3^\circ$$

Based on shearing stress $\tau_u = \frac{P_u}{2A_o} \sin \theta \cos \theta = \frac{P_u}{2A_o} \sin 2\theta$

$$\sin 2\theta = \frac{2A_o \tau_u}{P_u} = \frac{(2)(2.50)(1.3)}{7.2} = 0.90278$$

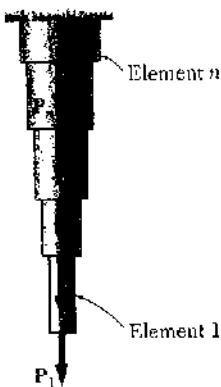
$$2\theta = 64.52^\circ$$

$$\theta = 32.3^\circ$$

$$\theta \leq 32.3^\circ$$

$$\text{Hence } 21.3^\circ \leq \theta \leq 32.3^\circ$$

PROBLEM 1.C1



1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by P_k , with the magnitude P_k of this load being assumed positive if P_k is directed downward as shown and negative otherwise. (a) Write a computer program that can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (b) Use this program to solve Probs. 1.1 and 1.3.

SOLUTION

FORCE IN ELEMENT i :

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^i P_k$$

AVERAGE STRESS IN ELEMENT i :

$$\text{Area} = A_i = \frac{1}{4} \pi d_i^2 \quad \text{Ave stress} = \frac{F_i}{A_i}$$

PROGRAM OUTPUTS

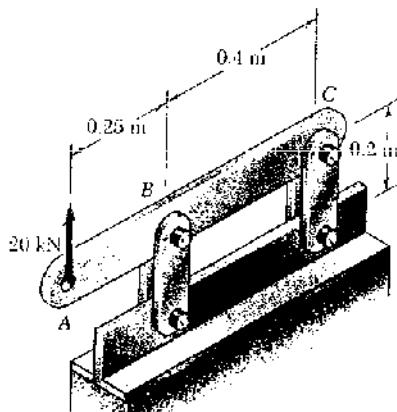
Problem 1.1
Element Stress (MPa)

1	42.441
2	35.651

Problem 1.3
Element Stress (ksi)

1	12.732
2	-2.829

PROBLEM 1.C2

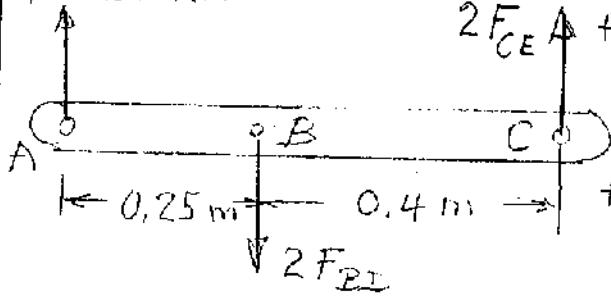


1.C2 A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (a) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (2) the average normal stress in the links connecting pins *C* and *E*, (3) the average shearing stress in pin *B*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *B* in member *ABC*, (6) the average bearing stress at *C* in member *ABC*. (b) Check your program by comparing the values obtained for *d* = 16 mm with the answers given for Probs. 1.9 and 1.27. (c) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively 150 MPa, 90 MPa, and 230 MPa. (d) Solve part c, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

FORCES IN LINKS

$$P = 20 \text{ kN}$$



(1) LINK BD

$$\begin{array}{l} \uparrow F_{BD} \\ \text{Thickness} = t_L \\ \text{Area } A_{BD} = t_L (w_L - d) \\ \text{Stress } \sigma_{BD} = + F_{BD} / A_{BD} \end{array}$$

(3) PIN B

$$\tau_B = F_{BD} / (\pi d^3 / 4)$$

(5) BEARING AT B

$$\text{Thickness of member } ABC = t_{AC}$$

$$\text{Stress at Pin } B = F_{BD} / (dt_{AC})$$

(6) BEARING STRESS AT C

$$\text{Stress at Pin } C = F_{CE} / (t_{AC})$$

F.B. DIAGRAM OF ABC:

$$2F_{CE} \uparrow + \sum M_C = 0: 2F_{BD}(BC) - P(AC) = 0$$

$$F_{BD} = P(AC)/2(BC) \quad (\text{TENSION})$$

$$+ \sum M_B = 0: 2F_{CE}(BC) - P(AB) = 0$$

$$F_{CE} = P(AB)/2(BC) \quad (\text{COMP.})$$

(2) LINK CE

$$\begin{array}{l} \downarrow F_{CE} \\ \text{Thickness} = t_L \\ \text{Area } A_{CE} = t_L w_L \\ \text{Stress } \sigma_{CE} = - F_{CE} / A_{CE} \end{array}$$

(4) PIN C

$$\tau_C = F_{CE} / (\pi d^3 / 4)$$

SHEARING STRESS IN ABC UNDER PIN B

$$\begin{array}{l} 2F_{BD} \downarrow \\ \uparrow F_B = \tau_{AC} t_{AC} (t_{AC}/2) \\ \sum F_y = 0: 2F_B = 2F_{BD} \\ \tau_{AC} = \frac{2F_{BD}}{t_{AC} (t_{AC})} \end{array}$$

(CONTINUED)

PROBLEM 1.C2 CONTINUED

PROGRAM OUTPUTS

INPUT DATA FOR PARTS (a), (b), (c): $P = 20 \text{ kN}$, $AB = 0.25 \text{ m}$, $BC = 0.40 \text{ m}$,
 $AC = 0.65 \text{ m}$, $TL = 8 \text{ mm}$, $WL = 36 \text{ mm}$, $TAC = 10 \text{ mm}$, $WAC = 50 \text{ mm}$

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	325.80	125.00
11.00	81.25	-21.70	170.99	65.77	295.45	113.64
12.00	84.64	-21.70	143.68	55.26	270.83	104.17
13.00	88.32	-21.70	122.43	47.09	250.00	96.15
14.00	92.33	-21.70	105.56	40.60	232.14	89.29
15.00	96.73	-21.70	91.96	35.37	216.67	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	156.25	-21.70	39.11	15.04	141.30	54.35
24.00	169.27	-21.70	35.92	13.82	135.42	52.08
25.00	184.66	-21.70	33.10	12.73	130.00	50.00
26.00	203.13	-21.70	30.61	11.77	125.00	48.08
27.00	225.69	-21.70	28.38	10.92	120.37	46.30
28.00	253.91	-21.70	26.39	10.15	116.07	44.64
29.00	290.18	-21.70	24.60	9.46	112.07	43.10
30.00	338.54	-21.70	22.99	8.84	108.33	41.67

(b)

(c) ANSWER: $16 \text{ mm} \leq d \leq 22 \text{ mm}$ (c)

CHECK: For $d = 22 \text{ mm}$, $\Tau_{AC} = 65 \text{ MPa} < 90 \text{ MPa}$ O.K.

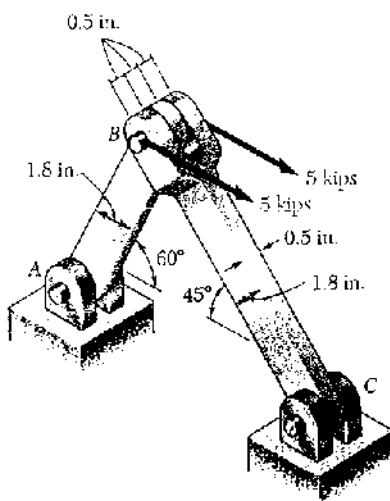
INPUT DATA FOR PART (d): $P = 20 \text{ kN}$, $AB = 0.25 \text{ m}$, $BC = 0.40 \text{ m}$,
 $AC = 0.65 \text{ m}$, $TL = 8 \text{ mm}$, $WL = 36 \text{ mm}$, $TAC = 8 \text{ mm}$, $WAC = 50 \text{ mm}$

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	406.25	156.25
11.00	81.25	-21.70	170.99	65.77	369.32	142.05
12.00	84.64	-21.70	143.68	55.26	338.84	130.21
13.00	88.32	-21.70	122.43	47.09	312.50	120.19
14.00	92.33	-21.70	105.56	40.60	290.18	111.61
15.00	96.73	-21.70	91.96	35.37	270.83	104.17
16.00	101.56	-21.70	80.82	31.08	253.91	97.66
17.00	106.91	-21.70	71.59	27.54	238.87	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	156.25	-21.70	39.11	15.04	176.63	67.93
24.00	169.27	-21.70	35.92	13.82	169.27	65.10
25.00	184.66	-21.70	33.10	12.73	162.50	62.50
26.00	203.13	-21.70	30.61	11.77	156.25	60.10
27.00	225.69	-21.70	28.38	10.92	150.46	57.87
28.00	253.91	-21.70	26.39	10.15	145.09	55.80
29.00	290.18	-21.70	24.60	9.46	140.09	53.88
30.00	338.54	-21.70	22.99	8.84	135.42	52.08

(d)

CHECK: For $d = 22 \text{ mm}$, $\Tau_{AC} = 81.25 \text{ MPa} < 90 \text{ MPa}$ O.K.

PROBLEM 1.C3

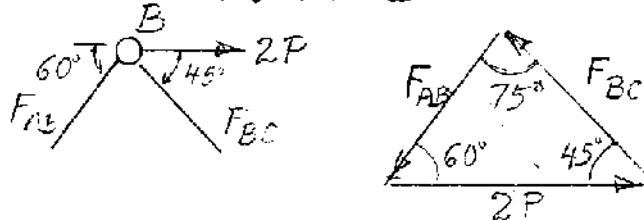


1.C3 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Each of the three pins at *A*, *B*, and *C* has the same diameter *d* and is double shear. (a) Write a computer program to calculate for values of *d* from 0.50 to 1.50 in., using 0.05-in. increments, (1) the maximum value of the average normal stress in member *AB*, (2) the average normal stress in member *BC*, (3) the average shearing stress in pin *A*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *A* in member *AB*, (6) the average bearing stress at *C* in member *BC*, (7) the average bearing stress at *B* in member *BC*. (b) Check your program by comparing the values obtained for *d* = 0.8 in. with the answers given for Probs. 1.10 and 1.28. (c) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used, are respectively, 22 ksi, 13 ksi, and 36 ksi. (d) Solve part *c*, assuming that a new design is being investigated, in which the thickness and width of the two members are changed, respectively, from 0.5 to 0.3 in. and from 1.8 to 2.4 in.

SOLUTION

FORCES IN MEMBERS AB AND BC

FREE BODY: PIN B



FROM FORCE TRIANGLE:

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{2P}{\sin 75^\circ}$$

$$F_{AB} = 2P(\sin 45^\circ / \sin 75^\circ)$$

$$F_{BC} = 2P(\sin 60^\circ / \sin 75^\circ)$$

(1) MAX. AVE. STRESS IN AB

$$A_{AB} = (w - d)t$$

$$\sigma_{AB} = F_{AB}/A_{AB}$$

(3) PIN A

$$\sigma_A = (F_{AB}/2)/(\pi d^2/4)$$

(5) BEARING STRESS AT A

$$\text{Sig Bear } A = F_{AB}/dt$$

(7) BEARING STRESS AT B
IN MEMBER BC

$$\text{Sig Bear } B = F_{BC}/2dt$$

(2) AVE. STRESS IN BC

$$A_{BC} = wt$$

$$\sigma_{BC} = F_{BC}/A_{BC}$$

(4) PIN C

$$\sigma_C = (F_{BC}/2)/(\pi d^2/4)$$

(6) BEARING STRESS AT C

$$\text{Sig Bear } C = F_{BC}/dt$$

(CONTINUED)

PROBLEM 1.C3 CONTINUED

PROGRAM OUTPUTS

INPUT DATA FOR PARTS (a), (b), (c): $P = 5$ kips, $w = 1.8$ in., $t = 0.5$ in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	11.262	-9.962	18.642	22.831	29.282	35.863	17.932
0.550	11.713	-9.962	15.406	18.869	26.620	32.603	16.301
0.600	12.201	-9.962	12.945	18.855	24.402	29.886	14.943
0.650	12.731	-9.962	11.030	18.510	22.525	27.587	13.793
0.700	13.310	-9.962	9.511	11.649	20.916	25.616	12.808
0.750	13.944	-9.962	8.285	10.147	19.521	23.909	11.954
0.800	14.641	-9.962	7.282	8.918	18.301	22.414	11.207
0.850	15.412	-9.962	6.450	7.900	17.225	21.096	10.548
0.900	16.268	-9.962	5.754	7.047	16.268	19.924	9.962
0.950	17.225	-9.962	5.164	6.324	15.412	18.875	9.438
1.000	18.301	-9.962	4.660	5.708	14.641	17.932	8.966
1.050	19.521	-9.962	4.227	5.177	13.944	17.078	8.539
1.100	20.916	-9.962	3.852	4.717	13.310	16.301	8.151
1.150	22.328	-9.962	3.524	4.316	12.731	15.593	7.796
1.200	24.402	-9.962	3.236	3.964	12.201	14.943	7.471
1.250	26.620	-9.962	2.983	3.653	11.713	14.345	7.173
1.300	29.782	-9.962	2.758	3.377	11.262	13.793	6.897
1.350	32.838	-9.962	2.557	3.132	10.845	13.283	6.641
1.400	36.803	-9.962	2.378	2.912	10.458	12.808	6.404
1.450	41.831	-9.962	2.217	2.715	10.097	12.367	6.183
1.500	46.803	-9.962	2.071	2.537	9.761	11.954	5.977

(b)

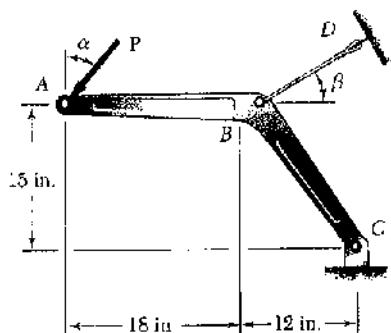
(c) ANSWER: $0.70 \text{ in.} \leq d \leq 1.10 \text{ in.}$ (c)INPUT DATA FOR PART (d): $P = 5$ kips, $w = 2.4$ in., $t = 0.3$ in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	12.843	-12.452	18.642	22.831	48.803	59.772	29.886
0.550	13.190	-12.452	15.406	18.869	41.367	54.338	27.169
0.600	13.556	-12.452	12.945	15.855	40.669	49.810	24.905
0.650	13.944	-12.452	11.030	13.510	37.541	48.978	22.989
0.700	14.354	-12.452	9.511	11.649	34.860	42.694	21.347
0.750	14.789	-12.452	8.285	10.147	32.536	39.848	19.924
0.800	15.251	-12.452	7.282	8.918	30.502	37.357	18.679
0.850	15.743	-12.452	6.450	7.900	28.708	35.160	17.580
0.900	16.268	-12.452	5.754	7.047	27.113	33.206	16.603
0.950	16.829	-12.452	5.164	6.324	25.686	31.459	15.729
1.000	17.430	-12.452	4.660	5.708	24.402	29.886	14.943
1.050	18.075	-12.452	4.227	5.177	23.240	28.463	14.231
1.100	18.771	-12.452	3.852	4.717	22.183	27.169	13.584
1.150	19.521	-12.452	3.524	4.316	21.219	25.988	12.994
1.200	20.335	-12.452	3.236	3.964	20.335	24.905	12.452
1.250	21.219	-12.452	2.983	3.653	19.521	23.909	11.954
1.300	22.183	-12.452	2.758	3.377	18.771	22.989	11.495
1.350	23.240	-12.452	2.557	3.132	18.075	22.138	11.069
1.400	24.402	-12.452	2.378	2.912	17.430	21.347	10.674
1.450	25.686	-12.452	2.217	2.715	16.829	20.611	10.305
1.500	27.113	-12.452	2.071	2.537	16.268	19.924	9.962

(d)

(d) ANSWER: $0.85 \text{ in.} \leq d \leq 1.25 \text{ in.}$

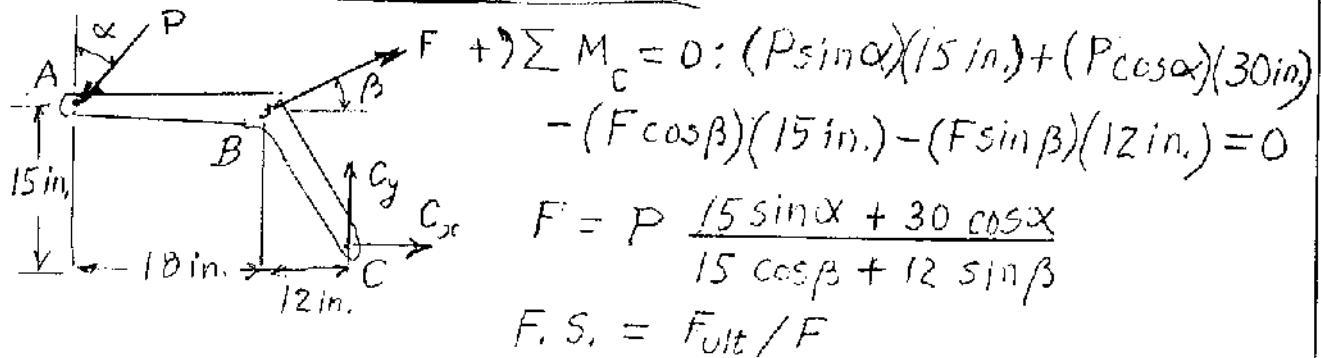
PROBLEM 1.C4



1.C4 A 4-kip force P forming an angle α with the vertical is applied as shown to member ABC , which is supported by a pin and bracket at C and by a cable BD forming an angle β with the horizontal. (a) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45° , using increments in α and β corresponding to 0.1 increments in $\tan \alpha$ and $\tan \beta$. (b) Check that for any given value of α the maximum value of the factor of safety is obtained for $\beta = 38.66^\circ$ and explain why. (c) Determine the smallest possible value of the factor of safety for $\beta = 38.66^\circ$, as well as the corresponding value of α , and explain the result obtained.

SOLUTION

(a) DRAW F.B. DIAGRAM OF ABC:



OUTPUT FOR $P = 4$ kips AND $F_{ult} = 20$ kips

VALUES OF FS

BETA

ALPHA	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.992	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

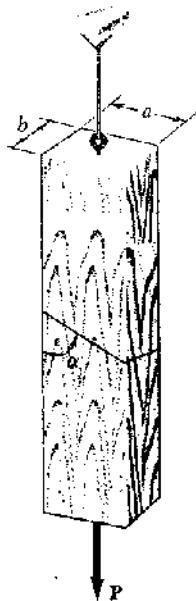
(b) When $\beta = 38.66^\circ$, $\tan \beta = 0.6$ and cable BD is perpendicular to the lever arm BC .

(c) $F.S. = 3.579$ for $\alpha = 26.6^\circ$; P is perpendicular to the lever arm AC .

NOTE:

The value $F.S. = 3.579$ is the smallest of the values of $F.S.$ corresponding to $\beta = 38.66^\circ$ and the largest of those corresponding to $\alpha = 26.6^\circ$. The point $\alpha = 26.6^\circ$, $\beta = 38.66^\circ$ is a "saddle point", or "minimax" of the function $F.S.(\alpha, \beta)$.

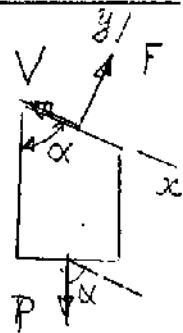
PROBLEM 1.C5



1.C5 A load P is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (a) Denoting by σ_u and τ_u , respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a , b , P , σ_u and τ_u , expressed in either SI or U.S. customary units, and for values of α from 5° to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (b) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that $\sigma_u = 1.26 \text{ MP}$ and $\tau_u = 1.50 \text{ MPa}$ for the glue used in Prob. 1.29, and that $\sigma_u = 150 \text{ psi}$ and $\tau_u = 214 \text{ psi}$ for the glue used in Prob. 1.31. (c) Verify in each of these two cases that the shearing stress is maximum for $\alpha = 45^\circ$.

SOLUTION

(1) and (2)



Draw the F.B. diagram of lower member:

$$\begin{aligned} \sum F_x &= 0: -V + P \cos \alpha = 0 & V &= P \cos \alpha \\ \sum F_y &= 0: F - P \sin \alpha = 0 & F &= P \sin \alpha \end{aligned}$$

$$\text{Area} = ab / \sin \alpha$$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/aB) \sin^2 \alpha$$

Shearing stress: $\tau = \frac{V}{\text{Area}} = (P/aB) \sin \alpha \cos \alpha$

(3) F.S. for tension (normal stresses)

$$FSN = \sigma_u / \sigma$$

(4) F.S. for shear:

$$FSS = \tau_u / \tau$$

(5) OVERALL F.S.:

FS = The smaller of FSN and FSS.

(CONTINUED)

PROBLEM 1.C5 CONTINUED**PROGRAM OUTPUTS****Problem 1.29**

$a = 150 \text{ mm}$
 $b = 75 \text{ mm}$
 $P = 11 \text{ kN}$
 $\sigma_{GU} = 1.26 \text{ MPa}$
 $\tau_{AU} = 1.50 \text{ MPa}$

ALPHA	SIG (MPa)	TAU (MPa)	FSN	FSS	FS
5	.007	.085	169.644	17.669	17.669
10	.029	.167	42.736	8.971	8.971
15	.065	.244	19.237	6.136	6.136
20	.114	.314	11.016	4.773	4.773
25	.175	.375	7.215	4.005	4.005
30	.244	.423	5.155	3.543	3.543
35	.322	.459	3.917	3.265	3.265
40	.404	.481	3.119	3.116	3.116
45	.489	.489	2.577	3.068	2.577
50	.574	.481	2.196	3.116	2.196
55	.656	.459	1.920	3.265	1.920
60	.733	.423	1.718	3.543	1.718
65	.803	.375	1.569	4.005	1.569
70	.863	.314	1.459	4.773	1.459
75	.912	.244	1.381	6.136	1.381
80	.948	.167	1.329	8.971	1.329
85	.970	.085	1.298	17.669	1.298

◀ (b), (c)**Problem 1.31**

$a = 5 \text{ in.}$
 $b = 3 \text{ in.}$
 $P = 1400 \text{ lb}$
 $\sigma_{GU} = 150 \text{ psi}$
 $\tau_{AU} = 214 \text{ psi}$

ALPHA	SIG (psi)	TAU (psi)	FSN	FSS	FS
5	.709	8.104	211.574	26.408	26.408
10	2.814	15.961	53.298	13.408	13.408
15	6.252	23.333	23.992	9.171	9.171
20	10.918	29.997	13.739	7.134	7.134
25	16.670	35.749	8.998	5.986	5.986
30	23.333	40.415	6.429	5.295	5.295
35	30.706	43.852	4.885	4.880	4.880
40	38.563	45.958	3.890	4.656	3.890
45	46.667	46.667	3.214	4.586	3.214
50	54.770	45.958	2.739	4.656	2.739
55	62.628	43.852	2.395	4.880	2.395
60	70.000	40.415	2.143	5.295	2.143
65	76.663	35.749	1.957	5.986	1.957
70	82.415	29.997	1.820	7.134	1.820
75	87.081	23.333	1.723	9.171	1.723
80	90.519	15.961	1.657	13.408	1.657
85	92.624	8.104	1.619	26.408	1.619

◀ (c)**◀ (b)**

Chapter 2

Problem 2.1

2.1 A steel rod that is 5.5 ft long stretches 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29 \times 10^6$ psi, determine (a) the smallest diameter rod that should be used, (b) the corresponding normal stress caused by the load.

$$L = 5.5 \text{ ft} = 66 \text{ in.}, S = 0.04 \text{ in.}, P = 2000 \text{ lb.} \quad S = \frac{PL}{AE}$$

$$A = \frac{PL}{ES} = \frac{(2000)(66)}{(29 \times 10^6)(0.04)} = 0.113793 \text{ in}^2$$

$$(a) A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.113793)}{\pi}} = 0.381 \text{ in.}$$

$$(b) \sigma = \frac{P}{A} = \frac{2000}{0.113793} = 17576 \text{ psi} \quad 17.56 \text{ ksi}$$

Problem 2.2

2.2 A 60-m-long steel wire is subjected to 6 kN tensile force. Knowing that $E = 200$ GPa and that the length of the rod increases by 48 mm, determine (a) the smallest diameter that may be selected for the wire, (b) the corresponding normal stress.

$$P = 6 \times 10^3 \text{ N}, S = 48 \times 10^{-3} \text{ m}, E = 200 \times 10^9 \text{ Pa} \quad S = \frac{PL}{AE}$$

$$A = \frac{PL}{ES} = \frac{(6 \times 10^3)(48 \times 10^{-3})}{(200 \times 10^9)(48 \times 10^{-3})} = 37.5 \times 10^{-6} \text{ m}^2$$

$$(a) A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(37.5 \times 10^{-6})}{\pi}} = 6.91 \times 10^{-3} \text{ m} \quad 6.91 \text{ mm}$$

$$(b) \sigma = \frac{P}{A} = \frac{6 \times 10^3}{37.5 \times 10^{-6}} = 160 \times 10^6 \text{ Pa} \quad 160.0 \text{ MPa}$$

Problem 2.3

2.3 A control rod made of yellow brass must not stretch more than $\frac{1}{8}$ in. when the tension in the wire is 800 lb. Knowing that $E = 15 \times 10^6$ psi and that the maximum allowable normal stress is 32 ksi, determine (a) the smallest diameter that can be selected for the rod, (b) the corresponding maximum length of the rod.

$$S = \frac{1}{8} \text{ in.} = 0.125 \text{ in.}$$

$$\sigma = 32 \text{ ksi} = 32 \times 10^3 \text{ psi}$$

$$\sigma = \frac{P}{A} \quad \text{or} \quad A = \frac{P}{\sigma} = \frac{800}{32 \times 10^3} = 25 \times 10^{-3} \text{ in}^2$$

$$(a) A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(25 \times 10^{-3})}{\pi}} = 0.1784 \text{ in.}$$

$$(b) S = \frac{PL}{AE} \quad L = \frac{AES}{P} = \frac{(25 \times 10^{-3})(15 \times 10^6)(0.125)}{800} = 58.6 \text{ in.}$$

Problem 2.4

2.4 Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod with $E = 73$ GPa and an ultimate strength of 140 MPa. Knowing that the distance between the gage marks is 250.28 mm after a load is applied, determine (a) the stress in the rod, (b) the factor of safety.

$$E = 73 \times 10^9 \text{ Pa}$$

$$S = 250.28 - 250.00 = 0.28 \text{ mm} = 0.28 \times 10^{-3} \text{ m}$$

$$L = 250 \text{ mm} = 250 \times 10^{-3} \text{ m}$$

$$(a) \sigma = E\varepsilon = \frac{ES}{L} = \frac{(73 \times 10^9)(0.28 \times 10^{-3})}{250 \times 10^{-3}} = 81.76 \times 10^6 \text{ Pa} \quad 81.8 \text{ MPa}$$

$$(b) F.S. = \frac{\sigma_u}{\sigma} = \frac{140 \times 10^6}{81.76 \times 10^6} = 1.712$$

Problem 2.5

2.5 A nylon thread is subjected to a 2-lb tension force. Knowing that $E = 0.7 \times 10^6$ psi, and that the length of the thread increases by 1.1 %, determine (a) the diameter of the thread, (b) the stress in the thread.

$$\epsilon = \frac{\delta}{L} = \frac{0.011 L}{L} = 0.011$$

$$\sigma = \frac{PL}{AE} \quad A = \frac{PL}{ES} = \frac{P}{E\epsilon} = \frac{2}{(0.7 \times 10^6)(0.011)} = 259.74 \times 10^{-6} \text{ in}^2$$

$$(a) \quad A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(259.74 \times 10^{-6})}{\pi}} = 0.01819 \text{ in.}$$

$$(b) \quad \sigma = \frac{P}{A} = \frac{2}{259.74 \times 10^{-6}} = 7.70 \times 10^3 \text{ psi} \quad 7.70 \text{ ksi}$$

Problem 2.6

2.6 A cast-iron tube is used to support a compressive load. Knowing that $E = 69$ GPa and that the maximum allowable change in length is 0.025 %, determine (a) the maximum normal stress in the tube, (b) the minimum wall thickness for a load of 7.2 kN if the outside diameter of the tube is 50 mm.

$$\epsilon = \frac{\delta}{L} = \frac{0.00025 L}{L} = 0.00025$$

$$(a) \quad \sigma = E\epsilon = (69 \times 10^9)(0.00025) = 17.25 \times 10^6 \text{ Pa} \quad 17.25 \text{ MPa}$$

$$(b) \quad \sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{7.2 \times 10^3}{17.25 \times 10^6} = 417.39 \times 10^{-6} \text{ m}^2 = 417.39 \text{ mm}^2$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) \quad d_i^2 = d_o^2 - \frac{4A}{\pi} = 50^2 - \frac{(4)(417.39)}{\pi} = 1968.56 \text{ mm}^2$$

$$d_i = 44.368 \text{ mm} \quad t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(50 - 44.368) = 2.82 \text{ mm}$$

Problem 2.7

2.7 A 28-ft length of 0.25-in.-diameter steel wire is to be used in a hanger. It is noted that the wire stretches 0.45 in. when a tensile force P is applied. Knowing that $E = 29 \times 10^6$ psi, determine (a) the magnitude of the force P , (b) the corresponding normal stress in the wire.

$$L = 28 \text{ ft} = 336 \text{ in.} \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.25)^2 = 49.087 \times 10^{-3} \text{ in}^2$$

$$(a) \quad \sigma = \frac{PL}{AE} \quad P = \frac{AES}{L} = \frac{(49.087 \times 10^{-3})(29 \times 10^6)(0.45)}{336} = 1.907 \times 10^3 \text{ lb} \quad 1.907 \text{ kips}$$

$$(b) \quad \sigma = E\epsilon = \frac{ES}{L} = \frac{(29 \times 10^6)(0.45)}{336} = 38.8 \times 10^3 \text{ psi} \quad 38.8 \text{ ksi}$$

Problem 2.8

$$\sigma = 120 \times 10^6 \text{ Pa}$$

$$E = 70 \times 10^9 \text{ Pa}$$

$$\delta = 1.4 \times 10^{-3} \text{ m}$$

$$(a) \delta = \frac{PL}{AE} = \frac{\sigma L}{E} \quad L = \frac{ES}{\sigma} = \frac{(70 \times 10^9)(1.4 \times 10^{-3})}{120 \times 10^6} = 0.817 \text{ m}$$

$$L = 817 \text{ mm}$$

$$(b) \delta = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{28 \times 10^3}{120 \times 10^6} = 233.333 \times 10^{-6} \text{ m}^2 = 233.333 \text{ mm}^2$$

$$A = a^2 \quad a = \sqrt{A} = \sqrt{233.333} = 15.28 \text{ mm}$$

Problem 2.9

2.9 A 9-kN tensile load will be applied to a 50-m length of steel wire with $E = 200 \text{ GPa}$. Determine the smallest diameter wire which can be used, knowing that the normal stress must not exceed 150 MPa and that the increase in the length of the wire should be at most 25 mm.

Considering allowable stress $\sigma = 150 \times 10^6 \text{ Pa}$

$$\sigma = \frac{P}{A} \therefore A = \frac{P}{\sigma} = \frac{9 \times 10^3}{150 \times 10^6} = 60 \times 10^{-6} \text{ m}^2$$

Considering allowable elongation $\delta = 25 \times 10^{-3} \text{ m}$

$$\delta = \frac{PL}{AE} \therefore A = \frac{PL}{E\delta} = \frac{(9 \times 10^3)(50)}{(200 \times 10^9)(25 \times 10^{-3})} = 90 \times 10^{-6} \text{ m}^2$$

Larger area governs $A = 90 \times 10^{-6} \text{ m}^2$

$$A = \frac{\pi d^2}{4} \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(90 \times 10^{-6})}{\pi}} = 10.70 \times 10^{-3} \text{ m}$$

$$= 10.70 \text{ mm}$$

Problem 2.10

2.10 A block of 250-mm length and $50 \times 40 \text{ mm}$ cross section is to support a centric compressive load P . The material to be used is a bronze for which $E = 95 \text{ GPa}$. Determine the largest load which can be applied, knowing that the normal stress must not exceed 80 MPa and that the decrease in length of the block should be at most 0.12% of its original length.

$$A = (50)(40) = 2000 \text{ mm}^2 = 2 \times 10^{-3} \text{ m}^2$$

$$\sigma_u = 80 \text{ MPa} = 80 \times 10^6 \text{ Pa}, \quad E = 95 \times 10^9 \text{ Pa}$$

Considering allowable stress

$$\sigma = \frac{P}{A} \quad P = A\sigma = (2 \times 10^{-3})(80 \times 10^6) = 160 \times 10^3 \text{ N}$$

Considering allowable deformation

$$\delta = \frac{PL}{EA} \quad P = EA\left(\frac{\delta}{L}\right) = (95 \times 10^9)(2 \times 10^{-3})(0.0012) = 228 \times 10^3 \text{ N}$$

Smaller value of P governs

$$P = 160 \times 10^3 \text{ N} = 160.0 \text{ kN}$$

Problem 2.11

$$L = 1.5 \text{ m}$$

$$S = 1 \times 10^{-3} \text{ m}, \quad \sigma = 40 \times 10^6 \text{ Pa}, \quad E = 70 \times 10^9 \text{ Pa}, \quad P = 3 \times 10^3 \text{ N}$$

$$\text{Stress: } \sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{3 \times 10^3}{40 \times 10^6} = 75 \times 10^{-6} \text{ m}^2 = 75 \text{ mm}^2$$

$$\text{Deformation: } S = \frac{PL}{AE}$$

$$A = \frac{PL}{ES} = \frac{(3 \times 10^3)(1.5)}{(70 \times 10^9)(1 \times 10^{-3})} = 64.29 \times 10^{-6} \text{ m}^2 = 64.29 \text{ mm}^2$$

Larger value of A governs.

$$A = 75 \text{ mm}^2$$

$$A = \frac{\pi d^2}{4} \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(75)}{\pi}} = 9.77 \text{ mm}$$

Problem 2.12

$$\sigma = 6 \text{ ksi} = 6 \times 10^3 \text{ psi}$$

2.12 A nylon thread is to be subjected to a 2.5-lb tensile load. Knowing that $E = 0.5 \times 10^6 \text{ psi}$, that the maximum allowable normal stress is 6 ksi, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

$$\text{Stress: } \sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{2.5}{6 \times 10^3} = 416.667 \times 10^{-6} \text{ in}^2$$

$$\text{Deformation: } S = \frac{PL}{AE}$$

$$A = \frac{P(L)}{E(S)} = \frac{2.5}{0.5 \times 10^6}(100) = 500 \times 10^{-6} \text{ in}^2$$

Larger value of A governs.

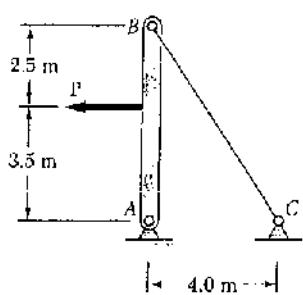
$$A = 500 \times 10^{-6} \text{ in}^2$$

$$A = \frac{\pi d^2}{4} \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(500 \times 10^{-6})}{\pi}} = 25.2 \times 10^{-3} \text{ in.}$$

$$0.0252 \text{ in.}$$

Problem 2.13

2.13 The 4-mm-diameter cable BC is made of a steel with $E = 200 \text{ GPa}$. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load P that can be applied as shown.

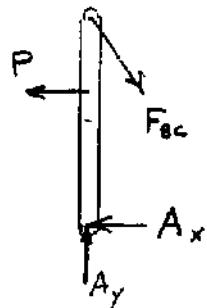


$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar AB as a free body

$$\sum M_A = 0 \quad 3.5P - (6)(\frac{4}{7.2111} F_{BC}) = 0$$

$$P = 0.9509 F_{BC}$$



Considering allowable stress $\sigma = 190 \times 10^6 \text{ Pa}$

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \therefore F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

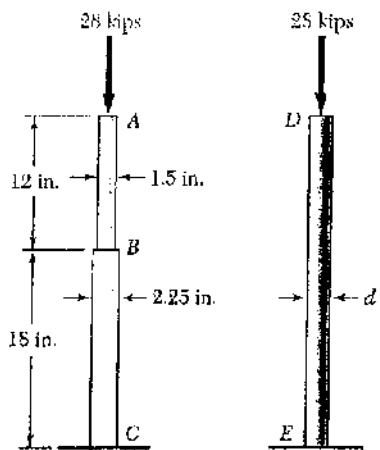
Considering allowable elongation $\delta = 6 \times 10^{-3} \text{ m}$

$$\delta = \frac{F_{BC} L_{BC}}{AE} \therefore F_{BC} = \frac{AES}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs $F_{BC} = 2.091 \times 10^3 \text{ N}$

$$P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N} = 1.988 \text{ kN}$$

Problem 2.14



2.14 The aluminum rod ABC ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions AB and BC , is to be replaced with a cylindrical steel rod DE ($E = 29 \times 10^6$ psi) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

Deformation of aluminum rod

$$S_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E} \left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right)$$

$$= \frac{28 \times 10^3}{10.1 \times 10^6} \left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2} \right)^2 = 0.031376 \text{ in}$$

Steel rod $S = 0.031376 \text{ in}$

$$S = \frac{PL}{EA} \therefore A = \frac{PL}{ES} = \frac{(28 \times 10^3)(30)}{(29 \times 10^6)(0.031376)} = 0.92317 \text{ in}^2$$

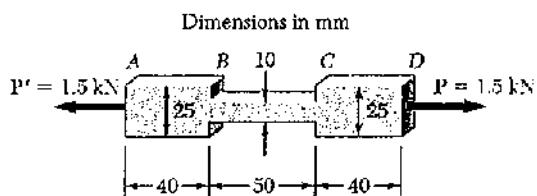
$$\sigma = \frac{P}{A} \therefore A = \frac{P}{\sigma} = \frac{28 \times 10^3}{24 \times 10^3} = 1.1667 \text{ in}^2$$

Required area is the larger value $A = 1.1667 \text{ in}^2$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(1.1667)}{\pi}} = 1.219 \text{ in.}$$

Problem 2.15

2.15 The specimen shown has been cut from a 5-mm-thick sheet of vinyl ($E = 3.10 \text{ GPa}$) and is subjected to a 1.5-kN tensile load. Determine (a) the total deformation of the specimen, (b) the deformation of its central portion BC .



$$S_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(1.5 \times 10^3)(40 \times 10^{-3})}{(3.10 \times 10^9)(25 \times 10^{-3})(5 \times 10^{-3})}$$

$$= 154.84 \times 10^{-6} \text{ m}$$

$$S_{BC} = \frac{PL_{BC}}{EA_{BC}} = \frac{(1.5 \times 10^3)(50 \times 10^{-3})}{(3.10 \times 10^9)(10 \times 10^{-3})(5 \times 10^{-3})}$$

$$= 483.87 \times 10^{-6} \text{ m}$$

$$S_{CD} = \frac{PL_{CD}}{EA_{CD}} = S_{AB} = 154.84 \times 10^{-6} \text{ m}$$

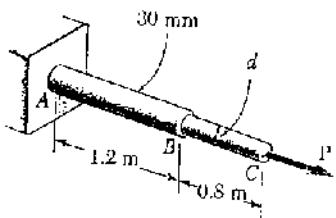
(a) Total deformation: $S = S_{AB} + S_{BC} + S_{CD} = 793.52 \times 10^{-6} \text{ m}$

0.794 mm

(b) Over BC $S_{BC} = 483.87 \times 10^{-6}$

0.484 mm

Problem 2.16



2.16 A single axial load of magnitude $P = 58 \text{ kN}$ is applied at end C of the brass rod ABC . Knowing that $E = 105 \text{ GPa}$, determine the diameter d of portion BC for which the deflection of point C will be 3 mm.

$$S_c = \sum \frac{P_i L_i}{A_i E} = \frac{P}{E} \left\{ \frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right\}$$

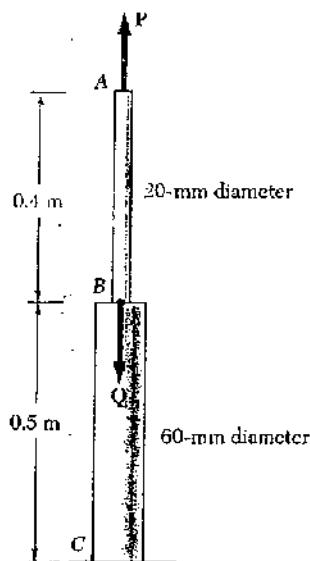
$$\frac{L_{BC}}{A_{BC}} = \frac{ES_c}{P} - \frac{L_{AB}}{A_{AB}} = \frac{(105 \times 10^9)(3 \times 10^{-3})}{58 \times 10^3} - \frac{1.2}{\frac{\pi}{4}(0.030)^2} = 3.7334 \times 10^3 \text{ m}^{-1}$$

$$A_{BC} = \frac{L_{BC}}{3.7334 \times 10^3} = \frac{0.8}{3.7334 \times 10^3} = 214.28 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 \therefore d_{BC} = \sqrt{\frac{4A_{BC}}{\pi}} = \sqrt{\frac{(4)(214.28 \times 10^{-6})}{\pi}} = 16.52 \times 10^{-3} \text{ m} \\ = 16.52 \text{ mm}$$

Problem 2.17

2.17 Both portions of the rod ABC are made of an aluminum for which $E = 70 \text{ GPa}$. Knowing that the magnitude of P is 4 kN, determine (a) the value of Q so that the deflection at A is zero, (b) the corresponding deflection of B .



$$(a) A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

Force in member AB is P tension

$$\text{Elongation } S_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{(4 \times 10^3)(0.4)}{(70 \times 10^9)(314.16 \times 10^{-6})} \\ = 72.756 \times 10^{-6} \text{ m}$$

Force in member BC is $Q - P$ compression

$$\text{Shortening } S_{BC} = \frac{(Q-P)L_{BC}}{E A_{BC}} = \frac{(Q-P)(0.5)}{(70 \times 10^9)(2.8274 \times 10^{-3})} \\ = 2.5263 \times 10^{-9} (Q-P)$$

$$\text{For zero deflection at } A \quad S_{BC} = S_{AB}$$

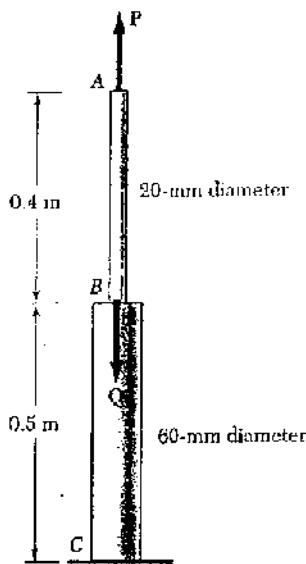
$$2.5263 \times 10^{-9} (Q-P) = 72.756 \times 10^{-6} \therefore Q - P = 28.8 \times 10^3 \text{ N}$$

$$Q = 28.8 \times 10^3 + 4 \times 10^3 = 32.8 \times 10^3 \text{ N} = 32.8 \text{ kN}$$

$$(b) S_{AB} = S_{BC} = S_B = 72.756 \times 10^{-6} \text{ m} = 0.0728 \text{ mm} \downarrow$$

Problem 2.18

2.18 The rod ABC is made of an aluminum for which $E = 70 \text{ GPa}$. Knowing that $P = 6 \text{ kN}$ and $Q = 42 \text{ kN}$, determine the deflection of (a) point A, (b) point B.



$$(a) A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (0.060)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

$$P_{AB} = P = 6 \times 10^3 \text{ N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3 \text{ N}$$

$$L_{AB} = 0.4 \text{ m} \quad L_{BC} = 0.5 \text{ m}$$

$$S_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)} = 109.135 \times 10^{-6} \text{ m}$$

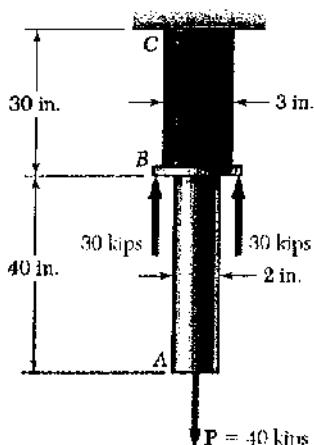
$$S_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)} = -90.947 \times 10^{-6} \text{ m}$$

$$S_A = S_{AB} + S_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6} \text{ m} = 18.19 \times 10^{-6} \text{ m} = 0.01819 \text{ mm} \quad \blacktriangleleft$$

$$(b) S_B = S_{BC} = -90.9 \times 10^{-6} \text{ m} = -0.0909 \text{ mm} \text{ or } 0.0919 \text{ mm} \quad \blacktriangleleft$$

Problem 2.19

2.19 Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel ($E = 29 \times 10^6 \text{ psi}$), and rod BC of brass ($E = 15 \times 10^6 \text{ psi}$). Determine (a) the total deformation of the composite rod ABC, (b) the deflection of point B.



Portion AB: $P_{AB} = 40 \times 10^3 \text{ lb}$, $L_{AB} = 40 \text{ in.}$, $d = 2 \text{ in.}$

$$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (2)^2 = 3.1416 \text{ in.}^2, E_{AB} = 29 \times 10^6 \text{ psi}$$

$$S_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(40 \times 10^3)(40)}{(29 \times 10^6)(3.1416)} = 17.5619 \times 10^{-3} \text{ in.}$$

Portion BC: $P_{BC} = -20 \times 10^3 \text{ lb}$, $L_{BC} = 30 \text{ in.}$, $d = 3 \text{ in.}$

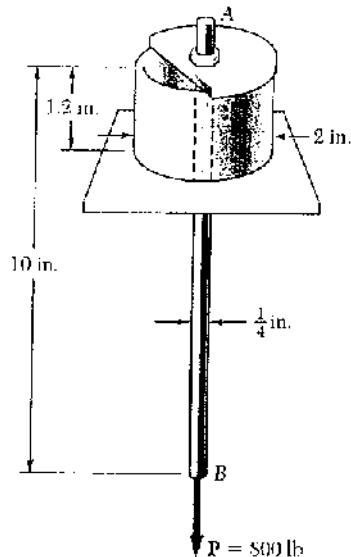
$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (3)^2 = 7.0686 \text{ in.}^2, E_{BC} = 15 \times 10^6 \text{ psi}$$

$$S_{BC} = \frac{P_{BC} L_{BC}}{E_{BC} A_{BC}} = \frac{(-20 \times 10^3)(30)}{(15 \times 10^6)(7.0686)} = -5.6588 \times 10^{-3} \text{ in.}$$

$$(a) S = S_{AB} + S_{BC} = 17.5619 \times 10^{-3} - 5.6588 \times 10^{-3} \quad S = 11.90 \times 10^{-3} \text{ in.} \quad \blacktriangleleft$$

$$(b) S_B = -S_{BC} \quad S_B = 5.66 \times 10^{-3} \text{ in.} \quad \blacktriangleleft$$

Problem 2.20



2.20 A $\frac{1}{8}$ -in.-thick hollow polystyrene cylinder ($E = 0.45 \times 10^6$ psi) and a rigid circular plate (only part of which is shown) are used to support a 10-in.-long steel rod AB ($E = 29 \times 10^6$ psi) of $\frac{1}{4}$ -in. diameter. If an 800-lb load P is applied at B , determine (a) the elongation of rod AB , (b) the deflection of point B , (c) the average normal stress in rod AB .

$$\text{Rod } AB: P_{AB} = 800 \text{ lb}, L_{AB} = 10 \text{ in. } d = \frac{1}{4} \text{ in}$$

$$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{1}{4}\right)^2 = 49.087 \times 10^{-3} \text{ in}^2$$

$$(a) S_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(800)(10)}{(29 \times 10^6)(49.087 \times 10^{-3})} = 5.62 \times 10^{-3} \text{ in.}$$

$$\text{Hollow cylinder: } d_o = 2 \text{ in. } d_i = 2 - (2)\left(\frac{1}{8}\right) = 1.75 \text{ in.}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(2^2 - 1.75^2) = 736.31 \times 10^{-3} \text{ in}^2$$

$$L = 1.2 \text{ in. }, P = 800 \text{ lb.}$$

$$S_{cyl} = \frac{PL}{EA} = \frac{(800)(1.2)}{(0.45 \times 10^6)(736.31 \times 10^{-3})} = 2.90 \times 10^{-3} \text{ in.}$$

(b) Deflection of point B

$$S_B = S_{AB} + S_{cyl} = 8.52 \times 10^{-3} \text{ in.}$$

(c) Stress in rod AB

$$\sigma = \frac{P}{A_{AB}} = \frac{800}{49.087 \times 10^{-3}} = 16.30 \times 10^3 \text{ psi} = 16.30 \text{ ksi.}$$

Problem 2.21

2.21 For the steel truss ($E = 29 \times 10^6$ psi) and loading shown, determine the deformations of members AB and AD , knowing that their cross-sectional areas are 4.0 in^2 and 2.8 in^2 , respectively.

Statics: Reactions at A and C. 25 kips ↑

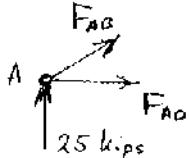
Member BD is a zero force member.

$$L_{AB} = \sqrt{13^2 + 8^2} = 15.2643 \text{ ft} = 183.172 \text{ in.}$$

$$L_{AD} = 13 \text{ ft} = 156 \text{ in.}$$

Use joint A as free body

$$+\uparrow \sum F_y = 0 : 25 + \frac{8}{15.2643} F_{AB} = 0 \quad F_{AB} = -47.701 \text{ kips}$$

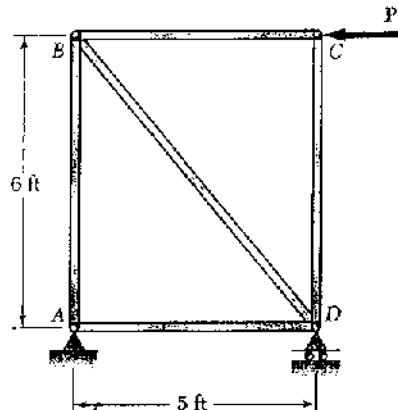


$$\pm \sum F_x = 0 : F_{AD} + \frac{13}{15.2643} F_{AB} = 0 \quad F_{AD} = 40.625 \text{ kips}$$

$$S_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-47.701 \times 10^3)(183.172)}{(29 \times 10^6)(4.0)} = -0.0753 \text{ in.}$$

$$S_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(40.625 \times 10^3)(156)}{(29 \times 10^6)(2.8)} = 0.0780 \text{ in.}$$

Problem 2.22



2.22 The steel frame ($E = 29 \times 10^6$ psi) shown has a diagonal brace BD with an area of 3.2 in^2 . Determine the largest allowable load P if the change in length of member BD is not to exceed $\frac{1}{16}$ in.

$$S_{BD} = \frac{1}{16} \text{ in.} = 0.0625 \text{ in.}$$

$$L_{BD} = \sqrt{5^2 + 6^2} = 7.8102 \text{ ft} = 93.723 \text{ in.}$$

$$S_{BD} = \frac{F_{BD} L_{BD}}{E A_{BD}} \quad F_{BD} = \frac{E A_{BD} S_{BD}}{L_{BD}}$$

$$F_{BD} = \frac{(29 \times 10^6)(3.2)(0.0625)}{93.723} = 61.884 \times 10^3 \text{ lb.}$$

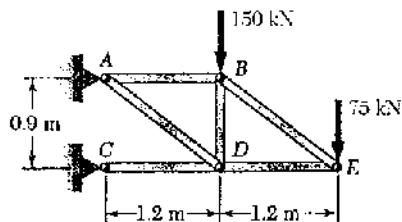
Use joint B as a free body.

$$\pm \sum F_x = 0 : -P + \frac{5}{7.8102} F_{BC} = 0$$

$$P = \frac{(5)(61.884 \times 10^3)}{7.8102} = 39.6 \times 10^3 \text{ lb} = 39.6 \text{ kips}$$

Problem 2.23

2.23 Members AB and BE of the truss shown consist of 25-mm-diameter steel rods ($E = 200 \text{ GPa}$). For the loading shown, determine the elongation of (a) rod AB , (b) rod BE .

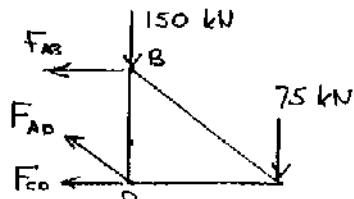


$$L_{BE} = \sqrt{1.2^2 + 0.9^2} = 1.5 \text{ m}$$

Use joint E as a free body.

$$+\uparrow \sum F_y = 0 : \frac{0.9}{1.5} F_{BE} - 75 = 0 \quad F_{BE} = 125 \text{ kN}$$

Use triangle BDE as a free body: $\rightarrow \sum M_B = 0$



$$0.9 F_{AB} - (1.2)(75) = 0$$

$$F_{AB} = 100 \text{ kN}$$

$$A_{AB} = A_{BE} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2 \\ = 490.87 \times 10^{-6} \text{ m}^2$$

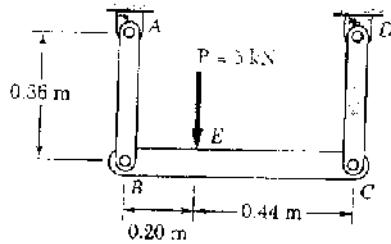
$$\bar{E} = 200 \times 10^9 \text{ Pa}$$

$$(a) S_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(100 \times 10^3)(1.2)}{(200 \times 10^9)(490.87 \times 10^{-6})} = 1.222 \times 10^{-3} \text{ m} \quad 1.222 \text{ mm}$$

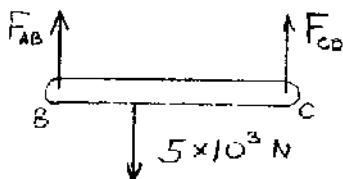
$$(b) S_{BE} = \frac{F_{BE} L_{BE}}{E A_{BE}} = \frac{(125 \times 10^3)(1.5)}{(200 \times 10^9)(490.87 \times 10^{-6})} = 1.910 \times 10^{-3} \text{ m} \quad 1.910 \text{ mm}$$

Problem 2.24

2.24 Each of the links AB and CD is made of aluminum ($E = 75 \text{ GPa}$) and has a cross-sectional area of 125 mm^2 . Knowing that they support the rigid member BC, determine the deflection of point E.



SOLUTION



Use member BC as a free body

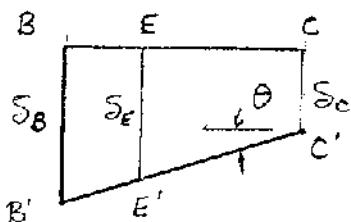
$$\textcircled{D} \sum M_C = 0 \quad -(0.64) F_{AB} + (0.44)(5 \times 10^3) = 0 \quad F_{AB} = 3.4375 \times 10^3 \text{ N}$$

$$\textcircled{D} \sum M_B = 0 \quad (0.64) F_{CD} - (0.20)(5 \times 10^3) = 0 \quad F_{CD} = 1.5625 \times 10^3 \text{ N}$$

For links AB and CD $A = 125 \text{ mm}^2 = 125 \times 10^{-6} \text{ m}^2$

$$S_{AB} = \frac{F_{AB} L_{AB}}{EA} = \frac{(3.4375 \times 10^3)(0.36)}{(75 \times 10^9)(125 \times 10^{-6})} = 132.00 \times 10^{-6} \text{ m} = S_B$$

$$S_{CD} = \frac{F_{CD} L_{CD}}{EA} = \frac{(1.5625 \times 10^3)(0.36)}{(75 \times 10^9)(125 \times 10^{-6})} = 60.00 \times 10^{-6} \text{ m} = S_C$$



$$\text{Slope } \theta = \frac{S_B - S_C}{l_{BC}} = \frac{72.00 \times 10^{-6}}{0.64} \\ = 112.5 \times 10^{-6} \text{ rad}$$

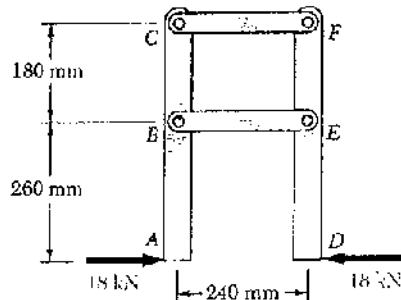
$$S_E = S_C + l_{BC} \theta$$

$$= 60.00 \times 10^{-6} + (0.44)(112.5 \times 10^{-6})$$

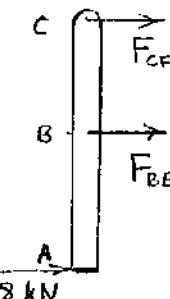
$$= 109.5 \times 10^{-6} \text{ m} = 0.1095 \text{ mm} \downarrow$$

Deformation diagram

Problem 2.25



2.25 Members *ABC* and *DEF* are joined with steel links ($E = 200 \text{ GPa}$). Each of the links is made of a pair of $25 \times 35\text{-mm}$ plates. Determine the change in length of (a) member *BE*, (b) member *CF*.



Use member *ABC* as a free body

$$\therefore \sum M_B = 0$$

$$(0.260)(18 \times 10^3) - (0.180)F_{CF} = 0$$

$$F_{CF} = \frac{(0.260)(18 \times 10^3)}{0.180} = 26 \times 10^3 \text{ N}$$

$$\therefore \sum M_c = 0 \quad (0.440)(18 \times 10^3) + (0.180)F_{BE} = 0$$

$$F_{BE} = \frac{(-0.440)(18 \times 10^3)}{0.180} = -44 \times 10^3 \text{ N}$$

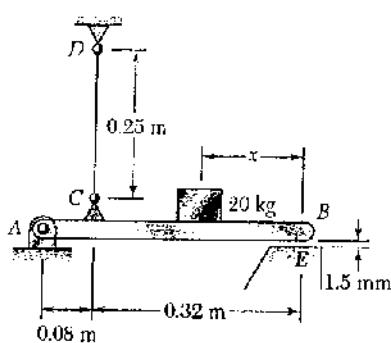
Area for link made
of two plates

$$A = (2)(0.025)(0.035) = 1.75 \times 10^{-3} \text{ m}^2$$

$$(a) \quad \delta_{BE} = \frac{F_{BE} L_{BE}}{EA} = \frac{(-44 \times 10^3)(0.240)}{(200 \times 10^9)(1.75 \times 10^{-3})} = -30.2 \times 10^{-6} \text{ m} = -0.0302 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \delta_{CF} = \frac{F_{CF} L_{CF}}{EA} = \frac{(26 \times 10^3)(0.240)}{(200 \times 10^9)(1.75 \times 10^{-3})} = 17.83 \times 10^{-6} \text{ m} = 0.01783 \text{ mm} \quad \blacktriangleleft$$

Problem 2.26



2.26 The length of the 2-mm-diameter steel wire CD has been adjusted so that with no load applied, a gap of 1.5 mm exists between the end B of the rigid beam ACB and a contact point E. Knowing that $E = 200 \text{ GPa}$, determine where a 20-kg block should be placed on the beam in order to cause contact between B and E.

Rigid rod ACE rotates through angle θ to close gap.

$$\theta = \frac{1.5 \times 10^{-3}}{0.40} = 3.75 \times 10^{-3} \text{ rad}$$

Point C moves downward.

$$S_c = 0.08 \theta = (0.08)(3.75 \times 10^{-3}) = 300 \times 10^{-6} \text{ m}$$

$$S_{co} = S_c = 300 \times 10^{-6} \text{ m}$$

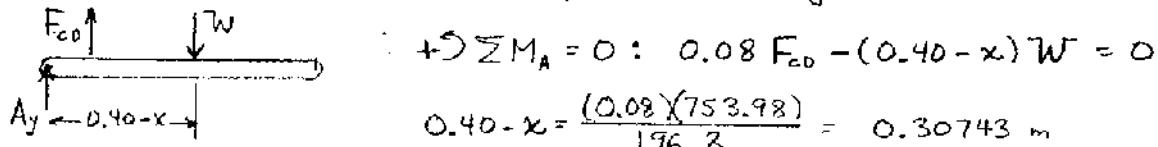
$$A_{co} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ mm}^2 = 3.1416 \times 10^{-6} \text{ m}^2$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$S_{co} = \frac{F_{co} L_{co}}{EA_{co}}$$

$$F_{co} = \frac{EA_{co}S_{co}}{L_{co}} = \frac{(200 \times 10^9)(3.1416 \times 10^{-6})(300 \times 10^{-6})}{0.25} = 753.98 \text{ N}$$

Use beam ACB as a free body. $W = mg = (20)(9.81) = 196.2 \text{ N}$

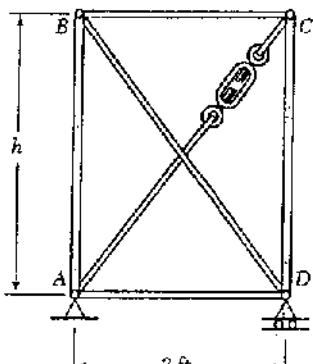


$$+\uparrow \sum M_A = 0 : 0.08 F_{co} - (0.40 - x)W = 0$$

$$0.40 - x = \frac{(0.08)(753.98)}{196.2} = 0.30743 \text{ m}$$

$$x = 0.0926 \text{ m} = 92.6 \text{ mm}$$

Problem 2.27



2.27 Members AB and CD are $1\frac{1}{8}$ -in.-diameter steel rods, and members BC and AD are $\frac{7}{8}$ -in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29 \times 10^6 \text{ psi}$, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

$$S_{AB} = S_{CD} = 0.04 \text{ in} \quad h = 4 \text{ ft.} = 48 \text{ in.} = L_{co}$$

$$A_{co} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$$

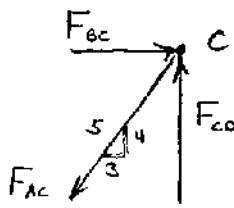
$$S_{co} = \frac{F_{co} L_{co}}{EA_{co}}$$

$$F_{co} = \frac{EA_{co}S_{co}}{L_{co}} = \frac{(29 \times 10^6)(0.99402)(0.04)}{48} = 24.022 \times 10^3 \text{ lb.}$$

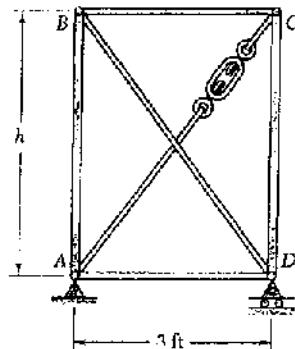
Use joint C as a free body

$$+\uparrow \sum F_y = 0 : F_{co} - \frac{4}{5} F_{ac} = 0 \quad \therefore F_{ac} = \frac{5}{4} F_{co}$$

$$F_{ac} = \frac{5}{4} (24.022 \times 10^3) = 30.0 \times 10^3 \text{ lb.} \\ = 30.0 \text{ kips}$$



Problem 2.28



2.28 For the structure in Prob. of 2.27, determine (a) the distance h so that the deformations in members AB , BC , CD and AD are equal, (b) the corresponding tension in member AC .

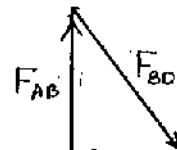
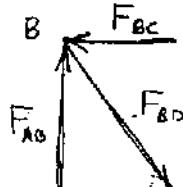
2.27 Members AB and CD are $1\frac{1}{8}$ -in.-diameter steel rods, and members BC and AD are $\frac{1}{8}$ -in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29 \times 10^6$ psi and $h = 4$ ft, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

(a) Statics: Use joint B as a free body

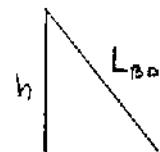
From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BC}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$



Force Triangle



Geometry

For equal deformations

$$S_{AB} = S_{BC} \therefore \frac{F_{AB} h}{E A_{AB}} = \frac{F_{BC} b}{E A_{BC}} \therefore F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

Equating expressions for F_{AB}

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC} \quad \frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{9/8}{7/8} = \frac{9}{7}$$

$$b = 3 \text{ ft} = 36 \text{ in.}$$

$$h = \frac{9}{7} b = \frac{9}{7} (3) = 3.86 \text{ ft} = 46.3 \text{ in.}$$

(b) Setting $S_{AB} = S_{BC} = 0.04$ in.

$$S_{BC} = \frac{F_{BC} b}{E A_{BC}} \therefore F_{BC} = \frac{E A_{BC} S_{BC}}{b} = \frac{(29 \times 10^6) \frac{\pi}{4} (\frac{7}{8})^2 (0.04)}{36} \\ = 19.376 \times 10^3 \text{ lb.}$$

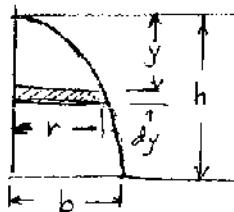
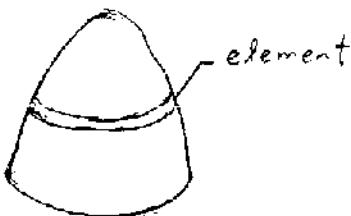
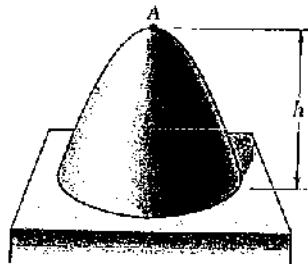
$$F_{AB} = \frac{h}{b} F_{BC} = \frac{9}{7} (19.376 \times 10^3) = 24.912 \times 10^3 \text{ lb.}$$

From the force triangle

$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 31.6 \times 10^3 \text{ lb.} = 31.6 \text{ kips}$$

Problem 2.29

2.29 Determine the deflection of the apex A of a homogeneous paraboloid of revolution of height h , density ρ , and modulus of elasticity E , due to its own weight.



For the element $r = b\left(\frac{y}{h}\right)^{1/2}$

$$A = \pi r^2 = \pi b^2 \frac{y}{h}$$

$$dP = \rho g A dy = \pi b^2 \rho g \frac{y}{h}$$

$$P = \int_0^y dP = \frac{\pi b^2 \rho g}{h} \int_0^y y dy = \frac{\pi b^2 \rho g y^2}{2h}$$

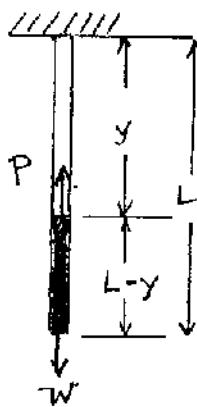
$$S = \sum \frac{P A_y}{E A} = \int_0^h \frac{P dy}{E A(y)} = \frac{\pi b^2 \rho g}{2Eh} \cdot \frac{h}{\pi b^2} \int_0^h y dy = \frac{\rho g h^2}{4E}$$

Let b = radius of the base and
 r = radius at section with coordinate y .

Problem 2.30

2.30 A homogeneous cable of length L and uniform cross section is suspended from one end. (a) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (b) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

SOLUTION



(a) For element at point identified by coordinate y

P = weight of portion below the point

$$= \rho g A (L-y)$$

$$dS = \frac{P dy}{E A} = \frac{\rho g A (L-y) dy}{E A} = \frac{\rho g (L-y)}{E} dy$$

$$S = \int_0^L \frac{\rho g (L-y)}{E} dy = \frac{\rho g}{E} \left(Ly - \frac{1}{2} y^2 \right) \Big|_0^L = \frac{\rho g}{E} \left(L^2 - \frac{L^2}{2} \right) = \frac{1}{2} \frac{\rho g L^2}{E}$$

(b) Total weight

$$W = \rho g A L$$

$$F = \frac{EAS}{L} = \frac{EA}{L} \cdot \frac{1}{2} \frac{\rho g L^2}{E} = \frac{1}{2} \rho g A L = \frac{1}{2} W$$

Problem 2.31

2.31 Denoting by ϵ the "engineering strain" in a tensile specimen, show that the true strain is $\epsilon_t = \ln(1 + \epsilon)$.

SOLUTION

$$\epsilon_t = \ln \frac{L}{L_0} = \ln \frac{L_0 + S}{L_0} = \ln \left(1 + \frac{S}{L_0}\right) = \ln(1 + \epsilon)$$

Thus

$$\epsilon_t = \ln(1 + \epsilon)$$

Problem 2.32

2.32 The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1 , show that when the diameter is d , the true strain is $\epsilon_t = 2 \ln(d_1/d)$.

SOLUTION

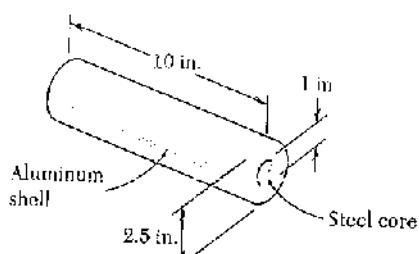
If the volume is constant $\frac{\pi}{4} d^2 L = \frac{\pi}{4} d_1^2 L_0$

$$\frac{L}{L_0} = \frac{d^2}{d_1^2} = \left(\frac{d_1}{d}\right)^2$$

$$\epsilon_t = \ln \frac{L}{L_0} = \ln \left(\frac{d_1}{d}\right)^2 = 2 \ln \frac{d_1}{d}$$

Problem 2.33

2.33 Compressive centric forces of 40 kips are applied at both ends of the assembly shown by means of rigid plates. Knowing that $E_a = 29 \times 10^6$ psi and $E_s = 10.1 \times 10^6$ psi, determine (a) the normal stresses in the steel core and the aluminum shell, (b) the deformation of the assembly.



Let P_a = portion of axial force carried by shell.

P_s = portion of axial force carried by core.

$$S = \frac{P_a L}{E_a A_a} \quad P_a = \frac{E_a A_a}{L} S$$

$$S = \frac{P_s L}{E_s A_s} \quad P_s = \frac{E_s A_s}{L} S$$

$$\text{Total Force } P = P_a + P_s = (E_a A_a + E_s A_s) \frac{S}{L}$$

$$\frac{S}{L} = \epsilon = \frac{P}{E_a A_a + E_s A_s}$$

$$\text{Data: } P = 40 \times 10^3 \text{ lb}$$

$$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(2.5^2 - 1.0^2) = 4.1233 \text{ in}^2$$

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ in}^2$$

$$\epsilon = \frac{-40 \times 10^3}{(10.1 \times 10^6)(4.1233) + (29 \times 10^6)(0.7854)} = -620.91 \times 10^{-6}$$

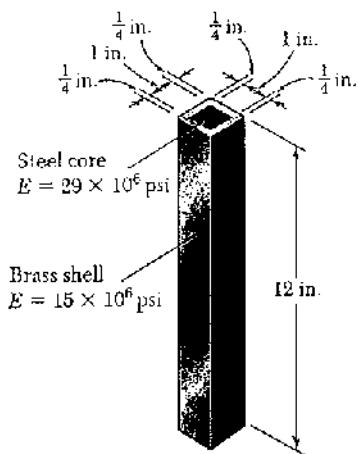
$$(a) \sigma_s = E_s \epsilon = (29 \times 10^6)(-620.91 \times 10^{-6}) = -18.01 \times 10^3 \text{ psi} = -18.01 \text{ ksi}$$

$$\sigma_a = E_a \epsilon = (10.1 \times 10^6)(-620.91 \times 10^{-6}) = -6.27 \times 10^3 \text{ psi} = -6.27 \text{ ksi}$$

$$(b) \Delta = L \epsilon = (10)(-620.91 \times 10^{-6}) = -6.21 \times 10^{-5} \text{ in.}$$

Problem 2.34

2.34 The length of the assembly decreases by 0.006 in. when an axial force is applied by means of rigid end plates. Determine (a) the magnitude of the applied force, (b) the corresponding stress in the steel core.



Let P_b = portion of axial force carried by brass shell.

P_s = portion of axial force carried by steel core.

$$S = \frac{P_s L}{A_b E_b} \quad P_b = \frac{E_b A_b S}{L}$$

$$S = \frac{P_s L}{A_s E_s} \quad P_s = \frac{E_s A_s S}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s) \frac{S}{L}$$

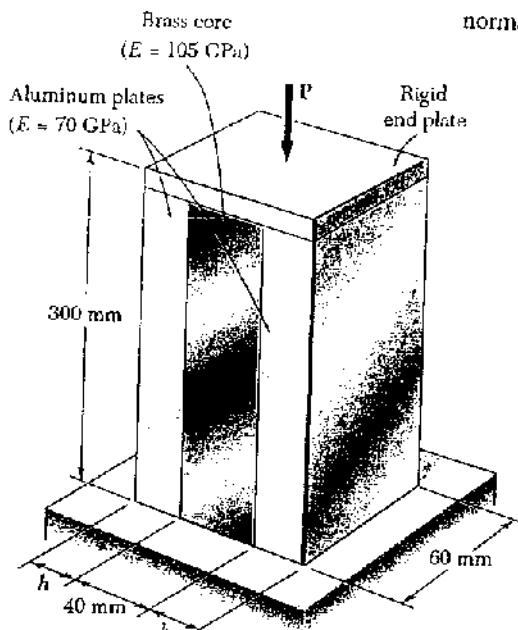
$$A_s = (1)(1) = 1.00 \text{ in}^2$$

$$A_b = (1.5)(1.5) - (1)(1) = 1.25 \text{ in}^2$$

$$(a) P = [(15 \times 10^6)(1.25) + (29 \times 10^6)(1.00)] \frac{-0.006}{12} = 23.9 \times 10^3 \text{ lb} \quad 23.9 \text{ kips} \blacktriangleleft$$

$$(b) \sigma_s = E_s \epsilon = E_s \frac{S}{L} = (29 \times 10^6) \frac{0.006}{12} = 14.50 \times 10^3 \text{ psi} \quad 14.50 \text{ ksi} \blacktriangleleft$$

Problem 2.35



2.35 An axial centric force of magnitude $P = 450 \text{ kN}$ is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10 \text{ mm}$, determine the normal stress in (a) the brass core, (b) the aluminum plates.

Let $P_b = \text{portion of axial force carried by brass core}$

$P_a = \text{portion carried by two aluminum plates}$

$$S = \frac{P_b L}{E_b A_b} \quad P_b = \frac{E_b A_b S}{L}$$

$$S = \frac{P_a L}{E_a A_a} \quad P_a = \frac{E_a A_a S}{L}$$

$$P = P_b + P_a = (E_b A_b + E_a A_a) \frac{S}{L}$$

$$\varepsilon = \frac{S}{L} = \frac{P}{E_b A_b + E_a A_a}$$

$$A_b = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

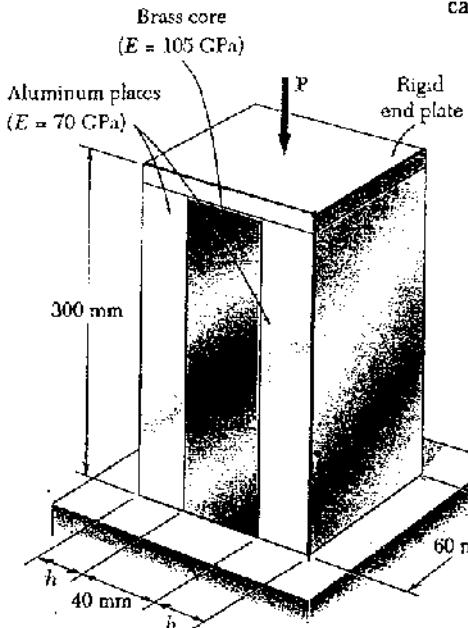
$$A_a = (2)(60)(10) = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

$$\varepsilon = \frac{450 \times 10^3}{(105 \times 10^9)(2400 \times 10^{-6}) + (70 \times 10^9)(1200 \times 10^{-6})} = 1.3393 \times 10^{-3}$$

$$(a) \sigma_b = E_b \varepsilon = (105 \times 10^9)(1.3393 \times 10^{-3}) = 140.6 \times 10^6 \text{ Pa} = 140.6 \text{ MPa} \blacktriangleleft$$

$$(b) \sigma_a = E_a \varepsilon = (70 \times 10^9)(1.3393 \times 10^{-3}) = 93.75 \times 10^6 \text{ Pa} = 93.75 \text{ MPa} \blacktriangleleft$$

Problem 2.36



2.36 For the composite block shown in Prob. 2.35, determine (a) the value of h if the portion of the load carried by the aluminum plates is half the portion of the load carried by the brass core, (b) the total load if the stress in the brass is 80 MPa.

Let P_b = portion of axial force carried by brass core

P_a = portion carried by the two aluminum plates

$$S = \frac{P_b L}{E_b A_b} \quad P_b = \frac{E_b A_b S}{L}$$

$$S = \frac{P_a L}{E_a A_a} \quad P_a = \frac{E_a A_a S}{L}$$

$$(a) \text{ Given } P_a = \frac{1}{2} P_b$$

$$\frac{E_a A_a S}{L} = \frac{1}{2} \frac{E_b A_b S}{L}$$

$$A_a = \frac{1}{2} \frac{E_b}{E_a} A_b$$

$$A_b = (40)(60) = 2400 \text{ mm}^2 = 2400 \times 10^{-4} \text{ m}^2$$

$$A_a = \frac{1}{2} \frac{105 \times 10^9}{70 \times 10^9} 2400 = 1800 \text{ mm}^2 = (2)(60)h$$

$$h = \frac{1800}{(2)(60)} = 15 \text{ mm}$$

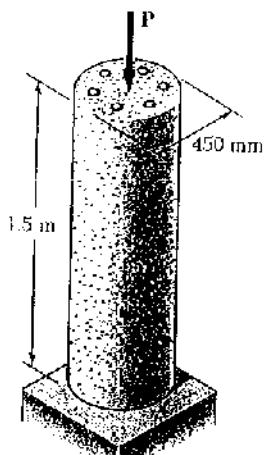
$$(b) \sigma_b = \frac{P_b}{A_b}$$

$$P_b = A_b \sigma_b = (2400 \times 10^{-4})(80 \times 10^6) = 192 \times 10^3 \text{ N}$$

$$P_a = \frac{1}{2} P_b = 96 \times 10^3 \text{ N}$$

$$P = P_b + P_a = 288 \times 10^3 \text{ N} = 288 \text{ kN}$$

Problem 2.37



2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200 \text{ GPa}$ and $E_c = 25 \text{ GPa}$, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force P is applied to the post.

Let $P_c = \text{portion of axial force carried by concrete}$
 $P_s = \text{portion carried by the six steel rods}$

$$S = \frac{P_c L}{E_c A_c} \quad P_c = \frac{E_c A_c S}{L}$$

$$S = \frac{P_s L}{E_s A_s} \quad P_s = \frac{E_s A_s S}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s) \frac{S}{L}$$

$$\varepsilon = \frac{S}{L} = \frac{P}{E_c A_c + E_s A_s}$$

$$A_s = 6 \cdot \frac{\pi}{4} d_s^2 = \frac{6\pi}{4} (28)^2 = 3.6945 \times 10^3 \text{ mm}^2 = 3.6945 \times 10^{-3} \text{ m}^2$$

$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4} (450)^2 - 3.6945 \times 10^3 = 155.349 \times 10^3 \text{ mm}^2$$

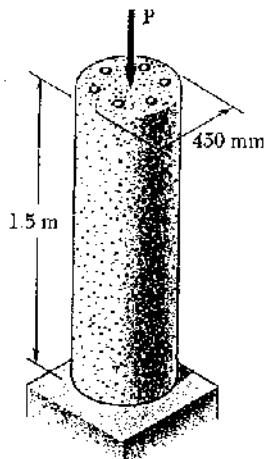
$$L = 1.5 \text{ m} \quad = 153.349 \times 10^{-3} \text{ m}^2$$

$$\varepsilon = \frac{1550 \times 10^3}{(25 \times 10^9)(155.349 \times 10^{-3}) + (200 \times 10^9)(3.6945 \times 10^{-3})} = 335.31 \times 10^{-6}$$

$$\sigma_s = E_s \varepsilon = (200 \times 10^9)(335.31 \times 10^{-6}) = 67.1 \times 10^6 \text{ Pa} \quad 67.1 \text{ MPa} \blacksquare$$

$$\sigma_c = E_c \varepsilon = (25 \times 10^9)(335.31 \times 10^{-6}) = 8.38 \times 10^6 \text{ Pa} \quad 8.38 \text{ MPa} \blacksquare$$

Problem 2.38



2.38 For the post of Prob. 2.37, determine the maximum centric force which may be applied if the allowable normal stress is 160 MPa in the steel and 18 MPa in the concrete.

2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200 \text{ GPa}$ and $E_c = 25 \text{ GPa}$, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force P is applied to the post.

Determine allowable strain in each material

$$\text{Steel: } \varepsilon_s = \frac{\sigma_s}{E_s} = \frac{160 \times 10^6}{200 \times 10^9} = 800 \times 10^{-6}$$

$$\text{Concrete: } \varepsilon_c = \frac{\sigma_c}{E_c} = \frac{18 \times 10^6}{25 \times 10^9} = 720 \times 10^{-6}$$

$$\text{Smaller value governs } \varepsilon = \frac{S}{L} = 720 \times 10^{-6}$$

Let P_c = portion of load carried by concrete

P_s = portion carried by six steel rods

$$S = \frac{P_c L}{E_c A_c}, \quad P_c = E_c A_c \frac{S}{L} = E_c A_c \varepsilon$$

$$S = \frac{P_s L}{E_s A_s}, \quad P_s = E_s A_s \frac{S}{L} = E_s A_s \varepsilon$$

$$P = P_c + P_s = (E_c A_c + E_s A_s) \varepsilon$$

$$A_s = 6 \cdot \frac{\pi}{4} d_s^2 = \frac{6\pi}{4} (28)^2 = 3.6945 \times 10^3 \text{ mm}^2 = 3.6945 \times 10^{-3} \text{ m}^2$$

$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4} (150)^2 - 3.6945 \times 10^3 = 155.349 \times 10^3 \text{ mm}^2 \\ = 155.349 \times 10^{-3} \text{ m}^2$$

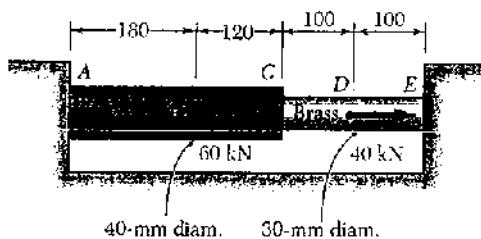
$$P = [(25 \times 10^9)(155.349 \times 10^{-3}) + (200 \times 10^9)(3.6945 \times 10^{-3})](720 \times 10^{-6})$$

$$= 3.33 \times 10^6 \text{ N}$$

3330 kN →

Problem 2.39

Dimensions in mm



2.39 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200 \text{ GPa}$ and $E_b = 105 \text{ GPa}$, determine (a) the reactions at A and E, (b) the deflection of point C.

$$A \text{ to } C: E = 200 \times 10^9 \text{ Pa}$$

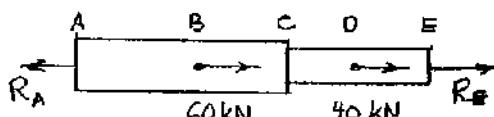
$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$EA = 251.327 \times 10^6 \text{ N}$$

$$C \text{ to } E: E = 105 \times 10^9 \text{ Pa}$$

$$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$EA = 74.220 \times 10^6 \text{ N}$$



$$A \text{ to } B: P = R_A \quad L = 180 \text{ mm} = 0.180 \text{ m}$$

$$S_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6} = 716.20 \times 10^{-12} R_A$$

$$B \text{ to } C: P = R_A - 60 \times 10^3 \quad L = 120 \text{ mm} = 0.120 \text{ m}$$

$$S_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6} = 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

$$C \text{ to } D: P = R_A - 60 \times 10^3 \quad L = 100 \text{ mm} = 0.100 \text{ m}$$

$$S_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 80.841 \times 10^{-6}$$

$$D \text{ to } E: P = R_A - 100 \times 10^3 \quad L = 100 \text{ mm} = 0.100 \text{ m}$$

$$S_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 134.735 \times 10^{-6}$$

$$A \text{ to } E: S_{AE} = S_{AB} + S_{BC} + S_{CD} + S_{DE} = 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6}$$

Since point E cannot move relative to A, $S_{AE} = 0$

$$(a) 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0 \quad R_A = 62.831 \times 10^3 \text{ N} \quad 62.8 \text{ kN} \leftarrow$$

$$R_E = R_A - 100 \times 10^3 = 62.8 \times 10^3 - 100 \times 10^3 = -37.2 \times 10^3 \text{ N} \quad 37.2 \text{ kN} \leftarrow$$

$$(b) S_C = S_{AB} + S_{BC} = 1.16367 \times 10^{-9} R_A - 26.848 \times 10^{-6}$$

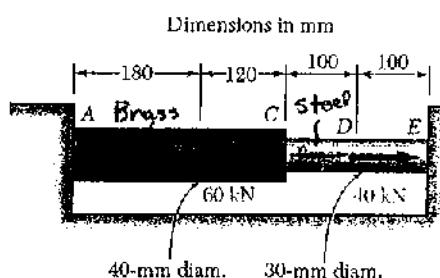
$$= (1.16367 \times 10^{-9})(62.831 \times 10^3) - 26.848 \times 10^{-6}$$

$$= 46.3 \times 10^{-6} \text{ m}$$

$$46.3 \mu\text{m} \rightarrow$$

Problem 2.40

2.40 Solve Prob. 2.39, assuming that rod *AC* is made of brass and rod *CE* is made of steel.



2.39 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_b = 200 \text{ GPa}$ and $E_s = 105 \text{ GPa}$, determine (a) the reactions at *A* and *E*, (b) the deflection of point *C*.

$$A \rightarrow C: E = 105 \times 10^9 \text{ Pa}$$

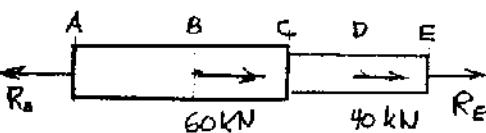
$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-5} \text{ m}^2$$

$$EA = 131.947 \times 10^6 \text{ N}$$

$$C \rightarrow E: E = 200 \times 10^9 \text{ Pa}$$

$$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$EA = 141.372 \times 10^6 \text{ N}$$



$$A \rightarrow B: P = R_A \quad L = 180 \text{ mm} = 0.180 \text{ m}$$

$$S_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947 \times 10^6} = 1.36418 \times 10^{-9} R_A$$

$$B \rightarrow C: P = R_A - 60 \times 10^3 \quad L = 120 \text{ mm} = 0.120 \text{ m}$$

$$S_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{131.947 \times 10^6} = 909.456 \times 10^{-12} R_A - 54.567 \times 10^{-6}$$

$$C \rightarrow D: P = R_A - 60 \times 10^3 \quad L = 100 \text{ mm} = 0.100 \text{ m}$$

$$S_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 42.441 \times 10^{-6}$$

$$D \rightarrow E: P = R_A - 100 \times 10^3 \quad L = 100 \text{ mm} = 0.100 \text{ m}$$

$$S_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 70.735 \times 10^{-6}$$

$$A \rightarrow E: S_{AE} = S_{AB} + S_{BC} + S_{CD} + S_{DE} = 3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6}$$

Since point *E* cannot move relative to *A*, $S_{AE} = 0$

$$(a) 3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6} = 0 \quad R_A = 45.479 \times 10^3 \text{ N} \quad 45.5 \text{ kN} \leftarrow$$

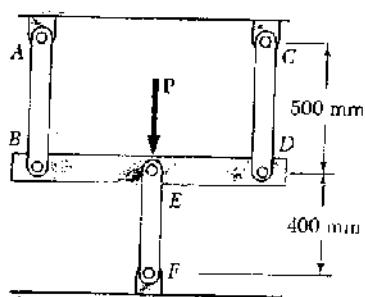
$$R_E = R_A - 100 \times 10^3 = 45.479 \times 10^3 - 100 \times 10^3 = -54.521 \times 10^3 \quad 54.5 \text{ kN} \leftarrow$$

$$(b) S_C = S_{AB} + S_{BC} = 2.27364 \times 10^{-9} R_A - 54.567 \times 10^{-6}$$

$$= (2.27364 \times 10^{-9})(45.479 \times 10^3) - 54.567 \times 10^{-6}$$

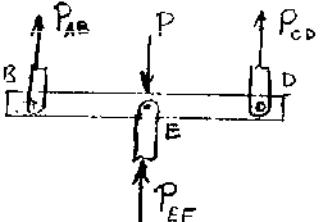
$$= 48.8 \times 10^{-6} \text{ m} \quad 48.8 \mu\text{m} \rightarrow$$

Problem 2.41



2.41 Three steel rods ($E = 200 \text{ GPa}$) support a 36-kN load P . Each of the rods AB and CD has a 200-mm^2 cross-sectional area and rod EF has a 625-mm^2 cross-sectional area. Determine the (a) the change in length of rod EF , (b) the stress in each rod.

Use member BED as a free body



By symmetry, or by $\sum M_E = 0$

$$P_{CD} = P_{AB}$$

$$\sum F_y = 0$$

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{EA_{AB}}, \quad \delta_{CD} = \frac{P_{CD} L_{CD}}{EA_{CD}}, \quad \delta_{EF} = \frac{P_{EF} L_{EF}}{EA_{EF}}$$

$$\text{Since } L_{AB} = L_{CD} \text{ and } A_{AB} = A_{CD}, \quad \delta_{AB} = \delta_{CD}$$

$$\text{Since points } A, C, \text{ and } E \text{ are fixed} \quad \delta_B = \delta_{AB}, \quad \delta_D = \delta_{CD}, \quad \delta_E = \delta_{EF}$$

$$\text{Since member BED is rigid} \quad \delta_E = \delta_B = \delta_C$$

$$\frac{P_{AB} L_{AB}}{EA_{AB}} = \frac{P_{EF} L_{EF}}{EA_{EF}} \quad \therefore P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF} \\ = 0.256 P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2)(0.256)P_{EF} + P_{EF} = 1.512 P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3 \text{ N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3 \text{ N}$$

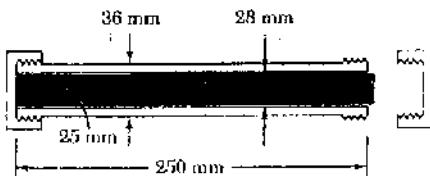
$$(a) \quad \delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m} \\ = 0.0762 \text{ mm}$$

$$\text{or} \quad \delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

$$(b) \quad \sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6 \text{ Pa} = 30.5 \text{ MPa}$$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6 \text{ Pa} = 38.1 \text{ MPa}$$

Problem 2.42



2.42 A 250-mm-long aluminum tube ($E = 70 \text{ GPa}$) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105 \text{ GPa}$) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

$$A_{\text{tube}} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\text{tube}} = \frac{PL}{E_{\text{tube}} A_{\text{tube}}} = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} = 8.8815 \times 10^{-9} P$$

$$\sigma_{\text{rod}} = -\frac{PL}{E_{\text{rod}} A_{\text{rod}}} = -\frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} = -4.8505 \times 10^{-9} P$$

$$\delta^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\sigma_{\text{tube}} = \sigma^* + \sigma_{\text{rod}} \quad \text{or} \quad \sigma_{\text{tube}} - \sigma_{\text{rod}} = \sigma^*$$

$$8.8815 \times 10^{-9} P + 4.8505 \times 10^{-9} P = 375 \times 10^{-6}$$

$$P = \frac{0.375 \times 10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308 \times 10^3 \text{ N}$$

$$(a) \sigma_{\text{tube}} = -\frac{P}{A_{\text{tube}}} = -\frac{27.308 \times 10^3}{402.12 \times 10^{-6}} = 67.9 \times 10^6 \text{ Pa} = 67.9 \text{ MPa}$$

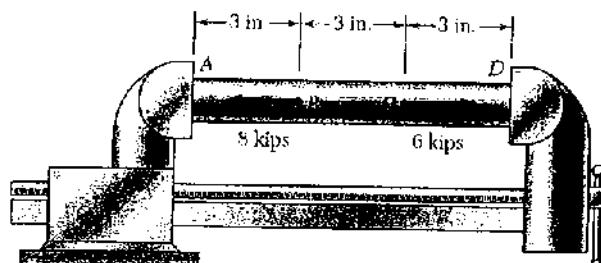
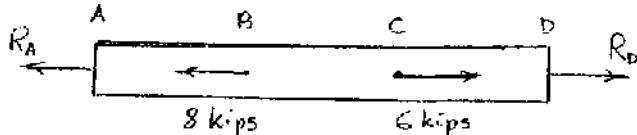
$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{27.308 \times 10^3}{490.87 \times 10^{-6}} = -55.6 \times 10^6 \text{ Pa} = -55.6 \text{ MPa}$$

$$(b) \delta_{\text{tube}} = (8.8815 \times 10^{-9})(27.308 \times 10^3) = 242.5 \times 10^{-6} \text{ m} = 0.2425 \text{ mm}$$

$$\delta_{\text{rod}} = -(4.8505 \times 10^{-9})(27.308 \times 10^3) = -132.5 \times 10^{-6} \text{ m} = -0.1325 \text{ mm}$$

Problem 2.43

2.43 A steel tube ($E = 29 \times 10^6$ psi) with a $1\frac{1}{4}$ in.-outer diameter and a $\frac{1}{8}$ in.-thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.008 in. Determine (a) the forces exerted by the vise on the tube at A and D , (b) the change in length of the portion BC of the tube.



$$\text{For the tube } d_o = 1\frac{1}{4} \text{ in.} = 1.25 \text{ in.}$$

$$d_i = d_o - 2t = 1.25 - (2)(\frac{1}{8}) = 1.00 \text{ in.}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(1.25^2 - 1.00^2) = 0.44179 \text{ in.}^2$$

$$\text{A to B: } P = R_A \text{ lb} \quad L = 3 \text{ in.}$$

$$S_{AB} = \frac{PL}{EA} = \frac{R_A(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A \text{ in.}$$

$$\text{B to C: } P = R_A + 8000 \text{ lb}, \quad L = 3 \text{ in.}$$

$$S_{BC} = \frac{PL}{EA} = \frac{(R_A + 8000)(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A + 1.87326 \times 10^{-8} \text{ in.}$$

$$\text{C to D: } P = R_A + 2000 \text{ lb}, \quad L = 3 \text{ in.}$$

$$S_{CD} = \frac{PL}{EA} = \frac{(R_A + 2000)(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A + 0.46831 \times 10^{-8} \text{ in.}$$

$$\text{A to D: } S_{AD} = S_{AB} + S_{BC} + S_{CD} = 702.471 \times 10^{-9} R_A + 2.34157 \times 10^{-8} \text{ in.}$$

$$\text{Given jaw movement} \quad S_{AD} = -0.008 \text{ in.}$$

$$(a) 702.471 \times 10^{-9} R_A + 2.34157 \times 10^{-8} = -0.008$$

$$R_A = -14.7217 \times 10^3 \text{ lb}$$

$$R_A = 14.72 \text{ kips} \rightarrow$$

$$R_B = -R_A + 2000 = -12.7217 \times 10^3 \text{ lb}$$

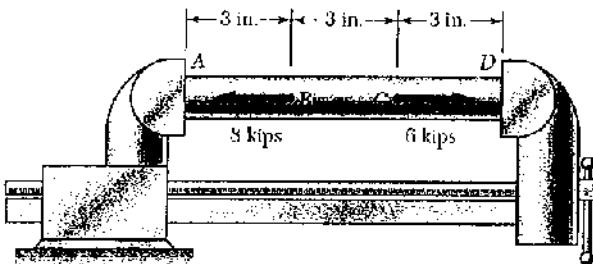
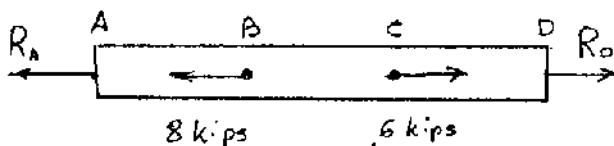
$$R_B = 12.72 \text{ kips} \leftarrow$$

$$(b) S_{BC} = (234.157 \times 10^{-9})(-14.7217 \times 10^3) + 1.87326 \times 10^{-8} = -1.574 \times 10^{-2} \text{ in.} \leftarrow$$

Problem 2.44

2.44 Solve Prob. 2.43, assuming that after the forces have been applied, the vise is adjusted to increase the distance between its jaws by 0.004 in.

2.43 A steel tube ($E = 29 \times 10^6$ psi) with a $1\frac{1}{4}$ in.-outer diameter and a $\frac{1}{8}$ in.-thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.008 in. Determine (a) the forces exerted by the vise on the tube at A and D , (b) the change in length of the portion BC of the tube.



$$\text{For the tube } d_o = 1\frac{1}{4} \text{ in.} = 1.25 \text{ in.}$$

$$d_i = d_o - 2t = 1.25 - (2 \times \frac{1}{8}) = 1.00 \text{ in.}$$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (1.25^2 - 1.00^2) = 0.44179 \text{ in}^2$$

$$A \text{ to } B: P = R_A \quad L = 3 \text{ in.}$$

$$S_{AB} = \frac{PL}{EA} = \frac{R_A(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A \text{ in.}$$

$$B \text{ to } C: P = R_A + 8000 \text{ lb} \quad L = 3 \text{ in.}$$

$$S_{BC} = \frac{PL}{EA} = \frac{(R_A + 8000)(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A + 1.87326 \times 10^{-3} \text{ in.}$$

$$C \text{ to } D: P = R_A + 2000 \text{ lb} \quad L = 3 \text{ in.}$$

$$S_{CD} = \frac{PL}{EA} = \frac{(R_A + 2000)(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A + 0.46831 \times 10^{-3} \text{ in.}$$

$$A \text{ to } D: S_{AD} = S_{AB} + S_{BC} + S_{CD} = 702.471 \times 10^{-9} R_A + 2.34157 \times 10^{-3} \text{ in.}$$

$$\text{Given jaw movement } S_{AD} = -0.004 \text{ in.}$$

$$(a) 702.471 \times 10^{-9} R_A + 2.34157 \times 10^{-3} = -0.004$$

$$R_A = -9.0275 \times 10^3 \text{ lb}$$

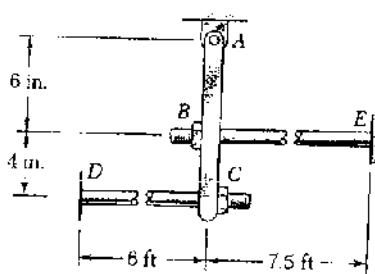
$$R_A = 9.03 \text{ kips} \rightarrow$$

$$R_o = R_A + 2000 = -7.0275 \times 10^3 \text{ lb}$$

$$R_o = 7.03 \text{ kips} \leftarrow$$

$$(b) S_{BC} = (234.157 \times 10^{-9})(-9.0275 \times 10^3) + 1.87326 \times 10^{-3} = -241 \times 10^{-6} \text{ in.}$$

Problem 2.45



2.45 The steel rods BE and CD each have a diameter of $\frac{5}{8}$ in. ($E = 29 \times 10^6$ psi). The ends are threaded with a pitch of 0.1 in. Knowing that after being snugly fit, the nut at B is tightened one full turn, determine (a) the tension in rod CD , (b) the deflection of point C of the rigid member ABC .

Let θ be the rotation of bar ABC as shown

$$\text{Then, } S_B = 6\theta \quad \text{and} \quad S_c = 10\theta$$

$$\text{But } S_B = S_{\text{turn}} - \frac{P_{BE} L_{BE}}{E_{BE} A_{BE}}$$

$$P_{BE} = (E_{BE} A_{BE})(S_{\text{turn}} - S_B) / L_{BE}$$

$$L_{BE} = 7.5 \text{ ft} = 90 \text{ in.}, \therefore S_{\text{turn}} = 0.1 \text{ in}$$

$$A_{BE} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$$

$$P_{BE} = \frac{(29 \times 10^6)(0.3068)}{90} (0.1 - 6\theta)$$

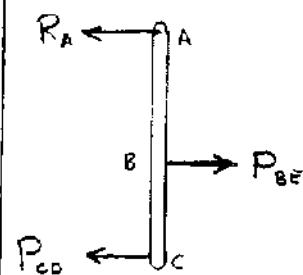
$$= 9.886 \times 10^3 - 593.15 \times 10^3 \theta$$

$$S_c = \frac{P_{CD} L_{CD}}{E A_{CD}} \quad ; \quad P_{CD} = \frac{E A S_c}{L_{CD}}$$

$$L_{CD} = 6 \text{ ft} = 72 \text{ in.}, \quad A_{CD} = 0.3068 \text{ in}^2$$

$$P_{CD} = \frac{(29 \times 10^6)(0.3068)}{72} (10\theta)$$

$$= 1.23572 \times 10^6 \theta$$



$$\textcircled{D} \sum M_A = 0 \quad 6P_{BE} - 10P_{CD} = 0$$

$$(6)(9.886 \times 10^3 - 593.16 \times 10^3 \theta) - (10)(1.23572 \times 10^6) \theta = 0$$

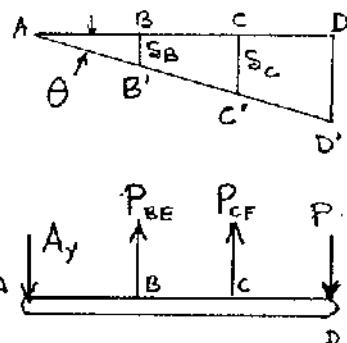
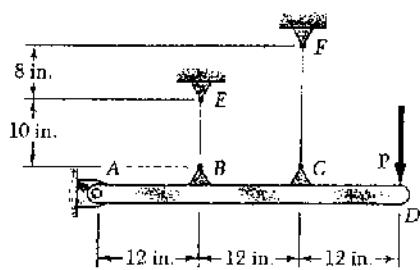
$$59.316 \times 10^3 - 15.916 \times 10^6 \theta = 0 \quad \theta = 3.7268 \times 10^{-3} \text{ rad}$$

$$(a) \quad P_{CD} = (1.23572 \times 10^6)(3.7268 \times 10^{-3}) = 4.61 \times 10^3 \\ = 4.61 \text{ kips}$$

$$(b) \quad S_c = 10\theta = (10)(3.7268 \times 10^{-3}) = 37.3 \times 10^{-2} \text{ in} \\ = 0.0373 \text{ in.}$$

Problem 2.46

2.46 The rigid bar AD is supported by two steel wires of $\frac{1}{16}$ -in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at D . Knowing that the wires were initially taught, determine (a) the additional tension in each wire when a 220-lb load P is applied at D , (b) the corresponding deflection of point D .



Let θ be the rotation of bar ABCD

$$\text{Then } S_B = 12\theta$$

$$S_C = 24\theta$$

$$S_B = \frac{P_{BE} L_{BE}}{AE}$$

$$P_{BE} = \frac{EA S_{BE}}{L_{BE}} = \frac{(29 \times 10^6) \frac{\pi}{4} (\frac{1}{16})^2 (12\theta)}{10} \\ = 106.77 \times 10^3 \theta$$

$$S_C = \frac{P_{CF} L_{CF}}{EA}$$

$$P_{CF} = \frac{EA S_{CF}}{L_{CF}} = \frac{(29 \times 10^6) \frac{\pi}{4} (\frac{1}{16})^2 (24\theta)}{18} \\ = 118.63 \times 10^3 \theta$$

Using free body ABCD

$$\text{D } \sum M_A = 0 \quad 12P_{BE} + 24P_{CF} - 36P = 0$$

$$(12)(106.77 \times 10^3 \theta) + (24)(118.63 \times 10^3 \theta) - (36)(220) = 0$$

$$4.1283 \times 10^6 \theta = (36)(220)$$

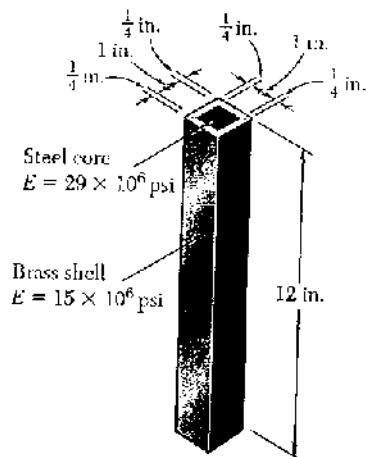
$$\theta = 1.9185 \times 10^{-3} \text{ rad}$$

$$(a) P_{BE} = (106.77 \times 10^3)(1.9185 \times 10^{-3}) = 204.8 \text{ lb.}$$

$$P_{CF} = (118.63 \times 10^3)(1.9185 \times 10^{-3}) = 227.6 \text{ lb.}$$

$$(b) S_D = 36\theta = (36)(1.9185 \times 10^{-3}) = 69.1 \times 10^{-3} \text{ in.} \\ = 0.0691 \text{ in.}$$

Problem 2.47



2.47 The brass shell ($\alpha_b = 11.6 \times 10^{-6}/^\circ\text{F}$) is fully bonded to the steel core ($\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 8 ksi.

Let P_s = axial force developed in the steel core

For equilibrium with zero total force, the compressive force in the brass shell is P_s .

$$\text{Strains } \epsilon_s = \frac{P_s}{E_s A_s} + \alpha_s (\Delta T)$$

$$\epsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b (\Delta T)$$

$$\text{Matching } \epsilon_s = \epsilon_b$$

$$\frac{P_s}{E_s A_s} + \alpha_s (\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b (\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b} \right) P_s = (\alpha_b - \alpha_s) (\Delta T) \quad (1)$$

$$A_b = (1.5)(1.5) - (1.0)(1.0) = 1.25 \text{ in}^2$$

$$A_s = (1.0)(1.0) = 1.0 \text{ in}^2$$

$$\alpha_b - \alpha_s = 5.1 \times 10^{-6} / ^\circ\text{F}$$

$$P_s = \sigma_s A_s = (8 \times 10^3)(1.0) = 8 \times 10^3 \text{ lb.}$$

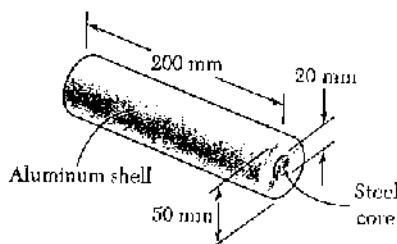
$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(29 \times 10^6)(1.0)} + \frac{1}{(15 \times 10^6)(1.25)} = 87.816 \times 10^{-9} \text{ lb}^{-1}$$

$$\text{From (1)} \quad (87.816 \times 10^{-9})(8 \times 10^3) = (5.1 \times 10^{-6})(\Delta T)$$

$$\Delta T = 137.8 \text{ } ^\circ\text{F}$$

Problem 2.48

2.48 The assembly shown consists of an aluminum shell ($E_a = 70 \text{ GPa}$, $\alpha_a = 23.6 \times 10^{-6}/\text{C}$) fully bonded to a steel core ($E_s = 200 \text{ GPa}$, $\alpha_s = 11.7 \times 10^{-6}/\text{C}$) and is unstressed at a temperature of 20°C . Considering only axial deformations, determine the stress in the aluminum shell when the temperature reaches 180°C .



$$A_s = \frac{\pi}{4}(20)^2 = 314.159 \text{ mm}^2 = 314.159 \times 10^{-6} \text{ m}^2$$

$$A_a = \frac{\pi}{4}(50^2 - 20^2) = 1.64934 \times 10^3 \text{ mm}^2 \\ = 1.64934 \times 10^{-3} \text{ m}^2$$

Let P_s be the axial force carried by the steel core and P_a that carried by the aluminum shell.

$$\text{Total axial force } P = P_a + P_s = 0$$

$$P_s = -P_a \quad (1)$$

Deformation.

$$S = \frac{P_a L}{E_a A_a} + L \alpha_a (\Delta T) = \frac{P_a L}{E_s A_s} + L \alpha_s (\Delta T)$$

$$\frac{P_a}{E_a A_a} - \frac{P_a}{E_s A_s} = (\alpha_s - \alpha_a)(\Delta T)$$

$$\text{Using (1)} \quad \left(\frac{1}{E_a A_a} + \frac{1}{E_s A_s} \right) P_a = (\alpha_s - \alpha_a)(\Delta T)$$

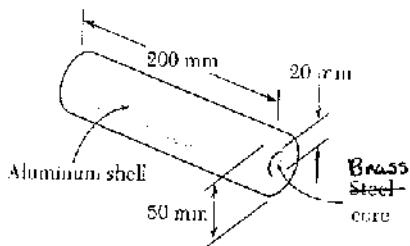
$$\left(\frac{1}{(70 \times 10^9)(1.64934 \times 10^{-3})} + \frac{1}{(200 \times 10^9)(314.159 \times 10^{-6})} \right) P_a \\ = (11.7 \times 10^{-6} - 23.6 \times 10^{-6})(180 - 20)$$

$$(24.577 \times 10^{-9}) P_a = -1.904 \times 10^{-3}$$

$$P_a = -77.471 \times 10^3 \text{ N}$$

$$\text{Stress in aluminum shell} \quad \sigma_a = \frac{P_a}{A_a} = \frac{-77.471 \times 10^3}{1.64934 \times 10^{-3}} = -47.0 \times 10^6 \text{ Pa} \\ = -47.0 \text{ MPa}$$

Problem 2.49



2.49 Solve Prob. 2.48, assuming that the core is made of brass ($E_b = 105 \text{ GPa}$, $\alpha_b = 20.9 \times 10^{-6}/\text{C}$).

2.48 The assembly shown consists of an aluminum shell ($E_a = 70 \text{ GPa}$, $\alpha_a = 23.6 \times 10^{-6}/\text{C}$) fully bonded to a steel core ($E_b = 200 \text{ GPa}$, $\alpha_b = 11.7 \times 10^{-6}/\text{C}$) and is unstressed at a temperature of 20°C . Considering only axial deformations, determine the stress in the aluminum shell when the temperature reaches 180°C .

$$A_b = \frac{\pi}{4} (20)^2 = 314.159 \text{ mm}^2 = 314.159 \times 10^{-6} \text{ m}^2$$

$$A_a = \frac{\pi}{4} (50^2 - 20^2) = 1.64934 \times 10^3 \text{ mm}^2 \\ = 1.64934 \times 10^{-3} \text{ m}^2$$

Let P_b be the axial force carried by the brass core and P_a that carried by the aluminum shell.

$$\text{Total axial force } P = P_a + P_b = 0$$

$$P_b = -P_a \quad (1)$$

Deformation.

$$\delta = \frac{P_a L}{E_a A_a} + L \alpha_a (\Delta T) = \frac{P_b L}{E_b A_b} + L \alpha_b (\Delta T)$$

$$\frac{P_a}{E_a A_a} + \frac{P_b}{E_b A_b} = (\alpha_b - \alpha_a)(\Delta T)$$

$$\text{Using (1), } \left(\frac{1}{E_a A_a} + \frac{1}{E_b A_b} \right) P_a = (\alpha_b - \alpha_a)(\Delta T)$$

$$\left(\frac{1}{(70 \times 10^9)(1.64934 \times 10^{-3})} + \frac{1}{(105 \times 10^9)(314.159 \times 10^{-6})} \right) P_a \\ = (20.9 \times 10^{-6} - 23.6 \times 10^{-6})(180 - 20)$$

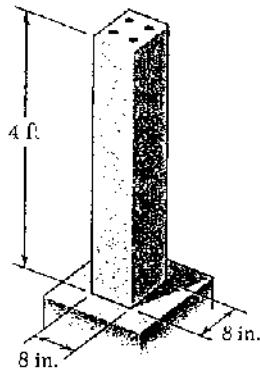
$$(38.977 \times 10^{-9}) P_a = -432 \times 10^{-6}$$

$$P_a = -11.0835 \times 10^3 \text{ N}$$

Stress in aluminum shell

$$\sigma_a = \frac{P_a}{A_a} = \frac{-11.0835 \times 10^3}{1.64934 \times 10^{-3}} = -6.72 \times 10^6 \text{ Pa} \\ -6.72 \text{ MPa}$$

Problem 2.50



2.50 A 4-ft concrete post is reinforced by four steel bars, each of $\frac{3}{4}$ in. diameter. Knowing that $E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$ and $E_c = 3.6 \times 10^6$ psi and $\alpha_c = 5.5 \times 10^{-6}/^\circ\text{F}$, determine the normal stresses induced in the steel and in the concrete by a temperature rise of 80°F .

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)(\frac{\pi}{4})(\frac{3}{4})^2 = 1.76715 \text{ in}^2$$

$$A_c = A - A_s = 8^2 - 1.76715 = 62.233 \text{ in}^2$$

Let P_c be the tensile in the concrete. For equilibrium with zero total force, the compressive force in the four steel rods is $-P_c$.

Strains:

$$\epsilon_s = \frac{P_s}{E_s A_s} + \alpha_s (\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$$

$$\epsilon_c = \frac{P_c}{E_c A_c} + \alpha_c (\Delta T)$$

Matching:

$$\epsilon_c = \epsilon_s \quad \frac{P_c}{E_c A_c} + \alpha_c (\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s} \right) P_c = (\alpha_s - \alpha_c) \Delta T$$

$$\left(\frac{1}{(3.6 \times 10^6)(62.233)} + \frac{1}{(29 \times 10^6)(1.76715)} \right) P_c$$

$$= (6.5 \times 10^{-4} - 5.5 \times 10^{-4})(80)$$

$$P_c = 8.3366 \times 10^3 \text{ lb} \quad P_s = -3.3366 \times 10^3 \text{ lb}$$

Stress in steel $\sigma_s = \frac{P_s}{A_s} = \frac{-3.3366 \times 10^3}{1.76715} = -1.888 \times 10^3 \text{ psi}$

Stress in concrete $\sigma_c = \frac{P_c}{A_c} = \frac{3.3366 \times 10^3}{62.233} = 53.6 \text{ psi}$

Problem 2.51

$$L = 39 \text{ ft} = 468 \text{ in.}$$

2.51 A steel railroad track ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$) was laid out at a temperature of 30°F . Determine the normal stress in the rails when the temperature reaches 125°F , assuming that the rails (a) are welded to form a continuous track, (b) are 39 ft long with $\frac{1}{4}$ -in. gaps between them.

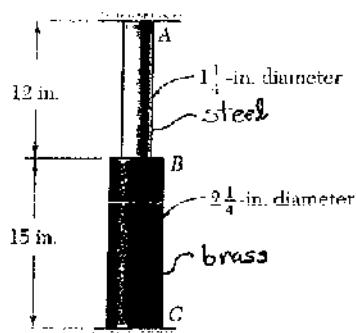
$$\delta_T = L \alpha (\Delta T) = (468)(6.5 \times 10^{-4})(125 - 30) = 288.99 \times 10^{-3} \text{ in.}$$

$$\delta_p = \frac{PL}{EA} = \frac{L}{E} \delta = \frac{468}{29 \times 10^6} \delta$$

(a) $\delta_p + \delta_T = 0 \quad \frac{468}{29 \times 10^6} \delta + 288.99 \times 10^{-3} = 0 \quad \delta = -17.91 \times 10^{-3} \text{ in.}$
 -17.91 ksi

(b) $\delta_p + \delta_T = \frac{1}{4} \quad \frac{468}{29 \times 10^6} \delta + 288.99 \times 10^{-3} = \frac{1}{4} \quad \delta = -2.42 \times 10^{-3} \text{ in.}$
 -2.42 ksi

Problem 2.52



2.52 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$) and portion *BC* is made of brass ($E_b = 17 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/^\circ\text{F}$). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 65°F , (b) the corresponding deflection of point *B*.

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2$$

Free thermal expansion

$$\begin{aligned} \delta_T &= L_{AB} \alpha_s (\Delta T) + L_{BC} \alpha_b (\Delta T) \\ &= (12)(6.5 \times 10^{-6})(65) + (15)(10.4 \times 10^{-6})(65) \\ &= 15.21 \times 10^{-3} \text{ in.} \end{aligned}$$

Shortening due to induced compressive force P

$$\begin{aligned} \delta_P &= \frac{PL_{AB}}{E_s A_{AB}} + \frac{PL_{BC}}{E_b A_{BC}} \\ &= \frac{12 P}{(29 \times 10^6)(1.2272)} + \frac{15 P}{(17 \times 10^6)(3.9761)} = 559.10 \times 10^{-9} P \end{aligned}$$

For zero net deflection $\delta_P = \delta_T$

$$(559.10 \times 10^{-9})P = 15.21 \times 10^{-3}$$

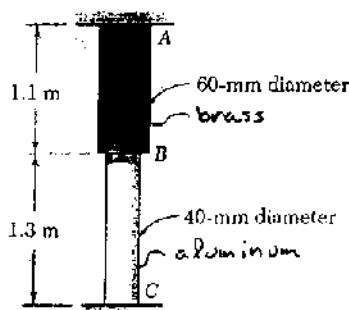
$$P = 27.204 \times 10^3 \text{ lb.}$$

$$(a) \sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{27.204 \times 10^3}{1.2272} = -22.2 \times 10^3 \text{ psi} = -22.2 \text{ ksi}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{27.204 \times 10^3}{3.9761} = -6.84 \times 10^3 \text{ psi} = -6.84 \text{ ksi}$$

$$\begin{aligned} (b) \delta_B &= +\frac{PL_{AB}}{E_s A_{AB}} - L_{AB} \alpha_s (\Delta T) \\ &= +\frac{(27.204 \times 10^3)(12)}{(29 \times 10^6)(1.2272)} - (12)(6.5 \times 10^{-6})(65) = +4.10 \times 10^{-3} \text{ in.} \\ &\quad \text{i.e. } 4.10 \times 10^{-3} \text{ in.} \uparrow \end{aligned}$$

Problem 2.53



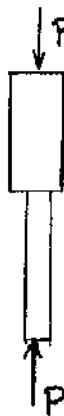
2.53 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of brass ($E_b = 105 \text{ GPa}$, $\alpha_b = 20.9 \times 10^{-6}/\text{C}$) and portion *BC* is made of aluminum ($E_a = 72 \text{ GPa}$, $\alpha_a = 23.9 \times 10^{-6}/\text{C}$). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 42°C , (b) the corresponding deflection of point *B*.

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(60)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-5} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(40)^2 = 1.2566 \times 10^3 \text{ mm}^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

Free thermal expansion

$$\begin{aligned} \delta_T &= L_{AB}\alpha_b(\Delta T) + L_{BC}\alpha_a(\Delta T) \\ &= (1.1)(20.9 \times 10^{-6})(42) + (1.3)(23.9 \times 10^{-6})(42) \\ &= 2.2705 \times 10^{-3} \text{ m} \end{aligned}$$



Shortening due to induced compressive force

$$\begin{aligned} \delta_P &= \frac{PL_{AB}}{E_b A_{AB}} + \frac{PL_{BC}}{E_a A_{BC}} \\ &= \frac{1.1 P}{(105 \times 10^9)(2.8274 \times 10^{-5})} + \frac{1.3 P}{(72 \times 10^9)(1.2566 \times 10^{-3})} \\ &= 18.074 \times 10^{-9} P \end{aligned}$$

For zero net deflection $\delta_P = \delta_T$

$$18.074 \times 10^{-9} P = 2.2705 \times 10^{-3}$$

$$P = 125.62 \times 10^3 \text{ N}$$

$$(a) \sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{125.62 \times 10^3}{2.8274 \times 10^{-5}} = -44.4 \times 10^6 \text{ Pa} = -44.4 \text{ MPa}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{125.62 \times 10^3}{1.2566 \times 10^{-3}} = -100.0 \times 10^6 \text{ Pa} = -100.0 \text{ MPa}$$

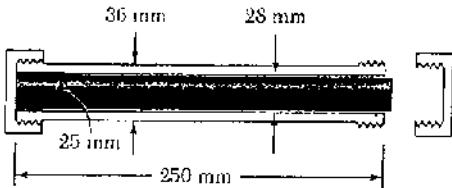
$$(b) \delta_B = +\frac{PL_{AB}}{E_b A_{AB}} - L_{AB}\alpha_b(\Delta T)$$

$$= \frac{(125.62 \times 10^3)(1.1)}{(105 \times 10^9)(2.8274 \times 10^{-5})} - (1.1)(20.9 \times 10^{-6})(42)$$

$$= -500 \times 10^{-6} \text{ m} = -0.500 \text{ mm}$$

i.e. $0.500 \text{ mm } \downarrow$

Problem 2.54



2.54 In Prob. 2.42, determine the average normal stress in the tube and the rod, assuming that the temperature was 15°C when the nuts were snugly fitted and that the final temperature is 55°C. (For aluminum, $\alpha_a = 23.6 \times 10^6/\text{C}$; for brass, $\alpha_b = 20.9 \times 10^6/\text{C}$).

2.42 A 250-mm-long aluminum tube ($E = 70 \text{ GPa}$) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105 \text{ GPa}$) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

$$A_{\text{tube}} = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\Delta T = 55 - 15 = 40^\circ\text{C}$$

$$\begin{aligned} \delta_{\text{tube}} &= \frac{PL}{E_{\text{tube}} A_{\text{tube}}} + L\alpha_{\text{tube}}(\Delta T) = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} + (0.250)(23.6 \times 10^6)(40) \\ &= 8.8815 \times 10^{-9} P + 236 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} \delta_{\text{rod}} &= -\frac{PL}{E_{\text{rod}} A_{\text{rod}}} + L\alpha_{\text{rod}}(\Delta T) = -\frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} + (0.250)(20.9 \times 10^6)(40) \\ &= -4.8505 \times 10^{-9} P + 209 \times 10^{-6} \end{aligned}$$

$$S^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{\text{tube}} = \delta_{\text{rod}} + S^*$$

$$8.8815 \times 10^{-9} P + 236 \times 10^{-6} = -4.8505 \times 10^{-9} P + 209 \times 10^{-6} + 375 \times 10^{-6}$$

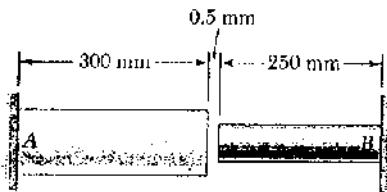
$$13.732 \times 10^{-9} P = 348 \times 10^{-6} \quad P = 25.342 \times 10^3 \text{ N}$$

$$\sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{25.342 \times 10^3}{402.12 \times 10^{-6}} = 63.0 \times 10^6 \text{ Pa} = 63.0 \text{ MPa}$$

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{25.342 \times 10^3}{490.87 \times 10^{-6}} = -51.6 \times 10^6 \text{ Pa} = -51.6 \text{ MPa}$$

Problem 2.55

2.55 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C , determine (a) the normal stress in the aluminum rod, (b) the change in length of the aluminum rod.



Aluminum	Stainless steel
$A = 2000 \text{ mm}^2$	$A = 800 \text{ mm}^2$
$E = 75 \text{ GPa}$	$E = 190 \text{ GPa}$
$\alpha = 23 \times 10^{-6}/^\circ\text{C}$	$\alpha = 17.3 \times 10^{-6}/^\circ\text{C}$

$$\Delta T = 140 - 20 = 120^\circ\text{C}$$

Free thermal expansion

$$\delta_T = L_a \alpha_a (\Delta T) + L_s \alpha_s (\Delta T)$$

$$= (0.300)(23 \times 10^{-6})(120) + (0.250)(17.3 \times 10^{-6})(120)$$

$$= 1.347 \times 10^{-3} \text{ m}$$

Shortening due to P to meet constraint.

$$\delta_P = 1.347 \times 10^{-3} - 0.5 \times 10^{-3} = 0.847 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \delta_P &= \frac{PL_a}{E_a A_a} + \frac{PL_s}{E_s A_s} = \left(\frac{L_a}{E_a A_a} + \frac{L_s}{E_s A_s} \right) P \\ &= \left(\frac{0.300}{(75 \times 10^9)(2000 \times 10^{-4})} + \frac{0.250}{(190 \times 10^9)(800 \times 10^{-4})} \right) P = 3.6447 \times 10^{-7} P \end{aligned}$$

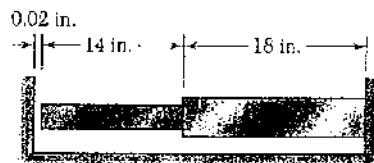
$$\text{Equating: } 3.6447 \times 10^{-7} P = 0.847 \times 10^{-3} \quad P = 232.39 \times 10^3 \text{ N}$$

$$(a) \sigma_a = -\frac{P}{A_a} = -\frac{232.39 \times 10^3}{2000 \times 10^{-4}} = -116.2 \times 10^6 \text{ Pa} \quad -116.2 \text{ MPa}$$

$$\begin{aligned} (b) \delta_a &= L_a \alpha_a (\Delta T) - \frac{PL_a}{E_a A_a} \\ &= (0.300)(23 \times 10^{-6})(120) - \frac{(232.39 \times 10^3)(0.300)}{(75 \times 10^9)(2000 \times 10^{-4})} = 363 \times 10^{-4} \text{ m} \end{aligned}$$

$$0.363 \text{ mm}$$

Problem 2.56



Bronze	Aluminum
$A = 2.4 \text{ in.}^2$	$A = 2.8 \text{ in.}^2$
$E = 15 \times 10^6 \text{ psi}$	$E = 10.6 \times 10^6 \text{ psi}$
$\alpha = 12 \times 10^{-6}/^\circ\text{F}$	$\alpha = 12.9 \times 10^{-6}/^\circ\text{F}$



2.56 Knowing that a 0.02-in. gap exists when the temperature is 75 °F, determine (a) the temperature at which the normal stress in the aluminum bar will be equal to -11 ksi, (b) the corresponding exact length of the aluminum bar.

$$\sigma_a = -11 \text{ ksi} = -11 \times 10^3 \text{ psi}$$

$$P = -\sigma_a A_a = (11 \times 10^3)(2.8) = 30.8 \times 10^3 \text{ lb.}$$

Shortening due to P

$$\begin{aligned} s_p &= \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a} \\ &= \frac{(30.8 \times 10^3)(14)}{(15 \times 10^6)(2.4)} + \frac{(30.8 \times 10^3)(18)}{(10.6 \times 10^6)(2.8)} \\ &= 30.657 \times 10^{-3} \text{ in.} \end{aligned}$$

Available elongation for thermal expansion

$$s_t = 0.02 + 30.657 \times 10^{-3} = 50.657 \times 10^{-3} \text{ in.}$$

$$\begin{aligned} \text{But } s_t &= L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T) \\ &= (14)(12 \times 10^{-6})(\Delta T) + (18)(12.9 \times 10^{-6})(\Delta T) = (400.2 \times 10^{-6}) \Delta T \end{aligned}$$

$$\text{Equating } (400.2 \times 10^{-6}) \Delta T = 50.657 \times 10^{-3} \quad \Delta T = 126.6^\circ\text{F}$$

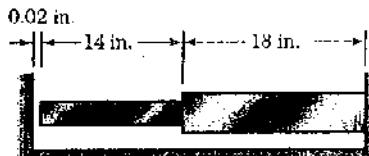
$$(a) \quad T_{hot} = T_{cold} + \Delta T = 75 + 126.6 = 201.6^\circ\text{F}$$

$$\begin{aligned} (b) \quad S_a &= L_a \alpha_a (\Delta T) - \frac{PL_a}{E_a A_a} \\ &= (18)(12.9 \times 10^{-6})(126.6) - \frac{(30.8 \times 10^3)(18)}{(10.6 \times 10^6)(2.8)} = 10.712 \times 10^{-3} \text{ in.} \end{aligned}$$

$$L_{exact} = 18 + 10.712 \times 10^{-3} = 18.0107 \text{ in.}$$

Problem 2.57

2.57 Determine (a) the compressive force in the bars shown after a temperature rise of 180°F , (b) the corresponding change in length of the bronze bar.



Bronze
 $A = 2.4 \text{ in.}^2$
 $E = 15 \times 10^6 \text{ psi}$
 $\alpha = 12 \times 10^{-6}/^{\circ}\text{F}$

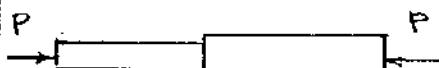
Aluminum
 $A = 2.8 \text{ in.}^2$
 $E = 10.6 \times 10^6 \text{ psi}$
 $\alpha = 12.9 \times 10^{-6}/^{\circ}\text{F}$

Thermal expansion if free of constraint

$$S_T = L_b d_b (\Delta T) + L_a \alpha_a (\Delta T)$$

$$= (14)(12 \times 10^{-6})(180) + (18)(12.9 \times 10^{-6})(180)$$

$$= 72.036 \times 10^{-3} \text{ in.}$$



Constrained expansion. $S = 0.02 \text{ in}$

Shortening due to induced compressive force P

$$S_p = 72.036 \times 10^{-3} - 0.02 = 52.036 \times 10^{-3} \text{ in.}$$

$$\text{But } S_p = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a} = \left(\frac{L_b}{E_b A_b} + \frac{L_a}{E_a A_a} \right) P$$

$$= \left(\frac{14}{(15 \times 10^6)(2.4)} + \frac{18}{(10.6 \times 10^6)(2.8)} \right) P = 995.36 \times 10^{-9} P$$

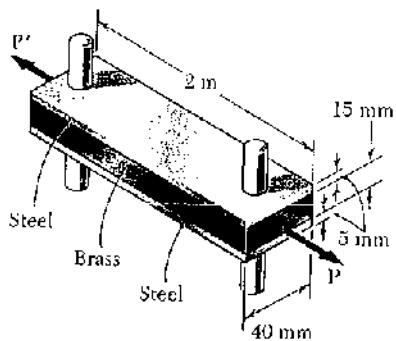
$$(a) \text{ Equating } 995.36 \times 10^{-9} P = 52.036 \times 10^{-3} \quad P = 52.279 \times 10^3 \text{ lb}$$

52.3 kips

$$(b) S_b = L_b d_b (\Delta T) - \frac{PL_b}{E_b A_b}$$

$$= (14)(12 \times 10^{-6})(180) - \frac{(52.279 \times 10^3)(14)}{(15 \times 10^6)(2.4)} = 9.91 \times 10^{-3} \text{ in.}$$

Problem 2.58



2.58 Two steel bars ($E_s = 200 \text{ GPa}$ and $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$) are used to reinforce a brass bar ($E_b = 105 \text{ GPa}$, $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$) which is subjected to a load $P = 25 \text{ kN}$. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

(a) Required temperature change for fabrication

$$\Delta L = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

(a) Temperature change required to expand steel bar by this amount

$$\Delta L = L \alpha_s \Delta T, \quad 0.5 \times 10^{-3} = (2.00)(11.7 \times 10^{-6})(\Delta T), \quad \Delta T =$$

$$0.5 \times 10^{-3} = (2)(11.7 \times 10^{-6})(\Delta T)$$

$$\Delta T = 21.368 \text{ } ^\circ\text{C}$$

$$21.4 \text{ } ^\circ\text{C}$$

(b) Once assembled, a tensile force P^* develops in the steel and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$(\delta_p)_s = \frac{P^* L}{A_s E_s} = \frac{P^*(2.00)}{(400 \times 10^{-6})(200 \times 10^9)} = 25 \times 10^{-9} P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$$(\delta_p)_b = \frac{P^* L}{A_b E_b} = \frac{P^*(2.00)}{(600 \times 10^{-6})(105 \times 10^9)} = 31.746 \times 10^{-9} P^*$$

But $(\delta_p)_s + (\delta_p)_b$ is equal to the initial amount of misfit

$$(\delta_p)_s + (\delta_p)_b = 0.5 \times 10^{-3}, \quad 56.746 \times 10^{-9} P^* = 0.5 \times 10^{-3}$$

$$P^* = 8.811 \times 10^3 \text{ N}$$

Stresses due to fabrication

$$\text{Steel: } \sigma_s^* = \frac{P^*}{A_s} = \frac{8.811 \times 10^3}{400 \times 10^{-6}} = 22.03 \times 10^6 \text{ Pa} = 22.03 \text{ MPa}$$

$$\text{Brass: } \sigma_b^* = -\frac{P^*}{A_b} = -\frac{8.811 \times 10^3}{600 \times 10^{-6}} = -14.68 \times 10^6 \text{ Pa} = -14.68 \text{ MPa}$$

To these stresses must be added the stresses due to the 25 kN load.

Continued

Problem 2.58 continued

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_b = \frac{A_s E_s}{L} \delta' = \frac{(400 \times 10^{-6})(200 \times 10^9)}{2.00} \delta' = 40 \times 10^6 \delta'$$

$$P_b = \frac{A_b E_b}{L} \delta' = \frac{(600 \times 10^{-6})(105 \times 10^9)}{2.00} \delta' = 31.5 \times 10^6 \delta'$$

$$\text{Total } P = P_s + P_b = 25 \times 10^3 \text{ N}$$

$$40 \times 10^6 \delta' + 31.5 \times 10^6 \delta' = 25 \times 10^3 \quad \delta' = 349.65 \times 10^{-6} \text{ m}$$

$$P_s = (40 \times 10^6)(349.65 \times 10^{-6}) = 13.986 \times 10^3 \text{ N}$$

$$P_b = (31.5 \times 10^6)(349.65 \times 10^{-6}) = 11.140 \times 10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986 \times 10^3}{400 \times 10^{-6}} = 34.97 \times 10^6 \text{ Pa}$$

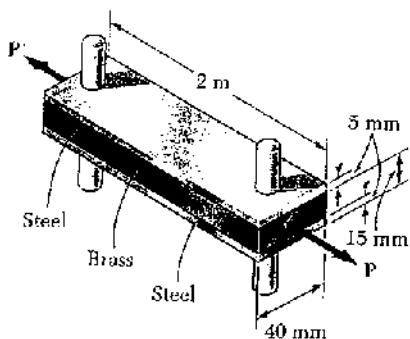
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140 \times 10^3}{600 \times 10^{-6}} = 18.36 \times 10^6 \text{ Pa}$$

Add stress due to fabrication

$$\sigma_s = 34.97 \times 10^6 + 22.03 \times 10^6 = 57.0 \times 10^6 \text{ Pa} = 57.0 \text{ MPa}$$

$$\sigma_b = 18.36 \times 10^6 - 14.68 \times 10^6 = 3.68 \times 10^6 \text{ Pa} = 3.68 \text{ MPa} \blacktriangleleft$$

Problem 2.59



2.59 Determine the maximum load P that may be applied to the brass bar of Prob. 2.58 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

2.58 Two steel bars ($E_s = 200 \text{ GPa}$ and $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$) are used to reinforce a brass bar ($E_b = 105 \text{ GPa}$, $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$) which is subjected to a load $P = 25 \text{ kN}$. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

See solution to Problem 2.58 to obtain the fabrication stresses

$$\sigma_s^* = 22.03 \text{ MPa} \quad \sigma_b^* = -14.68 \text{ MPa}$$

Allowable stresses: $\sigma_{s,\text{app}} = 30 \text{ MPa}$, $\sigma_{b,\text{all}} = 25 \text{ MPa}$

Available stress increase from load

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$

$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains

$$\epsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97 \times 10^6}{200 \times 10^9} = 39.85 \times 10^{-6}$$

$$\epsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68 \times 10^6}{105 \times 10^9} = 377.9 \times 10^{-6}$$

Smaller value governs: $\epsilon = 39.85 \times 10^{-6}$

$$\text{Areas: } A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$$

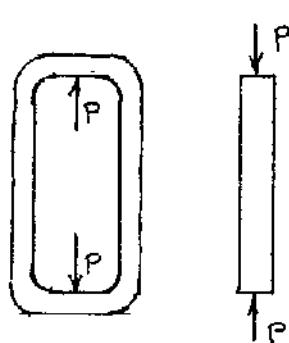
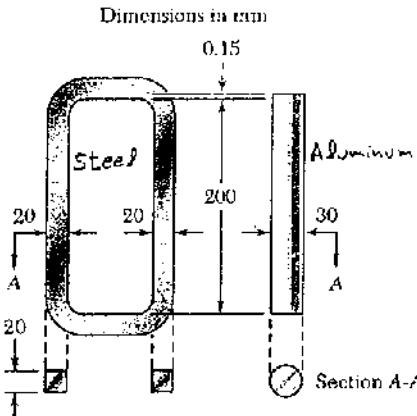
$$P_s = E_s A_s \epsilon = (200 \times 10^9)(400 \times 10^{-6})(39.85 \times 10^{-6}) = 3.188 \times 10^3 \text{ N}$$

$$P_b = E_b A_b \epsilon = (105 \times 10^9)(600 \times 10^{-6})(39.85 \times 10^{-6}) = 2.511 \times 10^3 \text{ N}$$

Total allowable additional force

$$P = P_s + P_b = 3.188 \times 10^3 + 2.511 \times 10^3 = 5.70 \times 10^3 \text{ N} \\ = 5.70 \text{ kN}$$

Problem 2.60



2.60 An aluminum rod ($E_a = 70 \text{ GPa}$, $\alpha_a = 23.6 \times 10^{-6}/\text{C}$) and a steel link ($E_s = 200 \text{ GPa}$, $\alpha_s = 11.7 \times 10^{-6}/\text{C}$) have the dimensions shown at a temperature of 20°C . The steel link is heated until the aluminum rod can be fitted freely into it. The temperature of the whole assembly is then raised to 150°C . Determine (a) the final stress in the rod, (b) in the link.

$$\Delta T = T_{\text{final}} - T_{\text{initial}} = 150^\circ\text{C} - 20^\circ\text{C} = 130^\circ\text{C}$$

Free thermal expansion of each part

$$\text{Aluminum rod: } (\delta_T)_a = L \alpha_a (\Delta T)$$

$$(\delta_T)_a = (0.200)(23.6 \times 10^{-6})(130) = 613.6 \times 10^{-6} \text{ m}$$

$$\text{Steel link: } (\delta_T)_s = L \alpha_s (\Delta T)$$

$$(\delta_T)_s = (0.200)(11.7 \times 10^{-6})(130) = 304.2 \times 10^{-6} \text{ m}$$

Let P be the compressive force developed in the aluminum rod. It is also the tensile force in the steel link.

$$\text{Aluminum rod: } A_a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$(\delta_p)_a = \frac{PL}{E_a A_a} = \frac{(P)(0.200)}{(70 \times 10^9)(706.86 \times 10^{-6})} = 4.04203 \times 10^{-9} P$$

$$\text{Steel link: } A_s = (2)(20)^2 = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$$

$$(\delta_p)_s = \frac{PL}{E_s A_s} = \frac{(P)(0.200)}{(200 \times 10^9)(800 \times 10^{-6})} = 1.25 \times 10^{-9} P$$

Matching the deformed lengths

$$0.200 + (\delta_T)_s + (\delta_p)_s = 0.200 + 0.15 \times 10^{-3} + (\delta_T)_a - (\delta_p)_a$$

$$(\delta_p)_s + (\delta_p)_a = 0.15 \times 10^{-3} + (\delta_T)_a - (\delta_T)_s$$

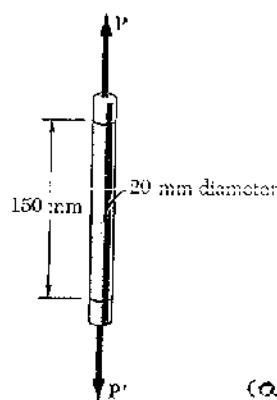
$$1.25 \times 10^{-9} P + 4.04203 \times 10^{-9} P = 0.15 \times 10^{-3} + 613.6 \times 10^{-6} - 304.2 \times 10^{-6}$$

$$P = 86.810 \times 10^3 \text{ N}$$

$$(a) \text{ Stress in rod} \quad \sigma_a = -\frac{P}{A_a} = -\frac{86.810 \times 10^3}{706.86 \times 10^{-6}} = -122.8 \times 10^6 \text{ Pa} = -122.8 \text{ MPa}$$

$$(b) \text{ Stress in link} \quad \sigma_s = \frac{P}{A_s} = \frac{86.810 \times 10^3}{800 \times 10^{-6}} = 108.5 \times 10^6 \text{ Pa} = 108.5 \text{ MPa}$$

Problem 2.61



2.61 In a standard tensile test an aluminum rod of 20-mm diameter is subjected to a tension force of $P = 30 \text{ kN}$. Knowing that $\nu = 0.35$ and $E = 70 \text{ GPa}$, determine (a) the elongation of the rod in an 150-mm gage length, (b) the change in diameter of the rod.

$$P = 30 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (20)^2 = 314.159 \text{ mm}^2 = 314.159 \times 10^{-6} \text{ m}^2$$

$$\sigma_y = \frac{P}{A} = \frac{30 \times 10^3}{314.159 \times 10^{-6}} = 95.493 \times 10^6 \text{ Pa}$$

$$\epsilon_y = \frac{\sigma_y}{E} = \frac{95.493 \times 10^6}{10 \times 10^9} = 1.36418 \times 10^{-3}$$

$$(a) \Delta L = L \epsilon_y = (150 \times 10^{-3})(1.36418 \times 10^{-3}) = 205 \times 10^{-6} \text{ m}$$

$$0.205 \text{ mm}$$

$$\epsilon_x = -\nu \epsilon_y = -(0.35)(1.36418 \times 10^{-3}) = -477.46 \times 10^{-6}$$

$$(b) \Delta x = d \epsilon_x = (20 \times 10^{-3})(-477.46 \times 10^{-6}) = -9.55 \times 10^{-6} \text{ m}$$

$$-0.00955 \text{ mm}$$

Problem 2.62

$$A = (1.6)(12) = 19.2 \text{ mm}^2$$

$$= 19.2 \times 10^{-6} \text{ m}^2$$

$$P = 2.75 \times 10^3 \text{ N}$$

$$\sigma_x = \frac{P}{A} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6 \text{ Pa}$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\epsilon_y = \epsilon_z = -\nu \epsilon_x = -(0.30)(716.15 \times 10^{-6}) = -214.84 \times 10^{-6}$$

$$(a) L = 0.050 \text{ m} \quad \Delta x = L \epsilon_x = (0.050)(716.15 \times 10^{-6}) = 35.81 \times 10^{-6} \text{ m}$$

$$0.0358 \text{ mm}$$

$$(b) W = 0.012 \text{ m} \quad \Delta y = W \epsilon_y = (0.012)(-214.84 \times 10^{-6}) = -2.578 \times 10^{-6} \text{ m}$$

$$-0.00258 \text{ mm}$$

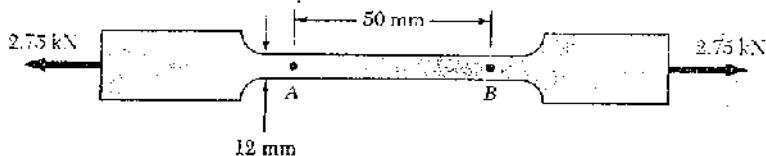
$$(c) t = 0.0016 \text{ m} \quad \Delta z = E \epsilon_z = (0.0016)(-214.84 \times 10^{-6}) = -3437 \times 10^{-9} \text{ m}$$

$$-0.0003437 \text{ mm}$$

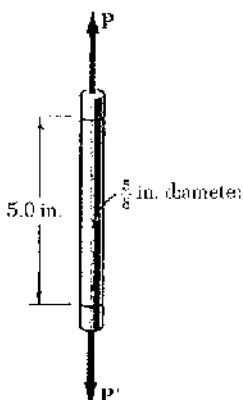
$$(d) A = w_0(1 + \epsilon_y)t_0(1 + \epsilon_z) = w_0t_0(1 + \epsilon_y + \epsilon_z + \epsilon_y \epsilon_z) \quad A_0 = w_0t_0$$

$$\Delta A = A - A_0 = w_0t_0(\epsilon_y + \epsilon_z + \text{negligible term}) = 2w_0t_0\epsilon_y$$

$$= (2)(0.012)(0.0016)(-214.84 \times 10^{-6}) = -8.25 \times 10^{-9} \text{ m}^2 = -0.00825 \text{ mm}^2$$



Problem 2.63



2.63 A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a $\frac{5}{8}$ -in.-diameter rod and it is subjected to a 800 lb tensile force. Knowing that an elongation of 0.45 in. and a decrease in diameter of 0.025 in. are observed in a 5-in. gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio of the material.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{5}{8}\right)^2 = 0.306796 \text{ in}^2$$

$$P = 800 \text{ lb}$$

$$\sigma_y = \frac{P}{A} = \frac{800}{0.306796} = 2.6076 \times 10^3 \text{ psi}$$

$$\epsilon_y = \frac{\delta_y}{L} = \frac{0.45}{5.0} = 0.090$$

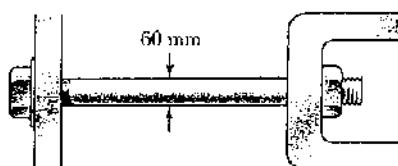
$$\epsilon_x = \frac{\delta_x}{d} = \frac{-0.025}{0.625} = -0.040$$

$$E = \frac{\sigma_y}{\epsilon_y} = \frac{2.6076 \times 10^3}{0.090} = 28.973 \times 10^3 \text{ psi} \quad E = 29.0 \times 10^3 \text{ psi}$$

$$\nu = -\frac{\epsilon_x}{\epsilon_y} = -\frac{-0.040}{0.090} = 0.44444 \quad \nu = 0.444$$

$$G = \frac{E}{2(1+\nu)} = \frac{28.973 \times 10^3}{2(1+0.44444)} = 10.029 \times 10^3 \text{ psi} \quad G = 10.03 \times 10^3 \text{ psi}$$

Problem 2.64



2.64 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 200 \text{ GPa}$ and $\nu = 0.29$, determine the internal force in the bolt, if the diameter is observed to decrease by 13 μm .

$$\delta_y = -13 \times 10^{-6} \text{ m} \quad d = 60 \times 10^{-3} \text{ m}$$

$$\epsilon_y = \frac{\delta_y}{d} = -\frac{13 \times 10^{-6}}{60 \times 10^{-3}} = -216.67 \times 10^{-6}$$

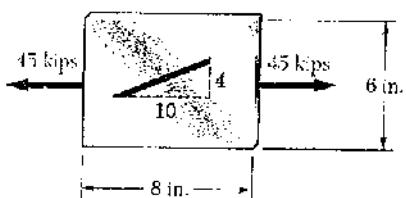
$$\nu = -\frac{\epsilon_y}{\epsilon_x} \therefore \epsilon_x = -\frac{\epsilon_y}{\nu} = \frac{216.67 \times 10^{-6}}{0.29} = 747.13 \times 10^{-6}$$

$$\sigma_x = E \epsilon_x = (200 \times 10^9) (747.13 \times 10^{-6}) = 149.43 \times 10^6 \text{ Pa}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (60)^2 = 2.827 \times 10^3 \text{ mm}^2 = 2.827 \times 10^{-3} \text{ m}^2$$

$$F = \sigma_x A = (149.43 \times 10^6) (2.827 \times 10^{-3}) = 422 \times 10^3 \text{ N} \\ = 422 \text{ kN}$$

Problem 2.65

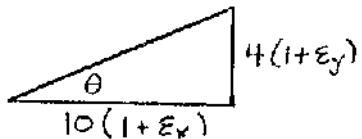


2.65 A line of slope 4:10 has been scribed on a cold-rolled yellow-brass plate, 6. in. wide and $\frac{1}{4}$ in. thick. Using the data available in Appendix B, determine the slope of the line when the plate is subjected as shown to a 45-kip centric axial load.

From Appendix B, $E = 15 \times 10^6$ psi, $G = 5.6 \times 10^6$ psi

$$G = \frac{E}{2(1+\nu)} \quad \nu = \frac{E}{2G} - 1 = \frac{15}{2(5.6)} - 1 = 0.3393$$

$$A = (6)(\frac{1}{4}) = 1.50 \text{ in}^2$$



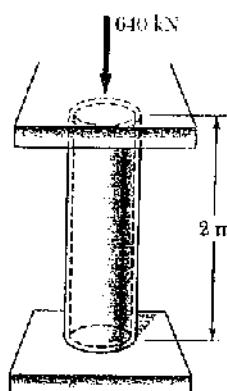
$$\sigma_x = \frac{P}{A} = \frac{45 \times 10^3}{1.50} = 30 \times 10^3 \text{ psi}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{30 \times 10^3}{15 \times 10^6} = 0.00200$$

$$\varepsilon_y = -\nu \varepsilon_x = -(0.3393)(0.00200) = -0.0006786$$

$$\tan \theta = \frac{4(1+\varepsilon_y)}{10(1+\varepsilon_x)} = \frac{(4)(1+0.0006786)}{(10)(1+0.00200)} = 0.39893$$

Problem 2.66



2.66 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (a) the change in length of the pipe, (b) the change in its outer diameter, (c) the change in its wall thickness.

$$d_o = 240 \text{ mm} \quad t = 10 \text{ mm} \quad d_i = d_o - 2t = 220 \text{ mm}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^{-3} \text{ mm}^2 \\ = 7.2257 \times 10^{-3} \text{ m}^2$$

$$P = 640 \times 10^3 \text{ N}$$

$$(a) S = -\frac{PL}{AE} = -\frac{(640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3} \text{ m} \\ = -2.43 \text{ mm}$$

$$\varepsilon = \frac{S}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$$

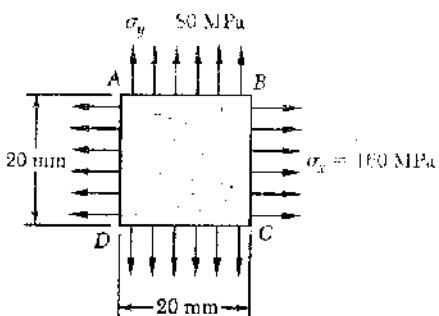
$$\varepsilon_{ext} = -\nu \varepsilon = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$$

$$(b) \Delta d_o = d_o \varepsilon_{ext} = (240)(400.4 \times 10^{-6}) = 0.0961 \text{ mm}$$

$$(c) \Delta t = t \varepsilon_{ext} = (10)(400.4 \times 10^{-6}) = 0.00400 \text{ mm}$$

Problem 2.67

2.67 A 20-mm square has been scribed on the side of a large steel pressure vessel. After pressurization, the biaxial stress condition of the square is as shown. Using the data available in Appendix B, for structural steel, determine the percent change in the slope of diagonal DB due to the pressurization of the vessel.



For structural steel Appendix B gives

$$E = 200 \text{ GPa}, \quad G = 77.2 \text{ GPa}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\nu = \frac{E}{2G} - 1 = \frac{200}{2(77.2)} - 1 = 0.2953$$

$$\sigma_x = 160 \times 10^6 \text{ Pa} \quad \sigma_y = 80 \times 10^6 \text{ Pa}$$

$$\sigma_z \approx 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z)$$

$$= \frac{1}{200 \times 10^9} [160 \times 10^6 - (0.2953)(80 \times 10^6)]$$

$$= 0.00068187$$

$$\epsilon_y = \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z)$$

$$= \frac{1}{200 \times 10^9} [-(0.2953)(160 \times 10^6) + 80 \times 10^6]$$

$$= 0.00016373$$

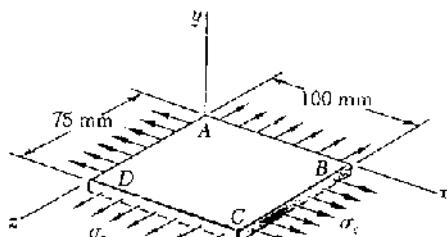
$$\text{Slope of } DB \quad \tan \theta = \frac{1 + \epsilon_y}{1 + \epsilon_x} = \frac{1 + 0.00016373}{1 + 0.00068187} = 0.9994822$$

$$\text{Change in slope} \quad 0.9994822 - 1 = -0.000518$$

$$\frac{\Delta \tan \theta}{\tan 45^\circ} \times 100\% = -0.0518\%$$

Problem 2.68

2.68 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120 \text{ MPa}$ and $\sigma_y = 160 \text{ MPa}$. Knowing that the properties of the fabric can be approximated as $E = 87 \text{ GPa}$ and $\nu = 0.34$, determine the change in length of (a) side AB , (b) side BC , (c) diagonal AC .



$$\sigma_x = 120 \times 10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160 \times 10^6 \text{ Pa}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z)$$

$$= \frac{1}{87 \times 10^9} [120 \times 10^6 - (0.34)(160 \times 10^6)]$$

$$= 754.02 \times 10^{-6}$$

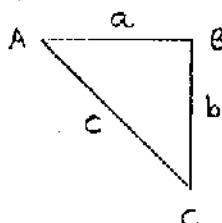
$$\epsilon_z = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_z) = \frac{1}{87 \times 10^9} [-(0.34)(120 \times 10^6) + 160 \times 10^6]$$

$$= 1.3701 \times 10^{-5}$$

$$(a) S_{AB} = (\overline{AB}) \epsilon_x = (100 \text{ mm})(754.02 \times 10^{-6}) = 0.0754 \text{ mm}$$

$$(b) S_{BC} = (\overline{BC}) \epsilon_z = (75 \text{ mm})(1.3701 \times 10^{-5}) = 0.1028 \text{ mm}$$

(c)



Labeled sides of right triangle ABC as a , b , and c .

$$c^2 = a^2 + b^2$$

Obtain differentials by calculus.

$$2c \, dc = 2a \, da + 2b \, db$$

$$dc = \frac{a}{c} da + \frac{b}{c} db$$

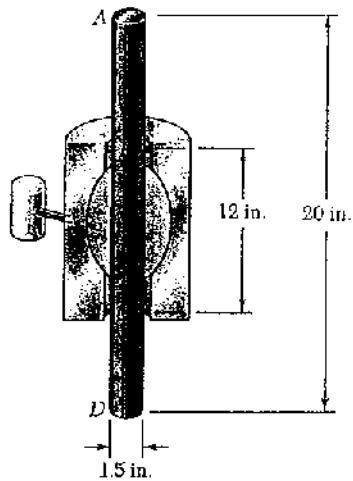
$$\text{But, } a = 100 \text{ mm}, \quad b = 75 \text{ mm}, \quad c = \sqrt{100^2 + 75^2} = 125 \text{ mm}$$

$$da = S_{AB} = 0.0754 \text{ mm} \quad db = S_{BC} = 0.1028 \text{ mm}$$

$$S_{AC} = dc = \frac{100}{125} (0.0754) + \frac{75}{125} (0.1028) = 0.1220 \text{ mm}$$

Problem 2.69

2.69 The aluminum rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion BC of the rod. Knowing that $E = 10.1 \times 10^6$ psi and $\nu = 0.36$, determine (a) the change in the total length AD , (b) the change in diameter at the middle of the rod.



$$\sigma_x = \sigma_z = -p = -6000 \text{ psi} \quad \sigma_y = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z)$$

$$= \frac{1}{10.1 \times 10^6} [-6000 - (0.36)(0) - (0.36)(-6000)]$$

$$= -380.198 \times 10^{-6}$$

$$\epsilon_y = \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z)$$

$$= \frac{1}{10.1 \times 10^6} [-(0.36)(-6000) + 0 - (0.36)(-6000)]$$

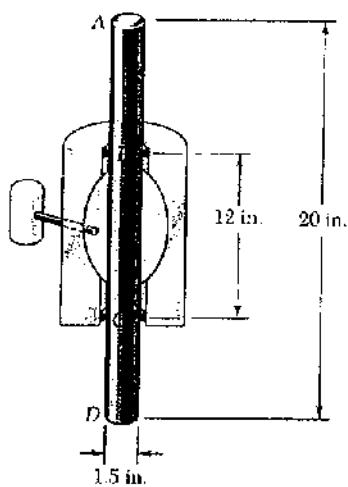
$$= 427.72 \times 10^{-6}$$

Length subject to strain ϵ_x : $L = 12 \text{ in.}$

$$(a) \Delta L = L \epsilon_x = (12)(427.72 \times 10^{-6}) = 5.13 \times 10^{-3} \text{ in.}$$

$$(b) \Delta s_x = d \epsilon_x = (1.5)(-380.198 \times 10^{-6}) = -0.570 \times 10^{-3} \text{ in.}$$

Problem 2.70



2.70 For the rod of Prob. 2.69, determine the forces that should be applied to the ends *A* and *D* of the rod (*a*) if the axial strain in portion *BC* of the rod is to remain zero as the hydrostatic pressure is applied, (*b*) if the total length *AD* of the rod is to remain unchanged.

2.69 The aluminum rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion *BC* of the rod. Knowing that $E = 10.1 \times 10^6$ psi and $\nu = 0.36$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

Over the pressurized portion *BC*

$$\sigma_x = \sigma_z = -p \quad \epsilon_y = \frac{\sigma_y}{E}$$

$$(\epsilon_y)_{BC} = \frac{1}{E} (-\sigma_x + \sigma_y - 2\sigma_z) \\ = \frac{1}{E} (2\nu p + \sigma_y)$$

$$(a) (\epsilon_y)_{BC} = 0 \quad 2\nu p + \sigma_y = 0$$

$$\sigma_y = -2\nu p = -(2)(0.36)(6000) \\ = -4320 \text{ psi}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1.5)^2 = 1.76715 \text{ in}^2$$

$$F = A \sigma_y = (1.76715)(-4320) = -7630 \text{ lb.}$$

i.e. ... 7630 lb compression

(b) Over unpressurized portions *AB* and *CD* $\sigma_x = \sigma_z = 0$

$$(\epsilon_y)_{AB} = (\epsilon_y)_{CD} = \frac{\sigma_y}{E}$$

For no change in length

$$S = L_{AB} (\epsilon_y)_{AB} + L_{BC} (\epsilon_y)_{BC} + L_{CD} (\epsilon_y)_{CD} = 0$$

$$(L_{AB} + L_{CD}) (\epsilon_y)_{AB} + L_{BC} (\epsilon_y)_{BC} = 0$$

$$(20 - 12) \frac{\sigma_y}{E} + \frac{12}{E} (2\nu p + \sigma_y) = 0$$

$$\sigma_y = -\frac{24\nu p}{20} = -\frac{(24)(0.36)(6000)}{20} = -2592 \text{ psi}$$

$$P = A \sigma_y = (1.76715)(-2592) = -4580 \text{ lb.}$$

i.e. 4580 lb compression

Problem 2.71

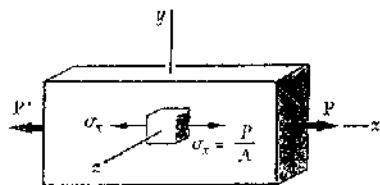


Fig 1.40 (a)

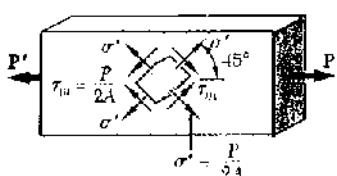
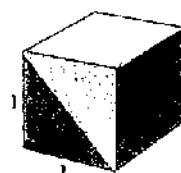
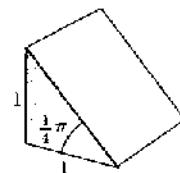


Fig 1.40 (b)

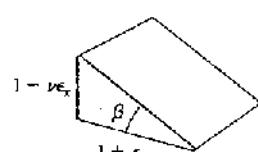
2.71 For a member under axial loading, express the normal strain ϵ' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ϵ_x by (a) comparing the hypotenuses of the triangles shown in Fig. 2.54, which represent respectively an element before and after deformation, (b) using the values of the corresponding stresses σ' and σ_x shown in Fig. 1.40, and the generalized Hooke's law.



(a)



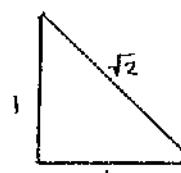
(b)



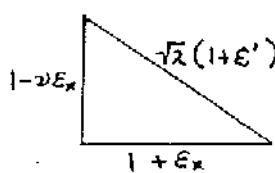
(c)

Fig 2.54

(a)



Before deformation



After deformation

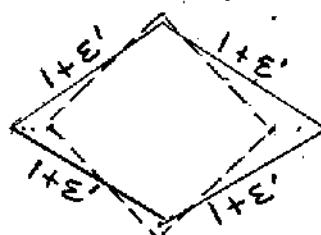
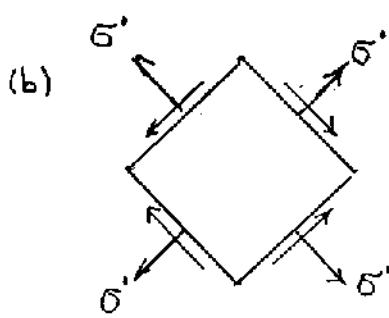
$$[\sqrt{1 + \epsilon'^2}]^2 = (1 + \epsilon_x)^2 + (1 - \nu \epsilon_x)^2$$

$$2(1 + 2\epsilon' + \epsilon'^2) = 1 + 2\epsilon_x + \epsilon_x^2 + 1 - 2\nu\epsilon_x + \nu^2\epsilon_x^2$$

$$4\epsilon' + 2\epsilon'^2 = 2\epsilon_x + \epsilon_x^2 - 2\nu\epsilon_x + \nu^2\epsilon_x^2$$

Neglect squares as small $4\epsilon' = 2\epsilon_x - 2\nu\epsilon_x$

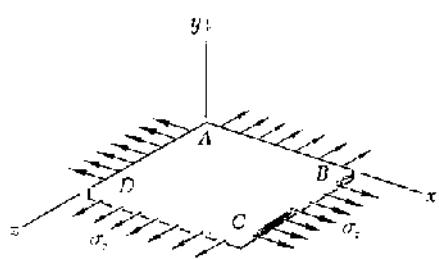
$$\epsilon' = \frac{1 - \nu}{2} \epsilon_x$$



$$\begin{aligned} \epsilon' &= \frac{\sigma'}{E} - \frac{\nu \sigma'}{E} \\ &= \frac{1 - \nu}{E} - \frac{P}{2A} \\ &= \frac{1 - \nu}{2E} \epsilon_x \end{aligned}$$

$$= \frac{1 - \nu}{2} \epsilon_x$$

Problem 2.72



2.72 The homogeneous plate $ABCD$ is subjected to a biaxial loading as shown. It is known that $\sigma_x = \sigma_z$ and that the change in length of the plate in the x direction must be zero, that is, $\epsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ratio, determine (a) the required magnitude of σ_x , (b) the ratio σ_x / ϵ_x .

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \epsilon_x = 0$$

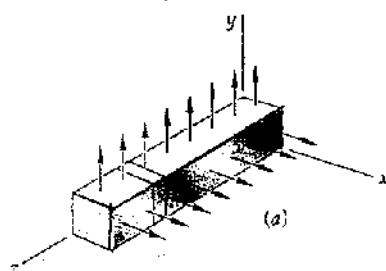
$$\epsilon_x = \frac{1}{E} (\epsilon_x - \nu \epsilon_y - \nu \epsilon_z) = \frac{1}{E} (\epsilon_x - \nu \epsilon_0)$$

$$(a) \quad \epsilon_x = \nu \epsilon_0$$

$$(b) \quad \epsilon_x = \frac{1}{E} (-\nu \epsilon_x - \nu \epsilon_y + \epsilon_z) = \frac{1}{E} (-\nu^2 \epsilon_0 - 0 + \epsilon_0) = \frac{1-\nu^2}{E} \epsilon_0$$

$$\frac{\sigma_0}{\epsilon_0} = \frac{E}{1-\nu^2}$$

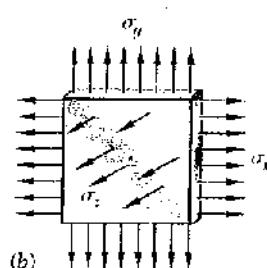
Problem 2.73



2.73 In many situations physical constraints prevent strain from occurring in a given direction; for example $\epsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express σ_z , ϵ_x and ϵ_y as follows:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\epsilon_x = \frac{1}{E} [(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y], \quad \epsilon_y = \frac{1}{E} [(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

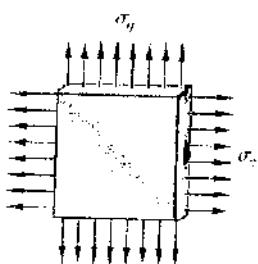


$$\epsilon_z = 0 = \frac{1}{E} (-\nu \epsilon_x - \nu \epsilon_y + \epsilon_z) \quad \text{or} \quad \epsilon_z = \nu(\epsilon_x + \epsilon_y)$$

$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\epsilon_x - \nu \epsilon_y - \nu \epsilon_z) = \frac{1}{E} [\epsilon_x - \nu \epsilon_y - \nu^2(\epsilon_x + \epsilon_y)] \\ &= \frac{1}{E} [(1-\nu^2)\epsilon_x - \nu(1+\nu)\epsilon_y] \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \epsilon_x + \epsilon_y - \nu \epsilon_z) = \frac{1}{E} [-\nu \epsilon_x + \epsilon_y - \nu^2(\epsilon_x + \epsilon_y)] \\ &= \frac{1}{E} [(1-\nu^2)\epsilon_y - \nu(1+\nu)\epsilon_x] \end{aligned}$$

Problem 2.74



2.74 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ϵ_x and ϵ_y have been determined experimentally, we can express σ_x , σ_y , and ϵ_z as follows:

$$\sigma_x = E \frac{\epsilon_x + \nu \epsilon_y}{1 - \nu^2} \quad \sigma_y = E \frac{\epsilon_y + \nu \epsilon_x}{1 - \nu^2} \quad \epsilon_z = -\frac{\nu}{1 - \nu} (\epsilon_x + \epsilon_y)$$

$$\epsilon_z = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) \quad (1) \quad \epsilon_y = \frac{1}{E} (-\nu \sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by ν and adding to (1)

$$\epsilon_x + \nu \epsilon_y = \frac{1 - \nu^2}{E} \sigma_x \quad \text{or} \quad \sigma_x = \frac{E}{1 - \nu^2} (\epsilon_x + \nu \epsilon_y)$$

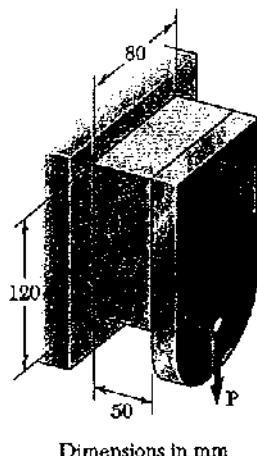
Multiplying (1) by ν and adding to (2)

$$\epsilon_y + \nu \epsilon_x = \frac{1 - \nu^2}{E} \sigma_y \quad \text{or} \quad \sigma_y = \frac{E}{1 - \nu^2} (\epsilon_y + \nu \epsilon_x)$$

$$\begin{aligned} \epsilon_z &= \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y) = -\frac{\nu}{E} \cdot \frac{E}{1 - \nu^2} (\epsilon_x + \nu \epsilon_y + \epsilon_y + \nu \epsilon_x) \\ &= -\frac{\nu(1 + \nu)}{1 - \nu^2} (\epsilon_x + \epsilon_y) = -\frac{\nu}{1 - \nu} (\epsilon_x + \epsilon_y) \end{aligned}$$

Problem 2.75

2.75 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load P is applied. Knowing that for the plastic used $G = 1050 \text{ MPa}$, determine the deflection of the plate.



Dimensions in mm



$$A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$P = 240 \times 10^3 \text{ N}$$

$$\Sigma = \frac{P}{A} = \frac{240 \times 10^3}{9.6 \times 10^{-3}} = 25 \times 10^6 \text{ Pa}$$

$$G = 1050 \times 10^6 \text{ Pa}$$

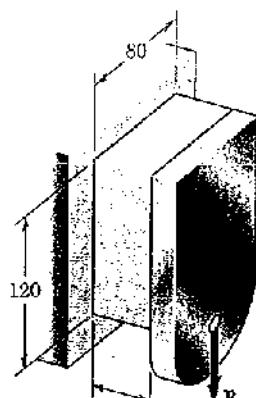
$$\gamma = \frac{\Sigma}{G} = \frac{25 \times 10^6}{1050 \times 10^6} = 23.810 \times 10^{-3}$$

$$h = 50 \text{ mm} = 0.050 \text{ m}$$

$$S = h \gamma = (0.050)(23.810 \times 10^{-3})$$

$$= 1.190 \times 10^{-3} \text{ m} \quad 1.091 \text{ mm} \downarrow$$

Problem 2.76



Dimensions in mm

2.76 What load P should be applied to the plate of Prob. 2.75 to produce a 1.5-mm deflection?

2.75 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load P is applied. Knowing that for the plastic used $G = 1050 \text{ MPa}$, determine the deflection of the plate.



$$S = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$h = 50 \text{ mm} = 50 \times 10^{-3} \text{ m}$$

$$\gamma = \frac{S}{h} = \frac{1.5 \times 10^{-3}}{50 \times 10^{-3}} = 30 \times 10^{-3}$$

$$G = 1050 \times 10^6 \text{ Pa}$$

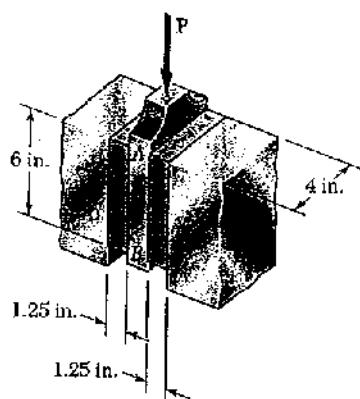
$$\tau = G\gamma = (1050 \times 10^6)(30 \times 10^{-3}) \\ = 31.5 \times 10^6 \text{ Pa}$$

$$A = (120)(80) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$P = \tau A = (31.5 \times 10^6)(9.6 \times 10^{-3}) = 302 \times 10^3 \text{ N} \quad 30.2 \text{ kN} \blacktriangleleft$$

Problem 2.77

2.77 A vibration isolation unit consists of two blocks of hard rubber bonded a plate AB and to rigid supports as shown. Knowing that a force of magnitude $P = 6 \text{ kips}$ causes a deflection $\delta = \frac{1}{16} \text{ in.}$ of plate AB , determine the modulus of rigidity of the rubber used.



$$F = \frac{1}{2}P = \frac{1}{2}(6000) = 3000 \text{ lb}$$

$$A = (6)(4) = 24 \text{ in}^2$$

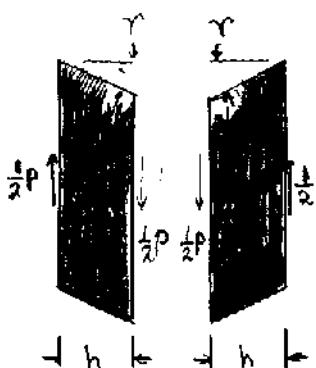
$$\tau = \frac{F}{A} = \frac{3000}{24} = 125 \text{ psi.}$$

$$S = \frac{1}{16} \text{ in.} = 0.0625 \text{ in.}$$

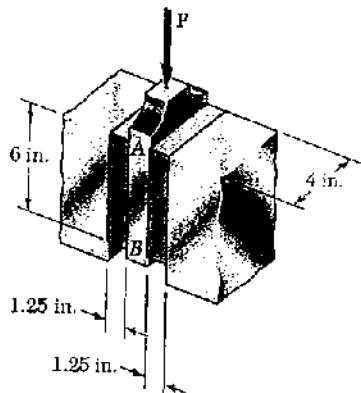
$$h = 1.25 \text{ in.}$$

$$\gamma = \frac{S}{h} = \frac{0.0625}{1.25} = 50 \times 10^{-3} \text{ in.}$$

$$G = \frac{\tau}{\gamma} = \frac{125}{50 \times 10^{-3}} = 2.50 \times 10^3 \text{ psi} \blacktriangleleft$$



Problem 2.78



2.78 A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 2.75 \text{ ksi}$ bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.

$$\text{Shearing strain} \quad \gamma = \frac{\delta}{h}$$

$$\text{Shearing stress} \quad \tau = G\gamma = \frac{G\delta}{h}$$

$$\text{Force} \quad \frac{1}{2}P = A\tau = \frac{GA\delta}{h}$$

$$P = \frac{2GAS}{h}$$

Effective spring constant

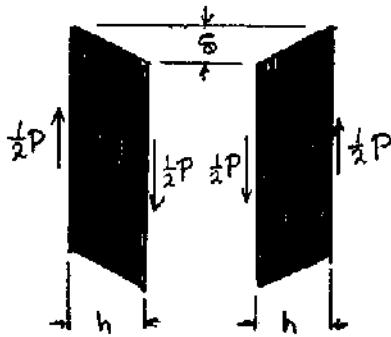
$$k = \frac{P}{\delta} = \frac{2GA}{h}$$

Data: $G = 2.75 \times 10^3 \text{ psi}$

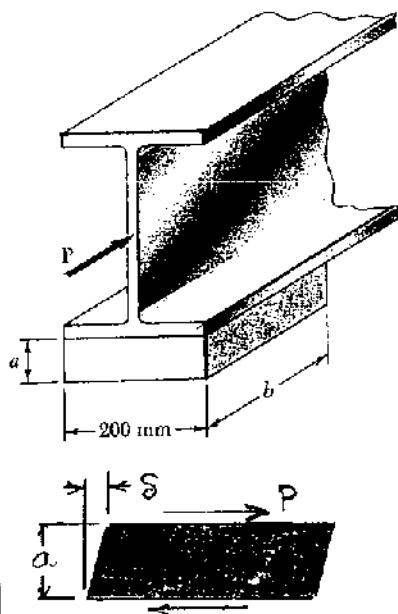
$$A = (6)(4) = 24 \text{ in}^2$$

$$h = 1.25 \text{ in.}$$

$$k = \frac{(2)(2.75 \times 10^3)(24)}{1.25} = 105.6 \times 10^3 \text{ lb/in}$$



Problem 2.79



2.79 An elastomeric bearing ($G = 0.9 \text{ MPa}$) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22 kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine (a) the smallest allowable dimension b , (b) the smallest required thickness a .

$$\text{Shearing force } P = 22 \times 10^3 \text{ N}$$

$$\text{Shearing stress } \tau = 420 \times 10^3 \text{ Pa}$$

$$\tau = \frac{S}{A} \therefore A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3} \text{ m}^2 \\ = 52.381 \times 10^3 \text{ mm}^2$$

$$A = (200 \text{ mm})(b)$$

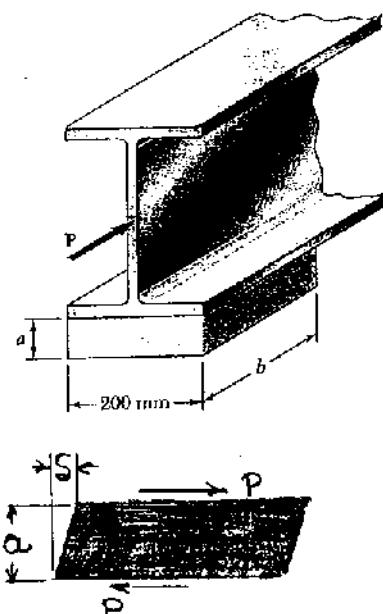
$$b = \frac{A}{200} = \frac{52.381 \times 10^3}{200} = 262 \text{ mm}$$

$$\gamma = \frac{\epsilon}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$$

$$\text{But } \gamma = \frac{\delta}{a} \therefore a = \frac{\delta}{\gamma} = \frac{10 \text{ mm}}{466.67 \times 10^{-3}} = 21.4 \text{ mm}$$

Problem 2.80

2.80 For the elastomeric bearing in Prob. 2.79 with $b = 220 \text{ mm}$ and $a = 30 \text{ mm}$, determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 19 \text{ kN}$ and a maximum displacement $\delta = 12 \text{ mm}$.



$$\text{Shearing force } P = 19 \times 10^3 \text{ N}$$

$$\text{Area } A = (200 \text{ mm})(220 \text{ mm}) = 44 \times 10^3 \text{ mm}^2 \\ = 44 \times 10^{-3} \text{ m}^2$$

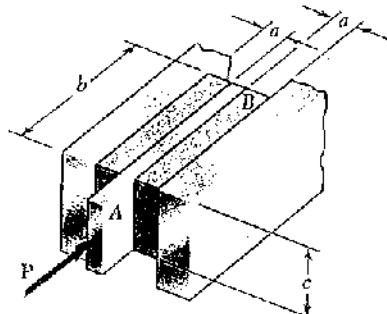
$$\tau = \frac{P}{A} = \frac{19 \times 10^3}{44 \times 10^{-3}} = 431.81 \times 10^3 \text{ Pa} \\ = 431 \text{ kPa}$$

$$\text{Shearing strain } \gamma = \frac{\delta}{a} = \frac{12 \text{ mm}}{30 \text{ mm}} = 0.400$$

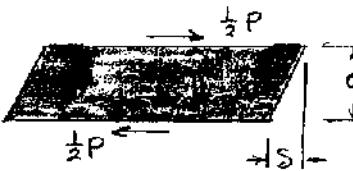
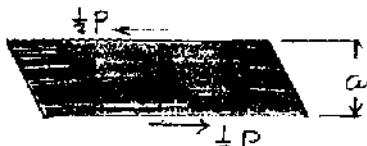
Shearing modulus

$$G = \frac{\tau}{\gamma} = \frac{431.81 \times 10^3}{0.4} = 1.080 \times 10^6 \text{ Pa} \\ = 1.080 \text{ MPa}$$

Problem 2.81



2.81 Two blocks of rubber with a modulus of rigidity $G = 1.75 \text{ ksi}$ are bonded to rigid supports and to a plate AB . Knowing that $c = 4 \text{ in.}$ and $P = 10 \text{ kips}$, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 200 psi and the deflection of the plate is to be at least $\frac{3}{16} \text{ in.}$



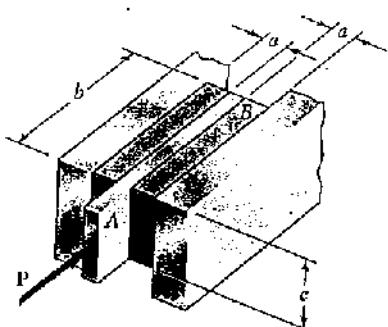
$$\text{Shearing strain } \gamma = \frac{s}{a} = \frac{\epsilon}{G}$$

$$a = \frac{Gs}{\gamma} = \frac{(1750)(\frac{3}{16})}{200} = 1.641 \text{ in.}$$

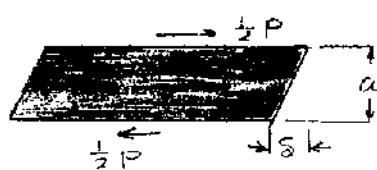
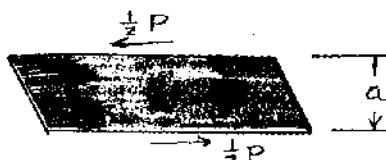
$$\text{Shearing stress } \tau = \frac{\frac{1}{2}P}{A} = \frac{P}{2bc}$$

$$b = \frac{P}{2c\tau} = \frac{10 \times 10^3}{(2)(4)(200)} = 6.25 \text{ in.}$$

Problem 2.82



2.82 Two blocks of rubber with a modulus of rigidity $G = 1.50 \text{ ksi}$ are bonded to rigid supports and to a plate AB . Knowing that $b = 8 \text{ in.}$ and $c = 5 \text{ in.}$, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 210 psi and the deflection of the plate is to be at least $\frac{1}{4} \text{ in.}$



$$\text{Shearing stress } \tau = \frac{\frac{1}{2}P}{A} = \frac{P}{2bc}$$

$$P = 2bc\tau = (2)(8)(5)(210) = 16.80 \times 10^3 \text{ lb} = 16.80 \text{ kips}$$

$$\text{Shearing strain } \gamma = \frac{s}{a} = \frac{\epsilon}{G}$$

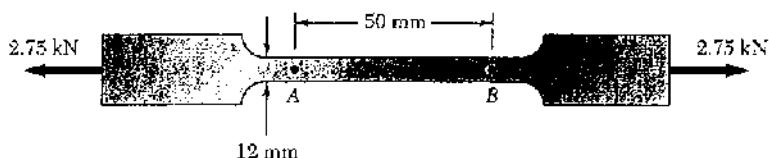
$$a = \frac{Gs}{\gamma} = \frac{(1500)(\frac{1}{4})}{210} = 1.786 \text{ in.}$$

Problem 2.83

*2.83 Determine the change in volume of the 50-mm gage length segment AB in, Prob. 2.62 (a) by computing the dilatation of the material, (b) by subtracting the original volume of portion AB from its final volume.

2.62 A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200 \text{ GPa}$, $\nu = 0.30$). Determine the resulting change (a) in the 50-mm gage length, (b) in the width of portion AB of the test coupon, (c) in the thickness of portion AB, (d) in the cross-sectional area of portion AB.

All calculations here
are in mm



$$(a) A_o = (12)(1.6) = 19.2 \text{ mm}^2 = 19.2 \times 10^{-6} \text{ m}^2$$

$$\text{Volume } V_o = L_o A_o = (50)(19.2) = 960 \text{ mm}^3$$

$$\sigma_x = \frac{P}{A_o} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6 \text{ Pa} \quad \sigma_y = \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\epsilon_y = \epsilon_z = -\nu \epsilon_x = -(0.30)(716.15 \times 10^{-6}) = -214.84 \times 10^{-6}$$

$$\epsilon = \epsilon_x + \epsilon_y + \epsilon_z = 286.46 \times 10^{-6}$$

$$\Delta V = V_o \epsilon = (960)(286.46 \times 10^{-6}) = 0.275 \text{ mm}^3$$

(b) From the solution to Problem 2.62

$$S_x = 0.03581 \text{ mm} \quad S_y = -0.002578 \text{ mm} \quad S_z = -0.0003437 \text{ mm}$$

The dimensions when under the 2.75 kN load are:

$$\text{length } L = L_o + S_x = 50 + 0.03581 = 50.03581 \text{ mm}$$

$$\text{width } w = w_o + S_y = 12 - 0.002578 = 11.997422 \text{ mm}$$

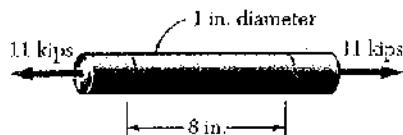
$$\text{thickness } t = t_o + S_z = 1.6 - 0.0003437 = 1.5996563 \text{ mm}$$

$$\text{volume } V = L w t = (50.03581)(11.997422)(1.5996563) = 960.275 \text{ mm}^3$$

$$\Delta V = V - V_o = 960.275 - 960 = 0.275 \text{ mm}^3$$

Problem 2.84

*2.84 Determine the dilatation ϵ and the change in volume of the 8-in. length of the rod shown if (a) the rod is made of steel with $E = 29 \times 10^6$ psi and $\nu = 0.30$, (b) the rod is made of aluminum with $E = 10.6 \times 10^6$ psi and $\nu = 0.35$.



$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1)^2 = 0.785398 \text{ in}^2$$

$$P = 11 \times 10^3 \text{ lb}$$

$$\text{Stresses: } \sigma_x = \frac{P}{A} = \frac{11 \times 10^3}{0.785398} = 14.0056 \times 10^3 \text{ psi}$$

$$\sigma_y = \sigma_z = 0$$

$$(a) \text{ Steel. } E = 29 \times 10^6 \text{ psi} \quad \nu = 0.30$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{\sigma_x}{E} = \frac{14.0056 \times 10^3}{29 \times 10^6} = 482.95 \times 10^{-6}$$

$$\epsilon_y = \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) = -\frac{\nu \sigma_x}{E} = -\nu \epsilon_x = -(0.30)(482.95 \times 10^{-6}) = -144.885 \times 10^{-6}$$

$$\epsilon_z = \frac{1}{E} (-\nu \sigma_y - \nu \sigma_z + \sigma_x) = -\frac{\nu \sigma_x}{E} = \epsilon_y = -144.885 \times 10^{-6}$$

$$\epsilon = \epsilon_x + \epsilon_y + \epsilon_z = 193.2 \times 10^{-6}$$

$$\Delta V = V \epsilon = A L \epsilon = (0.785398)(8)(193.2 \times 10^{-6}) = 1.214 \times 10^{-3} \text{ m}^3$$

$$(b) \text{ Aluminum. } E = 10.6 \times 10^6 \text{ psi} \quad \nu = 0.35$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{14.0056 \times 10^3}{10.6 \times 10^6} = 1.32128 \times 10^{-3}$$

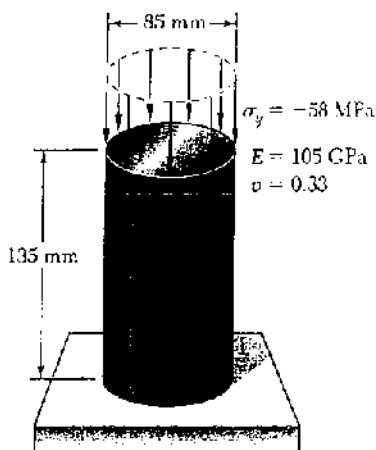
$$\epsilon_y = -\nu \epsilon_x = -(0.35)(1.32128 \times 10^{-3}) = -462.45 \times 10^{-6}$$

$$\epsilon_z = \epsilon_y = -462.45 \times 10^{-6}$$

$$\epsilon = \epsilon_x + \epsilon_y + \epsilon_z = 396 \times 10^{-6}$$

$$\Delta V = V \epsilon = A L \epsilon = (0.785398)(8)(396 \times 10^{-6}) = 2.49 \times 10^{-3} \text{ in}^3$$

Problem 2.85



*2.85 (a) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (b) Solve part a, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70 \text{ MPa}$.

$$h_o = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4} (85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_o = A_o h_o = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

$$(a) \quad \bar{\epsilon}_x = 0, \quad \bar{\epsilon}_y = -58 \times 10^6 \text{ Pa}, \quad \bar{\epsilon}_z = 0$$

$$\begin{aligned} \bar{\epsilon}_y &= \frac{1}{E} (-\nu \bar{\epsilon}_x + \bar{\epsilon}_y - \nu \bar{\epsilon}_z) = \frac{\bar{\epsilon}_y}{E} \\ &= -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6} \end{aligned}$$

$$\Delta h = h_o \bar{\epsilon}_y = (135 \text{ mm})(-552.38 \times 10^{-6}) = -0.0746 \text{ mm}$$

$$\begin{aligned} e &= \frac{1-2\nu}{E} (\bar{\epsilon}_x + \bar{\epsilon}_y + \bar{\epsilon}_z) = \frac{(1-2\nu)\bar{\epsilon}_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9} \\ &= -187.81 \times 10^{-6} \end{aligned}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) = -143.9 \text{ mm}^3$$

$$(b) \quad \bar{\epsilon}_x = \bar{\epsilon}_y = \bar{\epsilon}_z = -70 \times 10^6 \text{ Pa} \quad \bar{\epsilon}_x + \bar{\epsilon}_y + \bar{\epsilon}_z = -210 \times 10^6 \text{ Pa}$$

$$\begin{aligned} \bar{\epsilon}_y &= \frac{1}{E} (-\nu \bar{\epsilon}_x + \bar{\epsilon}_y - \nu \bar{\epsilon}_z) = \frac{1-2\nu}{E} \bar{\epsilon}_y \\ &= \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6} \end{aligned}$$

$$\Delta h = h_o \bar{\epsilon}_y = (135 \text{ mm})(-226.67 \times 10^{-6}) = -0.0306 \text{ mm}$$

$$e = \frac{1-2\nu}{E} (\bar{\epsilon}_x + \bar{\epsilon}_y + \bar{\epsilon}_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) = -521 \text{ mm}^3$$

Problem 2.86

*2.86 A 6-in. diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine (a) the decrease in diameter of the sphere, (b) the decrease in volume of the sphere, (c) the percent increase in the density of the sphere.

$$\text{For a solid sphere } V_o = \frac{\pi}{6} d_o^3 = \frac{\pi}{6} (6.00)^3 = 113.097 \text{ in}^3$$

$$\sigma_x = \sigma_y = \sigma_z = -p = -7.1 \times 10^3 \text{ psi}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - 2\nu\sigma_y - 2\nu\sigma_z) = -\frac{(1-2\nu)p}{E} = -\frac{(1-2(0.30))(7.1 \times 10^3)}{29 \times 10^6} = -97.93 \times 10^{-6}$$

$$\text{Likewise } \epsilon_y = \epsilon_z = -97.93 \times 10^{-6}$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = -293.79 \times 10^{-6}$$

$$(a) -\Delta d = -d_o \epsilon_x = -(6.00)(97.93 \times 10^{-6}) = 588 \times 10^{-6} \text{ in.}$$

$$(b) -\Delta V = -V_o e = -(113.097)(-293.79 \times 10^{-6}) = 33.2 \times 10^{-3} \text{ in}^3$$

(c) Let $m = \text{mass of sphere}$. $m = \text{constant.}$

$$m = \rho_0 V_o = \rho V = \rho V_o (1+e)$$

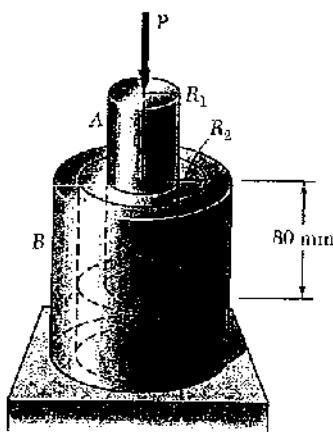
$$\frac{\rho - \rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{V_o(1+e)} \cdot \frac{V_e}{m} - 1 = \frac{1}{1+e} - 1 \\ = (1 - e + e^2 - e^3 + \dots) - 1 = -e + e^2 - e^3 + \dots$$

$$\approx -e = 293.79 \times 10^{-6}$$

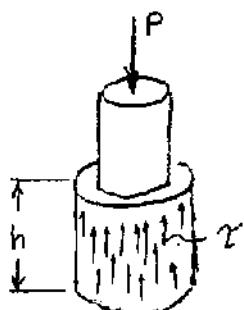
$$\frac{\rho - \rho_0}{\rho_0} \times 100\% = (293.79 \times 10^{-6})(100\%) = 0.0294\%$$

Problem 2.87

*2.87 A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_1 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93 \text{ MPa}$. Determine the required value of the ratio R_2/R_1 if a 10-kN force P is to cause a 2-mm deflection of rod A .



Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \leq r \leq R_2$



Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$

Shearing deformation over radial length dr

$$\frac{dS}{dr} = \gamma$$

$$dS = \gamma dr = \frac{P}{2\pi G h} \frac{dr}{r}$$

Total deformation

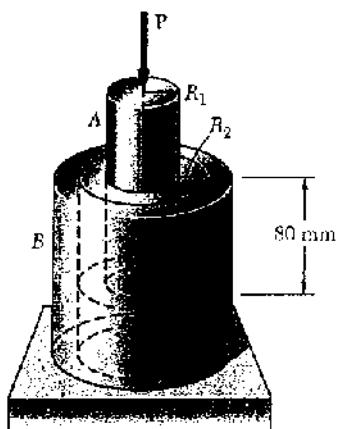
$$\begin{aligned} S &= \int_{R_1}^{R_2} dS = \frac{P}{2\pi G h} \int_{R_1}^{R_2} \frac{dr}{r} \\ &= \frac{P}{2\pi G h} \ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi G h} (\ln R_2 - \ln R_1) \\ &= \frac{P}{2\pi G h} \ln \frac{R_2}{R_1} \end{aligned}$$

$$\ln \frac{R_2}{R_1} = \frac{2\pi G h S}{P} = \frac{(2\pi)(10.93 \times 10^6)(0.080)(0.002)}{10 \times 10^3} = 1.0988$$

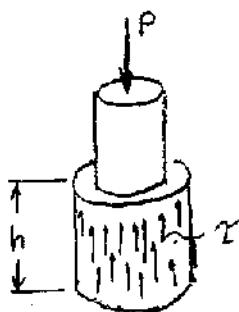
$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

Problem 2.88

*2.88 A vibration isolation support consists of a rod *A* of radius $R_1 = 10 \text{ mm}$ and a tube *B* of inner radius $R_2 = 25 \text{ mm}$ bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 12 \text{ MPa}$. Determine the largest allowable force *P* which may be applied to rod *A* if its deflection is not to exceed 2.50 mm.



Let *r* be a radial coordinate. Over the hollow rubber cylinder $R_1 \leq r \leq R_2$

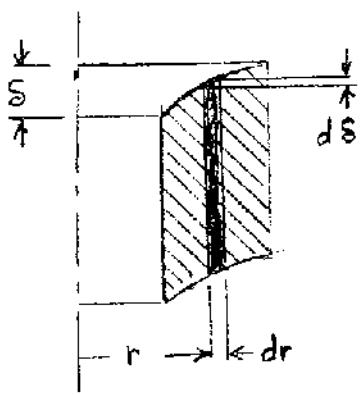


Shearing stress σ acting on a cylindrical surface of radius *r* is

$$\sigma = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\sigma}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length *dr*

$$\frac{dS}{dr} = \gamma$$

$$dS = \gamma dr = \frac{P}{2\pi G h} \frac{dr}{r}$$

Total deformation

$$S = \int_{R_1}^{R_2} dS = \frac{P}{2\pi G h} \int_{R_1}^{R_2} \frac{dr}{r}$$

$$= \frac{P}{2\pi G h} \ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi G h} (\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h} \ln \frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi G h S}{\ln(R_2/R_1)}$$

Data: $R_1 = 10 \text{ mm} = 0.010 \text{ m}$, $R_2 = 25 \text{ mm} = 0.025 \text{ m}$, $h = 80 \text{ mm} = 0.080 \text{ m}$

$$G = 12 \times 10^6 \text{ Pa} \quad S = 2.50 \times 10^{-3} \text{ m}$$

$$P = \frac{(2\pi)(12 \times 10^6)(0.080)(2.50 \times 10^{-3})}{\ln(0.025/0.010)} = 16.46 \times 10^8 \text{ N} \quad 16.46 \text{ kN} \blacksquare$$

Problem 2.89

*2.89 The material constants E , G , k , and ν are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (a) $k = GE/(9G - 3E)$ and (b) $\nu = (3k - 2G)/(6k + 2G)$.

$$k = \frac{E}{3(1-2\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}$$

$$(a) \quad 1+\nu = \frac{E}{2G} \quad \text{or} \quad \nu = \frac{E}{2G} - 1$$

$$k = \frac{E}{3[1 - 2(\frac{E}{2G} - 1)]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$= \frac{EG}{9G - 6E}$$

$$(b) \quad \frac{k}{G} = \frac{2(1+\nu)}{3(1-2\nu)}$$

$$3k - 6k\nu = 2G + 2G\nu$$

$$3k - 2G = 2G + \nu k$$

$$\nu = \frac{3k - 2G}{6k + 2G}$$

Problem 2.90

*2.90 Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [Hint: Refer to Eq. (2.43) and to Sec. 2.13.]

$$G = \frac{E}{2(1+\nu)} \quad \text{or} \quad \frac{E}{G} = 2(1+\nu)$$

Assume $\nu \geq 0$ for almost all materials and $\nu < \frac{1}{2}$ for a positive bulk modulus.

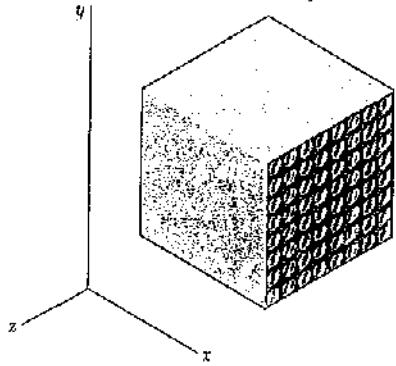
Applying the bounds $2 \leq \frac{E}{G} \leq 2(1+\frac{1}{2}) = 3$

Taking the reciprocals $\frac{1}{2} \geq \frac{G}{E} \geq \frac{1}{3}$

$$\text{or } \frac{1}{3} \leq \frac{E}{G} \leq \frac{1}{2}$$

Problem 2.91

$$\begin{aligned}E_x &= 50 \text{ GPa} & \nu_{xy} &= 0.254 \\E_y &= 15.2 \text{ GPa} & \nu_{yz} &= 0.254 \\E_z &= 15.2 \text{ GPa} & \nu_{xz} &= 0.428\end{aligned}$$



*2.91 A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (a) the change in the length of the cube in the x direction, (b) the stresses σ_x , σ_y , and σ_z .

Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1) \qquad \frac{\nu_{yy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{yz}\sigma_z}{E_z} \quad (2) \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{xy}}{E_z} \quad (5)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3) \qquad \frac{\nu_{xy}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y} \tilde{\sigma}_y - \frac{\nu_{xy}}{E_z} \tilde{\sigma}_z = \frac{\nu_{xy}}{E_x} \tilde{\sigma}_x \quad (7)$$

$$-\frac{\nu_{yz}}{E_y} \tilde{\sigma}_y + \frac{1}{E_z} \tilde{\sigma}_z = \frac{\nu_{xz}}{E_x} \tilde{\sigma}_x \quad (8)$$

$$\frac{1}{15.2} \tilde{\sigma}_y - \frac{0.428}{15.2} \tilde{\sigma}_z = \frac{0.254}{50} \tilde{\sigma}_x \quad \text{or} \quad \tilde{\sigma}_y - 0.428 \tilde{\sigma}_z = 0.077216 \tilde{\sigma}_x$$

$$-\frac{0.428}{15.2} \tilde{\sigma}_y + \frac{1}{15.2} \tilde{\sigma}_z = \frac{0.254}{50} \tilde{\sigma}_x \quad \text{or} \quad -0.428 \tilde{\sigma}_y + \tilde{\sigma}_z = 0.077216 \tilde{\sigma}_x$$

$$\text{Solving simultaneously} \quad \tilde{\sigma}_y = \tilde{\sigma}_z = 0.134993 \tilde{\sigma}_x$$

$$\text{Using (4) and (5) in (1)} \quad \varepsilon_x = \frac{1}{E_x} \tilde{\sigma}_x - \frac{\nu_{xy}}{E_y} \tilde{\sigma}_y - \frac{\nu_{xz}}{E_z} \tilde{\sigma}_z$$

$$\begin{aligned}\tilde{\sigma}_x &= \frac{1}{E_x} \left[1 - (0.254)(0.134993) - (0.254)(0.134993) \right] \tilde{\sigma}_x \\&= \frac{0.93142}{E_x} \tilde{\sigma}_x\end{aligned}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600 \times 10^{-6} \text{ m}^2$$

$$\tilde{\sigma}_x = \frac{P}{A} = \frac{65 \times 10^3}{1600 \times 10^{-6}} = 40.625 \times 10^6 \text{ Pa}$$

continued

Problem 2.91 continued

$$\varepsilon_x = \frac{(0.93142)(40.625 \times 10^3)}{50 \times 10^9} = 756.78 \times 10^{-6}$$

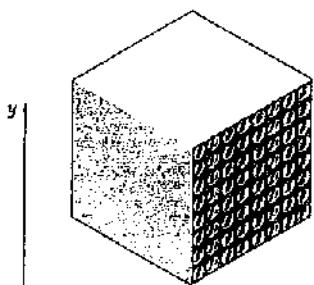
(a) $\Delta_x = L_x \varepsilon_x = (40 \text{ mm})(756.78 \times 10^{-6}) = 0.0303 \text{ mm}$

(b) $\sigma_x = 40.625 \times 10^6 \text{ Pa} = 40.6 \text{ MPa}$

$$\begin{aligned}\sigma_y &= \sigma_z = (0.134993)(40.625 \times 10^6) = 5.48 \times 10^6 \text{ Pa} \\ &= 5.48 \text{ MPa}\end{aligned}$$

Problem 2.92

*2.92 The composite cube of Prob. 2.91 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (a) the stresses σ_x , σ_y , and σ_z , (b) the change in the dimension in the y direction.



$$\begin{aligned}E_x &= 50 \text{ GPa} & \nu_{xz} &= 0.254 \\ E_y &= 15.2 \text{ GPa} & \nu_{xy} &= 0.254 \\ E_z &= 15.2 \text{ GPa} & \nu_{yz} &= 0.428\end{aligned}$$

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1)$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yz}}{E_y} \quad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{yz}\sigma_z}{E_z} \quad (2)$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{xy}}{E_x} \quad (5)$$

$$\varepsilon_z = -\frac{\nu_{zx}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3)$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xy}}{E_x} \quad (6)$$

Constraint condition $\varepsilon_z = 0$

Load condition $\sigma_y = 0$

From equation (3) $0 = -\frac{2\nu_{zx}}{E_x} \sigma_x + \frac{1}{E_z} \sigma_z$

$$\sigma_z = \frac{2\nu_{zx} E_z}{E_x} \sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216 \sigma_x$$

continued

Problem 2.92 continued

From equation (1) with $\epsilon_y = 0$

$$\begin{aligned}\epsilon_x &= \frac{1}{E_x} \sigma_x - \frac{\nu_{zx}}{E_z} \sigma_z = \frac{1}{E_x} \sigma_x - \frac{\nu_{zx}}{E_x} \sigma_z \\ &= \frac{1}{E_x} [\sigma_x - 0.254 \sigma_z] = \frac{1}{E_x} [1 - (0.254)(0.077216)] \sigma_x \\ &= \frac{0.98039}{E_x} \sigma_x \\ \sigma_x &= \frac{E_x \epsilon_x}{0.98039}\end{aligned}$$

But $\epsilon_x = \frac{\sigma_x}{L_x} = \frac{0.035 \text{ mm}}{40 \text{ mm}} = 875 \times 10^{-6}$

(a) $\sigma_x = \frac{(50 \times 10^9)(875 \times 10^{-6})}{0.98039} = 44.625 \times 10^3 \text{ Pa} = 44.6 \text{ MPa}$

$$\sigma_y = 0$$

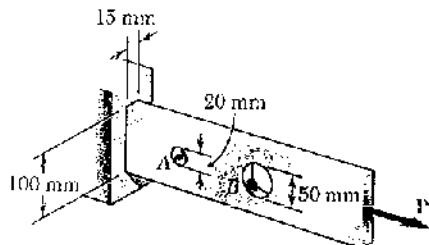
$$\sigma_z = (0.077216)(44.625 \times 10^3) = 3.446 \times 10^6 \text{ Pa} = 3.45 \text{ MPa}$$

From (2) $\epsilon_y = -\frac{\nu_{xy}}{E_x} \sigma_x + \frac{1}{E_y} \sigma_y - \frac{\nu_{zy}}{E_z} \sigma_z$
 $= -\frac{(0.254)(44.625 \times 10^3)}{50 \times 10^9} + 0 - \frac{(0.428)(3.446 \times 10^6)}{15.2 \times 10^9}$
 $= -323.73 \times 10^{-6}$

$$\delta_y = L_y \epsilon_y = (40 \text{ mm})(-323.73 \times 10^{-6}) = -0.0129 \text{ mm}$$

Problem 2.93

2.93 Knowing that $\sigma_{all} = 120 \text{ MPa}$, determine the maximum allowable value of the centric axial load P .



$$\text{At hole A } r = \frac{1}{2}(20) = 10 \text{ mm}$$

$$d = D - 2r = 100 - 20 = 80 \text{ mm}$$

$$A_{net} = dt = (80)(15) = 1200 \text{ mm}^2 = 1.20 \times 10^{-3} \text{ m}^2$$

$$\frac{r}{d} = \frac{10}{80} = 0.125$$

From Fig. 2.64a, $K = 2.65$

$$\sigma_{max} = \frac{KP}{A_{net}} \quad P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.20 \times 10^{-3})(120 \times 10^6)}{2.65} = 54.3 \times 10^3 \text{ N}$$

$$\text{At hole B } r = \frac{1}{2}(50) = 25 \text{ mm} \quad d = 100 - 50 = 50 \text{ mm}$$

$$A_{net} = (50)(15) = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2, \quad \frac{r}{d} = \frac{25}{50} = 0.50, \quad K = 2.16$$

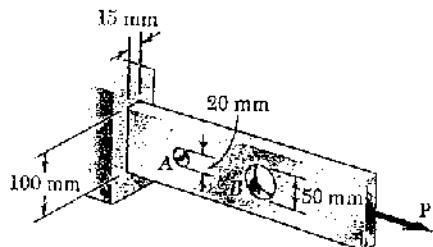
$$P = \frac{(750 \times 10^{-6})(120 \times 10^6)}{2.16} = 41.7 \times 10^3 \text{ N}$$

Allowable value of P is the smaller

$$P = 41.7 \times 10^3 \text{ N} = 41.7 \text{ kN}$$

Problem 2.94

2.94 Two holes have been drilled through a long steel bar that is subjected to a centric axial load as shown. For $P = 32 \text{ kN}$, determine the maximum value of the stress (σ) at A , (b) at B .



$$(a) \text{ At hole A } r = \frac{1}{2}(20) = 10 \text{ mm}$$

$$d = D - 2r = 100 - 20 = 80 \text{ mm}$$

$$A_{net} = dt = (80)(15) = 1200 \text{ mm}^2 = 1.20 \times 10^{-3} \text{ m}^2$$

$$\frac{r}{d} = \frac{10}{80} = 0.125$$

From Fig. 2.64a, $K = 2.65$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{(2.65)(32 \times 10^3)}{1.20 \times 10^{-3}} = 70.7 \times 10^6 \text{ Pa} \quad 70.7 \text{ MPa}$$

$$(b) \text{ At hole B } r = \frac{1}{2}(50) = 25 \text{ mm}, \quad d = 100 - 50 = 50 \text{ mm}$$

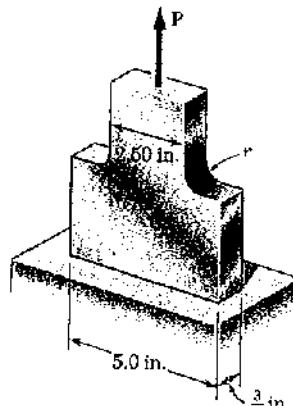
$$A_{net} = (50)(15) = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2$$

$$\frac{r}{d} = \frac{25}{50} = 0.50 \quad K = 2.16$$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{(2.16)(32 \times 10^3)}{750 \times 10^{-6}} = 92.2 \times 10^6 \text{ Pa} \quad 92.2 \text{ MPa}$$

Problem 2.95

2.95 Knowing that $P = 10 \text{ kips}$, determine the maximum stress when (a) $r = 0.50 \text{ in.}$, (b) $r = 0.625 \text{ in.}$



$$P = 10 \times 10^3 \text{ lb} \quad D = 5.0 \text{ in.} \quad d = 2.50 \text{ in.}$$

$$\frac{D}{d} = \frac{5.0}{2.50} = 2.00$$

$$A_{\min} = dt = (2.50)\left(\frac{3}{4}\right) = 1.875 \text{ in}^2$$

$$(a) r = 0.50 \text{ in.} \quad \frac{r}{d} = \frac{0.50}{2.50} = 0.20$$

From Fig. 2.64 b $K = 1.94$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{(1.94)(10 \times 10^3)}{1.875} = 10.35 \times 10^3 \text{ psi}$$

10.35 ksi

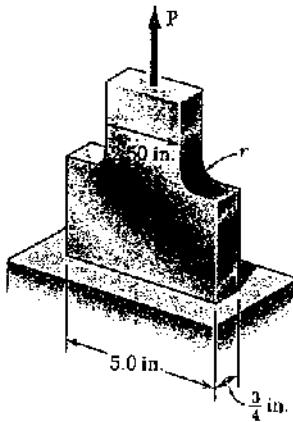
$$(b) r = 0.625 \quad \frac{r}{d} = \frac{0.625}{2.50} = 0.25 \quad K = 1.82$$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{(1.82)(10 \times 10^3)}{1.875} = 9.71 \times 10^3 \text{ psi}$$

9.71 ksi

Problem 2.96

2.96 Knowing that, for the plate shown, the allowable stress is 16 ksi, determine the maximum allowable value of P when (a) $r = \frac{3}{8} \text{ in.}$, (b) $r = \frac{3}{4} \text{ in.}$



$$\sigma_{\max} = 16 \times 10^3 \text{ psi}$$

$$D = 5.0 \text{ in.} \quad d = 2.50 \text{ in.} \quad \frac{D}{d} = 2.00$$

$$A_{\min} = dt = (2.50)\left(\frac{3}{4}\right) = 1.875 \text{ in}^2$$

$$(a) r = \frac{3}{8} \text{ in.} = 0.375 \text{ in.} \quad \frac{r}{d} = \frac{0.375}{2.50} = 0.15$$

From Fig. 2.64 b $K = 2.14$

$$\sigma_{\max} = \frac{KP}{A_{\min}}$$

$$P = \frac{A_{\min} \sigma_{\max}}{K} = \frac{(1.875)(16 \times 10^3)}{2.14} = 14,02 \times 10^3 \text{ lb}$$

14.02 kips

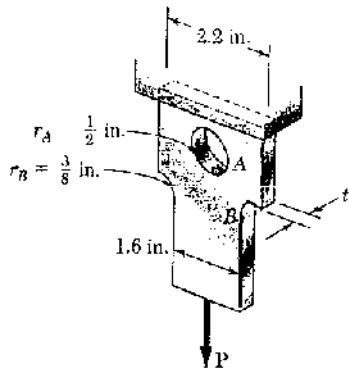
$$(b) r = \frac{3}{4} \text{ in.} = 0.75 \text{ in.} \quad \frac{r}{d} = \frac{0.75}{2.50} = 0.30 \quad K = 1.75$$

$$P = \frac{A_{\min} \sigma_{\max}}{K} = \frac{(1.875)(16 \times 10^3)}{1.75} = 17.14 \times 10^3 \text{ lb}$$

17.14 kips

Problem 2.97

2.97 For $P = 8.5$ kips, determine the minimum plate thickness t required if the allowable stress is 18 ksi.



At the hole: $r_A = \frac{1}{2}$ in. $d_A = 2.2 - 1.0 = 1.2$ in.

$$\frac{r_A}{d_A} = \frac{1/2}{1.2} = 0.417$$

From Fig 2.64 a $K = 2.22$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \therefore t = \frac{KP}{d_A \sigma_{max}}$$

$$t = \frac{(2.22)(8.5)}{(1.2)(18)} = 0.874 \text{ in.}$$

At the fillet $D = 2.2$ in., $d_B = 1.6$ in. $\frac{D}{d_B} = \frac{2.2}{1.6} = 1.375$

$$r_B = \frac{3}{8} = 0.375 \text{ in.} \quad \frac{r_B}{d_B} = \frac{0.375}{1.6} = 0.2344$$

From Fig 2.64 b $K = 1.70$ $\sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t}$

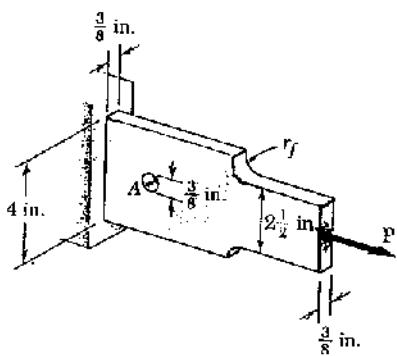
$$t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(8.5)}{(1.6)(18)} = 0.502 \text{ in.}$$

The larger value is the required minimum plate thickness

$$t = 0.874 \text{ in.}$$

Problem 2.98

2.98 Knowing that the hole has a diameter of $\frac{1}{8}$ -in., determine (a) the radius r_f of the fillets for which the same maximum stress occurs at the hole A and at the fillets, (b) the corresponding maximum allowable load P if the allowable stress is 15 ksi.



For the circular hole $r = (\frac{1}{2})(\frac{1}{8}) = 0.1875$ in.

$$d = 4 - \frac{1}{8} = 3.625 \text{ in.} \quad \frac{r}{d} = \frac{0.1875}{3.625} = 0.0517$$

$$A_{net} = d t = (3.625)(\frac{1}{8}) = 1.3594 \text{ in}^2$$

From Fig 2.64 a $K_{hole} = 2.82$

$$\sigma_{max} = \frac{K_{hole} P}{A_{net}}$$

$$(b) P = \frac{A_{net} \sigma_{max}}{K_{hole}} = \frac{(1.3594)(15)}{2.82} = 7.23 \text{ kips}$$

(a) For fillet $D = 4$ in., $d = 2.5$ in. $\frac{D}{d} = \frac{4.0}{2.5} = 1.60$

$$A_{min} = d t = (2.5)(\frac{1}{8}) = 0.9375 \text{ in}^2$$

$$\sigma_{max} = \frac{K_{fillet} P}{A_{min}} \therefore K_{fillet} = \frac{A_{min} \sigma_{max}}{P} = \frac{(0.9375)(15)}{7.23} = 1.945$$

$$\text{From Fig 2.64 b } \frac{r_f}{d} \approx 0.17 \therefore r_f \approx 0.17d = (0.17)(2.5) = 0.425 \text{ in.}$$

Problem 2.99

2.99 A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from 12 to 24 mm in 3-mm increments. (a) Determine the diameter d of the largest bit that can be used if the allowable load at the hole is to exceed that at the fillets. (b) If the allowable stress in the plate is 145 MPa, what is the corresponding allowable load P ?

At the fillets, $r = 9 \text{ mm}$ $d = 75 \text{ mm}$

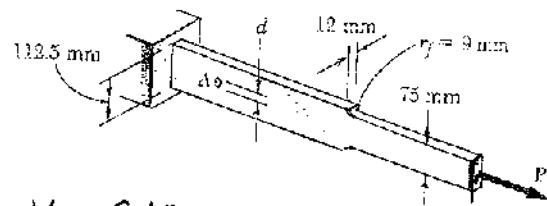
$$D = 112.5 \text{ mm} \quad \frac{D}{d} = \frac{112.5}{75} = 1.5$$

$$\frac{r}{d} = \frac{9}{75} = 0.12 \quad \text{From Fig 2.64 b} \quad K = 2.10$$

$$A_{\min} = (75)(12) = 900 \text{ mm}^2 = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\max} = K \frac{P_{all}}{A_{\min}} = \sigma_{all} \quad \therefore P_{all} = \frac{A_{\min} \sigma_{all}}{K} = \frac{(900 \times 10^{-6})(145 \times 10^6)}{2.10}$$

$$= 62.1 \times 10^3 \text{ N} = 62.1 \text{ kN}$$



At the hole: $A_{net} = (D - 2r)t$, $\frac{r}{d} = \frac{r}{D-2r}$

where $D = 112.5 \text{ mm}$ $r = \text{radius of circle}$ $t = 12 \text{ mm}$

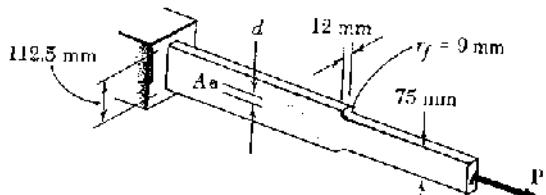
K is taken from Fig 2.64 a

$$\sigma_{\max} = K \frac{P}{A_{net}} = \sigma_{all} \quad \therefore P_{all} = \frac{A_{net} \sigma_{all}}{K}$$

Hole diam	r	$d = D - 2r$	r/d	K	$A_{net} (\text{m}^2)$	$P_{all}, (\text{N})$
12 mm	6.0 mm	100.5 mm	0.060	2.80	1206×10^{-6}	62.5×10^3
15 mm	7.5 mm	97.5 mm	0.077	2.75	1170×10^{-6}	61.7×10^3
18 mm	9.0 mm	94.5 mm	0.095	2.71	1134×10^{-6}	60.7×10^3
21 mm	10.5 mm	91.5 mm	0.115	2.67	1098×10^{-6}	59.6×10^3
24 mm	12.0 mm	88.5 mm	0.136	2.62	1062×10^{-6}	58.8×10^3

Problem 2.100

2.100 (a) For $P = 58 \text{ kN}$ and $d = 12 \text{ mm}$, determine the maximum stress in the plate shown. (b) Solve part a, assuming that the hole at A is not drilled.



Maximum stress at hole.

Use Fig. 2.64 a for values of K

$$\frac{r}{d} = \frac{9}{112.5 - 12} = 0.0597, \quad K = 2.80$$

$$A_{\text{net}} = (12)(112.5 - 12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\max} = K \frac{P}{A_{\text{net}}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6 \text{ Pa}$$

Maximum stress at fillets

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{75} = 0.12, \quad \frac{D}{d} = \frac{112.5}{75} = 1.50, \quad K = 2.10$$

$$A_{\min} = (12)(75) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\max} = K \frac{P}{A_{\min}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6 \text{ Pa}$$

(a) With hole and fillets $\sigma_{\max} = 134.7 \text{ MPa}$

(b) Without hole $\sigma_{\max} = 135.3 \text{ MPa}$

Problem 2.101



2.101 The cylindrical rod AB has a length $L = 5$ ft and a 0.75-in. diameter; it is made of mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_y = 36$ ksi. A force P is applied to the bar and then removed to give it a permanent set δ_p . Determine the maximum value of the force P and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (a) 0.1 in., (b) 0.2 in.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.75)^2 = 0.44179 \text{ in}^2 \quad L = 5 \text{ ft} = 60 \text{ in.}$$

$$S_y = L \varepsilon_y = \frac{L \sigma_y}{E} = \frac{(60)(36 \times 10^3)}{29 \times 10^3} = 0.074483 \text{ in.}$$

Where S_m exceeds S_y , thus causing permanent stretch δ_p , the maximum force is

$$P_m = A S_y = (0.44179)(36 \times 10^3) = 15.9043 \times 10^3 \text{ lb.}$$

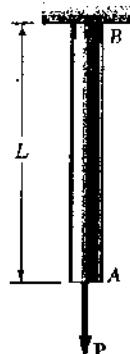
15.90 kips

$$\delta_p = \delta_m - S' = S_m - S_y \text{ so that } S_m = \delta_p + S_y$$

$$(a) \delta_p = 0.1 \text{ in.} \quad S_m = 0.1 + 0.074483 = 0.1745 \text{ in.}$$

$$(b) \delta_p = 0.2 \text{ in.} \quad S_m = 0.2 + 0.074483 = 0.274 \text{ in.}$$

Problem 2.102



2.102 The cylindrical rod AB has a length $L = 6$ ft and a 1.25-in. diameter; it is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_y = 36$ ksi. A force P is applied to the bar until end A has moved down by an amount δ_m . Determine the maximum value of the force P and the permanent set of the bar after the force has been removed, knowing that (a) $\delta_m = 0.125$ in., (b) $\delta_m = 0.250$ in.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1.25)^2 = 1.22718 \text{ in.}^2 \quad L = 6 \text{ ft} = 72 \text{ in.}$$

$$S_y = L \varepsilon_y = \frac{L \sigma_y}{E} = \frac{(72)(36 \times 10^3)}{29 \times 10^3} = 0.089379 \text{ in.}$$

$$\text{If } S_m \geq S_y, \quad P_m = A S_y = (1.22718)(36 \times 10^3) \\ = 44.179 \times 10^3 \text{ lb.} = 44.2 \text{ kips}$$

$$(a) S_m = 0.125 \text{ in.} > S_y \text{ so that } P_m = 44.2 \text{ kips}$$

$$S' = \frac{P_m L}{AE} = \frac{S_y L}{E} = S_y = 0.089379$$

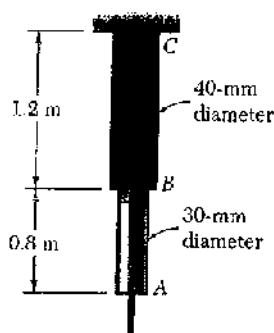
$$\delta_p = S_m - S' = 0.125 - 0.089379 = 0.0356 \text{ in.}$$

$$(b) S_m = 0.250 \text{ in.} > S_y \text{ so that } P_m = 44.2 \text{ kips}$$

$$S' = S_y$$

$$\delta_p = S_m - S' = 0.250 - 0.089379 = 0.1606 \text{ in.}$$

Problem 2.103



2.103 Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$. A force P is applied to the rod and then removed to give it a permanent set $\delta_p = 2 \text{ mm}$. Determine the maximum value of the force P and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

$$A_{AB} = \frac{\pi}{4} (30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

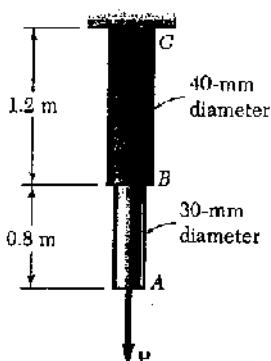
$$P_{max} = A_{min} \sigma_y = (706.86 \times 10^{-6})(250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

$$176.7 \text{ kN}$$

$$\begin{aligned} s' &= \frac{P' L_{AB}}{E A_{AB}} + \frac{P' L_{BC}}{E A_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})} \\ &= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm} \end{aligned}$$

$$s_p = s_m - s' \quad \text{or} \quad s_m = s_p + s' = 2 + 1.84375 = 3.84 \text{ mm}$$

Problem 2.104



2.104 Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$. A force P is applied to the rod until its end *A* has moved down by an amount $\delta_p = 5 \text{ mm}$. Determine the maximum value of the force P and the permanent set of the rod after the force has been removed.

$$A_{AB} = \frac{\pi}{4} (30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

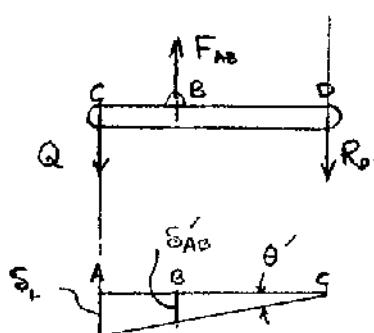
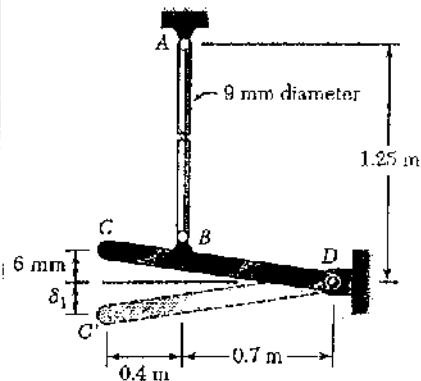
$$P_{max} = A_{min} \sigma_y = (706.86 \times 10^{-6})(250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

$$176.7 \text{ kN}$$

$$\begin{aligned} s' &= \frac{P' L_{AB}}{E A_{AB}} + \frac{P' L_{BC}}{E A_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})} \\ &= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm} \end{aligned}$$

$$s_p = s_m - s' = 5 - 1.84375 = 3.16 \text{ mm}$$

Problem 2.105



2.105 Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_y = 345 \text{ MPa}$. After the rod has been attached to the rigid lever CD, it is found that end C is 6 mm too high. A vertical force Q is then applied at C until this point has moved to position C'. Determine the required magnitude of Q and the deflection δ_1 if the lever is to snap back to a horizontal position after Q is removed.

$$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

Since rod AB is to be stretched permanently,

$$(F_{AB})_{max} = A_{AB}\sigma_y = (63.617 \times 10^{-6})(345 \times 10^6) \\ = 21.948 \times 10^3 \text{ N}$$

$$\rightarrow \sum M_D = 0: 1.1 Q - 0.7 F_{AB} = 0$$

$$Q_{max} = \frac{0.7}{1.1} (21.948 \times 10^3) = 13.967 \times 10^3 \text{ N} \\ 13.97 \text{ kN}$$

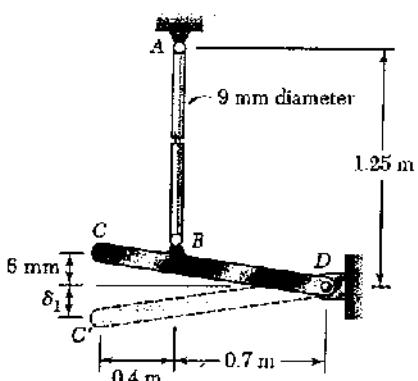
$$S_{AB}' = \frac{(F_{AB})_{max} L_{AB}}{E A_{AB}} = \frac{(21.948 \times 10^3)(1.25)}{(200 \times 10^9)(63.617 \times 10^{-6})} = 2.15625 \times 10^{-3} \text{ m}$$

$$\theta' = \frac{S_{AB}'}{0.7} = 3.0804 \times 10^{-3} \text{ rad}$$

$$S_1 = 1.1 \theta' = 3.39 \times 10^{-3} \text{ m}$$

3.39 mm

Problem 2.106



2.106 Solve Prob. 2.105, assuming that the yield point of the mild steel is 250 MPa.

2.105 Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_y = 345 \text{ MPa}$. After the rod has been attached to the rigid lever CD, it is found that end C is 6 mm too high. A vertical force Q is then applied at C until this point has moved to position C'. Determine the required magnitude of Q and the deflection δ_1 if the lever is to snap back to a horizontal position after Q is removed.

$$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

Since rod AB is to be stretched permanently,

$$(F_{AB})_{max} = A_{AB}\sigma_y = (63.617 \times 10^{-6})(250 \times 10^6) \\ = 15.9043 \times 10^3 \text{ N}$$

$$\rightarrow \sum M_D = 0 \quad 1.1 Q - 0.7 F_{AB} = 0$$

$$Q_{max} = \frac{0.7}{1.1} (15.9043 \times 10^3) = 10.12 \times 10^3 \text{ N} \\ 10.12 \text{ kN}$$

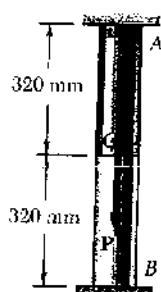
$$S_{AB}' = \frac{(F_{AB})_{max} L_{AB}}{E A_{AB}} = \frac{(15.9043 \times 10^3)(1.25)}{(200 \times 10^9)(63.617 \times 10^{-6})} = 1.5625 \times 10^{-3} \text{ m}$$

$$\theta' = \frac{S_{AB}'}{0.7} = 2.2321 \times 10^{-3} \text{ rad}$$

$$S_1 = 1.1 \theta' = 2.46 \times 10^{-3} \text{ m}$$

2.46 mm

Problem 2.107



2.107 Rod *AB* consists of two cylindrical portions *AC* and *BC*, each with a cross-sectional area of 2950 mm^2 . Portion *AC* is made of a mild steel with $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$, and portion *CB* is made of a high-strength steel with $E = 200 \text{ GPa}$ and $\sigma_y = 345 \text{ MPa}$. A load *P* is applied at *C* as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of *C* if *P* is gradually increased from zero to 1625 kN and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of *C*.

Displacement at *C* to cause yielding of *AC*

$$\delta_{c,r} = L_{AC} \epsilon_{y,AC} = \frac{L_{AC} \sigma_{y,AC}}{E} = \frac{(0.320)(250 \times 10^6)}{200 \times 10^9} = 0.400 \times 10^{-3} \text{ m}$$

Corresponding force $F_{AC} = A \epsilon_{y,AC} = (2950 \times 10^{-6})(250 \times 10^6)$
 $= 737.5 \times 10^3 \text{ N}$

$$F_{CB} = -\frac{EA S_c}{L_{CB}} = -\frac{(200 \times 10^9)(2950 \times 10^{-6})}{0.320}(0.400 \times 10^{-3}) = -737.5 \times 10^3 \text{ N}$$

$$F_{AC}$$

For equilibrium of element at *C*,

$$C \quad \begin{array}{l} \uparrow F_{AC} \\ \downarrow F_{CB} + P \end{array}$$

$$F_{AC} - (F_{CB} + P_r) = 0 \quad P_r = F_{AC} - F_{CB} = 1475 \times 10^3 \text{ N}$$

Since applied load $P = 1625 \times 10^3 \text{ N} > 1475 \times 10^3 \text{ N}$,
portion *AC* yields.

$$F_{CB} = F_{AC} - P = 737.5 \times 10^3 - 1625 \times 10^3 \text{ N} = -887.5 \times 10^3 \text{ N}$$

$$(a) S_c = -\frac{F_{CB} L_{CB}}{EA} = \frac{(887.5 \times 10^3)(0.320)}{(200 \times 10^9)(2950 \times 10^{-6})} = 0.48136 \times 10^{-3} \text{ m.}$$

$$= 0.481 \text{ mm}$$

$$(b) \text{ Maximum stresses.} \quad \sigma_{AC} = \sigma_{y,AC} = 250 \text{ MPa}$$

$$\sigma_{BC} = \frac{F_{CB}}{A} = \frac{-887.5 \times 10^3}{2950 \times 10^{-6}} = -300.81 \times 10^6 \text{ Pa} = -301 \text{ MPa}$$

(c) Deflection and forces for unloading.

$$S' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} \quad \therefore P_{CB}' = -P_{AC}' \frac{L_{AC}}{L_{CB}} = -P_{AC}'$$

$$P' = 1625 \times 10^3 = P_{AC}' - P_{CB}' = 2P_{AC}' \quad P_{AC}' = 812.5 \times 10^3 \text{ N}$$

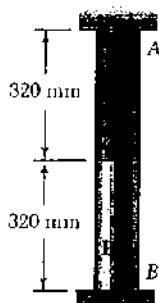
$$\delta' = \frac{(812.5 \times 10^3)(0.320)}{(200 \times 10^9)(2950 \times 10^{-6})} = 0.44068 \times 10^{-3} \text{ m}$$

$$\delta_p = S_m - S' = 0.48136 \times 10^{-3} - 0.44068 \times 10^{-3} = 0.04068 \times 10^{-3} \text{ m}$$

$$= 0.0407 \text{ mm} \downarrow$$

Problem 2.108

2.108 For the composite rod of Prob. 2.107, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.5$ mm and then decreased back to zero, determine, (a) the maximum value of P , (b) the maximum stress in each.



2.107 Rod AB consists of two cylindrical portions AC and BC , each with a cross-sectional area of 2950 mm^2 . Portion AC is made of a mild steel with $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$, and portion CB is made of a high-strength steel with $E = 200 \text{ GPa}$ and $\sigma_y = 345 \text{ MPa}$. A load P is applied at C as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of C if P is gradually increased from zero to 1625 kN and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C .

Displacement at C is $\delta_m = 0.30 \text{ mm}$. The corresponding strains are

$$\epsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.50 \text{ mm}}{320 \text{ mm}} = 1.5625 \times 10^{-3}$$

$$\epsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.50 \text{ mm}}{320 \text{ mm}} = -1.5625 \times 10^{-3}$$

Strains at initial yielding

$$\epsilon_{y,AC} = \frac{\sigma_{y,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \quad (\text{yielding})$$

$$\epsilon_{y,CB} = -\frac{\sigma_{y,CB}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \quad (\text{elastic})$$

(a) Forces: $F_{AC} = A\sigma_y = (2950 \times 10^{-6})(250 \times 10^6) = 737.5 \times 10^3 \text{ N}$

$$F_{CB} = EA\epsilon_{y,CB} = (200 \times 10^9)(2950 \times 10^{-6})(-1.725 \times 10^{-3}) = -921.875 \times 10^3 \text{ N}$$

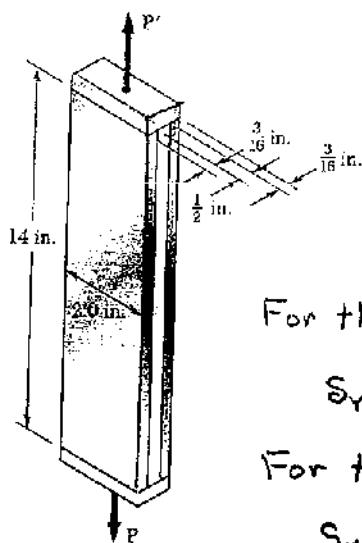
For equilibrium of element at C $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CB} = 737.5 \times 10^3 + 921.875 \times 10^3 = 1.659375 \times 10^6 \text{ N} = 1659 \text{ kN}$$

(b) Stresses: $AC \quad \sigma_{AC} = \sigma_{y,AC} = 250 \text{ MPa}$

$$CB \quad \sigma_{CB} = \frac{F_{CB}}{A} = -\frac{921.875 \times 10^3}{2950 \times 10^{-6}} = -312.5 \times 10^6 \text{ Pa} = -312.5 \text{ MPa}$$

Problem 2.109



2.109 Two tempered-steel bars, each $\frac{3}{16}$ -in. thick, are bonded to a $\frac{1}{2}$ -in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P . Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (a) the maximum value of P , (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

$$\text{For the mild steel } A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$$

$$S_{Y1} = \frac{L\delta_{Y1}}{E} = \frac{(14)(50 \times 10^3)}{29 \times 10^6} = 0.024138 \text{ in.}$$

$$\text{For the tempered steel } A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$$

$$S_{Y2} = \frac{L\delta_{Y2}}{E} = \frac{(14)(100 \times 10^3)}{29 \times 10^6} = 0.048276 \text{ in.}$$

$$\text{Total area: } A = A_1 + A_2 = 1.75 \text{ in}^2$$

$S_m < S_m < S_{Y2}$ The mild steel yields. Tempered steel is elastic.

$$(a) \text{ Forces } P_1 = A_1 S_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$$

$$P_2 = \frac{EA_2 S_m}{L} = \frac{(29 \times 10^3)(0.75)(0.04)}{14} = 62.14 \times 10^3 \text{ lb.}$$

$$P = P_1 + P_2 = 112.14 \times 10^3 \text{ lb} = 112.1 \text{ kips}$$

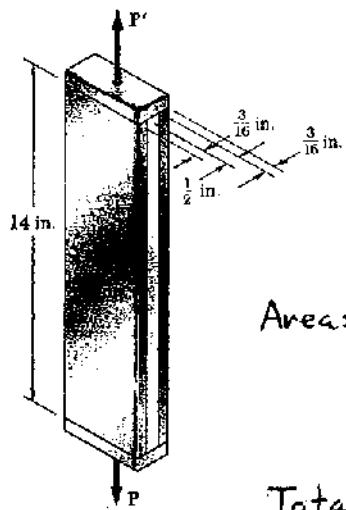
$$(b) \text{ Stresses } \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50 \times 10^3 \text{ psi} = 50 \text{ ksi}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14 \times 10^3}{0.75} = 82.86 \times 10^3 \text{ psi} = 82.86 \text{ ksi}$$

$$\text{Unloading } S' = \frac{PL}{EA} = \frac{(112.14 \times 10^3)(14)}{(29 \times 10^6)(1.75)} = 0.03094 \text{ in.}$$

$$(C) \text{ Permanent set } S_p = S_m - S' = 0.04 - 0.03094 \\ = 0.00906 \text{ in.}$$

Problem 2.110



2.110 For the composite bar of Prob. 2.109, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (a) the maximum deformation of the bar, (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

2.109 Two tempered-steel bars, each $\frac{3}{16}$ -in. thick, are bonded to a $\frac{1}{2}$ -in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P . Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

$$\text{Areas: Mild steel } A_1 = (\frac{1}{2})(\frac{\pi}{4}) = 1.00 \text{ in}^2$$

$$\text{Tempered steel } A_2 = 2(\frac{3}{16})(\frac{\pi}{4}) = 0.75 \text{ in}^2$$

$$\text{Total: } A = A_1 + A_2 = 1.75 \text{ in}^2$$

Total force to yield the mild steel

$$G_{Y_1} = \frac{P_Y}{A} \therefore P_Y = A G_{Y_1} = (1.75)(50 \times 10^3) = 87.50 \times 10^3 \text{ lb.}$$

$P > P_Y$, therefore mild steel yields.

Let P_1 = force carried by mild steel

P_2 = force carried by tempered steel

$$P_1 = A_1 G_1 = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98 \times 10^3 - 50 \times 10^3 = 48 \times 10^3 \text{ lb.}$$

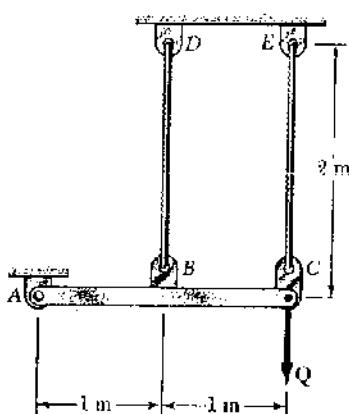
$$(a) \quad S_m = \frac{P_2 L}{E A_2} = \frac{(48 \times 10^3)(14)}{(29 \times 10^6)(0.75)} = 0.03090 \text{ in.}$$

$$(b) \quad G_2 = \frac{P_2}{A_2} = \frac{48 \times 10^3}{0.75} = 64 \times 10^3 \text{ psi} = 64 \text{ ksi}$$

$$\text{Unloading} \quad S' = \frac{P L}{E A} = \frac{(98 \times 10^3)(14)}{(29 \times 10^6)(1.75)} = 0.02703 \text{ in}$$

$$(c) \quad S_p = S_m - S' = 0.03090 - 0.02703 = 0.00387 \text{ in.}$$

Problem 2.111



2.111 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$. A force Q is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (a) the maximum stress that occurs in cable BD , (b) the maximum deflection of point C , (c) the final displacement of point C . (Hint: In part c, cable CE is not taut.)

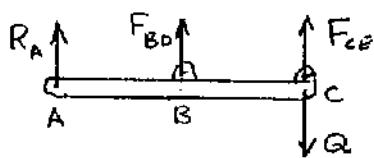
Elongation constraints for taut cables.

Let θ = rotation angle of rigid bar ABC

$$\theta = \frac{s_{BD}}{L_{AB}} = \frac{s_{CE}}{L_{AC}}$$

$$s_{BD} = \frac{L_{AB}}{L_{AC}} s_{CE} = \frac{1}{2} s_{CE} \quad (1)$$

Equilibrium of bar ABC .



$$\rightarrow M_A = 0 : L_{AB} F_{BD} + L_{AC} F_{CE} - L_{AC} Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}} F_{BD} = F_{CE} + \frac{1}{2} F_{BD} \quad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_y = (100 \times 10^{-6})(345 \times 10^6) = 34.5 \times 10^3 \text{ N}$

$$\text{From (2)} \quad F_{BD} = 2(Q - F_{CE}) = 2(50 \times 10^3 - 34.5 \times 10^3) = 31.0 \times 10^3 \text{ N}$$

Since $F_{BD} \leq A\sigma_y = 34.5 \times 10^3 \text{ N}$, cable BD is elastic when $Q = 50 \text{ kN}$.

(a) Maximum stresses.

$$\bar{\sigma}_{CE} = \sigma_y = 345 \text{ MPa}$$

$$\bar{s}_{BD} = \frac{F_{BD}}{A} = \frac{31.0 \times 10^3}{100 \times 10^{-6}} = 310 \times 10^6 \text{ Pa} \quad \bar{\sigma}_{BD} = 310 \text{ MPa}$$

(b) Maximum deflection of point C

$$s_{BD} = \frac{F_{BD} L_{BD}}{EA} = \frac{(31.0 \times 10^3)(2)}{(200 \times 10^9)(100 \times 10^{-6})} = 3.1 \times 10^{-3} \text{ m}$$

$$\text{From (1)} \quad s_c = s_{CE} = 2 s_{BD} = 6.2 \times 10^{-3} \text{ m} \quad 6.20 \text{ mm} \downarrow$$

Permanent elongation of cable CE : $(s_{CE})_p = (s_{CE}) - \frac{\sigma_y L_{CE}}{E}$

$$(s_{CE})_p = (s_{CE})_{max} - \frac{F_{CE} L_{CE}}{EA} = (s_{CE})_{max} - \frac{\sigma_y L_{CE}}{E} = \\ = 6.20 \times 10^{-3} - \frac{(345 \times 10^6)(2)}{200 \times 10^9} = 2.75 \times 10^{-3} \text{ m}$$

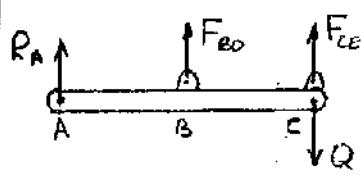
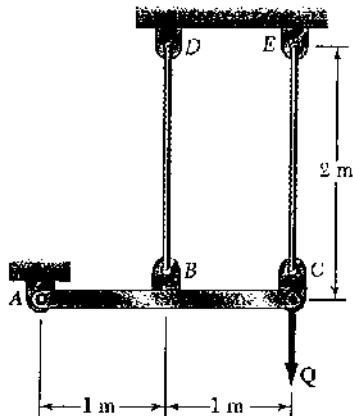
(c) Unloading. Cable CE is slack ($F_{CE} = 0$) at $Q = 0$.

$$\text{From (2)} \quad F_{BD} = 2(Q - F_{CE}) = 2(0 - 0) = 0$$

Since cable BD remained elastic $s_{BD} = \frac{F_{BD} L_{BD}}{EA} = 0$

Problem 2.112

2.112 Solve Prob. 2.111, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.



2.111 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$. A force Q is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (a) the maximum stress that occurs in cable BD , (b) the maximum deflection of point C , (c) the final displacement of point C . (Hint: In part c, cable CE is not taut.)

Elongation constraints.

Let θ = rotation angle of rigid bar ABC .

$$\theta = \frac{s_{BC}}{L_{AB}} = \frac{s_{CE}}{L_{AC}}$$

$$s_{BD} = \frac{L_{AB}}{L_{AC}} s_{CE} = \frac{1}{2} s_{CE} \quad (1)$$

Equilibrium of bar ABC

$$\rightarrow \sum M_A = 0: L_{AB} F_{BD} + L_{AC} F_{CE} - L_{AC} Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}} F_{BD} = F_{CE} + \frac{1}{2} F_{BD} \quad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_y = (100 \times 10^{-6})(345 \times 10^6) = 34.5 \times 10^3 \text{ N}$

$$\text{From (2)} \quad F_{BD} = 2(Q - F_{CE}) = 2(50 \times 10^3 - 34.5 \times 10^3) = 31.0 \times 10^3 \text{ N}$$

Since $F_{BD} \leq A\sigma_y = 34.5 \times 10^3 \text{ N}$, cable BD is elastic when $Q = 50 \text{ kN}$.

(a) Maximum stresses

$$\sigma_{CE} = \sigma_y = 345 \text{ MPa}$$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0 \times 10^3}{100 \times 10^{-6}} = 310 \times 10^6 \text{ Pa} \quad \sigma_{BD} = 310 \text{ MPa}$$

(b) Maximum deflection of point C

$$s_{BD} = \frac{F_{BD} L_{BD}}{EA} = \frac{(31.0 \times 10^3)(2)}{(200 \times 10^9)(100 \times 10^{-6})} = 3.1 \times 10^{-3} \text{ m}$$

$$\text{From (1)} \quad (s_c)_m = s_{CE} = 2 s_{BD} = 6.2 \times 10^{-3} \text{ m} \quad 6.20 \text{ mm}$$

Unloading. $Q' = 50 \times 10^3 \text{ N}$, $s_{CE}' = s_c$ From (1) $s_{BD}' = \frac{1}{2} s_c'$

$$\text{Elastic } F_{BD}' = \frac{EA s_{BD}'}{L_{BD}} = \frac{(200 \times 10^9)(100 \times 10^{-6})(\frac{1}{2}s_c')}{2} = 5 \times 10^6 s_c'$$

$$F_{CE}' = \frac{EA s_{CE}'}{L_{CE}} = \frac{(200 \times 10^9)(100 \times 10^{-6})(s_c')}{2} = 10 \times 10^6 s_c'$$

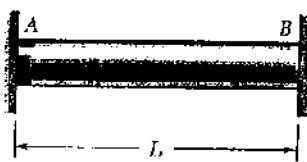
$$\text{From (2)} \quad Q' = F_{CE}' + \frac{1}{2} F_{BD}' = 12.5 \times 10^6 s_c'$$

$$\text{Equating expressions for } Q' \quad 12.5 \times 10^6 s_c' = 50 \times 10^3$$

$$s_c' = 4 \times 10^{-3} \text{ m}$$

$$(c) \text{ Final displacement. } s_c = (s_c)_m - s_c' = 6.2 \times 10^{-3} - 4 \times 10^{-3} = 2.2 \times 10^{-3} \text{ m} \quad 2.20 \text{ mm}$$

Problem 2.113



2.113 A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 45°F . The steel is assumed to be elastoplastic with $\sigma_y = 36 \text{ ksi}$ and $E = 29 \times 10^6 \text{ psi}$. Knowing that $\alpha = 6.5 \times 10^{-6}/^{\circ}\text{F}$, determine the stress in the bar (a) when the temperature is raised to 320°F , (b) after the temperature has returned to 45°F .

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$S = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_y L}{E} + L\alpha(\Delta T)_y = 0$$

$$(\Delta T)_y = \frac{\sigma_y}{E\alpha} = \frac{36 \times 10^3}{(29 \times 10^6)(6.5 \times 10^{-6})} = 190.98^{\circ}\text{F}$$

$$\text{But } \Delta T = 320 - 45 = 275^{\circ}\text{F} > (\Delta T_y)$$

(a) Yielding occurs

$$\sigma = -\sigma_y = -36 \text{ ksi}$$

$$\text{Cooling: } (\Delta T)' = 275^{\circ}\text{F}$$

$$S' = S_p' + S_t' = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)' =$$

$$= -(29 \times 10^6)(6.5 \times 10^{-6})(275) = -51.8375 \times 10^3 \text{ psi}$$

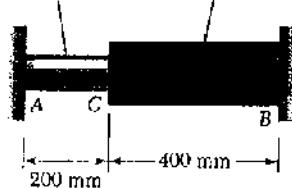
(b) Residual stress

$$\sigma_{res} = -\sigma_y - \sigma' = -36 \times 10^3 + 51.8375 \times 10^3 = 15.84 \times 10^3 \text{ psi}$$

$$15.84 \text{ ksi}$$

Problem 2.114

$$A = 450 \text{ mm}^2 \quad A = 600 \text{ mm}^2$$

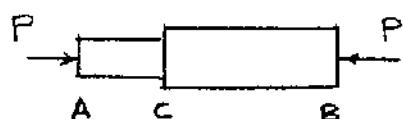


2.114 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 20°C . The steel is assumed elastoplastic, with $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$. The temperature of both portions of the rod is then raised to 120°C . Knowing that $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$, determine (a) the stress in portion *AC*, (b) the deflection of point *C*.

$$A_{AC} = 450 \times 10^{-6} \text{ m}^2 \quad L_{AC} = 0.200 \text{ m}$$

$$A_{CB} = 600 \times 10^{-6} \text{ m}^2 \quad L_{CB} = 0.400 \text{ m}$$

$$(S_{B/A})_p + (S_{B/A})_T = 0 \quad \text{constraint}$$



Determine ΔT to cause yielding in *AC*.

$$-\frac{PL_{AC}}{A_{AB}E} - \frac{PL_{CB}}{A_{CB}E} + L_{AB}\alpha(\Delta T)_y = 0$$

$$\Delta T = \frac{P}{L_{AB}Ed} \left(\frac{L_{AB}}{A_{AB}} + \frac{L_{CB}}{A_{CB}} \right)$$

$$\text{At yielding } P = A_{AC}\sigma_y$$

$$\begin{aligned} (\Delta T)_y &= \frac{A_{AC}\sigma_y}{L_{AB}Ed} \left(\frac{L_{AB}}{A_{AB}} + \frac{L_{AC}}{A_{AC}} \right) \\ &= \frac{(450 \times 10^{-6})(250 \times 10^6)}{(0.600)(200 \times 10^9)(11.7 \times 10^{-6})} \left(\frac{0.200}{450 \times 10^{-6}} + \frac{0.400}{600 \times 10^{-6}} \right) = 89.03^\circ\text{C} \end{aligned}$$

$$\text{Actual } \Delta T = 120 - 20 = 100^\circ\text{C} > (\Delta T)_y. \quad \text{Yielding occurs.}$$

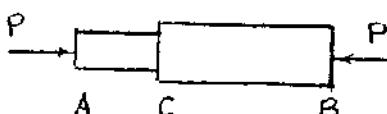
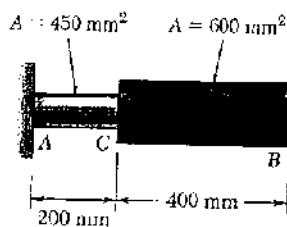
$$(a) \sigma_{AC} = -\sigma_y = -250 \text{ MPa}$$

$$P = A_{AC}\sigma_y = (450 \times 10^{-6})(250 \times 10^6) = 112.5 \times 10^3 \text{ N}$$

$$\begin{aligned} S_c &= -S_{c/y} = \frac{PL_{CB}}{EA_{CB}} - L_{CB}\alpha(\Delta T) \\ &= \frac{(112.5 \times 10^3)(0.400)}{(200 \times 10^9)(600 \times 10^{-6})} - (0.400)(11.7 \times 10^{-6})(100) \\ &= 375 \times 10^{-6} - 468 \times 10^{-6} = -93.0 \times 10^{-6} \text{ m} = -0.0930 \text{ mm} \end{aligned}$$

$$S_c = 0.0930 \text{ mm} \leftarrow$$

Problem 2.115



*2.115 Solve Prob. 2.114, assuming that the temperature of the rod is raised to 120 °C and then returned to 20 °C.

2.114 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 20°C. The steel is assumed elastoplastic, with $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$. The temperature of both portions of the rod is then raised to 120 °C. Knowing that $\alpha = 11.7 \times 10^{-6}/\text{°C}$, determine (a) the stress in portion *AC*, (b) the deflection of point *C*.

$$A_{AC} = 450 \times 10^{-6} \text{ m}^2 \quad L_{AC} = 0.200 \text{ m}$$

$$A_{CB} = 600 \times 10^{-6} \text{ m}^2 \quad L_{CB} = 0.400 \text{ m}$$

$$(S_{BA})_P + (S_{BA})_T = 0 \quad \text{constraint}$$

Determine ΔT to cause yielding in *AC*.

$$-\frac{PL_{AC}}{A_{AC}E} - \frac{PL_{CB}}{A_{CB}E} + L_{AB}\alpha(\Delta T) = 0$$

$$\Delta T = \frac{P}{L_{AB}Ed} \left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right) \quad \text{At yielding } P_r = A_{AC} \sigma_y$$

$$(\Delta T)_y = \frac{A_{AC}\sigma_y}{L_{AB}Ed} \left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right) = \frac{(450 \times 10^{-6})(250 \times 10^6)}{(0.600)(200 \times 10^9)(11.7 \times 10^{-6})} \left(\frac{0.200}{450 \times 10^{-6}} + \frac{0.400}{600 \times 10^{-6}} \right)$$

$$= 89.03 \text{ °C}$$

Actual (ΔT) = 120 - 20 = 100 °C > (ΔT)_y. Yielding occurs.

$$\text{At } 120^\circ\text{C} \quad \sigma_{AC} = -\sigma_y \quad P = A_{AC}\sigma_y = (450 \times 10^{-6})(250 \times 10^6) = 112.5 \times 10^3 \text{ N}$$

Cooling $\Delta T' = 100^\circ\text{C}$

$$P' = \frac{L_{AB}Ed(\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right)} = \frac{(0.600)(200 \times 10^9)(11.7 \times 10^{-6})(100)}{\left(\frac{0.200}{450 \times 10^{-6}} + \frac{0.400}{600 \times 10^{-6}} \right)} = 126.36 \times 10^3 \text{ N}$$

$$\text{Residual force } P = P - P' = 112.5 \times 10^3 - 126.36 \times 10^3 = -13.86 \times 10^3 \text{ N}$$

$$(a) \text{ Residual stress } \sigma_{AC} = -\frac{P}{A_{AC}} = \frac{13.86 \times 10^3}{450 \times 10^{-6}} = 30.8 \times 10^6 \text{ Pa}$$

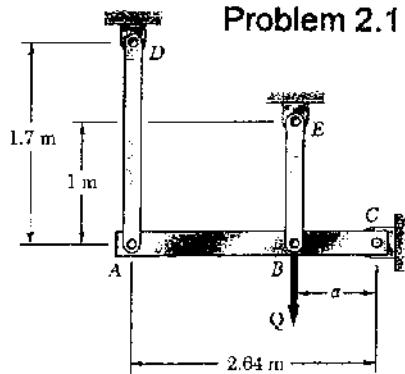
$$\sigma_{AC} = 30.8 \text{ MPa} \quad \blacksquare$$

$$(b) S_e = -S_{AC} = \frac{PL_{CB}}{EA_{CB}} = \frac{(-13.86 \times 10^3)(0.400)}{(200 \times 10^9)(600 \times 10^{-6})} = -46.2 \times 10^{-6} \text{ m}$$

$$= -0.0462 \text{ mm}$$

$$S_e = 0.0462 \text{ mm} \quad \blacksquare$$

Problem 2.116



2.116 The rigid bar ABC is supported by two links, AD and BE , of uniform 37.5×6 -mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$. The magnitude of the force Q applied at B is gradually increased from zero to 260 kN . Knowing that $a = 0.640 \text{ m}$, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B .

$$\text{Statics: } \sum M_c = 0 \quad 0.640(Q - P_{BE}) - 2.64 P_{AD} = 0$$

$$\text{Deformation: } S_A = 2.64\theta, \quad S_B = a\theta = 0.640\theta$$

Elastic Analysis:

$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}} S_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7} S_A = 26.47 \times 10^6 S_A \\ = (26.47 \times 10^6)(2.64\theta) = 69.88 \times 10^6 \theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$$

$$P_{BE} = \frac{EA}{L_{BE}} S_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0} S_B = 45 \times 10^6 S_B \\ = (45 \times 10^6)(0.640\theta) = 28.80 \times 10^6 \theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128 \times 10^9 \theta$$

$$\text{From Statics} \quad Q = P_{BE} + \frac{2.64}{0.640} P_{AD} = P_{BE} + 4.125 P_{AD}$$

$$= [28.80 \times 10^6 + (4.125)(69.88 \times 10^6)]\theta = 317.06 \times 10^6 \theta$$

$$\theta_y \text{ at yielding of link AD} \quad \sigma_{AD} = \sigma_y = 250 \times 10^6 = 310.6 \times 10^9 \theta$$

$$\theta_y = 804.89 \times 10^{-6}$$

$$Q_y = (317.06 \times 10^6)(804.89 \times 10^{-6}) = 255.2 \times 10^3 \text{ N}$$

$$\text{Since } Q = 260 \times 10^3 > Q_y, \text{ link AD yields.} \quad \sigma_{AD} = 250 \text{ MPa} \quad \blacktriangleleft$$

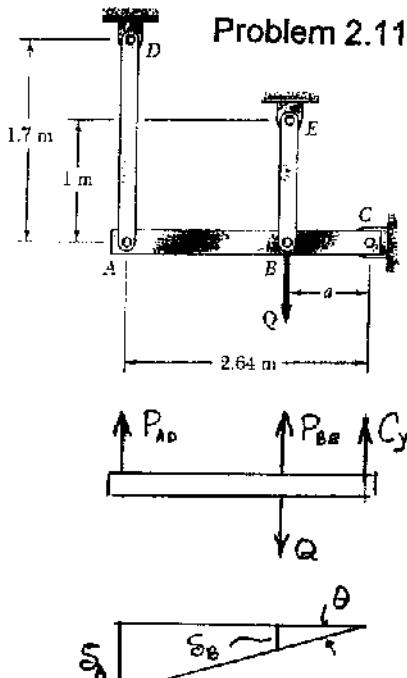
$$P_{AD} = A \sigma_y = (225 \times 10^{-6})(250 \times 10^6) = 56.25 \times 10^3 \text{ N}$$

$$\text{From Statics} \quad P_{BE} = Q - 4.125 P_{AD} = 260 \times 10^3 - (4.125)(56.25 \times 10^3)$$

$$P_{BE} = 27.97 \times 10^3 \text{ N} \quad \sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97 \times 10^3}{225 \times 10^{-6}} = 124.3 \times 10^6 \text{ Pa} \\ = 124.3 \text{ MPa} \quad \blacktriangleleft$$

$$S_B = \frac{P_{BE} L_{BE}}{EA} = \frac{(27.97 \times 10^3)(1.0)}{(200 \times 10^9)(225 \times 10^{-6})} = 621.53 \times 10^{-6} \text{ m} \\ = 0.622 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 2.117



2.117 Solve Prob. 2.116, knowing that $a = 1.76 \text{ m}$ and that the magnitude of the force Q applied at B is gradually increased from zero to 135 kN .

2.116 The rigid bar ABC is supported by two links, AD and BE , of uniform 37.5×6 -mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$. The magnitude of the force Q applied at B is gradually increased from zero to 260 kN . Knowing that $a = 0.640 \text{ m}$, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B .

$$\text{Statics: } \sum M_c = 0 \quad 1.76(Q - P_{BE}) - 2.64P_{AD} = 0$$

$$\text{Deformation: } S_A = 2.64\theta, \quad S_B = 1.76\theta$$

Elastic Analysis

$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}} S_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7} S_A = 26.47 \times 10^6 S_A \\ = (26.47 \times 10^6)(2.64\theta) = 69.88 \times 10^6 \theta$$

$$\bar{\epsilon}_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$$

$$P_{BE} = \frac{EA}{L_{BE}} S_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0} S_B = 45 \times 10^6 S_B = (45 \times 10^6)(1.76\theta) \\ = 79.2 \times 10^6 \theta \quad \bar{\epsilon}_{BE} = \frac{P_{BE}}{A} = 352 \times 10^9 \theta$$

$$\text{From Statics} \quad Q = P_{BE} + \frac{2.64}{1.76} P_{AD} = P_{BE} + 1.500 P_{AD} \\ = [73.8 \times 10^6 + (1.500)(69.88 \times 10^6)]\theta = 178.62 \times 10^6 \theta$$

$$\theta_y \text{ at yielding of link BE} \quad \sigma_{BE} = \sigma_y = 250 \times 10^6 = 352 \times 10^9 \theta_y$$

$$\theta_y = 710.23 \times 10^{-6}$$

$$Q_y = (178.62 \times 10^6)(710.23 \times 10^{-6}) = 126.86 \times 10^3 \text{ N}$$

Since $Q = 135 \times 10^3 \text{ N} > Q_y$, link BE yields $\bar{\epsilon}_{BE} = \bar{\epsilon}_y = 250 \text{ MPa}$

$$P_{BE} = A \bar{\epsilon}_y = (225 \times 10^{-6})(250 \times 10^6) = 56.25 \times 10^3 \text{ N}$$

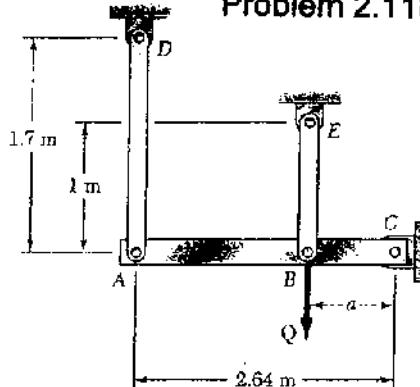
$$\text{From Statics} \quad P_{AD} = \frac{1}{1.500} (Q - P_{BE}) = 52.5 \times 10^3 \text{ N}$$

$$\bar{\epsilon}_{AD} = \frac{P_{AD}}{A} = \frac{52.5 \times 10^3}{225 \times 10^{-6}} = 233.3 \times 10^6 = 233 \text{ MPa}$$

$$\text{From elastic analysis of AD} \quad \theta = \frac{P_{AD}}{69.88 \times 10^6} = 751.29 \times 10^{-5} \text{ rad}$$

$$S_B = 1.76\theta = 1.322 \times 10^{-3} \text{ m} = 1.322 \text{ mm}$$

Problem 2.118



*2.118 Solve Prob. 2.116, assuming that the magnitude of the force Q applied at B is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (a) the residual stress in each link, (b) the final deflection of point B . Assume that the links are braced so that they can carry compressive forces without buckling.

2.116 The rigid bar ABC is supported by two links, AD and BE , of uniform 37.5×6 -mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_y = 250$ MPa. The magnitude of the force Q applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B .

See solution to Problem 2.116 for the normal stresses in each link and the deflection of point B after loading

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa} \quad \sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$

$$S_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to PROBLEM 2.121 applies to the unloading

$$Q = 317.06 \times 10^6 \text{ N}$$

$$\theta' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$$\sigma_{AD}' = 310.6 \times 10^9 \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$

$$\sigma_{BE}' = 128 \times 10^9 \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$

$$S_B' = 0.640 \theta' = 524.82 \times 10^{-6} \text{ m}$$

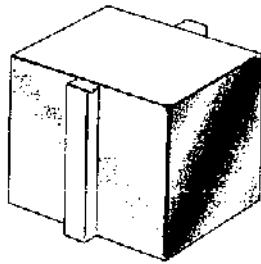
(a) Residual stresses

$$\sigma_{AD, \text{res}} = \sigma_{AD} - \sigma_{AD}' = 250 \times 10^6 - 254.70 \times 10^6 = -4.70 \times 10^6 \text{ Pa} \\ = -4.70 \text{ MPa}$$

$$\sigma_{BE, \text{res}} = \sigma_{BE} - \sigma_{BE}' = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} \\ = 19.34 \text{ MPa}$$

$$(b) S_{B,p} = S_B - S_B' = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} \\ = 96.71 \times 10^{-6} \text{ m} = 0.0967 \text{ mm} \downarrow$$

Problem 2.119



*2.119 A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at $T_1 = 70^\circ\text{F}$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1 , determine (a) the highest temperature T_2 that does not result in residual stresses, (b) the temperature T_2 that will result in a residual stress in the aluminum equal to 58 ksi. Assume $\alpha_a = 12.8 \times 10^{-6}/\text{F}$ for the aluminum and $\alpha_s = 6.5 \times 10^{-6}/\text{F}$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 10.9 \times 10^6 \text{ psi}$ and $\sigma_y = 58 \text{ ksi}$. (Hint: Neglect the small stresses in the plate.)

Determine temperature change to cause yielding

$$S = \frac{PL}{EA} + L\alpha_a(\Delta T)_y = L\alpha_s(\Delta T)_y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_y = -\sigma_y$$

$$(\Delta T_y) = \frac{\sigma_y L}{E(\alpha_a - \alpha_s)} = \frac{58 \times 10^6}{(10.9 \times 10^6)(12.8 - 6.5)(10^{-6})} = 844.62^\circ\text{F}$$

$$(a) T_{2y} = T_1 + (\Delta T)_y = 70 + 844.62 = 915^\circ\text{F}$$

After yielding

$$S = \frac{\sigma_y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$S' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{res} = \sigma_y - \frac{P'}{A} = \sigma_y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\text{Set } \sigma_{res} = -\sigma_y$$

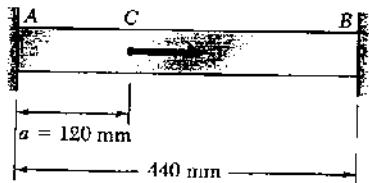
$$-\sigma_y = \sigma_y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_y}{E(\alpha_a - \alpha_s)} = \frac{(2)(58 \times 10^6)}{(10.9 \times 10^6)(12.8 - 6.5)(10^{-6})} = 1689^\circ\text{F}$$

$$(b) T_2 = T_1 + \Delta T = 70 + 1689 = 1759^\circ\text{F}$$

If $T_2 > 1759^\circ\text{F}$, the aluminum bar will most likely yield in compression.

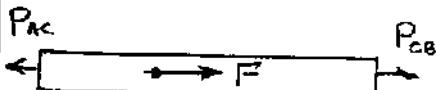
Problem 2.120



2.120 Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$. Knowing that the force F increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point C , (b) the residual stress in the bar.

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion AC : $P_{AC} = A\sigma_y = (1200 \times 10^{-6})(250 \times 10^6)$
 $= 300 \times 10^3 \text{ N}$



For equilibrium $F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$
 $= -220 \times 10^3 \text{ N}$

$$\delta_c = -\frac{P_{CB} L_{AC}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.293333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading

$$\delta'_c = \frac{P'_{AC} L_{AC}}{EA} = -\frac{P'_{CB} L_{CB}}{EA} = \frac{(F - P'_{AC}) L_{CB}}{EA}$$

$$P'_{AC} \left(\frac{L_{AC}}{EA} + \frac{L_{CB}}{EA} \right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.120)}{0.440} = 378.182 \times 10^3 \text{ N}$$

$$P'_{CB} = P'_{AC} - F = 378.182 \times 10^3 - 520 \times 10^3 = -141.818 \times 10^3 \text{ N}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{378.182 \times 10^3}{1200 \times 10^{-6}} = 315.152 \times 10^6 \text{ Pa}$$

$$\sigma'_{CB} = \frac{P'_{CB}}{A} = -\frac{141.818 \times 10^3}{1200 \times 10^{-6}} = -118.182 \times 10^6 \text{ Pa}$$

$$\delta'_c = \frac{(378.182)(0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.189091 \times 10^{-3} \text{ m}$$

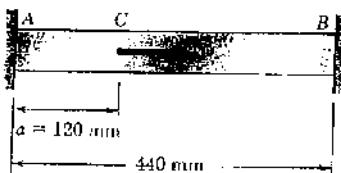
$$(a) \delta_{cp} = \delta_c - \delta'_c = 0.293333 \times 10^{-3} - 0.189091 \times 10^{-3} = 0.1042 \times 10^{-3} \text{ m}$$
 $= 0.1042 \text{ mm}$

$$(b) \sigma_{AC, res} = \sigma_y - \sigma'_{AC} = 250 \times 10^6 - 315.152 \times 10^6 = -65.2 \times 10^6 \text{ Pa}$$
 $= -65.2 \text{ MPa}$

$$\sigma_{CB, res} = \sigma_{CB} - \sigma'_{CB} = -183.333 \times 10^6 + 118.182 \times 10^6 = -65.2 \times 10^6 \text{ Pa}$$
 $= -65.2 \text{ MPa}$

Problem 2.121

2.121 Solve Prob. 2.120, assuming that $a = 180 \text{ mm}$.



2.120 Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$. Knowing that the force F increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point C, (b) the residual stress in the bar.

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

$$\text{Force to yield portion AC: } P_{Ac} = A\sigma_y = (1200 \times 10^{-6})(250 \times 10^6) \\ = 300 \times 10^3 \text{ N}$$

P_{Ac} For equilibrium $F + P_{cb} - P_{Ac} = 0$

$\longleftrightarrow F \longrightarrow P_{cb}$

$$P_{cb} = P_{Ac} - F = 300 \times 10^3 - 520 \times 10^3 \\ = -220 \times 10^3 \text{ N}$$

$$\delta_c = -\frac{P_{cb}L_{cb}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.238333 \times 10^{-3} \text{ m}$$

$$\sigma_{cb} = \frac{P_{cb}}{A} = -\frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading

$$\delta_c' = \frac{P_{Ac}'L_{Ac}}{EA} = -\frac{P_{cb}'L_{cb}}{EA} = \frac{(F - P_{Ac}')L_{cb}}{EA} : \quad P_{Ac}'\left(\frac{L_{Ac}}{EA} + \frac{L_{cb}}{EA}\right) = \frac{FL_{cb}}{EA}$$

$$P_{Ac}' = \frac{FL_{cb}}{L_{Ac} + L_{cb}} = \frac{(520 \times 10^3)(0.440 - 0.180)}{0.440} = 307.273 \times 10^3 \text{ N}$$

$$P_{cb}' = P_{Ac}' - F = 307.273 \times 10^3 - 520 \times 10^3 = -212.727 \times 10^3 \text{ N}$$

$$\delta_c' = \frac{(307.273 \times 10^3)(0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.230455 \times 10^{-3} \text{ m}$$

$$\sigma_{Ac}' = \frac{P_{Ac}'}{A} = \frac{307.273 \times 10^3}{1200 \times 10^{-6}} = 256.061 \times 10^6 \text{ Pa}$$

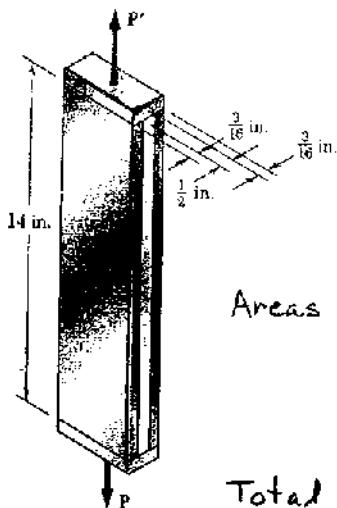
$$\sigma_{cb}' = \frac{P_{cb}'}{A} = \frac{-212.727 \times 10^3}{1200 \times 10^{-6}} = -177.273 \times 10^6 \text{ Pa}$$

$$(a) \quad \delta_{op} = \delta_c - \delta_c' = 0.238333 \times 10^{-3} - 0.230455 \times 10^{-3} = 0.00788 \times 10^{-3} \text{ m} \\ = 0.00788 \text{ mm}$$

$$(b) \quad \sigma_{Ac, res} = \sigma_{Ac}' - \sigma_{Ac} = 256.061 \times 10^6 - 250 \times 10^6 = -6.06 \times 10^6 \text{ Pa} \\ = -6.06 \text{ MPa}$$

$$\sigma_{cb, res} = \sigma_{cb}' - \sigma_{cb} = -177.273 \times 10^6 + 183.333 \times 10^6 = -6.06 \times 10^6 \text{ Pa} \\ = -6.06 \text{ MPa}$$

Problem 2.122



*2.122 For the composite bar of Prob. 2.109, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.

2.109 Two tempered-steel bars, each $\frac{3}{16}$ -in. thick, are bonded to a $\frac{1}{2}$ -in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P . Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero.

$$\text{Areas: Mild steel } A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$$

$$\text{Tempered steel } A_2 = (2)(\frac{3}{16})(2) = 0.75 \text{ in}^2$$

$$\text{Total: } A = A_1 + A_2 = 1.75 \text{ in}^2$$

Total force to yield the mild steel

$$\sigma_{y1} = \frac{P_y}{A} \therefore P_y = A\sigma_{y1} = (1.75)(50 \times 10^3) = 87.50 \times 10^3 \text{ lb.}$$

$P > P_y$; therefore mild steel yields

Let P_1 = force carried by mild steel

P_2 = force carried by tempered steel

$$P_1 = A_1 \sigma_{y1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98 \times 10^3 - 50 \times 10^3 = 48 \times 10^3 \text{ lb.}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{48 \times 10^3}{0.75} = 64 \times 10^3 \text{ psi}$$

$$\text{Unloading } \sigma' = \frac{P}{A} = \frac{98 \times 10^3}{1.75} = 56 \times 10^3 \text{ psi}$$

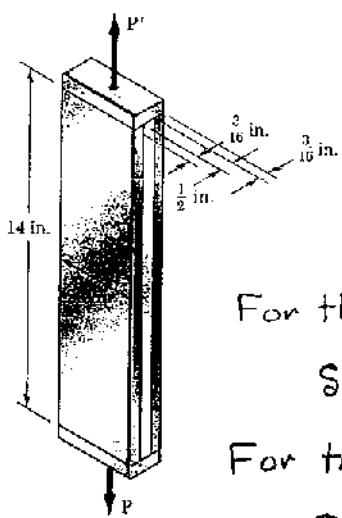
Residual stresses

$$\begin{aligned} \text{mild steel } \sigma_{y, \text{res}} &= \sigma_1 - \sigma' = 50 \times 10^3 - 56 \times 10^3 = -6 \times 10^3 \text{ psi} \\ &= -6 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{tempered steel } \sigma_{2, \text{res}} &= \sigma_2 - \sigma_1 = 64 \times 10^3 - 50 \times 10^3 \\ &= 8 \times 10^3 \text{ psi} = 8.00 \text{ ksi} \end{aligned}$$

Problem 2.123

*2.123 For the composite bar in Prob. 2.109, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.4$ in. and is then decreased back to zero.



2.109 Two tempered-steel bars, each $\frac{3}{16}$ -in. thick, are bonded to a $\frac{1}{2}$ -in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P . Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero.

$$\text{For the mild steel } A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$$

$$S_{Y1} = \frac{L\delta_{Y1}}{E} = \frac{(14)(50 \times 10^3)}{29 \times 10^6} = 0.024138 \text{ in}$$

$$\text{For the tempered steel } A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$$

$$S_{Y2} = \frac{L\delta_{Y2}}{E} = \frac{(14)(100 \times 10^3)}{29 \times 10^6} = 0.048276 \text{ in.}$$

$$\text{Total area: } A = A_1 + A_2 = 1.75 \text{ in}^2$$

$S_{Y1} < S_m < S_{Y2}$ The mild steel yields. Tempered steel is elastic.

$$\text{Forces } P_1 = A_1 S_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb}$$

$$P_2 = \frac{EA_2 S_m}{L} = \frac{(29 \times 10^6)(0.75)(0.04)}{14} = 62.14 \times 10^3 \text{ lb.}$$

$$\text{Stresses } \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50 \times 10^3 \text{ psi}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14 \times 10^3}{0.75} = 82.86 \times 10^3 \text{ psi}$$

$$\text{Unloading } \sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08 \times 10^3 \text{ psi}$$

Residual stresses

$$\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50 \times 10^3 - 64.08 \times 10^3 = -14.08 \times 10^3 \text{ psi}$$

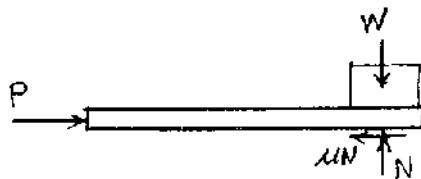
$$= -14.08 \text{ ksi}$$

$$\sigma_{2,\text{res}} = \sigma_2 - \sigma' = 82.86 \times 10^3 - 64.08 \times 10^3 = 18.78 \times 10^3 \text{ psi}$$

$$= 18.78 \text{ ksi}$$

Problem 2.124

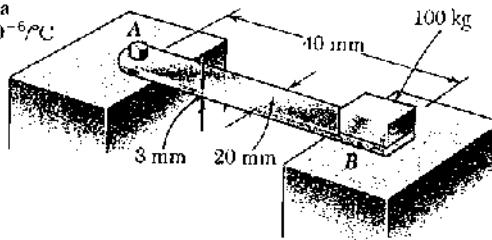
2.124 The brass strip *AB* has been attached to a fixed support at *A* and rests on a rough support at *B*. Knowing that the coefficient of friction is 0.60 between the strip and the support at *B*, determine the decrease in temperature for which slipping will impend.



Brass strip:

$$E = 105 \text{ GPa}$$

$$\alpha = 20 \times 10^{-6} \text{ }^{\circ}\text{C}$$



$$\uparrow \sum F_y = 0 : N - W = 0 \quad N = W$$

$$\pm \sum F_x = 0 \quad P - \mu N = 0 \quad P = \mu W = \mu mg$$

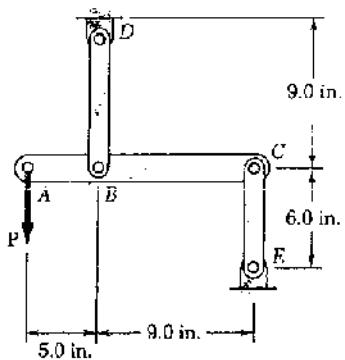
$$S = -\frac{PL}{EA} + L\alpha(\Delta T) = 0 \quad \Delta T = \frac{P}{EAd} = \frac{\mu mg}{EAd}$$

$$\text{Data: } \mu = 0.60 \quad A = (20)(3) = 60 \text{ mm}^2 = 60 \times 10^{-6} \text{ m}^2$$

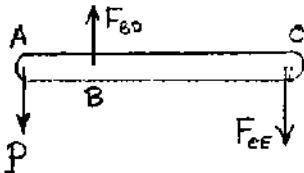
$$m = 100 \text{ kg} \quad g = 9.81 \text{ m/s}^2 \quad E = 105 \times 10^9 \text{ Pa}$$

$$\Delta T = \frac{(0.60)(100)(9.81)}{(105 \times 10^9)(60 \times 10^{-6})(20 \times 10^{-4})} = 4.67 \text{ }^{\circ}\text{C}$$

Problem 2.125



2.125 Link *BD* is made of brass ($E = 15 \times 10^6$ psi) and has a cross-sectional area of 0.40 in². Link *CE* is made of aluminum ($E = 10.4 \times 10^6$ psi) and has a cross-sectional area of 0.50 in². Determine the maximum force *P* that can be applied vertically at point *A* if the deflection of *A* is not to exceed 0.014 in.



Use member ABC
as a free body

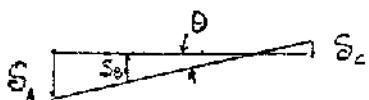
$$\therefore \sum M_C = 0, 14P - 9F_{BD} = 0, F_{BD} = 1.5556 P$$

$$\therefore \sum M_B = 0, 5P - 9F_{CE} = 0, F_{CE} = 0.5556 P$$

$$\varepsilon_B = S_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}} = \frac{(1.5556 P)(9.0)}{(15 \times 10^6)(0.40)} = 2.3333 \times 10^{-6} P \downarrow$$

$$S_c = S_{CE} = \frac{F_{CE} L_{CE}}{E_{CE} A_{CE}} = \frac{(0.5556 P)(6.0)}{(10.4 \times 10^6)(0.50)} = 0.6410 \times 10^{-6} P \uparrow$$

From the deformation diagram



$$\text{Slope } \theta = \frac{S_B + S_C}{l_{BC}} = \frac{2.9743 \times 10^{-6} P}{9} = 0.3305 \times 10^{-6} P$$

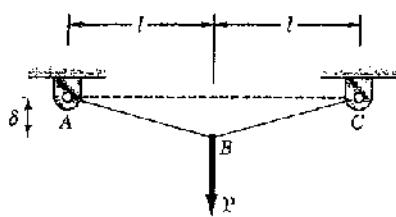
Deformation Diagram

$$\begin{aligned} S_A &= S_B + l_{AB} \theta \\ &= 2.3333 \times 10^{-6} P + (5)(0.3305 \times 10^{-6}) P \\ &= 3.9858 \times 10^{-6} P \end{aligned}$$

Apply displacement limit $S_A = 0.014 \text{ in} = 3.9858 \times 10^{-6} P$

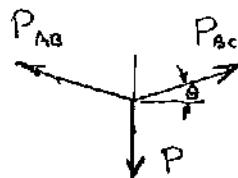
$$P = \frac{0.014}{3.9858 \times 10^{-6}} = 3.51 \times 10^3 \text{ lb} = 3.51 \text{ kips}$$

Problem 2.126



2.126 The uniform wire ABC , of unstretched length $2l$, is attached to the supports shown and a vertical load P is applied at the midpoint B . Denoting by A the cross-sectional area of the wire and by E the modulus of elasticity, show that, for $\delta \ll l$, the deflection at the midpoint B is

$$\delta = l \sqrt[3]{\frac{P}{AE}}$$



Use approximation

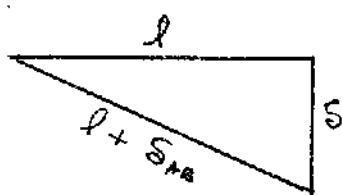
$$\sin \theta \approx \tan \theta \approx \frac{\delta}{l}$$

$$\text{Statics } \sum F_y = 0 \quad 2P_{AB} \sin \theta - P = 0$$

$$P_{AB} = \frac{P}{2 \sin \theta} \approx \frac{Pl}{2\delta}$$

$$\text{Elongation } S_{AB} = \frac{P_{AB}l}{AE} = \frac{Pl^2}{2AES}$$

Deflection



From the right triangle

$$(l + S_{AB})^2 = l^2 + \delta^2$$

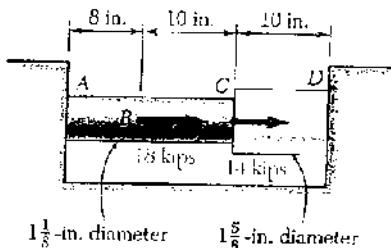
$$\delta^2 = l^2 + 2lS_{AB} + S_{AB}^2 - l^2$$

$$= 2lS_{AB} \left(1 + \frac{1}{2} \frac{S_{AB}}{l}\right) \approx 2lS_{AB}$$

$$\propto \frac{Pl^3}{AES}$$

$$S^3 \approx \frac{Pl^3}{AE} \therefore S \approx l \sqrt[3]{\frac{P}{AE}}$$

Problem 2.127



2.127 Two cylindrical rods, CD made of steel ($E = 29 \times 10^6$ psi) and AC made of aluminum ($E = 10.4 \times 10^6$ psi), are joined at C and restrained by rigid supports at A and D . Determine (a) the reactions at A and D , (b) the deflection of point C .

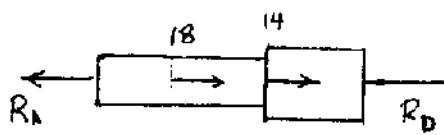
$$AB: P = R_A, L_{AB} = 8 \text{ in}$$

$$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$$

$$\sigma_{AB} = \frac{PL}{EA}$$

$$= \frac{R_A(8)}{(10.4 \times 10^6)(0.99402)}$$

$$= 0.77386 \times 10^{-6} R_A$$



$$BC: P = R_A - 18 \times 10^3, L = 10 \text{ in}, A = 0.99402 \text{ in}^2$$

$$\sigma_{BC} = \frac{PL}{EA} = \frac{(R_A - 18 \times 10^3)(10)}{(10.4 \times 10^6)(0.99402)} = 0.96732 \times 10^{-6} R_A = 17.412 \times 10^{-3}$$

$$CD: P = R_A - 18 \times 10^3 - 14 \times 10^3 = R_A - 32 \times 10^3$$

$$L = 10 \text{ in} \quad A = \frac{\pi}{4} d_{CD}^2 = \frac{\pi}{4}(1.625)^2 = 2.0739 \text{ in}^2$$

$$\sigma_{CD} = \frac{PL}{EA} = \frac{(R_A - 32 \times 10^3)(10)}{(29 \times 10^6)(2.0739)} = 0.16627 \times 10^{-6} R_A = 5.321 \times 10^{-3}$$

$$\sigma_{AD} = \sigma_{AB} + \sigma_{BC} + \sigma_{CD} = 1.9075 \times 10^{-6} R_A = 22.733 \times 10^{-3}$$

Since point D cannot move relative to A $\sigma_{AD} = 0$

$$(a) 1.9075 \times 10^{-6} R_A - 22.733 \times 10^{-3} = 0 \quad R_A = 11.92 \times 10^3 \text{ lb.} \leftarrow$$

$$R_D = 32 \times 10^3 - R_A = 20.08 \times 10^3 \text{ lb.} \leftarrow$$

$$(b) \sigma_c = \sigma_{AB} + \sigma_{CD}$$

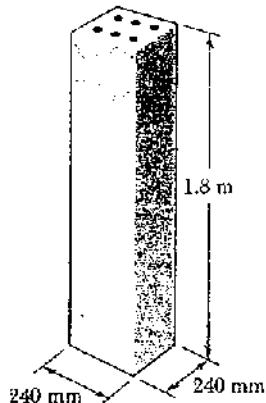
$$= 1.7412 \times 10^{-6} R_A - 17.412 \times 10^{-3}$$

$$= (1.7412 \times 10^{-6})(11.92 \times 10^3) - 17.412 \times 10^{-3} = 3.34 \times 10^{-3} \text{ in} \leftarrow$$

$$\text{or } \sigma_c = \frac{R_D L_{CD}}{E_{CD} A_{CD}} = \frac{(20.08 \times 10^3)(10)}{(29 \times 10^6)(2.0739)} = 3.34 \times 10^{-3} \text{ in} \leftarrow$$

Problem 2.128

2.128 The concrete post ($E_c = 25 \text{ GPa}$ and $\alpha_c = 9.9 \times 10^{-6}/^\circ\text{C}$) is reinforced with six steel bars, each of 22-mm diameter ($E_s = 200 \text{ GPa}$ and $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 35°C .



$$A_s = 6 \cdot \frac{\pi}{4} d^2 = 6 \cdot \frac{\pi}{4} (22)^2 = 2,280.8 \times 10^3 \text{ mm}^2 = 2.2808 \times 10^{-3} \text{ m}^2$$

$$A_c = 240^2 - A_s = 240^2 - 2.2808 \times 10^{-3} = 55.32 \times 10^3 \text{ mm}^2 \\ = 55.32 \times 10^{-3} \text{ m}^2$$

Let P_c = tensile force developed in the concrete

For equilibrium with zero total force, the compressive force in the six steel rods is P_c

Strains: $\varepsilon_s = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T) , \quad \varepsilon_c = \frac{P_c}{E_c A_c} + \alpha_c (\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s \quad \frac{P_c}{E_c A_c} + \alpha_c (\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s} \right) P_c = (\alpha_s - \alpha_c) (\Delta T)$$

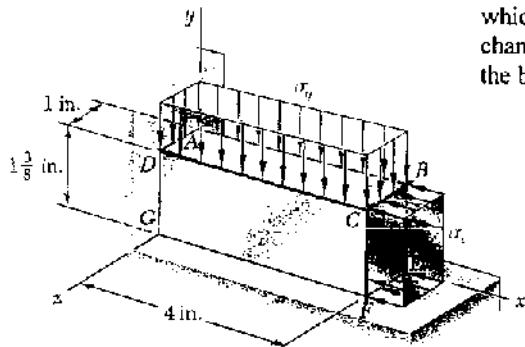
$$\left[\frac{1}{(25 \times 10^9) (55.32 \times 10^{-3})} + \frac{1}{(200 \times 10^9) (2.2808 \times 10^{-3})} \right] P_c = (1.8 \times 10^{-4}) (35)$$

$$P_c = 21.61 \times 10^3 \text{ N}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{21.61 \times 10^3}{55.32 \times 10^{-3}} = 0.391 \times 10^6 \text{ Pa} = 0.391 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_s = -\frac{P_c}{A_s} = \frac{21.61 \times 10^3}{2.2808 \times 10^{-3}} = -9.47 \times 10^6 \text{ Pa} = -9.47 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.129



2.129 The block shown is made of a magnesium alloy for which $E = 6.5 \times 10^6$ psi and $\nu = 0.35$. Knowing that $\sigma_z = -20$ ksi, determine (a) the magnitude of σ_y for which the change in the height of the block will be zero, (b) the corresponding change in the area of the face ABCD, (c) the corresponding change in the volume of the block.

$$\sigma_y = 0 \quad \epsilon_y = 0$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = 0$$

$$(a) \sigma_y = \nu \sigma_x = (0.35)(-20 \times 10^3)$$

$$= -7 \times 10^3 \text{ psi} = -7 \text{ ksi}$$

$$(b) \epsilon_z = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y) = -\frac{\nu (\sigma_x + \sigma_y)}{E}$$

$$= \frac{(0.35)(-20 \times 10^3 - 7 \times 10^3)}{6.5 \times 10^6} = 1.4538 \times 10^{-3}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{-20 \times 10^3 - (0.35)(-7 \times 10^3)}{6.5 \times 10^6}$$

$$= -2.7 \times 10^{-3}$$

$$A_o + \Delta A = L_x (1 + \epsilon_x) L_z (1 + \epsilon_z) = L_x L_z (1 + \epsilon_x + \epsilon_z + \epsilon_x \epsilon_z)$$

$$\text{But } A_o = L_x L_z$$

$$\Delta A = L_x L_z (\epsilon_x + \epsilon_z + \epsilon_x \epsilon_z)$$

$$= (4.0)(1.0)(1.4538 \times 10^{-3} - 2.7 \times 10^{-3} + \text{small term})$$

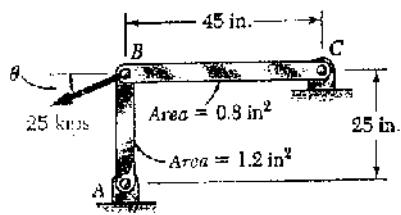
$$= -4.98 \times 10^{-3} \text{ in}^2 = -0.00498 \text{ in}^2$$

(c) Since L_y is constant

$$\Delta V = L_y (\Delta A) = (1.375)(-4.98 \times 10^{-3}) = -6.85 \times 10^{-3} \text{ in}^3$$

$$= -0.00685 \text{ in}^3$$

Problem 2.130



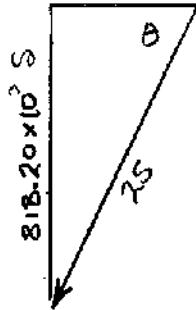
2.130 Knowing that $E = 29 \times 10^6$ psi, determine (a) the value of θ for which the deflection of point B is down and to the left along a line forming an angle of 36° with the horizontal, (b) the corresponding magnitude of the deflection of B.

$$S_{BC} = S \cos 36^\circ$$

$$P_{BC} = \frac{EA_{BC} S_{BC}}{L_{BC}} = \frac{(29 \times 10^6)(0.8) S \cos 36^\circ}{45}$$

$$= 417.09 \times 10^3 S$$

$$417.09 \times 10^3 S$$



$$S_{AC} = S \sin 36^\circ$$

$$P_{AC} = \frac{EA_{AC} S_{AC}}{L_{AC}} = \frac{(29 \times 10^6)(1.2) S \sin 36^\circ}{25}$$

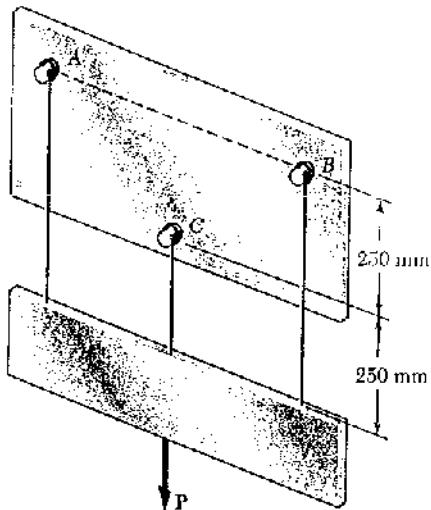
$$= 818.20 \times 10^3 S$$

$$\tan \theta = \frac{818.20 \times 10^3 S}{417.09 \times 10^3 S} = 1.9617 \quad \theta = 63.0^\circ$$

$$P = 25 \times 10^3 = \sqrt{(417.09 \times 10^3 S)^2 + (818.20 \times 10^3 S)^2} = 918.38 \times 10^3 S$$

$$S = \frac{25 \times 10^3}{918.38 \times 10^3} = 0.0272 \text{ in.}$$

Problem 2.131



2.131 Steel wires of 3.25-mm diameter are used at *A* and *B* while an aluminum wire of 2-mm diameter is used at *C*. Knowing that each wire is initially taut, determine the additional tension in each wire when a 900-N force *P* is applied to the midpoint of the lower edge of the plate. Use $E_s = 200 \text{ GPa}$ for steel and $E_a = 70 \text{ GPa}$ for aluminum.

Let P_A , P_B , and P_C be the wire tensions.

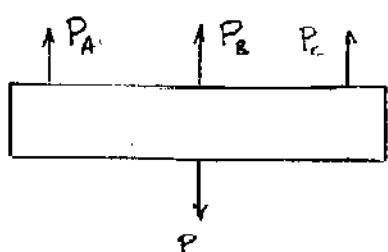
$$\text{By symmetry } P_A = P_B, S_A = S_B$$

$$\text{Also } S_C = S_A = S_B = S$$

$$A_A = \frac{\pi}{4}(3.25)^2 = 8.2958 \text{ mm}^2 = 8.2958 \times 10^{-6} \text{ m}^2$$

$$A_B = A_A$$

$$L_A = L_B = 500 \text{ mm} = 0.500 \text{ m}$$



$$P_A = P_B = \frac{E_s A_s S}{L_s} = \frac{(200 \times 10^9)(8.2958 \times 10^{-6})}{0.500} S$$

$$= 3.3183 \times 10^6 S$$

$$A_C = \frac{\pi}{4}(2)^2 = 3.1416 \text{ mm}^2 = 3.1416 \times 10^{-6} \text{ m}^2$$

$$L_C = 250 \text{ mm} = 0.250 \text{ m}$$

$$P_C = \frac{E_a A_a S}{L_a} = \frac{(70 \times 10^9)(3.1416 \times 10^{-6})}{0.250} S$$

$$= 0.87965 \times 10^6 S$$

$$\uparrow \sum F_y = 0 \quad P_A + P_B + P_C - P = 0$$

$$(3.3183 \times 10^6)S + (3.3183 \times 10^6)S + (0.87965 \times 10^6)S - 900 = 0$$

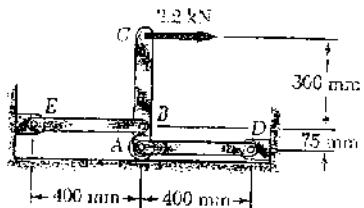
$$S = 119.74 \times 10^{-6} \text{ m}$$

$$P_A = P_B = (3.3183 \times 10^6)(119.74 \times 10^{-6}) = 397 \text{ N}$$

$$P_C = (0.87965 \times 10^6)(119.74 \times 10^{-6}) = 105.3 \text{ N}$$

Problem 2.132

2.132 The steel bars BE and AD each have a 6×18 -mm cross section. Knowing that $E = 200 \text{ GPa}$, determine the deflections of points A , B , and C of the rigid bar ABC .



Use rigid bar ABC as a free body

$$\textcircled{D} \sum M_B = 0 \quad (75) P_{AD} - (300)(3.2) = 0$$

$$P_{AD} = 12.8 \text{ kN}$$

$$\rightarrow \sum F_x = 0 \quad -P_{BE} + 3.2 + P_{AD} = 0$$

$$P_{BE} = 16 \text{ kN}$$

Deformations

$$A = (6)(18) = 108 \text{ mm}^2 = 108 \times 10^{-6} \text{ m}^2$$

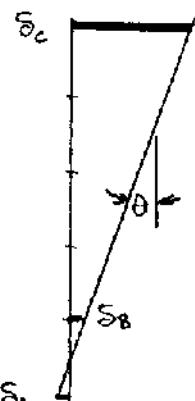
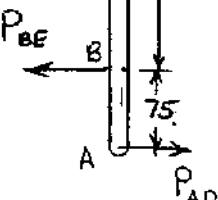
$$\leftarrow S_A = S_{AD} = \frac{P_{AD} L_{AD}}{EA} = \frac{(12.8 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})} \\ = 237.04 \times 10^{-6} \text{ m} = 0.237 \text{ mm} \leftarrow$$

$$\leftarrow S_B = S_{BE} = \frac{P_{BE} L_{BE}}{EA} = \frac{(16 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})} \\ = 296.30 \times 10^{-6} \text{ m} = 0.296 \text{ mm} \rightarrow$$

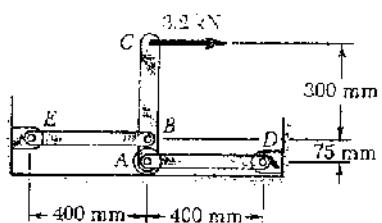
$$\Theta = \frac{S_A + S_B}{L_{AB}} = \frac{(237.04 + 296.30) \times 10^{-6}}{75 \times 10^{-3}} \\ = 7.1112 \times 10^{-3}$$

$$S_C = S_B + L_{BC} \Theta$$

$$= 296.30 \times 10^{-6} + (300 \times 10^{-3})(7.1112 \times 10^{-3}) \\ = 2.4297 \times 10^{-3} \text{ m} = 2.43 \text{ mm} \rightarrow$$



Problem 2.133



2.133 In Prob. 2.132, the 3.2-kN force caused point C to deflect to the right. Using $\alpha = 11.7 \times 10^6 / ^\circ\text{C}$, determine the (a) the overall change in temperature that causes point C to return to its original position, (b) the corresponding total deflection of points A and B.

2.132 The steel bars BE and AD each have a 6×18-mm cross section. Knowing that $E = 200 \text{ GPa}$, determine the deflections of points A, B, and C of the rigid bar ABC.

Use rigid ABC as a free body

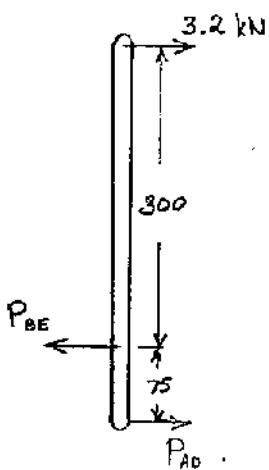
$$\rightarrow \sum M_B = 0 \quad 75 P_{AD} - (300)(3.2) = 0$$

$$P_{AD} = 12.8 \text{ kN}$$

$$\rightarrow \sum F_x = 0 \quad -P_{BE} + 3.2 + P_{AD} = 0$$

$$P_{BE} = 16 \text{ kN}$$

Deformations:



$$\leftarrow S_A = S_{AD} = \frac{P_{AD} L_{AD}}{E A} + L_{AD} \alpha (\Delta T)$$

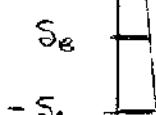
$$= \frac{(12.8 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})} + (400 \times 10^{-3})(11.7 \times 10^6)(\Delta T)$$

$$= 237.04 \times 10^{-6} + 4.68 \times 10^{-6}(\Delta T)$$

$$\rightarrow S_B = S_{BE} = \frac{P_{BE} L_{BE}}{E A} + L_{BE} \alpha (\Delta T)$$

$$= \frac{(16 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})} + (400 \times 10^{-3})(11.7 \times 10^6)(\Delta T)$$

$$= 296.30 \times 10^{-6} + 4.68 \times 10^{-6}(\Delta T)$$



$$S_c = 0 \quad S_B = 0.300 \theta \quad -S_A = 0.375 \theta$$

$$-S_A = \frac{0.375}{0.300} S_A = 1.25 S_A$$

$$-(237.04 \times 10^{-6} + 4.68 \times 10^{-6}(\Delta T)) = (1.25)[296.30 \times 10^{-6} + 4.68 \times 10^{-6}(\Delta T)]$$

$$-10.53 \times 10^{-6}(\Delta T) = 607.415 \times 10^{-6} \therefore \Delta T = -57.684 \text{ } ^\circ\text{C}$$

$$= -57.7 \text{ } ^\circ\text{C}$$

$$S_A = 237.04 \times 10^{-6} + (4.68 \times 10^{-6})(-57.684) = -32.92 \times 10^{-6} \text{ m}$$

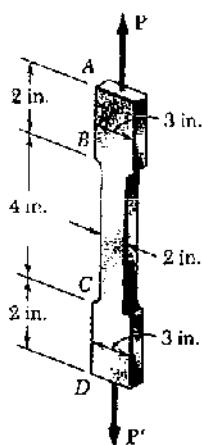
$$S_A = 0.0329 \text{ mm} \rightarrow$$

$$S_B = 296.30 \times 10^{-6} - (4.68 \times 10^{-6})(-57.684) = +26.34 \times 10^{-6} \text{ m}$$

$$S_B = 0.0263 \text{ mm} \rightarrow$$

Problem 2.134

2.134 The steel tensile specimen ABCD ($E = 29 \times 10^6$ psi and $\sigma_y = 50$ ksi) is loaded in tension until the maximum strain is $\epsilon = 0.0025$. (a) Neglecting the effect of the fillets on the change in length of the specimen, determine the resulting overall length AD of the specimen after the load is removed. (b) Following the removal of the load in part a, a compressive load is applied until the maximum compressive strain is $\epsilon = -0.0020$. Determine the resulting change in length AD after the compressive load is removed.



$$(a) \epsilon_y = \frac{\sigma_y}{E} = \frac{50 \times 10^3}{29 \times 10^6} = 0.001724$$

$\epsilon_{max} = 0.0025 > \epsilon_y$ Yielding occurs in portion BC

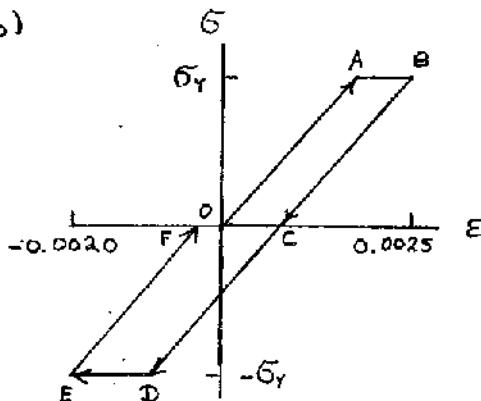
$$\sigma_{BC} = \sigma_y = 50 \times 10^3 \text{ psi}$$

Permanent strain in BC

$$\epsilon_{BC} = \epsilon_{max} - \epsilon_y = 0.0025 - 0.001724 = 0.000776$$

$$S_{BC} = L_{BC} \epsilon_B = (4)(0.000776) = 0.00310 \text{ in.}$$

(b)



In reversed loading, at point E on stress-strain plot

$$\epsilon = -0.0020$$

as given. During removal of the reversed load, the change in strain is $\epsilon_y/E = 0.001724$.

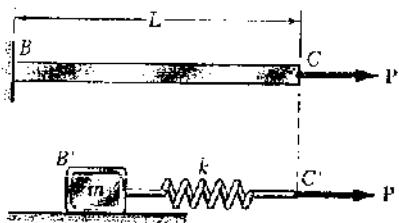
The permanent strain in BC is

$$\epsilon_{BC} = -0.0020 + 0.001724 = -0.000276$$

$$S_{BC} = L \epsilon_{BC} = (4)(-0.000276) = -0.001104 \text{ in.}$$

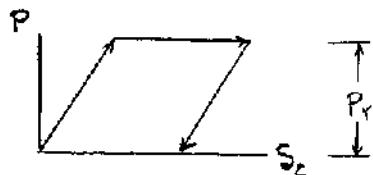
Note that portions AB and CD are always elastic, thus their deformations during loading and unloading do not contribute to any permanent deformation. Hence $S_{AD} = S_{BC}$ whenever the load P is zero.

Problem 2.135



2.135 The uniform rod BC has a cross-sectional area A and is made of a mild steel which can be assumed to be elastoplastic with a modulus of elasticity E and a yield strength σ_y . Using the block-and-spring system shown, it is desired to simulate the deflection of end C of the rod as the axial force P is gradually applied and removed, that is, the deflection of points C and C' should be the same for all values of P . Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (a) the required mass m of the block, (b) the required constant k of the spring.

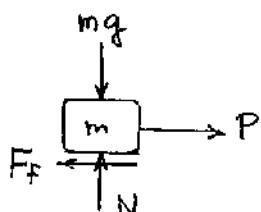
Force-deflection diagram for point C of rod BC .



$$\text{For } P < P_y = A\sigma_y$$

$$s_c = \frac{PL}{EA} \quad P = \frac{EA}{L} s_c$$

$$P_{max} = P_y = A\sigma_y$$



Force-deflection diagram for point C' of block and spring system.

$$+\uparrow \sum F_y = 0 : \quad N - mg = 0 \quad N = mg$$

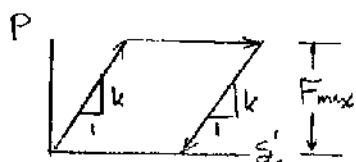
$$\pm \sum F_x = 0 : \quad P - F_f = 0 \quad P = F_f$$

If block does not move, i.e. $F_f < \mu N = \mu mg$ or $P < \mu mg$,

$$\text{then } s'_c = \frac{P}{k} \quad \text{or} \quad P = k s'_c$$

If $P = \mu mg$, then slip at $P = F_f = \mu mg$ occurs.

If the force P is removed, the spring returns to its initial length.

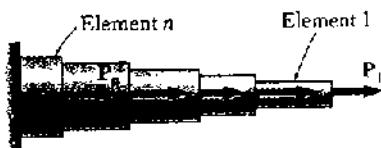


(a) Equating P_y and F_{max}

$$A\sigma_y = \mu mg \quad m = \frac{A\sigma_y}{\mu g}$$

(b) Equating slopes:

$$k = \frac{EA}{L}$$

PROBLEM 2.C1

2.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i , its cross-sectional area by A_i , modulus of elasticity by E_i , and the load applied to its right end by P_i , the magnitude P_i of this load being assumed to be positive if P_i is directed to the right and negative otherwise. (a) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (b) Use this program to solve Probs. 2.18 and 2.19.

SOLUTION

FOR EACH ELEMENT, ENTER

$$L_i, A_i, E_i$$

COMPUTE DEFORMATION

$$\text{UPDATE AXIAL LOAD } P = P + P_i$$

COMPUTE FOR EACH ELEMENT

$$\sigma_i = P/A_i$$

$$\delta_i = PL_i/A_iE_i$$

TOTAL DEFORMATION:UPDATE THROUGH n ELEMENTS

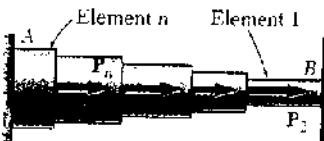
$$\delta = \delta + \delta_i$$

PROGRAM OUTPUT

Problem 2.18		
Element	Stress (MPa)	Deformation (mm)
1	19.0986	.1091
2	-12.7324	-.0909
Total Deformation =		.0182 mm

Problem 2.19		
Element	Stress (ksi)	Deformation (in.)
1	12.7324	.0176
2	-2.8294	-.0057
Total Deformation =		.01190 in.

PROBLEM 2.C2



2.C2 Rod AB is horizontal with both ends fixed; it consists of n elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element i is denoted by L_i , its cross-sectional area by A_i , its modulus of elasticity by E_i , and the load applied to its right end by P_i , the magnitude P_i of this load being assumed to be positive if P_i is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (a) Write a computer program which can be used to determine the reactions at A and B , the average normal stress in each element, and the deformation of each element. (b) Use this program to solve Probs. 2.39 and 2.40.

SOLUTION

WE CONSIDER THE REACTION AT B REDUNDANT AND RELEASE THE ROD AT B

COMPUTE δ_B WITH $R_B = 0$

FOR EACH ELEMENT, ENTER
 L_i, A_i, E_i

UPDATE AXIAL LOAD

$$P = P + P_i$$

COMPUTE FOR EACH ELEMENT

$$\sigma_i = P/A_i$$

$$\delta_i = PL_i/A_i E_i$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE δ_B DUE TO UNIT LOAD AT B

$$\text{UNIT } \sigma_i = 1/A_i$$

$$\text{UNIT } \delta_i = L_i/A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

SUPERPOSITION

FOR TOTAL DISPLACEMENT AT $B = \text{ZERO}$

$$\delta_B + R_B \text{ UNIT } \delta_B = 0$$

SOLVING:

$$R_B = -\delta_B / \text{UNIT } \delta_B$$

THEN:

$$R_A = \sum P_i + R_B$$

CONTINUED

PROBLEM 2.C2 CONTINUEDFOR EACH ELEMENT

$$\sigma = \sigma_i + R_B \text{ UNIT } \sigma_i$$

$$\delta = \delta_i + R_B \text{ UNIT } \delta_i$$

PROGRAM OUTPUT

Problem 2.39

RA = -62.809 kN

RB = -37.191 kN

Element Stress (MPa) Deformation (mm)

1	-52.615	-.05011
2	3.974	.00378
3	2.235	.00134
4	49.982	.04498

Problem 2.40

RA = -45.479 kN

RB = -54.521 kN

Element Stress (MPa) Deformation (mm)

1	-77.131	-.03857
2	-20.542	-.01027
3	-11.555	-.01321
4	36.191	.06204

PROBLEM 2.C3

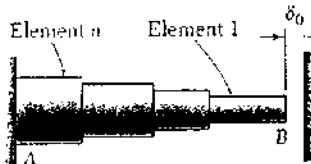


Fig. P2.C3

2.C3 Rod AB consists of n elements, each of which is homogeneous and of uniform cross section. End A is fixed, while initially there is a gap δ_0 between end B and the fixed vertical surface on the right. The length of element i is denoted by L_i , its cross-sectional area by A_i , its modulus of elasticity by E_i , and its coefficient of thermal expansion by α_i . After the temperature of the rod has been increased by ΔT , the gap at B is closed and the vertical surfaces exert equal and opposite forces on the rod. (a) Write a computer program which can be used to determine the magnitude of the reactions at A and B , the normal stress in each element, and the deformation of each element. (b) Use this program to solve Probs. 2.52, 2.53, 2.55, and 2.57.

SOLUTION

WE COMPUTE THE DISPLACEMENTS AT B
ASSUMING THERE IS NO SUPPORT AT B :

ENTER L_i, A_i, E_i, α_i

ENTER TEMPERATURE CHANGE T

COMPUTE FOR EACH ELEMENT

$$\delta_i = \alpha_i L_i T$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE δ_B DUE TO UNIT LOAD AT B

$$\text{UNIT } \delta_i = L_i / A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

COMPUTE REACTIONS

FROM SUPERPOSITION

$$R_B = (\delta_B - \delta_0) / \text{UNIT } \delta_B$$

THEN

$$R_A = -R_B$$

FOR EACH ELEMENT

$$\sigma_i = -R_B / A_i$$

$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

CONTINUED

PROBLEM 2.C3 CONTINUEDPROGRAM OUTPUT

Problem 2.52
R = 27.204 kips

Element	Stress (ksi)	Deform.(10 ⁻³ in.)
1	-22.168	-4.103
2	-6.842	4.103

Problem 2.53
R = 125.628 kN

Element	Stress (MPa)	Deform. (microm)
1	-44.432	.500
2	-99.972	-.500

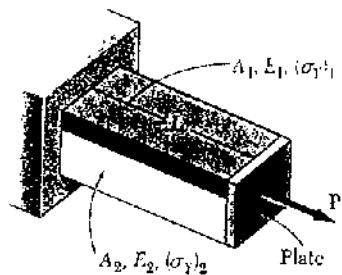
Problem 2.55
R = 232.390 kN

Element	Stress (MPa)	Deform. (microm)
1	-116.195	363.220
2	-290.487	136.780

Problem 2.57
R = 52.279 kips

Element	Stress (ksi)	Deform. (10 ⁻³ in.)
1	-21.783	9.909
2	-18.671	10.091

PROBLEM 2.C4



2.C4 Bar AB has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load P which is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (a) Write a computer program which, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (b) Use this program to solve Probs. 2.109, and 2.110.

SOLUTION

NOTE : THE FOLLOWING ASSUMES $(\sigma_y)_1 < (\sigma_y)_2$

DISPLACEMENT INCREMENT

$$\delta_m = 0.05 (\sigma_y)_2 L / E_2$$

DISPLACEMENTS AT YIELDING

$$\delta_A = (\sigma_y)_1 L / E_1, \quad \delta_B = (\sigma_y)_2 L / E_2$$

FOR EACH DISPLACEMENT

IF $\delta_m < \delta_A$:

$$\sigma_1 = \delta_m E_1 / L$$

$$\sigma_2 = \delta_m E_2 / L$$

$$P_m = (\delta_m / L) (A_1 E_1 + A_2 E_2)$$

IF $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_y)_1$$

$$\sigma_2 = \delta_m E_2 / L$$

$$P_m = A_1 \sigma_1 + (\delta_m / L) A_2 E_2$$

IF $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_y)_1, \quad \sigma_2 = (\sigma_y)_2$$

$$P_m = A_1 \sigma_1 + A_2 \sigma_2$$

PERMANENT DEFORMATIONS, RESIDUAL STRESSES

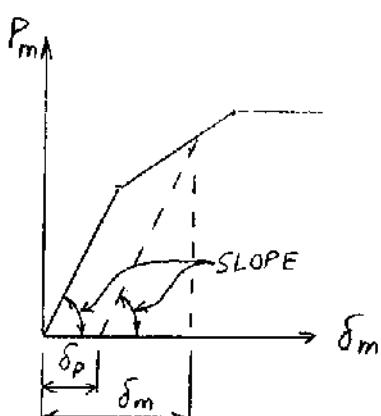
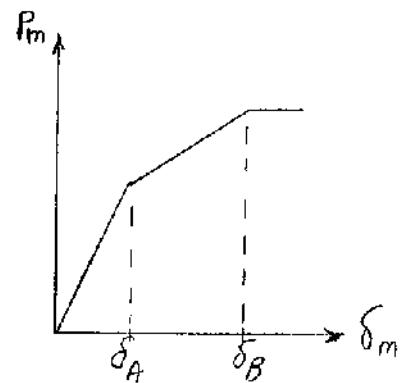
SLOPE OF FIRST (ELASTIC) SEGMENT

$$\text{SLOPE} = (A_1 E_1 + A_2 E_2) / L$$

$$\delta_p = \delta_m - (P_m / \text{SLOPE})$$

$$(\sigma_1)_{\text{res}} = \sigma_1 - (E_1 P_m / (L \text{ SLOPE}))$$

$$(\sigma_2)_{\text{res}} = \sigma_2 - (E_2 P_m / (L \text{ SLOPE}))$$



CONTINUED

PROBLEM 2.C4 CONTINUED**PROGRAM OUTPUT**

Problems 2.109 and 2.110

DM 10 ⁻³ in.	PM kips	SIGM(1) ksi	SIGM(2) ksi	DP 10 ⁻³ in.	SIGR(1) ksi	SIG(2) ksi
,000	,000	,000	,000	,000	,000	,000
2.414	8.750	5.000	5.000	,000	,000	,000
4.828	17.500	10.000	10.000	,000	,000	,000
7.241	26.250	15.000	15.000	,000	,000	,000
9.655	35.000	20.000	20.000	,000	,000	,000
12.069	43.750	25.000	25.000	,000	,000	,000
14.483	52.500	30.000	30.000	,000	,000	,000
16.897	61.250	35.000	35.000	,000	,000	,000
19.310	70.000	40.000	40.000	,000	,000	,000
21.724	78.750	45.000	45.000	,000	,000	,000
24.138	87.500	50.000	50.000	,000	,000	,000
26.552	91.250	50.000	55.000	1.379	-2.143	2.857
28.966	95.000	50.000	60.000	2.759	-4.286	5.714
31.379	98.750	50.000	65.000	4.138	-6.429	8.571
33.793	102.500	50.000	70.000	5.517	-8.571	11.429
36.207	106.250	50.000	75.000	6.897	-10.714	14.286
38.621	110.000	50.000	80.000	8.276	-12.857	17.143
41.034	113.750	50.000	85.000	9.655	-15.000	20.000
43.448	117.500	50.000	90.000	11.034	-17.143	22.857
45.862	121.250	50.000	95.000	12.414	-19.286	25.714
48.276	125.000	50.000	100.000	13.793	-21.429	28.571
50.690	125.000	50.000	100.000	16.207	-21.429	28.571
53.103	125.000	50.000	100.000	18.621	-21.429	28.571
55.517	125.000	50.000	100.000	21.034	-21.429	28.571
57.931	125.000	50.000	100.000	23.448	-21.429	28.571

2.110

2.109

PROBLEM 2.C5



2.C5 The plate has a hole centered across the width. The stress concentration factor for a flat bar under axial loading with a centric hole is:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. Write a computer program to determine the allowable load P for the given values of r , D , the thickness t of the bar, and the allowable stress σ_{all} of the material. Knowing that $t = \frac{1}{4}$ in., $D = 3.0$ in., and $\sigma_{all} = 16$ ksi., determine the allowable load P for values of r from 0.125 in. to 0.75 in., using 0.125 in. increments.

SOLUTION

ENTER

$$r, D, t, \sigma_{all}$$

COMPUTE K

$$RD = 2.0 \ r/D$$

$$K = 3.00 - 3.13 RD + 3.66 RD^2 - 1.53 RD^3$$

COMPUTE AVERAGE STRESS

$$\sigma_{ave} = \sigma_{all}/K$$

ALLOWABLE LOAD

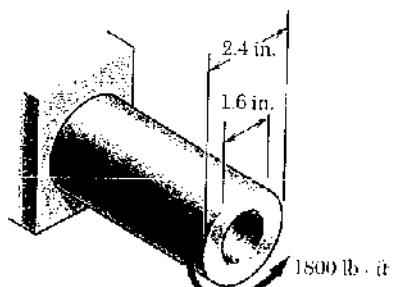
$$P_{all} = \sigma_{ave} (D - 2.0 r) t$$

PROGRAM OUTPUT

Radius (in.)	Allowable Load (kips)
.1250	3.9802
.2500	3.8866
.3750	3.7154
.5000	3.4682
.6250	3.1523
.7500	2.7794

Chapter 3

Problem 3.1



3.1 (a) For the hollow shaft and loading shown, determine the maximum shearing stress. (b) Determine the diameter of a solid shaft for which the maximum shearing stress in the loading shown is the same as in part a.

$$c_1 = \frac{1}{2}d_1 = (\frac{1}{2})(1.6) = 0.8 \text{ in}$$

$$c_2 = \frac{1}{2}d_2 = (\frac{1}{2})(2.4) = 1.2 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.2^4 - 0.8^4) = 2.6138 \text{ in}^3$$

$$T = 1800 \text{ lb-ft} = 21600 \text{ lb-in}$$

$$(a) \tau_{\max} = \frac{Tc}{J} = \frac{(21600)(1.2)}{2.6138} = 9917 \text{ psi}$$

$$\tau_{\max} = 9.92 \text{ ksi}$$

$$(b) \tau = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4 \quad \tau = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(21600)}{\pi(9917)} = 1.38666 \text{ in}^3$$

$$c = 1.115 \text{ in.} \quad d = 2c = 2.23 \text{ in.} \quad d = 2.23 \text{ in.}$$

Problem 3.2

3.2 (a) Determine the torque that can be applied to a solid shaft of 3.6-in. outer diameter without exceeding an allowable shearing stress of 10 ksi. (b) Solve part a, assuming that the solid shaft is replaced by a hollow shaft of the same mass and of 3.6-in. inner diameter.

$$(a) \text{For the solid shaft} \quad c = \frac{1}{2}d = (\frac{1}{2})(3.6) = 1.8 \text{ in.}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\pi}{2}(1.8)^3 = 9.1609 \text{ in}^5$$

$$\tau_{\max} = \frac{Tc}{J} \quad \text{or} \quad T = \frac{\tau_{\max} J}{c} = (10)(9.1609) = 91.609 \text{ kip-in}$$

$$T = 7.63 \text{ kip-ft}$$

$$(b) \text{Hollow shaft: } c_1 = \frac{1}{2}d_1 = (\frac{1}{2})(3.6) = 1.8 \text{ in.}$$

For equal masses the cross sectional areas must be equal.

$$A = \pi c^2 = \pi(c_2^2 - c_1^2) \quad \text{or} \quad c_2 = \sqrt{c_1^2 + c^2}$$

$$c_2 = \sqrt{(1.8)^2 + (1.8)^2} = 2.5456 \text{ in.}$$

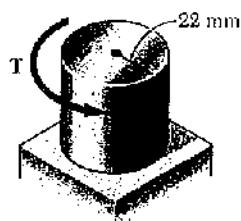
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 49.469 \text{ in}^4$$

$$T = \frac{\tau_{\max} J}{c_2} = \frac{(10)(49.469)}{2.5456} = 194.33 \text{ kip-in}$$

$$T = 16.19 \text{ kip-ft}$$

Problem 3.3

3.3 Determine the torque T that causes a maximum shearing stress of 80 MPa in the steel cylindrical shaft shown.



$$\tau_{max} = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4$$

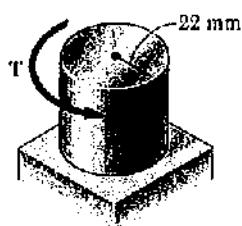
$$T = \frac{\pi}{2} c^3 \tau_{max} = \frac{\pi}{2} (0.022)^3 (80 \times 10^6)$$

$$= 1338.1 \text{ N}\cdot\text{m}$$

$$T = 133.8 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 3.4

3.4 For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T = 1.5 \text{ kN}\cdot\text{m}$.

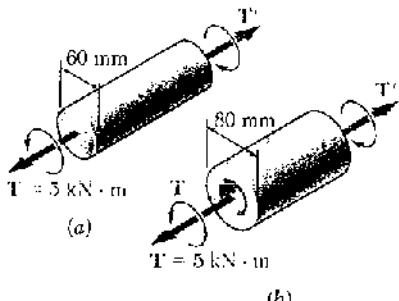


$$\tau_{max} = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4$$

$$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(1500)}{\pi (0.022)^3} = 89.682 \times 10^6$$

$$\tau_{max} = 89.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.5



3.5 (a) For the 60-mm-diameter solid cylinder and loading shown, determine the maximum shearing stress. (b) Determine the inner diameter of the hollow cylinder, of 80-mm outer diameter, for which the maximum stress is the same as in part a.

$$(a) \text{Solid shaft: } c = \frac{1}{2}d = \frac{1}{2}(0.060) = 0.030 \text{ m}$$

$$T_{max}^2 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(5000)}{\pi (0.030)^3} = 117.893 \times 10^6$$

$$\tau_{max} = 117.9 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \text{Hollow shaft: } c_2 = \frac{1}{2}d = \frac{1}{2}(0.080) = 0.040 \text{ m}$$

$$\frac{J}{c_2} = \frac{\frac{\pi}{2}(c_2^4 - c_1^4)}{c_2} = \frac{J}{2c_2}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi J_{max}} = (0.040)^4 - \frac{(2)(5000)(0.040)}{\pi (117.893 \times 10^6)} = 1.48 \times 10^{-6} \text{ m}^4$$

$$c_1 = 0.032237 \text{ m} = 34.879 \text{ mm}$$

$$d_1 = 2c_1 = 69.8 \text{ mm} \quad \blacktriangleleft$$

Problem 3.6

3.6 (a) Determine the torque that can be applied to a solid shaft of 20-mm diameter without exceeding an allowable shearing stress of 80 MPa. (b) Solve part a, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half of its own outer diameter.

$$(a) \text{Solid shaft: } c = \frac{1}{2}d = \frac{1}{2}(0.020) = 0.010 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.010)^4 = 15.7080 \times 10^{-9} \text{ m}^4$$

$$T = \frac{J \tau_{allow}}{c} = \frac{(15.7080 \times 10^{-9})(80 \times 10^6)}{0.010} = 125.664 \quad T = 125.7 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$(b) \text{Hollow shaft: } \text{Same area as solid shaft.}$$

$$A = \pi(c_2^2 - c_1^2) = \pi[c_2^2 - (\frac{1}{2}c_2)^2] = \frac{3}{4}\pi c_2^2 = \pi c_2^2$$

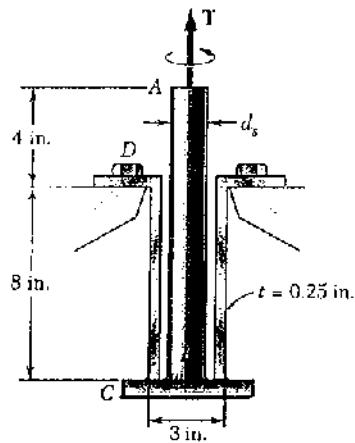
$$c_2 = \frac{2}{\sqrt{3}}c = \frac{2}{\sqrt{3}}(0.010) = 0.0115470 \text{ m}$$

$$c_1 = \frac{1}{2}c_2 = 0.0057735 \text{ m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0115470^4 - 0.0057735^4) = 26.180 \times 10^{-9} \text{ m}^4$$

$$T = \frac{\tau_{allow} J}{c_2} = \frac{(80 \times 10^6)(26.180 \times 10^{-9})}{0.0115470} = 181.38 \quad T = 181.4 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 3.7



3.7 The solid spindle AB has a diameter $d_s = 1.5$ in, and is made of a steel with an allowable shearing stress of 12 ksi, while sleeve CD is made of a brass with an allowable shearing stress of 7 ksi. Determine the largest torque T that can be applied at A.

$$\text{Solid spindle AB: } c = \frac{1}{2}d_s = \frac{1}{2}(1.5) = 0.75 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc}{J}$$

$$T_{AB} = \frac{J \tau_{\max}}{c} = \frac{(0.49701)(12)}{0.75} = 7.952 \text{ kip-in}$$

$$\text{Sleeve CD: } c_2 = \frac{1}{2}d_2 = \frac{1}{2}(3.0) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

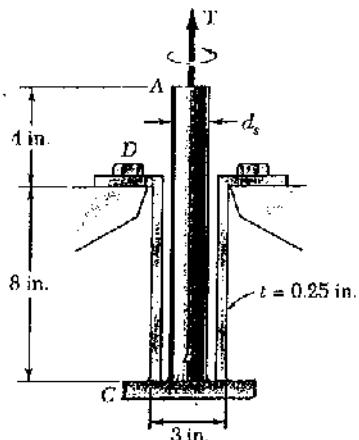
$$T_{CD} = \frac{J \tau_{\max}}{c_2} = \frac{(4.1172)(7)}{1.5} = 19.213 \text{ kip-in}$$

Allowable value of torque T is the smaller.

$$T = 7.95 \text{ kip-in} \quad \blacksquare$$

Problem 3.8

3.8 The solid spindle AB is made of a steel with an allowable shearing stress of 12 ksi, and sleeve CD is made of a brass with an allowable shearing stress of 7 ksi. Determine (a) the largest torque T that can be applied at A if the allowable shearing stress is not to be exceeded in sleeve CD, (b) the corresponding required value of the diameter d_s of spindle AB.



$$(a) \text{ Sleeve CD: } c_2 = \frac{1}{2}d_2 = \frac{1}{2}(3.0) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc_2}{J}$$

$$T_{CD} = \frac{J \tau_{\max}}{c_2} = \frac{(4.1172)(7)}{1.5} = 19.213 \text{ kip-in}$$

For equilibrium

$$T = 19.213 \text{ kip-in} \quad \blacksquare$$

$$(b) \text{ Solid spindle AB: } T = 19.213 \text{ kip-in}$$

$$\tau = \frac{TC}{J} = \frac{2T}{\pi c^3}$$

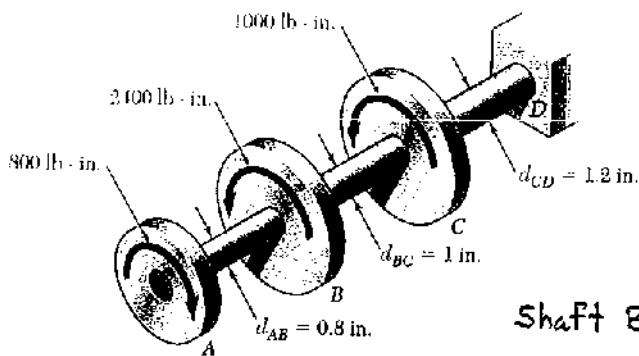
$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(19.213)}{\pi (12)}} = 1.0064 \text{ in.}$$

$$d_s = 2c = (2)(1.0064)$$

$$d_s = 2.01 \text{ in.} \quad \blacksquare$$

Problem 3.9

3.9 Knowing that each of the shafts AB, BC, and CD consist of solid circular rods, determine (a) the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress.



$$\text{Shaft AB: } T = 800 \text{ lb-in}$$

$$c = \frac{1}{2}d = 0.4 \text{ in.}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)(800)}{\pi (0.4)^3} = 7957 \text{ psi}$$

$$\text{Shaft BC: } T = -800 + 2400 = 1600 \text{ lb-in}$$

$$c = \frac{1}{2}d = 0.5 \text{ in.}$$

$$\tau_{\max} = \frac{(2)(1600)}{\pi (0.5)^3} = 8149 \text{ psi (largest)}$$

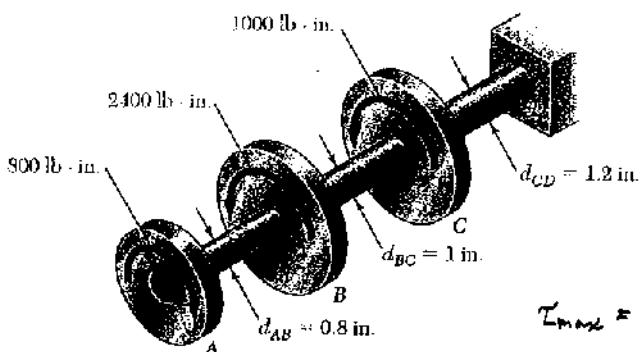
$$\text{Shaft CD: } T = -800 + 2400 + 1000 = 2600 \text{ lb-in} \quad c = \frac{1}{2}d = 0.6 \text{ in.}$$

$$\tau_{\max} = \frac{(2)(2600)}{\pi (0.6)^3} = 7663 \text{ psi}$$

Answers: (a) Shaft BC (b) 8.15 ksi

Problem 3.10

3.10 Knowing that a 0.40-in.-diameter hole has been drilled through each of the shafts AB, BC, and CD, determine (a) the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress.



$$\text{Hole: } c_1 = \frac{1}{2}d_1 = \frac{1}{2}(0.40) = 0.20 \text{ in.}$$

$$\text{Shaft AB: } T = 800 \text{ lb-in}$$

$$c_2 = \frac{1}{2}d_2 = 0.4 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.4^4 - 0.20^4) = 0.037699 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(800)(0.4)}{0.037699} = 8488 \text{ psi (largest)}$$

$$\text{Shaft BC: } T = -800 + 2400 = 1600 \text{ lb-in} \quad c_2 = \frac{1}{2}d_2 = 0.5 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.5^4 - 0.20^4) = 0.095661 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(1600)(0.5)}{0.095661} = 8363 \text{ psi}$$

$$\text{Shaft CD: } T = -800 + 2400 + 1000 = 2600 \text{ lb-in} \quad c_2 = \frac{1}{2}d_2 = 0.6 \text{ in.}$$

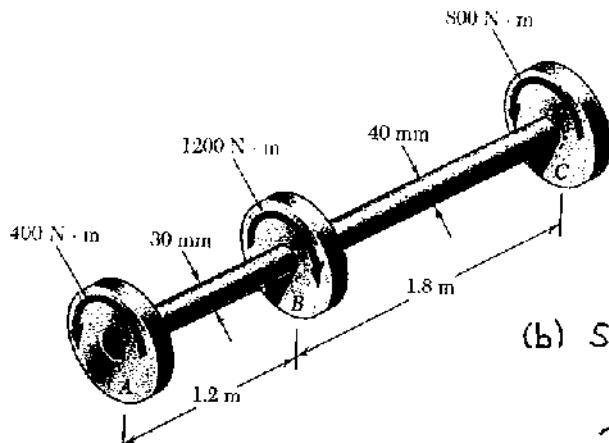
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.6^4 - 0.20^4) = 0.20106 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(2600)(0.6)}{0.20106} = 7759 \text{ psi}$$

Answers: (a) Shaft AB (b) 8.49 ksi

Problem 3.11

3.11 The torques shown are exerted on pulleys A, B, and C. Knowing that both shafts are solid, determine the maximum shearing stress in (a) shaft AB, (b) shaft BC.



(a) Shaft AB: $T = 400 \text{ N}\cdot\text{m}$

$$c = \frac{1}{2}d = \frac{1}{2}(0.030) = 0.015 \text{ m}$$

$$J = \frac{\pi}{2}C^4 \quad \tau_{max} = \frac{TC}{J} = \frac{2T}{\pi C^3}$$

$$\tau_{max} = \frac{(2)(400)}{\pi (0.015)^3} = 75.5 \times 10^6 \text{ Pa}$$

$$\tau_{max} = 75.5 \text{ MPa} \quad \blacktriangleleft$$

(b) Shaft BC: $T = 800 \text{ N}\cdot\text{m}$

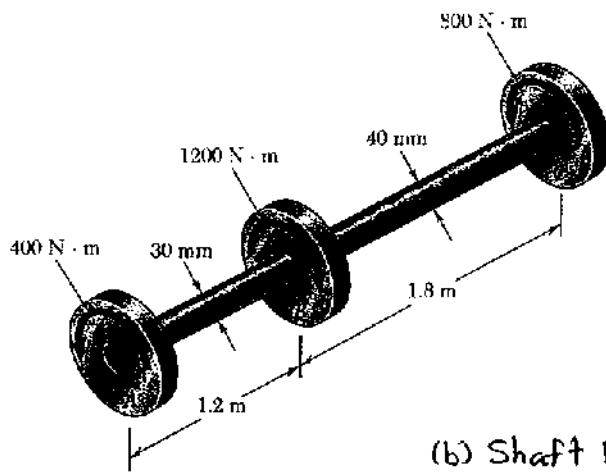
$$c = \frac{1}{2}d = 0.020 \text{ m}$$

$$\tau_{max} = \frac{2T}{\pi C^3} = \frac{(2)(800)}{\pi (0.020)^3} = 63.7 \times 10^6 \text{ Pa}$$

$$\tau_{max} = 63.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.12

3.12 The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 60 MPa, determine the smallest allowable diameter of (a) shaft AB, (b) shaft BC.



(a) Shaft AB: $T = 400 \text{ N}\cdot\text{m}$

$$\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

$$J = \frac{\pi}{2}C^4 \quad \tau_{max} = \frac{TC}{J} = \frac{2T}{\pi C^3}$$

$$C = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(400)}{\pi (60 \times 10^6)}}$$

$$= 16.19 \times 10^{-3} \text{ m} = 16.19 \text{ mm}$$

$$d_{AB} = 2C = 32.4 \text{ mm} \quad \blacktriangleleft$$

(b) Shaft BC: $T = 800 \text{ N}\cdot\text{m}$

$$\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

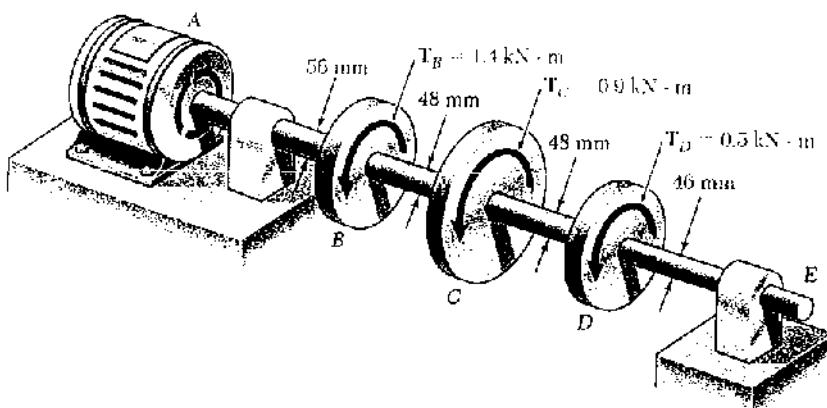
$$C = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(800)}{\pi (60 \times 10^6)}}$$

$$= 20.40 \times 10^{-3} \text{ m} = 20.40 \text{ mm}$$

$$d_{BC} = 2C = 40.8 \text{ mm} \quad \blacktriangleleft$$

Problem 3.13

3.13 Under normal operating conditions, the electric motor exerts a torque of 2.8 kN·m on shaft AB. Knowing that each shaft is solid, determine the maximum shearing stress in (a) shaft AB, (b) shaft BC, (c) shaft CD.



(a) Shaft AB: $T_{AB} = 2.8 \text{ kN}\cdot\text{m} = 2.8 \times 10^3 \text{ N}\cdot\text{m}$, $C = \frac{1}{2}d = 28 \text{ mm} = 0.028 \text{ m}$

$$\tau_{AB} = \frac{T_C}{J} = \frac{2T}{\pi C^3} = \frac{(2)(2.8 \times 10^3)}{\pi (0.028)^3} = 81.20 \times 10^6 \text{ Pa} \quad \tau_{AB} = 81.2 \text{ MPa}$$

(b) Shaft BC: $T_{BC} = 1.4 \text{ kN}\cdot\text{m} = 1.4 \times 10^3 \text{ N}\cdot\text{m}$, $C = \frac{1}{2}d = 24 \text{ mm} = 0.024 \text{ m}$

$$\tau_{BC} = \frac{2T}{\pi C^3} = \frac{(2)(1.4 \times 10^3)}{\pi (0.024)^3} = 64.47 \times 10^6 \text{ Pa} \quad \tau_{BC} = 64.5 \text{ MPa}$$

(c) Shaft CD: $T_{CD} = 0.5 \text{ kN}\cdot\text{m} = 0.5 \times 10^3 \text{ N}\cdot\text{m}$, $C = \frac{1}{2}d = 24 \text{ mm} = 0.024 \text{ m}$

$$\tau_{CD} = \frac{2T}{\pi C^3} = \frac{(2)(0.5 \times 10^3)}{\pi (0.024)^3} = 23.03 \times 10^6 \text{ Pa} \quad \tau_{CD} = 23.0 \text{ MPa}$$

Problem 3.14

3.14 In order to reduce the total mass of the assembly of Prob 3.13, a new design is being considered in which the diameter of shaft BC will be smaller. Determine the smallest diameter of shaft BC for which the maximum value of the shearing stress in the assembly will not be increased.

See the solution to Problem 3.13 for figure and for maximum shearing stresses in shafts AB, BC, and CD. The largest shearing occurs in shaft AB. Its magnitude is 81.2 MPa. Adjust the diameter of shaft BC so that its maximum shearing stress is 81.2 MPa.

$$\tau_{BC} = 81.2 \times 10^6 \text{ Pa}$$

$$T_{BC} = 1.4 \times 10^3 \text{ N}\cdot\text{m}$$

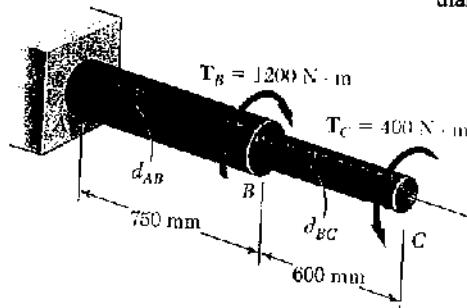
$$\tau_{BC} = \frac{T_C}{J} = \frac{2T}{\pi C^3}$$

$$C = \sqrt[3]{\frac{2T_{BC}}{\pi \tau_{BC}}} = \sqrt[3]{\frac{(2)(1.4 \times 10^3)}{\pi (81.2 \times 10^6)}} = 22.22 \times 10^{-3} \text{ m} = 22.22 \text{ mm}$$

$$d = 2C = 44.4 \text{ mm}$$

Problem 3.15

3.15 The solid shaft shown is formed of a brass for which the allowable shearing stress 55 MPa . Neglecting the effect of stress concentrations, determine smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.



$$\tau_{max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$\text{Shaft AB: } T_{AB} = 1200 - 400 = 800 \text{ N·m}$$

$$c = \sqrt[3]{\frac{(2)(800)}{\pi(55 \times 10^6)}} = 21.00 \times 10^{-3} \text{ m} = 21.0 \text{ mm}$$

$$\text{minimum } d_{AB} = 2c = 42.0 \text{ mm} \blacksquare$$

$$\text{Shaft BC: } T_{BC} = 400 \text{ N·m}$$

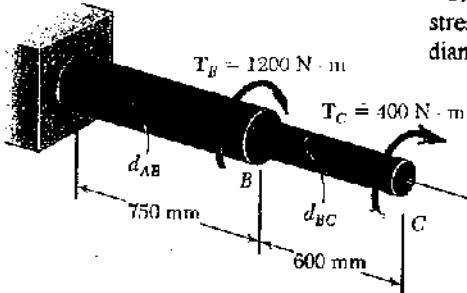
$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

$$\text{minimum } d_{BC} = 2c = 33.3 \text{ mm} \blacksquare$$

Problem 3.16

3.16 Solve Prob. 3.15 assuming that the direction of T_c is reversed.

3.15 The solid shaft shown is formed of a brass for which the allowable shearing stress 55 MPa . Neglecting the effect of stress concentrations, determine smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded..



Note that the direction of T_c has been reversed in the figure to the left.

$$\tau_{max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$\text{Shaft AB: } T_{AB} = 1200 + 400 = 1600 \text{ N·m}$$

$$c = \sqrt[3]{\frac{(2)(1600)}{\pi(55 \times 10^6)}} = 26.46 \times 10^{-3} \text{ m} = 26.46 \text{ mm}$$

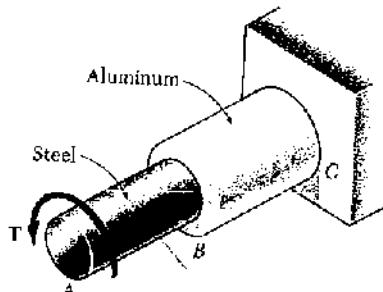
$$\text{minimum } d_{AB} = 2c = 52.9 \text{ mm} \blacksquare$$

$$\text{Shaft BC: } T_{BC} = 400 \text{ N·m}$$

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

$$\text{minimum } d_{BC} = 2c = 33.3 \text{ mm} \blacksquare$$

Problem 3.17



3.17 Shaft *AB* is made of a steel with an allowable shearing stress of 90 MPa and shaft *BC* is made of an aluminum with an allowable shearing stress of 60 MPa. Knowing that the diameter of shaft *BC* is 50 mm and neglecting the effect of stress concentrations, determine (a) the largest torque *T* that can be applied at *A* if the allowable stress is not to be exceeded in shaft *BC*, (b) the corresponding required diameter of shaft *AB*.

$$(a) \text{Shaft } BC: \tau_{max} = 60 \text{ MPa} = 90 \times 10^6 \text{ Pa}$$

$$d_{BC} = 50 \text{ mm} \quad C = \frac{1}{2} d = 25 \text{ mm} = 0.025 \text{ m}$$

$$\tau_{max} = \frac{TC}{J} = \frac{2T}{\pi C^3} \quad T = \frac{\pi}{2} \tau_{max} C^3$$

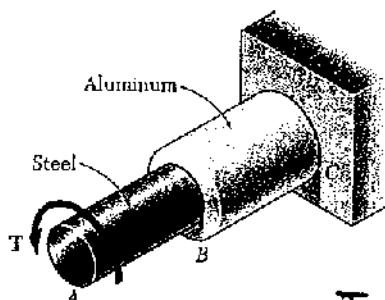
$$T = \frac{\pi}{2} (60 \times 10^6) (0.025)^3 = 1.47262 \times 10^3 \text{ N}\cdot\text{m} \quad T = 1.473 \text{ kN}\cdot\text{m}$$

$$(b) \text{Shaft } AB: \tau_{max} = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa} \quad T = 1.47262 \times 10^3 \text{ N}\cdot\text{m}$$

$$C = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(1.47262 \times 10^3)}{\pi (90 \times 10^6)}} = 21.84 \times 10^{-3} \text{ m} = 21.84 \text{ mm}$$

$$d_{AB} = 2C = 43.7 \text{ mm}$$

Problem 3.18



3.18 Shaft *AB* has a 30-mm diameter and is made of a steel with an allowable shearing stress of 90 MPa, while shaft *BC* has a 50-mm diameter and is made of an aluminum alloy with an allowable shearing stress of 60 MPa. Neglecting the effect of stress concentrations, determine the largest torque *T* that can be applied at *A*.

$$\text{Shaft } AB: C = \frac{1}{2} d_{AB} = \frac{1}{2}(30) = 15 \text{ mm} = 0.015 \text{ m}$$

$$\tau_{max} = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{TC}{J} = \frac{2T}{\pi C^3} \quad T = \frac{\pi}{2} \tau_{max} C^3$$

$$T_{AB} = \frac{\pi}{2} (90 \times 10^6) (0.015)^3 = 477 \text{ N}\cdot\text{m}$$

$$\text{Shaft } BC: C = \frac{1}{2} d_{BC} = \frac{1}{2}(50) = 25 \text{ mm} = 0.025 \text{ m}$$

$$\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

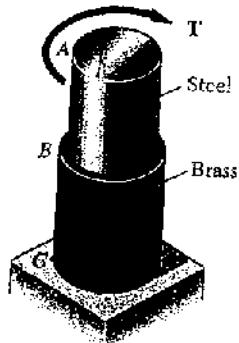
$$T_{BC} = \frac{\pi}{2} \tau_{max} C^3 = \frac{\pi}{2} (60 \times 10^6) (0.025)^3 = 1473 \text{ N}\cdot\text{m}$$

The allowable torque for the entire shaft is the smaller of T_{AB} and T_{BC}

$$T = T_{AB} = 477 \text{ N}\cdot\text{m}$$

Problem 3.19

3.19 The allowable shearing stress is 100 MPa in the 36-mm-diameter steel rod *AB* and 60 MPa in the 40-mm-diameter rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.



$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2} c^4, \quad T = \frac{\pi}{2} \tau_{max} c^3$$

$$\text{Shaft AB: } \tau_{max} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2} d_{AB} = \frac{1}{2}(36) = 18 \text{ mm} = 0.018 \text{ m}$$

$$T_{AB} = \frac{\pi}{2} (100 \times 10^6) (0.018)^3 = 916 \text{ N}\cdot\text{m}$$

$$\text{Shaft BC: } \tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2} d_{BC} = \frac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

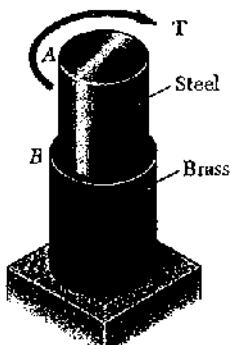
$$T_{BC} = \frac{\pi}{2} (60 \times 10^6) (0.020)^3 = 1.754 \text{ N}\cdot\text{m}$$

The allowable torque is the smaller of T_{AB} and T_{BC} .

$$T = 754 \text{ N}\cdot\text{m}$$

Problem 3.20

3.20 The allowable shearing stress is 100 MPa in the steel rod *AB* and 60 MPa in the brass rod *BC*. Knowing that a torque of magnitude $T = 900 \text{ N}\cdot\text{m}$ is applied at *A* and neglecting the effect of stress concentrations, determine the required diameter of (a) rod *AB*, (b) rod *BC*.



$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2} c^4, \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$\text{Shaft AB: } \tau_{max} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(100 \times 10^6)}} = 17.894 \times 10^{-3} \text{ m} = 17.894 \text{ mm}$$

$$d_{AB} = 2c = 35.8 \text{ mm}$$

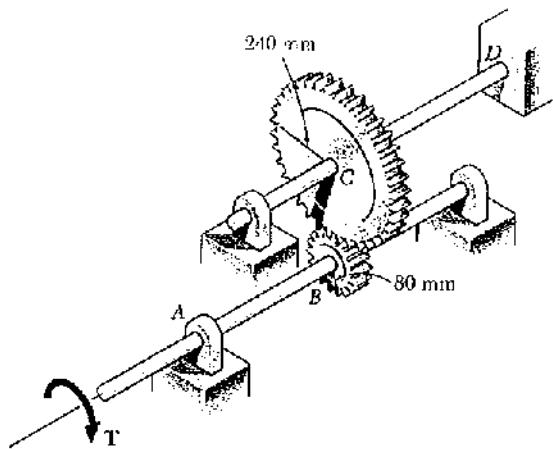
$$\text{Shaft BC: } \tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(60 \times 10^6)}} = 21.216 \times 10^{-3} \text{ m} = 21.216 \text{ mm}$$

$$d_{BC} = 2c = 42.4 \text{ mm}$$

Problem 3.21

3.21 Two solid steel shafts are connected by the gears shown. A torque of magnitude $T = 900 \text{ N}\cdot\text{m}$ is applied to shaft AB, knowing that the allowable shearing stress is 50 MPa and considering only stresses due to twisting, determine the required diameter of (a) shaft AB, (b) shaft CD.



$$T_{AB} = T = 900 \text{ N}\cdot\text{m}$$

$$T_{CD} = \frac{r_c}{r_b} T_{AB} = \frac{240}{80} (900) = 2700 \text{ N}\cdot\text{m}$$

$$(a) \text{Shaft AB: } \tau_{max} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}, \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(50 \times 10^6)}} = 22.55 \times 10^{-3} \text{ m} = 22.55 \text{ mm}$$

$$d_{AB} = 2c = 45.1 \text{ mm}$$

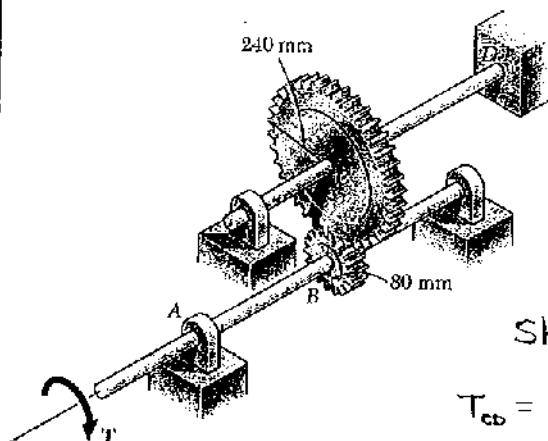
$$(b) \text{Shaft CD: } \tau_{max} = 50 \text{ MPa}$$

$$c = \sqrt[3]{\frac{(2)(2700)}{\pi(50 \times 10^6)}} = 32.52 \times 10^{-3} \text{ m} = 32.52 \text{ mm}$$

$$d_{CD} = 2c = 65.0 \text{ mm}$$

Problem 3.22

3.22 Shaft CD is made from a 66-mm-diameter rod and is connected to the 48-mm-diameter shaft AB as shown. Considering only stresses due to twisting and knowing that the allowable shearing stress is 60 MPa for each shaft, determine the largest torque T that can be applied.



$$\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

$$\text{Shaft AB: } c = \frac{1}{2} d_{AB} = 24 \text{ mm} = 0.024 \text{ m}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad T = \frac{\pi}{2} \tau_{max} c^3$$

$$T = \frac{\pi}{2} (60 \times 10^6) (0.024)^3 = 1303 \text{ N}\cdot\text{m}$$

$$\text{Shaft CD: } c = \frac{1}{2} d_{CD} = 33 \text{ mm} = 0.033 \text{ m}$$

$$T_{CD} = \frac{\pi}{2} (60 \times 10^6) (0.033)^3 = 3387 \text{ N}\cdot\text{m}$$

$$T = \frac{r_b}{r_c} T_{CD} = \frac{80}{240} (3387) = 1129 \text{ N}\cdot\text{m}$$

The allowable torque is the smaller of the two calculated values.

$$T = 1129 \text{ N}\cdot\text{m} = 1.129 \text{ kN}\cdot\text{m}$$

Problem 3.23

3.23 A torque of magnitude $T = 8 \text{ kip} \cdot \text{in}$. is applied at D as shown. Knowing that the diameter of shaft of shaft AB is 2.25 in. and that the diameter of shaft CD is 1.75 in., determine the maximum shearing stress in (a) shaft AB , (b) shaft CD .

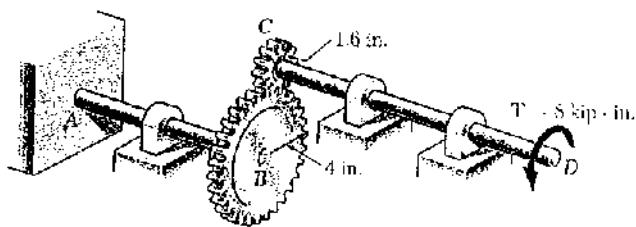
$$T_{co} = 8 \text{ kip} \cdot \text{in}$$

$$T_{AB} = \frac{r_e}{r_c} T_{co} = \frac{4}{1.6} (8) = 20 \text{ kip} \cdot \text{in}$$

$$(a) \text{Shaft } AB: c = \frac{1}{2} d_{AB} = 1.125 \text{ in.}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{max} = \frac{(2)(20)}{\pi (1.125)^3} = 8.94 \text{ ksi}$$



$$(a) \tau_{max} = 8.94 \text{ ksi}$$

$$(b) \text{Shaft } CD: c = \frac{1}{2} d_{CD} = 0.875 \text{ in.}$$

$$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(8)}{\pi (0.875)^3} = 7.60 \text{ ksi}$$

$$(b) \tau_{max} = 7.60 \text{ ksi}$$

Problem 3.24

3.24 A torque of magnitude $T = 8 \text{ kip} \cdot \text{in}$. is applied at D as shown. Knowing that the allowable shearing stress is 7.5 ksi in each shaft, determine the required diameter of (a) shaft AB , (b) shaft CD .

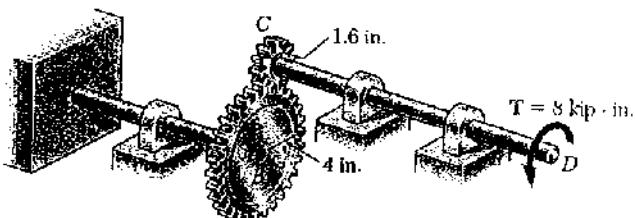
$$T_{co} = 8 \text{ kip} \cdot \text{in}$$

$$T_{AB} = \frac{r_e}{r_c} T_{co} = \frac{4}{1.6} (8) = 20 \text{ kip} \cdot \text{in}$$

$$\tau_{max} = 7.5 \text{ ksi}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$



$$(a) \text{Shaft } AB: c = \sqrt[3]{\frac{(2)(20)}{\pi (7.5)}} = 1.1929 \text{ in.}$$

$$d_{AB} = 2c = 2.39 \text{ in.}$$

$$(b) \text{Shaft } CD: c = \sqrt[3]{\frac{(2)(8)}{\pi (7.5)}} = 0.8789 \text{ in.}$$

$$d_{CD} = 2c = 1.758 \text{ in.}$$

Problem 3.25

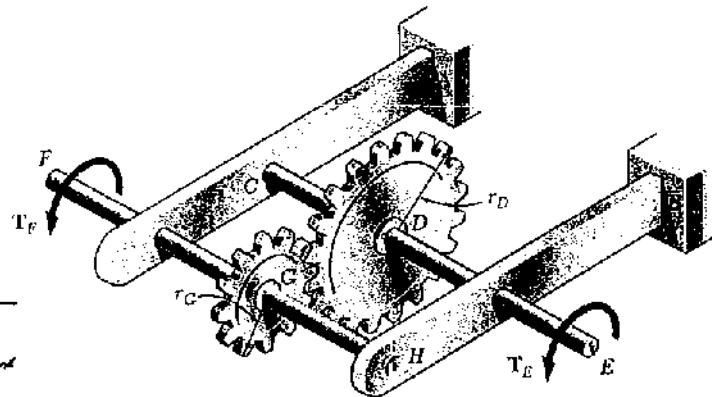
3.25 Under normal operating conditions a motor exerts a torque of magnitude $T_F = 150 \text{ N}\cdot\text{m}$ at F. The shafts are made of a steel for which the allowable shearing stress is 75 MPa . Knowing that for the gears $r_D = 200 \text{ mm}$ and $r_G = 75 \text{ mm}$, determine the required diameter of (a) shaft CDE, (b) shaft FGH.

$$T_{FG} = T_F = 150 \text{ N}\cdot\text{m}$$

$$T_{DE} = \frac{r_G}{r_D} T_{FG} = \frac{200}{75} (150) \\ = 400 \text{ N}\cdot\text{m}$$

$$\tau_{max} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}, \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$



(a) Shaft CDE:

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(75 \times 10^6)}} = 15.03 \times 10^{-3} \text{ m} = 15.03 \text{ mm} \quad d_{CDE} = 2c = 30.1 \text{ mm}$$

(b) Shaft FGH:

$$c = \sqrt[3]{\frac{(2)(150)}{\pi(75 \times 10^6)}} = 10.84 \times 10^{-3} \text{ m} = 10.84 \text{ mm} \quad d_{FGH} = 2c = 21.7 \text{ mm}$$

Problem 3.26

3.26 Under normal operating conditions a motor exerts a torque of magnitude T_F at F. The shafts are made of a steel for which the allowable shearing stress is 85 MPa and have diameters $d_{CDE} = 22 \text{ mm}$ and $d_{FGH} = 20 \text{ mm}$. Knowing that $r_D = 150 \text{ mm}$ and $r_G = 100 \text{ mm}$, determine the largest allowable value of T_F .

$$\tau_{max} = 85 \text{ MPa} = 85 \times 10^6 \text{ Pa}$$

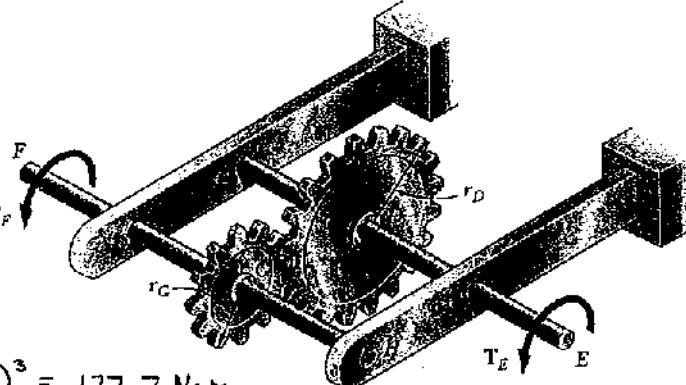
$$\text{Shaft FG: } c = \frac{1}{2}d = 10 \text{ mm} = 0.010 \text{ m}$$

$$T_F = \frac{J \tau_{max}}{c} = \frac{\frac{\pi}{2} c^3}{c} \tau_{max} = \frac{\pi}{2} (85 \times 10^6) (0.010)^3 = 133.5 \text{ N}\cdot\text{m}$$

$$\text{Shaft DE: } c = \frac{1}{2}d = 11 \text{ mm} = 0.011 \text{ m}$$

$$T_E = \frac{\pi}{2} c^3 \tau_{max} = \frac{\pi}{2} (85 \times 10^6) (0.011)^3 = 177.7 \text{ N}\cdot\text{m}$$

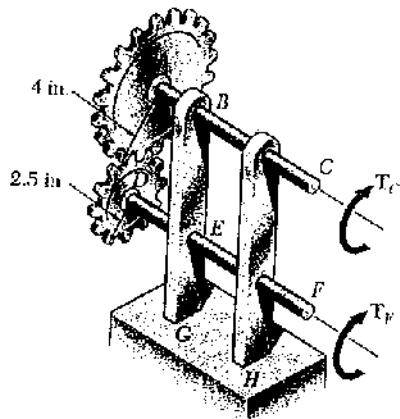
$$\text{By statics} \quad T_F = \frac{r_G}{r_D} T_E = \frac{100}{150} (177.7) = 118.5 \text{ N}\cdot\text{m}$$



The largest allowable value of T_F is the smaller of the two calculated values.

$$T_F = 118.5 \text{ N}\cdot\text{m}$$

Problem 3.27



3.27 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 8500 psi. Knowing that a torque of magnitude $T_c = 5$ kip · in. is applied at C and that the assembly is in equilibrium, determine the required diameter of (a) shaft BC, (b) shaft EF.

$$\tau_{max} = 8500 \text{ psi} = 8.5 \text{ ksi}$$

$$(a) \text{Shaft } BC: T_c = 5 \text{ kip-in}$$

$$T_{max} = \frac{T_c}{J} = \frac{2T}{\pi c^3} \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$c = \sqrt[3]{\frac{(2)(5)}{\pi(8.5)}} = 0.7208 \text{ in.}$$

$$d_{BC} = 2c = 1.442 \text{ in.}$$

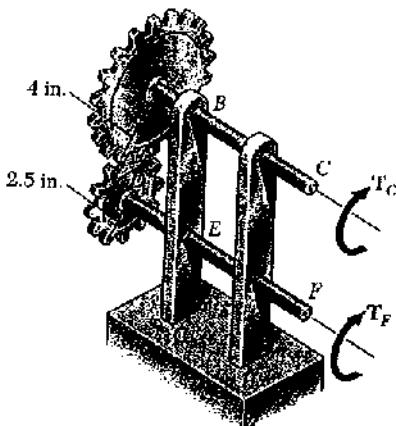
$$\text{Shaft } EF: T_f = \frac{r_b}{r_a} T_c = \frac{2.5}{4}(5) = 3.125 \text{ kip-in}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(3.125)}{\pi(8.5)}} = 0.6163 \text{ in.}$$

$$d_{EF} = 2c = 1.233 \text{ in.}$$

Problem 3.28

3.28 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 7000 psi. Knowing the diameters of the two shafts are, respectively, $d_{BC} = 1.6$ in. and $d_{EF} = 1.25$ in. determine the largest torque T_c that can be applied at C.



$$\tau_{max} = 7000 \text{ psi} = 7.0 \text{ ksi}$$

$$\text{Shaft } BC: d_{BC} = 1.6 \text{ in.} \quad c = \frac{d}{2} = 0.8 \text{ in.}$$

$$T_c = \frac{J \tau_{max}}{c} = \frac{\pi}{2} \tau_{max} c^3$$

$$= \frac{\pi}{2}(7.0)(0.8)^3 = 5.63 \text{ kip-in}$$

$$\text{Shaft } EF: d_{EF} = 1.25 \text{ in.} \quad c = \frac{d}{2} = 0.625 \text{ in.}$$

$$T_f = \frac{J \tau_{max}}{c} = \frac{\pi}{2} \tau_{max} c^3$$

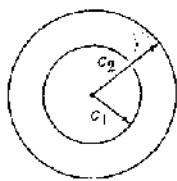
$$= \frac{\pi}{2}(7.0)(0.625)^3 = 2.684 \text{ kip-in}$$

$$\text{By statics } T_c = \frac{r_b}{r_a} T_f = \frac{4}{2.5} (2.684) = 4.30 \text{ kip-in}$$

Allowable value of T_c is the smaller.

$$T_c = 4.30 \text{ kip-in}$$

Problem 3.29



3.29 (a) For a given allowable stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (b) Denoting by $(T/w)_0$ the value of this ratio computed for a solid shaft of the same radius c_2 , express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2 .

w = weight per unit length, ρg = specific weight

W = total weight, L = length

$$w = \frac{W}{L} = \rho g L A = \rho g A = \rho g \pi (c_2^2 - c_1^2)$$

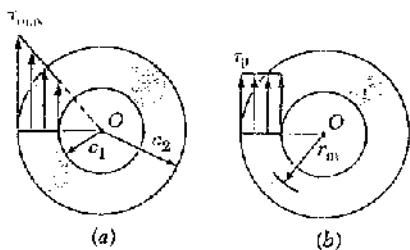
$$T_{al} = \frac{J \tau_{al}}{c_2} = \frac{\pi}{2} \frac{c_2^4 - c_1^4}{c_2} \tau_{al} = \frac{\pi}{2} \frac{(c_2^2 + c_1^2)(c_2^2 - c_1^2)}{c_2} \tau_{al}$$

$$\left(\frac{T}{w}\right)_h = \frac{(c_2^2 + c_1^2) \tau_{al}}{2 \rho g c_2} \quad (\text{hollow shaft})$$

$$c_1 = 0 \text{ for solid shaft} \quad \left(\frac{T}{w}\right)_0 = \frac{c_2 \tau_{al}}{2 \pi} \quad (\text{solid shaft})$$

$$\left(\frac{T}{w}\right)_h = 1 + \frac{c_1^2}{c_2^2} \quad \left(\frac{T}{w}\right)_h = \left(\frac{T}{w}\right)_0 \left(1 + \frac{c_1^2}{c_2^2}\right)$$

Problem 3.30



3.30 While the exact distribution of the shearing stresses in a hollow cylinder shaft is as shown in Fig. (1), an approximate value may be obtained for τ_{max} by assuming the stresses to be uniformly distributed over the area A of the cross section, as shown in Fig. (2), and then further assuming that all the elementary shearing forces act a distance from O equal to the mean radius $r_m = \frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value is $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2 respectively equal to 1.00, 0.95, 0.75, 0.50, and 0.

$$\text{For a hollow shaft: } \tau_{max} = \frac{TC_2}{J} = \frac{2TC_2}{\pi(c_2^4 - c_1^4)} = \frac{2TC_2}{\pi(c_2^2 - c_1^2)(c_2^2 + c_1^2)}$$

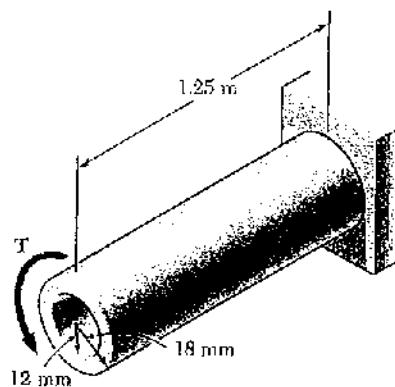
$$= \frac{2TC_2}{A(c_2^2 + c_1^2)}$$

$$\text{By definition } \tau_0 = \frac{T}{Ar_m} = \frac{2T}{A(c_2 + c_1)}$$

$$\text{Dividing } \frac{\tau_{max}}{\tau_0} = \frac{C_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2}$$

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

Problem 3.31



3.31 For the aluminum shaft shown ($G = 27 \text{ GPa}$), determine (a) the torque T what causes an angle of twist of 4° , (b) the angle of twist caused by the same torque T in a solid cylindrical shaft of the same length and cross sectional area.

$$(a) \varphi = \frac{TL}{GJ}, \quad T = \frac{GJ\varphi}{L}$$

$$\varphi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}, \quad L = 1.25 \text{ m}$$

$$G = 27 \text{ GPa} = 27 \times 10^9 \text{ Pa}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.018^4 - 0.012^4) = 132.324 \times 10^{-9} \text{ m}^4$$

$$T = \frac{(27 \times 10^9)(132.324 \times 10^{-9})(69.813 \times 10^{-3})}{1.25}$$

$$= 199.539 \text{ N}\cdot\text{m}$$

$$T = 199.5 \text{ N}\cdot\text{m} \blacksquare$$

(b) Matching areas:

$$A_i = \pi c^2 = \pi (c_2^2 - c_1^2)$$

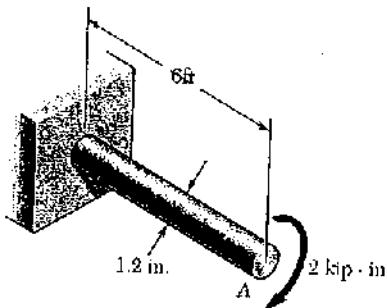
$$c = \sqrt{c_2^2 - c_1^2} = \sqrt{0.018^2 - 0.012^2} = 0.013416 \text{ m}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.013416)^4 = 50.894 \times 10^{-9} \text{ m}^4$$

$$\varphi = \frac{TL}{GJ} = \frac{(199.539)(1.25)}{(27 \times 10^9)(50.894 \times 10^{-9})} = 181.514 \times 10^{-3} \text{ rad} \quad \varphi = 10.40^\circ \blacksquare$$

Problem 3.32

3.32(a) For the solid steel shaft shown ($G = 11.2 \times 10^6 \text{ psi}$), determine the angle of twist at A. (b) Solve part (a), assuming that the steel shaft is hollow with a 1.2-in.-outer diameter and a 0.8-in.-inner diameter.



$$(a) c = \frac{1}{2}d = 0.6 \text{ in.} \quad J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.6)^4 = 0.203575 \text{ in}^4$$

$$L = 6 \text{ ft} = 72 \text{ in.}$$

$$T = 2 \text{ kip}\cdot\text{in} = 2000 \text{ lb}\cdot\text{in}$$

$$\varphi = \frac{(2000)(72)}{(11.2 \times 10^6)(0.203575)} = 63.157 \times 10^{-3} \text{ rad}$$

$$\varphi = 3.62^\circ \blacksquare$$

$$(b) c_2 = \frac{1}{2}d_2 = 0.6 \text{ in.} \quad c_1 = \frac{1}{2}d_1 = 0.4 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.6^4 - 0.4^4) = 0.163363 \text{ in}^4$$

$$\varphi = \frac{TL}{GJ} = \frac{(2000)(72)}{(11.2 \times 10^6)(0.163363)} = 78.703 \text{ rad}$$

$$\varphi = 4.51^\circ \blacksquare$$

Problem 3.33

3.33 Determine the largest allowable diameter of a 10-ft-long steel rod ($G = 11.2 \times 10^6$ psi) if the rod is to be twisted through 30° without exceeding a shearing stress of 12 ksi.

$$L = 10 \text{ ft} = 120 \text{ in.} \quad \phi = 30^\circ = \frac{30\pi}{180} = 0.52360 \text{ rad}$$

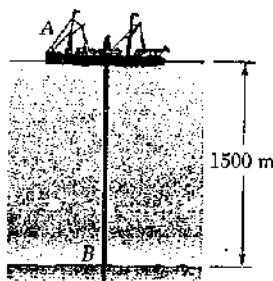
$$\tau = 12 \text{ ksi} = 12 \times 10^3 \text{ psi}$$

$$\phi = \frac{TL}{GJ}, \quad T = \frac{GJ\phi}{L}, \quad \tau = \frac{TC}{J} = \frac{GJ\phi c}{JL} = \frac{G\phi c}{L}, \quad c = \frac{\tau L}{G\phi}$$

$$c = \frac{(12 \times 10^3)(120)}{(11.2 \times 10^6)(0.52360)} = 0.24555 \text{ in.} \quad d = 2c = 0.491 \text{ in.}$$

Problem 3.34

3.34 The ship at A has just started to drill for oil on the ocean floor at a depth of 1500 m. Knowing that the top of the 200-mm-diameter steel drill pipe ($G = 77.2$ GPa) rotates through two complete revolutions before the drill bit at B starts to operate, determine the maximum shearing stress caused in the pipe by torsion.



$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ\phi}{L}$$

$$\tau = \frac{TC}{J} = \frac{GJ\phi c}{JL} = \frac{G\phi c}{L}$$

$$\phi = 2 \text{ rev} = 2(2\pi) \text{ rad} = 12.566 \text{ rad}$$

$$c = \frac{1}{2}d = 100 \text{ mm} = 0.100 \text{ m}$$

$$G = 77.2 \text{ GPa} = 77.2 \times 10^9 \text{ Pa}$$

$$\tau = \frac{(77.2 \times 10^9)(12.566)(0.100)}{1500} = 64.7 \times 10^6 \text{ Pa}$$

$$\tau = 64.7 \text{ MPa}$$

Problem 3.35

3.35 The torques shown are exerted on pulleys A, B, and C. Knowing that both shafts are solid and made of brass ($G = 39 \times 10^9 \text{ Pa}$), determine the angle of twist between (a) A and B, (b) A and C.

(a) Angle of twist between A and B

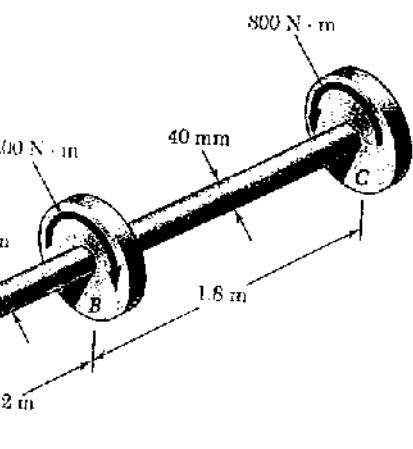
$$T_{AB} = 400 \text{ N}\cdot\text{m}, L_{AB} = 1.2 \text{ m}$$

$$c = \frac{1}{2}d = 0.015 \text{ m}, G = 39 \times 10^9 \text{ Pa}$$

$$J_{AB} = \frac{\pi}{2} c^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(400)(1.2)}{(39 \times 10^9)(79.52 \times 10^{-9})}$$

$$= 0.154772 \text{ rad } \square$$



$$\varphi_{A/B} = 8.87^\circ \square$$

(b) Angle of twist between A and C

$$T_{AC} = 800 \text{ N}\cdot\text{m}, L_{AC} = 1.8 \text{ m}, c = \frac{1}{2}d = 0.020 \text{ m}, G = 39 \times 10^9 \text{ Pa}$$

$$J_{AC} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.020)^4 = 251.327 \times 10^{-9} \text{ m}^4$$

$$\varphi_{A/C} = \frac{(800)(1.8)}{(39 \times 10^9)(251.327 \times 10^{-9})} = 0.146912 \text{ rad } \square$$

$$\varphi_{A/C} = \varphi_{A/B} + \varphi_{B/C} = 0.154772 + 0.146912 = 0.007850 \text{ rad } \square$$

$$\varphi_{C/A} = 0.450^\circ \square$$

Problem 3.36

3.36 The electric motor exerts a 500 N·m torque on the aluminum shaft ABCD when it is rotating at a constant speed. Knowing that $G = 27 \text{ GPa}$ and that the torques exerted on pulleys B and C are as shown, determine the angle twist between (a) B and C, (b) B and D.

(a) Angle of twist between B and C

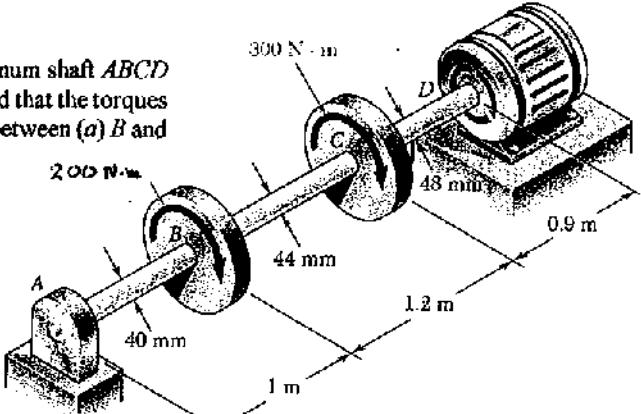
$$T_{BC} = 200 \text{ N}\cdot\text{m}, L_{BC} = 1.2 \text{ m}$$

$$c = \frac{1}{2}d = 0.022 \text{ m}, G = 27 \times 10^9 \text{ Pa}$$

$$J_{BC} = \frac{\pi}{2} c^4 = 367.97 \times 10^{-9} \text{ m}^4$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(200)(1.2)}{(27 \times 10^9)(367.97 \times 10^{-9})} = 24.157 \times 10^{-3} \text{ rad}$$

$$\varphi_{B/C} = 1.384^\circ \square$$



(b) Angle of twist between B and D.

$$T_{CD} = 500 \text{ N}\cdot\text{m}, L_{CD} = 0.9 \text{ m}, c = \frac{1}{2}d = 0.024 \text{ m}, G = 27 \times 10^9 \text{ Pa}$$

$$J_{CD} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.024)^4 = 521.153 \times 10^{-9} \text{ m}^4$$

$$\varphi_{C/D} = \frac{(500)(0.9)}{(27 \times 10^9)(521.153 \times 10^{-9})} = 31.980 \times 10^{-3} \text{ rad}$$

$$\varphi_{B/D} = \varphi_{B/C} + \varphi_{C/D} = 24.157 \times 10^{-3} + 31.980 \times 10^{-3} = 56.137 \times 10^{-3} \text{ rad}$$

$$\varphi_{B/D} = 3.22^\circ \square$$

Problem 3.37

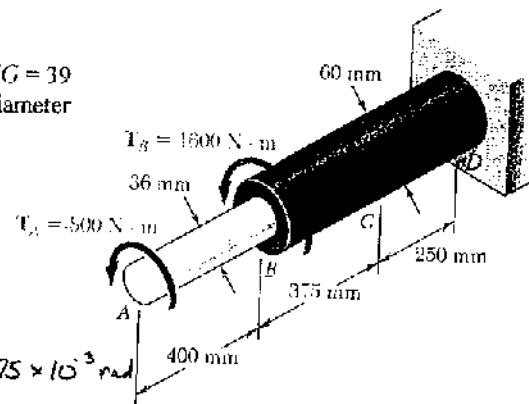
3.37 The aluminum rod AB ($G = 27 \text{ GPa}$) is bonded to the brass rod BD ($G = 39 \text{ GPa}$). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm determine the angle of twist at A .

Rod AB : $G = 27 \times 10^9 \text{ Pa}$, $L = 0.400 \text{ m}$

$$T = 800 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.018 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$



Part BC : $G = 39 \times 10^9 \text{ Pa}$, $L = 0.375 \text{ m}$, $c = \frac{1}{2}d = 0.030 \text{ m}$

$$T = 800 + 1600 = 2400 \text{ N} \cdot \text{m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\phi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3} \text{ rad}$$

Part CD : $c_1 = \frac{1}{2}d_1 = 0.020 \text{ m}$, $c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}$, $L = 0.250 \text{ m}$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6} \text{ m}^4$$

$$\phi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3} \text{ rad}$$

$$\text{Angle of twist at } A \quad \phi_A = \phi_{A/B} + \phi_{B/C} + \phi_{C/D}$$

$$= 105.080 \times 10^{-3} \text{ rad}$$

$$\phi_A = 6.02^\circ$$

Problem 3.38

3.38 Solve Prob. 3.37, assuming that portion BD is a solid 60-mm-diameter rod of length 625 mm.

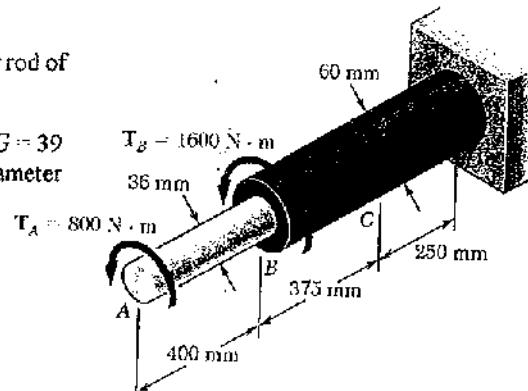
3.37 The aluminum rod AB ($G = 27 \text{ GPa}$) is bonded to the brass rod BD ($G = 39 \text{ GPa}$). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm determine the angle of twist at A .

Rod AB : $G = 27 \times 10^9 \text{ Pa}$, $L = 0.400 \text{ m}$

$$T = 800 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.18 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$



Rod BCD : $G = 39 \times 10^9 \text{ Pa}$, $L = 0.375 + 0.250 = 0.625 \text{ m}$, $c = \frac{1}{2}d = 0.030 \text{ m}$

$$T = 800 + 1600 = 2400 \text{ N} \cdot \text{m} \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\phi_{B/D} = \frac{TL}{GJ} = \frac{(2400)(0.625)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 30.229 \times 10^{-3} \text{ rad}$$

$$\text{Angle of twist at } A \quad \phi_A = \phi_{A/B} + \phi_{B/D}$$

$$= 102.104 \times 10^{-3} \text{ rad}$$

$$\phi_A = 5.85^\circ$$

Problem 3.39

3.39 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200$ N·m.

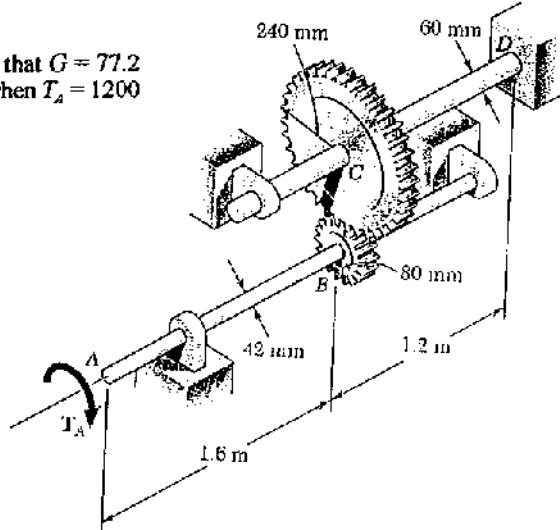
Calculation of torques.

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = 1200 \text{ N}\cdot\text{m}$$

$$T_{CD} = \frac{240}{80} (1200) = 3600 \text{ N}\cdot\text{m}$$



Twist in shaft CD: $c = \frac{1}{2}d = 0.030 \text{ m}$, $L = 1.2 \text{ m}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\phi_{CD} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(1.27234 \times 10^{-6})} = 43.981 \times 10^{-3} \text{ rad}$$

Rotation angle at C $\phi_C = \phi_{CD} = 43.981 \times 10^{-3} \text{ rad}$

Circumferential displacement at contact points of gears B and C

$$s = r_C \phi_C = r_B \phi_B$$

$$\text{Rotation angle at B} \quad \phi_B = \frac{r_C}{r_B} \phi_C = \frac{240}{80} (43.981 \times 10^{-3}) = 131.942 \times 10^{-3} \text{ rad}$$

Twist in shaft AB: $c = \frac{1}{2}d = 0.021 \text{ m}$, $L = 1.6 \text{ m}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.021)^4 = 305.49 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(305.49 \times 10^{-9})} = 81.412 \times 10^{-3} \text{ rad}$$

$$\text{Rotation angle at A} \quad \phi_A = \phi_B + \phi_{A/B} = 213.354 \times 10^{-3} \text{ rad}$$

$$\phi_A = 12.22^\circ$$

Problem 3.40

3.40 Solve Prob. 3.39, assuming that the diameter of each shaft is 54 mm.

3.39 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2 \text{ GPa}$ for each shaft, determine the angle through which end A rotates when $T_A = 1200 \text{ N}\cdot\text{m}$.

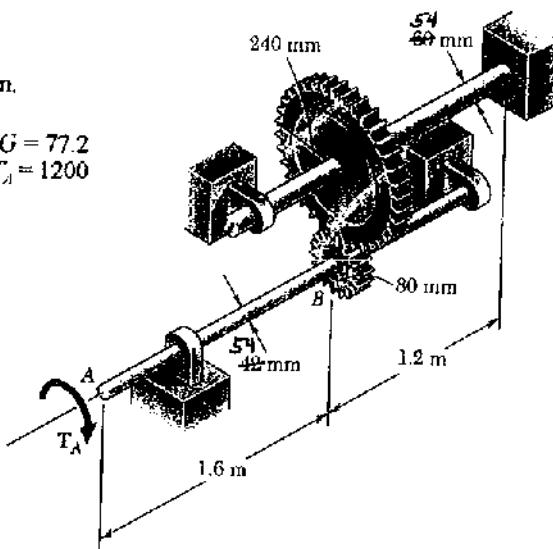
Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = 1200 \text{ N}\cdot\text{m}$$

$$T_{CD} = \frac{240}{80} (1200) = 3600 \text{ N}\cdot\text{m}$$



Twist in shaft CD: $c = \frac{1}{2}d = 0.027 \text{ m}$, $L = 1.2 \text{ m}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.027)^4 = 834.79 \times 10^{-9} \text{ m}^4$$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(834.79 \times 10^{-9})} = 67.033 \times 10^{-3} \text{ rad}$$

Rotation angle at C $\varphi_c = \varphi_{CD} = 67.033 \times 10^{-3} \text{ rad}$

Circumferential displacement at contact points of gears B and C

$$s = r_c \varphi_c = r_b \varphi_b$$

Rotation angle at B $\varphi_b = \frac{r_c}{r_b} \varphi_c = \frac{240}{80} (67.033 \times 10^{-3}) = 201.10 \times 10^{-3} \text{ rad}$

Twist in shaft AB: $c = \frac{1}{2}d = 0.027 \text{ m}$, $L = 1.6 \text{ m}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$J = 834.79 \times 10^{-9} \text{ m}$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(834.79 \times 10^{-9})} = 29.792 \times 10^{-3} \text{ rad}$$

Rotation angle at A $\varphi_A = \varphi_b + \varphi_{A/B} = 230.89 \times 10^{-3} \text{ rad}$

$$\varphi_A = 13.23^\circ$$

Problem 3.41

3.41 A coder F , used to record in digital form the rotation of shaft A , is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d . Two of the gears have a radius r and the other two a radius nr . If the rotation of the coder F is prevented, determine in terms of T , I , G , J , and n the angle through which end A rotates.

$$T_{AB} = T_A$$

$$T_{CD} = \frac{T_A}{T_B} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$$

$$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{\eta} = \frac{T_A}{\eta^2}$$

$$\Phi_E = \Phi_{EF} = \frac{T_{EF}l_{EF}}{G_F} = \frac{T_A l}{n^2 G_F}$$

$$\Phi_0 = \frac{r_0}{R_0} \Phi_E = \frac{\Phi_E}{n} = \frac{T_A l}{n^3 G J}$$

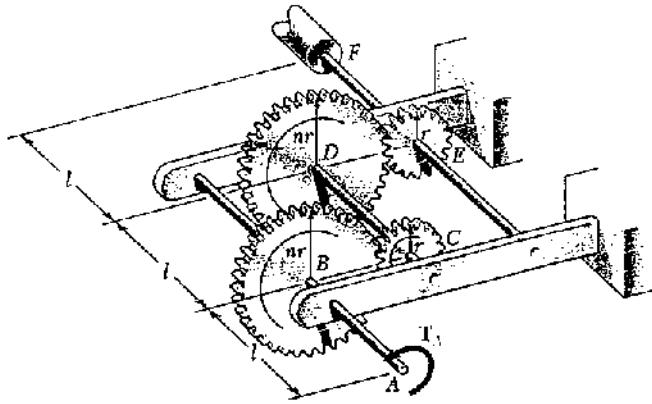
$$\phi_{cd} = \frac{T_{cd} l_{cd}}{GJ} = \frac{T_A l}{n GJ}$$

$$\Phi_c = \Phi_0 + \Phi_{cd} = \frac{T_{cd}}{n^3 G J} + \frac{T_{cd}}{n G J} = \frac{T_{cd}}{G J} \left(\frac{1}{n^3} + \frac{1}{n} \right)$$

$$\Phi_B = \frac{r_c}{r_a} \Phi_c = \frac{\Phi_c}{n} = \frac{T\ell}{GJ} \left(\frac{1}{n^4} + \frac{1}{n^2} \right)$$

$$\phi_{AB} = \frac{T_{AB, \text{lab}}}{GJ} = \frac{T_{AB}}{GJ}$$

$$\Phi_A = \Phi_B + \Phi_{AB} = \frac{T\Delta\ell}{GJ} \left(\frac{1}{n_1} + \frac{1}{n_2} + 1 \right)$$



Problem 3.42

See solution to PROBLEM
3.41 for development of
equation for ϕ .

3.42 For the gear train described in Prob. 3.41, determine the angle through which end A rotates when $T = 5 \text{ lb} \cdot \text{in.}$, $l = 2.4 \text{ in.}$, $d = 1/16 \text{ in.}$, $G = 11.2 \times 10^6 \text{ psi}$, and $n = 2$.

3.41 A coder F , used to record in digital form the rotation of shaft A , is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d . Two of the gears have a radius r and the other two a radius nr . If the rotation of the coder F is prevented, determine in terms of T , I , G , J , and n the angle through which end A rotates.

$$\varphi_n = \frac{1}{\zeta_0^2} \left(1 + \frac{1}{n^2} + \frac{1}{n^4} \right)$$

Data: $T = 5 \text{ lb-in}$, $\ell = 2.4 \text{ in}$, $C = \frac{1}{2}d = \frac{1}{32} \text{ in}$, $G = 11.2 \times 10^6 \text{ psi}$

$$n = 2, \quad J = \frac{\pi}{2} C^4 = \frac{\pi}{2} \left(\frac{1}{32}\right)^4 = 1.49803 \times 10^{-6} \text{ in}^4$$

$$\Phi_A = \frac{(5)(2.4)}{(11.2 \times 10^6)(1.49803 \times 10^{-5})} \left(1 + \frac{1}{4} + \frac{1}{16}\right) = 938.73 \times 10^{-3} \text{ rad}$$

$$\psi_A = 53.8^\circ$$

Problem 3.43

3.43 Two shafts, each of 7/8 in. diameter are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 1.2 kip · in. torque is applied at A.

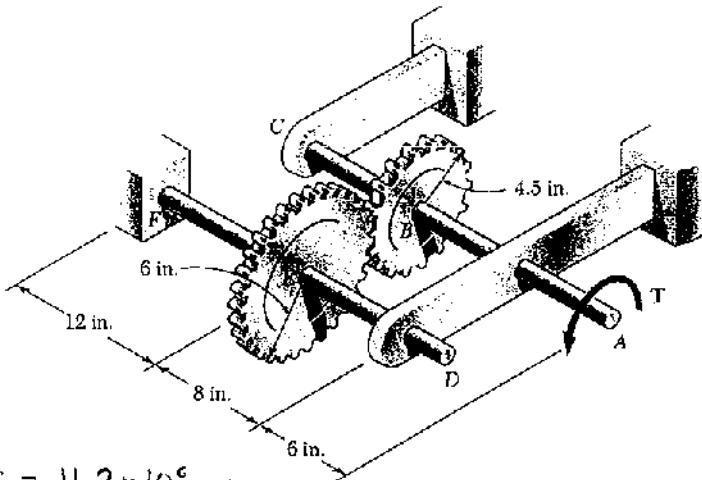
Calculation of torques,

Circumferential contact force between gears B and E.

$$F = \frac{T_{AB}}{r_E} = \frac{T_{EF}}{r_E} \quad T_{EF} = \frac{r_E}{r_B} T_{AB}$$

$$T_{AB} = 1.2 \text{ kip} \cdot \text{in} = 1200 \text{ lb} \cdot \text{in}$$

$$T_{EF} = \frac{6}{4.5} (1200) = 1600 \text{ lb} \cdot \text{in}$$



Twist in shaft FE.

$$L = 12 \text{ in.}, \quad c = \frac{1}{2}d = \frac{7}{16} \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{7}{16}\right)^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$\phi_{E/F} = \frac{TL}{GJ} = \frac{(1600)(12)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 29.789 \times 10^{-3} \text{ rad}$$

$$\text{Rotation at } E, \quad \phi_E = \phi_{E/F} = 29.789 \times 10^{-3} \text{ rad}$$

$$\text{Tangential displacement at gear circle} \quad S = r_E \phi_E = r_E \phi_B$$

$$\text{Rotation at } B \quad \phi_B = \frac{r_E}{r_B} \phi_E = \frac{6}{4.5} (29.789 \times 10^{-3}) = 39.718 \times 10^{-3} \text{ rad}$$

$$\text{Twist in shaft BA.} \quad L = 8 + 6 = 14 \text{ in.} \quad J = 57.548 \times 10^{-3} \text{ in}^4$$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(14)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 26.065 \times 10^{-3} \text{ rad}$$

$$\text{Rotation at } A. \quad \phi_A = \phi_B + \phi_{A/B} = 65.783 \times 10^{-3} \text{ rad} \quad \phi_A = 3.77^\circ$$

Problem 3.44

3.44 Solve Prob. 3.43, assuming that after a design change the radius of gear B is 6 in. and the radius of gear E is 4.5 in.

3.43 Two shafts, each of 7/8 in. diameter are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 1.2 kip-in. torque is applied at A.

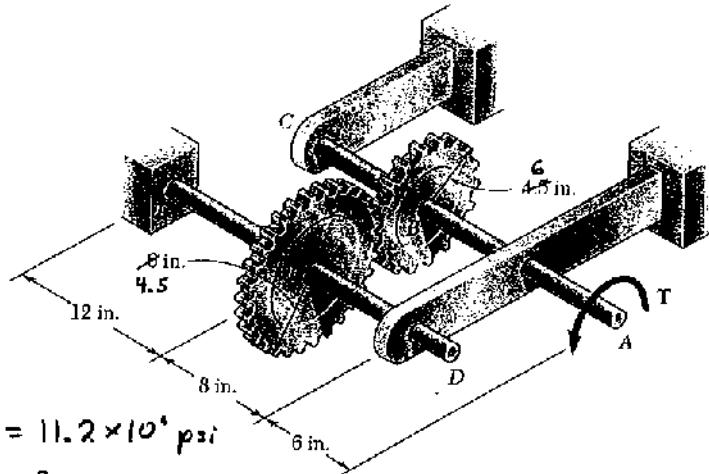
Calculation of torques.

Circumferential contact force between gears B and E.

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E} \quad T_{EF} = \frac{r_E}{r_B} T_{AB}$$

$$T_{AB} = 1.2 \text{ kip-in} = 1200 \text{ lb-in}$$

$$T_{EF} = \frac{4.5}{6}(1200) = 900 \text{ lb-in}$$



Twist in shaft FE

$$L = 12 \text{ in.} \quad c = \frac{1}{2} d = \frac{7}{16} \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{7}{16}\right)^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$\phi_{E/F} = \frac{TL}{GJ} = -\frac{(900)(12)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 16.756 \times 10^{-3} \text{ rad}$$

Rotation at E. $\phi_E = \phi_{E/F} = 16.756 \times 10^{-3} \text{ rad}$

Tangential displacement at gear circle. $s = r_E \phi_E = r_E \phi_E$

Rotation at B. $\phi_B = \frac{r_E}{r_B} \phi_E = \frac{4.5}{6}(16.756 \times 10^{-3}) = 12.567 \times 10^{-3} \text{ rad}$

Twist in shaft BA. $L = 8 + 6 = 14 \text{ in.} \quad J = 57.548 \times 10^{-3} \text{ in}^4$

$$\phi_{B/A} = \frac{TL}{GJ} = \frac{(1200)(14)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 26.065 \times 10^{-3} \text{ rad}$$

Rotation at A. $\phi_A = \phi_B + \phi_{B/A} = 38.632 \times 10^{-3} \text{ rad}$

$\phi_A = 2.21^\circ$

Problem 3.45

3.45 The design specifications of a 6-ft-long solid circular transmission shaft require that the angle of twist of the shaft not exceed 0.5° when a torque of 60 kip · in. is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 12.5 ksi and a modulus of rigidity of 11.2×10^6 psi.

$$T = 60 \text{ kip-in} = 60 \times 10^3 \text{ lb-in} \quad \phi = 0.5^\circ = 8.7266 \times 10^{-3} \text{ rad} \quad L = 6 \text{ ft} = 72 \text{ in.}$$

$$\tau = 12.5 \text{ ksi} = 12.5 \times 10^3 \text{ psi} \quad G = 11.2 \times 10^6 \text{ psi} \quad J = \frac{\pi}{2} c^4$$

$$\text{Based on angle of twist. } \phi = \frac{TL}{GJ} = \frac{2TL}{\pi Gc^4}$$

$$c = \sqrt[4]{\frac{2TL}{\pi G\phi}} = \sqrt[4]{\frac{(2)(60 \times 10^3)(72)}{\pi(11.2 \times 10^6)(8.7266 \times 10^{-3})}} = 2.303 \text{ in.}$$

$$\text{Based on shearing stress. } \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(60 \times 10^3)}{\pi(12.5 \times 10^3)}} = 1.451 \text{ in.}$$

Use larger value. $c = 2.303 \text{ in.}$

$$d = 2c = 4.61 \text{ in.} \blacksquare$$

Problem 3.46

3.46 The design specifications of a 1.2-m-long solid transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of 750 N · m is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77.2 GPa.

$$T = 750 \text{ N-m}, \quad \phi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}, \quad L = 1.2 \text{ m}, \quad J = \frac{\pi}{2} c^4$$

$$\tau = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa} \quad G = 77.2 \text{ GPa} = 77.2 \times 10^9 \text{ Pa}$$

$$\text{Based on angle of twist. } \phi = \frac{TL}{GJ} = \frac{2TL}{\pi Gc^4}$$

$$c = \sqrt[4]{\frac{2TL}{\pi G\phi}} = \sqrt[4]{\frac{(2)(750)(1.2)}{\pi(77.2 \times 10^9)(69.813 \times 10^{-3})}} = 18.06 \times 10^{-3} \text{ m}$$

$$\text{Based on shearing stress. } \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(750)}{\pi(90 \times 10^6)}} = 17.44 \times 10^{-3} \text{ m}$$

Use larger value $c = 18.06 \times 10^{-3} \text{ m} = 18.06 \text{ mm}$ $d = 2c = 36.1 \text{ mm} \blacksquare$

Problem 3.47

3.47 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD . It is further required that $\tau_{\max} \leq 9$ ksi and that the angle ϕ_D through which end D of shaft CD rotates not exceed 2° . Knowing that $G = 11.2 \times 10^6$ psi, determine the required diameter of the shafts.

$$T_{CD} = 5 \text{ kip-in} = 5 \times 10^3 \text{ lb-in}$$

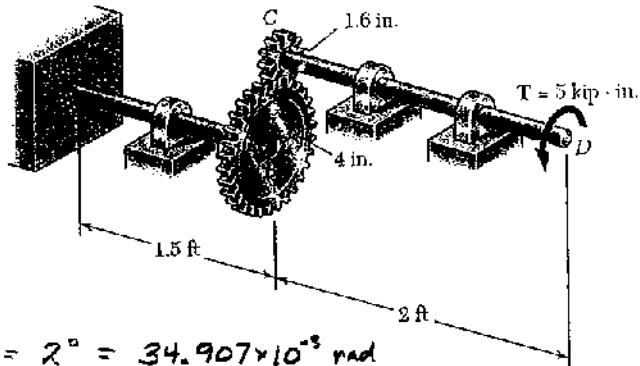
$$T_{AB} = \frac{r_a}{r_b} T_{CD} = \frac{4}{1.6} (5 \times 10^3) = 12.5 \times 10^3 \text{ lb-in}$$

For design based on stress use the larger value of torque. $T = 12.5 \times 10^3$ lb-in

$$\tau = 9 \text{ ksi} = 9 \times 10^6 \text{ psi}$$

$$c = \frac{T_c}{J} = \frac{2T}{\pi c^3} \quad c = \sqrt[3]{\frac{2T}{\pi \tau}}$$

$$c = \sqrt[3]{\frac{(2)(12.5 \times 10^3)}{\pi (9 \times 10^6)}} = 0.8842 \text{ in.}$$



Design based on rotation angle $\phi_B = 2^\circ = 34.907 \times 10^{-3}$ rad

$$\text{Shaft } AB: \quad \phi_B = \phi_{B/A} = \frac{T_{AB} L_{AB}}{GJ} \quad L_{AB} = 1.5 \text{ ft} = 18 \text{ in.}$$

$$\text{Gears: } S = r_b \phi_B = r_a \phi_c \quad \phi_c = \frac{r_b}{r_a} \phi_B$$

$$\phi_c = \frac{r_b}{r_a} \frac{T_{AB} L_{AB}}{GJ} = \frac{(4)(12.5 \times 10^3)(18)}{1.6 GJ} = \frac{562.5 \times 10^3}{GJ}$$

$$\text{Shaft } CD \quad \phi_{D/C} = \frac{T_{CD} L_{CD}}{GJ} \quad L_{CD} = 2 \text{ ft} = 24 \text{ in.}$$

$$\phi_{D/C} = \frac{(5 \times 10^3)(24)}{GJ} = \frac{120 \times 10^3}{GJ}$$

$$\text{Rotation at } D \quad \phi_D = \phi_c + \phi_{D/C} = \frac{682.5 \times 10^3}{GJ} = 34.907 \times 10^{-3}$$

$$GJ = \frac{\pi}{2} G c^4 = \frac{682.5 \times 10^3}{34.907 \times 10^{-3}} = 19.5520 \times 10^6 \text{ lb-in}$$

$$GJ = \frac{\pi}{2} G c^4$$

$$c = \sqrt[4]{\frac{2 G J}{\pi G}} = \sqrt[4]{\frac{(2)(19.5520 \times 10^6)}{\pi (11.2 \times 10^6)}} = 1.0267 \text{ in.}$$

Use the larger value $c = 1.0267 \text{ in.}$

$$d = 2c = 2.05 \text{ in.}$$

Problem 3.48

3.48 In the gear-and-shaft system shown the diameter of shafts are $d_{AB} = 2$ in. and $d_{CD} = 1.5$ in. Knowing that $G = 11.2 \times 10^6$ psi, determine the angle through which end D of shaft CD rotates.

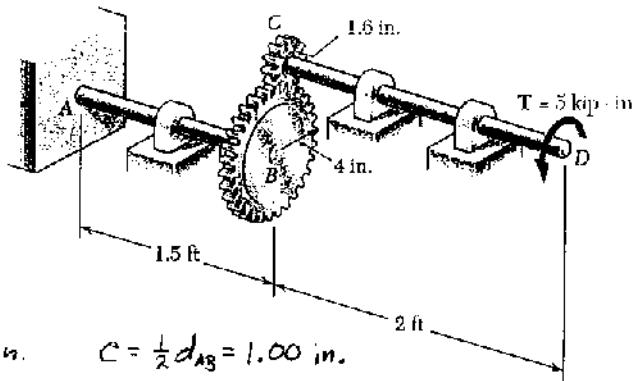
Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad T_{AB} = \frac{r_B}{r_C} T_{CD}$$

$$T_{CD} = 5 \text{ kip-in} = 5 \times 10^3 \text{ lb-in}$$

$$T_{AB} = \frac{4}{1.6} (5 \times 10^3) = 12.5 \times 10^3 \text{ kip-in}$$



Twist in shaft AB: $L = 1.5 \text{ ft} = 18 \text{ in.}$ $c = \frac{1}{2} d_{AB} = 1.00 \text{ in.}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (1.00)^4 = 1.5708 \text{ in}^4$$

$$\Phi_{B/A} = \frac{TL}{GJ} = \frac{(12.5 \times 10^3)(18)}{(11.2 \times 10^6)(1.5708)} = 12.789 \times 10^{-3} \text{ rad}$$

Rotation at B. $\phi_B = \phi_{B/A} = 12.789 \times 10^{-3} \text{ rad}$

Tangential displacement at gear circle $S = r_B \phi_B = r_c \phi_c$

Rotation at C $\phi_c = \frac{r_B}{r_c} \phi_B = \frac{4}{1.6} (12.789 \times 10^{-3}) = 31.973 \times 10^{-3} \text{ rad}$

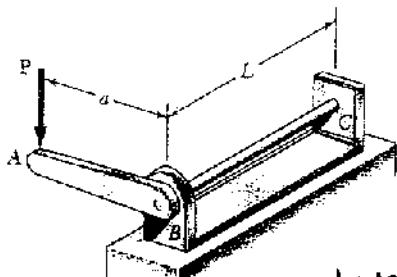
Twist in shaft CD: $L = 2 \text{ ft} = 24 \text{ in.}$ $c = \frac{1}{2} d_{CD} = 0.75 \text{ in.}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.75)^4 = 0.49701 \text{ in}^4$$

$$\Phi_{D/C} = \frac{TL}{GJ} = \frac{(5 \times 10^3)(24)}{(11.2 \times 10^6)(0.49701)} = 21.557 \times 10^{-3} \text{ rad}$$

Rotation at D. $\phi_D = \phi_c + \phi_{D/C} = 53.530 \times 10^{-3} \text{ rad}$ $\phi_D = 3.07^\circ$

Problem 3.49



3.49 The solid cylindrical rod BC is attached to the rigid lever AB and to the fixed support at C . The vertical force P applied at A causes a small displacement Δ at point A . Show that the corresponding maximum shearing stress in the rod is

$$\tau = \frac{Gd}{2La} \Delta$$

where d is the diameter of the rod and G its modulus of rigidity.

Lever AB turns through angle ϕ to position $A'B$ as shown in the auxiliary figure.

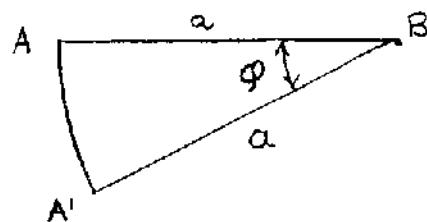
Vertical displacement is $\Delta = a \sin \phi$
from which $\phi = \arcsin \frac{\Delta}{a}$

The maximum shearing stress in rod BC is

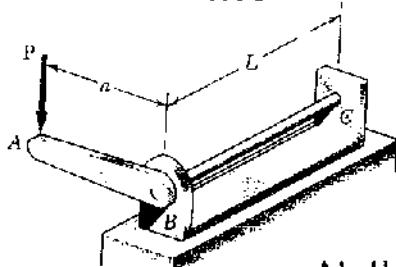
$$\tau_{\max} = G\gamma_{\max} = G \frac{C\phi}{L} = G \frac{d\phi}{2L} = \frac{Gd}{2L} \arcsin \frac{\Delta}{a}$$

For small $\frac{\Delta}{a}$, $\arcsin \frac{\Delta}{a} \approx \frac{\Delta}{a}$

$$\tau_{\max} = \frac{Gd\Delta}{2La}$$



Problem 3.50



3.50 and 3.51 The solid cylindrical rod BC of length $L = 24$ in. is attached to the rigid lever AB of length $a = 16$ in. and to the support at C . Design specifications require that the displacement of A not exceed 1 in. when a 100-lb force P is applied at A . For the material indicated determine the required diameter of the rod.

$$3.50 \text{ Steel: } \tau_{\max} = 12 \text{ ksi, } G = 11.2 \times 10^6 \text{ psi.}$$

$$\text{At the allowable twist angle } \sin \phi = \frac{\Delta}{a} = \frac{1}{16} = 0.0625$$

$$\phi = 3.5833^\circ = 0.062541 \text{ rad.}$$

$$T = Pa \cos \phi = (100)(16) \cos 3.5833^\circ = 1596.9 \text{ lb-in.}$$

$$\text{Based on twist } \phi = \frac{TL}{GJ} = \frac{2TL}{\pi G C^4} \therefore C^4 = \frac{2TL}{\pi G \phi}$$

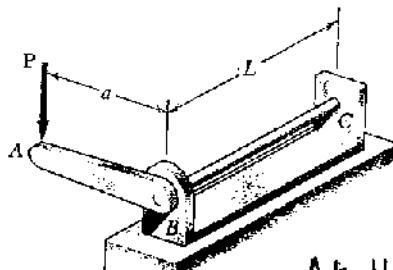
$$C^4 = \frac{(2)(1596.9)(24)}{\pi(11.2 \times 10^6)(0.062541)} = 34.832 \times 10^{-8} \text{ in}^4 \quad C = 0.4320 \text{ in.}$$

$$\text{Based on stress } \tau = \frac{TC}{J} = \frac{2T}{\pi C^3} \therefore C^3 = \frac{2T}{\pi \tau} \quad (\tau = 12000 \text{ psi})$$

$$C^3 = \frac{2(1596.9)}{\pi(12000)} = 84.718 \times 10^{-3} \text{ in}^3 \quad C = 0.4392 \text{ in.}$$

$$\text{Use larger value for design } C = 0.4392 \text{ in. } d = 2C = 0.878 \text{ in.} \blacksquare$$

Problem 3.51



3.50 and 3.51 The solid cylindrical rod BC of length $L = 24$ in. is attached to the rigid lever AB of length $a = 16$ in. and to the support at C . Design specifications require that the displacement of A not exceed 1 in. when a 100-lb force P is applied at A . For the material indicated determine the required diameter of the rod.

$$3.51 \text{ Aluminum: } \tau_{\max} = 9 \text{ ksi, } G = 3.9 \times 10^6 \text{ psi.}$$

$$\text{At the allowable twist angle } \sin \phi = \frac{\Delta}{a} = \frac{1}{16} = 0.0625$$

$$\phi = 3.5833^\circ = 0.062541 \text{ rad.}$$

$$T = Pa \cos \phi = (100)(16) \cos 3.5833^\circ = 1596.9 \text{ lb-in.}$$

$$\text{Based on twist } \phi = \frac{TL}{GJ} = \frac{2TL}{\pi G C^4} \therefore C^4 = \frac{2TL}{\pi G \phi}$$

$$C^4 = \frac{(2)(1596.9)(24)}{\pi(3.9 \times 10^6)(0.062541)} = 100.032 \times 10^{-8} \text{ in}^4 \quad C = 0.5624 \text{ in.}$$

$$\text{Based on stress } \tau = \frac{TC}{J} = \frac{2T}{\pi C^3} \therefore C^3 = \frac{2T}{\pi \tau} \quad (\tau = 9000 \text{ psi})$$

$$C^3 = \frac{(2)(1596.9)}{\pi(9000)} = 112.958 \times 10^{-3} \text{ in}^3 \quad C = 0.4834 \text{ in.}$$

$$\text{Use larger value for design } C = 0.5624 \text{ in. } d = 2C = 1.125 \text{ in.} \blacksquare$$

Problem 3.52

3.52 A torque of magnitude $T = 35 \text{ kip} \cdot \text{in}$. is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is $11.2 \times 10^6 \text{ psi}$ for the steel and $3.9 \times 10^6 \text{ psi}$ for the aluminum, determine (a) the maximum shearing stress in the steel core, (b) the maximum shearing stress in the aluminum jacket, (c) the angle of twist at A.

$$L = 8 \text{ ft} = 96 \text{ in.} \quad T = 35 \times 10^3 \text{ lb-in}$$

$$\text{Steel core: } C = \frac{1}{2}d = 1\frac{1}{8} \text{ in.} = 1.125 \text{ in.}$$

$$J_1 = \frac{\pi}{2}C^4 = 2.5161 \text{ in}^4$$

$$G_1 J_1 = (11.2 \times 10^6)(2.5161) = 28.180 \times 10^6 \text{ lb-in}$$

$$\text{Torque carried by steel core } T_1 = \frac{G_1 J_1 \phi}{L}$$

$$\text{Aluminum jacket: } C_1 = \frac{1}{2}d_1 = 1.125 \text{ in.}, \quad C_2 = \frac{1}{2}d_2 = 1.5 \text{ in.}$$

$$J_2 = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(1.5^4 - 1.125^4) = 5.4360 \text{ in}^4$$

$$G_2 J_2 = (3.9 \times 10^6)(5.4360) = 21.201 \times 10^6 \text{ lb-in}$$

$$\text{Torque carried by aluminum jacket } T_2 = \frac{G_2 J_2 \phi}{L}$$

$$\text{Total torque } T = T_1 + T_2 = (G_1 J_1 + G_2 J_2) \frac{\phi}{L}$$

$$\frac{\phi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{35 \times 10^3}{28.180 \times 10^6 + 21.201 \times 10^6} = 708.78 \times 10^{-6} \text{ rad/in}$$

(a) Maximum stress in steel core

$$\tau = G_1 \gamma = G_1 C \frac{\phi}{L} = (11.2 \times 10^6)(1.125)(708.78 \times 10^{-6}) \\ = 8.93 \times 10^3 \text{ psi}$$

$$\tau_{st} = 8.93 \text{ ksi}$$

(b) Maximum stress in aluminum jacket

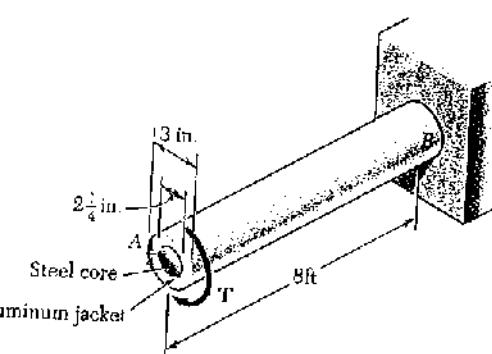
$$\tau = G_2 \gamma = G_2 C_2 \frac{\phi}{L} = (3.9 \times 10^6)(1.5)(708.78 \times 10^{-6}) \\ = 4.15 \times 10^3 \text{ psi}$$

$$\tau_{al} = 4.15 \text{ ksi}$$

(c) Angle of twist

$$\phi = L \frac{\theta}{L} = (96)(708.78 \times 10^{-6}) = 68.043 \times 10^{-3} \text{ rad}$$

$$\phi = 3.90^\circ$$



Problem 3.53

3.53 The composite shaft shown is to be twisted by applying a torque T at end A. Knowing that the modulus of rigidity is 11.2×10^6 psi for the steel and 3.9×10^6 psi for the aluminum, determine the largest angle through which end A can be rotated if the following allowable stresses are not to be exceeded $\tau_{\text{steel}} = 8500$ psi and $\tau_{\text{aluminum}} = 6500$ psi.

$$L = 8 \text{ ft} = 96 \text{ in.}$$

$$\tau_{\text{max}} = G \gamma_{\text{max}} = G C_{\text{max}} \frac{\phi}{L}$$

$$\frac{\phi_{\text{all}}}{L} = \frac{\tau_{\text{all}}}{G C_{\text{max}}} \text{ for each material}$$

Steel core: $\tau_{\text{all}} = 8500$ psi

$$C_{\text{max}} = \frac{1}{2} d = \frac{1}{2} \frac{1}{8} \text{ in.} = 1.125 \text{ in.}$$

$$\frac{\phi_{\text{all}}}{L} = \frac{8500}{(11.2 \times 10^6)(1.125)} = 674.60 \times 10^{-6} \text{ rad/in}$$

Aluminum jacket: $\tau_{\text{all}} = 6500$ psi $C_{\text{max}} = \frac{1}{2} d_2 = 1.5 \text{ in.}$

$$\frac{\phi_{\text{all}}}{L} = \frac{6500}{(3.9 \times 10^6)(1.5)} = 1.111 \times 10^{-6} \text{ rad/in}$$

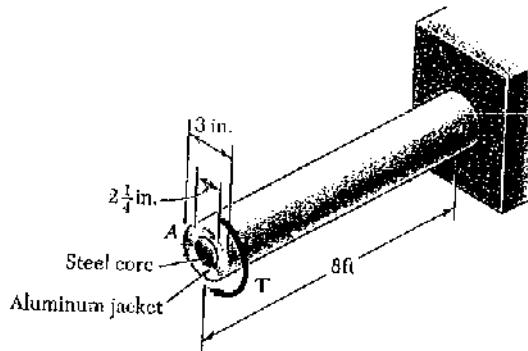
Smaller value of $\frac{\phi_{\text{all}}}{L}$ governs

$$\frac{\phi_{\text{all}}}{L} = 674.60 \times 10^{-6} \text{ rad/in}$$

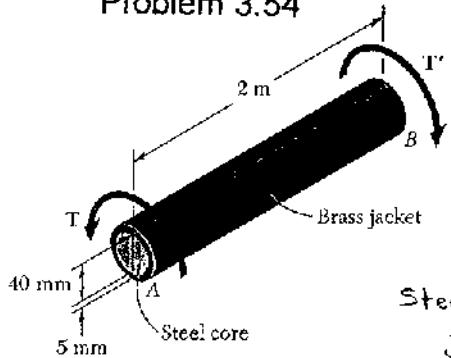
$$\text{Allowable angle of twist: } \phi_{\text{all}} = L \frac{\phi_{\text{all}}}{L} = (96)(674.60 \times 10^{-6})$$

$$= 64.76 \times 10^{-3} \text{ rad}$$

$$\phi_{\text{all}} = 3.71^\circ$$



Problem 3.54



3.54 The composite shaft shown consists of a 5-mm-thick brass jacket ($G_{\text{brass}} = 39 \text{ GPa}$) bonded to a 40-mm-diameter steel core ($G_{\text{steel}} = 77.2 \text{ GPa}$). Knowing that the shaft is subjected to a $600 \text{ N} \cdot \text{m}$ torque, determine (a) the maximum shearing stress in the brass jacket, (b) the maximum shearing stress in the steel core, (c) the angle of twist of B relative to A .

$$L = 2 \text{ m}$$

$$\text{Steel core: } c = \frac{1}{2}d = 20 \text{ mm} = 0.020 \text{ m}$$

$$J_1 = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

$$G_1 J_1 = (77.2 \times 10^9)(251.33 \times 10^{-9}) = 19.4025 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$\text{Torque carried by steel core } T_1 = G_1 J_1 \frac{\phi}{L}$$

$$\text{Brass jacket: } C_1 = \frac{1}{2}d = 20 \text{ mm} = 0.020 \text{ m} \quad C_2 = 20 + 5 = 25 \text{ mm} = 0.025 \text{ m}$$

$$J_2 = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.025^4 - 0.020^4) = 362.265 \times 10^{-9} \text{ m}^4$$

$$G_2 J_2 = (39 \times 10^9)(362.265 \times 10^{-9}) = 14.1283 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$\text{Torque carried by brass jacket } T_2 = G_2 J_2 \frac{\phi}{L}$$

$$\text{Total torque } T = T_1 + T_2 = (G_1 J_1 + G_2 J_2) \frac{\phi}{L}$$

$$\frac{\phi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{600}{19.4025 \times 10^3 + 14.1283 \times 10^3} = 17.894 \times 10^{-3} \text{ rad/m}$$

(a) Maximum shearing stress in brass jacket

$$\tau_{\text{max}} = G Y_{\text{max}} = G_2 C_2 \frac{\phi}{L} = (39 \times 10^9)(0.025)(17.894 \times 10^{-3})$$

$$= 17.45 \times 10^6 \text{ Pa}$$

$$\tau_{\text{brass}} = 17.45 \text{ MPa} \quad \blacktriangleleft$$

(b) Maximum shearing stress in steel core

$$\tau_{\text{max}} = G Y_{\text{max}} = G_1 C_1 \frac{\phi}{L} = (77.2 \times 10^9)(0.020)(17.894 \times 10^{-3})$$

$$= 27.6 \times 10^6 \text{ Pa}$$

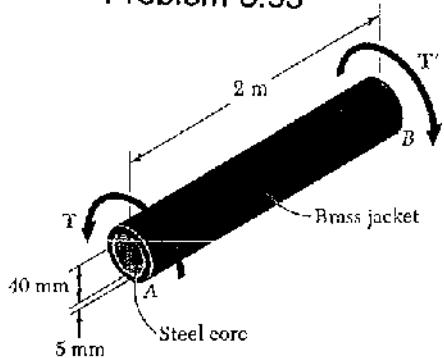
$$\tau_{\text{steel}} = 27.6 \text{ MPa} \quad \blacktriangleleft$$

(c) Angle of twist

$$\phi = L \frac{\phi}{L} = (2)(17.894 \times 10^{-3}) = 35.788 \times 10^{-3} \text{ rad}$$

$$\phi = 2.05^\circ$$

Problem 3.55



3.55 For the composite shaft of Prob. 3.54, the allowable shearing stress in the brass jacket is 20 MPa and 45 MPa in the steel core. Determine (a) the largest torque which can be applied to the shaft, (b) the corresponding angle of twist of B relative to A.

3.54 The composite shaft shown consists of a 5-mm-thick brass jacket ($G_{\text{brass}} = 39$ GPa) bonded to a 40-mm-diameter steel core ($G_{\text{steel}} = 77.2$ GPa). Knowing that the shaft is subjected to a 600 N·m torque, determine (a) the maximum shearing stress in the brass jacket, (b) the maximum shearing stress in the steel core, (c) the angle of twist of B relative to A.

$$\tau_{\text{max}} = G Y_{\text{max}} = G C_{\text{max}} \frac{\phi}{L}$$

$$\frac{\phi_{\text{all}}}{L} = \frac{\tau_{\text{all}}}{G C_{\text{max}}} \quad \text{for each material}$$

$$T_i = G_i J_i \frac{\phi}{L} \quad \text{for each material}$$

Brass jacket: $\tau_{\text{all}} = 20 \times 10^6 \text{ Pa}$, $C_1 = 20 \text{ mm} = 0.020 \text{ m}$, $C_2 = 20 + 5 = 25 \text{ mm} = 0.025 \text{ m}$

$$\frac{\phi_{\text{all}}}{L} = \frac{20 \times 10^6}{(39 \times 10^9)(0.025)} = 20.513 \times 10^{-3} \text{ rad/m}$$

$$J_2 = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.025^4 - 0.020^4) = 362.265 \times 10^{-9} \text{ m}^4$$

Steel core: $\tau_{\text{all}} = 45 \times 10^6 \text{ Pa}$, $C = 0.020 \text{ m}$

$$\frac{\phi_{\text{all}}}{L} = \frac{45 \times 10^6}{(77.2 \times 10^9)(0.020)} = 29.145 \times 10^{-3} \text{ rad/m}$$

$$J_1 = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Smaller value of $\frac{\phi_{\text{all}}}{L}$ governs $\frac{\phi_{\text{all}}}{L} = 20.513 \times 10^{-3} \text{ rad/m}$

Torque carried by brass sleeve

$$T_2 = G_2 J_2 \frac{\phi_{\text{all}}}{L} = (39 \times 10^9)(362.265 \times 10^{-9})(20.513 \times 10^{-3}) = 289.8 \text{ N.m}$$

Torque carried by steel core

$$T_1 = G_1 J_1 \frac{\phi_{\text{all}}}{L} = (77.2 \times 10^9)(251.33 \times 10^{-9})(20.513 \times 10^{-3}) = 398.0 \text{ N.m}$$

(a) Allowable torque $T = T_1 + T_2 = 687.8 \text{ N.m}$

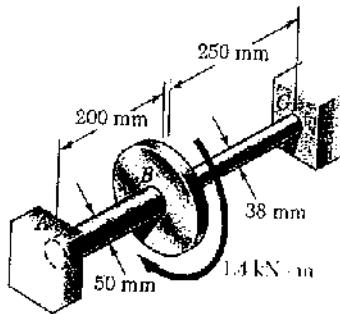
$$T_{\text{all}} = 688 \text{ N.m} \blacksquare$$

(b) Allowable twist angle

$$\phi_{\text{all}} = L \frac{\phi_{\text{all}}}{L} = (2)(20.513 \times 10^{-3}) = 41.026 \times 10^{-3} \text{ rad}$$

$$\phi_{\text{all}} = 2.35^\circ \blacksquare$$

Problem 3.56



3.56 Two solid steel shafts ($G = 77.2 \text{ GPa}$) are connected to a coupling disk B and to fixed supports at A and C . For the loading shown, determine (a) the reaction at each support, (b) the maximum shearing stress in shaft AB , (c) the maximum shearing stress in shaft BC .

Shaft AB

$$T = T_{AB}, L_{AB} = 0.200 \text{ m}, C = \frac{1}{2}d = 25 \text{ mm} = 0.025 \text{ m}$$

$$J_{AB} = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.025)^4 = 613.59 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L_{AB}}{G J_{AB}}$$

$$T_{AB} = \frac{G J_{AB}}{L_{AB}} \phi_B = \frac{(77.2 \times 10^9)(613.59 \times 10^{-9})}{0.200} \phi_B = 236.847 \times 10^3 \phi_B$$

Shaft BC

$$T = T_{BC}, L_{BC} = 0.250 \text{ m}, C = \frac{1}{2}d = 19 \text{ mm} = 0.019 \text{ m}$$

$$J_{BC} = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.019)^4 = 204.71 \times 10^{-9} \text{ m}^4 \quad \phi_B = \frac{T_{BC} L_{BC}}{G J_{BC}}$$

$$T_{BC} = \frac{G J_{BC}}{L_{BC}} \phi_B = \frac{(77.2 \times 10^9)(204.71 \times 10^{-9})}{0.250} = 63.214 \times 10^3 \phi_B$$

Equilibrium of coupling disk. $T = T_{AB} + T_{BC}$

$$1.4 \times 10^3 = 236.847 \times 10^3 \phi_B + 63.214 \times 10^3 \phi_B$$

$$\phi_B = 4.6657 \times 10^{-3} \text{ rad.}$$

$$T_{AB} = (236.847 \times 10^3)(4.6657 \times 10^{-3}) = 1.10506 \times 10^3 \text{ N}\cdot\text{m}$$

$$T_{BC} = (63.214 \times 10^3)(4.6657 \times 10^{-3}) = 294.94 \text{ N}\cdot\text{m}$$

(a) Reactions at supports

$$T_A = T_{AB} = 1105 \text{ N}\cdot\text{m}$$

$$T_C = T_{BC} = 294.94 \text{ N}\cdot\text{m}$$

(b) Maximum shearing stress in AB

$$\tau_{AB} = \frac{T_{AB} C}{J_{AB}} = \frac{(1.10506 \times 10^3)(0.025)}{613.59 \times 10^{-9}} = 45.0 \times 10^6 \text{ Pa}$$

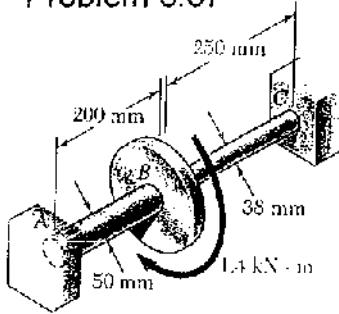
$$\tau_{AB} = 45.0 \text{ MPa}$$

(c) Maximum shearing stress in BC

$$\tau_{BC} = \frac{T_{BC} C}{J_{BC}} = \frac{(294.94)(0.019)}{204.71 \times 10^{-9}} = 27.4 \times 10^6 \text{ Pa}$$

$$\tau_{BC} = 27.4 \text{ MPa}$$

Problem 3.57



3.57 Solve Prob. 3.56, assuming that shaft *AB* is replaced by a hollow shaft of the same outer diameter and of 25-mm inner diameter.

3.56 Two solid steel shafts ($G = 77.2 \text{ GPa}$) are connected to a coupling disk *B* and to fixed supports at *A* and *C*. For the loading shown, determine (a) the reaction at each support, (b) the maximum shearing stress in shaft *AB*, (c) the maximum shearing stress in shaft *BC*.

Shaft AB

$$T = T_{AB}, \quad L_{AB} = 0.200 \text{ m}, \quad C_2 = 25 \text{ mm} = 0.025 \text{ m}$$

$$C_1 = 12.5 \text{ mm} = 0.0125 \text{ m}$$

$$J_{AB} = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.025^4 - 0.0125^4) = 575.243 \times 10^{-9} \text{ m}^4$$

$$T_{AB} = \frac{G J_{AB}}{L_{AB}} \phi_B = \frac{(77.2 \times 10^9)(575.243 \times 10^{-9})}{0.200} \phi_B = 222.044 \times 10^3 \phi_B$$

Shaft BC

$$T = T_{BC}, \quad L_{BC} = 0.250 \text{ m}, \quad C = \frac{1}{2}d = 19 \text{ mm} = 0.019 \text{ m}$$

$$J_{BC} = \frac{\pi}{2}C^4 = \frac{\pi}{2}(0.019)^4 = 204.71 \times 10^{-9} \text{ m}^4$$

$$T_{BC} = \frac{G J_{BC}}{L_{BC}} \phi_B = \frac{(77.2 \times 10^9)(204.71 \times 10^{-9})}{0.250} \phi_B = 63.214 \times 10^3 \phi_B$$

Equilibrium of coupling disk $T = T_{AB} + T_{BC}$

$$1.4 \times 10^3 = 222.044 \times 10^3 \phi_B + 63.214 \times 10^3 \phi_B$$

$$\phi_B = 4.9078 \times 10^{-3} \text{ rad}$$

$$T_{AB} = (222.044 \times 10^3)(4.9078 \times 10^{-3}) = 1.0897 \times 10^3 \text{ N·m}$$

$$T_{BC} = (63.214 \times 10^3)(4.9078 \times 10^{-3}) = 310.24 \text{ N·m}$$

(a) Reactions at supports

$$T_A = T_{AB} = 1090 \text{ N·m}$$

$$T_C = T_{BC} = 310 \text{ N·m}$$

(b) Maximum shearing stress in AB

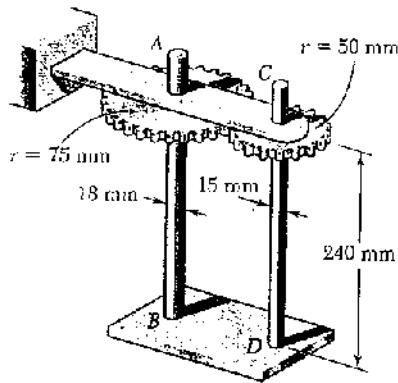
$$\tau_{AB} = \frac{T_{AB} C_2}{J_{AB}} = \frac{(1.0897 \times 10^3)(0.025)}{575.243 \times 10^{-9}} = 47.4 \times 10^6 \text{ Pa} \quad \tau_{AB} = 47.4 \text{ MPa}$$

(c) Maximum shearing stress in BC

$$\tau_{BC} = \frac{T_{BC} C}{J_{BC}} = \frac{(310.24)(0.019)}{204.71 \times 10^{-9}} = 28.8 \times 10^6 \text{ Pa} \quad \tau_{BC} = 28.8 \text{ MPa}$$

Problem 3.58

3.58 At a time when rotation is prevented at the lower end of each shaft, a 80-N·m torque is applied to end A of shaft AB. Knowing that $G = 77.2 \text{ GPa}$ for both shafts, determine (a) the maximum shearing stress in shaft CD, (b) the angle of rotation at A.



Let T_A = torque applied at A = 50 N·m

T_{AB} = torque in shaft AB

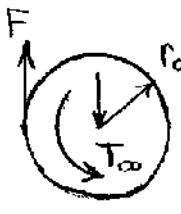
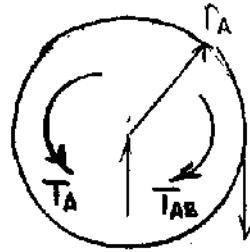
T_{CD} = torque in shaft CD

Statics

$$T_A - T_{AB} - Fr_A = 0$$

$$T_{CD} - Fr_C = 0$$

$$T_{CD} = \frac{r_c}{r_A} (T_A - T_{AB}) = \frac{2}{3} (T_A - T_{AB})$$



Gear C

$$\text{Kinematics: } r_A \phi_A = r_c \phi_c \quad \phi_A = \frac{r_c}{r_A} \phi_c = \frac{2}{3} \phi_c$$

$$\text{Angles of twist} \quad \phi_A = \frac{T_{AB} L}{G J_{AB}} \quad \phi_c = \frac{T_{CD} L}{G J_{CD}} = \frac{2 (T_A - T_{AB}) L}{G J_{CD}}$$

$$\frac{T_{AB} L}{G J_{AB}} = \frac{2}{3} \cdot \frac{2}{3} \frac{(T_A - T_{AB}) L}{G J_{CD}}$$

$$\left(\frac{4}{9} + \frac{T_{CD}}{J_{AB}} \right) T_{AB} = \left(\frac{4}{9} + \left(\frac{15}{18} \right)^4 \right) T_{AB} = \frac{4}{9} T_A$$

$$T_{AB} = 0. \quad T_A = (0.4796)(80) = 38.368 \text{ N}\cdot\text{m}$$

$$T_{CD} = \frac{2}{3} (80 - 38.368) = 27.755 \text{ N}\cdot\text{m}$$

(a) Maximum shearing stress in shaft CD

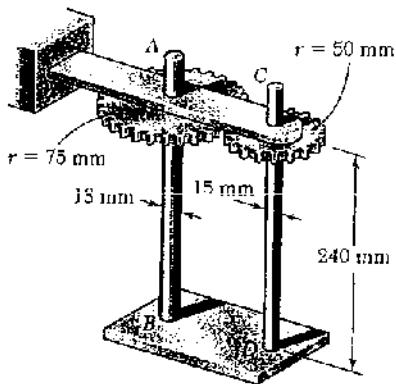
$$\tau_{cd} = \frac{T_{CD} C}{J_{CD}} = \frac{2 T_{CD}}{\pi C^3} = \frac{(2)(27.755)}{\pi (0.0075)^3} = 41.9 \times 10^6 \text{ Pa} \quad \tau_{cd} = 41.9 \text{ MPa}$$

(b) Angle of rotation at A

$$\phi_A = \frac{T_{AB} L}{G J_A} = \frac{2 T_{AB} L}{\pi G C^4} = \frac{(2)(38.368)(0.240)}{\pi (77.2 \times 10^9)(0.009)^4} = 11.57 \times 10^{-3} \text{ rad}$$

$$\phi_A = 0.663^\circ$$

Problem 3.59



3.59 Solve Prob. 3.58, assuming that the 80-N·m torque is applied to end C of shaft CD.

3.58 At a time when rotation is prevented at the lower end of each shaft, a 80-N·m torque is applied to end A of shaft AB. Knowing that $G = 77.2 \text{ GPa}$ for both shafts, determine (a) the maximum shearing stress in shaft CD, (b) the angle of rotation at A.

Let $T_c = \text{torque applied at } C = 80 \text{ N}\cdot\text{m}$

$T_{CD} = \text{torque in shaft } CD$

$T_{AB} = \text{torque in shaft } AB$

Statics.

$$T_{AB} - r_A F = 0$$

$$T_c - T_{CD} - r_c F = 0$$

$$T_{AB} = \frac{r_A}{r_c} (T_c - T_{CD}) = \frac{3}{2} (T_c - T_{CD})$$

$$\text{Kinematics: } r_A \phi_A = r_c \phi_c \quad \phi_c = \frac{r_A}{r_c} \phi_A = \frac{3}{2} \phi_A$$

$$\text{Angles of twist} \quad \phi_c = \frac{T_{CD} L}{G J_{CD}} \quad \phi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{3}{2} \frac{(T_c - T_{CD}) L}{G J_{AB}}$$

$$\frac{T_{CD} L}{G J_{CD}} = \frac{3}{2} \cdot \frac{3}{2} \frac{T_c - T_{CD}}{G J_{AB}}$$

$$\left(\frac{J_{AB}}{J_{CD}} + \frac{9}{4} \right) T_{CD} = \left(\left(\frac{18}{15} \right)^4 + \frac{9}{4} \right) T_{CD} = \frac{9}{4} T_c$$

$$T_{CD} = 0.4796 T_c = (0.52040)(80) = 41.632 \text{ N}\cdot\text{m}$$

$$T_{AB} = \frac{3}{2} (80 - 41.632) = 57.552 \text{ N}\cdot\text{m}$$

(a) Maximum shearing stress in CD

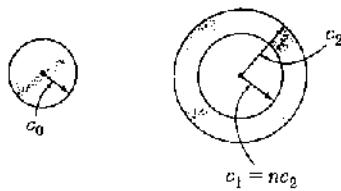
$$\tau_{cd} = \frac{T_{CD} c}{J_{CD}} = \frac{2 T_{CD}}{\pi c^3} = \frac{(2)(41.632)}{\pi (0.0075)^3} = 62.8 \times 10^6 \text{ Pa} \quad \tau_{cd} = 62.8 \text{ MPa}$$

(b) Angle of rotation at A

$$\phi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{2 T_{AB} L}{\pi G c^4} = \frac{(2)(57.552)(0.240)}{\pi (77.2 \times 10^9)(0.009)^4} = 17.36 \times 10^{-3} \text{ rad}$$

$$\phi_A = 0.995^\circ$$

Problem 3.60



3.60 A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by n the ratio c_1/c_2 , show that the ratio T_s/T_h of the torque T_s in the solid shaft to the torque T_h in the hollow shaft is (a) $\sqrt{1-n^2}/(1+n^2)$ if the maximum shearing stress is the same in each shaft, (b) $(1-n)/(1+n^2)$ if the angle of twist is the same for each shaft.

For equal weight and length, the areas are equal

$$\pi C_0^2 = \pi(c_2^2 - c_1^2) = \pi C_2^2(1-n^2) \therefore C_0 = C_2 (1-n^2)^{1/2}$$

$$J_s = \frac{\pi}{2} C_0^4 = \frac{\pi}{2} C_2^4 (1-n^2)^2 \quad J_h = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} C_2^4 (1-n^4)$$

(a) For equal stresses

$$\chi = \frac{T_s C_0}{J_s} = \frac{T_h C_2}{J_h}$$

$$\frac{T_s}{T_h} = \frac{J_h C_2}{J_s C_0} = \frac{\frac{\pi}{2} C_2^4 (1-n^2)^2 C_2}{\frac{\pi}{2} C_2^4 (1-n^4) C_0 (1-n^2)^{1/2}} = \frac{(1-n^2)}{(1+n^2)(1-n^2)^{1/2}} = \frac{(1-n^2)^{1/2}}{1+n^2}$$

(b) For equal angles of twist

$$\phi = \frac{T_s L}{G J_s} = \frac{T_h L}{G J_h}$$

$$\frac{T_s}{T_h} = \frac{J_h}{J_s} = \frac{\frac{\pi}{2} C_2^4 (1-n^2)^2}{\frac{\pi}{2} C_2^4 (1-n^4)} = \frac{(1-n^2)^2}{1-n^4} = \frac{1-n^2}{1+n^2}$$

Problem 3.61

3.61 A torque T is applied as shown to a solid tapered shaft AB . Show by integration that the angle of twist at A is

$$\phi = \frac{7TL}{12\pi Gc^4}$$

Introduce coordinate y as shown.

$$r = \frac{cy}{L}$$

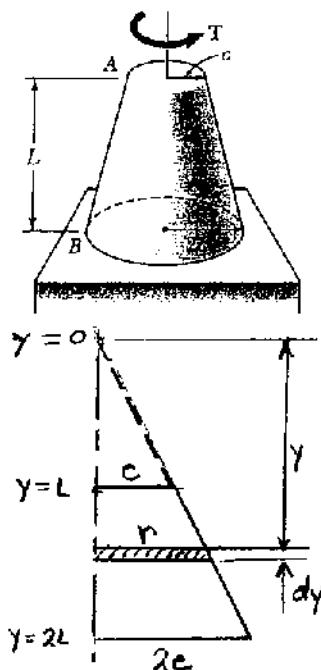
Twist in length dy

$$d\phi = \frac{T dy}{G J} = \frac{T dy}{G \frac{\pi}{2} r^4} = \frac{2TL^4 dy}{\pi G c^4 y^4}$$

$$\phi = \int_L^{2L} \frac{2TL^4}{\pi G c^4} \frac{dy}{y^4} = \frac{2TL}{\pi G c^4} \int_L^{2L} \frac{dy}{y^4}$$

$$= \frac{2TL^4}{\pi G c^4} \left\{ -\frac{1}{3y^3} \right\}_L^{2L} = \frac{2TL^4}{\pi G c^4} \left\{ -\frac{1}{24L^3} + \frac{1}{3L^3} \right\}$$

$$= \frac{2TL^4}{\pi G c^4} \left\{ \frac{7}{24L^3} \right\} = \frac{7TL}{12\pi G c^4}$$



Problem 3.62

3.62 An annular plate of thickness t and modulus of rigidity G is used to connect shaft AB of radius r_1 to tube CD of inner radius r_2 . Knowing that a torque T is applied to end A of shaft AB and that end D of tube CD is fixed, (a) determine the magnitude and location of the maximum shearing stress in the annular plate, (b) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

Use a free body consisting of shaft AB and an inner portion of the plate BC , the outer radius of this portion being ρ

The force per unit length of circumference is γt .

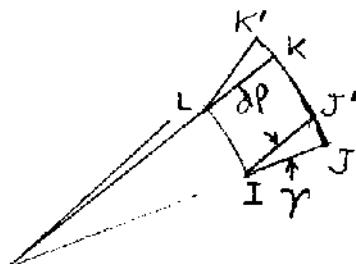
$$\sum M = 0$$

$$\gamma t (2\pi\rho)\rho - T = 0$$

$$\gamma = \frac{T}{2\pi t \rho^2}$$

(a) Maximum shearing stress occurs at $\rho = r_1$, $\tau_{max} = \frac{T}{2\pi t r_1^2}$ (1)

Shearing strain $\gamma = \frac{\epsilon}{G} = \frac{T}{2\pi G t \rho^2}$



The relative circumferential displacement in radial length $d\rho$ is

$$ds = \gamma d\rho = \rho d\phi$$

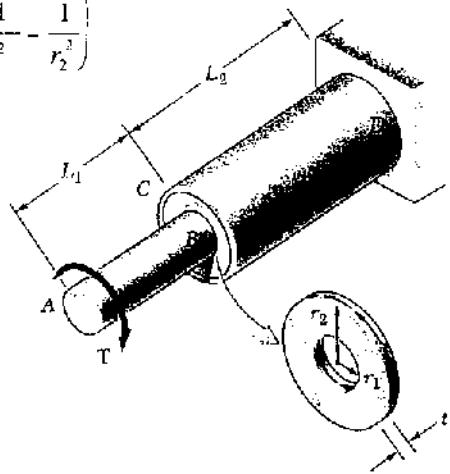
$$d\phi = \gamma \frac{d\rho}{\rho}$$

$$d\phi = \frac{T}{2\pi G t \rho^2} \frac{d\rho}{\rho} = \frac{T}{2\pi G t} \frac{d\rho}{\rho^3}$$

$$\phi_{AC} = \int_{r_1}^{r_2} \frac{T}{2\pi G t} \frac{d\rho}{\rho^3} = \frac{T}{2\pi G t} \int_{r_1}^{r_2} \frac{d\rho}{\rho^3} = \frac{T}{2\pi G t} \left\{ -\frac{1}{2\rho^2} \right\} \Big|_{r_1}^{r_2}$$

$$= \frac{T}{2\pi G t} \left\{ -\frac{1}{2r_2^2} + \frac{1}{2r_1^2} \right\} = \frac{T}{4\pi G t} \left\{ \frac{1}{r_1^2} - \frac{1}{r_2^2} \right\}$$

$$\phi_{B/C} = \frac{T}{4\pi G t} \left(\frac{1}{r_1^2} - \frac{1}{r_2^2} \right)$$



Problem 3.63

3.63 An annular brass plate ($G = 39 \text{ GPa}$), of thickness $t = 6 \text{ mm}$, is used to connect the brass shaft AB , of length $L_1 = 50 \text{ mm}$ and radius $r_1 = 30 \text{ mm}$, to the brass tube CD , of length $L_2 = 125 \text{ mm}$, inner radius $r_2 = 75 \text{ mm}$, and thickness $t = 3 \text{ mm}$. Knowing that a $2.8 \text{ kN} \cdot \text{m}$ torque T is applied to end A of shaft AB and that end D of tube CD is fixed, determine (a) the maximum shearing stress in the shaft-plate-tube system, (b) the angle through which end A rotates. (Hint: Use the formula derived in Prob. 3.62 to solve part b.)

Shaft AB

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi C^3} = \frac{2T}{\pi r_1^3} \\ = \frac{(2)(2800)}{\pi(0.030)^3} = 66.0 \times 10^6 \text{ Pa} \\ = 66.0 \text{ MPa}$$

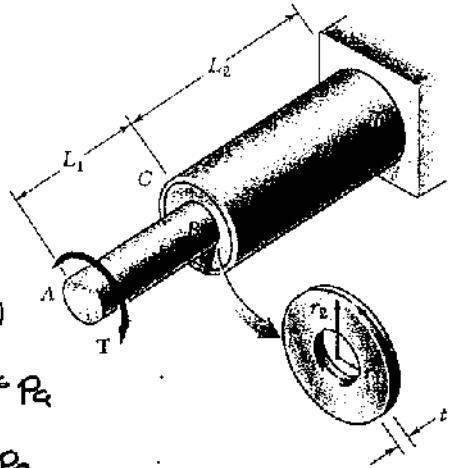


Plate BC (See PROBLEM 3.62 for derivation)

$$\tau = \frac{T}{2\pi Et r_1^2} = \frac{2800}{2\pi(0.006)(0.030)^2} = 82.5 \times 10^6 \text{ Pa} \\ = 82.5 \text{ MPa}$$

Shaft CD $C_1 = r_2 = 0.075 \text{ m}$, $C_2 = r_2 + t = 0.075 + 0.003 = 0.078 \text{ m}$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.078^4 - 0.075^4) = 8.4421 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{TC_2}{J} = \frac{(2800)(0.078)}{8.4421 \times 10^{-6}} = 25.9 \times 10^6 \text{ Pa} = 25.9 \text{ MPa}$$

(a) Largest stress

$$\tau_{\max} = 82.5 \text{ MPa} \quad \blacktriangleleft$$

Shaft AB $\phi_{AB} = \frac{TL_1}{GJ} = \frac{2TL_1}{\pi G C^4} = \frac{(2)(2800)(0.050)}{\pi(39 \times 10^9)(0.030)^4}$
 $= 2.821 \times 10^{-3} \text{ rad.}$

Plate BC (See Problem 3.62 for derivation)

$$\phi_{BC} = \frac{T}{4\pi Gt} \left\{ \frac{1}{r_1^2} - \frac{1}{r_2^2} \right\} = \frac{2800}{4\pi(39 \times 10^9)(0.006)} \left\{ \frac{1}{0.030^2} - \frac{1}{0.075^2} \right\} \\ = 0.889 \times 10^{-3} \text{ rad.}$$

Shaft CD $\phi_{CD} = \frac{TL_2}{GJ} = \frac{(2800)(0.125)}{(39 \times 10^9)(8.4421 \times 10^{-6})} = 1.063 \times 10^{-3} \text{ rad}$

(b) Total rotation angle $\phi_A = \phi_{AB} + \phi_{BC} + \phi_{CD} = 4.773 \times 10^{-3} \text{ rad}$

$$\phi_A = 0.273^\circ \quad \blacktriangleleft$$

Problem 3.64

3.64 Using an allowable shearing stress of 4.5 ksi, design a solid steel shaft to transmit 12 hp at speed of (a) 1200 rpm, (b) 2400 rpm.

$$\tau_{all} = 4.5 \text{ ksi} = 4500 \text{ psi}$$

$$P = 12 \text{ hp} = (12)(6600) = 79.2 \times 10^3 \text{ lb-in/s}$$

$$(a) f = \frac{1200}{60} = 20 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{79.2 \times 10^3}{2\pi(20)} = 630.25 \text{ lb-in}$$

$$C = \frac{Tc}{J} = \frac{2T}{\pi C^3}$$

$$C = \sqrt[3]{\frac{2T}{\pi C^3}} = \sqrt[3]{\frac{(2)(630.25)}{\pi(4500)}} = 0.44675 \text{ in.}$$

$$d = 2c = 0.893 \text{ in.}$$

$$(b) f = \frac{2400}{60} = 40 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{79.2 \times 10^3}{2\pi(40)} = 315.127 \text{ lb-in}$$

$$C = \sqrt[3]{\frac{(2)(315.127)}{\pi(4500)}} = 0.35458 \text{ in.}$$

$$d = 2c = 0.709 \text{ in.}$$

Problem 3.65

3.65 Using an allowable shearing stress of 50 MPa, design a solid steel shaft to transmit 15 kW at a frequency of (a) 30 Hz, (b) 60 Hz.

$$\tau_{all} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$$

$$P = 15 \text{ kW} = 15 \times 10^3 \text{ W}$$

$$(a) f = 30 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{15 \times 10^3}{2\pi(30)} = 79.577 \text{ N-m}$$

$$C = \frac{Tc}{J} = \frac{2T}{\pi C^3}$$

$$C = \sqrt[3]{\frac{2T}{\pi C^3}} = \sqrt[3]{\frac{(2)(79.577)}{\pi(50 \times 10^6)}} = 10.044 \times 10^{-3} \text{ m} = 10.044 \text{ mm}$$

$$d = 2c = 20.1 \text{ mm}$$

$$(b) f = 60 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{15 \times 10^3}{2\pi(60)} = 39.789 \text{ N-m}$$

$$C = \sqrt[3]{\frac{(2)(39.789)}{\pi(50 \times 10^6)}} = 7.972 \times 10^{-3} \text{ m} = 7.972 \text{ mm}$$

$$d = 2c = 15.94 \text{ mm}$$

Problem 3.66

3.66 Determine the maximum shearing stress in a solid shaft of 12-mm diameter as it transmits 2.5 kW at a frequency of (a) 25 Hz, (b) 50 Hz.

$$c = \frac{1}{2}d = 6 \text{ mm} = 0.006 \text{ m} \quad P = 2.5 \text{ kW} = 2500 \text{ W}$$

$$(a) f = 25 \text{ Hz} \quad T = \frac{P}{2\pi f} = \frac{2500}{2\pi(25)} = 15.9155 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(15.9155)}{\pi (0.006)^3} = 46.9 \times 10^6 \text{ Pa} \quad \tau = 46.9 \text{ MPa} \blacksquare$$

$$(b) f = 50 \text{ Hz} \quad T = \frac{2500}{2\pi(50)} = 7.9577 \text{ N}\cdot\text{m}$$

$$\tau = \frac{2(7.9577)}{\pi (0.006)^3} = 23.5 \times 10^6 \text{ Pa} \quad \tau = 23.5 \text{ MPa} \blacksquare$$

Problem 3.67

3.67 Determine the maximum shearing stress in a solid shaft of 1.5-in. diameter as it transmits 75 hp at a speed of (a) 750 rpm, (b) 1500 rpm.

$$c = \frac{1}{2}d = 0.75 \text{ in.} \quad P = 75 \text{ hp} = (75)(6600) = 495 \times 10^3 \text{ lb}\cdot\text{in}/\text{s}$$

$$(a) f = \frac{750}{60} = 12.5 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{495 \times 10^3}{2\pi(12.5)} = 6.3025 \times 10^3 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(6.3025 \times 10^3)}{\pi (0.75)^3} = 9.51 \times 10^3 \text{ psi} \quad \tau = 9.51 \text{ ksi} \blacksquare$$

$$(b) f = \frac{1500}{60} = 25 \text{ Hz}$$

$$T = \frac{495 \times 10^3}{2\pi(25)} = 3.1513 \times 10^3 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{(2)(3.1513 \times 10^3)}{\pi (0.75)^3} = 4.76 \times 10^3 \text{ psi} \quad \tau = 4.76 \text{ ksi} \blacksquare$$

Problem 3.68

3.68 A steel drive shaft is 6 ft long and its outer and inner diameters are respectively equal to 2.25 in. and 1.75 in. Knowing that the shaft transmits 240 hp while rotating at 1800 rpm, determine (a) the maximum shearing stress, (b) the angle of twist of the shaft ($G = 11.2 \times 10^6$ psi).

$$L = 6 \text{ ft} = 72 \text{ in.} \quad c_2 = \frac{1}{2} d_o = 1.125 \text{ in.} \quad c_1 = \frac{1}{2} d_i = 0.875 \text{ in.}$$

$$P = 240 \text{ hp} = (240)(6600) = 1.584 \times 10^6 \text{ lb-in/s}$$

$$f = \frac{1800}{60} = 30 \text{ Hz} \quad T = \frac{P}{2\pi f} = \frac{1.584 \times 10^6}{2\pi(30)} = 8.4034 \times 10^3 \text{ lb-in}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (1.125^4 - 0.875^4) = 1.59534 \text{ in}^4$$

$$(a) \text{ Maximum shearing stress.} \quad \tau = \frac{Tc_2}{J}$$

$$\tau = \frac{(8.4034 \times 10^3)(1.125)}{1.59534} = 5.9259 \times 10^3 \text{ psi} \quad \tau = 5.93 \text{ ksi}$$

$$(b) \text{ Angle of twist.} \quad \phi = \frac{TL}{GJ}$$

$$\phi = \frac{(8.4034 \times 10^3)(72)}{(11.2 \times 10^6)(1.59534)} = 33.86 \times 10^{-3} \text{ rad} \quad \phi = 1.940^\circ$$

Problem 3.69

3.69 One of two hollow drive shafts of a cruise ship is 40 m long, and its outer and inner diameters are 400 mm and 200 mm, respectively. The shaft is made of a steel for which $\tau_{all} = 60 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. Knowing that the maximum speed of rotation of the shaft is 160 rpm, determine (a) the maximum power that can be transmitted by one shaft to its propeller, (b) the corresponding angle of twist of the shaft.

$$L = 40 \text{ m}$$

$$c_2 = \frac{1}{2} d_o = 200 \text{ mm} = 0.200 \text{ m} \quad c_1 = \frac{1}{2} d_i = 100 \text{ mm} = 0.100 \text{ m}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (0.200^4 - 0.100^4) = 2.3562 \times 10^{-3} \text{ m}^4$$

$$\tau = \frac{Tc_{max}}{J} \quad T = \frac{J\tau}{G} = \frac{(2.3562 \times 10^{-3})(60 \times 10^6)}{0.200} = 706.86 \times 10^3 \text{ N-m}$$

$$f = \frac{160}{60} = 2.6667 \text{ Hz}$$

$$(a) \text{ Maximum power} \quad P = 2\pi f T$$

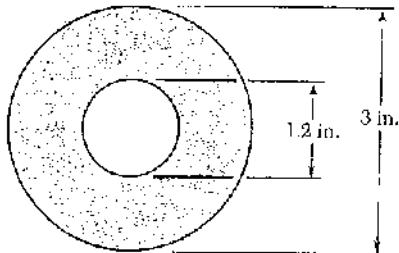
$$P = 2\pi (2.6667)(706.86 \times 10^3) = 11.844 \times 10^6 \text{ W} \quad P = 11.84 \text{ MW}$$

$$(b) \text{ Angle of twist} \quad \phi = \frac{TL}{GJ}$$

$$\phi = \frac{(706.86 \times 10^3)(40)}{(77.2 \times 10^9)(2.3562 \times 10^{-3})} = 155.44 \times 10^{-3} \text{ rad} \quad \phi = 8.91^\circ$$

Problem 3.70

3.70 While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is 2° in a 12 ft length. Using $G = 11.2 \times 10^6$ psi, determine the power being transmitted.



$$\varphi = 2^\circ = 34.907 \times 10^{-3} \text{ rad} \quad L = 12 \text{ ft} = 144 \text{ in.}$$

$$c_2 = \frac{1}{2}d_o = 1.5 \text{ in.} \quad c_1 = \frac{1}{2}d_i = 0.6 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.5^4 - 0.6^4) = 7.7486 \text{ in}^4$$

$$f = \frac{120}{60} = 2 \text{ Hz}$$

$$T = \frac{GJ\varphi}{L} = \frac{(11.2 \times 10^6)(7.7486)(34.907 \times 10^{-3})}{144} = 21.037 \times 10^3 \text{ lb-in}$$

$$P = 2\pi f T = 2\pi(2)(21.037 \times 10^3) = 264.36 \times 10^3 \text{ lb-in/s}$$

$$\frac{264.36}{6600} = 40.1$$

40.1 hp \blacktriangleleft

Problem 3.71

3.71 Determine the required thickness of the 50-mm tubular shaft of Example 3.07, if it is to transmit the same power while rotating at a frequency of 30 Hz.

From Example 3.07 $P = 100 \text{ kW} = 100 \times 10^3 \text{ W}$

$$\tau_{all} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa} \quad c_2 = \frac{1}{2}d = 25 \text{ mm} = 0.025 \text{ m}$$

Given $f = 30 \text{ Hz}$

$$T = \frac{P}{2\pi f} = 530.52 \text{ N-m}$$

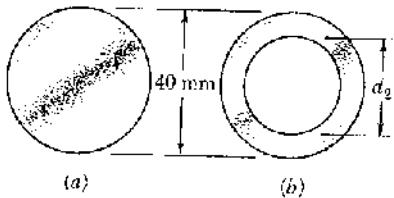
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) \quad \tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.025^4 - \frac{(2)(530.52)(0.025)}{\pi(60 \times 10^6)} = 249.90 \times 10^{-9} \text{ m}^4$$

$$c_1 = 22.358 \times 10^{-3} \text{ m} = 22.358 \text{ mm}$$

$$t = c_2 - c_1 = 25 \text{ mm} - 22.358 \text{ mm} = 2.642 \text{ mm} \quad t = 2.64 \text{ mm} \mathbf{\Delta}$$

Problem 3.72



3.72 The design of a machine element calls for a 40 mm-outer-diameter shaft to transmit 45 kW. (a) If the speed of rotation is 720 rpm, determine the maximum shearing stress in shaft *a*. (b) If the shaft of rotation can be increased 50% to 1080 rpm, determine the largest inner diameter of shaft *b* for which the maximum shearing stress will be the same in each shaft.

$$(a) f = \frac{720}{60} = 12 \text{ Hz} \quad P = 45 \text{ kW} = 45 \times 10^3 \text{ W}$$

$$T = \frac{P}{2\pi f} = \frac{45 \times 10^3}{2\pi(12)} = 596.83 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 20 \text{ mm} = 0.020 \text{ m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(596.83)}{\pi(0.020)^3} = 47.494 \times 10^6 \text{ Pa} \quad \tau_{max} = 47.5 \text{ MPa}$$

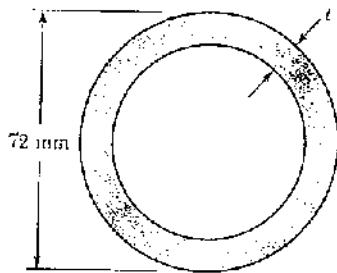
$$(b) f = \frac{1080}{60} = 18 \text{ Hz} \quad T = \frac{45 \times 10^3}{2\pi(18)} = 397.89 \text{ N}\cdot\text{m}$$

$$\tau = \frac{TC_2}{J} = \frac{2TC_2}{\pi(C_2^4 - C_1^4)} \quad C_1^4 = C_2^4 - \frac{2TC_2}{\pi\tau}$$

$$C_1^4 = 0.020^4 - \frac{(2)(397.89)(0.020)}{\pi(47.494 \times 10^6)} = 53.333 \times 10^{-9} \quad C_1 = 15.20 \times 10^{-3} \text{ m}$$

$$C_1 = 15.20 \times 10^{-3} \text{ m} = 15.20 \text{ mm} \quad d_2 = 2C_1 = 30.4 \text{ mm}$$

Problem 3.73



3.73 A steel pipe of 72-mm outer diameter is to be used to transmit a torque of 2500 N·m without exceeding an allowable shearing stress of 55 MPa. A series of 72-mm-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 4 mm to 10 mm in 2-mm increments, choose the lightest pipe that can be used.

$$C_2 = \frac{1}{2}d_o = 36 \text{ mm} = 0.036 \text{ m}$$

$$\tau = \frac{TC_2}{J} = \frac{2TC_2}{\pi(C_2^4 - C_1^4)}$$

$$C_1^4 = C_2^4 - \frac{2TC_2}{\pi\tau} = 0.036^4 - \frac{(2)(2500)(0.036)}{\pi(55 \times 10^6)} = 637.875 \times 10^{-9}$$

$$C_1 = 28.26 \times 10^{-3} \text{ m} = 28.26 \text{ mm}$$

$$t = C_2 - C_1 = 36 \text{ mm} - 28.26 \text{ mm} = 7.74 \text{ mm}$$

Use $t = 8 \text{ mm}$

Problem 3.74

3.74 A 2.5-m-long solid steel shaft is to transmit 10 kW at a frequency of 25 Hz. Determine the required diameter of the shaft, knowing that $G = 77.2 \text{ GPa}$, that the allowable shearing stress is 30 MPa, and that the angle of twist must not exceed 4° .

$$P = 10 \text{ kW} = 10 \times 10^3 \text{ W} \quad f = 25 \text{ Hz} \quad \phi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{2\pi(25)} = 63.662 \text{ N}\cdot\text{m}$$

$$\text{Stress requirement} \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(63.662)}{\pi(30 \times 10^6)}} = 11.055 \times 10^{-3} \text{ m} = 11.055 \text{ mm}$$

$$\text{Twist angle requirement} \quad \phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$c = \sqrt[4]{\frac{2TL}{\pi G \phi}} = \sqrt[4]{\frac{(2)(63.662)(2.5)}{\pi(77.2 \times 10^9)(69.813 \times 10^{-3})}} = 11.709 \text{ mm}$$

$$\text{Use the larger value} \quad c = 11.709 \text{ mm} \quad d = 2c = 23.4 \text{ mm} \quad \blacksquare$$

Problem 3.75

3.75 A 1.5-m-long solid steel shaft of 22-mm diameter is to transmit 12 kW. Determine the minimum frequency at which the shaft can rotate, knowing that $G = 77.2 \text{ GPa}$, that the allowable shearing stress is 30 MPa, and that the angle of twist must not exceed 3.5° .

$$L = 1.5 \text{ m} \quad c = \frac{d}{2} = 11 \text{ mm} = 0.011 \text{ m} \quad P = 12 \text{ kW} = 12 \times 10^3 \text{ W}$$

$$\phi = 3.5^\circ = 61.087 \times 10^{-3} \text{ rad} \quad G = 77.2 \times 10^9 \text{ Pa} \quad \tau = 30 \times 10^6 \text{ Pa}$$

$$\text{Stress requirement} \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$T = \frac{\pi}{2} \tau c^3 = \frac{\pi}{2} (30 \times 10^6)(0.011)^3 = 62.722 \text{ N}\cdot\text{m}$$

$$\text{Twist angle requirement} \quad \phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi}{2} \frac{G c^4 \phi}{L} = \frac{\pi(77.2 \times 10^9)(0.011)^4(61.087 \times 10^{-3})}{(2)(1.5)} = 72.305 \text{ N}\cdot\text{m}$$

Maximum allowable torque is the smaller value $T = 62.722 \text{ N}\cdot\text{m}$

Minimum frequency to transmit a power of 12 kW.

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{12 \times 10^3}{2\pi(62.722)} = 30.4 \text{ Hz}$$

$$f = 30.4 \text{ Hz} \quad \blacksquare$$

Problem 3.76

3.76 The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A* operating at a speed of 1260 rpm, to a machine tool at *D*. Knowing that each shaft has a diameter of 1 in., determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *CD*.

$$(a) \text{Shaft AB: } P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb-in/sec}$$

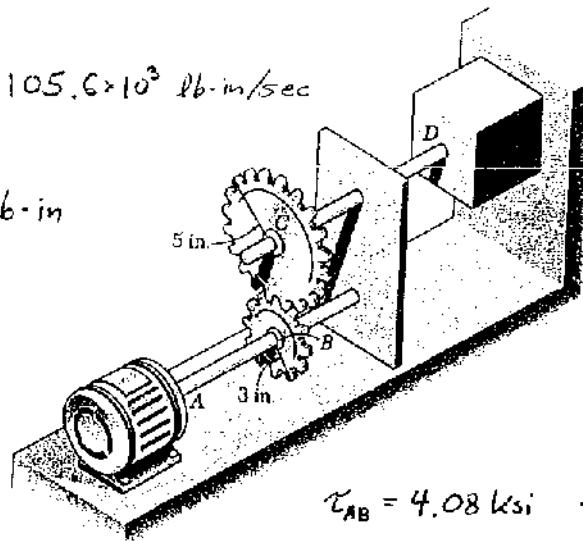
$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{105.6 \times 10^3}{2\pi(21)} = 800.32 \text{ lb-in}$$

$$C = \frac{1}{2}d = 0.5 \text{ in.}$$

$$\tau' = \frac{Tc}{J} = \frac{2T}{\pi C^3}$$

$$= \frac{(2)(800.32)}{\pi(0.5)^3} = 4.08 \times 10^3 \text{ psi}$$



$$\tau_{AB} = 4.08 \text{ ksi}$$

(b) Shaft CD

$$T_{CD} = \frac{r_c}{r_b} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb-in}$$

$$\tau' = \frac{2T}{\pi C^3} = \frac{(2)(1.33387 \times 10^3)}{\pi(0.5)^3} = 6.79 \times 10^3 \text{ psi}$$

$$\tau_{CD} = 6.79 \text{ ksi}$$

Problem 3.77

3.77 The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A* operating at a speed of 1260 rpm, to a machine tool at *D*. Knowing that the maximum allowable shearing stress is 8 ksi, determine the required diameter (*a*) of shaft *AB*, (*b*) of shaft *CD*.

$$(a) \text{Shaft AB: } P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb-in/sec}$$

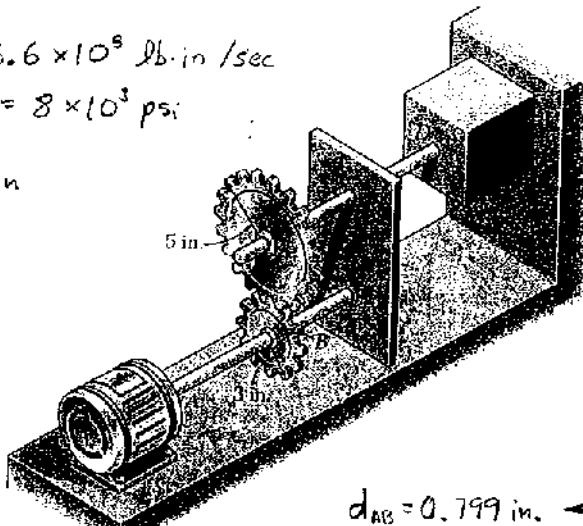
$$f = \frac{1260}{60} = 21 \text{ Hz} \quad \tau' = 8 \text{ ksi} = 8 \times 10^3 \text{ psi}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{105.6 \times 10^3}{2\pi(21)} = 800.32 \text{ lb-in}$$

$$\tau' = \frac{Tc}{J} = \frac{2T}{\pi C^3} \quad C = \sqrt[3]{\frac{2T}{\pi \tau'}}$$

$$C = \sqrt[3]{\frac{(2)(800.32)}{\pi(8 \times 10^3)}} = 0.399 \text{ in}$$

$$d_{AB} = 2C = 0.799 \text{ in.}$$



$$d_{AB} = 0.799 \text{ in.}$$

(b) Shaft CD

$$T_{CD} = \frac{r_c}{r_b} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb-in}$$

$$C = \sqrt[3]{\frac{2T}{\pi \tau'}} = \sqrt[3]{\frac{(2)(1.33387 \times 10^3)}{\pi(8 \times 10^3)}} = 0.473 \text{ in.}$$

$$d_{CD} = 2C = 0.947 \text{ in.}$$

$$d_{CD} = 0.947 \text{ in.}$$

Problem 3.78

3.78 A steel shaft must transmit 150 kW at speed of 360 rpm. Knowing that $G = 77.2 \text{ GPa}$, design a solid shaft so that the maximum stress will not exceed 50 MPa and the angle of twist in a 2.5-m length must not exceed 3° .

$$P = 150 \text{ kW} = 150 \times 10^3 \text{ W} \quad f = \frac{360}{60} = 6 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{150 \times 10^3}{2\pi (6)} = 3.9789 \times 10^3 \text{ N}\cdot\text{m}$$

Stress requirement $\sigma = 50 \times 10^6 \text{ Pa}$

$$\sigma = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \sigma}} = \sqrt[3]{\frac{(2)(3.9789 \times 10^3)}{\pi (50 \times 10^6)}} = 37.00 \times 10^{-3} \text{ m} = 37.00 \text{ mm}$$

Angle of twist requirement: $\phi = 3^\circ = 52.36 \times 10^{-3} \text{ rad}$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \quad G = 77.2 \times 10^9 \text{ Pa}, \quad L = 2.5 \text{ m}$$

$$c = \sqrt[4]{\frac{2TL}{\pi G \phi}} = \sqrt[4]{\frac{(2)(3.9789 \times 10^3)(2.5)}{\pi (77.2 \times 10^9)(52.36 \times 10^{-3})}} = 35.38 \times 10^{-3} \text{ m} = 35.38 \text{ mm}$$

Use larger value.

$$c = 37.00 \text{ mm}$$

$$d = 2c = 74.0 \text{ mm}$$

Problem 3.79

3.79 A 2.5-m-long steel shaft of 30-mm diameter rotates at a frequency of 30 Hz. Determine the maximum power that the shaft can transmit, knowing that $G = 77.2 \text{ GPa}$, that the allowable shearing stress is 50 MPa, and that the angle of twist must not exceed 7.5° .

$$c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m} \quad L = 2.5 \text{ m}$$

$$\text{Stress requirement } \sigma = 50 \times 10^6 \text{ Pa} \quad \sigma = \frac{Tc}{J}$$

$$T = \frac{\sigma J}{c} = \frac{\pi \sigma c^3}{2} = \frac{\pi}{2} (50 \times 10^6)(0.015)^3 = 265.07 \text{ N}\cdot\text{m}$$

$$\text{Twist angle requirement } \phi = 7.5^\circ = 130.90 \times 10^{-3} \text{ rad} \quad G = 77.2 \times 10^9 \text{ Pa}$$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi}{2} G c^4 \phi = \frac{\pi}{2} (77.2 \times 10^9)(0.015)^4 (130.90 \times 10^{-3}) = 803.60 \text{ N}\cdot\text{m}$$

Smaller value of T is the maximum allowable torque. $T = 265.07 \text{ N}\cdot\text{m}$

Power transmitted at $f = 30 \text{ Hz}$

$$P = 2\pi f T = 2\pi (30)(265.07) = 49.96 \times 10^3 \text{ W}$$

$$P = 50.0 \text{ kW}$$

Problem 3.80

3.80 A 1.5-in.-diameter steel shaft of length 4 ft will be used to transmit 60 hp between a motor and a pump. Knowing that $G = 11.2 \times 10^6$ psi, determine the lowest speed of rotation at which the stress does not exceed 8500 psi and the angle of twist does not exceed 2° .

$$c = \frac{1}{4}d = 0.75 \text{ in.} \quad L = 4 \text{ ft} = 48 \text{ in.} \quad P = 60 \text{ hp} = 396 \times 10^3 \text{ lb-in/s}$$

$$\text{Stress requirement} \quad \tau = 8500 \text{ psi} \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$T = \frac{\pi}{2} \tau c^3 = \frac{\pi}{2} (8500)(0.75)^3 = 5.6328 \times 10^3 \text{ lb-in}$$

$$\text{Twist angle requirement} \quad \phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad}$$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi G c^4 \phi}{2L} = \frac{\pi (11.2 \times 10^6)(0.75)^4 (34.907 \times 10^{-3})}{(2)(48)} = 4.0481 \times 10^3 \text{ lb-in}$$

Maximum allowable torque is the smaller value. $T = 4.0481 \times 10^3 \text{ lb-in}$

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{396 \times 10^3}{2\pi(4.0481 \times 10^3)} = 15.569 \text{ Hz} \quad f = 734 \text{ rpm}$$

Problem 3.81

3.81 A 5-ft-long solid steel shaft of 0.875-in. diameter is to transmit 18 hp. Determine the minimum speed at which the shaft can rotate, knowing that $G = 11.2 \times 10^6$ psi, that the allowable shearing stress is 4.5 ksi, and that the angle of twist must not exceed 3.5° .

$$L = 5 \text{ ft} = 60 \text{ in.} \quad c = \frac{1}{4}d = 0.4375 \text{ in.} \quad P = 18 \text{ hp} = 118.8 \times 10^3 \text{ lb-in/s}$$

$$\text{Stress requirement} \quad \tau = 4.5 \text{ ksi} = 4.5 \times 10^3 \text{ psi} \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$T = \frac{\pi}{2} \tau c^3 = \frac{\pi}{2} (4.5 \times 10^3)(0.4375)^3 = 591.92 \text{ lb-in}$$

$$\text{Twist angle requirement} \quad \phi = 3.5^\circ = 61.087 \times 10^{-3} \text{ rad} \quad \phi = \frac{TL}{GJ}$$

$$T = \frac{GJ\phi}{L} = \frac{\pi G c^4 \phi}{2L} = \frac{\pi (11.2 \times 10^6)(0.4375)^4 (61.087 \times 10^{-3})}{(2)(60)} \\ = 656.21 \text{ lb-in}$$

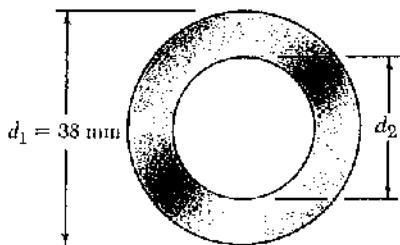
Maximum allowable torque is the smaller value $T = 591.92 \text{ lb-in}$

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{118.8 \times 10^3}{2\pi(591.92)} = 31.943 \text{ Hz} \quad f = 1917 \text{ rpm}$$

Problem 3.82

3.82 A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 is to be made of a steel for which $\tau_{al} = 65 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. Knowing that the angle of twist must not exceed 4° when the shaft is subjected to a torque of $600 \text{ N}\cdot\text{m}$, determine the largest inner diameter d_2 which can be specified in the design.



Stress requirement

$$C_1 = \sqrt[4]{C_2^4 - \frac{2TC_2}{\pi\tau}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(0.019)}{\pi(65 \times 10^6)}} = 11.689 \times 10^{-3} \text{ m} = 11.689 \text{ mm}$$

Twist angle requirement

$$C_1 = \sqrt[4]{C_2^4 - \frac{2TL\varphi}{\pi G}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(1.5)}{\pi(77.2 \times 10^9)(69.813 \times 10^{-3})}}$$

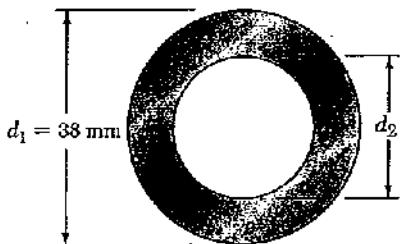
$$C_1 = 12.448 \times 10^{-3} \text{ m} = 12.448 \text{ mm}$$

Use smaller value of C_1 , $C_1 = 11.689 \text{ mm}$

$$d_2 = 2C_1 = 23.4 \text{ mm}$$

Problem 3.83

3.83 A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 100 kW between a turbine and a generator. Determine the minimum frequency at which the shaft can rotate, knowing that $G = 77.2 \text{ GPa}$, that the allowable shearing stress is 60 MPa, and that the angle of twist must not exceed 3° .



Stress requirement

$$T = \frac{J\tau}{C_2} = \frac{(125.186 \times 10^{-9})(60 \times 10^6)}{0.019} = 395.32 \text{ N}\cdot\text{m}$$

Twist angle requirement

$$T = \frac{GJ\varphi}{L} = \frac{(77.2 \times 10^9)(125.186 \times 10^{-9})(52.360 \times 10^{-3})}{1.5} = 337.35 \text{ N}\cdot\text{m}$$

Maximum allowable torque is the smaller value $T = 337.35 \text{ N}\cdot\text{m}$

Power transmitted $P = 100 \text{ kW} = 100 \times 10^3 \text{ W}$

$$P = 2\pi f T$$

$$\text{Frequency } f = \frac{P}{2\pi T} = \frac{100 \times 10^3}{2\pi(337.35)} = 47.2 \text{ Hz}$$

$$f = 47.2 \text{ Hz}$$

Problem 3.84

3.84 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 0.5$ in., determine that maximum power that can be transmitted without exceeding an allowable shearing stress of 7500 psi.



$$d = 5 \text{ in.} \quad D = 6 \text{ in.} \quad r = 0.5 \text{ in.}$$

$$\frac{D}{d} = \frac{6}{5} = 1.20 \quad \frac{r}{d} = \frac{0.5}{5} = 0.10 \quad \text{From Fig 3.32 } K = 1.33$$

$$\text{For the smaller side} \quad c = \frac{1}{2}d = 2.5 \text{ in.} \quad \chi = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \chi}{2K} = \frac{\pi (2.5)^3 (7500)}{(2)(1.33)} = 138.404 \times 10^3 \text{ lb-in.}$$

$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

$$\text{Power } P = 2\pi f T = 2\pi (7.5)(138.404 \times 10^3) = 6.52 \times 10^6 \text{ lb-in/s}$$

$$P = 988 \text{ hp}$$

Problem 3.85

3.85 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 0.2$ in., determine that maximum power that can be transmitted without exceeding an allowable shearing stress of 7500 psi.



$$d = 5 \text{ in.} \quad D = 6 \text{ in.} \quad r = 0.2 \text{ in.}$$

$$\frac{D}{d} = \frac{6}{5} = 1.20 \quad \frac{r}{d} = \frac{0.2}{5} = 0.04 \quad \text{From Fig. 3.32 } K = 1.55$$

$$\text{For smaller side} \quad c = \frac{1}{2}d = 2.5 \text{ in.} \quad \chi = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \chi}{2K} = \frac{\pi (2.5)^3 (7500)}{(2)(1.55)} = 118.76 \times 10^3 \text{ lb-in.}$$

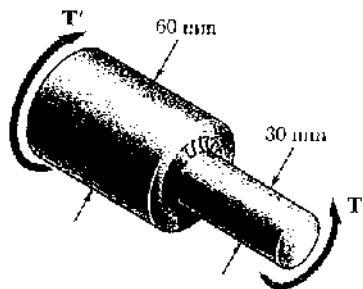
$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

$$\text{Power } P = 2\pi f T = 2\pi (7.5)(118.76 \times 10^3) = 5.60 \times 10^6 \text{ lb-in/s}$$

$$P = 848 \text{ hp}$$

Problem 3.86

3.86 Knowing that the stepped shaft shown must transmit 45 kW speed of 2100 rpm, determine the minimum radius r of the fillet if an allowable shearing stress of 50 MPa is not to be exceeded.



$$f = \frac{2100}{60} = 35 \text{ Hz}$$

$$P = 45 \times 10^3 \text{ W}$$

$$T = \frac{P}{2\pi f} = \frac{45 \times 10^3}{2\pi (35)} = 204.63 \text{ N}\cdot\text{m}$$

For smaller side $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$ $\chi = \frac{KTC}{J} = \frac{2KT}{\pi c^3}$

$$K = \frac{\pi \chi c^3}{2T} = \frac{\pi (50 \times 10^6) (0.015)^3}{(2)(204.63)} = 1.295$$

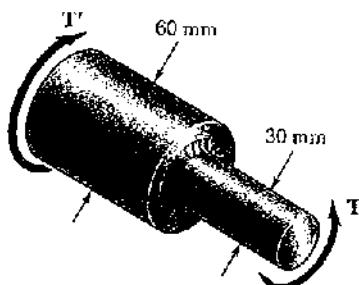
$$\frac{D}{d} = \frac{60}{30} = 2 \quad \text{From Fig. 3.32} \quad \frac{r}{d} = 0.17$$

$$r = 0.17 d = (0.17)(30) = 5.1 \text{ mm}$$

$$r = 5.1 \text{ mm} \quad \blacksquare$$

Problem 3.87

3.87 The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6 \text{ mm}$, determine the smallest permissible speed of the shaft.



$$\frac{r}{d} = \frac{6}{30} = 0.2 \quad \frac{D}{d} = \frac{60}{30} = 2$$

$$\text{From Fig. 3.32} \quad K = 1.26$$

For smaller side $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$

$$\chi = \frac{KTC}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \chi}{2K} = \frac{\pi (0.015)^3 (40 \times 10^6)}{(2)(1.26)} = 168.30 \times 10^3 \text{ N}\cdot\text{m}$$

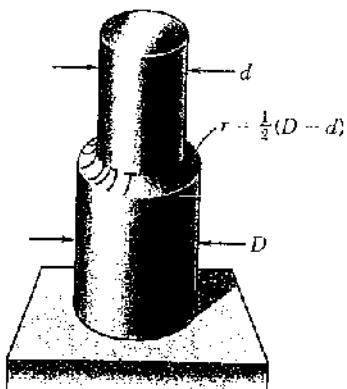
$$P = 45 \text{ kW} = 45 \times 10^3$$

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{45 \times 10^3}{2\pi (168.30 \times 10^3)} = 42.6 \text{ Hz}$$

$$f = 42.6 \text{ Hz} \quad \blacksquare$$

Problem 3.88



Full quarter-circular fillet extends to edge of larger shaft

3.88 A torque of magnitude $T = 200 \text{ lb} \cdot \text{in}$. is applied to the stepped shaft shown, which has a full quarter-circular fillet. Knowing that $D = 1 \text{ in}$, determine the maximum shearing stress in the shaft when (a) $d = 0.8 \text{ in}$, (b) $d = 0.9 \text{ in}$.

$$(a) \frac{D}{d} = \frac{1.0}{0.8} = 1.25 \quad r = \frac{1}{2}(D-d) = 0.1 \text{ in.}$$

$$\frac{r}{d} = \frac{0.1}{0.8} = 0.125$$

$$\text{From Fig. 3.32 } K = 1.31$$

$$\text{For smaller side } c = \frac{1}{2}d = 0.4 \text{ in.}$$

$$\tau = \frac{KTC}{J} = \frac{2KT}{\pi c^3} = \frac{(2)(1.31)(200)}{\pi(0.4)^3} = 2.61 \times 10^3 \text{ psi}$$

$$\tau = 2.61 \text{ ksi} \quad \blacksquare$$

$$(b) \frac{D}{d} = \frac{1.0}{0.9} = 1.11 \quad r = \frac{1}{2}(D-d) = 0.05$$

$$\frac{r}{d} = \frac{0.05}{1.0} = 0.05$$

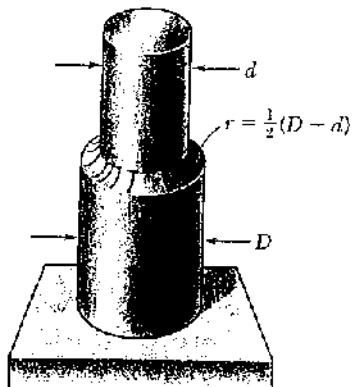
$$\text{From Fig. 3.32 } K = 1.44$$

$$\text{For smaller side } c = \frac{1}{2}d = 0.45 \text{ in}$$

$$T = \frac{2KT}{\pi c^3} = \frac{(2)(1.44)(200)}{\pi(0.45)^3} = 2.01 \times 10^3 \text{ psi}$$

$$\tau = 2.01 \text{ ksi} \quad \blacksquare$$

Problem 3.89



Full quarter-circular fillet extends to edge of larger shaft

3.89 In the stepped shaft shown, which has a full quarter-circular fillet, $D = 1.25 \text{ in}$. and $d = 1 \text{ in}$. Knowing that the speed of the shaft is 2400 rpm and that the allowable shearing stress is 7500 psi, determine the maximum power that can be transmitted by the shaft.

$$\frac{D}{d} = \frac{1.25}{1.0} = 1.25 \quad r = \frac{1}{2}(D-d) = 0.15 \text{ in.}$$

$$\frac{r}{d} = \frac{0.15}{1.0} = 0.15$$

$$\text{From Fig. 3.32 } K = 1.31$$

$$\text{For smaller side } c = \frac{1}{2}d = 0.5 \text{ in.}$$

$$\tau = \frac{KTC}{J} \quad T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$$

$$T = \frac{\pi (0.5)^3 (7500)}{(2)(1.31)} = 1.1241 \times 10^3 \text{ lb} \cdot \text{in}$$

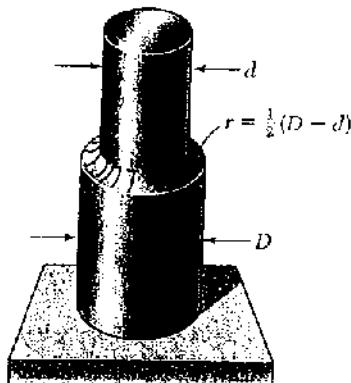
$$f = 2400 \text{ rpm} = 40 \text{ Hz}$$

$$P = 2\pi f T = 2\pi(40)(1.1241 \times 10^3) = 282.5 \times 10^3 \text{ lb} \cdot \text{in/s}$$

$$P = 42.8 \text{ hp} \quad \blacksquare$$

Problem 3.90

3.90 In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 80 MPa. Knowing that $D = 30 \text{ mm}$, determine the largest allowable torque that can be applied to the shaft is (a) $d = 26 \text{ mm}$, (b) $d = 24 \text{ mm}$.



Full quarter-circular fillet
extends to edge of larger shaft

$$\tau = 80 \times 10^6 \text{ Pa}$$

$$(a) \frac{D}{d} = \frac{30}{26} = 1.154 \quad r = \frac{1}{2}(D-d) = 2 \text{ mm}$$

$$\frac{r}{d} = \frac{2}{26} = 0.0768$$

From Fig. 3.32 $K = 1.36$

$$\text{Smaller side } c = \frac{1}{2}d = 13 \text{ mm} = 0.013 \text{ m}$$

$$\tau = \frac{Kt\sigma}{J} = \frac{2Kt}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.013)^3 (80 \times 10^6)}{(2)(1.36)}$$

$$= 203 \text{ N}\cdot\text{m}$$

$$T = 203 \text{ N}\cdot\text{m} \blacksquare$$

$$(b) \frac{D}{d} = \frac{30}{24} = 1.25 \quad r = \frac{1}{2}(D-d) = 3 \text{ mm} \quad \frac{r}{d} = \frac{3}{24} = 0.125$$

From Fig. 3.32 $K = 1.31$ $c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.012)^3 (80 \times 10^6)}{(2)(1.31)} = 165.8 \text{ N}\cdot\text{m} \quad T = 165.8 \text{ N}\cdot\text{m} \blacksquare$$

Problem 3.91

3.91 A 2-in.-diameter solid shaft is made of a mild steel that is assumed to be elastoplastic with $\tau_y = 20$ ksi. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) 30 kip·in., (b) 40 kip·in.

$$C = \frac{1}{2}d = 1.0 \text{ in. } \quad \tau_y = 20 \text{ ksi}$$

$$\text{Compute } T_y \quad T_y = \frac{\pi C^3 \tau_y}{4} = \frac{\pi}{2} C^3 \tau_y = \frac{\pi}{2} (1.0)^3 (20) = 31.416 \text{ kip·in.}$$

$$(a) T = 30 \text{ kip·in} < T_y \quad \text{elastic} \quad r = C = 1.0 \text{ in.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi C^3} = \frac{(2)(30)}{\pi (1.0)^3} = 19.10 \text{ ksi} \quad \tau_{max} = 19.10 \text{ ksi}$$

$$(b) T = 40 \text{ kip·in} > T_y \quad \text{plastic region with elastic core}$$

$$\text{The maximum shearing stress is} \quad \tau_{max} = \tau_y = 20 \text{ ksi}$$

$$T = \frac{4}{3} T_y \left(1 - \frac{r_y^3}{C^3} \right)$$

$$\frac{r_y^3}{C^3} = 4 - \frac{3T}{T_y} = 4 - \frac{(3)(40)}{31.416} = 0.18028 \quad \frac{r_y}{C} = 0.56492$$

$$r_y = 0.56492 C = (0.56492)(1.0) \quad r_y = 0.565 \text{ in.}$$

Problem 3.92

3.92 A 38-mm-diameter solid shaft is made of a mild steel that is assumed to be elastoplastic with $\tau_y = 145$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) 1.2 kN·m, (b) 1.8 kN·m.

$$C = \frac{1}{2}d = 0.019 \text{ m} \quad \tau_y = 150 \times 10^6 \text{ Pa}$$

$$\text{Compute } T_y \quad T_y = \frac{\pi}{2} C^3 \tau_y = \frac{\pi}{2} (0.019)^3 (145 \times 10^6) = 1.56224 \times 10^5 \text{ N·m}$$

$$(a) T = 1.2 \times 10^3 \text{ N·m} < T_y \quad \text{elastic} \quad r = C = 19 \text{ mm}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi C^3} = \frac{(2)(1.2 \times 10^3)}{\pi (0.019)^3} = 111.378 \times 10^6 \text{ Pa} \quad 111.4 \text{ MPa}$$

$$(b) T = 1.8 \times 10^3 \text{ N·m} > T_y \quad \text{plastic region with elastic core.}$$

$$\text{The maximum shearing stress is} \quad \tau_{max} = \tau_y = 145 \text{ MPa}$$

$$T = \frac{4}{3} T_y \left(1 - \frac{r_y^3}{C^3} \right)$$

$$\frac{r_y^3}{C^3} = 4 - \frac{3T}{T_y} = 4 - \frac{(3)(1.8 \times 10^3)}{1.56224 \times 10^5} = 0.54342 \quad \frac{r_y}{C} = 0.81604$$

$$r_y = 0.81604 C = (0.81604)(0.019) = 0.01550 \text{ m} \quad r_y = 15.50 \text{ mm}$$

Problem 3.93

3.93 It is observed that a straightened paper clip can be twisted through several revolutions by the application of a torque of approximately $60 \text{ mN} \cdot \text{m}$. Knowing that the diameter of the wire in the paper clip is 0.9 mm , determine the approximate value of the yield stress of the steel.

$$c = \frac{1}{2}d = 0.45 \text{ mm} = 0.45 \times 10^{-3} \text{ m} \quad T_p = 60 \text{ mN} \cdot \text{m} = 60 \times 10^{-3} \text{ N} \cdot \text{m}$$

$$T_p = \frac{4}{3} T_Y = \frac{4}{3} \frac{J \tau_Y}{c} = \frac{4}{3} \cdot \frac{\pi}{2} c^3 \tau_Y = \frac{2\pi}{3} c^3 \tau_Y$$

$$\tau_Y = \frac{3 T_p}{2\pi c^3} = \frac{(3)(60 \times 10^{-3})}{2\pi(0.45 \times 10^{-3})^3} = 314 \times 10^6 \text{ Pa} \quad \tau_Y = 314 \text{ MPa}$$

Problem 3.94

3.94 A 1.25-in. -diameter solid rod is made of an elastoplastic material with $\tau_p = 5 \text{ ksi}$. Knowing that the elastic core of the rod is of diameter 1 in. , determine the magnitude of the torque applied to the rod.

$$c = \frac{1}{2}d = 0.625 \text{ in.} \quad \tau_Y = 5 \times 10^3 \text{ psi} \quad \rho_Y = \frac{1}{2}d_Y = 0.5 \text{ in.}$$

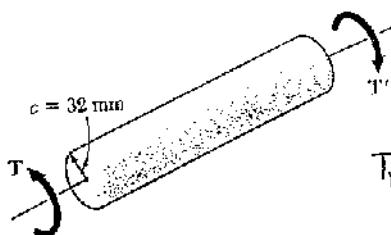
$$T_Y = \frac{J \tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2} (0.625)^3 (5 \times 10^3) = 1.91747 \times 10^3 \text{ lb-in}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_Y^3}{c^3}\right) = \frac{4}{3} (1.91747 \times 10^3) \left(1 - \frac{1}{4} \frac{0.5^3}{0.625^3}\right) = 2.23 \times 10^3 \text{ lb-in}$$

$$T = 2230 \text{ lb-in}$$

Problem 3.95

3.95 The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_p = 145 \text{ MPa}$. Determine the magnitude T of the applied torque when the plastic zone is (a) 16 mm deep, (b) 24 mm deep.



$$c = 32 \text{ mm} = 0.32 \text{ m} \quad \tau_Y = 145 \times 10^6 \text{ Pa}$$

$$T_Y = \frac{J \tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2} (0.032)^3 (145 \times 10^6)$$

$$= 7.4634 \times 10^3 \text{ N-m}$$

$$(a) \quad t_p = 16 \text{ mm} = 0.016 \text{ m} \quad \rho_Y = c - t_p = 0.032 - 0.016 = 0.016 \text{ m}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_Y^3}{c^3}\right) = \frac{4}{3} (7.4634 \times 10^3) \left(1 - \frac{1}{4} \frac{0.016^3}{0.032^3}\right)$$

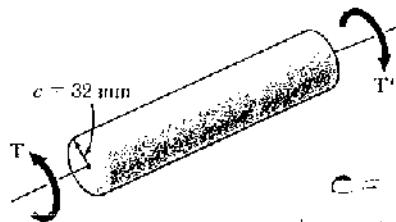
$$= 9.6402 \times 10^3 \text{ N-m} \quad T = 9.64 \text{ kN-m}$$

$$(b) \quad t_p = 24 \text{ mm} = 0.024 \text{ m} \quad \rho_Y = c - t_p = 0.032 - 0.024 = 0.008 \text{ m}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_Y^3}{c^3}\right) = \frac{4}{3} (7.4634 \times 10^3) \left(1 - \frac{1}{4} \frac{0.008^3}{0.032^3}\right)$$

$$= 9.9123 \times 10^3 \text{ N-m} \quad T = 9.91 \text{ kN-m}$$

Problem 3.96



3.96 For the shaft and loading of problem 3.95, assuming that $G = 77.2 \text{ GPa}$ determine the angle of twist in a 1.5-m length of the shaft.

3.95 The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_y = 145 \text{ MPa}$. Determine the magnitude T of the applied torque when the plastic zone is (a) 16 mm deep, (b) 24 mm deep.

$$c = 32 \text{ mm} = 0.032 \text{ m} \quad \tau_y = 145 \times 10^6 \text{ Pa} \quad G = 77.2 \times 10^9 \text{ Pa}$$

$$L = 1.5 \text{ m}$$

$$\gamma = \frac{P\phi}{L} \quad \gamma_y = \frac{c\phi_y}{L} \quad \phi_y = \frac{Ly}{c} = \frac{L\tau_y}{G} = \frac{(1.5)(145 \times 10^6)}{(0.032)(77.2 \times 10^9)} = 88.042 \times 10^{-3} \text{ rad}$$

$$\frac{P_y}{c} = \frac{\phi_y}{\phi} \quad \phi = \frac{c}{P_y} \phi_y$$

$$(a) t_p = 16 \text{ mm} = 0.016 \text{ m} \quad P_y = c - t_p = 0.016 \text{ m} \quad \frac{P_y}{c} = 0.500$$

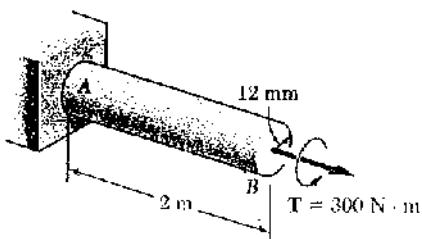
$$\phi = \frac{\phi_y}{P_y/c} = \frac{88.042 \times 10^{-3}}{0.500} = 176.08 \times 10^{-3} \text{ rad} \quad \phi = 10.09^\circ$$

$$(b) t_p = 24 \text{ mm} = 0.024 \text{ m} \quad P_y = c - t_p = 0.008 \text{ m} \quad \frac{P_y}{c} = 0.250$$

$$\phi = \frac{\phi_y}{P_y/c} = \frac{88.042 \times 10^{-3}}{0.250} = 352.168 \times 10^{-3} \text{ rad} \quad \phi = 20.2^\circ$$

Problem 3.97

3.97 The shaft AB is made of a material that is elastoplastic with $\tau_y = 90 \text{ MPa}$ and $G = 30 \text{ GPa}$. For the loading shown, determine (a) the radius of the elastic core of the shaft, (b) the angle of twist at end B .



$$(a) c = 12 \text{ mm} = 0.012 \text{ m} \quad \tau_y = 90 \times 10^6 \text{ Pa}$$

$$\tau_y = \frac{J\gamma_y}{c} = \frac{\pi c^3}{2} \gamma_y$$

$$= \frac{\pi}{2} (0.012)^3 (90 \times 10^6) = 244.29 \text{ N}\cdot\text{m}$$

$$T = 300 \text{ N}\cdot\text{m} > \tau_y \quad \text{plastic region with elastic core}$$

$$T = \frac{4}{3} \tau_y \left(1 - \frac{1}{4} \frac{P_y^3}{c^3} \right) \quad \frac{P_y^3}{c^3} = 1 - \frac{3T}{\tau_y} = 1 - \frac{(3)(300)}{244.29} = 0.31585$$

$$\frac{P_y}{c} = 0.68102 \quad P_y = (0.68102)(0.012) = 8.17 \times 10^{-3} \text{ m} \quad P_y = 8.17 \text{ nm}$$

$$(b) L = 2 \text{ m} \quad G = 30 \times 10^9 \text{ Pa}$$

$$\phi = \frac{T_y L}{J G} = \frac{2 T_y L}{\pi c^4 G} = \frac{(2)(244.29)(2)}{\pi (0.012)^4 (30 \times 10^9)} = 0.5000 \text{ rad}$$

$$\frac{\phi_y}{\phi} = \frac{P_y}{c} \quad \phi = \frac{\phi_y}{P_y/c} = \frac{0.5000}{0.68102} = 0.734 \text{ rad} \quad \phi = 42.1^\circ$$

Problem 3.98

3.98 A 0.75-in.-diameter solid circular shaft is made of material that is assumed to be elastoplastic with $\tau_y = 20 \text{ ksi}$ and $G = 11.2 \times 10^6 \text{ psi}$. For a 4-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 1800 lb-in. torque.

$$\tau_y = 20 \text{ ksi} = 20 \times 10^3 \text{ psi}, \quad c = \frac{1}{2} = 0.375 \text{ in.}, \quad L = 4 \text{ ft} = 48 \text{ in.}, \quad T = 1800 \text{ lb-in}$$

$$T_y = \frac{J\tau_y}{c} = \frac{\pi}{2} c^3 \tau_y = \frac{\pi}{2} (0.375)^3 (20 \times 10^3) = 1656.70 \text{ lb-in.}$$

$T > T_y$ Plastic region with elastic core $\tau_{max} = \tau_y = 20 \text{ ksi}$

$$\phi_y = \frac{T_y L}{GJ} = \frac{2T_y L}{\pi c^4 G} = \frac{(2)(1656.70)(48)}{\pi (0.375)^4 (11.2 \times 10^6)} = 0.22857 \text{ rad}$$

$$T = \frac{4}{3} T_y \left(1 - \frac{1}{4} \frac{\phi^3}{\phi_y^3}\right)$$

$$\left(\frac{\phi}{\phi_y}\right)^3 = 4 - \frac{3T}{T_y} = 4 - \frac{(3)(1800)}{1656.70} = 0.7451 \quad \frac{\phi}{\phi_y} = 0.90471$$

$$\phi = \frac{\phi_y}{0.90471} = \frac{0.22857}{0.90471} = 0.25264 \text{ rad} \quad \phi_y = 14.48^\circ$$

Problem 3.99

3.99 A solid circular rod is made of a material that is assumed to be elastoplastic. Denoting by τ_y and ϕ_y , respectively, the torque and the angle of twist at the onset of yield, determine the angle of twist if the torque is increased to (a) $T = 1.1T_y$, (b) $T = 1.25T_y$, (c) $T = 1.3T_y$.

$$T = \frac{4}{3} T_y \left(1 - \frac{1}{4} \frac{\phi^3}{\phi_y^3}\right)$$

$$\frac{\phi_y}{\phi} = \sqrt[3]{4 - \frac{3T}{T_y}} \quad \text{or} \quad \frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_y}}}$$

$$(a) \quad \frac{T}{T_y} = 1.10 \quad \frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - (3)(1.10)}} = 1.126 \quad \phi = 1.126 \phi_y$$

$$(b) \quad \frac{T}{T_y} = 1.25 \quad \frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - (3)(1.25)}} = 1.587 \quad \phi = 1.587 \phi_y$$

$$(c) \quad \frac{T}{T_y} = 1.3 \quad \frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - (3)(1.3)}} = 2.15 \quad \phi = 2.15 \phi_y$$

Problem 3.100

3.100 A 1.25-in. diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_y = 18 \text{ ksi}$ and $G = 11.2 \times 10^6 \text{ psi}$. For an 8-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 7.5 kip · in. torque.

$$C = \frac{1}{2}d = 0.625 \text{ in.}, G = 11.2 \times 10^6 \text{ psi}, \tau_y = 18 \text{ ksi} = 18000 \text{ psi}$$

$$L = 8 \text{ ft.} = 96 \text{ in.} \quad T = 7.5 \text{ kip} \cdot \text{in.} = 7.5 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T_y = \frac{\tau_y C^3}{G} = \frac{\pi}{2} C^3 \tau_y = \frac{\pi}{2} (0.625)^3 (18000) = 6.9029 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T > T_y \quad \text{plastic region with elastic core} \therefore \tau_{\max} = \tau_y = 18 \text{ ksi}$$

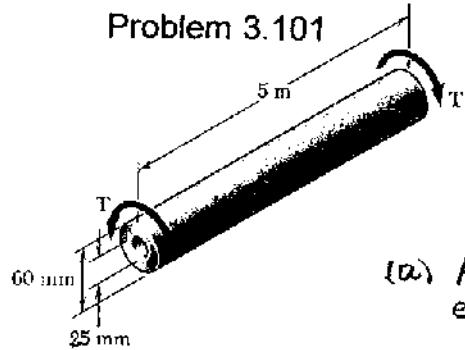
$$\gamma_y = \frac{C \phi_y}{L} \therefore \phi_y = \frac{L \gamma_y}{C} = \frac{L \tau_y}{C G} = \frac{(96)(18000)}{(0.625)(11.2 \times 10^6)} = 246.86 \times 10^{-3} \text{ rad}$$

$$T = T_y \left(1 - \frac{3T}{T_y} \right)$$

$$\frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_y}}} = \frac{1}{\sqrt[3]{4 - \frac{(3)(7.5 \times 10^3)}{6.9029 \times 10^3}}} = 1.10533$$

$$\phi = 1.10533 \phi_y = (1.10533)(246.86 \times 10^{-3}) = 272.86 \times 10^{-3} \text{ rad} \quad \phi = 15.63^\circ$$

Problem 3.101



3.101 The hollow shaft shown is made of steel which is assumed to be elastoplastic with $\tau_y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. Determine the magnitude T of the torque and the corresponding angle of twist (a) at the onset of yield, (b) when the plastic zone is 10 mm deep.

(a) At the onset of yield, the stress distribution is the elastic distribution with $\tau_{max} = \tau_y$

$$C_2 = \frac{1}{2}d_2 = 0.030 \text{ m}, C_1 = \frac{1}{2}d_1 = 0.0125 \text{ m}$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.030^4 - 0.0125^4) = 1.2340 \times 10^{-6} \text{ m}^4$$

$$\tau_{max} = \tau_y = \frac{T_r C_2}{J} \therefore T_y = \frac{J \tau_y}{C_2} = \frac{(1.2340 \times 10^{-6})(145 \times 10^6)}{0.030} = 5.9648 \times 10^3 \text{ N.m}$$

$$T_y = 5.96 \text{ kN.m} \blacksquare$$

$$\phi_y = \frac{T_r L}{GJ} = \frac{(5.9648 \times 10^3)(5)}{(77.2 \times 10^9)(1.2340 \times 10^{-6})} = 313.04 \times 10^{-3} \text{ rad} \quad \phi_y = 17.94^\circ \blacksquare$$

$$(b) t = 0.010 \text{ m} \quad p_r = C_2 - t = 0.030 - 0.010 = 0.020 \text{ m}$$

$$\gamma = \frac{P\Phi}{L} = \frac{P_r \Phi}{L} = \gamma_y = \frac{\tau_y}{G}$$

$$\phi = \frac{\gamma_y L}{G P_r} = \frac{(145 \times 10^6)(5)}{(77.2 \times 10^9)(0.020)} = 469.56 \times 10^{-3} \text{ rad} \quad \phi = 26.9^\circ \blacksquare$$

Torque T_1 carried by elastic portion $C_1 \leq p \leq p_r$

$$\tau = \tau_y \text{ at } p = p_r. \quad \tau_y = \frac{T_1 p_r}{J_1} \text{ where } J_1 = \frac{\pi}{2}(p_r^4 - C_1^4)$$

$$J_1 = \frac{\pi}{2}(0.020^4 - 0.0125^4) = 212.978 \times 10^{-9} \text{ m}^4$$

$$T_1 = \frac{J_1 \tau_y}{p_r} = \frac{(212.978 \times 10^{-9})(145 \times 10^6)}{0.020} = 1.5441 \times 10^3 \text{ N.m}$$

Torque T_2 carried by plastic portion

$$T_2 = 2\pi \int_{p_r}^{C_2} \tau_y p^2 dp = 2\pi \tau_y \frac{p^3}{3} \Big|_{p_r}^{C_2} = \frac{2\pi}{3} \tau_y (C_2^3 - p_r^3)$$

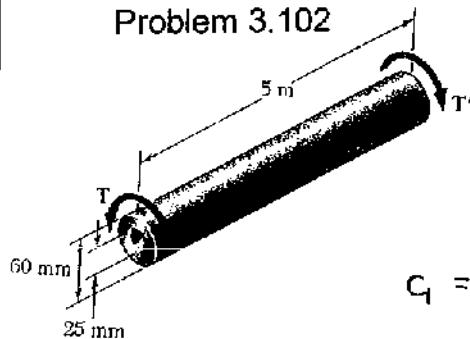
$$= \frac{2\pi}{3} (145 \times 10^6)(0.030^3 - 0.020^3) = 5.7701 \times 10^3 \text{ N.m}$$

Total torque

$$T = T_1 + T_2 = 1.5441 \times 10^3 + 5.7701 \times 10^3 = 7.3142 \times 10^3 \text{ N.m}$$

$$T = 7.31 \text{ kN.m} \blacksquare$$

Problem 3.102



3.102 For the shaft of Prob. 3.101, determine (a) the angle of twist at which the shaft first becomes fully plastic, (b) the corresponding magnitude T of the torque. Sketch the $T-\phi$ curve for the shaft.

3.101 The hollow shaft shown is made of steel which is assumed to be elastoplastic with $r_y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. Determine the magnitude T of the torque and the corresponding angle of twist (a) at the onset of yield, (b) when the plastic zone is 10 mm deep.

$$C_1 = \frac{1}{2} d_1 = 0.0125 \text{ m} \quad C_2 = \frac{1}{2} d_2 = 0.030 \text{ m}$$

(a) For onset of fully plastic yielding, $\rho_y = C_1$

$$\gamma = \gamma_y \Rightarrow \gamma = \frac{\gamma_y}{G} = \frac{\rho_y \Phi}{L} = \frac{C_1 \Phi}{L}$$

$$\Phi_y = \frac{L \gamma_y}{C_1 G} = \frac{(25)(145 \times 10^6)}{(0.0125)(77.2 \times 10^9)} = 7.51295 \times 10^{-3} \text{ rad} \quad \phi_y = 43.0^\circ$$

$$(b) T_p = 2\pi \int_{C_1}^{C_2} \gamma_y \rho^3 d\rho = 2\pi \gamma_y \frac{\rho^3}{3} \Big|_{C_1}^{C_2} = \frac{2\pi}{3} \gamma_y (C_2^3 - C_1^3)$$

$$= \frac{2\pi}{3} (145 \times 10^6) (0.030^3 - 0.0125^3) = 7.606 \times 10^3 \text{ N}\cdot\text{m}$$

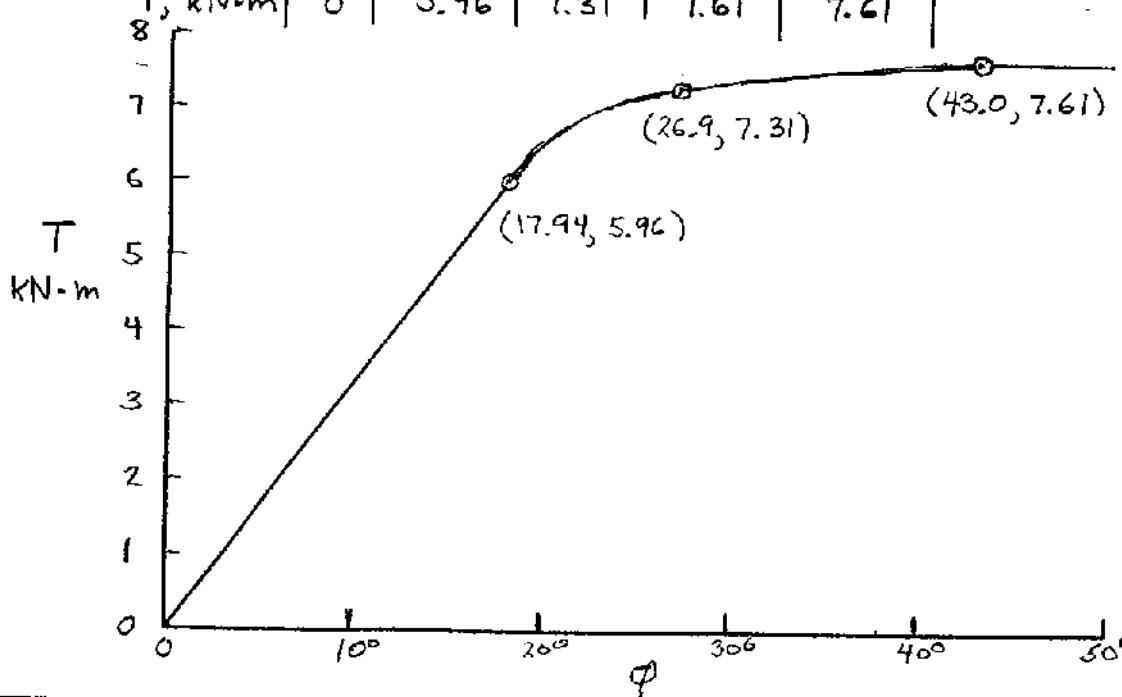
$$T_p = 7.61 \text{ kN}\cdot\text{m}$$

From Problem 3.101 $\Phi_y = 17.94^\circ$ $T_y = 5.96 \text{ kN}\cdot\text{m}$

Also from Problem 3.101 $\Phi = 26.9^\circ$ $T = 7.31 \text{ kN}\cdot\text{m}$

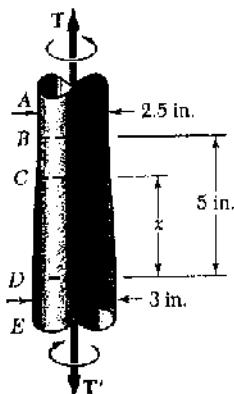
Plot T vs Φ using the following data.

$\Phi, \text{ deg}$	0	17.94	26.9	43.0	> 43.0
$T, \text{ kN}\cdot\text{m}$	0	5.96	7.31	7.61	7.61



Problem 3.103

3.103 A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 75 \text{ kip} \cdot \text{in}$. are applied. Assuming the steel to be elastoplastic with $\tau_y = 21 \text{ ksi}$ and $G = 11.2 \times 10^6 \text{ psi}$, determine (a) the radius of the elastic core in portion AB of the shaft, (b) the length of portion CD which remains fully elastic.



$$(a) \text{ In portion AB} \quad c = \frac{1}{2}d = 1.25 \text{ in.}$$

$$T_y = \frac{J_{AB} \tau_y}{c} = \frac{\pi}{2} C^3 \tau_y = \frac{\pi}{2} (1.25)^3 (21 \times 10^3) = 64.427 \times 10^3 \text{ lb-in}$$

$$T = \frac{4}{3} T_y (1 - \frac{P_y^3}{C^3})$$

$$\frac{P_y}{C} = \sqrt[3]{4 - \frac{3T}{T_y}} = \sqrt[3]{4 - \frac{(3)(75 \times 10^3)}{64.427 \times 10^3}} = 0.79775$$

$$P_y = 0.79775 C = (0.79775)(1.25) = 0.99718 \text{ in.}$$

$$P_y = 0.997 \text{ in.}$$

(b) For yielding at point C

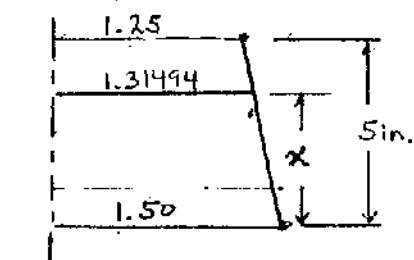
$$\tau = \tau_y, \quad C = C_x, \quad T = 75 \times 10^3 \text{ lb-in}$$

$$T = \frac{J_c \tau_y}{C_x} = \frac{\pi}{2} C_x^3 \tau_y \quad C_x = \sqrt[3]{\frac{2T}{\pi \tau_y}} = \sqrt[3]{\frac{(2)(75 \times 10^3)}{\pi (21 \times 10^3)}} = 1.31494 \text{ in.}$$

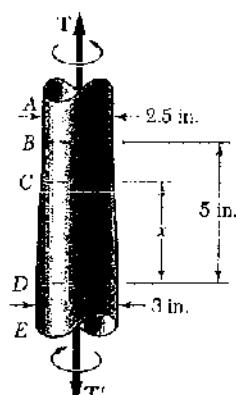
Using proportions from the sketch

$$\frac{1.50 - 1.31494}{1.50 - 1.25} = \frac{x}{5}$$

$$x = 3.70 \text{ in.}$$



Problem 3.104



3.104 If the torques applied to the tapered shaft of Prob. 3.103 are slowly increased, determine (a) the magnitude T of the largest torques which can be applied to the shaft, (b) the length of the portion CD which remains fully elastic.

3.103 A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 75 \text{ kip} \cdot \text{in}$. are applied. Assuming the steel to be elastoplastic with $\tau_y = 21 \text{ ksi}$ and $G = 11.2 \times 10^6 \text{ psi}$, determine (a) the radius of the elastic core in portion AB of the shaft, (b) the length of portion CD which remains fully elastic.

- (a) The largest torque that may be applied is that which makes portion AB fully plastic.

$$\text{In portion } AB \quad c = \frac{1}{2}d = 1.25 \text{ in.}$$

$$T_y = \frac{J\tau_y}{c} = \frac{\pi}{2} c^3 \tau_y = \frac{\pi}{2} (1.25)^3 (21 \times 10^3) = 64.427 \times 10^3 \text{ lb-in}$$

$$\text{For fully plastic shaft } \rho_y = 0 \quad T = \frac{4}{3} T_y \left(1 - \frac{1}{4} \frac{\rho_y^3}{c^3}\right) = \frac{4}{3} T_y$$

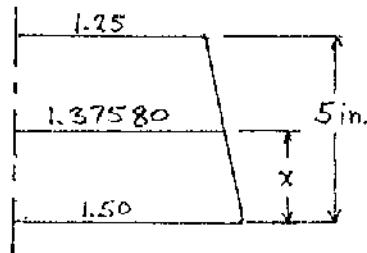
$$T = \frac{4}{3} (64.427 \times 10^3) = 85.903 \times 10^3 \text{ lb-in}$$

$$T = 85.9 \text{ kip-in} \quad \blacksquare$$

- (b) For yielding at point C , $\tau = \tau_y$, $c = c_x$, $T = 85.903 \times 10^3 \text{ lb-in}$

$$\tau_y = \frac{T c_x}{J_x} = \frac{2T}{\pi c_x^3}$$

$$c_x = \sqrt[3]{\frac{2T}{\pi \tau_y}} = \sqrt[3]{\frac{(2)(85.903 \times 10^3)}{\pi (21 \times 10^3)}} = 1.37580 \text{ in.}$$



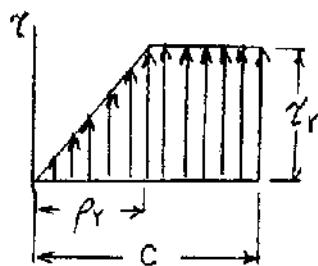
Using proportions from the sketch

$$\frac{1.50 - 1.37580}{1.50 - 1.25} = \frac{x}{5}$$

$$x = 2.48 \text{ in.} \quad \blacksquare$$

Problem 3.105

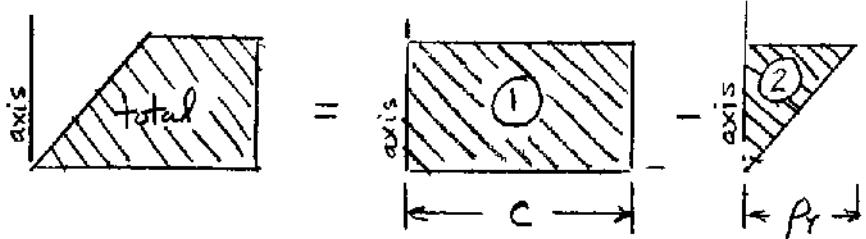
3.105 Considering the partially plastic shaft of Fig. 3.38a, derive Eq.(3.32) by recalling that the integral in Eq. (3.26) representing the second moment about the τ -axis of the area under the τ - ρ curve.



The stress is that shown on the left:

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi \int_0^c \rho^2 dA = 2\pi I$$

where $dA = \tau d\rho$ and $I = 2\text{nd moment about the } z\text{-axis.}$



$$I = I_1 - I_2$$

$$= \frac{1}{3} \sigma_y c^3 - \left\{ \frac{1}{36} \sigma_y r_y^3 + \frac{1}{2} \sigma_y r_y \left(\frac{1}{3} \sigma_y r_y \right)^2 \right\}$$

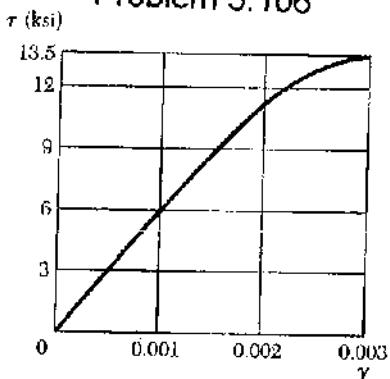
$$= \frac{1}{3} \sigma_y c^3 - \frac{1}{12} \sigma_y r_y^3 = \frac{1}{3} \sigma_y c^3 \left(1 - \frac{1}{4} \frac{r_y^3}{c^3} \right)$$

$$T = 2\pi I = \frac{2\pi}{3} \sigma_y c^3 \left(1 - \frac{1}{4} \frac{r_y^3}{c^3} \right)$$

$$\text{Recall that } T_y = \frac{J \sigma_y}{c} = \frac{\pi}{2} c^3 \sigma_y \quad \therefore \quad \frac{2\pi}{3} \sigma_y c^3 = \frac{4}{3} T_y$$

$$\text{Hence, } T = \frac{4}{3} T_y \left(1 - \frac{1}{4} \frac{r_y^3}{c^3} \right)$$

Problem 3.106



3.106 A solid brass rod of 1.2-in. diameter is subjected to a torque which causes a maximum shearing stress of 13.5 ksi in the rod. Using the τ - γ diagram shown for the brass used, determine (a) the magnitude of the torque, (b) the angle of twist in a 24-in. length of the rod.

$$(a) \quad \tau_{\max} = 13.5 \text{ ksi} \quad c = \frac{1}{2}d = 0.600 \text{ in}$$

From the stress-strain diagram $\gamma_{\max} = 0.003$

$$\text{Let } z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c}$$

$$T = 2\pi \int_0^c \rho^2 z \, d\rho = 2\pi c^3 \int_0^1 z^2 z \, dz = 2\pi c^3 I$$

where the integral I is given by $I = \int_0^1 z^2 z \, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 z$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below:

z	γ	τ , ksi	$z^2 z$, ksi	w	$wz^2 z$, ksi
0	0	0	0.000	1	0
0.25	0.00075	4.5	0.281	4	1.125
0.5	0.0015	8.6	2.15	2	4.30
0.75	0.00225	12.2	6.86	4	27.45
1.0	0.003	13.5	13.5	1	13.5
					46.375 $\leftarrow \sum wz^2 z$

$$I = \frac{(0.25)(46.375)}{3} = 3.865 \times 10^{-3} \text{ ksi}$$

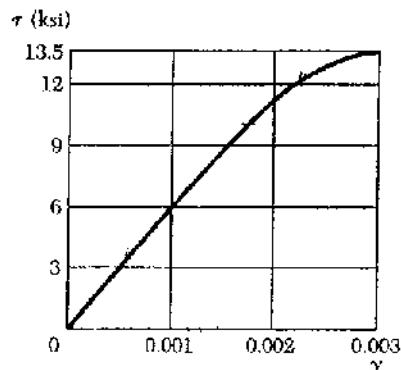
$$(a) \quad T = 2\pi c^3 I = 2\pi (0.600)^3 (3.865) = 5.24 \text{ kip.in} \quad T = 5.24 \text{ kip.in} \quad \blacksquare$$

$$(b) \quad \gamma_{\max} = \frac{c\phi}{L}$$

$$\phi = \frac{L\gamma_m}{c} = \frac{(24)(0.003)}{0.600} = 120 \times 10^{-3} \text{ rad} \quad \phi = 6.88^\circ \quad \blacksquare$$

Note: Answer may differ slightly due to reading of graph and choice of numerical integration formula.

Problem 3.107



3.107 A solid brass rod of 0.8-in. diameter and 30-in. length is twisted through an angle of 10° . Using the $\tau-\gamma$ diagram shown for the brass used, determine (a) the magnitude of the torque applied to the rod, (b) the maximum shearing stress in the rod.

$$(a) \quad \phi = 10^\circ = 174.53 \times 10^{-3} \text{ rad}$$

$$c = \frac{1}{2}d = 0.400 \text{ in.} \quad L = 30 \text{ in.}$$

$$\tau_{\max} = \frac{c\phi}{L} = \frac{(0.4)(174.53 \times 10^{-3})}{30}$$

$$= 0.00233$$

$$\text{Let } Z = \frac{\gamma}{\tau_{\max}} = \frac{\rho}{G_2}$$

$$T = 2\pi \int_0^c \rho^2 Z d\rho = 2\pi C_2^3 \int_0^1 Z^2 Z' dz = 2\pi C_2^3 I$$

$$\text{where the integral } I \text{ is given by } I = \int_{V_2}^1 Z^2 Z' dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 z'$$

where w is a weighting factor. Using $\Delta z = \frac{1}{6}$ we get the values given in the table below.

Z	γ	$Z, \text{ ksi}$	$Z^2 Z, \text{ ksi}$	w	$wZ^2 Z, \text{ ksi}$
0	0	0	0	1	0
0.25	0.000583	3.5	0.219	4	0.88
0.5	0.001165	7.0	1.75	2	3.50
0.75	0.001748	10.0	5.625	4	22.50
1	0.00233	12.2	12.6	1	12.60

$$39.48 \leftarrow \sum wZ^2 Z$$

$$I = \frac{(0.25)(39.48)}{3} = 3.29 \text{ ksi}$$

$$T = 2\pi C_2^3 I = 2\pi (0.400)^3 (3.29)$$

$$T = 1.322 \text{ kip-in.} \leftarrow$$

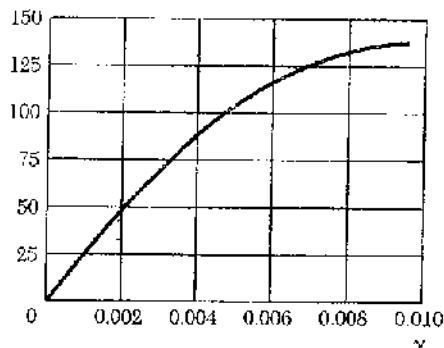
Note: Answer may differ slightly due to differences of opinion in reading the stress-strain curve.

(b) From the graph,

$$\tau_{\max} = 12.6 \text{ ksi} \leftarrow$$

Problem 3.108

τ (MPa)



3.108 A solid aluminum rod of 40-mm diameter is subjected to a torque which produces in the rod a maximum shearing strain of 0.008. Using the τ - γ diagram shown for the aluminum alloy used, determine (a) the magnitude of the torque applied to the rod, (b) the angle of twist in a 750-mm length of the rod.

$$\gamma_{\max} = 0.008 \quad c = \frac{1}{2}d = 0.020 \text{ m}$$

$$L = 750 \text{ mm} = 0.750 \text{ m}$$

$$(a) \text{ Let } z = \frac{\gamma}{\gamma_{\max}} = \frac{r}{c}$$

$$T = 2\pi \int_0^c r^2 \tau dr = 2\pi c^3 \int_0^1 z^2 \tau dz \\ = 2\pi c^3 I$$

where the integral I is given by $I = \int_0^1 z^2 \tau dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$ we get the values given in the table below.

z	γ	$\tau, \text{ MPa}$	$z^2 \tau, \text{ MPa}$	w	$w z^2 \tau, \text{ MPa}$
0	0	0	0	1	0
0.25	0.002	48	3.0	4	12.0
0.5	0.004	88	22.0	2	44.0
0.75	0.006	115	64.7	4	258.8
1	0.008	133	133.0	1	133.0
					447.8

$$\leftarrow \sum w z^2 \tau$$

$$I = \frac{(0.25)(447.8)}{3} = 37.3 \text{ MPa} = 37.3 \times 10^6 \text{ Pa}$$

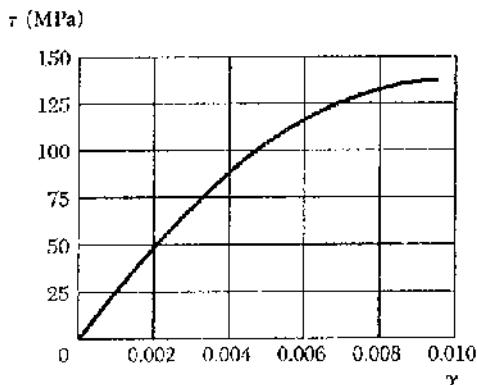
$$T = 2\pi (0.020)^3 (37.3 \times 10^6) = 1.876 \times 10^3 \text{ N}\cdot\text{m} \quad T = 1.876 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$(b) \gamma_{\max} = \frac{c \phi}{L}$$

$$\phi = \frac{L \gamma_{\max}}{c} = \frac{(0.750)(0.008)}{0.020} = 300 \times 10^{-3} \text{ rad}$$

$$\phi = 17.19^\circ \quad \blacktriangleleft$$

Problem 3.109



3.109 The curve shown in Fig. P3.108 can be approximated by the relation

$$\tau = 27.8 \times 10^9 \gamma + 1.390 \times 10^{12} \gamma^2$$

Using this relation and Eqs. (3.2) and (3.26), solve problem 3.108.

3.108 A solid aluminum rod of 40-mm diameter is subjected to a torque which produces in the rod a maximum shearing strain of 0.008. Using the τ - γ diagram shown for the aluminum alloy used, determine (a) the magnitude of the torque applied to the rod, (b) the angle of twist in a 750-mm length of the rod.

$$\gamma_{max} = 0.008 \quad C = \frac{1}{2}d = 0.020 \text{ m}$$

$$L = 750 \text{ mm} = 0.750 \text{ m}$$

$$(a) \text{ Let } z = \frac{\gamma}{\gamma_{max}} = \frac{\tau}{C}$$

$$T = 2\pi \int_0^C \rho^3 z d\rho = 2\pi C^3 \int_0^1 z^2 z dz$$

$$\text{The given stress-strain curve is } \tau = B\gamma + C\gamma^2 = B\gamma_{max}z + C\gamma_{max}^2 z^2$$

$$\text{where } B = 27.8 \times 10^9 \text{ and } C = -1.390 \times 10^{12}$$

$$T = 2\pi C^3 \int_0^1 z^2 (B\gamma_{max}z + C\gamma_{max}^2 z^2) dz$$

$$= 2\pi C^3 \left\{ B\gamma_{max} \int_0^1 z^3 dz + C\gamma_{max}^2 \int_0^1 z^4 dz \right\}$$

$$= 2\pi C^3 \left\{ \frac{1}{4} B\gamma_{max} + \frac{1}{5} C\gamma_{max}^2 \right\}$$

$$= 2\pi (0.020)^3 \left\{ \frac{1}{4} (27.8 \times 10^9) (0.008) + \frac{1}{5} (-1.390 \times 10^{12}) (0.008)^2 \right\}$$

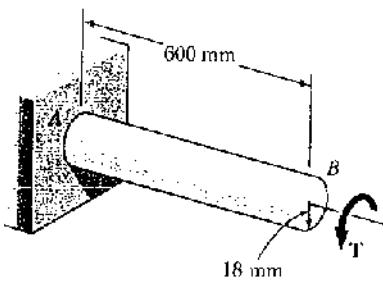
$$= 1.900 \times 10^3 \text{ N}\cdot\text{m}$$

$$T = 1.900 \text{ kN}\cdot\text{m}$$

$$(b) \gamma_{max} = \frac{C\phi}{L}$$

$$\phi = \frac{L\gamma_{max}}{C} = \frac{(0.750)(0.008)}{0.020} = 300 \times 10^3 \text{ rad} \quad \phi = 17.19^\circ$$

Problem 3.110



3.110 The solid circular shaft AB is made of a steel that is assumed to be elastoplastic with $\tau_r = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The torque T is increased until the radius of the elastic core is 6 mm. Determine the maximum residual shearing stress in the shaft after the torque T is removed.

$$C = 0.018 \text{ m} \quad \rho_r = 0.006 \text{ m} \quad \frac{\rho_r}{C} = \frac{0.006}{0.018} = \frac{1}{3}$$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.018)^4 = 164.896 \times 10^{-9} \text{ m}^4$$

$$\tau_r = \frac{J \tau_r}{C} = \frac{(164.896 \times 10^{-9})(145 \times 10^6)}{0.018} = 1.32833 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{At end of loading: } T = \frac{4}{3} \tau_r \left(1 - \frac{1}{4} \frac{\rho^3}{C^3} \right) = \frac{4}{3} (1.32833 \times 10^3) \left[1 - \frac{1}{4} \left(\frac{1}{3} \right)^3 \right] \\ = 1.7547 \times 10^3 \text{ N}\cdot\text{m}$$

The stresses are $\tau_{load} = 0$ at $\rho = 0$

$\tau_{load} = \tau_r = 145 \text{ MPa}$ at $\rho = 0.006 \text{ m}$

$\tau_{load} = \tau_r = 145 \text{ MPa}$ at $\rho = 0.018 \text{ m}$

Torque change during unloading $T = -1.7547 \times 10^3 \text{ N}\cdot\text{m}$

Stress changes during unloading $\tau' = \frac{T\rho}{J}$ (elastic)

$$\text{At } \rho = 0 \quad \tau' = 0$$

$$\text{At } \rho = 0.006 \text{ m} \quad \tau' = -\frac{(1.7547 \times 10^3)(0.006)}{164.896 \times 10^{-9}} = 63.8 \text{ MPa}$$

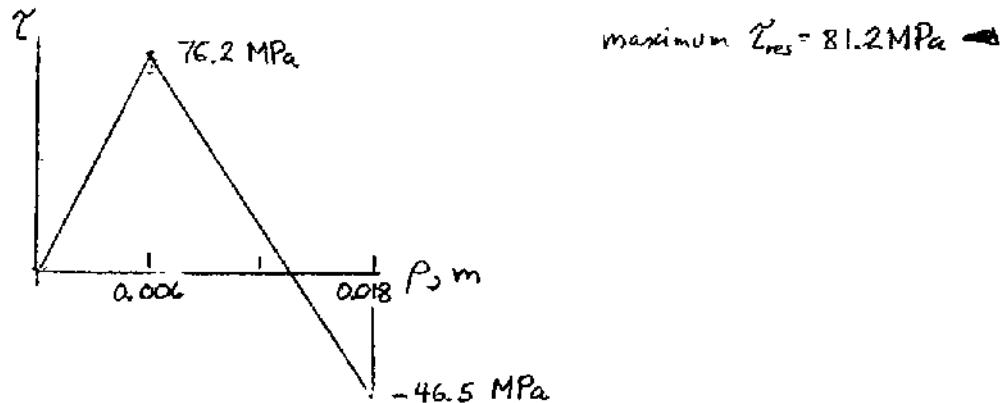
$$\text{At } \rho = 0.018 \text{ m} \quad \tau' = -\frac{(1.7547 \times 10^3)(0.018)}{164.896 \times 10^{-9}} = 191.5 \text{ MPa}$$

Residual stresses are found by adding $\tau_{res} = \tau_{load} - \tau'$

$$\text{At } \rho = 0 \quad \tau_{res} = 0$$

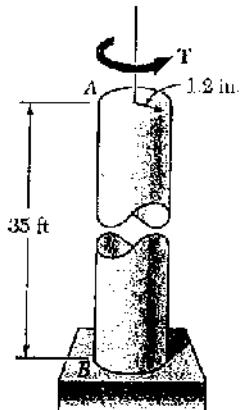
$$\text{At } \rho = 0.006 \text{ m} \quad \tau_{res} = 145 - 63.8 = 81.2 \text{ MPa}$$

$$\text{At } \rho = 0.018 \text{ m} \quad \tau_{res} = 145 - 191.5 = -46.5 \text{ MPa}$$



Problem 3.111

3.111 The solid circular drill rod AB is made of a steel that is assumed to be elastoplastic with $\tau_f = 22$ ksi and $G = 11.2 \times 10^6$ psi. Knowing that a torque $T = 75$ kip-in. is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.



SOLUTION

$$c = 1.2 \text{ in.} \quad L = 35 \text{ ft} = 420 \text{ in.}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (1.2)^4 = 3.2572 \text{ in}^4$$

$$T_y = \frac{J \tau_y}{c} = \frac{(3.2572)(22)}{1.2} = 59.715 \text{ kip-in}$$

Loading: $T = 75 \text{ kip-in}$

$$T = \frac{4}{3} T_y \left(1 - \frac{A^3}{c^3}\right)$$

$$\frac{\rho_y}{c^3} = 4 - \frac{3T}{T_y} = 4 - \frac{(3)(75)}{59.715} = 0.23213$$

$$\frac{\rho_y}{c} = 0.61458, \quad \rho_y = 0.61458 c = 0.73749 \text{ in.}$$

Unloading: $\tau' = \frac{T\rho}{J}$ where $T = 75 \text{ kip-in}$

$$\text{At } \rho = c \quad \tau' = \frac{(75)(0.73749)}{3.2572} = 27.63 \text{ ksi}$$

$$\text{At } \rho = \rho_y \quad \tau' = \frac{(75)(0.73749)}{3.2572} = 16.98 \text{ ksi}$$

Residual: $\tau_{res} = \tau_{load} - \tau'$

$$\text{At } \rho = c \quad \tau_{res} = 22 - 27.63 = -5.63 \text{ ksi}$$

$$\text{At } \rho = \rho_y \quad \tau_{res} = 22 - 16.98 = 5.02 \text{ ksi}$$

maximum $\tau_{res} = 5.63 \text{ ksi}$ —

Problem 3.112

3.112 In Prob. 3.111, determine the permanent angle of twist of the rod.

3.111 The solid circular drill rod AB is made of a steel that is assumed to be elastoplastic with $\tau_y = 22$ ksi and $G = 11.2 \times 10^6$ psi. Knowing that a torque $T = 75$ kip in. is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

From the solution to PROBLEM 3.111

$$C = 1.2 \text{ in.} \quad J = 3.2572 \text{ in}^4 \quad \frac{\rho_r}{c} = 0.61458, \quad \rho_r = 0.73749 \text{ in.}$$

$$\text{After loading} \quad \gamma = \frac{\rho \phi}{L} \therefore \phi = \frac{L \gamma}{\rho} = \frac{L \gamma_r}{\rho_r G} = \frac{L \gamma_r}{\rho_r G} \quad L = 35 \text{ ft} = 420 \text{ in.}$$

$$\Phi_{\text{load}} = \frac{(420)(22 \times 10^3)}{(0.73749)(11.2 \times 10^6)} = 1.11865 \text{ rad} = 64.09^\circ$$

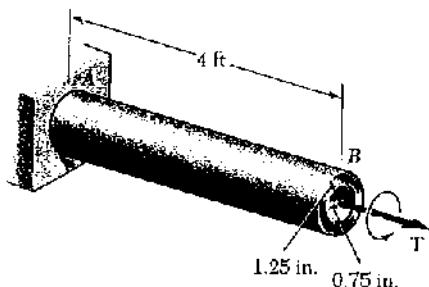
$$\text{During unloading} \quad \phi' = \frac{TL}{GJ} \quad (\text{elastic}) \quad T = 5 \times 10^3 \text{ N}\cdot\text{m}$$

$$\phi' = \frac{(75 \times 10^3)(420)}{(11.2 \times 10^6)(3.2572)} = 0.86347 \text{ rad} = 49.47^\circ$$

Permanent twist angle

$$\Phi_{\text{perm}} = \Phi_{\text{load}} - \phi = 1.11865 - 0.86347 = 0.25512 \quad \phi = 14.62^\circ$$

Problem 3.113



3.113 The hollow shaft AB is made of a mild steel that is assumed to be elastoplastic with $\tau_y = 18 \text{ ksi}$ and $G = 11.2 \times 10^6 \text{ psi}$. The magnitude T of the torque is slowly increased until the plastic zone first reaches the inner surface; the torque is then removed. Determine (a) the maximum residual shearing stress, (b) the permanent angle of twist.

$$C_1 = 0.75 \text{ in.} \quad C_2 = 1.25 \text{ in.} \quad L = 4 \text{ ft} = 48 \text{ in.}$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(1.25^4 - 0.75^4) = 3.3379 \text{ in}^4$$

$$\tau_y = 18 \text{ ksi} = 18 \times 10^3 \text{ psi}$$

Loading: When τ_y reaches the inner surface, then $\tau = \tau_y$ everywhere.

$$T_{\text{load}} = 2\pi \int_{C_1}^{C_2} \rho^3 \tau_y d\rho = 2\pi \tau_y \frac{\rho^3}{3} \Big|_{C_1}^{C_2} = \frac{2\pi}{3} \tau_y (C_2^3 - C_1^3)$$

$$= \frac{2\pi}{3} (18 \times 10^3)(1.25^3 - 0.75^3) = 57.727 \times 10^3 \text{ lb.in}$$

$$\gamma = \frac{\tau_y}{G} \text{ at } \rho = C_1 \quad \text{and} \quad \gamma = \frac{\rho \phi}{L} \quad \text{or} \quad \phi = \frac{L\gamma}{\rho} = \frac{L\tau_y}{C_1 G}$$

$$\phi_{\text{load}} = \frac{(48)(18 \times 10^3)}{(0.75)(11.2 \times 10^6)} = 0.102857 \text{ rad} = 5.8933^\circ$$

Unloading: $T = 57.727 \times 10^3 \text{ lb.in}$ Elastic $\tau' = \frac{T\rho}{J}$

$$\text{At } \rho = C_2 = 1.25 \text{ in.} \quad \tau' = \frac{(57.727 \times 10^3)(1.25)}{3.3379} = 21.62 \times 10^3 \text{ psi}$$

$$\text{At } \rho = C_1 = 0.75 \text{ in.} \quad \tau' = \frac{(57.727 \times 10^3)(0.75)}{3.3379} = 12.97 \times 10^3 \text{ psi}$$

$$\phi' = \frac{TL}{GJ} = \frac{(57.727 \times 10^3)(48)}{(11.2 \times 10^6)(3.3379)} = 0.074119 \text{ rad} = 4.2467^\circ$$

Residual: $\tau_{\text{res}} = \tau_{\text{load}} - \tau'$ $\phi_{\text{perm}} = \phi_{\text{load}} - \phi'$

$$(a) \text{ At } \rho = C_2 \quad \tau_{\text{res}} = 18 \times 10^3 - 21.62 \times 10^3 = -3.62 \times 10^3 \text{ psi}$$

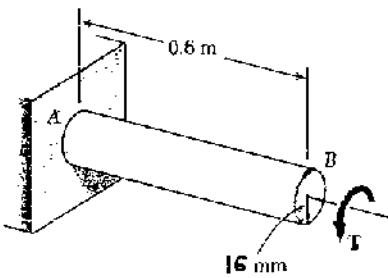
$$\tau_{\text{res}} = -3.62 \text{ ksi}$$

$$\text{At } \rho = C_1 \quad \tau_{\text{res}} = 18 \times 10^3 - 12.97 \times 10^3 = 5.03 \times 10^3 \text{ psi}$$

$$\text{maximum } \tau_{\text{res}} = 5.03 \text{ ksi}$$

$$(b) \quad \phi_{\text{perm}} = 0.102857 - 0.074119 = 0.028738 \text{ rad} \quad \phi_{\text{perm}} = 1.647^\circ$$

Problem 3.114



3.114 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The torque is increased in magnitude until the shaft has been twisted through 6° ; the torque is then removed. Determine (a) the magnitude and location of the maximum residual shearing stress, (b) the permanent angle of twist.

$$C = 0.016 \text{ m} \quad \varphi = 6^\circ = 104.72 \times 10^{-3} \text{ rad}$$

$$\gamma_{\max} = \frac{C\varphi}{L} = \frac{(0.016)(104.72 \times 10^{-3})}{0.6} = 0.0027925$$

$$\gamma_y = \frac{\tau_y}{G} = \frac{145 \times 10^6}{77.2 \times 10^9} = 0.0018782$$

$$\frac{p_y}{c} = \frac{\gamma_y}{\gamma_{\max}} = \frac{0.0018}{0.0027925} = 0.67260$$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.016)^4 = 102.944 \times 10^{-9} \text{ m}^4$$

$$T_y = \frac{J \tau_y}{C} = \frac{\pi}{2} C^3 \tau_y = \frac{\pi}{2} (0.016)^3 (145 \times 10^6) = 932.93 \text{ N}\cdot\text{m}$$

$$\text{At end of loading } T_{load} = \frac{4}{3} T_y \left(1 - \frac{1}{4} \frac{p_y^3}{C^3}\right) = \frac{4}{3} (932.93) \left[1 - \frac{1}{4} (0.67433)^3\right] \\ = 1.14855 \times 10^3 \text{ N}\cdot\text{m}$$

Unloading: elastic $T' = -1.14855 \times 10^3 \text{ N}\cdot\text{m}$

$$\text{At } p = c \quad \gamma' = \frac{T' C}{J} = \frac{(-1.14855 \times 10^3)(0.016)}{102.944 \times 10^{-9}} = -178.52 \times 10^6 \text{ Pa}$$

$$\text{At } p = p_y \quad \gamma' = \frac{T' C}{J} \frac{p_y}{c} = (-178.52 \times 10^6)(0.67433) = 120.38 \times 10^6 \text{ Pa}$$

$$\phi' = \frac{T' L}{G J} = \frac{(-1.14855 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = -86.71 \times 10^{-3} \text{ rad} = -4.97^\circ$$

Residual: $\gamma_{res} = \gamma_{load} - \gamma' \quad \phi_{perm} = \phi_{load} - \phi$

$$(a) \text{ At } p = c \quad \gamma_{res} = 145 \times 10^6 - 178.52 \times 10^6 = -33.52 \times 10^6 \text{ Pa} \\ = -33.5 \text{ MPa}$$

$$\text{At } p = p_y \quad \gamma_{res} = 145 \times 10^6 - 120.38 \times 10^6 = 24.62 \times 10^6 \text{ Pa} \\ = 24.6 \text{ MPa}$$

Maximum residual stress: 33.5 MPa at $p = 16 \text{ mm}$

$$(b) \quad \phi_{perm} = 104.72 \times 10^{-3} - 86.71 \times 10^{-3} = 17.78 \times 10^{-3} \text{ rad} \quad \phi_{perm} = 1.032^\circ$$

Problem 3.115

3.115 In Prob. 3.110, determine the permanent angle of twist of the shaft.

3.110 The solid circular shaft AB is made of a steel that is assumed to be elastoplastic with $\tau_y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The torque T is increased until the radius of the elastic core is 6 mm. Determine the maximum residual shearing stress in the shaft after the torque T is removed.

From the solution to PROBLEM 3.110 $C = 0.018 \text{ m}$, $J = 164.896 \times 10^{-9} \text{ m}^4$

After loading $T = 1.7547 \times 10^3 \text{ N}\cdot\text{m}$, $\rho_y = 0.006 \text{ m}$

$$\gamma = \frac{\rho \phi}{L} \quad \text{or} \quad \phi = \frac{LY}{P} = \frac{LY}{\rho_y G}$$

where $L = 600 \text{ mm} = 0.600 \text{ m}$, $\tau_y = 145 \times 10^6 \text{ Pa}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$\phi_{load} = \frac{(0.600)(145 \times 10^6)}{(0.006)(77.2 \times 10^9)} = 0.18782 \text{ rad} = 10.76^\circ$$

Unloading $T = 1.7547 \times 10^3 \text{ N}\cdot\text{m}$

$$\phi' = \frac{TL}{GJ} = \frac{(1.7547 \times 10^3)(0.600)}{(77.2 \times 10^9)(164.896 \times 10^{-9})} = 0.08270 \text{ rad} = 4.74^\circ$$

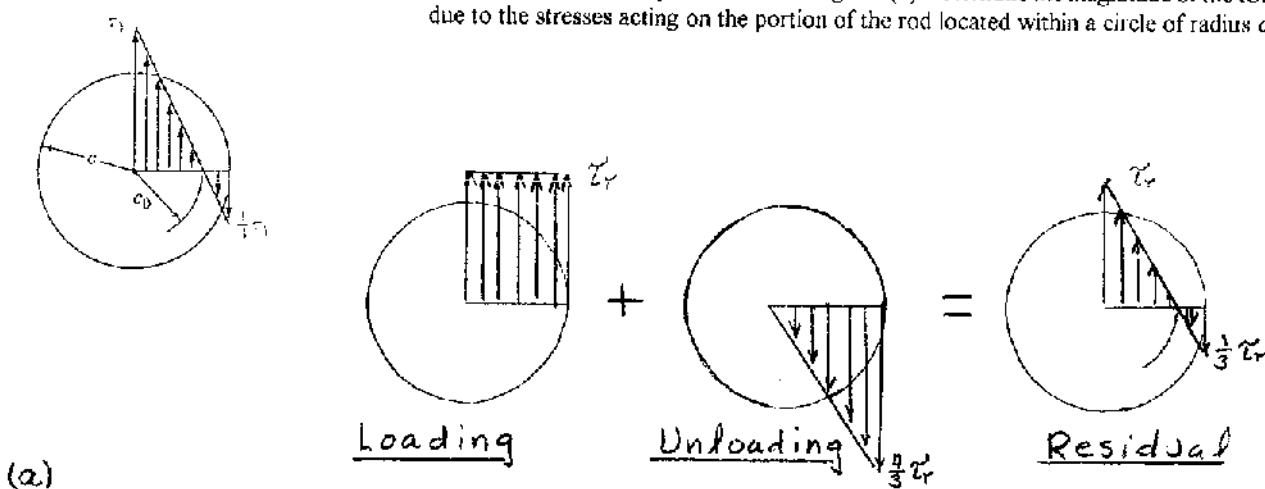
Permanent angle of twist $\phi_{perm} = \phi_{load} - \phi'$

$$\phi_{perm} = 0.18782 - 0.08270 = 0.10512 \text{ rad}$$

$$\phi_{perm} = 6.02^\circ$$

Problem 3.116

3.116 A torque T applied to a solid rod made of an elastoplastic material is increased until the rod is fully plastic and then removed. (a) Show that the distribution of residual shearing stresses is represented in the figure. (b) Determine the magnitude of the torque due to the stresses acting on the portion of the rod located within a circle of radius c_0 .



$$\text{After loading} \quad p_r = 0, \quad T_{\text{load}} = \frac{4}{3} T_r = \frac{4}{3} \frac{\pi}{2} C^3 \sigma_y = \frac{2\pi}{3} C^3 \sigma_y$$

$$\begin{aligned} \text{Unloading} \quad \sigma' &= \frac{T_r}{J} = \frac{2 T_r}{\pi C^3} = \frac{2 (T_{\text{load}})}{\pi C^3} = -\frac{4}{3} \sigma_y \text{ at } p = C \\ &\quad \sigma' = \frac{4}{3} \sigma_y \frac{p}{C} \end{aligned}$$

$$\text{Residual} \quad \sigma_{\text{res}} = \sigma_y - \frac{4}{3} \sigma_y \frac{p}{C} = \sigma_y \left(1 - \frac{4p}{3C}\right)$$

To find C_0 set $\sigma_{\text{res}} = 0$ and $p = C_0$

$$0 = 1 - \frac{4C_0}{3C} \quad \therefore C_0 = \frac{3}{4} C$$

$$\begin{aligned} (b) \quad T &= 2\pi \int_0^{C_0} p^2 \sigma' dp = 2\pi \int_0^{\frac{3}{4}C} p^2 \sigma_y \left(1 - \frac{4p}{3C}\right) dp \\ &= 2\pi \sigma_y \left(\frac{p^3}{3} - \frac{4}{3} \frac{p^4}{4C}\right) \Big|_0^{\frac{3}{4}C} = 2\pi \sigma_y C^3 \left\{ \frac{1}{3} \left(\frac{3}{4}\right)^3 - \left(\frac{4}{3}\right) \frac{1}{4} \left(\frac{3}{4}\right)^4 \right\} \\ &= 2\pi \sigma_y C^3 \left\{ \frac{9}{64} - \frac{27}{256} \right\} = \frac{9\pi}{128} \sigma_y C^3 = 0.2209 \sigma_y C^3 \end{aligned}$$

Problem 3.117

3.117 After the hollow shaft of Prob. 3.113 has been loaded and unloaded as described in that problem, a torque T_1 of sense opposite to the original torque T is applied to the shaft. Assuming no change in the value of τ_y , determine the magnitude T_1 of the torque T_1 required to initiate yield in this second loading and compare it with the magnitude T_y of the torque T that caused the shaft to yield in the original loading.

3.113 The hollow shaft AB is made of a mild steel that is assumed to be elastoplastic with $\tau_y = 18 \text{ ksi}$ and $G = 11.2 \times 10^6 \text{ psi}$. The magnitude T of the torque is slowly increased until the plastic zone first reaches the inner surface; the torque is then removed. Determine (a) the maximum residual shearing stress, (b) the permanent angle of twist.

From the solution of PROBLEM 3.113 $C_1 = 0.75 \text{ in.}$, $C_2 = 1.25 \text{ in.}$

$$\tau_y = 18 \text{ ksi} = 18 \times 10^3 \text{ psi} \quad J = 3.3379 \cdot \text{in}^4$$

$$\text{At } \rho = C_2 \quad \tau_{\text{res}} = -3.62 \text{ ksi} = -3.62 \times 10^3 \text{ psi}$$

Allowable reversed stress for reversed load

$$\tau_{\text{all}} = \tau_y + \tau_{\text{res}} = 18 \times 10^3 - 3.62 \times 10^3 = 14.38 \times 10^3 \text{ psi}$$

For elastic behavior $\tau = \frac{T C_2}{J}$

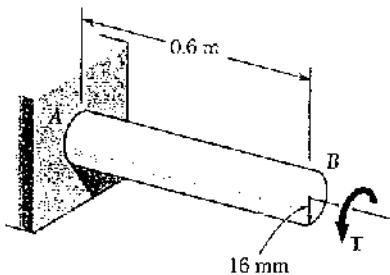
$$T_1 = \frac{J \tau_{\text{all}}}{C_2} = \frac{(3.3379)(14.38 \times 10^3)}{1.25} = 38.4 \times 10^3 \text{ lb-in.}$$

$$T_1 = 38.4 \text{ kip-in.} \quad \blacktriangleleft$$

$$T_y = \frac{J \tau_y}{C_2} = \frac{(3.3379)(18 \times 10^3)}{1.25} = 48.1 \times 10^3 \text{ lb-in.}$$

$$T_y = 48.1 \text{ kip-in.} \quad \blacktriangleleft$$

Problem 3.118



3.118 After the solid shaft of Prob. 3.114 has been loaded and unloaded as described in that problem, a torque T_1 of sense opposite to the original torque T is applied to the shaft. Assuming no change in the value of ϕ_y , determine the angle of twist ϕ_1 for which yield is initiated in this second loading and compare it with the angle ϕ_y for which the shaft started to yield in the original loading.

3.114 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The torque is increased in magnitude until the shaft has been twisted through 6° ; the torque is then removed. Determine (a) the magnitude and location of the maximum residual shearing stress, (b) the permanent angle of twist.

From the solution to PROBLEM 3.114 $C = 0.016 \text{ m}$, $L = 0.6 \text{ m}$

$$\tau_y = 145 \times 10^6 \text{ Pa}, \quad J = 102.944 \times 10^{-9} \text{ m}^4$$

The residual stress at $\rho = C$ is $\tau_{\text{res}} = 33.5 \text{ MPa}$

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_y - \tau_{\text{res}} = 145 \times 10^6 - 33.5 \times 10^6 = 111.5 \times 10^6 \text{ Pa}$$

$$\tau_1 = \frac{T_1 C}{J} \quad \therefore \quad T_1 = \frac{J \tau_1}{C} = \frac{(102.944 \times 10^{-9})(111.5 \times 10^6)}{0.016}$$

$$= 717 \text{ N}\cdot\text{m}$$

Angle of twist at yielding under reversed torque.

$$\phi_1' = \frac{T_1 L}{G J} = \frac{(717 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 54.16 \times 10^{-3} \text{ rad}$$

$$\phi_1' = 3.10^\circ \leftarrow$$

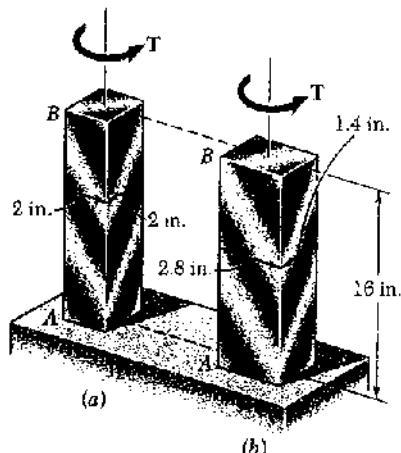
Angle of twist for yielding in original loading.

$$\gamma = \frac{\tau_y}{G} = \frac{C \phi_y}{L}$$

$$\phi_y = \frac{L \tau_y}{C G} = \frac{(0.6)(145 \times 10^6)}{(0.016)(77.2 \times 10^9)} = 70.434 \times 10^{-3} \text{ rad} \quad \phi_y = 4.04^\circ \leftarrow$$

Problem 3.119

3.119 Knowing that $T = 7 \text{ kip} \cdot \text{in}$. and that $G = 5.6 \times 10^6 \text{ psi}$, determine for each of the cold-rolled yellow brass bars shown the maximum shearing stress and the angle of twist of end B.



$$T = 7 \times 10^3 \text{ lb-in} \quad L = 16 \text{ in.}$$

$$(a) \quad a = 2 \text{ in.} \quad b = 2 \text{ in.} \quad \frac{a}{b} = 1.0$$

From Table 3.1, $C_1 = 0.208$, $C_2 = 0.1406$

$$\tau_{\max} = \frac{T}{C_1 a b^3} = \frac{7 \times 10^3}{(0.208)(2)(2)^3} = 4.21 \times 10^3 \text{ psi}$$

$$\tau_{\max} = 4.21 \text{ ksi} \quad \blacksquare$$

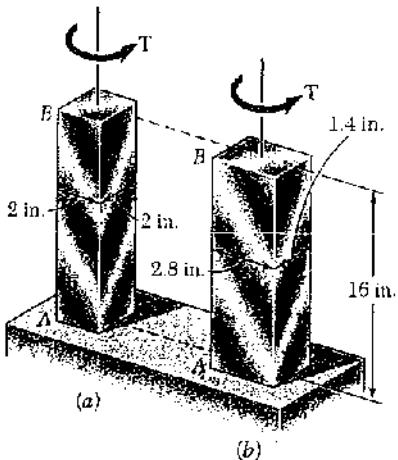
$$\varphi = \frac{TL}{C_2 a b^3 G} = \frac{(7 \times 10^3)(16)}{(0.1406)(2)(2)^3(5.6 \times 10^6)} \\ = 8.89 \times 10^{-3} \text{ rad} \quad \varphi = 0.509^\circ \quad \blacksquare$$

$$(b) \quad a = 2.8 \text{ in.} \quad b = 1.4 \text{ in.} \quad \frac{a}{b} = 2.0 \quad \text{From Table 3.1 } C_1 = 0.246, C_2 = 0.229$$

$$\tau_{\max} = \frac{T}{C_1 a b^3} = \frac{7 \times 10^3}{(0.246)(2.8)(1.4)^3} = 5.19 \times 10^3 \text{ psi} \quad \tau_{\max} = 5.19 \text{ ksi} \quad \blacksquare$$

$$\varphi = \frac{TL}{C_2 a b^3 G} = \frac{(7 \times 10^3)(16)}{(0.229)(2.8)(1.4)^3(5.6 \times 10^6)} = 11.37 \times 10^{-3} \text{ rad} \quad \varphi = 0.651^\circ \quad \blacksquare$$

Problem 3.120



3.120 Using $\tau_{all} = 7.5 \text{ ksi}$ and knowing that $G = 5.6 \times 10^6 \text{ psi}$, determine for each of the cold-rolled yellow brass bars shown the largest torque T that can be applied and the corresponding angle of twist at end B.

$$\tau_{all} = 7.5 \times 10^3 \text{ psi} \quad L = 16 \text{ in.}$$

$$(a) \quad a = 2 \text{ in.} \quad b = 2 \text{ in.} \quad \frac{a}{b} = 1.0$$

From Table 3.1 $C_1 = 0.208, C_2 = 0.1406$

$$\tau_{max} = \frac{T}{C_1 ab^3} \quad T = C_1 ab^2 \tau_{max}$$

$$T = (0.208)(2)(2)^2(7.5 \times 10^3) = 12.48 \times 10^3 \text{ lb-in}$$

$$T = 12.48 \text{ kip-in}$$

$$\phi = \frac{TL}{C_2 ab^3 G} = \frac{(12.48 \times 10^3)(16)}{(0.1406)(2)(2)^3(5.6 \times 10^6)} \\ = 15.85 \times 10^{-3} \text{ rad}$$

$$\phi = 0.908^\circ$$

$$(b) \quad a = 2.8 \text{ in.} \quad b = 1.4 \text{ in.} \quad \frac{a}{b} = 2.0 \quad \text{From Table 3.1} \quad C_1 = 0.246, C_2 = 0.229$$

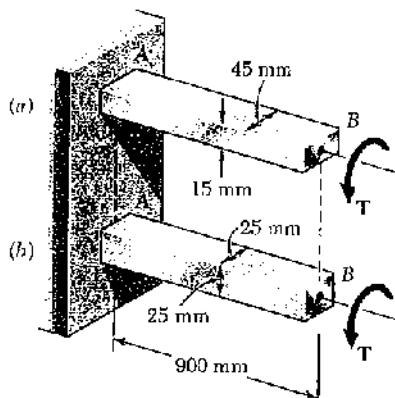
$$\tau_{max} = \frac{T}{C_1 ab^3} \quad T = C_1 ab^2 \tau_{max} = (0.246)(2.8)(1.4)^2(7.5 \times 10^3) = 10.13 \times 10^3 \text{ lb-in}$$

$$T = 10.31 \text{ kip-in}$$

$$\phi = \frac{TL}{C_2 ab^3 G} = \frac{(10.13 \times 10^3)(16)}{(0.229)(2.8)(1.4)^3(5.6 \times 10^6)} = 16.44 \times 10^{-3} \text{ rad} \quad \phi = 0.942^\circ$$

Problem 3.121

3.121 Knowing that the magnitude of the torque T is 200 N·m and that $G = 27$ GPa, determine for each of the aluminum bars shown that maximum shearing stress and the angle of twist at end B .



$$T = 200 \text{ N}\cdot\text{m} \quad L = 0.900 \text{ m} \quad G = 27 \times 10^9 \text{ Pa}$$

$$(a) \quad a = 45 \text{ mm} \quad b = 15 \text{ mm} \quad \frac{a}{b} = 3.0$$

$$\text{From Table 3.1} \quad c_1 = 0.267 \quad c_2 = 0.263$$

$$\tau_{\max} = \frac{T}{c_1 ab^2} = \frac{200}{(0.267)(0.045)(0.015)^2} = 74.0 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 74.0 \text{ MPa}$$

$$\phi = \frac{TL}{c_2 ab^3 G} = \frac{(200)(0.900)}{(0.263)(0.045)(0.015)^3 (27 \times 10^9)} = 166.9 \times 10^{-3} \text{ rad}$$

$$\phi = 9.56^\circ$$

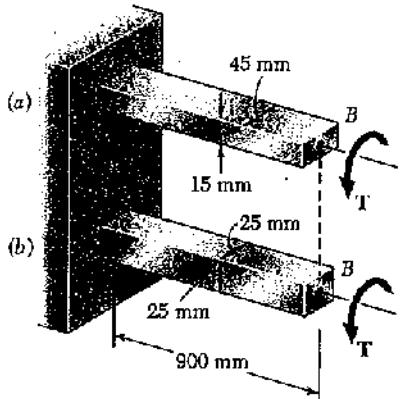
$$(b) \quad a = 25 \text{ mm} \quad b = 25 \text{ mm} \quad \frac{a}{b} = 1.0 \quad \text{From Table 3.1} \quad c_1 = 0.208, c_2 = 0.1406$$

$$\tau_{\max} = \frac{T}{c_1 ab^2} = \frac{200}{(0.208)(0.025)(0.025)^2} = 61.5 \times 10^6 \text{ Pa} \quad \tau_{\max} = 61.5 \text{ MPa}$$

$$\phi = \frac{TL}{c_2 ab^3 G} = \frac{(200)(0.900)}{(0.1406)(0.025)(0.025)^3 (27 \times 10^9)} = 121.4 \times 10^{-3} \text{ rad} \quad \phi = 6.95^\circ$$

Problem 3.122

3.122 Using $\tau_{\max} = 70 \text{ MPa}$ and $G = 27 \text{ GPa}$, determine for each of the aluminum bars shown the largest torque T that can be applied and the corresponding angle of twist at end B .



$$\tau_{\max} = 70 \times 10^6 \text{ Pa} \quad G = 27 \times 10^9 \text{ Pa} \quad L = 0.900 \text{ m}$$

$$(a) \quad a = 45 \text{ mm} \quad b = 15 \text{ mm} \quad \frac{a}{b} = 3.0$$

$$\text{From Table 3.1} \quad c_1 = 0.267, c_2 = 0.263$$

$$\tau_{\max} = \frac{T}{c_1 ab^2} \quad T = c_1 ab^2 \tau_{\max}$$

$$T = (0.267)(0.045)(0.015)^2 (70 \times 10^6) = 189.2 \text{ N}\cdot\text{m}$$

$$T = 189.2 \text{ N}\cdot\text{m}$$

$$\phi = \frac{TL}{c_2 ab^3 G} = \frac{(189.2)(0.900)}{(0.263)(0.045)(0.015)^3 (27 \times 10^9)} = 157.9 \times 10^{-3} \text{ rad} \quad \phi = 9.05^\circ$$

$$(b) \quad a = 25 \text{ mm} \quad b = 25 \text{ mm} \quad \frac{a}{b} = 1.0 \quad \text{From Table 3.1} \quad c_1 = 0.208, c_2 = 0.1406$$

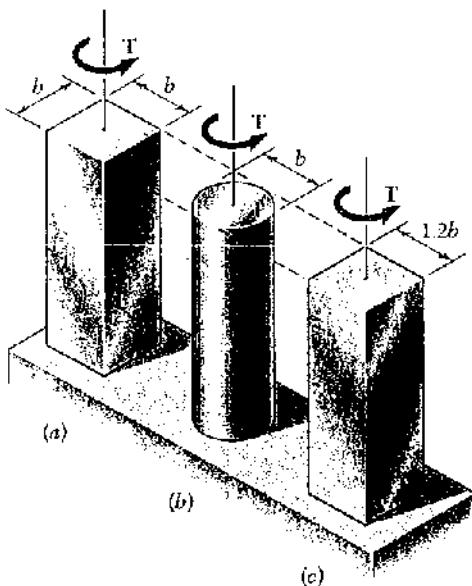
$$\tau_{\max} = \frac{T}{ab^2} \quad T = c_1 ab^2 \tau_{\max} = (0.208)(0.025)(0.025)^2 (70 \times 10^6)$$

$$= 227.5 \text{ N}\cdot\text{m}$$

$$T = 227.5 \text{ N}\cdot\text{m}$$

$$\phi = \frac{TL}{c_2 ab^3 G} = \frac{(227.5)(0.900)}{(0.1406)(0.025)(0.025)^3 (27 \times 10^9)} = 133.1 \times 10^{-3} \text{ rad} \quad \phi = 7.91^\circ$$

Problem 3.123



3.123 Each of the three steel bars shown is subjected to a torque of magnitude $T = 275 \text{ N} \cdot \text{m}$. Knowing that the allowable shearing stress is 50 MPa , determine the required dimension b for each bar.

$$T = 275 \text{ N} \cdot \text{m} \quad \tau_{\max} = 50 \times 10^6 \text{ Pa}$$

$$(a) \text{ Square: } a = b \quad \frac{a}{b} = 1.0$$

From Table 3.1 $C_1 = 0.208$

$$\tau_{\max} = \frac{T}{C_1 ab^2} = \frac{T}{C_1 b^3}$$

$$b = \sqrt[3]{\frac{T}{C_1 \tau_{\max}}} = \sqrt[3]{\frac{275}{(0.208)(50 \times 10^6)}} \\ = 29.8 \times 10^{-3} \text{ m}$$

$$b = 29.8 \text{ mm} \quad \blacktriangleleft$$

$$(b) \text{ Circle } C = \frac{1}{2} b \quad \tau_{\max} = \frac{TC}{J} = \frac{2T}{\pi C^3}$$

$$C = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(275)}{\pi (50 \times 10^6)}} = 15.18 \times 10^{-3} \text{ m}$$

$$C = 15.18 \text{ m}$$

$$b = 2C = 30.4 \text{ mm} \quad \blacktriangleleft$$

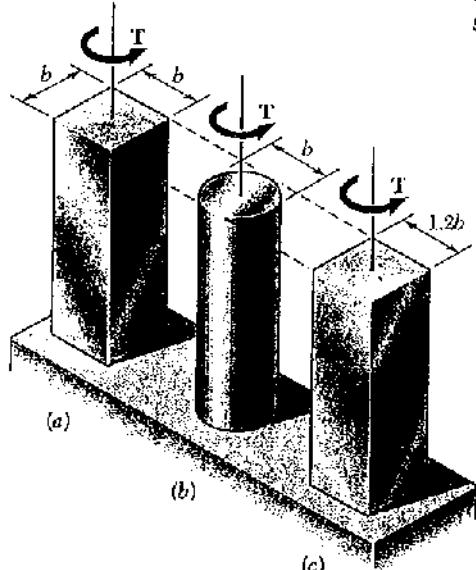
$$(c) \text{ Rectangle: } a = 1.2b \quad \text{From Table 3.1 } C_1 = 0.219 \quad \tau_{\max} = \frac{T}{C_1 ab^2} = \frac{T}{1.2C_1 b^3}$$

$$b = \sqrt[3]{\frac{T}{1.2C_1 \tau_{\max}}} = \sqrt[3]{\frac{275}{(1.2)(0.219)(50 \times 10^6)}} = 27.6 \times 10^{-3} \text{ m}$$

$$b = 27.6 \text{ mm} \quad \blacktriangleleft$$

Problem 3.124

3.124 Each of the three aluminum bars shown is to be twisted through an angle of 2° . Knowing that $b = 30 \text{ mm}$, $\tau_{\text{all}} = 50 \text{ MPa}$, and $G = 27 \text{ GPa}$, determine the shortest allowable length of each bar.



$$\Phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad} \quad Z' = 50 \times 10^6 \text{ Pa} \\ G = 27 \times 10^9 \text{ Pa} \quad b = 30 \text{ mm} = 0.030 \text{ m}$$

For square and rectangle

$$Z' = \frac{T}{C_1 ab^2} \quad \Phi = \frac{TL}{C_2 ab^3 G}$$

Divide to eliminate T ; then solve for L

$$\frac{\Phi}{Z'} = \frac{C_1 ab^2 L}{C_2 ab^3 G} \quad L = \frac{C_2 b G \Phi}{C_1 Z'}$$

$$(a) \text{ Square: } \frac{a}{b} = 1.0$$

$$\text{From Table 3.1} \quad C_1 = 0.208, C_2 = 0.1406$$

$$L = \frac{(0.1406)(0.030)(27 \times 10^9)(34.907 \times 10^{-3})}{(0.208)(50 \times 10^6)} = 382 \times 10^{-3} \text{ m} \quad L = 382 \text{ mm}$$

$$(b) \text{ Circle } a = \frac{1}{2}b = 0.015 \text{ m} \quad Z' = \frac{TC}{J} \quad \Phi = \frac{TL}{GJ}$$

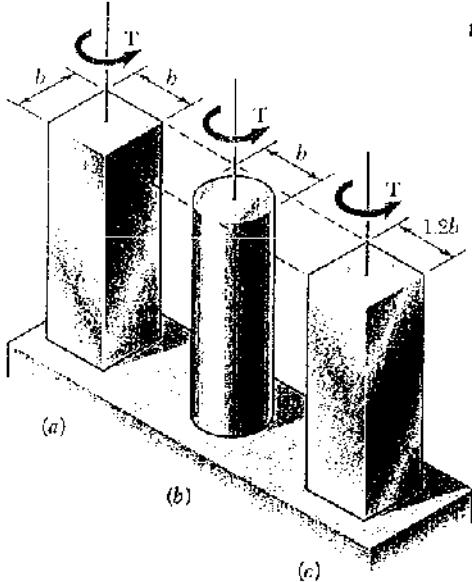
$$\text{Divide to eliminate } T; \text{ then solve for } L \quad \frac{\Phi}{Z'} = \frac{JL}{CGJ} = \frac{L}{CG}$$

$$L = \frac{CG\Phi}{Z'} = \frac{(0.015)(27 \times 10^9)(34.907 \times 10^{-3})}{50 \times 10^6} = 283 \times 10^{-3} \text{ m} \quad L = 283 \text{ mm}$$

$$(c) \text{ Rectangle: } a = 1.2b \quad \frac{a}{b} = 1.2 \quad \text{From Table 3.1} \quad C_1 = 0.219, C_2 = 0.1661$$

$$L = \frac{(0.1661)(0.030)(27 \times 10^9)(34.907 \times 10^{-3})}{(0.219)(50 \times 10^6)} = 429 \times 10^{-3} \text{ m} \quad L = 429 \text{ mm}$$

Problem 3.125



3.125 Each of the three steel bars shown is subjected to a torque of magnitude $T = 5 \text{ kip} \cdot \text{in}$. Knowing that the allowable shearing stress is 8 ksi, determine the required dimension b for each bar.

$$T = 5 \times 10^3 \text{ lb} \cdot \text{in} \quad \tau' = 8 \times 10^3 \text{ psi}$$

$$(a) \text{ Square: } a = b \quad \frac{a}{b} = 1.0$$

$$\text{From Table 3.1} \quad C_1 = 0.208$$

$$\tau' = \frac{T}{C_1 ab^2} = \frac{T}{C_1 b^3}$$

$$b = \sqrt[3]{\frac{T}{C_1 \tau'}} = \sqrt[3]{\frac{5 \times 10^3}{(0.208)(8 \times 10^3)}} \quad b = 1.443 \text{ in.}$$

$$(b) \text{ Circle: } \tau' = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

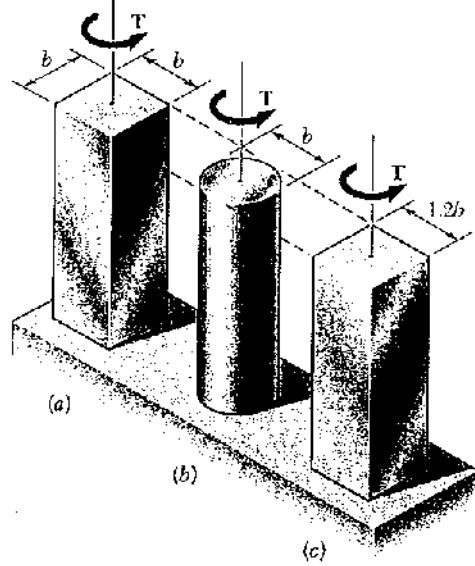
$$c = \sqrt[3]{\frac{2T}{\pi \tau'}} = \sqrt[3]{\frac{(2)(5 \times 10^3)}{\pi (8 \times 10^3)}} = 0.7355 \text{ in.}$$

$$b = 2c = 1.471 \text{ in.}$$

$$(c) \text{ Rectangle: } a = 1.2b \quad \frac{a}{b} = 1.2 \quad \text{From Table 3.1} \quad C_1 = 0.219$$

$$\tau' = \frac{T}{C_1 ab^2} = \frac{T}{1.2 C_1 b^3} \quad b = \sqrt[3]{\frac{T}{1.2 C_1 \tau'}} = \sqrt[3]{\frac{5 \times 10^3}{(1.2)(0.219)(8 \times 10^3)}} \quad b = 1.335 \text{ in.}$$

Problem 3.126



3.126 Each of the three aluminum bars shown is to be twisted through an angle of 1.25° . Knowing that $b = 1.5$ in., $\tau_u = 7.5$ ksi, and $G = 3.9 \times 10^6$ psi, determine the shortest allowable length of each bar.

$$\phi = 1.25^\circ = 21.817 \times 10^{-3} \text{ rad} \quad \tau = 7.5 \times 10^3 \text{ psi}$$

For square and rectangle

$$\tau = \frac{T}{c_1 ab^3} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

Divide to eliminate T ; then solve for L

$$\frac{\phi}{\tau} = \frac{c_1 ab^2 L}{c_2 ab^3 G} \quad L = \frac{c_2 b G \phi}{c_1 \tau}$$

(a) Square $\frac{a}{b} = 1.0$

From Table 3.1 $c_1 = 0.208$, $c_2 = 0.1406$

$$L = \frac{(0.1406)(1.5)(3.9 \times 10^6)(21.817 \times 10^{-3})}{(0.208)(7.5 \times 10^3)} \quad L = 11.50 \text{ in.}$$

(b) Circle: $c = \frac{1}{4}b = 0.75$ in. $\tau = \frac{Tc}{J} \quad \phi = \frac{TL}{GJ}$

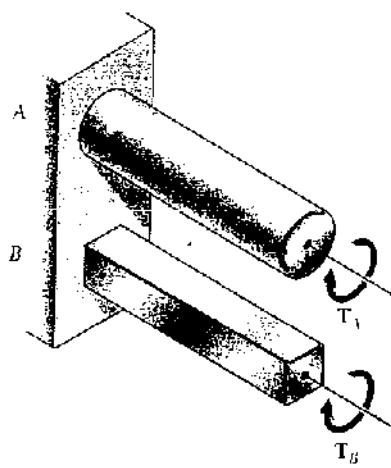
Divide to eliminate T ; then solve for L $\frac{\phi}{\tau} = \frac{JL}{cGJ} = \frac{L}{cG}$

$$L = \frac{cG\phi}{\tau} = \frac{(0.75)(3.9 \times 10^6)(21.817 \times 10^{-3})}{7.5 \times 10^3} \quad L = 8.51 \text{ in.}$$

(c) Rectangle: $a = 1.2b$ $\frac{a}{b} = 1.2$ From Table 3.1 $c_1 = 0.219$, $c_2 = 0.1661$

$$L = \frac{(0.1661)(1.5)(3.9 \times 10^6)(21.817 \times 10^{-3})}{(0.219)(7.5 \times 10^3)} \quad L = 12.91 \text{ in.}$$

Problem 3.127



3.127 Shafts *A* and *B* are made of the same material and have the same cross-sectional area, but *A* has a circular cross section and *B* has a square cross section. Determine the ratio of the maximum torques T_A and T_B that can be safely applied to *A* and *B*, respectively.

SOLUTION

Let c = radius of circular section *A* and b = side of square section *B*.

For equal areas $\pi c^2 = b^2$

$$c = \frac{b}{\sqrt{\pi}}$$

$$\text{Circle: } \chi_A = \frac{T_A c}{J} = \frac{2T_A}{\pi c^3} \therefore T_A = \frac{\pi}{2} c^3 \chi_A$$

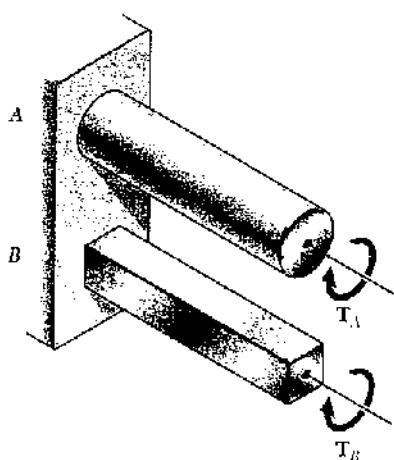
$$\text{Square: } C_1 = 0.208 \text{ from Table 3.1}$$

$$\chi_B = \frac{T_B}{C_1 ab^2} = \frac{T_B}{C_1 b^3} \therefore T_B = C_1 b^3 \chi_B$$

$$\text{Ratio: } \frac{T_A}{T_B} = \frac{\frac{\pi}{2} c^3 \chi_B}{C_1 b^3 \chi_B} = \frac{\frac{\pi}{2} \cdot \frac{b^3}{\sqrt{\pi}^3} \chi_B}{C_1 b^3 \chi_B} = \frac{1}{2C_1 \sqrt{\pi}} \frac{\chi_B}{\chi_B}$$

$$\text{For the same stresses } \chi_B = \chi_A \therefore \frac{T_A}{T_B} = \frac{1}{(2)(0.208)\sqrt{\pi}} = 1.356 \quad \blacksquare$$

Problem 3.128



3.128 Shafts *A* and *B* are made of the same material and have the same length and cross-sectional area, but *A* has a circular cross section and *B* has a square cross section. Determine the ratio of the maximum values of the angles ϕ_A and ϕ_B through which shafts *A* and *B*, respectively, can be twisted.

SOLUTION

Let c = radius of circular section *A* and b = side of square section *B*.

For equal areas $\pi c^2 = b^2 \therefore b = \sqrt{\pi} c$

$$\text{Circle: } \gamma_{max} = \frac{\chi_A}{G} = \frac{C \phi_A}{L} \therefore \phi_A = \frac{L \chi_A}{C G}$$

$$\text{Square: Table 3.1 } C_1 = 0.208, C_2 = 0.1406$$

$$\chi_B = \frac{T_B}{C_1 ab^2} = \frac{T_B}{0.208 b^3} \therefore T_B = 0.208 b^3 \chi_B$$

$$\phi_B = \frac{T_B L}{C_2 ab^3 G} = \frac{0.208 b^3 \chi_B L}{0.1406 b^4 G} = \frac{1.4794 L \chi_B}{b G}$$

$$\text{Ratio } \frac{\phi_A}{\phi_B} = \frac{L \chi_A}{C G} \cdot \frac{b G}{1.4794 L \chi_B} = 0.676 \frac{b \chi_A}{C \chi_B} = 0.676 \sqrt{\pi} \frac{\chi_A}{\chi_B}$$

$$\text{For equal stresses } \chi_A = \chi_B$$

$$\frac{\phi_B}{\phi_A} = 0.676 \sqrt{\pi} = 1.198 \quad \blacksquare$$

Problem 3.129

3.129 Determine the largest allowable square cross section of a steel shaft of length 6 m if the maximum shearing stress is not to exceed 120 MPa when the shaft is twisted through one complete revolution. Use $G = 77.2 \text{ GPa}$.

$$L = 6 \text{ m}, \quad \tau = 120 \times 10^6 \text{ Pa}, \quad G = 77.2 \times 10^9 \text{ Pa} \quad \phi = 1 \text{ rev} = 2\pi \text{ rad}$$

$$\tau = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

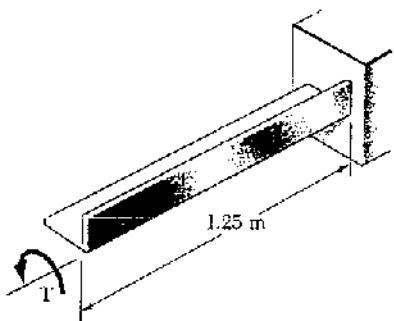
$$\text{Divide to eliminate } T; \text{ then solve for } b. \quad \frac{\phi}{\tau} = \frac{c_1 ab^2 L}{c_2 ab^3 G} = \frac{c_1 L}{c_2 b G}$$

$$b = \frac{c_1 L \tau}{c_2 G \phi} \quad \text{For a square cross section } \frac{a}{b} = 1.0$$

$$\text{From Table 3.1} \quad c_1 = 0.208, \quad c_2 = 0.1406$$

$$b = \frac{(0.208)(G)(120 \times 10^6)}{(0.1406)(77.2 \times 10^9)(2\pi)} = 2.20 \times 10^{-3} \text{ m} \quad b = 2.20 \text{ mm} \quad \blacksquare$$

Problem 3.130



3.130 A 1.25-m-long steel angle has an L127 × 76 × 6.4 cross section. From Appendix C we find that the thickness of the section is 6.4 mm and that its area is 1252 mm². Knowing that $\tau_u = 60 \text{ MPa}$ and that $G = 77.2 \text{ GPa}$, and ignoring the effect of stress concentration, determine (a) the largest torque T that can be applied, (b) the corresponding angle of twist.

$$A = 1252 \text{ mm}^2 \quad b = 6.4 \text{ mm} = 0.0064 \text{ m}$$

$$a = \frac{A}{b} = \frac{1252}{6.4} = 195.6 \text{ mm} = 0.1956 \text{ m}$$

$$\frac{a}{b} = \frac{195.6}{6.4} = 30.56$$

$$C_1 = C_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{a} \right) = 0.3265$$

$$\tau_{\max} = \frac{T}{C_1 ab^2} \therefore T = C_1 ab^2 \tau_{\max}$$

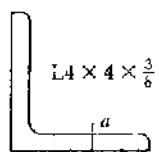
$$(a) T = (0.3265)(0.1956)(0.0064)^2(60 \times 10^6) = 156.95 \times 10^3 \text{ N}\cdot\text{m}$$

$$T = 157.0 \text{ kN}\cdot\text{m} \quad \blacksquare$$

$$(b) \varphi = \frac{TL}{C_1 ab^3 G} = \frac{C_1 ab^2 \tau_{\max} L}{C_1 ab^3 G} = \frac{C_1 \tau_{\max} L}{C_1 b G} = \frac{\tau_{\max} L}{b G}$$

$$\varphi = \frac{(60 \times 10^6)(1.25)}{(0.0064)(77.2 \times 10^9)} = 151.80 \times 10^{-3} \text{ rad} \quad \varphi = 8.70^\circ \quad \blacksquare$$

Problem 3.131



3.131 A 3000 lb·in. torque is applied to a 6-ft-long steel angle with an

$L \times 4 \times 4 \times \frac{3}{8}$ cross section. From Appendix C we find that the thickness of the section is $\frac{3}{8}$ in. and that its area is 2.86 in². Knowing that $G = 11.2 \times 10^6$ psi, determine (a) the maximum shearing stress along line a-a, (b) the angle of twist.

$$A = 2.86 \text{ in}^2, \quad b = \frac{3}{8} \text{ in} = 0.375 \text{ in}, \quad a = \frac{A}{b} = \frac{2.86}{0.375} = 7.627 \text{ in}$$

$$\frac{a}{b} = \frac{7.627}{0.375} = 20.34 \quad C_1 = C_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{a} \right) = 0.3230$$

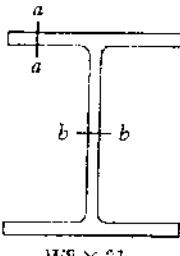
$$(a) \tau_{\max} = \frac{T}{C_1 ab^2} = \frac{3000}{(0.3230)(7.627)(0.375)^2} = 8.66 \times 10^3 \text{ psi} = 8.66 \text{ ksi} \quad \blacksquare$$

$$(b) \varphi = \frac{TL}{C_1 ab^3 G} = \frac{(3000)(72)}{(0.3230)(7.627)(0.375)^3(11.2 \times 10^6)} = 148.45 \times 10^{-3} \text{ rad}$$

$$\varphi = 8.51^\circ \quad \blacksquare$$

Note: $L = 6 \text{ ft.} = 72 \text{ in}$

Problem 3.132



3.132 An 8-ft-long steel member with a W8 × 31 cross section is subjected to a 5-kip · in. torque. The properties of the rolled-steel section are given in Appendix C. Knowing that $G = 11.2 \times 10^6$ psi, determine (a) the maximum shearing stress along line a-a, (b) the maximum shearing stress along line b-b, (c) the angle of twist. (Hint: consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

SOLUTION

$$\text{Flange: } a = 7.995 \text{ in}, \quad b = 0.435, \quad \frac{\alpha}{b} = \frac{7.995}{0.435} = 18.38$$

$$C_1 = C_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{\alpha} \right) = 0.3219 \quad \Phi_F = \frac{T_F L}{C_1 \alpha b^3 G}$$

$$T_F = C_2 \alpha b^3 \frac{G \Phi_F}{L} = K_F \frac{G \Phi}{L} \quad \text{where } K_F = C_2 \alpha b^3$$

$$K_F = (0.3219)(7.995)(0.435)^3 = 0.2138 \text{ in}^3$$

$$\text{Web: } a = 8.0 - (2)(0.435) = 7.13 \text{ in}, \quad b = 0.285 \text{ in}, \quad \frac{\alpha}{b} = \frac{7.13}{0.285} = 25.02$$

$$C_1 = C_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{\alpha} \right) = 0.3249 \quad \Phi_W = \frac{T_W L}{C_2 \alpha b^3 G}$$

$$T_W = C_2 \alpha b^3 \frac{G \Phi_W}{L} = K_W \frac{G \Phi}{L} \quad \text{where } K_W = C_2 \alpha b^3$$

$$K_W = (0.3249)(7.13)(0.285)^3 = 0.0563 \text{ in}^4$$

$$\text{For matching twist angles} \quad \Phi_F = \Phi_W = \Phi$$

$$\text{Total torque} \quad T = 2T_F + T_W = (2K_F + K_W) \frac{G \Phi}{L}$$

$$\frac{G \Phi}{L} = \frac{T}{2K_F + K_W} \quad \therefore T_F = \frac{K_F T}{2K_F + K_W}, \quad T_W = \frac{K_W T}{2K_F + K_W}$$

$$T_F = \frac{(0.2138)(5000)}{(2)(0.2138) + 0.0563} = 2221 \text{ lb-in}; \quad T_W = \frac{(0.0563)(5000)}{(2)(0.2138) + 0.0563} = 557 \text{ lb-in}$$

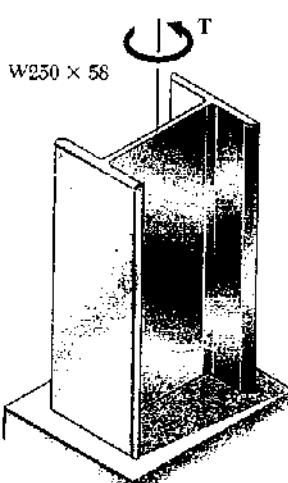
$$(a) \quad \tau_F' = \frac{T_F}{C_1 \alpha b^2} = \frac{2221}{(0.3219)(7.995)(0.435)^2} = 4570 \text{ psi} = 4.57 \text{ ksi}$$

$$(b) \quad \tau_W' = \frac{T_W}{C_2 \alpha b^2} = \frac{557}{(0.3249)(7.13)(0.285)^2} = 2960 \text{ psi} = 2.96 \text{ ksi}$$

$$(c) \quad \frac{G \Phi}{L} = \frac{T}{2K_F + K_W} \quad \therefore \Phi = \frac{TL^3}{G(2K_F + K_W)} \quad \text{where } L = 8 \text{ ft} = 96 \text{ in.}$$

$$\Phi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.2138) + 0.0563]} = 88.6 \times 10^{-3} \text{ rad} = 5.08^\circ$$

Problem 3.133



3.133 A 3-m-long steel member has a W250 × 58 cross section. Knowing that $G = 77.2 \text{ GPa}$ and that the allowable shearing stress is 35 MPa , determine (a) the largest torque T that can be applied, (b) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (See hint of Prob. 3.132.)

SOLUTION

$$\underline{\text{Flange}}: a = 203 \text{ mm}, b = 13.5 \text{ mm}, \frac{a}{b} = 15.04$$

$$C_1 = C_2 = \frac{1}{3}(1 - 0.630 \frac{b}{a}) = 0.3194$$

$$\Phi_F = \frac{T_F L}{C_2 a b^3 G} \quad \therefore T_F = C_2 a b^3 \frac{G \Phi}{L} = K_F \frac{G \Phi}{L}$$

$$K_F = (0.3194)(0.203)(0.0135)^3 = 159.53 \times 10^{-9} \text{ m}^4$$

$$\underline{\text{Web}}: a = 252 - (2)(13.5) = 225 \text{ mm}, b = 8 \text{ mm}$$

$$\frac{a}{b} = 28.13, \quad C_1 = C_2 = \frac{1}{3}(1 - 0.63 \frac{b}{a}) = 0.3259$$

$$\Phi_W = \frac{T_W L}{C_2 a b^3 G} \quad \therefore T_W = C_2 a b^3 \frac{G \Phi}{L} = K_W \frac{G \Phi}{L}$$

$$K_W = (0.3259)(0.225)(0.008)^3 = 37.54 \times 10^{-9} \text{ m}^4$$

$$\text{For matching twist angles } \Phi_F = \Phi_W = \Phi$$

$$\text{Total torque: } T = 2T_F + T_W = (2K_F + K_W) \frac{G \Phi}{L}$$

$$\frac{G \Phi}{L} = \frac{T}{2K_F + K_W} \quad , \quad T_F = \frac{K_F T}{2K_F + K_W} \quad \therefore T = \frac{2K_F + K_W}{K_F} T_F$$

$$T_W = \frac{K_W T}{2K_F + K_W} \quad \therefore T = \frac{2K_F + K_W}{K_W} T_W$$

Allowable value for T based on allowable value for T_F

$$T_F = C_1 a b^2 \bar{c} = (0.3194)(0.203)(0.0135)^2 (35 \times 10^6) = 413.6 \text{ N}\cdot\text{m}$$

$$T = \frac{(2)(159.53) + (37.54)}{159.53} (413.6) = 924.5 \text{ N}\cdot\text{m}$$

Allowable value for T based on allowable value for T_W

$$T_W = C_1 a b^2 \bar{c} = (0.3259)(0.225)(0.008)^2 (35 \times 10^6) = 164.25 \text{ N}\cdot\text{m}$$

$$T = \frac{(2)(159.53) + 37.54}{37.54} (164.25) = 1560 \text{ N}\cdot\text{m}$$

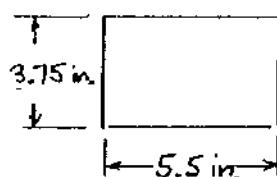
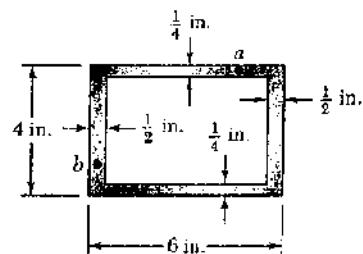
Choose smaller value

$$T = 925 \text{ N}\cdot\text{m}$$

$$\Phi = \frac{TL}{(2K_F + K_W)G} = \frac{(925)(3.00)}{(356.6 \times 10^9)(77.2 \times 10^9)} = 100.7 \times 10^{-3} \text{ rad} \quad \Phi = 5.77^\circ$$

Problem 3.134

3.134 A 5 kip · ft torque is applied to a hollow aluminum shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.



$$T = (5)(10^3)(12) = 60 \times 10^3 \text{ lb} \cdot \text{in}$$

Area bounded by center line

$$A = bh = (5.5)(3.75) = 20.625 \text{ in}^2$$

At point *a* $t = 0.25 \text{ in.}$

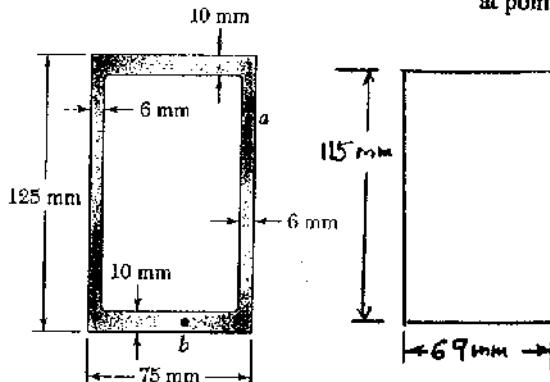
$$\tau = \frac{T}{2tA} = \frac{60 \times 10^3}{(2)(0.25)(20.625)} = 5.82 \times 10^3 \text{ psi} \quad \tau = 5.82 \text{ ksi}$$

At point *b* $t = 0.50 \text{ in.}$

$$\tau = \frac{T}{2tA} = \frac{60 \times 10^3}{(2)(0.50)(20.625)} = 2.91 \times 10^3 \text{ psi} \quad \tau = 2.91 \text{ ksi}$$

Problem 3.135

3.135 A 5 kN · m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.



$$T = 5 \times 10^3 \text{ N} \cdot \text{m}$$

Area bounded by center line

$$A = bh = (69)(115) = 7.935 \times 10^3 \text{ mm}^2 \\ = 7.935 \times 10^{-3} \text{ m}^2$$

At point *a* $t = 6 \text{ mm} = 0.006 \text{ m}$

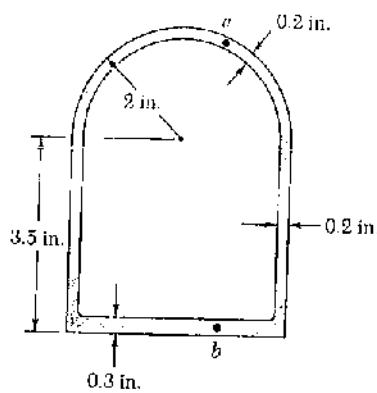
$$\tau = \frac{T}{2tA} = \frac{5 \times 10^3}{(2)(0.006)(7.935 \times 10^{-3})} \\ = 52.5 \times 10^6 \text{ Pa} \quad \tau = 52.5 \text{ MPa}$$

At point *b* $t = 10 \text{ mm} = 0.010 \text{ m}$

$$\tau = \frac{T}{2tA} = \frac{5 \times 10^3}{(2)(0.010)(7.935 \times 10^{-3})} = 31.5 \times 10^6 \text{ Pa} \quad \tau = 31.5 \text{ MPa}$$

Problem 3.136

3.136 A 50 kip · in. torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.



Area bounded by centerline.

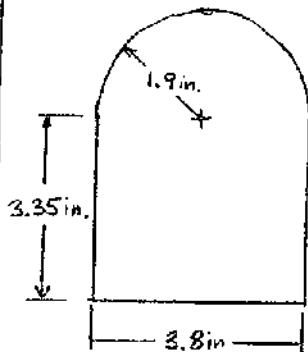
$$A = (3.8)(3.35) + \frac{\pi}{2}(1.9)^2 = 18.4006 \text{ in}^2$$

At point *a*. $t = 0.2 \text{ in}$

$$\tau = \frac{T}{2At} = \frac{50}{(2)(18.4006)(0.2)} = 6.79 \text{ ksi}$$

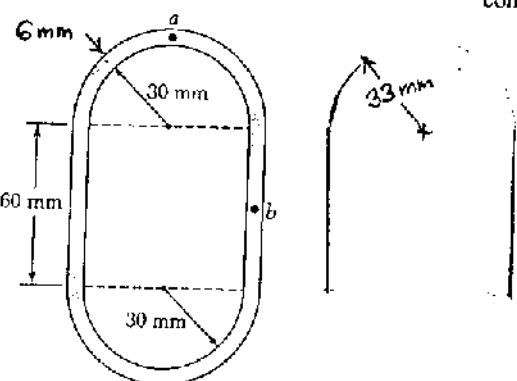
At point *b*. $t = 0.3 \text{ in}$

$$\tau = \frac{T}{2At} = \frac{50}{(2)(18.4006)(0.3)} = 4.53 \text{ ksi}$$



Problem 3.137

3.137 A 750 N · m torque is applied to a hollow shaft having the cross section shown and a uniform 6-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.



Area bounded by center line

$$A = 2 \frac{\pi}{2}(33)^2 + (60)(66) = 7381 \text{ mm}^2 \\ = 7381 \times 10^{-6} \text{ m}^2$$

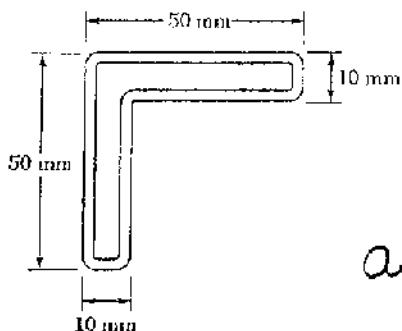
$t = 0.006 \text{ m}$ at both *a* and *b*.

Then at points *a* and *b*

$$\tau = \frac{T}{2tA} = \frac{750}{(2)(0.006)(7381 \times 10^{-6})} = 8.47 \times 10^6 \text{ Pa} = 8.47 \text{ MPa}$$

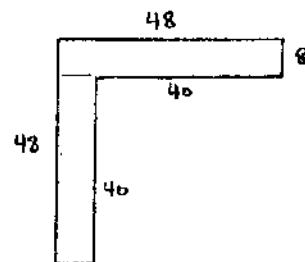
Problem 3.138

3.138 A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque that can be applied to the member.



Area bounded by centerline

$$A = (48)(8) + (40)(8) \\ = 704 \text{ mm}^2 = 704 \times 10^{-6} \text{ m}^2$$

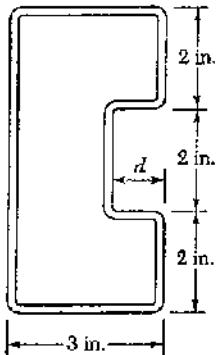


$$t = 0.002 \text{ m}$$

$$\tau = \frac{T}{2ta} \therefore T = 2ta\tau = (2)(0.002)(704 \times 10^{-6})(3 \times 10^6) \\ = 8.45 \text{ N}\cdot\text{m} \quad T = 8.45 \text{ N}\cdot\text{m} \blacksquare$$

Problem 3.139

3.139 and 3.140 A hollow member having the cross section shown is to be formed from sheet metal of 0.06-in. thickness. Knowing that a 1250 lb-in. torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

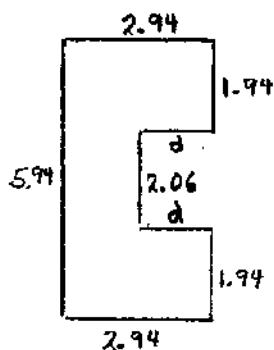


Area bounded by centerline

$$A = (5.94)(2.94) - 2.06d = 17.4636 - 2.06d$$

$$t = 0.06 \text{ in.}, \tau = 750 \text{ psi}, T = 1250 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{T}{2ta}$$

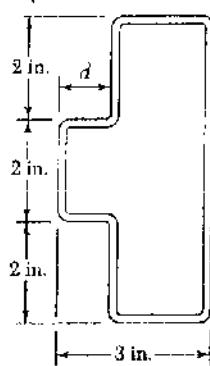


$$A = \frac{T}{2ta}$$

$$17.4636 - 2.06d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{2.06} = 1.735 \text{ in.} \quad d = 1.735 \text{ in.} \blacksquare$$

Problem 3.140



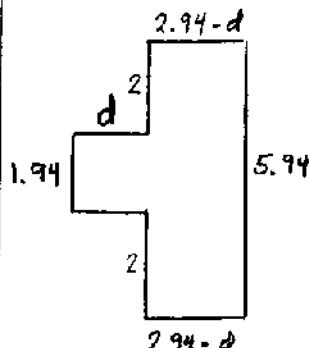
3.139 and 3.140 A hollow member having the cross section shown is to be formed from sheet metal of 0.06-in. thickness. Knowing that a 1250 lb·in. torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

Area bounded by center

$$A = (5.94)(2.94 - d) + 1.94d = 17.4636 - 4.00d$$

$$t = 0.06 \text{ in.}, \tau = 750 \text{ psi}, T = 1250 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{T}{2tA}$$

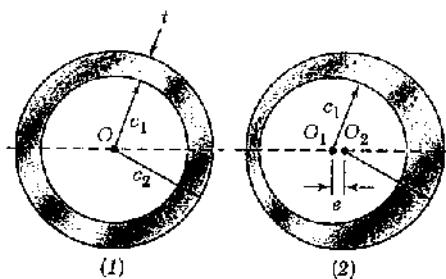


$$A = \frac{T}{2t\tau}$$

$$17.4636 - 4.00d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{4.00} = 0.894 \text{ in.} \quad d = 0.894 \text{ in.} \blacksquare$$

Problem 3.141



3.141 A hollow cylindrical shaft was designed with the cross section shown in Fig. (1) to withstand a maximum torque T_o . Defective fabrication, however, resulted in a slight eccentricity e between the inner and outer cylindrical surfaces of the shaft as shown in Fig. (2). (a) Express the maximum torque T which can be safely applied to the defective shaft in terms of T_o , e , and t . (b) Calculate the percent decrease in the allowable torque for values of the ratio e/t equal to 0.1, 0.5, and 0.9.

$$(a) \text{For both configurations: } A = \pi \left(\frac{c_1 + c_2}{2} \right)^2$$

Let $t = c_2 - c_1$ be the thickness of (1)

The minimum thickness of (2) is $t - e$.

$$\text{For (1)} \quad \tau_{al} = \frac{T_o}{2tA}$$

$$\text{For (2)} \quad \tau_{al} = \frac{T}{2(t-e)A}$$

$$\text{Forming the ratio } \frac{\tau_{al}}{\tau_{al}} = 1 = \frac{T}{2(t-e)A} \cdot \frac{2tA}{T_o} = \frac{Tt}{T_o(t-e)}$$

$$T = T_o \left(1 - \frac{e}{t} \right)$$

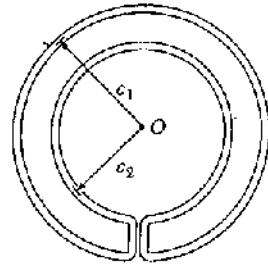
(b) Reduction in torque

$$\frac{T_o - T}{T_o} = \frac{e}{t}$$

$$\% \text{ reduction} = \frac{e}{t} \times 100\%$$

$\frac{e}{t}$	% red
0.1	10%
0.5	50%
0.9	90%

Problem 3.142



3.142 A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3-mm thickness. The radii $c_1 = 150 \text{ mm}$ and $c_2 = 100 \text{ mm}$ are measured to the center line of the sheet metal. Knowing that a torque of magnitude $T = 3 \text{ kN} \cdot \text{m}$ is applied to the tube, determine (a) the maximum shearing stress in the tube, (b) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

Area bounded by centerline

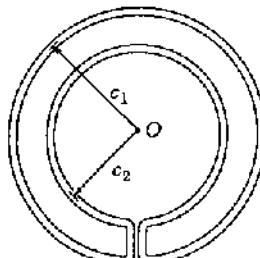
$$A = \pi(c_1^2 - c_2^2) = \pi(150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2 \\ = 39.27 \times 10^{-3} \text{ m}^2$$

$$t = 0.003 \text{ m}$$

$$(a) \tau = \frac{T}{2ta} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^5 \text{ Pa} \quad \boxed{\tau = 12.76 \text{ MPa}}$$

$$(b) T_i = (2\pi c_1 t \tau) c_1 = 2\pi c_1^2 t \tau \\ = 2\pi (0.150)^2 (0.003) (12.73 \times 10^5) = 5.46 \times 10^3 \text{ N} \cdot \text{m} \quad \boxed{T_i = 5.46 \text{ kN} \cdot \text{m}}$$

Problem 3.143



3.143 A cooling tube having the cross section shown is formed from a sheet of stainless steel of thickness t . The radii c_1 and c_2 are measured to the center line of the sheet metal. Knowing that a torque T is applied to the tube, determine in terms of T , c_1 , c_2 , and t the maximum shearing stress in the tube.

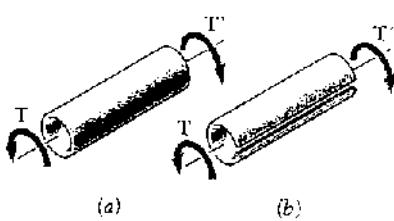
Area bounded by centerline

$$A = \pi(c_1^2 - c_2^2)$$

Shearing stress

$$\tau = \frac{T}{2ta} = \frac{T}{2\pi t(c_1^2 - c_2^2)} \quad \boxed{-}$$

Problem 3.144



3.144 Equal torques are applied to thin-walled tubes of the same length L , same thickness t , and same radius c . One of the tubes has been slit lengthwise as shown. Determine (a) the ratio τ_b/τ_a of the maximum shearing stresses in the tubes, (b) the ratio ϕ_b/ϕ_a of the angles of twist of the shafts.

Without slit

(a) (b)

$$\text{Area bounded by centerline: } A = \pi c^2$$

$$\tau_a = \frac{T}{2tA} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t$$

$$\phi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t G}$$

With slit: $A = 2\pi c, b = t, \frac{a}{b} = \frac{2\pi c}{t} \gg 1$

$$c_1 = c_2 = \frac{1}{3}$$

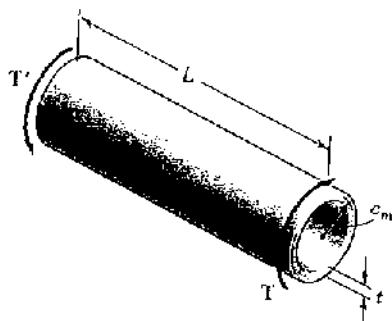
$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2}$$

$$\phi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

(a) Stress ratio: $\frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t} \quad \frac{\tau_b}{\tau_a} = \frac{3c}{t}$

(b) Twist ratio: $\frac{\phi_b}{\phi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t G}{TL} = \frac{3c^2}{t^2} \quad \frac{\phi_b}{\phi_a} = \frac{3c^2}{t^2}$

Problem 3.145



3.145 A hollow cylindrical shaft of length L , mean radius c_m , and uniform thickness t is subjected to torques of magnitude T . Consider, on the one hand, the values of the average shearing stress τ_{ave} and the angle of twist ϕ obtained from the elastic torsion formulas developed in Secs. 3.4 and 3.5 and, on the other hand, the corresponding values obtained from the formulas developed in Sec. 3.13 for thin-walled hollow shafts. (a) Show that the relative error introduced by using the thin-wall-shaft formulas rather than the elastic torsion formulas is the same for τ_{ave} and ϕ and that the relative error is positive and proportional to the square of the ratio t/c_m . (b) Compare the percent error corresponding to values of the ratio t/c_m equal 0.1, 0.2 and 0.4.

Let $c_2 = \text{outer radius} = c_m + \frac{1}{2}t$ and $c_1 = \text{inner radius} = c_m - \frac{1}{2}t$

$$\begin{aligned} J &= \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(c_2^2 + c_1^2)(c_2 + c_1)(c_2 - c_1) \\ &= \frac{\pi}{2}(c_m^2 + c_mt + \frac{1}{4}t^2 + c_m^2 - c_mt + \frac{1}{4}t^2)(2c_m)t \\ &= 2\pi(c_m^2 + \frac{1}{4}t^2)c_mt \end{aligned}$$

$$\tau_m = \frac{Tr}{J} = \frac{T}{2\pi(c_m^2 + \frac{1}{4}t^2)t}$$

$$\Phi_1 = \frac{TL}{JG} = \frac{TL}{2\pi(c_m^2 + \frac{1}{4}t^2)c_mt G}$$

Area bounded by centerline $A = \pi c_m^2$

$$\tau_{ave} = \frac{T}{2ta} = \frac{T}{2\pi c_m^2 t}$$

$$\Phi_2 = \frac{TL}{4Q^2 G} \int \frac{ds}{t} = \frac{TL(2\pi c_m/t)}{4(\pi c_m^2)^2 G} = \frac{TL}{2\pi c_m^3 t G}$$

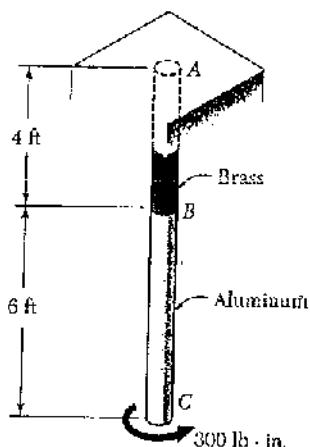
Ratios: $\frac{\tau_{ave}}{\tau_m} = \frac{T}{2\pi c_m^2 t} \cdot \frac{2\pi(c_m^2 + \frac{1}{4}t^2)t}{T} = 1 + \frac{1}{4} \frac{t^2}{c_m^2}$

$$\frac{\Phi_2}{\Phi_1} = \frac{TL}{2\pi c_m^3 t G} \cdot \frac{2\pi(c_m^2 + \frac{1}{4}t^2)c_mt G}{TL} = 1 + \frac{1}{4} \frac{t^2}{c_m^2}$$

$$\frac{\tau_{ave}}{\tau_m} - 1 = \frac{\Phi_2}{\Phi_1} - 1 = \frac{1}{4} \frac{t^2}{c_m^2}$$

$\frac{t}{c_m}$	0.1	0.2	0.4
$\frac{1}{4} \frac{t^2}{c_m^2}$	0.0025	0.01	0.04
0.25%	1%	4%	

Problem 3.146



3.146 The aluminum rod BC ($G = 3.9 \times 10^6$ psi) is bonded to the brass for AB ($G = 5.6 \times 10^6$ psi). Knowing that each rod is solid and has a diameter of 0.5 in., determine the angle of twist (a) at B , (b) at C .

$$\text{Both portions } C = \frac{1}{2}d = 0.25 \text{ in}$$

$$J = \frac{\pi}{2} C^4 = 6.1359 \times 10^{-3} \text{ in}^4 \quad T = 300 \text{ lb-in}$$

Shaft AB : $G_{AB} = 5.6 \times 10^6$ psi; $L_{AB} = 4 \text{ ft} = 48 \text{ in}$

$$\varphi_B = \varphi_{AB} = \frac{TL_{AB}}{G_{AB}J} = \frac{(300)(48)}{(5.6 \times 10^6)(6.1359 \times 10^{-3})}$$

$$= 0.419 \text{ rad} \quad \varphi_B 24.0^\circ$$

Shaft BC : $G = 3.9 \times 10^6$ psi; $L_{BC} = 6 \text{ ft} = 72 \text{ in}$

$$\varphi_{BC} = \frac{TL_{BC}}{G_{BC}J} = \frac{(300)(72)}{(3.9 \times 10^6)(6.1359 \times 10^{-3})}$$

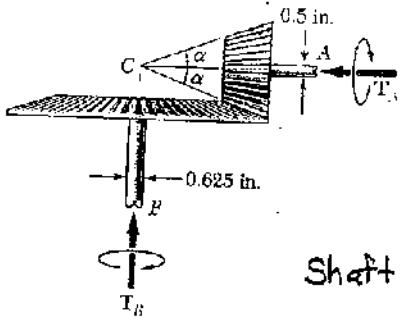
$$= 0.903 \text{ rad} = 51.7^\circ$$

$$\varphi_C = \varphi_B + \varphi_{BC}$$

$$= 0.417 + 0.903 = 1.320 \text{ rad} \quad \varphi_C 76.6^\circ$$

Problem 3.147

3.147 In the bevel-gear system shown, $\alpha = 18.43^\circ$. Knowing that the allowable shearing stress is 8 ksi in each shaft and that the system is in equilibrium, determine the largest torque T_A that can be applied at A .



Shaft A: $\tau = 8 \text{ ksi} \quad C = \frac{1}{2}d = 0.25 \text{ in}$

$$T_A = \frac{J\tau}{C} = \frac{\pi}{2} C^3 \tau = \frac{\pi}{2} (0.25)^3 (8) = 0.19635 \text{ kip-in}$$

Shaft B: $\tau = 8 \text{ ksi} \quad C = \frac{1}{2}d = 0.3125 \text{ in}$

$$T_B = \frac{J\tau}{C} = \frac{\pi}{2} C^3 \tau = \frac{\pi}{2} (0.3125)^3 (8) = 0.3835 \text{ kip-in}$$

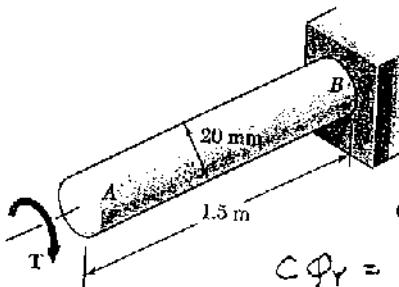
$$\text{From Statics: } T_A = \frac{T_B}{r_B} = (\tan \alpha) T_B = (\tan 18.43^\circ)(0.3835)$$

$$= 0.12779 \text{ kip-in}$$

Allowable value of T_A is the smaller

$$T_A = 0.1278 \text{ kip-in} = \quad T_A = 127.8 \text{ kip-in}$$

Problem 3.148



3.148 A torque T is applied to the 20-mm-diameter steel rod AB . Assuming that the steel is elastoplastic with $G = 77.2 \text{ GPa}$ and $\tau_y = 145 \text{ MPa}$, determine (a) the torque T when the angle of twist at A is 25° , (b) the corresponding diameter of the elastic core of the shaft.

$$C = \frac{1}{2}d = 0.010 \text{ m}, \quad L = 1.5 \text{ m}, \quad G = 77 \times 10^9 \text{ Pa}$$

$$C\phi_r = L\gamma_r = \frac{L\gamma_r}{G} \quad \phi_r = \frac{L\gamma_r}{GC}$$

$$\phi_r = \frac{(1.5)(145 \times 10^6)}{(77.2 \times 10^9)(0.010)} = 281.74 \times 10^{-3} \text{ rad}$$

$$T_r = \frac{J\gamma_r}{C} = \frac{\pi}{2} C^3 \gamma_r = \frac{\pi}{2} (0.010)^3 (145 \times 10^6) = 227.77 \text{ N}\cdot\text{m}$$

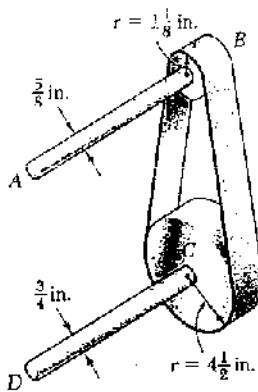
$$\phi = 25^\circ = 436.33 \times 10^{-3} \text{ rad} > \phi_r \quad \frac{\phi_r}{\phi} = \frac{281.74 \times 10^{-3}}{436.33 \times 10^{-3}} = 0.64569$$

$$(a) \quad T = \frac{4}{3} T_r \left(1 - \frac{1}{4} \frac{\phi_r^3}{\phi^3}\right) = \frac{4}{3} (227.77) \left[1 - \frac{1}{4} (0.64569)^3\right] \quad T = 283 \text{ N}\cdot\text{m} \quad \blacksquare$$

$$(b) \quad \frac{p_r}{c} = \frac{\phi_r}{\phi} = 0.64569 \quad p_r = 0.64737 c = (0.64737)(0.010)$$

$$p_r = 6.4569 \times 10^{-3} \text{ m} = 6.4569 \text{ mm} \quad d_r = 2p_r = 12.91 \text{ mm} \quad \blacksquare$$

Problem 3.149



3.149 The shaft-disk-belt arrangement shown is used to transmit 3 hp from point A to point D. (a) Using an allowable shearing stress of 9500 psi, determine the required speed of shaft AB. (b) Solve part a, assuming that the diameters of shafts AB and CD are respectively 0.75 in. and 0.625 in.

SOLUTION

$$\tau = 9500 \text{ psi}, P = 3 \text{ hp} = (3)(6600) = 19800 \text{ lb-in/s}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad T = \frac{\pi}{2} c^3 \tau$$

Allowable torques

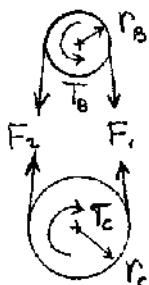
$\frac{5}{8}$ in. diameter shaft

$$c = \frac{5}{16} \text{ in}, T_{all} = \frac{\pi}{2} \left(\frac{5}{16}\right)^3 (9500) = 455.4 \text{ lb-in}$$

$\frac{3}{4}$ in diameter shaft

$$c = \frac{3}{8} \text{ in}, T_{all} = \frac{\pi}{2} \left(\frac{3}{8}\right)^3 (9500) = 786.9 \text{ lb-in}$$

Statics:



$$T_B = r_B (F_1 - F_2) \quad T_c = r_c (F_c - F_b)$$

$$T_B = \frac{r_B}{r_c} T_c = \frac{1.125}{4.5} T_c = 0.25 T_c$$

$$(a) \text{ Allowable torques } T_{B,all} = 455.4 \text{ lb-in}, T_{c,all} = 786.9 \text{ lb-in}$$

$$\text{Assume } T_c = 786.9 \text{ lb-in. Then } T_B = (0.25)(786.9) = 196.73 \text{ lb-in}$$

< 455.4 lb in (okay)

$$P = 2\pi f T \quad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(196.73)} \quad f_{AB} = 16.02 \text{ Hz}$$

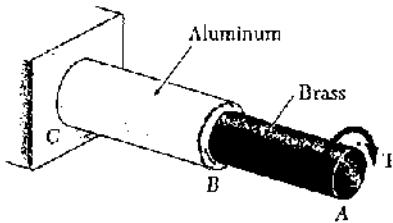
$$(b) \text{ Allowable torques } T_{B,all} = 786.9 \text{ lb-in}, T_{c,all} = 455.4 \text{ lb-in.}$$

$$\text{Assume } T_c = 455.4 \text{ lb-in. Then } T_B = (0.25)(455.4) = 113.85 \text{ lb-in}$$

< 786.9 lb-in

$$P = 2\pi f T \quad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(113.85)} \quad f_{AB} = 27.2 \text{ Hz}$$

Problem 3.150



3.150 The allowable stress is 50 MPa in the brass rod *AB* and 25 MPa in the aluminum rod *BC*. Knowing that a torque of magnitude $T = 1250 \text{ N} \cdot \text{m}$ is applied at *A*, determine the required diameter (a) of rod *AB*, (b) of rod *BC*.

$$I_{max} = \frac{TC}{J} \quad J = \frac{\pi}{2} C^4 \quad C^3 = \frac{2T}{\pi \tau_{max}}$$

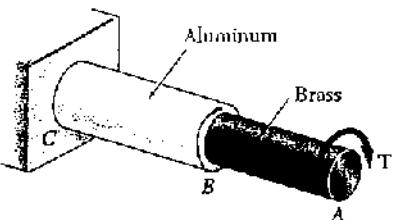
$$\text{Rod } AB: \quad C^3 = \frac{(2)(1250)}{\pi(50 \times 10^6)} = 15.915 \times 10^{-6} \text{ m}^3$$

$$C = 25.15 \times 10^{-3} \text{ m} = 25.15 \text{ mm} \quad d_{AB} = 2C = 50.3 \text{ mm}$$

$$\text{Rod } BC: \quad C^3 = \frac{(2)(1250)}{\pi(25 \times 10^6)} = 31.831 \times 10^{-6} \text{ m}^3$$

$$C = 31.69 \times 10^{-3} \text{ m} = 31.69 \text{ mm} \quad d_{BC} = 2C = 63.4 \text{ mm}$$

Problem 3.151



3.151 The solid rod *BC* has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is hollow and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (a) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (b) the largest torque that can be applied at *A*.

$$\text{Solid rod } BC: \quad \tau = \frac{TC}{J} \quad J = \frac{\pi}{2} C^4$$

$$\tau_{all} = 25 \times 10^6 \text{ Pa} \quad C = \frac{1}{2} d = 0.015 \text{ m}$$

$$T_{all} = \frac{\pi}{2} C^3 \tau_{all} = \frac{\pi}{2} (0.015)^3 (25 \times 10^6) = 132.536 \text{ N} \cdot \text{m}$$

$$\text{Hollow rod } AB: \quad \tau_{all} = 50 \times 10^6 \text{ Pa} \quad T_{all} = 132.536 \text{ N} \cdot \text{m}$$

$$C_2 = \frac{1}{2} d_2 = \frac{1}{2}(0.025) = 0.0125 \text{ m} \quad C_1 = ?$$

$$T_{all} = \frac{J \tau_{all}}{C_2} = \frac{\pi}{2} (C_2^4 - C_1^4) \frac{\tau_{all}}{C_2}$$

$$C_1^4 = C_2^4 - \frac{2 T_{all} C_2}{\pi \tau_{all}} = 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi (50 \times 10^6)} = 3.3203 \times 10^{-9} \text{ m}^4$$

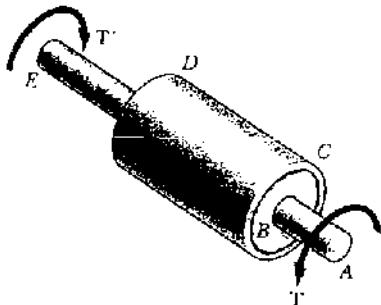
$$(a) \quad C_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm} \quad d_1 = 2C_1 = 15.18 \text{ mm}$$

(b) Allowable torque

$$T_{all} = 132.5 \text{ N} \cdot \text{m}$$

Problem 3.152

3.152 The steel jacket CD has been attached to the 40-mm-diameter steel shaft AE by means of rigid flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If 500 N·m torques are applied as shown, determine the maximum shearing stress in the jacket.



SOLUTION

$$\text{Solid shaft: } c = \frac{1}{2}d = 0.020 \text{ m}$$

$$J_s = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

$$\text{Jacket: } c_2 = \frac{1}{2}d = 0.040 \text{ m}$$

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.040^4 - 0.036^4) = 1.3829 \times 10^{-6} \text{ m}^4$$

$$\text{Torque carried by shaft} \quad T_s = G J_s \phi / L$$

$$\text{Torque carried by jacket} \quad T_J = G J_J \phi / L$$

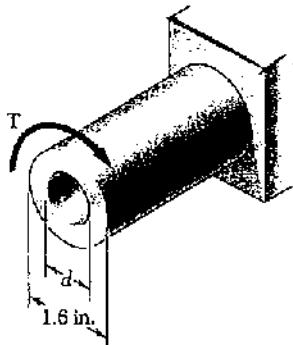
$$\text{Total torque} \quad T = T_s + T_J = (J_s + J_J) G \phi / L \quad \therefore \frac{G\phi}{L} = \frac{T}{J_s + J_J}$$

$$T_J = \frac{J_J}{J_s + J_J} T = \frac{(1.3829 \times 10^{-6})(500)}{1.3829 \times 10^{-6} + 251.33 \times 10^{-9}} = 423.1 \text{ N}\cdot\text{m}$$

Maximum shearing stress in jacket

$$\tau = \frac{T_J c}{J_c} = \frac{(423.1)(0.040)}{1.3829 \times 10^{-6}} = 12.24 \times 10^6 \text{ Pa} \quad 12.24 \text{ MPa} \blacksquare$$

Problem 3.153



3.153 Knowing that the internal diameter of the hollow shaft shown is $d = 0.9$ in., determine the maximum shearing stress caused by a torque of magnitude $T = 9$ kip-in.

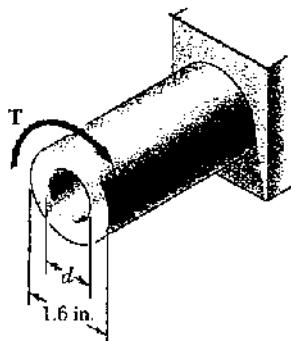
$$C_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in.} \quad C = 0.8 \text{ in.}$$

$$C_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(0.9) = 0.45 \text{ in.}$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4$$

$$\tau_{\max} = \frac{TC}{J} = \frac{(9)(0.8)}{0.5790} = 12.44 \text{ ksi} \quad \blacktriangleleft$$

Problem 3.154



3.154 Knowing that $d = 1.2$ in., determine the torque T that causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.

$$C_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in.} \quad C = 0.8 \text{ in.}$$

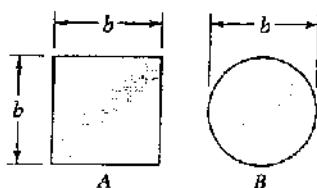
$$C_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(1.2) = 0.6 \text{ in.}$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.4398 \text{ in}^4$$

$$\tau_{\max} = \frac{TC}{J}$$

$$T = \frac{JC\tau_{\max}}{C} = \frac{(0.4398)(7.5)}{0.8} = 4.12 \text{ kip-in.} \quad \blacktriangleleft$$

Problem 3.155



3.155 Two shafts are made of the same material. The cross section of shaft A is a square of side b and that of shaft B is circle of diameter b . Knowing that the shafts are subjected to the same torque, determine the ratio τ_A/τ_B of the maximum shearing stresses occurring in the shafts.

A. square $\frac{a}{b} = 1$, $C_1 = 0.208$ (Table 3.1)

$$\tau_A = \frac{TC}{C_1 ab^2} = \frac{T}{0.208 b^3}$$

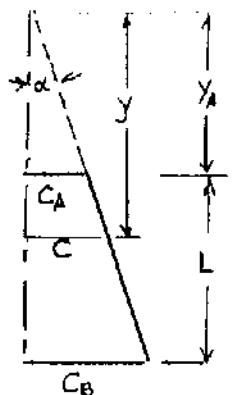
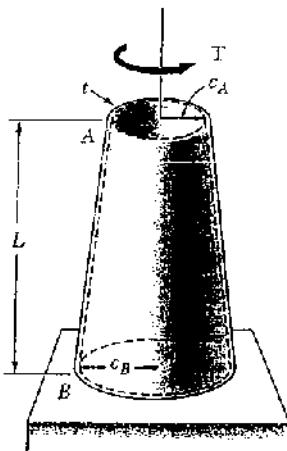
$$\text{B. circle } C = \frac{1}{2}b \quad \tau_B = \frac{TC}{J} = \frac{2T}{\pi C^3} = \frac{16T}{\pi b^3}$$

$$\text{Ratio } \frac{\tau_A}{\tau_B} = \frac{1}{0.208} \cdot \frac{\pi}{16} = 0.3005 \pi \quad \frac{\tau_A}{\tau_B} = 0.944 \quad \blacktriangleleft$$

Problem 3.156

3.156 The long, hollow, tapered shaft AB has a uniform thickness t . Denoting by G the modulus of rigidity, show that the angle of twist at end A is

$$\phi_A = \frac{TL}{4\pi Gt} \frac{c_A + c_B}{c_A^2 c_B^2}$$



From geometry

$$\tan \alpha = \frac{c_B - c_A}{L}$$

$$c = y \tan \alpha = \frac{c_B - c_A}{L} y$$

$$y_A = \frac{c_A}{\tan \alpha} = \frac{c_A L}{c_B - c_A}$$

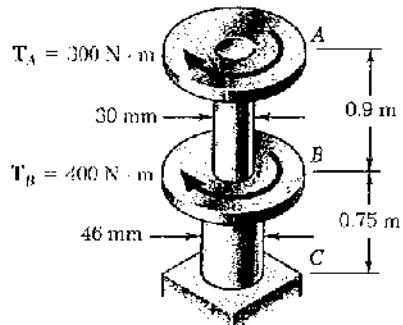
$$y_B = \frac{c_B}{\tan \alpha} = \frac{c_B L}{c_B - c_A}$$

$$J = 2\pi c^3 t = 2\pi \frac{(c_B - c_A)^3}{L^3} y^3 t$$

$$\begin{aligned} \phi &= \int_{y_A}^{y_B} \frac{T dy}{GJ} = \frac{TL^3}{2\pi(c_B - c_A)^3 t G} \int_{y_A}^{y_B} \frac{dy}{y^3} = \frac{TL^3}{2\pi(c_B - c_A)^3 t G} \left(-\frac{1}{2y^2} \right) \Big|_{y_A}^{y_B} \\ &= \frac{TL^3}{4\pi(c_B - c_A)^3 t G} \left\{ \frac{1}{y_A^2} - \frac{1}{y_B^2} \right\} = \frac{TL^3}{4\pi(c_B - c_A)^3 t G} \cdot \left\{ \frac{(c_B - c_A)^2}{L^2 c_A^2} - \frac{(c_B - c_A)^2}{L^2 c_B^2} \right\} \\ &= \frac{TL}{4\pi(c_B - c_A) t G} \left\{ \frac{1}{c_A^2} - \frac{1}{c_B^2} \right\} = \frac{TL (c_B^2 - c_A^2)}{4\pi(c_B - c_A) t G c_A^2 c_B^2} \\ &= \frac{TL (c_B + c_A)}{4\pi G t c_A^2 c_B^2} \end{aligned}$$

Problem 3.157

3.157 The torques shown are exerted on pulleys A and B. Knowing that the steel shafts are solid and that $G = 77.2 \text{ GPa}$, determine the angle of twist between (a) A and B, (b) A and C.



$$(a) T_{AB} = 300 \text{ N}\cdot\text{m}, L_{AB} = 0.9 \text{ m}, C_{AB} = \frac{1}{2}d = 0.015 \text{ m}$$

$$J_{AB} = \frac{\pi}{2}(0.015)^4 = 79.522 \times 10^{-9} \text{ m}^4$$

$$\varphi_{AB} = \frac{T_{AB} L_{AB}}{G J_{AB}} = \frac{(300)(0.9)}{(77.2 \times 10^9)(79.522 \times 10^{-9})} \\ = 43.98 \times 10^{-3} \text{ rad}$$

$$\varphi_{AB} = 2.52^\circ$$

$$(b) T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}, L_{BC} = 0.75 \text{ m}, C_{BC} = \frac{1}{2}d = 0.023 \text{ m}$$

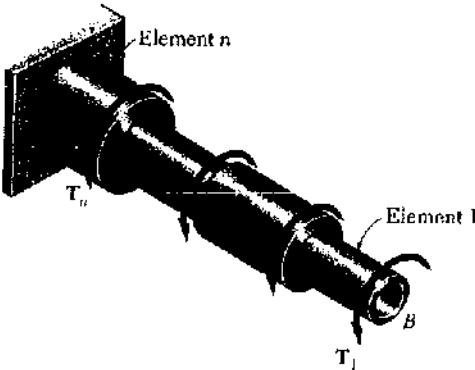
$$J_{BC} = \frac{\pi}{2}(0.023)^4 = 439.573 \times 10^{-9} \text{ m}^4$$

$$\varphi_{BC} = \frac{T_{BC} L_{BC}}{G J_{BC}} = \frac{(700)(0.75)}{(77.2 \times 10^9)(439.573 \times 10^{-9})} = 15.47 \times 10^{-3} \text{ rad}$$

$$\varphi_{AC} = \varphi_{AB} + \varphi_{BC} = 59.45 \times 10^{-3} \text{ rad}$$

$$\varphi_{AC} = 3.41^\circ$$

PROBLEM 3.C1



3.C1 Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Its end A is fixed, while its end B is free, and it is subjected to the loading shown. The length of element i is denoted by L_i , its outer diameter by OD_i , its inner diameter by ID_i , its modulus of rigidity by G_i , and the torque applied to its right end by T_i , the magnitude T_i of this torque being assumed to be positive if T_i is observed as counterclockwise from end B and negative otherwise. (Note that $ID_i = 0$ if the element is solid.) (a) Write a computer program that can be used to determine the maximum shearing stress in each element, the angle of twist of each element, and the angle of twist of the entire shaft. (b) Use this program to solve Probs. 3.36, 3.37, and 3.157.

SOLUTION

FOR EACH CYLINDRICAL ELEMENT, ENTER

$$L_i, OD_i, ID_i, G_i, T_i$$

AND COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

OUTLINE OF PROGRAM

$$\text{UPDATE TORQUE } T = T + T_i$$

AND COMPUTE

$$\tau_{AV_i} = T (OD_i/2)/J_i$$

$$\phi_{H_i} = TL_i/G_i J_i$$

ANGLE OF TWIST OF ENTIRE SHAFT, STARTING WITH $\Theta = 0$, UPDATE THROUGH n^{th} ELEMENT
 $\Theta = \Theta + \phi_{H_i}$

PROGRAM OUTPUT

Problem 3.36

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
---------	-------------------------	-----------------------------

1.0000	11.9575	1.3841
2.0000	23.0259	1.8323

Angle of twist for entire shaft = 3.2164 °

Problem 3.37

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
---------	-------------------------	-----------------------------

1.0000	87.3278	4.1181
2.0000	56.5884	1.0392
3.0000	70.5179	0.8633

Angle of twist for entire shaft = 6.0206 °

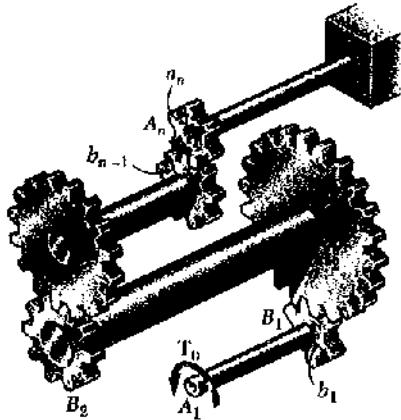
Problem 3.157

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
---------	-------------------------	-----------------------------

1.0000	56.5884	2.5199
2.0000	36.6264	0.8864

Angle of twist for entire shaft = 3.4063 °

PROBLEM 3.C2



3.C2 The assembly shown consists of n cylindrical shafts, which can be solid or hollow, connected by gears and supported by brackets (not shown). End A_1 of the first shaft is free and is subjected to a torque T_0 , while end B_n of the last shaft is fixed. The length of shaft A_iB_i is denoted by L_i , its outer diameter by OD_i , its inner diameter by ID_i , and its modulus of rigidity by G_i . (Note that $ID_i = 0$ if the element is solid.) The radius of gear A_i is denoted by a_i , and the radius of gear B_i by b_i . (a) Write a computer program that can be used to determine the maximum shearing stress in each shaft, the angle of twist of each shaft, and the angle through which end A_i rotates. (b) Use this program to solve Probs. 3.42, 3.43, and 3.44.

SOLUTION

TOQUE IN SHAFTS. ENTER $T_i = T_0$

$$T_{i+1} = T_i (A_{i+1}/B_i)$$

FOR EACH SHAFT, ENTER

$$L_i \quad OD_i \quad ID_i \quad G_i$$

COMPUTE: $J_i = (\pi/32)(OD_i^4 - ID_i^4)$

$$\text{TRU}_i = T_i (OD_i/2)/J_i$$

$$\text{PHI}_i = T_i L_i / G_i J_i$$

ANGLE OF ROTATION AT END A,

COMPUTE ROTATION AT THE "A" END OF EACH SHAFT

START WITH ANGLE = PHI_n, AND UPDATE

FROM n TO 1, AND ADD PHI_i

$$\text{ANGLE} = \text{ANGLE}(A_i) + \text{PHI}_{i-1}$$

PROGRAM OUTPUT

Problem 3.42

Shaft No.	Max Stress) (ksi)	Angle of Twist (degrees)
1	104.31	40.979
2	52.15	20.490
3	26.08	10.245

Angle through which A_1 rotates = 53.785°

Problem 3.43

Shaft No.	Max Stress (ksi)	Angle of Twist (degrees)
1	9.29	1.493
2	12.16	1.707

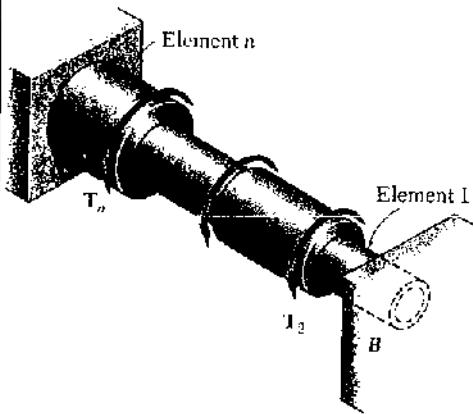
Angle through which A_1 rotates = 3.769°

Problem 3.44

Shaft No.	Max Stress) (ksi)	Angle of Twist (degrees)
1	9.12	1.493
2	6.84	0.960

Angle through which A_1 rotates = 2.213°

PROBLEM 3.C3



3.C3 Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Both of its ends are fixed, and it is subjected to the loading shown. The length of element i is denoted by L_i , its outer diameter by OD_i , its inner diameter by ID_i , its modulus of rigidity by G_i , and the torque applied to its right end by T_i , the magnitude T_i of this torque being assumed to be positive if T_i is observed as counterclockwise from end B and negative otherwise. Note that $ID_i = 0$ if the element is solid and also that $T_1 = 0$. Write a computer program that can be used to determine the reactions at A and B, the maximum shearing stress in each element, and the angle of twist of each element. Use this program (a) to solve Prob. 3.56, (b) to determine the maximum shearing stress in the shaft of Example 3.05.

SOLUTION WE CONSIDER THE REACTION AT B AS REDUNDANT AND RELEASE THE SHAFT AT B.
COMPUTE Θ_B WITH $T_B = 0$:

FOR EACH ELEMENT, ENTER

$$L_i, OD_i, ID_i, G_i, T_i \quad (\text{NOTE } T_1 = T_B = 0)$$

COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

UPDATE TORQUE

$$T = T + T_i$$

AND COMPUTE FOR EACH ELEMENT

$$\tau_{AV_i} = T(OD_i/2)/J_i$$

$$\phi_{HI_i} = TL_i/G_i J_i$$

COMPUTE Θ_B : STARTING WITH $\Theta = 0$ AND UPDATING THROUGH n ELEMENTS

$$\Theta_i = \Theta_{i-1} + \phi_{HI_i} \quad ; \quad \Theta_B = \Theta_n$$

COMPUTE Θ_B DUE TO UNIT TORQUE AT B

$$\text{UNIT } \tau_{AV_i} = OD_i/2J_i$$

$$\text{UNIT } \phi_{HI_i} = L_i/G_i J_i$$

FOR n ELEMENTS:

$$\text{UNIT } \Theta_B(n) = \text{UNIT } \theta_D / (L) + \text{UNIT } \phi_{HI_i}$$

SUPERPOSITION:

FOR TOTAL ANGLE AT B TO BE ZERO, $\Theta_B + T_B(\text{UNIT } \Theta_B(n)) = 0$

$$T_B = -\Theta_B / (\text{UNIT } \Theta_B(n))$$

THEN

$$T_B = \sum T_i(L) + T_B$$

FOR EACH ELEMENT: MAX STRESS: TOTAL $\tau_{AV_i} = \tau_{AV_i} + T_B(\text{UNIT } \tau_{AV_i})$
ANGLE OF TWIST: TOTAL $\phi_{HI_i} = \phi_{HI_i} + T_B(\text{UNIT } \phi_{HI_i})$

PROGRAM OUTPUT

Problem 3.56

$$\begin{aligned} TA &= -0.295 \text{ kN}\cdot\text{m} \\ TB &= -1.105 \text{ kN}\cdot\text{m} \end{aligned}$$

Element

tau max
(MPa)

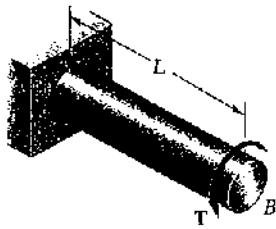
Angle of Twist
(degrees)

$$\begin{aligned} 1 & -45.024 & -0.267 \\ 2 & 27.375 & -.267 \end{aligned}$$

Problem 3.05

$$\begin{aligned} TA &= -51.733 \text{ lb}\cdot\text{ft} \\ TB &= -38.267 \text{ lb}\cdot\text{ft} \end{aligned}$$

PROBLEM 3.C4



3.C4 The homogeneous, solid cylindrical shaft AB has a length L , a diameter d , a modulus of rigidity G , and a yield strength τ_y . It is subjected to a torque T that is gradually increased from zero until the angle of twist of the shaft has reached a maximum value ϕ_m and then decreased back to zero. (a) Write a computer program that, for each of 16 values of ϕ_m equally spaced over a range extending from 0 to a value 3 times as large as the angle of twist at the onset of yield, can be used to determine the maximum value T_m of the torque, the radius of the elastic core, the maximum shearing stress, the permanent twist, and the residual shearing stress both at the surface of the shaft and at the interface of the elastic core and the plastic region. (b) Use this program to obtain approximate answers to Probs. 3.111, 3.112, 3.114.

SOLUTION

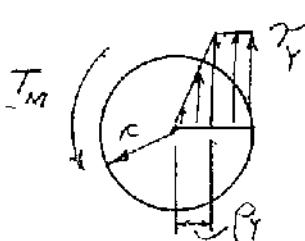


AT ONSET OF YIELD

$$T_y = \tau_y \frac{J}{c} = \frac{\pi}{2} \tau_y c^3$$

$$\phi_y = \frac{T_y L}{G J} = \left(\frac{\tau_y J}{c} \right) \frac{L}{G J} = \frac{\tau_y L}{c G}$$

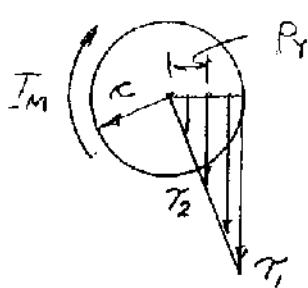
LOADING: $T_m > T_y$



$$T_m = \frac{4}{3} T_y \left[1 - \frac{1}{4} \left(\frac{\phi_y}{\phi_m} \right)^3 \right] \quad \text{EQ. (1)}$$

$$P_y = c \frac{4\tau_y}{\phi_m} \quad \text{EQ. (2)}$$

UNLOADING (ELASTIC)



$$\phi_u = \frac{T_m L}{G J}$$

ϕ_u = ANGLE OF TWIST FOR UNLOADING

$$\gamma_1 = T_m \frac{c}{J}$$

γ_1 = TAU AT $P = c$

$$\gamma_2 = \gamma_1 - \frac{P_y}{c}$$

γ_2 = TAU AT $P = P_y$

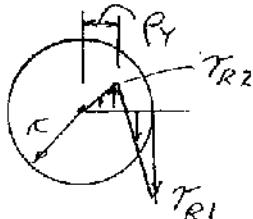
SUPERPOSE LOADING AND UNLOADING

For $\phi = 0$ to $\phi = 3\phi_y$ using 0.2 ϕ_y increments

$$\text{WHEN } \phi < \phi_y: \quad T_m = T_y \frac{\phi}{\phi_y} \quad P_y = \frac{1}{2} d \quad \phi_m = \phi_y \frac{\phi}{\phi_y}$$

WHEN $\phi > \phi_y$: T_m USE EQ(1) P_y USE EQ(2)

$$\text{RESIDUAL: } \phi_p = \phi_m - \phi_u \quad \tau_{R1} = \gamma_1 - \gamma_y \quad \tau_{R2} = \gamma_2 - \gamma_y$$



CONTINUED

PROBLEM 3.C4 - CONTINUED

Interpolate between values at the values of T_{max} or ϕ_{max} indicated

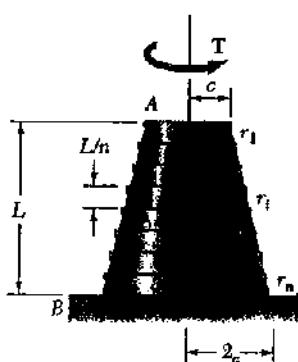
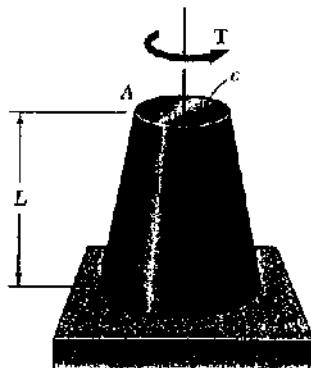
Problem 3.111 and 3.112

PHIM deg	TM kip*in.	RY in.	TAUM ksi	PHIP deg	TAUR1 ksi	TAUR2 ksi
0.000	0.000	1.200	0.000	0.000	0.000	0.000
7.878	11.943	1.200	4.400	0.000	0.000	0.000
15.756	23.886	1.200	8.800	0.000	0.000	0.000
23.635	35.829	1.200	13.200	0.000	0.000	0.000
31.513	47.772	1.200	17.600	0.000	0.000	0.000
39.391	59.715	1.200	22.000	0.000	0.000	0.000
47.269	68.101	1.000	22.000	2.346	1.092	-3.090
55.147	72.366	0.857	22.000	7.411	2.957	-4.661
63.025	74.761	0.750	22.000	13.710	4.786	-5.543
70.904	76.207	0.667	22.000	20.634	6.402	-6.076
78.782	77.132	0.600	22.000	27.902	7.792	-6.417
86.660	77.751	0.545	22.000	35.372	8.980	-6.645
94.538	78.181	0.500	22.000	42.967	9.999	-6.803
102.416	78.488	0.462	22.000	50.642	10.878	-6.916
110.294	78.714	0.429	22.000	58.371	11.643	-6.999
118.173	78.883	0.400	22.000	66.138	12.313	-7.062

Problem 3.114

PHIM deg	TM kN*m	RY mm	TAUM MPa	PHIP deg	TAUR1 MPa	TAUR2 MPa
0.000	0.000	16.000	0.000	0.000	0.000	0.000
0.807	0.187	16.000	29.000	0.000	0.000	0.000
1.614	0.373	16.000	58.000	0.000	0.000	0.000
2.421	0.560	16.000	87.000	0.000	0.000	0.000
3.228	0.746	16.000	116.000	0.000	0.000	0.000
4.036	0.933	16.000	145.000	0.000	0.000	0.000
4.843	1.064	13.333	145.000	0.240	7.198	-20.363
5.650	1.131	11.429	145.000	0.759	19.486	-30.719
6.457	1.168	10.000	145.000	1.405	31.542	-36.533
7.264	1.191	8.889	145.000	2.114	42.197	-40.046
8.071	1.205	8.000	145.000	2.859	51.354	-42.292
8.878	1.215	7.273	145.000	3.624	59.184	-43.794
9.685	1.221	6.667	145.000	4.402	65.901	-44.837
10.492	1.226	6.154	145.000	5.188	71.699	-45.583
11.300	1.230	5.714	145.000	5.980	76.739	-46.132
12.107	1.232	5.333	145.000	6.776	81.152	-46.543

PROBLEM 3.C5



3.C5 The exact expression is given in Prob. 3.64 for the angle of twist of the solid tapered shaft AB when a torque T is applied as shown. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical shafts of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \dots, n$. Using for T , L , G , and c values of your choice, determine the percentage error in the approximate expression when (a) $n = 4$, (b) $n = 8$, (c) $n = 20$, (d) $n = 100$.

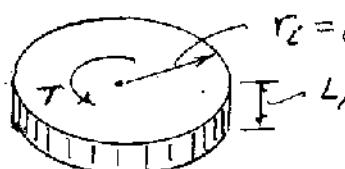
SOLUTION

FROM PROB. 3.64 EXACT EXPRESSION:

$$\phi = \frac{TTL}{12\pi G c^4}$$

$$\text{OR, } \phi = \left(\frac{T}{12\pi}\right) \frac{TL}{G c^4} = 0.18568 \frac{TL}{G c^4}$$

CONSIDER TYPICAL i^{th} SHAFT



$$r_i = (n + i - \frac{1}{2})(c/n)$$

$$J_i = \frac{\pi}{2} (r_i)^4$$

$$\Delta\phi = \frac{T(L/n)}{G J_i}$$

ENTER UNIT VALUES OF T , L , G , AND c .
(NOTE: SPECIFIC VALUES CAN BE ENTERED)

ENTER INITIAL VALUE OF ZERO FOR ϕ
ENTER n = NUMBER CYLINDRICAL SHAFTS

FOR $i = 1$ TO n , UPDATE ϕ

$$\phi = \phi + \Delta\phi$$

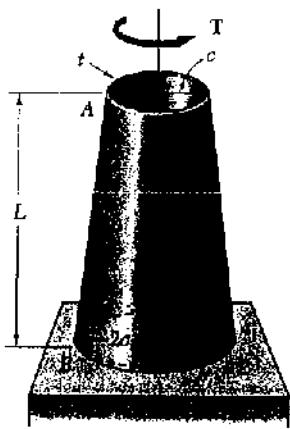
OUTPUT OF PROGRAM

COEFFICIENT OF TL/Gc^4

Exact coefficient from Prob. 3.64 is 0.18568
Number of elemental disks = n

n	approximate	exact	percent error
4	0.17959	0.18568	-3.28185
8	0.18410	0.18568	-0.85311
20	0.18542	0.18568	-0.13810
100	0.18567	0.18568	-0.00554

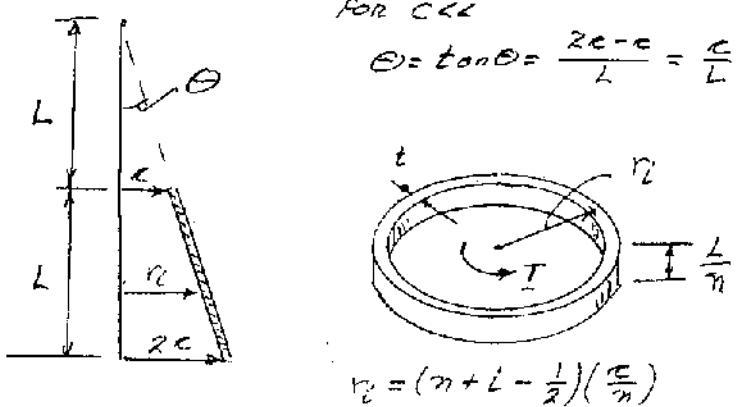
PROBLEM 3.C6



3.C6 A torque T is applied as shown to the long, hollow, tapered shaft AB of uniform thickness t . The exact expression for the angle of twist of the shaft can be obtained from the expression given in Prob. 3.153. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical rings of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \dots, n$. Using for T , L , G , c and t values of your choice, determine the percentage error in the approximate expression when (a) $n = 4$, (b) $n = 8$, (c) $n = 20$, (d) $n = 100$.

SOLUTION

SINCE THE SHAFT IS LONG $C \ll L$, THE ANGLE Θ IS SMALL AND WE CAN USE t AS THE THICKNESS OF THE n CYLINDRICAL RINGS.



$$J_i \approx (\text{AREA}) r_i^2 = (2\pi r_i t) r_i^2 = 2\pi t r_i^3$$

$$\Delta \phi = \frac{T (4L)}{G J_i}$$

ENTER UNIT VALUES FOR T , L , G , t , AND c
 (NOTE: SPECIFIC VALUES CAN BE ENTERED IF DESIRED)
ENTER INITIAL VALUE OF ZERO FOR ϕ
ENTER n = NUMBER OF CYLINDRICAL RINGS

For $i = 1$ to n , UPDATE ϕ
 $\phi = \phi + \Delta \phi$

OUTPUT OF PROGRAM

COEFFICIENT OF $TL/Gt c^3$

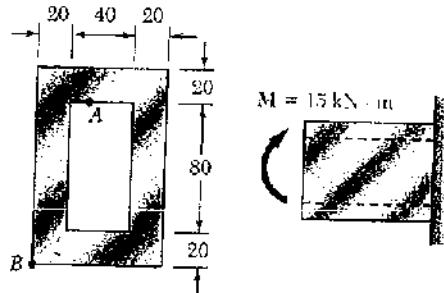
Exact coefficient from Prob. 3.153 is 0.05968
 Number of elemental disks = n

n	approximate	exact	percent error
4	0.058559	0.059683	-1.883078
8	0.059394	0.059683	-0.483688
20	0.059637	0.059683	-0.078022
100	0.059681	0.059683	-0.003127

Chapter 4

Problem 4.1

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.



Dimensions in mm

$$(a) y_A = 40 \text{ mm} = 0.040 \text{ m}$$

$$\sigma_A = -\frac{My_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$$

$$\sigma_A = -61.6 \text{ MPa}$$

$$(b) y_B = -60 \text{ mm} = -0.060 \text{ m}$$

$$\sigma_B = -\frac{My_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$$

$$\sigma_B = 91.7 \text{ MPa}$$

For rectangle $I = \frac{1}{12}bh^3$

Outside rectangle: $I_1 = \frac{1}{12}(80)(120)^3$

$$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$$

Cutout: $I_2 = \frac{1}{12}(40)(80)^3$

$$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

$$(a) y_A = 40 \text{ mm} = 0.040 \text{ m}$$

$$\sigma_A = -\frac{My_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$$

$$\sigma_A = -61.6 \text{ MPa}$$

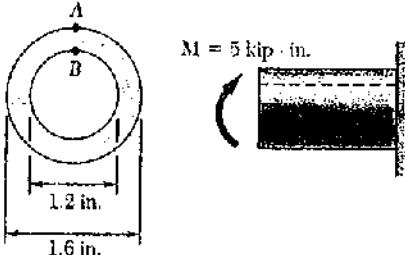
$$(b) y_B = -60 \text{ mm} = -0.060 \text{ m}$$

$$\sigma_B = -\frac{My_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$$

$$\sigma_B = 91.7 \text{ MPa}$$

Problem 4.2

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.



$$r_o = \frac{1}{2}(1.6) = 0.8 \text{ in.} \quad r_i = \frac{1}{2}(1.2) = 0.6 \text{ in.}$$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(0.8^4 - 0.6^4)$$

$$= 0.21991 \text{ in}^4$$

$$(a) y_A = 0.8 \text{ in.} \quad \sigma_A = -\frac{My_A}{I} = -\frac{(5 \times 10^3)(0.8)}{0.21991} = -18.19 \times 10^3 \text{ psi}$$

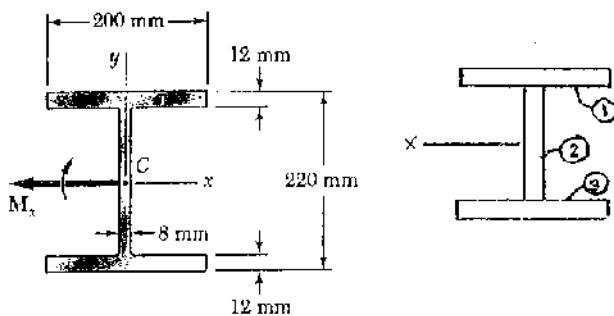
$$\sigma_A = -18.19 \text{ ksi}$$

$$(b) y_B = 0.6 \text{ in.} \quad \sigma_B = -\frac{My_B}{I} = -\frac{(5 \times 10^3)(0.6)}{0.21991} = -13.64 \times 10^3 \text{ psi}$$

$$\sigma_B = -13.64 \text{ ksi}$$

Problem 4.3

4.3 Using an allowable stress of 155 MPa, determine the largest bending moment M that can be applied to the wide flange beam shown. Neglect the effect of fillets.



Moment of inertia about x-axis

$$I_1 = \frac{1}{12}(200)(12)^3 + (200)(12)(104)^2 \\ = 25,9872 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(8)(196)^3 = 5.0197 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 25.9872 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 56.944 \times 10^6 \text{ mm}^4 = 56.944 \times 10^6 \text{ m}^4$$

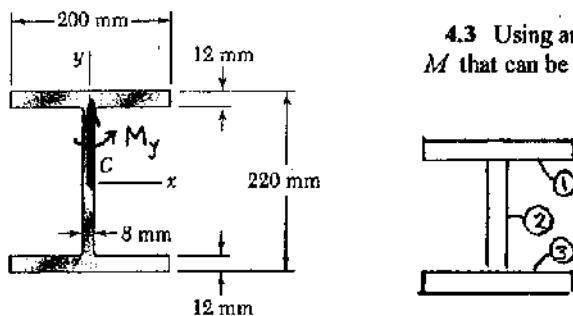
$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(220) = 110 \text{ mm} = 0.110 \text{ m}$$

$$M = \frac{I\sigma}{c} \quad \text{with} \quad \sigma = 155 \times 10^6 \text{ Pa}$$

$$M_x = \frac{(56.944 \times 10^6)(155 \times 10^6)}{0.110} = 80.2 \times 10^3 \text{ N}\cdot\text{m} \quad M_x = 80.2 \text{ kN}\cdot\text{m}$$

Problem 4.4

4.4 Solve Prob. 4.3, assuming that the wide-flange beam is bent about the y axis by a couple of moment M_y .



Moment of inertia about y-axis

$$I_1 = \frac{1}{12}(12)(200)^3 = 8 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(196)(8)^3 = 8.36 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 8 \times 10^6 \text{ mm}^4$$

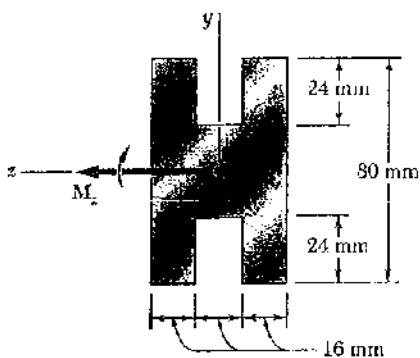
$$I = I_1 + I_2 + I_3 = 16.00836 \times 10^6 \text{ mm}^4 = 16.00836 \times 10^6 \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(200) = 100 \text{ mm} = 0.100 \text{ m}$$

$$M_y = \frac{I\sigma}{c} \quad \text{with} \quad \sigma = 155 \times 10^6 \text{ Pa}$$

$$M_y = \frac{(16.00836 \times 10^6)(155 \times 10^6)}{0.100} = 24.8 \times 10^3 \text{ N}\cdot\text{m} \quad M_y = 24.8 \text{ kN}\cdot\text{m}$$

Problem 4.5



4.5 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_y = 250 \text{ MPa}$ and $\sigma_c = 450 \text{ MPa}$. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.

$$\text{Allowable stress} = \frac{\sigma_u}{F.S.} = \frac{450}{3} = 150 \text{ MPa}$$

$$= 150 \times 10^6 \text{ Pa}$$

Moment of inertia about z axis

$$I_1 = \frac{1}{12}(16)(80)^3 = 682.67 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(16)(32)^3 = 43.69 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 682.67 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 1.40902 \times 10^6 \text{ mm}^4 = 1.40902 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with } c = \frac{1}{2}(80) = 40 \text{ mm} = 0.040 \text{ m}$$

$$M = \frac{I\sigma}{c} = \frac{(1.40902 \times 10^6)(150 \times 10^6)}{0.040} = 5.28 \times 10^3 \text{ N}\cdot\text{m} \quad M = 5.28 \text{ kN}\cdot\text{m}$$

Problem 4.6

4.6 Solve Prob. 4.5, assuming that the beam is bent about the y axis.

4.5 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_y = 250 \text{ MPa}$ and $\sigma_c = 450 \text{ MPa}$. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.

$$\text{Allowable stress} = \frac{\sigma_u}{F.S.} = \frac{450}{3.00} = 150 \text{ MPa}$$

$$= 150 \times 10^6 \text{ Pa}$$

Moment of inertia about y-axis

$$I_1 = \frac{1}{12}(80)(16)^3 + (80)(16)(16)^2 = 354.987 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(32)(16)^3 = 10.923 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 354.987 \times 10^3 \text{ mm}^4$$

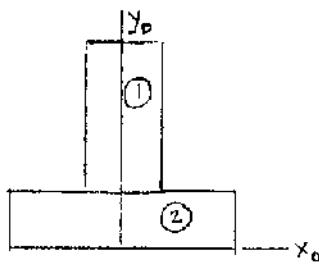
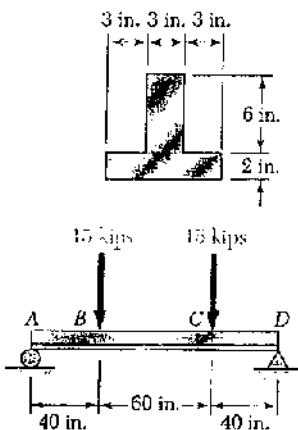
$$I = I_1 + I_2 + I_3 = 720.9 \times 10^3 \text{ mm}^4 = 720.9 \times 10^{-9} \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with } c = \frac{1}{2}(48) = 24 \text{ mm} = 0.024 \text{ m}$$

$$M = \frac{I\sigma}{c} = \frac{(720.9 \times 10^{-9})(150 \times 10^6)}{0.024} = 4.51 \times 10^3 \text{ N}\cdot\text{m} \quad M = 4.51 \text{ kN}\cdot\text{m}$$

Problem 4.7

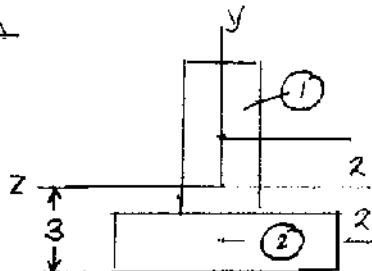
4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.



	A	\bar{y}_0	$A \bar{y}_0$
①	18	5	90
②	18	1	18
Σ	36		108

$$\bar{Y}_0 = \frac{108}{36} = 3 \text{ in}$$

Neutral axis lies 3 in. above the base.

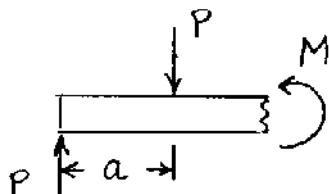


$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{top} = 5 \text{ in.} \quad y_{bot} = -3 \text{ in.}$$



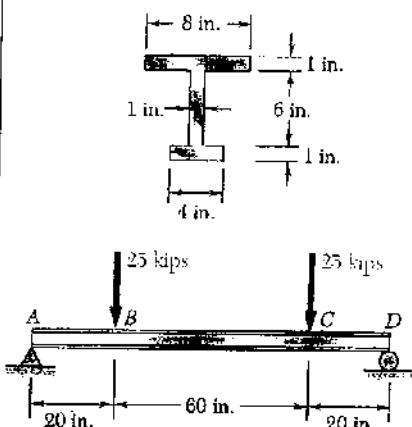
$$M - Pa = 0$$

$$M = Pa = (15)(40) = 600 \text{ kip.in.}$$

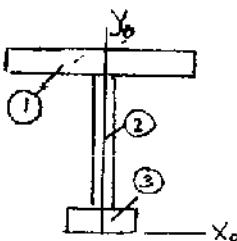
$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(600)(5)}{204} = -14.71 \text{ ksi} \quad (\text{compression})$$

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(600)(-3)}{204} = 8.82 \text{ ksi} \quad (\text{tension})$$

Problem 4.8



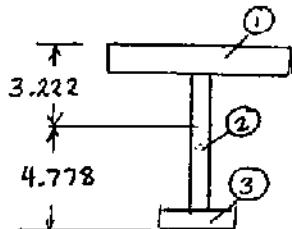
4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



	A	\bar{y}_0	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ		18	86

$$\bar{Y}_0 = \frac{86}{18} = 4.778 \text{ in}$$

Neutral axis lies 4.778 in above the base.



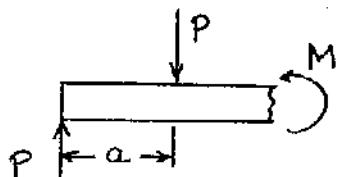
$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.722)^2 \\ = 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2 \\ = 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.54 = 155.16 \text{ in}^4$$

$$y_{top} = 3.222 \text{ in} \quad y_{bot} = -4.778 \text{ in}$$



$$M - Pa = 0$$

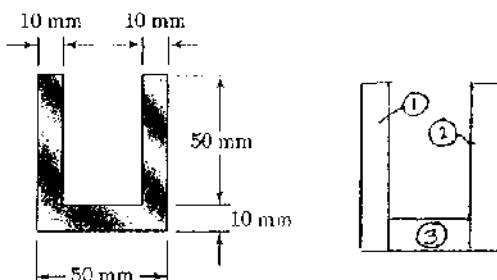
$$M = Pa = (25)(20) = 500 \text{ kip-in.}$$

$$\sigma_{top} = - \frac{My_{top}}{I} = - \frac{(500)(3.222)}{155.16} = - 10.38 \text{ ksi} \quad (\text{compression})$$

$$\sigma_{bot} = - \frac{My_{bot}}{I} = - \frac{(500)(-4.778)}{155.16} = 15.40 \text{ ksi} \quad (\text{tension})$$

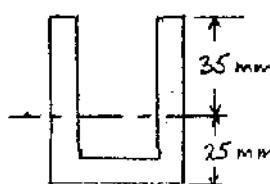
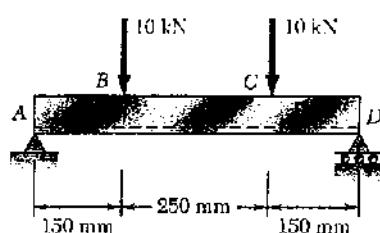
Problem 4.9

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.



	A, mm^2	\bar{y}_o, mm	$A\bar{y}_o, \text{mm}^3$
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3

$$\bar{Y}_o = \frac{37.5 \times 10^3}{1500} = 25 \text{ mm}$$



Neutral axis lies 25 mm above the base.

$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195 \times 10^3 \text{ mm}^4 \quad I_2 = I_1 = 195 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5 \times 10^3 \text{ mm}^4$$

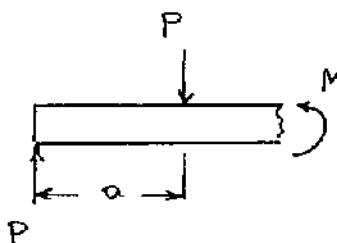
$$I = I_1 + I_2 + I_3 = 512.5 \times 10^3 \text{ mm}^4 = 512.5 \times 10^{-9} \text{ m}^4$$

$$y_{top} = 35 \text{ mm} = 0.035 \text{ m}$$

$$y_{bot} = -25 \text{ mm} = -0.025 \text{ m}$$

$$a = 150 \text{ mm} = 0.150 \text{ m} \quad P = 10 \times 10^3 \text{ N}$$

$$M = Pa = (10 \times 10^3)(0.150) = 1.5 \times 10^3 \text{ N} \cdot \text{m}$$



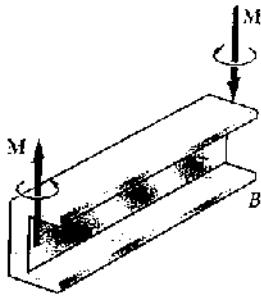
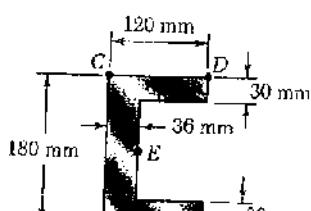
$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1.5 \times 10^3)(0.035)}{512.5 \times 10^{-9}} = -102.4 \times 10^6 \text{ Pa}$$

$$\sigma_{top} = -102.4 \text{ MPa} \quad (\text{compression})$$

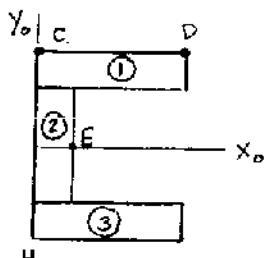
$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1.5 \times 10^3)(-0.025)}{512.5 \times 10^{-9}} = 73.2 \times 10^6 \text{ Pa}$$

$$\sigma_{bot} = 73.2 \text{ MPa} \quad (\text{tension})$$

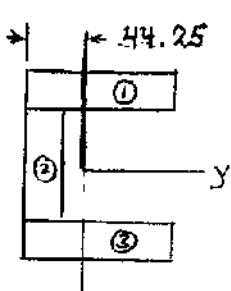
Problem 4.10



4.10 Two equal and opposite couples of magnitude $M = 25 \text{ kN} \cdot \text{m}$ are applied to the channel-shaped beam AB . Observing that the couples cause the beam to bend in a horizontal plane, determine the stress at (a) point C , (b) point D , (c) point E .



	A, mm^2	\bar{x}_o, mm	$A\bar{x}_o, \text{mm}^3$
①	3600	60	216×10^3
②	4320	18	77.76×10^3
③	3600	60	216×10^3
Σ	11520	509.76×10^3	



$$\bar{x} = \frac{509.76 \times 10^3}{11520} = 44.25 \text{ mm}$$

$$y_C = -44.25 \text{ mm} = -0.04425 \text{ m}$$

$$y_D = 120 - 44.25 = 75.75 \text{ mm} \\ = 0.07575 \text{ m}$$

$$y_E = 36 - 44.25 = -8.25 \text{ mm} \\ = -0.00825 \text{ m}$$

$$d_1 = 60 - 44.25 = 15.75 \text{ mm}$$

$$d_2 = 44.25 - 18 = 26.25 \text{ mm}$$

$$d_3 = d_1$$

$$I_1 = I_3 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(30)(120)^3 + (3600)(15.75)^2 = 5.2130 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(120)(36)^3 + (4320)(26.25)^2 = 3.4433 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187 \times 10^6 \text{ mm}^4 = 13.8694 \times 10^{-6} \text{ m}^4$$

$$M = 15 \times 10^3 \text{ N-m}$$

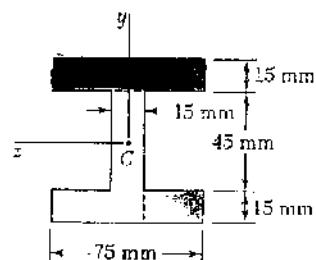
$$(a) \text{ Point } C: \sigma_C = -\frac{My_C}{I} = -\frac{(25 \times 10^3)(-0.04425)}{13.8694 \times 10^{-6}} = 79.8 \times 10^6 \text{ Pa} \\ = 79.8 \text{ MPa}$$

$$(b) \text{ Point } D: \sigma_D = -\frac{My_D}{I} = -\frac{(25 \times 10^3)(0.07575)}{13.8694 \times 10^{-6}} = -136.5 \times 10^6 \text{ Pa} \\ = -136.5 \text{ MPa}$$

$$(c) \text{ Point } E: \sigma_E = -\frac{My_E}{I} = -\frac{(25 \times 10^3)(0.00825)}{13.8694 \times 10^{-6}} = 14.87 \times 10^6 \text{ Pa} \\ = 14.87 \text{ MPa}$$

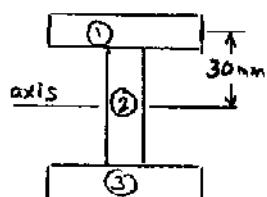
Problem 4.11

4.11 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kN · m, determine the total force acting on the top flange.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

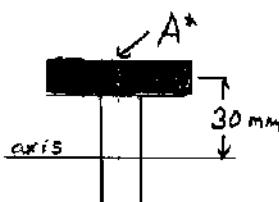
where \bar{y}^* is the centroidal coordinate of the shaded portion and A^* is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(15)^3 + (75)(15)(30)^2 = 1.0336 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(15)(45)^3 = 0.1139 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 1.0336 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 2.1811 \times 10^6 \text{ mm}^4 = 2.1811 \times 10^{-6} \text{ m}^4$$



$$A^* = (75)(15) = 1125 \text{ mm}^2 = 1125 \times 10^{-6} \text{ m}^2$$

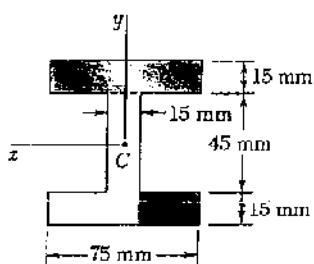
$$\bar{y}^* = 30 \text{ mm} = 0.030 \text{ m}$$

$$F = -\frac{My^*A}{I} = -\frac{(8 \times 10^3)(0.030)(1125 \times 10^{-6})}{2.1811 \times 10^{-6}}$$

$$= -123.8 \times 10^3 \text{ N} = -123.8 \text{ kN}$$

Problem 4.12

4.12 Knowing that a beam of the cross section shown is bent about a vertical axis and that the bending moment is 4 kN · m, determine the total force acting on the shaded portion of the lower flange.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

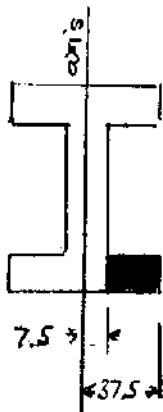
where \bar{y}^* is the centroidal coordinate of the shaded portion and A^* is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 = \frac{1}{12} (15)(75)^3 = 0.52734 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (45)(15)^3 = 0.01256 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 0.5273 \times 10^6$$

$$I = I_1 + I_2 + I_3 = 1.0672 \times 10^6 \text{ mm}^4 = 1.0672 \times 10^{-6} \text{ m}^4$$



$$A^* = (37.5 - 7.5)(15) = 450 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$$

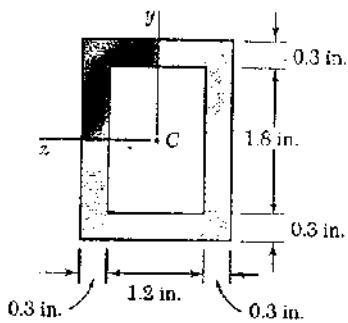
$$\bar{y}^* = \frac{1}{2}(37.5 + 7.5) = 22.5 \text{ mm} = 0.0225 \text{ m}$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(4 \times 10^3)(0.0225)(450 \times 10^{-6})}{1.0672 \times 10^{-6}}$$

$$= 37.9 \times 10^3 \text{ N} = 37.9 \text{ kN}$$

Problem 4.13

4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kip · in., determine the total force acting on the shaded portion of the beam.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

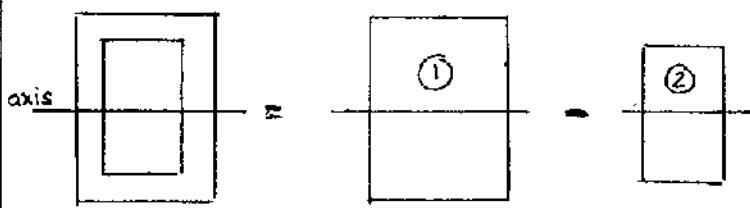
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

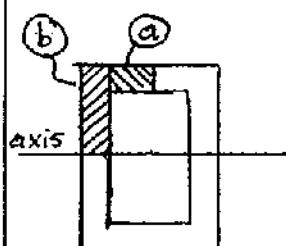
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

where \bar{y}^* is the centroidal coordinate of the shaded portion and A^* is its area.



$$\begin{aligned} I &= I_1 - I_2 \\ &= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3 \\ &= \frac{1}{12} (1.8)(2.4)^3 - \frac{1}{12} (1.2)(1.8)^3 \\ &= 1.4904 \text{ in}^4 \end{aligned}$$

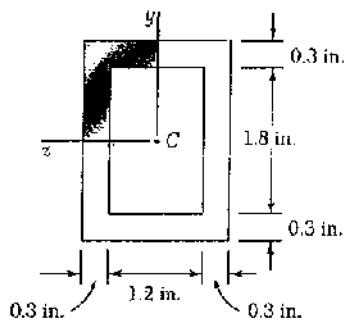


$$\begin{aligned} \bar{y}^* A^* &= \bar{y}_a A_a + \bar{y}_b A_b \\ &= (1.05)(0.9)(0.3) + (0.45)(0.3)(0.9) = 0.405 \text{ in}^2 \end{aligned}$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(8)(0.405)}{1.4904} = 2.17 \text{ kips}$$

Problem 4.14

4.14 Solve Prob. 4.13, assuming that the beam is bent about a vertical axis and that the bending moment is 8 kip · in.



4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kip · in., determine the total force acting on the shaded portion of the beam.

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

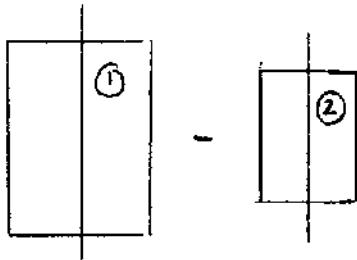
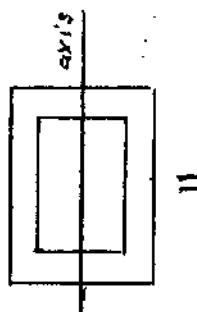
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

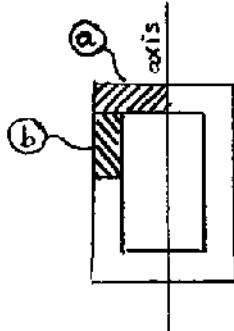
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

where \bar{y}^* is the centroidal coordinate of the shaded portion and A^* is its area.



$$\begin{aligned} I &= I_1 - I_2 \\ &= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3 \\ &= \frac{1}{12} (2.4)(1.8)^3 - \frac{1}{12} (1.8)(1.2)^3 \\ &= 0.9072 \text{ in}^4 \end{aligned}$$

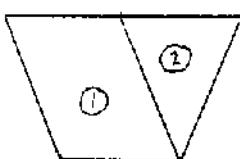
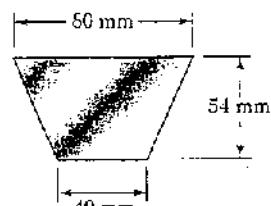


$$\begin{aligned} \bar{y}^* A^* &= \bar{y}_a A_a + \bar{y}_b A_b \\ &= (0.45)(0.3)(0.9) + (0.75)(0.9)(0.3) \\ &= 0.324 \text{ in}^3 \end{aligned}$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(8)(0.324)}{0.9072} = 2.89 \text{ kips.}$$

Problem 4.15

4.15 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple M that can be applied.



	A, mm^2	\bar{y}_c, mm	$A\bar{y}_c, \text{mm}^3$
①	2160	27	58320
②	1080	36	38880
Σ	3240		97200
$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$			

The neutral axis lies 30 mm above the bottom.

$$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m}$$

$$y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$$

$$I_2 = b_2 h_2^3 + A_2 d_2^2 = \frac{1}{36} (40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$$

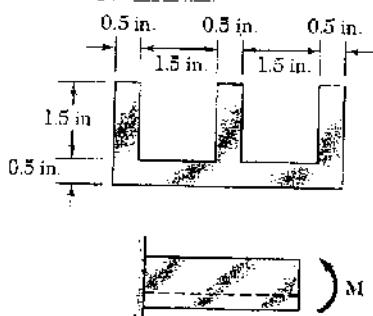
$$|S| = \left| \frac{My}{I} \right| \quad |M| = \left| \frac{S I}{y} \right|$$

top: tension side $M = \frac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

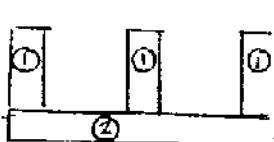
bottom: compression $M = \frac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

Choose the smaller as M_{all} $M_{all} = 3.7908 \times 10^3 \text{ N}\cdot\text{m} = 3.79 \text{ kN}\cdot\text{m} \blacktriangleleft$

Problem 4.16



4.16 Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple M that can be applied.



	A	\bar{y}_o	$A\bar{y}_o$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in}$$

The neutral axis lies 0.75 in. above bottom.

$$y_{top} = 2.0 - 0.75 = 1.25 \text{ in}, \quad y_{bot} = -0.75 \text{ in}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

$$|G| = \left| \frac{My}{I} \right| \quad M = \left| \frac{GJ}{Y} \right|$$

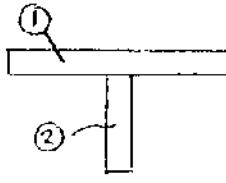
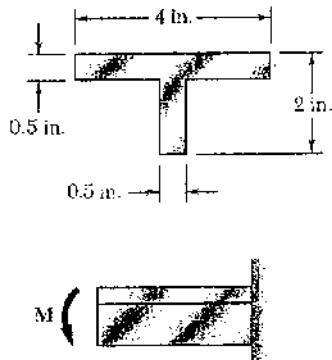
Top: compression $M = \frac{(16)(1.59375)}{1.25} = 20.4 \text{ kip-in}$

Bottom: tension $M = \frac{(12)(1.59375)}{0.75} = 25.5 \text{ kip-in}$

Choose the smaller as M_{all} $M_{all} = 20.4 \text{ kip-in}$

Problem 4.17

4.17 For the casting shown, determine the largest couple M that can be applied without exceeding either of the following allowable stresses: $\sigma_u = +6 \text{ ksi}$, $\sigma_d = -15 \text{ ksi}$.



	A, in^2	$\bar{y}_o, \text{in.}$	$A\bar{y}_o, \text{in}^3$
①	2.00	1.75	3.50
②	0.75	0.75	0.5625
Σ	2.75		4.0625

$$\bar{Y}_o = \frac{4.0625}{2.75} = 1.4773 \text{ in.}$$

The neutral axis lies 1.4773 in. above the bottom.

$$y_{top} = 2 - 1.4773 = 0.5227 \text{ in.}$$

$$y_{bot} = -1.4773 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(4)(0.5)^3 + (2.00)(0.2727)^2 = 0.19040 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(0.5)(1.5)^3 + (0.75)(0.7273)^2 = 0.53735 \text{ in}^4$$

$$I = I_1 + I_2 = 0.72775 \text{ in}^4$$

$$\sigma = \frac{My}{I} \quad M = \frac{\sigma I}{y}$$

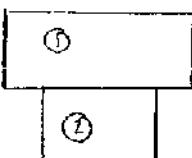
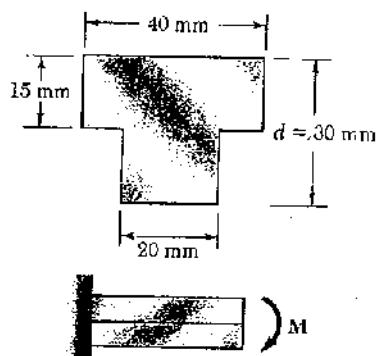
$$\text{Top: tension side} \quad M = \frac{(6 \times 10^6)(0.72775)}{0.5227} = 8.35 \times 10^3 \text{ lb-in}$$

$$\text{Bottom: compression} \quad M = \frac{(-15 \times 10^6)(0.72775)}{-1.4773} = 7.39 \times 10^3 \text{ lb-in}$$

Choose the smaller value $M = 7.39 \times 10^3 \text{ lb-in}$ $M = 7.39 \text{ kip-in}$

Problem 4.18

4.18 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple M that can be applied to the beam.



	A_i, mm^2	\bar{y}_i, mm	$A\bar{y}_i, \text{mm}^3$
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3

$$\bar{Y}_o = \frac{15.75 \times 10^3}{900} = 17.5 \text{ mm}$$

The neutral axis lies 17.5 mm above the bottom.

$$y_{top} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}, \quad y_{bot} = -17.5 \text{ mm} = -0.0175 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (40)(15)^3 + (600)(5)^2 = 26.25 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (20)(15)^3 + (300)(10)^2 = 35.625 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 61.875 \times 10^3 \text{ mm}^4 = 61.875 \times 10^{-9} \text{ m}^4$$

$$|M| = \left| \frac{My}{I} \right| \quad M = \left| \frac{yI}{y} \right|$$

$$\text{Top: tension side} \quad M = \frac{(24 \times 10^6)(61.875 \times 10^{-9})}{0.0125} = 118.8 \text{ N}\cdot\text{m}$$

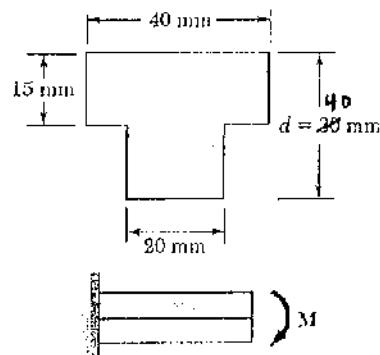
$$\text{Bottom: compression} \quad M = \frac{(30 \times 10^6)(61.875 \times 10^{-9})}{0.0175} = 106.1 \text{ N}\cdot\text{m}$$

Choose smaller value

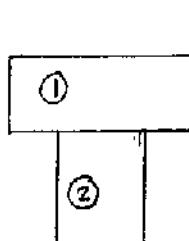
$$M = 106.1 \text{ N}\cdot\text{m}$$

Problem 4.19

4.19 Solve Prob. 4.18, assuming that $d = 40 \text{ mm}$.



4.18 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple M that can be applied to the beam.



	A, mm^2	\bar{y}_o, mm	$A\bar{y}_o, \text{mm}^3$
①	600	32.5	19.5×10^9
②	500	12.5	6.25×10^9
Σ	1100		25.75×10^9

$$\bar{Y}_o = \frac{25.75 \times 10^9}{1100} = 23.41 \text{ mm}$$

The neutral axis lies 23.41 mm above the bottom.

$$y_{top} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$$

$$y_{bot} = -23.41 \text{ mm} = -0.02341 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(9.09)^2 = 60182.7 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(20)(25)^3 + (500)(10.91)^2 = 185556 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 146.383 \times 10^3 \text{ mm}^4 = 146.383 \times 10^{-9} \text{ m}^4$$

$$|σ| = \left| \frac{M Y}{I} \right| \quad M = \left| \frac{G I}{Y} \right|$$

$$\text{Top: tension side} \quad M = \frac{(24 \times 10^6)(146.383 \times 10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$$

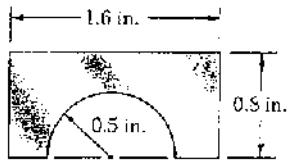
$$\text{Bottom: compression} \quad M = \frac{(30 \times 10^6)(146.383 \times 10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$$

Choose smaller value

$M = 187.6 \text{ N}\cdot\text{m}$

Problem 4.20

4.20 Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple M that can be applied.



① = rectangle ② = semi-circular cutout

$$A_1 = (1.6)(0.8) = 1.28 \text{ in}^2$$

$$A_2 = \frac{\pi}{2}(0.5)^2 = 0.3927 \text{ in}^2$$

$$A = 1.28 - 0.3927 = 0.8873 \text{ in}^2$$

$$\bar{y}_1 = 0.4 \text{ in.} \quad \bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.5)}{3\pi} = 0.2122 \text{ in}$$

$$\bar{Y} = \frac{\sum A \bar{y}}{\sum A} = \frac{(1.28)(0.4) - (0.3927)(0.2122)}{0.8873} = 0.4831 \text{ in.}$$

Neutral axis lies 0.4831 in above the bottom

Moment of inertia about the base

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(1.6)(0.8)^3 - \frac{\pi}{8}(0.5)^4 = 0.24852 \text{ in}^4$$

Centroidal moment of inertia

$$\bar{I} = I_b - A\bar{Y}^2 = 0.24852 - (0.8873)(0.4831)^2 \\ = 0.04144 \text{ in}^4$$

$$y_{top} = 0.8 - 0.4831 = 0.3169 \text{ in.}, \quad y_{bot} = -0.4831 \text{ in.}$$

$$|M| = \left| \frac{My}{I} \right| \quad M = \left| \frac{SI}{y} \right|$$

$$\text{Top: tension side} \quad M = \frac{(12)(0.04144)}{0.3169} = 1.569 \text{ kip-in}$$

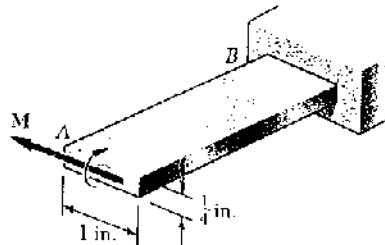
$$\text{Bottom: compression} \quad M = \frac{(16)(0.04144)}{0.4831} = 1.372 \text{ kip-in}$$

Choose the smaller value

$$M = 1.372 \text{ kip-in.}$$

Problem 4.21

4.21 Knowing that $\sigma_y = 24 \text{ ksi}$ for the steel strip AB, determine (a) the largest couple M that can be applied, (b) the corresponding radius of curvature. Use $E = 29 \times 10^6 \text{ psi}$.



$$I = \frac{1}{12} b h^3 = \left(\frac{1}{12}\right)(1)\left(\frac{1}{4}\right)^3 = 1.30208 \times 10^{-3} \text{ in}^4$$

$$\sigma = \frac{Mc}{I} \quad c = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = 0.125 \text{ in.}$$

$$(a) M = \frac{\sigma I}{c} = \frac{(24 \times 10^6)(1.30208 \times 10^{-3})}{0.125}$$

$$M = 250 \text{ lb-in.}$$

$$(b) \frac{\sigma}{\rho} = \frac{\sigma_{max}}{E} \quad \rho = \frac{Ec}{\sigma_{max}}$$

$$\rho = \frac{Ec}{\sigma_{max}} = \frac{(29 \times 10^6)(0.125)}{24 \times 10^6}$$

$$\rho = 151.0 \text{ in.}$$

Problem 4.22

4.22 Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine (a) the maximum stress in a coiled rod, (b) the corresponding bending moment in the rod. Use $E = 200 \text{ GPa}$.



Let D = inside diameter of the drum

d = diameter of rod, $c = \frac{1}{2}d$,

ρ = radius of curvature of center line of rods when bent,

$$\rho = \frac{1}{2}D - \frac{1}{2}d = \frac{1}{2}(1.25) - \frac{1}{2}(6 \times 10^{-3}) = 0.622 \text{ m}$$

$$I = \frac{\pi}{4} d^4 = \frac{\pi}{4} (0.003)^4 = 63.617 \times 10^{-12} \text{ m}^4$$

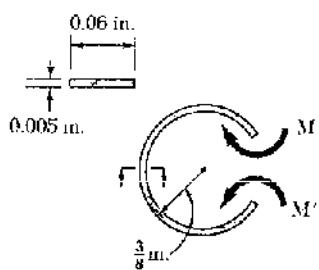
$$(a) \sigma_{max} = \frac{Ec}{\rho} = \frac{(200 \times 10^9)(0.003)}{0.622} = 965 \times 10^6 \text{ Pa}$$

$$\sigma = 965 \text{ MPa}$$

$$(b) M = \frac{EI}{\rho} = \frac{(200 \times 10^9)(63.617 \times 10^{-12})}{0.622} = 20.5 \text{ N-m}$$

$$M = 20.5 \text{ N-m}$$

Problem 4.23



4.23 It is observed that a thin steel strip of 0.06-in. width can be bent into a circle of $\frac{3}{8}$ -in. diameter without any resulting permanent deformation. Knowing that $E = 29 \times 10^6$ psi, determine (a) the maximum stress in the bent strip, (b) the magnitude of the couples required to bend the strip.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.06)(0.005)^3 = 625 \times 10^{-12} \text{ in}^4$$

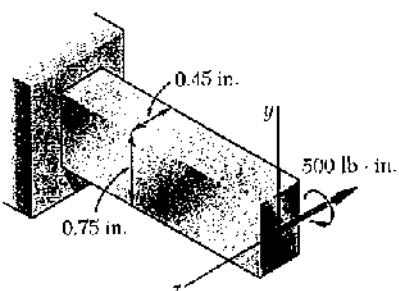
$$\rho = \frac{1}{2} D = \frac{1}{2} \left(\frac{3}{8}\right) = 0.375 \text{ in}$$

$$c = \frac{1}{2} h = 0.0025 \text{ in}$$

$$(a) \sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.0025)}{0.375} = 193.3 \times 10^3 \text{ psi} = 193.3 \text{ ksi}$$

$$(b) M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(625 \times 10^{-12})}{0.375} = 0.0483 \text{ lb-in.}$$

Problem 4.24



4.24 A 500 lb-in. couple is applied to the steel bar shown. (a) Assuming that the couple is applied about the z axis as shown, determine the maximum stress and the radius of curvature of the bar. (b) Solve part a, assuming that the couple is applied about the y axis. Use $E = 29 \times 10^6$ psi.

(a) Bending about z-axis.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.45)(0.75)^3 = 15.8203 \times 10^{-3} \text{ in}^4$$

$$c = \frac{1}{2} h = \frac{1}{2} (0.75) = 0.375 \text{ in.}$$

$$\sigma = \frac{Mc}{I} = \frac{(500)(0.375)}{15.8203 \times 10^{-3}} = 11.85 \times 10^3 \text{ psi}$$

$$\sigma = 11.85 \text{ ksi}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{500}{(29 \times 10^6)(15.8203 \times 10^{-3})} = 1.0898 \times 10^{-3} \text{ in}^{-1}$$

$$\rho = 918 \text{ in}$$

$$\rho = 76.5 \text{ ft}$$

(b) Bending about y-axis

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.75)(0.45)^3 = 5.6953 \times 10^{-3} \text{ in}^4$$

$$c = \frac{1}{2} h = \frac{1}{2}(0.45) = 0.225 \text{ in.}$$

$$\sigma = \frac{Mc}{I} = \frac{(500)(0.225)}{5.6953 \times 10^{-3}} = 19.75 \times 10^3 \text{ psi}$$

$$\sigma = 19.75 \text{ ksi}$$

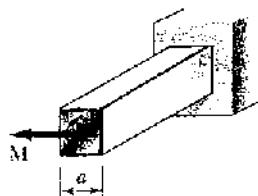
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{500}{(29 \times 10^6)(5.6953 \times 10^{-3})} = 3.0273 \times 10^{-3} \text{ in}^{-1}$$

$$\rho = 330 \text{ in.}$$

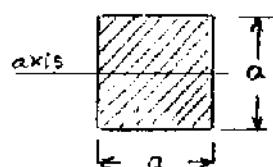
$$\rho = 27.5 \text{ ft}$$

Problem 4.25

4.25 A couple of magnitude M is applied to a square bar of side a . For each of the orientations shown, determine the maximum stress and the curvature of the bar.



(a)

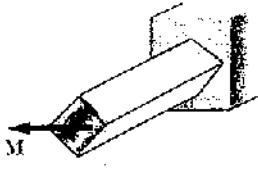


$$I = \frac{1}{12} b h^3 = \frac{1}{12} a a^3 = \frac{a^4}{12}$$

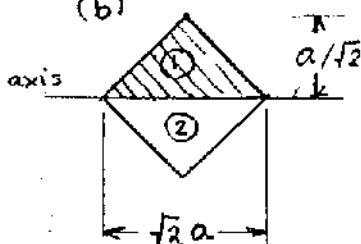
$$C = \frac{a}{2}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M \frac{a}{2}}{\frac{a^4}{12}} = \frac{6M}{a^3}$$

$$\frac{1}{R} = \frac{M}{EI} = \frac{M}{E \frac{a^4}{12}} = \frac{12M}{Ea^4}$$



(b)



For one triangle, the moment of inertia about its base is

$$I_1 = \frac{1}{12} b h^3 = \frac{1}{12} (\sqrt{2}a) \left(\frac{a}{\sqrt{2}}\right)^2 = \frac{a^4}{24}$$

$$I_2 = I_1 = \frac{a^4}{24}$$

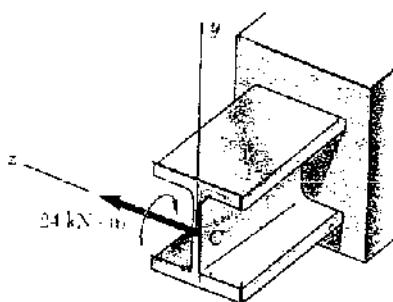
$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M \frac{a}{\sqrt{2}}}{\frac{a^4}{12}} = \frac{6\sqrt{2}M}{a^3}$$

$$\frac{1}{R} = \frac{M}{EI} = \frac{M}{E \frac{a^4}{12}} = \frac{12M}{Ea^4}$$

Problem 4.26



4.26 A 24 kN·m couple is applied to the W200×46.1 rolled-steel beam shown. (a) Assuming that the couple is applied about the z axis as shown, determine the maximum stress and the radius of curvature of the beam. (b) Solve part a, assuming that the couple is applied about the y axis. Use $E = 200$ GPa.

For W200×46.1 rolled steel section:

$$I_x = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$$

$$S_x = 448 \times 10^3 \text{ mm}^3 = 448 \times 10^{-6} \text{ m}^3$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$$

$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

$$(a) M_z = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{448 \times 10^{-6}} = 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}$$

$$\frac{1}{P} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(45.5 \times 10^{-6})} = 2.637 \times 10^{-3} \text{ m}^{-1}$$

$$P = 379 \text{ m}$$

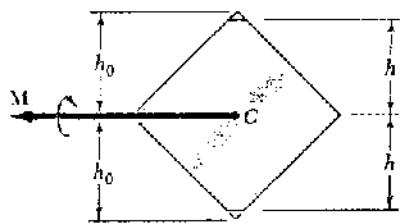
$$(b) M_y = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{151 \times 10^{-6}} = 158.9 \times 10^6 \text{ Pa} = 158.9 \text{ MPa}$$

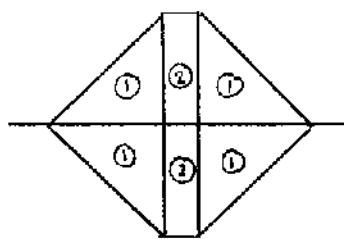
$$\frac{1}{P} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(15.3 \times 10^{-6})} = 7.84 \times 10^{-3} \text{ m}^{-1}$$

$$P = 127.5 \text{ m}$$

Problem 4.27



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple M . Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple M , and determine the value of k .



$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)h^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)(h^3)$$

$$= \frac{1}{3}h^4 + \frac{4}{3}h_0h^3 - \frac{4}{3}h^2h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h$$

$$\sigma_m = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$

For the original square $h = h_0$, $c = h$.

$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

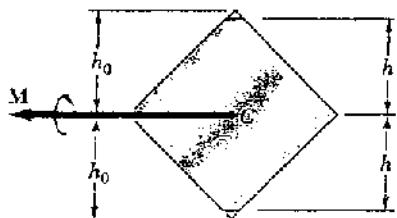
$$\frac{\sigma_m}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - 3)(0.9)h_0(0.9h_0^2)} = 0.950$$

$$\sigma = 0.950 \sigma_0$$

$$k = 0.950$$

Problem 4.28

4.28 In Prob. 4.27, determine (a) the value of h for which the maximum stress σ_m is as small as possible, (b) the corresponding value of k .



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple M . Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple M , and determine the value of k .

$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)h^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)h^3$$

$$= \frac{1}{3}h^4 - \frac{4}{3}h_0h^3 - \frac{4}{3}h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h \quad \frac{I}{c} = \frac{4}{3}h_0h^2 - h^3$$

$$\frac{I}{c} \text{ is maximum at } \frac{d}{dh}\left[\frac{4}{3}h_0h^2 - h^3\right] = 0$$

$$\frac{8}{3}h_0h + 3h^2 = 0 \quad h = \frac{8}{9}h_0$$

$$\frac{I}{c} = \frac{4}{3}h_0\left(\frac{8}{9}h_0\right)^2 - \left(\frac{8}{9}h_0\right)^3 = \frac{256}{729}h_0^3 \quad C = \frac{Mc}{I} = \frac{229}{256} \frac{M}{h_0^3}$$

$$\text{For the original square } h = h_0 \quad c = h_0 \quad \frac{I_0}{C_0} = \frac{1}{3}h_0^3$$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{229}{256} \cdot \frac{1}{3} = \frac{729}{768} \approx 0.949 \quad k = 0.949$$

Problem 4.29

4.29 For the bar and loading of Example 4.01, determine (a) the radius of curvature ρ , (b) the radius of curvature ρ' of a transverse cross section, (c) the angle between the sides of the bar that were originally vertical. Use $E = 29 \times 10^6$ psi and $\nu = 0.29$.

From Example 4.01 $M = 30 \text{ kip-in}$, $I = 1.042 \text{ in}^4$

$$(a) \frac{1}{\rho} = \frac{M}{EI} = \frac{(30 \times 10^3)}{(29 \times 10^6)(1.042)} = 993 \times 10^{-6} \text{ in}^{-1} \quad \rho = 1007 \text{ in.}$$

$$(b) \varepsilon' = \nu \varepsilon = \frac{\nu C}{\rho} = \nu \frac{C}{\rho},$$

$$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(993 \times 10^{-6}) \text{ in}^{-1} = 288 \text{ in}^{-1} \quad \rho' = 3470 \text{ in.}$$

$$(c) \Theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.8}{3470} = 230 \times 10^{-6} \text{ rad} = 0.01320^\circ$$

Problem 4.30

4.30 For the aluminum bar and loading of Sample Problem 4.1, determine (a) the radius of curvature ρ' of a transverse cross section, (b) the angle between the sides of the bar that were originally vertical. Use $E = 10.6 \times 10^6$ psi and $\nu = 0.33$.

From Sample Problem 4.1 $I = 12.97 \text{ in}^4$, $M = 103.8 \text{ kip-in}$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{103.8 \times 10^3}{(10.6 \times 10^6)(12.97)} = 7.55 \times 10^{-6} \text{ in}^{-1}$$

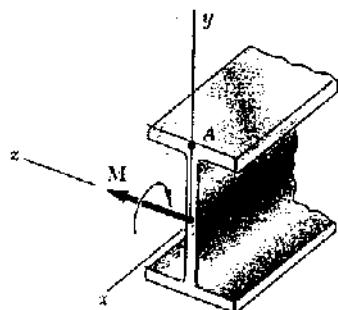
$$(a) \frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.33)(7.55 \times 10^{-6}) = 249 \times 10^{-6} \text{ in}^{-1}$$

$$\rho' = 4010 \text{ in.} = 334 \text{ ft.}$$

$$(b) \Theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{3.25}{4010} = 810 \times 10^{-6} \text{ rad} = 0.0464^\circ$$

Problem 4.31

4.31 A W200 × 31.3 rolled-steel beam is subjected to a couple M of moment 45 kN-m. Knowing that $E = 200 \text{ GPa}$ and $\nu = 0.29$, determine (a) the radius of curvature ρ , (b) the radius of curvature ρ' of a transverse cross section.



For W 200 × 31.3 rolled steel section

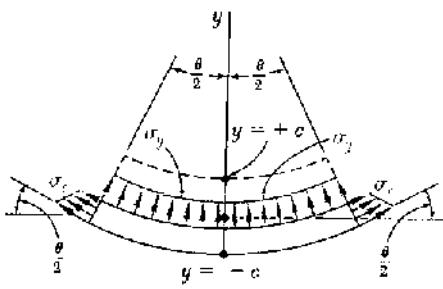
$$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$$

$$(a) \frac{1}{\rho} = \frac{M}{EI} = \frac{45 \times 10^3}{(200 \times 10^9)(31.4 \times 10^{-6})} = 7.17 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 139.6 \text{ m}$$

$$(b) \frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(7.17 \times 10^{-3}) = 2.07 \times 10^{-3} \text{ m}^{-1} \quad \rho' = 481 \text{ m}$$

Problem 4.32



4.32 It was assumed in Sec. 4.3 that the normal stresses σ_z in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (a) derive an approximate expression for σ_y as a function of y , (b) show that $(\sigma_y)_{\max} = -(c/2\rho)(\sigma_x)_{\max}$ and, thus, that σ_y can be neglected in all practical situations. (Hint: Consider the free-body diagram of the portion of beam located below the surface of ordinate y and assume that the distribution of the stress σ_x is still linear.)

Denote the width of the beam by b and the length by L .

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos \frac{\theta}{2} \approx 1$

$$\sum F_y = 0 \quad G_y b L + 2 \int_{-c}^y G_x b dy \sin \frac{\theta}{2} = 0$$

$$G_y = -\frac{2}{L} \sin \frac{\theta}{2} \int_{-c}^y G_x dy \approx -\frac{\theta}{L} \int_{-c}^y G_x dy = -\frac{1}{\rho} \int_{-c}^y G_x dy$$

But $G_x = -(\sigma_x)_{\max} \frac{y}{c}$

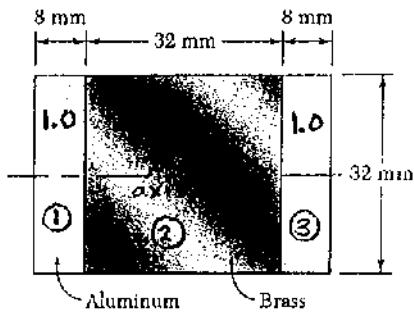
$$(a) \quad G_y = \frac{(\sigma_x)_{\max}}{\rho c} \int_{-c}^y y dy = \frac{(\sigma_x)_{\max}}{\rho c} \left[\frac{y^2}{2} \right]_{-c}^y = \frac{(\sigma_x)_{\max}}{2\rho c} (y^2 - c^2)$$

The maximum value G_y occurs at $y = 0$

$$(b) \quad (G_y)_{\max} = -\frac{(\sigma_x)_{\max} c^2}{2\rho c} = -\frac{(\sigma_x)_{\max} c}{2\rho}$$

Problem 4.33

4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.



	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum $n = 1.0$

For brass $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12} (8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12} (32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 174.7626 \times 10^3 \text{ mm}^4 = 174.7626 \times 10^{-9} \text{ m}^4$$

$$|M_y| = \left| \frac{n M_y}{I} \right| \quad M_y = \left| \frac{G I}{n y} \right|$$

$$\text{Aluminum: } n = 1.0, |y| = 16 \text{ mm} = 0.016 \text{ m}, G = 100 \times 10^6 \text{ Pa}$$

$$M_y = \frac{(100 \times 10^6)(174.7626 \times 10^{-9})}{(1.0)(0.016)} = 1.0923 \times 10^3 \text{ N}\cdot\text{m}$$

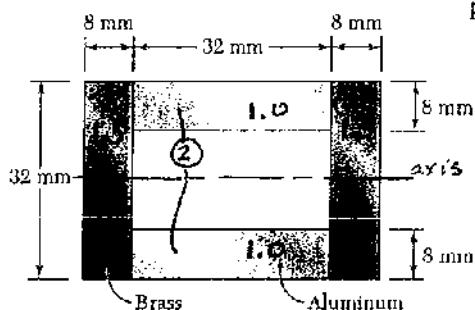
$$\text{Brass: } n = 1.5, |y| = 16 \text{ mm} = 0.016 \text{ m}, G = 160 \times 10^6 \text{ Pa}$$

$$M_y = \frac{(160 \times 10^6)(174.7626 \times 10^{-9})}{(1.5)(0.016)} = 1.1651 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.092 \times 10^3 \text{ N}\cdot\text{m}$

$M = 1.092 \text{ kN}\cdot\text{m}$

Problem 4.34



4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material

For aluminum $n = 1.0$

For brass $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12} (8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 (H_2^3 - h_2^3) = \frac{1.0}{12} (32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$$

$$M = \left| \frac{n My}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N}\cdot\text{m}$$

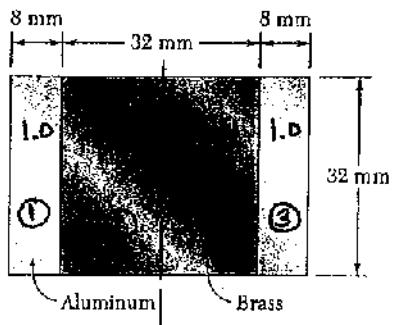
Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N}\cdot\text{m}$$

Choose the smaller value.

$$M = 887 \text{ N}\cdot\text{m}$$

Problem 4.35



4.35 and 4.36 For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

4.35 Bar of Prob. 4.33.

Aluminum	Brass
----------	-------

Modulus of elasticity	70 GPa	105 GPa
-----------------------	--------	---------

Allowable stress	100 MPa	160 MPa
------------------	---------	---------

Use aluminum as the reference material

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} h_1 b_1^3 + n_2 A_1 d_1^2 = \frac{1.0}{12} (32)(8)^3 + (1.0)[(32)(8)](20)^2 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.5}{12} (32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 338.58 \times 10^3 \text{ mm}^4 = 338.58 \times 10^{-9} \text{ m}^4$$

$$|M| = \frac{n My}{I} \quad M = \left| \frac{S I}{ny} \right|$$

Aluminum: $n = 1.0$, $|y| = 24 \text{ mm} = 0.024 \text{ m}$, $S = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(338.58 \times 10^{-9})}{(1.0)(0.024)} = 1.411 \times 10^3 \text{ N}\cdot\text{m}$$

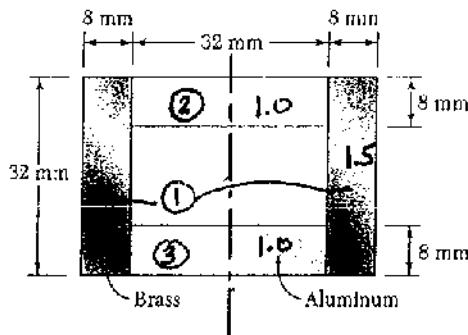
Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $S = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(338.58 \times 10^{-9})}{(1.5)(0.016)} = 2.257 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.411 \times 10^3 \text{ N}\cdot\text{m}$ $M = 1.411 \text{ kN}\cdot\text{m}$

Problem 4.36

4.35 and 4.36 For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.



4.36 Bar of Prob. 4.34

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum $n = 1.0$

For brass $n = E_b/E_a = 105/70 = 1.5$

Values of n and shown on the sketch.

For the transformed section

$$I_1 = \frac{n_1}{12} h_1 (B_1^3 - b_1^3) = \frac{1.5}{12} (32)(48^3 - 32^3) = 311.296 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.0}{12} (8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 354.99 \times 10^3 \text{ mm}^4 = 354.99 \times 10^{-9} \text{ m}^4$$

$$|M| = \left| \frac{n My}{I} \right| \quad M = \left| \frac{EI}{ny} \right|$$

$$\text{Aluminum: } n = 1.0, \quad ly = 16 \text{ mm} = 0.016 \text{ m} \quad \sigma = 100 \times 10^6 \text{ Pa}$$

$$M = \frac{(100 \times 10^6)(354.99 \times 10^{-9})}{(1.0)(0.016)} = 2.2187 \times 10^3 \text{ N}\cdot\text{m}$$

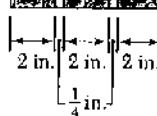
$$\text{Brass: } n = 1.5, \quad ly = 24 \text{ mm} = 0.024 \text{ m} \quad \sigma = 160 \times 10^6 \text{ Pa}$$

$$M = \frac{(160 \times 10^6)(354.99 \times 10^{-9})}{(1.5)(0.024)} = 1.57773 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.57773 \times 10^3 \text{ N}\cdot\text{m}$ $M = 1.578 \text{ kN}\cdot\text{m} \leftarrow$

Problem 4.37

4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity	2×10^6 psi	30×10^6 psi
Allowable stress	2000 psi	22,000 psi

Use wood as the reference material

$$\text{For wood } n_w = 1$$

$$\text{For steel } n = E_s/E_w = 30/2 = 15$$

Properties of the geometric section.

$$\text{Steel: } I_s = \frac{1}{12} \left(\frac{1}{4} + \frac{1}{4} \right) (10)^3 = 41.6667 \text{ in}^4$$

$$\text{Wood: } I_w = \frac{1}{12} (2 + 2 + 2)(10^3) = 500 \text{ in}^4$$

Transformed section

$$I_{\text{trans}} = n_s I_s + n_w I_w = (15)(41.6667) + (1)(500) = 1125 \text{ in}^4$$

$$15I = \left| \frac{n My}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

$$\text{Wood: } n = 1 \quad ly = 5 \text{ in.} \quad \sigma = 2000 \text{ psi}$$

$$M = \frac{(2000)(1125)}{(1)(5)} = 450 \times 10^3 \text{ lb-in}$$

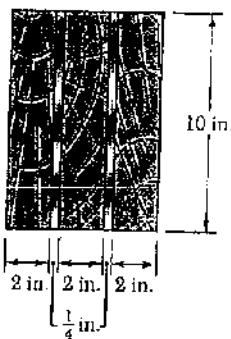
$$\text{Steel: } n = 15 \quad ly = 5 \text{ in.} \quad \sigma = 22,000 \text{ psi}$$

$$M = \frac{(22,000)(1125)}{(15)(5)} = 330 \times 10^3 \text{ lb-in}$$

$$\text{Choose the smaller value } M = 330 \times 10^3 \text{ lb-in}$$

$$M = 330 \text{ kip-in}$$

Problem 4.38



4.38 For the composite member of Prob. 4.37, determine the largest permissible bending moment when the member is bent about a vertical axis.

4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity	2×10^6 psi	30×10^6 psi
Allowable stress	2000 psi	22,000 psi

Use wood as the reference material

$$\text{For wood } n_w = 1$$

$$\text{For steel } n_s = E_s/E_w = 30/2 = 15$$

Properties of the geometric section

$$\text{Total: } I_t = \frac{1}{12} h b^3 = \frac{1}{12}(10)(6.5)^3 = 228.854 \text{ in}^4$$

$$\text{Steel: } I_s = \frac{1}{12} h (B^3 - b^3) = \frac{1}{12}(10)(2.5^3 - 2^3) = 6.3542 \text{ in}^4$$

$$\text{Wood: } I_w = I_t - I_s = 222.5 \text{ in}^4$$

Transformed section

$$I_{trans} = n_s I_s + n_w I_w = (15)(6.3542) + (1)(222.5) = 317.81 \text{ in}^4$$

$$|M| = \left| \frac{n My}{I} \right| \quad M = \left| \frac{S E}{ny} \right|$$

$$\text{Wood: } n = 1 \quad |y| = 3.25 \text{ in.} \quad \sigma = 2000 \text{ psi}$$

$$M = \frac{(2000)(317.81)}{(1)(3.25)} = 195.6 \times 10^3 \text{ lb-in}$$

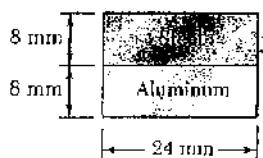
$$\text{Steel: } n = 15 \quad |y| = 1.25 \text{ in.} \quad \sigma = 22000 \text{ psi}$$

$$M = \frac{(22000)(317.81)}{(15)(1.25)} = 372.90 \times 10^3 \text{ lb-in}$$

Choose the smaller value. $M = 195.6 \times 10^3 \text{ lb-in}$

$M = 195.6 \text{ kip-in}$

Problem 4.39



4.39 and 4.40 A steel bar ($E_s = 210 \text{ GPa}$) and an aluminum bar ($E_a = 70 \text{ GPa}$) are bonded together to form the composite bar shown. Determine the maximum stress in (a) the aluminum, (b) the steel, when the bar is bent about a horizontal axis with $M = 60 \text{ N} \cdot \text{m}$.

Use aluminum as the reference material.

For aluminum $n = 1$

For steel $n = E_s/E_a = 210/70 = 3$

Transformed section

①	$n=3$
②	$n=1$

	A, mm^2	nA, mm^2	\bar{y}_o, mm	$nA\bar{y}_o, \text{mm}^3$
①	192	576	12	6912
②	192	192	4	768
Σ		768		7680

$$\bar{Y}_o = \frac{7680}{768} = 10 \text{ mm}$$

The neutral axis lies 10 mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{3}{12} (24)(8)^3 + (576)(2)^2 = 5.376 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12} (24)(8)^3 + (192)(6)^2 = 7.936 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 13.312 \times 10^3 \text{ mm}^4 = 13.312 \times 10^{-9} \text{ m}^4$$

$$M = 60 \text{ N} \cdot \text{m}$$

$$\sigma = -\frac{n My}{I}$$

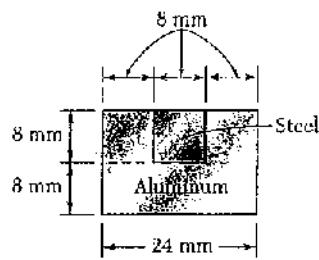
$$(a) \text{ Aluminum } n = 1, y = -10 \text{ mm} = -0.010 \text{ m}$$

$$\sigma_a = -\frac{(1)(60)(-0.010)}{13.312 \times 10^{-9}} = 45.072 \times 10^6 \text{ Pa} \quad \sigma_a = 45.1 \text{ MPa}$$

$$(b) \text{ Steel } n = 3, y = 6 \text{ mm} = 0.006 \text{ m}$$

$$\sigma_s = -\frac{(3)(60)(0.006)}{13.312 \times 10^{-9}} = 81.130 \times 10^6 \text{ Pa} \quad \sigma_s = 81.1 \text{ MPa}$$

Problem 4.40



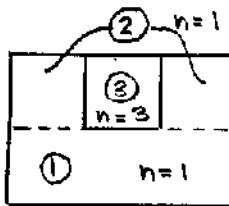
4.39 and 4.40 A steel bar ($E_s = 210 \text{ GPa}$) and an aluminum bar ($E_a = 70 \text{ GPa}$) are bonded together to form the composite bar shown. Determine the maximum stress in (a) the aluminum, (b) the steel, when the bar is bent about a horizontal axis with $M = 60 \text{ N} \cdot \text{m}$.

Use aluminum as the reference material.

For aluminum $n=1$

For steel $n = E_s/E_a = 210/70 = 3$

Transformed section



	A, mm^2	NA, mm^2	\bar{y}_o, mm	$nA\bar{y}_o, \text{mm}^3$
①	192	192	4	768
②	128	128	12	1536
③	64	192	12	2304
		512		4608

$$\bar{Y}_o = \frac{4608}{512} = 9 \text{ mm} \quad \text{The neutral axis lies } 9 \text{ mm above the bottom.}$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12} (24)(8)^3 + (192)(5)^2 = 5.824 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12} (16)(8)^3 + (128)(3)^2 = 1.83467 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 + n_3 A_3 d_3^2 = \frac{3}{12} (8)(8)^3 + (192)(3)^2 = 2.752 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 10.41067 \times 10^3 \text{ mm}^4 = 10.41067 \times 10^{-9} \text{ m}^4$$

$$M = 60 \text{ N} \cdot \text{m}$$

$$\sigma = - \frac{n M y}{I}$$

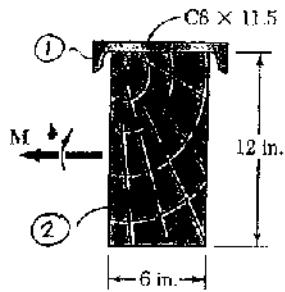
$$(a) \text{ Aluminum } n=1, y = -9 \text{ mm} = -0.009 \text{ m}$$

$$\sigma_a = - \frac{(1)(60)(-0.009)}{10.41067 \times 10^{-9}} = 51.870 \times 10^6 \text{ Pa} \quad \sigma_a = 51.9 \text{ MPa}$$

$$(b) \text{ Steel } n=3, y = 7 \text{ mm} = 0.007 \text{ m}$$

$$\sigma_s = - \frac{(3)(60)(0.007)}{10.41067 \times 10^{-9}} = -121.03 \times 10^6 \text{ Pa} \quad \sigma_s = -121.0 \text{ MPa}$$

Problem 4.41



4.41 and 4.42 The 6×20 -in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel 29×10^6 psi. knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip · in., determine the maximum stress in (a) the wood, (b) the steel.

Use wood as the reference material.

$$\text{For wood} \quad n = 1$$

$$\text{For steel} \quad n = E_s/E_w = 29/1.8 = 16.111$$

$$\text{For } C8 \times 11.5 \text{ channel section, } A = 3.38 \text{ in}^2$$

$$t_w = 0.220 \text{ in.}, \bar{x} = 0.571 \text{ in.}, I_y = 1.32 \text{ in}^4$$

For the composite section the centroid of the channel lies $12 + 0.220 - 0.571 = 11.649$ in. above the base. $\bar{y}_o = 11.649$ in. for channel.

Transformed section

$$\bar{Y}_o = \frac{1066.353}{126.456} \\ = 8.433 \text{ in.}$$

The neutral axis lies 8.433 in. above the bottom.

	$A, \text{ in}^2$	$nA, \text{ in}^2$	$\bar{y}_o, \text{ in.}$	$nA\bar{y}_o, \text{ in}^3$
①	3.38	54.456	11.649	634.353
②	72	72	6	432
		126.456		1066.353

$$I_1 = n_1 \bar{I}_x + n_1 A_1 d_1^2 = (16.111)(1.32) + (54.456)(11.649 - 8.433)^2 = 584.49 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(6)(12)^3 + (72)(8.433 - 6)^2 = 1290.20 \text{ in}^4$$

$$I = I_1 + I_2 = 1874.69 \text{ in}^4$$

$$M = 450 \text{ kip} \cdot \text{in} \quad \sigma = -\frac{n My}{I}$$

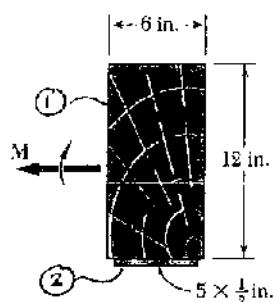
$$(a) \text{ Wood: } n = 1, y = -8.433 \text{ in.}$$

$$\sigma_w = -\frac{(450)(-8.433)}{1874.69} = 2.02 \text{ ksi} \quad \sigma_w = 2.02 \text{ ksi}$$

$$(b) \text{ Steel: } n = 16.111 \quad y = 12 + 0.220 - 8.433 = 3.787 \text{ in.}$$

$$\sigma_s = -\frac{(16.111)(450)(3.787)}{1874.69} = -14.65 \text{ ksi} \quad \sigma_s = -14.65 \text{ ksi}$$

Problem 4.42



4.41 and 4.42 The 6×20 -in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip · in., determine the maximum stress in (a) the wood, (b) the steel.

Use wood as the reference material.

$$\text{For wood} \quad n = 1$$

$$\text{For steel} \quad n = E_s/E_w = 29/1.8 = 16.111$$

Transformed section

	$A_i, \text{ in}^2$	$nA_i, \text{ in}^2$	\bar{y}_o	$nA\bar{y}_o, \text{ in}^3$
①	72	72	6	432
②	2.5	40.278	-0.25	-10.069
		112.278		421.931

The neutral axis lies 3.758 in. above the wood-steel interface.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(6)(12)^3 + (72)(6 - 3.758)^2 = 1225.91 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{16.111}{12}(5)(0.5)^3 + (40.278)(3.758 + 0.25)^2 = 647.87 \text{ in}^4$$

$$I = I_1 + I_2 = 1873.77 \text{ in}^4$$

$$M = 450 \text{ kip} \cdot \text{in} \quad \sigma = -\frac{nMy}{I}$$

$$(a) \text{ Wood: } n = 1 \quad y = 12 - 3.758 = 8.242 \text{ in.}$$

$$\sigma_w = -\frac{(1)(450)(8.242)}{1873.77} = -1.979 \text{ ksi} \quad \sigma_w = -1.979 \text{ ksi}$$

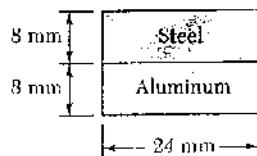
$$(b) \text{ Steel: } n = 16.111 \quad y = -3.758 - 0.5 = -4.258 \text{ in.}$$

$$\sigma_s = -\frac{(16.111)(450)(-4.258)}{1873.77} = 16.48 \text{ ksi} \quad \sigma_s = 16.48 \text{ ksi}$$

Problem 4.43

4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 60 N·m.

4.43 Bar of Prob. 4.39



See solution to Problem 4.39 for calculation of I .

$$I = 13.312 \times 10^{-9} \text{ m}^4 \quad E_a = 70 \times 10^9 \text{ Pa}$$

$$\frac{L}{P} = \frac{M}{EI} = \frac{60}{(70 \times 10^9)(13.312 \times 10^{-9})} = 64.39 \times 10^9 \text{ m}^{-1}$$

$$P = 15.53 \text{ m} \quad \blacksquare$$

Problem 4.44

4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 60 N·m.

4.44 Bar of Prob. 4.40

See solution to Problem 4.40 for calculation of I .

$$I = 10.4106 \times 10^{-9} \text{ m}^4 \quad E_a = 70 \times 10^9 \text{ Pa}$$

$$\frac{L}{P} = \frac{M}{EI} = \frac{60}{(70 \times 10^9)(10.4106 \times 10^{-9})} = 82.33 \text{ m}^{-1}$$

$$P = 12.15 \text{ m} \quad \blacksquare$$

Problem 4.45

4.45 and 4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip·in.

4.45 Bar of Prob. 4.41

See solution to Problem 4.41 for calculation of I .

$$I = 1874.69 \text{ in}^4 \quad E_w = 1.8 \times 10^6 \text{ psi}$$

$$M = 450 \text{ kip-in} = 450 \times 10^3 \text{ lb-in}$$

$$\frac{L}{P} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1874.69)} = 133.36 \times 10^{-6} \text{ in}^{-1}$$

$$P = 7499 \text{ in.} = 625 \text{ ft} \quad \blacksquare$$

Problem 4.46

4.45 and 4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip·in.

4.46 Bar of Prob. 4.42

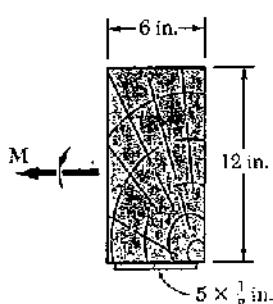
See solution to Problem 4.42 for calculation of I .

$$I = 1873.77 \text{ in}^4 \quad E_w = 1.8 \times 10^6 \text{ psi}$$

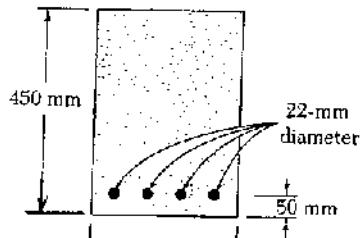
$$M = 450 \text{ kip-in} = 450 \times 10^3 \text{ lb-in}$$

$$\frac{L}{P} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1873.77)} = 133.42 \times 10^{-6} \text{ in}^{-1}$$

$$P = 7495 \text{ in.} = 625 \text{ ft} \quad \blacksquare$$



Problem 4.47

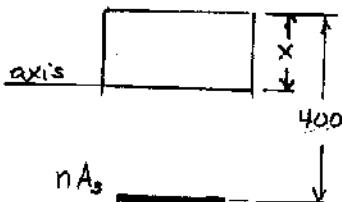


4.47 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)(\frac{\pi}{4})(22)^2 = 1.5205 \times 10^3 \text{ mm}^2$$

$$nA_s = 12.164 \times 10^3 \text{ mm}^2$$



'Locate the neutral axis

$$250 \times \frac{x}{2} - (12.164 \times 10^3)(400 - x) = 0$$

$$125x^2 + 12.164 \times 10^3 x - 4.8657 \times 10^6 = 0$$

Solving for x

$$x = \frac{-12.164 \times 10^3 + \sqrt{(12.164 \times 10^3)^2 + (4)(125)(4.8657 \times 10^6)}}{(2)(125)}$$

$$x = 154.55 \text{ mm}, \quad 400 - x = 245.45 \text{ mm}$$

$$\begin{aligned} I &= \frac{1}{3} 250 x^3 + (12.164 \times 10^3)(400 - x)^2 \\ &= \frac{1}{3} (250)(154.55)^3 + (12.164 \times 10^3)(245.45)^2 \\ &= 1.0404 \times 10^9 \text{ mm}^4 = 1.0404 \times 10^{-3} \text{ m}^4 \end{aligned}$$

$$\sigma = -\frac{n My}{I}$$

(a) Steel: $y = -245.45 \text{ mm} = -0.24545 \text{ m}$

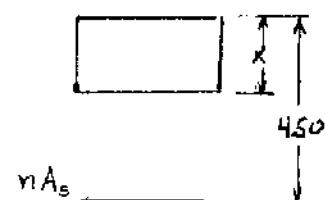
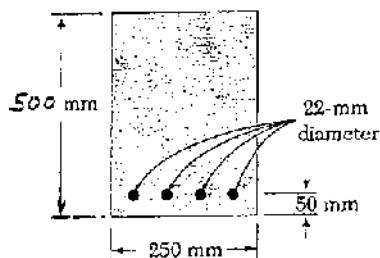
$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.24545)}{1.0404 \times 10^{-3}} = 330 \times 10^6 \text{ Pa} = 330 \text{ MPa}$$

(b) Concrete: $y = 154.55 \text{ mm} = 0.15455 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.15455)}{1.0404 \times 10^{-3}} = -26.0 \times 10^6 \text{ Pa} = -26.0 \text{ MPa}$$

Problem 4.48

4.48 Solve Prob. 4.47, assuming that the 450-mm depth is increased to 500 mm.



$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4) \left(\frac{\pi}{4} \right) (22)^2 = 1.5205 \times 10^3 \text{ mm}^2$$

$$nA_s = 12.164 \times 10^3 \text{ mm}^2$$

Locate the neutral axis

$$250 \times \frac{x}{2} - (12.164 \times 10^3)(450 - x) = 0$$

$$125x^2 + 12.164 \times 10^3 x - 5.4738 \times 10^6 = 0$$

Solving for x

$$x = \frac{-12.164 \times 10^3 + \sqrt{(12.164 \times 10^3)^2 + (4)(125)(5.4738 \times 10^6)}}{(2)(125)}$$

$$x = 166.19 \text{ mm}, \quad 450 - x = 283.81 \text{ mm}$$

$$\begin{aligned} I &= \frac{1}{3}(250)x^3 + (12.164 \times 10^3)(450 - x)^2 \\ &= \frac{1}{3}(250)(166.19)^3 + (12.164 \times 10^3)(283.81)^2 \\ &= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4 \end{aligned}$$

$$\sigma = -\frac{nM_y}{I}$$

$$(a) \text{ Steel: } y = -283.81 \text{ mm} = -0.28381 \text{ m}$$

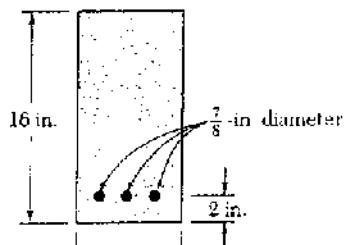
$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.28381)}{1.3623 \times 10^{-3}} = 292 \times 10^6 \text{ Pa} = 292 \text{ MPa} \blacktriangleleft$$

$$(b) \text{ Concrete: } y = 166.19 \text{ mm} = 0.16619 \text{ m}$$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.16619)}{1.3623 \times 10^{-3}} = -21.3 \times 10^6 \text{ Pa} = -21.3 \text{ MPa} \blacktriangleleft$$

Problem 4.49

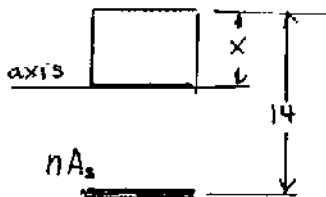
4.49 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 30×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.



$$n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3 \times 10^6} = 10$$

$$A_s = 3 \cdot \frac{\pi}{4} d^2 = 3 \left(\frac{\pi}{4}\right) \left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2$$

$$nA_s = 18.040 \text{ in}^2$$



Locate neutral axis.

$$8 \times \frac{x}{2} - (18.040)(14-x) = 0$$

$$4x^2 + 18.040x - 252.56 = 0$$

$$\text{Solve for } x \quad x = \frac{-18.040 + \sqrt{18.040^2 + (4)(4)(252.56)}}{(2)(4)} = 6.005 \text{ in.}$$

$$14 - x = 7.995 \text{ in}$$

$$I = \frac{1}{3} (8)x^3 + nA_s(14-x)^2 = \frac{1}{3}(8)(6.005)^3 + (18.040)(7.995)^2 \\ = 1730.4 \text{ in}^4$$

$$|M| = \left| \frac{nMy}{I} \right| \therefore M = \frac{6I}{ny}$$

$$\text{Concrete: } n = 1.0, \quad ly = 6.005 \text{ in}, \quad |M| = 1350 \text{ psi}$$

$$M = \frac{(1350)(1730.5)}{(1.0)(6.005)} = 389 \times 10^3 \text{ lb-in} = 389 \text{ kip-in}$$

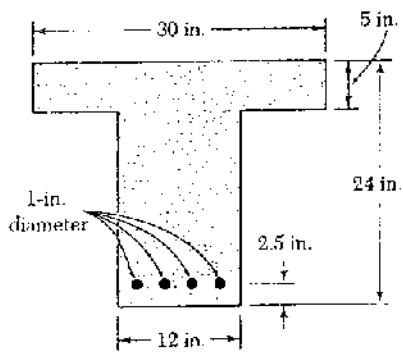
$$\text{Steel: } n = 10, \quad ly = 7.995, \quad \sigma = 20 \times 10^3 \text{ psi}$$

$$M = \frac{(20 \times 10^3)(1730.5)}{(10)(7.995)} = 433 \times 10^3 \text{ lb-in} = 433 \text{ kip-in}$$

$$\text{Choose the smaller value } M = 389 \text{ kip-in} = 32.4 \text{ kip-ft}$$

Problem 4.50

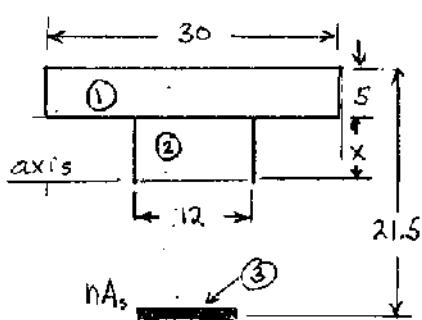
4.50 Knowing that the bending moment in the reinforced concrete beam is +150 kip·ft and that the modulus of elasticity is 3.75×10^6 psi for the concrete and 30×10^6 psi for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.



$$n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3.75 \times 10^6} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = 4 \left(\frac{\pi}{4}\right)(1)^2 = 3.1416 \text{ in}^2$$

$$nA_s = 25.133 \text{ in}^2$$



Locate the neutral axis

$$(30)(5)(x + 2.5) + 12 \times \frac{x}{2} - (25.133)(16.5 - x) = 0$$

$$150x + 375 + 6x^2 - 414.69 + 25.133x = 0$$

$$6x^2 + 175.133x - 39.69 = 0$$

Solve for x $x = \frac{-175.133 + \sqrt{(175.133)^2 + (4)(6)(39.69)}}{(2)(6)} = 0.225 \text{ in.}$

$$16.5 - x = 16.275 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h^3 + A_1 d_1^2 = \frac{1}{12}(30)(5)^3 + (30)(5)(2.725)^2 = 1426.3 \text{ in}^4$$

$$I_2 = \frac{1}{3} b_2 x^3 = \frac{1}{3}(12)(0.225)^3 = 0.1 \text{ in}^4$$

$$I_3 = nA_s d_3^2 = (25.133)(16.275)^2 = 6657.1 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 8083.5 \text{ in}^4$$

$$\sigma = -\frac{n M y}{I} \quad \text{where} \quad M = 150 \text{ kip-ft} = 1800 \text{ kip-in.}$$

(a) Steel $n = 8.0$, $y = -16.275 \text{ in.}$

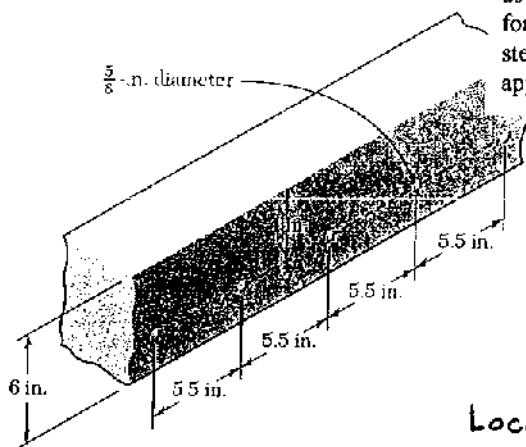
$$\sigma = -\frac{(8.0)(1800)(-16.275)}{8083.5} = 29.0 \text{ ksi}$$

(b) Concrete $n = 1.0$, $y = 5.225 \text{ in.}$

$$\sigma = -\frac{(1.0)(1800)(5.225)}{8083.5} = -1.163 \text{ ksi}$$

Problem 4.51

4.51 A concrete slab is reinforced by $\frac{5}{8}$ -in.-diameter rods placed on 5.5-in. centers as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1400 psi for the concrete and 20 ksi for the steel, determine the largest bending moment per foot of width that can be safely applied to the slab.



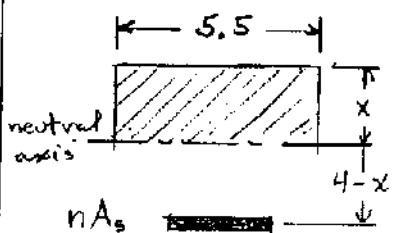
$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3 \times 10^6} = 9.6667$$

Consider a section 5.5 in. wide.

$$A_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4} \left(\frac{5}{8}\right)^2 = 0.8068 \text{ in}^2$$

$$nA_s = 2.9657 \text{ in}^2$$

Locate the neutral axis.



$$5.5x \frac{x}{2} - (4-x)(2.9657) = 0$$

$$2.75x^2 + 2.9657x - 11.8628 = 0$$

Solving for x

$$x = 1.6066 \text{ in.} \quad 4-x = 2.3934 \text{ in.}$$

$$I = \frac{1}{3}(5.5)x^3 + (2.9657)(4-x)^2$$

$$= \frac{1}{3}(5.5)(1.6066)^3 + (2.9657)(2.3934)^2$$

$$= 24.591 \text{ in}^4$$

$$|σ| = \left| \frac{nM_y}{I} \right|$$

$$M = \left| \frac{Iσ}{ny} \right|$$

Concrete: $n = 1$, $y = 1.6066 \text{ in.}$, $σ = 1400 \text{ psi}$

$$M = \frac{(24.591)(1400)}{(1.01)(1.6066)} = 21.429 \times 10^3 \text{ lb-in}$$

Steel: $n = 9.6667$, $y = 2.3934 \text{ in.}$, $σ = 20 \text{ ksi} = 20 \times 10^3 \text{ psi}$

$$M = \frac{(24.591)(20 \times 10^3)}{(9.6667)(2.3934)} = 21.258 \times 10^3 \text{ lb-in}$$

Choose the smaller value as the allowable moment for a 5.5 width.

$$M = 21.258 \times 10^3 \text{ lb-in}$$

For a 1 ft = 12 in. width

$$M = \frac{12}{5.5} (21.258 \times 10^3) = 46.38 \times 10^3 \text{ lb-in}$$

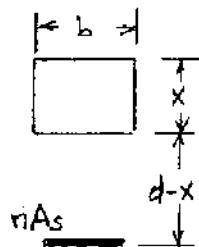
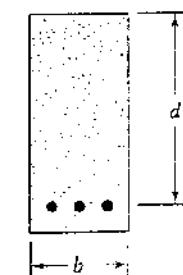
$$M = 46.38 \text{ kip-in} = 3.87 \text{ kip-ft}$$

Problem 4.52

4.52 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c . Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \frac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.



$$\sigma_s = \frac{n M (d - x)}{I} \quad \epsilon_c = \frac{M x}{\pm}$$

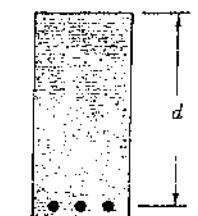
$$\frac{\sigma_s}{\sigma_c} = \frac{n (d - x)}{x} = n \frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n} \frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \frac{E_c \sigma_s}{E_s \sigma_c}}$$

Problem 4.53

4.53 For the concrete beam shown, the modulus of elasticity is 3.5×10^6 psi for the concrete and 29×10^6 psi for the steel. Knowing that $b = 8$ in. and $d = 22$ in., and using an allowable stress of 1800 psi for the concrete and 20 ksi for the steel, determine (a) the required area A_s of the steel reinforcement if the beam is to be balanced, (b) the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)



The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c .

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.5 \times 10^6} = 8.2857$$

$$\sigma_s = \frac{n M(d-x)}{I} \quad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n \frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n} \frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.2857} \cdot \frac{20 \times 10^3}{1800} = 2.3410$$

$$x = 0.42717 \text{ in.} \quad d = (0.42717)(22) = 9.398 \text{ in.}$$

$$d-x = 22 - 9.398 = 12.602 \text{ in.}$$

Locate neutral axis

$$b \times \frac{x}{2} - nA_s(d-x) = 0$$

$$(a) \quad A_s = \frac{b x^2}{2n(d-x)} = \frac{(8)(9.398)^2}{(2)(8.2857)(12.602)} = 3.3835 \text{ in}^2 \quad A_s = 3.38 \text{ in}^2$$

$$I = \frac{1}{3} b x^3 + nA_s(d-x)^2 = \frac{1}{3}(8)(9.398)^3 + (8.2857)(3.3835)(12.602)^2 \\ = 6665.6 \text{ in}^4$$

$$\sigma = \frac{n My}{I} \quad M = \frac{\sigma I}{n y}$$

$$\text{Concrete: } n = 1.0 \quad y = 9.398 \text{ in.} \quad \sigma = 1800 \text{ psi}$$

$$M = \frac{(1800)(6665.6)}{(1.0)(9.398)} = 1.277 \times 10^6 \text{ lb-in}$$

$$\text{Steel: } n = 8.2857 \quad |y| = 12.602 \text{ in.} \quad \sigma = 20 \times 10^3 \text{ psi}$$

$$M = \frac{(20 \times 10^3)(6665.6)}{(8.2857)(12.602)} = 1.277 \times 10^6 \text{ lb-in}$$

Note that both values are the same for balanced design

$$(b) \quad M = 1.277 \times 10^6 \text{ kip-in} = 106.4 \text{ kip-ft}$$

Problem 4.54

4.54 For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that $b = 200 \text{ mm}$ and $d = 450 \text{ mm}$ and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine (a) the required area A_s of the steel reinforcement if the beam is to be balanced, (b) the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)



$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{25 \times 10^9} = 8.0$$

$$\sigma_s = \frac{n M(d-x)}{I} \quad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n \frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n} \frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0} \cdot \frac{140 \times 10^6}{12.5 \times 10^6} = 2.40$$

$$x = 0.41667 d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis

$$bx \frac{x}{2} - n A_s (d-x)$$

$$(a) A_s = \frac{bx^2}{2n(d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2 \quad (a) A_s = 1674 \text{ mm}^2$$

$$I = \frac{1}{3} b x^3 + n A_s (d-x)^2 = \frac{1}{3} (200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4$$

$$\sigma = \frac{n My}{I} \quad M = \frac{I \sigma}{ny}$$

Concrete: $n = 1.0 \quad y = 187.5 \text{ mm} = 0.1875 \text{ m} \quad \sigma = 12.5 \times 10^6 \text{ Pa}$

$$M = \frac{(1.3623 \times 10^{-3})(12.5 \times 10^6)}{(1.0)(0.1875)} = 90.8 \times 10^3 \text{ N-m}$$

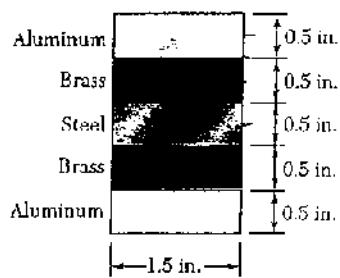
Steel: $n = 8.0 \quad y = 262.5 \text{ mm} = 0.2625 \text{ m} \quad \sigma = 140 \times 10^6 \text{ Pa}$

$$M = \frac{(1.3623 \times 10^{-3})(140 \times 10^6)}{(8.0)(0.2625)} = 90.8 \times 10^3 \text{ N-m}$$

Note that both values are the same for balanced design.

(b) $M = 90.8 \text{ kN-m}$

Problem 4.55



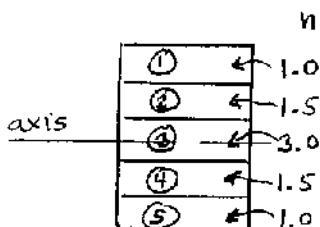
4.55 and 4.56 Five metal strips, each of 0.5×1.5 -in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment 12 kip · in., determine (a) the maximum stress in each of the three metals, (b) the radius of curvature of the composite beam.

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum.}$$



For the transformed section

$$I_1 = \frac{n_1 b_1 h_1^3}{12} + n_1 A_1 d_1^2 = \frac{1}{12}(1.5)(0.5)^3 + (0.75)(1.0)^2 = 0.7656 \text{ in}^4$$

$$I_2 = \frac{n_2 b_2 h_2^3}{12} + n_2 A_2 d_2^2 = \frac{1.5}{12}(1.5)(0.5)^3 + (1.5)(0.75)(0.5)^2 = 0.3047 \text{ in}^4$$

$$I_3 = \frac{n_3 b_3 h_3^3}{12} = \frac{3.0}{12}(1.5)(0.5)^3 = 0.0464 \text{ in}^4$$

$$I_4 = I_2 = 0.3047 \text{ in}^4$$

$$I_5 = I_1 = 0.7656 \text{ in}^4$$

$$I = \sum_i I_i = 2.1875 \text{ in}^4$$

$$(a) \text{ Aluminum: } \sigma = \frac{n M Y}{I} = \frac{(1.0)(12)(1.25)}{2.1875} = 6.86 \text{ ksi}$$

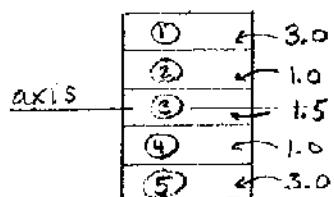
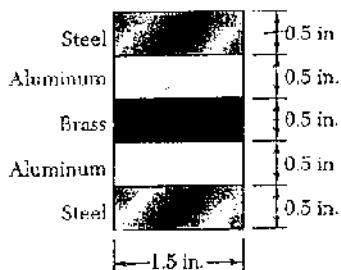
$$\text{Brass: } \sigma = \frac{n M Y}{I} = \frac{(1.5)(12)(0.75)}{2.1875} = 6.17 \text{ ksi}$$

$$\text{Steel: } \sigma = \frac{n M Y}{I} = \frac{(3.0)(12)(0.25)}{2.1875} = 4.11 \text{ ksi}$$

$$(b) \frac{1}{\rho} = \frac{M}{E_a I} = \frac{12 \times 10^3}{(10 \times 10^6)(2.1875)} = 548.57 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 1823 \text{ in} = 151.9 \text{ ft.}$$

Problem 4.56



4.55 and 4.56 Five metal strips, each of 0.5×1.5 -in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment 12 kip · in., determine (a) the maximum stress in each of the three metals, (b) the radius of curvature of the composite beam.

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum}$$

For the transformed section

$$\begin{aligned} I_1 &= \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 \\ &= \frac{3.0}{12} (1.5)(0.5)^3 + (3.0)(0.75)(1.0)^2 = 2.2969 \text{ in}^4 \end{aligned}$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12} (1.5)(0.5)^3 + (1.0)(0.75)(0.5)^2 = 0.2031 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12} (1.5)(0.5)^3 = 0.0234 \text{ in}^4$$

$$I_4 = I_2 = 0.2031 \text{ in}^4, \quad I_5 = I_1 = 2.2969 \text{ in}^4$$

$$I = \sum_i I_i = 5.0234 \text{ in}^4$$

$$(a) \text{ Steel: } \sigma = \frac{n My}{I} = \frac{(3.0)(12)(1.25)}{5.0234} = 8.96 \text{ ksi}$$

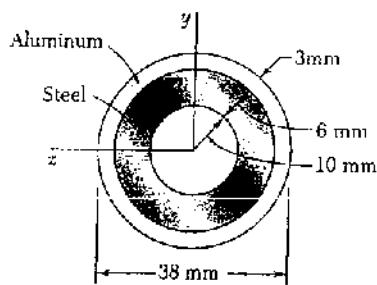
$$\text{Aluminum: } \sigma = \frac{n My}{I} = \frac{(1.0)(12)(0.75)}{5.0234} = 1.792 \text{ ksi}$$

$$\text{Brass: } \sigma = \frac{n My}{I} = \frac{(1.5)(12)(0.25)}{5.0234} = 0.896 \text{ ksi}$$

$$(b) \frac{1}{\rho} = \frac{M}{E_a I} = \frac{12 \times 10^3}{(10 \times 10^6)(5.0234)} = 238.89 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 4186 \text{ in.} = 349 \text{ ft}$$

Problem 4.57



4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (a) in the aluminum, (b) in the steel.

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel}$$

$$\text{Steel: } I_1 = n_1 \frac{\pi}{4} (r_o^4 - r_i^4) = (3.0) \frac{\pi}{4} (16^4 - 10^4) = 130.85 \times 10^3 \text{ mm}^4$$

$$\text{Aluminum: } I_2 = n_2 \frac{\pi}{4} (r_o^4 - r_i^4) = (1.0) \frac{\pi}{4} (19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 181.73 \times 10^3 \text{ mm}^4 = 181.73 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ Aluminum: } c = 19 \text{ mm} = 0.019 \text{ m}$$

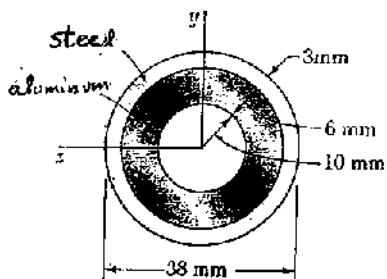
$$\sigma = \frac{n M c}{I} = \frac{(1.0)(500)(0.019)}{181.73 \times 10^{-9}} = 52.3 \times 10^6 \text{ Pa} = 52.3 \text{ MPa}$$

$$(b) \text{ Steel: } c = 16 \text{ mm} = 0.016 \text{ m}$$

$$\frac{n M c}{I} = \frac{(3.0)(500)(0.016)}{181.73 \times 10^{-9}} = 132.1 \times 10^6 \text{ Pa} = 132.1 \text{ MPa}$$

Problem 4.58

4.58 Solve Prob. 4.57, assuming that the 6-mm-thick inner pipe is made of aluminum and that the 3-mm-thick outer pipe is made of steel.



4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (a) in the aluminum, (b) in the steel.

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel.}$$

$$\text{Steel: } I_1 = n_1 \frac{\pi}{4} (r_o^4 - r_i^4) = (3.0) \frac{\pi}{4} (19^4 - 16^4) = 152.65 \times 10^3 \text{ mm}^4$$

$$\text{Aluminum: } I_2 = n_2 \frac{\pi}{4} (r_o^4 - r_i^4) = (1.0) \frac{\pi}{4} (16^4 - 10^4) = 43.62 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 196.27 \times 10^3 \text{ mm}^4 = 196.27 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ Aluminum: } c = 16 \text{ mm} = 0.016 \text{ m}$$

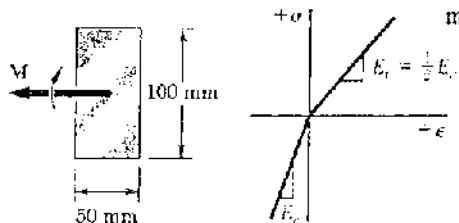
$$\sigma = \frac{n M c}{I} = \frac{(1.0)(500)(0.016)}{196.27 \times 10^{-9}} = 40.8 \times 10^6 \text{ Pa} = 40.8 \text{ MPa}$$

$$(b) \text{ Steel: } c = 19 \text{ mm} = 0.019 \text{ m}$$

$$\sigma = \frac{n M c}{I} = \frac{(3.0)(500)(0.019)}{196.27 \times 10^{-9}} = 145.2 \times 10^6 \text{ Pa} = 145.2 \text{ MPa}$$

Problem 4.59

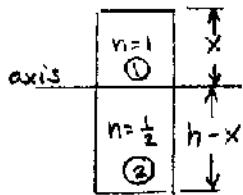
4.59 The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600 \text{ N} \cdot \text{m}$, determine the maximum (a) tensile stress, (b) compressive stress.



$n = \frac{1}{2}$ on the tension side of neutral axis

$n = 1$ on the compression side

Locate neutral axis.



$$n_1 b x \frac{x}{2} - n_2 b (h-x) \frac{h-x}{2} = 0$$

$$\frac{1}{2} b x^2 - \frac{1}{4} b (h-x)^2 = 0$$

$$x^2 = \frac{1}{2} (h-x)^2 \quad x = \frac{1}{\sqrt{2}} (h-x)$$

$$x = \frac{1}{\sqrt{2}+1} h = 0.41421 h = 41.421 \text{ mm}$$

$$h-x = 58.579 \text{ mm}$$

$$I_1 = n_1 \frac{1}{3} b x^3 = (1) \left(\frac{1}{3}\right) (50) (41.421)^3 = 1.1844 \times 10^6 \text{ mm}^4$$

$$I_2 = n_2 \frac{1}{3} b (h-x)^3 = \left(\frac{1}{2}\right) \left(\frac{1}{3}\right) (50) (58.579)^3 = 1.6751 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.8595 \times 10^6 \text{ mm}^4 = 2.8595 \times 10^{-6} \text{ m}^4$$

(a) tensile stress: $n = \frac{1}{2}$, $y = -58.579 \text{ mm} = -0.058579 \text{ m}$

$$\sigma = -\frac{n My}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595 \times 10^{-6}} = 6.15 \times 10^6 \text{ Pa} \\ = 6.15 \text{ MPa}$$

(b) compressive stress: $n = 1$, $y = 41.421 \text{ mm} = 0.041421 \text{ m}$

$$\sigma = -\frac{n My}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595 \times 10^{-6}} = -8.69 \times 10^6 \text{ Pa} \\ = -8.69 \text{ MPa}$$

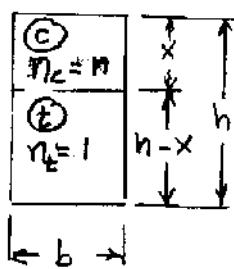
Problem 4.60

*4.60 A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$



Use E_t as the reference modulus.

Then $E_c = n E_t$

Locate neutral axis

$$nb \times \frac{x}{2} - b(h-x) \frac{h-x}{2} = 0$$

$$nx^2 - (h-x)^2 = 0 \quad \sqrt{n}x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \quad h-x = \frac{\sqrt{n}h}{\sqrt{n}+1}$$

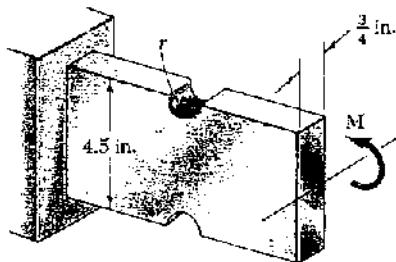
$$\begin{aligned} I_{trans} &= \frac{1}{3} b x^3 + \frac{1}{3} b (h-x)^3 = \left[\frac{h}{3} \left(\frac{1}{\sqrt{n}+1} \right)^3 + \left(\frac{\sqrt{n}h}{\sqrt{n}+1} \right)^3 \right] bh^3 \\ &= \frac{1}{3} \left(\frac{n+h}{\sqrt{n}+1} \right)^3 bh^3 = \frac{1}{3} \frac{n(1+\sqrt{n})}{(\sqrt{n}+1)^3} bh^3 = \frac{1}{3} \cdot \frac{n}{(\sqrt{n}+1)^2} bh^3 \end{aligned}$$

$$\frac{1}{\rho} = \frac{M}{E_t I_{trans}} = \frac{M}{E_r I} \quad \text{where } I = \frac{1}{12} bh^3$$

$$E_r I = E_t I_{trans}$$

$$\begin{aligned} E_r &= \frac{E_t I_{trans}}{I} = \frac{12}{bh^3} \cdot E_t \cdot \frac{n}{3(\sqrt{n}+1)^2} bh^3 \\ &= \frac{4 E_t E_c / E_t}{(\sqrt{E_c/E_t} + 1)^2} = \frac{4 E_t E_c}{(\sqrt{E_c} + \sqrt{E_c})^2} \end{aligned}$$

Problem 4.61



4.61 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 8 ksi, determine the largest bending moment that can be applied to the member when the radius r of the semicircular grooves is (a) $r = \frac{3}{8}$ in., (b) $r = \frac{3}{4}$ in.

$$(a) d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75 \text{ in.}$$

$$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$$

From Fig. 4.32 $K = 2.07$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (\frac{3}{4})(3.75)^3 = 3.296 \text{ in}^4, \quad c = \frac{1}{2} = 1.875 \text{ in}$$

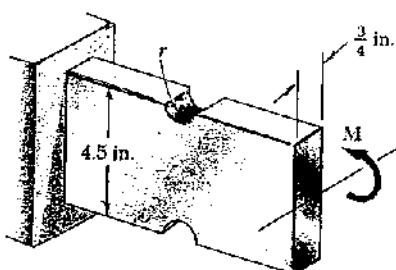
$$\sigma = K \frac{Mc}{I} \therefore M = \frac{K I}{c} = \frac{(8)(3.296)}{(2.07)(1.875)} = 6.79 \text{ kip-in.}$$

$$(b) d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.0, \quad \frac{D}{d} = \frac{4.5}{3.0} = 1.5, \quad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$$

$$\text{From Fig. 4.32 } K = 1.61, \quad I = \frac{1}{12} b h^3 = \frac{1}{12} (\frac{3}{4})(3.0)^3 = 1.6875 \text{ in}^4$$

$$c = \frac{1}{2} d = 1.5 \text{ in.} \quad M = \frac{\sigma I}{Kc} = \frac{(8)(1.6875)}{(1.61)(1.5)} = 5.59 \text{ kip-in.}$$

Problem 4.62



4.62 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 4$ kip · in., determine the maximum stress in the member when (a) $r = \frac{3}{8}$ in., (b) $r = \frac{3}{4}$ in.

$$(a) d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75 \text{ in.}$$

$$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$$

From Fig. 4.32 $K = 2.07$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (\frac{3}{4})(3.75)^3 = 3.296 \text{ in}^4, \quad c = \frac{1}{2} d = 1.875 \text{ in.}$$

$$\sigma = K \frac{Mc}{I} = \frac{(2.07)(4)(1.875)}{3.296} = 4.71 \text{ ksi}$$

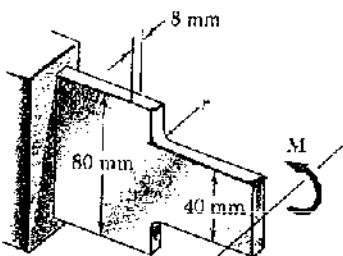
$$(b) d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.00 \text{ in.}, \quad \frac{D}{d} = \frac{4.5}{3.00} = 1.50, \quad \frac{r}{d} = \frac{0.75}{3.00} = 0.25$$

From Fig. 4.32 $K = 1.61$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (\frac{3}{4})(3.00)^3 = 1.6875 \text{ in}^4, \quad c = \frac{1}{2} d = 1.5 \text{ in.}$$

$$\sigma = K \frac{Mc}{I} = \frac{(1.61)(4)(1.5)}{1.6875} = 5.72 \text{ ksi}$$

Problem 4.63



4.63 Knowing that $M = 250 \text{ N}\cdot\text{m}$, determine the maximum stress in the beam shown when the radius r of the fillets is (a) 4 mm, (b) 8 mm.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$C = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

$$(a) \frac{r}{d} = \frac{4 \text{ mm}}{40 \text{ mm}} = 0.10 \quad \text{From Fig. 4.31} \quad K = 1.87$$

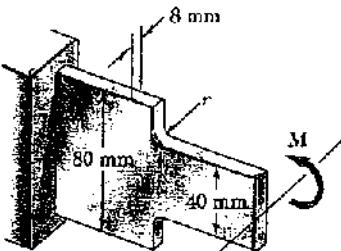
$$\sigma_{max} = K \frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667 \times 10^{-9}} = 219 \times 10^6 \text{ Pa} = 219 \text{ MPa}$$

$$(b) \frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.20 \quad \text{From Fig. 4.31} \quad K = 1.50$$

$$\sigma_{max} = K \frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667 \times 10^{-9}} = 176 \times 10^6 \text{ Pa} = 176 \text{ MPa}$$

Problem 4.64

4.64 Knowing that the allowable stress for the beam shown is 90 MPa, determine (a) the allowable bending moment M when the radius r of the fillets is (a) 8 mm, (b) 12 mm.



$$I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$C = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

$$(a) \frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.2 \quad \text{From Fig. 4.31} \quad K = 1.50$$

$$\sigma_{max} = K \frac{Mc}{I} \therefore M = \frac{\sigma_{max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)} \\ = 128 \text{ N}\cdot\text{m}$$

$$(b) \frac{r}{d} = \frac{12 \text{ mm}}{40 \text{ mm}} = 0.3 \quad \text{From Fig. 4.31} \quad K = 1.35$$

$$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)} = 142 \text{ N}\cdot\text{m}$$

Problem 4.65

4.65 The allowable stress used in the design of a steel bar is 80 MPa. Determine the largest couple M that can be applied to the bar (a) if the bar is designed with grooves having semicircular portions of radius $r = 15 \text{ mm}$, as shown in Fig. 4.65a, (b) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65b.

For both configurations

$$D = 150 \text{ mm}, d = 100 \text{ mm}.$$

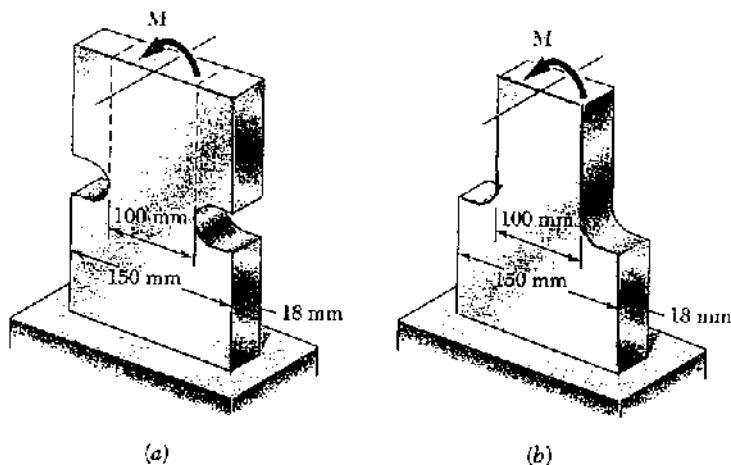
$$r = 15 \text{ mm}.$$

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (a), Fig 4.32 gives $K_a = 1.92$.

For configuration (b) Fig 4.31 gives $K_b = 1.57$.



$$I = \frac{1}{12} b h^3 = \frac{1}{12}(100)(150)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}d = 50 \text{ mm} = 0.050 \text{ m}$$

$$(a) \sigma = \frac{K_c M c}{I} \therefore M = \frac{K_c I}{c} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92)(0.05)} = 1.25 \times 10^3 \text{ N}\cdot\text{m} \\ = 1.25 \text{ kN}\cdot\text{m}$$

$$(b) M = \frac{K_c I}{c} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.53 \times 10^3 \text{ N}\cdot\text{m} = 1.53 \text{ kN}\cdot\text{m}$$

Problem 4.66

4.66 A couple of moment $M = 2 \text{ kN} \cdot \text{m}$ is to be applied to the end of a steel bar. Determine the maximum stress in the bar (a) if the bar is designed with grooves having semicircular portions of radius $r = 10 \text{ mm}$, as shown in Fig. 4.65a, (b) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65b.

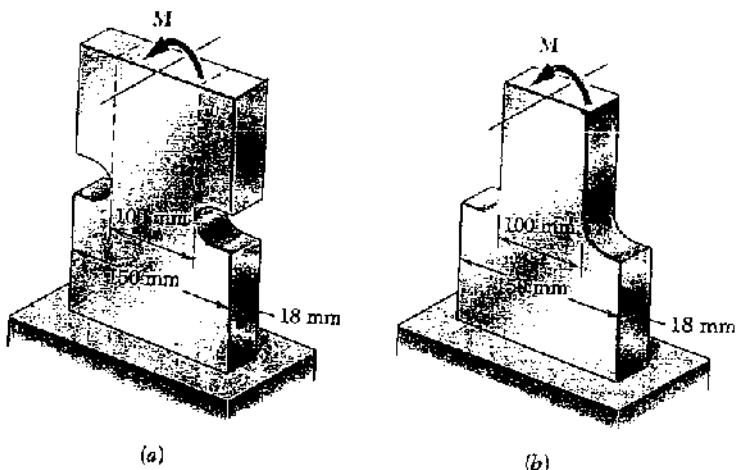
For both configurations

$$D = 150 \text{ mm}, \quad d = 100 \text{ mm}$$

$$r = 10 \text{ mm}.$$

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{10}{100} = 0.10$$



For configuration (a),

(a)

Fig 4.32 give $K_a = 2.21$

(b)

For configuration (b), Fig. 4.31 gives $K_b = 1.79$

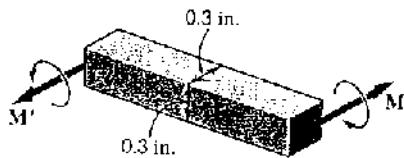
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 50 \text{ mm} = 0.05 \text{ m}$$

$$(a) \sigma = \frac{K M c}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147 \times 10^6 \text{ Pa} = 147 \text{ MPa}$$

$$(b) \sigma = \frac{K M c}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119 \times 10^6 \text{ Pa} = 119 \text{ MPa}$$

Problem 4.67



4.67 A bar having the cross section shown is made of a steel which is assumed to be elastoplastic with $E = 30 \times 10^6$ psi and $\sigma_y = 48$ ksi. Determine the bending moment M at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 0.09-in. thick.

$$(a) I = \frac{1}{12} b h^3 = \frac{1}{12} (0.3)(0.3)^3 = 675 \times 10^{-6} \text{ in}^4$$

$$c = \frac{1}{2} h = 0.15 \text{ in.}$$

$$M_y = \frac{G_y I}{c} = \frac{(48)(675 \times 10^{-6})}{0.15} = 0.216 \text{ kip-in}$$

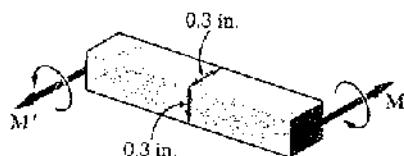
$$M_y = 216 \text{ lb-in}$$

$$(b) y_y = c - t_p = 0.15 - 0.09 = 0.06 \text{ in}$$

$$M_p = \frac{3}{2} M_y \left[1 - \frac{1}{3} \left(\frac{y_y}{c} \right)^2 \right] = \frac{3}{2} (216) \left[1 - \frac{1}{3} \left(\frac{0.06}{0.15} \right)^2 \right]$$

$$M_p = 307 \text{ lb-in}$$

Problem 4.68



4.68 For the steel bar of Prob. 4.67, determine the thickness of the plastic zones at the top and bottom of the bar when (a) $M = 250$ lb-in., (b) $M = 300$ lb-in.

4.67 A bar having the cross section shown is made of a steel which is assumed to be elastoplastic with $E = 30 \times 10^6$ psi and $\sigma_y = 48$ ksi. Determine the bending moment M at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 0.09-in. thick.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.3)(0.3)^3 = 675 \times 10^{-6} \text{ in}^4$$

$$c = \frac{1}{2} h = 0.15 \text{ in.}$$

$$M_y = \frac{G_y I}{c} = \frac{(48)(675 \times 10^{-6})}{0.15} = 0.216 \text{ kip-in}$$

$$M_y = 216 \text{ lb-in}$$

$$M_c = \frac{3}{2} M_y \left[1 - \frac{1}{3} \left(\frac{y_y}{c} \right)^2 \right]$$

$$\frac{y_y}{c} = \sqrt{3 - 2 \frac{M}{M_y}}$$

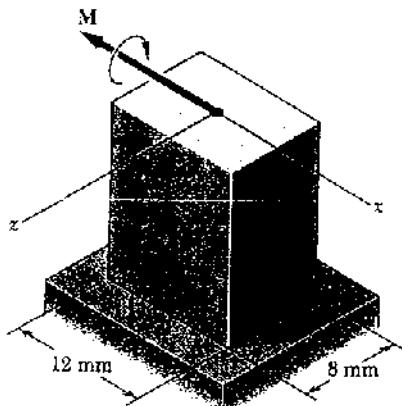
$$(a) M = 250 \text{ lb-in} \quad \frac{y_y}{c} = \sqrt{3 - 2 \left(\frac{250}{216} \right)} = 0.82215$$

$$y_y = (0.82215)(0.15) = 0.12316 \text{ in} \quad t_p = c - y_y = 0.0268 \text{ in.}$$

$$(b) M = 300 \text{ lb-in} \quad \frac{y_y}{c} = \sqrt{3 - 2 \left(\frac{300}{216} \right)} = 0.47140$$

$$y_y = (0.47140)(0.15) = 0.07071 \text{ in} \quad t_p = c - y_y = 0.0793 \text{ in.}$$

Problem 4.69



4.69 The prismatic bar shown is made of a steel which is assumed to be elastoplastic with $\sigma_y = 300 \text{ MPa}$ and is subjected to a couple M parallel to the x axis. Determine the moment M of the couple for which (a) yield first occurs, (b) the elastic core of the bar is 4 mm thick.

$$(a) I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(8)^3 = 512 \text{ mm}^4 \\ = 512 \times 10^{-12} \text{ m}^4$$

$$c = \frac{1}{2}h = 4 \text{ mm} = 0.004 \text{ m}$$

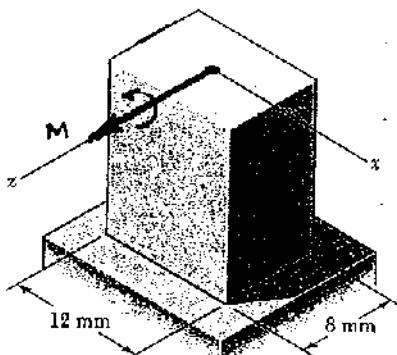
$$M_y = \frac{\sigma_y I}{c} = \frac{(300 \times 10^6)(512 \times 10^{-12})}{0.004} \\ = 38.4 \text{ N}\cdot\text{m} \quad M_y = 38.4 \text{ N}\cdot\text{m}$$

$$(b) y_r = \frac{1}{2}(4) = 2 \text{ mm} \quad \frac{y_r}{c} = \frac{2}{4} = 0.5$$

$$M = \frac{3}{2}M_y \left[1 - \frac{1}{3} \left(\frac{y_r}{c} \right)^2 \right] = \frac{3}{2}(38.4) \left[1 - \frac{1}{3}(0.5)^2 \right] = 52.8 \text{ N}\cdot\text{m}$$

$$M = 52.8 \text{ N}\cdot\text{m}$$

Problem 4.70



4.70 Solve Prob. 4.69, assuming that the couple M is parallel to the z axis.

4.69 The prismatic bar shown is made of a steel which is assumed to be elastoplastic with $\sigma_y = 300 \text{ MPa}$ and is subjected to a couple M parallel to the x axis. Determine the moment M of the couple for which (a) yield first occurs, (b) the elastic core of the bar is 4 mm thick.

$$(a) I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(12)^3 = 1.152 \times 10^3 \text{ mm}^4 \\ = 1.152 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}h = 4 \text{ mm} = 0.004 \text{ m}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(300 \times 10^6)(1.152 \times 10^{-9})}{0.004} \\ = 57.6 \text{ N}\cdot\text{m} \quad M_y = 57.6 \text{ N}\cdot\text{m}$$

$$(b) y_r = \frac{1}{2}(4) = 2 \text{ mm} \quad \frac{y_r}{c} = \frac{2}{4} = 0.5$$

$$M = \frac{3}{2}M_y \left[1 - \frac{1}{3} \left(\frac{y_r}{c} \right)^2 \right] = \frac{3}{2}(57.6) \left[1 - \frac{1}{3} \left(\frac{1}{2} \right)^2 \right] = 83.2 \text{ N}\cdot\text{m}$$

$$M = 83.2 \text{ N}\cdot\text{m}$$

Problem 4.71

4.71 A solid square rod of side 0.6 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_y = 48$ ksi. Knowing that a couple M is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is 6 ft.

$$I = \frac{1}{12} (0.6)(0.6)^3 = 10.8 \times 10^{-3} \text{ in}^4 \quad c = \frac{1}{2} h = 0.3 \text{ in.}$$

$$M_y = \frac{I \sigma_y}{c} = \frac{(10.8 \times 10^{-3})(48 \times 10^3)}{0.3} = 1728 \text{ lb-in.} \quad \rho = 6 \text{ ft} = 72 \text{ in.}$$

$$\epsilon_{max} = \frac{c}{\rho} = \frac{0.3}{72} = 4.16667 \times 10^{-3} \quad \epsilon_y = \frac{\sigma_y}{E} = \frac{48 \times 10^3}{29 \times 10^6} = 1.65517 \times 10^{-3}$$

$$\frac{y_y}{c} = \frac{\epsilon_y}{\epsilon_{max}} = \frac{1.65517 \times 10^{-3}}{4.16667 \times 10^{-3}} = 0.39724$$

$$M = \frac{3}{2} M_y \left[1 - \frac{1}{3} \left(\frac{y_y}{c} \right)^2 \right] = \frac{3}{2} (1728) \left[1 - \frac{1}{3} (0.39724)^2 \right]$$

$$M = 2460 \text{ lb-in.} \quad \blacksquare$$

Problem 4.72

4.72 For the solid square rod of Prob. 4.71, determine the moment M for which the radius of curvature is 3 ft.

4.71 A solid square rod of side 0.6 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_y = 48$ ksi. Knowing that a couple M is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is 6 ft.

$$I = \frac{1}{12} (0.6)(0.6)^3 = 10.8 \times 10^{-3} \text{ in}^4 \quad c = \frac{1}{2} h = 0.3 \text{ in.}$$

$$M_y = \frac{I \sigma_y}{c} = \frac{(10.8 \times 10^{-3})(48 \times 10^3)}{0.3} = 1728 \text{ lb-in.} \quad \rho = 3 \text{ ft} = 36 \text{ in.}$$

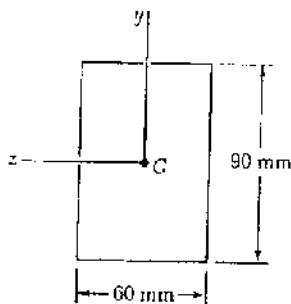
$$\epsilon_{max} = \frac{c}{\rho} = \frac{0.3}{36} = 8.3333 \times 10^{-3} \quad \epsilon_y = \frac{\sigma_y}{E} = \frac{48 \times 10^3}{29 \times 10^6} = 1.65517 \times 10^{-3}$$

$$\frac{y_y}{c} = \frac{\epsilon_y}{\epsilon_{max}} = \frac{1.65517 \times 10^{-3}}{8.3333 \times 10^{-3}} = 0.19862$$

$$M = \frac{3}{2} M_y \left[1 - \frac{1}{3} \left(\frac{y_y}{c} \right)^2 \right] = \frac{3}{2} (1728) \left[1 - \frac{1}{3} (0.19862)^2 \right]$$

$$M = 2560 \text{ lb-in.} \quad \blacksquare$$

Problem 4.73



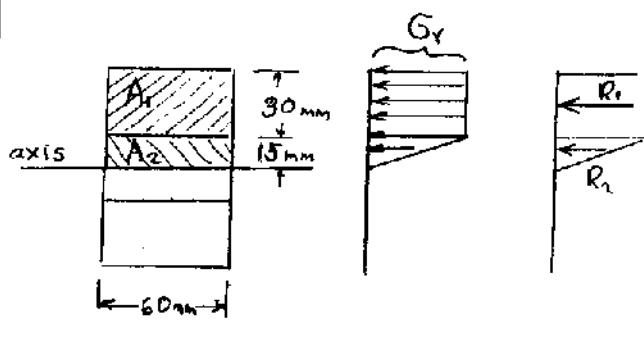
4.73 and 4.74 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 200 \text{ GPa}$ and $\sigma_y = 240 \text{ MPa}$. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 30-mm thick.

$$(a) I = \frac{1}{12} b h^3 = \frac{1}{12} (60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} h = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 19.44 \text{ kN}\cdot\text{m} \quad \blacksquare$$



$$R_1 = \sigma_y A_1 = (240 \times 10^6)(0.060)(0.030)$$

$$= 432 \times 10^3 \text{ N}$$

$$y_1 = 15 \text{ mm} + 15 \text{ mm} = 0.030 \text{ m}$$

$$R_2 = \frac{1}{2} \sigma_y A_2 = \frac{1}{2} (240 \times 10^6)(0.060)(0.015)$$

$$= 108 \times 10^3 \text{ N}$$

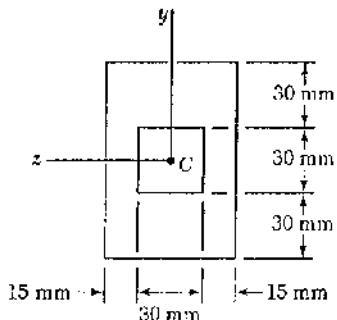
$$y_2 = \frac{2}{3} (15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$$

$$(b) M = 2(R_1 y_1 + R_2 y_2) = 2[(432 \times 10^3)(0.030) + (108 \times 10^3)(0.010)]$$

$$= 28.08 \times 10^3 \text{ N}\cdot\text{m} \quad M = 28.1 \text{ kN}\cdot\text{m} \quad \blacksquare$$

Problem 4.74

4.73 and 4.74 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 200 \text{ GPa}$ and $\sigma_y = 240 \text{ MPa}$. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 30-mm thick.



$$(a) I_{\text{rect}} = \frac{1}{12} b h^3 = \frac{1}{12} (60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4$$

$$I_{\text{outert}} = \frac{1}{12} b h^3 = \frac{1}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

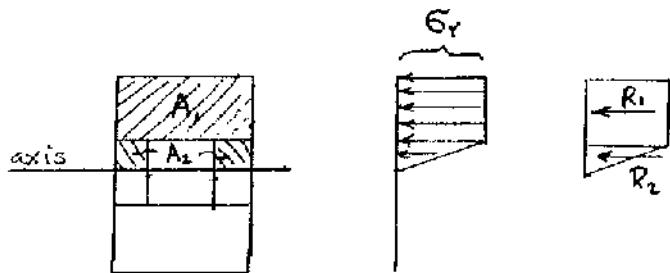
$$I = 3.645 \times 10^6 - 67.5 \times 10^3 = 3.5775 \times 10^6 \text{ mm}^4$$

$$= 3.5775 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} h = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(240 \times 10^6)(3.5775 \times 10^{-6})}{0.045}$$

$$= 19.08 \times 10^3 \text{ N}\cdot\text{m} = 19.08 \text{ kN}\cdot\text{m}$$



$$R_1 = \sigma_y A_1 = (240 \times 10^6)(0.060)(0.030) = 432 \times 10^3 \text{ N}$$

$$y_1 = 15 \text{ mm} + 15 \text{ mm} = 30 \text{ mm} = 0.030 \text{ m}$$

$$R_2 = \frac{1}{2} \sigma_y A_2 = \frac{1}{2} (240 \times 10^6)(0.030)(0.015) = 54 \times 10^3 \text{ N}$$

$$y_2 = \frac{2}{3} (15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$$

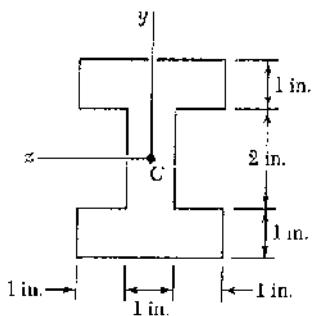
$$(b) M = 2(R_1 y_1 + R_2 y_2)$$

$$= 2[(432 \times 10^3)(0.030) + (54 \times 10^3)(0.010)]$$

$$= 27.00 \times 10^3 \text{ N}\cdot\text{m} = 27.0 \text{ kN}\cdot\text{m}$$

Problem 4.75

4.75 and 4.76 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 29 \times 10^6$ psi and $\sigma_y = 42$ ksi. For bending about the z axis, determine the bending moment and radius of curvature at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 1-in. thick.



$$(a) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(0.5)^2 = 7.0 \text{ in}^4$$

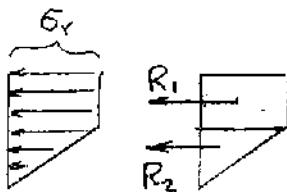
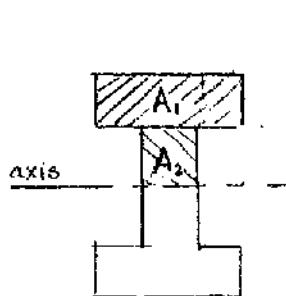
$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$$

$$I_3 = I_1 = 7.0 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 14.6667 \text{ in}^4$$

$$c = 2 \text{ in.}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(42)(14.6667)}{2} = 308 \text{ kip-in.}$$



$$R_1 = \sigma_y A_1 = (42)(3)(1) = 126 \text{ kip}$$

$$y_1 = 1.0 + 0.5 = 1.5 \text{ in.}$$

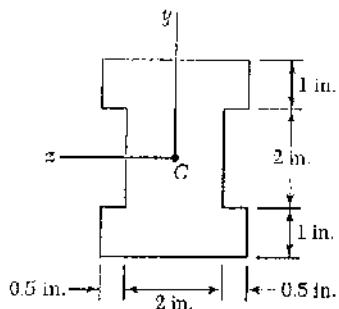
$$R_2 = \frac{1}{2} \sigma_y A_2 = \frac{1}{2}(42)(1)(1) = 21 \text{ kip}$$

$$y_2 = \frac{2}{3}(1.0) = 0.6667 \text{ in.}$$

$$(b) M = 2(R_1 y_1 + R_2 y_2) = 2[(126)(1.5) + (21)(0.6667)] = 406 \text{ kip-in.}$$

Problem 4.76

4.75 and 4.76 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 29 \times 10^6$ psi and $\sigma_y = 42$ ksi. For bending about the z axis, determine the bending moment and radius of curvature at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 1-in. thick.



$$(a) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (3)(1)^3 + (3)(1)(1.5)^2 = 17.0 \text{ in}^4$$

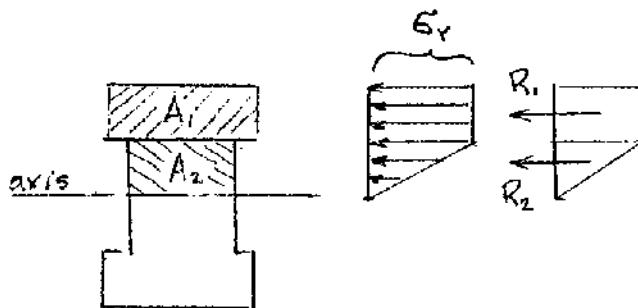
$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (2)(2)^3 = 1.3333 \text{ in}^4$$

$$I_3 = I_1 = 17.0 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 15.3333 \text{ in}^4$$

$$c = 2 \text{ in.}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(42)(15.3333)}{2} = 322 \text{ kip-in.}$$



$$R_1 = \sigma_y A_1 = (42)(8)(1) = 126 \text{ kip}$$

$$y_1 = 1.0 + 0.5 = 1.5 \text{ in.}$$

$$R_2 = \frac{1}{2} \sigma_y A_2 = \frac{1}{2}(42)(2)(1) = 42 \text{ kip}$$

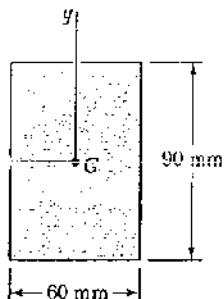
$$y_2 = \frac{2}{3}(1) = 0.6667 \text{ in.}$$

$$(b) M = 2(R_1 y_1 + R_2 y_2) = 2[(126)(1.5) + (42)(0.6667)] = 434 \text{ kip-in.}$$

Problem 4.77

4.77 through 4.80 For the beam indicated, determine (a) the fully plastic moment M_p , (b) the shape factor of the cross section.

4.77 Beam of Prob. 4.73.



From PROBLEM 4.73 $E = 200 \text{ GPa}$ and $\sigma_y = 240 \text{ MPa}$.

$$A_i = (60)(45) = 2700 \text{ mm}^2 \\ = 2700 \times 10^{-6} \text{ m}^2$$

$$R = \sigma_y A_i \\ = (240 \times 10^6)(2700 \times 10^{-6}) \\ = 648 \times 10^3 \text{ N}$$

$$d = 45 \text{ mm} = 0.045 \text{ m}$$

$$(a) M_p = R d = (648 \times 10^3)(0.045) = 29.16 \times 10^3 \text{ N}\cdot\text{m} = 29.2 \text{ kN}\cdot\text{m}$$

$$(b) I = \frac{1}{12} b h^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^5 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$$

$$c = 45 \text{ mm} = 0.045 \text{ m}$$

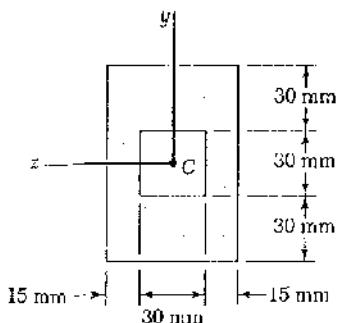
$$M_y = \frac{\sigma_y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3 \text{ N}\cdot\text{m}$$

$$k = \frac{M_p}{M_y} = \frac{29.16}{19.44} = 1.500$$

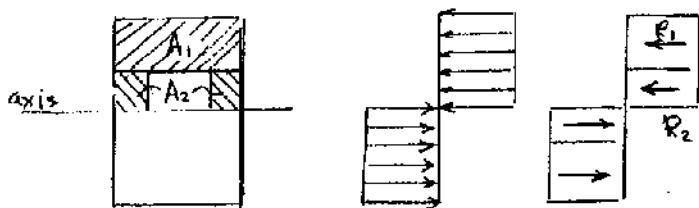
Problem 4.78

4.77 through 4.80 For the beam indicated, determine (a) the fully plastic moment M_p , (b) the shape factor of the cross section.

4.78 Beam of Prob. 4.74.



From PROBLEM 4.74 $E = 200 \text{ GPa}$ and $G_y = 240 \text{ MPa}$.



$$R_1 = G_y A_1$$

$$= (240 \times 10^6)(0.060)(0.030)$$

$$= 432 \times 10^3 \text{ N}$$

$$y_1 = 15\text{mm} + 15\text{mm} = 30\text{mm}$$

$$= 0.030 \text{ m}$$

$$R_2 = G_y A_2$$

$$= (240 \times 10^6)(0.030)(0.015)$$

$$= 10.8 \times 10^3 \text{ N}$$

$$y_2 = \frac{1}{2}(15) = 7.5\text{mm} = 0.0075 \text{ m}$$

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(432 \times 10^3)(0.030) + (10.8 \times 10^3)(0.0075)]$$

$$= 27.54 \times 10^3 \text{ N}\cdot\text{m} = 27.5 \text{ kN}\cdot\text{m}$$

$$(b) I_{rect} = \frac{1}{12} b h^3 = \frac{1}{12}(60)(70)^3 = 3.645 \times 10^6 \text{ mm}^4$$

$$I_{cutout} = \frac{1}{12} b h^3 = \frac{1}{12}(30)(80)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I = I_{rect} - I_{cutout} = 3.645 \times 10^6 - 67.5 \times 10^3 = 3.5775 \times 10^3 \text{ mm}^4$$

$$= 3.5775 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}h = 45\text{mm} = 0.045 \text{ m}$$

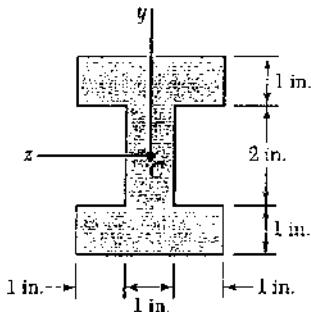
$$M_y = \frac{G_y I}{c} = \frac{(240 \times 10^6)(3.5775 \times 10^{-9})}{0.045} = 19.08 \times 10^3 \text{ N}\cdot\text{m}$$

$$k = \frac{M_p}{M_y} = \frac{27.54}{19.08} = 1.443$$

Problem 4.79

4.77 through 4.80 For the beam indicated, determine (a) the fully plastic moment M_p , (b) the shape factor of the cross section.

4.79 Beam of Prob. 4.75.



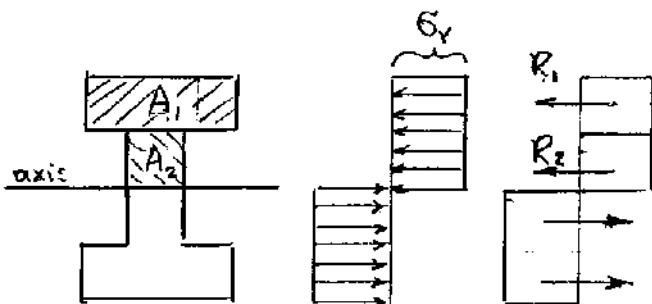
From PROBLEM 4.75 $E = 29 \times 10^6 \text{ psi}$ and $G_y = 42 \text{ ksi}$.

$$R_1 = G_y A_1 = (42)(3)(1) = 126 \text{ kip}$$

$$\bar{y}_1 = 1.0 + 0.5 = 1.5 \text{ in}$$

$$R_2 = G_y A_2 = (42)(1)(1) = 42 \text{ kip}$$

$$\bar{y}_2 = \frac{1}{2}(1) = 0.5 \text{ in}$$



$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(126)(1.5) + (42)(0.5)] = 420 \text{ kip-in}$$

$$(b) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(1.5)^2 = 7.0 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$$

$$I_3 = I_1 = 7.0 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 14.6667 \text{ in}^4$$

$$c = 2 \text{ in}$$

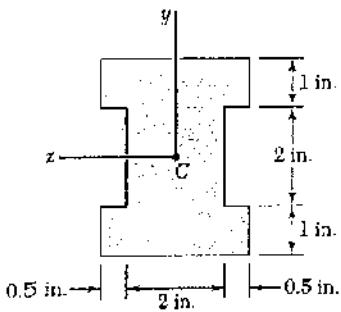
$$M_T = \frac{G_y I}{c} = \frac{(42)(14.6667)}{2} = 308 \text{ kip-in}$$

$$k = \frac{M_p}{M_T} = \frac{420}{308} = 1.364$$

Problem 4.80

4.77 through 4.80 For the beam indicated, determine (a) the fully plastic moment M_y , (b) the shape factor of the cross section.

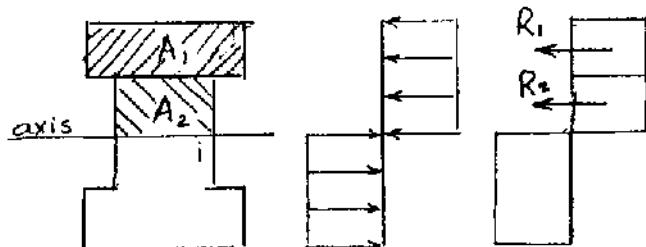
4.80 Beam of Prob. 4.76.



From PROBLEM 4.76 $E = 29 \times 10^6$ and $\sigma_y = 42$ ksi.

$$R_1 = \sigma_y A_1 = (42)(3)(1) = 126 \text{ kip}$$

$$y_1 = 1.0 + 0.5 = 1.5 \text{ in.}$$



$$R_2 = \sigma_y A_2 = (42)(2)(1) = 84 \text{ kip}$$

$$y_2 = \frac{1}{2}(1.0) = 0.5 \text{ in.}$$

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(126)(1.5) + (84)(0.5)] = 462 \text{ kip-in.}$$

$$(b) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(0.5)^2 = 7.0 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(2)(2)^3 = 1.3333 \text{ in}^4$$

$$I_3 = I_1 = 7.0 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 15.3333 \text{ in}^4$$

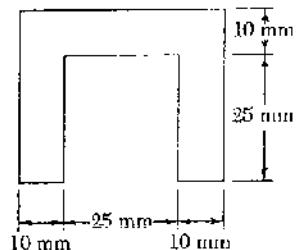
$$c = 2 \text{ in.}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(42)(15.333)}{2} = 322 \text{ kip-in.}$$

$$k = \frac{M_p}{M_y} = \frac{462}{322} = 1.435$$

Problem 4.81

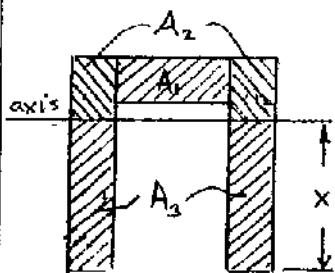
4.81 and 4.82 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.



$$\text{Total area } A = (25)(10) + (2)(10)(35) = 950 \text{ mm}^2$$

$$\frac{1}{2}A = 475 \text{ mm}^2$$

$$x = \frac{\frac{1}{2}A}{2b} = \frac{475}{20} = 23.75 \text{ mm} = 0.02375 \text{ m}$$



$$R_1 = \bar{G}_y A_1 = (240 \times 10^6)(0.025)(0.010) = 60 \times 10^3 \text{ N}$$

$$\bar{y}_1 = 30 - 23.75 = 6.25 \text{ m} = 0.00625 \text{ m}$$

$$R_2 = \bar{G}_y A_2 = (240 \times 10^6)(0.020)(0.01125) = 54 \times 10^3 \text{ N}$$

$$\bar{y}_2 = \frac{1}{2}(0.01125) = 0.005625 \text{ m}$$

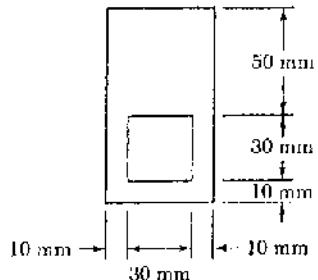
$$R_3 = \bar{G}_y A_3 = (240 \times 10^6)(0.020)(0.02375) = 114 \times 10^3 \text{ N}$$

$$\bar{y}_3 = \frac{1}{2}x = 0.011875 \text{ m}$$

$$\begin{aligned} M_p &= R_1 \bar{y}_1 + R_2 \bar{y}_2 + R_3 \bar{y}_3 \\ &= (60 \times 10^3)(0.00625) + (54 \times 10^3)(0.005625) + (114 \times 10^3)(0.011875) \\ &= 2.0325 \times 10^3 \text{ N}\cdot\text{m} = 2.03 \text{ kN}\cdot\text{m} \end{aligned}$$

Problem 4.82

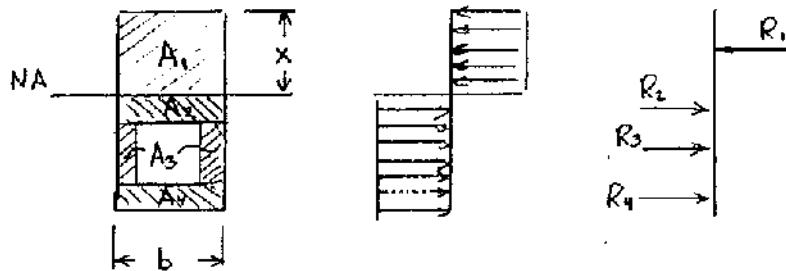
4.81 and 4.82 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.



$$\text{Total area } A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$$

$$\frac{1}{2}A = 1800 \text{ mm}^2$$

$$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$$



$$A_1 = (50)(36) = 1800 \text{ mm}^2, \bar{y}_1 = 18 \text{ mm} \quad A_1 \bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$$

$$A_2 = (50)(14) = 700 \text{ mm}^2, \bar{y}_2 = 7 \text{ mm} \quad A_2 \bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$$

$$A_3 = (20)(30) = 600 \text{ mm}^2, \bar{y}_3 = 29 \text{ mm} \quad A_3 \bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$$

$$A_4 = (50)(10) = 500 \text{ mm}^2, \bar{y}_4 = 49 \text{ mm} \quad A_4 \bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$$

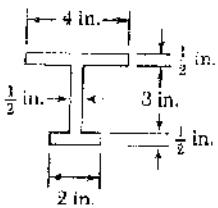
$$A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3 + A_4 \bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$$

$$M_p = 6_y \sum A_i \bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N} \cdot \text{m}$$

$$= 19.01 \text{ kN} \cdot \text{m}$$

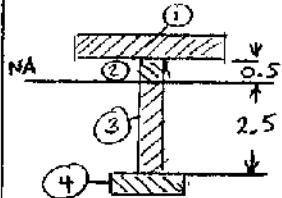
Problem 4.83

4.83 Determine the plastic moment M_p of the cross section shown assuming that the steel to be elastoplastic with a yield strength of 36 ksi.



$$\text{Total area: } A = (4)(\frac{1}{2}) + (\frac{1}{2})(3) + (2)(\frac{1}{2}) = 4.5 \text{ in}^2$$

$$\frac{1}{2}A = 2.25 \text{ in}^2$$



$$A_1 = 2.00 \text{ in}^2, \bar{y}_1 = 0.75, A_1\bar{y}_1 = 1.50 \text{ in}^3$$

$$A_2 = 0.25 \text{ in}^2, \bar{y}_2 = 0.25, A_2\bar{y}_2 = 0.0625 \text{ in}^3$$

$$A_3 = 1.25 \text{ in}^2, \bar{y}_3 = 1.25, A_3\bar{y}_3 = 1.5625 \text{ in}^3$$

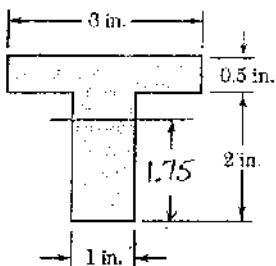
$$A_4 = 1.00 \text{ in}^2, \bar{y}_4 = 2.75, A_4\bar{y}_4 = 2.75 \text{ in}^3$$

$$M_p = 5_y (A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$$

$$= (36)(1.50 + 0.0625 + 1.5625 + 2.75) = 211.5 \text{ kip}\cdot\text{in}$$

Problem 4.84

4.84 Determine the plastic moment M_p of the cross section shown, assuming that the steel to be elastoplastic with a yield strength of 48 ksi.



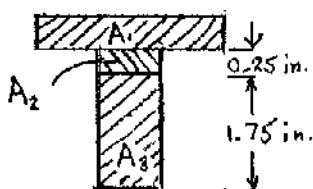
$$\text{Total area: } A = (3)(0.5) + (1)(2) = 3.5 \text{ in}^2$$

$$\frac{1}{2}A = 1.75 \text{ in}^2 \quad y_n = \frac{\frac{1}{2}A}{b} = \frac{1.75}{1} = 1.75 \text{ in.}$$

$$A_1 = (3)(0.5) = 1.5 \text{ in}^2, \bar{y}_1 = 0.5 \text{ in.}, A_1\bar{y}_1 = 0.75 \text{ in}^3$$

$$A_2 = (1)(0.25) = 0.25 \text{ in}^2, \bar{y}_2 = 0.125 \text{ in.}, A_2\bar{y}_2 = 0.03125 \text{ in}^3$$

$$A_3 = (1)(1.75) = 1.75 \text{ in}^2, \bar{y}_3 = 0.875 \text{ in.}, A_3\bar{y}_3 = 1.53125 \text{ in}^3$$

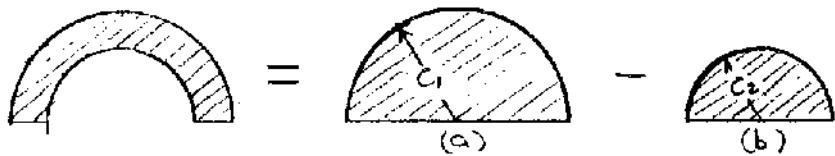
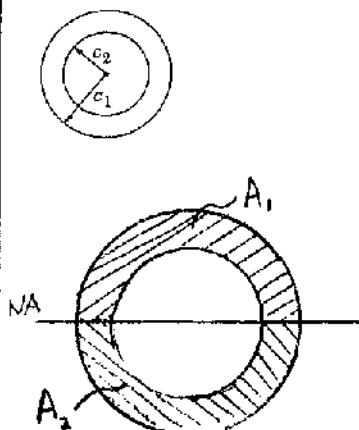


$$M_p = 5_y (A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3)$$

$$= 48 (0.75 + 0.03125 + 1.53125) = 111.0 \text{ kip}\cdot\text{in}$$

Problem 4.85

4.85 A thick-walled pipe of the cross section shown is made of a steel that is assumed to be elastoplastic with a yield strength σ_y . Derive an expression for the plastic moment M_p of the pipe in terms of c_1 , c_2 , and σ_y .



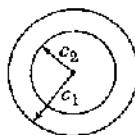
$$A_1 \bar{y}_1 = A_a \bar{y}_a - A_b \bar{y}_b \\ = \left(\frac{\pi}{2} c_1^2\right) \left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2} c_2^2\right) \left(\frac{4c_2}{3\pi}\right) \\ = \frac{2}{3} (c_1^3 - c_2^3)$$

$$A_2 \bar{y}_2 = A_1 \bar{y}_1 = \frac{2}{3} (c_1^3 - c_2^3)$$

$$M_p = \sigma_y (A_1 \bar{y}_1 + A_2 \bar{y}_2) = \frac{4}{3} \sigma_y (c_1^3 - c_2^3)$$

Problem 4.86

4.86 Determine the plastic moment M_p of a thick-walled pipe of the cross section shown, knowing that $c_1 = 60 \text{ mm}$, $c_2 = 40 \text{ mm}$, and $\sigma_y = 240 \text{ MPa}$.



See the solution to PROBLEM 4.85 for derivation of the following expression for M_p .

$$M_p = \frac{4}{3} \sigma_y (c_1^3 - c_2^3)$$

$$\text{Data: } \sigma_y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}$$

$$c_1 = 60 \text{ mm} = 0.060 \text{ m}$$

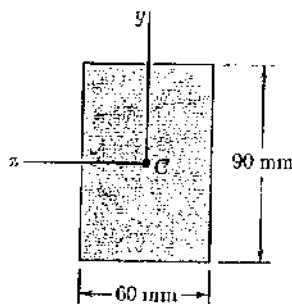
$$c_2 = 40 \text{ mm} = 0.040 \text{ m}$$

$$M_p = \frac{4}{3} (240 \times 10^6) (0.060^3 - 0.040^3) = 48.64 \times 10^3 \text{ N}\cdot\text{m} \\ = 48.6 \text{ kN}\cdot\text{m}$$

Problem 4.87

4.87 and 4.88 For the beam indicated a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

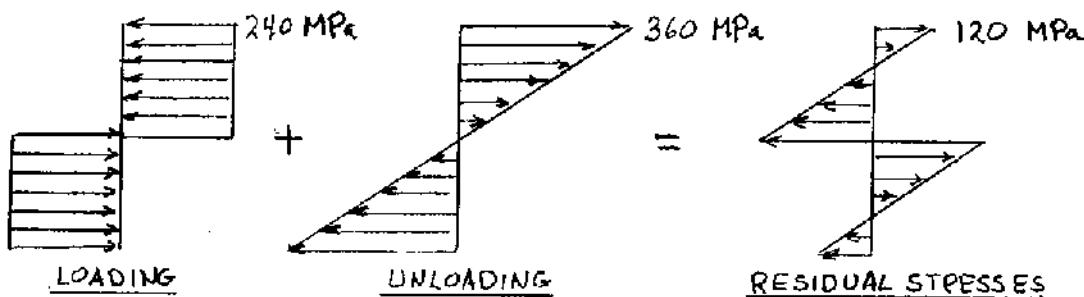
4.87 Beam of Prob. 4.73.



$$M_p = 29.16 \times 10^3 \text{ Nm} \quad (\text{See solutions to Problems 4.73 and 4.77})$$

$$I = 3.645 \times 10^{-6} \text{ m}^4, \quad c = 0.045 \text{ m}$$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{at } y = c = 45 \text{ mm.}$$

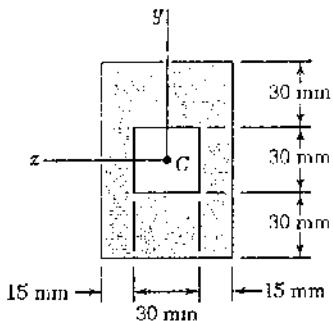


$$\sigma' = \frac{(29.16 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 360 \times 10^6 \text{ Pa}$$

$$\sigma_{res} = \sigma' - \sigma_y = 360 \times 10^6 - 240 \times 10^6 = 120 \times 10^6 \text{ Pa} = 120 \text{ MPa}$$

Problem 4.88

4.87 and 4.88 For the beam indicated a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45 \text{ mm}$.



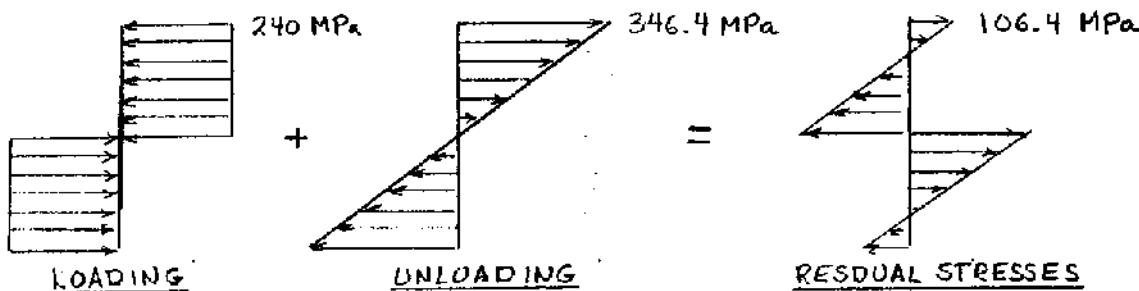
4.88 Beam of Prob. 4.74.

$$M_p = 27.54 \times 10^3 \text{ N-m} \text{ (See solutions to Problems 4.74 and 4.78)}$$

$$I = 3.5775 \times 10^{-6} \text{ m}^4, \quad c = 0.045 \text{ m}$$

$$\sigma' = \frac{M_{max}y}{I} = \frac{M_p c}{I} \quad \text{at } y = c.$$

$$\sigma' = \frac{(2254 \times 10^3)(0.045)}{3.5775 \times 10^{-6}} = 346.4 \times 10^6 \text{ Pa}$$

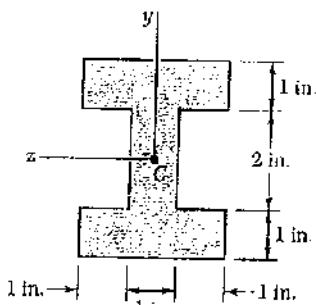


$$\sigma_{res} = \sigma' - \sigma_y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6 \text{ Pa} = 106.4 \text{ MPa}$$

Problem 4.89

4.89 and 4.90 For the beam indicated a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at (a) $y = 1$ in., (b) $y = 2$ in.

4.89 Beam of Prob. 4.75.

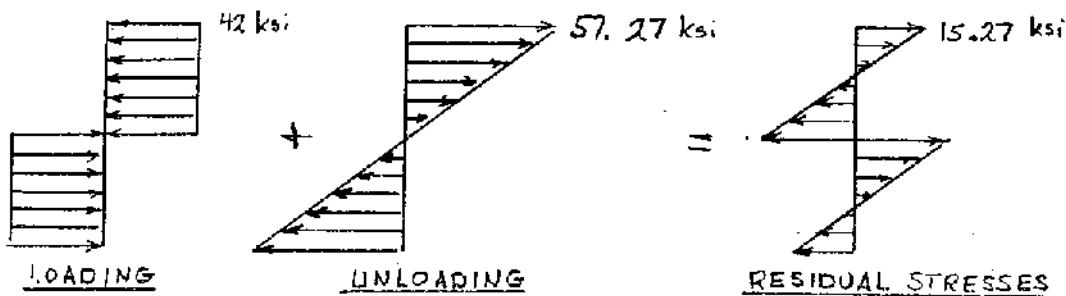


$$M_p = 420 \text{ kip-in} \quad (\text{See solutions to Problems 4.75 and 4.79})$$

$$I = 14.6667 \text{ in}^4 \quad c = 2 \text{ in.}$$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{at } y = c.$$

$$\sigma' = \frac{(420)(2)}{14.6667} = 57.27 \text{ ksi}$$



$$(a) \text{ At } y = 1 \text{ in.} = \frac{1}{2}c \quad \sigma' = \frac{1}{2}(57.27) = 28.64 \text{ ksi}$$

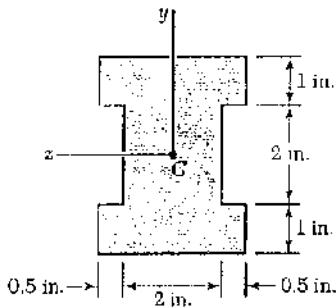
$$\sigma_{res} = -42 + 28.64 = -13.36 \text{ ksi}$$

$$(b) \text{ At } y = 2 \text{ in.} = c \quad \sigma' = 57.27 \text{ ksi}$$

$$\sigma_{res} = -42 + 57.27 = 15.27 \text{ ksi}$$

Problem 4.90

4.89 and 4.90 For the beam indicated a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at (a) $y = 1$ in., (b) $y = 2$ in.



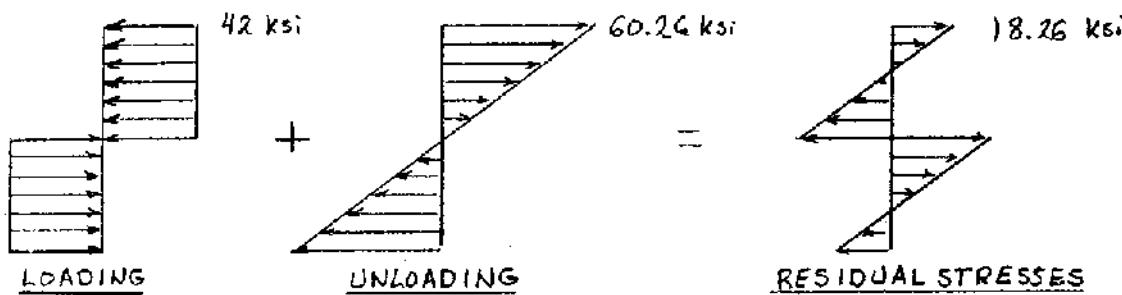
4.90 Beam of Prob. 4.76.

$$M_p = 462 \text{ kip-in.} \quad (\text{See solutions to Problems 4.76 and 4.80})$$

$$I = 15.3333 \text{ in}^4, \quad c = 2 \text{ in.}$$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{for } y = c$$

$$\sigma' = \frac{(462)(2)}{15.3333} = 60.26 \text{ ksi}$$



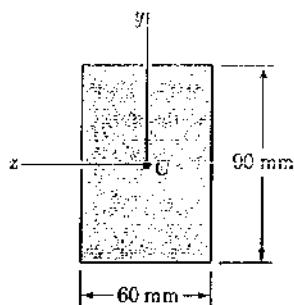
$$(a) \text{ At } y = 1 \text{ in.} = \frac{1}{2}c \quad \sigma' = \frac{1}{2}(60.26) = 30.13 \text{ ksi}$$

$$\sigma_{res} = -42 + 30.13 = -11.87 \text{ ksi}$$

$$(b) \text{ At } y = 2 \text{ in.} = c \quad \sigma' = 60.26 \text{ ksi}$$

$$\sigma_{res} = -42 + 60.26 = 18.26 \text{ ksi}$$

Problem 4.91



4.91 A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (a) the residual stress at $y = 45 \text{ mm}$, (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the beam.

See SOLUTION to PROBLEM 4.73 for bending couple and stress distribution during loading.

$$M = 28.08 \times 10^3 \text{ N-m} \quad y_r = 15 \text{ mm} = 0.015 \text{ m}$$

$$E = 200 \text{ GPa} \quad \sigma_r = 240 \text{ MPa}$$

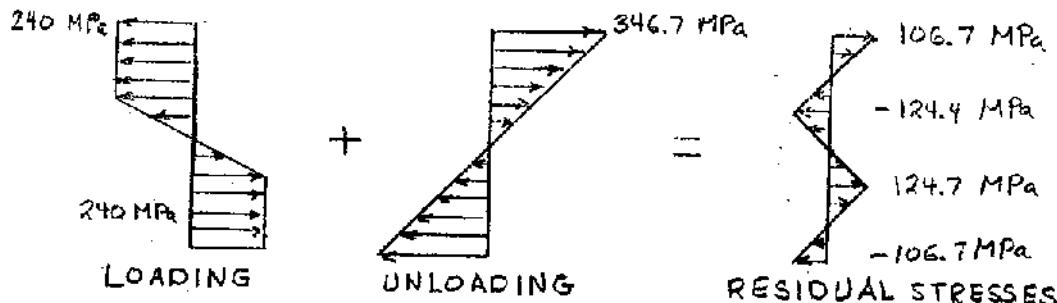
$$I = 3.645 \times 10^{-6} \text{ m}^4 \quad c = 0.045 \text{ m}$$

$$(a) \sigma' = \frac{Mc}{I} = \frac{(28.08 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 346.7 \times 10^6 \text{ Pa} = 346.7 \text{ MPa}$$

$$\sigma'' = \frac{My_r}{I} = \frac{(28.08 \times 10^3)(0.015)}{3.645 \times 10^{-6}} = 115.6 \times 10^6 \text{ Pa} = 115.6 \text{ MPa}$$

$$\text{At } y = c \quad \sigma_{\text{res}} = \sigma' - \sigma_r = 346.7 - 240 = 106.7 \text{ MPa}$$

$$\text{At } y = y_r \quad \sigma_{\text{res}} = \sigma'' - \sigma_r = 115.6 - 240 = -124.4 \text{ MPa}$$



$$(b) \sigma_{\text{res}} = 0 \quad \therefore \frac{My_0}{I} - \sigma_r = 0$$

$$y_0 = \frac{I \sigma_r}{M} = \frac{(3.645 \times 10^{-6})(240 \times 10^6)}{28.08 \times 10^3} = 31.15 \times 10^{-3} \text{ m} = 31.15 \text{ mm}$$

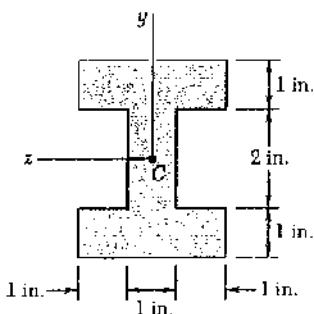
$$\text{ans. } y_0 = -31.15 \text{ mm}, 0, 31.15 \text{ mm}$$

$$(c) \text{ At } y = y_r, \quad \sigma_{\text{res}} = -124.4 \times 10^6 \text{ Pa}$$

$$\sigma = -\frac{Ey}{R} \quad \therefore R = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.015)}{-124.4 \times 10^6} = .24.1 \text{ m}$$

Problem 4.92

4.92 A bending couple is applied to the beam of Prob. 4.75, causing plastic zones 2 in. thick to develop at the top and bottom of the beam. After the couple has been removed, determine (a) the residual stress at $y = 2$ in., (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the beam.



See SOLUTION to PROBLEM 4.75 for bending couple and stress distribution

$$M = 406 \text{ kip-in} \quad y_r = 1.0 \text{ in}$$

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi} \quad \sigma_y = 42 \text{ ksi}$$

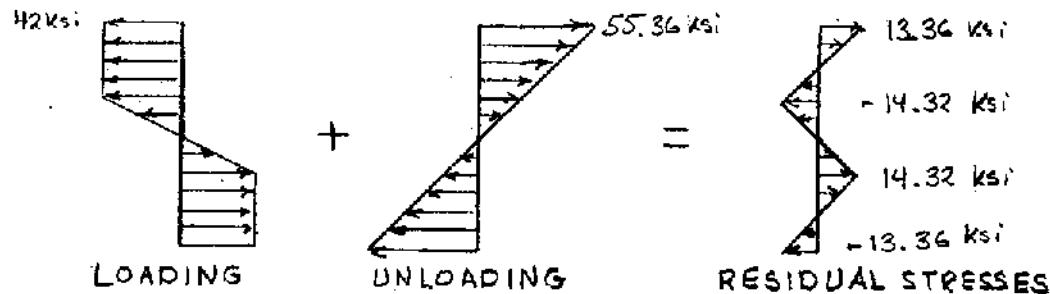
$$I = 14.6667 \text{ in}^4 \quad c = 2 \text{ in.}$$

$$(a) \sigma' = \frac{Mc}{I} = \frac{(406)(2)}{14.6667} = 55.36 \text{ ksi}$$

$$\sigma'' = \frac{My_r}{I} = \frac{(406)(1.0)}{14.6667} = 27.68 \text{ ksi}$$

$$\text{At } y = c \quad \sigma_{res} = \sigma' - \sigma_y = 55.36 - 42 = 13.36 \text{ ksi}$$

$$\text{At } y = y_r \quad \sigma_{res} = \sigma'' - \sigma_y = 27.68 - 42 = -14.32 \text{ ksi}$$



$$(b) \sigma_{res} = 0 \quad \therefore \frac{My_0}{I} - \sigma_y = 0$$

$$y_0 = \frac{I\sigma_y}{M} = \frac{(14.6667)(42)}{406} = 1.517 \text{ in}$$

$$\text{ans. } y_0 = -1.517 \text{ in.}, 0, 1.517 \text{ in}$$

$$(c) \text{ At } y = y_r, \quad \sigma_{res} = -14.32 \text{ ksi}$$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.0)}{14.32} = 2025 \text{ in} \\ = 168.8 \text{ ft}$$

Problem 4.93

4.93 A rectangular bar which is straight and unstressed is bent into an arc of circle of radius ρ by two couples of moment M . After the couples are removed, it is observed that the radius of curvature of the bar is ρ_r . Denoting by ρ_y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation:

$$\frac{1}{\rho_r} = \frac{1}{\rho} \left(1 - \frac{3}{2} \frac{\rho}{\rho_y} \left[1 - \frac{1}{3} \left(\frac{\rho}{\rho_y} \right)^2 \right] \right)$$

$$\frac{1}{R} = \frac{M_y}{EI}, \quad M = \frac{3}{2} M_y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_y^2} \right) \quad \text{Let } m \text{ denote } \frac{M}{M_y}$$

$$m = \frac{M}{M_y} = \frac{3}{2} \left(1 - \frac{\rho^2}{\rho_y^2} \right) \quad \therefore \quad \frac{\rho^2}{\rho_y^2} = 3 - 2m$$

$$\begin{aligned} \frac{1}{\rho_r} &= \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{m M_y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_y} \\ &= \frac{1}{\rho} \left\{ 1 - \frac{\rho}{\rho_y} m \right\} = \frac{1}{\rho} \left\{ 1 - \frac{3}{2} \frac{\rho}{\rho_y} \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_y^2} \right) \right\} \end{aligned}$$

Problem 4.94

4.94 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_y and ρ_y , respectively, the bending moment and radius of curvature at the onset of yield, determine (a) the radius of curvature when a couple of moment $M = 1.25 M_y$ is applied to the bar, (b) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.93.

$$(a) \frac{1}{\rho_y} = \frac{M_y}{EI}, \quad M = \frac{3}{2} M_y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_y^2} \right) \quad \text{Let } m = \frac{M}{M_y} = 1.25$$

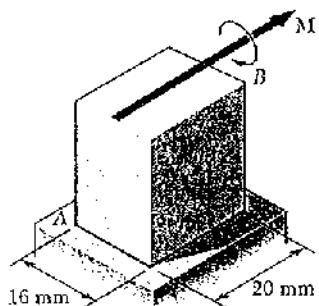
$$m = \frac{M}{M_y} = \frac{3}{2} \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_y^2} \right) \quad \frac{\rho}{\rho_y} = \sqrt{3 - 2m} = 0.70711$$

$$\rho = 0.70711 \rho_y$$

$$\begin{aligned} (b) \frac{1}{\rho_r} &= \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{m M_y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_y} = \frac{1}{0.70711 \rho_y} - \frac{1.25}{\rho_y} \\ &= \frac{0.16421}{\rho_y} \quad \rho_r = 6.09 \rho_y \end{aligned}$$

Problem 4.95

4.95 The prismatic bar AB is made of a steel which is assumed to be elastoplastic and for which $E = 200 \text{ GPa}$. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350 \text{ N} \cdot \text{m}$ is applied as shown, determine (a) the yield strength of the steel, (b) the thickness of the elastic core of the bar.



$$\begin{aligned} M &= \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_r^2} \right) \\ &= \frac{3}{2} \frac{G_r I}{c} \left(1 - \frac{1}{3} \frac{\rho^2 G_r^2}{E^2 c^2} \right) \\ &= \frac{3}{2} \frac{G_r b (2c)^3}{12 c} \left(1 - \frac{1}{3} \frac{\rho^2 G_r^2}{E^2 c^2} \right) \\ &= G_r b c^2 \left(1 - \frac{1}{3} \frac{\rho^2 G_r^2}{E^2 c^2} \right) \end{aligned}$$

(a) $b c^2 G_r \left(1 - \frac{\rho^2 G_r^2}{3 E^2 c^2} \right) = M \quad \text{Cubic equation for } G_r$

Data: $E = 200 \times 10^9 \text{ Pa}$, $M = 420 \text{ N} \cdot \text{m}$, $\rho = 2.4 \text{ m}$

$$b = 20 \text{ mm} = 0.020 \text{ m}, \quad c = \frac{1}{2} h = 8 \text{ mm} = 0.008 \text{ m}$$

$$(1.28 \times 10^{-6}) G_r \left[1 - 750 \times 10^{21} G_r^2 \right] = 350$$

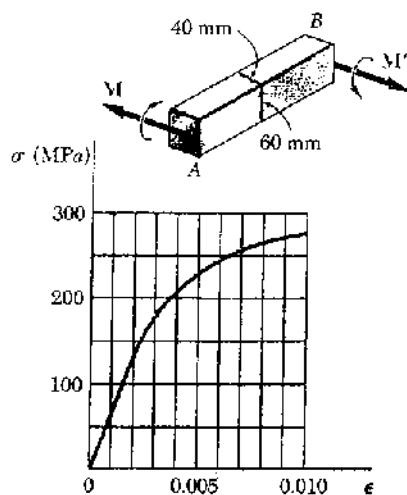
$$G_r \left[1 - 750 \times 10^{21} G_r^2 \right] = 273.44 \times 10^6$$

Solving by trial $G_r = 292 \times 10^6 \text{ Pa} = 292 \text{ MPa}$

(b) $y_r = \frac{G_r \rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3} \text{ m} = 3.504 \text{ mm}$

thickness of elastic core = $2y_r = 7.01 \text{ mm}$

Problem 4.96



4.96 The prismatic bar AB is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ - ϵ diagram is the same in compression as in tension, determine (a) the radius of curvature of the bar when the maximum stress is 250 MPa, (b) the corresponding value of the bending moment. (Hint: For part b, plot σ versus y and use an approximate method of integration.)

$$(a) \sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$$

$$E_m = 0.0064 \text{ from curve}$$

$$C = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$$

$$b = 40 \text{ mm} = 0.040 \text{ m}$$

$$\frac{1}{R} = \frac{E_m}{C} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1}$$

$$R = 4.69 \text{ m}$$

$$(b) \text{ Strain distribution } \epsilon = -\epsilon_m \frac{y}{C} = -\epsilon_m u \text{ where } u = \frac{y}{C}$$

Bending couple

$$M = - \int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 u |\sigma| du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u |\sigma| du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3} \sum w u |\sigma|$$

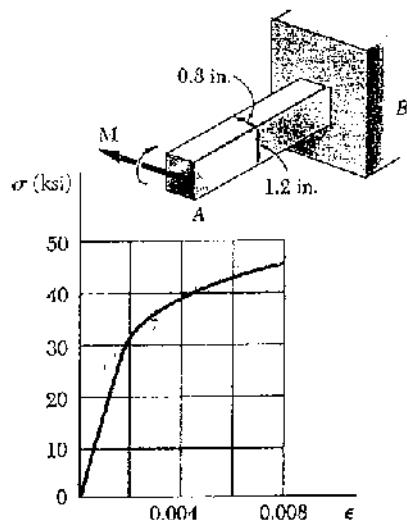
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$ EI $	$ \sigma , (\text{MPa})$	$u \sigma , (\text{MPa})$	w	$wu \sigma , (\text{MPa})$	$\sum wu \sigma $
0	0	0	0	1	0	
0.25	0.0016	110	27.5	4	110	
0.5	0.0032	180	90	2	180	
0.75	0.0048	225	168.75	4	675	
1.00	0.0064	250	250	1	250	
						1215
						$\sum wu \sigma $

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.080)^2(101.25 \times 10^6) = 7.29 \times 10^3 \text{ N}\cdot\text{m} = 7.29 \text{ kN}\cdot\text{m}$$

Problem 4.97



4.97 The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the $\sigma - \epsilon$ diagram is the same in compression as in tension, determine (a) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (b) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)

$$(a) \rho = 100 \text{ in}, b = 0.8 \text{ in}, C = 0.6 \text{ in.}$$

$$E_m = \frac{C}{\rho} = \frac{0.6}{100} = 0.006$$

$$\text{From the curve } \epsilon_m = 43 \text{ ksi}$$

$$(b) \text{ Strain distribution } \epsilon = -\epsilon_m \frac{y}{\rho} = -\epsilon_m u \text{ where } u = \frac{y}{\rho}$$

Bending couple

$$M = - \int_{-c}^c y \sigma \, dy = 2b \int_0^c y |\sigma| \, dy = 2bc^2 \int_0^1 u |\sigma| \, du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u |\sigma| \, du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3} \sum w u |\sigma|$$

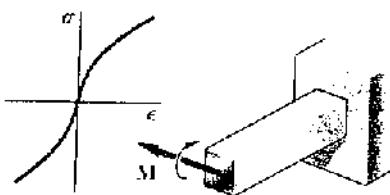
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$ E $	$ \sigma , \text{ksi}$	$u \sigma , \text{ksi}$	w	$w u \sigma , \text{ksi}$	
0	0	0	0	1	0	
0.25	0.0015	25	6.25	4	25	
0.5	0.003	36	18	2	36	
0.75	0.0045	40	30	4	120	
1.00	0.006	43	43	1	43	
					224	$\sum w u \sigma $

$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67) = 10.75 \text{ kip-in.}$$

Problem 4.98



4.98 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\epsilon = k\sigma^n$ for $\sigma > 0$ and $\epsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple M is applied to the bar show that the maximum stress is

$$\sigma_m = \frac{1 + 2n}{3n} \frac{Mc}{I}$$

Strain distribution $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = - \int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 \frac{y}{c} |\sigma| \frac{dy}{c}$$

$$= 2bc^2 \int_0^1 u |\sigma| du$$

For $\epsilon = K\sigma^n$, $\epsilon_m = K\sigma_m^n$

$$\frac{\epsilon}{\epsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \therefore |\sigma| = \sigma_m u^{1/n}$$

Then $M = 2bc^2 \int_0^1 u \sigma_m u^{1/n} du = 2bc^2 \sigma_m \int_0^1 u^{1+1/n} du$

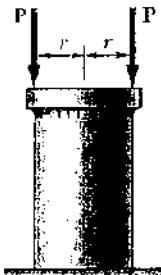
$$= 2bc^2 \sigma_m \left[\frac{u^{2+1/n}}{2+1/n} \right]_0^1 = \frac{2n}{2n+1} bc^2 \sigma_m$$

$$\sigma_m = \frac{2n+1}{2} \frac{M}{bc^2}$$

Recall $\frac{I}{c} = \frac{1}{12} \frac{b(2c)^3}{c} = \frac{2}{3} bc^2 \therefore \frac{1}{bc^2} = \frac{2}{3} \frac{c}{I}$

$$\sigma_m = \frac{2n+1}{3n} \frac{Mc}{I}$$

Problem 4.99



4.99 Two forces P can be applied separately or at the same time to a plate which is welded to a solid circular bar of radius r . Determine the largest compressive stress in the circular bar, (a) when both forces are applied, (b) when only one of the forces is applied.

For a solid circular section $A = \pi r^2$, $I = \frac{\pi}{4} r^4$, $c = r$

$$\begin{aligned}\text{Compressive stress } \sigma &= -\frac{F}{A} - \frac{Mc}{I} \\ &= -\frac{F}{\pi r^2} - \frac{4Mr}{\pi r^3}\end{aligned}$$

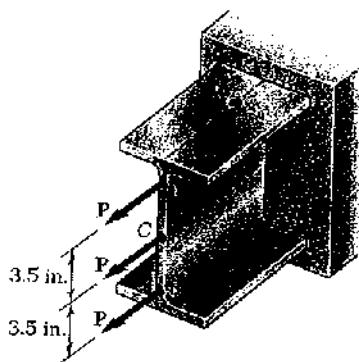
(a) Both forces applied. $F = 2P$, $M = 0$

$$\sigma = -2P/\pi r^2$$

(b) One force applied. $F = P$, $M = Pr$

$$\sigma = -\frac{F}{\pi r^2} - \frac{4Pr}{\pi r^3} \quad \sigma = -5P/\pi r^2$$

Problem 4.100



4.100 As many as three axial loads each of magnitude $P = 10$ kips can be applied to the end of a W 8 x 21 rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 1 and 2 only.

For W 8 x 21 Appendix C gives

$$A = 6.16 \text{ in}^2, d = 8.28 \text{ in.}, I_x = 75.3 \text{ in}^4$$

At point A $y = \frac{1}{2}d = 4.14 \text{ in.}$

$$\sigma = \frac{F}{A} - \frac{My}{I}$$

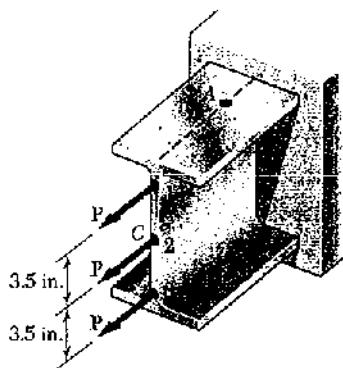
(a) Centric loading: $F = 30$ kips, $M = 0$

$$\sigma = \frac{30}{6.16} \quad \sigma = 4.87 \text{ ksi}$$

(b) Eccentric loading: $F = 2P = 20$ kips $M = -(10)(3.5) = -35 \text{ kip-in.}$

$$\sigma = \frac{20}{6.16} - \frac{(-35)(4.14)}{75.3} \quad \sigma = 5.17 \text{ ksi}$$

Problem 4.101



4.101 As many as three axial loads each of magnitude $P = 10$ kips can be applied to the end of a W 8 × 21 rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 2 and 3 only.

For W 8 × 21 Appendix C gives

$$A = 6.16 \text{ in}^2, d = 8.28 \text{ in}, I_x = 75.3 \text{ in}^4$$

At point A $y = \frac{1}{2}d = 4.14 \text{ in}$.

$$\sigma = \frac{F}{A} - \frac{My}{I}$$

(a) Centric loading: $F = 30 \text{ kips}, M = 0$

$$\sigma = \frac{30}{6.16}$$

$$\sigma = 4.87 \text{ ksi}$$

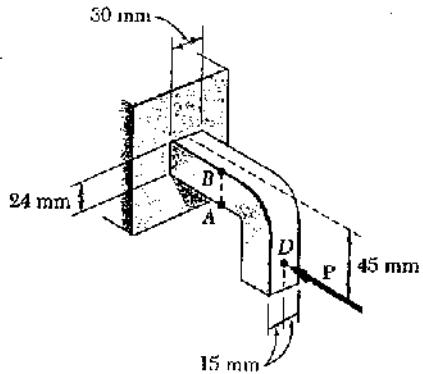
(b) Eccentric loading: $F = 2P = 20 \text{ kips}, M = (10)(3.5) = 35 \text{ kip-in}$

$$\sigma = \frac{20}{6.16} - \frac{(35)(4.14)}{75.3}$$

$$\sigma = 1.322 \text{ ksi}$$

Problem 4.102

4.102 Knowing that the magnitude of the horizontal force P is 8 kN, determine the stress at (a) point A, (b) point B.



$$A = (30)(24) = 720 \text{ mm}^2 = 720 \times 10^{-6} \text{ m}^2$$

$$e = 45 - 12 = 33 \text{ mm} = 0.033 \text{ m}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(30)(24)^3 = 34.56 \times 10^9 \text{ mm}^4 = 34.56 \times 10^{-9} \text{ m}^4$$

$$c = 24 \text{ mm} = 0.12 \text{ m} \quad P = 8 \times 10^3 \text{ N}$$

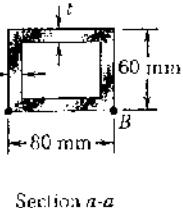
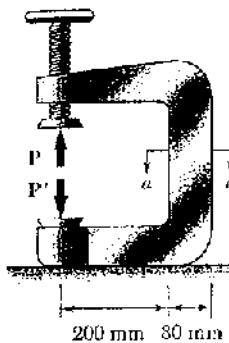
$$M = Pe = (8 \times 10^3)(0.033) = 264 \text{ N-m}$$

$$(a) \quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.12)}{34.56 \times 10^{-9}}$$

$$= -102.8 \times 10^6 \text{ Pa} = -102.8 \text{ MPa}$$

$$(b) \quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.12)}{34.56 \times 10^{-9}} = 80.6 \times 10^6 \text{ Pa} = 80.6 \text{ MPa}$$

Problem 4.103



4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10 \text{ mm}$. Knowing that the press has been tightened on wooden planks being glued together until $P = 20 \text{ kN}$, determine the stress at (a) point A, (b) point B.

Rectangular cutout is $60 \text{ mm} \times 40 \text{ mm}$

$$A = (80)(60) - (60)(40) = 2.4 \times 10^3 \text{ mm}^2 = 2.4 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84 \times 10^6 \text{ mm}^4 \\ = 1.84 \times 10^{-6} \text{ m}^4$$

$$c = 40 \text{ mm} = 0.040 \text{ m} \quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$

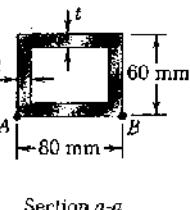
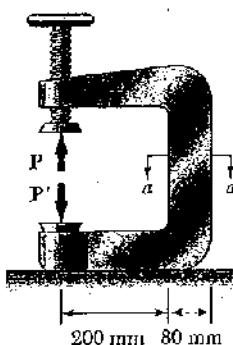
$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N-m}$$

$$(a) \sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{2.4 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = 112.7 \times 10^6 \text{ Pa} \\ \sigma_A = 112.8 \text{ MPa} \rightarrow$$

$$(b) \sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{2.4 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = -96.0 \times 10^6 \text{ Pa} \\ \sigma_B = -96.0 \text{ MPa} \rightarrow$$

Problem 4.104

4.104 Solve Prob. 4.103, assuming that $t = 8 \text{ mm}$.



4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10 \text{ mm}$. Knowing that the press has been tightened on wooden planks being glued together until $P = 20 \text{ kN}$, determine the stress at (a) point A, (b) point B.

Rectangular cutout is $64 \text{ mm} \times 44 \text{ mm}$

$$A = (80)(60) - (64)(44) = 1.984 \times 10^3 \text{ mm}^2 \\ = 1.984 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}(80)^3 - \frac{1}{12}(44)(64)^3 = 1.59881 \times 10^6 \text{ mm}^4 \\ = 1.59881 \times 10^{-6} \text{ m}^4$$

$$c = 40 \text{ mm} = 0.040 \text{ m} \quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$

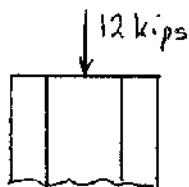
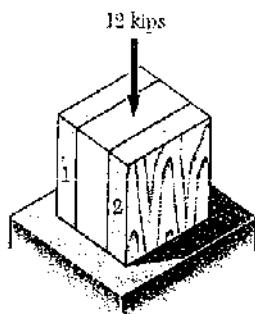
$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N-m}$$

$$(a) \sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = 130.2 \times 10^6 \text{ Pa} \\ \sigma_A = 130.2 \text{ MPa} \rightarrow$$

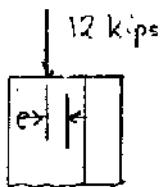
$$(b) \sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = -110.0 \times 10^6 \text{ Pa} \\ \sigma_B = -110.0 \text{ MPa} \rightarrow$$

Problem 4.105

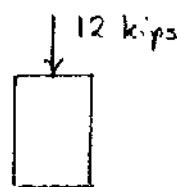
4.105 A short column is made by nailing two 1×4 -in. planks to a 2×4 -in. timber. Determine the largest compressive stress created in the column by a 12-kip load applied as shown at the center of the top section of the timber if (a) the column is as described, (b) plank 1 is removed, (c) both planks are removed.



(a)



(b)



(c)

(a) Centric loading: $4\text{ in} \times 4\text{ in}$ cross section $A = (4)(4) = 16 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{16} = -0.75 \text{ ksi}$$

(b) Eccentric loading: $4\text{ in} \times 3\text{ in}$ cross section $A = (4)(3) = 12 \text{ in}^2$

$$c = \left(\frac{l}{2}\right)(3) = 1.5 \text{ in} \quad e = 1.5 - 1.0 = 0.5 \text{ in.}$$

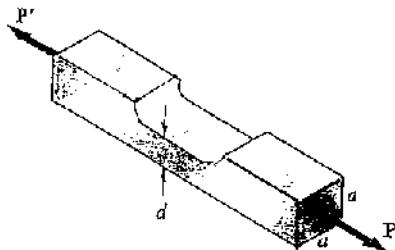
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(4)(3)^3 = 9 \text{ in}^4$$

$$\sigma = -\frac{P}{A} - \frac{Pec}{I} = -\frac{12}{12} - \frac{(12)(0.5)(1.5)}{9} = -2.00 \text{ ksi}$$

(c) Centric loading: $4\text{ in.} \times 2\text{ in.}$ cross section $A = (4)(2) = 8 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{8} = -1.50 \text{ ksi}$$

Problem 4.106



4.106 A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30 \text{ mm}$, $d = 20 \text{ mm}$, and $\sigma_{\text{all}} = 60 \text{ MPa}$, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \quad \text{where } K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

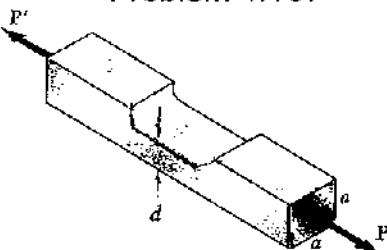
Data: $a = 30 \text{ mm} = 0.030 \text{ m}$ $d = 20 \text{ mm} = 0.020 \text{ m}$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60 \times 10^6}{4.1667 \times 10^3} = 14.40 \times 10^3 \text{ N}$$

$$P = 14.40 \text{ kN}$$

Problem 4.107



4.107 A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude P are applied at the centers of the ends of the bar. Knowing that $a = 30 \text{ mm}$ and $\sigma_{\text{all}} = 135 \text{ MPa}$, determine the smallest allowable depth d of the milled portion of the bar.

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P \frac{1}{2}(a-d) \frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{4ad^2}$$

$$\sigma = \frac{3P}{d^2} + \frac{2P}{ad} \quad \text{or} \quad \sigma d^2 + \frac{2P}{a} d - 3P = 0$$

$$\text{Solving for } d \quad d = \frac{1}{2\sigma} \left\{ \sqrt{\left(\frac{2P}{a}\right)^2 + (12P\sigma)} - \frac{2P}{a} \right\}$$

Data: $a = 0.030 \text{ m}$, $P = 18 \times 10^3 \text{ N}$, $\sigma = 135 \times 10^6 \text{ Pa}$

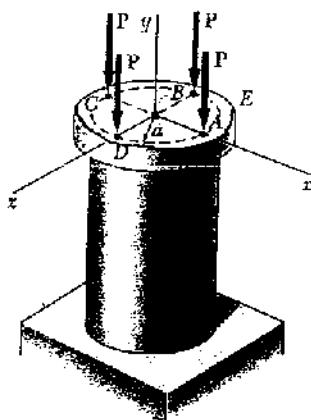
$$d = \frac{1}{(2)(135 \times 10^6)} \left\{ \sqrt{\left[\frac{(2)(18 \times 10^3)}{0.030}\right]^2 + 12(18 \times 10^3)(135 \times 10^6)} - \frac{(2)(18 \times 10^3)}{0.030} \right\}$$

$$= 16.04 \times 10^{-3}$$

$$d = 16.04 \text{ mm}$$

Problem 4.108

4.108 The four forces shown are applied to a rigid plate supported by a solid steel post of radius a . Knowing that $P = 100 \text{ kN}$ and $a = 40 \text{ mm}$, determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.



For a solid circular section of radius a

$$A = \pi a^2 \quad I = \frac{\pi}{4} a^4$$

(a) Centric force $F = 4P$, $M_x = M_z = 0$

$$\sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2}$$

(b) Force at D is removed.

$$F = 3P, \quad M_x = -Pa, \quad M_z = 0$$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4} a^2} = -\frac{7P}{\pi a^2}$$

(c) Forces at C and D are removed

$$F = 2P \quad M_x = -Pa, \quad M_z = -Pa$$

Resultant bending couple $M = \sqrt{M_x^2 + M_z^2} = \sqrt{2} Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2} Pa \cdot a}{\frac{\pi}{4} a^2} = -\frac{2+4\sqrt{2}}{\pi} \frac{P}{a^2} = -2.437 \frac{P}{a^2}$$

Numerical data: $P = 100 \times 10^3 \text{ N}$, $a = 0.040 \text{ m}$

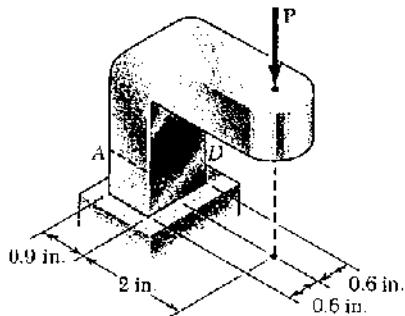
Answers: (a) $\sigma = \frac{(4)(100 \times 10^3)}{\pi (0.040)^2} = -79.6 \times 10^6 \text{ Pa} = -79.6 \text{ MPa}$

(b) $\sigma = -\frac{(7)(100 \times 10^3)}{\pi (0.040)^2} = -139.3 \times 10^6 \text{ Pa} = -139.3 \text{ MPa}$

(c) $\sigma = -\frac{(2.437)(100 \times 10^3)}{(0.040)^2} = -152.3 \times 10^6 \text{ Pa} = -152.3 \text{ MPa}$

Problem 4.109

4.109 Knowing that the allowable stress in section ABD is 10 ksi, determine the largest force P which can be applied to the bracket shown.



$$A = (1.2)(0.9) = 1.08 \text{ in}^2$$

$$I = \frac{1}{2}(1.2)(0.9)^3 = 72.9 \times 10^{-3} \text{ in}^4$$

$$c = \frac{1}{2}(0.9) = 0.45 \text{ in.}$$

$$e = 2 + 0.45 = 2.45 \text{ in.}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = PK$$

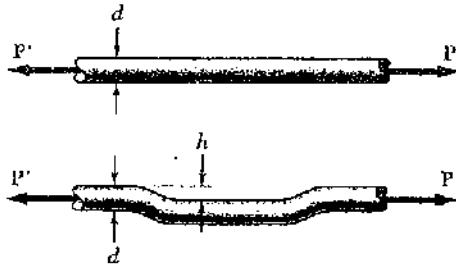
$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1.08} + \frac{(2.45)(0.45)}{72.9 \times 10^{-3}} = 16.049 \text{ in}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{10}{16.049} = 0.623 \text{ kip}$$

$$P = 623 \text{ lb.}$$

Problem 4.110

4.110 An offset h must be introduced into a solid circular rod of diameter d . Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.



$$\text{For centric loading } \sigma_c = \frac{P}{A}$$

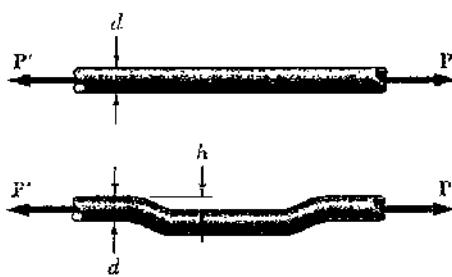
$$\text{For eccentric loading } \sigma_e = \frac{P}{A} + \frac{Phc}{I}$$

$$\text{Given } \sigma_e = 5\sigma_c$$

$$\frac{P}{A} + \frac{Phc}{I} = 5 \frac{P}{A}$$

$$\frac{Phc}{I} = 4 \frac{P}{A} \quad \therefore h = \frac{4I}{CA} = \frac{(4(\frac{\pi}{4}d^4))}{(\frac{\pi}{4}d^2)} = \frac{1}{2}d = 0.500d$$

Problem 4.111



4.111 An offset h must be introduced into a metal tube of 0.75-in. outer diameter and 0.08-in. wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the rod when it is straight, determine the largest offset that can be used.

$$c = \frac{1}{2}d = 0.375 \text{ in.}$$

$$c_i = c - t = 0.375 - 0.08 = 0.295 \text{ in}$$

$$A = \pi(c^2 - c_i^2) = \pi(0.375^2 - 0.295^2) \\ = 0.168389 \text{ in}^2$$

$$I = \frac{\pi}{4}(c^4 - c_i^4) = \frac{\pi}{4}(0.375^4 - 0.295^4) \\ = 9.5835 \times 10^{-3} \text{ in}^4$$

For centric loading $\sigma_{cen} = \frac{P}{A}$ For eccentric loading $\sigma_{ecc} = \frac{P}{A} + \frac{Phc}{I}$

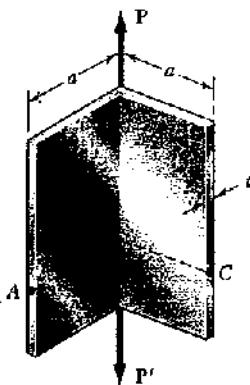
$$\sigma_{ecc} = 4\sigma_{cen} \text{ or } \frac{P}{A} + \frac{Phc}{I} = 4 \frac{P}{A}$$

$$\frac{hc}{I} = \frac{3}{A} \quad h = \frac{3I}{Ac} = \frac{(3)(9.5835 \times 10^{-3})}{(0.168389)(0.375)}$$

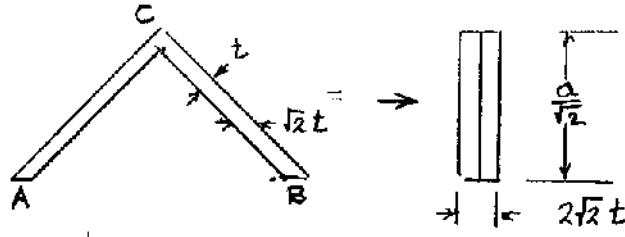
$$h = 0.455 \text{ in.}$$

Problem 4.112

4.112 The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (a) at A, (b) at B, (c) at C.



Moment of inertia about centroid



$$I = \frac{1}{12}(2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right)^3 \\ = \frac{1}{12}L^3 a^3$$

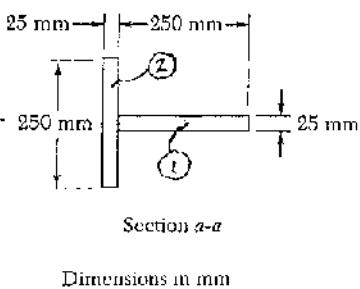
$$\text{Area } A = (2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right) = 2at \quad C = \frac{a}{2\sqrt{2}}$$

$$(a) \sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3} = -\frac{P}{2at}$$

$$(b) \sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3} = \frac{2P}{at}$$

$$(c) \sigma_C = \sigma_A = -\frac{P}{2at}$$

Problem 4.113



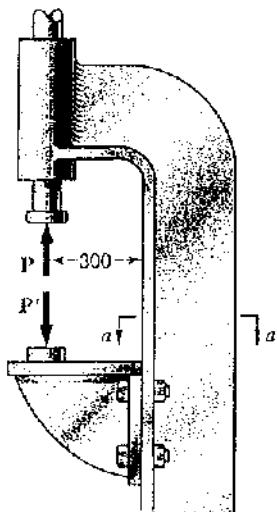
4.113 Knowing that the allowable stress in section a-a of the hydraulic press shown is 40 MPa in tension and 80 MPa in compression, determine the largest force P that can be exerted by the press.

Locate centroid of cross section.

	$A_i \text{ mm}^2$	$\bar{y}_i \text{ mm}$	$A\bar{y}_i \text{ mm}^3$
①	6250	18.5	781.25×10^3
②	6250	262.5	$1,640,625 \times 10^3$
Σ	12500		2.421875×10^6

$$\bar{Y}_o = \frac{2.421875 \times 10^6}{12500} = 193.75 \text{ mm}$$

The centroid lies at point C.



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(25)(250)^3 + (6250)(68.75)^2 \\ = 62.093 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(250)(25)^3 + (6250)(28.75)^2 \\ = 29.867 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 82.194 \times 10^6 \text{ mm}^4 = 91.960 \times 10^6 \text{ m}^4$$

$$A = 12500 \text{ mm}^2 = 12.5 \times 10^{-3} \text{ m}^2$$

$$\text{Eccentricity } e = 300 + 25 + 250 - 193.75 = 381.25 \text{ mm}$$

The bending moment about the centroid is $M = -Pe$.

$$\text{Stress: } \sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} + \frac{Pey}{I} = KP \quad \text{where } K = \frac{1}{A} + \frac{ey}{I}$$

$$\text{Then } P = \frac{\sigma}{K}$$

$$\text{Tensile stress: } \sigma = 40 \times 10^6 \text{ Pa}, \quad y = 81.25 \times 10^{-3} \text{ m}$$

$$K = \frac{1}{12.5 \times 10^{-3}} + \frac{(381.25 \times 10^{-3})(81.25 \times 10^{-3})}{91.960 \times 10^{-6}} = 416.85 \text{ m}^{-2}$$

$$P = \frac{40 \times 10^6}{416.85} = 96.0 \times 10^3 \text{ N}$$

$$\text{Compressive stress: } \sigma = -80 \times 10^6 \text{ Pa} \quad y = -193.75 \times 10^{-3} \text{ m}$$

$$K = \frac{1}{12.5 \times 10^{-3}} + \frac{(381.25 \times 10^{-3})(-193.75 \times 10^{-3})}{91.960 \times 10^{-6}} = -723.25 \text{ m}^{-2}$$

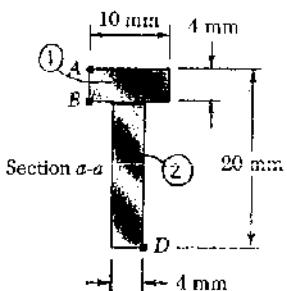
$$P = \frac{-80 \times 10^6}{-723.25} = 110.6 \times 10^3 \text{ N}$$

Choose the smaller value $P = 96.0 \times 10^3 \text{ N}$

$P = 96.0 \text{ kN}$

Problem 4.114

4.114 Knowing that the clamp shown has been tightened on wooden planks being glued together until $P = 400 \text{ N}$, determine in section $a-a$ (a) the stress at point A, (b) the stress at point D, (c) the location of the neutral axis.



Locate centroid.

Part	A, mm^2	\bar{y}_o, mm	$A\bar{y}_o, \text{mm}^3$
①	40	18	720
②	64	8	512
	104		1232

$$\bar{Y}_o = \frac{1232}{104} \\ = 11.846 \text{ mm}$$

The centroid lies 11.846 mm above point D.

$$\text{Eccentricity } e = (50 + 20 - 11.846) = -58.154 \text{ mm}$$

$$\begin{aligned} \text{Bending couple } M &= Pe = (400)(-58.154 \times 10^{-3}) \\ &= -23.262 \text{ N}\cdot\text{m} \end{aligned}$$

$$A = 104 \text{ mm}^2 = 104 \times 10^{-6} \text{ m}^2$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(10)(4)^3 + (40)(6.154)^2 = 1.5682 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4)(16)^3 + (64)(8.846)^2 = 2.3120 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 3.8802 \times 10^3 \text{ mm}^4 = 3.8802 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ Stress at point A: } y = 20 - 11.846 = 8.154 \text{ mm} = 8.154 \times 10^{-3} \text{ m}$$

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{400}{104 \times 10^{-6}} + \frac{(-23.262)(8.154 \times 10^{-3})}{3.8802 \times 10^{-9}} = 52.7 \times 10^6 \text{ Pa} \\ \sigma_A = 52.7 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \text{ Stress at point D. } y = +11.846 \text{ mm} = -11.846 \times 10^{-3} \text{ m}$$

$$\sigma_B = \frac{P}{A} + \frac{My}{I} = \frac{400}{104 \times 10^{-6}} + \frac{(-23.262)(-11.846 \times 10^{-3})}{3.8802 \times 10^{-9}} = -67.2 \times 10^6 \text{ Pa} \\ \sigma_B = -67.2 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \text{ Location of neutral axis. } \sigma = 0$$

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} + \frac{Pey}{I} = 0$$

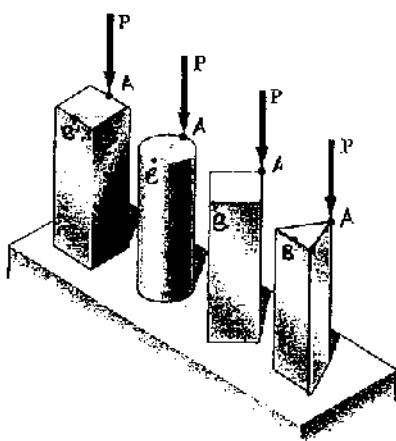
$$y = \frac{I}{Ae} = \frac{3.8802 \times 10^{-9}}{(104 \times 10^{-6})(-58.154 \times 10^{-3})} = -0.642 \times 10^{-3} \text{ m} \\ = -0.642 \text{ mm}$$

$$\text{Neutral axis lies } 11.846 - 0.642 = 11.204 \text{ mm}$$

11.20 mm above D \blacktriangleleft

Problem 4.115

4.115 The four bars shown have the same cross-sectional area. For the given loadings, show that (a) the maximum compressive stresses are in the ratio 4:5:7:9, (b) the maximum tensile stresses are in the ratio 2:3:5:3. (Note: the cross section of the triangular bar is an equilateral triangle.)



Stresses

$$\text{At } A \quad \sigma_A = -\frac{P}{A} - \frac{Pec_A}{I} \\ = -\frac{P}{A} \left(1 + \frac{Aec_A}{I} \right)$$

$$\text{At } B \quad \sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} \\ = \frac{P}{A} \left(\frac{Aec_B}{I} - 1 \right)$$

$$\left\{ \begin{array}{l} A_1 = a^2, \quad I_1 = \frac{1}{12} a^4, \quad c_A = c_B = \frac{1}{2} a, \quad e = \frac{1}{2} a \\ \sigma_A = -\frac{P}{A} \left(1 + \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^4} \right) = -4 \frac{P}{A}, \\ \sigma_B = \frac{P}{A} \left(\frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^4} - 1 \right) = 2 \frac{P}{A}. \end{array} \right.$$

$$\left\{ \begin{array}{l} A_2 = \pi c^2 = a^2 \quad \therefore c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4} c^4 \\ \sigma_A = -\frac{P}{A_2} \left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4} c^4} \right) = -5 \frac{P}{A_2}, \\ \sigma_B = \frac{P}{A_2} \left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4} c^4} - 1 \right) = 3 \frac{P}{A_2}. \end{array} \right.$$

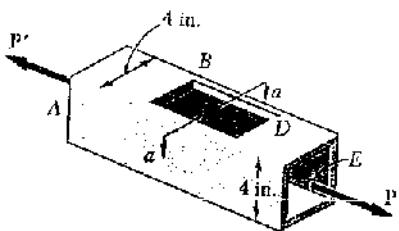
$$\left\{ \begin{array}{l} A_3 = a^2, \quad c = \frac{\sqrt{3}}{2} a, \quad I_3 = \frac{1}{12} a^4, \quad e = c \\ \sigma_A = -\frac{P}{A_3} \left(1 + \frac{(a^2)(\frac{\sqrt{3}}{2}a)(\frac{\sqrt{3}}{2}a)}{\frac{1}{12}a^4} \right) = -7 \frac{P}{A_3}, \\ \sigma_B = \frac{P}{A_3} \left(\frac{(a^2)(\frac{\sqrt{3}}{2}a)(\frac{\sqrt{3}}{2}a)}{\frac{1}{12}a^4} - 1 \right) = 5 \frac{P}{A_3}. \end{array} \right.$$

$$A_4 = \frac{1}{2}(s)(\frac{\sqrt{3}}{2}s) = \frac{\sqrt{3}}{4}s^2 \quad I_4 = \frac{1}{36}s(\frac{\sqrt{3}}{2}s)^3 = \frac{\sqrt{3}}{96}s^4$$

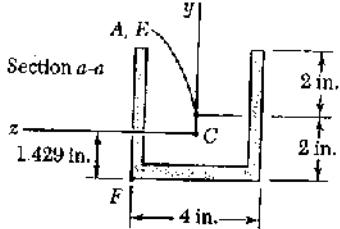
$$c_A = \frac{2}{3} \frac{\sqrt{3}}{2}s = \frac{s}{\sqrt{3}} = e \quad c_B = \frac{s}{2\sqrt{3}}$$

$$\left\{ \begin{array}{l} \sigma_A = -\frac{P}{A_4} \left(1 + \frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4} \right) = -9 \frac{P}{A_4}, \\ \sigma_B = \frac{P}{A_4} \left(\frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4} - 1 \right) = 3 \frac{P}{A_4}. \end{array} \right.$$

Problem 4.116



4.116 In order to provide access to the interior of a hollow square tube of 0.25-in. wall thickness, the portion *CD* of one side of the tube has been removed. Knowing that the loading of the tube is equivalent to two equal and opposite 15-kip forces acting at the geometric centers *A* and *E* of the ends of the tube, determine (a) the maximum stress in section *a-a*, (b) the stress at point *F*. Given: the centroid of the cross section is at *C* and $I_z = 4.81 \text{ in}^4$.



$$\text{Area } A = (2)(4)(0.25) + (3.5)(0.25) \\ = 2.875 \text{ in}^2$$

$$M = Pe = (15)(2 - 1.429) = 8.565 \text{ kip-in.}$$

- (a) Maximum stress occurs at top of section where $y = 4 - 1.429 = 2.571 \text{ in.}$

$$\sigma_{\max} = \frac{P}{A} + \frac{My}{I} = \frac{15}{2.875} + \frac{(8.565)(2.571)}{4.81}$$

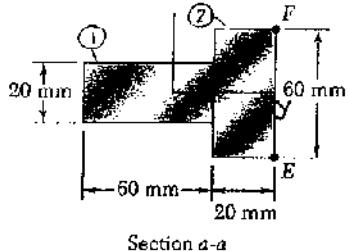
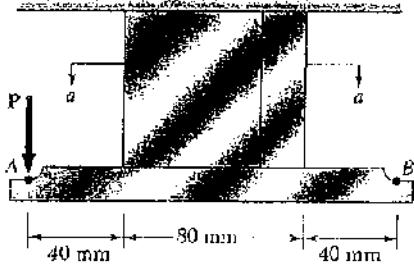
$$\sigma_{\max} = 9.80 \text{ ksi}$$

- (b) At point *F* $y = -1.429 \text{ in.}$

$$\sigma_F = \frac{P}{A} + \frac{My}{I} = \frac{15}{2.875} + \frac{(8.565)(-1.429)}{4.81} \quad \sigma_F = 2.67 \text{ ksi}$$

Problem 4.117

4.117 Knowing that the allowable stress is 150 MPa in section $a-a$ of the hanger shown, determine (a) the largest vertical force P that can be applied at point A , (b) the corresponding location of the neutral axis of section $a-a$.



Locate centroid

	A, mm^2	\bar{y}_o, mm	$A\bar{y}_o, \text{mm}^3$
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{y}_o = \frac{\sum A\bar{y}_o}{\sum A} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple $M = Pe$

$$e = 40 + 50 = 90 \text{ mm} = 0.090 \text{ m}$$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1,360 \times 10^6 \text{ mm}^4 = 1,360 \times 10^{-6} \text{ m}^4, \quad A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on tensile stress at left edge: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = KP$$

$$K = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(0.090)(-0.050)}{1.360 \times 10^{-6}} = 3.7255 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150 \times 10^6}{3.7255 \times 10^3} = 40.3 \times 10^3 \text{ N} = 40.3 \text{ kN}$$

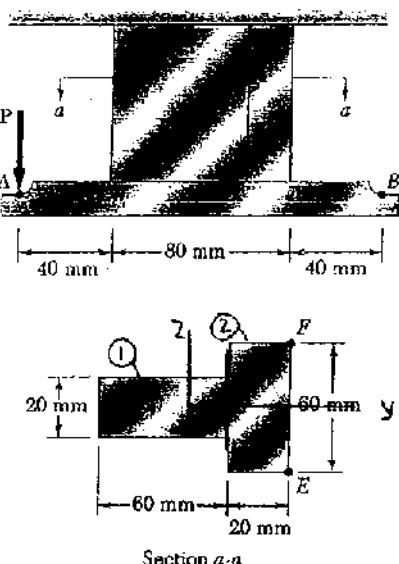
(b) Location of neutral axis: $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \quad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(0.090)} = 6.30 \times 10^{-3} \text{ m} = 6.30 \text{ mm}$$

The neutral axis lies 6.30 mm to the right of the centroid or 56.30 mm from the left face.

Problem 4.118



4.118 Solve Prob 4.117, assuming that the vertical force P is applied at point B .

4.117 Knowing that the allowable stress is 150 MPa in section $a-a$ of the hanger shown, determine (a) the largest vertical force P that can be applied at point A , (b) the corresponding location of the neutral axis of section $a-a$.

Locate centroid

	A, mm^2	\bar{y}_o, mm	$A\bar{y}_o, \text{mm}^3$
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\begin{aligned}\bar{y}_o &= \frac{\sum A \bar{y}_o}{\sum A} \\ &= \frac{120 \times 10^3}{2400} \\ &= 50 \text{ mm}\end{aligned}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple $M = Pe$

$$e = 50 - 120 = -70 \text{ mm} = -0.070 \text{ m}$$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360 \times 10^6 \text{ mm}^4 = 1.360 \times 10^{-6} \text{ m}^4, A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on stress at left edge of section: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_L P$$

$$K_L = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(-0.070)(-0.050)}{1.360 \times 10^{-6}} = -2.1569 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K_L} = \frac{-150 \times 10^6}{-2.1569 \times 10^3} = 69.6 \times 10^3 \text{ N}$$

Based on stress at right edge of section: $y = 30 \text{ mm} = 0.030 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_R P$$

$$K_R = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(-0.070)(0.030)}{1.360 \times 10^{-6}} = 1.9608 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K_R} = \frac{150 \times 10^6}{1.9608 \times 10^3} = 76.5 \times 10^3 \text{ N}$$

Choose the smaller value $P = 69.6 \times 10^3 \text{ N} = 69.6 \text{ kN}$

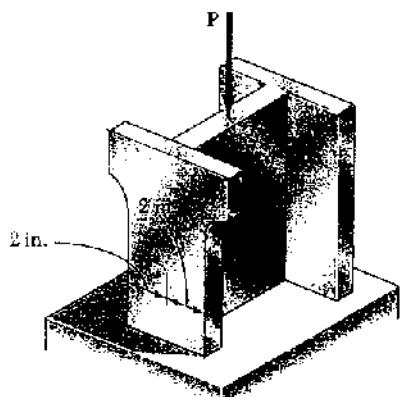
(b) Location of neutral axis: $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \quad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(-0.070)} = -8.10 \times 10^{-3} \text{ m} = -8.10 \text{ mm}$$

Neutral axis lies $50 - 8.10 = 41.9 \text{ mm}$ from left face.

Problem 4.119



4.119 Three steel plated, each of 1×6 -in cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 1-in. strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 15 ksi, determine the largest force P , (a) that could be applied to the original column, (b) that can be applied to the modified column.

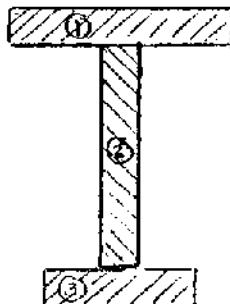
(a) Centric Loading

$$A = (3)(1)(6) = 18 \text{ in}^2$$

$$\sigma = -\frac{P}{A} \therefore P = \sigma A = (15)(18) = 270 \text{ kips}$$

(b) Eccentric Loading

Reduced cross section



	$A_i, \text{ in}^2$	$\bar{y}_{oi}, \text{ in}$	$A_i \bar{y}_{oi}, \text{ in}^3$
①	6	3.5	21.0
②	6	0	0
③	4	-3.5	-14.0
Σ	16		7.0

$$\begin{aligned}\bar{y}_o &= \frac{\sum A_i \bar{y}_o}{\sum A_i} \\ &= \frac{7.0}{16} \\ &= 0.4375 \text{ in}\end{aligned}$$

The centroid lies 0.4375 in. from the midpoint of the web.

$$I_1 = \frac{1}{12}(6)(1)^3 + (6)(3.0625)^2 = 56.773 \text{ in}^4$$

$$I_2 = \frac{1}{12}(1)(6)^3 + (6)(0.4375)^2 = 19.148 \text{ in}^4$$

$$I_3 = \frac{1}{12}(4)(1)^3 + (4)(3.9375)^2 = 62.349 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 138.27 \text{ in}^4, \quad c = 4.4375 \text{ in}$$

$$M = Pe \quad \text{where } e = 0.4375 \text{ in}$$

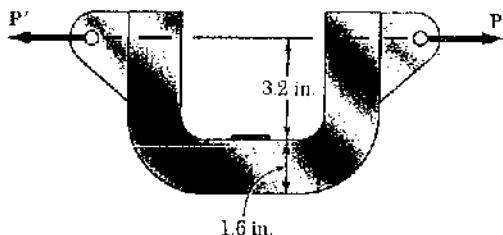
$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} + \frac{Pec}{I} = -K P$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{16} + \frac{(0.4375)(4.4375)}{138.27} = 0.076541 \text{ in}^{-2}$$

$$P = -\frac{\sigma}{K} = -\frac{-15}{0.076541} = 196.0 \text{ kips}$$

Problem 4.120

4.120 The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 1.6 in. and the strain on the inner edge was measured and found to be 450μ , determine the magnitude P of the forces. Use $E = 29 \times 10^6$ psi.



At the strain gage location

$$\sigma = E\varepsilon = (29 \times 10^6)(450 \times 10^{-6}) \\ = 13.05 \times 10^3 \text{ psi}$$

$$A = (1.6)(1.6) = 2.56 \text{ in}^2$$

$$I = \frac{1}{12}(1.6)(1.6)^3 = 0.54613 \text{ in}^4$$

$$e = 3.2 + 0.8 = 4.0 \text{ in.}$$

$$c = 0.8 \text{ in.}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

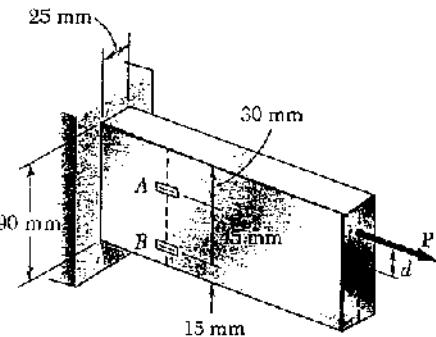
$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{2.56} + \frac{(4.0)(0.8)}{0.54613} = 6.25 \text{ in}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{13.05 \times 10^3}{6.25} = 2.088 \times 10^3 \text{ lb.}$$

$$P = 2.09 \text{ kips} \blacksquare$$

Problem 4.121

4.121 An eccentric force P is applied as shown to a steel bar of 25×90 -mm cross section. The strains at A and B have been measured and found to be



$$\epsilon_A = +350 \mu$$

$$\epsilon_B = -70 \mu$$

Knowing that $E = 200 \text{ GPa}$, determine (a) the distance d , (b) the magnitude of the force P .

$$h = 15 + 45 + 30 = 90 \text{ mm}$$

$$b = 25 \text{ mm}$$

$$c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 \\ = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \quad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B

$$\sigma_A = E\epsilon_A = (200 \times 10^9)(350 \times 10^{-6}) = 70 \times 10^6 \text{ Pa}$$

$$\sigma_B = E\epsilon_B = (200 \times 10^9)(-70 \times 10^{-6}) = -14 \times 10^6 \text{ Pa}$$

$$\tilde{\sigma}_A = \frac{P}{A} - \frac{My_A}{I} \quad (1)$$

$$\tilde{\sigma}_B = \frac{P}{A} - \frac{My_B}{I} \quad (2)$$

$$\text{Subtracting } \tilde{\sigma}_A - \tilde{\sigma}_B = - \frac{M(y_A - y_B)}{I}$$

$$M = - \frac{I(\tilde{\sigma}_A - \tilde{\sigma}_B)}{y_A - y_B} = - \frac{(1.51875 \times 10^{-6})(84 \times 10^6)}{0.045} = -2835 \text{ N.m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A \tilde{\sigma}_B - y_B \tilde{\sigma}_A = (y_A - y_B) \frac{P}{A}$$

$$P = \frac{A(y_A \tilde{\sigma}_B - y_B \tilde{\sigma}_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(-14 \times 10^6) - (-0.030)(-70 \times 10^6)]}{0.045} \\ = 94.5 \times 10^3 \text{ N}$$

$$(a) \quad M = -Pd \therefore d = -\frac{M}{P} = -\frac{-2835}{94.5 \times 10^3} = 0.030 \text{ m} = 30 \text{ mm} \quad \blacktriangleleft$$

(b)

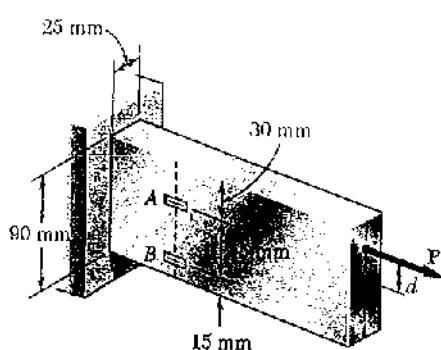
$$P = 94.5 \text{ kN} \quad \blacktriangleleft$$

Problem 4.122

4.122 Solve prob. 4.121, assuming that the measured strains are

$$\epsilon_A = +600 \mu$$

$$\epsilon_B = +420 \mu$$



4.121 An eccentric force P is applied as shown to a steel bar of 25×90 -mm cross section. The strains at A and B have been measured and found to be

$$\epsilon_A = +350 \mu$$

$$\epsilon_B = -70 \mu$$

Knowing that $E = 200 \text{ GPa}$, determine (a) the distance d , (b) the magnitude of the force P .

$$h = 15 + 45 + 30 = 90 \text{ mm}$$

$$b = 25 \text{ mm} \quad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-4} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \quad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B

$$\sigma_A = E\epsilon_A = (200 \times 10^9)(600 \times 10^{-6}) = 120 \times 10^6 \text{ Pa}$$

$$\sigma_B = E\epsilon_B = (200 \times 10^9)(420 \times 10^{-6}) = 84 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \quad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \quad (2)$$

$$\text{Subtracting} \quad \sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-4})(36 \times 10^6)}{0.045} = -1215 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A \sigma_B - y_B \sigma_A = (y_A - y_B) \frac{P}{A}$$

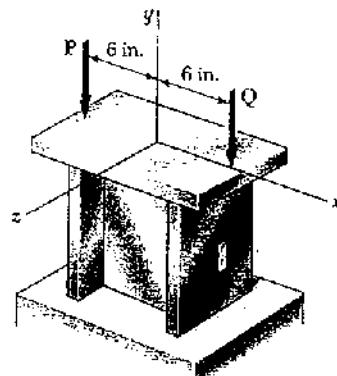
$$P = \frac{A(y_A \sigma_B - y_B \sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(84 \times 10^6) - (-0.030)(120 \times 10^6)]}{0.045} \\ = 243 \times 10^3 \text{ N}$$

$$M = -Pd$$

$$(a) \quad \therefore d = -\frac{M}{P} = -\frac{-1215}{243 \times 10^3} = 5 \times 10^{-3} \text{ m} = 5.00 \text{ mm}$$

$$(b) \quad P = 243 \text{ kN}$$

Problem 4.123



4.123 A short length of a rolled-steel column supports a rigid plate on which two loads P and Q are applied as shown. The strains at two point A and B on the center line of the outer faces of the flanges have been measured and found to be

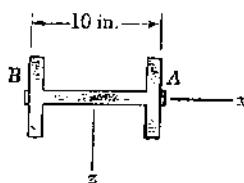
$$\epsilon_A = -400 \times 10^{-6} \text{ in./in.} \quad \epsilon_B = -300 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

Stresses at A and B from strain gages

$$\sigma_A = E\epsilon_A = (29 \times 10^6)(-400 \times 10^{-6}) = -11.6 \times 10^3 \text{ psi}$$

$$\sigma_B = E\epsilon_B = (29 \times 10^6)(-300 \times 10^{-6}) = -8.7 \times 10^3 \text{ psi}$$



Centric force $F = P + Q$

Bending couple $M = 6P - 6Q$

$$C = 5 \text{ in.}$$

$$A = 10.0 \text{ in}^2$$

$$I_x = 273 \text{ in}^4$$

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-11.6 \times 10^3 = +0.00989 P - 0.20989 Q \quad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

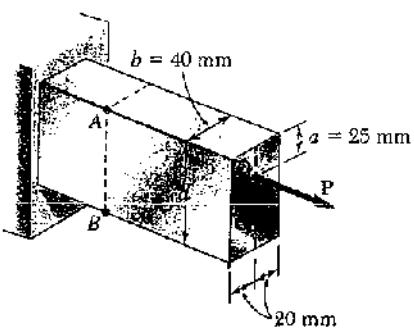
$$-8.7 \times 10^3 = -0.20989 P + 0.00989 Q \quad (2)$$

Solving (1) and (2) simultaneously

$$P = 44.2 \times 10^3 \text{ lb} = 44.2 \text{ kips}$$

$$Q = 57.3 \times 10^3 \text{ lb.} = 57.3 \text{ kips}$$

Problem 4.124



4.124 The eccentric axial force P acts at point D , which must be located 25 mm below the top surface of the steel bar shown. For $P = 60 \text{ kN}$, determine (a) the depth d of the bar for which the tensile stress at point A is maximum, (b) the corresponding stress at point A .

$$A = bd \quad I = \frac{1}{12} bd^3$$

$$C = \frac{1}{2}d \quad e = \frac{1}{2}d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

$$\sigma_A = \frac{P}{b} \left\{ \frac{1}{d} + \frac{12(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3} \right\} = \frac{P}{b} \left\{ \frac{4}{d} - \frac{6a}{d^2} \right\}$$

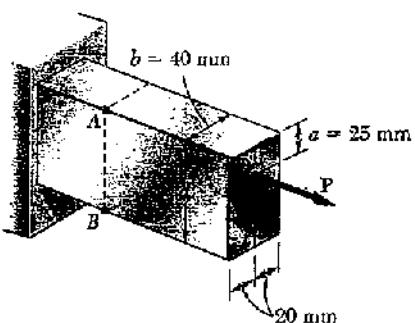
(a) Depth d for maximum σ_A . Differentiate with respect to d .

$$\frac{d\sigma_A}{dd} = \frac{P}{b} \left\{ -\frac{4}{d^2} + \frac{12a}{d^3} \right\} = 0 \quad d = 3a = 75 \text{ mm}$$

$$(b) \sigma_A = \frac{60 \times 10^3}{40 \times 10^{-3}} \left\{ \frac{4}{75 \times 10^{-3}} - \frac{(6)(25 \times 10^{-3})}{(75 \times 10^{-3})^2} \right\} = 40 \times 10^6 \text{ Pa} = 40 \text{ MPa}$$

Problem 4.125

4.125 For the bar and loading of Prob. 4.124, determine (a) the depth d of the bar for which the compressive stress at point B is maximum, (b) the corresponding stress at point B .



$$A = bd \quad I = \frac{1}{12} bd^3$$

$$C = \frac{1}{2}d \quad e = \frac{1}{2}d - a$$

$$\sigma_B = \frac{P}{A} - \frac{Pec}{I}$$

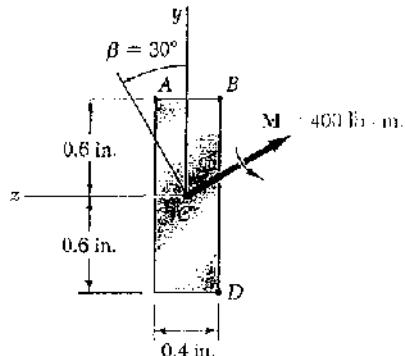
$$\sigma_B = \frac{P}{b} \left\{ \frac{1}{d} - \frac{(12)(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^2} \right\} = \frac{P}{b} \left\{ -\frac{2}{d} + \frac{6a}{d^2} \right\}$$

(a) Depth d for maximum σ_B : Differentiate with respect to d

$$\frac{d\sigma_B}{dd} = \frac{P}{b} \left\{ \frac{2}{d^2} - \frac{12a}{d^3} \right\} = 0 \quad d = 6a = 150 \text{ mm}$$

$$(b) \sigma_B = \frac{60 \times 10^3}{40 \times 10^{-3}} \left\{ -\frac{2}{150 \times 10^{-3}} + \frac{(6)(25 \times 10^{-3})}{(150 \times 10^{-3})^2} \right\} = -10 \times 10^6 \text{ Pa} = -10 \text{ MPa}$$

Problem 4.126



4.126 through 4.128 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A , (b) point B , (c) point D .

$$I_z = \frac{1}{12}(0.4)(1.2)^3 = 57.6 \times 10^{-3} \text{ in}^4$$

$$I_y = \frac{1}{12}(1.2)(0.4)^3 = 6.40 \times 10^{-3} \text{ in}^4$$

$$y_A = y_B = -y_D = 0.6 \text{ in}$$

$$z_A = -z_B = -z_D = (\frac{1}{2})(0.4) = 0.2 \text{ in.}$$

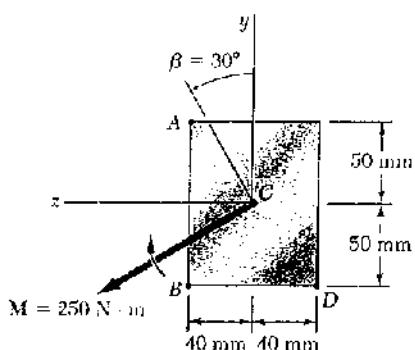
$$M_y = 400 \cos 60^\circ = 200 \text{ lb-in}, \quad M_z = -400 \sin 60^\circ = -346.41 \text{ lb-in}$$

$$(a) \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(346.41)(0.6)}{57.6 \times 10^{-3}} + \frac{(200)(0.2)}{6.40 \times 10^{-3}} \\ = 9.86 \times 10^3 \text{ psi} = 9.86 \text{ ksi}$$

$$(b) \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41)(0.6)}{57.6 \times 10^{-3}} + \frac{(200)(0.2)}{6.40 \times 10^{-3}} \\ = -2.64 \times 10^3 \text{ psi} = -2.64 \text{ ksi}$$

$$(c) \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41)(-0.6)}{57.6 \times 10^{-3}} + \frac{(200)(-0.2)}{6.40 \times 10^{-3}} \\ = -9.86 \times 10^3 \text{ psi} = -9.86 \text{ ksi}$$

Problem 4.127



4.126 through 4.128 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A , (b) point B , (c) point D .

$$I_z = \frac{1}{12}(80)(100)^3 = 6.6667 \times 10^6 \text{ mm}^4 = 6.6667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(100)(80)^3 = 4.2667 \times 10^6 \text{ mm}^4 = 4.2667 \times 10^{-6} \text{ m}^4$$

$$y_A = -y_B = -y_o = 50 \text{ mm}$$

$$z_A = z_B = -z_o = 40 \text{ mm}$$

$$M_y = -250 \sin 30^\circ = -125 \text{ N·m} \quad M_z = 250 \cos 30^\circ = 216.51 \text{ N·m}$$

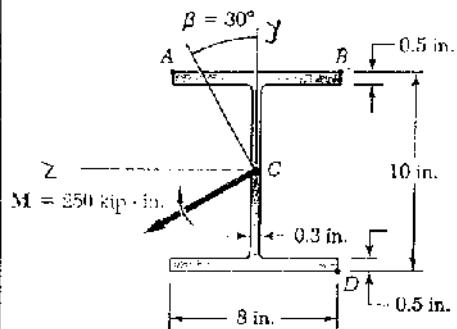
$$(a) \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(0.040)}{4.2667 \times 10^{-6}} \\ = -2.80 \times 10^6 \text{ Pa} \quad -2.80 \text{ MPa}$$

$$(b) \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(-0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(0.040)}{4.2667 \times 10^{-6}} \\ = 0.452 \times 10^3 \text{ Pa} \quad 0.452 \text{ MPa}$$

$$(c) \sigma_D = -\frac{M_z y_o}{I_z} + \frac{M_y z_o}{I_y} = -\frac{(216.51)(-0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(-0.040)}{4.2667 \times 10^{-6}} \\ = 2.80 \times 10^6 \text{ Pa} \quad 2.80 \text{ MPa}$$

Problem 4.128

4.126 through 4.128 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A , (b) point B , (c) point D .



$$\text{Flange: } I_z = \frac{1}{12}(8)(0.5)^3 + (8)(0.5)(4.75)^2 \\ = 90.333 \text{ in}^4$$

$$I_y = \frac{1}{12}(0.5)(8)^3 = 21.333 \text{ in}^4$$

$$\text{Web: } I_z = \frac{1}{12}(0.3)(9)^3 = 18.225 \text{ in}^4$$

$$I_y = \frac{1}{12}(9)(0.3)^3 = 0.02025 \text{ in}^4$$

$$\text{Total: } I_z = (2)(90.333) + 18.225 = 198.89 \text{ in}^4$$

$$I_y = (2)(21.333) + 0.02025 = 42.687 \text{ in}^4$$

$$y_A = y_B = -y_0 = 5 \text{ in.} ; z_A = -z_B = -z_0 = 4 \text{ in.}$$

$$M_z = 250 \cos 30^\circ = 216.51 \text{ kip-in}$$

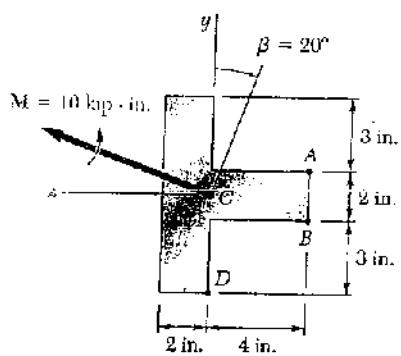
$$M_y = -250 \sin 30^\circ = -125 \text{ kip-in}$$

$$(a) \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(4)}{42.687} = -17.16 \text{ ksi}$$

$$(b) \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(-4)}{42.687} = 6.27 \text{ ksi}$$

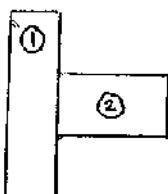
$$(c) \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-5)}{198.89} + \frac{(-125)(-4)}{42.687} = 17.16 \text{ ksi}$$

Problem 4.129



4.129 through 4.131 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.

Locate centroid



	$A, \text{ in}^2$	$\bar{z}, \text{ in}$	$A\bar{z}, \text{ in}^3$
①	16	-1	-16
②	8	2	16
Σ	24		0

The centroid lies at point C

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in.}, \quad y_D = -4 \text{ in.}$$

$$z_A = z_B = -4 \text{ in.}, \quad z_D = 0$$

$$M_z = 10 \cos 20^\circ = 9.3969 \text{ kip-in}$$

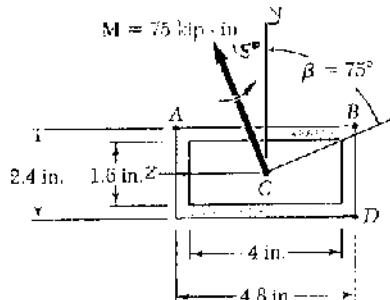
$$M_y = 10 \sin 20^\circ = 3.4202 \text{ kip-in.}$$

$$(a) \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(9.3969)(1)}{88} + \frac{(3.4202)(4)}{64} = 0.321 \text{ ksi}$$

$$(b) \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(9.3969)(-1)}{88} + \frac{(3.4202)(-4)}{64} = -0.107 \text{ ksi}$$

$$(c) \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64} = 0.427 \text{ ksi}$$

Problem 4.130



4.129 through 4.131 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A , (b) point B , (c) point D .

$$I_z = \frac{1}{12}(4.8)(2.4)^3 - \frac{1}{12}(4)(1.6)^3 = 4.1643 \text{ in}^4$$

$$I_y = \frac{1}{12}(2.4)(4.8)^3 - \frac{1}{12}(1.6)(4)^3 = 13.5851 \text{ in}^4$$

$$y_A = y_B = -y_D = 1.2 \text{ in.}$$

$$z_A = -z_B = -z_D = 2.4 \text{ in.}$$

$$M_z = 75 \sin 15^\circ = 19.4114 \text{ kip·in}$$

$$M_y = 75 \cos 15^\circ + 72.444 \text{ kip·in}$$

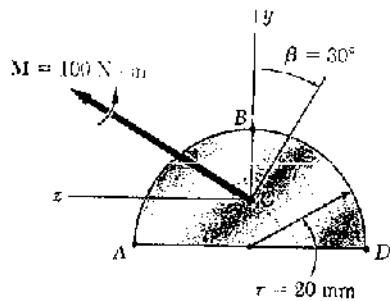
$$(a) \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(19.4114)(1.2)}{4.1643} + \frac{(72.444)(2.4)}{13.5851} = 7.20 \text{ ksi}$$

$$(b) \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(19.4114)(1.2)}{4.1643} + \frac{(72.444)(-2.4)}{13.5851} = -18.39 \text{ ksi}$$

$$(c) \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(19.4114)(-1.2)}{4.1643} + \frac{(72.444)(-2.4)}{13.5851} = -7.20 \text{ ksi}$$

Problem 4.131

4.129 through 4.131 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A , (b) point B , (c) point D .



$$I_z = \frac{\pi}{8} r^4 - \left(\frac{\pi}{2} r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(20)^4 = 17.5611 \times 10^3 \text{ mm}^4 = 17.5611 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{\pi}{8} r^4 = \frac{\pi (20)^4}{8} = 62.832 \times 10^3 \text{ mm}^4 = 62.832 \times 10^{-9} \text{ m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883 \text{ mm}$$

$$y_B = 20 - 8.4883 = 11.5117 \text{ mm}$$

$$z_A = -z_D = 20 \text{ mm} \quad z_B = 0$$

$$M_z = 100 \cos 30^\circ = 86.603 \text{ N·m}$$

$$M_y = 100 \sin 30^\circ = 50 \text{ N·m}$$

$$(a) \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883 \times 10^{-3})}{17.5611 \times 10^{-9}} + \frac{(50)(20 \times 10^{-3})}{62.832 \times 10^{-9}}$$

$$= 57.8 \times 10^6 \text{ Pa} \quad 57.8 \text{ MPa} \blacksquare$$

$$(b) \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117 \times 10^{-3})}{17.5611 \times 10^{-9}} + \frac{(50)(0)}{62.832 \times 10^{-9}}$$

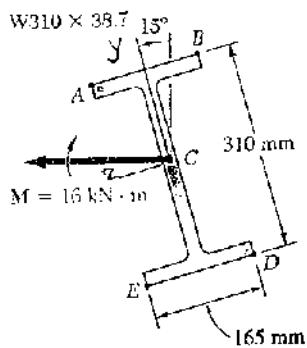
$$= -56.8 \times 10^6 \text{ Pa} \quad -56.8 \text{ MPa} \blacksquare$$

$$(c) \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883 \times 10^{-3})}{17.5611 \times 10^{-9}} + \frac{(50)(-20 \times 10^{-3})}{62.832 \times 10^{-9}}$$

$$= 25.9 \times 10^6 \text{ Pa} \quad 25.9 \text{ MPa} \blacksquare$$

Problem 4.132

4.132 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.



For W 310 x 38.7 rolled steel shape

$$I_z = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4$$

$$I_y = 7.27 \times 10^6 \text{ mm}^4 = 7.27 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = \frac{1}{2} y_o = -y_E = (\frac{1}{2}) 310 = 155 \text{ mm}$$

$$z_A = z_E = -z_B = -z_D = (\frac{1}{2})(165) = 82.5 \text{ mm}$$

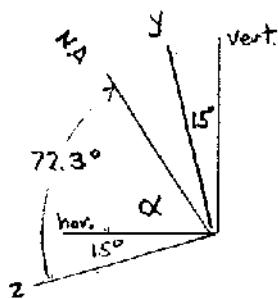
$$M_z = (16 \times 10^3) \cos 15^\circ = 15.455 \times 10^3 \text{ N·m}$$

$$M_y = (16 \times 10^3) \sin 15^\circ = 4.1411 \times 10^3 \text{ N·m}$$

$$(a) \tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{85.1 \times 10^{-6}}{7.27 \times 10^{-6}} \tan 15^\circ = 3.1365$$

$$\phi = 72.3^\circ$$

$$\alpha = 72.3 - 15 = 57.3^\circ$$



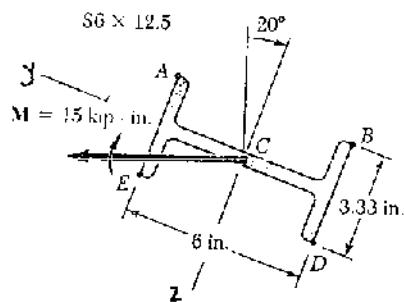
(b) Maximum tensile stress occurs at point E

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{85.1 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.27 \times 10^{-6}}$$

$$= 75.1 \times 10^6 \text{ Pa} = 75.1 \text{ MPa}$$

Problem 4.133

4.133 and 4.134 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.



For S6 x 12.5 rolled steel shape

$$I_z = 22.1 \text{ in}^4$$

$$I_y = 1.82 \text{ in}^4$$

$$Z_B = -Z_A = -Z_C = Z_D = \frac{1}{2}(3.33) = 1.665 \text{ in.}$$

$$y_A = y_B = -y_D = -y_E = \frac{1}{2}(6) = 3 \text{ in.}$$

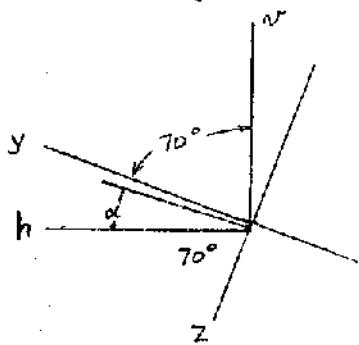
$$M_2 = 15 \sin 20^\circ = 5.1303 \text{ kip-in}$$

$$M_y = 15 \cos 20^\circ = 14.095 \text{ kip-in}$$

$$(a) \tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{22.1}{1.82} \tan(90^\circ - 20^\circ) = 33.36^\circ$$

$$\phi = 88.28^\circ$$

$$\alpha = 88.28^\circ - 70^\circ = 18.28^\circ$$

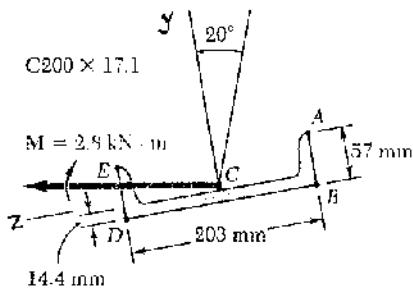


(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_2 y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(5.1303)(-3)}{22.1} + \frac{(14.095)(1.665)}{1.82} = 13.59 \text{ ksi}$$

Problem 4.134

4.133 and 4.134 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.



For C 200 x 17.1 rolled steel shape

$$I_z = 0.538 \times 10^6 \text{ mm}^4 = 0.538 \times 10^{-4} \text{ m}^4$$

$$I_y = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-4} \text{ m}^4$$

$$z_E = z_D = -z_A = -z_B = \frac{1}{2}(203) = 101.5 \text{ mm}$$

$$y_0 = y_E = -14.4 \text{ mm}$$

$$y_E = y_A = 57 - 14.4 = 42.6 \text{ mm}$$

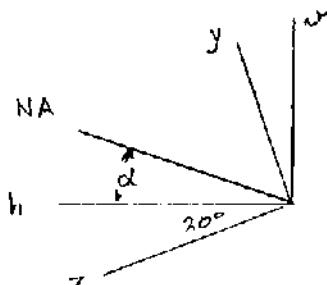
$$M_z = (2.8 \times 10^3) \cos 20^\circ = 2.6311 \times 10^3 \text{ N·m}$$

$$M_y = (2.8 \times 10^3) \sin 20^\circ = 957.66 \text{ N·m}$$

$$(a) \tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{0.538}{13.4} \tan 20^\circ = 0.014613$$

$$\phi = 0.837^\circ$$

$$\alpha = 20 - 0.837 = 19.16^\circ$$

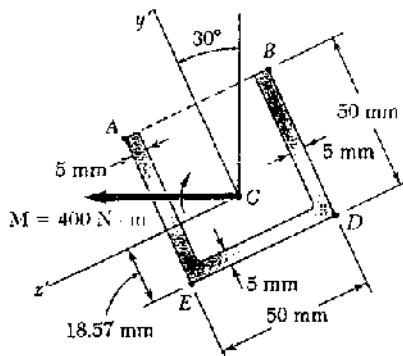


(b) Maximum tensile stress occurs at point D

$$\begin{aligned} \sigma_D &= -\frac{M_z y_0}{I_z} + \frac{M_y z_0}{I_y} = -\frac{(2.6311 \times 10^3)(-14.4 \times 10^{-3})}{0.538 \times 10^{-4}} + \frac{(957.66)(0.1015)}{13.4 \times 10^{-4}} \\ &= 70.423 \times 10^6 + 7.254 \times 10^6 = 77.7 \times 10^6 \text{ Pa} \\ &= 77.7 \text{ MPa} \end{aligned}$$

Problem 4.135

4.135 through 4.137 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.



$$I_y' = 281 \times 10^3 \text{ mm}^4$$

$$I_z' = 176.9 \times 10^3 \text{ mm}^4$$

$$I_z' = 176.9 \times 10^3 \text{ mm}^4 = 176.9 \times 10^{-9} \text{ m}^4$$

$$I_y' = 281 \times 10^3 \text{ mm}^4 = 281 \times 10^{-9} \text{ m}^4$$

$$y_E' = -18.57 \text{ mm}, \quad z_E = 25 \text{ mm}$$

$$M_z' = 400 \cos 30^\circ = 346.41 \text{ N}\cdot\text{m}$$

$$M_y' = 400 \sin 30^\circ = 200 \text{ N}\cdot\text{m}$$

$$(a) \tan \phi = \frac{I_z'}{I_y'} \tan \theta = \frac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30^\circ = 0.36346$$

$$\phi = 19.97^\circ$$

$$\alpha = 30^\circ - 19.97^\circ = 10.03^\circ$$

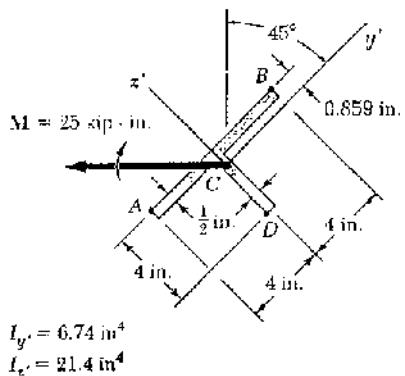
(b) Maximum tensile stress occurs at point E

$$\sigma_b = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$

$$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6 \text{ Pa}$$

$$= 54.2 \text{ MPa}$$

Problem 4.136



$$I_{y'} = 6.74 \text{ in}^4$$

$$I_{z'} = 21.4 \text{ in}^4$$

4.135 through 4.137 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.

$$I_z = 21.4 \text{ in}^4, I_{y'} = 6.74 \text{ in}^4$$

$$Z_A' = Z_B' = 0.859 \text{ in}$$

$$Z_D = -4 + 0.859 \text{ in} = -3.141 \text{ in}$$

$$y_A = -4 \text{ in}, y_B = 4 \text{ in}, y_D = -0.25 \text{ in}$$

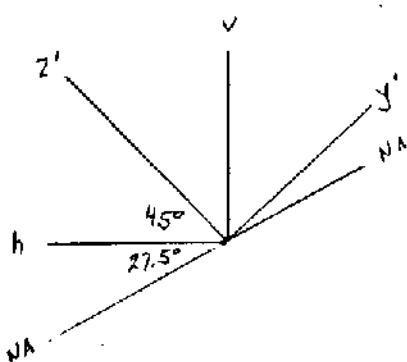
$$M_{y'} = -25 \sin 45^\circ = -17.678 \text{ kip·in}$$

$$M_{z'} = 25 \cos 45^\circ = 17.678 \text{ kip·in}$$

$$(a) \tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{21.4}{6.74} \tan (-45^\circ) = -3.1751$$

$$\phi = -72.5^\circ$$

$$\alpha = 72.5^\circ - 45^\circ = 27.5^\circ$$

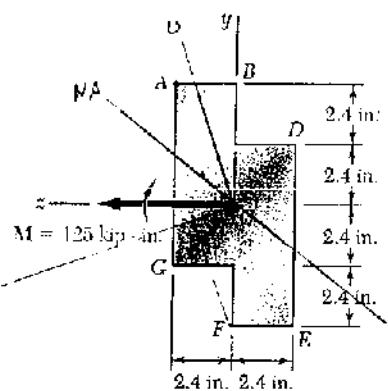


(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_{z'} y_D}{I_z} + \frac{M_{y'} z_D}{I_y} = -\frac{(-17.678)(-0.25)}{21.4} + \frac{(-17.678)(-3.141)}{6.74}$$

$$= 0.2065 + 8.238 = 8.44 \text{ ksi}$$

Problem 4.137



$$Y: (66.355 \text{ in}^4, 49.766 \text{ in}^4)$$

$$Z: (199.066 \text{ in}^4, -49.766 \text{ in}^4)$$

$$E: (132.710 \text{ in}^4, 0)$$

4.135 through 4.137 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.

$$I_y = 2 \left\{ \frac{1}{3} (7.2)(2.4)^3 \right\} = 66.355 \text{ in}^4$$

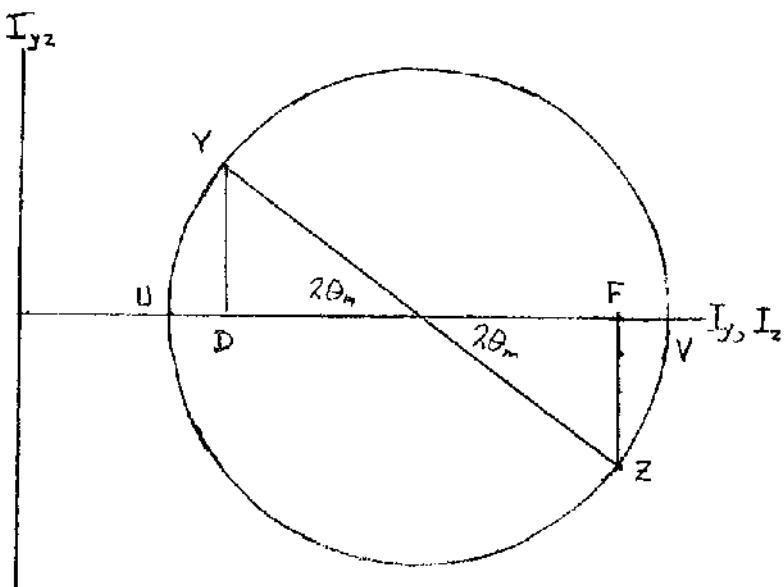
$$I_z = 2 \left\{ \frac{1}{12} (2.4)(7.2)^3 + (2.4)(7.2)(1.2)^2 \right\} = 199.066 \text{ in}^4$$

$$I_{yz} = 2 \{(2.4)(7.2)(1.2)\} = 49.766 \text{ in}^4$$

Using Mohr's circle determine the principal axes and principal moments of inertia.

$$\tan 2\theta_m = \frac{D Y}{D E} = \frac{49.766}{66.355}$$

$$2\theta_m = 36.87^\circ \quad \theta_m = 18.435^\circ$$



$$R = \sqrt{D E^2 + D Y^2} = 82.944 \text{ in}^4$$

$$I_u = 132.710 - 82.944 \\ = 49.766 \text{ in}^4$$

$$I_w = 132.710 + 82.944 \\ = 215.654 \text{ in}^4$$

$$\tan \theta_i = \frac{M_u}{M_w} = \frac{39.529}{118.585}$$

$$\theta_i = 18.435^\circ$$

$$\tan \phi = \frac{I_w}{I_u} \tan \theta_i = \frac{215.645}{49.766} \tan 18.435^\circ$$

$$\phi = 55.304^\circ$$

$$\beta = 55.304^\circ - 18.435^\circ = 36.869^\circ$$

$$\beta = 36.9^\circ \rightarrow$$

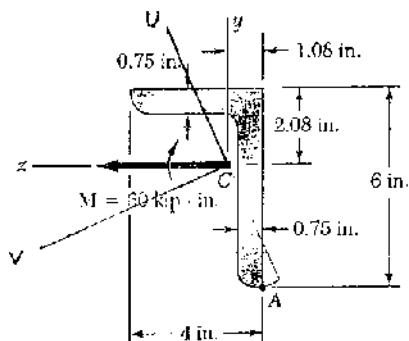
(b) Maximum tensile stress occurs at point F.

$$U_E = -4.8 \cos 18.435^\circ = -4.5537 \text{ in.} \quad N_E = 4.8 \sin 18.435^\circ = 1.5179 \text{ in.}$$

$$\sigma_p = -\frac{M_w U_E}{I_w} + \frac{M_u N_E}{I_u} = -\frac{(118.585)(-4.5537)}{215.654} + \frac{(39.529)(1.5179)}{49.766} = 3.71 \text{ ksi}$$

Problem 4.138

*4.138 and *4.139 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.



$$I_y = 8.7 \text{ in}^4$$

$$I_z = 24.5 \text{ in}^4$$

$$I_{yz} = +8.3 \text{ in}^4$$

$$Y(8.7, 8.3) \text{ in}^4$$

$$Z(24.5, -8.3) \text{ in}^4$$

$$E(16.6, 0) \text{ in}^4$$

$$EF = 7.9 \text{ in}^4$$

$$FZ = 8.3 \text{ in}^4$$

$$R = \sqrt{7.9^2 + 8.3^2} = 11.46 \text{ in}^4 \quad \tan 2\theta_m = \frac{FZ}{EF} = \frac{8.3}{7.9} = 1.0506$$

$$\theta_m = 23.2^\circ \quad I_o = 16.6 - 11.46 = 5.14 \text{ in}^4, \quad I_v = 16.6 + 11.46 = 28.06 \text{ in}^4$$

$$M_u = M \sin \theta_m = (60) \sin 23.2^\circ = 23.64 \text{ kip-in}$$

$$M_v = M \cos \theta_m = (60) \cos 23.2^\circ = 55.15 \text{ kip-in}$$

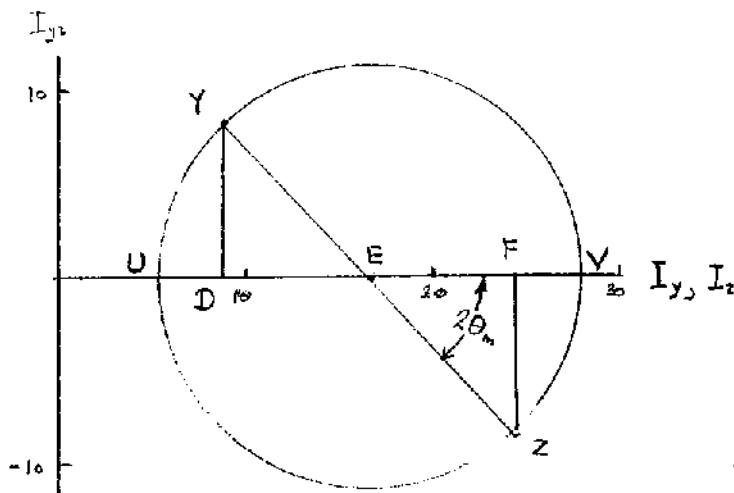
$$U_A = y_A \cos \theta_m + Z_A \sin \theta_m = -3.92 \cos 23.2^\circ + 1.08 \sin 23.2^\circ = -4.03 \text{ in.}$$

$$V_A = Z_A \cos \theta_m - y_A \sin \theta_m = -1.08 \cos 23.2^\circ + 3.92 \sin 23.2^\circ = 0.552 \text{ in.}$$

$$\sigma_A = -\frac{M_v U_A}{I_v} + \frac{M_u V_A}{I_o} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$$

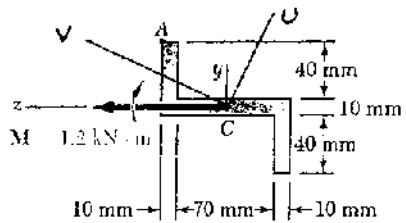
$$= 10.46 \text{ ksi}$$

Using Mohr's circle, determine the principal axes and principal moments of inertia



Problem 4.139

*4.138 and *4.139 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.



$$I_y = 1.894 \times 10^6 \text{ mm}^4$$

$$I_z = 0.614 \times 10^6 \text{ mm}^4$$

$$I_{yz} = +0.800 \times 10^6 \text{ mm}^4$$

$$Y(1.894, 0.800) \times 10^6 \text{ mm}^4$$

$$Z(0.614, 0.800) \times 10^6 \text{ mm}^4$$

$$E(1.254, 0) \times 10^6 \text{ mm}^4$$

$$R = \sqrt{EF^2 + FZ^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-4} = 1.0245 \times 10^6 \text{ mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6 \text{ mm}^4 = 0.2295 \times 10^6 \text{ mm}^4 = 0.2295 \times 10^{-6} \text{ m}^4$$

$$I_b = (1.254 + 1.0245) \times 10^6 \text{ mm}^4 = 2.2785 \times 10^6 \text{ mm}^4 = 2.2785 \times 10^{-6} \text{ m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \quad \theta_m = 25.67^\circ$$

$$M_v = M \cos \theta_m = (1.2 \times 10^3) \cos 25.67^\circ = 1.0816 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_u = -M \sin \theta_m = -(1.2 \times 10^3) \sin 25.67^\circ = -0.5198 \times 10^3 \text{ N} \cdot \text{m}$$

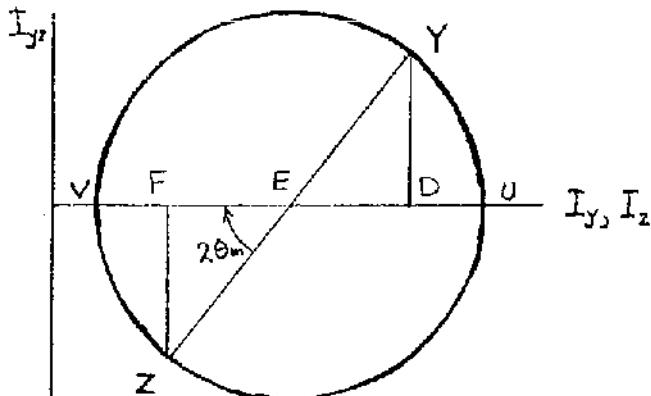
$$U_A = y_A \cos \theta_m - Z_A \sin \theta_m = 45 \cos 25.67^\circ - 45 \sin 25.67^\circ = 21.07 \text{ mm}$$

$$V_A = Z_A \cos \theta_m + y_A \sin \theta_m = 45 \cos 25.67^\circ + 45 \sin 25.67^\circ = 60.05 \text{ mm}$$

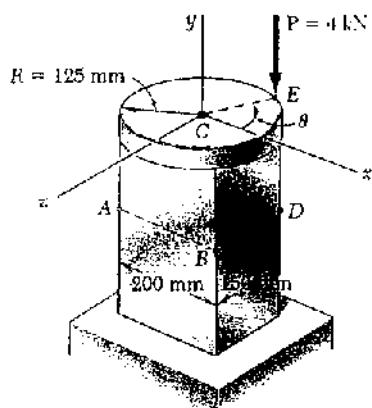
$$\sigma_A = -\frac{M_v U_A}{I_v} + \frac{M_u V_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6 \text{ Pa} = 113.0 \text{ MPa}$$

Using Mohr's circle, determine the principal axes and the principal moments of inertia.



Problem 4.140

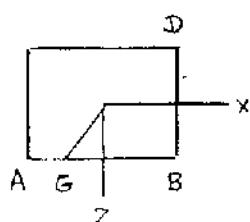


4.140 A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force P is applied at E with $\theta = 30^\circ$, determine (a) the stress at point A , (b) the stress at point B , (c) the point where the neutral axis intersects line ABD .

$$P = 4 \times 10^3 \text{ N (compression)}$$

$$M_x = -PR \sin 30^\circ \\ = -(4 \times 10^3)(125 \times 10^{-3}) \sin 30^\circ \\ = -250 \text{ N}\cdot\text{m}$$

$$M_z = -PR \cos 30^\circ \\ = -(4 \times 10^3)(125 \times 10^{-3}) \cos 30^\circ \\ = -433 \text{ N}\cdot\text{m}$$



$$I_x = \frac{1}{12}(200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12}(150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm}$$

$$z_A = z_B = 75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

$$(a) \sigma_A = -\frac{P}{A} + \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(-100 \times 10^{-3})}{100 \times 10^{-6}} \\ = 633 \times 10^3 \text{ Pa} = 633 \text{ kPa}$$

$$(b) \sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(100 \times 10^{-3})}{100 \times 10^{-6}} \\ = -233 \times 10^3 \text{ Pa} = -233 \text{ kPa}$$

(c) Let G be the point on AB where neutral axis intersects.

$$\sigma_G = 0 \quad z_G = 75 \text{ mm} \quad x_G = ?$$

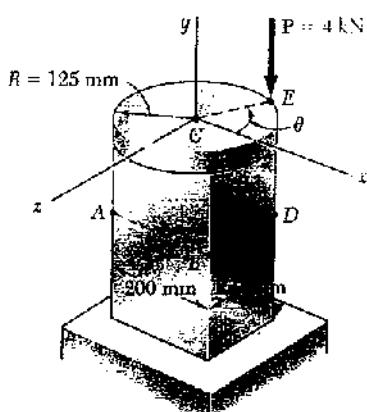
$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z} \left[\frac{P}{A} + \frac{M_x z_G}{I_x} \right] = \frac{100 \times 10^{-6}}{-433} \left\{ \frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} \right\}$$

$$= 46.2 \times 10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 146.2 mm from point A

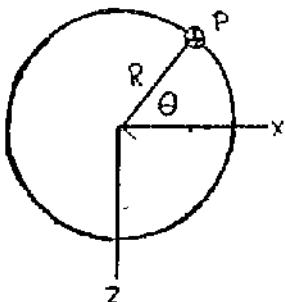
Problem 4.141



4.141 In Prob. 4.140, determine (a) the value of θ for which the stress at D reaches its largest value, (b) the corresponding values of the stress at A , B , C , and D .

4.140 A rigid circular plate of 125-mm radius is attached to a solid 150 \times 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force P is applied at E with $\theta = 30^\circ$, determine (a) the stress at point A , (b) the stress at point B , (c) the point where the neutral axis intersects line ABD .

(a)

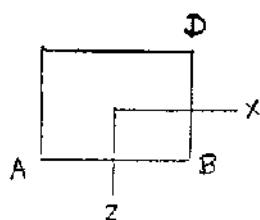


$$P = 4 \times 10^3 \text{ N}$$

$$PR = (4 \times 10^3)(125 \times 10^{-3}) = 500 \text{ N}\cdot\text{m}$$

$$M_x = -PR \sin \theta = -500 \sin \theta$$

$$M_z = -PR \cos \theta = -500 \cos \theta$$



$$I_x = \frac{1}{12}(200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12}(150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$x_D = 100 \text{ mm} \quad z_D = -75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P \left\{ \frac{1}{A} - \frac{Rz \sin \theta}{I_x} + \frac{Rx \cos \theta}{I_z} \right\}$$

For σ to be a maximum $\frac{d\sigma}{d\theta} = 0$ with $z = z_D$, $x = x_D$

$$\frac{d\sigma}{d\theta} = -P \left\{ 0 + \frac{Rz_D \cos \theta}{I_x} + \frac{Rx_D \sin \theta}{I_z} \right\} = 0$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100 \times 10^{-6})(-75 \times 10^{-3})}{(56.25 \times 10^{-6})(100 \times 10^{-3})} = \frac{4}{3}$$

$$\sin \theta = 0.8, \quad \cos \theta = 0.6, \quad \theta = 53.1^\circ$$

$$(b) \sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(500)(0.8)(16 \times 10^{-3})}{56.25 \times 10^{-6}} - \frac{(500)(0.6)(100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$= (-0.13333 + 0.53333 + 0.300) \times 10^6 \text{ Pa} = 0.700 \times 10^6 \text{ Pa} = 700 \text{ kPa}$$

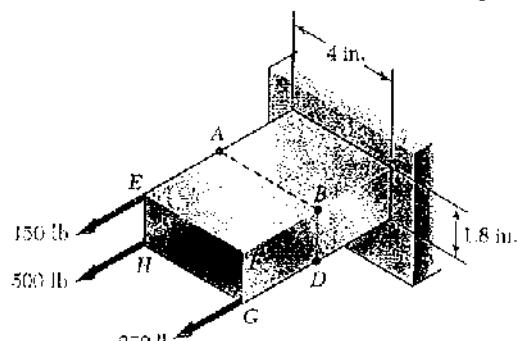
$$\sigma_B = (-0.13333 + 0.53333 - 0.300) \times 10^6 \text{ Pa} = 0.100 \times 10^6 \text{ Pa} = 100 \text{ kPa}$$

$$\sigma_C = (-0.13333 + 0 + 0) \times 10^6 \text{ Pa} = -0.13333 \times 10^6 \text{ Pa} = -133.3 \text{ kPa}$$

$$\sigma_D = (-0.13333 - 0.53333 - 0.300) \times 10^6 \text{ Pa} = -0.967 \times 10^6 \text{ Pa} = -967 \text{ kPa}$$

Problem 4.142

4.142 For the loading shown, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.



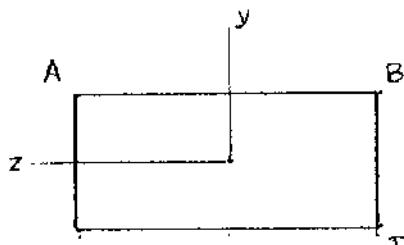
Add y and z axes as shown

$$A = (4)(1.8) = 7.2 \text{ in}^2$$

$$I_z = \frac{1}{12}(4)(1.8)^3 = 1.944 \text{ in}^4$$

$$I_y = \frac{1}{12}(1.8)(4)^3 = 9.6 \text{ in}^4$$

Resultant force and bending couples



$$P = 150 + 500 + 250 = 900 \text{ lb}$$

$$M_z = -(150)(0.9) + (500)(0.9) + (250)(0.9) \\ = 540 \text{ lb-in}$$

$$M_y = (150)(2) + (500)(2) - (250)(2) \\ = 800 \text{ lb-in}$$

$$(a) \sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{900}{7.2} - \frac{(540)(0.9)}{1.944} + \frac{(800)(2)}{9.6} = 41.7 \text{ psi}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{900}{7.2} - \frac{(540)(0.9)}{1.944} + \frac{(800)(-2)}{9.6} = 292 \text{ psi}$$

(b) Intersection of neutral axis with line AB or its extension.

$$\sigma = 0, \quad y = 0.9 \text{ in.}, \quad z = ?$$

$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{900}{7.2} - \frac{(540)(0.9)}{1.944} + \frac{800 z}{9.6} \\ - 125 + 83.333 z = 0 \quad z = 1.5 \text{ in.}$$

Intersects AB at 0.500 in. from A

Intersection of neutral axis with line BD or its extension.

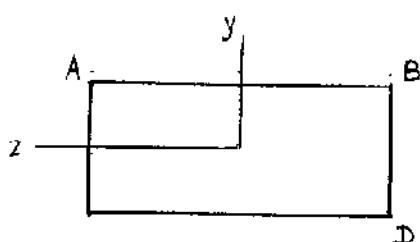
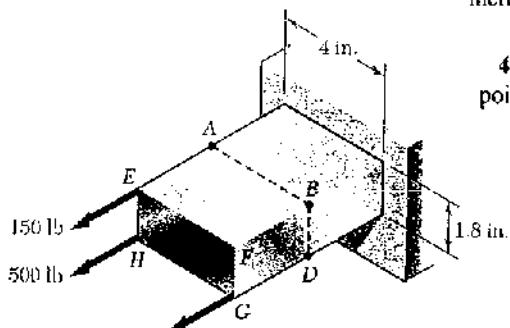
$$\sigma = 0 \quad z = -2 \text{ in.} \quad y = ?$$

$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{900}{7.2} - \frac{540 y}{1.944} + \frac{(800)(-2)}{9.6} \\ - 41.6667 - 277.778 y = 0 \quad y = -0.15 \text{ in.}$$

Intersects BD at 0.750 in. from D

Problem 4.143

4.143 Solve Prob. 4.142, assuming that the magnitude of the force applied at G is increased from 250 lb to 400 lb.



Add y and z axes as shown.

$$A = (4)(1.8) = 7.2 \text{ in}^2$$

$$I_z = \frac{1}{12}(4)(1.8)^3 = 1.944 \text{ in}^4$$

$$I_y = \frac{1}{12}(1.8)(4)^3 = 9.6 \text{ in}^4$$

Resultant force and bending couples

$$P = 150 + 500 + 400 = 1050 \text{ lb.}$$

$$M_z = -(150)(0.9) + (500)(0.9) + (400)(0.9) \\ = 675 \text{ lb-in}$$

$$M_y = (150)(2) + (500)(2) - (400)(2) \\ = 500 \text{ lb-in}$$

$$(a) \sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{1050}{7.2} - \frac{(675)(0.9)}{1.944} + \frac{(500)(2)}{9.6} = -62.5 \text{ psi}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{1050}{7.2} - \frac{(675)(0.9)}{1.944} + \frac{(500)(-2)}{9.6} = -271 \text{ psi}$$

(b) Intersection of neutral axis with line AB or its extension.

$$\sigma = 0, \quad y = 0.9 \text{ in.} \quad z = ?$$

$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{1050}{7.2} - \frac{(675)(0.9)}{1.944} + \frac{500 z}{9.6}$$

$$-166.667 + 52.083 z = 0 \quad z = 3.2 \text{ in.}$$

Does not intersect AB

Intersection of neutral axis with line BD or its extension.

$$\sigma = 0, \quad z = -2 \text{ in.} \quad y = ?$$

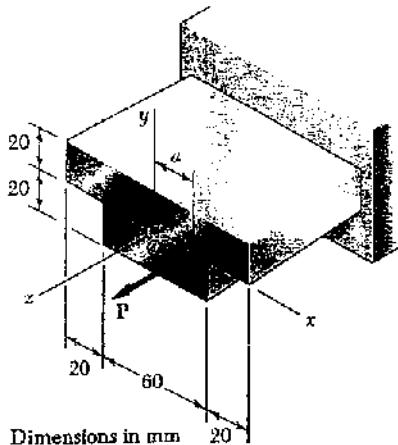
$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{1050}{7.2} - \frac{675 y}{1.944} + \frac{(500)(-2)}{9.6}$$

$$41.6667 - 347.222 y = 0 \quad y = 0.120 \text{ in.}$$

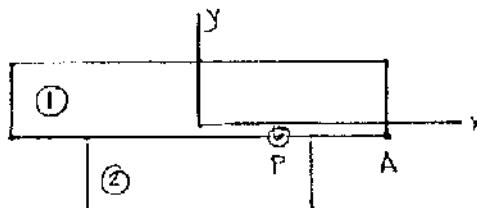
Intersects BD at 0.780 in. from B

Problem 4.144

4.144 A horizontal load P of magnitude 100 kN is applied to the beam shown. Determine the largest distance a for which the maximum tensile stress in the beam does not exceed 75 MPa.



Locate the centroid



	$A_i \text{ mm}^2$	$\bar{y}_i \text{ mm}$	$A\bar{y}_i \text{ mm}^3$
①	2000	10	20×10^3
②	1200	-10	-12×10^3
Σ	3200		8×10^3

$$\begin{aligned}\bar{y} &= \frac{\sum A\bar{y}}{\sum A} \\ &= \frac{8 \times 10^3}{3200} \\ &= 2.5 \text{ mm}\end{aligned}$$

Move coordinate origin to the centroid

Coordinates of load point: $x_P = a$, $y_P = -2.5 \text{ mm}$

$$\text{Bending couples} \quad M_x = y_P P \quad M_y = -aP$$

$$I_x = \frac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \frac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4 = 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(20)(100)^3 + \frac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} \quad \sigma_A = 75 \times 10^6 \text{ Pa}, \quad P = 100 \times 10^3 \text{ N}$$

$$M_y = \frac{I_y}{x} \left\{ \frac{P}{A} + \frac{M_x y}{I_x} - \sigma \right\} \quad \text{For point A} \quad x = 50 \text{ mm}, y = -2.5 \text{ mm}$$

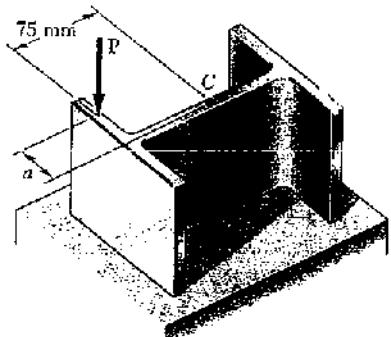
$$M_y = \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}} \left\{ \frac{100 \times 10^3}{3200 \times 10^{-6}} + \frac{(-2.5)(100 \times 10^3)(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} - 75 \times 10^6 \right\}$$

$$= \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}} \left\{ 31.25 + 1.537 - 75 \right\} \times 10^6 = -1.7111 \times 10^3 \text{ N-m}$$

$$a = -\frac{M_y}{P} = -\frac{(1.7111 \times 10^3)}{100 \times 10^3} = 17.11 \times 10^{-3} \text{ m} = 17.11 \text{ mm}$$

Problem 4.145

4.145 An axial load P of magnitude 50 kN is applied as shown to a short section of W 150 × 24 rolled-steel member. Determine the largest distance a for which the maximum compressive stress does not exceed 90 MPa.



Add y - and z axes.

For W 150 × 24 rolled-steel section

$$A = 3060 \text{ mm}^2 = 3060 \times 10^{-6} \text{ m}^2$$

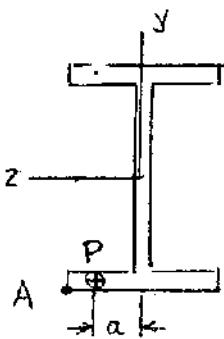
$$I_z = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$$

$$I_y = 1.83 \times 10^5 \text{ mm}^4 = 1.83 \times 10^{-5} \text{ m}^4$$

$$d = 160 \text{ mm}, \quad b_f = 102 \text{ mm}$$

$$y_A = -\frac{d}{2} = -80 \text{ mm}, \quad z_A = \frac{b_f}{2} = 51 \text{ mm}$$

$$P = 50 \times 10^3 \text{ N}$$



$$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3 \text{ N-m}$$

$$M_y = -Pa$$

$$\sigma_A = -\frac{P}{A} - \frac{M_z Y_A}{I_z} + \frac{M_y Z_A}{I_y} \quad \sigma_A = -90 \times 10^6 \text{ Pa}$$

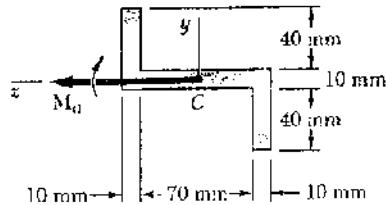
$$\begin{aligned} M_y &= \frac{I_y}{Z_A} \left\{ \frac{M_z Y_A}{I_z} + \frac{P}{A} + \sigma_A \right\} \\ &= \frac{1.83 \times 10^{-5}}{51 \times 10^{-3}} \left\{ \frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6) \right\} \\ &= \frac{1.83 \times 10^{-5}}{51 \times 10^{-3}} \left\{ +22.388 + 16.340 - 90 \right\} \times 10^6 \\ &= -1.8398 \times 10^3 \text{ N-m} \end{aligned}$$

$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3} \text{ m} = 36.8 \text{ mm}$$

Problem 4.146

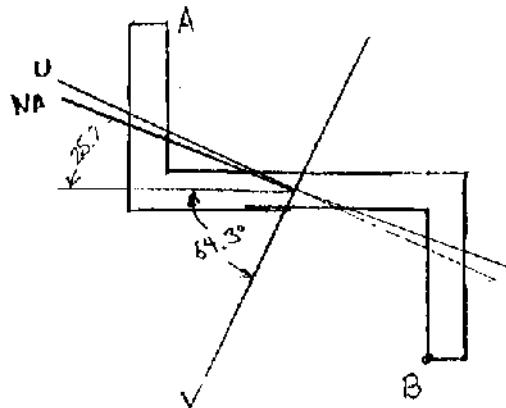
4.146 The Z section shown is subjected to a couple M_o acting in a vertical plane. Determine the largest permissible value of the moment M_o of the couple if the maximum stress is not to exceed 80 MPa. Given: $I_{max} = 2.28 \times 10^6 \text{ mm}^4$, $I_{min} = 0.23 \times 10^6 \text{ mm}^4$, principal axes 25.7° and 64.3° .

[[down to the right and up to the right]]



$$I_v = I_{max} = 2.28 \times 10^6 \text{ mm}^4 = 2.28 \times 10^{-6} \text{ m}^4$$

$$I_u = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^{-6} \text{ m}^4$$



$$M_v = M_o \cos 64.3^\circ$$

$$M_u = M_o \sin 64.3^\circ$$

$$\Theta = 64.3^\circ$$

$$\tan \phi = \frac{I_v}{I_u} \tan \Theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3^\circ = 20.597$$

$$\phi = 87.22^\circ$$

Points A and B are farthest from the neutral axis.

$$U_B = y_B \cos 64.3^\circ + z_B \sin 64.3^\circ = (-45) \cos 64.3^\circ + (-35) \sin 64.3^\circ \\ = -51.05 \text{ mm}$$

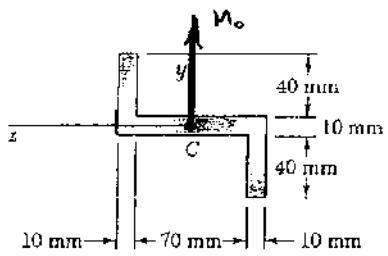
$$V_B = z_B \cos 64.3^\circ - y_B \sin 64.3^\circ = (-35) \cos 64.3^\circ - (-45) \sin 64.3^\circ \\ = +25.37 \text{ mm}$$

$$\sigma_B = -\frac{M_v U_B}{I_v} + \frac{M_u V_B}{I_u}$$

$$80 \times 10^6 = -\frac{(M_o \cos 64.3^\circ)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_o \sin 64.3^\circ)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}} \\ = 93.81 \times 10^3 \text{ N/m}$$

$$M_o = \frac{80 \times 10^6}{109.1 \times 10^3} = 733 \text{ N}\cdot\text{m}$$

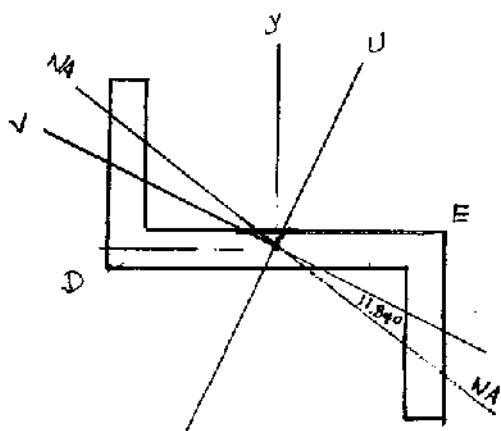
Problem 4.147



4.147 Solve Prob. 4.146 assuming that the couple M_o act in a horizontal plane.

4.146 The Z section shown is subjected to a couple M_o acting in a vertical plane. Determine the largest permissible value of the moment M_o of the couple if the maximum stress is not to exceed 80 MPa. Given: $I_{max} = 2.28 \times 10^6 \text{ mm}^4$, $I_{min} = 0.23 \times 10^6 \text{ mm}^4$, principal axes 25.7° and 64.3°

[[down to the right and up to the right]]



$$I_v = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^6 \text{ m}^4$$

$$I_o = I_{max} = 2.28 \times 10^6 \text{ mm}^4 = 2.28 \times 10^6 \text{ m}^4$$

$$M_v = M_o \cos 64.3^\circ$$

$$M_u = M_o \sin 64.3^\circ$$

$$\theta = 64.3^\circ$$

$$\tan \phi = \frac{I_v}{I_o} \tan \theta \\ = \frac{0.23 \times 10^{-6}}{2.28 \times 10^{-6}} \tan 64.3^\circ = 0.20961$$

$$\phi = 11.84^\circ$$

Points D and E are farthest from the neutral axis.

$$U_o = y_o \cos 25.7^\circ - z_o \sin 25.7^\circ = (-5) \cos 25.7^\circ - 45 \sin 25.7^\circ \\ = -24.02 \text{ mm}$$

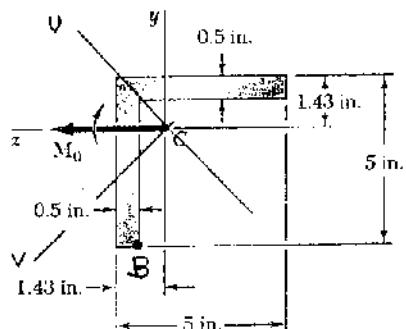
$$V_o = z_o \cos 25.7^\circ + y_o \sin 25.7^\circ = 45 \cos 25.7^\circ + (-5) \sin 25.7^\circ \\ = 38.38 \text{ mm}$$

$$\sigma_o = -\frac{M_v U_o}{I_v} + \frac{M_u V_o}{I_o} = -\frac{(M_o \cos 64.3^\circ)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}} + \frac{(M_o \sin 64.3^\circ)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$

$$80 \times 10^6 = 60.48 \times 10^3 M_o$$

$$M_o = 1.323 \times 10^3 \text{ N}\cdot\text{m} = 1.323 \text{ kN}\cdot\text{m}$$

Problem 4.148



4.148 A beam having the cross section shown is subjected to a couple M_o that acts in a vertical plane. Determine the largest permissible value of the moment M_o of the couple if the maximum stress in the beam is not to exceed 12 ksi. Given: $I_y = I_z = 11.3 \text{ in}^4$, $A = 4.75 \text{ in}^2$, $k_{min} = 0.983 \text{ in}$. (Hint: By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_y + I_z$)

$$M_u = M_o \sin 45^\circ = 0.70711 M_o$$

$$M_v = M_o \cos 45^\circ = 0.7071 M_o$$

$$I_{min} = Ak_{min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

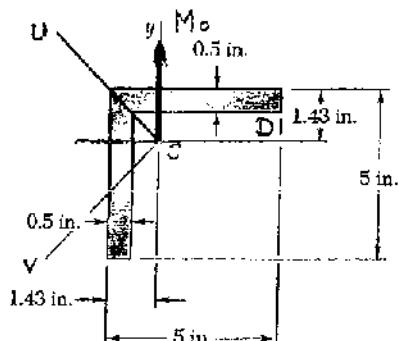
$$U_B = y_B \cos 45^\circ + z_B \sin 45^\circ = -3.57 \cos 45^\circ + 0.93 \sin 45^\circ = -1.866 \text{ in.}$$

$$V_B = z_B \cos 45^\circ - y_B \sin 45^\circ = -0.93 \cos 45^\circ - (-3.57) \sin 45^\circ = 3.182 \text{ in}$$

$$\begin{aligned} \sigma_B &= -\frac{M_u U_B}{I_v} + \frac{M_o V_B}{I_o} = -0.70711 M_o \left[-\frac{U_B}{I_{min}} + \frac{V_B}{I_{max}} \right] \\ &= 0.70711 M_o \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124 M_o \end{aligned}$$

$$M_o = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124} = 29.1 \text{ kip-in}$$

Problem 4.149



4.149 Solve Prob. 4.148, assuming that the couple M_o acts in a horizontal plane.

4.148 A beam having the cross section shown is subjected to a couple M_o that acts in a vertical plane. Determine the largest permissible value of the moment M_o of the couple if the maximum stress in the beam is not to exceed 12 ksi. Given: $I_y = I_z = 11.3 \text{ in}^4$, $A = 4.75 \text{ in}^2$, $k_{min} = 0.983 \text{ in}$. (Hint: By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_y + I_z$)

$$M_u = M_o \cos 45^\circ = 0.70711 M_o$$

$$M_v = -M_o \sin 45^\circ = -0.70711 M_o$$

$$I_{min} = A k_{min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$U_D = y_o \cos 45^\circ + z_o \sin 45^\circ = 0.93 \cos 45^\circ + (-3.57 \sin 45^\circ) = -1.866 \text{ in}$$

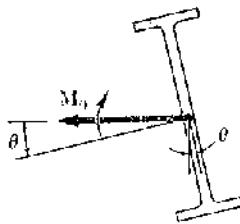
$$V_D = z_o \cos 45^\circ - y_o \sin 45^\circ = (-3.57) \cos 45^\circ - (0.93) \sin 45^\circ = 3.182 \text{ in.}$$

$$\begin{aligned}\tilde{\sigma}_D &= -\frac{M_v U_D}{I_v} + \frac{M_u V_D}{I_u} = 0.70711 M_o \left[-\frac{U_D}{I_{min}} + \frac{V_D}{I_{max}} \right] \\ &= 0.70711 M_o \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124 M_o\end{aligned}$$

$$M_o = \frac{\tilde{\sigma}_o}{0.4124} = \frac{12}{0.4124} = 29.1 \text{ kip.in}$$

Problem 4.150

4.150 A couple M_o acting in a vertical plane is applied to a W12 × 16 rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.



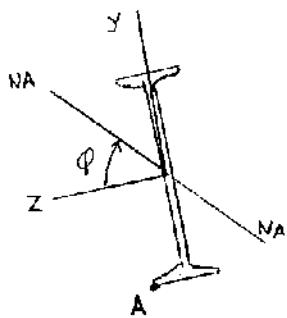
For W 12 × 16 rolled steel section

$$I_z = 103 \text{ in}^4 \quad I_y = 2.82 \text{ in}^4$$

$$d = 11.99 \text{ in} \quad b_F = 3.990 \text{ in.}$$

$$y_A = -\frac{d}{2} \quad z_A = \frac{b_F}{2}$$

$$\tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{103}{2.82} \tan \theta = 36.52 \tan \theta$$



$$M_y = M_o \sin \theta \quad M_z = M_o \cos \theta$$

$$\begin{aligned} \sigma_A &= -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_o d}{2 I_z} \cos \theta + \frac{M_o b_F}{2 I_y} \sin \theta \\ &= \frac{M_o d}{2 I_z} \left(1 + \frac{I_z b_F}{I_y d} \tan \theta \right) \end{aligned}$$

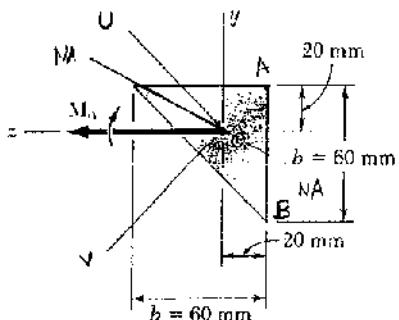
$$\text{For } \theta = 0 \quad \sigma_0 = \frac{M_o d}{2 I_z}$$

$$\sigma_A = \sigma_0 \left(1 + \frac{I_z b_F}{I_y d} \tan \theta \right) = 2\sigma_0$$

$$\tan \theta = \frac{I_z d}{I_y b_F} = \frac{(2.82)(11.99)}{(103)(3.990)} = 0.08273 \quad \theta = 4.70^\circ$$

Problem 4.151

4.151 A beam having the cross section shown is subjected to a couple M_o acting in a vertical plane. Determine the largest permissible value of the moment M_o of the couple if the maximum stress is not to exceed 100 MPa. Given: $I_y = I_z = b^4/36$ and $I_{yz} = b^4/72$.

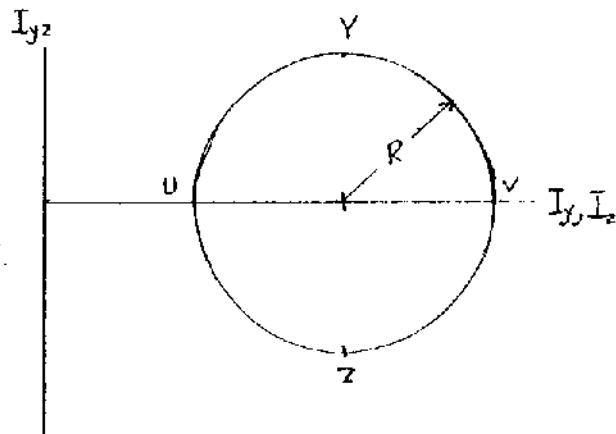


$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.

Using Mohr's circle determine the principal moments of inertia



$$R = \sqrt{I_y + I_z} = \sqrt{0.360 \times 10^6 + 0.180 \times 10^6} = 0.540 \times 10^6 \text{ mm}^4$$

$$I_1 = \frac{I_y + I_z}{2} + R$$

$$= \frac{0.360 \times 10^6 + 0.180 \times 10^6}{2} + 0.540 \times 10^6 = 0.540 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{I_y + I_z}{2} - R$$

$$= \frac{0.360 \times 10^6 - 0.180 \times 10^6}{2} = 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$$M_u = M_o \sin 45^\circ = 0.70711 M_o, \quad M_v = M_o \cos 45^\circ = 0.70711 M_o$$

$$\theta = 45^\circ \quad \tan \phi = \frac{I_v}{I_u} \tan \theta = \frac{0.540 \times 10^6}{0.180 \times 10^6} \tan 45^\circ = 3$$

$$\phi = 71.56^\circ$$

Point A: $U_A = 0, \quad V_A = -20\sqrt{2} \text{ mm}$

$$G_A = -\frac{M_u U_A}{I_v} + \frac{M_v V_A}{I_u} = 0 + \frac{(0.70711 M_o)(-20\sqrt{2} \times 10^{-3})}{0.180 \times 10^{-6}} = -11.11 \times 10^3 M_o$$

$$M_o = -\frac{G_A}{11.11 \times 10^3} = -\frac{-100 \times 10^6}{11.11 \times 10^3} = 900 \text{ N.m}$$

Point B: $U_B = -\frac{60}{72} \text{ mm}, \quad V_B = \frac{20}{72} \text{ mm}$

$$G_B = -\frac{M_u U_B}{I_v} + \frac{M_v V_B}{I_u} = -\frac{(0.70711 M_o)(-\frac{60}{72} \times 10^{-3})}{0.540 \times 10^{-6}} + \frac{(0.70711 M_o)(\frac{20}{72} \times 10^{-3})}{0.180 \times 10^{-6}}$$

$$= 111.11 \times 10^3 M_o$$

$$M_o = \frac{G_B}{111.11 \times 10^3} = \frac{100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N.m}$$

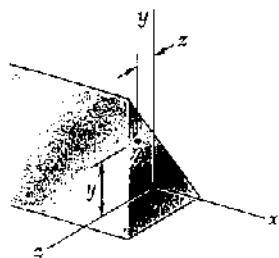
Choose the smaller value.

$$M_o = 900 \text{ N.m}$$

Problem 4.152

4.152 A beam of unsymmetric cross section is subjected to a couple M_0 acting in the vertical plane xy . Show that the stress at point A , of coordinates y and z is

$$\sigma_A = -\frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_0$$



where I_y , I_z , and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_0 the moment of the couple.

The stress σ_A varies linearly with the coordinates y and z . Since the axial force is zero, the y - and z -axes are centroidal axes.

$$\sigma_A = C_1 y + C_2 z \quad \text{where } C_1 \text{ and } C_2 \text{ are constants.}$$

$$M_y = \int z \sigma_A dA = C_1 \int yz dA + C_2 \int z^2 dA \\ = I_{yz} C_1 + I_y C_2 = 0$$

$$C_2 = -\frac{I_{yz}}{I_y} C_1$$

$$M_z = - \int y \sigma_A dz = -C_1 \int y^2 dz + C_2 \int yz dz \\ = -I_z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1$$

$$I_y M_z = -(I_y I_z - I_{yz}^2) C_1$$

$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2} \quad C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

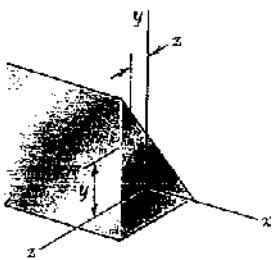
$$\sigma_A = -\frac{I_y z - I_{yz} y}{I_y I_z - I_{yz}^2} M_z$$

Problem 4.153

4.153 A beam of unsymmetric cross section is subjected to a couple M_y acting in the horizontal plane xz . Show that the stress at point A , of coordinates y and z is

$$\sigma_A = -\frac{zf_z - yf_y}{I_y I_z - I_{yz}^2} M_y$$

where I_y , I_z , and I_{yz} denote the moments of inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.



The stress σ_A varies linearly with the coordinates y and z . Since the axial force is zero, the y - and z -axes are centroidal axes.

$$\sigma_A = C_1 y + C_2 z \quad \text{where } C_1 \text{ and } C_2 \text{ are constants.}$$

$$\begin{aligned} M_z &= - \int y \sigma_A dA = -C_1 \int y^2 dA - C_2 \int yz dA \\ &= -I_z C_1 - I_{yz} C_2 = 0 \\ C_1 &= -\frac{I_{yz}}{I_z} C_2 \end{aligned}$$

$$\begin{aligned} M_y &= \int z \sigma_A dA = C_1 \int yz dA + C_2 \int z^2 dA \\ &= I_{yz} C_1 + I_y C_2 \\ &\quad - I_{yz} \frac{I_{yz}}{I_z} C_2 + I_y C_2 \end{aligned}$$

$$I_z M_y = (I_y I_z - I_{yz}^2) C_2$$

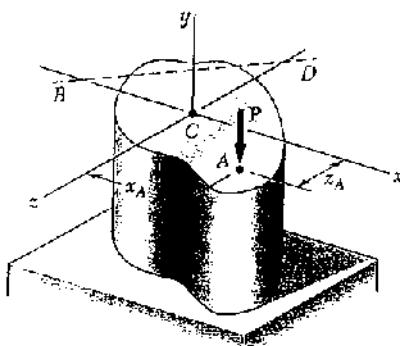
$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2} \quad C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{I_z y - I_{yz} z}{I_y I_z - I_{yz}^2} M_y$$

Problem 4.154

4.154 (a) Show that, if a vertical force P is applied at point A of the section shown, the equation of the neutral axis BD is

$$\left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$



where k_z and k_x denote the radius of gyration of the cross section with respect to the z axis and the x axis, respectively. (b) Further shown that, if a vertical force Q is applied at any point located on line BD , the stress at point A will be zero.

Definitions $k_z^2 = \frac{I_z}{A}$, $k_x^2 = \frac{I_x}{A}$

(a) $M_x = Pz_A$ $M_z = -Px_A$

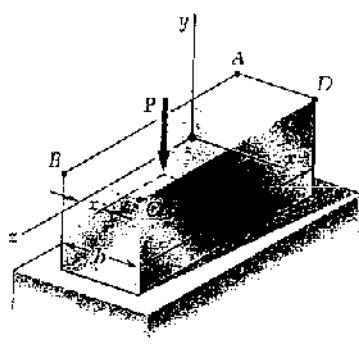
$$\begin{aligned} \sigma_E &= -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ak_z^2} - \frac{Pz_A z_E}{Ak_x^2} \\ &= -\frac{P}{A} \left[1 + \left(\frac{x_A}{k_z^2}\right) x_E + \left(\frac{z_A}{k_x^2}\right) z_E \right] = 0 \quad \text{if } E \text{ lies on neutral axis.} \end{aligned}$$

$$1 + \left(\frac{x_A}{k_z^2}\right) x + \left(\frac{z_A}{k_x^2}\right) z = 0, \quad \left(\frac{x_A}{k_z^2}\right) x + \left(\frac{z_A}{k_x^2}\right) z = -1$$

(b) $M_x = Pz_E$ $M_z = -Px_E$

$$\begin{aligned} \sigma_A &= -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} = -\frac{P}{A} - \frac{Px_E x_A}{Ak_z^2} - \frac{Pz_E z_A}{Ak_x^2} \\ &= 0 \quad \text{by equation from Part (a)} \end{aligned}$$

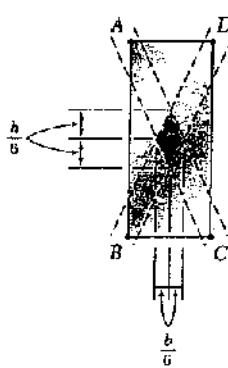
Problem 4.155



4.155 (a) Show that the stress at corner A of the prismatic member shown in Fig. P4.155a will be zero if the vertical force P is applied at a point located on the line

$$\frac{x}{h/6} + \frac{z}{h/6} = 1$$

(b) Further show that, if no tensile stress is to occur in the member, the force P must be applied at a point located within the area bounded by the line found in part a and three similar lines corresponding to the condition of zero stress at B, C, and D, respectively. This area, shown in Fig. 4.155b, is known as the *kern* of the cross section.



$$I_z = \frac{1}{12} hb^3 \quad I_x = \frac{1}{12} bh^3 \quad A = bh$$

$$Z_A = -\frac{h}{2} \quad X_A = -\frac{b}{2}$$

Let P be the load point

$$M_z = -Px_p \quad M_x = Pz_p$$

$$\sigma_A = -\frac{P}{A} + \frac{M_z X_A}{I_z} - \frac{M_x Z_A}{I_x}$$

$$= -\frac{P}{bh} + \frac{(-Px_p)(-\frac{b}{2})}{\frac{1}{12}hb^3} - \frac{Pz_p(-\frac{h}{2})}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh} \left[1 - \frac{X_p}{b/6} - \frac{Z_p}{h/6} \right]$$

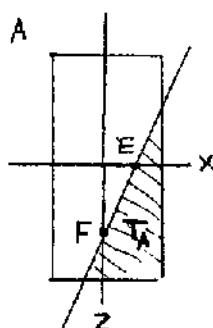
$$\text{For } \sigma_A = 0 \quad 1 - \frac{X_p}{b/6} - \frac{Z_p}{h/6} = 0, \quad \frac{X_p}{b/6} + \frac{Z_p}{h/6} = 1$$

$$\text{At point E} \quad z = 0 \quad \therefore x_E = b/6$$

$$\text{At point F} \quad x = 0 \quad \therefore z_F = h/6$$

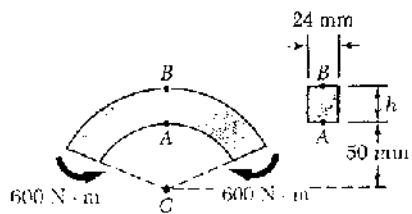
If the line of action (x_p, z_p) lies within the portion marked T_A , a tensile will occur at corner A.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.



Problem 4.156

4.156 For the curved bar shown, determine the stress at point A when (a) $h = 50$ mm, (b) $h = 60$ mm.



$$(a) \quad h = 50 \text{ mm}, \quad r_1 = 50 \text{ mm}, \quad r_2 = 100 \text{ mm}$$

$$A = (24)(50) = 1.200 \times 10^3 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{50}{\ln \frac{100}{50}} = 72.13475 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 75 \text{ mm}$$

$$e = \bar{r} - R = 2.8652 \text{ mm}$$

$$y_A = 72.13475 - 50 = 22.13475 \text{ mm} \quad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Ae^r_A} = -\frac{(600)(22.13475 \times 10^{-3})}{(1.200 \times 10^3)(2.8652 \times 10^{-3})(50 \times 10^{-3})} = -77.3 \times 10^6 \text{ Pa}$$

$$-77.3 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad h = 60 \text{ mm}, \quad r_1 = 50 \text{ mm}, \quad r_2 = 110 \text{ mm}, \quad A = (24)(60) = 1.440 \times 10^3 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{60}{\ln \frac{110}{50}} = 76.09796 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 80 \text{ mm} \quad e = \bar{r} - R = 3.90204 \text{ mm}$$

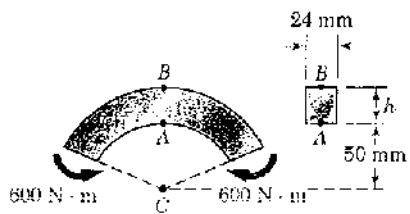
$$y_A = 76.09796 - 50 = 26.09796 \text{ mm} \quad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Ae^r_A} = -\frac{(600)(26.09796 \times 10^{-3})}{(1.440 \times 10^3)(3.90204 \times 10^{-3})(50 \times 10^{-3})} = -55.7 \times 10^6 \text{ Pa}$$

$$-55.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.157

4.157 For the curved bar shown, determine the stress at points A and B when $h = 55$ mm.



$$h = 55 \text{ mm} \quad r_1 = 50 \text{ mm} \quad r_2 = 105 \text{ mm}$$

$$A = (24)(55) = 1.320 \times 10^3 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{55}{\ln \frac{105}{50}} = 74.13025 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 77.5 \text{ mm}$$

$$e = \bar{r} - R = 3.36975 \text{ mm}$$

$$y_A = 74.13025 - 50 = 24.13025 \text{ mm} \quad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(24.13025 \times 10^{-3})}{(1.320 \times 10^3)(3.36975 \times 10^{-3})(50 \times 10^{-3})} = -65.1 \times 10^6 \text{ Pa}$$

- 65.1 MPa

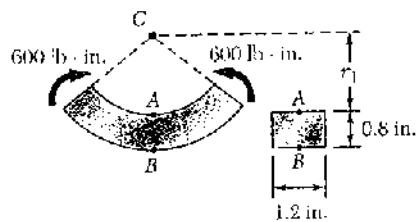
$$y_B = 74.13025 - 105 = -30.86975 \text{ mm} \quad r_B = 105 \text{ mm}$$

$$\sigma_B = -\frac{My_B}{Aer_B} = -\frac{(600)(-30.86975 \times 10^{-3})}{(1.320 \times 10^3)(3.36975 \times 10^{-3})(105 \times 10^{-3})} = 39.7 \times 10^6 \text{ Pa}$$

39.7 MPa

Problem 4.158

4.158 For the curved bar and loading shown, determine the stress points *A* and *B* when $r_1 = 1.6$ in.



$$h = 0.8 \text{ in.} \quad r_1 = 1.6 \text{ in.} \quad r_2 = 2.4 \text{ in.}$$

$$A = (1.2)(0.8) = 0.96 \text{ in}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{0.8}{\ln \frac{2.4}{1.6}} = 1.97304 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 2.0 \text{ in.}$$

$$e = \bar{r} - R = 0.02696 \text{ in.}$$

$$y_A = 1.97304 - 1.6 = 0.37304 \text{ in.} \quad r_A = 1.6 \text{ in.}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(0.37304)}{(0.96)(0.02696)(1.6)} = -5.405 \times 10^3 \text{ psi}$$

-5.40 ksi

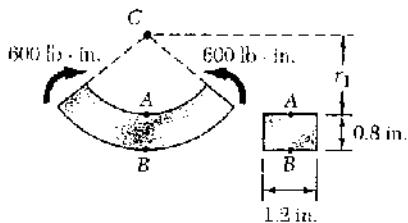
$$y_B = 1.97304 - 2.4 = -0.42696 \text{ in.} \quad r_B = 2.4 \text{ in.}$$

$$\sigma_B = -\frac{My_B}{Aer_B} = -\frac{(600)(-0.42696)}{(0.96)(0.02696)(2.4)} = 4.124 \times 10^3 \text{ psi}$$

4.12 ksi

Problem 4.159

4.159 For the curved bar and loading shown, determine the stress point A when (a) $r_1 = 1.2$ in., (b) $r_1 = 2$ in.



$$h = 0.8 \text{ in.} \quad A = (1.2)(0.8) = 0.96 \text{ in}^2$$

$$(a) \quad r_1 = 1.2 \text{ in.} \quad r_2 = 2.0 \text{ in.}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{0.8}{\ln \frac{2.0}{1.2}} = 1.56609 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 1.6 \text{ in.}$$

$$e = \bar{r} - R = 0.03391 \text{ in.}$$

$$y_A = 1.97304 - 1.6 = 0.36609 \text{ in.} \quad r_A = r_1 = 1.2 \text{ in.}$$

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(600)(0.36609)}{(0.96)(0.03391)(1.2)} = -5.623 \times 10^3 \text{ psi}$$

-5.62 ksi

$$(b) \quad r_1 = 2 \text{ in.} \quad r_2 = 2.8 \text{ in.}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{0.8}{\ln \frac{2.8}{2.0}} = 2.37761 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 2.4 \text{ in.}$$

$$e = \bar{r} - R = 0.02239 \text{ in.}$$

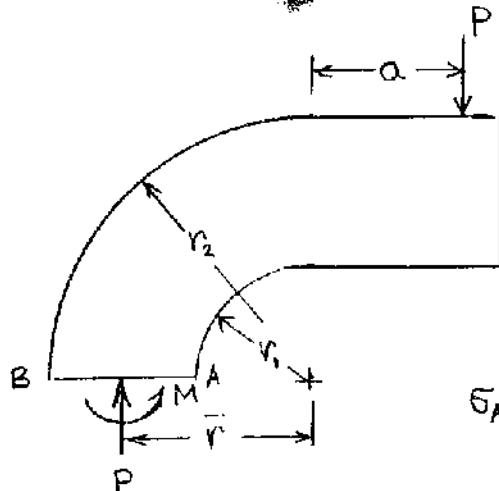
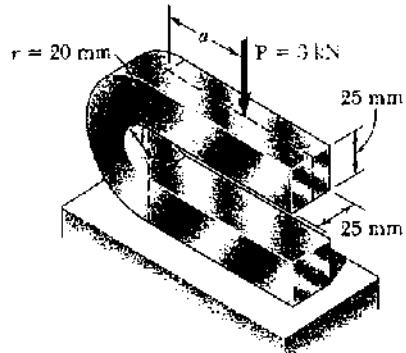
$$y_A = 2.37761 - 2 = 0.37761 \text{ in.} \quad r_A = r_1 = 2 \text{ in.}$$

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(600)(0.37761)}{(0.96)(0.02239)(2)} = -5.270 \times 10^3 \text{ psi}$$

5.27 ksi

Problem 4.160

4.160 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \quad \text{Also } e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Ae_r} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Ae_r}$$

$$= -K \frac{P}{A} \quad \text{with } y_A = R - r_1$$

$$\text{Thus, } K = 1 + \frac{(a + \bar{r})(R - r_1)}{er_1}$$

Data: $h = 25 \text{ mm}$, $r_1 = 20 \text{ mm}$, $r_2 = 45 \text{ mm}$, $\bar{r} = 32.5 \text{ mm}$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-4} \text{ m}^2$$

$$a = 60 \text{ mm}, a + \bar{r} = 92.5 \text{ mm}, R - r_1 = 10.8288 \text{ mm}$$

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968$$

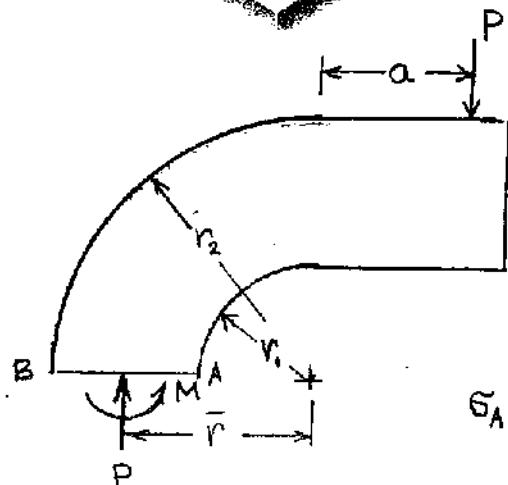
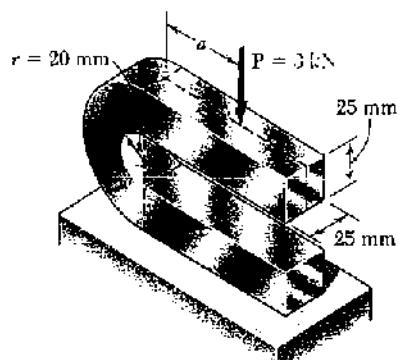
$$P = 3 \times 10^3 \text{ N}$$

$$\sigma_A = -\frac{KP}{A} = -\frac{(30.968)(3 \times 10^3)}{625 \times 10^{-4}} = -148.6 \times 10^6 \text{ Pa}$$

$$= -148.6 \text{ MPa}$$

Problem 4.161

4.161 Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies

$$R = \frac{h}{\ln \frac{R}{r_i}} \quad \text{Also } e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Ae_r} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Ae_r} \\ = -K \frac{P}{A} \quad \text{with } y_A = R - r_i$$

$$\text{Thus, } K = 1 + \frac{(a+\bar{r})(R-r_i)}{er_i}$$

Data: $h = 25 \text{ mm}$, $r_i = 20 \text{ mm}$, $r_2 = 45 \text{ mm}$, $\bar{r} = 32.5 \text{ mm}$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-4} \text{ m}^2$$

$$R - r_i = 10.8288 \text{ mm}$$

$$P = 3 \times 10^3 \text{ N} \cdot \text{m} \quad \sigma_A = -150 \times 10^6 \text{ Pa}$$

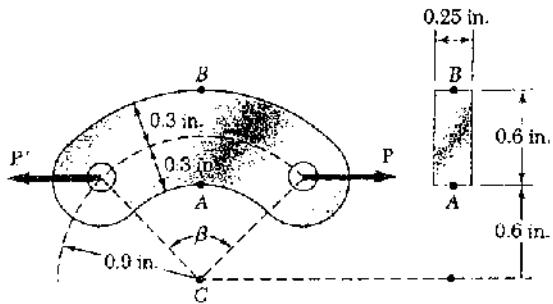
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-4})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)e_r}{R-r_i} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$$a = 93.37 - 32.5 = 60.8 \text{ mm}$$

Problem 4.162

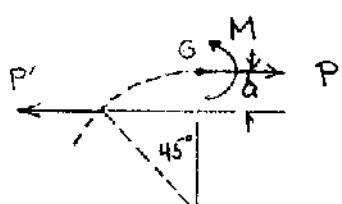
4.162 Steel links having the cross section shown are available with different central angles β . Knowing that the allowable stress is 15 ksi, determine the largest force P that can be applied to a link for which $\beta = 90^\circ$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}(1 - \cos \frac{\beta}{2})$$

The bending couple is $M = -Pa$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \quad \text{Also } e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Ae_r} = \frac{P}{A} + \frac{Pa y_A}{Ae_r} = \frac{P}{A} \left(1 + \frac{ay_A}{er} \right) = K \frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er}$ and $y_A = R - r$,

$$P = \frac{AG_A}{K}$$

Data: $\bar{r} = 0.9$ in., $r_1 = 0.6$ in., $y_2 = 1.2$ in., $h = 0.16$ in., $b = 0.25$ in.

$$A = (0.25)(0.6) = 0.15 \text{ in}^2, \quad R = \frac{0.6}{\ln \frac{1.2}{0.6}} = 0.86562 \text{ in.}$$

$$e = 0.9 - 0.86562 = 0.03438 \text{ in.}, \quad y_A = 0.86562 - 0.6 = 0.26562 \text{ in}$$

$$a = 0.9(1 - \cos 45^\circ) = 0.26360 \text{ in}$$

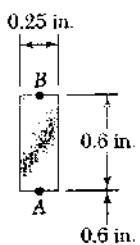
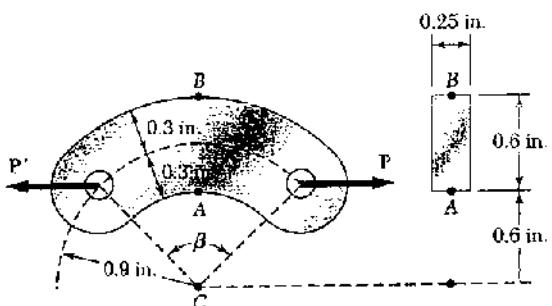
$$K = 1 + \frac{(0.26360)(0.26562)}{(0.03438)(0.6)} = 4.3943$$

$$P = \frac{(0.15)(15)}{4.3943} = 0.512 \text{ kips} = 512 \text{ lb.}$$

Problem 4.163

4.163 Solve Prob. 4.162, assuming that $\beta = 60^\circ$.

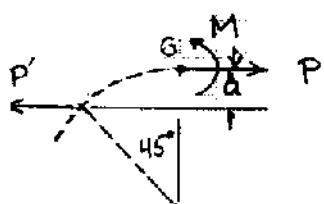
4.162 Steel links having the cross section shown are available with different central angles β . Knowing that the allowable stress is 15 ksi, determine the largest force P that can be applied to a link for which $\beta = 90^\circ$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$\alpha = \bar{r} (1 - \cos \frac{\beta}{2})$$

The bending couple is $M = -Pa$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \quad \text{Also } e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} + \frac{My_A}{Ae_r} = \frac{P}{A} + \frac{Pa y_A}{Ae_r} = \frac{P}{A} \left(1 + \frac{ay_A}{e_r} \right) = K \frac{P}{A}$$

where $K = 1 + \frac{ay_A}{e_r}$ and $y_A = R - r_1$

$$P = \frac{AG}{K}$$

Data: $\bar{r} = 0.9$ in., $r_1 = 0.6$ in., $r_2 = 1.2$ in., $h = 0.16$ in., $b = 0.25$

$$A = (0.15)(0.6) = 0.09 \text{ in}^2, \quad R = \frac{0.6}{\ln \frac{1.2}{0.6}} = 0.86562 \text{ in.}$$

$$e = 0.9 - 0.86562 = 0.03438 \text{ in.}, \quad y_A = 0.86562 - 0.6 = 0.26562$$

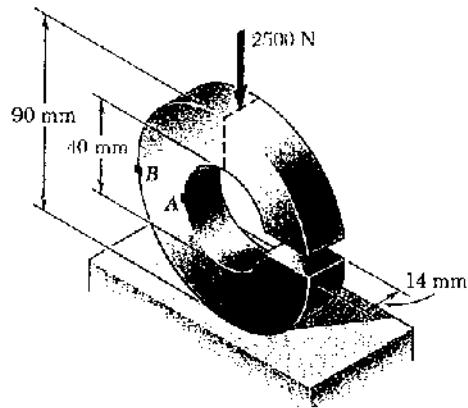
$$a = 0.9(1 - \cos 30^\circ) = 0.12058 \text{ in.}$$

$$K = 1 + \frac{(0.12058)(0.26562)}{(0.03438)(0.6)} = 2.5526$$

$$P = \frac{(0.15)(15)}{2.5526} = 0.881 \text{ kips} = 881 \text{ lb.}$$

Problem 4.164

4.164 For the split ring shown, determine the stress at (a) point A, (b) point B.



$$r_1 = \frac{1}{2}40 = 20 \text{ mm}, \quad r_2 = \frac{1}{2}(90) = 45 \text{ mm}$$

$$h = r_2 - r_1 = 25 \text{ mm}$$

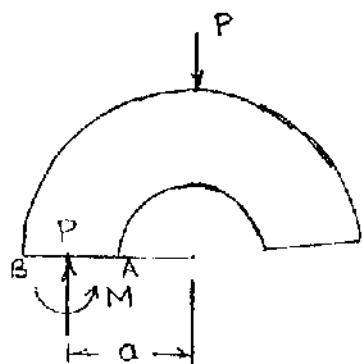
$$A = (14)(25) = 350 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{25}{\ln \frac{45}{20}} = 30.8288$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 32.5 \text{ mm}$$

$$e = \bar{r} - R = 1.6712 \text{ mm}$$

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the cross section. The bending couple is



$$M = Pa = P\bar{r}$$

$$= (2500)(32.5 \times 10^{-3}) = 81.25 \text{ N}\cdot\text{m}$$

$$(a) \text{ Point A: } r_A = 20 \text{ mm}$$

$$y_A = 30.8288 - 20 = 10.8288 \text{ mm}$$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{AeR} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(10.8288 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -82.4 \times 10^6 \text{ Pa} \quad -82.4 \text{ MPa}$$

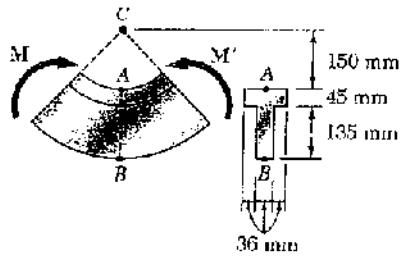
$$(b) \text{ Point B: } r_B = 45 \text{ mm} \quad y_B = 30.8288 - 45 = -14.1712 \text{ mm}$$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{AeR} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(-14.1712 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(45 \times 10^{-3})}$$

$$= 36.6 \times 10^6 \text{ Pa} \quad 36.6 \text{ MPa}$$

Problem 4.165

4.165 and 4.166 Knowing that $M = 20 \text{ kN} \cdot \text{m}$, determine the stress at (a) point A, (b) point B.



$$R = \frac{\sum A}{\sum S \div dA} = \frac{\sum b_i h_i}{\sum b_i l_i \ln \frac{r_{out}}{r_i}} = \frac{\sum A_i}{\sum b_i l_i \ln \frac{r_{out}}{r_i}}$$

$$\bar{r} = \frac{\sum A_i \bar{r}_i}{\sum A_i}$$

r_i, mm	Part	b_i, mm	h_i, mm	A_i, mm^2	$b_i l_i \ln \frac{r_{out}}{r_i}, \text{mm}$	\bar{r}_i, mm	$A \bar{r}_i, \text{mm}^3$
150	①	108	45	4860	28.3353	172.5	838.35×10^3
195							
330	②	36	135	4860	18.9394	262.5	1275.75×10^3
	Σ			9720	47.2747		2114.1×10^3

$$R = \frac{9720}{47.2747} = 205.606 \text{ mm} \quad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5 \text{ mm}$$

$$e = \bar{r} - R = 11.894 \text{ mm} \quad M = 20 \times 10^3 \text{ N} \cdot \text{m}$$

$$(a) y_A = R - r_1 = 205.606 - 150 = 55.606 \text{ mm}$$

$$\sigma_A = - \frac{M y_A}{A e r_1} = - \frac{(20 \times 10^3)(55.606 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -64.1 \times 10^6 \text{ Pa} = -64.1 \text{ MPa}$$

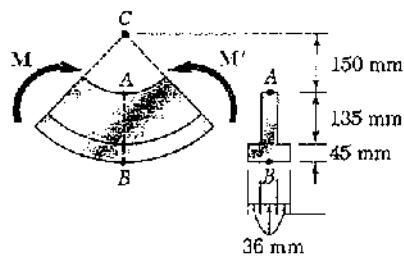
$$(b) y_B = R - r_2 = 205.606 - 330 = -124.394 \text{ mm}$$

$$\sigma_B = - \frac{M y_B}{A e r_2} = - \frac{(20 \times 10^3)(-124.394 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 65.2 \times 10^6 \text{ Pa} = 65.2 \text{ MPa}$$

Problem 4.166

4.165 and 4.166 Knowing that $M = 20 \text{ kN} \cdot \text{m}$, determine the stress at (a) point A, (b) point B.



$$R = \frac{\sum A}{\sum S \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\sum A_i \bar{r}_i}{\sum A_i}$$

r, mm	b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}, \text{mm}$	\bar{r}, mm	$A \bar{r}, \text{mm}^3$
150	(1)	36	135	4860	23.1067	217.5 1.05705×10^6
285	(2)	108	45	4860	15.8332	307.5 1.49445×10^6
330	Σ		9720	38.9399		2.5515×10^6

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \quad \bar{r} = \frac{2.5515 \times 10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \quad M = 20 \times 10^3 \text{ N} \cdot \text{m}$$

$$(a) \quad y_A = R - r_1 = 249.615 - 150 = 99.615 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(20 \times 10^3)(99.615 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(150 \times 10^{-3})}$$

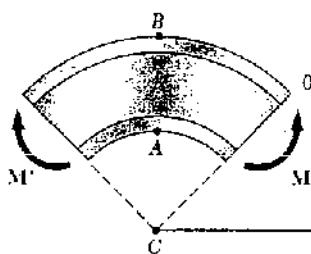
$$= -106.1 \times 10^6 \text{ Pa} = -106.1 \text{ MPa}$$

$$(b) \quad y_B = R - r_2 = 249.615 - 330 = -80.385 \text{ mm}$$

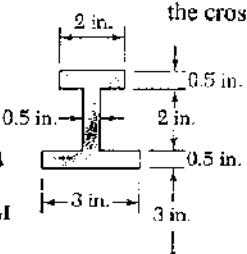
$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(20 \times 10^3)(-80.385 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 38.9 \times 10^6 \text{ Pa} = 38.9 \text{ MPa}$$

Problem 4.167



4.167 Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.



$$R = \frac{\sum A}{\sum S \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i l_i \frac{r_i}{r_{\bar{r}}}} = \frac{\sum A}{\sum b_i l_i \frac{r_i}{r_{\bar{r}}}}$$

$$\bar{r} = \frac{\sum A \bar{r}_i}{\sum A}$$

r	part	b	h	A	$b_i h_i \frac{r_i}{r_{\bar{r}}}$	\bar{r}_i	$A \bar{r}_i$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468	15.125	15.125

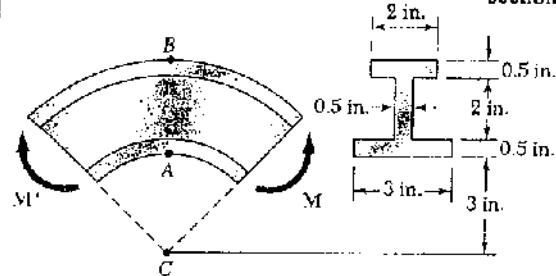
$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.}$$

0.263 in.

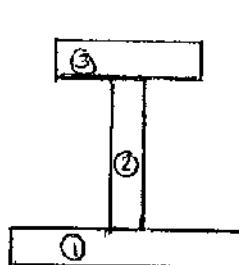
Problem 4.168

4.168 Three plates are welded together to form the curved beam shown. For $M = 8 \text{ kip} \cdot \text{in.}$, determine the stress at (a) point A, (b) point B, (c) the centroid of the cross section



$$R = \frac{\sum A}{\sum S \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i h_i \frac{r_i}{r_c}} = \frac{\sum A}{\sum b_i h_i \frac{r_i}{r_c}}$$

$$\bar{r} = \frac{\sum A \bar{r}_i}{\sum A}$$



r	part	b	h	A	$b_i h_i \frac{r_i}{r_c}$	\bar{r}	$A \bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468	15.125	

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.} \quad M = -8 \text{ kip} \cdot \text{in.}$$

$$(a) y_A = R - r_1 = 4.05812 - 3 = 1.05812 \text{ in.}$$

$$\sigma_A = -\frac{My_A}{Ae\bar{r}_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)} = 3.06 \text{ ksi}$$

$$(b) y_B = R - r_2 = 4.05812 - 6 = -1.94188 \text{ in.}$$

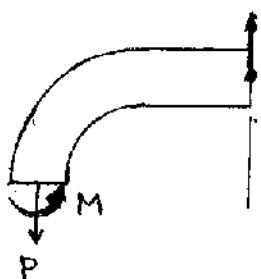
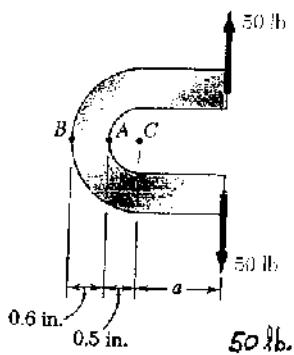
$$\sigma_B = -\frac{My_B}{Ae\bar{r}_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)} = -2.81 \text{ ksi}$$

$$(c) y_c = R - \bar{r} = -e =$$

$$\sigma_c = -\frac{My_c}{Ae\bar{r}} = -\frac{Me}{Ae\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-8}{(3.5)(4.32143)} = 0.529 \text{ ksi}$$

Problem 4.169

4.169 The bar shown has a *circular* cross section of 0.6-in. diameter. Knowing that $a = 1.2$ in., determine the stress at (a) point A, (b) point B.



$$c = \frac{1}{2}d = 0.3 \text{ in.}, \quad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in.}$$

$$R = \frac{1}{2} [\bar{r} + \sqrt{\bar{r}^2 - c^2}] = \frac{1}{2} [0.8 + \sqrt{0.8^2 - 0.3^2}] \\ = 0.77081 \text{ in.}$$

$$e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi (0.3)^2 = 0.28274 \text{ in.}^2$$

$$M = -P(a + \bar{r}) = -50(1.2 + 0.8) = -100 \text{ lb-in.}$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

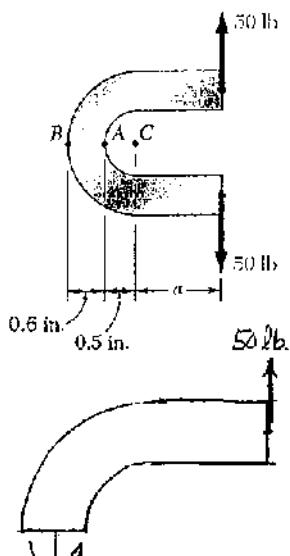
$$y_B = R - r_2 = 0.77081 - 1.1 = -0.32919 \text{ in.}$$

$$(a) \sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{50}{0.28274} - \frac{(-100)(0.27081)}{(0.28274)(0.02919)(0.5)} = 6.74 \times 10^3 \text{ psi} \\ = 6.74 \text{ ksi}$$

$$(b) \sigma_B = \frac{P}{A} - \frac{My_B}{Aer_2} = \frac{50}{0.28274} - \frac{(-100)(-0.32919)}{(0.28274)(0.02919)(1.1)} = -3.45 \times 10^3 \text{ psi} \\ = -3.45 \text{ ksi}$$

Problem 4.170

4.170 The bar shown has a *circular* cross section of 0.6-in. diameter. Knowing that the allowable stress is 8 ksi, determine the largest permissible distance a from the line of action of the 50-lb forces to the plane containing the center of curvature of the bar.



$$c = \frac{d}{2} = 0.3 \text{ in}, \quad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in}$$

$$R = \frac{1}{2} [\bar{r} + \sqrt{\bar{r}^2 - c^2}] = \frac{1}{2} [0.8 + \sqrt{0.8^2 - 0.3^2}] \\ = 0.77081 \text{ in.} \quad e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi (0.3)^2 = 0.28274 \text{ in}^2$$

$$M = -P(a + \bar{r})$$

$$y_A = R - r = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

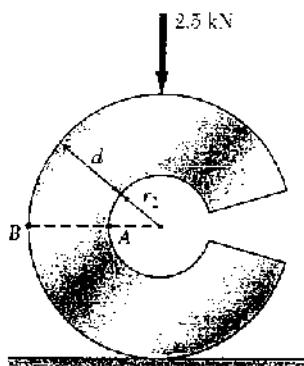
$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_i} = \frac{P}{A} + \frac{P(a+\bar{r})y_A}{Aer_i} = \frac{P}{A} \left[1 + \frac{(a+\bar{r})y_A}{er_i} \right] \\ = \frac{KP}{A} \quad \text{where} \quad K = 1 + \frac{(a+\bar{r})y_A}{er_i}$$

$$K = \frac{\sigma_A A}{P} = \frac{(8 \times 10^6)(0.28274)}{50} = 45.238$$

$$a + \bar{r} = \frac{(K-1)er_i}{y_A} = \frac{(45.238)(0.02919)(0.5)}{0.27081} = 2.384 \text{ in.}$$

$$a = 2.384 - 0.8 = 1.584 \text{ in.}$$

Problem 4.171



4.171 The split ring shown has an inner radius $r_1 = 20$ mm and a circular cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (a) point A, (b) point B.

$$c = \frac{1}{2}d = 16 \text{ mm} \quad r_1 = 20 \text{ mm}, \quad r_2 = r_1 + d = 52 \text{ mm}$$

$$\bar{r} = r_1 + c = 36 \text{ mm}$$

$$R = \frac{1}{2} [\bar{r} + \sqrt{\bar{r}^2 - c^2}] = \frac{1}{2} [36 + \sqrt{36^2 - 16^2}] \\ = 34.1245 \text{ mm.}$$

$$e = \bar{r} - R = 1.8755 \text{ mm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$P = 2.5 \times 10^3 \text{ N} \quad M = P\bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90 \text{ N}\cdot\text{m}$$

$$(a) \text{ Point A: } y_A = R - r_1 = 34.1245 - 20 = 14.1245 \text{ mm}$$

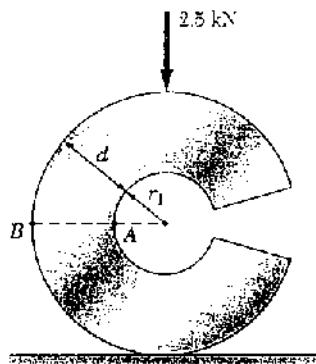
$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})} \\ = -45.2 \times 10^6 \text{ Pa} = -45.2 \text{ MPa}$$

$$(b) \text{ Point B: } y_B = R - r_2 = 34.1245 - 52 = -17.8755 \text{ mm}$$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})} \\ = 17.40 \times 10^6 \text{ Pa} = 17.40 \text{ MPa}$$

Problem 4.172

4.172 The split ring shown has an inner radius $r_1 = 16 \text{ mm}$ and a circular cross section of diameter $d = 32 \text{ mm}$. For the loading shown, determine the stress at (a) point A, (b) point B.



$$c = \frac{1}{2}d = 16 \text{ mm}, \quad r_1 = 16 \text{ mm}, \quad r_2 = r_1 + d = 48 \text{ mm}$$

$$\bar{r} = r_1 + c = 32 \text{ mm}$$

$$R = \frac{1}{2} [\bar{r} + \sqrt{\bar{r}^2 - c^2}] = \frac{1}{2} [32 + \sqrt{32^2 - 16^2}] \\ = 29.8564 \text{ mm}$$

$$e = \bar{r} - R = 2.1436 \text{ mm}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$P = 2.5 \times 10^3 \text{ N}$$

$$M = P\bar{r} = (2.5 \times 10^3)(32 \times 10^{-3}) = 80 \text{ N}\cdot\text{m}$$

$$(a) \text{ Point A: } y_A = R - r_1 = 29.8564 - 16 = 13.8564 \text{ mm}$$

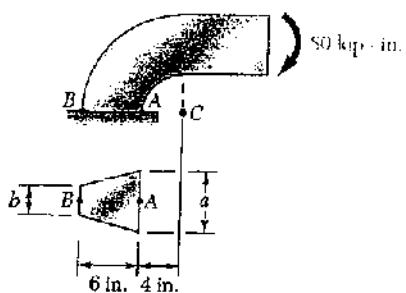
$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(13.8564 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(16 \times 10^{-3})} \\ = -43.3 \times 10^6 \text{ Pa} = -43.3 \text{ MPa}$$

$$(b) \text{ Point B: } y_B = R - r_2 = 29.8564 - 48 = -18.1436 \text{ mm.}$$

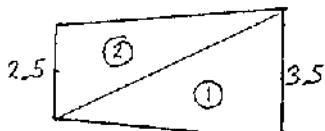
$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(-18.1436 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(48 \times 10^{-3})} \\ = 14.43 \times 10^6 \text{ Pa} = 14.43 \text{ MPa}$$

Problem 4.173

4.173 Knowing that the machine component shown has a trapezoidal cross section with $a = 3.5$ in. and $b = 2.5$ in., determine the stress at (a) point A, (b) point B.



Locate centroid



	$A_i \text{ in}^2$	$\bar{r}_i \text{ in}$	$A\bar{r}_i \text{ in}^3$
①	10.5	6	63
②	7.5	8	60
Σ	18		123

$$\bar{r} = \frac{123}{18} = 6.8333 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1\bar{r}_2 - b_2\bar{r}_1)\ln\frac{\bar{r}_2}{\bar{r}_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(3.5 + 2.5)}{[(3.5)(10) - (2.5)(4)]\ln\frac{10}{4} - (6)(3.5 - 2.5)} = 6.3878 \text{ in}$$

$$e = \bar{r} - R = 0.4452 \text{ in} \quad M = 80 \text{ kip-in.}$$

$$(a) y_A = R - r_1 = 6.3878 - 4 = 2.3878 \text{ in.}$$

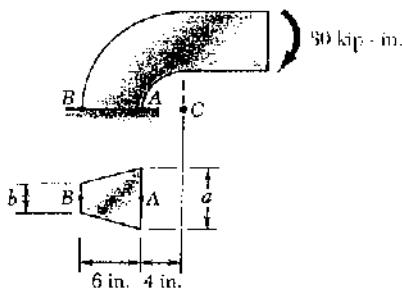
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.3878)}{(18)(0.4452)(4)} = -5.96 \text{ ksi}$$

$$(b) y_B = R - r_2 = 6.3878 - 10 = -3.6122 \text{ in.}$$

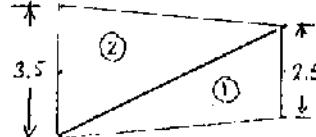
$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.6122)}{(18)(0.4452)(10)} = 3.61 \text{ ksi}$$

Problem 4.174

4.174 Knowing that the machine component shown has a trapezoidal cross section with $a = 2.5$ in. and $b = 3.5$ in., determine the stress at (a) point A, (b) point B.



Locate centroid



	$A_i \text{ in}^2$	$\bar{r}_i \text{ in}$	$A\bar{r}_i \text{ in}^3$
①	7.5	6	45
②	10.5	8	84
Σ	18		129

$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$\frac{(0.5)(6)^2(2.5 + 3.5)}{[(2.5)(10) - (3.5)(4)] \ln \frac{10}{4} - (6)(2.5 - 3.5)} = 6.7168 \text{ in}$$

$$e = \bar{r} - R = 0.4499 \text{ in}$$

$$M = 80 \text{ kip-in.}$$

$$(a) y_A = R - r_1 = 2.7168 \text{ in.}$$

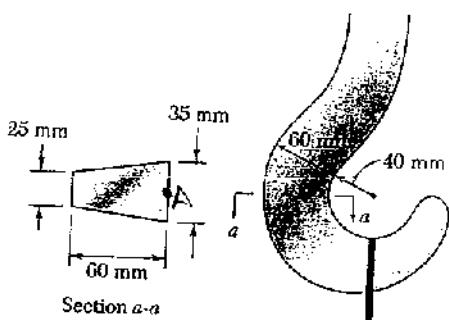
$$\sigma_A = - \frac{My_A}{Aer_1} = - \frac{(80)(2.7168)}{(18)(0.4499)(4)} = - 6.71 \text{ ksi}$$

$$(b) y_B = R - r_2 = -3.2832 \text{ in.}$$

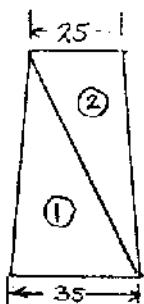
$$\sigma_B = - \frac{My_B}{Aer_2} = - \frac{(80)(-3.2832)}{(18)(0.4499)(10)} = 3.24 \text{ ksi}$$

Problem 4.175

4.175 For the crane hook shown, determine the largest tensile stress in section *a-a*.



Locate centroid



	A_i, mm^2	\bar{r}_i, mm	$A_i \bar{r}_i, \text{mm}^3$
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103 \times 10^3}{1800} = 68.333 \text{ mm}$$

Force-couple system at centroid: $P = 15 \times 10^3 \text{ N}$

$$M = -P\bar{r} = -(15 \times 10^3)(68.333 \times 10^{-3}) = -1.025 \times 10^3 \text{ N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(60)^2(35 + 25)}{[(35)(100) - (25)(40)] \ln \frac{100}{40} - (60)(35 + 25)} = 63.878 \text{ mm.}$$

$$e = \bar{r} - R = 4.452 \text{ mm.}$$

Maximum tensile stress occurs at point A

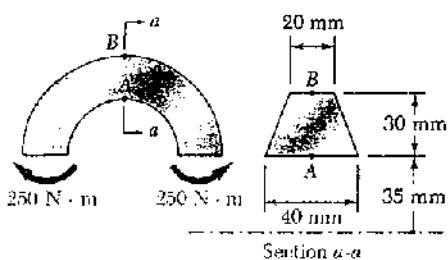
$$y_A = R - e = 23.878 \text{ mm.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer} = \frac{15 \times 10^3}{1800 \times 10^{-6}} - \frac{-(1.025 \times 10^3)(23.878 \times 10^{-3})}{(1800 \times 10^{-6})(4.452 \times 10^{-3})(40 \times 10^{-3})}$$

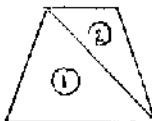
$$= 84.7 \times 10^6 \text{ Pa} = 84.7 \text{ MPa}$$

Problem 4.176

4.176 For the curved beam and loading shown, determine the stress at (a) point A, (b) point B.



Locate centroid.



	A_i, mm^2	\bar{r}_i, mm	$A\bar{r}, \text{mm}^3$
①	600	45	27×10^3
②	300	55	16.5×10^3
Σ	900		43.5×10^3

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1)2n\frac{r_e}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40 + 20)}{[(40)(65) - (20)(35)] \ln \frac{65}{35} - (30)(40 - 20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm}$$

$$M = -250 \text{ N·m}$$

$$(a) y_A = R - r_1 = 11.8608 \text{ mm.}$$

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa}$$

$$= 63.9 \text{ MPa} \blacksquare$$

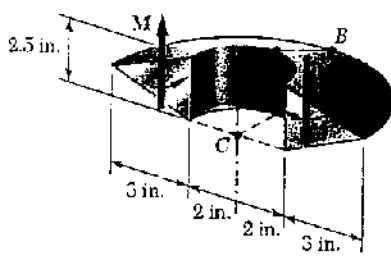
$$(b) y_B = R - r_2 = -18.1392 \text{ mm.}$$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa}$$

$$= -52.6 \text{ MPa} \blacksquare$$

Problem 4.177

4.177 and 4.178 Knowing that $M = 5 \text{ kip} \cdot \text{in.}$, determine the stress at (a) point A, (b) point B.



$$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 1 = 3.00000 \text{ in}$$

$$b_1 = 2.5 \text{ in.}, r_1 = 2 \text{ in.}, b_2 = 0, r_2 = 5 \text{ in.}$$

Use formula for trapezoid with $b_2 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5+0)}{[(2.5)(5) - (0)(2)]\ln\frac{5}{2} - (3)(2.5-0)} = 2.84548 \text{ in.}$$

$$e = \bar{r} - R = 0.15452 \text{ in.} \quad M = 5 \text{ kip} \cdot \text{in.}$$

$$(a) \quad y_A = R - r_1 = 0.84548 \text{ in.}$$

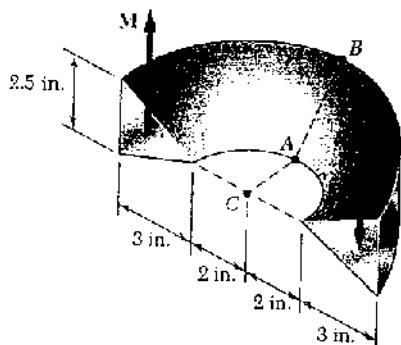
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)} = -3.65 \text{ ksi}$$

$$(b) \quad y_B = R - r_2 = -2.15452 \text{ in.}$$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)} = 3.72 \text{ ksi}$$

Problem 4.178

4.177 and 4.178 Knowing that $M = 5 \text{ kip} \cdot \text{in.}$, determine the stress at (a) point A, (b) point B.



$$A = \frac{1}{2} (2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 2 = 4.00000 \text{ in}$$

$$b_1 = 0, r_1 = 2 \text{ in.}, b_2 = 2.5 \text{ in.}, r_2 = 5 \text{ in.}$$

Use formula for trapezoid with $b_1 = 0$.

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(0+2.5)}{[(0)(5)-(2.5)(2)] \ln \frac{5}{2} - (3)(0-2.5)} = 3.85466 \text{ in}$$

$$e = \bar{r} - R = 0.14534 \text{ in.} \quad M = 5 \text{ kip} \cdot \text{in.}$$

$$(a) y_A = R - r_1 = 1.85466 \text{ in}$$

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)} = -8.51 \text{ ksi}$$

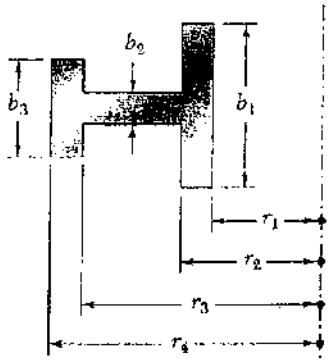
$$(b) y_B = R - r_2 = -1.14534 \text{ in.}$$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)} = 2.10 \text{ ksi}$$

Problem 4.179

4.179 Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln \left[\left(\frac{r_2}{r_1} \right)^{b_1} \left(\frac{r_3}{r_2} \right)^{b_2} \left(\frac{r_4}{r_3} \right)^{b_3} \right]}$$



where A is the total area of the cross section.

$$\begin{aligned} R &= \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{A}{\sum b_i \ln \frac{r_{i+1}}{r_i}} \\ &= \frac{A}{\sum \ln \left(\frac{r_{i+1}}{r_i} \right)^{b_i}} = \frac{A}{\ln \left[\left(\frac{r_2}{r_1} \right)^{b_1} \left(\frac{r_3}{r_2} \right)^{b_2} \left(\frac{r_4}{r_3} \right)^{b_3} \right]} \end{aligned}$$

Note that for each rectangle $\int \frac{1}{r} dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$

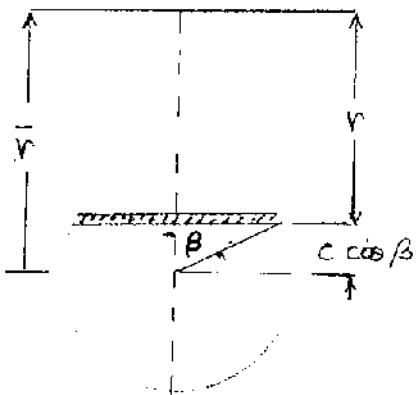
$$= b_i \int_{r_i}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$$

Problem 4.180

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.180 A circular cross section

Use polar coordinate β as shown



$$\text{width } w = 2c \sin \beta$$

$$r = \bar{r} - c \cos \beta$$

$$dr = c \sin \beta d\beta$$

$$dA = w dr = 2c^2 \sin^2 \beta d\beta$$

$$\int \frac{dA}{r} = \int_0^\pi \frac{2c^2 \sin^2 \beta}{\bar{r} - c \cos \beta} d\beta$$

$$\begin{aligned} \int \frac{dA}{r} &= \int_0^\pi \frac{c^2(1 - \cos^2 \beta)}{\bar{r} - c \cos \beta} d\beta = 2 \int_0^\pi \frac{\bar{r}^2 - c^2 \cos^2 \beta - (\bar{r}^2 - c^2)}{\bar{r} - c \cos \beta} d\beta \\ &= 2 \int_0^\pi (\bar{r} + c \cos \beta) d\beta - 2(\bar{r}^2 - c^2) \int_0^\pi \frac{dr}{\bar{r} - c \cos \beta} \\ &= 2\bar{r} \beta \Big|_0^\pi + 2c \sin \beta \Big|_0^\pi \\ &\quad - 2(\bar{r}^2 - c^2) \frac{2}{\sqrt{\bar{r}^2 - c^2}} \tan^{-1} \frac{\sqrt{\bar{r}^2 - c^2} + \tan \frac{1}{2}\beta}{\bar{r} + c} \Big|_0^\pi \\ &= 2\bar{r}(\pi - 0) + 2c(0 - 0) - 4\sqrt{\bar{r}^2 - c^2} \cdot \left(\frac{\pi}{2} - 0\right) \\ &= 2\pi \bar{r} - 2\pi \sqrt{\bar{r}^2 - c^2} \end{aligned}$$

$$A = \pi c^2$$

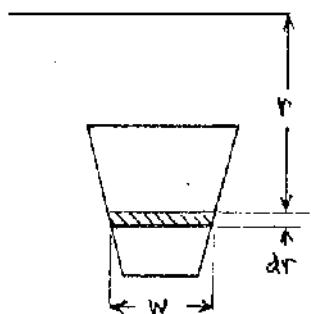
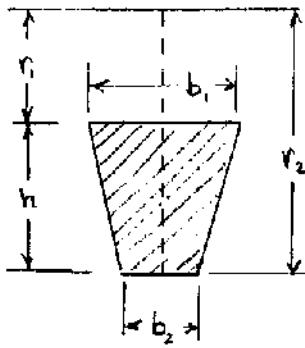
$$\begin{aligned} R &= \frac{A}{\int \frac{dA}{r}} = \frac{\pi c^2}{2\pi \bar{r} - 2\pi \sqrt{\bar{r}^2 - c^2}} \\ &= \frac{1}{2} \frac{c^2}{\bar{r} - \sqrt{\bar{r}^2 - c^2}} \cdot \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}} \\ &= \frac{1}{2} \frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2} \frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{c^2} \\ &= \frac{1}{2} (\bar{r} + \sqrt{\bar{r}^2 - c^2}) \end{aligned}$$

Problem 4.181

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.181 A trapezoidal cross section

The section width w varies linearly with r



$$w = C_0 + C_1 r$$

$$w = b_1 \text{ at } r = r_1 \text{ and } w = b_2 \text{ at } r = r_2$$

$$b_1 = C_0 + C_1 r_1$$

$$b_2 = C_0 + C_1 r_2$$

$$b_1 - b_2 = C_1(r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1) C_0 = h C_0$$

$$C_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$

$$\begin{aligned} \int \frac{dA}{r} &= \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{C_0 + C_1 r}{r} dr \\ &= C_0 \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2} \\ &= C_0 \ln \frac{r_2}{r_1} + C_1 (r_2 - r_1) \\ &= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h \\ &= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2) \end{aligned}$$

$$A = \frac{1}{2}(b_1 + b_2) h$$

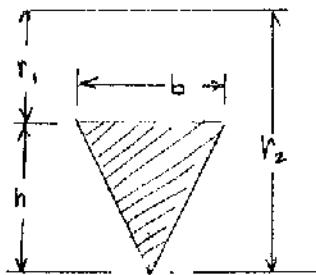
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

Problem 4.182

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.182 A triangular cross section

The section width w varies linearly with r



$$w = C_0 + C_1 r$$

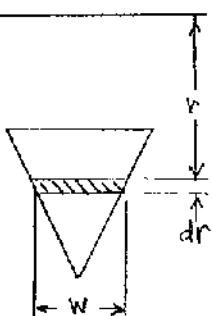
$$w = b \text{ at } r = r_1 \text{ and } w = 0 \text{ at } r = r_2$$

$$b = C_0 + C_1 r_1$$

$$0 = C_0 + C_1 r_2$$

$$b = C_1(r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b}{h} \text{ and } C_0 = -C_1 r_2 = \frac{b r_2}{h}$$



$$\begin{aligned} \int \frac{dA}{r} &= \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{C_0 + C_1 r}{r} dr \\ &= C_0 \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2} \\ &= C_0 \ln \frac{r_2}{r_1} + C_1 (r_2 - r_1) \\ &= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h \\ &= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b \left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right) \end{aligned}$$

$$A = \frac{1}{2} b h$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} b h}{b \left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

Problem 4.183

4.183 For a curved bar of rectangular cross section subjected to a bending couple M , show that the radius stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae} \left(1 - \frac{r_1}{R} - \ln \frac{R}{r_1} \right)$$

At radial distance r

$$\begin{aligned}\sigma_r &= \frac{M(r-R)}{Aer} \\ &= \frac{M}{Ae} - \frac{MR}{Aer}\end{aligned}$$

For portion above the neutral axis, the resultant force is

$$\begin{aligned}H &= \int G_r dA = \int_{r_1}^R G_r b dr \\ &= \frac{Mb}{Ae} \int_{r_1}^R dr - \frac{MRb}{Ae} \int_{r_1}^R \frac{dr}{r} \\ &= \frac{Mb}{Ae} (R - r_1) - \frac{MRb}{Ae} \ln \frac{R}{r_1} = \frac{MbR}{Ae} \left(1 - \frac{r_1}{R} - \ln \frac{R}{r_1} \right)\end{aligned}$$

Resultant of G_r

$$\begin{aligned}F_r &= \int G_r \cos \beta dA \\ &= \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} G_r \cos \beta b R d\beta \\ &= G_r b R \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \cos \beta d\beta \\ &= G_r b R \cdot \sin \beta \Big|_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \\ &= 2G_r b R \sin \frac{\theta}{2}\end{aligned}$$

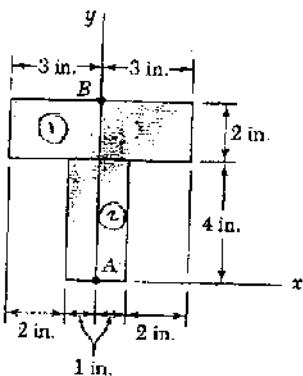
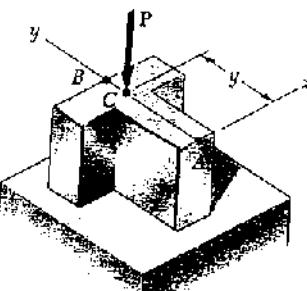
$F_r = \text{resultant of } G_r$

For equilibrium $F_r - 2H \sin \frac{\theta}{2} = 0$

$$2G_r b R \sin \frac{\theta}{2} - 2 \frac{MbR}{Ae} \left(1 - \frac{r_1}{R} - \ln \frac{R}{r_1} \right) = 0$$

$$G_r = \frac{M}{Ae} \left(1 - \frac{r_1}{R} - \ln \frac{R}{r_1} \right)$$

Problem 4.184



(b)

Locate centroid

Part	A, in^2	\bar{y}, in	$A\bar{y}, \text{in}^3$
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\sum A\bar{y}_i}{\sum A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $e = 5 - 3.8 = 1.2 \text{ in}$.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(a) Stress at A $c_A = 3.8 \text{ in}$

$$\sigma_A = -\frac{P}{A} + \frac{Pe}{I} = -\frac{20}{20} + \frac{(20)(1.2)(3.8)}{57.867} = 0.576 \text{ ksi}$$

(b) Stress at B $c_B = 6 - 3.8 = 2.2 \text{ in}$

$$\sigma_B = -\frac{P}{A} - \frac{Pe}{I} = -\frac{20}{20} - \frac{(20)(1.2)(2.2)}{57.867} = -1.912 \text{ ksi}$$

(c) Location of neutral axis: $\sigma = 0$

$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \therefore \frac{ea}{I} = \frac{1}{A}$$

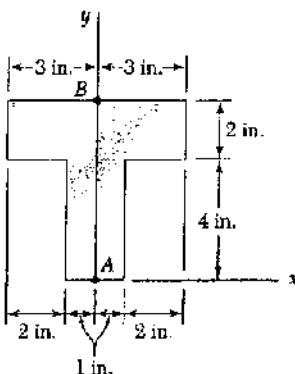
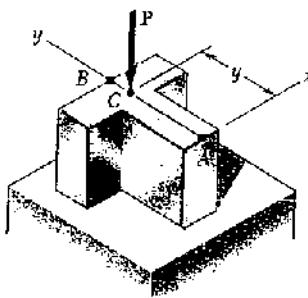
$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in}$$

Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411 = 1.389$ in above point A.

Answer 1.389 in from point A

Problem 4.185

4.185 A vertical force P of magnitude 20 kips is applied at a point C located on the axis of symmetry of the cross section of a short column. Determine the range of values of y for which tensile stresses do not occur in the column.



Locate centroid

	$A_i \text{ in}^2$	$\bar{y}_i \text{ in}$	$A_i \bar{y}_i \text{ in}^3$
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $e = y - 3.8 \text{ in}$
 $y = e + 3.8 \text{ in}$

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero. $c_A = 3.8 \text{ in}$

$$\sigma_A = -\frac{P}{A} + \frac{Pe c_A}{I} = 0 \quad : \quad \frac{e c_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{A c_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in} \quad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero. $c_B = 6 - 3.8 = 2.2 \text{ in}$

$$\sigma_B = -\frac{P}{A} - \frac{Pe c_B}{I} = 0 \quad : \quad \frac{e c_B}{I} = -\frac{1}{A}$$

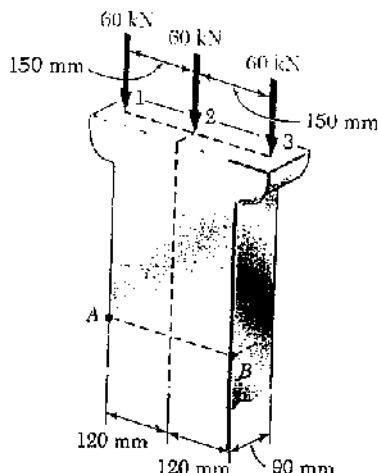
$$e = -\frac{I}{A c_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer: $2.485 \text{ in} < y < 4.561 \text{ in.}$

Problem 4.186

4.186 Determine the stress at points A and B, (a) for the loading shown, (b) if the 60-kN loads are applied at points 1 and 2 only.



(a) Loading is eccentric.

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-4} \text{ m}^2$$

$$\text{At } A \text{ and } B \quad \sigma = -\frac{P}{A} = -\frac{180 \times 10^3}{21.6 \times 10^{-4}} = -8.33 \times 10^6 \text{ Pa} \\ = -8.33 \text{ MPa} \quad \blacktriangleleft$$

(b) Eccentric loading

$$P = 120 \text{ kN} = 120 \times 10^3 \text{ N}$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N}\cdot\text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-4} \text{ m}^4$$

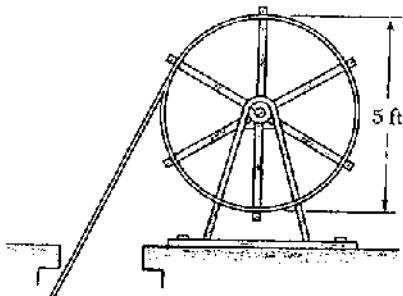
$$c = 120 \text{ mm} = 0.120 \text{ m}$$

$$\text{At } A \quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-4}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-4}} = -15.97 \times 10^6 \text{ Pa} = -15.97 \text{ MPa} \quad \blacktriangleleft$$

$$\text{At } B \quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-4}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-4}} = 4.86 \times 10^6 \text{ Pa} = 4.86 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.187

4.187 Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (a) the maximum stress in a rod, when the rod, which was initially straight, is wrapped on a spool, (b) the corresponding bending moment in the rod. Use $E = 29 \times 10^6 \text{ psi}$.



$$r = \frac{1}{2}d = \frac{1}{2}(0.30) = 0.15 \text{ in}$$

$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-8} \text{ in}^4$$

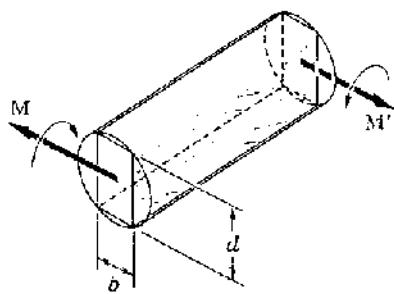
$$D = 5 \text{ ft} = 60 \text{ in} \quad \rho = \frac{1}{2}D = 30 \text{ in}$$

$$c = r = 0.15 \text{ in.}$$

$$(a) \quad \sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145 \times 10^3 \text{ psi} = 145 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-8})}{30} = 384 \text{ lb-in.} \quad \blacktriangleleft$$

Problem 4.188



4.188 A couple M will be applied to a beam of rectangular cross section that is to be sawed from a log of circular cross section. Determine the ratio d/b for which (a) the maximum stress σ_m will be as small as possible, (b) the radius of curvature of the beam will be maximum.

Let D be the diameter of the log.

$$D^2 = b^2 + d^2 \quad d^2 = D^2 - b^2$$

$$I = \frac{1}{12} bd^3 \quad c = \frac{1}{2} d \quad \frac{I}{c} = \frac{1}{6} bd^2$$

(a) σ_m is minimum when $\frac{I}{c}$ is maximum

$$\frac{I}{c} = \frac{1}{6} b(D^2 - b^2) = \frac{1}{6} D^2 b - \frac{1}{6} b^3$$

$$\frac{d}{db} \left(\frac{I}{c} \right) = \frac{1}{6} D^2 - \frac{2}{6} b^2 = 0 \quad b = \frac{1}{\sqrt{3}} D$$

$$d = \sqrt{D^2 - \frac{1}{3} D^2} = \sqrt{\frac{2}{3}} D \quad \frac{d}{b} = \sqrt{2} \quad \blacktriangleleft$$

$\rho = \frac{EI}{M}$ ρ is maximum when I is maximum.

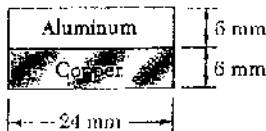
$\frac{1}{12} bd^3$ is maximum or $b^2 d^6$ is maximum

$(D^2 - d^2) d^6$ is maximum.

$$6D^2 d^5 - 8d^7 = 0 \quad d = \frac{\sqrt{3}}{2} D$$

$$b = \sqrt{D^2 - \frac{3}{4} D^2} = \frac{1}{2} D \quad \frac{d}{b} = \sqrt{3} \quad \blacktriangleleft$$

Problem 4.189



4.189 A copper strip ($E = 105 \text{ GPa}$) and an aluminum strip ($E = 75 \text{ GPa}$) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment $35 \text{ N} \cdot \text{m}$, determine the maximum stress in (a) the aluminum strip, (b) in the copper strip.

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$



$$n = E_c/E_a = 105/75 = 1.4 \text{ in copper}$$

Transformed section

	A_1, mm^2	nA_1, mm^2	\bar{y}_o, mm	$nA\bar{y}_o, \text{mm}^3$
①	144	144	9	1296
②	144	201.6	3	604.8
Σ		345.6		1900.8

$$\bar{Y}_o = \frac{1900.8}{345.6} = 5.50 \text{ mm}$$

The neutral axis lies 5.50 mm above the bottom

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12} (24)(6)^3 + (1.0)(24)(6)(3.5)^2 = 2196 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12} (24)(6)^3 + (1.4)(24)(6)(2.5)^2 = 1864.8 \text{ mm}^4$$

$$I = I_1 + I_2 = 4060.8 \text{ mm}^4 = 4.0608 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ Aluminum } n = 1.0 \quad y = 12 - 5.5 = 6.5 \text{ mm} = 0.0065 \text{ m}$$

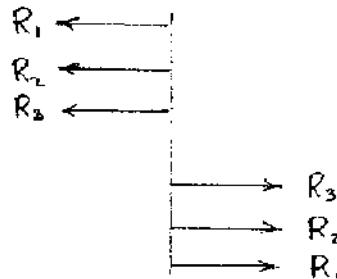
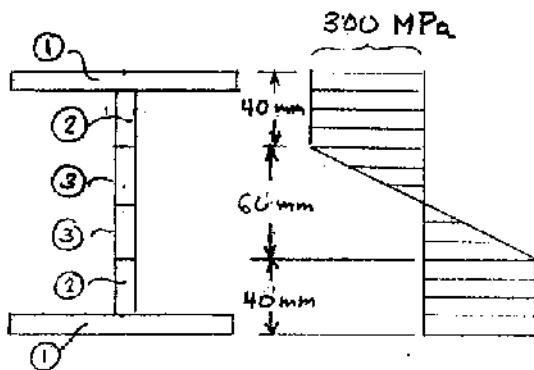
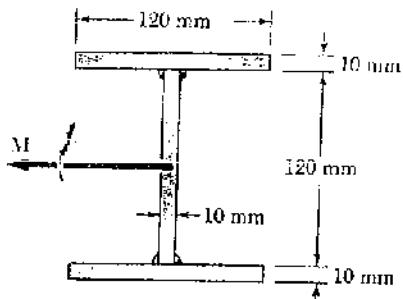
$$\sigma = -\frac{n My}{I} = -\frac{(1.0)(35)(0.0065)}{4.0608 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa}$$

$$(b) \text{ Copper } n = 1.4 \quad y = -5.5 \text{ mm} = -0.0055 \text{ m}$$

$$\sigma = -\frac{n My}{I} = -\frac{(1.4)(35)(-0.0055)}{4.0608 \times 10^{-9}} = 66.4 \times 10^6 \text{ Pa} = 66.4 \text{ MPa}$$

Problem 4.190

4.190 Three 120×10 -mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_y = 300$ MPa, determine (a) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (b) the corresponding radius of curvature of the beam.



$$A_1 = (120)(10) = 1200 \text{ mm}^2$$

$$R_1 = \sigma_y A_1 = (300 \times 10^6)(1200 \times 10^{-6}) = 360 \times 10^3 \text{ N}$$

$$A_2 = (30)(10) = 300 \text{ mm}^2$$

$$R_2 = \sigma_y A_2 = (300 \times 10^6)(300 \times 10^{-6}) = 90 \times 10^3 \text{ N}$$

$$A_3 = (30)(10) = 300 \text{ mm}^2$$

$$R_3 = \frac{1}{2} \sigma_y A_2 = \frac{1}{2} (300 \times 10^6)(300 \times 10^{-6}) = 45 \times 10^3 \text{ N}$$

$$y_1 = 65 \text{ mm} = 65 \times 10^{-3} \text{ m}$$

$$y_2 = 45 \text{ mm} = 45 \times 10^{-3} \text{ m}$$

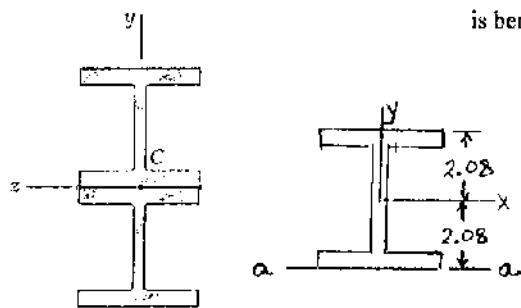
$$y_3 = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$$

$$(a) M = 2(R_1 y_1 + R_2 y_2 + R_3 y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\} = 56.7 \times 10^3 \text{ N}\cdot\text{m} = 56.7 \text{ kN}\cdot\text{m}$$

$$(b) \frac{y_r}{\rho} = \frac{\sigma_y}{E} \quad \rho = \frac{E y_r}{\sigma_y} = \frac{(200 \times 10^9)(30 \times 10^{-3})}{300 \times 10^6} = 20 \text{ m}$$

Problem 4.191

4.191 and 4.192 Two W4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_y = 36 \text{ ksi}$ and $\sigma_u = 58 \text{ ksi}$ and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



Properties of W 4×13 rolled section
See Appendix B

$$\text{Area} = 3.83 \text{ in}^2 \quad \text{Depth} = 4.16 \text{ in} \\ I_x = 11.3 \text{ in}^4$$

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

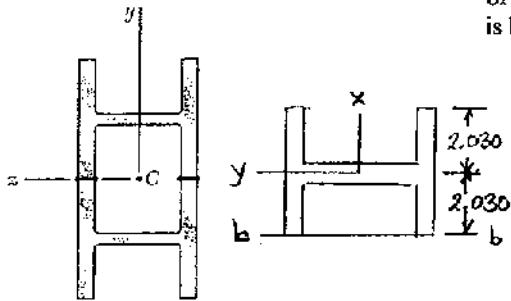
$$\text{For both sections } I_z = 2I_a = 55.74 \text{ in}^4$$

$$c = \text{depth} = 4.16 \text{ in}$$

$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \quad \bar{\sigma} = \frac{Mc}{I} \\ M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(55.74)}{4.16} = 259 \text{ kip-in.}$$

Problem 4.192

4.191 and 4.192 Two W4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_y = 36 \text{ ksi}$ and $\sigma_u = 58 \text{ ksi}$ and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



Properties of W 4×13 rolled section
See Appendix B

$$\text{Area} = 3.83 \text{ in}^2 \quad \text{Width} = 4.060 \text{ in} \\ I_y = 3.86 \text{ in}^4$$

For one rolled section, moment of inertia about axis b-b is

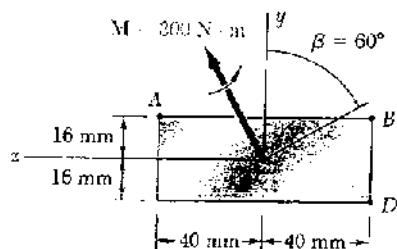
$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

$$\text{For both sections } I_z = 2I_b = 39.286 \text{ in}^4$$

$$c = \text{width} = 4.060 \text{ in}$$

$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \quad \bar{\sigma} = \frac{Mc}{I} \\ M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(39.286)}{4.060} = 187.1 \text{ kip-in.}$$

Problem 4.193



4.193 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A , (b) point B , (c) point D .

$$I_z = \frac{1}{12}(80)(32)^3 = 218.45 \times 10^3 \text{ mm}^4 = 218.45 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12}(32)(80)^3 = 1.36533 \times 10^6 \text{ mm}^4 = 1.36533 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 16 \text{ mm}$$

$$z_A = -z_B = -z_D = 40 \text{ mm}$$

$$M_y = 300 \cos 30^\circ = 259.81 \text{ N} \cdot \text{m}, M_z = 300 \sin 30^\circ = 150 \text{ N} \cdot \text{m}$$

$$(a) \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16 \times 10^{-3})}{218.45 \times 10^{-9}} + \frac{(259.81)(40 \times 10^{-3})}{1.36533 \times 10^{-6}}$$

$$= -3.37 \times 10^6 \text{ Pa} = -3.37 \text{ MPa}$$

$$(b) \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16 \times 10^{-3})}{218.45 \times 10^{-9}} + \frac{(259.81)(-40 \times 10^{-3})}{1.36533 \times 10^{-6}}$$

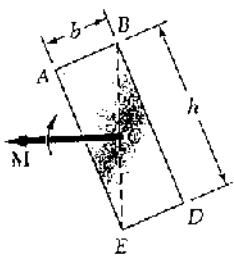
$$= -18.60 \times 10^6 \text{ Pa} = -18.60 \text{ MPa}$$

$$(c) \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16 \times 10^{-3})}{218.45 \times 10^{-9}} + \frac{(259.81)(-40 \times 10^{-3})}{1.36533 \times 10^{-6}}$$

$$= 3.37 \times 10^6 \text{ Pa} = 3.37 \text{ MPa}$$

Problem 4.194

4.194 Show that, if a solid beam having a rectangular cross section is bent by a couple applied in a plane containing one of diagonal of the cross section, the neutral axis will lie along the other diagonal.

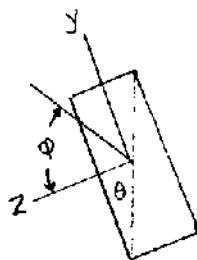


$$\tan \theta = \frac{b}{h}$$

$$M_z = M \cos \theta, \quad M_y = M \sin \theta$$

$$I_z = \frac{1}{12} b h^3 \quad I_y = \frac{1}{12} h b^3$$

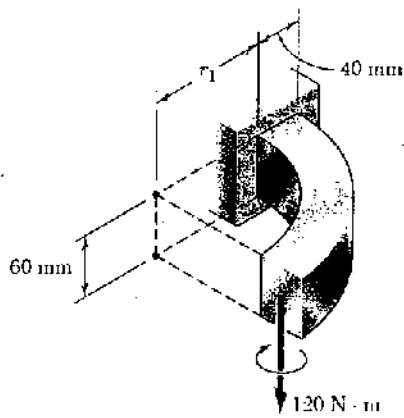
$$\tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{\frac{1}{12} b h^3}{\frac{1}{12} h b^3} \cdot \frac{b}{h} = \frac{h}{b}$$



Thus neutral axis passes through corner A

Problem 4.195

4.195 The curved bar shown has a cross section of 40×60 mm and an inner radius $r_1 = 15$ mm. For the loading shown determine the largest tensile and compressive stresses in the bar.



$$h = 40 \text{ mm}, \quad r_1 = 15 \text{ mm}, \quad r_2 = 55 \text{ mm}$$

$$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{55}{15}} = 30.786 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35 \text{ mm}$$

$$e = \bar{r} - R = 4.214 \text{ mm} \quad \sigma = -\frac{My}{AeR}$$

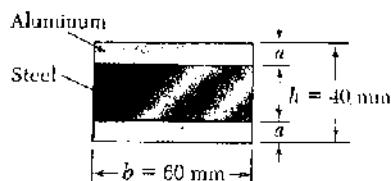
$$\text{At } r = 15 \text{ mm} \quad y = 30.786 - 15 = 15.786 \text{ mm}$$

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^6 \text{ Pa} \\ = -12.49 \text{ MPa} \quad (\text{compression})$$

$$\text{At } r = 55 \text{ mm} \quad y = 30.786 - 55 = -24.214 \text{ mm}$$

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^6 \text{ Pa} \\ = 5.22 \text{ MPa} \quad (\text{tension})$$

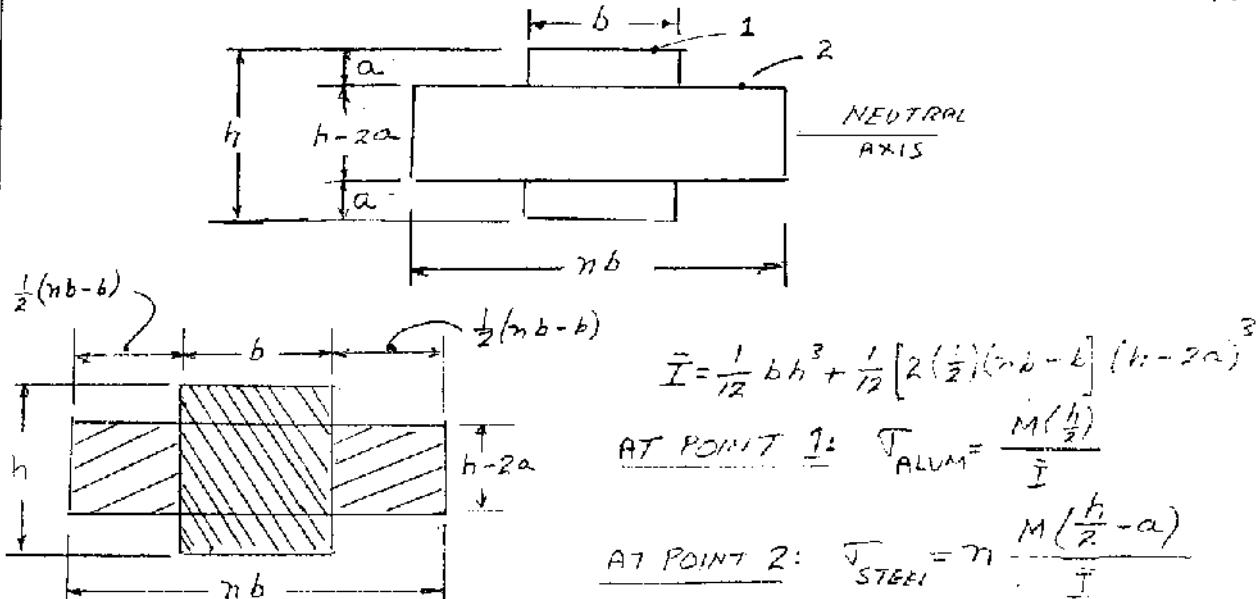
PROBLEM 4.C1



4.C1 Two aluminum strips and a steel strip are to be bonded together to form a composite member of width $b = 60 \text{ mm}$ and depth $h = 40 \text{ mm}$. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that $M = 1500 \text{ N} \cdot \text{m}$, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of a from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (a) the largest stress that can occur in the steel, (b) the corresponding value of a .

SOLUTION

TRANSFORMED SECTION (ALL STEEL) $n = \frac{E_{\text{STEEL}}}{E_{\text{ALUM}}}$



FOR $a = 0 \text{ TO } 20 \text{ mm}$ USING 2-mm INTERVALS COMPUTE: σ , I , σ_{ALUM} , σ_{STEEL} .

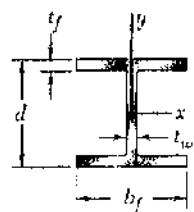
$b = 60 \text{ mm}$ $h = 40 \text{ mm}$ $M = 1500 \text{ N.m}$
Moduli of elasticity: Steel = 200 GPa Aluminum = 75 GPa

PROGRAM OUTPUT

a mm	I $\text{m}^4/10^6$	sigma aluminum MPa	sigma steel MPa
0.000	0.8533	35.156	93.750
2.000	0.7088	42.325	101.580
4.000	0.5931	50.585	107.914
6.000	0.5029	59.650	111.347
8.000	0.4352	68.934	110.294
10.000	0.3867	77.586	103.448
12.000	0.3541	84.714	90.361
14.000	0.3344	89.713	71.770
16.000	0.3243	92.516	49.342
18.000	0.3205	93.594	24.958
20.000	0.3200	93.750	0.000
<hr/>			
Find 'a' for max steel stress and the corresponding aluminum stress			
6.600	0.4804	62.447	111.572083
6.610	0.4800	62.494	111.572159
6.620	0.4797	62.540	111.572113

Max Steel Stress = 111.6 MPa occurs when $a = 6.61 \text{ mm}$
Corresponding Aluminum stress = 62.5 MPa

PROBLEM 4.C2



4.C2 A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_y and a modulus of elasticity E , is bent about the x axis. (a) Denoting by y_y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_y$. Neglect the effect of fillets. (b) Use this program to solve Prob. 4.190

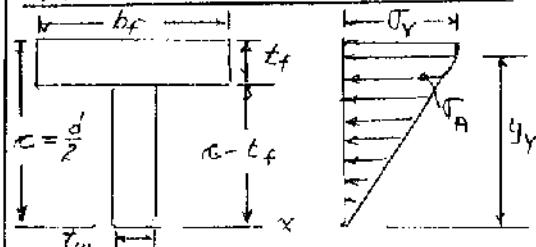
SOLUTION

COMPUTE MOMENT OF INERTIA I_x

$$I_x = \frac{1}{12} b_f d^3 - \frac{1}{12} (b_f - t_w)(d - 2t_w)^3$$

MAXIMUM ELASTIC MOMENT: $M_y = \sigma_y \frac{I_x}{(d/2)}$

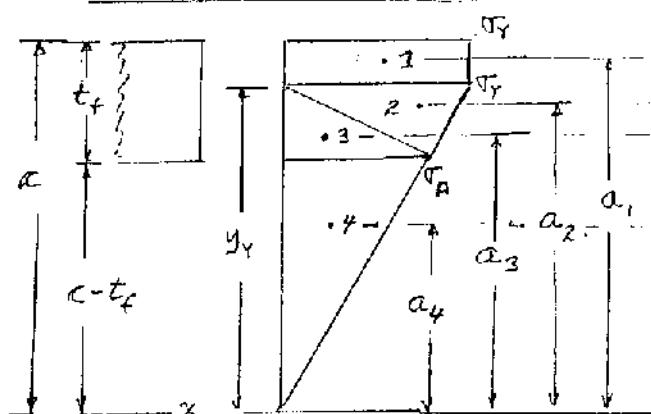
FOR YIELDING IN THE FLANGES: (CONSIDER UPPER HALF OF CROSS SECTION) $c = \frac{d}{2}$



STRESS AT JUNCTION OF WEB AND FLANGE:

$$\sigma_A = \frac{(d/2) - t_f}{y_y} \sigma_y$$

DETAIL OF STRESS DIAGRAM



RESULTANT FORCES

$$R_1 = \sigma_y [b_f(c - \sigma_y)]$$

$$R_2 = \frac{1}{2} \sigma_y b_f [y_y - (c - t_f)]$$

$$R_3 = \frac{1}{2} \sigma_A b_f [y_y - (c - t_f)]$$

$$E_3 = \frac{1}{2} \sigma_A t_w (c - t_f)$$

$$\alpha_1 = \frac{1}{2}(c + y_y)$$

$$\alpha_2 = y_y - \frac{1}{3}[y_y - (c - t_f)]$$

$$\alpha_3 = y_y - \frac{2}{3}[y_y - (c - t_f)]$$

$$\alpha_4 = \frac{2}{3}(c - t_f)$$

BENDING MOMENT

$$M = 2 \sum_{n=1}^4 R_n \alpha_n$$

RADIUS OF CURVATURE

$$y_y = \epsilon_y p = \frac{\sigma_y}{E} p; \quad p = \frac{y_y E}{\sigma_y}$$

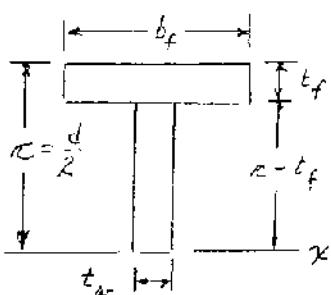
CONTINUED

PROBLEM 4.C2 - CONTINUED

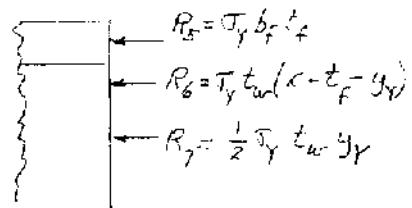
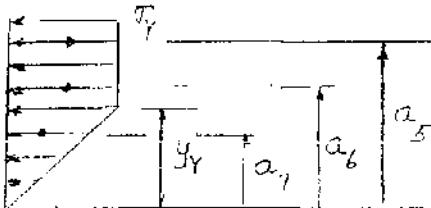
FOR YIELDING IN THE WEB

(CONSIDER UPPER HALF

OF FLANGE SECTION)



$$c = d/2$$



$$a_5 = c - \frac{1}{2}t_f$$

$$a_6 = \frac{1}{2}[y_y + (c - t_f)]$$

$$a_7 = \frac{2}{3} y_y$$

BENDING MOMENT

$$M = 2 \sum_{n=5}^7 R_n a_n$$

RADIUS OF CURVATURE

$$y_y = E_y p = \frac{\sigma_y}{E} p \quad p = \frac{y_y F}{\sigma_y}$$

PROGRAM: KEY IN EXPRESSIONS FOR a_n AND R_n FOR $n = 1$ TO 7

For $y_y = c$ TO $(c - t_f)$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2 \sum R_n a_n$, FOR $n = 1$ TO 4 AND $p = \frac{y_y F}{\sigma_y}$, THEN PRINT

For $y_y = (c - t_w)$ TO $c/3$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2 \sum R_n a_n$ FOR $n = 5$ TO 7 AND $p = \frac{y_y F}{\sigma_y}$, THEN PRINT

INPUT NUMERICAL VALUES AND RUN PROGRAM

PROGRAM OUTPUT

For a beam of Prob 4.190

Depth $d = 140.00$ mm Width of flange $b_f = 120.00$ mm
Thickness of flange $t_f = 10.00$ mm Thickness of web $t_w = 10.00$ mm

$I = 0.000011600$ m to the 4th

Yield strength of Steel $\sigma_{y,y} = 300$ MPa

Yield Moment $M_y = 49.71$ kip.in.

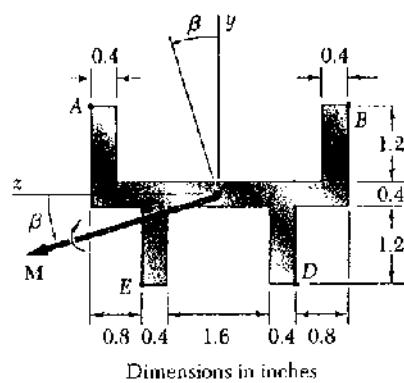
y_y(mm) M(kN.m) rho(m)
For yielding still in the flange.

70.000	49.71	46.67
65.000	52.59	43.33
60.000	54.00	40.00

For yielding in the web

60.000	54.00	40.00
55.000	54.58	36.67
50.000	55.10	33.33
45.000	55.58	30.00
40.000	56.00	26.67
35.000	56.38	23.33
30.000	56.70	20.00
25.000	56.97	16.67

PROBLEM 4.C3



4.C3 An 8 kip · in. couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at C and that the y and z axes are principal axes, write a computer program to calculate the stress at A , B , C , and D for values of β from 0 to 180° using 10° increments. (Given: $I_y = 6.23 \text{ in}^4$ and $I_z = 1.481 \text{ in}^4$.)

SOLUTION

INPUT COORDINATES OF F. B. C, D

$$\begin{aligned} z_A &= z(1) = 2 & y_A &= y(1) = 1.4 \\ z_E &= z(2) = -2 & y_E &= y(2) = 1.4 \\ z_C &= z(3) = -1 & y_C &= y(3) = -1.4 \\ z_D &= z(4) = 1 & y_D &= y(4) = -1.4 \end{aligned}$$

COMPONENTS OF M .

$$M_y = -M \sin \beta \quad M_z = M \cos \beta$$

$$\underline{\text{Eq } 4.55 \text{ page 273: }} \tau(n)_s = \frac{M_z y(n)}{I_z} + \frac{M_y z(n)}{I_y}$$

PROGRAM: FOR $\beta = 0$ TO 180° USING 10° INCREMENTS.

FOR $n = 1$ TO 4 USING UNIT INCREMENTS.

EVALUATE EQ 4.55 AND PRINT STRESSES

RETURN

RETURN

PROGRAM OUTPUT

Moment of couple $M = 8.00 \text{ kip-in.}$
Moments of inertia: $I_y = 6.23 \text{ in}^4$ $I_z = 1.481 \text{ in}^4$

Coordinates of points A, B, D, and E

Point A: $z(1) = 2$: $y(1) = 1.4$

Point B: $z(2) = -2$: $y(2) = 1.4$

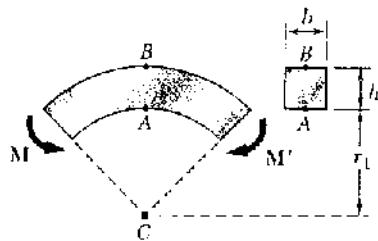
Point D: $z(3) = -1$: $y(3) = -1.4$

Point E: $z(4) = 1$: $y(4) = -1.4$

- - - Stress at Points - - -

beta °	A ksi	B ksi	D ksi	E ksi
0	-7.565	-7.565	7.565	7.565
10	-7.896	-7.004	7.673	7.227
20	-7.987	-6.230	7.548	6.669
30	-7.836	-5.267	7.193	5.909
40	-7.446	-4.144	6.621	4.970
50	-6.830	-2.895	5.846	3.879
60	-6.007	-1.558	4.895	2.670
70	-5.001	-0.174	3.794	1.381
80	-3.843	1.216	2.578	0.049
90	-2.569	2.569	1.284	-1.284
100	-1.216	3.843	-0.049	-2.578
110	0.174	5.001	-1.381	-3.794
120	1.558	6.007	-2.670	-4.895
130	2.895	6.830	-3.879	-5.846
140	4.144	7.446	-4.970	-6.621
150	5.267	7.836	-5.909	-7.193
160	6.230	7.987	-6.669	-7.548
170	7.004	7.896	-7.227	-7.673
180	7.565	7.565	-7.565	-7.565

PROBLEM 4.C4



4.C4 Couples of moment $M = 2 \text{ kN} \cdot \text{m}$ are applied as shown to a curved bar having a rectangular cross section with $h = 100 \text{ mm}$ and $b = 25 \text{ mm}$. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50 percent larger than the maximum stress in a straight bar of the same cross section.

SOLUTION INPUT: $h = 100 \text{ mm}$, $b = 25 \text{ mm}$, $M = 2.21 \text{ Nm}$

$$\text{FOR STRAIGHT BAR: } \sigma_{\text{STRAIGHT}} = \frac{M}{S} = \frac{6M}{bh^3} = 48 \text{ MPa}$$

FOLLOWING NOTATION OF SEC. 4.15, KEY IN THE FOLLOWING:

$$r_2 = h + r_1 ; R = h / \ln(r_2/r_1) ; \bar{r} = r_1 + r_2 ; e = \bar{r} - R ; A = bh = 2500 \quad \text{(I)}$$

$$\text{STRESSES: } \sigma_1 = \sigma_2 = M(r_1 - R) / (Ae^2) \quad \sigma_{\bar{r}} = \sigma_2 = M(r_2 - R) / (Ae^2) \quad \text{(II)}$$

SINCE $h = 100 \text{ mm}$, FOR $r_1/h = 10$, $r_1 = 1000 \text{ mm}$. ALSO $r_1/h = 10$, $r_1 = 100$

PROGRAM: For $r_1 = 1000$ TO 100 AT -100 DECREMENTS

USING EQUATIONS OF LINES I AND II EVALUATE r_2 , R , \bar{r} , e , σ_1 AND σ_2

ALSO EVALUATE: ratio = $\sigma_1 / \sigma_{\text{STRAIGHT}}$

RETURN AND REPEAT FOR $r_1 = 100$ TO 10 AT -10 DECREMENTS

PROGRAM OUTPUT

$M = \text{Bending Moment} = 2. \text{ kN.m}$ $h = 100.000 \text{ in.}$ $A = 2500.00 \text{ mm}^2$
Stress in straight beam = 48.00 MPa

r_1 mm	r_{bar} mm	R mm	e mm	σ_{max} MPa	σ_{max} MPa	r_1/h	ratio
1000	1050	1049	0.794	-49.57	46.51	10.000	-1.033
900	950	949	0.878	-49.74	46.36	9.000	-1.036
800	850	849	0.981	-49.95	46.18	8.000	-1.041
700	750	749	1.112	-50.22	45.95	7.000	-1.046
600	650	649	1.284	-50.59	45.64	6.000	-1.054
500	550	548	1.518	-51.08	45.24	5.000	-1.064
400	450	448	1.858	-51.82	44.66	4.000	-1.080
300	350	348	2.394	-53.03	43.77	3.000	-1.105
200	250	247	3.370	-55.35	42.24	2.000	-1.153
100	150	144	5.730	-61.80	38.90	1.000	-1.288
<hr/>							
100	150	144	5.730	-61.80	38.90	1.000	-1.288
90	140	134	6.170	-63.15	38.33	0.900	-1.316
80	130	123	6.685	-64.80	37.69	0.800	-1.350
70	120	113	7.299	-66.86	36.94	0.700	-1.393
60	110	102	8.045	-69.53	36.07	0.600	-1.449
50	100	91	8.976	-73.13	35.04	0.500	-1.523
40	90	80	10.176	-78.27	33.79	0.400	-1.631
30	80	68	11.803	-86.30	32.22	0.300	-1.798
20	70	56	14.189	-100.95	30.16	0.200	-2.103
10	60	42	18.297	-138.62	27.15	0.100	-2.888
<hr/>							

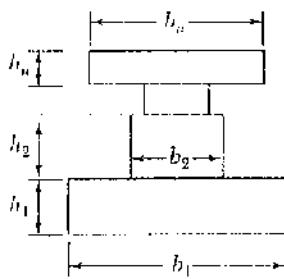
Find r_1/h for $(\sigma_{\text{max}})/(\sigma_{\text{straight}}) = 1.5$

52.70	103	94	8.703	-72.036	35.34	0.527	-1.501
52.80	103	94	8.693	-71.998	35.35	0.528	-1.500
52.90	103	94	8.683	-71.959	35.36	0.529	-1.499

Ratio of stresses is 1.5 for $r_1 = 52.8 \text{ mm}$ or $r_1/h = 0.529$

[Note: The desired ratio r_1/h is valid for any beam having a rectangular cross section.]

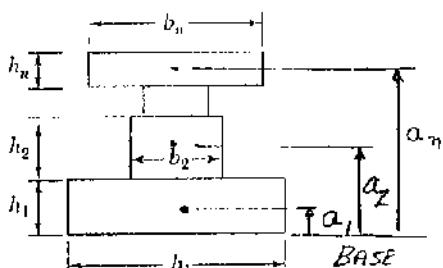
PROBLEM 4.C5



4.C5 The couple M is applied to a beam of the cross section shown.
 (a) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (b) Use this program to solve Probs. 4.7, 4.8, and 4.9.

SOLUTION

INPUT: BENDING MOMENT M



FOR $n=1$ TO n : ENTER b_n AND h_n

(PRINT)

$$\Delta \text{AREA} = b_n h_n$$

$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$
 [MOMENT OF RECTANGLE ABOUT BASE]

$$A_m = (\Delta \text{AREA}) a_n$$

[FOR WHOLE CROSS SECTION]

$$m = m + A_m ; \text{ AREA} = \text{AREA} + \Delta \text{AREA}$$

LOCATION OF CENTROID ABOVE BASE,

$$\bar{y} = m/\text{AREA}$$

(PRINT)

MOMENT OF INERTIA ABOUT HORIZONTAL CENTRAL AXIS

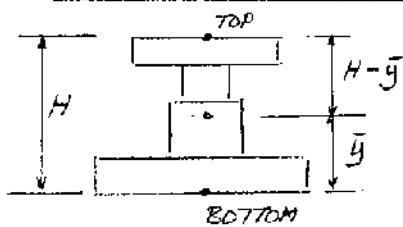
FOR $n=1$ TO n : $a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

$$\Delta I = b_n h_n^3 / 12 + (b_n h_n) (\bar{y} - a_n)^2$$

$$I = I + \Delta I$$

(PRINT)

COMPUTATION OF STRESSES



TOTAL HEIGHT: FOR $n=1$ TO n

$$H = H + h_n$$

STRESS AT TOP

$$\sigma_{\text{TOP}} = -M \frac{H - \bar{y}}{I}$$

(PRINT)

STRESS AT BOTTOM

$$\sigma_{\text{BOTTOM}} = M \frac{\bar{y}}{I}$$

(PRINT)

SET NEXT PAGE FOR PRINT OUTS

CONTINUED

PROBLEM 4.CS - CONTINUED

Problem 4.7

Summary of Cross Section Dimensions

Width (in.)	Height (in.)
9.00	2.00
3.00	6.00

Bending Moment = 600.000 kip.in.

Centroid is 3.000 in. above lower edge

Centroidal Moment of Inertia is 204.000 in⁴

Stress at top of beam = -14.706 ksi

Stress at bottom of beam = 8.824 ksi

Problem 4.8

Summary of Cross Section Dimensions

Width (in.)	Height (in.)
4.00	1.00
1.00	6.00
8.00	1.00

Bending Moment = 500.000 kip.in.

Centroid is 4.778 in. above lower edge

Centroidal Moment of Inertia is 155.111 in⁴

Stress at top of beam = -10.387 ksi

Stress at bottom of beam = 15.401 ksi

PROBLEM 4.9

Summary of Cross Section Dimensions

Width (mm)	Height (mm)
50	10
20	50

Bending Moment = 1500.0000 N.m

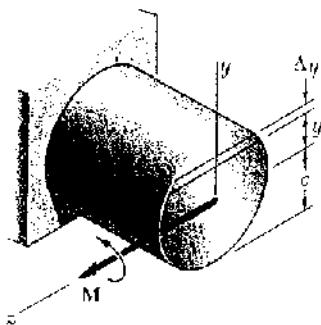
Centroid is 25.000 mm above lower edge

Centroidal Moment of Inertia is 512500 mm⁴

Stress at top of beam = -102.439 MPa

Stress at bottom of beam = 73.171 MPa

PROBLEM 4.C6



4.C6 A solid rod of radius $c = 1.2$ in. is made of a steel that is assumed to be elastoplastic with $E = 29,000$ ksi and $\sigma_y = 42$ ksi. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_y and then to the plastic moment M_p . Denoting by y_y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_y from 1.2 in. to 0 using 0.2-in. decrements. (Hint: Divide the cross section into 80 horizontal elements of 0.03-in. height.)

SOLUTION

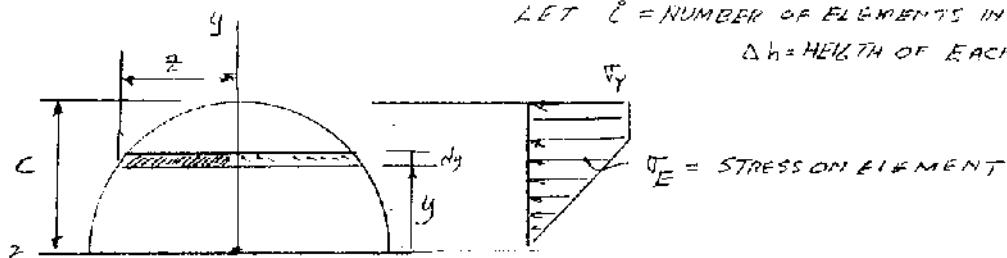
$$M_{y_y} = \sigma_y \frac{\pi}{4} c^3 = (42 \text{ ksi}) \frac{\pi}{4} (1.2 \text{ in.})^3 = 57 \text{ kip-in.}$$

$$M_p = \sigma_y \frac{4}{3} c^3 = (42 \text{ ksi}) \frac{4}{3} (1.2 \text{ in.})^3 = 96.8 \text{ kip-in.}$$

CONSIDER TOP HALF OF ROD

LET ℓ = NUMBER OF ELEMENTS IN TOP HALF

$$\Delta h = \text{HEIGHT OF EACH ELEMENT: } \Delta h = \frac{c}{\ell}$$



FOR $n=0$ TO $\ell-1$ STEP 1

$$y = n(\Delta h)$$

$$z = [c^2 - \{(n+0.5)\Delta h\}^2]^{1/2}$$

← z AT MIDHEIGHT OF ELEMENT

IF $y \geq y_y$ GO TO 100

$$\sigma_e = \sigma_y \frac{(n+0.5)\Delta h}{y_y}$$

← STRESS IN ELASTIC CORE

GOTO 200

$$\sigma_e = \sigma_y$$

← STRESS IN PLASTIC ZONE

$$\Delta \text{AREA} = \pi z (\Delta h)$$

$$\Delta \text{FORCE} = \sigma_e (\Delta \text{AREA})$$

$$\Delta \text{MOMENT} = \Delta \text{FORCE} (n+0.5) \Delta h$$

$$M = M + \Delta \text{MOMENT}$$

$$P = y_e E / T_y$$

PRINT y_y, M , AND P .

NEXT

REPEAT

FOR

$$y_y = 1.2 \text{ in.}$$

TO

$$y_y = 0$$

AT -0.2-in.

DECREMENTS

PROGRAM OUTPUT

Radius of rod = 1.2 in.

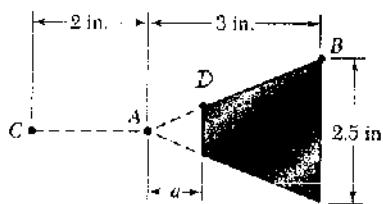
Yield point of steel = 42 ksi

Yield moment = 57.0 kip-in. Plastic moment = 96.8 kip.in.

Number of elements in half of the rod = 40

For $y_y = 1.20$ in.	$M = 57.1$ kip.in.	Radius of curvature = 828.57 in.
For $y_y = 1.00$ in.	$M = 67.2$ kip.in.	Radius of curvature = 690.48 in.
For $y_y = 0.80$ in.	$M = 76.9$ kip.in.	Radius of curvature = 552.38 in.
For $y_y = 0.60$ in.	$M = 85.2$ kip.in.	Radius of curvature = 414.29 in.
For $y_y = 0.40$ in.	$M = 91.6$ kip.in.	Radius of curvature = 276.19 in.
For $y_y = 0.20$ in.	$M = 95.5$ kip.in.	Radius of curvature = 138.10 in.
For $y_y = 0.00$ in.	$M = \text{infinite}$	Radius of curvature = zero

PROBLEM 4.C7



4.C7 The machine element of Prob. 4.178 to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width a will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of a from 0 to 1 in. using 0.1-in. increments. Using appropriate smaller increments, determine the distance a for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION SEE FIG 4.79 PAGE 289

$$M = 5 \text{ ft-lb/in. } r_2 = 5 \text{ in. } b_2 = 2.5 \text{ in}$$

For $a = 0$ to 1.0 at 0.1 intervals

$$h = 3 - a$$

$$r_1 = 2 + a$$

$$b_1 = b_2 (a/(h+a))$$

$$\text{AREA} = (b_1 + b_2)(h/2)$$

$$\bar{x} = a \left[\frac{1}{2} b_1 h \left(\frac{h}{3} \right) + \frac{1}{2} b_2 h \left(\frac{2h}{3} \right) \right] / \text{AREA}$$

$$\bar{r} = r_2 - (h - \bar{x})$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - \frac{1}{2} b_1^2) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$e = \bar{r} - R$$

$$\sigma_D = M(r_1 - R) / [\text{AREA}(e/r_1)]$$

$$\sigma_B = M(r_2 - R) / [\text{AREA}(e/r_2)]$$

PRINT AND RETURN

PROGRAM OUTPUT

a	R	σ_{D}	σ_{B}	b1	rbar	e
in.	in.	ksi	ksi			
0.00	3.855	-8.5071	2.1014	0.00	4.00	0.145
0.10	3.858	-7.7736	2.1197	0.08	4.00	0.144
0.20	3.869	-7.2700	2.1689	0.17	4.01	0.140
0.30	3.884	-6.9260	2.2438	0.25	4.02	0.134
0.40	3.904	-6.7004	2.3423	0.33	4.03	0.127
0.50	3.928	-6.5683	2.4641	0.42	4.05	0.119
0.60	3.956	-6.5143	2.6102	0.50	4.07	0.111
0.70	3.985	-6.5296	2.7828	0.58	4.09	0.103
0.80	4.018	-6.6098	2.9852	0.67	4.11	0.094
0.90	4.052	-6.7541	3.2220	0.75	4.14	0.086
1.00	4.089	-6.9647	3.4992	0.83	4.17	0.078

Determination of the maximum compressive stress that is as small as possible

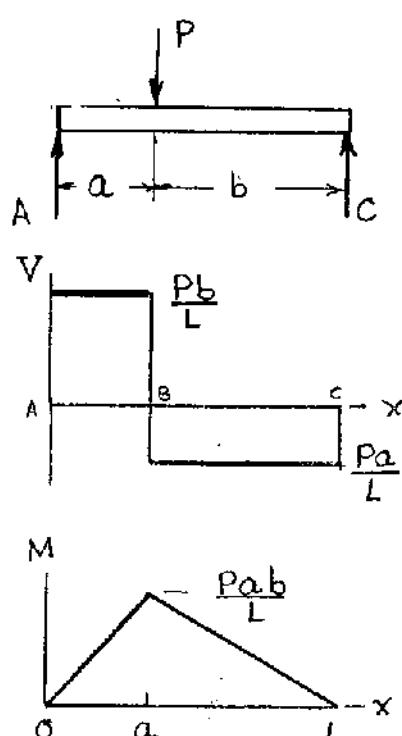
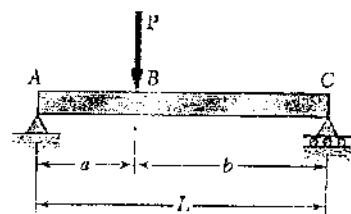
a	R	σ_{D}	σ_{B}	b1	rbar	e
in.	in.	ksi	ksi			
0.620	3.961	-6.51198	2.6425	0.52	4.07	0.109
0.625	3.963	-6.51185	2.6507	0.52	4.07	0.109
0.630	3.964	-6.51188	2.6591	0.52	4.07	0.109

ANSWER: When $a = 625$ in. the compressive stress is 6.51 ksi

Chapter 5

Problem 5.1

5.1 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



Reactions

$$\sum M_C = 0 \quad LA - bP = 0 \quad A = \frac{Pb}{L}$$

$$\sum M_A = 0 \quad LC - aP = 0 \quad C = \frac{Pa}{L}$$

From A to B $0 < x < a$

$$+ \uparrow \sum F_y = 0 \quad \frac{Pb}{L} - V = 0 \quad V = \frac{Pb}{L}$$

$$\therefore \sum M_J = 0 \quad M - \frac{Pb}{L}x = 0 \quad M = \frac{Pbx}{L}$$

From B to C $a < x < L$

$$+ \uparrow \sum F_y = 0 \quad V + \frac{Pa}{L} = 0 \quad V = -\frac{Pa}{L}$$

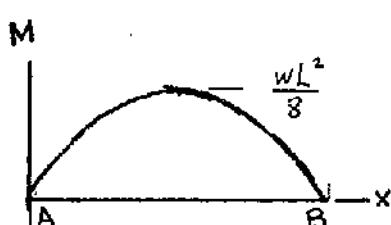
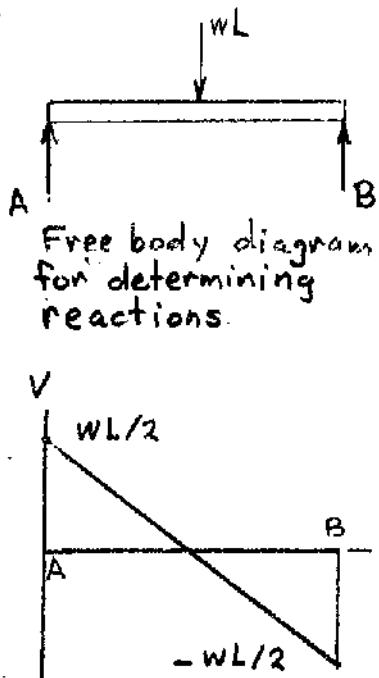
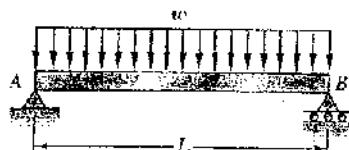
$$\therefore \sum M_K = 0 \quad -M + \frac{Pa}{L}(L-x) = 0 \quad M = \frac{Pa(L-x)}{L}$$

At section B

$$M = -\frac{Pab}{L}$$

Problem 5.2

5.2 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.

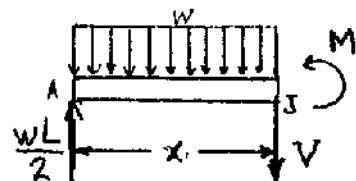


Reactions

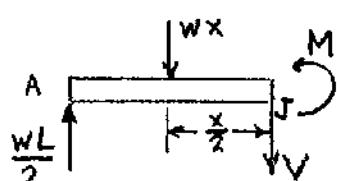
$$\textcircled{D} \sum M_B = 0 \quad -AL + WL \cdot \frac{L}{2} = 0 \quad A = \frac{WL}{2}$$

$$\textcircled{D} \sum M_A = 0 \quad BL - WL \cdot \frac{L}{2} = 0 \quad B = \frac{WL}{2}$$

Over whole beam $0 < x < L$



Place section at x .



Replace distributed load by equivalent concentrated load.

$$+\uparrow \sum F_y = 0 \quad \frac{WL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right)$$

$$\textcircled{D} \sum M_J = 0 \quad -\frac{WL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

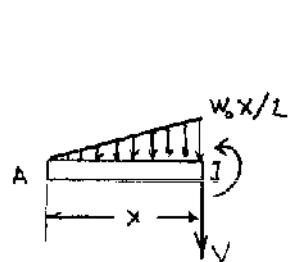
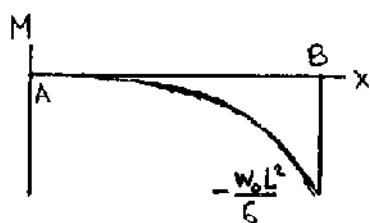
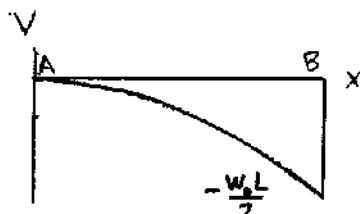
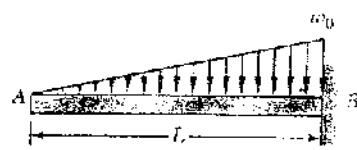
$$= \frac{w}{2}x(L-x)$$

Maximum bending moment occurs at $x = \frac{L}{2}$.

$$M_{max} = \frac{wL^2}{8}$$

Problem 5.3

5.3 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



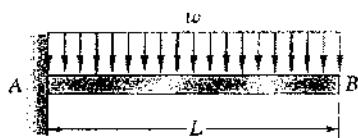
$$+\uparrow \sum F_y = 0 \quad -\frac{1}{2} \frac{w_0 x}{L} \cdot x - V = 0 \\ V = -\frac{w_0 x^2}{2L}$$

$$\textcircled{D} \sum M_J = 0 \quad \frac{1}{2} \frac{w_0 x}{L} \cdot x \cdot \frac{x}{3} + M = 0 \\ M = -\frac{w_0 x^3}{6L}$$

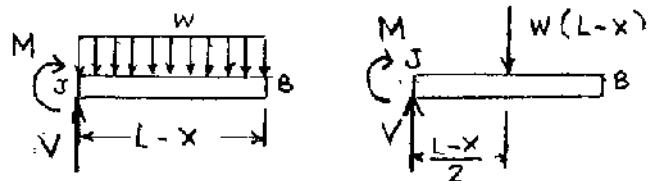
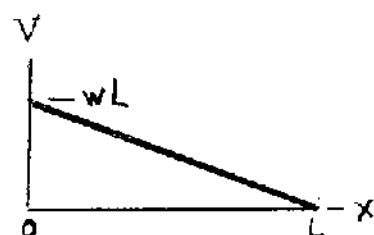
$$\text{At } x = L \quad V = -\frac{w_0 L}{2} \quad |V|_{\max} = \frac{w_0 L}{2} \\ M = -\frac{w_0 L^3}{6} \quad |M|_{\max} = \frac{w_0 L^2}{6}$$

Problem 5.4

5.4 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



Use portion to the right of the section as the free body.



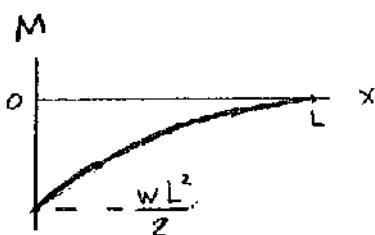
Replace distributed load by equivalent concentrated load.

$$\text{At } \sum F_y = 0 \quad V - w(L-x) = 0$$

$$V = w(L-x)$$

$$\text{At } \sum M_J = 0 \quad -M - w(L-x)\left(\frac{L-x}{2}\right) = 0$$

$$M = -\frac{w}{2}(L-x)^2$$



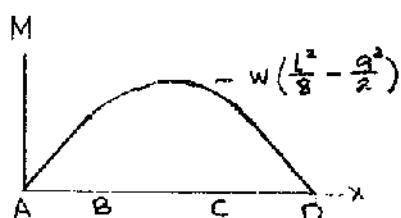
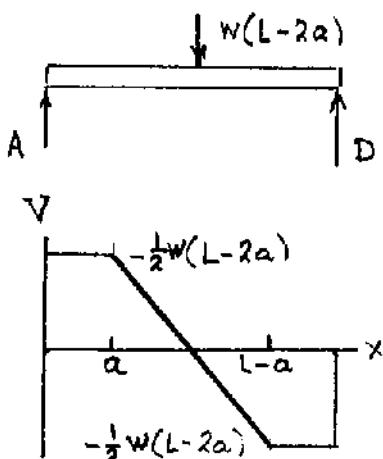
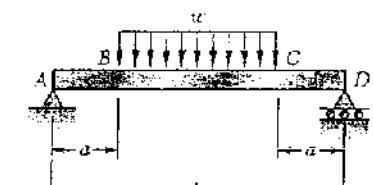
Largest negative bending moment occurs at $x = 0$.

$$M_{\min} = -\frac{wL^2}{2}$$

$$\text{Thus, } |M|_{\max} = \frac{wL^2}{2}$$

Problem 5.5

5.5 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



Calculate reactions after replacing distributed load by an equivalent concentrated load.

Reactions are

$$A = D = \frac{1}{2}w(L-2a)$$

From A to B $0 < x < a$

$$\begin{aligned} +\uparrow \sum F_y &= 0 & \frac{1}{2}w(L-2a) - V &= 0 \\ V &= \frac{1}{2}w(L-2a) & & \\ \therefore \sum M_A &= 0 & -\frac{1}{2}w(L-2a)x + M &= 0 \\ M &= \frac{1}{2}w(L-2a)x & & \end{aligned}$$

From B to C $a < x < L-a$

$$\begin{aligned} +\uparrow \sum F_y &= 0 & \frac{1}{2}w(L-2a) - w(x-a) - V &= 0 \\ V &= \frac{1}{2}w(L-2a) - w(x-a) & & \\ \therefore \sum M_B &= 0 & -\frac{1}{2}w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M &= 0 \\ M &= \frac{1}{2}w\left[(L-2a)x - (x-a)^2\right] & & \end{aligned}$$

Place section cut at x. Replace distributed load by equiv. conc. load.

$$+\uparrow \sum F_y = 0 \quad \frac{1}{2}w(L-2a) - w(x-a) - V = 0 \quad V = w\left(\frac{L}{2} - x\right)$$

$$\therefore M_x = 0 \quad -\frac{1}{2}w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M = 0$$

$$M = \frac{1}{2}w\left[(L-2a)x - (x-a)^2\right]$$

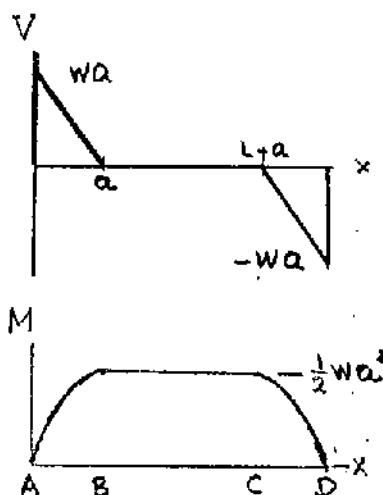
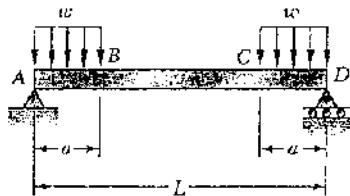
From C to D $L-a < x < L$

$$\begin{aligned} +\uparrow \sum F_y &= 0 & \frac{1}{2}w(L-2a) + V &= 0 \\ V &= -\frac{1}{2}w(L-2a) & & \\ \therefore \sum M_C &= 0 & -M + \frac{1}{2}w(L-2a)(L-x) &= 0 \\ M &= \frac{1}{2}w(L-2a)(L-x) & & \end{aligned}$$

$$\text{At } x = \frac{L}{2} \quad M_{\max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$$

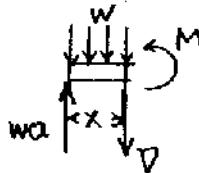
Problem 5.6

5.6 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



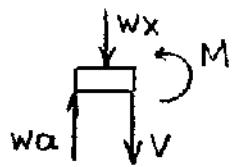
$$\text{Reactions: } A = D = wa$$

From A to B $0 < x < a$



$$+\uparrow \sum F_y = 0$$

$0 < x < a$



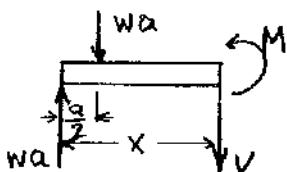
$$wa - wx - V = 0$$

$$V = w(a-x)$$

$$\text{④} \sum M_J = 0 \quad -wax + (wx)\frac{x}{2} + M = 0$$

$$M = w\left(ax - \frac{x^2}{2}\right)$$

From B to C $a < x < L-a$



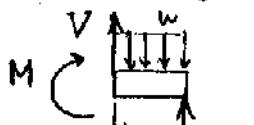
$$\sum F_y = 0$$

$$wa - wa - V = 0$$

$$V = 0$$

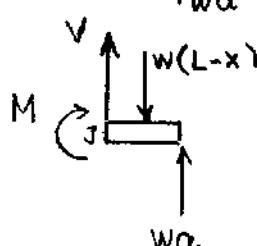
$$\text{④} \sum M_J = 0 \quad -wax + wa\left(x - \frac{a}{2}\right) + M = 0 \quad M = \frac{1}{2}wa^2$$

From C to D $L-a < x < L$



$$+\uparrow \sum F_y = 0 \quad V - w(L-x) + wa = 0$$

$$V = w(L-x - a)$$

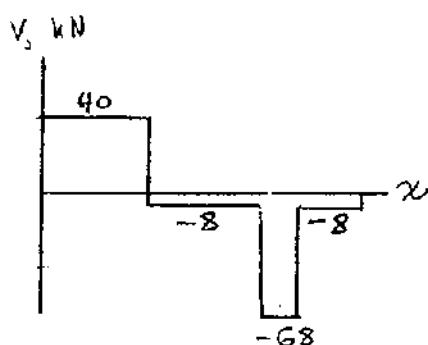
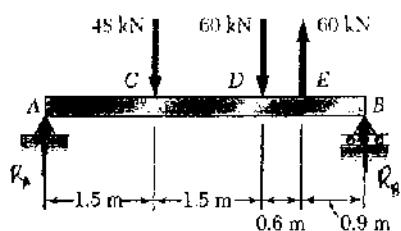


$$\text{④} \sum M_J = 0 \quad -M - w(L-x)\left(\frac{L-x}{2}\right) + wa(L-x) = 0$$

$$M = w\left[a(L-x) - \frac{1}{2}(L-x)^2\right]$$

Problem 5.7

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Reactions

$$+\circlearrowleft \sum M_B = 0$$

$$-4.5 R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

$$R_A = 40 \text{ kN}$$

$$+\circlearrowleft \sum M_A = 0$$

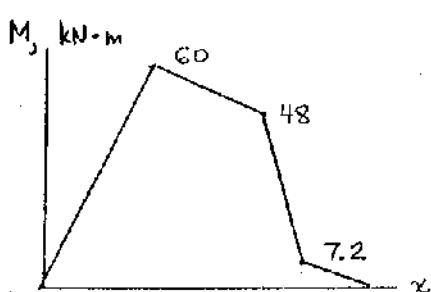
$$-(1.5)(48) - (3.0)(60) + (3.4)(60) + 4.5 R_B = 0$$

$$\text{At } A \text{ to } C \quad V = 48 \text{ kN}$$

$$\text{At } C \text{ to } D \quad V = -8 \text{ kN}$$

$$\text{At } D \text{ to } E \quad V = -68 \text{ kN}$$

$$\text{At } E \text{ to } B \quad V = -8 \text{ kN}$$

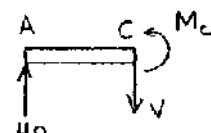


$$\text{At } A \text{ and } B \quad M = 0$$

$$\text{At } C \quad +\circlearrowleft \sum M_C = 0$$

$$-(1.5)(40) + M_C = 0$$

$$M_C = 60 \text{ kN}\cdot\text{m}$$



$$\text{At } D \quad +\circlearrowleft \sum M_D = 0$$

$$-(3.0)(40) + (1.5)(48) + M_D = 0$$

$$M_D = 48 \text{ kN}\cdot\text{m}$$



$$\text{At } E \quad +\circlearrowleft \sum M_E = 0$$

$$-M_E + (0.9)(8) = 0$$

$$M_E = 7.2 \text{ kN}\cdot\text{m}$$

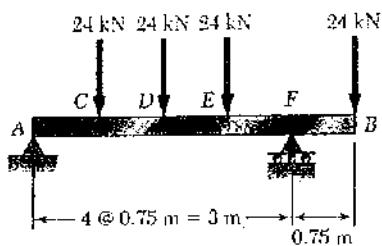


$$(a) |V|_{max} = 68.0 \text{ kN}$$

$$(b) |M|_{max} = 60.0 \text{ kN}\cdot\text{m}$$

Problem 5.8

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

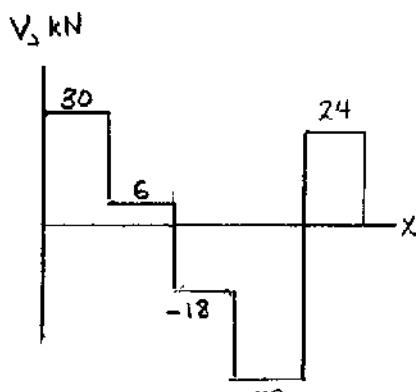


Reactions at A and F.

$$+\sum M_F = 0$$

$$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24) - (0.75)(24) = 0$$

$$R_A = 30 \text{ kN} \uparrow$$



$$+\sum M_A = 0$$

$$-(0.75)(24) - (1.50)(24) - (2.25)(24) + 3R_F - (3.75)(24) = 0$$

$$R_F = 66 \text{ kN} \uparrow$$

Shear diagram.

$$\text{A to C} \quad V = 30 \text{ kN}$$

$$\text{C to D} \quad V = 30 - 24 = 6 \text{ kN}$$

$$\text{D to E} \quad V = 6 - 24 = -18 \text{ kN}$$

$$\text{E to F} \quad V = -18 - 24 = -42 \text{ kN}$$

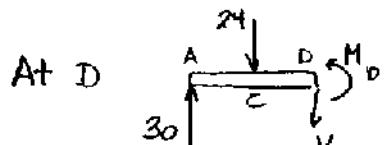
$$\text{F to B} \quad V = -42 + 66 = 24 \text{ kN}$$

$$\text{At A and B} \quad M_A = M_B = 0$$

$$\text{At C} \quad +\sum M_C = 0$$

$$-\text{M}_c + 30 = 0$$

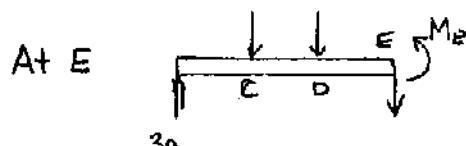
$$\text{M}_c = 22.5 \text{ kN}\cdot\text{m}$$



$$+\sum M_D = 0$$

$$-(1.50)(30) + (0.75)(24) + M_D = 0$$

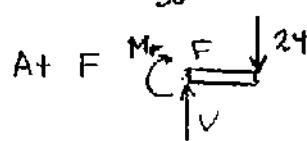
$$\text{M}_D = 27 \text{ kN}\cdot\text{m}$$



$$+\sum M_E = 0$$

$$-(2.25)(30) + (1.50)(24) + (0.75)(24) + M_E = 0$$

$$\text{M}_E = 13.5 \text{ kN}\cdot\text{m}$$



$$(a) |V|_{\max} = 42.0 \text{ kN}$$

$$+\sum M_F = 0$$

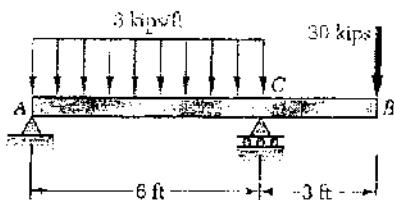
$$-M_F - (0.75)(24) = 0$$

$$M_F = -18 \text{ kN}\cdot\text{m}$$

$$(b) |M|_{\max} = 27.0 \text{ kN}\cdot\text{m}$$

Problem 5.9

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



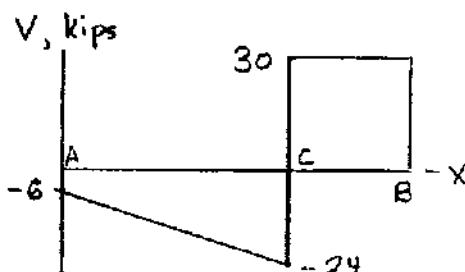
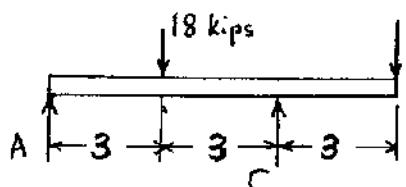
Reactions

$$\sum M_C = 0 \quad -6A + (3)(18) - (3)(30) = 0$$

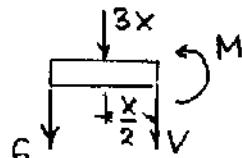
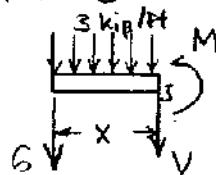
$$A = -6 \text{ kips} \quad (\text{i.e. } 6 \text{ kips down})$$

$$\sum M_A = 0 \quad 6C - (3)(18) - (9)(30) = 0$$

$$C = 54 \text{ kips}$$

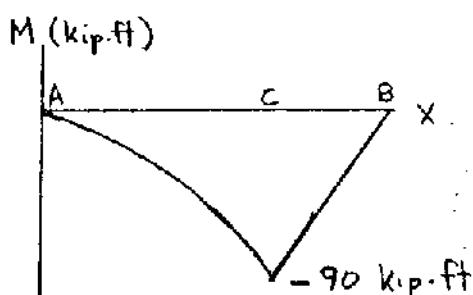


A to C $0 < x < 6 \text{ ft.}$



$$+\uparrow \sum F_y = 0 \quad -6 - 3x - V = 0$$

$$V = -6 - 3x \text{ kips.}$$

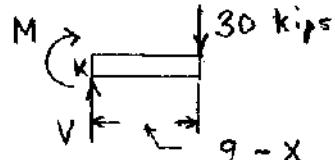


$$\sum M_J = 0 \quad -6x - (3x)\left(\frac{x}{2}\right) - M = 0$$

$$M = -6x - 1.5x^2 \text{ kip-ft}$$

C to B

$6 \text{ ft} < x < 9 \text{ ft}$



$$+\uparrow \sum F_y = 0$$

$$V - 30 = 0$$

$$V = 30 \text{ kips}$$

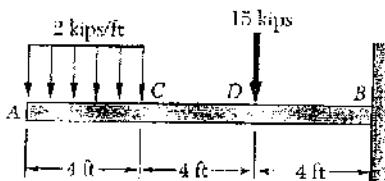
$$\sum M_K = 0 \quad -M - (9-x)(30) = 0$$

$$M = 30x - 270 \text{ kip-ft}$$

From the diagrams (a) $|V|_{\max} = 30.0 \text{ kips}$

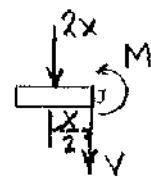
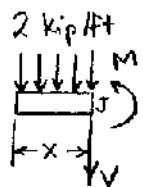
(b) $|M|_{\max} = 90.0 \text{ kip-ft}$

Problem 5.10

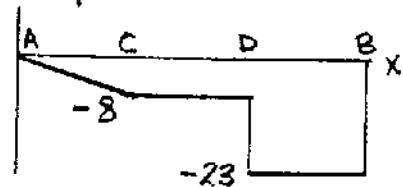


5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

A to C
 $0 < x < 4 \text{ ft}$



V (kips)



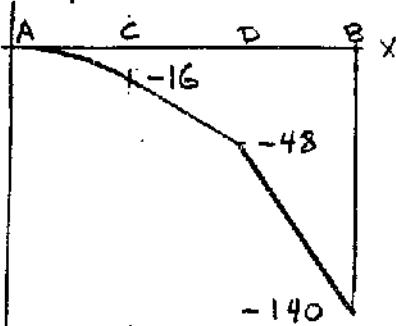
$$+\uparrow \sum F_y = 0 \quad -V - 2x = 0 \quad V = -2x \text{ kips}$$

$$\textcircled{D} \sum M_J = 0 \quad M + (2x)\left(\frac{x}{2}\right) = 0$$

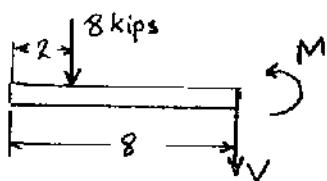
$$M = -x^2 \text{ kip-ft.}$$

$$\text{At } C \quad V = -8 \text{ kips} \quad M = -16 \text{ kip-ft.}$$

M (kip-ft)



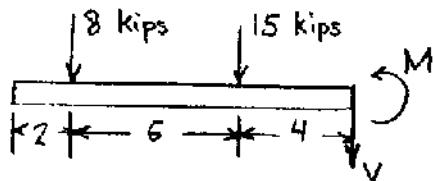
$$\text{At } D^-$$



$$+\uparrow \sum F_y = 0 \quad -8 - V = 0 \quad V = -8 \text{ kips}$$

$$\textcircled{D} \sum M_o = 0 \quad (6)(8) - M = 0 \quad M = -48 \text{ kip-ft}$$

$$\text{At } B^-$$



$$+\uparrow \sum F_y = 0 \quad -8 - 15 - V = 0$$

$$V = -23 \text{ kips}$$

$$\textcircled{D} \sum M_B = 0 \quad -(10)(8) - (4)(15) - M = 0$$

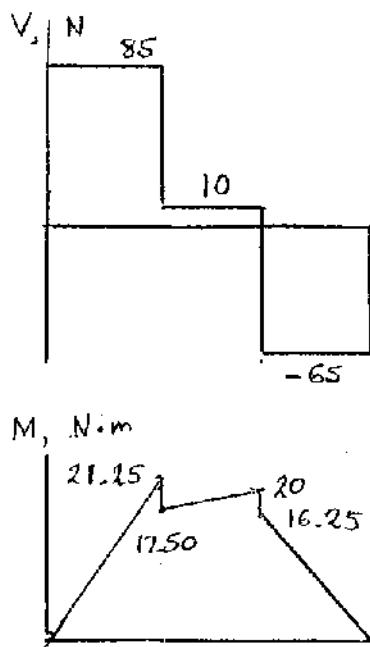
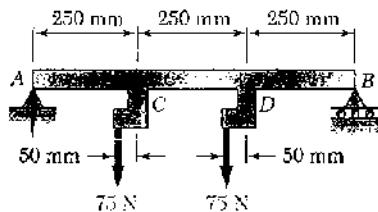
$$M = -140 \text{ kip-ft.}$$

From the diagrams (a) $|V|_{\max} = 23.0 \text{ kips}$ ←

(b) $|M|_{\max} = 140.0 \text{ kip-ft}$ ←

Problem 5.11

5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



$$\text{Reaction at } A, \quad +\sum M_B = 0$$

$$-0.750 R_A + (0.550)(75) + (0.300)(75) = 0$$

$$R_A = 85 \text{ N} \uparrow \quad \text{Also } R_B = 65 \text{ N} \uparrow$$

$$A \text{ to } C \quad V = 85 \text{ N}$$

$$C \text{ to } D \quad V = 10 \text{ N}$$

$$D \text{ to } B \quad V = -65 \text{ N}$$

$$\text{At } A \text{ and } B \quad M = 0$$

Just to the left of C

$$+\sum M_C = 0$$

$$-(0.25)(85) + M = 0$$

$$M = 21.25 \text{ N}\cdot\text{m}$$

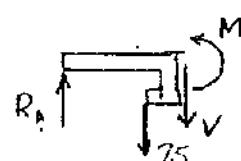


Just to the right of C

$$+\sum M_C = 0$$

$$-(0.25)(85) + (0.050)(75) + M = 0$$

$$M = 17.50 \text{ N}\cdot\text{m}$$



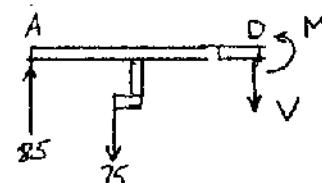
Just to the left of D

$$+\sum M_D = 0$$

$$-(0.50)(85) + (0.300)(75) + M = 0$$

$$+ M = 0$$

$$M = 20 \text{ N}\cdot\text{m}$$

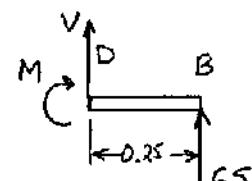


Just to the right of D

$$+\sum M_D = 0$$

$$- M + (0.25)(65) = 0$$

$$M = 16.25 \text{ kN}$$

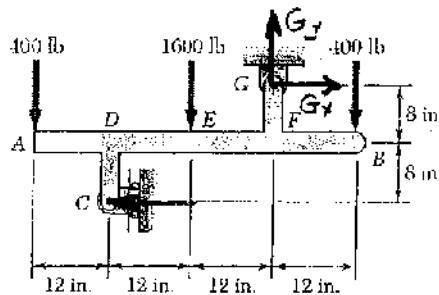


$$(a) \quad |V|_{\max} = 85.0 \text{ N}$$

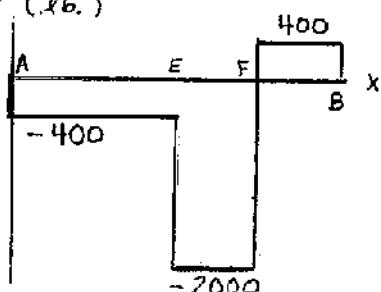
$$(b) \quad |M|_{\max} = 21.25 \text{ N}\cdot\text{m}$$

Problem 5.12

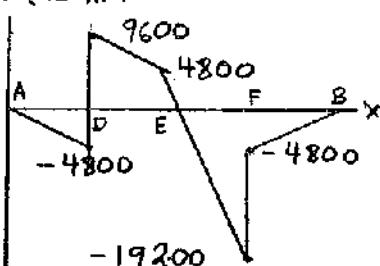
5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



V (lb.)



M (lb-in.)



$$\textcircled{D} \sum M_a = 0$$

$$-16G + (12)(400) + (12)(1600) - (12)(400) = 0 \\ G = 1800 \text{ lb.}$$

$$\textcircled{D} \sum F_x = 0 \quad -C + G_x = 0 \quad G_x = 1800 \text{ lb.}$$

$$\textcircled{+1} \sum F_y = 0 \quad -400 - 1600 + G_y - 400 = 0$$

$$G_y = 2400 \text{ lb.}$$

$$A \rightarrow E \quad V = -400 \text{ lb.}$$

$$E \rightarrow F \quad V = -2000 \text{ lb.}$$

$$F \rightarrow B \quad V = 400 \text{ lb.}$$

$$\text{At } A \text{ and } B \quad M = 0$$

$$\textcircled{D} \sum M_b = 0$$

$$(12)(400) + M = 0$$

$$M = -4800 \text{ lb-in.}$$

$$\textcircled{D} \sum M_d = 0$$

$$(12)(400) - (8)(1800) + M = 0$$

$$M = 9600 \text{ lb-in.}$$

$$\textcircled{D} \sum M_e = 0$$

$$(24)(400) - (8)(1800) + M = 0$$

$$M = 4800 \text{ lb-in.}$$

$$\textcircled{D} \sum M_f = 0$$

$$-M - (8)(1800) - (12)(400) = 0$$

$$M = -19200 \text{ lb-in.}$$

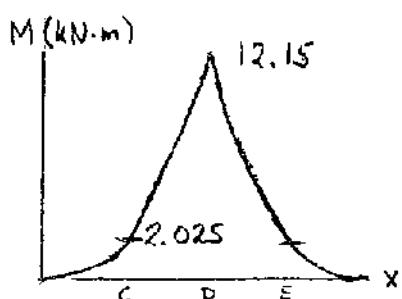
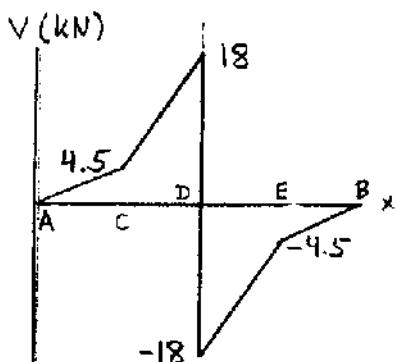
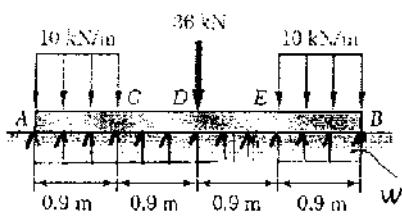
$$\sum M_F = 0$$

$$-M - (12)(400) = 0$$

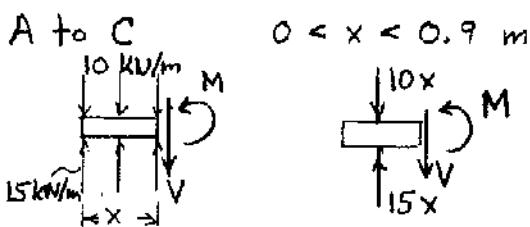
$$M = -4800 \text{ lb-in.}$$

Problem 5.13

5.13 and 5.14 Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value of (a) of the shear, (b) of the bending moment.



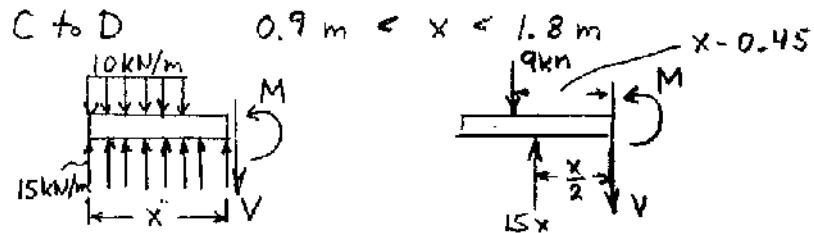
$$(a) \text{ Maximum } |V| = 18 \text{ kN}$$



$$+\uparrow \sum F_y = 0 \quad 15x - 10x - V = 0 \\ V = 5x$$

$$\textcircled{D} \sum M_J = 0 \quad -(15x)\frac{x}{2} + (10x)\frac{x}{2} + M = 0 \\ M = 2.5x^2$$

$$\text{At } x = C \quad V = 4.5 \text{ kN} \\ M = 2.025 \text{ kN}\cdot\text{m}$$



$$+\uparrow \sum F_y = 0 \quad 15x - 9 - V = 0 \\ V = 15x - 9$$

$$\textcircled{D} \sum M_J = 0 \quad -(15x)\left(\frac{x}{2}\right) + 9(x-0.45) + M = 0$$

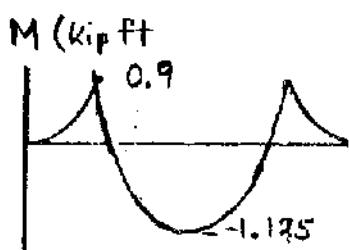
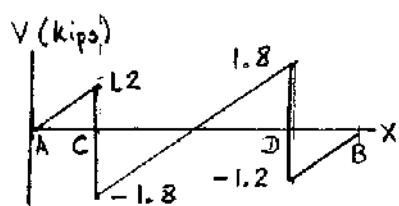
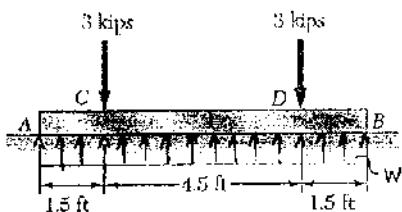
$$M = 7.5x^2 - 9x + 4.05 = 0$$

$$\text{At } D \quad V = 18 \text{ kN} \\ M = 12.15 \text{ kN}\cdot\text{m}$$

D to B Use symmetry to calculate the shear and bending moment.

Problem 5.14

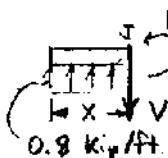
5.13 and 5.14 Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum absolute value of (a) of the shear, (b) of the bending moment.



Over the whole beam

$$+\uparrow \sum F_y = 0 \quad 7.5w - 3 - 3 = 0 \\ w = 0.8 \text{ kip/ft}$$

A to C

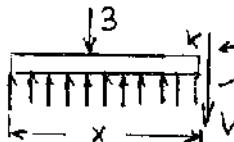


$0 < x < 1.5 \text{ ft.}$

$$+\uparrow \sum F_y = 0 \quad 0.8x - V = 0 \\ V = 0.8x \\ \text{ } \sum M_J = 0 \\ -(0.8x)(\frac{x}{2}) + M = 0 \\ M = 0.4x^2$$

At C⁺ $V = 1.2 \text{ kips}, \quad M = 0.9 \text{ kip-ft}$

C to D



$1.5 \text{ ft} < x < 6 \text{ ft}$

$$+\uparrow \sum F_y = 0 \\ 0.8x - 3 - V = 0 \\ V = 0.8x - 3$$

$$\sum M_K = 0 \quad -(0.8x)(\frac{x}{2}) + 3(x - 1.5) + M = 0 \\ M = 0.4x^2 - 3x + 4.5$$

At the center of the beam $x = 3.75 \text{ ft}$

$$V = 0, \quad M = -1.125 \text{ kip-ft}$$

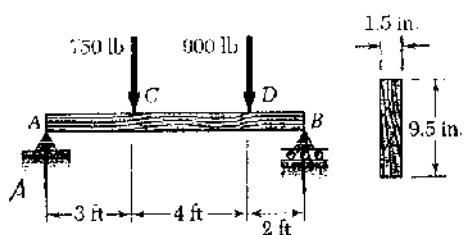
At C⁺ $V = -1.8 \text{ kip} \quad M = 0.9 \text{ kip-ft}$

(a) Maximum |V| = 1.8 kips

(b) Maximum |M| = 1.125 kip-ft

Problem 5.15

5.15 and 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



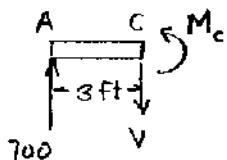
Reaction at A

$$\rightarrow \sum M_A = 0$$

$$-700(3) + (6)(750) + (2)(900) = 0$$

$$A = 700 \text{ lb.}$$

Using free body. A to C.



$$\textcircled{+} \sum M_C = 0 \quad -(700)(3) + M_c = 0$$

$$M_c = 2100 \text{ lb-ft} = 25.2 \times 10^3 \text{ lb-in} = 25.2 \text{ kip-in}$$

$$\text{For the cross section } I = \frac{1}{12}(0.5)(9.5)^3 = 107.172 \text{ in}^4$$

$$c = 4.75 \text{ in.}$$

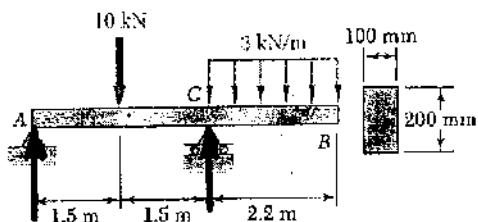
Maximum normal stress due to bending.

$$\sigma = \frac{Mc}{I} = \frac{(25.2)(4.75)}{107.172}$$

$$\sigma = 1.117 \text{ ksi}$$

Problem 5.16

5.15 and 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



Using CB as a free body

$$\textcircled{+} \sum M_C = 0$$

$$-M + (2.2)(3 \times 10^3)(1.1) = 0$$

$$M = 7.26 \times 10^3 \text{ N-m}$$

Section modulus for rectangle

$$S = \frac{1}{6} b h^2$$

$$= \frac{1}{6} (100)(200)^2 = 666.7 \times 10^3 \text{ mm}^3$$

$$= 666.7 \times 10^{-6} \text{ m}^3$$

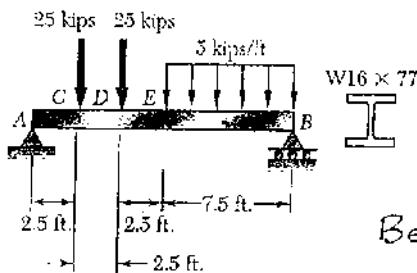
Normal stress

$$\sigma = \frac{M}{S} = \frac{7.26 \times 10^3}{666.7 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$$

$$\sigma = 10.89 \text{ MPa}$$

Problem 5.17

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



Reaction at A. $\rightarrow \sum M_A = 0$

$$-15.0 R_A + (12.5)(25) + (10.0)(25)$$

$$+ (317.5)(7.5)(5) = 0 \quad R_A = 46.875 \text{ kips}$$

Bending moment at C. $\rightarrow \sum M_C = 0$

$$-(2.5)(46.875) + M = 0$$

$$M = 117.1875 \text{ kip}\cdot\text{ft} = 1.40625 \times 10^3 \text{ kip}\cdot\text{in}$$



For W16x77 rolled steel section $S = 134 \text{ in}^3$

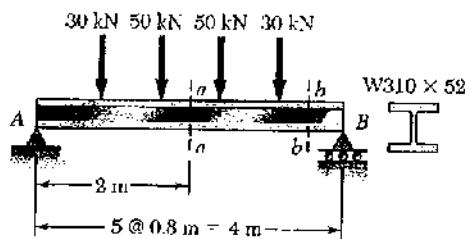
Normal stress at C

$$\sigma = \frac{M}{S} = \frac{1.40625 \times 10^3}{134} =$$

$$\sigma = 10.49 \text{ ksi}$$

Problem 5.18

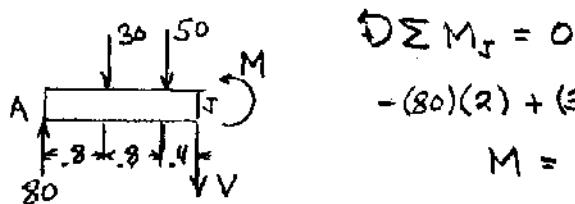
5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section a-a.



Reactions: By symmetry $A = B$

$$\uparrow \sum F_y = 0 \quad A = B = 80 \text{ kN}$$

Using left half of beam as free body



$\rightarrow \sum M_A = 0$

$$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$$

$$M = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$$

For W310x52

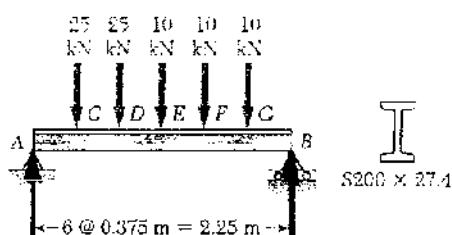
$$S = 748 \times 10^3 \text{ mm}^3$$

$$= 748 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress } \sigma = \frac{M}{S} = \frac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa} = 139.0 \text{ MPa}$$

Problem 5.19

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

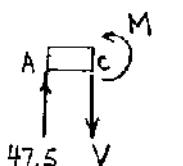


Use entire beam as free body

$$\textcircled{D} \sum M_B = 0$$

$$2.25 A - (1.875)(25) - (1.5)(25) - (1.125)(10) - (0.75)(10) - (0.375)(10) = 0$$

$$A = 47.5 \text{ kN}$$



Use portion AC as free body

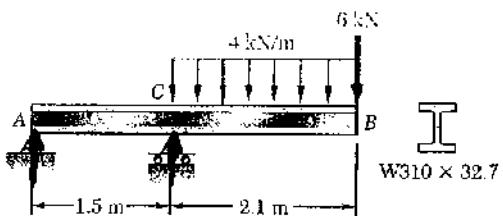
$$-(0.375)(47.5) + M = 0 \quad M = 17.8125 \text{ kN}\cdot\text{m}$$

$$\text{For } S 200 \times 27.4 \quad S = 235 \times 10^3 \text{ mm}^3 = 235 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress } \sigma = \frac{M}{S} = \frac{17.8125 \times 10^3}{235 \times 10^{-6}} = 75.8 \times 10^6 \text{ Pa} = 75.8 \text{ MPa} \blacksquare$$

Problem 5.20

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

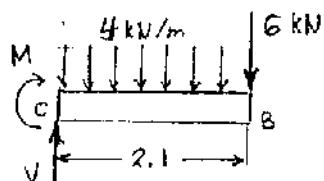


Use portion CB as free body.

$$\textcircled{D} \sum M_C = 0$$

$$-M + (4)(2.1)(1.05) + (6)(2.1) = 0$$

$$M = -21.42 \text{ kN}\cdot\text{m}$$

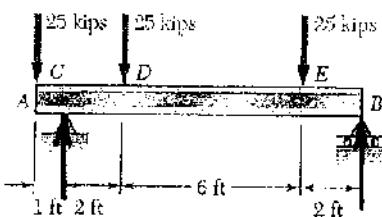


$$\text{For } W 310 \times 32.7 \quad S = 415 \times 10^3 \text{ mm}^3 = 415 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{21.42 \times 10^3}{415 \times 10^{-6}} = 51.6 \times 10^6 \text{ Pa} = 51.6 \text{ MPa} \blacksquare$$

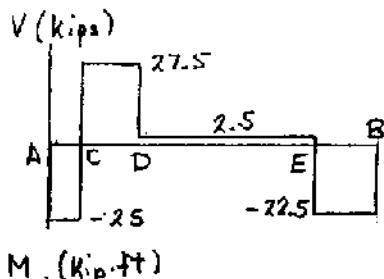
Problem 5.21

5.21 and 5.22 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



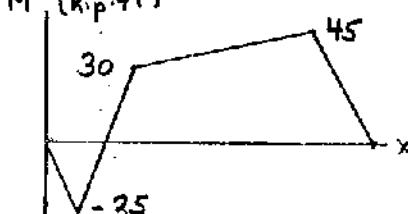
$$\textcircled{1} \sum M_B = 0 \\ (1)(25) - 10C + (8)(25) + (2)(25) = 0 \\ C = 52.5 \text{ kips}$$

$$\textcircled{2} \sum M_c = 0 \\ (1)(25) - (2)(25) - (8)(25) + 10B = 0 \\ B = 22.5 \text{ kips}$$



Shear

$$\begin{array}{ll} A \text{ to } C^- & V = -25 \text{ kips} \\ C^+ \text{ to } D^- & V = 27.5 \text{ kips} \\ D^+ \text{ to } E^- & V = 2.5 \text{ kips} \\ E^+ \text{ to } B & V = -22.5 \text{ kips} \end{array}$$



Bending moments

$$\text{At } C \quad \textcircled{1} \sum M_c = 0 \\ (1)(25) + M = 0 \\ M = -25 \text{ kip}\cdot\text{ft}$$

$$\text{At } D \quad \textcircled{2} \sum M_d = 0 \\ (3)(25) - (2)(52.5) + M = 0 \\ M = 30 \text{ kip}\cdot\text{ft}$$

At E

$$M \text{ (clockwise)} \quad \textcircled{3} \sum M_E = 0 \\ -M + (2)(22.5) = 0 \\ M = 45 \text{ kip}\cdot\text{ft}$$

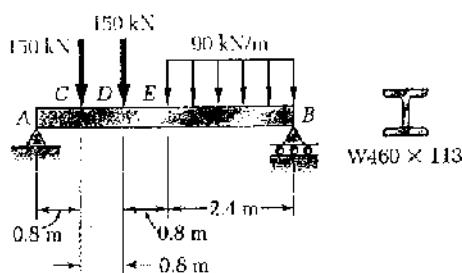
$$\max |M| = 45 \text{ kip}\cdot\text{ft} = 540 \text{ kip}\cdot\text{in}$$

$$\text{For } S12 \times 35 \text{ rolled steel section} \quad S = 38.2 \text{ in}^3$$

$$\text{Normal stress} \quad \sigma = \frac{|M|}{S} = \frac{540}{38.2} = 14.14 \text{ ksi}$$

Problem 5.22

5.21 and 5.22 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$\text{Reaction at } A \quad +\sum M_A = 0$$

$$-4.8 R_A + (4.0)(150) + (3.2)(150) + (1.2)(2.4)(90) = 0$$

$$R_A = 279 \text{ kN}$$

$$\text{Reaction at } B \quad +\sum M_B = 0$$

$$-(0.8)(150) - (1.6)(150) - (3.6)(2.4)(90) + 4.8 R_B = 0$$

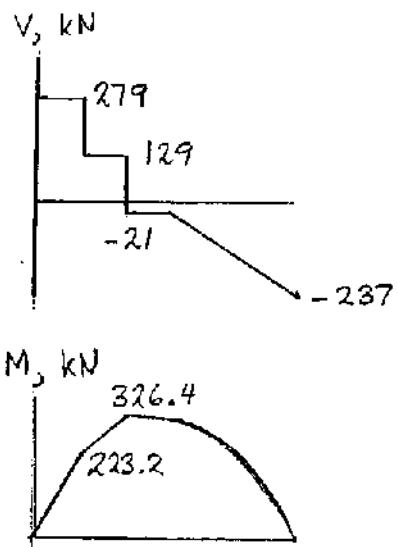
$$R_B = 237 \text{ kN}$$

$$\text{At A to C} \quad V = 279 \text{ kN}$$

$$\text{C to D} \quad V = 279 - 150 = 129 \text{ kN}$$

$$\text{D to E} \quad V = 279 - 150 - 150 = -21 \text{ kN}$$

$$\text{At B} \quad V = -237 \text{ kN}$$



From diagram

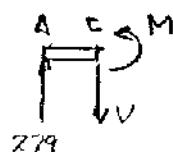
$$M_{max} = 326.4 \text{ kN}\cdot\text{m} \\ = 326.4 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{At A and B} \quad M = 0$$

$$\text{At C} \quad +\sum M_C = 0$$

$$-(0.8)(279) + M = 0$$

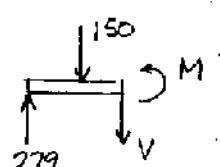
$$M = 223.2 \text{ kN}\cdot\text{m}$$



$$\text{At D} \quad +\sum M_D = 0$$

$$-(1.6)(279) + (0.8)(150) + M = 0$$

$$+ M = 0$$



$$M = 326.4 \text{ kN}\cdot\text{m}$$

$$\text{At E} \quad +\sum M_E = 0$$

$$-(2.4)(279) + (1.6)(150) + (0.8)(150) + M = 0$$

$$M = 309.6 \text{ kN}\cdot\text{m}$$

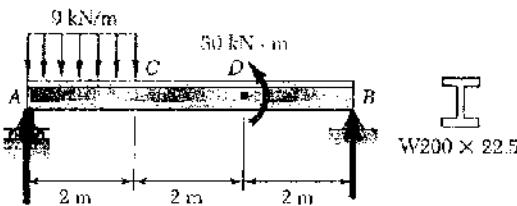
$$\text{For W 460x113 rolled steel section} \quad S = 2400 \times 10^3 \text{ mm}^3 \\ = 2400 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress} \quad \sigma_n = \frac{M_{max}}{S} = \frac{326.4 \times 10^3}{2400 \times 10^{-6}} = 136 \times 10^6 \text{ Pa}$$

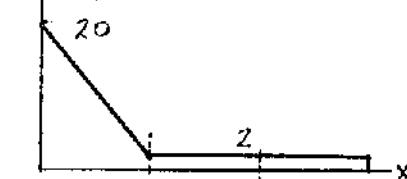
$$\sigma_n = 136.0 \text{ MPa}$$

Problem 5.23

5.23 and 5.24 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

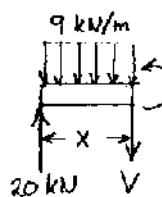


$V \text{ (kN)}$



A to C

$0 < x < 2 \text{ m}$



$$+\uparrow \sum F_y = 0$$

$$-6A + (2)(9)(1) + 30 = 0$$

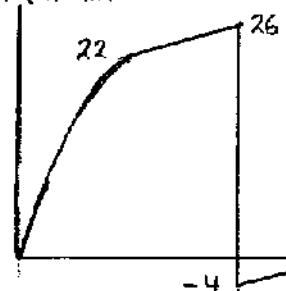
$$A = 20 \text{ kN}$$

$$+\odot \sum M_A = 0$$

$$-(2)(9)(1) + 30 + 6B = 0$$

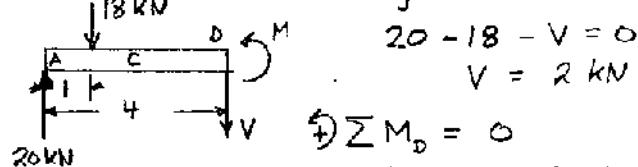
$$B = -2 \text{ kN} \text{ i.e. } 2 \text{ kN } \downarrow$$

$M \text{ (kN·m)}$



$$\text{At C } V = 2 \text{ kN} \quad M = 22 \text{ kN·m}$$

$$\text{At } D^- \quad +\uparrow \sum F_y = 0$$



$$20 - 18 - V = 0$$

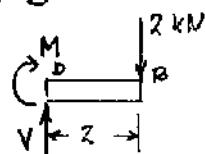
$$V = 2 \text{ kN}$$

$$+\odot \sum M_D = 0$$

$$-20x + (9x)\frac{x}{2} + M = 0$$

$$M = 20x - 4.5x^2$$

At D⁺



$$+\uparrow \sum F_y = 0 \quad V - 2 = 0$$

$$V = 2 \text{ kN}$$

$$+\odot \sum M_D = 0$$

$$-M - (2)(2) = 0$$

$$M = -4 \text{ kN·m}$$

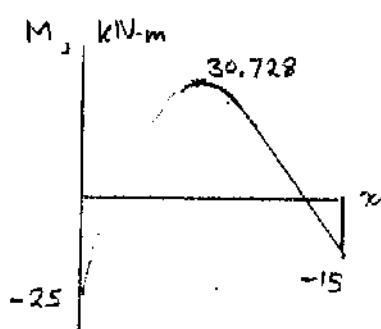
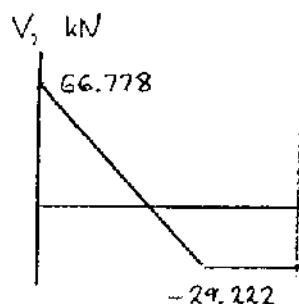
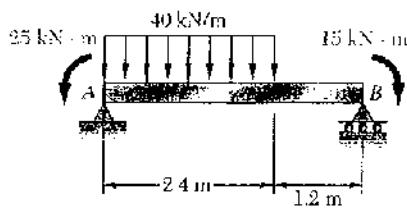
$$\max |M| = 26 \text{ kN·m} = 26 \times 10^3 \text{ N·m}$$

$$\text{For rolled steel section } W 200 \times 22.5 \quad S = 194 \times 10^3 \text{ mm}^3 \\ = 194 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{26 \times 10^3}{194 \times 10^{-6}} = 134.0 \times 10^6 \text{ Pa} = 134.0 \text{ MPa} \rightarrow$$

Problem 5.24

5.23 and 5.24 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$\text{Reaction at } A \quad \sum M_A = 0 \\ -3.6 R_A + 2(2.4)(2.4)(40) - 15 = 0$$

$$R_A = 66.778 \text{ kN}$$

$$\text{Reaction at } B \quad \sum M_B = 0 \\ 25 - (1.2)(2.4)(40) - 15 + 3.6 R_B = 0$$

$$R_B = 29.222 \text{ kN}$$

$$0 < x < 2.4 \text{ m}$$

$$\sum F_y = 0$$

$$66.778 - 40x - V = 0$$

$$V = 66.778 - 40x$$

$$V = 0 \text{ at } x = 1.6944 \text{ m}$$

$$\sum M = 0$$

$$25 + (40x)\frac{x}{2} - 66.778x + M = 0$$

$$M = -20x^2 + 66.778x - 25$$

$$M = -(20)(1.6944)^2 + (66.778)(1.6944) - 25 \\ = 30.728 \text{ kN}\cdot\text{m} \text{ at } x = 1.6944 \text{ m}$$

$$2.4 \leq x < 3.6 \quad V = -29.222 \text{ kN}$$

$$M = 29.222(3.6 - x) - 15$$

$$M = 20.067 \text{ kN}\cdot\text{m} \text{ at } x = 2.4 \text{ m}$$

$$M_{max} = 30.728 \text{ kN}\cdot\text{m} = 30.728 \times 10^3 \text{ N}\cdot\text{m}$$

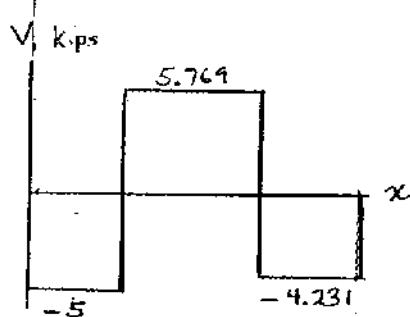
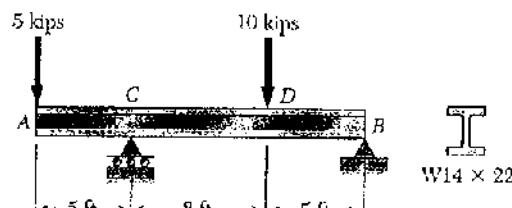
$$\text{For W310 x 38.7 rolled steel section } S = 549 \times 10^3 \text{ mm}^3 \\ = 549 \times 10^{-6} \text{ m}^3$$

$$\sigma_m = \frac{M_{max}}{S} = \frac{30.728 \times 10^3}{549 \times 10^{-6}} = 55.97 \times 10^6 \text{ Pa}$$

$$56.0 \text{ MPa} \blacktriangleleft$$

Problem 5.25

5.25 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



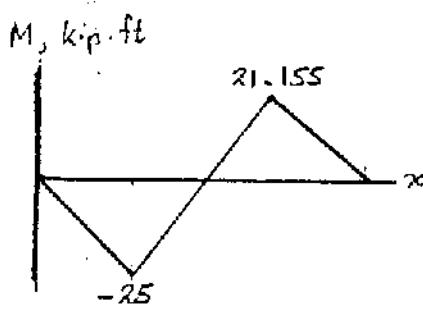
$$\text{Reaction at } C \quad \sum M_B = 0 \\ (18)(5) - 13C + (5)(10) = 0 \\ C = 10.769 \text{ kips}$$

$$\text{Reaction at } B \quad \sum M_C = 0 \\ (5)(5) - (8)(10) + 13B = 0 \\ B = 4.231 \text{ kips}$$

Shear diagram.

$$\begin{array}{ll} A \rightarrow C^- & V = -5 \text{ kips} \\ C^+ \rightarrow D^- & V = -5 + 10.769 = 5.769 \text{ kips} \\ D^+ \rightarrow B & V = 5.769 - 10 = -4.231 \text{ kips} \end{array}$$

$$\text{At } A \text{ and } B \quad M = 0$$



$$\text{At } C \quad \sum M_c = 0 \\ (5)(5) + M_c = 0 \\ M_c = -25 \text{ kip}\cdot\text{ft}$$

$$\text{At } D \quad \sum M_D = 0 \\ -M_D + (5)(4.231) \\ M_D = 21.155 \text{ kip}\cdot\text{ft}$$

$$|M|_{\max} \text{ occurs at } C \quad |M|_{\max} = 25 \text{ kip}\cdot\text{ft} = 300 \text{ kip}\cdot\text{in.}$$

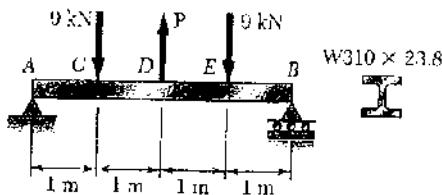
$$\text{For W14 x 22 rolled steel section} \quad S = 29.0 \text{ in}^3$$

$$\text{Normal stress} \quad \sigma = \frac{M}{S} = \frac{300}{29.0} = 10.34 \text{ ksi}$$

Problem 5.26

5.26 Knowing that $P = 10 \text{ kN}$, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.

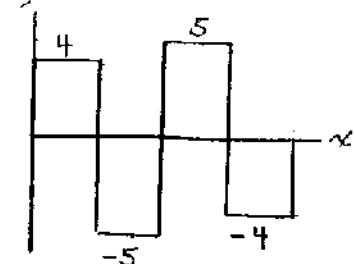
By symmetry the reactions at A and B are equal.



$$A = B$$

$$+\uparrow \sum F_y = 0 : A + B - 9 + 10 - 9 = 0 \\ A = B = 4 \text{ kN}$$

Shear diagram.



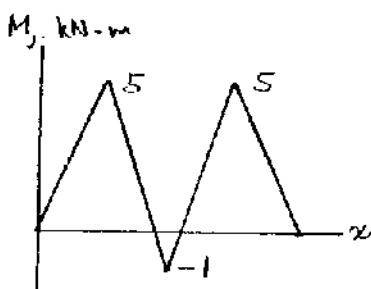
$$A \text{ to } C \quad V = 4 \text{ kN}$$

$$C \text{ to } D \quad V = 4 - 9 = -5 \text{ kN}$$

$$D \text{ to } E \quad V = -5 + 10 = 5 \text{ kN}$$

$$E \text{ to } B \quad V = 5 - 9 = -4 \text{ kN}$$

Bending moments. $M_A = M_B = 0$



$$+\circlearrowleft \sum M_c = 0 \\ -(1)(4) + M_c = 0 \\ M_c = 4 \text{ kN}\cdot\text{m}$$

$$+\circlearrowleft \sum M_D = 0 \\ -(2)(4) + (1)(9) + M_b = 0 \\ M_b = -1 \text{ kN}\cdot\text{m}$$

By symmetry $M_E = M_C = 4 \text{ kN}\cdot\text{m}$

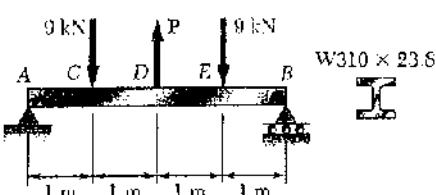
$$|M|_{\max} = M_c = 4 \text{ kN}\cdot\text{m} = 4 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{For W310x23.8} \quad S = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress} \quad \sigma = \frac{M}{S} = \frac{4 \times 10^3}{280 \times 10^{-6}} = 14.29 \times 10^6 \text{ Pa} \quad 14.29 \text{ MPa} \quad \blacksquare$$

Problem 5.27

5.27 Determine (a) the magnitude of the upward force P for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (Hint: Draw the bending-moment diagram and the equate the absolute values of the largest positive and negative bending moments obtained.)



By symmetry the reactions at A and B are equal.

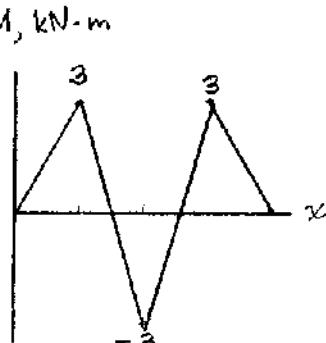
$$A = B$$

$$+\uparrow \sum F_y = 0: A - 9 + P - 9 + B = 0$$

$$A = B = 9 - \frac{1}{2}P \quad P = 18 - 2A$$

Also, by symmetry bending moments $M_C = M_E$.

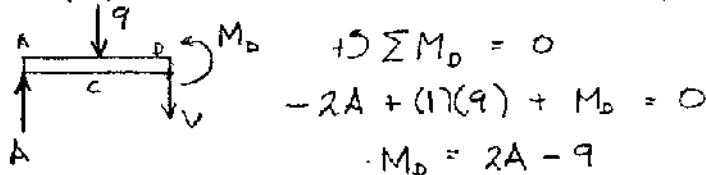
Using portion AC as a free body



$$+\circlearrowleft \sum M_c = 0$$

$$-1A + M_c = 0 \quad M_c = 1A$$

Using portion ACD as a free body



$$+\circlearrowleft \sum M_D = 0$$

$$-2A + (1)(9) + M_D = 0$$

$$M_D = 2A - 9$$

$$\text{Equate } M_c = -M_D$$

$$1A = 9 - 2A \quad A = 3 \text{ kN}$$

$$\text{Then } P = 18 - (2)(3) = 12 \text{ kN} \quad (\text{a}) 12.00 \text{ kN} \rightarrow$$

$$M_c = 3 \text{ kN}\cdot\text{m}$$

$$M_D = (2)(3) - 9 = -3 \text{ kN}\cdot\text{m}$$

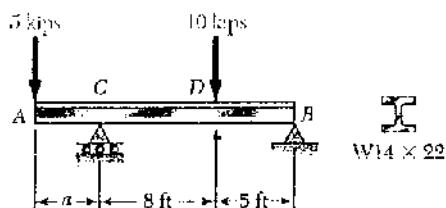
$$|M|_{\max} = 3 \text{ kN}\cdot\text{m} = 3 \times 10^3 \text{ N}\cdot\text{m}$$

For W310 x 23.4 rolled steel section $S = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{ m}^3$

$$\text{Normal stress } \sigma = \frac{M}{S} = \frac{3 \times 10^3}{280 \times 10^{-6}} = 10.71 \times 10^6 \text{ Pa} \quad (\text{b}) 10.71 \text{ MPa} \rightarrow$$

Problem 5.28

5.28 Determine (a) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27) (Hint: Draw the bending-moment diagram and the equate the absolute values of the largest positive and negative bending moments obtained.)



$$\text{Reaction at } B. \quad +\sum M_c = 0$$

$$5a - (8)(10) + 13R_B = 0$$

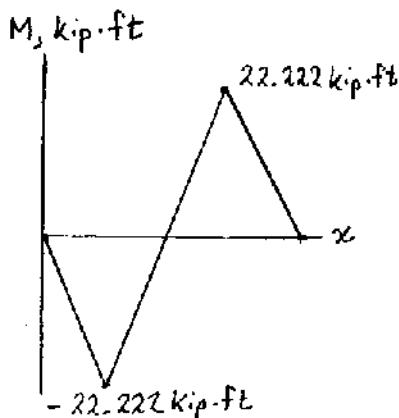
$$R_B = \frac{1}{13}(80 - 5a)$$

Bending moment at D.

$$+\sum M_D = 0$$

$$-M_D + 5R_B = 0$$

$$M_D = 5R_B = \frac{5}{13}(80 - 5a)$$



Bending moment at C

$$+\sum M_c = 0$$

$$5a + M_c = 0$$

$$M_c = -5a$$

$$\text{Equate } -M_c = M_D$$

$$5a = \frac{5}{13}(80 - 5a)$$

$$a = 4.4444 \text{ ft} \quad (a) \quad a = 4.44 \text{ ft} \quad \leftarrow$$

$$\text{Then } -M_c = M_D = (5)(4.4444) = 22.222 \text{ kip ft}$$

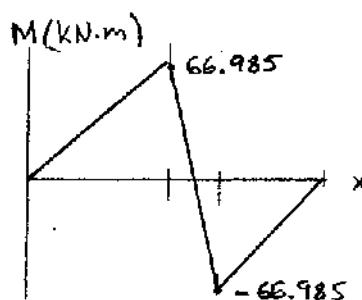
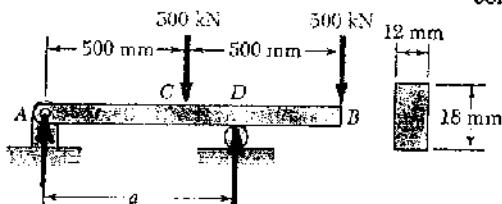
$$|M|_{\max} = 22.222 \text{ kip ft} = 266.67 \text{ kip in}$$

For W14 x 22 rolled steel section $S = 29.0 \text{ in}^3$

$$\text{Normal stress } \sigma = \frac{M}{S} = \frac{266.67}{29.0} = 9.20 \text{ ksi} \quad (b) \quad 9.20 \text{ ksi} \quad \leftarrow$$

Problem 5.29

5.29 For the beam and loading shown, determine (a) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27)



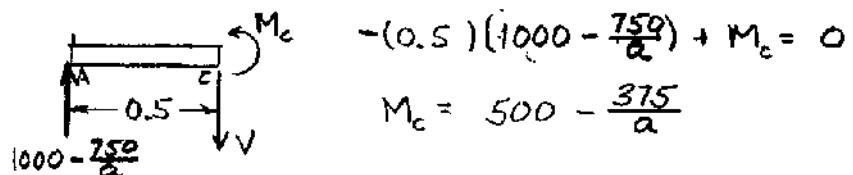
$$\text{Reaction at A} \quad \rightarrow \sum M_A = 0$$

$$-Aa + (500)(a - 0.5) - 500(1 - a) = 0$$

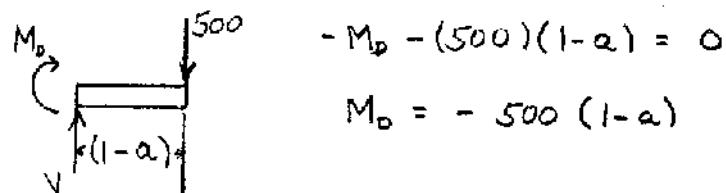
$$Aa = 1000a - 750$$

$$A = 1000 - \frac{750}{a}$$

$$\text{Bending moment at C} \quad \rightarrow \sum M_C = 0$$



$$\text{Bending moment at D} \quad \rightarrow \sum M_D = 0$$



$$(a) \text{ Equate } -M_D = M_C \quad 500(1-a) = 500 - \frac{375}{a}$$

$$a = 0.86603 \text{ m} \quad a = 866 \text{ mm} \quad \blacksquare$$

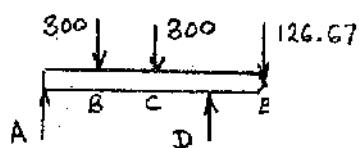
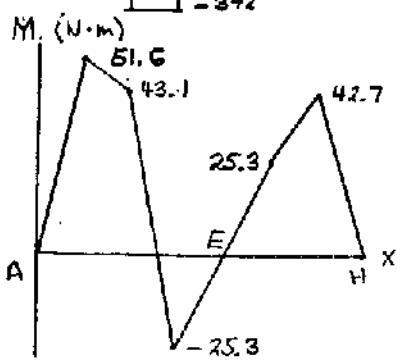
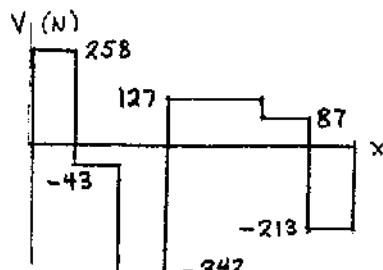
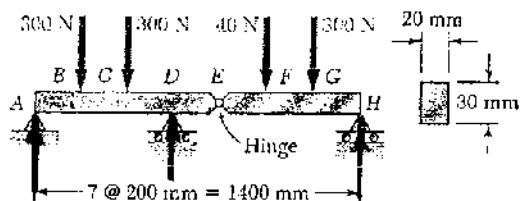
$$A = 133.98 \text{ kN} \quad M_c = 66.985 \text{ kN}\cdot\text{m} \quad M_D = -66.985 \text{ kN}\cdot\text{m}$$

$$\text{For rectangular cross section} \quad S = \frac{1}{6}bh^3 = \frac{1}{6}(12)(18)^3 = 11.664 \times 10^3 \text{ mm}^3 \\ = 11.664 \times 10^{-6} \text{ m}^3$$

$$(b) \text{ Normal stress} \quad \sigma = \frac{|M|}{S} = \frac{66.985 \times 10^3}{11.664 \times 10^{-6}} = 5.74 \times 10^6 \text{ Pa} \\ = 5.74 \text{ MPa} \quad \blacksquare$$

Problem 5.30

5.30 and 5.31 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

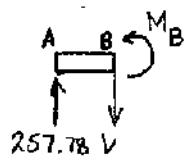


$$+\sum M_B = 0 \quad -0.6A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$$

$$A = 257.78 \text{ N}$$

$$+\sum M_A = 0 \quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6D = 0$$

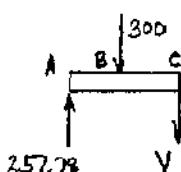
$$D = 468.89 \text{ N}$$



Bending moment at B

$$\sum M_B = 0 \quad -(0.2)(257.78) + M_B = 0$$

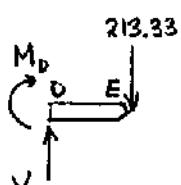
$$M_B = 51.56 \text{ N·m}$$



Bending moment at C

$$+\sum M_C = 0 \quad -(0.4)(257.78) + (0.2)(300) + M_C = 0$$

$$M_C = 43.11 \text{ N·m}$$



Bending moment at D

$$+\sum M_D = 0 \quad -M_D - (0.2)(213.33) = 0$$

$$M_D = -25.33 \text{ N·m}$$

Free body EFGH

Note that $M_E = 0$ due to hinge.

$$\sum M_E = 0$$

$$0.6H - (0.2)(40) - (0.4)(300) = 0$$

$$H = 213.33 \text{ N}$$

$$\sum F_y = 0 \quad V_E - 40 - 300 + 213.33 = 0$$

$$V_E = 126.67 \text{ N}$$

Shear: E to F $V = 126.67 \text{ N·m}$

F to G $V = 86.67 \text{ N·m}$

G to H $V = -213.33 \text{ N·m}$

Bending moment at F

$$\sum M_F = 0$$

$$M_F - (0.2)(126.67) = 0$$

$$M_F = 25.33 \text{ N·m}$$

Bending moment at G

$$\sum M_G = 0$$

$$-M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N·m}$$

Free body ABCDE

$$+\sum M_B = 0 \quad -0.6A + (0.4)(300) + (0.2)(300)$$

$$- (0.2)(126.67) = 0$$

$$A = 257.78 \text{ N}$$

$$+\sum M_A = 0 \quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6D = 0$$

$$+ 0.6D = 0$$

$$D = 468.89 \text{ N}$$

$$\max |M| = 51.56 \text{ N·m}$$

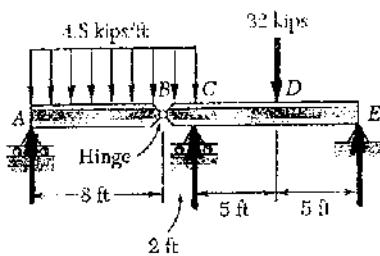
$$S = \frac{1}{6} b h^2 = \frac{1}{6} (20)(30)^2 = 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$$

Normal stress

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa} = 17.19 \text{ MPa}$$

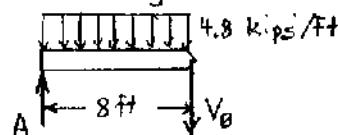
Problem 5.31

5.30 and 5.31 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

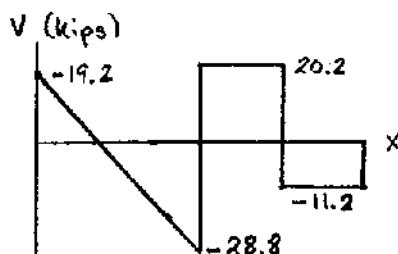


Free body
W12 x 40

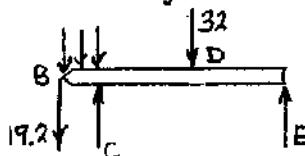
Free body AB



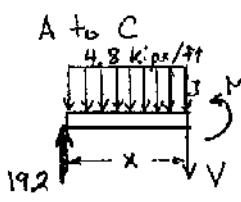
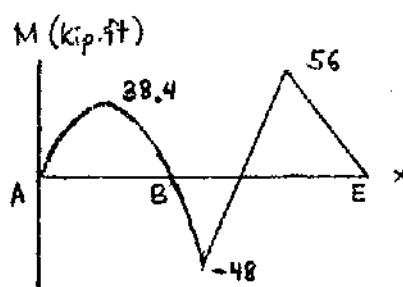
$$\begin{aligned} \textcircled{D} \sum M_A &= 0 \\ (4.8)(8)(4) - 8A &= 0 \\ A &= 19.2 \text{ kips} \\ \textcircled{D} \sum M_A &= 0 \\ -(4.8)(8)(4) - 8V_B &= 0 \\ V_B &= -19.2 \text{ kips} \end{aligned}$$



Free body BCDE



$$\begin{aligned} \textcircled{D} \sum M_E &= 0 \\ (19.2)(12) + (4.8)(2)(11) &= 10C + (32)(5) = 0 \\ C &= 49.2 \text{ kips} \\ \textcircled{D} \sum M_C &= 0 \\ (19.2)(2) + (4.8)(2)(1) &= -(32)(5) + 10E = 0 \\ E &= 11.2 \text{ kips} \end{aligned}$$



$$\begin{aligned} 0 < x < 10 \text{ ft.} \\ \textcircled{+} \sum F_y &= 0 \\ 19.2 - 4.8x - V &= 0 \\ V &= 19.2 - 4.8x \text{ kips.} \\ \textcircled{D} \sum M_J &= 0 \\ -19.2x + (4.8x)\frac{x}{2} + M &= 0 \\ M &= 19.2x - 2.4x^2 \text{ kip-ft} \end{aligned}$$

At C $x = 10$

$V = 19.2 - (4.8)(10) = -28.8 \text{ kips}$

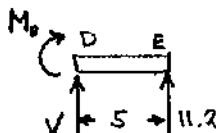
At C $x = 10$

$M_C = (19.2)(10) - (2.4)(10)^2 = -48 \text{ kip-ft}$

C to D $V = 19.2 - (4.8)(10) + 49.2 = 20.8 \text{ kips.}$

D to E $V = -11.2 \text{ kips}$

Bending moment at D



$$\begin{aligned} \textcircled{D} \sum M_D &= 0 \\ -M_D + (11.2)(5) &= 0 \\ M_D &= 56 \text{ kip-ft} \end{aligned}$$

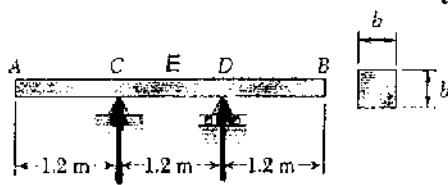
max |M| = 56 kip-ft = 672 kip-in

For W12 x 40 rolled steel section $S = 51.9 \text{ in}^3$

Normal stress $\sigma = \frac{|M|}{S} = \frac{672}{51.9} = 12.95 \text{ ksi}$

Problem 5.32

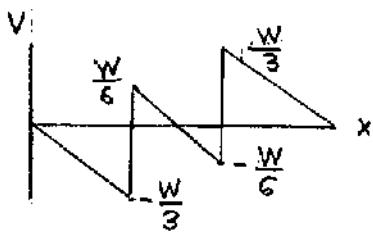
5.32 A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860 \text{ kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (a) 10 MPa , (b) 50 MPa .



$$\text{Weight density } \gamma = \rho g$$

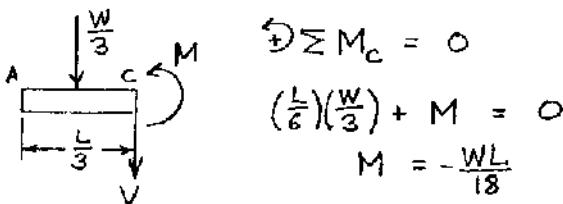
Let $L = \text{total length of beam}$

$$W = \gamma L = AL\gamma = b^2 L \rho g$$



$$\text{Reactions at } C \text{ and } D \quad C = D = \frac{W}{2}$$

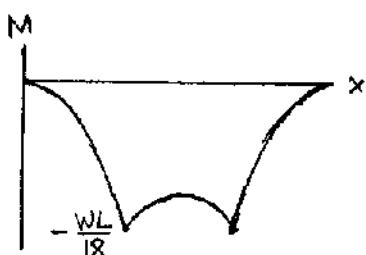
Bending moment at C



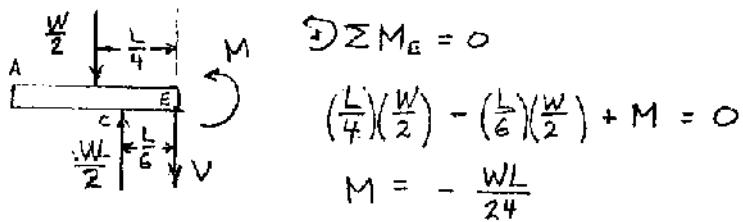
$$\sum M_C = 0$$

$$(\frac{L}{6})(\frac{W}{3}) + M = 0$$

$$M = -\frac{WL}{18}$$



Bending moment at center of beam



$$\sum M_E = 0$$

$$(\frac{L}{4})(\frac{W}{2}) - (\frac{L}{6})(\frac{W}{2}) + M = 0$$

$$M = -\frac{WL}{24}$$

$$\max |M| = \frac{WL}{18} = \frac{b^2 L^2 \rho g}{18}$$

$$\text{For a square section } S = \frac{1}{6} b^3$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{b^2 L^2 \rho g / 18}{b^3 / 6} = \frac{L^2 \rho g}{3b}$$

$$\text{Solve for } b \quad b = \frac{L^2 \rho g}{36}$$

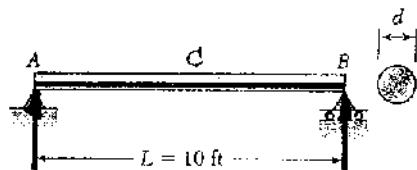
$$\text{Data: } L = 3.6 \text{ m} \quad \rho = 7860 \text{ kg/m}^3 \quad g = 9.81 \text{ m/s}^2$$

$$(a) \sigma = 10 \times 10^6 \text{ Pa} \quad (b) \sigma = 50 \times 10^6 \text{ Pa}$$

$$(a) b = \frac{(3.6)^2 (7860) (9.81)}{(3)(10 \times 10^6)} = 33.3 \times 10^{-3} \text{ m} = 33.3 \text{ mm}$$

$$(b) b = \frac{(3.6)^2 (7860) (9.81)}{(3)(50 \times 10^6)} = 6.66 \times 10^{-3} \text{ m} = 6.66 \text{ mm}$$

Problem 5.33



5.33 A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490 \text{ lb/ft}^3$, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 4 ksi.

Let $W = \text{total weight}$

$$W = g\gamma r = AL\gamma = \frac{\pi}{4}d^2L\gamma$$

Reaction at A

$$A = \frac{1}{2}W$$

Bending moment at center of beam

$$\begin{array}{l} \text{A} \downarrow \frac{W}{2} \text{ C} \curvearrowleft M \\ \text{---} \quad \text{---} \\ \text{V} \quad \frac{W}{2} \end{array} \quad \sum M_c = 0 \\ -\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0 \\ M = \frac{WL}{8} = \frac{\pi}{32}d^2L^2\gamma$$

For circular cross section ($c = \frac{1}{4}d$)

$$I = \frac{\pi}{4}c^4, \quad S = \frac{I}{c} = \frac{\pi}{4}c^3 = \frac{\pi}{32}d^3$$

Normal stress

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32}d^2L^2\gamma}{\frac{\pi}{32}d^3} = \frac{L^2\gamma}{d}$$

Solving for d $d = \frac{L^2\gamma}{\sigma}$

Data: $L = 10 \text{ ft} = (12)(10) = 120 \text{ in}$

$$\gamma = 490 \text{ lb/ft}^3 = \frac{490}{12^3} = 0.28356 \text{ lb/in}^3$$

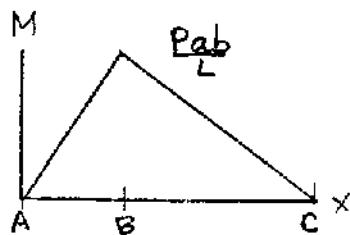
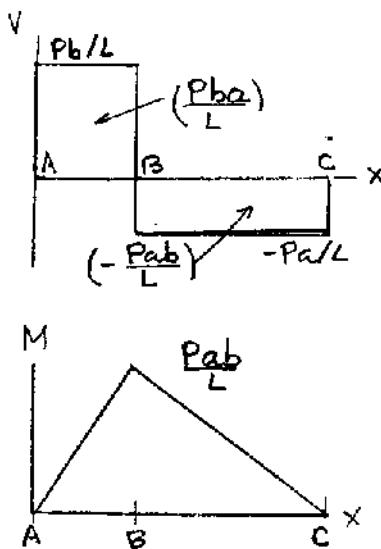
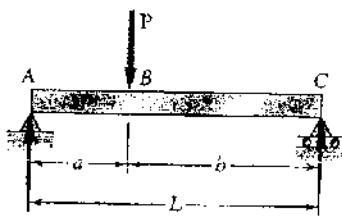
$$\sigma = 4 \text{ ksi} = 4000 \text{ lb/in}^2$$

$$d = \frac{(120)^2(0.28356)}{4000} = 1.021 \text{ in.}$$

Problem 5.34

5.34 Using the method of Sec. 5.3, solve Prob. 5.1a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$\text{At } \sum M_c = 0 \quad LA - bP = 0 \quad A = \frac{Pb}{L}$$

$$\text{At } \sum M_A = 0 \quad LC - aP = 0 \quad C = \frac{Pa}{L}$$

$$\text{At } A^+ \quad V = A = \frac{Pb}{L} \quad M = 0$$

$$\text{At } A \text{ to } B^- \quad 0 < x < a$$

$$w = 0 \quad \int_a^x w dx = 0$$

$$V_B - V_A = 0 \quad V_B = \frac{Pb}{L}$$

$$M_B - M_A = \int_a^x V dx = \int_a^x \frac{Pb}{L} dx = \frac{Pba}{L}$$

$$M_B = \frac{Pba}{L}$$

$$\text{At } B^+ \quad V = A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$$

$$B^+ \text{ to } C \quad a < x < L$$

$$w = 0 \quad \int_a^x w dx = 0$$

$$V_C - V_B = 0 \quad V = -\frac{Pa}{L}$$

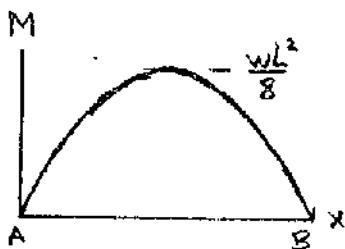
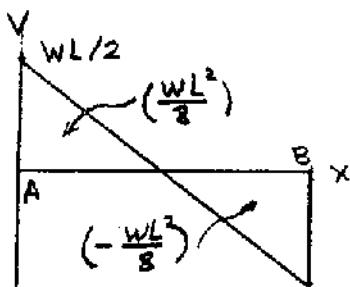
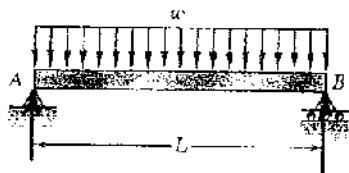
$$M_C - M_B = \int_a^L V dx = -\frac{Pa}{L}(L-a) = \frac{Pab}{L}$$

$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

Problem 5.35

5.35 Using the method of Sec. 5.3, solve Prob. 5.2a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$\text{④ } \sum M_B = 0 \quad -AL + WL \cdot \frac{L}{2} = 0 \quad A = \frac{WL}{2}$$

$$\text{⑤ } \sum M_A = 0 \quad BL - WL \cdot \frac{L}{2} = 0 \quad B = \frac{WL}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = - \int_0^x w dx = -wx$$

$$V = V_A - wx = A - wx = \frac{WL}{2} - wx$$

$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V dx = \int_0^x \left(\frac{WL}{2} - wx \right) dx \\ = \frac{WLx}{2} - \frac{wx^2}{2}$$

$$M = M_A + \frac{WLx}{2} - \frac{wx^2}{2} = \frac{WL}{2}(Lx - x^2)$$

Maximum M occurs at $x = \frac{L}{2}$ where

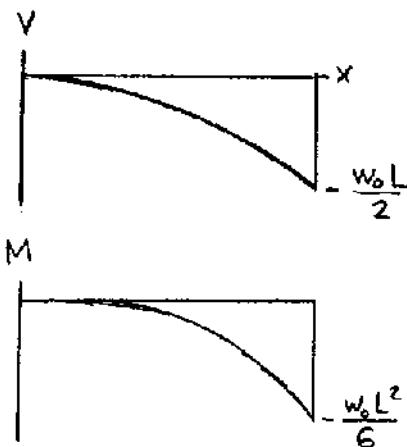
$$V = \frac{dM}{dx} = 0$$

$$\text{Max } M = \frac{WL^2}{8}$$

Problem 5.36

5.36 Using the method of Sec. 5.3, solve Prob. 5.3a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown



$$W = W_0 \frac{x}{L}$$

$$V_A = 0, \quad M_A = 0$$

$$\frac{dV}{dx} = -W = -\frac{W_0 x}{L}$$

$$V - V_A = - \int_0^x \frac{W_0 x}{L} dx = -\frac{W_0 x^2}{2L}$$

$$V = -\frac{W_0 x^2}{2L}$$

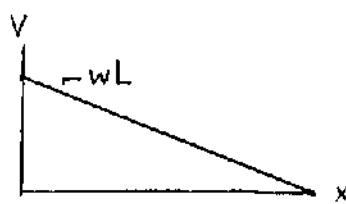
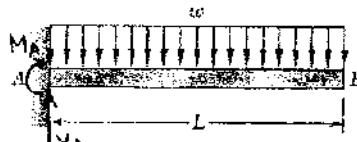
$$\frac{dM}{dx} = V = -\frac{W_0 x^2}{2L}$$

$$M - M_A = \int_0^x V dx = - \int_0^x \frac{W_0 x^2}{2L} dx \\ = -\frac{W_0 x^3}{6L}$$

Problem 5.37

5.37 Using the method of Sec. 5.3, solve Prob. 5.4a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$\text{At } \sum F_y = 0, \quad V_A - wL = 0 \quad V_A = wL$$

$$\text{At } \sum M_A = 0, \quad -M - (wL)(\frac{L}{2}) = 0 \quad M_A = -\frac{wL^2}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = - \int_0^x w dx = -wx$$

$$V = wL - wx$$

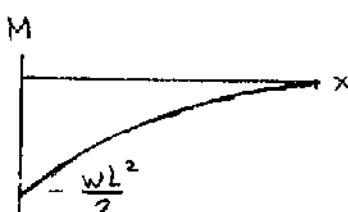
$$\frac{dM}{dx} = V = wL - wx$$

$$M - M_A = \int_0^x (wL - wx) dx = wLx - \frac{wx^2}{2}$$

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2}$$

$$\max |V| = wL$$

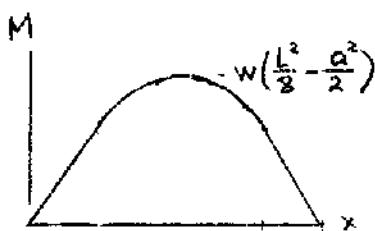
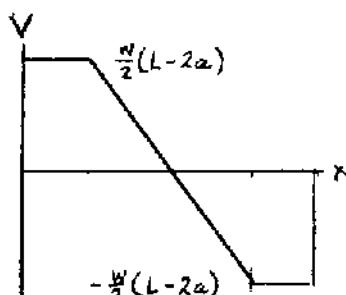
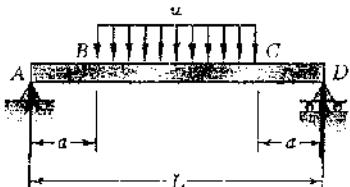
$$\max |M| = \frac{wL^2}{2}$$



Problem 5.38

5.38 Using the method of Sec. 5.3, solve Prob. 5.5a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$\text{Reactions } A = D = \frac{1}{2}w(L-2a)$$

$$\text{At } A \quad V_A = A = \frac{1}{2}w(L-2a), \quad M_A = 0$$

$$A \text{ to } B \quad 0 < x < a \quad w = 0$$

$$V_B - V_A = - \int_a^x w dx = 0$$

$$V_B = V_A = \frac{1}{2}w(L-2a)$$

$$M_B - M_A = \int_a^x V dx = \int_a^x \frac{1}{2}w(L-2a) dx$$

$$M_B = \frac{1}{2}w(L-2a)a$$

$$B \text{ to } C \quad a < x < L-a \quad w = w$$

$$V - V_B = - \int_a^x w dx = -w(x-a)$$

$$V = \frac{1}{2}w(L-2a) - w(x-a) = \frac{1}{2}w(L-2x)$$

$$\frac{dM}{dx} = V = \frac{1}{2}w(L-2x)$$

$$M - M_B = \int_a^x V dx = \frac{1}{2}w(Lx - x^2) \Big|_a^x$$

$$\therefore M = \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$\begin{aligned} M &= \frac{1}{2}w(L-2a)a + \frac{1}{2}w(Lx - x^2 - La + a^2) \\ &= \frac{1}{2}w(Lx - x^2 - a^2) \end{aligned}$$

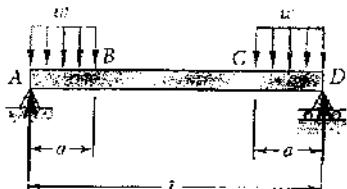
$$\text{At } C \quad x = L-a \quad V_C = -\frac{1}{2}w(L-2a) \quad M_C = \frac{1}{2}(L-2a)a$$

$$\text{C to D} \quad V = V_C = -\frac{1}{2}w(L-2a)$$

$$M_D = 0$$

$$\text{At } x = \frac{L}{2} \quad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$$

Problem 5.39



5.39 Using the method of Sec. 5.3, solve Prob. 5.6a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

$$\text{Reactions} \quad A = D = wa$$

$$A \text{ to } B \quad 0 < x < a \quad w = w$$

$$V_A = A = wa, \quad M_A = 0$$

$$V - V_A = - \int_0^x w dx = -wx$$

$$V = w(a-x) \quad V_B = 0$$

$$\frac{dM}{dx} = V = wa - wx$$

$$M - M_A = \int_0^x V dx = \int_0^x (wa - wx) dx \\ = wax - \frac{1}{2}wx^2$$

$$M_B = \frac{1}{2}wa^2 \quad \text{at } x=a.$$

$$B \text{ to } C \quad a < x < L-a \quad V = 0$$

$$\frac{dM}{dx} = V = 0$$

$$M - M_B = \int_a^{L-a} V dx = 0$$

$$M = M_B = \frac{1}{2}wa^2$$

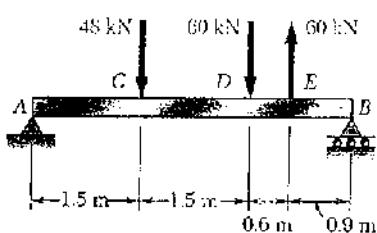
$$C \text{ to } D \quad V - V_C = - \int_{L-a}^x w dx = -w[x - (L-a)]$$

$$V = -w[x - (L-a)]$$

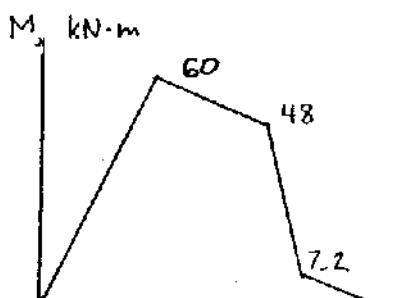
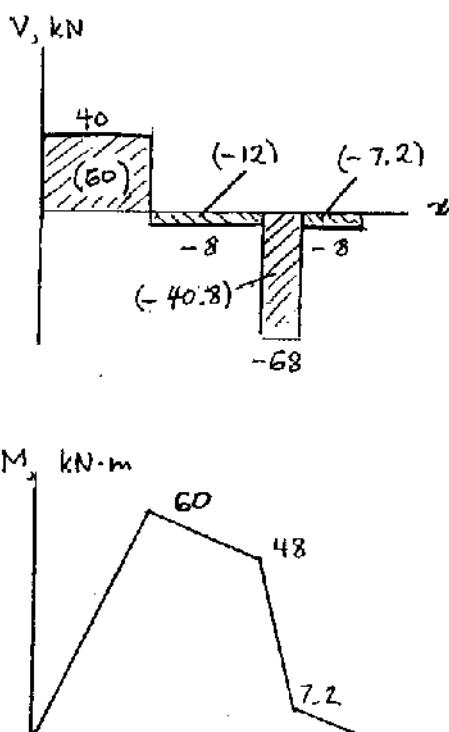
$$M - M_C = \int_{L-a}^x V dx = \int_{L-a}^x -w[x - (L-a)] dx \\ = -w\left[\frac{x^2}{2} - (L-a)x\right] \Big|_{L-a}^x \\ = -w\left[\frac{x^2}{2} - (L-a)x - \frac{(L-a)^2}{2} + (L-a)^2\right] \\ = -w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right] \\ M = \frac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$$

Problem 5.40

5.40 Using the method of Sec. 5.3, solve Prob. 5.7.



5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Reactions.

$$\rightarrow \sum M_B = 0$$

$$-1.5 R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

$$R_A = 40 \text{ kN}$$

Shear

$$A \text{ to } C^- \quad V = 40 \text{ kN}$$

$$C^+ \text{ to } D^- \quad V = 40 - 48 = -8 \text{ kN}$$

$$D^+ \text{ to } E^- \quad V = -8 - 60 = -68 \text{ kN}$$

$$E^+ \text{ to } B \quad V = -68 + 60 = -8 \text{ kN}$$

Areas of Shear Diagram.

$$A \text{ to } C \quad A_{AC} = (1.5)(40) = 60 \text{ kN}\cdot\text{m}$$

$$C \text{ to } D \quad A_{CD} = (1.5)(-8) = -12 \text{ kN}\cdot\text{m}$$

$$D \text{ to } E \quad A_{DE} = (0.6)(-68) = -40.8 \text{ kN}\cdot\text{m}$$

$$E \text{ to } B \quad A_{EB} = (0.9)(-8) = -7.2 \text{ kN}\cdot\text{m}$$

Bending moments.

$$M_A = 0$$

$$M_C = 0 + 60 = 60 \text{ kN}\cdot\text{m}$$

$$M_D = 60 - 12 = 48 \text{ kN}\cdot\text{m}$$

$$M_E = 48 - 40.8 = 7.2 \text{ kN}\cdot\text{m}$$

$$M_B = 7.2 - 7.2 = 0$$

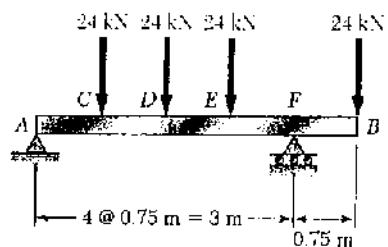
From the diagrams (a) $|V|_{max} = 68.0 \text{ kN}\cdot\text{m}$

(b) $|M|_{max} = 60.0 \text{ kN}\cdot\text{m}$

Problem 5.41

5.41 Using the method of Sec. 5.3, solve Prob. 5.8.

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

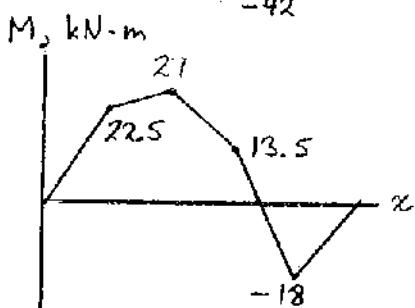
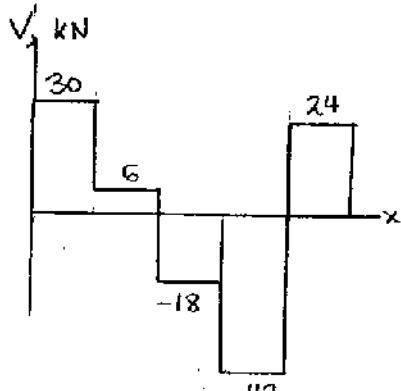


Reactions at A and F.

$$\rightarrow \sum M_F = 0$$

$$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24) - (0.75)(24) = 0$$

$$R_A = 30 \text{ kN} \uparrow$$



Shear diagram.

$$A \text{ to } C \quad V = 30 \text{ kN}$$

$$C \text{ to } D \quad V = 30 - 24 = 6 \text{ kN}$$

$$D \text{ to } E \quad V = 6 - 24 = -18 \text{ kN}$$

$$E \text{ to } F \quad V = -18 - 24 = -42 \text{ kN}$$

$$F \text{ to } B \quad V = -42 + 66 = 24 \text{ kN}$$

Areas of shear diagram

$$A_{AC} = (0.75)(30) = 22.5 \text{ kN}\cdot\text{m}$$

$$A_{CD} = (0.75)(6) = 4.5 \text{ kN}\cdot\text{m}$$

$$A_{DE} = (0.75)(-18) = -13.5 \text{ kN}\cdot\text{m}$$

$$A_{EF} = (0.75)(-42) = -31.5 \text{ kN}\cdot\text{m}$$

$$A_{FB} = (0.75)(24) = 18 \text{ kN}\cdot\text{m}$$

Bending moments.

$$M_A = 0$$

$$M_C = 0 + 22.5 = 22.5 \text{ kN}\cdot\text{m}$$

$$M_D = 22.5 + 4.5 = 27 \text{ kN}\cdot\text{m}$$

$$M_E = 27 - 13.5 = 13.5 \text{ kN}\cdot\text{m}$$

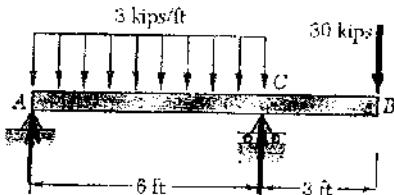
$$M_F = -13.5 - 31.5 = -45 \text{ kN}\cdot\text{m}$$

$$M_B = -45 + 18 = 0$$

$$(a) |V|_{max} = 42.0 \text{ kN}$$

$$(b) |M|_{max} = 27.0 \text{ kN}\cdot\text{m}$$

Problem 5.42



5.42 Using the method of Sec. 5.3, solve Prob. 5.9.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment

$$\rightarrow \sum M_C = 0 \quad -6A + (3)(18) - (3)(30) = 0$$

$$A = -6 \text{ kips} \quad \text{i.e. } 6 \text{ Kips } \downarrow$$

$$\rightarrow \sum M_A = 0 \quad 6C - (3)(18) - (9)(30) = 0$$

$$C = 54 \text{ kips } \uparrow$$

Shear

$$V_A = -6 \text{ kips}$$

$$\text{A to C} \quad 0 < x < 6 \text{ ft.} \quad w = -3 \text{ kips/ft}$$

$$V_B - V_A = - \int_0^6 w dx = - \int_0^6 3 dx = -18 \text{ kips}$$

$$V_B = -6 - 18 = -24 \text{ kips}$$

$$\text{C to B} \quad V = -24 + 54 = 30 \text{ kips.}$$

Areas under shear diagram

$$\text{A to C} \quad \int V dx = \left(\frac{1}{2} \right) (-6 - 24)(6) \\ = -90 \text{ kip ft.}$$

$$\text{C to B} \quad \int V dx = (3)(30) = 90 \text{ kip ft.}$$

Bending moments

$$M_A = 0$$

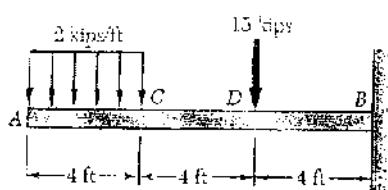
$$M_C = M_A + \int V dx = 0 - 90 = -90 \text{ kip ft.}$$

$$M_B = M_C + \int V dx = -90 + 90 = 0$$

$$\text{Maximum } |V| = 30 \text{ kips}$$

$$\text{Maximum } |M| = 90 \text{ kip ft.}$$

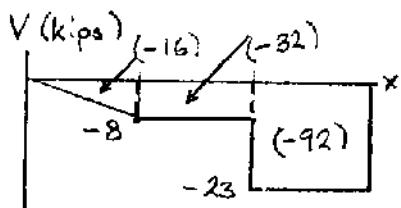
Problem 5.43



5.43 Using the method of Sec. 5.3, solve Prob. 5.10.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

Shear

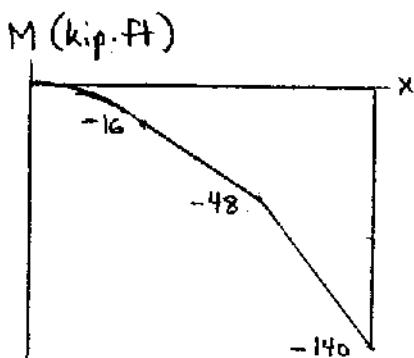


$$V_A = 0$$

$$V_B = V_A - \int_A^B w dx = 0 - (4)(2) = -8 \text{ kips}$$

$$\text{C to D} \quad V = -8 \text{ kips}$$

$$\text{D to B} \quad V = -8 - 15 = -23 \text{ kips}$$



Areas under shear diagram

$$\text{A to C} \quad \int V dx = (\frac{1}{2})(4)(-8) = -16 \text{ kip-ft}$$

$$\text{C to D} \quad \int V dx = (4)(-8) = -32 \text{ kip-ft}$$

$$\text{D to B} \quad \int V dx = (4)(-23) = -92 \text{ kip-ft}$$

Bending moments

$$M_A = 0$$

$$M_C = M_A + \int V dx = 0 - 16 = -16 \text{ kip-ft}$$

$$M_D = M_C + \int V dx = -16 - 32 = -48 \text{ kip-ft}$$

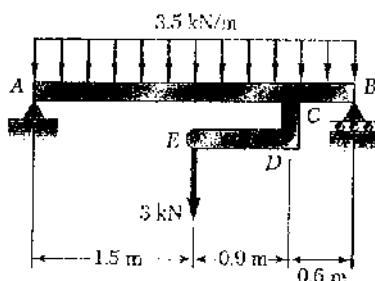
$$M_B = M_D + \int V dx = -48 - 92 = -140 \text{ kip-ft}$$

$$\text{Maximum } |V| = 23 \text{ kips}$$

$$\text{Maximum } |M| = 140 \text{ kip-ft}$$

Problem 5.44

5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



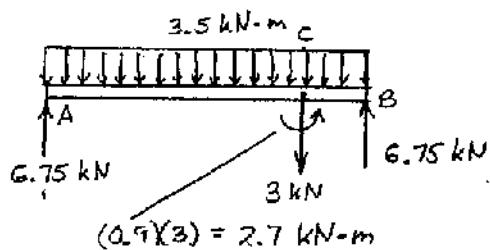
Reaction at A.

$$\rightarrow \sum M_A = 0$$

$$-3.0 A + (1.5)(3.0)(3.5) + (1.5)(3) = 0$$

$$A = 6.75 \text{ kN}$$

Reaction at B. $B = 6.75 \text{ kN}$



Beam ACB and loading. See sketch.

Areas of load diagram.

$$A \text{ to } C \quad (2.4)(3.5) = 8.4 \text{ kN}$$

$$C \text{ to } B \quad (0.6)(3.5) = 2.1 \text{ kN}$$

Shear diagram.

$$V_A = 6.75 \text{ kN}$$

$$V_C^- = 6.75 - 8.4 = -1.65 \text{ kN}$$

$$V_C^+ = -1.65 - 3 = -4.65 \text{ kN}$$

$$V_B = -4.65 - 2.1 = -6.75 \text{ kN}$$

$$\text{Over } A \text{ to } C \quad V = 6.75 - 3.5x$$

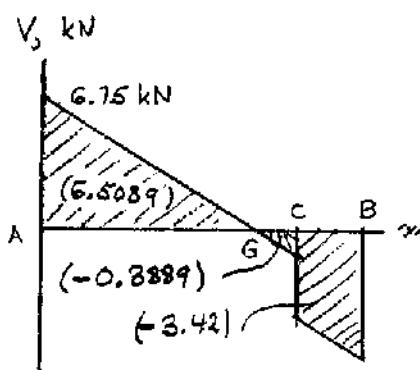
$$\text{At } G \quad V = 6.75 - 3.5x_G = 0 \quad x_G = 1.9286 \text{ m}$$

Areas of shear diagram.

$$A \text{ to } G \quad \frac{1}{2}(1.9286)(6.75) = 6.5089 \text{ kN}\cdot\text{m}$$

$$G \text{ to } C \quad \frac{1}{2}(0.4714)(-1.65) = -0.3889 \text{ kN}\cdot\text{m}$$

$$C \text{ to } B \quad \frac{1}{2}(0.6)(-4.65 - 6.75) = -3.42 \text{ kN}\cdot\text{m}$$



Bending moments.

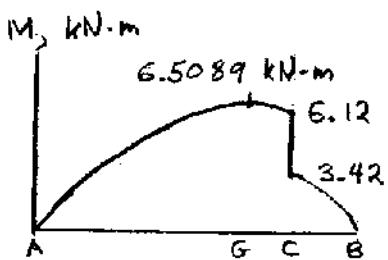
$$M_A = 0$$

$$M_G = 0 + 6.5089 = 6.5089 \text{ kN}\cdot\text{m}$$

$$M_C^- = 6.5089 - 0.3889 = 6.12 \text{ kN}\cdot\text{m}$$

$$M_C^+ = 6.12 - 2.7 = 3.42 \text{ kN}\cdot\text{m}$$

$$M_B = 3.42 - 3.42 = 0$$

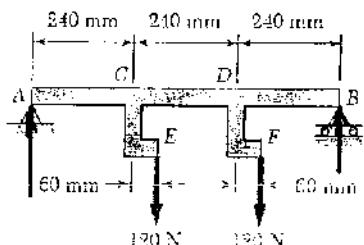


$$(a) |V|_{max} = 6.75 \text{ kN}$$

$$(b) |M|_{max} = 6.51 \text{ kN}\cdot\text{m}$$

Problem 5.45

5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

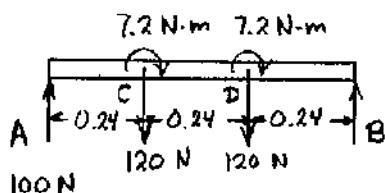


$$\text{At } B: \sum M_B = 0 \quad -0.72A + (0.48)(120) + (0.24)(120) \\ -7.2 - 7.2 = 0$$

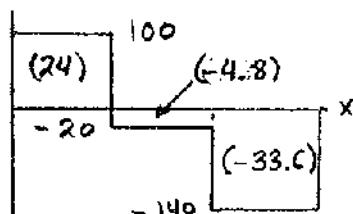
$$A = 100 \text{ N}$$

$$\text{At } A: \sum M_A = 0 \quad -(0.24)(120) - (0.48)(120) - 7.2 \\ -7.2 + 0.72B = 0$$

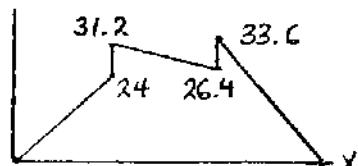
$$B = 140 \text{ N}$$



$V(N)$



$M(N\cdot m)$



Shear

$$A \text{ to } C \quad V = 100 \text{ N}$$

$$C \text{ to } D \quad V = 100 - 120 = -20 \text{ N}$$

$$D \text{ to } B \quad V = -20 - 120 = -140 \text{ N}$$

Areas under shear diagram

$$A \text{ to } C \quad \int V dx = (0.24)(100) = 24 \text{ N}\cdot\text{m}$$

$$C \text{ to } D \quad \int V dx = (0.24)(-20) = -4.8 \text{ N}\cdot\text{m}$$

$$D \text{ to } B \quad \int V dx = (0.24)(-140) = -33.6 \text{ N}\cdot\text{m}$$

Bending moments

$$M_A = 0$$

$$M_C^- = 0 + 24 = 24 \text{ N}\cdot\text{m}$$

$$M_C^+ = 24 + 7.2 = 31.2 \text{ N}\cdot\text{m}$$

$$M_D^- = 31.2 - 4.8 = 26.4 \text{ N}\cdot\text{m}$$

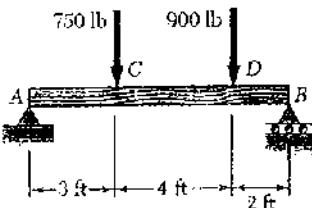
$$M_D^+ = 26.4 + 7.2 = 33.6 \text{ N}\cdot\text{m}$$

$$M_B = 33.6 - 33.6 = 0$$

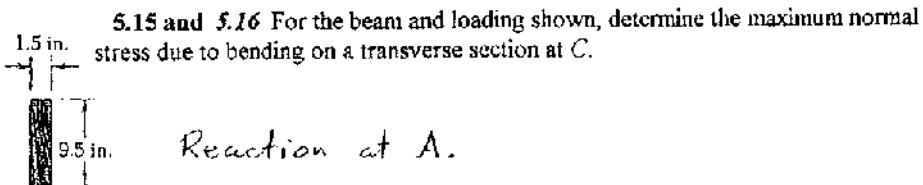
$$\text{Maximum } |V| = 140 \text{ N}$$

$$\text{Maximum } |M| = 33.6 \text{ N}\cdot\text{m}$$

Problem 5.46



5.46 Using the method of Sec. 5.3, solve Prob. 5.15.

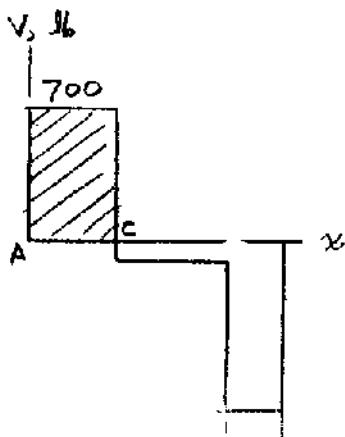


Reaction at A.

$$+D \sum M_B = 0$$

$$-9A + (6)(750) + (2)(900) = 0$$

$$A = 700 \text{ lb.}$$



Shear curve from A to C : $V = 700 \text{ lb.}$

Area of shear curve from A to C

$$A_{AC} = (3)(700) = 2100 \text{ lb}\cdot\text{ft}$$

Bending moments. $M_A = 0$

$$M_C = M_A + A_{AC} = 0 + 2100 = 2100 \text{ lb}\cdot\text{ft}$$

$$= 25.2 \times 10^3 \text{ lb}\cdot\text{in} = 25.2 \text{ kip}\cdot\text{in.}$$

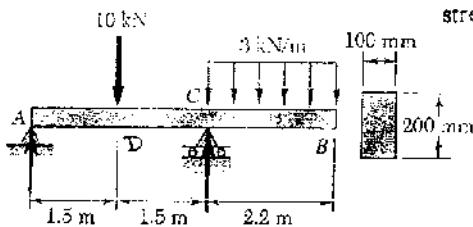
For the cross section $I = \frac{1}{12}(1.5)(9.5)^3 = 107.172 \text{ in}^4$
 $C = 4.75 \text{ in.}$

Maximum normal stress due to bending.

$$\sigma = \frac{Mc}{I} = \frac{(25.2)(4.75)}{107.172}$$

$$\sigma = 1.117 \text{ ksi}$$

Problem 5.47



5.47 Using the method of Sec. 5.3, solve Prob. 5.16.

5.15 and 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

$$\text{At } C: \sum M_C = 0$$

$$-3A + (1.5)(10) - (1.1)(2.2)(3) = 0$$

$$A = 2.58 \text{ kN}$$

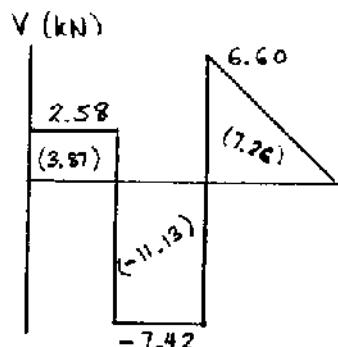
$$\text{At } A: \sum M_A = 0$$

$$-(1.5)(10) + 3C - (4.1)(2.2)(3) = 0$$

$$C = 14.02 \text{ kN}$$

Shear

A to D ⁻	V = 2.58 kN
D ⁺ to C ⁻	V = 2.58 - 10 = -7.42 kN
C ⁺	V = -7.42 + 14.02 = 6.60 kN
B	V = 6.60 - (2.2)(3) = 0



Areas under shear diagram

A to D	$\int V dx = (1.5)(2.58) = 3.87 \text{ kN}\cdot\text{m}$
D to C	$\int V dx = (1.5)(-7.42) = -11.13 \text{ kN}\cdot\text{m}$
C to B	$\int V dx = (\frac{1}{2})(2.2)(6.60) = 7.26 \text{ kN}\cdot\text{m}$

Bending moments

$$M_A = 0$$

$$M_D = 0 + 3.87 = 3.87 \text{ kN}\cdot\text{m}$$

$$M_C = 3.87 - 11.13 = -7.26 \text{ kN}\cdot\text{m}$$

$$M_B = 7.26 - 7.26 = 0$$

$$|M_C| = 7.26 \text{ kN}\cdot\text{m} = 7.26 \times 10^3 \text{ N}\cdot\text{m}$$

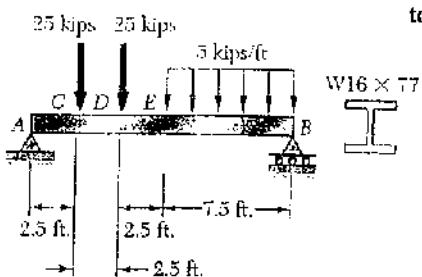
For rectangular cross section $S = \frac{1}{6}bh^2 = (\frac{1}{6})(100)(200)^2$

$$= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^3$$

Normal stress $\sigma = \frac{|M_c|}{S} = \frac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$

$$= 10.89 \text{ MPa}$$

Problem 5.48



5.4B Using the method of Sec. 5.3, solve Prob. 5.17.

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

$$\rightarrow \sum M_B = 0$$

$$-15A + (12.5)(25) + (10)(25) + (3.75)(7.5)(5) = 0$$

$$A = 46.875 \text{ kips}$$

$$\text{Shear } A \text{ to } C \quad V = 46.875 \text{ kips}$$

$$\text{Area under shear curve } A \text{ to } C \quad \int V dx = (2.5)(46.875) \\ = 117.1875 \text{ kip}\cdot\text{ft}$$

$$M_A = 0$$

$$M_c = 0 + 117.1875 = 117.1875 \text{ kip}\cdot\text{ft} = 1406.25 \text{ kip}\cdot\text{in}$$

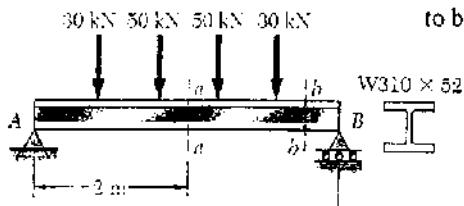
For W 16x77 rolled steel section $S = 134 \text{ in}^3$

$$\text{Normal stress } \sigma = \frac{M}{S} = \frac{1406.25}{134} = 10.49 \text{ ksi}$$

Problem 5.49

5.49 Using the method of Sec. 5.3, solve Prob. 5.18.

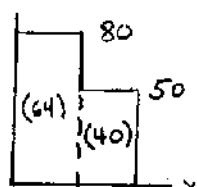
5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section $a-a$.



Reactions: By symmetry $A = B$

$$+\uparrow \sum F_y = 0 \quad A = B = 80 \text{ kN}$$

$V (\text{kN})$



Shear

$$\text{A to C} \quad V = 80 \text{ kN}$$

$$\text{C to D} \quad V = 80 - 30 = 50 \text{ kN}$$

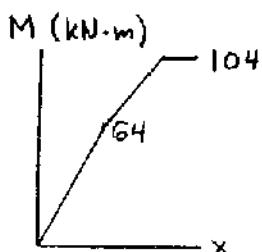
$$\text{D to E} \quad V = 50 - 50 = 0$$

Areas under shear diagram

$$\text{A to C} \quad \int V dx = (80)(0.8) = 64 \text{ kN}\cdot\text{m}$$

$$\text{C to D} \quad \int V dx = (50)(0.8) = 40 \text{ kN}\cdot\text{m}$$

$$\text{D to E} \quad \int V dx = 0$$



Bending moments

$$M_A = 0$$

$$M_C = 0 + 64 = 64 \text{ kN}\cdot\text{m}$$

$$M_D = 64 + 40 = 104 \text{ kN}\cdot\text{m}$$

$$M_E = 104 + 0 = 104 \text{ kN}\cdot\text{m}$$

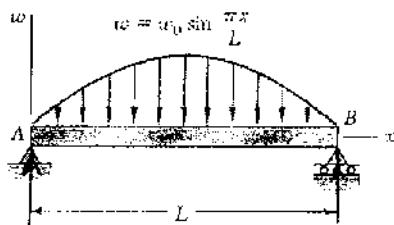
$$M_E = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{For } W 310 \times 52 \quad S = 748 \times 10^3 \text{ mm}^3 = 748 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress} \quad \sigma = \frac{|M|}{S} = \frac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa} = 139.0 \text{ MPa} \blacksquare$$

Problem 5.50

5.50 and 5.51 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = 0 \text{ at } x = L$$

$$0 = 0 + C_1 L + 0$$

$$C_1 = 0$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$$

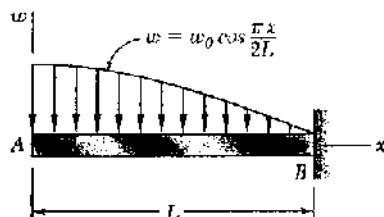
$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = 0 \text{ at } x = \frac{L}{2}$$

$$M_{max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2} = \frac{w_0 L^2}{\pi^2}$$

Problem 5.51

5.50 and 5.51 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\frac{2Lw_0}{\pi} \sin \frac{\pi x}{2L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{4L^2 w_0}{\pi^2} \cos \frac{\pi x}{2L} + C_1 x + C_2$$

$$V = 0 \text{ at } x = 0 \quad \text{Hence } C_1 = 0$$

$$M = 0 \text{ at } x = 0. \quad \text{Hence } C_2 = -\frac{4L^2 w_0}{\pi^2}$$

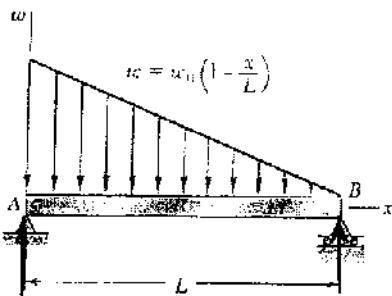
$$(a) \quad V = -(2Lw_0/\pi) \sin(\pi x/2L)$$

$$M = -(4L^2 w_0/\pi^2) [1 - \cos(\pi x/2L)]$$

$$(b) \quad |M|_{max} = 4w_0 L^2 / \pi^2$$

Problem 5.52

5.52 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.



$$w = w_0 \left(1 - \frac{x}{L}\right)$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{w_0 x}{L}$$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = 0 \text{ at } x = L \quad 0 = -\frac{w_0 L^2}{2} + \frac{w_0 L^3}{6} + C_1 L \therefore C_1 = \frac{w_0 L}{3}$$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + \frac{w_0 L}{3}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + \frac{w_0 L x}{3}$$

$$M \text{ is maximum where } \frac{dM}{dx} = V = 0$$

$$0 = -w_0 x_m + \frac{w_0 x_m^2}{2L} + \frac{w_0 L}{3}$$

$$\frac{1}{2} x_m^2 - L x_m + \frac{1}{3} L^2 = 0 \quad x_m = \frac{L \pm \sqrt{L^2 - (4)(\frac{1}{2})(\frac{1}{3}L^2)}}{(2)(\frac{1}{2})}$$

$$= (1 \pm \frac{\sqrt{3}}{3})L$$

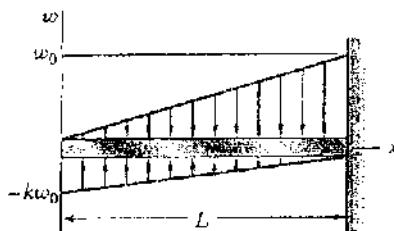
$$= 1.57735L, 0.42265L$$

$$M_{max} = \frac{-w_0 (0.42265L)^2}{2} + \frac{w_0 (0.42265L)^3}{6L} + \frac{w_0 L (0.42265L)}{3}$$

$$= 0.06415 w_0 L^2$$

Problem 5.53

5.53 For the beam and loading shown, determine the equations of the shear and bending-moment curves and the maximum absolute value of the bending moment in the beam, knowing that (a) $k = 1$, (b) $k = 0.5$.



$$w = \frac{w_0 x}{L} - \frac{k w_0 (L-x)}{L} = (1+k) \frac{w_0 x}{L} - k w_0$$

$$\frac{dV}{dx} = -w = k w_0 - (1+k) \frac{w_0 x}{L}$$

$$V = k w_0 x - (1+k) \frac{w_0 x^2}{2L} + C_1$$

$$V = 0 \text{ at } x = 0$$

$$C_1 = 0$$

$$\frac{dM}{dx} = V = k w_0 x - (1+k) \frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1+k) \frac{w_0 x^3}{6L} + C_2$$

$$M = 0 \text{ at } x = 0 \quad C_2 = 0$$

$$M = \frac{k w_0 x^2}{2} - \frac{(1+k) w_0 x^3}{6L}$$

$$(a) \quad k = 1$$

$$V = w_0 x - \frac{w_0 x^2}{L}$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L}$$

$$\text{Maximum } M \text{ occurs at } x = L \quad |M|_{\max} = \frac{w_0 L^2}{6}$$

$$(b) \quad k = \frac{1}{2}$$

$$V = \frac{w_0 x}{2} - \frac{3 w_0 x^2}{4L}$$

$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L}$$

$$V = 0 \text{ at } x = \frac{2}{3} L$$

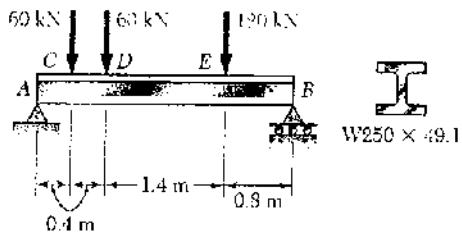
$$\text{At } x = \frac{2}{3} L \quad M = \frac{w_0 (\frac{2}{3} L)^2}{4} - \frac{w_0 (\frac{2}{3} L)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704 w_0 L^2$$

$$\text{At } x = L \quad M = 0$$

$$|M|_{\max} = \frac{w_0 L^2}{27}$$

Problem 5.54

5.54 and 5.55 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$+\odot \sum M_B = 0$$

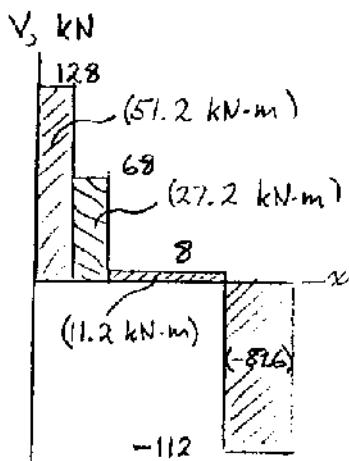
$$-3.0 A + (2.0)(60) + (2.2)(60) + (0.8)(120) = 0$$

$$A = 128 \text{ kN}$$

$$+\odot \sum M_A = 0$$

$$-(0.4)(60) - (0.8)(60) - (2.2)(120) + 3.0 B = 0$$

$$B = 112 \text{ kN}$$



Shear diagram.

$$A \text{ to } C \quad V = 128 \text{ kN}$$

$$C^+ \text{ to } D^- \quad V = 128 - 60 = 68 \text{ kN}$$

$$D^+ \text{ to } E^- \quad V = 68 - 60 = 8 \text{ kN}$$

$$E^+ \text{ to } B \quad V = 8 - 120 = -112 \text{ kN}$$

Areas of shear diagram.

$$A \text{ to } C \quad (0.4)(128) = 51.2 \text{ kN·m}$$

$$C \text{ to } D \quad (0.4)(68) = 27.2 \text{ kN·m}$$

$$D \text{ to } E \quad (1.4)(8) = 11.2 \text{ kN·m}$$

$$E \text{ to } B \quad (0.8)(-112) = -89.6 \text{ kN·m}$$

Bending moments

$$M_A = 0$$

$$M_C = 0 + 51.2 = 51.2 \text{ kN·m}$$

$$M_D = 51.2 + 27.2 = 78.4 \text{ kN·m}$$

$$M_E = 78.4 + 11.2 = 89.6 \text{ kN·m}$$

$$M_B = 89.6 - 89.6 = 0$$

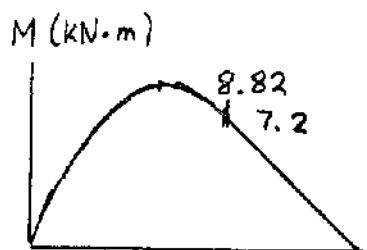
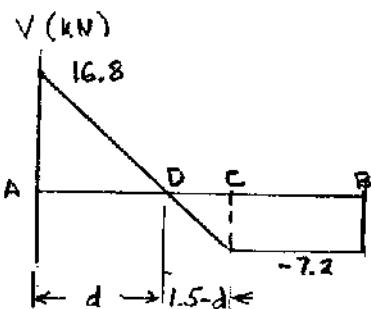
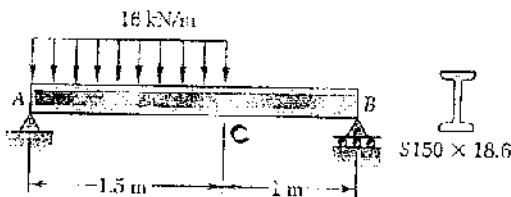
$$|M|_{\max} = 89.6 \text{ kN·m} = 89.6 \times 10^3 \text{ N·m}$$

For W250 x 49.1 rolled steel section $S = 572 \times 10^3 \text{ mm}^3 = 572 \times 10^{-6} \text{ m}^3$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{89.6 \times 10^3}{572 \times 10^{-6}} = 156.6 \times 10^6 \text{ Pa} = 156.6 \text{ MPa}$$

Problem 5.55

5.54 and 5.55 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$+\odot \sum M_B = 0 \\ -2.5A + (1.75)(1.5)(16) = 0$$

$$A = 16.8 \text{ kN}$$

$$+\odot \sum M_A = 0 \\ -(0.75)(1.5)(16) + 2.5B = 0 \\ B = 7.2 \text{ kN}$$

Shear

$$V_A = 16.8 \text{ kN}$$

$$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$$

$$V_B = -7.2 \text{ kN}$$

Locate point D where $V = 0$

$$\frac{d}{16.8} = \frac{1.5-d}{7.2} \quad 24d = 25.2 \\ d = 1.05 \text{ m} \quad 1.5-d = 0.45 \text{ m}$$

Areas under shear diagram

$$A \text{ to } D \quad \int V dx = (\frac{1}{2})(1.05)(16.8) = 8.82 \text{ kN}\cdot\text{m}$$

$$D \text{ to } C \quad \int V dx = (\frac{1}{2})(0.45)(-7.2) = -1.62 \text{ kN}\cdot\text{m}$$

$$C \text{ to } B \quad \int V dx = (1)(-7.2) = -7.2 \text{ kN}\cdot\text{m}$$

Bending moments

$$M_A = 0$$

$$M_D = 0 + 8.82 = 8.82 \text{ kN}\cdot\text{m}$$

$$M_C = 8.82 - 1.62 = 7.2 \text{ kN}\cdot\text{m}$$

$$M_B = 7.2 - 7.2 = 0$$

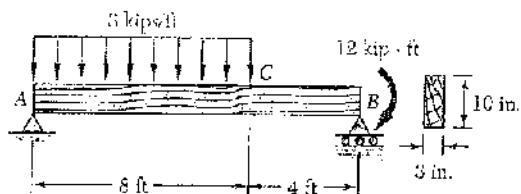
$$\text{Maximum } |M| = 8.82 \text{ kN}\cdot\text{m} = 8.82 \times 10^3 \text{ N}\cdot\text{m}$$

For S 150 x 18.6 rolled steel section $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa} = 73.5 \text{ MPa}$$

Problem 5.56

5.56 and 5.57 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$+\odot \sum M_B = 0 \\ -12A + (8)(8)(3) - 12 = 0 \\ A = 15 \text{ kips}$$

$$\textcircled{2} \sum M_A = 0 \\ -(4)(8)(3) + 12B - 12 = 0 \\ B = 9 \text{ kips}$$

Shear: $V_A = 15 \text{ kips}$

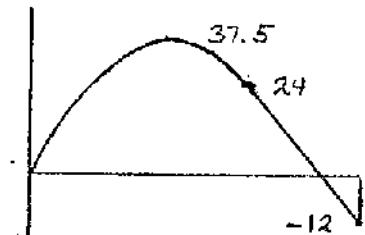
$$V_C = 15 - (8)(3) = -9 \text{ kips}$$

$$C \text{ to } B \quad V = -9 \text{ kips}$$

Locate point D where $V = 0$

$$\frac{d}{15} = \frac{8-d}{9} \quad 24d = 120 \\ d = 5 \text{ ft} \quad 8-d = 3 \text{ ft}$$

M (kip·ft)



Areas under shear diagram

$$A \text{ to } D \quad \int V dx = (\frac{1}{2})(5)(15) = 37.5 \text{ kip}\cdot\text{ft}$$

$$D \text{ to } C \quad \int V dx = (\frac{1}{2})(3)(-9) = -13.5 \text{ kip}\cdot\text{ft}$$

$$C \text{ to } B \quad \int V dx = (4)(-9) = -36 \text{ kip}\cdot\text{ft}$$

Bending moments: $M_A = 0$

$$M_D = 0 + 37.5 = 37.5 \text{ kip}\cdot\text{ft}$$

$$M_C = 37.5 - 13.5 = 24 \text{ kip}\cdot\text{ft}$$

$$M_B = 24 - 36 = -12 \text{ kip}\cdot\text{ft}$$

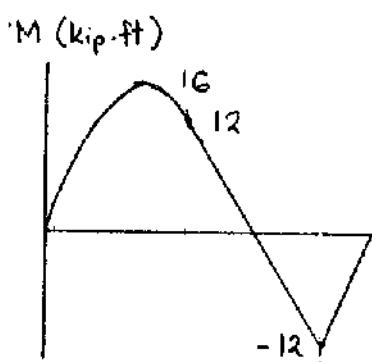
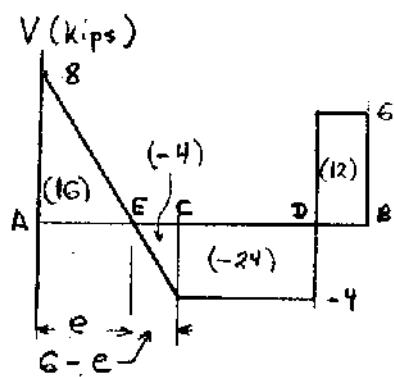
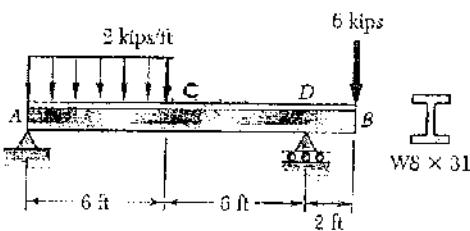
$$\text{Maximum } |M| = 37.5 \text{ kip}\cdot\text{ft} = 450 \text{ kip}\cdot\text{in}$$

$$\text{For rectangular cross section } S = \frac{1}{6}bh^2 = (\frac{1}{6})(3)(10)^2 = 50 \text{ in}^3$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{450}{50} = 9 \text{ ksi}$$

Problem 5.57

5.56 and 5.57 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$+\sum M_D = 0 \\ -12A + (9)(6)(2) - (2)(6) = 0 \\ A = 8 \text{ kips}$$

$$+\sum M_A = 0 \\ -(3)(6)(2) + 12D - (14)(6) = 0 \\ D = 10 \text{ kips}$$

Shear:

$$V_A = 8 \text{ kips}$$

$$V_C = 8 - (6)(2) = -4 \text{ kips}$$

$$\text{C to D} \quad V = -4 \text{ kips}$$

$$\text{D to B} \quad V = -4 + 10 = 6 \text{ kips}$$

Locate point E where $V = 0$

$$\frac{e}{8} = \frac{6-e}{4} \quad 12e = 48 \\ e = 4 \text{ ft} \quad 6-e = 3 \text{ ft}$$

Areas under shear diagram

$$\begin{aligned} \text{A to E} \quad S V dx &= (\frac{1}{2})(4)(8) = 16 \text{ kip-ft} \\ \text{E to C} \quad S V dx &= (\frac{1}{2})(3)(-4) = -4 \text{ kip-ft} \\ \text{C to D} \quad S V dx &= (6)(-4) = -24 \text{ kip-ft} \\ \text{D to B} \quad S V dx &= (2)(6) = 12 \text{ kip-ft} \end{aligned}$$

Bending moments: $M_A = 0$

$$M_E = 0 + 16 = 16 \text{ kip-ft}$$

$$M_C = 16 - 4 = 12 \text{ kip-ft}$$

$$M_D = 12 - 24 = -12 \text{ kip-ft}$$

$$M_B = -12 + 12 = 0$$

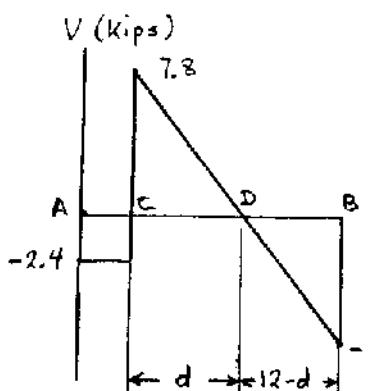
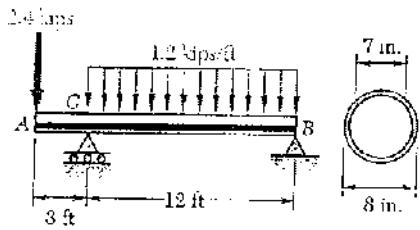
$$\text{Maximum } |M| = 16 \text{ kip-ft} = 192 \text{ kip-in.}$$

For W 8x31 rolled steel section $S = 27.5 \text{ in}^3$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{192}{27.5} = 6.98 \text{ ksi}$$

Problem 5.58

5.58 and 5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$+\circlearrowleft \sum M_C = 0 \\ (3)(2.4) - (6)(12)(1.2) + 12B = 0 \\ B = 6.6 \text{ kips}$$

$$+\circlearrowright \sum M_B = 0 \\ (15)(2.4) + (6)(12)(1.2) - 12C = 0 \\ C = 10.2 \text{ kips}$$

Shear: A to C $V = -2.4 \text{ kips}$
 $C^+ \quad V = -2.4 + 10.2 = 7.8 \text{ kips}$

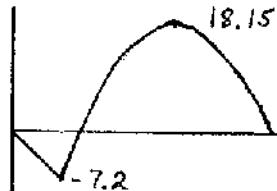
B $V_B = 7.8 - (12)(1.2) = -6.6 \text{ kips}$

Locate point D where $V = 0$

$$\frac{d}{7.8} = \frac{12-d}{6.6} \quad 14.4d = 93.6$$

$$d = 6.5 \text{ ft.} \quad 12-d = 5.5 \text{ ft.}$$

M (kip-ft)



Areas under shear diagram

A to C $\int V dx = (3)(-2.4) = -7.2 \text{ kip-ft}$

C to D $\int V dx = (\frac{1}{2})(6.5)(7.8) = 25.35 \text{ kip-ft}$

D to B $\int V dx = (\frac{1}{2})(5.5)(-6.6) = -18.15 \text{ kip-ft}$

Bending moments $M_A = 0$

$$M_C = 0 - 7.2 = -7.2 \text{ kip-ft}$$

$$M_D = -7.2 + 25.35 = 18.15 \text{ kip-ft}$$

$$M_B = 18.15 - 18.15 = 0$$

Maximum $|M| = 18.15 \text{ kip-ft} = 217.8 \text{ kip-in}$

For pipe $C_o = \frac{d_o}{2} = \frac{8}{2} = 4 \text{ in} \quad C_i = \frac{d_i}{2} = \frac{7}{2} = 3.5 \text{ in.}$

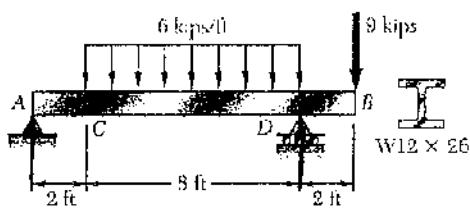
$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = \frac{\pi}{4}(4^4 - 3.5^4) = 82.20 \text{ in}^4$$

$$S = \frac{I}{C_o} = \frac{82.20}{4} = 20.80 \text{ in}^3$$

Normal stress $\sigma = \frac{|M|}{S} = \frac{217.8}{20.80} = 10.47 \text{ ksi}$

Problem 5.59

5.58 and 5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



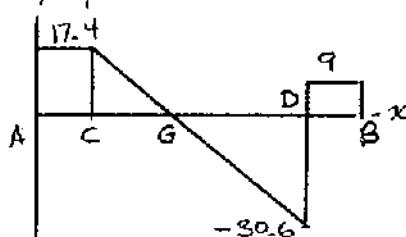
Reaction at A:

$$\sum M_A = 0 \\ -10A + (4)(8)(6) - (2)(9) = 0 \quad A = 17.4 \text{ kips}$$

Reaction at D:

$$\sum M_D = 0 \\ -(6)(8)(6) + 10D - (2)(9) = 0 \quad D = 39.6 \text{ kips}$$

V, kips



Areas of load diagram:

$$C \text{ to } D \quad (8)(6) = 48 \text{ kip} \cdot \text{ft}$$

Shear diagram:

$$A \text{ to } C \quad V = 17.4 \text{ kips}$$

$$V_B^- = 17.4 - 48 = -30.6 \text{ kips}$$

$$V_D^+ = -30.6 + 39.6 = 9 \text{ kips}$$

$$D \text{ to } B \quad V = 9 \text{ kips}$$

$$\text{Over } CD \quad V = 17.4 - 6(x-2)$$

$$\text{At } G \quad V = 17.4 - 6(x_G - 2) = 0 \quad x_G - 2 = 2.9 \text{ ft}$$

Areas of shear diagram:

$$A \text{ to } C \quad (2)(17.4) = 34.8 \text{ kip} \cdot \text{ft}$$

$$C \text{ to } G \quad \frac{1}{2}(2.9)(17.4) = 25.23 \text{ kip} \cdot \text{ft}$$

$$G \text{ to } D \quad \frac{1}{2}(5.1)(-30.6) = -78.03 \text{ kip} \cdot \text{ft}$$

$$D \text{ to } B \quad (2)(9) = 18 \text{ kip} \cdot \text{ft}$$

Bending moments. $M_A = 0$

$$M_C = 0 + 34.8 = 34.8 \text{ kip} \cdot \text{ft}$$

$$M_G = 34.8 + 25.23 = 60.03 \text{ kip} \cdot \text{ft}$$

$$M_D = 60.03 - 78.03 = -18 \text{ kip} \cdot \text{ft}$$

$$M_B = -18 + 18 = 0$$

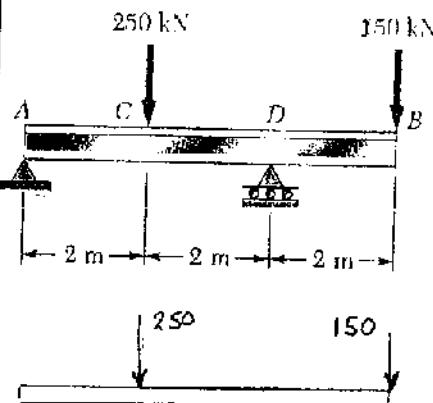
$$|M|_{\max} = M_G = 60.03 \text{ kip} \cdot \text{ft} = 720.36 \text{ kip} \cdot \text{in}$$

For W12 x 26 rolled steel section $S = 33.4 \text{ in}^3$

$$\text{Normal stress} \quad \sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{720.36}{33.4} \quad \sigma_{\max} = 21.6 \text{ ksi}$$

Problem 5.60

5.60 and 5.61 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$w = 0$$

$$\text{At } \sum M_A = 0$$

$$-4R_A + (2)(250) - (2)(150) = 0$$

$$R_A = 50 \text{ kN} \uparrow$$

$$\text{At } \sum M_B = 0$$

$$4R_B - (2)(250) - (6)(150) = 0$$

$$R_B = 350 \text{ kN} \uparrow$$

Shear: $V_A = 50 \text{ kN}$

A to C $V = 50 \text{ kN}$

C to D $V = 50 - 250 = -200 \text{ kN}$

D to B $V = -200 + 350 = 150 \text{ kN}$

Areas of shear diagram

A to C $\int V dx = (50)(2) = 100 \text{ kN}\cdot\text{m}$

C to D $\int V dx = (-200)(2) = -400 \text{ kN}\cdot\text{m}$

D to B $\int V dx = (150)(2) = 300 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$$M_C = M_A + \int V dx = 0 + 100 = 100 \text{ kN}\cdot\text{m}$$

$$M_D = M_C + \int V dx = 100 - 400 = -300 \text{ kN}\cdot\text{m}$$

$$M_B = M_D + \int V dx = -300 + 300 = 0$$

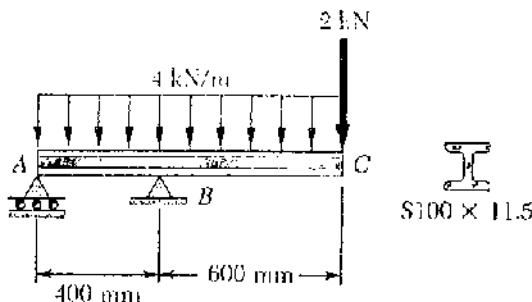
maximum $|M| = 300 \text{ kN}\cdot\text{m} = 300 \times 10^3 \text{ N}\cdot\text{m}$

For W410 x 114 rolled steel section $S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa} = 136.4 \text{ MPa}$$

Problem 5.61

5.60 and 5.61 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

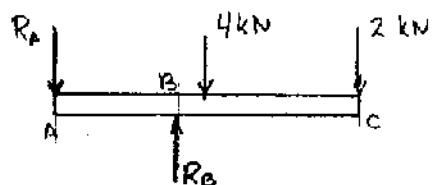


$$\textcircled{1} \sum M_B = 0 \quad (0.4)(R_A) - (0.1)(4) - (0.6)(2) = 0$$

$$R_A = 4 \text{ kN} \downarrow$$

$$\textcircled{2} \sum M_A = 0 \quad (0.4)(R_B) - (0.5)(4) - (1.1)(2) = 0$$

$$R_B = 10 \text{ kN} \uparrow$$



A to B $0 < x < 0.4 \text{ m}$

$$\frac{dV}{dx} = -w = -4 \text{ kN/m}$$

$$V = -4x - 4 \text{ kN}$$

$$\text{At } x = 0.4 \text{ m} \quad V_B = -5.6 \text{ kN}$$

$$\frac{dM}{dx} = -4x - 4$$

$$M = M_A - 2x^2 - 4x = 0 - 2x^2 - 4x$$

$$\text{At } x = 0.4 \text{ m} \quad M_B = 1.92 \text{ kN-m}$$

B to C $0.4 \text{ m} < x < 1.0 \text{ m}$

$$\frac{dV}{dx} = -w = -4 \text{ kN/m}$$

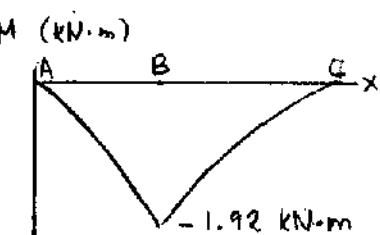
$$V = 4.4 - 4(x - 0.4) = 6 - 4x \text{ kN}$$

$$\frac{dM}{dx} = 6 - 4x$$

$$M = 6x - 2x^2 + C_1 \text{ kN-m}$$

$$M = 0 \text{ at } x = 1 \therefore C_1 = 4 \text{ kN-m}$$

$$M = 4 + 6x - 2x^2 \text{ kN-m}$$



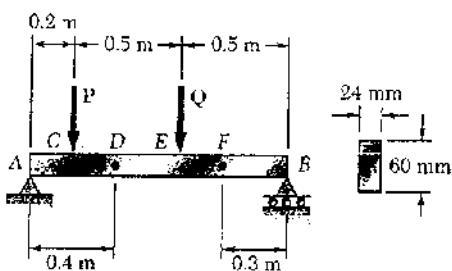
$$|M|_{\max} = 1.92 \text{ kN-m} = 1.92 \times 10^3 \text{ N-m}$$

For S 100 x 11.5 rolled steel section $S_x = 49.6 \times 10^3 \text{ mm}^3 = 49.6 \times 10^{-6} \text{ m}^3$

$$S_m = \frac{|M|_{\max}}{S_x} = \frac{1.92 \times 10^3}{49.6 \times 10^{-6}} = 38.7 \times 10^5 \text{ Pa} = 38.7 \text{ MPa}$$

Problem 5.62

*5.62 The beam AB supports two concentrated loads P and Q . The normal stress due to bending on the bottom edge of the beam is +55 MPa at D and +37.5 MPa at F . (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending which occurs in the beam.



$$I = \frac{1}{12}(24)(60)^3 = 432 \times 10^3 \text{ mm}^4$$

$$C = 30 \text{ mm}$$

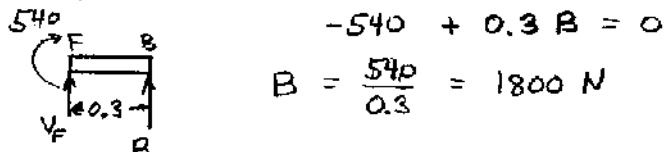
$$S = \frac{I}{C} = 14.4 \times 10^3 \text{ mm}^3 = 14.4 \times 10^{-6} \text{ m}^3$$

$$M = S \epsilon$$

$$\text{At } D \quad M_D = (14.4 \times 10^{-6})(55 \times 10^6) = 792 \text{ N}\cdot\text{m}$$

$$\text{At } F \quad M_F = (14.4 \times 10^{-6})(37.5 \times 10^6) = 540 \text{ N}\cdot\text{m}$$

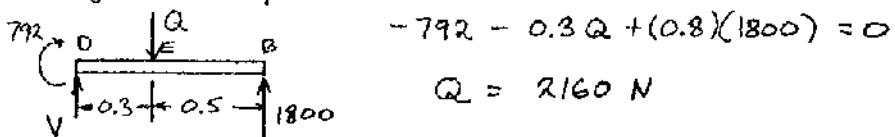
Using free body FB. $\sum M_F = 0$



$$-540 + 0.3 B = 0$$

$$B = \frac{540}{0.3} = 1800 \text{ N}$$

Using free body DEFB. $\sum M_D = 0$



$$-792 - 0.3 Q + (0.8)(1800) = 0$$

$$Q = 2160 \text{ N}$$

Using entire beam $\sum M_A = 0$



$$-0.2P - (0.7)(2160)$$

$$+(1.2)(1800) = 0$$

$$P = 3240 \text{ N}$$

$$+\sum F_y = 0 \quad A - 3240 - 2160 + 1800 = 0$$

$$A = 3600 \text{ N}$$

Shear diagram and its areas.

$$A \text{ to } C^- \quad V = 3600 \text{ N}$$

$$A_{AC} = (0.2)(3600) = 720 \text{ N}\cdot\text{m}$$

$$C^+ \text{ to } E^- \quad V = 3600 - 3240 = 360 \text{ N}$$

$$A_{CE} = (0.5)(360) = 180 \text{ N}\cdot\text{m}$$

$$E^+ \text{ to } B \quad V = 360 - 2160 = -1800 \text{ N}$$

$$A_{EB} = (0.15)(-1800) = -900 \text{ N}\cdot\text{m}$$

Bending moments

$$M_A = 0$$

$$|M|_{max} = 900 \text{ N}\cdot\text{m}$$

$$M_C = 0 + 720 = 720 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{|M|_{max}}{S} = \frac{900}{14.4 \times 10^{-6}} = 62.5 \times 10^6 \text{ Pa}$$

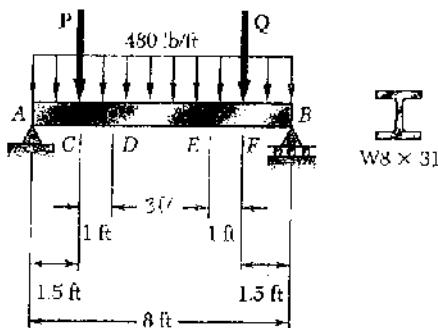
$$M_E = 720 + 180 = 900 \text{ N}\cdot\text{m}$$

$$M_B = 900 - 900 = 0$$

$$(b) \quad \sigma_{max} = 62.5 \text{ MPa}$$

Problem 5.63

*5.63 The beam AB supports a uniformly distributed load of 480 lb/ft and two concentrated loads P and Q . The normal stress due to bending on the bottom edge of the lower flange is +14.85 ksi at D and +10.65 ksi at E . (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending which occurs in the beam.



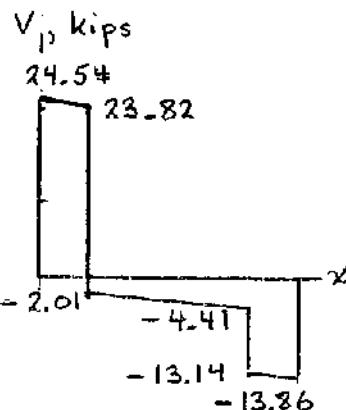
For $W8 \times 31$ rolled steel section $S = 27.5 \text{ in}^3$

$$M = S\sigma$$

$$\text{At } D \quad M_D = (27.5)(14.85) = 408.375 \text{ kip-in}$$

$$\text{At } E \quad M_E = (27.5)(10.65) = 292.875 \text{ kip-in.}$$

$$M_D = 34.03 \text{ kip-ft} \quad M_E = 24.41 \text{ kip-ft}$$

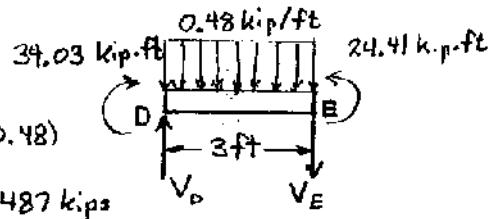


Use free body DE

$$+\int \sum M_E = 0$$

$$-34.03 + 24.41 + (1.5)(3)(0.48) - 3V_D = 0$$

$$V_D = -2.487 \text{ kips}$$



$$+\int \sum M_D = 0$$

$$-34.03 + 24.41 - (1.5)(3)(0.48) - 3V_E = 0$$

$$V_E = -3.927 \text{ kip}$$

Use free body ACD

$$+\int \sum M_A = 0$$

$$-1.5P + (1.25)(2.5)(0.48)$$

$$+(2.5)(2.487) + 34.03 = 0$$

$$P = 25.83 \text{ kips} \downarrow$$

$$+\int \sum F_y = 0$$

$$A - (2.5)(0.48) + 2.487 - 25.83 = 0$$

$$A = 24.54 \text{ kips} \uparrow$$

Use free body EFB

$$+\int \sum M_B = 0$$

$$1.5Q + (1.25)(2.5)(0.48)$$

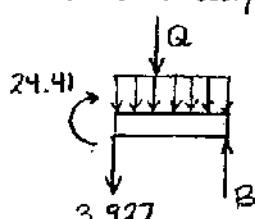
$$+(2.5)(3.927) - 24.41 = 0$$

$$Q = 8.728 \text{ kips}$$

$$+\int \sum F_y = 0$$

$$B - 3.927 - (2.5)(0.48) - 8.7 = 0$$

$$B = 13.855 \text{ kips.}$$



continued

Problem 5.63 continued

Areas of load diagram

$$A \text{ to } C \quad (1.5)(0.48) = 0.72 \text{ kip ft}$$

$$C \text{ to } F \quad (5)(0.48) = 2.4 \text{ kip ft}$$

$$F \text{ to } B \quad (1.5)(0.48) = 0.72 \text{ kip-ft}$$

Shear diagram

$$V_A = 24.54 \text{ kip}$$

$$V_C^- = 24.54 - 0.72 = 23.82 \text{ kips}$$

$$V_C^+ = 23.82 - 25.83 = -2.01 \text{ kips}$$

$$V_F^- = -2.01 - 2.4 = -4.41 \text{ kips}$$

$$V_F^+ = -4.41 - 8.728 = -13.14 \text{ kips}$$

$$V_B = -13.14 - 0.72 = -13.86 \text{ kips}$$

Areas of shear diagram

$$A \text{ to } C \quad \frac{1}{2}(1.5)(24.52 + 23.82) = 36.26 \text{ kip-ft}$$

$$C \text{ to } F \quad \frac{1}{2}(5)(-2.01 - 4.41) = -16.05 \text{ kip-ft}$$

$$F \text{ to } B \quad \frac{1}{2}(1.5)(-13.14 - 13.86) = -20.25 \text{ kip-ft}$$

Bending moments

$$M_A = 0$$

$$M_C = 0 + 36.26 = 36.26 \text{ kip-ft}$$

$$M_F = 36.26 - 16.05 = 20.21 \text{ kip-ft}$$

$$M_B = 20.21 - 20.25 \approx 0$$

Maximum $|M|$ occurs at C

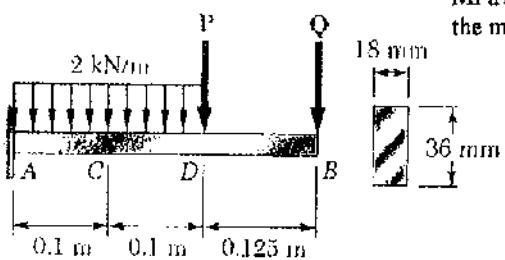
$$|M|_{\max} = 36.26 \text{ kip-ft}$$

$$= 435.1 \text{ kip-in}$$

Maximum stress

$$\sigma = \frac{|M|_{\max}}{S} = \frac{435.1}{27.5} = 15.82 \text{ ksi}$$

Problem 5.64

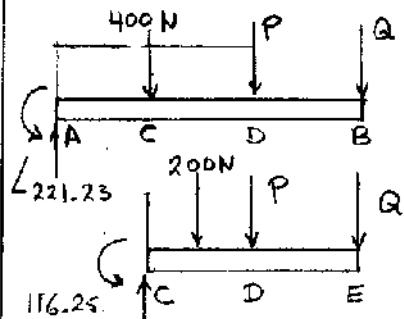


*5.64 Beam *AB* supports a uniformly distributed load of 2 kN/m and two concentrated loads *P* and *Q*. It has been experimentally determined that the normal stress due to bending in the bottom edge of the beam is -56.9 MPa at *A* and -29.9 MPa at *C*. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads *P* and *Q*.

$$I = \frac{1}{12}(18)(36)^3 = 69,984 \times 10^3 \text{ mm}^4$$

$$c = \frac{1}{2}d = 18 \text{ mm}$$

$$S = \frac{I}{c} = 3.888 \times 10^3 \text{ mm}^3 = 3.888 \times 10^{-6} \text{ m}^3$$



$$\text{At } A \quad M_A = S \tilde{e}_A$$

$$M_A = (3.888 \times 10^{-6})(-56.9) = -221.25 \text{ N} \cdot \text{m}$$

$$\text{At } C \quad M_C = S \tilde{e}_C$$

$$M_C = (3.888 \times 10^{-6})(-29.9) = -116.25 \text{ N} \cdot \text{m}$$

$$\therefore \sum M_A = 0$$

$$221.25 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \quad (1)$$

$$\therefore \sum M_C = 0$$

$$116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

$$0.1P + 0.225Q = 106.25 \quad (2)$$

Solving (1) and (2) simultaneously

$$P = 500 \text{ N}$$

$$Q = 250 \text{ N}$$

Reaction force at A

$$R_A - 400 - 500 - 250 = 0$$

$$R_A = 1150 \text{ N} \cdot \text{m}$$

$$V_A = 1150 \text{ N} \quad V_B = 250$$

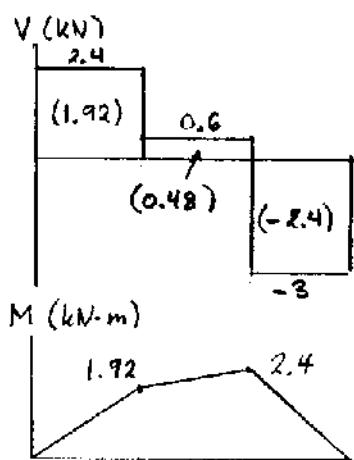
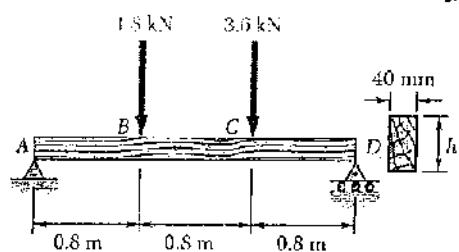
$$M_A = -221.25 \text{ N} \cdot \text{m}$$

$$M_C = -116.25 \text{ N} \cdot \text{m}$$

$$M_D = -31.25 \text{ N} \cdot \text{m}$$

Problem 5.65

5.65 and 5.66 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



$$+\rightarrow \sum M_D = 0 \\ -2.4A + (1.6)(1.8) + (0.8)(3.6) = 0 \\ A = 2.4 \text{ kN}$$

$$+\rightarrow \sum M_A = 0 \\ -(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0 \\ D = 3 \text{ kN}$$

Construct shear and bending moment diagrams

$$|M|_{\max} = 2.4 \text{ kN}\cdot\text{m} = 2.4 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{2.4 \times 10^3}{12 \times 10^6} = 200 \times 10^{-6} \text{ m}^3 \\ = 200 \times 10^3 \text{ mm}^3$$

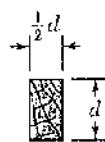
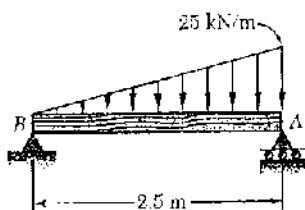
$$S = \frac{1}{6}bh^2 = \frac{1}{6}(40)h^2 = 200 \times 10^3$$

$$h^2 = \frac{(6)(200 \times 10^3)}{40} = 30 \times 10^3 \text{ mm}^2$$

$$h = 173.2 \text{ mm}$$

Problem 5.66

5.65 and 5.66 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



$$\text{Distributed load } w = \frac{25}{2.5} x = 10x \text{ kN/m}$$

$$\frac{dV}{dx} = -w = -10x$$

$$V = -5x^2 + C_1 = \frac{dM}{dx}$$

$$M = -\frac{5}{3}x^3 + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = 0 \text{ at } x = 2.5 \text{ m}$$

$$-\frac{5}{3}(2.5)^3 + C_1(2.5) = 0 \quad C_1 = 10.417 \text{ kN}$$

$$V = -5x^2 + 10.417$$

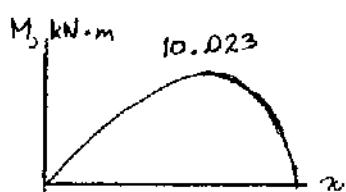
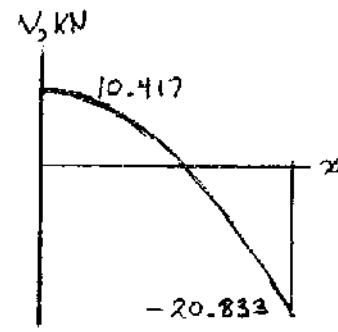
$$V = 0 \quad -5x^2 + 10.417 = 0 \quad x = 1.4434 \text{ m}$$

$$M_{\max} = -\frac{5}{3}(1.4434)^3 + (10.417)(1.4434)$$

$$= 10.023 \text{ kN-m} = 10.023 \times 10^3 \text{ N-m}$$

$$\text{Required } S = \frac{M_{\max}}{S_{\text{all}}} = \frac{10.023 \times 10^3}{12 \times 10^6} = 835.29 \times 10^{-6} \text{ m}^3$$

$$= 835.29 \times 10^3 \text{ mm}^3$$



For the rectangular section $I = \frac{1}{12}(\frac{1}{2}d)(d)^3$

$$c = \frac{1}{2}d \quad S = \frac{I}{c} = \frac{d^3}{12}$$

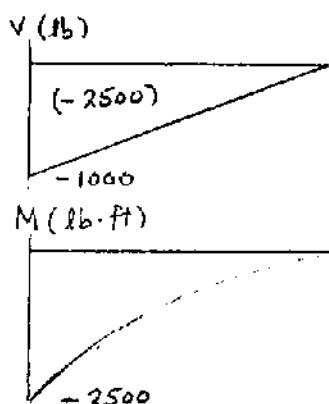
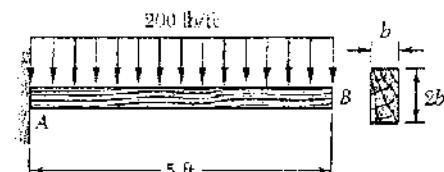
Equating expressions for S

$$\frac{d^3}{12} = 835.29 \times 10^3$$

$$d = 216 \text{ mm}$$

Problem 5.67

5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.



Construct shear and bending moment curves.

$$|M|_{\max} = 2500 \text{ lb}\cdot\text{ft} = 2.5 \text{ kip}\cdot\text{ft}$$

$$= 30 \text{ kip}\cdot\text{in.}$$

$$\sigma_{all} = 1.75 \text{ ksi}$$

$$S_{min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{30}{1.75} = 17.143 \text{ in}^3$$

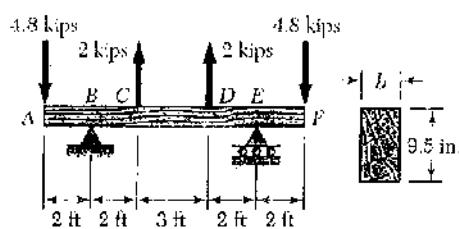
$$S = \frac{1}{6} b h^2 = \frac{1}{6} b (2b)^2 = \frac{2}{3} b^3 = 17.143$$

$$b^3 = \frac{(3)(17.143)}{2} = 25.7 \text{ in}^3$$

$$b = 2.95 \text{ in.}$$

Problem 5.68

5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

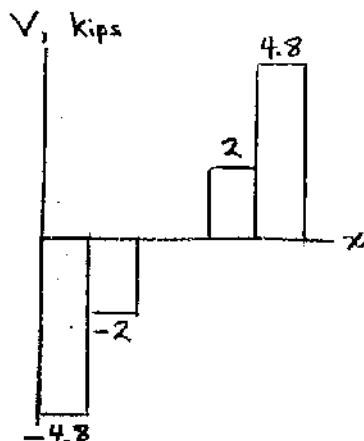


For equilibrium:

$$B = E = 2.8 \text{ kips}$$

Shear diagram:

$$\begin{aligned} A \text{ to } B^- & V = -4.8 \text{ kips} \\ B^+ \text{ to } C^- & V = -4.8 + 2.8 = -2 \text{ kips} \\ C^+ \text{ to } D^- & V = -2 + 2 = 0 \\ D^+ \text{ to } E^- & V = 0 + 2 = 2 \text{ kips} \\ E^+ \text{ to } F & V = 2 + 2.8 = 4.8 \text{ kips} \end{aligned}$$

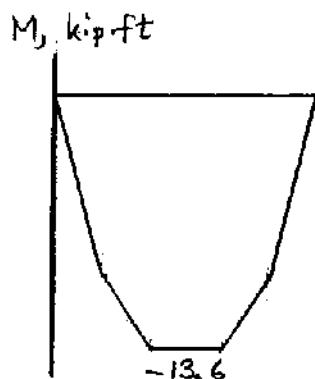


Areas of shear diagram:

$$\begin{aligned} A \text{ to } B & (2)(-4.8) = -9.6 \text{ kip-ft} \\ B \text{ to } C & (2)(-2) = -4 \text{ kip-ft} \\ C \text{ to } D & (3)(0) = 0 \\ D \text{ to } E & (2)(2) = 4 \text{ kip-ft} \\ E \text{ to } F & (2)(4.8) = 9.6 \text{ kip-ft} \end{aligned}$$

Bending moments:

$$\begin{aligned} M_A &= 0 \\ M_B &= 0 + 9.6 = 9.6 \text{ kip-ft} \\ M_C &= 9.6 - 4 = 13.6 \text{ kip-ft} \\ M_D &= 13.6 + 0 = 13.6 \text{ kip-ft} \\ M_E &= 13.6 + 4 = 17.6 \text{ kip-ft} \\ M_F &= 17.6 - 9.6 = 8 \text{ kip-ft} \end{aligned}$$



$$|M|_{\max} = 13.6 \text{ kip-ft} = 162.3 \text{ kip-in} = 162.3 \times 10^3 \text{ lb-in}$$

Required value for S:

$$S = \frac{|M|_{\max}}{\sigma_{all}} = \frac{162.3 \times 10^3}{1750} = 93.257 \text{ in}^3$$

For a rectangular section $I = \frac{1}{12}bh^3$, $c = \frac{1}{2}h$

$$S = \frac{I}{c} = \frac{bh^2}{6} = \frac{(b)(9.5)^2}{6} = 15.0417 b$$

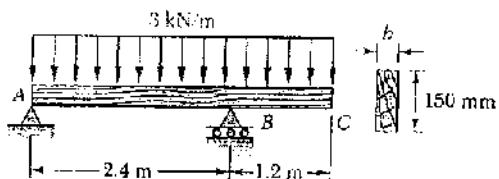
Equating the two expressions for S

$$15.0417 b = 93.257$$

$$b = 6.20 \text{ in.}$$

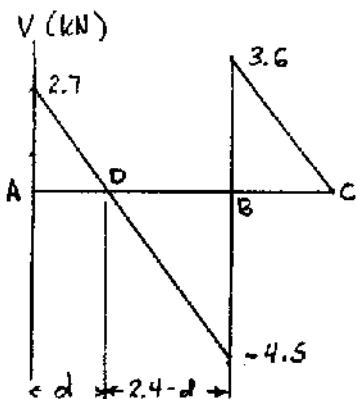
Problem 5.69

5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



$$\rightarrow M_B = 0 \\ -2.4A + (0.6)(3.6)(3) = 0 \quad A = 2.7 \text{ kN}$$

$$\rightarrow M_A = 0 \\ -(1.8)(3.6)(3) + 2.4B = 0 \quad B = 8.1 \text{ kN}$$



Shear: $V_A = 2.7 \text{ kN}$
 $V_B = 2.7 - (2.4)(3) = -4.5 \text{ kN}$
 $V_B = -4.5 + 8.1 = 3.6 \text{ kN}$
 $V_C = 3.6 - (1.2)(3) = 0$

Locate point D where $V = 0$

$$\frac{d}{2.7} = \frac{2.4-d}{4.5} \quad 7.2d = 6.48 \\ d = 0.9 \text{ m} \quad 2.4-d = 1.5 \text{ m}$$

Areas under shear curve

$$A \text{ to } D \quad \int V dx = \left(\frac{1}{2}\right)(0.9)(2.7) = 1.215 \text{ kN}\cdot\text{m}$$

$$D \text{ to } B \quad \int V dx = \left(\frac{1}{2}\right)(1.5)(-4.5) = -3.375 \text{ kN}\cdot\text{m}$$

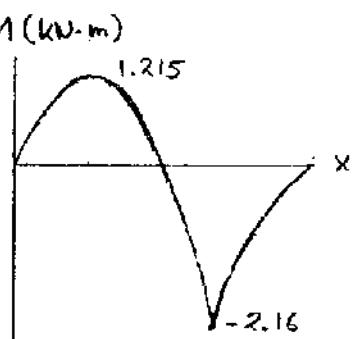
$$B \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(1.2)(3.6) = 2.16 \text{ kN}\cdot\text{m}$$

Bending moments: $M_A = 0$

$$M_D = 0 + 1.215 = 1.215 \text{ kN}\cdot\text{m}$$

$$M_B = 1.215 - 3.375 = -2.16 \text{ kN}\cdot\text{m}$$

$$M_C = -2.16 + 2.16 = 0$$



$$\text{Maximum } |M| = 2.16 \text{ kN}\cdot\text{m} = 2.16 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2.16 \times 10^3}{12 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

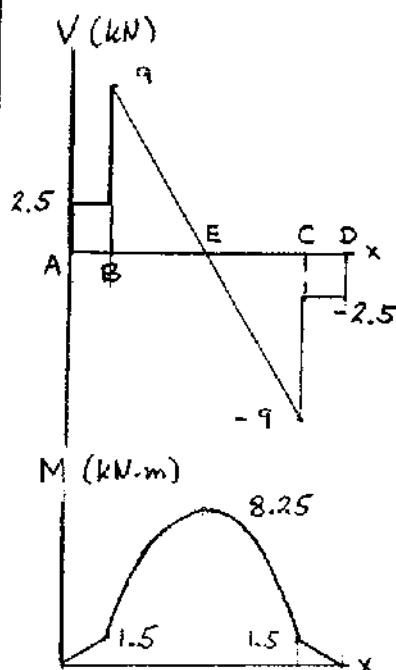
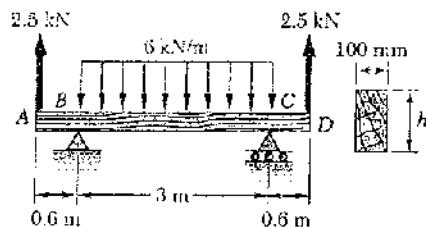
For rectangular section

$$S = \frac{1}{6} b h^2 = \frac{1}{6} b (150)^2 = 180 \times 10^3$$

$$b = \frac{(6)(180 \times 10^3)}{150^2} = 48 \text{ mm}$$

Problem 5.70

5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



By symmetry $B = C$

$$+\uparrow \sum F_y = 0 \quad B + C + 2.5 + 2.5 - (3)(c) = 0 \\ B = C = 6.5 \text{ kN}$$

Shear: A to B $V = 2.5 \text{ kN}$

$$V_{\text{at } E} = 2.5 + 6.5 = 9 \text{ kN}$$

$$V_{\text{at } C} = 9 - (3)(c) = -9 \text{ kN}$$

$$\text{'C to D} \quad V = -9 + 6.5 = -2.5 \text{ kN}$$

Areas under shear diagram

$$\text{A to B} \quad \int V dx = (0.6)(2.5) = 1.5 \text{ kN}\cdot\text{m}$$

$$\text{B to E} \quad \int V dx = (\frac{1}{2})(1.5)(9) = 6.75 \text{ kN}\cdot\text{m}$$

$$\text{E to C} \quad \int V dx = -6.75 \text{ kN}\cdot\text{m}$$

$$\text{C to D} \quad \int V dx = -1.5 \text{ kN}\cdot\text{m}$$

Bending moments $M_A = 0$

$$M_B = 0 + 1.5 = 1.5 \text{ kN}\cdot\text{m}$$

$$M_E = 1.5 + 6.75 = 8.25 \text{ kN}\cdot\text{m}$$

$$M_C = 8.25 - 6.75 = 1.5 \text{ kN}\cdot\text{m}$$

$$M_D = 1.5 - 1.5 = 0$$

$$\text{Maximum } |M| = 8.25 \text{ kN}\cdot\text{m} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$$

For a rectangular section $S = \frac{1}{6} b h^2$

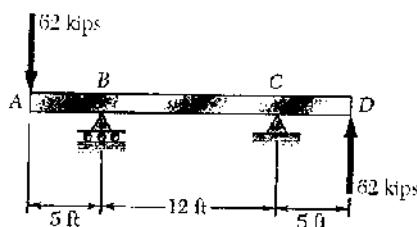
$$687.5 \times 10^3 = \left(\frac{1}{6}\right)(100) h^2$$

$$h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2$$

$$h = 203 \text{ mm}$$

Problem 5.71

5.71 and 5.72 Knowing that the allowable stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



$$+\odot \sum M_c = 0$$

$$(17)(62) - 12B + (5)(62) = 0$$

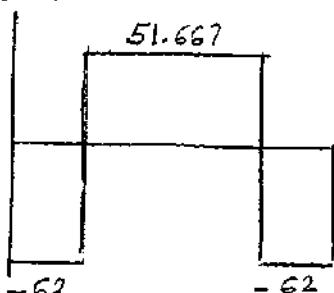
$$B = 113.667 \text{ kips} \uparrow$$

$$+\odot \sum M_B = 0$$

$$(5)(62) + 12C + (17)(62) = 0$$

$$C = -113.667 \text{ kips} \text{ or } C = 113.667 \text{ kips} \downarrow$$

V, kips



Shear diagram.

$$A \text{ to } B^-$$

$$V = -62 \text{ kips}$$

$$B^+ \text{ to } C^-$$

$$V = -62 + 113.667 = 51.667 \text{ kips}$$

$$C^+ \text{ to } D$$

$$V = 51.667 - 113.667 = -62 \text{ kips.}$$

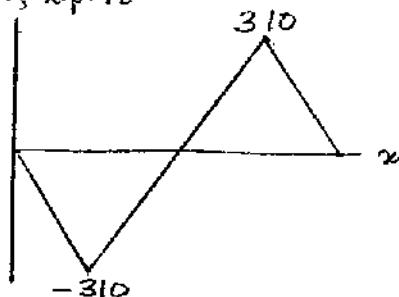
Areas of shear diagram.

$$A \text{ to } B \quad (5)(-62) = -310 \text{ kip-ft}$$

$$B \text{ to } C \quad (12)(51.667) = 620 \text{ kip-ft}$$

$$C \text{ to } D \quad (5)(-62) = -310 \text{ kip-ft}$$

$M, \text{kip-ft}$



Bending moments. $M_A = 0$

$$M_B = 0 - 310 = -310 \text{ kip-ft}$$

$$M_C = -310 + 620 = 310 \text{ kip-ft}$$

$$M_D = 310 - 310 = 0$$

$$|M|_{\max} = 310 \text{ kip-ft} = 3.72 \times 10^3 \text{ kip-in}$$

Required S_{min}

$$S_{min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{3.72 \times 10^3}{24} = 155 \text{ in}^3$$

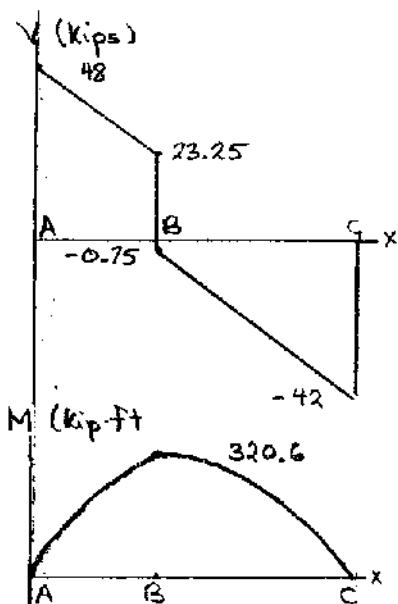
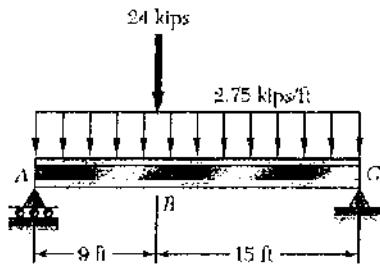
Shape	S, in^3
W 30 x 99	269
W 27 x 84	213
W 24 x 104	258
W 21 x 101	222
W 18 x 106	204
W 14 x 145	232

Lightest wide flange beam

W 27 x 84 @ 84 lb/ft

Problem 5.72

5.71 and 5.72 Knowing that the allowable stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



$$+\circlearrowleft \sum M_c = 0 \quad -24 A + (11.2)(24)(2.75) + (15)(24) = 0 \\ A = 48 \text{ kips.}$$

$$+\circlearrowleft \sum M_A = 0 \quad 24 C - (12.5)(24)(2.75) - (7)(24) = 0 \\ C = 42 \text{ kips.}$$

Shear: $V_A = 48$

$$V_B^- = 48 - (9)(2.75) = 23.25 \text{ kips}$$

$$V_B^+ = 23.25 - 24 = -0.75 \text{ kips}$$

$$V_C = -0.75 - (15)(2.75) = -42 \text{ kips}$$

Areas under shear diagram:

$$A \text{ to } B \quad \int V dx = (\frac{1}{2})(9)(48 + 23.25) = 320.6 \text{ kip} \cdot \text{ft.}$$

$$B \text{ to } C \quad \int V dx = (\frac{1}{2})(15)(-0.75 - 42) = -320.6 \text{ kip} \cdot \text{ft.}$$

Bending moments: $M_A = 0$

$$M_B = 0 + 320.6 = 320.6 \text{ kip} \cdot \text{ft}$$

$$M_C = 320.6 - 320.6 = 0$$

Maximum $|M| = 320.6 \text{ kip} \cdot \text{ft} = 3848 \text{ kip} \cdot \text{in}$

$$\sigma_{all} = 24 \text{ ksi}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3848}{24} = 160.3 \text{ in}^3$$

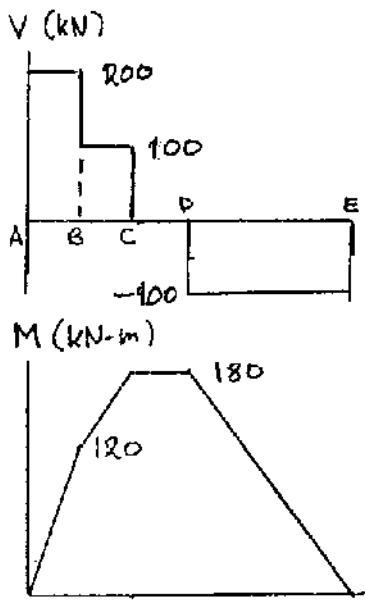
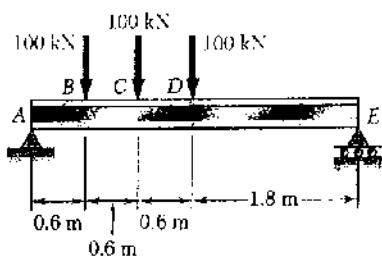
Shape	$S (\text{in}^3)$
W 30 x 99	269
W 27 x 84	213
W 24 x 104	258
W 21 x 101	227
W 18 x 106	204

Lightest wide flange beam

W 27 x 84 @ 84 lb/ft

Problem 5.73

5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.



$$+\odot \sum M_E = 0 \quad -3.6A + (3)(100) + (2.4)(100) + (1.8)(100) = 0$$

$$A = 200 \text{ kN}$$

$$+\odot M_A = 0 \quad 3.6E - (1.8)(100) - (1.2)(100) - (0.6)(100) = 0$$

$$E = 100 \text{ kN}$$

Shear: A to B $V = 200 \text{ kN}$

B to C $V = 200 - 100 = 100 \text{ kN}$

C to D $V = 100 - 100 = 0$

D to E $V = 0 - 100 = -100 \text{ kN}$

Areas under shear diagram

A to B $\int V dx = (0.6)(200) = 120 \text{ kN}\cdot\text{m}$

B to C $\int V dx = (0.6)(100) = 60 \text{ kN}\cdot\text{m}$

C to D $\int V dx = 0$

D to E $\int V dx = (1.8)(-100) = -162 \text{ kN}\cdot\text{m}$

Bending moments $M_A = 0$

$M_B = 0 + 120 = 120 \text{ kN}\cdot\text{m}$

$M_C = 120 + 60 = 180 \text{ kN}\cdot\text{m}$

$M_D = 180 + 0 = 180 \text{ kN}\cdot\text{m}$

$M_E = 180 - 180 = 0$

Maximum $|M| = 180 \text{ kN}\cdot\text{m} = 180 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{180 \times 10^3}{160 \times 10^6} = 1.125 \times 10^{-3} \text{ m}^3 = 1125 \times 10^3 \text{ mm}^3$$

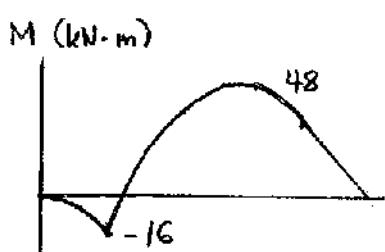
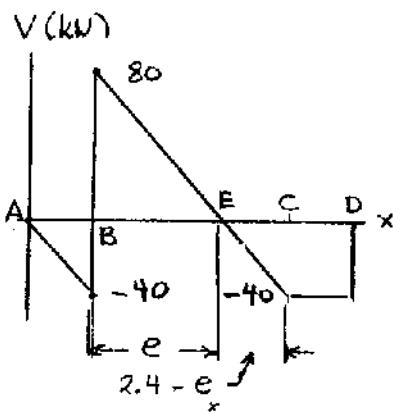
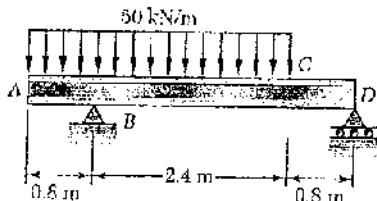
Shape	$S (10^3 \text{ mm}^3)$
W 530 x 66	1340
W 460 x 74	1460
W 360 x 79	1280
W 250 x 101	1240

Lightest wide flange beam

W 530 x 66 @ 66 kg/m

Problem 5.74

5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.



$$\sum M_D = 0 \quad -3.2B + (2.4)(3.2)(50) = 0 \\ B = 120 \text{ kN}$$

$$\sum M_B = 0 \quad 3.2D - (0.8)(3.2)(50) = 0 \\ D = 40 \text{ kN}$$

Shear: $V_A = 0$
 $V_B^- = 0 - (0.8)(50) = -40 \text{ kN}$
 $V_B^+ = -40 + 120 = 80 \text{ kN}$
 $V_C = 80 - (2.4)(50) = -40 \text{ kN}$
 $V_D = -40 + 0 = -40 \text{ kN}$

Locate point E where $V = 0$

$$\frac{e}{80} = \frac{2.4 - e}{40} \quad 120e = 192$$

$$e = 1.6 \text{ m} \quad 2.4 - e = 0.8 \text{ m}$$

Areas: A to B, $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN}\cdot\text{m}$
 B to E $\int V dx = (\frac{1}{2})(1.6)(80) = 64 \text{ kN}\cdot\text{m}$
 E to C $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN}\cdot\text{m}$
 C to D $\int V dx = (0.8)(-40) = -32 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$
 $M_B = 0 - 16 = -16 \text{ kN}\cdot\text{m}$
 $M_E = -16 + 64 = 48 \text{ kN}\cdot\text{m}$
 $M_C = 48 - 16 = 32 \text{ kN}\cdot\text{m}$
 $M_D = 32 - 32 = 0$

$$\text{Maximum } |M| = 48 \text{ kN}\cdot\text{m} = 48 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
W 310 x 32.7	415
W 250 x 28.4	308
W 200 x 35.9	342

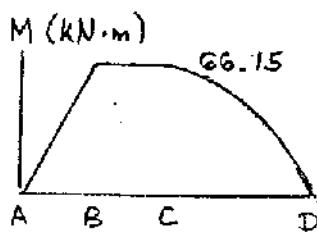
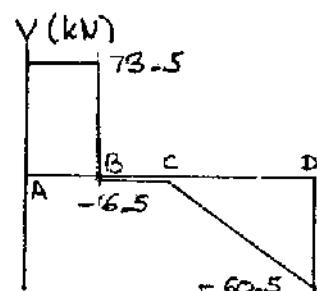
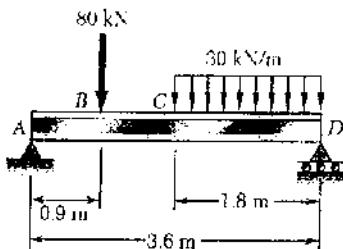
Lightest wide flange beam

W 250 x 28.4 @ 28.4 kg/m



Problem 5.75

5.75 and 5.76 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.



$$+\odot \sum M_D = 0 \quad -3.6 A + (2.7)(80) + (0.9)(1.8)(30) = 0 \\ A = 73.5 \text{ kN} \uparrow$$

$$+\odot \sum M_A = 0 \quad 3.6 D - (0.9)(80) - (2.7)(1.8)(30) = 0 \\ D = 60.5 \text{ kN} \uparrow$$

Shear: $A \rightarrow B \quad V = 73.5 \text{ kN}$
 $B \rightarrow C \quad V = 73.5 - 80 = -6.5 \text{ kN}$
 $V_B = -6.5 - (1.8)(30) = -60.5 \text{ kN}$

Areas under shear diagram
 $A \rightarrow B \quad \int V dx = (0.9)/73.5 = 66.15 \text{ kN}\cdot\text{m}$
 $B \rightarrow C \quad \int V dx = (0.9)/(-6.5) = -5.85 \text{ kN}\cdot\text{m}$
 $C \rightarrow D \quad (\frac{1}{2})(1.8)(-6.5 - 60.5) = -60.30 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$
 $M_B = 0 + 66.15 = 66.15 \text{ kN}\cdot\text{m}$
 $M_C = 66.15 - 5.85 = 60.30 \text{ kN}\cdot\text{m}$
 $M_D = 66.15 - 60.30 = 0$

$$\text{Maximum } |M| = 66.15 \text{ kN}\cdot\text{m} = 66.15 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.15 \times 10^3}{160 \times 10^6} = 413.4 \times 10^{-6} \text{ m}^3 = 413.4 \times 10^3 \text{ mm}^3$$

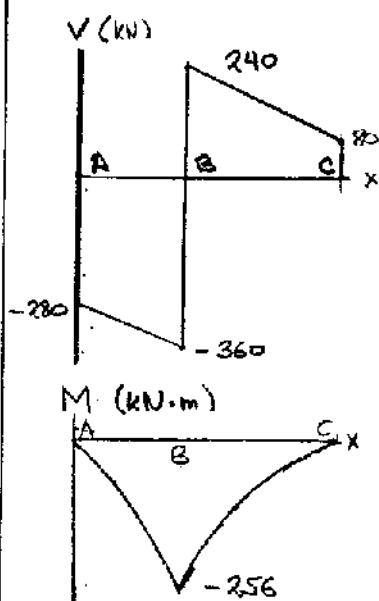
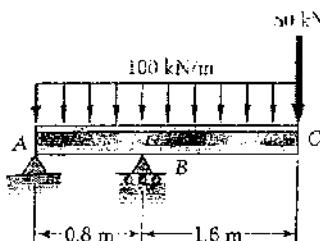
Shape	$S (10^3 \text{ mm}^3)$
S 380 x 64	971
S 310 x 47.3	593
S 250 x 52	482

Lightest S-section

S 310 x 47.3 @ 47.3 kg/m

Problem 5.76

5.75 and 5.76 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.



$$+\odot \sum M_B = 0 \quad 0.8 A - (0.4)(2.4)(100) - (1.6)(80) = 0 \\ A = 280 \text{ kN} \downarrow$$

$$+\odot \sum M_A = 0 \quad 0.8 B - (1.2)(2.4)(100) - (2.4)(80) = 0 \\ B = 600 \text{ kN} \uparrow$$

Shear: $V_A = -280 \text{ kN}$

$$V_{B-} = -280 - (0.8)(100) = -360 \text{ kN}$$

$$V_{B+} = -360 + 600 = 240 \text{ kN}$$

$$V_C = 240 - (1.6)(100) = 80 \text{ kN}$$

Areas under shear diagram

$$A \text{ to } B \quad (\frac{1}{2})(0.8)(-280 - 360) = -256 \text{ kN} \cdot \text{m}$$

$$B \text{ to } C \quad (\frac{1}{2})(1.6)(240 + 80) = 256 \text{ kN} \cdot \text{m}$$

Bending moments: $M_A = 0$

$$M_B = 0 - 256 = -256 \text{ kN} \cdot \text{m}$$

$$M_C = -256 + 256 = 0$$

$$\text{Maximum } |M| = 256 \text{ kN} \cdot \text{m} = 256 \times 10^3 \text{ N} \cdot \text{m}$$

$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^{-3} \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$
S 510 x 98.3	1950
S 460 x 104	1685

Lightest S-section

S 510 x 98.3

Problem 5.77

5.77 and 5.78 Knowing that the allowable stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

From statics the reactions at A and F are

$$A = 53 \text{ kips} \uparrow \quad B = 53 \text{ kips} \uparrow$$

Area of load diagram over CD $(6)(11) = 66 \text{ kips}$.

Shear diagram.

A to B	$V = 53 \text{ kips}$
B to C	$V = 53 - 20 = 33 \text{ kips}$
At D	$V = 33 + 66 = +33 \text{ kips}$
D to E	$V = -33 \text{ kips}$
E to F	$V = -33 - 20 = -53 \text{ kips}$

$$\text{Over } CD \quad V = 33 - 11(x-4) = 0$$

$$\text{At } G \quad V = 0 \quad x_G = 7 \text{ ft}$$

Areas of shear diagram.

A to B	$(2)(53) = 106 \text{ kip}\cdot\text{ft}$
B to C	$(2)(33) = 66 \text{ kip}\cdot\text{ft}$
C to G	$\frac{1}{2}(3)(33) = 49.5 \text{ kip}\cdot\text{ft}$
G to D	$\frac{1}{2}(3)(-33) = -49.5 \text{ kip}\cdot\text{ft}$
D to E	$(2)(-33) = -66 \text{ kip}\cdot\text{ft}$
E to F	$(2)(-53) = -106 \text{ kip}\cdot\text{ft}$

Bending moments. $M_A = 0$

$$M_B = 0 + 106 = 106 \text{ kip}\cdot\text{ft}$$

$$M_C = 106 + 66 = 172 \text{ kip}\cdot\text{ft}$$

$$M_G = 172 + 49.5 = 221.5 \text{ kip}\cdot\text{ft}$$

$$M_D = 221.5 - 49.5 = 172 \text{ kip}\cdot\text{ft}$$

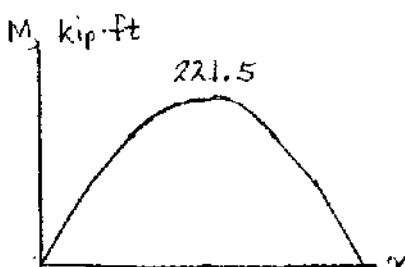
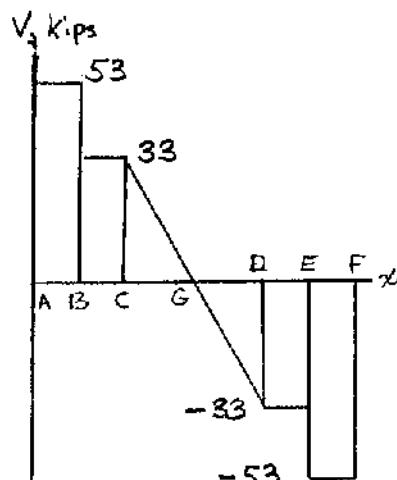
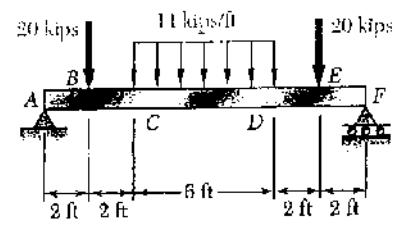
$$M_E = 172 - 66 = 106 \text{ kip}\cdot\text{ft}$$

$$M_F = 106 - 106 = 0$$

$$|M|_{\max} = 221.5 \text{ kip}\cdot\text{ft} = 2658 \text{ kip}\cdot\text{in.}$$

$$S_{\min} = \frac{|M|_{\max}}{F_{all}} = \frac{2658}{24} = 110.75 \text{ in}^3$$

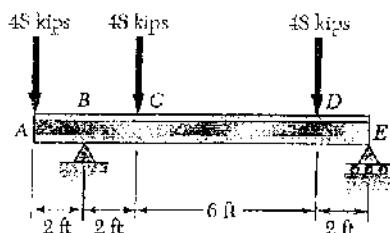
Use S 20 x 66



Shape	$S_3 \text{ in}^3$
S 20 x 66	119

Problem 5.78

5.77 and 5.78 Knowing that the allowable stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

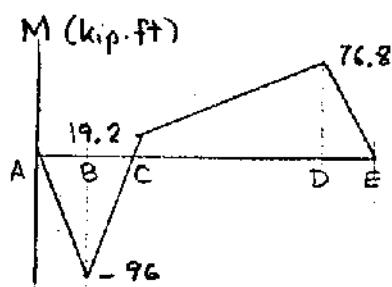
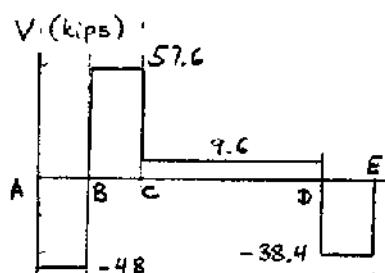


$$+\sum M_E = 0 \quad (12)(48) - 10B + (8)(48) + (2)(48) = 0$$

$$B = 105.6 \text{ kips}$$

$$+\sum M_B = 0 \quad (2)(48) - (2)(48) - (8)(48) + 10E = 0$$

$$E = 38.4 \text{ kips}$$



Shear: A to B $V = -48 \text{ kips}$

B to C $V = -48 + 105.6 = 57.6 \text{ kips}$

C to D $V = 57.6 - 48 = 9.6 \text{ kips}$

D to E $V = 9.6 - 48 = -38.4 \text{ kips}$

Areas: A to B $(2)(-48) = -96 \text{ kip}\cdot\text{ft}$

B to C $(2)(57.6) = 115.2 \text{ kip}\cdot\text{ft}$

C to D $(6)(9.6) = 57.6 \text{ kip}\cdot\text{ft}$

D to E $(2)(-38.4) = 76.8 \text{ kip}\cdot\text{ft}$

Bending moments: $M_A = 0$

$M_B = 0 - 96 = -96 \text{ kip}\cdot\text{ft}$

$M_C = -96 + 115.2 = 19.2 \text{ kip}\cdot\text{ft}$

$M_D = 19.2 + 57.2 = 76.8 \text{ kip}\cdot\text{ft}$

$M_E = 76.8 - 76.8 = 0$

Maximum |M| = 96 kip·ft. = 1152 kip·in

$\sigma_{all} = 24 \text{ ksi}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1152}{24} = 48 \text{ in}^3$$

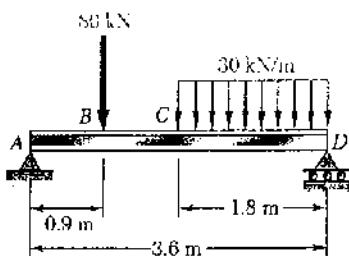
Shape	$S (\text{in}^3)$
S 15 x 42.9	59.6
S 12 x 50	50.8

Lightest S-shaped beam

S 15 x 42.9

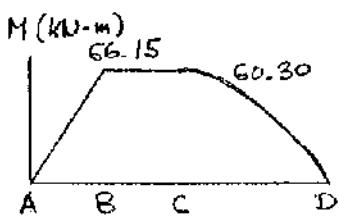
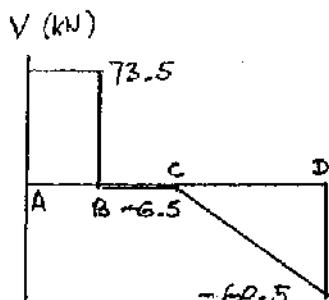
Problem 5.79

5.79 Two metric rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 200 MPa, determine the most economical channels that can be used.



$$+\sum M_p = 0 \quad -3.6A + (2.7)(80) + (0.9)(1.8)(30) = 0 \\ A = 73.5 \text{ kN} \uparrow$$

$$+\sum M_A = 0 \quad 3.6D - (0.9)(80) - (2.7)(1.8)(30) = 0 \\ D = 60.5 \text{ kN} \uparrow$$



Shear: A to B $V = 73.5 \text{ kN}$
 B to C $V = 73.5 - 80 = -6.5 \text{ kN}$
 $V_D = -6.5 - (1.8)(30) = -60.5 \text{ kN}$

Areas: A to B $(0.9)(73.5) = 66.15 \text{ kN}\cdot\text{m}$
 B to C $(0.9)(-6.5) = -5.85 \text{ kN}\cdot\text{m}$
 C to D $(\frac{1}{2})(1.8)(-6.5 - -60.5) = -60.3 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$
 $M_B = 0 + 66.15 = 66.15 \text{ kN}\cdot\text{m}$
 $M_C = 66.15 - 5.85 = 60.30 \text{ kN}\cdot\text{m}$
 $M_D = 60.30 - 60.3 = 0$

Maximum $|M| = 66.15 \text{ kN}\cdot\text{m} = 66.15 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma_{all} = 200 \text{ MPa} = 200 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.15 \times 10^3}{200 \times 10^6} = 330.75 \times 10^{-6} \text{ m}^3 \\ = 330.75 \times 10^3 \text{ mm}^3$$

For double channel

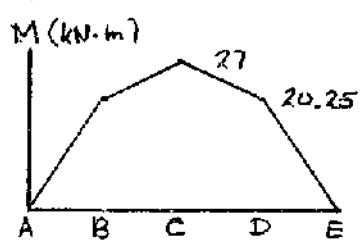
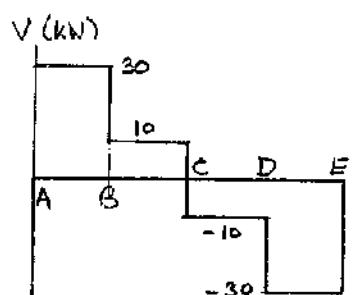
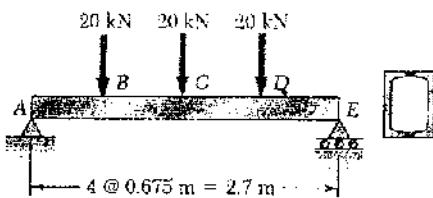
$$S_{min} = (\frac{1}{2})(330.75 \times 10^3) = 165.4 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
C 230 x 19.9	173
C 200 x 27.9	179

Lightest channel section
 C 230 x 19.9

Problem 5.80

5.80 Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.



By symmetry $A = E$

$$+\uparrow \sum F_y = 0 \quad A + E - 20 - 20 - 20 = 0$$

$$A = E = 30 \text{ kN}$$

Shear: A to B $V = 30 \text{ kN}$

$$B$$
 to C $V = 30 - 20 = 10 \text{ kN}$

$$C$$
 to D $V = 10 - 20 = -10 \text{ kN}$

$$D$$
 to E $V = -10 - 20 = -30 \text{ kN}$

Areas: A to B $(0.675)(30) = 20.25 \text{ kN}\cdot\text{m}$

$$B$$
 to C $(0.675)(10) = 6.75 \text{ kN}\cdot\text{m}$

$$C$$
 to D $(0.675)(-10) = -6.75 \text{ kN}\cdot\text{m}$

$$D$$
 to E $(0.675)(-30) = -20.25 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 + 20.25 = 20.25 \text{ kN}\cdot\text{m}$$

$$M_C = 20.25 + 6.75 = 27 \text{ kN}\cdot\text{m}$$

$$M_D = 27 - 6.75 = 20.25 \text{ kN}\cdot\text{m}$$

$$M_E = 20.25 - 20.25 = 0$$

$$\text{Maximum } |M| = 27 \text{ kN}\cdot\text{m} = 27 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 150 \text{ MPa} = 150 \times 10^6 \text{ Pa}$$

For a section consisting of two channels

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{27 \times 10^3}{150 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

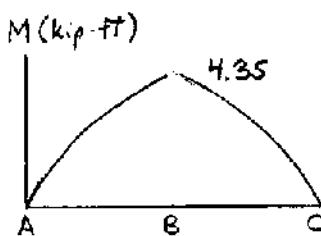
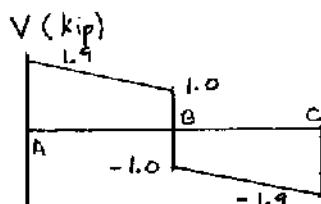
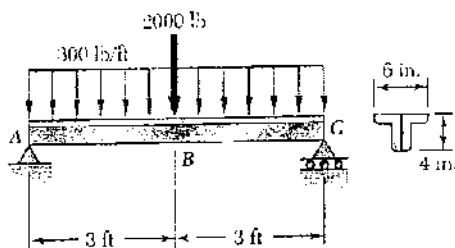
$$\text{For each channel } S_{min} = \left(\frac{1}{2}\right)(180 \times 10^3) = 90 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
C 180 x 14.6	99.2
C 150 x 19.3	93.6

Lightest channel section
C 180 x 14.6

Problem 5.81

5.81 Two L4 × 3 rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.



$$\text{By symmetry } A = C$$

$$+ \sum F_y = 0 \quad A + C - 2000 - (6)(300) = 0 \\ A = C = 1900 \text{ lb.}$$

$$\text{Shear: } V_A = 1900 \text{ lb.} = 1.9 \text{ kips}$$

$$V_B^- = 1900 - (3)(300) = 1000 \text{ lb.} = 1 \text{ kip}$$

$$V_B^+ = 1000 - 2000 = -1000 \text{ lb.} = -1 \text{ kip}$$

$$V_C = -1000 - (3)(300) = -1900 \text{ lb.} = -1.9 \text{ kip}$$

$$\text{Areas: } A \text{ to } B \quad (\frac{1}{2})(3)(1.9 + 1) = 4.35 \text{ kip}\cdot\text{ft}$$

$$B \text{ to } C \quad (\frac{1}{2})(3)(-1 - 1.9) = -4.35 \text{ kip}\cdot\text{ft}$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 + 4.35 = 4.35 \text{ kip}\cdot\text{ft}$$

$$M_C = 4.35 - 4.35 = 0$$

$$\text{Maximum } |M| = 4.35 \text{ kip}\cdot\text{ft} = 52.2 \text{ kip}\cdot\text{in}$$

$$\sigma_{all} = 24 \text{ ksi}$$

$$\text{For section consisting of two angles} \quad S_{min} = \frac{|M|}{\sigma_{all}} = \frac{52.2}{24} = 2.175 \text{ in}^3$$

$$\text{For each angle} \quad S_{min} = (\frac{1}{2})(2.175) = 1.0875 \text{ in}^3$$

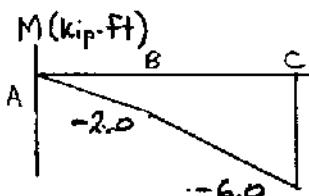
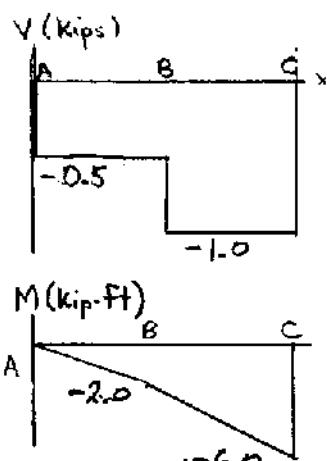
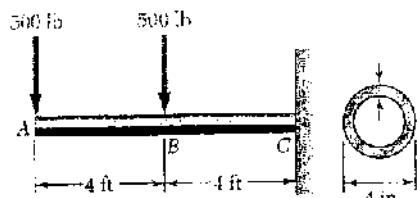
Angle section	$S (\text{in}^3)$
L 4 × 3 × $\frac{1}{2}$	1.89
L 4 × 3 × $\frac{3}{8}$	1.46
L 4 × 3 × $\frac{1}{4}$	1.00

Smallest allowable thickness

$$t = \frac{3}{8} \text{ in.}$$

Problem 5.82

5.82 A steel pipe of 4-in. diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from $\frac{1}{4}$ in. to 1 in. in $\frac{1}{8}$ -in. increments, and that the allowable normal stress for the steel used is 24 ksi, determine the minimum wall thickness t that can be used.



$$\text{Shear: } A \text{ to } B \quad V = -500 \text{ lb} = -0.5 \text{ kip}$$

$$B \text{ to } C \quad V = -500 - 500 = -1000 \text{ lb} = -1.0 \text{ kip}$$

$$\text{Areas: } A \text{ to } B \quad (4)(-0.5) = -2.0 \text{ kip}\cdot\text{ft}$$

$$B \text{ to } C \quad (4)(-1.0) = -4.0 \text{ kip}\cdot\text{ft}$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 - 2.0 = -2.0 \text{ kip}\cdot\text{ft}$$

$$M_C = -2.0 - 4.0 = -6.0 \text{ kip}\cdot\text{ft}$$

$$\text{Maximum } |M| = 6.0 \text{ kip}\cdot\text{ft} = 72 \text{ kip}\cdot\text{in}$$

$$\sigma_{all} = 24 \text{ ksi}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{72}{24} = 3 \text{ in}^3$$

$$I = \frac{\pi}{4}(C_2^4 - C_1^4) \quad C = C_2 \quad C_2 = \frac{1}{2}d = 2.0 \text{ in.}$$

$$S = \frac{I}{C} = \frac{\pi}{4} \frac{C_2^4 - C_1^4}{C_2} = \frac{\pi}{4} \frac{2^4 - C_1^4}{2} = 3 \text{ in}^3$$

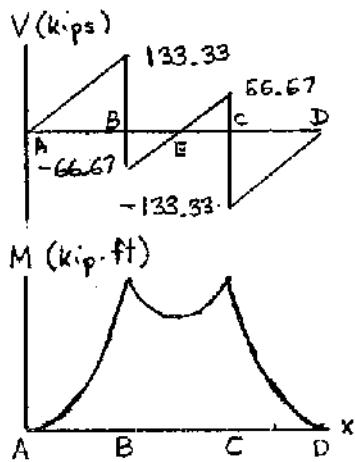
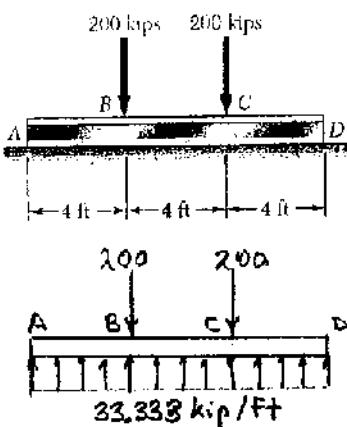
$$C_1^4 = 2^4 - \frac{(4)(2)(3)}{\pi} = 8.3606 \text{ in}^4 \quad C_1 = 1.7004 \text{ in.}$$

$$t_{min} = C_2 - C_1 = 2.0 - 1.7004 = 0.2996 \text{ in}$$

Using $\frac{1}{8}$ in. increments for design $t = \frac{3}{8}$ in. →

Problem 5.83

5.83 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



$$\text{Distributed reaction } q = \frac{400}{12} = 33.333 \text{ kip/ft}$$

$$\text{Shear: } V_A = 0$$

$$V_B^- = 0 + (4)(33.333) = 133.33 \text{ kips}$$

$$V_B^+ = 133.33 - 200 = -66.67 \text{ kips}$$

$$V_C^- = -66.67 + 4(33.333) = 66.67 \text{ kips}$$

$$V_C^+ = 66.67 - 200 = -133.33 \text{ kips}$$

$$V_D = -133.33 + (4)(33.333) = 0 \text{ kips}$$

$$\text{Areas: } A \text{ to } B \quad (\frac{1}{2})(4)(133.33) = 266.67 \text{ kip-ft}$$

$$B \text{ to } E \quad (\frac{1}{2})(2)(-66.67) = -66.67 \text{ kip-ft}$$

$$E \text{ to } C \quad (\frac{1}{2})(2)(66.67) = 66.67 \text{ kip-ft}$$

$$C \text{ to } D \quad (\frac{1}{2})(4)(-133.33) = -266.67 \text{ kip-ft}$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 + 266.67 = 266.67 \text{ kip-ft}$$

$$M_E = 266.67 - 66.67 = 200 \text{ kip-ft}$$

$$M_C = 200 + 66.67 = 266.67 \text{ kip-ft}$$

$$M_D = 266.67 - 266.67 = 0$$

$$\text{Maximum } |M| = 266.67 \text{ kip-ft} = 3200 \text{ kip-in}$$

$$\sigma_{all} = 24 \text{ ksi}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3200}{24} = 133.3 \text{ in}^3$$

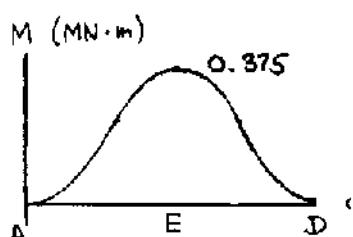
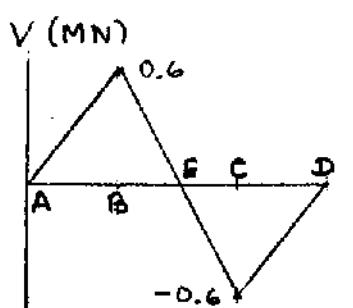
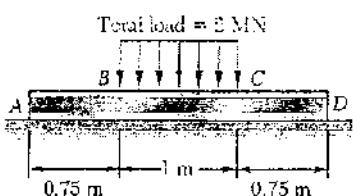
Shape	$S (\text{in}^3)$
W 27 x 84	213
W 24 x 68	154 ←
W 21 x 101	227
W 18 x 76	146
W 16 x 77	134
W 14 x 145	232

Lightest W-shaped section

W 24 x 68

Problem 5.84

5.84 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical wide-flange beam to support the loading shown.



$$\text{Downward distributed load } w = \frac{2}{1.0} = 2 \text{ MN/m}$$

$$\text{Upward distributed reaction } q_f = \frac{2}{2.5} = 0.8 \text{ MN/m}$$

$$\text{Net distributed load over BC} \quad 1.2 \text{ MN/m}$$

$$\text{Shear: } V_A = 0$$

$$V_B = 0 + (0.75)(0.8) = 0.6 \text{ MN}$$

$$V_C = 0.6 - (1.0)(1.2) = -0.6 \text{ MN}$$

$$V_D = -0.6 + (0.75)(0.8) = 0$$

$$\text{Areas: } A \text{ to } B \quad (\frac{1}{2})(0.75)(0.6) = 0.225 \text{ MN-m}$$

$$B \text{ to } E \quad (\frac{1}{2})(0.5)(0.6) = 0.150 \text{ MN-m}$$

$$E \text{ to } C \quad (\frac{1}{2})(0.5)(-0.6) = -0.150 \text{ MN-m}$$

$$C \text{ to } D \quad (\frac{1}{2})(0.75)(-0.6) = -0.225 \text{ MN-m}$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 + 0.225 = 0.225 \text{ MN-m}$$

$$M_E = 0.225 + 0.150 = 0.375 \text{ MN-m}$$

$$M_C = 0.375 - 0.150 = 0.225 \text{ MN-m}$$

$$M_D = 0.225 - 0.225 = 0$$

$$\text{Maximum } |M| = 0.375 \text{ MN-m} = 375 \times 10^3 \text{ N-m}$$

$$\sigma_{all} = 170 \text{ MPa} = 170 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3} \text{ m}^3 = 2206 \times 10^3 \text{ mm}^3$$

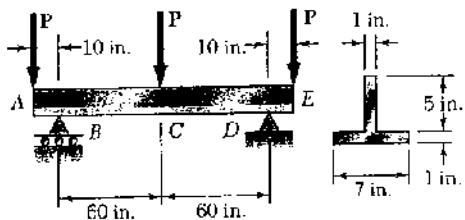
Shape	$S(10^3 \text{ mm}^3)$
W 690 x 125	3510
W 610 x 101	2530
W 530 x 150	3720
W 460 x 113	2400

Lightest wide flange section

W 610 x 101

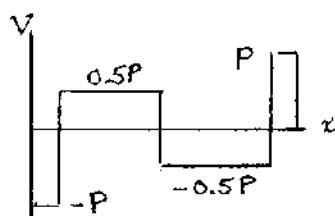
Problem 5.85

5.85 Determine the allowable value of P for the loading shown, knowing that the allowable normal stress is +8 ksi in tension and -18 ksi in compression.

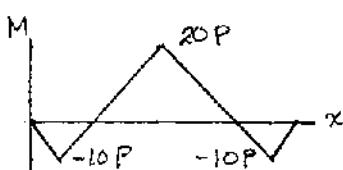


$$\text{Reactions. } B = D = 1.5 P \uparrow$$

$$\begin{array}{ll} \text{Shear diagram. } & A \text{ to } B^- \quad V = -P \\ & B^+ \text{ to } C^- \quad V = -P + 1.5P = 0.5P \\ & C^+ \text{ to } D^- \quad V = 0.5P - P = -0.5P \\ & D^+ \text{ to } E \quad V = -0.5P + 1.5P = P \end{array}$$



$$\begin{array}{ll} \text{Areas. } & A \text{ to } B \quad (10)(-P) = -10P \\ & B \text{ to } C \quad (60)(0.5P) = 30P \\ & C \text{ to } D \quad (60)(-0.5P) = -30P \\ & D \text{ to } E \quad (10)(P) = 10P \end{array}$$



$$\text{Bending moments. } M_A = 0$$

$$M_B = 0 - 10P = -10P$$

$$M_C = -10P + 30P = 20P$$

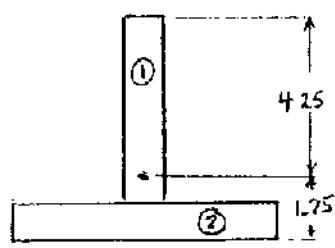
$$M_D = 20P - 30P = -10P$$

$$M_E = -10P + 10P = 0$$

Largest positive bending moment = $20P$

Largest negative bending moment = $-10P$

Centroid and moment of inertia.



Part	A_i, in^2	\bar{y}_i, in	$A\bar{y}_i, \text{in}^3$	d, in	Ad^2, in^4	I_g, in^4
①	5	3.5	17.5	1.75	15.3125	10.417
②	7	0.5	3.5	1.25	10.9375	0.583
Z	12		21		26.25	11.000

$$\text{Top: } y = 4.25 \text{ in.}$$

$$\text{Bottom: } y = -1.75 \text{ in.}$$

$$\bar{y} = \frac{21}{12} = 1.75 \text{ in.}$$

$$I = \sum Ad^2 + \sum I = 37.25 \text{ in}^4$$

$$\sigma = -\frac{My}{I}$$

$$\text{Top, Tension: } 8 = -\frac{(-10P)(4.25)}{37.25} \quad P = 7.01 \text{ kips}$$

$$\text{Top, Comp. } -18 = -\frac{(20P)(4.25)}{37.25} \quad P = 7.89 \text{ kips}$$

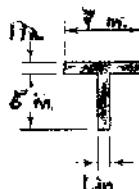
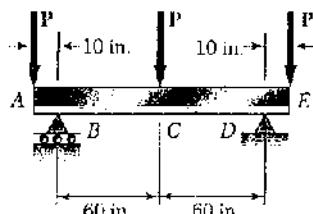
$$\text{Bot. Tension } 8 = -\frac{(20P)(-1.75)}{37.25} \quad P = 8.51 \text{ kips}$$

$$\text{Bot. Comp. } -18 = -\frac{(-10P)(-1.75)}{37.25} \quad P = 38.3 \text{ kips}$$

Smallest value of P is the allowable value

$$P = 7.01 \text{ kips}$$

Problem 5.86



5.86 Solve Prob. 5.85, assuming that the T-shaped beam is inverted.

5.85 Determine the allowable value of P for the loading shown, knowing that the allowable normal stress is +8 ksi in tension and -18 ksi in compression.

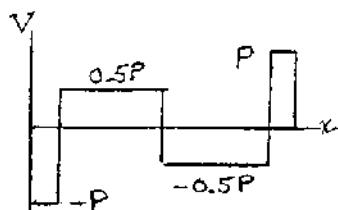
$$\text{Reactions. } B = D = 1.5 P \uparrow$$

$$\text{Shear diagram. } A \text{ to } B^+ \quad V = -P$$

$$B^+ \text{ to } C^- \quad V = -P + 1.5P = 0.5P$$

$$C^+ \text{ to } D^- \quad V = 0.5P - P = -0.5P$$

$$D^+ \text{ to } E \quad V = -0.5P + 1.5P = P$$

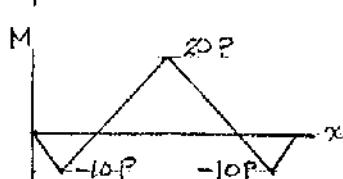


$$\text{Areas. } A \text{ to } B \quad (0)(-P) = -10P$$

$$B \text{ to } C \quad (60)(0.5P) = 30P$$

$$C \text{ to } D \quad (60)(-0.5P) = -30P$$

$$D \text{ to } E \quad (10)(P) = 10P$$



$$\text{Bending moments. } M_A = 0$$

$$M_B = 0 - 10P = -10P$$

$$M_C = -10P + 30P = 20P$$

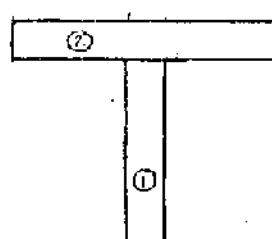
$$M_D = 20P - 30P = -10P$$

$$M_E = -10P + 10P = 0$$

$$\text{Largest positive bending moment} = 20P$$

$$\text{Largest negative bending moment} = -10P$$

Centroid and moment of inertia.



Part	$A_i \text{ in}^2$	$\bar{y}_i \text{ in.}$	$A\bar{y}_i \text{ in}^3$	$d_i \text{ in.}$	$Ad^3 \text{ in}^4$	$\bar{I} \text{ in}^4$
①	5	2.5	12.5	4.75	15.3125	10.417
②	7	5.5	38.5	1.25	10.9375	0.583
Σ	12		51		26.25	11.000

$$\text{Top: } y = 1.75 \text{ in.}$$

$$\text{Bottom: } y = -4.25 \text{ in.}$$

$$\sigma = -\frac{My}{I}$$

$$\bar{Y} = \frac{51}{12} = 4.25 \text{ in.}$$

$$I = \Sigma Ad^4 + \Sigma I = 37.25 \text{ in}^4$$

$$\text{Top, Tension} \quad \sigma = -\frac{(-10P)(4.75)}{37.25}$$

$$P = 17.03 \text{ kips}$$

$$\text{Top, Comp.} \quad -18 = -\frac{(20P)(4.75)}{37.25}$$

$$P = 19.46 \text{ kips}$$

$$\text{Bot. Tension} \quad \sigma = -\frac{(20P)(-4.25)}{37.25}$$

$$P = 3.51 \text{ kips}$$

$$\text{Bot. Comp.} \quad -18 = -\frac{(-10P)(-4.25)}{37.25}$$

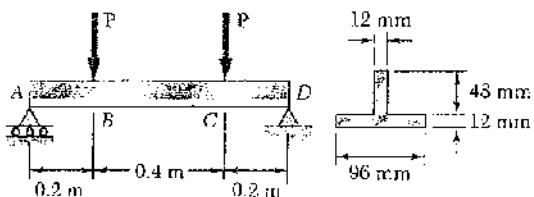
$$P = 15.78 \text{ kips}$$

Smallest value of P is the allowable value

$$P = 3.51 \text{ kips}$$

Problem 5.87

5.87 and 5.88 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and -140 MPa in compression.



$$\text{Reactions: } A = D = P$$

$$\text{Shear: } A \text{ to } B \quad V = P$$

$$B \text{ to } C \quad V = P - P = 0$$

$$C \text{ to } D \quad V = 0 - P = -P$$

$$\text{Areas: } A \text{ to } B \quad 0.2P$$

$$B \text{ to } C \quad 0$$

$$C \text{ to } D \quad -0.2P$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 + 0.2P = 0.2P$$

$$M_C = 0.2P + 0 = 0.2P$$

$$M_D = 0.2P - 0.2P = 0$$

Largest positive bending moment: $0.2P$

Largest negative bending moment: 0

Centroid and moment of inertia

Part	$A (\text{mm}^2)$	$\bar{y} (\text{mm})$	$A\bar{y} (\text{mm}^3)$	$d (\text{mm})$	$Ad^2 (\text{mm}^4)$	$\bar{I} \text{ mm}^4$
①	576	36	20736	20	230.4×10^3	110.6×10^3
③	1152	6	6912	10	115.2×10^3	13.8×10^3
2	1728		27648		345.6×10^3	124.4×10^3

$\bar{Y} = \frac{37648}{1728} = 16 \text{ mm}$

$$I = \sum Ad^2 + \sum \bar{I} = 470.6 \times 10^3 \text{ mm}^4$$

$$\text{Top } c = 44 \text{ mm} \quad S = \frac{I}{c} = \frac{470.6 \times 10^3}{44} = 10.68 \times 10^3 \text{ mm}^3 = 10.68 \times 10^{-6} \text{ m}^3$$

$$\text{Allowable pos. M. } M = 16_{\text{comp}} | S = (140 \times 10^6)(10.68 \times 10^{-6}) = 1495 \text{ N}\cdot\text{m}$$

$$\text{Bot. } c = 16 \text{ mm} \quad S = \frac{I}{c} = \frac{470.6 \times 10^3}{16} = 29.38 \times 10^3 \text{ mm}^3 = 29.38 \times 10^{-6} \text{ m}^3$$

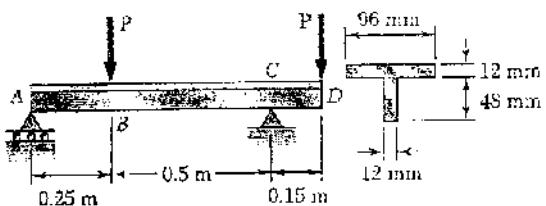
$$\text{Allowable pos. M } M = 16_{\text{tens}} | S = (80 \times 10^6)(29.38 \times 10^{-6}) = 2350 \text{ N}\cdot\text{m}$$

$$\text{Smaller value } M = 1495 \text{ N}\cdot\text{m}$$

$$\text{Allowable value of } P \quad 0.2P = 1495 \quad P = 7475 \text{ N} = 7.48 \text{ kN} \quad \blacktriangleleft$$

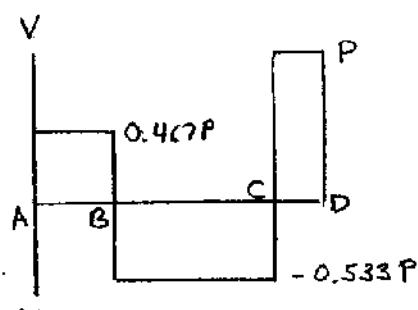
Problem 5.88

5.87 and 5.88 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and -140 MPa in compression.



$$\text{Clockwise } \sum M_C = 0 \quad -0.75A + 0.5P - 0.15P = 0 \\ A = 0.46667P$$

$$\text{Clockwise } \sum M_A = 0 \quad 0.75C - 0.25P - 0.9P = 0 \\ C = 1.53333P$$



$$\text{Shear: } A \text{ to } B \quad V = 0.46667P$$

$$B \text{ to } C \quad V = 0.46667P - P = -0.53333P$$

$$C \text{ to } D \quad V = -0.53333P + 1.53333P = P$$

$$\text{Areas: } A \text{ to } B \quad (0.25)(0.46667P) = 0.11667P$$

$$B \text{ to } C \quad (0.5)(-0.53333P) = -0.26667P$$

$$C \text{ to } D \quad (0.15)P = 0.15P$$

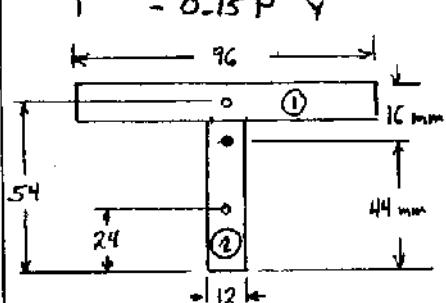
Bending moments: $M_A = 0$

$$M_B = 0 + 0.11667P = 0.11667P$$

$$M_C = 0.11667P - 0.26667P = -0.15P$$

$$M_D = -0.15P + 0.15P = 0$$

Centroid and moment of inertia.



Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	1152	54	62208	10	115200	13824
②	576	24	13824	20	230400	110592
Σ	1728		76032		345600	124416

$$\bar{Y} = \frac{76032}{1728} = 44 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 470016 \text{ mm}^4$$

$$\text{Top: } y = 16 \text{ mm} \quad \frac{I}{y} = \frac{470016}{16} = 29.376 \times 10^3 \text{ mm}^3 = 29.376 \times 10^{-6} \text{ m}^3$$

$$\text{Bottom: } y = -44 \text{ mm} \quad \frac{I}{y} = \frac{470016}{44} = 10.682 \times 10^3 \text{ mm}^3 = -10.682 \times 10^{-6} \text{ m}^3$$

$$\text{Bending moment limits: } M = -\frac{I}{y}$$

$$\text{Tension at B} = (-10.682 \times 10^{-6})(80 \times 10^6) = 854.56 \text{ N-m}$$

⇒ B

$$\text{Comp. at B} = -(29.376 \times 10^{-6})(-140 \times 10^6) = 4.1126 \times 10^3 \text{ N-m}$$

$$\text{Tension at C} = -(29.376 \times 10^{-6})(80 \times 10^6) = -2.35 \times 10^3 \text{ N-m}$$

$$\text{Comp. at C} = -(10.682 \times 10^{-6})(-140 \times 10^6) = -1.4955 \times 10^3 \text{ N-m}$$

⇒ C

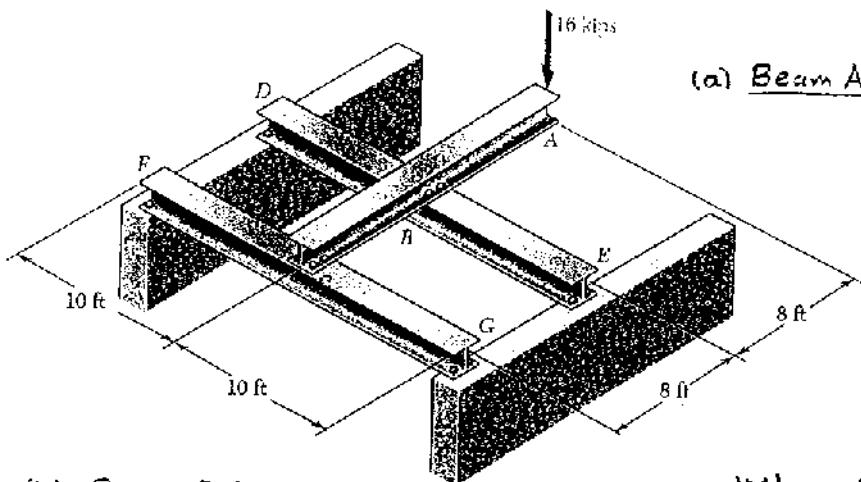
$$\text{Allowable load: } 0.11667P = 854.56 \quad P = 7.32 \times 10^3 \text{ N}$$

$$-0.15P = -1.4955 \times 10^3 \quad P = 9.97 \times 10^3 \text{ N}$$

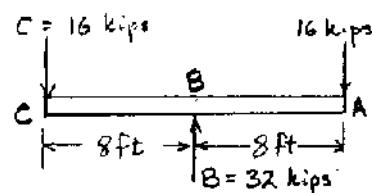
The smaller value is $P = 7.32 \text{ kN}$

Problem 5.89

5.89 Beam *ABC* is bolted to beams *DBE* and *FCG*. Knowing that the allowable normal stress is 24 ksi, select the most economical wide-flange shape that can be used
(a) for beam *ABC*, (b) for beam *DBE* (c) for beam *FCG*.



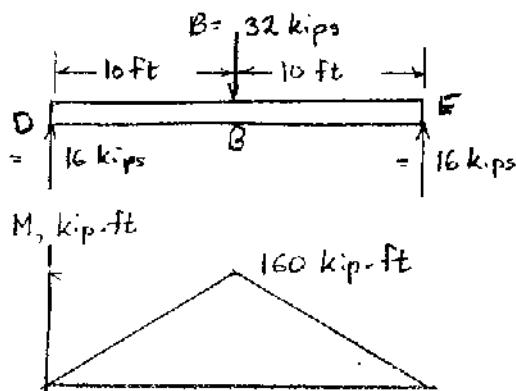
(a) Beam *ABC*



$$M_x \text{ kip-ft}$$

$$M_B = -(8)(16) = -128 \text{ kip-ft}$$

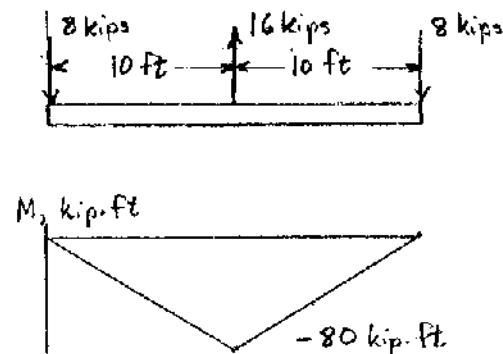
(b) Beam *DBE*



$$|M|_{\max} = 160 \text{ kip-ft} = 1920 \text{ kip-in}$$

$$S_{\min} = \frac{|M|_{\max}}{6 \text{ all}} = \frac{1920}{24} = 80 \text{ in}^3$$

(c) Beam *FCG*



$$|M|_{\max} = 80 \text{ kip-ft} = 960 \text{ kip-in}$$

$$S_{\min} = \frac{|M|_{\max}}{6 \text{ all}} = \frac{960}{24} = 40 \text{ in}^3$$

$$|M|_{\max} = 128 \text{ kip-ft} = 1536 \text{ kip-in}$$

$$S_{\min} = \frac{|M|_{\max}}{6 \text{ all}} = \frac{1536}{24} = 64 \text{ in}^3$$

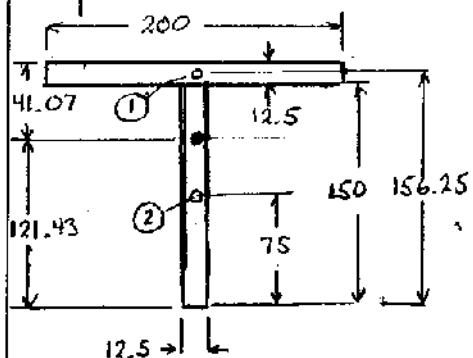
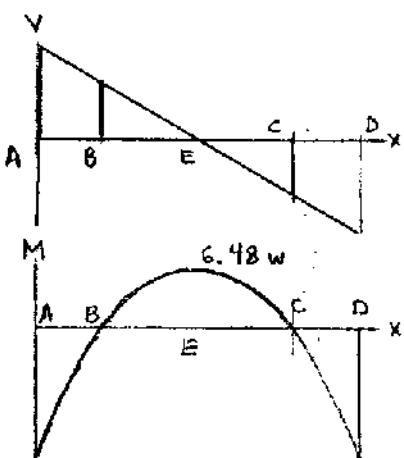
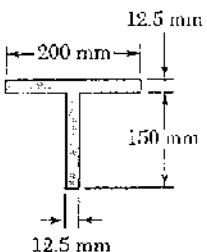
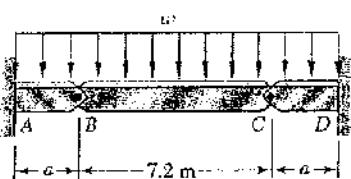
Shape	$S_x \text{ in}^3$
W 21 x 44	81.6
W 18 x 50	88.9
W 16 x 40	64.7 ← (a) W 16 x 40
W 14 x 53	77.8
W 12 x 50	64.7
W 10 x 68	75.7

Shape	$S_x \text{ in}^3$
W 21 x 44	81.6 ← (b) W 21 x 44
W 18 x 50	88.3
W 16 x 57	97.2
W 14 x 68	103
W 12 x 72	97.4
W 10 x 112	126

Shape	$S_x \text{ in}^3$
W 18 x 35	57.6
W 16 x 31	47.2
W 14 x 30	42.0 ← (c) W 14 x 30
W 12 x 35	45.6
W 10 x 39	42.1
W 8 x 48	43.3

Problem 5.90

5.90 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (a) the largest permissible value of w if beam BC is not to be overstressed, (b) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.



$$(a) M_B = M_C = 0 \\ V_B = -V_C = (\frac{1}{2})(7.2)w = 3.6w$$

$$\text{Area } B \text{ to } E \text{ of shear diagram} \\ (\frac{1}{2})(3.6)(3.6w) = 6.48w$$

$$M_E = 0 + 6.48w = 6.48w$$

Centroid and moment of inertia

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6
		$\bar{Y} = \frac{531250}{4375} = 121.43 \text{ mm}$				

$$I = \sum Ad^2 + \sum \bar{I} = 10.621 \times 10^6 \text{ mm}^4$$

Location	$y(\text{mm})$	$I/y(10^3 \text{ mm}^3)$	← also (10^{-6} m^3)
top	41.07	258.6	
bottom	-121.43	-87.47	

Bending moment limits $M = -6I/y$

$$\begin{aligned} \text{Tension at E: } & - (110 \times 10^6)(-87.47 \times 10^6) = 9.622 \times 10^9 \text{ N-m} \\ \text{Comp. at E: } & - (-150 \times 10^6)(258.6 \times 10^6) = 38.8 \times 10^9 \text{ N-m} \\ \text{Tension at A+D: } & - (110 \times 10^6)(258.6 \times 10^6) = -28.45 \times 10^9 \text{ N-m} \\ \text{Comp. at A+D: } & - (-150 \times 10^6)(-87.47 \times 10^6) = -13.121 \times 10^9 \text{ N-m} \end{aligned}$$

(a) Allowable load w

$$6.48w = 9.622 \times 10^9 \quad w = 1.485 \times 10^3 \text{ N/m} \\ = 1.485 \text{ kN/m} \quad \blacktriangleleft$$

Shear at A $V_A = (a + 3.6)w$

Area A to B of shear diagram $\frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a + 7.2)w$

Bending moment at A (also D) $M_A = \frac{1}{2}a(a + 7.2)w$

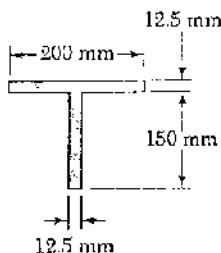
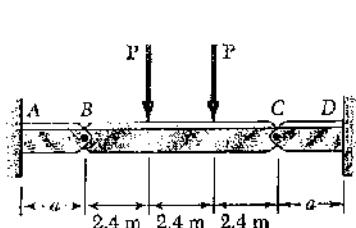
$$-\frac{1}{2}a(a + 7.2)(1.485 \times 10^3) = -13.121 \times 10^9$$

$$(b) \frac{1}{2}a^2 + 3.6a - 8.837 = 0$$

$$a = 1.935 \text{ m} \quad \blacktriangleleft$$

Problem 5.91

5.91 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (a) the largest permissible value of P if beam BC is not to be overstressed, (b) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed



$$(a) M_B = M_C = 0 \\ V_B = -V_C = P$$

Area B to E of shear diagram.
2.4 P

$$M_E = 0 + 2.4 P = 2.4 P = M_F$$

Centroid and moment of inertia

Part	A (mm ²)	\bar{y} (mm)	$A\bar{y}$ (mm ³)	d (mm)	Ad^2 (mm ⁴)	\bar{I} (mm ⁴)
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.51×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{y} = \frac{531250}{4375} = 121.43 \text{ mm}$$

$$I = \Sigma Ad^2 + \bar{I} = 10.621 \times 10^6 \text{ mm}^4$$

Location	y (mm)	I/y (10^3 mm^3)	← also (10^{-4} m^3)
top	41.07	258.6	
bottom	-121.43	-87.47	

Bending moment limits $M = -G I/y$

Tension at E & F: $-(110 \times 10^6)(-87.47 \times 10^6) = 9.622 \times 10^3 \text{ N}\cdot\text{m}$

Comp. at E & F: $-(-150 \times 10^6)(258.6 \times 10^6) = 38.8 \times 10^3 \text{ N}\cdot\text{m}$

Tension at A & D: $-(110 \times 10^6)(258.6 \times 10^6) = -28.45 \times 10^3 \text{ N}\cdot\text{m}$

Comp. at A & D: $-(-150 \times 10^6)(-87.47 \times 10^6) = -13.121 \times 10^3 \text{ N}\cdot\text{m}$

(a) Allowable load P.

$$2.4 P = 9.622 \times 10^3$$

$$P = 4.01 \times 10^3 \text{ N} \\ = 4.01 \text{ kN}$$

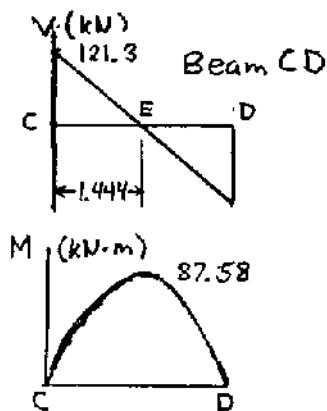
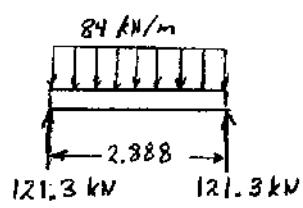
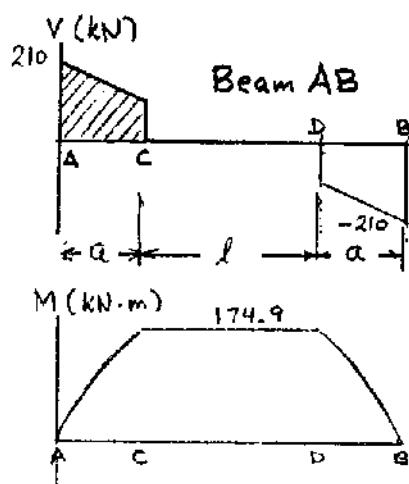
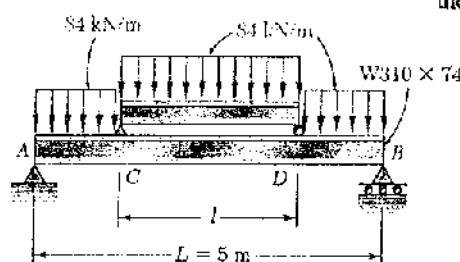
Shear at A $V_A = P$

Area A to B of shear diagram $\alpha V_A = \alpha P$

Bending moment at A $M_A = -\alpha P = -4.01 \times 10^3 \alpha$

(b) Distance a. $-4.01 \times 10^3 \alpha = -13.121 \times 10^3$ $\alpha = 3.27 \text{ m}$

Problem 5.92



5.92 A uniformly distributed load of 84 kN/m is to be supported over the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (a) the smallest allowable length l of beam CD if the W 310 × 74 beam AB is not to be overstressed, (b) the W shape which should be used for beam CD . Neglect the weight of both beams.

For rolled steel section W 310 × 74 of beam AB

$$S = 1060 \times 10^3 \text{ mm}^3 = 1060 \times 10^{-6} \text{ m}^3$$

$$\sigma_{all} = 165 \text{ MPa} = 165 \times 10^6 \text{ Pa}$$

Allowable bending moment

$$M_{all} = S \sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 174.9 \text{ kN}\cdot\text{m}$$

By symmetry reactions A and B are equal.

$$+\uparrow \sum F_y = 0 \quad A + B - (5)(84) = 0$$

$$A = B = 210 \text{ kN}$$

By symmetry, reaction at C and D are equal.

$$+\uparrow \sum F_y = 0 \quad C + D - 84l = 0$$

$$C = D = 42l$$

$$\text{Geometry} \quad a = \frac{1}{2}(5-l)$$

Beam AB: Area A to C of shear diagram

$$\frac{1}{2}(a)(A+C) = \frac{1}{2} \cdot \frac{1}{2}(5-l)(210 + 42l)$$

$$= \frac{1}{4}(1050 - 42l^2)$$

Bending moment at C $\frac{1}{4}(1050 - 42l^2)$

$$\frac{1}{4}(1050 - 42l^2) = 174.9 \quad l^2 = 8.3429 \text{ m}^2$$

$$(a) \quad l = 2.888 \text{ m}$$

$$C = D = 42l = 121.3 \text{ kN}$$

Beam CD (midpoint E)

Area C to E of shear diagram

$$\frac{1}{2}(1.444)(121.3) = 87.58 \text{ kN}\cdot\text{m}$$

Bending moment at E $M = 87.58 \text{ kN}\cdot\text{m} = 87.58 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma_{all} = 165 \times 10^6 \text{ Pa}$$

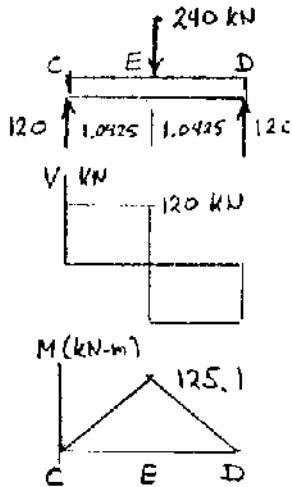
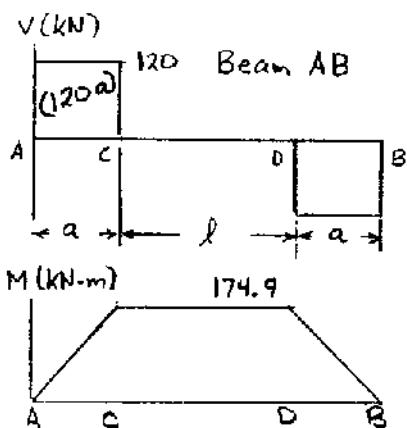
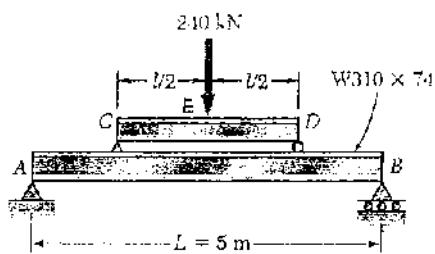
$$S_{min} = \frac{M}{\sigma_{all}} = \frac{87.58 \times 10^3}{165 \times 10^6} = 531 \times 10^{-6} \text{ m}^3 = 531 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
W 460 × 52	942
W 410 × 38.8	637
W 360 × 39	578
W 310 × 38.7	549
W 250 × 44.8	535
W 200 × 59	582

(b) Use W 310 × 38.7

Problem 5.93

5.93 A 240-kN load is to be supported at the center of the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (a) the smallest allowable length l of beam CD if the W310 × 74 beam AB is not to be overstressed, (b) the most economical W shape which can be used for beam CD. Neglect the weight of both beams.



For rolled steel section W 310 × 74 of beam AB

$$S = 1060 \times 10^3 \text{ mm}^3 = 1060 \times 10^{-6} \text{ m}^3$$

$$\sigma_{all} = 165 \text{ MPa} = 165 \times 10^6 \text{ Pa}$$

Allowable bending moment

$$M_{all} = S \sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 174.9 \text{ kN}\cdot\text{m}$$

(a) Beam AB

$$\text{Area } A \text{ to } C \text{ of shear diagram} = 120a$$

$$\text{Bending moment at } C = 120a$$

$$120a = 174.9 = 1.4575 \text{ m}$$

$$\text{Geometry: } 2a + l = 5 \quad l = 5 - 2a = 2.085 \text{ m} \blacksquare$$

(b) Beam CD (midpoint E)

$$\begin{aligned} \text{Area } C \text{ to } E \text{ of shear diagram} \\ = (1.0425)(120) = 125.1 \text{ kN}\cdot\text{m} \end{aligned}$$

Bending moment at E

$$M = 125.1 \text{ kN}\cdot\text{m} = 125.1 \times 10^3 \text{ N}\cdot\text{m}$$

$$S_{min} = \frac{M}{\sigma_{all}} = \frac{125.1 \times 10^3}{165 \times 10^6} = 758.2 \times 10^{-6} \text{ m}^3$$

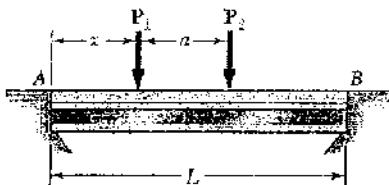
$$= 758.2 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
W 460 × 52	942
W 410 × 46.1	774 ←
W 360 × 52.8	899
W 310 × 60	851
W 250 × 67	809
W 200 × 86	853

Answer

W 410 × 46.1

Problem 5.94



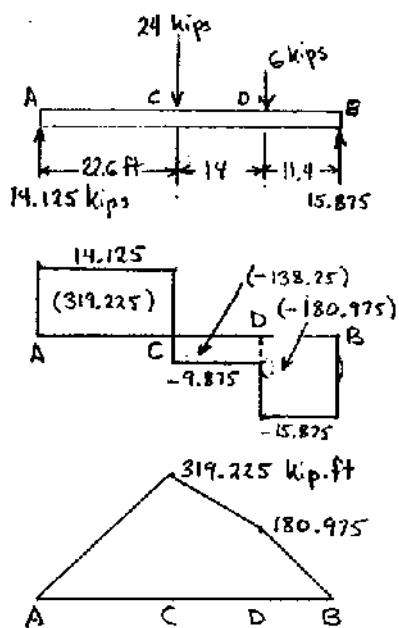
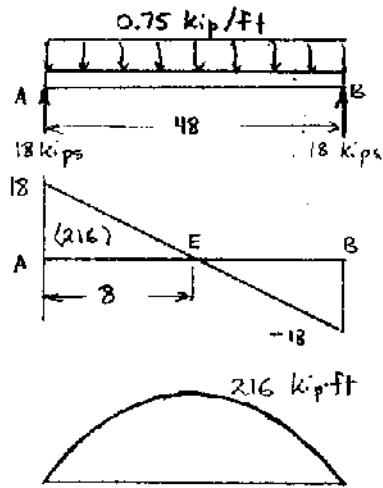
$$L = 48 \text{ ft}$$

$$a = 14 \text{ ft}$$

$$P_1 = 24 \text{ kips}$$

$$P_2 = 6 \text{ kips}$$

$$W = 0.75 \text{ kip/ft}$$



*5.94 A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_u = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads P_1 and P_2 exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [Hint. It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2(P_1 + P_2)$.]

$$\text{Dead load: } R_A = R_B = (\frac{1}{2})(48)(0.75) = 18 \text{ kips}$$

$$\text{Area A to E of shear diagram } (\frac{1}{2})(8)(18) = 216 \text{ kip}\cdot\text{ft}$$

$$M_{max} = 216 \text{ kip}\cdot\text{ft} = 2592 \text{ kip}\cdot\text{in. at point E}$$

$$\text{Live load: } U = \frac{aP_2}{2(P_1 + P_2)} = \frac{(14)(6)}{(2)(30)} = 1.4 \text{ ft}$$

$$x = \frac{L}{2} - U = 24 - 1.4 = 22.6 \text{ ft}$$

$$x + a = 22.6 + 14 = 36.6 \text{ ft}$$

$$L - x - a = 48 - 36.6 = 11.4 \text{ ft.}$$

$$\sum M_B = 0 - 48R_A + (25.4)(24) + (11.4)(6) = 0 \\ R_A = 14.125 \text{ kips}$$

$$\text{Shear: A to C } V = 14.125 \text{ kips}$$

$$\text{C to D } V = 14.125 - 24 = -9.875 \text{ kips}$$

$$\text{D to B } V = -15.875 \text{ kips}$$

$$\text{Area A to C } (22.6)(14.125) = 319.225 \text{ kip}\cdot\text{ft}$$

$$\text{Bending moment: } M_C = 319.225 \text{ kip}\cdot\text{ft} = 3831 \text{ kip}\cdot\text{in.}$$

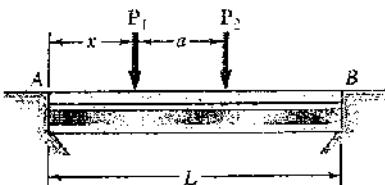
$$\text{Design: } \gamma_D M_0 + \gamma_L M_L = \phi M_u = \phi G_{ult} S_{min}$$

$$S_{min} = \frac{\gamma_D M_0 + \gamma_L M_L}{\phi G_{ult}} = \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)} \\ = 184.2 \text{ in}^3$$

Shape	S (in 3)
W 30x99	269
W 27x84	213 ←
W 24x104	258
W 21x101	227
W 18x106	204

W27x84

Problem 5.95



$$\begin{aligned}L &= 48 \text{ ft} \\a &= 14 \text{ ft} \\P_1 &= 24 \text{ kips} \\P_2 &= 6 \text{ kips} \\w &= 0.75 \text{ kip/ft}\end{aligned}$$

*5.95 Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.94, determine how much heavier a truck could safely cross the bridge designed in that problem.

*5.94 A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_u = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads P_1 and P_2 exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [Hint. It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2(P_1 + P_2)$.]

See solution to Problem 5.94 for calculation of the following:

$$M_D = 2592 \text{ kip-in} \quad M_L = 3831 \text{ kip-in}$$

$$\text{For rolled steel section W27x84} \quad S = 213 \text{ in}^3$$

Allowable live load moment M_L^*

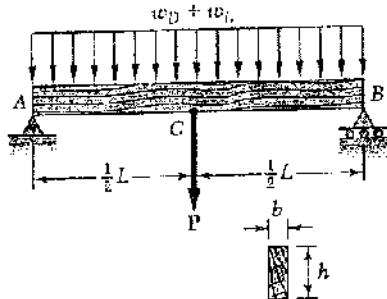
$$\gamma_D M_D + \gamma_L M_L^* = \phi M_u = \phi \sigma_u S$$

$$M_L^* = \frac{\phi \sigma_u S - \gamma_D M_D}{\gamma_L} = \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75} = 4721 \text{ kip-in}$$

$$\text{Ratio } \frac{M_L^*}{M_L} = \frac{4721}{3831} = 1.232 = 1 + 0.232$$

Increase 23.2 %

Problem 5.96



*5.96 A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16 \text{ m}$. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350 \text{ N/m}$. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600 \text{ N/m}$, and a 6-kN concentrated load P applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_u = 50 \text{ MPa}$ and that the width of the beam is $b = 75 \text{ mm}$, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

$$L = 16 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m} \\ w_L = 600 \text{ N/m} = 0.6 \text{ kN/m}, \quad P = 6 \text{ kN}$$

Dead load: $R_A = (\frac{1}{2})(16)(0.35) = 2.8 \text{ kN}$

Area A to C of shear diagram

$$(\frac{1}{2})(8)(2.8) = 11.2 \text{ kN}\cdot\text{m}$$

Bending moment at C: $11.2 \text{ kN}\cdot\text{m} = 11.2 \times 10^3 \text{ N}\cdot\text{m}$

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8 \text{ kN}$

Shear at C: $V = 7.8 - (8)(0.6) = 3 \text{ kN}$

Area A to C of shear diagram

$$(\frac{1}{2})(8)(7.8 + 3) = 43.2 \text{ kN}\cdot\text{m}$$

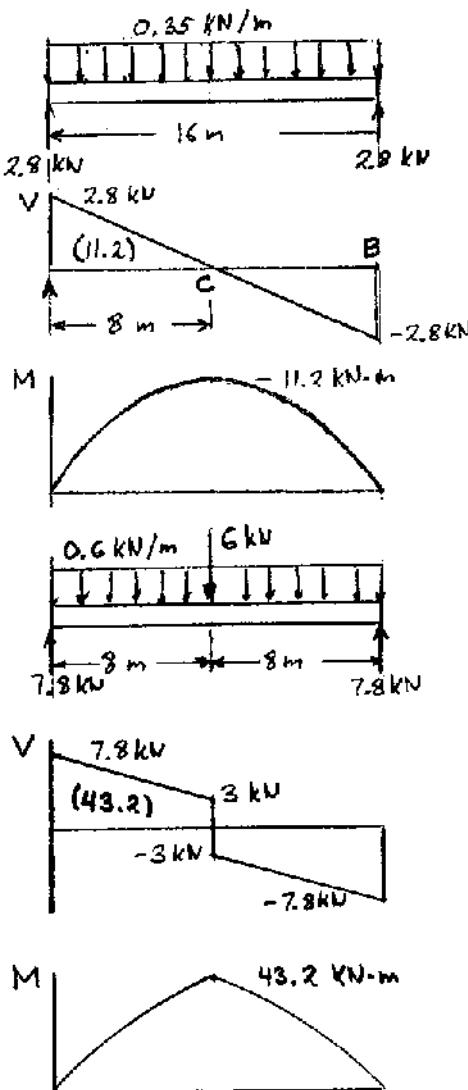
Bending moment at C: $43.2 \text{ kN}\cdot\text{m} = 43.2 \times 10^3 \text{ N}\cdot\text{m}$

Design $\gamma_D M_D + \gamma_L M_L = \phi M_u = \phi \sigma_u S$

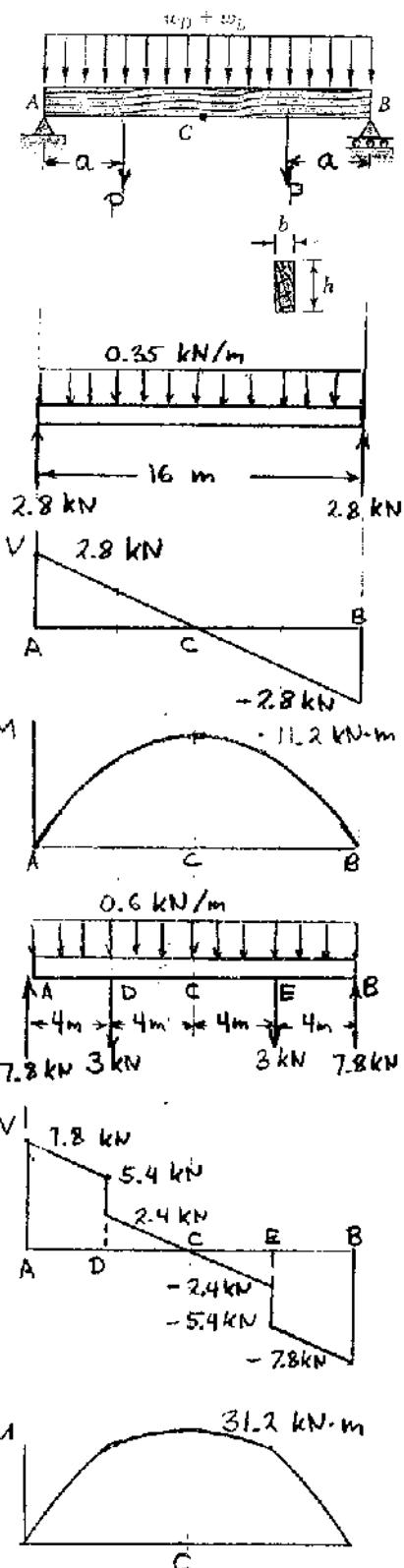
$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_u} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)} \\ = 1.8347 \times 10^{-3} \text{ m}^3 = 1.8347 \times 10^6 \text{ mm}^3$$

For a rectangular section $S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}} = 383 \text{ mm} \rightarrow$$



Problem 5.97



*5.97 Solve Prob. 5.96, assuming that the 6-kN concentrated load P applied to each beam is replaced by 3-kN concentrated loads P_1 and P_2 applied at a distance of 4 m from each end of the beams.

*5.96 A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load P applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_u = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

$$L = 16 \text{ m}, \quad a = 4 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m} \\ w_L = 600 \text{ N/m} = 0.6 \text{ kN/m} \quad P = 3 \text{ kN}$$

Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8 \text{ kN}$

Area A to C of shear diagram

$$\left(\frac{1}{2}\right)(8)(2.8) = 11.2 \text{ kN-m}$$

Bending moment at C: $11.2 \text{ kN-m} = 11.2 \times 10^3 \text{ N-m}$

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8 \text{ kN}$

Shear at D: $7.8 - (4)(0.6) = 5.4 \text{ kN}$

Shear at D': $5.4 - 3 = 2.4 \text{ kN}$

Area A to D: $\left(\frac{1}{2}\right)(4)(7.8 + 5.4) = 26.4 \text{ kN-m}$

Area D to C: $\left(\frac{1}{2}\right)(4)(2.4) = 4.8 \text{ kN-m}$

Bending moment at C = $26.4 + 4.8 = 31.2 \text{ kN-m}$
 $= 31.2 \times 10^3 \text{ N-m}$

Design $\gamma_D M_0 + \gamma_L M_L = \phi M_u = \phi G_{ult} S$

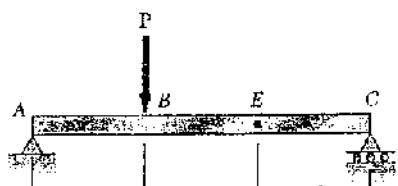
$$S = \frac{\gamma_D M_0 + \gamma_L M_L}{\phi G_{ult}} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(31.2 \times 10^3)}{(0.9)(50 \times 10^6)} \\ = 1.408 \times 10^{-3} \text{ m}^3 = 1.408 \times 10^6 \text{ mm}^3$$

For a rectangular section $S = \frac{1}{6} b h^2$

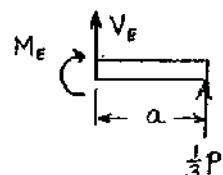
$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408 \times 10^6)}{75}} = 336 \text{ mm}$$

Problem 5.98

5.98 through 5.100 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of the beam to the right of E .



$$\textcircled{+} \sum M_c = 0 \quad -3aA + 2aP = 0 \quad A = \frac{2}{3}P$$



$$(a) V = \frac{2}{3}P - P(x-a)$$

$$M = \frac{2}{3}Px - P(x-a)$$

At point $E \quad x = 2a$

$$(b) M_E = \frac{2}{3}P(2a) - Pa = \frac{1}{3}Pa$$

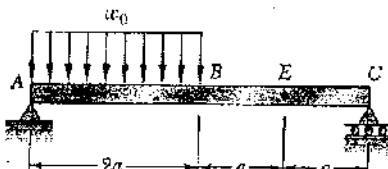
$$\sum M_A = 0 \quad 3aC - aP = 0 \quad C = \frac{1}{3}P$$

$$\textcircled{+} \sum M_E = 0 \quad -M_E + (a)(\frac{1}{3}P) = 0$$

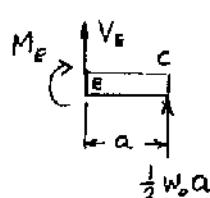
$$M_E = \frac{1}{3}Pa$$

Problem 5.99

5.98 through 5.100 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of the beam to the right of E .



$$\textcircled{+} \sum M_c = 0 \quad -4aA + (3a)(2a)w_0 = 0 \quad A = \frac{3}{2}w_0a$$



$$w = w_0 - w_0(x-2a)^\circ = -\frac{dw}{dx}$$

$$(a) V = -w_0x + w_0(x-2a) + \frac{3}{2}w_0a = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0x^2 + \frac{1}{2}w_0(x-2a)^2 + \frac{3}{2}w_0ax + 0$$

At point $E \quad x = 3a$

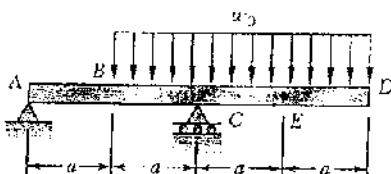
$$(b) M_E = -\frac{1}{2}w_0(3a)^2 + \frac{1}{2}w_0a^2 + \frac{3}{2}w_0a(3a) \\ = \frac{1}{2}w_0a^2$$

$$\textcircled{+} \sum M_A = 0 \quad 4aC - (2)(2aw_0) = 0 \quad C = \frac{1}{2}w_0a$$

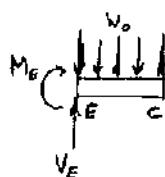
$$\textcircled{+} \sum M_E = 0 \quad -M_E + (a)(\frac{1}{2}w_0a) = 0$$

$$M_E = \frac{1}{2}w_0a^2$$

Problem 5.100



5.98 through 5.100 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of the beam to the right of E.



$$\rightarrow \sum M_C = 0 \quad -2aA + (\frac{a}{2})(3aw_0) = 0 \quad A = -\frac{3}{4}w_0a$$

$$\rightarrow \sum M_A = 0 \quad 2aC + (\frac{5a}{2})(3aw_0) = 0 \quad C = \frac{15}{4}w_0a$$

$$w = w_0(x-a)^0 = -\frac{dV}{dx}$$

$$(a) V = -w_0(x-a)^0 - \frac{3}{4}w_0a + \frac{15}{4}w_0a(x-2a)^0 = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0(x-a)^2 - \frac{3}{4}w_0ax + \frac{15}{4}w_0a(x-2a)^1 + 0$$

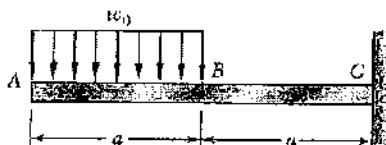
$$\text{At point } E \quad x = 3a$$

$$(b) M_E = -\frac{1}{2}w_0(2a)^2 - \frac{3}{4}w_0a(3a) + \frac{15}{4}w_0a(a) \\ = -\frac{1}{2}w_0a^2$$

$$\text{Check: } + \sum M_E = 0 \quad -M_E - \frac{a}{2}(w_0a) = 0$$

$$M_E = -\frac{1}{2}w_0a^2$$

Problem 5.101



5.101 through 5.103 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

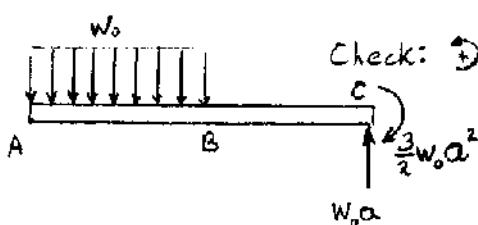
$$w = w_0 - w_0(x-a)^0 = -\frac{dV}{dx}$$

$$(a) V = -w_0x + w_0(x-a)^1 = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0x^2 + \frac{1}{2}w_0(x-a)^2$$

$$\text{At point } C \quad x = 2a$$

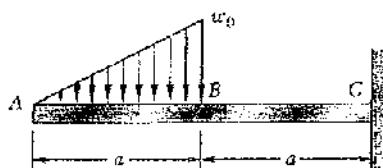
$$(b) M_c = -\frac{1}{2}w_0(2a)^2 + \frac{1}{2}w_0a^2 = -\frac{3}{2}w_0a^2$$



$$\text{Check: } \sum M_c = 0 \quad (\frac{3a}{2})(w_0a) + M_c = 0$$

$$M_c = -\frac{3}{2}w_0a^2$$

Problem 5.102



S.101 through 5.103 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

$$W = \frac{w_0 x}{a} - w_0 (x-a)^0 - \frac{w_0}{a} (x-a)^1 = -\frac{dV}{dx}$$

$$(a) V = -\frac{w_0 x^2}{2a} + w_0 (x-a)^1 + \frac{w_0}{2a} (x-a)^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2} (x-a)^2 + \frac{w_0}{6a} (x-a)^3$$

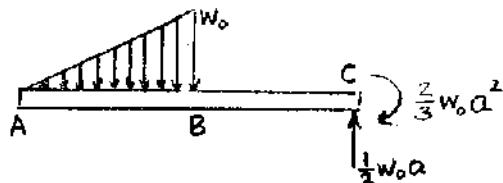
At point C $x = 2a$

$$(b) M_c = -\frac{w_0 (2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a} = -\frac{2}{3} w_0 a^2$$

Check: $\sum M_c = 0$

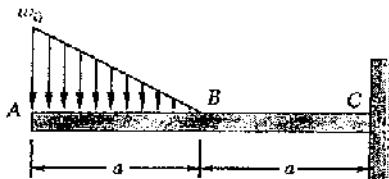
$$\left(\frac{4a}{3}\right)\left(\frac{1}{2}w_0 a\right) + M_c = 0$$

$$M_c = -\frac{2}{3} w_0 a^2$$



Problem 5.103

S.101 through 5.103 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.



$$W = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a} (x-a)^1 = -\frac{dV}{dx}$$

$$(a) V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a} (x-a)^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a} (x-a)^3$$

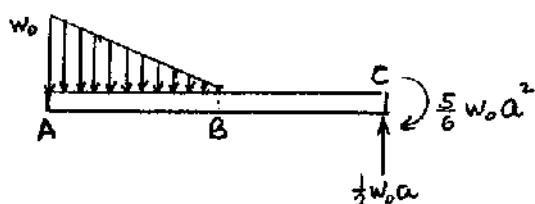
At point C $x = 2a$

$$(b) M_c = -\frac{w_0 (2a)^2}{2} + \frac{w_0 (2a)^3}{6a} - \frac{w_0 a^3}{6a} = -\frac{15}{6} w_0 a^2$$

Check: $\sum M_c = 0$

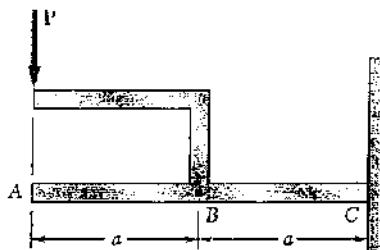
$$\left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_c = 0$$

$$M_c = -\frac{5}{6} w_0 a^2$$



Problem 5.104

5.104 (a) Using singularity functions, write the equations for the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for M to determine the bending moment just to the right of point B.



$$(a) V = -P(x-a)^0$$

$$\frac{dM}{dx} = -P(x-a)^0$$

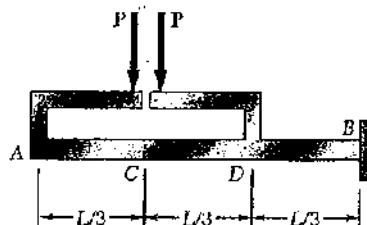
$$M = -P(x-a)^1 - Pa(x-a)^0$$

Just to the right of B $x = a^+$

$$(b) M = -0 - Pa = -Pa$$

Problem 5.105

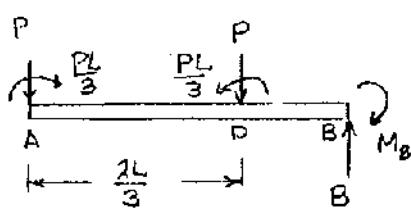
5.105 (a) Using singularity functions, write the equations for the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for M to determine the bending moment just to the right of point D.



$$(a) V = -P - P(x-\frac{2L}{3})^0 = \frac{dM}{dx}$$

$$M = -Px + \frac{PL}{3} - P(x-\frac{2L}{3})^1 - \frac{PL}{3}(x-\frac{2L}{3})^0$$

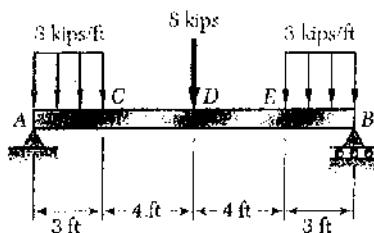
Just to the right of D $x = \frac{2L}{3}$



$$(b) M_D^+ = -P(\frac{2L}{3}) + \frac{PL}{3} - P(0) - \frac{PL}{3} = -\frac{4PL}{3}$$

Problem 5.106

5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



$$\textcircled{S} \sum M_A = 0$$

$$-14A + (12.5)(3)(3) + (7)(8) + (1.5)(3)(3) = 0$$

$$A = 13 \text{ kips} \uparrow$$

$$W = 3 - 3(x-3)^0 + 3(x-11)^0 = -\frac{dV}{dx}$$

$$(a) V = 13 - 3x + 3(x-3)^0 - 8(x-7)^0 - 3(x-11)^0 \text{ kips}$$

$$M = 13x - 1.5x^2 + 1.5(x-3)^2 - 8(x-7)^1 - 1.5(x-11)^2 \text{ kip-ft}$$

$$V_C^- = 13 - (3)(3) = 4 \text{ kips}$$

$$V_D^+ = 13 - (3)(7) + (3)(4) = 4 \text{ kips} \leftarrow \text{Changes sign}$$

$$V_E^- = 13 - (3)(7) + (3)(4) - 8 = -4 \text{ kips}$$

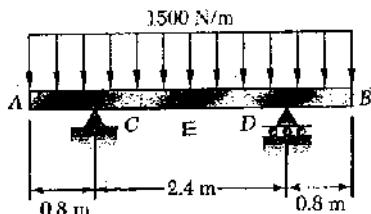
$$V_F^- = 13 - (3)(11) + (3)(8) - 8 = -4 \text{ kips}$$

$$V_B = 13 - (3)(14) + (3)(11) - 8 - (3)(3) = +13 \text{ kips}$$

$$(b) |M|_{\max} = M_b = (13)(7) - (1.5)(7)^2 + (1.5)(4)^2 - 0 - 0 = 41.5 \text{ kip-ft}$$

Problem 5.107

5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



$$W = 1.5 \text{ kN/m}$$

$$\text{By statics } C = D = 3 \text{ kN} \uparrow$$

$$(a) V = -1.5x + 3(x-0.8)^0 + 3(x-3.2)^0 \text{ kN}$$

$$M = -0.75x^2 + 3(x-0.8)^1 + 3(x-3.2)^1 \text{ kN-m}$$

Locate point E where $V=0$ Assume $x_C < x_E < x_D$

$$0 = -1.5x_C + 3(x_E - 0.8) + 0 \quad x_E = 2.0 \text{ m}$$

$$M_C = -(0.75)(0.8)^2 + 0 + 0 = -0.480 \text{ kN-m}$$

$$M_E = -(0.75)(2.0)^2 + (3)(1.2) + 0 = 0.600 \text{ kN-m}$$

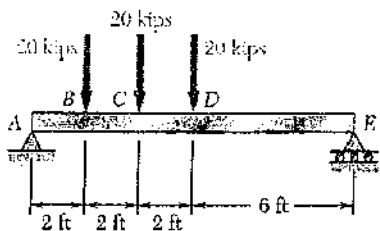
$$M_D = -(0.75)(3.2)^2 + (3)(2.4) + 0 = -0.480 \text{ kN-m}$$

$$(b) |M|_{\max} = 0.600 \text{ kN-m}$$

$$(b) 600 \text{ N-m}$$

Problem 5.108

5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



$$+\sum M_E = 0 \quad -12A + (10)(20) + (8)(20) + (6)(20) = 0 \\ A = 40 \text{ kips.}$$

$$(a) V = 40 - 20(x-2)^0 - 20(x-4)^0 - 20(x-6)^0 \text{ kips}$$

$$M = 40x - 20(x-2)' - 20(x-4)' - 20(x-6)' \text{ kip}\cdot\text{ft}$$

Values of V

$$A \text{ to } B \quad V = 40 \text{ kip}$$

$$B \text{ to } C \quad V = 40 - 20 = 20 \text{ kips}$$

$$C \text{ to } D \quad V = 40 - 20 - 20 = 0$$

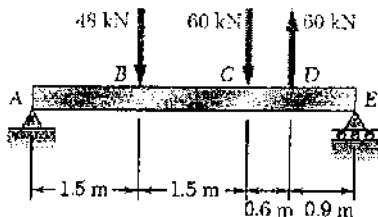
$$D \text{ to } E \quad V = 40 - 20 - 20 - 20 = -20 \text{ kip}$$

Bending moment is constant and maximum over C to D.

$$(b) \text{ At } C \quad x = 4 \text{ ft} \quad M = (40)(4) - (20)(2) - 0 - 0 = 120 \text{ kip}\cdot\text{ft}$$

Problem 5.109

5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



$$+\sum M_E = 0 \quad -4.5R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0 \\ R_A = 40 \text{ kN}$$

$$(a) V = 40 - 48(x-1.5)^0 - 60(x-3.0)^0 + 60(x-3.6)^0 \text{ kN}$$

$$M = 40x - 48(x-1.5)' - 60(x-3.0)' + 60(x-3.6)' \text{ kN}\cdot\text{m}$$

Pt. x (m) M (kN·m)

$$A \quad 0 \quad 0$$

$$B \quad 1.5 \quad (40)(1.5) = 60 \text{ kN}\cdot\text{m}$$

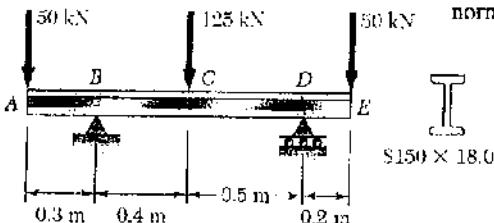
$$C \quad 3.0 \quad (40)(3.0) - (48)(1.5) = 48 \text{ kN}\cdot\text{m}$$

$$D \quad 3.6 \quad (40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2 \text{ kN}\cdot\text{m}$$

$$E \quad 4.5 \quad (40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$$

$$(b) M_{max} = 60 \text{ kN}\cdot\text{m}$$

Problem 5.110



5.110 and 5.111 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.

$$+\sum M_D = 0 \\ (1.2)(50) - 0.9B + (0.5)(125) - (0.2)(50) = 0 \\ B = 125 \text{ kN} \uparrow$$

$$-\sum M_B = 0 \\ (0.3)(50) - (0.4)(125) + 0.9D - (1.1)(50) = 0 \\ D = 100 \text{ kN} \uparrow$$

(a) $V = -50 + 125(x-0.3)^0 - 125(x-0.7)^0 + 100(x-1.2)^0 \text{ kN}$

$$M = -50x + 125(x-0.3)' - 125(x-0.7)' + 100(x-1.2)' \text{ kN.m}$$

Point x (m) M (kN-m)

$$B \quad 0.3 \quad -(50)(0.3) + 0 - 0 + 0 = -15 \text{ kN.m}$$

$$C \quad 0.7 \quad -(50)(0.7) + (125)(0.4) - 0 + 0 = 15 \text{ kN.m}$$

$$D \quad 1.2 \quad -(50)(1.2) + (125)(0.9) - (125)(0.5) + 0 = -10 \text{ kN.m}$$

$$E \quad 1.4 \quad -(50)(1.4) + (125)(1.1) - (125)(0.7) + (100)(0.2) = 0 \text{ checks.}$$

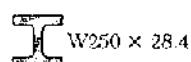
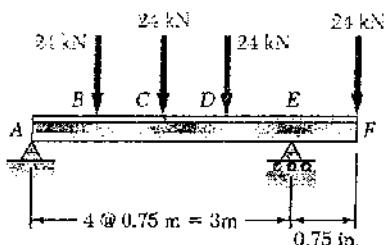
$$\text{Maximum } |M| = 15 \text{ kN.m} = 15 \times 10^3 \text{ N.m}$$

For S 150 x 18.0 rolled steel section $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

(b) Normal stress $\sigma = \frac{|M|}{S} = \frac{15 \times 10^3}{120 \times 10^{-6}} = 125 \times 10^6 \text{ Pa} = 125.0 \text{ MPa}$

Problem 5.111

5.110 and 5.111 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



$$\text{+} \sum M_E = 0 \\ -3R_A + (2.25)(24) - (1.5)(24) \\ - (0.75)(24) + (0.75)(24) = 0$$

$$R_A = 30 \text{ kips}$$

$$\text{+} \sum M_A = 0 \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0 \\ R_E = 66 \text{ kips}$$

$$(a) V = 30 - 24(x - 0.75)^0 - 24(x - 1.5)^0 - 24(x - 2.25)^0 + 66(x - 3)^0 \text{ kN}$$

$$M = 30x - 24(x - 0.75)' - 24(x - 1.5)' - 24(x - 2.25)' + 66(x - 3)' \text{ kN-m}$$

$$P_t \quad x(m) \quad M(kN \cdot m)$$

$$B \quad 0.75 \quad (30)(0.75) = 22.5 \text{ kN-m}$$

$$C \quad 1.5 \quad (30)(1.5) - (24)(0.75) = 27 \text{ kN-m}$$

$$D \quad 2.25 \quad (30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5 \text{ kN-m}$$

$$E \quad 3.0 \quad (30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18 \text{ kN-m}$$

$$F \quad 3.75 \quad (30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$$

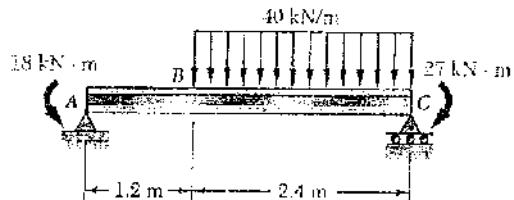
$$\text{Maximum } |M| = 27 \text{ kN-m} = 27 \times 10^3 \text{ N-m}$$

$$\text{For rolled steel section W250 x 28.4} \quad S = 308 \times 10^6 \text{ mm}^3 \\ = 308 \times 10^{-6} \text{ m}^3$$

$$(b) \text{ Normal stress } \sigma = \frac{|M|}{S} = \frac{27 \times 10^3}{308 \times 10^{-6}} = 87.7 \times 10^6 \text{ Pa} = 87.7 \text{ MPa}$$

Problem 5.112

5.112 and 5.113 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



I S310 × 52 $\rightarrow M_c = 0$

$$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$$

$$R_A = 29.5 \text{ kN}$$

$$V = 29.5 - 40(x - 1.2) \text{ KN}$$

$$\text{Point D} \quad V = 0 \quad 29.5 - 40(x_D - 1.2) = 0 \\ x_D = 1.9375 \text{ m}$$

$$M = -18 + 29.5x - 20(x - 1.2)^2 \text{ kN·m}$$

$$M_A = -18 \text{ kN·m}$$

$$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN·m}$$

$$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN·m}$$

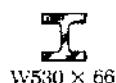
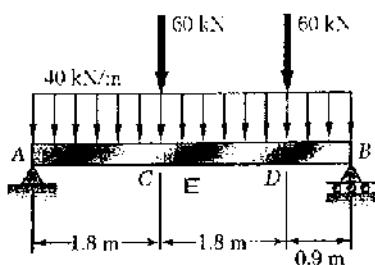
- (a) Maximum $|M| = 28.278 \text{ kN·m}$ at $x = 1.9375 \text{ m}$

For S310 × 52 rolled steel section $S = 625 \times 10^3 \text{ mm}^3$
 $= 625 \times 10^{-6} \text{ m}^3$

- (b) Normal stress $\sigma = \frac{|M|}{S} = \frac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa} = 45.2 \text{ MPa}$

Problem 5.113

5.112 and 5.113 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



$$\begin{aligned} \text{At } B: \sum M_B &= 0 \\ -4.5A + (2.25)(4.5)(40) + (2.7)(60) + (0.9)(60) &= 0 \\ A &= 138 \text{ kN} \uparrow \end{aligned}$$

$$\begin{aligned} \text{At } A: \sum M_A &= 0 \\ -(2.25)(4.5)(40) - (1.8)(60) - (3.6)(60) + 4.5B &= 0 \\ B &= 162 \text{ kN} \uparrow \end{aligned}$$

$$W = 40 \text{ kN} = \frac{dV}{dx}$$

$$V = -40x + 138 - 60(x-1.8)^0 - 60(x-3.6)^0 = \frac{dM}{dx}$$

$$M = -20x^2 - 138x - 60(x-1.8)^1 - 60(x-3.6)^1$$

$$V_C^+ = -(40)(1.8) + 138 - 60 = 6 \text{ kN}$$

$$V_D^- = -(40)(3.6) + 138 - 60 = -66 \text{ kN}$$

Locate point E where $V = 0$. It lies between C and D.

$$V_E = -40x_E + 138 - 60 + 0 = 0 \quad x_E = 1.95 \text{ m}$$

$$M_E = -(20)(1.95)^2 + (138)(1.95) - (60)(1.95 - 1.8) = 184 \text{ kN}\cdot\text{m}$$

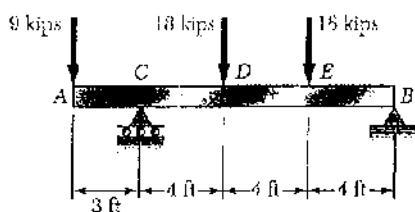
$$(a) |M|_{max} = 184 \text{ kN}\cdot\text{m} = 184 \times 10^3 \text{ N}\cdot\text{m} \text{ at } x = 1.950 \text{ m}$$

For W 530 x 66 rolled steel section $S = 1340 \times 10^3 \text{ mm}^3 = 1340 \times 10^{-6} \text{ m}^3$

$$(b) \text{ Normal stress } \sigma = \frac{|M|_{max}}{S} = \frac{184 \times 10^3}{1340 \times 10^{-6}} = 137.3 \times 10^6 \text{ Pa} = 137.3 \text{ MPa}$$

Problem 5.114

5.114 and 5.115 A beam is being designed to be supported and loaded as shown.
 (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.



$$\rightarrow \sum M_B = 0$$

$$(15)(9) - 12C + (8)(18) + (4)(18) = 0$$

$$C = 29.25 \text{ kips}$$

$$\rightarrow \sum M_C = 0$$

$$(3)(9) - (4)(18) - (8)(18) + 12B = 0$$

$$B = 15.75$$

$$V = -9 + 29.25(x-3)^0 - 18(x-7)^0 - 18(x-11)^0$$

$$M = -9x + 29.25(x-3)^1 - 18(x-7)^1 - 18(x-11)^1$$

$$\text{At } A \quad x = 0 \quad M_A = 0$$

$$\text{At } C \quad x = 3 \text{ ft} \quad M_C = -(9)(3) + 0 - 0 - 0 = -27 \text{ kip-ft}$$

$$\text{At } D \quad x = 7 \text{ ft} \quad M_D = -(9)(7) + (29.25)(4) - 0 - 0 = 54 \text{ kip-ft}$$

$$\text{At } E \quad x = 11 \text{ ft} \quad M_E = -(9)(11) + (29.25)(8) - (18)(4) - 0 = 63 \text{ kip-ft}$$

$$\text{At } B \quad x = 15 \text{ ft} \quad M_B = -(9)(15) + (29.25)(12) - (18)(8) - (18)(4) = 0 \text{ checks}$$

$$(a) |M|_{\max} = 63 \text{ kip-ft at } E$$

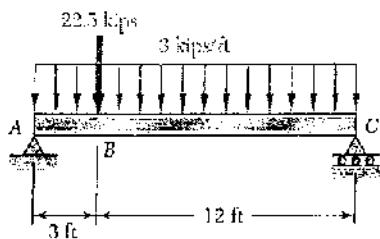
$$|M|_{\max} = 758 \text{ kip-in}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{758}{24} = 31.5 \text{ in}^3$$

Shape	S (in ³)
W 16 x 26	38.4
W 14 x 26	35.3
W 12 x 26	33.4
W 10 x 33	35.0

W 16 x 26 or W 14 x 26 or W 12 x 26

Problem 5.115



5.114 and 5.115 A beam is being designed to be supported and loaded as shown.
 (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

$$\rightarrow \sum M_c = 0 \quad -15 R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$R_A = 40.5 \text{ kips}$$

$$w = 3 \text{ kips/ft} = -\frac{dv}{dx}$$

$$V = 40.5 - 3x - 22.5(x-3)^0 \text{ kips}$$

Location of point D where $V = 0$. Assume $3 < x_D < 12$

$$0 = 40.5 - 3x_D - 22.5 \quad x_D = 6 \text{ ft}$$

$$M = 40.5x - 1.5x^2 - 22.5(x-3)^1 \text{ kip-ft}$$

$$\text{At point D } (x = 6 \text{ ft}) \quad M = (40.5)(6) - (1.5)(6)^2 - (22.5)(3)$$

$$= 121.5 \text{ kip-ft} = 1458 \text{ kip-in}$$

(a) Maximum $|M| = 121.5 \text{ kip-ft}$ at $x = 6 \text{ ft}$.

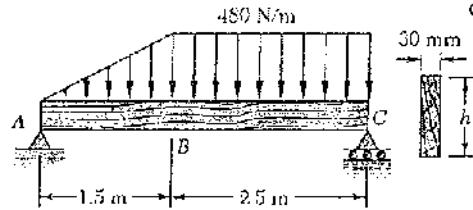
$$S_{min} = \frac{M}{\sigma_{all}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

Shape	$S (\text{in}^3)$
W 21 x 44	81.6
W 18 x 50	88.9
W 16 x 40	64.7
W 14 x 43	62.7
W 12 x 50	64.7
W 10 x 68	75.7

Answer W 16 x 40

Problem 5.116

5.116 and 5.117 A timber beam is being designed to be supported and loaded as shown. (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



$$480 \text{ N/m} = 0.48 \text{ kN/m}$$

$$\rightarrow \sum M_C = 0$$

$$-4 R_A + (3)(\frac{1}{2})(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$$

$$R_A = 0.645 \text{ kN}$$

$$W = \frac{0.48}{1.5} x - \frac{0.48}{1.5} \langle x - 1.5 \rangle^1 = 0.32x - 0.32 \langle x - 1.5 \rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16x^2 + 0.16 \langle x - 1.5 \rangle^2 \text{ kN}$$

Locate point D where $V = 0$. Assume $1.5 \text{ m} < x_D < 4 \text{ m}$

$$0 = 0.645 - 0.16x_D^2 + 0.16(x_D - 1.5)^2$$

$$= 0.645 - 0.16x_D^2 + 0.16x_D^2 - 0.48x_D + 0.36$$

$$x_D = 2.09375 \text{ m}$$

$$M = 0.645x - 0.05333x^3 + 0.05333 \langle x - 1.5 \rangle^3 \text{ kN-m}$$

(a) At point D

$$M_D = (0.645)(2.09375) - (0.05333)(2.09375)^3 + (0.05333)(0.59375)^3$$

$$= 0.87211 \text{ kN-m}$$

$$S_{min} = \frac{M_D}{S_{all}} = \frac{0.87211 \times 10^3}{12 \times 10^6} = 72.6758 \times 10^{-6} \text{ m}^3 = 72.6758 \times 10^3 \text{ mm}^3$$

For a rectangular cross section

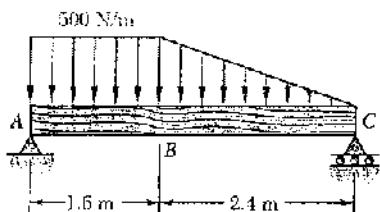
$$S = \frac{1}{4} b h^2 \quad h = \sqrt{\frac{6S}{b}}$$

$$h_{min} = \sqrt{\frac{(6)(72.6758 \times 10^3)}{30}} = 120.56 \text{ mm}$$

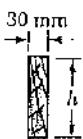
(b) At Next larger 10-mm increment

$$h = 130 \text{ mm}$$

Problem 5.117



5.116 and 5.117 A timber beam is being designed to be supported and loaded as shown. (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



$$\rightarrow \sum M_c = 0$$

$$500 \text{ N/m} = 0.5 \text{ kN/m}$$

$$-4R_A + (3.2)(1.6)(0.5) + (1.6)(\frac{1}{2})(2.4)(0.5) = 0$$

$$R_A = 0.880 \text{ kN}$$

$$W = 0.5 - \frac{0.5}{2.4}(x-1.6)^1 = 0.5 - 0.20833(x-1.6)^1 \text{ kN/m} - \frac{dV}{dx}$$

$$V = 0.880 - 0.5x + 0.104167(x-1.6)^2 \text{ kN}$$

$$V_A = 0.880 \text{ kN}$$

$$V_B = 0.880 - (0.5)(1.6) = 0.080 \text{ kN}$$

$$V_C = 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520 \text{ kN}$$

} sign change

Locate point D (Between B and C) where $V = 0$.

$$0 = 0.880 - 0.5x_D + 0.104167(x_D - 1.6)^2$$

$$0.104167x_D^2 - 0.83333x_D + 1.14667 = 0$$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - 4(0.104167)(1.14667)}}{2(0.104167)}$$

$$= 4.0 \pm 2.2342 = 5.2342, 1.7658 \text{ m}$$

$$M = 0.880x - 0.25x^2 + 0.347222(x-1.6)^3 \text{ kN-m}$$

$$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.347222)(0.1658)^3 = 0.776 \text{ kN-m}$$

(a) $M_{max} = 0.776 \text{ kN-m}$ at $x = 1.7658 \text{ m}$

$$S_{min} = \frac{M_{max}}{G_{all}} = \frac{0.776 \times 10^3}{12 \times 10^6} = 64.66 \times 10^{-6} \text{ m}^3 = 64.66 \times 10^3 \text{ mm}^3$$

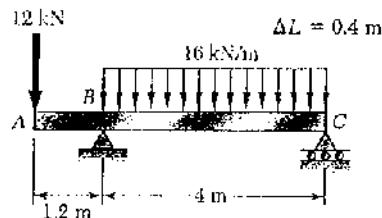
$$\text{For a rectangular cross section } S = \frac{1}{6}bh^2 \quad h = \frac{6S}{b}$$

$$h_{min} = \sqrt{\frac{(6)(64.66 \times 10^3)}{30}} = 113.7 \text{ mm}$$

(b) At next higher 10-mm increment $h = 120 \text{ mm}$

Problem 5.118

5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL , starting at point A and ending at the right-hand support.



$$+\rightarrow \sum M_C = 0$$

$$(5.2)(12) - 4B + (2)(4)(16) = 0$$

$$B = 47.6 \text{ kN} \uparrow$$

$$+\rightarrow \sum M_B = 0$$

$$(1.2)(12) - (2)(4)(16) + 4C = 0$$

$$C = 28.4 \text{ kN} \uparrow$$

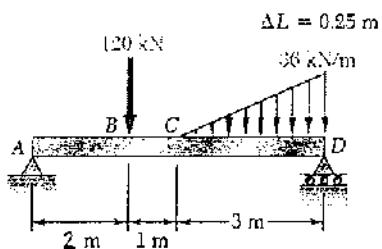
$$w = 16(x-1.2)^0 = -\frac{dw}{dx}$$

$$V = -16(x-1.2)^1 - 12 + 47.6(x-1.2)^0$$

$$M = -8(x-1.2)^2 - 12x + 47.6(x-1.2)^1$$

x m	V kN	M kN·m
0.0	-12.0	0.00
0.4	-12.0	-4.80
0.8	-12.0	-9.60
1.2	35.6	-14.40
1.6	29.2	-1.44
2.0	22.8	8.96
2.4	16.4	16.80
2.8	10.0	22.08
3.2	3.6	24.80
3.6	-2.8	24.96
4.0	-9.2	22.56
4.4	-15.6	17.60
4.8	-22.0	10.08
5.2	-28.4	-0.00

Problem 5.119



5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL , starting at point *A* and ending at the right-hand support.

$$\text{Ansatz: } \sum M_c = 0$$

$$- 6 R_A + (4)(120) + (1)(\frac{1}{2})(3)(36) = 0$$

$$R_A = 89 \text{ kN}$$

$$w = \frac{36}{3} (x-3)^3 = 12(x-3)^3$$

$$V = 89 - 120(x-2)^2 - 6(x-3)^2 \text{ kN}$$

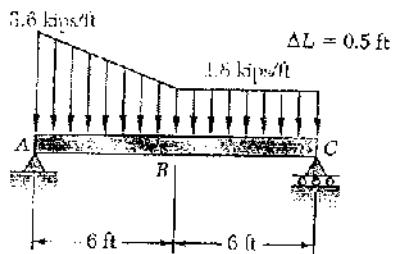
$$\begin{array}{c|c|c} x & V & M \\ \text{m} & \text{kN} & \text{kN}\cdot\text{m} \end{array}$$

$$M = 89x - 120(x-2)^3 - 2(x-3)^3 \text{ KN-m}$$

0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	-31.0	178.0
2.3	-31.0	170.3
2.5	-31.0	162.5
2.8	-31.0	154.8
3.0	-31.0	147.0
3.3	-31.4	139.2
3.5	-32.5	131.3
3.8	-34.4	122.9
4.0	-37.0	114.0
4.3	-40.4	104.3
4.5	-44.5	93.8
4.8	-49.4	82.0
5.0	-55.0	69.0
5.3	-61.4	54.5
5.5	-68.5	38.3
5.8	-76.4	20.2
6.0	-85.0	-0.0

Problem 5.120

5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL , starting at point A and ending at the right-hand support.



$$\rightarrow \sum M_c = 0$$

$$-12 R_A + (6)(12)(1.8) + (10)(\frac{1}{2})(6)(1.8) = 0$$

$$R_A = 15.3 \text{ Kips.}$$

$$W = 3.6 - \frac{1.8}{6}x + \frac{1.8}{6}(x-6)$$

$$= 3.6 - 0.3x + 0.3(x-6)$$

$$V = 15.3 - 3.6x + 0.15x^2 - 0.15(x-6)^2 \text{ kips} \quad \begin{matrix} \leftarrow \\ x \end{matrix}$$

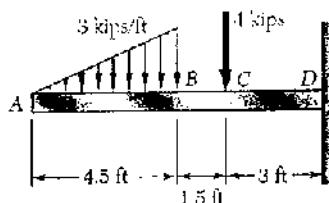
ft	kips	kip·ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	-0.90	37.8
6.5	-1.80	37.1
7.0	-2.70	36.0
7.5	-3.60	34.4
8.0	-4.50	32.4
8.5	-5.40	29.9
9.0	-6.30	27.0
9.5	-7.20	23.6
10.0	-8.10	19.8
10.5	-9.00	15.5
11.0	-9.90	10.8
11.5	-10.80	5.6
12.0	-11.70	0.0

$$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05(x-6)^3 \text{ kip·ft} \quad \begin{matrix} \leftarrow \\ x \end{matrix}$$

x ft	V kips	M kip·ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	-0.90	37.8
6.5	-1.80	37.1
7.0	-2.70	36.0
7.5	-3.60	34.4
8.0	-4.50	32.4
8.5	-5.40	29.9
9.0	-6.30	27.0
9.5	-7.20	23.6
10.0	-8.10	19.8
10.5	-9.00	15.5
11.0	-9.90	10.8
11.5	-10.80	5.6
12.0	-11.70	0.0

Problem 5.121

$\Delta L = 0.5 \text{ ft}$



5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL , starting at point A and ending at the right-hand support.

$$w = \frac{3}{4.5} x - 3(x-4.5)^0 - \frac{3}{4.5}(x-4.5)^1$$

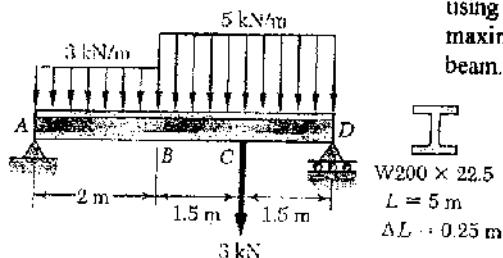
$$= \frac{2}{3}x - 3(x-4.5)^0 - \frac{2}{3}(x-4.5)^1 = -\frac{dv}{dx}$$

$$V = -\frac{1}{3}x^2 + 3(x-4.5)^1 + \frac{1}{3}(x-4.5)^2 - 4(x-6)^0$$

$$M = -\frac{1}{9}x^3 + \frac{3}{2}(x-4.5)^2 + \frac{1}{9}(x-4.5)^3 - 4(x-6)^1$$

x ft	V kips	M kip ft
0.0	0.00	0.00
0.5	-0.08	-0.01
1.0	-0.33	-0.11
1.5	-0.75	-0.38
2.0	-1.33	-0.89
2.5	-2.08	-1.74
3.0	-3.00	-3.00
3.5	-4.08	-4.76
4.0	-5.33	-7.11
4.5	-6.75	-10.13
5.0	-6.75	-13.50
5.5	-6.75	-16.88
6.0	-10.75	-20.25
6.5	-10.75	-25.63
7.0	-10.75	-31.00
7.5	-10.75	-36.38
8.0	-10.75	-41.75
8.5	-10.75	-47.13
9.0	-10.75	-52.50

Problem 5.122



5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (b) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

$$\sum M_o = 0$$

$$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$$

$$R_A = 10.2 \text{ kN}$$

$$W = 3 + 2(x-2)^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 10.2 - 3x - 2(x-2)^1 - 3(x-3.5)^0 \text{ kN}$$

$$M = 10.2x - 1.5x^2 - (x-2)^2 - 3(x-3.5)^1 \text{ kN.m}$$

x m	V kN	M kN·m	sigma MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.72	24.4
0.75	7.95	6.81	35.1
1.00	7.20	8.70	44.8
1.25	6.45	10.41	53.6
1.50	5.70	11.92	61.5
1.75	4.95	13.26	68.3
2.00	4.20	14.40	74.2
2.25	2.95	15.29	78.8
2.50	1.70	15.88	81.8
2.75	0.45	16.14	83.2
3.00	-0.80	16.10	83.0
3.25	-2.05	15.74	81.2
3.50	-6.30	15.07	77.7
3.75	-7.55	13.34	68.8
4.00	-8.80	11.30	58.2
4.25	-10.05	8.94	46.1
4.50	-11.30	6.27	32.3
4.75	-12.55	3.29	17.0
5.00	-13.80	-0.00	-0.0
2.83	0.05	16.164	83.3
2.84	0.00	16.164	83.3
2.85	-0.05	16.164	83.3

For rolled steel section
W 200 x 22.5

$$S = 194 \times 10^3 \text{ mm}^3$$

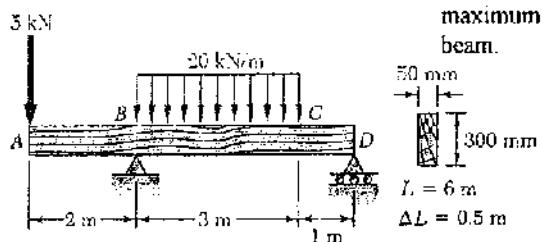
$$S = 194 \times 10^3 \text{ mm}^3 = 194 \times 10^{-6} \text{ m}^3$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{16164 \times 10^3}{194 \times 10^{-6}}$$

$$= 83.3 \times 10^6 \text{ Pa}$$

$$= 83.3 \text{ MPa}$$

Problem 5.123



5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔI , indicated, (b) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

$$+\sum M_D = 0$$

$$-4R_B + (6)(5) + (2.5)(3)(20) = 0$$

$$R_B = 45 \text{ kN}$$

$$w = 20(x-2)^0 - 20(x-5)^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = -5 + 45(x-2)^0 + 20(x-2)^1 + 20(x-5)^1 \text{ kN}$$

$$M = -5x + 45(x-2)^1 - 10(x-2)^2 + 10(x-5)^2 \text{ kN-m}$$

x m	V kN	M kN·m	sigma MPa
0.00	-5	0.00	0.0
0.50	-5	-2.50	-3.3
1.00	-5	-5.00	-6.7
1.50	-5	-7.50	-10.0
2.00	40	-10.00	-13.3
2.50	30	7.50	10.0
3.00	20	20.00	26.7
3.50	10	27.50	36.7
4.00	0	30.00	40.0
4.50	-10	27.50	36.7
5.00	-20	20.00	26.7
5.50	-20	10.00	13.3
6.00	-20	0.00	0.0

$$\text{Maximum } |M| = 30 \text{ kN·m}$$

at $x = 4.0 \text{ m}$

For rectangular cross section

$$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(50)(300)^2$$

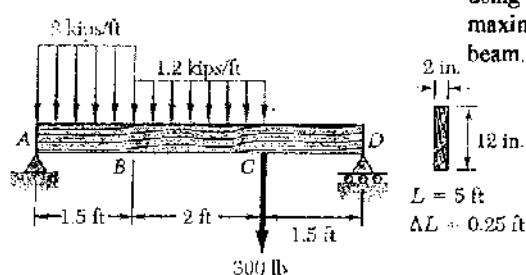
$$= 750 \times 10^3 \text{ mm}^3$$

$$= 750 \times 10^{-6} \text{ m}^3$$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{30 \times 10^3}{750 \times 10^{-6}}$$

$$= 40 \times 10^6 \text{ Pa} = 40 \text{ MPa}$$

Problem 5.124



5.124 and 5.125 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (b) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

$$+\sum M_D = 0$$

$$-5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$$

$$R_A = 3.84 \text{ kips}$$

$$300 \text{ lb} = 0.3 \text{ kips}$$

$$w = 2 - 0.8(x - 1.5)^0 - 1.2(x - 3.5)^0 \text{ kip/ft}$$

$$V = 3.84 - 2x + 0.8(x - 1.5)' + 1.2(x - 3.5)' - 0.3(x - 3.5)^0 \text{ kips}$$

$$M = 3.84x - x^2 + 0.4(x - 1.5)^2 + 0.6(x - 3.5)^2 - 0.3(x - 3.5)^0 \text{ kip-ft}$$

x ft	V kips	M kip·ft	sigma ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	-0.06	3.80	0.951
2.50	-0.36	3.75	0.937
2.75	-0.66	3.62	0.906
3.00	-0.96	3.42	0.855
3.25	-1.26	3.14	0.786
3.50	-1.86	2.79	0.697
3.75	-1.86	2.32	0.581
4.00	-1.86	1.86	0.465
4.25	-1.86	1.39	0.349
4.50	-1.86	0.93	0.232
4.75	-1.86	0.46	0.116
5.00	-1.86	-0.00	-0.000
2.10	0.12	3.80	0.949
2.20	0.00	3.80	0.951
2.30	-0.12	3.80	0.949

$$\text{Maximum } |M| = 3.804 \text{ kip-ft} \\ = 45.648 \text{ kip-in}$$

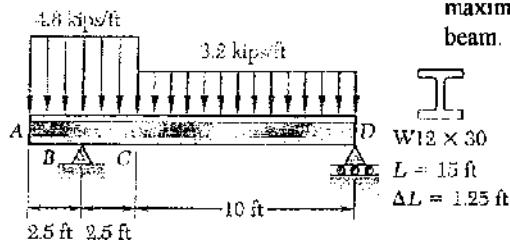
at $x = 2.20 \text{ ft}$
Rectangular section
2 in \times 12 in.

$$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(2)(12)^2 \\ = 48 \text{ in}^3$$

$$\sigma = \frac{M}{S} = \frac{45.648}{48} \\ = 0.951 \text{ ksi}$$

Problem 5.125

5.124 and 5.125 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (b) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.



$$\rightarrow \sum M_D = 0$$

$$-12.5 R_B + (12.5)(50)(4.8) + (5)(10)(3.2) = 0$$

$$R_B = 36.8 \text{ kips}$$

$$w = 4.8 - 1.6(x-5)^0 \text{ kips/ft}$$

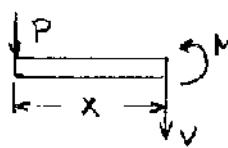
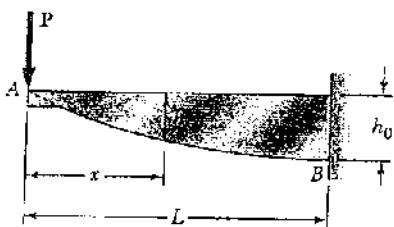
$$V = -4.8x + 36.8(x-2.5)^0 + 1.6(x-5)^1 \text{ kips}$$

$$M = -2.4x^2 + 36.8(x-2.5)^1 + 0.8(x-5)^2 \text{ kip-ft}$$

x ft	V kips	M kip·ft	sigma ksi	Maximum M = 57.6 kip·ft = 691.2 kip-in at x = 9.0 ft.
0.00	0.0	0.00	0.00	
1.25	-6.0	-3.75	-1.17	
2.50	24.8	-15.00	-4.66	For rolled steel section W12 x 30
3.75	18.8	12.25	3.81	
5.00	12.8	32.00	9.95	S = 38.6 in ³
6.25	8.8	45.50	14.15	
7.50	4.8	54.00	16.79	Maximum normal stress
8.75	0.8	57.50	17.88	
10.00	-3.2	56.00	17.41	$\sigma = \frac{M}{S} = \frac{691.2}{38.6} = 17.91 \text{ ksi}$
11.25	-7.2	49.50	15.39	
12.50	-11.2	38.00	11.81	
13.75	-15.2	21.50	6.68	
15.00	-19.2	0.00	0.00	
8.90	0.32	57.58	17.90	
9.00	-0.00	57.60	17.91 ←	
9.10	-0.32	57.58	17.90	

Problem 5.126

5.126 and 5.127 The beam AB , consisting of a cast-iron plate of uniform thickness b and length L , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 . (b) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.



$$V = -P$$

$$M = -Px \quad |M| = Px$$

$$S = \frac{|M|}{G_{all}} = \frac{Px}{G_{all}} x$$

For a rectangular cross section $S = \frac{1}{6}bh^2$

Equating $\frac{1}{6}bh^2 = \frac{Px}{G_{all}}$ $h = \left(\frac{6Px}{G_{all}b}\right)^{\frac{1}{2}}$ (1)

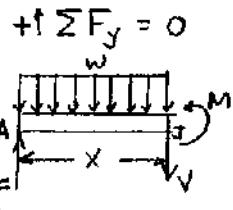
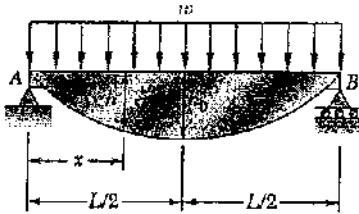
At $x = L$ $h = h_0 = \left\{\frac{6PL}{G_{all}b}\right\}^{\frac{1}{2}}$ (2)

(a) Divide Eq. (1) by Eq. (2) and solve for h $h = h_0(x/L)^{\frac{1}{2}}$

(b) Solving for P $P = \frac{6Gb h_0^2}{6L} = \frac{(24)(1.25)(12)^2}{(6)(36)} = 20 \text{ kips}$

Problem 5.127

5.126 and 5.127 The beam AB , consisting of a cast-iron plate of uniform thickness b and length L , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 . (b) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.



$$+ \uparrow \sum F_y = 0 \quad R_A + R_B - wL = 0 \quad R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \sum M_J = 0$$

$$\frac{wL}{2}x - wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{G_{all}} = \frac{wx(L-x)}{2G_{all}}$$

For a rectangular cross section $S = \frac{1}{6}bh^2$

Equating $\frac{1}{6}bh^2 = \frac{wx(L-x)}{2G_{all}}$

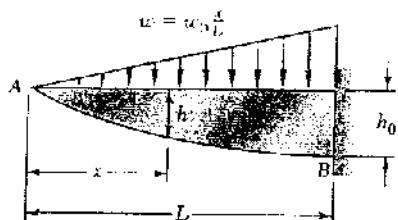
$$h = \left\{\frac{3wx(L-x)}{6G_{all}b}\right\}^{\frac{1}{2}}$$

(a) At $x = \frac{L}{2}$ $h = h_0 = \left\{\frac{3wL^2}{4G_{all}b}\right\}^{\frac{1}{2}}$

$$h = h_0 \left[\frac{x}{L} \left(1 - \frac{x}{L}\right)\right]^{\frac{1}{2}}$$

(b) Solving for w $w = \frac{4G_{all}b h_0^2}{3L^2} = \frac{(4)(24)(1.25)(12)^2}{(3)(36)^2} = 4.44 \text{ kip/in}$

Problem 5.128



5.128 and 5.129 The beam AB , consisting of a cast-iron plate of uniform thickness b and length L , is to support the distributed load $w(x)$ shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 . (b) Determine the smallest value of h_0 if $L = 750$ mm., $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \quad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{G_{all}} = \frac{w_0 x^3}{6L G_{all}}$$

For a rectangular cross section $S = \frac{1}{6} b h^2$

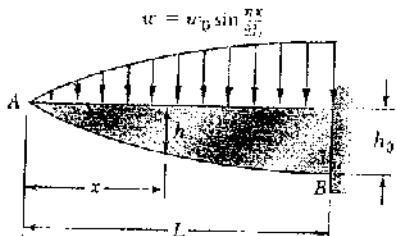
$$\text{Equating } \frac{1}{6} b h^2 = \frac{w_0 x^3}{6L G_{all}} \quad h = \sqrt{\frac{w_0 x^3}{6L G_{all} b}}$$

$$\text{At } x = L \quad h = h_0 = \sqrt{\frac{w_0 L^3}{6L G_{all} b}} \quad (a) \quad h = h_0 \left(\frac{x}{L}\right)^{3/2}$$

Data: $L = 750$ mm = 0.75 m, $b = 30$ mm = 0.030 m
 $w_0 = 300$ kN/m = 300×10^3 N/m, $G_{all} = 200$ MPa = 200×10^6 Pa

$$(b) \quad h_0 = \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 167.7 \times 10^{-3} \text{ m} = 167.7 \text{ mm}$$

Problem 5.129



5.128 and 5.129 The beam AB , consisting of a cast-iron plate of uniform thickness b and length L , is to support the distributed load $w(x)$ shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 . (b) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{2w_0 L}{\pi} \cos \frac{\pi x}{2L} + C_1$$

$$V = 0 \text{ at } x = 0 \rightarrow C_1 = -\frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi} \left(1 - \cos \frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0 L}{\pi} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right) \quad |M| = \frac{2w_0 L}{\pi} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

$$S = \frac{|M|}{G_{all}} = \frac{2w_0 L}{\pi G_{all}} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

For a rectangular cross section $S = \frac{1}{6}bh^2$

$$\text{Equating } \frac{1}{6}bh^2 = \frac{2w_0 L}{\pi G_{all}} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

$$h = \sqrt{\frac{12w_0 L}{\pi G_{all} b} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)}$$

$$\text{At } x = L \quad h = h_0 = \sqrt{\frac{12w_0 L^2}{\pi G_{all} b} \left(1 - \frac{2}{\pi}\right)} = 1.178 \sqrt{\frac{w_0 L^2}{G_{all} b}}$$

$$(a) \quad h = h_0 \left[\left(\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L} \right) / \left(1 - \frac{2}{\pi}\right) \right]^{1/2} = 1.659 h_0 \left[\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L} \right]^{1/2}$$

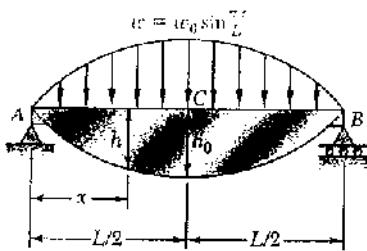
Data: $L = 750$ mm = 0.75 m, $b = 30$ mm = 0.030 m

$w_0 = 300$ kN/m = 300×10^3 N/m, $G_{all} = 200$ MPa = 200×10^6 Pa

$$(b) \quad h_0 = 1.178 \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 197.6 \times 10^{-3} \text{ m} = 197.6 \text{ mm}$$

Problem 5.130

5.130 and 5.131 The beam AB , consisting of an aluminum plate of uniform thickness b and length L , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800 \text{ mm}$, $h_0 = 200 \text{ mm}$, $b = 25 \text{ mm}$, and $\sigma_{all} = 72 \text{ MPa}$.



$$\frac{dy}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$\text{At } A, \quad x = 0 \quad M = 0 \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$\text{At } B, \quad x = L \quad M = 0 \quad 0 = \frac{w_0 L^2}{\pi^2} \sin \pi + C_1 L \quad C_1 = 0$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

$$\text{For constant strength} \quad S = \frac{|M|}{S_{all}} = \frac{w_0 L^2}{\pi^2 S_{all}} \sin \frac{\pi x}{L}$$

$$\text{For a rectangular section} \quad I = \frac{1}{12} b h^3, \quad c = \frac{h}{2}, \quad S = \frac{I}{c} = \frac{1}{6} b h^2$$

$$\text{Equating the two expressions for } S \quad \frac{1}{6} b h^2 = \frac{w_0 L^2}{\pi^2 S_{all}} \sin \frac{\pi x}{L} \quad (1)$$

$$\text{At } x = \frac{L}{2} \quad h = h_0 \quad \frac{1}{6} b h_0^2 = \frac{w_0 L^2}{\pi^2 S_{all}} \quad (2)$$

$$(a) \text{ Dividing Eq.(1) by Eq.2} \quad \frac{h^2}{h_0^2} = \sin \frac{\pi x}{L} \quad h = h_0 \left(\sin \frac{\pi x}{L} \right)^{1/2} \quad \square$$

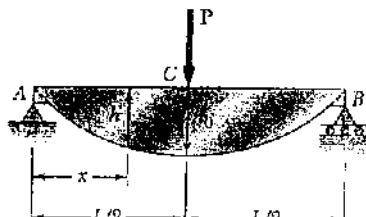
$$(b) \text{ Solving Eq. (1) for } w_0 \quad w_0 = \frac{\pi^2 S_{all} b h_0^2}{6 L^2}$$

Data: $S_{all} = 72 \times 10^6 \text{ Pa}$, $L = 800 \text{ mm} = 0.800 \text{ m}$, $h_0 = 200 \text{ mm} = 0.200 \text{ m}$, $b = 25 \text{ mm} = 0.025 \text{ m}$

$$w_0 = \frac{\pi^2 (72 \times 10^6)(0.025)(0.200)}{(6)(0.800)^2} = 185.1 \times 10^3 \text{ N/m}$$

$$185.1 \text{ kN/m} \quad \blacksquare$$

Problem 5.131



5.130 and 5.131 The beam AB , consisting of an aluminum plate of uniform thickness b and length L , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm., $h_0 = 200$ mm., $b = 25$ mm., and $\sigma_{all} = 72$ MPa.

$$R_A = R_B = \frac{P}{2}$$

$$\text{For } \sum M_J = 0 \\ -\frac{P}{2}x + M = 0 \\ M = \frac{Px}{2} \quad (0 < x < \frac{L}{2})$$

$$S = \frac{M}{\sigma_{all}} = \frac{Px}{2G_{all}}$$

$$\text{For a rectangular cross section } S = \frac{1}{6}bh^2$$

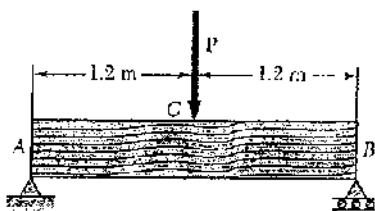
$$\text{Equating } \frac{1}{6}bh^2 = \frac{Px}{2G_{all}} \quad h = \sqrt{\frac{3Px}{G_{all}b}}$$

$$(a) \text{ At } x = \frac{L}{2} \quad h = h_0 = \sqrt{\frac{3PL}{2G_{all}b}} \quad h = h_0 \sqrt{\frac{2x}{L}}, \quad 0 < x < \frac{L}{2}$$

For $x > \frac{L}{2}$ replace x by $L-x$

$$(b) \text{ Solving for } P \quad P = \frac{2G_{all}bh_0^2}{3L} = \frac{(2)(72 \times 10^6)(0.025)(0.200)}{(3)(0.8)} = 60 \times 10^3 \text{ N} \\ = 60 \text{ kN}$$

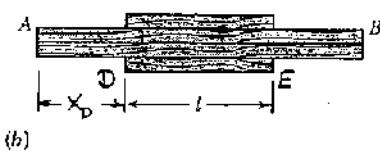
Problem 5.132



(a)

5.132 and 5.133 A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

$$R_A = R_B = \frac{P}{2}$$



(b)

$$\begin{aligned} & 0 < x < \frac{l}{2} \\ & \Delta \Sigma M_J = 0 \quad -\frac{P}{2}x + M = 0 \\ & M = \frac{Px}{2} \quad \text{or} \quad M = \frac{M_{max}x}{1.2} \end{aligned}$$

Bending moment diagram is two straight lines.

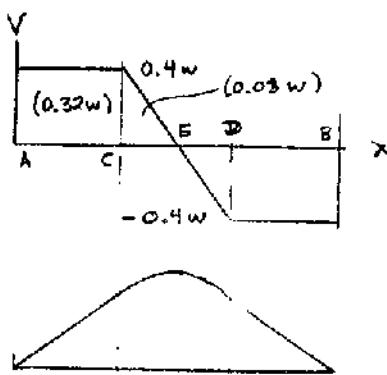
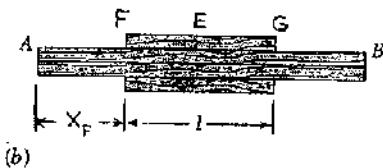
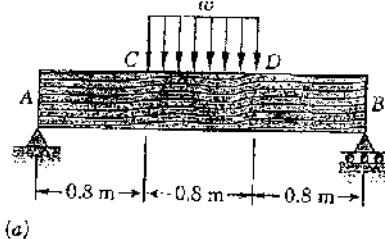
$$\text{At } C \quad S_c = \frac{1}{6} b h_c^2 \quad M_c = M_{max}$$

$$\text{At } D \quad S_d = \frac{1}{6} b h_d^2 \quad M_b = \frac{M_{max} x_d}{1.2}$$

$$\frac{S_d}{S_c} = \frac{h_d^2}{h_c^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}} \right)^2 = \frac{1}{4} = \frac{M_d}{M_c} = \frac{x_d}{1.2} \quad x_d = 0.3 \text{ m}$$

$$\frac{l}{2} = 1.2 - x_d = 0.9 \quad l = 1.800 \text{ m}$$

Problem 5.133



5.132 and 5.133 A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

$$R_A = R_B = \frac{0.8 w}{2} = 0.4 w$$

Shear: A to C $V = 0.4 w$
 D to B $V = -0.4 w$

Areas: A to C $(0.8)(0.4)w = 0.32 w$
 C to E $(\frac{1}{2})(0.4)(0.4)w = 0.08 w$

Bending moments.

At C $M_c = 0.40 w$

A to C $M = 0.40 w x$

At C $S_c = \frac{1}{6} b h_c^2$ $M_c = M_{max} = 0.40 w$

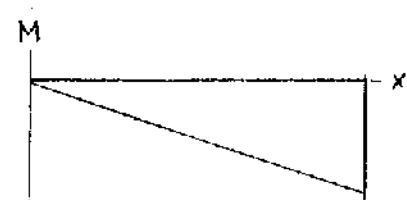
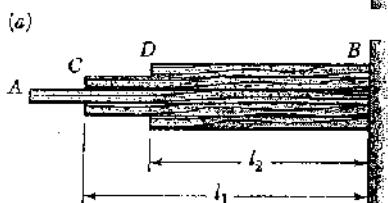
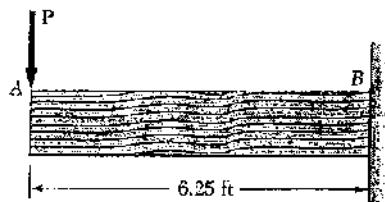
At F $S_F = \frac{1}{6} b h_F^2$ $M_F = 0.40 w x_F$

$$\frac{S_F}{S_c} = \frac{h_F^2}{h_c^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}} \right)^2 = \frac{1}{4} = \frac{M_F}{M_c} = \frac{0.40 w x_F}{0.40 w}$$

$$x_F = 0.25 \text{ m} \quad \frac{\ell}{2} = 1.2 - x_F = 0.95 \text{ m}$$

$$\ell = 1.900 \text{ m}$$

Problem 5.134



5.134 and 5.135 A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 10-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the factor of safety as the original design.

$$\text{At } B \quad \sum M_J = 0 \\ Px + M = 0 \quad M = -Px \\ |M| = Px$$

$$\begin{aligned} \text{At } B \quad |M|_B &= M_{\max} \\ \text{At } C \quad |M|_c &= M_{\max} x_c / 6.25 \\ \text{At } D \quad |M|_d &= M_{\max} x_d / 6.25 \end{aligned}$$

$$S_B = \frac{1}{6} b h^2 = \frac{1}{6} \cdot b (5b)^2 = \frac{25}{6} b^3$$

$$\text{At } C \quad S_c = \frac{1}{6} \cdot b (b)^2 = \frac{1}{6} b^3$$

$$\text{At } D \quad S_d = \frac{1}{6} b (3b)^2 = \frac{9}{6} b^3$$

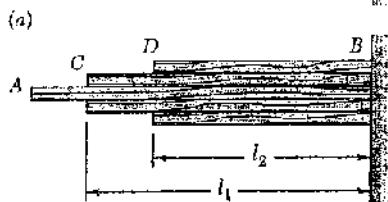
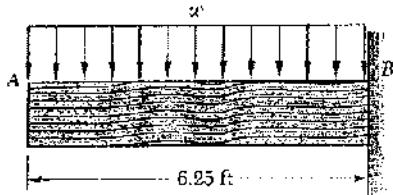
$$\frac{|M|_c}{|M|_B} = \frac{x_c}{6.25} = \frac{S_c}{S_B} = \frac{1}{25} \quad x_c = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$$

$$l_1 = 6.25 - 0.25 = 6.00 \text{ ft} \quad \blacktriangleleft$$

$$\frac{|M|_d}{|M|_B} = \frac{x_d}{6.25} = \frac{S_d}{S_B} = \frac{9}{25} \quad x_d = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$$

$$l_2 = 6.25 - 2.25 = 4.00 \text{ ft} \quad \blacktriangleleft$$

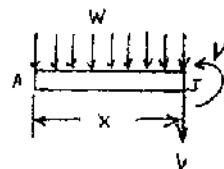
Problem 5.135



$$\frac{|M|_c}{|M|_B} = \left(\frac{x_c}{6.25}\right)^2 = \frac{S_c}{S_B} = \frac{1}{25}$$

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25}$$

5.134 and 5.135 A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 10-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the factor of safety as the original design.



$$\sum M_J = 0 \quad w \frac{x}{2} + M = 0$$

$$M = -\frac{wx^2}{2} \quad |M| = \frac{wx^2}{2}$$

$$\text{At } B \quad |M|_B = |M|_{\max}$$

$$\text{At } C \quad |M|_c = |M|_{\max} (x_c / 6.25)^2$$

$$\text{At } D \quad |M|_D = |M|_{\max} (x_D / 6.25)^2$$

$$\text{At } B \quad S_B = \frac{1}{6}bh^3 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$$

$$\text{At } C \quad S_c = \frac{1}{6}bh^3 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$$

$$\text{At } D \quad S_d = \frac{1}{6}bh^3 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$$

$$x_c = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$$

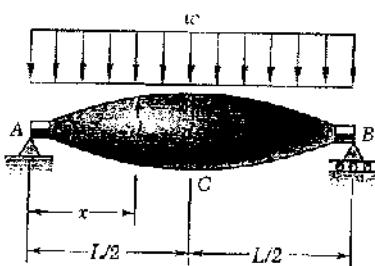
$$l_1 = 6.25 - 1.25 \text{ ft} = 5.00 \text{ ft}$$

$$x_D = \frac{6.25\sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$$

$$l_2 = 6.25 - 3.75 \text{ ft} = 2.50 \text{ ft}$$

Problem 5.136

5.136 and 5.137 A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x , L , and d_0 .



$$R_A = R_B = \frac{wL}{2}$$

$$\sum M_J = 0$$

$$-\frac{wL}{2}x + w \times \frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2G\mu}$$

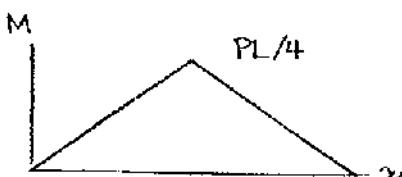
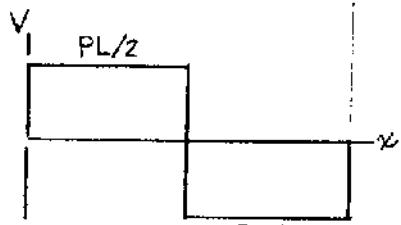
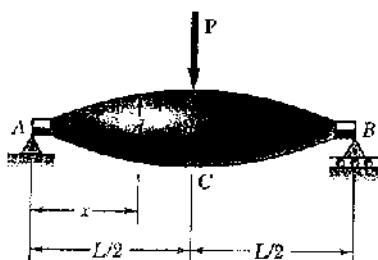
$$\text{For a solid circular cross section } c = \frac{d}{2} \quad I = \frac{\pi}{4}c^3 \quad S = \frac{I}{c} = \frac{\pi d^3}{32}$$

$$\text{Equating, } \frac{\pi d^3}{32} = \frac{wx(L-x)}{2G\mu} \quad d = \left\{ \frac{16wx(L-x)}{\pi G\mu} \right\}^{1/3}$$

$$\text{At } x = \frac{L}{2} \quad d = d_0 = \left\{ \frac{4wL^2}{\pi G\mu} \right\}^{1/3} \quad d = d_0 \left\{ 4 \frac{x}{L} \left(1 - \frac{x}{L}\right) \right\}^{1/3}$$

Problem 5.137

5.136 and 5.137 A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x , L , and d_0 .



Draw shear and bending moment diagrams.

$$0 \leq x \leq \frac{L}{2} \quad M = \frac{Px}{2}$$

$$\frac{L}{2} \leq x \leq L \quad M = \frac{P(L-x)}{2}$$

For a solid circular section $C = \frac{1}{2}d$

$$I = \frac{\pi}{4}C^4 = \frac{\pi}{64}d^4 \quad S = \frac{I}{c} = \frac{\pi}{32}d^3$$

For constant design, $\sigma = \text{constant}$

$$S = \frac{M}{\sigma}$$

$$\text{For } 0 \leq x \leq \frac{L}{2} \quad \frac{\pi}{32}d^3 = \frac{Px}{2} \quad (1a)$$

$$\text{For } \frac{L}{2} \leq x \leq L \quad \frac{\pi}{32}d^3 = \frac{P(L-x)}{2} \quad (1b)$$

$$\text{At point C} \quad \frac{\pi}{32}d_0^3 = \frac{PL}{4} \quad (2)$$

Dividing Eq.(1a) by Eq.(2)

$$0 \leq x \leq \frac{L}{2} \quad \frac{d^3}{d_0^3} = \frac{2x}{L}$$

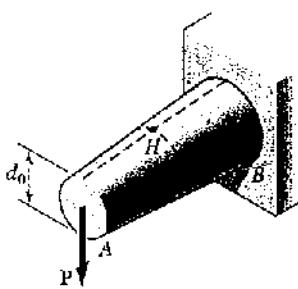
$$d = d_0(2x/L)^{1/3}$$

Dividing Eq.(1b) by Eq.(2)

$$\frac{L}{2} \leq x \leq L \quad \frac{d^3}{d_0^3} = \frac{2(L-x)}{L}$$

$$d = d_0[2(L-x)/L]^{1/3}$$

Problem 5.138



5.138 A transverse force P is applied as shown at end A of the conical taper AB . Denoting by d_0 the diameter of the taper at the A , show that the maximum normal stress occurs at point H , which is contained in a transverse section of diameter $d = 1.5 d_0$.

$$V = -P = \frac{dM}{dx} \quad M = -Px$$

$$\text{Let } d = d_0 + kx$$

$$\text{For a solid circular section } I = \frac{\pi}{4} C^4 = \frac{\pi}{64} d^4$$

$$C = \frac{d}{2} \quad S = \frac{I}{C} = \frac{\pi}{32} d^3 = \frac{\pi}{32} (d_0 + kx)^3$$

$$\frac{dS}{dx} = \frac{3\pi}{32} (d_0 + kx)^2 k = \frac{3\pi}{32} d^2 k$$

$$\text{Stress } \sigma = \frac{|M|}{S} = \frac{Px}{S}$$

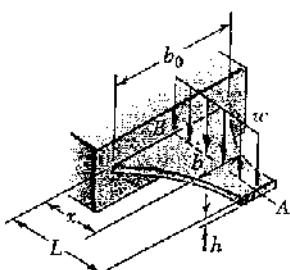
$$\text{At } H \quad \frac{d\sigma}{dx} = \frac{1}{S^2} (P S - P x \frac{dS}{dx}) = 0$$

$$S - x_H \frac{dS}{dx} = \frac{\pi}{32} d^3 - x_H \frac{3\pi}{32} d^2 k$$

$$kx_H = \frac{1}{3} d = \frac{1}{3} (d_0 + kx_H) \quad kx_H = \frac{1}{2} d_0$$

$$d = d_0 + \frac{1}{2} d_0 = \frac{3}{2} d_0 \quad d = 1.5 d_0 \blacksquare$$

Problem 5.139



5.139 A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support the distributed load w along its center line AB . (a) knowing that the beam is to be of constant strength, express b in terms of x , L , and b_0 . (b) Determine the maximum allowable value of w if $L = 15$ in., $b_0 = 8$ in., $h = 0.75$ in., and $\sigma_{all} = 24$ ksi.

$$\rightarrow \sum M_J = 0 \quad -M - w(L-x) \frac{L-x}{2} = 0$$

$$M = -\frac{w(L-x)^2}{2} \quad |M| = \frac{w(L-x)^2}{2}$$

$$S = \frac{|M|}{6\sigma_{all}} = \frac{w(L-x)^2}{26\sigma_{all}}$$

$$\text{For a rectangular cross section } S = \frac{1}{6} b h^2$$

$$\frac{1}{6} b h^2 = \frac{w(L-x)^2}{26\sigma_{all}} \quad b = \frac{3w(L-x)^2}{6\sigma_{all} h^2}$$

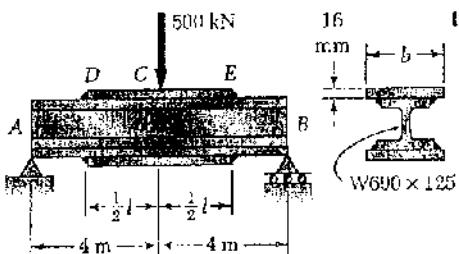
$$\text{At } x = 0 \quad b = b_0 = \frac{3wL^2}{6\sigma_{all} h^2} \quad b = b_0 (1 - \frac{x}{L})^2 \blacksquare$$

$$\text{Solving for } w \quad w = \frac{5\sigma_{all} b_0 h^2}{3L^2} = \frac{(24)(8)(0.75)^2}{(3)(15)^2} = 0.160 \text{ kip/in}$$

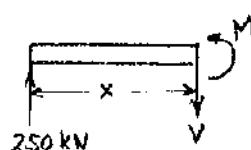
$$= 160 \text{ lb/in} \blacksquare$$

Problem 5.140

5.140 Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4 \text{ m}$ and $b = 285 \text{ mm}$, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



$$R_A = R_B = 250 \text{ kN}$$



$$\begin{aligned} \text{At } D: \sum M_D &= 0 \\ -250x + M &= 0 \\ M &= 250x \text{ KN}\cdot\text{m} \end{aligned}$$

At center of beam $x = 4 \text{ m}$ $M_c = (250)(4) = 1000 \text{ kN}\cdot\text{m}$

At D $x = \frac{1}{2}(8-l) = \frac{1}{2}(8-4) = 2 \text{ m}$ $M_d = 500 \text{ kN}\cdot\text{m}$

At center of beam $I = I_{\text{beam}} + 2I_{\text{plate}}$
 $= 1190 \times 10^6 + 2\{(285)(16)\left(\frac{678}{2} + \frac{16}{2}\right)^2 + \frac{1}{12}(285)(16)^3\}$
 $= 2288 \times 10^6 \text{ mm}^4$
 $C = \frac{678}{2} + 16 = 355 \text{ mm}$ $S = \frac{I}{C} = \frac{1190 \times 10^6}{355} \text{ mm}^3$
 $= 6445 \times 10^3 \text{ mm}^3$
 $= 6445 \times 10^{-6} \text{ m}^3$

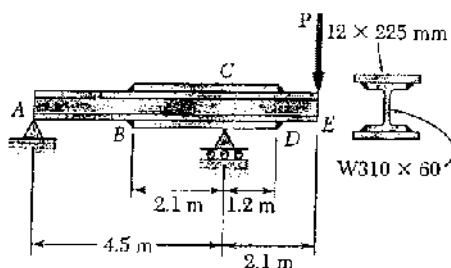
(a) Normal stress $\sigma = \frac{M}{S} = \frac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6 \text{ Pa} = 155.2 \text{ MPa}$

At D $S = 3510 \times 10^3 \text{ mm}^3 = 3510 \times 10^{-6} \text{ m}^3$

(b) Normal stress $\sigma = \frac{M}{S} = \frac{500 \times 10^3}{3510 \times 10^{-6}} = 142.4 \times 10^6 \text{ Pa} = 142.4 \text{ MPa}$

Problem 5.141

5.141 Knowing that $\sigma_{all} = 150 \text{ MPa}$, determine the largest concentrated load P that can be applied at end E of the beam shown.



$$\begin{aligned}\rightarrow \sum M_E &= 0 \quad -4.5 R_A - 2.1 P = 0 \\ R_A &= -0.46667 P \text{ ie } 0.46667 P \downarrow\end{aligned}$$

$$\begin{aligned}\rightarrow \sum M_A &= 0 \quad 4.5 R_c - 6.6 P = 0 \\ R_c &= 1.46667 P\end{aligned}$$

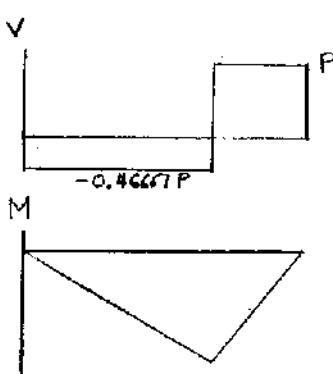
Shear: A to C $V = -0.46667 P$
 C to E $V = P$

Bending moments:

$$M_c = -(4.5)(0.46667 P) = -2.10 P \text{ kN-m}$$

$$M_B = \frac{2.4}{4.5} M_c = -1.12 P \text{ kN-m}$$

$$M_D = \frac{0.9}{2.1} M_c = -0.9 P \text{ kN-m}$$



At B and D $S = 851 \times 10^3 \text{ mm}^3 = 851 \times 10^{-6} \text{ m}^3$

$$G_{all} = \frac{|M|}{S} = \frac{1.120 P_{all}}{851 \times 10^{-6}} = 150 \times 10^6 \quad \text{at B}$$

$$P_{all} = 114.0 \text{ kN}$$

At C $I = I_{beam} + 2 I_{plate}$

$$\begin{aligned}&= 129 \times 10^6 + 2 \left\{ (225)(12) \left(\frac{310}{2} + \frac{12}{2} \right)^2 + \frac{1}{12}(225)(12)^3 \right\} \\&= 269 \times 10^6 \text{ mm}^4\end{aligned}$$

$$c = \frac{310}{2} + 12 = 167 \text{ mm}$$

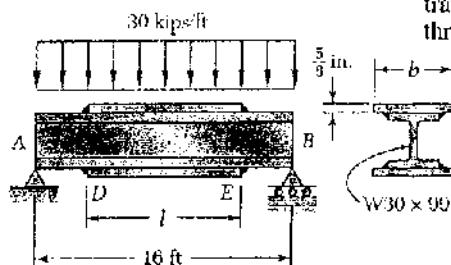
$$\begin{aligned}S &= \frac{I}{c} = 1611 \times 10^3 \text{ mm}^3 \\&= 1611 \times 10^{-6} \text{ m}^3\end{aligned}$$

$$G_{all} = \frac{|M|}{S} = \frac{2.10 P}{1611 \times 10^{-6}} = 150 \times 10^6$$

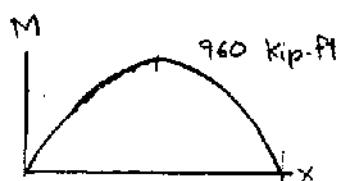
$$P_{all} = 115.1 \text{ kN}$$

Allowable load is the smaller value $P = 114.0 \text{ kN}$

Problem 5.142



5.142 Two cover plates, each $\frac{5}{8}$ -in. thick, are welded to a W30 × 99 beam as shown. Knowing that $l = 9$ ft and $b = 12$ in., determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



$$R_A = R_B = 240 \text{ kips}$$

$$\sum M_J = 0$$

$$-240x + 30 \times \frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip}\cdot\text{ft}$$

At center of beam $x = 8 \text{ ft}$

$$M_c = 960 \text{ kip}\cdot\text{ft} = 11520 \text{ kip}\cdot\text{in}$$

At point D, $x = \frac{1}{2}(16-9) = 3.5 \text{ ft}$

$$M_D = 656.25 \text{ kip}\cdot\text{ft} = 7875 \text{ kip}\cdot\text{in}$$

At center of beam $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$I = 3990 + 2 \left\{ (12)(0.625) \left(\frac{29.65}{2} + \frac{0.625}{2} \right)^2 + \frac{1}{12}(12)(0.625)^3 \right\} = 7428 \text{ in}^4$$

$$C = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

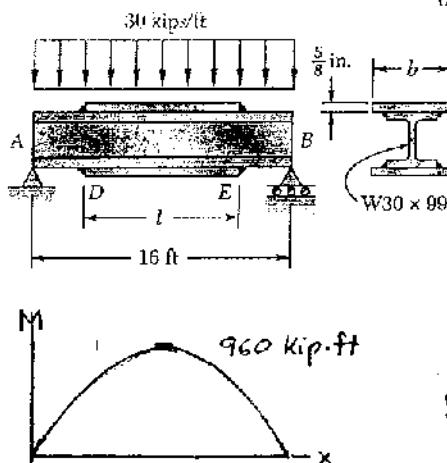
(a) Normal stress $\sigma = \frac{Mc}{I} = \frac{(11520)(15.45)}{7428} = 24.0 \text{ ksi}$

At point D $S = 269 \text{ in}^3$

(b) Normal stress $\sigma = \frac{M}{S} = \frac{7875}{269} = 29.3 \text{ ksi}$

Problem 5.143

5.143 Two cover plates, each $\frac{5}{8}$ -in. thick, are welded to a W30 × 99 beam as shown. Knowing that $\sigma_{all} = 22$ ksi for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.



$$R_A = R_B = 240 \text{ Kips}$$

$$\begin{aligned} & \text{For } \sum M_J = 0 \\ & -240x + 30 \times \frac{x}{2} + M = 0 \\ & M = 240x - 15x^2 \text{ kip-ft} \end{aligned}$$

For W 30 × 99 rolled steel section

$$S = 269 \text{ in}^3$$

Allowable bending moment

$$\begin{aligned} M_{all} &= G_{all} S = (22)(269) = 5918 \text{ kip-in.} \\ &= 493.167 \text{ kip-ft} \end{aligned}$$

To locate points D and E, set $M = M_{all}$

$$240x - 15x^2 = 493.167 \quad 15x^2 - 240x + 493.167 = 0$$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft}, 13.58 \text{ ft.}$$

$$(a) \quad l = x_E - x_D = 13.58 - 2.42 = 11.16 \text{ ft.}$$

$$\text{Center of beam} \quad M = 960 \text{ kip-ft} = 11520 \text{ kip-in.}$$

$$S = \frac{M}{G_{all}} = \frac{11520}{22} = 523.64 \text{ in}^3$$

$$C = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

$$\text{Required moment of inertia} \quad I = Sc = 8090 \text{ in}^4$$

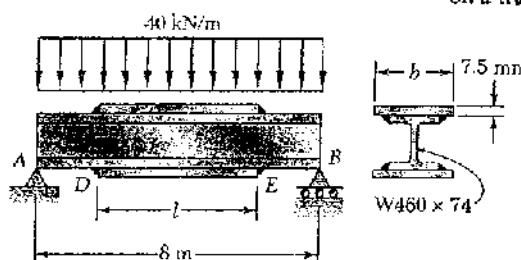
$$\text{But} \quad I = I_{beam} + 2I_{plate}$$

$$\begin{aligned} 8090 &= 3990 + 2 \left\{ (b)(0.625) \left(\frac{29.65}{2} + \frac{0.625}{2} \right)^2 + \frac{1}{12}(b)(0.625)^3 \right\} \\ &= 3990 + 286.47 b \end{aligned}$$

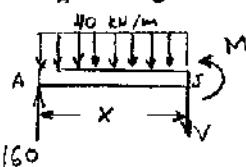
$$(b) \quad b = 14.31 \text{ in.}$$

Problem 5.144

5.144 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $l = 5 \text{ m}$ and $b = 200 \text{ mm}$, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



$$R_A = R_B = 160 \text{ kN}$$



$$+\sum M_J = 0$$

$$-160x + (40x)\frac{x}{2} + M = 0$$

$$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$$

At center of beam $x = 4 \text{ m}$ $M_c = 320 \text{ kN}\cdot\text{m}$

At D $x = \frac{1}{2}(8-l) = 1.5 \text{ m}$ $M_D = 195 \text{ kN}\cdot\text{m}$

At center of beam $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 333 \times 10^4 + 2 \left\{ (200)(7.5) \left(\frac{457}{2} + \frac{7.5}{2} \right)^2 + \frac{1}{12}(200)(7.5)^3 \right\}$$

$$= 494.8 \times 10^4 \text{ mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm} \quad S = \frac{I}{c} = 2097 \times 10^3 \text{ mm}^3 \\ = 2097 \times 10^{-6} \text{ m}^3$$

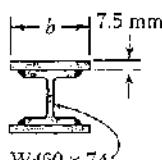
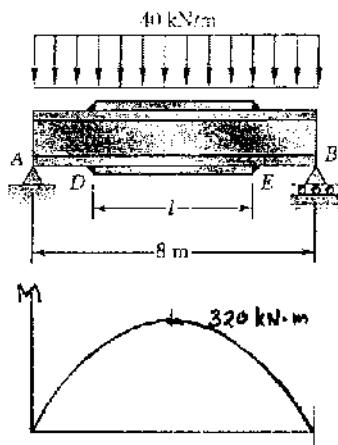
(a) Normal stress $\sigma = \frac{M}{S} = \frac{320 \times 10^3}{2097 \times 10^3} = 152.6 \times 10^6 \text{ Pa} \\ = 152.6 \text{ MPa}$

At D $S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$

(b) Normal stress $\sigma = \frac{M}{S} = \frac{195 \times 10^3}{1460 \times 10^{-6}} = 133.6 \times 10^6 \text{ Pa} \\ = 133.6 \text{ MPa}$

Problem 5.145

5.145 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $\sigma_{all} = 150 \text{ MPa}$ for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.



$$R_A = R_B = 160 \text{ kN}$$

$$\begin{aligned} & +\odot \sum M_J = 0 \\ & -160x + (40 \times \frac{x}{2}) + M = 0 \\ & M = 160x - 20x^2 \text{ kN}\cdot\text{m} \end{aligned}$$

For W 460 × 74 rolled steel beam

$$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$$

Allowable bending moment

$$\begin{aligned} M_{all} &= G_{all} S = (150 \times 10^6)(1460 \times 10^{-6}) \\ &= 219 \times 10^3 \text{ N}\cdot\text{m} = 219 \text{ kN}\cdot\text{m} \end{aligned}$$

To locate points D and E, set $M = M_{all}$

$$160x - 20x^2 = 219$$

$$20x^2 - 160x + 219 = 0$$

$$x = \frac{160 \pm \sqrt{160^2 - 4(20)(219)}}{(2)(20)} = \frac{1.753 \text{ m}}{6.247 \text{ m}}$$

$$(a) \quad x_D = 1.753 \text{ ft.} \quad x_E = 6.247 \text{ ft.} \quad l = x_E - x_D = 4.49 \text{ m}$$

At center of beam $M = 320 \text{ kN}\cdot\text{m} = 320 \times 10^3 \text{ N}\cdot\text{m}$

$$S = \frac{M}{G_{all}} = \frac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6} \text{ m}^3 = 2133 \times 10^3 \text{ mm}^3$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}^4$$

Required moment of inertia $I = Sc = 503.4 \times 10^6 \text{ mm}^4$

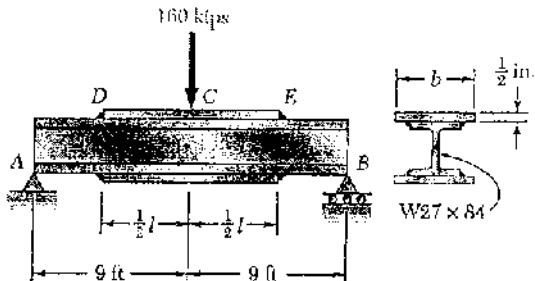
But $I = I_{beam} + 2I_{plate}$

$$\begin{aligned} 503.4 \times 10^6 &= 333 \times 10^6 + 2 \left\{ (b)(7.5) \left(\frac{457}{2} + \frac{7.5}{2} \right)^2 + \frac{1}{12}(b)(7.5)^3 \right\} \\ &= 333 \times 10^6 + 809.2 \times 10^3 b \end{aligned}$$

$$(b) \quad b = 211 \text{ mm}$$

Problem 5.146

5.146 Two cover plates, each $\frac{1}{2}$ -in. thick, are welded to a W27 x 84 beam as shown. Knowing that $L = 10$ ft and $b = 10.5$ in., determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.

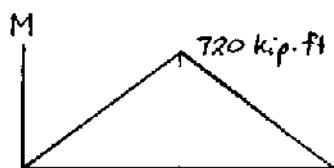
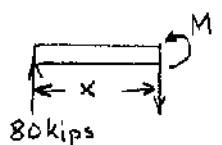


$$R_A = R_B = 80 \text{ kips}$$

$$\sum M_f = 0$$

$$-80x + M = 0$$

$$M = 80x \text{ kip-ft}$$



$$\begin{aligned} \text{At } C \quad x = 9 \text{ ft} \quad M_c &= 720 \text{ kip-ft} = 8640 \text{ kip-in} \\ \text{At } D \quad x = 9 - 5 = 4 \text{ ft} \quad M_b &= (80)(4) = 320 \text{ kip-ft} = 3840 \text{ kip-in} \end{aligned}$$

$$\text{At center of beam} \quad I = I_{\text{beam}} + 2I_{\text{plate}}$$

$$\begin{aligned} I &= 2850 + 2 \left\{ (10.5)(0.500) \left(\frac{26.71}{2} + \frac{0.500}{2} \right)^2 + \frac{1}{12}(10.5)(0.500)^3 \right\} \\ &= 4794 \text{ in}^3 \\ C &= \frac{26.71}{2} + 0.500 = 13.855 \text{ in} \end{aligned}$$

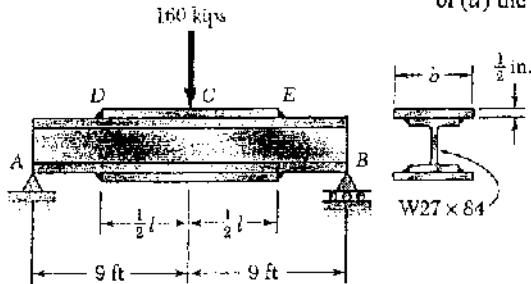
$$(a) \text{ Normal stress} \quad \sigma = \frac{Mc}{I} = \frac{(8640)(13.855)}{4794} = 25.0 \text{ ksi}$$

$$\text{At point D} \quad S = 213 \text{ in}^3$$

$$(b) \text{ Normal stress} \quad \sigma = \frac{M}{S} = \frac{3840}{213} = 18.03 \text{ ksi}$$

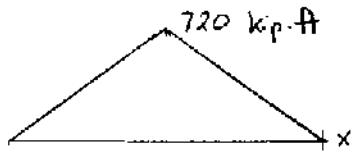
Problem 5.147

5.147 Two cover plates, each $\frac{1}{2}$ -in. thick, are welded to a W27 × 84 beam as shown. Knowing that $\sigma_{all} = 24$ ksi for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.



$$R_A = R_B = 80 \text{ Kips}$$

$$\begin{aligned} \sum M_J &= 0 \\ -80x + M &= 0 \\ M &= 80x \text{ kip-ft} \end{aligned}$$



$$\text{At D } S = 213 \text{ in}^3$$

Allowable bending moment

$$\begin{aligned} M_{all} &= \sigma_{all} S = (24)(213) = 5112 \text{ kip-in} \\ &= 426 \text{ kip} \end{aligned}$$

$$\text{Set } M_0 = M_{all}$$

$$80x_0 = 426 \quad x_0 = 5.325 \text{ ft}$$

$$l = 18 - 2x_0 = 7.35 \text{ ft}$$

(a)

$$\text{At center of beam } M = (80)(9) = 720 \text{ kip-ft} = 8640 \text{ kip-in.}$$

$$S = \frac{M}{\sigma_{all}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$C = \frac{26.71}{2} + 0.500 = 13.855 \text{ in}$$

$$\text{Required moment of inertia } I = Sc = 4987.8 \text{ in}^4$$

$$\text{But } I = I_{beam} + 2I_{plate}$$

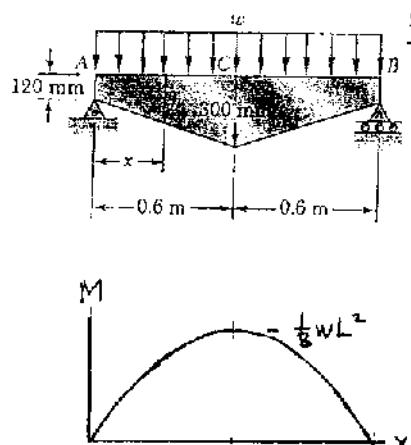
$$\begin{aligned} 4987.8 &= 2850 + 2 \left\{ (b)(0.500) \left(\frac{26.71}{2} + \frac{0.500}{2} \right)^2 + \frac{1}{12}(b)(0.500)^3 \right\} \\ &= 2850 + 185.12 b \end{aligned}$$

(b)

$$b = 11.55 \text{ in.}$$

Problem 5.148

5.148 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs. (b) the largest distributed load w that can be applied, knowing that $\sigma_{\text{all}} = 140 \text{ MPa}$.



$$R_A = R_C = \frac{1}{2}WL \quad L = 1.2 \text{ m}$$

$$\sum M_J = 0$$

$$-\frac{1}{2}WL + WX \frac{X}{2} + M = 0$$

$$M = \frac{W}{2}(LX - X^2)$$

$$= \frac{W}{2}X(L - X)$$

For the tapered beam $h = a + kx$

$$a = 120 \text{ mm} \quad k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For rectangular cross section $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

$$\text{Bending stress } \sigma = \frac{M}{S} = \frac{3w}{b} \frac{Lx - x^2}{(a+kx)^2}$$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\begin{aligned} \frac{d\sigma}{dx} &= \frac{3w}{b} \frac{d}{dx} \left\{ \frac{Lx - x^2}{(a+kx)^2} \right\} = \frac{3w}{b} \left\{ \frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^4} \right\} \\ &= \frac{3w}{b} \left\{ \frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3} \right\} \\ &= \frac{3w}{b} \left\{ \frac{aL + kLx - 2ax - 2kx^2 - 2KLx + 2kx^2}{(a+kx)^3} \right\} \\ &= \frac{3w}{b} \left\{ \frac{aL - (2a+KL)x}{(a+kx)^3} \right\} = 0 \end{aligned}$$

$$(a) x_m = \frac{aL}{2a+KL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24 \text{ m}$$

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

$$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$$

$$\text{Allowable value of } M_m \quad M_m = S_m \sigma_{\text{all}} = (122.88 \times 10^{-6})(140 \times 10^6)$$

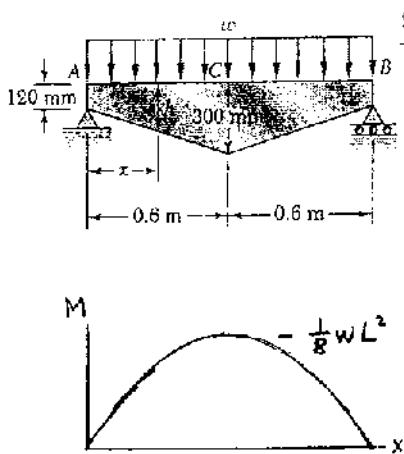
$$= 17.2032 \times 10^3 \text{ N-m}$$

$$(b) \text{ Allowable value of } w \quad w = \frac{2M_m}{X_m(L-x_m)} = \frac{(2)(17.2032 \times 10^3)}{(0.24)(0.96)}$$

$$= 149.3 \times 10^3 \text{ N/m} = 149.3 \text{ kN/m}$$

Problem 5.149

5.149 For the tapered beam shown, knowing that $w = 160 \text{ kN/m}$, determine (a) the transverse section in which the maximum normal stress occurs. (b) the corresponding value of the normal stress.



$$R_A = R_B = \frac{1}{2} wL$$

$$\sum M_J = 0$$

$$-\frac{1}{2} wL x + w \times \frac{x}{2} + M = 0$$

$$M = \frac{w}{2} (Lx - x^2)$$

$$= \frac{w}{2} x (L - x)$$

where $w = 160 \text{ kN/m}$ and $L = 1.2 \text{ m}$.

For the tapered beam $h = a + kx$

$$a = 120 \text{ mm} \quad k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For a rectangular cross section $S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

$$\text{Bending stress } \sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a + kx)^2}$$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b} \frac{d}{dx} \left\{ \frac{Lx - x^2}{(a + kx)^2} \right\} = \frac{3w}{b} \left\{ \frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^4} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a + kx)^3} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{aL - 2ax - kLx}{(a + kx)^3} \right\} = 0$$

$$(a) x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24 \text{ m}$$

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

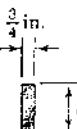
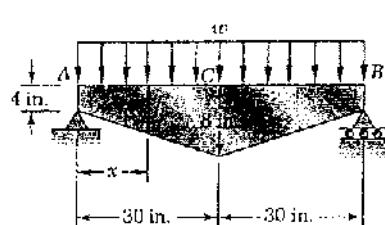
$$S_m = \frac{1}{6} b h_m^2 = \frac{1}{6} (20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$$

$$M_m = \frac{w}{2} x_m (L - x_m) = \frac{160 \times 10^3}{2} (0.24)(0.96) = 18.432 \times 10^3 \text{ N-m}$$

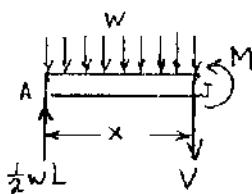
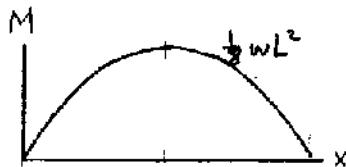
$$(b) \text{ Maximum bending stress } \sigma_m = \frac{M_m}{S_m} = \frac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6 \text{ Pa} \\ = 150 \text{ MPa}$$

Problem 5.150

5.150 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs. (b) the largest distributed load w that can be applied, knowing that $\sigma_{all} = 24 \text{ ksi}$.



$$R_A = R_B = \frac{1}{2} wL \quad L = 60 \text{ in}$$



$$\sum M_J = 0$$

$$-\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L-x)$$

For the tapered beam $h = a + kx$

$$a = 4 \text{ in} \quad k = \frac{8-4}{30} = \frac{2}{15} \text{ in/in.}$$

For a rectangular cross section $S = \frac{1}{6}bh^3 = \frac{1}{6}b(a+kx)^3$

$$\text{Bending stress } \sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a+kx)^2}$$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\begin{aligned} \frac{d\sigma}{dx} &= \frac{3w}{b} \frac{d}{dx} \left\{ \frac{Lx - x^2}{(a+kx)^2} \right\} = \frac{3w}{b} \left\{ \frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^4} \right\} \\ &= \frac{3w}{b} \left\{ \frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3} \right\} \\ &= \frac{3w}{b} \left\{ \frac{aL + kLx - 2ax - 2kx^2 - 2KLx + 2kx^2}{(a+kx)^3} \right\} \\ &= \frac{3w}{b} \left\{ \frac{aL - (2a+KL)x}{(a+kx)^3} \right\} = 0 \end{aligned}$$

$$(a) \quad x_m = \frac{aL}{2a+KL} = \frac{(4)(60)}{(2)(4) + (\frac{2}{15})(60)} = 15 \text{ in.}$$

$$h_m = a + kx_m = 4 + (\frac{2}{15})(15) = 6.00 \text{ in.}$$

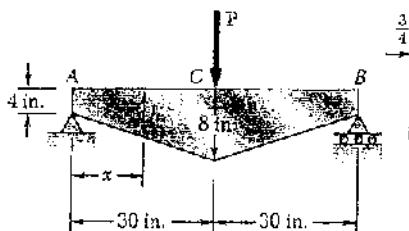
$$S_m = \frac{1}{6}bh_m^3 = (\frac{1}{6})(\frac{3}{4})(6.00)^2 = 4.50 \text{ in}^3$$

$$\text{Allowable value of } M_m = S_m \sigma_{all} = (4.50)(24) = 108.0 \text{ kip-in}$$

$$(b) \text{ Allowable value of } w \quad w = \frac{2M_m}{x_m(L-x_m)} = \frac{(2)(108.0)}{(15)(45)} = 0.320 \text{ kip/in} \\ = 320 \text{ lb/in}$$

Problem 5.151

5.151 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs. (b) the largest concentrated load P that can be applied, knowing that $\sigma_{all} = 24$ ksi.



$$R_A = R_B = \frac{P}{2}$$

$$\begin{aligned} \text{Free body diagram: } & M \\ & P \quad x \quad V \\ & -\frac{Px}{2} + M = 0 \\ & M = \frac{Px}{2} \quad (0 < x < \frac{L}{2}) \end{aligned}$$

$$\text{For a tapered beam} \quad h = a + kx$$

$$\text{For a rectangular cross section} \quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$$

$$\text{Bending stress} \quad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\begin{aligned} \frac{d\sigma}{dx} &= \frac{3P}{b} \frac{d}{dx} \left\{ \frac{x}{(a+kx)^2} \right\} = \frac{3P}{b} \frac{(a+kx)^2 - x \cdot 2(a+kx)k}{(a+kx)^4} \\ &= \frac{3P}{b} \frac{a - kx}{(a+kx)^3} = 0 \quad x_m = \frac{a}{k} \end{aligned}$$

$$\text{Data: } a = 4 \text{ in.}, \quad k = \frac{8-4}{30} = 0.13333 \text{ in/in}$$

$$(a) \quad x_m = \frac{4}{0.13333} = 30 \text{ in.}$$

$$h_m = a + kx_m = 8 \text{ in}$$

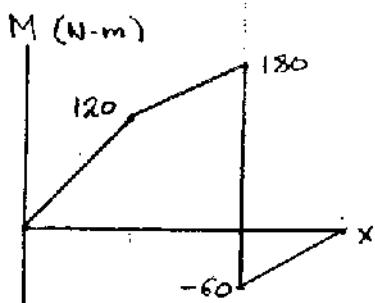
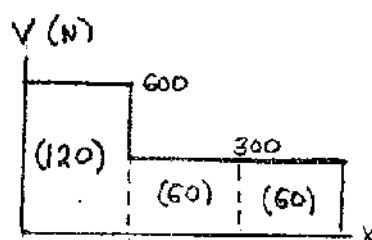
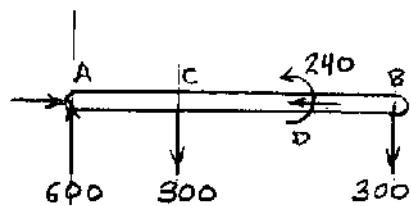
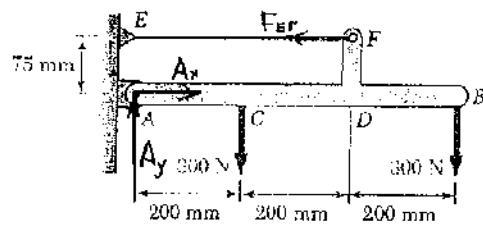
$$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$$

$$M_m = G_m S_m = (24)(8) = 192 \text{ kip-in}$$

$$(b) \quad P = \frac{2M_m}{X_m} = \frac{(2)(192)}{30} = 12.8 \text{ kips}$$

Problem 5.152

5.152 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



$$\textcircled{+} \sum M_A = 0$$

$$0.075 F_{EF} - (0.2)(300) - (0.6)(300) = 0$$

$$F_{EF} = 3.2 \times 10^3 \text{ N}$$

$$\textcircled{+} \sum F_x = 0 \quad A_x - F_{EF} = 0 \quad A_x = 3.2 \times 10^3 \text{ N}$$

$$\textcircled{+} \sum F_y = 0 \quad A_y - 300 - 300 = 0$$

$$A_y = 600 \text{ N}$$

$$\text{Couple at D} \quad M_D = (0.075)(3.2 \times 10^3)$$

$$= 240 \text{ N}\cdot\text{m}$$

Shear

$$A \text{ to } C \quad V = 600 \text{ N}$$

$$C \text{ to } B \quad V = 600 - 300 = 300 \text{ N}$$

Areas under shear diagram

$$A \text{ to } C \quad \int V dx = (0.2)(600) = 120 \text{ N}\cdot\text{m}$$

$$C \text{ to } D \quad \int V dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$$

$$D \text{ to } B \quad \int V dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$$

Bending moments

$$M_A = 0$$

$$M_C = 0 + 120 = 120 \text{ N}\cdot\text{m}$$

$$M_D^- = 120 + 60 = 180 \text{ N}\cdot\text{m}$$

$$M_D^+ = 180 - 240 = -60 \text{ N}\cdot\text{m}$$

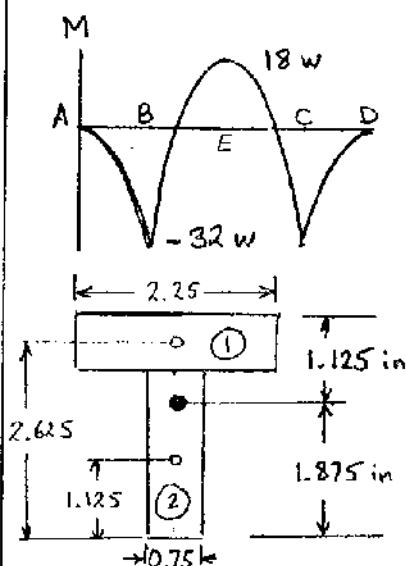
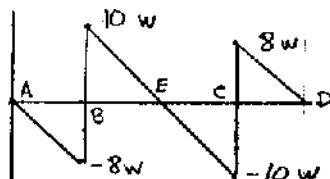
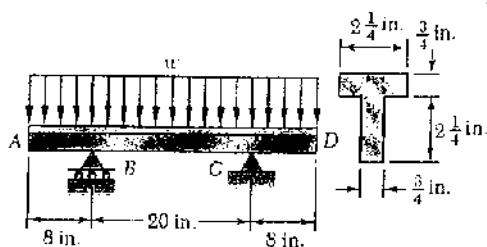
$$M_B = -60 + 60 = 0$$

$$(a) \quad \text{Maximum } |V| = 600 \text{ N}$$

$$(b) \quad \text{Maximum } |M| = 180 \text{ N}\cdot\text{m}$$

Problem 5.153

5.153 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and -29.5 ksi in compression.



$$\text{Top: } y = 1.125$$

$$\text{Bottom: } y = -1.875$$

$$I/y = 2.3906 \text{ in}^3$$

$$I/y = -1.4343 \text{ in}^3$$

Bending moment limits

$$M = -5I/y$$

Tension at B and C

$$-(12)(2.3906) = -28.687 \text{ kip-in}$$

Comp. at B and C

$$-(-19.5)(-1.4343) = -27.969 \text{ kip-in}$$

Tension at E

$$-(12)(-1.4343) = 17.212 \text{ kip-in}$$

Compression at E

$$-(-19.5)(2.3906) = 46.6 \text{ kip-in}$$

Allowable load w

$$\begin{array}{lll} B \neq C & -32w = -27.969 & w = 0.874 \text{ kip/in} \\ E & 18w = 17.212 & w = 0.956 \text{ kip/in} \end{array}$$

$$\text{Smallest } w = 0.874 \text{ kip/in} = 10.49 \text{ kip/ft.}$$

$$\text{Reactions: } B + C - 36w = 0 \quad B = C = 18w$$

$$\text{Shear: } V_A = 0$$

$$V_{B-} = 0 - 8w = -8w$$

$$V_{B+} = -8w + 18w = 10w$$

$$V_{C-} = 10w - 20w = -10w$$

$$V_{C+} = -10w + 18w = 8w$$

$$V_D = 8w - 8w = 0$$

$$\text{Areas: } A \text{ to } B \quad (\frac{1}{2})(8)(4w) = -32w$$

$$B \text{ to } E \quad (\frac{1}{2})(10)(10w) = 50w$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 - 32w = -32w$$

$$M_E = -32w + 50w = 18w$$

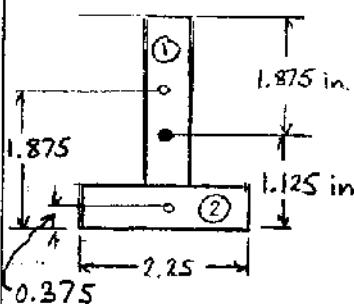
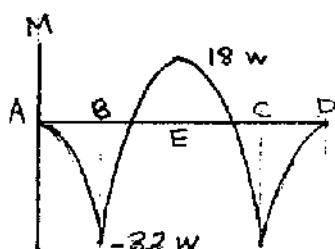
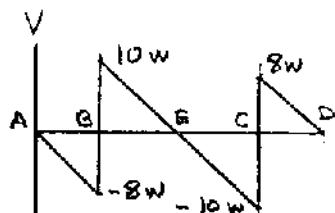
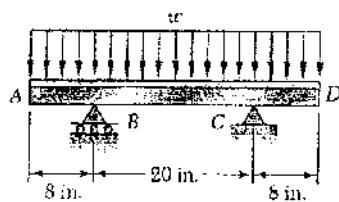
Centroid and moment of inertia

Part	$A (\text{in}^2)$	$\bar{y} (\text{in})$	$A\bar{y} (\text{in}^3)$	$d \text{ in}^2$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
①	1.6875	2.625	44.4297	0.75	0.9492	0.0791
②	1.6875	1.125	1.8984	0.75	0.9492	0.7119
Σ	3.375		6.3281		1.8984	0.7910

$$\bar{y} = \frac{6.3281}{3.375} = 1.875 \text{ in}$$

$$I = \sum Ad^2 + \sum \bar{I} = 2.6894 \text{ in}^4$$

Problem 5.154



Top: $y = 1.875$ in
Bottom: $y = -1.125$

Centroid and moment of inertia

Part	$A (\text{in}^2)$	$\bar{y} (\text{in})$	$A\bar{y} (\text{in}^3)$	$d (\text{in})$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
①	1.6875	1.875	3.1641	0.75	0.9492	0.7119
②	1.6875	0.375	0.6328	0.75	0.9492	0.0791
Σ	3.375		3.7969		1.8984	0.7910

$$\bar{Y} = \frac{3.7969}{3.375} = 1.125 \text{ in.}$$

$$I = \sum Ad^2 + \bar{I} = 2.6894 \text{ in}^4$$

$$\begin{aligned} I/y &= 1.4343 \text{ in}^3 \\ -I/y &= -2.3906 \text{ in}^3 \end{aligned}$$

Bending moment limits $M = -6I/y$

Tension at B and C

$$-(12)(1.4343) = -17.212 \text{ kip-in} \leftarrow$$

Comp. at B and C

$$-(+19.5)(-2.3906) = -46.6 \text{ kip-in}$$

Tension at E

$$-(12)(-2.3906) = 28.687 \text{ kip-in}$$

Compression at E

$$-(+19.5)(1.4343) = 27.969 \text{ kip-in} \leftarrow$$

Allowable load w :

$$B+C: -32w = -17.212 \quad w = 0.539 \text{ kip/in}$$

$$E: 18w = 27.969 \quad w = 1.554 \text{ kip/in}$$

Smallest $w = 0.539 \text{ kip/in} = 6.45 \text{ kip/ft} \leftarrow$

5.154 Solve Prob. 5.153, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at B and C.

5.153 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and -29.5 ksi in compression.

$$\text{Reactions: } B + C - 36w = 0 \quad B = C = 18w$$

Shear: $V_A = 0$

$$V_G = 0 - 8w = -8w$$

$$V_B^+ = -8w + 18w = 10w$$

$$V_C^- = 10w - 20w = -10w$$

$$V_C^+ = -10w + 18w = 8w$$

$$V_D = 8w - 8w = 0$$

$$\text{Areas: } A \text{ to } B: \frac{1}{2}(8)(-8w) = -32w$$

$$B \text{ to } E: \frac{1}{2}(10)(10w) = 50w$$

Bending moments: $M_A = 0$

$$M_B = 0 - 32w = -32w$$

$$M_E = -32w + 50w = 18w$$

Centroid and moment of inertia

Part	$A (\text{in}^2)$	$\bar{y} (\text{in})$	$A\bar{y} (\text{in}^3)$	$d (\text{in})$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
①	1.6875	1.875	3.1641	0.75	0.9492	0.7119
②	1.6875	0.375	0.6328	0.75	0.9492	0.0791
Σ	3.375		3.7969		1.8984	0.7910

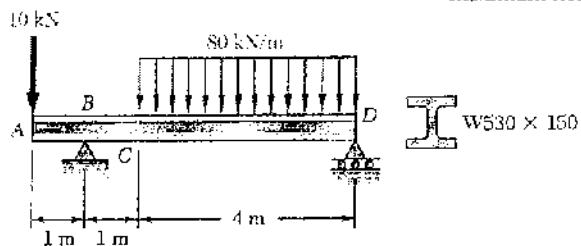
$$\bar{Y} = \frac{3.7969}{3.375} = 1.125 \text{ in.}$$

$$I = \sum Ad^2 + \bar{I} = 2.6894 \text{ in}^4$$

$$\begin{aligned} I/y &= 1.4343 \text{ in}^3 \\ -I/y &= -2.3906 \text{ in}^3 \end{aligned}$$

Problem 5.155

5.155 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



$$+ \oint M_D = 0 \\ (6)(10) - 5R_B + (2)(4)(80) = 0 \\ R_B = 140 \text{ kN}$$

$$W = 80(x-2)^0 \text{ kN/m} = -dV/dx$$

$$V = -10 + 140(x-1)^0 - 80(x-2)' \text{ kN}$$

$$\text{A to B} \quad V = -10 \text{ kN}$$

$$\text{B to C} \quad V = -10 + 140 = 130 \text{ kN}$$

$$\text{D } (x=6) \quad V = -10 + 140 - 80(4) = -190 \text{ kN}$$

V changes sign at B and at point E ($x=x_E$) between C and D.

$$V = 0 = -10 + 140(x_E-1)^0 - 80(x_E-2)' \\ = -10 + 140 - 80(x_E-2) \quad x_E = 3.625 \text{ m}$$

$$M = -10x + 140(x-1)' - 40(x-2)^2 \text{ kN-m}$$

$$\text{At pt. B} \quad x=1 \quad M_B = -(10)(1) = -10 \text{ kN-m}$$

$$\text{At pt. E} \quad x=3.625$$

$$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6 \text{ kN-m}$$

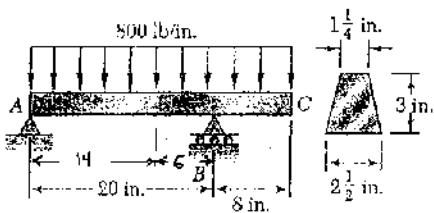
$$(a) |M|_{\max} = 225.6 \text{ kN-m} \quad \text{at } x = 3.625 \text{ m}$$

$$\text{For W530 x 150} \quad S = 3720 \times 10^3 \text{ mm}^3 = 3720 \times 10^{-6} \text{ m}^3$$

$$(b) \text{ Normal stress} \quad \sigma = \frac{|M|}{S} = \frac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6 \text{ Pa} \\ = 60.6 \text{ MPa}$$

Problem 5.156

5.156 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

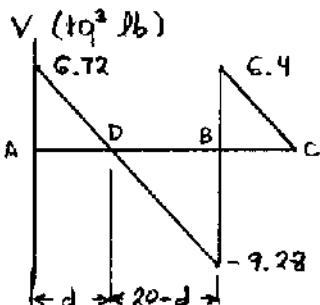


$$\textcircled{+} \sum M_B = 0 \quad -20A + (6)(28)(800) = 0$$

$$A = 6.72 \times 10^3 \text{ lb.}$$

$$\textcircled{-} \sum M_A = 0 \quad 20B - (14)(28)(800) = 0$$

$$B = 15.68 \times 10^3 \text{ lb.}$$



$$\text{Shear: } V_A = 6.72 \times 10^3 \text{ lb.}$$

$$B^- \quad V_B^- = 6.72 \times 10^3 - (20)(800) = -9.28 \times 10^3 \text{ lb.}$$

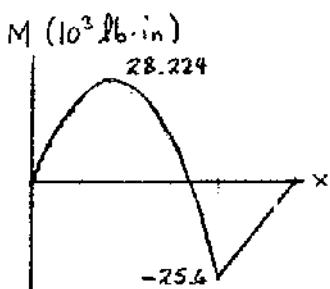
$$B^+ \quad V_B^+ = -9.28 \times 10^3 + (15.68 \times 10^3) = 6.4 \times 10^3 \text{ lb.}$$

$$C \quad V_C = 6.4 \times 10^3 - (8)(800) = 0$$

Locate point D where $V = 0$

$$\frac{d}{6.72} = \frac{20-d}{9.28} \quad 16d = 134.4$$

$$d = 8.4 \text{ in} \quad 20-d = 11.6 \text{ in.}$$



Areas under shear diagram

$$A \text{ to } D \quad \int V dx = (\frac{1}{2})(8.4)(6.72 \times 10^3) = 28.224 \times 10^3 \text{ lb-in}$$

$$D \text{ to } B \quad \int V dx + (\frac{1}{2})(11.6)(-9.28 \times 10^3) = -53.824 \times 10^3 \text{ lb-in}$$

$$B \text{ to } C \quad \int V dx = (\frac{1}{2})(8)(6.4 \times 10^3) = 25.6 \times 10^3 \text{ lb-in.}$$

Bending moments: $M_A = 0$

$$M_D = 0 + 28.224 \times 10^3 = 28.224 \times 10^3 \text{ lb-in}$$

$$M_B = 28.224 \times 10^3 - 53.824 \times 10^3 = -25.6 \times 10^3 \text{ lb-in.}$$

$$M_C = -25.6 \times 10^3 + 25.6 \times 10^3 = 0$$

$$\text{Maximum } |M| = 28.224 \times 10^3 \text{ lb-in.}$$

Locate centroid of cross section

$$\bar{Y} = \frac{7.5}{5.625} = 1.3333 \text{ in. from bottom}$$

$$\text{For each triangle } \bar{I} = \frac{1}{36} b h^3$$

Moment of inertia

$$I = \sum \bar{I} + \sum Ad^2$$

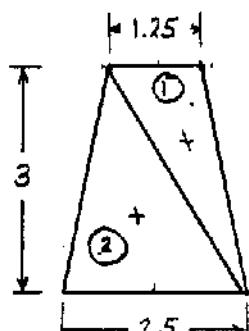
$$= 1.25 + 2.8125 = 4.0625 \text{ in}^4$$

Normal stress

$$\sigma = \frac{Mc}{I} = \frac{(28.224 \times 10^3)(1.6667)}{4.0625}$$

$$= 11.58 \times 10^3 \text{ psi}$$

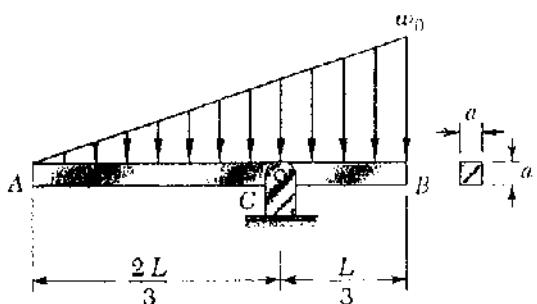
$$= 11.58 \text{ ksi}$$



Part	$A_i \text{ in}^2$	$\bar{y}_i \text{ in}$	$A\bar{y}_i \text{ in}^3$	$d_i \text{ in}$	$A d_i^2 \text{ in}^4$	$\bar{I} \text{ in}^4$
①	1.975	2	3.75	0.6667	0.8333	0.9375
②	3.75	1	3.75	0.3333	0.4167	1.875
Σ	5.625	7.5			1.25	2.8125

Problem 5.157

5.157 Beam AB , of length L and square cross section of side a , is supported by a pivot at C and loaded as shown. (a) Check that the beam is in equilibrium. (b) Show that the maximum stress due to bending occurs at C and is equal to $w_0 L^2 / (1.5a)^3$.



Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

For the triangular distribution the centroid lies at $x = \frac{2L}{3}$.

$$W = \frac{1}{2} w_0 L$$

$$\sum F_y = 0 \quad R_o - W = 0 \quad R_o = \frac{1}{2} w_0 L$$

$$\sum M_c = 0 \quad 0 = 0 \quad \text{equilibrium}$$

$$V = 0, M = 0 \quad \text{at } x = 0$$

$$0 < x < \frac{2L}{3}$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L} + C_1 = -\frac{w_0 x^2}{2L}$$

$$M = -\frac{w_0 x^3}{6L} + C_2 = -\frac{w_0 x^3}{6L}$$

Just to the left of C

$$V = -\frac{w_0 (2L/3)^2}{2L} = -\frac{2}{9} w_0 L$$

Just to the right of C

$$V = -\frac{2}{9} w_0 L + R_o = \frac{5}{18} w_0 L$$

Note sign change. Maximum $|M|$ occurs at C .

$$M_c = -\frac{w_0 (2L/3)^3}{6L} = -\frac{4}{81} w_0 L^2$$

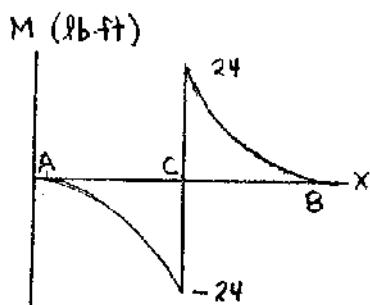
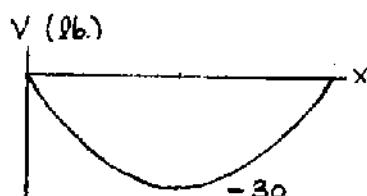
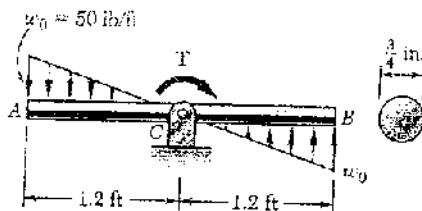
$$\text{Maximum } |M| = \frac{4}{81} w_0 L^2$$

$$\text{For square cross section} \quad I = \frac{1}{12} a^4 \quad c = \frac{1}{2} a$$

$$G_m = \frac{|M|_{\max} c}{I} = \frac{4}{81} \frac{w_0 L^2}{a^3} \cdot \frac{a}{2} = \frac{8}{27} \frac{w_0 L^2}{a^3} = \left(\frac{2}{3}\right)^3 \frac{w_0 L^2}{a^3} = \frac{w_0 L^2}{(1.5a)^3}$$

Problem 5.158

5.158 Knowing that rod AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



A to C $0 < x < 1.2 \text{ ft}$

$$w = 50 \left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$$

$$\frac{dV}{dx} = -w = 41.667x - 50$$

$$V = V_A + \int_0^x (41.667x - 50) dx \\ = 0 + 20.833x^2 - 50x = \frac{dM}{dx}$$

$$M = M_A + \int_0^x V dx$$

$$= 0 + \int_0^x (20.833x^2 - 50x) dx$$

$$= 6.944x^3 - 25x^2$$

$$\text{At } x = 1.2 \text{ ft}, \quad V = -30 \text{ lb.} \\ M = -24 \text{ lb-in.}$$

C to B Use symmetry conditions.

$$\text{Maximum } |M| = 24 \text{ lb-ft} = 288 \text{ lb-in.}$$

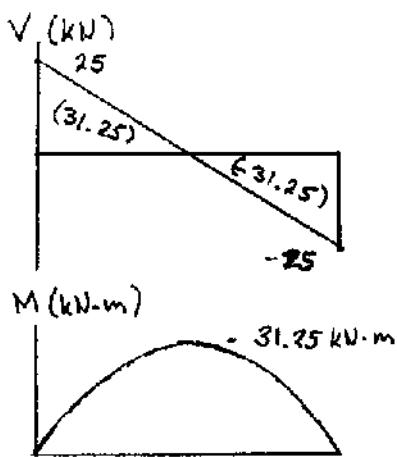
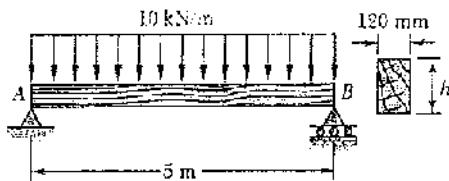
$$\text{Cross section} \quad c = \frac{d}{2} = \left(\frac{1}{2}\right)(0.75) = 0.375 \text{ in}$$

$$I = \frac{\pi}{4} c^4 = \left(\frac{\pi}{4}\right)(0.375)^4 = 15.532 \times 10^{-3} \text{ in}^4$$

$$\text{Normal stress} \quad \sigma = \frac{|M|c}{I} = \frac{(288)(0.375)}{15.532 \times 10^{-3}} = 6.95 \times 10^3 \text{ psi} \\ = 6.95 \text{ ksi}$$

Problem 5.159

5.159 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



Reactions: $A = B$ by symmetry

$$\uparrow \sum F_y = 0 \quad A + B - (5)(10) = 0$$

$$A = B = 25 \text{ kN}$$

From bending moment diagram

$$|M|_{\max} = 31.25 \text{ kN}\cdot\text{m} = 31.25 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{31.25 \times 10^3}{12 \times 10^6} = 2.604 \times 10^{-3} \text{ m}^3 \\ = 2.604 \times 10^6 \text{ mm}^3$$

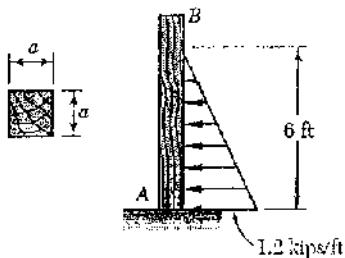
$$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(120) h^2 = 2.604 \times 10^6$$

$$h^2 = \frac{(6)(2.604 \times 10^6)}{120} = 130.21 \times 10^3 \text{ mm}^2$$

$$h = 361 \text{ mm}$$

Problem 5.160

5.160 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.



Equivalent concentrated load

$$P = \left(\frac{1}{2}\right)(6)(1.2) = 3.6 \text{ kips}$$

Bending moment at A

$$M_A = (2)(3.6) = 7.2 \text{ kip}\cdot\text{ft} = 86.4 \text{ kip}\cdot\text{in}$$

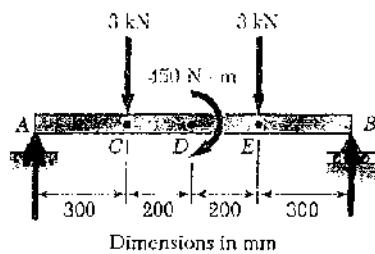
$$S_{min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3$$

For a square section $S = \frac{1}{6} a^3$

$$a = \sqrt[3]{6 \cdot 49.37}$$

$$a_{min} = \sqrt[3]{(6)(49.37)} = 6.67 \text{ in.}$$

Problem 5.161



5.161 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

$$\text{At } B \quad \sum M_B = 0 \quad (100)(3) - 450 + (300)(3) - 1000A = 0 \\ A = 2.55 \text{ kN} \uparrow$$

$$\sum M_A = 0 \quad -(300)(3) - 450 - (700)(3) + 1000B = 0 \\ B = 3.45 \text{ kN} \uparrow$$

At A $V = 2.55 \text{ kN}$ $M = 0$

At C $V = 2.55 \text{ kN}$

At C $\sum M_C = 0$
 $-(300)(2.55) + M = 0$
 $M = 765 \text{ N}\cdot\text{m}$

C to E $V = -0.45 \text{ N}\cdot\text{m}$

At D⁻ $\sum M_D = 0$
 $-(500)(2.55) + (200)(3) + M = 0$
 $M = 675 \text{ N}\cdot\text{m}$

At D⁺ $\sum M_D = 0$
 $-(500)(2.55) + (200)(3) - 450 + M = 0$
 $M = 1125 \text{ N}\cdot\text{m}$

E to B $V = -3.45 \text{ kN}$

At E $\sum M_E = 0$
 $-M + (300)(3.45) = 0$
 $M = 1035 \text{ N}\cdot\text{m}$

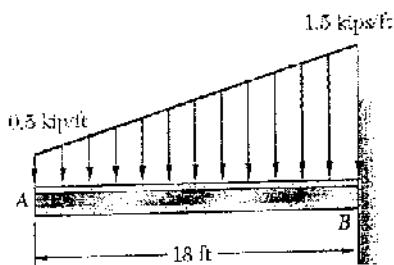
At B $V = 3.45 \text{ kN}$, $M = 0$

(a) Maximum $|V| = 3.45 \text{ kN}$

(b) Maximum $|M| = 1125 \text{ N}\cdot\text{m}$

Problem 5.162

5.162 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



$$W = 0.5 + \frac{(1.5 - 0.5)x}{18} = 0.5 + 0.05555 x$$

$$\frac{dV}{dx} = -W = -0.5 - 0.05556 x$$

$$V = 0 - 0.5x - 0.02778 x^2 = \frac{dM}{dx}$$

$$M = 0 - 0.25x^2 - 0.009259 x^3$$

Maximum $|M|$ occurs at $x = 18 \text{ ft}$.

$$|M|_{\max} = (0.25)(18)^2 + (0.009259)(18)^3 = 135 \text{ kip-ft} = 1620 \text{ kip-in}$$

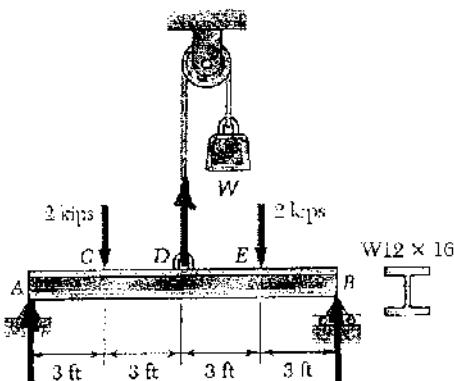
$$\sigma_{all} = 24 \text{ ksi}$$

$$S_{min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{1620}{24} = 67.5 \text{ in}^3$$

Shape	$S (\text{in}^3)$	
W21 x 44	81.6	Lightest wide flange beam
W18 x 50	88.9	←
W16 x 57	92.2	
W14 x 53	77.8	W18 x 50 @ 50 lb/ft
W12 x 72	97.4	→
W10 x 68	75.7	

Problem 5.163

5.163 Determine (a) the magnitude of the counterweight W for which the maximum value of the bending moment in the beam is as small as possible, (b) the corresponding maximum stress due to bending. (See hint of Prob. 5.27.)



$$\text{By symmetry } A = B$$

$$+\uparrow \sum F_y = 0 \quad A - 2 + W - 2 + B = 0$$

$$A = B = 2 - \frac{W}{2}$$

$$\text{Bending moment at } C \quad \circlearrowleft \sum M_c = 0$$

$$\begin{array}{l} A \quad c \leftarrow M_c \\ \text{Diagram: A horizontal beam segment from A to C. At point } c, \text{ there is a clockwise moment } M_c. \text{ At the left end } A, \text{ there is an upward reaction force } 2 - \frac{W}{2}. \text{ At the right end } C, \text{ there is a downward reaction force } 2. \end{array}$$

$$-(3)(2 - \frac{W}{2}) + M_c = 0$$

$$M_c = 6 - 1.5W$$

$$\text{Bending moment at } D \quad \circlearrowleft \sum M_D = 0$$

$$\begin{array}{l} \text{Diagram: A horizontal beam segment from A to D. At point } D, \text{ there is a clockwise moment } M_D. \text{ At the left end } A, \text{ there is an upward reaction force } 2 - \frac{W}{2}. \text{ At the right end } D, \text{ there is a downward reaction force } 2. \end{array}$$

$$-(6)(2 - \frac{W}{2}) + (3)(2) + M_D = 0$$

$$M_D = 6 - 3W$$

$$(a) \text{ Equate } -M_D = M_c$$

$$3W - 6 = 6 - 1.5W \quad W = 2.667 \text{ kips}$$

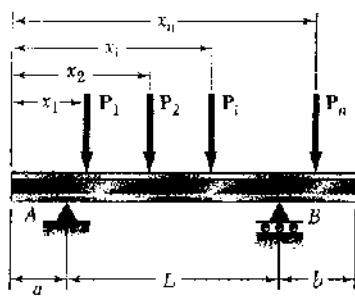
$$M_c = 2.0 \text{ kip-ft} \quad M_D = -2.0 \text{ kip-ft}$$

$$\max |M| = 2.0 \text{ kip-ft} = 24 \text{ kip-in}$$

For W12 x 16 rolled steel section $S = 17.1 \text{ in}^3$

$$(b) \text{ Normal stress } \sigma = \frac{|M|}{S} = \frac{24}{17.1} = 1.404 \text{ ksi}$$

PROBLEM 5.C1



5.C1 Several concentrated loads $P_i (i = 1, 2, \dots, n)$ can be applied to a beam as shown. Write a computer program that can be used to calculate the shear, bending moment, and normal stress at any point of the beam for a given loading of the beam and a given value of its section modulus. Use this program to solve Probs. 5.18, 5.21, and 5.25. (Hint: Maximum values will occur at a support or under a load.)

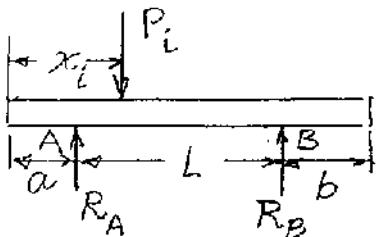
SOLUTION

REACTIONS AT A AND B

$$\rightarrow \sum M_A = 0: R_B L - \sum_i P_i (x_i - a)$$

$$R_B = (1/L) \sum_i P_i (x_i - a)$$

$$R_A = \sum_i P_i - R_B$$



WE USE STEP FUNCTIONS (See bottom of page 348 of text.)

WE DEFINE: IF $x \geq a$ THEN STPA=1 ELSE STPA=0

IF $x \geq a+L$ THEN STPB=1 ELSE STPB=0

IF $x \geq x_i$ THEN STP(i)=1 ELSE STP(i)=0

$$V = R_A STPA + R_B STPB - \sum_i P_i STP(i)$$

$$M = R_A(x-a)STPA + R_B(x-a-L)STPB - \sum_i P_i(x-x_i)STP(i)$$

$\sigma = M/S$, where S is obtained from Appendix C.

PROGRAM OUTPUTS

Problem 5.18

$$RA=80.0 \text{ kN} \quad RB=80.0 \text{ kN}$$

X m	V kN	M kN.m	Sigma MPa
2.00	0.00	104.00	139.0

Problem 5.25

$$R1 = 10.77 \text{ kips} \quad R2 = 4.23 \text{ kips}$$

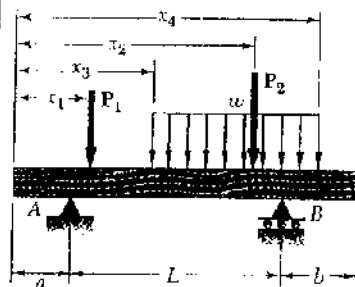
X ft	V kips	M kip.in.	Sigma ksi
0	-5.00	0	0
5	5.69	-300.00	-10.34
13	-4.23	253.8	8.75
18	-4.23	0	0

Problem 5.21

$$R1 = 52.5 \text{ kips} \quad R2 = 22.5 \text{ kips}$$

X ft	V kips	M kip.ft	Sigma ksi
0.00	-25.00	0.00	0.00
1.00	27.50	-25.00	-7.85
3.00	2.50	30.00	9.42
9.00	-22.50	45.00	14.14
11.00	0.00	0.00	0.00

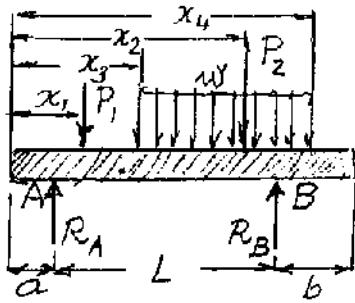
PROBLEM 5.C2



5.C2 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value σ_{all} . Write a computer program that can be used to calculate at given intervals ΔL the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals ΔL indicated: (a) Prob. 5.65 ($\Delta L = 0.1$ m), (b) Prob. 5.69 ($\Delta L = 0.2$ m), (c) Prob. 5.80 ($\Delta L = 0.3$ m).

SOLUTION

REACTIONS AT A AND B



$$\rightarrow \sum M_A = 0: R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3) \left(\frac{x_4 + x_3}{2} - a \right) = 0$$

$$R_B = \frac{1}{L} [P_1(x_1 - a) + P_2(x_2 - a) + \frac{1}{2} w(x_4 - x_3)(x_4 + x_3 - 2a)]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

$$\text{SET } x_L = (a+b+L)/\Delta L$$

$$\text{FOR } l = 0 \text{ TO } n: x = (\Delta L)l$$

WE DEFINE: IF $x \geq a$ THEN STPA = 1 ELSE STPA = 0

IF $x \geq a + L$, THEN STPB = 1 ELSE, STB = 0

IF $x \geq x_1$, THEN STP1 = 1 ELSE, STP1 = 0

IF $x \geq x_2$, THEN STP2 = 1 ELSE, STP2 = 0

IF $x \geq x_3$, THEN STP3 = 1 ELSE, STP3 = 0

IF $x \geq x_4$, THEN STP4 = 1 ELSE, STP4 = 0

$$V = R_A \text{STPA} + R_B \text{STPB} - P_1 \text{STP1} - P_2 \text{STP2} \\ - w(x - x_3) \text{STP3} + w(x - x_4) \text{STP4}$$

$$M = R_A(x - a) \text{STPA} + R_B(x - a - L) \text{STPB} - P_1(x - x_1) \text{STP1} \\ - P_2(x - x_2) \text{STP2} - \frac{1}{2} w(x - x_3)^2 \text{STP3} + \frac{1}{2} w(x - x_4)^2 \text{STP4}$$

$$S_{min} = |M| / G_{all}$$

IF UNKNOWN DIMENSION IS h :

$$\text{From } S = \frac{1}{6} t h^2, \text{ we have } h = \sqrt{6S/t}$$

IF UNKNOWN DIMENSION IS t :

$$\text{From } S = \frac{1}{6} t h^2, \text{ we have } t = 6S/h^2$$

(CONTINUED)

PROBLEM 5.C2 CONTINUED

PROGRAM OUTPUTS

Problem 5.65

RA =	2.40 kN	RB =	3.00 kN
X	V	M	H
m	kN	kN.m	mm
0.00	2.40	0.000	0.00
0.10	2.40	0.240	54.77
0.20	2.40	0.480	77.46
0.30	2.40	0.720	94.87
0.40	2.40	0.960	109.54
0.50	2.40	1.200	122.47
0.60	2.40	1.440	134.16
0.70	2.40	1.680	144.91
0.80	0.60	1.920	154.92
0.90	0.60	1.980	157.32
1.00	0.60	2.040	159.69
1.10	0.60	2.100	162.02
1.20	0.60	2.160	164.32
1.30	0.60	2.220	166.58
1.40	0.60	2.280	168.82
1.50	0.60	2.340	171.03
1.60	-3.00	2.400	173.21
1.70	-3.00	2.100	162.02
1.80	-3.00	1.800	150.00
1.90	-3.00	1.500	136.93
2.00	-3.00	1.200	122.47
2.10	-3.00	0.900	106.07
2.20	-3.00	0.600	86.60
2.30	-3.00	0.300	61.24
2.40	0.00	0.000	0.05

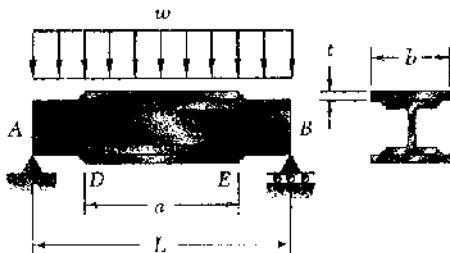
Problem 5.69

RA =	2.70 kN	RB =	8.10 kN
X	V	M	T
m	kN	kN.m	mm
0.00	2.70	0.000	0.00
0.20	2.10	0.480	10.67
0.40	1.50	0.840	18.67
0.60	0.90	1.080	24.00
0.80	0.30	1.200	26.67
1.00	-0.30	1.200	26.67
1.20	-0.90	1.080	24.00
1.40	-1.50	0.840	18.67
1.60	-2.10	0.480	10.67
1.80	-2.70	0.000	0.00
2.00	-3.30	-0.600	13.33
2.20	-3.90	-1.320	29.33
2.40	3.60	-2.160	48.00
2.60	3.00	-1.500	33.33
2.80	2.40	-0.960	21.33
3.00	1.80	-0.540	12.00
3.20	1.20	-0.240	5.33
3.40	0.60	-0.060	1.33
3.60	0.00	-0.000	0.00

Problem 5.70

RA =	6.50 kN	RB =	6.50 kN
X	V	M	H
m	kN	kN.m	mm
0.00	2.50	0.000	0.00
0.30	2.50	0.750	61.24
0.60	9.00	1.500	86.60
0.90	7.20	3.930	140.18
1.20	5.40	5.820	170.59
1.50	3.60	7.170	189.34
1.80	1.80	7.980	199.75
2.10	-0.00	8.250	203.10
2.40	-1.80	7.980	199.75
2.70	-3.60	7.170	189.34
3.00	-5.40	5.820	170.59
3.30	-7.20	3.930	140.18
3.60	-2.50	1.500	86.60
3.90	-2.50	0.750	61.24
4.20	0.00	0.000	0.06

PROBLEM 5.C3



5.C3 Two cover plates, each of thickness t , are to be welded to a wide-flange beam of length L , which is to support a uniformly distributed load w . Denoting by σ_{all} the allowable normal stress in the beam and in the plates, by d the depth of the beam, and by I_b and S_b , respectively, the moment of inertia and the section modulus of the cross section of the unreinforced beam about a horizontal centroidal axis, write a computer program that can be used to calculate the required value of (a) the length a of the plates, (b) the width b of the plates. Use this program to solve Probs. 5.143 and 5.145.

SOLUTION

(a) Required Length of Plates

$$FB = AD; \sum M_D = 0: M_D + wZ\left(\frac{L}{2}\right) - R_A x = 0$$

But: $R_A = \frac{1}{2}wL$ and $M_D = 5G_{all}$ Divide by $\frac{1}{2}w$:

$$R_A = \frac{1}{2}wL \quad x^2 - Lx + (2.5G_{all}/w) = 0 \quad \text{Set } k = \frac{2.5G_{all}}{w}$$

$$x^2 - Lx + k = 0$$

Solving the quadratic, $x = \frac{L - \sqrt{L^2 - 4k}}{2}$

Compute x and

$$a = L - 2x$$

(b) Required Width of Plates

At midpoint C of beam:

$$FB = AC; \sum M_C = 0: M_C + \frac{wL}{2} \cdot \frac{L}{4} - \frac{wL}{2} \cdot \frac{L}{2} = 0$$

Compute $M_C = \frac{1}{8}wL^2$

Compute: $C = t + \frac{1}{2}d$

From $G_{all} = \frac{M_C C}{I}$ compute $I = \frac{M_C C}{G_{all}}$

$R_A = \frac{1}{2}wL$

t

$\frac{1}{2}(d+t)$

But $I = I_{beam} + I_{plates} = I_b + 2 \left[\frac{1}{12}bt^3 + bt(d+\frac{t}{2})^2 \right]$

Solving for b : $b = \frac{6(I - I_b)}{t[t^2 + 3(d+t)^2]}$

PROGRAM OUTPUTS

PROB. 5.155: W 460 x 74, $G_{all} = 150 \text{ MPa}$
 $w = 40 \text{ kN/m}$, $L = 8 \text{ m}$, $t = 7.5 \text{ mm}$
 $d = 457 \text{ mm}$, $I_b = 333 \times 10^6 \text{ mm}^4$, $S = 1460 \times 10^3 \text{ mm}^3$

PROB. 5.157: W 30 x 99, $G_{all} = 22 \text{ ksi}$
 $w = 30 \text{ kips/ft}$, $L = 16 \text{ ft}$, $t = 5/8 \text{ in.}$
 $d = 29.65 \text{ in.}$, $I_b = 3990 \text{ in}^4$, $S = 269 \text{ in}^3$

Problem 5.145

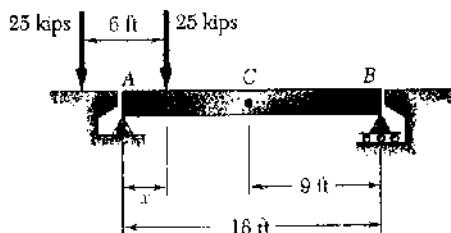
$$a = 4.49 \text{ m}$$

$$b = 211 \text{ mm}$$

Problem 5.143

$$a = 11.16 \text{ ft}$$

$$b = 14.31 \text{ in.}$$

PROBLEM 5.C4


5.C4 Two 25-kip loads are maintained 6 ft apart as they are moved slowly across the 18-ft beam AB. Write a computer program and use it to calculate the bending moment under each load and at the midpoint C of the beam for values of x from 0 to 24 ft at intervals $\Delta x = 1.5 \text{ ft}$.

SOLUTION

NOTATION: Length of beam = $L = 18 \text{ ft}$

Loads: $P_1 = P_2 = P = 25 \text{ kips}$

Distance between loads = $d = 6 \text{ ft}$

We note that $d < L/2$

(1) FROM $x=0$ TO $x=d$:

$$\rightarrow \sum M_B = 0: P(L-x) - R_A L = 0.$$

$$R_A = P(L-x)/L$$

Under P_1 : $M_1 = R_A x$

At C: $M_C = R_A (\frac{L}{2}) - P(\frac{L}{2} - x)$

(2) FROM $x=d$ TO $x=L$:

$$\rightarrow \sum M_B = 0: P(L-x) + P(L-x+d) - R_A L = 0. \quad \text{"DI"}$$

$$R_A = P(2L-2x+d)/L$$

Under P_1 : $M_1 = R_A x - Pd$

Under P_2 : $M_2 = R_A (x-d)$

(2A) FROM $x=d$ TO $x=L/2$:

$$M_C = R_A (\frac{L}{2}) - P(\frac{L}{2} - x) - P(\frac{L}{2} - x + d) \\ = R_A (L/2) - P(L-2x+d)$$

(2B) FROM $x=L/2$ TO $x=L/2+d$:

$$M_C = R_A (L/2) - P(\frac{L}{2} - x + d)$$

(2C) FROM $x=L/2+d$ TO $x=L$:

$$M_C = R_A L/2$$

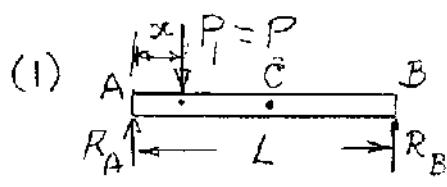
(3) FROM $x=L$ TO $x=L+d$:

$$\rightarrow \sum M_B = 0: P(L-x+d) - R_A L = 0$$

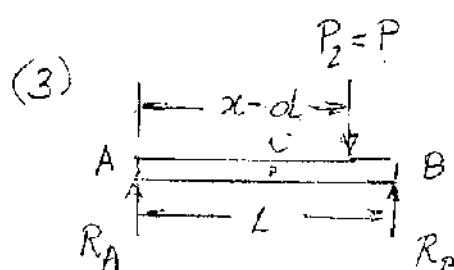
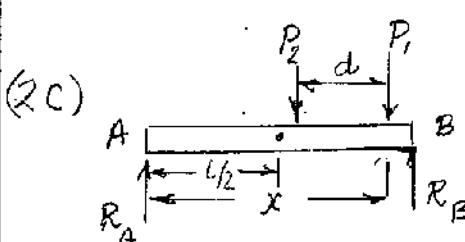
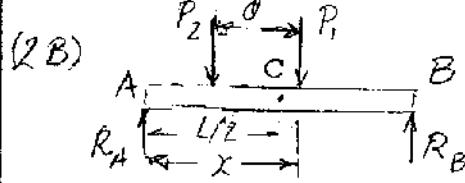
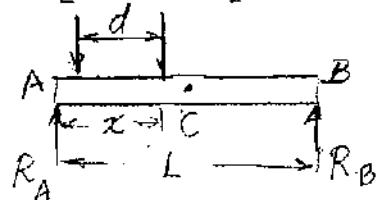
$$R_A = P(L-x+d)/L$$

Under P_2 : $M_2 = R_A (x-d)$

At C: $M_C = R_A (L/2)$



$$(2A) P_2 = P, P_1 = P$$



(CONTINUED)

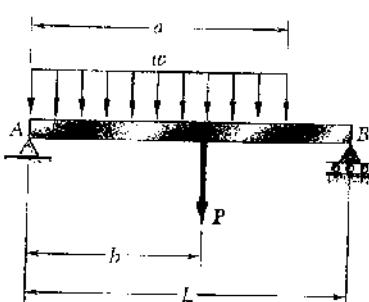
PROBLEM 5.C4 CONTINUED

PROGRAM OUTPUT

P = 25 kips, L = 18 ft, D = 6 ft

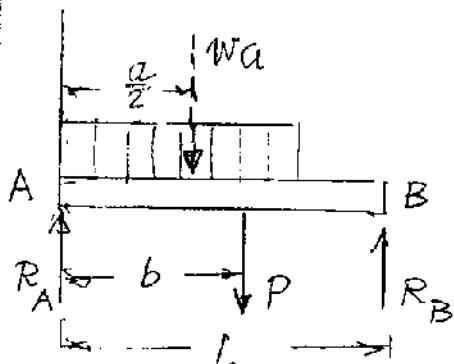
x ft	MC kip.ft	M1 kip.ft	M2 kip.ft
0.0	0.00	0.00	0.00
1.5	18.75	34.38	0.00
3.0	37.50	62.50	0.00
4.5	56.25	84.38	0.00
6.0	75.00	100.00	0.00
7.5	112.50	131.25	56.25
9.0	150.00	150.00	100.00
10.5	150.00	156.25	131.25
12.0	150.00	150.00	150.00
13.5	150.00	131.25	156.25
15.0	150.00	100.00	150.00
16.5	112.50	56.25	131.25
18.0	75.00	0.00	100.00
19.5	56.25	0.00	84.38
21.0	37.50	0.00	62.50
22.5	18.75	0.00	34.38
24.0	0.00	0.00	0.00

PROBLEM 5.C5



5.C5 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.2$ ft to the beam and loading of (a) Prob. 5.72, (b) Prob. 5.115.

SOLUTION



REACTIONS AT A AND B

USING FB DIAGRAM OF BEAM:

$$+\Downarrow \sum M_A = 0: R_B L - Pb - wa(a/2) = 0$$

$$R_B = (1/L)(Pb + \frac{1}{2}wa^2)$$

$$R_A = P + wa - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text).

SET $n = L/\Delta L$. FOR $i = 0$ TO n : $x = (\Delta L)i$

WE DEFINE: IF $x \geq a$ THEN $STPA = 1$ ELSE $STPA = 0$
IF $x \geq b$ THEN $STPB = 1$ ELSE $STPB = 0$.

$$V = R_A - wxc + w(x-a) STPA - P STPB$$

$$M = R_A x - \frac{1}{2}wxc^2 + \frac{1}{2}w(x-a)^2 STPA - P(x-b) STPB$$

LOCATE AND PRINT (x, V) AND (x, M)

SEE NEXT PAGES FOR PROGRAM OUTPUTS

(CONTINUED)

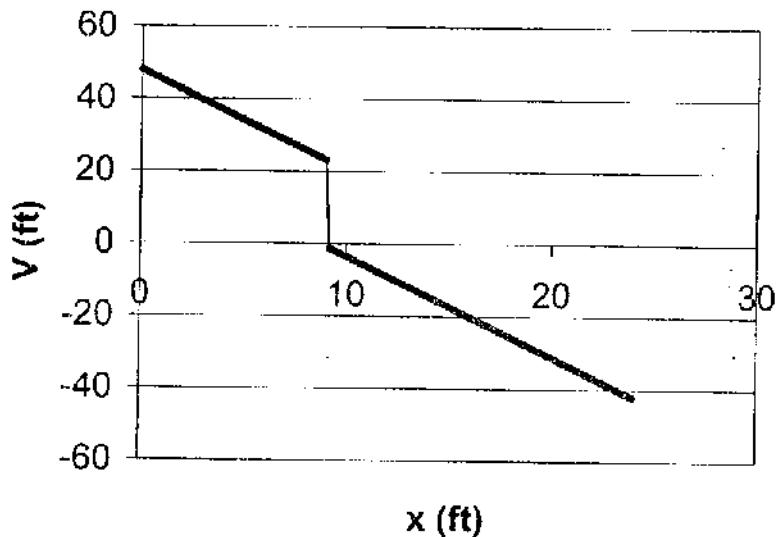
PROBLEM 5.C5 CONTINUED

PROGRAM OUTPUT

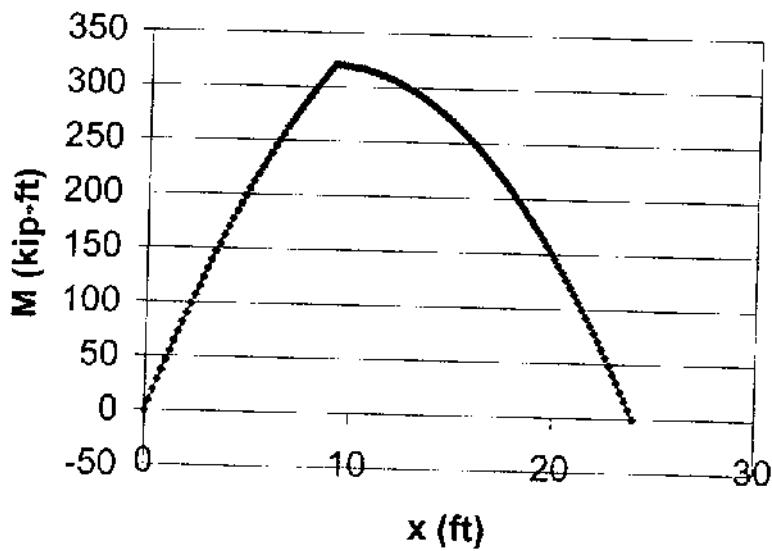
Problem 5.72

$$RA = 48.00 \text{ kips} \quad RB = 42.00 \text{ kips}$$

Shear Diagram



Moment Diagram



(CONTINUED)

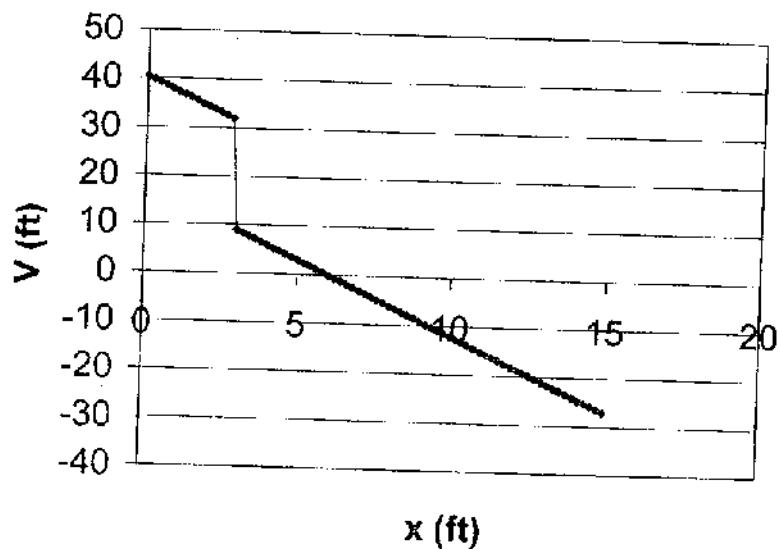
PROBLEM 5.C5 CONTINUED

PROGRAM OUTPUT FOR

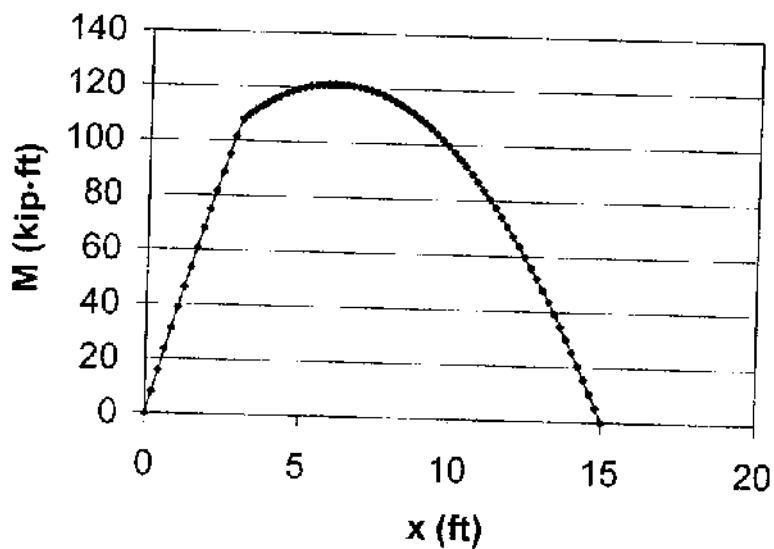
Problem 5.115

RA = 40.50 kips RB = 27.00 kips

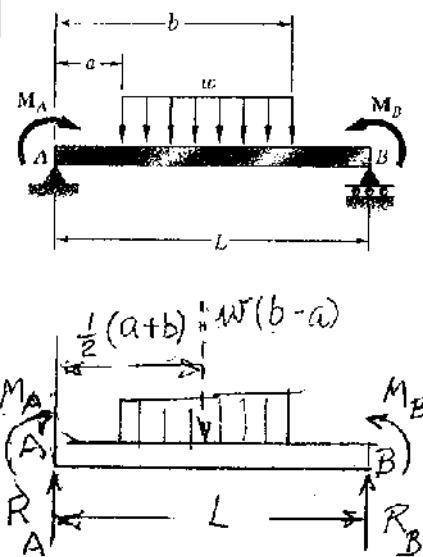
Shear Diagram



Moment Diagram



PROBLEM 5.C6



5.C6 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.025 \text{ m}$ to the beam and loading of Prob. 5.112

SOLUTION

REACTIONS AT A AND B

$$+\uparrow \sum M_A = 0:$$

$$R_B L + M_B - M_A - w(b-a)\frac{1}{2}(a+b) = 0$$

$$R_B = (1/L)[M_A - M_B + \frac{1}{2}w(b^2 - a^2)]$$

$$R_A = w(b-a) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET $n = L/\Delta L$, FOR $i = 0$ TO n : $x = (\Delta L)i$

WE DEFINE: IF $x \geq a$ THEN $STPA = 1$ ELSE $STPA = 0$

IF $x \geq b$ THEN $STPB = 1$ ELSE $STPB = 0$

$$V = R_A - w(x-a)STPA + w(x-b)STPB$$

$$M = M_A + R_A x - \frac{1}{2}w(x-a)^2 STPA + \frac{1}{2}w(x-b)^2 STPB$$

LOCATE AND PRINT (x, V) AND (x, M)

PROGRAM OUTPUT ON NEXT PAGE

(CONTINUED)

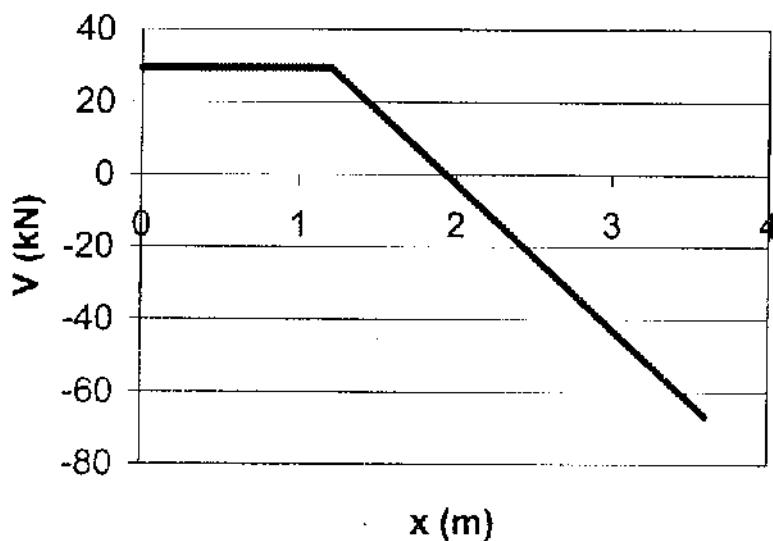
PROBLEM 5.C6 CONTINUED

PROGRAM OUTPUT

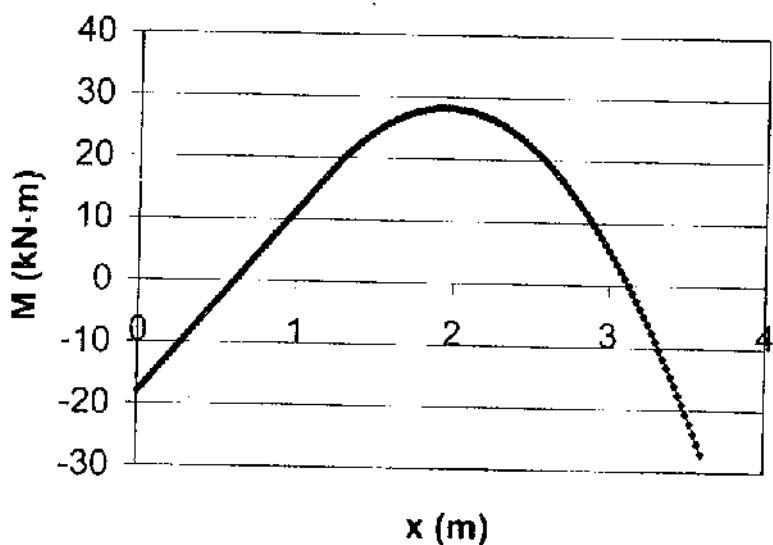
Problem 5.112

RA = 29.50 kips RB = 66.50 kips

Shear Diagram

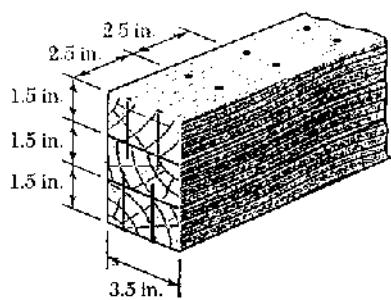


Moment Diagram



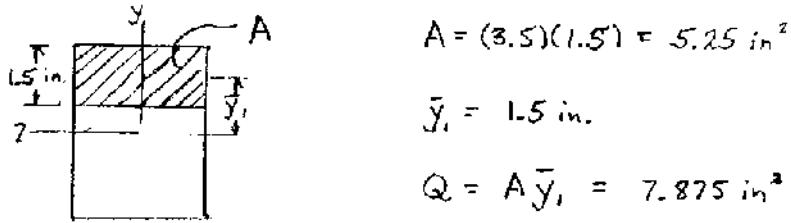
Chapter 6

Problem 6.1



6.1 Three boards, each of 1.5×3.5 -in. rectangular cross section, are nailed together to form a beam that is subjected to a vertical shear of 250 lb. Knowing that the spacing between each pair of nails is 2.5 in., determine the shearing force in each nail.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (3.5)(4.5)^3 = 26.578 \text{ in}^4$$



$$A = (3.5)(1.5) = 5.25 \text{ in}^2$$

$$\bar{y}_1 = 1.5 \text{ in.}$$

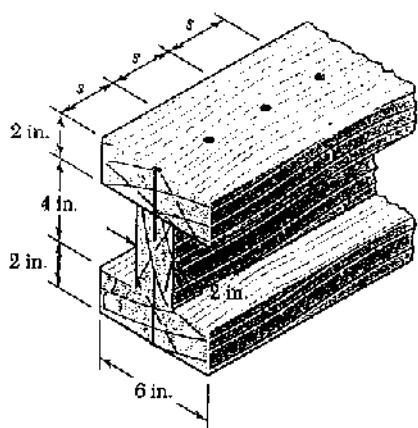
$$Q = A \bar{y}_1 = 7.875 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(250)(7.875)}{26.578} = 74.074 \text{ lb/in}$$

$$q s = 2 F_{nail}$$

$$F_{nail} = \frac{q s}{2} = \frac{(74.074)(2.5)}{2} = 92.6 \text{ lb.}$$

Problem 6.2



6.2 Three boards, each 2 in. thick, are nailed together to form a beam that is subjected to a vertical shear. Knowing that the allowable shearing force in each nail is 150 lb, determine the allowable shear if the spacing s between the nails is 3 in.

$$I_1 = \frac{1}{12} b h^3 + A d^2$$

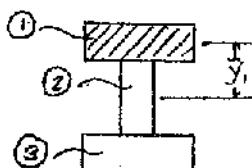
$$= \frac{1}{12} (6)(2)^3 + (6)(2)(3)^2 = 112 \text{ in}^4$$

$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12} (2)(4)^3 = 10.667 \text{ in}^4$$

$$I_3 = I_1 = 112 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 234.667 \text{ in}^4$$

$$Q = A \bar{y}_1 = (6)(2)(3) = 36 \text{ in}^3$$



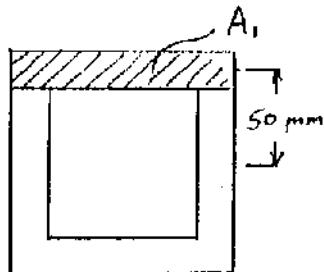
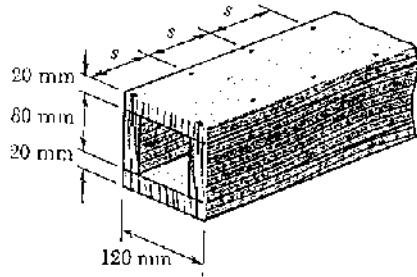
$$q s = F_{nail} \quad (1)$$

$$q = \frac{VQ}{I} \quad (2)$$

$$\text{Dividing Eq (2) by Eq (1)} \quad \frac{1}{s} = \frac{VQ}{F_{nail} I}$$

$$V = \frac{F_{nail} I}{Q s} = \frac{(150)(234.667)}{(36)(3)} = 326 \text{ lb.}$$

Problem 6.3



6.3 A square box beam is made of two 20×80 -mm planks and two 20×120 -mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 50$ mm and that the allowable shearing force in each nail is 300 N, determine (a) the largest allowable vertical shear in the beam, (b) the corresponding maximum shearing stress in the beam.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3 \\ = \frac{1}{12} (120)(120)^3 - \frac{1}{12} (80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4 \\ = 13.8667 \times 10^{-6} \text{ m}^4$$

$$(a) A_1 = (120)(20) = 2400 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

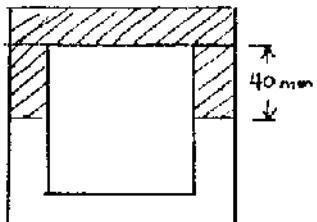
$$Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$$

$$q_{\text{all}} = \frac{2F_{\text{nail}}}{s} = \frac{(2)(300)}{50 \times 10^{-3}} = 12 \times 10^3 \text{ N}$$

$$\sigma = \frac{VQ}{I}$$

$$V = \frac{q I}{Q} = \frac{(12 \times 10^3)(13.8667 \times 10^{-6})}{120 \times 10^{-6}}$$

$$= 1.38667 \times 10^3 \text{ N} = 1.387 \text{ kN}$$



$$(b) Q = Q_1 + (2)(20)(40)(20)$$

$$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$$

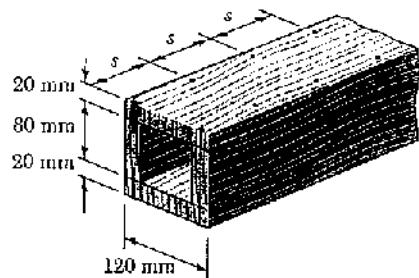
$$= 152 \times 10^{-6} \text{ m}^3$$

$$\sigma_{\text{max}} = \frac{VQ}{It} = \frac{(1.38667 \times 10^3)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^3)}$$

$$= 380 \times 10^3 \text{ Pa}$$

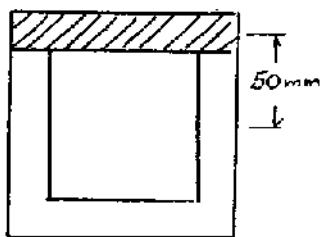
$$380 \text{ kPa}$$

Problem 6.4



6.4 A square box beam is made of two 20×80 -mm planks and two 20×120 -mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 30$ mm and that the vertical shear in the beam is $V = 1200$ N, determine (a) the shearing force in each nail, (b) the maximum shearing stress in the beam.

$$\begin{aligned} I &= \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3 \\ &= \frac{1}{12} (120)(120)^3 - \frac{1}{12} (80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4 \\ &= 13.8667 \times 10^{-6} \text{ m}^4 \end{aligned}$$



$$(a) A_i = (120)(20) = 2400 \text{ mm}^2$$

$$\bar{y}_i = 50 \text{ mm}$$

$$Q_i = A_i \bar{y}_i = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{I} = \frac{(1200)(120 \times 10^{-6})}{13.8667 \times 10^{-6}} = 10.385 \times 10^3 \text{ N/m}$$

$$q_s = 2 F_{nail}$$

$$F_{nail} = \frac{q_s}{2} = \frac{(10.385 \times 10^3)(30 \times 10^{-3})}{2} = 155.8 \text{ N}$$

$$(b) Q = Q_i + (2)(20)(40)(20)$$

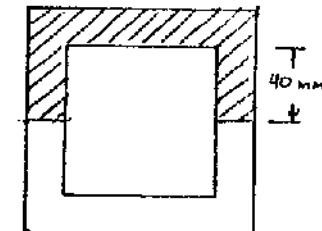
$$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$$

$$= 152 \times 10^{-6} \text{ m}^3$$

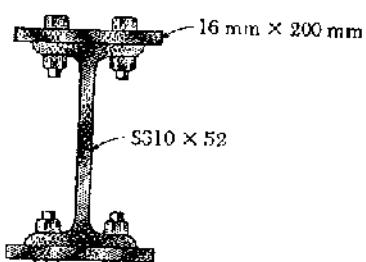
$$\tau_{max} = \frac{VQ}{IT} = \frac{(1200)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$$

$$= 329 \times 10^3 \text{ Pa}$$

329 kPa



Problem 6.5



6.5 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 × 200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

Calculate moment of inertia.

Part	$A (\text{mm}^2)$	$d(\text{mm})$	$Ad^2 (10^6 \text{ mm}^4)$	$\bar{I} (10^6 \text{ mm}^4)$
Top plate	3200	*160.5	82.43	0.07
S310x52	6650	0		95.3
Bot. plate	3200	*160.5	82.43	0.07
Σ			164.86	95.44

$$* d = \frac{305}{2} + \frac{16}{2} \\ = 160.5 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{\text{plate}} d_{\text{plate}} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} (18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$F_{\text{bolt}} = \gamma_{\text{all}} A_{\text{bolt}} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

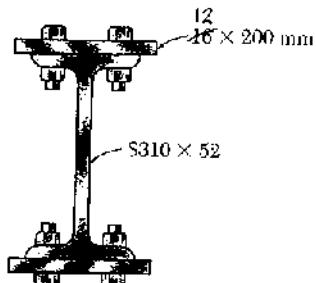
$$q_s = 2 F_{\text{bolt}} \quad q = \frac{2 F_{\text{bolt}}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \quad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^6)(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N}$$

$$= 193.5 \text{ kN}$$

Problem 6.6

6.6 Solve Prob. 6.5, assuming that the reinforcing plates are only 12 mm thick.



$$d = \frac{305}{2} + \frac{12}{2} \\ = 158.5 \text{ mm}$$

Calculate moment of inertia.

Part	$A (\text{mm}^2)$	$d (\text{mm})$	$Ad^2 (10^6 \text{ mm}^4)$	$\bar{I} (10^6 \text{ mm}^4)$
Top plate	2400	* 158.5	60.29	0.03
S310 x 52	6650	0	0	95.3
Bot. plate	2400	158.5	60.29	0.03
Σ			120.58	95.36

$$I = \sum Ad^2 + \sum \bar{I} = 215.94 \times 10^6 \text{ mm}^4 = 215.94 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate} d_{plate} = (200)(12)(158.5) = 380.4 \times 10^3 \text{ mm}^3 = 380.4 \times 10^{-6} \text{ m}^3$$

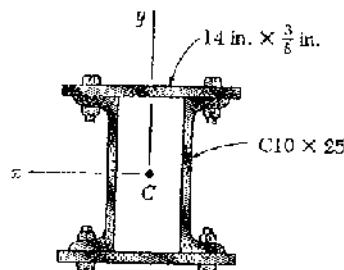
$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} (18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$F_{bolt} = 2 \cdot \text{all } A_{bolt} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.902 \times 10^3 \text{ N}$$

$$q_s = 2 F_{bolt} \quad q = \frac{2 F_{bolt}}{s} = \frac{(2)(22.903 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \quad V = \frac{I q}{Q} = \frac{(215.94 \times 10^6)(381.7 \times 10^3)}{380.4 \times 10^6} = 217 \times 10^3 \text{ N} \\ = 217 \text{ kN}$$

Problem 6.7



6.7 and 6.8 A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{8}$ -in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.

Calculate moment of inertia.

Part	A (in ²)	d (in)	Ad ² (in ⁴)	\bar{I} (in ⁴)
Top plate	5.25	*5.1875	141.28	0.06
C10 x 25	7.35	0		91.2
C10 x 25	7.35	0		91.2
Bot. plate	5.25	*5.1875	141.28	0.06
Σ			282.56	182.52

$$* d = \frac{10}{2} + \frac{1}{2}\left(\frac{3}{8}\right) \\ = 5.1875 \text{ in.} \\ = \bar{y}_1$$

$$I = \sum Ad^2 + \sum \bar{I} = 282.56 + 182.52 = 465.08 \text{ in}^4$$

$$Q = A_{plate} \bar{y}_1 = (5.25)(5.1875) = 27.234 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(30)(27.234)}{465.08} = 1.7567 \text{ kips/in}$$

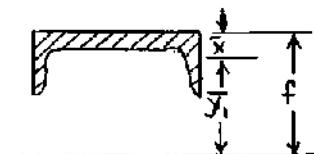
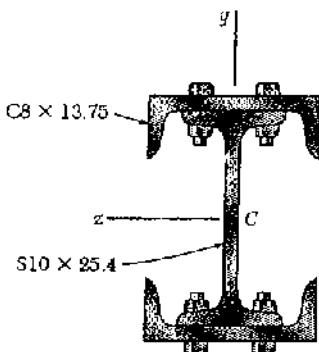
$$F_{ult} = \frac{1}{2} q S = \left(\frac{1}{2}\right)(1.7567)(5) = 4.392 \text{ kips}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 = 0.4418 \text{ in}^2$$

$$\sigma_{bolt} = \frac{F_{ult}}{A_{bolt}} = \frac{4.392}{0.4418} = 9.94 \text{ ksi}$$

Problem 6.8

6.7 and 6.8 A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$ -in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.



Geometry

$$f = \left(\frac{d}{2}\right)_s + (t_w)_c$$

$$= \frac{10}{2} + 0.303 = 5.303 \text{ in.}$$

$$\bar{x} = 0.533 \text{ in}$$

$$\bar{y}_i = f - \bar{x} = 5.303 - 0.533 = 4.770 \text{ in.}$$

Determine moment of inertia.

Part	$A (\text{in}^2)$	$d (\text{in})$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
C8 x 13.75	4.04	4.770	91.92	1.53
S10 x 25.4	7.46	0	0	124
C8 x 13.75	4.04	4.770	91.92	1.53
Σ			183.84	127.06

$$I = \sum Ad^2 + \sum \bar{I} = 183.84 + 127.06 = 310.9 \text{ in}^4$$

$$Q = A \bar{y}_i = (4.04)(4.770) = 19.271 \text{ in}^3$$

$$q_s = \frac{VQ}{I} = \frac{(30)(19.271)}{310.9} = 1.8595 \text{ kip/in.}$$

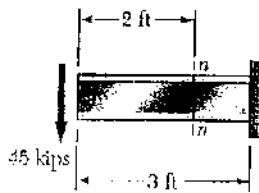
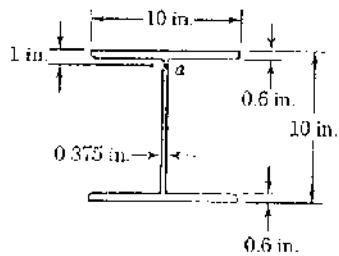
$$F_{bolts} = \frac{1}{2} q_s s = \left(\frac{1}{2}\right)(1.8595)(5) = 4.649 \text{ kip}$$

$$A_{bolts} = \frac{\pi}{4} d_{bolts}^2 = \frac{\pi}{4} \left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\sigma_{bolts} = \frac{F_{bolts}}{A_{bolts}} = \frac{4.649}{0.4418} = 10.52 \text{ ksi}$$

Problem 6.9

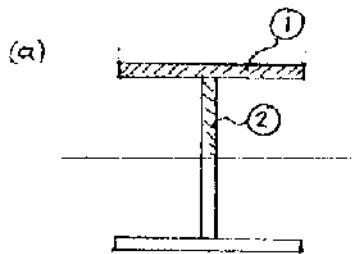
6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (a) the largest shearing stress in that section, (b) the shearing stress at point *a*.



$$V = 45 \text{ kips}$$

Part	$A (\text{in}^2)$	$z(\text{in.})$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
Flange	6.00	4.7	132.54	0.18
Web	3.30	0	0	21.296
Flange	6.00	4.7	132.54	0.18
Σ			265.08	21.656

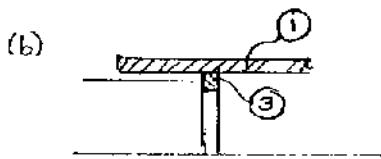
$$\begin{aligned} I &= \sum Ad^2 + \sum \bar{I} \\ &= 265.08 + 21.656 \\ &= 286.736 \text{ in}^4 \end{aligned}$$



$$\begin{aligned} Q &= A_1 \bar{y}_1 + A_2 \bar{y}_2 \\ &= (6.00)(4.7) + (0.375)(4.4)(2.2) = 31.83 \text{ in}^3 \end{aligned}$$

$$t = 0.375 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(45)(31.83)}{(286.736)(0.375)} = 13.32 \text{ ksi}$$

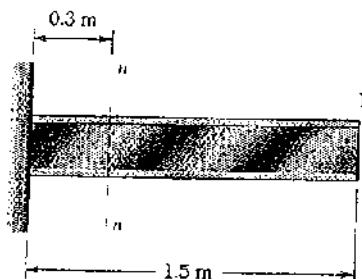


$$\begin{aligned} Q_a &= A_1 \bar{y}_1 + A_3 \bar{y}_3 \\ &= (6.00)(4.7) + (0.375)(0.4) \left(\frac{4.4 + 4.0}{2} \right) \\ &= 28.83 \text{ in}^3 \end{aligned}$$

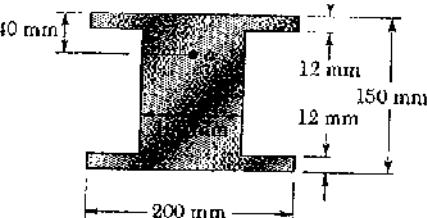
$$t = 0.375 \text{ in.}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(45)(28.83)}{(286.736)(0.375)} = 12.07 \text{ ksi}$$

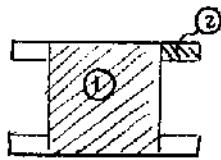
Problem 6.10



6.9 through 6.12 For the beam and loading shown, consider section $n-n$ and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a .

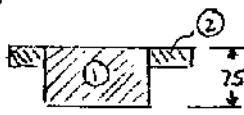


At section $n-n$
 $V = 10 \text{ kN}$



$$\begin{aligned} I &= I_1 + 4I_2 \\ &= \frac{1}{12} b_1 h_1^3 + 4 \left(\frac{1}{12} b_2 h_2^3 + A_2 d_2^2 \right) \\ &= \frac{1}{12} (100)(150)^3 + 4 \left[\frac{1}{12} (50)(12)^3 + (50)(12)(69)^2 \right] \\ &= 28.125 \times 10^6 + 4 [0.0072 \times 10^6 + 2.8566 \times 10^6] \\ &= 39.58 \times 10^6 \text{ mm}^4 = 39.58 \times 10^{-6} \text{ m}^4 \end{aligned}$$

(a)



$$\begin{aligned} Q &= A_1 \bar{y}_1 + 2A_2 \bar{y}_2 \\ &= (100)(75)(37.5) + (2)(50)(12)(69) \\ &= 364.05 \times 10^3 \text{ mm}^3 = 364.05 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$t = 100 \text{ mm} = 0.100 \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10 \times 10^3)(364.05 \times 10^{-6})}{(39.58 \times 10^{-6})(0.100)} = 920 \times 10^3 \text{ Pa} = 920 \text{ kPa}$$

(b)



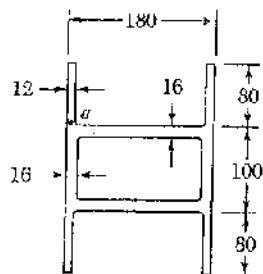
$$\begin{aligned} Q &= A_1 \bar{y}_1 + 2A_2 \bar{y}_2 \\ &= (100)(40)(55) + (2)(50)(12)(69) \\ &= 302.8 \times 10^3 \text{ mm}^3 = 302.8 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$t = 100 \text{ mm} = 0.100 \text{ m}$$

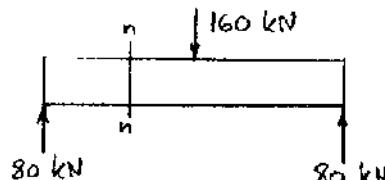
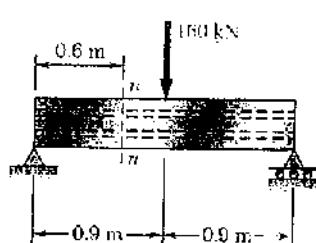
$$\tau = \frac{VQ}{It} = \frac{(10 \times 10^3)(302.8 \times 10^{-6})}{(39.58 \times 10^{-6})(0.100)} = 765 \times 10^3 \text{ Pa} = 765 \text{ kPa}$$

Problem 6.11

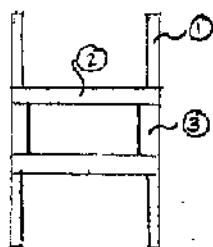
6.9 through 6.12 For the beam and loading shown, consider section $n-n$ and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a .



Dimensions in mm



At section $n-n$ $V = 80 \text{ kN}$



Consider cross section as composed of rectangles of types ①, ②, and ③.

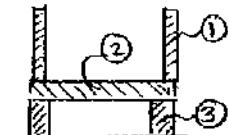
$$I_1 = \frac{1}{12}(12)(80)^3 + (12)(80)(90)^2 = 8.268 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(180)(16)^3 + (180)(16)(42)^2 = 5.14176 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(16)(68)^3 = 419.24 \times 10^3 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + 2I_3 = 44.274 \times 10^6 \text{ mm}^4 = 44.274 \times 10^{-4} \text{ m}^4$$

(a) Calculate Q at neutral axis.



$$Q_1 = (12)(80)(90) = 86.4 \times 10^6 \text{ mm}^4$$

$$Q_2 = (180)(16)(42) = 120.96 \times 10^3 \text{ mm}^4$$

$$Q_3 = (16)(34)(17) = 9.248 \times 10^3 \text{ mm}^4$$

$$Q = 2Q_1 + Q_2 + 2Q_3 = 312.256 \times 10^3 \text{ mm}^3 = 312.256 \times 10^{-6} \text{ m}^3$$

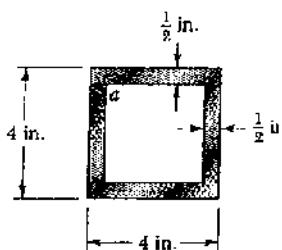
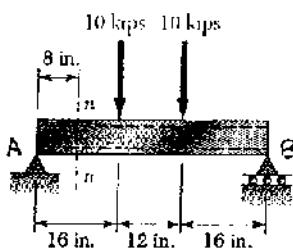
$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(312.256 \times 10^{-6})}{(44.274 \times 10^{-4})(2 \times 16 \times 10^{-3})} = 17.63 \times 10^6 \text{ Pa} = 17.63 \text{ MPa}$$

(b) At point a , $Q = Q_1 = 86.4 \times 10^6 \text{ mm}^4 = 86.4 \times 10^{-6} \text{ m}^4$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(86.4 \times 10^{-6})}{(44.274 \times 10^{-4})(12 \times 10^{-3})} = 13.01 \times 10^6 \text{ Pa} = 13.01 \text{ MPa}$$

Problem 6.12

6.9 through 6.12 For the beam and loading shown, consider section $n-n$ and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a .

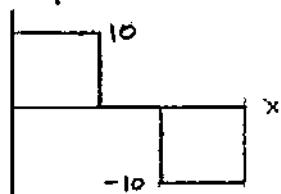


$$\text{By symmetry } R_A = R_B$$

$$\rightarrow \sum F_y = 0: R_A + R_B - 10 - 10 = 0$$

$$R_A = R_B = 10 \text{ kips}$$

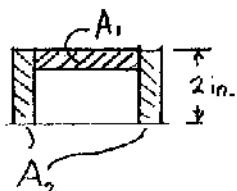
V (kips)



From the shear diagram, $V = 10$ kips at $n-n$.

$$\begin{aligned} I &= \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3 \\ &= \frac{1}{12} (4)(4)^3 - \frac{1}{12} (3)(3)^3 = 14.583 \text{ in}^4 \end{aligned}$$

(a)

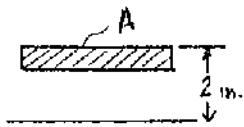


$$\begin{aligned} Q &= A_1 \bar{y}_1 + A_2 \bar{y}_2 = (3)\left(\frac{1}{2}\right)(1.75) + (2)\left(\frac{1}{2}\right)(2)(1) \\ &= 4.625 \text{ in}^3 \end{aligned}$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)} = 3.17 \text{ ksi}$$

(b)

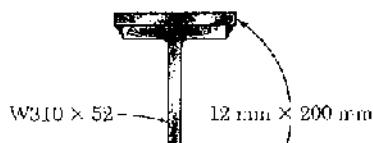


$$Q = A \bar{y} = (4)\left(\frac{1}{2}\right)(1.75) = 3.5 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)} = 2.40 \text{ ksi}$$

Problem 6.13



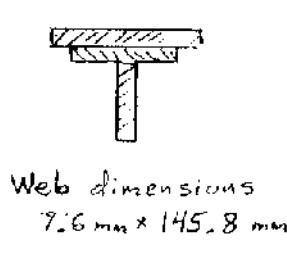
$$d = \frac{318}{2} + \frac{12}{2} \\ = 165 \text{ mm}$$

6.13 Two steel plates of 12×200 -mm rectangular cross section are welded to the W 310 × 52 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

Calculate moment of inertia.

Part	$A(\text{mm}^2)$	$d(\text{mm})$	$\bar{A}d^2(10^6 \text{ mm}^4)$	$\bar{I}(10^6 \text{ mm}^4)$
Top plate	2400	* 165	65.34	0.0288
W 310 × 52	6670	0	0	119.
Bot. plate	2400	* 165	65.34	0.0288
Σ			130.68	119.0576

$$I = \sum A d^2 + \sum \bar{I} = 249.74 \times 10^6 \text{ mm}^4 = 249.74 \times 10^6 \text{ m}^4$$



Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3 \text{ mm}^3)$
Plate	2400	165.	396.
Top flange	2204.4	152.4	335.95
Half web	1108.08	72.9	80.78
Σ			812.73

$$Q = \sum A \bar{y} = 812.73 \times 10^3 \text{ mm}^3 = 812.73 \times 10^6 \text{ m}^3$$

$$t = 7.6 \text{ mm} = 7.6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} \quad V = \frac{It\tau}{Q} = \frac{(249.74 \times 10^6)(7.6 \times 10^{-3})(90 \times 10^6)}{812.73 \times 10^6} \\ = 210 \times 10^3 \text{ N} \quad 210 \text{ kN}$$

Problem 6.14



$$d = \frac{318}{2} + \frac{8}{2} \\ = 163 \text{ mm}$$

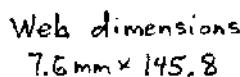
6.14 Solve Prob. 6.13, assuming that the two steel plates are (a) replaced by 8×200 -mm plates, (b) removed.

6.13 Two steel plates of 12×200 -mm rectangular cross section are welded to the W310 x 52 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

(a) Calculate moment of inertia.

Part	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(10^6 \text{mm}^4)$	$\bar{I}(10^6 \text{mm}^4)$
Top plate	1600	* 163	42.5104	0.008
W310x52	6670	0		119.
Bot. plate	1600	* 163	42.5104	0.008
			85.0208	119.016

$$I = \sum Ad^2 + \sum \bar{I} = 204.04 \times 10^6 \text{ mm}^4 = 204.04 \times 10^{-6} \text{ m}^4$$



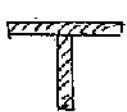
Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3 \text{mm}^3)$
Plate	1600	163	260.8
Top flange	2204.4	152.4	335.95
Half web	1168.08	72.9	80.78
Σ			677.53

$$Q = \sum A\bar{y} = 677.53 \times 10^3 \text{ mm}^3 = 677.53 \times 10^{-6} \text{ m}^3$$

$$t = 7.6 \text{ mm} = 7.6 \times 10^{-3} \text{ m}$$

$$\gamma = \frac{VQ}{It} \quad V = \frac{It\gamma}{Q} = \frac{(204.04 \times 10^6)(7.6 \times 10^{-3})(90 \times 10^6)}{677.53 \times 10^{-6}} \\ = 206 \times 10^3 \text{ N} \quad 206 \text{ kN} \rightarrow$$

$$(b) I = 119 \times 10^6 \text{ mm}^4 = 119 \times 10^{-6} \text{ m}^4$$



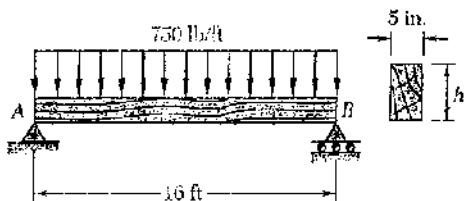
$$Q = (335.93 + 80.78) \times 10^3 = 416.71 \times 10^3 \text{ mm}^3 \\ = 416.71 \times 10^{-6} \text{ m}^3$$

$$t = 7.6 \times 10^{-3} \text{ m}$$

$$V = \frac{It\gamma}{Q} = \frac{(119 \times 10^6)(7.6 \times 10^{-3})(90 \times 10^6)}{416.71 \times 10^{-6}} \\ = 195.3 \times 10^3 \text{ N} \quad 195.3 \text{ kN} \rightarrow$$

Problem 6.15

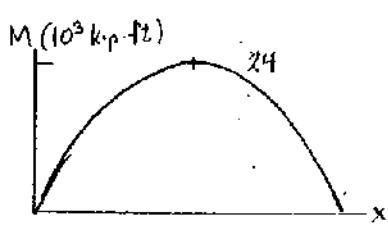
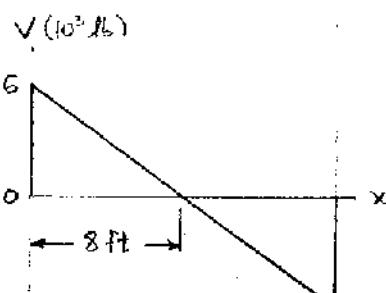
6.15 For the beam and loading shown, determine the minimum required depth h , knowing that for the grade of timber used, $\sigma_{all} = 1750 \text{ psi}$ and $\tau_{all} = 130 \text{ psi}$.



$$\text{Total load } (750 \text{ lb/ft})(16 \text{ ft}) = 12 \times 10^3 \text{ lb}$$

$$\text{Reaction at A } R_A = 6 \times 10^3 \text{ lb}$$

$$V_{max} = 6 \times 10^3 \text{ lb}$$

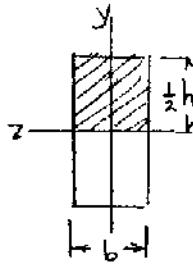


Bending. $S = \frac{1}{6}bh^2$ for rectangular section.

$$S = \frac{M_{max}}{\sigma_{all}} = \frac{288 \times 10^3}{1750} = 164.57 \text{ in}^3$$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(164.57)}{5}} = 14.05 \text{ in}$$

Shear. $I = \frac{1}{12}bh^3$ for rectangular section.



$$A = \frac{1}{2}bh$$

$$\bar{y} = \frac{1}{4}h$$

$$Q = A\bar{y} = \frac{1}{8}bh^2$$

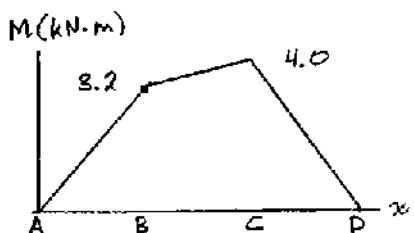
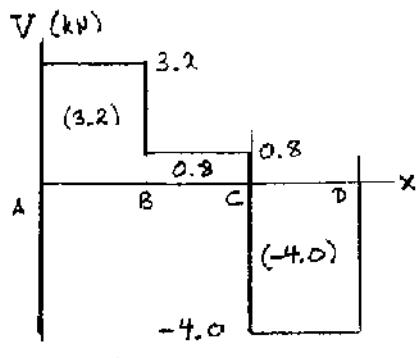
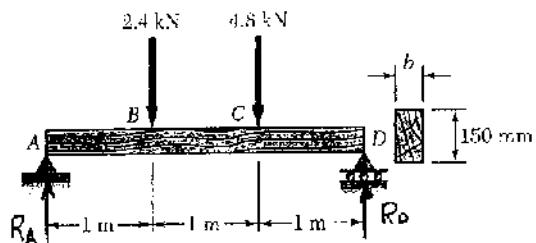
$$\tau_{max} = \frac{VQ}{Ib} = \frac{3V_{max}}{2bh}$$

$$h_r = \frac{3V_{max}}{2b\tau_{max}} = \frac{(3)(6 \times 10^3)}{(2)(5)(130)} = 13.85 \text{ in.}$$

The larger value of h is the minimum required depth. $h = 14.05 \text{ in.}$

Problem 6.16

6.16 For the beam and loading shown, determine the minimum required width b , knowing that for the grade of timber used $\sigma_{all} = 12 \text{ MPa}$ and $\tau_{all} = 825 \text{ kPa}$.



$$\sum M_D = 0 : -3R_A + (2)(2.4) + (1)(4.8) = 0$$

$$R_A = 3.2 \text{ kN}$$

Draw shear and bending moment diagrams.

$$|V|_{max} = 4.0 \text{ kN} \quad |M|_{max} = 4.0 \text{ kN}\cdot\text{m}$$

Bending. $S = \frac{|M|_{max}}{6h} = \frac{4.0 \times 10^3}{12 \times 10^6} = 333.33 \times 10^{-6} \text{ m}^3 = 333.33 \times 10^3 \text{ mm}^3$

For a rectangular cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{\frac{1}{2}h} = \frac{1}{6}bh^2$$

$$b = \frac{6S}{h^2} = \frac{(6)(333.33 \times 10^3)}{150^2} = 88.9 \text{ mm}$$

Shear.

$$A = \frac{1}{2}bh, \quad \bar{y} = \frac{1}{4}h$$

$$Q = A\bar{y} = \frac{1}{8}bh^2, \quad I = \frac{1}{12}bh^3$$

$$Z = \frac{VQ}{It} = \frac{3V}{2bh}$$

$$bh = \frac{3}{2} \frac{V}{Z} = \frac{3}{2} \frac{4.0 \times 10^3}{825 \times 10^3}$$

$$= 7.2727 \times 10^3 \text{ m}^2 = 7.2727 \times 10^3 \text{ mm}^2$$

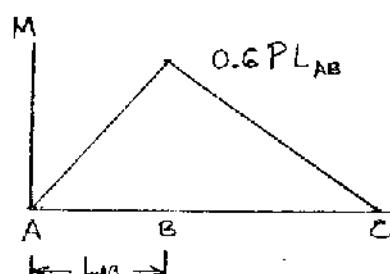
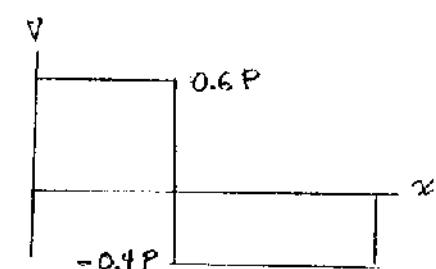
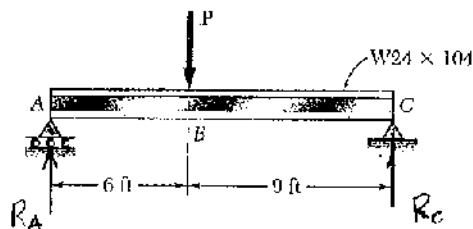
$$b = \frac{bh}{h} = \frac{7.2727 \times 10^3}{150} = 48.5 \text{ mm}$$

The required value for b is the larger one.

$$b = 88.9 \text{ mm} \blacksquare$$

Problem 6.17

6.17 For the wide-flange beam with the loading shown, determine the largest load P that can be applied, knowing that the maximum normal stress is 24 ksi and the largest shearing stress using the approximation $\tau_m = V/A_{\text{web}}$ is 14.5 ksi.



$$\rightarrow \sum M_C = 0 = -15R_A + 7P = 0$$

$$R_A = 0.6P$$

Draw shear and bending moment diagrams.

$$|V|_{\max} = 0.6P \quad |M|_{\max} = 0.6PL_{AB}$$

$$L_{AB} = 6 \text{ ft} = 72 \text{ in.}$$

Bending. For W24x104, $S = 258 \text{ in}^3$

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{0.6PL_{AB}}{\sigma_{\text{all}}}$$

$$P = \frac{\sigma_{\text{all}} S}{0.6L_{AB}} = \frac{(24)(258)}{(0.6)(72)} = 143.3 \text{ kips}$$

Shear. $A_{\text{web}} = d t_w$

$$= (24.06)(0.500)$$

$$= 12.03 \text{ in}^2$$

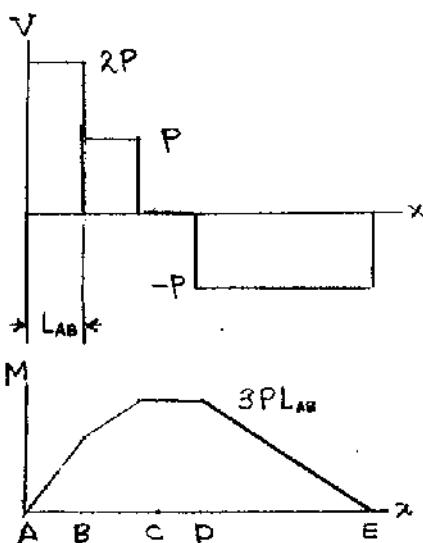
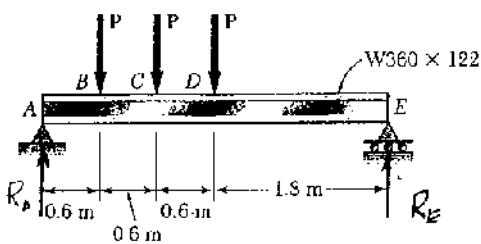
$$\tau = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{0.6P}{A_{\text{web}}}$$

$$P = \frac{\tau A_{\text{web}}}{0.6} = \frac{(14.5)(12.03)}{0.6} = 291 \text{ kips}$$

The smaller value of P is the allowable value. $P = 143.3 \text{ kips}$ \blacktriangleleft

Problem 6.18

6.18 For the wide-flange beam with the loading shown, determine the largest load P that can be applied, knowing that the maximum normal stress is 160 MPa and the largest shearing stress using the approximation $\tau_m = V/A_{web}$ is 100 MPa.



$$\rightarrow \sum M_E = 0 : -3.6R_A + 3.0P + 2.4P + 1.8P = 0$$

$$R_A = 2P$$

Draw shear and bending moment diagrams.

$$M_B = 2PL_{AB}, M_C = M_D = 3PL_{AB}$$

$$|V|_{max} = 2P \quad |M|_{max} = 3PL_{AB}$$

Bending. For W360 x 122 $S = 2010 \times 10^3 \text{ mm}^3$
 $= 2010 \times 10^{-6} \text{ m}^3$

$$\frac{|M|_{max}}{S_{all}} = \frac{3PL_{AB}}{S_{all}} = S$$

$$P = \frac{G_{ad}S}{3I_{AB}} = \frac{(160 \times 10^6)(2010 \times 10^3)}{(3)(0.6)} = 178.7 \times 10^3 \text{ N}$$

Shear. $A_{web} = d t_w = (363)(13.0)$
 $= 4.719 \times 10^3 \text{ mm}^2 = 4.719 \times 10^{-3} \text{ m}^2$

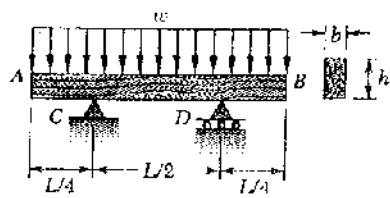
$$\tau' = \frac{|V|_{max}}{A_{web}} = \frac{2P}{A_{web}}$$

$$P = \frac{2' A_{web}}{2} = \frac{(100 \times 10^6)(4.719 \times 10^{-3})}{2} = 235 \times 10^3 \text{ N}$$

The smaller value of P is the allowable one.

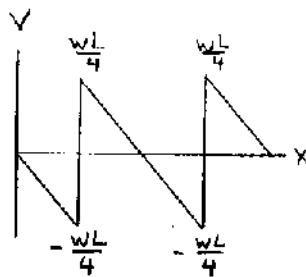
$$P = 178.7 \text{ kN} \blacksquare$$

Problem 6.19



6.19 A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown. (a) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (b) Determine the depth h and the width b of the beam, knowing that $L = 5 \text{ m}$, $w = 8 \text{ kN/m}$, $\tau_m = 1.08 \text{ MPa}$, and $\sigma_m = 12 \text{ MPa}$.

$$R_A = R_B = \frac{wL}{2}$$

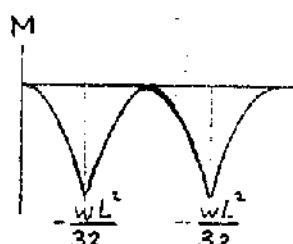


$$\text{From shear diagram } |V|_m = \frac{wl}{4} \quad (1)$$

$$\text{For rectangular section } A = bh \quad (2)$$

$$\tau_m = \frac{3}{2} \frac{V_m}{A} = \frac{3wl}{8bh} \quad (3)$$

From bending moment diagram



$$|M|_m = \frac{wl^2}{32} \quad (4)$$

For a rectangular cross section

$$S = \frac{1}{6}bh^2 \quad (5)$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wl^2}{16bh^2} \quad (6)$$

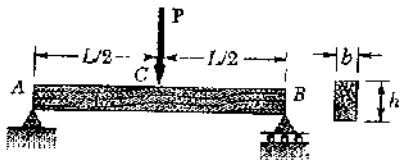
$$(a) \text{ Dividing eq. (3) by eq. (6)} \quad \frac{\tau_m}{\sigma_m} = \frac{2h}{L}$$

$$(b) \text{ Solving for } h \quad h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08 \times 10^6)}{(2)(12 \times 10^6)} = 225 \times 10^{-3} \text{ m} \\ = 225 \text{ mm}$$

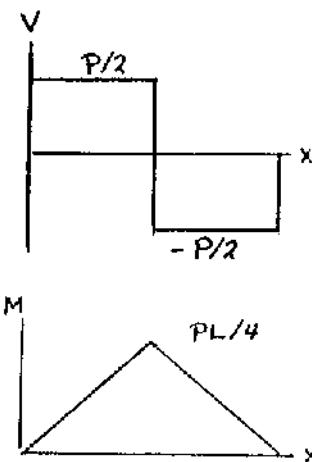
$$\text{Solving eq. (3) for } b \quad b = \frac{3wl}{8h\tau_m} = \frac{(3)(8 \times 10^3)(5)}{(8)(225 \times 10^{-3})(1.08 \times 10^6)} \\ = 61.7 \times 10^{-3} \text{ m} = 61.7 \text{ mm}$$

Problem 6.20

6.20 A timber beam AB of length L and rectangular cross section carries a single concentrated load P at its midpoint C . (a) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (b) Determine the depth h and the width b of the beam, knowing that $L = 2 \text{ m}$, $P = 40 \text{ kN}$, $\tau_m = 960 \text{ kPa}$, and $\sigma_m = 12 \text{ MPa}$.



$$\text{Reactions: } R_A = R_B = P/2$$



$$(1) V_{\max} = R_A = \frac{P}{2}$$

(2) $A = bh$ for rectangular section.

$$(3) \tau_m = \frac{3}{2} \frac{V_{\max}}{A} = \frac{3P}{4bh} \quad \text{for rectangular section.}$$

$$(4) M_{\max} = \frac{PL}{4}$$

$$(5) S = \frac{1}{6} bh^2 \quad \text{for rectangular section.}$$

$$(6) \sigma_m = \frac{M_{\max}}{S} = \frac{3PL}{2bh^2}$$

$$(a) \frac{\tau_m}{\sigma_m} = \frac{h}{2L}$$

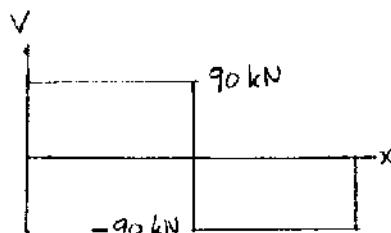
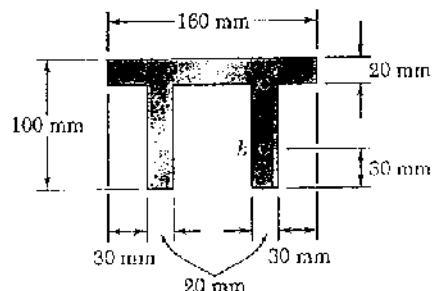
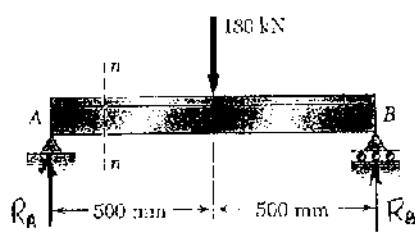
$$(b) \text{ Solving for } h: \quad h = \frac{2L \tau_m}{\sigma_m} = \frac{(2)(2)(960 \times 10^3)}{12 \times 10^6} = 320 \times 10^{-3} \text{ m} \\ = 320 \text{ mm}$$

Solving equation (3) for b ,

$$b = \frac{3P}{4h \tau_m} = \frac{(3)(40 \times 10^3)}{(4)(320 \times 10^{-3})(960 \times 10^3)} = 97.7 \times 10^{-3} \text{ m} \\ = 97.7 \text{ mm}$$

Problem 6.21

6.21 and 6.22 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (a) point *a*, (b) point *b*.



Draw the shear diagram. $|V|_{max} = 90 \text{ kN}$

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	-25	1.000	0.8533
③	1600	40	64	-25	1.000	0.8533
Σ	6400		416		4.000	1.8133

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

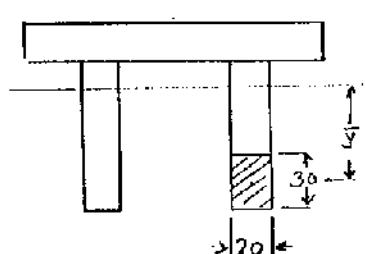
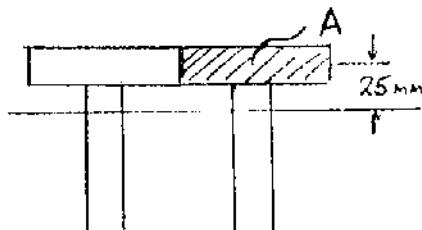
$$I = \sum Ad^2 + \sum \bar{I} = (4.000 + 1.8133) \times 10^6 \text{ mm}^4 \\ = 5.8133 \times 10^6 \text{ mm}^4 = 5.8133 \times 10^{-6} \text{ m}^4$$

$$(a) A = (80)(20) = 1600 \text{ mm}^2$$

$$\bar{y} = 25 \text{ mm}$$

$$Q_a = A\bar{y} = 40 \times 10^3 \text{ mm}^3 = 40 \times 10^{-6} \text{ m}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(90 \times 10^3)(40 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^{-3})} = 31.0 \times 10^6 \text{ Pa} \\ 31.0 \text{ MPa} \blacksquare$$



$$(b) A = (30)(20) = 600 \text{ mm}^2$$

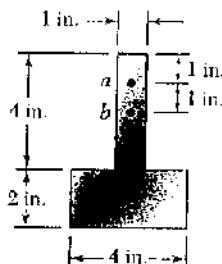
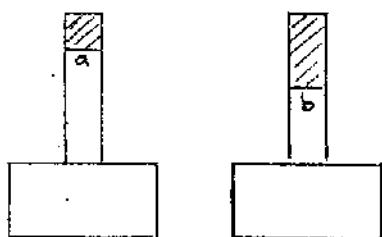
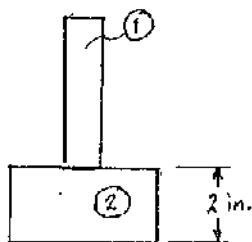
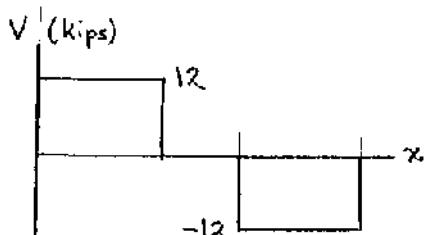
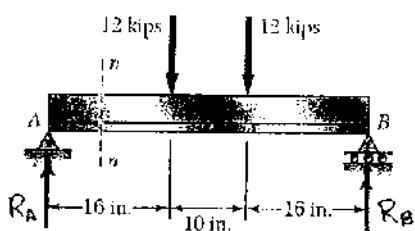
$$\bar{y} = 65 - 15 = 50 \text{ mm}$$

$$Q_b = A\bar{y} = 30 \times 10^3 \text{ mm}^3 = 30 \times 10^{-6} \text{ m}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(90 \times 10^3)(30 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^{-3})} = 23.2 \times 10^6 \text{ Pa} \\ 23.2 \text{ MPa} \blacksquare$$

Problem 6.22

6.21 and 6.22 For the beam and loading shown, consider section $n-n$ and determine the shearing stress at (a) point a , (b) point b .



$$R_A = R_B = 12 \text{ kips}$$

Draw shear diagram.

$$V = 12 \text{ kips}$$

Determine section properties.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4	4	16	2	16	5.333
②	8	1	8	-1	8	2.667
Σ	12		24		24	8.000

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{24}{12} = 2 \text{ in.}$$

$$I = \sum Ad^2 + \sum \bar{I} = 32 \text{ in}^4$$

$$(a) \quad A = 1 \text{ in}^2 \quad \bar{y} = 3.5 \text{ in.} \quad Q_a = A\bar{y} = 3.5 \text{ in}^3 \\ t = 1 \text{ in.}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(12)(3.5)}{(32)(1)} = 1.3125 \text{ ksi}$$

$$(b) \quad A = 2 \text{ in}^2 \quad \bar{y} = 3 \text{ in.} \quad Q_b = A\bar{y} = 6 \text{ in}^3 \\ t = 1 \text{ in.}$$

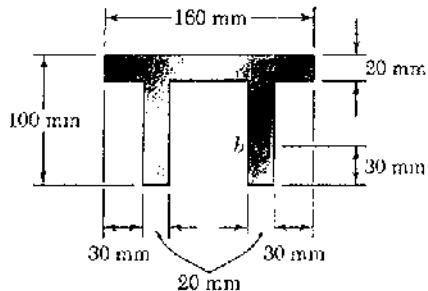
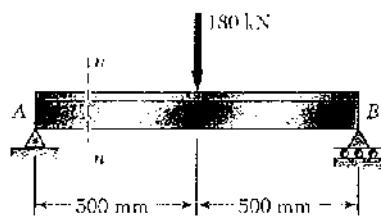
$$\tau_b = \frac{VQ_b}{It} = \frac{(12)(6)}{(32)(1)} = 2.25 \text{ ksi}$$

(a)

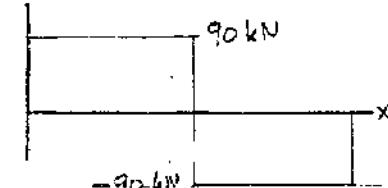
(b)

Problem 6.23

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section *n-n*.



V

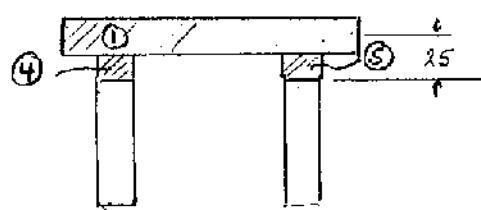
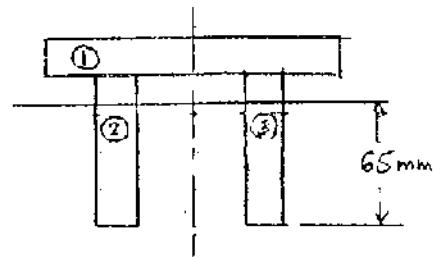


Draw the shear diagram. $|V|_{max} = 90 \text{ kN}$

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3 \text{mm}^3)$	$d(\text{mm})$	$Ad^2(10^6 \text{mm}^4)$	$\bar{I}(10^6 \text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	-25	1.000	0.8533
③	1600	40	64	-25	1.000	0.8533
Σ	6400		416		4.000	1.8133

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

$$I = \sum Ad^2 + \sum I = (4.000 + 1.8133) \times 10^6 \text{ mm}^4 \\ = 5.8133 \times 10^6 \text{ mm}^4 = 5.8133 \times 10^{-6} \text{ m}^4$$



Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3 \text{mm}^3)$
①	3200	25	80
④	300	7.5	2.25
⑤	300	7.5	2.25
Σ			84.5

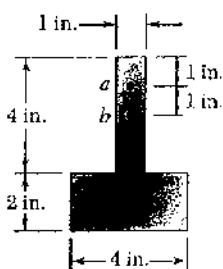
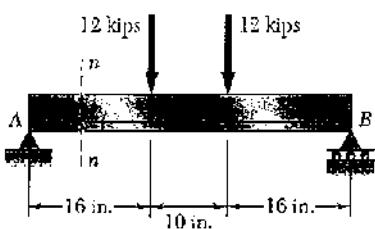
$$Q = \sum A\bar{y} = 84.5 \times 10^3 \text{ mm}^3 = 84.5 \times 10^{-6} \text{ m}^3$$

$$t = (2)(20) = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(90 \times 10^3)(84.5 \times 10^{-6})}{(5.8133 \times 10^{-6})(40 \times 10^{-3})} = 32.7 \times 10^6 \text{ Pa} = 32.7 \text{ MPa} \blacktriangleleft$$

Problem 6.24

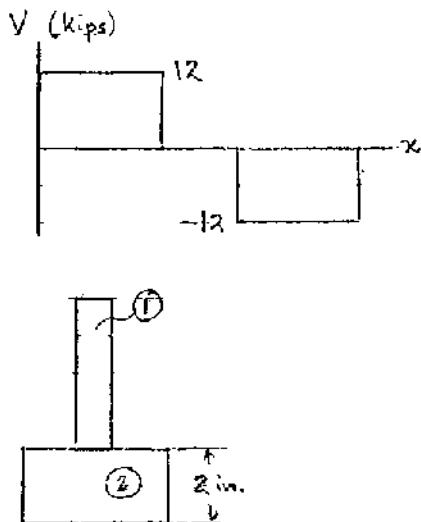
6.23 and 6.24 For the beam and loading shown, determine the largest shoring stress in section $n-n$.



$$R_A = R_B = 12 \text{ kips}$$

Draw shear diagram.

$$V = 12 \text{ kips}$$

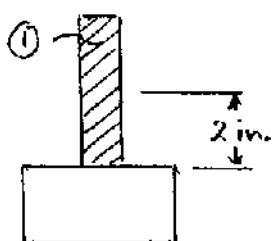


Determine section properties.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4	4	16	2	16	5.333
②	8	1	8	-1	8	2.667
Σ	12		24		24	8.000

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{24}{12} = 2 \text{ in.}$$

$$I = \sum Ad^2 + \bar{I} = 32 \text{ in}^4$$



$$Q = A_1 \bar{y}_1 = (4)(2) = 8 \text{ in}^2$$

$$t = 1 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(12)(8)}{(32)(1)} = 3.00 \text{ ksi}$$

Problem 6.25



6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear V . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k \frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$I = \frac{\pi}{4} C^4 \quad \text{and} \quad A = \pi C^2$$



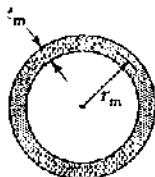
$$\text{For semicircle, } A_s = \frac{\pi}{2} C^2 \quad \bar{y} = \frac{4C}{3\pi}$$

$$Q = A_s \bar{y} = \frac{\pi}{2} C^2 \cdot \frac{4C}{3\pi} = \frac{2}{3} C^3$$

(a) τ_{\max} occurs at center where $t = 2C$.

$$(b) \tau_{\max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3} C^3}{\frac{\pi}{4} C^4 \cdot 2C} = \frac{4V}{3\pi C^2} = \frac{4}{3} \frac{V}{A} \quad k = \frac{4}{3} = 1.333$$

Problem 6.26



6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear V . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k \frac{V}{A}$$

where A is the cross-sectional area of the beam.

For a thin walled circular section, $A = 2\pi r_m t_m$

$$J = Ar_m^2 = 2\pi r_m^3 t_m, \quad I = \frac{1}{2} J = \pi r_m^3 t_m$$



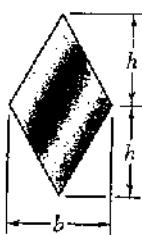
$$\text{For a semicircular arc, } \bar{y} = \frac{2r_m}{\pi}$$

$$A_s = \pi r_m t_m \quad Q = A_s \bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$$

(a) $t = 2t_m$ at neutral axis where maximum occurs.

$$(b) \tau_{\max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A} \quad k = 2.00$$

Problem 6.27

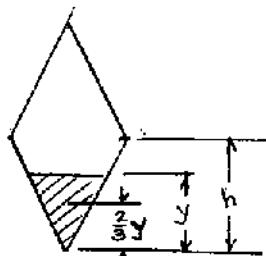


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear V . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k \frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$A = 2\left(\frac{1}{2}bh\right) = bh \quad I = 2\left(\frac{1}{12}bh^3\right) = \frac{1}{6}bh^3$$



For a cut at location y , where $y \leq h$

$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{2} - \frac{by^3}{3h}$$

$$\tau(y) = \frac{VQ}{It}$$

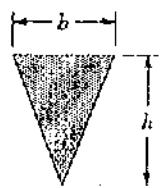
$$\tau(y) = \frac{VQ}{It} = V \frac{6}{bh^3} \cdot \frac{h}{by} \cdot \frac{by^2}{2} - \frac{by^3}{3h} = \frac{V}{bh} \left[3\left(\frac{y}{h}\right) - 2\left(\frac{y}{h}\right)^2 \right]$$

(a) To find the location of maximum of τ , set $\frac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{V}{bh^2} \left[3 - 4\frac{y}{h} \right] = 0 \quad \frac{y}{h} = \frac{3}{4} \quad \text{i.e. } \pm \frac{3}{4}h \text{ from neutral axis.}$$

$$(b) \tau(y_m) = \frac{V}{bh} \left[3\left(\frac{3}{4}\right) - 2\left(\frac{3}{4}\right)^2 \right] = \frac{9}{8} \frac{V}{bh} = 1.125 \frac{V}{A} \quad k = 1.125$$

Problem 6.28

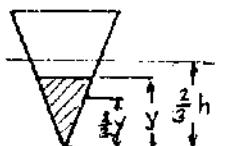


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear V . Determine (a) the horizontal line along which the shearing stress is maximum. (b) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k \frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$A = \frac{1}{2}bh \quad I = \frac{1}{36}bh^3$$



For a cut at location y ,

$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \frac{2}{3}h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h-y)$$

$$\tau(y) = \frac{VQ}{IT} = \frac{V \cdot \frac{by^2}{3}(h-y)}{\left(\frac{1}{36}bh^3\right)\frac{by}{h}} = \frac{12Vy(h-y)}{bh^3} = \frac{12V}{bh^2}(hy - y^2)$$

(a) To find location of maximum of τ , set $\frac{d\tau}{dy} = 0$,

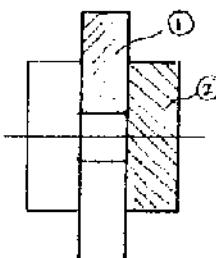
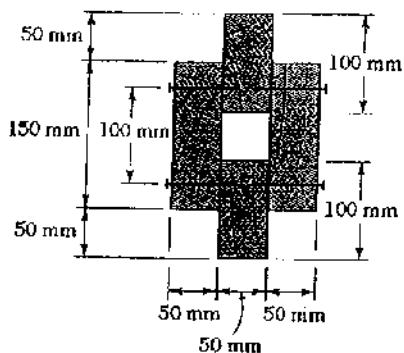
$$\frac{d\tau}{dy} = \frac{12V}{bh^2}(h-2y_m) = 0 \quad y_m = \frac{1}{2}h \quad \text{i.e. at mid-height} \quad \blacktriangleleft$$

(b) $\tau_m = \frac{12V}{bh^2}(hy_m - y_m^2) = \frac{12V}{bh^2} \left[\frac{1}{2}h^2 - \left(\frac{1}{2}h\right)^2 \right] = \frac{3V}{bh^2} = \frac{3V}{A} \quad \times$

$$k = \frac{3}{2} = 1.500 \quad \blacktriangleleft$$

Problem 6.29

6.29 The built-up timber beam is subjected to a 6-kN vertical shear. Knowing that the longitudinal spacing of the nails is $s = 60 \text{ mm}$ and that each nail is 90 mm long, determine the shearing force in each nail.



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 \\ = \frac{1}{12} (50)(100)^3 + (50)(100)(75)^2 \\ = 32.292 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12} (50)(150)^3 \\ = 14.0625 \times 10^6 \text{ mm}^4$$

$$I = 2I_1 + 2I_2 = 92.71 \times 10^6 \text{ mm}^4 = 92.71 \times 10^{-6} \text{ m}^4$$

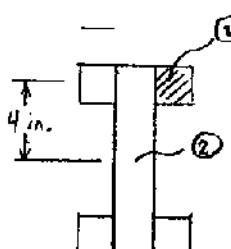
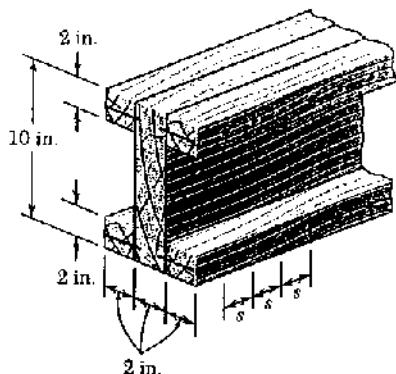
$$Q = Q_i = A_i \bar{y}_i = (50)(100)(75) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(6 \times 10^3)(375 \times 10^{-6})}{92.71 \times 10^{-6}} = 24.27 \times 10^3 \text{ N/m} \quad s = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

$$2F_{\text{nail}} = qs \quad F_{\text{nail}} = \frac{1}{2}qs = \frac{1}{2}(24.27 \times 10^3)(60 \times 10^{-3}) = 728 \text{ N}$$

Problem 6.30

6.30 The built-up timber beam is subjected to a vertical shear of 1200 lb. Knowing that the allowable shearing force in the nails is 75 lb, determine the largest permissible spacing s of the nails.



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 \\ = \frac{1}{12}(2)(2)^3 + (2)(2)(4)^2 = 65.333 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(2)(10)^3 = 16.667 \text{ in}^4$$

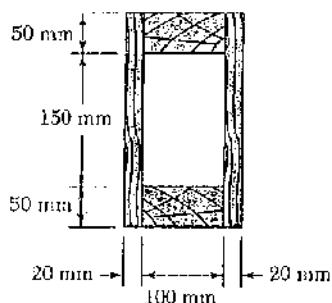
$$I = 4I_1 + I_2 = 42.8 \text{ in}^4$$

$$Q = Q_i = A_i \bar{y}_i = (2)(2)(4) = 16 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(1200)(16)}{42.8} = 44.86 \text{ lb/in}$$

$$F_{\text{nail}} = qs \quad s = \frac{F_{\text{nail}}}{q} = \frac{75}{44.86} = 1.672 \text{ in.}$$

Problem 6.31

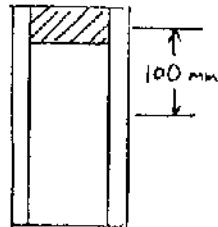


6.31 The built-up beam shown is made by gluing together two 20×250 -mm plywood strips and two 50×100 -mm planks. Knowing that the allowable average shearing stress in the glued joints is 350 kPa, determine the largest permissible vertical shear in the beam.

$$I = \frac{1}{12}(140)(250)^3 - \frac{1}{12}(100)(150)^3 = 154,167 \times 10^6 \text{ mm}^4 \\ = 154,167 \times 10^{-6} \text{ m}^4$$

$$Q = A\bar{y} = (100 \times 50)(100) = 500 \times 10^3 \text{ mm}^3 \\ = 500 \times 10^{-6} \text{ m}^3$$

$$t = 50 \text{ mm} + 50 \text{ mm} = 100 \text{ mm} = 100 \times 10^{-3} \text{ m}$$

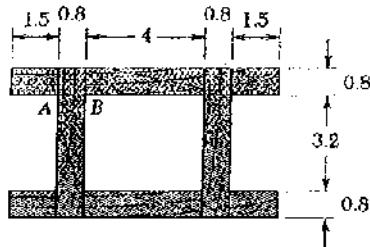


$$\tau = \frac{VQ}{It}$$

$$V = \frac{It\tau}{Q} = \frac{(154,167 \times 10^{-6})(100 \times 10^{-3})(350 \times 10^3)}{500 \times 10^{-6}} \\ = 10.79 \times 10^3 \text{ N} \quad V = 10.79 \text{ kN}$$

Problem 6.32

6.32 The built-up beam was made by gluing together several wooden planks. Knowing that the beam is subjected to a 1200 -lb shear, determine the average shearing stress in the glued joint (a) at A, (b) at B.



Dimensions in inches

$$I = 2 \left[\frac{1}{12}(0.8)(4.8)^3 + \frac{1}{12}(7)(0.8)^3 + (7)(0.8)(2.0)^2 \right] \\ = 60.143 \text{ in}^4$$

$$(a) A_a = (1.5)(0.8) = 1.2 \text{ in}^2 \quad \bar{y}_a = 2.0 \text{ in.}$$

$$Q_a = A_a \bar{y}_a = 2.4 \text{ in}^3$$

$$t_a = 0.8 \text{ in.}$$

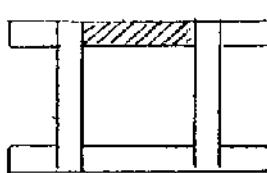
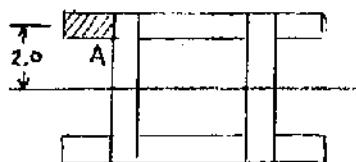
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(1200)(2.4)}{(60.143)(0.8)} = 59.9 \text{ psi}$$

$$(b) A_b = (4)(0.8) = 3.2 \text{ in}^2 \quad \bar{y}_b = 2.0 \text{ in.}$$

$$Q_b = A_b \bar{y}_b = (3.2)(2.0) = 6.4 \text{ in}^3$$

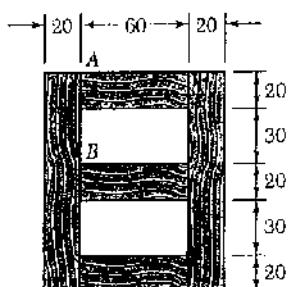
$$t_b = (2)(0.8) = 1.6 \text{ in.}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(1200)(6.4)}{(60.143)(1.6)} = 79.8 \text{ psi}$$

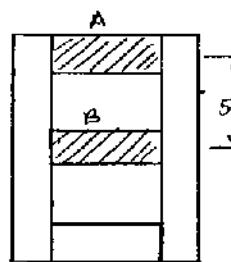


Problem 6.33

6.33 Several planks are glued together to form the box beam shown. Knowing that the beam is subjected to a vertical shear of 3 kN, determine the average shearing stress in the glued joint (a) at A, (b) at B.



Dimensions in mm



$$I_A = \frac{1}{12} b h^3 + Ad^2 = \frac{1}{12}(60)(20)^3 + (60)(20)(50)^2 \\ = 3.04 \times 10^6 \text{ mm}^4$$

$$I_B = \frac{1}{12} b h^3 = \frac{1}{12}(60)(20)^3 = 0.04 \times 10^6 \text{ mm}^4$$

$$I_c = \frac{1}{12} b h^3 = \frac{1}{12}(20)(120)^3 = 2.88 \times 10^6 \text{ mm}^4$$

$$I = 2I_A + I_B + 2I_c = 11.88 \times 10^6 \text{ mm}^4 \\ = 11.88 \times 10^{-6} \text{ m}^4$$

$$Q_A = A\bar{y} = (60)(20)(50) = 60 \times 10^3 \text{ mm}^3 = 60 \times 10^{-6} \text{ m}^3$$

$$t = 20 \text{ mm} + 20 \text{ mm} = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

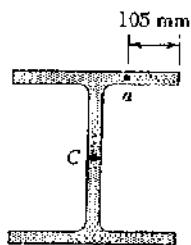
$$(a) \tau_A = \frac{VQ}{It} = \frac{(3 \times 10^3)(60 \times 10^{-6})}{(11.88 \times 10^{-6})(40 \times 10^{-3})} = 379 \times 10^3 \text{ Pa} \\ = 379 \text{ kPa}$$

$$Q_B = 0$$

$$(b) \tau_B = \frac{VQ_B}{It} = 0$$

Problem 6.34

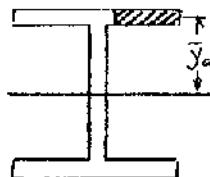
6.34 Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (a) at point A, (b) at the centroid C of the section.



For W360 × 122, $d = 363 \text{ mm}$, $b_f = 25.7 \text{ mm}$, $t_f = 21.70 \text{ mm}$, $t_w = 13.0 \text{ mm}$

$$I = 365 \times 10^6 \text{ mm}^4 = 365 \times 10^{-6} \text{ m}^4$$

(a)



$$A_a = (105)(21.70) = 2278.5 \text{ mm}^2$$

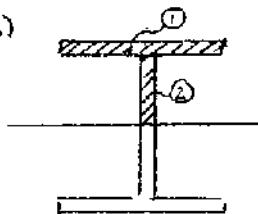
$$\bar{y}_a = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$$

$$Q_a = A_a \bar{y}_a = 2278.5 \times 10^3 \text{ mm}^3 = 2278.5 \times 10^{-6} \text{ m}^3$$

$$t_a = t_f = 21.70 \text{ mm} = 21.7 \times 10^{-3} \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(2278.5 \times 10^{-6})}{(365 \times 10^{-6})(21.7 \times 10^{-3})} = 12.27 \times 10^6 \text{ Pa} = 12.27 \text{ MPa}$$

(b)



$$A_1 = b_f t_f = (25.7)(21.70) = 5577 \text{ mm}^2$$

$$\bar{y}_1 = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$$

$$A_2 = t_w \left(\frac{d}{2} - t_f\right) = (13.0)(159.8) = 2077 \text{ mm}^2$$

$$\bar{y}_2 = \frac{1}{2} \left(\frac{d}{2} - t_f\right) = 79.9 \text{ mm}$$

$$Q_c = \sum A_i \bar{y}_i = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3 \text{ mm}^3$$

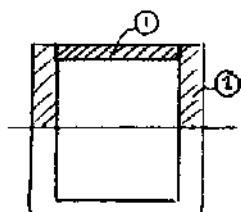
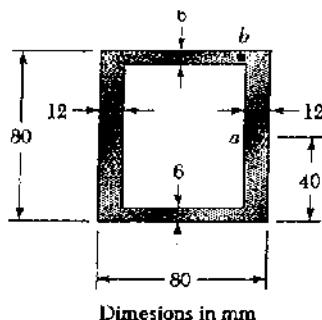
$$= 1117.7 \times 10^{-6} \text{ m}^3$$

$$t_c = t_w = 13.0 \text{ mm} = 13 \times 10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(365 \times 10^{-6})(13 \times 10^{-3})} = 58.9 \times 10^6 \text{ Pa} = 58.9 \text{ MPa}$$

Problem 6.35

6.35 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (a) point *a*, (b) point *b*.



$$I = \frac{1}{12}(80)(80)^3 - \frac{1}{12}(56)(68)^3 = 1.9460 \times 10^6 \text{ mm}^4$$

$$= 1.946 \times 10^{-6} \text{ m}^4$$

$$(a) Q_a = A_1 \bar{y}_1 + 2A_2 \bar{y}_2$$

$$= (56)(6)(37) + (2)(12)(40)(20) = 31.632 \times 10^3 \text{ mm}^3$$

$$= 31.632 \times 10^{-6} \text{ m}^3$$

$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150 \times 10^3)(31.632 \times 10^{-6})}{(1.946 \times 10^{-6})(0.024)} = 101.6 \times 10^6 \text{ Pa}$$

$$= 101.6 \text{ MPa}$$

$$(b) Q_b = A_1 \bar{y}_1 = (56)(6)(37) = 12.432 \times 10^3 \text{ mm}^3$$

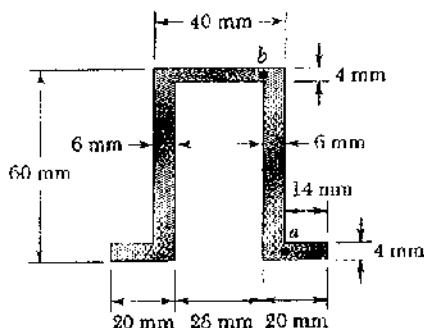
$$= 12.432 \times 10^{-6} \text{ m}^3$$

$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150 \times 10^3)(12.432 \times 10^{-6})}{(1.946 \times 10^{-6})(0.012)} = 79.9 \times 10^6 \text{ Pa} = 79.9 \text{ MPa}$$

Problem 6.36

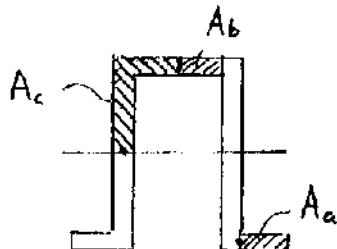
6.36 Knowing that a given vertical shear V causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress at (a) point a , (b) point b .



Neutral axis lies 30 mm above bottom.

$$\tau_c = \frac{VQ_c}{It}, \quad \tau_a = \frac{VQ_a}{It_a}, \quad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a}, \quad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$



$$Q_c = (5)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$$

$$t_c = 6 \text{ mm}$$

$$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$$

$$t_a = 4 \text{ mm}$$

$$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$$

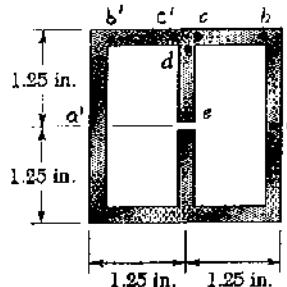
$$t_b = 4 \text{ mm}$$

$$\tau_c = 75 \text{ MPa}$$

$$\tau_a = \frac{Q_a}{Q_c} \cdot \frac{t_c}{t_a} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

$$\tau_b = \frac{Q_b}{Q_c} \cdot \frac{t_c}{t_b} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

Problem 6.37



6.37 and 6.38 The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

Add symmetric points c' , b' , and a' .

$$Q_e = 0$$

$$Q_d = (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.07910 \text{ in}^3 \quad t_d = 0.125 \text{ in.}$$

$$Q_c = Q_e + (0.125)^2(1.1875) = 0.09765 \text{ in}^3 \quad t_c = 0.25 \text{ in.}$$

$$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41308 \text{ in}^3 \quad t_b = 0.25 \text{ in.}$$

$$Q_a = Q_b + (2)(0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.60839 \text{ in}^3 \quad t_a = 0.25 \text{ in.}$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.60839)}{(1.2382)(0.25)} = 3.93 \text{ ksi} \quad \blacksquare$$

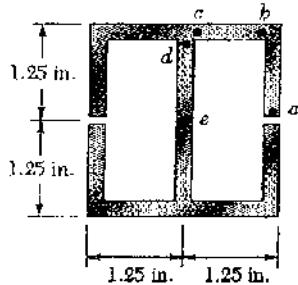
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41308)}{(1.2382)(0.25)} = 2.67 \text{ ksi} \quad \blacksquare$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.09765)}{(1.2382)(0.25)} = 0.63 \text{ ksi} \quad \blacksquare$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.07910)}{(1.2382)(0.125)} = 1.02 \text{ ksi} \quad \blacksquare$$

$$\tau_e = \frac{VQ_e}{It_e} = 0 \quad \blacksquare$$

Problem 6.38



6.37 and 6.38 The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.

$$I = \frac{1}{12} (2.50)(2.50)^3 - \frac{1}{12} (2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

$t = 0.125 \text{ in. at all sections.}$

$$V = 2 \text{ kips}$$

$$Q_a = 0 \quad \tau_a = \frac{VQ_a}{It} = 0$$

$$Q_b = (0.125)(1.25) \left(\frac{1.25}{2}\right) = 0.09766 \text{ in}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.09766)}{(1.2382)(0.125)} = 1.26 \text{ ksi}$$

$$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in}^3$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.2382)(0.125)} = 3.30 \text{ ksi}$$

$$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$$

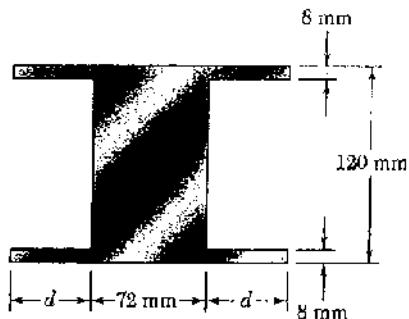
$$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.2382)(0.125)} = 6.84 \text{ ksi}$$

$$Q_e = Q_d + (0.125)(1.125) \left(\frac{1.125}{2}\right) = 0.60839$$

$$\tau_e = \frac{VQ_e}{It} = \frac{(2)(0.60839)}{(1.2382)(0.125)} = 7.86 \text{ ksi}$$

Problem 6.39

6.39 The vertical shear is 25 kN in a beam having the cross section shown. Knowing that $d = 50 \text{ mm}$, determine the shearing stress at (a) point *a*, (b) point *b*.

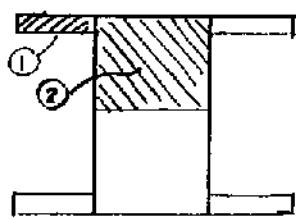


$$I_1 = \frac{1}{12}(50)(8)^3 + (50)(8)(56)^2 = 1.25653 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{3}(72)(60)^3 = 5.184 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 = 15.3933 \times 10^6 \text{ mm}^4 = 15.3933 \times 10^{-6} \text{ m}^4$$

$$Q_1 = A_1 \bar{y}_1 = (50)(8)(56) = 22.4 \times 10^3 \text{ mm}^3 = 22.4 \times 10^{-6} \text{ m}^3$$



$$(a) Q_a = Q_1, t_a = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(25 \times 10^3)(22.4 \times 10^{-6})}{(15.3933 \times 10^{-6})(8 \times 10^{-3})} = 4.55 \times 10^6 \text{ Pa} = 4.55 \text{ MPa}$$

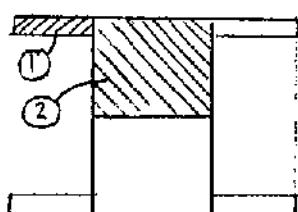
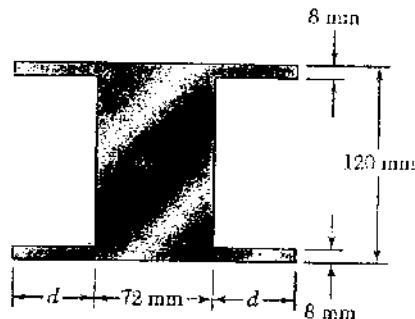
$$(b) Q_b = 2Q_1 + Q_2 = 174.4 \times 10^{-6} \text{ m}^3$$

$$t_b = 72 \text{ mm} = 72 \times 10^{-3} \text{ m}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(25 \times 10^3)(174.4 \times 10^{-6})}{(15.3933 \times 10^{-6})(72 \times 10^{-3})} = 3.93 \times 10^6 \text{ Pa} = 3.93 \text{ MPa}$$

Problem 6.40

6.40 The vertical shear is 25 kN in a beam having the cross section shown. Determine (a) the distance d for which $\tau_a = \tau_b$, (b) the corresponding shearing stress at points a and b .



$$Q_1 = (d)(8)(56) = 448 d \text{ mm}^3$$

$$Q_2 = (72)(60)(30) = 129.6 \times 10^3 \text{ mm}^3$$

$$Q_a = Q_1 = 448 d \quad t_a = 8 \text{ mm}$$

$$Q_b = 2Q_1 + Q_2 = 896 d + 129.6 \times 10^3 \text{ mm}^3$$

$$t_b = 72 \text{ mm}$$

$$\tau_a = \frac{VQ_a}{It_a} \quad \tau_b = \frac{VQ_b}{It_b}$$

$$\text{Since } \tau_a = \tau_b \quad \frac{Q_a}{t_a} = \frac{Q_b}{t_b}$$

$$(a) \quad \frac{448 d}{8} = \frac{896 d + 129.6 \times 10^3}{72} \quad d = 41.3 \text{ mm}$$

$$(b) \quad I_1 = \frac{1}{12}(d)(8)^3 + (d)(8)(56)^2 = 25.131 d = 1.03696 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{3}(72)(60)^3 = 5.184 \times 10^6 \text{ mm}^4$$

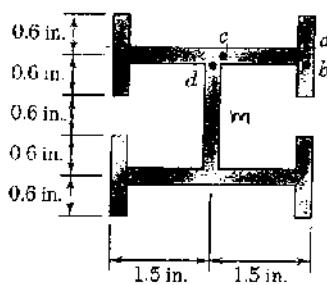
$$I = 4I_1 + 2I_2 = 14.5158 \times 10^6 \text{ mm}^4 = 14.5158 \times 10^{-6} \text{ m}^4$$

$$Q_a = 448 d = (448)(41.263) = 18.4857 \times 10^3 \text{ mm}^2 = 18.4857 \times 10^{-6} \text{ m}^3$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(25 \times 10^3)(18.4857 \times 10^{-6})}{(14.5158 \times 10^{-6})(8 \times 10^{-3})} = 3.98 \times 10^4 \text{ Pa}$$

$$= 3.98 \text{ MPa}$$

Problem 6.41



6.41 An extruded beam has the cross section shown and a uniform wall thickness of 0.20 in. Knowing that a given vertical shear V causes a maximum shearing stress $\tau = 9$ ksi, determine the shearing stress at the four points indicated.

$$Q_a = (0.2)(0.5)(0.5 - 0.25) = 0.125 \text{ in}^3$$

$$Q_b = (0.2)(0.5)(0.3 + 0.25) = 0.055 \text{ in}^3$$

$$Q_c = Q_a + Q_b + (1.4)(0.2)(0.9) = 0.432 \text{ in}^3$$

$$Q_d = 2Q_a + 2Q_b + (3.0)(0.2)(0.9) = 0.900 \text{ in}^3$$

$$Q_m = Q_d + (0.2)(0.8)(0.4) = 0.964 \text{ in}^3$$

$$\tau = \frac{VQ}{It}$$

Since V , I , and t are constant, τ is proportional to Q .

$$\frac{\tau_a}{0.125} = \frac{\tau_b}{0.055} = \frac{\tau_c}{0.432} = \frac{\tau_d}{0.900} = \frac{\tau_m}{0.964} = \frac{q}{0.964}$$

$$\tau_a = 1.167 \text{ ksi}; \tau_b = 0.513 \text{ ksi}; \tau_c = 4.03 \text{ ksi}; \tau_d = 8.40 \text{ ksi}$$

Problem 6.42

6.42 Solve Prob. 6.41 assuming that the beam is subjected to a horizontal shear V .

6.41 An extruded beam has the cross section shown and a uniform wall thickness of 0.20 in. Knowing that a given vertical shear V causes a maximum shearing stress $\tau = 9$ ksi, determine the shearing stress at the four points indicated.

$$Q_a = (0.5)(0.2)(1.4) = 0.140 \text{ in}^3$$

$$Q_b = (0.5)(0.2)(1.4) = 0.140 \text{ in}^3$$

$$Q_c = Q_a + Q_b + (0.2)(1.4)(0.8) = 0.504 \text{ in}^3$$

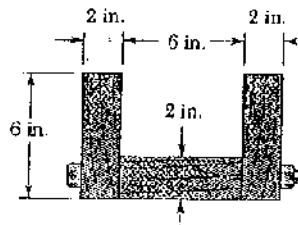
$$Q_d = 0 \quad Q_m = Q_c = 0.504 \text{ in}^3$$

$$\tau = \frac{VQ}{It} \quad \text{Since } V, I, \text{ and } t \text{ are constant, } \tau \text{ is proportional to } Q.$$

$$\frac{\tau_a}{0.140} = \frac{\tau_b}{0.140} = \frac{\tau_c}{0.504} = \frac{\tau_d}{0} = \frac{\tau_m}{Q_m} = \frac{q}{0.504}$$

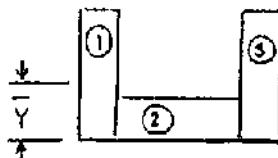
$$\tau_a = 2.50 \text{ ksi}; \tau_b = 2.50 \text{ ksi}, \tau_c = 9.00 \text{ ksi}, \tau_d = 0$$

Problem 6.43



6.43 A beam consists of three planks connected as shown by $\frac{3}{8}$ -in.-diameter bolts spaced every 12 in. along the longitudinal axis of the beam. Knowing that the beam is subjected to a 2500-lb vertical shear, determine the average shearing stress in the bolts.

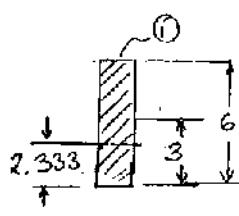
Locate neutral axis and compute moment of inertia.



Part	$A (\text{in}^2)$	$\bar{y} (\text{in.})$	$A\bar{y} \text{ in}^3$	$d (\text{in.})$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
①	12	3	36	0.667	5.333	36
②	12	1	12	1.333	21.333	4
③	12	3	36	0.667	5.333	36
Σ	36		84		32	76

$$\bar{y} = \frac{\sum A \bar{y}}{\sum A} = \frac{84}{36} = 2.333 \text{ in.}$$

$$I = \sum Ad^2 + \sum \bar{I} = 108 \text{ in}^4$$



$$Q = A_1 \bar{y}_1 = (2)(6)(3 - 2.333) = 8 \text{ in}^3$$

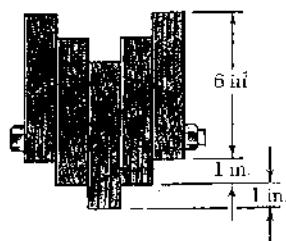
$$q = \frac{VQ}{I} = \frac{(2500)(8)}{108} = 185.2 \text{ lb/in}$$

$$F_{bolt} = q s = (185.2)(12) = 2.222 \times 10^3 \text{ lb.}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2$$

$$T_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.222 \times 10^3}{0.1104} = 20.1 \times 10^3 \text{ psi} = 20.1 \text{ ksi}$$

Problem 6.44



6.44 A beam consists of five planks of 1.5×6 -in. cross section connected by steel bolts with a longitudinal spacing of 9 in. Knowing that the shear in the beam is vertical and equal to 2000 lb. and that the allowable average shearing stress in each bolt is 7500 psi, determine the smallest permissible bolt diameter that may be used.

Part	$A (\text{in}^2)$	$\bar{y}_o (\text{in})$	$A\bar{y}_o (\text{in}^2)$	$\bar{y} (\text{in})$	$A\bar{y}^2 (\text{in}^4)$	$\bar{I} \text{ in}^4$
①	9	5	45	0.8	5.76	27
②	9	4	36	-0.2	0.36	27
③	9	3	27	-1.2	12.96	27
④	9	4	36	-0.2	0.36	27
⑤	9	5	45	0.8	5.76	27
Σ	45		189		25.20	135

$$\bar{Y}_o = \frac{\sum A_y}{\sum A} = \frac{189}{45} = 4.2 \text{ in.}$$

$$I = \sum Ad^2 + \sum \bar{I} = 160.2 \text{ in}^4$$

Between ① and ②: $Q_{12} = Q_1 = A\bar{y}_1 = (9)(0.8) = 7.2 \text{ in}^3$

Between ② and ③: $Q_{23} = Q_1 + A\bar{y}_2 = 7.2 + (9)(-0.2) = 5.4 \text{ in}^3$

$$q_f = \frac{VQ}{I} \quad \text{Maximum } q \text{ is based on } Q_{12} = 7.2 \text{ in}^3$$

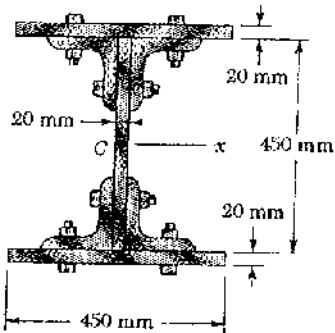
$$q_f = \frac{(2000)(7.2)}{160.2} = 89.888 \text{ lb/in}$$

$$F_{bolt} = q_s = (89.888)(9) = 809 \text{ lb}$$

$$Z_{bolt} = \frac{F_{bolt}}{A_{bolt}} \quad A_{bolt} = \frac{F_{bolt}}{Z_{bolt}} = \frac{809}{7500} = 0.1079 \text{ in}^2$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 \quad d_{bolt} = \sqrt{\frac{4A_{bolt}}{\pi}} = \sqrt{\frac{(4)(0.1079)}{\pi}} = 0.371 \text{ in.}$$

Problem 6.45



6.45 Two 20×450 -mm steel plates are bolted to four L 152 x 152 x 19.0 angles to form a beam with the cross section shown. The bolts have a 22-mm diameter and are spaced longitudinally every 125 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shear in the beam. (Given: $I_x = 1896 \times 10^6 \text{ mm}^4$).

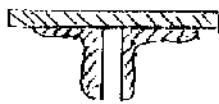
$$\text{Flange: } I_f = \frac{1}{12} (450)(20)^3 + (450)(20)(235)^2 = 497.3 \times 10^6 \text{ mm}^4$$

$$\text{Web: } I_w = \frac{1}{12} (20)(450)^3 = 151.9 \times 10^6 \text{ mm}^4$$

$$\text{Angle: } \bar{I} = 11.6 \times 10^6 \text{ mm}^4, \quad A = 5420 \text{ mm}^2 \\ y = 44.9 \text{ mm} \quad d = 225 - 44.9 = 180.1 \text{ mm}$$

$$I_a = \bar{I} + Ad^2 \\ = 11.6 \times 10^6 + (5420)(180.1)^2 = 187.4 \times 10^6 \text{ mm}^4$$

$$I = 2I_f + I_w + 4I_a = 1896 \times 10^6 \text{ mm}^4 = 1896 \times 10^{-6} \text{ m}^4$$



$$Q_f = (450)(20)(235) = 2115 \times 10^3 \text{ mm}^3$$

$$Q_a = (5420)(180.1) = 976 \times 10^3 \text{ mm}^3$$

$$Q = Q_f + 2Q_a = 4067 \times 10^3 \text{ mm}^3 = 4067 \times 10^{-6} \text{ m}^3$$

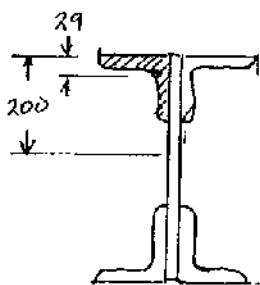
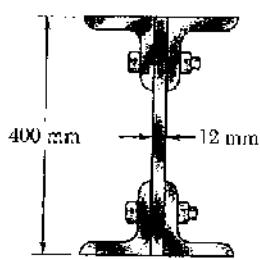
$$A_{b,H} = \frac{\pi}{4} d_{b,H}^2 = \frac{\pi}{4} (22)^2 = 380.1 \text{ mm}^2 = 380.1 \times 10^{-6} \text{ m}^2$$

$$F_{bult} = 2A_{b,H} A_b \cdot (2)(90 \times 10^6) (380.1 \times 10^{-6}) = 68.42 \times 10^3 \text{ N}$$

$$q_{all} = \frac{F_{bult}}{S} = \frac{68.42 \times 10^3}{0.125} = 547.36 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \quad V_{all} = \frac{I q_{all}}{Q} = \frac{(1896 \times 10^{-6})(547.36 \times 10^3)}{4067 \times 10^{-6}} = 255 \times 10^3 \text{ N} \\ = 255 \text{ kN} \quad \blacktriangleleft$$

Problem 6.46



6.46 Four 102×9.5 steel angles shapes and a 12×400 -mm steel plate are bolted together to form a beam with the cross section shown. The bolts are of 22-mm diameter and are spaced longitudinally every 120 mm. Knowing that the beam is subjected to a vertical shear of 240 kN, determine the average shearing stress in each bolt.

$$\text{Angle : } A = 1850 \text{ mm}^2, \bar{I} = 1.83 \times 10^6 \text{ mm}^4, y = 29 \text{ mm}$$

$$d = 200 - 29 = 171 \text{ mm}$$

$$I_a = \bar{I} + Ad^2 = 55.726 \times 10^6 \text{ mm}^4$$

$$\text{Plate : } I_p = \frac{1}{12}(12)(400)^3 = 64 \times 10^6 \text{ mm}^4$$

$$I = 4I_a + I_p = 287.7 \times 10^6 \text{ mm}^4 = 287.7 \times 10^{-6} \text{ m}^4$$

$$Q = (1850)(171) = 316.35 \times 10^3 \text{ mm}^3 = 316.35 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(240 \times 10^3)(316.35 \times 10^3)}{287.7 \times 10^{-6}} = 263.9 \times 10^3 \text{ N/m}$$

$$F_{b,lt} = qS = (263.9 \times 10^3)(120 \times 10^{-3}) = 31.668 \times 10^3 \text{ N}$$

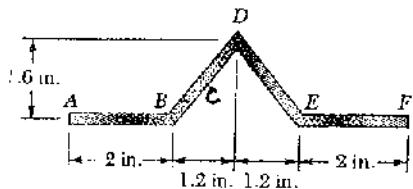
$$A_{bolt} = \frac{\pi d^2}{4} = \frac{\pi}{4}(22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$\tau_{bolt} = \frac{F_{b,lt}}{A_{bolt}} = \frac{31.668 \times 10^3}{380.13 \times 10^{-6}} = 83.3 \times 10^6 \text{ Pa}$$

83.3 MPa

Problem 6.47

6.47 A plate of $\frac{1}{4}$ -in. thickness is corrugated as shown and then used as a beam. For a vertical shear of 1.2 kips, determine (a) the maximum shearing stress in the section, (b) the shearing stress at point B. Also sketch the shear flow in the cross section.



$$l_{eo} = \sqrt{(1.2)^2 + (0.6)^2} = 2.0 \text{ in.}$$

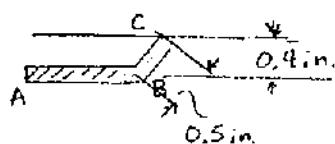
$$A_{eo} = (0.25)(2.0) = 0.5 \text{ in}^2$$

Locate neutral axis and compute moment of inertia.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in})$	$A\bar{y}(\text{in}^3)$	$d(\text{in})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$	$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{0.8}{2.0} = 0.4 \text{ in.}$
AB	0.5	0	0	0.4	0.080	neglect	
BD	0.5	0.8	0.4	0.4	0.080	* 0.1067	* $\frac{1}{12} A_{eo} h^3 = \frac{1}{12} (0.5)(1.6)^3 = 0.1067 \text{ in}^4$
DE	0.5	0.8	0.4	0.4	0.080	* 0.1067	
EF	0.5	0	0	0.4	0.080	neglect	
Σ	2.0	0.8			0.320	0.2133	$I = \sum Ad^2 + \bar{I}$ $= 0.5333 \text{ in}^4$

(a)

$$Q_m = Q_{AB} + Q_{BC}$$



$$Q_{AB} = (2)(0.25)(0.4) = 0.2 \text{ in}^3$$

$$Q_{BC} = (0.5)(0.25)(0.2) = 0.025 \text{ in}^3$$

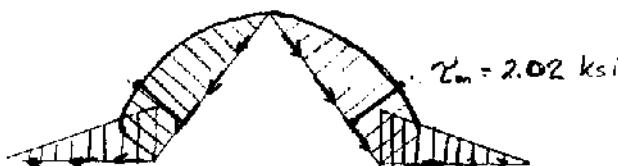
$$Q_m = 0.225 \text{ in}^3$$

$$\tau_m = \frac{V Q_m}{I t} = \frac{(1.2)(0.225)}{(0.5333)(0.25)} = 2.025 \text{ ksi}$$

$$(b) Q_B = Q_{AB} = 0.2 \text{ in}^3$$

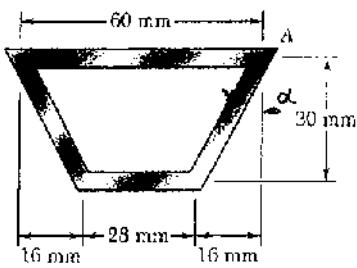
$$\tau_B = \frac{V Q_B}{I t} = \frac{(1.2)(0.2)}{(0.5333)(0.25)} = 1.80 \text{ ksi}$$

$$\tau_d = 0$$



Problem 6.48

6.48 An extruded beam has the cross section shown and a uniform wall thickness of 3 mm. For a vertical shear of 10 kN, determine (a) the shearing stress at point A, (b) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.



$$\tan \alpha = \frac{16}{30} \quad \alpha = 28.07^\circ$$

$$\text{Side: } A = (3 \sec \alpha)(30) = 102 \text{ mm}^2$$

$$\bar{I} = \frac{1}{12}(3 \sec \alpha)(30)^3 = 7.6498 \times 10^3 \text{ mm}^4$$

Part	$A(\text{mm}^2)$	$\bar{y}_s(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
Top	180	30	5.4	11.932	.25.627	neglect
Side	102	15	1.53	3.077	0.966	7.6498
Side	102	15	1.53	3.077	0.966	7.6498
Bot	84	0	0	18.077	27.449	neglect
Σ	468		8.46		55.008	15.7996

$$\bar{Y}_o = \frac{\sum A\bar{y}}{\sum A} = \frac{8.46 \times 10^3}{468} = 18.077 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 70.31 \times 10^3 \text{ mm}^4 = 70.31 \times 10^{-9} \text{ m}^4$$

$$(a) Q_A = (180)(11.932) = 2.14776 \times 10^3 \text{ mm}^3 = 2.14776 \times 10^{-6} \text{ m}^3$$

$$t = (2)(3 \times 10^{-3}) = 6 \times 10^{-3} \text{ m}$$

$$\tau_A = \frac{VQ}{It} = \frac{(10 \times 10^3)(2.14776 \times 10^{-6})}{(70.31 \times 10^{-9})(6 \times 10^{-3})} = 50.9 \times 10^6 \text{ Pa} = 50.9 \text{ MPa}$$

(b)

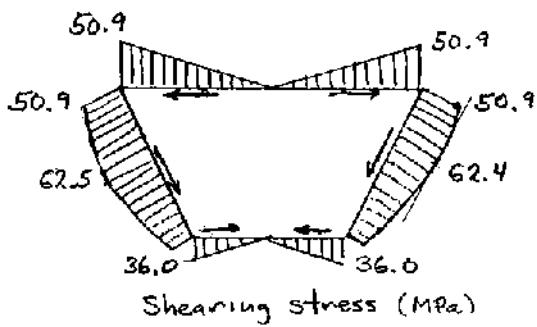
$$Q_m = Q_A + (2)(3 \sec \alpha)(11.932)(\frac{1}{2} \times 11.932)$$

$$= 2.14776 \times 10^3 + 484.06 = 2.6318 \times 10^3 \text{ mm}^3$$

$$= 2.6318 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau_m = \frac{VQ_m}{It} = \frac{(10 \times 10^3)(2.6318 \times 10^{-6})}{(70.31 \times 10^{-9})(6 \times 10^{-3})} = 62.4 \times 10^6 \text{ Pa} = 62.4 \text{ MPa}$$

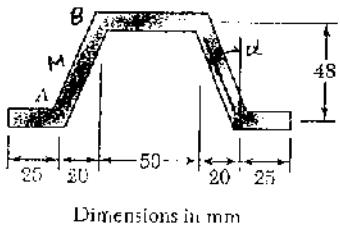


$$Q_B = (28)(3)(18.077) = 1.51847 \times 10^3 \text{ mm}^3$$

$$\tau_B = \frac{Q_B}{Q_A} \tau_A = \frac{1.51847 \times 10^3}{2.14776 \times 10^3} (50.9) \\ = 36.0 \text{ MPa}$$

Problem 6.49

6.49 A plate of 4-mm thickness is bent as shown and then used as a beam. For a vertical shear of 12 kN, determine (a) the shearing stress at point A, (b) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.



$$\tan \alpha = \frac{20}{48} \quad \alpha = 22.62^\circ$$

$$\text{Slanted side: } A_s = (4 \text{ sec } \alpha)(48) = 208 \text{ mm}^2$$

$$\bar{I}_s = \frac{1}{12}(4 \text{ sec } \alpha)(48)^3 = 39.936 \times 10^3 \text{ mm}^4$$

$$\text{Top: } I_T = \frac{1}{12}(50)(4)^3 + (50)(4)(24)^2 = 115.46 \times 10^3 \text{ mm}^4$$

$$\text{Bottom: } I_B = I_T = 115.46 \times 10^3 \text{ mm}^4$$

$$I = 2\bar{I}_s + I_T + I_B = 310.8 \times 10^3 \text{ mm}^4 = 310.8 \times 10^{-9} \text{ m}^4$$

$$(a) Q_A = (25)(4)(24) = 2.4 \times 10^3 \text{ mm}^3 = 2.4 \times 10^{-6} \text{ m}^3$$

$$t = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{(12 \times 10^3)(2.4 \times 10^{-6})}{(310.8 \times 10^{-9})(4 \times 10^{-3})} = 23.2 \times 10^6 \text{ Pa} = 23.2 \text{ MPa}$$

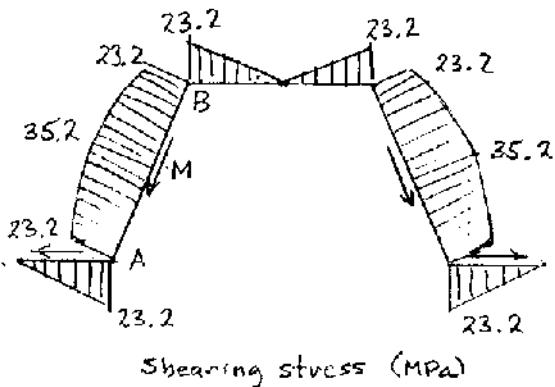
(b) Maximum shearing occurs at point M, 24 mm above the bottom.

$$Q_M = Q_A + (4 \text{ sec } \alpha)(24)(12) = 2.4 \times 10^3 + 1.248 \times 10^3 = 3.648 \times 10^3 \text{ mm}^3$$

$$= 3.648 \times 10^{-6} \text{ m}^3$$

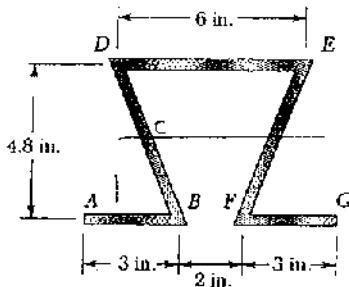
$$\tau_M = \frac{VQ_M}{It} = \frac{(12 \times 10^3)(3.648 \times 10^{-6})}{(310.8 \times 10^{-9})(4 \times 10^{-3})} = 35.2 \times 10^6 \text{ Pa} = 35.2 \text{ MPa}$$

$$Q_B = Q_A \quad \tau_B = \tau_A = 23.2 \text{ MPa}$$



Problem 6.50

6.50 A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 600 lb, determine (a) the thickness t for which the maximum shearing stress is 300 psi, (b) the corresponding shearing stress at point E. Also sketch the shear flow in the cross section.



$$L_{BD} = L_{EF} = \sqrt{4.8^2 + 2^2} = 5.2 \text{ in.}$$

Neutral axis lies at 2.4 in. above AB.

Calculate I.

$$I_{AB} = (3t)(2.4)^2 = 17.28 t$$

$$I_{BD} = \frac{1}{12}(5.2t)(4.8)^2 = 9.984 t$$

$$I_{DG} = (6t)(2.4)^2 = 34.56 t$$

$$I_{EF} = I_{DG} = 9.984 t$$

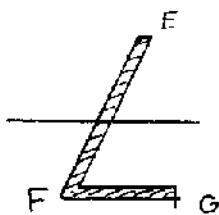
$$I_{FG} = I_{AB} = 17.28 t$$

$$I = \sum I = 89.09 t$$

(a) At point C: $Q_c = Q_{AB} + Q_{BC} = (3t)(2.4) + (2.6t)(1.2) = 10.32 t$

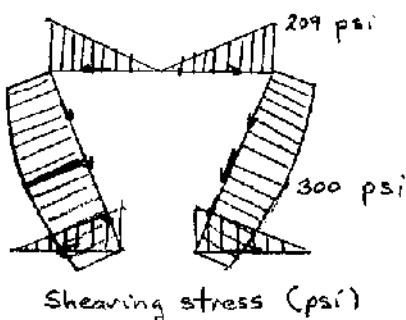
$$\tau = \frac{VQ}{It} \therefore t = \frac{VQ}{\tau I} = \frac{(600)(10.32t)}{(300)(89.09t)} = 0.23168 \text{ in.}$$

(b) $I = (89.09)(0.23168) = 20.64 \text{ in}^3$



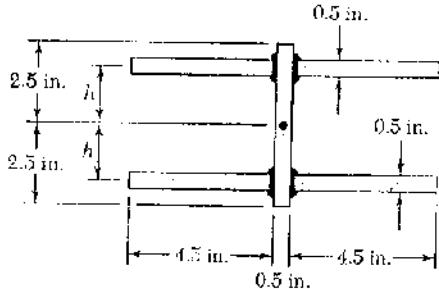
$$Q_E = Q_{EF} + Q_{FG} \\ = 0 + (3)(0.23168)(2.4) = 1.668 \text{ in}^2$$

$$\tau_E = \frac{VQ_E}{It} = \frac{(600)(1.668)}{(20.64)(0.23168)} = 209 \text{ psi}$$



Problem 6.51

6.51 The design of a beam requires welding four horizontal plates to a vertical 0.5 × 5-in. plate as shown. For a vertical shear V , determine the dimension h for which the shear flow through the welded surface is maximum.



Horizontal plate:

$$I_h = \frac{1}{12}(4.5)(0.5)^3 + (4.5)(0.5)h^2 \\ = 0.046875 + 2.25h^2$$

$$\text{Vertical plate: } I_{av} = \frac{1}{12}(0.5)(5)^3 = 5.2083 \text{ in}^4$$

$$\text{Whole section: } I = 4I_h + I_{av} = 9h^2 + 5.39583 \text{ in}^4$$

$$\text{For one horizontal plat: } Q = (4.5)(0.5)h = 2.25h \text{ in}^3$$

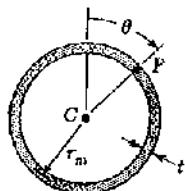
$$\tau = \frac{VQ}{I} = \frac{2.25Vh}{9h^2 + 5.39583}$$

To maximize τ set $\frac{d\tau}{dh} = 0$

$$2.25V \frac{(9h^2 + 5.39583) - 18h^2}{(9h^2 + 5.39583)^2} = 0 \quad h = 0.774 \text{ in.}$$

Problem 6.52

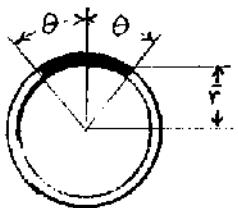
6.52 (a) Determine the shearing stress at point P of a thin-walled pipe of the cross section shown caused by a vertical shear V . (b) Show that the maximum shearing stress occurs for $\theta = 90^\circ$ and is equal to $2V/A$, where A is the cross-sectional area of the pipe.



$$A = 2\pi r_m t \quad J = Ar_m^2 = 2\pi r_m^3 t \quad I = \frac{1}{2}J = \pi r_m^3 t$$

$$\bar{r} = \frac{\sin \theta}{\theta} \text{ for a circular arc}$$

$$A_p = 2r\theta t$$



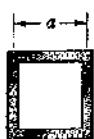
$$Q_p = A_p \bar{r} = 2rt \sin \theta$$

$$(a) \tau_p = \frac{VQ_p}{I(2t)} = \frac{(V)(2rt \sin \theta)}{(\pi r_m^3 t)(2t)} = \frac{V \sin \theta}{\pi r_m^3 t}$$

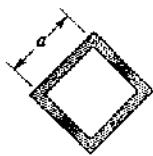
$$(b) \tau_m = \frac{2V \sin \frac{\pi}{2}}{2\pi r_m t} = \frac{2V}{A}$$

Problem 6.53

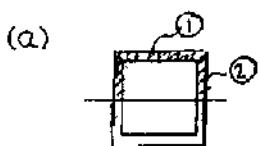
6.53 and 6.54 An extruded beam has a uniform wall thickness t . Denoting by V the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{\max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



(a)



(b)



$$I_1 = (at)\left(\frac{a}{2}\right)^2 \\ = \frac{1}{4}a^3t$$

$$I_2 = \frac{1}{3}t\left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3t$$

$$I = 2I_1 + 4I_2 = \frac{2}{3}a^3t$$

$$Q_1 = (at)\left(\frac{a}{2}\right) = \frac{1}{2}a^2t$$

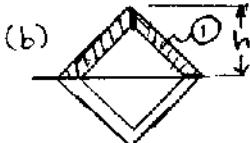
$$Q_2 = \left(\frac{1}{2}at\right)\left(\frac{a}{4}\right) = \frac{1}{8}a^2t$$

$$Q = Q_1 + 2Q_2 = \frac{3}{4}a^2t$$

$$\tau_{\max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{3}{4}a^2t\right)}{\left(\frac{2}{3}a^3t\right)(2t)} =$$

$$= \frac{9}{16} \frac{V}{at} = \frac{9}{4} \frac{V}{4at} = \frac{9}{4} \frac{V}{A}$$

$$= k \frac{V}{A} \quad \therefore \quad k = \frac{9}{4} = 2.25 \quad \blacktriangleleft$$



$$h = \frac{1}{2}\sqrt{2}a$$

$$I_1 = \frac{1}{3}A_1 h^2 = \left(\frac{1}{3}at\right)\left(\frac{\sqrt{2}}{2}a\right)^2 \\ = \frac{1}{6}a^3t$$

$$I = 4I_1 = \frac{2}{3}a^3t$$

$$Q_1 = at\left(\frac{h}{2}\right) = \frac{1}{4}\sqrt{2}a^2t$$

$$Q = 2Q_1 = \frac{1}{2}\sqrt{2}a^2t$$

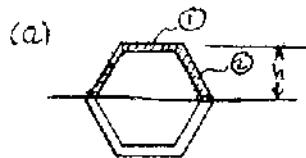
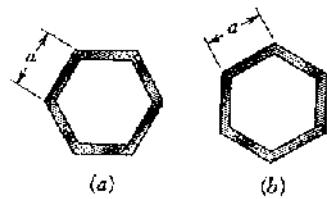
$$\tau_{\max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{1}{2}\sqrt{2}a^2t\right)}{\left(\frac{2}{3}a^3t\right)(2t)}$$

$$= \frac{3\sqrt{2}}{8} \frac{V}{at} = \frac{3\sqrt{2}}{2} \frac{V}{4at}$$

$$= \frac{3\sqrt{2}}{2} \frac{V}{A} = k \frac{V}{A}$$

$$k = \frac{3\sqrt{2}}{2} = 2.12 \quad \blacktriangleleft$$

Problem 6.54



$$h = \frac{\sqrt{3}}{2}a$$

$$A_1 = A_2 = at$$

$$I_1 = A_1 h^2 = at h^2 = \frac{3}{4}a^3 t$$

$$I_2 = \frac{1}{3}A_2 h^2 = \frac{1}{3}at \frac{3}{4}a^2 = \frac{1}{4}a^3 t$$

$$I = 2I_1 + 4I_2 = \frac{5}{2}a^3 t$$

$$Q_1 = A_1 h = \frac{\sqrt{3}}{2}a^2 t$$

$$Q_2 = A_2 \frac{h}{2} = \frac{\sqrt{3}}{4}a^2 t$$

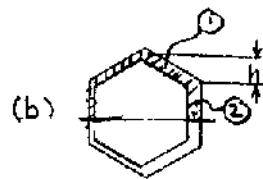
$$Q_m = Q_1 + 2Q_2 = \sqrt{3}a^2 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V\sqrt{3}a^2 t}{(\frac{5}{2}a^3 t)8t} = \frac{\sqrt{3}}{5} \frac{V}{at}$$

$$= \frac{6\sqrt{3}}{5} \frac{V}{6at} = \frac{6\sqrt{3}}{5} \frac{V}{A} = k \frac{V}{A}$$

$$k = \frac{6\sqrt{3}}{5} = 2.08 \quad \blacktriangleleft$$

6.53 and 6.54 An extruded beam has a uniform wall thickness t . Denoting by V the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



$$(b) \quad h = \frac{a}{2}$$

$$A_1 = at \quad A_2 = \frac{1}{2}at$$

$$I_1 = \bar{I}_1 + A_1 d^2$$

$$= \frac{1}{12}at h^2 + at \left(\frac{a}{2} + \frac{h}{2}\right)^2$$

$$= \frac{1}{48}a^3 t + \frac{9}{16}a^3 t = \frac{7}{12}a^3 t$$

$$I_2 = \frac{1}{3}t \left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3 t$$

$$I = 4I_1 + 4I_2 = \frac{5}{2}a^3 t$$

$$Q_1 = at \left(\frac{a}{2} + \frac{h}{2}\right) = \frac{3}{4}a^2 t$$

$$Q_2 = (\frac{1}{2}at)\left(\frac{a}{4}\right) = \frac{1}{8}a^2 t$$

$$Q = 2Q_1 + 2Q_2 = \frac{7}{4}a^2 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \cdot \frac{7}{4}a^2 t}{(\frac{5}{2}a^3 t)(2t)}$$

$$= \frac{7}{20} \frac{V}{at} = \frac{42}{20} \frac{V}{6at} = \frac{21}{10} \frac{V}{A}$$

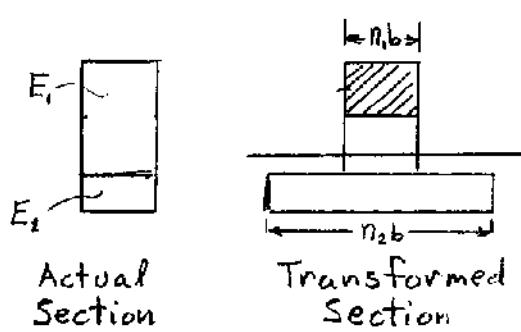
$$= K \frac{V}{A} \quad K = \frac{21}{10} = 2.10 \quad \blacktriangleleft$$

Problem 6.55

6.55 For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

$$\tau_{ave} = \frac{VQ}{It}$$

remains valid provided that both Q and I are computed by using the transformed section of the beam (see Sect 4.6) and provided further that t is the actual width of the beam where τ_{ave} is computed.



Let E_{ref} be a reference modulus of elasticity

$$n_1 = \frac{E_1}{E_{ref}} \rightarrow n_2 = \frac{E_2}{E_{ref}}, \text{ etc.}$$

Widths b of actual section are multiplied by n 's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{n My}{I}$$

where I is the moment of inertia of the transformed cross section and y is measured from the centroid of the transformed section.

The horizontal shearing force over length Δx is

$$\Delta H = - \int (\Delta \sigma_x) dA = \int \frac{n(\Delta M)y}{I} dA = \frac{(\Delta M)}{I} \int ny dA = \frac{Q(\Delta M)}{I}$$

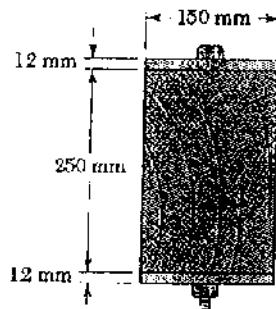
$Q = \int ny dA$ = first moment of transformed section.

$$\text{Shear flow: } q = \frac{\Delta H}{\Delta x} = \frac{\Delta M}{\Delta x} \frac{Q}{I} = \frac{VQ}{I}$$

q is distributed over actual width t , thus $\tau = \frac{q}{t}$.

$$\tau = \frac{VQ}{It}$$

Problem 6.56



6.56 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (a) the average shearing stress in the bolts, (b) the shearing stress at the center of the cross section. (Hint: Use the method indicated in Prob. 6.55.)

$$\text{Let } E_{\text{ref}} = E_s = 200 \text{ GPa}$$

$$n_s = 1 \quad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = \frac{1}{20}$$

Widths of transformed section:

$$\begin{aligned} b_s &= 150 \text{ mm} \quad b_w = \left(\frac{1}{20}\right)(150) = 7.5 \text{ mm} \\ I &= 2 \left[\frac{1}{12}(150)(12)^3 + (150)(12)(125+6)^2 \right] \\ &\quad + \frac{1}{12}(7.5)(250)^3 \\ &= 2 \left[0.0216 \times 10^6 + 30.890 \times 10^6 \right] + 9.766 \times 10^6 \\ &\quad 71.589 \times 10^6 \text{ mm}^4 = 71.589 \times 10^6 \text{ m}^4 \end{aligned}$$

$$(a) \quad Q_1 = (150)(12)(125+6) = 235.8 \times 10^3 \text{ mm}^3 \\ = 235.8 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ_1}{I} = \frac{(4 \times 10^3)(235.8 \times 10^{-6})}{71.589 \times 10^{-6}} = 13.175 \times 10^3 \text{ N/m}$$

$$F_{\text{bolt}} = q_s s = (13.175 \times 10^3)(200 \times 10^{-3}) = 2.635 \times 10^3 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi d_{\text{bolt}}^2}{4} = \frac{\pi}{4}(12)^2 = 113.1 \text{ mm}^2 = 113.1 \times 10^{-6} \text{ m}^2$$

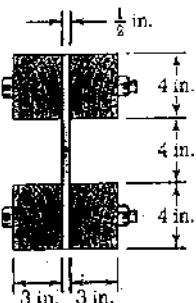
$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{2.635 \times 10^3}{113.1 \times 10^{-6}} = 23.3 \times 10^6 \text{ Pa} = 23.3 \text{ MPa}$$

$$(b) \quad Q_2 = Q_1 + (7.5)(125)(62.5) = 235.8 \times 10^3 + 58.594 \times 10^3 = 294.4 \times 10^3 \text{ mm}^3 \\ = 294.4 \times 10^{-6} \text{ m}^3$$

$$t = 150 \text{ mm} = 150 \times 10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_2}{I t} = \frac{(4 \times 10^3)(294.4 \times 10^{-6})}{(71.589 \times 10^{-6})(150 \times 10^{-3})} = 109.7 \times 10^3 \text{ Pa} = 109.7 \text{ kPa}$$

Problem 6.57



6.57 A composite beam is made by attaching the timber and steel portions shown with bolts of $\frac{5}{8}$ -in. diameter spaced longitudinally every 8 in. The modulus of elasticity is 1.9×10^6 psi for the wood and 29×10^6 psi for the steel. For a vertical shear of 4000 lb, determine (a) the average shearing stress in the bolts, (b) the shearing stress at the center of the cross section. (Hint: Use the method indicated in Prob. 6.55.)

$$\text{Let } E_{ref} = E_w = 1.9 \times 10^6 \text{ psi}$$

$$n_s = \frac{E_s}{E_w} = \frac{29}{1.9} = 15.263$$

$$\text{For one timber, } I_w = \frac{1}{12}(3)(4)^3 + (3)(4)(4)^2 = 208 \text{ in}^4$$

$$\text{For steel plate, } I_s = \frac{1}{12}(\frac{1}{2})(12)^3 = 72 \text{ in}^4$$

$$\text{For transformed section, } I = n_s I_s + 4 I_w = 1930.9 \text{ in}^4$$

$$(a) Q_1 = (3)(4)(4) = 48 \text{ in}^3$$

$$q = \frac{VQ_1}{I} = \frac{(4000)(48)}{1930.9} = 99.435 \text{ lb/in}$$

$$F_{bolt} = q_s = (99.435)(8) = 795.5 \text{ lb.}$$

$$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (\frac{5}{8})^2 = 0.30678 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{795.5}{0.30678} = 2590 \text{ psi} = 2.59 \text{ ksi}$$

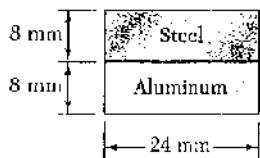
$$(b) Q_2 = 2Q_1 + (15.263)(\frac{1}{2})(6)(3) = 196 + 137.367 = 233.367 \text{ in}^3$$

$$t = \frac{1}{2} \text{ in.} = 0.5 \text{ in.}$$

$$\tau_c = \frac{VQ_2}{It} = \frac{(4 \times 10^3)(233.367)}{(1930.9)(0.5)} = .967 \text{ psi}$$

Problem 6.58

6.58 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 6 kN and that the modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum, determine (a) the average stress at the bonded surface, (b) the maximum shearing stress in the beam. (Hint: Use the method indicated in Prob. 6.55.)



$$n = 1 \text{ in aluminum}$$

$$n = \frac{210}{70} = 3 \text{ in steel}$$

Part	$A(\text{mm}^2)$	$nA(\text{mm}^2)$	$\bar{y}(\text{mm})$	$nA\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$nAd^2(\text{mm}^4)$	$n\bar{I}(\text{mm})^4$
Steel	192	576	12	6912	2	2304	3072
Alum.	192	192	4	768	-6	6912	1024
Σ		768		7680		9216	4096

For transformed section, $\bar{Y}_o = \frac{\sum nA\bar{y}}{\sum nA} = \frac{7680}{768} = 10 \text{ mm above bottom.}$

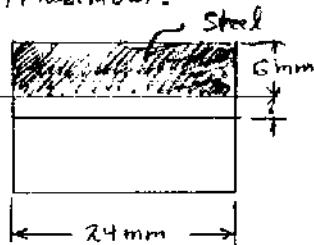
$$\bar{I} = \sum nAd^2 + \sum n\bar{I} = 13312 \text{ mm}^4 = 13312 \times 10^{-12} \text{ m}^4$$

(a) At bond: $Q_1 = (576)(2) = 1152 \text{ mm}^3 = 1152 \times 10^{-9} \text{ m}^3$

$$t = 24 \text{ mm} = 24 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(6 \times 10^3)(1152 \times 10^{-9})}{(13312 \times 10^{-12})(24 \times 10^{-3})} = 21.6 \times 10^6 \text{ Pa} = 21.6 \text{ MPa} \blacktriangleleft$$

(b) Maximum:



$$nA = (3)(24)(6) = 432 \text{ mm}^2$$

$$\bar{y} = 3 \text{ mm}$$

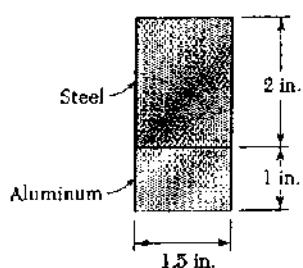
$$Q_2 = nA\bar{y} = (432)(3) = 1296 \text{ mm}^3 = 1296 \times 10^{-9} \text{ m}^3$$

$$t = 24 \text{ mm} = 24 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ_2}{It} = \frac{(6 \times 10^3)(1296 \times 10^{-9})}{(13312 \times 10^{-12})(24 \times 10^{-3})}$$

$$= 24.3 \times 10^6 \text{ Pa} = 24.3 \text{ MPa} \blacktriangleleft$$

Problem 6.59



6.59 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (a) the average stress at the bonded surface, (b) the maximum shearing stress in the beam. (Hint: Use the method indicated in Prob. 6.55.)

$$n = 1 \text{ in aluminum} \quad n = \frac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358 \text{ in steel}$$

Part	$nA (\text{in}^2)$	$\bar{y} (\text{in})$	$nA\bar{y} (\text{in}^3)$	$d (\text{in})$	$nAd^2 (\text{in}^4)$	$n\bar{I} (\text{in}^4)$
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum.	1.5	0.5	0.75	1.2682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608

$$\bar{Y} = \frac{\sum nA\bar{y}}{\sum A} = \frac{17.1648}{9.7074} = 1.7682 \text{ in}$$

$$I = \sum nAd^2 + \sum n\bar{I} = 5.7143 \text{ in}^4$$

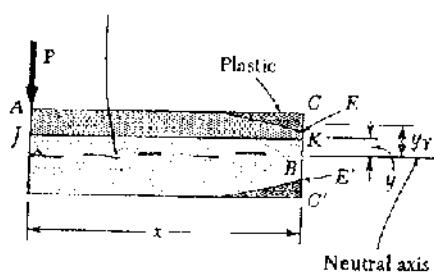
$$(a) \text{ At the bonded surface: } Q = (1.5)(1.2682) = 1.9023 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} = 0.888 \text{ ksi}$$

$$(b) \text{ At the neutral axis: } Q = (2.7358)(1.5)(1.2318 \times \frac{1.2318}{2}) = 3.1133 \text{ in}^3$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} = 1.453 \text{ ksi}$$

Problem 6.60



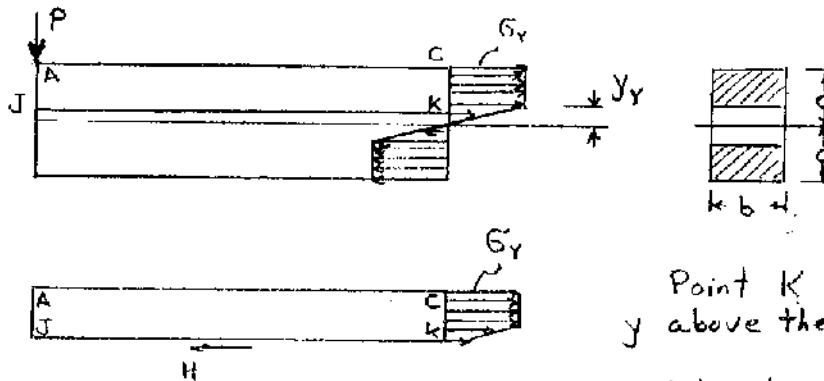
6.60 Consider the cantilever beam AB discussed in Sec. 6.8 and the portion $ACKJ$ of the beam that is located to the left of the transverse section CC' and above the horizontal plane JK , where K is a point at a distance $y < y_r$ above the neutral axis (Fig. 6.60). (a) Recalling that $\sigma_x = \sigma_y$ between C and E and $\sigma_x = (\sigma_y/y_r)y$ between E and K , show that the magnitude of the horizontal shearing force H exerted on the lower face of the portion of beam $ACKJ$ is

$$H = \frac{1}{2} b \sigma_y \left(2c - y_r - \frac{y^2}{y_r} \right)$$

(b) Observing that the shearing stress at K is

$$\tau_{xy} = \lim_{\Delta x \rightarrow 0} \frac{\Delta H}{\Delta A} = \lim_{\Delta x \rightarrow 0} \frac{1}{b} \frac{\Delta H}{\Delta x} = \frac{1}{b} \frac{\partial H}{\partial x}$$

and recalling that y_r is a function of x defined by Eq. (6.14), derive Eq. (6.15).



Point K is located a distance y above the neutral axis.

The stress distribution is given by

$$\sigma = \sigma_y \frac{y}{y_r} \quad \text{for } 0 \leq y < y_r \quad \text{and} \quad \sigma = \sigma_y \quad \text{for } y_r \leq y \leq c.$$

For equilibrium of horizontal forces acting on $ACKJ$,

$$\begin{aligned} H &= \int \sigma dA = \int_y^{y_r} \frac{\sigma_y y b}{y_r} dy + \int_{y_r}^c \sigma_y b dy = \frac{\sigma_y b}{y_r} \left(\frac{y_r^2 - y^2}{2} \right) + \sigma_y b (c - y_r) \\ &= \frac{1}{2} b \sigma_y \left(2c - y_r - \frac{y^2}{y_r} \right) \end{aligned} \quad \blacktriangleleft (a)$$

Note that y_r is a function of x .

$$\tau_{xy} = \frac{1}{b} \frac{\partial H}{\partial x} = \frac{1}{2} \sigma_y \left(-\frac{\partial y_r}{\partial x} + \frac{y^2}{y_r^2} \frac{dy_r}{dx} \right) = -\frac{1}{2} \sigma_y \left(1 - \frac{y^2}{y_r^2} \right) \frac{dy_r}{dx}$$

$$\text{But } M = P_x = \frac{3}{2} M_y \left(1 - \frac{y^2}{c^2} \right)$$

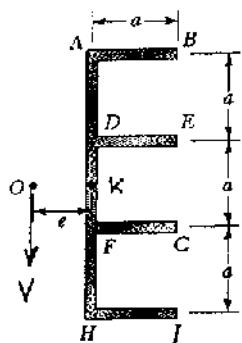
$$\text{Differentiating, } \frac{dM}{dx} = P = \frac{3}{2} M_y \left(-\frac{2}{3} \frac{y_r}{c^2} \frac{dy_r}{dx} \right)$$

$$\frac{dy_r}{dx} = -\frac{P c^2}{y_r M_y} = -\frac{P c^2}{y_r \frac{2}{3} \sigma_y b c^2} = -\frac{3 P}{2 \sigma_y b y_r}$$

$$\text{Then, } \tau_{xy} = \frac{1}{2} \sigma_y \left(1 - \frac{y^2}{y_r^2} \right) \frac{3}{2} \frac{P}{\sigma_y b y_r} = \frac{3 P}{4 b y_r} \left(1 - \frac{y^2}{y_r^2} \right) \quad \blacktriangleleft (b)$$

Problem 6.61

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = I_{HJ} = at \left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

$$I_{DE} = I_{FG} = a^2 \left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^3$$

$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3 \quad I = \sum I = \frac{29}{4}ta^3$$

$$\text{Part AB: } A = tx \quad \bar{y} = \frac{3a}{2} \quad Q = \frac{3}{2}atx$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{3}{2}atx}{\frac{29}{4}ta^3 t} = \frac{6Vx}{29a^2 t}$$

$$F_1 = \int \tau dA = \int_0^a \frac{6Vx}{29a^2 t} t dx = \frac{6V}{29a^2} \int_0^a x dx = \frac{3}{29} V$$

$$\text{Part DE: } A = tx \quad \bar{y} = \frac{a}{2} \quad Q = \frac{1}{2}atx$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}atx}{\frac{29}{4}ta^3 t} = \frac{2Vx}{29a^2 t}$$

$$F_2 = \int \tau dA = \int_0^a \frac{2Vx}{29a^2 t} t dx = \frac{2V}{29a^2} \int_0^a x dx = \frac{1}{29} V$$

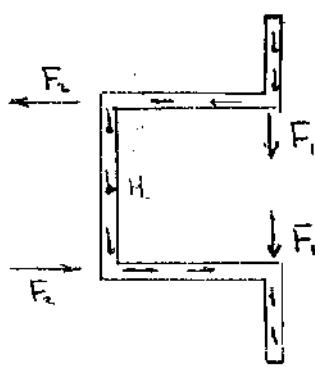
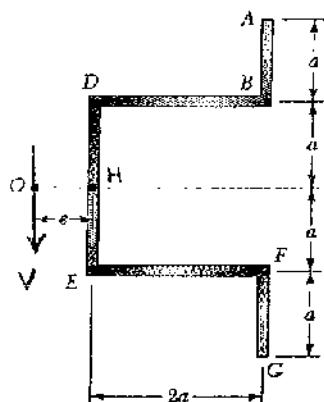
$$\Rightarrow \sum M_k = \sum M_r:$$

$$Ve = F_1(3a) + F_2(a) = \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a = 0.345a$$

Problem 6.62

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

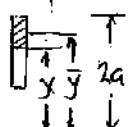


$$I_{AB} = I_{FG} = \frac{1}{12} t a^3 + (ta) \left(\frac{3a}{2}\right)^2 = \frac{7}{3} t a^3$$

$$I_{DB} = I_{EF} = (2at)a^2 + \frac{1}{12}(2a)t^3 \approx 2a^3 t$$

$$I_{DG} = \frac{1}{12} t (2a)^3 = \frac{2}{3} t a^3 \quad I = \sum I = \frac{28}{3} t a^3$$

$$\text{Part AB: } A = t(2a - y), \bar{y} = \frac{2a + y}{2}$$



$$Q = A\bar{y} = \frac{1}{2}t(2a - y)(2a + y) \\ = \frac{1}{2}t(4a^2 - y^2)$$

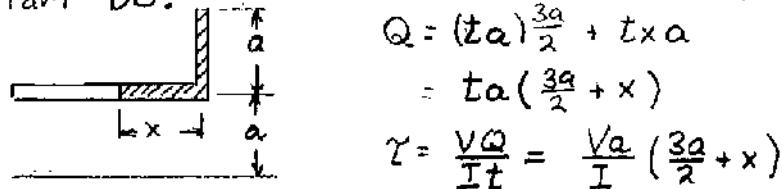
$$\bar{z} = \frac{VQ}{It} = \frac{V}{2I}(4a^2 - y^2)$$

$$F_z = \int \bar{z} dA = \int_0^{2a} \frac{V}{2I} (4a^2 - y^2) t dy$$

$$= \frac{Vt}{2I} \left(4a^2y - \frac{y^3}{3}\right) \Big|_0^{2a} = \frac{Vt a^3}{2I} \left[(4)(2) - \left(\frac{2}{3}\right)^3 - (4)(1) + \left(\frac{1}{3}\right) \right]$$

$$= \frac{5}{6} \frac{Vt a^3}{I} = \frac{5}{56} V$$

$$\text{Part DB:}$$



$$Q = (ta) \frac{3a}{2} + txa \\ = ta \left(\frac{3a}{2} + x \right)$$

$$\bar{z} = \frac{VQ}{It} = \frac{Va}{I} \left(\frac{3a}{2} + x \right)$$

$$F_z = \int \bar{z} dA = \int_0^{2a} \frac{Va}{I} \left(\frac{3a}{2} + x \right) t dx = \frac{Vta}{I} \int_0^{2a} \left(\frac{3a}{2} + x \right) dx$$

$$= \frac{Vta}{I} \left(\frac{3ax}{2} + \frac{x^2}{2} \right) \Big|_0^{2a} = \frac{Vta^3}{I} \left[\frac{(3)(2)^2}{2} + \frac{(2)^2}{2} \right]$$

$$= 5 \frac{Vta^3}{I} = \frac{15}{28} V$$

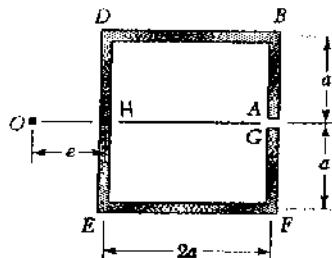
$$\therefore \sum M_H = \sum M_H:$$

$$Ve = F_x(2a) - 2F_z(2a) = \frac{30}{28} Va - \frac{10}{56} Va = \frac{5}{7} Va$$

$$e = \frac{5}{7} a$$

Problem 6.63

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

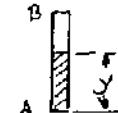


$$I_{AB} = I_{FG} = \frac{1}{3}ta^3 \quad I_{DB} = I_{EP} = 2ata^3 + \frac{1}{12}2at^3 \approx 2ta^3$$

$$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \quad I = \sum I = \frac{16}{3}ta^3$$

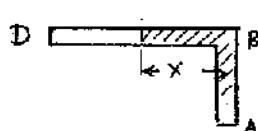
$$\text{Part AB: } A = ty \quad \bar{y} = \frac{y}{2} \quad Q = \frac{1}{2}ty^2$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}ty^2}{\frac{16}{3}ta^3 t} = \frac{3V}{32a^2 t} y^2$$



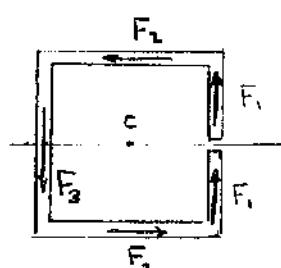
$$F_1 = \int \tau dA = \int_0^a \tau t dy = \frac{3V}{32a^2} \int_0^a y^2 dt = \frac{1}{32}V$$

Part BD:



$$Q = Q_B + t \times a = \frac{1}{2}ta^2 + tax$$

$$\tau = \frac{VQ}{It} = \frac{Vt}{\frac{16}{3}a^2 t} (\frac{1}{2}a^2 + ax) - \frac{3V}{32a^2} (a + 2x)$$



$$F_2 = \int \tau dA = \int_0^{2a} \frac{3V}{32a^2} (a+2x) dx - \frac{3V}{32a^2} (ax+x^2) \Big|_0^{2a} = \frac{3V}{32a^2} (2a^2 + 4a^2) = \frac{9}{16}V$$

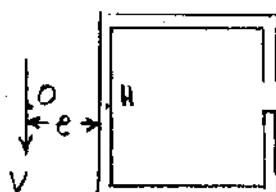
$$\therefore \sum M_H = \sum M_H:$$

$$Ve = (2a)(2F_1) + (2a)(F_2)$$

$$= \frac{1}{8}Va + \frac{9}{8}Va = \frac{5}{4}Va$$

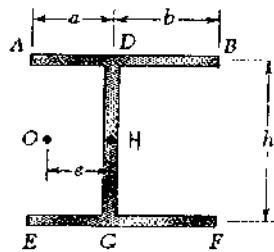
$$e = \frac{5}{4}a$$

$$= 1.250a \leftarrow$$



Problem 6.64

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{DG} = \frac{1}{12}th^3 \quad I = \sum I = \frac{1}{12}t(6a+6b+h)h^2$$

$$\text{Part AD: } Q = t \times \frac{b}{2} = \frac{1}{2}thx$$

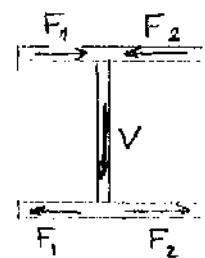
$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_1 = \int \tau dA = \int_0^a \frac{Vhx}{2I} t dx = \frac{Vht}{2I} \int_0^a x dx \\ = \frac{Vht}{2I} \left. \frac{x^2}{2} \right|_0^a = \frac{Vhta^2}{4I}$$

$$\text{Part BD: } Q = t \times \frac{h}{2} = \frac{1}{2}thx$$

$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

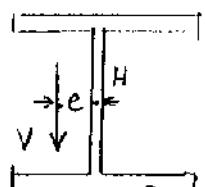
$$F_2 = \int \tau dA = \int_0^b \frac{Vhx}{2I} t dx = \frac{Vht}{2I} \int_0^b x dx \\ = \frac{Vht}{2I} \left. \frac{x^2}{2} \right|_0^b = \frac{Vhtb^2}{4I}$$



$$\sum M_H = \sum M_H:$$

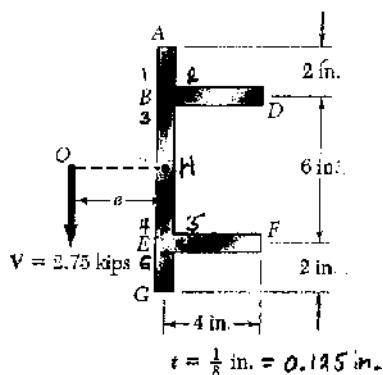
$$Ve = F_2 h - F_1 h = \frac{Vht(b^2 - a^2)}{4I} \\ = \frac{Vh^2t(b^2 - a^2)}{4 \cdot \frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2 - a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2 - a^2)}{6(a+b) + h}$$



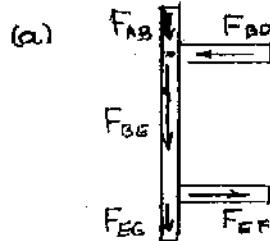
Problem 6.65

6.65 and 6.66 An extruded beam has the cross section shown. Determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by the 2.75-kip vertical shearing force applied at O .

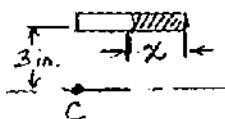


Part	$A (\text{in}^2)$	$d (\text{in})$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
BD	0.50	3	4.50	≈ 0
ABEG	1.25	0	0	10.417
EF	0.50	3	4.50	≈ 0
Σ	2.25		9.00	10.417

$$I = \sum Ad^2 + \sum \bar{I} = 19.417 \text{ in}^4$$



Part BD



$$Q(x) = 3tx$$

$$q_f(x) = \frac{VQ(x)}{I} = \frac{V(3tx)}{I}$$

$$F_{BD} = \frac{3Vt}{I} \int_0^4 x dx = \frac{3Vt}{I} (8) = \frac{24Vt}{I}$$

$$\text{Its moment about } H: (M_{BD})_H = 3F_{BD} = \frac{72Vt}{I}$$

$$\text{Part EF: By same method, } F_{EF} = \frac{24Vt}{I} \quad (M_{EF})_H = \frac{72Vt}{I}$$

Moments of F_{AB} , F_{BE} , and F_{EG} about H are zero.

$$V_e = \sum M_H = \frac{72Vt}{I} + \frac{72Vt}{I} = \frac{144Vt}{I}$$

$$e = \frac{144t}{I} = \frac{(144)(0.125)}{19.417} = 0.927 \text{ in.}$$

$$(b) \text{ At A, D, F, and G: } Q = 0 \quad \tau_A = \tau_D = \tau_F = \tau_G = 0$$

$$\text{Just above B: } Q_1 = Q_{AB} = (2t)(4) = 8t$$

$$\tau_1 = \frac{VQ_1}{It} = \frac{(2.75)(8t)}{(19.417)t} = 1.133 \text{ ksi}$$

$$\text{Just to the right of B: } Q_2 = Q_{BD} = (3)t(4) = 12t$$

$$\tau_2 = \frac{VQ_2}{It} = \frac{(2.75)(12t)}{(19.417)t} = 1.700 \text{ ksi}$$

$$\text{Just below B: } Q_3 = Q_1 + Q_2 = 20t$$

$$\tau_3 = \frac{VQ_3}{It} = \frac{(2.75)(20t)}{(19.417)t} = 2.83 \text{ ksi}$$

Continued on next page.

Problem 6.65 continued.

$$\text{At H (neutral axis): } Q_{Ht} = Q_3 + Q_{BH} = 20t + t(3)(1.5) = 24.5t$$

$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(24.5t)}{(19.417)t} = 3.47 \text{ ksi}$$

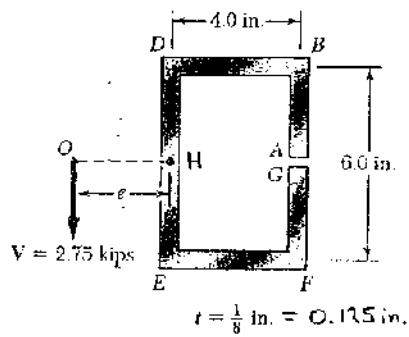
$$\text{By symmetry: } \tau_4 = \tau_3 = 2.83 \text{ ksi}$$

$$\tau_5 = \tau_2 = 1.700 \text{ ksi}$$

$$\tau_6 = \tau_1 = 1.133 \text{ ksi}$$

Problem 6.66

6.65 and 6.66 An extruded beam has the cross section shown. Determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by the 2.75-kip vertical shearing force applied at O .



$$I_{AB} = \frac{1}{3}(0.125)(3)^3 = 1.125 \text{ in}^4$$

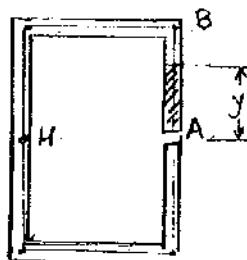
$$I_{BD} = \frac{1}{12}(4)(0.125)^3 + (4)(0.125)(3)^2 = 4.50065 \text{ in}^4$$

$$I_{DE} = \frac{1}{12}(0.125)(5)^3 = 2.25 \text{ in}^4$$

$$I_{EF} = I_{BG} = 4.50065 \text{ in}^4$$

$$I_{FG} = I_{AB} = 1.125 \text{ in}^4$$

$$I = \sum I = 13.50 \text{ in}^4$$



$$(a) \text{ Part AB: } Q(y) = ty \frac{y}{2} = 0.5ty^2$$

$$q(y) = \frac{Vq(y)}{I} = \frac{0.5Vt}{I} y^2$$

$$F_{AB} = \int_0^3 q(y) dy = \frac{0.5Vt}{I} \int_0^3 y^2 dy = 4.5 \frac{Vt}{I} \uparrow$$

$$\text{Its moment about H is } 4F_{AB} = 18 \frac{Vt}{I} \leftarrow$$

$$Q_B = (0.5)(t)(3)^2 = 4.5t$$

$$\text{Part BD: } Q(x) = Q_B + xt(3) = (4.5 + 3x)t$$

$$q(x) = \frac{Vq(x)}{I} = \frac{Vt}{I} (4.5 + 3x)$$

$$F_{BD} = \int_0^4 q(x) dx = \frac{Vt}{I} \int_0^4 (4.5 + 3x) dx = 42 \frac{Vt}{I} \leftarrow$$

$$\text{Its moment about H: } 3F_{BD} = 126 \frac{Vt}{I} \leftarrow$$

$$Q_D = [4.5 + 3(4)]t = 16.5t$$

Continued on next page.

Problem 6.66 continued.

Part EF: By symmetry with part BD, $F_{EF} = 42 \frac{Vt}{I} \rightarrow$

Its moment about H is $3F_{EF} = 126 \frac{Vt}{I} \downarrow$

Part FG: By symmetry with part AB, $F_{FG} = 4.5 \frac{Vt}{I} \uparrow$

Its moment about H is $4F_{FG} = 18 \frac{Vt}{I} \leftarrow$

Moment about H of force in part DE is zero.

$$Ve = \sum M_H = \frac{Vt}{I} (18 + 126 + 0 + 126 + 18) = \frac{144 Vt}{I}$$

$$e = \frac{144 t}{I} = \frac{(2.88)(0.125)}{13.50} = 2.67 \text{ in.}$$

(b) $Q_A = Q_G = 0 \quad \gamma_A = \gamma_G = 0$

$$Q_B = Q_D = 4.5t$$

$$\gamma_B = \gamma_F = \frac{VQ_B}{It} = \frac{(2.75)(4.5t)}{13.50 t} = 0.917 \text{ ksi}$$

$$Q_D = Q_E = 16.5t$$

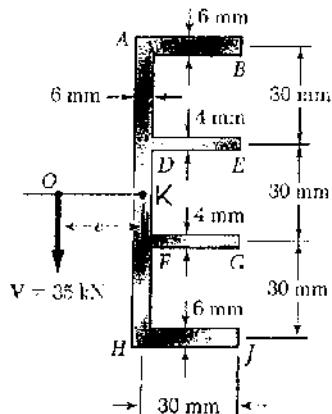
$$\gamma_D = \gamma_E = \frac{VQ_D}{It} = \frac{(2.75)(16.5t)}{13.50 t} = 3.36 \text{ ksi}$$

At H (neutral axis): $Q_H = Q_D + t(3)(1.5) = 21t$

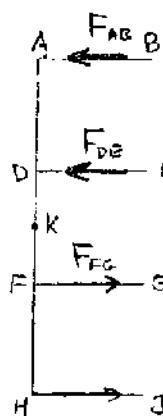
$$\gamma_H = \frac{VQ_H}{It} = \frac{(2.75)(21t)}{13.50 t} = 4.28 \text{ ksi}$$

Problem 6.67

6.67 and 6.68 For an extruded beam having the cross section shown, determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by the vertical shearing force V shown applied at O .



$$I_2 = 1.149 \times 10^6 \text{ mm}^4$$



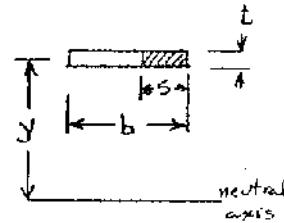
(a) For a typical flange,

$$A(s) = t s$$

$$Q(s) = y t s$$

$$q(s) = \frac{V Q(s)}{I} = \frac{V y t s}{I}$$

$$F = \int_0^b q(s) ds = \frac{V y t b^2}{2I}$$



$$\text{Flange AB: } F_{AB} = \frac{V(45)(6)(30^2)}{(2)(1.14882 \times 10^6)} = 0.10576 V \leftarrow$$

$$\text{Flange DE: } F_{DE} = \frac{V(15)(4)(30)^2}{(2)(1.14882 \times 10^6)} = 0.023502 V \leftarrow$$

$$\text{Flange FG: } F_{FG} = 0.023502 V \rightarrow$$

$$\text{Flange HJ: } F_{HJ} = 0.10576 V \rightarrow$$

$$+\sum M_K = +\sum M_K:$$

$$Ve = 45 F_{AB} + 15 F_{DE} + 15 F_{FG} + 45 F_{HJ} = 10.223 V$$

Dividing by V ,

$$e = 10.22 \text{ mm}$$

(b) Calculation of shearing stresses.

$$V = 35 \times 10^3 \text{ N} \quad I = 1.14882 \times 10^6 \text{ mm}^2$$

At B, E, G, and J:

$$\gamma = 0$$

$$\text{At A and H: } Q = (30)(6)(45) = 8.1 \times 10^3 \text{ mm}^3 = 8.1 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\gamma = \frac{VQ}{It} = \frac{(35 \times 10^3)(8.1 \times 10^{-6})}{(1.14882 \times 10^6)(6 \times 10^{-3})} = 41.1 \times 10^4 \text{ Pa} = 41.1 \text{ MPa}$$

Continued on next page.

Problem 6.67 continued

Just above D and just below F:

$$Q = 8.1 \times 10^3 + (6)(30)(30) = 13.5 \times 10^3 \text{ mm}^3 = 13.5 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(13.5 \times 10^{-6})}{(1.14882 \times 10^{-2})(6 \times 10^{-3})} = 68.5 \times 10^6 \text{ Pa} = 68.5 \text{ MPa} \blacksquare$$

Just to right of D and just to the right of F:

$$Q = (30)(4)(15) = 1.8 \times 10^3 \text{ mm}^3 = 1.8 \times 10^{-6} \text{ m}^3 \quad t = 4 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(1.8 \times 10^{-6})}{(1.14882 \times 10^{-2})(4 \times 10^{-3})} = 13.71 \times 10^6 \text{ Pa} = 13.71 \text{ MPa} \blacksquare$$

Just below D and just above F:

$$Q = 13.5 \times 10^3 + 1.8 \times 10^3 = 15.3 \times 10^3 \text{ mm}^3 = 15.3 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

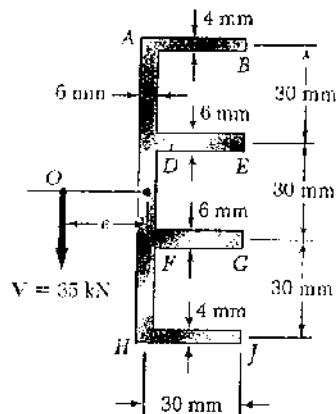
$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(15.3 \times 10^{-6})}{(1.14882 \times 10^{-2})(6 \times 10^{-3})} = 77.7 \times 10^6 \text{ Pa} = 77.7 \text{ MPa} \blacksquare$$

At K: $Q = 15.3 \times 10^3 + (6)(15)(7.5) = 15.975 \times 10^3 \text{ mm}^3 = 15.975 \times 10^{-6} \text{ m}^3$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(15.975 \times 10^{-6})}{(1.14882 \times 10^{-2})(6 \times 10^{-3})} = 81.1 \times 10^6 \text{ Pa} = 81.1 \text{ MPa} \blacksquare$$

Problem 6.68

6.67 and 6.68 For an extruded beam having the cross section shown, determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by the vertical shearing force V shown applied at O .



$$I_z = 0.933 \times 10^6 \text{ mm}^4$$

$$I_{AB} = I_{HJ} = \frac{1}{12}(30)(4)^3 + (30)(4)(45)^2 = 0.24316 \times 10^6 \text{ mm}^4$$

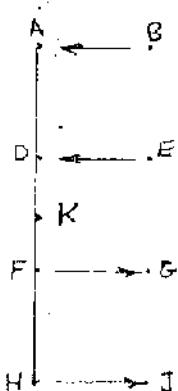
$$I_{DE} = I_{FG} = \frac{1}{12}(30)(6)^3 + (30)(6)(15)^2 = 0.04104 \times 10^6 \text{ mm}^4$$

$$I_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645 \times 10^6 \text{ mm}^4$$

$$I = \sum I = 0.9329 \times 10^6 \text{ mm}^4$$

(a) For a typical flange, $A(s) = ts$

$$\begin{aligned} & \text{Flange AB: } Q(s) = yts \\ & \quad \text{Flange DE: } q(s) = \frac{VQ(s)}{I} = \frac{Vyt s}{I} \\ & \quad F = \int_0^b q(s) ds \\ & \quad = \frac{Vyt b^2}{2I} \end{aligned}$$



$$\text{Flange AB: } F_{AB} = \frac{V(45)(4)(30)^2}{(2)(0.9329 \times 10^6)} = 0.086826 V \leftarrow$$

$$\text{Flange DE: } F_{DE} = \frac{V(15)(6)(30)^2}{(2)(0.9329 \times 10^6)} = 0.043413 V \leftarrow$$

$$\text{Flange FG: } F_{FG} = 0.043413 V \rightarrow$$

$$\text{Flange HJ: } F_{HJ} = 0.086826 V \rightarrow$$

$$+\sum M_k = +\sum M_k:$$

$$Ve = 45 F_{AB} + 15 F_{DE} + 15 F_{FG} + 45 F_{HJ} = 9.1167 V$$

$$\text{Dividing by } V, \quad e = 9.12 \text{ mm}$$

(b) Calculation of shearing stresses.

$$V = 35 \times 10^3 \text{ N} \quad I = 0.9329 \times 10^{-6} \text{ m}^4$$

At B, E, G, and J: $\tau = 0$

$$\text{At A and H: } Q = (30)(4)(45) = 5.4 \times 10^3 \text{ mm}^3 = 5.4 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(35 \times 10^3)(5.4 \times 10^{-6})}{0.9329 \times 10^{-6}} = 202.59 \times 10^3 \text{ N/m}$$

Just to the right of A and H: $t = 4 \times 10^{-3} \text{ m}$

$$\tau = \frac{q}{t} = \frac{202.59 \times 10^3}{4 \times 10^{-3}} = 50.6 \times 10^6 \text{ Pa} = 50.6 \text{ MPa}$$

Continued on next page.

Problem 6.68 continued

Just below A and just above H: $t = 6 \times 10^{-3} \text{ m}$

$$\gamma = \frac{q}{t} = \frac{202.59 \times 10^3}{6 \times 10^{-3}} = 33.8 \times 10^6 \text{ Pa} = 33.8 \text{ MPa}$$

Just above D and just below F: $t = 6 \times 10^{-3} \text{ m}$

$$Q = 5.4 \times 10^3 + (6)(30)(30) = 10.8 \times 10^3 \text{ mm}^3 = 10.8 \times 10^{-6} \text{ m}^3$$

$$\gamma = \frac{VQ}{It} = \frac{(35 \times 10^3)(10.8 \times 10^{-6})}{(0.9329 \times 10^{-6})(6 \times 10^{-3})} = 67.5 \times 10^6 \text{ Pa} = 67.5 \text{ MPa}$$

Just to the right of D and E: $t = 6 \times 10^{-3} \text{ m}$

$$Q = (30)(6)(15) = 2.7 \times 10^3 \text{ mm}^3 = 2.7 \times 10^{-6} \text{ m}^3$$

$$\gamma = \frac{VQ}{It} = \frac{(35 \times 10^3)(2.7 \times 10^{-6})}{(0.9329 \times 10^{-6})(6 \times 10^{-3})} = 16.88 \times 10^6 \text{ Pa} = 16.88 \text{ MPa}$$

Just below D and just above F: $t = 6 \times 10^{-3}$

$$Q = 10.8 \times 10^3 + 2.7 \times 10^3 = 13.5 \times 10^3 \text{ mm}^3 = 13.5 \times 10^{-6} \text{ m}^3$$

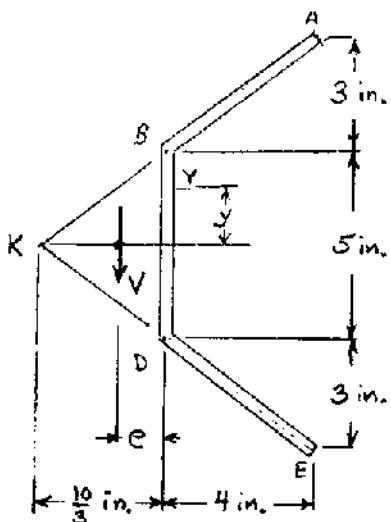
$$\gamma = \frac{VQ}{It} = \frac{(35 \times 10^3)(13.5 \times 10^{-6})}{(0.9329 \times 10^{-6})(6 \times 10^{-3})} = 84.4 \times 10^6 \text{ Pa} = 84.4 \text{ MPa}$$

At K: $Q = 13.5 \times 10^3 + (6)(15)(7.5) = 14.175 \times 10^3 \text{ mm}^3 = 14.175 \times 10^{-6} \text{ m}^3$

$$\gamma = \frac{VQ}{It} = \frac{(35 \times 10^3)(14.175 \times 10^{-6})}{(0.9329 \times 10^{-6})(6 \times 10^{-3})} = 88.6 \times 10^6 \text{ Pa} = 88.6 \text{ MPa}$$

Problem 6.69

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$L_{AB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \quad A_{AB} = 5t$$

$$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12}(5t)(3)^2 + (5t)(4)^2 = 83.75 t \text{ in}^4$$

$$I_{BD} = \frac{1}{12}(t)(5)^3 = 10.417 t \text{ in}^4$$

$$I = 2I_{AB} + I_{BD} = 177.917 t \text{ in.}$$

$$\text{In part BD: } Q = Q_{AB} + Q_{BD}$$

$$Q = (5t)(4) + (2.5 - y)t(\frac{1}{2})(2.5 + y) = 20t + 3.125t - \frac{1}{2}ty^2$$

$$= (23.125 - \frac{1}{2}y^2)t$$

$$Z = \frac{VQ}{It}$$

$$F_{BD} = \int z \, dA = \int_{-2.5}^{2.5} \frac{V(23.125 - \frac{1}{2}y^2)t}{It} \cdot t \, dy$$

$$= \frac{Vt}{I} \int_{-2.5}^{2.5} (23.125 - \frac{1}{2}y^2) \, dy = \frac{Vt}{I} [23.125y - \frac{1}{6}y^3]_{-2.5}^{2.5}$$

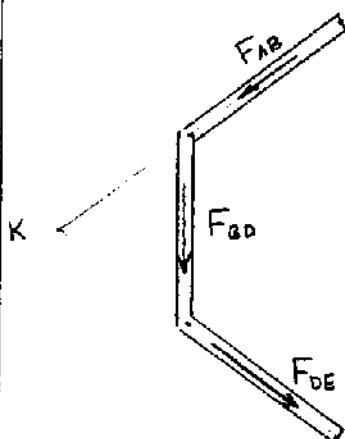
$$= \frac{Vt}{I} \cdot 2 \left[(23.125)(2.5) - \frac{(2.5)^3}{6} \right] = \frac{Vt (110.417)}{177.917 t}$$

$$= 0.62061 V$$

$$\sum M_K = \sum M_k: \quad -V \left(\frac{10}{3} - e \right) = -\frac{10}{3} (0.62061 V)$$

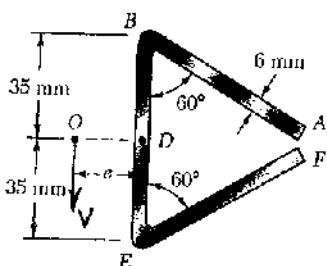
$$e = \frac{10}{3} [1 - 0.62061] = 1.265 \text{ in.}$$

Note that the lines of action of F_{AB} and F_{DE} pass through point K. Thus, these forces have zero moment about point K.



Problem 6.70

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

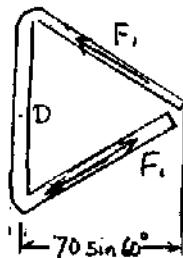


$$I_{DB} = \frac{1}{3}(6)(35)^3 = 85.75 \times 10^3 \text{ mm}^4$$

$$L_{AB} = 70 \text{ mm} \quad A_{AB} = (70)(6) = 420 \text{ mm}^2$$

$$I_{AC} = \frac{1}{3}A_{AB}h^2 = \left(\frac{1}{3}\right)(420)(35)^2 = 171.5 \times 10^3 \text{ mm}^4$$

$$I = (2)(85.75 \times 10^3) + (2)(171.5 \times 10^3) = 514.5 \times 10^3 \text{ mm}^4$$



$$\text{Part AB: } A = ts = 6s$$

$$\bar{y} = \frac{1}{2}s \sin 30^\circ = \frac{1}{4}s$$

$$Q = A\bar{y} = \frac{3}{2}s^2$$

$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$$

$$F_r = \int \tau dA = \int_0^{70} \frac{3Vs^2}{2It} ds = \frac{3V}{I} \int_0^{70} s^2 ds$$

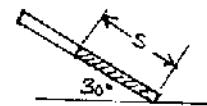
$$= \frac{(3)(70)^3}{(2)(3)I} V = \frac{1}{3}V$$

$$+\sum M_o = +\sum M_o: \quad Ve = 2(F_r \cos 60^\circ)(70 \sin 60^\circ)$$

$$= 20.2 V$$

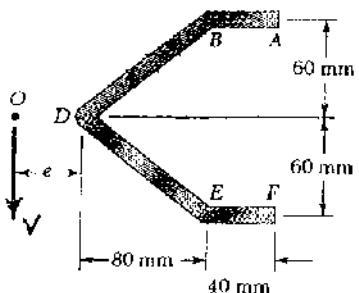
Dividing by V ,

$$e = 20.2 \text{ mm}$$



Problem 6.71

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

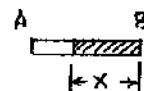
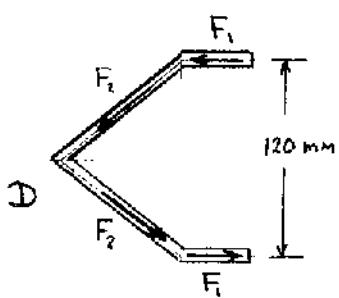


$$I_{AB} = (40t)(60)^3 = 144 \times 10^3 t$$

$$L_{DB} = \sqrt{80^2 + 60^2} = 100 \text{ mm} \quad A_{DB} = 100t$$

$$I_{DB} = \frac{1}{3} A_{DB} h^2 = \frac{1}{3} (100t)(60)^2 = 120 \times 10^3 t$$

$$I = 2I_{AB} + 2I_{DB} = 528 \times 10^3 t$$



$$\text{Part AB: } A = t \times \bar{y} = 60t \text{ mm}^2$$

$$Q = A\bar{y} = 60t \times \text{mm}^3$$

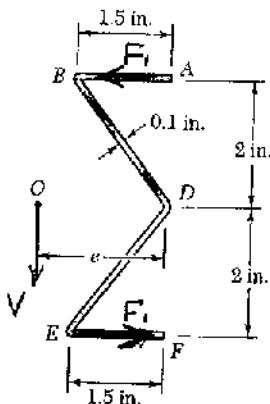
$$\gamma = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60Vx}{I}$$

$$F_v = \int \gamma' dA = \int_0^{120} \frac{60Vx}{I} t dx = \frac{60Vt}{I} \int_0^{120} x dx \\ = \frac{60Vt}{I} \frac{x^2}{2} \Big|_0^{120} = \frac{(60)(30)^2 Vt}{(2)(528 \times 10^3)t} = 0.051136 V$$

$$\therefore \sum M_o = \sum M_p : \quad V e = (0.051136 V)(120) \quad e = (0.051136)(120) = 6.14 \text{ mm} \rightarrow$$

Problem 6.72

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = \frac{1}{12}(1.5)(0.1)^3 + (1.5)(0.1)(2)^2 = 0.600125 \text{ in}^4$$

$$L_{BD} = \sqrt{1.5^2 + 2^2} = 2.5 \text{ in.} \quad A_{BD} = (2.5)(0.1) = 0.25 \text{ in}^2$$

$$I_{BD} = \frac{1}{3}A_{BD}h^2 = \frac{1}{3}(0.25)(2)^2 = 0.33333 \text{ in}^4$$

$$I = 2I_{AB} + 2I_{BD} = 1.86692 \text{ in}^4$$

Part AB: $A(x) = 2x = 0.1x, \bar{y} = 2 \text{ in.}$

$$Q(x) = A(x)\bar{y} = 0.2x \text{ in}^3$$

$$q(x) = \frac{VQ(x)}{I} = \frac{0.2Vx}{I}$$

$$F_1 = \int_0^{1.5} q(x)dx = \frac{0.2V}{I} \int_0^{1.5} x dx \\ = \frac{(0.2)(1.5)^3 V}{2 I} = 0.225 \frac{V}{I}$$

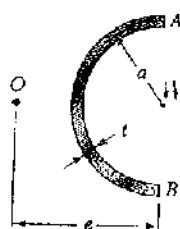
Likewise, by symmetry, in part EF: $F_1 = 0.225 \frac{V}{I}$

$$+ \sum M_D = + \sum M_D: \quad Ve = 4F_1 = 9 \frac{V}{I} = 0.482 V$$

Dividing by V , $e = 0.482 \text{ in.}$

Problem 6.73

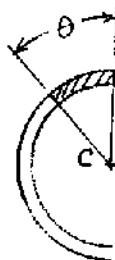
6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



For a thin-walled hollow circular cross section $A = 2\pi at$

$$J = a^2 A = 2\pi a^3 t \quad I = \frac{1}{2} J = \pi a^3 t$$

$$\text{For the half-pipe section} \quad I = \frac{\pi}{2} a^3 t$$



Use polar coordinate θ for partial cross section

$$A = st = a\theta t \quad s = \text{arc length}$$

$$\bar{r} = a \frac{\sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{\theta}{2}$$

$$\bar{y} = \bar{r} \cos \alpha = a \frac{\sin \alpha \cos \alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t a \frac{\sin \alpha \cos \alpha}{\alpha} = a^2 t (2 \sin \alpha \cos \alpha) \\ = a^2 t \sin 2\alpha = a^2 t \sin \theta$$

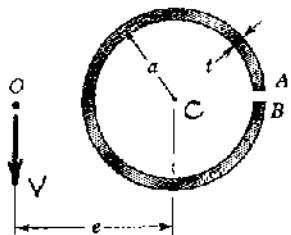
$$\gamma = \frac{VQ}{It} = \frac{Va^2}{I} \sin \theta$$

$$M_H = \int a \gamma dA = \int_0^\pi a \frac{Va^2}{I} \sin \theta t a d\theta = \left[\frac{Va^3 t}{I} - \cos \theta \right]_0^\pi \\ = 2 \frac{Va^3 t}{I} = \frac{4}{\pi} Va$$

$$\text{But } M_H = Ve, \quad \text{hence} \quad e = \frac{4}{\pi} a = 1.273 a$$

Problem 6.74

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

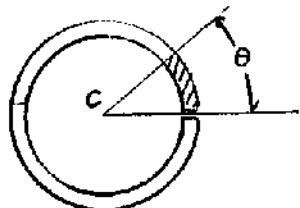


For whole cross section, $A = 2\pi a t$

$$J = Aa^2 = 2\pi a^3 t \quad I = \frac{1}{2} J = \pi a^3 t$$

Use polar coordinate θ for partial cross section.

$$A = st = a\theta t \quad s = \text{arc length}$$



$$\bar{r} = a \frac{\sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{1}{2}\theta$$

$$\bar{y} = \bar{r} \sin \alpha = a \frac{\sin^2 \alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t a \frac{\sin^2 \alpha}{\alpha} = a^2 t 2 \sin^2 \alpha \\ = a^2 t 2 \sin^2 \frac{\theta}{2} = a^2 t (1 - \cos \theta)$$

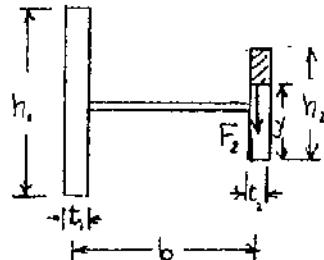
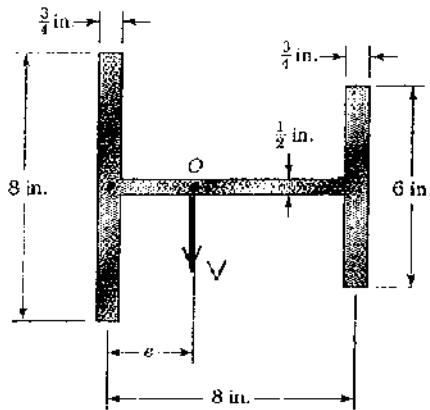
$$\tau = \frac{VQ}{It} = \frac{Va^2}{I} (1 - \cos \theta)$$

$$M_e = \int a \tau dA = \int_0^{2\pi} \frac{Va^3}{I} (1 - \cos \theta) t \alpha d\theta = \left[\frac{Va^3 t}{I} (\theta - \sin \theta) \right]_0^{2\pi} \\ = \frac{2\pi Va^3 t}{\pi a^3 t} = 2aV$$

$$\text{But } M_c = Ve; \quad \text{hence} \quad e = 2a$$

Problem 6.75

6.75 and 6.76 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.



$$I = \frac{1}{12} t_1 h_1^3 + \frac{1}{12} t_2 h_2^3$$

Right flange:

$$A = (\frac{1}{2} h_2 - y) t_2$$

$$\bar{y} = \frac{1}{2} (\frac{1}{2} h_2 + y) t_2$$

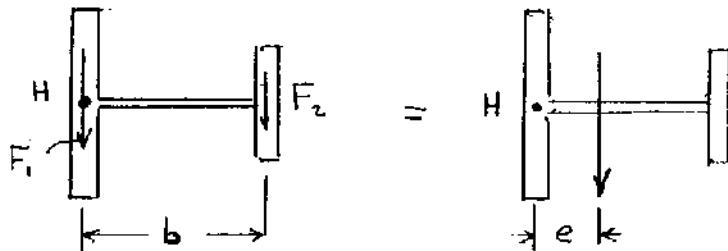
$$\begin{aligned} Q &= A\bar{y} \\ &= \frac{1}{2} (\frac{1}{2} h_2 - y) (\frac{1}{2} h_2 + y) t_2 \\ &= \frac{1}{2} (\frac{1}{4} h_2^2 - y^2) t_2 \end{aligned}$$

$$\gamma = \frac{VQ}{It_2} = \frac{V}{2It_2} (\frac{1}{4} h_2^2 - y^2) t_2$$

$$\begin{aligned} F_2 &= \int \gamma dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2It_2} (\frac{1}{4} h_2^2 - y^2) t_2 dy = \frac{Vt_2}{2I} \left(\frac{1}{4} h_2^2 y - \frac{y^3}{3} \right) \Big|_{-h_2/2}^{h_2/2} \\ &= \frac{Vt_2}{2I} \left\{ \frac{1}{4} h_2^2 \frac{h_2}{2} - \frac{1}{3} \left(\frac{h_2}{2}\right)^3 + \frac{1}{4} h_2^2 \frac{h_2}{2} - \frac{1}{3} \left(\frac{h_2}{2}\right)^3 \right\} = \frac{Vt_2 h_2^3}{12I} = \frac{Vt_2 h_2^3}{t_1 h_1^3 + t_2 h_2^3} \end{aligned}$$

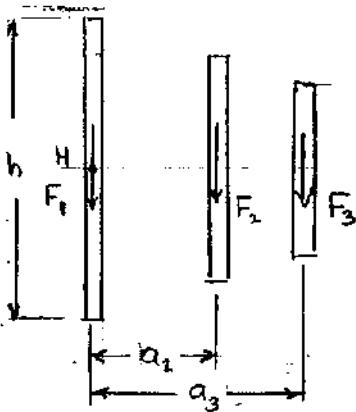
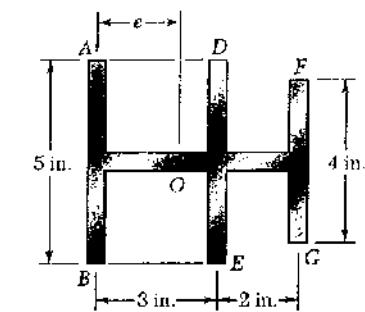
$$\text{④ } M_H = -M_H: \quad -Ve = -F_2 b = -V \frac{t_2 h_2^3 b}{t_1 h_1^3 + t_2 h_2^3}$$

$$e = \frac{t_2 h_2^3 b}{t_1 h_1^3 + t_2 h_2^3} = \frac{(0.75)(6)^3 (8)}{(0.75)(8)^3 + (0.75)(6)^3} = 2.37 \text{ in.}$$



Problem 6.76

6.75 and 6.76 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.



Let $h_1 = \overline{AB}$, $h_2 = \overline{DE}$, and $h_3 = \overline{FG}$.

$$I = \frac{1}{12} t (h_1^3 + h_2^3 + h_3^3)$$

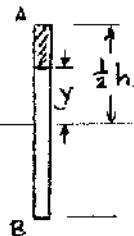
$$\text{Part AB: } A = (\frac{1}{2}h_1 - y)t$$

$$\bar{y} = \frac{1}{2}(\frac{1}{2}h_1 + y)$$

$$Q = A\bar{y} = \frac{1}{2}t(\frac{1}{2}h_1 - y)(\frac{1}{2}h_1 + y) \\ = \frac{1}{2}t(\frac{1}{4}h_1^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)$$

$$F_1 = \int \tau dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)t dy \\ = \frac{Vt}{2I} \left(\frac{1}{4}h_1^2 y - \frac{y^3}{3} \right) \Big|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \\ = \frac{Vt}{I} \left(\frac{1}{4}h_1^2 \cdot \frac{1}{2}h_1 - \frac{1}{3}(\frac{h_1}{2})^3 \right) = \frac{Vt h_1^3}{12 I} \\ = \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$



Likewise, for Part DE:

$$F_2 = \frac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$$

and for Part FG:

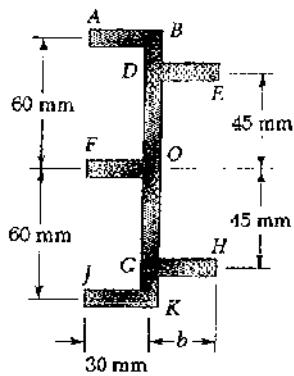
$$F_3 = \frac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$$

$$\rightarrow \sum M_H = \rightarrow \sum M_V: \quad Ve = a_2 F_2 + a_3 F_3 = \frac{a_2 h_2^3 + a_3 h_3^3}{h_1^3 + h_2^3 + h_3^3} V$$

$$\text{Dividing by } V, \quad e = \frac{a_2 h_2^3 + a_3 h_3^3}{h_1^3 + h_2^3 + h_3^3} = \frac{(3)(5)^3 + (5)(4)^3}{5^3 + 5^3 + 4^3} = 2.21 \text{ in.}$$

Problem 6.77

6.77 and 6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



$$\text{Part AB: } A = t \times \bar{y} = 60 \text{ mm}$$

$$Q = A\bar{y} = 60t \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{60Vt}{I}$$

$$F_1 = \int \tau' dA = \int_0^{30} \frac{60Vx}{I} t dx = \frac{60Vt}{I} \int_0^{30} x dx \\ = \frac{60Vt}{I} \left[\frac{x^2}{2} \right]_0^{30} = \frac{(60)(30)^2}{2} \frac{Vt}{I} = 27 \times 10^3 \frac{Vt}{I}$$

$$\text{Part DE: } A = t \times \bar{y} = 45 \text{ mm}$$

$$Q = A\bar{y} = 45t \text{ mm}^3$$

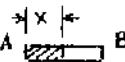
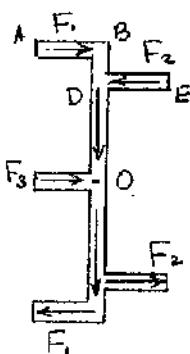
$$\tau = \frac{VQ}{It} = \frac{45Vt}{I}$$

$$F_2 = \int \tau dA = \int_0^b \frac{45Vx}{I} t dx = \frac{45Vt}{I} \int_0^b x dx = \frac{45b^2 Vt}{2I}$$

$$\therefore \sum M_O = +\sum M_O \quad O = (2)(45)F_2 - (2)(60)F_1$$

$$[(45)^2 b^2 - (2)(60)(27 \times 10^3)] \frac{Vt}{I} = 0$$

$$b^2 = \frac{(2)(60)(27 \times 10^3)}{45^2} = 1600 \text{ mm}^2 \quad b = 40 \text{ mm}$$



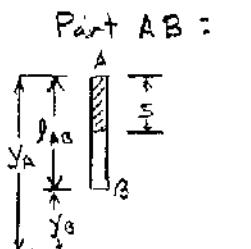
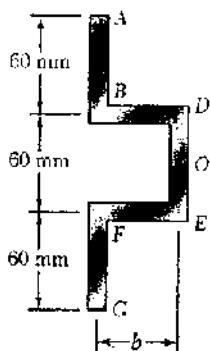
Note that the pair of F_1 forces form a couple.

Likewise, the pair of F_2 forces form a couple.

The lines of action of the forces in BDOGK pass through point O.

Problem 6.78

6.77 and 6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



$$A(s) = ts$$

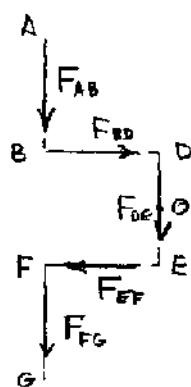
$$\bar{y}(s) = y_A - \frac{1}{2}s$$

$$Q(s) = A(s)\bar{y}(s) = tsy_A - \frac{1}{2}ts^2$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vt}{I}(sy_A - \frac{1}{2}s^2)$$

$$F_{AB} = \int_0^{l_{AB}} q(s) ds$$

$$= \frac{Vt}{I} \left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6} \right) \downarrow$$



$$\text{At } B: Q_B = ty_A l_{AB} - \frac{1}{2}tl_{AB}^2$$

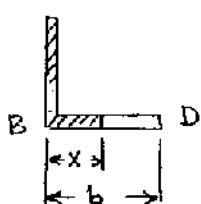
$$\text{By symmetry, } F_{FG} = F_{AB}$$

$$\text{Part BD: } A(x) = tx$$

$$Q(x) = Q_B + y_B A(x) = Q_B + ty_B x$$

$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(Q_B + ty_B x)$$

$$F_{BD} = \int_0^b q(x) dx = \frac{V}{I} (Q_B b + \frac{1}{2}ty_B b^2) \rightarrow$$



$$\text{By symmetry, } F_{EF} = F_{BD}$$

F_{DE} is not required, since its moment about is zero.

$$\Rightarrow \sum M_O = 0 \therefore b(F_{AB} + F_{FG}) - y_B F_{BD} + y_F F_{EF} = 0$$

$$2bF_{AB} - 2y_B F_{BD} = 0$$

$$2b \cdot \frac{Vt}{I} \left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6} \right) - 2y_B \frac{V}{I} (Q_B b + \frac{1}{2}ty_B b^2)$$

$$\frac{2Vt}{I} \left\{ \frac{1}{2}y_A l_{AB}^2 - \frac{1}{6}l_{AB}^3 \right\} b - \frac{2Vt}{I} \left\{ (y_A l_{AB} - \frac{1}{2}l_{AB}^2)y_B b - \frac{1}{2}y_B^2 b^2 \right\} = 0$$

Dividing by $\frac{2Vt}{I}$ and substituting numerical data,

$$\left\{ \frac{1}{2}(90)(60)^2 - \frac{1}{6}(60)^3 \right\} b - [(90)(60) - \frac{1}{2}(60)^2](30)b + \frac{1}{2}(30)^2 b^2 = 0$$

$$126 \times 10^3 b - 108 \times 10^3 b + 450 b^2 = 0$$

$$18 \times 10^3 b - 450 b^2 = 0$$

$$b = 0 \text{ and } b = 40.0 \text{ mm}$$

Problem 6.79

6.79 For the angle shape and loading of Sample Prob. 6.5, check that $\int q dz = 0$ along the horizontal leg of the angle and $\int q dy = P$ along its vertical leg.

Refer to Sample Prob. 6.5.

$$\text{Along horizontal leg: } z_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3} (a^2 - 4az + 3z^2)$$

$$\begin{aligned} \int q dz &= \int_0^a z_f t dz = \frac{3P}{4ta^3} \left\{ (a^2 - 4az + 3z^2) dz \right\}_0^a = \frac{3P}{4a^3} (az - 4a \frac{z^2}{2} + 3 \frac{z^3}{3}) \Big|_0^a \\ &= \frac{3P}{4a^3} (a^3 - 2a^3 + a^3) = 0 \end{aligned}$$

$$\text{Along vertical leg: } z_e = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3} (a^2 + 4ay - 5y^2)$$

$$\begin{aligned} \int q dy &= \int_0^a z_e t dy = \frac{3P}{4a^3} \int_0^a (a^2 + 4ay - 5y^2) dy = \frac{3P}{4a^3} (a^2 y + 4a \frac{y^2}{2} - 5 \frac{y^3}{3}) \Big|_0^a \\ &= \frac{3P}{4a^3} (a^3 + 2a^3 - \frac{5}{3}a^3) = \frac{3P}{4a^3} \cdot \frac{4}{3}a^3 = P \end{aligned}$$

Problem 6.80

6.80 For the angle shape and loading of Sample Prob. 6.5, (a) determine the points where the shearing stress is maximum and the corresponding values of the stress, (b) verify that the points obtained are located on the neutral axis corresponding to the given loading.

Refer to Sample Prob. 6.5.

$$(a) \text{ Along vertical leg: } \tau_c = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$$

$$\frac{d\tau_c}{dy} = \frac{3P}{4ta^3}(4a - 10y) = 0 \quad y = \frac{2}{5}a$$

$$\tau_m = \frac{3P}{4ta^3}[a^2 + (4a)(\frac{2}{5}a) - (5)(\frac{2}{5}a)^2] = \frac{3P}{4ta^3}(\frac{9}{5}a^2) = \frac{27}{20}\frac{P}{ta}$$

$$\text{Along horizontal leg: } \tau_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3}(a^2 - 4az + 3z^2)$$

$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3}(-4a + 6z) = 0 \quad z = \frac{2}{3}a$$

$$\tau_m = \frac{3P}{4ta^3}[a^2 - (4a)(\frac{2}{3}a) + (3)(\frac{2}{3}a)^2] = \frac{3P}{4ta^3}(-\frac{5}{3}a^2) = -\frac{15}{4}\frac{P}{ta}$$

$$\text{At the corner: } y = 0, z = 0 \quad \tau = \frac{3}{4}\frac{P}{ta}$$

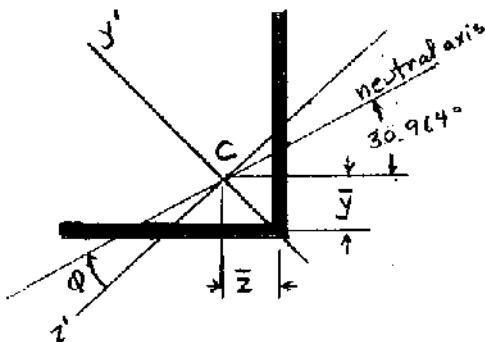
$$(b) I_{y'} = \frac{1}{3}ta^3 \quad I_{z'} = \frac{1}{12}ta^3 \quad \theta = 45^\circ$$

$$\tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{1}{4} \quad \phi = 14.036^\circ$$

$$\theta - \phi = 45 - 14.036 = 30.964^\circ$$

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

$$\bar{z} = \frac{\sum A\bar{z}}{\sum A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$



Neutral axis intersects vertical leg at

$$y = \bar{y} + \bar{z} \tan 30.964^\circ$$

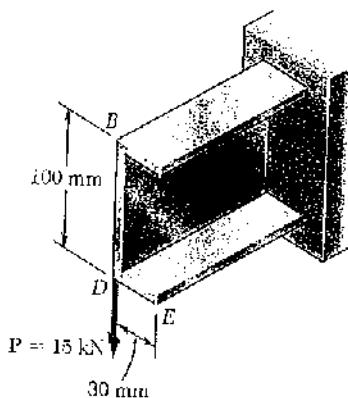
$$= (\frac{1}{4} + \frac{1}{4} \tan 30.964^\circ)a = 0.400a \\ = \frac{2}{5}a$$

Neutral axis intersects horizontal leg at

$$z = \bar{z} + \bar{y} \tan(45^\circ + \phi)$$

$$= (\frac{1}{4} + \frac{1}{4} \tan 59.036^\circ)a = 0.6667a \\ = \frac{2}{3}a$$

Problem 6.81



*6.81 A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load P acts at a point in the midplane of the web of the channel, determine (a) the torque T that would cause the channel to twist in the same way that it does under the load P , (b) the maximum shearing stress in the channel caused by the load P .

Use results of Example 6.06 with $b = 30 \text{ mm}$, $h = 100 \text{ mm}$, and $t = 8 \text{ mm}$.

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} t h^3 (6b + h) = \frac{1}{12} (8)(100)^3 [(6)(30) + 100] \\ = 1.86667 \times 10^6 \text{ mm}^4 = 1.86667 \times 10^{-6} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$

$$(a) T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3}) = 144.64 \text{ N}\cdot\text{m}$$

Stress at neutral axis due to V

$$Q = bt \frac{h}{2} + t(\frac{h}{2})(\frac{h}{4}) \\ = \frac{1}{8} t h (h + 4b) \\ = \frac{1}{8}(8)(100)[100 + (4)(30)] \\ = 22 \times 10^3 \text{ mm}^3 = 22 \times 10^{-6} \text{ m}^3$$

$$t = 8 \times 10^{-3} \text{ m}$$

$$\gamma_v = \frac{VQ}{It} = \frac{(15 \times 10^3)(22 \times 10^{-6})}{(1.86667 \times 10^{-6})(8 \times 10^{-3})} = 22.10 \times 10^6 \text{ Pa} = 22.10 \text{ MPa}$$

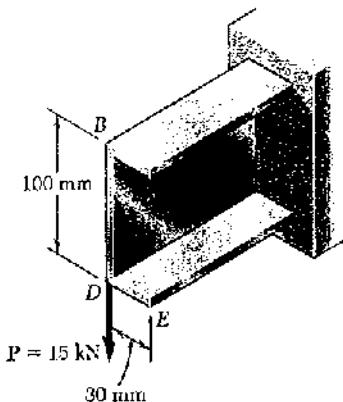
Stress due to T $a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_1 = \frac{1}{3} \left(1 - 0.630 \frac{t}{a} \right) = \frac{1}{3} \left[1 - (0.630) \frac{8}{0.160} \right] = 0.32281$$

$$\gamma_T = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.32281)(0.160)(8 \times 10^{-3})^2} = 43.76 \times 10^6 \text{ Pa} \\ = 43.76 \text{ MPa}$$

$$(b) \text{ By superposition, } \gamma_{\max} = \gamma_v + \gamma_T = 65.9 \text{ MPa}$$

Problem 6.82



*6.82 Solve Prob. 6.81, assuming that a 6-mm-thick plate is bent to form the channel shown.

*6.81 A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load P acts at a point in the midplane of the web of the channel, determine (a) the torque T that would cause the channel to twist in the same way that it does under the load P , (b) the maximum shearing stress in the channel caused by the load P .

Use results of Example 6.06 with $b = 30 \text{ mm}$

$h = 100 \text{ mm}$, and $t = 6 \text{ mm}$

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} t h^3 (6b + h) = \frac{1}{12} (6)(100)^2 [(6)(30) + 100] \\ = 1.400 \times 10^6 \text{ mm}^4 = 1.400 \times 10^{-4} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$

$$(a) T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3}) = 144.64 \text{ N}\cdot\text{m}$$

Stress at neutral axis due to V

$$Q = bt \frac{h}{2} + t \left(\frac{h}{2}\right) \left(\frac{h}{4}\right) \\ = \frac{1}{8} th(h + 4b) \\ = \frac{1}{8} (6)(100)[100 + (4)(30)] \\ = 16.5 \times 10^3 \text{ mm}^3 = 16.5 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau_v = \frac{VQ}{It} = \frac{(15 \times 10^3)(16.5 \times 10^{-6})}{(1.400 \times 10^{-4})(6 \times 10^{-3})} = 29.46 \times 10^6 \text{ Pa} = 29.46 \text{ MPa}$$

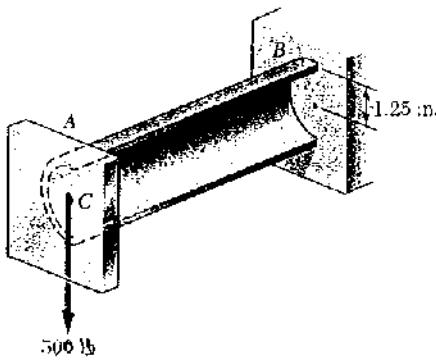
Stress due to T $a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_i = \frac{1}{3} \left(1 - 0.630 \frac{t}{a}\right) = \frac{1}{3} \left[1 - (0.630) \frac{6}{0.160}\right] = 0.32546$$

$$\tau_T = \frac{T}{G_1 a t^2} = \frac{144.64}{(0.32546)(0.160)(6 \times 10^{-3})^2} = 77.16 \times 10^6 \text{ Pa} = 77.16 \text{ MPa}$$

$$(b) \text{ By superposition } \tau_{\max} = \tau_v + \tau_T = 106.6 \text{ MPa}$$

Problem 6.83



*6.83 The cantilever beam AB , consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{1}{8}$ -in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum shearing stress in the beam. (Hint: The shear center O of this cross section was shown in Prob. 6.73 to be located twice as far from its vertical diameter as its centroid C .)

From the solution to Problem 6.73,

$$I = \frac{\pi}{4} a^3 t \quad Q = a^2 t \sin \theta$$

$$e = \frac{4}{\pi} a$$

$$Q_{max} = a^2 t$$

For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$

At each section of the beam, the shearing force V is equal to P . Its line of action passes through the centroid C . The moment arm of its moment about the shear center O is

$$d = e - \bar{x} = \frac{4}{\pi} a - \frac{2}{\pi} a = \frac{2}{\pi} a$$

(a) Equivalent force-couple system at O .

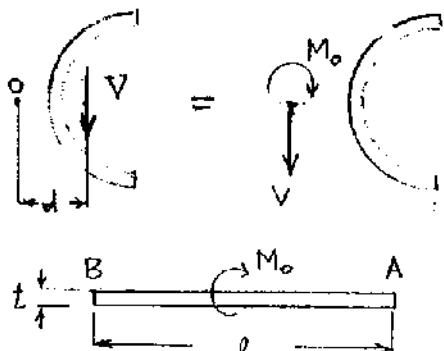
$$V = P \quad M_o = Vd = \frac{2}{\pi} Pa$$

Data: $P = 500 \text{ lb}$ $a = 1.25 \text{ in.}$

$$V = 500 \text{ lb}$$

$$M_o = \left(\frac{2}{\pi}\right)(500)(1.25)$$

$$M_o = 398 \text{ lb-in}$$



(b) Shearing stresses

$$(1) \text{ Due to } V. \quad \tau_v = \frac{V Q_{max}}{I t}$$

$$\tau_v = \frac{(P)(a^2 t)}{\left(\frac{\pi}{4} a^3 t\right)t} = \frac{2P}{\pi a t} = \frac{(2)(500)}{\pi(1.25)(0.375)} = 679 \text{ psi}$$

(2) Due to M_o , the torque.

For a long rectangular section of length l and width t the shearing stress due to torque M_o is

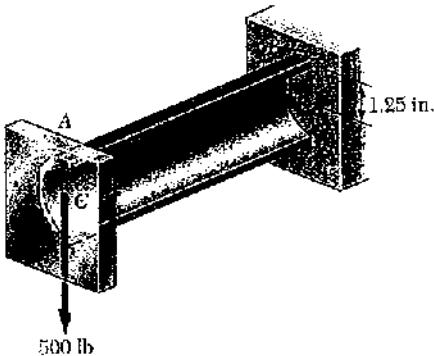
$$\tau_m = \frac{M_o}{C_1 l t^2} \quad \text{where} \quad C_1 = \frac{1}{3} \left(1 - 0.630 \frac{t}{l}\right)$$

Data: $l = \pi a = \pi(1.25) = 3.927 \text{ in.}$ $t = 0.375 \text{ in.}$ $C_1 = 0.31328$

$$\tau_m = \frac{397.9}{(0.31328)(3.927)(0.375)^2} = 2300 \text{ psi}$$

By superposition $\tau = \tau_v + \tau_m = 679 \text{ psi} + 2300 \text{ psi} = 2980 \text{ psi}$

Problem 6.84



*6.84 Solve Prob. 6.83, assuming that the thickness of the beam is reduced to $\frac{1}{4}$ in.

*6.83 The cantilever beam AB , consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{1}{8}$ -in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum shearing stress in the beam. (Hint: The shear center O of this cross section was shown in Prob. 6.73 to be located twice as far from its vertical diameter as its centroid C .)

From the solution to Problem 6.73

$$I = \frac{\pi}{4} a^3 t$$

$$Q = a^2 t \sin \theta$$

$$e = \frac{4}{\pi} a$$

$$Q_{max} = a^2 t$$

For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$

At each section of the beam, the shearing force V is equal to P . Its line of action passes through the centroid C . The moment arm of its moment about the shear center O is

$$d = e - \bar{x} = \frac{4}{\pi} a - \frac{2}{\pi} a = \frac{2}{\pi} a$$

(a) Equivalent force-couple system at O .

$$V = P \quad M_o = Vd = \frac{2}{\pi} Pa$$

$$\text{Data: } P = 500 \text{ lb} \quad a = 1.25 \text{ in.}$$

$$V = 500 \text{ lb} \quad \blacktriangleleft$$

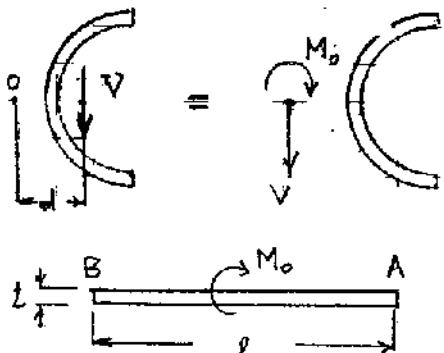
$$M_o = 398 \text{ lb-in.} \quad \blacktriangleleft$$

(b) Shearing stresses

$$(1) \text{ Due to } V. \quad \tau_v = \frac{V Q_{max}}{I t}$$

$$\tau_v = \frac{(P)(a^2 t)}{(\frac{\pi}{4} a^3 t) t} = \frac{2P}{\pi a t} = \frac{(2)(500)}{\pi (1.25)(0.250)} = 1019 \text{ psi}$$

(2) Due to M_o , the torque.



For a long rectangular section of length l and width t the shearing stress due to torque M_o is

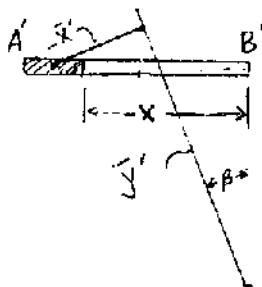
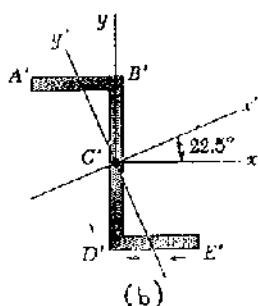
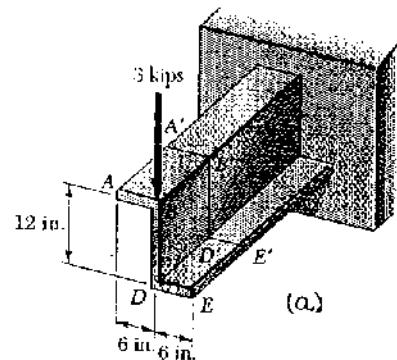
$$\tau_m = \frac{M_o}{c_1 l t^2} \quad \text{where} \quad c_1 = \frac{1}{3} \left(1 - 0.630 \frac{t}{l} \right)$$

$$\text{Data: } l = \pi a = \pi (1.25) = 3.927 \text{ in.} \quad t = 0.375 \text{ in.} \quad c_1 = 0.31996$$

$$\tau_m = \frac{392.9}{(0.31996)(3.927)(0.250)^2} = 5067 \text{ psi}$$

$$\text{By superposition} \quad \tau = \tau_v + \tau_m = 1019 \text{ psi} + 5067 \text{ psi} = 6086 \text{ psi}$$

Problem 6.85



*6.85 The cantilever beam shown consists of a Z shape of $\frac{1}{4}$ -in. thickness. For the given loading, determine the distribution of the shearing stresses along line A'B' in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_x' = 166.3 \text{ in}^4$ and are $I_y' = 13.61 \text{ in}^4$.

$$V = 3 \text{ kips} \quad \beta = 22.5^\circ$$

$$V_{x'} = V \sin \beta \quad V_{y'} = V \cos \beta$$

In upper horizontal leg use coordinate x ($-6 \text{ in} \leq x \leq 0$)

$$A = \frac{1}{4}(6+x) \text{ in.}$$

$$\bar{x} = \frac{1}{2}(-6+x) \text{ in.}$$

$$\bar{y} = 6 \text{ in.}$$

$$\bar{x}' = \bar{x} \cos \beta + \bar{y} \sin \beta$$

$$\bar{y}' = \bar{y} \cos \beta - \bar{x} \sin \beta$$

$$\text{Due to } V_{x'}: \quad \tau_1 = \frac{V_{x'} A \bar{x}'}{I_y t}$$

$$\tau_1 = \frac{(V \sin \beta)(\frac{1}{4})(-6+x)[\frac{1}{2}(-6+x) \cos \beta + 6 \sin \beta]}{(13.61)(\frac{1}{4})}$$

$$= -0.084353(6+x)(-0.47554 + 0.46194x)$$

$$\text{Due to } V_{y'}: \quad \tau_2 = \frac{V_{y'} A \bar{y}'}{I_x t} = \frac{(V \cos \beta)(\frac{1}{4})(6+x)[6 \cos \beta - \frac{1}{2}(-6+x) \sin \beta]}{(166.3)(\frac{1}{4})}$$

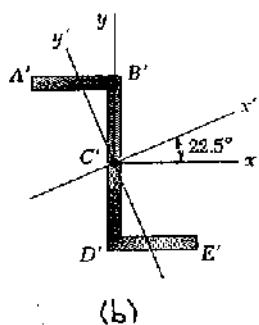
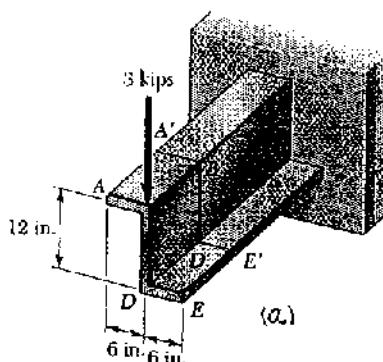
$$= 0.0166665(6+x)[6.69132 - 0.19134x]$$

$$\text{Total: } \tau_1 + \tau_2 = (6+x)[0.07141 + 0.085396x]$$

$x \text{ (in)}$	-6	-5	-4	-3	-2	-1	0
$\tau \text{ (ksi)}$	0	-0.105	-0.140	-0.104	0.003	0.180	0.428

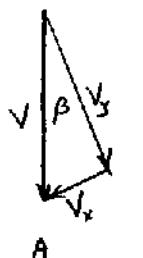
Problem 6.86

*6.86 For the cantilever beam and loading of Prob. 6.85, determine the distribution of the shearing stress along line $B'D'$ in the vertical web of the Z shape.



*6.85 The cantilever beam shown consists of a Z shape of $\frac{1}{4}$ -in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3 \text{ in}^4$ and are $I_{y'} = 13.61 \text{ in}^4$.

$$V = 3 \text{ kips} \quad \beta = 22.5^\circ$$



$$V_{y'} = V \sin \beta \quad V_{x'} = V \cos \beta$$

$$\text{For part } AB': \quad A = \left(\frac{1}{4}\right)(6) = 1.5 \text{ in}^2$$

$$\bar{x} = -3 \text{ in}, \bar{y} = 6 \text{ in.}$$

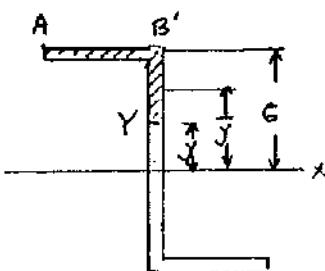
For part $B'D'$

$$A = \frac{1}{4}(6-y)$$

$$\bar{x} = 0 \quad \bar{y} = \frac{1}{2}(6+y)$$

$$x' = x \cos \beta + y \sin \beta$$

$$y' = y \cos \beta - x \sin \beta$$



Due to $V_{x'}$: $\tau_1 = \frac{V_x(A_{AB'}\bar{x}'_{AB'} + A_{B'D'}\bar{x}'_{B'D'})}{I_{y'} t}$

$$\tau_1 = \frac{(V \sin \beta)[(1.5)(-3 \cos \beta + 6 \sin \beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y) \sin \beta]}{(13.61)(\frac{1}{4})}$$

$$= \frac{(V \sin \beta)[-0.7133 + 1.7221 - 0.047885 y^2]}{8.4025} = 0.3404 - 0.01614 y^2$$

Due to $V_{y'}$: $\tau_2 = \frac{V_{y'}(A_{AB'}\bar{y}'_{AB'} + A_{B'D'}\bar{y}'_{B'D'})}{I_{x'} t}$

$$\tau_2 = \frac{(V \cos \beta)[(1.5)(6 \cos \beta + 3 \sin \beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y) \cos \beta]}{(166.3)(\frac{1}{4})}$$

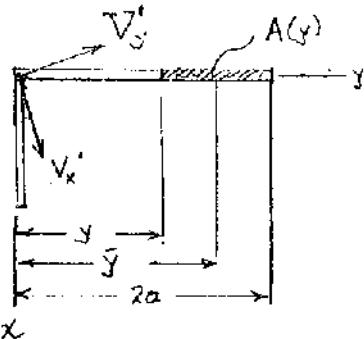
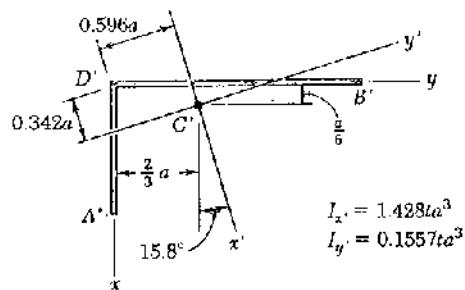
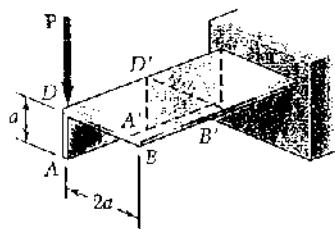
$$= \frac{(V \cos \beta)[10.037 + 4.1575 - 0.11548 y^2]}{(166.3)(\frac{1}{4})} = 0.9463 - 0.00770 y^2$$

Total: $\tau_1 + \tau_2 = 1.2867 - 0.02384 y^2$

y (in)	0	± 2	± 4	± 6
τ (ksi)	1.287	1.191	0.905	0.428

Problem 6.87

*6.87 Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.



$$\beta = 15.8^\circ \quad V_x' = P \cos \beta \quad V_y' = -P \sin \beta$$

$$A(y) = (2a - y)t \quad \bar{y} = \frac{1}{2}(2a + y), \quad \bar{x} = 0$$

Coordinate transformation.

$$y' = (y - \frac{2}{3}a) \cos \beta - (x - \frac{1}{6}a) \sin \beta$$

$$x' = (x - \frac{1}{6}a) \cos \beta + (y - \frac{2}{3}a) \sin \beta$$

In particular,

$$\bar{y}' = (\bar{y} - \frac{2}{3}a) \cos \beta - (\bar{x} - \frac{1}{6}a) \sin \beta$$

$$= (\frac{1}{2}\bar{y} + \frac{1}{3}a) \cos \beta - (+\frac{1}{6}a) \sin \beta$$

$$= 0.48111 \bar{y} + 0.36612 a$$

$$\bar{x}' = (\bar{x} - \frac{1}{6}a) \cos \beta + (\bar{y} - \frac{2}{3}a) \sin \beta$$

$$= (-\frac{1}{6}a) \cos \beta + (\frac{1}{2}\bar{y} + \frac{1}{3}a) \sin \beta$$

$$= 0.13614 \bar{y} - 0.06961 a$$

$$\rightarrow \tau: \quad \frac{V_x' A \bar{x}'}{I_y' t} + \frac{V_y' A \bar{y}'}{I_x' t}$$

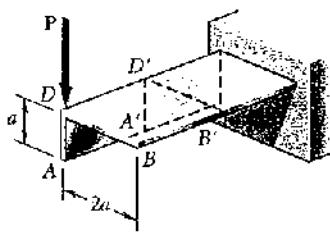
$$= \frac{(P \cos \beta)(2a - y)(t)(0.13614 \bar{y} - 0.06961 a)}{(0.1557 t a^3)(t)}$$

$$+ \frac{(-P \sin \beta)(2a - y)(0.48111 \bar{y} + 0.36612 a)}{(1.428 a^3 t)(t)}$$

$$= \frac{P(2a - y)(0.750 \bar{y} - 0.500 a)}{t a^3}$$

$y(a)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$-\tau \left(\frac{P}{at}\right)$	-1.000	-0.417	0	0.250	0.833	0.250	0

Problem 6.88



*6.88 For the angle shape and loading of Prob. 6.87, determine the distribution of the shearing stresses along line $D'A'$ in the vertical leg.

*6.87 Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.

$$\beta = 15.8^\circ \quad V_x' = P \cos \beta \quad V_y' = -P \sin \beta$$

$$A(x) = (a-x)t \quad \bar{x} = \frac{1}{2}(a+x), \bar{y} = 0$$

Coordinate transformation.

$$y' = (\bar{y} - \frac{2}{3}a) \cos \beta - (\bar{x} - \frac{1}{6}a) \sin \beta$$

$$x' = (\bar{x} - \frac{1}{6}a) \cos \beta + (\bar{y} - \frac{2}{3}a) \sin \beta$$

In particular,

$$y' = (\bar{y} - \frac{2}{3}a) \cos \beta - (\bar{x} - \frac{1}{6}a) \sin \beta$$

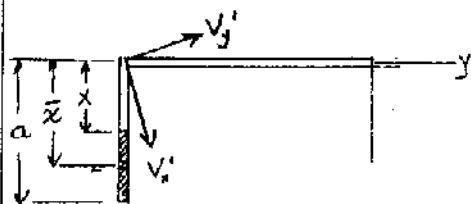
$$= (-\frac{2}{3}a) \cos \beta - (\frac{1}{2}\bar{x} + \frac{1}{3}a) \sin \beta$$

$$= -0.13614\bar{x} - 0.73224a$$

$$x' = (\bar{x} - \frac{1}{6}a) \cos \beta + (\bar{y} - \frac{2}{3}a) \sin \beta$$

$$= (\frac{1}{2}\bar{x} + \frac{1}{3}a) \cos \beta + (-\frac{2}{3}a) \sin \beta$$

$$= 0.48111\bar{x} + 0.13922a$$



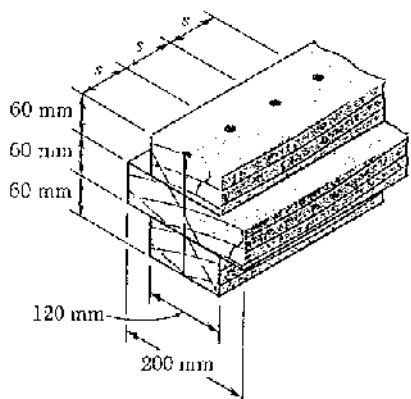
$$\begin{aligned} \tau &= \frac{V_x' A(x) \bar{x}'}{I_y' t} - \frac{V_y' A(x) \bar{y}'}{I_{x'} t} \\ &= \frac{(P \cos \beta)(a-x)(t)(0.48111\bar{x} + 0.13922a)}{(0.1557 ta^3)(t)} \\ &\quad + \frac{(-P \sin \beta)(a-x)(t)(-0.13614\bar{x} - 0.73224a)}{(1.428 ta^3)(t)} \end{aligned}$$

$$= \frac{P(a-x)(3.00x + 1.000a)}{ta^3}$$

$x(a)$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
τ	1.000	1.250	1.333	1.250	1.000	0.583	0

Problem 6.89

6.89 Three boards are nailed together to form the beam shown, which is subjected to a vertical shear. Knowing that the spacing between the nails is $s = 75 \text{ mm}$ and that the allowable shearing force in each nail is 400 N , determine the allowable shear.



Part	$A (\text{mm}^2)$	$d (\text{mm})$	$Ad^3 (\text{10}^6 \text{mm}^4)$	$\bar{I} (\text{10}^6 \text{mm}^4)$
Top plank	7200	60	25.92	2.16
Middle plank	12000	0	0	3.60
Bottom plank	7200	60	25.92	2.16
Σ			51.84	7.92

$$I = \bar{z}Ad^2 + \sum \bar{I} = 59.76 \times 10^4 \text{ mm}^4 = 59.76 \times 10^{-6} \text{ m}^4$$

$$Q = (7200)(60) = 432 \times 10^3 \text{ mm}^3 = 432 \times 10^{-6} \text{ m}^3$$

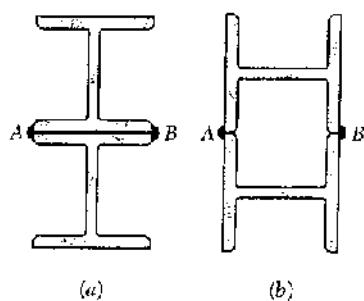
$$q_f = \frac{VQ}{I} \quad F_{max} = q_f s$$

$$g = \frac{F_{grav}}{s} \quad V = \frac{I g}{Q} = \frac{I F_{grav}}{Qs}$$

$$V = \frac{(59.76 \times 10^{-6})(400)}{(432 \times 10^{-6})(75 \times 10^{-3})} = 738 \text{ N}$$

Problem 6.90

6.90 Two W 8 × 31 rolled sections may be welded at A and B in either of the two ways shown in order to form a composite beam. Knowing that for each weld the allowable horizontal shearing force is 3000 lb per inch of weld, determine the maximum allowable vertical shear in the composite beam for each of the two arrangements shown.



Since there are two welds, the allowable shear force on a beam is 6000 lb/in.

$$q_f = \frac{VQ}{I} \quad V = \frac{Tq_f}{Q}$$

For W 8 × 31 rolled steel section

$$A = 9.13 \text{ in}^2, \quad d = 8.00 \text{ in.}, \quad b = 7.995 \text{ in.}$$

$$I_x = 110 \text{ in}^4 \quad I_y = 37.1 \text{ in}^4$$

$$(a) \quad I = 2 \left[110 + (9.13) \left(\frac{8.00}{2} \right)^2 \right] = 512.16 \text{ in}^4$$

$$Q = (9.13) \left(\frac{8.00}{2} \right) = 36.52 \text{ in}^3$$

$$V = \frac{(512.16)(6000)}{36.52} = .841 \times 10^3 \text{ lb} \quad 84.1 \text{ kips}$$

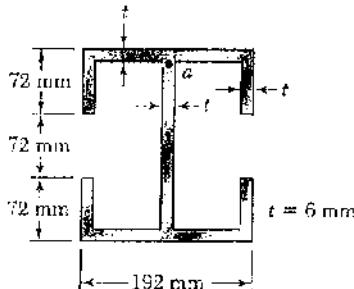
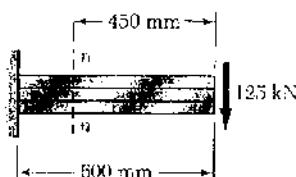
$$(b) \quad I = 2 \left[37.1 + (9.13) \left(\frac{7.995}{2} \right)^2 \right] = 365.99 \text{ in}^4$$

$$Q = (9.13) \left(\frac{7.995}{2} \right) = 36.50 \text{ in}^3$$

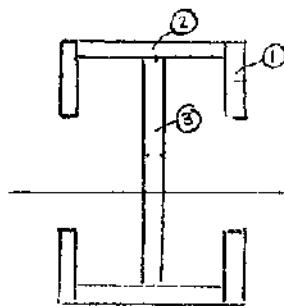
$$V = \frac{(365.99)(6000)}{36.50} = 60.2 \times 10^3 \text{ lb} \quad 60.2 \text{ kips}$$

Problem 6.91

6.91 For the beam and loading shown, consider section *n-n* and determine (a) the largest shearing stress in that section, (b) the shearing stress at point *a*.



At section *n-n* $V = 125 \text{ kN}$



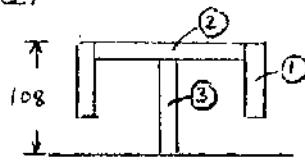
$$I_1 = \frac{1}{12}(6)(72)^3 + (6)(72)(72)^2 = 2.4261 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(180)(6)^3 + (180)(6)(105)^2 = 11.910 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(6)(204)^3 = 4.2448 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + I_3 = 37.77 \times 10^6 \text{ mm}^4 = 37.77 \times 10^{-4} \text{ m}^4$$

(a)



$$\begin{aligned} Q &= 2A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 \\ &= (2)(6)(72)(72) + (180)(6)(105) + (6)(102)(51) \\ &= 206.82 \times 10^3 \text{ mm}^3 = 206.82 \times 10^{-6} \text{ m}^4 \end{aligned}$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(125 \times 10^3)(206.82 \times 10^{-6})}{(37.77 \times 10^{-4})(6 \times 10^{-3})} = 114.1 \times 10^6 \text{ Pa} = 114.1 \text{ MPa} \quad \blacksquare$$

(b) $Q = 2A_1\bar{y}_1 + A_2\bar{y}_2$

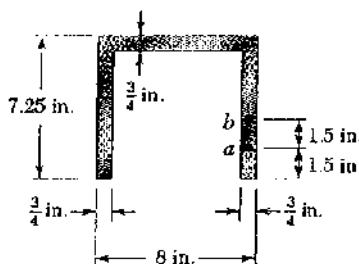
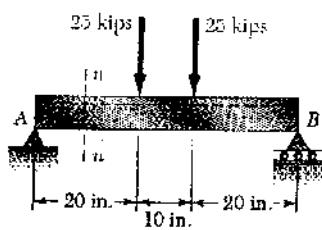
$$= (2)(6)(72) + (180)(6)(105) = 175.61 \times 10^3 \text{ mm}^3 = 175.61 \times 10^{-6} \text{ m}^4$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(125 \times 10^3)(175.61 \times 10^{-6})}{(37.77 \times 10^{-4})(6 \times 10^{-3})} = 96.9 \times 10^6 \text{ Pa} = 96.9 \text{ MPa} \quad \blacksquare$$

Problem 6.92

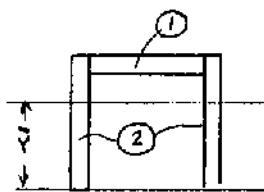
6.92 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (a) point *a*, (b) the shearing stress at point *b*.



$$R_A = R_B = 25 \text{ kips}$$

At section *n-n* $V = 25 \text{ kips}$.

Locate centroid and calculate moment of inertia.

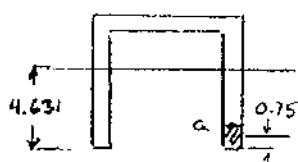


Part	$A (\text{in}^2)$	$\bar{y} (\text{in})$	$A\bar{y} (\text{in}^3)$	$d (\text{in})$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \sum Ad^2 + \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

(a)

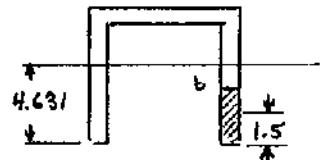


$$Q_a = A\bar{y} = \left(\frac{3}{4}\right)(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3$$

$$t = \frac{3}{4} = 0.75 \text{ in.}$$

$$T_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.42)(0.75)} = 1.745 \text{ ksi}$$

(b)



$$Q_b = A\bar{y} = \left(\frac{3}{4}\right)(3)(4.631 - 1.5) = 7.045 \text{ in}^3$$

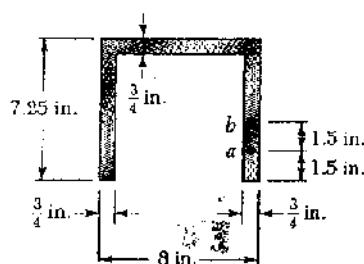
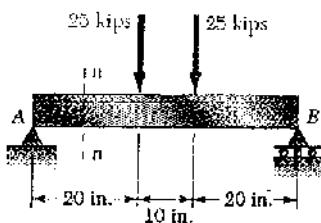
$$t = 0.75 \text{ in.}$$

$$T_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.42)(0.75)} = 2.82 \text{ ksi}$$

Problem 6.93

6.93 For the beam and loading shown in Prob. 6.92, determine the largest shearing stress in section *n-n*.

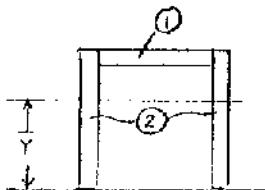
6.92 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (a) point *a*, (b) the shearing stress at point *b*.



$$R_A = R_B = 25 \text{ kips}$$

At section *n-n* $V = 25 \text{ kips}$

Locate centroid and calculate moment of inertia.

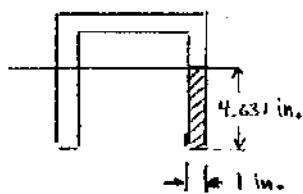


Part	$A (\text{in}^2)$	$\bar{y} (\text{in.})$	$A\bar{y} (\text{in}^3)$	$d (\text{in.})$	$Ad^2 (\text{in}^4)$	$\bar{I} (\text{in}^4)$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \sum Ad^2 + \sum \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



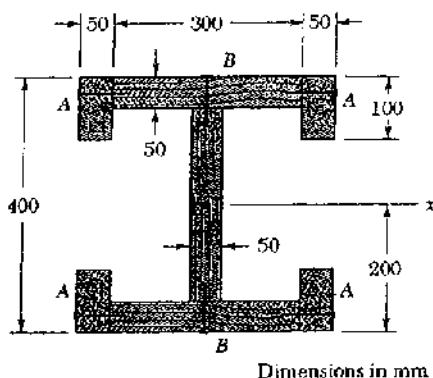
$$Q = A\bar{y} = \left(\frac{3}{4}\right)(4.631) \left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

$$t = \frac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.42)(0.75)} = 3.21 \text{ ksi}$$

Problem 6.94

6.94 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at *A* and every 25 mm at *B*, determine the shearing force in the nails (*a*) at *A*, (*b*) at *B*. (Given: $I_c = 1.504 \times 10^9 \text{ mm}^4$.)



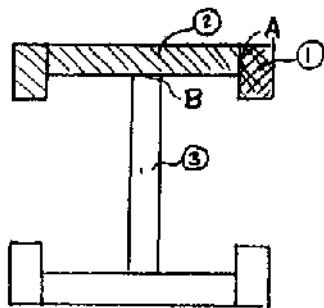
$$I_x = 1.504 \times 10^4 \text{ mm}^4 = 1504 \times 10^{-2} \text{ m}^4$$

$$S_A = 60 \text{ mm} = 0.060 \text{ m}$$

$$S_B = 25 \text{ mm} = 0.025 \text{ m}$$

$$(a) Q_A = Q_1 = A_1 \bar{y}_1 = (50)(100)(150) = 750 \times 10^3 \text{ mm}^3$$

$$= 750 \times 10^{-6} \text{ m}^3$$



$$F_A = \frac{VQ_1 S_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}} = 239 N$$

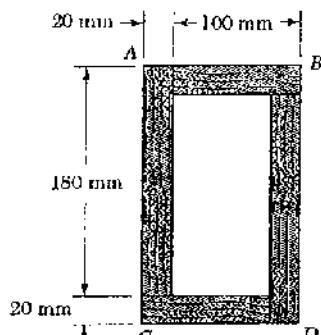
$$(b) \quad Q_2 = A_2 \bar{y}_2 = (300)(50)(175) = 2625 \times 10^3 \text{ mm}^3$$

$$Q_B = 2Q_1 + Q_2 = 4125 \times 10^3 \text{ mm}^3$$

$$= 4125 \times 10^{-6} \text{ m}^3$$

$$F_B = q_B s_B = \frac{V Q_B s_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-2}} = 549 \text{ N}$$

Problem 6.95



6.95 Two 20×100 -mm and two 20×180 -mm boards are glued together as shown to form a 120×200 -mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN , determine the average shearing stress in the glued joint (a) at A, (b) at B.

$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \text{ mm}^4 \\ = 52.693 \times 10^{-6} \text{ m}^4$$

$$(a) Q_A = (80)(20)(90) = 144 \times 10^3 \text{ mm}^3 \\ = 144 \times 10^{-6} \text{ m}^3$$

$$t_A = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$$

$$\tau_A' = \frac{VQ_A}{It_A} = \frac{(3.5 \times 10^3)(144 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} \\ = 239 \times 10^3 \text{ Pa} = 239 \text{ kPa} \quad \blacksquare$$

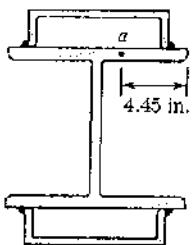
$$(b) Q_B = (120)(20)(90) = 216 \times 10^3 \text{ mm}^3 = 216 \times 10^{-6} \text{ m}^3$$

$$t_B = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$$

$$\tau_B' = \frac{VQ_B}{It_B} = \frac{(3.5 \times 10^3)(216 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} \\ = 359 \times 10^3 \text{ Pa} = 359 \text{ kPa} \quad \blacksquare$$

Problem 6.96

6.96 The composite beam shown is made by welding C 8 × 11.5 rolled-steel channels to the flanges of a W 10 × 54 wide-flange rolled-steel shape. Knowing that the beam is subjected to a vertical shear of 45 kips, determine (a) the horizontal shearing force per inch at each weld, (b) the shearing stress at point *a* of the flange of the wide-flange shape.



For W 10 × 54 rolled steel shape

$$d = 10.09 \text{ in.}, t_f = 0.615 \text{ in.}, I_x = 303 \text{ in.}^4$$

For C 8 × 11.5 rolled steel shape

$$A = 3.38 \text{ in.}^2, b_f = 2.260 \text{ in.}, t_f = 0.390 \text{ in.}$$

$$I_y = 1.32 \text{ in.}^4, \bar{x} = 0.571$$

Compute moment of inertia of the composite beam.

$$I = 303 + 2 \left[1.32 + (3.38) \left(\frac{10.09}{2} + 2.260 - 0.571 \right)^2 \right] \\ = 612.18 \text{ in.}^4$$



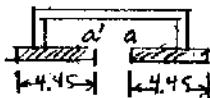
(a) For one channel $\bar{y} = \frac{10.09}{2} + 2.260 - 0.571 = 6.734 \text{ in.}$

$$Q = A\bar{y} = (3.38)(6.734) = 22.761 \text{ in.}^3$$

$$\frac{V}{I} = \frac{VQ}{I} = \frac{(45)(22.761)}{612.18} = 1.673 \text{ kips/in.}$$

Since there are two welds, the horizontal shearing force per inch at each weld is

$$\frac{V}{2} = \frac{1.673}{2} = 0.837 \text{ kips/in.}$$



(b) For cuts at *a* and *a'*,

$$Q = 22.761 + 2 \left[(4.45)(0.615) \left(\frac{10.09}{2} - \frac{0.615}{2} \right) \right] \\ = 48.692 \text{ in.}^3$$

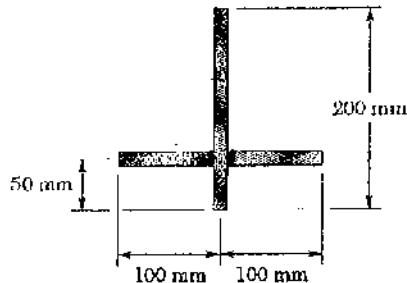
Since there are cuts at points *a* and *a'*,

$$t = 2t_f = (2)(0.615) = 1.230 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(45)(48.692)}{(612.18)(1.230)} = 2.91 \text{ ksi}$$

Problem 6.97

6.97 Three plates, each 12 mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.



Locate neutral axis.

$$\sum A = (12)(200) + (2)(94)(12) = 4656 \text{ mm}^2$$

$$\sum A\bar{y} = (12)(200)(100) + (2)(94)(12)(50) = 352.8 \times 10^3 \text{ mm}^3$$

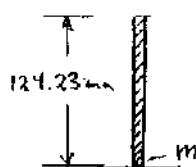
$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = 75.77 \text{ mm}$$

$$I = \frac{1}{12}(12)(200)^3 + (12)(200)(24.23)^2 + 2 \left[\frac{1}{12}(94)(12)^3 + (94)(12)(25.77)^2 \right] = 10.934 \times 10^6 \text{ mm}^4 = 10.934 \times 10^{-6} \text{ m}^4$$

$$Q = (94)(12)(25.77) = 29.07 \times 10^3 \text{ mm}^3 = 29.07 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(100 \times 10^3)(29.07 \times 10^{-6})}{10.934 \times 10^{-6}} = 266 \times 10^3 \text{ N/m} = 266 \text{ kN/m}$$

The maximum shear flow in the cross section occurs at the neutral axis.



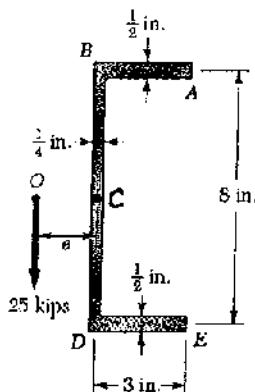
$$Q_m = (12)(124.23)\left(\frac{124.23}{2}\right) = 92.748 \times 10^3 \text{ mm}^3 = 92.748 \times 10^{-6} \text{ m}^3$$

$$f_m = \frac{VQ_m}{I} = \frac{(100 \times 10^3)(92.748 \times 10^{-6})}{(10.934 \times 10^{-6})} = 848 \times 10^3 \text{ N/m}$$

848 kN/m

Problem 6.98

6.98 An extruded beam has the cross section shown. Determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by a 25-kip vertical shearing force applied at O .



$$I = 2 \left[\frac{1}{12} \cdot (3) \left(\frac{1}{2} \right)^3 + (3) \left(\frac{1}{2} \right) (4)^2 \right] + \frac{1}{12} \cdot \left(\frac{1}{4} \right) (8)^3 = 58.729 \text{ in}^4$$

$$\text{Part AB: } A = \frac{1}{2} \times 3, \bar{y} = 4, Q = A\bar{y} = 2x$$

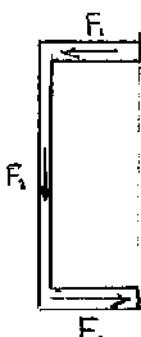
$$\tau = \frac{VQ}{It} = \frac{(25)(2x)}{(58.729)(\frac{1}{2})} = 1.7027x$$

$$\text{Point A: } x = 0 \quad \tau = 0$$

$$\text{Point B: } x = 3 \text{ in.} \quad \tau = 5.11 \text{ ksi}$$

$$F_v = \int \tau dA = \int_0^3 1.7027x \cdot \frac{1}{2} dx = \frac{1.7027}{4} x^2 \Big|_0^3 \\ = \frac{(1.7027)(3)^2}{4} = 3.8311 \text{ kips}$$

$$\sum M_H = \sum M_H: 25e = (F_v)(8) \\ e = \frac{(3.8311)(8)}{25} = 1.226 \text{ in.}$$



$$\text{Part BD: } Q = (2)(3) + (\frac{1}{4}(4-y)(\frac{4+y}{2}))$$

$$= 6 + \frac{1}{8}(16 - y^2) = 8 - \frac{1}{8}y^2$$

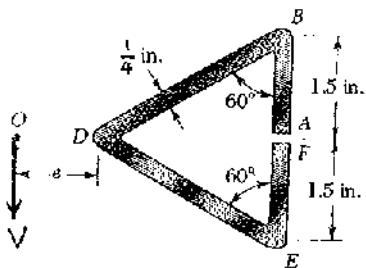
$$\tau = \frac{VQ}{It} = \frac{25(8 - \frac{1}{8}y^2)}{(58.729)(\frac{1}{4})} \\ = 13.622 - 0.2128y^2$$

$$\text{Point B: } y = 4 \text{ in.} \quad \tau = 10.22 \text{ ksi}$$

$$\text{Point C: } y = 0 \quad \tau = 13.62 \text{ ksi}$$

Problem 6.99

6.99 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = \frac{1}{3} \left(\frac{1}{4}\right) (1.5)^3 = 0.28125 \text{ in}^4$$

$$L_{BD} = 3 \text{ in. } A_{BD} = (3)\left(\frac{1}{4}\right) = 0.75 \text{ in}^2$$

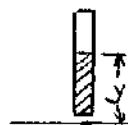
$$I_{BD} = \frac{1}{3} A_{BD} h^2 = \frac{1}{3}(0.75)(1.5)^2 = 0.5625 \text{ in}^4$$

$$I = (2)(0.28125) + (2)(0.5625) = 1.6875 \text{ in}^4$$

$$\text{Part AB: } A = \frac{1}{4}y \quad \bar{y} = \frac{1}{2}y \quad Q = A\bar{y} = \frac{1}{8}y^2$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{8}y^2}{(8)(1.6875)(0.25)} = \frac{V y^2}{3.375}$$

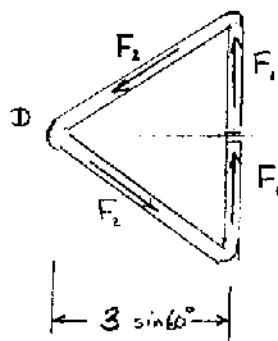
$$F_1 = \int \tau dA = \int_0^{1.5} \frac{V y^2}{3.375} \cdot (0.25 dy) \\ = \frac{(0.25)V}{3.375} \left[\frac{y^3}{3} \right]_0^{1.5} = \frac{(0.25)(1.5)^3}{(3.375)(3)} \\ = 0.08333 V$$



$$\odot M_D = \odot M_D: \quad V e = 2 F_1 (3 \sin 60^\circ)$$

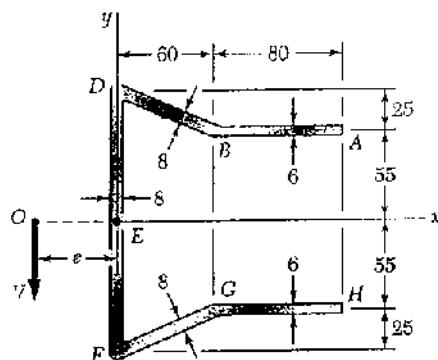
$$V e = (2)(0.08333)V(3 \sin 60^\circ)$$

$$e = (2)(0.08333)(3 \sin 60^\circ) = 0.433 \text{ in.}$$



Problem 6.100

6.100 For an extruded beam having the cross section shown, determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by a vertical 2.5-kN force V applied at O .



$$I_{AB} = I_{GH} = \frac{1}{12}(80)(5)^3 + (80)(6)(55)^3 \\ = 1.45344 \times 10^6 \text{ mm}^4$$

$$I_{DE} = \frac{1}{12}(6)(160)^3 = 2.73067 \times 10^6 \text{ mm}^4$$

Parts BD and FG:

$$l_{BD} = l_{FG} = \sqrt{60^2 + 25^2} = 65 \text{ mm}$$

$$\sin \alpha = \frac{25}{65} \quad \cos \alpha = \frac{60}{65}$$

$$A_{BD} = A_{FG} = (65)(8) = 520 \text{ mm}^2$$

$$\bar{y}_{BD} = -\bar{y}_{FG} = \frac{55+80}{2} = 67.5 \text{ mm}$$

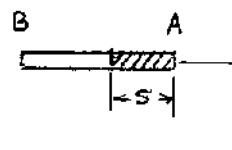
$$\bar{I}_{BD} = \bar{I}_{FG} = \frac{1}{12} \frac{8}{\sin \alpha} (25)^3 = 27.083 \times 10^3 \text{ mm}^4$$

$$I_{BD} = I_{FG} = (520)(67.5)^2 + 27.083 \times 10^3 \\ = 2.39633 \times 10^6 \text{ mm}^4$$

$$I = \sum I = 10.4302 \times 10^6 \text{ mm}^4$$

By symmetry, $F_5 = F_1$ and $F_4 = F_2$.

Calculation of F_1 and F_5 .



$$A(s) = t_{AB} s$$

$$Q(s) = \bar{y}_{AB} t_{AB} s$$

$$q(s) = \frac{V}{I} Q(s) = V \frac{\bar{y}_{AB} t_{AB} s}{I}$$

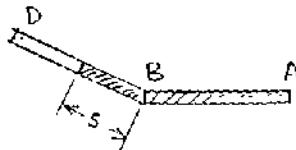
$$F_1 = \int_0^l q(s) ds = V \frac{\bar{y}_{AB} t_{AB}}{I} \int_0^{l_{AB}} s ds = V \frac{\bar{y}_{AB} t_{AB} l_{AB}^2}{2I} \\ = V \frac{(55)(6)(80)^2}{(2)(10.4302 \times 10^6)} = 0.101244 V \leftarrow$$

$$F_5 = 0.101244 V \rightarrow$$

$$\text{At point B: } Q_B = \bar{y}_{AB} t_{AB} l_{AB} = (55)(6)(80) = 26.4 \times 10^3 \text{ mm}^3$$

Continued on next page.

Calculation of F_2 and F_4 .



$$\text{In part BD} \quad A(s) = t_{BD} s$$

$$\bar{y}(s) = \bar{y}_{AB} + \frac{1}{2}s \sin\alpha$$

$$Q(s) = Q_B + A(s) \bar{y}(s) = Q_B + t_{BD} s (\bar{y}_{AB} + \frac{1}{2}s \sin\alpha)$$

$$= Q_B + t_{BD} \bar{y}_{AB} s + \frac{1}{2}s^2 \sin\alpha$$

$$q(s) = \frac{V Q(s)}{I} = \frac{V}{I} (Q_B + t_{BD} \bar{y}_{AB} s + \frac{1}{2}t_{BD} s^2 \sin\alpha)$$

$$F_2 = \int_0^{l_{BD}} = \frac{V}{I} \left\{ Q_B l_{BD} + \frac{1}{2} t_{BD} \bar{y}_{AB} l_{BD}^2 + \frac{1}{6} t_{BD} l_{BD}^3 \sin\alpha \right\}$$

$$= \frac{V}{10,4302 \times 10^6} \left\{ (26.4 \times 10^3)(65) + \frac{1}{2}(8)(55)(65)^2 + \frac{1}{6}(8)(65)^3 \left(\frac{25}{65}\right) \right\}$$

$$= 0.267141 V \sin\alpha$$

$$F_4 = 0.267141 V \cos\alpha$$

$$\text{At point D} \quad Q_D = 26.4 \times 10^3 + (520)(67.5) = 61.5 \times 10^3 \text{ mm}^3$$

F_3 need not be calculated since its moment about E is zero.

$$+\Sigma M_E = +\Sigma M_E \quad \text{by statics.}$$

$$V_e = 55 F_1 + 80 F_2 \cos\alpha + 80 F_4 \cos\alpha + 55 F_5$$

$$= 110 F_1 + 160 F_2 \cos\alpha$$

$$= [(110)(0.101244) + (160)(0.267141) \frac{65}{65}] V = 50.6 V$$

Dividing by V,

$$e = 50.6 \text{ mm}$$

$$\text{At point E: } Q_E = Q_D + A_{DE} \bar{y}_{DE} = 61.5 \times 10^3 + (8)(80)(40) = 87.1 \times 10^3 \text{ mm}^3$$

Continued on next page.

Calculation of shearing stresses.

$$V = 2.5 \times 10^3 \text{ N} \quad I = 10.4302 \times 10^{-6} \text{ m}^4$$

At points A and H: $\tau = 0$

At points B and G: $Q = 26.4 \times 10^{-6} \text{ m}^3$

$$\tau = \frac{VQ}{I} = \frac{(2.5 \times 10^3)(26.4 \times 10^{-6})}{10.4302 \times 10^{-6}} = 6.3278 \times 10^3 \text{ N/m}$$

On the right: $t = 6 \times 10^{-3} \text{ m}$

$$\tau = \frac{\tau}{t} = \frac{6.3278 \times 10^3}{6 \times 10^{-3}} = 1.054 \times 10^6 \text{ Pa} = 1.054 \text{ MPa}$$

On the left: $t = 8 \times 10^{-3} \text{ m}$

$$\tau = \frac{\tau}{t} = \frac{6.3278 \times 10^3}{8 \times 10^{-3}} = 0.791 \times 10^6 \text{ Pa} = 0.791 \text{ MPa}$$

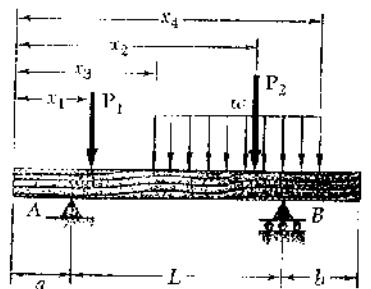
At points D and F: $Q = 51.5 \times 10^{-6} \text{ m}^3$, $t = 8 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{It} = \frac{(2.5 \times 10^3)(51.5 \times 10^{-6})}{(10.4302 \times 10^{-6})(8 \times 10^{-3})} = 1.843 \times 10^6 \text{ Pa} = 1.843 \text{ MPa}$$

At point E: $Q = 87.1 \times 10^{-6} \text{ m}^3$, $t = 8 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{It} = \frac{(2.5 \times 10^3)(87.1 \times 10^{-6})}{(10.4302 \times 10^{-6})(8 \times 10^{-3})} = 2.61 \times 10^6 \text{ Pa} = 2.61 \text{ MPa}$$

PROBLEM 6.C1



6.C1 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values σ_{all} and τ_{all} . Measuring x from end A and using either SI or U.S. customary units, write a computer program to calculate for successive cross sections, from $x = 0$ to $x = L$, and using given increments Δx , the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement, (2) the allowable shearing stress requirement. Use this program to design the beams of uniform cross section of the following problems, assuming $\sigma_{all} = 12 \text{ MPa}$ and $\tau_{all} = 825 \text{ kPa}$, and using the increments indicated: (a) Prob. 5.65 ($\Delta x = 0.1 \text{ m}$), (b) Prob. 5.159 ($\Delta x = 0.2 \text{ m}$).

SOLUTION

See solution of P 5.C2 for the determination of R_A , R_B , $V(x)$, and $M(x)$. We recall that

$$V(x) = R_A S T P A + R_B S T P B - P_1 S T P 1 - P_2 S T P 2 \\ - w(x - x_3) S T P 3 + w(x - x_4) S T P 4$$

$$M(x) = R_A (x - a) S T P A + R_B (x - a - L) S T P B - P_1 (x - x_1) S T P 1 \\ - P_2 (x - x_2) S T P 2 - \frac{1}{2} w (x - x_3)^2 S T P 3 + \frac{1}{2} w (x - x_4)^2 S T P 4$$

where $STPA$, $STPB$, $STP1$, $STP2$, $STP3$, and $STP4$ are step functions defined in P 5.C2.

(1) TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:
If unknown dimension is h :

$$\sigma_{min} = |M| / I_{all} . \text{ From } S = \frac{1}{6} t h^2, \text{ we have } h_0 = h = \sqrt{6S/t}$$

If unknown dimension is t :

$$\sigma_{min} = |M| / I_{all} . \text{ From } S = \frac{1}{6} t h^2, \text{ we have } t_0 = t = 6S/h^2$$

(2) TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378: $\tau_{max} = \frac{3I_A}{2A} = \frac{3M}{2t h}$

If unknown dimension is h : $h_0 = h = \frac{3M}{2t \tau_{all}}$

If unknown dimension is t : $t_0 = t = \frac{3M}{2h \tau_{all}}$

(CONTINUED)

PROBLEM 6.CI CONTINUED

PROGRAM OUTPUTS

Problem 5.65

RA = 2.40 kN RB = 3.00 kN

X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	2.40	0.000	0.00	109.09
0.10	2.40	0.240	54.77	109.09
0.20	2.40	0.480	77.46	109.09
0.30	2.40	0.720	94.87	109.09
0.40	2.40	0.960	109.54	109.09
0.50	2.40	1.200	122.47	109.09
0.60	2.40	1.440	134.16	109.09
0.70	2.40	1.680	144.91	109.09
0.80	0.60	1.920	154.92	27.27
0.90	0.60	1.980	157.32	27.27
1.00	0.60	2.040	159.69	27.27
1.10	0.60	2.100	162.02	27.27
1.20	0.60	2.160	164.32	27.27
1.30	0.60	2.220	166.58	27.27
1.40	0.60	2.280	168.82	27.27
1.50	0.60	2.340	171.03	27.27
1.60	-3.00	2.400	173.21	136.36
1.70	-3.00	2.100	162.02	136.36
1.80	-3.00	1.800	150.00	136.36
1.90	-3.00	1.500	136.93	136.36
2.00	-3.00	1.200	122.47	136.36
2.10	-3.00	0.900	106.07	136.36
2.20	-3.00	0.600	86.60	136.36
2.30	-3.00	0.300	61.24	136.36
2.40	0.00	0.000	0.05	0.00

Problem 5.159

RA = 25.00 kN RB = 25.00 kN

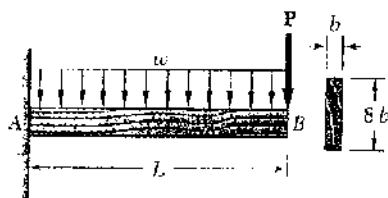
X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	25.00	0.000	0.00	378.79
0.20	23.00	4.800	141.42	348.48
0.40	21.00	9.200	195.79	318.18
0.60	19.00	13.200	234.52	287.88
0.80	17.00	16.800	264.58	257.58
1.00	15.00	20.000	288.68	227.27
1.20	13.00	22.800	308.22	196.97
1.40	11.00	25.200	324.04	166.67
1.60	9.00	27.200	336.65	136.36
1.80	7.00	28.800	346.41	106.06
2.00	5.00	30.000	353.55	75.76
2.20	3.00	30.800	358.24	45.45
2.40	1.00	31.200	360.56	15.15
2.60	-1.00	31.200	360.56	15.15
2.80	-3.00	30.800	358.24	45.45
3.00	-5.00	30.000	353.55	75.76
3.20	-7.00	28.800	346.41	106.06
3.40	-9.00	27.200	336.65	136.36
3.60	-11.00	25.200	324.04	166.67
3.80	-13.00	22.800	308.22	196.97
4.00	-15.00	20.000	288.68	227.27
4.20	-17.00	16.800	264.58	257.58
4.40	-19.00	13.200	234.52	287.88
4.60	-21.00	9.200	195.79	318.18
4.80	-23.00	4.800	141.42	348.48
5.00	0.00	0.000	0.00	0.00

The smallest allowable value of h is the largest of the values shown in the last two columns.

For Prob. 5.65, $h = h_G = 173.2$ mm.

For Prob. 5.159 $h = h_G = 379$ mm

PROBLEM 6.C2



6.C2 A cantilever timber beam AB of length L and of uniform rectangular section shown supports a concentrated load P at its free end and a uniformly distributed load w along its entire length. Write a computer program to determine the length L and the width b of the beam for which both the maximum normal stress and the maximum shearing stress in the beam reach their largest allowable values. Assuming $\sigma_{all} = 1.8$ ksi and $\tau_{all} = 120$ psi, use this program to determine the dimensions L and b when (a) $P = 1000$ lb and $w = 0$, (b) $P = 0$ and $w = 12.5$ lb/in., (c) $P = 500$ lb and $w = 12.5$ lb/in.

SOLUTION

Both the maximum shear and the maximum bending moment occur at A. We have

$$V_A = P + wL$$

$$M_A = PL + \frac{1}{2} wL^2$$

TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

$$\sigma_{all} = \frac{M_A}{S} = \frac{M_A}{\frac{1}{6} b(8b)^2} = \frac{3M_A}{32b^3} \quad b_0 = b = \left[\frac{3}{32} \frac{M_A}{\sigma_{all}} \right]^{1/3}$$

TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378:

$$\tau_{all} = \frac{3V}{2A} = \frac{3}{2} \frac{V_A}{b(8b)} = \frac{3V_A}{16b^2} \quad b_C = b = \left[\frac{3}{16} \frac{V_A}{\tau_{all}} \right]^{1/2}$$

PROGRAM

For $L=0$, $V_A = P$ and $b_C > 0$, while $M_A = 0$ and $b_0 = 0$.

Starting with $L=0$ and using increments $\Delta L=0.001$ in., we increase L until b_0 and b_C become equal. We then print L and b .

PROGRAM OUTPUTS

For $P = 1000$ lb, $w = 0.0$ lb/in.

Increment = 0.0010 in.

$L = 37.5$ in., $b = 1.250$ in.

For $P = 0$ lb, $w = 12.5$ lb/in.

Increment = 0.0010 in.

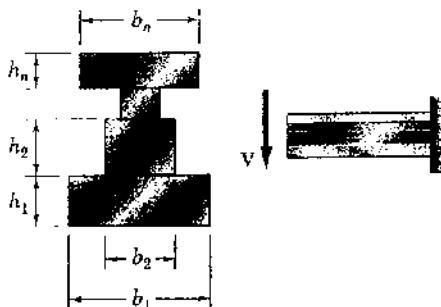
$L = 70.3$ in., $b = 1.172$ in.

For $P = 500$ lb, $w = 12.5$ lb/in.

Increment = 0.0010 in.

$L = 59.8$ in., $b = 1.396$ in.

PROBLEM 6.C3



6.C3 A beam having the cross section shown is subjected to a vertical shear V . Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (a) Prob. 6.10, (b) Prob. 6.12, (c) Prob. 6.21.

SOLUTION

1. Enter V and the number n of rectangles.
2. For $i = 1$ to n , enter the dimensions b_i and h_i .
3. Determine the area $A_i = b_i h_i$ of each rectangle.
4. Determine the elevation of the centroid of each rectangle:

$$\bar{y}_i = \frac{\sum_{k=1}^i h_k}{k} - 0.5h_i$$

and the elevation \bar{y} of the centroid of the entire section:

$$\bar{y} = \frac{(\sum_i A_i \bar{y}_i)}{(\sum_i A_i)}$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[\frac{1}{12} b_i h_i^3 + A_i (\bar{y}_i - \bar{y})^2 \right]$$

6. For each surface separating two rectangles i and $i+1$, determine Q_i of the area below that surface:

$$Q_i = \sum_{k=1}^i A_k (\bar{y}_k - \bar{y})$$

7. Select for t_i the smaller of b_i and b_{i+1} .

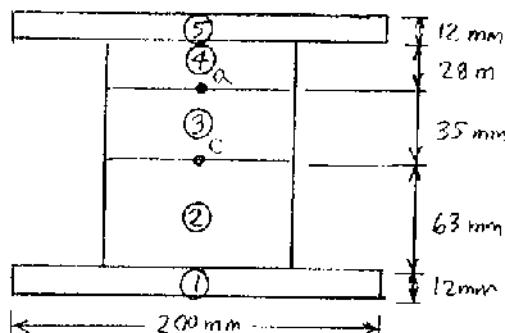
The shearing stress on the surface between the rectangles i and $i+1$ is

$$\tau_i = \frac{V Q_i}{I t_i}$$

(CONTINUED)

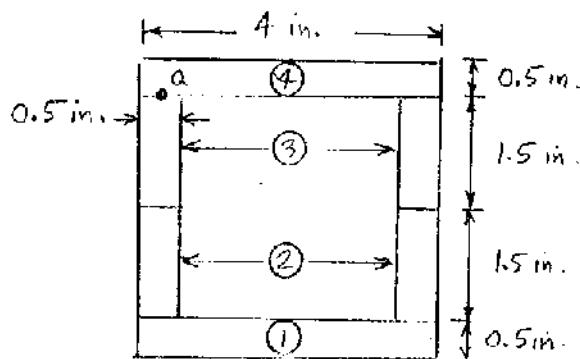
PROBLEM 6.C3 CONTINUED

PROGRAM OUTPUTS



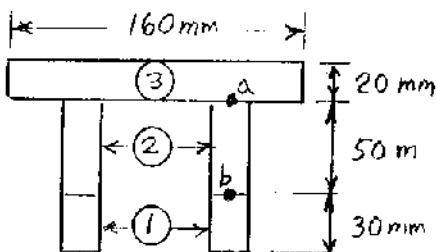
Problem 6.10

Shearing force = 10 kN
 YBAR = 75.000 mm above base
 $I = 39.560 \times 10^{-6} \text{ mm}^4$
 Between elements 1 and 2:
 $\tau_{AU} = 418.39 \text{ kPa}$
 Between elements 2 and 3:
 $\tau_{AU} = 919.78 \text{ kPa}$ $\blacktriangleleft (a)$
 Between elements 3 and 4:
 $\tau_{AU} = 765.03 \text{ kPa}$ $\blacktriangleleft (b)$
 Between elements 4 and 5:
 $\tau_{AU} = 418.39 \text{ kPa}$



Problem 6.12

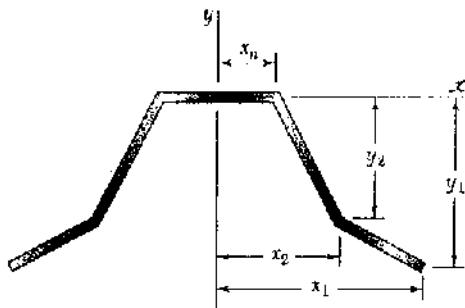
Shearing force = 10 kips
 YBAR = 2.000 in.
 $I = 14.58 \text{ in}^4$
 Between elements 1 and 2:
 $\tau_{AU} = 2.400 \text{ ksi}$
 Between elements 2 and 3:
 $\tau_{AU} = 3.171 \text{ ksi}$ $\blacktriangleleft (a)$
 Between elements 3 and 4:
 $\tau_{AU} = 2.400 \text{ ksi}$ $\blacktriangleleft (b)$



Problem 6.21

Shearing force = 90 kN
 YBAR = 65.000 mm
 $I = 58.133 \times 10^{-6} \text{ mm}^4$
 Between elements 1 and 2:
 $\tau_{AU} = 23.222 \text{ MPa}$ $\blacktriangleleft (b)$
 Between elements 2 and 3:
 $\tau_{AU} = 30.963 \text{ MPa}$ $\blacktriangleleft (a)$

PROBLEM 6.C4



6.C4 A plate of uniform thickness t is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the distribution of shearing stresses caused by a vertical shear V . Use this program (a) to solve Prob. 6.47, (b) to find the shearing stress at a point E for the shape and load of Prob. 6.50, assuming a thickness $t = \frac{1}{4}$ in.

SOLUTION

For each element on the right-hand side, we compute (for $i=1$ to n):

$$\text{Length of element} = L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$$

$$\text{Area of element} = A_i = t L_i \quad \text{where } t = \frac{1}{4} \text{ in.}$$

$$\text{Distance from } x \text{ axis to centroid of element} = \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$$

$$\text{Distance from } x \text{ axis to centroid of section:}$$

$$\bar{y} = (\sum A_i \bar{y}_i) / \sum A_i$$

Note that $y_n = 0$ and that $x_{n+1} = y_{n+1} = 0$

Moment of inertia of section about centroidal axis:

$$I = 2 \sum A_i \left[\frac{1}{12} (y_i - \bar{y}_{i+1})^2 + (\bar{y}_i - \bar{y})^2 \right]$$

Computation of Q at point P where stress is desired

$Q = \sum A_i (\bar{y}_i - \bar{y})$ where sum extends to the areas located between one end of section and point P .

Shearing stress at P :

$$\tau = \frac{VQ}{It}$$

NOTE: τ_{\max} occurs on neutral axis, i.e., for $y_P = \bar{y}$

PROGRAM OUTPUTS

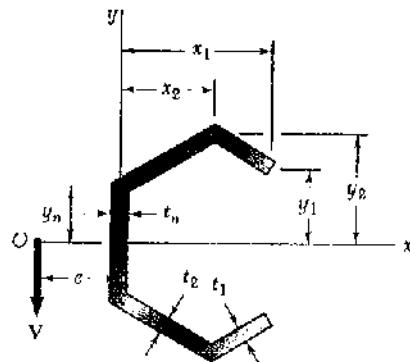
Part (a):

$$\begin{aligned} I &= 0.5333 \text{ in}^4 \\ \text{Tau}_{\max} &= 2.02 \text{ ksi} \\ \text{Tau}_B &= 1.800 \text{ ksi} \end{aligned}$$

Part (b):

$$\begin{aligned} I &= 22.27 \text{ in}^4 \\ \text{Tau}_E &= 194.0 \text{ psi} \end{aligned}$$

PROBLEM 6.C5



6.C5 The cross section of an extruded beam is symmetric with respect to the x axis and consists of several straight segments as shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine (a) the location of the shear center O , (b) the distribution of shearing stresses caused by a vertical force applied at O . Use this program to solve Probs. 6.66 and 6.70.

SOLUTION

SHIPS CENTER IS SYMMETRIC WITH x AXIS,
COMPUTATION WILL BE DONE FOR TOP
HALF.

FOR $L = 1$ TO $n+1$ (NOTE: $n+1$ IS THE ORIGIN)
ENTER t_i , x_i , y_i

COMPUTE LENGTH OF EACH SEGMENT

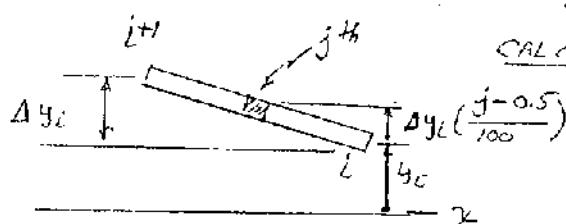
FOR $L = 1$ TO n

$$\Delta x_L = x_{L+1} - x_L$$

$$\Delta y_L = y_{L+1} - y_L$$

$$L = (\Delta x_L^2 + \Delta y_L^2)^{1/2}$$

CALCULATE MOMENT OF INERTIA I_x



CONSIDER EACH SEGMENT AS MADE
OF 100 EQUAL PARTS

FOR $L = 1$ TO n

$$\text{AREA} = t_i L / 100$$

FOR $j = 1$ TO 100

$$y = y_i + \Delta y_i (j - 0.5) / 100$$

$$\Delta I = (\text{AREA}) y^2$$

$$I_x = I_x + \Delta I$$

SINCE ONLY TOP HALF WAS USED

$$I_x = 2 I_x$$

CALCULATE SHEARING STRESS AT ENDS OF
SEGMENTS AND SHEAR FORCES IN SEGMENTS

FOR $L = 1$ TO n

$$\text{AREA} = L t_i / 100, \quad T_{new} = T_{next}$$

FOR $j = 1$ TO 100

$$y = y_i + \Delta y_i (j - 0.5) / 100$$

$$\Delta Q = (\text{AREA}) y$$

$$T_{old} = T_{new}, \quad Q = Q + \Delta Q$$

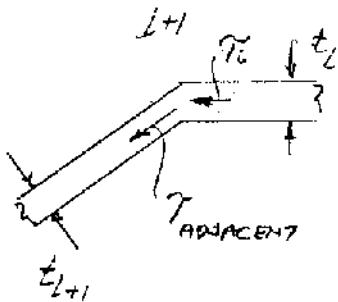
$$T_{new} = VQ / I_x t_i$$

$$\tau_{ave} = 0.5(T_{old} + T_{new})$$

$$\tau = \tau + \tau_{ave}$$

CONTINUED

PROBLEM 6.C5 - CONTINUED



$$\text{Force}_i = T(\text{AREA})$$

$$T_i = VQ/I_x t_i$$

$$(T_{\text{ADJACENT}})_L = VQ/I_y t_{i+1}$$

$$Q_L = Q$$

$$T_{\text{max}} = (T_{\text{ADJACENT}})_L$$

LOCATION OF SHEAR CENTER
CALCULATE MOMENT OF SHEAR
FORCES ABOUT ORIGIN

For $L = 1 \text{ to } n$

$$(F_x)_L = \text{Force}_L (\Delta x_L)/L_L$$

$$(F_y)_L = \text{Force}_L (\Delta y_L)/L_L$$

$$\text{MOMENT}_L = -(F_x)_L y_L + (F_y)_L x_L$$

$$\text{MOMENT} = \text{MOMENT}_1 + \text{MOMENT}_2 + \dots + \text{MOMENT}_n$$

For whole section $\text{MOMENT} = 2(\text{MOMENT})$
 SHEAR CENTER IS AT

$$e = \text{MOMENT}/V$$

PROGRAM OUTPUT

Prob. 6.66

	T (K) in.	X (K) in.	Y (K) in.	L (K) in.
1	.13	4.00	.00	3.00
2	.13	4.00	3.00	4.00
3	.13	.00	3.00	3.00
4	.13	.00	.00	
Moment of inertia:	$I_x = 13.4999 \text{ in}^{**4}$		Shear = 2.750	
kips				

Junction of segments	Q in ^{**3}	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	.56	.92	.92	.12
2 and 3	2.06	3.36	3.36	1.07
3 and 4	2.63	4.28	4.28	1.49

Moment of shear forces about origin: $M = 7.338 \text{ kip-in.}$
 + counterclockwise

Distance from origin to shear center: $e = 2.6684 \text{ in.}$

CONTINUED

PROBLEM 6.C5 - PROGRAM PRINTOUTS CONTINUED

Prob. 6.70

	T (K) mm	X (K) mm	Y (K) mm	L (K) mm
1	6.00	60.62	.00	70.00
2	6.00	.00	35.00	35.00
3	6.00	.00	.00	

Moment of inertia: $I_x = 514487.$ mm **4 Shear = 1000.000 N

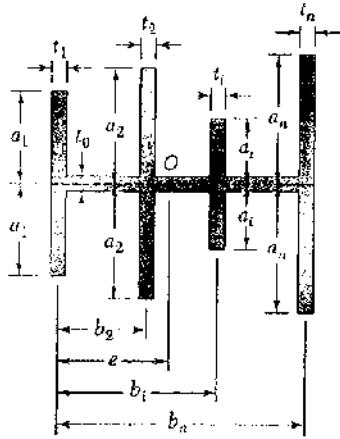
Junction of segments	Q mm **3	Tau Before MPa	Tau After MPa	Force in segment kN
-------------------------	------------------	----------------------	---------------------	---------------------------

1 and 2	7350.00	2.38	2.38	335.01
2 and 3	11025.00	3.57	3.57	666.27

Moment of shear forces about origin: $M = 20.309$ N·m
+ counterclockwise

Distance from origin to shear center: $e = 20.309$ mm

PROBLEM 6.C6



6.C6 A thin-walled beam has the cross section shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the location of the shear center O of the cross section. Use the program to solve Prob. 6.75.

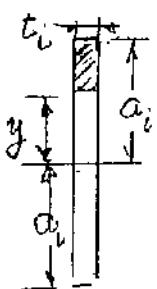
SOLUTION

Distribution of shearing stresses in element i

Let V = shear in cross section

\bar{I} = Centroidal moment of inertia of section

We have for shaded area



$$Q = A \bar{y} = t_i (a_i - y) \frac{a_i + y}{2}$$

$$= \frac{1}{2} t_i (a_i^2 - y^2)$$

$$\bar{c} = \frac{QV}{\bar{I} t_i} = \frac{V}{2\bar{I}} (a_i^2 - y^2)$$

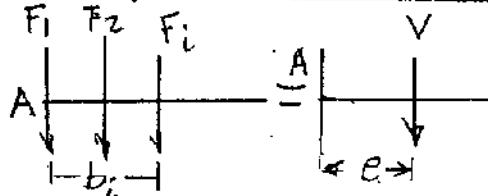
Force exerted on element i



$$F_i = \int_{-a_i}^{a_i} \bar{c} (t_i dy) = \frac{V t_i}{2\bar{I}} \int_{-a_i}^{a_i} (a_i^2 - y^2) dy$$

$$= \frac{V t_i}{\bar{I}} \int_0^{a_i} (a_i^2 - y^2) dy = \frac{V t_i}{\bar{I}} \left(a_i^3 - \frac{1}{3} a_i^3 \right) = \frac{2}{3} \frac{V}{\bar{I}} t_i a_i^3$$

The system of the forces F_i must be equivalent to V at shear center.



$$\sum F_i = \sum F: \frac{2}{3} \frac{V}{\bar{I}} \sum t_i a_i^3 = V \quad (1)$$

$$\sum M_A = \sum M_A: \frac{2}{3} \frac{V}{\bar{I}} \sum t_i a_i^3 b_i = eV \quad (2)$$

$$\text{Divide (2) by (1): } e = \frac{\sum t_i a_i^3 b_i}{\sum t_i a_i^3}$$



PROGRAM OUTPUT:

Problem 6.75

For element 1:

$t = 0.75$ in., $a = 4$ in., $b = 0$

For element 2:

$t = 0.75$ in., $a = 3$ in., $b = 8$ in.

Answer: $e = 2.37$ in.



PROBLEM SOLUTIONS

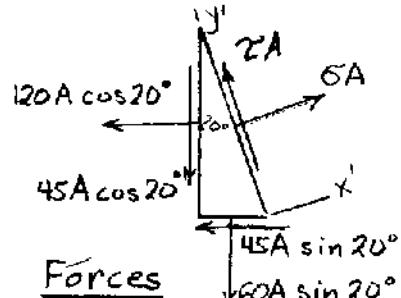
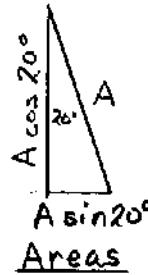
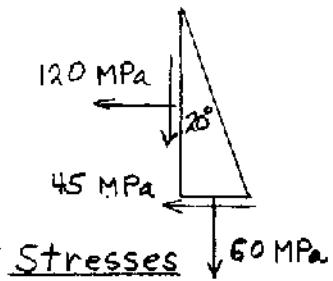
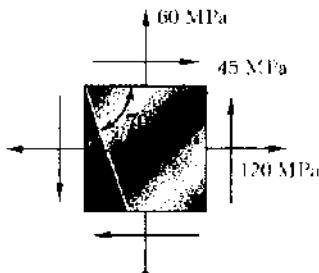
by

Dean P. Updike

Chapter 7

Problem 7.1

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



$$\rightarrow \sum F = 0$$

$$5A - 120A \cos 20^\circ \cos 20^\circ - 45A \cos 20^\circ \sin 20^\circ - 45A \sin 20^\circ \cos 20^\circ - 60A \sin 20^\circ \sin 20^\circ = 0$$

$$\sigma = 120 \cos^2 20^\circ + 45 \cos 20^\circ \sin 20^\circ + 45 \sin 20^\circ \cos 20^\circ + 60 \sin^2 20^\circ = 14.19 \text{ MPa} \quad \blacktriangleleft$$

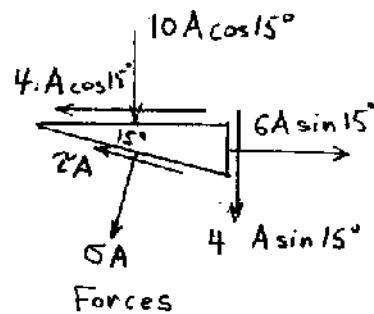
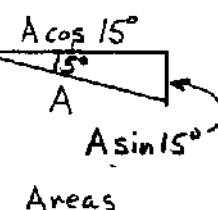
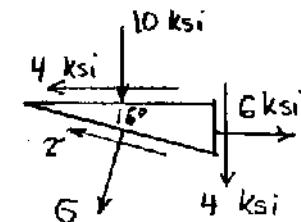
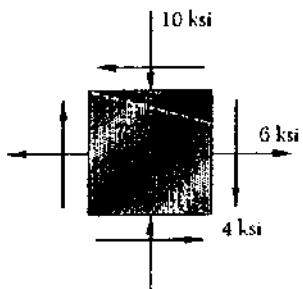
$$\uparrow \sum F = 0$$

$$2A + 120A \cos 20^\circ \sin 20^\circ - 45A \cos 20^\circ \cos 20^\circ + 45A \sin 20^\circ \sin 20^\circ - 60A \sin 20^\circ \cos 20^\circ = 0$$

$$\tau = -120 \cos 20^\circ \sin 20^\circ + 45(\cos^2 20^\circ - \sin^2 20^\circ) + 60 \sin 20^\circ \cos 20^\circ = 15.19 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.2

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



$$\rightarrow \sum F = 0$$

$$5A + 4A \cos 15^\circ \sin 15^\circ + 10A \cos 15^\circ \cos 15^\circ - 6A \sin 15^\circ \sin 15^\circ + 4A \sin 15^\circ \cos 15^\circ = 0$$

$$\sigma = -4 \cos 15^\circ \sin 15^\circ - 10 \cos^2 15^\circ + 6 \sin^2 15^\circ - 4 \sin 15^\circ \cos 15^\circ = 10.93 \text{ ksi} \quad \blacktriangleleft$$

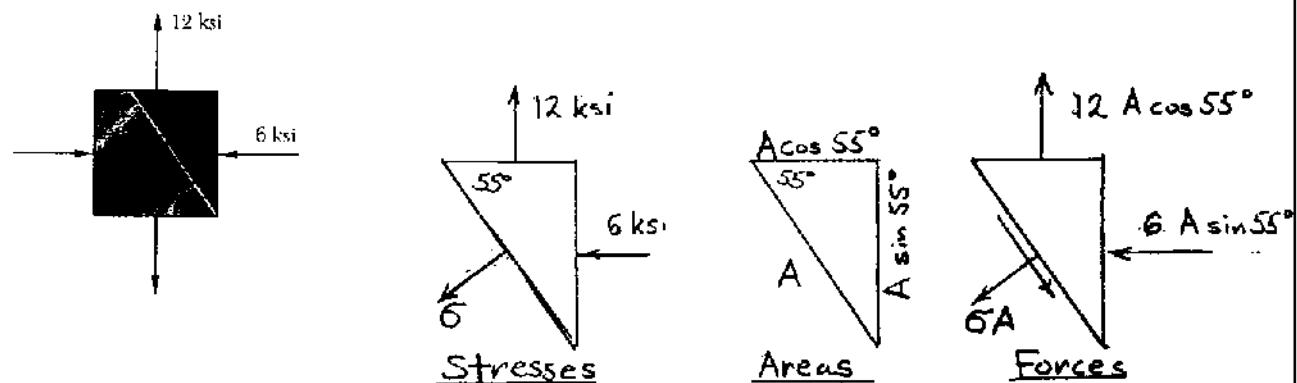
$$\uparrow \sum F = 0$$

$$2A + 4A \cos 15^\circ \cos 15^\circ - 10A \cos 15^\circ \sin 15^\circ - 6A \sin 15^\circ \cos 15^\circ - 4A \sin 15^\circ \sin 15^\circ = 0$$

$$\tau = -4(\cos^2 15^\circ - \sin^2 15^\circ) + (10 + 6) \cos 15^\circ \sin 15^\circ = 0.536 \text{ ksi} \quad \blacktriangleleft$$

Problem 7.3

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



$$+\nabla \sum F = 0$$

$$\sigma A - 12 A \cos 55^\circ \cos 55^\circ + 6 A \sin 55^\circ \sin 55^\circ = 0$$

$$\sigma = 12 \cos^2 55^\circ - 6 \sin^2 55^\circ = -0.078 \text{ ksi}$$

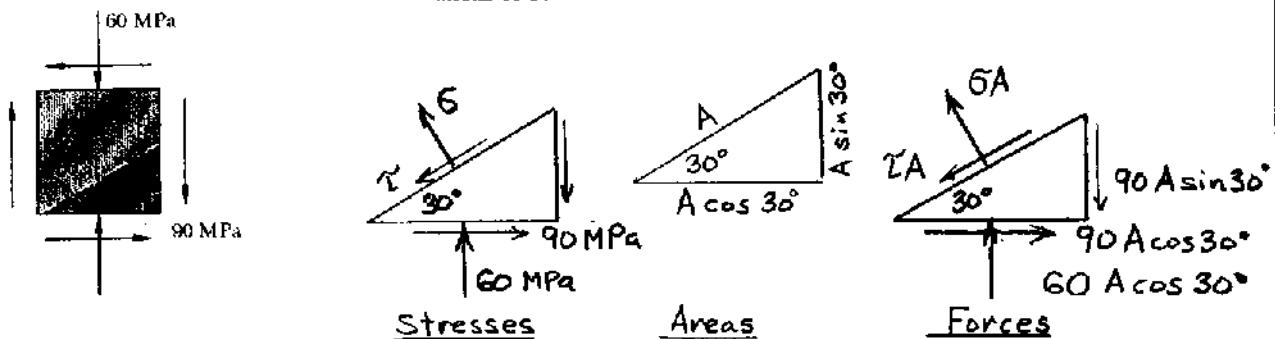
$$+\nabla \sum F = 0$$

$$\tau A - 12 A \cos 55^\circ \sin 55^\circ - 6 A \sin 55^\circ \cos 55^\circ$$

$$\tau = 18 \cos 55^\circ \sin 55^\circ = 8.46 \text{ ksi}$$

Problem 7.4

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



$$+\nabla \sum F = 0$$

$$6A - 90 A \sin 30^\circ \cos 30^\circ - 90 A \cos 30^\circ \sin 30^\circ + 60 A \cos 30^\circ \cos 30^\circ = 0$$

$$\sigma = 180 \sin 30^\circ \cos 30^\circ - 60 \cos^2 30^\circ = 32.9 \text{ MPa}$$

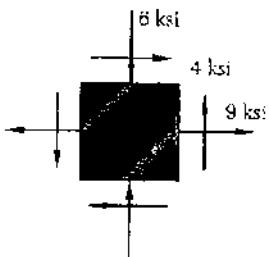
$$+\nabla \sum F = 0$$

$$\tau A + 90 A \sin 30^\circ \sin 30^\circ - 90 A \cos 30^\circ \cos 30^\circ - 60 A \cos 30^\circ \sin 30^\circ = 0$$

$$\tau = 90 (\cos^2 30^\circ - \sin^2 30^\circ) + 60 \cos 30^\circ \sin 30^\circ = 71.0 \text{ MPa}$$

Problem 7.5

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



$$\sigma_x = 9 \text{ ksi} \quad \sigma_y = -6 \text{ ksi} \quad \tau_{xy} = 4 \text{ ksi}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(4)}{9 + 6} = 0.5333$$

$$2\theta_p = 28.07^\circ \quad \theta_p = 14.04^\circ, 104.04^\circ$$

$$(b) \sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{9 - 6}{2} \pm \sqrt{\left(\frac{9 + 6}{2}\right)^2 + (8)^2}$$

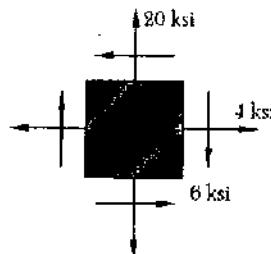
$$= 1.5 \pm 8.5 \text{ ksi}$$

$$\sigma_{max} = 10 \text{ ksi}$$

$$\sigma_{min} = -7 \text{ ksi}$$

Problem 7.6

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



$$\sigma_x = 4 \text{ ksi} \quad \sigma_y = 20 \text{ ksi} \quad \tau_{xy} = -6 \text{ ksi}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-6)}{4 - 20} = 0.750$$

$$2\theta_p = 36.87^\circ \quad \theta_p = 18.43^\circ, 108.43^\circ$$

$$(b) \sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4 + 20}{2} \pm \sqrt{\left(\frac{4 - 20}{2}\right)^2 + (-6)^2}$$

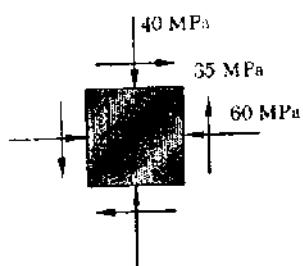
$$= 12 \pm 10 \text{ ksi}$$

$$\sigma_{max} = 22 \text{ ksi}$$

$$\sigma_{min} = 2 \text{ ksi}$$

Problem 7.7

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35)}{-60 + 40} = -3.50$$

$$2\theta_p = -74.05^\circ \quad \theta_p = -37.03^\circ, 52.97^\circ$$

$$(b) \sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-60 - 40}{2} \pm \sqrt{\left(\frac{-60 + 40}{2}\right)^2 + (35)^2}$$

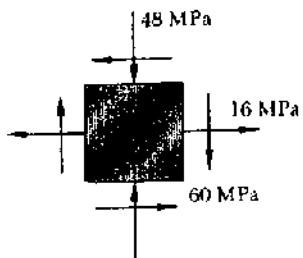
$$= -50 \pm 36.4 \text{ MPa}$$

$$\sigma_{max} = -13.60 \text{ MPa}$$

$$\sigma_{min} = -86.4 \text{ MPa}$$

Problem 7.8

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



$$\sigma_x = 16 \text{ MPa} \quad \sigma_y = -48 \text{ MPa} \quad \tau_{xy} = -60 \text{ MPa}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{16 + 48} = -1.875$$

$$2\theta_p = -61.93^\circ \quad \theta_p = -30.96^\circ, 59.04^\circ$$

$$(b) \sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16 - 48}{2} \pm \sqrt{\left(\frac{16 + 48}{2}\right)^2 + (-60)^2}$$

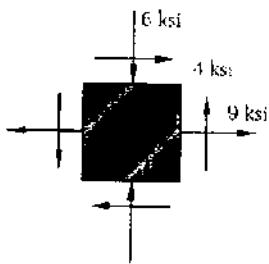
$$= -16 \pm 68$$

$$\sigma_{max} = 52 \text{ MPa}$$

$$\sigma_{min} = -84 \text{ MPa}$$

Problem 7.9

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$\bar{\sigma}_x = 9 \text{ ksi}$$

$$\bar{\sigma}_y = -6 \text{ ksi}$$

$$\tau_{xy} = 4 \text{ ksi}$$

$$(a) \tan 2\theta_s = -\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2\tau_{xy}} = -\frac{9 + 6}{(2)(4)} = -1.875$$

$$2\theta_s = -61.93^\circ \quad \theta_s = -30.96^\circ, 59.04^\circ$$

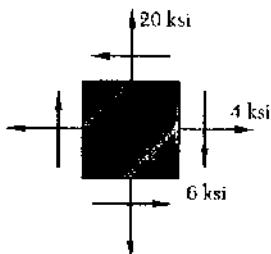
$$(b) \tau_{max} = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{9 + 6}{2}\right)^2 + (4)^2} = 8.5 \text{ ksi}$$

$$(c) \bar{\sigma}' = \bar{\sigma}_{ave} = \frac{\bar{\sigma}_x + \bar{\sigma}_y}{2} = \frac{9 - 6}{2} = 1.5 \text{ ksi}$$

Problem 7.10

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$\bar{\sigma}_x = 4 \text{ ksi}$$

$$\bar{\sigma}_y = 20 \text{ ksi}$$

$$\tau_{xy} = -6 \text{ ksi}$$

$$(a) \tan 2\theta_s = -\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2\tau_{xy}} = -\frac{4 - 20}{(2)(-6)} = -1.3333$$

$$2\theta_s = -53.13^\circ \quad \theta_s = -26.57^\circ, 63.43^\circ$$

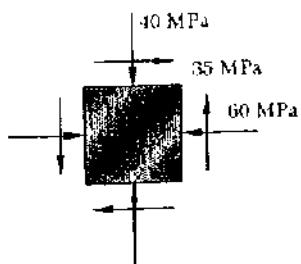
$$(b) \tau_{max} = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{4 - 20}{2}\right)^2 + (-6)^2} = 10 \text{ ksi}$$

$$(c) \bar{\sigma}' = \bar{\sigma}_{ave} = \frac{\bar{\sigma}_x + \bar{\sigma}_y}{2} = \frac{4 + 20}{2} = 12 \text{ ksi}$$

Problem 7.11

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{-60 + 40}{(2)(35)} = 0.2857$$

$$2\theta_s = 15.95^\circ \quad \theta_s = 7.97^\circ, 97.97^\circ$$

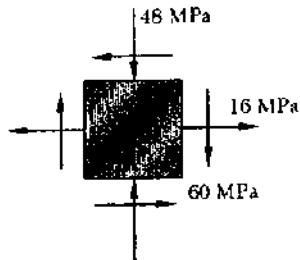
$$(b) \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-60 + 40}{2}\right)^2 + (35)^2} = 36.4 \text{ MPa}$$

$$(c) \sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{-60 - 40}{2} = -50 \text{ MPa}$$

Problem 7.12

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$\sigma_x = 16 \text{ MPa} \quad \sigma_y = -48 \text{ MPa} \quad \tau_{xy} = -60 \text{ MPa}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{16 + 48}{(2)(-60)} = 0.5333$$

$$2\theta_s = 28.07^\circ \quad \theta_s = 14.04^\circ, 104.04^\circ$$

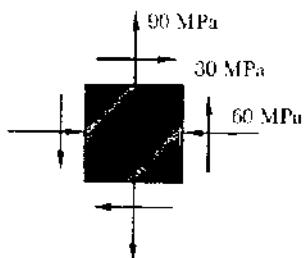
$$(b) \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{16 + 48}{2}\right)^2 + (-60)^2} = 68 \text{ MPa}$$

$$(c) \sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{16 - 48}{2} = -16 \text{ MPa}$$

Problem 7.13

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = 90 \text{ MPa} \quad \tau_{xy} = 30 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = -75 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{xy'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$(a) \theta = -25^\circ \quad 2\theta = -50^\circ$$

$$\sigma_{x'} = 15 - 75 \cos(-50^\circ) + 30 \sin(-50^\circ) = -56.2 \text{ MPa}$$

$$\tau_{xy'} = +75 \sin(-50^\circ) + 30 \cos(-50^\circ) = -38.2 \text{ MPa}$$

$$\sigma_{y'} = 15 + 75 \cos(-50^\circ) - 30 \sin(-50^\circ) = 86.2 \text{ MPa}$$

$$(b) \theta = 10^\circ \quad 2\theta = 20^\circ$$

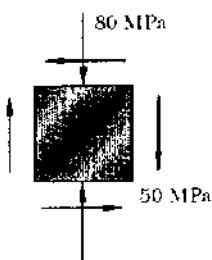
$$\sigma_{x'} = 15 - 75 \cos(20^\circ) + 30 \sin(20^\circ) = -45.2 \text{ MPa}$$

$$\tau_{xy'} = +75 \sin(20^\circ) + 30 \cos(20^\circ) = 53.8 \text{ MPa}$$

$$\sigma_{y'} = 15 + 75 \cos(20^\circ) - 30 \sin(20^\circ) = 75.2 \text{ MPa}$$

Problem 7.14

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$\sigma_x = 0$$

$$\sigma_y = -80 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$(a) \theta = -25^\circ \quad 2\theta = -50^\circ$$

$$\sigma_{x'} = -40 + 40 \cos(-50^\circ) - 50 \sin(-50^\circ) = 24.0 \text{ MPa}$$

$$\tau_{x'y'} = -40 \sin(-50^\circ) + 50 \cos(-50^\circ) = -1.5 \text{ MPa}$$

$$\sigma_{y'} = -40 - 40 \cos(-50^\circ) + 50 \sin(-50^\circ) = -104.0 \text{ MPa}$$

$$(b) \theta = 10^\circ \quad 2\theta = 20^\circ$$

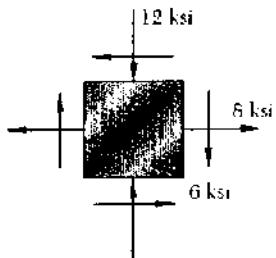
$$\sigma_{x'} = -40 + 40 \cos(20^\circ) - 50 \sin(20^\circ) = -19.5 \text{ MPa}$$

$$\tau_{x'y'} = -40 \sin(20^\circ) - 50 \cos(20^\circ) = -60.7 \text{ MPa}$$

$$\sigma_{y'} = -40 - 40 \cos(20^\circ) + 50 \sin(20^\circ) = -60.5 \text{ MPa}$$

Problem 7.15

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$\sigma_x = 8 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = -6 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi} \quad \frac{\sigma_x - \sigma_y}{2} = 10 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{xy'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$(a) \quad \theta = -25^\circ \quad 2\theta = -50^\circ$$

$$\sigma_{x'} = -2 + 10 \cos(-50^\circ) - 6 \sin(-50^\circ) = 9.02 \text{ ksi} \quad \blacksquare$$

$$\tau_{xy'} = -10 \sin(-50^\circ) - 6 \cos(-50^\circ) = 3.80 \text{ ksi} \quad \blacksquare$$

$$\sigma_{y'} = -2 - 10 \cos(-50^\circ) + 6 \sin(-50^\circ) = -13.02 \text{ ksi} \quad \blacksquare$$

$$(b) \quad \theta = 10^\circ \quad 2\theta = 20^\circ$$

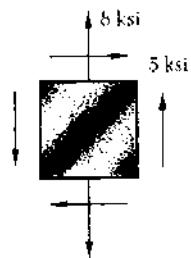
$$\sigma_{x'} = -2 + 10 \cos(20^\circ) - 6 \sin(20^\circ) = 5.34 \text{ ksi} \quad \blacksquare$$

$$\tau_{xy'} = -10 \sin(20^\circ) - 6 \cos(20^\circ) = -9.06 \text{ ksi} \quad \blacksquare$$

$$\sigma_{y'} = -2 - 10 \cos(20^\circ) + 6 \sin(20^\circ) = -9.34 \text{ ksi} \quad \blacksquare$$

Problem 7.16

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$\sigma_x = 0 \quad \sigma_y = 8 \text{ ksi} \quad \tau_{xy} = 5 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi} \quad \frac{\sigma_x - \sigma_y}{2} = -4 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{xy'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$(a) \quad \theta = -25^\circ \quad 2\theta = -50^\circ$$

$$\sigma_{x'} = 4 - 4 \cos(-50^\circ) + 5 \sin(-50^\circ) = -2.40 \text{ ksi} \quad \blacksquare$$

$$\tau_{xy'} = 4 \sin(-50^\circ) + 5 \cos(-50^\circ) = 0.15 \text{ ksi} \quad \blacksquare$$

$$\sigma_{y'} = 4 + 4 \cos(-50^\circ) - 5 \sin(-50^\circ) = 10.40 \text{ ksi} \quad \blacksquare$$

$$(b) \quad \theta = 10^\circ \quad 2\theta = 20^\circ$$

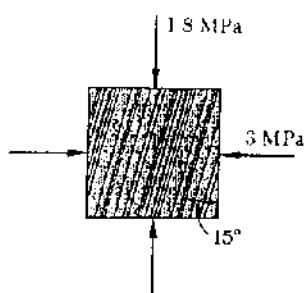
$$\sigma_{x'} = 4 - 4 \cos(20^\circ) + 5 \sin(20^\circ) = 1.95 \text{ ksi} \quad \blacksquare$$

$$\tau_{xy'} = 4 \sin(20^\circ) + 5 \cos(20^\circ) = 6.07 \text{ ksi} \quad \blacksquare$$

$$\sigma_{y'} = 4 + 4 \cos(20^\circ) - 5 \sin(20^\circ) = 6.05 \text{ ksi} \quad \blacksquare$$

Problem 7.17

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.



$$\sigma_x = -3 \text{ MPa} \quad \sigma_y = -1.8 \text{ MPa} \quad \tau_{xy} = 0$$

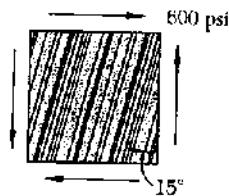
$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$(a) \tau'_{xy} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \sin 2\theta \\ = -\frac{-3 - 1.8}{2} \sin(-30^\circ) + 0 \\ = -0.300 \text{ MPa}$$

$$(b) \sigma'_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ = \frac{-3 - 1.8}{2} + \frac{-3 + 1.8}{2} \cos(-30^\circ) + 0 \\ = -2.92 \text{ MPa}$$

Problem 7.18

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.



$$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = 600 \text{ psi}$$

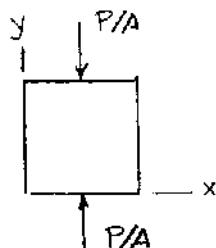
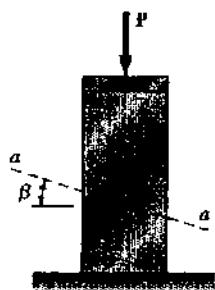
$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$(a) \tau'_{xy} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ = -0 + 600 \cos(-30^\circ) \\ = 520 \text{ psi}$$

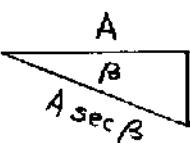
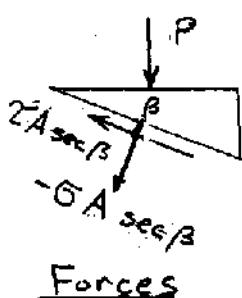
$$(b) \sigma'_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ = 0 + 0 + 600 \sin(-30^\circ) \\ = -300 \text{ psi}$$

Problem 7.19

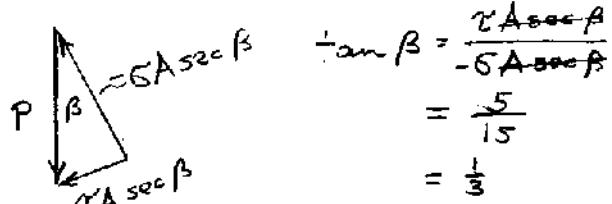
7.19 The centric force P is applied to a short post as shown. Knowing that the stresses on plane $a-a$ are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (a) the angle β that plane $a-a$ forms with the horizontal, (b) the maximum compressive stress in the post.



$$\begin{aligned}\sigma_x &= 0 \\ \tau_{xy} &= 0 \\ \sigma_y &= \sigma_{\text{max comp.}} = -\frac{P}{A}\end{aligned}$$



Areas



$$\begin{aligned}\tan \beta &= \frac{7A \sec \beta}{6A \sec \beta} \\ &= \frac{7}{6} \\ &= \frac{1}{3}\end{aligned}$$

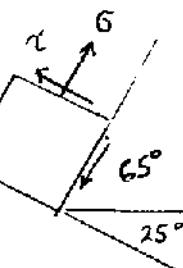
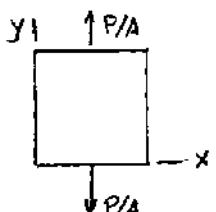
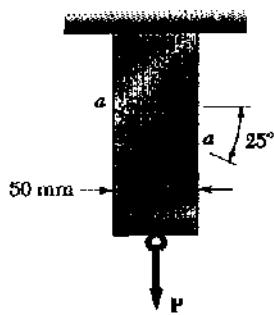
$$\begin{aligned}(a) \beta &= \arctan \frac{1}{3} \\ &= 18.4^\circ\end{aligned}$$

Forces

$$(b) P = (-6A \sec \beta)(\sec \beta) \quad \frac{P}{A} = \frac{-6}{\cos^2 \beta} = \frac{15}{\cos^2 18.4^\circ} = 16.67 \text{ ksi}$$

Problem 7.20

7.20 Two members of uniform cross section 50×80 mm are glued together along plane $a-a$, that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load P that can be applied.



For plane $a-a$ $\theta = 65^\circ$

$$\sigma_x = 0 \quad \tau_{xy} = 0 \quad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta = 0 + \frac{P}{A} \sin^2 65^\circ + 0$$

$$P = \frac{A \sigma}{\sin^2 65^\circ} = \frac{(50 \times 10^{-3})(80 \times 10^{-3})(800 \times 10^3)}{\sin^2 65^\circ} = 3.90 \times 10^3 \text{ N}$$

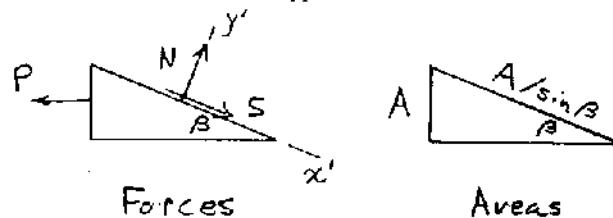
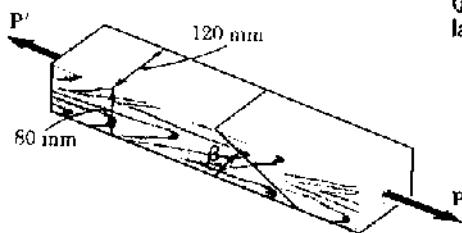
$$\tau = -(\sigma_x - \sigma_y) \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta) = \frac{P}{A} \sin 65^\circ \cos 65^\circ + 0$$

$$P = \frac{A \tau}{\sin 65^\circ \cos 65^\circ} = \frac{(50 \times 10^{-3})(80 \times 10^{-3})(600 \times 10^3)}{\sin 65^\circ \cos 65^\circ} = 6.27 \times 10^3 \text{ N}$$

Allowable value of P is the smaller one. $P = 3.90 \times 10^3 \text{ N} = 3.90 \text{ kN}$

Problem 7.21

7.21 Two wooden members of 80×120 -mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22^\circ$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load P that can be applied.



$$A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$\sigma_{all} = \sigma_{ult} A / \sin \beta = \frac{(400 \times 10^3)(9.6 \times 10^{-3})}{\sin 22^\circ} = 10.251 \times 10^3 \text{ N}$$

$$+/\sum F_{y'} = 0: \quad N - P \sin \beta = 0 \quad P = \frac{N}{\sin \beta} = \frac{10.251 \times 10^3}{\sin 22^\circ} = 27.4 \times 10^3 \text{ N}$$

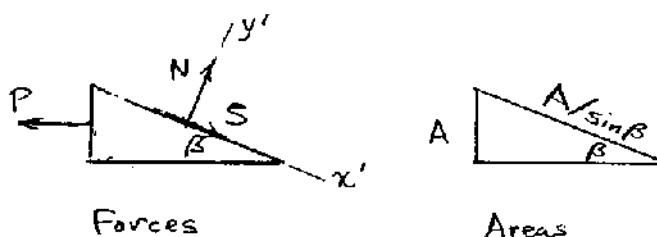
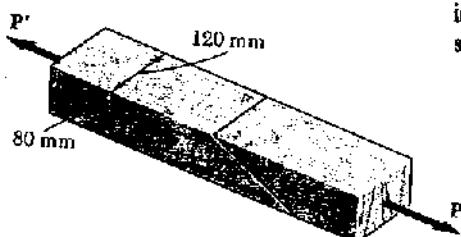
$$\tau_{all} = \tau_{ult} A / \sin \beta = \frac{(600 \times 10^3)(9.6 \times 10^{-3})}{\sin 22^\circ} = 15.376 \times 10^3 \text{ N}$$

$$+\star \sum F_{x'} = 0: \quad S - P \cos \beta = 0 \quad P = \frac{S}{\cos \beta} = \frac{15.376 \times 10^3}{\cos 22^\circ} = 16.58 \times 10^3 \text{ N}$$

The smaller value for P governs. $P = 16.58 \times 10^3 \text{ N} = 16.58 \text{ kN}$

Problem 7.22

7.22 Two wooden members of 80×120 -mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25^\circ$ and that centric loads of magnitude $P = 10 \text{ kN}$ are applied to the members as shown, determine (a) the in-plane shearing stress parallel to the splice, (b) the normal stress perpendicular to the splice.



$$A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$(b) +/\sum F_{y'} = 0: \quad N - P \sin \beta = 0 \quad N = P \sin \beta = (10 \times 10^3) \sin 25^\circ = 4.226 \times 10^3 \text{ N}$$

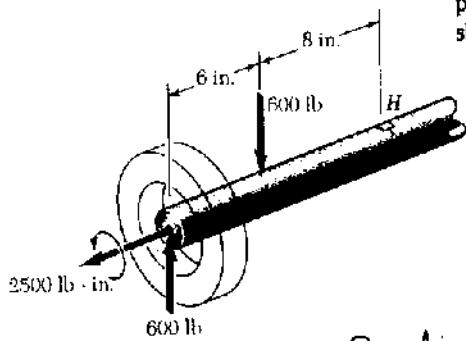
$$\sigma = \frac{N}{A / \sin \beta} = \frac{(4.226 \times 10^3) \sin 25^\circ}{9.6 \times 10^{-3}} = 186.0 \times 10^3 \text{ Pa} = 186.0 \text{ kPa}$$

$$(a) +\star \sum F_{x'} = 0: \quad S - P \cos \beta = 0 \quad S = P \cos \beta = (10 \times 10^3) \cos 25^\circ = 9.063 \times 10^3 \text{ N}$$

$$\tau = \frac{S}{A / \sin \beta} = \frac{(9.063 \times 10^3) \sin 25^\circ}{9.6 \times 10^{-3}} = 399 \times 10^3 \text{ Pa} = 399 \text{ kPa}$$

Problem 7.23

7.23 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 1.25 in., determine (a) the principal planes and principal stresses at point H located on top of the axle, (b) the maximum shearing stress at the same point.



$$c = \frac{1}{2}d = \frac{1}{2}(1.25) = 0.625 \text{ in}^4$$

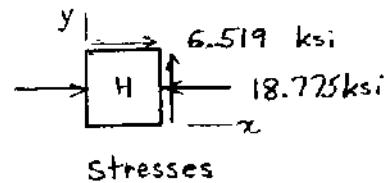
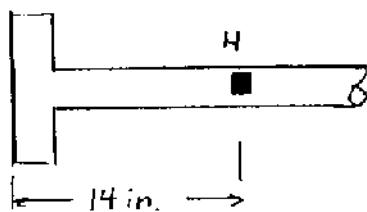
$$\text{Tension: } \sigma = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\sigma = \frac{(2)(2500)}{\pi(0.625)^3} = 6.519 \times 10^3 \text{ psi} = 6.519 \text{ ksi}$$

$$\text{Bending: } I = \frac{\pi}{4}c^4 = 119.842 \times 10^{-8} \text{ in}^4$$

$$M = (6)(600) = 3600 \text{ lb-in} \quad \sigma = -\frac{My}{I} = -\frac{(3600)(0.625)}{119.842 \times 10^{-8}} = -18.775 \times 10^3 \text{ psi} = -18.775 \text{ ksi}$$

Top View



$$\bar{\sigma}_x = -18.775 \text{ ksi} \quad \bar{\sigma}_y = 0 \quad \tau_{xy} = 6.519 \text{ ksi}$$

$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) = -9.387 \text{ ksi}$$

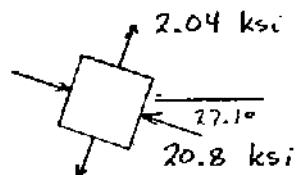
$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-9.387)^2 + (6.519)^2} = 11.429 \text{ ksi}$$

$$(a) \sigma_1 = \bar{\sigma}_{ave} + R = -9.387 + 11.429 = 2.04 \text{ ksi}$$

$$\sigma_2 = \bar{\sigma}_{ave} - R = -9.387 - 11.429 = -20.8 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\bar{\sigma}_x - \bar{\sigma}_y} = \frac{(2)(6.519)}{-9.387} = -1.3889$$

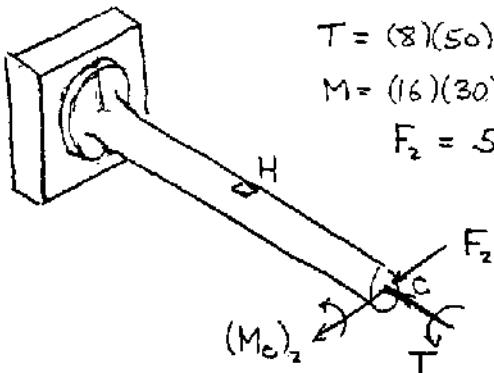
$$\theta_p = -27.1^\circ \text{ and } 62.9^\circ$$



$$(b) \tau_{max} = R = 11.43 \text{ ksi}$$

Problem 7.24

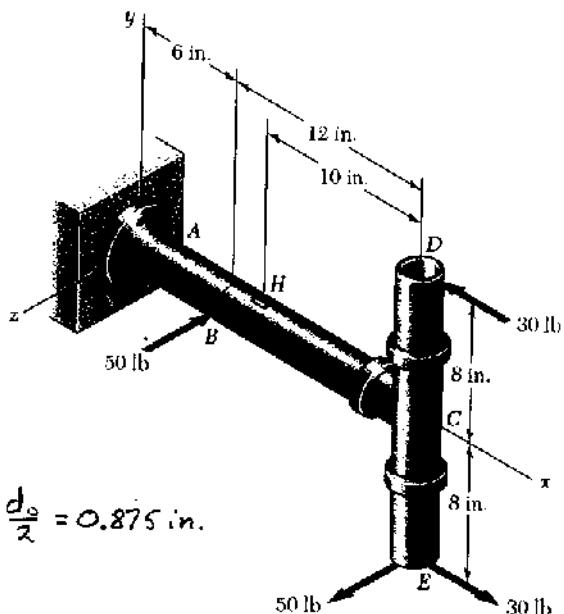
Replace forces on pipe DCE by an equivalent force-couple system at C.



$$T = (8)(50) = 400 \text{ lb-in}$$

$$M = (16)(30) = 480 \text{ lb-in}$$

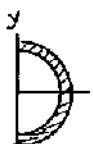
$$F_z = 50 \text{ lb}$$



$$\text{Cross section. } C_1 = \frac{d_1}{2} = 0.750 \text{ in. } C_2 = \frac{d_2}{2} = 0.875 \text{ in.}$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 0.42376 \text{ in}^4$$

$$I = \frac{1}{2}J = 0.21188 \text{ in.}$$



$$Q_y = \frac{2}{3}(C_2^3 - C_1^3) = \frac{2}{3}(0.875^3 - 0.750^3) = 0.16536 \text{ in}^3$$

$$t = C_2 - C_1 = 0.125 \text{ in.}$$

At the section containing element H

$$T = 400 \text{ lb-in}, \quad M_2 = 480 \text{ lb-in}, \quad V_2 = 50 \text{ lb.}$$

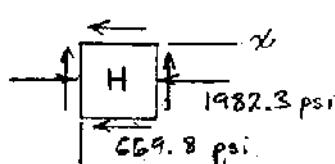
Stresses.

$$\text{Tension: } \sigma_{zx} = -\frac{Tz}{J} = -\frac{(400)(0.875)}{0.42376} = -825.9 \text{ psi}$$

$$\text{Bending: } \sigma_x = -\frac{My}{I} = -\frac{(480)(0.875)}{0.21188} = -1982.3 \text{ psi}$$

$$\text{Transverse Shear: } \tau_{zx} = \frac{VQ}{I(2t)} = \frac{(50)(0.16536)}{(0.21188)(0.250)} = 156.1 \text{ psi}$$

$$\text{Total: } \sigma_x = 0, \quad \sigma_x = -1982.3 \text{ psi}, \quad \tau_{zx} = -825.9 + 156.1 = -669.8 \text{ psi}$$



$$(a) \tan 2\theta_p = \frac{2\tau_{zx}}{\sigma_x - \sigma_z} = \frac{(2)(-669.8)}{0 + 1982.3} = -0.67578$$

$$\theta_p = -17.0^\circ \text{ and } 73.0^\circ \text{ from z-axis.}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_z + \sigma_x) = \frac{1}{2}(0 - 1982.3) = -991.1 \text{ psi}$$

$$R = \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} = \sqrt{991.1^2 + 669.8^2} = 1196.2 \text{ psi}$$

$$\sigma_1 = \sigma_{ave} + R = 205 \text{ psi at } 17.0^\circ \text{ from z-axis.}$$

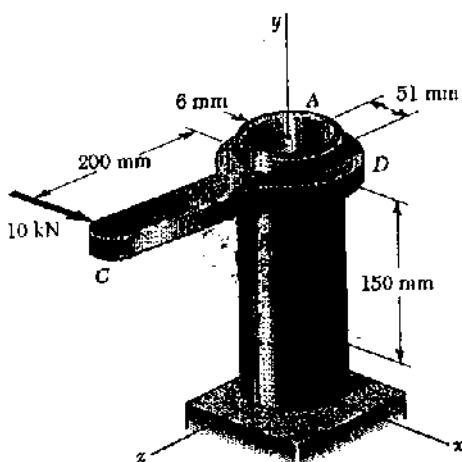
$$\sigma_2 = \sigma_{ave} - R = -2187 \text{ psi at } 73.0^\circ \text{ from z-axis.}$$

$$(b) \tau_{max} = R$$

$$\tau_{max} = 1196 \text{ psi}$$

Problem 7.25

7.25 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point H .



$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2} J = 2.0927 \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K .

$$F_x = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$

Torsion

$$T = M_y = 2000 \text{ N}\cdot\text{m}$$

$$C = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6 \text{ Pa}$$



Transverse Shear

For semicircle

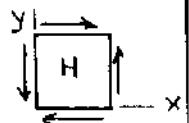
$$A = \frac{\pi}{2} r^2 \quad \bar{y} = \frac{4}{3\pi} r$$

$$Q = A\bar{y} = \frac{2}{3} r^3$$

$$Q_s = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = 27.684 \times 10^3 \text{ mm}^3 = 27.684 \times 10^{-6} \text{ m}^3$$

$$V = F_x = 10 \times 10^3 \text{ N} \quad t = (2)(6 \text{ mm}) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \times 10^6 \text{ Pa}$$



Bending: Point H lies on neutral axis. $\sigma_y = 0$

Total stresses at point H : $\sigma_x = 0$, $\sigma_y = 0$

$$\tau_{xy} = 24.37 \times 10^6 + 11.02 \times 10^6 = 35.39 \times 10^6 \text{ Pa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0 \quad R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = 35.39 \times 10^6 \text{ Pa}$$

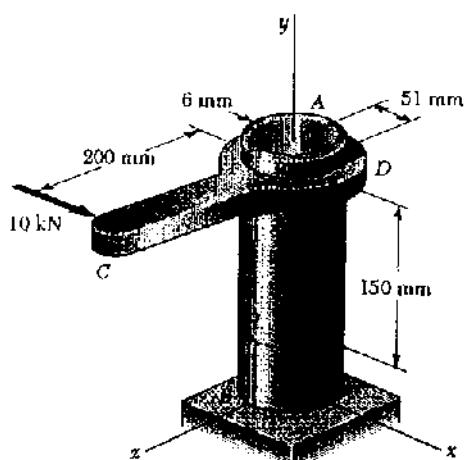
$$\sigma_{max} = \sigma_{ave} + R = 35.39 \times 10^6 \text{ Pa} = 35.4 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -35.39 \times 10^6 \text{ Pa} = -35.4 \text{ MPa}$$

$$\tau_{max} = R = 35.4 \text{ MPa}$$

Problem 7.26

7.26 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.



$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

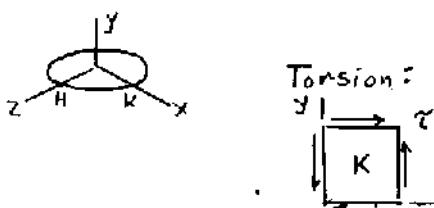
$$I = \frac{1}{2}J = 2.0927 \times 10^{-6} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K

$$F_x = 10 \text{ kN} = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$



At point K, place local x-axis in negative global z-direction

$$T = M_y = 2000 \text{ N}\cdot\text{m} \quad C = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^6} = 24.37 \times 10^6 \text{ Pa} = 24.37 \text{ MPa}$$

Transverse Shear: Stress due to transverse shear $V = F_x$ is zero at pt. K.

$$\text{Bending: } 1G_y I = \frac{1M_z c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \times 10^6 \text{ Pa} = 36.56 \text{ MPa}$$

Point K lies on compression side of neutral axis: $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K $\sigma_x = 0 \quad \sigma_y = -36.56 \text{ MPa}, \tau_{xy} = 24.37 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

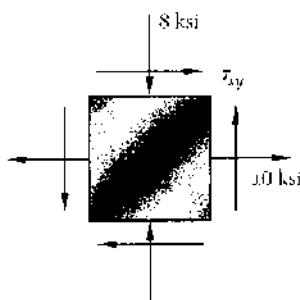
$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 = +12.18 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 = -48.74 \text{ MPa}$$

$$\tau_{max} = R = 30.46 \text{ MPa}$$

Problem 7.27

7.27 For the state of plane stress shown, determine (a) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (b) the corresponding principal stresses.



$$\bar{\sigma}_x = 10 \text{ ksi}, \bar{\sigma}_y = -8 \text{ ksi}, \tau_{xy} = ?$$

$$\begin{aligned}\tau_{max} = R &= \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10 - (-8)}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{q^2 + \tau_{xy}^2} = 12 \text{ ksi}\end{aligned}$$

$$(a) \quad \tau_{xy} = \sqrt{12^2 - q^2} = 7.94 \text{ ksi}$$

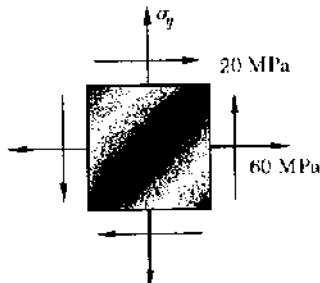
$$(b) \quad \sigma_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) = 1 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 1 + 12 = 13 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = 1 - 12 = -11 \text{ ksi}$$

Problem 7.28

7.28 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.



$$\bar{\sigma}_x = 60 \text{ MPa}, \bar{\sigma}_y = ?, \tau_{xy} = 20 \text{ MPa}$$

$$\text{Let } v = \frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}. \quad \text{Then } \bar{\sigma}_y = \bar{\sigma}_x - 2v$$

$$R = \sqrt{v^2 + \tau_{xy}^2} \approx 75 \text{ MPa}$$

$$v = \pm \sqrt{R^2 - \tau_{xy}^2} = \pm \sqrt{75^2 - 20^2} = 72.284 \text{ MPa}$$

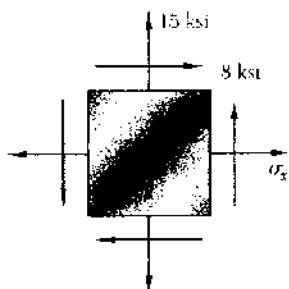
$$\bar{\sigma}_y = \bar{\sigma}_x - 2v = 60 \mp (2)(72.284) = -84.6 \text{ MPa or } 205 \text{ MPa}$$

Largest value of $\bar{\sigma}_y$ is required.

$$\bar{\sigma}_y = 205 \text{ MPa}$$

Problem 7.29

7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.



$$\bar{\sigma}_x = ? , \bar{\sigma}_y = 15 \text{ ksi} , \tau_{xy} = 8 \text{ ksi}$$

$$\text{Let } u = \frac{\bar{\sigma}_x - \bar{\sigma}_y}{2} \quad \sigma_x = \bar{\sigma}_y + 2u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 10 \text{ ksi}$$

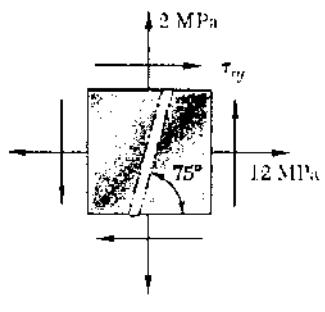
$$u = \pm \sqrt{R^2 - \tau_{xy}^2} = \pm \sqrt{10^2 - 8^2} = \pm 6 \text{ ksi}$$

$$\sigma_x = \bar{\sigma}_y + 2u = 15 \pm (2)(6) = 27 \text{ ksi or } 3 \text{ ksi}$$

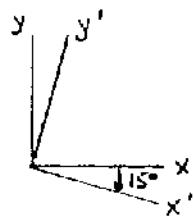
Allowable range $3 \text{ ksi} \leq \sigma_x \leq 27 \text{ ksi}$

Problem 7.30

7.30 For the state of plane stress shown, determine (a) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (b) the corresponding principal stresses.



$$\bar{\sigma}_x = 12 \text{ MPa} , \bar{\sigma}_y = 2 \text{ MPa} , \tau_{xy} = ?$$



Since $\tau_{xy'} = 0$, x' -direction is a principal direction.

$$\theta_p = -15^\circ$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\bar{\sigma}_x - \bar{\sigma}_y}$$

$$(a) \tau_{xy} = \frac{1}{2}(\bar{\sigma}_x - \bar{\sigma}_y) \tan 2\theta_p = \frac{1}{2}(12 - 2) \tan(-30^\circ) = -2.89 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735 \text{ MPa}$$

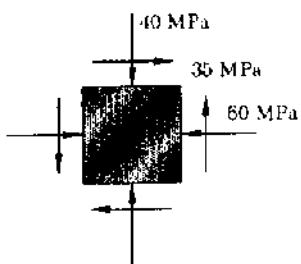
$$\sigma_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) = 7 \text{ MPa}$$

$$(b) \sigma_a = \sigma_{ave} + R = 7 + 5.7735 = 12.77 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 7 - 5.7735 = 1.226 \text{ MPa}$$

Problem 7.31

7.31 Solve Probs. 7.7 and 7.11, using Mohr's circle.



7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$$

Points

$$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (-50 \text{ MPa}, 0)$$

$$\tan \beta = \frac{G_X}{CG} = \frac{35}{10} = 3.500$$

$$\beta = 74.05^\circ$$

$$\theta_B = -\frac{1}{2}\beta = -37.03^\circ$$

$$\alpha = 180^\circ - \beta = 105.95^\circ$$

$$\theta_A = \frac{1}{2}\alpha = 52.97^\circ$$

$$R = \sqrt{CG^2 + G_X^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -50 - 36.4 = -86.4 \text{ MPa}$$

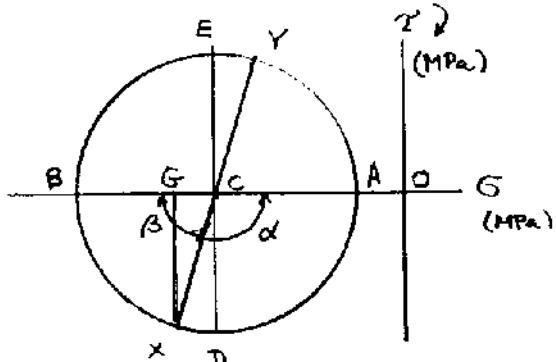
$$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4 = -13.6 \text{ MPa}$$

$$\theta_D = \theta_B + 45^\circ = 7.97^\circ$$

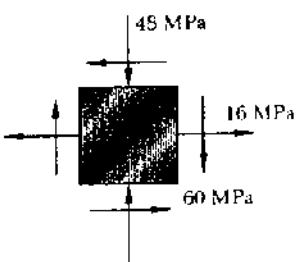
$$\theta_E = \theta_A + 45^\circ = 97.97^\circ$$

$$\tau_{max} = R = 36.4 \text{ MPa}$$

$$\sigma' = \sigma_{ave} = -50 \text{ MPa}$$



Problem 7.32



7.32 Solve Probs. 7.8 and 7.12, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

$$\sigma_x = 16 \text{ MPa} \quad \sigma_y = -48 \text{ MPa} \quad \tau_{xy} = -60 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -16 \text{ MPa}$$

Points:

$$X: (\sigma_x, -\tau_{xy}) = (16 \text{ MPa}, 60 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (-48 \text{ MPa}, -60 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (-16 \text{ MPa}, 0)$$

$$\tan \alpha = \frac{FX}{CF} = \frac{60}{32} = 1.875$$

$$\alpha = 61.93^\circ$$

$$\theta_A = -\frac{1}{2}\alpha = -30.96^\circ$$

$$\beta = 180^\circ - \alpha = 118.07^\circ$$

$$\theta_B = \frac{1}{2}\beta = 59.04^\circ$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{32^2 + 60^2} = 68 \text{ MPa}$$

$$\sigma_a = \sigma_{max} = \sigma_{ave} + R = -16 + 68 = 52 \text{ MPa}$$

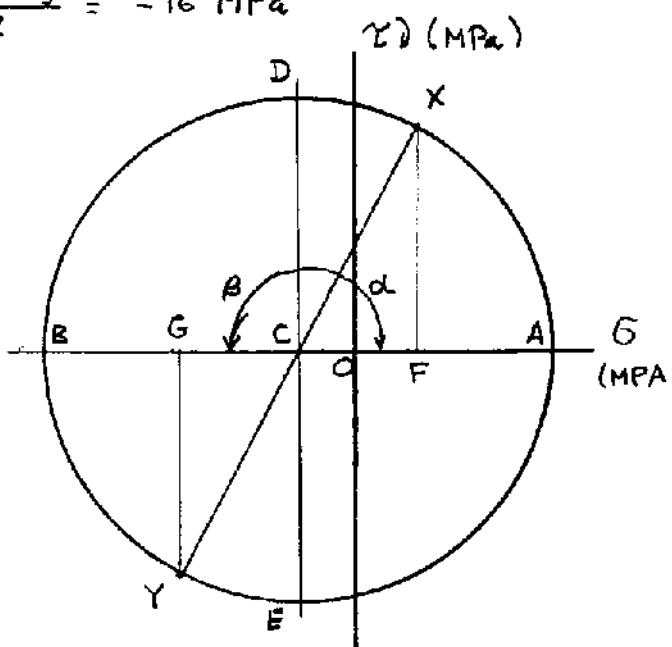
$$\sigma_b = \sigma_{min} = \sigma_{ave} - R = -16 - 68 = -84 \text{ MPa}$$

$$\theta_d = \theta_a + 45^\circ = 14.04^\circ$$

$$\theta_e = \theta_b + 45^\circ = 104.04^\circ$$

$$\tau_{max} = R = 68 \text{ MPa}$$

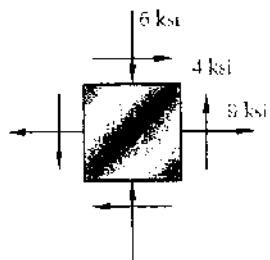
$$\sigma' = \sigma_{ave} = -16 \text{ MPa}$$



Problem 7.33

7.33 Solve Prob. 7.9, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$\sigma_x = 9 \text{ ksi} \quad \sigma_y = -6 \text{ ksi} \quad \tau_{xy} = 4 \text{ ksi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 1.5 \text{ ksi}$$

Points

$$X: (\sigma_x, -\tau_{xy}) = (9 \text{ ksi}, -4 \text{ ksi})$$

$$Y: (\sigma_y, \tau_{xy}) = (-6 \text{ ksi}, 4 \text{ ksi})$$

$$C: (\sigma_{ave}, 0) = (1.5 \text{ ksi}, 0)$$

$$\tan \alpha = \frac{F_x}{C_F} = \frac{4}{7.5} = 0.5333$$

$$\alpha = 28.07^\circ$$

$$\theta_a = \frac{1}{2}\alpha = 14.04^\circ$$

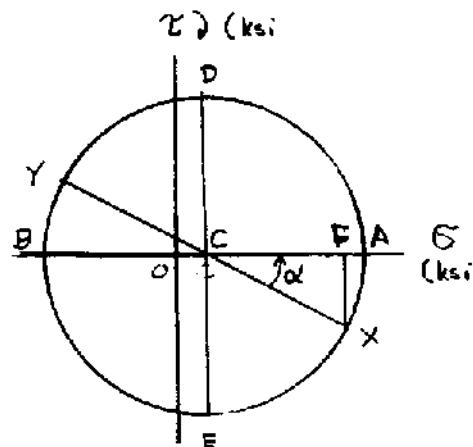
$$\theta_d = \theta_a + 45^\circ = 59.04^\circ$$

$$\theta_e = \theta_a - 45^\circ = -30.96^\circ$$

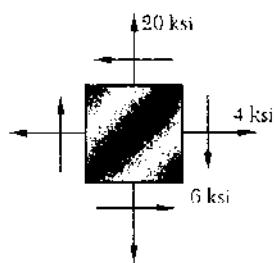
$$R = \sqrt{C_F^2 + F_x^2} = \sqrt{7.5^2 + 4^2} = 8.5 \text{ ksi}$$

$$\tau_{max} = R = 8.5 \text{ ksi}$$

$$\sigma' = \sigma_{ave} = 1.5 \text{ ksi}$$



Problem 7.34



7.34 Solve Prob. 7.10, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

$$\sigma_x = 4 \text{ ksi} \quad \sigma_y = 20 \text{ ksi} \quad \tau_{xy} = -6 \text{ ksi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{4 + 20}{2} = 12 \text{ ksi}$$

Points

$$X: (\sigma_{x_0}, \tau_{xy}) = (4 \text{ ksi}, 6 \text{ ksi})$$

$$Y: (\sigma_y, \tau_{xy}) = (20 \text{ ksi}, -6 \text{ ksi})$$

$$C: (\sigma_{ave}, 0) = (12 \text{ ksi}, 0)$$

$$\tan \alpha = \frac{F_x}{F_c} = \frac{6}{8} = 0.75$$

$$\alpha = 36.87^\circ$$

$$\theta_b = \frac{1}{2}\alpha = 18.43^\circ$$

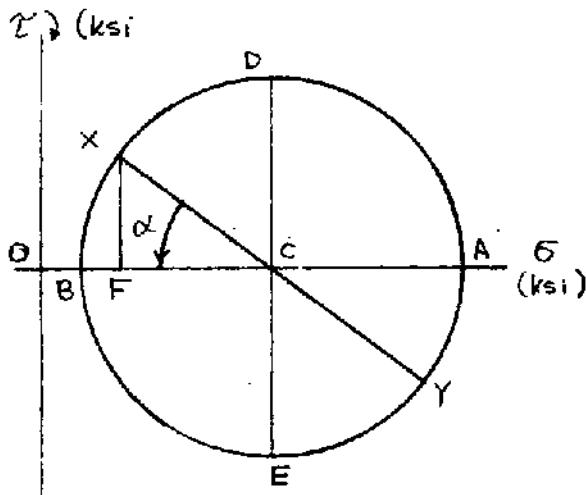
$$\theta_d = \theta_b - 45^\circ = -26.57^\circ$$

$$\theta_e = \theta_b + 45^\circ = 63.43^\circ$$

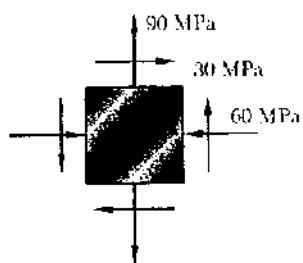
$$R = \sqrt{CF^2 + F_x^2} = \sqrt{8^2 + 6^2} = 10 \text{ ksi}$$

$$\tau_{max} = R = 10 \text{ ksi}$$

$$\sigma' = \sigma_{ave} = 12 \text{ ksi}$$



Problem 7.35

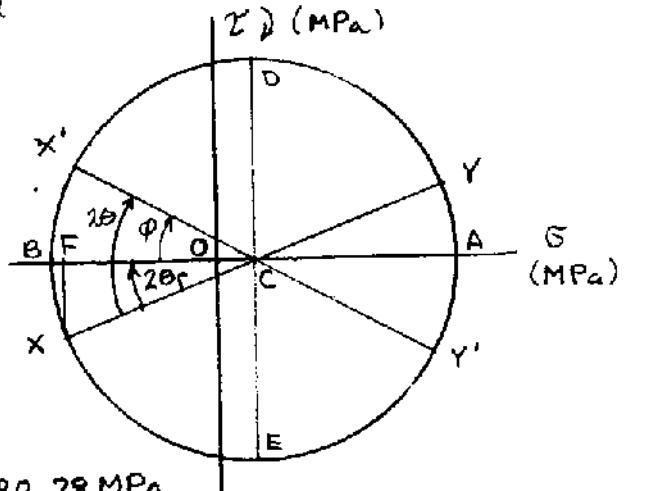


7.35 Solve Prob. 7.13, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

$$\bar{\sigma}_x = -60 \text{ MPa} \quad \bar{\sigma}_y = 90 \text{ MPa} \quad \tau_{xy} = 30 \text{ MPa}$$

$$\bar{\sigma}_{ave} = \frac{\bar{\sigma}_x + \bar{\sigma}_y}{2} = 15 \text{ MPa}$$



Points

$$X: (-60 \text{ MPa}, -30 \text{ MPa})$$

$$Y: (90 \text{ MPa}, 30 \text{ MPa})$$

$$C: (15 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{F_x}{F_c} = \frac{30}{75} = 0.4$$

$$2\theta_p = 21.80^\circ \quad \theta_p = 10.90^\circ$$

$$R = \sqrt{F_c^2 + F_x^2} = \sqrt{75^2 + 30^2} = 80.78 \text{ MPa}$$

$$(a) \theta = 25^\circ \quad 2\theta = 50^\circ$$

$$\phi = 2\theta - 2\theta_p = 50^\circ - 21.80^\circ = 28.20^\circ$$

$$\sigma_{x'} = \bar{\sigma}_{ave} - R \cos \phi = -56.2 \text{ MPa}$$

$$\tau_{x'y'} = -R \sin \phi = -38.2 \text{ MPa}$$

$$\sigma_{y'} = \bar{\sigma}_{ave} + R \cos \phi = 86.2 \text{ MPa}$$

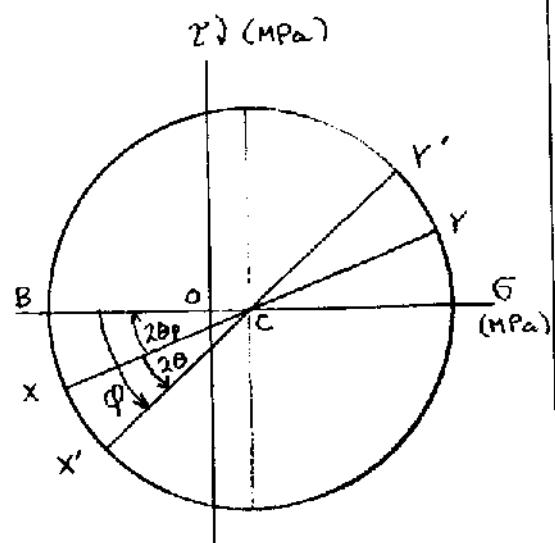
$$(b) \theta = 10^\circ \quad 2\theta = 20^\circ$$

$$\phi = 2\theta_p + 2\theta = 21.80^\circ + 20^\circ = 41.80^\circ$$

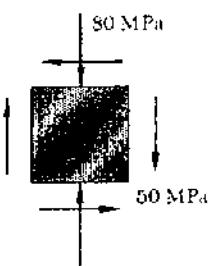
$$\sigma_{x'} = \bar{\sigma}_{ave} - R \cos \phi = -45.2 \text{ MPa}$$

$$\tau_{x'y'} = R \sin \phi = 53.8 \text{ MPa}$$

$$\sigma_{y'} = \bar{\sigma}_{ave} + R \cos \phi = 75.2 \text{ MPa}$$



Problem 7.36



7.36 Solve Prob. 7.14, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

$$\sigma_x = 0 \quad \sigma_y = -80 \text{ MPa} \quad \tau_{xy} = -50 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$$

Points

$$X: (0, 50 \text{ MPa})$$

$$Y: (-80 \text{ MPa}, -50 \text{ MPa})$$

$$C: (-40 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{F_x}{CF} = \frac{50}{40} = 1.25$$

$$2\theta_p = 51.34^\circ$$

$$R = \sqrt{CF^2 + F_x^2} = \sqrt{40^2 + 50^2} = 64.03 \text{ MPa}$$

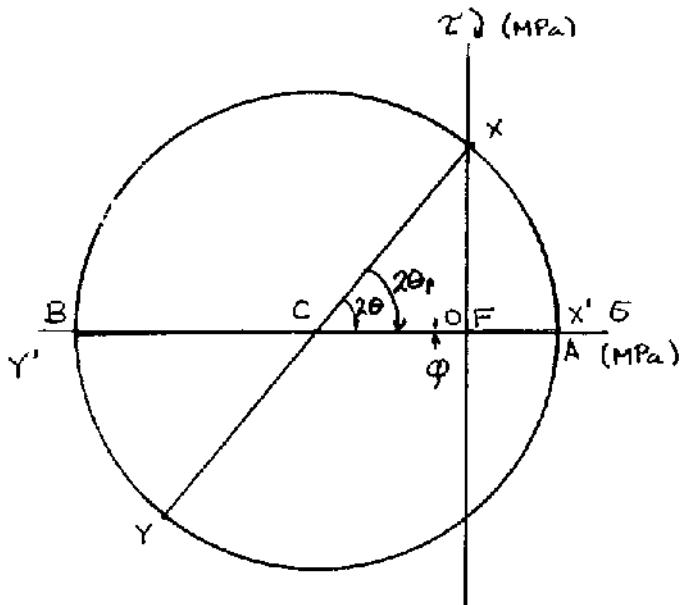
$$(a) \theta = 25^\circ \rightarrow 2\theta = 50^\circ$$

$$\phi = 51.34^\circ - 50^\circ = 1.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = 24.0 \text{ MPa}$$

$$\tau_{x'y'} = -R \sin \phi = -1.5 \text{ MPa}$$

$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -104.0 \text{ MPa}$$



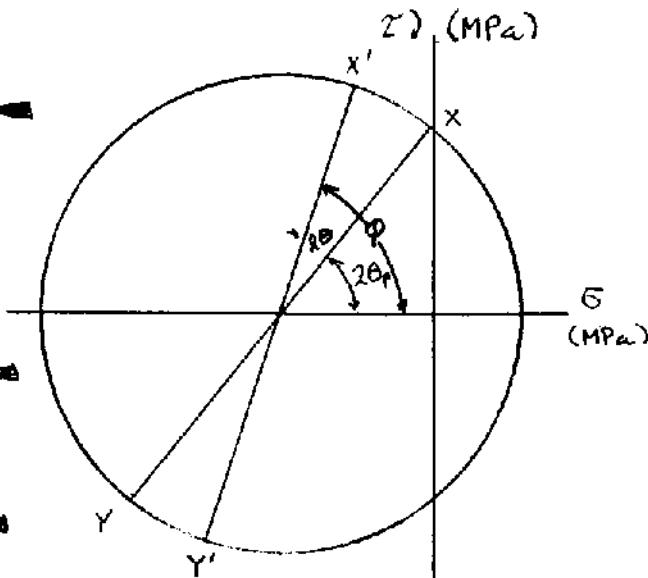
$$(b) \theta = 10^\circ \rightarrow 2\theta = 20^\circ$$

$$\phi = 51.34^\circ + 20^\circ = 71.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = -19.5 \text{ MPa}$$

$$\tau_{x'y'} = +R \sin \phi = -60.7 \text{ MPa}$$

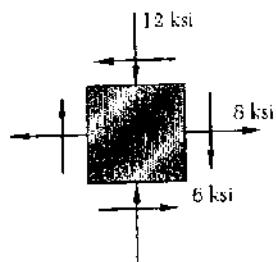
$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -60.5 \text{ MPa}$$



Problem 7.37

7.37 Solve Prob. 7.15, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$\sigma_x = 8 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = -6 \text{ ksi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi}$$

Points

$$X: (8 \text{ ksi}, 6 \text{ ksi})$$

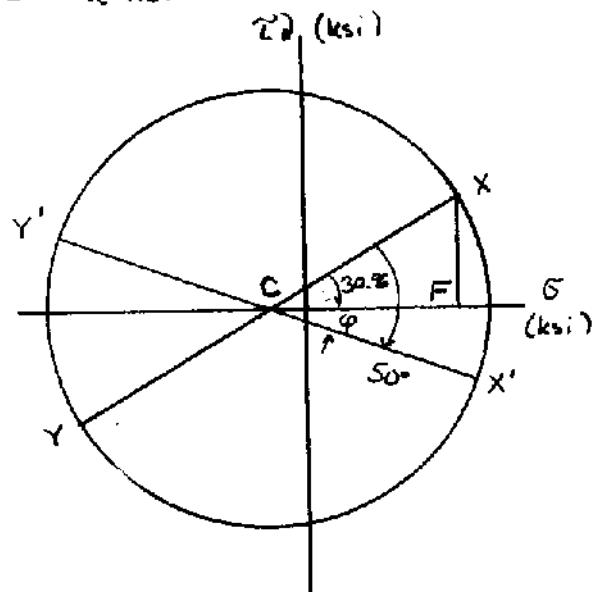
$$Y: (-12 \text{ ksi}, -6 \text{ ksi})$$

$$C: (-2 \text{ ksi}, 0)$$

$$\tan 2\theta_p = \frac{F_x}{C_F} = \frac{6}{10} = 0.6$$

$$2\theta_p = 30.96^\circ$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{10^2 + 6^2} = 11.66 \text{ ksi}$$



$$(a) \theta = 25^\circ \quad 2\theta = 50^\circ$$

$$\phi = 50^\circ - 30.96^\circ = 19.04^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = 9.02 \text{ ksi}$$

$$\tau_{xy'} = R \sin \phi = 3.80 \text{ ksi}$$

$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -13.02 \text{ ksi}$$

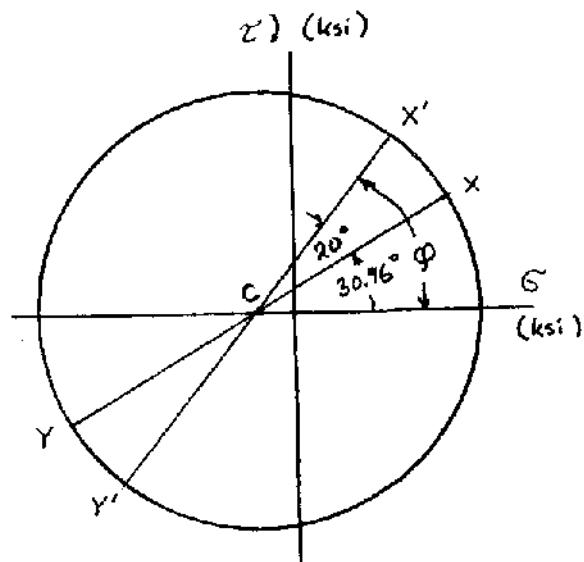
$$(b) \theta = 10^\circ \quad 2\theta = 20^\circ$$

$$\phi = 30.96^\circ + 20^\circ = 50.96^\circ$$

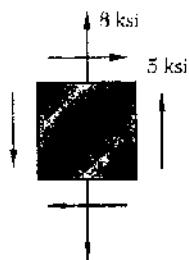
$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = 5.34 \text{ ksi}$$

$$\tau_{xy'} = -R \sin \phi = -9.06 \text{ ksi}$$

$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -9.34 \text{ ksi}$$



Problem 7.38



7.38 Solve Prob. 7.16, using Mohr's circle:

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

$$\sigma_x = 0 \quad \sigma_y = 8 \text{ ksi} \quad \tau_{xy} = 5 \text{ ksi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi}$$

Points:

$$X: (0, -5 \text{ ksi})$$

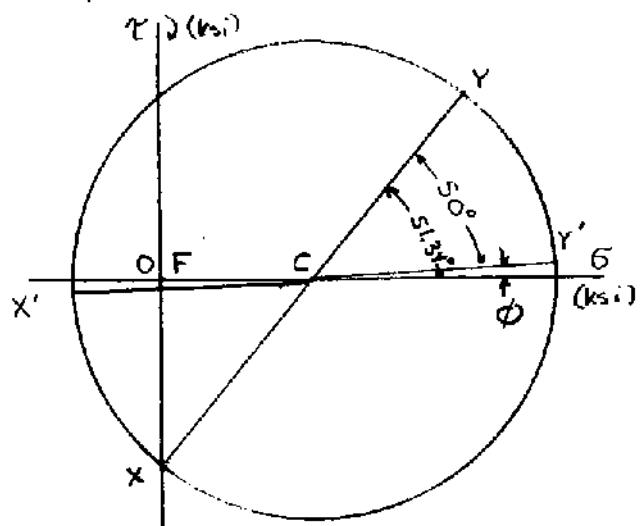
$$Y: (8 \text{ ksi}, 5 \text{ ksi})$$

$$C: (4 \text{ ksi}, 0)$$

$$\tan 2\theta_p = \frac{F_x}{F_C} = \frac{5}{4} = 1.25$$

$$2\theta_p = 51.34^\circ$$

$$R = \sqrt{F_C^2 + F_x^2} = \sqrt{4^2 + 5^2} \\ = 6.40 \text{ ksi}$$



$$(a) \theta = 25^\circ \rightarrow 2\theta = 50^\circ$$

$$\phi = 51.34^\circ - 50^\circ = 1.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} - R \cos \phi = -2.40 \text{ ksi}$$

$$\tau_{xy'} = R \sin \phi = 0.15 \text{ ksi}$$

$$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 10.40 \text{ ksi}$$

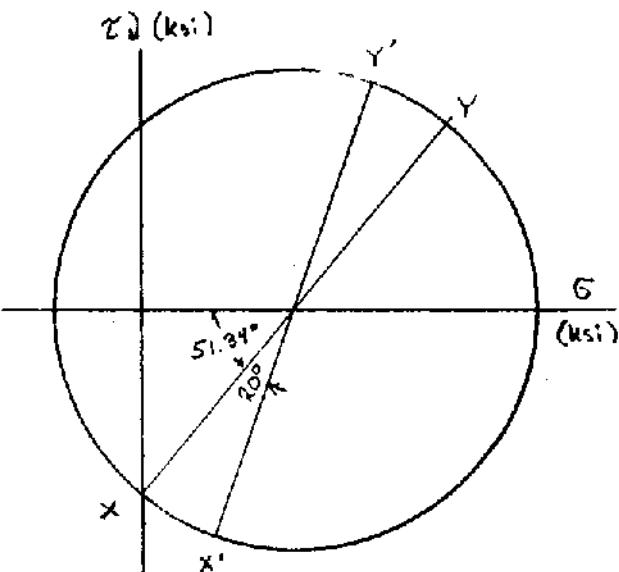
$$(b) \theta = 10^\circ \rightarrow 2\theta = 20^\circ$$

$$\phi = 51.34^\circ + 20^\circ = 71.34^\circ$$

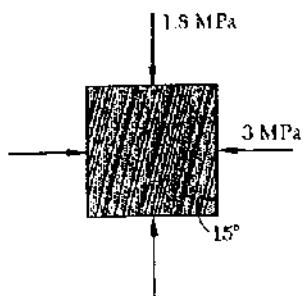
$$\sigma_{x'} = \sigma_{ave} - R \cos \phi = 1.95 \text{ ksi}$$

$$\tau_{yy'} = R \sin \phi = 6.07 \text{ ksi}$$

$$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 6.05 \text{ ksi}$$



Problem 7.39



7.39 Solve Prob. 7.17, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

$$\sigma_x = -3 \text{ MPa} \quad \sigma_y = -1.8 \text{ MPa} \quad \tau_{xy} = 0$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2.4 \text{ MPa}$$

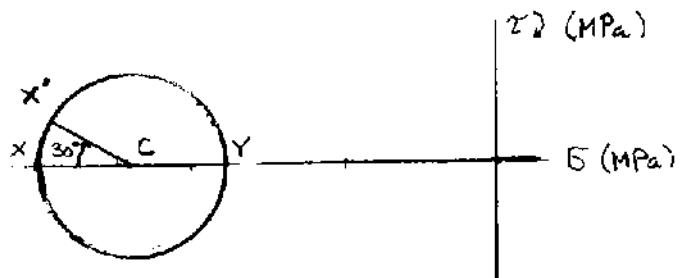
Points

$$X: (\sigma_x, -\tau_{xy}) = (-3 \text{ MPa}, 0)$$

$$Y: (\sigma_y, \tau_{xy}) = (-1.8 \text{ MPa}, 0)$$

$$C: (\sigma_{ave}, 0) = (-2.4 \text{ MPa}, 0)$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

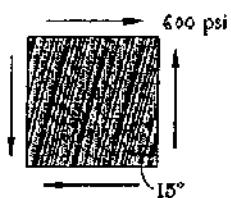


$$\bar{C}x = 0.6 \text{ MPa} \quad R = 0.6 \text{ MPa}$$

$$(a) \tau_{x'y'} = -\bar{C}x' \sin 30^\circ = -R \sin 30^\circ = -0.6 \sin 30^\circ = -0.3 \text{ MPa}$$

$$(b) \sigma_{x'} = \sigma_{ave} - \bar{C}x' \cos 30^\circ = -2.4 - 0.6 \cos 30^\circ = -2.92 \text{ MPa}$$

Problem 7.40



7.40 Solve Prob. 7.18, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

$$\sigma_x = \sigma_y = 0 \quad \tau_{xy} = 400 \text{ psi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Points

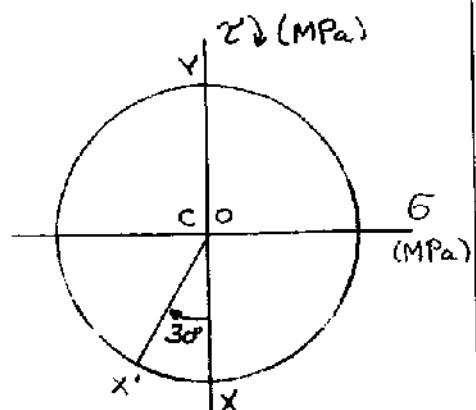
$$X: (\sigma_x, -\tau_{xy}) = (0, -600 \text{ psi})$$

$$Y: (\sigma_y, \tau_{xy}) = (0, 600 \text{ psi})$$

$$C: (\sigma_{ave}, 0) = (0, 0)$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$\bar{C}x = R = 600 \text{ psi}$$



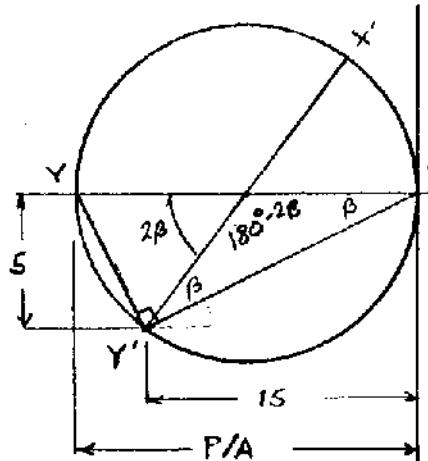
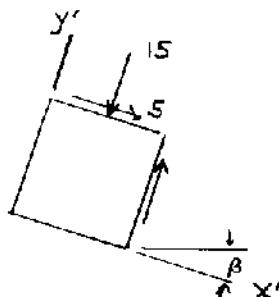
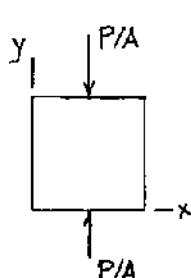
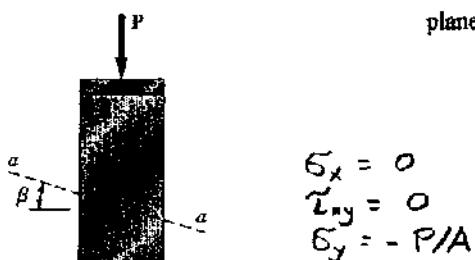
$$(a) \tau_{x'y'} = R \cos 30^\circ = 600 \cos 30^\circ = 520 \text{ psi}$$

$$(b) \sigma_{x'} = \sigma_{ave} - R \sin 30^\circ = -600 \sin 30^\circ = -300 \text{ psi}$$

Problem 7.41

7.41 Solve Prob. 7.19, using Mohr's circle.

7.19 The centric force P is applied to a short post as shown. Knowing that the stresses on plane $a-a$ are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (a) the angle β that plane $a-a$ forms with the horizontal, (b) the maximum compressive stress in the post.



From the Mohr's circle

$$\tan \beta = \frac{5}{15} = 0.3333 \quad \beta = 18.4^\circ$$

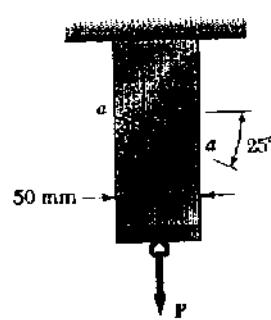
$$-\sigma = \frac{P}{2A} + \frac{P}{2A} \cos 2\beta$$

$$\begin{aligned} \frac{P}{A} &= \frac{2(-5)}{1 + \cos 2\beta} = \frac{(2)(15)}{1 + \cos 2\beta} \\ &= 16.67 \text{ ksi} \end{aligned}$$

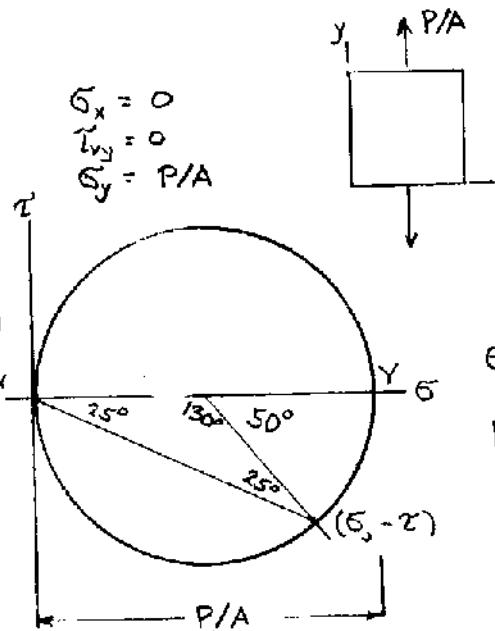
Problem 7.42

7.42 Solve Prob. 7.20, using Mohr's circle.

7.20 Two members of uniform cross section $50 \times 80 \text{ mm}$ are glued together along plane $\alpha-\alpha'$, that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800 \text{ kPa}$ and $\tau = 600 \text{ kPa}$, determine the largest centric load P that can be applied.



$$A = (50 \times 10^{-3})(80 \times 10^{-3}) \\ = 4 \times 10^{-3} \text{ m}^2$$



$$\sigma = \frac{P}{2A} (1 + \cos 50^\circ)$$

$$P = \frac{2AG}{1 + \cos 50^\circ}$$

$$P \leq \frac{(2)(4 \times 10^{-3})(800 \times 10^3)}{1 + \cos 50^\circ}$$

$$P \leq 3.90 \times 10^3 \text{ N}$$

$$\tau = \frac{P}{2A} \sin 50^\circ$$

$$P = \frac{2A\tau}{\sin 50^\circ} \leq \frac{(2)(4 \times 10^{-3})(600 \times 10^3)}{\sin 50^\circ} = 6.27 \times 10^3 \text{ N}$$

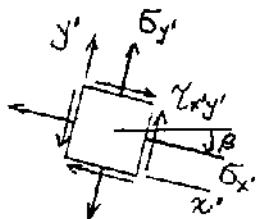
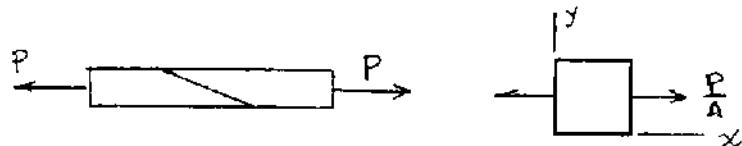
Choosing the smaller value $P \leq 3.90 \times 10^3 \text{ N} = 3.90 \text{ kN}$

Problem 7.43

7.43 Solve Prob. 7.21, using Mohr's circle.



7.21 Two wooden members of 80×120 -mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22^\circ$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load P that can be applied.



$$\sigma_x = \frac{P}{A}, \quad \sigma_y = 0, \quad \tau_{xy} = 0$$

Plotted points

$$X: \left(\frac{P}{A}, 0\right), \quad Y: (0, 0)$$

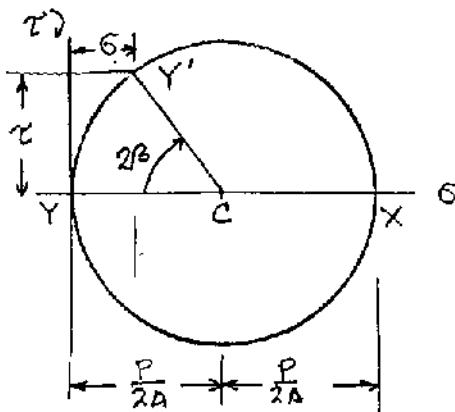
$$C: \left(\frac{P}{2A}, 0\right)$$

$$R = \overline{CX} = \frac{P}{2A}$$

Coordinates of point Y'

$$\sigma = \frac{P}{2A}(1 - \cos 2\beta), \quad \tau = \frac{P}{2A} \sin 2\beta$$

$$\text{Data: } A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$



If $\sigma = 400$ kPa = 400×10^3 Pa,

$$P = \frac{2A\sigma}{1 - \cos 2\beta} = \frac{(2)(9.6 \times 10^{-3})(400 \times 10^3)}{(1 - \cos 44^\circ)}$$

$$= 27.4 \times 10^3 \text{ N} = 27.4 \text{ kN}$$

$$\text{If } \tau = 600 \text{ kPa} = 600 \times 10^3 \text{ Pa,} \quad P = \frac{2A\tau}{\sin 2\beta} = \frac{(2)(9.6 \times 10^{-3})(600 \times 10^3)}{\sin 44^\circ}$$

$$= 16.58 \times 10^3 \text{ N} = 16.58 \text{ kN}$$

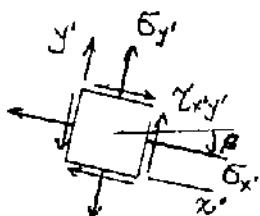
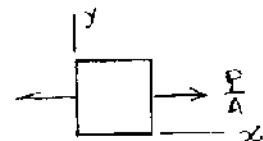
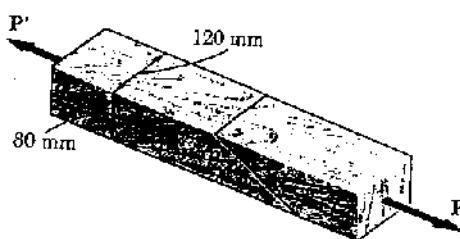
The smaller value of P governs.

$$P = 16.58 \text{ kN}$$

Problem 7.44

7.44 Solve Prob. 7.22, using Mohr's circle.

7.22 Two wooden members of 80×120 -mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25^\circ$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (a) the in-plane shearing stress parallel to the splice, (b) the normal stress perpendicular to the splice.



$$\sigma_x = \frac{P}{A}, \quad \sigma_y = 0, \quad \tau_{xy} = 0$$

Plotted points

$$X: \left(\frac{P}{A}, 0\right), \quad Y: (0, 0)$$

$$C: \left(\frac{P}{2A}, 0\right)$$

$$R = \overline{CX} = \frac{P}{2A}$$

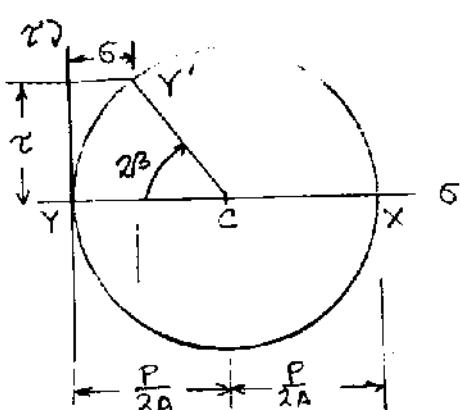
Coordinates of point Y'

$$\sigma = \frac{P}{2A}(1 - \cos 2\beta), \quad \tau = \frac{P}{2A} \sin 2\beta$$

$$\text{Data: } A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$(a) \tau = \frac{(10 \times 10^3) \sin 50^\circ}{(2)(9.6 \times 10^{-3})} = 399 \times 10^3 \text{ Pa} = 399 \text{ kPa}$$

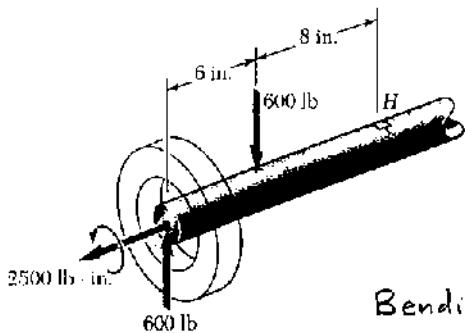
$$(b) \sigma = \frac{(10 \times 10^3)(1 - \cos 50^\circ)}{(2)(9.6 \times 10^{-3})} = 186.0 \times 10^3 \text{ Pa} = 186.0 \text{ kPa}$$



Problem 7.45

7.45 Solve Prob. 7.23, using Mohr's circle.

7.23 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 1.25 in., determine (a) the principal planes and principal stresses at point H located on top of the axle, (b) the maximum shearing stress at the same point.



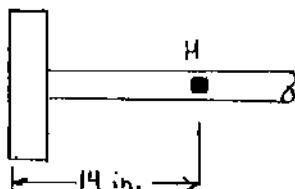
$$c = \frac{1}{2}d = \frac{1}{2}(1.25) = 0.625 \text{ in}$$

$$\begin{aligned} \text{Torsion: } \tau &= \frac{Tc}{J} = \frac{2T}{\pi c^3} \\ &= \frac{(2)(2500)}{\pi(0.625)^3} = 6.519 \times 10^3 \text{ psi} = 6.519 \text{ ksi} \end{aligned}$$

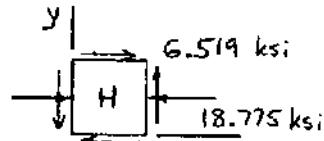
$$\text{Bending: } I = \frac{\pi c^4}{4} = 119.842 \times 10^{-3} \text{ in}^4$$

$$M = (6)(600) = 3600 \text{ lb-in.} \quad \sigma = -\frac{My}{I} = -\frac{(3600)(0.625)}{119.842 \times 10^{-3}} = -18.775 \times 10^3 \text{ psi} = -18.775 \text{ ksi}$$

Top view

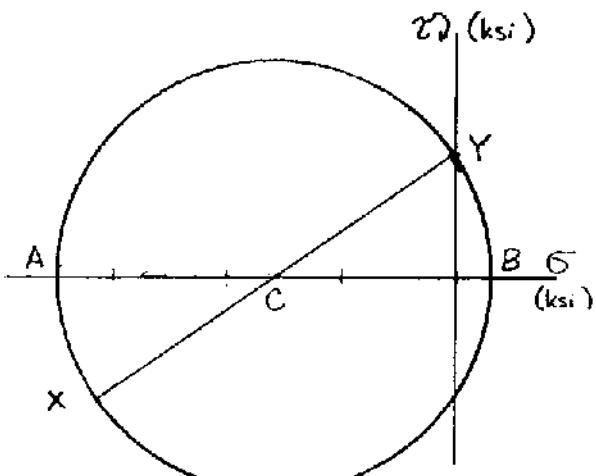


Stresses



$$\sigma_x = -18.775 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = 6.519 \text{ ksi}$$

Plotted points. $X: (-18.775, -6.519)$; $Y: (0, 6.519)$; $C(-9.3875, 0)$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -9.3875 \text{ ksi}$$

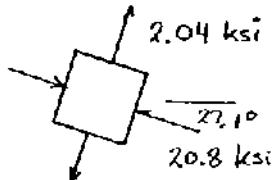
$$\begin{aligned} R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{\left(\frac{-18.775}{2}\right)^2 + (6.519)^2} \\ &= 11.429 \text{ ksi} \end{aligned}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6.519)}{-18.775} = -1.3889$$

$$(a) \quad \theta_a = 27.1^\circ, \quad \theta_b = 62.9^\circ$$

$$\sigma_a = \sigma_{ave} - R = -9.3875 - 11.429 = -20.8 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} + R = -9.3875 + 11.429 = 2.04 \text{ ksi}$$

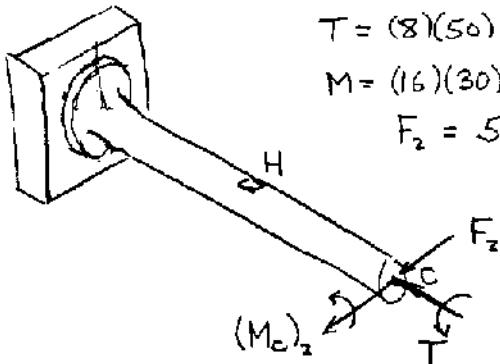


$$(b) \quad \tau_{max} = R = 11.43 \text{ ksi}$$

Problem 7.46

7.46 Solve Prob. 7.24, using Mohr's circle.

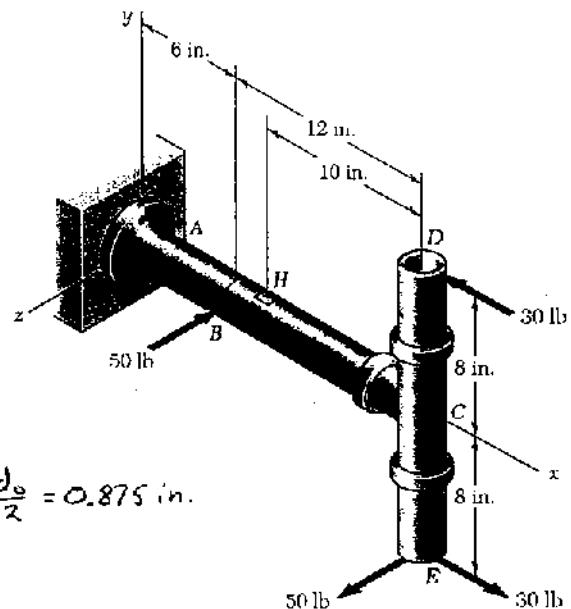
Replace forces on pipe DCE by an equivalent force-couple system at C.



$$T = (8)(50) = 400 \text{ lb-in}$$

$$M = (16)(30) = 480 \text{ lb-in}$$

$$F_z = 50 \text{ lb}$$



$$\text{Cross section. } c_1 = \frac{d_i}{2} = 0.750 \text{ in. } c_2 = \frac{d_o}{2} = 0.875 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 0.42376 \text{ in}^4$$

$$I = \frac{1}{2}J = 0.21188 \text{ in.}$$

$$Q_y = \frac{2}{3}(c_2^3 - c_1^3) = \frac{2}{3}(0.875^3 - 0.750^3) = 0.16536 \text{ in}^3$$

$$t = c_2 - c_1 = 0.125 \text{ in.}$$

At the section containing element H

$$T = 400 \text{ lb-in}, \quad M_z = 480 \text{ lb-in}, \quad V_z = 50 \text{ lb.}$$

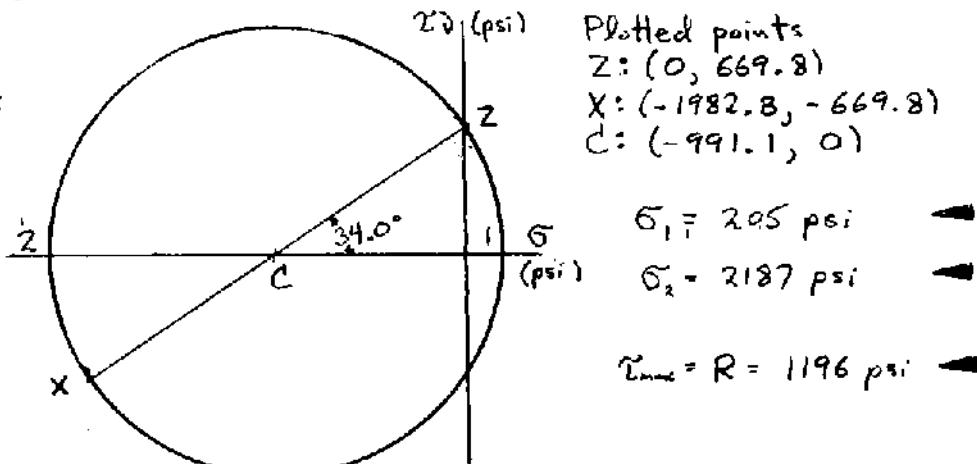
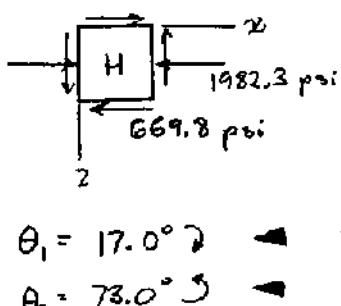
Stresses.

$$\text{Tension: } \sigma_{zx} = -\frac{Te}{J} = -\frac{(400)(0.875)}{0.42376} = -825.9 \text{ psi}$$

$$\text{Bending: } \sigma_x = -\frac{M_z y}{I} = -\frac{(480)(0.875)}{0.21188} = -1982.3 \text{ psi}$$

$$\text{Transverse Shear: } \tau_{zx} = \frac{V_z Q_y}{I(2t)} = \frac{(50)(0.16536)}{(0.21188)(0.250)} = 156.1 \text{ psi}$$

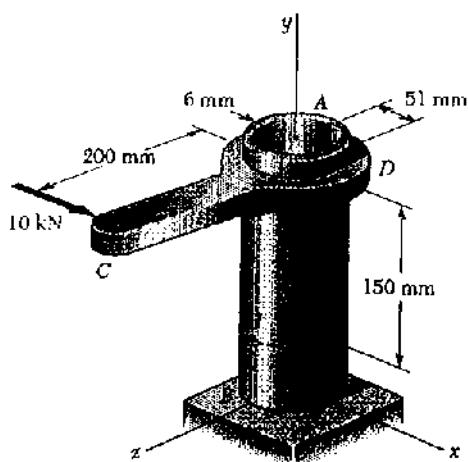
$$\text{Total: } \sigma_z = 0, \quad \sigma_x = -1982.3, \quad \tau_{zx} = -825.9 + 156.1 = -669.8 \text{ psi}$$



Problem 7.47

7.47 Solve Prob. 7.25, using Mohr's circle.

7.25 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point H .



$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{4}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

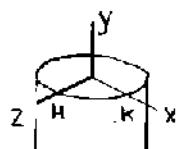
$$I = \frac{1}{2}J = 2.0927 \times 10^{-6} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K

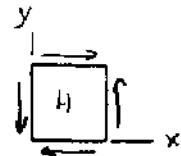
$$F_x = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$



Torsion: $T = M_y = 2000 \text{ N}\cdot\text{m}$
 $C = r_o = 51 \times 10^{-3} \text{ m}$



$$\tau_{xy} = \frac{TC}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \text{ MPa}$$



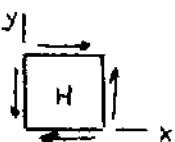
Transverse Shear:

For semicircle $A = \frac{\pi}{2} r^2$ $\bar{y} = \frac{4}{3\pi} r$
 $Q = A \bar{y} = \frac{2}{3} r^3$

For pipe $Q = Q_o - Q_i = \frac{4}{3} r_o^3 - \frac{2}{3} r_i^3 = 27.684 \times 10^3 \text{ mm}^3 = 27.684 \times 10^{-6} \text{ m}^3$

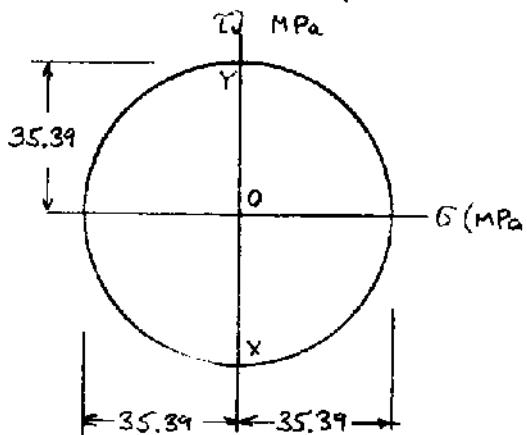
$$V = F_x = 10 \times 10^3 \text{ N} \quad t = (2)(6) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \text{ MPa}$$



Bending: Point H lies on neutral axis $\sigma_y = 0$

Total stresses at point H $\sigma_x = 0, \sigma_y = 0, \tau_{xy} = 24.37 + 11.02 = 35.39 \text{ MPa}$



$$\sigma_{ave} = 0$$

$$R = 35.39 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = 35.39 \text{ MPa}$$

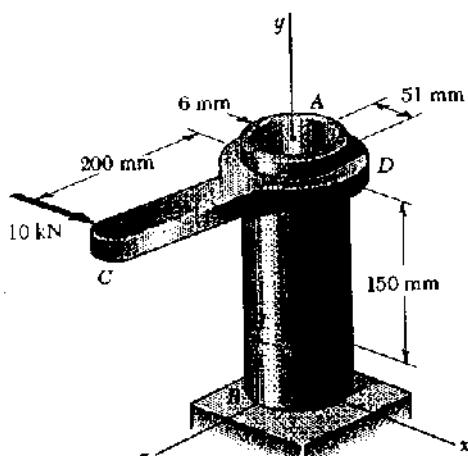
$$\sigma_{min} = \sigma_{ave} - R = -35.39 \text{ MPa}$$

$$\tau_{max} = R = 35.39 \text{ MPa}$$

Problem 7.48

7.48 Solve Prob. 7.26, using Mohr's circle.

7.26 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K .



$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-4} \text{ m}^4$$

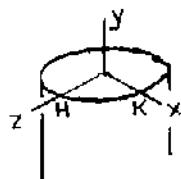
$$I = \frac{1}{2} J = 2.0927 \times 10^{-4} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K

$$F_x = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

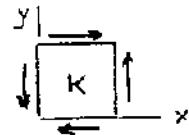
$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$



$$\text{Torsion: } T = M_y = 2000 \text{ N}\cdot\text{m}$$

$$C = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-4}} = 24.37 \text{ MPa}$$



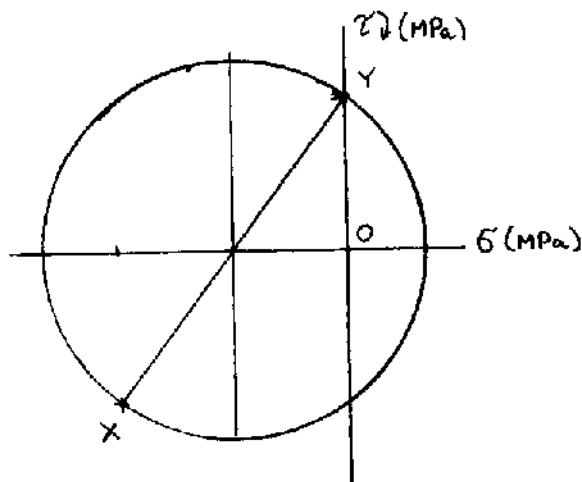
Note that local x -axis is taken along negative global z -direction.

Transverse Shear: Stress due to $V = F_x$ is zero at point K.

$$\text{Bending: } |\sigma_y| = \frac{|M_z|C}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-4}} = 36.56 \text{ MPa}$$

Point K lies on compression side of neutral axis. $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K $\sigma_x = 0$, $\sigma_y = -36.56 \text{ MPa}$, $\tau_{xy} = 24.37 \text{ MPa}$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

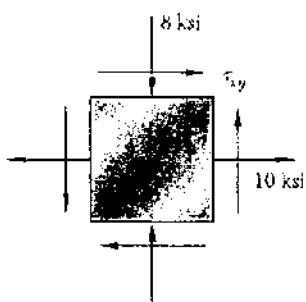
$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 = 12.18 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 = -48.74 \text{ MPa}$$

$$\tau_{max} = R = 30.46 \text{ MPa}$$

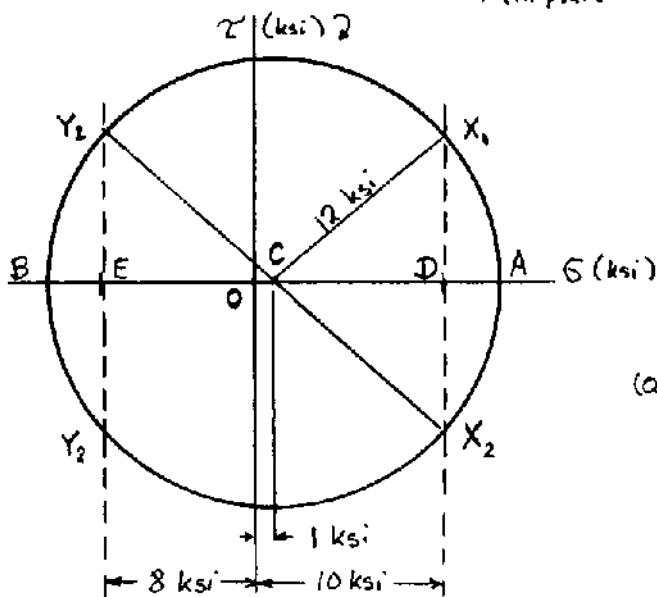
Problem 7.49

7.49 Solve Prob. 7.27, using Mohr's circle.



7.27 For the state of plane stress shown, determine (a) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (b) the corresponding principal stresses.

The center of the Mohr's circle lies at point C with coordinates $(\frac{\sigma_x + \sigma_y}{2}, 0) = (\frac{10 - 8}{2}, 0)$
 $= (1, 0)$ ksi, 0). The radius of the circle is
 $\tau_{max(in-plane)} = 12$ ksi.



The stress point $(\sigma_x, -\tau_{xy})$ lie along the line X_1X_2 of the Mohr circle diagram. The extreme points with $R \leq 12$ ksi are X_1 and X_2 .

(a) The largest allowable value of τ_{xy} is obtained from triangle CDX_1 ,

$$DX_1^2 = DX_2^2 = \sqrt{CX_1^2 - CD^2}$$

$$\tau_{xy}^2 = \sqrt{12^2 - 9^2} = 7.94 \text{ ksi}$$

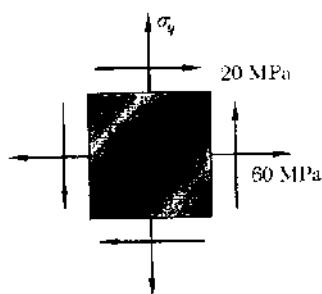
(b) The principal stresses are $\sigma_a = 1 + 12 = 13$ ksi

$$\sigma_b = 1 - 12 = -11 \text{ ksi}$$

Problem 7.50

7.50 Solve Prob. 7.28, using Mohr's circle.

7.28 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.



$$\text{Given: } \tau_{max} = R = 75 \text{ MPa}$$

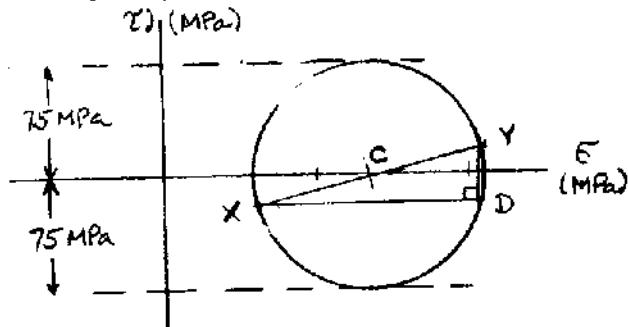
$$\bar{XY} = 2R = 150 \text{ MPa}$$

$$\bar{DY} = (2)(\tau_{xy}) = 40 \text{ MPa}$$

$$\bar{XD} = \sqrt{\bar{XY}^2 - \bar{DY}^2} = \sqrt{150^2 - 40^2} = 144.6 \text{ MPa}$$

$$\sigma_y = \sigma_x + \bar{XD} = 60 + 144.6 = 204.6 \text{ MPa}$$

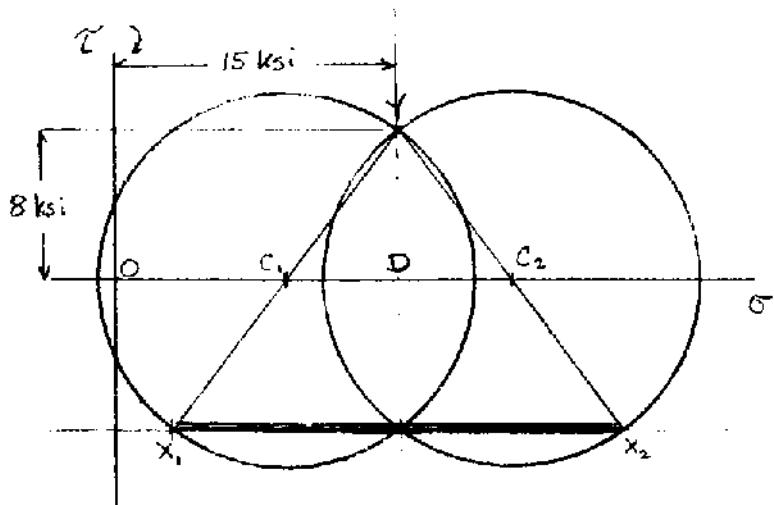
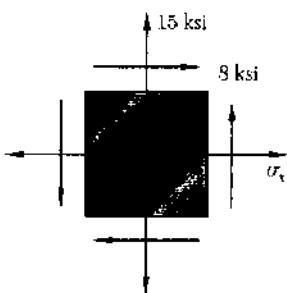
$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$



Problem 7.51

7.51 Solve Prob. 7.29, using Mohr's circle.

7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.



For the Mohr's circle, point Y lies at (15 ksi, 8 ksi).

The radius of limiting circles is $R = 10$ ksi.

Let C_1 be the location of the left most limiting circle and C_2 be that of the right most one.

$$\overline{CY} = 10 \text{ ksi}$$

$$\overline{C_2Y} = 10 \text{ ksi}$$

Noting right triangles C_1DY and C_2DY

$$\overline{C_1D}^2 + \overline{DY}^2 = \overline{CY}^2 \quad \overline{C_1D}^2 + 8^2 = 10^2 \quad \overline{C_1D} = 6 \text{ ksi}$$

Coordinates of point C_1 are $(0, 15 - 6) = (0, 9 \text{ ksi})$.

Likewise, coordinates of point C_2 are $= (0, 15 + 6) = (0, 21 \text{ ksi})$.

Coordinates of point X_1 $(9 - 6, -8) = (3 \text{ ksi}, -8 \text{ ksi})$.

Coordinates of point X_2 $(21 + 6, -8) = (27 \text{ ksi}, -8 \text{ ksi})$.

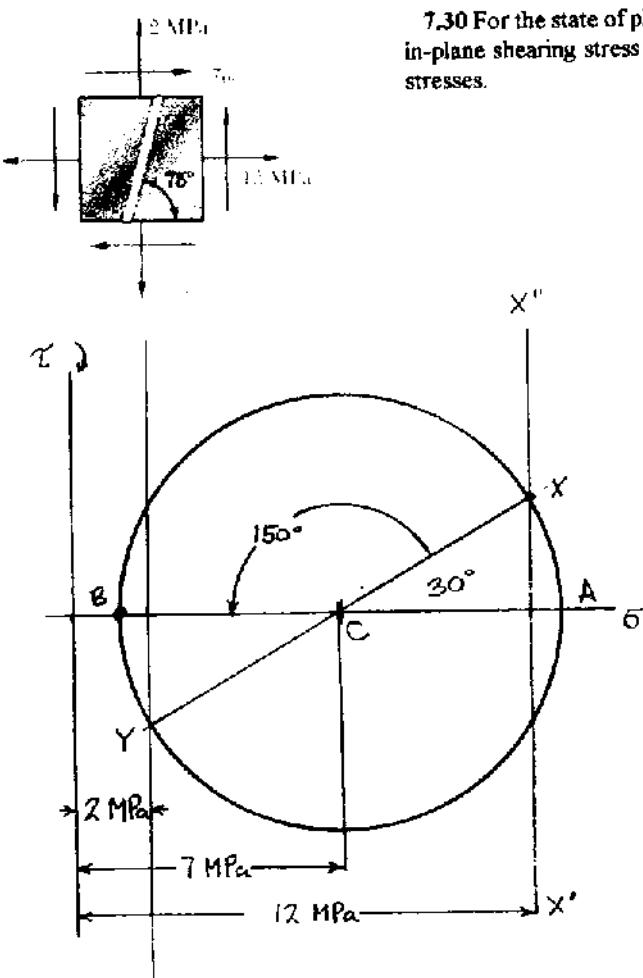
The point $(\bar{\sigma}_x, -\bar{\tau}_{xy})$ must lie on the line X_1X_2 .

Thus $3 \text{ ksi} \leq \bar{\sigma}_x \leq 27 \text{ ksi}$

Problem 7.52

7.52 Solve Prob. 7.30, using Mohr's circle.

7.30 For the state of plane stress shown, determine (a) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (b) the corresponding principal stresses.



Point X of Mohr's circle must lie on $X'X''$ so that $\sigma_x = 12 \text{ MPa}$. Likewise, point Y lies on line $Y'Y''$ so that $\sigma_y = 2 \text{ MPa}$. The coordinates of C are $\frac{2+12}{2}, 0 = (7 \text{ MPa}, 0)$.

Counterclockwise rotation through 150° brings line CX to CB, where $\gamma = 0$.

$$\begin{aligned} R &= \frac{\sigma_x - \sigma_y}{2} \sec 30^\circ \\ &= \frac{12 - 2}{2} \sec 30^\circ \\ &= 5.77 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \tau_{xy} &= \frac{\sigma_x - \sigma_y}{2} \tan 30^\circ \\ &= \frac{12 - 2}{2} \tan 30^\circ \\ &= -2.89 \text{ MPa} \end{aligned}$$

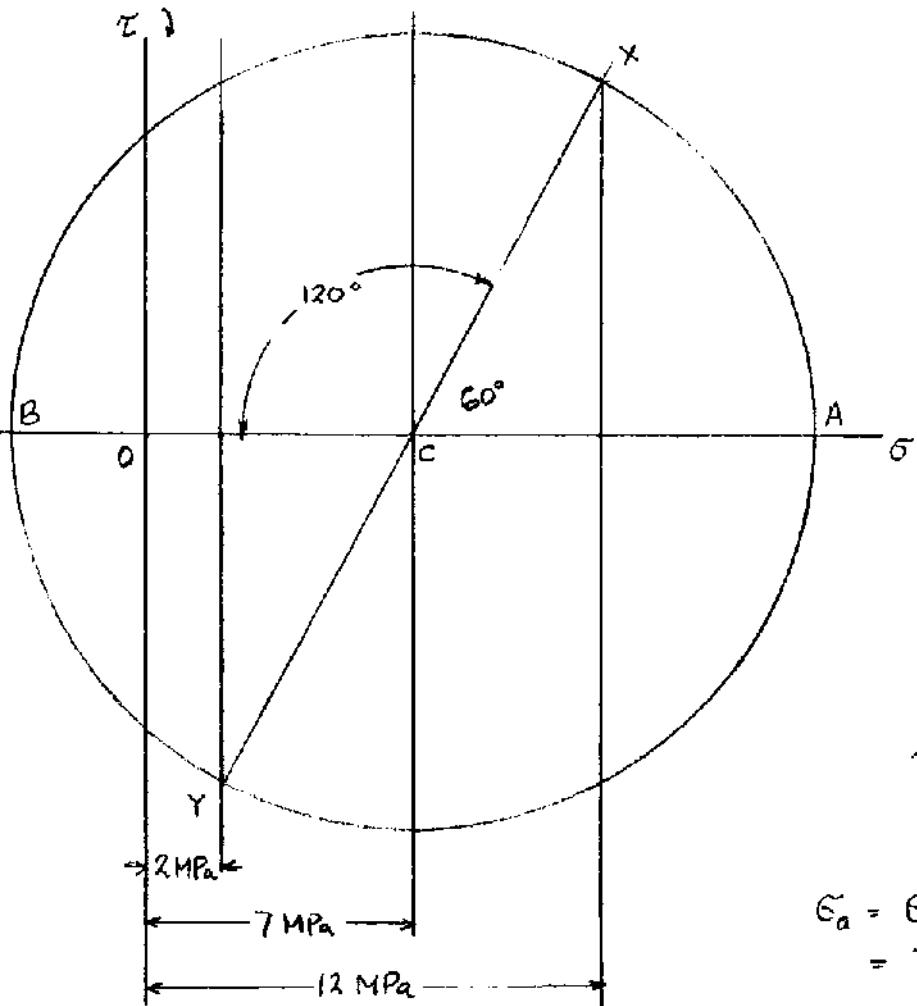
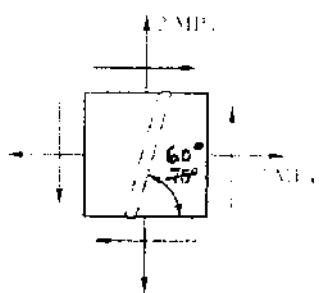
$$\sigma_A = \sigma_{ave} + R = 7 + 5.77 = 12.77 \text{ MPa}$$

$$\sigma_B = \sigma_{ave} - R = 7 - 5.77 = 1.23 \text{ MPa}$$

Problem 7.53

7.53 Solve Prob. 7.30, using Mohr's circle and assuming that the weld forms an angle of 60° with the horizontal.

7.30 For the state of plane stress shown, determine (a) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (b) the corresponding principal stresses.



$$\text{Locate point C at } \sigma = \frac{12+2}{2} = 7 \text{ MPa}$$

with $\gamma = 0$.

$$\text{Angle } XCB = 120^\circ$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{12 - 2}{2} = 5 \text{ MPa}$$

$$R = 5 \sec 60^\circ \\ = 10 \text{ MPa}$$

$$\tau_{xy} = -5 \tan 60^\circ \\ = -8.66 \text{ MPa}$$

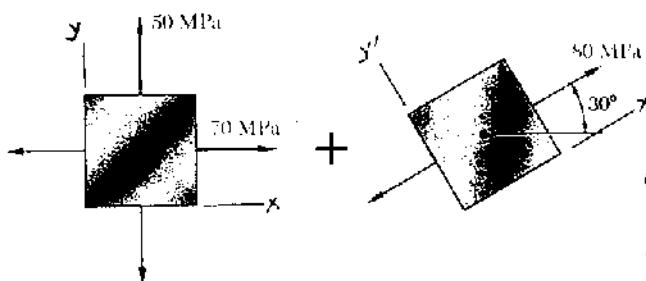
$$\tilde{\epsilon}_a = \tilde{\epsilon}_{ave} + R \\ = 7 + 10 = 17 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R$$

$$= 7 - 10 = -3 \text{ MPa}$$

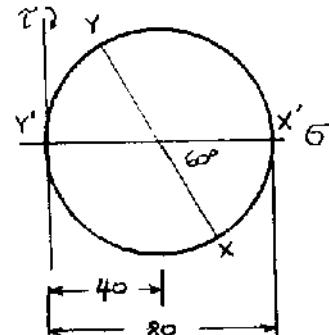
Problem 7.54

7.54 and 7.55 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.



Mohr's circle for 2nd stress state

$$\begin{aligned}\sigma_x &= 40 + 40 \cos 60^\circ \\ &= 60 \text{ MPa} \\ \sigma_y &= 40 - 40 \cos 60^\circ \\ &= 20 \text{ MPa} \\ \tau_{xy} &= 40 \sin 60^\circ \\ &= 34.64 \text{ MPa}\end{aligned}$$



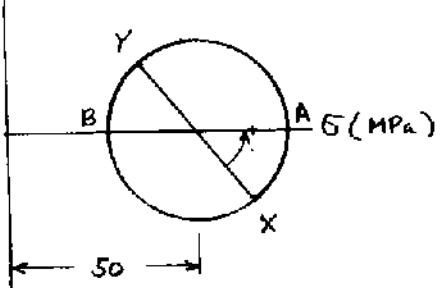
Resultant stresses

$$\sigma_x = 70 + 60 = 130 \text{ MPa}$$

$$\sigma_y = 50 + 20 = 70 \text{ MPa}$$

$$\tau_{xy} = 0 + 34.64 = 34.64 \text{ MPa}$$

σ (MPa)



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(130 + 70) = 100 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(34.64)}{130 - 70} = 1.1547$$

$$2\theta_p = 49.11^\circ \quad \theta_a = 24.6^\circ \quad \theta_b = 114.6^\circ$$

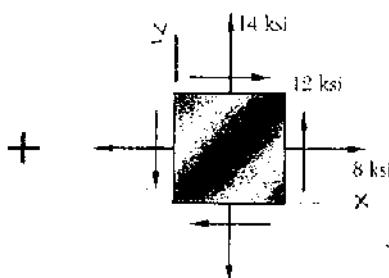
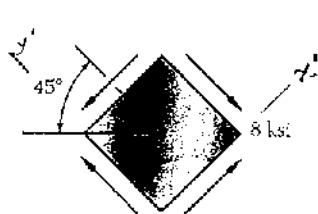
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 45.8 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 145.8 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 54.2 \text{ MPa}$$

Problem 7.55

7.54 and 7.55 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.



Mohr's circle for 1st stress state.

$$\sigma_x = 8 \text{ ksi}$$

$$\sigma_y = -8 \text{ ksi}$$

$$\tau_{xy} = 0$$

Resultant stresses

$$\sigma_x = 8 + 8 = 16 \text{ ksi}$$

$$\sigma_y = -8 + 14 = 6 \text{ ksi}$$

$$\tau_{xy} = 12 + 0 = 12 \text{ ksi}$$

90°

22 (ksi)

6 (ksi)

8

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 11 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(12)}{16 - 6} = 2.4$$

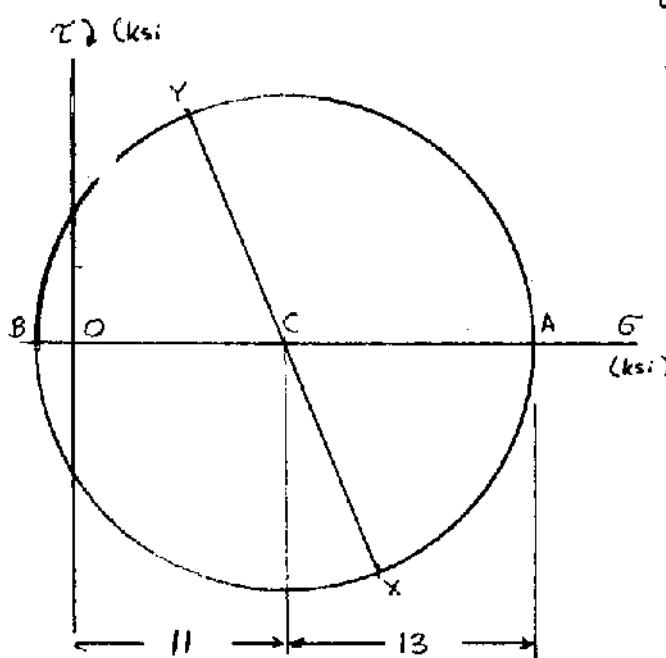
$$2\theta_p = 67.38^\circ \quad \theta_a = 33.69^\circ \quad \theta_b = 123.69^\circ$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{11^2 - 12^2} = 13 \text{ ksi}$$

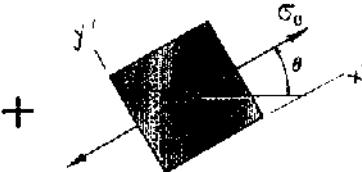
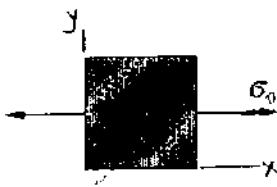
$$\sigma_a = \sigma_{ave} + R = 24 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -2 \text{ ksi}$$



Problem 7.56

7.56 and 7.57 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.

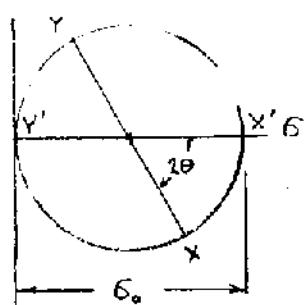


Mohr's circle for 2nd stress state

$$\sigma_x = \frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = \frac{1}{2}\sigma_0 \sin 2\theta$$



Resultant stresses

$$\sigma_x = \sigma_0 + \frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta = \frac{3}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = 0 + \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta = \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = 0 + \frac{1}{2}\sigma_0 \sin 2\theta = \frac{1}{2}\sigma_0 \sin 2\theta$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0 \cos 2\theta}$$

$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$$

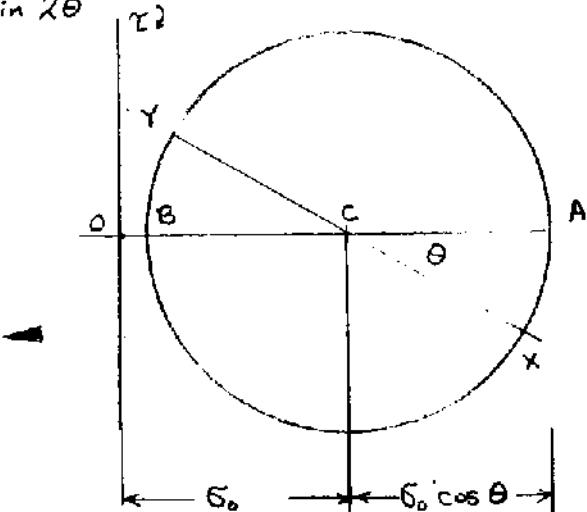
$$\theta_p = \frac{1}{2}\theta$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} =$$

$$= \sqrt{\left(\frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta\right)^2 + \left(\frac{1}{2}\sigma_0 \sin 2\theta\right)^2}$$

$$= \frac{1}{2}\sigma_0 \sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}$$

$$= \frac{\sqrt{2}}{2}\sigma_0 \sqrt{1 + \cos 2\theta} = \sigma_0 |\cos \theta|$$

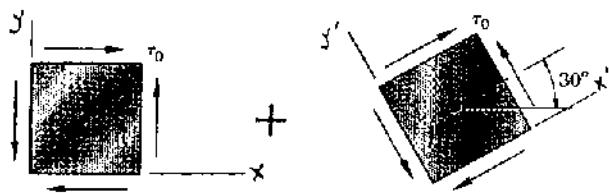


$$\sigma_a = \sigma_{ave} + R = \sigma_0 + \sigma_0 \cos \theta$$

$$\sigma_b = \sigma_{ave} - R = \sigma_0 - \sigma_0 \cos \theta$$

Problem 7.57

7.56 and 7.57 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress.



Mohr's circle for 2nd state of stress

$$\begin{aligned}\sigma_{x'} &= 0 \\ \sigma_{y'} &= 0 \\ \tau_{xy'} &= \tau_0\end{aligned}$$

$$\sigma_x = -\tau_0 \sin 60^\circ = -\frac{\sqrt{3}}{2} \tau_0$$

$$\sigma_y = \tau_0 \sin 60^\circ = \frac{\sqrt{3}}{2} \tau_0$$

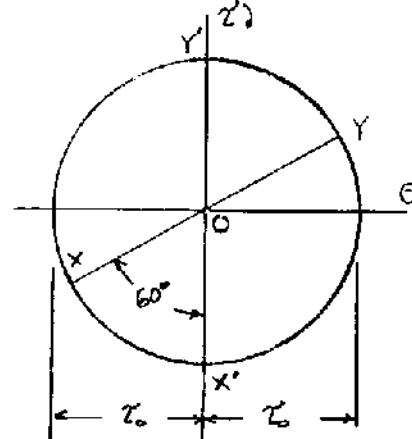
$$\tau_{xy} = \tau_0 \cos 60^\circ = \frac{1}{2} \tau_0$$

Resultant stresses

$$\sigma_x = 0 - \frac{\sqrt{3}}{2} \tau_0 = -\frac{\sqrt{3}}{2} \tau_0$$

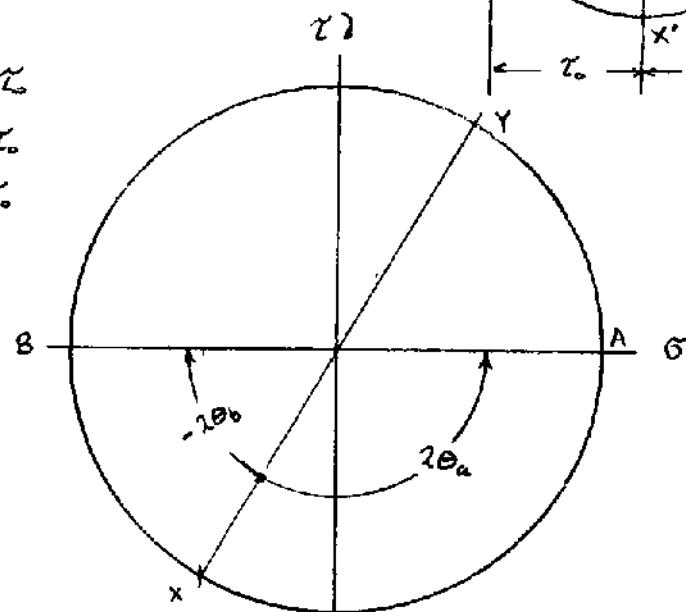
$$\sigma_y = 0 + \frac{\sqrt{3}}{2} \tau_0 = \frac{\sqrt{3}}{2} \tau_0$$

$$\tau_{xy} = \tau_0 + \frac{1}{2} \tau_0 = \frac{3}{2} \tau_0$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$\begin{aligned}R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{\left(\frac{\sqrt{3}}{2} \tau_0\right)^2 + \left(\frac{3}{2} \tau_0\right)^2} \\ &= \sqrt{3} \tau_0\end{aligned}$$

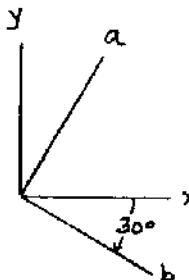


$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \cdot \frac{3}{2}}{-\sqrt{3}} = -\sqrt{3}$$

$$2\theta_p = -60^\circ \quad \theta_b = -30^\circ \quad \theta_a = 60^\circ$$

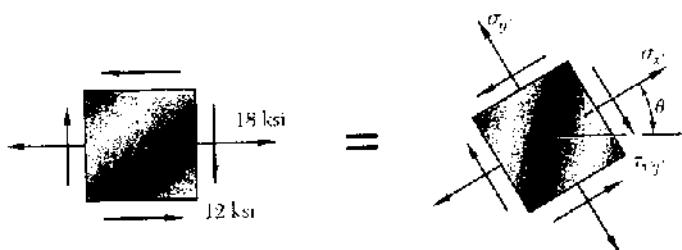
$$\sigma_a = \sigma_{ave} + R = \sqrt{3} \tau_0$$

$$\sigma_b = \sigma_{ave} - R = -\sqrt{3} \tau_0$$



Problem 7.58

7.58 For the state of stress shown, determine the range of values of θ for which the normal stress σ_x is equal to or less than 20 ksi.



$$\sigma_x = 18 \text{ ksi}, \tau_y = 0 \\ \tau_{xy} = -12 \text{ ksi}$$

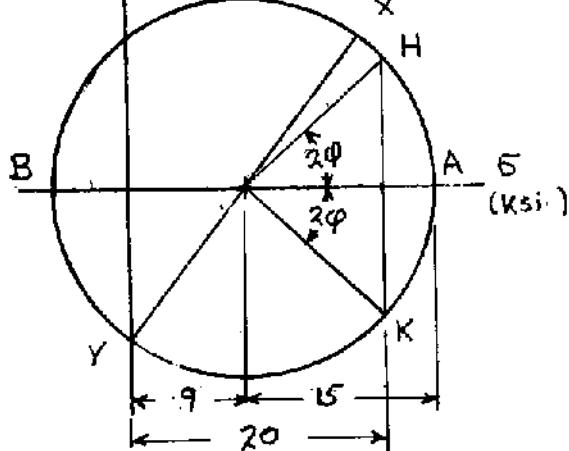
$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 9 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ = \sqrt{9^2 + 12^2} = 15 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-12)}{18 - 9} = -\frac{4}{3}$$

$$2\theta_p = -53.13^\circ$$

$$\theta_a = -26.565^\circ$$



$\sigma_x \leq 20 \text{ ksi}$ for states of stress corresponding to arc HBK of Mohr's circle. From the circle

$$R \cos 2\phi = 20 - 9 = 11 \text{ ksi}$$

$$\cos 2\phi = \frac{11}{15} = 0.73333$$

$$2\phi = 42.833^\circ \quad \phi = 21.417^\circ$$

$$\theta_h = \theta_a + \phi = -26.565^\circ + 21.417^\circ = -5.15^\circ$$

$$2\theta_k = 2\theta_h + 360^\circ - 4\phi = -10.297^\circ + 360^\circ - 85.666^\circ = 264.037^\circ$$

$$\theta_k = 132.02^\circ$$

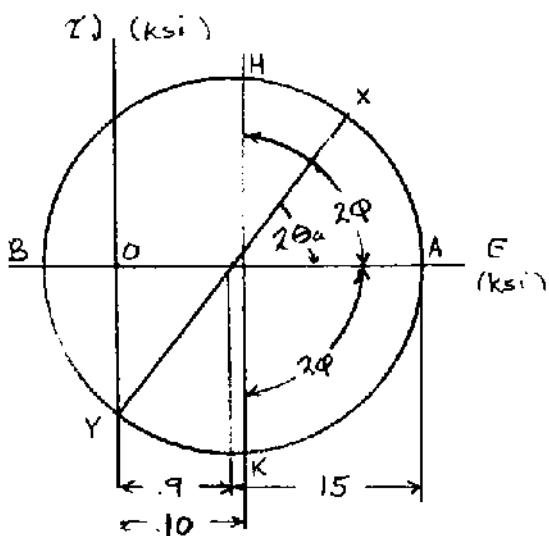
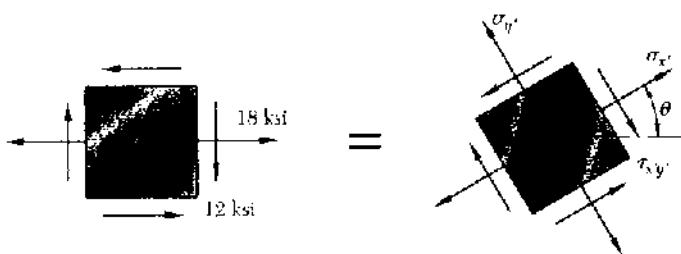
Permissible range of θ is $\theta_h \leq \theta \leq \theta_k$

$$-5.15^\circ \leq \theta \leq 132.02^\circ$$

$$\text{Also } 174.85^\circ \leq \theta \leq 312.02^\circ$$

Problem 7.59

7.59 For the state of stress shown, determine the range of values of θ for which the normal stress σ_z is equal to or less than 10 ksi.



$$\sigma_x = 18 \text{ ksi}, \quad \sigma_y = 0 \\ \tau_{xy} = -12 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = 9 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{9^2 + 12^2} = 15 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-12)}{18} = -\frac{4}{3}$$

$$2\theta_p = -53.13^\circ$$

$$\theta_a = -26.565^\circ$$

$\sigma_z \leq 10 \text{ ksi}$ for states of stress corresponding to the arc HK of Mohr's circle. From the circle

$$R \cos 2\phi = 10 - 9 = 1 \text{ ksi}$$

$$\cos 2\phi = \frac{1}{15} = 0.066667$$

$$2\phi = 86.177^\circ \quad \phi = 43.089^\circ$$

$$\theta_H = \theta_a + \phi = -26.565^\circ + 43.089^\circ = 16.524^\circ$$

$$2\theta_K = 2\theta_H + 360^\circ - 4\phi = 32.524^\circ + 360^\circ - 172.355^\circ = 220.169^\circ$$

$$\theta_K = 110.085^\circ$$

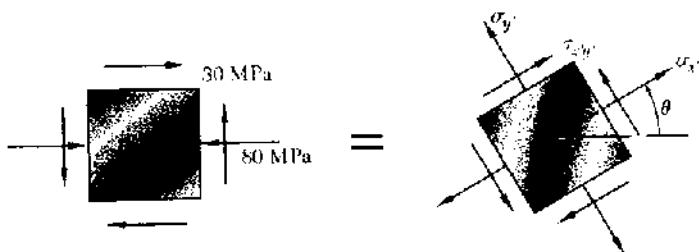
Permissible range of θ is $\theta_H \leq \theta \leq \theta_K$

$$16.524^\circ \leq \theta \leq 110.085^\circ$$

$$\text{Also } 196.524^\circ \leq \theta \leq 290.085^\circ$$

Problem 7.60

7.60 For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress τ_{xy} is equal to or less than 40 MPa.



$$\sigma_x = -80 \text{ MPa}, \quad \sigma_y = 0 \\ \tau_{xy} = 30 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = -40 \text{ MPa}$$

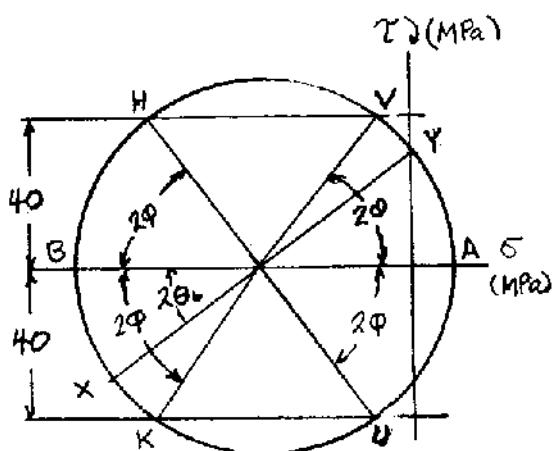
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-40)^2 + 30^2} = 50 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(30)}{-80} = -0.75$$

$$2\theta_p = -36.870^\circ$$

$$\theta_b = -18.435^\circ$$



$|\tau_{xy}| \leq 40 \text{ MPa}$ for states of stress corresponding to arcs HBK and UAV of Mohr's circle. The angle ϕ is calculated from

$$R \sin 2\phi = 40$$

$$\sin 2\phi = \frac{40}{50} = 0.8$$

$$2\phi = 53.130^\circ \quad \phi = 26.565^\circ$$

$$\theta_H = \theta_b - \phi = -18.435^\circ - 26.565^\circ = -45^\circ$$

$$\theta_K = \theta_b + \phi = -18.435 + 26.565^\circ = 8.13^\circ$$

$$\theta_U = \theta_H + 90^\circ = 45^\circ$$

$$\theta_V = \theta_K + 90^\circ = 98.13^\circ$$

Permissible ranges of θ $\theta_H \leq \theta \leq \theta_K$

$$-45^\circ \leq \theta \leq 8.13^\circ$$

$$\theta_U \leq \theta \leq \theta_V$$

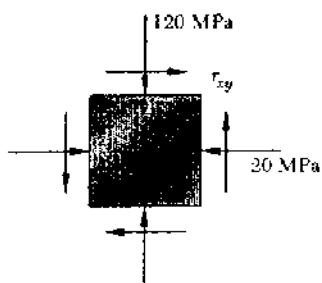
$$45^\circ \leq \theta \leq 98.13^\circ$$

$$\text{Also } 135^\circ \leq \theta \leq 188.13^\circ$$

$$225^\circ \leq \theta \leq 278.13^\circ$$

Problem 7.61

7.61 For the element shown, determine the range of values of τ_{xy} for which the maximum tensile stress is equal to or less than 60 MPa.



$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -70 \text{ MPa}$$

$$\text{Set } \sigma_{max} = 60 \text{ MPa} = \sigma_{ave} + R$$

$$R = \sigma_{max} - \sigma_{ave} = 130 \text{ MPa}$$

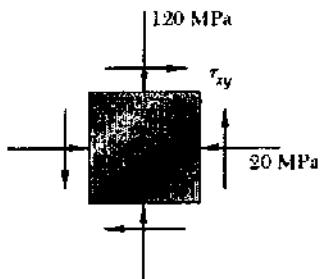
$$\text{But } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{130^2 - 50^2} = 120 \text{ MPa}$$

$$\text{Range of } \tau_{xy} \quad -120 \text{ MPa} \leq \tau_{xy} \leq 120 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.62

7.62 For the element shown, determine the range of values of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 150 MPa.



$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = 50 \text{ MPa}$$

$$\text{Set } \tau_{max(\text{in-plane})} = R = 150 \text{ MPa}$$

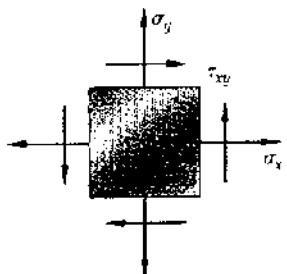
$$\text{But } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{150^2 - 50^2} = 141.4 \text{ MPa}$$

$$\text{Range of } \tau_{xy} \quad -141.4 \text{ MPa} \leq \tau_{xy} \leq 141.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.63

7.63 For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 14 \text{ ksi}$, $\sigma_y = 9 \text{ ksi}$, and $\sigma_{xy} = 5 \text{ ksi}$. Determine (a) the orientation of the principal planes, (b) the principal stress σ_{max} , (c) the maximum in-plane shearing stress.



$$\bar{\sigma}_x = 14 \text{ ksi}, \bar{\sigma}_y = 9 \text{ ksi}, \bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) = 11.5 \text{ ksi}$$

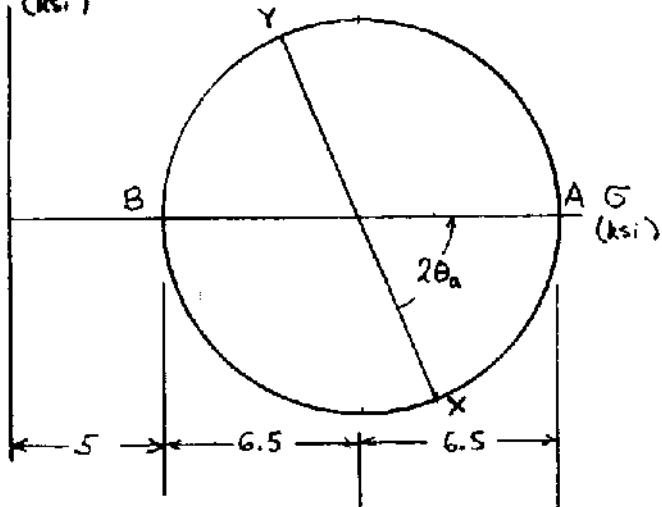
$$\bar{\sigma}_{min} = \bar{\sigma}_{ave} - R \quad \therefore R = \bar{\sigma}_{ave} - \bar{\sigma}_{min} \\ = 11.5 - 5 = 6.5 \text{ ksi}$$

$$R = \sqrt{(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2})^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm \sqrt{R^2 - (\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2})^2} = \pm \sqrt{6.5^2 - 2.5^2} = \pm 6 \text{ ksi}$$

But it is given that τ_{xy} is positive, thus $\tau_{xy} = +6 \text{ ksi}$

(a)



$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\bar{\sigma}_x - \bar{\sigma}_y} \\ = \frac{(2)(6)}{5} = 2.4$$

$$2\theta_p = 67.38^\circ$$

$$\theta_a = 33.69^\circ$$

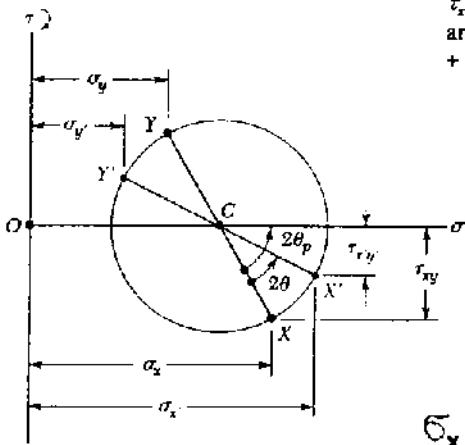
$$\theta_b = 123.69^\circ$$

$$(b) \sigma_{max} = \bar{\sigma}_{ave} + R \\ = 18 \text{ ksi}$$

$$(c) \tau_{max(in-plane)} = R \\ = 6.5 \text{ ksi}$$

Problem 7.64

7.64 The Mohr circle shown corresponds to the state of stress given in Fig. 7.5a and b. Noting that $\sigma_x = OC + (CX) \cos(2\theta_p - 2\theta)$ and that $\tau_{xy} = (CX') \sin(2\theta_p - 2\theta)$, derive the expressions for σ_x and τ_{xy} given in Eqs. (7.5) and (7.6), respectively. [Hint: Use $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and $\cos(A+B) = \cos A \cos B - \sin A \sin B$.]



$$\overline{OC} = \frac{1}{2}(\sigma_x + \sigma_y) \quad \overline{CX}' = \overline{CX}$$

$$\overline{CX}' \cos 2\theta_p = \overline{CX} \cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2}$$

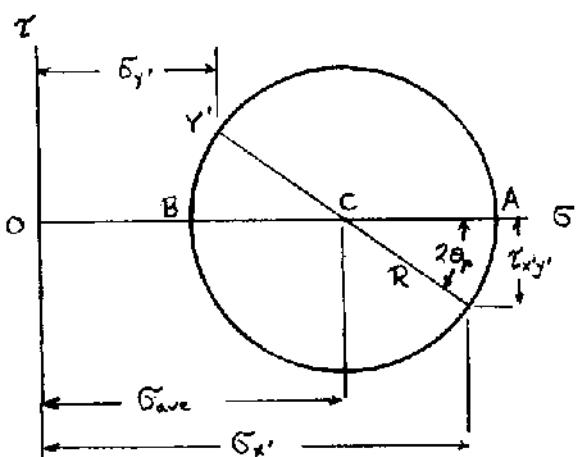
$$\overline{CX}' \sin 2\theta_p = \overline{CX} \sin 2\theta_p = \tau_{xy}$$

$$\begin{aligned}\sigma_x &= \overline{OC} + \overline{CX}' \cos(2\theta_p - 2\theta) \\ &= \overline{OC} + \overline{CX}' (\cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta) \\ &= \overline{OC} + \overline{CX}' \cos 2\theta_p \cos 2\theta + \overline{CX}' \sin 2\theta_p \sin 2\theta \\ &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta\end{aligned}$$

$$\begin{aligned}\tau_{xy} &= \overline{CX}' \sin(2\theta_p - 2\theta) = \overline{CX}' (\sin 2\theta_p \cos 2\theta - \cos 2\theta_p \sin 2\theta) \\ &= \overline{CX}' \sin 2\theta_p \cos 2\theta - \overline{CX}' \cos 2\theta_p \sin 2\theta \\ &= \tau_{xy} \cos 2\theta - \frac{\sigma_x - \sigma_y}{2} \sin 2\theta\end{aligned}$$

Problem 7.65

7.65 (a) Prove that the expression $\sigma_x \sigma_y - \tau_{xy}^2$. Where σ_x , σ_y , and τ_{xy} are components of the stress along the rectangular axes x' and y' , is independent of the orientation of these axes. Also, show that the given expression represents the square of the tangent drawn from the origin of coordinates of Mohr's circle. (b) Using the invariance property established in part (a), express the shearing stress τ_{xy} in terms of σ_x , σ_y , and the principal stresses σ_a and σ_b .



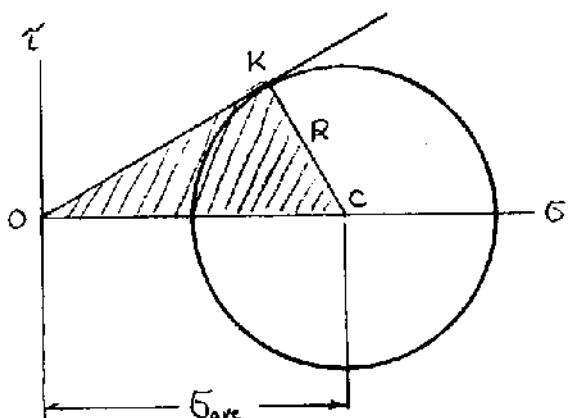
(a) From Mohr's circle

$$\tau_{xy'} = R \sin 2\theta_p$$

$$\sigma_{x'} = \sigma_{ave} + R \cos 2\theta_p$$

$$\sigma_{y'} = \sigma_{ave} - R \cos 2\theta_p$$

$$\begin{aligned}\sigma_x \sigma_y - \tau_{xy}^2 &= \sigma_{ave}^2 - R^2 \cos^2 2\theta_p - R^2 \sin^2 2\theta_p \\ &= \sigma_{ave}^2 - R^2 \\ &= \sigma_{ave}^2 - R^2; \text{ independent of } \theta_p.\end{aligned}$$



Draw line \overline{OK} from origin tangent to the circle at K . Triangle OCK is a right triangle

$$\overline{OC}^2 = \overline{OK}^2 + \overline{EK}^2$$

$$\overline{OK}^2 = \overline{OC}^2 - \overline{EK}^2$$

$$= \sigma_{ave}^2 - R^2$$

$$= \sigma_x \sigma_y - \tau_{xy}^2$$

(b) Applying above to σ_x , σ_y , and τ_{xy} and to σ_a , σ_b ,

$$\sigma_x \sigma_y - \tau_{xy}^2 = \sigma_a \sigma_b - \tau_{ab}^2 = \sigma_{ave}^2 - R^2$$

But $\tau_{ab} = 0$, $\sigma_a = \sigma_{max}$, $\sigma_b = \sigma_{min}$

$$\sigma_x \sigma_y - \tau_{xy}^2 = \sigma_{max} \sigma_{min}$$

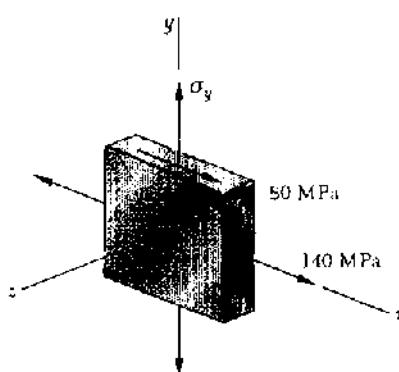
$$\tau_{xy}^2 = \sigma_x \sigma_y - \sigma_{max} \sigma_{min}$$

$$\tau_{xy} = \pm \sqrt{\sigma_x \sigma_y - \sigma_{max} \sigma_{min}}$$

The sign cannot be determined from above equations.

Problem 7.66

7.66 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_y = 20 \text{ MPa}$, (b) $\sigma_y = 140 \text{ MPa}$. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



$$(a) \quad \sigma_x = 140 \text{ MPa}, \sigma_y = 20 \text{ MPa}$$

$$\tau_{xy} = 80 \text{ MPa}$$

$$\begin{aligned} \sigma_{ave} &= \frac{1}{2}(\sigma_x + \sigma_y) \\ &= 80 \text{ MPa} \end{aligned}$$

$$\begin{aligned} R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{60^2 + 80^2} = 100 \text{ MPa} \end{aligned}$$

$$\sigma_a = \sigma_{ave} + R = 80 + 100 = 180 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = 80 - 100 = -20 \text{ MPa} \quad (\text{min})$$

$$\sigma_c = 0$$

$$\tau_{\text{max(in-plane)}} = \frac{1}{2}(\sigma_a - \sigma_b) = 100 \text{ MPa}$$

$$\tau_{\text{max}} = \frac{1}{2}(\sigma_{\text{max}} - \sigma_{\text{min}}) = 100 \text{ MPa}$$

$$(b) \quad \sigma_x = 140 \text{ MPa}, \quad \sigma_y = 140 \text{ MPa}$$

$$\tau_{xy} = 80 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 140 \text{ MPa}$$

$$\begin{aligned} R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{0 + 80^2} = 80 \text{ MPa} \end{aligned}$$

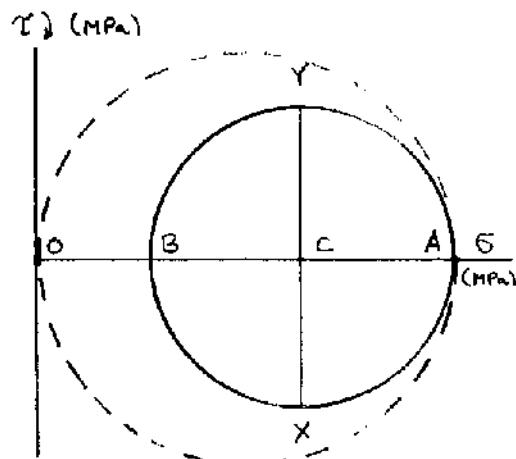
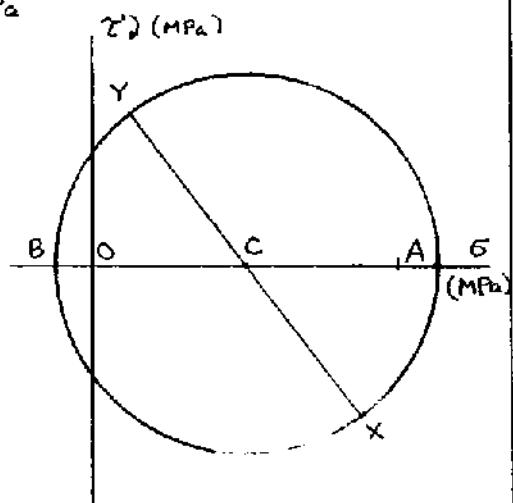
$$\sigma_a = \sigma_{ave} + R = 220 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = 60 \text{ MPa}$$

$$\sigma_c = 0 \quad (\text{min})$$

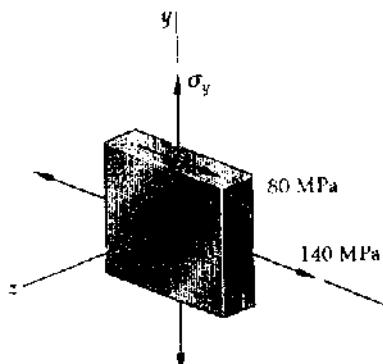
$$\tau_{\text{max(in-plane)}} = \frac{1}{2}(\sigma_a - \sigma_b) = 80 \text{ MPa}$$

$$\tau_{\text{max}} = \frac{1}{2}(\sigma_{\text{max}} - \sigma_{\text{min}}) = 110 \text{ MPa}$$



Problem 7.67

7.67 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_y = 40 \text{ MPa}$, (b) $\sigma_y = 120 \text{ MPa}$. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



$$(a) \bar{\sigma}_x = 140 \text{ MPa} \quad \bar{\sigma}_y = 40 \text{ MPa} \quad \tau_{xy} = 80 \text{ MPa}$$

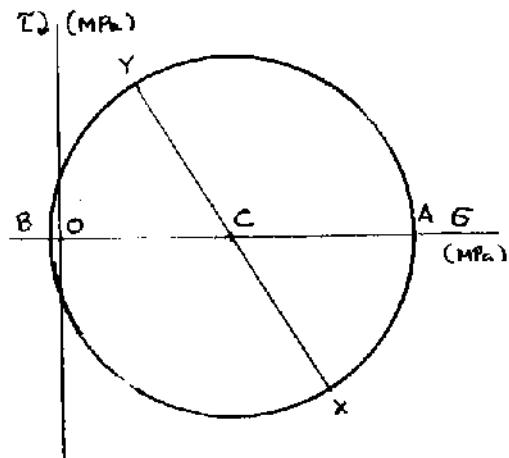
$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) \\ = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} \\ = \sqrt{50^2 + 80^2} \\ = 94.34 \text{ MPa}$$

$$\sigma_a = \bar{\sigma}_{ave} + R = 184.34 \text{ MPa} \quad (\max)$$

$$\sigma_b = \bar{\sigma}_{ave} - R = -4.34 \text{ MPa} \quad (\min)$$

$$\sigma_c = 0$$



$$\tau_{max(in-plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.34 \text{ MPa} \quad \blacksquare$$

$$(b) \bar{\sigma}_x = 140 \text{ MPa}, \bar{\sigma}_y = 120 \text{ MPa}, \tau_{xy} = 80 \text{ MPa}$$

$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} \\ = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$

$$\sigma_a = \bar{\sigma}_{ave} + R = 210.62 \text{ MPa} \quad (\max)$$

$$\sigma_b = \bar{\sigma}_{ave} - R = 49.38 \text{ MPa}$$

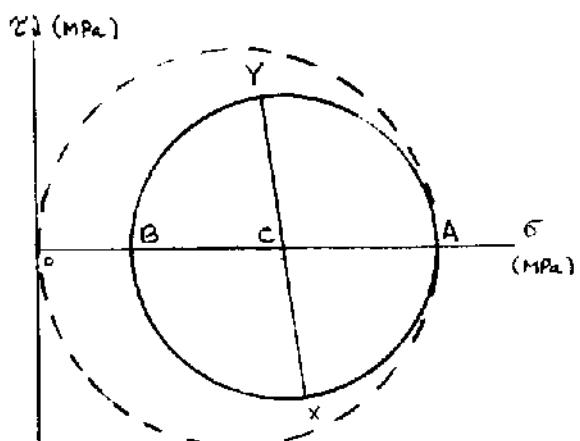
$$\sigma_c = 0 \quad (\min)$$

$$\sigma_{max} = \sigma_a = 210.62 \text{ MPa}$$

$$\sigma_{min} = \sigma_c = 0$$

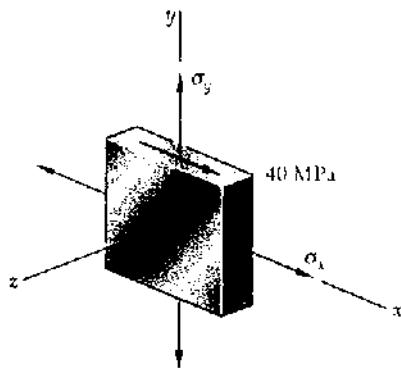
$$\tau_{max(in-plane)} = R = 80.62 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 105.31 \text{ MPa} \quad \blacksquare$$



Problem 7.68

7.68 For the state of plane stress shown, determine the maximum shearing stress when (a) $\sigma_x = 30 \text{ MPa}$ and $\sigma_y = 90 \text{ MPa}$, (b) $\sigma_x = 70 \text{ MPa}$ and $\sigma_y = 10 \text{ MPa}$. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



$$(a) \bar{\sigma}_x = 30 \text{ MPa} \quad \bar{\sigma}_y = 90 \text{ MPa} \quad \tau_{xy} = 40 \text{ MPa}$$

$$\bar{\sigma}_{ave} = \frac{1}{2} (\bar{\sigma}_x + \bar{\sigma}_y)$$

$$= 60 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{30^2 + 40^2}$$

$$= 50 \text{ MPa}$$

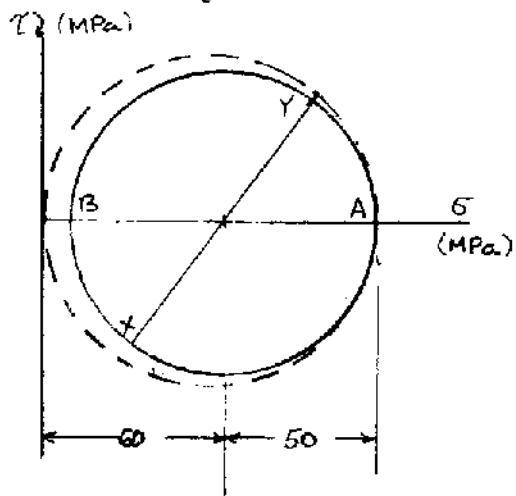
$$\sigma_a = \bar{\sigma}_{ave} + R = 60 + 50 = 110 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \bar{\sigma}_{ave} - R = 60 - 50 = 10 \text{ MPa}$$

$$\sigma_c = 0 \quad (\text{min})$$

$$\tau_{\text{max(in-plane)}} = R = 50 \text{ MPa}$$

$$\tau_{\text{max}} = \frac{1}{2} (\sigma_{\text{max}} - \sigma_{\text{min}}) = 55 \text{ MPa} \quad \blacktriangleleft$$



$$(b) \bar{\sigma}_x = 70 \text{ MPa} \quad \bar{\sigma}_y = 10 \text{ MPa} \quad \tau_{xy} = 40 \text{ MPa}$$

$$\bar{\sigma}_{ave} = \frac{1}{2} (\bar{\sigma}_x + \bar{\sigma}_y) = 40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{30^2 + 40^2} = 50 \text{ MPa}$$

$$\sigma_a = \bar{\sigma}_{ave} + R = 90 \text{ MPa} \quad (\text{max})$$

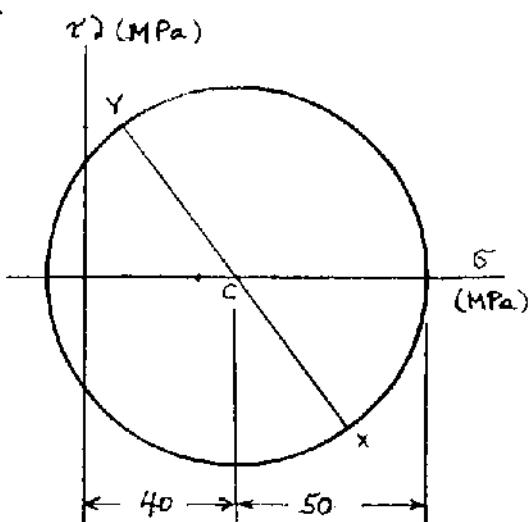
$$\sigma_b = \bar{\sigma}_{ave} - R = -10 \text{ MPa} \quad (\text{min})$$

$$\sigma_c = 0$$

$$\sigma_{\text{max}} = 90 \text{ MPa}$$

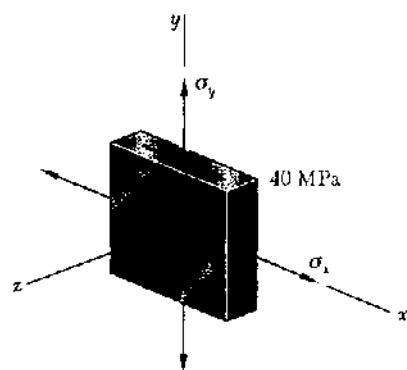
$$\sigma_{\text{min}} = -10 \text{ MPa}$$

$$\tau_{\text{max}} = \frac{1}{2} (\sigma_{\text{max}} - \sigma_{\text{min}}) = 50 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.69

7.69 For the state of plane stress shown, determine the maximum shearing stress when (a) $\sigma_x = 0$ and $\sigma_y = 60 \text{ MPa}$, (b) $\sigma_x = 105 \text{ MPa}$ and $\sigma_y = 45 \text{ MPa}$. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



$$(a) \bar{\sigma}_x = 0, \bar{\sigma}_y = 60 \text{ MPa}, \tau_{xy} = 40 \text{ MPa}$$

$$\bar{\sigma}_{ave} = \frac{1}{2} (\bar{\sigma}_x + \bar{\sigma}_y) \\ = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} \\ = \sqrt{(-30)^2 + 40^2} \\ = 50 \text{ MPa}$$

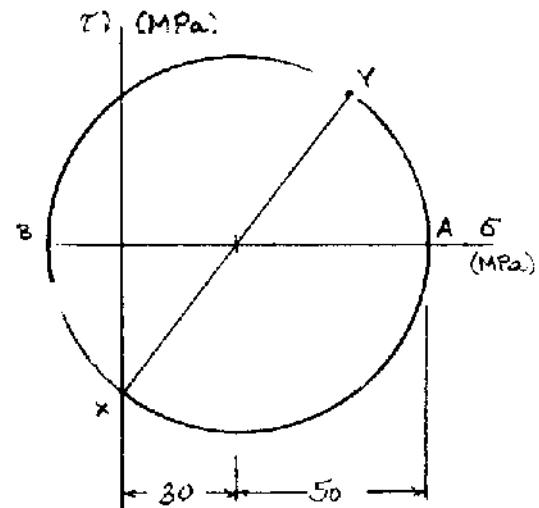
$$\sigma_a = \bar{\sigma}_{ave} + R = 80 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \bar{\sigma}_{ave} - R = -20 \text{ MPa} \quad (\text{min})$$

$$\sigma_c = 0$$

$$\sigma_{\text{max}} = 80 \text{ MPa} \quad \sigma_{\text{min}} = -20 \text{ MPa}$$

$$\tau_{\text{max}} = \frac{1}{2} (\sigma_{\text{max}} - \sigma_{\text{min}}) = 50 \text{ MPa} \quad \blacktriangleleft$$



$$(b) \bar{\sigma}_x = 105 \text{ MPa} \quad \bar{\sigma}_y = 45 \text{ MPa} \quad \tau_{xy} = 40 \text{ MPa}$$

$$\bar{\sigma}_{ave} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} \\ = \sqrt{(-30)^2 + 40^2} = 50 \text{ MPa}$$

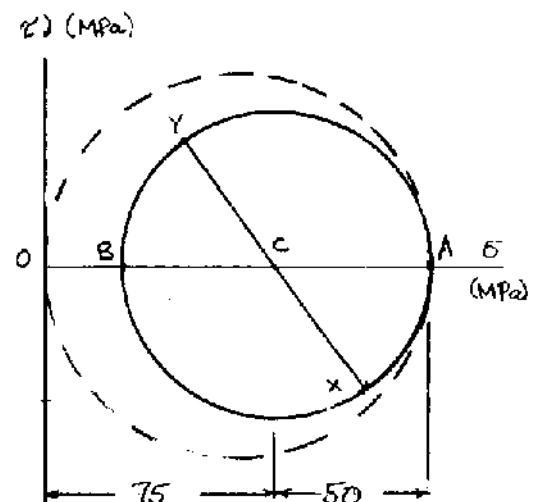
$$\sigma_a = \bar{\sigma}_{ave} + R = 125 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \bar{\sigma}_{ave} - R = 25 \text{ MPa}$$

$$\sigma_c = 0 \quad (\text{min})$$

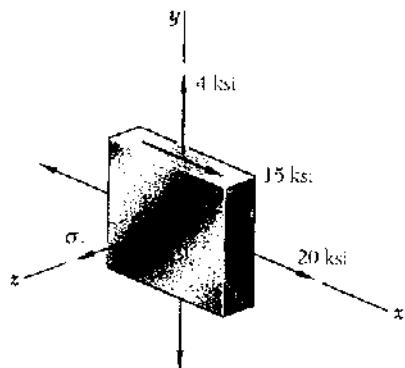
$$\sigma_{\text{max}} = 125 \text{ MPa}, \quad \sigma_{\text{min}} = 0$$

$$\tau_{\text{max}} = \frac{1}{2} (\sigma_{\text{max}} - \sigma_{\text{min}}) = 62.5 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.70

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = 0$, (b) $\sigma_z = +9$ ksi, (c) $\sigma_z = -9$ ksi.



$$\bar{\sigma}_x = 20 \text{ ksi}$$

$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) \\ = 12 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} \\ = \sqrt{8^2 + 15^2} \\ = 17 \text{ ksi}$$

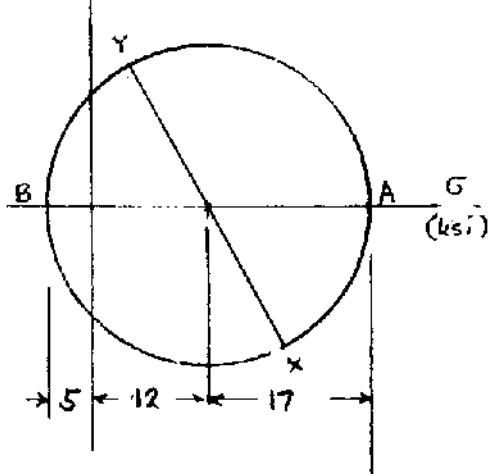
$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = 29 \text{ ksi}$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R = -5 \text{ ksi}$$

$$\bar{\sigma}_y = 4 \text{ ksi}$$

$$\bar{\tau}_{xy} = 15 \text{ ksi}$$

$$\tau_z (\text{ksi})$$



$$(a) \bar{\sigma}_z = 0, \bar{\sigma}_a = 29 \text{ ksi}, \bar{\sigma}_b = -5 \text{ ksi}$$

$$\sigma_{max} = 29 \text{ ksi}, \sigma_{min} = -5 \text{ ksi}, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 17 \text{ ksi}$$

$$(b) \sigma_z = +9 \text{ ksi}, \bar{\sigma}_a = 29 \text{ ksi}, \bar{\sigma}_b = -5 \text{ ksi}$$

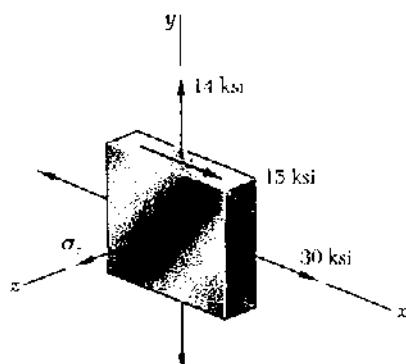
$$\sigma_{max} = 29 \text{ ksi}, \sigma_{min} = -5 \text{ ksi}, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 17 \text{ ksi}$$

$$(c) \sigma_z = -9 \text{ ksi}, \bar{\sigma}_a = 29 \text{ ksi}, \bar{\sigma}_b = -5 \text{ ksi}$$

$$\sigma_{max} = 29 \text{ ksi}, \sigma_{min} = -9 \text{ ksi}, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 19 \text{ ksi}$$

Problem 7.71

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = 0$, (b) $\sigma_z = +9$ ksi, (c) $\sigma_z = -9$ ksi.



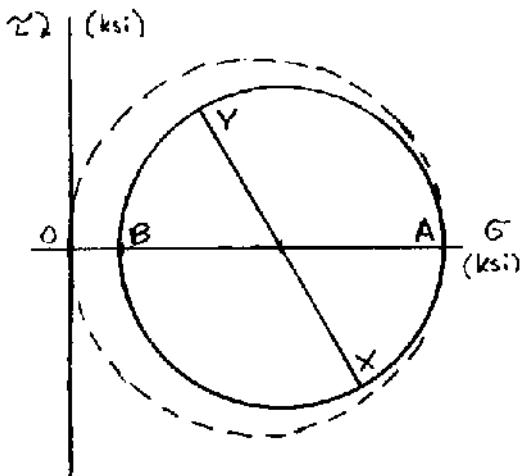
$$\bar{\sigma}_x = 30 \text{ ksi}, \bar{\sigma}_y = 14 \text{ ksi}, \tau_{xy} = 15 \text{ ksi}$$

$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) \\ = 22 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{30 - 14}{2}\right)^2 + 15^2} \\ = 17 \text{ ksi}$$

$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = 39 \text{ ksi}$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R = 5 \text{ ksi}$$



$$(a) \bar{\sigma}_z = 0, \bar{\sigma}_a = 39 \text{ ksi}, \bar{\sigma}_b = 5 \text{ ksi}$$

$$\bar{\sigma}_{max} = 39 \text{ ksi}, \bar{\sigma}_{min} = 0, \tau_{max} = \frac{1}{2}(\bar{\sigma}_{max} - \bar{\sigma}_{min}) = 19.5 \text{ ksi}$$

$$(b) \bar{\sigma}_z = 9 \text{ ksi}, \bar{\sigma}_a = 39 \text{ ksi}, \bar{\sigma}_b = 5 \text{ ksi}$$

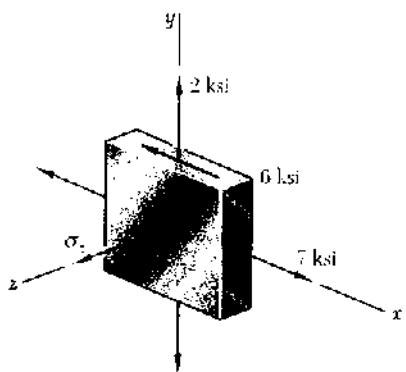
$$\bar{\sigma}_{max} = 39 \text{ ksi}, \bar{\sigma}_{min} = 5 \text{ ksi}, \tau_{max} = \frac{1}{2}(\bar{\sigma}_{max} - \bar{\sigma}_{min}) = 17 \text{ ksi}$$

$$(c) \bar{\sigma}_z = -9 \text{ ksi}, \bar{\sigma}_a = 39 \text{ ksi}, \bar{\sigma}_b = 5 \text{ ksi}$$

$$\bar{\sigma}_{max} = 39 \text{ ksi}, \bar{\sigma}_{min} = -9 \text{ ksi}, \tau_{max} = \frac{1}{2}(\bar{\sigma}_{max} - \bar{\sigma}_{min}) = 24 \text{ ksi}$$

Problem 7.72

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = +4$ ksi, (b) $\sigma_z = -4$ ksi, (c) $\sigma_z = 0$.



$$\sigma_x = 7 \text{ ksi}, \sigma_y = 2 \text{ ksi}, \tau_{xy} = -6 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 4.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{2.5^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 11 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -2 \text{ ksi}$$

$$(a) \sigma_z = 4 \text{ ksi}, \sigma_a = 11 \text{ ksi}, \sigma_b = -2 \text{ ksi}$$

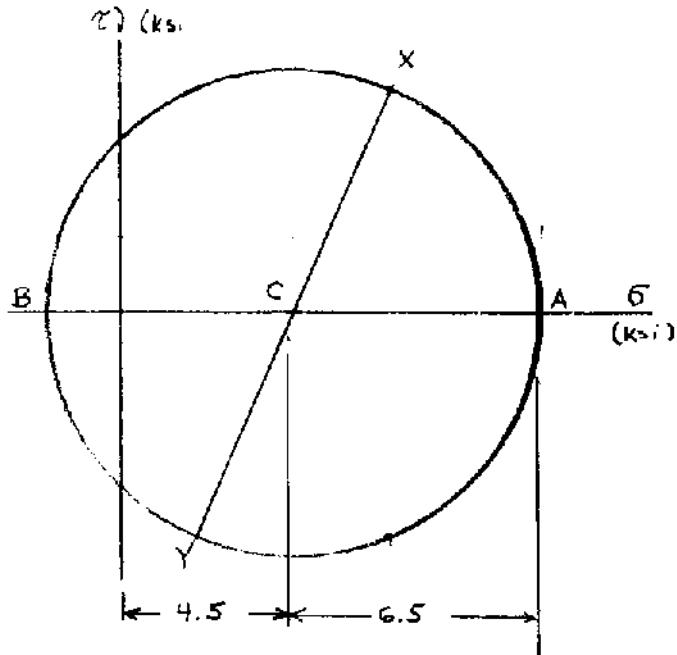
$$\sigma_{max} = 11 \text{ ksi}, \sigma_{min} = -2 \text{ ksi}, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5 \text{ ksi}$$

$$(b) \sigma_z = -4 \text{ ksi}, \sigma_a = 11 \text{ ksi}, \sigma_b = -2 \text{ ksi}$$

$$\sigma_{max} = 11 \text{ ksi}, \sigma_{min} = -4 \text{ ksi}, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5 \text{ ksi}$$

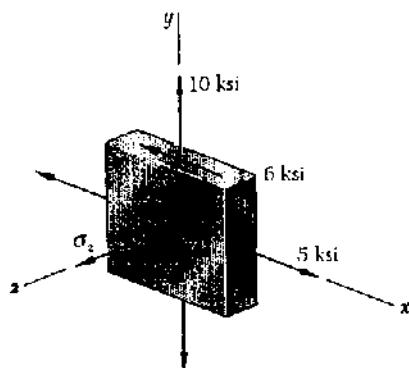
$$(c) \sigma_z = 0, \sigma_a = 11 \text{ ksi}, \sigma_b = -2 \text{ ksi}$$

$$\sigma_{max} = 11 \text{ ksi}, \sigma_{min} = -2 \text{ ksi}, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5 \text{ ksi}$$



Problem 7.73

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = +4 \text{ ksi}$, (b) $\sigma_z = -4 \text{ ksi}$, (c) $\sigma_z = 0$.



$$\sigma_x = 5 \text{ ksi}, \quad \sigma_y = 10 \text{ ksi}, \quad \tau_{xy} = -6 \text{ ksi}$$

$$\begin{aligned}\bar{\sigma}_{\text{ave}} &= \frac{1}{2}(\sigma_x + \sigma_y) = 7.5 \text{ ksi} \\ R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{(5 - 10)^2 + (-6)^2} = 6.5 \text{ ksi}\end{aligned}$$

$$\sigma_a = \bar{\sigma}_{\text{ave}} + R = 14 \text{ ksi}$$

$$\sigma_b = \bar{\sigma}_{\text{ave}} - R = 1 \text{ ksi}$$

$$(a) \quad \sigma_z = +4 \text{ ksi}, \quad \sigma_a = 14 \text{ ksi}, \quad \sigma_b = 1 \text{ ksi}$$

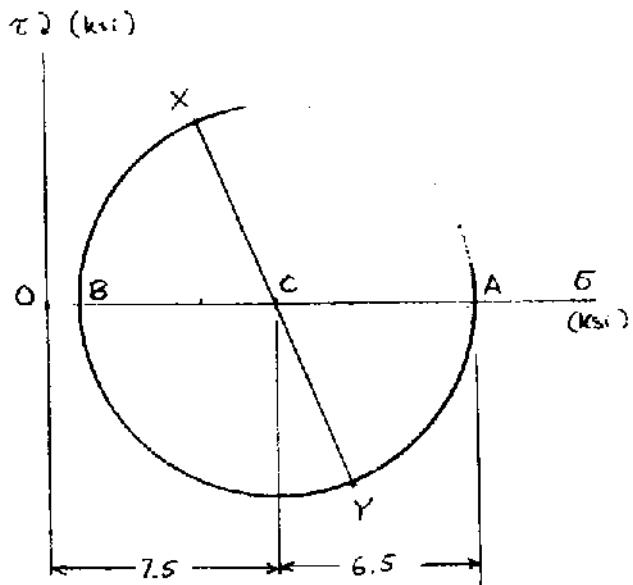
$$\sigma_{\max} = 14 \text{ ksi}, \quad \sigma_{\min} = 1 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 6.5 \text{ ksi}$$

$$(b) \quad \sigma_z = -4 \text{ ksi}, \quad \sigma_a = 14 \text{ ksi}, \quad \sigma_b = 1 \text{ ksi}$$

$$\sigma_{\max} = 14 \text{ ksi}, \quad \sigma_{\min} = -4 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 9 \text{ ksi}$$

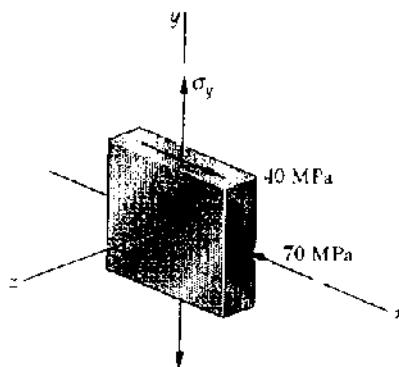
$$(c) \quad \sigma_z = 0, \quad \sigma_a = 14 \text{ ksi}, \quad \sigma_b = 1 \text{ ksi}$$

$$\sigma_{\max} = 14 \text{ ksi}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 7 \text{ ksi}$$



Problem 7.74

7.74 For the state of stress shown, determine two values of σ_z for which the maximum shearing stress is 75 MPa.



$$\sigma_x = -70 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa}$$

$$\text{Let } u = \frac{\sigma_y - \sigma_x}{2} \quad \sigma_y = 2u + \sigma_x$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} \quad u = \pm \sqrt{R^2 - \tau_{xy}^2}$$

$$\text{Case 1} \quad \tau_{max} = R = 75 \text{ MPa}, \quad u = \pm \sqrt{75^2 - 40^2} = \pm 63.44 \text{ MPa}$$

$$(1a) \quad u = +63.44 \text{ MPa} \quad \sigma_y = 2u + \sigma_x = 56.88 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.56 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 68.44 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -81.56 \text{ MPa}$$

$$\sigma_c = 0 \quad \sigma_{max} = 68.44 \text{ MPa}, \quad \sigma_{min} = -81.56 \text{ MPa} \quad \tau_{max} = 75 \text{ MPa}$$

$$(1b) \quad u = -63.44 \text{ MPa} \quad \sigma_y = 2u + \sigma_x = -196.88 \text{ MPa} \quad (\text{reject})$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -133.44 \text{ MPa} \quad \sigma_a = \sigma_{ave} + R = -58.44 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -208.44 \text{ MPa}, \quad \sigma_c = 0, \quad \sigma_{max} = 0$$

$$\sigma_{min} = -208.44 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 104.22 \text{ MPa} \neq 75 \text{ MPa}$$

$$\text{Case (2)} \quad \text{Assume} \quad \sigma_{max} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 75 \text{ MPa}$$

$$\sigma_{min} = -150 \text{ MPa} = \sigma_b$$

$$\sigma_b = \sigma_{ave} - R = \sigma_x + u = \sqrt{u^2 + \tau_{xy}^2}$$

$$\sqrt{u^2 + \tau_{xy}^2} = \sigma_x + u - \sigma_b$$

$$u^2 + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)u + u^2$$

$$2u = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} = \frac{(40)^2 - (-70 + 150)^2}{-70 + 150} = -160 \text{ MPa}$$

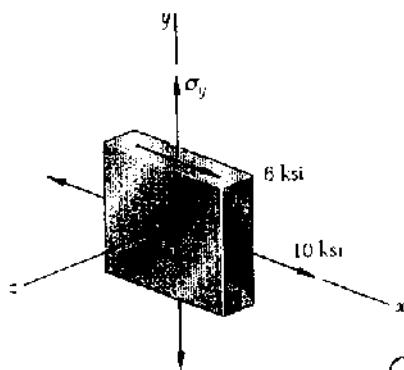
$$u = -30 \text{ MPa} \quad \sigma_y = 2u + \sigma_x = -130 \text{ MPa}$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\sigma_a = \sigma_b + 2R = -150 + 100 = -50 \text{ MPa} \quad \text{O.K.}$$

Problem 7.75

7.75 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 7.5 ksi.



$$\sigma_x = 10 \text{ ksi}, \quad \tau_{xy} = 6 \text{ ksi}, \quad \tau_{max} = 7.5 \text{ ksi}$$

$$\text{Let } u = \frac{\sigma_y - \sigma_x}{2} \quad \sigma_y = 2u + \sigma_x$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} \quad u = \pm \sqrt{R^2 - \tau_{xy}^2}$$

$$\text{Case 1} \quad \tau_{max} = R = 7.5 \text{ ksi}, \quad u = \pm 4.5 \text{ ksi}$$

$$(1a) \quad u = +4.5 \text{ ksi} \quad \sigma_y = 2u + \sigma_x = 19 \text{ ksi} \quad \text{reject}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 14.5 \text{ ksi}, \quad \sigma_a = \sigma_{ave} + R = 22 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 7 \text{ ksi}$$

$$\sigma_{max} = 22 \text{ ksi}, \quad \sigma_{min} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 11 \text{ ksi} \neq 7.5 \text{ ksi}$$

$$(1b) \quad u = -4.5 \text{ ksi} \quad \sigma_y = 2u + \sigma_x = 1 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5 \text{ ksi}, \quad \sigma_a = \sigma_{ave} + R = 13 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = -2 \text{ ksi}$$

$$\sigma_{max} = 13 \text{ ksi}, \quad \sigma_{min} = -2 \text{ ksi}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5 \text{ ksi} \quad \text{OK.}$$

$$\text{Case 2} \quad \text{Assume } \sigma_{min} = 0 \quad \sigma_{max} = 2\tau_{max} = 15 \text{ ksi} = \sigma_a$$

$$\sigma_a = \sigma_{ave} + R = \sigma_v + u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2$$

$$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + u^2 = u^2 + \tau_{xy}^2$$

$$2u = \frac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \frac{(15 - 10)^2 - 6^2}{15 - 10} = -2.2 \text{ ksi}$$

$$u = -1.1 \text{ ksi}$$

$$\sigma_y = 2u + \sigma_x = 7.8 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_v + \sigma_y) = 8.9 \text{ ksi} \quad R = \sqrt{u^2 + \tau_{xy}^2} = 6.1 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 15 \text{ ksi} \quad \checkmark$$

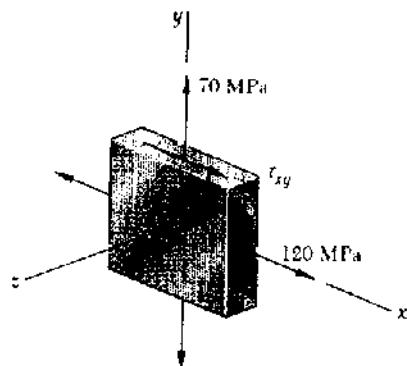
$$\sigma_b = \sigma_{ave} - R = 2.8 \text{ ksi}$$

$$\sigma_{max} = 15 \text{ ksi}, \quad \sigma_{min} = 0$$

$$\tau_{max} = 7.5 \text{ ksi} \quad \checkmark$$

Problem 7.76

7.76 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is 80 MPa.



$$\sigma_x = 120 \text{ MPa} \quad \sigma_y = 70 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = 95 \text{ MPa}$$

$$\sigma_x - \sigma_y = \frac{120 - 70}{2} = 25 \text{ MPa}$$

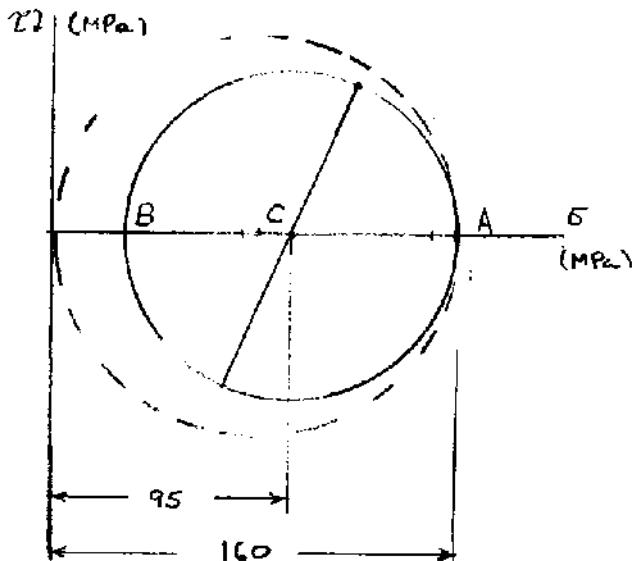
$$\text{Assume} \quad \sigma_{min} = 0 \quad \sigma_{max} = 2\sigma_{ave} = 160 \text{ MPa}$$

$$\sigma_a = \sigma_{max} = \sigma_{ave} + R \quad R = \sigma_{max} - \sigma_{ave} = 160 - 95 = 65 \text{ MPa}$$

$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \quad \tau_{xy}^2 = R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 = 65^2 - 25^2 = 60^2$$

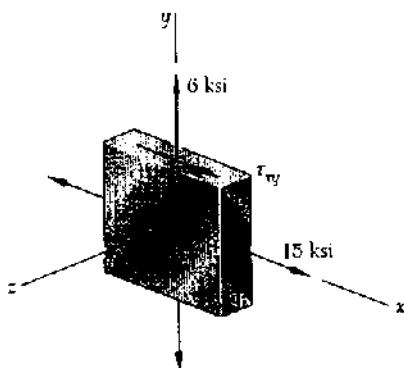
$$\tau_{xy} = \pm 60 \text{ ksi}$$

$$\sigma_b = \sigma_a - 2R = 160 - 130 = 30 \text{ MPa} \geq 0 \quad \text{O.K.}$$



Problem 7.77

7.77 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (a) 9 ksi, (b) 12 ksi.



$$\sigma_x = 15 \text{ ksi} \quad \sigma_y = 6 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = 10.5 \text{ ksi}$$

$$U = \frac{\sigma_x - \sigma_y}{2} = 4.5 \text{ ksi}$$

$$\tau_{xy} (\text{ksi})$$

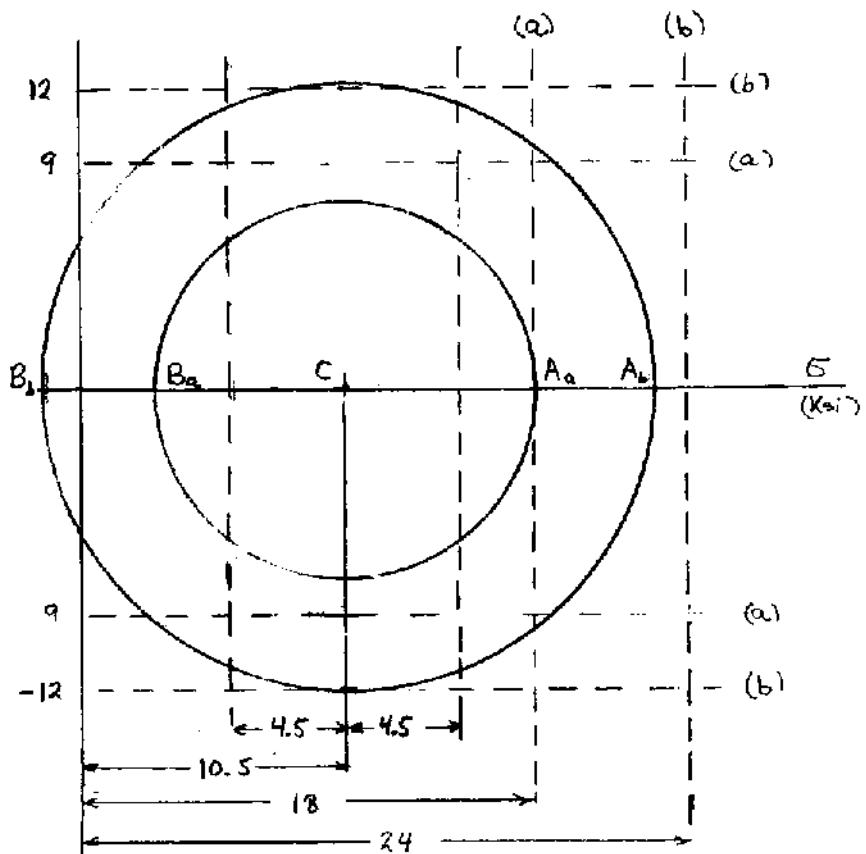
$$(a) \text{ For } \tau_{max} = 9 \text{ ksi}$$

Center of Mohrs circle lies at point C. Lines marked (a) show the limits on τ_{max} . Limit on σ_{max} is $\sigma_{max} = 2\tau_{max} = 18 \text{ ksi}$. For the Mohr's circle $\sigma_a = \sigma_{max}$ corresponds to point A_a.

$$R = \sigma_a - \sigma_{ave} \\ = 18 - 10.5 = 7.5 \text{ ksi}$$

$$R = \sqrt{U^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm \sqrt{R^2 - U^2} \\ = \pm \sqrt{7.5^2 - 4.5^2} \\ = \pm 6 \text{ ksi}$$



$$(b) \text{ For } \tau_{max} = 12 \text{ ksi.}$$

Center of Mohr's circle lies at point C. $R = 12 \text{ ksi}$

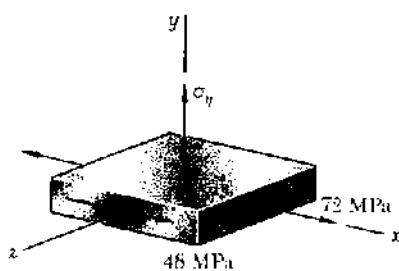
$$\tau_{xy}' = \pm \sqrt{R^2 - U^2} = \pm 11.24 \text{ ksi}$$

$$\text{Checking} \quad \sigma_a = 10.5 + 12 = 22.5 \text{ ksi} \quad \sigma_b = 10.5 - 12 = -1.5 \text{ ksi} \\ \sigma_c = 0$$

$$\tau_{max} = \frac{1}{2} (\sigma_{max} - \sigma_{min}) = 12 \text{ ksi} \quad O.K.$$

Problem 7.78

7.78 For the state of stress shown, determine two values of σ , for which the maximum shearing stress is 64 MPa.



$$\bar{\sigma}_x = 72 \text{ MPa} \quad \bar{\sigma}_z = 0 \quad \tau_{xz} = 48 \text{ MPa}$$

Mohr's circle for stresses in zx-plane

$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_z) = 36 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{36^2 + 48^2} = 60 \text{ MPa}$$

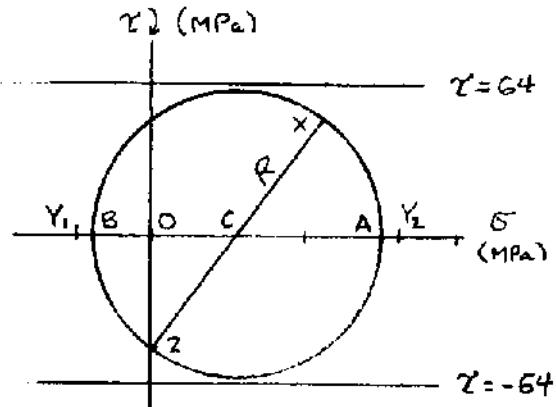
$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = 96 \text{ MPa}, \quad \bar{\sigma}_b = \bar{\sigma}_{ave} - R = -24 \text{ MPa}$$

Assume $\bar{\sigma}_{max} = \bar{\sigma}_a = 96 \text{ MPa}$

$$\begin{aligned} \bar{\sigma}_y &= \bar{\sigma}_{min} = \bar{\sigma}_{max} - 2\tau_{max} \\ &= 96 - (2)(64) = -32 \text{ MPa} \end{aligned}$$

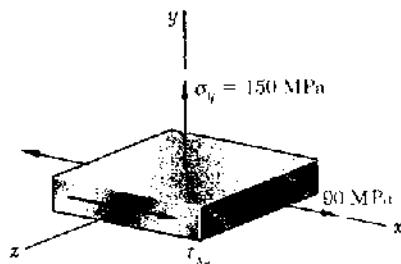
Assume $\bar{\sigma}_{min} = \bar{\sigma}_b = -24 \text{ MPa}$

$$\begin{aligned} \bar{\sigma}_y &= \bar{\sigma}_{max} = \bar{\sigma}_{min} + 2\tau_{max} \\ &= -24 + (2)(64) = 104 \text{ MPa} \end{aligned}$$



Problem 7.79

7.79 For the state of stress shown, determine the range of values of τ_{xz} for which the maximum shearing stress is equal to or less than 90 MPa.



$$\sigma_x = 90 \text{ MPa}, \quad \sigma_z = 0, \quad \sigma_y = 150 \text{ MPa}$$

For Mohr's circle of stresses in zx -plane

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 45 \text{ MPa}$$

$$U = \frac{\sigma_x - \sigma_z}{2} = 45$$

$$\text{Assume } \sigma_{max} = \sigma_y = 150 \text{ MPa}$$

$$\begin{aligned}\sigma_{min} &= \sigma_b = \sigma_{max} - 2U \\ &= 150 - (2)(45) = -30 \text{ MPa}\end{aligned}$$

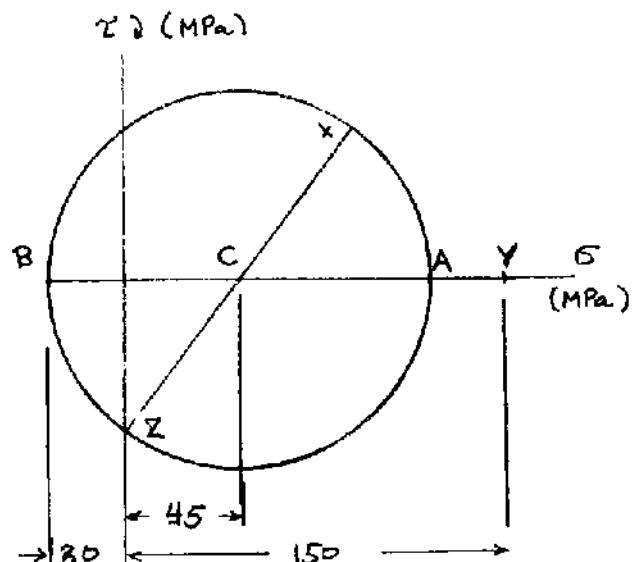
$$\begin{aligned}R &= \sigma_{ave} - \sigma_b \\ &= 45 - (-30) = 75 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\sigma_a &= \sigma_{ave} + R \\ &= 45 + 75 = 120 \text{ MPa} < \sigma_y \\ &\text{O.K.}\end{aligned}$$

$$R = \sqrt{U^2 + \tau_{xz}^2}$$

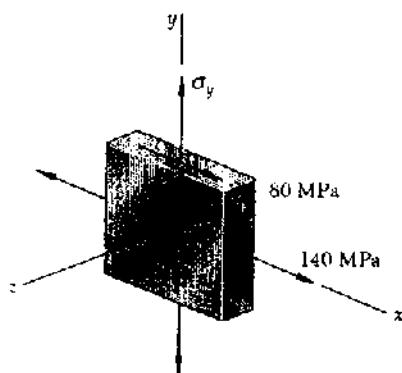
$$\begin{aligned}\tau_{xz} &= \pm \sqrt{R^2 - U^2} \\ &= \pm \sqrt{75^2 - 45^2} = \pm 60 \text{ MPa}\end{aligned}$$

$$-60 \text{ MPa} \leq \tau_{xz} \leq 60 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.80

*7.80 For the state of stress of Prob. 7.66, determine (a) the value of σ_y for which the maximum shearing stress is as small as possible, (b) the corresponding value of the shearing stress.



$$\text{Let } u = \frac{\sigma_x - \sigma_y}{2} \quad \bar{\sigma}_y = \sigma_x - 2u$$

$$\bar{\sigma}_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$$

Assume τ_{max} is the in-plane shearing stress $\tau_{max} = R$

Then $\tau_{max(\text{in-plane})}$ is minimum if $u = 0$

$$\bar{\sigma}_y = \sigma_x - 2u = \sigma_x = 140 \text{ MPa}, \quad \bar{\sigma}_{ave} = \sigma_x - u = 140 \text{ MPa}$$

$$R = |\tau_{xy}| = 80 \text{ MPa}$$

$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = 140 + 80 = 220 \text{ MPa}$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R = 140 - 80 = 60 \text{ MPa}$$

$$\sigma_{max} = 220 \text{ MPa}, \quad \sigma_{min} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 110 \text{ MPa}$$

Assumption is incorrect

$$\text{Assume } \sigma_{max} = \bar{\sigma}_a = \bar{\sigma}_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_{min} = 0 \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}\bar{\sigma}_a$$

$$\frac{d\bar{\sigma}_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0 \quad (\text{no minimum})$$

Optimum value for u occurs when $\tau_{max(\text{out-of-plane})} = \tau_{max(\text{in-plane})}$

$$\frac{1}{2}(\bar{\sigma}_a + R) = R \text{ or } \bar{\sigma}_a = R \text{ or } \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + u^2 = u^2 + \tau_{xy}^2$$

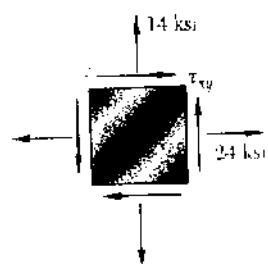
$$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3 \text{ MPa} \quad u = 47.14 \text{ MPa}$$

$$(a) \quad \bar{\sigma}_y = \sigma_x - 2u = 140 - 94.3 = 45.7 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 92.9 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.81

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_y = 30$ ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a) $\tau_{xy} = 6$ ksi, (b) $\tau_{xy} = 12$ ksi, (c) $\tau_{xy} = 14$ ksi. If yield does not occur, determine the corresponding factor of safety.



$$\sigma_x = 24 \text{ ksi} \quad \sigma_y = 14 \text{ ksi} \quad \sigma_z = 0$$

$$\text{For stresses in } xy\text{-plane} \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 19 \text{ ksi}$$

$$\frac{\sigma_x - \sigma_y}{2} = 5 \text{ ksi}$$

$$(a) \quad \tau_{xy} = 6 \text{ ksi} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (6)^2} = 7.810 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 26.810 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 11.190 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 23.324 \text{ ksi} < 30 \text{ ksi} \quad (\text{No yielding})$$

$$\text{F.S.} = \frac{30}{23.324} = 1.286$$

$$(b) \quad \tau_{xy} = 12 \text{ ksi} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (12)^2} = 13 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 32 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 8 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 29.462 \text{ ksi} < 30 \text{ ksi} \quad (\text{No yielding})$$

$$\text{F.S.} = \frac{30}{29.462} = 1.018$$

$$(c) \quad \tau_{xy} = 14 \text{ ksi} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (14)^2} = 14.866 \text{ ksi}$$

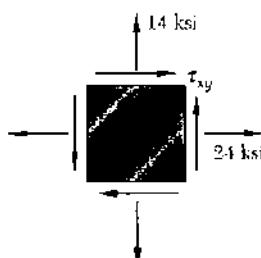
$$\sigma_a = \sigma_{ave} + R = 33.866, \quad \sigma_b = \sigma_{ave} - R = 4.134 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 32.00 \text{ ksi} > 30 \text{ ksi} \quad (\text{Yielding occurs})$$

Problem 7.82

7.82 Solve Prob. 7.81, using the maximum-shearing-stress criterion.

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_y = 30$ ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a) $\tau_{xy} = 6$ ksi, (b) $\tau_{xy} = 12$ ksi, (c) $\tau_{xy} = 14$ ksi. If yield does not occur, determine the corresponding factor of safety.



$$\sigma_x = 24 \text{ ksi} \quad \sigma_y = 14 \text{ ksi} \quad \sigma_z = 0$$

For stresses in xy-plane $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 19 \text{ ksi}$

$$\frac{\sigma_x - \sigma_y}{2} = 5 \text{ ksi}$$

(a) $\tau_{xy} = 6 \text{ ksi}$ $R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = 7.810 \text{ ksi}$

$$\sigma_a = \sigma_{ave} + R = 26.810 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 11.190 \text{ ksi}$$

$$\sigma_{max} = 26.810 \text{ ksi}, \quad \sigma_{min} = 0$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 26.810 \text{ ksi} < 30 \text{ ksi} \quad (\text{No yielding})$$

$$\text{F.S.} = \frac{30}{26.810} = 1.119$$

(b) $\tau_{xy} = 12 \text{ ksi}$ $R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = 13 \text{ ksi}$

$$\sigma_a = \sigma_{ave} + R = 32 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 6 \text{ ksi}$$

$$\sigma_{max} = 32 \text{ ksi}, \quad \sigma_{min} = 0$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 32 \text{ ksi} > 30 \text{ ksi} \quad (\text{Yielding occurs})$$

(c) $\tau_{xy} = 14 \text{ ksi}$ $R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = 14.866 \text{ ksi}$

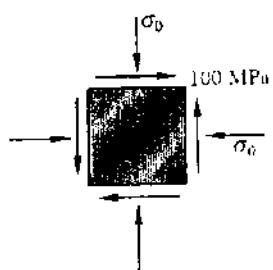
$$\sigma_a = \sigma_{ave} + R = 33.866 \text{ ksi} \quad \sigma_b = \sigma_{ave} - R = 4.134 \text{ ksi}$$

$$\sigma_{max} = 33.866 \text{ ksi}, \quad \sigma_{min} = 0$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 33.866 \text{ ksi} > 30 \text{ ksi} \quad (\text{Yielding occurs})$$

Problem 7.83

7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_y = 325 \text{ MPa}$. Using the maximum-distortion-energy criterion, determine whether yield will occur when (a) $\sigma_o = 200 \text{ MPa}$, (b) $\sigma_o = 240 \text{ MPa}$, (c) $\sigma_o = 280 \text{ MPa}$. If yield does not occur, determine the corresponding factor of safety.



$$\sigma_{ave} = -\sigma_o \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a) $\sigma_o = 200 \text{ MPa} \quad \sigma_{ave} = -200 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -100 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -300 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 264.56 \text{ MPa} < 325 \text{ MPa} \quad (\text{No yielding})$$

$$\text{F.S.} = \frac{325}{264.56} = 1.228$$

(b) $\sigma_o = 240 \text{ MPa} \quad \sigma_{ave} = -240 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -140 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -340 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 295.97 \text{ MPa} < 325 \text{ MPa} \quad (\text{No yielding})$$

$$\text{F.S.} = \frac{325}{295.97} = 1.098$$

(c) $\sigma_o = 280 \text{ MPa} \quad \sigma_{ave} = -280 \text{ MPa}$

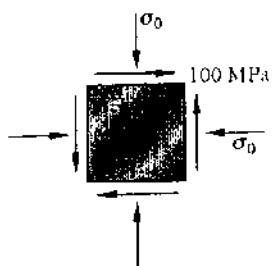
$$\sigma_a = \sigma_{ave} + R = -180 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -380 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 329.24 \text{ MPa} > 325 \text{ MPa} \quad (\text{Yielding occurs})$$

Problem 7.84

7.84 Solve Prob. 7.83, using the maximum-shearing-stress criterion.

7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_y = 325 \text{ MPa}$. Using the maximum-distortion-energy criterion, determine whether yield will occur when (a) $\sigma_0 = 200 \text{ MPa}$, (b) $\sigma_0 = 240 \text{ MPa}$, (c) $\sigma_0 = 280 \text{ MPa}$. If yield does not occur, determine the corresponding factor of safety.



$$\sigma_{ave} = -\sigma_0 \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a) $\sigma_0 = 200 \text{ MPa}, \quad \sigma_{ave} = -200 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -100 \text{ MPa} \quad \sigma_b = \sigma_{ave} - R = -300 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -300 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 300 \text{ MPa} < 325 \text{ MPa} \quad (\text{No yielding})$$

$$\text{F.S.} = \frac{325}{300} = 1.083$$

(b) $\sigma_0 = 240 \text{ MPa}, \quad \sigma_{ave} = -240 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -140 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -340 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -340 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 340 \text{ MPa} > 325 \text{ MPa} \quad (\text{Yielding occurs})$$

(c) $\sigma_0 = 280 \text{ MPa}, \quad \sigma_{ave} = -280 \text{ MPa}$

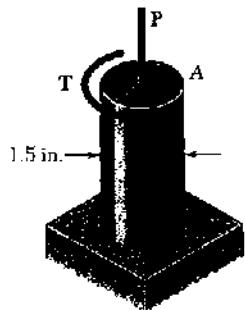
$$\sigma_a = \sigma_{ave} + R = -180 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -380 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -380 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 380 \text{ MPa} > 325 \text{ MPa} \quad (\text{Yielding occurs})$$

Problem 7.85

7.85 The 1.5-in.-diameter shaft *AB* is made of a grade of steel for which the yield strength is σ_y is 42 ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the torque *T* for which yield occurs when $P = 60$ kips.



$$P = 60 \text{ kips} \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1.5)^2 = 1.7671 \text{ in}^2$$

$$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953 \text{ ksi}$$

$$\sigma_y = 0 \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_y$$

$$4\tau_{xy}^2 = \sigma_y^2 - \sigma_x^2 \quad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{42^2 - 33.953^2} \\ = 12.361 \text{ ksi}$$

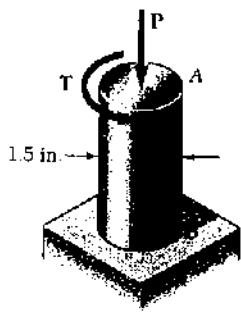
$$\text{From torsion} \quad \tau_{xy} = \frac{Tc}{J} \quad T = \frac{J\tau_{xy}}{c}$$

$$c = \frac{1}{2}d = 0.75 \text{ in} \quad J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$$

$$T = \frac{(0.49701)(12.361)}{0.75} = 8.19 \text{ kip-in}$$

Problem 7.86

7.86 Solve Prob. 7.85, using the maximum-distortion-energy criterion.



7.85 The 1.5-in.-diameter shaft *AB* is made of a grade of steel for which the yield strength is σ_y is 42 ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the torque *T* for which yield occurs when $P = 60$ kips.

$$P = 60 \text{ kips} \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1.5)^2 = 1.7671 \text{ in}^2$$

$$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953 \text{ ksi}$$

$$\sigma_y = 0 \quad \sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = \frac{1}{2} \sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4} \sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R \quad \sigma_b = \sigma_{ave} - R$$

$$\begin{aligned} \sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b &= (\sigma_{ave} + R)^2 + (\sigma_{ave} - R)^2 - (\sigma_{ave} + R)(\sigma_{ave} - R) \\ &= \sigma_{ave}^2 + 2\sigma_{ave}R + R^2 + \sigma_{ave}^2 - 2\sigma_{ave}R + R^2 - \sigma_{ave}^2 + R^2 \\ &= \sigma_{ave}^2 + 3R^2 \\ &= \frac{1}{4} \sigma_x^2 + 3\left(\frac{1}{4} \sigma_x^2 + \tau_{xy}^2\right) = \sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2 \end{aligned}$$

$$3\tau_{xy}^2 = \sigma_y^2 - \sigma_x^2 \quad \tau_{xy} = \frac{1}{\sqrt{3}} (\sigma_y^2 - \sigma_x^2) = \frac{1}{\sqrt{3}} \sqrt{42^2 - 33.953^2} \\ = 14.273 \text{ ksi}$$

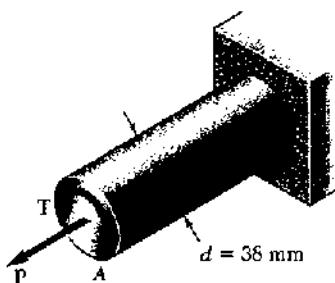
$$\text{From torsion} \quad \tau_{xy} = \frac{Tc}{J} \quad T = \frac{J \tau_{xy}}{c}$$

$$c = \frac{1}{2} d = 0.75 \text{ in.} \quad J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.75)^4 = 0.49701 \text{ in}^3$$

$$T = \frac{(0.49701)(14.273)}{0.75} = 9.46 \text{ kip-in}$$

Problem 7.87

7.87 The 38-mm-diameter shaft *AB* is made of a grade of steel for which the yield strength is $\sigma_y = 250 \text{ MPa}$. Using the maximum-shearing-stress criterion, determine the magnitude of the torque *T* for which yield occurs when $P = 240 \text{ kN}$.



$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$$

$$\sigma_y = 0 \quad \sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = \frac{1}{2} \sigma_x$$

$$R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4} \sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_y$$

$$4\tau_{xy}^2 = \sigma_y^2 - \sigma_x^2 \quad \tau_{xy} = \frac{1}{2} \sqrt{\sigma_y^2 - \sigma_x^2} = \frac{1}{2} \sqrt{250^2 - 211.6^2}$$

$$= 66.568 \text{ MPa} = 66.568 \times 10^6 \text{ Pa}$$

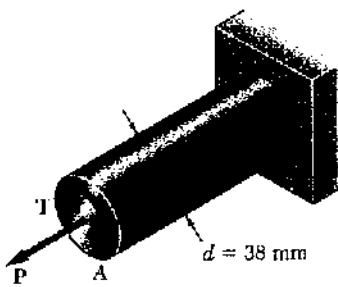
From torsion $\tau_{xy} = \frac{Tc}{J}$ $T = \frac{J \tau_{xy}}{c}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} d = 19 \times 10^{-3} \text{ m}$$

$$T = \frac{(204.71 \times 10^{-9})(66.568 \times 10^6)}{19 \times 10^{-3}} = 717 \text{ N} \cdot \text{m}$$

Problem 7.88



7.88 Solve Prob. 7.87, using the maximum-distortion-energy criterion.

7.87 The 38-mm-diameter shaft *AB* is made of a grade of steel for which the yield strength is $\sigma_y = 250 \text{ MPa}$. Using the maximum-shearing-stress criterion, determine the magnitude of the torque T for which yield occurs when $P = 240 \text{ kN}$.

$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$$

$$\sigma_y = 0 \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\begin{aligned} \sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b &= \frac{1}{4}\sigma_x^2 + \sigma_x \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &\quad + \frac{1}{4}\sigma_x^2 - \sigma_x \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &\quad - \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &= \sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2 \end{aligned}$$

$$\tau_{xy}^2 = \frac{1}{3}(\sigma_y^2 - \sigma_x^2)$$

$$\tau_{xy} = \frac{1}{\sqrt{3}} \sqrt{250^2 - 211.6^2} = 76.867 \text{ MPa} = 76.867 \times 10^6 \text{ Pa}$$

$$\text{From torsion} \quad \tau_{xy} = \frac{Tc}{J} \quad T = \frac{J \tau_{xy}}{c}$$

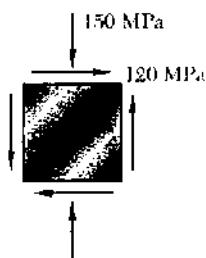
$$J = \frac{\pi}{32} c^4 = \frac{\pi}{32} \left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}d = 19 \times 10^{-3} \text{ m}$$

$$T = \frac{(204.71 \times 10^{-9})(76.867 \times 10^6)}{19 \times 10^{-3}} = 828 \text{ N} \cdot \text{m}$$

Problem 7.89

7.89 and 7.90 The state of plane stress shown is expected in a cast-iron base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160 \text{ MPa}$ and $\sigma_{UC} = 320 \text{ MPa}$ and using Mohr's criterion, determine whether rupture of the component will occur.



$$\bar{\sigma}_x = 0 \quad \bar{\sigma}_y = -150 \text{ MPa} \quad \tau_{xy} = 120 \text{ MPa}$$

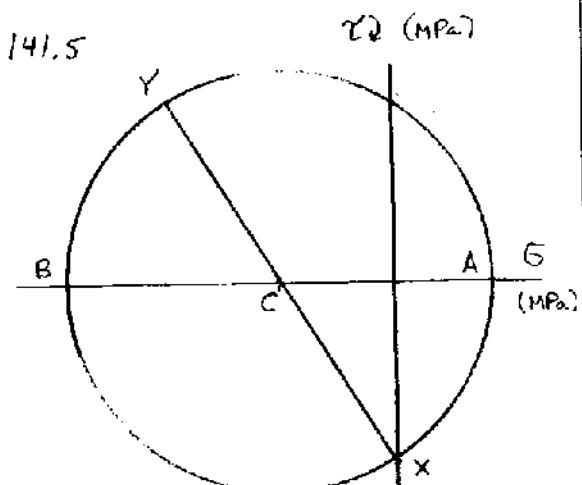
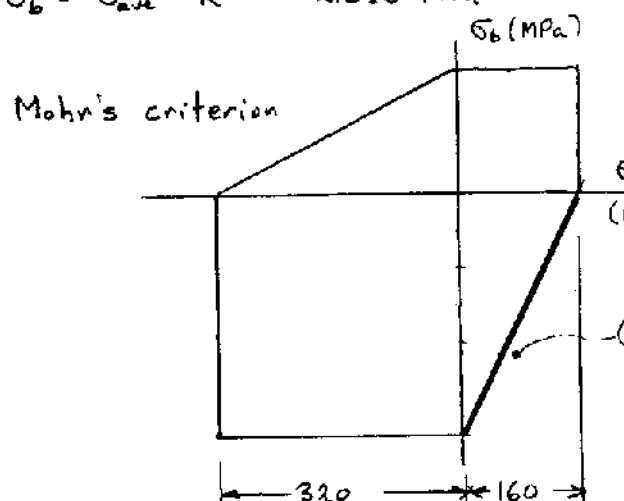
$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) = -75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} = 141.5 \text{ MPa}$$

$$= \sqrt{75^2 + 120^2} = 141.5$$

$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = 66.5 \text{ MPa}$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R = -216.5 \text{ MPa}$$



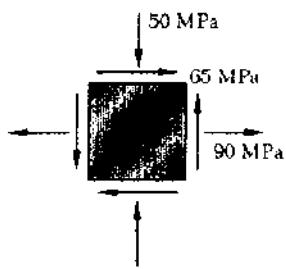
Equation of 4th quadrant boundary is $\frac{\bar{\sigma}_a}{\sigma_{UT}} - \frac{\bar{\sigma}_b}{\sigma_{UC}} = 1$

$$\frac{66.5}{160} - \frac{-216.5}{320} = 1.092 > 1$$

Rupture occurs.

Problem 7.90

7.89 and 7.90 The state of plane stress shown is expected in a cast-iron base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160 \text{ MPa}$ and $\sigma_{UC} = 320 \text{ MPa}$ and using Mohr's criterion, determine whether rupture of the component will occur.



$$\bar{\sigma}_x = 90 \text{ MPa}, \quad \bar{\sigma}_y = -50 \text{ MPa}, \quad \tau_{xy} = 65 \text{ MPa}$$

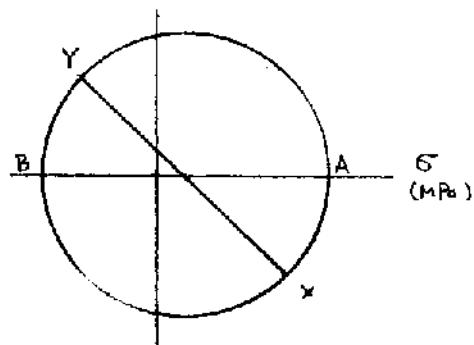
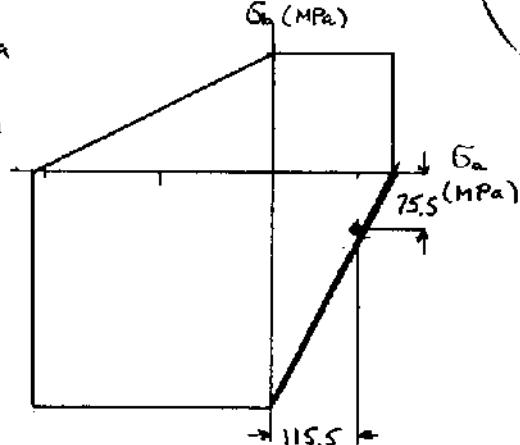
$$\bar{\sigma}_{ave} = \frac{1}{2} (\bar{\sigma}_x + \bar{\sigma}_y) = 20 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{70^2 + 65^2} = 95.5 \text{ MPa}$$

$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = 115.5 \text{ MPa}$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R = -75.5 \text{ MPa}$$

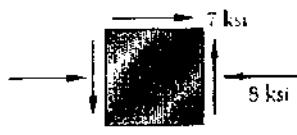


$$\text{Equation of 4th quadrant boundary} \quad \frac{\bar{\sigma}_a}{\sigma_{UT}} - \frac{\bar{\sigma}_b}{\sigma_{UC}} = 1$$

$$\frac{115.5}{160} - \frac{(-75.5)}{320} = 0.958 < 1 \quad \text{No rupture}$$

Problem 7.91

7.91 and 7.92 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{ut} = 10 \text{ ksi}$ and $\sigma_{uc} = 30 \text{ ksi}$ and using Mohr's criterion, determine whether rupture of the component will occur.



$$\sigma_x = -8 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = 7 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -4 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + 7^2} = 8.062 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = -4 + 8.062 = 4.062 \text{ ksi}$$

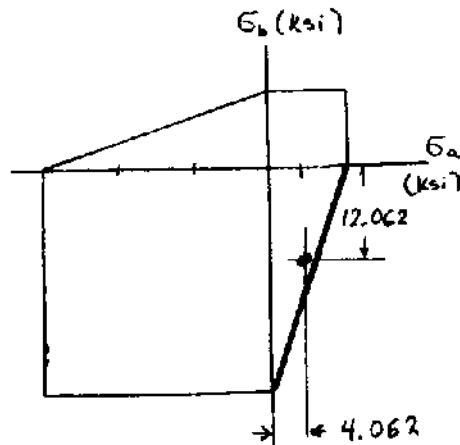
$$\sigma_b = \sigma_{ave} - R = -4 - 8.062 = -12.062 \text{ ksi}$$

Equation of 4 th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{ut}} - \frac{\sigma_b}{\sigma_{uc}} = 1$$

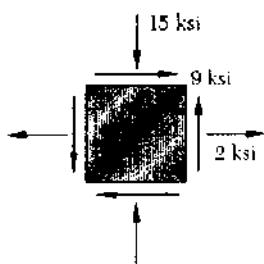
$$\frac{4.062}{10} - \frac{(-12.062)}{30} = 0.808 < 1$$

(No rupture)



Problem 7.92

7.91 and 7.92 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{ut} = 10 \text{ ksi}$ and $\sigma_{uc} = 30 \text{ ksi}$ and using Mohr's criterion, determine whether rupture of the component will occur.



$$\sigma_x = 2 \text{ ksi} \quad \sigma_y = -15 \text{ ksi} \quad \tau_{xy} = 9 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{8.5^2 + 9^2} = 12.379 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 5.879 \text{ ksi}$$

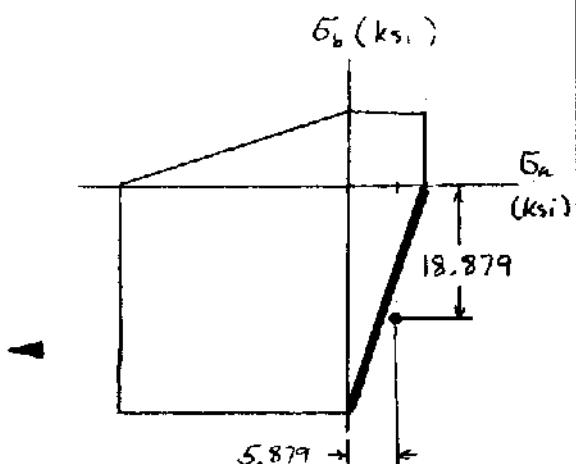
$$\sigma_b = \sigma_{ave} - R = -18.879 \text{ ksi}$$

Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{ut}} - \frac{\sigma_b}{\sigma_{uc}} = 1$$

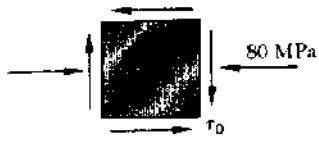
$$\frac{5.879}{10} - \frac{(-18.879)}{30} = 1.217 > 1$$

Rupture will occur.



Problem 7.93

7.93 The state of plane stress shown will occur at a critical point in a pipe made of an aluminum alloy for which $\sigma_{UT} = 75 \text{ MPa}$ and $\sigma_{IC} = 150 \text{ MPa}$. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



$$\sigma_x = -80 \text{ MPa} \quad \sigma_y = 0 \quad \tau_{xy} = -\tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2} \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R, \quad \sigma_b = \sigma_{ave} - R, \quad \tau_0 = \pm \sqrt{R^2 - 40^2}$$

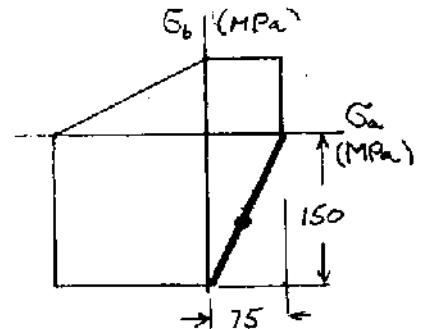
Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{IC}} = 1$$

$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$

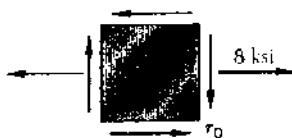
$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$

$$R = 63.33 \text{ MPa}, \quad \tau_0 = \pm \sqrt{63.33^2 - 40^2} = \pm 49.1 \text{ MPa}$$



Problem 7.94

7.94 The state of plane stress shown will occur at a critical point in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 10 \text{ ksi}$ and $\sigma_{UC} = 25 \text{ ksi}$. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



$$\bar{\sigma}_x = 8 \text{ ksi}, \quad \bar{\sigma}_y = 0, \quad \tau_{xy} = \tau_0$$

$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) = 4 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + \tau_0^2}, \quad \tau_0 = \pm \sqrt{R^2 - 4^2}$$

$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = (4 + R) \text{ ksi} \quad \bar{\sigma}_b = \bar{\sigma}_{ave} - R = (4 - R) \text{ ksi}$$

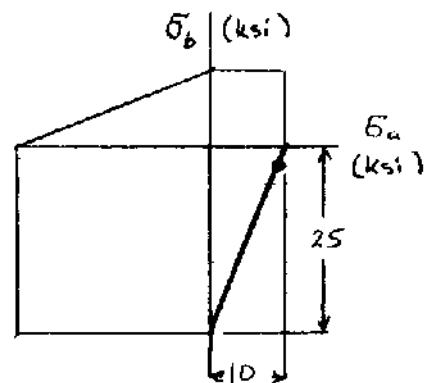
Since $|\bar{\sigma}_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\bar{\sigma}_a}{\sigma_{UT}} - \frac{\bar{\sigma}_b}{\sigma_{UC}} = 1$$

$$\frac{4+R}{10} - \frac{4-R}{25} = 1$$

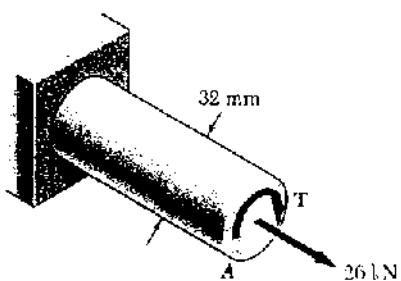
$$\left(\frac{1}{10} + \frac{1}{25}\right)R = 1 - \frac{4}{10} + \frac{4}{25}$$

$$R = 5.429 \text{ ksi} \quad \tau_0 = \pm \sqrt{5.429^2 - 4^2} = \pm 3.67 \text{ ksi}$$



Problem 7.95

7.95 The case-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 60 \text{ MPa}$ and $\sigma_{UC} = 120 \text{ MPa}$. Using Mohr's criterion, determine the magnitude of the torque T for which failure should be expected.



$$P = 26 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} (32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{26 \times 10^3}{804.25 \times 10^{-6}} = 32.328 \times 10^6 \text{ Pa} \\ = 32.328 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(32.328 + 0) = 16.164 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(32.328 - 0) = 16.164 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 16.164 + R \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 16.164 - R \text{ MPa}$$

Since $|\sigma_{ave}| < R$, stress point lies in the 4th quadrant. Equation of the 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{16.164 + R}{60} - \frac{16.164 - R}{120} = 1$$

$$\left(\frac{1}{60} + \frac{1}{120}\right)R = 1 - \frac{16.164}{60} + \frac{16.164}{120}$$

$$R = 34.612 \text{ MPa}$$

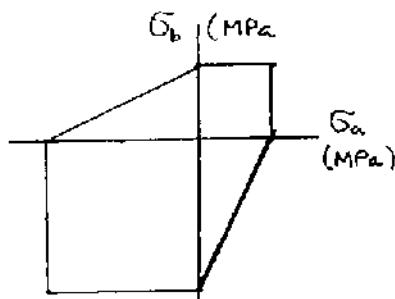
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{34.612^2 - 16.164^2} = 30.606 \text{ MPa}$$

$$= 30.606 \times 10^6 \text{ Pa}$$

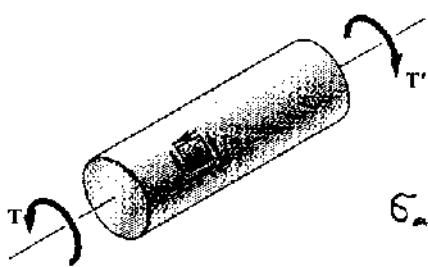
$$\text{For torsion } \tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \text{where} \quad c = \frac{1}{2}d = 16 \text{ mm} = 16 \times 10^{-3} \text{ m}$$

$$T = \frac{\pi}{2} c^3 \tau_{xy} = \frac{\pi}{2} (16 \times 10^{-3})^3 (30.606 \times 10^6) = 196.9 \text{ N} \cdot \text{m}$$



Problem 7.96

7.96 The case-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 70 \text{ MPa}$ and $\sigma_{uc} = 175 \text{ MPa}$. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_o that should be expected at rupture.



$$\epsilon_x = 0, \quad \epsilon_y = 0 \quad \tau_{xy} = -\tau_o$$

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$$

$$\epsilon_a = \epsilon_{ave} + R = R$$

$$\sigma_b = \epsilon_{ave} - R = -R$$

Since $|\epsilon_{ave}| < R$, stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is

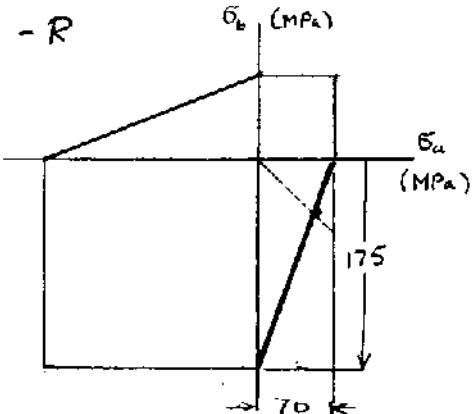
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{uc}} = 1$$

$$\frac{R}{70} - \frac{-R}{175} = 1$$

$$\left(\frac{1}{70} + \frac{1}{175}\right)R = 1$$

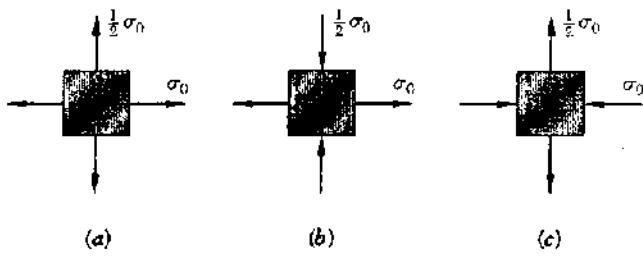
$$R = 50 \text{ MPa}$$

$$|\tau_{xy}| = \tau_o = 50 \text{ MPa}$$



Problem 7.97

7.97 A machine component is made of a grade of cast iron for which $\sigma_{UT} = 8 \text{ ksi}$ and $\sigma_{UC} = 20 \text{ ksi}$. For each of the states of stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which rupture of the component should be expected.



$$(a) \quad \sigma_a = \sigma_0, \quad \sigma_b = \frac{1}{2}\sigma_0$$

Stress point lies in 1st quadrant.

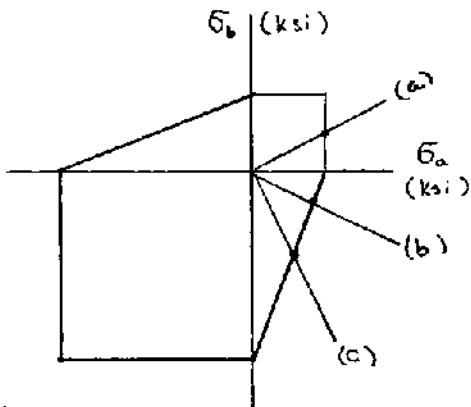
$$\sigma_a = \sigma_0 = \sigma_{UT} = 8 \text{ ksi}$$

$$(b) \quad \sigma_a = \sigma_0, \quad \sigma_b = -\frac{1}{2}\sigma_0$$

Stress point lies in 4th quadrant.
Equation of 4th quadrant boundary is.

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{8} - \frac{-\frac{1}{2}\sigma_0}{20} = 1 \quad \sigma_0 = 6.67 \text{ ksi}$$



$$(c) \quad \sigma_a = \frac{1}{2}\sigma_0, \quad \sigma_b = -\sigma_0, \quad 4\text{th quadrant}$$

$$\frac{\frac{1}{2}\sigma_0}{8} - \frac{-\sigma_0}{20} = 1 \quad \sigma_0 = 8.89 \text{ ksi}$$

Problem 7.98

7.98 A spherical gas container made of steel has a 6-m outer diameter and a wall thickness of 9 mm. Knowing that the internal pressure is 500 kPa, determine the maximum normal stress and the maximum shearing stress in the container.

$$r = \frac{1}{2}d - t = \frac{6}{2} - 9 \times 10^{-3} = 2.991 \text{ m} \quad t = 9 \times 10^{-3} \text{ m}$$

$$P = 500 \times 10^3 \text{ Pa}$$

$$\sigma_1 = \sigma_2 = \frac{Pr}{2t} = \frac{(500 \times 10^3)(2.991)}{(2)(9 \times 10^{-3})} = 83.1 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = \sigma_1$$

$$\sigma_{\max} = 83.1 \text{ MPa}$$

$$\sigma_{\min} \approx -P = -0.5 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 41.8 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 41.8 \text{ MPa}$$

Problem 7.99

7.99 The maximum gage pressure is known to be 10 MPa in a spherical steel pressure vessel having a 200-mm outer diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_u = 400 \text{ MPa}$, determine the factor of safety with respect to tensile failure.

$$r = \frac{1}{2}d - t = \frac{1}{2}(200) - 6 = 94 \text{ mm} = 94 \times 10^{-3} \text{ m}$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \sigma_2 = \frac{Pr}{2t} = \frac{(10 \times 10^6)(94 \times 10^{-3})}{(2)(6)} = 78.3 \times 10^6 \text{ Pa} = 78.3 \text{ MPa}$$

$$\sigma_{\max} = \sigma_1 = 78.3 \text{ MPa}$$

$$\text{F.S.} = \frac{\sigma_u}{\sigma_{\max}} = \frac{400}{78.3} = 5.11$$

Problem 7.100

7.100 Determine the normal stress in a basketball of 9.5-in. outer diameter and 0.125-in. wall thickness that is inflated to a gage pressure of 9 psi.

$$r = \frac{1}{2}d - t = \frac{1}{2}(9.5) - 0.125 = 4.625 \text{ in.}$$

$$\sigma_1 = \sigma_2 = \frac{Pr}{2t} = \frac{(9)(4.625)}{(2)(0.125)} = 166.5 \text{ psi}$$

Problem 7.101

7.101 A spherical gas container having an outer diameter of 15 ft and a wall thickness of 0.90 in. is made of a steel for which $E = 29 \times 10^6$ psi and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 250 psi, determine (a) the maximum normal stress in the container, (b) the increase in the diameter of the container.

$$r = \frac{1}{2}d - t = \frac{1}{2}(15)(12) - 0.90 = 89.1 \text{ in.} \quad t = 0.90 \text{ in.}$$

$$(a) \sigma_1 = \sigma_2 = \frac{Pr}{2t} = \frac{(250)(89.1)}{(2)(0.90)} = 12.38 \times 10^3 \text{ psi} \quad 12.38 \text{ ksi}$$

$$(b) \epsilon_1 = \epsilon_2 = \frac{1}{E} (\sigma_1 - 2\sigma_2) = \frac{1-2\nu}{E} \sigma_1 \\ = \frac{1-0.29}{29 \times 10^6} (12.38 \times 10^3) = 302.97 \times 10^{-6}$$

$$\Delta d = (d)(\epsilon_1) = (15)(12)(302.97 \times 10^{-6}) = 0.0545 \text{ in.}$$

Problem 7.102

7.102 A spherical pressure vessel has an outer diameter of 3 m and a wall thickness of 12 mm. Knowing that for the steel used $\sigma_u = 80$ MPa, $E = 200$ GPa, and $\nu = 0.29$, determine (a) the allowable gage pressure, (b) the corresponding increase in the diameter of the vessel.

$$r = \frac{1}{2}d - t = \frac{1}{2}(3000) - 12 = 1488 \text{ mm}$$

$$\sigma_1 = \sigma_2 = \sigma_{allow} = 8 \text{ MPa}$$

$$(a) \sigma_1 = \sigma_2 = \frac{Pr}{2t} \quad P = \frac{2t\sigma_1}{r} = \frac{(2)(12)(80)}{1488} = 1.290 \text{ MPa}$$

$$\epsilon_1 = \frac{1}{E} (\sigma_1 - 2\sigma_2) = \frac{1-2\nu}{E} \sigma_1 = \frac{1-0.29}{200 \times 10^9} (8 \times 10^6) = 28.4 \mu$$

$$(b) \Delta d = d \epsilon_1 = (3000)(28.4 \times 10^{-6}) = 85.2 \times 10^{-3} \text{ mm} = 0.0852 \text{ mm}$$

Problem 7.103

7.103 A spherical pressure vessel of 750-mm outer diameter is to be fabricated from a steel having an ultimate stress $\sigma_u = 400$ MPa. Knowing that a factor of safety of 4 is desired and that the gage pressure can reach 4.2 MPa, determine the smallest wall thickness that should be used.

$$P = 4.2 \times 10^6 \text{ Pa} \quad r = \frac{1}{2}d - t = \frac{0.750}{2} - t = 0.375 - t \text{ m}$$

$$\sigma_{max} = \sigma_1 = \sigma_2 = \frac{Pr}{2t}$$

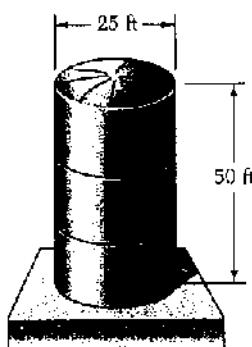
$$F.S. = \frac{\sigma_u}{\sigma_{max}} = \frac{2t\sigma_1}{Pr}$$

$$2\sigma_u t = (F.S.) Pr$$

$$(2)(400 \times 10^6) t = (4)(4.2 \times 10^6)(0.375 - t)$$

$$816.8 \times 10^6 t = 6.3 \times 10^6 \quad t = 7.71 \times 10^{-3} \text{ m} \quad t = 7.71 \text{ mm}$$

Problem 7.104



7.104 When filled to capacity, the unpressurized storage tank shown contains water to a height of 48 ft above its base. Knowing that the lower portion of the tank has a wall thickness of 0.625 in., determine the maximum normal stress and the maximum shearing stress in the tank. (Specific weight of water = 62.4 lb/ft³.)

$$\sigma_{max} = \gamma h - (62.4 \text{ lb/ft}^3)(48 \text{ ft}) = 2995.2 \text{ lb/ft}^2 \\ = 20.8 \text{ lb/in}^2 = 20.8 \text{ psi}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(25)(12) - 0.625 = 149.375 \text{ in.}$$

$$\bar{\sigma}_i = \frac{Pr}{t} = \frac{(20.8)(149.375)}{0.625} = 4.97 \times 10^3 \text{ psi}$$

$$\bar{\sigma}_{max} = \bar{\sigma}_i = 4.97 \times 10^3 \text{ psi} \quad \bar{\sigma}_{min} = 4.97 \text{ ksi}$$

$$\sigma_{min} \approx -P = -250 \text{ psi},$$

$$\tau_{max} = 2.496 \times 10^3 \text{ psi}$$

$$\tau_{max} = 2.50 \text{ ksi}$$

Problem 7.105

7.105 Determine the largest internal pressure that can be applied to a cylindrical tank of 1.75-m outer diameter and 16-mm wall thickness if the ultimate normal stress of the steel used is 450 MPa and a factor of safety of 5.0 is desired.

$$\bar{\sigma}_i = \frac{\sigma_u}{F.S.} = \frac{450 \text{ MPa}}{5} = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{1.75}{2} - 16 \times 10^{-3} = 0.859 \text{ m}$$

$$\bar{\sigma}_i = \frac{Pr}{t} \quad P = \frac{t\bar{\sigma}_i}{r} = \frac{(16 \times 10^{-3})(90 \times 10^6)}{0.859} = 1.676 \times 10^6 \text{ Pa}$$

$$1.676 \text{ MPa}$$

Problem 7.106

7.106 A storage tank contains liquified propane under a pressure of 210 psi at a temperature of 100°F. Knowing that the tank has an outer diameter of 12.6 in and a wall thickness of 0.11 in. Determine the maximum normal stress and the maximum shearing stress in the tank

$$r = \frac{1}{2}d - t = \frac{1}{2}(12.6) - 0.11 = 6.19 \text{ in.} \quad t = 0.11 \text{ in.}$$

$$\bar{\sigma}_i = \frac{Pr}{t} = \frac{(210)(6.19)}{0.11} = 11.82 \times 10^3 \text{ psi}$$

$$\sigma_{max} = \bar{\sigma}_i = 11.82 \times 10^3 \text{ psi} \quad \sigma_{min} = 11.82 \text{ ksi}$$

$$\sigma_{min} \approx -P = -210 \text{ psi}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.01 \times 10^3 \text{ psi}$$

$$\tau_{max} = 6.01 \text{ ksi}$$

Problem 7.107

7.107 The bulk storage tank shown in Fig. 7.49 has an outer diameter of 3.5 m and a wall thickness of 20 mm. At a time when the internal pressure of the tank is 1.2 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

$$r = \frac{1}{2}d - t = \frac{1}{2}(3.5) - 20 \times 10^{-3} = 1.73 \text{ m} \quad t = 20 \times 10^{-3} \text{ m}$$

$$\sigma_i = \frac{pr}{t} = \frac{(1.2 \times 10^6)(1.73)}{20 \times 10^{-3}} = 103.8 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = \sigma_i = 103.8 \times 10^6 \text{ Pa}$$

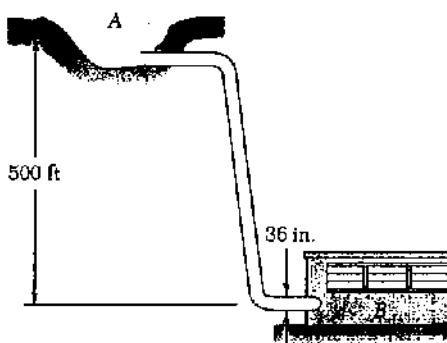
$$\sigma_{\max} = 103.8 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{\min} \approx -p = 1.2 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 52.5 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 52.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.108



7.108 A steel penstock has a 36-in. outer diameter, a 0.5-in. wall thickness, and connects a reservoir at *A* with a generating station at *B*. Knowing that the specific weight of water is 62.4 lb/ft³, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.

$$r = \frac{1}{2}d - t = \frac{1}{2}(36) - 0.5 = 17.5 \text{ in.}$$

$$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2 \times 10^3 \text{ lb/ft}^2 \\ = 216.67 \text{ psi}$$

$$\sigma_i = \frac{pr}{t} = \frac{(216.67)(17.5)}{0.5} = 7583 \text{ psi}$$

$$\sigma_{\max} = \sigma_i = 7583 \text{ psi}$$

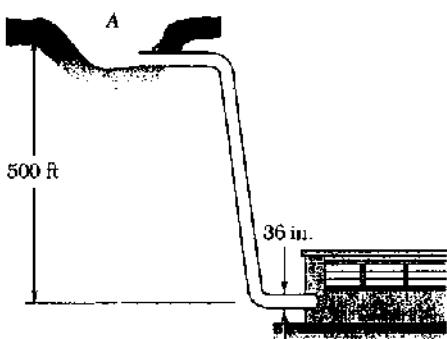
$$\sigma_{\max} = 7.58 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{\min} \approx -p = -216.67 \text{ psi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 3900 \text{ psi} \quad \tau_{\max} = 3.90 \text{ ksi} \quad \blacktriangleleft$$

Problem 7.109

7.109 A steel penstock has a 36-in. outer diameter, a 0.5-in. wall thickness, and connects a reservoir at *A* with a generating station at *B*. Knowing that the specific weight of water is 62.4 lb/ft³ and that the allowable normal stress in the steel is 12.5 ksi, determine the smallest thickness that can be used for the penstock.



$$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2 \times 10^3 \text{ lb/ft}^2 \\ = 216.67 \text{ psi}$$

$$\sigma_i = 12.5 \text{ ksi} = 12.5 \times 10^3 \text{ psi}$$

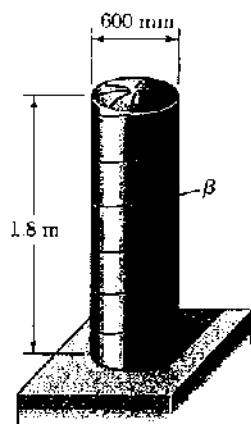
$$r = \frac{1}{2}d - t = 18 - t$$

$$\sigma_i = \frac{pr}{t}, \quad \frac{r}{t} = \frac{\sigma_i}{p}, \quad \frac{18 - t}{t} = \frac{12.5 \times 10^3}{216.67} = 57.692$$

$$\frac{18}{t} = 58.692$$

$$t = 0.307 \text{ in.} \quad \blacktriangleleft$$

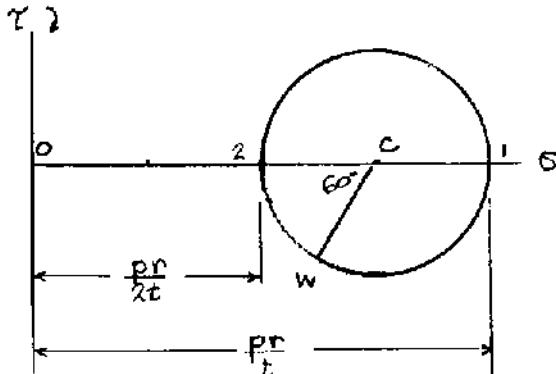
Problem 7.110



7.110 The cylindrical portion of the compressed air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30^\circ$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.

$$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 8 = 292 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t}, \quad \sigma_2 = \frac{1}{2} \frac{Pr}{t}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{Pr}{t}$$

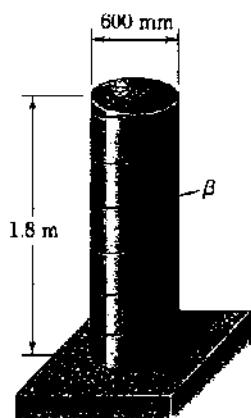
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{Pr}{t}$$

$$\sigma_w = \sigma_{ave} + R \cos 60^\circ \\ = \frac{5}{8} \frac{Pr}{t}$$

$$P = \frac{8}{5} \frac{\sigma_w t}{r}$$

$$P = \frac{8}{5} \frac{(75)(8)}{292} = 3.29 \text{ MPa} \quad \blacksquare$$

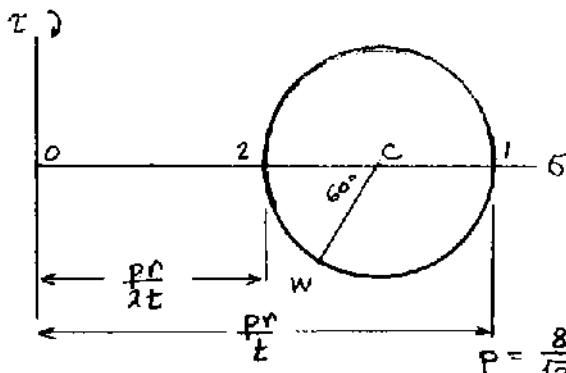
Problem 7.111



7.111 The cylindrical portion of the compressed air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30^\circ$ with the horizontal. Determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.

$$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 8 = 292 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t}, \quad \sigma_2 = \frac{1}{2} \frac{Pr}{t}$$



$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{Pr}{t}$$

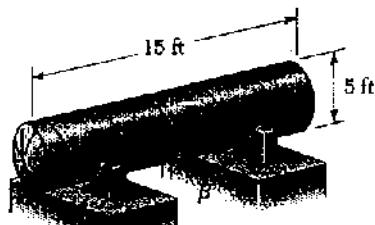
$$\tau_w = R \sin 60^\circ$$

$$= \frac{\sqrt{3}}{8} \frac{Pr}{t}$$

$$P = \frac{8}{\sqrt{3}} \frac{\tau_w t}{R}$$

$$P = \frac{8}{\sqrt{3}} \frac{(30)(8)}{292} = 3.80 \text{ MPa} \quad \blacksquare$$

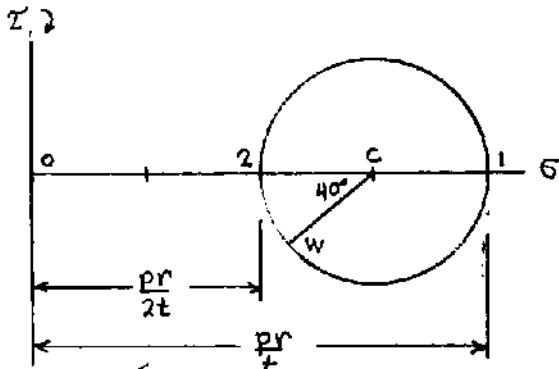
Problem 7.112



7.112 The pressure tank shown has a 0.375-in. wall thickness and butt welded seams forming an angle $\beta \approx 20^\circ$ with a transverse plane. For a gage pressure of 85 psi, determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

$$d = 5 \text{ ft} = 60 \text{ in.} \quad r = \frac{1}{2}d - t = 30 - 0.375 = 29.625 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715 \text{ psi}$$



$$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5 \text{ psi}$$

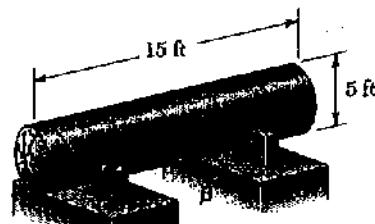
$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 5036.25 \text{ psi}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75 \text{ psi}$$

$$(a) \sigma_w = \sigma_{ave} - R \cos 40^\circ = 3750 \text{ psi}$$

$$(b) \tau_w = R \sin 40^\circ = 1078 \text{ psi}$$

Problem 7.113



7.113 The pressure tank shown has a 0.375-in. wall thickness and butt welded seams forming an angle β with a transverse plane. Determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 1350 psi when the gage pressure is 85 psi.

$$d = 5 \text{ ft} = 60 \text{ in.} \quad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715 \text{ psi}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5 \text{ psi}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75$$

$$\tau_w = R \sin 2\beta = \tau_w$$

$$\sin 2\beta_a = \frac{\tau_w}{R} = \frac{1350}{1678.75} = 0.80417$$

$$2\beta_a = -53.53^\circ$$

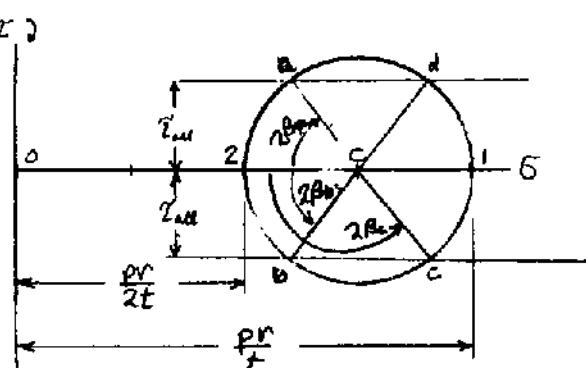
$$\left. \begin{array}{l} \beta_a \approx -26.8^\circ \\ \beta_b = 26.8^\circ \end{array} \right\} -26.8^\circ \leq \beta \leq 26.8^\circ$$

$$2\beta_b = +53.53^\circ$$

$$2\beta_c = -53.53^\circ + 180^\circ = 126.47^\circ$$

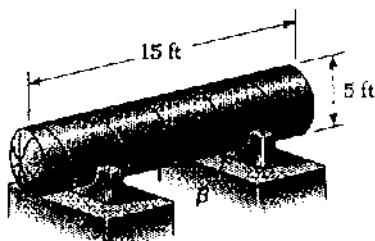
$$2\beta_d = 53.53^\circ + 180^\circ = 233.53^\circ$$

$$\left. \begin{array}{l} \beta_c = 63.2^\circ \\ \beta_d = 116.8^\circ \end{array} \right\} 63.2^\circ \leq \beta \leq 116.8^\circ$$

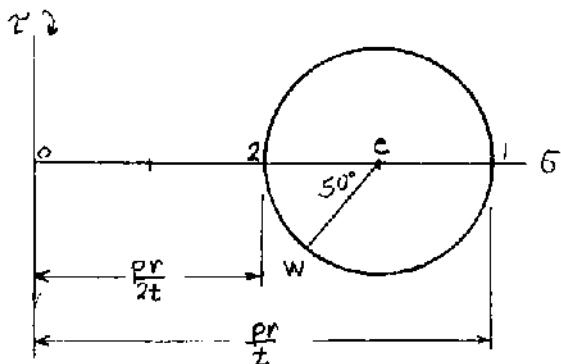


Problem 7.114

7.114 The pressure tank shown has a 0.375-in. wall thickness and butt welded seams forming an angle $\beta = 25^\circ$ with a transverse plane. Determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 18 ksi and the allowable shearing stress parallel to the weld is 10 ksi.



$$d = 5 \text{ ft} = 60 \text{ in.} \quad r = \frac{1}{2}d - t = 30 - 0.375 = 29.625 \text{ in.}$$



$$\sigma_1 = \frac{Pr}{t}$$

$$\sigma_2 = \frac{Pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{Pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{Pr}{t}$$

$$\sigma_w = \sigma_{ave} - R \cos 50^\circ$$

$$= \left(\frac{3}{4} - \frac{1}{4} \cos 50^\circ \right) \frac{Pr}{t}$$

$$= 0.5893 \frac{Pr}{t}$$

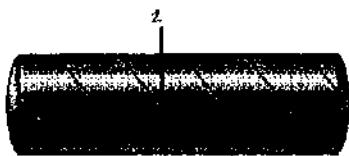
$$P = \frac{\sigma_w t}{0.5893 r} = \frac{(18)(0.375)}{(0.5893)(29.625)} = 0.387 \text{ ksi} = 387 \text{ psi}$$

$$\tau_w = R \sin 50^\circ = 0.19151 \frac{Pr}{t}$$

$$P = \frac{\tau_w t}{0.19151 r} = \frac{(10)(0.375)}{(0.19151)(29.625)} = 0.661 \text{ ksi} = 661 \text{ psi}$$

Allowable gage pressure is the smaller value. $P = 387 \text{ psi}$ ■

Problem 7.115



7.115 The steel pressure tank shown has a 30-in. inner diameter and a 0.375-in. wall thickness. Knowing that the butt welded seams form an angle $\beta = 50^\circ$ with the longitudinal axis of the tank and that the gage pressure in the tank is 200 psi, determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

50°

$$r = \frac{1}{2}d = 15 \text{ in.}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(200)(15)}{0.375} = 8000 \text{ psi}$$

$\tau_w \text{ (psi)}$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 4000 \text{ psi}$$

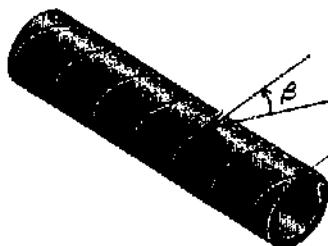
$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 6000 \text{ psi}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 2000 \text{ psi}$$

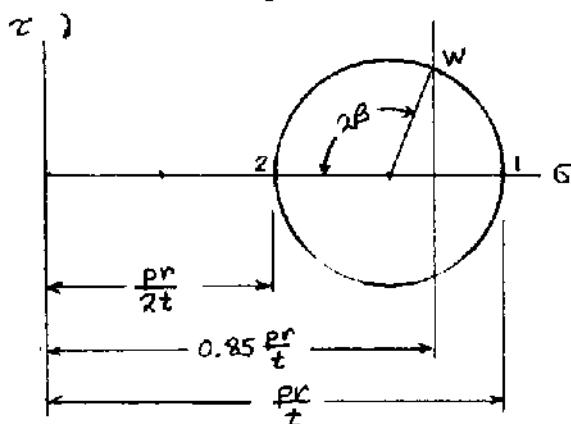
$$(a) \sigma_w = \sigma_{ave} + R \cos 100^\circ \\ = 5652 \text{ psi}$$

$$(b) \tau_w = R \sin 100^\circ \\ = 1970 \text{ psi}$$

Problem 7.116



7.116 The pressurized tank shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not to be larger than 85 percent of the maximum stress in the tank.



$$\sigma_1 = \frac{Pr}{t}$$

$$\sigma_2 = \frac{Pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{Pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{Pr}{t}$$

$$\sigma_w = \sigma_{ave} - R \cos 2\beta$$

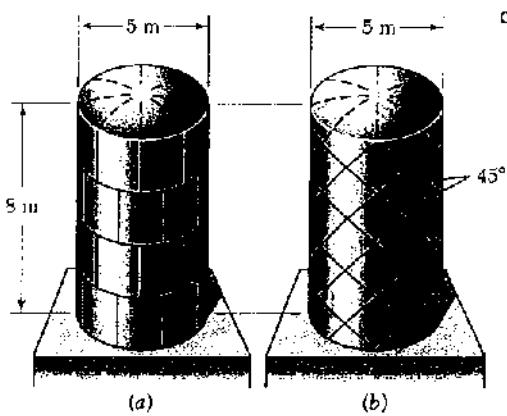
$$0.85 \frac{Pr}{t} = \left(\frac{3}{4} - \frac{1}{4} \cos 2\beta \right) \frac{Pr}{t}$$

$$\cos 2\beta = -4 \left(0.85 - \frac{3}{4} \right) = -0.4$$

$$2\beta = 113.6^\circ \quad \beta = 56.8^\circ$$

Problem 7.117

7.117 Square plates, each of 16-mm thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed air tank. Knowing that the allowable normal stress perpendicular to the weld is 65 MPa, determine the largest allowable gage pressure in each case.



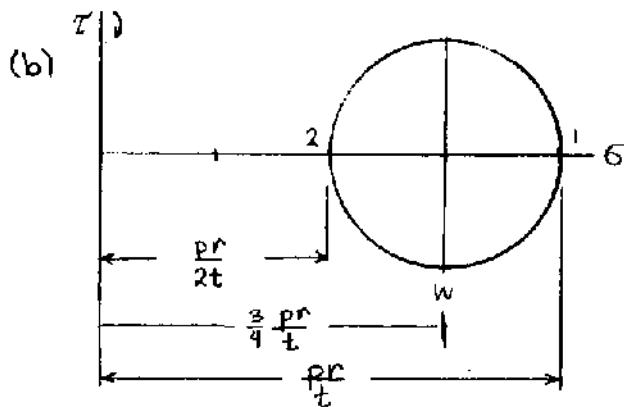
$$r = \frac{1}{2}d - t = \frac{1}{2}(5) - 16 \times 10^{-3} = 2.484 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$(a) \quad \sigma_1 = 65 \text{ MPa} = 65 \times 10^6 \text{ Pa}$$

$$p = \frac{\sigma_1 t}{r} = \frac{(65 \times 10^6)(16 \times 10^{-3})}{2.484} = 419 \times 10^3 \text{ Pa}$$

$$419 \text{ kPa}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

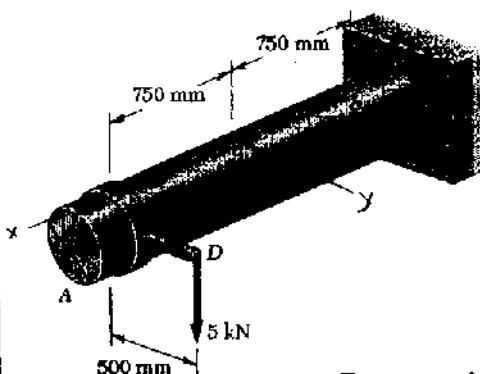
$$\beta = \pm 45^\circ$$

$$\sigma_w = \sigma_{ave} + R \cos \beta \\ = \frac{3}{4} \frac{pr}{t}$$

$$p = \frac{4\sigma_w t}{3r} = \frac{(4)(65 \times 10^6)(16 \times 10^{-3})}{(3)(2.484)} = 558 \times 10^3 \text{ Pa} \quad 558 \text{ kPa}$$

Problem 7.118

7.118 The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point *a* on the top of the tank.



$$r = \frac{1}{2}d = 225 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$$

$$\sigma_2 = \frac{Pr}{2t} = 22.5 \text{ MPa}$$

$$\text{Torsion: } C_1 = 225 \text{ mm}, \quad C_2 = 225 + 6 = 231 \text{ mm}$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N-m}$$

$$\tau = \frac{TC}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6 \text{ Pa} = 1.292 \text{ MPa}$$

Transverse shear: $\tau = 0$ at point *a*.

$$\text{Bending: } I = \frac{1}{2}J = 223.45 \times 10^{-6} \text{ m}^4, \quad c = 231 \times 10^{-3} \text{ m}$$

$$\text{At point } a \quad M = (5 \times 10^3)(750 \times 10^{-3}) = 3750 \text{ N-m}$$

$$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88 \text{ MPa}$$

Total stresses (MPa)

$$\text{Longitudinal} \quad \sigma_x = 22.5 + 3.88 = 26.38$$

$$\text{Circumferential} \quad \sigma_y = 45$$

$$\text{Shear} \quad \tau_{xy} = 1.292$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40 \text{ MPa}$$

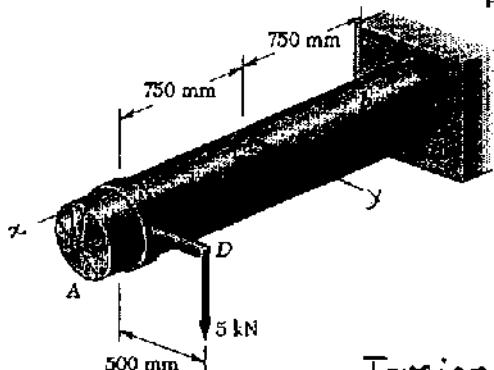
$$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max\text{ (in-plane)}} = R = 9.40 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.119

7.119 For the compressed-air tank and loading of Prob. 7.118, determine the maximum normal stress and the maximum in-plane shearing stress at point *b* on the top of the tank.

7.118 The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point *a* on the top of the tank.



$$r = \frac{1}{2}d = 225 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$$

$$\sigma_2 = \frac{Pr}{2t} = 22.5 \text{ MPa}$$

$$\text{Torsion: } C_1 = 225 \text{ mm}, \quad C_2 = 225 + 6 = 231 \text{ mm}$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N}\cdot\text{m}$$

$$\tau = \frac{TC}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6 \text{ Pa} = 1.292 \text{ MPa}$$

Transverse shear: $\tau = 0$ at point

$$\text{Bending: } I = \frac{1}{2}J = 223.45 \times 10^{-6} \text{ m}^4, \quad c = 231 \times 10^{-3} \text{ m}$$

$$\text{At point } b \quad M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(7500)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 7.75 \text{ MPa}$$

Total stresses (MPa)

$$\text{Longitudinal: } \sigma_x = 22.5 + 7.75 = 30.25 \text{ MPa}$$

$$\text{Circumferential: } \sigma_y = 45 \text{ MPa}$$

$$\text{Shear } \tau_{xy} = 1.292 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.625 \text{ MPa}$$

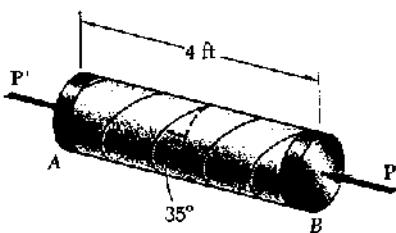
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.487 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa}$$

$$\tau_{max(in-plane)} = R = 7.49 \text{ MPa}$$

Problem 7.120

7.120 A pressure vessel of 10-in. inner diameter and 0.25 in. wall thickness is fabricated from a 4-ft section of spirally welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric axial forces P and P' are applied to the end plates. Determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.



$$r = \frac{1}{2}d = \frac{1}{2}(10) = 5 \text{ in.} \quad t = 0.25 \text{ in.}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(300)(5)}{0.25} = 6000 \text{ psi} = 6 \text{ ksi}$$

$$\sigma_2 = \frac{Pr}{2t} = \frac{(300)(5)}{(2)(0.25)} = 3000 \text{ psi} = 3 \text{ ksi}$$

$$r_o = r + t = 5 + 0.25 = 5.25 \text{ in.}$$

$$A = \pi(r_o^2 - r^2) = \pi(5.25^2 - 5.00^2) = 8.0503 \text{ in.}^2$$

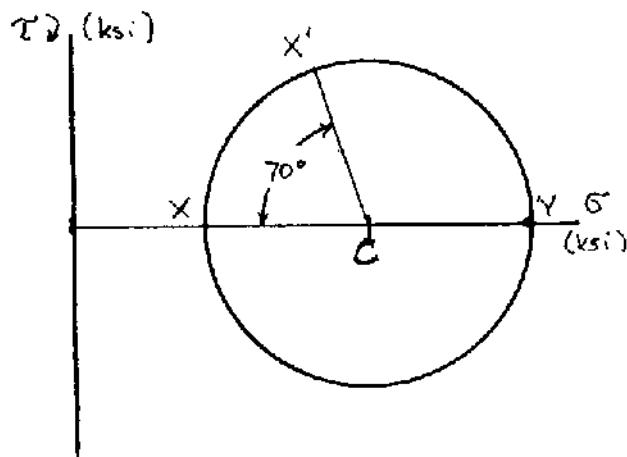
$$\sigma = -\frac{P}{A} = -\frac{10 \times 10^3}{8.0503} = -1242 \text{ psi} = -1.242 \text{ ksi}$$

Total stresses. Longitudinal $\sigma_x = 3 - 1.242 = 1.758 \text{ ksi}$

Circumferential $\sigma_y = 6 \text{ ksi}$

Shear $\tau_{xy} = 0$

Plotted points for Mohr's circle.



$$X: (1.758, 0)$$

$$Y: (6, 0)$$

$$C: (2.121, 0)$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 3.879 \text{ ksi}$$

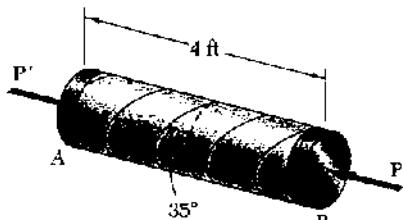
$$R = \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2} \\ = \sqrt{\left[\frac{(6 - 1.758)}{2}\right]^2 + 0} = 2.121 \text{ ksi}$$

$$(a) \sigma_{x'} = \sigma_{ave} + R \cos 70^\circ = 3.879 - 2.121 \cos 70^\circ = 3.15 \text{ ksi}$$

$$(b) |\tau_{xy}| = R \sin 70^\circ = 2.121 \sin 70^\circ = 1.993 \text{ ksi}$$

Problem 7.121

7.121 Solve Prob. 7.120, assuming that the magnitude P of the two forces is increased to 30 kips.



7.120 A pressure vessel of 10-in. inner diameter and 0.25 in. wall thickness is fabricated from a 4-ft section of spirally welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric axial forces P and P' are applied to the end plates. Determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

$$r = \frac{1}{2}d = \frac{1}{2}(10) = 5 \text{ in.} \quad t = 0.25 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(300)(5)}{0.25} = 6000 \text{ psi} = 6 \text{ ksi}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(300)(5)}{(2)(0.25)} = 3000 \text{ psi} = 3 \text{ ksi}$$

$$r_o = r + t = 5 + 0.25 = 5.25 \text{ in.}$$

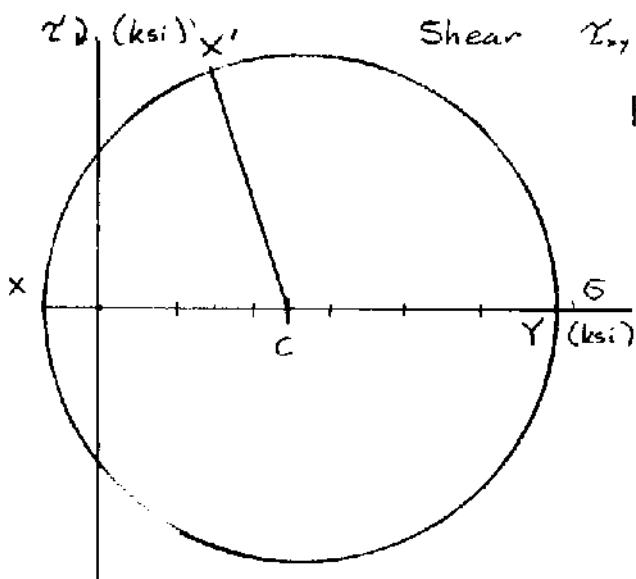
$$A = \pi(r_o^2 - r^2) = \pi(5.25^2 - 5^2) = 8.0503 \text{ in}^2$$

$$\sigma = -\frac{P}{A} = -\frac{30 \times 10^3}{8.0503} = -3727 \text{ psi} = -3.727 \text{ ksi}$$

Total stresses. Longitudinal $\sigma_x = 3 - 3.727 = -0.727 \text{ ksi}$

Circumferential $\sigma_y = 6 \text{ ksi}$

Shear $\tau_{xy} = 0$



Plotted points for Mohr's circle.

$$X: (-0.727, 0)$$

$$Y: (6, 0)$$

$$C: (2.6365, 0)$$

$$\bar{\sigma}_{ave} = \frac{1}{2}(\bar{\sigma}_x + \bar{\sigma}_y) = 2.6365 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

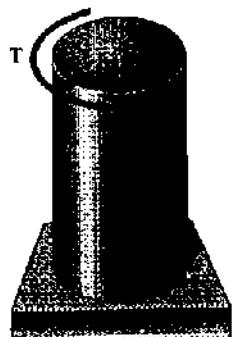
$$= \sqrt{\left(\frac{-0.727 - 6}{2}\right)^2 + 0} = 3.3635 \text{ ksi}$$

$$(a) \sigma_{x'} = \bar{\sigma}_{ave} - R \cos 70^\circ = 2.6365 - 3.3635 \cos 70^\circ = 1.486 \text{ ksi}$$

$$(b) |\tau_{x'y'}| = R \sin 70^\circ = 3.3635 \sin 70^\circ = 3.16 \text{ ksi}$$

Problem 7.122

7.122 A torque of magnitude $T = 12 \text{ kN} \cdot \text{m}$ is applied to the end of a tank containing compressed air under a pressure to 8 MPa. Knowing that the tank has a 180-mm inner diameter and a 12-mm wall thickness, Determine the maximum normal stress and the maximum shearing stress in the tank.



$$d = 180 \text{ mm} \quad r = \frac{1}{2}d = 90 \text{ mm} \quad t = 12 \text{ mm}$$

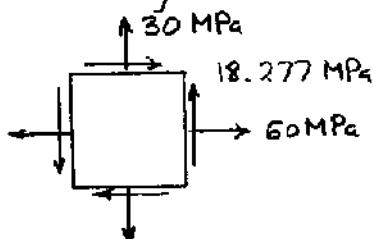
$$\text{Torsion: } c_1 = 90 \text{ mm} \quad c_2 = 90 + 12 = 102 \text{ mm}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau' = \frac{Tc}{J} = \frac{(12 \times 10^3)(102 \times 10^{-3})}{66.968 \times 10^{-6}} = 18.277 \text{ MPa}$$

$$\text{Pressure: } \sigma_i = \frac{Pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa} \quad \sigma_o = \frac{Pr}{2t} = 30 \text{ MPa}$$

Summary of stresses $\sigma_x = 60 \text{ MPa}, \sigma_y = 30 \text{ MPa}, \tau_{xy} = 18.277 \text{ MPa}$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 68.64 \text{ MPa}$$

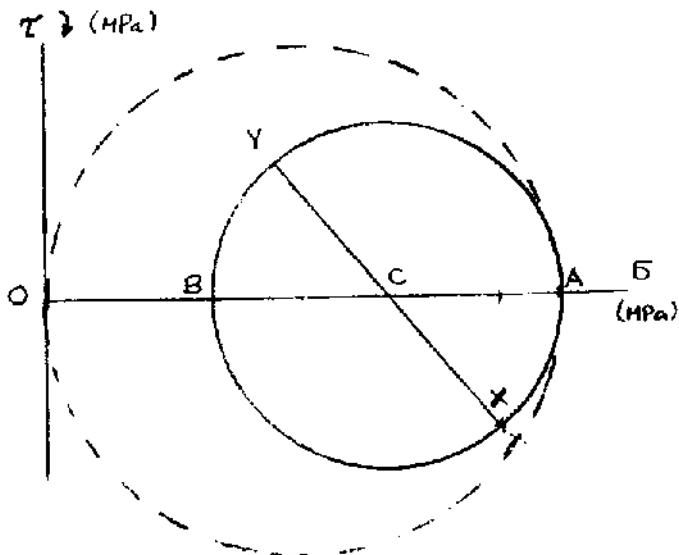
$$\sigma_b = \sigma_{ave} - R = 21.36 \text{ MPa}$$

$$\sigma_c \approx 0$$

$$\sigma_{max} = 68.64 \text{ MPa} \quad \blacksquare$$

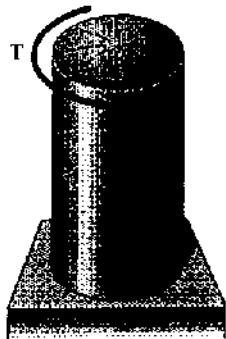
$$\sigma_{min} = 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 34.32 \text{ MPa} \quad \blacksquare$$



Problem 7.123

7.123 The tank shown has a 180-mm inner diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress is 75 MPa.



$$r = \frac{1}{2}d = \left(\frac{1}{2}\right)(180) = 90 \text{ mm} \quad t = 12 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa} \quad \sigma_2 = \frac{Pr}{2t} = 30 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 45 \text{ MPa}$$

$$\sigma_{max} = 75 \text{ MPa} \quad R = \sigma_{max} - \sigma_{ave} = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98 \text{ MPa}$$

$$= 25.98 \times 10^6 \text{ Pa}$$

$$\text{Tension: } c_1 = 90 \text{ mm} \quad c_2 = 90 + 12 = 102 \text{ mm}$$

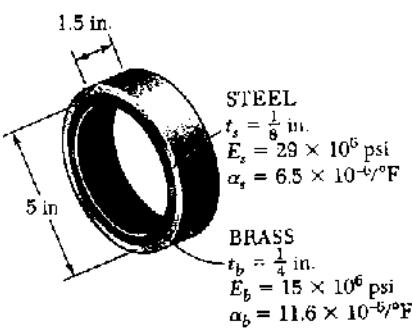
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau_{xy} = \frac{Tc}{J} \quad T = \frac{J \tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 17.06 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 7.124

7.124 A brass ring of 5-in. outer diameter and 0.25-in. thickness fits exactly inside a steel ring of 5-in. inner diameter and 0.125-in. thickness when the temperature of both rings is 50°F. Knowing that the temperature of both rings is then raised to 125°F, determine (a) the tensile stress in the steel ring, (b) the corresponding pressure exerted by the brass ring on the steel ring.



Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p_s $\sigma_s = \frac{pr}{t_s}$ (1)

Corresponding strain $\epsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$

Strain due to temperature change $\epsilon_{st} = \alpha_s \Delta T$

Total strain $\epsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \epsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p_s $\sigma_b = -\frac{pr}{t_b}$

Corresponding strains $\epsilon_{bp} = -\frac{pr}{E_b t_b}$, $\epsilon_{bt} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \epsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b $\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \quad (2)$$

Data: $\Delta T = 125^\circ\text{F} - 50^\circ\text{F} = 75^\circ\text{F}$

$$r = \frac{1}{2} d = \frac{1}{2}(5) = 2.5 \text{ in.}$$

From Eq. (2) $\left[\frac{2.5}{(29 \times 10^6)(0.125)} + \frac{2.5}{(15 \times 10^6)(0.25)} \right] p = (11.6 - 6.5)(10^{-6})(75)$

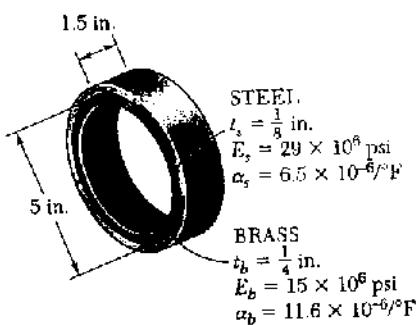
$$1.35632 \times 10^{-6} p = 382.5 \times 10^{-6} \quad p = 282.0 \text{ psi}$$

From Eq. (1) $\sigma_s = \frac{pr}{t_s} = \frac{(282.0)(2.5)}{0.125} = 5.64 \times 10^3 \text{ psi}$

(a) $\sigma_s = 5.64 \text{ ksi}$

(b) $p = 282 \text{ psi}$

Problem 7.125



7.125 Solve Prob. 7.124, assuming that the brass ring is 0.125 in. thick and the steel ring is 0.25 in. thick.

7.124 A brass ring of 5-in. outer diameter and 0.25-in. thickness fits exactly inside a steel ring of 5-in. inner diameter and 0.125-in. thickness when the temperature of both rings is 50°F. Knowing that the temperature of both rings is then raised to 125°F, determine (a) the tensile stress in the steel ring, (b) the corresponding pressure exerted by the brass ring on the steel ring.

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p_s $\sigma_s = \frac{pr}{t_s}$ (1)

Corresponding strain $\epsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$

Strain due to temperature change $\epsilon_{st} = \alpha_s \Delta T$

Total strain $\epsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \epsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p_b $\sigma_b = -\frac{pr}{t_b}$

Corresponding strains $\epsilon_{bp} = -\frac{pr}{E_b t_b}$, $\epsilon_{bt} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \epsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b $\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \quad (2)$$

Data: $\Delta T = 125^{\circ}\text{F} - 50^{\circ}\text{F} = 75^{\circ}\text{F}$

$$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5 \text{ in.}$$

From Eq.(2) $\left[\frac{2.5}{(29 \times 10^6)(0.25)} + \frac{2.5}{(15 \times 10^6)(0.125)} \right] p = (11.6 - 6.5)(10^{-6})(75)$

$$1.67816 \times 10^{-6} p = 382.5 \times 10^{-6} \quad p = 227.93 \text{ psi}$$

From Eq.(1) $\sigma_s = \frac{pr}{t_s} = \frac{(227.93)(2.5)}{0.25} = 2279 \text{ psi}$

(a) $\sigma_s = 2.28 \text{ ksi}$

(b) $p = 228 \text{ psi}$

Problem 7.126

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ .

$$7.126 \quad \epsilon_x = +240 \mu, \quad \epsilon_y = +160 \mu, \quad \gamma_{xy} = +150 \mu, \quad \theta = 60^\circ \Rightarrow -60^\circ$$

$$\frac{\epsilon_x + \epsilon_y}{2} = +200 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -40 \mu$$

$$\begin{aligned} \epsilon_{x'} &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ 200 + 40 \cos(-120^\circ) + \frac{150}{2} \sin(-120^\circ) \right\} \mu = 115.0 \mu \end{aligned}$$

$$\begin{aligned} \epsilon_{y'} &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ 200 - 40 \cos(-120^\circ) - \frac{150}{2} \sin(-120^\circ) \right\} \mu = 285 \mu \end{aligned}$$

$$\begin{aligned} \gamma_{xy'} &= -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \\ &= \left\{ -(240 - 160) \sin(-120^\circ) + 150 \cos(-120^\circ) \right\} \mu = -5.72 \mu \end{aligned}$$

Problem 7.127

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ .

$$7.127 \quad \epsilon_x = 0, \quad \epsilon_y = +320 \mu, \quad \gamma_{xy} = -100 \mu, \quad \theta = 30^\circ \Rightarrow +30^\circ$$

$$\frac{\epsilon_x + \epsilon_y}{2} = 160 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -160 \mu$$

$$\begin{aligned} \epsilon_{x'} &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ 160 - 160 \cos 60^\circ - \frac{100}{2} \sin 60^\circ \right\} \mu = +36.7 \mu \end{aligned}$$

$$\begin{aligned} \epsilon_{y'} &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ 160 + 160 \cos 60^\circ + \frac{100}{2} \sin 60^\circ \right\} \mu = +283 \mu \end{aligned}$$

$$\begin{aligned} \gamma_{xy'} &= -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \\ &= \left\{ -(0 - 320) \sin 60^\circ - 100 \cos 60^\circ \right\} \mu = +227 \mu \end{aligned}$$

Problem 7.128

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ .

$$7.128 \quad \epsilon_x = -800 \mu, \quad \epsilon_y = +450 \mu, \quad \gamma_{xy} = +200 \mu, \quad \theta = 25^\circ \text{ or } -25^\circ$$

$$\frac{\epsilon_x + \epsilon_y}{2} = -175 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -625 \mu$$

$$\begin{aligned} \epsilon_{x'} &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ -175 - 625 \cos(-50^\circ) + \frac{200}{2} \sin(-50^\circ) \right\} \mu = -653 \mu \end{aligned}$$

$$\begin{aligned} \epsilon_{y'} &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ -175 + 625 \cos(-50^\circ) - \frac{200}{2} \sin(-50^\circ) \right\} \mu = +303 \mu \end{aligned}$$

$$\begin{aligned} \gamma_{x'y'} &= -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \\ &= \left\{ -(-800 - 450) \sin(-50^\circ) + 200 \cos(-50^\circ) \right\} \mu = -829 \mu \end{aligned}$$

Problem 7.129

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ .

$$7.129 \quad \epsilon_x = +500 \mu, \quad \epsilon_y = -300 \mu, \quad \gamma_{xy} = 0, \quad \theta = 25^\circ \text{ or } +25^\circ$$

$$\frac{\epsilon_x + \epsilon_y}{2} = 100 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = 400 \mu$$

$$\begin{aligned} \epsilon_{x'} &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ 100 + 400 \cos 50^\circ + 0 \right\} \mu = 357 \mu \end{aligned}$$

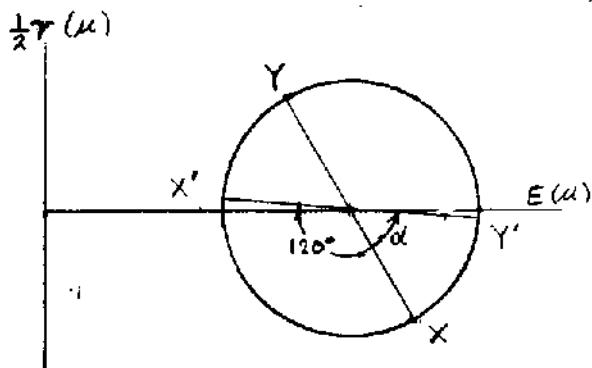
$$\begin{aligned} \epsilon_{y'} &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ 100 - 400 \cos 50^\circ - 0 \right\} \mu = -157.1 \mu \end{aligned}$$

$$\begin{aligned} \gamma_{x'y'} &= -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \\ &= \left\{ -(500 - (-300)) \sin 50^\circ + 0 \right\} \mu = -613 \mu \end{aligned}$$

Problem 7.130

7.130 through 7.133 For the given state of plane strain, Use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

7.130 $E_x = +240 \mu, E_y = +160 \mu, \gamma_{xy} = +150 \mu, \theta = 60^\circ$



Plotted points

$$X: (+240 \mu, -75 \mu)$$

$$Y: (+160 \mu, 75 \mu)$$

$$C: (200 \mu, 0)$$

$$\tan \alpha = \frac{75}{40} = 1.875 \quad \alpha = 61.93^\circ$$

$$R = \sqrt{(40 \mu)^2 + (75 \mu)^2} = 85 \mu$$

$$\beta = 2\theta - \alpha = -120^\circ - 61.93^\circ = -181.93^\circ$$

$$E_{x'} = E_{ave} + R \cos \beta = 200 \mu + (85 \mu) \cos(-181.93^\circ) \\ = 115.0 \mu$$

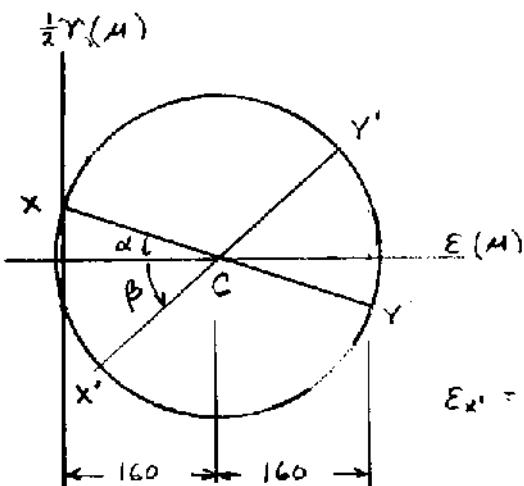
$$E_{y'} = E_{ave} - R \cos \beta = 200 \mu - (85 \mu) \cos(-181.93^\circ) \\ = 285 \mu$$

$$\frac{1}{2}\gamma_{y'} = -R \sin \beta = -85 \sin(-181.93^\circ) = -2.86 \mu \\ \gamma_{x'y'} = -5.7 \mu$$

Problem 7.131

7.130 through 7.133 For the given state of plane strain, Use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

7.131 $E_x = 0, E_y = +320 \mu, \gamma_{xy} = -100 \mu, \theta = 30^\circ$



Plotted points

$$X: (0, 50 \mu)$$

$$Y: (320 \mu, -50 \mu)$$

$$C: (160 \mu, 0)$$

$$\tan \alpha = \frac{50}{160} \quad \alpha = 17.35^\circ$$

$$R = \sqrt{(160 \mu)^2 + (50 \mu)^2} = 167.63 \mu$$

$$\beta = 2\theta - \alpha = 60^\circ - 17.35^\circ = 42.65^\circ$$

$$E_{x'} = E_{ave} - R \cos \beta = 160 \mu - 167.63 \mu \cos 42.65^\circ \\ = 36.7 \mu$$

$$E_{y'} = E_{ave} + R \cos \beta = 160 \mu + 167.63 \mu \cos 42.65^\circ \\ = 283 \mu$$

$$\frac{1}{2}\gamma_{y'} = R \sin \beta = 167.63 \mu \sin 42.65^\circ$$

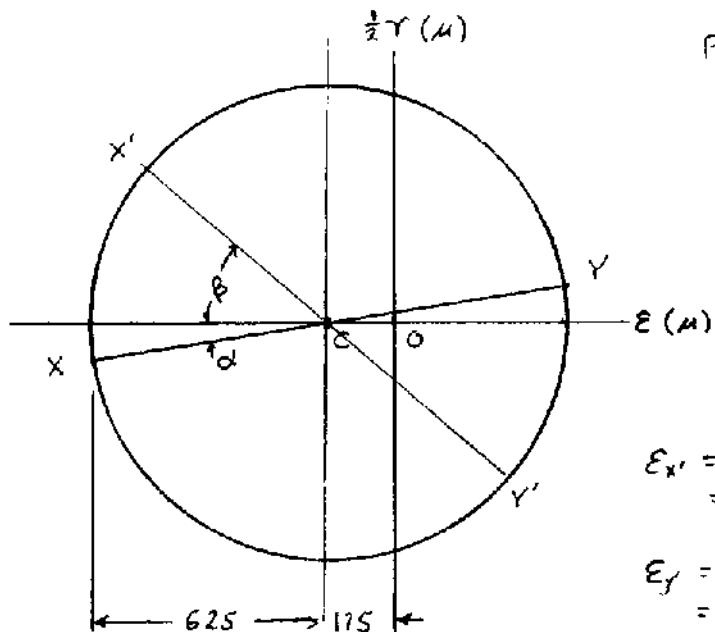
$$\gamma_{x'y'} = -227 \mu$$

Problem 7.132

7.132

7.130 through 7.133 For the given state of plane strain, Use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$\epsilon_x = -800 \mu, \epsilon_y = 450 \mu, \gamma_{xy} = +200 \mu, \theta = 25^\circ$$



Plotted points

$$X: (-800 \mu, -100 \mu)$$

$$Y: (+450 \mu, +100 \mu)$$

$$C: (-175 \mu, 0)$$

$$\tan \alpha = \frac{100}{625} \quad \alpha = 9.09^\circ$$

$$R = \sqrt{(625 \mu)^2 + (100 \mu)^2} = 632.95 \mu$$

$$\beta = 2\theta - \alpha = 50^\circ - 9.09^\circ = 40.91^\circ$$

$$\begin{aligned} \epsilon_{x'} &= \epsilon_{ave} - R \cos \beta = -175 \mu - 632.95 \mu \cos 40.91^\circ \\ &= -653 \mu \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \epsilon_{ave} + R \cos \beta = -175 \\ &= +303 \mu \end{aligned}$$

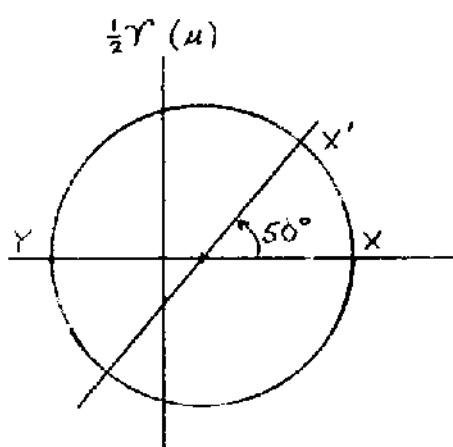
$$\frac{1}{2}\gamma_{xy} = -R \sin \beta = -632.95 \mu \sin 40.91^\circ \quad \gamma_{xy} = -829 \mu$$

Problem 7.133

7.133

7.130 through 7.133 For the given state of plane strain, Use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$\epsilon_x = +500 \mu, \epsilon_y = -300 \mu, \gamma_{xy} = 0, \theta = 25^\circ$$



Plotted points

$$X: (500 \mu, 0)$$

$$Y: (-300 \mu, 0)$$

$$C: (100 \mu, 0)$$

$$\tan \alpha = 0 \quad \alpha = 0$$

$$R = 400 \mu \quad \beta = 2\theta - \alpha = 50^\circ$$

$$\begin{aligned} \epsilon_{x'} &= \epsilon_{ave} + R \cos \beta = 100 \mu + (400 \mu) \cos 50^\circ \\ &= 357 \mu \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \epsilon_{ave} - R \cos \beta = 100 \mu - (400 \mu) \cos 50^\circ \\ &= -157.1 \mu \end{aligned}$$

$$\frac{1}{2}\gamma_{xy} = -R \sin \beta = -400 \sin 50^\circ = -306.4 \mu \quad \gamma_{xy} = -613 \mu$$

Problem 7.134

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

7.134

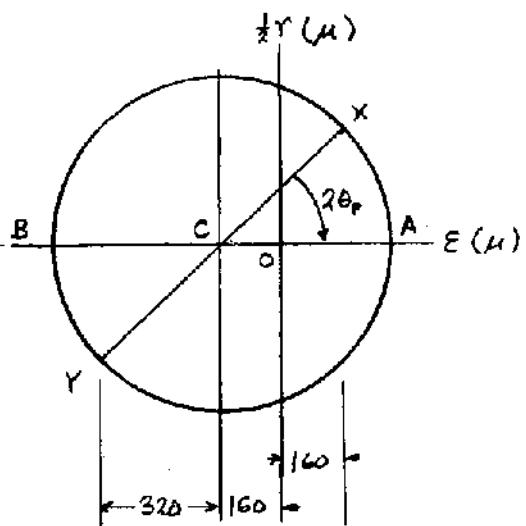
$$\varepsilon_x = +160 \mu \quad \varepsilon_y = -480 \mu \quad \gamma_{xy} = -600 \mu$$

(a) For Mohr's circle of strain, plot points

$$X: (160 \mu, 300 \mu)$$

$$Y: (-480 \mu, -300 \mu)$$

$$C: (-160 \mu, 0)$$



$$(a) \tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-300}{320} = -0.9375$$

$$2\theta_p = -43.15^\circ \quad \theta_p = -21.58^\circ \quad \text{and } -21.58 + 90 = 68.42^\circ$$

$$\theta_a = -21.58^\circ$$

$$\theta_b = 68.42^\circ$$

$$R = \sqrt{(320 \mu)^2 + (300 \mu)^2} = 438.6 \mu$$

$$\varepsilon_a = \varepsilon_{ave} + R = -160 \mu + 438.6 \mu = +278.6 \mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = -160 \mu - 438.6 \mu = -598.6 \mu$$

$$(b) \frac{1}{2} \gamma_{(\max, \text{in-plane})} = R \quad \gamma_{(\max, \text{in-plane})} = 2R = 877 \mu$$

$$(c) \varepsilon_c = -\frac{\nu}{1-\nu} (\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu} (\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3} (160 \mu - 480 \mu) = 160 \mu$$

$$\varepsilon_{\max} = 278.6 \mu \quad \varepsilon_{\min} = -598.6 \mu$$

$$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 278.6 \mu + 598.6 \mu = 877 \mu$$

Problem 7.135

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

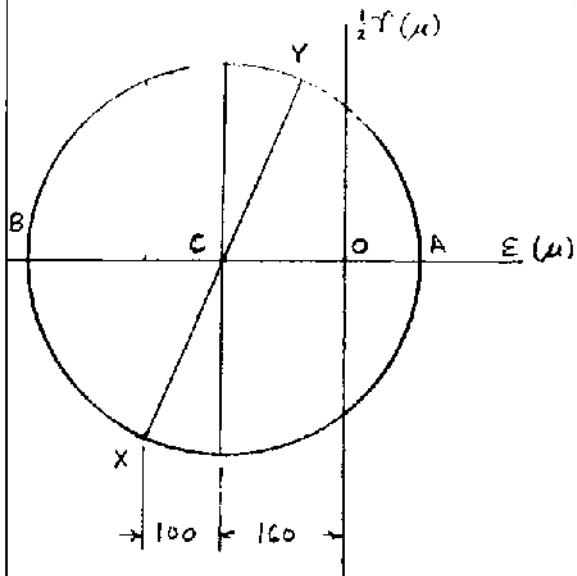
$$7.135 \quad \epsilon_x = -260 \mu \quad \epsilon_y = -60 \mu \quad \gamma_{xy} = +480 \mu$$

For Mohr's circle of strain plot points

$$X: (-260 \mu, -240 \mu)$$

$$Y: (-60 \mu, 240 \mu)$$

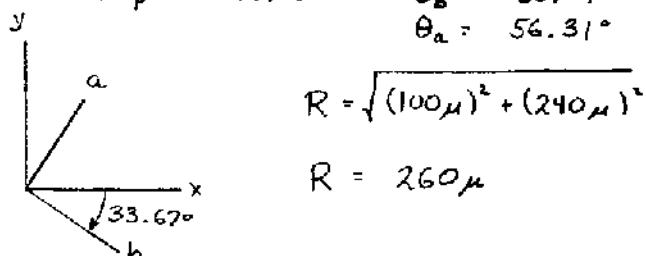
$$C: (-160 \mu, 0)$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{480}{-260 + 60} = -2.4$$

$$2\theta_p = -67.38^\circ \quad \theta_b = -33.67^\circ$$

$$\theta_a = 56.31^\circ$$



$$R = \sqrt{(100 \mu)^2 + (240 \mu)^2}$$

$$R = 260 \mu$$

$$(a) \quad \epsilon_a = \epsilon_{av} + R = -160 \mu + 260 \mu = 100 \mu$$

$$\epsilon_b = \epsilon_{av} - R = -160 \mu - 260 \mu = -420 \mu$$

$$(b) \quad \frac{1}{2} \gamma_{max, (in-plane)} = R \quad \gamma_{max, (in-plane)} = 2R = 520 \mu$$

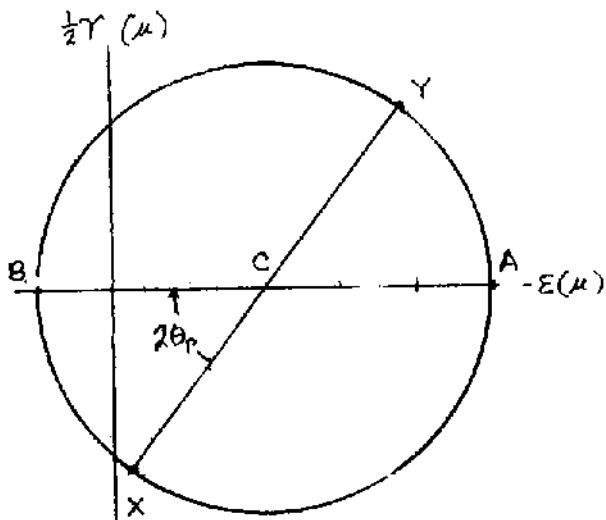
$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3} (-260 - 60) \\ = 160 \mu$$

$$\epsilon_{max} = 160 \mu \quad \epsilon_{min} = -420 \mu$$

$$(c) \quad \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 160 \mu + 420 \mu = 580 \mu$$

Problem 7.136

7.136 $\epsilon_x = +300 \mu, \epsilon_y = +570 \mu, \gamma_{xy} = +720 \mu$



Plotted points.

$$X: (30 \mu, -360 \mu)$$

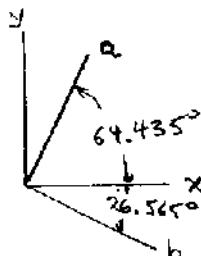
$$Y: (570 \mu, +360 \mu)$$

$$C: (300 \mu, 0)$$

$$\tan 2\theta_p = \frac{-360}{720} = -1.3333$$

$$2\theta_p = -53.13^\circ$$

(a) $\theta_b = -26.565^\circ$
 $\theta_a = 64.435^\circ$



$$R = \sqrt{(270 \mu)^2 + (360 \mu)^2} = 450 \mu$$

$$\epsilon_a = \epsilon_{ave} + R = 300 \mu + 450 \mu = 750 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = 300 \mu - 450 \mu = -150 \mu$$

(b) $\gamma_{max(in-plane)} = 2R = 900 \mu$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{1/3}{2/3} (750 \mu - 150 \mu) = -300 \mu$$

$$\epsilon_{max} = \epsilon_a = 750 \mu, \quad \epsilon_{min} = \epsilon_c = -300 \mu$$

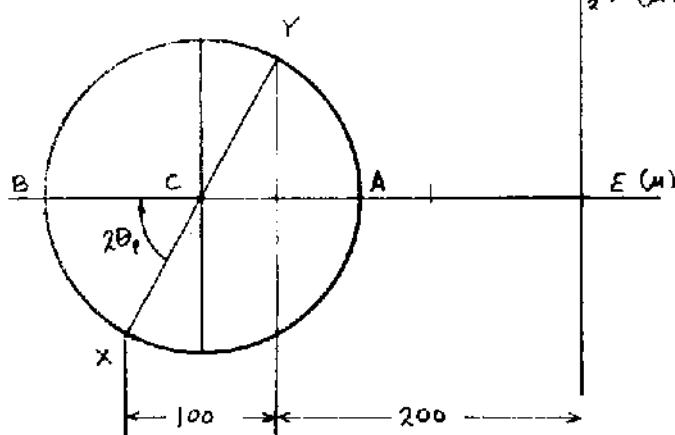
(c) $\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 750 \mu - (-300 \mu) = 1050 \mu$

Problem 7.137

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

7.137

$$\epsilon_x = -600 \mu \quad \epsilon_y = -400 \mu \quad \gamma_{xy} = +350 \mu$$



Plotted points

$$X: (-600 \mu, -175 \mu)$$

$$Y: (-400 \mu, +175 \mu)$$

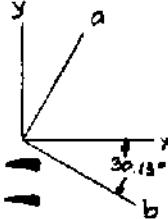
$$C: (-500 \mu, 0)$$

$$\tan 2\theta_p = -\frac{175}{100}$$

$$2\theta_p = -60.26$$

$$\theta_b = -30.13^\circ$$

$$\theta_a = 59.87^\circ$$



$$R = \sqrt{(100 \mu)^2 + (175 \mu)^2} \\ = 201.6 \mu$$

$$(a) \epsilon_a = \epsilon_{ave} + R = -500 \mu + 201.6 \mu = -298 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -500 \mu - 201.6 \mu = -702 \mu$$

$$(b) \gamma_{max(in-plane)} = 2R = 403 \mu$$

$$\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu}(\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3}(-600 \mu - 400 \mu) \\ = +500 \mu$$

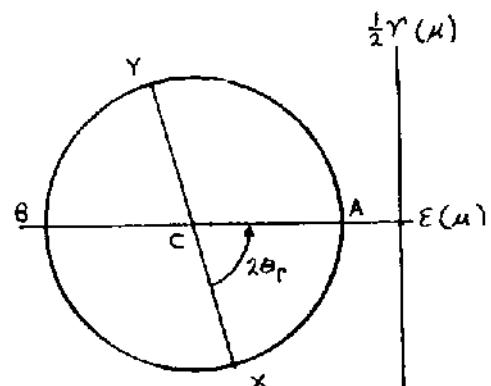
$$\epsilon_{max} = 500 \mu \quad \epsilon_{min} = -702 \mu$$

$$(c) \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 500 \mu + 702 \mu = 1202 \mu$$

Problem 7.138

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$7.138 \quad \epsilon_x = -180\mu, \quad \epsilon_y = -260\mu, \quad \gamma_{xy} = +315\mu$$



Plot points

$$X: (-180\mu, -157.5\mu), \quad Y: (-260\mu, +157.5\mu)$$

$$C: (-220\mu, 0)$$

$$(a) \tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{315}{80} = 3.9375$$

$$2\theta_p = 75.75^\circ \quad \theta_a = 37.87^\circ$$

$$\theta_b = 127.87^\circ$$

$$R = \sqrt{(40\mu)^2 + (157.5\mu)^2} = 162.5\mu$$

$$\epsilon_a = \epsilon_{ave} + R = -220\mu + 162.5\mu = -57.5\mu$$

$$\epsilon_b = \epsilon_{ave} - R = -220\mu - 162.5 = -382.5\mu$$

$$(b) \gamma_{max(inplane)} = 2R = 325\mu$$

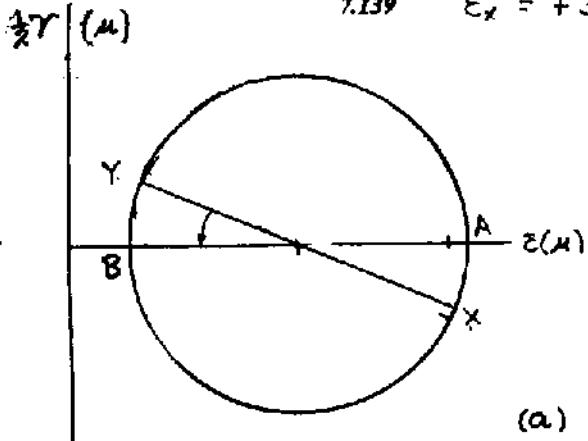
$$(c) \epsilon_c = 0 \quad \epsilon_{max} = 0, \quad \epsilon_{min} = -382.5\mu$$

$$\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 0 + 382.5\mu = 382.5\mu$$

Problem 7.139

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$7.139 \quad \epsilon_x = +300\mu, \quad \epsilon_y = +60\mu, \quad \gamma_{xy} = +100\mu$$



$$X: (300\mu, -50\mu), \quad Y: (60\mu, 50\mu)$$

$$C: (180\mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{100}{300 - 60} = 22.62^\circ$$

$$\theta_a = 11.31^\circ \quad \theta_b = 101.31^\circ$$

$$R = \sqrt{(120\mu)^2 + (50\mu)^2} = 130\mu$$

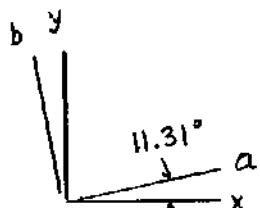
$$(a) \epsilon_a = \epsilon_{ave} + R = 180\mu + 130\mu = 310\mu$$

$$\epsilon_b = \epsilon_{ave} - R = 180\mu - 130\mu = 50\mu$$

$$(b) \gamma_{max(inplane)} = 2R = 260\mu$$

$$(c) \epsilon_c = 0 \quad \epsilon_{max} = 310\mu \quad \epsilon_{min} = 0$$

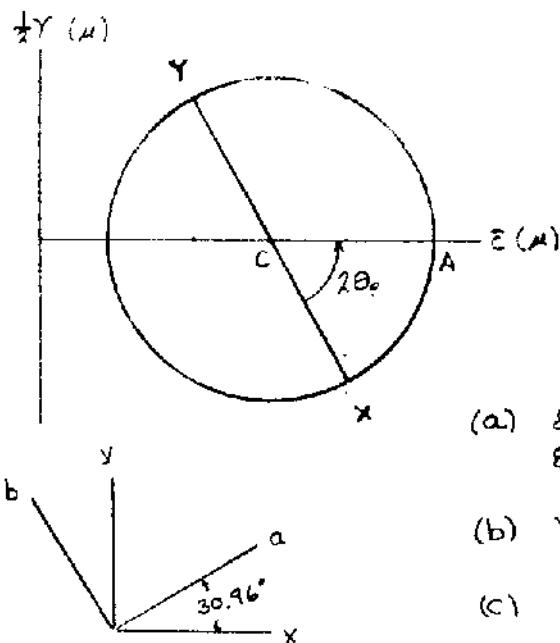
$$\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 310\mu$$



Problem 7.140

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$7.140 \quad \varepsilon_x = +400 \mu \quad \varepsilon_y = +200 \mu \quad \gamma_{xy} = +375 \mu$$



Plotted points

$$X: (+400 \mu, -187.5 \mu)$$

$$Y: (+200 \mu, +187.5 \mu)$$

$$C: (+300 \mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{375}{400 - 200} = 1.875$$

$$2\theta_p = 61.93^\circ \quad \theta_a = 30.96^\circ, \quad \theta_b = 120.96^\circ$$

$$R = \sqrt{(100 \mu)^2 + (187.5 \mu)^2} = 212.5 \mu$$

$$(a) \quad \varepsilon_a = \varepsilon_{ave} + R = 300 \mu + 212.5 \mu = 512.5 \mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = 300 \mu - 212.5 \mu = 87.5 \mu$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 425 \mu$$

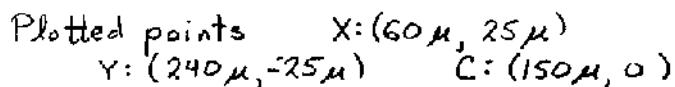
$$(c) \quad \varepsilon_c = 0 \quad \varepsilon_{max} = 512.5 \mu \quad \varepsilon_{min} = 0$$

$$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 512.5 \mu$$

Problem 7.141

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$7.141 \quad \varepsilon_x = +60 \mu \quad \varepsilon_y = +240 \mu \quad \gamma_{xy} = -50 \mu$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-50}{60 - 240} = 0.277778$$

$$2\theta_p = 15.52^\circ \quad \theta_b = 7.76^\circ \quad \theta_a = 97.76^\circ$$

$$R = \sqrt{(90 \mu)^2 + (25 \mu)^2} = 93.4 \mu$$

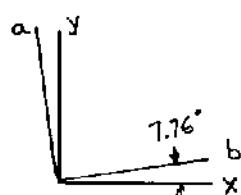
$$(a) \quad \varepsilon_a = \varepsilon_{ave} + R = 150 \mu + 93.4 \mu = 243.4 \mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = 150 \mu - 93.4 \mu = 56.6 \mu$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 186.8 \mu$$

$$(c) \quad \varepsilon_c = 0 \quad \varepsilon_{max} = 243.4 \mu \quad \varepsilon_{min} = 0$$

$$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 243.4$$



Problem 7.142

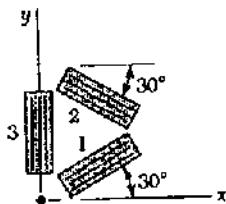
7.142 The strains determined by the use of the rosette shown during the test of a machine element are

$$\epsilon_1 = +600 \mu$$

$$\epsilon_2 = +450 \mu$$

$$\epsilon_3 = -75 \mu$$

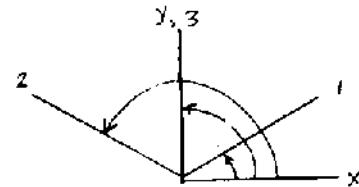
Determine (a) the in-plane principal strains, (b) the in-plane maximum shearing strain.



$$\theta_1 = 30^\circ$$

$$\theta_2 = 150^\circ$$

$$\theta_3 = 90^\circ$$



$$\epsilon_x \cos^2 \theta_1 + \epsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \epsilon_1$$

$$0.75 \epsilon_x + 0.25 \epsilon_y + 0.43301 \gamma_{xy} = 600 \mu \quad (1)$$

$$\epsilon_x \cos^2 \theta_2 + \epsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \epsilon_2$$

$$0.75 \epsilon_x + 0.25 \epsilon_y - 0.43301 \gamma_{xy} = 450 \mu \quad (2)$$

$$\epsilon_x \cos^2 \theta_3 + \epsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \epsilon_3$$

$$0 + \epsilon_y = -75 \mu \quad (3)$$

Solving (1), (2), and (3) simultaneously

$$\epsilon_x = 725 \mu, \quad \epsilon_y = -75 \mu, \quad \gamma_{xy} = 173.21 \mu$$

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 325 \mu$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725 - 75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3 \mu$$

$$(a) \epsilon_a = \epsilon_{ave} + R = 734 \mu$$

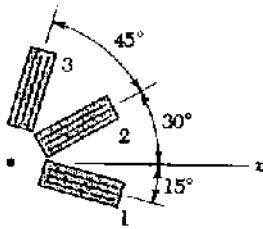
$$\epsilon_b = \epsilon_{ave} - R = -84.3 \mu$$

$$(b) \gamma_{max(in-plane)} = 2R = 819 \mu$$

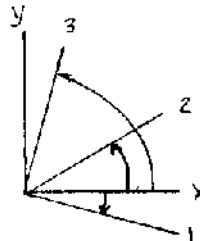
Problem 7.143

7.143 Determine the strain ϵ_3 knowing that the following strains have been determined by use of the rosette shown:

$$\epsilon_1 = +480 \times 10^{-6} \text{ in./in.} \quad \epsilon_2 = -120 \times 10^{-6} \text{ in./in.} \quad \epsilon_3 = +80 \times 10^{-6} \text{ in./in.}$$



$$\begin{aligned}\theta_1 &= -15^\circ \\ \theta_2 &= 30^\circ \\ \theta_3 &= 75^\circ\end{aligned}$$



$$\epsilon_x \cos^2 \theta_1 + \epsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \epsilon_1$$

$$0.9330 \epsilon_x + 0.06694 \epsilon_y - 0.25 \gamma_{xy} = 480 \times 10^{-6} \quad (1)$$

$$\epsilon_x \cos^2 \theta_2 + \epsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \epsilon_2$$

$$0.75 \epsilon_x + 0.25 \epsilon_y + 0.4330 \gamma_{xy} = -120 \times 10^{-6} \quad (2)$$

$$\epsilon_x \cos^2 \theta_3 + \epsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \epsilon_3$$

$$0.06699 \epsilon_x + 0.9330 \epsilon_y + 0.25 \gamma_{xy} = 80 \times 10^{-6} \quad (3)$$

Solving (1), (2), and (3) simultaneously

$$\epsilon_x = 253 \times 10^{-6} \text{ in/in}, \quad \epsilon_y = 307 \times 10^{-6} \text{ in/in}, \quad \gamma_{xy} = -893 \times 10^{-6} \text{ in/in}$$

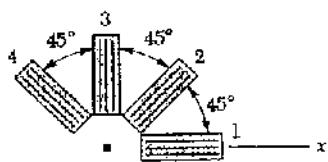
$$\epsilon_x = 253 \times 10^{-6} \text{ in/in}$$

Problem 7.144

7.144 The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$$\epsilon_1 = +420 \mu \quad \epsilon_2 = -45 \mu \quad \epsilon_4 = +165 \mu$$

(a) What should be the reading of gage 3? (b) Determine the principal strains and the maximum in-plane shearing strain.



(a) Gages 2 and 4 are 90° apart $\epsilon_{ave} = \frac{1}{2}(\epsilon_2 + \epsilon_4)$

$$\epsilon_{ave} = \frac{1}{2}(-45\mu + 165\mu) = 60\mu$$

Gages 1 and 3 are also 90° apart $\epsilon_{ave} = \frac{1}{2}(\epsilon_1 + \epsilon_3)$

$$\epsilon_3 = 2\epsilon_{ave} - \epsilon_1 = (2)(60\mu) - 420\mu = -300\mu$$

(b) $\epsilon_x = \epsilon_1 \approx 420 \mu \quad \epsilon_y = \epsilon_3 = -300 \mu$

$$\gamma_{xy} = 2\epsilon_2 - \epsilon_x - \epsilon_y = (2)(-45\mu) - 420\mu + 300\mu \\ = -210\mu$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\mu + 300\mu}{2}\right)^2 + \left(\frac{-210\mu}{2}\right)^2} \\ = 375\mu$$

$$\epsilon_a = \epsilon_{ave} + R = 60\mu + 375\mu = 435\mu$$

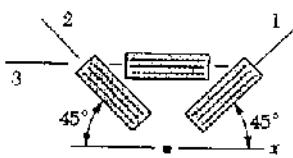
$$\epsilon_b = \epsilon_{ave} - R = 60\mu - 375\mu = -315\mu$$

$$\gamma_{max(in-plane)} = 2R = 750\mu$$

Problem 7.145

7.145 Determine the largest in-plane normal strain, knowing that the following strains have been obtained by the use of the rosette shown:

$$\epsilon_1 = -50 \times 10^{-6} \text{ in./in.} \quad \epsilon_2 = +360 \times 10^{-6} \text{ in./in.} \quad \epsilon_3 = +315 \times 10^{-6} \text{ in./in.}$$



$$\theta_1 = 45^\circ \quad \theta_2 = -45^\circ \quad \theta_3 = 0$$

$$\epsilon_x \cos^2 \theta_1 + \epsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \epsilon_1 \\ 0.5 \epsilon_x + 0.5 \epsilon_y + 0.5 \gamma_{xy} = -50 \times 10^{-6} \quad (1)$$

$$\epsilon_x \cos^2 \theta_2 + \epsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \epsilon_2 \\ 0.5 \epsilon_x + 0.5 \epsilon_y - 0.5 \gamma_{xy} = 360 \times 10^{-6} \quad (2)$$

$$\epsilon_x \cos^2 \theta_3 + \epsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \epsilon_3 \\ \epsilon_x + 0 + 0 = 315 \times 10^{-6} \quad (3)$$

$$\text{From (3)} \quad \epsilon_x = 315 \times 10^{-6} \text{ in/in.}$$

$$\text{Eq.(1) - Eq.(2)} \quad \gamma_{xy} = -50 \times 10^{-6} - 360 \times 10^{-6} = -410 \times 10^{-6} \text{ in/in}$$

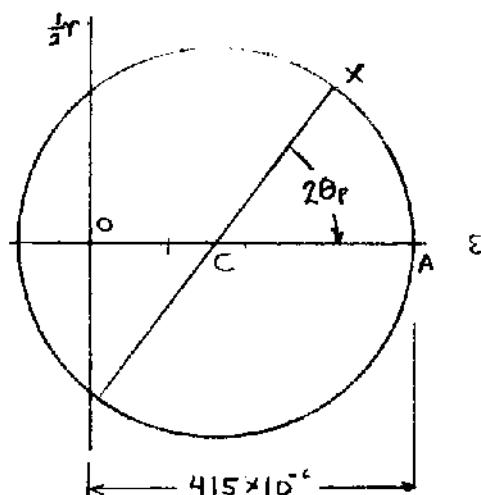
$$\text{Eq(1) + Eq(2)} \quad \epsilon_x + \epsilon_y = \epsilon_1 + \epsilon_2$$

$$\epsilon_y = \epsilon_1 + \epsilon_2 - \epsilon_x = -50 \times 10^{-6} + 360 \times 10^{-6} - 315 \times 10^{-6} = -5 \times 10^{-6} \text{ in/in.}$$

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 155 \times 10^{-6} \text{ in/in.}$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{315 \times 10^{-6} + 5 \times 10^{-6}}{2}\right)^2 + \left(\frac{-410 \times 10^{-6}}{2}\right)^2} \\ = 260 \times 10^{-6} \text{ in/in.}$$

$$\epsilon_{max} = \epsilon_{ave} + R = 415 \times 10^{-6} \text{ in/in.}$$



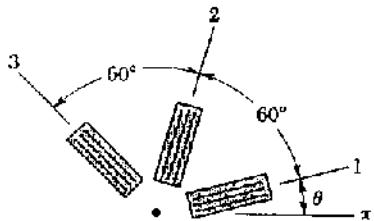
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = -1.323$$

$$2\theta_p = -52.0^\circ$$

$$\theta_p = -26.0^\circ$$

Problem 7.146

7.146 Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to
 $\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{ave}$
where ϵ_{ave} is the abscissa of the center of the corresponding Mohr's circle.



$$\epsilon_1 = \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \quad (1)$$

$$\begin{aligned}\epsilon_2 &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos (2\theta + 120^\circ) + \frac{\gamma_{xy}}{2} \sin (2\theta + 120^\circ) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} (\cos 120^\circ \cos 2\theta - \sin 120^\circ \sin 2\theta) \\ &\quad + \frac{\gamma_{xy}}{2} (\cos 120^\circ \sin 2\theta + \sin 120^\circ \cos 2\theta) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \left(-\frac{1}{2} \cos 2\theta - \frac{\sqrt{3}}{2} \sin 2\theta \right) \\ &\quad + \frac{\gamma_{xy}}{2} \left(-\frac{1}{2} \sin 2\theta + \frac{\sqrt{3}}{2} \cos 2\theta \right) \quad (2)\end{aligned}$$

$$\begin{aligned}\epsilon_3 &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos (2\theta + 240^\circ) + \frac{\gamma_{xy}}{2} \sin (2\theta + 240^\circ) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} (\cos 240^\circ \cos 2\theta - \sin 240^\circ \sin 2\theta) \\ &\quad + \frac{\gamma_{xy}}{2} (\cos 240^\circ \sin 2\theta + \sin 240^\circ \cos 2\theta) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \left(-\frac{1}{2} \cos 2\theta + \frac{\sqrt{3}}{2} \sin 2\theta \right) \\ &\quad + \frac{\gamma_{xy}}{2} \left(-\frac{1}{2} \sin 2\theta - \frac{\sqrt{3}}{2} \cos 2\theta \right) \quad (3)\end{aligned}$$

Adding (1), (2), and (3)

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{ave} + 0 + 0$$

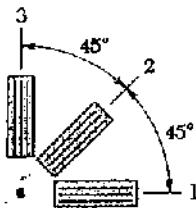
$$3\epsilon_{ave} = \epsilon_1 + \epsilon_2 + \epsilon_3$$

Problem 7.147

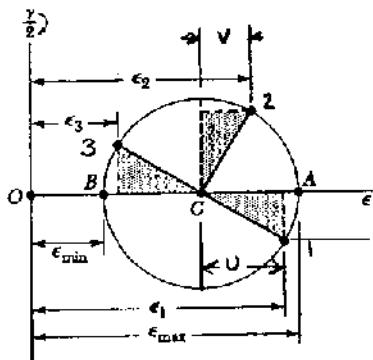
7.147 Using a 45° rosette, the strains ϵ_1 , ϵ_2 , and ϵ_3 have been determined at a given point. Using Mohr's circle, show that the principal strains are:

$$\epsilon_{\max, \min} = \frac{1}{2}(\epsilon_1 + \epsilon_3) \pm \frac{1}{\sqrt{2}}[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2]^{1/2}$$

(Hint: The shaded triangles are congruent.)



Since gage directions 1 and 3 are 90° apart



$$\epsilon_{ave} = \frac{1}{2}(\epsilon_1 + \epsilon_3)$$

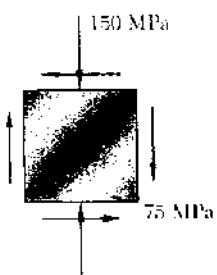
$$\text{Let } u = \epsilon_1 - \epsilon_{ave} = \frac{1}{2}(\epsilon_1 - \epsilon_3)$$

$$v = \epsilon_2 - \epsilon_{ave} = \epsilon_2 - \frac{1}{2}(\epsilon_1 + \epsilon_3)$$

$$\begin{aligned} R^2 &= u^2 + v^2 \\ &= \frac{1}{4}(\epsilon_1 - \epsilon_3)^2 + \epsilon_2^2 - \epsilon_2(\epsilon_1 + \epsilon_3) + \frac{1}{4}(\epsilon_1 + \epsilon_3)^2 \\ &= \frac{1}{4}\epsilon_1^2 - \frac{1}{2}\epsilon_1\epsilon_3 + \frac{1}{4}\epsilon_3^2 \\ &\quad + \epsilon_2^2 - \epsilon_2\epsilon_1 - \epsilon_2\epsilon_3 \\ &\quad + \frac{1}{4}\epsilon_1^2 + \frac{1}{2}\epsilon_1\epsilon_3 + \frac{1}{4}\epsilon_3^2 \\ &= \frac{1}{2}\epsilon_1^2 - \epsilon_2\epsilon_1 + \epsilon_2^2 - \epsilon_2\epsilon_3 + \frac{1}{2}\epsilon_3^2 \\ &= \frac{1}{2}(\epsilon_1 - \epsilon_2)^2 + \frac{1}{2}(\epsilon_2 - \epsilon_3)^2 \\ R &= \frac{1}{\sqrt{2}} [(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2]^{1/2} \end{aligned}$$

$$\epsilon_{\max, \min} = \epsilon_{ave} \pm R \quad \text{gives the required formula.}$$

Problem 7.148



7.148 The given state of plane stress is known to exist on the surface of a machine component. Knowing that $E = 200 \text{ GPa}$ and $G = 77 \text{ GPa}$, determine the direction and magnitude of the three principal strains (*a*) by determining the corresponding state of strain [use Eq. (2.43) and Eq. (2.38)] and then using Mohr's circle for strain, (*b*) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

$$(a) \quad \sigma_x = 0, \quad \sigma_y = -150 \times 10^6 \text{ Pa}, \quad \tau_{xy} = -75 \times 10^6 \text{ Pa} \\ E = 200 \times 10^9 \text{ Pa} \quad G = 77 \times 10^9 \text{ Pa}$$

$$G = \frac{E}{2(1+\nu)} \quad \nu = \frac{E}{2G} - 1 = 0.2987$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{200 \times 10^9} [0 + (0.2987)(-150 \times 10^6)] = 224 \mu$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{200 \times 10^9} [(-150 \times 10^6) - 0] = -750 \mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-75 \times 10^6}{77 \times 10^9} = -974 \mu \quad \frac{\gamma_{xy}}{2} = -487.0 \mu$$

$$\epsilon_{ave} = \frac{1}{2} (\epsilon_x + \epsilon_y) = -263 \mu$$

$$\epsilon_x - \epsilon_y = 974 \mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-974}{974} = -1.000$$

$$2\theta_a = -45^\circ \quad \theta_a = -22.5^\circ$$

$$R = \sqrt{(\frac{\epsilon_x - \epsilon_y}{2})^2 + (\frac{\gamma_{xy}}{2})^2} = 689 \mu$$

$$\epsilon_a = \epsilon_{ave} + R = 426 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -952 \mu$$

$$\epsilon_c = -\frac{\nu}{E} (\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 150 \times 10^6)}{200 \times 10^9} \\ = -224 \mu$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = -75 \text{ MPa}$$

$$R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \sqrt{(\frac{0 + 150}{2})^2 + 75^2} \\ = 106.07 \text{ MPa}$$

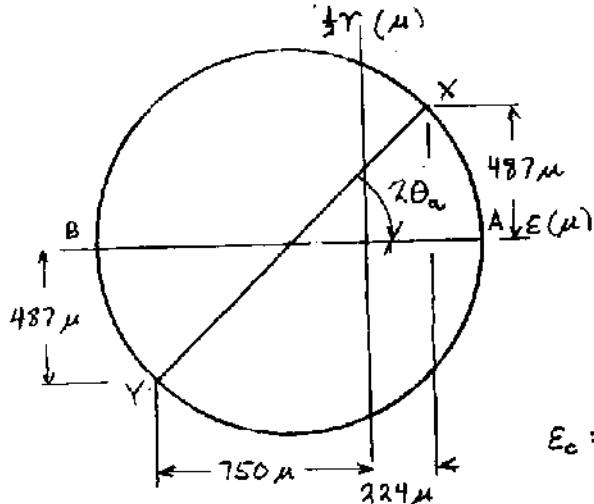
$$\sigma_a = \sigma_{ave} + R = 31.07 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -181.07 \text{ MPa}$$

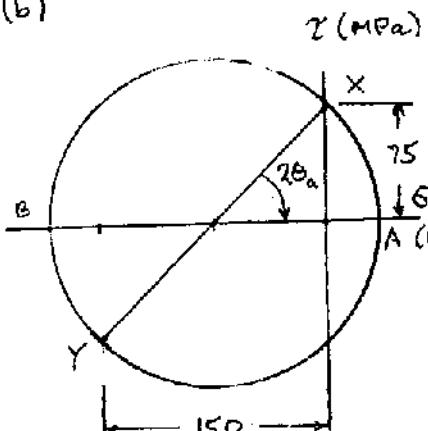
$$\epsilon_a = \frac{1}{E} (\sigma_a - \nu \sigma_b)$$

$$= \frac{1}{200 \times 10^9} [31.07 \times 10^6 - (0.2987)(-181.07 \times 10^6)] \\ = 426 \times 10^{-6} = 426 \mu$$

$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} = -1.000 \quad 2\theta_a = -45^\circ \quad \theta_a = -22.5^\circ$$



(b)



Problem 7.149

7.149 The following state of strain has been determined on the surface of a cast-iron machine part:

$$\epsilon_1 = -720 \times 10^{-6} \text{ in./in.} \quad \epsilon_2 = -400 \times 10^{-6} \text{ in./in.} \quad \epsilon_3 = +660 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 10 \times 10^6 \text{ psi}$ and $G = 4 \times 10^6 \text{ psi}$, determine the principal planes and principal stresses (σ) by determining the corresponding state of plane stress [use Eq. (2.36), Eq.(2.43), and the first two equations of Prob.2.74] and then using Mohr's circle for stress, (b) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and the determining the corresponding stresses.

$$G = \frac{E}{2(1+\nu)} \quad \nu = \frac{E}{2G} - 1 = \frac{10}{2(4)} - 1 = 0.25$$

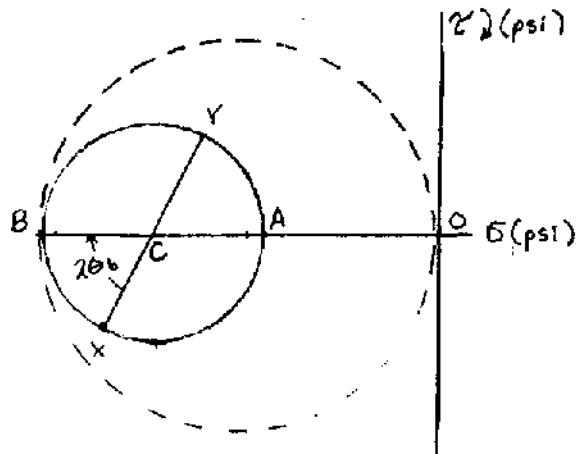
$$\frac{E}{1-\nu^2} = \frac{10 \times 10^6}{1-0.25^2} = 10.667 \times 10^6 \text{ psi}$$

Note that the 3rd
principal stress
 $\sigma_c = 0$

$$(a) \quad \sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = 10.667 \times 10^6 [-720 \times 10^{-6} + (0.25)(-400 \times 10^{-6})] \\ = -8746.7 \text{ psi}$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = 10.667 \times 10^6 [-400 \times 10^{-6} + (0.25)(-720 \times 10^{-6})] \\ = -6186.7 \text{ psi}$$

$$\tau = GY = (4 \times 10^6)(660 \times 10^{-6}) = 2640 \text{ psi}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = -7466.7 \text{ psi}$$

$$\tan 2\theta_b = \frac{2\tau}{\sigma_1 - \sigma_2} = -2.0625$$

$$2\theta_b = -64.1^\circ \quad \theta_b = -32.1^\circ \quad \theta_a = 57.9^\circ$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} = 2934 \text{ psi}$$

$$\sigma_a = \sigma_{ave} + R = -4533 \text{ psi}$$

$$\sigma_b = \sigma_{ave} - R = -10400 \text{ psi}$$

$$(b) \quad \epsilon_{ave} = \frac{1}{2}(\epsilon_1 + \epsilon_2) = -560 \times 10^{-6}$$

$$\tan 2\theta_b = \frac{\gamma}{\epsilon_1 - \epsilon_2} = \frac{660}{-720 + 400} = -2.0625$$

$$2\theta_b = -64.1^\circ \quad \theta_b = -32.1^\circ \quad \theta_a = 57.9^\circ$$

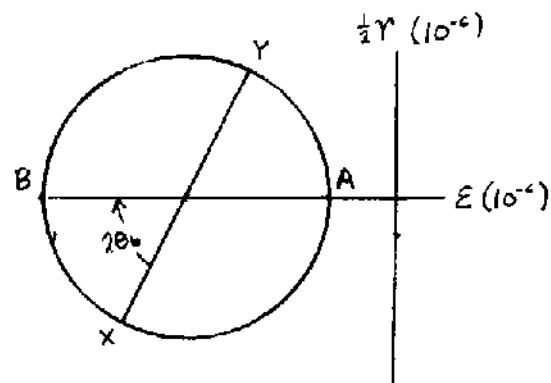
$$R = \sqrt{\left(\frac{\epsilon_1 - \epsilon_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2} = 366.74 \times 10^{-6}$$

$$\epsilon_a = \epsilon_{ave} + R = -193.26 \times 10^{-6}$$

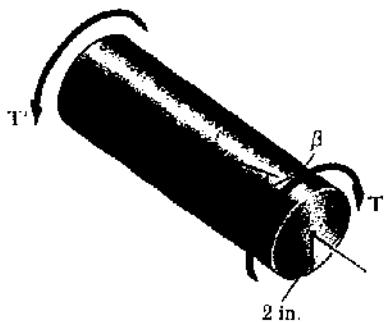
$$\epsilon_b = \epsilon_{ave} - R = -926.74 \times 10^{-6}$$

$$\sigma_a = \frac{E}{1-\nu^2} (\epsilon_a + \nu \epsilon_b) = -4533 \text{ psi}$$

$$\sigma_b = \frac{E}{1-\nu^2} (\epsilon_b + \nu \epsilon_a) = -10400 \text{ psi}$$



Problem 7.150



7.150 A single gage is cemented to a solid 4-in.-diameter steel shaft at an angle $\beta = 25^\circ$ with a line parallel to the axis of the shaft. Knowing that $G = 11.5 \times 10^6$ psi, determine the torque T indicated by a gage reading of 300×10^{-6} in./in.

$$\text{For torsion, } \epsilon_x = \epsilon_y = 0, \gamma_{xy} = \gamma_0$$



$$\epsilon_x = \frac{1}{E} (\epsilon_x - \nu \epsilon_y) = 0$$

$$\epsilon_y = \frac{1}{E} (\epsilon_y - \nu \epsilon_x) = 0$$

$$\gamma_{xy} = \frac{\gamma_0}{G} \quad \frac{1}{2} \gamma_{xy} = \frac{\gamma_0}{2G}$$

Draw the Mohr's circle for strain

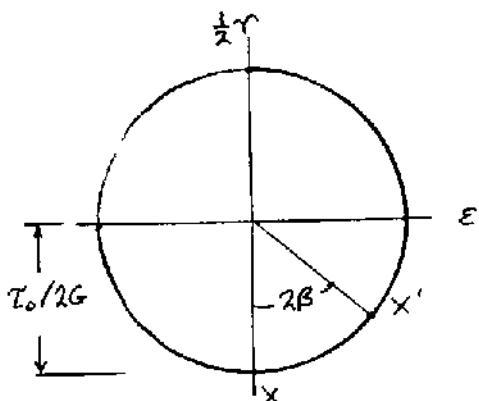
$$R = \frac{\gamma_0}{2G}$$

$$\epsilon_{x'} = R \sin 2\beta = \frac{\gamma_0}{2G} \sin 2\beta$$

$$\text{But } \gamma_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G \epsilon_{x'}}{\sin 2\beta}$$

$$T = \frac{\pi c^3 G \epsilon_{x'}}{\sin 2\beta} = \frac{\pi (2)^3 (11.5 \times 10^6) (300 \times 10^{-6})}{\sin 50^\circ}$$

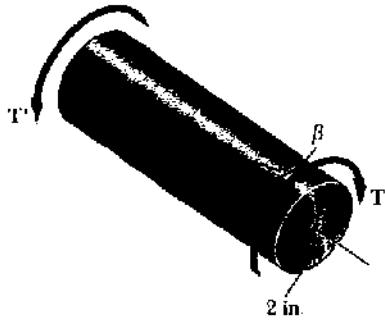
$$= 113.2 \times 10^3 \text{ lb-in} = 113.2 \text{ kip-in}$$



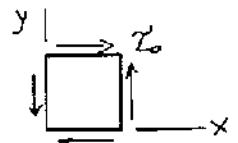
Problem 7.151

7.151 Solve Prob. 7.150, assuming that the gage forms an angle $\beta = 35^\circ$ with a line parallel to the axis of the shaft.

7.150 A single gage is cemented to a solid 4-in.-diameter steel shaft at an angle $\beta = 25^\circ$ with a line parallel to the axis of the shaft. Knowing that $G = 11.5 \times 10^6$ psi, determine the torque T indicated by a gage reading of 300×10^{-6} in./in.



$$\text{For torsion } \epsilon_x = 0, \epsilon_y = 0, \gamma_{xy} = \gamma_0$$



$$\epsilon_x = \frac{1}{E} (\epsilon_x - \nu \epsilon_y) = 0$$

$$\epsilon_y = \frac{1}{E} (\epsilon_y - \nu \epsilon_x) = 0$$

$$\gamma_{xy} = \frac{\gamma_0}{G} \quad \frac{1}{2} \gamma_{xy} = \frac{\gamma_0}{2G}$$

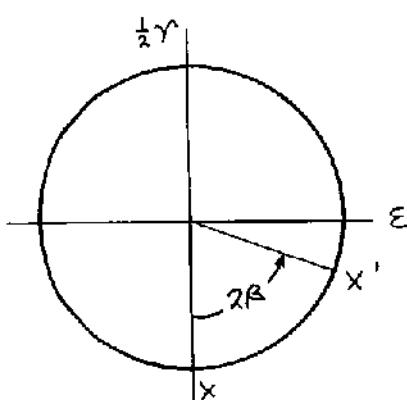
Draw Mohr's circle for strain

$$R = \frac{\gamma_0}{2G} \quad \epsilon_{x'} = R \sin 2\beta = \frac{\gamma_0}{2G} \sin 2\beta$$

$$\text{But } \gamma_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G \epsilon_{x'}}{\sin 2\beta}$$

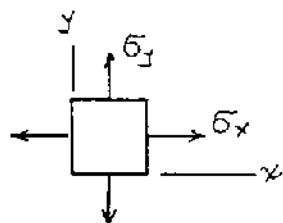
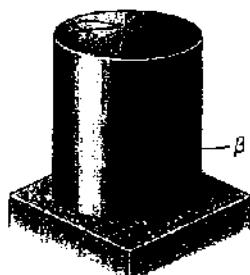
$$T = \frac{\pi c^3 G \epsilon_{x'}}{\sin 2\beta} = \frac{\pi (2)^3 (11.5 \times 10^6) (300 \times 10^{-6})}{\sin 70^\circ}$$

$$= 92.3 \times 10^3 \text{ lb-in} = 92.3 \text{ kip-in}$$



Problem 7.152

7.152 A single strain gage forming an angle $\beta = 18^\circ$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200 \text{ GPa}$ and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ .

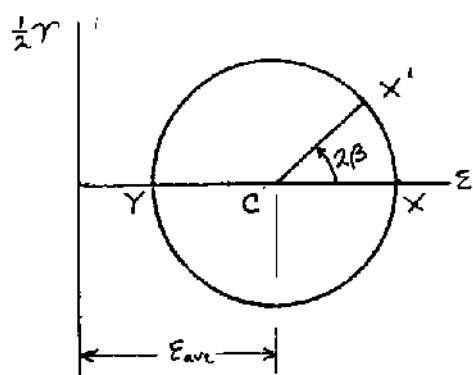


$$\sigma_x = \sigma_z = \frac{Pr}{t} \Rightarrow \sigma_y = \frac{1}{2}\sigma_x, \sigma_z \approx 0$$

$$\epsilon_x = \frac{1}{E}(\sigma_x - 2\nu\sigma_y - 2\nu\sigma_z) = (1 - \frac{\nu}{2})\frac{\sigma_x}{E} \\ = 0.85 \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{1}{E}(-2\sigma_x + \sigma_y - 2\nu\sigma_z) = (\frac{1}{2} - \nu)\frac{\sigma_x}{E} \\ = 0.20 \frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



Draw Mohr's circle for strain

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0.525 \frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\epsilon_x - \epsilon_y) = 0.325 \frac{\sigma_x}{E}$$

$$\epsilon_x' = \epsilon_{ave} + R \cos 2\beta = (0.525 + 0.325 \cos 36^\circ) \frac{\sigma_x}{E}$$

$$P = \frac{t\sigma_x}{r} = \frac{t E \epsilon_x'}{r(0.525 + 0.325 \cos 36^\circ)}$$

Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

$$t = 6 \times 10^{-3} \text{ mm} \quad E = 200 \times 10^9 \text{ Pa}, \quad \epsilon_x = 280 \times 10^{-6}$$

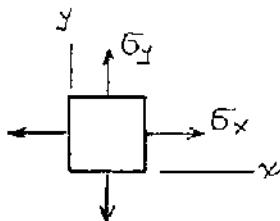
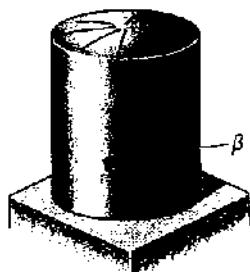
$$\beta = 18^\circ$$

$$P = \frac{(6 \times 10^{-3})(200 \times 10^9)(280 \times 10^{-6})}{(0.300)(0.525 + 0.325 \cos 36^\circ)} = 1.421 \times 10^6 \text{ Pa}$$

$$P = 1.421 \text{ MPa} \blacksquare$$

Problem 7.153

7.153 Solve Prob. 7.152, assuming that the gage forms an angle $\beta = 35^\circ$ with a horizontal plane.



$$\sigma_x = \sigma_1 = \frac{Pr}{t} \Rightarrow \sigma_y = \frac{1}{2}\sigma_x, \sigma_z \approx 0$$

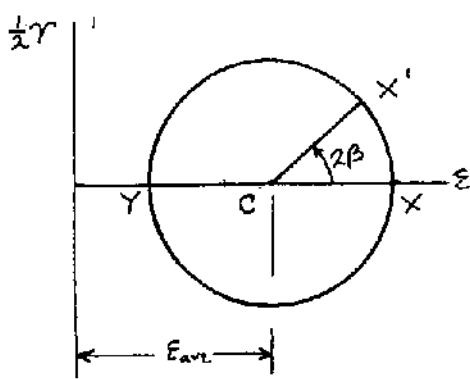
$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = (1 - \frac{\nu}{2})\frac{\sigma_x}{E}$$

$$= 0.85 \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = (\frac{1}{2} - \nu)\frac{\sigma_x}{E}$$

$$= 0.20 \frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\epsilon_{xy}}{G} = 0$$



Draw Mohr's circle for strain

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0.525 \frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\epsilon_x - \epsilon_y) = 0.325 \frac{\sigma_x}{E}$$

$$\epsilon_x' = \epsilon_{ave} + R \cos 2\beta = (0.525 + 0.325 \cos 2\beta) \frac{\sigma_x}{E}$$

$$P = \frac{t\sigma_x}{r} = \frac{t E \epsilon_x'}{r(0.525 + 0.325 \cos 2\beta)}$$

Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

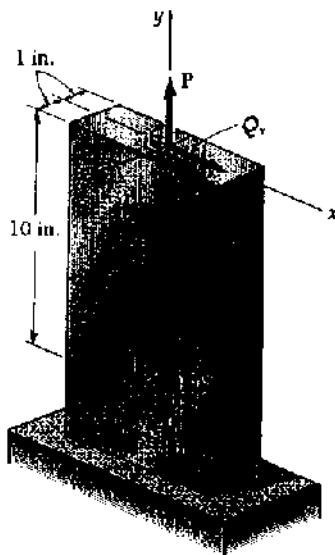
$$t = 6 \times 10^{-3} \text{ m} \quad E = 200 \times 10^9 \text{ Pa} \quad \epsilon_x' = 280 \times 10^{-6}$$

$$\beta = 35^\circ$$

$$P = \frac{(6 \times 10^{-3})(200 \times 10^9)(280 \times 10^{-6})}{(0.300)(0.525 + 0.325 \cos 70^\circ)} = 1.761 \times 10^6 \text{ Pa}$$

$$P = 1.761 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.154



7.154 A centric axial force P and a horizontal force Q_x are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$\epsilon_1 = -75 \times 10^{-6}$ in./in. $\epsilon_2 = +300 \times 10^{-6}$ in./in. $\epsilon_3 = +250 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of P and Q_x .

$$\epsilon_x = \epsilon_1 = -75 \times 10^{-6}$$

$$\epsilon_y = \epsilon_3 = 250 \times 10^{-6}$$

$$\gamma_{xy} = 2\epsilon_2 - \epsilon_1 - \epsilon_3 = 425 \times 10^{-6}$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{29}{1-0.3^2} [-75 + (0.3)(250)] \\ = 0$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{29}{1-0.3^2} [250 + (0.3)(-75)] \\ = 7.25 \times 10^3 \text{ psi}$$

$$\frac{P}{A} = \sigma_y \quad P = A\sigma_y = (2)(6)(7.25 \times 10^3) \\ = 87.0 \times 10^3 \text{ lb} = 87.0 \text{ kips}$$

$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{2(1.3)} = 11.154 \times 10^6 \text{ psi}$$

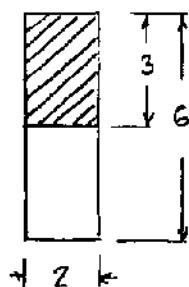
$$\tau_{xy} = G\gamma_{xy} = (11.154)(425) = 4.740 \times 10^3 \text{ psi}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (2)(6)^3 = 36 \text{ in}^4$$

$$Q = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \quad z = 2 \text{ in}$$

$$\tau_{xy} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(4.74 \times 10^3)}{9} = 37.9 \times 10^3 \text{ lb.}$$

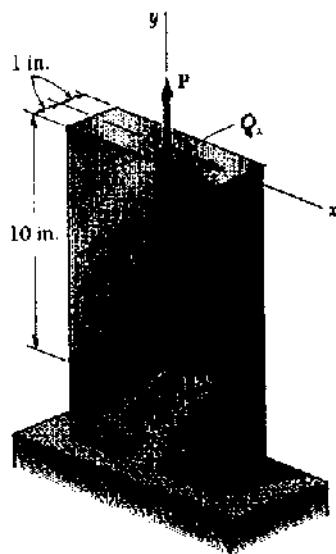


$$Q_x = V = 37.9 \times 10^3 \text{ lb} = 37.9 \text{ kips}$$

Problem 7.155

7.155 Solve Prob. 7.154, assuming that the rosette at point A indicates the following strains:

$$\epsilon_1 = -60 \times 10^{-6} \text{ in./in.} \quad \epsilon_2 = +410 \times 10^{-6} \text{ in./in.} \quad \epsilon_3 = +200 \times 10^{-6} \text{ in./in.}$$



7.154 A centric axial force P and a horizontal force Q_x are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$$\epsilon_1 = -75 \times 10^{-6} \text{ in./in.} \quad \epsilon_2 = +300 \times 10^{-6} \text{ in./in.} \quad \epsilon_3 = +250 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of P and Q_x .

$$\epsilon_x = \epsilon_1 = -60 \times 10^{-6} \quad \epsilon_y = \epsilon_3 = 200 \times 10^{-6}$$

$$\gamma_{xy} = 2\epsilon_2 - \epsilon_1 - \epsilon_3 = 680 \times 10^{-6}$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{29}{1-0.3^2} [-60 + (0.3)(200)] \\ = 0$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{29}{1-0.3^2} [200 + (0.3)(-60)] \\ = 5.800 \times 10^3 \text{ psi}$$

$$\frac{P}{A} = \sigma_y \quad P = A\sigma_y = (2)(6)(5.800 \times 10^3) \\ = 69.6 \times 10^3 \text{ lb} = 69.6 \text{ kips}$$

$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6 \text{ psi}$$

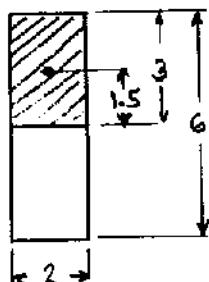
$$\tau_{xy} = G \gamma_{xy} = (11.154)(680) = 7.585 \times 10^3 \text{ psi}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$Q = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \quad t = 2 \text{ in.}$$

$$\tau_{xy} = \frac{VQ}{It}$$

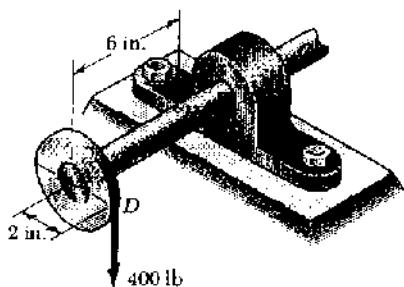
$$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(7.585 \times 10^3)}{9} = 60.7 \times 10^3 \text{ lb.}$$



$$Q_x = V = 60.7 \times 10^3 \text{ lb.} = 60.7 \text{ kips}$$

Problem 7.156

7.156 A 400-lb vertical force is applied at D to a gear attached to the solid one-inch diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.



Equivalent force-couple system at center of shaft in section at point H.

$$V = 400 \text{ lb.} \quad M = (400)(6) = 2400 \text{ lb-in.} \\ T = (400)(2) = 800 \text{ lb-in.}$$

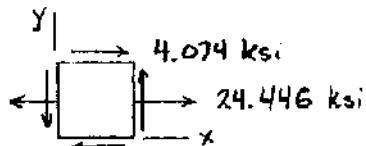
Shaft cross section.

$$d = \text{in} \quad C = \frac{1}{2}d = 0.5 \text{ in} \\ J = \frac{\pi}{2} C^4 = 0.098175 \text{ in}^4 \quad I = \frac{1}{2}J = 0.049087 \text{ in}^4$$

Torsion: $\tau = \frac{Tc}{J} = \frac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$

Bending: $\sigma = \frac{Mc}{I} = \frac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$

Transverse shear: Stress at point H is zero.



$$\sigma_x = 24.446 \text{ ksi} \quad \sigma_y = 0 \quad \tau_{xy} = 4.074 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(12.223)^2 + (4.074)^2} \\ = 12.884 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 25.107 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -0.661 \text{ ksi}$$

$$\tau_{max} = R = 12.884 \text{ ksi}$$

Problem 7.157

7.157 Determine the largest internal pressure that can be applied to a cylindrical tank of 5.5-ft outer-diameter and $\frac{5}{8}$ -in. wall thickness if the ultimate normal stress of the steel used is 65 ksi and a factor of safety of 5.0 is desired.

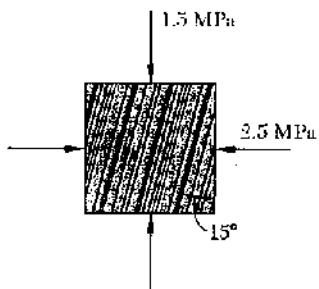
$$\bar{\sigma}_i = \frac{\sigma_u}{F.S.} = \frac{65}{5.0} = 13 \text{ ksi} \quad d = 5.5 \text{ ft} = 66 \text{ in}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(66) - 0.625 = 32.375 \text{ in}$$

$$\sigma_i = \frac{Pr}{t} \quad P = \frac{\sigma_i t}{r} = \frac{(13)(0.625)}{32.375} = 0.251 \text{ ksi} = 251 \text{ psi}$$

Problem 7.158

7.158 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.



$$\sigma_x = -2.5 \text{ MPa} \quad \sigma_y = -1.5 \text{ MPa} \quad \tau_{xy} = 0$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

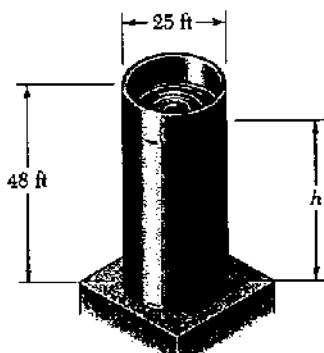
$$(a) \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ = -\frac{-2.5 - (-1.5)}{2} \sin(-30^\circ) + 0 \\ = -0.250 \text{ MPa}$$

$$(b) \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ = \frac{-2.5 + (-1.5)}{2} + \frac{-2.5 - (-1.5)}{2} \cos(-30^\circ) + 0 = -2.43 \text{ MPa}$$

Problem 7.159

7.159 The unpressurized cylindrical storage tank shown has a $\frac{3}{16}$ -in. wall

thickness and is made of a steel having a 60-ksi ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Specific weight of water = 62.4 lb/ft³.)



$$d_o = (25)(12) = 300 \text{ in.}$$

$$r = \frac{1}{2}d - t = 150 - \frac{3}{16} = 149.81 \text{ in.}$$

$$\sigma_{ult} = \frac{\sigma_u}{F.S.} = \frac{60 \text{ ksi}}{4.0} = 15 \text{ ksi} = 15 \times 10^3 \text{ psi}$$

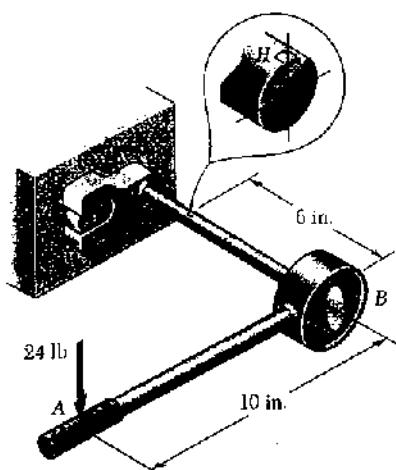
$$\sigma_{act} = \frac{Pr}{t}$$

$$P F \frac{t \sigma_{act}}{r} = \frac{\left(\frac{3}{16}\right)(15 \times 10^3)}{149.81} = 18.77 \text{ psi} = 2703 \text{ lb/ft}^2$$

$$\text{But } p = \gamma h \quad h = \frac{P}{\gamma} = \frac{2703 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} = 43.3 \text{ ft}$$

Problem 7.160

7.160 A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical force of 24-lb at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown on top of the 0.75-in. diameter shaft.



Equivalent force-couple system at center of shaft in section at point *H*.

$$V = 24 \text{ lb}, \quad M = (24)(6) = 144 \text{ lb-in} \\ T = (24)(10) = 240 \text{ lb-in}$$

Shaft cross section: $d = 0.75 \text{ in}$, $c = \frac{1}{4}d = 0.375 \text{ in}$.
 $J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \quad I = \frac{1}{4}J = 0.015532 \text{ in}^4$

Torsion: $\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897 \text{ ksi}$

Bending: $\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477 \text{ ksi}$

Transverse Shear: At point *H* stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477 \text{ ksi}$, $\sigma_y = 0$, $\tau_{xy} = 2.897 \text{ ksi}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{1.738^2 + 2.897^2} = 3.378 \text{ ksi}$$

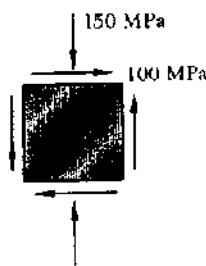
$$\sigma_a = \sigma_{ave} + R = 5.116 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -1.640 \text{ ksi}$$

$$\tau_{max} = R = 3.378 \text{ ksi}$$

Problem 7.161

7.161 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160 \text{ MPa}$ and $\sigma_{UC} = 320 \text{ MPa}$ and using Mohr's criterion, determine whether rupture of the component will occur.



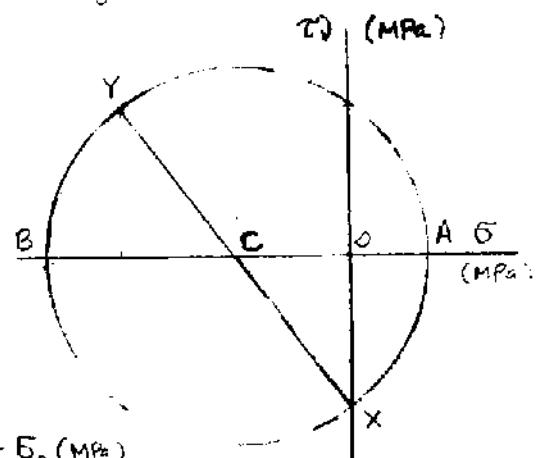
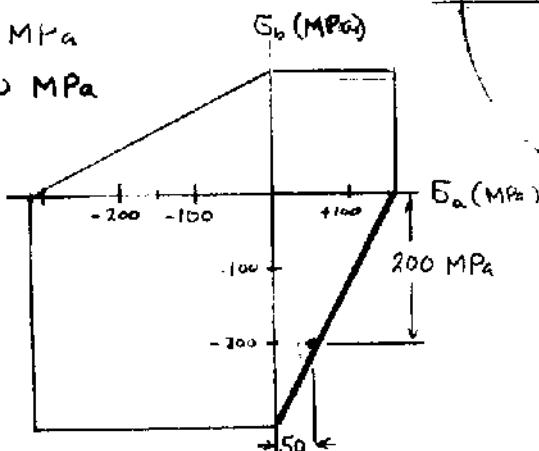
$$\sigma_x = 0 \quad \sigma_y = -150 \text{ MPa} \quad \tau_{xy} = 100 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ = \sqrt{75^2 + 100^2} = 125 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 50 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -200 \text{ MPa}$$

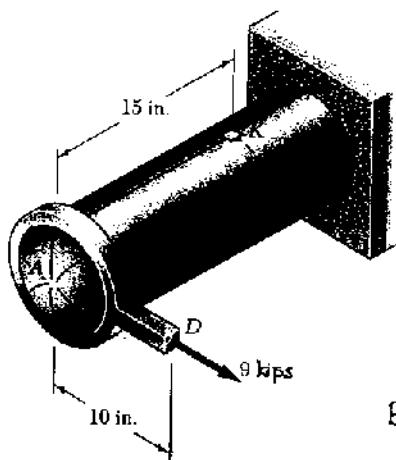


Equation of the 4th quadrant boundary is $\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$$\frac{50}{160} - \frac{(-200)}{320} = 0.9375 < 1 \quad , \quad \text{No rupture.}$$

Problem 7.162

7.162 The cylindrical tank AB has an 8-in. inner diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point K located on the top of the tank.



$$r_i = \frac{d}{2} = 4 \text{ in} \quad r_o = r_i + t = 4.32 \text{ in.}$$

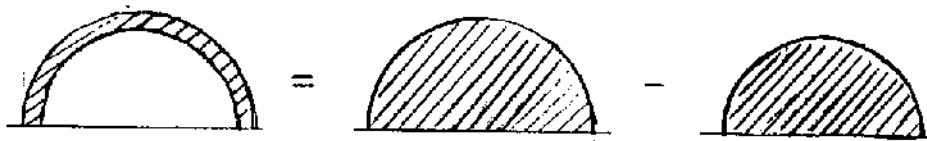
$$\sigma_i = \frac{Pr_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$$

$$\sigma_2 = \frac{1}{2}\sigma_i = 3.75 \text{ ksi}$$

Torsion: No applied torque

Bending: Point K lies on neutral axis.

Transverse shear: $V = 9 \text{ kips}$



For semicircle

$$A = \frac{\pi}{2} r^2$$

$$\bar{y} = \frac{4r}{3\pi}$$

$$Q = \frac{2}{3} r^3$$

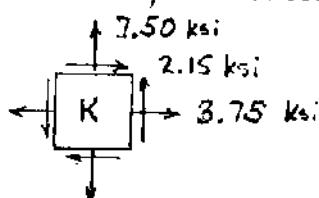
$$Q = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = \frac{2}{3} (4.32^3 - 4^3) = 11.081 \text{ in}^3$$

$$t = (2)(0.32) = 0.64 \text{ in}$$

$$I = \frac{\pi}{4} (r_o^4 - r_i^4) = \frac{\pi}{4} (4.32^4 - 4^4) = 72.481 \text{ in}^4$$

$$\tau = \frac{VQ}{It} = \frac{(9)(11.081)}{(72.481)(0.64)} = 2.15 \text{ ksi}$$

Summary of stresses:



Longitudinal

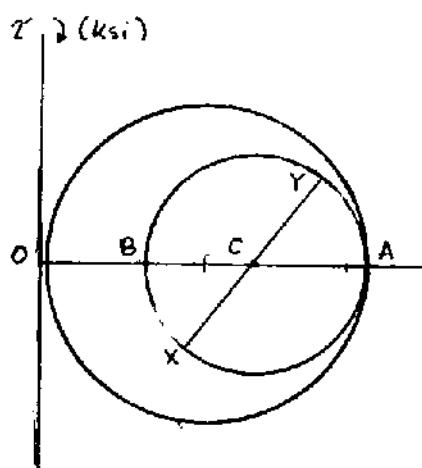
$$\sigma_x = \sigma_i = 3.75 \text{ ksi}$$

Circumferential

$$\sigma_y = \sigma_2 = 7.50 \text{ ksi}$$

Shear

$$\tau_{xy} = 2.15 \text{ ksi}$$



$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = 5.625 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ = 2.853 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 8.48 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = 2.77 \text{ ksi}$$

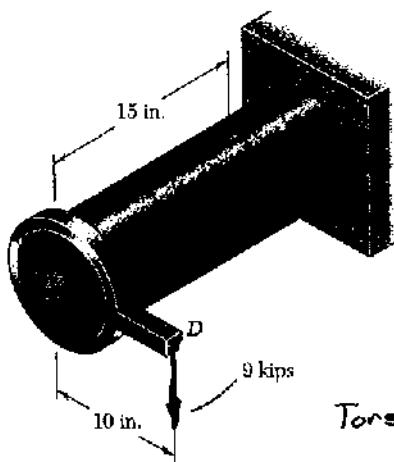
$$\sigma_c = 0$$

$$\sigma_{max} = 8.48 \text{ ksi}$$

$$\sigma_{min} = 0$$

$$\tau_{max} = \frac{1}{2} (\sigma_{max} - \sigma_{min}) = 4.24 \text{ ksi}$$

Problem 7.163



7.163 Solve Prob. 7.162, assuming that the 9-kip force applied at point D is directed vertically downward.

7.162 The cylindrical tank AB has an 8-in. inner diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point K located on the top of the tank.

$$r_i = \frac{d_i}{2} = 4 \text{ in.} \quad r_o = r_i + t = 4.32 \text{ in}$$

$$\sigma_i = \frac{\rho r_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$$

$$\sigma_2 = \frac{1}{2} \sigma_i = 3.75 \text{ ksi}$$

$$\text{Torsion: } J = \frac{\pi}{2} (r_o^4 - r_i^4) = 144.96 \text{ in}^4 \quad c = r_o = 4.32 \text{ in}$$

$$T = (9)(10) = 90 \text{ kip-in}$$

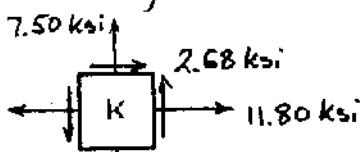
$$\tau' = \frac{Tc}{J} = \frac{(90)(4.32)}{144.96} = 2.68 \text{ ksi}$$

$$\text{Bending: } I = \frac{1}{2} J = 72.48 \text{ in}^4 \quad c = r_o = 4.32 \text{ in}$$

$$M = (9)(15) = 135 \text{ kip-in} \quad \sigma_m = \frac{Mc}{I} = \frac{(135)(4.32)}{72.48} = 8.05 \text{ ksi}$$

Transverse shear: At point K, $VQ/Ic = 0$

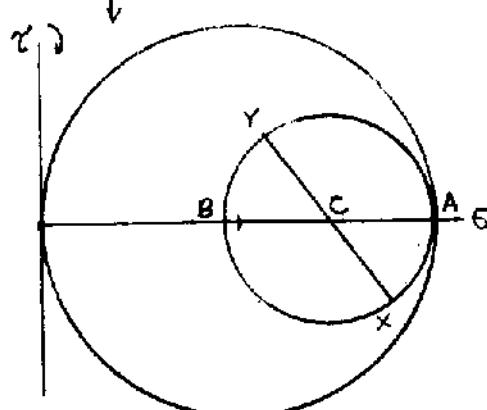
Summary of stresses:



$$\text{Longitudinal: } \sigma_x = \sigma_i = 3.75 + 8.05 = 11.80 \text{ ksi}$$

$$\text{Circumferential: } \sigma_y = \sigma_i = 7.50 \text{ ksi}$$

$$\text{Shear: } \tau_{xy} = 2.68 \text{ ksi}$$



$$\bar{\sigma}_{ave} = \frac{1}{2} (11.80 + 7.50) = 9.65 \text{ ksi}$$

$$R = \sqrt{\left(\frac{11.80 - 7.50}{2}\right)^2 + (2.68)^2} = 3.44 \text{ ksi}$$

$$\sigma_a = \bar{\sigma}_{ave} + R = 13.09 \text{ ksi}$$

$$\sigma_b = \bar{\sigma}_{ave} - R = 6.21 \text{ ksi}$$

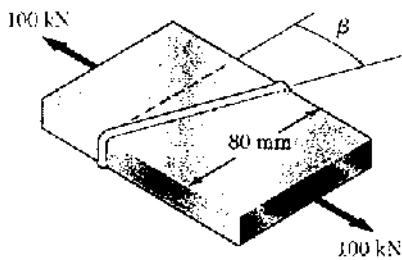
$$\sigma_z = 0$$

$$\sigma_{max} = 13.09 \text{ ksi}$$

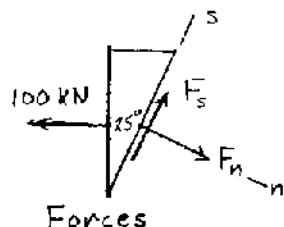
$$\sigma_{min} = 0$$

$$\tau_{max} = \frac{1}{2} (\sigma_{max} - \sigma_{min}) = 6.54 \text{ ksi}$$

Problem 7.164



7.164 Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25^\circ$, determine (a) the in-plane shearing stress parallel to the weld, (b) the normal stress perpendicular to the weld.



Area of weld

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos 25^\circ} = 882.7 \times 10^{-6} \text{ m}^2$$

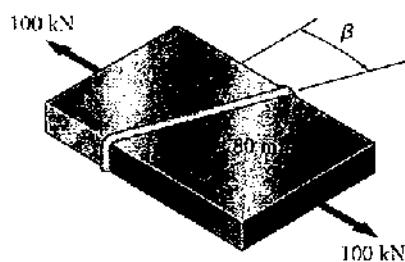
$$(a) \sum F_s = 0 \quad F_s - 100 \sin 25^\circ = 0 \quad F_s = 42.26 \text{ kN}$$

$$\tau_w = \frac{F_s}{A_w} = \frac{42.26 \times 10^3}{882.7 \times 10^{-6}} = 47.9 \times 10^6 \text{ Pa} = 47.9 \text{ MPa}$$

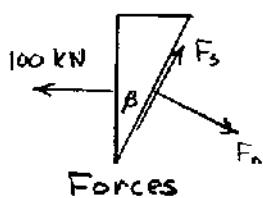
$$(b) \sum F_n = 0 \quad F_n - 100 \cos 25^\circ = 0 \quad F_n = 90.63 \text{ kN}$$

$$\sigma_w = \frac{F_n}{A_w} = \frac{90.63 \times 10^3}{882.7 \times 10^{-6}} = 102.7 \times 10^6 \text{ Pa} = 102.7 \text{ MPa}$$

Problem 7.165



7.165 Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (a) the angle β , (b) the corresponding normal stress perpendicular to the weld.



Area of weld

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos \beta} = \frac{800 \times 10^{-6}}{\cos \beta} \text{ m}^2$$

$$(a) \sum F_s = 0 \quad F_s - 100 \sin \beta = 0 \quad F_s = 100 \sin \beta \text{ kN} = 100 \times 10^3 \sin \beta \text{ N}$$

$$\tau_w = \frac{F_s}{A_w} = \frac{100 \times 10^3 \sin \beta}{800 \times 10^{-6} / \cos \beta} = 125 \times 10^6 \sin \beta \cos \beta$$

$$\sin \beta \cos \beta = \frac{1}{2} \sin 2\beta = \frac{30 \times 10^3}{125 \times 10^6} = 0.240 \quad \beta = 14.34^\circ$$

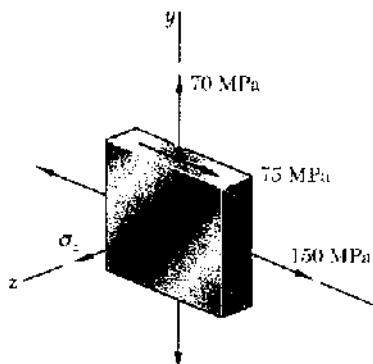
$$(b) \sum F_n = 0 \quad F_n - 100 \cos \beta = 0 \quad F_n = 100 \cos 14.34^\circ = 96.88 \text{ kN}$$

$$A_w = \frac{800 \times 10^{-6}}{\cos 14.34^\circ} = 825.74 \times 10^{-6} \text{ m}^2$$

$$\sigma_w = \frac{F_n}{A_w} = \frac{96.88 \times 10^3}{825.74 \times 10^{-6}} = 117.3 \times 10^6 \text{ Pa} = 117.3 \text{ MPa}$$

Problem 7.166

7.166 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = 0$, (b) $\sigma_z = +45 \text{ MPa}$, (c) $\sigma_z = -45 \text{ MPa}$.



$$\sigma_x = 150 \text{ MPa}, \sigma_y = 70 \text{ MPa}, \tau_{xy} = 75 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$$

$$= 110 \text{ MPa}$$

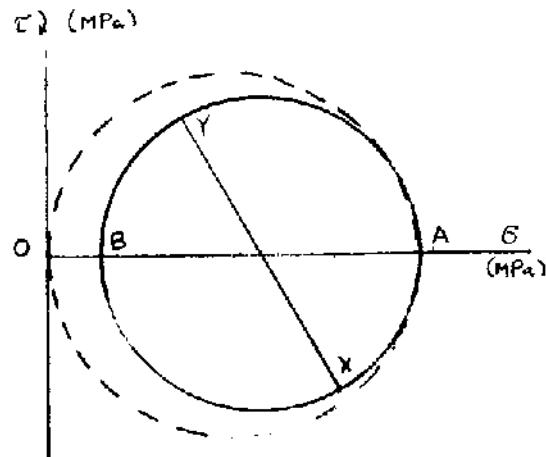
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2}$$

$$= 85 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 195 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 25 \text{ MPa}$$



$$(a) \sigma_z = 0, \sigma_a = 195 \text{ MPa}, \sigma_b = 25 \text{ MPa}$$

$$\sigma_{max} = 195 \text{ MPa}, \sigma_{min} = 0, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 97.5 \text{ MPa}$$

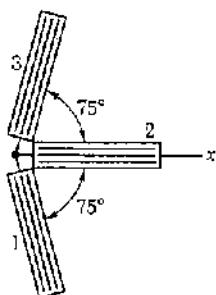
$$(b) \sigma_z = +45 \text{ MPa}, \sigma_a = 195 \text{ MPa}, \sigma_b = 25 \text{ MPa}$$

$$\sigma_{max} = 195 \text{ MPa}, \sigma_{min} = 25 \text{ MPa}, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85 \text{ MPa}$$

$$(c) \sigma_z = -45 \text{ MPa}, \sigma_a = 195 \text{ MPa}, \sigma_b = 25 \text{ MPa}$$

$$\sigma_{max} = 195 \text{ MPa}, \sigma_{min} = -45 \text{ MPa}, \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 120 \text{ MPa}$$

Problem 7.167



7.167 The strains determined by the use of a rosette attached as shown to the surface of a machine element are

$\epsilon_1 = -93.1 \times 10^{-6}$ in/in. $\epsilon_2 = +385 \times 10^{-6}$ in/in. $\epsilon_3 = +210 \times 10^{-6}$ in/in.
Determine (a) the orientation and magnitude of the principal strains in the plane of the rosette, (b) the maximum in-plane shearing stress.

$$\text{Use } \epsilon_{x1} = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

where $\theta = -75^\circ$ for gage 1, $\theta = 0^\circ$ for gage 2, and $\theta = +75^\circ$ for gage 3.

$$\epsilon_1 = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos(-150^\circ) + \frac{\gamma_{xy}}{2} \sin(-150^\circ) \quad (1)$$

$$\epsilon_2 = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos 0^\circ + \frac{\gamma_{xy}}{2} \sin 0^\circ \quad (2)$$

$$\epsilon_3 = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos(150^\circ) + \frac{\gamma_{xy}}{2} \sin(150^\circ) \quad (3)$$

$$\text{From Eq. (2)} \quad \epsilon_x = \epsilon_2 = 385 \times 10^{-6} \text{ in/in}$$

Adding Eq's (1) and (3)

$$\epsilon_1 + \epsilon_3 = (\epsilon_x + \epsilon_y) + (\epsilon_x - \epsilon_y) \cos 150^\circ$$

$$= \epsilon_x (1 + \cos 150^\circ) + \epsilon_y (1 - \cos 150^\circ)$$

$$\epsilon_y = \frac{\epsilon_1 + \epsilon_3 - \epsilon_x (1 + \cos 150^\circ)}{(1 - \cos 150^\circ)} = \frac{-93.1 \times 10^{-6} + 210 \times 10^{-6} - 385 \times 10^{-6} (1 + \cos 150^\circ)}{1 - \cos 150^\circ}$$

$$= 35.0 \times 10^{-6} \text{ in/in}$$

Subtracting Eq (1) from Eq (3)

$$\epsilon_3 - \epsilon_1 = \gamma_{xy} \sin 150^\circ$$

$$\gamma_{xy} = \frac{\epsilon_3 - \epsilon_1}{\sin 150^\circ} = \frac{210 \times 10^{-6} - (-93.1 \times 10^{-6})}{\sin 150^\circ} = 606.2 \times 10^{-6} \text{ in/in.}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{606.2 \times 10^{-6}}{385 \times 10^{-6} - 35.0 \times 10^{-6}} = 1.732 \quad (\text{a}) \theta_a = 30.0^\circ, \theta_b = 120.0^\circ$$

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = \frac{1}{2}(385 \times 10^{-6} + 35.0 \times 10^{-6}) = 210 \times 10^{-6} \text{ in/in}$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{385 \times 10^{-6} - 35.0 \times 10^{-6}}{2}\right)^2 + \left(\frac{606.2 \times 10^{-6}}{2}\right)^2} = 350.0 \times 10^{-6}$$

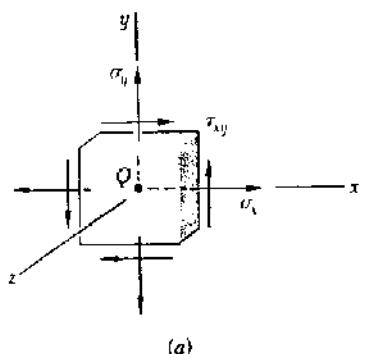
$$\epsilon_a = \epsilon_{ave} + R = 210 \times 10^{-6} + 350.0 \times 10^{-6} = 560 \times 10^{-6} \text{ in/in}$$

$$\epsilon_b = \epsilon_{ave} - R = 210 \times 10^{-6} + 350.0 \times 10^{-6} = -140.0 \times 10^{-6} \text{ in/in}$$

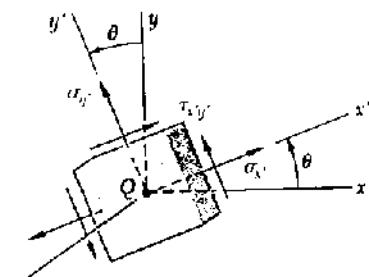
$$(b) \frac{\gamma_{max(in\ plane)}}{2} = R = 350.0 \times 10^{-6} \text{ in/in}$$

$$\gamma_{max(in\ plane)} = 700 \times 10^{-6} \text{ in/in}$$

PROBLEM 7.C1



(a)



(b)

7.C1 A state of plane stress is defined by the stress components σ_x , σ_y , and τ_{xy} associated with the element shown in Fig. P7.C1a. (a) Write a computer program that can be used to calculate the stress components σ_x' , σ_y' , and τ_{xy}' associated with the element after it has rotated through an angle θ about the z axis (Fig. P7.C1b). (b) Use this program to solve Probs. 7.13 through 7.16.

SOLUTION

PROBLEMS INVOLVING EQUATIONS

$$EQ(7.5, p427) \quad \sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

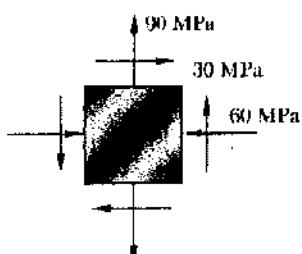
$$EQ(7.7, p427) \quad \sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_y - \sigma_x}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$EQ(7.6, p427) \quad \tau_{xy'} = -\frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

ENTER σ_x , σ_y , τ_{xy} AND θ

POINT VALUES OBTAINED FOR σ_x' , σ_y' AND $\tau_{xy'}$

Problem 7.13a



Sigma x = 90 MPa
Sigma y = 30 MPa
Tau xy = 60 MPa

Rotation of element
(+ counterclockwise)
theta = -25 degrees

Sigma x' = -56.19 MPa
Sigma y' = 86.19 MPa
Tau $x'y'$ = -38.17 MPa

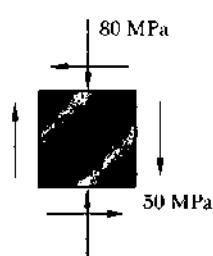
Problem 7.13b

Sigma x = -60 MPa
Sigma y = 90 MPa
Tau xy = 30 MPa

Rotation of element
(+ counterclockwise)
theta = 10 degrees

Sigma x' = -45.22 MPa
Sigma y' = 75.22 MPa
Tau $x'y'$ = 53.84 MPa

Problem 7.14a



Sigma x = 80 MPa
Sigma y = 50 MPa
Tau xy = -50 MPa

Rotation of element
(+ counterclockwise)
theta = -25 degrees

Sigma x' = 24.01 MPa
Sigma y' = -104.01 MPa
Tau $x'y'$ = -1.50 MPa

Problem 7.14b

Sigma x = 0 MPa
Sigma y = -80 MPa
Tau xy = -50 MPa

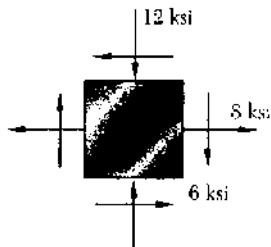
Rotation of element
(+ counterclockwise)
theta = 10 degrees

Sigma x' = -19.51 MPa
Sigma y' = -60.49 MPa
Tau $x'y'$ = -60.67 MPa

CONTINUED

PROBLEM 7.C1 - CONTINUED

PROGRAM OUTPUT

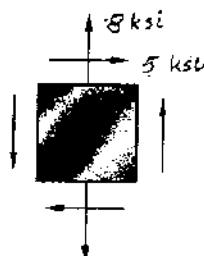


Problem 7.15a

$\Sigma \sigma_x = 8 \text{ ksi}$
 $\Sigma \sigma_y = -12 \text{ ksi}$
 $\tau_{xy} = -6 \text{ ksi}$
 Rotation of element
 (+ counterclockwise)
 $\theta = -25 \text{ degrees}$
 $\Sigma \sigma_{x'} = 9.02 \text{ ksi}$
 $\Sigma \sigma_{y'} = -13.02 \text{ ksi}$
 $\tau_{x'y'} = 3.80 \text{ ksi}$

Problem 7.15b

$\Sigma \sigma_x = 8 \text{ ksi}$
 $\Sigma \sigma_y = -12 \text{ ksi}$
 $\tau_{xy} = -6 \text{ ksi}$
 Rotation of element
 (+ counterclockwise)
 $\theta = 10 \text{ degrees}$
 $\Sigma \sigma_{x'} = 5.34 \text{ ksi}$
 $\Sigma \sigma_{y'} = -9.34 \text{ ksi}$
 $\tau_{x'y'} = -9.06 \text{ ksi}$



Problem 7.16a

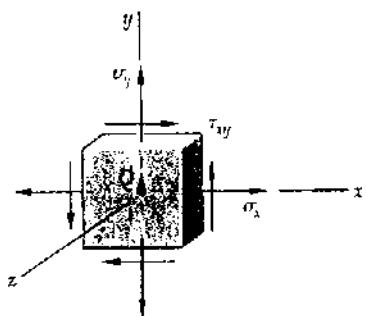
$\Sigma \sigma_x = 0 \text{ ksi}$
 $\Sigma \sigma_y = 8 \text{ ksi}$
 $\tau_{xy} = 5 \text{ ksi}$
 Rotation of element
 (+ counterclockwise)
 $\theta = -25 \text{ degrees}$

$\Sigma \sigma_{x'} = -2.40 \text{ ksi}$
 $\Sigma \sigma_{y'} = 10.40 \text{ ksi}$
 $\tau_{x'y'} = 0.15 \text{ ksi}$

Problem 7.16b

$\Sigma \sigma_x = 0 \text{ ksi}$
 $\Sigma \sigma_y = 8 \text{ ksi}$
 $\tau_{xy} = 5 \text{ ksi}$
 Rotation of element
 (+ counterclockwise)
 $\theta = 10 \text{ degrees}$
 $\Sigma \sigma_{x'} = 1.95 \text{ ksi}$
 $\Sigma \sigma_{y'} = 6.05 \text{ ksi}$
 $\tau_{x'y'} = 6.07 \text{ ksi}$

PROBLEM 7.C2



7.C2 A state of plane stress is defined by the stress components σ_x , σ_y , and τ_{xy} associated with the element shown in Fig. P7.C1a. (a) Write a computer program that can be used to determine the principal axes, the principal stresses, the maximum in-plane shearing stress, and the maximum shearing stress. (b) Use this program to solve Probs. 7.7, 7.11, 7.66, and 7.67.

SOLUTION

PROGRAM EQUATIONS AND SOLUTIONS

$$EQ.(7.10) \quad \bar{\tau}_{ave} = \frac{\tau_x + \tau_y}{2} : R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$EQ.(7.14) \quad \tau_{max} = \bar{\tau}_{ave} + R$$

$$\tau_{min} = \bar{\tau}_{ave} - R$$

$$EQ.(7.12) \quad \Theta_p = \tan^{-1} \frac{2\tau_{xy}}{\tau_x - \tau_y}$$

$$EQ.(7.15) \quad \Theta_s = \tan^{-1} - \frac{\tau_x - \tau_y}{2\tau_{xy}}$$

SHEARING STRESS $\text{IF } \tau_{max} > 0 \text{ and } \tau_{min} < 0:$

THEN $\tau_{max}(\text{in-plane}) = R ; \tau_{max}(\text{out-of-plane}) = R$

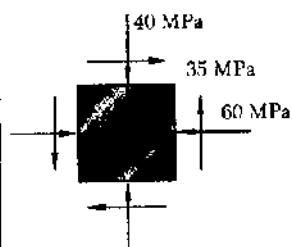
IF $\tau_{max} > 0$ and $\tau_{min} > 0$:

THEN $\tau_{max}(\text{in-plane}) = R ; \tau_{max}(\text{out-of-plane}) = \frac{1}{2}\tau_{max}$

IF $\tau_{max} < 0$ and $\tau_{min} < 0$:

THEN $\tau_{max}(\text{in-plane}) = R ; \tau_{max}(\text{out-of-plane}) = \frac{1}{2}|\tau_{min}|$

PROGRAM OUTPUT



Problems 7.7 AND 7.11

$\Sigma x = -60.00 \text{ MPa}$
 $\Sigma y = -40.00 \text{ MPa}$
 $\tau_{xy} = 35.00 \text{ MPa}$

Angle between xy axes and principal axes
(+ counterclockwise)

$\theta_p = -37.03 \text{ deg. and } 52.97 \text{ deg.}$

$\sigma_{max} = 13.60 \text{ MPa}$

$\sigma_{min} = -86.40 \text{ MPa}$

Angle between xy axis and planes of maximum in-plane shearing stress
(+ counterclockwise)

$\theta_s = 7.97 \text{ deg. and } 97.97 \text{ deg.}$

$\tau_{max}(\text{in plane}) = 36.40 \text{ MPa}$

$\tau_{max} = 43.20 \text{ MPa}$

CONTINUED

PROBLEM 7.C2 - CONTINUED

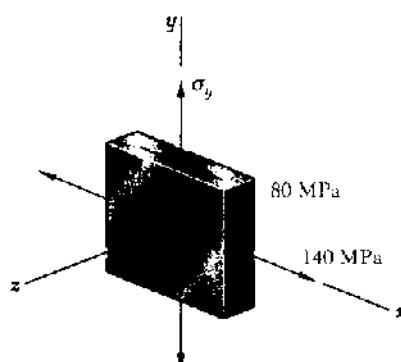


Fig. P7.66 and P7.67

Problem 7.66a: $\Sigma x = 140.00 \text{ MPa}$
 $\Sigma y = 20.00 \text{ MPa}$
 $\tau_{xy} = 80.00 \text{ MPa}$

Angle between xy axes and principal axes
(+ counterclockwise)
 $\theta_p = 26.57 \text{ deg. and } 116.57 \text{ deg.}$
 $\Sigma \max = 180.00 \text{ MPa}$
 $\Sigma \min = -20.00 \text{ MPa}$
Angle between xy axis and planes of maximum in-plane
in-plane shearing stress (+ counterclockwise)
 $\theta_s = 71.57 \text{ deg. and } 161.57 \text{ deg.}$
 $\tau_{\max \text{ (in-plane)}} = 100.00 \text{ MPa}$
 $\tau_{\max \text{ (out-of-plane)}} = 100.00 \text{ MPa}$

Problem 7.66b: $\Sigma x = 140.00 \text{ MPa}$
 $\Sigma y = 140.00 \text{ MPa}$
 $\tau_{xy} = 80.00 \text{ MPa}$

Angle between xy axes and principal axes
(+ counterclockwise)
 $\theta_p = 45.00 \text{ deg. and } 135.00 \text{ deg.}$
 $\Sigma \max = 220.00 \text{ MPa}$
 $\Sigma \min = 60.00 \text{ MPa}$
Angle between xy axis and planes of maximum in-plane
in-plane shearing stress (+ counterclockwise)
 $\theta_s = 90.00 \text{ deg. and } 180.00 \text{ deg.}$
 $\tau_{\max \text{ (in-plane)}} = 80.00 \text{ MPa}$
 $\tau_{\max \text{ (out-of-plane)}} = 110.00 \text{ MPa}$

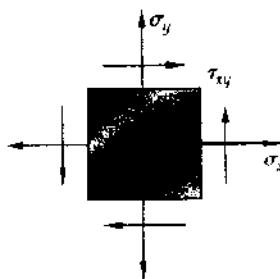
Problem 7.67a: $\Sigma x = 140.00 \text{ MPa}$
 $\Sigma y = 40.00 \text{ MPa}$
 $\tau_{xy} = 80.00 \text{ MPa}$

Angle between xy axes and principal axes
(+ counterclockwise)
 $\theta_p = 29.00 \text{ deg. and } 119.00 \text{ deg.}$
 $\Sigma \max = 184.34 \text{ MPa}$
 $\Sigma \min = -4.34 \text{ MPa}$
Angle between xy axis and planes of maximum in-plane
in-plane shearing stress (+ counterclockwise)
 $\theta_s = 74.00 \text{ deg. and } 164.00 \text{ deg.}$
 $\tau_{\max \text{ (in-plane)}} = 94.34 \text{ MPa}$
 $\tau_{\max \text{ (out-of-plane)}} = 94.34 \text{ MPa}$

Problem 7.67b: $\Sigma x = 140.00 \text{ MPa}$
 $\Sigma y = 120.00 \text{ MPa}$
 $\tau_{xy} = 80.00 \text{ MPa}$

Angle between xy axes and principal axes
(+ counterclockwise)
 $\theta_p = 41.44 \text{ deg. and } 131.44 \text{ deg.}$
 $\Sigma \max = 210.62 \text{ MPa}$
 $\Sigma \min = 49.38 \text{ MPa}$
Angle between xy axis and planes of maximum in-plane
in-plane shearing stress (+ counterclockwise)
 $\theta_s = 86.44 \text{ deg. and } 176.44 \text{ deg.}$
 $\tau_{\max \text{ (in-plane)}} = 80.62 \text{ MPa}$
 $\tau_{\max \text{ (out-of-plane)}} = 105.31 \text{ MPa}$

PROBLEM 7.C3



7.C3 (a) Write a computer program that, for a given state of plane stress and a given yield strength of a ductile material, can be used to determine whether the material will yield. The program should use both the maximum-shearing-stress criterion and the maximum-distortion-energy criterion. It should also print the values of the principal stresses and, if the material does not yield, calculate the factor of safety. (b) Use this program to solve Probs. 7.81 through 7.84.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{\text{max}} = \frac{\sigma_x + \sigma_y}{2} + R ; \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{max}} + R$$

$$\sigma_b = \sigma_{\text{max}} - R$$

$$\text{MAXIMUM-SHEARING-STRESS CRITERION} \quad \tau_y = \frac{1}{2} \sigma_y$$

$$\text{IF } \tau_a \text{ AND } \tau_b \text{ HAVE SAME SIGN, } \tau_{\text{max}} = \frac{1}{2} \tau_y$$

IF $\tau_{\text{max}} > \tau_y$, YIELDING OCCURS

IF $\tau_{\text{max}} < \tau_y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\tau_y}{\tau_{\text{max}}}$$

MAXIMUM-DISTORTION-ENERGY CRITERION

$$\text{COMPUTE RADICAL} = \sqrt{\tau_a^2 - \tau_a \tau_b + \tau_b^2}$$

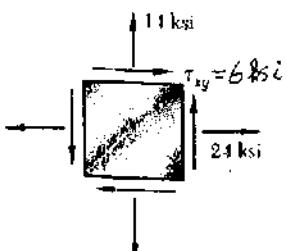
IF RADICAL > σ_y , YIELDING OCCURS

IF RADICAL < σ_y , NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\tau_y}{\text{RADICAL}}$$

PROGRAM OUTPUT

Problems 7.81a and 7.82a Yield strength = 30 ksi



Sigma x = 24.00 ksi

Sigma y = 14.00 ksi

Tau xy = 6.00 ksi

Sigma max = 26.81 ksi

Sigma min = 11.19 ksi

(a) Using the maximum-shearing-stress criterion:

Material will not yield

F.S. = 1.119

(b) Using the maximum-distortion-energy criterion:

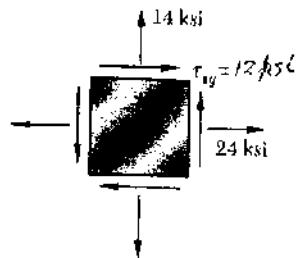
Material will not yield

F.S. = 1.286

CONTINUED

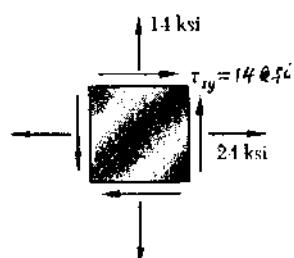
PROBLEM 7.C3 - CONTINUED

Problems 7.81b and 7.82b Yield strength = 30 ksi



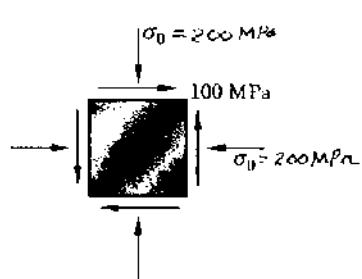
$\Sigma \sigma_x = 24.00 \text{ ksi}$
 $\Sigma \sigma_y = 14.00 \text{ ksi}$
 $\tau_{xy} = 12.00 \text{ ksi}$
 $\Sigma \sigma_{max} = 32.00 \text{ ksi}$
 $\Sigma \sigma_{min} = 6.00 \text{ ksi}$
 (a) Using the maximum-shearing-stress criterion:
 Material will yield
 (b) Using the maximum-distortion-energy criterion:
 Material will not yield
 F.S. = 1.018

Problems 7.81c and 7.82c Yield strength = 30 ksi



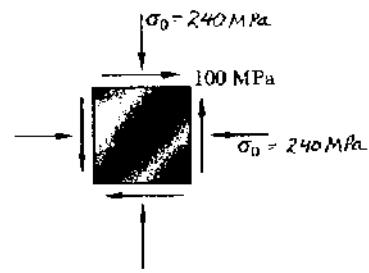
$\Sigma \sigma_x = 24.00 \text{ ksi}$
 $\Sigma \sigma_y = 14.00 \text{ ksi}$
 $\tau_{xy} = 14.00 \text{ ksi}$
 $\Sigma \sigma_{max} = 33.87 \text{ ksi}$
 $\Sigma \sigma_{min} = 4.13 \text{ ksi}$
 (a) Using the maximum-shearing-stress criterion:
 Material will yield
 (b) Using the maximum-distortion-energy criterion:
 Material will yield

Problems 7.83a and 7.84a Yield strength = 325 MPa



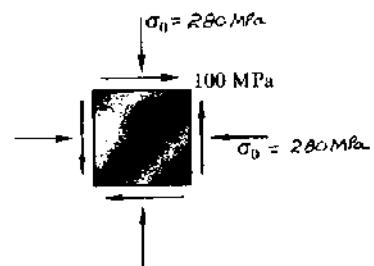
$\Sigma \sigma_x = -200.00 \text{ MPa}$
 $\Sigma \sigma_y = -200.00 \text{ MPa}$
 $\tau_{xy} = 100.00 \text{ MPa}$
 $\Sigma \sigma_{max} = -100.00 \text{ MPa}$
 $\Sigma \sigma_{min} = -300.00 \text{ MPa}$
 Using the maximum-shearing-stress criterion:
 Material will not yield
 F.S. = 1.083
 Using the maximum-distortion-energy criterion:
 Material will not yield
 F.S. = 1.228

Problems 7.83b and 7.84b Yield strength = 325 MPa



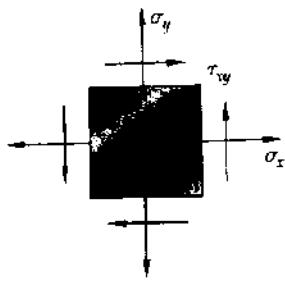
$\Sigma \sigma_x = -240.00 \text{ MPa}$
 $\Sigma \sigma_y = -240.00 \text{ MPa}$
 $\tau_{xy} = 100.00 \text{ MPa}$
 $\Sigma \sigma_{max} = -140.00 \text{ MPa}$
 $\Sigma \sigma_{min} = -340.00 \text{ MPa}$
 Using the maximum-shearing-stress criterion:
 Material will yield
 Using the maximum-distortion-energy criterion:
 Material will not yield
 F.S. = 1.098

Problems 7.83c and 7.84c Yield strength = 325 MPa



$\Sigma \sigma_x = -280.00 \text{ MPa}$
 $\Sigma \sigma_y = -280.00 \text{ MPa}$
 $\tau_{xy} = 100.00 \text{ MPa}$
 $\Sigma \sigma_{max} = -180.00 \text{ MPa}$
 $\Sigma \sigma_{min} = -380.00 \text{ MPa}$
 Using the maximum-shearing-stress criterion:
 Material will yield
 Using the maximum-distortion-energy criterion:
 Material will yield

PROBLEM 7.C4



7.C4 (a) Write a computer program based on Mohr's fracture criterion for brittle materials that, for a given state of plane stress and given values of the ultimate strength of the material in tension and in compression, can be used to determine whether rupture will occur. The program should also print the values of the principal stresses. (b) Use this program to solve Probs. 7.91 and 7.92 and to check the answers given for Probs. 7.93 and 7.94.

SOLUTION

PRINCIPAL STRESSES

$$\bar{\sigma}_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R$$

Mohr's Fracture Criterion

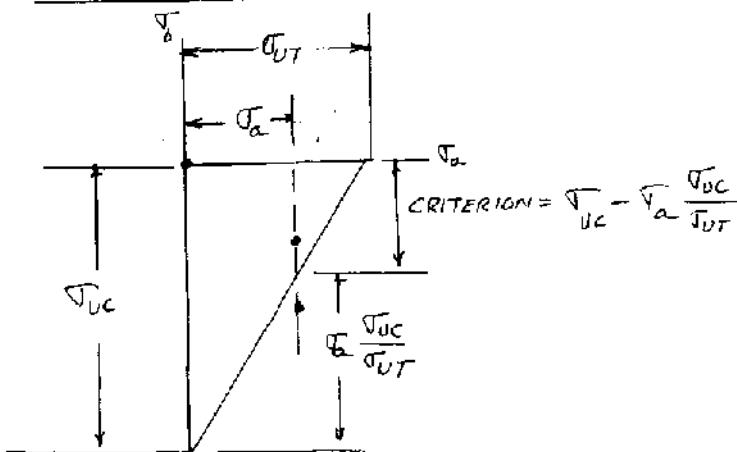
IF $\bar{\sigma}_a$ AND $\bar{\sigma}_b$ HAVE SAME SIGN, AND

$\bar{\sigma}_a < \sigma_{UT}$ AND $\bar{\sigma}_b < \sigma_{UC}$, NO FAILURE

$\bar{\sigma}_a > \sigma_{UT}$ OR $\bar{\sigma}_b > \sigma_{UC}$, FAILURE

IF $\bar{\sigma}_a > 0$ AND $\bar{\sigma}_b < 0$:

CONSIDER FOURTH QUADRANT OF FIG. 7.47



FOR NO RUPTURE TO OCCUR:

POINT $(\bar{\sigma}_a, \bar{\sigma}_b)$ MUST LIE WITHIN
MOHR'S ENVELOPE (FIG. 7.47)

IF $\bar{\sigma}_b >$ CRITERION,
THEN RUPTURE OCCURS

IF $\bar{\sigma}_b <$ CRITERION,
THEN NO RUPTURE OCCURS

PROGRAM OUTPUT



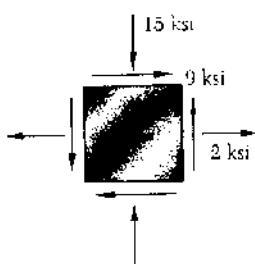
Fig. P7.91

Problem 7.91 Sigma x = -8.00 ksi
 Sigma y = 0.00 ksi
 Tau xy = 7.00 ksi
Ultimate strength in tension = 10 ksi
Ultimate strength in compression = 30 ksi

Sigma max = Sigma a = 4.06 ksi
Sigma min = Sigma b = -12.06 ksi
Rupture will not occur

CONTINUED

PROBLEM 7.C4 - CONTINUED



Problem 7.92

Sigma x = 2.00 ksi
Sigma y = -15.00 ksi
Tau xy = 9.00 ksi

Ultimate strength in tension = 10 ksi
Ultimate strength in compression = 30 ksi

Sigma max = Sigma a = 5.88 ksi
Sigma min = Sigma b = -18.88 ksi
Rupture will occur

Fig. P7.92

TO CHECK ANSWERS TO FOLLOWING PROBLEMS, IF CHECK FOR
ADJACENT VALUE AND SUBTRACT ANSWERS IN ALL ADJACENT VALUE.

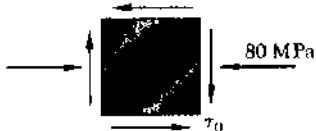


Fig. P7.93

ANSWER:
Rupture occurs for $T_0 = 47.1 \text{ MPa}$

Problem 7.93

Sigma x = -80.00 MPa
Sigma y = 0.00 MPa
Tau xy = 49.10 MPa

Ultimate strength in tension = 75 MPa
Ultimate strength in compression = 150 MPa

Sigma max = Sigma a = 23.33 MPa
Sigma min = Sigma b = -103.33 MPa
Rupture will not occur

Problem 7.93

Sigma x = -80.00 MPa
Sigma y = 0.00 MPa
Tau xy = 49.20 MPa

Ultimate strength in tension = 75 MPa
Ultimate strength in compression = 150 MPa

Sigma max = Sigma a = 23.41 MPa
Sigma min = Sigma b = -103.41 MPa
Rupture will occur

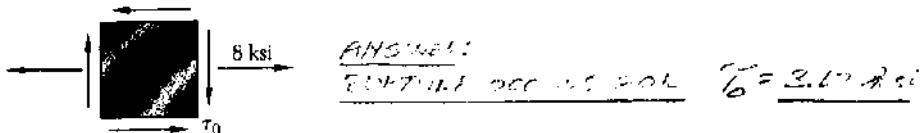


Fig. P7.94

Problem 7.94

Sigma x = 8.00 ksi
Sigma y = 0.00 ksi
Tau xy = 3.67 ksi

Ultimate strength in tension = 10 ksi
Ultimate strength in compression = 25 ksi

Sigma max = Sigma a = 9.43 ksi
Sigma min = Sigma b = -1.43 ksi
Rupture will not occur

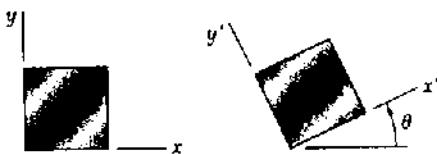
Problem 7.94

Sigma x = 8.00 ksi
Sigma y = 0.00 ksi
Tau xy = 3.68 ksi

Ultimate strength in tension = 10 ksi
Ultimate strength in compression = 25 ksi

Sigma max = Sigma a = 9.44 ksi
Sigma min = Sigma b = -1.44 ksi
Rupture will occur

PROBLEM 7.C5



7.C5 A state of plane strain is defined by the strain components ϵ_x , ϵ_y , and γ_{xy} associated with the x and y axes. (a) Write a computer program that can be used to calculate the strain components ϵ_x' , ϵ_y' , and $\gamma_{x'y'}$ associated with the frame of reference $x'y'$ obtained by rotating the x and y axes through an angle θ . (b) Use this program to solve Probs. 7.126 through 7.129.

SOLUTION

PROGRAM FOR CONVERSION EQUATIONS

$$EQ(7.44) \quad \epsilon_x' = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{1}{2} (\gamma_{xy}) \sin 2\theta$$

$$EQ(7.45) \quad \epsilon_y' = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \sin 2\theta - \frac{1}{2} (\gamma_{xy}) \cos 2\theta$$

$$EQ(7.46) \quad \gamma_{x'y'} = -(\epsilon_x - \epsilon_y) \sin 2\theta + (\gamma_{xy}) \cos 2\theta$$

ENTER ϵ_x , ϵ_y , γ_{xy} , θ

PRINT VALUES OBTAINED FOR ϵ_x' , ϵ_y' , $\gamma_{x'y'}$

PROGRAM OUTPUT

Problem 7.126

Epsilon x = 240 micro meters

Epsilon y = 160 micro meters

Gamma xy = 150 micro radians

Rotation of element, in degrees (+ counterclockwise)

Theta = -60 degrees

Epsilon x' = 115.05 micro meters

Epsilon y' = 284.95 micro meters

Gamma x'y' = -5.72 micro radians

Problem 7.127

Epsilon x = 0 micro meters

Epsilon y = 320 micro meters

Gamma xy = -100 micro radians

Rotation of element, in degrees (+ counterclockwise)

Theta = 30 degrees

Epsilon x' = 36.70 micro meters

Epsilon y' = 283.30 micro meters

Gamma x'y' = 227.13 micro radians

Problem 7.128

Epsilon x = -800 micro meters

Epsilon y = 450 micro meters

Gamma xy = 200 micro radians

Rotation of element, in degrees (+ counterclockwise)

Theta = -25 degrees

Epsilon x' = -653.35 micro meters

Epsilon y' = 303.35 micro meters

Gamma x'y' = -829.00 micro radians

Problem 7.129

Epsilon x = 500 micro meters

Epsilon y = -300 micro meters

Gamma xy = 0 micro radians

Rotation of element, in degrees (+ counterclockwise)

Theta = 25 degrees

Epsilon x' = 357.12 micro meters

Epsilon y' = -157.12 micro meters

Gamma x'y' = -612.84 micro radians

PROBLEM 7.C6

7.C6 A state of strain is defined by the strain components ϵ_x , ϵ_y , and γ_{xy} associated with the x and y axes. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.134 through 7.137.

SOLUTION PROGRAM FOLLOWING EQUATIONS

$$\text{EQ(7.50)} \quad \epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2} \quad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\text{EQ(7.51)} \quad \epsilon_{max} = \epsilon_{ave} + R \quad \epsilon_{min} = \epsilon_{ave} - R$$

$$\text{EQ(7.17)} \quad \rho_p = \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max(in-plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

$$\text{LET } \epsilon_a = \epsilon_{max}$$

$$\epsilon_b = \epsilon_{min}$$

$$\text{CALCULATE } \epsilon_c = -\frac{V}{1-V} (\epsilon_a + \epsilon_b)$$

$$\text{IF } \epsilon_a > \epsilon_b > \epsilon_c : \gamma_{out-of-plane} = \epsilon_a - \epsilon_c$$

$$\text{IF } \epsilon_a > \epsilon_c > \epsilon_b : \gamma_{out-of-plane} = \epsilon_a - \epsilon_b = 2R$$

$$\text{IF } \epsilon_c > \epsilon_a > \epsilon_b : \gamma_{out-of-plane} = \epsilon_c - \epsilon_b$$

PROBLEM 7.134

Problem 7.134 $\epsilon_x = 160$ micro meters
 $\epsilon_y = -480$ micro meters
 $\gamma_{xy} = -600$ micro radians
 $\nu = 0.333$

Angle between xy axes and principal axes (+ = counterclockwise)

$$\theta_p = -21.58 \text{ degrees}$$

$$\epsilon_a = 278.63 \text{ micro meters}$$

$$\epsilon_b = -598.63 \text{ micro meters}$$

$$\epsilon_c = 159.98 \text{ micro meters}$$

$\gamma_{max (in plane)} = 877.27$ micro radians
 $\gamma_{max} = 877.27$ micro radians

CONTINUED

PROBLEM 7.C6 - CONTINUED

Problem 7.135

Epsilon x = -260 micro meters
Epsilon y = -60 micro meters
Gamma xy = 480 micro radians
 $\nu = 0.333$

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = -33.69 degrees
Epsilon a = 100.00 micro meters
Epsilon b = -420.00 micro meters
Epsilon c = 159.98 micro meters

Gamma max (in plane) = 520.00 micro radians
Gamma max = 579.98 micro radians

Problem 7.136

Epsilon x = 30 micro meters
Epsilon y = 570 micro meters
Gamma xy = 720 micro radians
 $\nu = 0.333$

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = -26.57 degrees
Epsilon a = 750.00 micro meters
Epsilon b = -150.00 micro meters
Epsilon c = -300.00 micro meters

Gamma max (in plane) = 900.00 micro radians
Gamma max = 1050.00 micro radians

Problem 7.137

Epsilon x = -600 micro meters
Epsilon y = -400 micro meters
Gamma xy = 350 micro radians
 $\nu = 0.333$

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = -30.13 degrees
Epsilon a = -298.44 micro meters
Epsilon b = -701.56 micro meters
Epsilon c = 500.00 micro meters

Gamma max (in plane) = 403.11 micro radians
Gamma max = 1201.56 micro radians

PROBLEM 7.C7

7.C7 A state of plane strain is defined by the strain components ϵ_x , ϵ_y , and γ_{xy} measured at a point. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.138 through 7.141.

SOLUTIONPROGRAM FOLLOWING EQUATIONS

$$EQ(7.50) \quad \epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2} \quad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$EQ(7.51) \quad \epsilon_{max} = \epsilon_{ave} + R \quad \epsilon_{min} = \epsilon_{ave} - R$$

$$EQ(7.52) \quad \theta_p = \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINSMAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{xy} (\text{in-plane}) = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

$$\text{LET } \epsilon_a = \epsilon_{xy},$$

$$\epsilon_b = \epsilon_{yy}$$

$$\epsilon_c = 0 \quad (\text{PLAIN STRAIN})$$

$$\text{IF } \epsilon_a > \epsilon_b > \epsilon_c : \gamma_{\text{OUT-OF-PLANE}} = \epsilon_a - \epsilon_c$$

$$\text{IF } \epsilon_a < \epsilon_b > \epsilon_c : \gamma_{\text{OUT-OF-PLANE}} = \epsilon_a - \epsilon_b = 2R$$

$$\text{IF } \epsilon_c > \epsilon_a > \epsilon_b : \gamma_{\text{OUT-OF-PLANE}} = \epsilon_c - \epsilon_b$$

PROGRAM PRINTOUTProblem 7.138

Epsilon x = -180 micro meters
 Epsilon y = -260 micro meters
 Gamma xy = 315 micro radians
 nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 37.87 and -52.13 degrees

Epsilon a = -57.50 micro meters

Epsilon b = -382.50 micro meters

Epsilon c = 0.00 micro meters

Gamma max (in plane) = 325.00 micro radians

Gamma max = 382.50 micro radians

CONTINUED

PROBLEM 7.C7 - CONTINUED

Problem 7.139

Epsilon x = 300 micro meters
Epsilon y = 60 micro meters
Gamma xy = 100 micro radians
 ν = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 11.31 and -78.69 degrees
Epsilon a = 310.00 micro meters
Epsilon b = 50.00 micro meters
Epsilon c = 0.00 micro meters

Gamma max (in plane) = 260.00 micro radians
Gamma max = 310.00 micro radians

Problem 7.140

Epsilon x = 400 micro meters
Epsilon y = 200 micro meters
Gamma xy = 375 micro radians
 ν = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 30.96 and -59.04 degrees
Epsilon a = 512.50 micro meters
Epsilon b = 87.50 micro meters
Epsilon c = 0.00 micro meters

Gamma max (in plane) = 425.00 micro radians
Gamma max = 512.50 micro radians

Problem 7.141

Epsilon x = 60 micro meters
Epsilon y = 240 micro meters
Gamma xy = -50 micro radians
 ν = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 7.76 and -82.24 degrees
Epsilon a = 243.41 micro meters
Epsilon b = 56.59 micro meters
Epsilon c = 0.00 micro meters

Gamma max (in plane) = 186.82 micro radians
Gamma max = 243.41 micro radians

PROBLEM 7.C8

7.C8 A rosette consisting of three gages forming, respectively, angles θ_1 , θ_2 , and θ_3 with the x axis is attached to the free surface of a machine component made of a material with a given Poisson's ratio v . (a) Write a computer program that, for given readings ϵ_1 , ϵ_2 , and ϵ_3 of the gages, can be used to calculate the strain components associated with the x and y axes and to determine the orientation and magnitude of the three principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.142 through 7.145.

SOLUTION

FOR $n=1$ TO 3, ENTER θ_n AND ϵ_n
ENTER: MU = V

SOLVE Eqs. (7.60) FOR ϵ_x , ϵ_y , AND γ_{xy} USING
METHOD OF DETERMINATES OR ANY OTHER
METHOD,

$$\text{ENTH} \quad \epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2}; \quad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \gamma_{xy}^2}$$

$$\epsilon_a = \epsilon_{max} = \epsilon_{ave} + R$$

$$\epsilon_b = \epsilon_{min} = \epsilon_{ave} - R$$

$$\epsilon_c = -\frac{V}{1-V} (\epsilon_a + \epsilon_b)$$

$$\theta_p = \frac{1}{2} \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max(\text{in-plane})} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN,
AND CHECK WHETHER IT IS THE MAXIMUM
SHEARING STRAIN,

$$\text{IF } \epsilon_c < \epsilon_b : \gamma_{out-of-plane} = \epsilon_a - \epsilon_c$$

$$\text{IF } \epsilon_c > \epsilon_a : \gamma_{out-of-plane} = \epsilon_c - \epsilon_b$$

$$\text{OTHERWISE: } \gamma_{out-of-plane} = 2R$$

PROGRAM OUTPUT

Problem 7.142

Gage	theta degrees	epsilon micro meters
1	30	600
2	-30	450
3	90	-75

Epsilon x = 725.000 micro meters

Epsilon y = -75.000 micro meters

Gamma xy = 173.205 micro radians

Epsilon a = 734.268 micro meters

Epsilon b = -84.268 micro meters

Gamma max (in plane) = 818.535 micro radians

CONTINUED

PROBLEM 7.C8 - CONTINUED

Problem 7.143

Gage	theta degrees	epsilon in./in.
------	------------------	--------------------

1	-15	480
2	30	-120
3	75	80

Epsilon x = 253.21 micro meters
 Epsilon y = 306.79 micro meters
 Gamma xy = -892.82 micro radians

Epsilon a = 727.21 micro meters
 Epsilon b = -167.21 micro meters
 Gamma max (in plane) = 894.43 micro radians

Problem 7.144

OBSERVE THAT GAGE 3 IS ORIENTATED ALONG THE Y AXIS. THEREFORE

ENTER ϵ_3 AND ϵ_4 AS ϵ_3 AND ϵ_4 ,
 THE VALUE OF ϵ_4 THAT IS OBTAINED
 IS ALSO THE EXPECTED READING OF GAGE 3.

Gage	theta degrees	epsilon micro meters
------	------------------	-------------------------

1	0	420
2	45	-45
4 → X	135	165

Epsilon x = 420.00 micro meters
 Epsilon y = -300.00 micro meters
 Gamma xy = -210.00 micro radians

Epsilon a = 435.00 micro meters
 Epsilon b = -315.00 micro meters
 Gamma max (in plane) = 750.00 micro radians

Problem 7.145

Gage	theta degrees	epsilon in./in.
------	------------------	--------------------

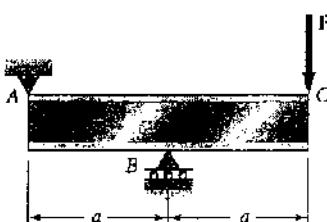
1	45	-50
2	-45	360
3	0	315

Epsilon x = 315.000 in./in.
 Epsilon y = -5.000 in./in.
 Gamma xy = -410.000 micro radians

Epsilon a = 415.048 in./in.
 Epsilon b = -105.048 in./in.
 Gamma max (in plane) = 520.096 micro radians

Chapter 8

Problem 8.1

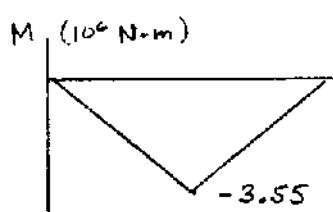
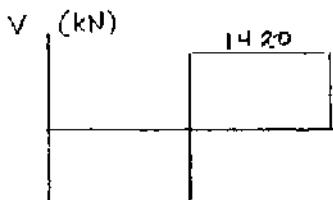


8.1 An overhanging W920 x 446 rolled-steel beam supports a load P as shown. Knowing that $P = 1420 \text{ kN}$, $a = 2.5 \text{ m}$, and $\sigma_{all} = 200 \text{ MPa}$, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

$$|V|_{max} = 1420 \text{ kN} = 1420 \times 10^3 \text{ N}$$

$$|M|_{max} = (1420 \times 10^3)(2.5) = 3.55 \times 10^6 \text{ N}\cdot\text{m}$$

For W920 x 446 rolled steel beam



$$d = 933 \text{ mm} \quad b_f = 423 \text{ mm} \quad t_f = 42.70 \text{ mm}$$

$$t_w = 24.0 \text{ mm} \quad I_x = 8470 \times 10^8 \text{ mm}^4 \quad S_x = 18200 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 466.5 \text{ mm} \quad y_b = c - t_f = 423.8 \text{ mm}$$

$$(a) \sigma_m = \frac{|M|_{max}}{S_x} = \frac{3.55 \times 10^6}{18200 \times 10^3} = 195.1 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{423.8}{466.5} (195.1) = 177.2 \text{ MPa}$$

$$A_f = b_f t_f = 18.062 \times 10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 445.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 18.062 \times 10^3 \text{ mm}^3 = 8040.3 \times 10^3 \text{ mm}^3 = 8040.3 \times 10^{-6} \text{ m}^4$$

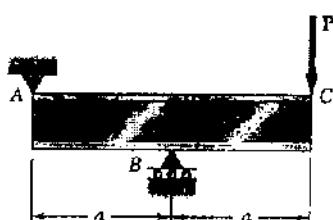
$$\sigma_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(1420 \times 10^3)(8040.3 \times 10^3)}{(8470 \times 10^8)(24.0 \times 10^{-3})} = 56.2 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \sigma_{xy}^2} = \sqrt{\left(\frac{177.2}{2}\right)^2 + 56.2^2} = 104.9 \text{ MPa}$$

$$(b) \sigma_{max} = \frac{\sigma_b}{2} + R = 193.5 \text{ MPa}$$

(c) Since $193.5 \text{ MPa} < \sigma_{all}$, W920 x 446 is acceptable.

Problem 8.2



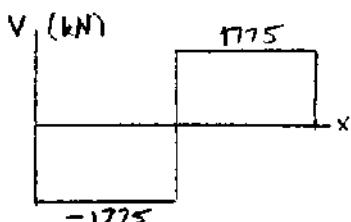
8.2 Solve Prob. 8.1, assuming that $P = 1775 \text{ kN}$ and $a = 2.0 \text{ m}$.

8.1 An overhanging W920 × 446 rolled-steel beam supports a load \mathbf{P} as shown. Knowing that $P = 1420 \text{ kN}$, $a = 2.5 \text{ m}$, and $\sigma_{\text{all}} = 200 \text{ MPa}$, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

$$|V|_{\text{max}} = 1775 \text{ kN} = 1775 \times 10^3 \text{ N}$$

$$|M|_{\text{max}} = (1775 \times 10^3)(2.0) = 3.55 \times 10^6 \text{ N}\cdot\text{m}$$

For W920 × 446 rolled steel section



$$d = 933 \text{ mm} \quad b_f = 423 \text{ mm} \quad t_f = 42.70 \text{ mm}$$

$$t_w = 24.0 \text{ mm} \quad I_x = 8470 \times 10^6 \text{ mm}^4 \quad S_x = 18200 \times 10^3 \text{ mm}^3$$

$$C = \frac{1}{2}d = 466.5 \text{ mm} \quad y_b = C - t_f = 423.8 \text{ mm}$$

$$(a) \sigma_m = \frac{|M|_{\text{max}}}{S_x} = \frac{3.55 \times 10^6}{18200 \times 10^3} = 195.1 \text{ MPa} \rightarrow$$

$$\sigma_b = \frac{y_b}{C} \sigma_m = \frac{423.8}{466.5} (195.1) = 177.2 \text{ MPa}$$

$$A_f = b_f t_f = 18.062 \times 10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(C + y_b) = 445.5 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8040.3 \times 10^3 \text{ mm}^3 = 8040.3 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = \frac{|V|_{\text{max}} Q_b}{I_x t_w} = \frac{(1775 \times 10^3)(8040.3 \times 10^{-6})}{(8470 \times 10^6)(24.0 \times 10^{-3})} = 70.2 \text{ MPa}$$

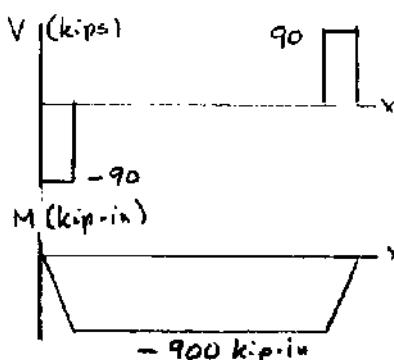
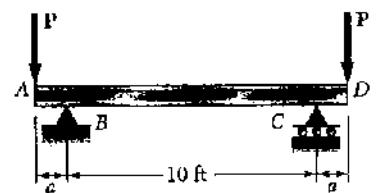
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{177.2}{2}\right)^2 + 70.2^2} = 113.0 \text{ MPa}$$

$$(b) \sigma_{\text{max}} = \frac{\sigma_b}{2} + R = 201.6 \text{ MPa} \rightarrow$$

(c) Since $201.6 \text{ MPa} > \sigma_{\text{all}}$ W920 × 446 is not acceptable \rightarrow

Problem 8.3

8.3. An overhanging W10 x 39 rolled-steel beam supports a load P as shown. Knowing that $P = 90$ kips, $a = 10$ in., and $\sigma_{all} = 36$ ksi, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.



$$|V|_{max} = 90 \text{ kips}$$

$$|M|_{max} = (90)(10) = 900 \text{ kip-in}$$

For W10 x 39 rolled steel section

$$d = 9.92 \text{ in. } b_f = 7.985 \text{ in. } t_f = 0.530 \text{ in.}$$

$$t_w = 0.315 \text{ in. } I_x = 209 \text{ in}^4 \quad S_x = 42.1 \text{ in}^3$$

$$c = \frac{1}{2}d = 4.96 \text{ in. } y_b = c - t_f = 4.43 \text{ in.}$$

$$(a) \sigma_m = \frac{|M|_{max}}{S_x} = \frac{900}{42.1} = 21.37 \text{ ksi}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{4.43}{4.96} \right) (21.37) = 19.09 \text{ ksi}$$

$$A_f = b_f t_f = 4.23205 \text{ in}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695 \text{ in.}$$

$$Q_b = A_f \bar{y}_f = 19.869 \text{ in}^3$$

$$\chi_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(90)(19.869)}{(209)(0.315)} = 27.16 \text{ ksi}$$

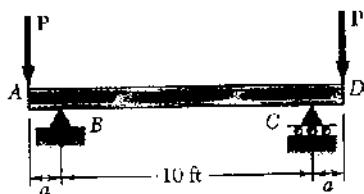
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \chi_{xy}^2} = 28.79 \text{ ksi}$$

$$(b) \sigma_{max} = \frac{\sigma_b}{2} + R = 38.3 \text{ ksi}$$

(c) Since $\sigma_{max} > 36$ ksi, W10 x 39 is not acceptable.

Problem 8.4

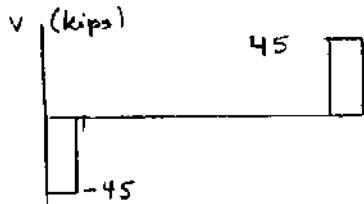
8.4 Solve Prob. 8.3, assuming that $P = 45$ kips, $a = 20$ in..



8.3 An overhanging W10 x 39 rolled-steel beam supports a load P as shown. Knowing that $P = 90$ kips, $a = 10$ in., and $\sigma_{all} = 36$ ksi, determine (a) the maximum value of the normal stress σ_n in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

$$|V|_{max} = 45 \text{ kips}$$

$$|M|_{max} = (45)(20) = 900 \text{ kip-in}$$

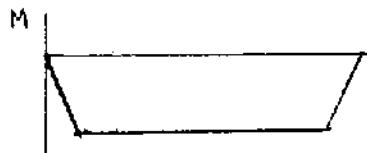


For W10 x 39 rolled steel section

$$d = 9.92 \text{ in.} \quad b_f = 7.985 \text{ in.} \quad t_f = 0.530 \text{ in.}$$

$$t_w = 0.315 \text{ in.} \quad I_x = 209 \text{ in}^4 \quad S_x = 42.1 \text{ in}^3$$

$$C = \frac{1}{2}d = 4.96 \text{ in.} \quad y_b = C - t_f = 4.43 \text{ in.}$$



$$(a) \sigma_m = \frac{|M|_{max}}{S_x} = \frac{900}{42.1} = 21.37 \text{ ksi}$$

$$\sigma_b = \frac{y_b}{C} \sigma_m = \left(\frac{4.43}{4.96} \right) (21.37) = 19.09 \text{ ksi}$$

$$A_f = b_f t_f = 4.23205 \text{ in}^2$$

$$\bar{y}_f = \frac{1}{2}(C + y_b) = 4.695 \text{ in.}$$

$$Q_b = A_f \bar{y}_f = 19.869 \text{ in}^3$$

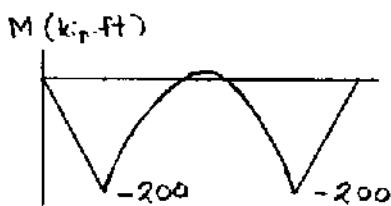
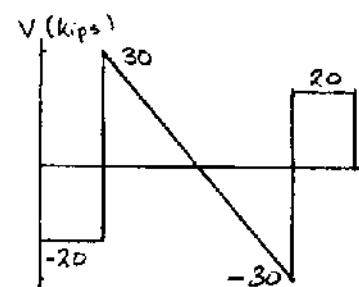
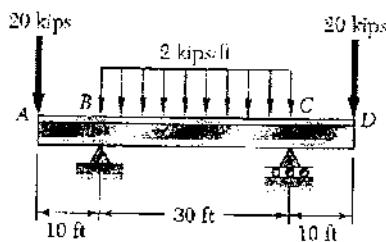
$$\chi'_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(45)(19.869)}{(209)(0.315)} = 13.58 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \chi'_{xy}^2} = 16.60$$

$$(b) \sigma_{max} = \frac{\sigma_b}{2} + R = 26.14 \text{ ksi}$$

(c) Since $\sigma_{max} < 36$ ksi, W10 x 39 is acceptable.

Problem 8.5



$$C = \frac{1}{2}d = \frac{20.99}{2} = 10.495 \text{ in.}$$

8.5 and 8.6 (a) Knowing that $\sigma_{all} = 24 \text{ ksi}$ and $\tau_{all} = 14.5 \text{ ksi}$, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

$$R_A = 50 \text{ kips } \uparrow$$

$$R_D = 50 \text{ kips } \uparrow$$

$$|V|_{max} = 30 \text{ kips}$$

$$|M|_{max} = 200 \text{ kip-ft} = 2400 \text{ kip-in.}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2400}{24} = 100 \text{ in}^3$$

Shape	S (in ³)
W 24 x 68	154
W 21 x 62	127
W 18 x 76	146
W 16 x 77	134
W 12 x 96	103
W 10 x 112	131

(a) Use

W 21 x 62.

$$d = 20.99 \text{ in.}$$

$$t_f = 0.615 \text{ in.}$$

$$t_w = 0.400 \text{ in.}$$

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{2400}{127} = 18.90 \text{ ksi}$$

$$\tau_m = \frac{|V|_{max}}{d t_w} = \frac{30}{(20.99)(0.400)} = 3.57 \text{ ksi}$$

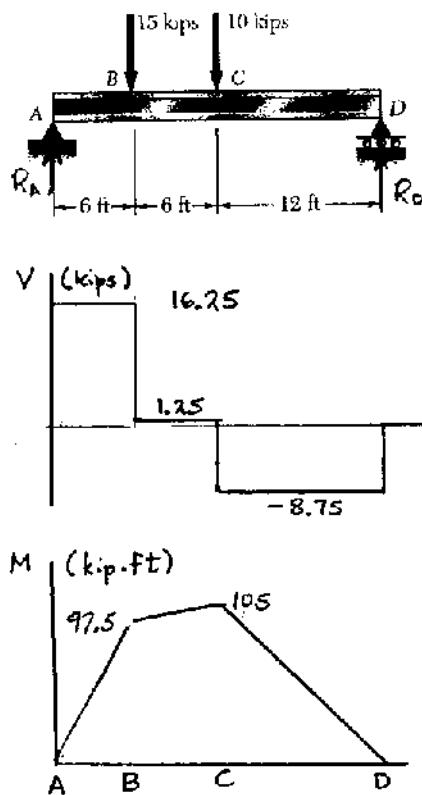
$$\sigma_b = C - t_f = 10.495 - 0.615 = 9.88 \text{ in.}$$

$$\sigma_b = \frac{y_b}{C} \sigma_m = \left(\frac{9.88}{10.495} \right) (18.90) = 17.79 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(8.896)^2 + (3.57)^2} = 9.586 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.896 + 9.586 = 18.48 \text{ ksi}$$

Problem 8.6



8.5 and 8.6 (a) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

$$+\Sigma M_D = 0: -24R_A + (18)(15) + (12)(10) = 0$$

$$R_A = 16.25 \text{ kips } \uparrow$$

$$|V|_{max} = 16.25 \text{ kips}$$

$$|M|_{max} = 105 \text{ kip-ft} = 1260 \text{ kip-in}$$

$$S_{min} = \frac{|M|_{max}}{6_{all}} = \frac{1260}{24} = 52.5 \text{ in}^3$$

Shape	$S_x (\text{in}^3)$
W 21 x 44	81.6
W 18 x 35	57.6
W 16 x 40	64.7
W 14 x 38	54.6
W 12 x 50	64.7
W 10 x 54	60.0

(a) Use
W 18 x 35
 $d = 17.70 \text{ in.}$
 $t_f = 0.425 \text{ in.}$
 $t_w = 0.300 \text{ in.}$

(a) Use W 18 x 35 with $S = 57.6 \text{ in}^3$

$$c = \frac{1}{2}d = 8.85 \text{ in.} \quad y_b = c - t_f = 8.425 \text{ in.}$$

$$\text{At C: } M = (105)(12) = 1260 \text{ kip-in.} \quad \sigma_m = \frac{M}{S} = \frac{1260}{57.6} = 21.875 \text{ ksi}$$

$$|V| = 8.75 \text{ kips} \quad \tau_m = \frac{V}{dt_w} = \frac{8.75}{(17.70)(0.300)} = 1.65 \text{ ksi}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{8.425}{8.85}\right)(21.875) = 20.82 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{19.33}{2}\right)^2 + 1.65^2} = 10.54 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 20.95 \text{ ksi}$$

$$\text{At B: } M = (97.5)(12) = 1170 \text{ kip-in.} \quad \sigma_m = \frac{1170}{57.6} = 20.31 \text{ ksi}$$

$$|V| = 16.25 \text{ kips} \quad \tau_m = \frac{V}{dt_w} = \frac{16.25}{(17.70)(0.300)} = 3.06 \text{ ksi}$$

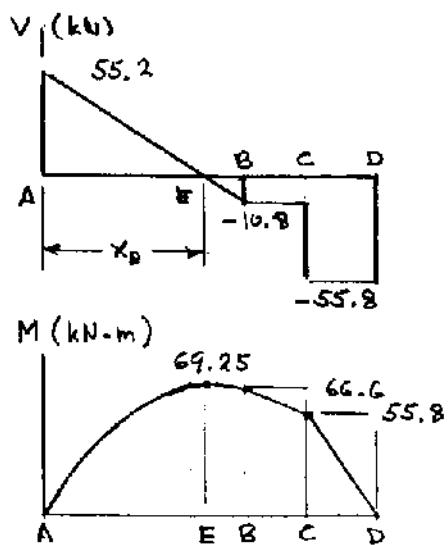
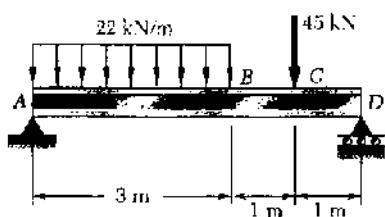
$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{8.425}{8.85}\right)(20.31) = 19.34 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{19.34}{2}\right)^2 + 3.06^2} = 10.14 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 19.81 \text{ ksi}$$

Problem 8.7

8.7 and 8.8 (a) Knowing that $\sigma_{\text{all}} = 160 \text{ MPa}$ and $\tau_{\text{all}} = 100 \text{ MPa}$, select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$$\rightarrow \sum M_D = 0 \quad -5R_A + (3.5)(22) + (1)(45) = 0$$

$$R_A = 55.2 \text{ kN} \uparrow \quad R_D = 55.8 \text{ kN} \uparrow$$

Draw shear and bending moment diagrams.

Locate point where $V = 0$ and M_E .

$$55.2 - 22x_E = 0 \quad x_E = 2.5091 \text{ m.}$$

$$M_E = \frac{1}{2}(55.2)(2.5091) = 69.25 \text{ kNm}$$

$$S_{\min} = \frac{IM_{\max}}{O_{\text{all}}} = \frac{69.25 \times 10^3}{160 \times 10^6} = 432.8 \times 10^{-6} \text{ m}^3 \\ = 432.8 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
W 410 x 38.8	637
→ W 360 x 32.9	474
W 310 x 38.7	549
W 250 x 44.8	535
W 200 x 46.1	448

(a) Use
W 360 x 32.9

$$\text{For W } 360 \times 32.9 \quad S = 474 \times 10^3 \text{ mm}^3 = 474 \times 10^{-6} \text{ m}^3$$

$$A_{\text{web}} = d t_w = (349)(5.8) = 2024.2 \text{ mm}^2 = 2024.2 \times 10^{-6} \text{ m}^2$$

$$\text{At point E: } \sigma_m = \frac{M_E}{S} = \frac{69.25 \times 10^3}{474 \times 10^{-6}} = 146.1 \text{ MPa}$$

$$\text{At point C: } \sigma_m = \frac{M_c}{S} = \frac{55.8 \times 10^3}{474 \times 10^{-6}} = 117.7 \text{ MPa}$$

$$\tau_m = \frac{|V|}{A_{\text{web}}} = \frac{55.8 \times 10^3}{2024.2 \times 10^{-6}} = 27.6 \text{ MPa}$$

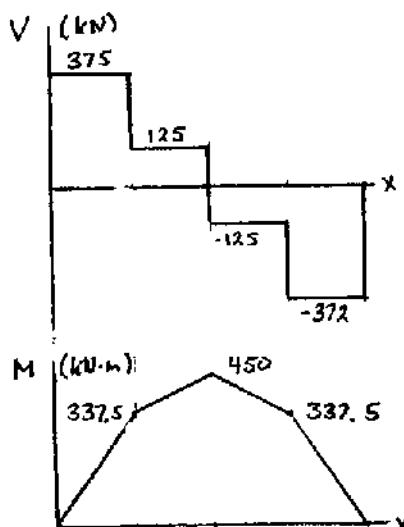
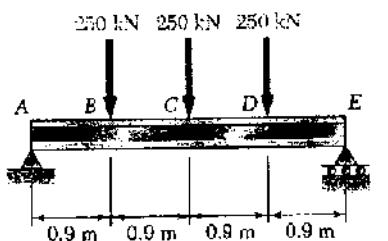
$$c = \frac{1}{2}d = \frac{1}{2}(349) = 174.5 \text{ mm} \quad y_b = c - t_f = 166 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{166}{174.5}\right)(117.7) = 112.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = 62.4 \text{ MPa}$$

$$\sigma_{\text{max}} = \frac{\sigma_b}{2} + R = 118.4 \text{ MPa}$$

Problem 8.8



8.7 and 8.8 (a) Knowing that $\sigma_{all} = 160 \text{ MPa}$ and $\tau_{all} = 100 \text{ MPa}$, select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

$$\text{Reactions: } R_A = 375 \text{ kN } \downarrow, \quad R_E = 375 \text{ kN } \uparrow$$

$$|V|_{max} = 375 \text{ kN}$$

$$|M|_{max} = 450 \text{ kN-m}$$

$$|V| \text{ at point C: } 125 \text{ kN}$$

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{450 \times 10^3}{160 \times 10^6} = 2.8125 \times 10^{-3} \text{ m}^3 \\ = 2812.5 \times 10^3 \text{ mm}^3$$

Shape	$S_x (10^3 \text{ mm}^3)$
W 840 x 176	5890
W 760 x 147	4410
W 690 x 125	3510
W 610 x 155	4220
W 530 x 150	3720
W 460 x 158	3340
W 360 x 216	3800

(a) Use
W 690 x 125.
 $d = 678 \text{ mm}$
 $t_f = 16.30 \text{ mm}$
 $t_w = 11.7 \text{ mm}$

$$(b) \sigma_m = \frac{|M|_{max}}{S_x} = \frac{450 \times 10^3}{3510 \times 10^{-3}} = 128.2 \times 10^6 \text{ Pa} = 128.2 \text{ MPa}$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V| L_w}{d t_w} = \frac{375 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 47.3 \times 10^6 \text{ Pa} = 47.3 \text{ MPa}$$

$$\text{At point C, } \tau_w = \frac{|V|}{A_w} = \frac{125 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 15.76 \times 10^6 \text{ Pa} \\ = 15.76 \text{ MPa}$$

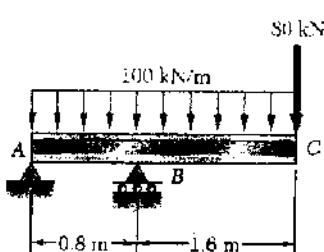
$$C = \frac{1}{2} d = \frac{678}{2} = 339 \text{ mm} \quad y_b = C - t_f = 339 - 16.30 = 322.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{C} \sigma_m = \left(\frac{322.7}{339} \right) (128.2) = 122.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(61.0)^2 + (15.76)^2} = 63.0 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 61.0 + 63.0 = 124.0 \text{ MPa}$$

Problem 8.9



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.9 Loading of Prob. 5.76 and selected S510 × 98.3 shape.

From Problem 5.76 $\sigma_{all} = 160 \text{ MPa}$

$$|M|_{max} = 256 \text{ kN}\cdot\text{m} \text{ at point B}$$

$$|V| = 360 \text{ kN at B}$$

For S 510 × 98.3 rolled steel section

$$d = 508 \text{ mm}, b_f = 159 \text{ mm}, t_f = 20.2 \text{ mm}$$

$$t_w = 12.8 \text{ mm}, I_g = 495 \times 10^6 \text{ mm}^4, S_g = 1950 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 254 \text{ mm}$$

$$(a) \sigma_m = \frac{|M|_{max}}{S_g} = \frac{256 \times 10^3}{1950 \times 10^3} = 131.3 \text{ MPa}$$

$$y_b = c - t_f = 233.8$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 120.9 \text{ MPa} \quad \frac{\sigma_m}{2} = 60.45 \text{ MPa}$$

$$A_f = b_f t_f = 3212 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 243.9 \text{ mm}$$

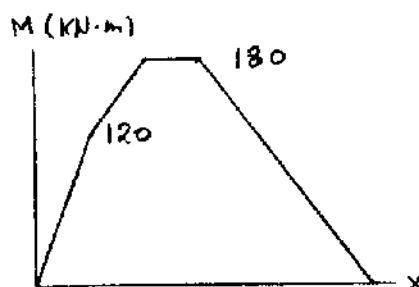
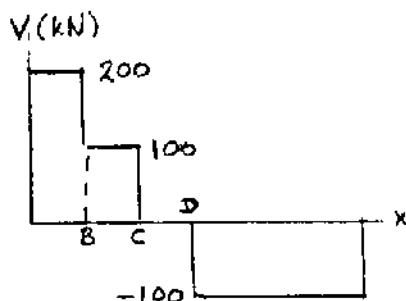
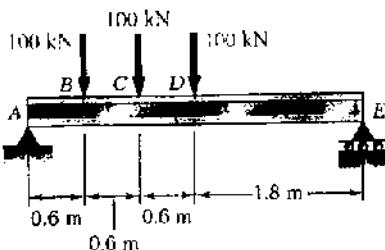
$$Q = A_f \bar{y} = 783.4 \times 10^3 \text{ mm}^3$$

$$\tau_b = \frac{V Q}{I t_w} = \frac{(360 \times 10^3)(783.4 \times 10^3)}{(495 \times 10^6)(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

$$(b) \sigma_{max} = \frac{\sigma_m}{2} + R = 60.45 + 75.06 = 135.5 \text{ MPa}$$

Problem 8.10



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.10 Loading of Prob. 5.73 and selected W460 × 74 shape.

From Problem 5.73 $\sigma_{all} = 160 \text{ MPa}$

$$|M|_{max} = 180 \text{ kN}\cdot\text{m} \text{ at } C \text{ and } D$$

$$|V| = 100 \text{ kN} \text{ at } C \text{ and } D$$

For W 460 × 74 rolled steel section

$$d = 457 \text{ mm}, \quad b_f = 190 \text{ mm}, \quad t_f = 14.50 \text{ mm}$$

$$t_w = 9.0 \text{ mm}, \quad I_z = 333 \times 10^6 \text{ mm}^4, \quad S_z = 1460 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 228.5 \text{ mm}$$

$$(a) \sigma_m = \frac{|M|_{max}}{S} = \frac{180 \times 10^3}{1460 \times 10^3} = 123.3 \text{ MPa}$$

$$y_b = c - t_f = 214 \text{ mm}$$

$$\sigma_b = \frac{Y_b}{c} \sigma_m = 115.5 \text{ MPa}$$

$$A_f = b_f t_f = 2755 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 221.25 \text{ mm}$$

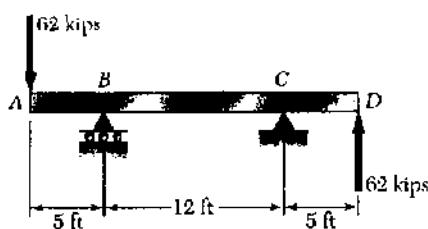
$$Q = A_f \bar{y} = (2755)(221.25) = 609.5 \times 10^3 \text{ mm}^3$$

$$\tau_b = \frac{VQ}{I t_w} = \frac{(100 \times 10^3)(609.5 \times 10^3)}{(333 \times 10^6)(9.0 \times 10^{-3})} = 20.3 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{57.75^2 + 20.3^2} = 61.21 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 57.75 + 61.21 = 119.0 \text{ MPa}$$

Problem 8.11

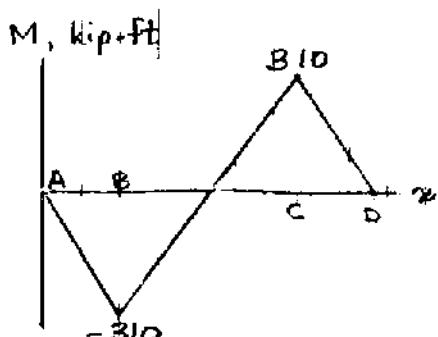
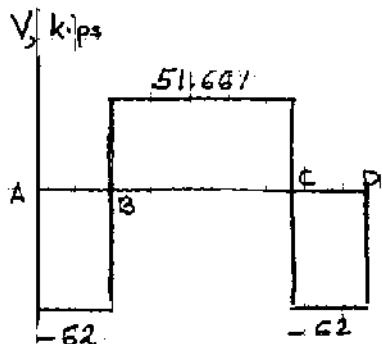


8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m < \sigma_{ul}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.11 Loading of Prob. 5.71 and selected W27 x 84 shape.

From Problem 5.71 $\sigma_{all} = 24 \text{ ksi}$.

$$|M|_{max} = 310 \text{ kip-ft} = 3720 \text{ kip-in at section B.}$$



$$(a) \sigma_m = \frac{|M|_{max}}{S} = \frac{3720}{213} = 17.46 \text{ ksi}$$

$$y_b = c - t_f = 12.715 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 16.628 \text{ ksi}$$

$$A_f = b_f t_f = 6.3744 \text{ in}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 13.035 \text{ in}$$

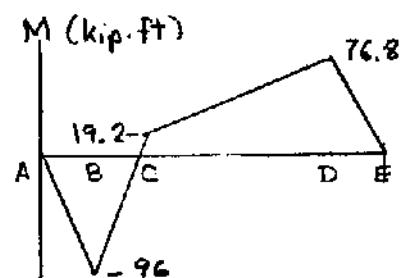
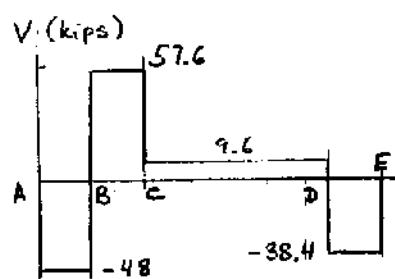
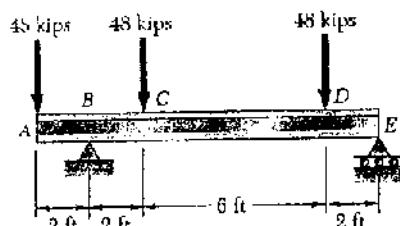
$$Q = A_f \bar{y} = (6.3744)(13.035) = 83.09 \text{ in}^3$$

$$\tau_b = \frac{VQ}{It_w} = \frac{(62)(83.09)}{(2850)(0.460)} = 3.930 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.314^2 + 3.930^2} = 9.196 \text{ ksi}$$

$$(b) \sigma_{max} = \frac{\sigma_b}{2} + R = 17.51 \text{ ksi}$$

Problem 8.12



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.12 Loading of Prob. 5.78 and selected S15 × 42.9 shape.

From Problem 5.78 $\sigma_{all} = 24$ ksi

$$|M|_{max} = 96 \text{ kip}\cdot\text{ft} = 1152 \text{ kip}\cdot\text{in. at section B.}$$

$$|V| = 57.6 \text{ kips at B.}$$

For S 15 × 42.9 rolled steel shape

$$d = 15.00 \text{ in. } b_f = 5.501 \text{ in. } t_f = 0.622 \text{ in.}$$

$$t_w = 0.411 \text{ in. } I_z = 447 \text{ in}^4 \quad S = 59.6 \text{ in}^3$$

$$c = \frac{1}{2}d = 7.50 \text{ in.}$$

$$(a) \sigma_m = \frac{|M|_{max}}{S} = \frac{1152}{59.6} = 19.33 \text{ ksi}$$

$$y_b = c - t_f = 6.878 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 17.726 \text{ ksi}$$

$$A_F = b_f t_f = 3.4216 \text{ in}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 7.189 \text{ in.}$$

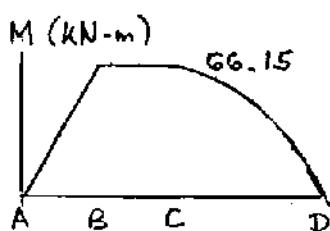
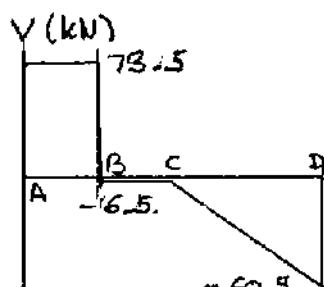
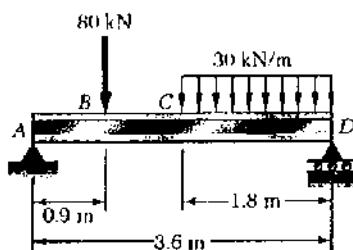
$$Q = A_F \bar{y} = (3.4216)(7.189) = 24.60 \text{ in}^3$$

$$\gamma_b = \frac{VQ}{I t_w} = \frac{(57.6)(24.60)}{(447)(0.411)} = 7.712 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \gamma_b^2} = \sqrt{8.863^2 + 7.712^2} = 11.749 \text{ ksi}$$

$$(b) \sigma_{max} = \frac{\sigma_b}{2} + R = 20.6 \text{ ksi}$$

Problem 8.13



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.13 Loading of Prob. 5.75 and selected S310 × 47.3 shape.

From Problem 5.75 $\sigma_{all} = 160 \text{ MPa}$

$$|M|_{max} = 66.15 \text{ kN-m} \text{ at section B.}$$

$$|V| = 73.5 \text{ kN at B}$$

For S310 × 47.3 rolled steel section

$$d = 305 \text{ mm} \quad b_p = 127 \text{ mm} \quad t_p = 13.8 \text{ mm}$$

$$t_w = 8.9 \text{ mm} \quad I = 90.6 \times 10^6 \text{ mm}^4 \quad S = 593 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 152.5 \text{ mm}$$

$$(a) \sigma_m = \frac{|M|_{max}}{S} = \frac{66.15 \times 10^3}{593 \times 10^3} = 111.6 \text{ MPa}$$

$$y_b = c - t_p = 138.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 101.5 \text{ MPa}$$

$$A_f = b_p t_p = 1752.6 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 145.6 \text{ mm}$$

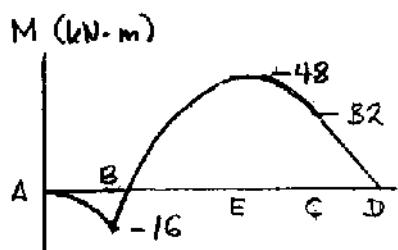
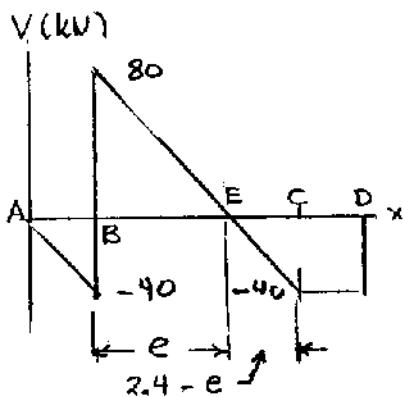
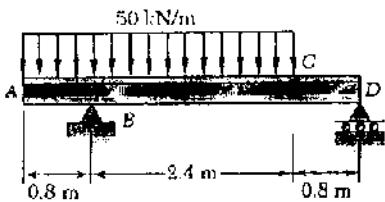
$$Q = A_f \bar{y} = 255.18 \times 10^3 \text{ mm}^3 = 255.18 \times 10^{-6} \text{ m}^3$$

$$\tau_b = \frac{VQ}{I I_w} = \frac{(73.5 \times 10^3)(255.18 \times 10^{-6})}{(90.6 \times 10^{-6})(8.9 \times 10^{-3})} = 23.26 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{50.75^2 + 23.26^2} = 55.83 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 106.6 \text{ MPa}$$

Problem 8.14



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.14 Loading of Prob. 5.74 and selected W250 × 28.4 shape.

From Problem 5.74 $\sigma_{all} = 160 \text{ MPa}$

$|M|_{max} = 48 \text{ kN}\cdot\text{m}$ at section E, which lies 1.6 m to the right of B.

$$|V| = 0$$

For W 250 × 28.4 rolled steel section

$$d = 260 \text{ mm} \quad b_f = 102 \text{ mm} \quad t_f = 10.0 \text{ mm}$$

$$t_w = 6.4 \text{ mm} \quad I = 40.0 \times 10^6 \text{ mm}^4 \quad S = 308 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 130 \text{ mm}$$

$$(a) \sigma_m = \frac{|M|_{max}}{S} = \frac{48 \times 10^3}{308 \times 10^3} = 155.8 \text{ MPa}$$

$$y_b = c - t_f = 120 \text{ mm}$$

$$A_f = b_f t_f = 1020 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 125 \text{ mm}$$

$$Q = A_f \bar{y} = 127.5 \times 10^3 \text{ mm}^3 = 127.5 \times 10^{-6} \text{ m}^3$$

$$\text{At section E} \quad V = 0 \quad M = M_{max}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 143.8 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = 0 \quad (b) \quad \sigma_{max} = \sigma_b = 143.8 \text{ MPa}$$

$$\text{At section C.} \quad Nl = 40 \text{ kN} \quad M = 32 \text{ kN}\cdot\text{m}$$

$$\sigma_b = \frac{My_b}{I} = \frac{(32 \times 10^3)(120 \times 10^3)}{40.0 \times 10^6} = 96 \text{ MPa}$$

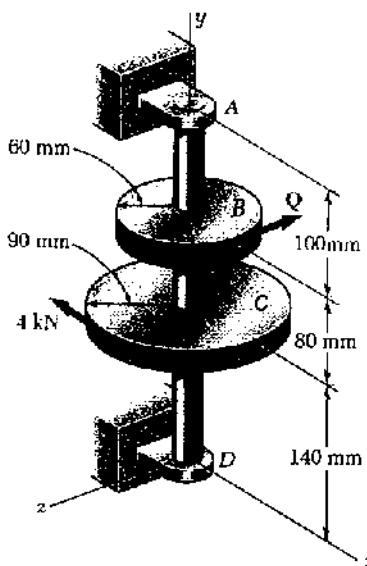
$$\tau_b = \frac{VQ}{It} = \frac{(40 \times 10^3)(127.5 \times 10^{-6})}{(40.0 \times 10^6)(6.4 \times 10^{-3})} = 19.92 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{48^2 + 19.92^2} = 51.97 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 100.0 \text{ MPa}, \text{ which is less than } \sigma_{all} \text{ at E.}$$

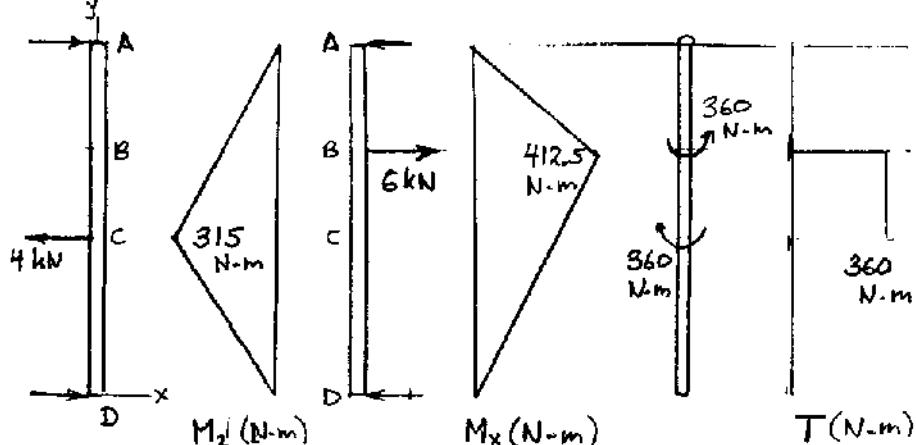
Problem 8.15

8.15 The 4-kN force is parallel to the x axis, and the force Q is parallel to the z axis. The shaft AD is hollow. Knowing that the inner diameter is half the outer diameter and that $\tau_{all} = 60 \text{ MPa}$, determine the smallest permissible outer diameter of the shaft.



$$\sum M_y = 0 : 60 \times 10^3 Q - (90 \times 10^3)(4 \times 10^3) = 0 \\ Q = 6 \times 10^3 \text{ N} = 6 \text{ kN}$$

Bending moment and torque diagrams.



In xy plane $(M_z)_{max} = 315 \text{ N}\cdot\text{m}$ at C .

In yz plane $(M_x)_{max} = 412.5 \text{ N}\cdot\text{m}$ at B .

About z -axis $T_{max} = 360 \text{ N}\cdot\text{m}$ between B and C .

$$\text{At } B \quad M_z = \left(\frac{100}{180}\right)(315) = 175 \text{ N}\cdot\text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{175^2 + 412.5^2 + 360^2} = 574.79 \text{ N}\cdot\text{m}$$

$$\text{At } C \quad M_x = \left(\frac{140}{220}\right)(412.5) = 262.5 \text{ N}\cdot\text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{315^2 + 262.5^2 + 360^2} = 545.65 \text{ N}\cdot\text{m}$$

Largest value is $574.79 \text{ N}\cdot\text{m}$

$$\frac{J}{C} = \frac{\max \sqrt{M_x^2 + M_z^2 + T^2}}{J} C$$

$$\frac{J}{C} = \frac{\max \sqrt{M_x^2 + M_z^2 + T^2}}{J_{max}} = \frac{574.79}{60 \times 10^6} = 9.5798 \times 10^{-6} \text{ m}^3 = 9.5798 \times 10^3 \text{ mm}^3$$

$$\frac{J}{C} = \frac{\frac{\pi}{2}(C_o^4 - C_i^4)}{C_o} = \frac{\pi}{2} C_o^3 \left(1 - \frac{C_i^4}{C_o^4}\right) = \frac{\pi}{2} C_o^3 \left[1 - \left(\frac{1}{2}\right)^4\right] = 1.47262 C_o^3$$

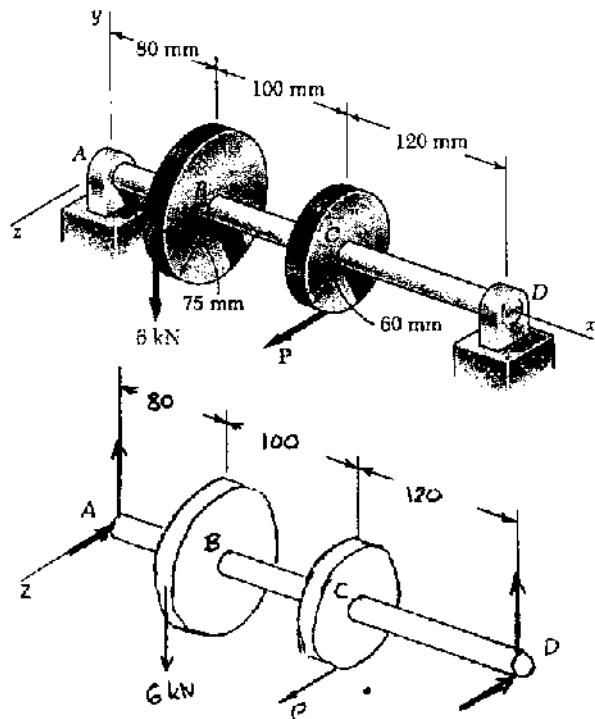
$$1.47262 C_o^3 = 9.5798 \times 10^3$$

$$C_o = 18.67 \text{ mm}$$

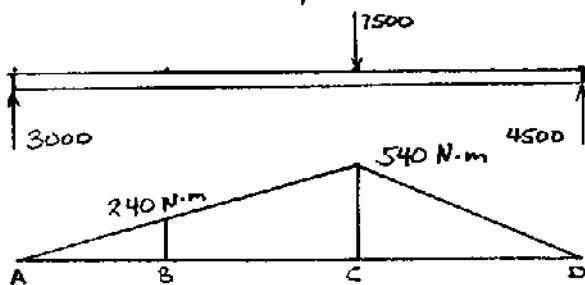
$$d_o = 2C_o = 37.3 \text{ mm}$$

Problem 8.16

8.16 The 6-kN force is vertical and the force P is parallel to the z axis. Knowing that $\tau_{all} = 60 \text{ MPa}$, determine the smallest permissible diameter of the solid shaft AD .



Forces in horizontal plane:



$$\sum M_x = 0$$

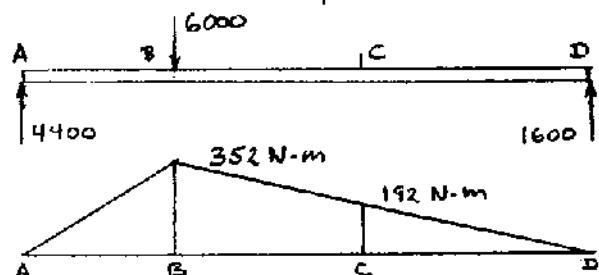
$$(6 \times 10^3)(75 \times 10^{-3}) - (60 \times 10^3)P = 0$$

$$P = 7.5 \times 10^3 \text{ N}$$

Over portion BC

$$T = (6 \times 10^3)(75 \times 10^{-3}) = 450 \text{ N}\cdot\text{m}$$

Forces in vertical plane:



Bending moments:

$$\text{At } B: M = \sqrt{352^2 + 240^2} \\ = 426.0 \text{ N}\cdot\text{m}$$

$$\text{At } C: M = \sqrt{540^2 + 192^2} \\ = 573.1 \text{ N}\cdot\text{m}$$

Critical section is just to the left of gear C.

$$M = 573.1 \text{ N}\cdot\text{m} \quad T = 450 \text{ N}\cdot\text{m} \quad \sqrt{M^2 + T^2} = 728.67 \text{ N}\cdot\text{m}$$

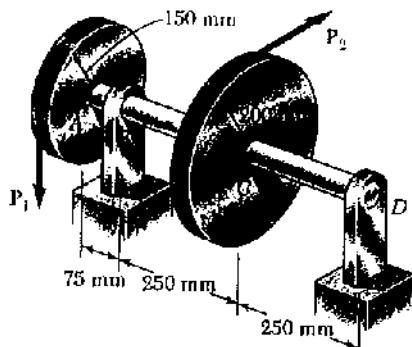
$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\frac{\pi}{2} C^3}{\frac{\pi}{2} C^4} = \frac{(\sqrt{M^2 + T^2})_{max}}{\tau_{all}} = \frac{728.67}{60 \times 10^6} = 12.145 \times 10^{-6} \text{ m}^3$$

$$C = 19.77 \times 10^{-3} \text{ m} \quad d = 2C = 39.5 \times 10^{-3} \text{ m} = 39.5 \text{ mm}$$

Problem 8.17

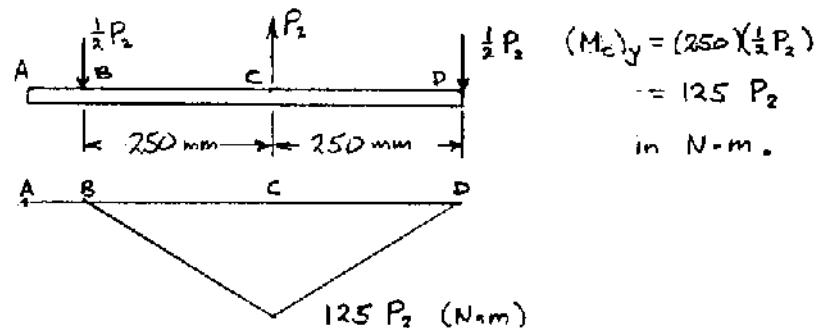
&J7 The vertical force P_1 and the horizontal force P_2 are applied as shown to disks welded to the solid shaft AD. Knowing that the diameter of the shaft is 40 mm and that $\tau_{all} = 55 \text{ MPa}$, determine the largest permissible magnitude of the force P_2 .



Let the dimensions of P_2 be kN.

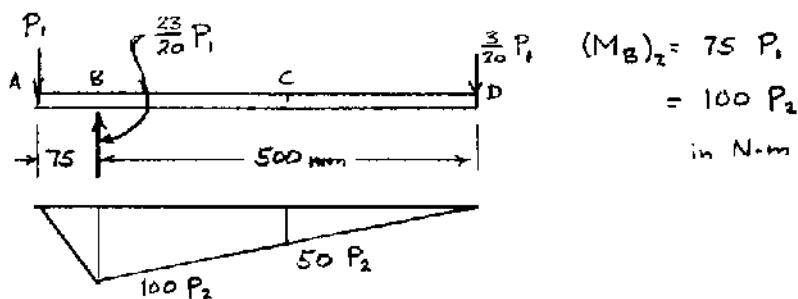
$$\sum M_{shaft} = 0: 150 P_1 - 200 P_2 = 0 \quad P_1 = \frac{4}{3} P_2$$

Torque over portion ABC: $T = 200 P_2$ in N·m.



Bending in horizontal plane.

Bending in vertical plane.



$$\text{At } B: \sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{(100 P_2)^2 + 0 + (200 P_2)^2} = 223.61 P_2$$

$$\text{At } C: \sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{(125 P_2)^2 + (50 P_2)^2 + (200 P_2)^2} = 241.09 P_2$$

$$d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} \quad c = \frac{1}{2} d = 20 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (20 \times 10^{-3})^4 = 251.33 \times 10^{-9} \text{ m}^4$$

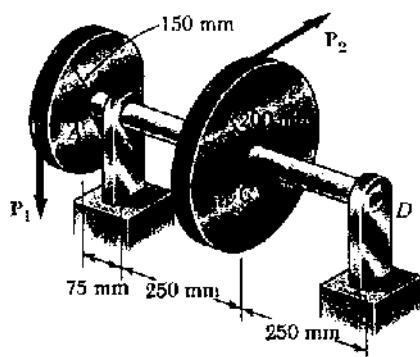
$$\chi_{max} = \frac{\max \sqrt{M_y^2 + M_z^2 + T^2}}{J} c$$

$$55 \times 10^6 = \frac{(241.09 P_2)(20 \times 10^{-3})}{251.33 \times 10^{-9}} \quad P_2 = 2.87 \text{ kN}$$

Problem 8.18

8.18 Solve Prob. 8.17, assuming that the solid shaft AD has been replaced by a hollow shaft of the same material and of inner diameter 35 mm and outer diameter 45 mm.

8.17 The vertical force P_1 and the horizontal force P_2 are applied as shown to disks welded to the solid shaft AD. Knowing that the diameter of the shaft is 40 mm and that $\tau_{allow} = 55 \text{ MPa}$, determine the largest permissible magnitude of the force P_2 .

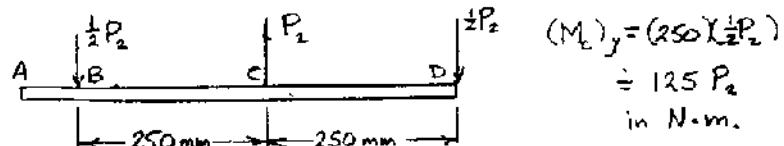


Let the dimensions of P_2 be kN.

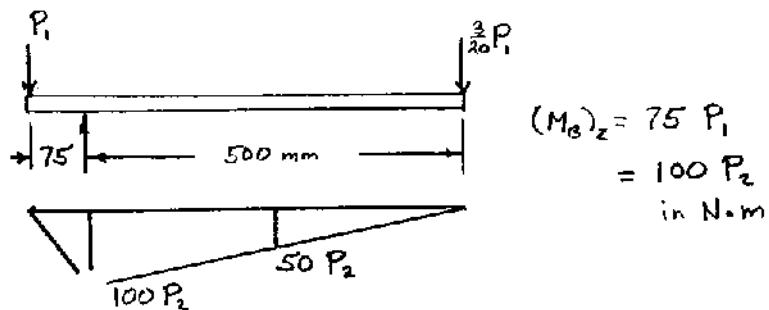
$$\sum M_{\text{shaft}} = 0: 150P_1 - 200P_2 = 0 \quad P_1 = \frac{4}{5}P_2$$

Torque of portion ABC: $T = 200P_2$ in N·m.

Bending in horizontal plane.



Bending in vertical plane.



$$\text{At } B: \sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{D + (100P_2)^2 + (200P_2)^2} = 223.61 P_2$$

$$\text{At } C: \sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{(125P_2)^2 + (50P_2)^2 + (200P_2)^2} = 241.09 P_2$$

$$C_o = \frac{1}{2}d_o = 22.5 \text{ mm} = 22.5 \times 10^{-3} \text{ m} \quad C_i = \frac{1}{2}d_i = 17.5 \text{ mm} = 17.5 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2}(C_o^4 - C_i^4) = \frac{\pi}{2}(22.5^4 - 17.5^4) = 255.25 \times 10^3 \text{ mm}^4 = 255.25 \times 10^{-9} \text{ m}^4$$

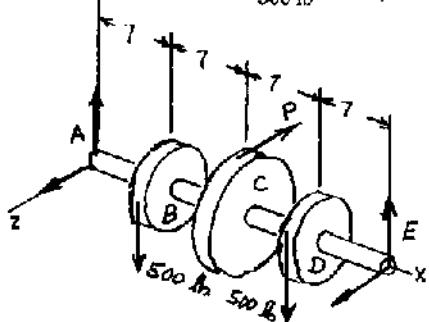
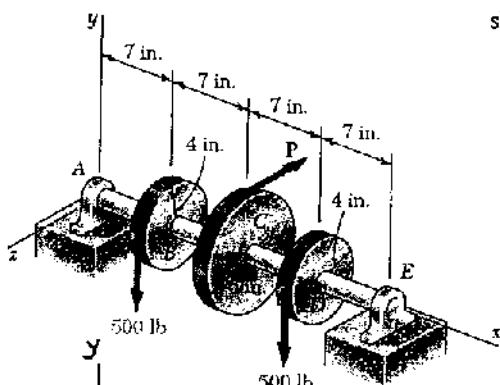
$$\tau_{\text{max}} = \frac{\max \sqrt{M_y^2 + M_z^2 + T^2}}{J} C_o$$

$$55 \times 10^6 = \frac{(241.09 P_2)(22.5 \times 10^{-3})}{255.25 \times 10^{-9}}$$

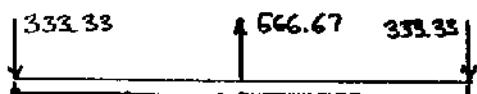
$$P_2 = 2.59 \text{ kN}$$

Problem 8.19

8.19 The two 500-lb forces are vertical and the force P is parallel to the z axis. Knowing that $\tau_{all} = 8$ ksi, determine the smallest permissible diameter of the solid shaft AE .



Forces in horizontal plane:



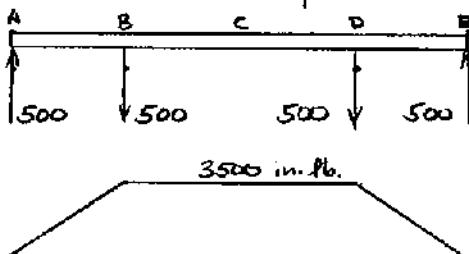
$$\sum M_x = 0 \quad (4)(500) - 6P + (4)(500) = 0$$

$$P = 666.67 \text{ lb}$$

Torques:

AB:	$T = 0$
BC:	$T = -(4)(500) = -2000 \text{ in-lb}$
CD:	$T = (4)(500) = 2000 \text{ in-lb}$
DE:	$T = 0$

Forces in vertical plane:



Critical sections are either side of disk C.

$$T = 2000 \text{ in-lb} \quad M_z = 3500 \text{ in-lb}$$

$$M_y = 4667 \text{ in-lb}$$

$$\tau_{all} = \frac{C}{J} \sqrt{M_y^2 + M_z^2 + T^2}$$

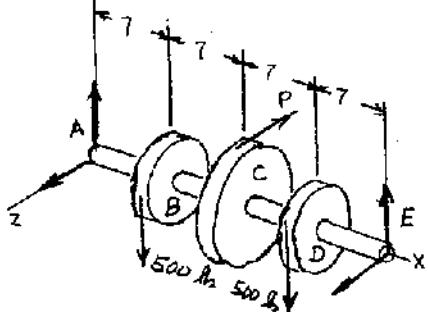
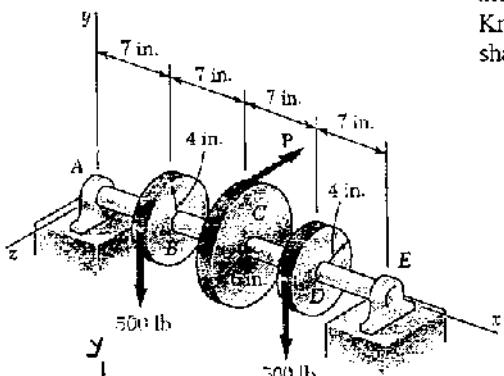
$$\frac{C}{J} = \frac{\pi}{2} C^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{Z_{all}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3} = 0.77083 \text{ in}^3$$

$$C = 0.7888 \text{ in.} \quad d = 2C = 1.578 \text{ in.}$$

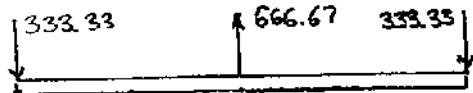
Problem 8.20

8.20 For the gear-and-shaft system and loading of Prob. 8.19, determine the smallest permissible diameter of shaft AE, knowing that the shaft is hollow and has an inner diameter that is $\frac{2}{3}$ the outer diameter.

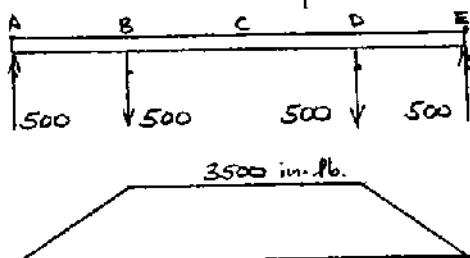
8.19 The two 500-lb forces are vertical and the force P is parallel to the z axis. Knowing that $\tau_{all} = 8 \text{ ksi}$, determine the smallest permissible diameter of the solid shaft AE.



Forces in horizontal plane:



Forces in vertical planes:



Critical sections are either side of disk C.

$$T = 2000 \text{ in-lb} \quad M_z = 3500 \text{ in-lb}$$

$$M_y = 4667 \text{ in-lb}$$

$$\tau_{all} = \frac{C}{T} \sqrt{M_y^2 + M_z^2 + T^2}$$

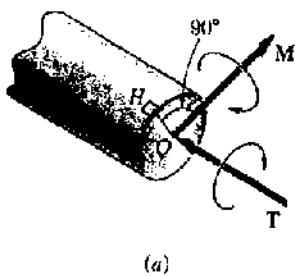
$$\frac{J}{C} = \frac{\pi}{2C} [C_o^4 - C_i^4] = \frac{\pi}{2C} [C^4 - (\frac{2}{3}C)^4] = \frac{\pi}{2C} \frac{CSC^4}{81} = \frac{65\pi}{162} C^3$$

$$\frac{65\pi}{162} C^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{T_{all}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3} = 0.77083 \text{ in}^3$$

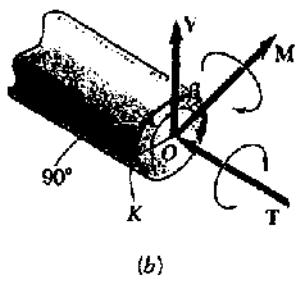
$$C = 0.8488 \text{ in.}$$

$$d = 2C = 1.698 \text{ in.}$$

Problem 8.21



(a)



(b)

8.21 It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point *H* (Fig. P8.21a) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J} \sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point *K* (Fig. P8.21b), where the effect of the shear *V* is greatest, can be expressed as

$$\tau_K = \frac{c}{J} \sqrt{(M \cos \beta)^2 + (\frac{2}{3} c V + T)^2}$$

where β is the angle between the vectors *V* and *M*. It is clear that the effect of the shear *V* cannot be ignored when $\tau_K > \tau_H$. (Hint: Only the component of *M* along *V* contributes to the shearing stress at *K*.)

Shearing stress at point *K*.

Due to *V*:

$$\text{For a semicircle } Q = \frac{2}{3} c^3$$

$$\text{For a circle cut across its diameter } t = d = 2c$$

$$\text{For a circular section } I = \frac{1}{2} J$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)(\frac{2}{3} c^3)}{(\frac{1}{2} J)(2c)} = \frac{2}{3} \frac{Vc^2}{J}$$

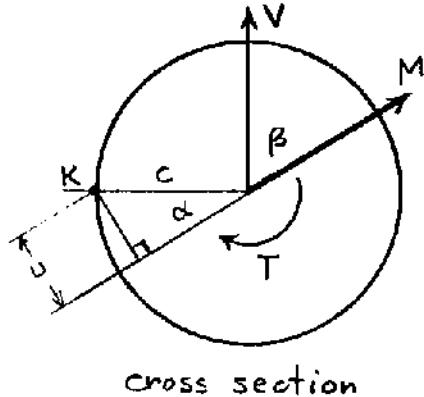
$$\text{Due to } T: \quad \tau_{xy} = \frac{Ic}{J}$$

Since these shearing stresses have the same orientation,

$$\tau_{xy} = \frac{c}{J} \left(\frac{2}{3} Vc + T \right)$$

$$\text{Bending stress at point } K: \quad \sigma_x = \frac{Mu}{I} = \frac{2Mu}{J}$$

where *u* is distance between point *K* and the neutral axis,



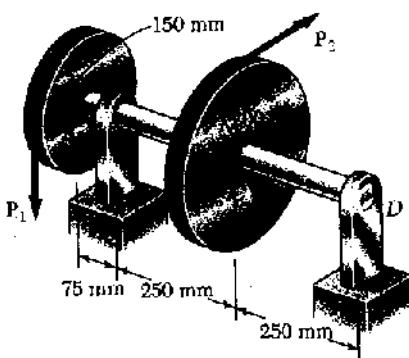
$$u = c \sin \alpha = c \sin(\frac{\pi}{2} - \beta) = c \cos \beta$$

$$\sigma_x = \frac{2Mc \cos \beta}{J}$$

Using Mohr's circle.

$$\begin{aligned} \tau_x &= R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{c}{J} \sqrt{(M \cos \beta)^2 + \left(\frac{2}{3} Vc + T\right)^2} \end{aligned}$$

Problem 8.22



8.22 Assuming that the magnitudes of the forces applied to disks A and C of Prob. 8.17 are, respectively $P_1 = 4.8 \text{ kN}$ and $P_2 = 3.6 \text{ kN}$, and using the expressions given in Prob. 8.21, determine the values of τ_K and τ_H in a section (a) just to the left of B, (b) just to the left of C.

8.17 The vertical force P_1 and the horizontal force P_2 are applied as shown to disks welded to the solid shaft AD. Knowing that the diameter of the shaft is 40 mm and that $\tau_{all} = 55 \text{ MPa}$, determine the largest permissible magnitude of the force P_2 .

$$\text{From Problem 8.17 } d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

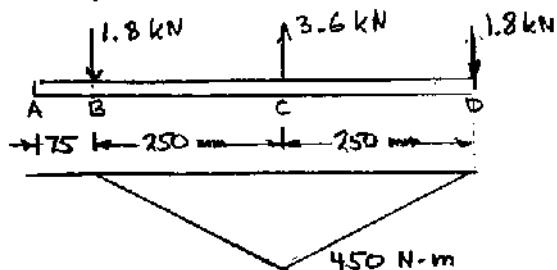
$$C = \frac{1}{2}d = 20 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (20 \times 10^{-3})^4 = 251.33 \times 10^{-9} \text{ m}^4$$

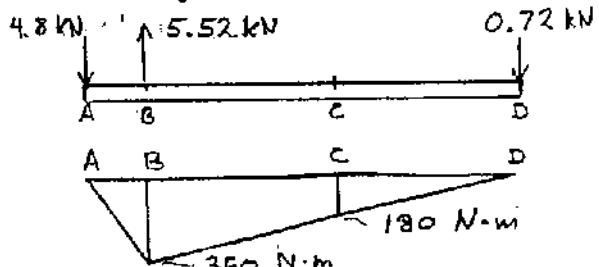
Torque over portion ABC of shaft.

$$T = (150 \times 10^{-3})(4.8 \times 10^3) = 720 \text{ N}\cdot\text{m}$$

Bending in horizontal plane.



Bending in vertical plane.



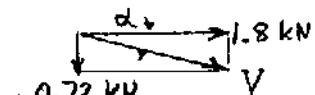
(a) Just to the left of section B. $V = 4.8 \times 10^3 \text{ N}$, $M = 360 \text{ N}\cdot\text{m}$

$$\beta = 90^\circ \quad T = 720 \text{ N}\cdot\text{m}$$

$$\tau_H = \frac{C}{J} \sqrt{M^2 + T^2} = \frac{20 \times 10^{-3}}{251.33 \times 10^{-9}} \sqrt{360^2 + 720^2} \\ = 64.1 \times 10^6 \text{ Pa} \quad 64.1 \text{ MPa}$$

$$\tau_K = \frac{C}{J} \sqrt{(M \cos \beta)^2 + (\frac{1}{3} V c + T)^2} = \frac{C}{J} |Vc + T| \\ = \frac{20 \times 10^{-3}}{251.33 \times 10^{-9}} \left[\frac{2}{3} (4.8 \times 10^3) (20 \times 10^{-3}) + 720 \right] \\ = 62.4 \times 10^6 \text{ Pa} \quad 62.4 \text{ MPa}$$

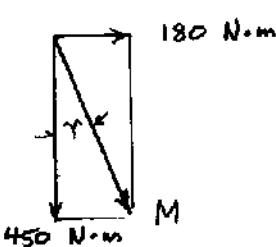
(b) Just to the left of section C.



Shear

$$V = \sqrt{(1.8)^2 + (0.72)^2} = 1.9387 \text{ kN}$$

$$\alpha = \tan^{-1} \left(\frac{0.72}{1.8} \right) = 21.8^\circ$$



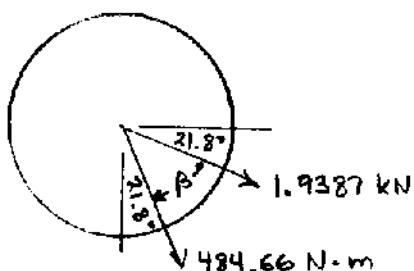
Bending moment

$$M = \sqrt{450^2 + 180^2} = 484.66 \text{ N}\cdot\text{m}$$

$$\gamma = \tan^{-1} \left(\frac{180}{450} \right) = 21.8^\circ$$

continued.

Problem 8.22 continued



$$\beta = 90^\circ - d - \gamma = 90^\circ - 21.8^\circ - 21.8^\circ = 46.4^\circ$$

$$T_H = \frac{C}{J} \sqrt{M^2 + T^2} = \frac{20 \times 10^3}{251.33 \times 10^9} \sqrt{484.66^2 + 720^2}$$

$$= 69.1 \times 10^6 \text{ Pa} \quad 69.1 \text{ MPa} \blacktriangleleft$$

$$M \cos \beta = 334.23 \text{ N} \cdot \text{m}$$

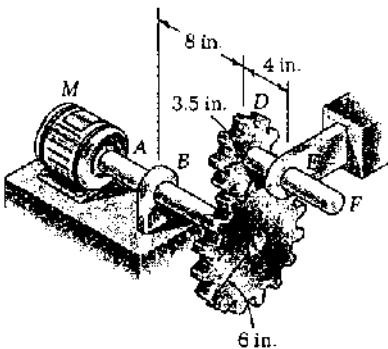
$$\frac{2}{3} V_C + T = \frac{2}{3} (1.9387 \times 10^3) (20 \times 10^3) + 720$$

$$= 745.85 \text{ N} \cdot \text{m}$$

$$T_K = \frac{C}{J} \sqrt{(M \cos \beta)^2 + (\frac{2}{3} V_C + T)^2} = \frac{20 \times 10^3}{251.33 \times 10^9} \sqrt{(334.23)^2 + (745.85)^2}$$

$$= 65.0 \times 10^6 \text{ Pa} \quad 65.0 \text{ MPa} \blacktriangleleft$$

Problem 8.23



8.23 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\sigma_{all} = 7.5$ ksi, determine the smallest permissible diameter of (a) shaft *ABC*, (b) shaft *DEF*.

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in-lb/s}$$

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

$$(a) \text{ Shaft } ABC: T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(4)} = 5252 \text{ lb-in}$$

$$\text{Gear } C \quad F_{cd} = \frac{T}{r_c} = \frac{5252}{6} = 875.4 \text{ lb.}$$

$$\text{Bending moment at } B: M_B = (8)(875.4) = 7003 \text{ lb-in}$$

$$\chi_{all} = \frac{c}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\chi_{all}} = \frac{\sqrt{(5252)^2 + (7003)^2}}{7500} = 1.1671 \text{ in}^3$$

$$c = 0.9057 \text{ in} \quad d = 2c = 1.811 \text{ in}$$

$$(b) \text{ Shaft } DEF: T = r_p F_{cd} = (3.5)(875.4) = 3064 \text{ lb-in}$$

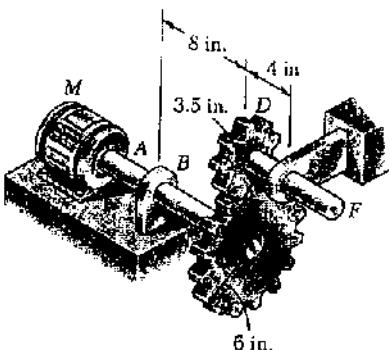
$$\text{Bending moment at } E: M_E = (4)(875.4) = 3502 \text{ lb-in}$$

$$\chi_{all} = \frac{c}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\chi_{all}} = \frac{\sqrt{(3502)^2 + (3064)^2}}{7500} = 0.6204 \text{ in}^3$$

$$c = 0.7337 \text{ in.} \quad d = 2c = 1.467 \text{ in.}$$

Problem 8.24



8.24 Solve Prob. 8.23, assuming that the motor rotates at 360 rpm.

8.23 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (a) shaft *ABC*, (b) shaft *DEF*.

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in-lb/s}$$

$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$(a) \text{Shaft } ABC: T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(6)} = 3501 \text{ lb-in}$$

$$\text{Gear } C \quad F_{co} = \frac{T}{r_c} = \frac{3501}{6} = 583.6 \text{ lb.}$$

$$\text{Bending moment at } B: M_B = (8)(583.6) = 4669 \text{ lb-in}$$

$$\tau_{ab} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\frac{\pi}{2} C^3}{\frac{C}{J}} = \frac{\sqrt{M^2 + T^2}}{7500} = \frac{\sqrt{4669^2 + 3501^2}}{7500} = 0.77806 \text{ in}^3$$

$$C = 0.791 \text{ in.} \quad d = 2C = 1.582 \text{ in.}$$

$$(b) \text{Shaft } DEF: T = r_o F_{co} = (3.5)(583.6) = 2043 \text{ lb-in}$$

$$\text{Bending moment at } E: M_E = (4)(583.6) = 2334 \text{ lb-in}$$

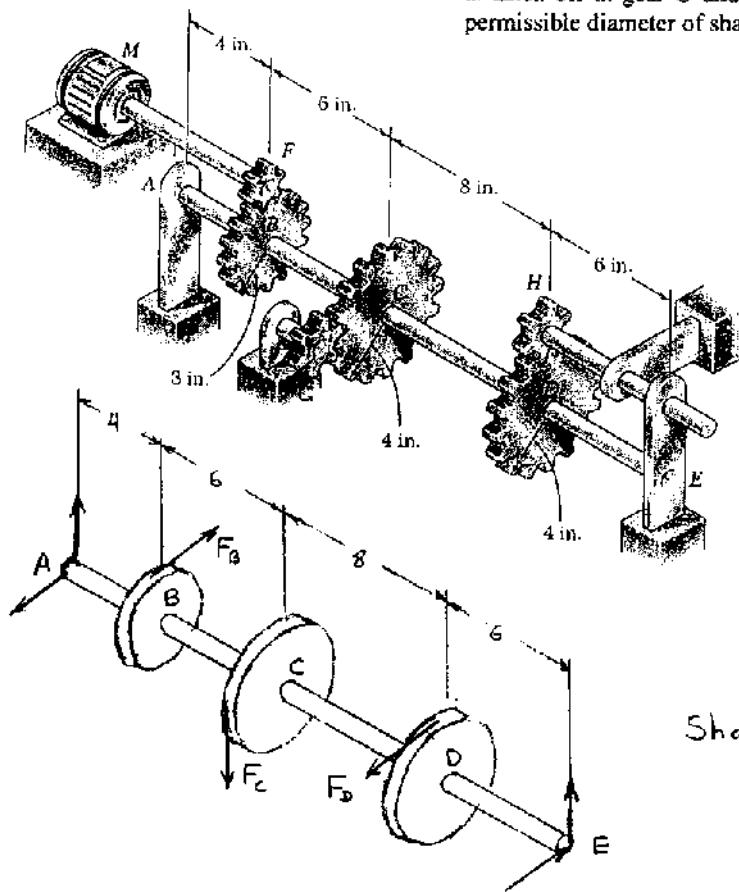
$$\tau_{de} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\frac{\pi}{2} C^3}{\frac{C}{J}} = \frac{\sqrt{M^2 + T^2}}{7500} = \frac{\sqrt{2334^2 + 2043^2}}{7500} = 0.41362 \text{ in}^3$$

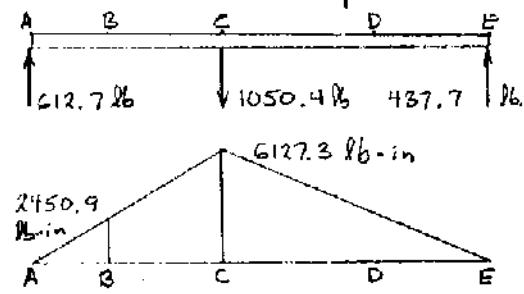
$$C = 0.6410 \text{ in.} \quad d = 2C = 1.282 \text{ in.}$$

Problem 8.25

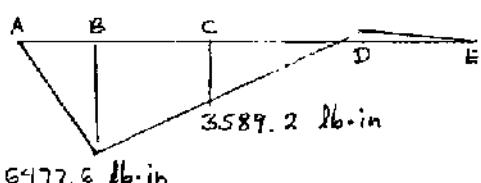
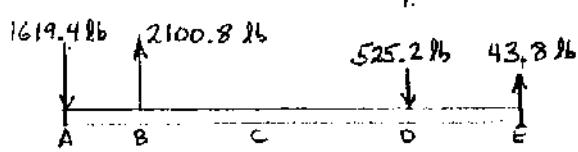
8.25 The solid shaft AE rotates at 600 rpm and transmits 60 hp from the motor M to machine tools connected to gears G and H . Knowing that $\tau_{\text{all}} = 8 \text{ ksi}$ and that 40 hp is taken off at gear G and 20 hp is taken off at gear H , determine the smallest permissible diameter of shaft AE .



Forces in vertical plane



Forces in horizontal plane



$$60 \text{ hp} = (60)(6600) \\ = 396 \times 10^3 \text{ in-lb/sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear B.

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} \\ = 6302.5 \text{ lb-in.}$$

Torques on gears C and D:

$$T_C = \frac{40}{60} T_B = 4201.7 \text{ lb-in}$$

$$T_D = \frac{20}{60} T_B = 2100.8 \text{ lb-in}$$

Shaft torques:

$$AB: T_{AB} = 0$$

$$BC: T_{BC} = 6302.5 \text{ lb-in}$$

$$CD: T_{CD} = 2100.8 \text{ lb-in}$$

$$DE: T_{DE} = 0$$

Gear forces:

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb.}$$

$$F_C = \frac{T_C}{r_C} = \frac{4201.7}{4} = 1050.4 \text{ lb.}$$

$$F_D = \frac{T_D}{r_D} = \frac{2100.8}{4} = 525.2 \text{ lb.}$$

$$\text{At } B^+, \sqrt{M_z^2 + M_y^2 + T^2} \\ = \sqrt{2450.9^2 + 6477.6^2 + 6302.5^2} \\ = 9364 \text{ lb-in}$$

$$\text{At } C^-, \sqrt{M_z^2 + M_y^2 + T^2} \\ = \sqrt{6127.3^2 + 3589.2^2 + 6302.5^2} \\ = 9495 \text{ lb-in. (maximum)}$$

$$I_{\text{all}} = \frac{c}{J} (\sqrt{M_z^2 + M_y^2 + T^2})_{\text{max}}$$

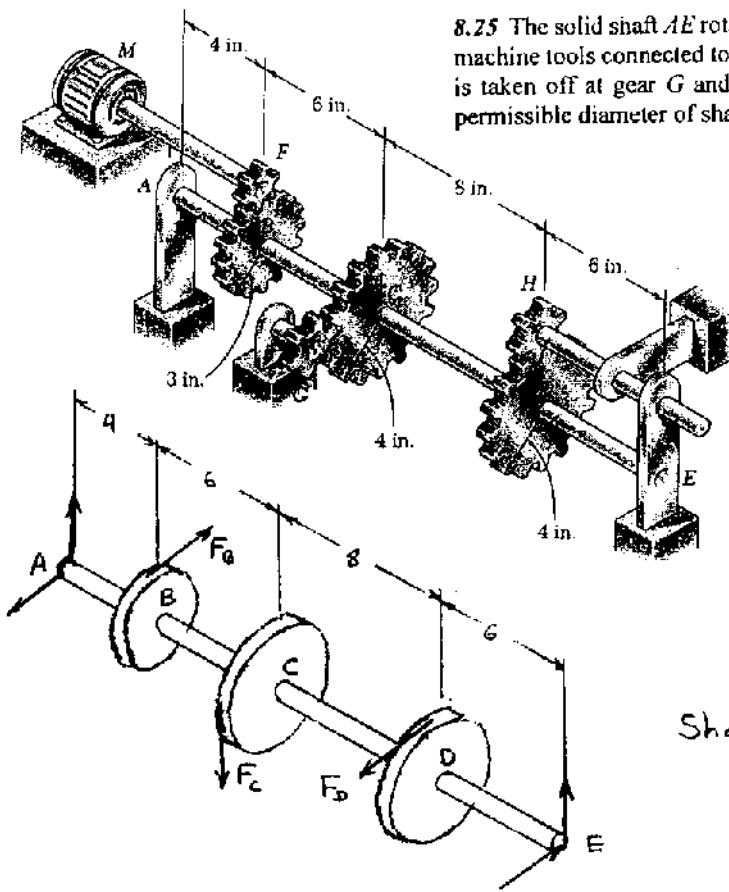
$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{(\sqrt{M_z^2 + M_y^2 + T^2})_{\text{max}}}{I_{\text{all}}} = \frac{9495}{8 \times 10^3} = 1.1868 \text{ in}^3$$

$$c = 0.911 \text{ in.}$$

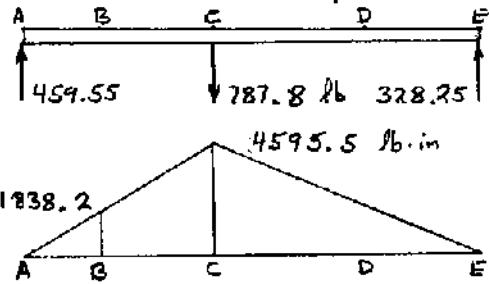
$$d = 2c = 1.822 \text{ in.}$$

Problem 8.26

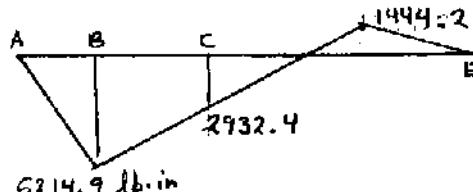
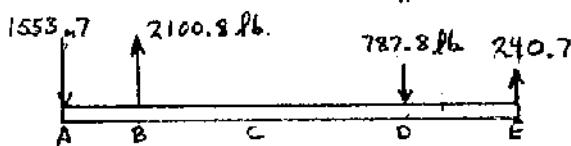
8.26 Solve Prob. 8.25, assuming that 30 hp is taken off at gear G and 30 hp is taken off at gear H.



Forces in vertical plane



Forces in horizontal plane



8.25 The solid shaft AE rotates at 600 rpm and transmits 60 hp from the motor M to machine tools connected to gears G and H. Knowing that $\sigma_{all} = 8 \text{ ksi}$ and that 40 hp is taken off at gear G and 20 hp is taken off at gear H, determine the smallest permissible diameter of shaft AE.

$$60 \text{ hp} = (60)(6600) \\ = 396 \times 10^3 \text{ in-lb/sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear B

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} \\ = 6302.5 \text{ lb-in}$$

Torques on gears C and D

$$T_C = \frac{T_B}{60} = 3151.3 \text{ lb-in}$$

$$T_D = \frac{T_B}{60} = 3151.3 \text{ lb-in}$$

Shaft torques

$$AB: T_{AB} = 0$$

$$BC: T_{BC} = 6302.5 \text{ lb-in}$$

$$CD: T_{CD} = 3151.3 \text{ lb-in}$$

$$DE: T_{DE} = 0$$

Gear forces

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb.}$$

$$F_C = \frac{T_C}{r_C} = \frac{3151.3}{4} = 787.8 \text{ lb.}$$

$$F_D = \frac{T_D}{r_D} = \frac{3151.3}{4} = 787.8 \text{ lb.}$$

$$\text{At } B^+ \sqrt{M_z^2 + M_y^2 + T^2}$$

$$= \sqrt{1838.2^2 + 6214.9^2 + 6302.5^2} \\ = 9040.2 \text{ lb-in (maximum)}$$

$$\text{At } C^- \sqrt{M_z^2 + M_y^2 + T^2}$$

$$= \sqrt{4595.5^2 + 2932.4^2 + 6302.5^2} \\ = 8333.0 \text{ lb-in}$$

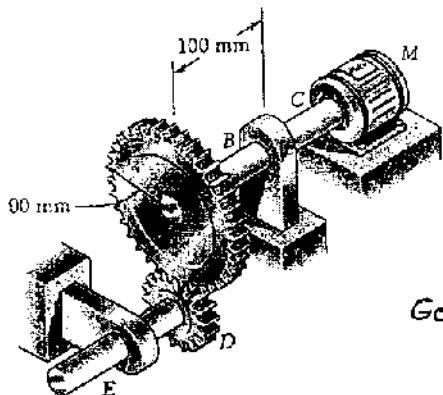
$$T_{all} = \frac{C}{J} (\sqrt{M_z^2 + M_y^2 + T^2})_{max}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{(\sqrt{M_z^2 + M_y^2 + T^2})_{max}}{T_{all}} = \frac{9040.2}{8 \times 10^3} = 1.1300 \text{ in}^3$$

$$C = 0.8960 \text{ in.}$$

$$d = 2C = 1.792 \text{ in.} \quad \blacktriangleleft$$

Problem 8.27

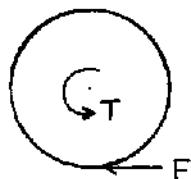


8.27 The solid shaft ABC and the gears shown are used to transmit 10 kW from the motor M to a machine tool connected to gear D . Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60 \text{ MPa}$, determine the smallest permissible diameter of shaft ABC .

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A.



$$Fr_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B: $M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$

$$\chi_{ul} = \frac{C}{J} \sqrt{M^2 + T^2}$$

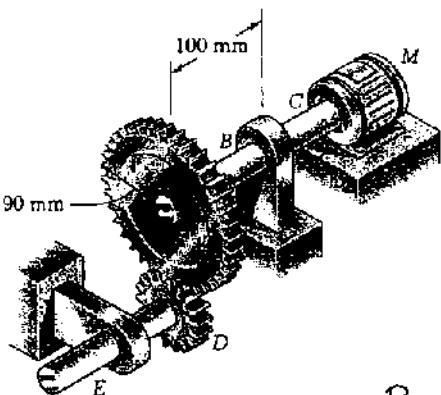
$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M^2 + T^2}}{\chi_{ul}}$$

$$C^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\chi_{ul}} = \frac{(2) \sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)} = 6.3108 \times 10^{-6} \text{ m}^3$$

$$C = 18.479 \times 10^{-3} \text{ m}$$

$$d = 2C = 37.0 \times 10^{-3} \text{ m} = 37.0 \text{ mm}$$

Problem 8.28



8.28 Assuming that shaft *ABC* of Prob. 8.27 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.

8.27 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60 \text{ MPa}$, determine the smallest permissible diameter of shaft *ABC*.

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N-m}$$

Gear A.



$$Fr_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B: $M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N-m}$

$$\tau_{all} = \frac{C_o}{J} \sqrt{M^2 + T^2} \quad C_o = \frac{1}{2} d_o = 25 \times 10^{-3} \text{ m}$$

$$\frac{T}{C_o} = \frac{\pi}{2} \frac{(C_o^4 - C_i^4)}{C_o} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

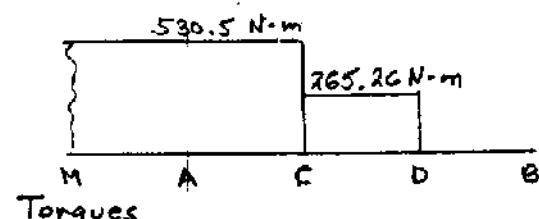
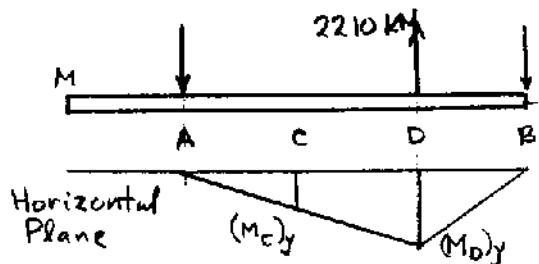
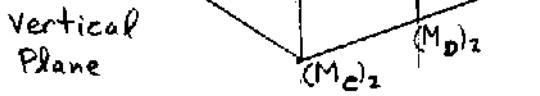
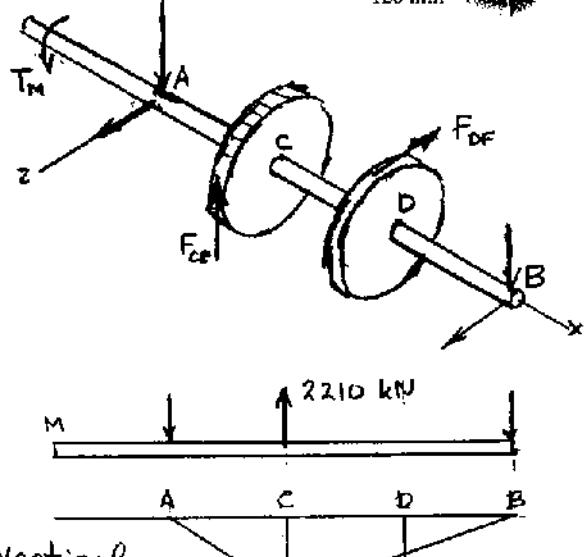
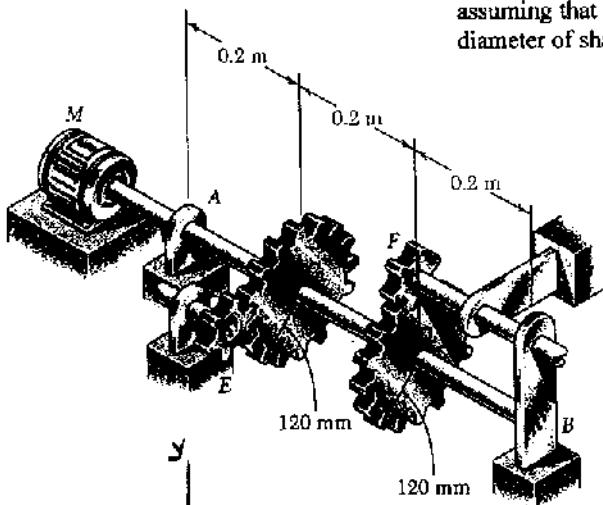
$$C_i^4 = C_o^4 - \frac{2C_o \sqrt{M^2 + T^2}}{\pi \tau_{all}} = (25 \times 10^{-3})^4 - \frac{(2)(25 \times 10^{-3}) \sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)}$$

$$= 390.625 \times 10^{-9} - 157.772 \times 10^{-9} = 232.85 \times 10^{-9}$$

$$C_i = 21.967 \times 10^{-3} \text{ m} \quad d_i = 2C_i = 43.93 \times 10^{-3} \text{ m} = 43.9 \text{ mm}$$

Problem 8.29

8.29 The solid shaft AB rotates at 360 rpm and transmits 20 kW from the motor M to machine tools connected to gears E and F . Knowing that $\tau_{\text{all}} = 45 \text{ MPa}$ and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft AB .



$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$T_M = \frac{B_m}{2\pi f} = \frac{20 \times 10^3}{2\pi (6)} = 530.5 \text{ N·m}$$

$$T_E = \frac{\Theta_E}{2\pi f} = \frac{10 \times 10^3}{2\pi (6)} = 265.26 \text{ N·m}$$

$$T_D = \frac{\Theta_D}{2\pi f} = \frac{10 \times 10^3}{2\pi (6)} = 265.26 \text{ N·m}$$

$$F_{ce} = \frac{T_E}{r_c} = \frac{265.26}{120 \times 10^{-3}} = 2.210 \times 10^3 \text{ N}$$

$$F_{Dc} = \frac{T_D}{r_d} = \frac{265.26}{120 \times 10^{-3}} = 2.210 \times 10^3 \text{ N}$$

$$(M_C)_z = \frac{(0.2)(0.4)(2.210 \times 10^3)}{0.6} = 294.7 \text{ N·m}$$

$$(M_D)_z = \frac{1}{2}(M_B)_z = 147.37 \text{ N·m}$$

$$(M_C)_y = \frac{(0.4)(0.2)(2.210 \times 10^3)}{0.6} = 294.7 \text{ N·m}$$

$$(M_D)_y = \frac{1}{2}(M_B)_y = 147.37 \text{ N·m}$$

Torques in shaft.

$$T_{MAC} = 530.5 \text{ N·m}$$

$$T_{CD} = 265.26 \text{ N·m}$$

$$T_{BB} = 0$$

Just to the left of gear C.

$$\max \sqrt{M_y^2 + M_z^2 + T^2} = 624.5 \text{ N·m}$$

$$\frac{T}{C} = \frac{\pi}{2} C^3 = \frac{\max \sqrt{M_y^2 + M_z^2 + T^2}}{T_{\max}}$$

$$= \frac{624.5}{45 \times 10^6} = 13.878 \times 10^{-6} \text{ m}^3$$

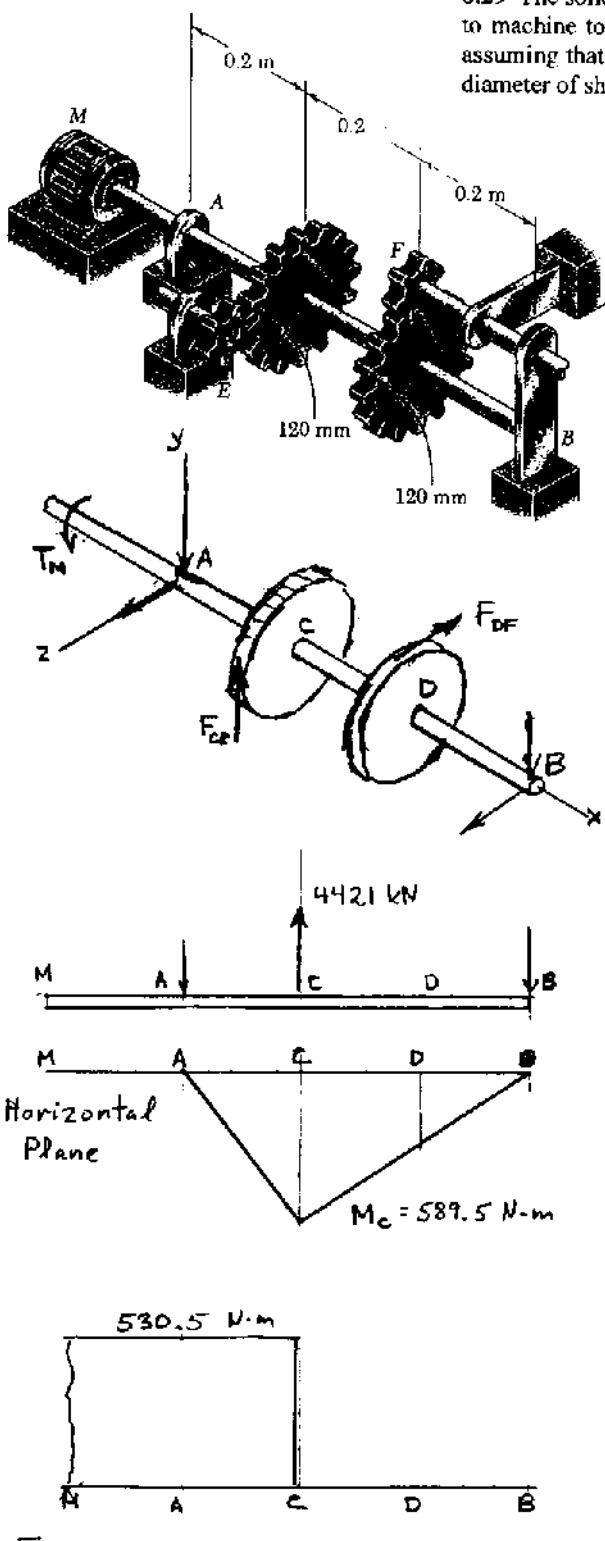
$$C = 20.67 \times 10^{-3} \text{ m} = 20.67 \text{ mm}$$

$$d = 2C = 41.3 \text{ mm}$$

Problem 8.30

8.30 Solve Prob. 8.29, assuming that the entire 20 kW is taken off at gear E.

8.29 The solid shaft AB rotates at 360 rpm and transmits 20 kW from the motor M to machine tools connected to gears E and F. Knowing that $\tau_{all} = 45 \text{ MPa}$ and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft AB.



$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$T_M = \frac{\sigma_M}{2\pi f} = \frac{20 \times 10^3}{2\pi(6)} = 530.5 \text{ N·m}$$

$$T_c = \frac{\sigma_c}{2\pi f} = \frac{20 \times 10^3}{2\pi(6)} = 530.5 \text{ N·m}$$

$$T_D = \frac{\sigma_D}{2\pi f} = 0$$

$$F_{ce} = \frac{T_c}{r_c} = \frac{530.5}{120 \times 10^{-3}} = 4.421 \times 10^3 \text{ N}$$

$$F_{bf} = \frac{T_D}{r_D} = 0$$

Since $F_{bf} = 0$, there is no bending in the horizontal plane.

$$M_c = \frac{(0.2)(0.4)(4.421 \times 10^3)}{0.6} = 589.5 \text{ N·m}$$

Torques in shaft

$$T_{MAC} = 530.5 \text{ N·m}$$

$$T_{CDE} = 0$$

Just to the left of gear C

$$\begin{aligned} m_{avg} &\sqrt{M_y^2 + M_z^2 + T^2} \\ &= \sqrt{0 + 589.5^2 + 530.5^2} \\ &= 793.1 \text{ N·m} \end{aligned}$$

$$\begin{aligned} \frac{J}{c} &= \frac{\pi}{2} c^3 = \frac{m_{avg} \sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}} \\ &= \frac{793.1}{45 \times 10^6} = 17.624 \times 10^{-6} \text{ m}^3 \end{aligned}$$

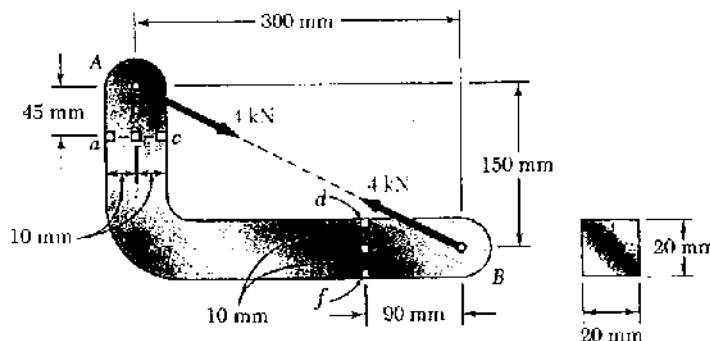
$$c = 22.39 \times 10^{-3} = 22.39 \text{ mm}$$

$$d = 2c = 44.8 \text{ mm}$$

Torques

Problem 8.31

8.31 Two 4-kN forces are applied to an L-shaped machine element AB as shown. Determine the normal and shearing stresses at (a) point a , (b) point b , (c) point c .



Let β be the slope angle of line AB .

$$\tan \beta = \frac{150}{300} \quad \beta = 26.565^\circ$$

Draw free body sketch of the portion of the machine element lying above section abc.

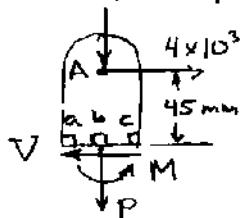
$$P = -(4 \times 10^3) \sin \beta \\ = -1.78885 \times 10^3 \text{ N}$$

$$4 \times 10^3 \sin \beta$$

$$V = (4 \times 10^3) \cos \beta = 3.5777 \times 10^3 \text{ N}$$

$$4 \times 10^3 \cos \beta$$

$$M = (45 \times 10^{-3})(4 \times 10^3) \cos \beta = 160.997 \text{ N}\cdot\text{m}$$



$$\text{Section properties: } A = (20)(20) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$$

$$I_z = \frac{1}{12}(20)(20)^3 = 13.3333 \times 10^3 \text{ mm}^4 = 13.3333 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ Point } a. \quad \sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885 \times 10^3}{400 \times 10^{-6}} - \frac{(160.997)(-10 \times 10^{-3})}{13.3333 \times 10^{-9}}$$

$$= 116.3 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = 0 \quad \blacktriangleleft$$

$$(b) \text{ Point } b. \quad \sigma = \frac{P}{A} = -\frac{1.78885 \times 10^3}{400 \times 10^{-6}} \quad = -4.47 \text{ MPa} \quad \blacktriangleleft$$

$$Q = (20)(10)(5) = 1000 \text{ mm}^3 = 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{IT} = \frac{(3.5777)(10^{-6})}{(13.3333 \times 10^{-9})(20 \times 10^{-3})} = 13.42 \text{ MPa} \quad \blacktriangleleft$$

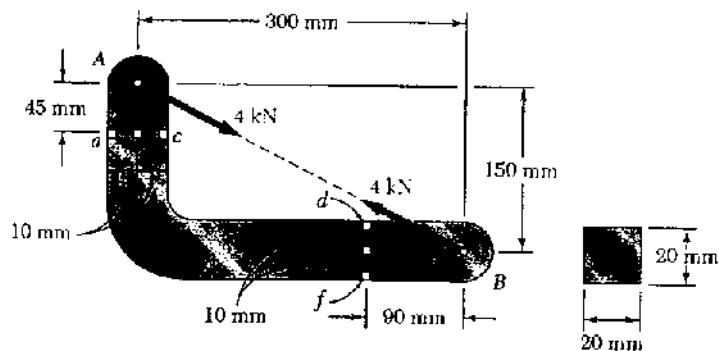
$$(c) \text{ Point } c. \quad \sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885 \times 10^3}{400 \times 10^{-6}} - \frac{(160.997)(10 \times 10^{-3})}{13.3333 \times 10^{-9}}$$

$$= -125.2 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = 0 \quad \blacktriangleleft$$

Problem 8.32

8.32 Two 4-kN forces are applied to an L-shaped machine element AB as shown. Determine the normal and shearing stresses at (a) point d , (b) point e , (c) point f .

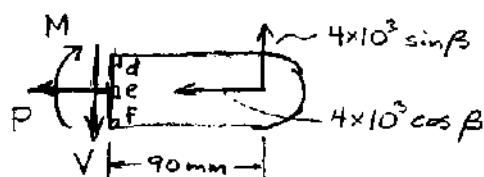


Let β be the slope angle of line AB .

$$\tan \beta = \frac{150}{300} \quad \beta = 26.565^\circ$$

Draw free body sketch of the portion of the machine element lying to the right of section def.

$$P = -(4 \times 10^3) \cos \beta \\ = -3.5777 \times 10^3 \text{ N}$$



$$V = (4 \times 10^3) \sin \beta = 1.78885 \times 10^3 \text{ N}$$

$$M = (90 \times 10^{-3})(4 \times 10^3) \sin \beta = 160.997 \text{ N}\cdot\text{m}$$

Section properties. $A = (20)(20) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$I = \frac{1}{12}(20)(20)^3 = 13.3333 \times 10^3 \text{ mm}^4 = 13.3333 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ Point } d. \quad \sigma = \frac{P}{A} - \frac{My}{I} = \frac{-3.5777 \times 10^3}{400 \times 10^{-6}} - \frac{(160.997)(10 \times 10^{-3})}{13.3333 \times 10^{-9}}$$

$$= -129.7 \text{ MPa} \quad \blacktriangleright$$

$$\tau = 0 \quad \blacktriangleright$$

$$(b) \text{ Point } e. \quad \sigma = \frac{P}{A} = -\frac{3.5777 \times 10^3}{400 \times 10^{-6}} \quad = -8.94 \text{ MPa} \quad \blacktriangleright$$

$$Q = (20)(10)(5) = 1000 \text{ mm}^3 = 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(1.78885 \times 10^3)(10^{-6})}{(13.3333 \times 10^{-9})(20 \times 10^{-3})} = 6.71 \text{ MPa} \quad \blacktriangleright$$

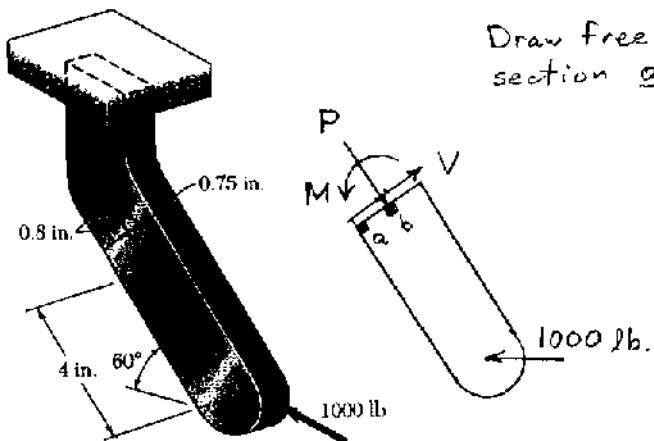
$$(c) \text{ Point } f. \quad \sigma = \frac{P}{A} - \frac{My}{I} = \frac{-3.5777 \times 10^3}{400 \times 10^{-6}} - \frac{(160.997)(-10 \times 10^{-3})}{13.3333 \times 10^{-9}}$$

$$= 111.8 \text{ MPa} \quad \blacktriangleright$$

$$\tau = 0 \quad \blacktriangleright$$

Problem 8.33

8.33 For the bracket and loading shown, determine the normal and shearing stresses at (a) point *a*, (b) point *b*.



Draw free body diagram of portion below section *ab*.

From statics,

$$P = 1000 \cos 60^\circ = 500 \text{ lb}$$

$$V = 1000 \sin 60^\circ = 866 \text{ lb.}$$

$$\begin{aligned} M &= (4)(1000) \sin 60^\circ \\ &= 3464 \text{ lb-in.} \end{aligned}$$

Section properties.

$$A = (0.75)(1.6) = 1.2 \text{ in}^2$$

$$I = \frac{1}{12}(0.75)(1.6)^3 = 0.256 \text{ in}^4$$

(a) Point *a* $\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{500}{1.2} - \frac{(3464)(0.8)}{0.256} = -11.24 \text{ ksi}$

$$\tau = 0$$

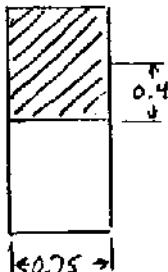
(b) Point *b* $\sigma = -\frac{P}{A} = -\frac{500}{1.2} = -0.417 \text{ ksi}$

$$A = (0.75)(0.8) = 0.6 \text{ in}^2$$

$$\bar{y} = 0.4 \text{ in.}$$

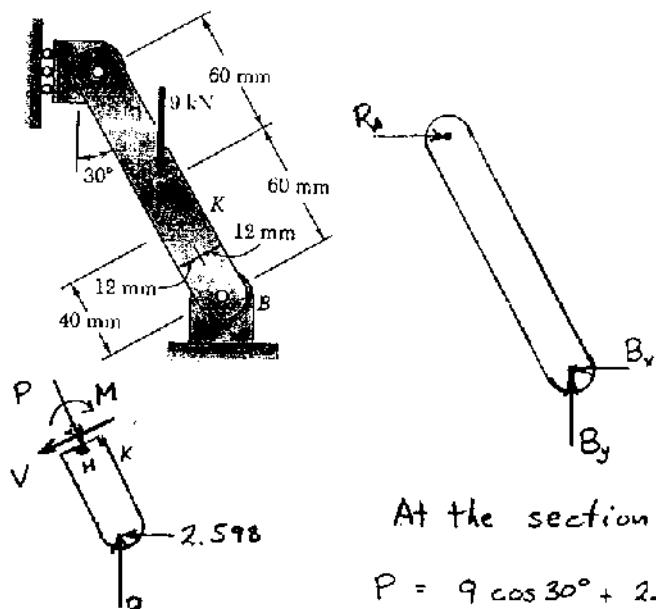
$$Q = A\bar{y} = (0.6)(0.4) = 0.24 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(866)(0.24)}{(0.256)(0.75)} = 1.083 \text{ ksi}$$



Problem 8.34

8.34 and 8.35 Member *AB* has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stresses (a) point *H*, (b) point *K*.



$$\textcircled{3} \sum M_B = 0$$

$$(120 \cos 30^\circ) R_A - (60 \sin 30^\circ)(q) = 0$$

$$R_A = 2.598 \text{ kN}$$

$$\textcircled{4} \sum F_y = 0 \quad B_y - q = 0 \quad B_y = q \text{ kN}$$

$$\textcircled{5} \sum F_x = 0 \quad 2.598 - B_x = 0$$

$$B_x = 2.598 \text{ kN} \leftarrow$$

At the section containing points *H* and *K*,

$$P = q \cos 30^\circ + 2.598 \sin 30^\circ = 9.093 \text{ kN}$$

$$V = q \sin 30^\circ - 2.598 \cos 30^\circ = 2.25 \text{ kN}$$

$$M = (9.093 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) - (2.598 \times 10^3)(40 \times 10^{-3} \cos 30^\circ) = 90 \text{ N.m}$$

$$A = 10 \times 240 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ At point } H, \quad \sigma_x = -\frac{P}{A} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} = -37.9 \text{ MPa}$$

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$$

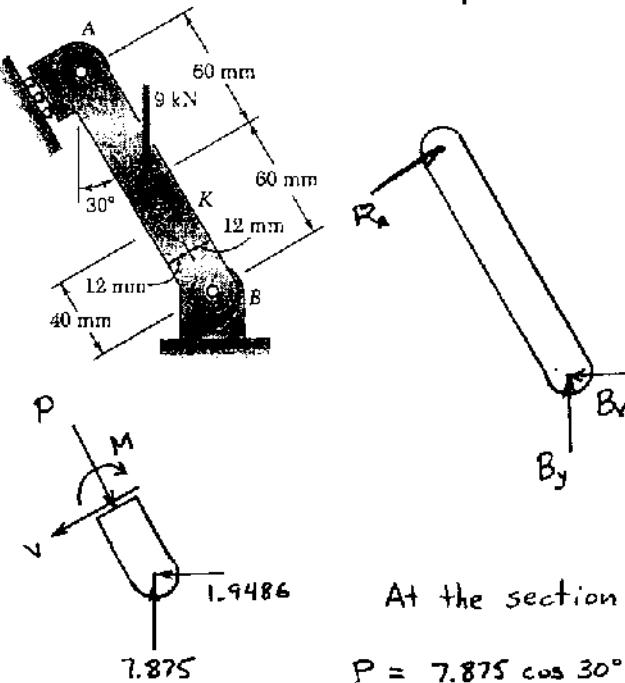
$$(b) \text{ At point } K, \quad \sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

$$= -131.6 \text{ MPa}$$

$$\tau_{xy} = 0$$

Problem 8.35

8.34 and 8.35 Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (a) point H, (b) point K.



$$\text{At } \sum M_B = 0$$

$$(9)(60 \sin 30^\circ) - 120 R_A = 0$$

$$R_A = 2.25 \text{ kN}$$

$$\text{At } \sum F_x = 0 \quad 2.25 \cos 30^\circ - B_x = 0$$

$$B_x = 1.9486 \text{ kN} \leftarrow$$

$$\text{At } \sum F_y = 0$$

$$2.25 \sin 30^\circ - 9 + B_y = 0$$

$$B_y = 7.875 \text{ kN} \uparrow$$

At the section containing points H and K,

$$P = 7.875 \cos 30^\circ + 1.9486 \sin 30^\circ = 7.794 \text{ kN}$$

$$V = 7.875 \sin 30^\circ - 1.9486 \cos 30^\circ = 2.25 \text{ kN}$$

$$M = (7.875 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) - (1.9486 \times 10^3)(40 \times 10^{-3} \cos 30^\circ) = 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^8 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

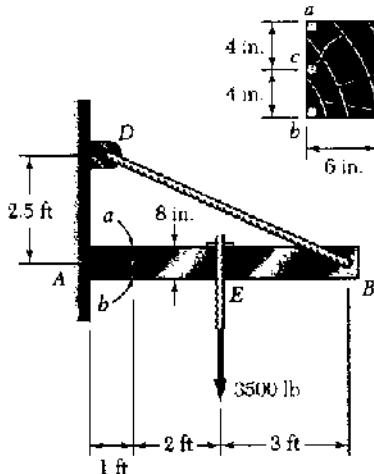
$$(a) \text{ At point H, } \sigma_x = -\frac{P}{A} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} = -32.5 \text{ MPa}$$

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$$

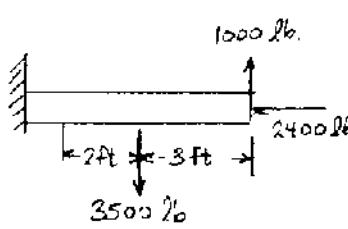
$$(b) \text{ At point K, } \sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}} = -126.2 \text{ MPa}$$

$$\tau_{xy} = 0$$

Problem 8.36



8.36 The cantilever beam AB has a rectangular cross section of 6×8 in. Knowing that the tension in the cable BD is 2600 lb and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.



$$DE = \sqrt{(6)^2 + (2.5)^2} = 6.5 \text{ ft}$$

Vertical component of T_{DE}

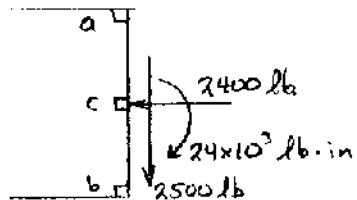
$$\left(\frac{2.5}{6.5}\right)(2600) = 1000 \text{ lb.}$$

Horizontal component of T_{DE} = $\left(\frac{6}{6.5}\right)(2600) = 2400 \text{ lb.}$

At the section containing points a , b , and c .

$$P = 2400 \text{ lb.} \quad V = 3500 - 1000 = 2500 \text{ lb}$$

$$M = (2+3)(1000) - (2)(3500) = -2000 \text{ lb-ft} \\ = -24 \times 10^3 \text{ lb-in}$$



Section properties

$$A = (6)(8) = 48 \text{ in}^3$$

$$I = \frac{1}{12}(6)(8)^3 = 256 \text{ in}^4$$

$$c = 4 \text{ in.}$$

$$\text{At point } a. \quad \sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2400}{48} - \frac{(-24 \times 10^3)(4)}{256} = 325 \text{ psi}$$

$$\tau_{xy} = 0$$

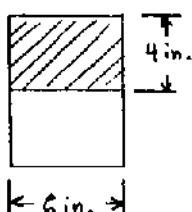
$$\text{At point } b. \quad \sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{2400}{48} + \frac{(-24 \times 10^3)(4)}{256} = -425 \text{ psi}$$

$$\tau_{xy} = 0$$

$$\text{At point } c. \quad \sigma_x = -\frac{P}{A} = -\frac{2400}{48} = -50 \text{ psi}$$

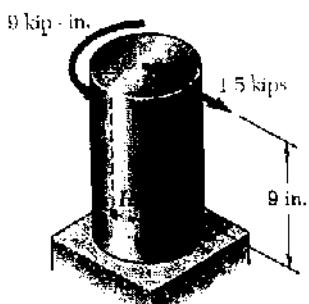
$$A = (6)(4) = 24 \text{ in}^2, \quad \bar{y} = 2 \text{ in.} \quad Q = A\bar{y} = 48 \text{ in}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(2500)(48)}{(256)(6)} = -78.1 \text{ psi}$$



Problem 8.37

8.37 A 1.5-kip force and a 9-kip in. couple are applied at the top of the 2.5-in.-diameter cast-iron post shown. Determine the normal and shearing stresses at (a), point H, (b) point K.



diameter = 2.5 in.

At the section containing points H and K.

$$P = 0 \quad V = 1.5 \text{ kips}$$

$$T = 9 \text{ kip-in} \quad M = (1.5)(9) = 13.5 \text{ kip-in}$$

$$d = 2.5 \text{ in.} \quad c = \frac{1}{2}d = 1.25 \text{ in.}$$

$$A = \pi C^2 = 4.909 \text{ in}^2 \quad I = \frac{\pi}{4} C^4 = 1.9175 \text{ in}^4 \quad J = 2I = 3.835 \text{ in}^4$$

$$\text{For a semicircle, } Q = \frac{2}{3} C^3 = 1.3021 \text{ in}^3$$

$$(a) \text{ At point H, } \sigma_H = 0$$

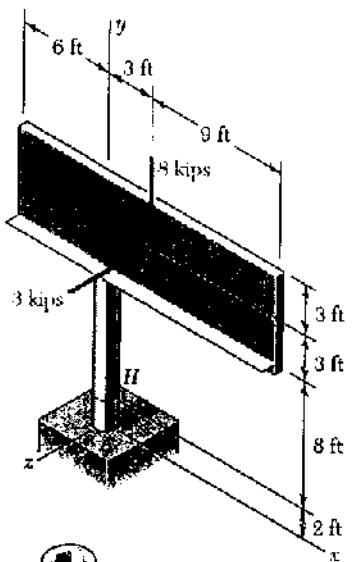
$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(9)(1.25)}{3.835} + \frac{(1.5)(1.3021)}{(1.9175)(2.5)} = 2.934 + 0.407 \\ = 3.34 \text{ ksi}$$

$$(b) \text{ At point K, } \sigma_K = -\frac{Mc}{I} = -\frac{(13.5)(1.25)}{1.9175} = -8.80 \text{ ksi}$$

$$\tau_K = \frac{Tc}{J} = \frac{(9)(1.25)}{3.835} = 2.93 \text{ ksi}$$

Problem 8.38

8.38 The billboard shown weights 8000 lb and is supported by a structural tube that has a 15-in. outer diameter and a 0.5-in. wall thickness. At a time when the resultant of the wind pressure is 3 kips located at the center C of the billboard, determine the normal and shearing stresses at point H.



At section containing point H,

$$P = 8 \text{ kips (compression)}$$

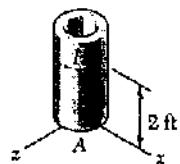
$$T = (3)(3) = 9 \text{ kip-ft} = 108 \text{ kip-in}$$

$$M_x = -(11)(3) = -33 \text{ kip-ft} = -396 \text{ kip-in}$$

$$M_z = -(3)(8) = -24 \text{ kip-ft} = -288 \text{ kip-in}$$

$$V = 3 \text{ kip}$$

Section properties.



$$d_o = 15 \text{ in. } C_o = \frac{1}{2} d_o = 7.5 \text{ in. } C_i = C_o - t = 7.0 \text{ in.}$$

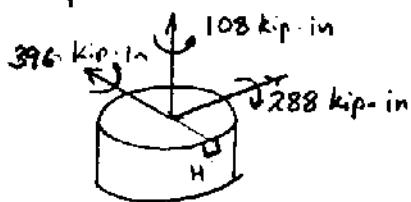
$$A = \pi (C_o^2 - C_i^2) = 22.777 \text{ in}^2$$

$$I = \frac{\pi}{4} (C_o^4 - C_i^4) = 599.31 \text{ in}^4$$

$$J = 2I = 1198.62 \text{ in}^4$$

$$Q = \frac{2}{3} (C_o^3 - C_i^3) = 52.583 \text{ in}^3$$

Couples

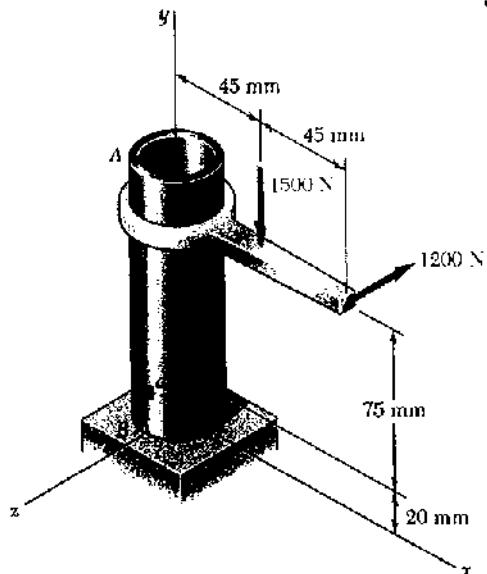


$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8}{22.777} - \frac{(288)(7.5)}{599.31} = -0.351 - 3.604 = -3.96 \text{ ksi} \quad \blacksquare$$

$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(108)(7.5)}{1198.62} + \frac{(3)(52.583)}{(599.31)(1.0)} = 0.675 + 0.268 = 0.938 \text{ ksi} \quad \blacksquare$$

Problem 8.39

8.39 Two forces are applied to the pipe AB as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (a) point a , (b) point b .

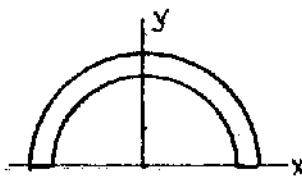


$$C_o = \frac{d_o}{2} = 21 \text{ mm}, \quad C_i = \frac{d_i}{2} = 17.5 \text{ mm}$$

$$A = \pi(C_o^2 - C_i^2) = 423.33 \text{ mm}^2$$

$$J = \frac{\pi}{2}(C_o^4 - C_i^4) = 158.166 \times 10^3 \text{ mm}^4$$

$$I = \frac{1}{2}J = 79.083 \times 10^3 \text{ mm}^4$$



For semicircle with
semicircular cutout

$$Q = \frac{2}{3}(C_o^3 - C_i^3)$$

$$Q = 2.6011 \times 10^3 \text{ mm}^3$$

At the section containing points a and b

$$P = -1500 \text{ N}$$

$$V_z = -1200 \text{ N}$$

$$V_x = 0$$

$$M_z = -(45 \times 10^{-3})(1500) = -67.5 \text{ N}\cdot\text{m}$$

$$M_x = -(75 \times 10^{-3})(1200) = -90 \text{ N}\cdot\text{m}$$

$$T = (90 \times 10^{-3})(1200) = 108 \text{ N}\cdot\text{m}$$

$$(a) \sigma = \frac{P}{A} - \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}} = 20.4 \text{ MPa}$$

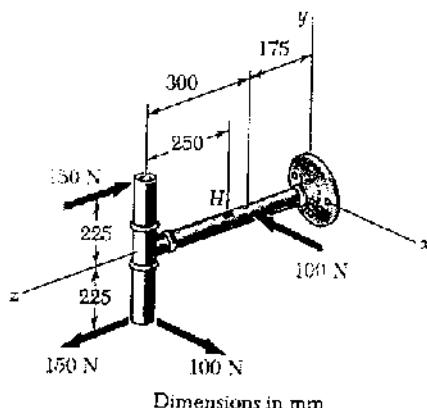
$$\tau = \frac{Tc}{J} + \frac{V_x Q}{I t} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0 = 14.34 \text{ MPa}$$

$$(b) \sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}} = -21.5 \text{ MPa}$$

$$\tau = \frac{Tc}{J} + \frac{|V_z| Q}{I t} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^6)}{(79.083 \times 10^{-9})(7 \times 10^{-3})} = 19.98 \text{ MPa}$$

Problem 8.40

8.40 Several forces are applied to the pipe assembly shown. Knowing that each section of pipe has inner and outer diameters equal to 36 and 42 mm, respectively, determine the normal and shearing stresses at point H located at the top of the outer surface of the pipe.



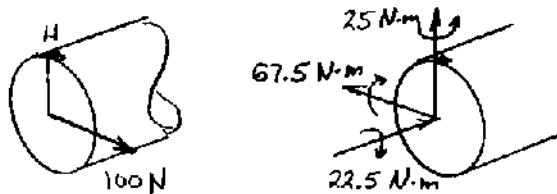
At the section containing point H,

$$P = 0, \quad V_x = 100 \text{ N}, \quad V_y = 0$$

$$M_x = -(0.450)(150) = -67.5 \text{ N}\cdot\text{m}$$

$$M_y = (0.250)(100) = 25 \text{ N}\cdot\text{m}$$

$$M_z = -(0.225)(100) = -22.5 \text{ N}\cdot\text{m}$$



$$d_o = 42 \text{ mm} \quad d_i = 32 \text{ mm}$$

$$c_o = 21 \text{ mm} \quad c_i = 18 \text{ mm}$$

$$t = c_o - c_i = 3 \text{ mm}$$

$$A = \pi(c_o^2 - c_i^2) = 367.57 \text{ mm}^2 = 367.57 \times 10^{-6} \text{ m}^2$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 70.30 \times 10^3 \text{ mm}^4 = 70.30 \times 10^{-9} \text{ m}^4, \quad J = 2I = 140.59 \times 10^{-9} \text{ m}^4$$

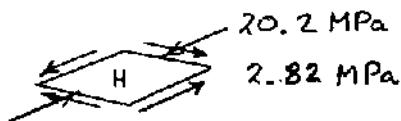
$$\text{For half-pipe, } Q = \frac{2}{3}(c_o^3 - c_i^3) = 2.286 \times 10^3 \text{ mm}^3 = 2.286 \times 10^{-6} \text{ m}^3$$

$$\sigma_H = \frac{M_x y}{I_x} = \frac{(-67.5)(21 \times 10^{-3})}{70.30 \times 10^{-9}} = -20.2 \text{ MPa}$$

$$\text{Due to torque: } (\tau_H)_T = \frac{Tc}{J} = \frac{(22.5)(21 \times 10^{-3})}{140.59 \times 10^{-9}} = 3.36 \text{ MPa}$$

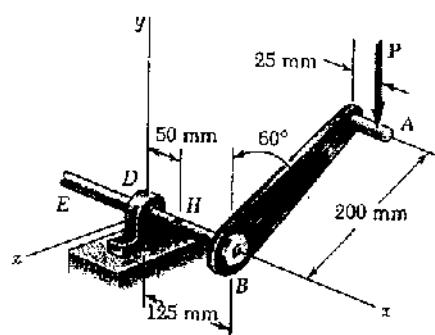
$$\text{Due to shear: } (\tau_H)_V = \frac{VQ}{IT} = \frac{(100)(2.286 \times 10^{-6})}{(70.30 \times 10^{-9})(6 \times 10^{-3})} = 0.54 \text{ MPa}$$

$$\text{Net: } \tau_H = 3.36 - 0.54 = 2.82 \text{ MPa}$$



Problem 8.41

8.41 A vertical force P of magnitude 250 N is applied to the crank at point A . Knowing that the shaft BDE has a diameter of 18 mm, determine the principal stresses and the maximum shearing stress at point H located at the top of the shaft, 50 mm to the right of support D .

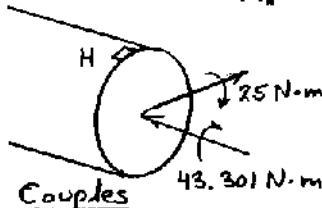
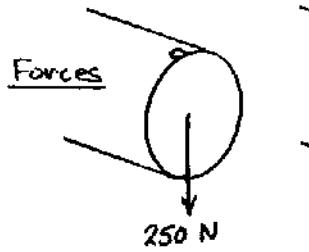


Force-couple system at the centroid of the section containing point H .

$$F_x = 0, \quad V_y = -250 \text{ N}, \quad V_z = 0$$

$$M_z = -(125 - 50 + 25)(10^{-3})(250) = -25 \text{ N}\cdot\text{m}$$

$$M_w = -(200 \sin 60^\circ)(10^{-3})(250) = -43.301 \text{ N}\cdot\text{m}$$



$$d = 18 \text{ mm} \quad C = \frac{1}{2}d = 9 \text{ mm}$$

$$I = \frac{\pi}{4}C^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$$

$$J = 2I = 10.306 \times 10^{-9} \text{ m}^4$$

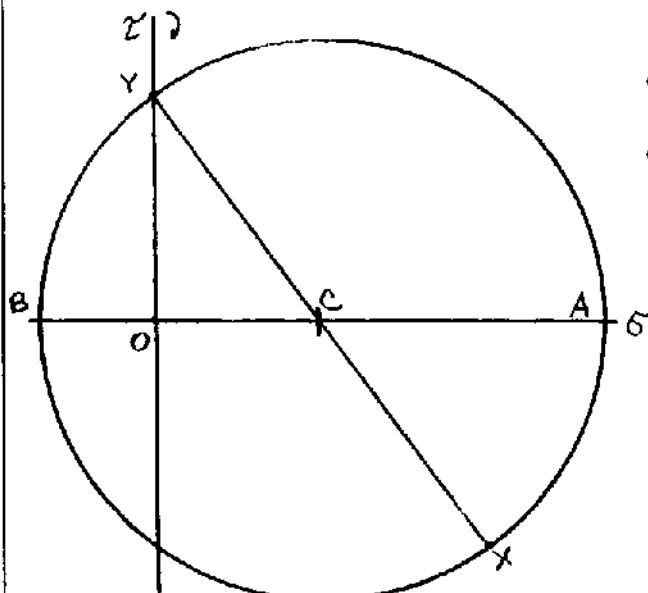
$$\text{At point } H, \quad \sigma_H = -\frac{M_{zy}}{I_w} = -\frac{(-25)(9 \times 10^{-3})}{5.153 \times 10^{-9}} = 43.66 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} = \frac{(43.301)(9 \times 10^{-3})}{10.306 \times 10^{-9}} = 37.81 \text{ MPa}$$

Use Mohr's circle.

$$\sigma_c = \frac{1}{2}\sigma_H = 21.83 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + (\tau_H)^2} = 43.66 \text{ MPa}$$



$$\sigma_a = \sigma_c + R = 65.5 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -21.8 \text{ MPa}$$

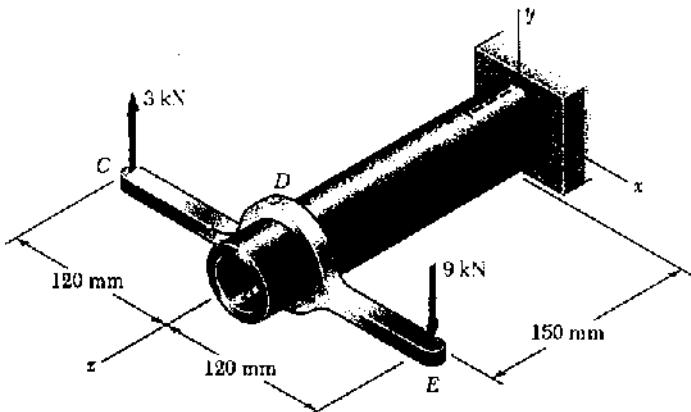
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{75.62}{43.66} = 1.7320$$

$$\theta_a = 30^\circ, \quad \theta_b = 120^\circ$$

$$\tau_{max} = R = 43.7 \text{ MPa}$$

Problem 8.42

8.42 The steel pipe *AB* has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that the arm *CDE* is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shearing stress at point *H*.



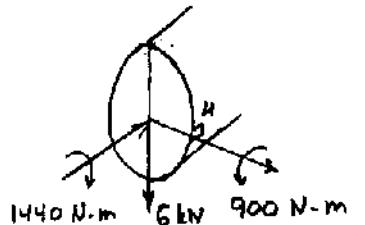
Replace the forces at *C* and *E* by an equivalent force-couple system at *D*.

$$F_D = 9 - 3 = 6 \text{ kN} \downarrow$$

$$\begin{aligned} T_D &= (9 \times 10^3)(120 \times 10^{-3}) \\ &\quad + (3 \times 10^3)(120 \times 10^{-3}) \\ &= 1440 \text{ N}\cdot\text{m} \end{aligned}$$

At the section containing point *H*,

$$P = 0, V = 6 \text{ kN}, T = 1440 \text{ N}\cdot\text{m}$$



$$M = (6 \times 10^3)(150 \times 10^{-3}) = 900 \text{ N}\cdot\text{m}$$

Section properties: $d_o = 72 \text{ mm}$ $C_o = \frac{1}{4} d_o = 36 \text{ mm}$ $C_i = C_o - t = 31 \text{ mm}$

$$A = \pi(C_o^2 - C_i^2) = 1.0524 \times 10^3 \text{ mm}^2 = 1.0524 \times 10^{-3} \text{ m}^2$$

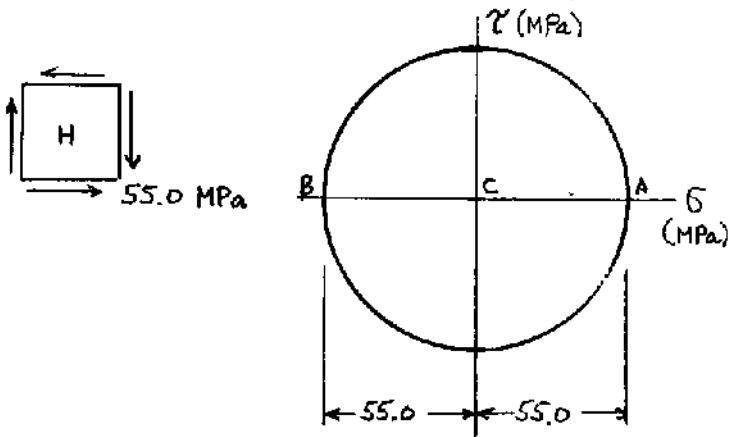
$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = 593.84 \times 10^{-9} \text{ mm}^4 = 593.84 \times 10^{-9} \text{ m}^4$$

$$J = 2I = 1.1877 \times 10^{-8} \text{ m}^4$$

$$\text{For half-pipe, } Q = \frac{2}{3}(C_o^3 - C_i^3) = 11.243 \times 10^3 \text{ mm}^3 = 11.243 \times 10^{-6} \text{ m}^3$$

At point *H*. Point *H* lies on the neutral axis of bending. $\sigma_H = 0$.

$$\tau_H = \frac{TC}{J} + \frac{VQ}{It} = \frac{(1440)(36 \times 10^{-3})}{1.1877 \times 10^{-8}} + \frac{(6 \times 10^3)(11.243 \times 10^{-6})}{(593.84 \times 10^{-9})(10 \times 10^{-3})} = 55.0 \text{ MPa}$$



Use Mohr's circle.

$$\sigma_c = 0$$

$$R = 55.0 \text{ MPa}$$

$$\sigma_a = \sigma_c + R = 55.0 \text{ MPa}$$

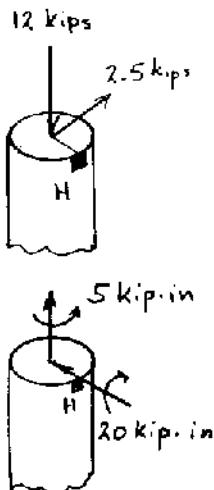
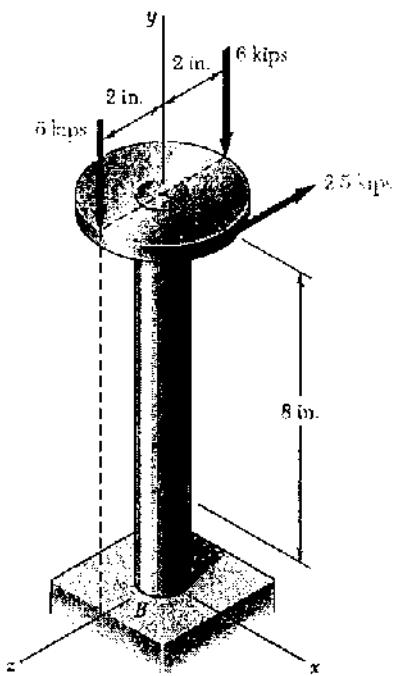
$$\sigma_b = \sigma_c - R = -55.0 \text{ MPa}$$

$$\theta_a = -45^\circ, \theta_b = +45^\circ$$

$$\tau_{max} = R = 55.0 \text{ MPa}$$

Problem 8.43

8.43 Three forces are applied to a 4-in.-diameter plate that is attached to the solid 1.8-in. diameter shaft AB . At point H , determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



At the section containing point H ,

$$P = 12 \text{ kips (compression)}$$

$$V = 2.5 \text{ kips}$$

$$T = (2)(2.5) = 5 \text{ kip-in}$$

$$M = (8)(2.5) = 20 \text{ kip-in.}$$

$$d = 1.8 \text{ in} \quad c = \frac{1}{2}d = 0.9 \text{ in.}$$

$$A = \pi C^2 = 2.545 \text{ in}^2$$

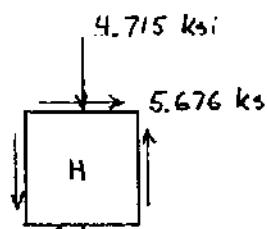
$$I = \frac{\pi}{4} C^4 = 0.5153 \text{ in}^4$$

$$J = 2I = 1.0306 \text{ in}^4$$

For a semicircle, $Q = \frac{2}{3}C^3 = 0.486 \text{ in}^3$

Point H lies on neutral axis of bending. $\sigma_H = \frac{P}{A} = -\frac{12}{2.545} = -4.715 \text{ ksi}$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5)(0.9)}{1.0306} + \frac{(2.5)(0.486)}{(0.5153)(1.8)} = 5.676 \text{ ksi}$$



Use Mohr's circle.

$$\sigma_c = \frac{1}{2}(-4.715) = -2.3575 \text{ ksi}$$

$$R = \sqrt{\left(\frac{4.715}{2}\right)^2 + 5.676^2} = 6.1461$$

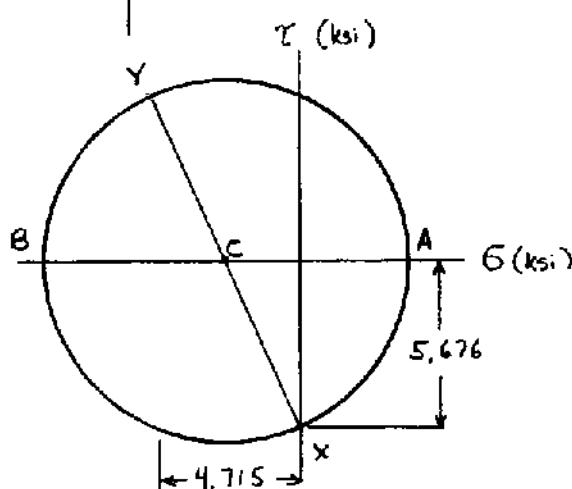
$$(a) \sigma_a = \sigma_c + R = 3.79 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -8.50 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2(5.676)}{4.715} = 2.408$$

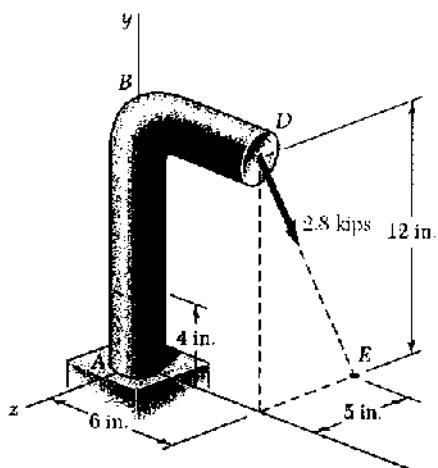
$$\theta_a = 33.7^\circ \quad \theta_b = 123.7^\circ$$

$$(b) \tau_{max} = R = 6.15 \text{ ksi}$$



Problem 8.44

8.44 A 2.8-kip force is applied as shown to the 2.4-in.-diameter cast-iron post *ABD*. At point *H*, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



$$DE = \sqrt{5^2 + 12^2} = 13 \text{ in.}$$

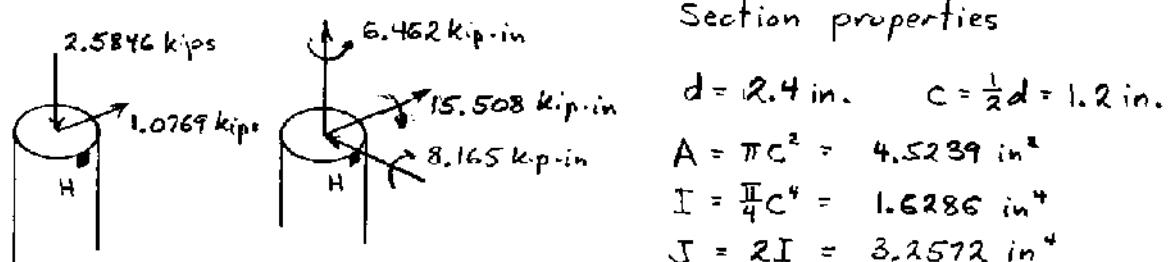
$$\text{At point D} \quad F_x = 0$$

$$F_y = -\left(\frac{12}{13}\right)(2.8) = -2.5846 \text{ kips}$$

$$F_z = -\left(\frac{5}{13}\right)(2.8) = -1.0769 \text{ kips}$$

Moment of equivalent force-couple system at C, the centroid of the section containing point H

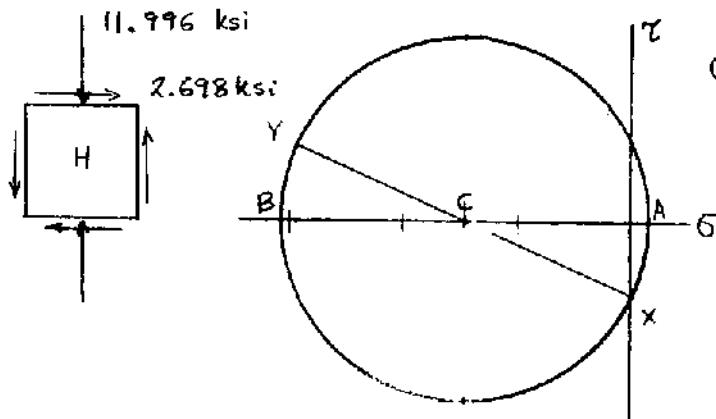
$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 8 & 0 \\ 0 & -2.5846 & -1.0769 \end{vmatrix} = 8.165 \hat{i} + 6.462 \hat{j} - 15.508 \hat{k} \text{ kip-in}$$



$$\text{For a semicircle} \quad Q = \frac{2}{3} c^3 = 1.152 \text{ in}^3$$

$$\text{At point H} \quad \sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2.5846}{4.5239} - \frac{(15.508)(1.2)}{1.6286} = -11.996 \text{ ksi}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(6.462)(1.2)}{3.2572} + \frac{(1.0769)(1.152)}{(1.6286)(2.4)} = 2.698 \text{ ksi}$$



$$(a) \quad \sigma_c = \frac{\sigma_H}{2} = -5.998 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 5.677 \text{ ksi}$$

$$\sigma_a = \sigma_c + R = 0.579 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -12.58 \text{ ksi}$$

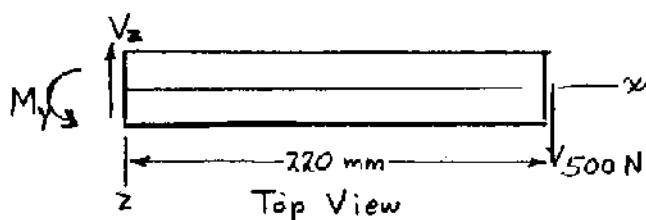
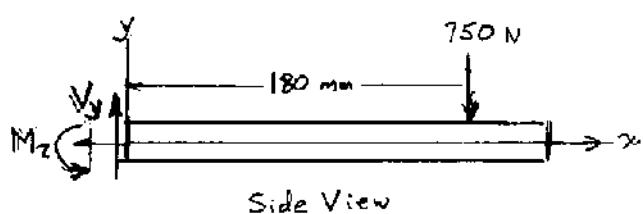
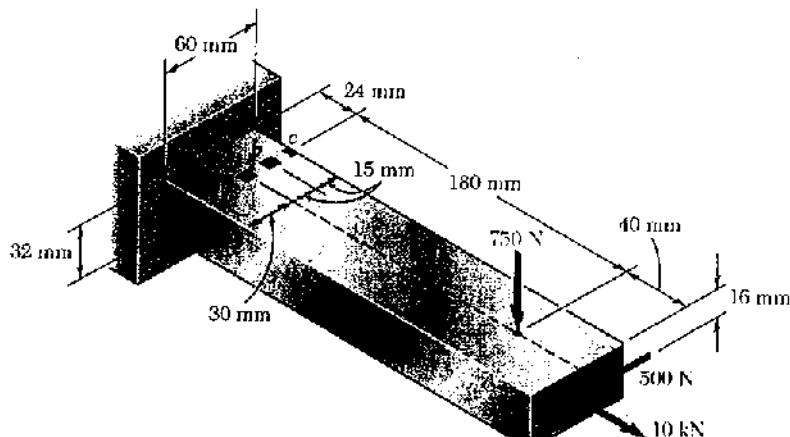
$$\tan 2\theta_p = \frac{2\tau_H}{10\sigma_c} = 0.4497$$

$$\theta_a = 12.1^\circ, \theta_b = 102.1^\circ$$

$$(b) \quad \tau_{max} = R = 5.677 \text{ ksi}$$

Problem 8.45

8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point *a*, (b) point *b*, (c) point *c*.



$$(a) \text{ Point } a. \quad y = 16 \text{ mm}, z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3 \text{ mm}^3$$

$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - 0 = 18.39 \text{ MPa}$$

$$\tau = \frac{(500)(14.4 \times 10^{-3})}{(576 \times 10^{-9})(32 \times 10^{-3})} = 0.391 \text{ MPa}$$

$$(b) \text{ Point } b. \quad y = 16 \text{ mm}, z = -15 \text{ mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3 \text{ mm}^3$$

$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}} = 21.3 \text{ MPa}$$

$$\tau = \frac{(500)(10.8 \times 10^{-3})}{(576 \times 10^{-9})(32 \times 10^{-3})} = 0.293 \text{ MPa}$$

$$(c) \text{ Point } c. \quad y = 16 \text{ mm}, z = -30 \text{ mm}, \quad Q = 0$$

$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-3})}{576 \times 10^{-9}} = 24.1 \text{ MPa}$$

$$\tau = 0$$

$$A = (60)(32) = 1920 \text{ mm}^2 \\ = 1920 \times 10^{-6} \text{ m}^2$$

$$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4 \\ = 163.84 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12}(32)(60)^3 = 576 \times 10^3 \text{ mm}^4 \\ = 576 \times 10^{-9} \text{ m}^4$$

At the section containing points *a*, *b*, and *c*

$$P = 10 \text{ kN}$$

$$V_y = 750 \text{ N}, V_z = 500 \text{ N}$$

$$M_z = (180 \times 10^{-3})(750) \\ = 135 \text{ N} \cdot \text{m}$$

$$M_y = (220 \times 10^{-3})(500) \\ = 110 \text{ N} \cdot \text{m}$$

$$T = 0$$

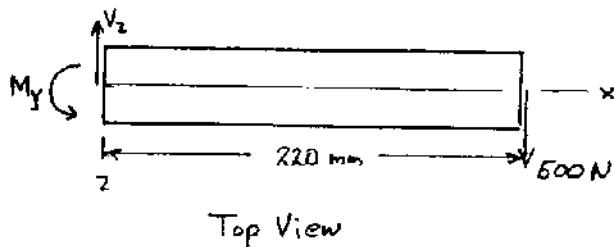
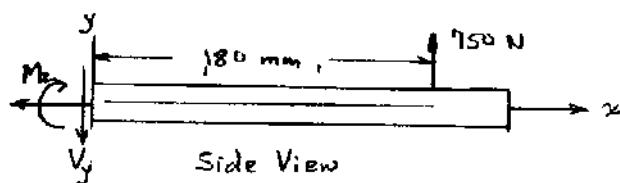
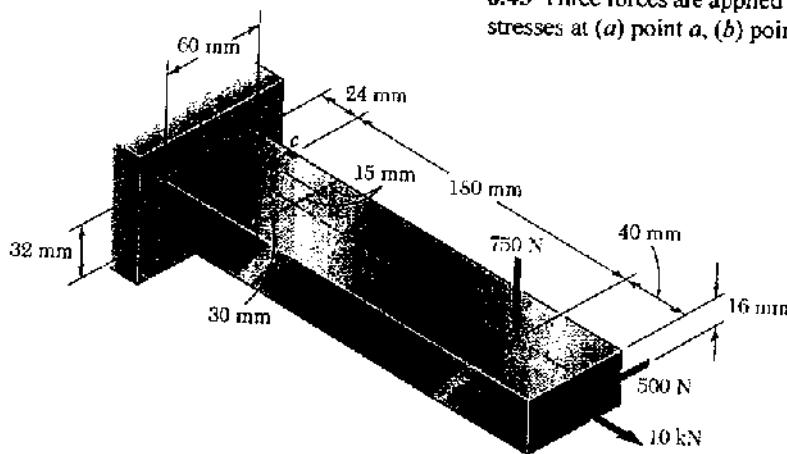
$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$

$$\tau = \frac{V_z Q}{I_t}$$

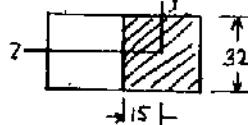
Problem 8.46

8.46 Solve Prob. 8.45, assuming that the 750-N force is directed vertically upward.

8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point *a*, (b) point *b*, (c) point *c*.



$$(a) \text{ Point } a \quad y = 16 \text{ mm}, \quad z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3 \text{ mm}^3$$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - 0 = -7.98 \text{ MPa}$$

$$\tau = \frac{(500)(14.4 \times 10^{-6})}{(163.84 \times 10^{-9})(32 \times 10^{-3})} = 0.391 \text{ MPa}$$

$$(b) \text{ Point } b \quad y = 16 \text{ mm}, \quad z = -15 \text{ mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3 \text{ mm}^3$$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}} = -5.11 \text{ MPa}$$

$$\tau = \frac{(500)(10.8 \times 10^{-6})}{(163.84 \times 10^{-9})(32 \times 10^{-3})} = 0.293 \text{ MPa}$$

$$(c) \text{ Point } c \quad y = 16 \text{ mm}, \quad z = -30 \text{ mm}, \quad Q = 0$$

$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-3})}{576 \times 10^{-9}} = -2.25 \text{ MPa}$$

$$\tau = 0$$

$$A = (60)(32) = 1920 \text{ mm}^2 \\ = 1920 \times 10^{-6} \text{ m}^2$$

$$I_2 = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4 \\ = 163.84 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12}(32)(60)^3 = 576 \times 10^3 \text{ mm}^4 \\ = 576 \times 10^{-9} \text{ m}^4$$

At the section containing points *a*, *b*, and *c*

$$P = 10 \text{ kN}, \quad T = 0$$

$$V_y = 750 \text{ N}, \quad V_z = 500 \text{ N}$$

$$M_x = (180 \times 10^{-3})(750) \\ = 135 \text{ N}\cdot\text{m}$$

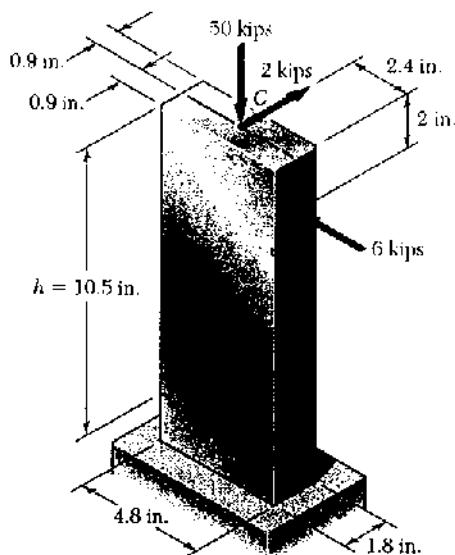
$$M_y = (220 \times 10^{-3})(500) \\ = 110 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{P}{A} - \frac{M_y y}{I_2} - \frac{M_z z}{I_y}$$

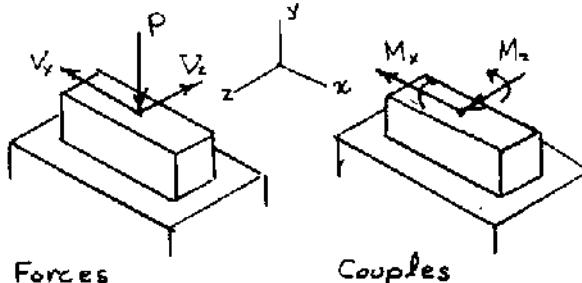
$$\tau = \frac{V_z Q}{I_2 t}$$

Problem 8.47

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point *a*, (b) point *b*, (c) point *c*.



Calculate forces and couples at section containing points *a*, *b*, and *c*. $h = 10.5$ in.



$$P = 50 \text{ kips} \quad V_x = 6 \text{ kips} \quad V_z = 2 \text{ kips}$$

$$M_z = (10.5 - 2)(6) = 51 \text{ kip-in}$$

$$M_x = (10.5)(2) = 21 \text{ kip-in}$$

Section properties. $A = (1.8)(4.8) = 8.64 \text{ in}^2$

$$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4 \quad I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$$

Stresses $\sigma = -\frac{P}{A} + \frac{M_z z}{I_z} + \frac{M_x z}{I_x} \quad z = \frac{V_x Q}{I_z E}$

(a) Point *a*: $x = 0$, $z = 0.9$ in., $Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(21)(0.9)}{2.3328} = 2.31 \text{ ksi}$$

$$\tau = \frac{(\sigma)(5.184)}{(16.5888)(1.8)} = 1.042 \text{ ksi}$$

(b) Point *b*: $x = 1.2$ in., $z = 0.9$ in., $Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + \frac{(51)(1.2)}{16.5888} + \frac{(21)(0.9)}{2.3328} = 6.00 \text{ ksi}$$

$$\tau = \frac{(\sigma)(3.888)}{(16.5888)(1.8)} = 0.781 \text{ ksi}$$

(c) Point *c*: $x = 2.4$ in., $z = 0.9$ in., $Q = 0$

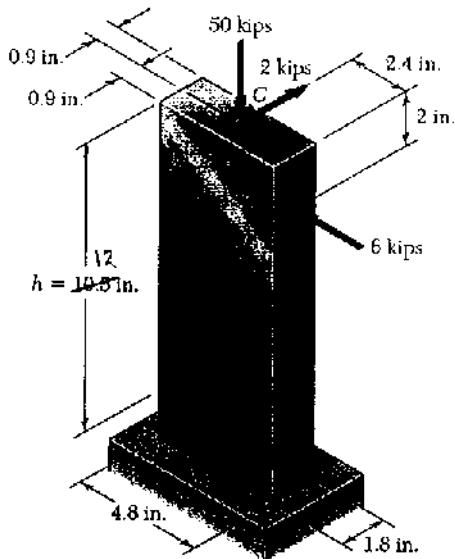
$$\sigma = -\frac{50}{8.64} + \frac{(51)(2.4)}{16.5888} + \frac{(21)(0.9)}{2.3328} = 9.69 \text{ ksi}$$

$$\tau = 0$$

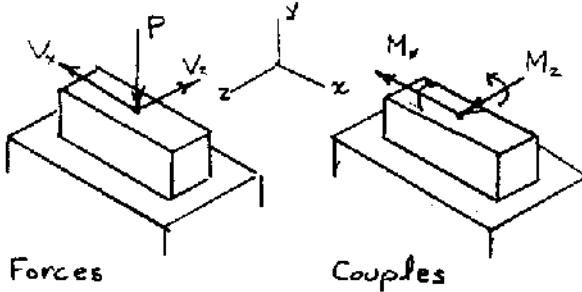
Problem 8.48

8.48 Solve Prob. 8.47, assuming that $h = 12$ in.

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point a , (b) point b , (c) point c .



Calculate forces and couples at section containing points a , b , and c . $h = 12$ in.



$$P = 50 \text{ kips} \quad V_x = 6 \text{ kips} \quad V_z = 2 \text{ kips}$$

$$M_z = (12 - 2)(6) = 60 \text{ kip-in}$$

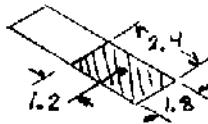
$$M_x = (12)(2) = 24 \text{ kip-in}$$

Section properties. $A = (1.8)(4.8) = 8.64 \text{ in}^2$

$$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4 \quad I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$$

Stresses $\sigma = -\frac{P}{A} + \frac{M_z z}{I_z} + \frac{M_x z}{I_x} \quad \tau = \frac{V_x Q}{I_z t}$

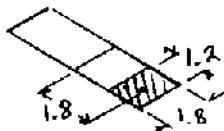
(a) Point a : $x = 0$, $z = 0.9$ in., $Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(24)(0.9)}{2.3328} = 3.47 \text{ ksi}$$

$$\tau = \frac{(G)(5.184)}{(16.5888)(1.8)} = 1.042 \text{ ksi}$$

(b) Point b : $x = 1.2$ in., $z = 0.9$ in., $Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + \frac{(60)(1.2)}{16.5888} + \frac{(24)(0.9)}{2.3328} = 7.81 \text{ ksi}$$

$$\tau = \frac{(G)(3.888)}{(16.5888)(1.8)} = 0.781 \text{ ksi}$$

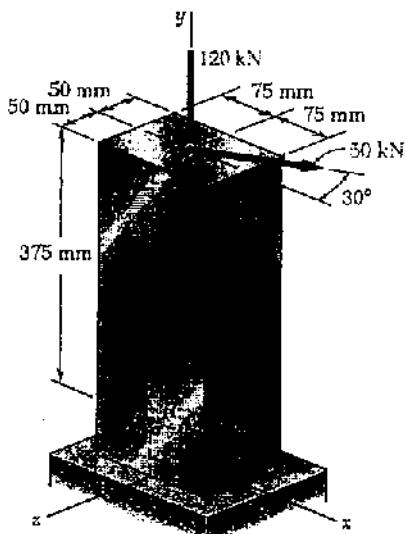
(c) Point c : $x = 2.4$ in., $z = 0.9$ in., $Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(60)(2.4)}{16.5888} + \frac{(24)(0.9)}{2.3328} = 12.15 \text{ ksi}$$

$$\tau = 0$$

Problem 8.49

8.49 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point H.



Components of force at point C.

$$F_x = 50 \cos 30^\circ = 43.301 \text{ kN}$$

$$F_z = -50 \sin 30^\circ = -25 \text{ kN}, \quad F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points H and K.

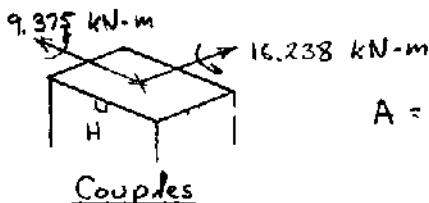
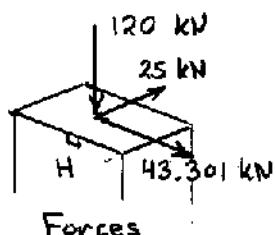
$$P = 120 \text{ kN} \text{ (compression)}$$

$$V_x = 43.301 \text{ kN}, \quad V_z = -25 \text{ kN}$$

$$M_x = -(25)(0.375) = -9.375 \text{ kN-m}$$

$$M_y = 0$$

$$M_2 = -(43.301)(0.375) = -16.238 \text{ kN-m}$$



$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 = 15 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6 \text{ mm}^4 = 12.5 \times 10^{-6} \text{ m}^4$$

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120 \times 10^3)}{15 \times 10^{-3}} - \frac{(-9.375 \times 10^3)(50 \times 10^3)}{12.5 \times 10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2} \frac{V_x}{A} = \frac{3}{2} \frac{43.301 \times 10^3}{15 \times 10^{-3}} = 4.33 \text{ MPa}$$

Use Mohr's circle.

$$\sigma_c = \frac{1}{2} \sigma_H = 14.75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 15.37 \text{ MPa}$$

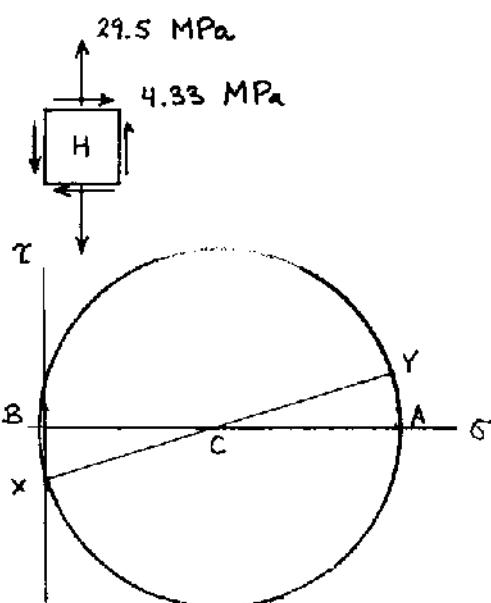
$$\sigma_a = \sigma_c + R = 30.1 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -0.62 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$$

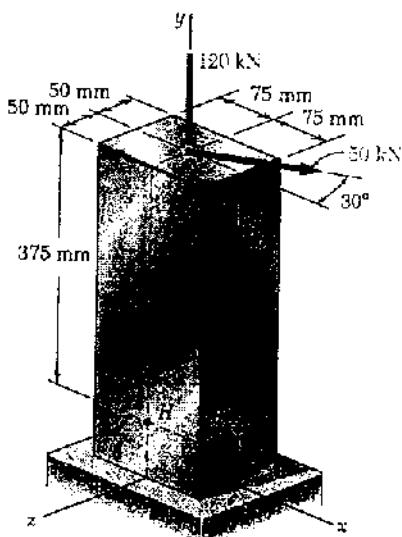
$$\theta_a = -8.2^\circ \quad \theta_b = 81.8^\circ$$

$$\tau_{max} = R = 15.37 \text{ MPa}$$



Problem 8.50

8.50 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point K.



Components of force at point C.

$$F_x = 50 \cos 30^\circ = 43.301 \text{ kN}$$

$$F_z = -50 \sin 30^\circ = -25 \text{ kN} \quad F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points H and K.

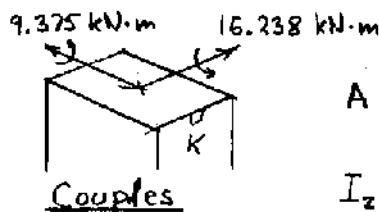
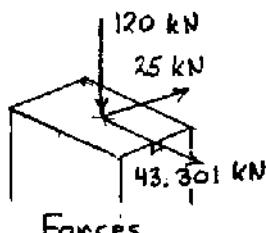
$$P = 120 \text{ kN} \text{ (compression)}$$

$$V_x = 43.301 \text{ kN}, \quad V_z = -25 \text{ kN}$$

$$M_x = -(25)(0.375) = -9.375 \text{ kN}\cdot\text{m}$$

$$M_y = 0$$

$$M_z = -(43.301)(0.375) = -16.238 \text{ kN}\cdot\text{m}$$



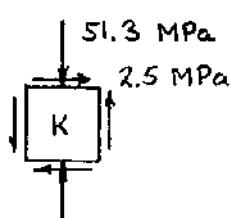
$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 \\ = 15 \times 10^{-3} \text{ m}^2$$

$$I_z = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4 \\ = 28.125 \times 10^{-4} \text{ m}^4$$

Stresses at point K.

$$\sigma_K = -\frac{P}{A} + \frac{M_z X}{I_z} = -\frac{120 \times 10^3}{15 \times 10^{-3}} + \frac{(-16.238 \times 10^3)(75 \times 10^{-3})}{28.125 \times 10^{-4}} = -51.3 \text{ MPa}$$

$$\tau_K = \frac{3}{2} \frac{V_z}{A} = \frac{3}{2} \frac{25 \times 10^3}{15 \times 10^{-3}} = 2.5 \text{ MPa}$$



Use Mohr's circle.

$$\sigma_c = \frac{1}{2} \sigma_K \mp -25.65 \text{ MPa}$$

$$R = \sqrt{\left(\frac{51.3}{2}\right)^2 + (2.5)^2} = 25.77 \text{ MPa}$$

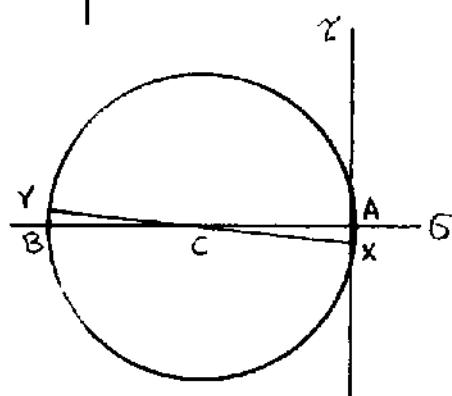
$$\sigma_a = \sigma_c + R = 0.12 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -51.4 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_K}{-\sigma_K} = 0.09747$$

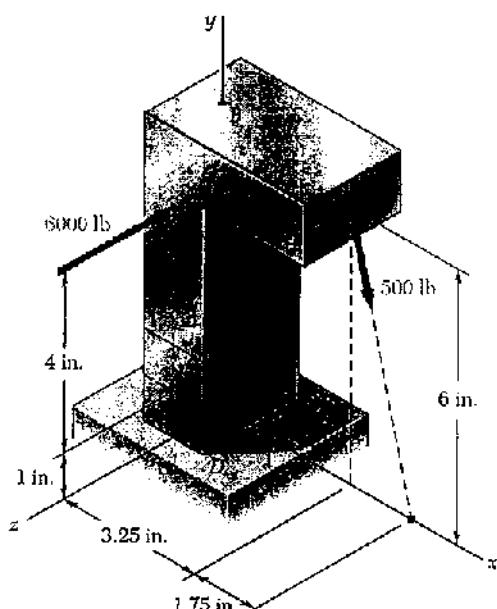
$$\theta_a = 2.8^\circ \quad \theta_b = 92.8^\circ$$

$$\tau_{max} = R = 25.8 \text{ MPa}$$



Problem 8.51

8.51 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5×2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.



Components of 500 lb-force.

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb.}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb.}$$

Moment arm of 500 lb-force.

$$\vec{r} = 3.25 \hat{i} + (6-1) \hat{j}$$

Moment of 500 lb-force.

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260 \hat{k} \text{ lb-in}$$

At the section containing point *H*, $P = -480 \text{ lb.}$ $V_x = 140 \text{ lb.}$

$$V_z = -6000 \text{ lb.}, M_z = -2260 \text{ lb-in}, M_x = -(4)(6000) = -24000 \text{ lb-in.}$$

$$A = (1.5)(2.4) = 3.6 \text{ in}^2 \quad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2} \frac{V_z}{A} = \frac{3}{2} \frac{6000}{3.6} = 2500 \text{ psi}$$

Use Mohr's circle.

$$\sigma_c = -\frac{2644}{2} = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (2500)^2} = 2828 \text{ psi}$$

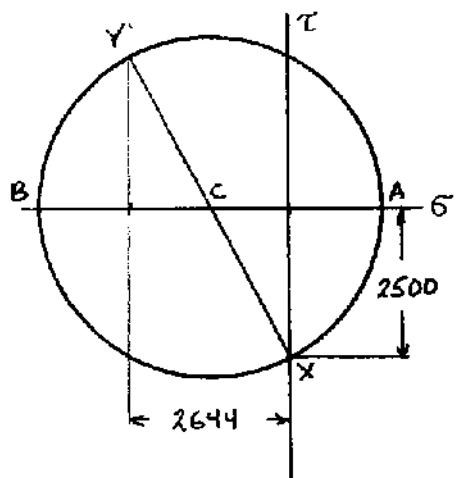
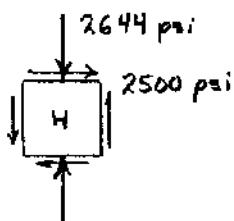
$$\sigma_a = \sigma_c + R = 1506 \text{ psi}$$

$$\sigma_b = \sigma_c - R = -4150 \text{ psi}$$

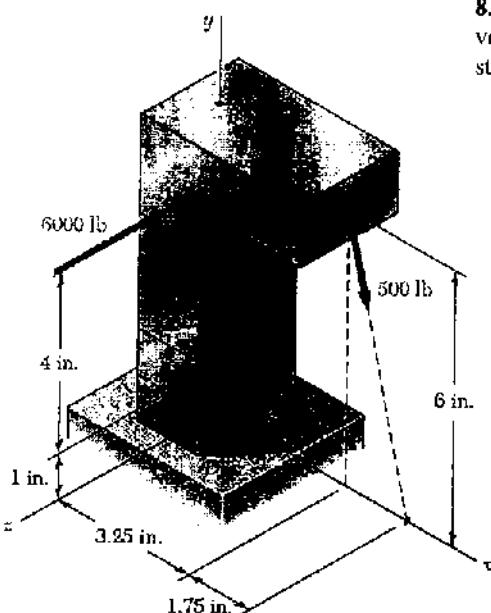
$$\tan 2\theta_p = \frac{2\tau_p}{1506} = \frac{(2)(2500)}{2644} = 1.891$$

$$\theta_a = 34.1^\circ, \theta_b = 121.1^\circ$$

$$\tau_{max} = R = 2828 \text{ psi}$$



Problem 8.52



8.52 Solve Prob. 8.51, assuming that the magnitude of the 6000-lb force is reduced to 1500 lb.

8.51 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5×2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

Components of 500 lb. force.

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb}$$

Moment arm of 500 lb. force.

$$\vec{r} = 3.25 \hat{i} + (6-1) \hat{j}$$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260 \hat{k} \text{ lb-in}$$

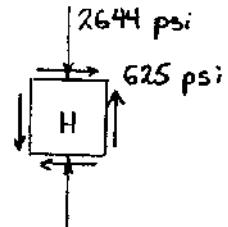
At the section containing point *H*, $P = -480 \text{ lb}$, $V_x = 140 \text{ lb}$.

$$V_z = -1500 \text{ lb}, M_z = -2260 \text{ lb-in}, M_x = -(4)(1500) = -6000 \text{ lb-in}$$

$$A = (1.5)(2.4) = 3.6 \text{ in}^2 \quad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2} \frac{V_z}{A} = \frac{3}{2} \frac{1500}{3.6} = 625 \text{ psi}$$



Use Mohr's circle.

$$\sigma_c = \frac{1}{2} \sigma_H = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (625)^2} = 1462 \text{ psi}$$

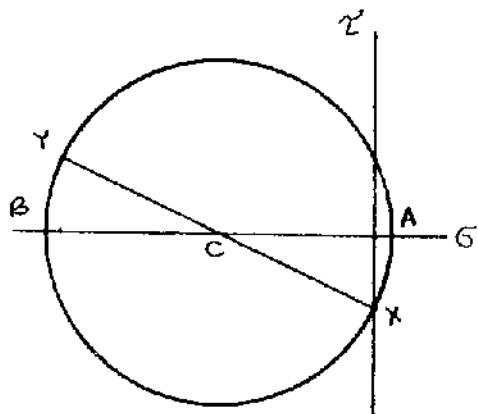
$$\sigma_a = \sigma_c + R = 140 \text{ psi}$$

$$\sigma_b = \sigma_c - R = -2784 \text{ psi}$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(625)}{2644} = 0.4728$$

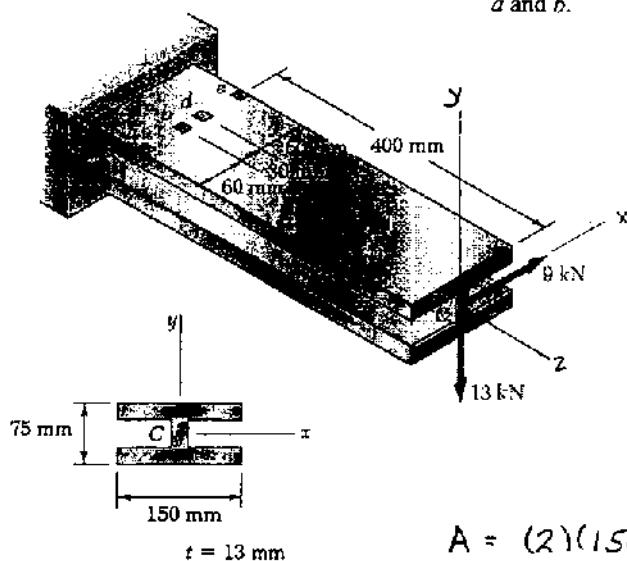
$$\theta_a = 12.7^\circ \quad \theta_b = 102.7^\circ$$

$$\tau_{max} = R = 1462 \text{ psi}$$



Problem 8.53

8.53 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.



Equivalent force - couple system at section containing points *a* and *b*.

$$F_x = 9 \text{ kN}, F_y = -13 \text{ kN}, F_z = 0$$

$$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N}\cdot\text{m}$$

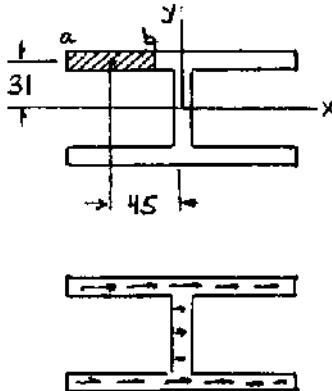
$$M_y = 0.400(9 \times 10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75 - 26) = 4537 \text{ mm}^2 \\ = 4537 \times 10^{-6} \text{ m}^2$$

$$I_x = 2 \left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5 - 6.5)^2 \right] + \frac{1}{12}(13)(75 - 26)^3 = 3.9303 \times 10^6 \text{ mm}^4 \\ = 3.9303 \times 10^{-6} \text{ m}^4$$

$$I_y = 2 \cdot \frac{1}{12}(13)(150)^3 + \frac{1}{12}(75 - 26)(13)^3 = 7.3215 \times 10^6 \text{ mm}^4 = 7.3215 \times 10^{-6} \text{ m}^4$$



$$\text{For point } a, Q_x = 0 \quad Q_y = 0$$

$$\text{For point } b, A^* = (60)(13) = 780 \text{ mm}^2 \\ \bar{x} = -45 \text{ mm} \quad \bar{y} = 31 \text{ mm}$$

$$Q_x = A^* \bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$$

$$Q_y = A^* \bar{x} = -35.1 \times 10^{-3} \text{ mm}^3 = -35.1 \times 10^{-6} \text{ m}^3$$

$$\text{At point } a, \sigma_a = \frac{M_y y}{I_x} - \frac{M_x y}{I_y} \\ = \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(-75 \times 10^{-3})}{7.3215 \times 10^{-6}} = 86.5 \text{ MPa} \quad \blacksquare$$

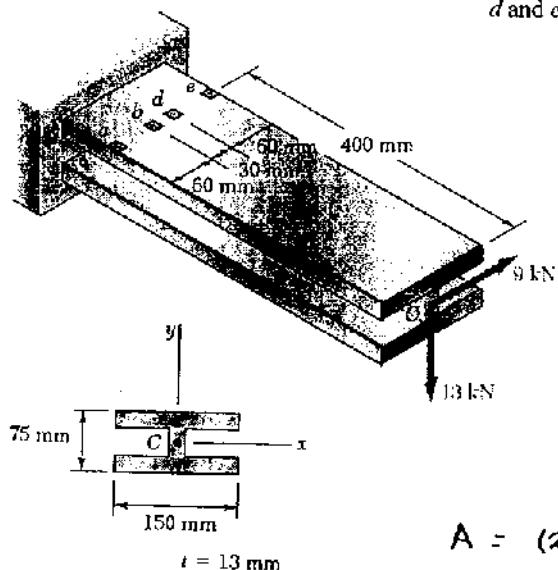
$$\tau_a = 0 \quad \blacksquare$$

$$\text{At point } b, \sigma_b = \frac{M_y y}{I_x} - \frac{M_x y}{I_y} \\ = \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(-15 \times 10^{-3})}{7.3215 \times 10^{-6}} = 57.0 \text{ MPa} \quad \blacksquare$$

$$\tau_b = \frac{|V_x|Q_y}{I_y t} + \frac{N_z|Q_x|}{I_x t} = \frac{(9 \times 10^3)(35.1 \times 10^{-6})}{(7.3215 \times 10^{-6})(13 \times 10^{-3})} + \frac{(13 \times 10^3)(24.18 \times 10^{-6})}{(3.9303 \times 10^{-6})(13 \times 10^{-3})} \\ = 3.32 \text{ MPa} + 6.15 \text{ MPa} = 9.47 \text{ MPa} \quad \blacksquare$$

Problem 8.54

8.54 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.



Equivalent force-couple system at section containing points *a* and *b*.

$$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N}\cdot\text{m}$$

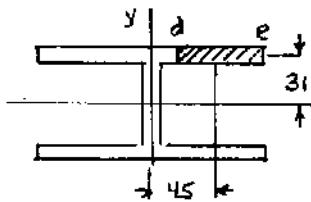
$$M_y = (0.400)(9 \times 10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75 - 26) = 4537 \text{ mm}^2 \\ = 4537 \times 10^{-6} \text{ m}^2$$

$$I_x = 2 \left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5 - 6.5)^2 \right] + \frac{1}{12}(13)(75 - 26)^3 = 3.9803 \times 10^6 \text{ mm}^4 \\ = 3.9803 \times 10^{-6} \text{ m}^4$$

$$I_y = 2 \left[\frac{1}{12}(13)(150)^3 \right] + \frac{1}{12}(75 - 26)(13)^3 = 7.3215 \times 10^6 \text{ mm}^4 = 7.3215 \times 10^{-6} \text{ m}^4$$



$$\text{For point } d, \quad A^* = (60)(13) = 780 \text{ mm}^2 \quad \bar{x} = 45 \text{ mm} \quad \bar{y} = 31 \text{ mm}$$

$$Q_x = A^* \bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$$

$$Q_y = A^* \bar{x} = 35.1 \times 10^3 \text{ mm}^3 = 35.1 \times 10^{-6} \text{ m}^3$$

$$\text{For point } e, \quad Q_x = 0, \quad Q_y = 0$$



$$\text{At point } d, \quad \sigma_d = \frac{M_x y}{I_y} - \frac{M_y x}{I_x} \\ = \frac{(5200)(37.5 \times 10^{-3})}{3.9803 \times 10^{-6}} - \frac{(3600)(15 \times 10^{-3})}{7.3215 \times 10^{-6}} = 42.2 \text{ MPa} \rightarrow$$



$$\text{Due to } V_x: \quad \tau_d = \frac{|V_x| Q_y}{I_y t} = \frac{(9000)(35.1 \times 10^{-6})}{(7.3215 \times 10^{-6})(13 \times 10^{-3})} = 3.32 \text{ MPa} \rightarrow$$

$$\text{Due to } V_y: \quad \tau_d = \frac{|V_y| Q_x}{I_x t} = \frac{(13000)(24.18 \times 10^{-6})}{(3.9803 \times 10^{-6})(13 \times 10^{-3})} = 6.15 \text{ MPa} \leftarrow$$

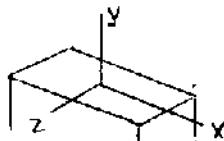
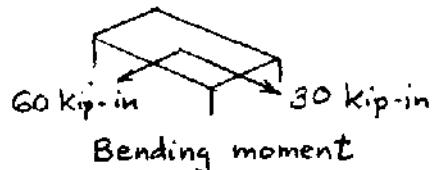
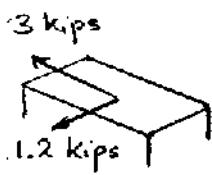
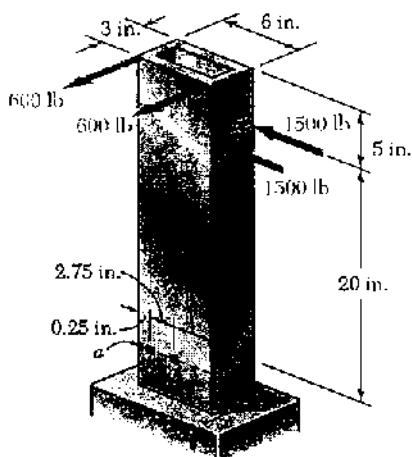
$$\text{Net} \quad \tau_d = 2.83 \text{ MPa} \rightarrow$$

$$\text{At point } e, \quad \sigma_e = \frac{M_x y}{I_y} - \frac{M_y x}{I_x} = \frac{(5200)(37.5 \times 10^{-3})}{3.9803 \times 10^{-6}} - \frac{(3600)(75 \times 10^{-3})}{7.3215 \times 10^{-6}} \\ = 12.74 \text{ MPa} \rightarrow$$

$$\tau_e = 0 \rightarrow$$

Problem 8.55

8.55 Knowing that the structural tube shown has a uniform wall thickness of 0.25 in., determine the normal and shearing stresses at the three points indicated.



$$b_o = 6 \text{ in.} \quad b_i = b_o - 2t = 5.5 \text{ in.} \\ h_o = 3 \text{ in.} \quad h_i = h_o - 2t = 2.5 \text{ in.}$$

$$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 6.3385 \text{ in}^4 \quad I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 19.3385 \text{ in}^4$$

Normal stresses

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

$$\frac{(60)(-3)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -16.41 \text{ ksi}$$

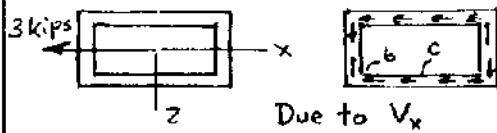
$$\frac{(60)(-2.75)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -15.63 \text{ ksi}$$

$$\frac{(60)(0)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -7.10 \text{ ksi}$$

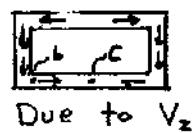
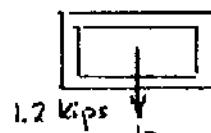
Shearing stresses

Point a is an outside corner; $\tau_a = 0$

Direction of shearing stresses

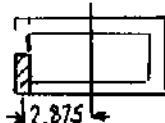


Due to V_x



Due to V_z

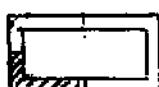
At point b



$$Q_{zb} = (1.5)(0.25)(2.875) = 1.0781 \text{ in}^3$$

$$\tau_{b,V_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(1.0781)}{(19.3385)(0.25)} = 0.669 \text{ ksi}$$

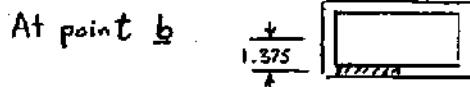
At point c



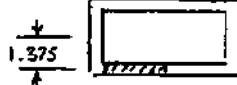
$$Q_{zc} = Q_{zb} + (2.75)(0.25)(\frac{3.25}{2})$$

$$= 2.0234 \text{ in}^3$$

$$\tau_{c,V_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(2.0234)}{(19.3385)(0.25)} = 1.256 \text{ ksi}$$



At point b



$$Q_{xb} = (2.75)(0.25)(1.375) = 0.9453 \text{ in}^3$$

$$\tau_{b,V_z} = \frac{V_z Q_x}{I_x t} = \frac{(1.2)(0.9453)}{(6.3385)(0.25)} = 0.716 \text{ ksi}$$

At point c (symmetry axis)

$$\tau_{c,V_z} = 0$$

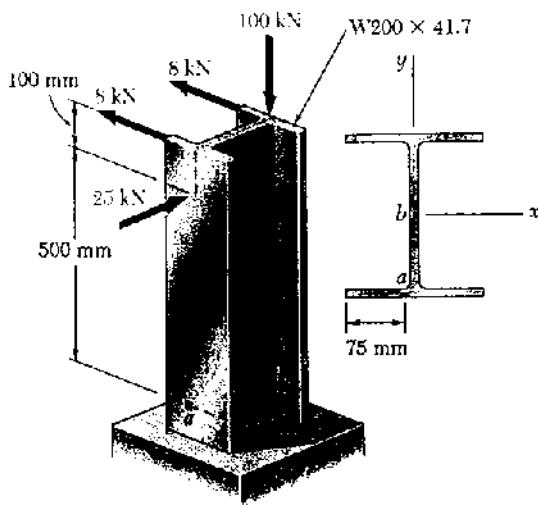
Net shearing stress at points b and c

$$\tau_b = 0.716 - 0.669 = 0.047 \text{ ksi}$$

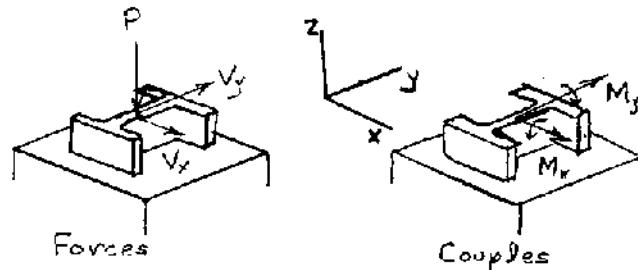
$$\tau_c = 1.256 \text{ ksi}$$

Problem 8.56

8.56 Four forces are applied to a W 200 × 41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point a .



Calculate forces and couples at section containing point a .



Section properties.

$$A = 5310 \text{ mm}^2 \quad d = 205 \text{ mm}$$

$$b_f = 166 \text{ mm} \quad t_f = 11.8 \text{ mm}$$

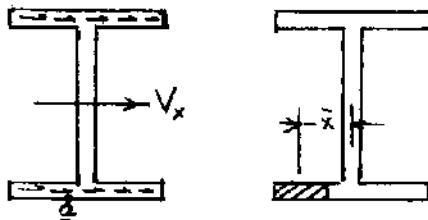
$$t_w = 7.2 \text{ mm}$$

$$I_x = 40.9 \times 10^6 \text{ mm}^4 = 40.9 \times 10^{-6} \text{ m}^4, \quad I_y = 9.01 \times 10^6 \text{ mm}^4 = 9.01 \times 10^{-6} \text{ m}^4$$

$$\text{Point } a: \quad x_a = -\frac{166}{2} + 75 = -8 \text{ mm}, \quad y_a = -\frac{205}{2} = -102.5 \text{ mm}$$

$$\begin{aligned} \sigma_a &= -\frac{P}{A} + \frac{M_x y_a}{I_x} + \frac{M_y x_a}{I_y} \\ &= -\frac{100 \times 10^3}{5310 \times 10^{-6}} + \frac{(-22.75 \times 10^3)(-102.5 \times 10^{-3})}{40.9 \times 10^{-6}} + \frac{(9.6 \times 10^3)(-8 \times 10^{-3})}{9.01 \times 10^{-6}} \\ &= -18.83 \times 10^6 + 57.01 \times 10^6 - 8.52 \times 10^6 = 29.66 \text{ MPa} \end{aligned}$$

Shearing stress at point a due to V_x



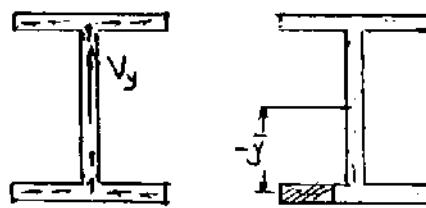
$$A = (75)(11.8) = 885 \text{ mm}^2$$

$$\bar{x} = -\frac{166}{2} + \frac{75}{2} = -45.5 \text{ mm}$$

$$Q = -A \bar{x} = 40.2675 \times 10^3 \text{ mm}^3$$

$$\tau_{x2} = \frac{V_x Q}{I_y t} = \frac{(-16 \times 10^3)(40.2675 \times 10^3)}{(9.01 \times 10^{-6})(11.8 \times 10^{-3})} = -6.060 \text{ MPa}$$

Shearing stress at point a due to V_y



$$A = (75)(11.8) = 885 \text{ mm}^2$$

$$\bar{y} = -\frac{205}{2} + \frac{11.8}{2} = -96.6 \text{ mm}$$

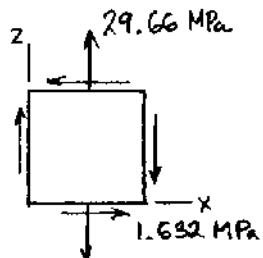
$$Q = -A \bar{y} = -85.491 \times 10^3 \text{ mm}^3$$

$$\tau_{y2} = \frac{V_y Q}{I_x t} = \frac{(25 \times 10^3)(-85.491 \times 10^3)}{(40.9 \times 10^{-6})(11.8 \times 10^{-3})} = 4.428 \text{ MPa}$$

Continued

Problem 8.56 continued

Combined shearing stress $\tau_g = -6.060 + 4.428 = -1.632 \text{ MPa}$



$$\sigma_{ave} = \frac{29.66 + 0}{2} = 14.83 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.66 - 0}{2}\right)^2 + (-1.632)^2} = 14.92 \text{ MPa}$$

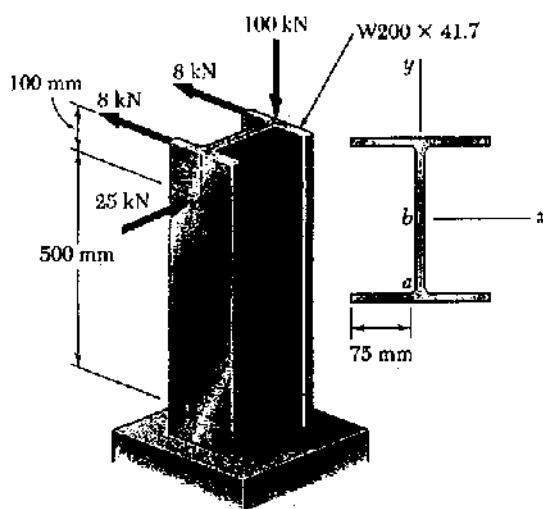
$$\sigma_{max} = \sigma_{ave} + R = 29.75 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -0.09 \text{ MPa}$$

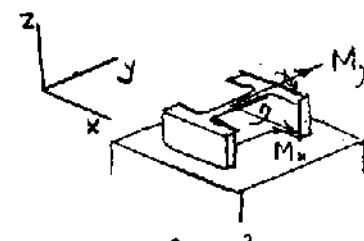
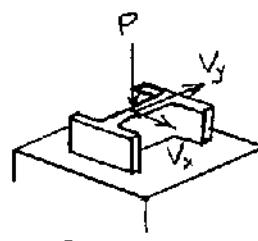
$$\tau_{max} = R = 14.92 \text{ MPa}$$

Problem 8.57

8.57 Four forces are applied to a W 200 × 41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point b.



Calculate forces and couples at section containing point b.



$$P = 100 \text{ kN} \quad V_x = -16 \text{ kN} \quad V_y = 25 \text{ kN}$$

$$M_x = -(500 \times 10^3)(25 \times 10^3) \\ - \left(\frac{205}{2} \times 10^3\right)(100 \times 10^3) \\ = -22.75 \times 10^3 \text{ N·m}$$

$$M_y = (500 + 100)(10^3)(16 \times 10^3) \\ = 9.6 \times 10^3 \text{ N·m}$$

Section properties.

$$A = 5310 \text{ mm}^2 \quad d = 205 \text{ mm}$$

$$b_f = 166 \text{ mm} \quad t_f = 11.8 \text{ mm}$$

$$t_w = 7.2 \text{ mm}$$

$$I_x = 40.9 \times 10^6 \text{ mm}^4 = 40.9 \times 10^6 \text{ m}^4, \quad I_y = 9.01 \times 10^6 \text{ mm}^4 = 9.01 \times 10^6 \text{ m}^4$$

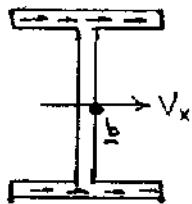
Point b: $x_b = 0, y_b = 0$

$$\sigma_b = -\frac{P}{A} + \frac{M_x y_b}{I_x} + \frac{M_y x_b}{I_y} = -\frac{100 \times 10^3}{5310 \times 10^6} = -18.83 \text{ MPa}$$

continued

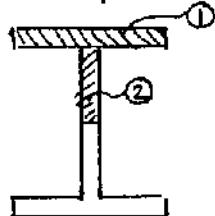
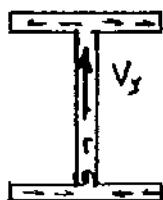
Problem 8.57 continued

Shearing stress at point b due to V_x .



$$\tau_{xz} = 0$$

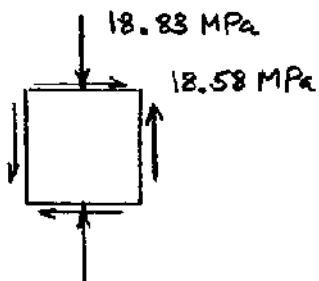
Shearing stress at point b due to V_y



	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3 \text{ mm}^3)$
①	1958.8	96.6	189.220
②	653.04	45.35	29.615
Z			218.835

$$Q = \sum A\bar{y} = 218.835 \times 10^3 \text{ mm}^3 = 218.835 \times 10^{-6} \text{ m}^3$$

$$\tau_b' = \frac{V_y Q}{I_x t_w} = \frac{(25 \times 10^3)(218.835 \times 10^{-6})}{(40.9 \times 10^{-6})(7.2 \times 10^{-3})} = 18.58 \text{ MPa}$$



$$\bar{\sigma}_{ave} = \frac{-18.83 + 0}{2} = -9.415 \text{ MPa}$$

$$R = \sqrt{\left(\frac{-18.83 - 0}{2}\right)^2 + (18.58)^2} = 20.829 \text{ MPa}$$

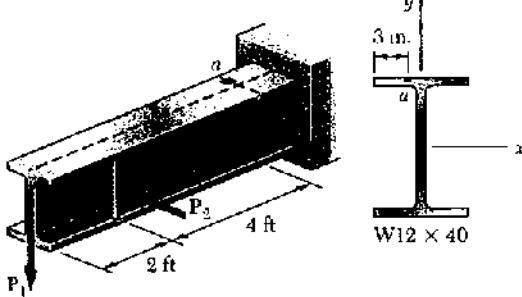
$$\bar{\sigma}_{max} = \bar{\sigma}_{ave} + R = 11.41 \text{ MPa}$$

$$\bar{\sigma}_{min} = \bar{\sigma}_{ave} - R = -30.2 \text{ MPa}$$

$$\tau_{max} = R = 20.8 \text{ MPa}$$

Problem 8.58

8.58 Two forces P_1 and P_2 , are applied as shown in directions perpendicular to the longitudinal axis of a W 12 × 40 beam. Knowing that $P_1 = 5$ kips and $P_2 = 3$ kips, determine the principal stresses and the maximum shearing stress at point a .



At the section containing point a

$$M_x = (6)(5) = 30 \text{ kip-ft} = 360 \text{ kip-in}$$

$$M_y = -(4)(3) = -12 \text{ kip-ft} = -144 \text{ kip-in}$$

$$V_x = -3 \text{ kips} \quad V_y = -5 \text{ kips}$$

For W 12 × 40 rolled steel section

$$d = 11.94 \text{ in.} \quad b_f = 8.005 \text{ in.} \quad t_f = 0.515 \text{ in.} \quad t_w = 0.295 \text{ in.}$$

$$I_x = 310 \text{ in}^4 \quad I_y = 44.1 \text{ in}^4$$

Normal stress at point a . $x = -\frac{b_f}{2} + 3 = -1.0025 \text{ in.} \quad y = \frac{d}{2} = 5.97 \text{ in}$

$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(360)(5.97)}{310} - \frac{(-144)(-1.0025)}{44.1} \\ = 6.933 - 3.273 = 3.660 \text{ ksi}$$

Shearing stress at point a .

$$\tau_{xz} = \frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$

$$A^* = (3)(0.515) = 1.545 \text{ in.}$$

$$\bar{x} = \frac{8.005}{2} + \frac{3}{2} = -2.5025 \text{ in.}$$

$$\bar{y} = \frac{11.94}{2} - \frac{0.515}{2} = 5.7125 \text{ in.}$$

$$\tau_{xz} = \frac{(-3)(1.545)(-2.5025)}{(44.1)(0.515)} - \frac{(-5)(1.545)(5.7125)}{(310)(0.515)} \\ = 0.5107 - 0.2764 = 0.234 \text{ ksi}$$

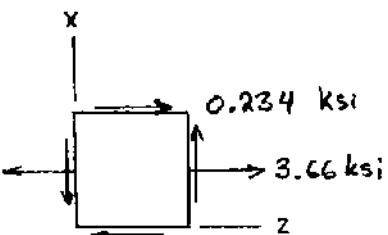
$$\sigma_{ave} = \frac{3.66 + 0}{2} = 1.830 \text{ ksi}$$

$$R = \sqrt{\left(\frac{3.66 - 0}{2}\right)^2 + 0.234^2} = 1.845 \text{ ksi}$$

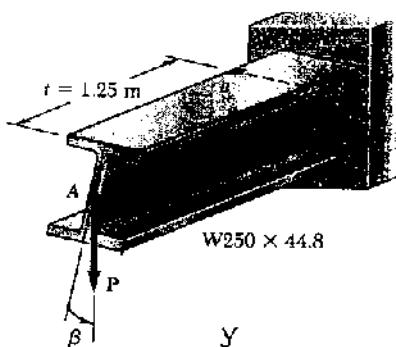
$$\sigma_{max} = \sigma_{ave} + R = 1.830 + 1.845 = 3.675 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = 1.830 - 1.845 = -0.015 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{max} = R = 1.845 \text{ ksi} \quad \blacktriangleleft$$



Problem 8.59



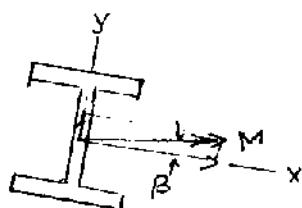
8.59 A vertical force P is applied at the center of the free end of cantilever beam AB .
 (a) If the beam is installed with the web vertical ($\beta = 0$) and with its longitudinal axis AB horizontal, determine the magnitude of the force P for which the normal stress at point a is $+120 \text{ MPa}$. (b) Solve part a, assuming that the beam is installed with $\beta = 3^\circ$.

For W250 x 44.8 rolled steel section

$$S_x = 535 \times 10^3 \text{ mm}^3 = 535 \times 10^{-6} \text{ m}^3$$

$$S_y = 95.0 \times 10^3 \text{ mm} = 95.0 \times 10^{-6} \text{ m}^3$$

At the section containing point a .



Stress at a

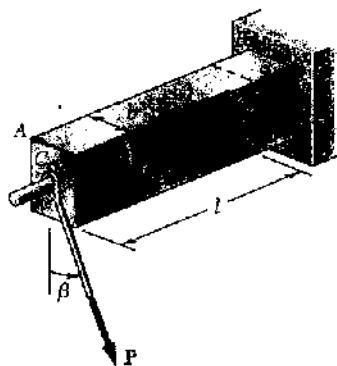
$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl \cos \beta}{S_x} + \frac{Pl \sin \beta}{S_y}$$

Allowable load. $P_{all} = \frac{G_{all}}{l} \left[\frac{\cos \beta}{S_x} + \frac{\sin \beta}{S_y} \right]^{-1}$

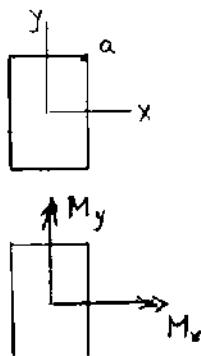
(a) $\beta = 0$: $P_{all} = \frac{120 \times 10^6}{1.25} \left[\frac{1}{535 \times 10^{-6}} + 0 \right]^{-1} = 51.4 \times 10^3 \text{ N} = 51.4 \text{ kN}$ —

(b) $\beta = 3^\circ$: $P_{all} = \frac{120 \times 10^6}{1.25} \left[\frac{\cos 3^\circ}{535 \times 10^{-6}} + \frac{\sin 3^\circ}{95.0 \times 10^{-6}} \right]^{-1} = 39.7 \text{ kN}$ —

Problem 8.60



8.60 A force P is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that P acts in a direction perpendicular to the longitudinal axis of the beam, determine (a) the normal stress at point a in terms of P , b , h , l , and β , (b) the values of β for which the normal stress at a is zero.



$$I_x = \frac{1}{12} b h^3 \quad I_y = \frac{1}{12} h b^3$$

$$\sigma = \frac{M_x(b/2)}{I_x} - \frac{M_y(b/2)}{I_y}$$

$$= \frac{6M_x}{bh^3} - \frac{6M_y}{hb^3}$$

$$\vec{P} = P \sin \beta \hat{i} - P \cos \beta \hat{j} \quad \vec{r} = l \hat{k}$$

$$\vec{M} = \vec{r} \times \vec{P} = l \hat{k} \times (P \sin \beta \hat{i} - P \cos \beta \hat{j}) = Pl \cos \beta \hat{i} + Pl \sin \beta \hat{j}$$

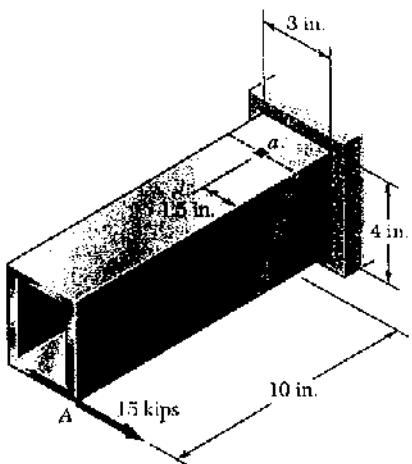
$$M_x = Pl \cos \beta \quad M_y = Pl \sin \beta$$

$$(a) \quad \sigma = \frac{6Pl \cos \beta}{bh^3} - \frac{6Pl \sin \beta}{hb^3} = \frac{6Pl}{bh} \left[\frac{\cos \beta}{h} - \frac{\sin \beta}{b} \right]$$

$$(b) \quad \sigma = 0 \quad \frac{\cos \beta}{h} - \frac{\sin \beta}{b} = 0 \quad \tan \beta = \frac{b}{h}$$

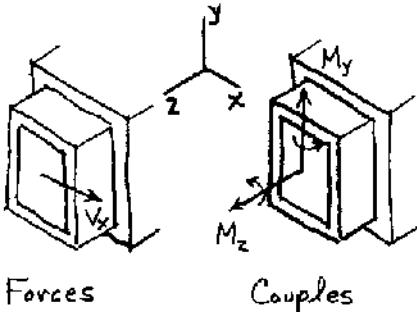
$$\beta = \tan^{-1}\left(\frac{b}{h}\right)$$

Problem 8.61



*8.61 The structural tube shown has a uniform wall thickness of 0.3 in. Knowing that the 15-kip load is applied 0.15 in. above the base of the tube, determine the shearing stress at (a) point a, (b) point b.

Calculate forces and couples at section containing points a and b.



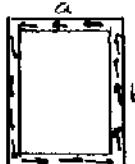
$$V_x = 15 \text{ kips}$$

$$M_z = (2 - 0.15)(15) = 27.75 \text{ kip-in}$$

$$M_y = (10)(15) \approx 150 \text{ kip-in}$$

Shearing stresses due to torque $T = M_z$

$$Q = [3 - (2)(0.15)][4 - (2)(0.15)] = 9.99 \text{ in}^2$$



$$\tau = \frac{M_z}{2Q} = \frac{27.75}{(2)(9.99)} = 1.3889 \text{ kip/in}$$

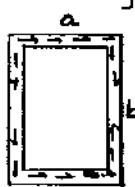
At point a. $t = 0.3 \text{ in.}$

$$\tau_a = \frac{\tau}{2} = \frac{1.3889}{0.3} = 4.630 \text{ ksi} \leftarrow$$

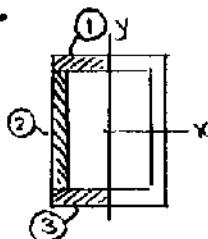
At point b. $t = 0.3 \text{ in.}$

$$\tau_b = \frac{\tau}{2} = \frac{1.3889}{0.3} = 4.630 \text{ ksi} \uparrow$$

Shearing stresses due to V_x .



At point a



Part	$A (\text{in}^2)$	$\bar{x} (\text{in.})$	$A \bar{x} (\text{in}^3)$
①	0.45	-0.75	-0.3375
②	1.02	-1.35	-1.377
③	0.45	-0.75	-0.3375
Σ			-2.052

$$Q = |\sum A \bar{x}| = 2.052 \text{ in}^3 \quad t = (2)(0.3) = 0.6 \text{ in.}$$

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832$$

$$\tau_a = \frac{V_x Q}{I_y t} = \frac{(15)(2.052)}{(5.0832)(0.6)} = 10.092 \text{ ksi}$$

At point b $\tau_b = 0$

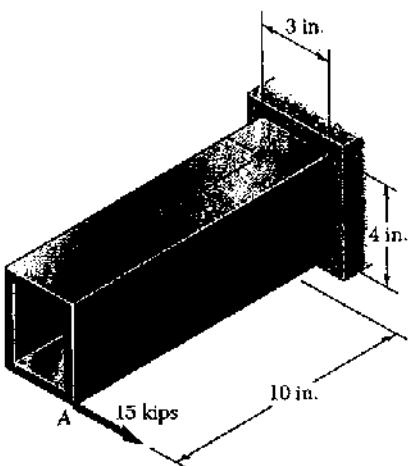
Combined shearing stresses.

$$\text{At point } \underline{a}. \quad \tau_a = 4.630 \leftarrow + 10.092 \rightarrow = 5.46 \text{ ksi} \rightarrow$$

$$\text{At point } \underline{b}. \quad \tau_b = 4.630 \uparrow + 0 = 4.63 \text{ ksi} \uparrow$$

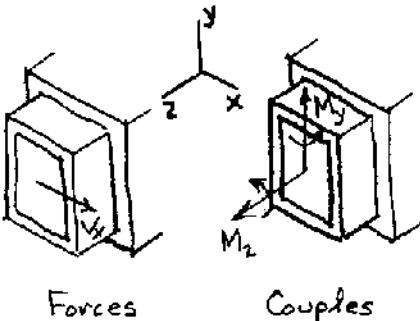
Problem 8.62

*8.62 For the tube and loading of Prob. 8.61, determine the principal stresses and the maximum shearing stress at point b.



*8.61 Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the shearing stress at points a and b.

Calculate forces and couples at section containing point b.



$$V_x = 15 \text{ kips}$$

$$M_z = (2 - 0.15)(15) \\ = 27.75 \text{ kip-in}$$

$$M_y = (10)(15) \\ = 150 \text{ kip-in}$$

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832 \text{ in}^4$$

$$\sigma_b = -\frac{M_y x_b}{I_y} = -\frac{(150)(1.5)}{5.0832} = -44.26 \text{ ksi}$$

Shearing stress at point b due to torque M_z .

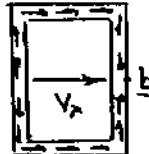
$$Q = [3 - (2)(0.15)][4 - (2)(0.15)] = 9.99 \text{ in}^3$$

$$q = \frac{M_z}{2Q} = \frac{27.75}{(2)(9.99)} = 1.38889 \text{ kip/in}$$

$$\tau = \frac{q}{t} = \frac{1.38889}{0.3} = 4.630 \text{ ksi}$$

Shearing at point b due to V_x .

$$\tau = 0$$



Calculation of principal stresses and maximum shearing stress.

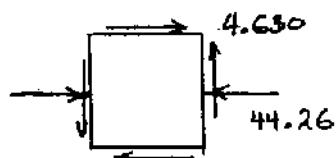
$$\sigma_{ave} = \frac{-44.26 + 0}{2} = -22.13 \text{ ksi}$$

$$R = \sqrt{\left(\frac{-44.26 - 0}{2}\right)^2 + (4.630)^2} = 22.61 \text{ ksi}$$

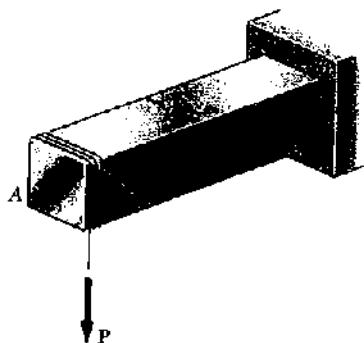
$$\sigma_{max} = \sigma_{ave} + R = 0.48 \text{ ksi}$$

$$\sigma_{min} = \sigma_{ave} - R = -44.7 \text{ ksi}$$

$$\tau_{max} = R = 22.6 \text{ ksi}$$



Problem 8.63



*8.63 A 5-kN force P is applied to a wire that is wrapped around bar AB as shown. Knowing that the cross section of the bar is a square of side $d = 40 \text{ mm}$, determine the principal stresses and the maximum shearing stress at point a .

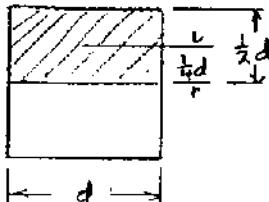
Bending: Point a lies on the neutral axis.

$$\sigma = 0$$

Torsion: $\tau = \frac{T}{c_1 ab^2}$ where $a = b = d$ and
 $c_1 = 0.208$ for a square section.

$$\text{Since } T = \frac{Pd}{2}, \quad \tau_T = \frac{P}{0.416 d^2} = 2.404 \frac{P}{d^2}$$

Transverse shear: $V = P$ $I = \frac{1}{12} d^4$



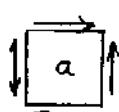
$$A = \frac{1}{2} d^2 \quad \bar{y} = \frac{1}{4} d \quad Q = A\bar{y} = \frac{1}{8} d^3$$

$$t = d$$

$$\tau_V = \frac{VQ}{It} = 1.5 \frac{P}{d^2}$$

$$\text{By superposition} \quad \tau = \tau_T + \tau_V = 3.904 \frac{P}{d^2}$$

$$\tau = \frac{(3.904)(5 \times 10^3)}{(40 \times 10^{-3})^2} = 12.2 \times 10^6 \text{ Pa} \quad 12.2 \text{ MPa.}$$



12.2 MPa

By Mohr's circles

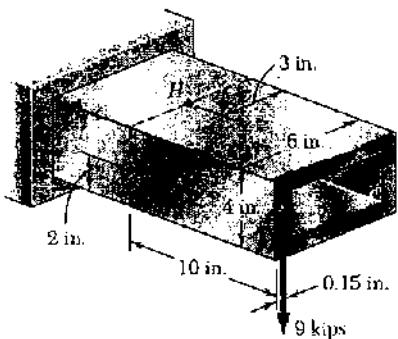
$$\sigma_{max} = 12.2 \text{ MPa}$$

$$\sigma_{min} = -12.2 \text{ MPa}$$

$$\tau_{max} = 12.2 \text{ MPa}$$

Problem 8.64

*8.64 Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the principal stresses, principal planes, and maximum shearing stress at (a) point H, (b) point K.



At the section containing points H and K

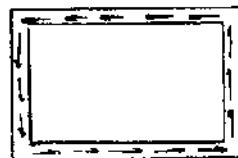
$$V = 9 \text{ kips} \quad M = (9)(10) = 90 \text{ kip-in.}$$

$$T = (9)(3 - 0.15) = 25.65 \text{ kip-in.}$$

Tension:

$$\alpha = (5.7)(3.7) = 21.09 \text{ in}^2$$

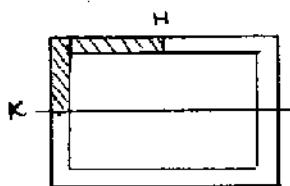
$$\gamma = \frac{T}{2t\alpha} = \frac{25.65}{(2)(0.3)(21.09)} = 2.027 \text{ ksi}$$



Transverse shear:

$$Q_H = 0$$

$$Q_K = (3)(2)(1) - (2.7)(1.7)(0.85) \\ = 2.0985 \text{ in}^3$$

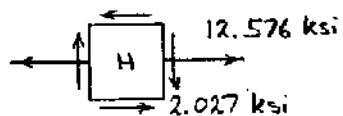


$$I = \frac{1}{12}(6)(4)^3 - \frac{1}{12}(5.4)(3.4)^3 = 14.3132 \text{ in}^4$$

$$\gamma_H = 0 \quad \gamma_K = \frac{VQ_H}{It} = \frac{(9)(2.0985)}{(14.3132)(0.3)} = 4.398 \text{ ksi}$$

$$\text{Bending: } \sigma_H = \frac{Mc}{I} = \frac{(90)(2)}{14.3132} = 12.576 \text{ ksi}, \quad \sigma_K = 0$$

(a) Point H:



$$\sigma_c = \frac{12.576}{2} = 6.288 \text{ ksi}$$

$$R = \sqrt{\left(\frac{12.576}{2}\right)^2 + (2.027)^2} = 6.607 \text{ ksi}$$

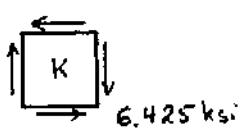
$$\sigma_{max} = \sigma_c + R = 12.90 \text{ ksi}$$

$$\sigma_{min} = \sigma_c - R = -0.32 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2T}{\sigma} = -0.3224 \quad \theta_p = -8.9^\circ, 81.1^\circ$$

$$\tau_{max} = R = 6.61 \text{ ksi}$$

(b) Point K: $\sigma = 0$ $\gamma = 2.027 + 4.398 = 6.425 \text{ ksi}$



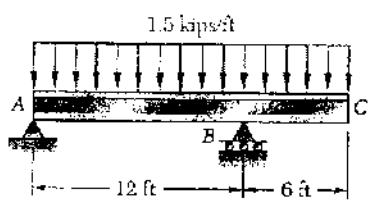
$$\sigma_{max} = 6.43 \text{ ksi}$$

$$\sigma_{min} = -6.43 \text{ ksi}$$

$$\theta_p = \pm 45^\circ$$

$$\tau_{max} = 6.43 \text{ ksi}$$

Problem 8.65



8.65 (a) Knowing that $\sigma_{all} = 24 \text{ ksi}$ and $\tau_{all} = 14.5 \text{ ksi}$, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

$$\textcircled{d} \sum M_A = 0 \quad -12R_A + (1.5)(18)(3) = 0 \quad R_A = 6.75 \text{ kips} \uparrow$$

$$\textcircled{d} \sum M_B = 0 \quad 12R_B + (1.5)(18)(9) = 0 \quad R_B = 20.25 \text{ kips} \uparrow$$

$$|V|_{max} = 11.25 \text{ kips}$$

$$|M|_{max} = 27 \text{ kip}\cdot\text{ft} = 324 \text{ kip}\cdot\text{in}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{324}{24} = 13.5 \text{ in}^3$$

Shape	$S (\text{in}^3)$
W 12 x 16	17.1
W 10 x 15	13.8
W 8 x 18	15.2
W 6 x 20	13.4

(a) Use

W 10 x 15

$$d = 9.99 \text{ in.}$$

$$t_f = 0.270 \text{ in.}$$

$$t_w = 0.230 \text{ in.}$$

$$(b) \sigma_m = \frac{|M|_{max}}{S} = \frac{324}{13.8} = 23.5 \text{ ksi}$$

$$\tau_m = \frac{|V|_{max}}{d t_w} = \frac{11.25}{(9.99)(0.230)} = 4.90 \text{ ksi}$$

$$c = \frac{1}{2}d = \frac{9.99}{2} = 4.995 \text{ in.}$$

$$y_b = c - t_f = 4.995 - 0.270 = 4.725 \text{ in.}$$

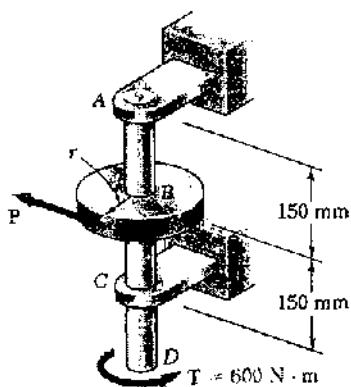
$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{4.725}{4.995}\right)(23.5) = 22.2 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{22.2}{2}\right)^2 + (4.90)^2} = 12.1 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{22.2}{2} + 12.1 = 23.2 \text{ ksi}$$

Problem 8.66

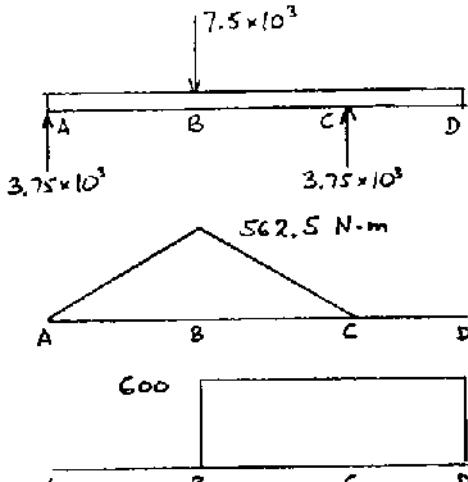
8.66 Determine the smallest allowable diameter of the solid shaft $ABCD$, knowing that $\tau_{all} = 60 \text{ MPa}$ and that the radius of disk B is $r = 80 \text{ mm}$.



$$\sum M_{axis} = 0$$

$$T - Pr = 0$$

$$P = \frac{T}{r} = \frac{600}{80 \times 10^{-3}} = 7.5 \times 10^3 \text{ N}$$



$$R_A = R_C = \frac{1}{2}P$$

$$= 3.75 \times 10^3 \text{ N}$$

$$M_B = (3.75 \times 10^3)(150 \times 10^{-3})$$

$$= 562.5 \text{ N·m}$$

Bending moment.

Torque.

Critical section lies at point B. $M = 562.5 \text{ N·m}$, $T = 600 \text{ N·m}$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{(\sqrt{M^2 + T^2})_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi} \frac{\sqrt{(562.5)^2 + (600)^2}}{60 \times 10^6} = 8.726 \times 10^{-6} \text{ m}^3$$

$$c = 20.58 \times 10^{-3} \text{ m} \quad d = 2c = 41.2 \times 10^{-3} \text{ m} = 41.2 \text{ mm}$$

Problem 8.67

8.67 Using the notation Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a cylindrical shaft can be expressed as

$$\sigma_{\max} = \frac{C}{J} [(M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}}]_{\max}$$

Maximum bending stress

$$\sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2} c}{I}$$

Maximum torsional stress

$$\tau_m = \frac{Tc}{J}$$

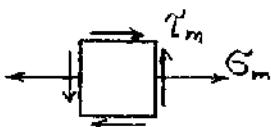
$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2} c}{2I} = \frac{c}{J} \sqrt{M_y^2 + M_z^2}$$

Using Mohr's circle,

$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2} (M_y^2 + M_z^2) + \frac{T^2 c^2}{J^2}}$$

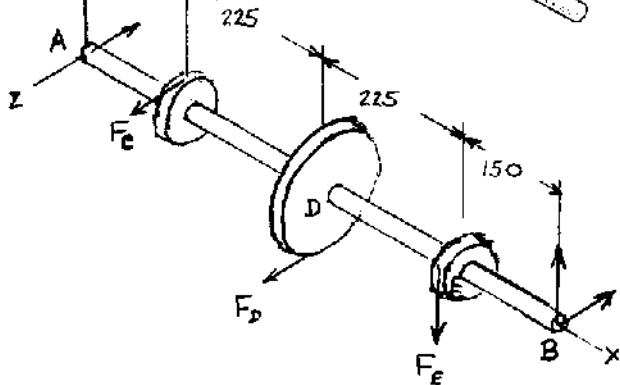
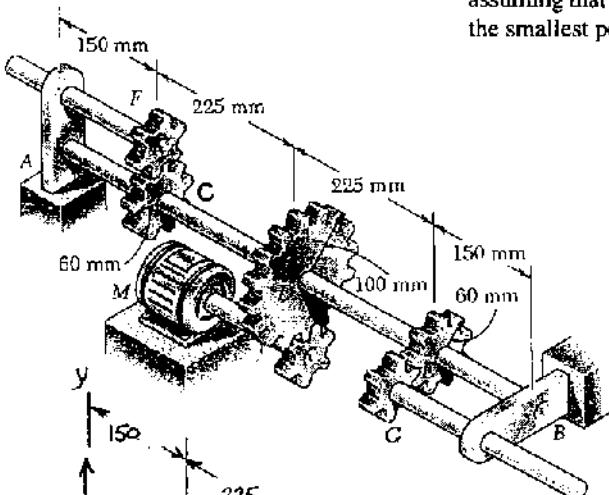
$$= \frac{c}{J} \sqrt{M_y^2 + M_z^2 + T^2}$$

$$\begin{aligned} \sigma_{\max} &= \frac{\sigma_m}{2} + R = \frac{c}{J} \sqrt{M_y^2 + M_z^2} + \frac{c}{J} \sqrt{M_y^2 + M_z^2 + T^2} \\ &= \frac{c}{J} \left[(M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}} \right] \end{aligned}$$

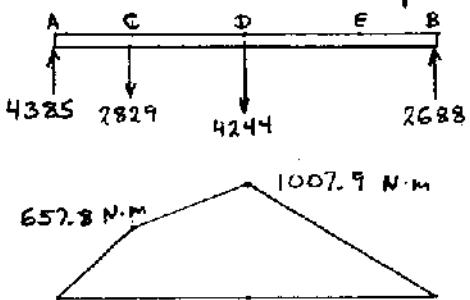


Problem 8.68

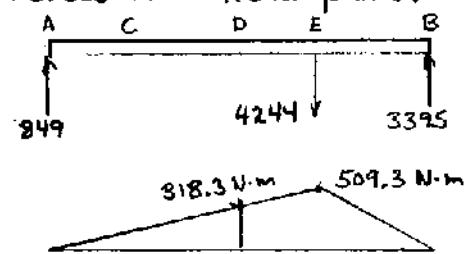
8.68 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G . Knowing that $\tau_{\text{all}} = 55 \text{ MPa}$ and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G , determine the smallest permissible diameter of shaft AB .



Forces in horizontal plane.



Forces in vertical plane.



$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D:

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)} = 424.41 \text{ N}\cdot\text{m}$$

Torques on gears C and E:

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N}\cdot\text{m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N}\cdot\text{m}$$

Forces on gears:

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60 \times 10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60 \times 10^{-3}} = 4244 \text{ N}$$

Torques in various parts

$$AC: T = 0$$

$$CD: T = 169.76 \text{ N}\cdot\text{m}$$

$$DE: T = 254.65 \text{ N}\cdot\text{m}$$

$$EB: T = 0$$

Critical point lies just to the right of D.

$$T = 254.65 \text{ N}\cdot\text{m}$$

$$M_y = 1007.9 \text{ N}\cdot\text{m}$$

$$M_z = 318.3 \text{ N}\cdot\text{m}$$

$$(\sqrt{M_y^2 + M_z^2 + T^2})_{\text{max}} = 1087.2 \text{ N}\cdot\text{m}$$

$$T_{\text{all}} = \frac{C}{J} (\sqrt{M_y^2 + M_z^2 + T^2})_{\text{max}}$$

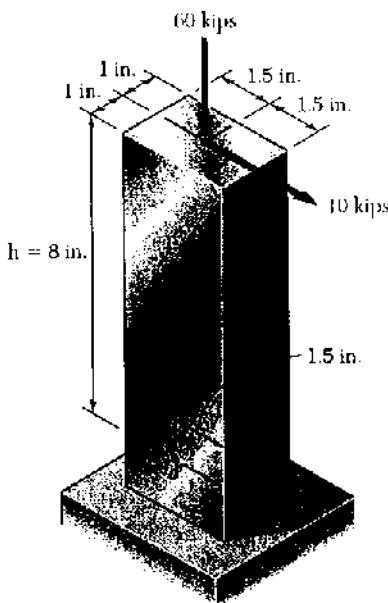
$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{(\sqrt{M_y^2 + M_z^2 + T^2})_{\text{max}}}{T_{\text{all}}} = \frac{1087.2}{55 \times 10^6} = 19.767 \times 10^{-3} \text{ m}$$

$$C = 23.26 \times 10^{-3} \text{ m}$$

$$d = 2C = 46.5 \times 10^{-3} \text{ m} = 46.5 \text{ mm}$$

Problem 8.69

8.69 Two forces are applied to the bar shown. At point *a*, determine (a) the principal stresses and principal planes, (b) the maximum shearing stresses.



At the section containing point *a* and *b*.

$$V = 10 \text{ kips} \quad P = 60 \text{ kips (compression)}$$

$$M = (8)(10) = 80 \text{ kip-in}$$

$$A = (2)(3) = 6 \text{ in}^2$$

$$I = \frac{1}{12}(2)(3)^3 = 4.5 \text{ kip-in}^2$$

$$\text{At point } a \quad \sigma_y = -\frac{P}{A} = -\frac{60}{6} = -10 \text{ ksi}$$

$$\tau = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{(10)}{6} = 2.5 \text{ ksi}, \quad \sigma_x = 0$$

Use Mohr's circle.

$$\sigma_c = -5 \text{ ksi}$$

$$R = \sqrt{5^2 + 2.5^2} = 5.590 \text{ ksi}$$

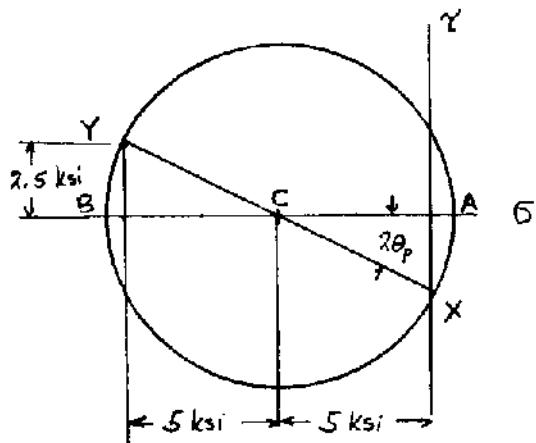
$$\sigma_a = \sigma_c + R = 0.590 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -10.59 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2.5}{5} = 0.5$$

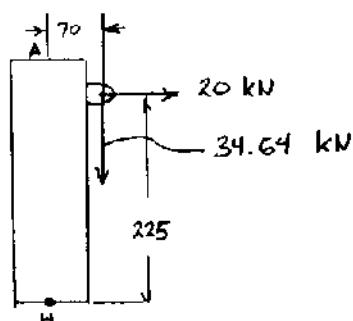
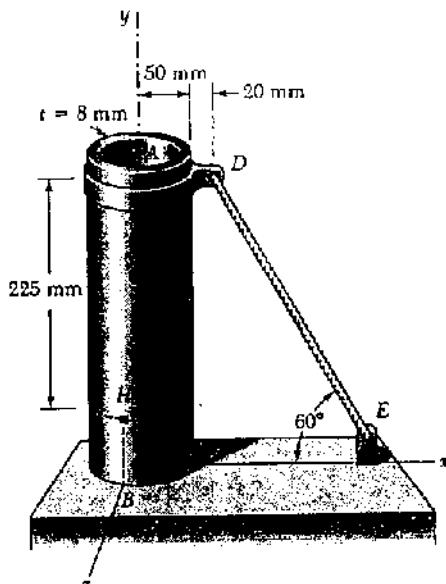
$$\theta_p = 13.3^\circ, 103.3^\circ$$

$$\tau_{max} = R = 5.59 \text{ ksi}$$



Problem 8.70

8.70 The steel pile AB has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stress at point H .



$$\begin{aligned} \text{Vertical force} \\ 40 \cos 30^\circ &= 34.64 \text{ kN} \\ \text{Horizontal Force} \\ 40 \sin 30^\circ &= 20 \text{ kN} \end{aligned}$$

Point H lies on neutral axis of bending

Section properties.

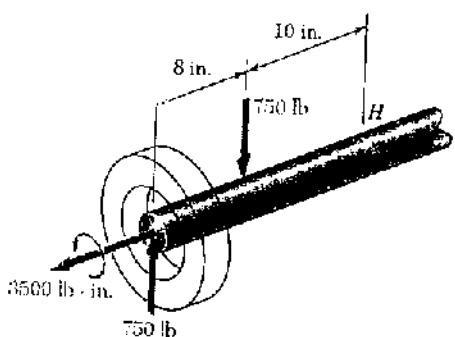
$$d_o = 100 \text{ mm} \quad C_o = \frac{1}{2} d_o = 50 \text{ mm} \quad C_i = C_o - t = 42 \text{ mm}$$

$$A = \pi(C_o^2 - C_i^2) = 2.312 \times 10^3 \text{ mm}^2 = 2.312 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{34.64 \times 10^3}{2.312 \times 10^{-3}} = -14.98 \text{ MPa}$$

$$\text{For thin pipe, } \tau = 2 \frac{V}{A} = \frac{(2)(20 \times 10^3)}{2.314 \times 10^{-3}} = 17.29 \text{ MPa}$$

Problem 8.71



8.71 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 1.42 in., determine the normal and shearing stresses at point H located on the top of the axle.

The bending moment causing normal stress at point H is

$$M = (8)(750) = 6000 \text{ lb-in.}$$

$$c = \frac{1}{2}d = 0.71 \text{ in.}$$

$$I = \frac{\pi}{4} c^4 = 0.19958 \text{ in}^4, \quad J = 2I = 0.39916 \text{ in}^4$$

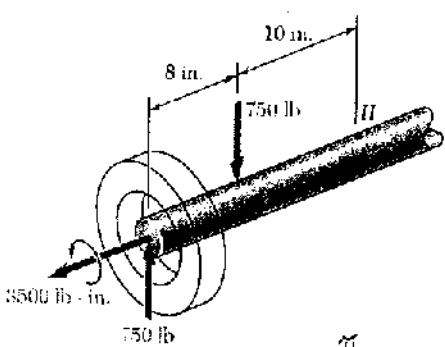
Normal stress at H: $\sigma_H = -\frac{Mc}{I} = -\frac{(6000)(0.71)}{0.19958} = -21.3 \times 10^3 \text{ psi}$
 $= -21.3 \text{ ksi}$

At the section containing point H, $V = 0, T = 3500 \text{ lb-in}$

$$\tau_H = \frac{Tc}{J} = \frac{(3500)(0.71)}{0.39916} = 6.23 \text{ ksi}$$

Problem 8.72

8.72 For the truck axle and loading of Prob. 8.71, determine the principal stresses and the maximum shearing stress at point H.

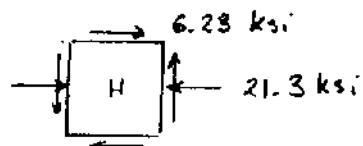


8.71 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 1.42 in., determine the normal and shearing stresses at point H located on the top of the axle.

From the solution of Prob. 8.71

$$\sigma_H = -21.3 \text{ ksi}$$

$$\tau_H = 6.23 \text{ ksi}$$



Use Mohr's circle.

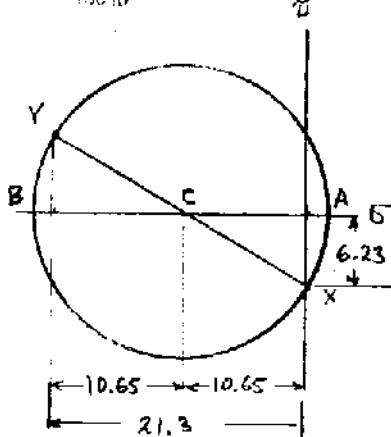
$$\sigma_c = -\frac{21.3}{2} = -10.65 \text{ ksi}$$

$$R = \sqrt{\left(\frac{21.3}{2}\right)^2 + (6.23)^2} = 12.34 \text{ ksi}$$

$$\sigma_a = \sigma_c + R = 1.69 \text{ ksi}$$

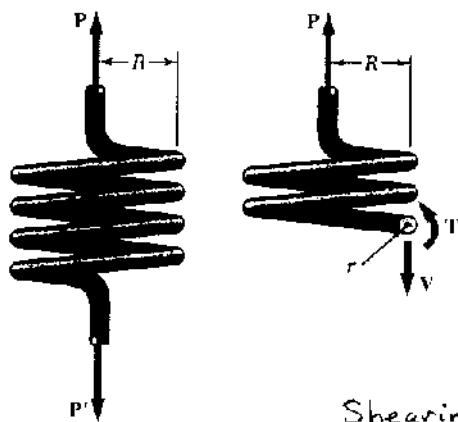
$$\sigma_b = \sigma_c - R = -23.0 \text{ ksi}$$

$$\tau_{max} = R = 12.34 \text{ ksi}$$



Problem 8.73

8.73 A close-coiled spring is made of a circular wire of radius r that is formed into a helix of radius R . Determine the maximum shearing stress produced by the two equal and opposite forces P and P' . (Hint: First determine the shear V and the torque T in a transverse cross section.)



$$\uparrow \sum F_y = 0 : \quad P - V = 0 \quad V = P$$

$$\rightarrow \sum M_c = 0 : \quad T - PR = 0 \quad T = PR$$

Shearing stress due to T

$$\tau_T = \frac{Tr}{J} = \frac{2T}{\pi r^3} = \frac{2PR}{\pi r^3}$$

Shearing stress due to V

$$\text{For semicircle, } Q = \frac{2}{3}\pi r^3, \quad t = d = 2r$$

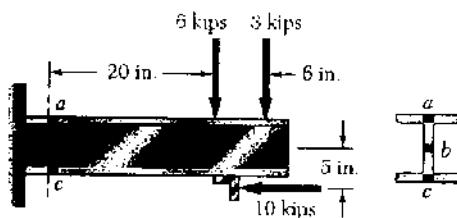
$$\text{For solid circular section, } I = \frac{1}{2}J = \frac{\pi}{4}r^4$$

$$\tau_V = \frac{VQ}{It} = \frac{V(\frac{2}{3}\pi r^3)}{\frac{\pi}{4}r^4(2r)} = \frac{4V}{3\pi r^2} = \frac{4P}{3\pi r^2}$$

$$\text{By superposition } \tau_{\max} = \tau_T + \tau_V = P(2R + 4r/3)/\pi r^3$$

Problem 8.74

8.74 Three forces are applied to a W 6 × 20 rolled beam as shown. Determine the normal and shearing stresses at points *a*, *b*, and *c*. (Note: Points *a* and *c* are located at the top and bottom surfaces of the flanges, respectively).

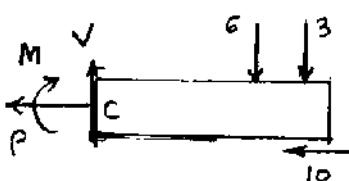


$$+\sum M_c = 0 : M + (20)(6) + (26)(3) + (5)(10) = 0$$

$$M = -248 \text{ kip in}$$

$$\pm \sum F_y = 0 : -P - 10 = 0 \quad P = -10 \text{ kips}$$

$$+\uparrow \sum F_y = 0 \quad V - 6 - 3 = 0 \quad V = 9 \text{ kips}$$

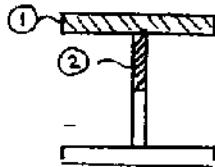


For W 6 × 20 rolled steel shape, $A = 5.87 \text{ in}^2$,
 $d = 6.20 \text{ in.}$, $b_f = 6.020 \text{ in.}$, $t_f = 0.365 \text{ in.}$
 $t_w = 0.260 \text{ in.}$, $I_x = 41.4 \text{ in}^4$, $S_x = 13.4 \text{ in}^3$.

$$\text{Point a. } \sigma = \frac{P}{A} - \frac{M}{S_x} = \frac{-10}{5.87} - \frac{-248}{13.4} = 16.80 \text{ ksi}$$

$$\tau = 0$$

$$\text{Point b. } \sigma = \frac{P}{A} = \frac{-10}{5.87} = -1.704 \text{ ksi}$$



Part	$A (\text{in}^2)$	$\bar{y} (\text{in.})$	$A\bar{y} (\text{in}^3)$
①	2.1973	2.9175	6.4106
②	0.7111	1.3675	0.9724
Z			7.383

$$Q = 7.383 \text{ in}^3$$

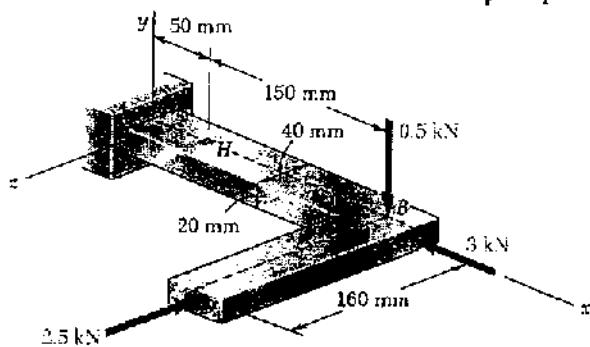
$$\tau = \frac{VQ}{I_x t_w} = \frac{(9)(7.383)}{(41.4)(0.260)} = 6.17 \text{ ksi}$$

$$\text{Point c. } \sigma = \frac{P}{A} + \frac{M}{S} = \frac{-10}{5.87} + \frac{-248}{13.4} = -20.2 \text{ ksi}$$

$$\tau = 0$$

Problem 8.75

8.75 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a $20 \times 40, determine the principal stresses and the maximum shearing stress at point *H*.$

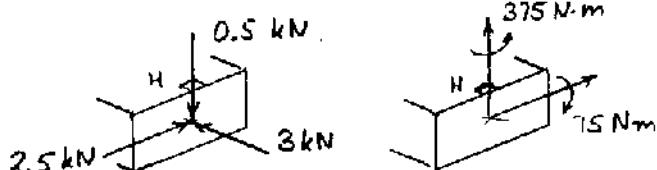


Equivalent force-couple system at section containing point *H*:

$$F_x = -3 \text{ kN}, F_y = -0.5 \text{ kN}, F_z = -2.5 \text{ kN}$$

$$M_x = 0, M_y = (0.150)(2500) = 375 \text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75 \text{ N}\cdot\text{m}$$



$$A = (20)(40) = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$$

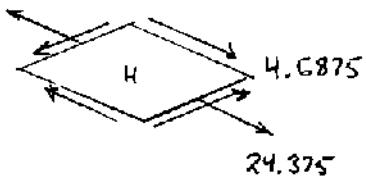
$$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3 \text{ mm}^4 = 26.667 \times 10^{-9} \text{ m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z Y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375 \text{ MPa}$$

$$\tau_H = \frac{3}{2} \frac{|V_2|}{A} = \frac{3}{2} \cdot \frac{2500}{800 \times 10^{-6}} = 4.6875 \text{ MPa}$$

Use Mohr's circle.

$$\sigma_c = \frac{1}{2} \sigma_H = 12.1875 \text{ MPa}$$



$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2} = 13.0579 \text{ MPa}$$

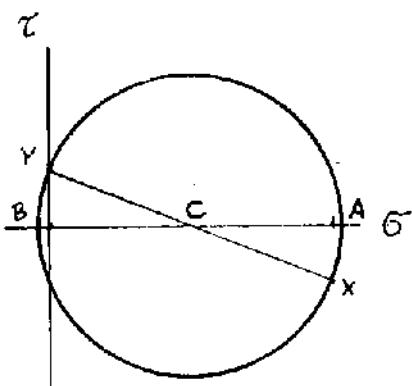
$$\sigma_a = \sigma_c + R = 25.2 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -0.87 \text{ MPa}$$

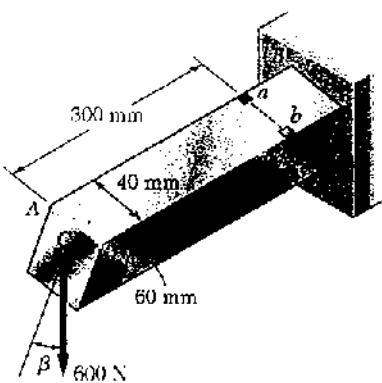
$$\tan 2\theta_p = \frac{2T_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$$

$$\theta_a = 10.5^\circ, \theta_b = 100.5^\circ$$

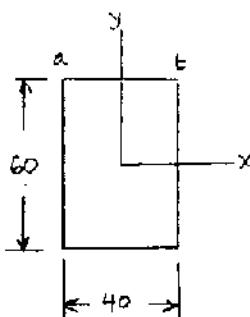
$$\tau_{max} = R = 13.06 \text{ MPa}$$



Problem 8.76



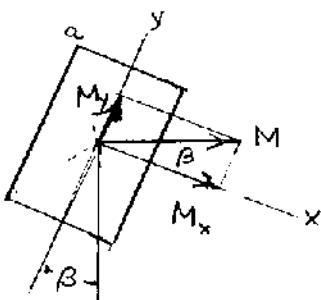
8.76 The cantilever beam AB will be installed so that the 60-mm side forms an angle β between 0 and 90° with the vertical. Knowing that the 600-N vertical forces is applied at the center of the free end of the beam, determine the normal stress at point a when (a) $\beta = 0$, (b) $\beta = 90^\circ$. (c) Also, determine the value of β for which the normal stress at point a is a maximum and the corresponding value of that stress.



$$S_x = \frac{1}{6}(40)(60)^2 = 24 \times 10^3 \text{ mm}^3 \\ = 24 \times 10^{-6} \text{ m}^3$$

$$S_y = \frac{1}{6}(60)(40)^2 = 16 \times 10^3 \text{ mm}^3 \\ = 16 \times 10^{-6} \text{ m}^3$$

$$M = P l = (600)(300 \times 10^{-3}) = 180 \text{ N}\cdot\text{m}$$



$$M_x = M \cos \beta = 180 \cos \beta$$

$$M_y = M \sin \beta = 180 \sin \beta$$

$$\sigma_a = \frac{M_x}{S_x} + \frac{M_y}{S_y} : \frac{180 \cos \beta}{24 \times 10^{-6}} + \frac{180 \sin \beta}{16 \times 10^{-6}} \\ = (7.5 \times 10^6)(\cos \beta + \frac{3}{2} \sin \beta) \text{ Pa} \\ = 7.5 (\cos \beta + \frac{3}{2} \sin \beta) \text{ MPa}$$

$$(a) \beta = 0 \quad \sigma_a = 7.50 \text{ MPa}$$

$$(b) \beta = 90^\circ \quad \sigma_a = 11.25 \text{ MPa}$$

$$(c) \frac{d\sigma_a}{d\beta} = 7.5 (-\sin \beta + \frac{3}{2} \cos \beta) = 0$$

$$\sin \beta = \frac{3}{2} \cos \beta \quad \tan \beta = \frac{3}{2} \quad \beta = 56.3^\circ$$

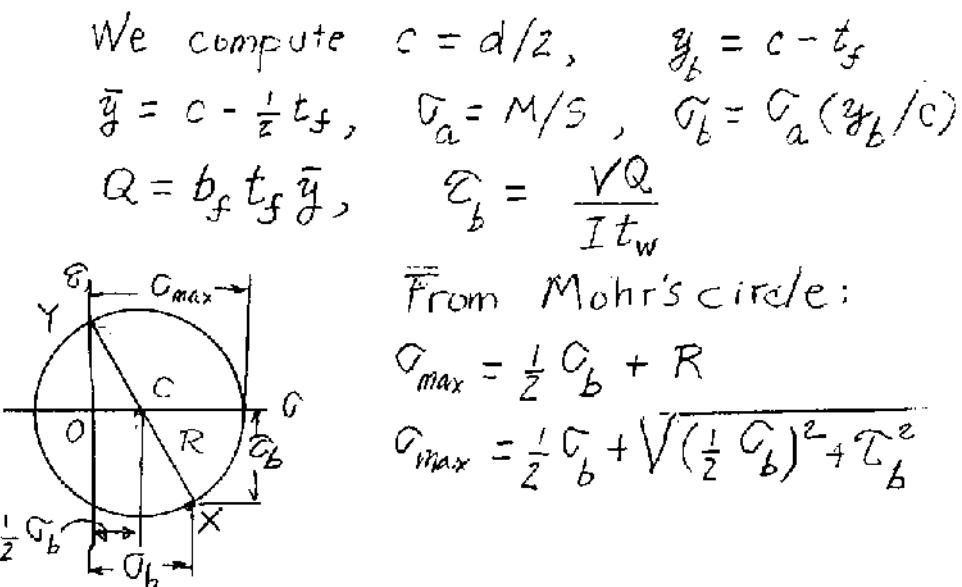
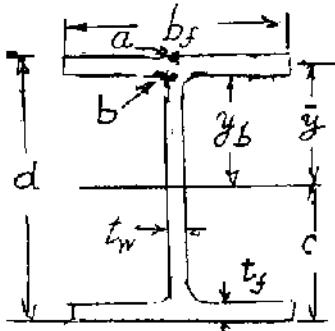
$$\sigma_a = 7.5 (\cos 56.3^\circ + \frac{3}{2} \sin 56.3^\circ) = 13.52 \text{ MPa}$$

PROBLEM 8.C1

8.C1 Let us assume that the shear V and the bending moment M have been determined in a given section of a rolled-steel beam. Write a computer program to calculate in that section, from the data available in Appendix C, (a) the maximum normal stress σ_m , (b) the principal stress σ_{max} at the junction of a flange and the web. Use this program to solve parts a and b of the following problems:
 (1) Prob. 8.1 (Use $V = 1420$ kN and $M = 3550$ kN·m)
 (2) Prob. 8.2 (Use $V = 1775$ kN and $M = 3550$ kN·m)
 (3) Prob. 8.3 (Use $V = 90$ kips and $M = 900$ kip·in.)
 (4) Prob. 8.4 (Use $V = 45$ kips and $M = 900$ kip·in.)

SOLUTION

We enter the given values of V and M obtain from Appendix C the values of a , b_f , t_f , t_w , I , and S for the given WF shape.



PROGRAM OUTPUTS

Problem 8.1

Given Data:
 $V = 1420$ kN, $M = 3550$ kN·m
 $d = 930$ mm, $b_f = 423$ mm
 $t_f = 43$ mm, $t_w = 24$ mm
 $I = 8470$ (10^6 mm 4)
 $S = 18200$ (10^3 mm 3)

Answers:

- (a) $\sigma_{GA} = 195.05$ MPa
- (b) $\sigma_{GM} = 193.50$ MPa

Problem 8.2

Given Data:
 $V = 1775$ kN, $M = 3550$ kN·m
 $d = 930$ mm, $b_f = 423$ mm
 $t_f = 43$ mm, $t_w = 24$ mm
 $I = 8470$ (10^6 mm 4)
 $S = 18200$ (10^3 mm 3)

Answers:

- (a) $\sigma_{GA} = 195.05$ MPa
- (b) $\sigma_{GM} = 201.64$ MPa

Problem 8.3

Given Data:
 $V = 90$ kips, $M = 900$ kip·in.
 $d = 9.92$ in., $b_f = 7.985$ in.
 $t_f = .530$ in., $t_w = .315$ in.
 $I = 209.00$ in 4 , $S = 42.10$ in 3

Answers:

- (a) $\sigma_{GA} = 21377.67$ psi
- (b) $\sigma_{GM} = 38338.15$ psi

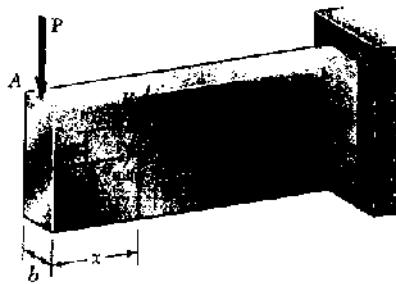
Problem 8.4

Given Data:
 $V = 45$ kips, $M = 900$ kip·in.
 $d = 9.92$ in., $b_f = 7.985$ in.
 $t_f = .530$ in., $t_w = .315$ in.
 $I = 209.00$ in 4 , $S = 42.10$ in 3

Answers:

- (a) $\sigma_{GA} = 21377.67$ psi
- (b) $\sigma_{GM} = 26147.63$ psi

PROBLEM 8.C2



8.C2 A cantilever beam AB with a rectangular cross section of width b and depth $2c$ supports a single concentrated load P at its end A . Write a computer program to calculate, for any values of x/c and y/c , (a) the ratios σ_{\max}/σ_m and σ_{\min}/σ_m where σ_{\max} and σ_{\min} are the principal stresses at point $K(x, y)$ and σ_m the maximum normal stress in the same transverse section, (b) the angle θ_p that the principal planes at K form with a transverse and a horizontal plane through K . Use this program to check the values shown in Fig. 8.8 and to verify that σ_{\max} exceeds σ_m if $x \leq 0.544c$, as indicated in the second footnote on page 499.

SOLUTION

Since the distribution of the normal stresses is linear, we have $\sigma = \sigma_m (y/c)$ (1)

$$\text{where } \sigma_m = \frac{Mc}{I} = \frac{Pzc}{I} \quad (2)$$

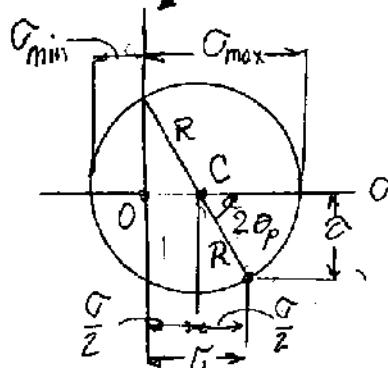
$$\text{We use Eq. (8.4), page 498: } \tau = \frac{3}{2} \frac{P}{A} \left(1 - \frac{y^2}{c^2}\right) \quad (3)$$

$$\text{Dividing (3) by (2): } \frac{\tau}{\sigma_m} = \frac{3}{2} \frac{I}{A} \frac{1 - (y/c)^2}{xc}$$

$$\text{or, since } \frac{I}{A} = \frac{\frac{1}{2}b(2c)^3}{b(2c)} = \frac{1}{3}c^2; \quad \frac{\tau}{\sigma_m} = \frac{1}{2} \frac{1 - (y/c)^2}{x/c} \quad (4)$$

Letting $X = x/c$ and $Y = y/c$, Eqs. (1) and (4) yield

$$\sigma = \sigma_m Y \quad \tau = \sigma_m \frac{1 - Y^2}{2X} \quad \rightarrow$$



Using Mohr's circle, we calculate

$$R = \sqrt{\left(\frac{1}{2}\sigma\right)^2 + \tau^2}$$

$$= \frac{1}{2}\sigma_m \sqrt{Y^2 + \left(\frac{1 - Y^2}{X}\right)^2}$$

$$\frac{\sigma_{\max}}{\sigma_m} = \frac{1}{2}Y + R \quad \frac{\sigma_{\min}}{\sigma_m} = \frac{1}{2}Y - R \quad \blacksquare$$

$$\tan 2\theta_p = \frac{\tau}{\sigma/2} = \frac{1 - Y^2}{2X(Y/2)} = \frac{1 - Y^2}{XY} \quad \theta_p = \frac{1}{2} \tan^{-1} \left(\frac{1 - Y^2}{XY} \right) \quad \blacksquare$$

NOTE

For $y > 0$, the angle θ_p is π , which is opposite to what was arbitrarily assumed in Fig. P8.C2.

PROBLEM 8.C2 CONTINUED

PROGRAM OUTPUTSFor $x/c = 2$:

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta ^o
1.0	0.000	1.000	0.00
0.8	-0.010	0.810	6.24
0.6	-0.040	0.640	14.04
0.4	-0.090	0.490	23.20
0.2	-0.150	0.360	33.69
0.0	-0.250	0.250	45.00
-0.2	-0.360	0.160	33.69
-0.4	-0.490	0.090	-23.20
-0.6	-0.640	0.040	-14.04
-0.8	-0.810	0.010	-6.34
-1.0	-1.000	0.000	-0.00

For $x/c = 8$:

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta ^o
1.0	0.000	1.000	0.00
0.8	-0.001	0.801	1.61
0.6	-0.003	0.603	3.80
0.4	-0.007	0.407	7.35
0.2	-0.017	0.217	15.48
0.0	-0.062	0.063	45.00
-0.2	-0.217	0.017	-15.48
-0.4	-0.407	0.007	-7.35
-0.6	-0.603	0.003	-3.80
-0.8	-0.801	0.001	-1.61
-1.0	-1.000	0.000	-0.00

To check that $G_{\max} > G_m$ if $x \leq 0.544c$, we run the program for $x/c = 0.544$ and for $x/c = 0.545$ and observe that G_{\max}/G_m exceeds 1 for several values of y/c in the first case, but does not exceed 1 in the second case.

For $x/c = 0.544$:

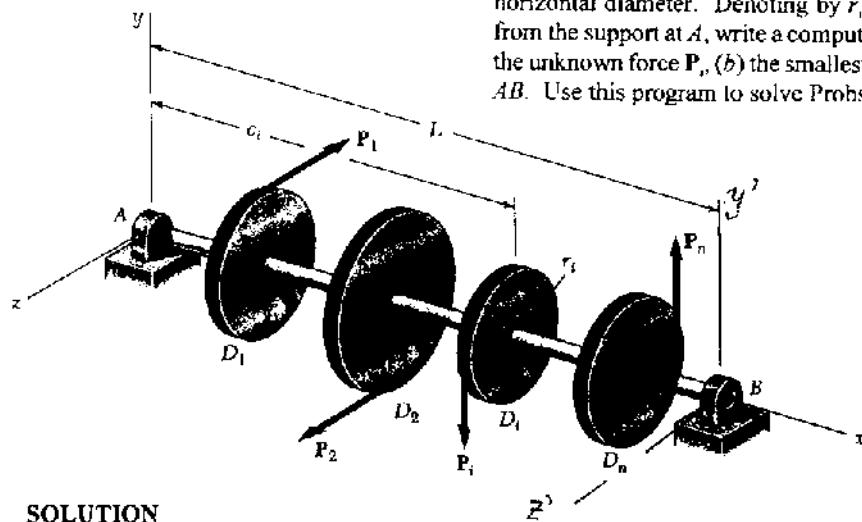
y/c	Sigmin/Sigm	Sigmax/Sigm	Theta ^o
0.30	-0.700	0.9997	39.92
0.31	-0.690	1.0001	39.72
0.32	-0.680	1.0004	39.51
0.33	-0.670	1.0005	39.30
0.34	-0.660	1.0005	39.09
0.35	-0.650	1.0003	38.88
0.36	-0.640	1.0000	38.66
0.37	-0.630	0.9996	38.44
0.38	-0.619	0.9990	38.21
0.39	-0.608	0.9983	37.98
0.40	-0.598	0.9975	37.74

For $x/c = 0.545$:

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta ^o
0.30	-0.698	0.9982	39.91
0.31	-0.689	0.9986	39.71
0.32	-0.679	0.9989	39.50
0.33	-0.669	0.9990	39.29
0.34	-0.659	0.9990	39.08
0.35	-0.649	0.9988	38.87
0.36	-0.639	0.9986	38.65
0.37	-0.628	0.9982	38.42
0.38	-0.618	0.9976	38.20
0.39	-0.607	0.9970	37.96
0.40	-0.596	0.9962	37.73

PROBLEM 8.C3

8.C3 Disks D_1, D_2, \dots, D_n are attached to the solid shaft AB of length L , uniform diameter d , and allowable shearing stress τ_{all} . Forces P_1, P_2, \dots, P_n of known magnitude (except for one of them) are applied to the disks, either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. Denoting by r_i the radius of disk D_i and by c_i its distance from the support at A , write a computer program to calculate (a) the magnitude of the unknown force P_i , (b) the smallest permissible value of the diameter d of shaft AB . Use this program to solve Probs. 8.15 and 8.19.



SOLUTION

1. Determine the unknown force P_i by equating to zero the sum of their torques T_i about the x axis.

2. Determine the components $(F_y)_i$ and $(F_z)_i$ of all forces.

3. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:

$$\sum M_{z'} = 0: -A_y L - \sum (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L} \sum (F_y)_i (L - c_i)$$

$$\sum M_{y'} = 0: A_z L + \sum (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L} \sum (F_z)_i (L - c_i)$$

4. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of disk D_i :

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k < c_i - c_k >$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k < c_i - c_k >$$

$$T_i = \sum_k T_k < c_i - c_k >$$

where $< >$ indicates a singularity function.

5. The minimum diameter d required to the left of D_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{C_{all}}$$

(CONTINUED)

PROBLEM 8.C3 CONTINUED

6. Recalling that $J = \frac{1}{2} \pi c^4$ and, thus, that $(\frac{J}{c})_i = \frac{1}{2} \pi c_i^3$,

We have $c_i = \frac{2}{\pi} (\frac{J}{c})_i^{1/3}$ and $d_i = \frac{4}{\pi} (\frac{J}{c})_i^{1/3}$

This is the required diameter just to the left of disk D_i .

7. The required diameter just to the right of disk D_i is obtained by replacing T_i with T_{i+1} in the above computation.

8. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for d_i .

PROGRAM OUTPUTS

Problem 8.15

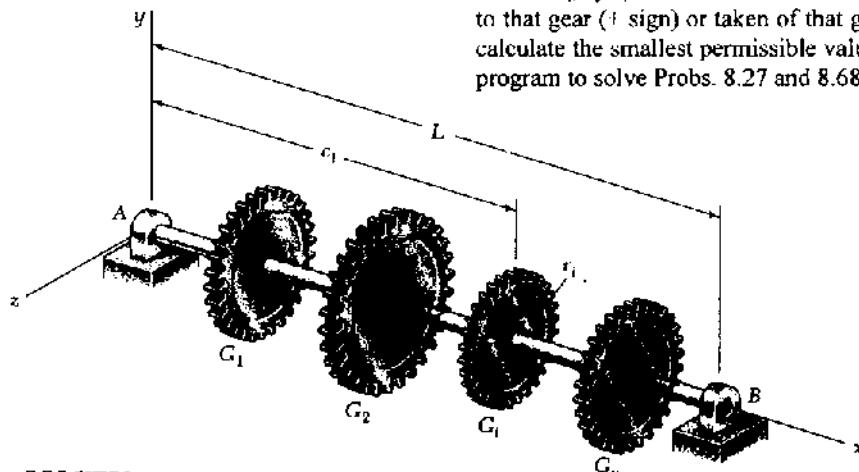
Length of shaft = 300 mm
 TAU = 60 MPa
 For Disk 1
 Force = 6.000 kN
 Radius of disk = 75 mm
 Distance from A in mm = 80
 For Disk 2
 Force = 0.000 kN
 Radius of disk = 60 mm
 Distance from A in mm = 180
 Unknown force = -7.500 kN
 AY = 4.400 kN, AZ = -3.000 kN
 BY = 1.600 kN, BZ = -4.500 kN
 Just to the left of Disk 1
 MY = -240.00 Nm
 MZ = -352.00 Nm
 T = 0.00 Nm
 Diameter must be at least 33.07 mm
 Just to the right of Disk 1
 T = 450.00 Nm
 Diameter must be at least 37.47 mm
 Just to the left of Disk 2
 MY = -540.00 Nm
 MZ = -192.00 Nm
 T = 450.00 Nm
 Diameter must be at least 39.55 mm
 Just to the right of Disk 2
 T = 0.00 Nm
 Diameter must be at least 36.51 mm

Problem 8.19

Length of shaft = 28 in.
 TAU (ksi) = 8
 For Disk 1
 Force = 0.500 kips
 Radius of disk = 4.0 in.
 Distance from A = 7.0 in.
 For Disk 2
 Force = 0.000 kips
 Radius of disk = 6.0 in.
 Distance from A = 14.0 in.
 For Disk 3
 Force = 0.500 kips
 Radius of disk = 4.0 in.
 Distance from A = 21.0 in.
 Unknown force = -0.667 kips
 AY = 0.500 kips, AZ = 0.333 kips
 BY = 0.500 kips, BZ = 0.333 kips
 Just to the left of Disk 1
 MY = 2.3333 kip.in.
 MZ = -3.5000 kip.in.
 T = 0.0000 kip.in.
 Diameter must be at least 1.389 in.
 Just to the right of Disk 1
 T = 2.00 kip.in.
 Diameter must be at least 1.437 in.
 Just to the left of Disk 2
 MY = 4.6667 kip.in.
 MZ = -3.5000 kip.in.
 T = 2.0000 kip.in.
 Diameter must be at least 1.578 in.
 Just to the right of Disk 2
 T = -2.00 kip.in.
 Diameter must be at least 1.578 in.
 Just to the left of Disk 3
 MY = 2.3333 kip.in.
 MZ = -3.5000 kip.in.
 T = -2.0000 kip.in.
 Diameter must be at least 1.437 in.
 Just to the right of Disk 3
 T = 0.00 kip.in.
 Diameter must be at least 1.389 in.

PROBLEM 8.C4

8.C4 The solid shaft AB of length L , uniform diameter d , and allowable shearing stress τ_{all} rotates at a given speed expressed in rpm (Fig. 8.C4). Gears G_1, G_2, \dots, G_n are attached to the shaft and each of these gears meshes with another gear (not shown), either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. One of these other gears is connected to a motor and the rest of them to various machine tools. Denoting by r_i the radius of disk G_i , by c_i its distance from the support at A , and by P_i the power transmitted to that gear (+ sign) or taken of that gear (- sign), write a computer program to calculate the smallest permissible value of the diameter d of shaft AB . Use this program to solve Probs. 8.27 and 8.68.



SOLUTION

1. Enter ω in rpm and determine frequency $f = \omega/60$.
 2. For each gear, determine the torque $T_i = P_i / 2\pi f$, where P_i is the power input (+) or output (-) at the gear.
 3. For each gear, determine the force $F_i = T_i / r_i$ exerted on the gear and its components $(F_y)_i$ and $(F_z)_i$.
 4. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:
- $$\sum M_{z'} = 0 : -A_y L - \sum (F_y)_i (L - c_i) = 0, A_y = -\frac{1}{L} \sum (F_y)_i (L - c_i)$$
- $$\sum M_{y'} = 0 : A_z L + \sum (F_z)_i (L - c_i) = 0, A_z = -\frac{1}{L} \sum (F_z)_i (L - c_i)$$
5. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of gear G_i :

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k < c_i - c_k >$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k < c_i - c_k >$$

$$T_i = \sum_k T_k < c_i - c_k >$$

Where $< >$ indicates a singularity function.

(CONTINUED)

PROBLEM 8.C4 CONTINUED

6. The minimum diameter d required to the left of G_i is obtained by first computing $(J/c)_i$ from Eq.(8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\epsilon_{all}}$$

7. Recalling that $J = \frac{1}{2} \pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2} \pi c_i^3$, we have $c_i = \frac{2}{\pi} \left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi} \left(\frac{J}{c}\right)_i^{1/3}$

This is the required diameter just to the left of gear G_i .

8. The required diameter just to the right of gear G_i is obtained by replacing T_i with T_{i+1} in the above computation.

9. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for d_i .

PROBLEM 8.C4 CONTINUED

PROGRAM OUTPUTS

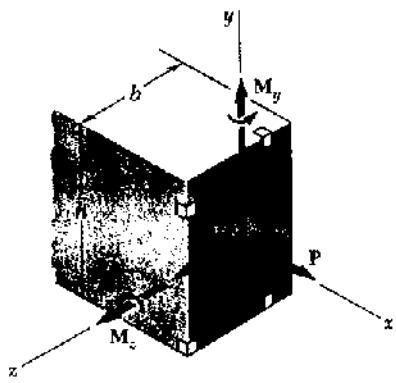
Problem 8.27

Omega = 600 rpm
 Number of Gears: 3
 Length of shaft = 24 in.
 Tau = 8 ksi
 For Gear 1
 Power input = 60.00 hp
 Radius of gear= 3.00 in.
 Distance from A in inches = 4.0
 FY= 0
 FZ = 2.100845
 For Gear 2
 Power input = -40.00 hp
 Radius of gear= 4.00 in.
 Distance from A in inches = 10.0
 FY= 1.050423
 FZ = 0
 For Gear 3
 Power input = -20.00 hp
 Radius of gear= 4.00 in.
 Distance from A in inches = 18.0
 FY= 0
 FZ = -.5252113
 AY=-0.6127 kips, AZ=-1.6194 kips
 BY=-0.4377 kips, BZ= 0.0438 kips
 Just to the left of Gear 1
 MY= -6.478 kip.in.
 MZ= 2.451 kip.in.
 T= 0.000 kip.in.
 Diameter must be at least 1.640 in.
 Just to the right of Gear 1
 T= 6.3025 kip.in.
 Diameter must be at least 1.813 in.
 Just to the left of Gear 2
 MY= -3.589 kip.in.
 MZ= 6.127 kip.in.
 T= 6.303 kip.in.
 Diameter must be at least 1.622 in.
 Just to the right of Gear 2
 T= 2.1008 kip.in.
 Diameter must be at least 1.677 in.
 Just to the left of Gear 3
 MY= 0.263 kip.in.
 MZ= 2.626 kip.in.
 T= 2.101 kip.in.
 Diameter must be at least 1.290 in.
 Just to the right of Gear 3
 T= 0.0000 kip.in.
 Diameter must be at least 1.189 in.

Problem 8.68

Omega = 450 rpm
 Number of Gears: 3
 Length of shaft = 750 mm
 Tau = 55 MPa
 For Gear 1
 Power input = -8.00 kW
 Radius of gear= 60 mm
 Distance from A in mm = 150
 For Gear 2
 Power input = 20.00 kW
 Radius of gear=100 mm
 Distance from A in mm = 375
 For Gear 3
 Power input = -12.00 kW
 Radius of gear= 60 mm
 Distance from A in mm = 600
 $AY = -0.849 \text{ kN}$, $AZ = 4.386$
 $BY = -3.395 \text{ kN}$, $BZ = 2.688$
 Just to the left of Gear 1
 $MY = 657.84 \text{ Nm}$
 $MZ = 127.32 \text{ Nm}$
 $T = 0.00 \text{ Nm}$
 Diameter must be at least 39.59 mm
 Just to the right of Gear 1
 $T = -169.77 \text{ Nm}$
 Diameter must be at least 40.00 mm
 Just to the left of Gear 2
 $MY = 1007.98 \text{ Nm}$
 $MZ = 318.31 \text{ Nm}$
 $T = -169.77 \text{ Nm}$
 Diameter must be at least 46.28 mm
 Just to the right of Gear 2
 $T = 254.65 \text{ Nm}$
 Diameter must be at least 46.52 mm
 Just to the left of Gear 3
 $MY = 403.19 \text{ Nm}$
 $MZ = 509.30 \text{ Nm}$
 $T = 254.65 \text{ Nm}$
 Diameter must be at least 40.13 mm
 Just to the right of Gear 3
 $T = 0.00 \text{ Nm}$
 Diameter must be at least 39.18 mm

PROBLEM 8.C5



8.C5 Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates y and z located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (a) Prob. 8.45a, (b) Prob. 8.47b.

SOLUTION

ENTER: b AND h

$$\underline{\text{PREISKIRK:}} \quad A = b h \quad I_y = b^3 h / 12 \quad I_z = b h^3 / 12$$

FOR POINT ON SURFACE, ENTER y AND z

NOTE y AND z MUST SATISFY ONE OF FOLLOWING:

$$y^2 = h^2/4 \quad \text{AND} \quad z^2 \leq b^2/4 \quad (1)$$

$$\text{OR} \quad z^2 = b^2/4 \quad \text{AND} \quad y^2 \leq h^2/4 \quad (2)$$

IF EITHER (1) OR (2) ARE SATISFIED, COMPUTE

$$\tau = \frac{P}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

IF $z^2 = b^2/4$, THEN POINT IS ON VERTICAL SURFACE AND

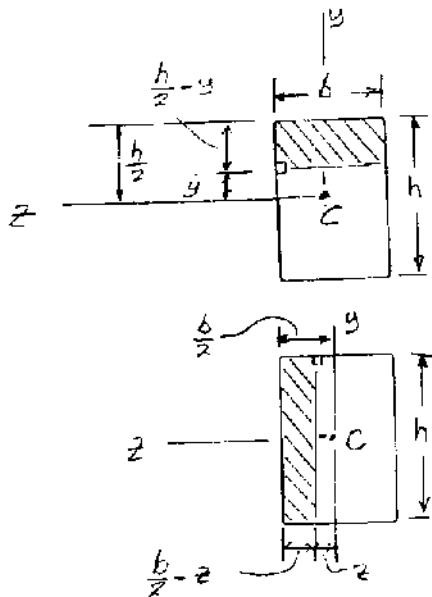
$$Q_z = b \left(\frac{h}{2} - y \right) \left(\frac{h}{2} + z \right) \frac{1}{2} = b \left(\frac{h^2}{8} - \frac{y^2}{2} \right)$$

$$\tau = \frac{V_y Q_z}{I_z b}$$

IF $y^2 = h^2/4$, THE POINT IS ON HORIZONTAL SURFACE, AND

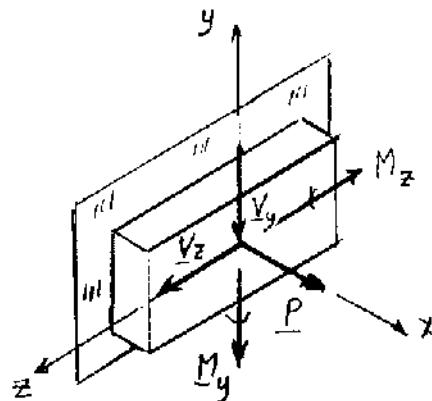
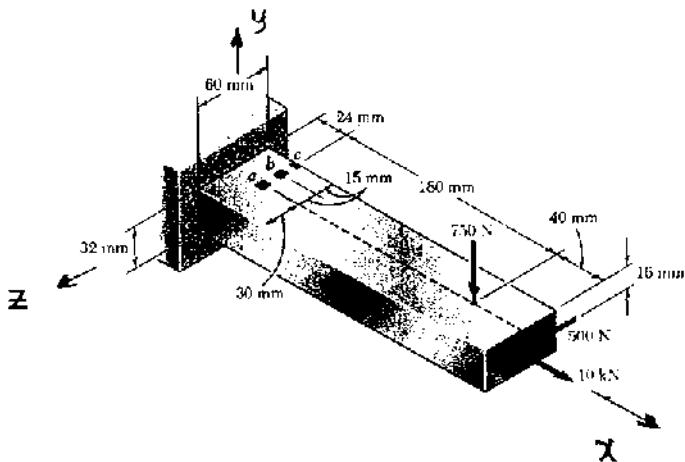
$$Q_y = h \left(\frac{b}{2} - z \right) \left(\frac{b}{2} + z \right) \frac{1}{2} = h \left(\frac{b^2}{8} - \frac{z^2}{2} \right)$$

$$\tau = \frac{V_z Q_y}{I_y h}$$



CONTINUED

PROBLEM 8.C5 CONTINUED



FORCE-COUPLE SYSTEM

$$P = 10 \text{ kN} \quad V_y = -750 \text{ N} \quad V_z = 500 \text{ N}$$

$$M_y = (500 \text{ N})(220 \text{ mm}) = 110 \text{ N}\cdot\text{m} \quad M_z = (750 \text{ N})(180 \text{ mm}) = 135 \text{ N}\cdot\text{m}$$

Problem 8.45 a

Force-Couple at Centroid

P = 10000.00 N

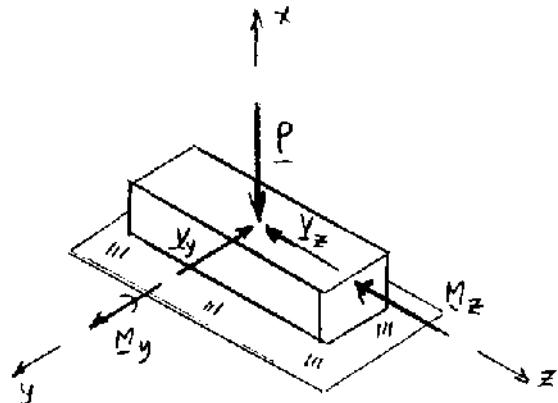
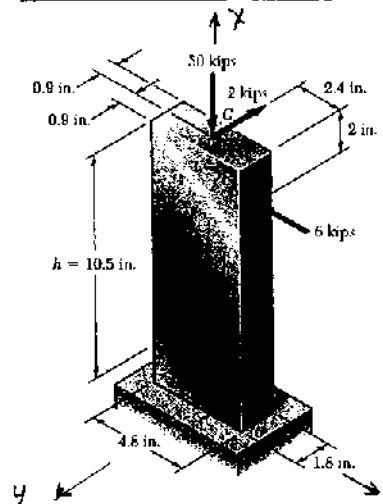
$$M_Y = 110.00 \text{ N}\cdot\text{m} \quad M_Z = -135.00 \text{ N}\cdot\text{m}$$

VY = 750.00 N VZ = 500.00 N

At point of coordinates: y = 16.00 mm z = 0.00 mm
sigma = 19.303 MPa

sigma = 18.392 MPa

$$\tau_u = 0.391 \text{ MPa}$$



FORCE-COUPLE SYSTEM

$$P = -50 \text{ kips} \quad V_z = -6 \text{ kips} \quad V_y = -2 \text{ kips}$$

$$M_u = (6 \text{ kips})(8.5 \text{ in.}) = 51 \text{kip} \cdot \text{in.}$$

$$M_2 = (2 \text{ kips})(10.5 \text{ in.}) = 21 \text{ kip-in}$$

Problem 8.47 b

Force-Couple at Centroid

$$P = -50.00 \text{ kips}$$

MY = 51.00 kip-in. MZ = -21.00 kip-in.

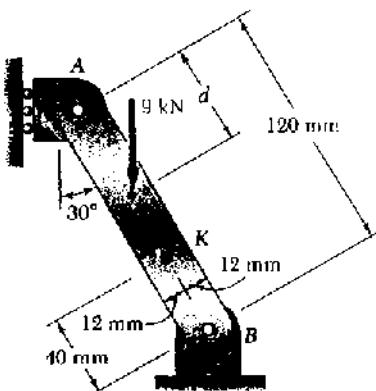
VY = -2.00 kips VZ = -6.00 kips

At point of coordinates: y = 0.90 in. z = 1.20 in.

sigma = 6.004 kb

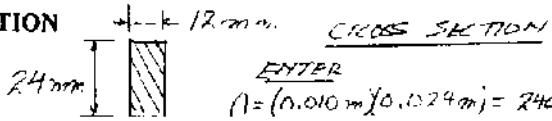
tau = -0.781 ks

PROBLEM 8.C6



8.C6 Member *AB* has a rectangular cross section of 10×24 mm. For the loading shown, write a computer program that can be used to determine the normal and shearing stresses at points *H* and *K* for values of *d* from 0 to 120 mm, using 15-mm increments. Use this program to solve Prob. 8.34.

SOLUTION

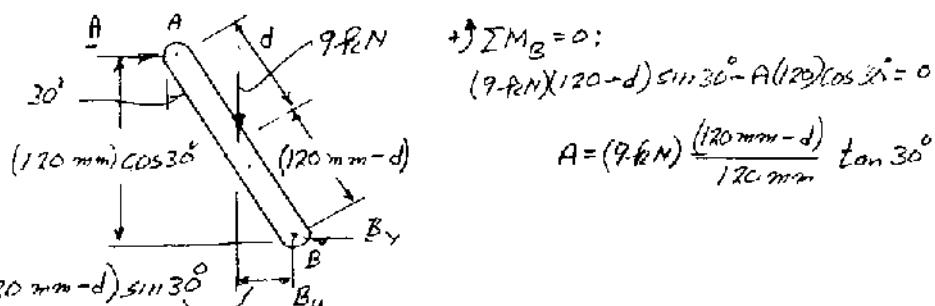


$$A = (0.010 \text{ m})(0.024 \text{ m}) = 240 \times 10^{-6} \text{ m}^2$$

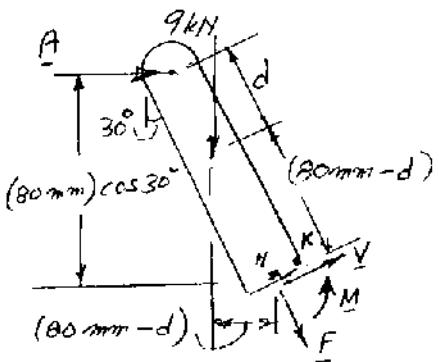
$$I = (0.010 \text{ m})(0.024 \text{ m})^3 / 12 = 138.24 \times 10^{-9} \text{ m}^4$$

$$R = 0.5(0.024 \text{ m}) = 12 \times 10^{-3} \text{ m}$$

COMPUTE REACTION AT A.



FREE BODY FROM A TO SECTION CONTAINING POINTS H AND K.



DEFINE: IF $d < 80$ mm THEN $STP = 1$ ELSE $STP = 0$

PROGRAM FORCE-COUPLE SYSTEM

$$F = -A \sin 30^\circ - (9 \text{ kN}) \cos 30^\circ (\text{STP})$$

$$V = -A \cos 30^\circ + (9 \text{ kN}) \sin 30^\circ (\text{STP})$$

$$M = A(80 \text{ mm}) \cos 30^\circ - (9 \text{ kN})(80 \text{ mm} - d) \sin 30^\circ (\text{STP})$$

AT POINT H:

$$\sigma_H = +F/A \quad \tau_H = \frac{3}{2} V/A$$

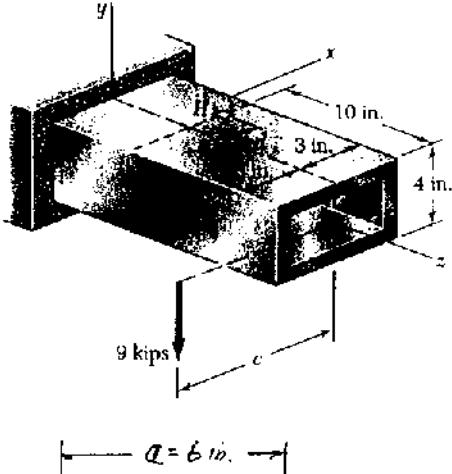
AT POINT K:

$$\sigma_K = +F/A - Mc/I \quad \tau_K = 0$$

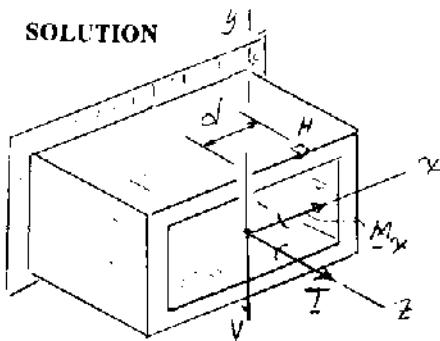
PROGRAM OUTPUT

Problem 8.34

d mm	Stresses in MPa			
	σ_{H}	τ_{H}	σ_{K}	τ_{K}
0.0	-43.30	0.00	-43.30	0.00
15.0	-41.95	3.52	-65.39	0.00
30.0	-40.59	7.03	-87.47	0.00
45.0	-39.24	10.55	-109.55	0.00
60.0	-37.89	14.06	-131.64	0.00
75.0	-36.54	17.58	-153.72	0.00
90.0	-2.71	-7.03	-96.46	0.00
105.0	-1.35	-3.52	-48.23	0.00
120.0	0.00	0.00	0.00	0.00

PROBLEM 8.C7


*8.C7 The structural tube shown has a uniform wall thickness of 0.3 in. A 9-kip force is applied at a bar (not shown) that is welded to the end of the tube. Write a computer program that can be used to determine, for any given value of c , the principal stresses, principal planes, and maximum shearing stress at point H for values of d from -3 in. to 3 in., using one-inch increments. Use this program to solve Prob. 8.64a.

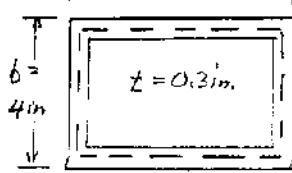
SOLUTION

FORCE-COUPLE SYSTEM

ENTER:

$$V = 9 \text{ kips} \downarrow$$

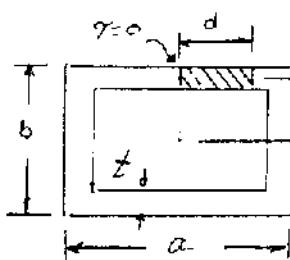
$$M_x = (9 \text{ kips})(10 \text{ in.}) \\ = 90 \text{ kip-in.}$$

$$T = 17.2 \text{ kips} \leftarrow$$


AREA ENCLOSED

$$A = (a-t)(b-t)$$

$$\tau_t = \frac{T}{I} \frac{c}{2+t} = \frac{9c}{2t(a-t)}$$



$$Q = dt\left(\frac{b}{2} - \frac{t}{2}\right)$$

$$I_t = ab^3/12 - (a-2t)(b-2t)^3/12$$

$$\tau_y = \frac{VQ}{It}$$

 $\tau_t = \text{SHEARING STRESS DUE TO TORSION}$
 $\tau_y = \text{SHEARING STRESS DUE TO V}$

BENDING: $\sigma_H = \frac{Mx(b)}{I}$

$$\tau_{total} = \tau_t + \tau_y$$

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{1}{2}\sigma_H + R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{total}^2}$$

$$\sigma_{max} = \sigma_{ave} + R; \quad \sigma_{min} = \sigma_{ave} - R; \quad \theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{total}}{\sigma_{ave}}\right); \quad \gamma_{max} = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{total}^2}$$

 Rectangular tube of uniform thickness $t = 0.3$ in.
 Outside dimensions

 Horizontal width $a = 6$ in.

 Vertical depth $b = 4$ in.

 Vertical load $P = 9$ kips; line of action at $x = -c$

Find normal and shearing stresses at

 Point H ($x = d$, $y = b/2$)

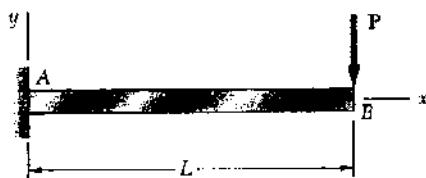
 Problem 8.64a Program Output for Value of $c = 2.85$ in.

d in.	sigma ksi	tauV ksi	tauT ksi	tauTotal ksi	sigmaMax ksi	sigmaMin ksi	tauMax ksi	theta p degrees
-3.00	12.58	-3.49	-2.03	-5.52	14.65	-2.08	8.36	-18.49
-2.00	12.58	-2.33	-2.03	-4.35	13.94	-1.36	7.65	-16.00
-1.00	12.58	-1.16	-2.03	-3.19	13.34	-0.76	7.05	-12.78
0.00	12.58	0.00	-2.03	-2.03	12.89	-0.32	6.61	-8.73
1.00	12.58	1.16	-2.03	-0.86	12.63	-0.06	6.35	-3.89
2.00	12.58	2.33	-2.03	0.30	12.58	-0.01	6.30	1.36
3.00	12.58	3.49	-2.03	1.46	12.74	-0.17	6.46	6.46

Chapter 9

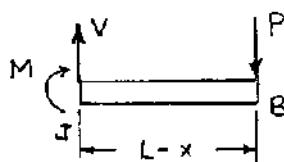
Problem 9.1

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB , (b) the deflection at the free end, (c) the slope at the free end.



$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



$$\sum M_J = 0 : -M - M + P(L-x) = 0$$

$$M = -P(L-x)$$

$$EI \frac{d^2y}{dx^2} = -P(L-x) = -PL + Px$$

$$EI \frac{dy}{dx} = -PLx + \frac{1}{2}Px^2 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 = -0 + 0 + C_1 \quad C_1 = 0$$

$$EIy = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_2x + C_3$$

$$[x=0, y=0]$$

$$0 = -0 + 0 + 0 + C_3 \quad C_3 = 0$$

(a) Elastic curve.

$$y = -\frac{Px^2}{6EI}(3L-x)$$

$$\frac{dy}{dx} = -\frac{Px}{2EI}(2L-x)$$

(b) $y @ x=L$.

$$y_B = -\frac{PL^2}{6EI}(3L-L) = -\frac{PL^3}{3EI}$$

$$y_B = \frac{PL^3}{3EI} \downarrow$$

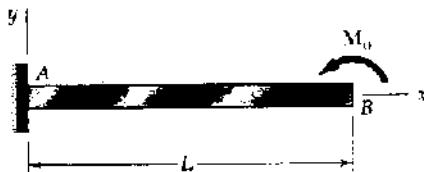
(c) $\frac{dy}{dx} @ x=L$:

$$\left. \frac{dy}{dx} \right|_B = -\frac{PL}{2EI}(2L-L) = -\frac{PL^2}{2EI}$$

$$\theta_B = \frac{PL^2}{2EI} \swarrow$$

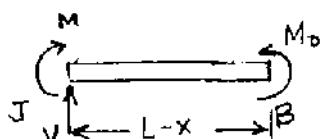
Problem 9.2

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB , (b) the deflection at the free end, (c) the slope at the free end.



$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



$$\rightarrow \sum M_J = 0 : M_0 - M = 0 \quad M = M_0$$

$$EI \frac{d^2y}{dx^2} = M_0$$

$$EI \frac{dy}{dx} = M_0 x + C_1$$

$$EI y = \frac{1}{2} M_0 x^2 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 = 0 + C_1 \quad C_1 = 0$$

$$EI y = \frac{1}{2} M_0 x^2 + C_2$$

$$[x=0, y=0]$$

$$0 = 0 + C_2$$

$$C_2 = 0$$

(a) Elastic curve.

$$y = \frac{M_0 x^2}{2EI}$$

$$\frac{dy}{dx} = \frac{M_0 x}{EI}$$

(b) y @ $x=L$.

$$y_B = \frac{M_0 L^2}{2EI}$$

$$y_B = \frac{M_0 L^2}{2EI}$$

(c) $\frac{dy}{dx}$ @ $x=L$.

$$\left. \frac{dy}{dx} \right|_B = \frac{M_0 L}{EI}$$

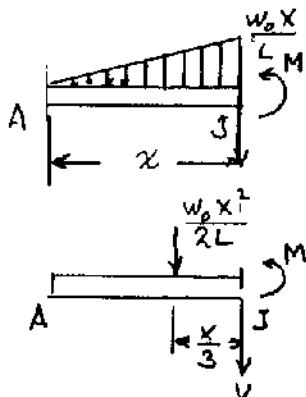
$$\theta_B = \frac{M_0 L}{EI}$$

Problem 9.3



$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



$$[x=L, y=0]$$

$$EI y = -\frac{1}{120} \frac{w_0 x^5}{L} + C_1 x + C_2$$

$$C_1 = \frac{1}{24} w_0 L^3$$

$$C_2 = \frac{1}{30} w_0 L^4$$

(a) Elastic curve.

$$y = -\frac{w_0}{120 EI L} (x^5 - 5L^4 x + 4L^5)$$

$$\frac{dy}{dx} = \frac{w_0}{24 EI L} (-x^4 + L^4)$$

(b) $y @ x=0:$

$$y_A = -\frac{w_0 L^4}{30 EI}$$

(c) $\frac{dy}{dx} @ x=0:$

$$\left. \frac{dy}{dx} \right|_A = \frac{w_0 L^3}{24 EI} \quad \theta_A = \frac{w_0 L^4}{24 EI}$$

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB , (b) the deflection at the free end, (c) the slope at the free end.

Use free body AJ

$$+\sum M_J = 0 : M + \frac{w_0 x^2}{2L} \cdot \frac{x}{3} = 0$$

$$M = -\frac{1}{6} \frac{w_0 x^3}{L}$$

$$\frac{d^2 y}{dx^2} = -\frac{1}{6} \frac{w_0 x^3}{L}$$

$$EI \frac{dy}{dx} = -\frac{1}{24} \frac{w_0 x^4}{L} + C_1$$

$$EI y = -\frac{1}{120} \frac{w_0 x^5}{L} + C_1 x + C_2$$

$$[x=L, \frac{dy}{dx}=0]: -\frac{1}{24} w_0 L^3 + C_1$$

$$C_1 = \frac{1}{24} w_0 L^3$$

$$EI y = -\frac{1}{120} w_0 L^4 + \frac{1}{24} w_0 L^4 + C_2 = 0$$

$$C_2 = \frac{1}{30} w_0 L^4$$

(a) Elastic curve.

$$y = -\frac{w_0}{120 EI L} (x^5 - 5L^4 x + 4L^5)$$

$$\frac{dy}{dx} = \frac{w_0}{24 EI L} (-x^4 + L^4)$$

(b) $y @ x=0:$

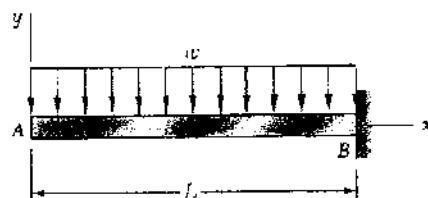
$$y_A = -\frac{w_0 L^4}{30 EI}$$

(c) $\frac{dy}{dx} @ x=0:$

$$\left. \frac{dy}{dx} \right|_A = \frac{w_0 L^3}{24 EI} \quad \theta_A = \frac{w_0 L^4}{24 EI}$$

Problem 9.4

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB , (b) the deflection at the free end, (c) the slope at the free end.



$$\sum M_J = 0: \quad (wx)\frac{x}{2} + M = 0$$

$$M = -\frac{1}{2}wx^2$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$$[x=L, \frac{dy}{dx}=0]$$

$$0 = -\frac{1}{6}wL^3 + C_1$$

$$C_1 = \frac{1}{6}wL^3$$

$$EI \frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EI y = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$$[x=L, y=0]$$

$$0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$$

$$C_2 = (\frac{1}{24} - \frac{1}{6})wL^4 = -\frac{3}{24}wL^4$$

(a) Elastic curve.

$$y = -\frac{w}{24EI} (x^4 - 4L^3x + 3L^4)$$

(b) $y @ x=0$:

$$y_A = -\frac{3wL^4}{24EI} = -\frac{wL^4}{8EI}$$

$$y_A = \frac{wL^4}{8EI} \downarrow$$

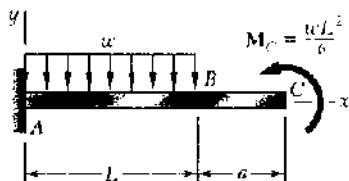
(c) $\frac{dy}{dx} @ x=0$:

$$\left. \frac{dy}{dx} \right|_A = \frac{wL^3}{6EI}$$

$$\theta_A = \frac{wL^3}{6EI} \nearrow$$

Problem 9.5

9.5 and 9.6 For the cantilever beam and loading shown, determine (a) the equation of the elastic curve for portion AB of the beam, (b) the deflection at B, (c) the slope at B.



$$[x=0, y=0]$$

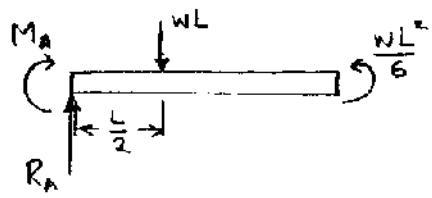
$$[x=0, \frac{dy}{dx}=0]$$

Using ABC as a free body

$$+\uparrow \sum F_y = 0 : R_A - wL = 0 \quad R_A = wL$$

$$+\rightarrow \sum M_A = 0 : -M_A - (wL)(\frac{L}{2}) + \frac{wL^2}{6} = 0$$

$$M_A = -\frac{1}{3}wL$$



Using AJ as a free body (Portion AB only)

$$+\rightarrow M_J = 0 : M + (wx)(\frac{x}{2}) - R_A x - M_A = 0$$

$$M = -\frac{1}{2}wx^2 + R_A x + M_A$$

$$= -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI \frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI \frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}wLx^2 - \frac{1}{3}wLx + C_1$$

$$[x=0, \frac{dy}{dx}=0] : -0 + 0 - 0 + C_1 = 0 \quad C_1 = 0$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wLx^3 - \frac{1}{6}wLx^2 + C_2$$

$$[x=0, y=0] \quad -0 + 0 - 0 + C_2 = 0 \quad C_2 = 0$$

(a) Elastic curve over AB. $y = \frac{w}{24EI}(-x^4 + 4Lx^3 - 4L^2x^2)$

$$\frac{dy}{dx} = \frac{w}{6EI}(-x^3 + 2Lx^2 - L^2x)$$

(b) y at $x=L$:

$$y_B = -\frac{wL^4}{24EI}$$

$$y_B = \frac{wL^4}{24EI} \downarrow$$

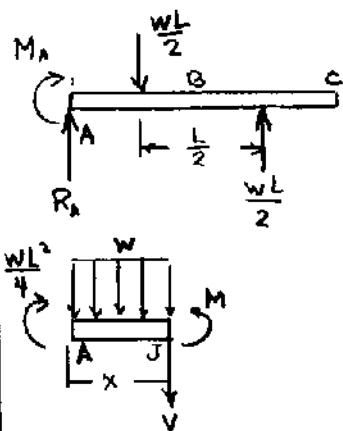
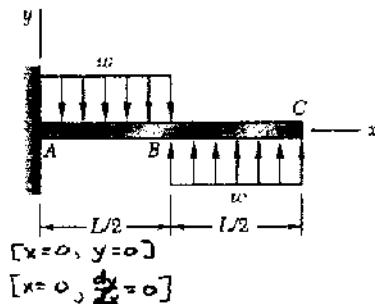
(c) $\frac{dy}{dx}$ at $x=L$:

$$\left. \frac{dy}{dx} \right|_B = 0$$

$$\theta_B = 0$$

Problem 9.6

9.5 and 9.6 For the cantilever beam and loading shown, determine (a) the equation of the elastic curve for portion AB of the beam, (b) the deflection at B, (c) the slope at B.



Using ABC as a free body.

$$+\uparrow \sum F_y = 0 : R_A - \frac{wL}{2} + \frac{wL}{2} = 0 \quad R_A = 0$$

$$+\rightarrow \sum M_A = 0 : -M_A + (\frac{wL}{2})(\frac{L}{2}) = 0 \quad M_A = \frac{wL^2}{4}$$

Using AJ as a free body (Portion AB only)

$$+\rightarrow \sum M_J = 0 : -\frac{wL^2}{4} + (wx)\frac{L}{2} + M = 0$$

$$M = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{4}wL^2x - \frac{1}{6}wx^3 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 = 0 - 0 + C_1, \quad C_1 = 0$$

$$EI y = \frac{1}{8}wL^2x^2 - \frac{1}{24}wx^4 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2, \quad C_2 = 0$$

(a) Elastic curve.

$$y = \frac{w}{EI} \left(\frac{1}{8}L^2x^2 - \frac{1}{24}x^4 \right)$$

$$\frac{dy}{dx} = \frac{w}{EI} \left(\frac{1}{4}L^2x - \frac{1}{6}x^3 \right)$$

(b) y at $x = \frac{L}{2}$:

$$y_B = \frac{w}{EI} \left\{ \frac{1}{8}L^2 \left(\frac{L}{2}\right)^2 - \frac{1}{24} \left(\frac{L}{2}\right)^4 \right\} = \frac{wL^5}{EI} \left\{ \frac{1}{32} - \frac{1}{384} \right\}$$

$$= \frac{11wL^5}{384EI}$$

$$y_B = \frac{11wL^5}{384EI}$$

(c) $\frac{dy}{dx}$ at $x = \frac{L}{2}$:

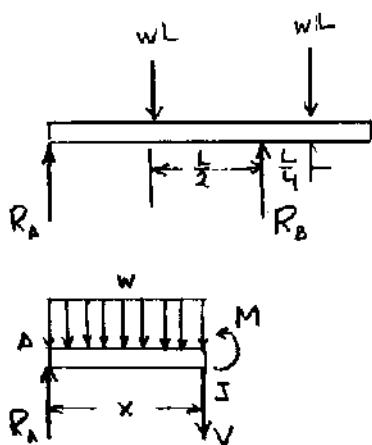
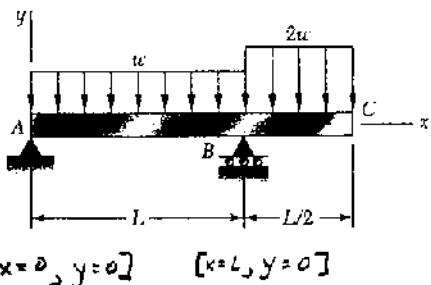
$$\theta_B = \frac{w}{EI} \left\{ \frac{1}{4}L^2 \left(\frac{L}{2}\right) - \frac{1}{6} \left(\frac{L}{2}\right)^3 \right\} = \frac{wL^3}{EI} \left\{ \frac{1}{8} - \frac{1}{48} \right\}$$

$$= \frac{5wL^3}{48EI}$$

$$\theta_B = \frac{5wL^3}{48EI}$$

Problem 9.7

9.7 For the beam and loading shown, determine (a) the equation of the elastic curve for portion AB of the beam, (b) the slope at A, (c) the slope at B.



Using free body ABC

$$\rightarrow \sum M_B = 0 : -R_A L + (wL)(\frac{L}{2}) - (wL)(\frac{L}{4}) = 0 \\ R_A = \frac{1}{4} wL$$

For portion AB ($0 < x < L$)

$$\rightarrow \sum M_J = 0 \quad M - R_A x + (wx)(\frac{x}{2}) = 0 \\ M = \frac{1}{4} wLx - \frac{1}{2} wx^2$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{4} wLx - \frac{1}{2} wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{8} wLx^2 - \frac{1}{6} wx^3 + C_1$$

$$EI y = \frac{1}{24} wLx^3 - \frac{1}{24} wx^4 + C_1 x + C_2$$

$$[x=0, y=0] : 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] : 0 = \frac{1}{24} wL^4 - \frac{1}{24} wL^4 + C_1 L + 0 = 0 \quad C_1 = 0$$

(a) Elastic curve ($0 \leq x \leq L$). $y = \frac{w}{24EI} (Lx^3 - x^4)$

$$\frac{dy}{dx} = \frac{w}{24EI} (3Lx^2 - 4x^3)$$

(b) $\frac{dy}{dx}$ at $x=0$

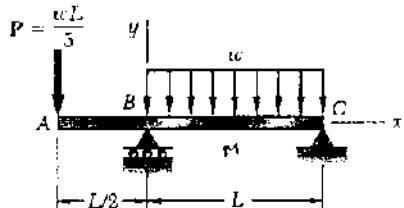
$$\left. \frac{dy}{dx} \right|_A = 0 \quad \theta_A = 0$$

(c) $\frac{dy}{dx}$ at $x=L$

$$\left. \frac{dy}{dx} \right|_B = -\frac{wL^3}{24EI} \quad \theta_B = \frac{wL^3}{24EI}$$

Problem 9.8

9.8 For the beam and loading shown, determine (a) the equation of the elastic curve for portion BC of the beam, (b) the deflection at midspan, (c) the slope at B.

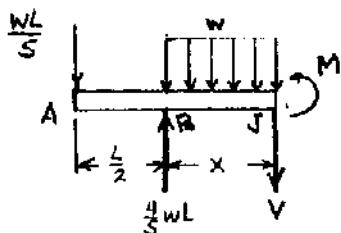


$$x = -\frac{L}{2} \quad [x=0] \quad [x=L] \quad [y=0]$$

Using ABC as a free body

$$\rightarrow \sum M_C = 0 : \quad (\frac{wL}{5})(\frac{3L}{2}) + R_B L + (wL)(\frac{L}{2}) = 0$$

$$R_B = \frac{4}{5}wL$$



For portion BC only ($0 < x < L$)

$$\rightarrow \sum M_J = 0 : \quad \frac{wL}{5}(\frac{L}{2} + x) - \frac{4}{5}wLx + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{3}{5}wLx - \frac{1}{2}wx^2 - \frac{1}{10}wL^2$$

$$EI \frac{d^2y}{dx^2} = \frac{3}{5}wLx - \frac{1}{2}wx^2 - \frac{1}{10}wL^2$$

$$EI \frac{dy}{dx} = \frac{3}{10}wLx^2 - \frac{1}{6}wx^3 - \frac{1}{10}wL^2x + C_1$$

$$EI y = \frac{1}{10}wLx^3 - \frac{1}{24}wx^4 - \frac{1}{20}wL^2x^2 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = (\frac{1}{10} - \frac{1}{24} - \frac{1}{20})wL^4 + C_1L + 0 \quad C_1 = -\frac{1}{120}wL^3$$

$$(a) \text{ Elastic curve.} \quad y = \frac{w}{EI} \left(\frac{1}{10}Lx^3 - \frac{1}{24}x^4 - \frac{1}{20}L^2x^2 - \frac{1}{120}L^3x \right)$$

$$\frac{dy}{dx} = \frac{w}{EI} \left(\frac{3}{10}Lx^2 - \frac{1}{6}x^3 - \frac{1}{10}L^2x - \frac{1}{120}L^3 \right)$$

$$(b) \quad y @ x = \frac{L}{2} : \quad y_M = \frac{w}{EI} \left\{ \frac{1}{10}L\left(\frac{L}{2}\right)^3 - \frac{1}{24}\left(\frac{L}{2}\right)^4 - \frac{1}{20}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{120}L^3\left(\frac{L}{2}\right) \right\}$$

$$= \frac{wL^4}{EI} \left\{ \frac{1}{80} - \frac{1}{384} - \frac{1}{80} - \frac{1}{240} \right\} = -\frac{13wL^4}{1920EI}$$

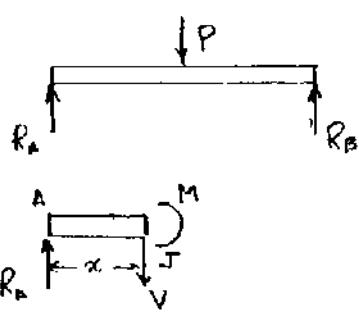
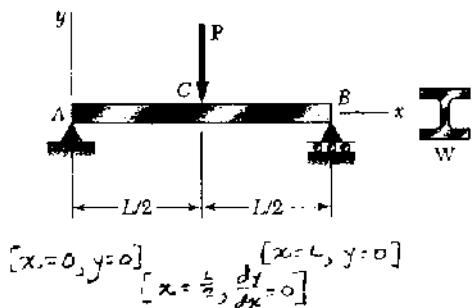
$$y_M = \frac{13wL^4}{1920EI} \downarrow$$

$$(c) \quad \frac{dy}{dx} @ x = 0 : \quad \left. \frac{dy}{dx} \right|_B = \frac{w}{EI} \left(0 - 0 - 0 - \frac{1}{120}L^3 \right) = -\frac{wL^3}{120EI}$$

$$\theta_B = \frac{wL^3}{120EI}$$

Problem 9.9

9.9 Knowing that beam AB is a W 130 × 23.8 rolled shape and that $P = 50 \text{ kN}$, $L = 1.25 \text{ m}$, and $E = 200 \text{ GPa}$, determine (a) the slope at A , (b) the deflection at C .



Use symmetry boundary condition at C .

$$\text{By symmetry } R_A = R_B = \frac{1}{2}P$$

$$\text{Using free body AJ } 0 \leq x \leq \frac{L}{2}$$

$$+\sum M_J = 0 \quad M - R_A x = 0$$

$$M = R_A x = \frac{1}{2}Px$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{2}Px$$

$$EI \frac{dy}{dx} = \frac{1}{4}Px^2 + C_1$$

$$EIy = \frac{1}{12}Px^3 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0] \quad 0 = \frac{1}{4}P\left(\frac{L}{2}\right)^2 + C_1 \quad C_1 = -\frac{1}{16}PL^2$$

$$\text{Elastic curve. } y = \frac{PL}{48EI} (4x^3 - 3L^2x)$$

$$\frac{dy}{dx} = \frac{PL}{16EI} (4x^2 - L^2)$$

$$\text{Slope at } x=0. \quad \frac{dy}{dx} \Big|_A = -\frac{PL^3}{16EI} \quad \Theta_A = \frac{PL^2}{16EI} \leftarrow$$

$$\text{Deflection at } x=\frac{L}{2}. \quad y_c = -\frac{PL^3}{48EI} \quad y_c = \frac{PL^3}{48EI} \downarrow$$

$$\text{Data: } P = 50 \times 10^3 \text{ N}, \quad I = 8.80 \times 10^6 \text{ mm}^4 = 8.80 \times 10^{-6} \text{ m}^4$$

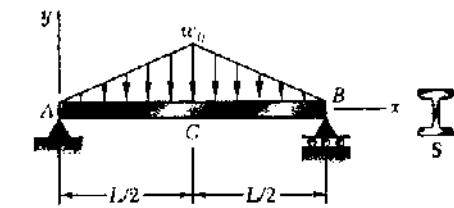
$$E = 200 \times 10^9 \text{ Pa} \quad EI = 1.76 \times 10^6 \text{ N.m}^2 \quad L = 1.25 \text{ m}$$

$$(a) \quad \Theta_A = \frac{(50 \times 10^3)(1.25)^2}{(16)(1.76 \times 10^6)} = 2.77 \times 10^{-3} \text{ rad} \leftarrow$$

$$(b) \quad y_c = \frac{(50 \times 10^3)(1.25)^3}{(48)(1.76 \times 10^6)} = 1.156 \times 10^{-3} \text{ m} = 1.156 \text{ mm} \downarrow$$

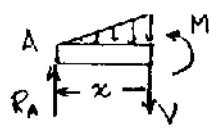
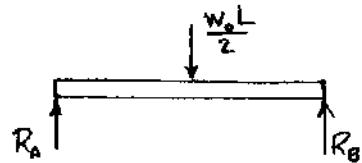
Problem 9.10

9.10 Knowing that beam AB is an S 8 × 18.4 rolled shape and that $w_0 = 4 \text{ kips/ft}$, $L = 9 \text{ ft}$, and $E = 29 \times 10^6 \text{ psi}$, determine (a) the slope at A , (b) the deflection at C .



$$[x=0, y=0] \quad [x=L, y=0]$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0]$$



Use symmetry boundary condition at C .

Using free body ACB and symmetry

$$R_A = R_B = \frac{1}{4} w_0 \frac{L}{2}$$

$$\text{For } 0 < x < \frac{L}{2} \quad w = \frac{2w_0 x}{L}$$

$$\frac{dV}{dx} = -w = -\frac{2w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{L} + R_A = \frac{w_0}{L} \left(\frac{1}{4} L^2 - x^2 \right)$$

$$M = \frac{w_0}{L} \left(\frac{1}{4} L^2 x - \frac{1}{3} x^3 \right) + C_M$$

But $M = 0$ at $x=0$; hence $C_M = 0$

$$EI \frac{d^2y}{dx^2} = \frac{w_0}{L} \left(\frac{1}{4} L^2 x - \frac{1}{3} x^3 \right)$$

$$EI \frac{dy}{dx} = \frac{w_0}{L} \left(\frac{1}{8} L^2 x^2 - \frac{1}{12} x^4 \right) + C_1$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0]$$

$$0 = \frac{w_0}{L} \left(\frac{1}{32} L^4 - \frac{1}{192} L^4 \right) + C_1 = 0 \quad C_1 = -\frac{5}{192} w_0 L^3$$

$$EIy = \frac{w_0}{L} \left(\frac{1}{24} L^2 x^3 - \frac{1}{120} x^5 \right) - \frac{5}{192} w_0 L^3 x + C_2$$

$$[x=0, y=0]$$

$$0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

Elastic curve.

$$y = \frac{w_0}{EIL} \left(\frac{1}{24} L^2 x^3 - \frac{1}{60} x^5 - \frac{5}{192} L^4 x \right)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL} \left(\frac{1}{8} L^2 x^2 - \frac{1}{12} x^4 - \frac{5}{192} L^4 \right)$$

Data: $w_0 = 4 \text{ kips/ft}$ $E = 29 \times 10^6 \text{ psi}$, $I = 57.6 \text{ in}^4$

$$EI = (29 \times 10^6)(57.6) = 1.6704 \times 10^9 \text{ lb-in}^2 = 11.6 \times 10^3 \text{ kip-ft}^2$$

$$L = 9 \text{ ft}$$

$$(a) \text{ Slope at } x=0. \quad \frac{dy}{dx} = \frac{4}{(11.6 \times 10^3)(9)} \left[-\left(\frac{5}{192}\right)(9)^4 \right] = -6.55 \times 10^{-3}$$

$$\theta_A = 6.55 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

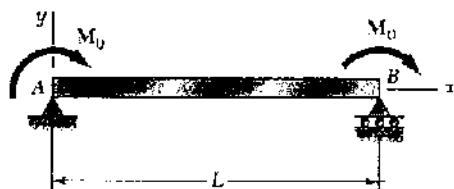
(b) Deflection at $x=4.5 \text{ ft}$.

$$y = \frac{4}{(11.6 \times 10^3)(9)} \left[\frac{1}{24}(9)^2(4.5)^3 - \frac{1}{60}(4.5)^5 - \frac{5}{192}(9)^4(4.5) \right] = -18.853 \times 10^{-3} \text{ ft}$$

$$= 0.226 \text{ in.} \downarrow$$

Problem 9.11

9.11 (a) Determine the location and magnitude of the maximum absolute deflection in AB between A and the center of the beam. (b) Assuming that beam AB is a W 460 x 113, $M_o = 224 \text{ kN}\cdot\text{m}$ and $E = 200 \text{ GPa}$, determine the maximum allowable length L so that the maximum deflection does not exceed 1.2 mm.



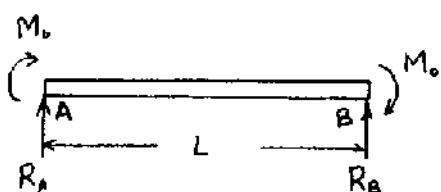
$$[x=0, y=0]$$

$$[x=L, y=0]$$

Using AB as a free body

$$\sum M_B = 0 : -2M_o - R_A L = 0$$

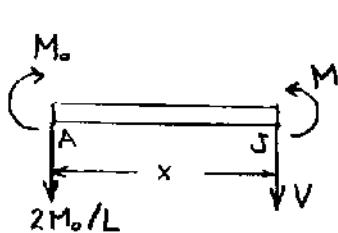
$$R_A = -\frac{2M_o}{L}$$



Using portion AJ as a free body

$$\sum M_J = 0 : -M_o + \frac{2M_o}{L}x + M = 0$$

$$M = \frac{M_o}{L}(L - 2x)$$



$$EI \frac{d^2y}{dx^2} = \frac{M_o}{L}(L - 2x)$$

$$EI \frac{dy}{dx} = \frac{M_o}{L}(Lx - x^2) + C_1$$

$$EI y = \frac{M_o}{L}(\frac{1}{2}Lx^2 - \frac{1}{3}x^3) + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{M_o}{L}(\frac{1}{2}L^3 - \frac{1}{3}L^3) + C_1 L + 0 \quad C_1 = -\frac{1}{6}M_o L^2$$

$$y = \frac{M_o}{EI L}(\frac{1}{2}Lx^2 - \frac{1}{3}x^3 - \frac{1}{6}L^2 x) \quad \frac{dy}{dx} = \frac{M_o}{EI L}(Lx - x^2 - \frac{1}{6}L^2)$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$

$$x_m^2 - Lx_m - \frac{1}{6}L^2 = 0 \quad x_m = \frac{L - \sqrt{L^2 - (4)(\frac{1}{6}L^2)}}{2} = \frac{1}{2}(1 - \frac{\sqrt{3}}{3})L = 0.21132 L$$

$$y_m = \frac{M_o L^2}{EI} \left\{ (\frac{1}{2})(0.21132)^2 - (\frac{1}{3})(0.21132)^3 - (\frac{1}{6})(0.21132) \right\} = -0.0160375 \frac{M_o L^2}{EI}$$

$$|y_m| = 0.0160375 \frac{M_o L^2}{EI}$$

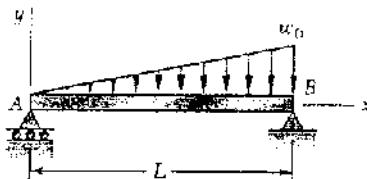
$$\text{Solving for } L, \quad L = \left\{ \frac{EI |y_m|}{0.0160375 M_o} \right\}^{1/2}$$

$$\text{Data: } E = 200 \times 10^9 \text{ Pa}, \quad I = 556 \times 10^6 \text{ mm}^4 = 556 \times 10^{-6} \text{ m}^4$$

$$|y_m| = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}, \quad M_o = 224 \times 10^3 \text{ N}\cdot\text{m}$$

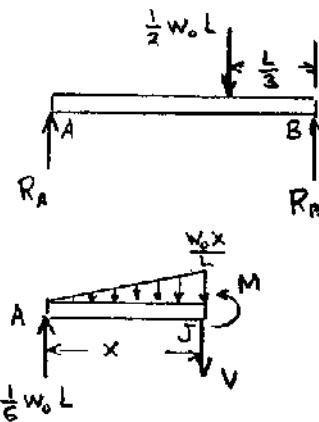
$$L = \left\{ \frac{(200 \times 10^9)(556 \times 10^{-6})(1.2 \times 10^{-3})}{(0.0160375)(224 \times 10^3)} \right\}^{1/2} = 6.09 \text{ m}$$

Problem 9.12



$$[x=0, y=0] \quad [x=L, y=0]$$

9.12 For the beam and loading shown, (a) express the magnitude and location of the maximum deflection in terms of w_0 , L , E , and I . (b) Calculate the value of the maximum deflection, assuming that beam AB is a W 18 × 50 rolled shape and that $w_0 = 4.5$ kips/ft, $L = 18$ ft, and $E = 29 \times 10^6$ psi.



Using entire beam as a free body,

$$\therefore \sum M_B = 0: \quad -R_A L + (\frac{1}{2} w_0 L)(\frac{L}{3}) = 0 \\ R_A = \frac{1}{6} w_0 L$$

Using AJ as a free body, $\sum M_J = 0:$

$$-\frac{1}{6} w_0 L x + (\frac{1}{2} \frac{w_0 x^2}{L})(\frac{x}{3}) + M = 0 \\ M = \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3$$

$$EI \frac{dy}{dx} = \frac{1}{12} w_0 L x^2 - \frac{1}{24} \frac{w_0}{L} x^4 + C_1$$

$$EI y = \frac{1}{36} w_0 L x^3 - \frac{1}{120} \frac{w_0}{L} x^5 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{1}{36} w_0 L^3 - \frac{1}{120} w_0 L^5 + C_1 L + 0 \quad C_1 = -\frac{7}{360} \frac{w_0 L^3}{L}$$

$$\text{Elastic curve.} \quad y = \frac{w_0}{EI} \left\{ \frac{1}{36} L x^3 - \frac{1}{120} \frac{x^5}{L} - \frac{7}{360} L^3 x \right\}$$

$$\frac{dy}{dx} = \frac{w_0}{EI} \left\{ \frac{1}{12} L x^2 - \frac{1}{24} \frac{x^4}{L} - \frac{7}{360} L^3 \right\}$$

To find location of maximum deflection, set $\frac{dy}{dx} = 0$

$$15x_m^4 - 30L^2x_m^2 + 7L^4 = 0 \quad x_m^2 = \frac{30L^2 - \sqrt{900L^4 - 420L^4}}{30}$$

$$x_m^2 = (1 - \sqrt{\frac{6}{15}})L^2 = 0.2697 L^2 \quad x_m = 0.5193 L$$

$$y_m = \frac{w_0}{EI} \left\{ \frac{1}{36} L (0.5193 L)^3 - \frac{1}{120} \frac{(0.5193 L)^5}{L} - \frac{7}{360} L^3 (0.5193 L) \right\}$$

$$= -0.00652 \frac{w_0 L^4}{EI} \quad \text{or} \quad 0.00652 \frac{w_0 L^4}{EI} \downarrow$$

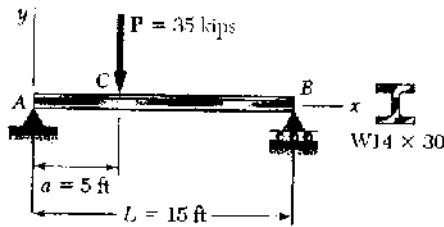
$$\text{Data: } w_0 = 4.5 \text{ kips/ft} = \frac{4500}{12} = 375 \text{ lb/in.}, \quad L = 18 \text{ ft} = 216 \text{ in.}$$

$$I = 800 \text{ in}^4 \text{ for W18x50}$$

$$y_m = \frac{(0.00652)(375)(216)^4}{(29 \times 10^6)(800)} = 0.229 \text{ in.} \downarrow$$

Problem 9.13

9.13 For the beam and loading shown, determine the deflection at point C. Use $E = 29 \times 10^6$ psi.



$$[x=0, y=0] \quad [x=L, y=0]$$

$$\begin{bmatrix} [x=a, y=y] \\ [x=a, \frac{dy}{dx}=\frac{dy}{dx}] \end{bmatrix}$$

$$\text{Let } b = L - a$$

$$\text{Reactions: } R_A = \frac{Pb}{L} \uparrow, \quad R_B = \frac{Pa}{L} \uparrow$$

Bending moments

$$0 < x < a \quad M = \frac{Pb}{L} x$$

$$a < x < L \quad M = \frac{P}{L} [bx - L(x - a)]$$

$$0 < x < a$$

$$EI \frac{d^2y}{dx^2} = \frac{P}{L} (bx)$$

$$EI \frac{dy}{dx} = \frac{P}{L} \left(\frac{1}{2} bx^2 \right) + C_1 \quad (1)$$

$$EI y = \frac{P}{L} \left(\frac{1}{6} bx^3 \right) + C_1 x + C_2 \quad (2)$$

$$a < x < L$$

$$EI \frac{d^2y}{dx^2} = \frac{P}{L} [bx - L(x - a)]$$

$$EI \frac{dy}{dx} = \frac{P}{L} \left[\frac{1}{2} bx^2 - \frac{1}{2} L(x - a)^2 \right] + C_3 \quad (3)$$

$$EI y = \frac{P}{L} \left[\frac{1}{6} bx^3 - \frac{1}{6} L(x - a)^3 \right] + C_3 x + C_4 \quad (4)$$

$$[x=0, y=0] \quad E_q(2) \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad Eqs. (1) \text{ and } (3) \quad \frac{P}{L} \left(\frac{1}{2} ba^2 \right) + C_1 = \frac{P}{L} \left[\frac{1}{2} ba^2 + 0 \right] + C_3 \therefore C_3 = C_1$$

$$[x=a, y=y] \quad Eqs. (2) \text{ and } (4) \quad \frac{P}{L} \left(\frac{1}{6} ba^3 \right) + C_1 a + C_2 \\ = \frac{P}{L} \left[\frac{1}{6} ba^3 + 0 \right] + C_1 a + C_4 \quad C_4 = C_2 = 0$$

$$[x=L, y=0] \quad E_q(4) \quad \frac{P}{L} \left[\frac{1}{6} bL^3 - \frac{1}{6} L(L-a)^3 \right] + C_3 L = 0$$

$$C_1 = C_3 = \frac{P}{L} \left[\frac{1}{6} (L-a)^3 - \frac{1}{6} bL^2 \right] = \frac{P}{L} \left(\frac{1}{6} b^3 - \frac{1}{6} bL^2 \right)$$

Make $x = a$ in Eq. (2).

$$y_c = \frac{P}{EI L} \left[\frac{1}{6} ba^3 + \frac{1}{6} b^3 a - \frac{1}{6} bL^2 a \right] = \frac{P(ba^3 + b^3 a - L^2 ab)}{6EI L}$$

$$\text{Data: } P = 35 \text{ kips}, \quad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ kips/in}^2$$

$$L = 15 \text{ ft}, \quad a = 5 \text{ ft}, \quad b = 10 \text{ ft.}$$

$$I = 291 \text{ in}^4, \quad EI = 8.439 \times 10^6 \text{ kip-in}^2 = 58.604 \times 10^3 \text{ kip-ft}^2$$

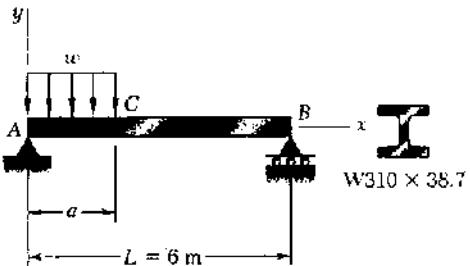
$$y_c = \frac{35}{(6)(58.604 \times 10^3)(15)} \left[(10)(5)^3 + (10^3)(5) - (15)^2(5)(10) \right]$$

$$= -33.179 \times 10^{-3} \text{ ft} = -0.398 \text{ in.}$$

$$y_c = 0.398 \text{ in.} \downarrow$$

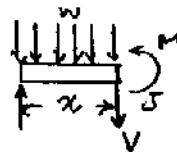
Problem 9.14

9.14 For the beam and loading shown, knowing that $a = 2 \text{ m}$, $w = 50 \text{ kN/m}$, and $E = 200 \text{ GPa}$, determine (a) the slope at support A, (b) the deflection at point C.



$$\begin{aligned} [x=0, y=0] \quad & [x=L, y=0] \\ [x=a, y=y] \quad & [x=a, \frac{dy}{dx} = \frac{dy}{dx}] \end{aligned}$$

$$0 \leq x \leq a$$



$$+\sum M_J = 0: \quad M - R_A x + (wx) \left(\frac{x}{2}\right) = 0$$

$$M = R_A x - \frac{1}{2}wx^2$$

$$EI \frac{d^2y}{dx^2} = R_A x - \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$$EI y = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$EI y = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + C_1 = -\frac{1}{2}R_B (2a)^2 + C_3$$

$$C_3 = C_1 + \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + \frac{1}{2}R_B (2a)^2 = C_1 + \frac{7}{12}wa^3$$

$$[x=a, y=y] \quad \frac{1}{6}R_A a^3 - \frac{1}{24}wa^4 + C_1 a = \frac{1}{6}R_B (2a)^3 - (C_1 + \frac{7}{12}wa^3)(2a)$$

$$3C_1 a = -\frac{1}{6}R_A a^3 + \frac{1}{24}wa^4 + \frac{1}{6}R_B (2a)^3 - \frac{7}{12}wa^2(2a) = -\frac{25}{24}wa^4$$

$$C_1 = -\frac{25}{72}wa^3$$

$$\text{For } 0 \leq x \leq a, \quad EI y = \frac{5}{36}wx^3 - \frac{1}{24}wx^4 - \frac{25}{72}wa^3 x$$

$$EI \frac{dy}{dx} = \frac{5}{12}wx^2 - \frac{1}{6}wx^3 - \frac{25}{72}wa^3$$

Using ACB as a free body and noting that $L = 3a$,

$$\begin{aligned} A & \begin{array}{c} w \\ \downarrow \\ \frac{w}{2} \\ \downarrow \end{array} \\ R_A & \leftarrow \quad L \quad \rightarrow R_B \\ +\sum M_A = 0: \quad R_B L - (wa) \left(\frac{w}{2}\right) & = 0 \\ R_B = (wa) \frac{a}{2L} = \frac{1}{6}wa & \end{aligned}$$

$$+\sum F_y = 0: \quad R_A + R_B - wa = 0 \quad R_A = \frac{5}{6}wa$$

$$0 \leq x \leq L$$

$$\begin{array}{c} M \uparrow \\ \curvearrowleft \\ L-x \end{array} \quad R_B$$

$$+\sum M_K = 0 \quad -M + R_B(L-x) = 0$$

$$M = R_B(L-x)$$

$$EI \frac{d^2y}{dx^2} = R_B(L-x)$$

$$EI \frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + C_3$$

$$EI y = \frac{1}{6}R_B(L-x)^3 + C_3 x + C_4$$

$$[x=L, y=0] \quad 0 = 0 + C_3 L + C_4$$

$$C_4 = -C_3 L$$

$$EI y = \frac{1}{6}R_B(L-x)^3 - C_3(L-x)$$

$$EI \frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + C_3$$

continued

Problem 9.14 continued

Data: $w = 50 \times 10^3 \text{ N} \cdot \text{m}$, $a = 2 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$

$$I = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4, \quad EI = 17.02 \times 10^6 \text{ N} \cdot \text{m}^2$$

(a) Slope at $x = 0$.

$$17.02 \times 10^6 \frac{dy}{dx} \Big|_A = 0 - 0 - \frac{25}{72} (50 \times 10^3)(2)^3$$

$$\frac{dy}{dx} \Big|_A = \Theta_A = -8.16 \times 10^{-3}$$

$$\Theta_A = 8.16 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

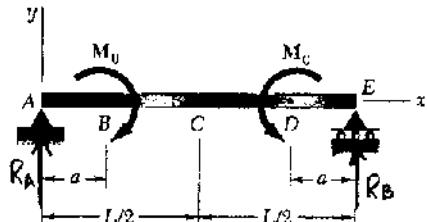
(b) Deflection at $x = 2 \text{ m}$.

$$EI y_c = \frac{5}{36} wa^4 - \frac{1}{24} wa^4 - \frac{25}{72} wa^4 = -\frac{1}{4} wa^4$$

$$17.02 \times 10^6 y_c = -\frac{1}{4} (50 \times 10^3)(2)^4 \quad y_c = -11.75 \times 10^{-3} \text{ m} \quad y_c = 11.75 \text{ mm} \downarrow$$

Problem 9.15

9.15 Knowing that beam AE is a W 360 × 101 rolled shape and that $M_0 = 310 \text{ kN}\cdot\text{m}$, $L = 2.4 \text{ m}$, $a = 0.5 \text{ m}$ and $E = 200 \text{ GPa}$, determine (a) the equation of the elastic curve for portion BD , (b) the deflection at point C .



Use continuity boundary condition at B and symmetry boundary condition at C .

From Statics, $R_A = R_B = 0$

$$[x=0, y=a]$$

$$[x=a, y=y]$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = 0]$$

$$0 \leq x \leq a$$

$$M = 0$$

$$EI \frac{d^2y}{dx^2} = 0$$

$$EI \frac{dy}{dx} = C_1$$

$$EI y = C_1 x + C_2$$

$$a \leq x \leq L-a$$

$$M = M_0$$

$$EI \frac{d^2y}{dx^2} = M_0$$

$$EI \frac{dy}{dx} = M_0 x + C_3$$

$$EI y = \frac{1}{2} M_0 x^2 + C_3 x + C_4$$

$$[x=0, y=0] \quad 0 = 0 + C_2 \quad C_2 = 0$$

$$[x = \frac{L}{2}, \frac{dy}{dx} = 0] \quad 0 = \frac{1}{2} M_0 L + C_3 \quad C_3 = -\frac{1}{2} M_0 L$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad C_1 = M_0 a + C_3 = -M_0 (\frac{1}{2} L - a)$$

$$[x=a, y=y] \quad -M_0 (\frac{1}{2} L - a) a + 0 = \frac{1}{2} M_0 a^2 - \frac{1}{2} M_0 L a + C_4 \\ C_4 = \frac{1}{2} M_0 a^2$$

$$(a) \text{Elastic curve } (a \leq x \leq L-a). \quad EI y = \frac{1}{2} M_0 x^2 - \frac{1}{2} M_0 L x + \frac{1}{2} M_0 a^2$$

$$y = \frac{M_0}{2EI} (x^2 - Lx + a^2)$$

$$(b) \text{Deflection at } x = \frac{L}{2}.$$

$$y_c = \frac{M_0}{2EI} \left[\left(\frac{L}{2}\right)^2 - (L)\left(\frac{L}{2}\right) + a^2 \right] = -\frac{M_0}{8EI} (L^2 - 4a^2)$$

$$\text{Data: } M_0 = 310 \times 10^3 \text{ N}\cdot\text{m}, \quad L = 2.4 \text{ m}, \quad a = 0.5 \text{ m}, \quad E = 200 \times 10^9 \text{ Pa}$$

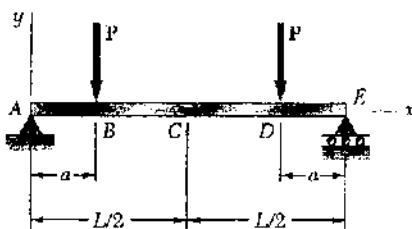
$$I = 302 \times 10^4 \text{ mm}^4 = 302 \times 10^{-4} \text{ m}^4 \quad EI = 60.4 \times 10^6 \text{ N}\cdot\text{m}^2$$

$$y_c = -\frac{310 \times 10^3}{(8)(60.4 \times 10^6)} \left[(2.4)^2 - 4(0.5)^2 \right] = -3.05 \times 10^{-3}$$

$$y_c = 3.05 \text{ mm} \downarrow$$

Problem 9.16

9.16 Knowing that beam AE is an S 200 × 27.4 rolled shape and that $P = 17.5 \text{ kN}$, $L = 2.5 \text{ m}$, $a = 0.8 \text{ m}$ and $E = 200 \text{ GPa}$, determine (a) the equation of the elastic curve for portion BD , (b) the deflection at the center C of the beam.



Consider portion ABC or DE , and consider symmetry about C .

$$\text{Reactions } R_A = R_E = P$$

Boundary conditions: $[x=0, y=0]$, $[x=a, y=y]$, $[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$, $[x=\frac{L}{2}, \frac{dy}{dx} = 0]$

$$0 < x < a$$

$$EI \frac{d^2y}{dx^2} = M = Px$$

$$EI \frac{dy}{dx} - \frac{1}{2}Px^2 + C_1 \quad (1)$$

$$EIy = \frac{1}{6}Px^3 + C_1x + C_2 \quad (2)$$

$$[x=0, y=0] \rightarrow C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \quad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$$

$$[x=\frac{L}{2}, y=y] \quad \frac{1}{6}Pa^3 + (\frac{1}{2}Pa^2 - \frac{1}{2}PaL)a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L + C_2 \\ C_2 = \frac{1}{6}Pa^3$$

$$a < x < L-a$$

$$EI \frac{d^2y}{dx^2} = M = Pa$$

$$EI \frac{dy}{dx} = Pax + C_3$$

$$EIy = \frac{1}{2}Pax^2 + C_3x + C_4$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = 0] \rightarrow C_3 = -\frac{1}{2}PaL$$

(a) Elastic curve for portion BD .

$$y = \frac{1}{EI}(\frac{1}{2}Pax^2 + C_3x + C_4) \\ = \frac{P}{EI}(\frac{1}{2}ax^2 - \frac{1}{2}aLx + \frac{1}{6}a^3)$$

For deflection at C , set $x = \frac{L}{2}$.

$$y_C = \frac{P}{EI}(\frac{1}{8}aL^2 - \frac{1}{4}aL^2 + \frac{1}{6}a^3) = -\frac{Pa}{EI}(\frac{1}{8}L^2 - \frac{1}{6}a^2)$$

Data: $I = 23.9 \times 10^6 \text{ mm}^4 = 23.9 \times 10^6 \text{ m}^4$, $E = 200 \times 10^9 \text{ Pa}$

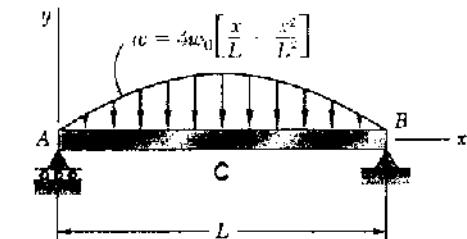
$P = 17.5 \times 10^3 \text{ N}$, $L = 2.5 \text{ m}$, $a = 0.8 \text{ m}$

$$(b) y_C = -\frac{(17.5 \times 10^3)(0.8)}{(200 \times 10^9)(23.9 \times 10^6)} \left\{ \frac{2.5^2}{8} - \frac{0.8^2}{6} \right\} = -1.976 \times 10^{-3} \text{ m}$$

$$y_C = 1.976 \text{ mm}$$

Problem 9.17

9.17 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection at the midpoint of the span.



Boundary conditions at A and B are noted.

$$w = \frac{w_0}{L^2} (4Lx - 4x^2)$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2} (4x^2 - 4Lx)$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2} \left(\frac{4}{3}x^3 - 2Lx^2 \right) + C_1$$

$$M = \frac{w_0}{L^2} \left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 \right) + C_1x + C_2$$

$$[x=0, M=0] \quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, M=0] \quad 0 = \frac{w_0}{L^2} \left(\frac{1}{3}L^4 - \frac{2}{3}L^4 \right) + C_1L + 0 \quad C_1 = \frac{1}{3}w_0L^2$$

$$EI \frac{d^2y}{dx^2} = M = \frac{w_0}{L^2} \left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 + \frac{1}{3}L^3x \right)$$

$$EI \frac{dy}{dx} = \frac{w_0}{L^2} \left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2 \right) + C_3$$

$$EI y = \frac{w_0}{L^2} \left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3 \right) + C_3x + C_4$$

$$[x=0, y=0] \quad 0 = 0 + 0 + 0 + 0 + C_4 \quad C_4 = 0$$

$$[x=L, y=0] \quad 0 = \frac{w_0}{L^2} \left(\frac{1}{90}L^6 - \frac{1}{30}L^5 + \frac{1}{18}L^6 \right) + C_3L + 0 \quad C_3 = -\frac{1}{30}w_0L^3$$

$$(a) \text{ Elastic curve.} \quad y = \frac{w_0}{EI L^2} \left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3 - \frac{1}{30}L^5x \right)$$

$$\frac{dy}{dx} = \frac{w_0}{EI L^2} \left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2 - \frac{1}{30}L^5 \right)$$

$$(b) \text{ Slope at end A.} \quad \text{Set } x=0 \text{ in } \frac{dy}{dx}. \quad \left. \frac{dy}{dx} \right|_A = -\frac{1}{30} \frac{w_0 L^3}{EI}$$

$$\theta_A = \frac{1}{30} \frac{w_0 L^3}{EI}$$

$$(c) \text{ Deflection at midpoint.} \quad \text{Set } x = \frac{L}{2} \text{ in } y.$$

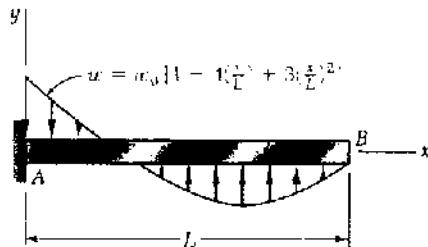
$$y_C = \frac{w_0 L^4}{EI} \left\{ \left(\frac{1}{90} \right) \left(\frac{1}{2} \right)^6 - \left(\frac{1}{30} \right) \left(\frac{1}{2} \right)^5 + \frac{1}{18} \left(\frac{1}{2} \right)^3 - \frac{1}{30} \left(\frac{1}{2} \right)^2 \right\}$$

$$= \frac{w_0 L^4}{EI} \left\{ \frac{1}{5760} - \frac{1}{960} + \frac{1}{144} - \frac{1}{60} \right\} = -\frac{61}{5760} \frac{w_0 L^4}{EI}$$

$$y_C = \frac{61}{5760} \frac{w_0 L^4}{EI}$$

Problem 9.18

9.18 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at the free end.



Boundary conditions are shown at left.

$$\begin{cases} u=0, y=0 \\ x=0, \frac{dy}{dx}=0 \end{cases}$$

$$\begin{cases} x=L, v=0 \\ x=L, M=0 \end{cases}$$

$$\frac{dV}{dx} = -w = -w_0 \left[1 - 4\left(\frac{x}{L}\right) + 3\left(\frac{x}{L}\right)^2 \right]$$

$$V = -w_0 \left[x - \frac{2x^2}{L} + \frac{x^3}{L^2} \right] + C_V$$

$$[x=L, V=0]:$$

$$0 = -w_0 [L - 2L + L] + C_V = 0 \quad C_V = 0$$

$$\frac{dM}{dx} = V = -w_0 \left[x - \frac{2x^2}{L} + \frac{x^3}{L^2} \right]$$

$$M = -w_0 \left[\frac{x^2}{2} - \frac{2x^3}{3L} + \frac{x^4}{4L^2} \right] + C_M$$

$$[x=L, M=0]: \quad 0 = -w_0 \left[\frac{1}{2}L^2 - \frac{2}{3}L^3 + \frac{1}{4}L^4 \right] + C_M \quad C_M = \frac{1}{12}w_0 L^2$$

$$EI \frac{d^2y}{dx^2} = M = -w_0 \left[\frac{1}{2}x^2 - \frac{2}{3}\frac{x^3}{L} + \frac{1}{4}\frac{x^4}{L^2} - \frac{1}{12}L^2 \right]$$

$$EI \frac{dy}{dx} = -w_0 \left[\frac{1}{6}x^3 - \frac{1}{6}\frac{x^4}{L} + \frac{1}{20}\frac{x^5}{L^2} - \frac{1}{12}L^2 x \right] + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad \rightarrow \quad C_1 = 0$$

$$EIy = -w_0 \left[\frac{1}{24}x^4 - \frac{1}{30}\frac{x^5}{L} + \frac{1}{120}\frac{x^6}{L^2} - \frac{1}{24}L^2 x^2 \right] + C_2$$

$$[x=0, y=0] \quad \rightarrow \quad C_2 = 0$$

$$(a) \text{ Elastic curve. } y = -\frac{w_0}{EI L^2} \left(\frac{1}{24}L^2 x^4 - \frac{1}{30}L x^5 + \frac{1}{120}x^6 - \frac{1}{24}L^2 x^2 \right)$$

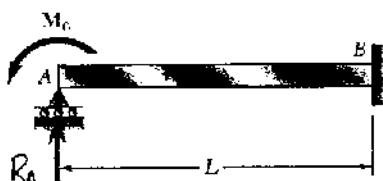
(b) Deflection at $x=L$.

$$y_B = -\frac{w_0}{EI L^2} \left(\frac{1}{24}L^6 - \frac{1}{30}L^6 + \frac{1}{120}L^6 - \frac{1}{24}L^6 \right) = -\frac{w_0 L^4}{40 EI}$$

$$y_B = \frac{w_0 L^4}{40 EI} \downarrow$$

Problem 9.19

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$

Using free body AJ

$$+\rightarrow \sum M_J = 0 \quad M_o - R_A x + M = 0$$

$$M = R_A x - M_o$$

$$EI \frac{d^2y}{dx^2} = R_A x - M_o$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_o x + C_1$$

$$[x=L, \frac{dy}{dx}=0]$$

$$0 = \frac{1}{2} R_A L^2 - M_o L + C_1$$

$$C_1 = M_o L - \frac{1}{2} R_A L^2$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_o x^2 + C_1 x + C_2$$

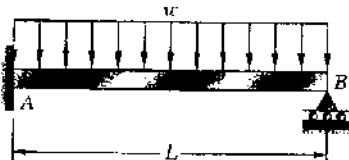
$$[x=0, y=0] \rightarrow C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{1}{6} R_A L^3 - \frac{1}{2} M_o L^2 + (M_o L - \frac{1}{2} R_A L^2) L + 0$$

$$R_A = \frac{3}{2} \frac{M_o}{L}$$

Problem 9.20

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



$$[x=0, y=0]$$

$$[\alpha=0, \frac{dy}{dx}=0]$$

$$[x=0, y=0]$$

$$[\alpha=0, \frac{d^2y}{dx^2}=0]$$

Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body KB

$$\rightarrow \sum M_K = 0 : R_B(L-x) - w(L-x)\left(\frac{L-x}{2}\right) - M = 0$$

$$M = R_B(L-x) - \frac{1}{2}w(L-x)^2$$

$$EI \frac{d^2y}{dx^2} = R_B(L-x) - \frac{1}{2}w(L-x)^2$$

$$EI \frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + \frac{1}{6}w(L-x)^3 + C_1$$

$$[x=0, \frac{dy}{dx}=0] : 0 = -\frac{1}{2}R_B L^2 + \frac{1}{6}wL^3 + C_1 \quad C_1 = \frac{1}{2}R_B L^2 - \frac{1}{6}wL^3$$

$$EIy = \frac{1}{6}R_B(L-x)^3 - \frac{1}{24}w(L-x)^3 + C_1 x + C_2$$

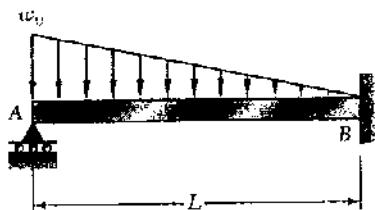
$$[x=0, y=0] : 0 = \frac{1}{6}R_B L^3 - \frac{1}{24}wL^3 + C_2 \quad C_2 = -\frac{1}{6}R_B L^3 + \frac{1}{24}wL^4$$

$$[x=L, y=0] \quad 0 = 0 - 0 + C_1 L + C_2$$

$$\frac{1}{2}R_B L^3 - \frac{1}{6}wL^4 - \frac{1}{6}R_B L^3 + \frac{1}{24}wL^4 = 0 \quad R_B = \frac{3}{8}wL \uparrow$$

Problem 9.21

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



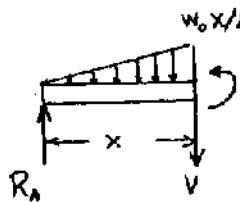
Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



$$w = \frac{w_0}{L} (L-x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L} (L-x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L} (Lx - \frac{1}{2}x^2) + R_A$$

$$M = -\frac{w_0}{L} (\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + R_A x$$

$$EI \frac{d^2y}{dx^2} = -\frac{w_0}{L} (\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + R_A x$$

$$EI \frac{dy}{dx} = -\frac{w_0}{L} (\frac{1}{6}Lx^3 - \frac{1}{24}x^4) + \frac{1}{2}R_A x^2 + C_1$$

$$EI y = -\frac{w_0}{L} (\frac{1}{24}Lx^3 - \frac{1}{120}x^5) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$$[x=0, y=0]$$

$$0 = 0 + 0 + 0 + C_2$$

$$C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0]$$

$$-\frac{w_0}{L} (\frac{1}{6}L^4 - \frac{1}{24}L^4) + \frac{1}{2}R_A L^2 + C_1 = 0$$

$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

$$[x=L, y=0] \quad -\frac{w_0}{L} (\frac{1}{24}L^4 - \frac{1}{120}L^4) + \frac{1}{6}R_A L^3 + (\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2)L = 0$$

$$(\frac{1}{2} - \frac{1}{6})R_A = (\frac{1}{8} - \frac{1}{24} + \frac{1}{120})w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L$$

$$R_A = \frac{11}{40}w_0 L$$

Problem 9.22

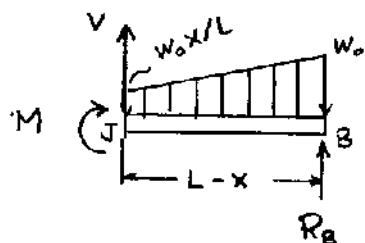
9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body J-B $\sum M_J = 0$:

$$-M + R_B(L-x) + \frac{1}{2}w_0(L-x)\frac{2}{3}(L-x)$$

$$+ \frac{1}{2}\frac{w_0x}{L}(L-x)\frac{1}{3}(L-x) = 0$$

$$M = R_B(L-x) - \frac{w_0}{6L}[2L(L-x)^2 + x(L-x)^2]$$

$$= R_B(L-x) - \frac{w_0}{6L}[2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2Lx^2 + x^3]$$

$$= R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI \frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI \frac{dy}{dx} = R_B(Lx - \frac{1}{2}x^2) - \frac{w_0}{6L}(\frac{1}{4}x^4 - \frac{3}{2}L^2x^2 + 2L^2x) + C_1$$

$$EIy = R_B(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) - \frac{w_0}{6L}(\frac{1}{20}x^5 - \frac{1}{2}L^2x^3 + L^2x^2) + C_1x + C_2$$

$$[x=0, y=0] \rightarrow C_2 = 0$$

$$[x=0, \frac{dy}{dx}=0] \rightarrow C_1 = 0$$

$$[x=L, y=0] \quad 0 = R_B L^3 (\frac{1}{2} - \frac{1}{6}) - \frac{w_0 L^4}{6} (\frac{1}{20} - \frac{1}{2} + 1)$$

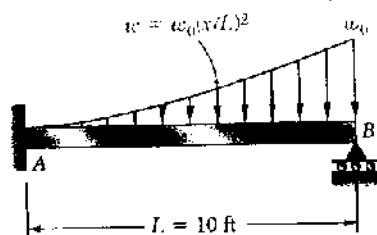
$$\frac{1}{3}R_B = (\frac{1}{6})(\frac{11}{20})w_0 L$$

$$R_B = \frac{11}{40}w_0 L \uparrow$$

—

Problem 9.23

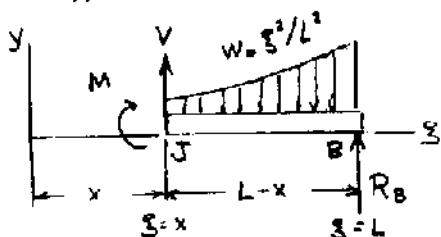
9.23 For the beam shown determine the reaction at the roller support when $w_0 = 1.4$ kips/ft.



$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



Reactions are statically indeterminate.
Boundary conditions are shown at left.

Using free body JB, $\sum M_J = 0$:

$$\begin{aligned} -M + \int_x^L \frac{w_0}{L^2} s^2 (s-x) ds + R_B (L-x) &= 0 \\ M &= \frac{w_0}{L^2} \int_x^L s^2 (s-x) ds - R_B (L-x) \\ &= \frac{w_0}{L^2} \left(\frac{1}{4}s^4 - \frac{1}{3}s^3 \right) \Big|_x^L - R_B (L-x) \\ &= \frac{w_0}{L^2} \left(\frac{1}{4}L^4 - \frac{1}{3}L^3 x + \frac{1}{12}x^4 \right) - R_B (L-x) \end{aligned}$$

$$EI \frac{d^2y}{dx^2} = \frac{w_0}{L^2} \left(\frac{1}{4}L^4 - \frac{1}{3}L^3 x + \frac{1}{12}x^4 \right) - R_B (L-x)$$

$$EI \frac{dy}{dx} = \frac{w_0}{L^2} \left(\frac{1}{4}L^4 x - \frac{1}{6}L^3 x^2 + \frac{1}{60}x^5 \right) - R_B (Lx - \frac{1}{2}x^2) + C_1$$

$$EI y = \frac{w_0}{L^2} \left(\frac{1}{8}L^4 x^2 - \frac{1}{18}L^3 x^3 + \frac{1}{360}x^6 \right) - R_B \left(\frac{1}{2}Lx^3 - \frac{1}{6}x^5 \right) + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 = 0 + 0 + C_1 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad \left(\frac{1}{8} - \frac{1}{18} + \frac{1}{360} \right) w_0 L^4 - \left(\frac{1}{2} - \frac{1}{6} \right) R_B L^3 = 0$$

$$\frac{13}{180} w_0 L^4 - \frac{1}{3} R_B L^3 = 0 \quad R_B = \frac{13}{60} w_0 L$$

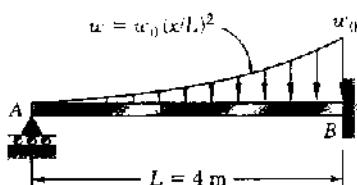
Data: $w_0 = 1.4$ kips/ft, $L = 10$ ft,

$$R_B = \frac{13}{60} (1.4)(10) = 3.03 \text{ kips}$$

3.03 kips ↑

Problem 9.24

9.24 For the beam shown determine the reaction at the roller support when $w_0 = 65 \text{ kN/m}$.



Reactions are statically indeterminate.

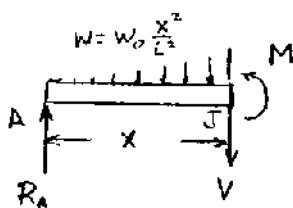
Boundary conditions are shown at left.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$

$$w = w_0 \frac{x^2}{L^2}$$



$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2} \frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2} \frac{x^4}{12} + R_A x$$

$$EI \frac{d^2y}{dx^2} = -\frac{w_0}{L^2} \frac{x^3}{12} + R_A x$$

$$EI \frac{dy}{dx} = -\frac{w_0}{L^2} \frac{x^5}{60} + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = -\frac{w_0}{L^2} \frac{x^6}{360} + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad -\frac{1}{60} w_0 L^3 + \frac{1}{2} R_A L^2 + C_1 = 0 \quad C_1 = \frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2$$

$$[x=L, y=0] \quad -\frac{1}{360} w_0 L^6 + \frac{1}{6} R_A L^5 + (\frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2)L = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_A = (\frac{1}{60} - \frac{1}{360}) w_0 L$$

$$\frac{1}{3} R_A = \frac{1}{72} w_0 L$$

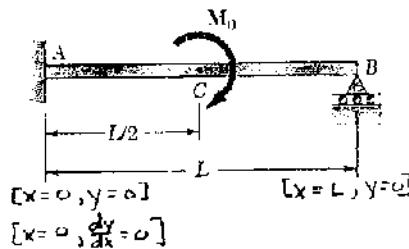
$$R_A = \frac{1}{18} w_0 L$$

Data: $w_0 = 65 \text{ kN/m}$, $L = 4 \text{ m}$

$$R_A = \frac{1}{18}(65)(4) = 14.44 \text{ kN} \uparrow$$

Problem 9.25

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

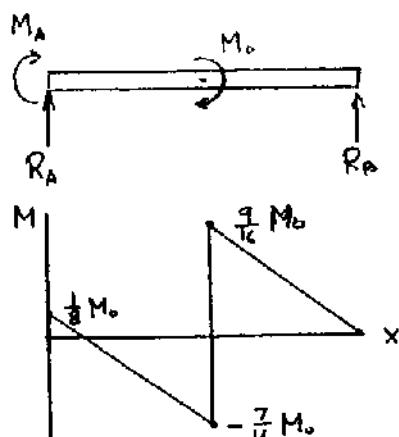


Reactions are statically indeterminate.

$$\uparrow \sum F_y = 0: R_A + R_B = 0 \quad R_A = -R_B$$

$$\Rightarrow \sum M_A = 0: -M_A - M_o + R_B L = 0$$

$$M_A = R_B L - M_o$$



$$0 < x < \frac{L}{2}$$

$$M = R_B x + M_A = -M_o + R_B L - R_B x$$

$$EI \frac{d^2y}{dx^2} = -M_o + R_B (L-x)$$

$$EI \frac{dy}{dx} = -M_o x + R_B (Lx - \frac{1}{2}x^2) + C_1$$

$$EI y = -\frac{1}{2}M_o x^2 + R_B (\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + C_1 x + C_2$$

$$\frac{L}{2} < x < L \quad M = R_B (L-x)$$

$$EI \frac{d^2y}{dx^2} = R_B (L-x)$$

$$EI \frac{dy}{dx} = R_B (Lx - \frac{1}{2}x^2) + C_3$$

$$EI y = R_B (\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0]$$

$$0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$[x=0, y=0]$$

$$0 + 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}]$$

$$-M_o \frac{L}{2} + R_B (\frac{1}{2}L^2 - \frac{1}{6}L^2) = R_B (\frac{1}{2}L^2 - \frac{1}{6}L^2) + C_3 \quad C_3 = -\frac{M_o L}{2}$$

$$[x=\frac{L}{2}, y=y]$$

$$-\frac{1}{2}M_o (\frac{L}{2})^2 + R_B (\frac{1}{8}L^3 - \frac{1}{48}L^3) = R_B (\frac{1}{8}L^3 - \frac{1}{48}L^3) + C_3 \frac{L}{2} + C_4$$

$$C_4 = -\frac{1}{8}M_o L^2 - \frac{1}{2}C_3 L = (-\frac{1}{8} + \frac{1}{4})M_o L^2 = \frac{1}{8}M_o L^2$$

$$[x=L, y=0]$$

$$R_B (\frac{1}{2}L^3 - \frac{1}{6}L^3) + \frac{M_o L}{2} L + \frac{1}{8}M_o L^2 = 0$$

$$(\frac{1}{2} - \frac{1}{6})R_B L^3 = (\frac{1}{2} - \frac{1}{8})M_o L^2 \quad \frac{1}{3}R_B = \frac{3}{8} \frac{M_o}{L}$$

$$R_B = \frac{9}{8} \frac{M_o}{L} \uparrow$$

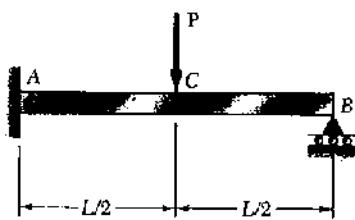
$$M_A = \frac{9}{8}M_o - M_o = \frac{1}{8}M_o$$

$$M_{c-} = -M_o + \frac{9}{8} \frac{M_o}{L} \frac{L}{2} = -\frac{7}{16}M_o$$

$$M_{c+} = R_B (L - \frac{L}{2}) = \frac{9}{8} \frac{M_o}{L} (\frac{L}{2}) = \frac{9}{16}M_o$$

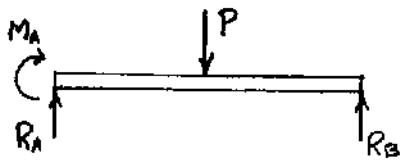
Problem 9.26

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



$$[x=0, y=0]$$

$$[x=L, y=0]$$



$$\frac{1}{2}L < x < L \quad M = M_A + R_A x - P(x - \frac{1}{2}L)$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - P(x - \frac{1}{2}L)$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P(x - \frac{1}{2}L)^2 + C_1$$

$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P(x - \frac{1}{2}L)^3 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x = \frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2}M_A L + \frac{1}{8}R_A L^2 + 0 = \frac{1}{2}M_A + \frac{1}{8}R_A L - 0 + C_3 \quad C_3 = 0$$

$$[x = \frac{L}{2}, y = y] \quad \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 + 0 + 0 \\ = \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 - 0 + 0 + C_4 \quad C_4 = 0$$

$$[x=L, y=0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{48}PL^3 + 0 + 0 = 0$$

$$\frac{1}{2}(R_B L - \frac{1}{2}P)L^3 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0 \quad R_B = \frac{5}{16}P \uparrow$$

$$R_A = P - \frac{5}{16}P$$

$$R_A = \frac{7}{16}P \uparrow$$

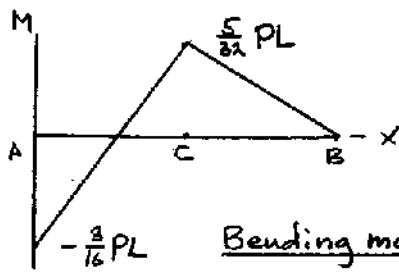
$$M_A = \frac{5}{16}PL - \frac{1}{2}PL$$

$$M_A = -\frac{3}{16}PL$$

$$M_C = R_B(\frac{L}{2}) = (\frac{5}{16}P)(\frac{L}{2})$$

$$M_C = \frac{5}{32}PL$$

$$M_B = 0$$



Bending moment diagram

Reactions are statically indeterminate.

$$+\uparrow \sum F_y = 0: \quad R_A + R_B - P = 0 \quad R_A = P - R_B$$

$$+\rightarrow \sum M_A = 0: \quad -M_A + \frac{1}{2}PL - R_B L = 0$$

$$M_A = R_B L - \frac{1}{2}PL$$

$$0 < x < \frac{1}{2}L \quad M = M_A + R_A x$$

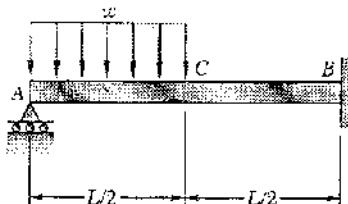
$$EI \frac{d^2y}{dx^2} = M_A + R_A x$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

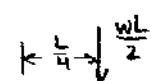
$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

Problem 9.27

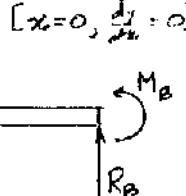
9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



$$[x=0, y=0]$$



$$[x=0, y=0]$$



Reactions are statically indeterminate.

$$\rightarrow \sum F_y = 0 : R_A + R_B - \frac{1}{2}wL = 0 \quad R_B = \frac{1}{2}wL - R_A$$

$$\rightarrow \sum M_A = 0 : -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) + M_B = 0$$

$$M_B = R_A L - \frac{9}{8}wL^2$$

$$0 < x \leq \frac{L}{2} \quad M = R_A x - \frac{1}{2}wx^2$$

$$EI \frac{d^2y}{dx^2} = R_A x - \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$$EI y = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$$\frac{L}{2} \leq x < L \quad M = R_A x - \frac{wL}{2}(x - \frac{L}{4})$$

$$EI \frac{d^2y}{dx^2} = R_A x - WL\left(\frac{1}{2}x - \frac{1}{8}L\right)$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - WL\left(\frac{1}{4}x^2 - \frac{1}{16}Lx\right) + C_3$$

$$EI y = \frac{1}{6}R_A x^3 - WL\left(\frac{1}{12}x^3 - \frac{1}{48}Lx^2\right) + C_3 x + C_4$$

$$[x=0, y=0]$$

$$0 - 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$\frac{1}{8}R_A L^2 - \frac{1}{48}wL^3 + C_1 = \frac{1}{8}R_A L^2 + 0 + C_3$$

$$C_3 = C_1 - \frac{1}{48}wL^3$$

$$[x=\frac{L}{2}, y=y]$$

$$\frac{1}{48}R_A L^3 - \frac{1}{384}wL^4 + \frac{1}{2}C_1 L + 0$$

$$= \frac{1}{48}R_A L^3 - WL\left(\frac{1}{96}L^3 - \frac{1}{64}L^2\right) + (C_1 - \frac{1}{48}wL^3)\left(\frac{L}{2}\right) + C_4$$

$$C_4 = \frac{1}{384}wL^4$$

$$[x=L, \frac{dy}{dx} = 0]$$

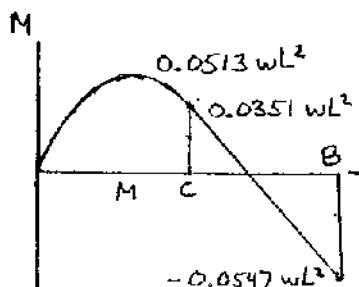
$$\frac{1}{2}R_A L^2 - WL\left(\frac{1}{4}L^2 - \frac{1}{8}L^2\right) + C_3 = 0$$

$$C_3 = \frac{1}{8}wL^3 - \frac{1}{2}R_A L^2$$

$$[x=L, y=0]$$

$$\frac{1}{6}R_A L^3 - WL\left(\frac{1}{12}L^3 - \frac{1}{16}L^3\right) + \left(\frac{1}{8}wL^3 - \frac{1}{2}R_A L^2\right)L + \frac{1}{384}wL^4 = 0$$

$$R_A = \frac{41}{128}wL \uparrow$$



$$R_B = \frac{1}{2}wL - \frac{41}{128}wL$$

$$R_B = \frac{23}{128}wL \uparrow$$

$$M_B = \frac{41}{128}wL^2 - \frac{3}{8}wL^2$$

$$M_B = -\frac{7}{128}wL^2 = -0.0547 wL^2$$

$$\text{Over } 0 < x < \frac{L}{2}, \quad V = R_A - wx = \frac{41}{128}wL - wx$$

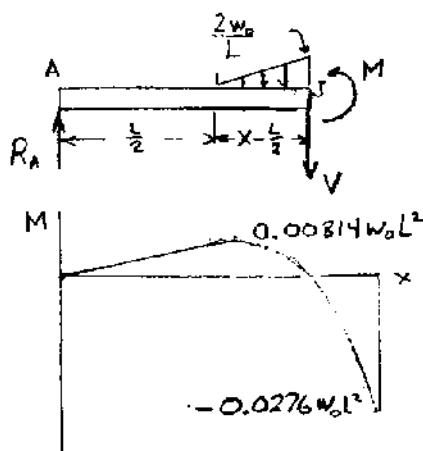
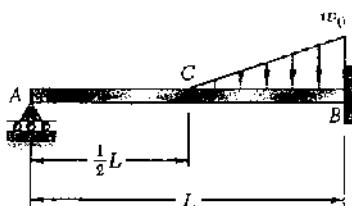
$$V = 0 \text{ at } x = x_m = \frac{41}{128}L$$

$$M = \frac{41}{128}wLx - \frac{1}{2}wx^2$$

$$M_C = \frac{41}{256}wL^2 - \frac{1}{8}wL^2 = 0.0351 wL^2, \quad M_m = \frac{41}{128}wL\left(\frac{41}{128}L\right) - \frac{1}{2}w\left(\frac{41}{128}L\right)^2 = 0.0513 wL^2$$

Problem 9.28

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



Reactions are statically indeterminate.

$$0 \leq x \leq \frac{L}{2}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x \quad (1)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1 \quad (2)$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2 \quad (3)$$

$$\frac{L}{2} \leq x \leq L \quad \sum M_J = 0$$

$$-R_A x + \frac{1}{2} \frac{2w_0}{L} (x - \frac{L}{2})(x - \frac{L}{2}) \frac{1}{3}(x - \frac{L}{2}) + M = 0$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - \frac{1}{3} \frac{w_0}{L} (x - \frac{L}{2})^3 \quad (4)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{12} \frac{w_0}{L} (x - \frac{L}{2})^4 + C_3 \quad (5)$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{60} \frac{w_0}{L} (x - \frac{L}{2})^5 + C_3 x + C_4 \quad (6)$$

$$[x=0, y=0] \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x = \frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2} R_A (\frac{L}{2})^2 + C_1 = \frac{1}{2} R_A (\frac{L}{2})^2 - 0 + C_3 \quad C_1 = C_3$$

$$[x = \frac{L}{2}, y = y] \quad \frac{1}{6} R_A (\frac{L}{2})^3 + C_1 \frac{L}{2} + C_2 = \frac{1}{6} R_A (\frac{L}{2})^3 - 0 + C_3 \frac{L}{2} + C_4 \quad C_4 = C_2 = 0$$

$$[x = L, \frac{dy}{dx} = 0] \quad \frac{1}{2} R_A L^2 - \frac{1}{12} \frac{w_0}{L} (\frac{L}{2})^4 + C_3 = 0 \quad C_3 = \frac{1}{192} w_0 L^3 - \frac{1}{2} R_A L^2$$

$$[x = L, y = 0] \quad \frac{1}{6} R_A L^3 - \frac{1}{60} \frac{w_0}{L} (\frac{L}{2})^5 + \frac{1}{48} w_0 L^4 - \frac{1}{2} R_A L^3 + 0 = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_A L^3 = (\frac{1}{192} - \frac{1}{1920}) w_0 L^4 \quad \frac{1}{3} R_A = \frac{3}{640} w_0 L \quad R_A = \frac{9}{640} w_0 L \quad 1$$

$$\text{From (1), with } x = \frac{L}{2} \quad M_C = R_A \frac{L}{2} = \frac{9}{1280} w_0 L^2 = 0.007031 w_0 L^2 \quad 1$$

$$\text{From (4), with } x = L \quad M_B = \frac{9}{640} w_0 L^3 - \frac{1}{3} \frac{w_0}{L} (\frac{L}{2})^3 = -\frac{53}{1920} w_0 L^2 \\ = -0.02761 w_0 L^2 \quad 1$$

Location of maximum positive M in portion CB.

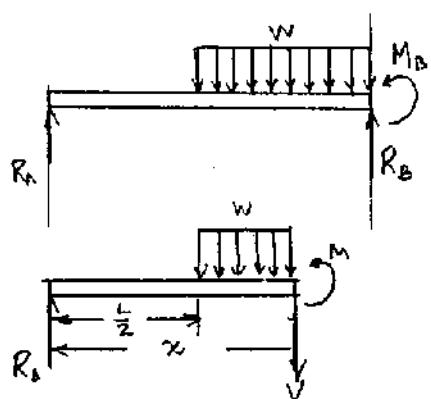
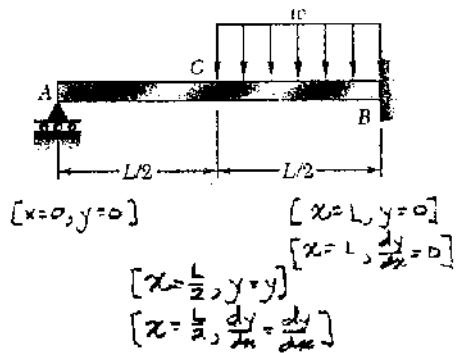
$$\frac{dM}{dx} = R_A - \frac{w_0}{L} (x - \frac{L}{2})^2 = 0 \quad x_m - \frac{L}{2} = \sqrt{\frac{R_A L}{w_0}} = \sqrt{\frac{9}{640}} L = 0.1186 L$$

$$x_m = 0.5L + 0.1186 L = 0.6186 L$$

$$\text{From (4), with } x = x_m, \quad M_m = R_A (0.6186 L) - \frac{1}{3} \frac{w_0}{L} (0.6186 L)^3 \\ = 0.008143 w_0 L^2 \quad 1$$

Problem 9.29

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point C.



$$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{d^2y}{dx^2}] \quad \frac{1}{8}R_A L^2 + C_1 = \frac{1}{8}R_A x^2 + C_3 \quad C_3 = C_1$$

$$[x=\frac{L}{2}, y=y] \quad \frac{1}{48}R_A L^3 + \frac{1}{2}C_1 L + 0 = \frac{1}{48}R_A x^3 - 0 + \frac{1}{2}C_1 x + C_4 \quad C_4 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad \frac{1}{6}R_A L^3 - \frac{1}{24}wL^3 + C_3 = 0 \quad C_3 = -\left(\frac{1}{6}R_A L^3 - \frac{1}{48}wL^3\right)$$

$$[x=L, y=0] \quad \frac{1}{6}R_A L^3 - \frac{1}{384}wL^4 - \left(\frac{1}{6}R_A - \frac{1}{48}wL^3\right)L + 0$$

$$R_A = \frac{7}{128}wL \uparrow \quad \blacktriangleleft$$

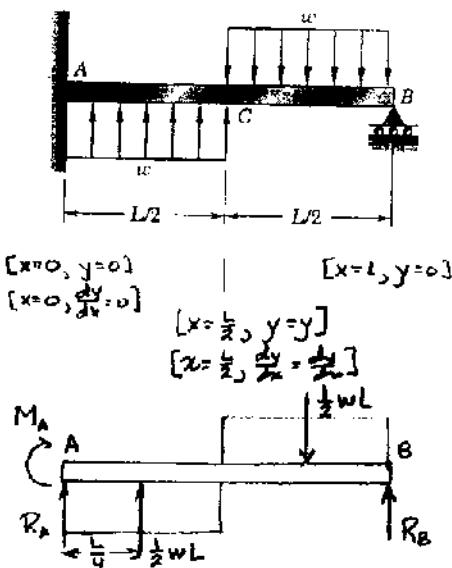
$$C_1 = C_3 = -\left(\frac{7}{256}wL^3 - \frac{1}{48}wL^3\right) = -\frac{5}{768}wL^3$$

$$\text{At } x = \frac{L}{2} \quad EIy_C = \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + \left(-\frac{5}{768}wL^3\right)\frac{L}{2} + 0 = -\frac{13}{6144}wL^4$$

$$y_C = \frac{13}{6144} \frac{wL^4}{EI} \downarrow \quad \blacktriangleleft$$

Problem 9.30

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point C.



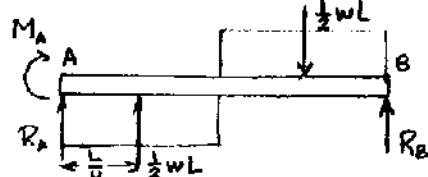
$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=\frac{L}{2}, y=y]$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}]$$

$$[x=L, y=0]$$



Reactions are statically indeterminate

$$\text{+}\sum F_y = 0 \quad R_A + \frac{1}{2}wL - \frac{1}{2}wL + R_B = 0 \quad R_A = -R_B$$

$$\text{+}\sum M_A = 0 \quad -M_A - (\frac{1}{2}wL) \frac{L}{2} + R_B L = 0$$

$$M_A = R_B L - \frac{1}{4}wL^2$$

From A to C $0 < x \leq \frac{L}{2}$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}w x^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}w x^3 + C_1$$

$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}w x^4 + C_1 x + C_2$$

From C to B $\frac{L}{2} \leq x \leq L$

$$EI \frac{d^3y}{dx^3} = M = M_A + R_A x + \frac{1}{2}wL(x - \frac{L}{4}) - \frac{1}{2}w(x - \frac{L}{2})^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}wL(x - \frac{L}{4})^2 - \frac{1}{6}w(x - \frac{L}{2})^3 + C_3$$

$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wL(x - \frac{L}{4})^3 - \frac{1}{24}(x - \frac{L}{2})^4 + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \quad M_A \frac{L}{2} - \frac{1}{2}R_A (\frac{L}{2})^2 + \frac{1}{6}w(\frac{L}{2})^3 = M_A \frac{L}{2} + \frac{1}{2}R_A (\frac{L}{2})^2 + \frac{1}{6}wL(\frac{L}{4})^2 - 0 + C_3$$

$$C_3 = (\frac{1}{48} - \frac{1}{64}) wL^3 = \frac{1}{192} wL^3$$

$$[x=\frac{L}{2}, y=y] \quad \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{6}R_A (\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4 \\ = \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{6}R_A (\frac{L}{2})^3 + \frac{1}{12}wL(\frac{L}{4})^3 - 0 + \frac{1}{192}wL^3(\frac{L}{2}) + C_4$$

$$C_4 = (\frac{1}{384} - \frac{1}{768} + \frac{1}{384}) wL^4 = -\frac{1}{768} wL^4$$

$$[x=L, y=0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 + \frac{1}{24}wL(\frac{3L}{4})^2 - \frac{1}{24}w(\frac{L}{2})^4 + \frac{1}{192}wL^3(L) - \frac{1}{768}wL^4 = 0$$

$$\frac{1}{2}(R_B L - \frac{1}{4}wL^2)L^2 + \frac{1}{6}(-R_B)L^3 + (\frac{27}{768} - \frac{1}{384} + \frac{1}{192} - \frac{1}{768})wL^4 = 0$$

$$(\frac{1}{2} - \frac{1}{6})R_B L^3 = -(\frac{1}{8} - \frac{7}{192})wL^4 \quad \frac{1}{3}R_B = \frac{17}{192}wL \quad R_B = \frac{17}{64}wL \uparrow$$

$$R_A = -R_B = -\frac{17}{64}wL$$

$$M_A = R_B L - \frac{1}{4}wL^2 = (\frac{17}{64} - \frac{1}{4})wL^2 = \frac{1}{64}wL^2$$

(b) Deflection at C. (y at $x = \frac{L}{2}$)

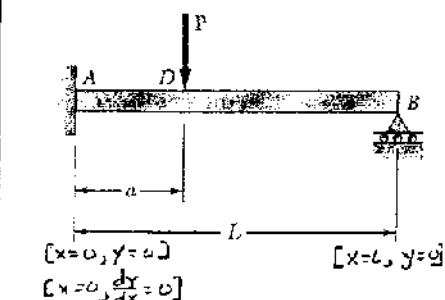
$$EI y_C = \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{6}R_A (\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4 = \frac{1}{2}(\frac{1}{64}wL^2)(\frac{L}{2})^2 + \frac{1}{6}(-\frac{17}{64}wL)(\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4$$

$$= (\frac{1}{512} - \frac{17}{3072} + \frac{1}{384})wL^4 = -\frac{1}{1024}wL^4$$

$$y_C = \frac{1}{1024} \frac{wL^4}{EI}$$

Problem 9.31

9.31 and 9.32 Determine the reaction at the roller support and the deflection at point D if α is equal to $L/3$.



Reactions are statically indeterminate.

$$+\uparrow \sum F_y = 0: R_A + R_B - P = 0 \quad R_A = P - R_B$$

$$+\rightarrow \sum M_A = 0: -M_A - Pa - R_B L = 0$$

$$M_A = R_B L - Pa$$

$$0 < x < a \quad M = M_A + R_A x$$

$$EI \frac{d^2Y}{dx^2} = M = M_A + R_A x$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$a < x < L \quad M = M_A + R_A x - P(x-a)$$

$$EI \frac{d^2Y}{dx^2} = M = M_A + R_A x - P(x-a)$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P(x-a)^2 + C_3$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P(x-a)^3 + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad M_A a + \frac{1}{2} R_A a^2 + C_1 = M_A a + \frac{1}{2} R_A a^2 - 0 + C_3 \quad C_3 = C_1 = 0$$

$$[x=a, y=y] \quad \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 + C_3 a + C_4 \\ = \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 - 0 + C_3 a + C_4 \quad C_4 = C_2 = 0$$

$$[x=L, y=0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P(L-a)^3 + 0 + 0 = 0$$

$$\frac{1}{2} (R_B L - Pa) L^2 + \frac{1}{6} (P - R_B) L^3 - \frac{1}{6} P(L-a)^3 = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_B L^3 = P \left[\frac{1}{2} a L^2 - \frac{1}{6} L^3 + \frac{1}{6} (L-a)^3 \right]$$

$$\frac{1}{3} R_B L^3 = P \left[\frac{1}{2} a t^2 - \frac{1}{6} t^3 + \frac{1}{6} t^3 - \frac{1}{2} t^2 a + \frac{1}{2} t a^2 - \frac{1}{6} a^3 \right] \\ = P a^2 (\frac{1}{2} L - \frac{1}{6} a)$$

$$R_B = \frac{Pa^2}{2L^3} (3L-a) = \frac{P(L/3)^2}{2L^3} (3L - \frac{L}{3}) = \frac{4}{27} P \uparrow$$

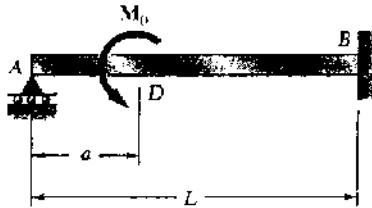
Deflection at D. (y at $x=a = \frac{L}{3}$)

$$y_D = \frac{1}{EI} \left\{ \frac{1}{2} M_A \left(\frac{L}{3} \right)^2 + \frac{1}{6} R_A \left(\frac{L}{3} \right)^3 \right\} = \frac{1}{EI} \left\{ \frac{1}{18} (R_B L - P \frac{L}{3}) L^2 + \frac{1}{162} (P - R_B) L^3 \right\}$$

$$= \frac{PL^3}{EI} \left\{ \frac{1}{18} \left(\frac{4}{27} - \frac{1}{3} \right) + \frac{1}{162} \left(1 - \frac{4}{27} \right) \right\} = -\frac{11}{2187} \frac{PL^3}{EI}, \quad y_D = \frac{11}{2187} \frac{PL^3}{EI} \downarrow$$

Problem 9.32

9.31 and 9.32 Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.



Reactions are statically indeterminate.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$[x=a, y=y]$$

$$0 < x < a \quad M = R_A x$$

$$EI \frac{d^2y}{dx^2} = M = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$a < x < L \quad M = R_A x - M_0$$

$$EI \frac{d^2y}{dx^2} = R_A x - M_0$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_0(x-a) + C_3$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_0(x-a)^2 + C_3 x + C_4$$

$$[x=0, y=0]$$

$$0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$\frac{1}{2} R_A a^2 + C_1 = \frac{1}{2} R_A a^2 - 0 + C_5$$

$$C_1 = C_5$$

$$[x=a, y=y]$$

$$\frac{1}{6} R_A a^3 + C_1 a + C_2 = \frac{1}{6} R_A a^3 + 0 + C_3 a + C_4$$

$$C_2 = C_4 = 0$$

$$[x=L, \frac{dy}{dx}=0]$$

$$\frac{1}{2} R_A L^2 - M_0(L-a) + C_3 = 0 \quad C_3 = M_0(L-a) - \frac{1}{2} R_A L^2$$

$$[x=L, y=0]$$

$$\frac{1}{6} R_A L^3 - \frac{1}{2} M_0(L-a)^2 + [M_0(L-a) - \frac{1}{2} R_A L^2] L + 0 = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_A L^3 = M_0 [(L-a)L - \frac{1}{2}(L-a)^2]$$

$$\frac{1}{3} R_A L^3 = M_0 [L^2 - aL - \frac{1}{2}L^2 + La - \frac{1}{2}a^2] = \frac{1}{2} M_0 (L^2 - a^2)$$

$$R_A = \frac{3}{2} \frac{M_0}{L^3} (L^2 - a^2) = \frac{3}{2} \frac{M_0}{L^3} \left[L^2 - \left(\frac{L}{3} \right)^2 \right] = \frac{4}{3} \frac{M_0}{L} \uparrow$$

Deflection at D. (y at $x = a = \frac{L}{3}$)

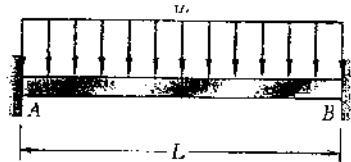
$$y_D = \frac{1}{EI} \left\{ \frac{1}{6} R_A \left(\frac{L}{3} \right)^3 + C_1 \left(\frac{L}{3} \right) \right\} = \frac{1}{EI} \left\{ \frac{1}{6} \left(\frac{4}{3} \frac{M_0}{L} \right) \left(\frac{L}{3} \right)^3 + C_3 \left(\frac{L}{3} \right) \right\}$$

$$= \frac{1}{EI} \left\{ \frac{4}{48} M_0 L^2 + [M_0(L - \frac{L}{3}) - \frac{1}{2} \cdot \frac{4}{3} \frac{M_0}{L} L^2] \frac{L}{3} \right\}$$

$$= \frac{M_0 L^2}{EI} \left(\frac{4}{48} + \frac{2}{9} - \frac{4}{18} \right) = \frac{2}{243} \frac{M_0 L^4}{EI} \uparrow$$

Problem 9.33

9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

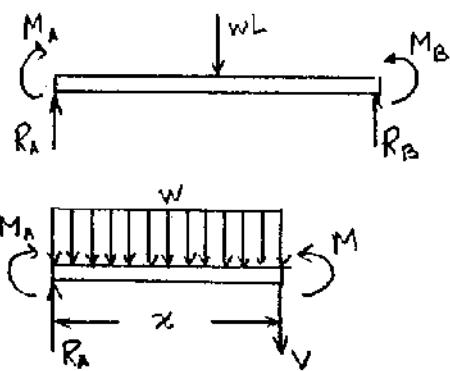


$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



$$[x=\frac{L}{2}, \frac{dy}{dx}=0]$$

$$\frac{1}{2}M_A L + \frac{1}{16}WL^3 + \frac{1}{48}WL^3 + 0 = 0$$

$$M_A = -\frac{1}{12}WL^2$$

$$M = -\frac{1}{12}WL^2 + \frac{1}{2}WLx - \frac{1}{2}wx^2$$

$$M = \frac{WL}{12}[6x(L+x) - L^2]$$

Reactions are statically indeterminate.

By symmetry, $R_B = R_A$; $M_B = M_A$

$$\frac{dy}{dx} = 0 \text{ at } x = \frac{L}{2}$$

$$+\uparrow \sum F_y = 0: R_A + R_B - wL = 0$$

$$R_B = R_A = \frac{1}{2}wL \rightarrow$$

Over entire beam, $M = M_A + R_Ax - \frac{1}{2}wx^2$

$$EI \frac{d^2y}{dx^2} = M_A + \frac{1}{2}wLx - \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{4}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 - 0 + C_1 = 0$$

$$C_1 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0]$$

$$\frac{1}{2}M_A L + \frac{1}{16}WL^3 + \frac{1}{48}WL^3 + 0 = 0$$

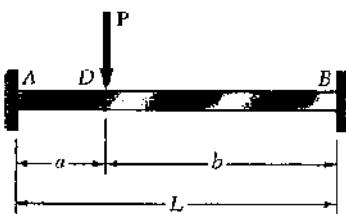
$$M_A = -\frac{1}{12}WL^2$$

$$M = -\frac{1}{12}WL^2 + \frac{1}{2}WLx - \frac{1}{2}wx^2$$

$$M = \frac{WL}{12}[6x(L+x) - L^2]$$

Problem 9.34

9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

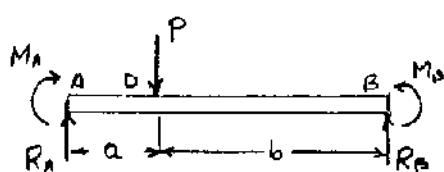


$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



Reactions are statically indeterminate.

$$0 \leq x \leq a$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \rightarrow C_1 = 0$$

$$[x=0, y=0] \rightarrow C_2 = 0$$

$$a \leq x \leq L$$

$$EI \frac{d^2y}{dx^2} = M_A + R_A x - P(x-a)$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P(x-a)^2 + C_3$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P(x-a)^3 + C_3 x + C_4$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad M_A a + \frac{1}{2} R_A a^2 = M_A a + \frac{1}{2} R_A a^2 - 0 + C_3 \quad C_3 = 0$$

$$[x=a, y=y] \quad \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 = \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 - 0 + 0 + C_4 \quad C_4 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad M_A L + \frac{1}{2} R_A L^2 - \frac{1}{2} P b^2 = 0 \quad (1)$$

$$[x=L, y=0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P b^3 = 0 \quad (2)$$

Solving (1) and (2) simultaneously, $M_A = -\frac{Pb^2(L-b)}{L^2} = -\frac{Pab^2}{L^2}$

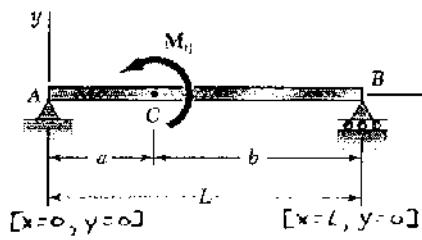
$$R_A = \frac{Pb^2}{L^2} - \frac{2M_A}{L} = \frac{Pb^2}{L^2}(1 + 2\frac{a}{L}) = \frac{Pb^2(3a+b)}{L^3}$$

$$M_B = -M_A + R_A L - Pb = -\frac{Pab^2}{L^2} + \frac{Pb^2(3a+b)(a+b)}{L^3} - Pb = -\frac{Pba^2}{L^2}$$

$$M_B = M_A + R_A a = -\frac{Pab^2}{L^2} + \frac{Pb^2(3a+b)a}{L^3} = \frac{2Pab^2}{L^3}$$

Problem 9.35

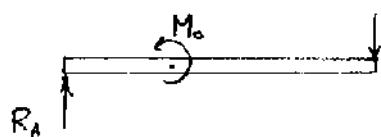
9.35 and 9.36 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection of point C.



$$\text{Reactions} \quad R_A = \frac{M_0}{L} \uparrow, \quad R_B = \frac{M_0}{L} \downarrow$$

$$0 < x < a \quad M = R_A x$$

$$a < x < L \quad M = R_A x - M_0$$



Using singularity functions

$$EI \frac{d^2y}{dx^2} = M = R_A x - M_0(x-a)^0$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_0(x-a)^1 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_0(x-a)^2 + C_1 x + C_2$$

$$[x=0, y=0]$$

$$0 = 0 - 0 + 0 + C_2$$

$$C_2 = 0$$

$$[x=L, y=0]$$

$$\frac{1}{6} R_A L^3 - \frac{1}{2} M_0 (L-a)^2 + C_1 L + 0 = 0$$

$$C_1 L = -\frac{1}{6} \frac{M_0}{L} L^3 + \frac{1}{2} M_0 b^2 \quad C_1 = -\frac{M_0}{6L} (3b^2 - L^2)$$

$$(a) \text{ Elastic curve.} \quad y = \frac{1}{EI} \left\{ \frac{1}{6} \frac{M_0}{L} x^3 - \frac{1}{2} M_0 (x-a)^2 + \frac{M_0}{6L} (3b^2 - L^2) x \right\}$$

$$= \frac{M_0}{6EI} \left\{ x^3 - 3L(x-a)^2 + (3b^2 - L^2)x \right\}$$

$$\frac{dy}{dx} = \frac{M_0}{6EI} \left\{ 3x^2 - 6L(x-a)' + (3b^2 - L^2) \right\}$$

$$(b) \text{ Slope at A.} \quad (\frac{dy}{dx} \text{ at } x=0)$$

$$\theta_A = \frac{M_0}{6EI} \left\{ 0 - 0 + 3Lb^2 - L^3 \right\} = \frac{M_0}{6EI} (3b^2 - L^2)$$

$$(c) \text{ Deflection at C.} \quad (y \text{ at } x=a)$$

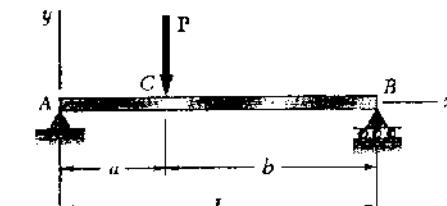
$$y_C = \frac{M_0}{6EI} \left\{ a^3 - 0 + (3b^2 - L^2)a \right\} = \frac{M_0 a}{6EI} \left\{ a^2 + 3b^2 - (a+b)^2 \right\}$$

$$= \frac{M_0 a}{6EI} \left\{ a^2 + 3b^2 - a^2 - 2ab - b^2 \right\} = \frac{M_0 a}{6EI} \left\{ 2b^2 - 2ab \right\}$$

$$= \frac{M_0 ab}{3EI} (b-a) \uparrow$$

Problem 9.36

9.35 and 9.36 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection of point C.

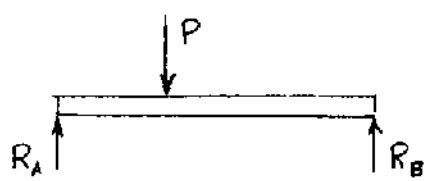


$$[x=0, M=0]$$

$$[x=a, y=0]$$

$$[x=L, M=0]$$

$$[x=L, y=0]$$



$$[x=L, y=0]$$

$$\frac{Pb}{6L} L^3 - \frac{1}{6} F(L-a)^3 + C_1 L = 0$$

$$C_1 = -\frac{1}{6} \frac{Pb}{L} (bL^2 - b^3) = -\frac{1}{6} \frac{Pb}{L} (L^2 - b^2)$$

(a) Elastic curve.

$$y = \frac{P}{EI} \left\{ \frac{b}{6L} x^3 - \frac{1}{6} (x-a)^3 - \frac{1}{6} \frac{b}{L} (L^2 - b^2) x \right\}$$

$$y = \frac{P}{6EI} \left\{ bx^3 - L(x-a)^3 - b(L^2 - b^2) x \right\}$$

(b) Slope at end A.

$$EI \frac{dy}{dx} \Big|_{x=0} = C_1 = -\frac{Pb}{6L} (L^2 - b^2)$$

$$\theta_A = -\frac{Pb}{6EI} (L^2 - b^2)$$

$$\theta_A = \frac{Pb}{6EI} (L^2 - b^2)$$

(c) Deflection at C.

$$EI y_C = \frac{Pb}{6L} a^3 + C_1 a = \frac{Pba^3}{6L} - \frac{Pb}{6L} (L^2 - b^2) a$$

$$= \frac{Pba}{6L} (a^2 - L^2 - b^2)$$

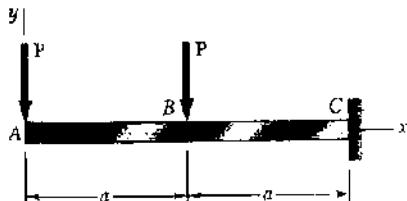
$$y_C = -\frac{Pab}{6EI} (L^2 - a^2 - b^2) = -\frac{Pab}{6EI} \{ a^2 + 2ab + b^2 - a^2 - b^2 \}$$

$$= -\frac{Pa^2 b^2}{3EI}$$

$$y_C = \frac{Pa^2 b^2}{3EI}$$

Problem 9.37

9.37 and 9.38 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at the free end, (c) the deflection of the free end.



$$\frac{dM}{dx} = V = -P - P(x-a)$$

$$EI \frac{d^2y}{dx^2} = M = -Px - P(x-a)$$

$$EI \frac{dy}{dx} = -\frac{1}{2}Px^2 - \frac{1}{2}P(x-a)^2 + C_1$$

$$EIy = -\frac{1}{6}Px^3 - \frac{1}{6}P(x-a)^3 + C_1x + C_2$$

$$[x=2a, \frac{dy}{dx}=0] \quad -\frac{1}{2}P(2a)^2 - \frac{1}{2}P(a)^2 + C_1 = 0 \quad C_1 = \frac{5}{2}Pa^2$$

$$[x=2a, y=0] \quad -\frac{1}{6}P(2a)^3 - \frac{1}{6}P(a)^3 + \frac{5}{2}Pa^2(2a) + C_2 = 0 \quad C_2 = -\frac{7}{2}Pa^3$$

$$EIy = -\frac{1}{6}Px^3 - \frac{1}{6}P(x-a)^3 + \frac{5}{2}Pa^2x - \frac{7}{2}Pa^3$$

(a) Elastic curve. $y = \frac{P}{EI} \left\{ -\frac{1}{6}x^3 - \frac{1}{6}(x-a)^3 + \frac{5}{2}a^2x - \frac{7}{2}a^3 \right\}$

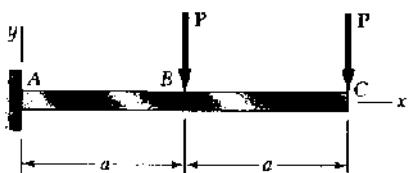
$$\frac{dy}{dx} = \frac{P}{EI} \left\{ -\frac{1}{2}x^2 - \frac{1}{2}(x-a)^2 + \frac{5}{2}a^2 \right\}$$

(b) Slope at A. ($\frac{dy}{dx}$ at $x=0$) $\frac{dy}{dx}|_A = \frac{5Pa^2}{2EI}$ $\theta_A = \frac{5Pa^2}{2EI}$

(c) Deflection at A. (y at $x=0$) $y_A = -\frac{7Pa^3}{2EI}$ $y_A = \frac{7Pa^3}{2EI}$

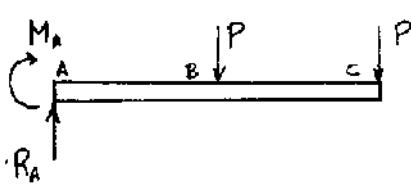
Problem 9.38

9.37 and 9.38 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at the free end, (c) the deflection of the free end.



$$+\uparrow \sum F_y = 0: R_A - P - P = 0 \quad R_A = 2P$$

$$+\rightarrow \sum M_A = 0: -M_a - Pa - P(2a) = 0 \quad M_a = -3Pa$$



$$\frac{dM}{dx} = V = 2P - P(x-a)$$

$$EI \frac{d^2y}{dx^2} = M = 2Px - P(x-a)^2 - 3Pa$$

$$EI \frac{dy}{dx} = Px^2 - \frac{1}{2}P(x-a)^2 - 3Pax + C_1$$

$$[x=0, \frac{dy}{dx}=0]$$

$$0 - 0 - 0 + C_1 = 0 \quad C_1 = 0$$

$$EIy = \frac{1}{3}Px^3 - \frac{1}{6}P(x-a)^3 - \frac{3}{2}Pax^2 + C_2$$

$$[x=0, y=0]$$

$$0 - 0 - 0 + C_2 = 0 \quad C_2 = 0$$

$$EIy = \frac{1}{3}Px^3 - \frac{1}{6}P(x-a)^3 - \frac{3}{2}Pax^2$$

(a) Elastic curve,

$$y = \frac{P}{EI} \left\{ \frac{1}{3}x^3 - \frac{1}{6}(x-a)^3 - \frac{3}{2}ax^2 \right\}$$

$$\frac{dy}{dx} = \frac{P}{EI} \left\{ x^2 - \frac{1}{2}(x-a)^2 - 3ax \right\}$$

(b) Slope at end C.

$$\frac{dy}{dx} \text{ at } x=2a$$

$$\left. \frac{dy}{dx} \right|_A = \frac{P}{EI} \left\{ (2a)^2 - \frac{1}{2}a^2 - (3a)(2a) \right\} = -\frac{5Pa^2}{2EI} \quad \theta_c = \frac{5Pa^2}{2EI}$$

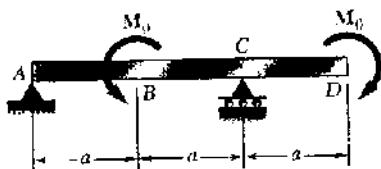
(c) Deflection at end C.

$$(y \text{ at } x=2a)$$

$$y_A = \frac{P}{EI} \left\{ \frac{1}{3}(2a)^3 - \frac{1}{6}a^3 - \left(\frac{3}{2}a\right)(2a)^2 \right\} = -\frac{7Pa^3}{2EI} \quad y_c = \frac{7Pa^3}{2EI}$$

Problem 9.39

9.39 and 9.40 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point B, (c) the deflection at end D.



Loading is self-equilibrated. $R_A = R_C = 0$

$$EI \frac{d^2y}{dx^2} = M = -M_0(x-a)$$

$$EI \frac{dy}{dx} = -M_0(x-a) + C_1$$

$$EIy = -\frac{1}{2}M_0(x-a)^2 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 + 0 + C_2 = 0 \quad , C_2 = 0$$

$$[x=2a, y=0] \quad -\frac{1}{2}M_0a^2 + 0 + C_1(2a) \quad C_1 = \frac{1}{4}M_0a$$

$$EIy = -\frac{1}{2}M_0(x-a)^2 + \frac{1}{4}M_0ax$$

$$\text{Elastic curve. } y = \frac{M_0}{EI} \left\{ -\frac{1}{2}(x-a)^2 + \frac{1}{4}ax \right\}$$

$$\frac{dy}{dx} = \frac{M_0}{EI} \left\{ -2(x-a) + \frac{1}{4}a \right\}$$

$$(a) \text{ Slope at end A. } \left(\frac{dy}{dx} \text{ at } x=0 \right)$$

$$\left. \frac{dy}{dx} \right|_A = \frac{M_0a}{4EI} \quad \theta_A = \frac{M_0a}{4EI} \leftarrow$$

$$(b) \text{ Deflection at point B. } (y \text{ at } x=a)$$

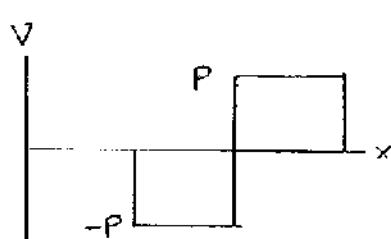
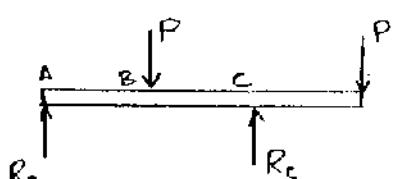
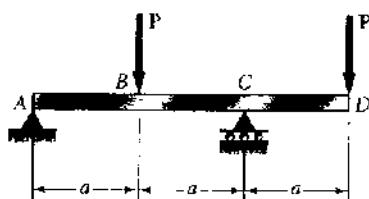
$$y_B = \frac{M_0}{EI} \left\{ -0 + \left(\frac{1}{4}a\right)(a) \right\} \quad y_B = \frac{M_0a^2}{4EI} \uparrow$$

$$(c) \text{ Deflection at end D. } (y \text{ at } x=3a)$$

$$y_D = \frac{M_0}{EI} \left\{ -\frac{1}{2}(2a)^2 + \left(\frac{1}{4}a\right)(3a) \right\} = -\frac{5M_0a^2}{4EI} \quad y_D = \frac{5M_0a^2}{4EI} \downarrow$$

Problem 9.40

9.39 and 9.40 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point B, (c) the deflection at end D.



$$+\Sigma \sum M_c = 0: \quad -2aR_a + 2P - aP = 0 \quad R_a = 0$$

$$+\sum F_y = 0: \quad R_a + R_c - P - P = 0 \quad R_c = 2P \uparrow$$

$$\frac{dM}{dx} = V = -P(x-a)^0 + 2P(x-2a)^0$$

$$EI \frac{d^2y}{dx^2} = M = -P(x-a)^1 + 2P(x-2a)^1$$

$$EI \frac{dy}{dx} = -\frac{1}{2}P(x-a)^2 + P(x-2a)^2 + C_1$$

$$EIy = -\frac{1}{6}P(x-a)^3 + \frac{1}{3}P(x-2a)^3 + C_1x + C_2$$

$$[x=0, y=0] \quad -0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=2a, y=0] \quad -\frac{1}{6}Pa^3 + 0 + 0 + C_1(2a) + 0 = 0$$

$$C_1 = \frac{1}{12}Pa^2$$

$$EIy = -\frac{1}{6}P(x-a)^3 + \frac{1}{3}R(x-2a)^3 + \frac{1}{12}Pa^2x$$

Elastic curve. $y = \frac{P}{EI} \left\{ -\frac{1}{6}(x-a)^3 + \frac{1}{3}(x-2a)^3 + \frac{1}{12}a^2x \right\}$

$$\frac{dy}{dx} = \frac{P}{EI} \left\{ -\frac{1}{2}(x-a)^2 + (x-2a)^2 + \frac{1}{12}a^2 \right\}$$

(a) Slope at end A. ($\frac{dy}{dx}$ at $x=0$)

$$\left. \frac{dy}{dx} \right|_A = \frac{P}{EI} \left\{ -0 + 0 + \frac{1}{12}a^2 \right\} = \frac{Pa^2}{12EI} \quad \theta_A = \frac{Pa^2}{12EI} \quad \blacktriangleleft$$

(b) Deflection at point B. (y at $x=a$)

$$y_B = \frac{P}{EI} \left\{ -0 + 0 + \frac{1}{12}a^3 \right\} = \frac{Pa^3}{12EI} \quad y_B = \frac{Pa^3}{12EI} \quad \blacktriangleleft$$

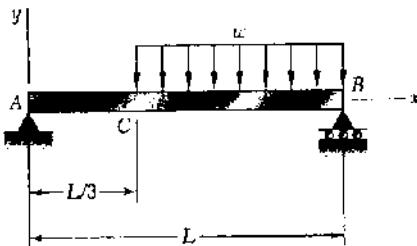
(c) Deflection at end D. (y at $x=3a$)

$$y_D = \frac{P}{EI} \left\{ -\frac{1}{6}(2a)^3 + \frac{1}{3}a^3 + \left(\frac{1}{12}a^2\right)(3a) \right\} = -\frac{3Pa^3}{4EI}$$

$$y_D = \frac{3Pa^3}{4EI} \quad \downarrow \quad \blacktriangleleft$$

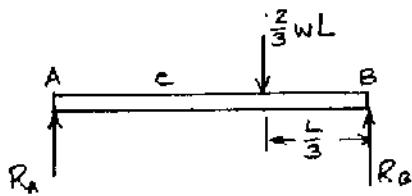
Problem 9.41

9.41 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at point A, (c) the deflection at point C.



"Use free body ACB with the distributed load replaced by equivalent concentrated load."

$$\rightarrow M_B = 0 : -R_A L + \left(\frac{2}{3}wL\right)\left(\frac{L}{3}\right) = 0 \\ R_A = \frac{2}{9}wL$$



$$\frac{dV}{dx} = -w = -w(x - \frac{L}{3})^0$$

$$\frac{dM}{dx} = V = \frac{2}{9}wL - w(x - \frac{L}{3})^1$$

$$EI \frac{dy}{dx^2} = M = \frac{2}{9}wLx - \frac{1}{2}w(x - \frac{L}{3})^2$$

$$EI \frac{dy}{dx} = \frac{1}{9}wLx^2 - \frac{1}{6}w(x - \frac{L}{3})^3 + C_1$$

$$EIy = \frac{1}{27}wLx^3 - \frac{1}{24}w(x - \frac{L}{3})^4 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 = 0 + 0 + C_2 \Rightarrow C_2 = 0$$

$$[x=L, y=0] \quad (\frac{1}{27}wL)L^3 - \frac{1}{24}(\frac{2L}{3})^4 + C_1L + 0 = 0$$

$$C_1 = -\frac{7}{243}wL^3$$

$$EIy = \frac{1}{27}wLx^3 - \frac{1}{24}w(x - \frac{L}{3})^4 - \frac{7}{243}wL^3x$$

(a) Elastic curve.

$$y = \frac{w}{EI} \left\{ \frac{1}{27}Lx^3 - \frac{1}{24}(x - \frac{L}{3})^4 - \frac{7}{243}L^3x^2 \right\}$$

$$\frac{dy}{dx} = \frac{w}{EI} \left\{ \frac{1}{9}Lx^2 - \frac{1}{6}(x - \frac{L}{3})^3 - \frac{7}{243}L^3x \right\}$$

(b) Slope at point A. ($\frac{dy}{dx}$ at $x=0$)

$$\left. \frac{dy}{dx} \right|_A = \frac{w}{EI} \left\{ 0 - 0 - \frac{7}{243}L^3 \right\} = -\frac{7wL^3}{243EI} \quad \theta_A = \frac{7wL^3}{243EI}$$

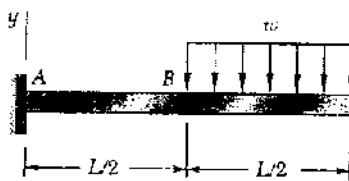
(c) Deflection at point C. (y at $x = \frac{L}{3}$)

$$y_C = \frac{w}{EI} \left\{ \left(\frac{1}{27}L\right)\left(\frac{L}{3}\right)^3 - 0 - \left(\frac{7}{243}L^3\right)\left(\frac{L}{3}\right) \right\} = -\frac{2wL^4}{243EI}$$

$$y_C = \frac{2wL^4}{243EI}$$

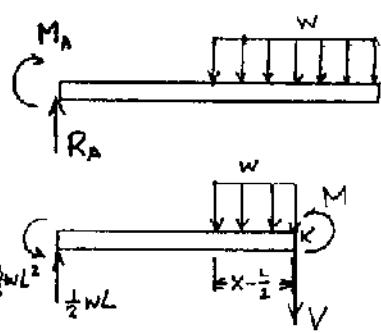
Problem 9.42

9.42 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at point B, (c) the deflection at point C.



$$[x=0, \frac{dy}{dx}=0]$$

$$[x=0, y=0]$$



$$\uparrow \sum F_y = 0: R_A - \frac{1}{2}wL = 0 \quad R_A = \frac{1}{2}wL$$

$$\leftarrow \sum M_A = 0: -M_A - (\frac{1}{2}wL)(\frac{3}{4}L) = 0 \quad M_A = -\frac{3}{8}wL^2$$

$$0 < x < \frac{L}{2} \quad M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx$$

$$\frac{L}{2} < x < L \quad (\text{See free body diagram})$$

$$\leftarrow \sum M_K = 0$$

$$\frac{3}{8}wL^2 - \frac{1}{2}wLx + \frac{1}{2}w(x - \frac{L}{2})^2 + M = 0$$

$$M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx - \frac{1}{2}w(x - \frac{L}{2})^2$$

Using singularity functions,

$$EI \frac{d^2y}{dx^2} = M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx - \frac{1}{2}w(x - \frac{L}{2})^2$$

$$EI \frac{dy}{dx} = -\frac{3}{8}wL^2x + \frac{1}{2}wLx^2 - \frac{1}{6}w(x - \frac{L}{2})^3 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad -0 + 0 - 0 + C_1 = 0 \quad C_1 = 0$$

$$EIy = -\frac{3}{16}wL^2x^2 + \frac{1}{12}wLx^3 - \frac{1}{24}w(x - \frac{L}{2})^4 + C_1x + C_2$$

$$[x=0, y=0] \quad -0 + 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

(a) Elastic curve. $y = \frac{w}{EI} \left\{ -\frac{3}{16}L^2x^2 + \frac{1}{12}Lx^3 - \frac{1}{24}(x - \frac{L}{2})^4 \right\}$

$$\frac{dy}{dx} = \frac{w}{EI} \left\{ -\frac{3}{8}L^2x + \frac{1}{4}Lx^2 - \frac{1}{6}(x - \frac{L}{2})^3 \right\}$$

(b) Deflection at B. (y at $x = \frac{L}{2}$)

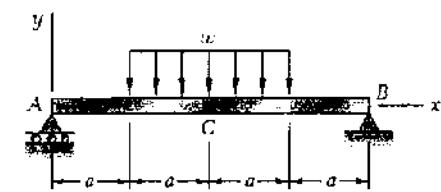
$$y_B = \frac{wL^4}{EI} \left\{ -\frac{3}{16}(\frac{1}{2})^2 + \frac{1}{12}(\frac{1}{2})^3 - 0 \right\} = -\frac{7}{192} \frac{wL^4}{EI}, \quad y_B = \frac{7}{192} \frac{wL^4}{EI} \downarrow$$

(c) Deflection at C. (y at $x = L$)

$$y_C = \frac{wL^4}{EI} \left\{ -\frac{3}{16}(1)^2 + \frac{1}{12}(1)^3 - \frac{1}{24}(1)^4 \right\} = -\frac{41}{384} \frac{wL^4}{EI}, \quad y_C = \frac{41}{384} \frac{wL^4}{EI} \downarrow$$

Problem 9.43

9.43 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at the midpoint C.



$$\text{By symmetry, } R_A = R_B \\ +\uparrow \sum F_y = 0: \quad R_A + R_B - 2wa = 0 \quad R_A = wa$$

$$w(x) = w(x-a)^0 - w(x-3a)^0$$

$$\frac{dV}{dx} = -w(x) = -w(x-a)^0 + w(x-3a)^0$$

$$\frac{dM}{dx} = V = R_A - w(x-a)^0 + w(x-3a)^0$$

$$M = M_A + R_A x - \frac{1}{2}w(x-a)^2 + \frac{1}{2}(x-3a)^2 \quad \text{with } M_A = 0$$

$$EI \frac{d^2y}{dx^2} = M = wax - \frac{1}{2}w(x-a)^2 + \frac{1}{2}w(x-3a)^2$$

$$EI \frac{dy}{dx} = \frac{1}{2}wax^2 - \frac{1}{6}w(x-a)^3 + \frac{1}{6}w(x-3a)^3 + C_1$$

$$EI y = \frac{1}{6}wax^3 - \frac{1}{24}w(x-a)^4 + \frac{1}{24}w(x-3a)^4 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 - 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=4a, y=0] \quad \frac{1}{6}wa(4a)^3 - \frac{1}{24}w(3a)^4 + \frac{1}{24}a^4 + C_1(4a) = 0$$

$$4C_1 = wa^3 \left\{ -\frac{64}{6} + \frac{81}{24} - \frac{1}{24} \right\} = -\frac{22}{3}wa^3 \quad C_1 = -\frac{11}{6}wa^3$$

(a) Equation of elastic curve.

$$y = \frac{w}{EI} \left\{ \frac{1}{6}ax^3 - \frac{1}{24}(x-a)^4 + \frac{1}{24}(x-3a)^4 - \frac{11}{6}a^3x \right\}$$

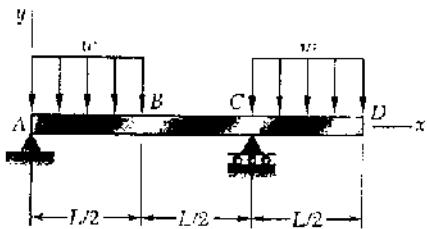
(b) Deflection at C. (y at $x = 2a$)

$$y_C = \frac{wa^4}{EI} \left\{ \frac{1}{6}(2)^3 - \frac{1}{24}(1)^4 + 0 - \frac{11}{6}(2) \right\} = -\frac{19}{8} \frac{wa^4}{EI}$$

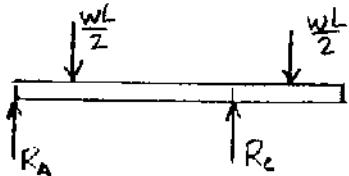
$$y_C = \frac{19}{8} \frac{wa^4}{EI}$$

Problem 9.44

9.44 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at point B, (c) the deflection at point D.



Use free body ABCD with the distributed loads replaced by equivalent concentrated loads.



$$+\sum M_c = 0: -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{1}{4}wL$$

$$+\sum M_A = 0: R_C L - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{5L}{4}\right) = 0$$

$$R_C = \frac{3}{4}wL$$

$$\frac{dV}{dx} = -w = -w + w\langle x - \frac{L}{2} \rangle^0 - w\langle x - L \rangle^0$$

Integrating and adding terms to account for the reactions,

$$\frac{dM}{dx} = V = -wx + w\langle x - \frac{L}{2} \rangle' - w\langle x - L \rangle' + R_A + R_C\langle x - L \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2 + \frac{1}{2}w\langle x - \frac{L}{2} \rangle^2 - \frac{1}{2}w\langle x - L \rangle^2 + R_A x + R_C\langle x - L \rangle'$$

$$EI \frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}w\langle x - \frac{L}{2} \rangle^3 - \frac{1}{6}w\langle x - L \rangle^3 + \frac{1}{2}R_A x^2 + \frac{1}{2}R_C\langle x - L \rangle^2 + C_1$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\langle x - \frac{L}{2} \rangle^4 - \frac{1}{24}w\langle x - L \rangle^4 + \frac{1}{6}R_A x^3 + \frac{1}{6}R_C\langle x - L \rangle^3 + C_1 x + C_2$$

$$[x=0, y=0] \quad -0 + 0 - 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, y=0] \quad -\frac{1}{24}wL^4 + \frac{1}{24}w\left(\frac{L}{2}\right)^4 - 0 + \frac{1}{6}\left(\frac{wL}{4}\right)L^3 + 0 + C_1 L + 0 = 0 \quad C_1 = -\frac{1}{384}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\langle x - \frac{L}{2} \rangle^4 + \frac{1}{24}w\langle x - L \rangle^4 + \frac{1}{6}\left(\frac{wL}{4}\right)x^3 + \frac{1}{6}\left(\frac{3wL}{4}\right)\langle x - L \rangle^3 - \frac{1}{384}wL^3x$$

(a) Elastic curve.

$$y = \frac{w}{24EI} \left\{ -x^4 + \langle x - \frac{L}{2} \rangle^4 - \langle x - L \rangle^4 + Lx^3 + 3L\langle x - L \rangle^3 - \frac{1}{16}L^3x \right\}$$

(b) Deflection at B. (y at $x = \frac{L}{2}$)

$$y_B = \frac{w}{24EI} \left\{ -\left(\frac{L}{2}\right)^4 + 0 - 0 + \left(L\right)\left(\frac{L}{2}\right)^3 + 0 - \left(\frac{1}{16}L^3\right)\left(\frac{L}{2}\right) \right\} = \frac{wL^4}{768EI}$$

(c) Deflection at D. (y at $x = \frac{3L}{2}$)

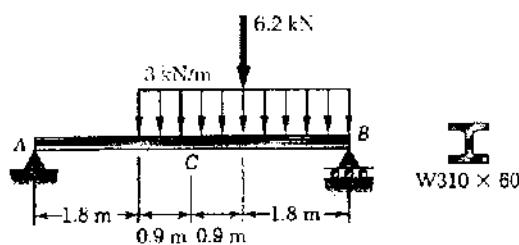
$$y_D = \frac{w}{24EI} \left\{ -\left(\frac{3L}{2}\right)^4 + L^4 - \left(\frac{L}{2}\right)^4 + \left(L\right)\left(\frac{3L}{2}\right)^3 + \left(3L\right)\left(\frac{L}{2}\right)^3 - \left(\frac{1}{16}L\right)\left(\frac{3L}{2}\right) \right\}$$

$$= -\frac{5wL^4}{256EI}$$

$$y_D = \frac{5wL^4}{256EI}$$

Problem 9.45

9.45 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 200 \text{ GPa}$.

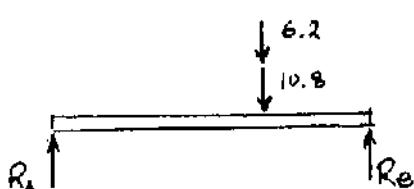


Units: Forces in kN, Lengths in meters.

$$+\odot \sum M_B = 0:$$

$$-5.4 R_A - (1.8)(6.2 + 10.8) = 0$$

$$R_A = 5.6667 \text{ kN}$$



$$w(x) = 3(x - 1.8)^0$$

$$\frac{dV}{dx} = -w(x) = -3(x - 1.8)^0$$

$$\frac{dM}{dx} = V = 5.6667 - 3(x - 1.8)^1 - 6.2(x - 3.6)^0$$

$$EI \frac{d^2y}{dx^2} = M = 5.6667 x - \frac{3}{2}(x - 1.8)^2 - 6.2(x - 3.6)^1 \quad \text{kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = 2.8333 x^2 - \frac{1}{2}(x - 1.8)^3 - 3.1(x - 3.6)^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI y = 0.9444 x^3 - \frac{1}{8}(x - 1.8)^4 - 1.0833(x - 3.6)^3 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$$[x=0, y=0] \quad 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=5.4, y=0] \quad (0.9444)(5.4)^3 - \frac{1}{8}(3.6)^4 - 1.0833(1.8)^3 + C_1(5.4) + 0 = 0$$

$$C_1 = -22.535 \text{ kN}\cdot\text{m}^2$$

$$\text{Data: } E = 200 \times 10^9 \text{ Pa}, \quad I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N}\cdot\text{m}^2 = 25.8 \times 10^3 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A. ($\frac{dy}{dx}$ at $x = 0$)

$$EI \frac{dy}{dx} = 0 - 0 - 0 - 22.535 \text{ kN}\cdot\text{m}^2$$

$$\Theta_A = -\frac{22.535}{25.8 \times 10^3} = -873 \times 10^{-6} = 0.873 \times 10^{-3} \text{ rad} \rightarrow$$

(b) Deflection at C. (y at $x = 2.7 \text{ m}$)

$$EI y_C = (0.9444)(2.7)^3 - \frac{1}{8}(0.9)^4 - 0 - (22.535)(2.7) + 0$$

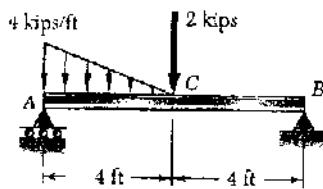
$$= -42.337 \text{ kN}\cdot\text{m}^3$$

$$y_C = -\frac{42.337}{25.8 \times 10^3} = -1.641 \times 10^{-3} \text{ m}$$

$$y_C = 1.641 \text{ mm} \downarrow$$

Problem 9.46

9.46 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 29 \times 10^6$ psi.



I
S6 x 12.5

Distributed loads: ① $w_1(x) = w_0 - kx$
② $w_2(x) = kx$

Data: $a = 4 \text{ ft}$, $w_0 = 4 \text{ kips}/\text{ft}$, $k = 1 \text{ kip}/\text{ft}^2$
 $P = 2 \text{ kips}$

$$+\sum \Sigma M_B = 0: -8R_A + (8)(6\frac{2}{3}) + (2)(4) = 0 \quad R_A = \frac{23}{3} \text{ kips}$$

$$w(x) = w_0 - kx + k(x-4)$$

$$= 4 - x + (x-4)$$

$$\frac{dV}{dx} = -w = -4 + x - (x-4)$$

$$\frac{dM}{dx} = V = \frac{23}{3} - 4x + \frac{1}{2}x^2 - \frac{1}{2}(x-4)^2 - 2(x-4)$$

$$EI \frac{dy}{dx^2} = M = \frac{23}{3}x - 2x^2 + \frac{1}{6}x^3 - \frac{1}{6}(x-4)^3 - 2(x-4) \quad \text{kip}\cdot\text{in}$$

$$EI \frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{24}(x-4)^4 - (x-4)^3 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EI y = \frac{23}{18}x^3 - \frac{1}{6}x^4 + \frac{1}{120}x^5 - \frac{1}{120}(x-4)^5 - \frac{1}{3}(x-4)^3 + C_1x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$$[x=0, y=0] \quad 0 - 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=8, y=0] \quad (\frac{23}{18})(8)^3 - \frac{1}{6}(8)^4 + \frac{1}{120}(8)^5 - \frac{1}{120}(4)^5 - \frac{1}{3}(4)^3 + C_1(8) = 0$$

$$C_1 = -26.844 \quad \text{kip}\cdot\text{ft}^2$$

Data: $E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$ $I = 22.1 \text{ in}^4$

$$EI = (29 \times 10^3)(22.1) = 640.9 \times 10^3 \text{ kip}\cdot\text{in}^2 = 4451 \text{ kip}\cdot\text{ft}^2$$

(a) Slope at A. $(\frac{dy}{dx} \text{ at } x=0)$

$$EI \theta_A = 0 + 0 + 0 + 0 + 0 = 26.844 \quad \text{kip}\cdot\text{ft}^2$$

$$\theta_A = -\frac{26.844}{4451} = -6.03 \times 10^{-5} \text{ rad} \quad 6.03 \times 10^{-5} \text{ rad} \quad \blacksquare$$

(b) Deflection at C. (y at $x = 4 \text{ ft}$)

$$EI y_C = \frac{23}{18}(4)^3 - \frac{1}{6}(4)^4 + \frac{1}{120}(4)^5 - 0 - 0 - (26.844)(4) + 0$$

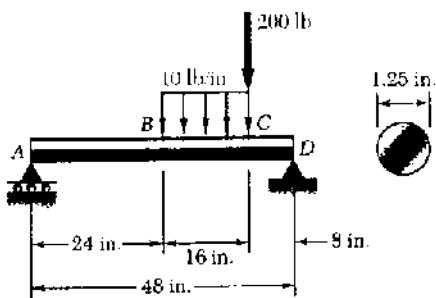
$$= -51.73 \quad \text{kip}\cdot\text{ft}^3$$

$$y_C = -\frac{51.73}{4451} = -13.42 \times 10^{-5} \text{ ft}$$

$$= -0.1610 \text{ in.} \downarrow \quad \blacksquare$$

Problem 9.47

9.47 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point B. Use $E = 29 \times 10^6$ psi.



Units: Forces in lb. Lengths in inches

$$c = \frac{1}{2}d = (\frac{1}{2})(1.25) = 0.625 \text{ in.}$$

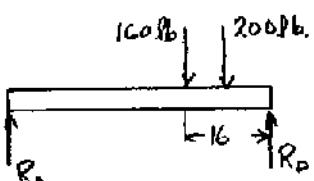
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.625)^4 = 119.84 \times 10^{-8} \text{ in}^4$$

$$EI = (29 \times 10^6)(119.84 \times 10^{-8}) = 3.4754 \times 10^6 \text{ lb-in}^2$$

Use entire beam ABCD as free body.

$$\rightarrow \sum M_A = 0 : -48R_A + (16)(60) + (8)(200) = 0$$

$$R_A = 86.667 \text{ lb.} \leftarrow$$



$$w(x) = 10(x-24)^\circ - 10(x-40)^\circ \text{ lb/in.}$$

$$\frac{dv}{dx} = -w = -10(x-24)^\circ + 10(x-40)^\circ \text{ lb/in}$$

$$\frac{dM}{dx} = V = -10(x-24)' + 10(x-40)' + 86.667 - 200(x-40)^\circ \text{ lb-in}$$

$$EI \frac{d^2y}{dx^2} = M = -5(x-24)^2 + 5(x-40)^2 + 86.667x - 200(x-40)' \text{ lb-in}$$

$$EI \frac{dy}{dx} = -\frac{5}{3}(x-24)^3 + \frac{5}{3}(x-40)^3 + 43.333x^2 - 100(x-40)^2 + C_1 \text{ lb-in}^2$$

$$EIy = -\frac{5}{12}(x-24)^4 + \frac{5}{12}(x-40)^4 + 14.4444x^3 - \frac{100}{3}(x-40)^3 + C_1x + C_2 \text{ lb-in}^3$$

$$[x=0, y=0] \quad -0+0+0+0+C_2=0 \quad C_2=0$$

$$[x=48, y=0] \quad -(\frac{5}{12})(24)^3 + (\frac{5}{12})(8)^4 + (14.4444)(48)^3 - (\frac{100}{3})(8)^3 + 48C_1 = 0 \\ C_1 = -30.08 \times 10^3 \text{ lb-in}^2$$

(a) Slope at end A. ($\frac{dy}{dx}$ at $x=0$)

$$EI \left(\frac{dy}{dx} \right)_A = -0+0+0+C_1$$

$$\left(\frac{dy}{dx} \right)_A = \frac{C_1}{EI} = \frac{-30.08 \times 10^3}{3.4754 \times 10^6} = -8.66 \times 10^{-3} \text{ rad} \leftarrow$$

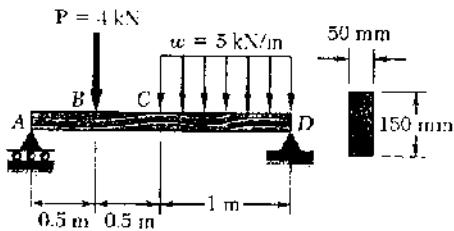
(b) Deflection at point B. (y at $x=24$ in.)

$$EIy_B = -0+0+(14.4444)(24)^3 - 0 + (-30.08 \times 10^3)(24) \\ = -522.24 \times 10^3 \text{ lb-in}^3$$

$$y_B = \frac{-522.24 \times 10^3}{3.4754 \times 10^6} = -0.1503 \text{ in.} \quad y_B = 0.1503 \text{ in.} \leftarrow$$

Problem 9.48

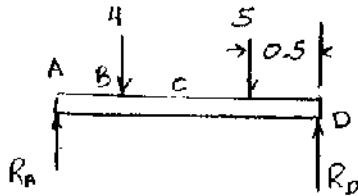
9.48 For the timber beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 12 \text{ GPa}$.



Units: Forces in kN, lengths in meters.

$$I = \frac{1}{12}(50)(150)^3 = 14.0625 \times 10^6 \text{ mm}^4 = 14.0625 \times 10^{-6} \text{ m}^4$$

$$EI = (12 \times 10^9)(14.0625 \times 10^{-6}) = 168.75 \times 10^3 \text{ N} \cdot \text{m}^2 = 168.75 \text{ kN} \cdot \text{m}^2$$



$$+\Sigma M_D = 0: -2R_A + (1.5)(4) + (0.5)(5) = 0$$

$$R_A = 4.25 \text{ kN}$$

$$w(x) = 5(x-1)^0 \text{ kN} \cdot \text{m}$$

$$\frac{dV}{dx} = -w = -5(x-1)^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = -5(x-1)^1 + 4.25 - 4(x-0.5)^0 \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{5}{2}(x-1)^2 + 4.25x - 4(x-0.5)^1 \text{ kN} \cdot \text{m}$$

$$EI \frac{dy}{dx} = -\frac{5}{6}(x-1)^3 + 2.125x^2 - 2(x-0.5)^2 + C_1 \text{ kN} \cdot \text{m}^2$$

$$EIy = -\frac{5}{24}(x-1)^4 + \frac{2.125}{3}x^3 - \frac{2}{3}(x-0.5)^3 + C_1x + C_2 \text{ kN} \cdot \text{m}^3$$

$$[x=0, y=0] \quad -0+0-0+0+C_2=0 \quad C_2=0$$

$$[x=2, y=0] \quad -\left(\frac{5}{24}\right)(1)^4 + \left(\frac{2.125}{3}\right)(2)^3 - \left(\frac{2}{3}\right)(1.5)^3 + 2C_1 = 0 \\ C_1 = -1.60417 \text{ kN} \cdot \text{m}^2$$

(a) Slope at end A. ($\frac{dy}{dx}$ at $x=0$)

$$EI \left(\frac{dy}{dx} \right)_A = -0+0-0+C_1$$

$$\left(\frac{dy}{dx} \right)_A = \frac{C_1}{EI} = \frac{-1.60417}{168.75} = -9.51 \times 10^{-3} \quad \theta_A = 9.51 \times 10^{-3} \text{ rad} \rightarrow$$

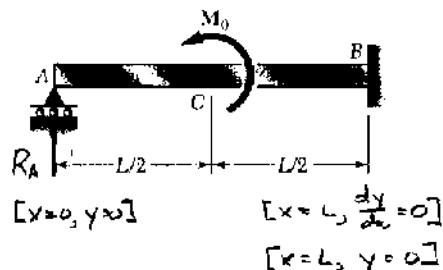
(b) Deflection at midpoint C. (y at $x=1 \text{ m}$)

$$EIy_C = -0 + \left(\frac{2.125}{3}\right)(1)^3 - \left(\frac{2}{3}\right)(0.5)^3 + (-1.60417)(1) = -979.17 \times 10^{-3} \text{ kN} \cdot \text{m}^3$$

$$y_C = \frac{-979.17 \times 10^{-3}}{168.75} = -5.80 \times 10^{-3} \text{ m} \quad y_C = 5.80 \text{ mm} \downarrow$$

Problem 9.49

9.49 and 9.50 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point C.



$$\text{For } 0 \leq x \leq \frac{L}{2}, \quad M = R_A x$$

$$\text{For } \frac{L}{2} \leq x \leq L, \quad M = R_A x - M_0$$

$$\text{Then } EI \frac{d^2y}{dx^2} = M = R_A x - M_0 \left(x - \frac{L}{2}\right)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_0 \left(x - \frac{L}{2}\right)^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_0 \left(x - \frac{L}{2}\right)^3 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad \frac{1}{2} R_A L^2 - M_0 \left(\frac{L}{2}\right) + C_1 = 0 \quad C_1 = \frac{1}{2} (M_0 L - R_A L^2)$$

$$[x=L, y=0] \quad \frac{1}{6} R_A L^3 - \frac{1}{2} M_0 \left(\frac{L}{2}\right)^2 + \frac{1}{2} (M_0 L - R_A L^2) L + 0 = 0$$

$$-\frac{1}{3} R_A L^3 + \frac{3}{8} M_0 L^2 = 0$$

$$(a) \text{ Reaction at A.} \quad R_A = \frac{9 M_0}{8 L} \quad M_A = 0$$

$$C_1 = \frac{1}{2} [M_0 L - (\frac{9 M_0}{8 L})(L^2)] = -\frac{1}{16} M_0 L$$

$$EI y = \frac{1}{6} \left(\frac{9 M_0}{8 L}\right) x^3 - \frac{1}{2} M_0 \left(x - \frac{L}{2}\right)^2 - \frac{1}{16} M_0 L x + 0$$

$$\text{Elastic curve.} \quad y = \frac{M_0}{EI L} \left\{ \frac{9}{8} x^3 - \frac{1}{2} L \left(x - \frac{L}{2}\right)^2 - \frac{1}{16} L^2 x \right\}$$

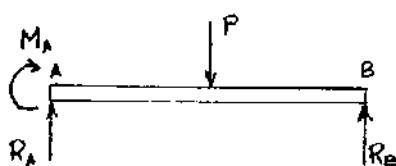
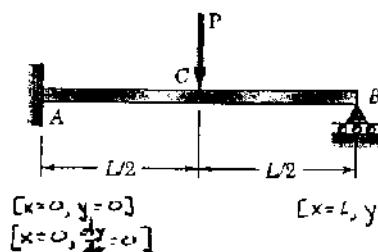
$$(b) \text{ Deflection at point C.} \quad (y \text{ at } x = \frac{L}{2})$$

$$y_C = \frac{M_0}{EI L} \left\{ \left(\frac{1}{6}\right) \left(\frac{9}{8}\right) \left(\frac{L}{2}\right)^3 - 0 - \left(\frac{1}{16} L^2\right) \left(\frac{L}{2}\right) \right\} = -\frac{M_0 L^2}{128 EI}$$

$$y_C = \frac{M_0 L^2}{128 EI} \downarrow$$

Problem 9.50

9.49 and 9.50 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point C.



$$\begin{aligned} \uparrow \sum F_y &= 0: \quad R_A + R_B - P = 0 & R_A = P - R_B \\ \rightarrow \sum M_A &= 0: \quad -M_A - P \frac{L}{2} + R_B L = 0 & M_A = R_B L - \frac{1}{2} PL \end{aligned}$$

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - P\left(x - \frac{L}{2}\right)$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - P\left(x - \frac{L}{2}\right)$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P\left(x - \frac{L}{2}\right)^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P\left(x - \frac{L}{2}\right)^3 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, y=0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P\left(\frac{L}{2}\right)^3 + 0 + 0 = 0$$

$$\frac{1}{2} (R_B L - \frac{1}{2} PL) L^2 + \frac{1}{6} (P - R_B) L^3 - \frac{1}{48} PL^3 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_B L^3 = \left(\frac{1}{4} - \frac{1}{6} + \frac{1}{48}\right) PL^3 \quad \frac{1}{3} R_B = \frac{5}{48} P \quad R_B = \frac{5}{16} P$$

$$R_A = P - \frac{5}{16} P = \frac{11}{16} P$$

$$M_A = \frac{5}{16} PL - \frac{1}{2} PL = -\frac{3}{16} PL$$

(b) Deflection at C. (y at $x = \frac{L}{2}$)

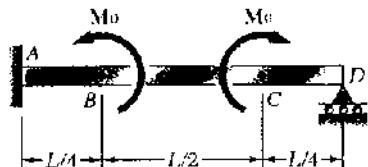
$$y_C = \frac{1}{EI} \left\{ \frac{1}{2} M_A \left(\frac{L}{2}\right)^2 + \frac{1}{6} R_A \left(\frac{L}{2}\right)^3 + 0 + 0 + 0 \right\}$$

$$= \frac{PL^3}{EI} \left\{ \left(\frac{1}{2}\right) \left(-\frac{3}{16}\right) \left(\frac{1}{4}\right) + \left(\frac{1}{3}\right) \left(\frac{11}{16}\right) \left(\frac{1}{8}\right) \right\} = -\frac{7}{168} \frac{PL^3}{EI}$$

$$y_C = \frac{7}{168} \frac{PL^3}{EI}$$

Problem 9.51

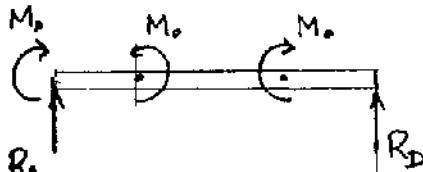
9.51 and 9.52 For the beam and loading shown, determine (a) the reaction at the roller support. (b) the deflection at point B.



$$+\uparrow \sum F_y = 0: R_A + R_D = 0 \quad R_A = -R_D$$

$$+\rightarrow \sum M_A = 0: -M_A + M_o + M_o + RL = 0$$

$$M_A = R_D L$$



$$M(x) = M_A + R_D x - M_o \left(x - \frac{L}{4}\right)^0 + M_o \left(x - \frac{3L}{4}\right)^0$$

$$EI \frac{d^3y}{dx^3} = R_D L - R_D x - M_o \left(x - \frac{L}{4}\right)^0 + M_o \left(x - \frac{3L}{4}\right)^0$$

$$EI \frac{dy}{dx} = R_D L x - \frac{1}{2} R_D x^2 - M_o \left(x - \frac{L}{4}\right)^1 + M_o \left(x - \frac{3L}{4}\right)^1 + C_1$$

$$[x=0, \frac{dy}{dx} = 0] \quad 0 - 0 - 0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$EIy = \frac{1}{2} R_D L x^2 - \frac{1}{6} R_D x^3 - \frac{1}{2} M_o \left(x - \frac{L}{4}\right)^1 + \frac{1}{2} M_o \left(x - \frac{3L}{4}\right)^1 + C_2$$

$$[x=0, y=0] \quad 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, y=0] \quad \frac{1}{2} R_D L^3 - \frac{1}{6} R_D L^3 - \frac{1}{2} M_o \left(\frac{3L}{4}\right)^2 + \frac{1}{2} M_o \left(\frac{L}{4}\right)^2 = 0$$

(a) Reaction at D.

$$R_D = \frac{3 M_o}{4 L} \uparrow$$

$$EIy = \frac{1}{2} \left(\frac{3 M_o}{4 L} \right) L x^2 - \frac{1}{6} \left(\frac{3 M_o}{4 L} \right) x^3 - \frac{1}{2} M_o \left(x - \frac{L}{4}\right)^2 + \frac{1}{2} M_o \left(x - \frac{3L}{4}\right)^2$$

Elastic curve.

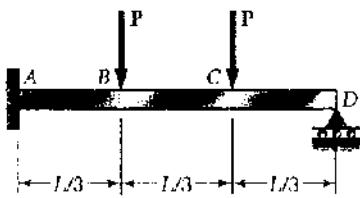
$$y = \frac{M_o}{E I L} \left\{ \frac{3}{8} L x^2 - \frac{1}{8} x^3 - \frac{1}{2} L \left(x - \frac{L}{4}\right)^2 + \frac{1}{2} L \left(x - \frac{3L}{4}\right)^2 \right\}$$

(b) Deflection at point B. (y at $x = \frac{L}{4}$)

$$y_B = \frac{M_o}{E I L} \left\{ \left(\frac{3}{8} L\right) \left(\frac{L}{4}\right)^2 - \left(\frac{1}{8} \left(\frac{L}{4}\right)^3 - 0 + 0\right) \right\} = \frac{11 M_o L^2}{512 E I} \uparrow$$

Problem 9.52

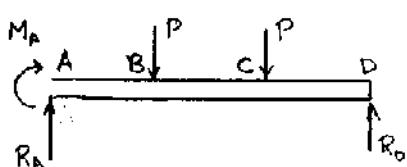
9.51 and 9.52 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point B.



$$\uparrow \sum F_y = 0: R_A - P - P + R_D = 0 \quad R_A = 2P - R_D$$

$$\rightarrow \sum M_A = 0: -M_A - \frac{PL}{3} - \frac{2PL}{3} + R_D L = 0$$

$$M_A = R_D L - PL$$



$$\frac{dM}{dx} = V = R_A - P(x - \frac{L}{3}) - P(x - \frac{2L}{3})$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - P(x - \frac{L}{3}) - P(x - \frac{2L}{3})$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P(x - \frac{L}{3})^2 - \frac{1}{2} P(x - \frac{2L}{3})^2 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 - 0 - 0 + C_1 = 0 \quad C_1 = 0$$

$$EIy = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P(x - \frac{L}{3})^3 - \frac{1}{6} P(x - \frac{2L}{3})^3 + C_2$$

$$[x=L, y=0] \quad 0 + 0 - 0 - 0 + C_2 = 0 \quad C_2 = 0$$

$$EIy = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P(x - \frac{L}{3})^3 - \frac{1}{6} P(x - \frac{2L}{3})^3$$

$$[x=L, y=0] \quad \frac{1}{2}(R_D L - PL)L^2 + \frac{1}{6}(2P - R_D)L^3 - \frac{1}{6}P(\frac{2L}{3})^3 - \frac{1}{6}P(\frac{L}{3})^3 = 0$$

$$\frac{1}{3}R_D L^3 - \frac{2}{9}PL^3 = 0$$

(a) Reaction at D.

$$R_D = \frac{2}{3}P \uparrow$$

$$M_A = \frac{2}{3}PL - PL = -\frac{1}{3}PL \quad R_A = 2P - \frac{2}{3}P = \frac{4}{3}P$$

$$EIy = \frac{1}{2}(-\frac{1}{3}PL)x^2 + \frac{1}{6}(\frac{4}{3}P)x^3 - \frac{1}{6}P(x - \frac{L}{3})^3 - \frac{1}{6}(x - \frac{2L}{3})^3$$

Elastic curve.

$$y = \frac{P}{EI} \left\{ -\frac{1}{6}Lx^2 + \frac{2}{9}x^3 - \frac{1}{6}(x - \frac{L}{3})^3 - \frac{1}{6}(x - \frac{2L}{3})^3 \right\}$$

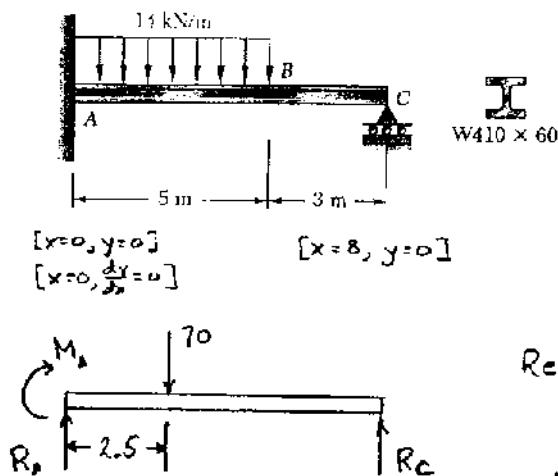
(b) Deflection at B. (y at $x = \frac{L}{3}$)

$$y_B = \frac{P}{EI} \left\{ -(\frac{L}{6})(\frac{L}{3})^2 + \frac{2}{9}(\frac{L}{3})^3 - 0 - 0 \right\} = -\frac{5PL^3}{486EI}$$

$$y_B = \frac{5PL^3}{486EI} \downarrow$$

Problem 9.53

9.53 For the beam and loading shown, determine (a) the reaction at point C, (b) the deflection at point B. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m.

$$+\uparrow \sum F_y = 0: R_A - 70 + R_C = 0$$

$$R_A = 70 - R_C \text{ kN}$$

$$+\rightarrow \sum M_A = 0: -M_A - (70)(2.5) + 8R_C = 0$$

$$M_A = 8R_C - 175 \text{ kN.m}$$

Reactions are statically indeterminate.

$$w(x) = 14 - 14(x-5)^0 \text{ kN/m}$$

$$\frac{dV}{dx} = -w = -14 + 14(x-5)^1 \text{ kN/m}$$

$$\frac{dM}{dx} = V = R_A - 14x + 14(x-5)^1 \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7(x-5)^2 \text{ kN.m}$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{7}{3} x^3 + \frac{7}{3}(x-5)^3 + C_1 \text{ kN.m}^2$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{7}{12} x^4 + \frac{7}{12}(x-5)^4 + C_1 x + C_2 \text{ kN.m}^3$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=8, y=0] \quad \frac{1}{2} M_A (8)^2 + \frac{1}{6} R_A (8)^3 - \frac{7}{12} (8)^4 + \frac{7}{12} (3)^4 + 0 + 0 = 0$$

$$32(8R_C - 175) + \frac{512}{6} (70 - R_C) - \frac{28105}{12} = 0$$

$$170.667 R_C = 5600 - \frac{35840}{6} + \frac{28105}{12} = 1968.75 \quad R_C = 11.536 \text{ kN} \uparrow$$

$$M_A = (8)(11.536) - 175 = -82.715 \text{ kN.m}$$

$$R_A = 70 - 11.536 = 58.464 \text{ kN}$$

$$\text{Data: } E = 200 \times 10^9 \text{ Pa} \quad I = 216 \times 10^4 \text{ mm}^4 = 216 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(216 \times 10^{-6}) = 43.2 \times 10^6 \text{ N.m}^2 = 43200 \text{ kN.m}^2$$

(b) Deflection at B. (y at $x=5 \text{ m}$)

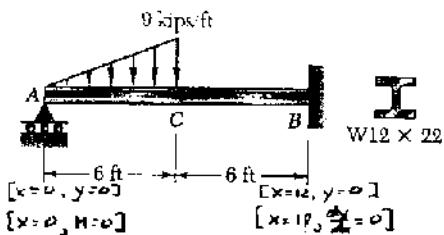
$$EI y_B = \frac{1}{2} (-82.715)(5)^3 + \frac{1}{6} (58.464)(5)^3 - \frac{7}{12} (5)^4 = -180.52 \text{ kN.m}^3$$

$$y_B = -\frac{180.52}{43200} = -4.18 \times 10^{-3} \text{ m}$$

$$y_B = 4.18 \text{ mm} \downarrow$$

Problem 9.54

9.54 and 9.55 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at point C. Use $E = 29 \times 10^6$ psi.



Units: Forces in Kips, lengths in ft.

$$k = \frac{9 \text{ kips}/\text{ft}}{6 \text{ ft}} = 1.5 \text{ kips}/\text{ft}^2$$

$$w(x) = 1.5x - 9(x-6)^0 - 1.5(x-6)^1$$

$$\frac{dV}{dx} = -w(x) = -1.5x + 9(x-6)^0 + 1.5(x-6)^1$$

$$\frac{dM}{dx} = V = R_A - 0.75x^2 + 9(x-6)^1 + 0.75(x-6)^2$$

$$EI \frac{dy}{dx^2} = M = R_A x - 0.25x^3 + 4.5(x-6)^2 + 0.25(x-6)^3$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - 0.0625x^4 + 1.5(x-6)^3 + 0.0625(x-6)^4 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EIy = \frac{1}{6}R_A x^3 - 0.0125x^5 + 0.375(x-6)^4 + 0.0125(x-6)^5 + C_1 x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=12, \frac{dy}{dx}=0] \quad \frac{1}{2}(R_A)(12)^2 - (0.0625)(12)^4 + (1.5)(6)^3 + (0.0625)(6)^4 + C_1 = 0$$

$$C_1 = 891 - 72R_A = 0 \quad \text{kip}\cdot\text{ft}^2$$

$$[x=12, y=0] \quad \frac{1}{6}R_A(12)^3 - (0.0125)(12)^5 + (0.375)(6)^4 + (0.0125)(6)^5 + (891 - 72R_A)(12) + 0 = 0$$

$$(864 - 288)R_A = 8164.8 \quad R_A = 14.175 \text{ kips}$$

$$C_1 = 891 - (72)(14.175) = -129.6 \text{ kip}\cdot\text{ft}^2$$

Data: $E = 29 \times 10^6$ psi = 29×10^3 ksi $I = 156 \text{ in}^4$

$$EI = (29 \times 10^3)(156) = 4.524 \times 10^6 \text{ kip}\cdot\text{in}^2 = 31417 \text{ kip}\cdot\text{ft}^2$$

(b) Deflection at C. (y at $x=6$)

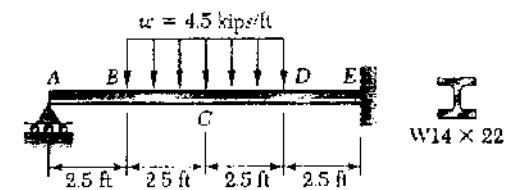
$$EIy_C = \frac{1}{6}(14.175)(6)^3 - (0.0125)(6)^5 + 0 + 0 - (129.6)(6) + 0 \\ = -364.5 \text{ kip}\cdot\text{ft}^2$$

$$y_C = -\frac{364.5}{31417} = -11.60 \times 10^{-3} \text{ ft}$$

$$y_C = 0.1392 \text{ in.}$$

Problem 9.55

9.54 and 9.55 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at point C. Use $E = 29 \times 10^6$ psi.



Units: Forces in kips, lengths in ft.

$$\begin{aligned} [x=0, y=0] \\ [x=0, M=0] \end{aligned}$$

$$\begin{aligned} [x=10, y=0] \\ [x=10, \frac{dy}{dx}=0] \end{aligned}$$

$$w(x) = 4.5(x-2.5)^0 - 4.5(x-7.5)^0$$

$$\frac{dV}{dx} = -w(x) = -4.5(x-2.5)^0 + 4.5(x-7.5)^0 \text{ kip/ft}$$

$$\frac{dM}{dx} = V = R_A - 4.5(x-2.5)^1 + 4.5(x-7.5)^1 \text{ kips}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - 2.25(x-2.5)^2 + 2.25(x-7.5)^2 \text{ kip-ft}$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{2.25}{3}(x-2.5)^3 + \frac{2.25}{3}(x-7.5)^3 + C_1 \text{ kip-ft}^2$$

$$EI y = \frac{1}{6}R_A x^3 - \frac{2.25}{12}(x-2.5)^4 + \frac{2.25}{12}(x-7.5)^4 + C_1 x + C_2 \text{ kip-ft}^3$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=10, \frac{dy}{dx}=0] \quad \frac{1}{2}R_A(10)^2 - \frac{2.25}{3}(7.5)^3 + \frac{2.25}{3}(2.5)^3 + C_1 = 0$$

$$C_1 = 304.69 - 50R_A \text{ kip-ft}^2$$

$$[x=10, y=0] \quad \frac{1}{6}R_A(10)^3 - \frac{2.25}{12}(7.5)^4 + \frac{2.25}{12}(2.5)^4 + (304.69 - 50R_A)(10) + 0 = 0$$

$$(500 - \frac{1000}{6})R_A = 24609 \quad R_A = 7.3833 \text{ kips} \uparrow$$

$$C_1 = 304.69 - (50)(7.3833) = -64.45 \text{ kip-ft}^2$$

$$\text{Data: } E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}, \quad I = 199 \text{ in}^4$$

$$EI = (29 \times 10^3)(199) = 5.771 \times 10^6 \text{ kip-in}^2 = 40076 \text{ kip-ft}^2$$

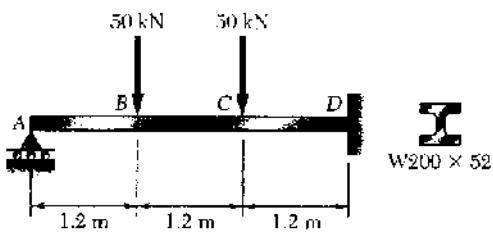
(b) Deflection at C. (y at $x = 5$ ft)

$$EI y_C = \frac{1}{6}(7.3833)(5)^3 - \frac{2.25}{12}(2.5)^4 + 0 - (64.45)(5) + 0 = -175.76 \text{ kip-ft}^2$$

$$y_C = -\frac{175.76}{40076} = -4.3856 \times 10^{-3} \text{ ft} \quad y_C = 0.0526 \text{ in.} \downarrow$$

Problem 9.56

9.56 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at point B. Use $E = 200 \text{ GPa}$.

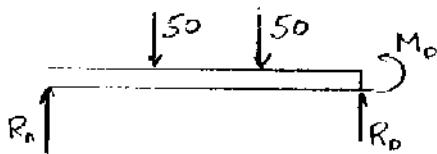


Units: Forces in kN, lengths in meters.

$$I = 52.7 \times 10^6 \text{ mm}^4 = 52.7 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(52.7 \times 10^{-6}) = 10.54 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$= 10540 \text{ kN} \cdot \text{m}^2$$



$$\frac{dM}{dx} = V = R_A - 50(x-1.2)^\circ - 50(x-2.4)^\circ \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - 50(x-1.2)^1 - 50(x-2.4)^1 \text{ kN} \cdot \text{m}$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - 25(x-1.2)^2 - 25(x-2.4)^2 + C_1 \text{ kN} \cdot \text{m}^2$$

$$EIy = \frac{1}{6} R_A x^3 - \frac{25}{3}(x-1.2)^3 - \frac{25}{3}(x-2.4)^3 + C_1 x + C_2 \text{ kN} \cdot \text{m}^3$$

$$[x=0, y=0]$$

$$0 = 0 - 0 + 0 + C_2 \Rightarrow C_2 = 0$$

$$[x=3.6, \frac{dy}{dx}=0]$$

$$\frac{1}{2} R_A (3.6)^2 - (25)(2.4)^2 - (25)(1.2)^2 + C_1 = 0$$

$$C_1 = 180 - 6.48 R_A \text{ kN} \cdot \text{m}^2$$

$$[x=3.6, y=0]$$

$$\frac{1}{6} R_A (3.6)^3 - \frac{25}{3}(2.4)^3 - \frac{25}{3}(1.2)^3 + (180 - 6.48 R_A)(3.6) = 0$$

$$-15.552 R_A + 518.4 = 0$$

(a) Reaction at A.

$$R_A = \frac{100}{3} \text{ kN}$$

$$R_A = 33.3 \text{ kN} \uparrow$$

$$C_1 = 180 - (6.48)(33.333) = -36 \text{ kN} \cdot \text{m}^2$$

$$EIy = \frac{1}{6} \left(\frac{100}{3} \right) x^3 - \frac{25}{3} (x-1.2)^3 - \frac{25}{3} (x-2.4)^3 - 36x \text{ kN} \cdot \text{m}^3$$

(b) Deflection at point B. (y at $x = 1.2 \text{ m}$)

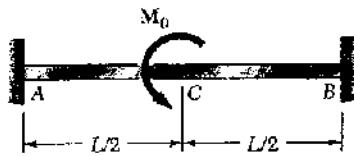
$$EIy_B = \left(\frac{1}{6} \right) \left(\frac{100}{3} \right) (1.2)^3 - 0 - 0 - (36)(1.2) = -33.6 \text{ kN} \cdot \text{m}^3$$

$$y_B = \frac{-33.6}{10540} = -3.19 \times 10^{-3} \text{ m}$$

$$y_B = 3.19 \text{ mm} \downarrow$$

Problem 9.57

9.57 For the beam and loading shown, determine (a) the reaction at point A, (b) the slope at point C.



Reactions are statically indeterminate.

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - M_0 \left(x - \frac{L}{2}\right)^0$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - M_0 \left(x - \frac{L}{2}\right)^1 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{2} M_0 \left(x - \frac{L}{2}\right)^2 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx} = 0]$$

$$C_1 = 0$$

$$[x=0, y = 0]$$

$$C_2 = 0$$

$$[x=L, \frac{dy}{dx} = 0] \quad M_A L + \frac{1}{2} R_A L^2 - M_0 \frac{L}{2} = 0 \quad M_A = \frac{1}{2} M_0 - \frac{1}{2} R_A L$$

$$[x=L, y = 0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{2} M_0 \left(\frac{L}{2}\right)^2 = 0$$

$$\frac{1}{2} \left(\frac{1}{2} M_0 - \frac{1}{2} R_A L \right) L^2 + \frac{1}{6} R_A L^3 - \frac{1}{8} M_0 L^2 = 0$$

$$\left(\frac{1}{4} - \frac{1}{6} \right) R_A L^3 = \left(\frac{1}{4} - \frac{1}{8} \right) M_0 L^2 \quad R_A = \frac{3}{2} \frac{M_0}{L} \uparrow$$

$$M_A = \frac{1}{2} M_0 - \frac{1}{2} \frac{3}{2} \frac{M_0}{L} L = -\frac{1}{4} M_0$$

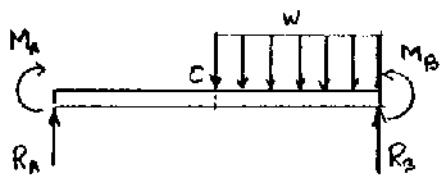
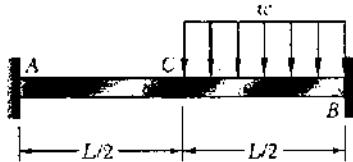
(b) Slope at C. ($\frac{dy}{dx}$ at $x = \frac{L}{2}$)

$$\Theta_C = \frac{1}{EI} \left\{ \left(-\frac{1}{4} M_0\right) \frac{L}{2} + \frac{1}{2} \left(\frac{3}{2} \frac{M_0}{L}\right) \left(\frac{L}{2}\right)^2 + 0 + 0 \right\} - \frac{1}{16} \frac{M_0 L}{EI}$$

$$\Theta_C = \frac{1}{16} \frac{M_0 L}{EI}$$

Problem 9.58

9.58 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at midpoint C.



$$w(x) = w(x - \frac{L}{2})^0$$

$$\frac{dV}{dx} = -w(x) = -w(x - \frac{L}{2})^0$$

$$\frac{dM}{dx} = V = R_A - w(x - \frac{L}{2})^1$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{1}{2} w(x - \frac{L}{2})^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{6} w(x - \frac{L}{2})^3 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{24} w(x - \frac{L}{2})^4 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0]$$

$$0 + 0 - 0 + C_1 = 0$$

$$C_1 = 0$$

$$[x=0, y=0]$$

$$0 + 0 - 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0]$$

$$M_A L + \frac{1}{2} R_A L^2 - \frac{1}{6} w(\frac{L}{2})^3 = 0$$

(1)

$$[x=L, y=0]$$

$$\frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{24} w(\frac{L}{2})^4 = 0$$

(2)

Solving equations (1) and (2) simultaneously,

$$(a) R_A = \frac{3wL}{32} \quad M_A = -\frac{5wL^2}{192}$$

$$R_A = \frac{3wL}{32} \uparrow$$

$$M_A = \frac{5wL^2}{192} \leftarrow$$

$$EI y = -\frac{5}{384} w L^2 x^2 + \frac{3}{192} w L x^3 - \frac{1}{24} w(x - \frac{L}{2})^4$$

Elastic curve.

$$y = \frac{w}{EI} \left\{ -\frac{5}{384} L^2 x^2 + \frac{3}{192} L x^3 - \frac{1}{24} (x - \frac{L}{2})^4 \right\}$$

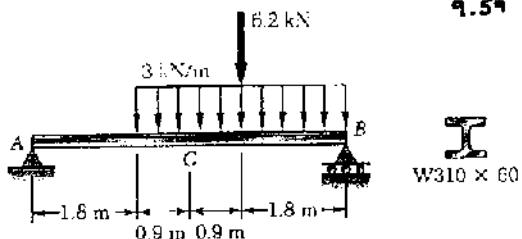
(b) Deflection at midpoint C. (y at $x = \frac{L}{2}$)

$$y_C = \frac{w}{EI} \left\{ \left(-\frac{5}{384} L^2 \right) \left(\frac{L}{2} \right)^2 + \left(\frac{3}{192} L \right) \left(\frac{L}{2} \right)^3 - 0 \right\}$$

$$= -\frac{WL^4}{768 EI}$$

$$y_B = \frac{WL^4}{768 EI} \downarrow$$

Problem 9.59



9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.59 Beam and loading of Prob. 9.45.

See solution to Prob. 9.45 for the derivation of the equations used in the following.

$$EI = 25.8 \times 10^3 \text{ kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}(x-1.8)^2 - 3.1(x-3.6)^2 - 22.535$$

$$EI y = 0.9444x^3 - \frac{1}{6}(x-1.8)^3 - 1.03333(x-3.6)^3 - 22.535x$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $1.8 < x_m < 3.6$

$$EI \frac{dy}{dx} = 2.8333x_m^2 - \frac{1}{2}(x_m-1.8)^2 - 22.535 = 0$$

Solving by iteration: $x_m = 3, 2.86, 2.855$ $x_m = 2.855 \text{ m}$ ←
 $df/dx = 15.8, 15.15$

$$EI y_m = 0.9444x_m^3 - \frac{1}{8}(x_m-1.8)^3 - 22.535x_m$$

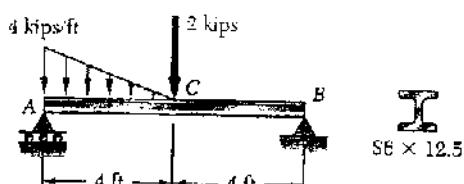
$$= (0.9444)(2.855)^3 - \frac{1}{8}(2.855-1.8)^3 - (22.535)(2.855) = -42.507 \text{ kN}\cdot\text{m}^3$$

$$y_m = -\frac{42.507}{25.8 \times 10^3} = -1.648 \times 10^{-3} \text{ m} \quad y_m = 1.648 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.60

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.60 Beam and loading of Prob. 9.46.



I
S6 x 12.5

See solution to Prob. 9.46 for the derivation of the equations used in the following.

$$EI = 4451 \text{ kip}\cdot\text{ft}^2$$

$$EI \frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{24}(x-4)^4 - (x-4)^2 - 26.844 \quad \text{kip}\cdot\text{ft}^2$$

$$EIy = \frac{23}{18}x^3 - \frac{1}{6}x^4 + \frac{1}{120}x^5 - \frac{1}{120}(x-4)^5 - \frac{1}{3}(x-4)^3 - 26.844x \quad \text{kip}\cdot\text{ft}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0 < x < 4 \text{ ft}$.

$$EI \frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - 26.844 = 0 \quad f.$$

Solve by iteration: $x_m = 4.0 \quad 3.73 \quad 3.735 \quad x_m = 3.735 \text{ ft.}$
 $df/dx = 9.33 \quad 9.42$

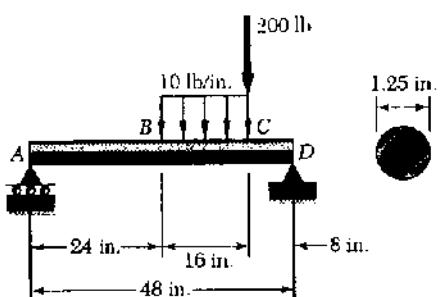
$$EIy_m = \frac{23}{18}(3.735)^3 - \frac{1}{6}(3.735)^4 + \frac{1}{120}(3.735)^5 - (26.844)(3.735) = -60.06 \text{ kip}\cdot\text{ft}^3$$

$$y_m = -\frac{60.06}{4451} = -13.49 \times 10^{-5} \text{ ft.} \quad y_m = 0.1619 \text{ in. } \downarrow$$

Problem 9.61

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.61 Beam and loading of Prob. 9.47.



See solution to Prob. 7.47 for the derivation of equations used in the following.

$$EI = 3.4754 \times 10^6 \text{ lb-in}^2$$

$$C_1 = -30.08 \times 10^3 \text{ lb-in}^2, \quad C_2 = 0$$

$$EI \frac{dy}{dx} = -\frac{5}{3}(x-24)^3 + \frac{5}{3}(x-40)^3 + 43.333 x^2 - 100(x-40)^2 + C_1 \text{ lb-in}^2$$

$$EI y = -\frac{5}{12}(x-24)^4 + \frac{5}{12}(x-40)^4 + 14.4444 x^3 - \frac{100}{3}(x-40)^3 + C_1 x + C_2 \text{ lb-in}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $24 < x_m < 40$.

$$EI \frac{dy}{dx} = -\frac{5}{3}(x_m-24)^3 + 0 + 43.333 x^2 - 0 - 30.08 \times 10^3 = 0$$

$$f = -\frac{5}{3}(x_m-24)^3 + 43.333 x^2 - 30.08 \times 10^3$$

$$\frac{df}{dx_m} = -5(x_m-24)^2 + 86.667 x$$

Solve by iteration:	$x_m = 24$	26.46	26.35
•	$f_i = -5.12 \times 10^3$	249	-14

$$df/dx = 2.08 \times 10^2 \quad 2.26 \times 10^3$$

$$x_m = 26.4 \text{ in.}$$

$$EI y_m = -\frac{5}{12}(26.35-24)^4 + 0 + (14.4444)(26.35)^3 - (30.08 \times 10^3)(26.35)$$

$$= -528.35 \times 10^3 \text{ lb-in}^3$$

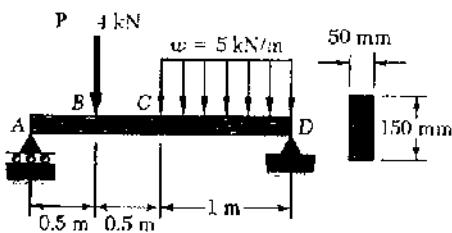
$$y_m = \frac{-528.35 \times 10^3}{3.4754 \times 10^6} = -0.1520 \text{ in.}$$

$$y_m = 0.1520 \text{ in.}$$

Problem 9.62

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.62 Beam and loading of Prob. 9.48.



See solution to Prob. 9.48 for the derivation of the equations used in the following.

$$EI = 168.75 \text{ kN}\cdot\text{m}^2$$

$$C_1 = -1.60417 \text{ kN}\cdot\text{m}^2$$

$$C_2 = 0$$

$$EI \frac{dy}{dx} = -\frac{5}{6}(x-1)^3 + 2.125x^2 - 2(x-0.5)^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI y = -\frac{5}{24}(x-1)^4 + \frac{2.125}{3}x^3 - \frac{2}{3}(x-0.5)^3 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$$

Compute slope at C. ($\frac{dy}{dx}$ at $x = 1 \text{ m}$)

$$EI \left(\frac{dy}{dx} \right)_c = 0 + (2.125)(1)^2 - 2(0.5)^2 - 1.60417 = 20.83 \times 10^{-3} \text{ kN}\cdot\text{m}^2$$

Since the slope at C is positive, the largest deflection occurs in portion BC, where

$$EI \frac{dy}{dx} = 2.125x^2 - 2(x-0.5)^2 - 1.60417$$

$$EI y = \frac{2.125}{3}x^3 - \frac{2}{3}(x-0.5)^3 - 1.60417 x$$

To find the location of the largest downward deflection, set $\frac{dy}{dx} = 0$.

$$\begin{aligned} 2.125x_m^2 - 2(x_m^2 - x_m + 0.25) - 1.60417 \\ = 0.125x_m^2 + 2x_m - 2.10417 = 0 \end{aligned}$$

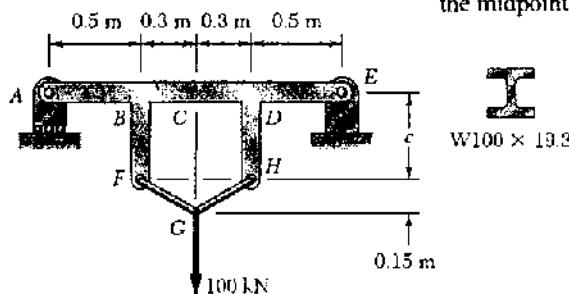
$$x_m = 1.0521 - 0.0625x_m^2$$

Solve by iteration. $x_m = 1, 0.989, 0.991, \dots, x_m = 0.991 \text{ m}$

$$\begin{aligned} EI y_m &= \left(\frac{2.125}{3}\right)(0.991)^3 - \frac{2}{3}(0.991 - 0.5)^3 - (1.60417)(0.991) \\ &= -0.97927 \text{ kN}\cdot\text{m}^3 \end{aligned}$$

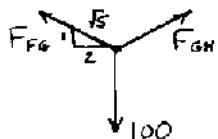
$$y_m = -\frac{0.97927}{168.75} = -5.80 \times 10^{-5} \text{ m} \quad y_m = 5.80 \text{ mm} \downarrow$$

Problem 9.63



9.63 The rigid bars *BF* and *DH* are welded to the rolled-steel beam *AE* as shown. Determine for the loading shown (a) the deflection at point *B*, (b) the deflection at the midpoint *C* of the beam. Use $E = 200 \text{ GPa}$.

Using joint *G* as a free body,



By symmetry
 $F_{GH} = F_{FG}$

$$2 F_{Bxy} - 100 = 0 \quad F_{Bxy} = 50 \text{ kN}$$

$$F_{Bx} = 2 F_{Bxy} = 100 \text{ kN.}$$

Forces in kN. Lengths in m.

$$V = 50 - 50(x-0.5)^0 - 50(x-1.1)^0 \quad \text{kN}$$

$$M = 50x - 50(x-0.5)' - 50(x-1.1)' + 40(x-0.5)^0 - 40(x-1.1) \quad \text{kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = 25x^2 - 25(x-0.5)^2 - .25(x-1.1)^2 - 40(x-0.5)' + 40(x-1.1)' + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI y = \frac{25}{3}x^3 - \frac{25}{3}(x-0.5)^3 - \frac{25}{3}(x-1.1)^3 - 20(x-0.5)^2 + 20(x-1.1)^2 + C_1x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$$[x=0, y=0] \quad C_2 = 0$$

$$[x=1.6, y=0]$$

$$\left(\frac{25}{3}\right)(1.6)^3 - \left(\frac{25}{3}\right)(1.1)^3 - \left(\frac{25}{3}\right)(0.5)^3 - (20)(1.1)^2 + (20)(0.5)^2 + C_1(1.6) + 0 = 0$$

$$C_1 = -1.75 \quad \text{kN}\cdot\text{m}^3$$

$$\text{For } EI y_B, \quad x = 0.5 \text{ m}$$

$$EI y_B = \left(\frac{25}{3}\right)(0.5)^3 - 0 - 0 + 0 - 0 - (1.75)(0.5) = 0.1667 \text{ kN}\cdot\text{m}^3$$

$$\text{For } EI y_C, \quad x = 0.8 \text{ m}$$

$$EI y_C = \left(\frac{25}{3}\right)(0.8)^3 - \left(\frac{25}{3}\right)(0.3)^3 - 0 - (20)(0.3)^2 - 0 - (1.75)(0.8) + 0 = -0.8417 \text{ kN}\cdot\text{m}^3$$

$$\text{For W 100 x 19.3 rolled steel section} \quad I = 4.77 \times 10^6 \text{ mm}^4 = 4.77 \times 10^{-6} \text{ m}^4$$

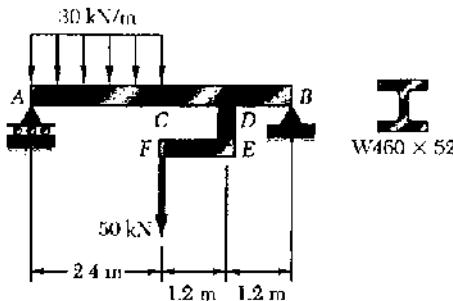
$$EI = (200 \times 10^9)(4.77 \times 10^{-6}) = 954 \times 10^3 \text{ N}\cdot\text{m}^2 = 954 \text{ kN}\cdot\text{m}^2$$

$$(a) y_B = \frac{0.1667}{954} = 0.175 \times 10^{-3} \text{ m} = 0.175 \text{ mm} \uparrow$$

$$(b) y_C = \frac{0.8417}{954} = 0.882 \times 10^{-3} \text{ m} = 0.882 \text{ mm} \uparrow$$

Problem 9.64

9.64 The rigid bar DEF is welded at point D to the rolled-steel beam AB. For the loading shown, determine (a) the slope at point A, (b) the deflection at midpoint C of the beam. Use $E = 200 \text{ GPa}$.



Units: Forces in kN; lengths in meters.

$$+\sum M_B = 0 :$$

$$-4.8 R_A + (30)(2.4)(3.6) + (50)(2.4) = 0$$

$$R_A = 79 \text{ kN} \uparrow$$

$$I = 212 \times 10^6 \text{ mm}^4 = 212 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(212 \times 10^{-6}) = 42.4 \times 10^6 \text{ N} \cdot \text{m}^2 \\ = 42400 \text{ kN} \cdot \text{m}^2$$

$$w(x) = 30 - 30(x-2.4)^0$$

$$\frac{dV}{dx} = -w = -30 + 30(x-2.4)^0 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = 79 - 30x + 30(x-2.4)^1 - 50(x-3.6)^0 \quad \text{kN}$$

$$EI \frac{d^2y}{dx^2} = M = 79x - 15x^2 + 15(x-2.4)^2 - 50(x-3.6)^1 - 60(x-3.6)^0 \quad \text{kN} \cdot \text{m}$$

$$EI \frac{dy}{dx} = \frac{79}{2}x^2 - 5x^3 + 5(x-2.4)^3 - 25(x-3.6)^2 - 60(x-3.6)^1 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}(x-2.4)^4 - \frac{25}{3}(x-3.6)^3 - 30(x-3.6)^2 + C_1x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x=0, y=0] \quad 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=4.8, y=0] \quad (\frac{79}{6})(4.8)^3 - (\frac{5}{4})(4.8)^4 + (\frac{5}{4})(2.4)^4 - (\frac{25}{3})(1.2)^3 - (30)(1.2)^2 + 4.8C_1 = 0$$

$$C_1 = -161.76 \text{ kN} \cdot \text{m}^2$$

(a) Slope at point A. ($\frac{dy}{dx}$ at $x=0$)

$$EI \left(\frac{dy}{dx} \right)_A = 0 - 0 + 0 - 0 - 0 - 161.76 = -161.76 \text{ kN} \cdot \text{m}^2$$

$$\left(\frac{dy}{dx} \right)_A = \frac{-161.76}{42400} = -3.82 \times 10^{-3} \quad \theta_A = 3.82 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

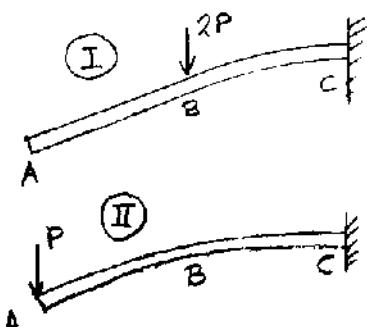
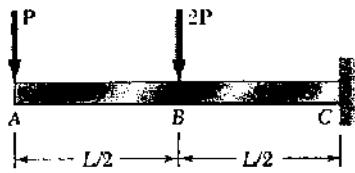
(b) Deflection at midpoint C. (y at $x=2.4$)

$$EIy_C = (\frac{79}{6})(2.4)^3 - (\frac{5}{4})(2.4)^4 + 0 - 0 - 0 - (161.76)(2.4) + 0 = -247.68 \text{ kN} \cdot \text{m}^3$$

$$y_C = \frac{-247.68}{42400} = -5.84 \times 10^{-3} \text{ m} \quad y_C = 5.84 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.65

9.65 and 9.66 For the cantilever beam and loading shown, determine the slope and deflection at the free end.



Loading I: $2P$ downward at B.

Case I of Appendix D applied to portion BC.

$$\theta_B' = \frac{(2P)(L/2)^2}{2EI} = \frac{1}{4} \frac{PL^2}{EI}$$

$$y_B' = \frac{(2P)(L/2)^3}{3EI} = \frac{1}{12} \frac{PL^3}{EI}$$

AB remains straight.

$$\theta_A' = \theta_B' = \frac{1}{4} \frac{PL^2}{EI}$$

$$y_A' = y_B' - \left(\frac{L}{2}\right) \theta_B' = -\frac{1}{12} \frac{PL^3}{EI} - \frac{1}{3} \frac{PL^3}{EI} = -\frac{5}{24} \frac{PL^3}{EI}$$

Loading II: P downward at A. Case I of Appendix D.

$$\theta_A'' = \frac{PL^2}{2EI} \quad , \quad y_A'' = -\frac{PL^3}{3EI}$$

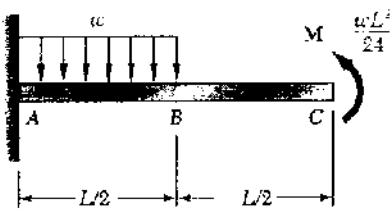
By superposition,

$$\theta_A = \theta_A' + \theta_A'' = \frac{1}{4} \frac{PL^2}{EI} + \frac{1}{2} \frac{PL^2}{EI} = \frac{3}{4} \frac{PL^2}{EI} \quad \frac{3PL^2}{4EI} \curvearrowleft$$

$$y_A = y_A' + y_A'' = -\frac{5}{24} \frac{PL^3}{EI} - \frac{1}{3} \frac{PL^3}{EI} = -\frac{13}{24} \frac{PL^3}{EI} \quad \frac{13PL^3}{24EI} \downarrow$$

Problem 9.66

9.65 and 9.66 For the cantilever beam and loading shown, determine the slope and deflection at the free end.



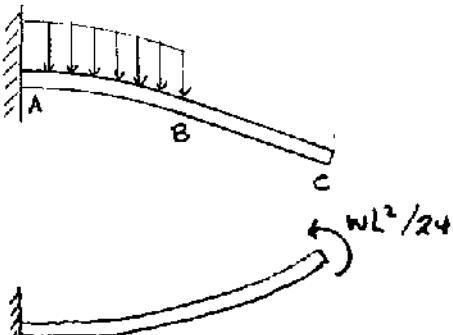
Loading I: Downward distributed load w applied to portion AB.

Case 2 of Appendix D applied to portion AB.

$$\theta_a' = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48} \frac{wL^3}{EI}$$

$$y_a' = -\frac{w(L/2)^4}{8EI} = -\frac{1}{128} \frac{wL^4}{EI}$$

Portion BC remains straight.



$$\theta_c' = \theta_a' = -\frac{1}{48} \frac{wL^3}{EI}$$

$$y_c' = y_a' + \left(\frac{L}{2}\right) \theta_a' = -\frac{1}{128} \frac{wL^4}{EI} - \frac{1}{96} \frac{wL^4}{EI} = -\frac{7}{384} \frac{wL^4}{EI}$$

Loading II: Counterclockwise couple $\frac{WL^2}{24}$ applied at C.

Case 3 of Appendix D

$$\theta_c'' = \frac{(WL^2/24)L}{EI} = \frac{1}{24} \frac{wL^3}{EI}$$

$$y_c'' = \frac{(WL^2/24)L^2}{2EI} = \frac{1}{48} \frac{wL^4}{EI}$$

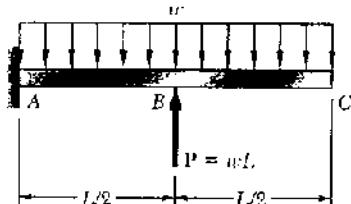
By superposition,

$$\theta_c = \theta_c' + \theta_c'' = -\frac{1}{48} \frac{wL^3}{EI} + \frac{1}{24} \frac{wL^3}{EI} = \frac{1}{48} \frac{wL^3}{EI}$$

$$y_c = y_c' + y_c'' = -\frac{7}{384} \frac{wL^4}{EI} + \frac{1}{48} \frac{wL^4}{EI} = \frac{1}{384} \frac{wL^4}{EI}$$

Problem 9.67

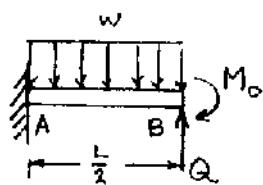
9.67 and 9.68 For the cantilever beam and loading shown, determine the slope and deflection at point B.



Consider portion AB as a cantilever beam subjected to the three loadings shown.

$$\text{By statics: } Q = P - \frac{wL}{2} = \frac{wL}{2}$$

$$M_0 = \left(\frac{wL}{2}\right)\left(\frac{L}{2} - \frac{L}{2}\right) = \frac{wL^2}{8}$$



$$\text{Slope at B. } \theta_B = (\theta_B)_w + (\theta_B)_Q + (\theta_B)_M$$

$$\text{Case 2 of App. D. } (\theta_B)_w = -\frac{w}{6EI} \left(\frac{L}{2}\right)^3 = -\frac{wL^3}{48EI}$$

$$\text{Case 1 of App. D. } (\theta_B)_Q = +\frac{(wL/2)}{2EI} \left(\frac{L}{2}\right)^2 = \frac{wL^3}{16EI}$$

$$\text{Case 3 of App. D. } (\theta_B)_M = -\frac{(wL^2/8)}{EI} \left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$$

$$\theta_B = -\frac{wL^3}{48EI}$$

$$\theta_B = \frac{wL^3}{48EI}$$

Deflection at B.

$$y_B = (y_B)_w + (y_B)_Q + (y_B)_M$$

$$\text{Case 2 of App. D. } (y_B)_w = -\frac{w}{8EI} \left(\frac{L}{2}\right)^4 = -\frac{wL^4}{128EI}$$

$$\text{Case 1 of App. D. } (y_B)_Q = +\frac{(wL/2)}{3EI} \left(\frac{L}{2}\right)^3 = \frac{wL^4}{48EI}$$

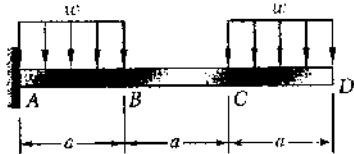
$$\text{Case 3 of App. D. } (y_B)_M = -\frac{(wL^2/8)}{2EI} \left(\frac{L}{2}\right)^2 = -\frac{wL^4}{64EI}$$

$$y_B = -\frac{wL^4}{384EI}$$

$$y_B = \frac{wL^4}{384EI}$$

Problem 9.68

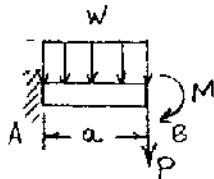
9.67 and 9.68 For the cantilever beam and loading shown, determine the slope and deflection at point B.



Consider portion AB as a cantilever beam subjected to the three loadings shown.

By statics: $P = wa$

$$M = (wa)(a + \frac{a}{2}) = \frac{3}{2}wa^2$$



$$\text{Slope at } B. \quad \theta_B = (\theta_B)_w + (\theta_B)_P + (\theta_B)_M$$

$$\text{Case 2 of App. D.} \quad (\theta_B)_w = -\frac{wa^3}{6EI}$$

$$\text{Case 1 of App. D.} \quad (\theta_B)_P = -\frac{(wa)a^2}{2EI} = -\frac{wa^3}{2EI}$$

$$\text{Case 3 of App. D.} \quad (\theta_B)_M = -\frac{(\frac{3}{2}wa^2)a}{EI} = -\frac{3wa^3}{2EI}$$

$$\theta_B = -\frac{13wa^3}{6EI}$$

$$\theta_B = \frac{13wa^3}{6EI}$$

Deflection at B.

$$y_B = (y_B)_w + (y_B)_P + (y_B)_M$$

$$\text{Case 2 of App. D.} \quad (y_B)_w = -\frac{wa^4}{8EI}$$

$$\text{Case 1 of App. D.} \quad (y_B)_P = -\frac{(wa)a^3}{3EI} = -\frac{wa^4}{3EI}$$

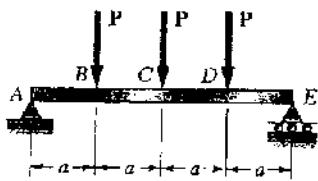
$$\text{Case 3 of App. D.} \quad (y_B)_M = -\frac{(\frac{3}{2}wa^2)a}{2EI} = -\frac{3wa^4}{4EI}$$

$$y_B = -\frac{29wa^4}{24EI}$$

$$y_B = \frac{29wa^4}{24EI}$$

Problem 9.69

9.69 through 9.72 For the beam and loading shown, determine (a) the deflection at the C, (b) the slope at end A.



Loading I: Load at B

Case 5 in Appendix D.

$$L = 4a, \quad a = a, \quad b = 3a, \quad x = 2a$$

For $x > a$, replace x by $L-x$ and interchange a and b in expression for elastic curve given.

$$y = \frac{Pa}{6EI} [(L-x)^3 - (L^2 - a^2)(L-x)]$$

$$y_c = \frac{Pa}{6EI(4a)} [(2a)^3 - (16a^2 - a^2)(2a)] = -\frac{11}{12} \cdot \frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EI L} = -\frac{P(3a)(16a^2 - 9a^2)}{6EI(4a)} = -\frac{7}{8} \cdot \frac{Pa^2}{EI}$$

Loading II: Load at C. Case 4 of Appendix D with $L = 4a$.

$$y_c = -\frac{PL^3}{48EI} = -\frac{P(4a)^3}{48EI} = -\frac{4}{3} \cdot \frac{Pa^3}{EI}$$

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{P(4a)^2}{16EI} = -\frac{Pa^2}{EI}$$

Loading III: Load at D. Case 5 of Appendix D.

$$L = 4a, \quad a = 3a, \quad b = a, \quad x = 2a \text{ at point C.}$$

$$y_c = \frac{Pb}{6EI} [x^3 - (L^2 - b^2)x] = \frac{Pa}{6EI(4a)} [(2a)^3 - (16a^2 - a^2)(2a)] \\ = -\frac{11}{12} \cdot \frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EI L} = -\frac{Pa(16a^2 - a^2)}{6EI(4a)} = -\frac{5}{8} \cdot \frac{Pa^3}{EI}$$

(a) Deflection at C. $y_c = -\frac{11}{12} \cdot \frac{Pa^3}{EI} - \frac{4}{3} \cdot \frac{Pa^3}{EI} - \frac{11}{12} \cdot \frac{Pa^3}{EI} = -\frac{19}{6} \cdot \frac{Pa^3}{EI}$

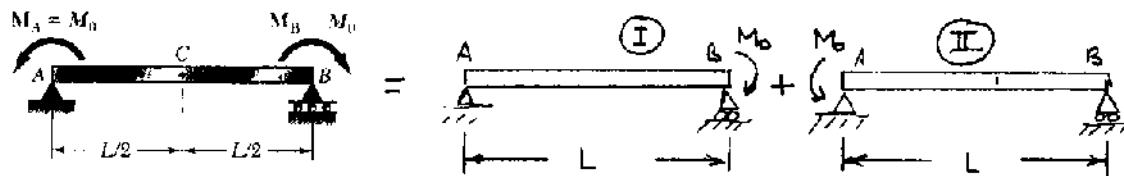
$$y_c = \frac{19}{6} \cdot \frac{Pa^3}{EI} \downarrow$$

(b) Slope at A. $\theta_A = -\frac{7}{8} \cdot \frac{Pa^2}{EI} - \frac{Pa^2}{EI} - \frac{5}{8} \cdot \frac{Pa^2}{EI} = -\frac{5}{2} \cdot \frac{Pa^2}{EI}$

$$\theta_A = \frac{5}{2} \cdot \frac{Pa^2}{EI} \checkmark$$

Problem 9.70

9.69 through 9.72 For the beam and loading shown, determine (a) the deflection at the C, (b) the slope at end A.



Loading I: Case 7 of Appendix D. $y = -\frac{M_0}{6EI} (x^3 - L^2 x)$

$$\text{At } C, x = \frac{L}{2} \quad (y_C)_1 = -\frac{M_0}{6EI} \left[\left(\frac{L}{2}\right)^3 - L^2 \left(\frac{L}{2}\right) \right] = \frac{M_0 L^2}{16EI}$$

$$(\theta_A)_1 = \frac{M_0 L}{6EI} \quad (\theta_B)_1 = -\frac{M_0 L}{3EI}$$

Loading II: Mirror image of Loading I.

$$(y_C)_2 = (y_C)_1 \quad (\theta_A)_2 = -(\theta_A)_1$$

(a) Deflection at C.

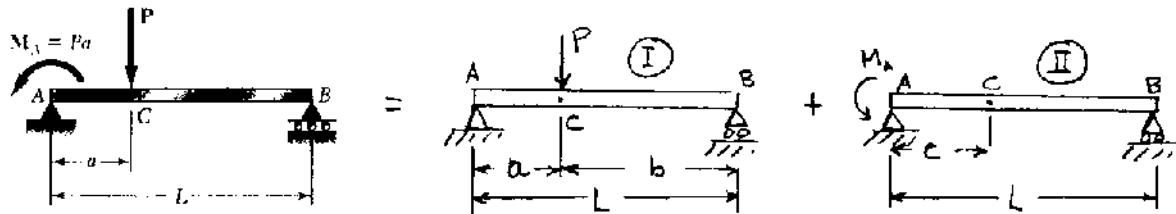
$$\text{By superposition, } y_C = (y_C)_1 + (y_C)_2 = \frac{M_0 L^2}{8EI} \quad y_C = \frac{M_0 L^2}{8EI} \uparrow$$

(b) Slope at A.

$$\text{By superposition, } \theta_A = (\theta_A)_1 + (\theta_A)_2 = -\frac{M_0 L}{2EI} \quad \theta_A = -\frac{M_0 L}{2EI} \leftarrow$$

Problem 9.71

9.69 through 9.72 For the beam and loading shown, determine (a) the deflection at the C, (b) the slope at end A.



Loading I: Case 5 of Appendix D. $b = L-a$

$$\text{For } x=a: \quad (y_c)_1 = -\frac{Pa^2 b^3}{3EI L} = -\frac{Pa^2 (L-a)^2}{3EI L}$$

$$\begin{aligned} \text{For end A: } (\theta_A)_1 &= -\frac{Pb(L^2-b^2)}{6EI L} = -\frac{P(L-a)(2La-a^2)}{6EI L} \\ &= -\frac{Pa(2L^2-3La+a^2)}{6EI L} \end{aligned}$$

Loading II: Mirror image of Case 5 of Appendix D.

Replace x by $(L-x)$ in expression for y .

$$y = -\frac{Ma}{6EI L} [(L-x)^3 - L^2(L-x)] = -\frac{Ma x}{6EI L} (3Lx - 2L^2 - x^2)$$

$$\text{With } x=a, \text{ and } M_a = Pa, \quad (y_c)_2 = -\frac{Pa^2}{6EI L} (3La - 2L^2 - a^2)$$

$$(\theta_A)_2 = \frac{M_a L}{3EI} = \frac{Pa L}{3EI}$$

(a) Deflection at C. $y_c = (y_c)_1 + (y_c)_2$

$$y_c = -\frac{Pa^2}{6EI L} [2(L-a)^2 + 3La - 2L^2 - a^2] = -\frac{Pa^2 (L-a)}{6EI L} \uparrow$$

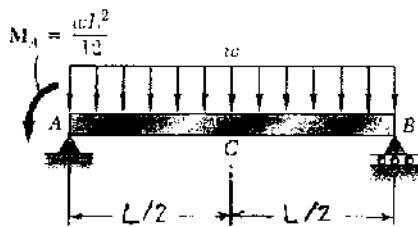
(b) Slope at end A. $\theta_A = (\theta_A)_1 + (\theta_A)_2$

$$\theta_A = -\frac{Pa}{6EI L} [2L^2 - 3La + a^2 - 2L^2] = +\frac{Pa^2 (3L-a)}{6EI L}$$

$$\theta_A = \frac{Pa^2 (3L-a)}{6EI L} \leftarrow \uparrow$$

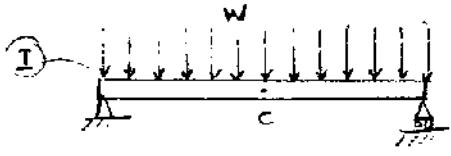
Problem 9.72

9.69 through 9.72 For the beam and loading shown, determine (a) the deflection at the C, (b) the slope at end A.



Loading I: Case G in Appendix D.

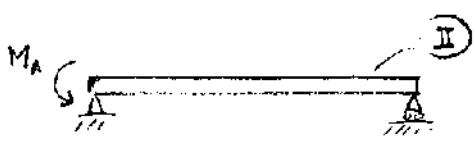
$$y_C = -\frac{5}{384} \frac{wL^4}{EI}; \quad \theta_A = -\frac{1}{24} \frac{wL^3}{EI}$$



Loading II: Case 7 of Appendix D.

Note that center deflection is

$$\begin{aligned} y_C &= -\frac{M_A}{6EI_L} \left[\left(\frac{L}{2}\right)^3 - L^2 \left(\frac{L}{2}\right) \right] \\ &= \frac{1}{16} \frac{M_A L}{EI} \end{aligned}$$



$$\theta_A = \frac{M_A L}{3EI}$$

$$\text{with } M_A = \frac{wL^2}{12}, \quad y_C = \frac{1}{192} \frac{wL^4}{EI}, \quad \theta_A = \frac{1}{36} \frac{wL^3}{EI}$$

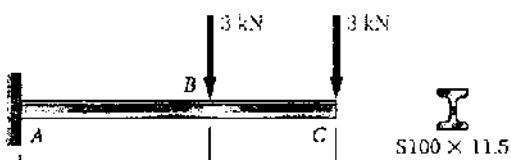
$$\begin{aligned} (\text{a}) \text{ Deflection at C.} \quad y_C &= -\frac{5}{384} \frac{wL^4}{EI} + \frac{1}{192} \frac{wL^4}{EI} = -\frac{1}{128} \frac{wL^4}{EI} \\ &\quad y_C = \frac{1}{128} \frac{wL^4}{EI} \downarrow \end{aligned}$$

$$\begin{aligned} (\text{b}) \text{ Slope at A.} \quad \theta_A &= -\frac{1}{24} \frac{wL^3}{EI} + \frac{1}{36} \frac{wL^3}{EI} = -\frac{1}{72} \frac{wL^3}{EI} \end{aligned}$$

$$\theta_A = \frac{1}{72} \frac{wL^3}{EI} \quad \checkmark$$

Problem 9.73

9.73 For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m.

Loading I: Concentrated load at B

Case I of Appendix D applied to portion AB.

$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight

$$\theta_C' = \theta_B' = -\frac{0.84375}{EI}$$

$$y_C' = y_B' - (0.5)\theta_B' = -\frac{0.84375}{EI}$$

Loading II: Concentrated load at C. Case I of Appendix D.

$$\theta_A'' = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y_A'' = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition, $\theta_A = \theta_A' + \theta_A'' = -\frac{3.1875}{EI}$

$$y_A = y_A' + y_A'' = -\frac{2.796875}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

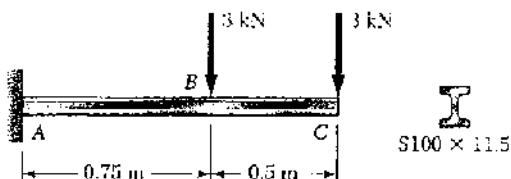
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N}\cdot\text{m}^2 = 506 \text{ kN}\cdot\text{m}^2$$

Slope at C. $\theta_C = -\frac{3.1875}{506} = -6.30 \times 10^{-3} \text{ rad} = 6.30 \times 10^{-3} \text{ rad}$ \rightarrow

Deflection at C. $y_C = -\frac{2.796875}{506} = -5.53 \times 10^{-3} \text{ m} = 5.53 \text{ mm}$ \downarrow \rightarrow

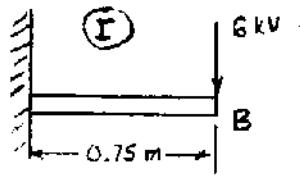
Problem 9.74

9.74 For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200 \text{ GPa}$.

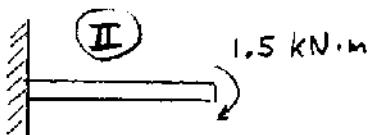


Units: Forces in kN, lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.



Reducing the force at C to an equivalent force-couple system at B and adding the force already at B gives the loadings I and II shown.



Loading I: Case 1 of Appendix D

$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D

$$\theta_B'' = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y_B'' = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{EI} = -\frac{0.421875}{EI}$$

By superposition,

$$\theta_B = \theta_B' + \theta_B'' = -\frac{2.8125}{EI}$$

$$y_B = y_B' + y_B'' = -\frac{1.265625}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

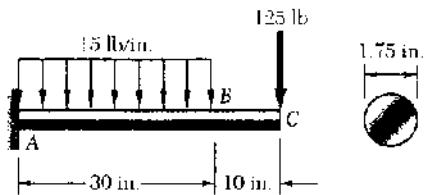
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N}\cdot\text{m}^2 = 506 \text{ kN}\cdot\text{m}^2$$

Slope at B. $\theta_B = -\frac{2.8125}{506} = -5.56 \times 10^{-3} \text{ rad} = 5.56 \times 10^{-3} \text{ rad}$

Deflection at B. $y_B = -\frac{1.265625}{506} = -2.50 \times 10^{-3} \text{ m} = 2.50 \text{ mm}$

Problem 9.75

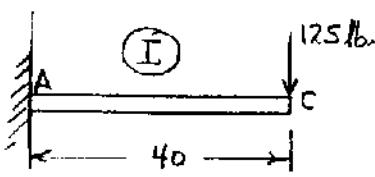
9.75 For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 29 \times 10^6$ psi.



$$c = \frac{1}{2}d = \frac{1}{2}(1.75) = 0.875 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.875)^4 = 0.460386 \text{ in}^4$$

$$EI = (29 \times 10^6)(0.460386) = 13.351 \times 10^6 \text{ lb-in}^2$$

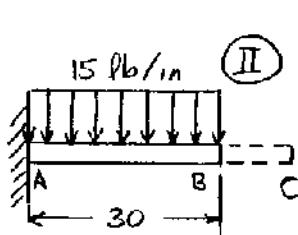


Loading I. Case 1 of Appendix D.

$$P = 125 \text{ lb}, L = 40 \text{ in.}$$

$$(y_c)_1 = -\frac{PL^3}{3EI} = -\frac{(125)(40)^3}{(3)(13.351 \times 10^6)} = -0.199735 \text{ in.}$$

$$(\theta_c)_1 = -\frac{PL^2}{2EI} = -\frac{(125)(40)^2}{(2)(13.351 \times 10^6)} = -7.4901 \times 10^{-3}$$



Loading II. Treat portion AB as a cantilever beam. (Case 2)

$$w = 15 \text{ lb/in}, L = 30 \text{ in.}$$

$$(y_B)_2 = -\frac{WL^4}{8EI} = -\frac{(15)(30)^4}{(8)(13.351 \times 10^6)} = -0.113755 \text{ in.}$$

$$(\theta_B)_2 = -\frac{WL^3}{6EI} = -\frac{(15)(30)^3}{(6)(13.351 \times 10^6)} = -5.0558 \times 10^{-3}$$

Portion BC remains straight for loading II.

$$L_{BC} = 10 \text{ in.}$$

$$(y_c)_2 = (y_B)_2 + L_{BC}(\theta_B)_2 = -0.164313 \text{ in.}$$

$$(\theta_c)_2 = (\theta_B)_2 = -5.0558 \times 10^{-3}$$

Slope at end C. By superposition, $\theta_c = (\theta_c)_1 + (\theta_c)_2$

$$\theta_c = -12.5459 \times 10^{-3}$$

$$\theta_c = 12.55 \times 10^{-3} \text{ rad. } \leftarrow$$

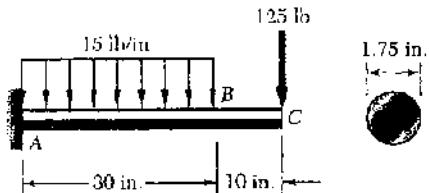
Deflection at end C. By superposition, $y_c = (y_c)_1 + (y_c)_2$

$$y_c = -0.364048 \text{ in.}$$

$$y_c = 0.364 \text{ in. } \downarrow \leftarrow$$

Problem 9.76

9.76 For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 29 \times 10^6$ psi.



$$c = \frac{1}{2}d = \frac{1}{2}(1.75) = 0.875 \text{ in.}$$

$$I = \frac{\pi c^4}{4} = \frac{\pi}{4}(0.875)^4 = 0.460386 \text{ in}^4$$

$$EI = (29 \times 10^6)(0.460386) = 13.351 \times 10^6 \text{ lb-in}^2$$

Loading I: Case 1 of Appendix D.

$$P = 125 \text{ lb. } L = 40 \text{ in. } x = 30 \text{ in.}$$

$$y_1 = \frac{P}{6EI} (x^3 - 3Lx^2)$$

$$\theta_1 = \frac{dy}{dx} = \frac{P}{2EI} (x^2 - 2Lx)$$

$$(y_B)_1 = \frac{125}{(6)(13.351 \times 10^6)} [(30)^3 - (3)(40)(30)^2] \\ = -0.126345 \text{ in.}$$

$$(\theta_B)_1 = \frac{125}{(2)(13.351 \times 10^6)} [(30)^2 - (2)(40)(30)] \\ = -7.0219 \times 10^{-3}$$

Loading II. Case 2 of Appendix D.

$$w = 15 \text{ lb/in, } L = 30 \text{ in}$$

$$(y_B)_2 = -\frac{WL^4}{8EI} = -\frac{(15)(30)^4}{(8)(13.351 \times 10^6)} = -0.113755 \text{ in.}$$

$$(\theta_B)_2 = -\frac{WL^3}{6EI} = -\frac{(15)(30)^3}{(6)(13.351 \times 10^6)} = -5.0558 \times 10^{-3}$$

$$\text{Slope at point B. } \theta_B = (\theta_B)_1 + (\theta_B)_2$$

$$\theta_B = -12.0777 \times 10^{-3}$$

$$\theta_B = 12.08 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

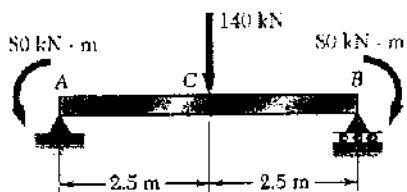
$$\text{Deflection at point B. } y_B = (y_B)_1 + (y_B)_2$$

$$y_B = -0.24015 \text{ in.}$$

$$y_B = 0.240 \text{ in.} \downarrow \quad \blacktriangleleft$$

Problem 9.77

9.77 and 9.78 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point C. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m.

Loading I: Moment at B.

Case 7 of Appendix D. $M = 80 \text{ kN}\cdot\text{m}$, $L = 5.0 \text{ m}$, $x = 2.5 \text{ m}$

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EI} (x^3 - L^2 x) = -\frac{80}{6EI(5.0)} [2.5^3 - (5.0)^2(2.5)] = \frac{125}{EI}$$

Loading II: Moment at A. Case 7 of Appendix D.

$M = 80 \text{ kN}\cdot\text{m}$, $L = 5.0 \text{ m}$, $x = 2.5 \text{ m}$

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \quad (\text{Same as loading I})$$

Loading III: 140 kN concentrated load at C. $P = 140 \text{ kN}$

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 156 \times 10^6 \text{ mm}^4 = 156 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(156 \times 10^{-6}) = 31.2 \times 10^6 \text{ N}\cdot\text{m}^2 = 31200 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A. $\theta_A = \frac{-66.667 + 133.333 - 218.75}{31200} = -0.601 \times 10^{-3} \text{ rad}$

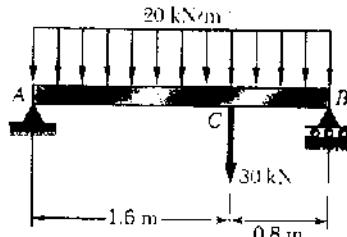
$$\theta_A = 0.601 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

(b) Deflection at C. $y_C = \frac{125 + 125 - 364.583}{31200} = -3.67 \times 10^{-3} \text{ m}$

$$y_C = 3.67 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.78

9.77 and 9.78 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point C. Use $E = 200 \text{ GPa}$.

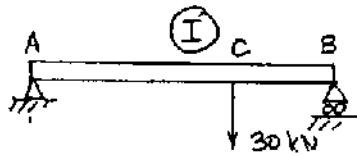


W150 x 24

Units: Forces in kN. Lengths in meters.

$$\text{For W150 x 24} \quad I = 13.4 \times 10^4 \text{ mm}^4 \\ = 13.4 \times 10^{-6} \text{ m}^4$$

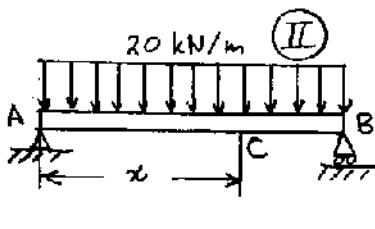
$$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^6 \text{ N} \cdot \text{m} \\ = 2680 \text{ kN} \cdot \text{m}^2$$



Loading I: Case 5 of Appendix D.

$$P = 30 \text{ kN}, L = 2.4 \text{ m}, a = 1.6 \text{ m}, b = 0.8 \text{ m}$$

$$(\theta_A)_1 = -\frac{Pb(l^2 - b^2)}{6EI L} = -\frac{(30)(0.8)(2.4^2 - 0.8^2)}{(6)(2680)(2.4)} \\ = -3.1841 \times 10^{-3}$$



$$(y_C)_1 = -\frac{Pa^2 b^2}{3EI L} = -\frac{(30)(1.6)^2(0.8)^2}{(3)(2680)(2.4)} \\ = -2.54726 \times 10^{-3} \text{ m}$$

Loading II: Case 6 of Appendix D.

$$w = 20 \text{ kN/m}, L = 2.4 \text{ m}, x = 1.6 \text{ m at point C.}$$

$$(\theta_A)_2 = -\frac{WL^3}{24EI} = -\frac{(20)(2.4)^3}{(24)(2680)} = -4.2985 \times 10^{-3}$$

$$y_2 = -\frac{w}{24EI} (x^4 - 2Lx^3 + L^3 x)$$

$$(y_C)_2 = -\frac{20}{(24)(2680)} [(1.6)^4 - (2)(2.4)(1.6)^3 + (2.4)^3(1.6)] = -2.80199 \times 10^{-3} \text{ m}$$

$$(a) \text{ Slope at end A.} \quad \theta_A = (\theta_A)_1 + (\theta_A)_2$$

$$\theta_A = -7.4826 \times 10^{-3}$$

$$\theta_A = 7.48 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

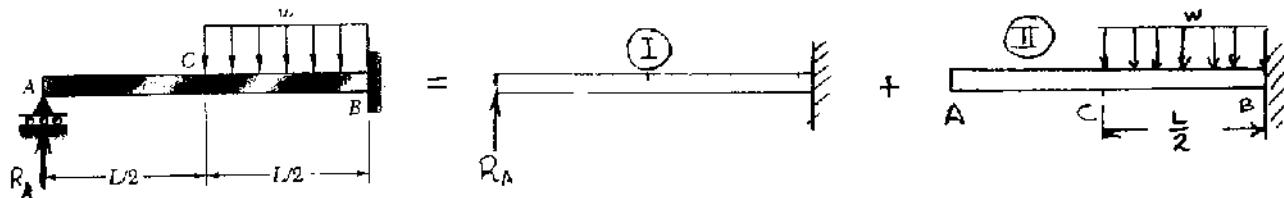
$$(b) \text{ Deflection at point C.} \quad y_C = (y_C)_1 + (y_C)_2$$

$$y_C = -5.34925 \times 10^{-3} \text{ m}$$

$$y_C = 5.35 \text{ mm} \quad \blacktriangleleft$$

Problem 9.79

9.79 and 9.80 For the uniform beam shown, determine (a) the reaction at A, (b) the reaction at B.



Consider R_A as redundant and replace loading system by I and II.

Loading I. (Case 1 of Appendix D) $(y_A)_I = \frac{R_A L^3}{3EI}$

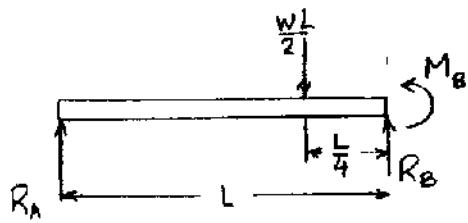
Loading II. Portion CB. (Case 2 of Appendix D)

$$(y_B)_2 = -\frac{w(L/2)^4}{8EI} = -\frac{wL^4}{128EI}, \quad (\theta_B)_2 = \frac{w(L/2)^3}{6EI} = \frac{wL^3}{48EI}$$

Portion AC is straight. $(y_A)_2 = (y_B)_2 - \frac{L}{2}(\theta_B)_2 = -\frac{7wL^4}{384EI}$

Superposition and constraint. $y_A = (y_A)_I + (y_A)_2 = 0$

(a) $\frac{R_A L^3}{3EI} - \frac{7wL^4}{384EI} = 0 \quad R_A = \frac{7wL}{128} \uparrow$



Using entire beam as a free body

$$+\uparrow \sum F_y = 0: R_A + R_B - \frac{wL}{2} = 0$$

$$R_B = \frac{wL}{2} - \frac{7wL}{128} \quad R_B = \frac{57wL}{128} \uparrow$$

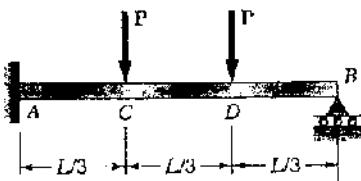
$$+\rightarrow \sum M_B = 0: M_B - R_A L - \frac{wL}{2} \cdot \frac{L}{4} = 0$$

$$M_B = R_A L - \frac{wL^2}{8} = -\frac{9wL^2}{128}$$

$$M_B = \frac{9wL^2}{128} \uparrow$$

Problem 9.80

9.79 and 9.80 For the uniform beam shown, determine (a) the reaction at A, (b) the reaction at B.



Consider R_B as redundant and replace loading system by I, II, and III.

Loading I. Case I of Appendix D applied to AB.

$$(y_s)_I = \frac{R_B L^3}{3EI}$$

Loading II. Case I applied to portion AC.

$$(\theta_c)_{II} = -\frac{P(L/3)^2}{2EI} = -\frac{1}{18} \frac{PL^2}{EI}$$

$$(y_c)_{II} = -\frac{P(L/3)^3}{3EI} = -\frac{1}{81} \frac{PL^3}{EI}$$

Portion CB remains straight.

$$(y_s)_{II} = (y_c)_{II} + \frac{2L}{3} (\theta_c)_{II} = -\frac{4}{81} \frac{PL^3}{EI}$$

Loading III Case I applied to portion AD.

$$(\theta_c)_{III} = \frac{P(2L/3)^2}{2EI} = -\frac{2}{9} \frac{PL^2}{EI}$$

$$(y_c)_{III} = \frac{P(2L/3)^3}{3EI} = -\frac{8}{81} \frac{PL^3}{EI}$$

Portion DB remains straight.

$$(y_s)_{III} = (y_c)_{III} + \frac{L}{3} (\theta_c)_{III} = -\frac{14}{81} \frac{PL^3}{EI}$$

Superposition and constraint: $y_B = (y_s)_I + (y_s)_{II} + (y_s)_{III} = 0$

$$\frac{1}{3} R_B L^3 - \frac{4}{81} \frac{PL^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = \frac{1}{3} \frac{R_B L^3}{EI} - \frac{2}{9} \frac{PL^3}{EI} = 0 \quad (b) \quad R_B = \frac{2}{3} P \uparrow$$

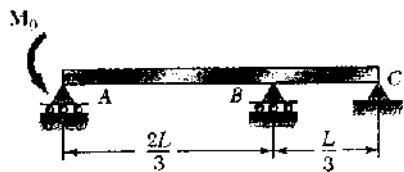
Statics:

$$+\uparrow \sum F_y = 0: \quad R_A - P - P + \frac{2}{3} P = 0 \quad (a) \quad R_A = \frac{4}{3} P \uparrow$$

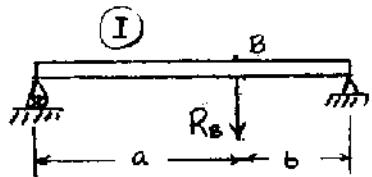
$$0 \sum M_A = 0: \quad M_A - P\left(\frac{L}{3}\right) - P\left(\frac{2L}{3}\right) + \left(\frac{2}{3}P\right)(L) = 0 \quad M_A = \frac{4}{3} PL \curvearrowright$$

Problem 9.81

9.81 and 9.82 For the uniform beam shown, determine the reaction at each of the three supports.



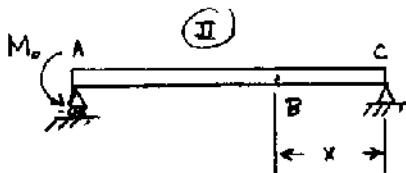
Beam is statically indeterminate to first degree. Consider R_B to be the redundant reaction, and replace the loading by loadings I and II.



Loading I: Case 5 of Appendix D.

$$(y_B)_I = -\frac{R_B a^2 b^2}{3EI L} = -\frac{R_B (2L/3)^2 (L/3)^2}{3EI L} = -\frac{4}{243} \frac{R_B L^3}{EI}$$

Loading II: Case 7 of Appendix D.

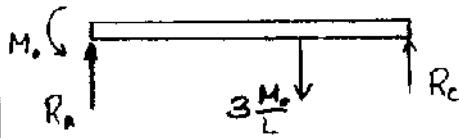


$$(y_B)_{II} = -\frac{M_o}{6EI L} (x^3 - L^2 x) = -\frac{M_o}{6EI L} \left[\left(\frac{L}{3}\right)^3 - L^2 \left(\frac{L}{3}\right) \right] \\ = \frac{4}{81} \frac{M_o L^2}{EI}$$

Superposition and constraint.

$$y_B = (y_B)_I + (y_B)_{II} = 0$$

$$-\frac{4}{243} \frac{R_B L^3}{EI} + \frac{4}{81} \frac{M_o L^2}{EI} = 0 \quad R_B = 3 \frac{M_o}{L} + 1$$



Statics:

$$\textcircled{D} \sum M_c = 0:$$

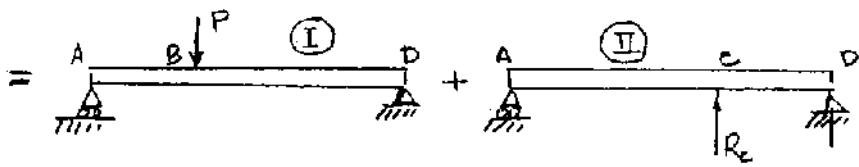
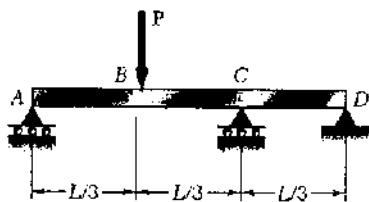
$$-R_A L + M_o + 3 \frac{M_o}{L} \cdot \frac{L}{3} = 0 \quad R_A = 2 \frac{M_o}{L} \uparrow$$

$$+\uparrow \sum F_y = 0:$$

$$2 \frac{M_o}{L} - 3 \frac{M_o}{L} + R_C = 0 \quad R_C = \frac{M_o}{L} \uparrow$$

Problem 9.82

9.81 and 9.82 For the uniform beam shown, determine the reaction at each of the three supports.



Consider R_c as redundant and replace loading system by I and II.

Loading I. (Case 5 of Appendix D) $a = \frac{2L}{3}$, $b = \frac{L}{3}$, $x = \frac{L}{3}$ at C.

$$(y_e)_1 = \frac{Pb}{6EI} [x^3 - (L^2 - b^2)x] = \frac{P(L/3)}{6EI} \left[\left(\frac{L}{3}\right)^3 + \left\{ L^2 - \left(\frac{L}{3}\right)^2 \right\} \frac{L}{3} \right]$$

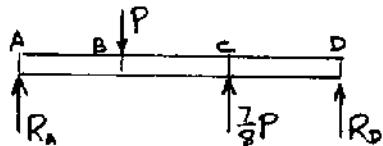
$$= -\frac{7PL^3}{486EI}$$

Loading II. (Case 5 of Appendix D) $a = \frac{2L}{3}$, $b = \frac{L}{3}$

$$(y_e)_2 = \frac{R_c x^2 b^2}{3EI} = \frac{R_c (L/3)^2 (2L/3)^2}{3EI} = -\frac{4R_c L^3}{243EI}$$

Superposition and constraint. $y_e = (y_e)_1 + (y_e)_2 = 0$

$$-\frac{7PL^3}{486EI} + \frac{4R_c}{243EI} = 0 \quad R_c = \frac{7}{8}P \uparrow$$



$$+\uparrow \sum M_D = 0:$$

$$-R_A L + P(\frac{3L}{3}) - (\frac{7}{8}P)(\frac{L}{3}) = 0$$

$$R_A = \frac{3}{8}P \uparrow$$

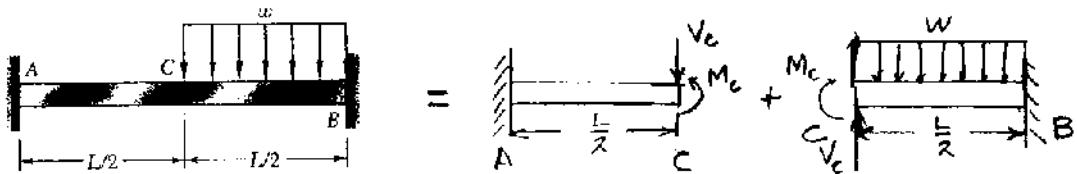
$$+\uparrow \sum F_y = 0: \quad R_A + R_D - P + \frac{7}{8}P = 0$$

$$R_D = P - \frac{7}{8}P - \frac{3}{8}P = -\frac{1}{4}P$$

$$R_D = \frac{1}{4}P \downarrow$$

Problem 9.83

9.83 and 9.84 For the beam shown, determine the reaction at B.



Portion AC: Superposition of Cases 3 and 1 of Appendix D.

$$y_c = \frac{M_c(L/2)^2}{2EI} - \frac{V_c(L/2)^3}{3EI} = \frac{M_c L^2}{8EI} - \frac{V_c L^3}{24EI}$$

$$\theta_c = \frac{M_c(L/2)}{EI} - \frac{V_c(L/2)^2}{2EI} = \frac{M_c L}{2EI} - \frac{V_c L^2}{8EI}$$

Portion CB: Superposition of Cases 3, 1, and 2 of Appendix D.

$$y_c = \frac{M_c(L/2)^2}{2EI} + \frac{V_c(L/2)^3}{3EI} - \frac{w(L/2)^4}{8EI}$$

$$= \frac{M_c L^2}{8EI} + \frac{V_c L^3}{24EI} - \frac{wL^4}{128EI}$$

$$\theta_c = -\frac{M_c(L/2)}{EI} - \frac{V_c(L/2)^3}{2EI} + \frac{w(L/2)^3}{6EI}$$

$$= -\frac{M_c L}{2EI} - \frac{V_c L^3}{8EI} + \frac{wL^3}{48EI}$$

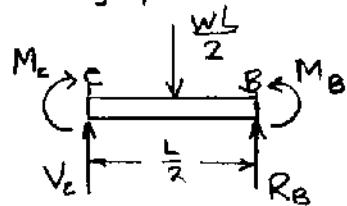
Matching expressions for y_c ,

$$\frac{M_c L^2}{8EI} - \frac{V_c L^3}{24EI} = \frac{M_c L^2}{8EI} + \frac{V_c L^3}{24EI} - \frac{wL^4}{128EI} \quad V_c = \frac{3}{32} wL$$

Matching expressions for θ_c ,

$$\frac{M_c L}{2EI} - \frac{V_c L^3}{8EI} = -\frac{M_c L}{2EI} - \frac{V_c L^3}{8EI} + \frac{wL^3}{48EI} \quad M_c = \frac{1}{48} wL^2$$

Using portion CB as a free body,



$$+\uparrow \sum F_y = 0: R_B + V_c - \frac{wL}{2} = 0$$

$$R_B = \frac{wL}{2} - \frac{3}{32} wL = \frac{13}{32} wL \uparrow$$

$$+\sum M_B = 0: M_B - M_c - V_c \frac{L}{2} + \frac{wL}{2} \cdot \frac{L}{4} = 0$$

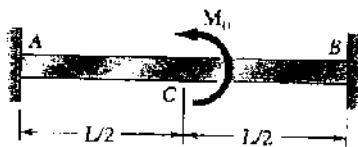
$$M_B = \frac{1}{48} wL^2 + \left(\frac{3}{32} wL\right)\left(\frac{L}{2}\right) - \frac{wL^2}{8}$$

$$M_B = -\frac{11}{192} wL^2$$

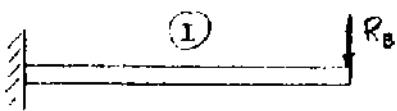
$$M_B = \frac{11}{192} wL^2$$

Problem 9.84

9.83 and 9.84 For the beam shown, determine the reaction at B.



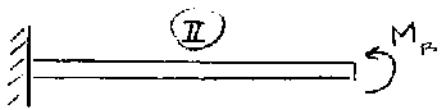
Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.



Loading I. Case 1 of Appendix D.

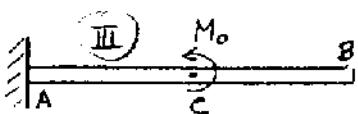
$$(y_B)_I = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_I = -\frac{R_B L^2}{2EI}$$

Loading II. Case 3 of Appendix D.



$$(y_B)_{II} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{II} = \frac{M_B L}{EI}$$

Loading III. Case 3 applied to portion AC.



$$(y_c)_{III} = \frac{M_o (L/2)^2}{2EI} = \frac{M_o L^2}{8EI}$$

$$(\theta_c)_{III} = \frac{M_o (L/2)}{EI} = \frac{M_o L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{III} = (y_c)_{III} + \frac{L}{2}(\theta_c)_{III} = \frac{3}{8} \frac{M_o L^2}{EI}$$

$$(\theta_B)_{III} = (\theta_c)_{III} = \frac{1}{2} \frac{M_o L}{EI}$$

Superposition and constraint:

$$\begin{aligned} y_B &= (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0 \\ -\frac{L^3}{3EI} R_B + \frac{L^2}{2EI} M_B + \frac{3}{8} \frac{M_o L^2}{EI} &= 0 \end{aligned} \quad (1)$$

$$\begin{aligned} \theta_B &= (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0 \\ -\frac{L^2}{2EI} R_B + \frac{L}{EI} M_B + \frac{1}{2} \frac{M_o L}{EI} &= 0 \end{aligned} \quad (2)$$

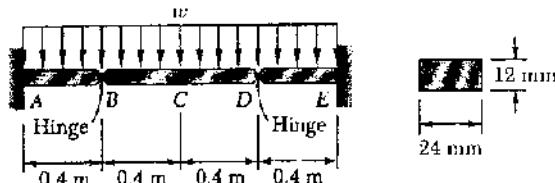
Solving (1) and (2) simultaneously,

$$R_B = \frac{3}{2} \frac{M_o}{L} \downarrow$$

$$M_B = \frac{1}{4} M_o \curvearrowright$$

Problem 9.85

9.85 A central beam BD is joined at hinges to two cantilever beams AB and DE . All beams have the cross section shown. For the loading shown, determine the largest w so that the deflection at C does not exceed 3 mm. Use $E = 200 \text{ GPa}$.



$$\text{Let } a = 0.4 \text{ m}$$

Cantilever beams AB and CD.

Cases 1 and 2 of Appendix D, $y_c = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = \frac{11}{24} \frac{wa^4}{EI}$

Beam BCD, with $L = 0.8 \text{ m}$, assuming that points B and D do not move.

Case 6 of Appendix D.

$$y_c' = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points B and D.

$$y_c'' = y_B = y_D = -\frac{11}{24} \frac{wa^4}{EI}$$

Total deflection at C. $y_c = y_c' + y_c''$

$$y_c = -\frac{w}{EI} \left\{ \frac{5L^4}{384} + \frac{11a^4}{24} \right\}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = \frac{1}{12}(24)(12)^3 = 3.456 \times 10^{-3} \text{ mm}^4 = 3.456 \times 10^{-9} \text{ m}^4$

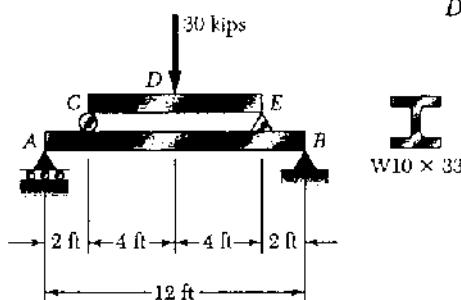
$$EI = (200 \times 10^9)(3.456 \times 10^{-9}) = 691.2 \text{ N}\cdot\text{m}^2$$

$$y_c = -3 \times 10^{-3} \text{ m}$$

$$-3 \times 10^{-3} = -\frac{w}{691.2} \left\{ \frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24} \right\} = -24.69 \times 10^{-6} w$$

$$w = 121.5 \text{ N/m}$$

Problem 9.86

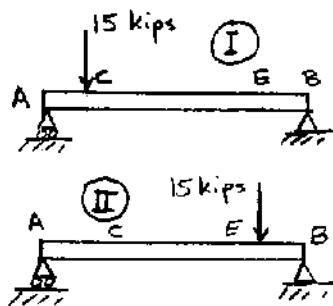


9.86 Beam CE rests on beam AB , as shown. Knowing that a $W10 \times 33$ rolled-steel shape is used for each beam, determine for the loading shown the deflection at point D . Use $E = 29 \times 10^6$ psi.

$$\text{For } W10 \times 33, I = 170 \text{ in}^4$$

$$EI = (29 \times 10^6)(170) = 4.93 \times 10^9 \text{ lb.in}^2 \\ = 34236 \text{ kip.ft}^2$$

Beam AB : 15 kip downward loads at C and E . Refer to Case 5 of Appendix D.



$$\text{Loading I. } (y_c)_1 = -\frac{Pa^2b}{3EI L}$$

$$\text{with } a = 2 \text{ ft}, b = 10 \text{ ft}, L = 12 \text{ ft}$$

$$(y_c)_1 = -\frac{(15)(2)^2(10)^2}{(3)(34236)(12)} = -4.8682 \times 10^{-3} \text{ ft}$$

$$\text{Loading II. } (y_c)_2 = \frac{Pb[x^3 - (L^2 - b^2)x]}{6EI L}$$

$$\text{with } b = 2 \text{ ft}, x = 2 \text{ ft}, L = 12 \text{ ft}$$

$$(y_c)_2 = \frac{(15)(2)[2^3 - (12^2 - 2^2)(2)]}{(6)(34236)(12)}$$

$$= -3.3104 \times 10^{-3} \text{ ft}$$

$$y_c = (y_c)_1 + (y_c)_2 = -8.1786 \times 10^{-3} \text{ ft}$$

$$\text{By symmetry } y_E = y_c$$

Beam CDE . 30 kip downward load at D .

Refer to Case 4 of Appendix D.

$$y_{D/c} = -\frac{PL^3}{48EI}$$

$$\text{with } P = 30 \text{ kips and } L = 8 \text{ ft.}$$

$$y_{D/c} = -\frac{(30)(8)^3}{(48)(34236)} = -9.3469 \times 10^{-3} \text{ ft}$$

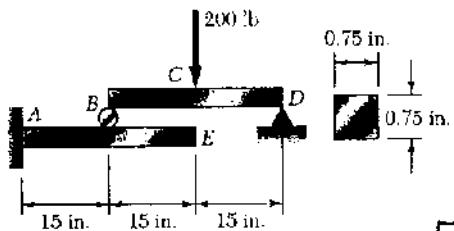
Total deflection at D .

$$y_D = y_c + y_{D/c} = 17.5255 \times 10^{-3} \text{ ft}$$

$$= 0.210 \text{ in. } \downarrow$$

Problem 9.87

9.87 Beam BD rests on the cantilever beam AE , as shown. Knowing that a square rod of side 0.75 in. is used for each beam, determine for the loading shown (a) the deflection at point C , (b) the deflection at point E . Use $E = 29 \times 10^6$ psi.

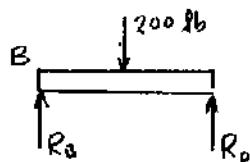


$$I = \frac{1}{12}(0.75)(0.75)^3 = 26.367 \times 10^{-3} \text{ in}^4$$

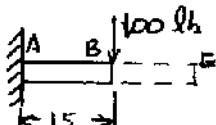
$$EI = (29 \times 10^6)(26.367 \times 10^{-3}) = 764.65 \times 10^3 \text{ lb-in}^2$$

For free body BCD , $\sum M_B = 0$:

$$-30 R_B + (15)(200) = 0 \quad R_B = 100 \text{ lb.}$$



Cantilever ABE . Portion AB.



Case 1 of Appendix D.

$$y_B = -\frac{PL^3}{3EI} = -\frac{(100)(15)^3}{(3)(764.65 \times 10^3)} = -0.14713 \text{ in.}$$

$$\theta_B = -\frac{PL^2}{2EI} = -\frac{(100)(15)^2}{(2)(764.65 \times 10^3)} = -14.713 \times 10^{-3}$$

Deflection at point C if point B does not move.

Apply Case 4 of Appendix D to beam BCD .

$$(y_C)_1 = -\frac{PL^3}{48EI} = -\frac{(200)(30)^3}{(48)(764.65 \times 10^3)} = -0.14713 \text{ in.}$$

Addition deflection at point C due to movement of point B .

$$(y_C)_2 = \frac{15}{30} y_B = -0.07356 \text{ in.}$$

$$(a) \text{ Total deflection at point } C. \quad y_C = (y_C)_1 + (y_C)_2 = -0.22069 \text{ in.}$$

$$y_C = 0.221 \text{ in.} \quad \blacktriangleleft$$

$$(b) \text{ Deflection at point } E. \quad y_E = y_B + L_{BE}\theta_B$$

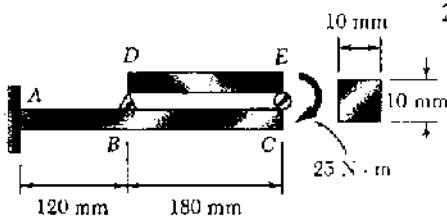
$$= -0.14713 + (15)(-14.713 \times 10^{-3})$$

$$= -0.3678 \text{ in.}$$

$$y_E = 0.368 \text{ in.} \quad \blacktriangleleft$$

Problem 9.88

9.88 Beam DE rests on the cantilever beam AC as shown. Knowing that a square rod of side 10 mm is used for each beam, determine the deflection at end C if the 25-N·m couple is applied (a) to end E of beam DE, (b) to end C of beam AC. Use $E = 200 \text{ GPa}$.



$$E = 200 \times 10^9 \text{ Pa}$$

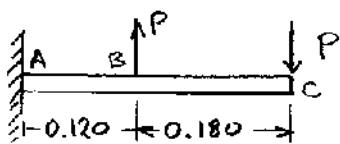
$$I = \frac{1}{12}(10)(10)^3 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = 166.667 \text{ N}\cdot\text{m}^2$$

(a) Couple applied to beam DE.

Free body DE. $\rightarrow \sum M = 0:$

$$0.180 P - 25 = 0 \quad P = 138.889 \text{ N}$$



Loads on cantilever beam ABC are $P\uparrow$ at point B and $P\downarrow$ at point C as shown.

Due to $P\uparrow$ at point B.

Using portion AB and applying Case 1 of Appendix D,

$$(y_B)_1 = \frac{PL^3}{3EI} = \frac{(138.889)(0.120)^3}{(3)(166.667)} = 0.480 \times 10^{-3} \text{ m}$$

$$(\theta_B)_1 = \frac{PL^2}{2EI} = \frac{(138.889)(0.120)^2}{(2)(166.667)} = 5.00 \times 10^{-3}$$

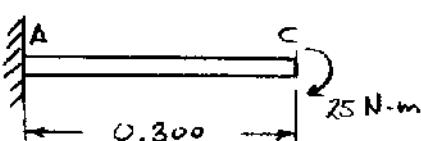
$$(y_C)_1 = (y_B)_1 + L_{BC}(\theta_B)_1 = 0.480 \times 10^{-3} + (0.180)(5.00 \times 10^{-3}) = 1.56 \times 10^{-3} \text{ m}$$

Due to load $P\downarrow$ at point C. Case 1 of App. D applied to ABC.

$$(y_C)_2 = -\frac{PL^3}{3EI} = -\frac{(138.889)(0.120+0.180)^3}{(3)(166.667)} = -7.50 \times 10^{-3} \text{ m}$$

Total deflection at point C. $y_C = (y_C)_1 + (y_C)_2 = -5.94 \times 10^{-3} \text{ m}$

$$y_C = 5.94 \text{ mm} \downarrow$$



(b) Couple applied to beam AC.

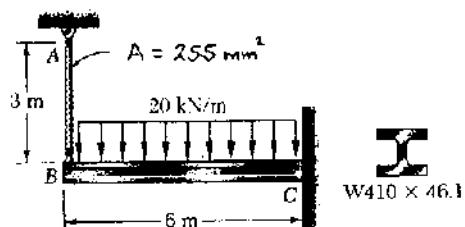
Case 3 of Appendix D.

$$y_C = -\frac{ML^2}{2EI} = -\frac{(25)(0.300)^2}{(2)(166.667)} = -6.75 \times 10^{-3} \text{ m}$$

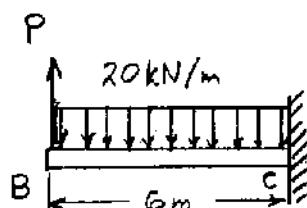
$$y_C = 6.75 \text{ mm} \downarrow$$

Problem 9.89

9.89 The cantilever beam BC is attached to the steel cable AB as shown. Knowing that the cable is initially taught, determine the tension in the cable caused by the distributed load shown. Use $E = 200 \text{ GPa}$.



Let P be the tension developed in member AB
and δ_B be the elongation of that member.



$$\delta = \frac{PL}{EA} = \frac{(P)(3)}{(200 \times 10^9)(254.47 \times 10^{-6})} \\ = 58.946 \times 10^{-3} P$$

Beam BC : $I = 156 \times 10^6 \text{ mm}^4 = 156 \times 10^{-6} \text{ m}^4$
 $EI = (200 \times 10^9)(156 \times 10^6) = 31.2 \times 10^6 \text{ N} \cdot \text{m}^2$

Loading I. 20 kN/m downward.

Refer to Case 2 of Appendix D.

$$(y_B)_1 = -\frac{WL^4}{8EI} = -\frac{(20 \times 10^3)(6)^4}{(8)(31.2 \times 10^6)} \\ = -103.846 \times 10^{-3} \text{ m}$$

Loading II. Upward force P at point B.

Refer to Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(6)^3}{(3)(31.2 \times 10^6)} = 2.3077 \times 10^{-6} P$$

By superposition, $y_B = (y_B)_1 + (y_B)_2$

Also, matching the deflection at B, $y_B = -\delta$

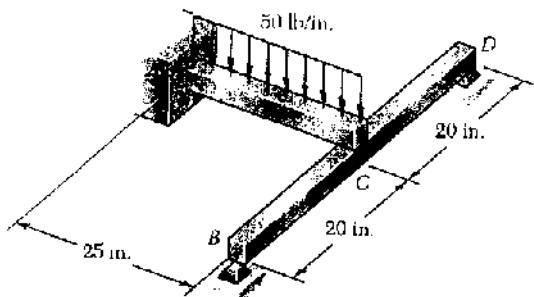
$$-103.846 \times 10^{-3} + 2.3077 \times 10^{-6} P = -58.946 \times 10^{-3} P$$

$$2.3666 \times 10^{-6} P = -103.846 \times 10^{-3} \quad P = 43.9 \times 10^3 \text{ N}$$

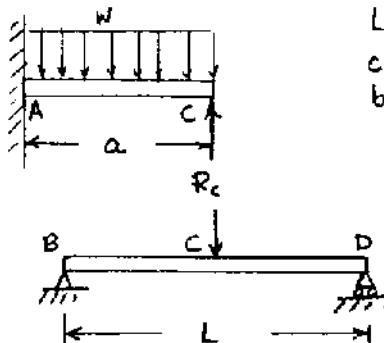
$$P = 43.9 \text{ kN} \quad \blacktriangleleft$$

Problem 9.90

9.90 For the loading shown, knowing that beams *AC* and *BD* have the same flexural rigidity, determine the reaction at *B*.



Consider the two beams shown below.



Let R_c be the contact force between beams *AC* and *BCD*.

Applying Cases 1 and 2 of Appendix D to cantilever beam *AC*,

$$y_c = \frac{R_c a^3}{3EI} - \frac{w a^4}{8EI}$$

Applying Case 4 of Appendix D to simply supported beam *BCD*,

$$y_c = -\frac{R_c L^3}{48EI}$$

Equating expressions for y_c ,

$$\frac{R_c a^3}{3EI} - \frac{w a^4}{8EI} = -\frac{R_c L^3}{48EI}$$

$$(16a^3 + L^3)R_c = 6wa^4$$

$$R_c = \frac{6wa}{16 + L^3/a^3}$$

Data: $w = 50 \text{ lb/in.}$, $a = 25 \text{ in.}$, $L = 20 + 20 = 40 \text{ in.}$

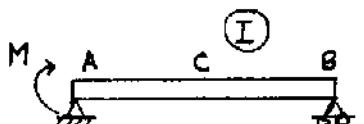
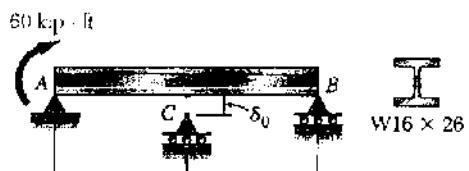
$$R_c = \frac{(6)(50)(25)}{16 + (40/25)^3} = 373.21 \text{ lb}$$

Using beam *BCD* as a free body,

$$\textcircled{D} \sum M_D = 0: -R_B L + R_c \frac{L}{2} = 0 \quad R_B = \frac{1}{2} R_c = 186.6 \text{ lb} \uparrow$$

Problem 9.91

9.91 Before the 60 kip·ft load is applied, a gap, $\delta_0 = 0.05$ in. existed between the W 16 × 26 beam and the support at C. Knowing that $E = 29 \times 10^6$ psi, determine the reaction at each support after the couple is applied.



Units: Forces in kips, lengths in ft.

$$\delta_0 = 0.05 \text{ in} = 4.1667 \times 10^{-3} \text{ ft}$$

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 301 \text{ in}^4$$

$$EI = 8.729 \times 10^6 \text{ kip-in}^2 = 60618 \text{ kip-ft}^2$$

Loading I. Case 7 of Appendix D.

$$y = -\frac{M}{6EI} (x^3 - L^2 x)$$

$$\text{with } M = 60 \text{ kip-ft}, L = 13 \text{ ft}, x = 6.5 \text{ ft}$$

$$(y_c)_1 = -\frac{(60)[6.5^3 - (13)^2(6.5)]}{(6)(60618)(13)} = -10.454 \times 10^{-3} \text{ ft}$$

Loading II. Case 4 of Appendix D.

$$(y_c)_2 = \frac{R_e L^3}{48 EI} = \frac{(13)^3 R_e}{(48)(60618)}$$

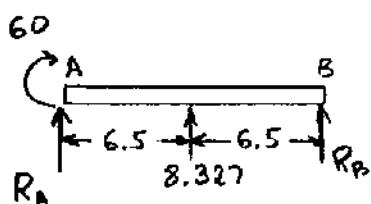
$$= 755.07 \times 10^{-6} R_e$$

Deflection at C.

$$y_c = (y_c)_1 + (y_c)_2 = -\delta_0$$

$$-10.454 \times 10^{-3} + 755.07 \times 10^{-6} R_e = -4.1667 \times 10^{-3}$$

$$R_e = 8.327 \text{ kips} \uparrow$$



Statics:

$$\rightarrow \sum M_B = 0:$$

$$-13 R_A - 60 - (6.5)(8.327) = 0$$

$$R_A = -8.779 \text{ kips} \quad R_A = 8.779 \text{ kips} \downarrow$$

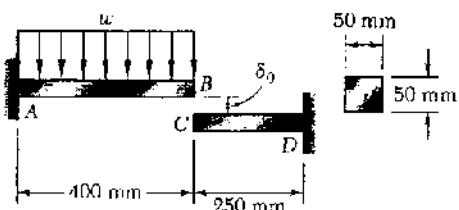
$$\leftarrow \sum M_A = 0:$$

$$(13 R_A - 60 + (6.5)(8.327)) = 0$$

$$R_B = 0.452 \text{ kips} \uparrow$$

Problem 9.92

9.92 Before the uniformly distributed load w is applied, a gap, $\delta_0 = 1.2 \text{ mm}$, exists between the ends of the cantilever bars AB and CD . Knowing that $E = 105 \text{ GPa}$ and $w = 30 \text{ kN/m}$, determine (a) the reaction at A , (b) the reaction at D .



$$I = \frac{1}{12}(50)(50)^3 = 520.833 \times 10^3 \text{ mm}^4 = 520.833 \times 10^{-6} \text{ m}^4$$

$$EI = (105 \times 10^9)(520.833 \times 10^{-6}) = 54.6875 \times 10^3 \text{ N}\cdot\text{m}^2$$

$$= 54.6875 \text{ kN}\cdot\text{m}^2$$

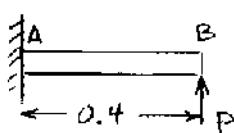
Units: Forces in kN. Lengths in meters.

Compute deflection at B due to w . Case 8 of Appendix D.

$$(y_B)_i = -\frac{wL^4}{8EI} = -\frac{(30)(0.400)^4}{(8)(54.6875)} = -1.75543 \times 10^{-3} = -1.7553 \text{ mm}$$

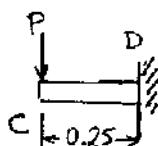
The displacement is more than δ_0 , the gap closes.

Let P be the contact force between points B and C .



Compute deflection of B due to P . Use Case 1 of Appendix D.

$$(y_B)_x = \frac{PL^3}{3EI} = \frac{P(0.4)^3}{(3)(54.6875)} = 390.095 \times 10^{-6} P \text{ m}$$



Compute deflection of C due to P .

$$y_C = -\frac{PL^3}{3EI} = -\frac{P(0.25)^3}{(3)(54.6875)} = -95.238 \times 10^{-6} P \text{ m}$$

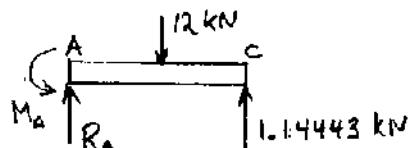
Displacement condition. $y_B + \delta_0 = y_C$

Using superposition, $(y_B)_i + (y_B)_x - \delta_0 = y_C$

$$-1.75543 \times 10^{-3} + 390.095 \times 10^{-6} P + 1.2 \times 10^{-3} = -95.238 \times 10^{-6} P$$

$$485.333 \times 10^{-6} P = 0.55543 \times 10^{-3} \quad P = 1.14443 \text{ kN.}$$

(a) Reaction at A .



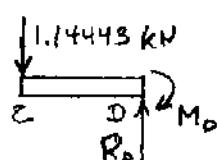
$$+\uparrow \sum F_y = 0: \quad R_A - 12 + 1.14443 = 0$$

$$R_A = 10.86 \text{ kN} \uparrow$$

$$+\rightarrow \sum M_A = 0: \quad M_A - (0.2)(12) + (0.4)(1.14443) = 0$$

$$M_A = 1.942 \text{ kN}\cdot\text{m} \curvearrowright$$

(b) Reaction at D .



$$+\uparrow \sum F_y = 0: \quad R_D - 1.14443 = 0$$

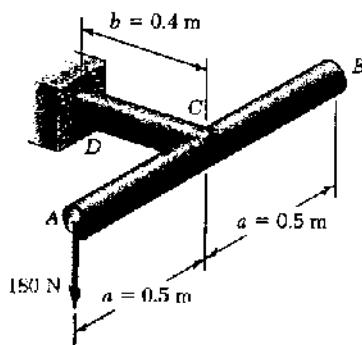
$$R_D = 1.144 \text{ kN} \uparrow$$

$$+\rightarrow \sum M_D = 0: \quad -M_D + (0.25)(1.14443) = 0$$

$$M_D = 0.286 \text{ kN}\cdot\text{m} \curvearrowleft$$

Problem 9.93

9.93 Two 24-mm-diameter aluminum rods are welded together to form the T-shaped hanger shown. Knowing that $E = 70 \text{ GPa}$ and $G = 26 \text{ GPa}$, determine the deflection at (a) end A, (b) end B.

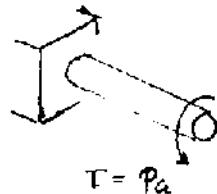


Consider tension of rod CD. $(180 \text{ N} = P)$

$$\Phi_c = \frac{TL}{GJ} = \frac{(Pa)b}{GJ}$$

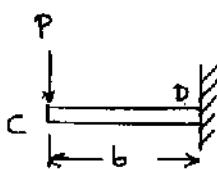
$$(y_A)_I = -a\Phi_c = -\frac{Pa^2b}{GJ}$$

$$(y_B)_I = a\Phi_c = \frac{Pa^2b}{GJ}$$



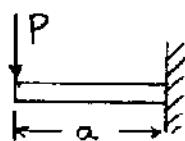
Consider bending of rod CD.

$$(y_A)_{II} = (y_B)_{II} = (y_C)_{II} = -\frac{Pb^3}{3EI} \quad (\text{Case I, App D.})$$



Consider bending of rod portion AC.

$$(y_A)_{III} = -\frac{Pa^3}{3EI}$$



By superposition,

$$y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III}$$

$$= P \left\{ -\frac{a^2b}{GJ} - \frac{b^3}{3EI} - \frac{a^3}{3EI} \right\}$$

$$y_B = (y_B)_I + (y_B)_{II}$$

$$= P \left\{ \frac{a^2b}{GJ} - \frac{b^3}{3EI} \right\}$$

$$\text{Data: } G = 26 \times 10^9 \text{ Pa}, \quad J = \frac{\pi}{4}c^4 = \frac{\pi}{4}(12)^4 = 32.572 \times 10^3 \text{ mm}^4 = 32.572 \times 10^{-9} \text{ m}^4$$

$$E = 70 \times 10^9 \text{ Pa}, \quad I = \frac{1}{4}J = 16.286 \times 10^{-9} \text{ m}^4$$

$$GJ = 846.87 \text{ N}\cdot\text{m}^2 \quad EI = 1140.02 \text{ N}\cdot\text{m}^2$$

$$a = 0.5 \text{ m}, \quad b = 0.4 \text{ m}$$

$$y_A = 180 \left\{ -\frac{(0.5)^2(0.4)}{846.87} - \frac{(0.4)^3}{(3)(1140.02)} - \frac{(0.5)^3}{(3)(1140.02)} \right\} = -31.2 \times 10^{-3} \text{ m}$$

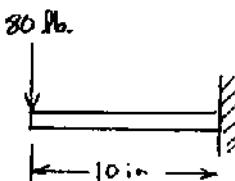
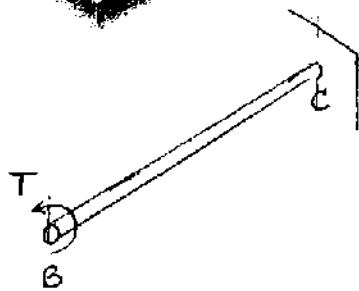
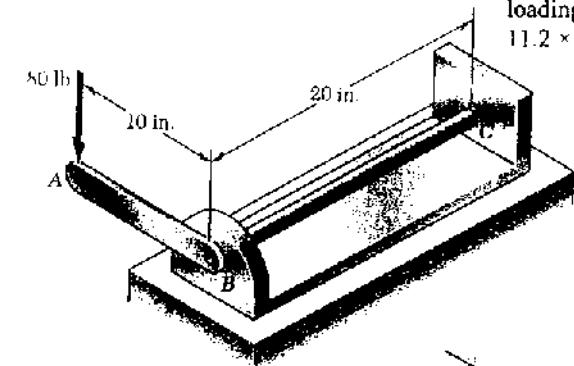
$$= 31.2 \text{ mm} \downarrow \quad (a)$$

$$y_B = 180 \left\{ \frac{(0.5)^2(0.4)}{846.87} - \frac{(0.4)^3}{(3)(1140.02)} \right\} = 17.89 \times 10^{-3} \text{ m}$$

$$17.89 \text{ mm} \uparrow \quad (b)$$

Problem 9.94

9.94 A $\frac{7}{8}$ -in.-diameter rod BC is attached to the lever AB and to the fixed support at C. Lever AB has a uniform cross section $\frac{3}{8}$ in. thick and 1 in. deep. For the loading shown, determine the deflection of point A. Use $E = 29 \times 10^6$ psi and $G = 11.2 \times 10^6$ psi.



Deformation of rod BC. (Torsion)

$$c = \frac{1}{2}d = \frac{1}{2}\left(\frac{7}{8}\right) = 0.4375 \text{ in}$$

$$J = \frac{\pi}{2}c^3 = 57.548 \times 10^{-3} \text{ in}^4$$

$$T = Pa = (80)(10) = 800 \text{ lb-in.}$$

$$L = 20 \text{ in.}$$

$$\varphi_B = \frac{TL}{GJ} = \frac{(800)(20)}{(11.2 \times 10^6)(57.548 \times 10^{-3})}$$

$$= 24.824 \times 10^{-3} \text{ rad.}$$

Deflection of point A assuming lever AB to be rigid.

$$(y_A)_1 = a\varphi_B = (10)(24.824 \times 10^{-3})$$

$$= 0.24824 \text{ in. } \downarrow$$

Additional deflection due to bending of lever AB.

Refer to Case 1 of Appendix D.

$$I = \frac{1}{12}\left(\frac{3}{8}\right)(1)^3 = 31.25 \times 10^{-3} \text{ in}^4$$

$$(y_A)_2 = \frac{PL^3}{3EI} = \frac{(80)(10)^3}{(3)(29 \times 10^6)(31.25 \times 10^{-3})}$$

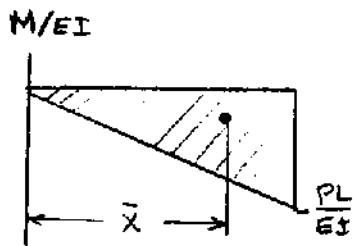
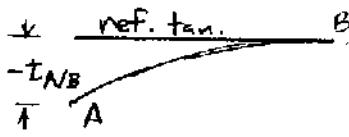
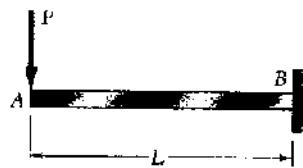
$$= 0.02943 \text{ in. } \downarrow$$

Total deflection at point A.

$$y_A = (y_A)_1 + (y_A)_2 = 0.278 \text{ in. } \downarrow$$

Problem 9.95

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine
(a) the slope at the free end, (b) the deflection at the free end.



(b) Deflection at A.

Place reference tangent at B..

Draw $\frac{M}{EI}$ diagram.

$$A = \frac{1}{2} \left(-\frac{PL}{EI} \right) L = -\frac{PL^2}{2EI}$$

$$\bar{x} = \frac{2}{3} L$$

$$\theta_{B/A} = A = -\frac{PL^2}{2EI}$$

$$t_{A/B} = A \bar{x} = \left(-\frac{PL^2}{2EI} \right) \left(\frac{2}{3} L \right) = -\frac{PL^3}{3EI}$$

(a) Slope at end A.

$$\theta_B = \theta_A + A$$

$$0 = \theta_A - \frac{PL^2}{2EI}$$

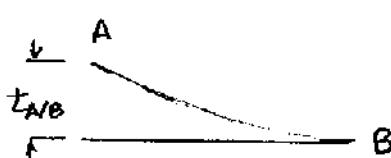
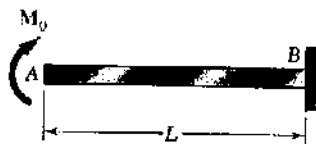
$$\theta_A = \frac{PL^2}{2EI}$$

$$y_A = t_{A/B} = -\frac{PL^3}{3EI}$$

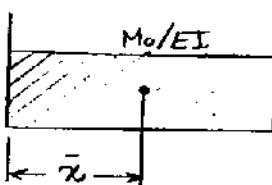
$$y_A = \frac{PL^3}{3EI}$$

Problem 9.96

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine
(a) the slope at the free end, (b) the deflection at the free end.



M/EI



(b) Deflection at A.

Place reference tangent at B..

Draw $\frac{M}{EI}$ diagram.

$$A = \left(\frac{M_0}{EI} \right) L = \frac{M_0 L}{EI}$$

$$\bar{x} = \frac{1}{2} L$$

$$\theta_{B/A} = A = \frac{M_0 L}{EI}$$

$$t_{B/A} = A \bar{x} = \left(\frac{M_0 L}{EI} \right) \left(\frac{1}{2} L \right) = \frac{M_0 L^2}{2EI}$$

(a) Slope at end A.

$$\theta_B = \theta_A + A \quad 0 = \theta_A + \frac{M_0 L}{EI}$$

$$\theta_A = -\frac{M_0 L}{EI}$$

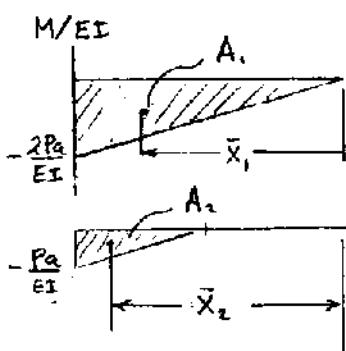
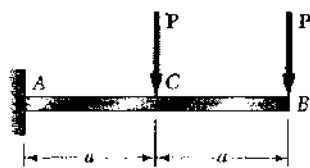
$$\theta_A = \frac{1}{2} \frac{M_0 L}{EI}$$

$$y_A = t_{A/B} = \frac{M_0 L^2}{2EI}$$

$$y_A = \frac{M_0 L^2}{2EI}$$

Problem 9.97

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine
(a) the slope at the free end, (b) the deflection at the free end.



Place reference tangent at A. $\theta_A = 0$

Draw $\frac{M}{EI}$ diagram by parts (two triangles).

$$A_1 = \frac{1}{2} \left(-\frac{2Pa}{EI} \right) (2a) = -\frac{2Pa^2}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(2a) = \frac{4}{3}a$$

$$A_2 = \frac{1}{2} \left(-\frac{Pa}{EI} \right) a = -\frac{1}{2} \frac{Pa^2}{EI}$$

$$\bar{x}_2 = a + \frac{2}{3}a = \frac{5}{3}a$$

By first moment-area theorem,

$$\theta_{B/A} = A_1 + A_2 = -\frac{2Pa^2}{EI} - \frac{1}{2} \frac{Pa^2}{EI} = -\frac{5}{2} \frac{Pa^2}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{5}{2} \frac{Pa^2}{EI} = \frac{5}{2} \frac{Pa^2}{EI}$$

By second moment area theorem,

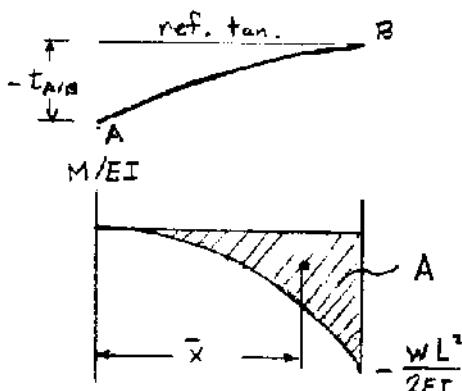
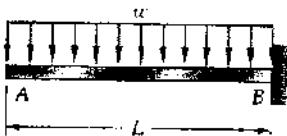
$$t_{B/A} = A_1 \bar{x}_1 + A_2 \bar{x}_2$$

$$= \left(-\frac{2Pa^2}{EI} \right) \left(\frac{4}{3}a \right) + \left(-\frac{1}{2} \frac{Pa^2}{EI} \right) \left(\frac{5}{3}a \right) = -\frac{7}{2} \frac{Pa^3}{EI}$$

$$y_B = t_{B/A} = -\frac{7}{2} \frac{Pa^3}{EI} = \frac{7Pa^3}{2EI}$$

Problem 9.98

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.



Place reference tangent at B. $\theta_B = 0$

Draw $\frac{M}{EI}$ curve as parabola.

$$A = -\frac{1}{3} \left(\frac{wL^2}{2EI} \right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$

By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{6} \frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6} \frac{wL^3}{EI} = \frac{1}{6} \frac{wL^3}{EI}$$

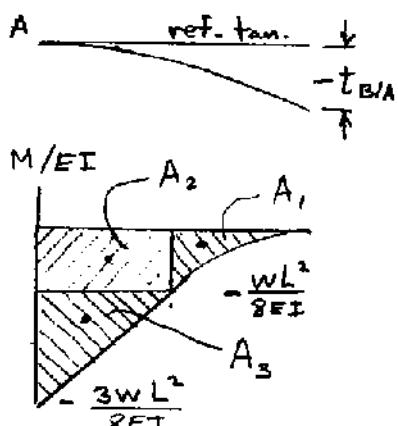
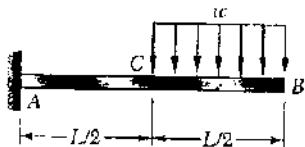
By second moment-area theorem,

$$t_{A/B} = \bar{x} A = \left(\frac{3}{4}L \right) \left(-\frac{1}{6} \frac{wL^3}{EI} \right) = -\frac{1}{8} \frac{wL^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{8} \frac{wL^4}{EI} = \frac{wL^4}{8EI} \downarrow$$

Problem 9.99

9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (a) the slope and deflection at (a) point B, (b) point C.



Place reference tangent at A.

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{3} \left(-\frac{wL^2}{8EI} \right) \left(\frac{L}{2} \right) = -\frac{wL^3}{48EI}$$

$$A_2 = \left(-\frac{wL^2}{8EI} \right) \left(\frac{L}{2} \right) = -\frac{wL^3}{16EI}$$

$$A_3 = \frac{1}{2} \left(-\frac{2wL^2}{8EI} \right) \left(\frac{L}{2} \right) = -\frac{wL^3}{16EI}$$

$$\theta_A = 0 \rightarrow y_A = 0$$

(a) Slope at B.

$$\theta_B = \theta_A + A_1 + A_2 + A_3$$

$$= -\frac{7wL^3}{48EI} \quad \theta_B = \frac{7wL^3}{48EI}$$

Deflection at B.

$$y_B = t_{B/A} = A_1 \left(\frac{3}{4} \cdot \frac{L}{2} \right) + A_2 \left(\frac{L}{2} + \frac{1}{2} \cdot \frac{L}{2} \right) + A_3 \left(\frac{L}{2} + \frac{2}{3} \cdot \frac{L}{2} \right)$$

$$= -\frac{wL^4}{128EI} - \frac{3wL^4}{64EI} - \frac{5wL^4}{96EI} = -\frac{41wL^4}{384EI}$$

$$y_B = \frac{41wL^4}{384EI}$$

(b) Slope at C.

$$\theta_C = \theta_A + A_2 + A_3 = -\frac{wL^3}{8EI}$$

$$\theta_C = \frac{wL^3}{8EI}$$

Deflection at C.

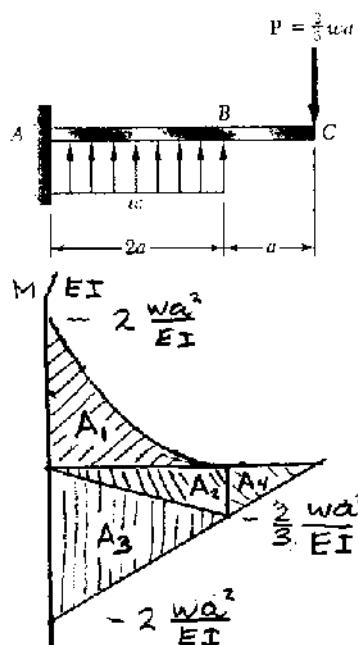
$$y_C = t_{C/A} = A_2 \left(\frac{1}{2} \cdot \frac{L}{2} \right) + A_3 \left(\frac{2}{3} \cdot \frac{L}{2} \right)$$

$$= -\frac{7wL^4}{192EI}$$

$$y_C = \frac{7wL^4}{192EI}$$

Problem 9.100

9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (a) the slope and deflection at (a) point B, (b) point C.



Draw $\frac{M}{EI}$ diagram by parts and divide into areas A_1, A_2, A_3 , and A_4 .

$$A_1 = \frac{1}{3}(2a)\left(2\frac{w\alpha^2}{EI}\right) = \frac{4}{3}\frac{w\alpha^3}{EI}$$

$$A_2 = \frac{1}{2}(2a)\left(-\frac{2w\alpha^2}{3EI}\right) = -\frac{2}{3}\frac{w\alpha^3}{EI}$$

$$A_3 = \frac{1}{2}(2a)\left(-\frac{2w\alpha^2}{EI}\right) = -2\frac{w\alpha^3}{EI}$$

$$A_4 = \frac{1}{2}(a)\left(-\frac{2}{3}\frac{w\alpha^2}{EI}\right) = -\frac{1}{3}\frac{w\alpha^3}{EI}$$

(a) Slope at B.

$$\Theta_B = A_1 + A_2 + A_3 = -\frac{4}{3}\frac{w\alpha^3}{EI} = \frac{4w\alpha^3}{3EI}$$

Deflection at B. $y_B = t_{B/A}$

$$\begin{aligned} y_B = t_{B/A} &= A_1\left(\frac{3}{4} \cdot 2a\right) + A_2\left(\frac{1}{3} \cdot 2a\right) + A_3\left(\frac{2}{3} \cdot 2a\right) \\ &= 2\frac{w\alpha^4}{EI} - \frac{4}{9}\frac{w\alpha^4}{EI} - \frac{8}{3}\frac{w\alpha^4}{EI} = -\frac{10}{9}\frac{w\alpha^4}{EI} \end{aligned}$$

(b) Slope at C.

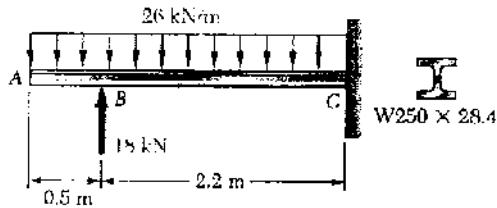
$$\Theta_C = \Theta_B + A_4 = -\frac{4}{3}\frac{w\alpha^3}{EI} - \frac{1}{3}\frac{w\alpha^3}{EI} = -\frac{5}{3}\frac{w\alpha^3}{EI}$$

Deflection at C.

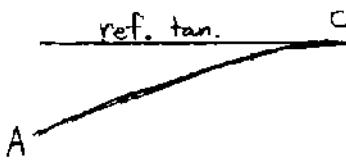
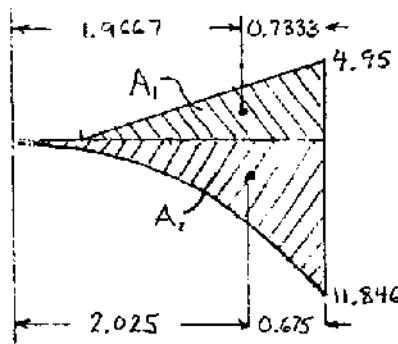
$$\begin{aligned} y_C &= y_B + \Theta_B a + t_{C/B} \\ &= -\frac{10}{9}\frac{w\alpha^4}{EI} + \left(-\frac{4}{3}\frac{w\alpha^3}{EI}\right)(a) + \left(-\frac{1}{3}\frac{w\alpha^3}{EI}\right)\left(\frac{2a}{3}\right) = -\frac{8}{3}\frac{w\alpha^4}{EI} \end{aligned}$$

Problem 9.101

9.101 For the cantilever beam and loading shown, determine (a) the slope at point A, (b) the deflection at point A. Use $E = 200 \text{ GPa}$.



$$10^3 \text{ M/EI}$$



Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 40.0 \times 10^6 \text{ mm}^4 = 40.0 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(40.0 \times 10^{-6}) = 8.00 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$= 8000 \text{ kN} \cdot \text{m}^2$$

Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{(18)(2.2)}{8000} = 4.95 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(4.95 \times 10^{-3})(2.2) = 5.445 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(2.2) = 0.7333 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8000)} = -11.846 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \frac{1}{3}(-11.846 \times 10^{-3})(2.7) = -10.662 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.7) = 0.675 \text{ m}$$

Draw reference tangent at C.

$$\theta_c = \theta_A + \theta_{c/A} = \theta_A + A_1 + A_2 = 0$$

$$\theta_A = -A_1 - A_2 = -5.445 \times 10^{-3} + 10.662 \times 10^{-3}$$

$$= 5.22 \times 10^{-3} \text{ rad}$$

$$\theta_A = 5.22 \times 10^{-3}$$

$$y_A = y_c - \theta_c L + t_{AC}$$

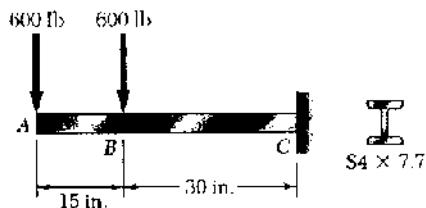
$$= 0 - 0 + A_1 \bar{x}_1 + A_2 \bar{x}_2$$

$$= 0 - 0 + (5.445 \times 10^{-3})(1.9667) - (10.662 \times 10^{-3})(2.025)$$

$$= -10.88 \times 10^{-3} \text{ m} = 10.88 \text{ mm } \downarrow$$

Problem 9.102

9.102 For the cantilever beam and loading shown, determine the slope and deflection at (a) end A, (b) point B. Use $E = 29 \times 10^6$ psi.



$$I = 6.08 \text{ in}^4$$

$$EI = (29 \times 10^6)(6.08) = 176.32 \times 10^6 \text{ lb-in}^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = -\frac{1}{2}(30) \frac{18 \times 10^3}{EI} = -1.5313 \times 10^{-5}$$

$$A_2 = -\frac{1}{2}(30) \frac{27 \times 10^3}{EI} = -2.2970 \times 10^{-5}$$

$$A_3 = -\frac{1}{2}(30) \frac{9 \times 10^3}{EI} = -0.7657 \times 10^{-5}$$

$$A_4 = -\frac{1}{2}(15) \frac{9 \times 10^3}{EI} = -0.3828 \times 10^{-5}$$

Place reference tangent at C.

$$\theta_c = 0 \quad y_c = 0$$

(a) Slope at A. $\theta_A = -\theta_{c/A} = -A_1 - A_2 - A_3 - A_4 = 4.98 \times 10^{-5}$

Deflection at A. $y_A = t_{A/C}$

$$y_A = A_1(15+20) + A_2(15+20) + A_3(15+10) + A_4(10) = -0.1570 \text{ in.}$$

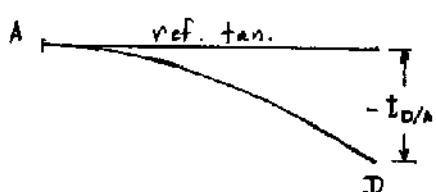
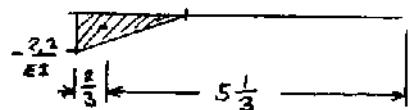
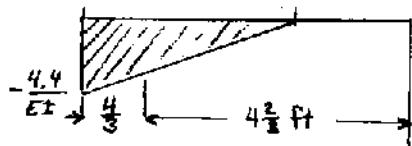
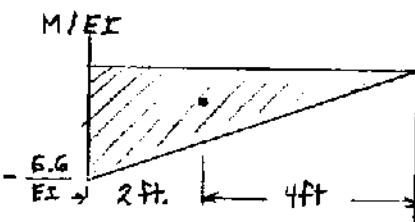
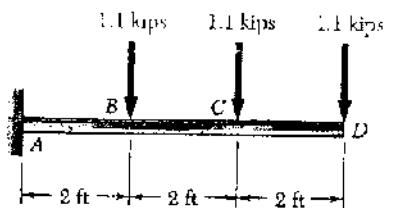
(b) Slope at B. $\theta_B = -\theta_{B/A} = -A_1 + A_2 + A_3 = 4.59 \times 10^{-5}$

Deflection at B. $y_B = t_{B/C}$

$$y_B = A_1(20) + (A_2(20) + A_3(10)) = -0.0842 \text{ in.}$$

Problem 9.103

9.103 Two C 6 × 8.2 channels are welded back to back and loaded as shown. Knowing that $E = 29 \times 10^6$ psi., determine (a) the slope at point D, (b) the deflection at point D.



Units: Forces in kips; lengths in ft.

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = (2)(13.1) = 26.2 \text{ in}^4$$

$$\begin{aligned} EI &= (29 \times 10^3)(26.2) = 759.8 \times 10^3 \text{ kip-in}^2 \\ &= 5276 \text{ kip-ft}^2 \end{aligned}$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = -\frac{(1.1)(6)}{EI} = -\frac{6.6}{EI} \text{ ft}^{-1}$$

$$A_1 = \frac{1}{2} \left(\frac{6.6}{EI} \right)(6) = -\frac{19.8}{EI}$$

$$\bar{x}_1 = \frac{1}{3}(6) = 2 \text{ ft.}$$

$$\frac{M_2}{EI} = -\frac{(1.1)(4)}{EI} = -\frac{4.4}{EI} \text{ ft}^{-1}$$

$$A_2 = \frac{1}{2} \left(-\frac{4.4}{EI} \right)(4) = -\frac{8.8}{EI}$$

$$\bar{x}_2 = \frac{1}{3}(4) = \frac{4}{3} \text{ ft}$$

$$\frac{M_3}{EI} = -\frac{(1.1)(2)}{EI} = -\frac{2.2}{EI} \text{ ft}^{-1}$$

$$A_3 = \frac{1}{2} \left(-\frac{2.2}{EI} \right)(2) = -\frac{2.2}{EI}$$

$$\bar{x}_3 = \frac{1}{3}(2) = \frac{2}{3} \text{ ft.}$$

Place reference tangent at A. $\theta_A = 0$

$$\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{30.8}{EI} = -\frac{30.8}{5276} = -5.84 \times 10^{-3} \text{ rad.}$$

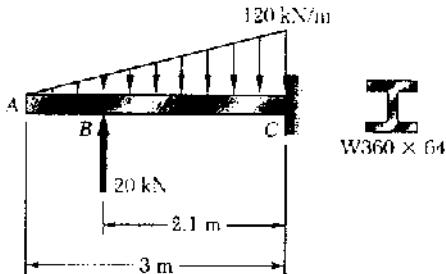
$$\theta_D = \theta_A + \theta_{D/A} = -5.84 \times 10^{-3} \text{ rad.}$$

$$t_{D/A} = \left(-\frac{19.8}{EI} \right)(4) + \left(-\frac{8.8}{EI} \right)(4\frac{2}{3}) + \left(-\frac{2.2}{EI} \right)(5\frac{1}{3}) = -\frac{132.0}{EI} = -\frac{132.0}{5276} = 25.02 \times 10^{-3} \text{ ft}$$

$$y_D = t_{D/A} = 25.02 \times 10^{-3} \text{ ft} = 0.300 \text{ in. } \downarrow$$

Problem 9.104

9.104 For the cantilever beam and loading shown, determine (a) the slope at point A, (b) the deflection at point A. Use $E = 200 \text{ GPa}$.



Units: Forces in kN; lengths in meters.

$$I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(178 \times 10^{-6}) = 35600 \text{ kN-m}^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{(20)(2.1)}{35600} = 1.17978 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = (\frac{1}{2})(2.1)(1.17978 \times 10^{-3}) = 1.23876 \times 10^{-3}$$

$$M_2 = -\frac{(\frac{1}{2})(120)(3)(\frac{2}{3})}{35600} = -5.0562 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = (\frac{1}{4})(3)(-5.0562 \times 10^{-3}) = -3.7921 \times 10^{-3}$$

Place reference tangent at C. $\theta_c = 0$

(a) Slope at A.

$$\theta_A = -\theta_{c/A} = -A_1 - A_2 = 2.55 \times 10^{-3}$$

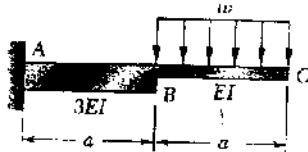
(b) Deflection at A.

$$y_A = \sum A_i / c$$

$$y_A = A_1(3 - 0.7) + A_2(3)(\frac{2}{3}) = -6.25 \times 10^{-3} \text{ m} = 6.25 \text{ mm} \downarrow$$

Problem 9.105

9.105 For the cantilever beam ABC , determine the deflection at (a) point B , (b) end C .



Draw $\frac{M}{EI}$ diagram.

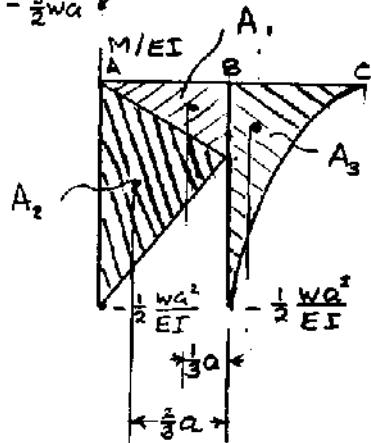
$$A_1 = \frac{1}{2} \left(-\frac{1}{6} \frac{w a^2}{E I} \right) a = -\frac{1}{12} \frac{w a^3}{E I}$$

$$A_2 = \frac{1}{2} \left(-\frac{1}{2} \frac{w a^2}{E I} \right) a = -\frac{1}{4} \frac{w a^3}{E I}$$

$$A_3 = \frac{1}{3} \left(-\frac{1}{2} \frac{w a^2}{E I} \right) a = -\frac{1}{6} \frac{w a^3}{E I}$$

Place reference tangent at A.

(a) Deflection at B.



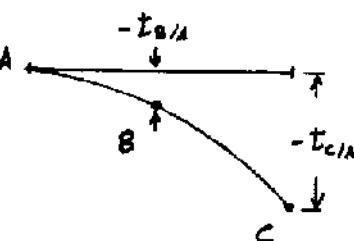
$$\begin{aligned} t_{B/A} &= A_1 \left(\frac{1}{3} a \right) + A_2 \left(\frac{2}{3} a \right) \\ &= \left(-\frac{1}{12} \frac{w a^3}{E I} \right) \left(\frac{1}{3} a \right) + \left(-\frac{1}{4} \frac{w a^3}{E I} \right) \left(\frac{2}{3} a \right) \\ &= -\frac{7}{36} \frac{w a^4}{E I} \end{aligned}$$

$$y_B = t_{B/A} = -\frac{7}{36} \frac{w a^4}{E I}$$

(b) Deflection at C.

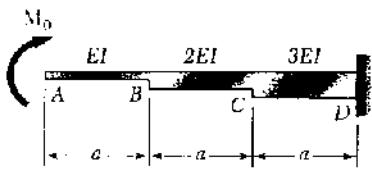
$$\begin{aligned} t_{C/A} &= A_1 \left(a + \frac{1}{3} a \right) + A_2 \left(a + \frac{2}{3} a \right) + A_3 \left(a - \frac{1}{4} a \right) \\ &= \left(-\frac{1}{12} \frac{w a^2}{E I} \right) \left(\frac{4}{3} a \right) + \left(-\frac{1}{4} \frac{w a^2}{E I} \right) \left(\frac{5}{3} a \right) \\ &\quad - \left(\frac{1}{6} \frac{w a^2}{E I} \right) \left(\frac{3}{4} a \right) = -\frac{47}{72} \frac{w a^4}{E I} \end{aligned}$$

$$y_C = t_{C/A} = -\frac{47}{72} \frac{w a^4}{E I}$$

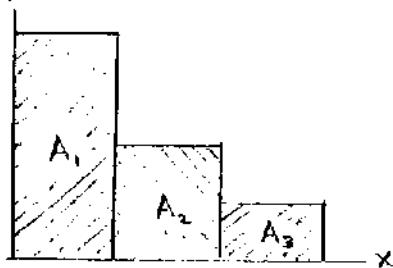


Problem 9.106

9.106 For the cantilever beam and loading shown, determine the deflection and slope at end A caused by the moment M_0 .



EI/a^2



Draw $\frac{M}{EI}$ diagram.

$$A_1 = + \frac{M_0 a}{EI}$$

$$A_2 = + \frac{M_0 a}{2EI}$$

$$A_3 = + \frac{M_0 a}{3EI}$$

$$\theta_D = 0, y_D = 0$$

Place reference tangent at D.

Deflection at A. $y_A = t_{A/D}$

$$y_A = A_1(\frac{1}{2}a) + A_2(\frac{3}{2}a) + A_3(\frac{5}{2}a) = \frac{25M_0 a^2}{12EI} \uparrow$$

Slope at A. $\theta_A = -\theta_{c/A}$

$$\theta_A = -A_1 - A_2 - A_3 = -\frac{11M_0 a}{6EI}$$

$$\theta_A = \frac{11M_0 a}{6EI} A$$

A ref. tan D

$-t_{A/D}$



↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

↓

↑

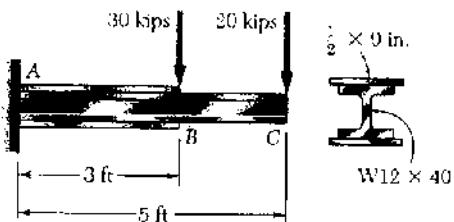
↓

↑

↓

Problem 9.107

9.107 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 29 \times 10^6$ psi., determine the slope and deflection at end C.



Use units of kips and ft.

Over portion BC $I = 310 \text{ in}^4$

$$EI = (29 \times 10^6)(310) = 8.99 \times 10^9 \text{ lb-in}^2$$

$$= 62431 \text{ kip-ft}^2$$

Portion AB	$A(\text{in}^2)$	$d(\text{in.})$	$Ad^2(\text{in}^6)$	$\bar{I}(\text{in}^4)$
Top plate	4.5	6.22	174.1	0.05
W12 x 40		0	0	310
Bot. plate	4.5	6.22	174.1	0.05
Σ			348.2	310.1

$$I = 348.2 + 310.1 = 658.3 \text{ in}^4$$

$$EI = (29 \times 10^6)(658.3) = 19.091 \times 10^9 \text{ lb-in}^2$$

$$= 132574 \text{ kip-ft}^2$$

Draw $\frac{M}{EI}$ diagram.

$$\frac{M_1}{EI} = -\frac{(30)(3)}{132574} = -678.86 \times 10^{-6} \text{ ft}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(20)(5)}{132574} = -754.30 \times 10^{-6} \text{ ft}^{-1}$$

$$\frac{M_4}{EI} = -\frac{(20)(2)}{62431} = -640.71 \times 10^{-6} \text{ ft}^{-1}$$

$$A_1 = \left(\frac{1}{2}\right)(3)(-678.86 \times 10^{-6}) = -1.01829 \times 10^{-3}$$

$$A_2 = \left(\frac{1}{2}\right)(3)(-754.30 \times 10^{-6}) = -1.13145 \times 10^{-3}$$

$$A_3 = \frac{2}{5} A_2 = -0.45248 \times 10^{-3}$$

$$A_4 = \left(\frac{1}{2}\right)(2)(-640.71 \times 10^{-6}) = -0.64071 \times 10^{-3}$$

Place reference tangent at A.

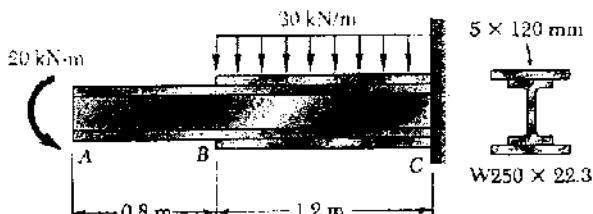
$$\text{Slope at C. } \theta_C = \theta_{CA} = A_1 + A_2 + A_3 + A_4 = -3.24 \times 10^{-3}$$

$$\text{Deflection at C. } y_C = t_{C/A}$$

$$y_C = (4)(A_1) + (4)(A_2) + (3)(A_3) + \left(\frac{4}{3}\right)(A_4) = -10.81 \times 10^{-3} \text{ ft} = -0.1297 \text{ in.}$$

Problem 9.108

9.108 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200 \text{ GPa}$, determine the (a) slope at end A, (b) the deflection at end A.



Units: Forces in kN, lengths in m.

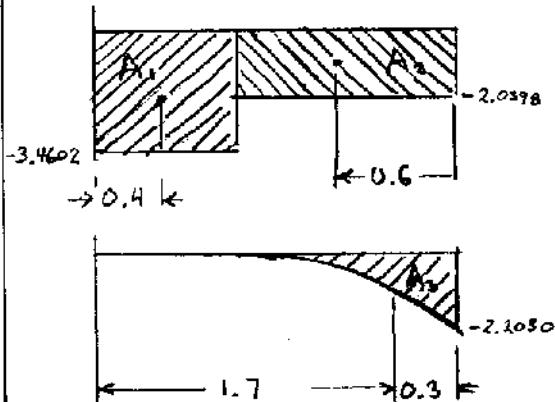
$$E = 200 \times 10^9 \text{ Pa}$$

$$\text{From A to B } I = 28.9 \times 10^6 \text{ mm}^4 \\ = 28.9 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(28.9 \times 10^{-6})$$

$$= 5.78 \times 10^6 \text{ N} \cdot \text{m}^2 = 5780 \text{ kN} \cdot \text{m}^2$$

$10^3 M/EI$



$$\text{From B to C } I = I_w + 2A_p d^2 + 2\bar{I}_p$$

$$A_p = 5 \times 120 = 600 \text{ mm}^2$$

$$d = \frac{254}{2} + \frac{5}{2} = 129.5 \text{ mm}$$

$$A_d^2 = 10.062 \times 10^6 \text{ mm}^4$$

$$\bar{I}_p = \frac{1}{12}(120)(5)^3 = 0.00125 \times 10^6 \text{ mm}^4$$

$$I = [28.9 + (2)(10.062) + (2)(0.00125)] \times 10^6 \text{ mm}^4 \\ = 49.03 \times 10^6 \text{ mm}^4 = 49.03 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(49.03 \times 10^{-6}) = 9.805 \times 10^5 \text{ N} \cdot \text{m}^2 \\ = 9805 \text{ kN} \cdot \text{m}^2$$

Draw M/EI diagram by parts.

$$A \text{ to } B \quad \frac{M_k}{EI} = -\frac{20}{5780} = -3.4602 \times 10^{-3} \text{ m}^{-1}$$

$$B \text{ to } C \quad \frac{M_k}{EI} = -\frac{20}{9805} = -2.0398 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_k}{EI} = -\frac{(30)(1.2)^2}{(2)(9805)} = -2.2030 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = (-3.4602 \times 10^{-3})(0.8) = -2.7682 \times 10^{-3}$$

Place reference tangent
at C. $\theta_c = 0$

$$A_2 = (-2.0398 \times 10^{-3})(1.2) = -2.4478 \times 10^{-3}$$

$$A_3 = \frac{1}{3}(-2.2030 \times 10^{-3})(1.2) = -0.8812 \times 10^{-3}$$

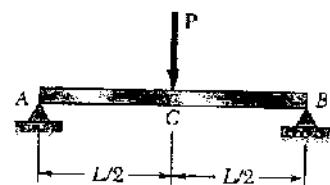
$$(a) \text{ Slope at A, } \theta_A = \theta_c - \theta_{AC} = 0 - (A_1 + A_2 + A_3) = 6.10 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

(b) Deflection at A.

$$y_A = t_{AC} = (-2.7682 \times 10^{-3})(0.4) + (-2.4478 \times 10^{-3})(1.4) + (-0.8812 \times 10^{-3})(1.7) \\ = -6.03 \times 10^{-3} \text{ m} = 6.03 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.109

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

Place reference tangent at C.

$$\theta_c = 0, \quad y_c = -t_{A/C}$$

$$\text{Reactions } R_A = R_B = \frac{1}{2}P$$

$$\text{Bending moment at } C. \quad M_c = \frac{1}{4}PL$$

$$A = \frac{1}{2} \left(\frac{1}{4} \frac{PL}{EI} \right) \left(\frac{L}{2} \right) = \frac{1}{16} \frac{PL^2}{EI}$$

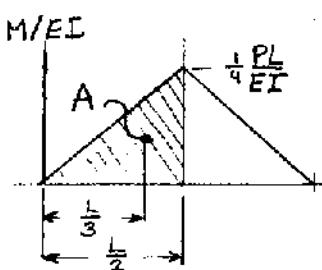
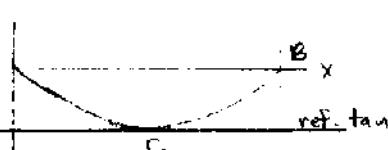
$$(a) \text{ Slope at } A. \quad \theta_A = \theta_c - \theta_{c/A}$$

$$\theta_A = 0 - \frac{1}{16} \frac{PL^2}{EI} = -\frac{1}{16} \frac{PL^2}{EI}$$

(b) Deflection at C.

$$y_c = -t_{A/C} = -A \left(\frac{L}{3} \right) = -\left(\frac{1}{16} \frac{PL^2}{EI} \right) \left(\frac{L}{3} \right)$$

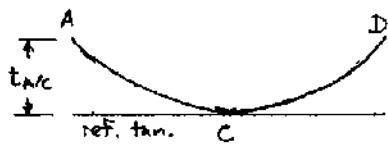
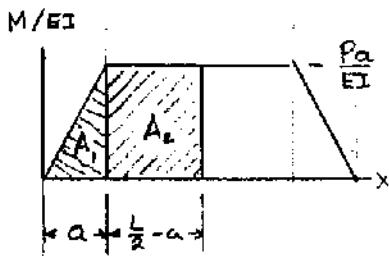
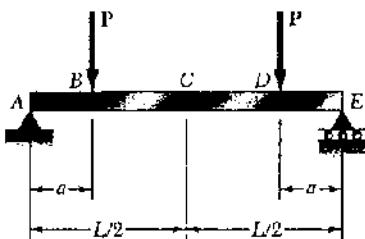
$$y_c = \frac{1}{48} \frac{PL^3}{EI}$$



PROPRIETARY MATERIAL. © 2006 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

Problem 9.110

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam with symmetric loading.

$$\theta_c = 0 \quad \text{Place reference tangent at } C.$$

Draw $\frac{M}{EI}$ diagram.

$$A_1 = -\frac{1}{2}a\left(\frac{Pa}{EI}\right) = \frac{Pa^2}{2EI}$$

$$A_2 = \left(\frac{L}{2}-a\right)\left(-\frac{Pa}{EI}\right) = \frac{Pa(L-2a)}{2EI}$$

$$\text{Slope at end A.} \quad \theta_A = -\theta_{c/A}$$

$$\theta_A = A_1 + A_2 = -\frac{Pa(L-a)}{2EI}$$

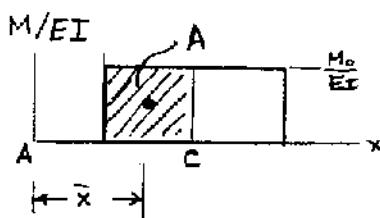
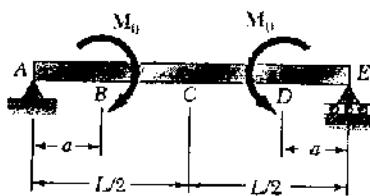
$$\text{Deflection at } C. \quad y_c = -t_{A/C}$$

$$\begin{aligned} t_{A/C} &= \left(\frac{2}{3}a\right)A_1 + \left[\frac{1}{2}\left(\frac{L}{2}+a\right)\right]A_2 \\ &= \frac{2}{3}\frac{Pa^2}{2EI} + \left(\frac{L+2a}{4}\right)\frac{Pa(L-2a)}{2EI} \\ &= \frac{Pa}{EI} \left\{ \frac{1}{3}a^2 + \frac{1}{8}(L^2 - 4a^2) \right\} \\ &= \frac{Pa}{24EI} (2L^2 - 4a^2) \end{aligned}$$

$$y_c = -\frac{Pa}{24EI} (3L^2 - 4a^2)$$

Problem 9.111

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

Place reference tangent at C. $\theta_c = 0$.

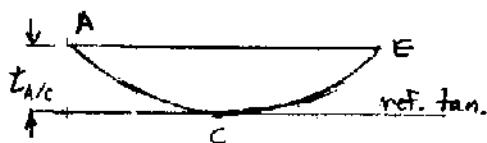
Draw $\frac{M}{EI}$ diagram.

(a) Slope at A. θ_A

$$A = \frac{M_0}{EI} \left(\frac{L}{2} - a \right) = \frac{1}{2} \frac{M_0}{EI} (L - 2a)$$

$$\theta_A = \theta_c - \theta_{c/A} = 0 - A =$$

$$= -\frac{1}{2} \frac{M_0}{EI} (L - 2a)$$



(b) Deflection at C.

$$\bar{x} = a + \frac{1}{2} \left(\frac{L}{2} - a \right) = \frac{1}{4} (L + 2a)$$

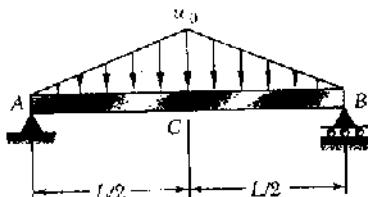
$$y_c = -t_{c/A} = A \bar{x}$$

$$= -\frac{1}{2} \frac{M_0}{EI} (L - 2a) \frac{1}{4} (L + 2a)$$

$$= -\frac{1}{8} \frac{M_0}{EI} (L^2 - 4a^2)$$

Problem 9.112

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

$$\theta_c = 0 \quad \text{Place reference tangent at } C.$$

$$\text{Reactions.} \quad R_A = R_B = \frac{w_0 L}{4}$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{R_A L}{2} = \frac{w_0 L^2}{8 EI}$$

$$A_1 = \frac{1}{2} \left(\frac{L}{2} \right) \left(\frac{M_1}{EI} \right) = \frac{w_0 L^3}{32 EI}$$

$$\frac{M_2}{EI} = \frac{1}{EI} \cdot \frac{1}{2} \left(\frac{w_0 L}{2} \right) \left(\frac{1}{3} \cdot \frac{L}{2} \right) = -\frac{w_0 L^2}{24 EI}$$

$$A_2 = \frac{1}{4} \left(\frac{L}{2} \right) \left(-\frac{w_0 L^2}{24 EI} \right) = -\frac{w_0 L^3}{192 EI}$$

$$\text{Slope at } A. \quad \theta_A = -\theta_{C/A}$$

$$\begin{aligned} \theta_A &= -A_1 - A_2 = \left(-\frac{1}{32} + \frac{1}{192} \right) \frac{w_0 L^2}{EI} \\ &= -\frac{5 w_0 L^3}{192 EI} \end{aligned}$$

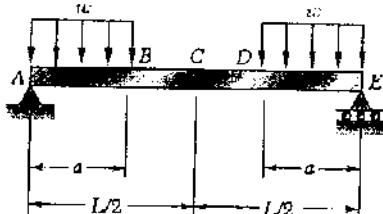
$$\text{Deflection at } C. \quad y_C = t_{AC}$$

$$t_{AC} = A_1 \left[\left(\frac{2}{3} \right) \left(\frac{L}{2} \right) \right] + A_2 \left[\left(\frac{4}{5} \times \frac{L}{2} \right) \right] = \left[\left(\frac{1}{3} \times \frac{L}{32} \right) - \left(\frac{2}{5} \times \frac{1}{192} \right) \right] \frac{w_0 L^4}{EI} = \frac{w_0 L^4}{120 EI}$$

$$y_C = -\frac{w_0 L^4}{120 EI}$$

Problem 9.113

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

Place reference tangent at C. $\theta_c = 0$

Reactions. $R_A = R_E = wa$

Bending moment

$$\text{Over AB} \quad M = wax - \frac{1}{2}wa^2$$

$$\text{Over BD} \quad M = \frac{1}{2}wa^2$$

Draw $\frac{M}{EI}$ diagram by parts.

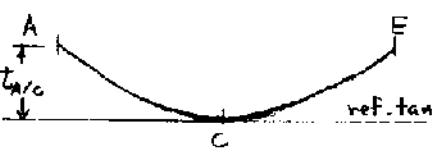
$$\frac{M_1}{EI} = \frac{w\alpha^2}{EI} \quad \frac{M_2}{EI} = -\frac{1}{2} \frac{w\alpha^2}{EI}$$

$$\frac{M_3}{EI} = \frac{1}{2} \frac{w\alpha^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} \alpha = \frac{1}{2} \frac{w\alpha^3}{EI}$$

$$A_2 = -\frac{1}{3} \frac{M_2}{EI} \alpha = -\frac{1}{6} \frac{w\alpha^3}{EI}$$

$$A_3 = \frac{M_3}{EI} \left(\frac{L}{2} - \alpha \right) = \frac{1}{4} \frac{w\alpha^2}{EI} (L - 2\alpha)$$



$$(a) \text{ Slope at } A. \quad \theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{1}{2} \frac{w\alpha^3}{EI} + \frac{1}{6} \frac{w\alpha^3}{EI} - \frac{1}{4} \frac{w\alpha^2}{EI} (L - 2\alpha) = -\frac{w\alpha^2}{EI} \left(\frac{1}{4}L - \frac{1}{6}\alpha \right)$$

$$= -\frac{1}{12} \frac{w\alpha^2}{EI} (3L - 2\alpha)$$

$$(b) \text{ Deflection at } C. \quad y_c = -t_{c/A}$$

$$\bar{x}_1 = \frac{2}{3}\alpha, \quad \bar{x}_2 = \frac{3}{4}\alpha, \quad \bar{x}_3 = \alpha + \frac{1}{2}\left(\frac{L}{2} - \alpha\right) = \frac{1}{4}(L + 2\alpha)$$

$$y_c = -t_{c/A} = -A_1 \bar{x}_1 - A_2 \bar{x}_2 - A_3 \bar{x}_3$$

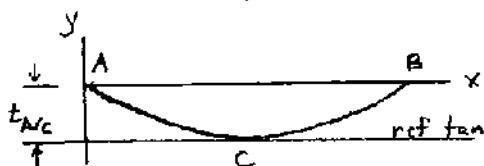
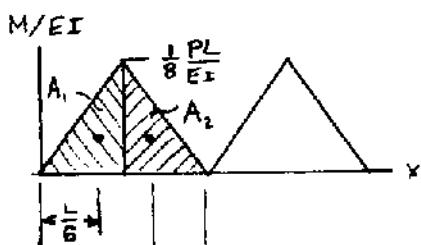
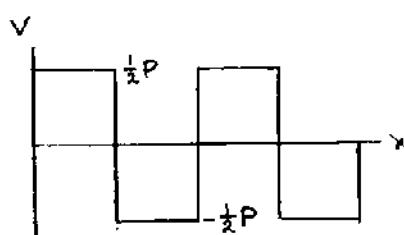
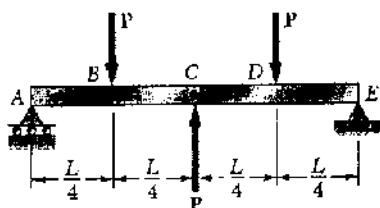
$$= -\left(\frac{1}{2} \frac{w\alpha^3}{EI}\right)\left(\frac{2}{3}\alpha\right) + \left(\frac{1}{6} \frac{w\alpha^3}{EI}\right)\left(\frac{3}{4}\alpha\right) - \frac{1}{4} \left(\frac{w\alpha^2}{EI}\right)(L - 2\alpha) \frac{1}{4}(L + 2\alpha)$$

$$= -\frac{1}{3} \frac{w\alpha^3}{EI} + \frac{1}{8} \frac{w\alpha^3}{EI} - \frac{1}{16} \frac{w\alpha^2}{EI} (L^2 - 4\alpha^2)$$

$$= -\frac{w\alpha^2}{EI} \left(\frac{1}{16}L^2 - \frac{1}{24}\alpha^2 \right) = -\frac{1}{48} \frac{w\alpha^2}{EI} (3L^2 - 2\alpha^2)$$

Problem 9.114

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

Place reference tangent at C. $\theta_c = 0$

$$\text{Reactions } R_A = R_E = \frac{1}{2}P$$

Draw V (shear) and M/EI diagrams.

$$A_1 = A_2 = \frac{1}{2} \left(\frac{1}{8} \frac{PL}{EI} \right) \frac{L}{4} = \frac{1}{64} \frac{PL^2}{EI}$$

(a) Slope at A.

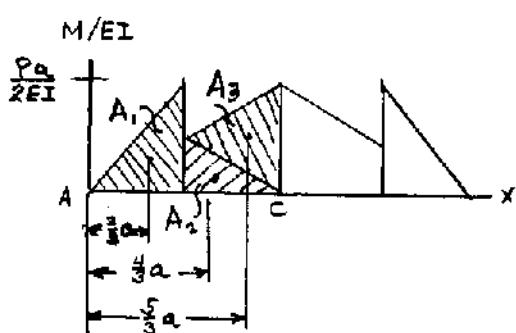
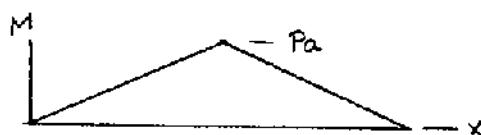
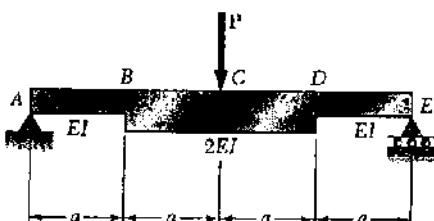
$$\begin{aligned} \theta_A &= \theta_c - \theta_{A/C} = 0 - A_1 - A_2 \\ &= -\frac{1}{32} \frac{PL^2}{EI} \end{aligned}$$

(b) Deflection at C.

$$\begin{aligned} y_C &= -t_{A/C} = -(A_1 \frac{L}{3} + A_2 \frac{L}{3}) \\ &= -\left(\frac{1}{64} \frac{PL^3}{EI} \cdot \frac{L}{6} + \frac{1}{64} \frac{PL^3}{EI} \cdot \frac{L}{3} \right) \\ &= -\frac{1}{128} \frac{PL^3}{EI} \end{aligned}$$

Problem 9.115

9.115 and 9.116 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading. $R_A = R_E = \frac{1}{2}P$

$$M_{max} = \left(\frac{1}{2}P\right)(2a) = Pa$$

Draw M and $\frac{M}{EI}$ diagrams.

$$A_1 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{Pa}{4EI}\right)a = \frac{1}{8}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

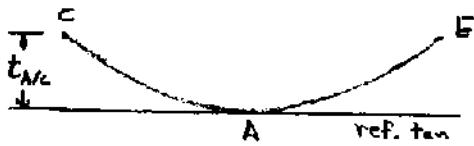
Place reference tangent at C. $\theta_c = 0$

(a) Slope at A.

$$\begin{aligned} \theta_A &= \theta_c - \theta_{cA} = 0 - (A_1 + A_2 + A_3) \\ &= -\frac{5}{8}\frac{Pa^2}{EI} \end{aligned}$$

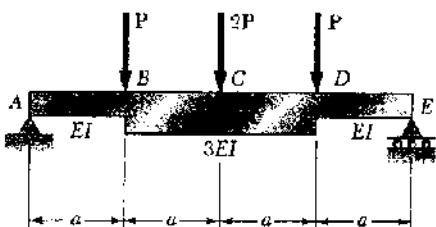
(b) Deflection at C.

$$\begin{aligned} |y_C| &= t_{AC} = A_1\left(\frac{2}{3}a\right) + A_2\left(\frac{4}{3}a\right) + A_3\left(\frac{5}{3}a\right) \\ &= \frac{1}{6}\frac{Pa^3}{EI} + \frac{1}{6}\frac{Pa^3}{EI} + \frac{5}{12}\frac{Pa^3}{EI} \\ &= \frac{3}{4}\frac{Pa^3}{EI} \end{aligned}$$



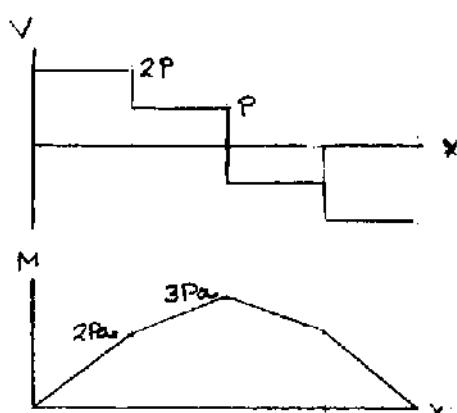
Problem 9.116

9.115 and 9.116 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading. $R_A = R_B = 2P$.

Draw V, M, and $\frac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2} \left(\frac{2Pa}{EI} \right) a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2} \left(\frac{2}{3} \frac{Pa}{EI} \right) a = \frac{1}{3} \frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2} \left(\frac{Pa}{EI} \right) a = \frac{1}{2} \frac{Pa^2}{EI}$$

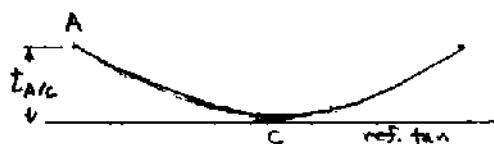
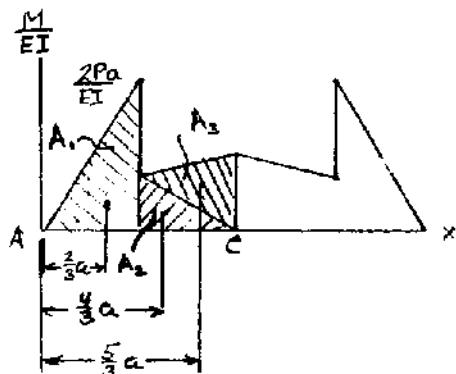
Place reference tangent at C. $\theta_c = 0$

(a) Slope at A.

$$\begin{aligned} \theta_A &= \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3) \\ &= -\frac{11}{6} \frac{Pa^2}{EI} \end{aligned}$$

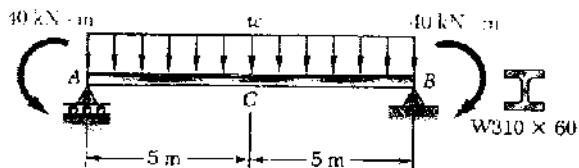
(b) Deflection at C.

$$\begin{aligned} |t_{AC}| &= t_{AC} = A_1 \left(\frac{2}{3}a \right) + A_2 \left(\frac{4}{3}a \right) + A_3 \left(\frac{5}{3}a \right) \\ &= \frac{35}{18} \frac{Pa^3}{EI} \end{aligned}$$



Problem 9.117

9.117 For the beam and loading shown and knowing that $w = 8 \text{ kN/m}$, determine (a) the slope at end A, (b) the deflection at midpoint C. Use $E = 200 \text{ GPa}$.



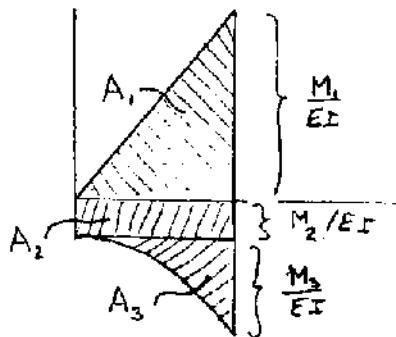
$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$= 25800 \text{ kN} \cdot \text{m}^2$$

M/EI



Symmetrical beam and loading.

$$R_A = R_B = \frac{1}{2}(8)(10) = 40 \text{ kN}$$

Bending moment

$$M = 40x - 40 - \frac{1}{2}(8)x^2$$

$$\text{At } x = 5$$

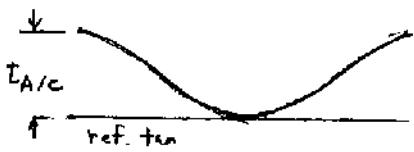
$$M = 200 - 40 - 100$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{200}{25800} = 7.7519 \times 10^{-5} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = \frac{-40}{25800} = -1.5504 \times 10^{-5} \text{ m}^{-1}$$

$$\frac{M_3}{EI} = \frac{-100}{25800} = -3.8760 \times 10^{-5} \text{ m}^{-1}$$



$$A_1 = \frac{1}{2}(7.7519 \times 10^{-5})(5) = 19.380 \times 10^{-5} \quad \bar{x}_1 = (\frac{2}{3})(5) = 3.3333 \text{ m}$$

$$A_2 = -1.5504(5) = -7.7520 \times 10^{-5} \quad \bar{x}_2 = (\frac{1}{2})(5) = 2.5 \text{ m}$$

$$A_3 = -\frac{1}{3}(3.8760)(5) = -6.4600 \times 10^{-5} \quad \bar{x}_3 = (\frac{2}{3})(5) = 3.75 \text{ m}$$

Place reference tangent at C. $\theta_c = 0$

$$(a) \text{ Slope at A. } \theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3)$$

$$\theta_A = -(19.380 \times 10^{-5} - 7.7520 \times 10^{-5} - 6.4600 \times 10^{-5}) = -5.17 \times 10^{-5} \text{ rad}$$

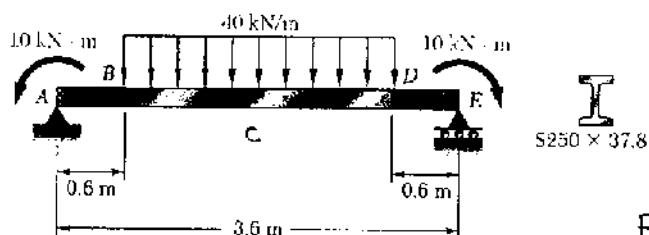
$$(b) \text{ Deflection at C. } |y_c| = t_{A/C}$$

$$= (19.380 \times 10^{-5})(3.3333) - (7.7520 \times 10^{-5})(2.5) - (6.4600 \times 10^{-5})(3.75)$$

$$= 21.0 \times 10^{-5} \text{ m} \quad = 21.0 \text{ mm} \downarrow$$

Problem 9.118

9.118 and 9.119 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint of the beam. Use $E = 200 \text{ GPa}$.

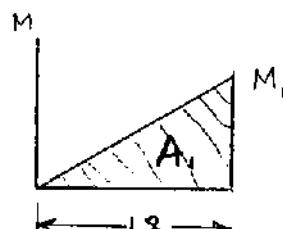


Use units of kN and m.

For $E = 200 \times 10^9 \text{ N/m}^2$

$$I = 51.1 \times 10^6 \text{ mm}^4 = 51.1 \times 10^{-6} \text{ m}^4$$

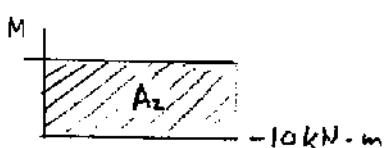
$$EI = (200 \times 10^9)(51.1 \times 10^{-6}) \\ = 10.22 \times 10^5 \text{ N} \cdot \text{m}^2 = 10220 \text{ kN} \cdot \text{m}^2$$



Place reference tangent at midpoint C.

$$\text{Reactions: } R_A = R_E = \frac{1}{2}(40)(3.6 - 1.2) = 48 \text{ kN}$$

Draw bending moment diagram of left half of beam by parts.



$$M_1 = (48)(1.8) = 86.4 \text{ kN} \cdot \text{m}$$

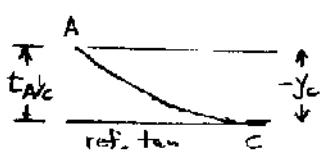
$$A_1 = \frac{1}{2}(1.8)(86.4) = 77.76 \text{ kN} \cdot \text{m}^2$$

$$A_2 = (1.8)(-10) = -18 \text{ kN} \cdot \text{m}^2$$

$$M_3 = \frac{1}{2}(40)(1.8 - 0.6)^2 = -28.8 \text{ kN} \cdot \text{m}$$

$$A_3 = \frac{1}{3}(1.2)(-28.8) = -11.52 \text{ kN} \cdot \text{m}^2$$

$$\bar{x} = \frac{1}{4}(1.2) = 0.3 \text{ m}$$



Slope at end A. $\theta_A = -\theta_{A/C}$

$$\theta_A = \frac{1}{EI} \left\{ -A_1 - A_2 - A_3 \right\} \\ = \frac{-77.76 + 18 + 11.52}{10220} = -4.72 \times 10^{-3}$$

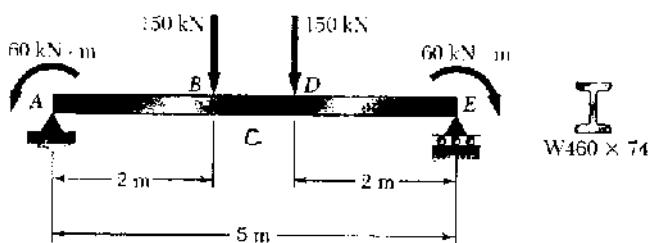
Deflection at midpoint C. $y_C = -t_{A/C}$

$$t_{A/C} = \frac{1}{EI} \left\{ 1.2 A_1 + 0.9 A_2 + (1.8 - 0.3) A_3 \right\} \\ = \frac{(1.2)(77.76) - (0.9)(18) - (1.5)(11.52)}{10220} = 5.85 \times 10^{-3} \text{ m}$$

$$y_C = -5.85 \times 10^{-3} \text{ m} \quad y_C = 5.85 \text{ mm} \downarrow$$

Problem 9.119

9.118 and 9.119 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint of the beam. Use $E = 200 \text{ GPa}$.

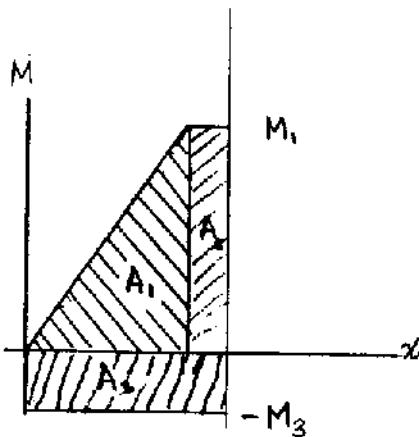


Use units of kN and m.

For W460 x 74

$$I = 333 \times 10^6 \text{ mm}^4 = 333 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(333 \times 10^{-6}) \\ = 66.6 \times 10^6 \text{ N}\cdot\text{m}^2 = 66600 \text{ kN}\cdot\text{m}^2$$



Symmetric beam and loading. Place reference tangent at midpoint C where $\theta_c = 0$.

Reactions. $R_A = R_E = 150 \text{ kN} \uparrow$

Draw bending moment diagram of left half of beam by parts

$$M_1 = (2)(150) = 300 \text{ kN}\cdot\text{m}$$

$$A_1 = (\frac{1}{2})(2)(300) = 300 \text{ kN}\cdot\text{m}^2$$

$$A_2 = (0.5)(300) = 150 \text{ kN}\cdot\text{m}^2$$

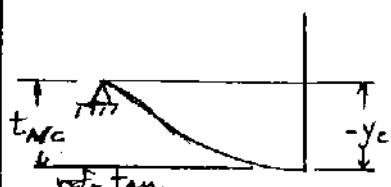
$$M_2 = -60 \text{ kN}\cdot\text{m}$$

$$A_3 = (2.5)(-60) = -150 \text{ kN}\cdot\text{m}^2$$

Slope at end A. $\theta_A = -\theta_{c/A}$

$$\theta_A = \frac{1}{EI} \{-A_1 - A_2 - A_3\}$$

$$= \frac{-300 - 150 + 150}{66600} = -4.50 \times 10^{-3}$$



Deflection at midpoint C.

$$y_c = -t_{nc}$$

$$t_{nc} = \frac{1}{EI} \left\{ \left(\frac{2}{3} \cdot 2\right) A_1 + \left(2 + \frac{0.5}{2}\right) A_2 + \left(\frac{2.5}{2}\right) A_3 \right\}$$

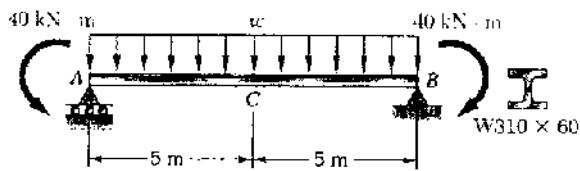
$$= \frac{400 + 337.5 - 187.5}{66600} = 8.26 \times 10^{-3} \text{ m}$$

$$y_c = -8.26 \times 10^{-3} \text{ m}$$

$$y_c = 8.26 \text{ mm} \downarrow$$

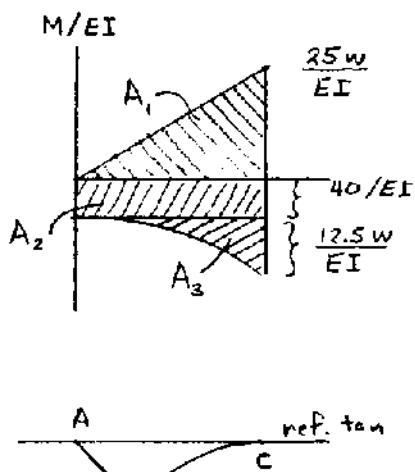
Problem 9.120

9.120 For the beam and loading of Prob. 9.117, determine the value of w for which the deflection is zero at the midpoint C of the beam. Use $E = 200 \text{ GPa}$.



Symmetric beam and loading.

$$R_A = R_B = 5w \quad (\text{w in kN/m})$$



Bending moment in $\text{kN}\cdot\text{m}$.

$$M = 5wx - 40 - \frac{1}{2}wx^2$$

At $x = 5 \text{ m}$

$$M = 25w - 40 - 12.5w$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{25w}{EI} \right) (5) = \frac{62.5w}{EI}$$

$$A_2 = - \frac{(40)(5)}{EI} = - \frac{200}{EI}$$

$$A_3 = - \frac{1}{3} \left(\frac{12.5w}{EI} \right) (5) = - \frac{20.833w}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(5) = 3.3333 \text{ m}$$

$$\bar{x}_2 = \frac{1}{2}(5) = 2.5 \text{ m}$$

$$\bar{x}_3 = \frac{3}{4}(5) = 3.75 \text{ m}$$

Place reference tangent at C .

Deflection at C is zero. $\delta_{AC} = y_A - y_C = 0$

$$A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3 = 0$$

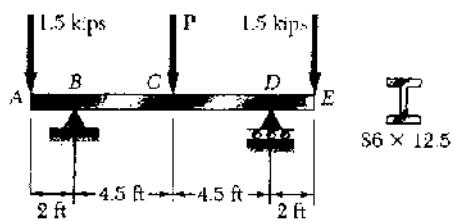
$$\left(\frac{62.5w}{EI} \right) (3.3333) - \left(\frac{200}{EI} \right) (2.5) - \left(\frac{20.833w}{EI} \right) (3.75) = 0$$

$$\frac{130.21w}{EI} - \frac{500}{EI} = 0$$

$$w = \frac{500}{130.21} = 3.84 \text{ kN/m}$$

Problem 9.121

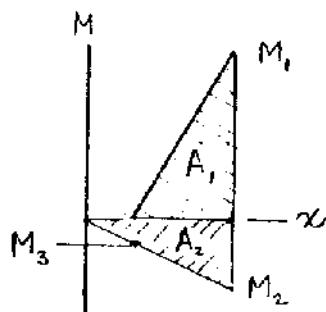
9.121 Knowing that the magnitude of the load P is 7 kips, determine (a) the slope at end A , (b) the deflection at end A , (c) the deflection at midpoint C of the beam. Use $E = 29 \times 10^6$ psi.



Use units of kips and ft. $P = 7$ kips

For SG x 12.5 $I = 22.1 \text{ in}^4$

$$EI = (20 \times 10^6)(22.1) = 640.9 \times 10^6 \text{ lb-in}^2 \\ = 4451 \text{ kip-ft}^2$$



Symmetric beam with symmetric loading. Place reference tangent at midpoint C where $\theta_c = 0$.

$$R_B = R_D = \frac{1}{2}(1.5 + 7 + 1.5) = 5 \text{ kips } \uparrow$$

Draw the bending moment diagram by parts for the left half of the beam.

$$M_1 = (4.5)(5) = 22.5 \text{ kip-ft}$$

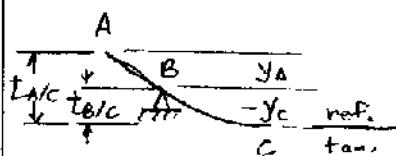
$$A_1 = \frac{1}{2}(4.5)(22.5) = 50.625 \text{ kip-ft}^2$$

$$M_2 = -(2+4.5)(1.5) = -9.75 \text{ kip-ft}$$

$$A_2 = \frac{1}{2}(6.5)(-9.75) = -31.6875 \text{ kip-ft}^2$$

$$M_3 = -(2)(1.5) = -3 \text{ kip-ft}$$

$$A_3 = \frac{1}{2}(2)(-3) = -3 \text{ kip-ft}^2$$



Formulas. $\theta_A = -\theta_{C/A}$, $y_A - y_C = t_{A/C}$, $y_C = y_A - t_{A/C}$

$$y_B = y_A - 2\theta_A + t_{B/A} = 0 \quad y_A = -2\theta_A - t_{B/A}$$

$$\theta_{C/A} = \frac{1}{EI} \{ A_1 + A_2 \} = \frac{50.625 - 31.6875}{4451} = 4.25466 \times 10^{-3}$$

$$t_{A/C} = \frac{1}{EI} \{ (2+3)A_1 + \frac{2}{3}(6.5)A_2 \} = \frac{115.8125}{4451} = 26.0194 \times 10^{-3} \text{ ft}$$

$$t_{B/A} = \frac{1}{EI} \{ \frac{1}{3}(2)A_3 \} = \frac{-2}{4451} = -0.44934 \times 10^{-3} \text{ ft}$$

(a) Slope at end A .

$$\theta_A = -4.25 \times 10^{-3}$$

(b) Deflection at A .

$$y_A = -(2)(-4.25466 \times 10^{-3}) - (-0.44934 \times 10^{-3}) \\ = 8.95866 \times 10^{-3} \text{ ft}$$

$$y_A = 0.1075 \text{ in. } \uparrow$$

(c) Deflection at C

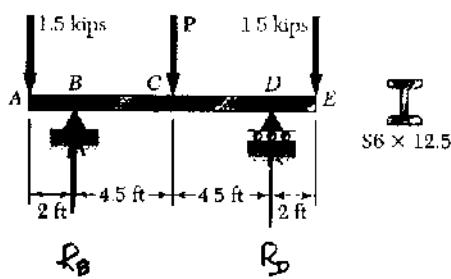
$$y_C = 8.95866 \times 10^{-3} - 26.0194 \times 10^{-3} = -17.06074 \times 10^{-3} \text{ ft}$$

$$y_C = -0.205 \text{ in.}$$

$$y_C = 0.205 \text{ in. } \downarrow$$

Problem 9.122

9.122 For the beam and loading shown, determine (a) the load P for which the deflection is zero at the midpoint C of the beam, (b) the corresponding deflection at end A . Use $E = 29 \times 10^6$ psi.



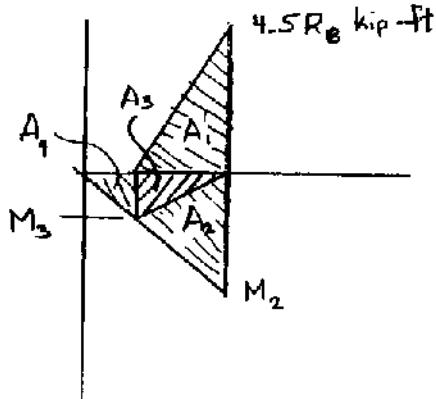
Use units of kips and ft.

$$\text{For } S6 \times 12.5 \quad I = 22.1 \text{ in}^4$$

$$EI = (29 \times 10^6)(22.1) = 640.9 \times 10^6 \text{ lb-in}^2 \\ = 4451 \text{ kip-ft}^2$$

Symmetric beam with symmetric loading. Place reference tangent at midpoint C where $\theta_C = 0$.

Draw the bending moment diagram by parts for the left half of the beam.



$$A_1 = \frac{1}{2}(4.5)(4.5)R_B = 10.125 R_B \text{ kip-ft}^2$$

$$M_2 = -(2+4.5)(1.5) = -9.75 \text{ kip-ft}$$

$$A_2 = \frac{1}{2}(4.5)(-9.75) = -21.9375 \text{ kip-ft}^2$$

$$M_3 = -(2)(1.5) = -3 \text{ kip-ft}$$

$$A_3 = \frac{1}{2}(4.5)(-3) = -6.75 \text{ kip-ft}^2$$

$$A_4 = \frac{1}{2}(2)(-3) = -3 \text{ kip-ft}^2$$

$$(a) \quad t_{B/C} = 0: \quad \frac{1}{EI} \left\{ \frac{2}{3}(4.5)A_1 + \frac{2}{3}(4.5)A_2 + \frac{1}{3}(4.5)A_3 \right\} = \\ = \frac{30.375 R_B - 75.9375}{EI} = 0 \quad R_B = 2.5 \text{ kips}$$

$$\text{By statics, } -(2)(1.5) + 2R_B - P = 0 \quad P = 2.00 \text{ kips}$$

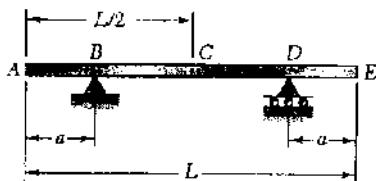
$$A_1 = (10.125)(2.5) = 25.3125 \text{ kip-ft}^2$$

$$(b) \quad y_A = t_{A/C} = \frac{1}{EI} \left\{ (2+3)A_1 + (2+3)A_2 + (2+1.5)A_3 + \left(\frac{2}{3} \cdot 2\right)A_4 \right\} \\ = \frac{(5)(25.3125) + (5)(-21.9375) + (3.5)(-6.75) + (\frac{4}{3})(-3)}{4451} \\ = -2.4152 \times 10^{-3} \text{ ft} = -0.0290 \text{ in.}$$

$$y_C = 0.0290 \text{ in.}$$

Problem 9.123

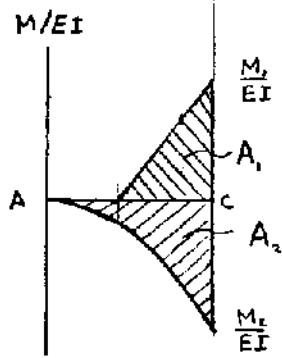
9.123 A uniform rod AE is to be supported at two points B and D . Determine the distance a for which the slope at ends A and E is zero



Let w = weight per unit length of rod.

Symmetric beam and loading.

$$R_B = R_D = \frac{1}{2} wL$$



Bending moment

$$\text{Over } AB \quad M = -\frac{1}{2}wx^2$$

$$\text{Over } BCD \quad M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2} \frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4} \frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = \frac{1}{2} \frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8} \frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} \left(\frac{1}{2} - a \right) = \frac{1}{16} \frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3} \left(\frac{M_2}{EI} \right) \frac{1}{2} = -\frac{1}{48} \frac{wL^3}{EI}$$



Place reference tangent at C . $\theta_c = 0$

$$\theta_A = \theta_c - \theta_{cA} = 0 - (A_1 + A_2) = 0$$

$$-\frac{1}{16} \frac{wL(L-2a)^2}{EI} + \frac{1}{48} \frac{wL^3}{EI} = 0$$

Let $u = \frac{a}{L}$ and divide by $\frac{wL^3}{48EI}$.

$$1 - 3(1-2u)^2 = 0$$

$$1 - 2u = \frac{\sqrt{3}}{3}$$

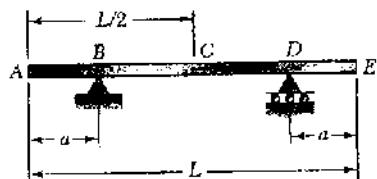
$$u = \frac{1}{2} \left(1 - \frac{\sqrt{3}}{3} \right) = 0.21132$$

$$\frac{a}{L} = 0.211$$

$$a = 0.211 L$$

Problem 9.124

*9.124 A uniform rod AE is to be supported at two points B and D . Determine the distance a from the ends of the rod to the points of support, if the downward deflections of points A , C , and E are to be equal.

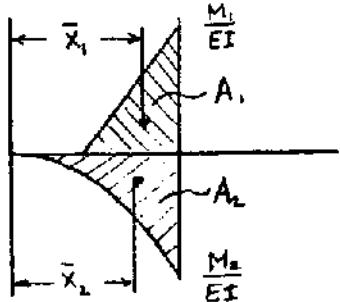


Let w = weight per unit length of rod.

Symmetric beam and loading.

$$R_B = R_D = \frac{1}{2} wL$$

M/EI



Bending moment:

$$\text{Over } AB \quad M = -\frac{1}{2}wx^2$$

$$\text{Over } BCD \quad M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2} \frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4} \frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2} \frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8} \frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} (\frac{L}{2}-a) = \frac{1}{16} \frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3} \left(\frac{M_2}{EI} \right) \left(\frac{L}{2} \right) = -\frac{1}{48} \frac{wL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3} \left(\frac{L}{2} - a \right) = \frac{1}{3} (L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4} \left(\frac{L}{2} \right) = \frac{3}{8} L$$

A C

Place reference tangent at C.

$$y_C - y_c = t_{AC} = 0$$

$$A_1 \bar{x}_1 + A_2 \bar{x}_2 = 0$$

$$\frac{1}{16} \frac{wL(L-2a)^2}{EI} \frac{1}{3}(L+a) - \frac{1}{48} \frac{wL^3}{EI} \frac{3}{8}L = 0$$

$$\text{Let } u = a/L. \text{ Divide by } \frac{WL^3}{48EI} -$$

$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

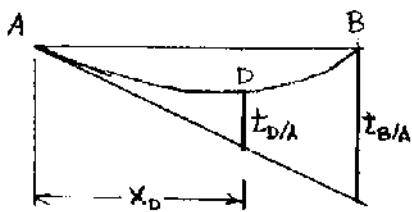
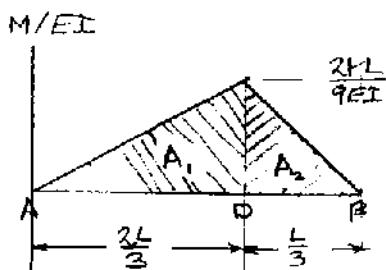
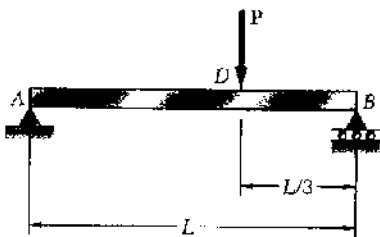
$$\text{Solving for } u, \quad u = 0.22315$$

$$\frac{a}{L} = 0.223 \quad a = 0.223 L$$

◀

Problem 9.125

9.125 through 9.128 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



$$\text{From statics } R_A = \frac{1}{3}P \uparrow, R_B = \frac{2}{3}P \uparrow$$

$$M_0 = R_A \left(\frac{2}{3}L\right) = \frac{2}{9}PL$$

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2} \left(\frac{2L}{3}\right) \left(\frac{2PL}{9EI}\right) = \frac{2PL^2}{27EI}$$

$$A_2 = \frac{1}{2} \left(\frac{L}{3}\right) \left(\frac{2PL}{9EI}\right) = \frac{PL^2}{27EI}$$

Place reference tangent at A.

$$t_{B/A} = A_1 \left(\frac{L}{3} + \frac{1}{3} \cdot \frac{2L}{3}\right) + A_2 \left(\frac{2}{3} \cdot \frac{L}{3}\right) = \frac{4PL^3}{81EI}$$

$$t_{D/A} = A_1 \left(\frac{1}{3} \cdot \frac{2L}{3}\right) = \frac{4PL^3}{243EI}$$

(a) Deflection at D.

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$$

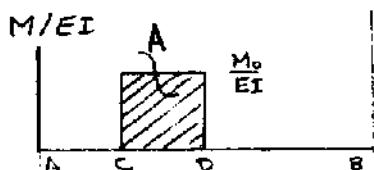
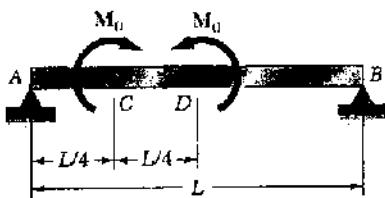
$$y_D = \frac{4PL^3}{243EI} - \left(\frac{2}{3}\right) \frac{4PL^3}{81EI}$$

$$= - \frac{4PL^3}{243EI} \quad y_D = \frac{4PL^3}{243EI} \downarrow$$

(b) Slope at end A. $\theta_A = -\frac{t_{B/A}}{L} = -\frac{4PL^2}{81EI} \quad \theta_0 = \frac{4PL^2}{81EI} \leftarrow$

Problem 9.126

9.125 through 9.128 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



From Statics $R_A = R_B = 0$.

Draw $\frac{M}{EI}$ diagram.

$$A = \left(\frac{M_0}{EI}\right)\left(\frac{L}{4}\right) = \frac{1}{4} \frac{M_0 L}{EI}$$

Place reference tangent at A.

$$t_{B/A} = A\left(\frac{L}{2} + \frac{L}{8}\right) = \frac{5}{32} \frac{M_0 L^2}{EI}$$

$$t_{D/A} = A\left(\frac{L}{8}\right) = \frac{1}{32} \frac{M_0 L^2}{EI}$$

(b) Slope at A.

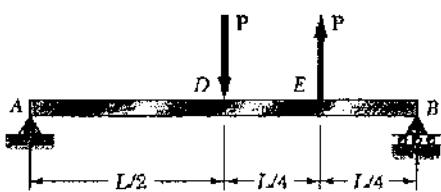
$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{32} \frac{M_0 L}{EI} = \frac{5M_0 L}{32EI}$$

(a) Deflection at D.

$$\begin{aligned} y_D &= t_{D/A} - \frac{x_0}{L} t_{B/A} = t_{D/A} - \frac{1}{2} t_{B/A} \\ &= \frac{1}{32} \frac{M_0 L^2}{EI} - \frac{5}{64} \frac{M_0 L^2}{EI} = -\frac{3}{64} \frac{M_0 L^2}{EI} \\ &= \frac{3M_0 L^2}{64EI} \end{aligned}$$

Problem 9.127

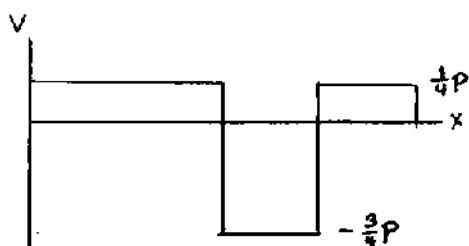
9.125 through 9.128 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



$$\sum \text{M}_B = 0 \quad -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \quad R_A = \frac{1}{4}P \uparrow$$

$$\sum \text{M}_A = 0 \quad -\frac{PL}{2} + P \frac{3L}{4} + R_B L = 0 \quad R_B = \frac{1}{4}P \downarrow$$

Draw V (shear) diagram and $\frac{M}{EI}$ diagram.

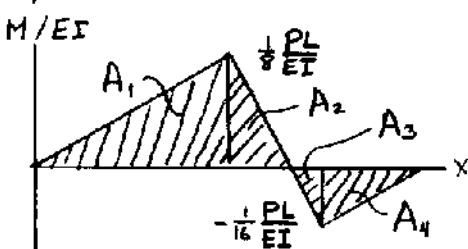


$$A_1 = \frac{1}{2} \left(\frac{1}{8} \frac{PL}{EI} \right) \left(\frac{L}{2} \right) = \frac{1}{32} \frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2} \left(\frac{1}{8} \frac{PL}{EI} \right) \left(\frac{L}{2} \right) = \frac{1}{96} \frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2} \left(-\frac{1}{16} \frac{PL}{EI} \right) \left(\frac{L}{12} \right) = -\frac{1}{384} \frac{PL^2}{EI}$$

$$A_4 = \frac{1}{2} \left(-\frac{1}{16} \frac{PL}{EI} \right) \left(\frac{L}{4} \right) = -\frac{1}{128} \frac{PL^2}{EI}$$



Place reference tangent at A.

$$t_{B/A} = \left(\frac{1}{32} \frac{PL^2}{EI} \right) \left(\frac{2L}{3} \right) + \left(\frac{1}{96} \frac{PL^2}{EI} \right) \left(\frac{L}{2} - \frac{1}{3} \cdot \frac{L}{6} \right) + \left(-\frac{1}{384} \frac{PL^2}{EI} \right) \left(\frac{L}{4} + \frac{1}{3} \cdot \frac{L}{12} \right) + \left(-\frac{1}{128} \frac{PL^2}{EI} \right) \left(\frac{2}{3} \cdot \frac{L}{4} \right)$$

$$= \frac{1}{48} \frac{PL^3}{EI} + \frac{1}{216} \frac{PL^3}{EI} - \frac{5}{6912} \frac{PL^3}{EI} - \frac{1}{768} \frac{PL^3}{EI}$$

$$= \frac{3}{128} \frac{PL^3}{EI}$$

$$t_{D/A} = \left(\frac{1}{32} \frac{PL^2}{EI} \right) \left(\frac{1}{3} \cdot \frac{L}{2} \right) = \frac{1}{192} \frac{PL^3}{EI}$$

(a) Deflection at D.

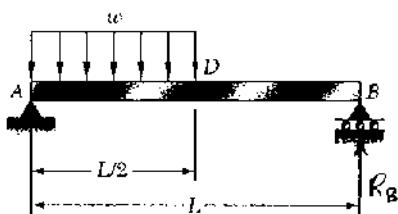
$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{1}{192} \frac{PL^3}{EI} - \frac{1}{2} \left(\frac{3}{128} \frac{PL^3}{EI} \right) = -\frac{5}{768} \frac{PL^3}{EI}$$

(b) Slope at A.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{128} \frac{PL^2}{EI}$$

Problem 9.128

9.125 through 9.128 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



$$+\rightarrow \sum M_A = 0 : R_B L - \frac{wL}{2} \left(\frac{L}{4}\right) = 0$$

$$R_B = \frac{1}{8} w L^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{R_B L}{EI} = \frac{wL^2}{8EI}$$

$$\frac{M_2}{EI} = -\frac{wL^3}{8EI}$$

$$A_1 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right) \frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_2 = \left(\frac{1}{3}\right)\left(\frac{L}{2}\right) \frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \left(\frac{1}{4}\right)\left(\frac{L}{2}\right) \frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_4 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right) \frac{wL^2}{8EI} = \frac{wL^3}{32EI}$$

(a) Deflection at D.

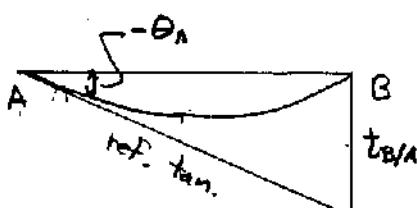
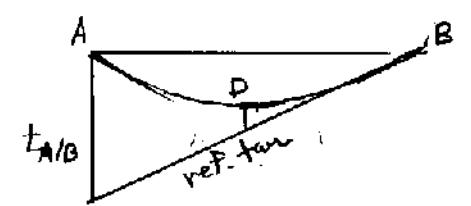
Place reference tangent at B.

$$y_D = t_{D/B} - \frac{L}{2} t_{A/B}$$

$$t_{D/A} = \left(\frac{1}{3} - \frac{L}{2}\right) A_1 = \frac{wL^4}{384EI}$$

$$t_{B/A} = \frac{L}{3} (A_1 + A_3 + A_4) + \left(\frac{L}{4} - \frac{L}{2}\right) A_4 = \frac{7wL^4}{384EI}$$

$$y_D = \frac{wL^4}{384EI} - \frac{1}{2} \cdot \frac{7wL^4}{384EI} = -\frac{5wL^4}{768EI}$$

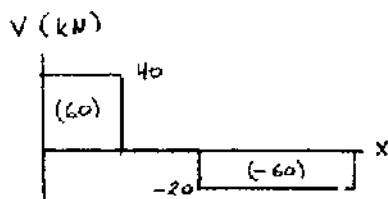
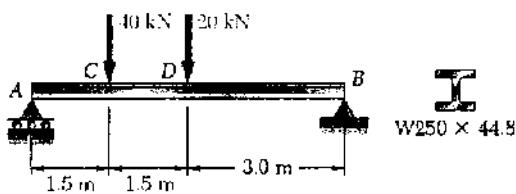


(b) Slope at A. Place reference tangent at A.

$$\begin{aligned} \theta_A &= -\frac{1}{L} t_{B/A} = -\left(\frac{1}{L}\right) \left\{ \left(\frac{2L}{3}\right) (A_1 + A_3 + A_4) + \left(L - \frac{1}{4} - \frac{L}{2}\right) A_2 \right\} \\ &= -\frac{3wL^2}{128EI} \end{aligned}$$

Problem 9.129

9.129 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D. Use $E = 200 \text{ GPa}$.



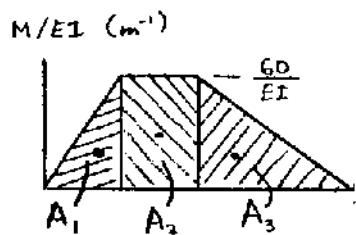
$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 71.1 \times 10^6 \text{ mm}^4 = 71.1 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(71.1 \times 10^{-6}) = 14220 \times 10^4 \text{ N} \cdot \text{m}^2 \\ = 14220 \text{ kN} \cdot \text{m}^2$$

$$\rightarrow \sum M_B = 0 : -6R_A + (4.5)(40) + (3)(20) = 0 \\ R_A = 40 \text{ kN.}$$

Draw shear and $\frac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2} \left(\frac{60}{EI} \right) (1.5) = \frac{45}{EI}$$

$$A_2 = \left(\frac{60}{EI} \right) (1.5) = \frac{90}{EI}$$

$$A_3 = \frac{1}{2} \left(\frac{60}{EI} \right) (3) = \frac{90}{EI}$$

Place reference tangent at A.



$$t_{B/A} = A_1(4.5 + 0.5) + A_2(3 + 0.75) \\ + A_3(2.0) = \frac{742.5}{EI} \text{ m.}$$

$$t_{D/A} = A_1(1.5 + 0.5) + A_2(0.75) \\ = \frac{157.5}{EI} \text{ m}$$

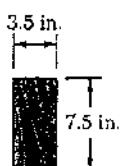
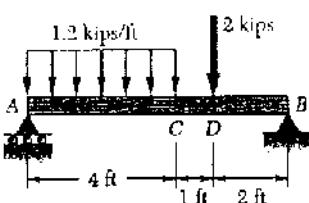
$$(a) \text{ Slope at } A. \quad \theta_A = -\frac{t_{B/A}}{L} = -\frac{742.5}{6EI} = -\frac{123.75}{EI} = -\frac{123.75}{14220} \\ = -8.70 \times 10^{-3} \text{ rad.}$$

(b) Deflection at D.

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{157.5}{EI} - \left(\frac{3}{6} \right) \left(\frac{742.5}{EI} \right) = -\frac{213.75}{EI} \\ = -\frac{213.75}{14220} = -15.03 \times 10^{-3} \text{ m} \\ = 15.03 \text{ mm} \downarrow$$

Problem 9.130

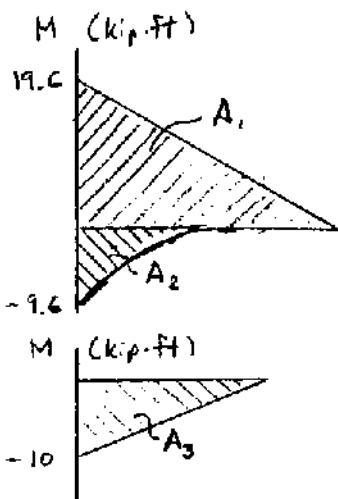
9.130 For the timber beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D. Use $E = 1.5 \times 10^6$ psi.



$$I = \frac{1}{12}(3.5)(7.5)^3 = 123.047 \text{ in}^4$$

$$E = 1.5 \times 10^6 \text{ psi} = 1.5 \times 10^3 \text{ ksi}$$

$$EI = 184.57 \times 10 \text{ kip-in}^2 = 1281.7 \text{ kip-ft}^2$$



$$\sum M_A = 0 \quad 7R_B - (2)(5) - (1.2)(4)(2) = 0$$

$$R_B = 2.8 \text{ kip}$$

Draw bending moment diagram by parts.

$$M_1 = (2.8)(7) = 19.6 \text{ kip-ft}$$

$$M_2 = -(1.2)(4)(2) = -9.6 \text{ kip-ft}$$

$$M_3 = -(2)(5) = -10 \text{ kip-ft}$$

$$A_1 = \frac{1}{2}(7)(19.6) = 68.6 \text{ kip-ft}^2$$

$$A_2 = \frac{1}{2}(4)(-9.6) = -12.8 \text{ kip-ft}^2$$

$$A_3 = \frac{1}{2}(5)(-10) = -25.0 \text{ kip-ft}^2$$



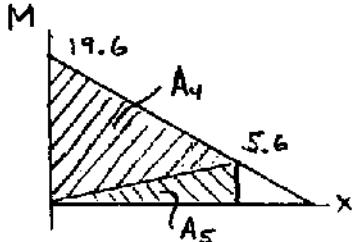
Draw reference tangent at A.

$$\theta_A = -\frac{t_{B/A}}{L}$$

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$$

$$EI t_{B/A} = A_1(7 - \frac{7}{3}) + A_2(7 - 1) + A_3(7 - \frac{5}{3}) = 110.0 \text{ kip-ft}^2$$

$$(a) \quad \theta_A = -\frac{EI t_{B/A}}{EIL} = -\frac{110.0}{(1281.7)(7)} = -12.26 \times 10^{-3} \text{ rad}$$



$$A_4 = \frac{1}{2}(19.6)(5) = 49 \text{ kip-ft}^2$$

$$A_5 = \frac{1}{2}(5.6)(5) = 14 \text{ kip-ft}^2$$

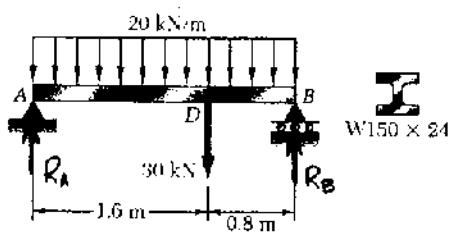
$$EI t_{D/A} = A_4(5 - \frac{5}{3}) + A_5(\frac{5}{3}) + A_2(5 - 1) + A_3(5 - \frac{5}{3}) = 52.133 \text{ kip-ft}^3$$

$$EI y_D = 52.133 - \frac{5}{3}(110.0) = -26.438 \text{ kip-ft}^3$$

$$(b) \quad y_D = -\frac{26.438}{1281.7} = -20.63 \times 10^{-3} \text{ ft} = 0.248 \text{ in.}$$

Problem 9.131

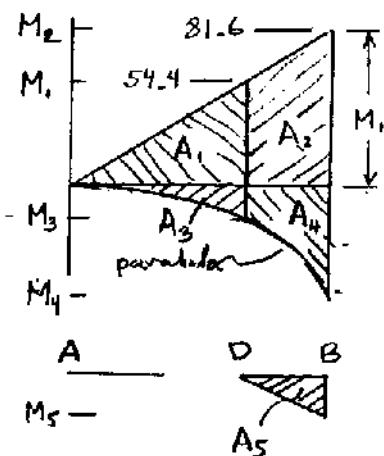
9.131 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point D. Use $E = 200 \text{ GPa}$.



Units: Forces in kN. Lengths in meters.

$$\text{For W150 x 24} \quad I = 13.4 \times 10^6 \text{ mm}^4 \\ = 13.4 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^6 \text{ N} \cdot \text{m}^2 \\ = 2680 \text{ kN} \cdot \text{m}^2$$



$$+\sum M_B = 0: -2.4 R_A + (0.8)(30) + (1.2)(2.4)(20) = 0$$

$$R_A = 34 \text{ kN} \uparrow$$

Draw bending moment diagram by parts.

$$M_1 = (1.6)(34) = 54.4 \text{ kN} \cdot \text{m}$$

$$M_2 = (2.4)(34) = 81.6 \text{ kN} \cdot \text{m}$$

$$M_3 = \frac{1}{3}(20)(1.6)^2 = -25.6 \text{ kN} \cdot \text{m}$$

$$M_4 = -\frac{1}{3}(20)(2.4)^2 = -57.6 \text{ kN} \cdot \text{m}$$

$$M_5 = -(0.8)(30) = -24 \text{ kN} \cdot \text{m}$$

$$A_1 = \frac{1}{2}(1.6)(54.4) = 43.52 \text{ kN} \cdot \text{m}^2$$

$$A_1 + A_2 = \frac{1}{2}(2.4)(81.6) = 97.92 \text{ kN} \cdot \text{m}^2$$

$$A_3 = \frac{1}{3}(1.6)(-25.6) = -13.6533 \text{ kN} \cdot \text{m}^2$$

$$A_3 + A_4 = \frac{1}{3}(2.4)(-57.6) = -46.08 \text{ kN} \cdot \text{m}^2$$

$$A_5 = \frac{1}{2}(0.8)(-24) = -9.6 \text{ kN} \cdot \text{m}^2$$

(a) Slope at A. Place reference tangent at A. $\theta_A = -\frac{1}{L} t_{BA}$

$$t_{BA} = \frac{1}{EI} \left\{ (A_1 + A_2) \left(\frac{1}{3}\right)(2.4) + (A_3 + A_4) \left(\frac{1}{3}\right)(2.4) + A_5 \left(\frac{1}{3}\right)(0.8) \right\}$$

$$= \frac{48.128}{2680} = 17.9582 \times 10^{-3} \text{ m}$$

$$\theta_A = -\frac{17.9582 \times 10^{-3}}{2.4} = -7.48258 \times 10^{-3} \quad \theta_A = 7.48 \times 10^{-3} \text{ rad} \rightarrow$$

(b) Deflection at point D. $y_D = t_{D/A} + \theta_A x_D$

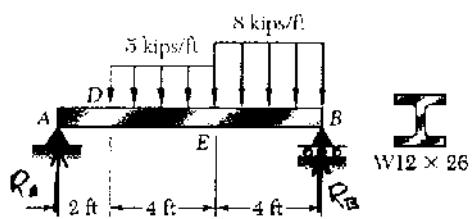
$$t_{D/A} = \frac{1}{EI} \left\{ A_1 \left(\frac{1}{3}\right)(1.6) + A_2 \left(\frac{1}{4}\right)(1.6) \right\} = \frac{17.7493}{2680} = 6.62289 \times 10^{-3} \text{ m}$$

$$y_D = 6.62289 \times 10^{-3} + (-7.48258 \times 10^{-3})(1.6) = -5.3492 \times 10^{-3} \text{ m}$$

$$y_D = 5.35 \text{ mm} \downarrow \rightarrow$$

Problem 9.132

9.132 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point E. Use $E = 29 \times 10^6$ psi.



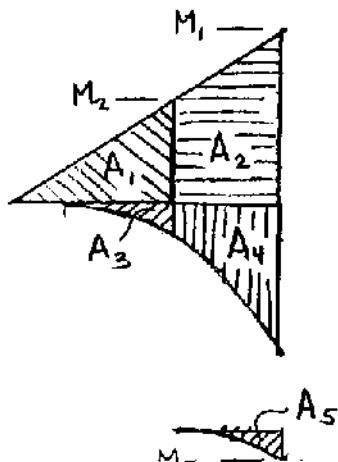
Units: Forces in kips, lengths in ft.

$$\text{For } W12 \times 26, \quad I = 204 \text{ in}^4$$

$$EI = (29 \times 10^6)(204) = 5.916 \times 10^9 \text{ lb-in}^2 \\ = 41083 \text{ kip-ft}^2$$

$$\rightarrow \sum M_B = 0:$$

$$\leftarrow -10R_A + (6)(4)(5) + (2)(4)(8) = 0 \rightarrow R_A = 18.4 \text{ kips} \uparrow$$



Consider loading as 5 kips/ft from D to B plus 3 kips/ft from E to B. Draw bending moment diagram by parts.

$$M_1 = 10R_A = 184 \text{ kip-ft}$$

$$M_2 = -6R_A = -10.4 \text{ kip-ft}$$

$$M_3 = -\frac{1}{2}(5)(8)^2 = -160 \text{ kip-ft}$$

$$M_4 = -\frac{1}{2}(5)(4)^2 = -40 \text{ kip-ft}$$

$$M_5 = -\frac{1}{2}(3)(4)^2 = -24 \text{ kip-ft}$$

$$A_1 + A_2 = \frac{1}{2}(10)(184) = 920 \text{ kip-ft}^2$$

$$A_1 = \frac{1}{3}(6)(10.4) = 331.2 \text{ kip-ft}^2$$

$$A_3 + A_4 = \frac{1}{3}(8)(-160) = -426.667 \text{ kip-ft}^2$$

$$A_3 = \frac{1}{3}(4)(-40) = -53.333 \text{ kip-ft}^2$$

$$A_5 = \frac{1}{3}(4)(-24) = -32 \text{ kip-ft}^2$$

$$(a) \text{ Slope at } A. \quad y_B = y_A + \theta_A L + t_{B/A}. \quad y_A = y_B = 0$$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \frac{1}{EI} \{ (A_1 + A_2) \left(\frac{1}{3} \right) (10) + (A_3 + A_4) \left(\frac{1}{4} \right) (8) + (A_5) \left(\frac{1}{4} \right) (4) \}$$

$$= \frac{2181.33}{41083} = 53.096 \times 10^{-3} \text{ ft}$$

$$\theta_A = -\frac{53.096 \times 10^{-3}}{10} = -5.3096 \times 10^{-3}$$

$$\theta_A = -5.31 \text{ rad} \leftarrow$$

$$(b) \text{ Deflection at } E. \quad y_E = x_E \theta_A + t_{E/A}$$

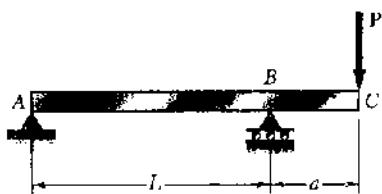
$$t_{E/A} = \frac{1}{EI} \{ (A_1) \left(\frac{1}{3} \right) (6) + (A_3) \left(\frac{1}{4} \right) (4) \} = \frac{609.067}{41083} = 14.8253 \times 10^{-3} \text{ ft}$$

$$y_E = (6)(-5.3096 \times 10^{-3}) + 14.8253 \times 10^{-3} = -17.0323 \times 10^{-3} \text{ ft}$$

$$= 0.204 \text{ in.} \downarrow$$

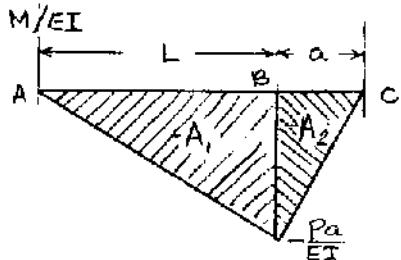
Problem 9.133

9.133 For the beam and loading shown, determine (a) the slope at point C, (b) the deflection at point C.



$$A_1 = -\frac{PaL}{2EI}$$

$$A_2 = -\frac{Pa^2}{2EI}$$



$$t_{A/B} = A_1 \left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

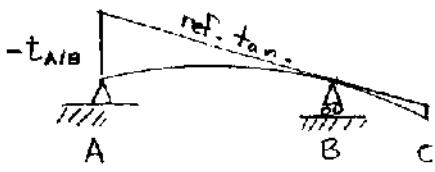
$$\theta_B = \frac{t_{A/B}}{L} = -\frac{PaL}{3EI}$$

$$(a) \text{ Slope at } C. \quad \theta_C = \theta_B + \theta_{c/B}$$

$$\theta_C = -\frac{PaL}{3EI} - \frac{Pa^2}{2EI}$$

$$= -\frac{Pa(2L+3a)}{6EI}$$

$$\theta_C = \frac{Pa(2L+3a)}{6EI}$$



Deflection at point C.

$$y_C = a\theta_B + t_{c/B}$$

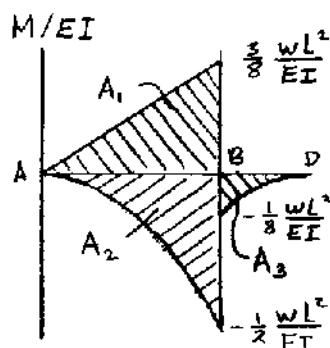
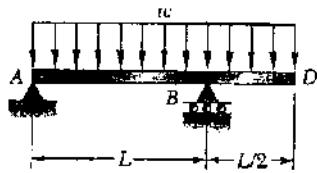
$$y_C = -\frac{Pa^2L}{3EI} + \left(-\frac{Pa^2}{2EI}\right)\left(\frac{2}{3}a\right)$$

$$= -\frac{Pa^2(L+a)}{3EI}$$

$$y_C = \frac{Pa^2(L+a)}{3EI}$$

Problem 9.134

9.134 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D.



$$\rightarrow \sum M_A = 0: -R_A L + (\frac{3}{2} w L)(\frac{1}{4} L) = 0 \quad R_A = \frac{3}{8} w L$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{3}{8} \frac{w L^2}{E I} \right) L = \frac{3}{16} \frac{w L^3}{E I}$$

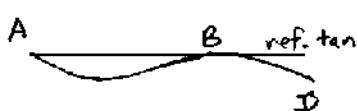
$$A_2 = -\frac{1}{3} \left(\frac{1}{2} \frac{w L^2}{E I} \right) L = -\frac{1}{6} \frac{w L^3}{E I}$$

$$A_3 = -\frac{1}{3} \left(\frac{1}{8} \frac{w L^2}{E I} \right) \frac{L}{2} = -\frac{1}{48} \frac{w L^4}{E I}$$

Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= A_1 \frac{L}{3} + A_2 \frac{L}{4} \\ &= \frac{1}{16} \frac{w L^4}{E I} - \frac{1}{24} \frac{w L^4}{E I} = -\frac{1}{48} \frac{w L^4}{E I} \end{aligned}$$

(a) Slope at A.



$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{1}{48} \frac{w L^3}{E I}$$

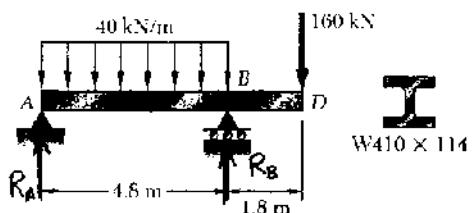
$$\begin{aligned} t_{D/A} &= A_1 \left(\frac{L}{3} + \frac{L}{2} \right) + A_2 \left(\frac{L}{4} + \frac{L}{2} \right) + A_3 \left(\frac{3}{4} \cdot \frac{L}{2} \right) \\ &= \frac{5}{32} \frac{w L^4}{E I} - \frac{1}{8} \frac{w L^4}{E I} - \frac{1}{128} \frac{w L^4}{E I} = \frac{3}{128} \frac{w L^4}{E I} \end{aligned}$$

(b) Deflection at D.

$$\begin{aligned} y_D &= t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{3}{128} \frac{w L^4}{E I} - \frac{3}{2} \cdot \frac{1}{48} \frac{w L^4}{E I} = -\frac{1}{128} \frac{w L^4}{E I} \\ y_D &= \frac{1}{128} \frac{w L^4}{E I} \downarrow \end{aligned}$$

Problem 9.135

9.135 For the beam and loading shown, determine (a) the slope at point B, (b) the deflection at point D. Use $E = 200 \text{ GPa}$.



Units: Forces in kN; lengths in meters.

$$I = 462 \times 10^8 \text{ mm}^4 = 462 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(462 \times 10^{-6})$$

$$= 92.4 \times 10^6 \text{ N} \cdot \text{m}^2 = 92400 \text{ kN} \cdot \text{m}^2$$

$$+\sum M_B = 0: -4.8 R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36 \text{ kN}$$

Draw bending moment diagram by parts.

$$A_1 = \frac{1}{2}(4.8)(172.8) = 414.72 \text{ kN} \cdot \text{m}^2$$

$$A_2 = \frac{1}{3}(4.8)(-460.8) = -737.28 \text{ kN} \cdot \text{m}^2$$

$$A_3 = \frac{1}{2}(1.8)(-288) = -259.2 \text{ kN} \cdot \text{m}^2$$

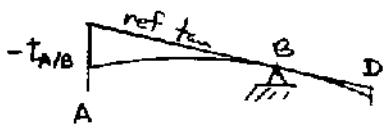
Place reference tangent at B.

$$(a) \text{ Slope at } B. \quad y_A = y_B - L\theta_B + t_{A/B}$$

$$\theta_B = \frac{t_{B/A}}{L} = \frac{1}{EI} \left\{ A_1 \left(\frac{2}{3}\right)(4.8) + A_2 \left(\frac{3}{4}\right)(4.8) \right\}$$

$$= \frac{-1327.104}{(92400)(4.8)} = -2.9922 \times 10^{-3}$$

$$\theta_B = 2.99 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$



$$(b) \text{ Deflection at } D. \quad y_D = y_B + a\theta_B + t_{D/B}$$

$$= 0 + (1.8)(-2.9922 \times 10^{-3})$$

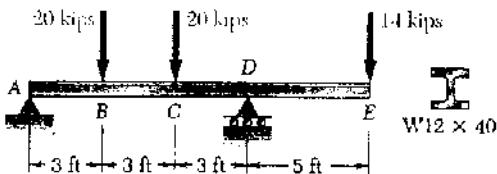
$$- \frac{1}{EI} \left\{ A_3 \left(\frac{2}{3}\right)(1.8) \right\}$$

$$= -5.3860 \times 10^{-3} - \frac{311.04}{92400}$$

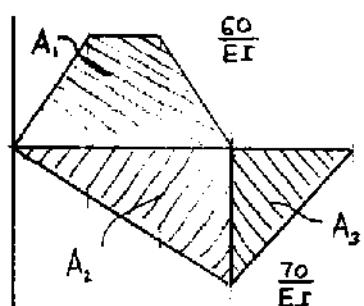
$$= -8.75 \times 10^{-3} \text{ m} \quad y_D = -8.75 \text{ mm} \quad \blacktriangleleft$$

Problem 9.136

9.136 For the beam and loading shown, determine (a) the slope at point D, (b) the deflection at point E. Use $E = 29 \times 10^6$ psi.



M/EI



$$A_1 = [2 \cdot \frac{1}{2}(3)(60) + (3)(60)]/EI = 360/EI$$

$$A_2 = \frac{1}{2}(9)(70)/EI = -315/EI$$

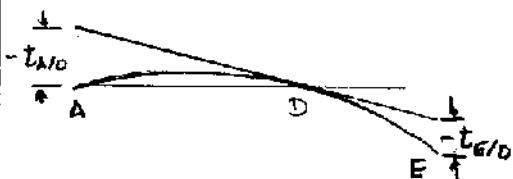
$$A_3 = \frac{1}{2}(5)(70) = -175/EI$$

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 310 \text{ in}^4$$

$$EI = (29 \times 10^3)(310) = 8.99 \times 10^6 \text{ kip-in}^2$$

$$= 62430 \text{ kip-ft}^2$$



Place reference tangent at D.

$$t_{A/D} = A_1(4.5) + A_2(6) = -270/EI \text{ ft.}$$

$$(a) \text{ Slope at } D. \quad \theta_D = \frac{t_{A/D}}{L} = -\frac{270}{9EI} = -\frac{30}{EI}$$

$$= -0.48054 \times 10^{-3} = 0.481 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

$$t_{E/D} = A_3(\frac{2}{3} \cdot 5) = -583.333/EI = -9.3438 \times 10^{-3} \text{ ft}$$

(b) Deflection at E.

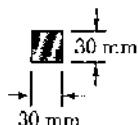
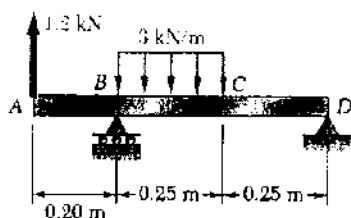
$$y_E = L_{DE} \theta_D + t_{E/D}$$

$$= -(5)(0.48054 \times 10^{-3}) - 9.3438 \times 10^{-3} = -11.75 \times 10^{-3} \text{ ft}$$

$$= 0.1410 \text{ in. } \downarrow \quad \blacktriangleleft$$

Problem 9.137

9.137 Knowing the beam AD is made of a solid steel bar, determine the (a) slope at point B , (c) the deflection at point A . Use $E = 200 \text{ GPa}$.

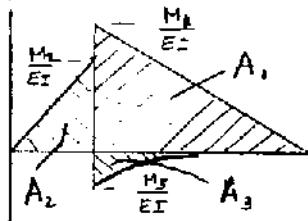


$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \frac{1}{12}(30)(30)^3 = 67.5 \times 10^6 \text{ mm}^4 \\ = 67.5 \times 10^{-7} \text{ m}^4$$

$$EI = (200 \times 10^9)(67.5 \times 10^{-7}) = 13500 \text{ N}\cdot\text{m}^2 \\ = 13.5 \text{ kN}\cdot\text{m}^2$$

M/EI



$$\textcircled{D} \sum M_B = 0 \quad -(0.2)(1.2) - (3)(0.25)(0.125) + 5R_o = 0 \\ R_o = 0.6675 \text{ kN}$$

Draw $\frac{M}{EI}$ diagram by parts.

$$M_1 = (0.6675)(0.5) = 0.33375 \text{ kN}\cdot\text{m}$$

$$M_2 = (1.2)(0.2) = 0.240 \text{ kN}\cdot\text{m}$$

$$M_3 = -\frac{1}{2}(3)(0.25)^2 = -0.09375 \text{ kN}\cdot\text{m}$$

$$A_1 = \frac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$$

$$A_2 = \frac{1}{2}(0.240)(0.2)/EI = 0.024/EI$$

$$A_3 = \frac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$$

Place reference tangent at B .

$$t_{D/B} = A_1(\frac{2}{3}(0.5)) + A_3(\frac{3}{4}(0.25) + 0.25) = 0.024395/EI$$

$$(a) \text{ Slope at } B, \quad \theta_B = -\frac{t_{D/B}}{L} = -\frac{0.024395}{0.5 EI} = -\frac{0.048789}{EI} \\ = -3.6140 \times 10^{-3} \quad 3.61 \times 10^{-3} \text{ rad} \quad \blacksquare$$

$$t_{A/B} = A_2(\frac{2}{3}(0.20)) = 0.0032/EI = 0.23704 \times 10^{-3} \text{ m}$$

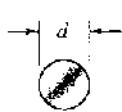
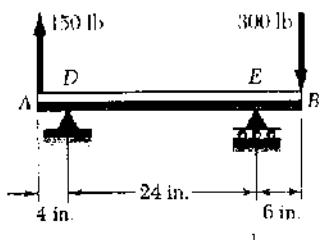
(b) Deflection at A .

$$y_A = t_{A/B} - L_{AB} \theta_B$$

$$= 0.23704 \times 10^{-3} - (0.2)(-3.6140 \times 10^{-3}) = 0.960 \times 10^{-3} \text{ m} \\ = 0.960 \text{ mm} \quad \blacksquare$$

Problem 9.138

9.138 Knowing that the beam AB is made of a solid steel rod of diameter $d = 0.75$ in., determine for the loading shown (a) the slope at point D , (b) the deflection at point A . Use $E = 29 \times 10^6$ psi.

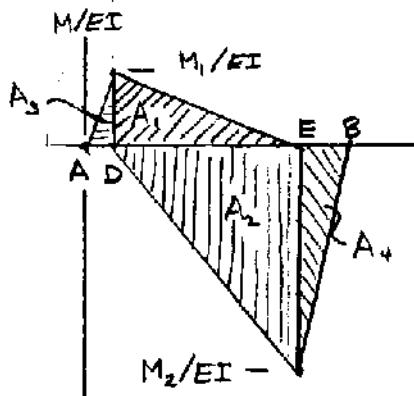


Units: Forces in lb; lengths in inches.

$$c = \frac{d}{2} = \frac{1}{2}(0.75) = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4}(0.375)^4 = 0.0155316 \text{ in}^4$$

$$EI = (29 \times 10^6)(0.0155316) = 450.4 \times 10^6 \text{ lb-in}^2$$



Draw $\frac{M}{EI}$ diagram by parts by considering the bending moment diagram due to each of the applied loads.

$$\frac{M_1}{EI} = \frac{(150)(4)}{450.4 \times 10^6} = 1.33215 \times 10^{-3} \text{ in}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(300)(6)}{450.4 \times 10^6} = -3.99645 \times 10^{-3} \text{ in}^{-1}$$

$$A_1 = \frac{1}{2}(24)(1.33215 \times 10^{-3}) = 15.9858 \times 10^{-3}$$

$$A_2 = \frac{1}{2}(24)(-3.99645 \times 10^{-3}) = -47.9574 \times 10^{-3}$$

$$A_3 = \frac{1}{2}(4)(1.33215 \times 10^{-3}) = 2.6643 \times 10^{-3}$$

Place reference tangent at D .

(a) Slope at point D . $y_E = y_D + L\theta_D + t_{E/D}$ $\theta_D = -t_{E/D}/L$

$$t_{E/A} = 16A_1 + 8A_2 = -127.8864 \times 10^{-3} \text{ in.}$$

$$\theta_D = \frac{-127.8864 \times 10^{-3}}{24} = 5.3286 \times 10^{-3} \quad \theta_D = 5.33 \times 10^{-3} \text{ rad} \leftarrow$$

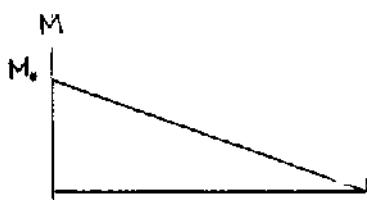
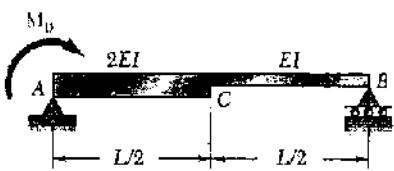
(b) Deflection at A . $y_A = y_D - a\theta_D + t_{A/D} = t_{A/D} - a\theta_D$

$$y_A = A_3(\frac{2}{3})(4) - (4)(5.3286 \times 10^{-3}) = -14.21 \times 10^{-3} \text{ in.}$$

$$y_A = -0.01421 \text{ in.} \downarrow$$

Problem 9.139

9.139 and 9.140 For the beam and loading shown, determine (a) the slope at end A, (b) the slope at end B, (c) the deflection at the midpoint C.



Draw bending moment and $\frac{M}{EI}$ diagrams.

$$A_1 = \frac{1}{2} \left(\frac{M_o}{2EI} \right) \left(\frac{L}{2} \right) = \frac{1}{8} \frac{M_o L}{EI}$$

$$A_2 = \frac{1}{2} \left(\frac{M_o}{4EI} \right) \left(\frac{L}{2} \right) = \frac{1}{16} \frac{M_o L}{EI}$$

$$A_3 = \frac{1}{2} \left(\frac{M_o}{2EI} \right) \left(\frac{L}{2} \right) = \frac{1}{8} \frac{M_o L}{EI}$$

Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= A_1 \left(\frac{L}{2} + \frac{2}{3} \frac{L}{2} \right) + A_2 \left(\frac{L}{2} + \frac{1}{3} \frac{L}{2} \right) + A_3 \left(\frac{2}{3} \frac{L}{2} \right) \\ &= \left(\frac{1}{8} \frac{M_o L}{EI} \right) \left(\frac{5}{2} L \right) + \left(\frac{1}{16} \frac{M_o L}{EI} \right) \left(\frac{2}{3} L \right) + \left(\frac{1}{8} \frac{M_o L}{EI} \right) \left(\frac{1}{3} L \right) \\ &= \frac{3}{16} \frac{M_o L^2}{EI} \end{aligned}$$

(a) Slope at A.

$$\theta_A = - \frac{t_{B/A}}{L} = - \frac{3}{16} \frac{M_o L}{EI}$$



(b) Slope at B.

$$\begin{aligned} \theta_B &= \theta_A + \theta_{B/A} = \theta_A + A_1 + A_2 + A_3 \\ &= - \frac{3}{16} \frac{M_o L}{EI} + \frac{1}{8} \frac{M_o L}{EI} + \frac{1}{16} \frac{M_o L}{EI} + \frac{1}{8} \frac{M_o L}{EI} \\ &= \frac{1}{8} \frac{M_o L}{EI} \end{aligned}$$

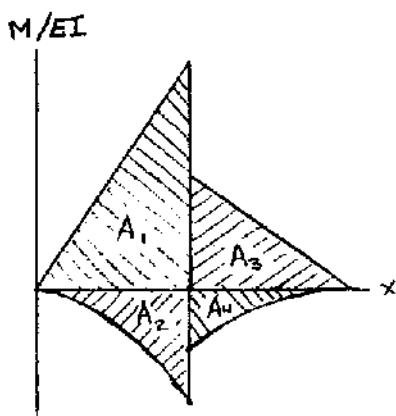
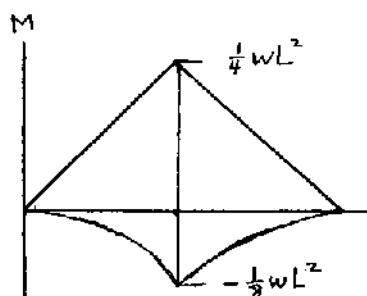
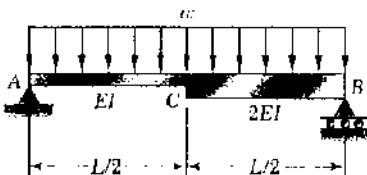
$$t_{C/A} = A_1 \left(\frac{2}{3} \frac{L}{2} \right) + A_2 \left(\frac{1}{3} \frac{L}{2} \right) = \left(\frac{1}{8} \frac{M_o L}{EI} \right) \left(\frac{1}{3} L \right) + \left(\frac{1}{16} \frac{M_o L}{EI} \right) \left(\frac{1}{6} L \right) = \frac{5}{48} \frac{M_o L^2}{EI}$$

(c) Deflection at C.

$$y_C = t_{C/A} + \frac{L}{2} \theta_A = \frac{5}{48} \frac{M_o L^2}{EI} + \frac{3}{32} \frac{M_o L^2}{EI} = - \frac{1}{24} \frac{M_o L^2}{EI} = \frac{1}{24} \frac{M_o L^2}{EI}$$

Problem 9.140

9.139 and 9.140 For the beam and loading shown, determine (a) the slope at end A, (b) the slope at end B, (c) the deflection at the midpoint C.



$$\text{Reactions: } R_A = R_B = \frac{1}{2} wL$$

Draw bending moment and M/EI diagrams by parts as shown.

$$A_1 = \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{wL^2}{4EI} = \frac{wL^5}{16EI}$$

$$A_2 = -\frac{1}{3} \cdot \frac{L}{2} \cdot \frac{wL^2}{8EI} = -\frac{wL^5}{48EI}$$

$$A_3 = \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{wL^2}{8EI} = \frac{wL^5}{32EI}$$

$$A_4 = -\frac{1}{3} \cdot \frac{L}{2} \cdot \frac{wL^2}{16EI} = -\frac{wL^5}{96EI}$$

Place reference tangent at A.

$$(a) \text{ Slope at end A. } y_B = y_A + L\theta_A + t_{B/A}$$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \left(\frac{L}{2} + \frac{L}{6}\right)A_1 + \left(\frac{L}{2} + \frac{L}{3}\right)A_2 + \frac{L}{3}A_3 + \frac{3L}{8}A_4 \\ = \frac{WL^4}{EI} \left(\frac{1}{24} - \frac{5}{384} + \frac{1}{96} - \frac{1}{256} \right) = \frac{9WL^4}{256EI}$$

$$\theta_A = -\frac{9WL^4}{256EI} \cdot \frac{1}{L} = -\frac{9WL^3}{256EI} \quad \theta_A = \frac{9WL^3}{256EI}$$

$$(b) \text{ Slope at end B. } \theta_B = \theta_A + \theta_{B/A} = -\frac{9WL^3}{256EI} + A_1 + A_2 + A_3 + A_4$$

$$\theta_B = \frac{7WL^3}{256EI}$$

$$\theta_B = \frac{7WL^3}{256EI}$$

$$(c) \text{ Deflection at midpoint C. } y_C = y_C + \frac{L}{2}\theta_A + t_{C/A}$$

$$t_{C/A} = \left(\frac{L}{6}\right)A_1 + \left(\frac{L}{8}\right)A_2 = \frac{WL^4}{128EI}$$

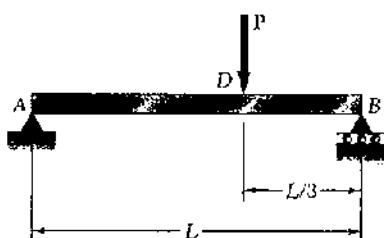
$$y_C = 0 + \left(\frac{L}{2}\right)\left(-\frac{9WL^3}{256EI}\right) + \frac{WL^4}{128EI} = -\frac{5WL^4}{512EI}$$

$$y_C = \frac{5WL^4}{512EI}$$

Problem 9.141

9.141 through 9.144 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.141 Beam and loading of Prob. 9.125

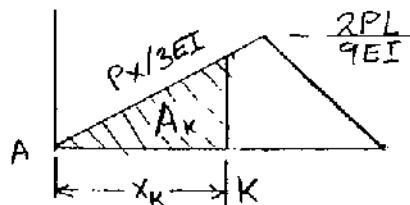


From the solution to Problem 9.125

$$R_A = \frac{1}{3}P \uparrow, \quad \theta_A = -\frac{4PL^2}{8EI}$$

$$\text{Over portion AD, } \frac{M}{EI} = \frac{R_A x}{EI} = \frac{Px}{3EI}$$

M/EI



$$A_k = \frac{1}{2} x_k \left(\frac{Px_k}{3EI} \right) = \frac{Px_k^2}{6EI}$$

$$\theta_k = \theta_A + A_k = 0$$

$$-\frac{4PL^2}{8EI} + \frac{Px_k^2}{6EI} = 0$$

$$x_k^2 = \frac{8}{27} L^2$$

$$x_k = 0.54433 L$$

$$A_k = -\theta_A = \frac{4PL^2}{8EI}$$

$$t_{A/K} = \left(\frac{2}{3} x_k \right) A_k = 0.01792 \frac{PL^3}{EI}$$

$$y_A = y_k + t_{A/K} = 0$$

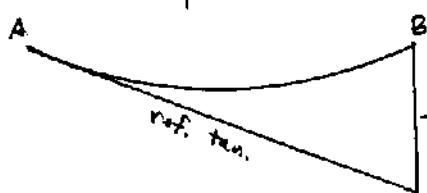
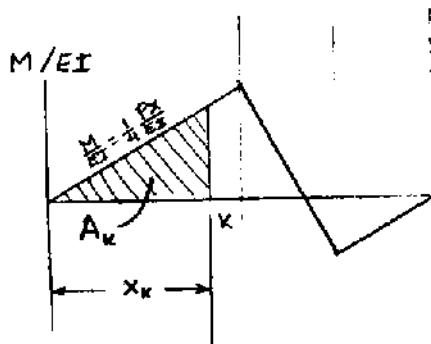
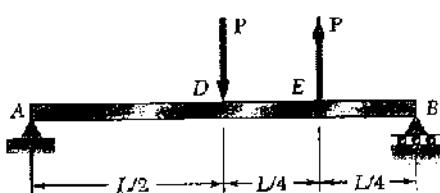
$$y_k = -t_{A/K} = -0.01792 \frac{PL^3}{EI}$$

$$y_k = 0.01792 \frac{PL^3}{EI} \downarrow$$

$$x_k = 0.544 L$$

Problem 9.142

9.141 through 9.144 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.
 9.142 Beam and loading of Prob. 9.127



$$y_k = t_{KA} - \frac{x_k}{L} t_{KA} = \frac{\sqrt{3}}{512} \frac{PL^3}{EI} - \left(\frac{1}{4}\sqrt{3}\right) \frac{3}{128} \frac{PL^3}{EI} = -\frac{\sqrt{3}}{256} \frac{PL^3}{EI}$$

$$= 0.00677 \frac{PL^3}{EI}$$

Referring to the solution of Prob. 9.127

$$R_A = \frac{1}{4}P, \quad t_{BA} = \frac{3}{128} \frac{PL^3}{EI}, \quad \theta_A = -\frac{3}{128} \frac{PL^2}{EI}$$

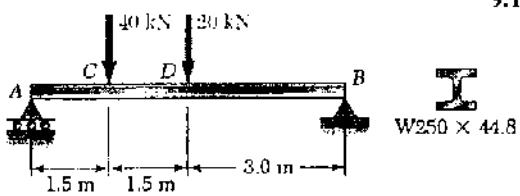
$$\begin{aligned} \theta_K &= \theta_A + \theta_{K/A} \\ &= -\frac{3}{128} \frac{PL^2}{EI} + A_k \\ &= -\frac{3}{128} \frac{PL^2}{EI} + \frac{1}{2} \left(\frac{1}{4} \frac{Px_k}{EI} \right) x_k \\ &= \frac{P}{EI} \left(-\frac{3}{128} L^2 + \frac{1}{8} x_k^2 \right) = 0 \end{aligned}$$

$$x_k = \sqrt{\frac{3}{16}} L = \frac{1}{4} \sqrt{3} L = 0.433 L$$

$$\begin{aligned} t_{KA} &= A_k \left(\frac{1}{3} x_k \right) = \frac{1}{2} \left(\frac{1}{4} \frac{Px_k}{EI} \right) \frac{x_k}{3} \\ &= \frac{1}{24} \frac{Px_k^3}{EI} = \frac{\sqrt{3}}{512} \frac{PL^3}{EI} \end{aligned}$$

$$\begin{aligned} y_k &= t_{KA} - \frac{x_k}{L} t_{BA} = \frac{\sqrt{3}}{512} \frac{PL^3}{EI} - \left(\frac{1}{4}\sqrt{3}\right) \frac{3}{128} \frac{PL^3}{EI} = -\frac{\sqrt{3}}{256} \frac{PL^3}{EI} \\ &= 0.00677 \frac{PL^3}{EI} \end{aligned}$$

Problem 9.143



9.141 through 9.144 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.143 Beam and loading of Prob. 9.129

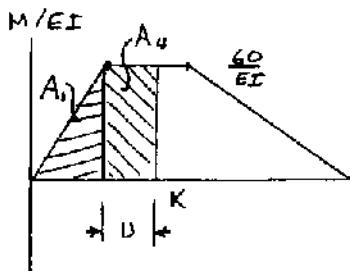
Referring to the solution to Prob.
9.129,

$$EI = 14220 \text{ kN}\cdot\text{m}^2$$

$$R_A = 40 \text{ kN}, A_1 = \frac{45}{EI}$$

$$t_{B/A} = \frac{742.5}{EI} \text{ m}$$

$$\theta_A = -\frac{123.75}{EI}$$



Let K be the location of the maximum deflection. Assume that K lies between C and D.



$$\begin{aligned} \theta_K &= \theta_A + \theta_{K/A} \\ &= -\frac{123.75}{EI} + A_1 + A_4 \\ &= -\frac{123.75}{EI} + \frac{45}{EI} + \frac{60/4}{EI} = 0 \\ u &= \frac{123.75 - 45}{60} = 1.3125 \text{ m.} \end{aligned}$$

$$x_K = 1.5 + u = 2.8125 \text{ m}$$

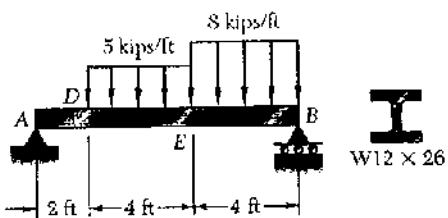
$$\begin{aligned} t_{K/A} &= A_1(u + 0.5) + A_4(\frac{1}{2}u) \\ &= \frac{45}{EI}(1.8125) + \frac{(60)(1.3125)}{EI}(\frac{1}{2})(1.3125) = \frac{133.242}{EI} \end{aligned}$$

$$\begin{aligned} y_K &= t_{K/A} - \frac{x_K}{L} t_{B/A} \\ &= \frac{133.242}{EI} - \frac{2.8125}{3} \left(\frac{742.5}{EI} \right) = -\frac{214.80}{EI} = -\frac{214.80}{14220} \\ &= -15.11 \times 10^{-3} \text{ m} = 1.511 \text{ mm} \end{aligned}$$

Problem 9.144

9.141 through 9.144 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.144 Beam and loading of Prob. 9.132



From the solution to Problem 9.132

$$EI = 41083 \text{ kip}\cdot\text{ft}^2$$

$$R_A = 18.4 \text{ kips}$$

$$A_1 = 331.2 \text{ kip}\cdot\text{ft}^2$$

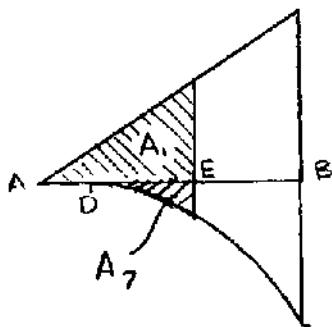
$$A_3 = -53.333 \text{ kip}\cdot\text{ft}^2$$

$$\theta_A = -5.3096 \times 10^{-3}$$

$$\text{Slope at } E. \quad \theta_E = \theta_A + \theta_{E/A}$$

$$\theta_{E/A} = \frac{1}{EI} \{ A_1 + A_3 \} = \frac{278.767}{41083} = 6.7855 \times 10^{-3}$$

$$\theta_E = 1.4759 \times 10^{-2}$$



Since $\theta_E > 0$, the point K of zero slope lies to the left of point E. Let x_K be the coordinate of point K.

$$A_6 = \frac{1}{2} R_A x_K^2 = 9.2 x_K^2$$

$$A_7 = -\frac{1}{6}(5)(x_K - 2)^3$$

$$\theta_K = \theta_A + \theta_{KA} = \theta_A + \frac{1}{EI} \{ A_6 + A_7 \} = 0$$

$$A_6 + A_7 + EI \theta_A = 0$$

$$f(x_K) = 9.2 x_K^2 - \frac{5}{6}(x_K - 2)^3 - 218.134 = 0$$

$$\frac{df}{dx_K} = 18.4 x_K - 2.5(x_K - 2)^2$$

Solve for x_K by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx_K}$$

x_K	5	5.1473	5.1525	$x_K = 5.1525 \text{ ft}$
f	-10.634	-0.362	0.001	
df/dx_K	72.2	70.131		

$$A_6 = 244.244 \text{ kip}\cdot\text{ft}^2, \quad A_7 = -26.198 \text{ kip}\cdot\text{ft}^2$$

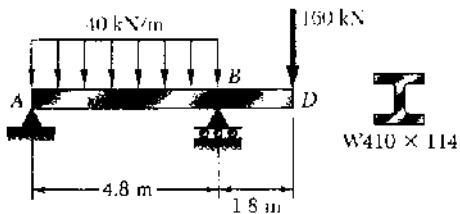
Maximum deflection. $y_A = y_K + t_{AK} = 0 \quad y_K = -t_{AK}$

$$\bar{x}_6 = \frac{2}{3}x_K, \quad \bar{x}_7 = 2 + \frac{3}{4}(x_K - 2) = \frac{3x_K + 2}{4} \quad x_K = 5.15 \text{ ft}$$

$$y_K = -\frac{1}{EI} \{ A_6 \bar{x}_6 + A_7 \bar{x}_7 \} = -\frac{725.033}{41083} = -17.648 \times 10^{-3} \text{ ft} \quad y_K = 0.212 \text{ in.}$$

Problem 9.145

9.145 For the beam and loading of Prob. 9.135, determine the largest upward deflection in span AB.

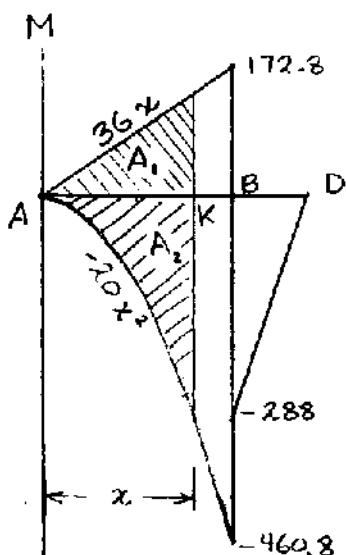


Units: Forces in kN. Lengths in meters.

$$I = 462 \times 10^6 \text{ mm}^4 = 462 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9) (462 \times 10^{-6}) = 92.4 \times 10^6 \text{ N.m}^2 = 92400 \text{ kN.m}$$

$$\rightarrow M_B = 0 : -4.8 R_A + (40)(4.8)(2.4) - (160)(1.8) = 0 \\ R_A = 36 \text{ kN}$$



$$A_1 = \frac{1}{2} x (36x) = 18x^2$$

$$A_2 = \frac{1}{3} x (-20x^2) = -\frac{20}{3}x^3$$

Place reference tangent at A.

$$y_B = y_A + L \theta_A + t_{B/A} = 0$$

$$\theta_A = -\frac{t_{B/A}}{L}$$

$$(A_1)_B = (18)(4.8)^2 = 414.72 \text{ kN.m}^2$$

$$(A_2)_B = \left(\frac{20}{3}\right)(4.8)^3 = -737.28 \text{ kN.m}^2$$

$$\theta_A = -\frac{1}{EI L} \left\{ (A_1)_B \left(\frac{1}{3}\right)(4.8) + (A_2)_B \left(\frac{1}{4}\right)(4.8) \right\}$$

$$= -\frac{-221.184}{(92400)(4.8)} = 0.49870 \times 10^{-3}$$

Locate point K of maximum deflection. $\theta_K = \theta_A + \theta_{K/A} = 0$

$$EI \theta_A + A_1 + A_2 = 0$$

$$f = 46.08 + 18x_K^2 - \frac{20}{3}x_K^3 = 0 \quad \frac{df}{dx} = 36x_K - 20x_K^2$$

Solve by iteration.

$$x_K \begin{vmatrix} 3 & 3.39 & 3.327 & 3.3251 & 3.32514 \end{vmatrix} \leftarrow \\ \frac{f}{df/dx} \begin{vmatrix} -72 & -107.8 & -101.6 & -101.42 & \end{vmatrix} \\ f \begin{vmatrix} 28.08 & -6.78 & -0.188 & 0.005 & \end{vmatrix}$$

Place reference tangent at K. $y_A = y_K + t_{A/K}$

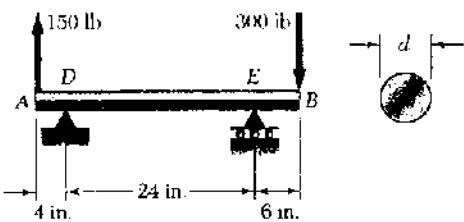
$$y_K - y_A = -t_{A/K} = -\frac{1}{EI} \left\{ (A_1) \left(\frac{2}{3}x_K\right) + A_2 \left(\frac{3}{4}x_K\right) \right\} = -\frac{1}{EI} \left\{ 12x_K^3 - 5x_K^4 \right\}$$

$$= -\frac{170.064}{92400} = -1.841 \times 10^{-3} \text{ m}$$

$$y_K = 1.841 \text{ mm} \uparrow \leftarrow$$

Problem 9.146

9.146 For the beam and loading of Prob. 9.138, determine the largest upward deflection in span DE.



Units: Forces in lbs; lengths in inches.

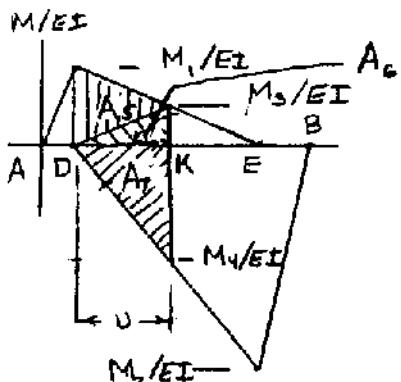
From the solution to Problem 9.138

$$EI = 450.4 \times 10^3 \text{ lb-in}$$

$$\frac{M_1}{EI} = 1.33215 \times 10^{-3} \text{ in}^{-1}$$

$$\frac{M_2}{EI} = -3.99645 \times 10^{-3} \text{ in}^{-1}$$

$$\theta_D = 5.3286 \times 10^{-3}$$



Location of maximum deflection.

$$\frac{M_3}{EI} = \frac{M_1}{EI} \left(1 - \frac{u}{24}\right)$$

$$\frac{M_4}{EI} = \frac{M_2}{EI} \frac{u}{24}$$

$$A_6 = \frac{1}{2} \frac{M_1}{EI} \cdot u = 0.666075 \times 10^{-3} u$$

$$A_5 = \frac{1}{2} \frac{M_2}{EI} \left(\frac{u}{24}\right) = 0.666075 \times 10^{-3} \left(1 - \frac{u}{24}\right) u$$

$$A_7 = \frac{1}{2} \frac{M_4}{EI} \frac{u}{24} = -1.998225 \times 10^{-3} \left(\frac{u}{24}\right)^2 u$$

$$\theta_K = \theta_D + A_5 + A_6 + A_7 = 0 \quad \text{Multiply by } 10^3.$$

$$5.3286 + 0.666075 u + 0.666075 \left(1 - \frac{u}{24}\right) u - (1.998225) \frac{u}{24} u = 0$$

$$5.3286 + 1.33215 u - 0.1110125 u^2 = 0$$

$$u = 15.16515 \text{ in.}$$

$$A_5 = 10.10113 \times 10^{-3}, \quad A_6 = 3.71842 \times 10^{-3}, \quad A_7 = -19.14814 \times 10^{-3}$$

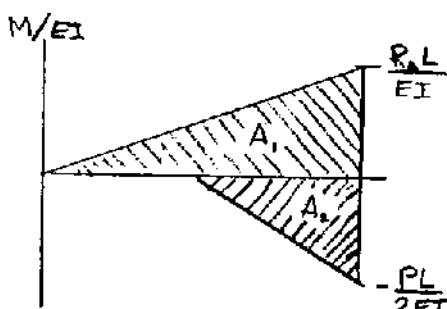
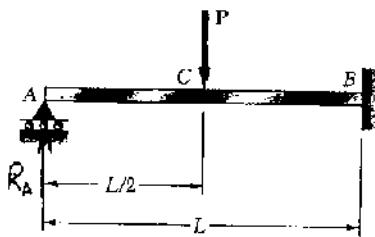
Maximum deflection in portion DE. $y_D = y_K + t_{DK} = 0$

$$y_K = -t_{DK} = -\left\{ A_5 \left(\frac{u}{3}\right) + A_6 \left(\frac{2u}{3}\right) + A_7 \left(\frac{u}{3}\right) \right\}$$

$$= -\left\{ -0.1049 \right\} = 0.1049 \text{ in. } \uparrow$$

Problem 9.147

9.147 through 1.150 For the beam and loading shown, determine the reaction at the roller support.



Remove support A and treat R_A as redundant.

Draw the $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = -\frac{1}{2} \frac{L}{2} \frac{PL}{2} = -\frac{PL^2}{8EI}$$

Place reference tangent at B.

$$y_A = y_B - \theta_B L + t_{A/B} = 0$$

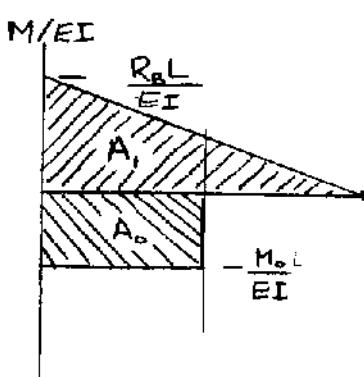
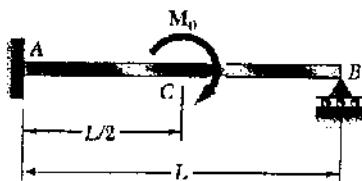
$$t_{A/B} = 0$$

$$A_1 \left(\frac{2L}{3}\right) + A_2 \left(\frac{L}{2} + \frac{L}{3}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{5PL^3}{48EI} = 0 \quad R_A = \frac{5}{16}P \uparrow$$

Problem 9.148

9.147 through 1.150 For the beam and loading shown, determine the reaction at the roller support.



Remove support B and treat R_B as redundant.

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = \frac{L}{2} \cdot \frac{M_0 L}{EI} = \frac{M_0 L^2}{2EI}$$

Place reference tangent at A.

$$y_B = y_A + L \theta_A + t_{B/A} = 0$$

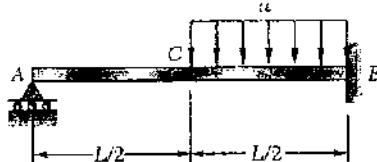
$$t_{B/A} = 0$$

$$A_1 \left(\frac{3L}{4}\right) + A_2 \left(\frac{L}{2} + \frac{L}{4}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{3M_0 L^2}{8EI} = 0 \quad R_A = \frac{9}{8} \frac{M_0}{L} \uparrow$$

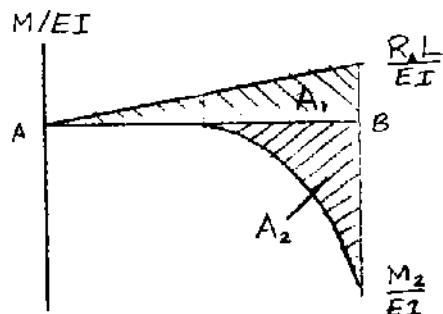
Problem 9.149

9.147 through 1.150 For the beam and loading shown, determine the reaction at the roller support.



Remove support A and treat R_A as redundant.

Draw M/EI diagram for loads R_A and w.



$$M_2 = -\frac{1}{2} w \left(\frac{L}{2}\right)^2 = -\frac{1}{8} w L^2$$

$$A_1 = \frac{1}{2} \left(\frac{R_A L}{EI}\right) L = \frac{1}{2} \frac{R_A L^2}{EI}$$

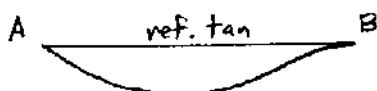
$$A_2 = \frac{1}{3} \left(-\frac{1}{8} \frac{w L^2}{EI}\right) \left(\frac{L}{2}\right) = -\frac{1}{48} \frac{w L^3}{EI}$$

Place reference tangent at B.

$$t_{A/B} = A_1 \left(\frac{2}{3} L\right) + A_2 \left(\frac{L}{2} + \frac{3}{4} \frac{L}{2}\right)$$

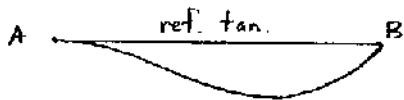
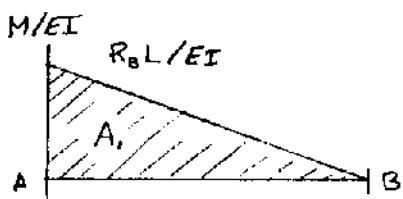
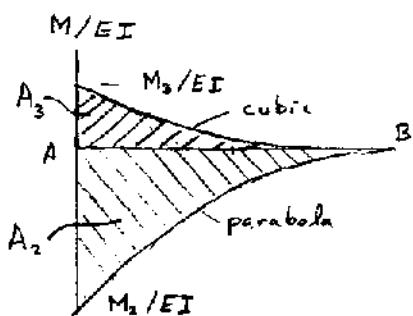
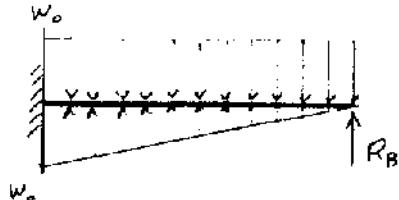
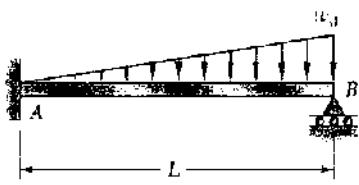
$$= \frac{1}{3} \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} = 0$$

$$R_A = \frac{7}{128} w L \uparrow$$



Problem 9.150

9.147 through 9.150 For the beam and loading shown, determine the reaction at the roller support.



Remove support B and treat R_B as redundant.

Replace loading by equivalent shown at left.

Draw M/EI diagram for load w_0 and R_B .
Use parts as shown.

$$A_1 = \frac{1}{2} \left(\frac{R_B L}{EI} \right) (L) = \frac{1}{2} \frac{R_B L^2}{EI}$$

$$M_2 = -\frac{1}{2} w_0 L^2$$

$$A_2 = \frac{1}{3} \left(-\frac{1}{2} \frac{w_0 L^2}{EI} \right) L = -\frac{1}{6} \frac{w_0 L^3}{EI}$$

$$M_3 = \frac{1}{6} \frac{w_0}{L} L^3 = \frac{1}{6} w_0 L^2$$

$$A_3 = \frac{1}{4} \left(\frac{1}{6} \frac{w_0 L^2}{EI} \right) L = \frac{1}{24} \frac{w_0 L^3}{EI}$$

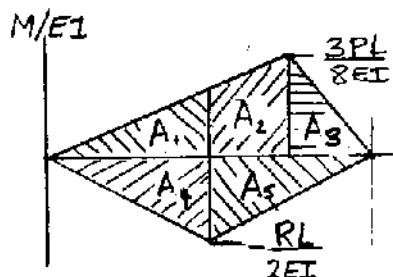
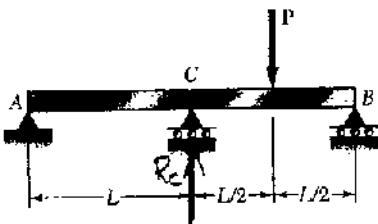
Place reference tangent at A.

$$\begin{aligned} T_{B/A} &= A_1 \left(\frac{2}{3} L \right) + A_2 \left(\frac{3}{4} L \right) + A_3 \left(\frac{4}{5} L \right) \\ &= \frac{1}{3} \frac{R_B L^3}{EI} - \frac{1}{8} \frac{w_0 L^4}{EI} + \frac{1}{30} \frac{w_0 L^4}{EI} = 0 \end{aligned}$$

$$R_B = \frac{11}{40} w_0 L = 0.275 w_0 L \uparrow$$

Problem 9.151

9.151 and 9.152 For the beam and loading shown, determine the reaction at each support.



Remove support C and add reaction R_c .

Draw $\frac{M}{EI}$ due to each of the loads (P and R_c).

$$A_1 = \frac{1}{2} \cdot L \cdot \frac{2}{3} \cdot \frac{3PL}{8EI} = \frac{PL^2}{8EI}$$

$$A_1 + A_2 = \frac{1}{2} \cdot \frac{3L}{2} \cdot \frac{3PL}{8EI} = \frac{9PL^2}{32EI}$$

$$A_3 = \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{3PL}{8EI} = \frac{3PL^2}{32EI}$$

$$A_4 = \frac{1}{2} \cdot L \left(-\frac{R_c L}{2EI} \right) = -\frac{R_c L^2}{4EI}$$

$$A_4 + A_5 = \frac{1}{2} (2L) \left(-\frac{R_c L}{2EI} \right) = -\frac{R_c L^2}{2EI}$$

Place reference tangent at A. $y_A = 0$

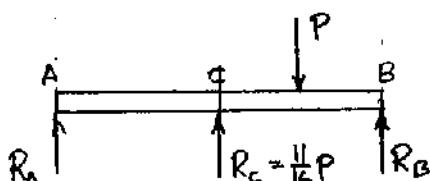
$$y_C = L\theta_A + t_{C/A} = 0 \quad \theta_A = -\frac{t_{C/A}}{L}$$

$$y_B = 2L\theta_A + t_{B/A} = 0 \quad -2t_{C/A} + t_{B/A} = 0$$

$$-2 \left[A_1 \cdot \frac{L}{3} + A_4 \cdot \frac{L}{3} \right] + [(A_1 + A_2) \left(\frac{L}{2} + \frac{2}{3} \cdot \frac{3L}{2} \right) + A_3 \cdot \frac{2}{3} \cdot \frac{L}{2} + (A_4 + A_5) \cdot L] = 0$$

$$-2 \left[\frac{PL^3}{24EI} - \frac{R_c L^3}{12EI} \right] + \left[\frac{9PL^3}{32EI} + \frac{PL^3}{32EI} - \frac{R_c L^3}{2EI} \right] = 0$$

$$-\frac{1}{3} \frac{R_c L^3}{EI} + \frac{11}{48} \frac{PL^3}{EI} = 0 \quad R_c = \frac{11}{16} P \uparrow$$



$\rightarrow \sum M_B = 0$:

$$-2LR_A - LR_c + \frac{L}{2}P = 0$$

$$R_A = \frac{P}{4} - \frac{1}{2}R_c = -\frac{3}{32}P$$

$$R_A = \frac{3}{32}P \downarrow$$

$\rightarrow \sum M_A = 0$:

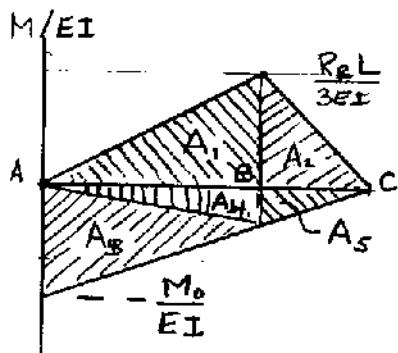
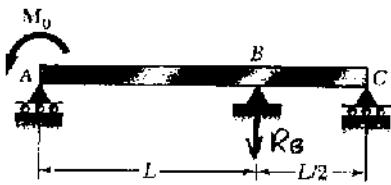
$$2LR_B + LR_c - \frac{3L}{2}P = 0$$

$$R_B = \frac{3P}{4} - \frac{1}{2}R_c = \frac{13}{32}P$$

$$R_B = \frac{13}{32}P \uparrow$$

Problem 9.152

9.151 and 9.152 For the beam and loading shown, determine the reaction at each support.



Choose $R_B \downarrow$ as the redundant reaction.

Draw $\frac{M}{EI}$ diagram for the loads R_B and M_0 .

$$A_1 = \frac{1}{2}(L)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{6EI}$$

$$A_2 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{12EI}$$

$$A_3 = \frac{1}{2}(L)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{2EI}$$

$$A_4 = \frac{1}{2}(L)\left(\frac{1}{3}\right)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{6EI}$$

$$A_3 + A_4 + A_5 = \frac{1}{2}\left(\frac{3L}{2}\right)\left(-\frac{M_0}{EI}\right) = -\frac{3M_0 L}{4EI}$$

$$y_B = y_A + L\theta_A + t_{B/A} \quad \theta_A = -t_{B/A}/L$$

$$y_C = y_A + \frac{3L}{2}\theta_A + t_{C/A} = 0 \quad -\frac{3}{2}t_{B/A} + t_{C/A} = 0$$

$$t_{B/A} = (A_1)\left(\frac{L}{3}\right) + A_3\left(\frac{2L}{3}\right) + A_4\left(\frac{L}{3}\right) = \frac{R_B L^3}{18EI} - \frac{7M_0 L^2}{18EI}$$

$$t_{C/A} = (A_1)\left(\frac{L}{2} + \frac{L}{3}\right) + A_2\left(\frac{L}{3}\right) + (A_3 + A_4 + A_5)(L) = \frac{R_B L^3}{6EI} - \frac{3M_0 L^2}{4EI}$$

$$-\frac{3}{2}t_{B/A} + t_{C/A} = \frac{R_B L^3}{12EI} - \frac{M_0 L^2}{6EI} = 0 \quad R_B = \frac{2M_0}{L} \downarrow \rightarrow$$

$$+\sum M_C = 0: \quad M_0 + \frac{1}{2}R_B - \frac{3L}{2}R_A = 0$$

$$R_A = \frac{2}{3L}[M_0 + M_0]$$

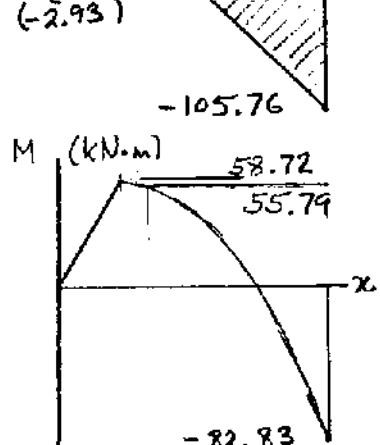
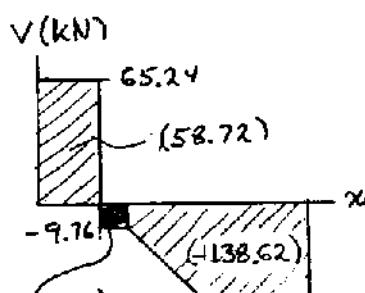
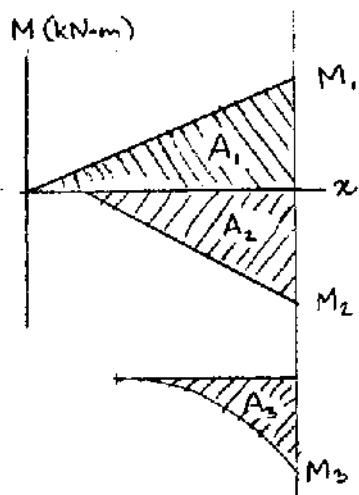
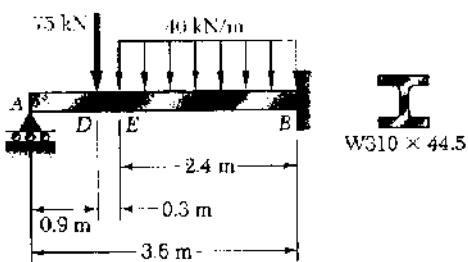
$$R_A = \frac{4M_0}{3L} \uparrow \rightarrow$$

$$+\uparrow \sum F_y = 0: \quad R_A + R_B + R_C = 0 \quad \frac{4}{3}\frac{M_0}{L} - \frac{2M_0}{L} + R_C = 0$$

$$R_C = \frac{2M_0}{3L} \uparrow \rightarrow$$

Problem 9.153

9.153 and 9.154 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Units: Forces in kN. Lengths in meters.

Let R_A be the redundant reaction.

Remove support at A and add reaction $R_A \uparrow$.

Draw bending moment diagram by parts.

$$M_1 = 3.6 R_A \text{ kN·m}$$

$$M_2 = -(75)(0.3 + 2.4) = -202.5 \text{ kN·m}$$

$$M_3 = -\frac{1}{2}(40)(2.4)^2 = -115.2 \text{ kN·m}$$

$$A_1 = \frac{1}{2}(3.6)(3.6 R_A) = 6.48 \text{ kN·m}^2$$

$$A_2 = \frac{1}{2}(2.7)(-202.5) = -273.375 \text{ kN·m}^2$$

$$A_3 = \frac{1}{3}(2.4)(-115.2) = -92.16 \text{ kN·m}^2$$

Place reference tangent at B, where

$$\theta_B = 0 \text{ and } y_B = 0$$

$$\text{Then, } y_A = \dot{\theta}_{AB} = 0$$

$$\begin{aligned} t_{AB} &= \frac{1}{EI} \left[\left(\frac{2}{3} \cdot 3.6 \right) A_1 + (0.9 + \frac{2}{3} \cdot 2.7) A_2 \right. \\ &\quad \left. + (0.9 + 0.3 + \frac{2}{3} \cdot 2.4) A_3 \right] \\ &= \frac{1}{EI} \left\{ 15.552 R_A - 1014.5925 \right\} = 0 \end{aligned}$$

$$R_A = 65.24 \text{ kN}$$

Draw shear diagram.

$$A \rightarrow D \quad V = R_A = 65.24 \text{ kN}$$

$$D \rightarrow E \quad V = 65.24 - 75 = -9.76 \text{ kN}$$

$$E \rightarrow B \quad V = -9.76 - 40(x - 1.2) \text{ kN}$$

$$At B \quad V_B = -105.76 \text{ kN.}$$

Bending moment diagram. $M_A = 0$

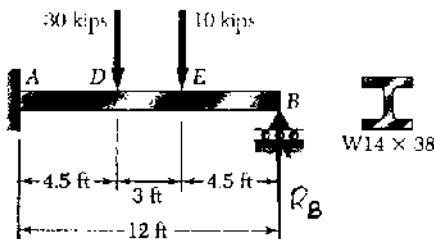
$$M_D = M_A + 58.72 = 58.72 \text{ kN·m}$$

$$M_E = 58.72 - 2.93 = 55.79 \text{ kN·m}$$

$$M_B = 55.79 - 138.62 = -82.83 \text{ kN·m}$$

Problem 9.154

9.153 and 9.154 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Units - Forces in kips - Lengths in feet.

Let R_B be the redundant reaction.

Remove support B and add load R_B .

Draw bending moment diagram by parts.

$$M_1 = 12 R_B \text{ kip-ft}$$

$$M_2 = -(4.5 + 3)(10) = -75 \text{ kip-ft}$$

$$M_3 = -(4.5)(30) = -135 \text{ kip-ft}$$

$$A_1 = \frac{1}{2}(12)(12R_B) = 72R_B \text{ kip-ft}^2$$

$$A_2 = \frac{1}{2}(7.5)(-75) = -281.25 \text{ kip-ft}^2$$

$$A_3 = \frac{1}{2}(4.5)(-135) = -303.75 \text{ kip-ft}^2$$

$$y_B = y_A + 12\theta_A + t_{B/A} = 0$$

$$t_{B/A} = 0$$

$$t_{B/A} = \frac{1}{EI} \left\{ (72R_B)(8) + (-281.25)(4.5+5) + (-303.75)(7.5+3) \right\} = 0$$

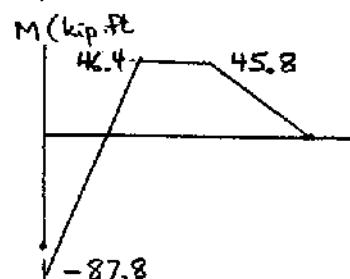
$$576R_B - 5861.25 = 0 \quad R_B = 10.18 \text{ kips} \uparrow$$

Draw shear diagram working from right to left.

$$\text{B to E} \quad V = -R_B = -10.176 \text{ kips}$$

$$\text{E to D} \quad V = -10.176 + 10 = -0.176 \text{ kips}$$

$$\text{D to A} \quad V = -0.176 + 30 = 29.824 \text{ kips}$$



Areas of shear diagram.

$$A_{AD} = (4.5)(29.824) = 134.21 \text{ kip-ft}$$

$$A_{DE} = (3)(-0.176) = -0.53 \text{ kip-ft}$$

$$A_{EB} = (4.5)(-10.176) = 45.79 \text{ kip-ft}$$

Bending moments.

$$M_A = M_1 + M_2 + M_3 = -87.89 \text{ kip-ft}$$

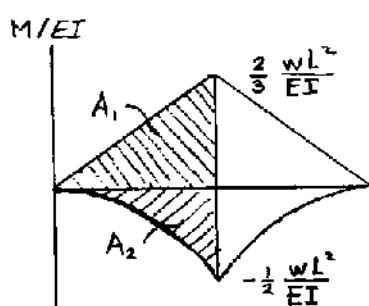
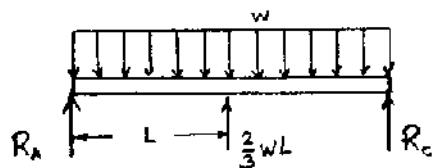
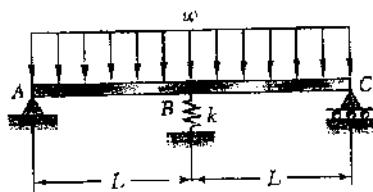
$$M_D = M_A + A_{AD} = 46.32 \text{ kip-ft}$$

$$M_E = M_D + A_{DE} = 45.79 \text{ kip-ft}$$

$$M_B = M_E + A_{EB} = 0$$

Problem 9.155

9.155 For the beam and loading shown, determine the spring constant k for which the force in the spring is equal to one-third of the total load on the beam.



Symmetric beam and loading. $R_C = R_A$

$$\text{Spring force: } F = \frac{1}{3}(2wL) = \frac{2}{3}wL$$

$$+\uparrow \sum F_y = 0: R_A + F - 2wL + R_C = 0$$

$$R_A = R_C = \frac{2}{3}wL$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{2}{3} \frac{wL^2}{EI} \right) L = \frac{1}{3} \frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{2} \left(\frac{1}{2} \frac{wL^2}{EI} \right) L = -\frac{1}{4} \frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$

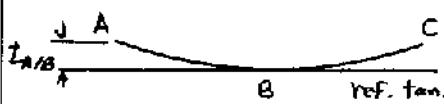
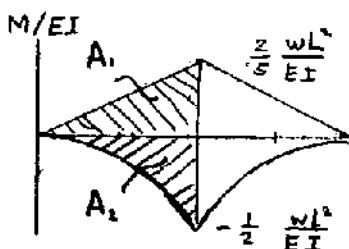
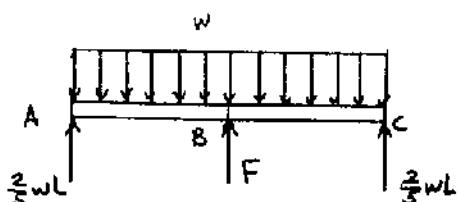
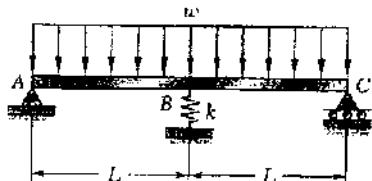
$$\begin{aligned} y_B &= -t_{A/B} \\ &= -(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L) \\ &= -\frac{7}{72} \frac{wL^4}{EI} \end{aligned}$$

$$F = -k y_B$$

$$k = -\frac{E}{y_B} = \frac{\frac{2}{3}wL}{\frac{7}{72} \frac{wL^4}{EI}} = \frac{48}{7} \frac{EI}{L^3}$$

Problem 9.156

9.156 For the beam and loading shown, determine the spring constant k for which the bending moment at B is $M_B = -wL^2/10$.



Using free body AB,
 $\sum M_B = 0$:
 $R_A L + (wL)(\frac{L}{2}) - \frac{1}{10}wL^2 = 0$
 $R_A = \frac{2}{5}wL \uparrow$

Symmetric beam and loading. $R_C = R_A$

Using free body ABC,
 $\sum F_y = 0$:
 $\frac{2}{5}wL + F + \frac{3}{5}wL - 2wL = 0$
 $F = \frac{6}{5}wL$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{2}{5} \frac{wL^2}{EI} \right) L = \frac{1}{5} \frac{WL^3}{EI}$$

$$A_2 = -\frac{1}{3} \left(\frac{1}{2} \frac{wL^2}{EI} \right) L = -\frac{1}{6} \frac{WL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$

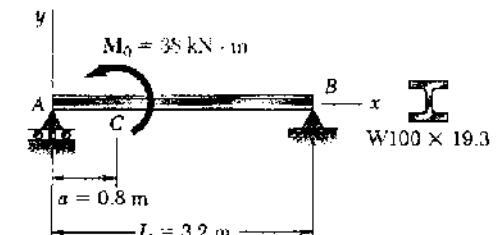
$$\begin{aligned} y_B &= -t_{AB} \\ &= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right) \\ &= -\frac{1}{120} \frac{WL^4}{EI} \end{aligned}$$

$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{6}{5}wL}{\frac{1}{120} \frac{WL^4}{EI}} = 144 \frac{EI}{L^3}$$

Problem 9.157

9.157 For the beam and loading shown, determine the deflection at point C. Use $E = 200 \text{ GPa}$.



$$\begin{aligned} [x=0, y=0] \\ [x=a, y=y] \\ [x=a, \frac{dy}{dx} = \frac{dy}{dx}] \end{aligned}$$

$$\text{Reactions: } R_A = M_o/L \uparrow, R_B = M_o/L \downarrow$$

$$0 < x < a \quad \rightarrow \sum M_j = 0:$$

$$- \frac{M_o}{L} x + M = 0$$

$$M = \frac{M_o}{L} x$$

$$a < x < L \quad + \sum M_k = 0:$$

$$- \frac{M_o}{L} x + M_o + M = 0$$

$$M = \frac{M_o}{L} (x - L)$$

$$0 < x < a$$

$$EI \frac{d^2y}{dx^2} = \frac{M_o}{L} x$$

$$EI \frac{dy}{dx} = \frac{M_o}{L} (\frac{1}{2}x^2) + C_1 \quad (1)$$

$$EI y = \frac{M_o}{L} (\frac{1}{6}x^3) + C_1 x + C_2 \quad (2)$$

$$a < x < L$$

$$EI \frac{d^2y}{dx^2} = \frac{M_o}{L} (x - L)$$

$$EI \frac{dy}{dx} = \frac{M_o}{L} (\frac{1}{2}x^2 - Lx) + C_3 \quad (3)$$

$$EI y = \frac{M_o}{L} (\frac{1}{6}x^3 - \frac{1}{2}Lx^2) + C_3 x + C_4 \quad (4)$$

$$[x=0, y=0] \quad Eq. (2): \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad Eqs.(1) \& (3): \quad \frac{M_o}{L} (\frac{1}{2}a^2) + C_1 = \frac{M_o}{L} (\frac{1}{2}a^3 - La) + C_3$$

$$C_3 = C_1 + M_o a$$

$$[x=a, y=y] \quad Eqs.(2) \& (4) \quad \frac{M_o}{L} (\frac{1}{6}a^3) + C_2 a = \frac{M_o}{L} (\frac{1}{6}a^3 - \frac{1}{2}La^2) + (C_3 + M_o a)a + C_4$$

$$C_4 = -\frac{1}{2}M_o a^2$$

$$[x=L, y=0] \quad Eq. (1) \quad \frac{M_o}{L} (\frac{1}{6}L^3 - \frac{1}{2}L^2) + (C_1 + M_o a)L - \frac{1}{2}M_o a^2 = 0$$

$$C_1 = \frac{M_o}{L} (\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)$$

$$\text{Elastic curve for } 0 < x < a. \quad y = \frac{M_o}{EI L} \left[\frac{1}{6}x^3 + (\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)x \right]$$

$$\text{Make } x = a. \quad y_a = \frac{M_o}{EI L} \left[\frac{1}{6}a^3 + \frac{1}{3}L^2 a + \frac{1}{2}a^3 - a^2 L \right] = \frac{M_o}{EI L} \left[\frac{2}{3}a^3 + \frac{1}{3}L^2 a - La^2 \right]$$

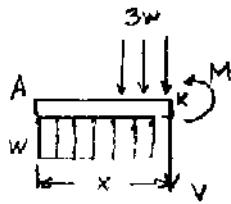
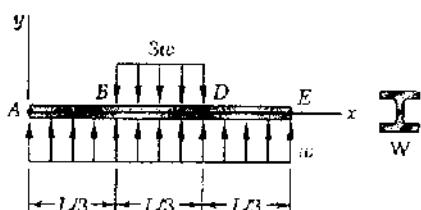
$$\text{Data: } E = 200 \times 10^9 \text{ Pa}, \quad I = 4.77 \times 10^6 \text{ mm}^4 = 4.77 \times 10^{-6} \text{ m}^4, \quad M_o = 38 \times 10^3 \text{ N}\cdot\text{m}$$

$$y_a = \frac{(38 \times 10^3) \left[(2)(0.8)^3 / 3 + (3.2)^2 (0.8) / 3 - (3.2)(0.8)^2 \right]}{(200 \times 10^9)(4.77 \times 10^{-6})(3.2)} = 12.75 \times 10^{-3} \text{ m}$$

$$= 12.75 \text{ mm} \uparrow$$

Problem 9.158

9.158 Uniformly distributed loads are applied to beam AE as shown. (a) Selecting the x axis through the centers A and E of the end sections of the beam, determine the equation of the elastic curve for portion AB of the beam. (b) Knowing that the beam is a W 200 × 35.9 rolled shape and that $L = 3 \text{ m}$, $w = 5 \text{ kN/m}$, and $E = 200 \text{ GPa}$, determine the distance of the center of the beam from the x axis.



$$0 < x < \frac{L}{3} \quad \text{and} \quad \sum M_J = 0:$$

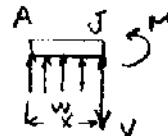
$$-(wx)(\frac{x}{2}) + M = 0$$

$$M = \frac{1}{2}wx^2$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{24}wx^4 + C_1x + C_2$$



$$\frac{L}{3} \leq x \leq \frac{2L}{3} \quad \Rightarrow \quad \sum M_K = 0: \quad -(wx)(\frac{x}{2}) + 3w(x - \frac{L}{3})(\frac{x - \frac{L}{3}}{2}) + M = 0$$

$$M = \frac{1}{2}wx^2 - \frac{3}{2}w(x - \frac{L}{3})^2$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{3}{2}w(x - \frac{L}{3})^2$$

$$EI \frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{1}{2}w(x - \frac{L}{3})^3 + C_3$$

$$EIy = \frac{1}{24}wx^4 - \frac{1}{8}w(x - \frac{L}{3})^4 + C_3x + C_4$$

$$[x=0, y=0] \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x = \frac{L}{3}, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{6}w(\frac{L}{3})^3 + C_3 = \frac{1}{6}w(\frac{L}{3})^3 + 0 + C_3 \quad C_1 = C_3$$

$$[x = \frac{L}{3}, y = y] \quad \frac{1}{24}w(\frac{L}{3})^4 + C_1 \frac{L}{3} + C_2 = \frac{1}{24}w(\frac{L}{3})^4 + 0 + C_3 \frac{L}{3} + C_4$$

$$C_4 = C_2 = 0$$

$$\text{Symmetry boundary condition} \quad [x = \frac{L}{2}, \frac{dy}{dx} = 0]$$

$$\frac{1}{6}w(\frac{L}{2})^3 - \frac{1}{2}w(\frac{L}{2} - \frac{L}{3})^3 + C_3 = 0 \quad C_3 = -\left(\frac{1}{48} - \frac{1}{432}\right)wL^3 = -\frac{1}{54}wL^3$$

(a) Elastic curve for portion AB .

$$y = \frac{1}{EI} \left\{ \frac{1}{24}wx^4 + C_1x + C_2 \right\} = \frac{w}{EI} \left(\frac{1}{24}x^4 - \frac{1}{54}L^3x \right)$$

$$(b) \text{ Deflection at center.} \quad y_c = \frac{1}{EI} \left\{ \frac{1}{24}w(\frac{L}{2})^4 - \frac{1}{8}w(\frac{L}{2} - \frac{L}{3})^4 - \frac{1}{54}wL^3(\frac{L}{2}) + 0 \right\}$$

$$= \frac{wL^4}{EI} \left\{ \frac{1}{384} - \frac{1}{10368} - \frac{1}{108} \right\} = -\frac{35}{5184} \frac{wL^4}{EI}$$

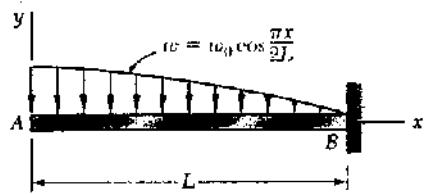
$$\text{Data: } I = 34.4 \times 10^6 \text{ mm}^4 = 34.4 \times 10^{-6} \text{ m}^4, \quad E = 200 \times 10^9 \text{ Pa}, \quad L = 3 \text{ m}$$

$$w = 5 \times 10^3 \text{ N/m}$$

$$y_c = -\frac{35}{5184} \frac{(5 \times 10^3)(3)^4}{(200 \times 10^9)(34.4 \times 10^{-6})} = -397 \times 10^{-6} \text{ m, ie } 0.397 \text{ mm } \downarrow$$

Problem 9.159

9.159 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at the free end, (c) the deflection at the free end.



$$[x=0, V=0]$$

$$[x=L, \frac{dy}{dx}=0]$$

$$[x=0, M=0]$$

$$[x=L, y=0]$$

$$\frac{dV}{dx} = -W = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\frac{2w_0 L}{\pi} \sin \frac{\pi x}{2L} + C_1$$

$$[x=0, V=0] \quad 0 = 0 + C_1 \quad C_1 = 0$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi} \sin \frac{\pi x}{2L}$$

$$M = \frac{4w_0 L^2}{\pi^2} \cos \frac{\pi x}{2L} + C_2$$

$$[x=0, M=0] \quad C_2 = -\frac{4w_0 L^2}{\pi^2}$$

$$EI \frac{d^2Y}{dx^2} = M = \frac{4w_0 L^2}{\pi^2} \left(\cos \frac{\pi x}{2L} - 1 \right)$$

$$EI \frac{dy}{dx} = \frac{4w_0 L^2}{\pi^2} \left(\frac{2L}{\pi} \sin \frac{\pi x}{2L} - x \right) + C_3$$

$$[x=L, \frac{dy}{dx}=0] \quad \frac{4w_0 L^2}{\pi^2} \left(\frac{2L}{\pi} - L \right) + C_3 = 0 \quad C_3 = \frac{4w_0 L^3}{\pi^3} (\pi - 2)$$

$$EIy = \frac{4w_0 L^2}{\pi^2} \left[-\frac{4L^2}{\pi^2} \cos \frac{\pi x}{2L} - \frac{1}{2}x^2 \right] + C_3 x + C_4$$

$$[x=L, y=0] \quad \frac{4w_0 L^2}{\pi^2} \left(-\frac{1}{2}L^2 \right) + C_3 L + C_4 = 0$$

$$C_4 = \frac{2w_0 L^4}{\pi^2} - C_3 L$$

$$(a) \text{ Elastic curve. } y = \frac{w_0}{EI} \left\{ -\frac{16L^4}{\pi^4} \cos \frac{\pi x}{2L} - \frac{2L^2 x^2}{\pi^2} + \frac{4L^3}{\pi^3} (\pi - 2)(x - L) + \frac{2L^4}{\pi^2} \right\}$$

$$y = \frac{2w_0 L^4}{\pi^4 EI} \left\{ -8 \cos \frac{\pi x}{2L} - \pi^2 \frac{x^2}{L^2} + 2\pi(\pi - 2) \frac{x}{L} + \pi(4 - \pi) \right\} \quad \blacktriangleleft$$

(b) Slope at free end. ($x=0$)

$$EI \frac{dy}{dx} \Big|_{x=0} = C_3 = \frac{4(\pi - 2)}{\pi^3} w_0 L^3$$

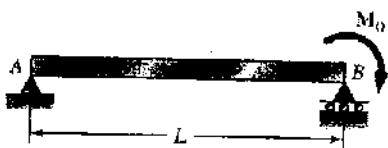
$$\frac{dy}{dx} \Big|_A = \frac{4(\pi - 2)}{\pi^3} \frac{w_0 L^3}{EI} = 0.14727 \frac{w_0 L^3}{EI} \quad \blacktriangleleft$$

(c) Deflection at free end. ($x=0$)

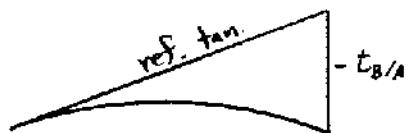
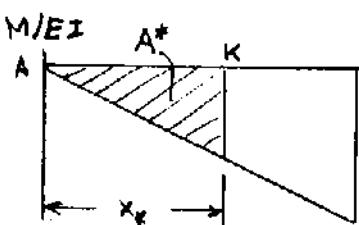
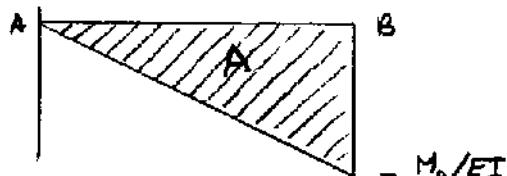
$$y_A = \frac{2w_0 L^4}{\pi^4 EI} \left\{ -8 + \pi(4 - \pi) \right\} = -0.10889 \frac{w_0 L^4}{EI} = 0.1089 \frac{w_0 L^4}{EI} \quad \blacktriangleleft$$

Problem 9.160

9.160 For the beam and loading shown, determine the magnitude and location of the maximum deflection.



M/EI



Draw $\frac{M}{EI}$ diagram.

Place reference tangent at A.

$$A = \frac{1}{2} \left(-\frac{M_0}{EI} \right) L = -\frac{1}{2} \frac{M_0 L}{EI}$$

$$t_{B/A} = A \left(\frac{L}{3} \right) = -\frac{1}{6} \frac{M_0 L^2}{EI}$$

$$\Theta_A = -\frac{t_{B/A}}{L} = \frac{1}{6} \frac{M_0 L}{EI}$$

$$A^* = \frac{1}{2} \left(\frac{M_0}{EI} \frac{x_k}{L} \right) x_k = -\frac{1}{2} \frac{M_0 x_k^2}{EIL}$$

$$\Theta_K = \Theta_A + \Theta_{K/A} = \Theta_A + A^* = 0$$

$$\frac{1}{6} \frac{M_0 L}{EI} - \frac{1}{2} \frac{M_0 x_k^2}{EIL} = 0$$

$$x_k = \frac{\sqrt{3}}{3} L = 0.577 L$$

$$t_{K/A} = A^* \left(\frac{1}{3} x_k \right)$$

$$= -\frac{1}{2} \frac{M_0 x_k^2}{EIL} \left(\frac{1}{3} x_k \right)$$

$$= -\frac{1}{6} \frac{M_0 x_k^3}{EIL}$$

Maximum deflection.

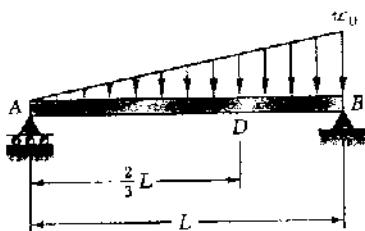
$$y_k = t_{K/A} - \frac{x_k}{L} t_{B/A} = -\frac{1}{6} \frac{M_0 x_k^2}{EIL} - \frac{x_k}{L} \left(-\frac{M_0 L^2}{6EI} \right)$$

$$= \frac{M_0 x_k}{6EI} (L^2 - x_k^2) = \frac{\sqrt{3}}{18} \frac{M_0}{EI} \left(L^2 - \frac{1}{3} L^2 \right) = \frac{\sqrt{3}}{27} \frac{M_0 L^2}{EI}$$

$$= 0.0642 \frac{M_0 L^2}{EI}$$

Problem 9.161

9.161 For the prismatic beam and loading shown, determine the magnitude and location of the largest downward deflection.



$$\rightarrow \sum M_B = 0 \quad -R_A L + (\frac{1}{2} w_0 L)(\frac{1}{3} L) = 0 \quad R_A = \frac{1}{6} w_0 L$$

$$\text{Bending moment} \quad M = R_A x - \frac{1}{6} \frac{w_0}{L} x^3 \\ = \frac{1}{6} \frac{w_0}{L} (L^2 x - x^3)$$

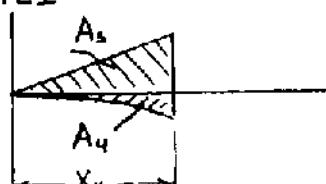
$$M/EI \quad \frac{1}{6} \frac{w_0 L^2}{EI} \quad \text{At } x = L \quad M = \frac{1}{6} w_0 L^2 - \frac{1}{6} w_0 L^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{1}{6} \frac{w_0 L^2}{EI} \right) L = \frac{1}{12} \frac{w_0 L^3}{EI} \quad \bar{x}_1 = \frac{1}{3} L$$

$$A_2 = \frac{1}{4} \left(-\frac{1}{6} \frac{w_0 L^2}{EI} \right) L = -\frac{1}{24} \frac{w_0 L^3}{EI} \quad \bar{x}_2 = \frac{1}{5} L$$

M/EI



Place reference tangent at A.

$$t_{B/A} = A_1 \bar{x}_1 + A_2 \bar{x}_2 \\ = \frac{1}{36} \frac{w_0 L^4}{EI} - \frac{1}{120} \frac{w_0 L^4}{EI} = \frac{7}{360} \frac{w_0 L^4}{EI}$$

$$\text{Slope at A.} \quad \Theta_A = -1 \frac{t_{B/A}}{L} = -\frac{7}{360} \frac{w_0 L^3}{EI}$$

$$A_3 = A_1 \left(\frac{x_k}{L} \right)^2 = \frac{1}{12} \frac{w_0 L^3}{EI} u^2$$

$$A_4 = A_2 \left(\frac{x_k}{L} \right)^4 = -\frac{1}{24} \frac{w_0 L^3}{EI} u^4$$

$$\Theta_{K/A} = A_3 + A_4 = \frac{w_0 L^3}{EI} \left(\frac{1}{12} u^2 - \frac{1}{24} u^4 \right) = -\Theta_A = \frac{7}{360} \frac{w_0 L^3}{EI}$$

$$u^4 - 2u^2 + \frac{7}{15} = 0 \quad \text{Solving for } u \quad u = 0.51933$$

$$x_k = 0.51933 L$$

$$A_3 = \frac{1}{12} \frac{w_0 L^3}{EI} (0.51933)^2 = 0.0224753 \frac{w_0 L^3}{EI}, \quad \bar{x}_3 = \frac{1}{3} (0.51933) L$$

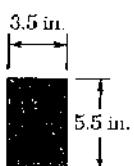
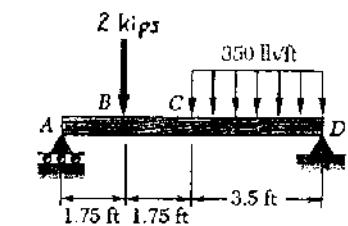
$$A_4 = -\frac{1}{24} \frac{w_0 L^3}{EI} (0.51933)^4 = -0.0030308 \frac{w_0 L^3}{EI}, \quad \bar{x}_4 = \frac{1}{5} (0.51933) L$$

$$t_{K/A} = A_3 \bar{x}_3 + A_4 \bar{x}_4 = 0.0035759 \frac{w_0 L^4}{EI}$$

$$y_k = t_{K/A} - \frac{x_k}{L} t_{B/A} = 0.0035759 \frac{w_0 L^4}{EI} - (0.51933) \left(\frac{7}{360} \frac{w_0 L^4}{EI} \right) \\ = -0.00652 \frac{w_0 L^4}{EI} = 0.00652 \frac{w_0 L^4}{EI} \downarrow$$

Problem 9.162

9.162 For the timber beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 1.6 \times 10^6$ psi.



Units: Forces in kips; lengths in ft.

$$\rightarrow \sum M_B = 0:$$

$$-7R_A + (2)(5.25) + (1.225)(1.75) = 0$$

$$R_A = 1.80625 \text{ kips}$$

$$w(x) = 0.350(x - 3.5)^0$$

$$\frac{dV}{dx} = -w = -0.35(x - 3.5)^0$$

$$\frac{dM}{dx} = V = 1.05625 - 1(x - 1.75)^0 - 0.35(x - 3.5)^0$$

$$EI \frac{d^2y}{dx^2} = M = 1.80625x - 2(x - 1.75)^1 - 0.175(x - 3.5)^2 \quad \text{kip}\cdot\text{ft}$$

$$EI \frac{dy}{dx} = 0.903125x^2 - 1(x - 1.75)^2 - 0.05833(x - 3.5)^3 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EI y = 0.301042x^3 - \frac{1}{3}(x - 1.75)^3 - 0.014583(x - 3.5)^4 + C_1x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$$[x=0, y=0] \quad C_2 = 0$$

$$[x=7, y=0] \quad (0.301042)(7)^3 - \frac{1}{3}(5.25)^3 - 0.014583(3.5)^4 + C_1(7) + 0 = 0$$

$$C_1 = -7.54779 \text{ kip}\cdot\text{ft}^2$$

$$\text{Data: } E = 1.6 \times 10^6 \text{ psi} = 1.6 \times 10^3 \text{ ksi}$$

$$I = \frac{1}{12}(3.5)(5.5)^3 = 48.526 \text{ in}^4$$

$$EI = (1.6 \times 10^3)(48.526) = 77.6417 \text{ kip}\cdot\text{in}^2 = 539.18 \text{ kip}\cdot\text{ft}^2$$

(a) Slope at A. ($\frac{dy}{dx}$ at $x = 0$)

$$EI \frac{dy}{dx} = 0 - 0 - 0 = 7.54779 \text{ kip}\cdot\text{ft}^2$$

$$\theta_A = -\frac{7.54779}{539.18} = -14.00 \times 10^{-3} \text{ rad} = 14.00 \times 10^{-3} \text{ rad}$$

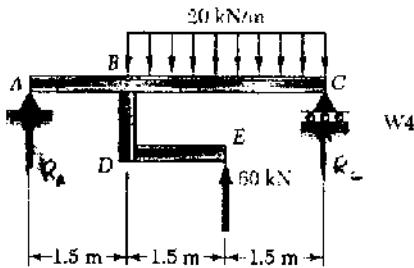
(b) Deflection at C. (y at $x = 3.5 \text{ ft}$)

$$EI y_C = (0.301042)(3.5)^3 - \frac{1}{3}(1.75)^3 - 0 - (7.54779)(3.5) + 0 \\ = -15.297 \text{ kip}\cdot\text{ft}^3$$

$$y_C = -\frac{15.297}{539.18} = -28.37 \times 10^{-3} \text{ ft} = 0.340 \text{ in.}$$

Problem 9.163

9.163 The rigid bar BDE is welded at point B to the rolled steel beam AC . For the loading shown, determine (a) the slope at point A , (b) the deflection at point B . Use $E = 200 \text{ GPa}$.



$$\text{At } M_C = 0:$$

$$-4.5R_A + (20)(3)(1.5) - (60)(1.5) = 0$$

$$R_A = 0$$

Units: Forces in kN; lengths in m.

$$EI \frac{d^2y}{dx^2} = M = 60(x-1.5)' - 90(x-1.5)^0 - \frac{1}{2}(20)(x-1.5)^2$$

$$EI \frac{dy}{dx} = 30(x-1.5)^2 - 90(x-1.5)' - (\frac{1}{6})(20)(x-1.5)^3 + C_1$$

$$EIy = 10(x-1.5)^3 - 45(x-1.5)^2 - \frac{1}{24}(20)(x-1.5)^4 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=4.5, y=0] \quad (10)(3)^3 - (45)(3)^2 - \frac{1}{24}(20)(3)^4 + 4.5C_1 + 0 = 0$$

$$C_1 = 45 \text{ kN}\cdot\text{m}^2$$

$$\text{Data: } E = 200 \times 10^9 \text{ Pa}, \quad I = 315 \times 10^6 \text{ mm}^4 = 315 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(315 \times 10^{-6}) = 63 \times 10^6 \text{ N}\cdot\text{m}^2 = 63000 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A. ($\frac{dy}{dx}$ at $x=0$)

$$EI\theta_A = C_1 = 45 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = \frac{45}{63000} = 0.714 \times 10^{-3} \text{ rad} \quad \theta_A = 0.714 \times 10^{-3} \text{ rad} \leftarrow$$

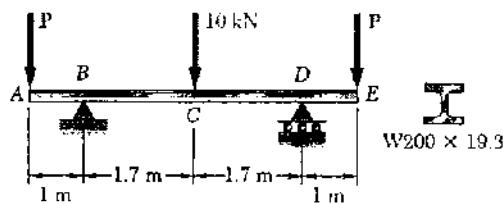
(b) Deflection at B. (y at $x=1.5$)

$$EIy_B = (C_1)(1.5) = (45)(1.5) = 67.5 \text{ kN}\cdot\text{m}^3$$

$$y_B = \frac{67.5}{63000} = 1.071 \times 10^{-3} \text{ m} = 1.071 \text{ mm} \uparrow$$

Problem 9.164

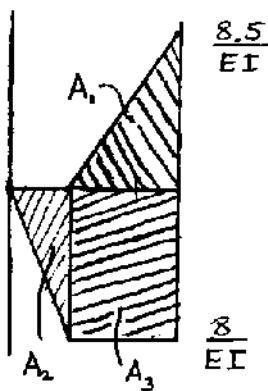
9.164 Knowing that $P = 8 \text{ kN}$, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 200 \text{ GPa}$.



$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 16.6 \times 10^6 \text{ mm}^4 = 16.6 \times 10^4 \text{ m}^4$$

$$EI = (200 \times 10^9)(16.6 \times 10^4) = 3.32 \times 10^{13} \text{ N} \cdot \text{m}^2 \\ = 3320 \text{ kN} \cdot \text{m}^2$$



Symmetric beam and loading.

$$R_A = R_B = P + S = 8 + 5 = 13 \text{ kN}$$

Bending moment

$$\text{Over } AB \quad M = -Px = -8x$$

$$\text{Over } BC \quad M = -8x + 13(x-1) \\ = 5(x-1) - 8$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{8.5}{EI} \right) (1.7) = \frac{7.225}{EI}$$

$$A_2 = -\frac{1}{2} \left(\frac{8}{EI} \right) (1) = -\frac{4}{EI}$$

$$A_3 = -\left(\frac{8}{EI} \right) (1.7) = -\frac{13.600}{EI}$$

Place reference tangent at C $\theta_c = 0$

$$(a) \text{ Slope at } A. \quad \theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3)$$

$$\theta_A = -\left(\frac{7.225}{EI} - \frac{4}{EI} - \frac{13.600}{EI} \right) = \frac{10.375}{EI} = \frac{10.375}{3320} = 3.125 \times 10^{-3} \text{ rad}$$

$$(b) \text{ Deflection at } C. \quad y_c = -t_{B/C}$$

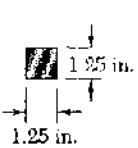
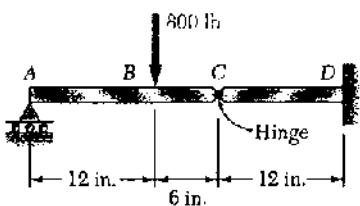
$$= -(A_1 \bar{x}_1 + A_3 \bar{x}_3)$$

$$= -\left[\left(\frac{7.225}{EI} \right) \left(\frac{1}{3}(1.7) \right) - \left(\frac{13.600}{EI} \right) \left(\frac{1.7}{2} \right) \right] = \frac{3.3717}{EI} = \frac{3.3717}{3320}$$

$$= 1.016 \times 10^{-3} \text{ m} = 1.016 \text{ mm} \uparrow$$

Problem 9.165

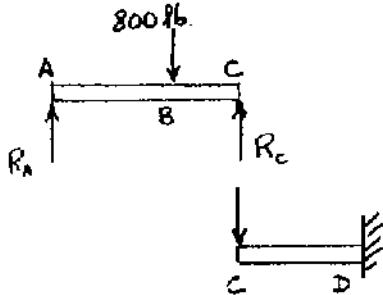
9.165 The two beams shown have the same cross section and are joined by a hinge at C. For the loading shown, determine (a) the slope at point A, (b) the deflection at point B. Use $E = 29 \times 10^6$ psi.



Using free body ABC,

$$\textcircled{D} \sum M_A = 0: 18 R_c - (12)(800) = 0$$

$$R_c = 533.33 \text{ lb.}$$



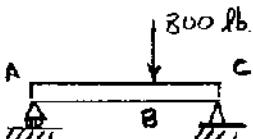
$$E = 29 \times 10^6 \text{ psi}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(1.25)(1.25)^3 = 0.20345 \text{ in}^4$$

$$EI = (29 \times 10^6)(0.20345) = 5.900 \times 10^6 \text{ lb-in}^2$$

Using cantilever beam CD with load R_C ,

Case I of Appendix D.



$$y_C = -\frac{R_c L_{co}^3}{3EI} = -\frac{(533.33)(12)^3}{(3)(5.900 \times 10^6)} = -52.067 \times 10^{-3} \text{ in.}$$

Calculation of θ_A' and y_B' assuming that point C does not move.

Case 5 of Appendix D $P = 800 \text{ lb.}$, $L = 18 \text{ in.}$, $a = 12 \text{ in.}$, $b = 6 \text{ in.}$

$$\theta_A' = -\frac{Pb(l^2 - b^2)}{6EI L} = -\frac{(800)(6)(18^2 - 6^2)}{(6)(5.900 \times 10^6)(18)} = -2.1695 \times 10^{-3} \text{ rad.}$$

$$y_B' = -\frac{Pb^2 a^2}{3EI L} = -\frac{(800)(6)^2(12)^2}{(3)(5.900 \times 10^6)(18)} = -13.017 \times 10^{-3} \text{ in.}$$

Additional slope and deflection due to movement of point C.

$$\theta_A'' = \frac{y_C}{L_{AC}} = -\frac{52.067 \times 10^{-3}}{18} = -2.8926 \times 10^{-3} \text{ rad.}$$

$$y_B'' = \frac{a}{L} y_C = -\frac{(12)(52.067 \times 10^{-3})}{18} = -34.711 \times 10^{-3} \text{ in.}$$

$$(a) \text{ Slope at } A. \quad \theta_A = \theta_A' + \theta_A'' = -2.1695 \times 10^{-3} - 2.8926 \times 10^{-3}$$

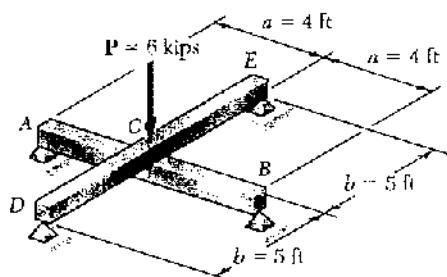
$$= -5.06 \times 10^{-3} \text{ rad} = 5.06 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

$$(b) \text{ Deflection at } B. \quad y_B = y_B' + y_B'' = -13.017 \times 10^{-3} - 34.711 \times 10^{-3}$$

$$= -47.7 \times 10^{-3} \text{ in.} = 0.0477 \text{ in.} \downarrow \quad \blacktriangleleft$$

Problem 9.166

9.166 For the loading shown, and knowing that beams *AB* and *DE* have the same flexural rigidity, determine the reaction (a) at *B*, (b) at *E*.



Units: Forces in kips; lengths in ft.

For beam *ACB*, using Case 4 of Appendix D.

$$(y_c)_1 = -\frac{R_c(2a)^3}{48EI}$$

For beam *DCE*, using Case 4 of Appendix D.

$$(y_c)_2 = \frac{(R_c - P)(2b)^3}{48EI}$$

Matching deflections at *C*,

$$-\frac{R_c(2a)^3}{48EI} = \frac{(R_c - P)(2b)^3}{48EI}$$

$$R_c = \frac{Pb^3}{a^3 + b^3} = \frac{(6)(5)^3}{4^3 + 5^3} = 3.968 \text{ kips}$$

$$P - R_c = 6 - 3.968 = 2.032 \text{ kips}$$

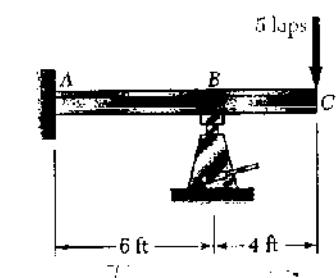
Using free body *ACB*, $\sum M_A = 0: 2aR_B - aR_c = 0$

$$(a) R_B = \frac{1}{2}R_c = 1.984 \text{ kips} \uparrow$$

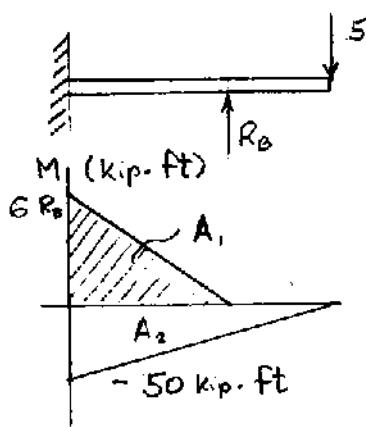
Using free body *DCE*, $\sum M_D = 0: 2bR_E - b(P - R_c) = 0$

$$(b) R_E = \frac{1}{2}(P - R_c) = 1.016 \text{ kips} \uparrow$$

Problem 9.167



9.167 A hydraulic jack can be used to raise point B of the cantilever beam *ABC*. The beam was originally straight, horizontal, and unloaded. A 5-kip load was then applied at point *C*, causing this point to move down. Determine (a) how much point *B* should be raised to return point *C* to its original position, (b) the final value of the reaction at *B*. Use $E = E = 29 \times 10^6$ psi.



$$\text{For } W5 \times 16 \quad I_y = 21.3 \text{ in}^4$$

$$E = 29 \times 10^6 \text{ psi}$$

$$EI = (29 \times 10^6)(21.3) = 617.7 \times 10^6 \text{ lb-in}^2$$

$$= 4.2896 \times 10^5 \text{ kip-ft}^2$$

Let R_B be the jack force in Kips.

$$A_1 = \frac{1}{2}(6 R_B)(6) = 18 R_B$$

$$A_2 = \frac{1}{2}(-50)(10) = -250 \text{ kip-ft}^2$$

$$EI t_{C/A} = (4 + 4) A_1 + \left(\frac{2}{3} \cdot 10\right) A_2$$

$$= 144 R_B - 1666.67$$

$$R_B = \frac{1666.67}{144} = 11.5741 \text{ kips}$$

$$A_1 = 208.333 \text{ kip-ft}^2$$

$$A_3 = \frac{1}{2}(-50)(6) = -150$$

$$A_4 = \frac{1}{2}(-20)(6) = -60$$

$$EI t_{B/A} = 4 A_1 + 4 A_3 + 2 A_4$$

$$= 113.333 \text{ kip-ft}^3$$

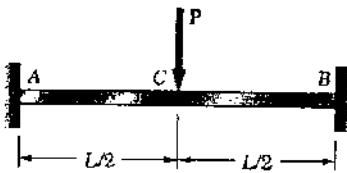
$$(a) y_a = t_{B/A} = \frac{EI t_{B/A}}{EI} = \frac{113.333}{4.2896 \times 10^5} = 26.42 \times 10^{-3}$$

$$= 26.42 \times 10^{-3} \text{ ft} = 0.317 \text{ in.} \uparrow$$

$$(b) R_B = 11.57 \text{ kips} \uparrow$$

Problem 9.168

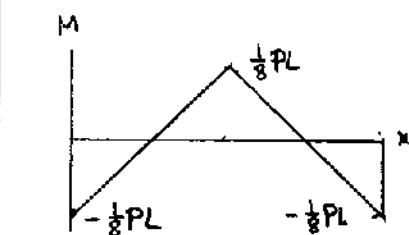
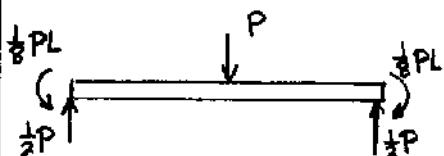
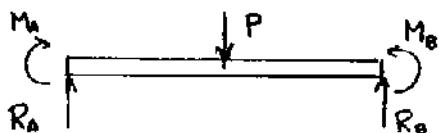
9.168 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0]$$



By symmetry, $R_A = R_B$ and $\frac{dy}{dx} = 0$ at $x = \frac{L}{2}$.

$$+\uparrow \sum F_y = 0 \quad R_A + R_B - P = 0 \quad R_A = R_B = \frac{1}{2}P \quad \rightarrow$$

Moment reaction is statically indeterminate.

$$0 < x < \frac{L}{2} \quad M = M_A + R_A x = M_A + \frac{1}{2}Px$$

$$EI \frac{d^2y}{dx^2} = M_A + \frac{1}{2}Px$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 - 0 + C_1 = 0 \quad C_1 = 0$$

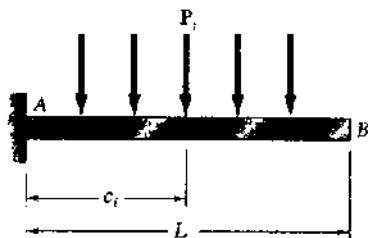
$$[x=\frac{L}{2}, \frac{dy}{dx}=0] \quad M_A \frac{L}{2} + \frac{1}{4}P\left(\frac{L}{2}\right)^2 + 0 = 0$$

$$M_A = -\frac{1}{8}PL \quad M_A = \frac{1}{8}PL \quad \rightarrow$$

$$\text{By symmetry, } M_B = M_A = \frac{1}{8}PL \quad \rightarrow$$

$$M_C = M_A + \frac{1}{2}P \frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL = \frac{1}{8}PL \quad \rightarrow$$

PROBLEM 9.C1



9.C1 Several concentrate loads can be applied to the cantilever beam AB . Write a computer program to calculate the slope and deflection of beam AB from $x = 0$ to $x = L$, using given increments Δx . Apply this program with increments $\Delta x = 50 \text{ mm}$ to the beam and loading of Probs. 9.73 and Prob. 9.74.

SOLUTION

FOR EACH LOAD, ENTER

$$P_i, c_i$$

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS

$$R_A = R_A + P_i$$

$$M_A = M_A - P_i c_i$$

COMPUTE SLOPE AND DEFLECTION

USE METHOD OF INTEGRATION

STARTING WITH $x=0$ AND UPDATING
THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A:

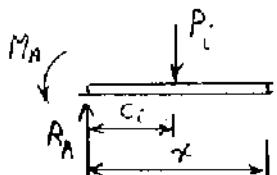
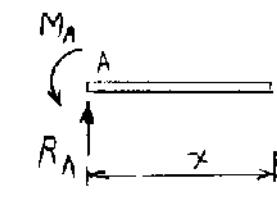
$$\Theta = (1/EI)(R_A x^2/2.0 + M_A x)$$

$$y = (1/EI)(R_A x^3/6.0 + M_A x^2/2.0)$$

(2) DUE TO EACH LOAD WITH $c_i < x$:

$$\Theta = -(1/EI)(P_i/2.0)(x - c_i)^2$$

$$y = -(1/EI)(P_i/6.0)(x - c_i)^3$$



$$\text{AT } x = 0, y = \frac{dy}{dx} = 0$$

∴ THE CONSTANTS OF
INTEGRATION EQUAL ZERO

CONTINUED

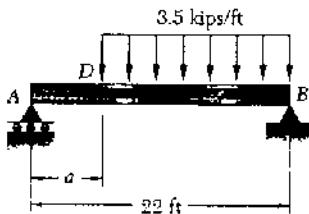
PROBLEM 9.C1 CONTINUEDPROGRAM OUTPUT

Problem 9.73 and 9.74

At A: Force = 6.0 kN Couple = -6.0 kN·m

x m	Slope radians	Deflection m
.00	.000000	.000000
.05	-.000578	-.000015
.10	-.001126	-.000057
.15	-.001645	-.000127
.20	-.002134	-.000221
.25	-.002594	-.000340
.30	-.003024	-.000480
.35	-.003424	-.000642
.40	-.003794	-.000822
.45	-.004135	-.001021
.50	-.004447	-.001235
.55	-.004728	-.001465
.60	-.004980	-.001708
.65	-.005203	-.001962
.70	-.005395	-.002227
.75	-.005558	-.002501
.80	-.005699	-.002783
.85	-.005825	-.003071
.90	-.005936	-.003365
.95	-.006033	-.003664
1.00	-.006114	-.003968
1.05	-.006181	-.004275
1.10	-.006233	-.004586
1.15	-.006270	-.004898
1.20	-.006292	-.005213
1.25	-.006299	-.005527

PROBLEM 9.C2



9.C2 The 22-ft beam AB consists of a W 21× 62 rolled-steel shape and supports a 3.5-kip/ft distributed load as shown. Write a computer program and use it to calculate for values of a from 0 to 22 ft, using 1-ft increments, (a) the slope and deflection at D , (b) the location and magnitude of the maximum deflection. Use $E = 29 \times 10^6$ psi.

SOLUTION

ENTER LOAD w , LENGTH L , α

COMPUTE REACTION AT A

$$R_A = w(L - \alpha)^2 / (2.0L)$$

COMPUTE SLOPE AND DEFLECTION AT D

USING SINGULARITY FUNCTIONS:

$$C_1 = -\frac{w}{24L} (L - \alpha)^4 - \frac{1}{6} R_A L^2$$

$$\theta = (1/EI) (R_A \alpha^2 / 2.0 + C_1)$$

$$y = (1/EI) (R_A \alpha^3 / 6.0 + C_1 \alpha)$$

$$EI \frac{d^2y}{dx^2} = R_A x - \frac{w}{2} (x - \alpha)^2$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{w}{6} (x - \alpha)^3 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{w}{24} (x - \alpha)^4 + C_1 x + C_2$$

FROM BOUNDARY CONDITIONS:

$$C_2 = 0$$

$$C_1 = -\frac{w}{24L} (L - \alpha)^4 - \frac{1}{6} R_A L^2$$

COMPUTE LOCATION AND MAGNITUDE OF MAXIMUM DEFLECTION

MAXIMUM y AT $\theta = 0$:

$$0 = \frac{1}{2} R_A x^2 - \frac{w}{6} (x - \alpha)^3 + C_1$$

IF $x_{max} \leq \alpha$

$$\frac{1}{2} R_A x^2 + C_1 = 0$$

$$x_{max} = \sqrt{\frac{-2.0 C_1}{R_A}}$$

$$y_{max} = \frac{1}{6} R_A x_{max}^3 + C_1 x_{max}$$

ASSUME $\gamma < A$:

$$x_{max} = (-2.0 C_1 / R_A)^{\frac{1}{2}}$$

IF $x_{max} < \alpha$, THEN

$$y_{max} = (1/EI) (\frac{1}{6} R_A x_{max}^3 + C_1 x_{max})$$

IF $x_{max} > \alpha$, THEN

BEGIN WITH $x = \alpha$

$$\theta = (1/EI) (\frac{1}{2} R_A x - \frac{1}{6} (x - \alpha)^3 + C_1)$$

INCREASE x BY SMALL AMOUNT
UNTIL θ IS APPROXIMATELY 0

$$y_{max} = (1/EI) (\frac{1}{6} R_A x^3 - \frac{w}{24} (x - \alpha)^4 + C_1 x)$$

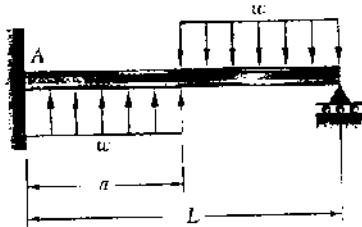
CONTINUED

PROBLEM 9.C2 CONTINUED

PROGRAM OUTPUT

	a ft	theta D radians	yD in.	xm ft	ym in.
0.	.00580	.000000	11.000	+.478290	
1.	-.00569	-.068798	11.008	-.475922	
2.	-.00539	-.133047	11.030	-.468860	
3.	-.00494	-.189440	11.068	-.457231	
4.	-.00439	-.235551	11.121	-.441245	
5.	-.00378	-.269927	11.189	-.421192	
6.	-.00314	-.291944	11.272	-.397443	
7.	-.00250	-.301695	11.370	-.370441	
8.	-.00188	-.299889	11.481	-.340699	
9.	-.00131	-.287738	11.606	-.308795	
10.	-.00080	-.266855	11.742	-.275364	
11.	-.00036	-.239145	11.885	-.241090	
12.	-.00001	-.206699	12.028	-.206700	
13.	.00025	-.171684	12.159	-.172954	
14.	.00043	-.136240	12.275	-.140603	
15.	.00052	-.102374	12.376	-.110339	
16.	.00054	-.071846	12.463	-.082792	
17.	.00049	-.046069	12.537	-.056515	
18.	.00039	-.026001	12.596	-.037987	
19.	.00027	-.012036	12.643	-.021604	
20.	.00014	-.003896	12.675	-.009677	
21.	.00004	-.000530	12.695	-.002431	
22.	.00000	.000000	12.702	.000000	

PROBLEM 9.C3



9.C3 The cantilever beam AB carries the distributed loads shown. Write a computer program to calculate the slope and deflection of beam AB from $x = 0$ to $x = L$ using given increments Δx . Apply this program with increments $\Delta x = 100$ mm, assuming that $L = 2.4$ m, $w = 36$ kN/m, and (a) $a = 0.6$ m, (b) $a = 1.2$ m, (c) $a = 1.8$ m. Use $E = 200$ GPa.

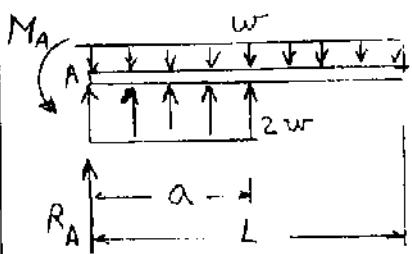
SOLUTION

ENTER w, a, L

COMPUTE REACTION AT A

$$R_A = wL - 2.0wa$$

$$M_A = \frac{1}{2}wL^2 - \frac{1}{2}wa^2$$



COMPUTE SLOPE AND DEFLECTION

USE EQUATION OF ELASTIC CURVE

STARTING WITH $x=0$ AND UPDATING THROUGH INCREMENTS; SUPERPOSE:

(1) DUE TO REACTIONS AT A

$$\theta = (1/EI) \left(\frac{1}{2} R_A x^2 + M_A x \right)$$

$$y = (1/EI) \left(\frac{1}{6} R_A x^3 + \frac{1}{2} M_A x^2 \right)$$

(2) DUE TO LOAD w

$$\theta = -(1/EI) \left(\frac{1}{6} w x^3 \right)$$

$$y = -(1/EI) \left(\frac{1}{24} w x^4 \right)$$

(3) DUE TO LOAD $2w$

IF $x \leq a$

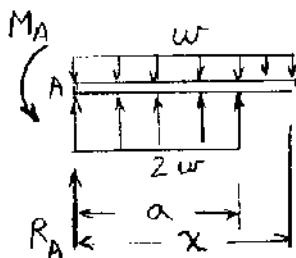
$$\theta = (1/EI) \left(\frac{1}{3} w x^3 \right)$$

$$y = (1/EI) \left(\frac{1}{12} w x^4 \right)$$

IF $x > a$

$$\theta = (1/EI) \left(\frac{1}{3} w x^3 - \frac{1}{3} w (x-a)^3 \right)$$

$$y = (1/EI) \left(\frac{1}{12} w x^4 - \frac{1}{12} w (x-a)^4 \right)$$



$$\text{AT } x=0, y = \frac{dy}{dx} = 0$$

\therefore THE CONSTANTS OF INTEGRATION ARE ZERO

CONTINUED

PROBLEM 9.C3 CONTINUED

PROGRAM OUTPUT

Problem 9.C3 (a) $a = 0.6 \text{ m}$

At A: Force = 43.2 kN Couple = -90.7 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000905	-.000046
.20	-.001762	-.000179
.30	-.002567	-.000396
.40	-.003318	-.000691
.50	-.004009	-.001058
.60	-.004638	-.001491
.70	-.005202	-.001983
.80	-.005703	-.002529
.90	-.006145	-.003122
1.00	-.006533	-.003756
1.10	-.006868	-.004427
1.20	-.007156	-.005128
1.30	-.007399	-.005856
1.40	-.007602	-.006607
1.50	-.007769	-.007376
1.60	-.007902	-.008160
1.70	-.008006	-.008955
1.80	-.008083	-.009760
1.90	-.008139	-.010571
2.00	-.008177	-.011387
2.10	-.008199	-.012206
2.20	-.008211	-.013027
2.30	-.008215	-.013848
2.40	-.008216	-.014669

Problem 9.C3 (b) $a = 1.2 \text{ m}$

At A: Force = 0.0 kN Couple = -51.8 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000529	-.000026
.20	-.001055	-.000106
.30	-.001574	-.000237
.40	-.002081	-.000420
.50	-.002574	-.000653
.60	-.003048	-.000934
.70	-.003500	-.001262
.80	-.003926	-.001633
.90	-.004323	-.002046
1.00	-.004687	-.002497
1.10	-.005014	-.002982
1.20	-.005301	-.003498
1.30	-.005544	-.004041
1.40	-.005747	-.004606
1.50	-.005913	-.005189
1.60	-.006047	-.005787
1.70	-.006150	-.006398
1.80	-.006228	-.007017
1.90	-.006284	-.007642
2.00	-.006321	-.008273
2.10	-.006344	-.008906
2.20	-.006356	-.009541
2.30	-.006360	-.010177
2.40	-.006361	-.010813

CONTINUED

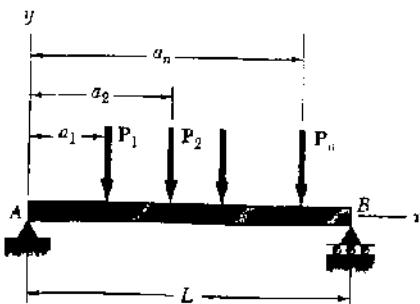
PROBLEM 9.C3 PROGRAM OUTPUTS CONTINUED

Problem 9.C3 (c) $a = 1.8 \text{ m}$

At A: Force = -43.2 kN Couple = 13.0 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	.000111	.000006
.20	.000182	.000021
.30	.000215	.000041
.40	.000216	.000063
.50	.000187	.000083
.60	.000133	.000099
.70	.000056	.000109
.80	-.000039	.000110
.90	-.000149	.000101
1.00	-.000270	.000080
1.10	-.000398	.000046
1.20	-.000530	.000000
1.30	-.000662	-.000060
1.40	-.000790	-.000132
1.50	-.000911	-.000217
1.60	-.001021	-.000314
1.70	-.001116	-.000421
1.80	-.001193	-.000537
1.90	-.001248	-.000659
2.00	-.001286	-.000786
2.10	-.001309	-.000916
2.20	-.001320	-.001047
2.30	-.001325	-.001179
2.40	-.001325	-.001312

PROBLEM 9.C4



9.C4 The simple beam AB is of constant flexural rigidity EI and carries several concentrated loads as shown. Using the *Method of Integration*, write a computer program that can be used to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx . Apply this program to the beam and loading of (a) Prob. 9.13 with $\Delta x = 1$ ft, (b) Prob. 9.16 with $\Delta x = 0.05$ m, (c) Prob. 9.129 $\Delta x = 0.25$ m.

SOLUTION

FOR EACH LOAD, ENTER P_i, a_i

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS:

$$M_A = M_A + P_i a_i$$

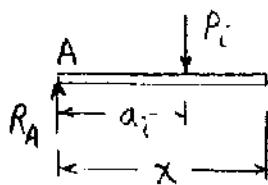
$$\text{LOAD} = \text{LOAD} + P_i$$

THEN:

$$R_B = M_A / L$$

$$R_A = \text{LOAD} - R_B$$

FOR LOAD P_i :



FOR $x < a_i$:

$$EI \frac{d^2y}{dx^2} = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

FOR $x > a_i$:

$$EI \frac{d^2y}{dx^2} = R_A x - P_i(x - a_i)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P_i(x - a_i)^2 + C_3$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{6} (x - a_i)^3 + C_3 x + C_4$$

FROM BOUNDARY CONDITIONS

$$C_2 = C_4 = 0$$

$$C_1 = C_3 = \frac{P_i}{6L} (L - a_i)^3 - \frac{1}{6} R_A L^2$$

NOTE: R_A FOR LOAD P_i

COMPUTE SLOPE AND DEFLECTION

STARTING WITH $x = 0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A

$$\theta = (1/EI) \left(\frac{1}{2} R_A x^2 \right)$$

$$y = (1/EI) \left(\frac{1}{6} R_A x^3 \right)$$

(2) DUE TO LOADS - CONSTANT PART

$$\text{CONST}_1 = -\frac{1}{6} R_A L^2$$

FOR i TO NUMBER LOADS

$$\text{CONST}_2 = \frac{1}{6L} P_i (L - a_i)^3 + \text{CONST}_1$$

THEN, TOTAL CONTRIBUTION FOR CONSTANT

$$\text{CONST} = (1/EI) (\text{CONST}_1 + \text{CONST}_2)$$

(3) DUE TO LOADS - REMAINING PART

IF $x \leq a_i$

$$\theta = (1/EI) \left(\frac{1}{2.0} R_A x^2 \right)$$

$$y = (1/EI) \left(\frac{1}{6.0} R_A x^3 \right)$$

IF $x > a_i$

$$\theta = (1/EI) \left(\frac{1}{2.0} R_A x^2 - \frac{1}{2.0} P_i (x - a_i)^2 \right)$$

$$y = (1/EI) \left(\frac{1}{6.0} R_A x^3 - \frac{1}{6.0} P_i (x - a_i)^3 \right)$$

CONTINUED

PROBLEM 9.C4 CONTINUED

PROGRAM OUTPUT

Problem 9.13

x in.	theta rad*10***3	y in.
.00	-8.2948	.000000
12.00	-8.0957	-.098742
24.00	-7.4985	-.192705
36.00	-6.5031	-.277113
48.00	-5.1096	-.347188
60.00	-3.3179	-.398151
72.00	-1.4267	-.426420
84.00	.2654	-.433189
96.00	1.7585	-.420846
108.00	3.0525	-.391781
120.00	4.1474	-.348383
132.00	5.0433	-.293039
144.00	5.7400	-.228141
156.00	6.2377	-.156075
168.00	6.5363	-.079232
180.00	6.6359	.000000

Problem 9.16

x m	theta rad*10***3	y mm
.000	-2.490	.000
.050	-2.485	-.124
.100	-2.471	-.248
.150	-2.448	-.371
.200	-2.416	-.493
.250	-2.375	-.613
.300	-2.325	-.730
.350	-2.265	-.845
.400	-2.197	-.957
.450	-2.119	-1.065
.500	-2.032	-1.168
.550	-1.936	-1.268
.600	-1.831	-1.362
.650	-1.716	-1.451
.700	-1.593	-1.533
.750	-1.460	-1.610
.800	-1.318	-1.679
.850	-1.172	-1.741
.900	-1.025	-1.796
.950	-.879	-1.844

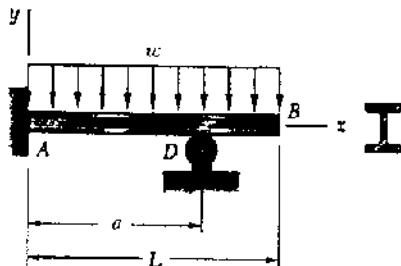
Problem 9.129

x m	theta rad*10***3	y mm	(a)
.000	-8.703	.000	
.250	-8.615	-2.168	
.500	-8.351	-4.293	
.750	-7.911	-6.329	
1.000	-7.296	-8.234	
1.250	-6.505	-9.962	
1.500	-5.538	-11.472	
1.750	-4.483	-12.724	
2.000	-3.428	-13.713	
2.250	-2.373	-14.438	
2.500	-1.319	-14.900	
2.750	-.264	-15.098	
3.000	.791	-15.032	
3.250	1.802	-14.706	
3.500	2.725	-14.138	
3.750	3.560	-13.350	
4.000	4.307	-12.365	
4.250	4.967	-11.204	
4.500	5.538	-9.889	
4.750	6.021	-8.442	
5.000	6.417	-6.886	
5.250	6.725	-5.241	
5.500	6.944	-3.531	
5.750	7.076	-1.776	
6.000	7.120	.000	

(b)

1.000	-.732	-1.884
1.050	-.586	-1.917
1.100	-.439	-1.943
1.150	-.293	-1.961
1.200	-.146	-1.972
1.250	.000	-1.976
1.300	.146	-1.972
1.350	.293	-1.961
1.400	.439	-1.943
1.450	.586	-1.917
1.500	.732	-1.884
1.550	.879	-1.844
1.600	1.025	-1.796
1.650	1.172	-1.741
1.700	1.318	-1.679
1.750	1.460	-1.610
1.800	1.593	-1.533
1.850	1.716	-1.451
1.900	1.831	-1.362
1.950	1.936	-1.268
2.000	2.032	-1.168
2.050	2.119	-1.065
2.100	2.197	-.957
2.150	2.265	-.845
2.200	2.325	-.730
2.250	2.375	-.613
2.300	2.416	-.493
2.350	2.448	-.371
2.400	2.471	-.248
2.450	2.485	-.124
2.500	2.490	.000

PROBLEM 9.C5



9.C5 The supports of beam *AB* consist of a fixed support at end *A* and a roller support located at point *D*. Write a computer program that can be used to calculate the slope and deflection at the free end of the beam for values of *a* from 0 to *L* using given increments Δa . Apply this program to calculate the slope and deflection at point *B* for each of the following cases:

<i>L</i>	ΔL	<i>w</i>	<i>E</i>	Shape
(a) 12 ft	0.5 ft	1.6 k/ft	29×10^6 psi	W 16x57
(b) 3 m	0.2 m	18 kN/m	200 GPa	W460x113

SOLUTION

BEAM IS INDETERMINATE

USE APPENDIX B AND SUPERPOSITION

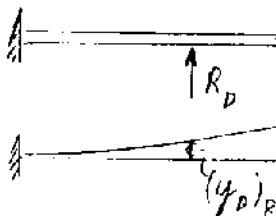
DETERMINE REACTION AT *D*

DUE TO DISTRIBUTED LOAD

$$(y_D)_w = -\frac{w}{24EI} (a^4 - 4La^3 + 6L^2a^2)$$

DUE TO REDUNDANT LOAD:

$$(y_D)_R = \frac{R_D L^3}{3EI}$$



REDUNDANT REACTION:

$$\text{SINCE } (y_D)_w + (y_D)_R = 0:$$

$$R_D = \frac{3EI}{L^3} (y_D)_w$$

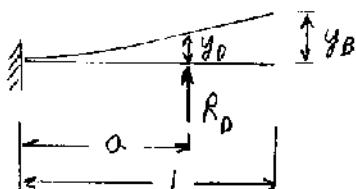
COMPUTE SLOPE AND DEFLECTION AT *B*

SUPERPOSE:

DUE TO DISTRIBUTED LOAD:

$$\Theta_B = -\frac{wL^3}{6EI}$$

$$y_B = -\frac{wL^4}{8EI}$$



DUE TO R_D :

$$\Theta_B = \frac{Pa^2}{2EI}$$

$$y_B = \frac{Pa^3}{3EI} + (L-a) \frac{Pa^2}{2EI}$$

$$\Theta_B = \Theta_D$$

$$y_B = y_D + (L-a)\Theta_D$$

CONTINUED

PROBLEM 9.C5 CONTINUED**PROGRAM OUTPUT**

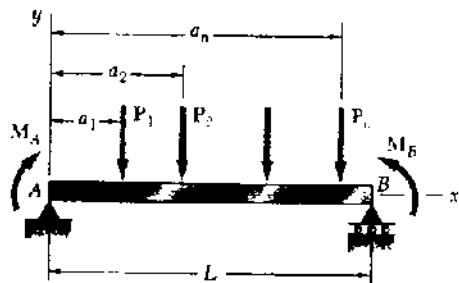
Problem 9.C5 (a)

a ft	theta B rad*10^-3	y at B in.
.0	-3.019	-.3260
.5	-2.743	-.2869
1.0	-2.483	-.2511
1.5	-2.238	-.2183
2.0	-2.007	-.1885
2.5	-1.790	-.1614
3.0	-1.586	-.1369
3.5	-1.395	-.1149
4.0	-1.216	-.0953
4.5	-1.049	-.0778
5.0	-.893	-.0624
5.5	-.748	-.0490
6.0	-.613	-.0374
6.5	-.488	-.0274
7.0	-.373	-.0191
7.5	-.266	-.0122
8.0	-.168	-.0067
8.5	-.077	-.0025
9.0	.006	.0006
9.5	.082	.0027
10.0	.152	.0037
10.5	.216	.0039
11.0	.274	.0033
11.5	.328	.0020
12.0	.377	.0000

Problem 9.C5 (b)

a m	theta B rad*10^-3	y at B mm
.0	-.728	-1.6389
.2	-.624	-1.3324
.4	-.529	-1.0663
.6	-.442	-.8374
.8	-.364	-.6426
1.0	-.293	-.4789
1.2	-.230	-.3435
1.4	-.174	-.2338
1.6	-.124	-.1472
1.8	-.079	-.0813
2.0	-.040	-.0337
2.2	-.006	-.0024
2.4	.023	.0149
2.6	.049	.0198
2.8	.072	.0143

PROBLEM 9.C6



9.C6 For the beam and loading shown, use the *Moment-Area Method* to write a computer to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx . Apply this program to calculate the slope and deflection at each concentrated load for the beam of (a) Prob. 9.77 with $\Delta x = 0.5$ m, (b) Prob. 9.119 with $\Delta x = 0.5$ m.

SOLUTION

ENTER M_A AND M_B

FOR EACH LOAD ENTER P_i AND a_i

DETERMINE REACTION AT A

DUE TO MOMENTS AT ENDS:

$$(R_A)_1 = -(M_A - M_B)/L$$

DUE TO LOADS P_i :

FOR $i = 1$ TO NUMBER OF LOADS

$$R_B = R_B + P_i a_i / L$$

LOAD = LOAD + P_i

$$(R_A)_2 = LOAD - R_B$$

$$R_A = (R_A)_1 + (R_A)_2$$

DETERMINE SLOPE AT A

USE SECOND MOMENT-AREA THEOREM
TO GET TANGENTIAL DEVIATION AT B

DUE TO M_A :

$$t_{B/A} = M_A L^3 / (2.0 EI)$$

DUE TO R_A :

$$t_{B/A} = R_A L^3 / (6.0 EI)$$

DUE TO LOADS P_i :

FOR $i = 1$ TO NUMBER OF LOADS

$$t_{B/A} = -P_i (L - a_i)^3 / (6.0 EI)$$

SUM $t_{B/A}$:

$$\theta_A = -t_{B/A} / L$$

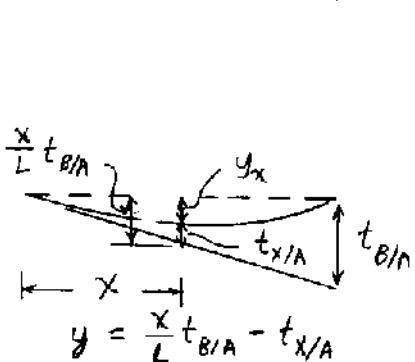
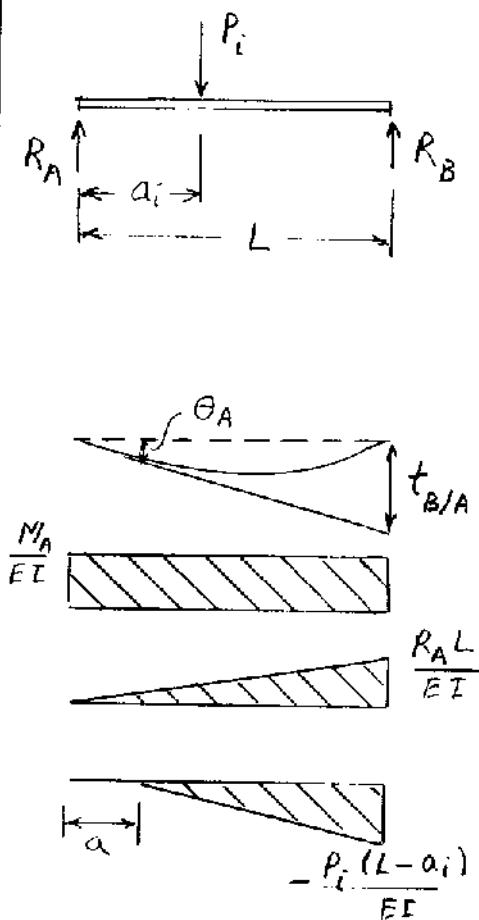
DETERMINE SLOPE AND DEFLECTIONS

FOR $x = 0$ TO L , SUPERPOSE:

DUE TO M_A AND R_A :

$$\theta_x = \theta_A + (M_A x + R_A x^2 / 2.0) / EI$$

CONTINUED



PROBLEM 9.C6 CONTINUED

$$y_x = \frac{x}{L} t_{B/A} - M_A x^2 / (2.0 EI) - R_A x^3 / (6.0 EI)$$

DUE TO LOADS P_i :

DO FOR ALL LOADS WITH $a_i < x$

$$\theta_x = P_i (x - a_i)^2 / (2.0 EI)$$

$$y_x = P_i (x - a_i)^3 / (6.0 EI)$$

PROGRAM OUTPUT

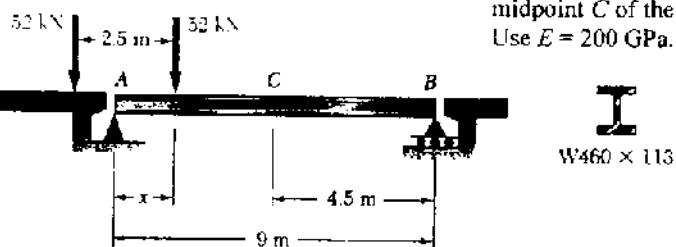
Problem 9.77

x m	theta rad*1000	y at x mm	
.000	-.600962	.000000	◀ (a)
.500	-1.602564	.574252	
1.000	-2.043269	1.509081	
1.500	-1.923077	2.524039	
2.000	-1.241987	3.338675	
2.500	.000000	3.672543	◀ (b)
3.000	1.241987	3.338676	
3.500	1.923077	2.524039	
4.000	2.043269	1.509082	
4.500	1.602564	.574253	
5.000	.600962	.000000	

Problem 9.119

x m	theta rad*1000	y at x mm	
.000	-4.504505	.000000	◀ (a)
.500	-4.673423	2.317943	
1.000	-4.279279	4.579579	
1.500	-3.322072	6.503378	
2.000	-1.801802	7.807808	
2.500	.000000	8.258258	◀ (b)
3.000	1.801802	7.807808	
3.500	3.322072	6.503378	
4.000	4.279279	4.579579	
4.500	4.673423	2.317943	
5.000	4.504505	.000000	

PROBLEM 9.C7



9.C7 Two 52 kN loads are maintained 2.5 m apart as they are moved slowly across beam AB. Write a computer program to calculate the deflection at the midpoint C of the beam for values of x from 0 to 9 m, using 0.5-m increments. Use $E = 200 \text{ GPa}$.

SOLUTION

ENTER LOAD P , BEAM LENGTH L AND SPACE BETWEEN LOADS D .

WILL SOLVE WITH MOMENT-AREA METHOD

DETERMINE DEFLECTION AT C

FOR $x = 0$ TO L

IF $0 \leq x \leq D$:

HAVE ONE LOAD TO LEFT OF C

$$R_B = Px/L$$

$$t_{A/B} = (R_B L^3 - P x^3) / (6.0 EI)$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

$$t_{C/B} = R_B L^3 / (48.0 EI)$$



IF $D < x \leq L/2$

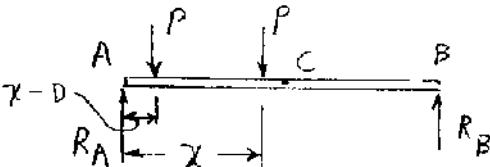
HAVE TWO LOADS TO LEFT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - P x^3 - P(x-D)^3) / (6.0 EI)$$

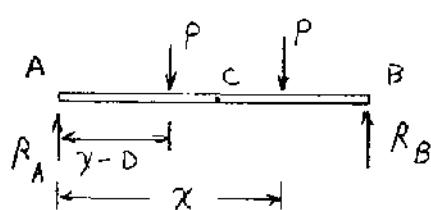
$$t_{C/B} = R_B L^3 / (48.0 EI)$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



IF $L/2 < x \leq (L/2 + D)$

HAVE ONE LOAD TO LEFT OF C AND ONE TO RIGHT OF C OR AT C



CONTINUED

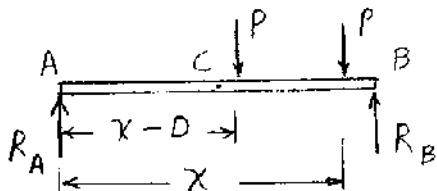
PROBLEM 9.C7 CONTINUED

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - P x^3 - P(x-D)^3)/(6.0 EI)$$

$$t_{C/B} = (R_B L^3/48.0 - P(x - \frac{L}{2})^3/6.0)/EI$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



IF $(L/2 + D) < x < L$

HAVE BOTH LOADS TO RIGHT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - P x^3 - P(x-D)^3)/(6.0 EI)$$

$$t_{C/B} = (R_B L^3/48.0 - P(x - \frac{L}{2})^3/6.0$$

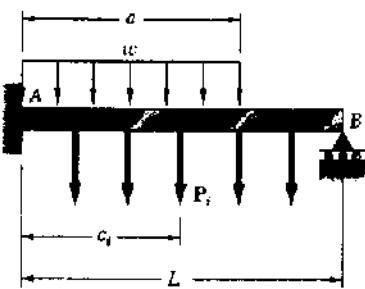
$$- P(\gamma - D - \frac{L}{2})^3/6.0)/EI$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

PROGRAM OUTPUT

x m	R _B kN	Theta _B rad	Y _C mm
.0	.000	.00000	.00000
.5	2.889	.00315	1.17881
1.0	5.778	.00624	2.32839
1.5	8.667	.00921	3.41951
2.0	11.556	.01200	4.42296
2.5	14.444	.01456	5.30950
3.0	20.222	.01998	7.22872
3.5	26.000	.02499	8.94335
4.0	31.778	.02947	10.39493
4.5	37.556	.03331	11.52503
5.0	43.333	.03639	12.28492
5.5	49.111	.03859	12.66487
6.0	54.889	.03980	12.66487
6.5	60.667	.03989	12.28492
7.0	66.444	.03876	11.52503
7.5	72.222	.03629	10.39493
8.0	78.000	.03235	8.94335
8.5	83.778	.02684	7.22872
9.0	89.556	.01963	5.30950

PROBLEM 9.C8



9.C8 A uniformly distributed load w and several distributed loads P_i may be applied to the cantilever beam AB . Write a computer program to determine the reaction at the roller support and apply this program to the beam and loading of (a) Prob. 9.53a, (b) Prob. 9.154.

SOLUTION

THE BEAM IS INDETERMINATE

USE EQUATION OF ELASTIC CURVE

ENTER w AND FOR EACH LOAD P_i AND c_i

COMPUTE DISPLACEMENT AT B DUE TO LOADS

REACTION AT A:

DUE TO w

$$R_A = wa$$

$$M_A = \frac{1}{2} w a^2$$

FOR $i = 1$ TO NUMBER LOADS P_i

$$R_A = R_A - P_i$$

$$M_A = M_A - P_i c_i$$

FOR DISPLACEMENT AT B, SUPERPOSE:

DUE TO REACTION AT A

$$EI y_B = \frac{1}{6} R_A L^3 + \frac{1}{2} M_A L^2$$

DUE TO DISTRIBUTED LOADS

$$EI y_B = \frac{1}{24} (-wL^4 + w(L-a)^4)$$

DUE TO P_i

FOR $i = 1$ TO NUMBER LOADS

$$EI y_B = \frac{1}{6} P_i (L-c_i)^3$$

COMPUTE DISPLACEMENT AT B DUE TO UNIT R_B

$$EI(y_B)_{\text{UNIT}} = \frac{1}{3} L^3$$

COMPUTE REACTION AT B

$$\text{FROM } EI y_B + R_B EI(y_B)_{\text{UNIT}} = 0$$

$$R_B = -y_B / (y_B)_{\text{UNIT}}$$

$$\begin{aligned} EI \frac{d^2y}{dx^2} &= -x + L \\ EI \frac{dy}{dx} &= -\frac{1}{2} x^2 + Lx + C_1 \\ EI y &= -\frac{1}{6} x^3 + \frac{1}{2} Lx^2 + C_1 x + C_2 \end{aligned}$$

BOUNDARY CONDITIONS GIVE $C_1 = C_2 = 0$

CONTINUED

PROBLEM 9.C8 CONTINUED

PROGRAM OUTPUT

Problem 9.53 (a)

Reaction at Roller Support = 11.5356 kN

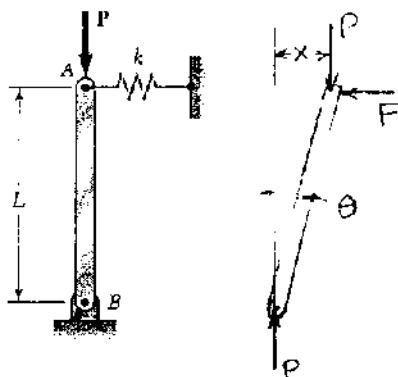
Problem 9.154

Reaction at Roller Support = 10.1758 kN

Chapter 10

Problem 10.1

10.1 Knowing that the spring at A is of constant k and that the bar AB is rigid, determine the critical load P_{cr} .



Let θ be the angle change of bar AB.

$$F = kx = kL \sin\theta$$

$$\therefore \sum M_B = 0: \quad Fl \cos\theta - Px = 0$$

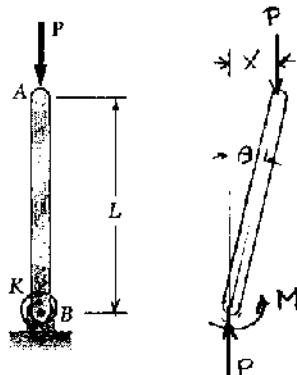
$$KL^2 \sin\theta \cos\theta - PL \sin\theta = 0$$

Using $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, $KL^2\theta - PL\theta = 0$

$$(KL^2 - PL)\theta = 0 \quad P_{cr} = KL$$

Problem 10.2

10.2 Knowing that the torsional spring at B is of constant K and that the bar AB is rigid, determine the critical load P_{cr} .



Let θ be the angle change of bar AB.

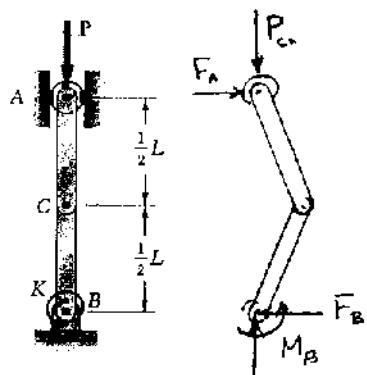
$$M = K\theta, \quad x = L \sin\theta \approx L\theta$$

$$\therefore M_B = 0 \quad M - Px = 0 \quad K\theta - PL\theta = 0$$

$$(K - PL)\theta = 0 \quad P_{cr} = K/L$$

Problem 10.3

10.3 Two rigid bars AC and BC are connected by a pin at C as shown. Knowing that the torsional spring at B is of constant K , determine the critical load P_{cr} for the system.



Let θ be the angle change of each bar.

$$M_B = K\theta$$

$$\therefore \sum M_B = 0 \quad K\theta - F_A L = 0 \quad F_A = \frac{K\theta}{L}$$

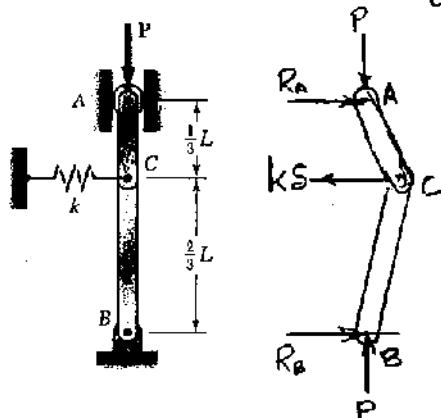
$$\text{Bar } AC \quad \therefore \sum M_C = 0:$$

$$P_{cr} \frac{1}{2}L\theta - \frac{1}{2}L F_A = 0$$

$$P_{cr} = \frac{F_A}{\theta} = \frac{K}{L}$$

Problem 10.4

10.4 Two rigid bars AC and BC are connected as shown to a spring of constant k . Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.



Let S be the deflection of point C .

Using free body AC and $\sum M_C = 0$:

$$-\frac{1}{3}LR_A + PS = 0 \quad R_A = \frac{3PS}{L}$$

Using free body BC and $\sum M_C = 0$:

$$\frac{2}{3}LR_B - PS = 0 \quad R_B = \frac{3PS}{2L}$$

Using both free bodies together,

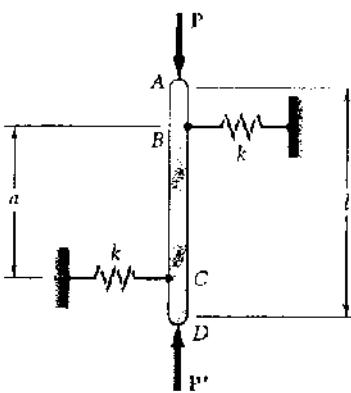
$$\sum F_x = 0: \quad R_A + R_B - kS = 0$$

$$\frac{3PS}{L} + \frac{3PS}{2L} - kS = 0$$

$$\left(\frac{9}{2}\frac{P}{L} - k\right)S = 0$$

$$P_{cr} = \frac{2kL}{9}$$

Problem 10.5



10.5 The rigid bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Knowing that the equal and opposite loads P and P' remain vertical, determine the magnitude P_c of the critical load for the system. Each spring can act in either tension or compression.

Let x_B and x_C be the deflections of points B and C, positive \rightarrow .

$$\text{Then, } F_B = -kx_B \quad \text{and} \quad F_C = -kx_C$$

$$\pm \sum F_x = 0: \quad -F_B - F_C = 0 \quad F_C = -F_B$$

$$x_C = -x_B \quad F_B \text{ and } F_C \text{ form a couple } \curvearrowleft.$$

$$\text{Let } \theta \text{ be the angle change. } y_B = -y_C = \frac{1}{2}a \sin \theta$$

$$s = l \sin \theta$$

$$P \text{ and } P' \text{ form a couple } PS \curvearrowright$$

$$+\sum M = 0: \quad k\left(\frac{1}{2}a \sin \theta\right)a \cos \theta - Pl \sin \theta = 0$$

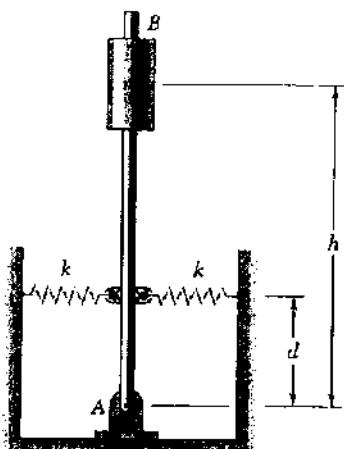
$$P = \frac{ka^2 \cos \theta}{2l}$$

$$\text{Let } \theta \rightarrow 0$$

$$P_{cr} = \frac{ka^2}{2l}$$

Problem 10.6

10.6 If $m = 125 \text{ kg}$, $h = 700 \text{ mm}$, and the constant of each spring is $k = 2.8 \text{ kN/m}$, determine the range of values of the distance d for which the equilibrium of the rigid rod AB is stable in the position shown. Each spring can act in either tension or compression.



$$h = 700 \text{ mm.} = 700 \times 10^{-3} \text{ m}$$

Let θ be the small rotation of AB .

$$x = d\theta \quad F = kx = kd\theta$$

$$+\sum M_A = 0: \quad 2Fd - mgh\theta = 0$$

$$2kd^2\theta - mgh\theta = 0$$

$$d_{cr}^2 = \frac{mgh}{2k}$$

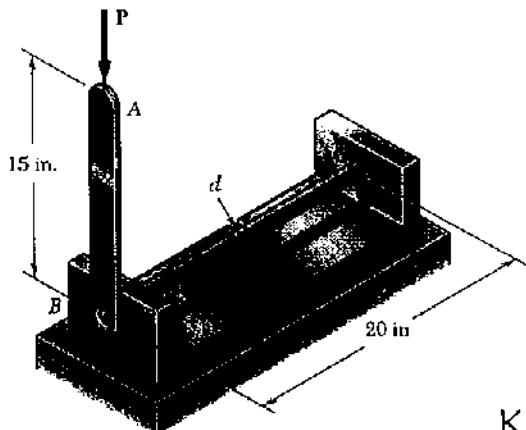
$$d_{cr} = \sqrt{\frac{mgh}{k}} = \sqrt{\frac{(125)(9.81)(700 \times 10^{-3})}{(2)(2.8 \times 10^3)}}$$

$$= 0.392 \text{ m} = 392 \text{ mm}$$

$d > 392 \text{ mm}$ for stability

Problem 10.7

10.7 The steel rod BC is attached to the rigid bar AB and to the fixed support at C . Knowing that $G = 11.2 \times 10^6$ psi, determine the critical load P_{cr} of the system when $d = \frac{1}{2}$ in.



Look at torsion spring BC .

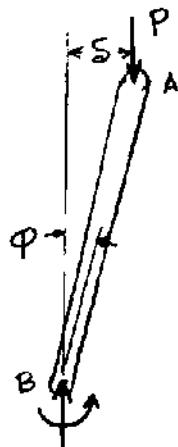
$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ}{L} \phi = K\phi$$

$$G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{2} \left(\frac{1}{4}\right)^4 = 6.1359 \times 10^{-3} \text{ in}^4$$

$$L = 20 \text{ in.}$$

$$K = \frac{GJ}{L} = \frac{(11.2 \times 10^6)(6.1359 \times 10^{-3})}{20} = 3436.1 \text{ lb-in/rad}$$



$$S = l \sin \phi = 15 \sin \phi$$

$$+ \sum M_B = 0: -PS + T = 0$$

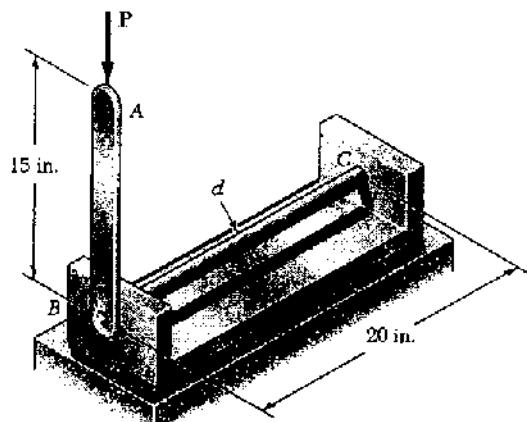
$$P = \frac{T}{S} = \frac{K\phi}{l \sin \phi} = \frac{3436.1 \phi}{15 \sin \phi} = 229 \frac{\phi}{\sin \phi} \text{ lb.}$$

Let $\phi \rightarrow 0$.

$$P_{cr} = 229 \text{ lb.} \quad \blacksquare$$

Problem 10.8

10.8 The steel rod BC is attached to the rigid bar AB and to the fixed support at C . Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of rod BC for which the critical load P_{cr} of the system is 80 lb.



Look at torsion spring BC .

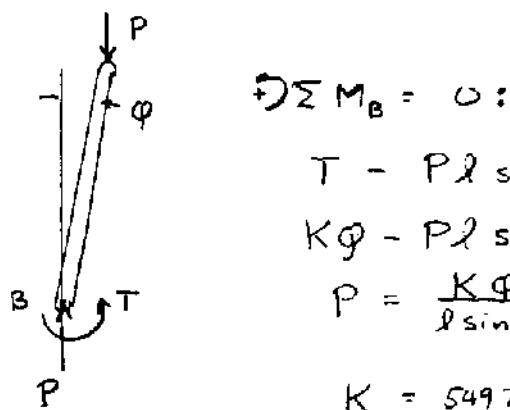
$$\varphi = \frac{TL}{GJ} \quad T = \frac{GJ}{L} \varphi = K\varphi$$

$$G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2} C^3 = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

$$L = 20 \text{ in}$$

$$K = \frac{(11.2 \times 10^6) \pi d^4}{(20)(32)} = 54978 d^4$$



$$\sum M_B = 0:$$

$$T - Pl \sin \varphi = 0$$

$$K\varphi - Pl \sin \varphi = 0$$

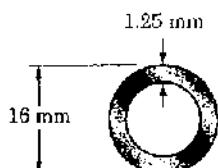
$$P = \frac{K\varphi}{l \sin \varphi} \quad P_{cr} = \frac{K}{l}$$

$$K = 54978 d^4 = P_{cr}l = (80)(15) = 1200$$

$$d = \sqrt[4]{\frac{1200}{54978}} = 0.384 \text{ in.}$$

Problem 10.9

10.9 Determine the critical load of an aluminum tube that is 1.5 m long and has a 16-mm outer diameter and a 1.25 mm wall thickness. Use $E = 70 \text{ GPa}$.



$$c_o = \frac{1}{2} d_o = \frac{1}{2}(16) = 8 \text{ mm} \quad c_i = c_o - t = 8 - 1.25 = 6.75 \text{ mm}$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}(8^4 - 6.75^4) = 1.58655 \times 10^9 \text{ mm}^4 \\ = 1.58655 \times 10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(70 \times 10^9)(1.58655 \times 10^9)}{(1.5)^2}$$

$$= 487$$

$$P_{cr} = 487 \text{ N}$$

Problem 10.10

10.10 Determine the critical load of a round wooden dowel that is 48 in. long and has a diameter of (a) 0.375 in., (b) 0.5 in. Use $E = 1.6 \times 10^6 \text{ psi}$.

$$(a) \quad c = \frac{1}{2} d = 0.1875 \text{ in} \quad I = \frac{\pi}{4} c^4 = 970.7 \times 10^{-6} \text{ in}^4$$

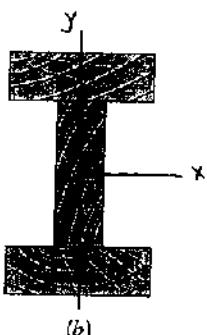
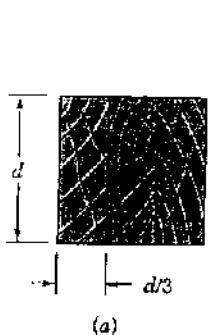
$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(1.6 \times 10^6)(970.7 \times 10^{-6})}{(48)^2} = 6.65 \text{ lb.}$$

$$(b) \quad c = \frac{1}{2} d = 0.25 \text{ in} \quad I = \frac{\pi}{4} c^4 = 3.068 \times 10^{-3} \text{ in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(1.6 \times 10^6)(3.068 \times 10^{-3})}{(48)^2} = 21.0 \text{ lb.}$$

Problem 10.11

10.11 A column of effective length L can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement *a* to the critical load using the arrangement *b*.



Arrangement (a).

$$I_a = \frac{1}{12} d^4$$

$$P_{cr,a} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d^4}{12 L_e^2}$$

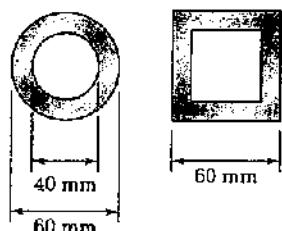
Arrangement (b). $I_{min} = I_y = \frac{1}{12} \left(\frac{d}{3} \right)^3 + \frac{1}{12} (d) \left(\frac{d}{3} \right)^3 + \frac{1}{12} \left(\frac{d}{3} \right) (d)^3 = \frac{19}{324} d^4$

$$P_{cr,b} = \frac{\pi^2 EI}{L_e^2} = \frac{19 \pi^2 E d^4}{324 L_e^2}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = \frac{1}{12} \cdot \frac{324}{19} = \frac{27}{19} = 1.421$$

Problem 10.12

10.12 Two brass rods used as compression members, each of 3-m effective length, have the cross sections shown. (a) Determine the wall thickness of the hollow square rod for which the rods have the same cross-sectional area. (b) Using $E = 105$ GPa, determine the critical load of each rod.



(a) Same area: $\frac{\pi}{4} (d_o^2 - d_i^2) = b_o^2 - b_i^2$

$$b_i^2 = b_o^2 - \frac{\pi}{4} (d_o^2 - d_i^2) \\ = 60^2 - \frac{\pi}{4} (60^2 - 40^2) = 2.0292 \text{ mm}^2$$

$$b_i = 45.047 \text{ mm} \quad t = \frac{1}{2} (b_o - b_i) = 7.48 \text{ mm}$$

(b) Circular: $I = \frac{\pi}{64} (d_o^4 - d_i^4) = 510.51 \times 10^3 \text{ mm}^4 = 510.51 \times 10^{-9} \text{ m}^4$

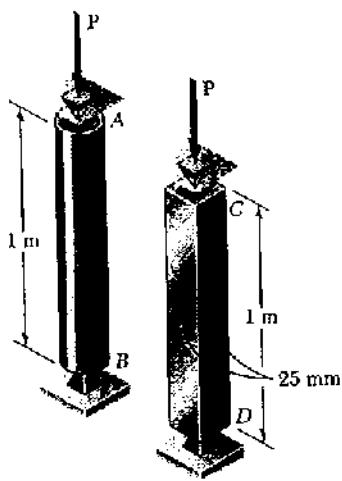
$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (105 \times 10^9) (510.51 \times 10^{-9})}{(3.0)^2} = 58.8 \times 10^3 \text{ N} = 58.8 \text{ kN}$$

Square: $I = \frac{1}{12} (b_o^4 - b_i^4) = 736.85 \times 10^3 \text{ mm}^4 = 736.85 \times 10^{-9} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (105 \times 10^9) (736.85 \times 10^{-9})}{(3.0)^2} = 84.8 \times 10^3 \text{ N} = 84.8 \text{ kN}$$

Problem 10.13

10.13 Determine the radius of the round strut so that the round and square struts have the same cross-sectional area and compute the critical load for each strut. Use $E = 200 \text{ GPa}$.



$$\text{For square strut } A = 25^2 = 625 \text{ mm}^2$$

$$I = \frac{1}{12}(25)^4 = 32.552 \times 10^3 \text{ mm}^4 = 32.552 \times 10^{-9} \text{ m}^4$$

$$\text{For round strut } \pi d^2 = A$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(625)}{\pi}} = 28.2 \text{ mm}$$

$$c = \frac{1}{2}d = 14.10 \text{ mm}$$

$$I = \frac{\pi}{4} c^4 = 31.085 \times 10^3 \text{ mm}^4 = 31.085 \times 10^{-9} \text{ m}^4$$

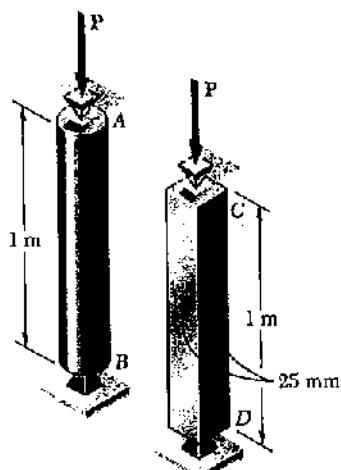
$$\text{Critical loads } P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\text{Round strut. } P_{cr} = \frac{\pi^2 (200 \times 10^9) (31.085 \times 10^{-9})}{(1)^2} = 61.4 \times 10^3 \text{ N} = 61.4 \text{ kN}$$

$$\text{Square strut. } P_{cr} = \frac{\pi^2 (200 \times 10^9) (32.552 \times 10^{-9})}{(1)^2} = 64.3 \times 10^3 \text{ N} = 64.3 \text{ kN}$$

Problem 10.14

10.14 Determine (a) the critical load for the square strut, (b) the radius of the round strut for which both struts have the same critical load. (c) Express the cross-sectional area of the square strut as a percentage of the cross-sectional area of the round strut. Use $E = 200 \text{ GPa}$.



$$(a) \text{ Square strut. } A = (25)^2 = 625 \text{ mm}^2$$

$$I = \frac{1}{12}(25)^4 = 32.552 \times 10^3 \text{ mm}^4 = 32.552 \times 10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (32.552 \times 10^{-9})}{(1)^2}$$

$$= 64.3 \times 10^3 \text{ N} = 64.3 \text{ kN}$$

(b) Round strut. For the same critical load, modulus of elasticity, and length, the moments of inertia must be equal.

$$\frac{\pi}{4} c^4 = I$$

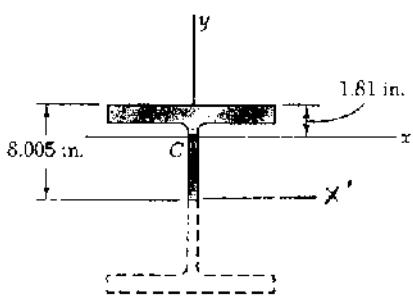
$$c = \sqrt[4]{\frac{4I}{\pi}} = \sqrt[4]{\frac{(4)(32.552 \times 10^3)}{\pi}} = 14.27 \text{ mm}$$

$$d = 2c = 28.5 \text{ mm}$$

$$(c) \text{ Area. } A = \pi c^2 = 639.58 \text{ mm}^2. \text{ Ratio} = \frac{625}{639.58} = 0.977 = 97.7\%$$

Problem 10.15

10.15 A column of 26-ft effective length is made from half of a W16 x 40 rolled-steel shape. Knowing that the centroid of the cross section is located as shown, determine the factor of safety if the allowable centric load is 20 kips. Use $E = 29 \times 10^6$ psi.



$$\text{For } W16 \times 40: \quad A = 11.8 \text{ in}^2 \\ I_x = 518 \text{ in}^4, \quad I_y = 28.9 \text{ in}^4$$

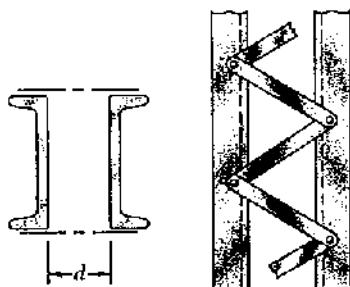
$$\text{Half } W16 \times 40: \\ A = \frac{1}{2}(11.8) = 5.90 \text{ in}^2 \\ I_x = \frac{1}{2}(518) - (5.90)(8.005 - 1.81)^2 = 32.57 \text{ in}^4 \\ I_y = \frac{1}{2}(28.9) = 14.45 \text{ in}^4 = I_{min}$$

$$P_{cn} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(14.45)}{(26 \times 12)^2} = 42.5 \times 10^3 \text{ lb} = 42.5 \text{ kips.}$$

$$P_{all} = \frac{P_{cn}}{F.S.} \quad F.S. = \frac{P_{cn}}{P_{all}} = \frac{42.5}{20} = 2.125$$

Problem 10.16

10.16 A single compression member of 27-ft effective length is obtained by connecting two C 8 x 11.5 steel channels with lacing bars as shown. Knowing that the factor of safety is 1.85, determine the allowable centric load for the member. Use $E = 29 \times 10^6$ psi and $d = 4$ in.



For C 8 x 11.5 rolled steel shape, $A = 3.38 \text{ in}^2$

$$\bar{I}_x = 32.6 \text{ in}^4, \quad \bar{I}_y = 1.32 \text{ in}^4, \quad \bar{x} = 0.571 \text{ in.}$$

For the fabricated column

$$I_x = 2\bar{I}_x = (2)(32.6) = 65.2 \text{ in}^4$$

$$I_y = 2[\bar{I}_y + A(\frac{d}{2} + \bar{x})^2] \\ = 2[1.32 + (3.38)(2 + 0.571)^2] = 47.32 \text{ in}^4$$

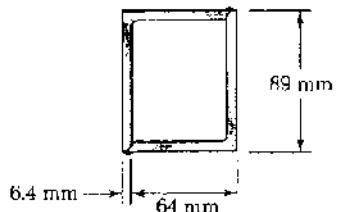
$$I_{min} = I_y = 47.32 \text{ in}^4, \quad L_e = 27 \text{ ft} = 324 \text{ in.}$$

$$P_{cn} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(47.32)}{(324)^2} = 129 \times 10^3 \text{ lb.}$$

$$P_{all} = \frac{P_{cn}}{F.S.} = \frac{129 \times 10^3}{1.85} = 69.7 \times 10^3 \text{ lb} = 69.7 \text{ kips}$$

Problem 10.17

10.17 A column of 4-m effective length is made by welding together two $89 \times 64 \times 6.4$ -mm angles as shown. Using $E = 200$ GPa, determine the factor of safety with respect to buckling for a centric load of 80 kN.



$$b_o = 89 \text{ mm} \quad h_o = 64 + 6.4 = 70.4 \text{ mm}$$

$$b_i = b_o - 2t = 76.2 \text{ mm} \quad h_i = h_o - 2t = 57.6 \text{ mm}$$

$$I_{min} = \frac{1}{12} (b_o h_o^3 - b_i h_i^3) = 1.37427 \times 10^6 \text{ mm}^4$$

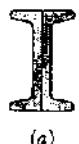
$$= 1.37427 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9)(1.37427 \times 10^{-6})}{(4)^2} = 169.54 \times 10^3 \text{ N} = 169.54 \text{ kN}$$

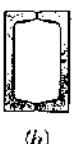
$$\text{F.S.} = \frac{P_{cr}}{P} = \frac{169.54}{80} = 2.12$$

Problem 10.18

10.18 A column of 3-m effective length is to be made by welding together two C130 \times 13 rolled-steel channels. Using $E = 200$ GPa, determine for each arrangement shown the allowable centric load if a factor of safety of 2.4 is required.



(a)



(b)

For channel C130 \times 13 $A = 1710 \text{ mm}^2$ $b_f = 48 \text{ mm}$

$$I_x = 3.70 \times 10^6 \text{ mm}^4 \quad I_y = 0.264 \times 10^6 \text{ mm}^4 \quad \bar{x} = 12.2 \text{ mm}$$

$$\text{Arrangement (a).} \quad I_x = 2(3.70 \times 10^6) = 7.40 \times 10^6 \text{ mm}^4$$

$$I_y = 2[0.264 \times 10^6 + (1710)(12.2)^2] = 1.0370 \times 10^6 \text{ mm}^4$$

$$I_{min} = I_y = 1.0370 \times 10^6 \text{ mm}^4 = 1.0370 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI_{min}}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(1.0370 \times 10^{-6})}{(3.0)^2} = 227 \times 10^3 \text{ N} = 227 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{\text{F.S.}} = \frac{227}{2.4} = 94.8 \text{ kN}$$

$$\text{Arrangement (b).} \quad I_x = 2(3.70 \times 10^6) \text{ mm}^4$$

$$I_y = 2[0.264 \times 10^6 + (1710)(48 - 12.2)^2] = 4.911 \times 10^6 \text{ mm}^4$$

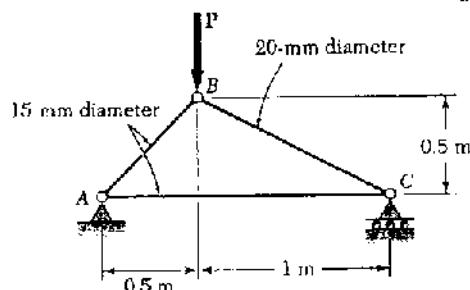
$$I_{min} = I_y = 4.911 \times 10^6 \text{ mm}^4 = 4.911 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI_{min}}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(4.911 \times 10^{-6})}{(3.0)^2} = 1077 \times 10^3 \text{ N} = 1077 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{\text{F.S.}} = \frac{1077}{2.4} = 449 \text{ kN}$$

Problem 10.19

10.19 Knowing that a factor of safety of 2.6 is required, determine the largest load P that can be applied to the structure shown. Use $E = 200 \text{ GPa}$ and consider only buckling in the plane of the structure.



$$\text{BC: } L_{BC} = \sqrt{1^2 + 0.5^2} = 1.1180 \text{ m}$$

$$I = \frac{\pi}{64} d_{BC}^4 = \frac{\pi}{64} (20)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (7.854 \times 10^{-9})}{(1.1180)^2} = 12.403 \times 10^3 \text{ N} = 12.403 \text{ kN}$$

$$F_{BC,all} = \frac{P_{cr}}{F.S.} = \frac{12.403}{2.6} = 4.770 \text{ kN}$$

$$\text{AB: } L_{AB} = \sqrt{0.5^2 + 0.5^2} = 0.70711 \text{ m}$$

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (15)^4 = 2.485 \times 10^3 \text{ mm}^4 = 2.485 \times 10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (2.485 \times 10^{-9})}{(0.70711)^2} = 9.8106 \times 10^3 \text{ N} = 9.8106 \text{ kN}$$

$$F_{AB,all} = \frac{P_{cr}}{F.S.} = \frac{9.8106}{2.6} = 3.773 \text{ kN}$$

Joint B

$$\therefore \sum F_x = 0: \quad \frac{0.5}{0.70711} F_{AB} - \frac{1.0}{1.1180} F_{BC} = 0$$

$$F_{BC} = 0.79057 F_{AB}$$

$$+ \sum F_y = 0: \quad \frac{0.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} + P = 0$$

$$0.70711 F_{AB} + (0.44721)(0.79057 F_{AB}) - P = 0$$

$$P = 1.06066 F_{AB}$$

$$P = (1.06066) \frac{F_{BC}}{0.79057} = 1.3416 F_{BC}$$

Allowable value for P .

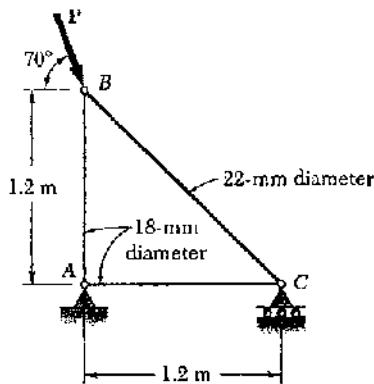
$$P < 1.06066 F_{AB,all} = (1.06066)(3.773) = 4.00 \text{ kN}$$

$$P < 1.3416 F_{BC,all} = (1.3416)(4.770) = 6.40 \text{ kN}$$

$$P_{all} = 4.00 \text{ kN}$$

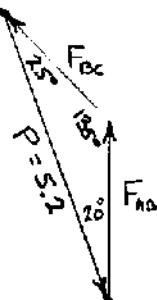
Problem 10.20

30.20 Knowing that $P = 5.2 \text{ kN}$, determine the factor of safety for the structure shown. Use $E = 200 \text{ GPa}$ and consider only buckling in the plane of the structure.



Joint B:

From force triangle,



$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$F_{AB} = 3.1079 \text{ kN (comp)}$$

$$F_{BC} = 2.5152 \text{ kN (comp)}$$

$$\text{Member AB: } I_{AB} = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{18}{2}\right)^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$$

$$F_{AB,cr} = \frac{\pi^2 E I_{AB}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9) (5.153 \times 10^{-9})}{(1.2)^2} = 7.0636 \times 10^3 \text{ N} = 7.0636 \text{ kN}$$

$$\text{F.S.} = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

$$\text{Member BC: } I_{BC} = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{22}{2}\right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$$

$$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$$

$$F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-9})}{2.88} = 7.8813 \times 10^3 \text{ N} = 7.8813 \text{ kN}$$

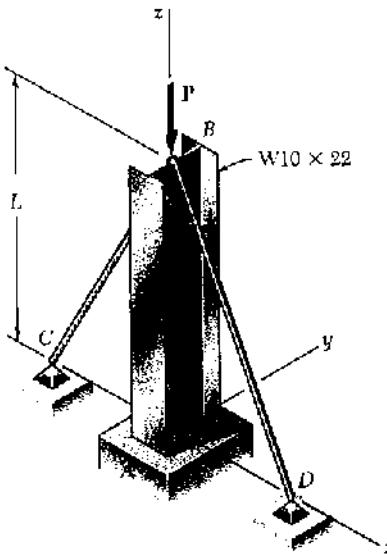
$$\text{F.S.} = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest F.S. governs.

F.S. = 2.27



Problem 10.21



10.21 Column AB carries a centric load P of magnitude 15 kips. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L . Use $E = 29 \times 10^6$ psi.

$$W10 \times 22 : I_x = 118 \text{ in}^4, I_y = 11.4 \text{ in}^4$$

$$P = 15 \times 10^3 \text{ lb.}$$

$$P_{cr} = (F.S.)P = (2.2)(15 \times 10^3) = 33 \times 10^3 \text{ lb}$$

$$\underline{\text{Buckling in } xz\text{-plane.}} \quad L_e = 0.7L$$

$$P_{cr} = \frac{\pi^2 EI_y}{(0.7L)^2} \quad L = \frac{\pi}{0.7} \sqrt{\frac{EI_y}{P_{cr}}}$$

$$L = \frac{\pi}{0.7} \sqrt{\frac{(29 \times 10^6)(11.4)}{33 \times 10^3}} = 449.2 \text{ in.}$$

$$\underline{\text{Buckling in } yz\text{-plane.}} \quad L_e = 2L$$

$$P_{cr} = \frac{\pi^2 EI_x}{(2L)^2} \quad L = \frac{\pi}{2} \sqrt{\frac{EI_x}{P_{cr}}} = \frac{\pi}{2} \sqrt{\frac{(29 \times 10^6)(118)}{33 \times 10^3}} = 505.8 \text{ in.}$$

Smaller value for L governs. $L = 449.2 \text{ in.} = 37.4 \text{ ft}$

Problem 10.22

10.22 A W8 x 21 rolled-steel shape is used with the support and cable arrangement shown in Prob. 10.21. Knowing that $L = 24 \text{ ft}$, determine the allowable centric load P if a factor of safety of 2.2 is required. Use $E = 29 \times 10^6$ psi.

10.21 Column AB carries a centric load P of magnitude 15 kips. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L . Use $E = 29 \times 10^6$ psi.

$$W8 \times 21 : I_x = 75.3 \text{ in}^4, I_y = 9.77 \text{ in}^4$$

$$L = 24 \text{ ft} = 288 \text{ in.}$$

$$\underline{\text{Buckling in } xz\text{-plane:}} \quad L_e = 0.7 = 201.6 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI_y}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(9.77)}{(201.6)^2} = 68.80 \times 10^3 \text{ lb.}$$

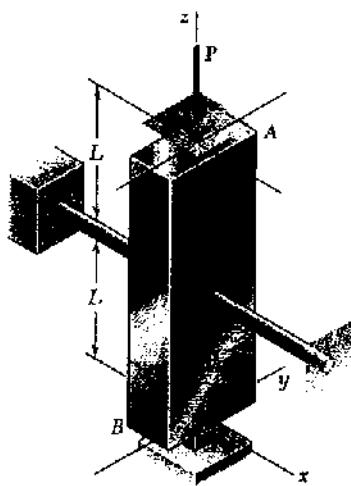
$$\underline{\text{Buckling in } yz\text{-plane:}} \quad L_e = 2L = 576 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI_x}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(75.3)}{(576)^2} = 64.96 \times 10^3 \text{ lb.}$$

The smaller value governs. $P_c = 64.96 \times 10^3 \text{ lb.} = 64.96 \text{ kips}$

$$P_{all} = \frac{P_c}{F.S.} = \frac{64.96}{2.2} = 29.5 \text{ kips}$$

Problem 10.23



10.23 Column *ABC* has a uniform rectangular cross section and is braced in the *xz*-plane at its midpoint *C*. (a) Determine the ratio b/d for which the factor of safety is the same with respect to buckling in the *xz* and *yz* planes. (b) Using the ratio found in part *a*, design the cross section of the column so that the factor of safety will be 3.0 when $P = 4.4$ kN, $L = 1$ m, and $E = 200$ GPa.

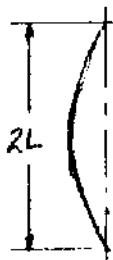
Buckling in *xz*-plane. $L_e = L$, $I = \frac{bd^3}{12}$



$$(P_{cr})_1 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d b^3}{12 L^2}$$

$$(\text{F.S.})_1 = \frac{(P_{cr})_1}{P} = \frac{\pi^2 E d b^3}{12 PL^2}$$

Buckling in *yz*-plane. $L_e = 2L$, $I = \frac{bd^3}{12}$



$$(P_{cr})_2 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E b d^3}{12 (2L)^2}$$

$$(\text{F.S.})_2 = \frac{(P_{cr})_2}{P} = \frac{\pi^2 E b d^3}{48 PL^2}$$

(a) Equating the two factors of safety,

$$\frac{\pi^2 E d b^3}{12 PL^2} = \frac{\pi^2 E b d^3}{48 PL^2}$$

$$b^2 = \frac{1}{4} d^2 \quad b/d = 1/2$$

Then, $(\text{F.S.}) = \frac{\pi^2 E d^4}{96 PL^2}$

$$d^4 = \frac{96 (\text{F.S.}) PL^2}{\pi^2 E} = \frac{(96)(3.0)(4.4 \times 10^3)(1)^2}{\pi^2 (200 \times 10^9)} = 641.97 \times 10^{-9} \text{ m}^4$$

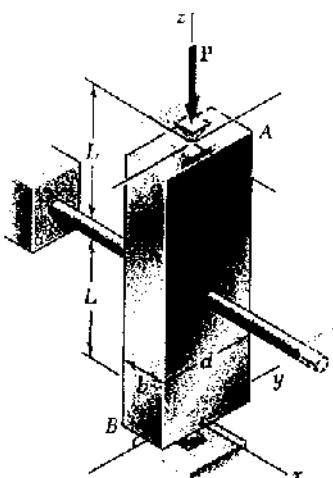
(b) $d = 28.3 \times 10^{-3} \text{ m}$

$$d = 28.3 \text{ mm}$$

$$b = 14.15 \text{ mm}$$

Problem 10.24

10.24 Column ABC has a uniform rectangular cross section with $b = 12 \text{ mm}$ and $d = 22 \text{ mm}$. The column is braced in the xz plane at its midpoint C and carries a centric load P of magnitude 3.8 kN . Knowing that a factor of safety of 3.2 is required, determine the largest allowable length L . Use $E = 200 \text{ GPa}$.



$$P_{cr} = (\text{F.S.})P = (3.2)(3.8 \times 10^3) = 12.16 \times 10^3 \text{ N}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad L_e = \pi \sqrt{\frac{EI}{P_{cr}}}$$

Buckling in xz -plane. $L = L_e = \pi \sqrt{\frac{EI}{P_{cr}}}$



$$I = \frac{1}{12} db^3 = \frac{1}{12}(22)(12)^3 = 3.168 \times 10^3 \text{ mm}^4$$

$$= 3.168 \times 10^{-9} \text{ m}^4$$

$$L = \pi \sqrt{\frac{(200 \times 10^9)(3.168 \times 10^{-9})}{12.16 \times 10^3}} = 0.717 \text{ m}$$

Buckling in yz -plane. $L_e = 2L \quad L = \frac{L_e}{2} = \frac{\pi}{2} \sqrt{\frac{EI}{P_{cr}}}$

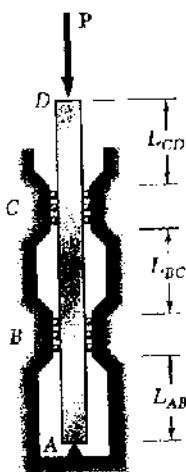
$$I = \frac{1}{12} bd^3 = \frac{1}{12}(12)(22)^3 = 10.648 \times 10^3 \text{ mm}^4 = 10.648 \times 10^{-9} \text{ m}^4$$

$$L = \frac{\pi}{2} \sqrt{\frac{(200 \times 10^9)(10.648 \times 10^{-9})}{12.16 \times 10^3}} = 0.657 \text{ m}$$

The smaller length governs. $L = 0.657 \text{ m} = 657 \text{ mm}$ ■

Problem 10.25

10.25 A 1-in.-square aluminum strut is maintained in the position shown by a pin support at *A* and by sets of rollers at *B* and *C* that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 3$ ft, $L_{BC} = 4$ ft, and $L_{CD} = 1$ ft, determine the allowable load P using a factor of safety with respect to buckling of 3.2. Consider only buckling in the plane of the figure and use $E = 10.4 \times 10^6$ psi.



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1)(1)^3 = 0.083333 \text{ in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad P_{all} = \frac{(P_{cr})_{min}}{F.S.} = \frac{\pi^2 EI}{(F.S.)(L_e)_{max}^2}$$

$$\text{Portion AB: } L_e = 0.7L_{AB} = (0.7)(3) = 2.1 \text{ ft}$$

$$\text{Portion BC: } L_e = 0.5L_{BC} = (0.5)(4) = 2.0 \text{ ft}$$

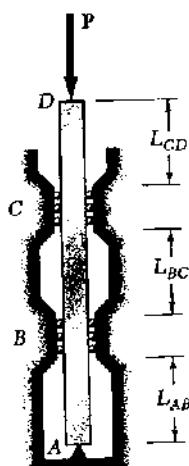
$$\text{Portion CD: } L_e = 2L_{CD} = (2)(1) = 2.0 \text{ ft}$$

$$(L_e)_{max} = 2.1 \text{ ft} = 25.2 \text{ in.}$$

$$P_{all} = \frac{\pi^2(10.4 \times 10^6)(0.083333)}{(3.2)(25.2)^2} = 4.21 \times 10^3 \text{ lb} = 4.21 \text{ kips}$$

Problem 10.26

10.26 For the strut of Prob. 10.25, knowing that $L_{AB} = 3$ ft, determine (a) the largest values of L_{BC} and L_{CD} that may be used if the allowable load P is to be as large as possible, (b) the magnitude of the corresponding allowable load.



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1)(1)^3 = 0.083333 \text{ in}^4$$

$$(a) \text{ Equivalent lengths: } AB \quad L_e = 0.7L_{AB} = 2.1 \text{ ft} = 25.2 \text{ in.}$$

$$BC \quad L_e = 0.5L_{BC}$$

$$L_{BC} = \frac{2.1}{0.5} = 4.2 \text{ ft}$$

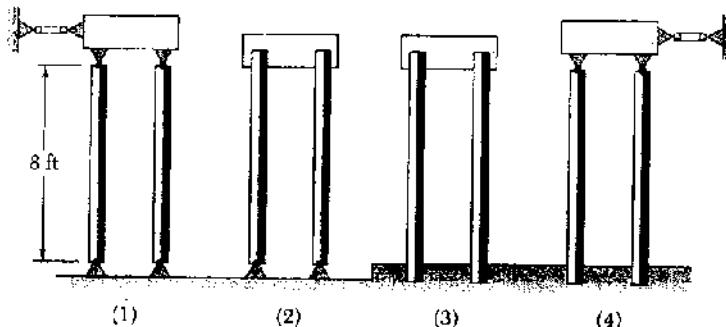
$$CD \quad L_e = 2L_{CD}$$

$$L_{CD} = \frac{2.1}{2} = 1.05 \text{ ft}$$

$$(b) \quad P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{(F.S.)L_e^2} = \frac{\pi^2(10.4 \times 10^6)(0.083333)}{(3.2)(25.2)^2} = 4.21 \times 10^3 \text{ lb} = 4.21 \text{ kips}$$

Problem 10.27

10.27 Two columns are used to support a block weighing 3.25 kips in each of the four ways shown. (a) Knowing that the column of Fig. (1) is made of steel with a 1.25-in.-diameter, determine the factor of safety with respect to buckling for the loading shown. (b) Determine the diameter of each of the other columns for which the factor of safety is the same as the factor of safety obtained in part a. Use $E = 29 \times 10^6$ psi.

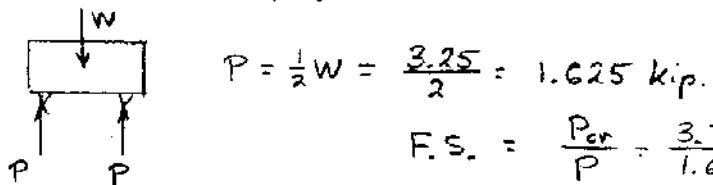


$$(a) I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{1.25}{2}\right)^4 = 0.119842 \text{ in}^4$$

$$L = 8 \text{ ft} = 96 \text{ in}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 (29 \times 10^6)(0.119842)}{(96)^2} = 3722 \text{ lb} = 3.722 \text{ kip, for one column.}$$



$$F.S. = \frac{P_{cr}}{P} = \frac{3.722}{1.625} = 2.29$$

$$P_{cr(1)} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr(2)} = \frac{\pi^2 EI_a}{(L_{e,a})^2}$$

$$\frac{P_{cr(2)}}{P_{cr(1)}} = 1 \quad \frac{I_a}{I_{(1)}} \cdot \frac{L^2}{L_{e,a}^2} = 1 \quad \left(\frac{d_a}{d_{(1)}}\right)^4 \left(\frac{L_e}{L}\right)^2 = 1$$

$$d_a = d_{(1)} \sqrt{\frac{L_{e,a}}{L}}$$

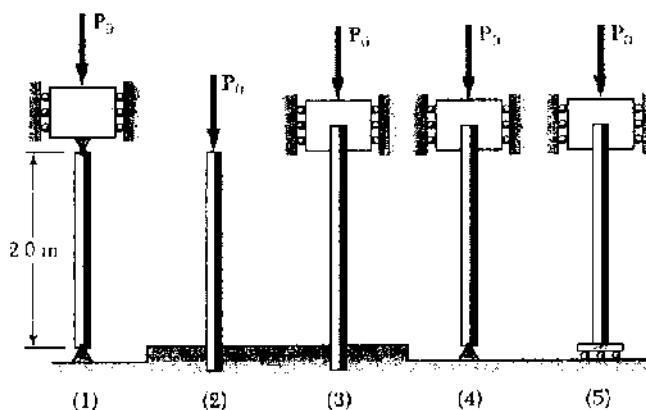
$$(b) (2) L_{e,a}/L = 2.0 \quad d_{(2)} = 1.25 \sqrt{20} = 1.768 \text{ in.}$$

$$(3) L_{e,a}/L = 1.0 \quad d_{(3)} = 1.25 \text{ in.}$$

$$(4) L_{e,a}/L = 0.7 \quad d_{(4)} = 1.25 \sqrt{0.7} = 1.046 \text{ in.}$$

Problem 10.28

10.28 Each of the five struts consists of an aluminum tube that has a 32-mm outer diameter and a 4-mm wall thickness. Using $E = 70 \text{ GPa}$ and a factor of safety of 2.3, determine the allowable load P_0 for each support condition shown.



$$c_o = \frac{1}{2} d_o = \frac{1}{2}(32) = 16 \text{ mm}$$

$$c_i = c_o - t = 16 - 4 = 12 \text{ mm}$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = 35.1858 \times 10^8 \text{ mm}^4$$

$$= 35.1858 \times 10^{-6} \text{ m}^4$$

$$\pi^2 EI = \pi^2 (70 \times 10^9)(35.1858 \times 10^{-6})$$

$$= 24309 \text{ N}\cdot\text{m}^2$$

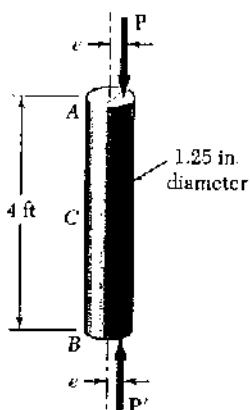
$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{24309}{L_e^2}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{10569}{L_e^2}$$

(1)	$L_e = (1)(2.0) = 2.0 \text{ m}$,	$P_{all} = 2642 \text{ N} = 2.64 \text{ kN}$	1
(2)	$L_e = (2)(2.0) = 4.0 \text{ m}$,	$P_{all} = 661 \text{ N} = 0.661 \text{ kN}$	1
(3)	$L_e = (\frac{1}{2})(2.0) = 1.0 \text{ m}$,	$P_{all} = 10569 \text{ N} = 10.57 \text{ kN}$	1
(4)	$L_e = (0.7)(2.0) = 1.4 \text{ m}$,	$P_{all} = 5392 \text{ N} = 5.39 \text{ kN}$	1
(5)	$L_e = (1.0)(2.0) = 2.0 \text{ m}$,	$P_{all} = 2642 \text{ N} = 2.64 \text{ kN}$	1

Problem 10.29

10.29 An axial load P is applied to the 1.25-in.-diameter steel rod AB as shown. For $P = 8.6$ kips and $e = \frac{1}{16}$ in., determine (a) the deflection at the midpoint C of the rod, (b) the maximum stress in the rod. Use $E = 29 \times 10^6$ psi.



$$c = \frac{1}{2}d = \frac{1.25}{2} = 0.625 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.625)^4 = 0.119842 \text{ in}^4$$

$$L_e = L = 4 \text{ ft} = 48 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29 \times 10^6)(0.119842)}{(48)^2} = 14.8876 \times 10^3 \text{ lb.}$$

$$\frac{P}{P_{cr}} = \frac{8.6 \times 10^3}{14.8876 \times 10^3} = 0.57766$$

$$(a) y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = \frac{1}{16} \left[1.71691 \right]$$

$$= 0.1073 \text{ in.} \quad \blacksquare$$

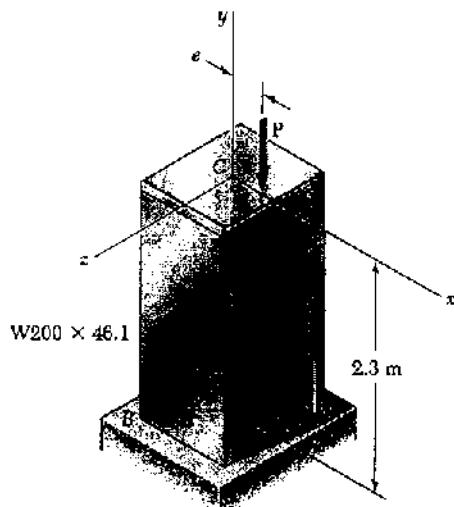
$$M_{max} = P(e + y_{max}) = (8.6 \times 10^3) \left(\frac{1}{16} + 0.1073 \right) = 1.46033 \times 10^3 \text{ lb-in}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.25)^2 = 1.22718 \text{ in}^2$$

$$(b) \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{8.6 \times 10^3}{1.22718} + \frac{(1.46033 \times 10^3)(0.625)}{0.119842} = 14.62 \times 10^3 \text{ psi} \\ = 14.62 \text{ ksi} \quad \blacksquare$$

Problem 10.30

10.30 An axial load P of magnitude 560 kN is applied at a point on the x axis at a distance $e = 8 \text{ mm}$ from the geometric axis of the W 200 × 46.1 rolled-steel column BC. Using $E = 200 \text{ GPa}$, determine (a) the horizontal deflection of end C, (b) the maximum stress in the column.



$$L_e = 2L = (2)(2.3) = 4.6 \text{ m} \quad e = 8 \times 10^{-3} \text{ m}$$

$$\text{W } 200 \times 46.1 \quad A = 5860 \text{ mm}^2 = 5860 \times 10^{-6} \text{ m}^2$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-4} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(15.3 \times 10^{-4})}{(4.6)^2}$$

$$= 1.42727 \times 10^6 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{560 \times 10^3}{1.42727 \times 10^6} = 0.39236$$

$$(a) \quad y_m = e \left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = (8 \times 10^{-3}) \left[\sec\left(\frac{\pi}{2}\sqrt{0.39236}\right) - 1 \right]$$

$$= (8 \times 10^{-3}) [\sec(0.98393) - 1] = (8 \times 10^{-3}) [1.8058 - 1]$$

$$= 6.447 \times 10^{-3} \text{ m} \quad = 6.45 \text{ mm}$$

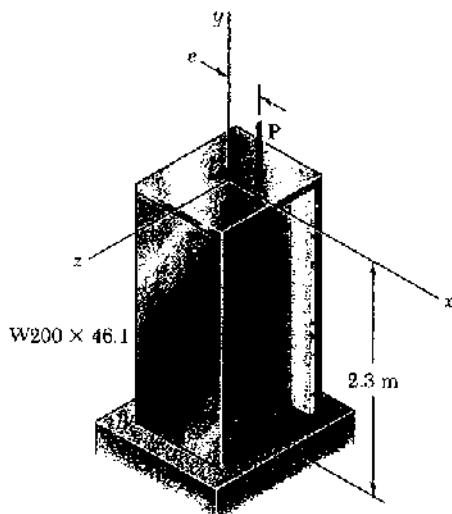
$$M_{max} = P(y_m + e) = (560 \times 10^3)(8 \times 10^{-3} + 6.447 \times 10^{-3}) = 8.090 \times 10^3 \text{ N} \cdot \text{m}$$

$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{M}{S_y} = \frac{560 \times 10^3}{5860 \times 10^{-6}} + \frac{8.090 \times 10^3}{151 \times 10^{-6}} = 149.1 \times 10^6 \text{ Pa} = 149.1 \text{ MPa}$$

Problem 10.31

10.31 Solve Prob. 10.30 if the load P is applied parallel to the geometric axis of the column AB so that it intersects the x axis at $e = 6 \text{ mm}$.



10.30 An axial load P of magnitude 560 kN is applied at a point on the x axis at a distance $e = 8 \text{ mm}$ from the geometric axis of the W 200 × 46.1 rolled-steel column BC . Using $E = 200 \text{ GPa}$, determine (a) the horizontal deflection of end C , (b) the maximum stress in the column.

$$L_e = 2L = (2)(2.3) = 4.6 \text{ m} \quad e = 6 \times 10^{-3} \text{ m}$$

$$\text{W } 200 \times 46.1 \quad A = 5860 \text{ mm}^2 = 5860 \times 10^{-6} \text{ m}^2$$

$$I_y = 15.3 \times 10^4 \text{ mm}^4 = 15.3 \times 10^4 \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (15.3 \times 10^4)}{(4.6)^2}$$

$$= 1.42727 \times 10^6 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{560 \times 10^3}{1.42727 \times 10^6} = 0.39236$$

$$(a) \quad y_m = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = (6 \times 10^{-3}) \left[\sec \left(\frac{\pi}{2} \sqrt{0.39236} \right) - 1 \right]$$

$$= (6 \times 10^{-3}) \left[\sec (0.98393) - 1 \right] = (6 \times 10^{-3})(1.8058 - 1)$$

$$= 4.835 \times 10^{-3} \text{ m} \quad y_m = 4.84 \text{ mm}$$

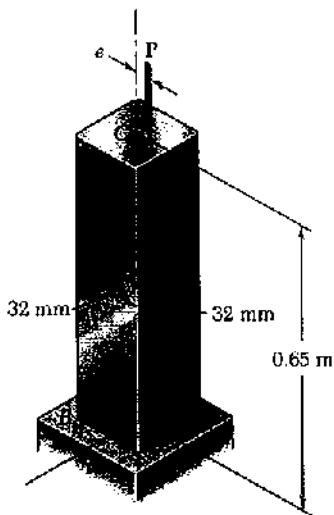
$$M_{max} = P(y_m + e) = (560 \times 10^3)(4.835 \times 10^{-3} + 6 \times 10^{-3}) = 6.0676 \times 10^3 \text{ N-m}$$

$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{M}{S_y} = \frac{560 \times 10^3}{5860 \times 10^{-6}} + \frac{6.0676 \times 10^3}{151 \times 10^{-6}} = 1351.7 \times 10^6 \text{ Pa} = 135.7 \text{ MPa}$$

Problem 10.32

10.32 An axial load P is applied to the 32-mm-square aluminum bar BC as shown. When $P = 24 \text{ kN}$, the horizontal deflection at end C is 4 mm. Using $E = 70 \text{ GPa}$, determine (a) the eccentricity e of the load, (b) the maximum stress in the rod.



$$I = \frac{1}{12} (32)^4 = 87.3813 \times 10^3 \text{ mm}^4 = 87.3813 \times 10^{-9} \text{ m}^4$$

$$A = (32)^2 = 1.024 \times 10^3 \text{ mm}^2 = 1.024 \times 10^{-3} \text{ m}^2$$

$$L_e = 2L = (2)(0.65) = 1.30 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (70 \times 10^9) (87.3813 \times 10^{-9})}{(1.30)^2} = 35.7215 \times 10^3 \text{ N} = 35.7215 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{24}{35.7215} = 0.67186$$

$$(a) y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = e \left[\sec \left(\frac{\pi}{2} \sqrt{0.67186} \right) - 1 \right] = e [\sec 1.28754 - 1] = 2.5780 e$$

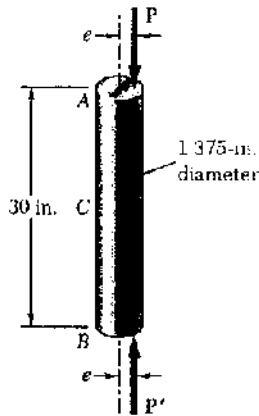
$$e = \frac{y_{max}}{2.5780} = \frac{4}{2.5780} = 1.552 \text{ mm}$$

$$(b) M_{max} = P(e + y_{max}) = (24 \times 10^3) [1.552 \times 10^{-3} + 4 \times 10^{-3}] = 133.24 \text{ N} \cdot \text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} e}{I} = \frac{24 \times 10^3}{1.024 \times 10^{-3}} + \frac{(133.24)(0.016)}{87.3813 \times 10^{-9}} = 47.8 \times 10^6 \text{ Pa} = 47.8 \text{ MPa}$$

Problem 10.33

10.33 An axial load P is applied to the 1.375-in.-diameter steel rod AB as shown. When $P = 21$ kips, it is observed that the horizontal deflection of the midpoint C is 0.03 in. Using $E = 29 \times 10^6$ psi, determine (a) the eccentricity e of the load, (b) the maximum stress in the rod.



$$c = \frac{1}{2}d = 0.6875 \text{ in. } A = \pi c^2 = 1.4849 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 0.175461 \text{ in}^4$$

$$L_e = L = 30 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(0.175461)}{(30)^2} = 55.800 \times 10^3 \text{ lb.}$$

$$\frac{P}{P_{cr}} = \frac{21 \times 10^3}{55.800 \times 10^3} = 0.37634$$

$$(a) y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = e [\sec 0.96363 - 1] \\ = 0.75272 e$$

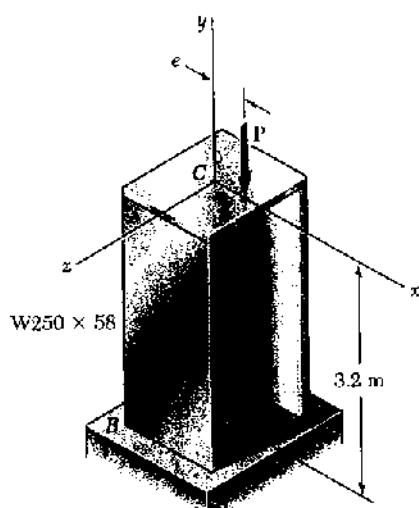
$$e = \frac{y_{max}}{0.75272} = \frac{0.03}{0.75272} = 0.0399 \text{ in.}$$

$$(b) M_{max} = P(e + y_{max}) = (21 \times 10^3)(0.0399 + 0.03) = 1.467 \times 10^3 \text{ lb-in.}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{21 \times 10^3}{1.4849} + \frac{(1.467 \times 10^3)(0.6875)}{0.175461} = 19.89 \times 10^3 \text{ psi} \\ = 19.89 \text{ ksi}$$

Problem 10.34

10.34 The axial load P is applied at a point located on the x axis at a distance e from the geometric axis of the rolled-steel column BC . When $P = 350 \text{ kN}$, the horizontal deflection of the top of the column is 5 mm. Using $E = 200 \text{ GPa}$, determine (a) the eccentricity e of the load, (b) the maximum stress in the column.



$$W 250 \times 58 \quad A = 7420 \text{ mm}^2 = 7420 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.8 \times 10^6 \text{ mm}^4 = 18.8 \times 10^{-6} \text{ m}^4$$

$$S_y = 185 \times 10^3 \text{ mm}^3 = 185 \times 10^{-6} \text{ m}^3$$

$$L_e = 2L = (2)(3.2) = 6.4 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.8 \times 10^{-6})}{(6.4)^2} \\ = 906 \times 10^3 \text{ N} = 906 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{350}{906} = 0.38631$$

$$(a) y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = e \left[\sec 0.97632 - 1 \right] = 0.78546 e$$

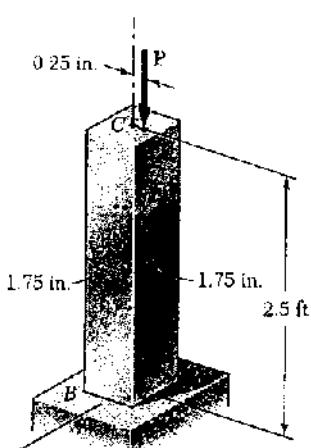
$$e = \frac{y_{max}}{0.78546} = \frac{5}{0.78546} = 6.37 \text{ mm}$$

$$(b) M_{max} = P(e + y_{max}) = (350 \times 10^3) (6.37 \times 10^{-3} + 5 \times 10^{-3}) = 3.978 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{350 \times 10^3}{7420 \times 10^{-6}} + \frac{3.978 \times 10^3}{185 \times 10^{-6}} = 68.7 \times 10^6 \text{ Pa} = 68.7 \text{ MPa}$$

Problem 10.35

10.35 An axial load P is applied at a point D that is 0.25 in. from the geometric axis of the square aluminum bar BC . Using $E = 10.1 \times 10^6$ psi, determine (a) the load P for which the horizontal deflection of end C is 0.50 in., (b) the corresponding maximum stress in the column.



$$I = \frac{1}{12} b h^3 = \frac{1}{12} (1.75)(1.75)^3 = 0.78157 \text{ in.}^4$$

$$A = (1.75)^2 = 3.0625 \text{ in.}^2 \quad c = \frac{1}{2}(1.75) = 0.875 \text{ in.}$$

$$L = 2.5 \text{ ft} = 30 \text{ in.} \quad L_e = 2L = 60 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (10.1 \times 10^6)(0.78157)}{(60)^2} = 21.641 \text{ kips.}$$

$$y_{max} = e [\sec(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}) - 1]$$

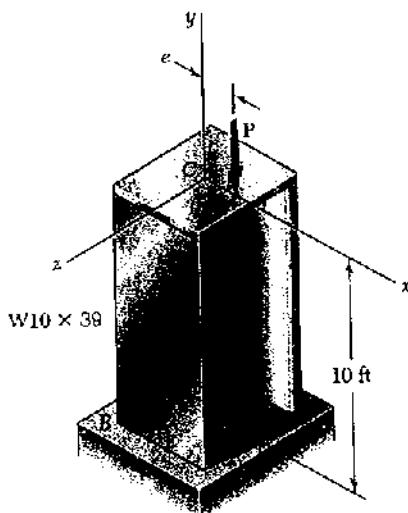
$$\sec(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}) = \frac{y_{max} + e}{e}, \quad \cos(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}) = \frac{e}{y_{max} + e}$$

$$(a) \quad \frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{e + y_{max}} \right]^2 = \left[\frac{2}{\pi} \arccos \frac{0.25}{0.25 + 0.50} \right]^2 \\ = 0.61411 \quad P = 0.61411 P_{cr} = 13.29 \text{ kips.}$$

$$(b) M_{max} = P(e + y_{max}) = (13.29)(0.25 + 0.50) = 9.9675 \text{ kip-in.}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{13.29}{3.0625} + \frac{(9.9675)(0.875)}{0.78157} = 15.50 \text{ ksi}$$

Problem 10.36



10.36 An axial load P is applied at a point located on the x axis at a distance $e = 0.5$ in. from the geometric axis of the W 10 × 39 rolled-steel column BC . Using $E = 29 \times 10^6$ psi, determine (a) the load P for which the horizontal deflection of the top of the column is 0.6 in., (b) the corresponding maximum stress in the column.

$$W 10 \times 39 \quad A = 11.5 \text{ in}^2, \quad I_y = 45.0 \text{ in}^4, \quad S_y = 11.3 \text{ in}^3$$

$$L_e = 2L = (2)(10) = 20 \text{ ft} = 240 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(45.0)}{(240)^2}$$

$$= 223.61 \times 10^3 \text{ lb} = 223.61 \text{ kips}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos\left(\frac{e}{y_{max} + e}\right) \right]^2 = \left[\frac{2}{\pi} \arccos\left(\frac{0.5}{0.6 + 0.5}\right) \right]^2$$

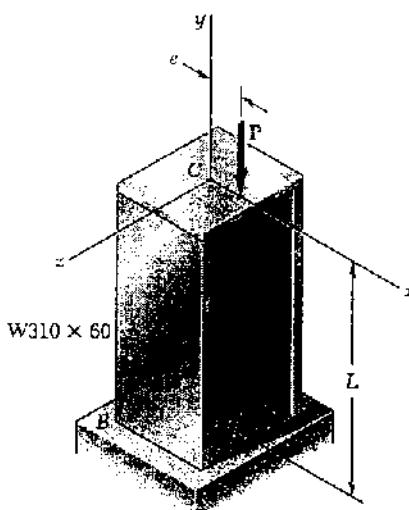
$$= 0.48944$$

$$(a) \quad P = (0.48944)(223.61) = 109.4 \text{ kips}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (109.4)(0.5 + 0.6) = 120.39 \text{ kip-in}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{109.4}{11.5} + \frac{120.39}{11.3} = 20.2 \text{ ksi}$$

Problem 10.37



10.37 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 × 60 rolled-steel column BC . Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection at end C is 15 mm, (b) the corresponding maximum stress in the column.

W310 × 60

$$A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-4} \text{ m}^4$$

$$S_y = 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3$$

$$L = 3.5 \text{ m}$$

$$L_e = 2L = 7.0 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.3 \times 10^{-4})}{(7.0)^2}$$

$$= 737.2 \times 10^3 \text{ N} = 737.2 \text{ kN}$$

$$y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] \quad \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{y_{max} + e}{e} \quad \cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2 = \left[\frac{2}{\pi} \arccos \frac{12}{15 + 12} \right]^2 = 0.49957$$

$$(a) \quad P = 0.49957 P_{cr} = 368.28 \text{ kN} \quad 368 \text{ kN} \quad \blacksquare$$

$$M_{max} = P(e + y_{max}) = (368.28 \times 10^3)(12 + 15)(10^{-3}) = 9944 \text{ N-m}$$

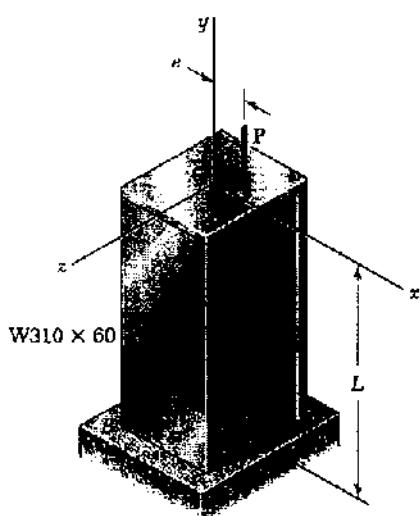
$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{368.28 \times 10^3}{7590 \times 10^{-6}} + \frac{9944}{180 \times 10^{-6}} = 103.8 \times 10^6 \text{ Pa}$$

$$= 103.8 \text{ MPa} \quad \blacksquare$$

Problem 10.38

10.38 Solve Prob. 10.37, assuming that L is 4.5 m.

10.37 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 × 60 rolled-steel column BC . Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection at end C is 15 mm, (b) the corresponding maximum stress in the column.



$$\text{W } 310 \times 60 \quad A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-4} \text{ m}^4$$

$$S_y = 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3$$

$$L = 4.5 \text{ m} \quad L_e = 2L = 9.0 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.3 \times 10^{-4})}{(9.0)^2}$$

$$= 445.96 \times 10^3 \text{ N} = 445.96 \text{ kN}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \quad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2 = \left[\frac{2}{\pi} \arccos \frac{12}{15 + 12} \right]^2 = 0.49957$$

$$(a) \quad P = 0.49957 P_{cr} = 222.79 \text{ kN} \quad 223 \text{ kN}$$

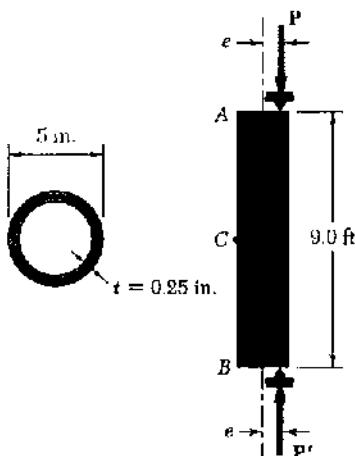
$$M_{max} = P(e + y_{max}) = (222.79 \times 10^3)(12 + 15)(10^{-3}) = 6015 \text{ N} \cdot \text{m}$$

$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{222.79 \times 10^3}{7590 \times 10^{-6}} + \frac{6015}{180 \times 10^{-6}} = 62.8 \times 10^6 \text{ Pa}$$

$$= 62.8 \text{ MPa}$$

Problem 10.39

10.39 A brass pipe having the cross section shown has an axial load P applied 0.15 in. from its geometric axis. Using $E = 17 \times 10^6$ psi, determine (a) the load P for which the horizontal deflection at the midpoint C is 0.20 in., (b) the corresponding maximum stress in the column.



$$c_o = \frac{1}{2}d_o = \frac{1}{2}(5) = 2.5 \text{ in.} \quad c_z = c_o - \frac{e}{2} = 2.25 \text{ in.}$$

$$I = \frac{\pi}{4}(c_o^4 - c_z^4) = \frac{\pi}{4}(2.5^4 - 2.25^4) = 10.551 \text{ in.}^4$$

$$L_e = L = 9.0 \text{ ft} = 108 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(17 \times 10^6)(10.551)}{(108)^2} = 151.77 \times 10^3 \text{ lb} \\ = 151.77 \text{ kips}$$

$$(a) y_{max} = e \left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{e + y_{max}} \quad \frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{0.15}{0.20 + 0.15} \right]^2 = 0.51557 \quad P = 0.51557 P_{cr} = 78.249 \text{ kips}$$

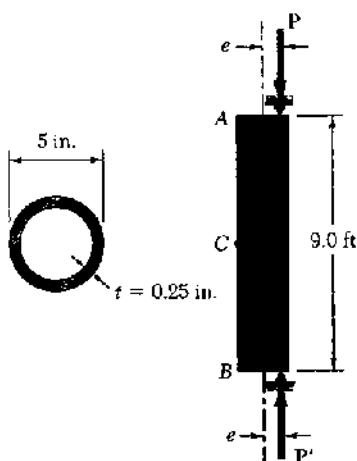
$$(b) M_{max} = P(e + y_{max}) = (78.249)(0.15 + 0.20) = 27.387 \text{ kip-in}$$

$$A = \pi(c_o^2 - c_z^2) = \pi(2.5^2 - 2.25^2) = 3.7306 \text{ in.}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{78.249}{3.7306} + \frac{(27.387)(2.5)}{10.551} = 27.5 \text{ ksi}$$

Problem 10.40

10.40 Solve Prob. 10.39, assuming that the axial load P is applied 0.3 in. from the geometric axis of the column.



10.39 A brass pipe having the cross section shown has an axial load P applied 0.15 in. from its geometric axis. Using $E = 17 \times 10^6$ psi, determine (a) the load P for which the horizontal deflection at the midpoint C is 0.20 in., (b) the corresponding maximum stress in the column.

$$c_o = \frac{1}{2} d_o = \frac{1}{2}(5) = 2.5 \text{ in.} \quad c_x = c_o - t = 2.25 \text{ in.}$$

$$I = \frac{\pi}{4} (c_o^4 - c_x^4) = \frac{\pi}{4} (2.5^4 - 2.25^4) = 10.551 \text{ in}^4$$

$$L_e = L = 9.0 \text{ ft} = 108 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (17 \times 10^6)(10.551)}{(108)^2} = 151.77 \times 10^3 \text{ lb} \\ = 151.77 \text{ kips}$$

$$(a) y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] \quad \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{y_{max} + e}{e} \quad \cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2 = \left[\frac{2}{\pi} \arccos \frac{0.3}{0.20 + 0.3} \right]^2 = 0.34849$$

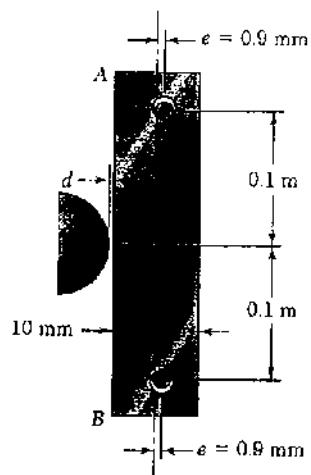
$$P = 0.34849 P_{cr} = 52.891 \text{ kips}$$

$$(b) M_{max} = P(e + y_{max}) = (52.891)(0.3 + 0.20) = 26.446 \text{ kip-in}$$

$$A = \pi (c_o^2 - c_x^2) = \pi (2.5^2 - 2.25^2) = 3.7306 \text{ in}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{52.891}{3.7306} + \frac{(26.446)(2.5)}{10.551} = 20.4 \text{ ksi}$$

Problem 10.41



10.41 The steel bar AB has a 10×10 -mm square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.9$ mm from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.3$ mm. Use $E = 200$ GPa and the coefficient of thermal expansion $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$.

$$A = (10)^2 = 100 \text{ mm}^2 = 100 \times 10^{-4} \text{ m}^2$$

$$I = \frac{1}{12}(10)^4 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = (200 \times 10^9)(833.33 \times 10^{-12}) = 166.667 \text{ N}\cdot\text{m}^2$$

$$L_e = L = 0.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(166.667)}{(0.2)^2} = 41.1234 \times 10^3 \text{ N}$$

Calculate P using the secant formula

$$y_{max} = d = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] \quad \text{or} \quad \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 1 + \frac{d}{e} =$$

$$\cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{e}{e+d} = \frac{0.9}{0.9+0.3} = 0.75$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = \cos^{-1}(0.75) = 0.72273$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.72273) \right]^2 = 0.21170 \quad P = 0.21170 P_{cr} = 8.7058 \times 10^3 \text{ N}$$

If the effects of eccentricity and the shortening due to bending are neglected, the shortening of the bar, if not constrained by the pins would be

$$\delta = \frac{PL}{EA} = \frac{(8.7058 \times 10^3)(0.2)}{(200 \times 10^9)(100 \times 10^{-4})} = 87.058 \times 10^{-6} \text{ m}$$

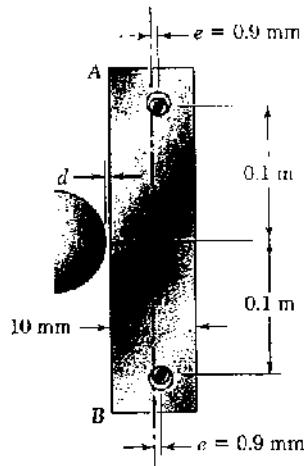
This mechanical shortening is compensated by thermal expansion

$$L\alpha(\Delta T) - \delta = 0$$

$$\Delta T = \frac{\delta}{L\alpha} = \frac{87.058 \times 10^{-6}}{(0.2)(11.7 \times 10^{-6})} = 37.2 {}^\circ\text{C}$$

Problem 10.42

10.42 For the bar of Prob. 10.41, determine the required distance d for which the bar will just make contact with point C when the temperature increases by 60°C .



10.41 The steel bar AB has a 10×10 -mm square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.9 \text{ mm}$ from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.3 \text{ mm}$. Use $E = 200 \text{ GPa}$ and the coefficient of thermal expansion $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$.

$$A = (10)^2 = 100 \text{ mm}^2 = 100 \times 10^{-4} \text{ m}^2$$

$$I = \frac{1}{12}(10)^4 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = (200 \times 10^9)(833.33 \times 10^{-12}) = 166.667 \text{ N}\cdot\text{m}^2$$

$$L_e = L = 0.2 \text{ m}$$

$$P_{cn} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (166.667)}{(0.2)^2} = 41.1234 \times 10^3 \text{ N}$$

Free thermal expansion of the bar:

$$\delta = L\alpha(\Delta T) = (0.2)(11.7 \times 10^{-6})(60) = 140.4 \times 10^{-6} \text{ m}$$

The free expansion is prevented by constraint of the pins.

If eccentricity and shortening due to bending are neglected, the constraining force P is given by

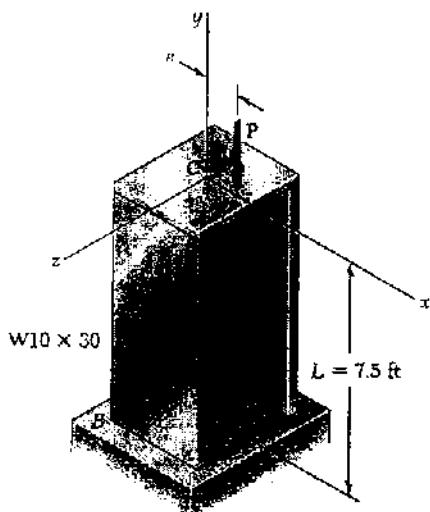
$$P = \frac{EAS}{L} = \frac{(200 \times 10^9)(100 \times 10^{-4})(140.4 \times 10^{-6})}{0.2} = 14.04 \times 10^3 \text{ N}$$

Using the secant formula,

$$\begin{aligned} y_{max} = d &= e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cn}}} \right) - 1 \right] \\ &= (9 \times 10^{-3}) \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{14.04 \times 10^3}{41.1234 \times 10^3}} \right) - 1 \right] = 5.81 \times 10^{-3} \text{ m} \end{aligned}$$

$$d = 5.81 \text{ mm}$$

Problem 10.43



10.43 An axial load P is applied to the W10 × 30 rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 0.5$ in. and that for the grade of steel used $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi, determine (a) the magnitude of P of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (b) the ratio of the load found in part (a) to the magnitude of the allowable eccentric load for the column. (*Hint:* Since the factor of safety must be applied to the load P , not to the stress, use Fig. 10.24 to determine P_y).

$$W10 \times 30 \quad A = 8.84 \text{ in}^2 \quad r_y = 1.37 \text{ in}$$

$$c = \frac{b_f}{2} = \frac{5.810}{2} = 2.905 \text{ in} \quad I_y = 16.7 \text{ in}^4$$

$$L = 7.5 \text{ ft} = 90 \text{ in} \quad L_e = 2L = 180 \text{ in}$$

$$\frac{L_e}{r} = \frac{180}{1.37} = 131.39 \quad e = 0.5 \text{ in}$$

$$\frac{ec}{r^2} = \frac{(0.5)(2.905)}{(1.37)^2} = 0.7739$$

$$\text{Using Fig 10.24} \quad \frac{P}{A} = 10.47 \text{ ksi}$$

$$P = (10.47)(8.84) = 92.6 \text{ kips}$$

$$(a) \text{ Using factor of safety, } P_{all} = \frac{92.6}{2.4} = 38.6 \text{ kips}$$

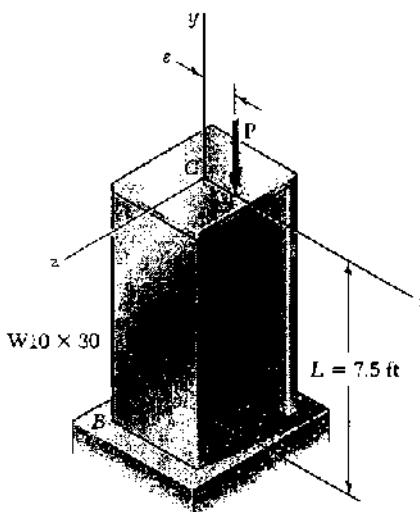
$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29000)(16.7)}{(180)^2} = 147.5 \text{ kips}$$

$$\text{Using factor of safety, } P_{all} = \frac{147.5}{2.4} = 61.5 \text{ kips}$$

$$(b) \text{ ratio} = \frac{38.6}{61.5} = 0.628$$

Problem 10.44

10.44 Solve Prob. 10.43, assuming that the length of the column is reduced to 5.0 ft.



10.43 An axial load P is applied to the W10 × 30 rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 0.5$ in. and that for the grade of steel used $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi., determine (a) the magnitude of P of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (b) the ratio of the load found in part (a) to the magnitude of the allowable centric load for the column. (Hint: Since the factor of safety must be applied to the load P , not to the stress, use Fig. 10.24 to determine P_y).

$$\begin{aligned} \text{W10} \times 30 \quad A &= 8.84 \text{ in}^2 \quad I_y = 16.7 \text{ in}^4 \\ r_y &= 1.37 \text{ in} \quad c = \frac{b_e}{2} = \frac{5.810}{2} = 2.905 \text{ in} \\ L &= 5.0 \text{ ft} = 60 \text{ in} \quad L_e = 2L = 120 \text{ in} \\ \frac{L_e}{r} &= \frac{120}{1.37} = 87.6 \\ \frac{ec}{r^2} &= \frac{(0.5)(2.905)}{(1.37)^2} = 0.7739 \end{aligned}$$

Using Fig 10.24 $\frac{P}{A} = 14.90$ ksi $P = (14.90)(8.84) = 131.7$ kips

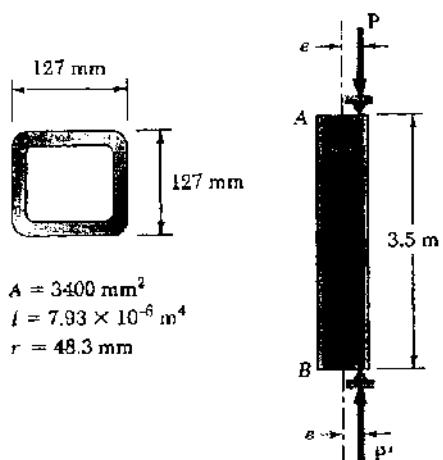
(a) Using factor of safety, $P_{all} = \frac{131.7}{2.4} = 54.9$ kips

$$P_{per} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29000)(16.7)}{(120)^2} = 332 \text{ kips}$$

Using factor of safety, $P_{all} = \frac{332}{2.4} = 138.3$ kips

(b) ratio = $\frac{54.9}{138.3} = 0.397$

Problem 10.45



10.45 A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load P when the eccentricity e is (a) 15 mm, (b) 7.5 mm. (See hint of Prob. 10.43).

$$A = 3400 \times 10^{-6} \text{ m}^2 \quad r = 48.3 \times 10^{-3} \text{ m}$$

$$L_e = 3.5 \text{ m} \quad \frac{L_e}{r} = \frac{3.5}{48.3 \times 10^{-3}} = 72.46$$

$$C = \frac{127}{2} = 63.5 \text{ mm.}$$

$$(a) \quad e = 15 \text{ mm.} \quad \frac{ec}{r^2} = \frac{(15)(63.5)}{(48.3)^2} = 0.40829$$

Using Fig. 10.24 with $L_e/r = 72.46$ and $ec/r^2 = 0.40829$,

$$P/A = 144.75 \text{ MPa} = 144.75 \times 10^6 \text{ Pa}$$

$$P = (144.75 \times 10^6)(3400 \times 10^{-6}) = 492 \times 10^3 \text{ N}$$

$$\text{Using factor of safety, } P_{all} = \frac{492 \times 10^3}{2.6} = 189 \times 10^3 \text{ N} = 189 \text{ kN}$$

$$(b) \quad e = 7.5 \text{ mm.} \quad \frac{ec}{r^2} = \frac{(7.5)(63.5)}{(48.3)^2} = 0.20415$$

Using Fig. 10.24 with $L_e/r = 72.46$ and $ec/r^2 = 0.20415$,

$$P/A = 175.2 \text{ MPa} = 175.2 \times 10^6 \text{ Pa}$$

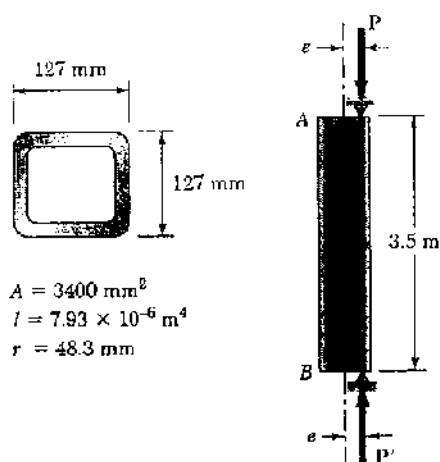
$$P = (175.2 \times 10^6)(3400 \times 10^{-6}) = 596 \times 10^3 \text{ N}$$

$$\text{Using factor of safety, } P_{all} = \frac{596 \times 10^3}{2.6} = 229 \times 10^3 \text{ N} = 229 \text{ kN}$$

Problem 10.46

10.46 Solve Prob. 10.45, assuming that the length of the steel tube is increased to 5 m.

10.45 A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load P when the eccentricity e is (a) 15 mm, (b) 7.5 mm. (See hint of Prob. 10.43).



$$A = 3400 \times 10^{-6} \text{ m}^2 \quad r = 48.3 \times 10^{-3} \text{ m}$$

$$L_e = 5 \text{ m} \quad \frac{L_e}{r} = \frac{5}{48.3 \times 10^{-3}} = 103.52$$

$$c = \frac{127}{2} = 63.5 \text{ mm}$$

$$(a) \underline{e = 15 \text{ mm}}. \quad \frac{ec}{r^2} = \frac{(15)(63.5)}{(48.3)^2} = 0.40829$$

Using Fig. 10.24 with $\frac{L_e}{r} = 103.52$

and $\frac{ec}{r^2} = 0.40829$ gives $\frac{P}{A} = 112.75 \text{ MPa} = 112.75 \times 10^6 \text{ Pa}$

$$P = (112.75 \times 10^6) (3400 \times 10^{-6}) = 383 \times 10^3 \text{ N}$$

Using factor of safety $P_{all} = \frac{383 \times 10^3}{2.6} = 147 \times 10^3 \text{ N} = 147 \text{ kN}$

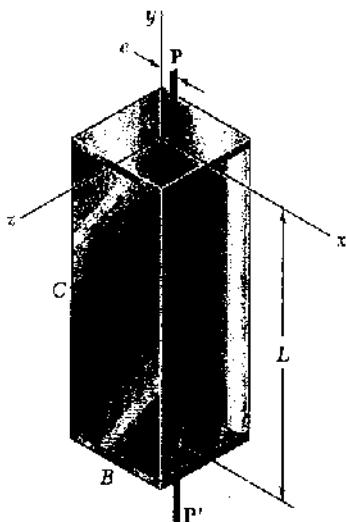
$$(b) \underline{e = 7.5 \text{ mm}}. \quad \frac{ec}{r^2} = \frac{(7.5)(63.5)}{(48.3)^2} = 0.20415$$

Using Fig. 10.24 gives $\frac{P}{A} = 133.2 \text{ MPa} = 133.2 \times 10^6 \text{ Pa}$

$$P = (133.2 \times 10^6) (3400 \times 10^{-6}) = 453 \times 10^3 \text{ N}$$

Using factor of safety, $P_{all} = \frac{453 \times 10^3}{2.6} = 174 \times 10^3 \text{ N} = 174 \text{ kN}$

Problem 10.47



10.47 Axial loads of magnitude $P = 20$ kips are applied parallel to the geometric axis of the W 8 × 15 rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{all} = 12$ ksi and $E = 29 \times 10^6$ psi, determine the largest permissible length L when (a) $e = 0.25$ in., (b) $e = 0.5$ in.

$$\text{Data: } P = 20 \text{ kips} \quad E = 29 \times 10^6 \text{ ksi}$$

$$\text{W } 8 \times 15 \quad A = 4.44 \text{ in}^2, \quad c = \frac{bf}{2} = 2.0075 \text{ in.}$$

$$I_y = 3.41 \text{ in}^4, \quad r_y = 0.876 \text{ in.}$$

$$\sigma_{all} = \sigma_{max} = 12 \text{ ksi}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec} \left[\frac{A\sigma_{max}}{P} - 1 \right]$$

$$(a) \quad e = 0.25 \text{ in.} \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(0.876)^2}{(0.25)(2.0075)} \left[\frac{(4.44)(12)}{20} - 1 \right] = 2.5443$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.39304 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.16686$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (1.16686) \right]^2 = 0.55182$$

$$P_{cr} = \frac{P}{0.55182} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.55182 \pi^2 EI}{P} = \frac{0.55182 \pi^2 (29 \times 10^6)(3.41)}{20} = 26.926 \times 10^5 \text{ in}^2$$

$$L_e = 164.1 \text{ in.}$$

$$L = L_e = 164.1 \text{ in.} = 13.68 \text{ ft}$$

$$(b) \quad e = 0.5 \text{ in.} \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(0.876)^2}{(0.5)(2.0075)} \left[\frac{(4.44)(12)}{20} - 1 \right] = 1.27214$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.78608 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.66636$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.66636) \right]^2 = 0.179962$$

$$P_{cr} = \frac{P}{0.179962} = \frac{\pi^2 EI}{L_e^2}$$

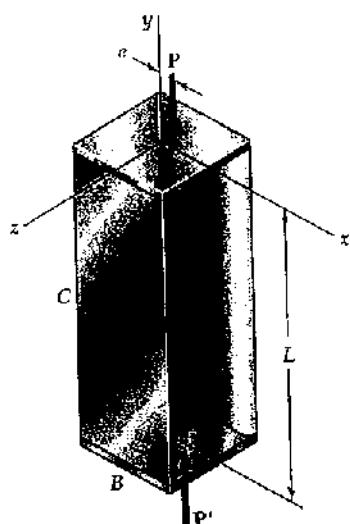
$$L_e^2 = \frac{0.179962 \pi^2 EI}{P} = \frac{0.179962 \pi^2 (29 \times 10^6)(3.41)}{20} = 8.7822 \times 10^3 \text{ in}^2$$

$$L_e = 93.713 \text{ in.}$$

$$L = L_e = 93.713 \text{ in.} = 7.81 \text{ ft}$$

Problem 10.48

10.48 Axial loads of magnitude $P = 135$ kips are applied parallel to the geometric axis of the W 10 × 54 rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{all} = 12$ ksi and $E = 29 \times 10^6$ psi, determine the largest permissible length L when (a) $e = 0.25$ in., (b) $e = 0.5$ in.



$$\text{Data: } P = 135 \text{ kips} \quad E = 29 \times 10^6 \text{ ksi}$$

$$\text{W } 10 \times 54 \quad A = 15.8 \text{ in}^2 \quad c = \frac{b_e}{2} = 5.015 \text{ in.}$$

$$I_y = 103 \text{ in}^4 \quad r_y = 2.56 \text{ in.}$$

$$\sigma_{all} = \sigma_{max} = 12 \text{ ksi}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec} \left[\frac{A\sigma_{max}}{P} - 1 \right]$$

$$(a) \underline{e = 0.25 \text{ in.}} \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(2.56)^2}{(0.25)(5.015)} \left[\frac{(15.8)(12)}{135} - 1 \right] = 2.11411$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.47301 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.07809$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (1.07809) \right]^2 = 0.47105$$

$$P_{cr} = \frac{P}{0.47105} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.47105 \pi^2 EI}{P} = \frac{0.47105 \pi^2 (29 \times 10^6)(103)}{135} = 102.866 \times 10^3 \text{ in}^2$$

$$L_e = 320.73 \text{ in.}$$

$$L = L_e = 320.73 \text{ in.} = 26.7 \text{ ft}$$

$$(b) \underline{e = 0.5 \text{ in.}} \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(2.56)^2}{(0.5)(5.015)} \left[\frac{(15.8)(12)}{135} - 1 \right] = 1.05706$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.94602 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.33006$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.33006) \right]^2 = 0.044151$$

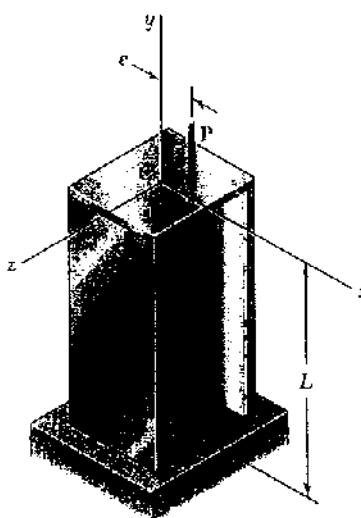
$$P_{cr} = \frac{P}{0.044151} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.044151 \pi^2 EI}{P} = \frac{0.044151 \pi^2 (29 \times 10^6)(103)}{135} = 9.6414 \times 10^3 \text{ in}^2$$

$$L_e = 98.19 \text{ in.}$$

$$L = L_e = 98.19 \text{ in.} = 8.18 \text{ ft}$$

Problem 10.49



10.49 A 250-kN axial load P is applied to the W200 × 35.9 rolled-steel column BC which is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 6\text{mm}$, determine the largest permissible length L if the allowable stress in the column is 80 MPa. Use $E = 200 \text{ GPa}$.

$$\text{W}200 \times 35.9 \quad A = 4580 \text{ mm}^2 = 4580 \times 10^{-6} \text{ m}^2$$

$$b_f = 165 \text{ mm} \quad c = \frac{b_f}{2} = 82.5 \text{ mm}$$

$$I_y = 7.64 \times 10^6 \text{ mm}^4 = 7.64 \times 10^{-6} \text{ m}^4 \quad r_y = 40.8 \text{ mm}$$

$$\sigma_{max} = 80 \times 10^6 \text{ Pa} \quad P = 250 \times 10^3 \text{ N}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

$$\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{r_y^2}{ec} \left(\frac{A\sigma_{max}}{P} - 1 \right) = \frac{(40.8)^2}{(6)(82.5)} \left[\frac{(4580 \times 10^{-6})(80 \times 10^6)}{250 \times 10^3} - 1 \right] \\ = 1.56657$$

$$\cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 0.63866 \quad \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.87804$$

$$\frac{P}{P_{cr}} = \left[\frac{\pi}{2} (0.87804) \right]^2 = 0.31245$$

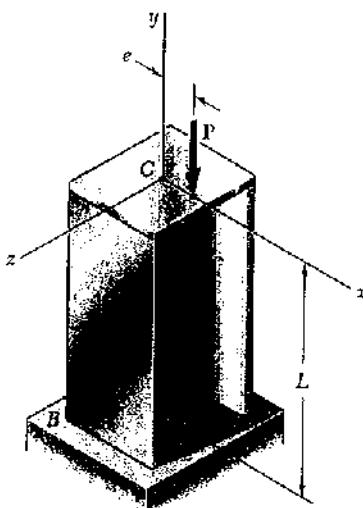
$$P_{cr} = \frac{P}{0.31245} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.31245 \pi^2 EI}{P} = \frac{0.31245 \pi^2 (200 \times 10^9)(7.64 \times 10^{-6})}{250 \times 10^3} = 18.848 \text{ m}^2$$

$$L_e = 4.34 \text{ m} = 2L$$

$$L = 2.17 \text{ m}$$

Problem 10.50



10.50 A 100-kN axial load P is applied to the W150 × 18 rolled-steel column BC which is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 6\text{mm}$, determine the largest permissible length L if the allowable stress in the column is 80 MPa. Use $E = 200 \text{ GPa}$.

$$\text{W}150 \times 18 \quad A = 2290 \text{ mm}^2 = 2290 \times 10^{-6} \text{ m}^2$$

$$b_f = 102 \text{ mm} \quad c = \frac{b_f}{2} = 51 \text{ mm}$$

$$I_y = 1.26 \times 10^6 \text{ mm}^4 = 1.26 \times 10^{-6} \text{ m}^4 \quad r_y = 23.5 \text{ mm}$$

$$\sigma_{max} = 80 \times 10^6 \text{ Pa} \quad P = 100 \times 10^3 \text{ N}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

$$\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{r_y^2}{ec} \left(\frac{A\sigma_{max}}{P} - 1 \right) = \frac{(23.5)^2}{(6)(51)} \left[\frac{(2290 \times 10^{-6})(80 \times 10^6)}{100 \times 10^3} - 1 \right]$$

$$= 1.50154$$

$$\cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 0.66598 \quad \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.84199$$

$$\frac{P}{P_{cr}} = \left[\frac{\pi}{2} (0.84199) \right]^2 = 0.28732$$

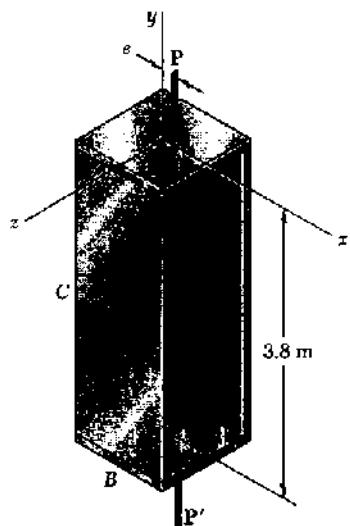
$$P_{cr} = \frac{P}{0.28732} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.28732 \pi^2 EI}{P} = \frac{0.28732 \pi^2 (200 \times 10^9)(1.26 \times 10^{-6})}{100 \times 10^3} = 7.1461 \text{ m}^2$$

$$L_e = 2.6732 = 2L$$

$$L = 1.337 \text{ m}$$

Problem 10.51



10.51 Axial loads of magnitude $P = 175 \text{ kN}$ are applied parallel to the geometric axis of a W250 × 44.8 rolled-steel column AB and intersect the axis at a distance $e = 12 \text{ mm}$ from its geometric axis. Knowing that $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the factor of safety with respect to yield. (Hint: Since the factor of safety must be applied to the load P, not to the stresses, use Fig. 10.24 to determine P_y)

$$\text{For W } 250 \times 44.8 \quad A = 5720 \text{ mm}^2, \quad r_y = 35.1 \text{ mm}$$

$$L_e = 3800 \text{ mm} \quad L_e/r = 108.26$$

$$C = \frac{b_f}{2} = \frac{148}{2} = 74 \text{ mm} \quad e = 12 \text{ mm}$$

$$\frac{ec}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$$

Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{ec}{r^2} = 0.72077$,

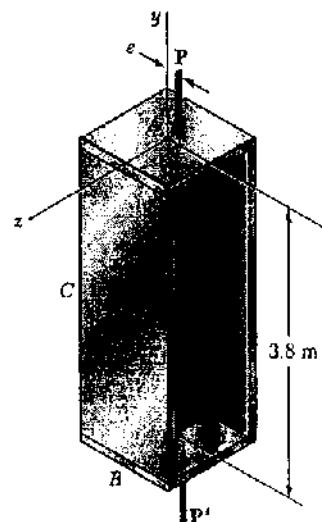
$$P_y/A = 90.37 \text{ MPa} = 90.37 \times 10^6 \text{ N/m}^2$$

$$P_y = A P_y/A = (5720 \times 10^{-6})(90.37 \times 10^6) = 517 \times 10^3 \text{ N} = 517 \text{ kN}$$

$$\text{F.S.} = \frac{P_y}{P} = \frac{517}{175} = 2.95$$

Problem 10.52

10.52 Solve Prob. 10.51, assuming that $e = 0.16 \text{ mm}$ and $P = 155 \text{ kN}$.



10.51 Axial loads of magnitude $P = 175 \text{ kN}$ are applied parallel to the geometric axis of a W250 × 44.8 rolled-steel column AB and intersect the axis at a distance $e = 12 \text{ mm}$ from its geometric axis. Knowing that $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the factor of safety with respect to yield. (Hint: Since the factor of safety must be applied to the load P, not to the stresses, use Fig. 10.24 to determine P_y)

$$\text{For W } 250 \times 44.8 \quad A = 5720 \text{ mm}^2, \quad r_y = 35.1 \text{ mm}$$

$$L_e = 3800 \text{ mm} \quad L_e/r = 108.26$$

$$C = \frac{b_f}{2} = \frac{148}{2} = 74 \text{ mm} \quad e = 16 \text{ mm}$$

$$\frac{ec}{r^2} = \frac{(16)(74)}{(35.1)^2} = 0.96103$$

Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{ec}{r^2} = 0.96103$,

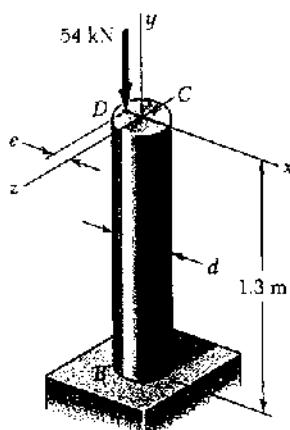
$$P_y/A = 81.17 \text{ MPa} = 81.17 \text{ N/m}^2$$

$$P_y = A(P_y/A) = (5720 \times 10^{-6})(81.17 \times 10^6) = 464 \times 10^3 \text{ N} = 464 \text{ kN}$$

$$\text{F.S.} = \frac{P_y}{P} = \frac{464}{155} = 3.00$$

Problem 10.53

10.53 A 54 kN axial load is applied with an eccentricity $e = 10 \text{ mm}$ to the circular steel rod BC that is free at its top C and fixed at its base B . Knowing that the stock of rods available for use have diameters in increments of 4 mm from 44 mm to 72 mm, determine the lightest rod that may be used if $\sigma_{all} = 110 \text{ MPa}$. Use $E = 200 \text{ GPa}$.



For a solid circular section $c = \frac{1}{2}d$

$$A = \pi c^2 = \frac{\pi}{4} d^2 \quad I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$$

$$r^2 = \frac{I}{A} = \frac{1}{16} d^2 \quad \frac{ec}{r^2} = \frac{8e}{d}$$

$$P = 54 \times 10^3 \text{ N} \quad e = 10 \text{ mm} = 0.010 \text{ m}$$

$$L_e = 2L = (2)(1.3) = 2.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P}{P_{cr}} = \frac{PL_e^2}{\pi^2 EI} = \frac{(54 \times 10^3)(2.6)^2(64)}{\pi^2(200 \times 10^9)\pi d^4} = 3.7674 \times 10^{-6} d^{-4}$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 3.0489 \times 10^{-3} d^{-2}$$

$$\frac{P}{A} = \frac{(54 \times 10^3)(4)}{\pi d^2} = 68.755 \times 10^3 d^{-2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right] \quad \frac{ec}{r^2} = \frac{0.08}{d}$$

$$\sigma_{max} = 68.755 \times 10^3 d^{-2} \left[1 + 0.08 d^{-1} \sec (3.0489 \times 10^{-3} d^{-2}) \right]$$

$d \text{ (mm)}$	$\sigma_{max} \text{ (MPa)}$
44	-159.19 ($P/P_{cr} > 1$)
48	232
52	116.6
56	77.5 ←
60	57.5
64	45.3
68	37.0
72	31.0

The lightest section
with $\sigma_{max} < 110 \text{ MPa}$
is $d = 56 \text{ mm}$.

Checking: $d = 56 \text{ mm} = 0.056 \text{ m} \quad c = 0.028 \text{ m}$

$$A = \frac{\pi}{4} d^2 = 2.4630 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{64} d^4 = 482.75 \times 10^{-9} \text{ m}^4$$

$$r^2 = \frac{I}{A} = 196 \times 10^{-6} \quad \frac{ec}{r^2} = \frac{(0.010)(0.028)}{196 \times 10^{-6}} = 1.4286$$

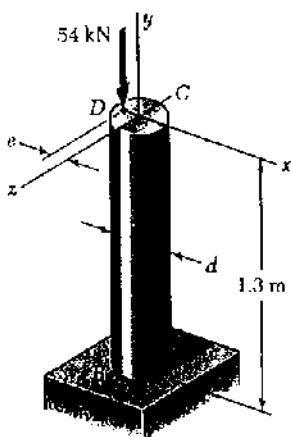
$$P_{cr} = \frac{\pi^2 (200 \times 10^9) (482.75 \times 10^{-9})}{(2.6)^2} = 140.96 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{54 \times 10^3}{140.96 \times 10^3} = 0.38308$$

$$\sigma_{max} = \frac{54 \times 10^3}{2.4630 \times 10^{-3}} \left[1 + 1.4286 \sec \left(\frac{\pi}{2} \sqrt{0.38308} \right) \right] = 77.5 \times 10^6 \text{ Pa}$$

Problem 10.54

10.54 Solve Prob. 10.53, assuming that the 54 kN axial load will be applied to the rod with an eccentricity $e = \frac{1}{2} d$.



10.53 A 54 kN axial load is applied with an eccentricity $e = 10 \text{ mm}$ to the circular steel rod BC that is free at its top C and fixed at its base B . Knowing that the stock of rods available for use have diameters in increments of 4 mm from 44 mm to 72 mm, determine the lightest rod that may be used if $\sigma_{all} = 110 \text{ MPa}$. Use $E = 200 \text{ GPa}$.

For a solid circular section $c = \frac{1}{2}d$

$$A = \pi c^2 = \frac{\pi}{4}d^2 \quad I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4$$

$$r^2 = \frac{I}{A} = \frac{1}{16}d^2$$

$$\text{Use } e = \frac{1}{2}d \quad \frac{ec}{r^2} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(16) = 4$$

$$P = 54 \times 10^3 \text{ N}$$

$$L_e = 2L = (2)(1.3) = 2.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P}{P_{cr}} = \frac{PL_e^2}{\pi^2 EI} = \frac{(54 \times 10^3)(2.6)^2(64)}{\pi^2(200 \times 10^9) \pi d^4} = 3.7674 \times 10^{-6} d^{-4}$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 3.0489 \times 10^{-3} d^{-2}$$

$$\frac{P}{A} = \frac{(54 \times 10^3)(4)}{\pi d^2} = 68.755 \times 10^3 d^{-2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

$$\sigma_{max} = 68.755 \times 10^3 d^{-2} \left[1 + 4 \sec(3.0489 \times 10^{-3} d^{-2}) \right]$$

$d (\text{mm})$	$\sigma_{max} (\text{MPa})$
44	-35.1
48	517
52	263
56	177.6
60	134.4
64	108.1 ←
68	90.1
72	72.0

The lightest section
with $\sigma_{max} \leq 110 \text{ MPa}$
is $d = 64 \text{ mm}$

Checking $A = \frac{\pi}{4}(0.064)^2 = 3.217 \times 10^{-3} \text{ m}^2, \quad I = \frac{\pi}{64}(0.064)^4 = 823.55 \times 10^{-9} \text{ m}^4$

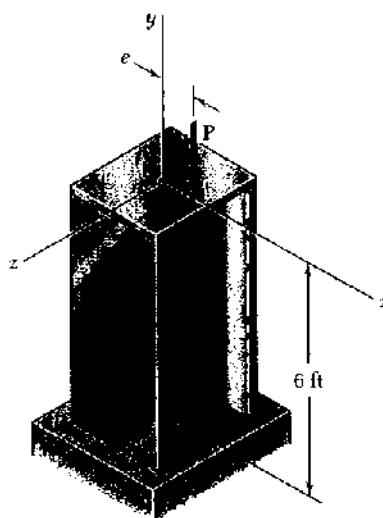
$$P_{cr} = \frac{\pi^2 (200 \times 10^9) (823.55 \times 10^{-9})}{(2.6)^2} = 240.48 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{54 \times 10^3}{240.48 \times 10^3} = 0.22455$$

$$\sigma_{max} = \frac{54 \times 10^3}{3.217 \times 10^{-3}} \left[1 + 4 \sec \left(\frac{\pi}{2} \sqrt{0.22455} \right) \right] = 108.1 \times 10^6 \text{ Pa}$$

Problem 10.55

10.55 An axial load of magnitude $P = 50$ kips is applied at a point located on the x axis at a distance $e = 0.25$ in. from the geometric axis of the wide-flange column BC . Knowing that $E = 29 \times 10^6$ psi, chose the lightest W8 shape that can be used if $\sigma_{all} = 18$ ksi.



$$P = 50 \text{ kips} \quad L = 6 \text{ ft}$$

$$E = 29 \times 10^3 \text{ ksi}$$

$$L_e = 2L = 12 \text{ ft} = 144 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI_x}{L_e^2} = \frac{\pi^2 (29 \times 10^3) I_y}{(144)^2}$$

$$= 13.803 I_y$$

$$c = \frac{1}{2} b_f \quad \frac{ec}{r_y^2} = \frac{(0.25) b_f}{(r_y)^2 (2)} = \frac{0.125 b_f}{r_y^2}$$

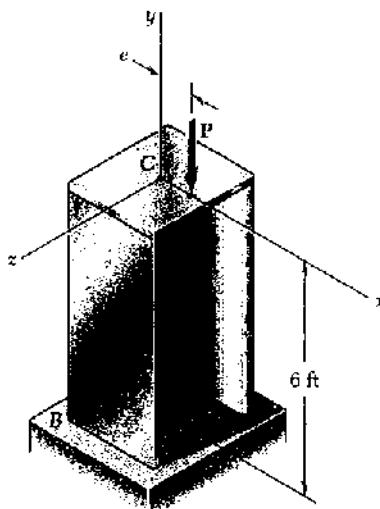
$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

Shape	$A (\text{in}^2)$	$b_f (\text{in.})$	$I_y (\text{in}^4)$	$r_y (\text{in.})$	$P_{cr} (\text{kips})$	$\frac{ec}{r_y^2}$	$\sigma_{max} (\text{ksi})$
W8 x 58	17.1	8.220	75.1	2.10	1037	0.233	3.65
W8 x 48	14.1	8.110	60.9	2.08	841	0.234	4.44
W8 x 40	11.7	8.070	49.1	2.04	678	0.243	5.41
W8 x 35	10.3	8.020	42.6	2.03	588	0.243	6.17
W8 x 31	9.13	7.995	37.1	2.02	512	0.245	7.00
W8 x 28	8.25	6.535	21.7	1.62	299.5	0.311	8.41
W8 x 24	7.08	6.495	18.5	1.61	255.4	0.313	9.94
W8 x 21	6.16	5.270	9.77	1.26	134.85	0.415	13.96
W8 x 18	5.26	5.250	7.97	1.23	110.01	0.484	17.93
W8 x 15	4.44	4.015	3.41	0.876	47.07	0.654	X
W8 x 13	3.84	4.000	2.73	0.843	37.68	0.704	X

Use W8 x 18 $\sigma_{max} = 17.93$ ksi

Problem 10.56

10.56 Solve Prob. 10.55, assuming that the magnitude of the axial load is $P = 78$ kips.



10.55 An axial load of magnitude $P = 50$ kips is applied at a point located on the x axis at a distance $e = 0.25$ in. from the geometric axis of the wide-flange column BC . Knowing that $E = 29 \times 10^6$ psi, chose the lightest W8 shape that can be used if $\sigma_{all} = 18$ ksi.

$$P = 78 \text{ kips} \quad L = 6 \text{ ft} \quad E = 29 \times 10^6 \text{ ksi}$$

$$L_e = 2L = 12 \text{ ft} = 144 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 (29 \times 10^6) I_y}{(144)^2} \\ = 13.803 I_y$$

$$c = \frac{1}{2} b_F \quad \frac{ec}{r_y^2} = \frac{(0.25) b_F}{(r_y)^2 (2)} = \frac{0.125 b_F}{r_y^2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

Shape	$A (\text{in}^2)$	$b_F (\text{in.})$	$I_y (\text{in}^4)$	$r_y (\text{in.})$	$P_{cr} (\text{kips})$	$\frac{ec}{r_y^2}$	$\sigma_{max} (\text{ksi})$
W 8×58	17.1	8.220	75.1	2.10	1037	0.233	5.73
W 8×48	14.1	8.110	60.9	2.08	841	0.234	6.99
W 8×40	11.7	8.070	49.1	2.04	678	0.243	8.55
W 8×35	10.3	8.020	42.6	2.03	588	0.243	9.76
W 8×31	9.13	7.995	37.1	2.02	512	0.245	11.10
W 8×28	8.25	6.535	21.7	1.62	299.5	0.311	13.68
W 8×24	7.08	6.495	18.3	1.61	252.6	0.313	16.38
W 8×21	6.16	5.270	9.77	1.26	134.85	0.415	27.0
W 8×18	5.26	5.250	7.97	1.23	110.01	0.434	41.0
W 8×15	4.44	4.015	3.41	0.876	47.07	0.654	X
W 8×13	3.84	4.000	2.73	0.843	37.68	0.704	X

Use W 8×24 $\sigma_{max} = 16.38$ ksi

Problem 10.57

10.57 A W8 × 31 rolled-steel shape is used to form a column of 21-ft effective length. Using allowable stress design, determine the allowable centric load if the yield strength of the grade of steel used is (a) $\sigma_y = 36$ ksi, (b) $\sigma_y = 50$ ksi. Use $E = 29 \times 10^6$ psi.

$$\text{Steel: } E = 29000 \text{ ksi} \quad W8 \times 31 \quad A = 9.13 \text{ in}^2 \quad r_{min} = 2.02 \text{ in}$$

$$L_e = 21 \text{ ft} = 252 \text{ in} \quad L_e/r = 124.75$$

$$(a) \sigma_y = 36 \text{ ksi} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

$$L_e/r < C_c \quad \frac{L_e/r}{C_c} = 0.98932$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.98932) - \frac{1}{8}(0.98932)^3 = 1.91662$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.91662} \left[1 - \frac{1}{2} (0.98932)^2 \right] = 9.59 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (9.59)(9.13) = 87.6 \text{ kips}$$

$$(b) \sigma_y = 50 \text{ ksi} \quad C_c = \sqrt{\frac{2\pi^2 (29000)}{50}} = 107.00$$

$$L_e/r > C_c \quad \sigma_{all} = \frac{\pi^2 E}{\frac{23}{12} (L_e/r)^2} = 9.59 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (9.59)(9.13) = 87.6 \text{ kips}$$

Problem 10.58

10.58 Using allowable stress design, determine the allowable centric load for a column of 6-m effective length that is made from the following rolled-steel shape: (a) W200 × 35.9, (b) W200 × 86. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.664$$

$$(a) W200 \times 35.9 \quad A = 4580 \times 10^{-6} \text{ m}^2 \quad r_{min} = 40.8 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{6}{40.8 \times 10^{-3}} = 147.06 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(147.06)^2} = 47.539 \times 10^6 \text{ Pa}$$

$$P_{all} = A \sigma_{all} = (4580 \times 10^{-6})(47.539 \times 10^6) = 218 \times 10^3 \text{ N} = 218 \text{ kN}$$

$$(b) W200 \times 86 \quad A = 11000 \times 10^{-6} \text{ m}^2 \quad r_{min} = 53.2 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{6}{53.2 \times 10^{-3}} = 112.782 < 125.664 \quad \frac{L_e/r}{C_c} = 0.89749$$

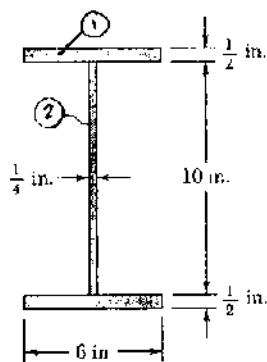
$$F.S. = \frac{5}{3} + \frac{3}{8}(0.89749) - \frac{1}{8}(0.89749)^3 = 1.91219$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.91219} \left[1 - 0.89749^2 \right] = 78.058 \times 10^6 \text{ Pa}$$

$$P_{all} = A \sigma_{all} = (11000 \times 10^{-6})(78.058 \times 10^6) = 859 \times 10^3 \text{ N} = 859 \text{ kN}$$

Problem 10.59

10.59 A column with the cross section shown has a 13.5-ft effective length. Using allowable stress design, determine the largest centric load that can be applied to the column. Use $\sigma_y = 36 \text{ ksi}$ and $E = 29 \times 10^6 \text{ psi}$.



$$A = 2A_1 + A_2 = (2)(\frac{1}{2})(6) + (10)(\frac{1}{4}) = 8.5 \text{ in}^2$$

$$I_y = 2I_1 + I_2 = (2)(\frac{1}{12})(\frac{1}{2})(6)^3 + (\frac{1}{12})(10)(\frac{1}{4})^3 = 18.013 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{18.013}{8.5}} = 1.4557 \text{ in.}$$

$$L_e = 13.5 \text{ ft} = 162 \text{ in} \quad \frac{L_e}{r} = \frac{162}{1.4557} = 111.29 < C_c$$

$$\text{Steel: } E = 29000 \text{ ksi}, \sigma_y = 36 \text{ ksi} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

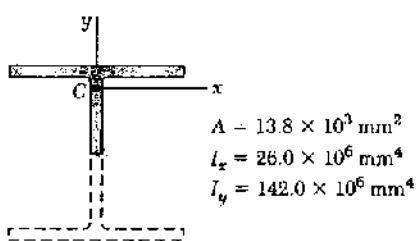
$$\frac{L_e/r}{C_c} = 0.8826 \quad \text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.8826) - \frac{1}{8}(0.8826)^3 = 1.912$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} = \frac{\sigma_y}{1.912} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.912} \left[1 - \frac{1}{2} (0.8826)^2 \right] = 11.49 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (11.49)(8.5) = 97.7 \text{ kips}$$

Problem 10.60

10.60 A column is made from half of a W360 × 216 rolled-steel shape, with the geometric properties as shown. Using allowable stress design, determine the allowable centric load if the effective length of the column is (a) 4.0 m, (b) 6.5 m. Use $\sigma_y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$.



$$A = 13.8 \times 10^3 \text{ mm}^2$$

$$I_x = 26.0 \times 10^6 \text{ mm}^4$$

$$I_y = 142.0 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{26.0 \times 10^6}{13.8 \times 10^3}} = 43.406 \text{ mm} = 43.406 \times 10^{-3} \text{ m}$$

$$A = 13.8 \times 10^{-3} \text{ m}^2$$

$$\text{Steel } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{345 \times 10^6}} = 106.97$$

$$(a) L_e = 4.0 \text{ m. } \frac{L_e}{r} = 92.153 < C_c \quad \frac{L_e/r}{C_c} = 0.86149$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.86149) - \frac{1}{8}(0.86149)^3 = 1.9098$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} = \frac{345 \times 10^6}{1.9098} \left[1 - \frac{1}{2} (0.86149)^2 \right] = 113.61 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (113.61 \times 10^6)(13.8 \times 10^{-3}) = 1568 \times 10^3 \text{ N} = 1568 \text{ kN}$$

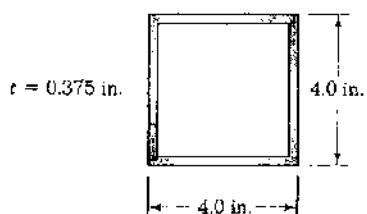
$$(b) L_e = 6.5 \text{ m. } \frac{L_e}{r} = 149.75 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(149.75)^2} = 45.845 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (45.845 \times 10^6)(13.8 \times 10^{-3}) = 633 \times 10^3 \text{ N} = 633 \text{ kN}$$

Problem 10.61

10.61 A compression member has the cross section shown and an effective length of 5 ft. Knowing that the aluminum alloy used is 2014-T6, determine the allowable centric load.



$$b_o = 4.0 \text{ in.}, \quad b_i = b_o - 2t = 3.25 \text{ in.}$$

$$A = (4.0)^2 - (3.25)^2 = 5.4375 \text{ in}^2$$

$$I = \frac{1}{12}[(4.0)^4 - (3.25)^4] = 12.036 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{12.036}{5.4375}} = 1.488 \text{ in.} \quad L_e 5 \text{ ft} = 60 \text{ in.}$$

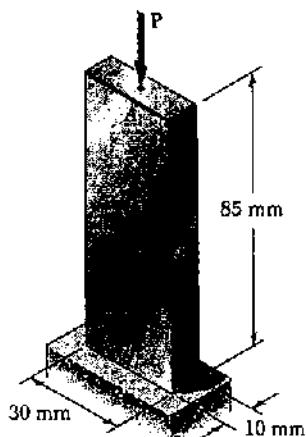
$$\frac{L}{r} = \frac{60}{1.488} = 40.33 < 55 \text{ for 2014-T6 aluminum alloy.}$$

$$\sigma_{all} = 30.7 - 0.23(L/r) = 30.7 - (0.23)(40.33) = 21.42 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (21.42)(5.4375) = 116.5 \text{ kips}$$

Problem 10.62

10.62 Bar AB is free at its end A and fixed at its base B. Determine the allowable centric load P if the aluminum alloy is (a) 6061-T6, (b) 2014-T6.



$$A = (30)(10) = 300 \text{ mm}^2 = 300 \times 10^{-6} \text{ m}^2$$

$$I_{min} = \frac{1}{12}(30)(10)^3 = 2.50 \times 10^3 \text{ mm}^4$$

$$r_{min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.50 \times 10^3}{300}} = 2.887 \text{ mm}$$

$$L_e = 2L = (2)(85) = 170 \text{ mm} \quad \frac{L_e}{r_m} = 58.88$$

(a) 6061-T6 $L/r < 66$

$$\sigma_{all} = 139 - 0.868(L/r) = 139 - (0.868)(58.88)$$

$$= 87.9 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (87.9 \times 10^6)(300 \times 10^{-6}) = 26.4 \times 10^3 \text{ N}$$

$$= 26.4 \text{ kN}$$

(b) 2014-T6 $L/r > 55$

$$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(58.88)^2} = 107.3 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (107.3 \times 10^6)(300 \times 10^{-6}) = 32.2 \times 10^3 \text{ N} = 32.2 \text{ kN}$$

Problem 10.63

10.63 A sawn lumber column with a 7.5×5.5 -in. cross section has an 18-ft effective length. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 1220$ psi and that $E = 1.3 \times 10^6$ psi, determine the maximum allowable centric load for the column.

Sawn lumber: $C = 0.8$, $\sigma_c = 1220$ psi, $E = 1.3 \times 10^6$ psi, $K_{ce} = 0.3$

$$A = (7.5)(5.5) = 41.25 \text{ in}^2 \quad d = 5.5 \text{ in.} \quad L = 18 \text{ ft} = 216 \text{ in}$$

$$L/d = 216/5.5 = 39.273$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{(0.3)(1.3 \times 10^6)}{(39.273)^2} = 252.86 \text{ ps} \quad \frac{\sigma_{ce}}{\sigma_c} = 0.20726$$

$$U = \frac{1 + \sigma_{ce}/\sigma_c}{2C} = \frac{1.20726}{(2)(0.8)} = 0.754537 \quad V = \frac{\sigma_c/\sigma_{ce}}{C} = 0.259075$$

$$C_p = U - \sqrt{U^2 - V} = 0.197535$$

$$\sigma_{ul} = \sigma_c C_p = (1220)(0.197535) = 241.0 \text{ psi}$$

$$P_{ul} = \sigma_{ul} A = (241.0)(41.25) = 9.94 \times 10^3 \text{ lb} = 9.94 \text{ kips} \quad \blacktriangleleft$$

Problem 10.64

10.64 A column having a 3.5-m effective length is made of sawn lumber with a 114 \times 140-mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 7.6$ MPa and $E = 10$ GPa, determine the maximum allowable centric load for the column.

Sawn lumber: $C = 0.8$, $\sigma_c = 7.6$ MPa, $K_{ce} = 0.3$, $E = 10000$ MPa

$$A = (114)(140) = 15960 \text{ mm}^2 = 15960 \times 10^{-6} \text{ m}^2$$

$$d = 114 \text{ mm} = 114 \times 10^{-3} \text{ m}$$

$$L/d = 3.5/114 \times 10^{-3} = 30.70$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{(0.3)(10000)}{(30.70)^2} = 3.1827 \text{ MPa} \quad \frac{\sigma_{ce}}{\sigma_c} = 0.41878$$

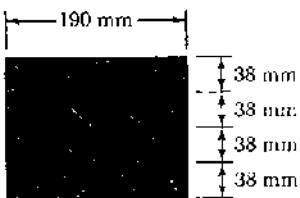
$$U = \frac{1 + \sigma_{ce}/\sigma_c}{2C} = \frac{1.41878}{(2)(0.8)} = 0.88673 \quad V = \frac{\sigma_c/\sigma_{ce}}{C} = 0.523475$$

$$C_p = U - \sqrt{U^2 - V} = 0.37408$$

$$\sigma_{ul} = \sigma_c C_p = (7.6)(0.37408) = 2.84 \text{ MPa}$$

$$P_{ul} = \sigma_{ul} A = (2.84 \times 10^6)(15960 \times 10^{-6}) = 45.4 \times 10^3 \text{ N} = 45.4 \text{ kN} \quad \blacktriangleleft$$

Problem 10.65



10.65 The glue laminated column shown is made from four planks, each of 38×190 -mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 10 \text{ MPa}$ and $E = 12 \text{ GPa}$, determine the maximum allowable centric load if the effective length of the column is (a) 7 m, (b) 3 m.

Glued laminated column. $C = 0.9$, $K_{ce} = 0.418$

$$\sigma_c = 10 \text{ MPa} \quad E = 12000 \text{ MPa}$$

$$4 \times 38 = 152 \text{ mm} = d \quad b = 190 \text{ mm}$$

$$A = (152)(190) = 28.88 \times 10^3 \text{ mm}^2 = 28.88 \times 10^{-3} \text{ m}^2$$

$$(a) L = 7 \text{ m} = 7000 \text{ mm} \quad L/d = 46.053$$

$$G_{ce} = \frac{K_{ce}E}{(L/d)^2} = \frac{(0.418)(12000)}{(46.053)^2} = 8.36510 \text{ MPa} \quad \frac{\sigma_{ce}}{\sigma_c} = 0.836510$$

$$U = \frac{1 + \sigma_{ce}/\sigma_c}{2C} = \frac{1.19709}{(2)(0.9)} = 0.68695 \quad \frac{\sigma_{ce}/\sigma_c}{C} = 0.262788$$

$$C_p = U - \sqrt{U^2 - V} = 0.22966$$

$$\sigma_{all} = \sigma_c C_p = (10)(0.22966) = 2.2966 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (2.2966 \times 10^6)(28.88 \times 10^{-3}) = 66.3 \times 10^3 \text{ N} = 66.3 \text{ kN} \quad \blacktriangleleft$$

$$(b) L = 3 \text{ m} = 3000 \text{ mm} \quad L/d = 19.7368$$

$$G_{ce} = \frac{K_{ce}E}{(L/d)^2} = \frac{(0.418)(12000)}{(19.7368)^2} = 12.8766 \text{ MPa} \quad \frac{\sigma_{ce}}{\sigma_c} = 1.28766$$

$$U = \frac{1 + \sigma_{ce}/\sigma_c}{2C} = \frac{2.28766}{(2)(0.9)} = 1.27092 \quad V = \frac{\sigma_{ce} K_c}{C} = 1.43074$$

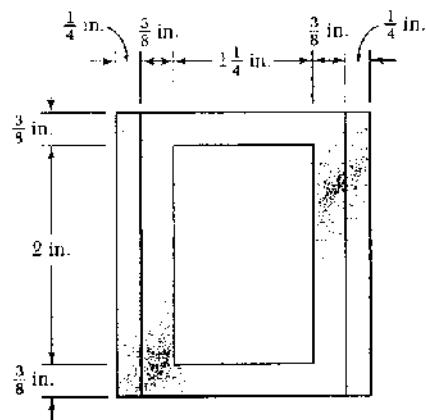
$$C_p = U - \sqrt{U^2 - V} = 0.84138$$

$$\sigma_{all} = \sigma_c C_p = (10)(0.84138) = 8.4138 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (8.4138 \times 10^6)(28.88 \times 10^{-3}) = 243 \times 10^3 \text{ N} = 243 \text{ kN} \quad \blacktriangleleft$$

Problem 10.66

10.66 An aluminum structural tube is reinforced by riveting two plates to it as shown for use as a column of 5.6 ft effective length. Knowing that all material is aluminum alloy 2014-T6, determine the maximum allowable centric load.



$$b_o = \frac{1}{4} + \frac{3}{8} + 1\frac{1}{4} + \frac{3}{8} + \frac{1}{4} = 2\frac{1}{2} \text{ in.} = 2.5 \text{ in.}$$

$$b_i = 1\frac{1}{4} \text{ in.} = 1.25 \text{ in.}$$

$$h_o = \frac{3}{8} + 2 + \frac{3}{8} = 2\frac{3}{4} = 2.75 \text{ in.}$$

$$h_i = 2 \text{ in.}$$

$$\begin{aligned} A &= b_o h_o - b_i h_i = (2.5)(2.75) - (1.25)(2) \\ &= 4.375 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} I_x &= \frac{1}{12} [b_o h_o^3 - b_i h_i^3] = \frac{1}{12} [(2.5)(2.75)^3 - (1.25)(2)^3] \\ &= 3.49935 \text{ in}^4 \end{aligned}$$

$$I_y = \frac{1}{12} [h_o b_o^3 - h_i b_i^3] = \frac{1}{12} [(2.75)(2.5)^3 - (2)(1.25)^3] = 3.25521 \text{ in}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{3.25521}{4.375}} = 0.86258 \text{ in.} \quad L_e = 5.6 \text{ ft} = 67.2 \text{ in.}$$

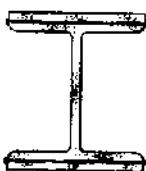
$$\frac{L_e}{r} = \frac{67.2}{0.86258} = 77.906 \geq 55$$

$$S_{all} = \frac{54000}{(L_e/r)^2} = 8.8972 \text{ ksi}$$

$$P_{all} = S_{all} A = (8.8972)(4.375) = 38.9 \text{ kips}$$

Problem 10.67

10.67 and 10.68 A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a W250 × 80 rolled-steel shape as shown. Knowing that $\sigma_y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$ and using allowable stress design, determine the allowable centric load for the compression member.



$$\text{For W } 250 \times 80 \quad A = 10200 \text{ mm}^2, \quad d = 256 \text{ mm}, \quad b_p = 255 \text{ mm} \\ I_x = 126 \times 10^6 \text{ mm}^4, \quad I_y = 43.1 \times 10^6 \text{ mm}^4$$

$$\text{For one plate: } A = (255)(10) = 2550 \text{ mm}^2 \\ I_x = \frac{1}{12}(255)(10)^3 + (2550)\left(\frac{256}{2} + \frac{10}{2}\right)^2 = 45.128 \times 10^6 \text{ mm}^4 \\ I_y = \frac{1}{12}(10)(255)^3 = 13.818 \times 10^6 \text{ mm}^4$$

$$\text{For column: } A = 10200 + (2)(2550) = 15300 \text{ mm}^2 = 15.3 \times 10^{-3} \text{ m}^2 \\ I_x = 126 \times 10^6 + (2)(45.128 \times 10^6) = 216.256 \times 10^6 \text{ mm}^4$$

$$I_y = 43.1 \times 10^6 + (2)(13.818 \times 10^6) = 70.736 \times 10^6 \text{ mm}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{70.736 \times 10^6}{15300}} = 67.995 \text{ mm} = 67.995 \times 10^{-3} \text{ m}$$

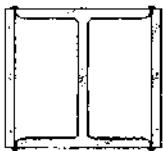
$$\frac{L_e}{r} = \frac{9}{67.995 \times 10^{-3}} = 132.363$$

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{345 \times 10^6}} = 106.972 < \frac{L_e}{r}$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(132.363)^2} = 58.681 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (58.681 \times 10^6)(15.3 \times 10^{-3}) = 898 \times 10^3 \text{ N} = 898 \text{ kN}$$

Problem 10.68



10.67 and 10.68 A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a W250 × 80 rolled-steel shape as shown. Knowing that $\sigma_y = 345 \text{ MPa}$, and $E = 200 \text{ GPa}$ and using allowable stress design, determine the allowable centric load for the compression member.

$$\text{For } W250 \times 80 \quad A = 10200 \text{ mm}^2, d = 256 \text{ mm}, b_f = 255 \text{ mm} \\ I_x = 126 \times 10^6 \text{ mm}^4, \quad I_y = 43.1 \times 10^6 \text{ mm}^4$$

$$\text{For one plate: } A = (256)(10) = 2560 \text{ mm}^4 \\ I_x = \frac{1}{12}(10)(256)^3 = 13.981 \times 10^6 \text{ mm}^4 \\ I_y = \frac{1}{12}(256)(10)^3 + (2560)\left(\frac{255}{2} + \frac{10}{2}\right)^2 = 44.965 \times 10^6 \text{ mm}^4$$

$$\text{For column: } A = 10200 + (2)(2560) = 15.32 \times 10^3 \text{ mm}^2 = 15.32 \times 10^{-3} \text{ m}^2$$

$$I_x = 126 \times 10^6 + (2)(13.981 \times 10^6) = 153.962 \times 10^6 \text{ mm}^4$$

$$I_y = 43.1 \times 10^6 + (2)(44.965 \times 10^6) = 133.03 \times 10^6 \text{ mm}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{133.03 \times 10^6}{15.32 \times 10^3}} = 93.185 \text{ mm} = 93.185 \times 10^{-3} \text{ m}$$

$$\frac{l_e}{r} = \frac{9}{93.185 \times 10^{-3}} = 96.582$$

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(200 \times 10^9)}{345 \times 10^6}} = 106.972 > \frac{l_e}{r}$$

$$\frac{l_e/r}{C_c} = \frac{96.582}{106.972} = 0.90287$$

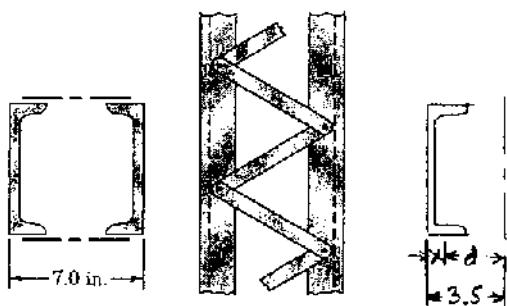
$$\text{F.S.} = \frac{E}{3} + \frac{3}{8}(0.90287) - \frac{1}{8}(90287)^3 = 1.91324$$

$$\sigma_{all} = 345 \left[1 - \frac{1}{2}(0.90287)^2 \right] = 204.382 \text{ MPa} = 204.382 \times 10^6 \text{ Pa}$$

$$P_{all} = \frac{\sigma_{all} A}{\text{F.S.}} = \frac{(204.382 \times 10^6)(15.32 \times 10^{-3})}{1.91324} = 1.637 \times 10^6 \text{ N} = 1637 \text{ kN} \blacksquare$$

Problem 10.69

10.69 A column of 21-ft effective length is obtained by connecting two C10 × 20 steel channels with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_y = 36 \text{ ksi}$ and $E = 29 \times 10^6 \text{ psi}$.



$$\begin{aligned} \text{C10} \times 20 \quad A &= 5.88 \text{ in}^2 & x &= 0.606 \text{ in} \\ I_x &= 78.9 \text{ in}^4 & I_y &= 2.81 \text{ in}^4 \\ d &= 3.5 - x = 2.894 \text{ in} \\ \text{For the column: } A &= (2)(5.88) = 11.76 \text{ in}^2 \\ I_x &= (2)(78.9) = 157.8 \text{ in}^4 \\ I_y &= 2[2.81 + (5.88)(2.894)^2] = 104.11 \text{ in}^4 \end{aligned}$$

$$r = \sqrt{\frac{I_{\text{min}}}{A}} = \sqrt{\frac{104.11}{11.76}} = 2.975 \text{ in.} \quad L_e = 21 \text{ ft} = 252 \text{ in.}$$

$$\frac{L_e}{r} = 84.69 \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

$$\frac{L_e}{r} < C_c \quad \frac{L_e/r}{C_c} = 0.67165$$

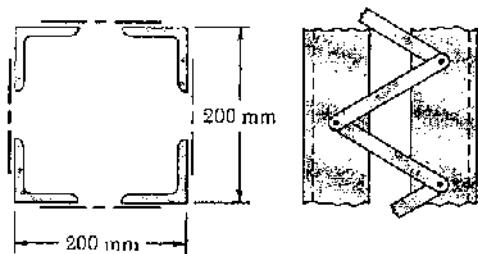
$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.67165) - \frac{1}{8}(0.67165)^3 = 1.8807$$

$$\sigma_{\text{all}} = \frac{\sigma_y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e}{r} \right)^2 \right] = \frac{36}{1.8807} \left[1 - \frac{1}{2} (0.67165)^2 \right] = 14.82 \text{ ksi}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (14.82)(11.76) = 174.3 \text{ kips}$$

Problem 10.70

10.70 A column of 6.4-m effective length is obtained by connecting four $89 \times 89 \times 9.5$ -mm steel angles with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$.



$$\text{Steel: } C_c = \frac{2\pi^2 E}{\sigma_y} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{345 \times 10^6}} = 106.97$$

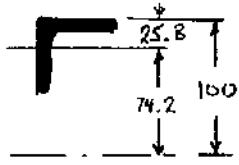
$89 \times 89 \times 9.5 \text{ mm angle}$

$$A_L = 1600 \text{ mm}^2$$

$$x = 25.8 \text{ mm}$$

$$I_x = 1.19 \times 10^6 \text{ mm}^4$$

$$d = 100 - x = 74.2 \text{ mm}$$



$$I = 4(Ad^2 + I_x) = 4[(1600)(74.2)^2 + 1.19 \times 10^6] \\ = 39.996 \times 10^6 \text{ mm}^4$$

$$A = 4A_L = 6400 \text{ mm}^2 = 6400 \times 10^{-6} \text{ m}^2$$

$$r = \sqrt{\frac{I}{A}} = 79.053 \text{ mm} = 79.053 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{6.4}{79.053 \times 10^{-3}} = 80.958 < C_c \quad \frac{L_e/r}{C_c} = 0.75683$$

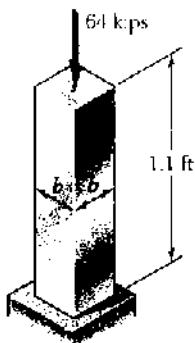
$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.75683) - \frac{1}{8}(0.75683)^3 = 1.8963$$

$$\sigma_{all} = \frac{\sigma_y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e}{r} \right)^2 \right] = \frac{345 \times 10^6}{1.8963} \left[1 - \frac{1}{2} (0.75683)^2 \right] = 129.83 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (129.83 \times 10^6)(6400 \times 10^{-6}) = 831 \times 10^3 \text{ N} = 831 \text{ kN} \blacksquare$$

Problem 10.71

10.71 A 64-kip centric load is applied to the column shown, which is free at its top *A* and fixed at its base *B*. Using aluminum alloy 2014-T6, select the smallest square cross section that can be used.



$$L_e = 2L = (2)(1.1) = 2.2 \text{ ft} = 26.4 \text{ in.}$$

$$A = b^2 \quad I = \frac{1}{12} b^4 \quad r = \sqrt{\frac{I}{A}} = \frac{b}{\sqrt{12}}$$

$$\frac{L_e}{r} = \frac{26.4\sqrt{12}}{b} = \frac{91.452}{b}$$

2014-T6 aluminum alloy.

$$\text{Assume } \frac{L_e}{r} < 55.$$

$$\sigma_{all} = 30.7 - 0.23(L_e/r) = 30.7 - \frac{21.034}{b} \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (30.7 - \frac{21.034}{b})b^2 = 64$$

$$30.7 b^2 - 21.034 b - 64 = 0$$

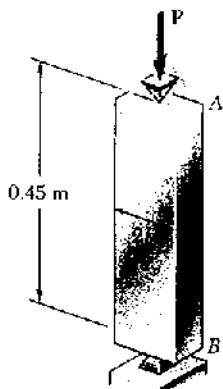
$$b = \frac{21.034 + \sqrt{(21.034)^2 - (4)(30.7)(-64)}}{(2)(30.7)} = 1.8265 \text{ in.}$$

$$\frac{L_e}{r} = \frac{91.452}{1.8265} = 50.069 < 55 \text{ as assumed.}$$

$$b = 1.827 \text{ in. } \blacktriangleleft$$

Problem 10.72

10.72 A 72-kN centric load must be supported by an aluminum column as shown. Using the aluminum alloy 6061-T6, determine the minimum dimension b that can be used.



$$L_e = L = 0.45 \text{ m}, \quad A = 2b^2 \quad I_{min} = \frac{1}{12}(2b)(b)^3 = \frac{1}{6}b^4$$

$$r = \sqrt{\frac{I_{min}}{A}} = \frac{b}{\sqrt{12}} \quad \frac{L_e}{r} = \frac{0.45\sqrt{12}}{b} = \frac{1.55885}{b}$$

6061-T6 aluminum alloy. Assume $\frac{L_e}{r} < 66$.

$$\begin{aligned} \sigma_{all} &= 139 - 0.868(L_e/r) = 139 - \frac{(0.868)(1.55885)}{b} \\ &= 139 - \frac{1.35308}{b} \text{ MPa} \end{aligned}$$

$$P_{all} = \sigma_{all} A = \left(139 - \frac{1.35308}{b}\right)(10^6)(2b^2)$$

$$72 \times 10^3 = 278 \times 10^6 b^2 \sim 2.70616 \times 10^6 b$$

$$278 b^2 - 2.70616 b - 0.072 = 0$$

$$b = \frac{2.70616 + \sqrt{(2.70616)^2 - (4)(278)(-0.072)}}{2(278)} = 21.680 \times 10^{-3} \text{ m}$$

$$r = 6.2586 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{0.45}{6.2586 \times 10^{-3}} = 71.9 \quad \text{Assumption is not verified.}$$

$$\text{Assume } \frac{L_e}{r} > 66. \quad \sigma_{all} = \frac{351 \times 10^3}{(L_e/r)^2} = \frac{351 \times 10^3 b^2}{(1.55885)^2} = 144.444 \times 10^3 b^2 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (144.444 \times 10^3 b^2)(10^6)(2b^2) = 288.89 \times 10^9 b^4 \text{ N}$$

$$72 \times 10^3 = 288.89 \times 10^9 b^4 \quad b^4 = 249.23 \times 10^{-9} \text{ m}^4$$

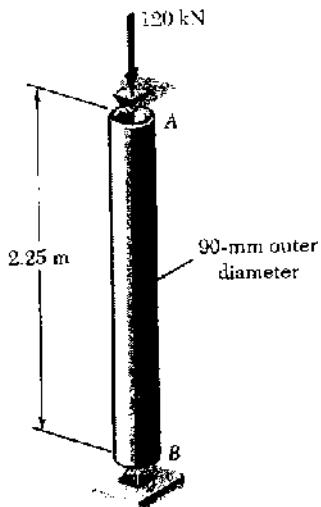
$$b = 22.3 \times 10^{-3} \text{ m} \quad r = 6.45 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = 69.77 > 66$$

Assumption is verified.

$$b = 22.3 \times 10^{-3} \text{ m} = 22.3 \text{ mm} \quad \blacksquare$$

Problem 10.73

10.73 An aluminum tube of 90-mm outer diameter is to carry a centric load of 120 kN. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and with wall thicknesses in increments of 3 mm from 6 mm to 15 mm, determine the lightest tube that can be used.



$$L = 2250 \text{ mm}, P = 120 \times 10^3 \text{ N} \quad r_o = 45 \text{ mm}$$

$$r_i = r_o - t \quad A = \pi(r_o^2 - r_i^2) \quad I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$r = \sqrt{I/A}$$

For 2014-T6 aluminum alloy

$$\sigma_{all} = 212 - 1.585(L/r) \text{ MPa if } L/r < 55$$

$$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} \text{ MPa if } L/r > 55$$

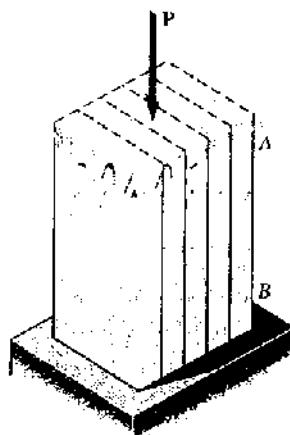
$$P_{all} = \sigma_{all} A$$

Calculate P_{all} for each thickness.

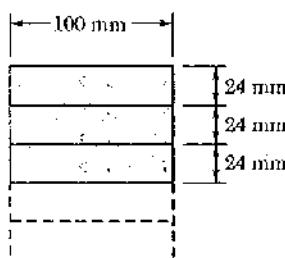
t mm	r_i mm	A mm^2	I 10^6 mm^4	r mm	L/r	σ_{all} MPa	P_{all} kN
6	39	1583	1.404	29.78	75.56	65.16	103.1
9	36	2290	1.901	28.82	78.08	61.01	139.7
12	33	2941	2.289	27.90	80.65	57.20	168.2
15	30	3534	2.584	27.04	83.20	53.74	189.9

Since P_{all} must be greater than 120 kN, use $t = 9 \text{ mm}$

Problem 10.74



10.74 A glue laminated column of 3-m effective length is to be made from boards of 24×100 -mm cross section. Knowing that for the grade of wood used, $E = 11 \text{ GPa}$ and the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 9 \text{ MPa}$, determine the number of boards that must be used to support the centric load shown when (a) $P = 34 \text{ kN}$, (b) $P = 17 \text{ kN}$.



$$\text{One board: } A_1 = (100)(24) = 2400 \text{ mm}^2 = 2.4 \times 10^{-3} \text{ m}^2$$

$$n \text{ boards: } A = (2.4 \times 10^{-3})n$$

$$1 \text{ to } 4 \text{ boards: } d = 24n \text{ mm} = 24 \times 10^{-3} n \text{ m}$$

$$\frac{L}{d} = \frac{3}{24 \times 10^{-3}} = \frac{3}{24 \times 10^{-3}} = \frac{125}{n}$$

$$\text{Since } \frac{L}{d} < 50, \quad n \geq 3$$

$$5 \text{ or more boards, } d = 100 \text{ mm} = 0.1 \text{ m}$$

$$\frac{L}{d} = \frac{3}{0.1} = 30$$

Since $n \geq 3$, the column is laminated.

$$c = 0.9 \quad K_{ce} = 0.418$$

For 3 or 4 boards,

$$\begin{aligned} \sigma_{ce} &= \frac{K_{ce} E}{(L/d)^2} = \frac{(0.418)(11 \times 10^9)}{(125)^2} n^2 \\ &= 294.272 \times 10^3 n^2 \text{ Pa} \end{aligned}$$

$$\sigma_{ce}/\sigma_c = \frac{294.272 \times 10^3 n^2}{9 \times 10^6} = 32.697 \times 10^{-3} n^2$$

$$\text{For 5 or more boards, } \sigma_{ce} = \frac{(0.418)(11 \times 10^9)}{(30)^2} = 5.1089 \times 10^6 \text{ Pa}$$

$$\sigma_{ce}/\sigma_c = \frac{5.1089 \times 10^6}{9 \times 10^6} = 0.56765$$

$$\text{Let } u = \frac{1 + \sigma_{ce}/\sigma_c}{2c} \text{ and } v = \frac{\sigma_{ce}/\sigma_c}{c}.$$

$$\text{Then, } C_p = u - \sqrt{u^2 - v} \quad \sigma_{ce} = \sigma_c C_p = 9 \times 10^6 C_p$$

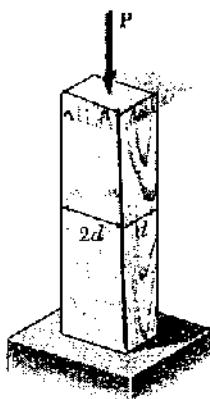
n	u	v	C_p	$\sigma_{ce} (\text{MPa})$	$A (\text{m}^2)$	$P_{\text{all}} (\text{kN})$
3	0.71904	0.36330	0.32697	2.9427	7.2×10^{-3}	21.19
4	0.84619	0.58128	0.47910	4.3119	9.6×10^{-3}	41.39
5 or more	0.87092	0.63072	0.51345	4.6210	$2.4 \times 10^{-3} n$	11.09 n

(a) $P = 34 \text{ kN}$ Use 4 boards

(b) $P = 17 \text{ kN}$ Use 3 boards

Problem 10.75

10.75 An 18-kip centric load is applied to a rectangular sawn lumber column of 22-ft effective length. Using sawn lumber for which the adjusted allowable stress for compression parallel to the grain is $\sigma_{ce} = 1050 \text{ psi}$ and knowing that $E = 1.5 \times 10^6 \text{ psi}$, determine the smallest cross section that may be used. Use $b = 2d$.



$$\text{Sawn lumber: } c = 0.8 \quad K_{ce} = 0.3$$

$$\sigma_c = 1050 \text{ psi} \quad E = 10 \times 10^6 \text{ psi}$$

$$A = 2d^2 \quad L = 22 \text{ ft} = 264 \quad L/d = \frac{264}{d}$$

$$\text{Assumed } C_p = 0.5$$

$$\sigma_{all} = \sigma_c C_p = (1050)(0.5) = 525 \text{ psi}$$

$$P_{all} = \sigma_{all} A = 2\sigma_{all} d^2$$

$$d = \sqrt{\frac{P_{all}}{2\sigma_{all}}} = \sqrt{\frac{18000}{2 \cdot 525}} = \frac{94.868}{\sqrt{1050}} = 4.14 \text{ in}$$

$$L/d = 63.76 \quad \sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{(0.3)(1.5 \times 10^6)}{(L/d)^2} = \frac{450 \times 10^3}{(L/d)^2} = 110.69 \text{ psi}$$

$$\sigma_{ce}/\sigma_c = 0.1054$$

$$\text{Checked: } C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + (\sigma_{ce}/\sigma_c)}{2c}\right)^2 - \frac{\sigma_{ce}/\sigma_c}{c}} = 0.0966$$

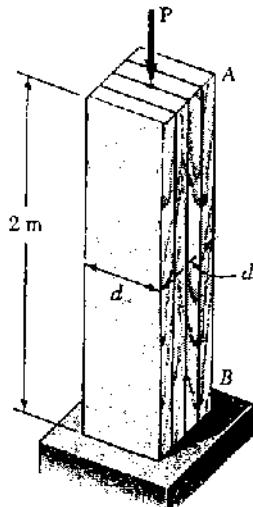
Results of similar trials are summarized in the table below:

Assumed C_p	σ_{all} (psi)	d (in)	L/d	σ_{ce} (psi)	σ_{ce}/σ_c	Checked C_p	ΔC_p
0.5	525	4.14	63.76	110.68	0.1054	0.0966	-0.4034
0.2983	313.2	5.36	49.25	185.52	0.1767	0.1697	-0.3286
0.2340	245.2	6.05	43.62	236.50	0.2252	0.2136	-0.0204
0.2238	235.0	6.19	42.66	247.28	0.2355	0.2227	-0.0011
0.2233	234.4	6.20	42.61	247.87	0.2361	0.2232	-0.0001

$$d = 6.20 \text{ in.}$$

Problem 10.76

10.76 The glued laminated column shown is free at its top *A* and fixed at its base *B*. Using wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 9.2 \text{ MPa}$ and a modulus of elasticity $E = 12 \text{ GPa}$, determine the smallest cross section that can support a centric load of 62 kN.



Glued laminated column: $C = 0.7 \quad K_{ce} = 0.418$

$$L_e = 2L = (2)(2.0) = 4 \text{ m} = 4000 \text{ mm}$$

$$\sigma_c = 9.2 \text{ MPa} \quad E = 12000 \text{ MPa}$$

$$A = d^2 \quad P_{av} = 62000 \text{ N}$$

$$L_e/d = 4000/d \quad \text{with } d \text{ in mm}$$

$$\sigma_{all} = \sigma_c C_p$$

$$\text{Assume } C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{\sigma_{all}}{9.2} = 9.2 C_p \text{ MPa}$$

$$d = \sqrt{\frac{P_{av}}{\sigma_{all}}} = \sqrt{\frac{62000}{9.2}} = \frac{249}{\sqrt{9.2}}$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{(0.418)(12000)}{(L/d)^2} = \frac{5016}{(L/d)^2}$$

$$\text{Checking, } C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2C} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{2C}\right)^2 - \frac{\sigma_{ce}/\sigma_c}{C}}$$

Calculations are carried out in the following table:

C_p (assumed)	σ_{all} (MPa)	d mm	L/d	σ_{ce} (MPa)	σ_{ce}/σ_c	C_p	ΔC_p
0.5	4.6	116.1	34.45	4.225	0.4593	0.4274	-0.0726
0.4	3.68	129.8	30.82	5.282	0.5741	0.5183	+0.1183
0.46	4.232	121.0	33.05	4.593	0.4992	0.4600	+0.0000

$$\sigma_{all} = (0.4600)(9.2) = 4.232 \text{ MPa}$$

$$d = \frac{249}{\sqrt{4.232}} = 121 \text{ mm}$$

Problem 10.77

10.77 A column of 4.5-m effective length must carry a centric load of 900 kN. Knowing that $\sigma_y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$, use allowable stress design to select the wide-flange shape of 250-mm nominal depth that should be used.

$$\text{Steel: } E = 200000 \text{ MPa} \quad \sigma_y = 345 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = 106.97$$

$$P < \frac{\sigma_y}{(F.S.)A} \quad A > \frac{(F.S.)P}{\sigma_y} = \frac{(5/3)(900 \times 10^3)}{345 \times 10^6} = 4348 \times 10^{-6} \text{ m}^2 = 4348 \text{ mm}^2$$

$$P < \frac{\pi^2 EI}{(F.S.)L_e^2} \quad I > \frac{(F.S.)P L_e^2}{\pi^2 E} = \frac{(1.907)(900 \times 10^3)(4.5)^2}{\pi^2 200 \times 10^9} = 17.73 \times 10^{-6} \text{ m}^4 = 17.73 \times 10^{-6} \text{ mm}^4$$

Try W250x58. $A = 7420 \text{ mm}^2 \quad r_y = 50.3 \text{ mm}$

$$\frac{L_e}{r_y} = \frac{4.5}{50.3 \times 10^{-3}} = 89.46 < 106.97 \quad \frac{L_e/r_y}{C_c} = 0.8363$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.8363) - \frac{1}{8}(0.8363)^2 = 1.907$$

$$\sigma_{ult} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r_y}{C_c} \right)^2 \right] = \frac{345}{1.907} \left[1 - \frac{1}{2} (0.8363)^2 \right] = 117.6 \text{ MPa}$$

$$P_{ult} = A \sigma_{ult} = (7420 \times 10^{-6})(117.6 \times 10^6) = 873 \times 10^3 \text{ N} = 873 \text{ kN} < 900 \text{ kN}$$

(not acceptable)

Try W250x67. $A = 8580 \text{ mm}^2 \quad r_y = 51.0 \text{ mm}$

$$\frac{L_e}{r_y} = \frac{4.5}{51.0 \times 10^{-3}} = 88.24 < 106.97 \quad \frac{L_e/r_y}{C_c} = 0.8249$$

$$F.S. = 1.906 \quad \sigma_{ult} = \frac{345}{1.906} \left[1 - \frac{1}{2} (0.8249)^2 \right] = 119.4 \text{ MPa}$$

$$P_{ult} = A \sigma_{ult} = (8580 \times 10^{-6})(119.4 \times 10^6) = 1025 \times 10^3 \text{ N} = 1025 \text{ kN} > 900 \text{ kN}$$

(acceptable)

Use W250x67. →

Problem 10.78

10.78 A column of 22.5-ft effective length must carry a centric load of 288 kips. Using allowable stress design, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_y = 50$ ksi and $E = 29 \times 10^6$ psi.

$$P < \frac{\sigma_y A}{F.S.} \quad A > \frac{(F.S.)P}{\sigma_y} = \frac{(5/3)(288)}{50} = 9.6 \text{ in}^2$$

$$L_e = 22.5 \text{ ft} = 270 \text{ in} \quad E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2} \quad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(288)(270)^2}{\pi^2 (29000)} = 140.8 \text{ in}^4$$

$$\text{Try W } 14 \times 82. \quad A = 24.1 \text{ in}^2, \quad I_{min} = 148 \text{ in}^4, \quad r = 2.48 \text{ in}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{50}} = 107.00$$

$$\frac{L_e}{r} = \frac{270}{2.48} = 108.87 > 107.00$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L/r)^2} = \frac{\pi^2 (29000)}{(1.92)(108.87)^2} = 12.58 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (12.58)(24.1) = 303 \text{ kips} > 288 \text{ kips}$$

Use W 14 × 82.

Problem 10.79

10.79 A column of 17-ft effective length must carry a centric load of 235 kips. Using allowable stress design, select the wide-flange shape of 10-in. nominal depth that should be used. Use $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi.

$$P < \frac{\sigma_y A}{F.S.} \quad A > \frac{(F.S.)P}{\sigma_y} = \frac{(5/3)(235)}{36} = 10.88 \text{ in}^2$$

$$L_e = 17 \text{ ft} = 204 \text{ in} \quad E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2} \quad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(235)(204)^2}{\pi^2 (29000)} = 65.6 \text{ in}^4$$

$$\text{Try W } 10 \times 54. \quad A = 15.8 \text{ in}^2 \quad I_y = 103 \text{ in}^4 \quad r = 2.56 \text{ in}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

$$\frac{L_e}{r} = \frac{204}{2.56} = 79.69 < C_c \quad \frac{L_e/r}{C_c} = \frac{79.69}{126.10} = 0.63194$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.63194) - \frac{1}{8}(0.63194)^3 = 1.8721$$

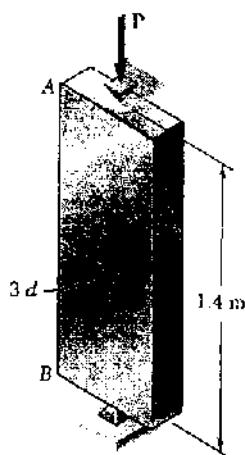
$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e}{C_c} \right)^2 \right] = \frac{36}{1.8721} \left[1 - \frac{1}{2} (0.63194)^2 \right] = 15.39 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (15.39)(15.8) = 243 \text{ kips} > 235 \text{ kips}$$

Use W 10 × 54.

Problem 10.80

10.80 A centric load P must be supported by the steel bar AB . Using allowable stress design, determine the smallest dimension d of the cross section that can be used when (a) $P = 108 \text{ kN}$, (b) $P = 166 \text{ kN}$. Use $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$.



$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250}} = 125.66$$

$$L_e = L = 1.4 \text{ m}$$

$$A = (3d)(d) = 3d^2$$

$$I = \frac{1}{2}(3d)(d)^3 = \frac{1}{4}d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} = 0.288675 d$$

$$(a) P = 108 \times 10^3 \text{ N.} \quad \text{Assume } \frac{L_e}{r} > C_c.$$

$$P_{ult} = \frac{\pi^2 EI}{1.92 L_e^2} \quad I = \frac{(1.92) P_{ult} L_e^2}{\pi^2 E} = \frac{1}{4}d^4$$

$$d^4 = \frac{(4)(1.92) P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(108 \times 10^3)(1.4)^2}{\pi^2 (200 \times 10^9)} = 823.59 \times 10^{-7} \text{ m}^4$$

$$d = 30.125 \times 10^{-3} \text{ m} \quad r = 8.696 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{8.696 \times 10^{-3}} = 160.99 > 125.66 \quad d = 30.1 \text{ mm}$$

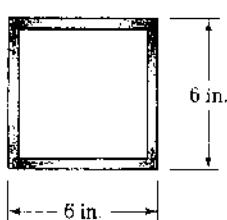
$$(b) P = 166 \times 10^3 \text{ N.} \quad \text{Assume } \frac{L_e}{r} > C_c$$

$$d^4 = \frac{(4)(1.92) P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(166 \times 10^3)(1.4)^2}{\pi^2 (200 \times 10^9)} = 1.26588 \times 10^{-6} \text{ m}^4$$

$$d = 33.543 \times 10^{-3} \text{ m} \quad r = 9.68295 \times 10^{-3}$$

$$\frac{L_e}{r} = \frac{1.4}{9.68295 \times 10^{-3}} = 144.58 > 125.66 \quad d = 33.5 \text{ mm}$$

Problem 10.81



10.81 A square structural tube having the cross section shown is used as a column of 26-ft effective length to carry a centric load of 65 kips. Knowing that the tubes available for use are made with wall thicknesses of $\frac{1}{16}$ in. from $\frac{1}{4}$ in. to $\frac{3}{4}$ in., use allowable stress design to determine the lightest tube that can be used. Use $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi.

$$b_o = 6 \text{ in.} \quad b_i = b_o - 2t \quad A = b_o^2 - b_i^2$$

$$I = \frac{1}{12} (b_o^4 - b_i^4)$$

$$L_e = 26 \text{ ft} = 312 \text{ in.} \quad P = 65 \text{ kips}$$

$$\sigma_y = 36 \text{ ksi} \quad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 29 \times 10^3}{36}} = 126.099$$

$$\text{Try } t = \frac{1}{2} \text{ in.} = 0.5 \text{ in.} \quad b_i = 5.0 \text{ in.} \quad A = 6^2 - 5.0^2 = 11 \text{ in}^2$$

$$I = \frac{1}{12} [(6)^4 - (5.0)^4] = 55.9167 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.2546 \text{ in.}$$

$$\frac{L_e}{r} = \frac{312}{2.2546} = 138.382 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = 7.7846 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (7.7846)(11) = 85.7 \text{ kips.}$$

P_{all} is approximately proportional to I .

$$\frac{t}{0.5} = \frac{65}{85.7} \quad t \approx 0.3795 \text{ in.}$$

$$\text{Try } t = \frac{3}{8} \text{ in.} = 0.375 \text{ in.} \quad b_i = 5.25 \text{ in.} \quad A = 8.4375 \text{ in}^2$$

$$I = \frac{1}{12} [(6)^4 - (5.25)^4] = 44.6924 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.3015 \text{ in.}$$

$$\frac{L_e}{r} = \frac{312}{2.3015} = 135.564 > C_c$$

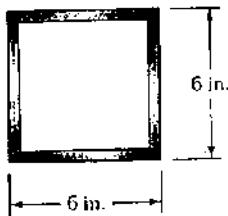
$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = 8.1116 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (8.1116)(8.4375) = 68.4 \text{ kips.} > 65 \text{ kips}$$

Use $t = 3/8$ in.

Problem 10.82

10.82 Solve Prob. 10.81, assuming that the effective length of the column is decreased to 20 ft.



10.81 A square structural tube having the cross section shown is used as a column of 26-ft effective length to carry a centric load of 65 kips. Knowing that the tubes available for use are made with wall thicknesses of $\frac{1}{16}$ in. from $\frac{1}{4}$ in. to $\frac{3}{4}$ in., use allowable stress design to determine the lightest tube that can be used. Use $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi.

$$b_o = 6 \text{ in.} \quad b_i = b_o - 2t \quad A = b_o^2 - b_i^2$$

$$I = \frac{1}{12} (b_o^4 - b_i^4)$$

$$L_e = 20 \text{ ft} = 240 \text{ in.} \quad P = 65 \text{ kips}$$

$$\sigma_y = 36 \text{ ksi} \quad E = 29 \times 10^6 \text{ psi} = 29 \times 10^9 \text{ ksi}$$

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29 \times 10^9)}{36}} = 126.099$$

$$\text{Try } t = \frac{1}{2} \text{ in.} = 0.5 \text{ in.} \quad b_i = 5.0 \text{ in.} \quad A = 6^2 - 5.0^2 = 11 \text{ in.}^2$$

$$I = \frac{1}{12} [(6)^4 - (5.0)^4] = 55.9167 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.2546 \text{ in.}$$

$$\frac{L_e}{r} = \frac{240}{2.2546} = 106.449 < C_c \quad \frac{L_e/r}{C_c} = 0.84417$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.84417) - \frac{1}{8}(0.84417)^3 = 1.908$$

$$\sigma_{all} = \frac{\sigma_y}{\text{F.S.}} [1 - \frac{1}{2} (\frac{L_e/r}{C_c})^2] = \frac{36}{1.908} [1 - \frac{1}{2} (0.84417)^2] = 12.145 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (12.145)(11) = 133.6 \text{ kips.}$$

P_{all} is approximately proportional to t .

$$\frac{t}{0.5} \approx \frac{65}{133.6} \quad t \approx 0.243 \text{ in.}$$

$$\text{Try } t = \frac{1}{4} \text{ in.} = 0.25 \text{ in.} \quad b_i = 5.5 \text{ in.} \quad A = 5.75 \text{ in.}^2$$

$$I = \frac{1}{12} [(6)^4 - (5.5)^4] = 31.745 \text{ in.}^4 \quad r = \sqrt{\frac{I}{A}} = 2.3496 \text{ in.}$$

$$\frac{L_e}{r} = \frac{240}{2.3496} = 102.143 < C_c \quad \frac{L_e/r}{C_c} = 0.81002$$

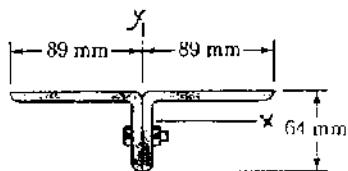
$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.81002) - \frac{1}{8}(0.81002)^3 = 1.904$$

$$\sigma_{all} = \frac{36}{1.904} [1 - \frac{1}{2} (0.81002)^2] = 12.705 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (12.705)(5.75) = 73.1 \text{ kips} > 65 \text{ kips}$$

Use $t = 1/4$ in. ◀

Problem 10.83



10.83 Two 89×64 -mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 180 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$.

$$\text{Steel: } \sigma_y = 250 \times 10^6 \text{ Pa}, \quad E = 200 \times 10^9 \text{ Pa}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.664$$

Use x and y axes as added above.

$$\text{Try } L 89 \times 64 \times 9.5 \quad A = (2)(1360) = 2720 \text{ mm}^2 = 2720 \times 10^{-6} \text{ m}^2$$

$$r_x = 18.6 \text{ mm} = 18.6 \times 10^{-3} \text{ m} \quad (r_y \text{ in Appendix C})$$

$$L_e/r = 2.4/18.6 \times 10^{-3} = 129.032 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(129.032)^2} = 61.749 \times 10^6 \text{ Pa}$$

$$P_{all} = A \sigma_{all} = (2720 \times 10^{-6})(61.749 \times 10^6) = 168.0 \times 10^3 \text{ N} = 168.0 \text{ kN}$$

$P_{all} < P = 180 \text{ kN}$. Do not use.

$$\text{Try } L 89 \times 64 \times 12.7 \quad A = (2)(1780) = 3960 \text{ mm}^2 = 3960 \times 10^{-6} \text{ m}^2$$

$$r_x = 18.1 \text{ mm} = 18.1 \times 10^{-3} \text{ m}$$

$$L_e/r = 2.4/18.1 \times 10^{-3} = 132.597 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(132.597)^2} = 58.474 \times 10^6 \text{ Pa}$$

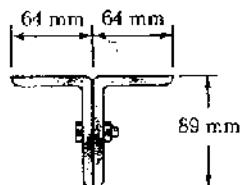
$$P_{all} = A \sigma_{all} = (3960 \times 10^{-6})(58.474 \times 10^6) = 232 \times 10^3 \text{ N} = 232 \text{ kN}$$

$P_{all} > P$

Use $L 89 \times 64 \times 12.7$. ◀

Problem 10.84

10.84 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 325 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$.



$$\text{Steel: } \bar{\sigma}_y = 250 \times 10^6 \text{ Pa} \quad E = 200 \times 10^9 \text{ Pa}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\bar{\sigma}_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.64$$

Try L 89×64×9.5.

$$A = (2)(1360) = 2720 \text{ mm}^2 = 2720 \times 10^{-6} \text{ m}^2$$

$$I_x = (2)(1.07 \times 10^6) = 2.14 \times 10^6 \text{ mm}^4$$

$$I_y = (2)[0.463 \times 10^6 + (1360)(16.9)^2] = 1.70285 \times 10^6 \text{ mm}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{1.70285 \times 10^6}{2720}} = 25.021 \text{ mm} = 25.021 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.021 \times 10^{-3}} = 95.919 < C_c \quad \frac{L_e/r}{C_c} = 0.76330$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.76330) - \frac{1}{8}(0.76330)^3 = 1.8973$$

$$\bar{\sigma}_{all} = \frac{5\bar{\sigma}_y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.8973} \left[1 - \frac{1}{2} (0.76330)^2 \right] = 93.38 \times 10^6 \text{ Pa}$$

$$P_{all} = \bar{\sigma}_{all} A = (93.38 \times 10^6)(2720 \times 10^{-6}) = 254 \times 10^3 \text{ N} = 254 \text{ kN} < 325 \text{ kN}$$

Do not use.

Try L 89×64×12.7. $A = (2)(1780) = 3560 \text{ mm}^2 = 3560 \times 10^{-6} \text{ m}^2$

$$I_x = (2)(1.36 \times 10^6) = 2.72 \times 10^6 \text{ mm}^4$$

$$I_y = (2)[0.581 \times 10^6 + (1780)(18.1)^2] = 2.3283 \times 10^6 \text{ mm}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.3283 \times 10^6}{3560}} = 25.574 \text{ mm} = 25.574 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.574 \times 10^{-3}} = 93.846 < C_c \quad \frac{L_e/r}{C_c} = 0.7468$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.7468) - \frac{1}{8}(0.7468)^3 = 1.89465$$

$$\bar{\sigma}_{all} = \frac{250 \times 10^6}{1.89465} \left[1 - \left(\frac{1}{2} \right) (0.7468)^2 \right] = 95.155 \times 10^6 \text{ Pa}$$

$$P_{all} = \bar{\sigma}_{all} A = (95.155 \times 10^6)(3560 \times 10^{-6}) = 339 \times 10^3 \text{ N} = 339 \text{ kN} > 325 \text{ kN}$$

Use L 89×64×12.7.

Problem 10.85

10.85 A column with a 5.8-m effective length supports a centric load, with ratio of dead to live load equal to 1.35. The dead load factor is $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$, and the resistance factor $\phi = 0.85$. Use load and resistance factor design to determine the allowable centric dead and live loads if the column is made of the following rolled-steel shape: (a) W250 x 67, (b) W 360 x 101. Use $\sigma_y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$.

$$(a) \underline{\text{W250 x 67}} \quad A = 8580 \text{ mm}^2 = 8580 \times 10^{-6} \text{ m}^2, \quad r_y = 51.0 \text{ mm} = 51.0 \times 10^{-3} \text{ m}$$

$$L_e/r_y = \frac{5.8}{51.0 \times 10^{-3}} = 113.725$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_y}{E}} = \frac{113.725}{\pi} \sqrt{\frac{345 \times 10^6}{200 \times 10^9}} = 1.5035 > 1.5$$

$$P_u = A \left(\frac{0.877}{\lambda_c^2} \right) \bar{\sigma}_y = (8580 \times 10^{-6}) \left(\frac{0.877}{1.5035^2} \right) (345 \times 10^6) = 1.1484 \times 10^6 \text{ N}$$

$$\gamma_D P_d + \gamma_L P_L = \phi P_u$$

$$(1.2) P_d + \left(\frac{1.6}{1.35} \right) P_d = (0.85)(1.1484 \times 10^6) \quad P_d = 409.26 \times 10^3 \text{ N}$$

$$P_L = \frac{409.26 \times 10^3}{1.35} = 303.16 \times 10^3 \text{ N}$$

$$P_d = 409 \text{ kN}, \quad P_L = 303 \text{ kN}$$

$$(b) \underline{\text{W 360 x 101}} \quad A = 12900 \text{ mm}^2 = 12900 \times 10^{-6} \text{ m}^2, \quad r_y = 62.6 \text{ mm} = 62.6 \times 10^{-3} \text{ m}$$

$$L_e/r_y = \frac{5.8}{62.6 \times 10^{-3}} = 92.652$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_y}{E}} = \frac{92.652}{\pi} \sqrt{\frac{345 \times 10^6}{200 \times 10^9}} = 1.22489 < 1.5$$

$$P_u = A (0.658 \lambda^*) \bar{\sigma}_y = (12900 \times 10^{-6}) (0.658)^{1.22489} (345 \times 10^6)$$

$$= 2.3751 \times 10^6 \text{ N}$$

$$\gamma_D P_d + \gamma_L P_L = \phi P_u$$

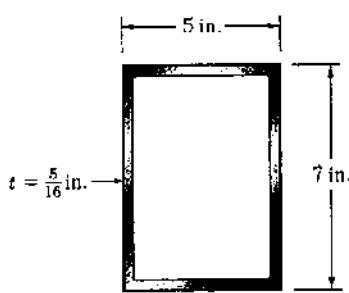
$$(1.2) P_d + \left(\frac{1.6}{1.35} \right) P_d = (0.85)(2.3751 \times 10^6) \quad P_d = 846.41 \times 10^3 \text{ N}$$

$$P_L = \frac{846.41 \times 10^3}{1.35} = 626.97 \times 10^3 \text{ N}$$

$$P_d = 846 \text{ kN}, \quad P_L = 627 \text{ kN}$$

Problem 10.86

10.86 A rectangular tube having the cross section shown is used as a column of 14.5-ft effective length. Knowing that $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi., use load and resistance factor design to determine the largest centric live load that can be applied if the centric dead load is 54 kips. Use a dead load factor $\gamma_D = 1.2$, a live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.



$$L = 14.5 \text{ ft} = 174 \text{ in.}$$

$$b_o = 7 \text{ in.} \quad b_i = 7 - (2)(\frac{5}{16}) = 6\frac{3}{8} \text{ in.}$$

$$h_o = 5 \text{ in.} \quad h_i = 5 - (2)(\frac{5}{16}) = 4\frac{3}{8} \text{ in.}$$

$$A = (7)(5) - (6\frac{3}{8})(4\frac{3}{8}) = 7.109375 \text{ in}^2$$

$$I = \frac{1}{12}[(7)(5)^3 - (6\frac{3}{8})(4\frac{3}{8})^3] = 28.42967 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.99972 \text{ in.}$$

$$\frac{L_e}{r} = \frac{174}{1.99972} = 87.012$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{E}{G_y}} = \frac{87.012}{\pi} \sqrt{\frac{36}{29000}} = 0.97585 < 1.5$$

$$\lambda_c^2 = 0.952276$$

$$P_v = A (0.658)^{\lambda_c^2} G_y = (7.109375)(0.658)^{0.952276}(36) = 171.805 \text{ kips.}$$

$$\gamma_D P_d + \gamma_L P_L = \phi P_v$$

$$(1.2)(54) + (1.6)P_L = (0.85)(171.805)$$

$$P_L = 50.8 \text{ kips}$$

Problem 10.87

10.87 A column of 5.5-m effective length must carry a centric dead load of 310 kN and a centric live load of 375 kN. Knowing that $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$, use load and resistance factor design to select the wide-flange shape of 310-mm nominal depth that should be used. The dead load factor $\gamma_d = 1.2$, the live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.

$$\gamma_d P_d + \gamma_L P_L = \phi P_u$$

$$\text{Required. } P_u = \frac{\gamma_d P_d + \gamma_L P_L}{\phi} = \frac{(1.2)(310) + (1.6)(375)}{0.85} = 1143 \text{ kN}$$

Preliminary calculations.

$$P_u < \sigma_y A \quad \therefore A > \frac{P_u}{\sigma_y} = \frac{1143 \times 10^3}{250 \times 10^6} = 4.572 \times 10^{-4} \text{ m}^2 = 4572 \text{ mm}^2$$

$$P_u < \frac{\pi^2 EI}{L^2} \quad \therefore I > \frac{P_u L^2}{\pi^2 E} = \frac{(1143 \times 10^3)(5.5)^2}{\pi^2 (200 \times 10^9)} = 17.52 \times 10^{-6} \text{ m}^4 = 17.52 \times 10^6 \text{ mm}^4$$

$$\text{Try W 310x60.} \quad A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2 \\ I_y = 18.3 \times 10^4 \text{ mm}^4, \quad r_y = 49.1 \text{ mm} = 49.1 \times 10^{-3} \text{ m}$$

$$\lambda_c = \frac{L_e}{\pi r} \sqrt{\frac{\sigma_y}{E}} = \frac{5.5}{\pi(49.1 \times 10^{-3})} \sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 1.2606 < 1.5$$

$$\lambda_c^2 = 1.5892$$

$$P_u = A (0.658)^{\lambda_c^2} \sigma_y = (7590 \times 10^{-6})(0.658)^{1.5892} (250 \times 10^6) \\ = 975 \times 10^3 \text{ N} = 975 \text{ kN} < 1143 \text{ kN} \\ \text{Too light. Do not use.}$$

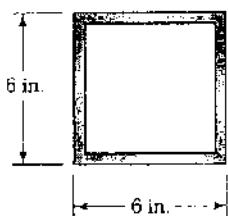
$$\text{Try W 310x74.} \quad A = 9480 \text{ mm}^2 = 9480 \times 10^{-6} \text{ m}^2 \\ r_y = 49.7 \text{ mm} = 49.7 \times 10^{-3} \text{ m}$$

$$\lambda_c = \frac{5.5}{\pi(49.7 \times 10^{-3})} \sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 1.2454 \quad \lambda_c^2 = 1.5510$$

$$P_u = (9480 \times 10^{-6})(0.658)^{1.5510} (250 \times 10^6) = 1238 \times 10^3 \text{ N} \\ = 1238 \text{ kN} > 1143 \text{ kN}$$

Use W 310x74.

Problem 10.88



10.88 The structural tube having the cross section shown is used as a column of 15-ft effective length to carry a centric dead load of 51 kips and a centric live load of 58 kips. Knowing that the tubes available for use are made with wall thicknesses in increments of $\frac{1}{16}$ in. from $\frac{3}{16}$ in. to $\frac{1}{8}$ in., use load and resistance factor design to determine the lightest tube that can be used. Use $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.

$$L_e = 15 \text{ ft} = 180 \text{ in}$$

$$\gamma_D P_0 + \gamma_L P_L = \phi P_u$$

$$\text{Required } P_u = \frac{\gamma_D P_0 + \gamma_L P_L}{\phi} = \frac{(1.2)(51) + (1.6)(58)}{0.85} = 181.2 \text{ kips}$$

$$\text{Try } t = \frac{1}{4} \text{ in} = 0.25 \text{ in. } b_o = 6.0 \text{ in. } b_i = b_o - 2t = 5.5 \text{ in.}$$

$$A = b_o^2 - b_i^2 = (6)^2 - (5.5)^2 = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = \frac{1}{12}[(6)^4 - (5.5)^4] = 31.74 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.74}{5.75}} = 2.3496 \text{ in} \quad \frac{L_e}{r} = \frac{180}{2.3496} = 76.61$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_y}{E}} = \frac{76.61}{\pi} \sqrt{\frac{36}{29000}} = 0.85916 < 1.5 \quad \lambda_c^2 = 0.73815$$

$$P_u = A (0.658)^{\lambda_c} \sigma_y = (5.75)(0.658)^{0.73815} (36) = 152.0 \text{ kips} < 181.2 \text{ kips}$$

Thickness is too small.

Since P_u is approximately proportional to thickness, the required thickness is approximately

$$\frac{t_{\text{req}}}{0.25} \approx \frac{P_u(\text{req})}{152} = \frac{181.18}{152} \quad t_{\text{req}} \approx 0.296 \text{ in.}$$

$$\text{Try } t = \frac{5}{16} \text{ in.} = 0.3125 \text{ in. } b_i = 5.375 \text{ in.}$$

$$A = 7.1094 \text{ in}^2, I = 38.44 \text{ in}^4, r = 2.3254 \text{ in} \quad \frac{L_e}{r} = 77.41$$

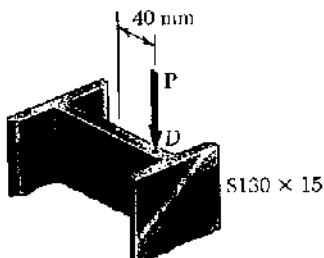
$$\lambda_c = \frac{77.41}{\pi} \sqrt{\frac{36}{29000}} = 0.86811 < 1.5 \quad \lambda_c^2 = 0.75361$$

$$P_u = (7.1094)(0.658)^{0.75361} (36) = 186.7 \text{ kips} > 181.2 \text{ kips}$$

$$\text{Use } t = \frac{5}{16} \text{ in.}$$

Problem 10.89

10.89 A steel compression member of 2.75-m effective length supports an eccentric load as shown. Using the allowable-stress method and assuming $e = 40$ mm, determine the load P . Use $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$.



$$\text{Steel: } E = 200 \times 10^9 \text{ Pa}, \sigma_y = 250 \times 10^6 \text{ Pa}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.64$$

$$\text{S } 130 \times 15: A = 1890 \text{ mm}^2 = 1890 \times 10^{-6} \text{ m}^2$$

$$r_{min} = r_y = 16.3 \text{ mm} = 16.3 \times 10^{-3} \text{ m}$$

$$S_x = 79.8 \times 10^5 \text{ mm}^3 = 79.8 \times 10^{-6} \text{ m}^3$$

$$e_x = 0$$

$$e_y = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r_{min}} = \frac{2.75}{16.3 \times 10^{-3}} = 168.712 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(168.713)^2} = 36.119 \times 10^6 \text{ Pa}$$

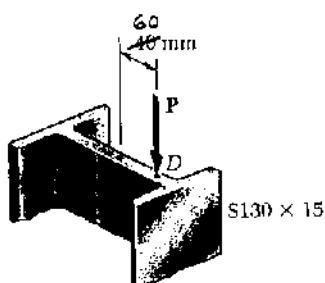
$$\frac{P}{A} + \left| \frac{M_x}{S_x} \right| + \left| \frac{M_y}{S_y} \right| = \frac{P}{A} + \frac{Pe_y}{S_x} + \frac{Pe_x}{S_y} = P \left(\frac{1}{A} + \frac{e_y}{S_x} + \frac{e_x}{S_y} \right) = KP$$

$$KP \leq \sigma_{all} \quad P \leq \frac{\sigma_{all}}{K}$$

$$K = \frac{1}{1890 \times 10^{-6}} + \frac{40 \times 10^{-3}}{79.8 \times 10^{-6}} + 0 = 1.03035 \times 10^3 \text{ m}^{-2}$$

$$P \leq \frac{36.119 \times 10^6}{1.03035 \times 10^3} = 35.1 \times 10^3 \text{ N} = 35.1 \text{ kN}$$

Problem 10.90



10.90 Solve Prob. 10.89, using $e = 60 \text{ mm}$.

10.89 A steel compression member of 2.75-m effective length supports an eccentric load as shown. Using the allowable-stress method and assuming $e = 40 \text{ mm}$, determine the load P . Use $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$.

$$\text{Steel: } E = 200 \times 10^9 \text{ Pa}, \quad G_y = 250 \times 10^6 \text{ Pa}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{G_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.66 \text{ m}$$

$$S130 \times 15 \quad A = 1890 \text{ mm}^2 = 1890 \times 10^{-6} \text{ m}^2$$

$$r_{min} = r_y = 16.3 \text{ mm} = 16.3 \times 10^{-3} \text{ m}$$

$$S_x = 79.8 \times 10^3 \text{ mm}^3 = 79.8 \times 10^{-6} \text{ m}^3$$

$$e_x = 0$$

$$e_y = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r_{min}} = \frac{2.75}{16.3 \times 10^{-3}} = 168.712 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(168.712)^2} = 36.119 \times 10^6 \text{ Pa}$$

$$\frac{P}{A} + \left| \frac{M_x}{S_x} \right| + \left| \frac{M_y}{S_y} \right| = \frac{P}{A} + \frac{Pe_y}{S_x} + \frac{Pe_x}{S_y} = P \left(\frac{1}{A} + \frac{e_y}{S_x} + \frac{e_x}{S_y} \right) = KP$$

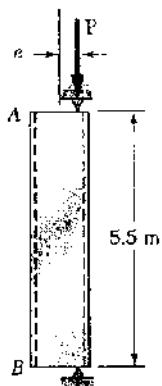
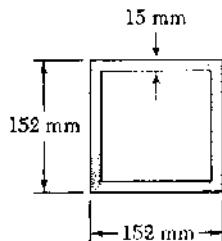
$$KP \leq \sigma_{all} \quad P \leq \frac{\sigma_{all}}{K}$$

$$K = \frac{1}{1890 \times 10^{-6}} + \frac{60 \times 10^{-3}}{79.8 \times 10^{-6}} + 0 = 1.28098 \times 10^3 \text{ m}^{-2}$$

$$P \leq \frac{36.119 \times 10^6}{1.28098 \times 10^3} = 28.2 \times 10^3 \text{ N} = 28.2 \text{ kN}$$

Problem 10.91

10.91 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load P , knowing that the eccentricity is (a) $e = 0$, (b) $e = 40$ mm.



$$b_o = 152 \text{ mm} \quad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{5.5}{56.26 \times 10^{-3}} = 97.76 > 55$$

$$\sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(97.76)^2} = 38.92 \text{ MPa} \text{ for centric loading}$$

$$\frac{P}{A \sigma_{all,c}} + \frac{P e c}{I \sigma_{all,b}} = 1$$

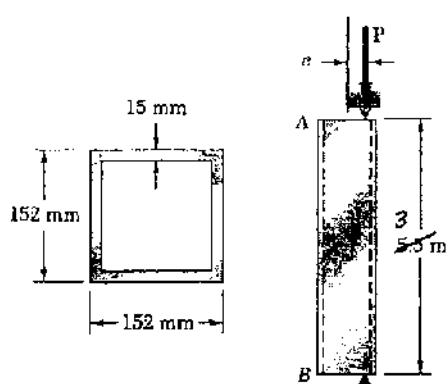
$$(a) \quad e = 0 \quad P = A \sigma_{all,c} = (8220 \times 10^{-6})(38.92 \times 10^6) = 320 \times 10^3 \text{ N} = 320 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 40 \times 10^{-3} \text{ m} \quad c = \frac{1}{2}(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$$

$$\frac{P}{(8220 \times 10^{-6})(38.92 \times 10^6)} + \frac{P (40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^6)(220 \times 10^6)} = 3.6568 \times 10^{-6} P = 1$$

$$P = 273 \times 10^3 \text{ N} = 273 \text{ kN} \quad \blacktriangleleft$$

Problem 10.92



10.92 Solve Prob. 10.91, assuming that the effective length of the column is 3.0 m.

10.91 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load P , knowing that the eccentricity is (a) $e = 0$, (b) $e = 40$ mm.

$$b_o = 152 \text{ mm} \quad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8200 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{3.0}{56.26 \times 10^{-3}} = 53.32 < 55$$

$$\sigma_{ult,c} = 212 - 1.585(L/r) = 212 - (1.585)(53.32) = 127.5 \text{ MPa}$$

$$\frac{P}{A\sigma_{ult,c}} + \frac{Pec}{I\sigma_{ult,b}} = 1$$

$$(a) \quad e = 0 \quad P = A\sigma_{ult,c} = (8220 \times 10^{-6})(127.5 \times 10^6) = 1048 \times 10^3 \text{ N} = 1048 \text{ kN}$$

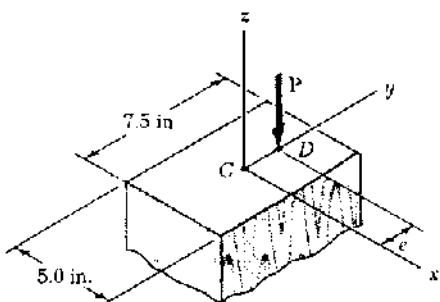
$$(b) \quad e = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} \quad c = (\frac{1}{2})(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$$

$$\frac{P}{(8220 \times 10^{-6})(127.5 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^6)(220 \times 10^6)} = 1.4852 \times 10^{-6} P = 1$$

$$P = 673 \times 10^3 \text{ N} = 673 \text{ kN}$$

Problem 10.93

10.93 A sawn-lumber column of 5.0×7.5 -in. cross section has an effective length of 8.5 ft. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 1180$ psi and a modulus of elasticity $E = 1.2 \times 10^6$ psi. Using the allowable-stress method, determine the largest eccentric load P that can be applied when (a) $e = 0.5$ in., (b) $e = 1.0$ in.



$$\text{Sawn lumber: } \sigma_c = 1180 \text{ psi} \quad E = 1.2 \times 10^6 \text{ psi}$$

$$c = 0.8 \quad K_{ce} = 0.300$$

$$L_e = 8.5 \text{ ft} = 102 \text{ in}$$

$$b = 7.5 \text{ in.}, \quad d = 5.0 \text{ in.}, \quad c = \frac{b}{2} = 3.75 \text{ in.}$$

$$A = b d = (7.5)(5.0) = 37.5 \text{ in}^2 \quad I_x = \frac{1}{12}(5.0)(7.5)^3 = 175.78 \text{ in}^4$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{K_{ce} Ed^2}{L^2} = \frac{(0.300)(1.2 \times 10^6)(5.0)^2}{(102)^2} = 865 \text{ psi}$$

$$\sigma_{ce}/\sigma_c = 865/1180 = 0.7331$$

$$C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{ce}/\sigma_c}{c}} = 0.5763$$

$$\sigma_{all} = \sigma_c C_p = (1180)(0.5763) = 680 \text{ psi}$$

$$\frac{P_{all}}{A} + \frac{P_{ue}ec}{I_x} = \sigma_{all}$$

$$P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{ec}{I_x}}$$

$$(a) \quad e = 0.5 \text{ in.}$$

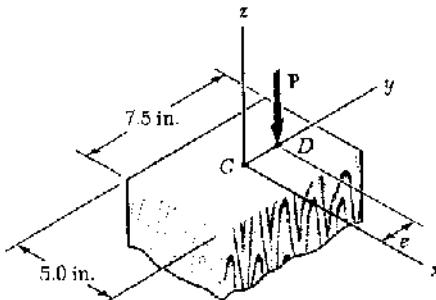
$$P_{all} = \frac{680}{\frac{1}{37.5} + \frac{(0.5)(3.75)}{175.78}} = 18210 \text{ lb.} = 18.21 \text{ kips}$$

$$(b) \quad e = 1.0 \text{ in.}$$

$$P_{all} = \frac{680}{\frac{1}{37.5} + \frac{(1.0)(3.75)}{175.78}} = 14170 \text{ lb.} = 14.17 \text{ kips}$$

Problem 10.94

10.94 Solve Prob. 10.93 using the interaction method and an allowable stress in bending of 1300 psi.



10.93 A sawn-lumber column of 5.0×7.5 -in. cross section has an effective length of 8.5 ft. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 1180$ psi and a modulus of elasticity $E = 1.2 \times 10^6$ psi. Using the allowable-stress method, determine the largest eccentric load P that can be applied when (a) $e = 0.5$ in., (b) $e = 1.0$ in.

$$\text{Sawn lumber: } \sigma_c = 1180 \text{ psi} \quad E = 1.2 \times 10^6 \text{ psi}$$

$$c = 0.8 \quad K_{ce} = 0.300$$

$$L_e = 8.5 \text{ ft} = 102 \text{ in.}$$

$$b = 7.5 \text{ in.}, \quad d = 5.0 \text{ in.}, \quad c = \frac{b}{2} = 3.75 \text{ in.}$$

$$A = bd = (7.5)(5.0) = 37.5 \text{ in.}^2 \quad I_x = \frac{1}{12}(5.0)(7.5)^3 = 175.78 \text{ in.}^4$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{K_{ce} E d^2}{L^2} = \frac{(0.300)(1.2 \times 10^6)(5.0)^2}{(102)^2} = 865 \text{ psi}$$

$$\sigma_{ce}/\sigma_c = 865/1180 = 0.7331$$

$$C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{ce}/\sigma_c}{c}} = 0.5763$$

$$\sigma_{all} = \sigma_c C_p = (1180)(0.5763) = 680 \text{ psi}$$

$$\frac{P_{all}}{A\sigma_{all}} + \frac{P_{ec}}{I_x(\sigma_{all})_b} = BP \leq 1 \quad P \leq \frac{1}{B}$$

(a) $e = 0.5$ in.

$$B = \frac{1}{(37.5)(680)} + \frac{(0.5)(3.75)}{(175.78)(1300)} = 47.421 \times 10^{-6} \text{ lb}^{-1}$$

$$P_{all} = \frac{1}{47.421 \times 10^{-6}} = 21.1 \times 10^3 \text{ lb} = 21.1 \text{ kips}$$

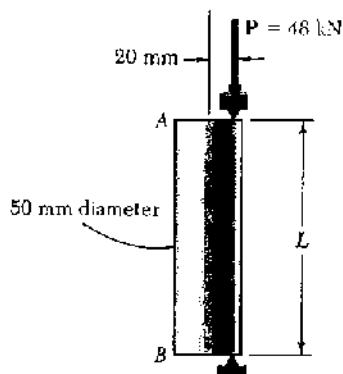
(b) $e = 1.0$ in.

$$B = \frac{1}{(37.5)(680)} + \frac{(1.0)(3.75)}{(175.78)(1300)} = 55.626 \times 10^{-6} \text{ lb}^{-1}$$

$$P_{all} = \frac{1}{55.626 \times 10^{-6}} = 17.98 \times 10^3 \text{ lb} = 17.98 \text{ kips}$$

Problem 10.95

10.95 An eccentric load $P = 48 \text{ kN}$ is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.



$$c = \frac{1}{2}d = 25 \text{ mm} \quad A = \pi c^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 306.8 \times 10^6 \text{ mm}^4 \quad r = \sqrt{\frac{I}{A}} = 12.5 \text{ mm}$$

$$e = 20 \text{ mm} \quad \sigma_{all,b} = 145 \times 10^6 \text{ Pa}$$

$$\frac{P}{AG_{all,c}} + \frac{Pec}{IG_{all,b}} = 1$$

$$\frac{P}{AG_{all,c}} = 1 - \frac{Pec}{IG_{all,b}}$$

$$\frac{1}{G_{all,c}} = \frac{A}{P} \left[1 - \frac{Pec}{IG_{all,b}} \right] = \frac{1.9635 \times 10^3}{48 \times 10^3} \left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^6)(145 \times 10^6)} \right]$$

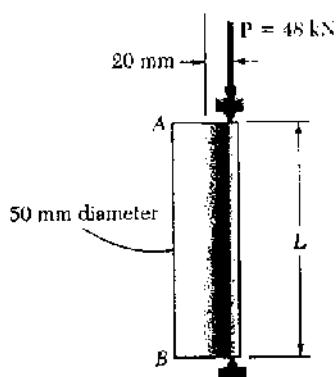
$$= 18.838 \times 10^3 \text{ Pa}^{-1} \quad G_{all,c} = 53.086 \times 10^6 \text{ Pa} = 53.086 \text{ MPa}$$

$$\text{Assume } \frac{L}{r} > 66. \quad \sigma_{all,c} = \frac{351 \times 10^3}{(L/r)^2}$$

$$\frac{L}{r} = \sqrt{\frac{351 \times 10^3}{G_{all,c}}} = \sqrt{\frac{351 \times 10^3}{53.086}} = 81.314 > 66$$

$$L = 81.314 r = (81.314)(12.5 \times 10^{-3}) = 1.016 \text{ m}$$

Problem 10.96



10.96 Solve Prob. 10.95, assuming that the aluminum alloy used is 2014-T6 and that the allowable stress in bending is 180 MPa.

10.95 An eccentric load $P = 48 \text{ kN}$ is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.

$$c = \frac{1}{2}d = 25 \text{ mm} \quad A = \pi c^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 306.8 \times 10^3 \text{ mm}^4 \quad r = \sqrt{\frac{I}{A}} = 12.5 \text{ mm}$$

$$e = 20 \text{ mm} \quad \sigma_{all,b} = 180 \times 10^6 \text{ Pa}$$

$$\frac{P}{AG_{all,c}} + \frac{Pec}{IG_{all,b}} = 1$$

$$\frac{P}{AG_{all,c}} = 1 - \frac{Pec}{IG_{all,b}}$$

$$\frac{1}{G_{all,c}} = \frac{A}{P} \left[1 - \frac{Pec}{IG_{all,b}} \right] = \frac{1.9635 \times 10^{-3}}{48 \times 10^3} \left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^{-9})(180 \times 10^6)} \right]$$

$$= 23.129 \times 10^{-9} \text{ Pa}^{-1} \quad G_{all,c} = 43.236 \times 10^6 \text{ Pa} = 43.236 \text{ MPa}$$

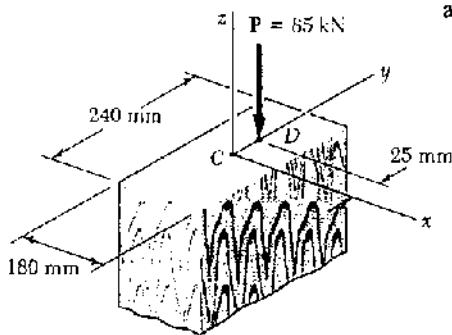
$$\text{Assume } \frac{L}{r} > 55. \quad G_{all,c} = \frac{372 \times 10^3}{(L/r)^2}$$

$$\frac{L}{r} = \sqrt{\frac{372 \times 10^3}{G_{all,c}}} = \sqrt{\frac{372 \times 10^3}{43.236}} = 92.757 > 55$$

$$L = 92.757 r = (92.757)(12.5 \times 10^{-3}) = 1.159 \text{ m}$$

Problem 10.97

10.97 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8.3 \text{ MPa}$ and a modulus of elasticity $E = 11100 \text{ MPa}$. Using the allowable-stress method, determine the largest allowable effective length L that can be used.



$$d = 180 \text{ mm} = 0.180 \text{ m} \quad b = 240 \text{ mm} = 0.240 \text{ m}$$

$$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \quad E = 11100 \text{ MPa}$$

$$I_x = \frac{1}{12} db^3 = \frac{1}{12} (0.180)(0.240)^3 \\ = 207.36 \times 10^{-6} \text{ m}^4$$

$$e = 25 \text{ mm} = 0.025 \text{ m} \quad c = \frac{b}{2} = 0.120 \text{ m}$$

$$\frac{P}{A} + \frac{Pec}{I} \leq \sigma_{all} \quad \sigma_{all} \geq \frac{85 \times 10^3}{43.2 \times 10^{-3}} + \frac{(85 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa} \\ = \text{ MPa}$$

$$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.9496 \times 10^6}{8.3} = 0.38522 = y \quad \text{Let } x = \sigma_{ce}/\sigma_c$$

$$y = \frac{1+x}{2c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \quad \text{where } c = 0.8 \text{ for sawn lumber}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c}\right)^2 - y\left(\frac{1+x}{2c}\right) + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

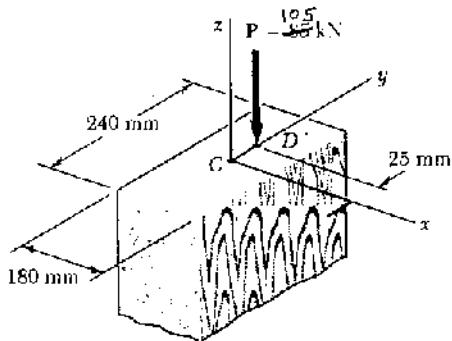
$$x = y \frac{(1-c)y}{1-y} = (0.38522) \frac{1-(0.8)(0.38522)}{1-0.38522} = 0.43350$$

$$\sigma_{ce} = \sigma_c (0.43350) = (8.3)(0.43350) = 3.598 \text{ MPa}$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} \quad L^2 = \frac{K_{ce} E d^2}{\sigma_{ce}} \quad \text{where } K_{ce} = 0.300$$

$$L = d \sqrt{\frac{K_{ce} E}{\sigma_{ce}}} = (0.180) \sqrt{\frac{(0.300)(11100)}{3.598}} = 5.48 \text{ m}$$

Problem 10.98



10.98 Solve Prob. 10.97, assuming that $P = 105 \text{ kN}$.

10.97 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8.3 \text{ MPa}$ and a modulus of elasticity $E = 11100 \text{ MPa}$. Using the allowable-stress method, determine the largest allowable effective length L that can be used.

$$d = 180 \text{ mm} = 0.180 \text{ m} \quad b = 240 \text{ mm} = 0.240 \text{ m}$$

$$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \quad E = 11100 \text{ MPa}$$

$$I_y = \frac{1}{12} d b^3 = \frac{1}{12} (0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$$

$$e = 25 \text{ mm} = 0.025 \text{ m} \quad c = \frac{b}{2} = 0.120 \text{ m}$$

$$\frac{P}{A} + \frac{\sigma_{ec}}{I_x} \leq \sigma_{au} \quad \sigma_{au} \geq \frac{105 \times 10^3}{43.2 \times 10^{-3}} + \frac{(105 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa}$$

$$= 3.9496 \text{ MPa}$$

$$C_p = \frac{\sigma_{au}}{\sigma_c} = \frac{3.9496}{8.3} = 0.47586 = y \quad \text{Let } x = \sigma_{ce}/\sigma_c$$

$$y = \frac{1+x}{c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \quad \text{where } c = 0.8 \text{ for sawn lumber}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c} - y\right)^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$\left(\frac{1+x}{2c}\right)^2 - \frac{1+x}{c} y + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

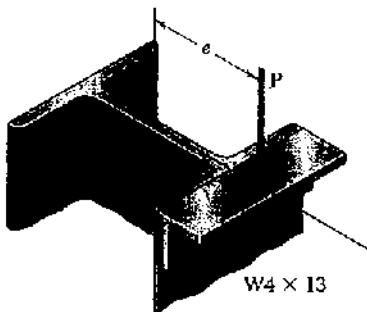
$$x = y \frac{1-cy}{1-y} = (0.47586) \frac{1 - (0.8)(0.47586)}{1 - 0.47586} = 0.56227$$

$$\sigma_{ce} = \sigma_c (0.56227) = (8.3)(0.56227) = 4.6668 \text{ MPa}$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} \quad L^2 = \frac{K_{ce} E d^2}{\sigma_{ce}} \quad \text{where } K_{ce} = 0.300$$

$$L = d \sqrt{\frac{K_{ce} E}{\sigma_{ce}}} = 0.180 \sqrt{\frac{(0.300)(11100)}{4.6668}} = 4.81 \text{ m}$$

Problem 10.99



10.99 A steel compression member of 9-ft effective length supports an eccentric load as shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 30$ kips, (b) $P = 18$ kips. Use $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi.

$$W\ 4 \times 13 \quad A = 3.83 \text{ in}^2, \quad S_x = 5.46 \text{ in}^3$$

$$r_y = 1.00 \text{ in.} \quad L_e = 9 \text{ ft} = 108 \text{ in.} \quad L_e/r_y = 108$$

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29 \times 10^6)}{36}} = 126.10$$

$$\frac{L_e/r}{C_c} = \frac{108}{126.10} = 0.85647$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.85647) - \frac{1}{8}(0.85647)^3 = 1.9093$$

$$\sigma_{all} = \frac{S_x}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.9093} \left[1 - \left(\frac{1}{2} \right) (0.85647)^2 \right] = 11.9396 \text{ ksi}$$

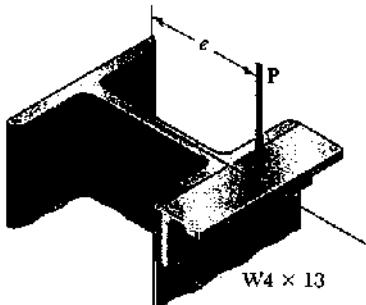
$$\frac{P}{A} + \frac{P_{ec}}{I_x} = \frac{P}{A} + \frac{Pe}{S_x} = \sigma_{all} \quad e = \frac{S_x}{P} \left(\sigma_{all} - \frac{P}{A} \right)$$

$$(a) P = 30 \text{ kips} \quad e = \frac{5.46}{30} \left(11.9396 - \frac{30}{3.83} \right) = 0.747 \text{ in.}$$

$$(b) P = 18 \text{ kips} \quad e = \frac{5.46}{18} \left(11.9396 - \frac{18}{3.83} \right) = 2.20 \text{ in.}$$

Problem 10.100

10.100 Solve Prob. 10.99, assuming that the effective length of the column is increased to 12 ft and that (a) $P = 20$ kips, (b) $P = 15$ kips.



10.99 A steel compression member of 9-ft effective length supports an eccentric load as shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 30$ kips, (b) $P = 18$ kips. Use $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi.

$$W\ 4 \times 13 \quad A = 3.83 \text{ in}^2, \quad S_x = 5.46 \text{ in}^3$$

$$r_y = 1.00 \text{ in.} \quad L_e = 12 \text{ ft} = 144 \text{ in.} \quad L_e/r_y = 144$$

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29 \times 10^6)}{36}} = 126.10$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (29 \times 10^6)}{(1.92)(144)^2} = 7.1891 \text{ ksi}$$

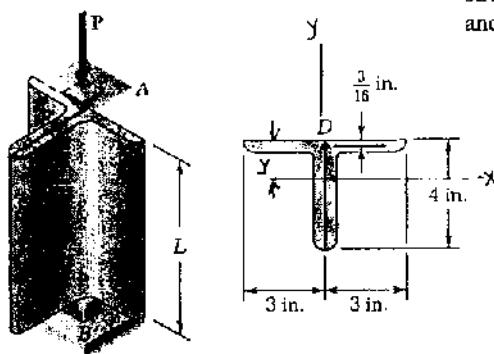
$$\frac{P}{A} + \frac{P_{ec}}{I_x} = \frac{P}{A} + \frac{Pe}{S_x} = \sigma_{all} \quad e = \frac{S_x}{P} \left(\sigma_{all} - \frac{P}{A} \right)$$

$$(a) P = 20 \text{ kips} \quad e = \frac{5.46}{20} \left(7.1891 - \frac{20}{3.83} \right) = 0.537 \text{ in.}$$

$$(b) P = 15 \text{ kips} \quad e = \frac{5.46}{15} \left(7.1891 - \frac{15}{3.83} \right) = 1.191 \text{ in.}$$

Problem 10.101

10.101 Two $4 \times 3 \times \frac{3}{8}$ -in. steel angles are welded together to form the column AB . An axial load P of magnitude 14 kips is applied at point D . Using the allowable-stress method, determine the largest allowable length L . Assume $E = 29 \times 10^6$ psi. and $\sigma_y = 36$ ksi.



$$\text{One angle: } L = 4 \times 3 \times \frac{3}{8} \text{ in.} \quad A = 2.48 \text{ in.}^2$$

$$I_x = 3.96 \text{ in.}^4, S_x = 1.46 \text{ in.}^3, r_x = 1.26 \text{ in.}, y = 1.28 \text{ in.}$$

$$I_y = 1.92 \text{ in.}^4, r_y = 0.879 \text{ in.}, x = 0.782 \text{ in.}$$

$$\text{Two angles: } A = 2(2.48) = 4.96 \text{ in.}^2$$

$$I_x = 2(3.96) = 7.92 \text{ in.}^4, S_x = 2(1.46) = 2.92 \text{ in.}^3, r_x = 1.26, y = 1.28 \text{ in.}$$

$$I_y = 2[I_{y_0} + Ax^2] = 2[1.92 + (2.48)(0.782)^2] = 6.873 \text{ in.}^4 = I_{min}$$

$$r_{min} = \sqrt{\frac{I_{min}}{A}} = 1.177 \text{ in.} \quad e = y - \frac{3}{16} = 1.28 - \frac{3}{16} = 1.0925 \text{ in.}$$

$$P = 14 \text{ kips} \quad \sigma_{all} = \frac{P}{A} + \frac{Pe}{I_x} = \frac{14}{4.96} + \frac{(14)(1.0925)(1.28)}{7.92} = 5.294 \text{ ksi}$$

$$E = 29000 \text{ ksi} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.1$$

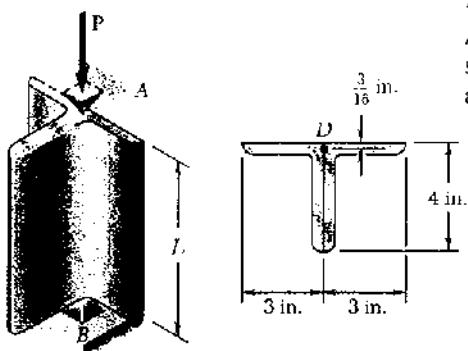
$$\text{Assume } \frac{L_e}{r} > C_c \quad \sigma_{all} = \frac{\pi^2 E}{1.92(L/r_{min})^2} \quad \left(\frac{L}{r_{min}}\right)^2 = \frac{\pi^2 E}{1.92 \sigma_{all}}$$

$$\frac{L}{r_{min}} = \sqrt{\frac{\pi^2 E}{1.92 \sigma_{all}}} = \sqrt{\frac{\pi^2 (29000)}{(1.92)(5.294)}} = 167.8 > C_c$$

$$L = 167.8 r_{min} = (167.8)(1.177) = 197.5 \text{ in.} = 16.46 \text{ ft}$$

Problem 10.102

10.102 Solve Prob. 10.101, using the interaction method with $P = 18$ kips and an allowable stress in bending of 22 ksi.



10.101 Two $4 \times 3 \times \frac{3}{8}$ -in. steel angles are welded together to form the column AB. An axial load P of magnitude 14 kips is applied at point D. Using the allowable-stress method, determine the largest allowable length L. Assume $E = 29 \times 10^6$ psi. and $\sigma_y = 36$ ksi.

$$\text{One angle } L 4 \times 3 \times \frac{3}{8} \quad A = 2.48 \text{ in}^2$$

$$I_x = 3.96 \text{ in}^4, S_x = 1.46 \text{ in}^3, r_x = 1.26 \text{ in}, y = 1.28 \text{ in.}$$

$$I_y = 1.92 \text{ in}^4, r_y = 0.879 \text{ in.}, x = 0.782 \text{ in.}$$

$$\text{Two angles } A = (2)(2.48) = 4.96 \text{ in}^2$$

$$I_x = (2)(3.96) = 7.92 \text{ in}^4, S_y = (2)(1.46) = 2.92 \text{ in}^3, r_x = 1.26 \text{ in.}, y = 1.28 \text{ in.}$$

$$I_y = 2 [I_{y(1)} + Ax^2] = (2)[1.92 + (2.48)(0.782)^2] = 6.873 \text{ in}^4 = I_{min}$$

$$r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{6.873}{4.96}} = 1.177 \text{ in.} \quad e = y - \frac{3}{16} = 1.28 - \frac{3}{16} = 1.0925 \text{ in.}$$

$$P = 18 \text{ kips.} \quad \frac{P}{A\sigma_{all,c}} + \frac{Pey}{I_x\sigma_{all,b}} = 1 \quad \frac{1}{\sigma_{all,c}} = \frac{A}{P} \left(1 - \frac{Pey}{I_x\sigma_{all,b}} \right)$$

$$\frac{1}{\sigma_{all,c}} = \frac{4.96}{18} \left[1 - \frac{(18)(1.0925)(1.28)}{(7.92)(22)} \right] = 0.23575(\text{ksi})^{-1}$$

$$\sigma_{all,c} = 4.2418 \text{ ksi} \quad E = 29000 \text{ ksi;}$$

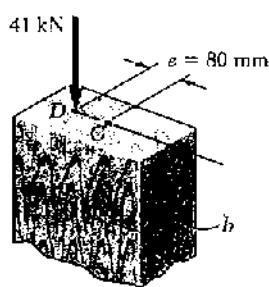
$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.1$$

$$\text{Assume } \frac{L_e}{r} > C_c. \quad \sigma_{all,c} = \frac{\pi^2 E}{1.92(L_e/r)^2}$$

$$\frac{L_e}{r} = \sqrt{\frac{\pi^2 E}{1.92\sigma_{all,c}}} = \sqrt{\frac{\pi^2 (29000)}{(1.92)(4.2418)}} = 187.466 > C_c$$

$$L_e = 187.466 r_{min} = (187.466)(1.177) = 220.65 \text{ in.} = 18.39 \text{ ft} \quad \blacktriangleleft$$

Problem 10.103



10.103 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41 kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8.1 \text{ MPa}$ and $E = 8.3 \text{ GPa}$. Use the allowable-stress method to determine the lightest section that can be used.

$$\text{Sawn lumber: } \sigma_c = 8.1 \text{ MPa} \quad E = 8.3 \text{ GPa} \\ c = 0.8 \quad K_{ce} = 0.300 \\ L_e = 2.2 \text{ m}$$

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all}$$

$$e = 80 \times 10^{-3} \text{ m}, \quad c = \frac{1}{2}(190) = 95 \text{ mm} = 95 \times 10^{-3} \text{ m}$$

$$A = 0.190 b \text{ m}^2 \quad I_x = \frac{1}{12} b (0.190)^3 = 571.58 \times 10^{-6} b \text{ m}^4$$

$$\frac{P_{all}}{0.190 b} + \frac{P_{all}(80 \times 10^{-3})(95 \times 10^{-3})}{571.58 \times 10^{-6} b} = \frac{18.56 P_{all}}{b} = \sigma_{all}$$

$$P_{all} = 0.05388 \sigma_{all} b$$

$d = 0.190 \text{ m}$ or b , whichever is smaller.

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5 d^2 \text{ MPa}$$

$$\sigma_{ce}/\sigma_c = \frac{514.5 d^2}{8.1} = 63.51 d^2$$

$$C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{ce}/\sigma_c}{c}}$$

$$= \frac{1 + \sigma_{ce}/\sigma_c}{1.6} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{1.6}\right)^2 - \frac{\sigma_{ce}/\sigma_c}{0.8}}$$

$$\sigma_{all} = \sigma_c C_p = (8.1 \times 10^6) C_p$$

$$P_{all} = (0.05388 b) (8.1 \times 10^6) C_p = 472.47 \times 10^3 b C_p$$

Calculate P_{all} for all four values of b . See table below.

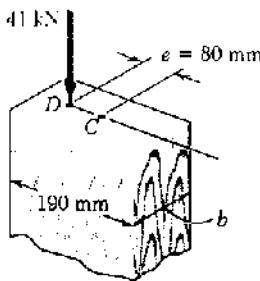
$b, (\text{m})$	$d, (\text{m})$	σ_{ce}/σ_c	C_p	$P_{all}, (\text{kN})$
0.090	0.090	0.51443	0.44367	18.87
0.140	0.140	1.24480	0.76081	50.3
0.190	0.190	2.2927	0.8878	79.7
0.240	0.190	2.2927	0.8878	100.7

$$\rightarrow P = 41 \text{ kN}$$

$$\text{Use } b = 0.140 \text{ m} \\ = 140 \text{ mm}$$

Problem 10.104

10.104 Solve Prob. 10.103, assuming that $e = 40 \text{ mm}$.



10.103 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41 kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8.1 \text{ MPa}$ and $E = 8.3 \text{ GPa}$. Use the allowable-stress method to determine the lightest section that can be used.

$$\text{Sawn lumber: } \sigma_{all} = 8.1 \text{ MPa} \quad E = 8.3 \text{ GPa} \\ c = 0.8 \quad K_{ce} = 0.300 \\ L_e = 2.2 \text{ m}$$

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all}$$

$$e = 40 \times 10^{-3} \text{ m}, \quad c = \frac{1}{2}(190) = 95 \text{ mm} = 95 \times 10^{-3} \text{ m}$$

$$A = 0.190 \text{ b m}^2 \quad I_x = \frac{1}{12} b (0.190)^3 = 571.58 \times 10^{-6} \text{ b m}^4$$

$$\frac{P_{all}}{0.190b} + \frac{P_{all}(40 \times 10^{-3})(95 \times 10^{-3})}{571.58 \times 10^{-6} b} = \frac{11.911 P_{all}}{b} = \sigma_{all}$$

$$P_{all} = 0.083953 \sigma_{all} b$$

$d = 0.190 \text{ m}$ or b , whichever is smaller.

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5 d^2 \text{ MPa}$$

$$\sigma_{ce}/\sigma_c = \frac{514.5 d^2}{8.1} = 63.51 d^2$$

$$C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{ce} K_{ce}}{c}} \\ = \frac{1 + \sigma_{ce}/\sigma_c}{1.6} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{1.6}\right)^2 - \frac{\sigma_{ce} K_{ce}}{0.8}}$$

$$\sigma_{all} = \sigma_c C_p = (8.1 \times 10^6) C_p$$

$$P_{all} = (0.083953 b)(8.1 \times 10^6) C_p = 680.02 \times 10^3 b C_p$$

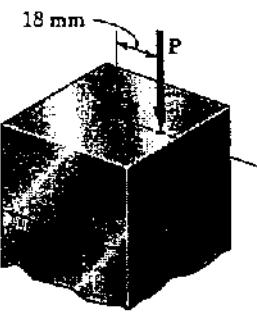
Calculate P_{all} for all four values of b . See table below.

$b(\text{m})$	$d(\text{m})$	σ_{ce}/σ_c	C_p	$P_{all} (\text{kN})$
0.090	0.090	0.51443	0.44367	27.2
0.140	0.140	1.24480	0.76081	72.4
0.190	0.190	2.2927	0.8878	114.7
0.240	0.190	2.2927	0.8878	144.9

$\rightarrow P = 41 \text{ kN}$

Use $b = 0.140 \text{ m}$
= 140 mm

Problem 10.105



10.105 A compression member made of steel has a 1.62 m effective length and must support the 128-kN load P as shown. For the material used, $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

Using dimensions in meters,

$$A = 40 \times 10^{-3} d \quad L_e = 1.62 \text{ m}$$

$$I_x = \frac{1}{12} (40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12} (40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-3} d^3$$

$$l_{x1} = \frac{1}{2} d, \quad l_{y1} = 20 \text{ mm} = 20 \times 10^{-3} \text{ m} \quad |l_{ex}| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

Steel: $\sigma_y = 250 \text{ MPa}$ $E = 200000 \text{ MPa}$

$$C_c = \sqrt{\frac{2\pi^2 E}{G_y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

Assume $d > 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$. Then, $I_{min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{3.3333 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = 140.29 > C_c$$

$$\sigma_{all, \text{centric}} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (200000)}{(1.92)(140.29)^2} = 52.236 \text{ MPa} \quad \sigma_{all, \text{bending}} = 150 \text{ MPa}$$

$$\frac{P}{A \sigma_{all, \text{centric}}} + \frac{P e_x}{I_y \sigma_{all, \text{bending}}} = 1$$

$$\frac{128 \times 10^3}{(40 \times 10^{-3} d)(52.236 \times 10^6)} + \frac{(128 \times 10^3)(18 \times 10^{-3})(\frac{1}{2}d)}{(3.3333 \times 10^{-3} d^3)(150 \times 10^6)} = 1$$

$$\frac{61.260 \times 10^{-3}}{d} + \frac{2.304 \times 10^{-3}}{d^2} = 1$$

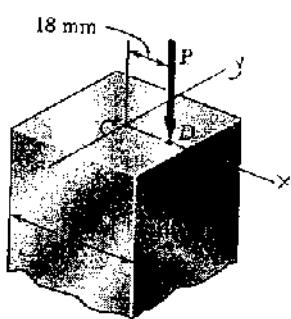
$$d^2 - 61.260 \times 10^{-3} d - 2.304 \times 10^{-3} = 0$$

$$d = \frac{1}{2} \left\{ 61.260 \times 10^{-3} + \sqrt{(61.260 \times 10^{-3})^2 + (4)(2.304 \times 10^{-3})} \right\}$$

$$= 87.6 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m}$$

$$d = 87.6 \text{ mm}$$

Problem 10.106



10.106 Solve Prob. 10.105, assuming that the effective length is 720 mm and that the magnitude P of the eccentric load is 198 kN.

10.105 A compression member made of steel has a 1.62 m effective length and must support the 128-kN load P as shown. For the material used, $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

Using dimensions in meters,

$$A = 40 \times 10^{-3} d \quad L_e = 720 \text{ mm} = 0.720 \text{ m}$$

$$I_x = \frac{1}{12} (40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12} (40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-3} d^3$$

$$|x| = \frac{d}{2}, \quad |y| = 20 \text{ mm} = 0.020 \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$\text{Steel: } \sigma_y = 250 \text{ MPa} \quad E = 200000 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$$

$$C_c = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

Assume $d > 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$. Then, $I_{min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{40 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m}, \quad \frac{L_e}{r} = 62.35 < C_c$$

$$\frac{L_e/r}{C_c} = 0.49621 \quad \text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.49621) - \frac{1}{8}(0.49621)^3 = 1.83747$$

$$\sigma_{all, \text{cyclic}} = \frac{\sigma_y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.83747} \left[1 - \frac{1}{2} (0.49621)^2 \right] = 119.31 \text{ MPa}$$

$$\sigma_{all, \text{bending}} = 150 \text{ MPa}$$

$$\frac{P}{A \sigma_{all, \text{cyclic}}} + \frac{P e_x x}{I_y \sigma_{all, \text{bending}}} = 1$$

$$\frac{198 \times 10^3}{(40 \times 10^{-3} d)(119.31 \times 10^6)} + \frac{(198 \times 10^3)(18 \times 10^{-3})(\frac{1}{2}d)}{(3.3333 \times 10^{-3} d^3)(150 \times 10^6)} = 1$$

$$\frac{41.489 \times 10^{-3}}{d} + \frac{3.5640 \times 10^{-3}}{d^2} = 1$$

$$d^2 - 41.489 \times 10^{-3} d - 3.5640 \times 10^{-3} = 0$$

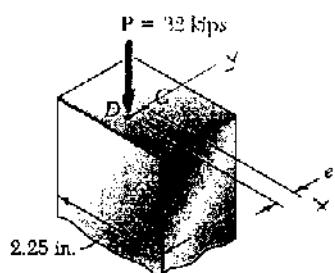
$$d = \frac{1}{2} \left\{ 41.489 \times 10^{-3} + \sqrt{(41.489 \times 10^{-3})^2 + (4)(3.5640 \times 10^{-3})} \right\}$$

$$= 83.9 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m}$$

$$d = 83.9 \text{ mm}$$

Problem 10.107

10.107 A compression member of rectangular cross section has an effective length of 36 in. and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 24 ksi. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 0.4$ in.



$$A = 2.25 d \quad c = \frac{1}{2}d \quad e = 0.4 \text{ in.} \quad L_e = 36 \text{ in.}$$

$$\sigma_{all,b} = 24 \text{ ksi} \quad P = 32 \text{ kips}$$

$$I_x = \frac{1}{12} (2.25)d^3 \quad r_x = \frac{d}{\sqrt{12}}$$

Assume $r_x = r_{min}$, i.e. $d < 2.25$ in.

$$L_e/r_{min} = \sqrt{12} L_e/d$$

$$\text{Assume } L_e/r_{min} > 55. \quad \sigma_{all,c} = \frac{54000}{(L_e/r_x)^2} = \frac{54000 d^2}{12 L_e^2} = \frac{54000}{(12)(36)^2} d^2 = 3.47222 d^2$$

$$\frac{P}{A G_{el,c}} + \frac{P_{ec}}{I G_{el,b}} = \frac{32}{(2.25d)(3.47222d^2)} + \frac{(12)(32)(0.4)(\frac{1}{2}d)}{(2.25d^3)(24)} = 1$$

$$\frac{4.096}{d^3} + \frac{1.42222}{d^2} = 1 \quad \text{Let } x = \frac{1}{d} \quad 4.096x^3 + 1.42222x^2 = 1$$

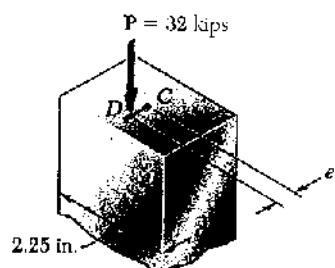
Solving for x , $x = 0.528118$, $d = \frac{1}{x} = 1.894$ in. < 2.25 in.

$$L_e/r_x = (\sqrt{12})(36)/1.894 = 65.8 > 55 \quad d = 1.894 \text{ in.} \blacksquare$$

Problem 10.108

10.108 Solve Prob. 10.107, assuming that $e = 0.2$ in.

10.107 A compression member of rectangular cross section has an effective length of 36 in. and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 24 ksi. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 0.4$ in.



$$A = 2.25 d \quad c = \frac{1}{2}d \quad e = 0.2 \text{ in.} \quad L_e = 36 \text{ in.}$$

$$\sigma_{all,b} = 24 \text{ ksi} \quad P = 32 \text{ kips}$$

$$I_x = \frac{1}{12} (2.25)d^3 \quad r_x = \frac{d}{\sqrt{12}}$$

Assume $r_x = r_{min}$, i.e. $d < 2.25$ in.

$$L_e/r_{min} = \sqrt{12} L_e/d$$

$$\text{Assume } L_e/r_{min} > 55. \quad \sigma_{all,c} = \frac{54000}{(L_e/r_x)^2} = \frac{54000 d^2}{12 L_e^2} = \frac{54000 d^2}{(12)(36)^2} = 3.47222 d^2$$

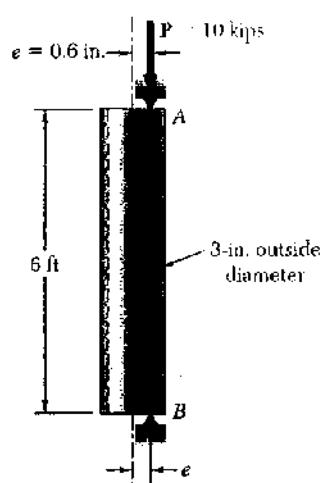
$$\frac{P}{A G_{el,c}} + \frac{P_{ec}}{I G_{el,b}} = \frac{32}{(2.25d)(3.47222d^2)} + \frac{(12)(32)(0.2)(\frac{1}{2}d)}{(2.25d^3)(24)} = 1$$

$$\frac{4.096}{d^3} + \frac{0.71111}{d^2} = 1 \quad \text{Let } x = \frac{1}{d} \quad 4.096x^3 + 0.71111x^2 = 1$$

Solving for x , $x = 0.57216$ in. $d = \frac{1}{x} = 1.748$ in. < 2.25 in.

$$L_e/r_x = (\sqrt{12})(36)/1.748 = 71.4 > 55 \quad d = 1.748 \text{ in.} \blacksquare$$

Problem 10.109



10.109 An aluminum tube of 3-in. outside diameter is to carry a load of 10 kips having an eccentricity $e = 0.6$ in. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and have wall thicknesses in increments of $\frac{1}{16}$ in. up to $\frac{1}{2}$ in., determine the lightest tube that may be used. Use the allowable-stress method of design.

$$L_e = 6 \text{ ft} = 72 \text{ in.} \quad c_o = \frac{1}{2} d_o = 1.500 \text{ in.} = c$$

$$c_i = c_o - t = 1.500 - t$$

$$A = \pi (c_o^2 - c_i^2) \quad I = \frac{\pi}{4} (c_o^4 - c_i^4)$$

$$\text{Try } t = \frac{1}{4} \text{ in.} = 0.25 \text{ in.} \quad c_i = 1.250 \text{ in.}$$

$$A = 2.1598 \text{ in}^2 \quad I = 2.0586 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 0.97629 \text{ in}$$

$$\frac{L}{r} = \frac{72}{0.97629} = 73.748 > 55.$$

$$G_{all} = \frac{54000}{(L/r)^2} = \frac{54000}{(73.748)^2} = 9.9285 \text{ ksi}$$

$$\frac{P_{all}}{A} + \frac{Mc}{I} = \frac{P_{all}}{A} + \frac{P_{all}ec}{I} = \left[\frac{1}{2.1598} + \frac{(0.6)(1.500)}{2.0586} \right] P_{all} = 0.9002 P_{all}$$

$$P_{all} = \frac{G_{all}}{0.9002} = \frac{9.9285}{0.9002} = 11.03 \text{ kips.} > 10 \text{ kips}$$

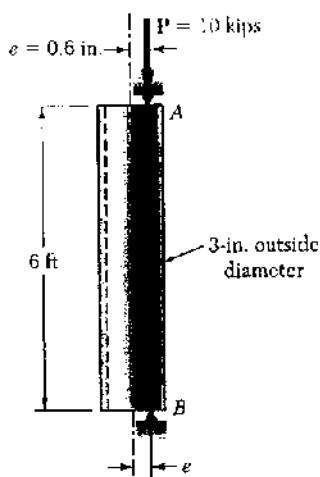
Since P_{all} is nearly proportional to the thickness t ,

$$\frac{t}{0.25} \propto \frac{10}{11.03} \quad t \approx 0.227 \text{ in.} > \frac{3}{16} \text{ in.} = 0.1875 \text{ in.}$$

$\frac{3}{16}$ in. thickness would be too light. Use $t = \frac{1}{4}$ in. ■

Problem 10.110

10.110 Solve Prob. 10.109, using the interaction method of design with an allowable stress in bending of 25 ksi.



10.109 An aluminum tube of 3-in. outside diameter is to carry a load of 10 kips having an eccentricity $e = 0.6$ in. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and have wall thicknesses in increments of $\frac{1}{16}$ in. up to $\frac{1}{2}$ in., determine the lightest tube that may be used. Use the allowable-stress method of design.

$$L_e = 6 \text{ ft} = 72 \text{ in.} \quad c_o = \frac{1}{2} d_o = 1.500 \text{ in.} = c$$

$$c_i = c_o - t = 1.500 - t$$

$$A = \pi(c_o^2 - c_i^2) \quad I = \frac{\pi}{4}(c_o^4 - c_i^4)$$

$$\text{Try } t = \frac{1}{4} \text{ in.} = 0.25 \text{ in.} \quad c_i = 1.250 \text{ in.}$$

$$A = 2.1598 \text{ in}^2 \quad I = 2.0586 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 0.97629 \text{ in.}$$

$$\frac{L}{r} = \frac{72}{0.97629} = 73.748 > 55$$

$$\sigma_{all,c} = \frac{54000}{(L/r)^2} = \frac{54000}{(73.748)^2} = 9.9285 \text{ ksi}$$

$$\frac{P_{all}}{AG_{all,c}} + \frac{Mc}{IG_{all,b}} = \frac{P_{all}}{A\sigma_{all,c}} + \frac{Pe_ec}{I\sigma_{all,b}}$$

$$= \left[\frac{1}{(2.1598)(9.9285)} + \frac{(0.6)(1.500)}{(2.0586)(25)} \right] P_{all} = 0.06412 P_{all} = 1$$

$$P_{all} = \frac{1}{0.06412} = 15.60 \text{ kips} > 10 \text{ kips.}$$

Since P_{all} is nearly proportional to the thickness t ,

$$\frac{t}{0.25} \propto \frac{10}{15.60} \quad t \approx 0.1603 \text{ in.} < \frac{3}{16} \text{ in.} = 0.1875 \text{ in.}$$

$$\text{Try } t = \frac{3}{16} = 0.1875 \text{ in.} \quad c_i = 1.3125 \text{ in.}$$

$$A = 1.6567 \text{ in}^2, \quad I = 1.64537 \text{ in}^4, \quad r = 0.99658 \text{ in.}$$

$$\frac{L}{r} = 72.247 > 55 \quad \sigma_{all,c} = 10.345 \text{ ksi}$$

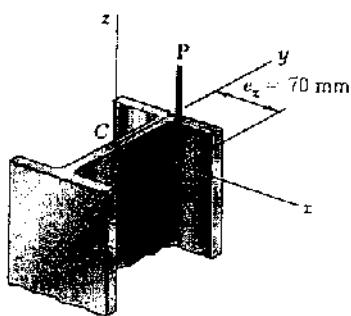
$$\left[\frac{1}{(1.6567)(10.345)} + \frac{(0.6)(1.500)}{(1.64537)(25)} \right] P_{all} = 0.080225 P_{all} = 1$$

$$P_{all} = \frac{1}{0.080225} = 12.46 \text{ kips.}$$

$$\frac{t}{0.1875} \propto \frac{10}{12.46} \quad t \approx 0.1504 \text{ in.} < \frac{3}{16} \text{ in.}$$

$$\text{Use } t = \frac{3}{16} \text{ in.} \quad \rightarrow$$

Problem 10.111



10.111 A steel column of 7.2-m effective length is to support an 83-kN eccentric load P at a point D located on the x axis as shown. Using the allowable-stress method, select the wide-flange shape of 250-mm nominal depth that should be used. Use $E = 200 \text{ GPa}$, $\sigma_y = 250 \text{ MPa}$.

$$\text{Steel: } E = 200000 \text{ MPa} \quad \sigma_y = 250 \text{ MPa}$$

$$C_e = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

$$L_e = 7.2 \text{ m}$$

Try $W 250 \times 49.1$. $A = 6250 \times 10^{-6}$

$$b_f = 202 \times 10^{-3} \text{ m}, \quad c = 101 \times 10^{-3} \text{ m}, \quad I_y = 15.1 \times 10^{-8} \text{ m}^4, \quad r_y = 49.2 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r_y} = \frac{7.2}{49.2 \times 10^{-3}} = 146.34 > C_e$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r_y)^2} = \frac{\pi^2 (200000)}{(1.92)(146.34)^2} = 48.01 \text{ MPa}$$

$$\begin{aligned} \frac{P}{A} + \frac{P e_x c}{I_y} &= \frac{83 \times 10^3}{6250 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})(101 \times 10^{-3})}{15.1 \times 10^{-8}} \\ &= 13.28 \times 10^6 + 38.86 \times 10^6 = 52.14 \text{ MPa} > 48.01 \text{ MPa} \\ &\quad (\text{not allowed}) \end{aligned}$$

$$\text{Required area} \quad A \approx \frac{52.14}{48.01} (6250 \text{ mm}^2) = 6788 \text{ mm}^2$$

Try $W 250 \times 58$.

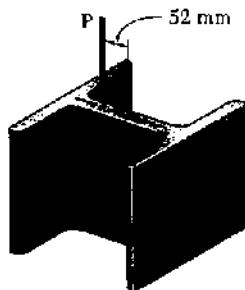
$$\frac{L_e}{r_y} = \frac{7.2}{50.3 \times 10^{-3}} = 143.14 \quad \sigma_{all} = \frac{\pi^2 (200000)}{(1.92)(143.14)^2} = 50.18 \text{ MPa}$$

$$\begin{aligned} \frac{P}{A} + \frac{P e_x c}{I_y} &= \frac{83 \times 10^3}{7420 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})(101.5 \times 10^{-3})}{18.8 \times 10^{-8}} \\ &= 11.19 \times 10^6 + 31.37 \times 10^6 = 42.56 \text{ MPa} < 50.18 \text{ MPa} \end{aligned}$$

Use $W 250 \times 58$.

Problem 10.112

10.112 A steel column of 6.3-m effective length must carry a load of 360 kN with an eccentricity of 52 mm as shown. Using the interaction method, select the wide-flange shape of 310-mm nominal depth that should be used. Use $E = 200 \text{ GPa}$, $\sigma_y = 250 \text{ MPa}$ and $\sigma_{all} = 160 \text{ MPa}$ in bending.



$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.66$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pe}{I_y \sigma_{all,b}} < 1 \quad (1)$$

Assume $\frac{L}{r_y} > C_c = 125.66$.

$$\text{Then } G_{all,c} = \frac{\pi^2 E}{1.92(L/r_y)^2} = \frac{\pi^2 E r_y^2}{1.92 L^2}$$

The first term of inequality (1) becomes

$$\frac{1.92 PL^2}{\pi^2 E A r_y^2} = \frac{1.92 PL^2}{\pi^2 E I_y} = \frac{\alpha}{I_y} \quad \text{where } \alpha = \frac{1.92 PL^2}{\pi^2 E}$$

$$\alpha = \frac{(1.92)(360 \times 10^3)(6.3)^2}{\pi^2 (200 \times 10^9)} = 13.898 \times 10^{-6} \text{ m}^4 = 13.898 \times 10^6 \text{ mm}^4$$

The second term of inequality (1) is $\frac{Pe}{S_x \sigma_{all,b}} = \frac{\beta}{S_x}$ where

$$\beta = \frac{Pe}{E_{all,b}} = \frac{(360 \times 10^3)(52 \times 10^{-3})}{160 \times 10^6} = 117 \times 10^{-6} \text{ m}^3 = 117 \times 10^3 \text{ mm}^3$$

Inequality (1) is then $\frac{\alpha}{I_y} + \frac{\beta}{S_x} < 1$

Obviously, $I_y > \alpha = 13.898 \times 10^6 \text{ mm}^4$

$$S_x > \beta = 117 \times 10^3 \text{ mm}^3$$

Try W 310 × 60. $I_y = 18.3 \times 10^6 \text{ mm}^4$, $S_x = 851 \times 10^3 \text{ mm}^2$

$$\frac{\alpha}{I_y} + \frac{\beta}{S_x} = \frac{13.898}{18.3} + \frac{117}{851} = 0.897 < 1 \quad \left. \right\} \text{safe}$$

$$r_y = 49.1 \text{ mm} \quad \frac{L}{r_y} = \frac{6.3}{49.1 \times 10^{-3}} = 128.3 > C_c \quad \left. \right\} \text{safe}$$

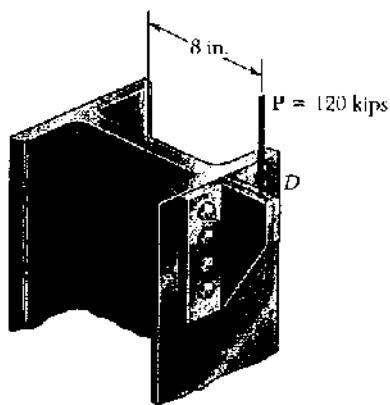
Try W 310 × 52. $I_y = 10.3 \times 10^6 \text{ mm}^4$, $S_x = 748 \times 10^3 \text{ mm}^2$

$$\frac{\alpha}{I_y} + \frac{\beta}{S_x} = \frac{13.898}{10.3} + \frac{117}{748} = 1.504 > 1 \quad \left. \right\} \text{unsafe}$$

$$r_y = 39.3 \text{ mm} \quad \frac{L}{r_y} = \frac{6.3}{39.3 \times 10^{-3}} = 160.3 > C_c \quad \left. \right\} \text{unsafe}$$

Use W 310 × 60. \blacktriangleleft

Problem 10.113



10.113 A steel column having a 24-ft effective length is loaded eccentrically as shown. Using the allowable stress method, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_y = 36 \text{ ksi}$ and $E = 29 \times 10^6 \text{ psi}$.

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.1$$

$$L = 24 \text{ ft} = 288 \text{ in.}$$

If column is short and loading is centric

$$F.S. = \frac{5}{3} \quad \bar{\sigma}_{all} = \frac{\sigma_y}{F.S.} = \frac{36}{5/3} = 21.6 \text{ ksi}$$

$$A > \frac{P}{\bar{\sigma}_{all}} = \frac{120}{21.6} = 5.56 \text{ in}^2$$

If column is long and loading is centric, $F.S. = 1.92$

$$P < \frac{P_{cr}}{1.92} = \frac{\frac{\pi^2 EI_y}{L^2}}{1.92 L^2}, \quad I_y > \frac{1.92 PL^2}{\pi^2 E} = \frac{(1.92)(120)(288)^2}{\pi^2 (29000)} = 66.8 \text{ in}^4$$

$$\frac{P}{A} + \frac{Pe}{I_x} = -\frac{P}{A} + \frac{Pe}{S_x} < \bar{\sigma}_{all}$$

Try W14x68. $A = 20.0 \text{ in}^2$, $I_y = 121 \text{ in}^4$, $r_y = 2.46 \text{ in.}$, $S_x = 103 \text{ in}^3$

$$\frac{L}{r_y} = \frac{288}{2.46} = 117.07 < C_c \quad \frac{L/r_y}{C_c} = 0.9284$$

$$F.S. \approx \frac{5}{3} + \frac{2}{3}(0.9284) - \frac{1}{3}(0.9284)^2 = 1.9148$$

$$\bar{\sigma}_{all} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L/r_y}{C_c} \right)^2 \right] = \frac{36}{1.9148} \left[1 - \frac{1}{2} (0.9248)^2 \right] = 10.698 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pe}{S_x} = \frac{120}{20.0} + \frac{(120)(8)}{103} = 15.32 \text{ ksi} > \bar{\sigma}_{all} \quad \underline{\text{unsafe}}$$

Assuming that $\bar{\sigma}_{max}$ is approximately proportional to $\frac{1}{A}$, the required area is

$$A = (20.0) \frac{15.32}{10.698} = 28.64 \text{ in}^2$$

Try W14x145. $A = 42.7 \text{ in}^2$, $S_x = 232 \text{ in}^3$, $r_y = 3.98 \text{ in}$

$$\frac{L}{r_y} = 72.36 \quad \frac{L/r_y}{C_c} = 0.5738 \quad F.S. = 1.8582 \quad \bar{\sigma}_{all} = 16.18 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pe}{S_x} = 6.95 \text{ ksi} < 16.18 \text{ ksi} \quad \underline{\text{safe}}$$

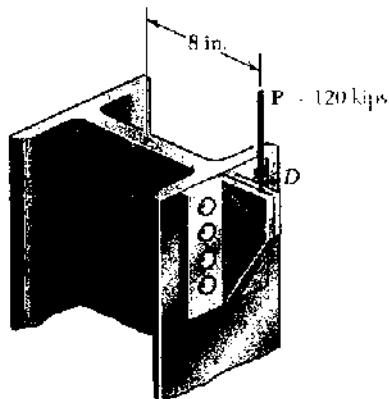
Try W14x82. $A = 24.1 \text{ in}^2$, $S_x = 123 \text{ in}^3$, $r_y = 2.48 \text{ in.}$

$$\frac{L}{r_y} = 116.13 \quad \frac{L/r_y}{C_c} = 0.9209 \quad F.S. = 1.9144 \quad \bar{\sigma}_{all} = 10.83 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pe}{S_x} = 12.78 \text{ ksi} > 10.83 \text{ ksi} \quad \underline{\text{unsafe}}$$

Use W14x145.

Problem 10.114



10.114 Solve Prob. 10.113 using the interaction method, assuming that $\sigma_y = 50$ ksi and an allowable stress in bending of 30 ksi.

10.113 A steel column having a 24-ft effective length is loaded eccentrically as shown. Using the allowable stress method, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_y = 36$ ksi and $E = 29 \times 10^6$ psi.

$$\text{Steel: } C_e = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{50}} = 107.1$$

$$L_e = 24 \text{ ft} = 288 \text{ in.}$$

$$\text{Assume } \frac{L}{r_y} > 107.1$$

$$r_y < \frac{L}{107.1} = \frac{288}{107.1} = 2.69 \text{ in.}$$

This criterion is met by all W14's except for W14x370 and W14x145.

$$G_{all,c} = \frac{\pi^2 E}{1.92(L/r_y)^2} = \frac{\pi^2 E r_y^2}{1.92 L^2}$$

$$\text{Interaction formula: } \frac{P}{A G_{all,c}} + \frac{P e c}{I_x G_{all,b}} < 1 \quad (1)$$

The first term of inequality (1) becomes

$$\frac{1.92 PL^2}{\pi^2 E A r_y^2} = \frac{1.92 PL^2}{\pi^2 E I_y} = \frac{(1.92)(120)(288)^2}{\pi^2 (29000) I_y} = \frac{66.768}{I_y}$$

The second term of inequality (1) becomes

$$\frac{P e c}{S_x G_{all,b}} = \frac{(120)(8)(30)}{(S_x)(30)} = \frac{32}{S_x}$$

$$\text{Then inequality (1) becomes } \frac{66.768}{I_y} + \frac{32}{S_x} = \beta < 1$$

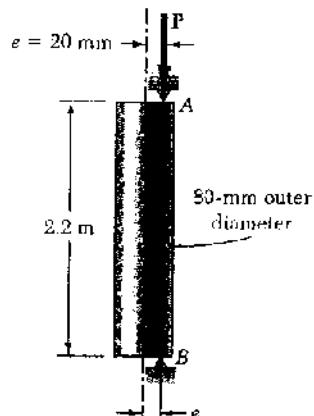
Obviously $I_y > 66.768 \text{ in}^4$ and $S_x > 32 \text{ in}^3$

Shape	$I_y (\text{in}^4)$	$S_x (\text{in}^3)$	β
W14x82	148	128	0.701
W14x68	121	103	0.862
W14x53	57.7	77.8	2.49

W14x68 is the lightest shape with $\beta < 1$

Use W14x68.

Problem 10.115



10.115 A steel tube of 80-mm outer diameter is to carry a 93-kN load P with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable stress method, determine the lightest tube that can be used. Assume $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$.

$$r_o = \frac{\pi d_o}{4} = 40 \text{ mm}, \quad r_c = r_o - t$$

$$A = \pi (r_o^2 - r_c^2), \quad I = \frac{\pi}{4} (r_o^4 - r_c^4) \quad r = \sqrt{\frac{I}{A}}$$

t mm	r_o mm	A mm^2	I 10^6 mm^4	r mm
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$$L_e = 2.2 \text{ m}$$

$$P = 93 \times 10^3 \text{ N}$$

$$\text{Steel: } E = 200000 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

$$\text{Try } t = 9 \text{ mm.} \quad \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < C_c \quad \frac{L_e/r}{C_c} = 0.6917$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.6917) - \frac{1}{8}(0.6917)^3 = 1.885$$

$$\sigma_{ult} = \frac{\sigma_y}{\text{F.S.}} \left[1 - \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.885} \left[1 - \frac{1}{2}(0.6917)^2 \right] = 100.9 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2007 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.285 \times 10^{-6}} = 104.2 \text{ MPa} > 100.9 \text{ MPa} \quad (\text{not allowed})$$

$$\text{Approximate required area: } \left(\frac{104.2}{100.9} \right) (2007 \times 10^{-6}) = 2073 \times 10^{-6} \text{ m}^2 = 2073 \text{ mm}^2$$

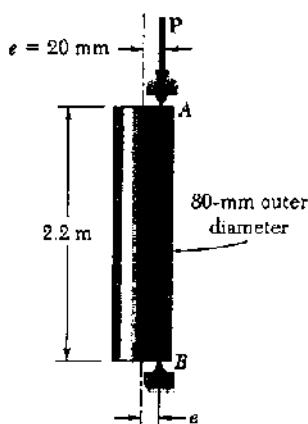
$$\text{Try } t = 12 \text{ mm.} \quad \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < C_c \quad \frac{L_e/r}{C_c} = 0.7172$$

$$\text{F.S.} = 1.890 \quad \sigma_{ult} = 98.3 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2564 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.528 \times 10^{-6}} = 85.0 \text{ MPa} < 98.3 \text{ MPa}$$

Use $t = 12 \text{ mm.}$

Problem 10.116



10.116 Solve Prob. 10.115, using the interaction method, with $P = 165 \text{ kN}$, $e = 15 \text{ mm}$, and an allowable stress in bending of 150 MPa .

10.115 A steel tube of 80-mm outer diameter is to carry a 93-kN load P with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable stress method, determine the lightest tube that can be used. Assume $E = 200 \text{ GPa}$ and $\sigma_y = 250 \text{ MPa}$.

$$r_o = \frac{1}{2} d_o = 40 \text{ mm} \quad r_i = r_o - t \\ A = \pi(r_o^2 - r_i^2) \quad I = \frac{\pi}{4}(r_o^4 - r_i^4) \quad r = \sqrt{\frac{I}{A}}$$

t mm	r_i mm	A mm^2	I 10^6 mm^4	r mm
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$$L_e = 2.2 \text{ m}$$

$$P = 165 \times 10^3 \text{ N}$$

$$\sigma_{\text{allow, bending}} = 150 \text{ MPa}$$

$$\text{Steel: } E = 200000 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

$$\text{Try } t = 9 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < C_c \quad \frac{L_e/r}{C_c} = 0.6917$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.6917) - \frac{1}{8}(0.6917)^3 = 1.885$$

$$\sigma_{\text{all, eccentric}} = \frac{\sigma_r}{\text{F.S.}} [1 - \frac{1}{2}(\frac{L_e/r}{C_c})^2] = \frac{250}{1.885} [1 - \frac{1}{2}(0.6917)^2] = 100.9 \text{ MPa}$$

$$\frac{P}{A\sigma_{\text{all, eccentric}}} + \frac{Pec}{I\sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(2007 \times 10^{-4})(100.9 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.285 \times 10^{-4})(150 \times 10^6)} \\ = 0.815 + 0.514 = 1.329 > 1 \quad (\text{not allowed})$$

$$\text{Approximate required area: } A = (1.329)(2007) = 2667 \text{ mm}^2$$

$$\text{Try } t = 12 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < C_c \quad \frac{L_e/r}{C_c} = 0.7172$$

$$\text{F.S.} = 1.890 \quad \sigma_{\text{all, eccentric}} = 98.3 \text{ MPa}$$

$$\frac{P}{A\sigma_{\text{all, eccentric}}} + \frac{Pec}{I\sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(2564 \times 10^{-4})(98.3 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.528 \times 10^{-4})(150 \times 10^6)} \\ = 0.655 + 0.432 = 1.087 > 1 \quad (\text{not allowed})$$

$$\text{Try } t = 15 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{23.59 \times 10^{-3}} = 93.26 < C_c \quad (L_e/r)/C_c = 0.7422$$

$$\text{F.S.} = 1.894 \quad \sigma_{\text{all, eccentric}} = 95.64 \text{ MPa}$$

$$\frac{P}{A\sigma_{\text{all, eccentric}}} + \frac{Pec}{I\sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(3063 \times 10^{-4})(95.64 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.704 \times 10^{-4})(150 \times 10^6)} \\ = 0.563 + 0.387 = 0.950 < 1 \quad (\text{allowed})$$

Use $t = 15 \text{ mm.}$

Problem 10.117

10.117 Supports A and B of the pin-ended column shown are at a fixed distance L from each other. Knowing that at a temperature T_0 the force in the column is zero and that buckling occurs when the temperature is $T_1 = T_0 + \Delta T$, express ΔT in terms of b, L, and the coefficient of thermal expansion α .



Let P be the compressive force in the column.

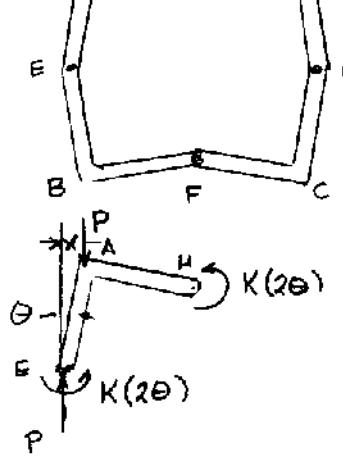
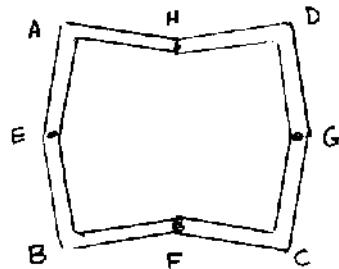
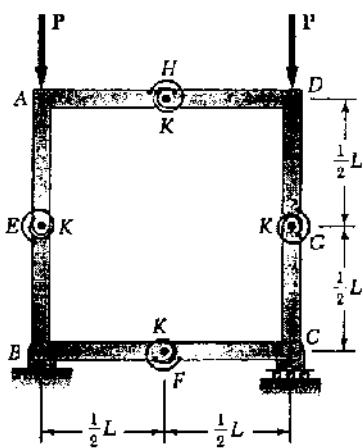
$$L\alpha(\Delta T) - \frac{PL}{EA} = 0 \quad P = EA\alpha(\Delta T)$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = P = EA\alpha(\Delta T)$$

$$\Delta T = \frac{\pi^2 EI}{L^2 EA\alpha} = \frac{\pi^2 E b^4 / 12}{L^2 E b^2} = \frac{\pi^2 b^2}{12 L^2 \alpha}$$

Problem 10.118

10.118 A frame consists of four L-shaped members connected by four torsional springs, each of constant K. Knowing that equal loads P are applied at points A and D as shown, determine the critical value P_{cr} of the loads applied to the frame.



Let Θ be the rotation of each L-shaped member.

Angle change across each torsional spring is 2θ .

$$x = \frac{1}{2}L \sin \theta \approx \frac{1}{2}L\theta$$

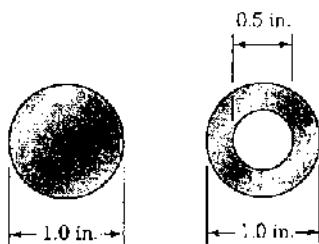
$$\sum M_E = 0:$$

$$K(2\theta) + K(2\theta) - Px = 0$$

$$P_{cr} = \frac{4K\theta}{x} = \frac{8K}{L}$$

Problem 10.119

10.119 A compression member of 20-in. effective length consists of a solid 1.0-in.-diameter aluminum rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (a) the percent reduction in the critical load, (b) the value of the critical load for the hollow rod. Use $E = 10.6 \times 10^6$ psi.



$$\text{Solid: } A_s = \frac{\pi}{4} d_o^2 \quad I_s = \frac{\pi}{4} \left(\frac{d_o}{2}\right)^4 = \frac{\pi}{64} d_o^4$$

$$\text{Hollow: } A_h = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{3}{4} A_s = \frac{3}{4} \frac{\pi}{64} d_o^4$$

$$d_i^2 = \frac{1}{4} d_o^2 \quad d_i = \frac{1}{2} d_o = 0.5 \text{ in.}$$

$$\text{Solid rod: } I_s = \frac{\pi}{64} (1.0)^4 = 0.049087 \text{ in}^4$$

$$P_{cr} = \frac{\pi^2 E I_s}{L^2} = \frac{\pi^2 (10.6 \times 10^6)(0.049087)}{(20)^2} = 12.839 \times 10^3 \text{ lb.}$$

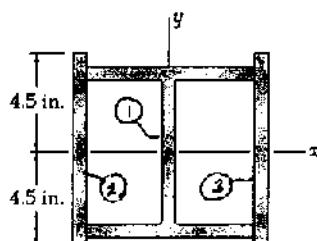
$$\text{Hollow rod: } I_h = \frac{\pi}{64} (d_o^4 - d_i^4) = \frac{\pi}{64} \left[(1)^4 - \left(\frac{1}{2}\right)^4 \right] = 0.046019 \text{ in}^4$$

$$(b) \quad P_{cr} = \frac{\pi^2 E I_h}{L^2} = \frac{\pi^2 (10.6 \times 10^6)(0.046019)}{(20)^2} = 12.036 \times 10^3 \text{ lb.} = 12.04 \text{ kips} \quad \blacksquare$$

$$(a) \quad \frac{P_s - P_h}{P_s} = \frac{12.839 \times 10^3 - 12.036 \times 10^3}{12.839 \times 10^3} = 0.0625 = 6.25\% \quad \blacksquare$$

Problem 10.120

10.120 A column of 22-ft effective length is to be made by welding two 9×0.5 in. plates to a W8 \times 35 as shown. Determine the allowable centric load if a factor of safety of 2.3 is required. Use $E = 29 \times 10^6$ psi.



$$\textcircled{1} \quad \text{W } 8 \times 35 \quad I_x = 127 \text{ in}^4 \quad I_y = 42.6 \text{ in}^4$$

$$b_f = 8.02 \text{ in}$$

$$\textcircled{2} \text{ and } \textcircled{3} \quad \text{For each plate: } A = (0.5)(9.0) = 4.5 \text{ in}^2$$

$$I_x = \frac{1}{12} (0.5)(9)^3 = 30.375 \text{ in}^4$$

$$I_y = \frac{1}{12} (9)(0.5)^3 + (4.5) \left(\frac{8.02}{2} + \frac{0.5}{2} \right)^2 = 81.758 \text{ in}^4$$

$$\text{Column: } I_x = 127 + (2)(30.375) = 187.75 \text{ in}^4 = I_{min}$$

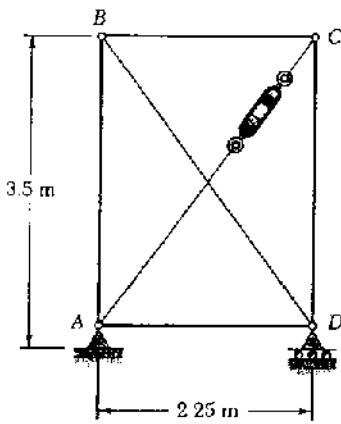
$$I_y = 42.6 + (2)(81.758) = 206.12 \text{ in}$$

$$L = 22 \text{ ft} = 264 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(187.75)}{264^2} = 771.0 \times 10^3 \text{ lb} = 771 \text{ kips}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{771}{2.3} = 335 \text{ kips} \quad \blacksquare$$

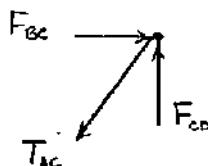
Problem 10.121



10.121 Members *AB* and *CD* are 30-mm-diameter steel rods, and members *BC* and *AD* are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that a factor of safety with respect to buckling of 2.75 is required, determine the largest allowable tension in *AC*. Use $E = 200 \text{ GPa}$ and consider only buckling in the plane of the structure.

$$L_{AC} = \sqrt{(3.5)^2 + (2.25)^2} = 4.1608 \text{ m}$$

Joint C



$$\pm \sum F_x = 0: F_{BC} - \frac{2.25}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.84926 F_{BC}$$

$$+ \sum F_y = 0: F_{CD} - \frac{3.5}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.1888 F_{CD}$$

Members *BC* and *AD*: $I_{BC} = \frac{\pi}{4} \left(\frac{d_{BC}}{2}\right)^4 = \frac{\pi}{4} \left(\frac{22}{2}\right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-7} \text{ m}^4$

$$L_{BC} = 2.25 \text{ m} \quad F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-7})}{(2.25)^2} = 4.4836 \times 10^3 \text{ N}$$

$$F_{BC,al} = \frac{F_{BC,cr}}{F.S.} = 1.6304 \times 10^3 \text{ N} \quad T_{AC,al} = 3.02 \times 10^3 \text{ N}$$

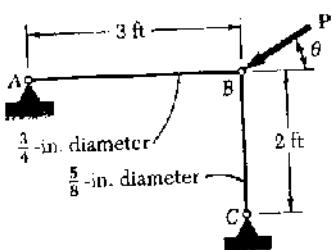
Members *AB* and *CD*: $I_{CD} = \frac{\pi}{4} \left(\frac{d_{CD}}{2}\right)^4 = \frac{\pi}{4} \left(\frac{30}{2}\right)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$

$$L_{CD} = 3.5 \text{ m} \quad F_{CD,cr} = \frac{\pi^2 E I_{CD}}{L_{CD}^2} = \frac{\pi^2 (200 \times 10^9) (39.761 \times 10^{-9})}{(3.5)^2} = 6.4069 \times 10^3 \text{ N}$$

$$F_{CD,al} = \frac{F_{CD,cr}}{F.S.} = 2.3298 \times 10^3 \text{ N} \quad T_{AC,al} = 2.77 \times 10^3 \text{ N}$$

Smaller value for $T_{AC,al}$ governs. $T_{AC,al} = 2.77 \times 10^3 \text{ N} = 2.77 \text{ kN}$

Problem 10.122



10.122 (a) Considering only buckling in the plane of the structure shown and using Euler's formula, determine the value of θ between 0 and 90° for which the allowable magnitude of the load P is maximum. (b) Determine the corresponding maximum value of P knowing that a factor of safety of 3.2 is required. Use $E = 29 \times 10^6$ psi.

Strut AB. $L = 3 \text{ ft} = 36 \text{ in.}$

$$c = \frac{1}{2}d = \frac{1}{2}\left(\frac{3}{4}\right) = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = 15.5316 \times 10^{-3} \text{ in}^4$$

$$(P_{AB})_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29 \times 10^6)(15.5316 \times 10^{-3})}{(36)^2} \\ = 3.4301 \times 10^3 \text{ lb.}$$

Strut BC. $L = 2 \text{ ft} = 24 \text{ in.}, c = \frac{1}{2}\left(\frac{5}{8}\right) = 0.3125 \text{ in.}, I = 7.4901 \times 10^{-5} \text{ in}^4$

$$(P_{BC})_{cr} = \frac{\pi^2 (29 \times 10^6)(7.4901 \times 10^{-5})}{(24)^2} = 3.7219 \times 10^3 \text{ lb.}$$

For the structure, P is maximum if both struts buckle simultaneously.

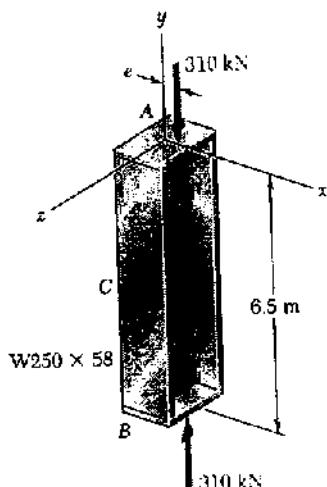
$$(a) \tan \theta = \frac{(P_{BC})_{cr}}{(P_{AB})_{cr}} = 1.08507 \quad \theta = 47.2^\circ$$

$$(b) P_{cr} = \sqrt{(P_{AB})_{cr}^2 + (P_{BC})_{cr}^2} = 5.0614 \times 10^3 \text{ lb} = 5.0614 \text{ kips}$$

With F.S. = 3.2, $P = \frac{5.0614}{3.2} = 1.582 \text{ kips}$

Problem 10.123

10.123 The line of action of the 310-kN axial load is parallel to the geometric axis of the column AB and intersects the x axis at $x = e$. Using $E = 200 \text{ GPa}$, determine (a) the eccentricity e when the deflection of the midpoint C of the column is 9 mm, (b) the corresponding maximum stress in the column.



For W250x58

$$A = 7420 \text{ mm}^2 = 7420 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.8 \times 10^6 \text{ mm}^4 = 18.8 \times 10^{-6} \text{ m}^4$$

$$S_y = 185 \times 10^3 \text{ mm}^3 = 185 \times 10^{-6} \text{ m}^3$$

$$L_e = 6.5 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.8 \times 10^{-6})}{(6.5)^2} = 878.3 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{310 \times 10^3}{878.3 \times 10^3} = 0.35294$$

$$y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = 0.67990 e$$

$$(a) e = \frac{y_{max}}{0.67990} = \frac{9 \times 10^{-3}}{0.67990} = 13.24 \times 10^{-3} \text{ m} = 13.24 \text{ mm}$$

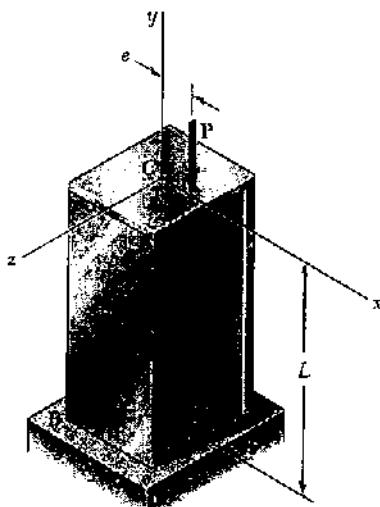
$$(b) M_{max} = P(e + y_{max}) = (310 \times 10^3)(9 + 13.24)(10^{-3}) = 6893.5 \text{ N-m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{310 \times 10^3}{7420 \times 10^{-6}} + \frac{6893.5}{185 \times 10^{-6}}$$

$$= 41.78 \times 10^6 + 37.26 \times 10^6 = 79.04 \times 10^6 \text{ Pa} = 79.0 \text{ MPa}$$

Problem 10.124

10.124 A 26-kip axial load P is applied to a W6 × 12 rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 0.25$ in., determine the largest permissible length L if the allowable stress in the column is 14 ksi. Use $E = 29 \times 10^6$ psi.



Data: $P = 26$ kips, $e = 0.25$ in

$$E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$\text{W } 6 \times 12: A = 3.55 \text{ in}^2 \quad b_f = 4.000 \text{ in}$$

$$c = \frac{b_f}{2} = 2.000 \text{ in}, \quad I_y = 2.99 \text{ in}^4, \quad r_y = 0.918 \text{ in.}$$

$$\sigma_{max} = 14 \text{ ksi}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec} \left(\frac{A\sigma_{max}}{P} - 1 \right) = \frac{(0.918)^2}{(0.25)(2.000)} \left[\frac{(3.55)(14)}{26} - 1 \right] = 1.53635$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.65089 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.86204$$

$$\frac{P}{P_{cr}} = \left[\frac{\pi}{2}(0.86204) \right]^2 = 0.30117$$

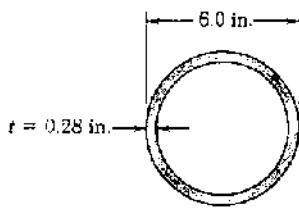
$$P_{cr} = \frac{P}{0.30117} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.30117 \pi^2 EI}{P} = \frac{0.30117 \pi^2 (29000)(2.99)}{26} = 9.913 \times 10^3 \text{ in}^2$$

$$L_e = 99.56 \text{ in} = 2L$$

$$L = 49.78 \text{ in.} = 4.15 \text{ ft}$$

Problem 10.125



10.125 A steel pipe having the cross section shown is used as a column. Using the AISC allowable stress design formulas, determine the allowable centric load if the effective length of the column is (a) 18 ft, (b) 26 ft. Use $\sigma_y = 36 \text{ ksi}$ and $E = 29 \times 10^6 \text{ psi}$.

$$C_o = \frac{d_o}{2} = 3.0 \text{ in} \quad C_c = C_o - t = 2.72 \text{ in}$$

$$A = \pi (C_o^2 - C_c^2) = 5.0316 \text{ in}^2 \quad r = \sqrt{\frac{I}{A}} = 2.0247 \text{ in}$$

$$I = \frac{\pi}{4} (C_o^4 - C_c^4) = 20.627 \text{ in}^4$$

$$\text{Steel: } E = 29000 \text{ ksi} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

$$(a) L_e = 18 \text{ ft} = 216 \text{ in} \quad L_e/r = 106.68 < C_c \quad \frac{L_e/r}{C_c} = 0.84601$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.84601) - \frac{1}{8}(0.84601)^3 = 1.9082$$

$$\sigma_{all} = \frac{\sigma_y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.9082} \left[1 - \frac{1}{2} (0.84601)^2 \right] = 12.11 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (12.11)(5.0316) = 61.0 \text{ kips}$$

$$(b) L_e = 26 \text{ ft} = 312 \text{ in} \quad L_e/r = 154.097 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = \frac{\pi^2 (29000)}{(1.92)(154.097)^2} = 6.28 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (6.28)(5.0316) = 31.6 \text{ kips}$$

Problem 10.126

10.126 A column of 4.6-m effective length must carry a centric load of 525 kN. Knowing that $\sigma_y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$, use the AISC allowable stress design formulas to select the wide-flange shape of 200-mm nominal depth that should be used.

$$P < \frac{\sigma_y A}{\text{F.S.}}$$

$$A > \frac{(\text{F.S.}) P}{\sigma_y} = \frac{(5/3)(525 \times 10^3)}{345 \times 10^6} = 2.54 \times 10^{-3} \text{ m}^2 = 2540 \text{ mm}^2$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2}$$

$$I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(525 \times 10^3)(4.6)^2}{\pi^2 (200 \times 10^9)} = 10.89 \times 10^{-6} \text{ m}^4 = 10.89 \times 10^4 \text{ mm}^4$$

$$\text{Try W 200 x 46.1. } A = 5860 \text{ mm}^2, I_{min} = 15.3 \times 10^4 \text{ mm}^4, r = 51.1 \times 10^{-3} \text{ m}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{3\pi^2 (200 \times 10^9)}{345 \times 10^6}} = 106.97$$

$$\frac{L_e}{r} = \frac{4.6}{51.1 \times 10^{-3}} = 90.02 < C_c \quad \frac{L_e/r}{C_c} = 0.84154$$

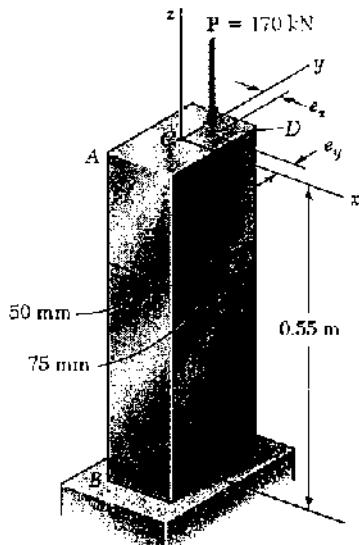
$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.84154) - \frac{1}{8}(0.84154)^3 = 1.9077$$

$$\sigma_{all} = \frac{\sigma_y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{345 \times 10^6}{1.9077} \left[1 - \frac{1}{2} (0.84154)^2 \right] = 116.8 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (116.8 \times 10^6)(5860 \times 10^{-6}) = 684 \text{ kN} > 525 \text{ kN}$$

Use W 200 x 46.1.

Problem 10.127



10.127 The compression member *AB* is made of a steel for which $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$. It is free at its top *A* and fixed at its base *B*. Using the interaction method with an allowable bending stress equal to 120 MPa and knowing that the eccentricities e_x and e_y are equal, determine their largest allowable common value.

$$\text{Steel: } \sigma_y = 250 \text{ MPa} \quad E = 200000 \text{ MPa}$$

$$C_c : \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

$$A = (75 \times 10^{-3})(50 \times 10^{-3}) = 3750 \times 10^{-6} \text{ m}^2$$

$$I_y = \frac{1}{12}(75 \times 10^{-3})(50 \times 10^{-3})^3 = 781.25 \times 10^{-9} \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = 14.434 \times 10^{-3} \text{ m}$$

$$I_x = \frac{1}{12}(50 \times 10^{-3})(75 \times 10^{-3}) = 1.7578 \times 10^{-6} \text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = 21.651 \times 10^{-3} \text{ m}$$

$$L_e = 2L = (2)(0.55) = 1.10 \text{ m} \quad L_e/r_{min} = 1.10 / 14.434 \times 10^{-3} = 76.21 < C_c$$

$$\frac{L_e/r_{min}}{C_c} = \frac{76.21}{125.66} = 0.6065 \quad \text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.6065) - \frac{1}{8}(0.6065)^3 = 1.8662$$

$$\sigma_{all(\text{concrete})} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r_{min}}{C_c} \right)^2 \right] = \frac{250}{1.8662} \left[1 - \frac{1}{2} (0.6065)^2 \right] = 109.32 \text{ MPa}$$

$$\sigma_{all(\text{bending})} = 120 \text{ MPa}$$

$$\frac{P}{A\sigma_{all(\text{concrete})}} + \frac{Pe_{xy}}{I_x \sigma_{all(\text{bending})}} + \frac{Pe_y x}{I_y \sigma_{all(\text{bending})}} = 1 \quad \text{with } e_x = e_y$$

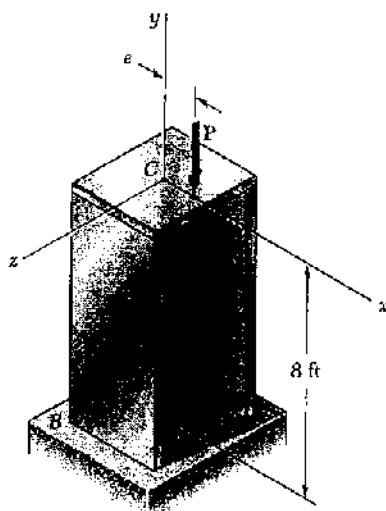
$$\frac{P}{\sigma_{all(\text{bending})}} \left(\frac{y}{I_x} + \frac{x}{I_y} \right) e = 1 - \frac{P}{A\sigma_{all(\text{concrete})}}$$

$$\frac{170 \times 10^3}{120 \times 10^6} \left(\frac{37.5 \times 10^{-3}}{1.7578 \times 10^{-6}} + \frac{25 \times 10^{-3}}{781.25 \times 10^{-9}} \right) e = 1 - \frac{170 \times 10^3}{(3750 \times 10^{-6})(109.32 \times 10^6)}$$

$$75.556 e = 1 - 0.41468$$

$$e = 7.75 \times 10^{-3} \text{ m} = 7.75 \text{ mm}$$

Problem 10.128



10.128 A 43-kip axial load P is applied to the rolled-steel column BC at a point on the x axis at a distance $e = 2.5$ in. from the geometric axis of the column. Using the allowable-stress method, select the wide-flange shape of 8-in. nominal depth that should be used. Use $E = 29 \times 10^6$ psi. and $\sigma_y = 36$ ksi.

$$\text{Steel: } E = 29 \times 10^6 \text{ ksi} \quad \sigma_y = 36 \text{ ksi}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

$$L = 8 \text{ ft} = 96 \text{ in} \quad L_e = 2L = 192 \text{ in}$$

$$\text{Try } \underline{W 8 \times 31}. \quad r_y = 2.02 \text{ in}, \quad \frac{L_e}{r_y} = 95.05 < C_c$$

$$\frac{L_e/r_y}{C_c} = 0.754$$

$$\text{F.S.} = \frac{5}{3} + \frac{2}{3}(0.754) - \frac{1}{3}(0.754)^2 = 1.896$$

$$\sigma_{all} = \frac{\sigma_y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r_y}{C_c} \right)^2 \right] = \frac{36}{1.896} \left[1 - \frac{1}{2} (0.754)^2 \right] \\ = 13.59 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{9.13} + \frac{(43)(2.5)(\frac{1}{4} \cdot 7.995)}{37.1} = 4.71 + 11.58 = 16.29 \text{ ksi} \\ > 13.59 \text{ ksi}$$

(not allowed)

$$\text{Approximate required area} = \frac{16.29}{13.59} \times (9.13) = 10.9 \text{ in}^2$$

$$\text{Try } \underline{W 8 \times 35}. \quad r_y = 2.03 \quad \frac{L_e}{r_y} = 94.58 < C_c \quad \frac{L_e/r_y}{C_c} = 0.750$$

$$\text{F.S.} = 1.895 \quad \sigma_{all} = 13.65 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{10.3} + \frac{(43)(2.5)(\frac{1}{4} \cdot 8.020)}{42.6} = 14.29 \text{ ksi} > 13.65 \text{ ksi} \quad (\text{not allowed})$$

$$\text{Try } \underline{W 8 \times 40}. \quad r_y = 2.04 \quad \frac{L_e}{r_y} = 94.12 < C_c \quad \frac{L_e/r_y}{C_c} = 0.746$$

$$\text{F.S.} = 1.895 \quad \sigma_{all} = 13.71 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{11.7} + \frac{(43)(2.5)(\frac{1}{4} \cdot 8.07)}{49.1} = 12.51 \text{ ksi} < 13.71 \text{ ksi} \quad (\text{allowed})$$

Use $\underline{W 8 \times 40}$.

PROBLEM 10.C1

10.C1 A solid steel rod having an effective length of 500 mm is to be used as a compression strut to carry a centric load P . For the grade of steel used $E = 200$ GPa and $\sigma_y = 245$ MPa. Knowing that a factor of safety of 2.8 is required and using Euler's formula, write a computer program and use it to calculate the allowable centric load P_{all} for values of the radius of the rod from 6 mm to 24 mm, using 2-mm increments.

SOLUTION

ENTER RADIUS RAD, EFFECTIVE LENGTH L_e
AND FACTOR OF SAFETY FS

COMPUTE RADIUS OF GYRATION

$$A = \pi R^2$$

$$I = \frac{1}{4} \pi R^4$$

$$r = \sqrt{\frac{I}{A}}$$

DETERMINE ALLOWABLE CENTRIC LOADCRITICAL STRESS:

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

LET σ EQUAL SMALLER OF σ_{cr} AND σ_y

$$P_{all} = \frac{\sigma A}{FS}$$

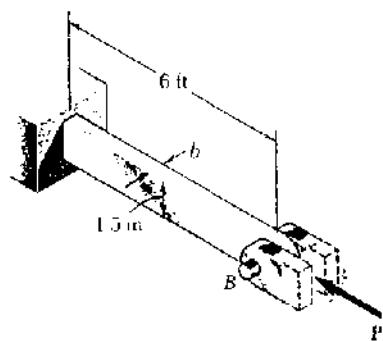
PROGRAM OUTPUT

Radius of rod m	Critical stress MPa	Allowable load kN
.006	71.1	2.87
.008	126.3	9.07
.010	197.4	22.15

.012	284.2	39.58
.014	386.9	53.88
.016	505.3	70.37
.018	639.6	89.06
.020	789.6	109.96
.022	955.4	133.05
.024	1137.0	158.34

Below the dashed line we have:
 $\sigma_{cr} > \sigma_y$

PROBLEM 10.C2



10.C2 An aluminum bar is fixed at end *A* and supported at end *B* so that it is free to rotate about a horizontal axis through the pin. Rotation about a vertical axis at end *B* is prevented by the brackets. Knowing that $E = 10.1 \times 10^6$ psi, use Euler's formula with a factor of safety of 2.5 to determine the allowable centric load P for values of b from 0.75 in. to 1.5 in., using 0.125-in. increments.

SOLUTION

ENTER E , LENGTH L AND FACTOR OF SAFETY FS

FOR $b = 0.75$ TO 1.5 WITH 0.125 INCREMENTS

COMPUTE RADIOS OF GYRATION

$$A = 1.5 b$$

$$I_x = \frac{1}{12} b 1.5^3$$

$$I_y = \frac{1}{8} b^3$$

$$r_x = \sqrt{\frac{I_x}{A}}$$

$$r_y = \sqrt{\frac{I_y}{A}}$$

COMPUTE CRITICAL STRESSES

$$(\sigma_{cr})_x = \frac{\pi^2 E}{(0.7L/r_x)^2}$$

$$(\sigma_{cr})_y = \frac{\pi^2 E}{(0.5L/r_y)^2}$$

LET σ_{cr} EQUAL SMALLER STRESS

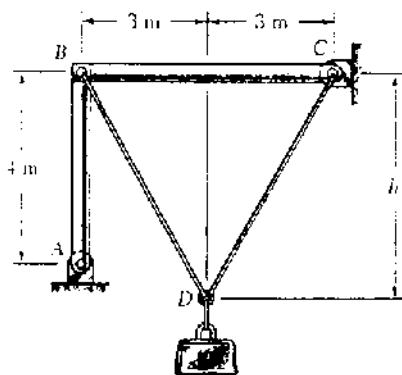
COMPUTE ALLOWABLE CENTRIC LOAD

$$P_{all} = \frac{\sigma_{cr} A}{FS}$$

PROGRAM OUTPUT

b in.	Critical stress x axis ksi	Critical stress y axis ksi	Allowable load kips
.750	7.358	3.6	1.62
.875	7.358	4.9	2.58
1.000	7.358	6.4	3.85
1.125	7.358	8.1	4.97
1.250	7.358	10.0	5.52
1.375	7.358	12.1	6.07
1.500	7.358	14.4	6.62

PROBLEM 10.C3



10.C3 The pin-connected members AB and BC consist of sections of aluminum pipe of 120-mm outer diameter and 10-mm wall thickness. Knowing that a factor of safety of 3.5 is required, determine the mass m of the largest block that can be supported by the cable arrangement shown for values of h from 4 m to 8 m, using 0.25-m increments. Use $E = 70 \text{ GPa}$ and consider only buckling in the plane of the structure.

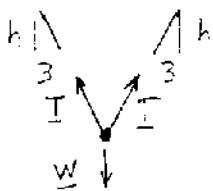
SOLUTION

COMPUTE MOMENT OF INERTIA

$$I = \frac{\pi}{4} (0.36^4 - 0.05^4)$$

FOR $h = 4$ TO 8 USING 0.25 INCREMENTS

JOINT 0:



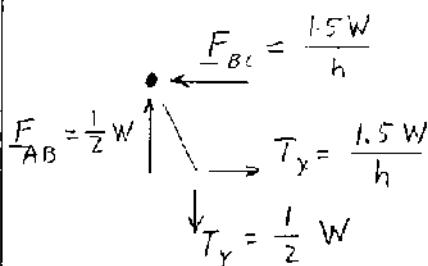
$\sum F_y = 0$ YIELDS

$$T_y = \frac{1}{2}W$$

$$\frac{T_x}{T_y} = \frac{3}{h} \text{ YIELDS}$$

$$T_x = \frac{1.5W}{h}$$

JOINT 2:



$$F_{Bc} = \frac{1.5W}{h}$$

$$F_{AB} = \frac{1}{2}W$$

$$T_x = \frac{1.5W}{h}$$

$$T_y = \frac{1}{2}W$$

CHECKS ALLOWABLE LOADS FOR JUMPS

$$(F_{AB})_{cr} = \frac{\pi^2 EI}{3.5(4)^2}; (F_{Bc})_{cr} = \frac{\pi^2 EI}{3.5(6)^2}$$

DETERMINE ALLOWABLE W

$$(W_{all})_1 = 2(F_{AB})_{cr}; (W_{all})_2 = \frac{h}{1.5}(F_{Bc})_{cr}$$

W_{all} EQUALS SMALLER VALUE

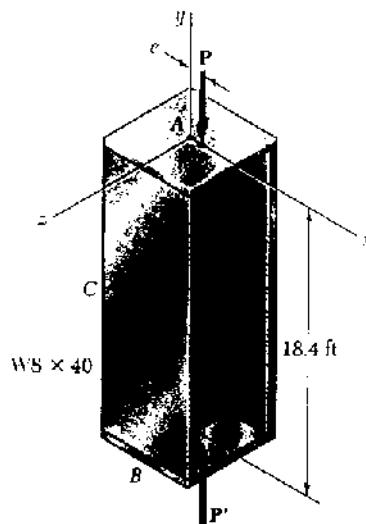
COMPUTE MASS m

$$m = \frac{W_{all}}{9.81}$$

PROGRAM OUTPUT

h m	Weight critical stress AB kN	Weight critical stress BC kN	mass kg
4.00	455.11	269.7	7854.88
4.25	455.11	286.6	8345.80
4.50	455.11	303.4	8836.74
4.75	455.11	320.3	9327.66
5.00	455.11	337.1	9818.59
5.25	455.11	354.0	10309.52
5.50	455.11	370.8	10800.45
5.75	455.11	387.7	11291.38
6.00	455.11	404.5	11782.31
6.25	455.11	421.4	12273.24
6.50	455.11	438.3	12764.17
6.75	455.11	455.1	13255.10
7.00	455.11	472.0	13255.10
7.25	455.11	488.8	13255.10
7.50	455.11	505.7	13255.10
7.75	455.11	522.5	13255.10
8.00	455.11	539.4	13255.10

PROBLEM 10.C4



10.C4 An axial load P is applied at a point located on the x axis at a distance $e = 0.5$ in. from the geometric axis of the W8 \times 40 rolled-steel column AB . Using $E = 29 \times 10^6$ psi, write a computer program and use it to calculate for values of P from 25 to 75 kips, using 5-kip increments, (a) the horizontal deflection at the midpoint C , (b) the maximum stress in the column.

SOLUTION

ENTER LENGTH L , ECCENTRICITY e

ENTER PROPERTIES A, I_y, r_y, b_f

COMPUTE CRITICAL LOAD

$$P_{cr} = \frac{\pi^2 E I_y}{L^2}$$

FOR $P = 25$ TO 75 IN INCREMENTS OF 5

COMPUTE HORIZONTAL DEFLECTION AT C

$$y_c = e \left(\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1.0 \right)$$

COMPUTE MAXIMUM STRESS

$$\sigma_{max} = \frac{P}{A} \left(1 + \frac{e b_f}{2 r_y^2} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

PROGRAM OUTPUT

Load kip	maximum deflection in.	maximum stress kips
25.0	.059	3.29
30.0	.072	3.99
35.0	.086	4.69
40.0	.100	5.41
45.0	.115	6.14
50.0	.130	6.88
55.0	.146	7.65
60.0	.163	8.43
65.0	.181	9.22
70.0	.199	10.04
75.0	.219	10.88

PROBLEM 10.C5

10.C5 A column of effective length L is made from a rolled-steel shape and carries a centric axial load P . The yield strength for the grade of steel used is denoted by σ_y , the modulus of elasticity by E , the cross-sectional area of the selected shape by A , and its smallest radius of gyration by r_y . Using the AISC design formulas for allowable stress design, write a computer program that can be used with either SI or U.S. customary units to determine the allowable load P . Use this program to solve (a) Prob. 10.57, (b) Prob. 10.58, (c) Prob. 10.60.

SOLUTION

ENTER L, E, σ_y

ENTER PROPERTIES: A, r_y

DETERMINE ALLOWABLE STRESS

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$$

IF $L/r_y \geq C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r_y)^2}$$

IF $L/r_y < C_c$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L/r_y}{C_c}\right) - \frac{1}{8}\left(\frac{L/r_y}{C_c}\right)^3$$

$$\sigma_{all} = \frac{\sigma_y}{FS} \left(1 - \frac{(L/r_y)^2}{2C_c^2}\right)$$

CACULATE ALLOWABLE LOAD:

$$P_{all} = \sigma_{all} A$$

CONTINUED

PROBLEM 10.CS CONTINUED

PROGRAM OUTPUT

Problem 10.57 a

Effective Length = 21.00 ft
A = 9.130 in**2
ry = 2.020 in.
Yield strength = 36.0 ksi
E = 29000 ksi

Allowable centroid load: P = 87.566 kips

Problem 10.57 b

Effective Length = 21.00 ft
A = 9.130 in**2
ry = 2.020 in.
Yield strength = 50.0 ksi
E = 29000 ksi

Allowable centroid load: P = 87.452 kips

Problem 10.58 a

Effective Length = 6.00 m
A = 4580.0 mm**2
ry = 40.8 mm
Yield strength = 250.0 MPa
E = 200 GPa

Allowable centroid load: P = 217.727 kN

Problem 10.58 b

Effective Length = 6.00 m
A = 11000.0 mm**2
ry = 53.2 mm
Yield strength = 250.0 MPa
E = 200 GPa

Allowable centroid load: P = 858.637 kN

Problem 10.60 a

Effective Length = 4.00 m
A = 13.8 mm**2
ry = 43.4 mm
Yield strength = 345.0 MPa
E = 200 GPa

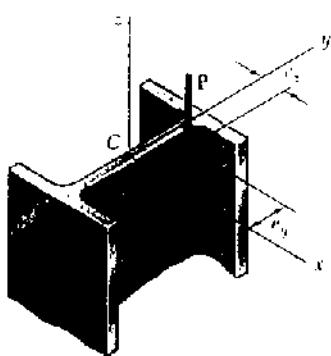
Allowable centroid load: P = 1568 kN

Problem 10.60 b

Effective Length = 6.50 m
A = 13800.0 mm**2
ry = 43.4 mm
Yield strength = 345.0 MPa
E = 200 GPa

Allowable centroid load: P = 632.667 kN

PROBLEM 10.C6



10.C6 A column of effective length L is made from a rolled-steel shape and is loaded eccentrically as shown. The yield strength of the grade of steel used is denoted by σ_y , the allowable stress in bending by σ_{all} , the modulus of elasticity by E , the cross-sectional area of the selected shape by A , and its smallest radius of gyration by r_y . Write a computer program that can be used with either SI or U.S. customary units to determine the allowable load P , using either the allowable-stress method or the interaction method. Use this program to check the given answer for (a) Prob. 10.113, (b) Prob. 10.114.

SOLUTION

ENTER L , E , σ_y , $(\sigma_{all})_{\text{bending}}$, e_x , e_y

ENTER PROPERTIES A , s_x , s_y , r_y

DETERMINE ALLOWABLE STRESS

$$C_c = \sqrt{\frac{2T^2 E}{\sigma_y}}$$

$$\text{IF } L/r_y \geq C_c$$

$$J_{eff} = \frac{\pi^2 E}{1.92(L/r)^2}$$

$$\text{IF } L/r_y < C_c$$

$$FS = \frac{5}{3} + \frac{3}{2} \left(\frac{L/r_y}{C_c} \right) - \frac{1}{8} \left(\frac{L/r_y}{C_c} \right)^3$$

$$\sigma_{all} = \frac{\sigma_y}{FS} \left(1 - \frac{(L/r_y)^2}{2C_c^2} \right)$$

FOR ALLOWABLE-STRESS METHOD

$$COFF = \frac{1}{A} + \frac{e_x}{s_x} + \frac{e_y}{s_y}$$

$$P_{all} = \frac{J_{eff}}{COFF}$$

FOR INTERACTION METHOD

$$COFF = \frac{1}{A \sigma_{all}} + \frac{(e_x/s_x) + (e_y/s_y)}{(\sigma_{all})_{\text{bending}}}$$

$$P_{all} = \frac{1.0}{COFF}$$

CONTINUED

PROBLEM 10.C6 CONTINUED

PROGRAM OUTPUT

Problem 10.113

Effective Length = 24.00 ft
A = 42.700 in**2
RY = 3.980 in.
SX = 232.00 in**3
Yield strength = 36.0 ksi
E = 29×10^3 ksi

Using Allowable-Stress Method
Allowable load: P = 279.496 kips

Problem 10.114

Effective Length = 24.00 ft
A = 20.000 in**2
RY = 2.460 in.
SX = 103.00 in**3
Yield strength = 50.0 ksi
E = 29×10^3 ksi

Using Interaction Method
Allowable load: P = 139.157 kips

Chapter 11

Problem 11.1

11.1 Determine the modulus of resilience for each of the following grades of structural steel:

- (a) ASTM A709 Grade 50: $\sigma_y = 50 \text{ ksi}$
- (b) ASTM A913 Grade 65: $\sigma_y = 65 \text{ ksi}$
- (c) ASTM A709 Grade 100: $\sigma_y = 100 \text{ ksi}$

Structural steel $E = 29 \times 10^6 \text{ psi}$ for all three steels given.

(a) $\sigma_y = 50 \text{ ksi} = 50 \times 10^3 \text{ psi}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(50 \times 10^3)^2}{(2)(29 \times 10^6)} = 43.1 \text{ in. lb/in}^3$$

(b) $\sigma_y = 65 \text{ ksi} = 65 \times 10^3 \text{ psi}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(65 \times 10^3)^2}{(2)(29 \times 10^6)} = 72.8 \text{ in. lb/in}^3$$

(c) $\sigma_y = 100 \text{ ksi} = 100 \times 10^3 \text{ psi}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(100 \times 10^3)^2}{(2)(29 \times 10^6)} = 172.4 \text{ in. lb/in}^3$$

Problem 11.2

11.2 Determine the modulus of resilience for each of the following aluminum alloys:

- (a) 1100-H14: $E = 70 \text{ GPa}$ $\sigma_y = 55 \text{ MPa}$
- (b) 2014-T6 $E = 75 \text{ GPa}$ $\sigma_y = 220 \text{ MPa}$
- (c) 6061-T6 $E = 70 \text{ GPa}$ $\sigma_y = 150 \text{ MPa}$

Aluminum alloys

(a) $E = 70 \times 10^9 \text{ Pa}$, $\sigma_y = 55 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(55 \times 10^6)^2}{(2)(70 \times 10^9)} = 21.6 \times 10^3 \text{ N.m/m}^2 = 21.6 \text{ kJ/m}^3$$

(b) $E = 75 \times 10^9 \text{ Pa}$, $\sigma_y = 220 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(220 \times 10^6)^2}{(2)(75 \times 10^9)} = 323 \times 10^3 \text{ N.m/m}^2 = 323 \text{ kJ/m}^3$$

(c) $E = 70 \times 10^9 \text{ Pa}$, $\sigma_y = 150 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(150 \times 10^6)^2}{(2)(70 \times 10^9)} = 160.7 \times 10^3 \text{ N.m/m}^2 = 160.7 \text{ kJ/m}^3$$

Problem 11.3

11.3 Determine the modulus of resilience for each of the following metals:

(a) Stainless steel

AISI 302 (annealed): $E = 190 \text{ GPa}$ $\sigma_y = 260 \text{ MPa}$

(b) Stainless steel 2014-T6

AISI 302 (cold-rolled): $E = 190 \text{ GPa}$ $\sigma_y = 520 \text{ MPa}$ (c) Malleable cast iron: $E = 165 \text{ GPa}$ $\sigma_y = 230 \text{ MPa}$

(a) $E = 190 \times 10^9 \text{ Pa}, \quad \sigma_y = 260 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(260 \times 10^6)^2}{(2)(190 \times 10^9)} = 177.9 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 177.9 \text{ kJ}/\text{m}^3$$

(b) $E = 190 \times 10^9 \text{ Pa}, \quad \sigma_y = 520 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(520 \times 10^6)^2}{(2)(190 \times 10^9)} = 712 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 712 \text{ kJ}/\text{m}^3$$

(c) $E = 165 \times 10^9 \text{ Pa}, \quad \sigma_y = 230 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(230 \times 10^6)^2}{(2)(165 \times 10^9)} = 160.3 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 160.3 \text{ kJ}/\text{m}^3$$

Problem 11.4

11.4 Determine the modulus of resilience for each of the following alloys:

(a) Titanium: $E = 16.5 \times 10^6 \text{ psi}$ $\sigma_y = 125 \text{ ksi}$ (b) Magnesium: $E = 6.5 \times 10^6 \text{ psi}$ $\sigma_y = 30 \text{ ksi}$ (c) Cupronickel (annealed): $E = 20 \times 10^6 \text{ psi}$ $\sigma_y = 18 \text{ ksi}$

(a) $E = 16.5 \times 10^6 \text{ psi}, \quad \sigma_y = 125 \times 10^3 \text{ psi}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(125 \times 10^3)^2}{(2)(16.5 \times 10^6)} = 473 \text{ in-lb/in}^3$$

(b) $E = 6.5 \times 10^6 \text{ psi}, \quad \sigma_y = 30 \times 10^3 \text{ psi}$

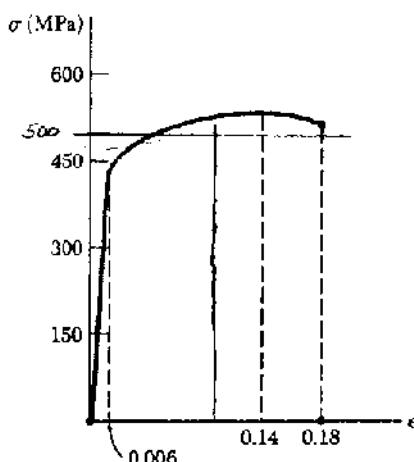
$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(30 \times 10^3)^2}{(2)(6.5 \times 10^6)} = 69.2 \text{ in-lb/in}^3$$

(c) $E = 20 \times 10^6 \text{ psi}, \quad \sigma_y = 18 \times 10^3 \text{ psi}$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(18 \times 10^3)^2}{(2)(20 \times 10^6)} = 8.10 \text{ in-lb/in}^3$$

Problem 11.5

11.5 The stress-strain diagram shown has been drawn from data obtained during a tensile test of an aluminum alloy. Using $E = 72 \text{ GPa}$, determine (a) the modulus of resilience of the alloy, (b) the modulus of toughness of the alloy.



$$(a) \sigma_y = E \epsilon_y$$

$$U_y = \frac{\sigma_y^2}{2E} = \frac{1}{2} E \epsilon_y^2 = \frac{1}{2} (72 \times 10^9) (0.006)^2 \\ = 1296 \times 10^3 \text{ N} \cdot \text{m}/\text{m}^3 = 1296 \text{ kJ/m}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

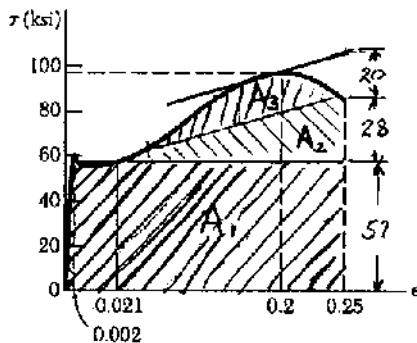
The average ordinate of the stress-strain curve is $500 \text{ MPa} = 500 \times 10^6 \text{ N/m}^2$

The area under the curve is $A = (500 \times 10^6)(0.18) = 90 \times 10^6 \text{ N/m}^2$

$$\text{modulus of toughness} = 90 \times 10^6 \text{ J/m}^3 = 90 \text{ MJ/m}^3$$

Problem 11.6

11.6 The stress-strain diagram shown has been drawn from data obtained during a tensile test of a specimen of structural steel. Using $E = 29 \times 10^6 \text{ psi}$, determine (a) the modulus of resilience of the steel, (b) the modulus of toughness of the steel.



$$(a) \sigma_y = E \epsilon_y$$

$$U_y = \frac{\sigma_y^2}{2E} = \frac{1}{2} E \epsilon_y^2 = \frac{1}{2} (29 \times 10^6) (0.002)^2 \\ = 58.0 \text{ in-lb/in}^2$$

(b) Modulus of toughness = total area under the stress-strain curve

$$A_1 = (57)(0.25 - 0.002) = 14.14 \text{ kips/in}^2 = 14.14 \text{ in-kip/in}^3$$

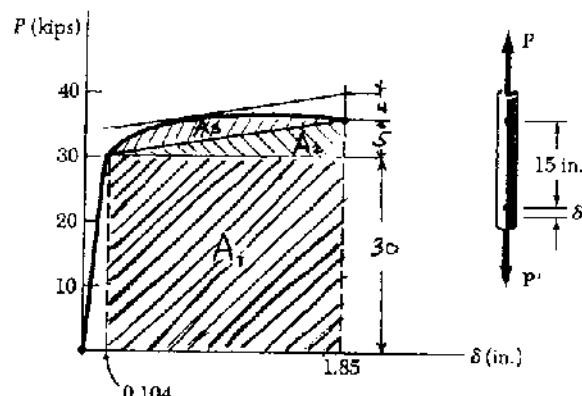
$$A_2 = \frac{1}{2}(28)(0.25 - 0.021) = 3.21 \text{ kips/in}^2 = 3.21 \text{ in-kip/in}^3$$

$$A_3 = \frac{2}{3}(20)(0.25 - 0.075) = 2.33 \text{ kips/in}^2 = 2.33 \text{ in-kip/in}^3$$

$$\text{modulus of toughness} = U_y + A_1 + A_2 + A_3 \approx 20 \text{ in-kip/in}^3$$

Problem 11.7

11.7 The load-deformation diagram shown has been drawn from data obtained during a tensile test of a 0.875-in.-diameter rod of an aluminum alloy. Knowing that the deformation was measured using a 15-in. gage length, determine (a) the modulus of resilience of the alloy, (b) the modulus of toughness of the alloy.



Volume of stressed material involved in the measurement:

$$V = \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4} (0.875)^2 (15) = 9.0198 \text{ in}^3$$

(a) Modulus of resilience.

$$P_r = 30 \text{ kips}, \quad S_r = 0.104 \text{ in}$$

$$U_r = \frac{1}{2} P_r S_r = \frac{1}{2} (30)(0.104) = 1.56 \text{ in-kip} = 1560 \text{ in-lb}$$

$$\text{modulus of resilience} \quad U_r = \frac{U_r}{V} = \frac{1560}{9.0198} = 173.0 \text{ in-lb/in}^3$$

(b) Modulus of toughness.

$$A_1 = (30)(1.85 - 0.104) = 52.38 \text{ kip-in} = 52380 \text{ in-lb/in}^2$$

$$A_2 = \frac{1}{2}(5)(1.85 - 0.104) = 4.365 \text{ kip-in} = 4365 \text{ in-lb/in}^2$$

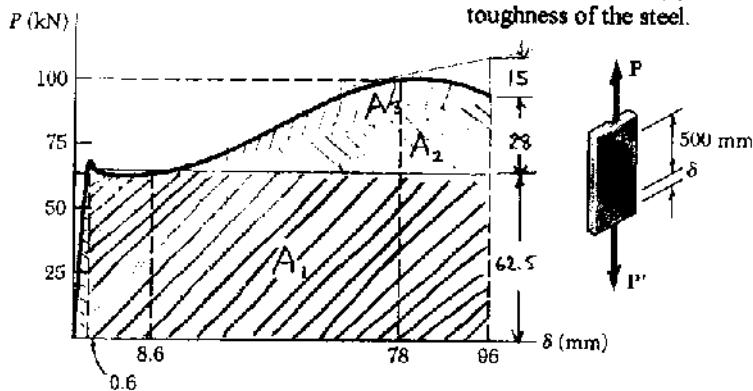
$$A_3 = \frac{2}{3}(4)(1.85 - 0.104) = 4.656 \text{ kip-in} = 4656 \text{ in-lb/in}$$

$$U = U_r + A_1 + A_2 + A_3 = 62961 \text{ in-lb/in}^2$$

$$\text{modulus of toughness} \quad \frac{U}{V} = \frac{62961}{9.0198} = 6980 \text{ in-lb/in}^2$$

Problem 11.8

11.8 The load-deformation diagram shown has been drawn from data obtained during a tensile test of structural steel. Knowing that the cross-sectional area of the specimen is 250 mm^2 and that the deformation was measured using a 500-mm gage length, determine (a) the modulus of resilience of the steel, (b) the modulus of toughness of the steel.



Assuming that yielding occurs at $P = 62.5 \text{ kN}$
and $\delta = 0.6 \text{ mm}$

$$\begin{aligned} U_Y &= \frac{1}{2} (62.5 \times 10^3)(0.6 \times 10^{-3}) \\ &= 18.75 \text{ N}\cdot\text{m} \\ &= 18.75 \text{ J} \end{aligned}$$

Volume of stressed material: $V = AL = (250)(500) = 125 \times 10^3 \text{ mm}^3$
 $= 125 \times 10^{-6} \text{ m}^3$

$$U_Y = \frac{U_Y}{V} = \frac{18.75}{125 \times 10^{-6}} = 150 \times 10^3 = 150 \text{ kJ/m}^3$$

$$A_1 = (62.5 \times 10^3)(96 \times 10^{-3}) = 6 \times 10^3 \text{ N}\cdot\text{m} = 6 \times 10^3 \text{ J}$$

$$A_2 = \frac{1}{2} (28 \times 10^3)(96 - 8.6) \times 10^{-3} = 1.22 \times 10^3 \text{ N}\cdot\text{m} = 1.22 \times 10^3 \text{ J}$$

$$A_3 = \frac{2}{3} (15 \times 10^3)(61 \times 10^{-3}) = 0.61 \times 10^3 \text{ N}\cdot\text{m} = 0.61 \times 10^3 \text{ J}$$

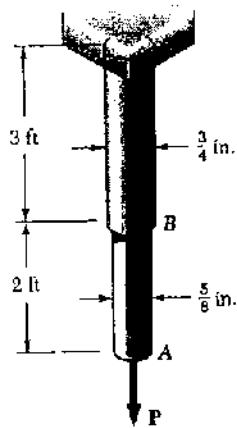
Total energy $U = U_Y + A_1 + A_2 + A_3 = 7.85 \times 10^3 \text{ J}$

$$\text{modulus of toughness} = \frac{U}{V} = \frac{7.85 \times 10^3}{125 \times 10^{-6}} = 63 \times 10^6 \text{ J/m}^3$$

$$= 63 \text{ MJ/m}^3$$

Problem 11.9

11.9 Using $E = 29 \times 10^6$ psi, determine (a) the strain energy of the steel rod ABC when $P = 8$ kips, (b) the corresponding strain energy density in portions AB and BC of the rod.



$$P = 8 \text{ kips}, E = 29 \times 10^6 \text{ ksi}$$

$$A = \frac{\pi d^2}{4}, V = AL, \sigma = \frac{P}{A}, U = \frac{\sigma^2}{2E}$$

$$U = UV$$

Portion	d in.	L in.	A in ²	V in ³	σ ksi	U in-kip/in ³	U in-kip
AB	0.625	24	0.3608	7.363	26.08	11.72×10^{-3}	86.32×10^{-3}
BC	0.75	36	0.4418	15.904	18.11	5.65×10^{-3}	89.92×10^{-3}
Σ							176.24×10^{-3}

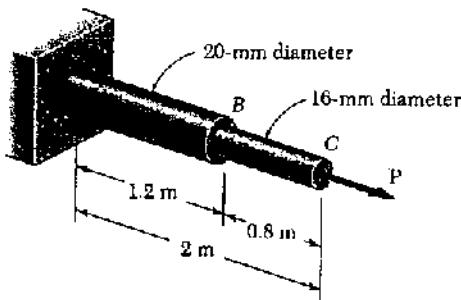
$$(a) U = 176.2 \times 10^{-3} \text{ in-kip} = 176.2 \text{ in-lb} \rightarrow$$

$$(b) \text{ In AB, } u = 11.72 \times 10^{-3} \text{ in-kip/in}^3 = 11.72 \text{ in-lb/in}^3 \rightarrow$$

$$\text{In BC, } u = 5.65 \times 10^{-3} \text{ in-kip/in}^3 = 5.65 \text{ in-lb/in}^3 \rightarrow$$

Problem 11.10

11.10 Using $E = 200 \text{ GPa}$, determine (a) the strain energy of the steel rod ABC when $P = 25 \text{ kN}$, (b) the corresponding strain-energy density in portions AB and BC of the rod.



$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$P = 25 \times 10^3 \text{ N}$$

$$U = \sum \frac{P^2 L}{2EA}$$

$$= \frac{(25 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(314.16 \times 10^{-6})}$$

$$+ \frac{(25 \times 10^3)^2 (0.8)}{(2)(200 \times 10^9)(201.06 \times 10^{-6})}$$

$$(a) U = 5.968 + 6.213 = 12.18 \text{ N}\cdot\text{m} = 12.18 \text{ J}$$

$$(b) \sigma_{AB} = \frac{P}{A_{AB}} = \frac{25 \times 10^3}{314.16 \times 10^{-6}} = 79.58 \times 10^6 \text{ Pa}$$

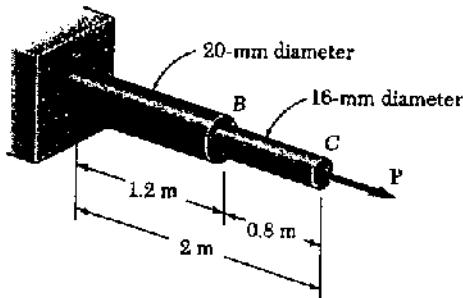
$$U_{AB} = \frac{\sigma_{AB}^2}{2E} = \frac{(79.58 \times 10^6)^2}{(2)(200 \times 10^9)} = 15.83 \times 10^3 = 15.83 \text{ kJ/m}^3$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{25 \times 10^3}{201.06 \times 10^{-6}} = 124.28 \times 10^6 \text{ Pa}$$

$$U_{BC} = \frac{\sigma_{BC}^2}{2E} = \frac{(124.28 \times 10^6)^2}{(2)(200 \times 10^9)} = 38.6 \times 10^3 = 38.6 \text{ kJ/m}^3$$

Problem 11.11

11.11 Rod ABC is made of a steel for which the yield strength is $\sigma_y = 250 \text{ MPa}$ and the modulus of elasticity is $E = 200 \text{ GPa}$. Determine, for the loading shown, the maximum strain energy that can be acquired by the rod without causing any permanent deformation.



$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$$

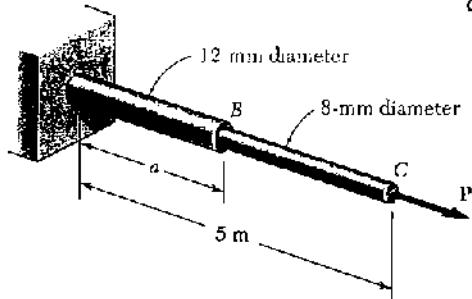
$$P = \sigma_y A_{min} = (250 \times 10^6)(201.06 \times 10^{-6}) \\ = 50.265 \times 10^3 \text{ Pa}$$

$$U = \sum \frac{P^2 L}{2EA} = \frac{(50265)^2 (1.2)}{(2)(200 \times 10^9)(314.16 \times 10^{-6})} + \frac{(50265)^2 (0.8)}{(2)(200 \times 10^9)(201.06 \times 10^{-6})}$$

$$\approx 24.13 + 25.13 = 49.3 \text{ J}$$

Problem 11.12

11.12 Rods AB and BC are made of a steel for which the yield strength is $\sigma_y = 300 \text{ MPa}$ and the modulus of elasticity is $E = 200 \text{ GPa}$. Determine the maximum strain energy that can be acquired by the assembly without causing permanent deformation when the length a of rod AB is (a) 2 m, (b) 4 m.



$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(8)^2 = 50.265 \text{ mm}^2 = 50.265 \times 10^{-6} \text{ m}^2$$

$$P = \sigma_y A_{min} = (300 \times 10^6)(50.265 \times 10^{-6}) \\ = 15.08 \times 10^3 \text{ N}$$

$$U = \sum \frac{P^2 l}{2EA} = \frac{P^2}{2E} \sum \frac{l}{A}$$

$$(a) \quad a = 2 \text{ m}, \quad L-a = 5-2 = 3 \text{ m}$$

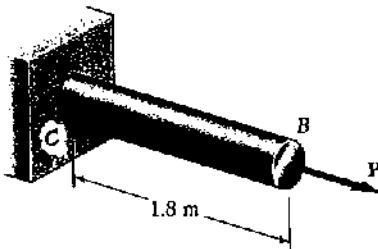
$$U = \frac{(15.08 \times 10^3)^2}{(2)(200 \times 10^9)} \left[\frac{2}{113.097 \times 10^{-6}} + \frac{3}{50.265 \times 10^{-6}} \right] = 44.0 \text{ N}\cdot\text{m} = 44.0 \text{ J} \quad \blacksquare$$

$$(b) \quad a = 4 \text{ m} \quad L-a = 1 \text{ m}$$

$$U = \frac{(15.08 \times 10^3)^2}{(2)(200 \times 10^9)} \left[\frac{4}{113.097 \times 10^{-6}} + \frac{1}{50.265 \times 10^{-6}} \right] = 31.4 \text{ N}\cdot\text{m} = 31.4 \text{ J} \quad \blacksquare$$

Problem 11.13

11.13 Rod BC is made of a steel for which the yield strength is $\sigma_y = 300 \text{ MPa}$ and the modulus of elasticity is $E = 200 \text{ GPa}$. Knowing that a strain energy of 10 J must be acquired by the rod when the axial load P is applied, determine the diameter of the rod for which the factor of safety with respect to permanent deformation is six.



For factor of safety of six on the energy,

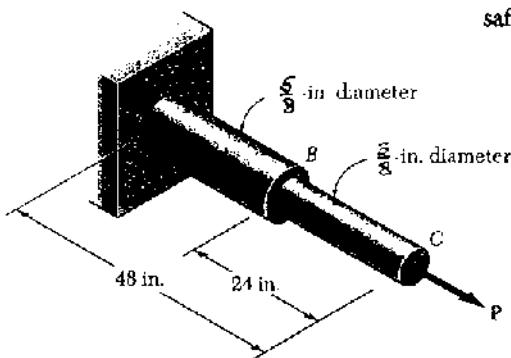
$$U_Y = (6)(10) = 60 \text{ J}$$

$$U_Y = \frac{\sigma_y^2}{2E} = \frac{(300 \times 10^6)^2}{(2)(200 \times 10^9)} = 225 \times 10^3 \text{ J/m}^3$$

$$A = \frac{U_Y}{L U_Y} = \frac{60}{1.8(225 \times 10^3)} = 148.148 \times 10^{-6} \text{ m}^2$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(148.148 \times 10^{-6})}{\pi}} = 13.73 \times 10^{-3} \text{ m} \\ = 13.73 \text{ mm} \quad \blacksquare$$

Problem 11.14



11.14 Rod *AB* is made of a steel for which the yield strength is $\sigma_y = 65$ ksi and the modulus of elasticity is $E = 29 \times 10^6$ psi. Knowing that a strain energy of 60 in \cdot lb must be acquired by the rod as the axial load P is applied, determine the factor of safety of the rod with respect to permanent deformation.

$$A_{AB} = \frac{\pi}{4} \left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 = 0.11045 \text{ in}^2$$

$$P_y = \sigma_y A_{min} = (65 \times 10^3)(0.11045) = 7185 \text{ lb.}$$

$$U_r = \sum \frac{P_r^2 \ell}{EA} = \frac{P_y^2}{2E} \sum \frac{\ell}{A}$$

$$U_r = \frac{(7185)^2}{(2Y(29 \times 10^6))} \left[\frac{48 - 24}{0.3068} + \frac{24}{0.11045} \right] = 263 \text{ in. lb}$$

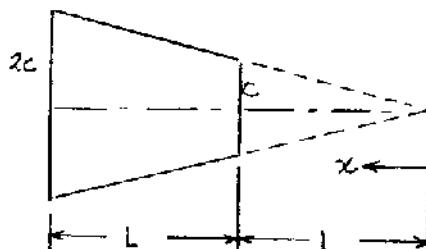
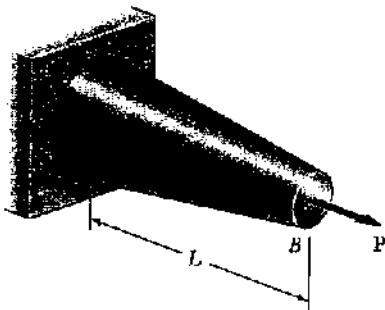
$$\text{F. S.} = \frac{U_r}{U_{design}} = \frac{263}{60} = 4.38$$

Problem 11.15

11.15 Show by integration that the strain energy of the tapered rod *AB* is

$$U = \frac{1}{4} \frac{P^2 L}{EA_{min}}$$

where A_{min} is the cross-sectional area at end *B*.



$$\text{radius } r = \frac{Cx}{L} \quad A_{min} = \pi c^2$$

$$A = \pi r^2 = \frac{\pi c^2}{L^2} x^2$$

$$U = \int_L^{2L} \frac{P^2 dx}{2EA} = \frac{P^2}{2E} \int_L^{2L} \frac{L^2}{\pi c^2} \frac{dx}{x^2}$$

$$= \frac{P^2 L^2}{2E \pi c^2} \left(-\frac{1}{x}\right) \Big|_L^{2L}$$

$$= \frac{P^2 L^2}{2E A_{min}} \left(-\frac{1}{2L} + \frac{1}{L}\right) = \frac{P^2 L^2}{4E A_{min}}$$

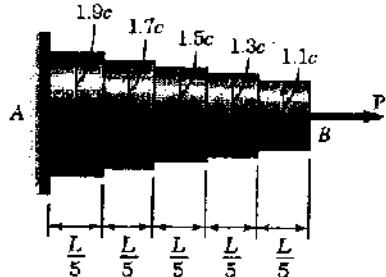
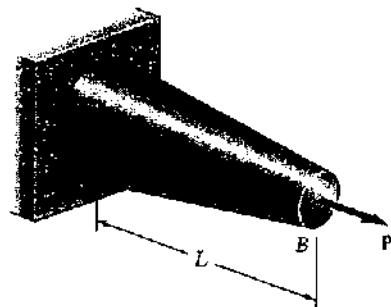
Problem 11.16

11.16 Solve, Prob. 11.15, using the stepped rod shown as an approximation of the tapered rod. What is the percentage error in the answer obtained.

11.15 Show by integration that the strain energy of the tapered rod AB is

$$U = \frac{1}{4} \frac{P^2 L}{EA_{\min}}$$

where A_{\min} is the cross-sectional area at end B .



$$A_i = \pi r_i^2 \quad A_{\min} = \pi c^2$$

$$U = \sum \frac{P^2 l_i}{2E A_i} = \frac{P^2 (L/5)}{2E} \sum \frac{1}{A_i}$$

$$= \frac{P^2 L}{10\pi E} \sum \frac{1}{r_i^2}$$

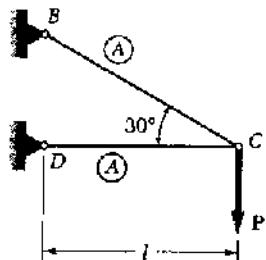
$$= \frac{P^2 L}{10\pi E} \left\{ \frac{1}{(1.9c)^2} + \frac{1}{(1.7c)^2} + \frac{1}{(1.5c)^2} + \frac{1}{(1.3c)^2} + \frac{1}{(1.1c)^2} \right\}$$

$$= \frac{P^2 L}{10 E (\pi c^2)} \{ 2.4856 \} = 0.24856 \frac{P^2 L}{E A_{\min}}$$

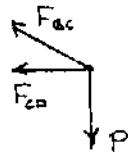
$$\% \text{ error} = \frac{0.24856 - 0.25}{0.25} \times 100\% = -0.575\%$$

Problem 11.17

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.



Joint C



$$+\uparrow \sum F_y = 0:$$

$$\frac{1}{2} F_{Co} - P = 0 \quad F_{Co} = 2P$$

$$+\rightarrow \sum F_x = 0$$

$$-F_{Co} - \frac{\sqrt{3}}{2} F_{Bc} = 0 \quad F_{Bc} = -\sqrt{3}P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

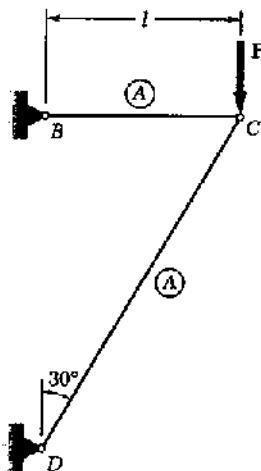
Member	F	L	A	$F^2 L / A$
BC	$2P$	$\frac{2}{\sqrt{3}}l$	A	$\frac{8}{\sqrt{3}}P^2 l / A$
CD	$-\sqrt{3}P$	l	A	$3P^2 l / A$
Σ				$7.62P^2 l / A$

$$U = \frac{1}{2E} \left(7.62 \frac{P^2 l}{A} \right)$$

$$= 3.81 \frac{P^2 l}{EA}$$

Problem 11.18

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.



Joint C

$$+\uparrow \sum F_y = 0:$$

$$-\frac{\sqrt{3}}{2} F_{Co} - P = 0 \quad F_{Co} = -\frac{2}{\sqrt{3}}P$$

$$+\rightarrow \sum F_x = 0$$

$$-F_{Bc} - \frac{1}{2} F_{Co} = 0 \quad F_{Bc} = \frac{1}{\sqrt{3}}P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

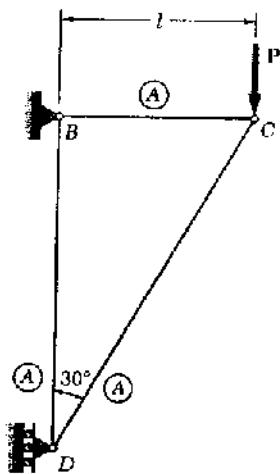
Member	F	L	A	$F^2 L / A$
BC	$\frac{1}{\sqrt{3}}P$	l	A	$\frac{1}{3}P^2 l / A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2 l / A$
Σ				$3P^2 l / A$

$$U = \frac{1}{2E} \left(3 \frac{P^2 l}{A} \right)$$

$$= 1.5 \frac{P^2 l}{EA}$$

Problem 11.19

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.



Joint C

$$\begin{aligned} \uparrow \sum F_y &= 0: -\frac{\sqrt{3}}{2} F_{cd} - P = 0 \\ F_{cd} &= -\frac{2}{\sqrt{3}} P \end{aligned}$$

$$\begin{aligned} \rightarrow \sum F_x &= 0: -F_{bc} - \frac{1}{2} F_{cd} = 0 \\ F_{bc} &= -\frac{1}{2} P \end{aligned}$$

Joint D

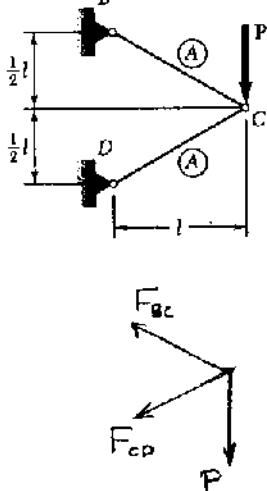
$$\begin{aligned} \uparrow \sum F_y &= 0: F_{bd} + \frac{\sqrt{3}}{2} F_{cd} = 0 \\ F_{bd} &= -P \end{aligned}$$

Member	F	L	A	$F^2 L / A$
BC	$\frac{1}{\sqrt{3}} P$	l	A	$\frac{1}{3} P^2 l / A$
CD	$-\frac{2}{\sqrt{3}} P$	$2l$	A	$\frac{4}{3} P^2 l / A$
BD	P	$\sqrt{3}l$	A	$\sqrt{3} P^2 l / A$
Σ				$4.732 P^2 l / A$

$$\begin{aligned} U &= \sum \frac{1}{2} \frac{F^2 L}{EA} \\ &= \frac{1}{2E} \sum \frac{F^2 L}{A} \\ &= \frac{1}{2E} \left(4.732 \frac{P^2 l}{A} \right) \\ &= 2.37 \frac{P^2 l}{EA} \end{aligned}$$

Problem 11.20

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.



$$L_{BC} = L_{CD} = \sqrt{l^2 + (\frac{1}{2}l)^2} = \frac{\sqrt{5}}{2} l$$

Joint C

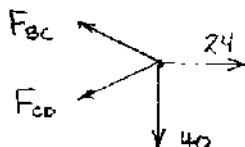
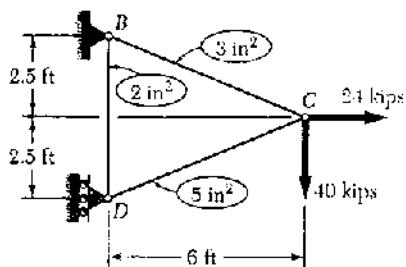
$$\begin{aligned} \pm \sum F_x &= 0: -\frac{2}{\sqrt{5}} F_{bc} - \frac{2}{\sqrt{5}} F_{cd} = 0 \\ F_{cd} &= -F_{bc} \end{aligned}$$

$$\begin{aligned} \uparrow \sum F_y &= 0: \frac{1}{\sqrt{5}} F_{bc} - \frac{1}{\sqrt{5}} F_{cd} - P = 0 \\ F_{bc} &= \frac{\sqrt{5}}{2} P \quad F_{cd} = -\frac{\sqrt{5}}{2} P \end{aligned}$$

$$\begin{aligned} U &= \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \left[F_{bc}^2 L_{bc} + F_{cd}^2 L_{cd} \right] \\ &= \frac{1}{2EA} \left[\left(\frac{\sqrt{5}}{2} P \right)^2 \left(\frac{\sqrt{5}}{2} l \right) + \left(\frac{\sqrt{5}}{2} P \right)^2 \left(\frac{\sqrt{5}}{2} l \right) \right] \\ &= 1.398 \frac{P^2 l}{EA} \end{aligned}$$

Problem 11.21

11.21 Each member of the truss shown is made of aluminum and has the cross-sectional area shown. Using $E = 10.5 \times 10^6$ psi, determine the strain energy of the truss for the loading shown.



$$L_{BC} = L_{CD} = \sqrt{6^2 + 2.5^2} = 6.5 \text{ ft} = 78 \text{ in.}$$

Joint C

$$\pm \sum F_x = 0 : -\frac{6}{6.5} F_{Bc} - \frac{6}{6.5} F_{cd} + 24 = 0 \quad (1)$$

$$+\uparrow \sum F_y = 0 : \frac{2.5}{6.5} F_{Bc} - \frac{2.5}{6.5} F_{cd} - 40 = 0 \quad (2)$$

Solving (1) and (2) simultaneously,

$$F_{Bc} = 65 \text{ kips} \quad F_{cd} = -39 \text{ kips}$$

Joint D



$$+\uparrow \sum F_y = 0 : F_{BD} + \frac{2.5}{6.5} F_{cd} = 0 \quad F_{BD} = 15 \text{ kips}$$

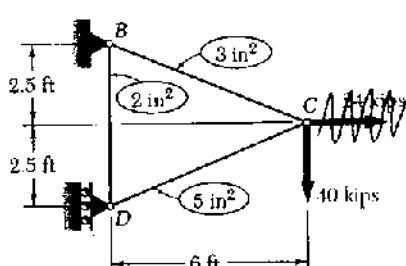
$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	$F (10^3 lb)$	$L (in.)$	$A (in^2)$	$F^2 L / A (10^9 lb^2/in)$
BC	65	78	3	109.85
BD	15	60	2	6.75
CD	-39	78	5	23.73
Σ	-			140.33

$$U = \frac{140.33 \times 10^9}{(2)(10.5 \times 10^6)} = 6682 \text{ lb-in} = 6.68 \text{ kip-in} \blacksquare$$

Problem 11.22

11.22 Solve Prob. 11.21, assuming that the 24-kip load is removed.



$$L_{ec} = L_{eo} = \sqrt{6^2 + 2.5^2} = 6.5 \text{ ft} = 78 \text{ in.}$$

Joint C.

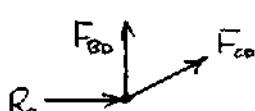
$$\pm \sum F_x = 0 : -\frac{6}{6.5} F_{BC} - \frac{6}{6.5} F_{AB} = 0$$

$$+\uparrow \sum F_y = 0 : \quad \frac{2.5}{6.5} F_{bc} - \frac{2.5}{6.5} F_{cd} - 40 = 0 \quad (9)$$

Solving (1) and (2) simultaneously,

$$F_{bc} = 52 \text{ kips} \quad F_{cd} = -52 \text{ kips}$$

Joint D.



$$+\uparrow \sum F_y = 0: \quad F_{80} + \frac{2.5}{6.5} F_{c0} = 0 \quad F_{80} = -20 \text{ kips}$$

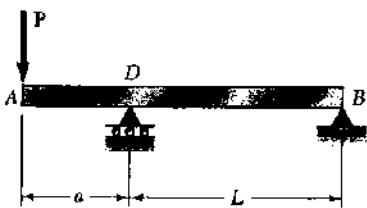
$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	$F (10^3 \text{ lb})$	$L (\text{in.})$	$A (\text{in}^2)$	$FL^3/A (10^9 \text{ lb}^2/\text{in.})$
BC	52	78	3	70.304
BD	20	60	2	12.0
CD	-52	78	5	42.1824
Σ				124.4864

$$U = \frac{124.4864 \times 10^9}{(2)(10.5 \times 10^6)} = 5928 \text{ lb.in} = 5.93 \text{ kip.in} \quad \blacktriangleleft$$

Problem 11.23

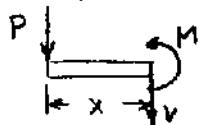
11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\text{At } \sum M_D = 0:$$

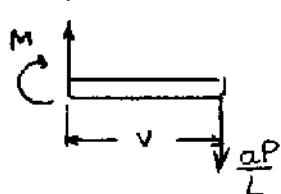
$$aP + LR_B = 0 \quad R_B = -\frac{aP}{L} = \frac{aP}{L} \downarrow$$

Over portion AD : $M = -Px$



$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a P^2 x^2 dx = \frac{P^2}{2EI} \frac{x^3}{3} \Big|_0^a = \frac{P^2 a^3}{6EI}$$

Over portion DB : $M = -\frac{aP}{L} v$

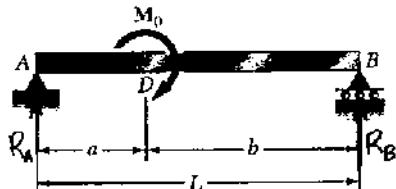


$$U_{DB} = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \frac{a^2 P^2}{L^2} v^2 dv = \frac{P^2 a^2}{2EI L^2} \int_0^L v^2 dv = \frac{P^2 a^2}{2EI L^2} \frac{v^3}{3} \Big|_0^L = \frac{P^2 a^2 L}{6EI}$$

$$\text{Total } U = U_{AD} + U_{DB} = \frac{P^2 a^2}{6EI} (a+L)$$

Problem 11.24

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\text{At } \sum M_B = 0: -R_A L - M_0 = 0 \quad R_A = \frac{M_0}{L} \downarrow$$

$$\text{At } \sum M_A = 0: R_B L - M_0 = 0 \quad R_B = \frac{M_0}{L} \uparrow$$

$$\text{A to D } \text{At } \sum M_J = 0: \frac{M_0 x}{L} + M = 0$$

$$M = -\frac{M_0 x}{L}$$

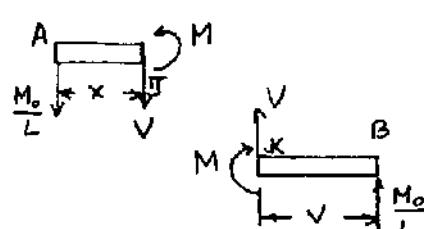
$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^a x^2 dx = \frac{M_0^2 a^3}{6EI L^2}$$

$$\text{D to B } \text{At } \sum M_k = 0: -M + \frac{M_0 v}{L}$$

$$M = \frac{M_0 v}{L}$$

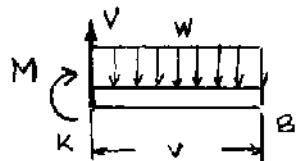
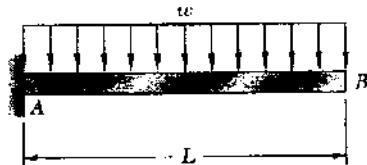
$$U_{DB} = \int_a^b \frac{M^2}{2EI} dv = \frac{M_0^2}{2EI L^2} \int_a^b v^2 dv = \frac{M_0^2 b^3}{6EI L^2}$$

$$\text{Total } U = U_{AD} + U_{DB} = \frac{M_0^2 (a^3 + b^3)}{6EI L^2}$$



Problem 11.25

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



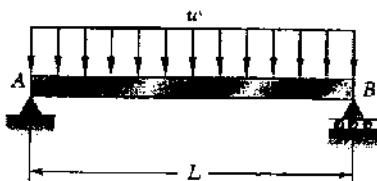
$$\text{④ } \sum M_k = 0; \quad -M - (wv) \left(\frac{v}{2}\right) = 0$$

$$M = -\frac{1}{2} w v^2$$

$$\begin{aligned} U &= \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \left(\frac{1}{2} w v^2\right)^2 dv \\ &= \frac{w^2}{8EI} \int_0^L v^4 dv = \frac{w^2}{8EI} \left.\frac{v^5}{5}\right|_0^L \\ &= \frac{w^2 L^5}{40 EI} \end{aligned}$$

Problem 11.26

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\text{④ } \sum M_B = 0; \quad -R_A L + (wL) \left(\frac{L}{2}\right) = 0 \quad R_A = \frac{wL}{2}$$

$$\begin{aligned} \text{Bending moment} \quad M &= R_A x - \frac{1}{2} w x^2 \\ &= \frac{w}{2} (Lx - x^2) \end{aligned}$$

$$\begin{aligned} U &= \int_0^L \frac{M^2}{2EI} dx = \frac{w^2}{8EI} \int_0^L (Lx - x^2)^2 dx \\ &= \frac{w^2}{8EI} \int_0^L (L^2 x^2 - 2Lx^3 + x^4) dx = \frac{w^2}{8EI} \left[\frac{L^2 x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right]_0^L \\ &= \frac{w^2 L^5}{8EI} \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right] = \frac{w^2 L^5}{240 EI} \end{aligned}$$

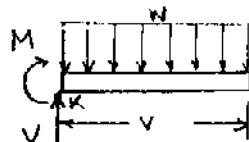
Problem 11.27

11.27 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is



$$u_{\max} = 15 \frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.



$$\sum M_A = 0 \Rightarrow -M - (wv) \frac{L}{2} = 0 \\ M = -\frac{1}{2} w v^2$$

$$U = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L (\frac{1}{2} w v^2)^2 dv = \frac{w^2}{8EI} \int_0^L v^4 dv = \frac{w^2}{8EI} \cdot \frac{v^5}{5} \Big|_0^L = \frac{w^2 L^5}{40EI}$$

$$M_{\max} = \frac{1}{2} w L^2 \quad \sigma_{\max} = \frac{M_{\max} c}{I}$$

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{M_{\max}^2 c^2}{2EI^2} = \frac{\frac{1}{4} w^2 L^4 c^2}{2EI^2} = \frac{w^2 L^4 c^2}{8EI^2}$$

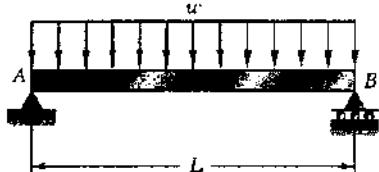
$$\frac{U}{U_{\max}} = \frac{L I}{5 c^2} = \frac{L (\frac{1}{12} b d^3)}{5 (\frac{d}{2})^2} = \frac{1}{15} L b d = \frac{1}{15} V$$

$$U_{\max} = 15 \frac{U}{V}$$



Problem 11.28

11.28 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is



$$U_{\max} = \frac{45}{8} \frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.

$$\therefore M_B = 0; -R_A L + (wL) \frac{L}{2} = 0 \quad R_A = \frac{1}{2} wL$$

$$M = R_A x - \frac{1}{2} wL^2 = \frac{1}{2} w(Lx - x^2)$$

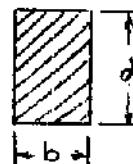
$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{w^2}{8EI} \int_0^L (L^2 x^2 - 2Lx^3 + x^4) dx = \frac{w^2}{8EI} \left[\frac{L^2 x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right]_0^L \\ = \frac{w^2 L^5}{8EI} \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right] = \frac{w^2 L^5}{240EI}$$

$$M_{\max} = \frac{1}{2} w \left[L \cdot \frac{L}{2} - \left(\frac{L}{2} \right)^2 \right] = \frac{1}{8} wL^2 \quad \sigma_{\max} = \frac{M_{\max} c}{I} = \frac{wL^2 c}{8I}$$

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{w^2 L^4 C^2}{128 EI^2}$$

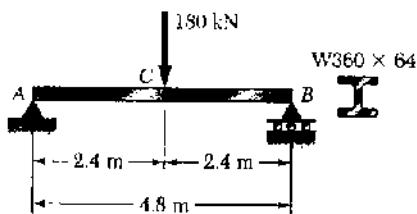
$$\frac{U}{U_{\max}} = \frac{8L I}{15C^2} = \frac{8L \left(\frac{1}{2} bd^3 \right)}{15 \left(\frac{1}{3} \right)^2} = \frac{8}{45} Lbd = \frac{8}{45} V$$

$$U_{\max} = \frac{45}{8} \frac{U}{V}$$



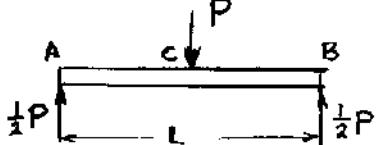
Problem 11.29

11.29 and 11.30 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



Over portion AC: $M = \frac{1}{2}Px$

$$U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx = \frac{P^2}{8EI} \int_0^{\frac{L}{2}} x^2 dx \\ = \frac{P^2}{8EI} \left. \frac{x^3}{3} \right|_0^{\frac{L}{2}} = \frac{P^2 L^3}{192EI}$$



By symmetry, $U_{CB} = U_{AC} = \frac{P^2 L^3}{192EI}$

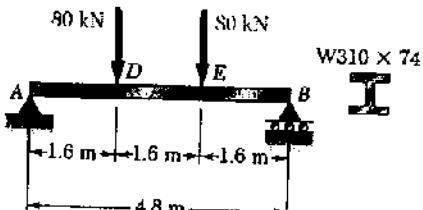
$$\text{Total: } U = U_{AC} + U_{CB} = \frac{P^2 L^3}{96EI}$$

Data: $P = 180 \times 10^3 \text{ N}$, $L = 4.8 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$
 $I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6} \text{ m}^4$

$$U = \frac{(180 \times 10^3)^2 (4.8)^3}{(96)(200 \times 10^9)(178 \times 10^{-6})} = 1048 \text{ N}\cdot\text{m} = 1048 \text{ J}$$

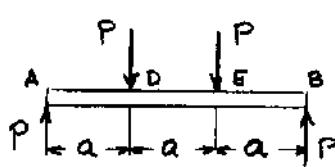
Problem 11.30

11.29 and 11.30 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



Over portion AD: $M = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a (Px)^2 dx \\ = \frac{P^2}{2EI} \left. \frac{x^3}{3} \right|_0^a = \frac{P^2 a^3}{6EI}$$



Over portion DE: $M = Pa$

$$U_{DE} = \frac{(Pa)^2 a}{2EI} = \frac{P^2 a^3}{2EI}$$

By symmetry, $U_{EB} = U_{AD} = \frac{P^2 a^3}{6EI}$

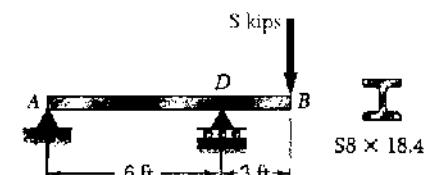
$$U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{6} \frac{P^2 a^3}{EI}$$

Data: $P = 80 \times 10^3 \text{ N}$, $a = 1.6 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$
 $I = 165 \times 10^6 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$

$$U = \frac{5}{6} \frac{(80 \times 10^3)^2 (1.6)^3}{(200 \times 10^9)(165 \times 10^{-6})} = 662 \text{ N}\cdot\text{m} = 662 \text{ J}$$

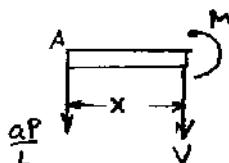
Problem 11.31

11.31 and 11.32 Using $E = 29 \times 10^6 \text{ psi}$, determine the strain energy due to bending for the steel beam and loading shown.



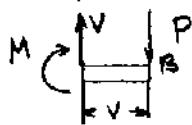
$$\sum M_D = 0 \quad -R_A L - aP = 0 \quad R_A = \frac{aP}{L} \downarrow$$

$$\text{Over portion AD: } M = -\frac{aP}{L}x$$



$$\begin{aligned} U_{AD} &= \int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^L \left(\frac{aP}{L}x\right)^2 dx \\ &= \frac{P^2 a^4}{2EI L^3} \int_0^L x^2 dx \\ &= \frac{P^2 a^4 L}{6EI} \end{aligned}$$

$$\text{Over portion DB: } M = -Px$$



$$U_{DB} = \int_0^a \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^a P^2 x^2 dx = \frac{P^2 a^3}{6EI}$$

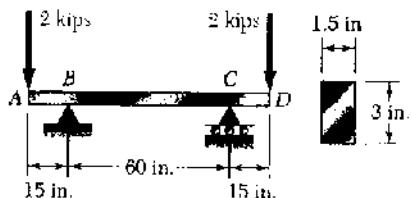
$$\text{Total: } U = U_{AD} + U_{DB} = \frac{P^2 a^2}{6EI} (a + L)$$

Data: $P = 8000 \text{ lb.}$, $L = 6 \text{ ft.} = 72 \text{ in.}$, $a = 3 \text{ ft.} = 36 \text{ in.}$, $E = 29 \times 10^6 \text{ psi}$
 $I = 57.6 \text{ in.}^4$

$$U = \frac{(8000)^2 (36)^2 (72 + 36)}{(6)(29 \times 10^6)(57.6)} = 894 \text{ in-lb.}$$

Problem 11.32

11.31 and 11.32 Using $E = 29 \times 10^6 \text{ psi}$, determine the strain energy due to bending for the steel beam and loading shown.



Over A to B: $M = -Px$

$$U_{AB} = \int_0^a \frac{M^2 dx}{2EI} = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over B to C: $-M = Pa = \text{constant}$

$$U_{BC} = \frac{M^2 b}{2EI} = \frac{P^2 a^2 b}{2EI}$$

$$\text{By symmetry, } U_{CD} = U_{AB} = \frac{P^2 a^3}{6EI}$$

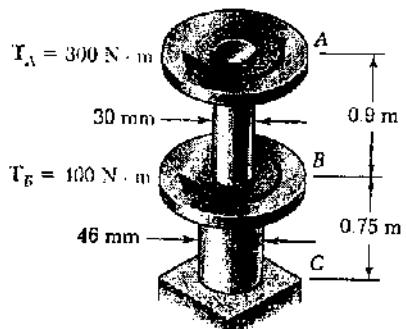
$$\text{Total } U = U_{AB} + U_{BC} + U_{CD} = \frac{P^2 a^2 (2a + 3b)}{6EI}$$

Data: $P = 2 \times 10^3 \text{ lb}$, $a = 15 \text{ in.}$, $b = 60 \text{ in.}$, $I = \frac{1}{12}(1.5)(3)^3 = 3.375 \text{ in.}^4$

$$U = \frac{(2 \times 10^3)^2 (15)^2 [(2)(15) + (3)(60)]}{(6)(29 \times 10^6)(3.375)} = 322 \text{ in.-lb}$$

Problem 11.33

11.33 In the assembly shown torques T_A and T_B are exerted on disks A and B respectively. Knowing that both shafts are solid and made of aluminum ($G = 73 \text{ GPa}$), determine the total strain energy acquired by the assembly.



Over portion AB:

$$T_{AB} = T_A = 300 \text{ N}\cdot\text{m}$$

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{30}{2}\right)^4 = 79.52 \times 10^9 \text{ mm}^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = 0.9 \text{ m}$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2G J_{AB}} = \frac{(300)^2 (0.9)}{(2)(73 \times 10^9)(79.52 \times 10^{-9})} = 6.977 \text{ J}$$

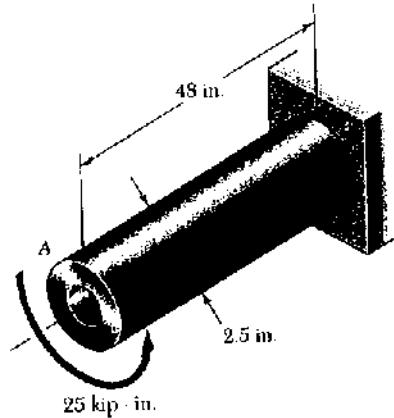
Over portion BC: $T_{BC} = T_A + T_B = 300 + 400 = 700 \text{ N}\cdot\text{m}$, $L_{BC} = 0.75 \text{ m}$

$$J_{BC} = \frac{\pi}{2} \left(\frac{46}{2}\right)^4 = 439.57 \times 10^9 \text{ mm}^4 = 439.57 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T_{BC}^2 L_{BC}}{2G J_{BC}} = \frac{(700)^2 (0.75)}{(2)(73 \times 10^9)(439.57 \times 10^{-9})} = 5.726 \text{ J}$$

$$\text{Total, } U = U_{AB} + U_{BC} = 6.977 + 5.726 = 12.70 \text{ J}$$

Problem 11.34



11.34 The design specifications for the steel shaft *AB* require that the shaft acquire a strain energy of 400 in · lb as the 25-kip · in. torque is applied. Using $G = 11.2 \times 10^6$ psi, determine (a) the largest inner diameter of the shaft that can be used, (b) the corresponding maximum shearing stress in the shaft.

$$U = 400 \text{ in} \cdot \text{lb} \quad T = 25 \text{ kip} \cdot \text{in} = 25 \times 10^3 \text{ lb} \cdot \text{in}$$

$$L = 48 \text{ in}$$

$$U = \frac{T^2 L}{2 G J}$$

$$J = \frac{T^2 L}{2 G U} = \frac{(25 \times 10^3)^2 (48)}{(2)(11.2 \times 10^6)(400)} = 3.3482 \text{ in}^4$$

$$\text{But } J = \frac{\pi}{2} \left[\left(\frac{d_o}{2} \right)^4 - \left(\frac{d_i}{2} \right)^4 \right] = \frac{\pi}{32} (d_o^4 - d_i^4)$$

$$(a) d_i^4 = d_o^4 - \frac{32}{\pi} J = 2.5^4 - \frac{32}{\pi} (3.3482) = 4.95787 \text{ in}^4$$

$$d_i = 1.492 \text{ in.}$$

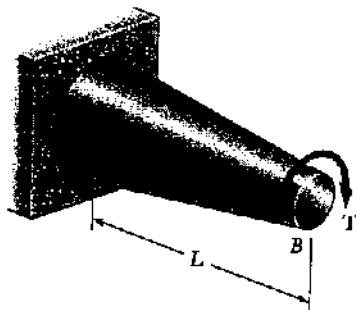
$$(b) \tau = \frac{T c_o}{J} = \frac{(25 \times 10^3)(1.25)}{2.5112} = 9.33 \times 10^3 \text{ psi} = 9.33 \text{ ksi}$$

Problem 11.35

11.35 Show by integration that the strain energy in the tapered rod *AB* is

$$U = \frac{7}{48} \frac{T^2 L}{G J_{\min}}$$

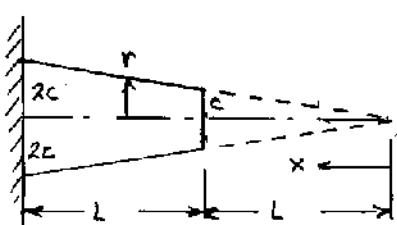
where J_{\min} is the polar moment of inertia of the rod at end *B*.



$$r = \frac{c x}{L}$$

$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2} \frac{c^4}{L^4} x^4, \quad J_{\min} = \frac{\pi}{2} c^4$$

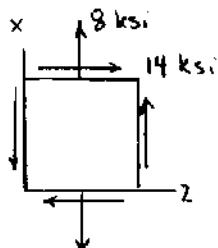
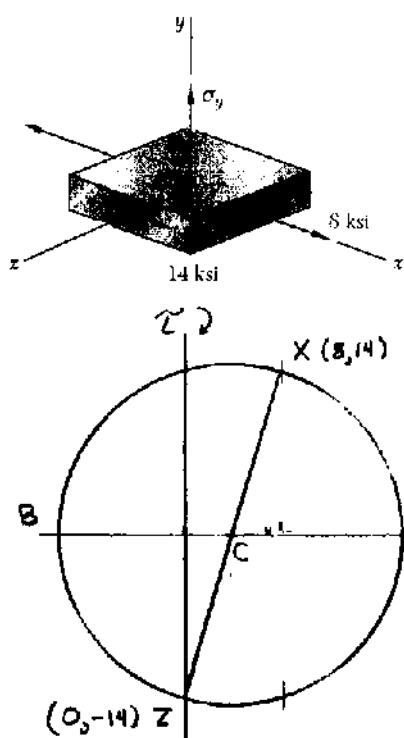
$$\begin{aligned} U &= \int_L^{2L} \frac{T^2 dx}{2 G J} = \int_L^{2L} \frac{T^2}{2 G \left(\frac{\pi}{2} \frac{c^4}{L^4} x^4 \right)} dx \\ &= \frac{T^2 L^4}{2 G J_{\min}} \int_L^{2L} \frac{dx}{x^4} \\ &= \frac{T^2 L^4}{2 G J_{\min}} \left(-\frac{1}{3x^3} \right) \Big|_L^{2L} \end{aligned}$$



$$U = \frac{T^2 L^2}{2 G J_{\min}} \left(-\frac{1}{3(2L)^3} + \frac{1}{3L^3} \right) = \frac{7}{48} \frac{T^2 L}{G J_{\min}}$$

Problem 11.36

11.36 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_y = 65$ ksi. Using the maximum-distortion-energy criterion, determine the range of values of σ_y for which the factor of safety associated with the yield strength is equal to or larger than 2.2.



$$\bar{\sigma}_{ave} = \frac{1}{2}(0 + 8) = 4 \text{ ksi}$$

$$\frac{\bar{\sigma}_x - \bar{\sigma}_z}{2} = \frac{8 - 0}{2} = 4 \text{ ksi}$$

$$\tau_{xz} = 14 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \sqrt{4^2 + 14^2} = 14.56 \text{ ksi}$$

$$\sigma_a = \bar{\sigma}_{ave} + R = 18.56 \text{ ksi}$$

$$\sigma_b = \bar{\sigma}_{ave} - R = -10.56 \text{ ksi}$$

$$\sigma_c = \bar{\sigma}_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_y}{F.S.}\right)^2$$

$$(18.56 + 10.56)^2 + (-10.56 - \sigma_y)^2 + (\sigma_y - 18.56)^2 = 2\left(\frac{65}{2.2}\right)^2$$

$$847.97 + (111.51 + 21.12\sigma_y + \sigma_y^2) + (\sigma_y^2 - 37.12\sigma_y + 344.47) = 1745.87$$

$$2\sigma_y^2 - 16\sigma_y - 441.92 = 0$$

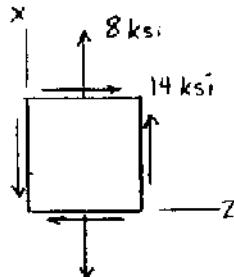
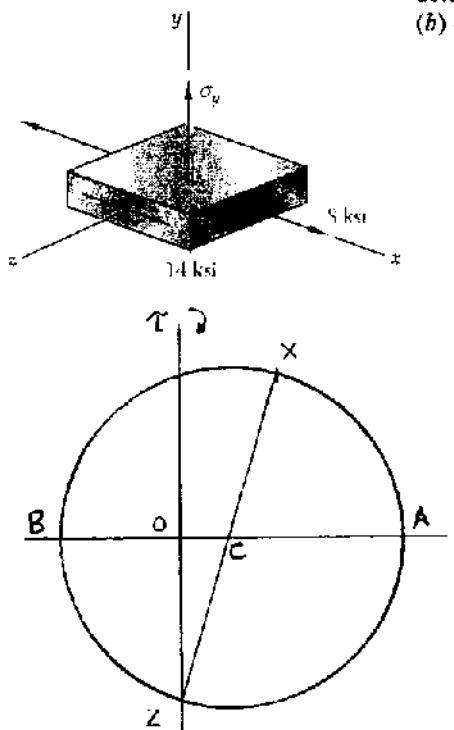
$$\sigma_y = \frac{16 \pm \sqrt{16^2 + (4)(2)(441.92)}}{(2)(2)} = 4 \pm 15.39$$

$$\sigma_y = 19.39 \text{ ksi}, -11.39 \text{ ksi}$$

$$-11.39 \text{ ksi} \leq \sigma_y \leq 19.39 \text{ ksi}$$

Problem 11.37

11.37 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_y = 65$ ksi. Using the maximum-distortion-energy criterion, determine the factor of safety associated with the yield strength when (a) $\sigma_y = +16$ ksi, (b) $\sigma_y = -16$ ksi.



$$\bar{\sigma}_{ave} = \frac{1}{2}(0 + 8) = 4 \text{ ksi}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{8 - 0}{2} = 4 \text{ ksi}$$

$$2\tau_{xz} = 14 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + 2\tau_{xz}^2} = \sqrt{4^2 + 14^2} = 14.56 \text{ ksi}$$

$$\sigma_a = \bar{\sigma}_{ave} + R = 18.56$$

$$\sigma_b = \bar{\sigma}_{ave} - R = -10.56$$

$$\sigma_c = \sigma_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_y}{F.S.}\right)^2$$

$$(a) \quad \sigma_c = \sigma_y = 16 \text{ ksi}$$

$$(18.56 + 10.56)^2 + (-10.56 - 16)^2 + (16 - 18.56)^2 = 2\left(\frac{\sigma_y}{F.S.}\right)^2$$

$$847.97 + 705.43 + 6.55 = \frac{8450}{(F.S.)^2} \quad F.S. = 2.33$$

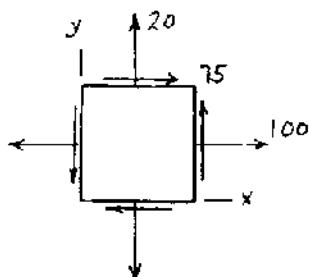
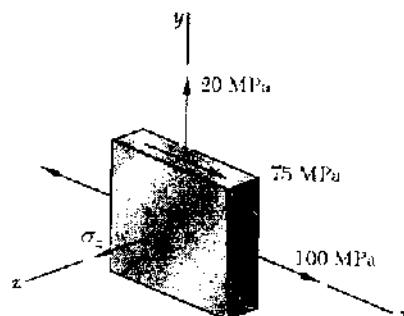
$$(b) \quad \sigma_c = \sigma_y = -16 \text{ ksi}$$

$$(18.56 + 10.56)^2 + (-10.56 + 16)^2 + (-16 - 18.56)^2 = 2\left(\frac{\sigma_y}{F.S.}\right)^2$$

$$847.97 + 29.59 + 1194.39 = \frac{8450}{(F.S.)^2} \quad F.S. = 2.02$$

Problem 11.38

11.38 The state of stress shown occurs in a machine component made of a brass for which $\sigma_y = 160 \text{ MPa}$. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a) $\sigma_z = +45 \text{ MPa}$, (b) $\sigma_z = -45 \text{ MPa}$.

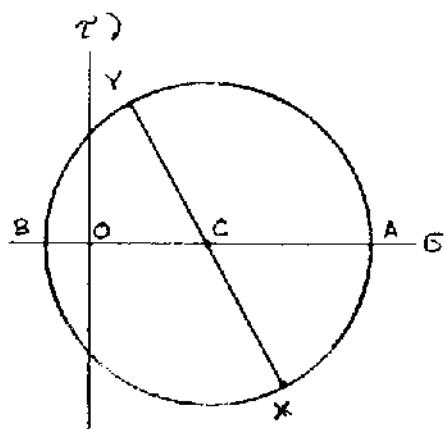


$$\bar{\sigma}_{ave} = \frac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\bar{\sigma}_{x-\bar{\sigma}_y} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\tau_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\bar{\sigma}_x - \bar{\sigma}_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$



$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = 145 \text{ MPa}$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R = -25 \text{ MPa}$$

$$\bar{\sigma}_c = \bar{\sigma}_x$$

$$(\bar{\sigma}_a - \bar{\sigma}_b)^2 + (\bar{\sigma}_b - \bar{\sigma}_c)^2 + (\bar{\sigma}_c - \bar{\sigma}_a)^2 \stackrel{?}{\leq} 2\sigma_y^2$$

$$(a) \bar{\sigma}_c = \bar{\sigma}_z = +45 \text{ MPa}$$

$$(145 + 25)^2 + (-25 - 45)^2 + (45 - 145)^2 \stackrel{?}{\leq} 2(160)^2 = 51200$$

$$28900 + 4900 + 10000 = 43800 < 51200 \quad (\text{No yield})$$

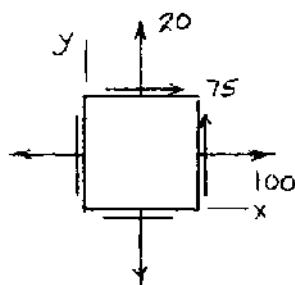
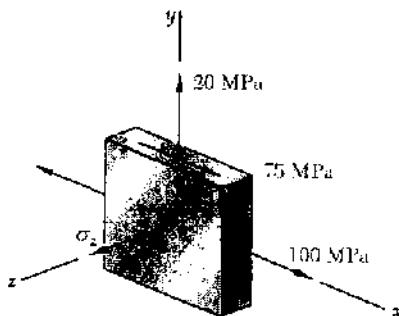
$$(b) \bar{\sigma}_c = \bar{\sigma}_z = -45 \text{ MPa}$$

$$(145 + 25)^2 + (-25 + 45)^2 + (-45 - 145)^2 \stackrel{?}{\leq} 51200$$

$$28900 + 400 + 36100 = 65400 > 51200 \quad (\text{Yield occurs})$$

Problem 11.39

11.39 The state of stress shown occurs in a machine component made of a brass for which $\sigma_y = 160 \text{ MPa}$. Using the maximum-distortion-energy criterion, determine the range of values of σ_z for which yield does not occur.

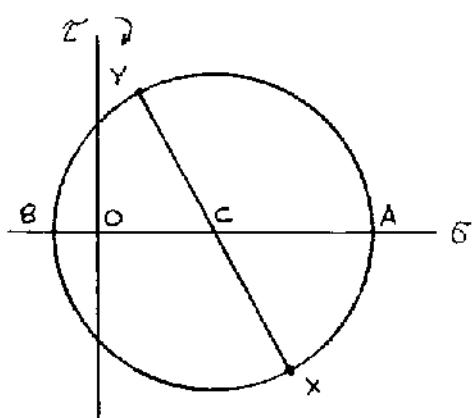


$$\bar{\sigma}_{ave} = \frac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \bar{\sigma}_{ave}}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\bar{\sigma}_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \bar{\sigma}_{ave}}{2}\right)^2 + \bar{\sigma}_{xy}^2} = \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$



$$\bar{\sigma}_a = \bar{\sigma}_{ave} + R = 145 \text{ MPa}$$

$$\bar{\sigma}_b = \bar{\sigma}_{ave} - R = -25 \text{ MPa}$$

$$\bar{\sigma}_c = \bar{\sigma}_z$$

$$(\bar{\sigma}_a - \bar{\sigma}_b)^2 + (\bar{\sigma}_b - \bar{\sigma}_c)^2 + (\bar{\sigma}_c - \bar{\sigma}_a)^2 = 2\bar{\sigma}_r^2$$

$$(145 + 25)^2 + (-25 - \bar{\sigma}_z)^2 + (\bar{\sigma}_z - 145)^2 = (2)(160)^2$$

$$28900 + (625 + 50\bar{\sigma}_z + \bar{\sigma}_z^2) + (\bar{\sigma}_z^2 - 290\bar{\sigma}_z + 21025) = 51200$$

$$2\bar{\sigma}_z^2 - 240\bar{\sigma}_z - 650 = 0$$

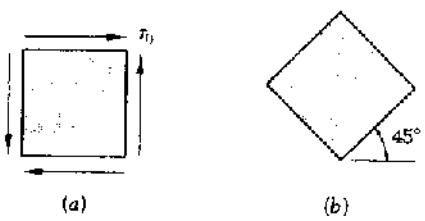
$$\bar{\sigma}_z = \frac{240 \pm \sqrt{240^2 + 4(2)(650)}}{2(2)} = 60 \pm 62.65$$

$$\bar{\sigma}_z = 122.65 \text{ MPa}, -2.65 \text{ MPa}$$

$$-2.65 \text{ MPa} < \bar{\sigma}_z < 122.65 \text{ MPa}$$

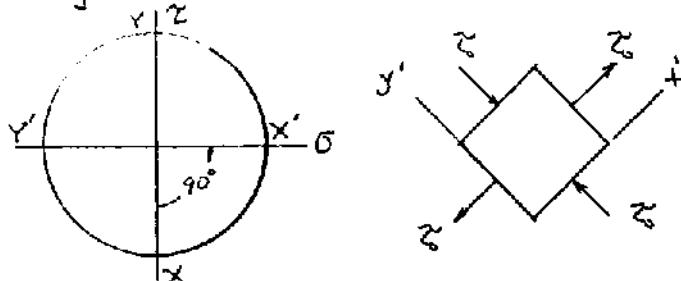
Problem 11.40

11.40 For the state of stress shown in Fig. a, determine the stresses in an element oriented as shown in Fig. b. Compare the strain energy density in the given state by first using Fig. a and then by using Fig. b. Equating the two results obtained, show that



$$G = \frac{E}{2(1+\nu)}$$

Using Mohr's circle



$$(a) \sigma_x = 0, \sigma_y = 0, \tau_{xy} = \tau_0$$

$$U = \frac{1}{2E} (\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{1}{2G} \tau_{xy}^2 = \frac{\tau_0^2}{2G}$$

$$(b) \sigma_{x'} = \tau_0, \sigma_{y'} = -\tau_0, \tau_{x'y'} = 0$$

$$U = \frac{1}{2E} (\sigma_{x'}^2 + \sigma_{y'}^2 - 2\nu\sigma_{x'}\sigma_{y'}) + \frac{1}{2G} \tau_{x'y'}^2 = \frac{(2+2\nu)\tau_0^2}{2E}$$

$$\text{Equate } \frac{\tau_0^2}{2G} = \frac{(2+2\nu)\tau_0^2}{2E} \quad G = \frac{E}{2(1+\nu)}$$

Problem 11.41

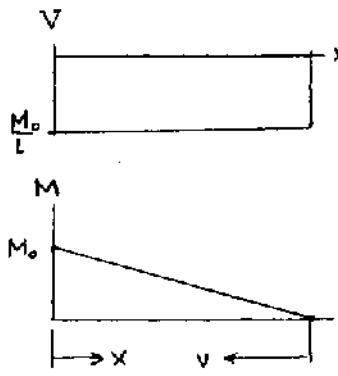
11.41 Determine the strain energy of the prismatic beam AB , taking into account the effect of both normal and shearing stresses.



$$\text{Reactions} \quad R_A = \frac{M_0}{L} \downarrow, \quad R_B = \frac{M_0}{L} \uparrow$$

$$\text{Shear: } V = -\frac{M_0}{L}$$

$$\text{Bending moment: } M = \frac{M_0}{L} v$$



For bending

$$U_1 = \int_0^L \frac{M^2}{2EI} dv = \frac{M_0^2}{2EI L^2} \int_0^L v^2 dv = \frac{M_0^2 L^3}{6EI^2}$$

$$= \frac{M_0^2 L}{6EI}$$

For shear

$$T_{xy} = \frac{3}{2} \frac{V}{A} \left(1 - \frac{y^2}{c^2} \right) \quad c = \frac{1}{2} d$$

$$U = \frac{T_{xy}^2}{2G} = \frac{9V^2}{8GA^2} \left(1 - \frac{y^2}{c^2} \right)^2 = \frac{9M_0^2}{8G(bd)^2 L^2} \left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4} \right)$$

$$U_2 = \int U dv = \int_0^L \int_{-c}^c u b dy dx = \frac{9M_0^2 b}{8Gb^2 d^2 L^2} \int_0^L \int_{-c}^c \left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4} \right) dy dx$$

$$= \frac{9M_0^2}{8Gb d^2 L^2} \int_0^L \left(y - \frac{2}{3} \frac{y^3}{c^2} + \frac{1}{5} \frac{y^5}{c^4} \right) \Big|_{-c}^c dx = \frac{9M_0^2}{8Gb d^2 L^2} \int_0^L \left(2c - \frac{4}{3}c + \frac{2}{5}c \right) dx$$

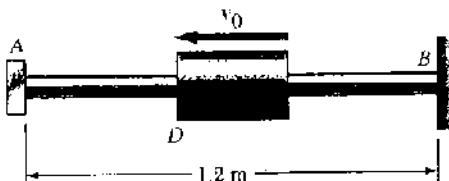
$$= \frac{9M_0^2}{8Gb d^2 L^2} \left(\frac{16}{15}c \right) L = \frac{6}{5} \frac{M_0^2 c}{Gb d^2 L} = \frac{3}{5} \frac{M_0^2}{Gb d L}$$

$$\text{Total } U = U_1 + U_2 = \frac{M_0^2 L}{6EI} + \frac{3}{5} \frac{M_0^2}{Gb d L}$$

$$\text{With } I = \frac{1}{12} b d^3$$

$$U = \frac{2M_0^2 L}{E b d^3} + \frac{3}{5} \frac{M_0^2}{Gb d L} = \frac{2M_0^2 L}{Eb d^3} \left\{ 1 + \frac{3}{10} \cdot \frac{E}{G} \frac{d^2}{L^2} \right\}$$

Problem 11.42



11.42 A 5-kg collar D moves along the uniform rod AB and has a speed $v_0 = 6 \text{ m/s}$ when it strikes a small plate attached to end A of the rod. Using $E = 200 \text{ GPa}$ and knowing that the allowable stress in the rod is 250 MPa , determine the smallest diameter which can be used for the rod.

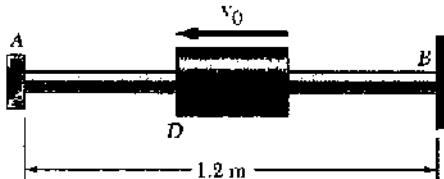
$$U_m = \frac{1}{2}mv_0^2 = \frac{1}{2}(5)(6)^2 = 90 \text{ J}$$

$$U_m = \frac{P_m^2 L}{2EA} = \frac{(A\sigma_{max})^2 L}{2EA}$$

$$A = \frac{2E U_m}{\epsilon_{max}^2 L} = \frac{(2)(200 \times 10^9)(90)}{(250 \times 10^6)^2 (1.2)} = 480 \times 10^{-6} \text{ m}^2$$

$$\frac{\pi}{4}d^2 = A \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{14(480 \times 10^{-6})}{\pi}} = 24.7 \times 10^{-3} \text{ m} = 24.7 \text{ mm}$$

Problem 11.43



11.43 A 6-kg collar has a speed $v_0 = 4.5 \text{ m/s}$ when it strikes a small plate attached to end A of the 20-mm-diameter rod AB . Using $E = 200 \text{ GPa}$, determine (a) the equivalent static load, (b) the maximum stress in the rod (c) the maximum deflection of the A .

$$U_m = \frac{1}{2}mv_0^2 = \frac{1}{2}(6)(4.5)^2 = 60.75 \text{ J}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.159 \times 10^{-6} \text{ m}^2$$

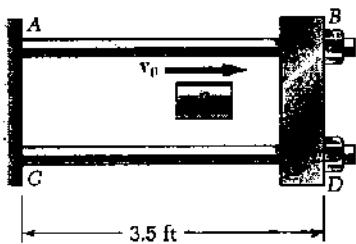
$$U_m = \frac{P^2 L}{2EA}$$

$$(a) P = \sqrt{\frac{2EAU_m}{L}} = \sqrt{\frac{(2)(200 \times 10^9)(314.159 \times 10^{-6})(60.75)}{1.2}} = 79.76 \times 10^3 \text{ N} \quad P = 79.8 \text{ kN}$$

$$(b) \sigma_{max} = \frac{P}{A} = \frac{79.76 \times 10^3}{314.159 \times 10^{-6}} = 2.54 \times 10^6 \text{ Pa} \quad \sigma_{max} = 254 \text{ MPa}$$

$$(c) S = \frac{PL}{EA} = \frac{(79.76 \times 10^3)(1.2)}{(200 \times 10^9)(314.159 \times 10^{-6})} = 1.523 \times 10^{-3} = 1.523 \text{ mm}$$

Problem 11.44



11.44 The cylindrical block E has a speed $v_0 = 16 \text{ ft/s}$ when it strikes squarely the yoke BD that is attached to the $\frac{7}{8}$ -in.-diameter rods AB and CD . Knowing that the rods are made of a steel for which $\sigma_y = 50 \text{ ksi}$ and $E = 29 \times 10^6 \text{ psi}$, determine the weight of the block E for which the factor of safety is five with respect to permanent deformation of the rods.

At the onset of yielding, the force in each rod is

$$F = \sigma_y A$$

Corresponding strain energy •

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_y^2 A^2 L}{2EA} = \frac{\sigma_y^2 AL}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_y^2 AL}{2E}$$

$$U_m = U_{AB} + U_{CD} = \frac{\sigma_y^2 AL}{E}$$

$$U_m = \left(\frac{1}{2} m v_0^2 \right) (\text{F.S.}) = \left(\frac{1}{2} \frac{W}{g} v_0^2 \right) (\text{F.S.})$$

$$\text{Solving for } W, \quad W = \frac{2g U_m}{v_0^2 (\text{F.S.})} = \frac{2g \sigma_y^2 AL}{v_0^2 (\text{F.S.}) E}$$

$$\text{Data: } g = 32.17 \text{ ft/sec}^2 = 386 \text{ in/sec}^2, \quad \sigma_y = 50 \times 10^6 \text{ psi},$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2 \quad E = 29 \times 10^6 \text{ psi}$$

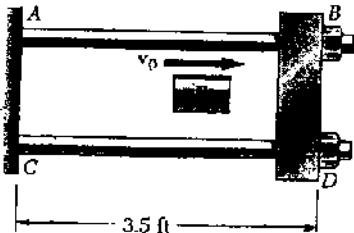
$$L = 3.5 \text{ ft} = 42 \text{ in}$$

$$\text{F.S.} = 5$$

$$v_0 = 16 \text{ ft/sec} = 192 \text{ in/sec}$$

$$W = \frac{(2)(386)(50 \times 10^6)^2 (0.60132)(42)}{(192)^2 (5) (29 \times 10^6)} = 9.12 \text{ lb.}$$

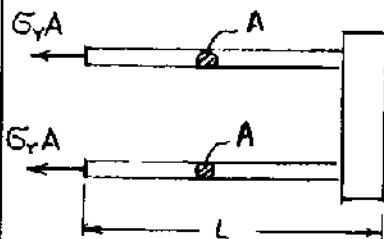
Problem 11.45



11.45 The 18-lb cylindrical block E has a horizontal velocity v_0 when it strikes squarely the yoke BD that is attached to the $\frac{7}{8}$ -in.-diameter rods AB and CD . Knowing that the rods are made of a steel for which $\sigma_y = 50$ ksi and $E = 29 \times 10^6$ psi, determine the maximum allowable speed v_0 if the rods are not to be permanently deformed.

At the onset of yielding the force in each rod is

$$F = \sigma_y A$$



Corresponding strain energy.

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_y^2 A^2 L}{2EA} = \frac{\sigma_y^2 AL}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_y^2 AL}{2E}$$

$$\text{Total } U_m = U_{AB} + U_{CD} = \frac{\sigma_y^2 AL}{E}$$

$$U_m = \frac{1}{2} m V_0^2 = \frac{1}{2} \frac{W}{g} V_0^2$$

$$\text{Solving for } V_0^2, \quad V_0^2 = \frac{2g U_m}{W} = \frac{2g \sigma_y^2 AL}{EW}$$

$$V_0 = \sqrt{\frac{2g \sigma_y^2 AL}{EW}}$$

$$\text{Data: } g = 32.17 \text{ ft/sec}^2 = 386 \text{ in/sec}^2, \quad \sigma_y = 50 \times 10^3 \text{ psi}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (\frac{7}{8})^2}{4} = 0.60132 \text{ in}^2, \quad E = 29 \times 10^6 \text{ psi}$$

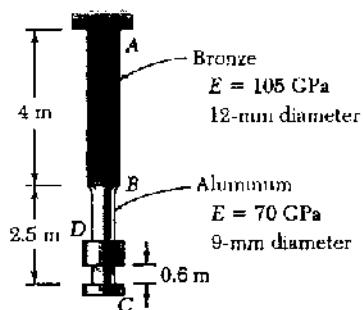
$$L = 3.5 \text{ ft} = 42 \text{ in} \quad W = 18 \text{ lb.}$$

$$V_0 = \sqrt{\frac{(2)(386)(50 \times 10^3)^2 (0.60132)(42)}{(29 \times 10^6)(18)}} = 305.6 \text{ in/sec}$$

$$= 25.5 \text{ ft/sec} \quad \blacksquare$$

Problem 11.46

11.46 Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC. Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.



$$\text{Portion BC: } \sigma_m = 125 \times 10^6 \text{ Pa}$$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952 \text{ N}$$

Corresponding strain energy.

$$U_{BC} = \frac{P_m^2 L_{BC}}{2E_{BC} A_{BC}} = \frac{(7952)^2 (2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2E_{AB} A_{AB}} = \frac{(7952)^2 (4)}{(2)(105 \times 10^9)(113.907 \times 10^{-6})} = 10.574 \text{ J}$$

$$U_m = U_{BC} + U_{AB} = 28.324 \text{ J}$$

Corresponding elongation Δ_m . $\frac{1}{2} P_m \Delta_m = U_m$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(28.324)}{7952} = 7.12 \times 10^{-3} \text{ m}$$

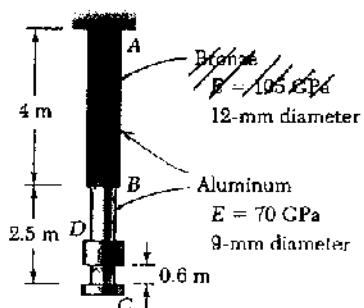
Falling distance $h = 0.6 + 7.12 \times 10^{-3} = 0.60712 \text{ m}$

Work of weight = U_m $W h = mgh = U_m$

$$m = \frac{U_m}{g h} = \frac{28.324}{(9.81)(0.60712)} = 4.76 \text{ kg}$$

Problem 11.47

11.47 Solve Prob. 11.46, assuming that both portions of rod ABC are made of aluminum.



11.46 Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC. Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.

$$\text{Portion BC: } \sigma_m = 125 \times 10^6 \text{ Pa}$$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952 \text{ N}$$

Corresponding strain energy.

$$U_{BC} = \frac{P_m^2 L_{BC}}{2E A_{BC}} = \frac{(7952)^2 (2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2E A_{AB}} = \frac{(7952)^2 (4)}{(2)(70 \times 10^9)(113.907 \times 10^{-6})} = 15.861 \text{ J}$$

$$\text{Total } U_m = U_{BC} + U_{AB} = 33.611 \text{ J}$$

Corresponding elongation Δ_m . $\frac{1}{2} P_m \Delta_m = U_m$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(33.611)}{7952} = 8.45 \times 10^{-3} \text{ m}$$

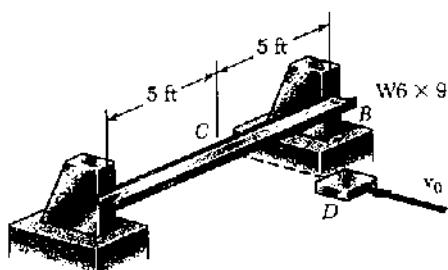
$$\text{Falling distance. } h = 0.6 + \Delta_m = 0.60845 \text{ m}$$

$$\text{Work of weight} = U_m. \quad Wh = mgh = U_m$$

$$m = \frac{U_m}{gh} = \frac{33.611}{(9.81)(0.60845)} = 5.63 \text{ kg}$$

Problem 11.48

11.48 The steel beam AB is struck squarely at its midpoint C by a 100-lb block moving horizontally with a speed $v_0 = 7 \text{ ft/s}$. Using $E = 29 \times 10^6 \text{ psi}$, determine (a) the equivalent static load, (b) the maximum normal stress in the beam, (c) the maximum deflection of the midpoint C of the beam.



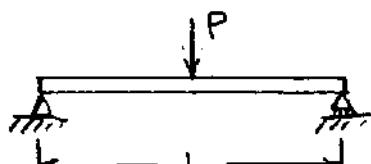
From Appendix C for W6 x 9

$$I_x = 10.4 \text{ in}^4 \quad S_x = 5.56 \text{ in}^3$$

$$\text{For moving block, } m = \frac{W}{g}$$

$$\text{Kinetic energy: } T = \frac{1}{2} m v_0^2 = \frac{1}{2} \frac{W}{g} v_0^2$$

$$T = \frac{1}{2} \frac{(100 \text{ lb})(7 \text{ ft/s})^2}{32.2 \text{ ft/sec}^2} = 76.087 \text{ ft-lb} = 913.04 \text{ in-lb.}$$



From Appendix D, Case 4

$$|y_m| = \frac{PL^3}{48EI} \quad M_{max} = \frac{PL}{4}$$

$$U_m = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96EI} = T$$

$$L = 5 \text{ ft} + 5 \text{ ft} = 10 \text{ ft} = 120 \text{ in.}$$

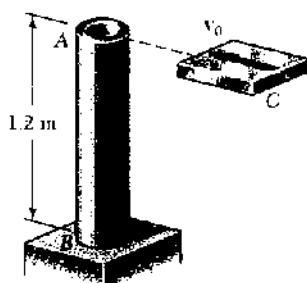
$$(a) \quad P_m = \sqrt{\frac{96EI T}{L^3}} = \sqrt{\frac{(96)(29 \times 10^6)(16.4)(913.04)}{(120)^3}} = 4.9117 \times 10^3 \text{ lb} \\ = 4.91 \text{ kips} \quad \blacktriangleleft$$

$$(b) \quad M_{max} = \frac{(4.9117 \times 10^3)(120)}{4} = 147.35 \times 10^3 \text{ lb-in.}$$

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{147.35 \times 10^3}{5.56} = 26.5 \times 10^3 \text{ psi} = 26.5 \text{ ksi} \quad \blacktriangleleft$$

$$(c) \quad y_m = \frac{2U_m}{P_m} = \frac{2(913.04)}{4.9117 \times 10^3} = 0.372 \text{ in.} \quad \blacktriangleleft$$

Problem 11.49



11.49 The post *AB* consists of a steel pipe of 80-mm-diameter and 6-mm wall thickness. A 6-kg block *C* moving horizontally with at velocity v_0 hits the post squarely at *A*. Using $E = 200$ GPa, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 180 MPa.

$$C_o = \frac{1}{2}d_o = \frac{1}{2}(80) = 40 \text{ mm} \quad C_i = C_o - t = 20 - 6 = 34 \text{ mm}$$

$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = \frac{\pi}{4}(40^4 - 34^4) = 961.06 \times 10^9 \text{ mm}^4 \\ = 961.06 \times 10^{-9} \text{ m}^4$$

$$\sigma_m = 180 \times 10^6 \text{ Pa} = \frac{M_m c}{I}$$

$$M_m = \frac{I \sigma_m}{c} = \frac{(961.06 \times 10^9)(180 \times 10^6)}{40 \times 10^{-3}} = 4.325 \times 10^3 \text{ N}\cdot\text{m}$$

$$P_m = \frac{M_m}{L} = \frac{4.325 \times 10^3}{1.2} = 3.604 \times 10^3 \text{ N}$$

By Appendix D, Case 1

$$y_m = \frac{P_m L^3}{3EI} = \frac{(3.604 \times 10^3)(1.2)^3}{(3)(200 \times 10^9)(961.06 \times 10^9)} = 10.8 \times 10^{-3} \text{ m}$$

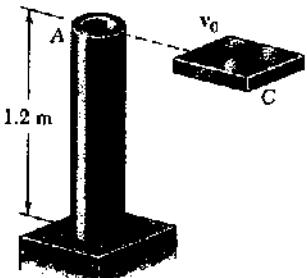
$$U_m = \frac{1}{2}P_m y_m = \frac{1}{2}(3.604 \times 10^3)(10.8 \times 10^{-3}) = 19.4616 \text{ J}$$

$$\frac{1}{2}mv_0^2 = U_m$$

$$v_0 = \sqrt{\frac{2U_m}{m}} = \sqrt{\frac{(2)(19.4616)}{6}} = 2.55 \text{ m/s}$$

Problem 11.50

11.50 Solve Prob. 11.49, assuming that the post *AB* consists of a solid steel rod of 80-mm diameter.



11.49 The post *AB* consists of a steel pipe of 80-mm-diameter and 6-mm wall thickness. A 6-kg block *C* moving horizontally with at velocity v_0 hits the post squarely at *A*. Using $E = 200$ GPa, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 180 MPa.

$$C = \frac{1}{2}d^2 = \frac{1}{2}(80)^2 = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$I = \frac{\pi}{4}C^4 = \frac{\pi}{4}(40 \times 10^{-3})^4 = 2.01062 \times 10^{-12} \text{ m}^4$$

$$\sigma_m = 180 \times 10^6 \text{ Pa} = \frac{M_m c}{I}$$

$$M_m = \frac{I \sigma_m}{c} = \frac{(2.01062 \times 10^{-12})(180 \times 10^6)}{40 \times 10^{-3}} = 9.048 \times 10^3 \text{ N}\cdot\text{m}$$

$$P_m = \frac{M_m}{L} = \frac{9.048 \times 10^3}{1.2} = 7.5398 \times 10^3 \text{ N}$$

By Appendix D, Case 1 $y_m = \frac{P_m L^3}{3EI}$

$$y_m = \frac{(7.5398 \times 10^3)(1.2)^3}{(3)(200 \times 10^9)(2.01062 \times 10^{-12})} = 10.8 \times 10^{-3} \text{ m}$$

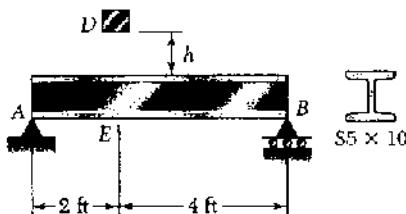
$$U_m = \frac{1}{2}P_m y_m = \frac{1}{2}(7.5398 \times 10^3)(10.8 \times 10^{-3}) = 40.715 \text{ J}$$

$$\frac{1}{2}mv_0^2 = U_m$$

$$v_0 = \sqrt{\frac{2U_m}{m}} = \sqrt{\frac{(2)(40.715)}{6}} = 3.68 \text{ m/s}$$

Problem 11.51

11.51 The 45-lb block D is dropped from a height $h = 0.6$ ft onto the steel beam AB. Knowing that $E = 29 \times 10^6$ psi, determine (a) the maximum deflection at point E, (b) the maximum normal stress in the beam.

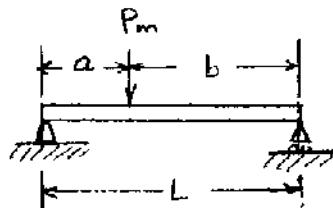


$$I_x = 12.3 \text{ in}^4 \quad S_x = 4.92 \text{ in}^3$$

$$a = 2 \text{ ft} = 24 \text{ in.} \quad b = 4 \text{ ft} = 48 \text{ in}$$

$$L = 6 \text{ ft} = 72 \text{ in.} \quad h \leq 0.6 \text{ ft} = 7.2 \text{ in.}$$

$$R_A = \frac{P_m b}{L}, \quad R_B = \frac{P_m a}{L}, \quad M_m = M_E = \frac{P_m ab}{L}$$



From Appendix D Case 5

$$y_E = \frac{P_m a^2 b^2}{3EI L} = \frac{P_m (24)^2 (48)^2}{(3)(29 \times 10^6)(12.3)(72)} = 17.2245 \times 10^{-6} P_m$$

$$P_m = 58057 y_E$$

$$U_m = \frac{1}{2} P_m y_m = 29028 y_E^2$$

$$\text{Work of falling weight: } W(h + y_E) = 45(7.2 + y_E) = 324 + 45y_E$$

$$\text{Equating work and energy, } 324 + 45y_E = 29028 y_E^2$$

$$y_E^2 - 1.5502 \times 10^{-3} y_E - 11.1615 \times 10^{-3} = 0$$

$$(a) y_E = \frac{1}{2} \left[1.5502 \times 10^{-3} + \sqrt{(1.5502 \times 10^{-3})^2 - (4)(-11.1615 \times 10^{-3})} \right] = 0.1064 \text{ in.}$$

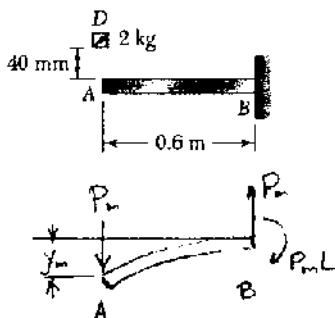
$$(b) P_m = (58057)(0.1064) = 6.179 \times 10^3 \text{ lb.}$$

$$M_m = \frac{(6.179 \times 10^3)(24)(48)}{72} = 98.86 \times 10^3 \text{ lb-in}$$

$$\sigma_m = \frac{M_m}{S_x} = \frac{98.86 \times 10^3}{4.92} = 20.1 \times 10^3 \text{ psi} = 20.1 \text{ ksi}$$

Problem 11.52

11.52 and 11.53 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200 \text{ GPa}$, determine (a) the maximum deflection of end A, (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.2170 \times 10^3 \text{ mm}^4 = 3.2170 \times 10^{-9} \text{ m}^4$$

$$C = \frac{d}{2} = 8 \text{ mm} = 8 \times 10^{-3} \text{ m} \quad L_{AB} = 0.6 \text{ m}$$

Appendix D, Case 1

$$y_m = \frac{P_m L_{AB}^3}{3EI} \quad M_m = P_m L_{AB}$$

$$P_m = \frac{3EI}{L_{AB}^3} y_m = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3} = 8.9361 \times 10^3 \text{ N}$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2} (8.9361 \times 10^3) y_m^2 = 4.4681 \times 10^3 y_m^2$$

$$\begin{aligned} \text{Work of dropped weight: } mg(h + y_m) &= (2)(9.81)(0.040 + y_m) \\ &= 0.7848 + 19.62 y_m \end{aligned}$$

Equating work and energy,

$$0.7848 + 19.62 y_m = 4.4681 \times 10^3 y_m^2$$

$$y_m^2 - 4.3911 \times 10^{-3} y_m - 175.645 \times 10^{-6} = 0$$

$$(a) \quad y_m = \frac{1}{2} \left\{ 4.3911 \times 10^{-3} + \sqrt{(4.3911 \times 10^{-3})^2 + (4)(175.645 \times 10^{-6})} \right\} \\ = 15.629 \times 10^{-3} \text{ m} = 15.63 \text{ mm}$$

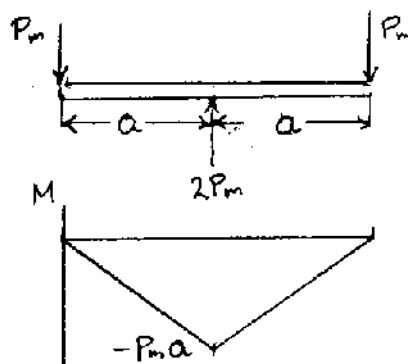
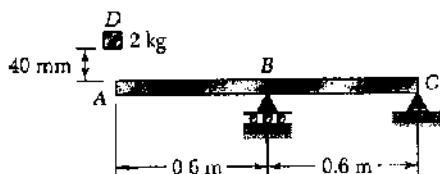
$$P_m = (8.9361 \times 10^3)(15.629 \times 10^{-3}) = 139.66 \text{ N}$$

$$(b) \quad M_m = -P_m L_{AB} = -(139.66)(0.6) = -83.8 \text{ N-m}$$

$$(c) \quad \sigma_m = \frac{M_m C}{I} = \frac{(83.8)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 208 \times 10^6 \text{ Pa} = 208 \text{ MPa}$$

Problem 11.53

11.52 and 11.53 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200 \text{ GPa}$, determine (a) the maximum deflection of end A, (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.2170 \times 10^3 \text{ mm}^4 = 3.2170 \times 10^{-9} \text{ m}^4$$

$$c = \frac{d}{2} = 8 \text{ mm} = 8 \times 10^{-3} \text{ m} \quad a = 0.6 \text{ m}$$

$$\text{Over AB: } M = -P_m x \quad M_m = -P_m a$$

$$U_{AB} = \int_0^a \frac{P_m^2 x^2}{2EI} dx = \frac{P_m^2 a^3}{6EI}$$

$$= \frac{(0.6)^3}{(6)(200 \times 10^9)(3.2170 \times 10^{-9})} P_m^2$$

$$= 55.953 \times 10^{-6} P_m^2$$

By symmetry of bending moment diagram,

$$U_{BC} = U_{AB} = 55.953 \times 10^{-6} P_m^2$$

$$U_m = U_{AB} + U_{BC} = 111.906 \times 10^{-6} P_m^2$$

$$\frac{1}{2} P_m y_m = U_m = 111.906 \times 10^{-6} P_m^2 \quad P_m = 4.4681 \times 10^3 y_m$$

$$U_m = \frac{1}{2} P_m y_m = 2.2340 \times 10^3 y_m^2$$

$$\text{Work of dropped weight: } mg(h + y_m) = (2)(9.81)(0.040 + y_m)$$

$$= 0.7848 + 19.62 y_m$$

Equating work and energy,

$$0.7848 + 19.62 y_m = 2.2340 \times 10^3 y_m^2$$

$$y_m^2 - 8.7825 \times 10^3 y_m - 351.298 \times 10^{-6} = 0$$

$$(a) y_m = \frac{1}{2} \left\{ 8.7825 \times 10^3 + \sqrt{(8.7825 \times 10^3)^2 + (4)(351.298 \times 10^{-6})} \right\}$$

$$= 23.636 \times 10^{-3} \text{ m} = 23.6 \text{ mm}$$

$$P_m = (4.4681 \times 10^3)(23.636 \times 10^{-3}) = 105.61 \text{ N}$$

$$(b) M_m = -(105.61)(0.6) = -64.4 \text{ N-m}$$

$$(c) \sigma_m = \frac{|M_m|c}{I} = \frac{(64.4)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 157.6 \times 10^6 \text{ Pa}$$

$$= 157.6 \text{ MPa}$$

Problem 11.54

11.54 A block of weight W is placed in contact with a beam at some given point D and released. Show that the resulting maximum deflection at point D is twice as large as the deflection due to a static load W applied at D .

Consider dropping the weight from a height h above the beam. The work done by the weight is

$$\text{Work} = W(h + y_m)$$

$$\text{Strain energy: } U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2$$

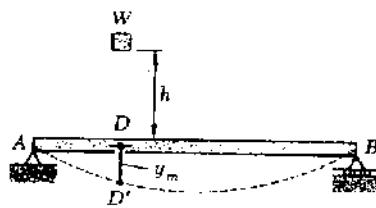
where k is the spring constant of the beam for loading at point D .

$$\text{Equating work and energy, } W(h + y_m) = \frac{1}{2} k y_m^2$$

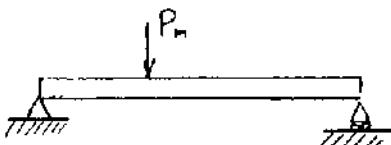
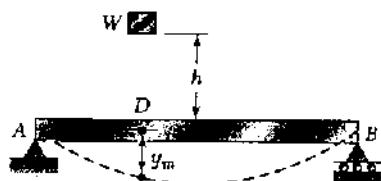
$$\text{Setting } h = 0, \quad W y_m = \frac{1}{2} k y_m^2, \quad y_m = \frac{2W}{k}$$

The static deflection at point D due to weight applied at D is

$$S_{st} = \frac{W}{k}. \quad \text{Thus, } y_m = 2 S_{st}$$



Problem 11.55



$$\text{Work of falling weight: } \text{Work} = W(h + y_m)$$

$$\text{Strain energy: } U = \frac{1}{2}P_m y_m = \frac{1}{2}k y_m^2$$

where k is the spring constant for a load applied at point D.

Equating work and energy,

$$W(h + y_m) = \frac{1}{2}k y_m^2$$

$$y_m^2 - \frac{2W}{k} y_m - \frac{2W}{k} h = 0$$

$$y_m^2 - 2y_{st} y_m - 2y_{st} h = 0 \quad \text{where } y_{st} = \frac{W}{k}$$

$$(a) \quad y_m = \frac{2y_{st} + \sqrt{4y_{st}^2 + 8y_{st}h}}{2} = y_{st} \left(1 + \sqrt{1 + \frac{2h}{y_{st}}} \right)$$

$$\text{For Prob. 11.52} \quad W = mg = (2)(9.81) = 19.62 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa} \quad I = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$$

$$L = 0.6 \text{ m} \quad h = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\text{Using Appendix D Case 1} \quad y_{st} = \frac{WL^3}{3EI}$$

$$y_{st} = \frac{(19.62)(0.6)^3}{(3)(200 \times 10^9)(3.217 \times 10^{-9})} = 2.196 \times 10^{-5} \text{ m}$$

$$\frac{2h}{y_{st}} = \frac{(2)(40 \times 10^{-3})}{2.196 \times 10^{-5}} = 36.44$$

$$(b) \quad \text{impact factor} = 1 + \sqrt{1 + 36.44} = 7.12$$

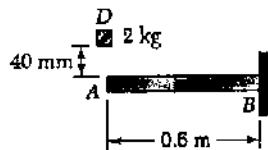
11.55 A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D . (a) Show that the maximum deflection y_m at point D can be

expressed as

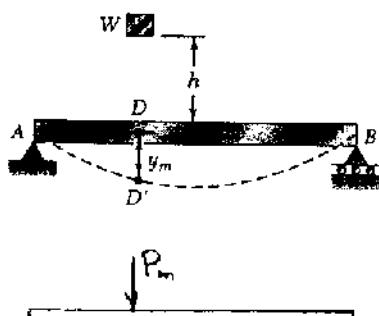
$$y_m = y_{st} \left(1 + \sqrt{1 + \frac{2h}{y_{st}}} \right)$$

where y_m represents the deflection at D caused by a static load W applied at that point and where the quantity in parenthesis is referred to as the *impact factor*. (b) Compute the impact factor for the beam and the impact of Prob. 11.52.

11.52 and 11.53 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end A , (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.

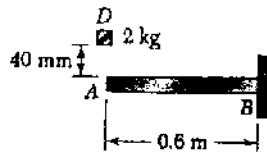


Problem 11.56



11.56 A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D . (a) Denoting by y_m the exact value of the maximum deflection at D and by y'_m the value obtained by neglecting the effect of this deflection on the change in potential energy of the block, show that the absolute value of the relative error is $(y'_m - y_m)/y_m$ never exceeds $y'_m/2h$. (b) Check the result obtained in part a by solving part A of Prob. 11.52 without taking y_m into account when determining the change in potential energy of the load, and comparing the answer obtained in this way with the exact answer to that problem.

11.52 and 11.53 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end A , (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



$$U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2 \quad \text{where } k \text{ is the spring constant for a load at point D.}$$

$$\begin{aligned} \text{Work of falling weight:} \quad & \text{exact: Work} = W(h + y_m) \\ & \text{approximate: Work} \propto Wh \end{aligned}$$

$$\begin{aligned} \text{Equating work and energy,} \quad & \frac{1}{2} k y_m^2 = W(h + y_m) \quad (1) \text{ exact} \\ & \frac{1}{2} k y_m'^2 = Wh \quad (2) \text{ approximate} \end{aligned}$$

where y'_m is the approximate value for y_m .

$$\text{Subtracting} \quad \frac{1}{2} k (y_m^2 - y_m'^2) = W y_m$$

$$y_m^2 - y_m'^2 = (y_m - y_m')(y_m + y_m') = \frac{2W}{k} y_m$$

$$\text{Relative error: } \frac{y_m - y_m'}{y_m} = \frac{2W}{k(y_m + y_m')}$$

$$\text{But, } \frac{2W}{k} = \frac{y_m'^2}{h} \text{ from equation (2).}$$

$$(a) \text{ Relative error} = \frac{y_m - y_m'}{y_m} = \frac{y_m'^2}{h(y_m + y_m')} < \frac{y_m'}{2h}$$

$$(b) \text{ From the solution to Prob. 11.62 } y_m = 15.63 \text{ mm}$$

$$\text{Approximate solution: } W = mg = (2)(9.81) = 19.62 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa} \quad I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^9 \text{ m}^4$$

$$L = 0.6 \text{ m}, \quad h = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$K = \frac{3EI}{L^3} = \frac{(3)(200 \times 10^9)(3.217 \times 10^9)}{(0.6)^3} = 8.936 \times 10^3 \text{ N/m}$$

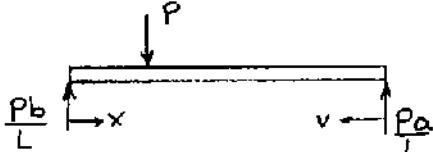
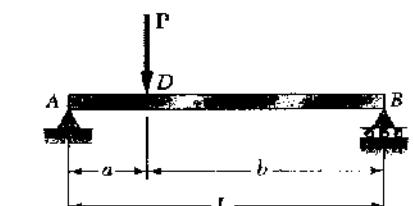
$$y_m'^2 = \frac{2Wh}{k} = \frac{(2)(19.62)(40 \times 10^{-3})}{8.936 \times 10^3} = 175.65 \times 10^{-6} \text{ m}^2$$

$$y_m' = 13.25 \times 10^{-3} \text{ m} = 13.25 \text{ mm}$$

$$\text{relative error} = \frac{15.63 - 13.25}{15.63} = 0.152 \quad \frac{y_m'}{2h} = 0.166$$

Problem 11.57

11.57 and 11.58 Using the method of work and energy, determine the deflection at point D caused by the load P.



$$\text{Reactions: } R_A = \frac{Pb}{L}, \quad R_B = \frac{Pa}{L}$$

$$\text{Over AD: } M = R_A x = \frac{Pbx}{L}$$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2 b^2}{2EI L^2} \int_0^a x^2 dx \\ = \frac{P^2 b^2 a^3}{6EI L^2}$$

$$\text{Over DB: } M = R_B v = \frac{Pav}{L}$$

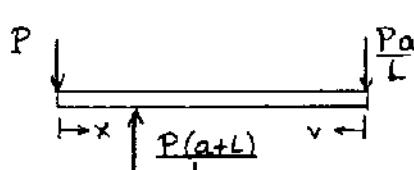
$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^b v^2 dv \\ = \frac{P^2 a^2 b^3}{6EI L^2}$$

$$\text{Total } U = U_{AD} + U_{DB} = \frac{P^2 a^2 b^2 (a+b)}{6EI L^2} = \frac{P^2 a^2 b^2}{6EI L}$$

$$\frac{1}{2} PS_0 = U \quad S_0 = \frac{2U}{P} = \frac{P a^2 b^2}{3EI L} \downarrow$$

Problem 11.58

11.57 and 11.58 Using the method of work and energy, determine the deflection at point D caused by the load P.



$$\text{Over portion DA: } M = -Px$$

$$U_{DA} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

$$\text{Over portion AB: } M = -\frac{Pav}{L}$$

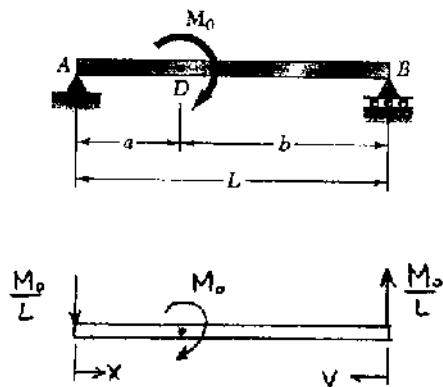
$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^L v^2 dv = \frac{P a^2 L}{6EI}$$

$$\text{Total } U = U_{DA} + U_{AB} = \frac{P^2 a^2 (a+L)}{6EI}$$

$$\frac{1}{2} PS_0 = U \quad S_0 = \frac{2U}{P} = \frac{P a^2 (a+L)}{3EI} \downarrow$$

Problem 11.59

11.59 and 11.60 Using the method of work and energy, determine the slope at point D caused by the couple M_0 .



$$\text{Reactions } R_A = \frac{M_0}{L} \downarrow \quad R_B = \frac{M_0}{L} \uparrow$$

$$\text{Over portion AD } M = -\frac{M_0 x}{L}$$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^a x^2 dx \\ = \frac{M_0^2 a^3}{6EI L^2}$$

$$\text{Over portion DB } M = \frac{M_0 v}{L}$$

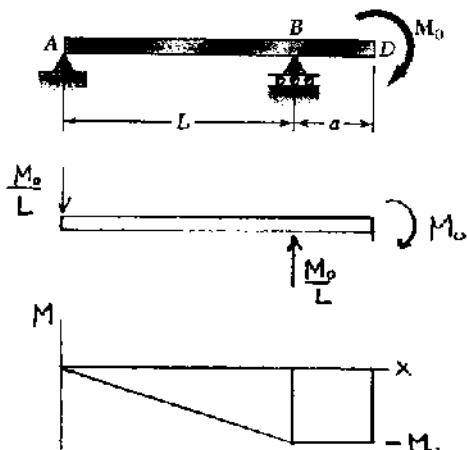
$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{M_0^2}{2EI L^2} \int_0^b v^2 dv \\ = \frac{M_0^2 b^3}{6EI L^2}$$

$$\text{Total } U = U_{AD} + U_{DB} = \frac{M_0^2 (a^3 + b^3)}{6EI L^2}$$

$$\frac{1}{2} M_0 \theta_D = U \quad \theta_D = \frac{2U}{M_0} = \frac{M_0 (a^3 + b^3)}{3EI L^2}$$

Problem 11.60

11.59 and 11.60 Using the method of work and energy, determine the slope at point D caused by the couple M_0 .



$$\text{Reactions } R_A = \frac{M_0}{L} \downarrow \quad R_B = \frac{M_0}{L} \uparrow$$

$$\text{Over portion AB } M = -\frac{M_0 x}{L}$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^L x^2 dx \\ = \frac{M_0^2 L}{6EI}$$

$$\text{Over portion BD } M = -M_0$$

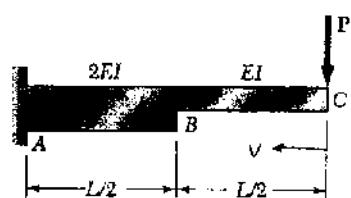
$$U_{BD} = \frac{M_0^2 a}{2EI}$$

$$\text{Total } U = U_{AB} + U_{BD} = \frac{M_0^2 (L + 3a)}{6EI}$$

$$\frac{1}{2} M_0 \theta_D = U \quad \theta_D = \frac{2U}{M_0} = \frac{M_0 (L + 3a)}{3EI}$$

Problem 11.61

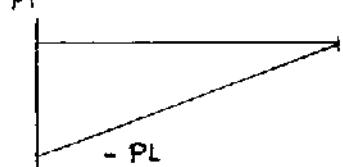
11.61 and 11.62 Using the method of work and energy, determine the deflection at point C caused by the load P.



Bending moment. $M = -Pv$

Over AB:

$$U_{AB} = \int_{\frac{L}{2}}^L \frac{M^2}{4EI} dv = \frac{P^2}{4EI} \int_{\frac{L}{2}}^L v^2 dv \\ = \frac{P^2}{12EI} \left[L^3 - \left(\frac{L}{2}\right)^3 \right] = \frac{7}{96} \frac{P^2 L^3}{EI}$$



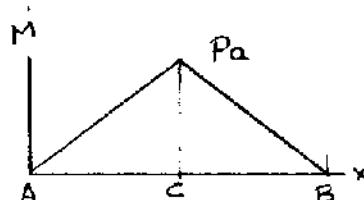
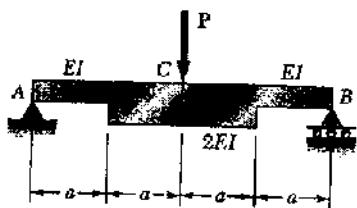
$$\text{Over BC} \quad U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_0^{\frac{L}{2}} v^3 dv \\ = \frac{1}{48} \frac{P^2 L^3}{EI}$$

$$\text{Total} \quad U = U_{AB} + U_{BC} = \frac{9}{32} \frac{P^2 L^3}{EI}$$

$$\frac{1}{2} P S_c = U \quad S_c = \frac{2U}{P} = \frac{3}{16} \frac{P L^3}{EI} \downarrow$$

Problem 11.62

11.61 and 11.62 Using the method of work and energy, determine the deflection at point C caused by the load P.



Symmetric beam and loading. $R_A = R_B = \frac{1}{2} P$

From A to C $M = R_A x = \frac{1}{2} Px$

$$U_{AC} = \int_0^a \frac{M^2}{2EI} dx + \int_a^{2a} \frac{M^2}{4EI} dx \\ = \frac{P^2}{8EI} \int_0^a x^2 dx + \frac{P^2}{16EI} \int_a^{2a} x^2 dx \\ = \frac{P^2 a^3}{24EI} + \frac{P^2}{48EI} [(2a)^3 - a^3] = \frac{3}{16} \frac{P^2 a^3}{EI}$$

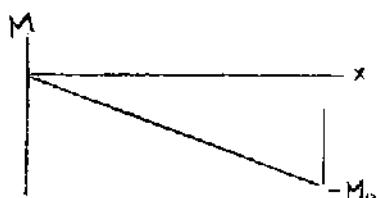
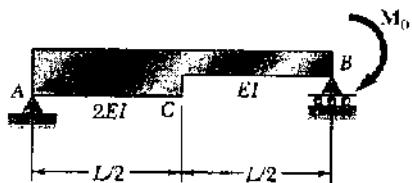
$$\text{By symmetry} \quad U_{CB} = U_{AB} = \frac{3}{16} \frac{P^2 a^3}{EI}$$

$$\text{Total} \quad U = U_{AB} + U_{BC} = \frac{3}{8} \frac{P^2 a^3}{EI}$$

$$\frac{1}{2} P S_c = U \quad S_c = \frac{2U}{P} = \frac{3}{4} \frac{P a^3}{EI} \downarrow$$

Problem 11.63

11.63 Using the method of work and energy, determine the slope at point *B* caused by the couple M_o .



$$\sum M_o = 0: -R_A L - M_o = 0 \quad R_A = -\frac{M_o}{L}$$

$$M = R_A x = -\frac{M_o}{L} x$$

$$\text{Over portion AC: } U_{Ac} = \int_0^{\frac{L}{2}} \frac{M^2}{2(2EI)} dx$$

$$U_{Ac} = \frac{M_o^2}{4EI L^2} \int_0^{\frac{L}{2}} x^2 dx = \frac{1}{96} \frac{M_o^2 L}{EI}$$

$$\text{Over portion CB: } U_{cb} = \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dx$$

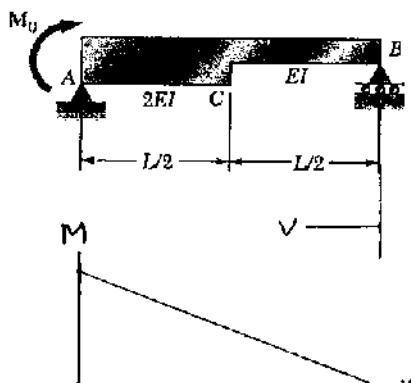
$$U_{cb} = \frac{M_o^2}{2EI L^2} \int_{\frac{L}{2}}^L x^2 dx = \frac{M_o^2}{6EI L^2} [L^3 - (\frac{L}{2})^3] \\ = \frac{7}{48} \frac{M_o^2 L}{EI}$$

$$\text{Total } U = U_{Ac} + U_{cb} = \frac{5}{32} \frac{M_o^2 L}{EI}$$

$$\frac{1}{2} M_o \theta_B = U \quad \theta_B = \frac{2U}{M_o} = \frac{5}{16} \frac{M_o L}{EI}$$

Problem 11.64

11.64 Using the method of work and energy, determine the slope at point *A* caused by the couple M_o .



$$R_B = \frac{M_o}{L}$$

$$M = R_B v = \frac{M_o}{L} v$$

$$\text{Over AC: } U_{Ac} = \int_{\frac{L}{2}}^L \frac{M^2}{2(2EI)} dv$$

$$U_{Ac} = \frac{M_o^2}{4EI L^2} \int_{\frac{L}{2}}^L v^2 dv = \frac{M_o^2}{12EI L^2} [L^3 - (\frac{L}{2})^3] \\ = \frac{7}{96} \frac{M_o^2 L}{EI}$$

$$\text{Over CB: } U_{cb} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$$

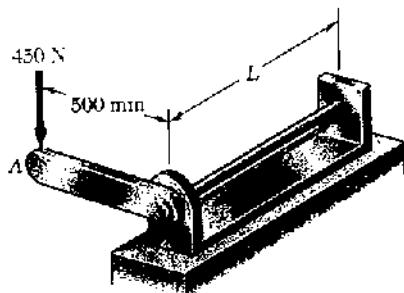
$$U_{cb} = \frac{M_o^2}{2EI L^2} \int_0^{\frac{L}{2}} v^2 dv = \frac{1}{48} \frac{M_o^2 L}{EI}$$

$$\text{Total } U = U_{Ac} + U_{cb} = \frac{3}{32} \frac{M_o^2 L}{EI}$$

$$\frac{1}{2} M_o \theta_A = U \quad \theta_A = \frac{2U}{M_o} = \frac{3}{16} \frac{M_o L}{EI}$$

Problem 11.65

11.65 The 20-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C . The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the deflection of point A when $L = 600$ mm. Use $E = 200$ GPa and $G = 77.2$ GPa.



Member AB. (Bending)

$$I = \frac{1}{2}(10)(30)^3 = 22.5 \times 10^5 \text{ mm}^4 = 22.5 \times 10^{-9} \text{ m}^4$$

$$a = 500 \text{ mm} = 0.500 \text{ m}$$

$$M_B = Pa = (450)(0.500) = 225 \text{ N}\cdot\text{m}$$

$$M = Px$$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \int_0^a \frac{P^2 x^2}{2EI} dx = \frac{P^2 a^3}{6EI}$$

$$= \frac{(450)^2 (0.500)^3}{(6)(200 \times 10^9)(22.5 \times 10^{-9})} = 0.9375 \text{ J}$$

Member BC (Torsion)

$$T = M_B = 225 \text{ N}\cdot\text{m} \quad c = \frac{1}{2}d = 10 \text{ mm} \quad J = \frac{\pi}{2}c^4 = 15.708 \times 10^3 \text{ mm}^4 = 15.708 \times 10^{-9} \text{ m}^4$$

$$L = 600 \text{ mm} = 0.600 \text{ m}$$

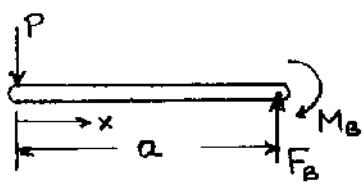
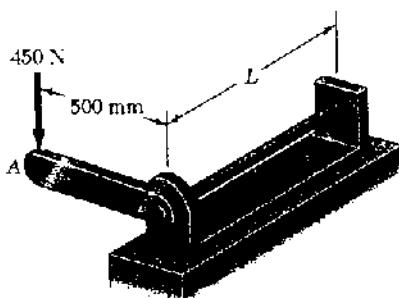
$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(225)^2 (0.600)}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 12.5242 \text{ J}$$

$$\text{Total } U = U_{AB} + U_{BC} = 0.9375 + 12.5242 = 13.4617 \text{ J}$$

$$\frac{1}{2}PS_A = U \quad S_A = \frac{2U}{P} = \frac{(2)(13.4617)}{450} = 59.8 \times 10^{-3} \text{ m} = 59.8 \text{ mm} \quad \blacktriangleleft$$

Problem 11.66

JI.66 The 20-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C. The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the length L of the rod BC for which the deflection at point A is 40 mm. Use $E = 200 \text{ GPa}$ and $G = 77.2 \text{ GPa}$.



Member AB. (Bending)

$$I = \frac{1}{12}(10)(30)^3 = 22.5 \times 10^3 \text{ mm}^4 = 22.5 \times 10^{-9} \text{ m}^4$$

$$a = 500 \text{ mm} = 0.500 \text{ m}$$

$$M_B = Pa = (450)(0.500) = 225 \text{ N}\cdot\text{m}$$

$$M = Px$$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \int_0^a \frac{P^2 x^2 dx}{2EI} = \frac{P^2 a^3}{6EI}$$

$$= \frac{(450)^2 (0.500)^3}{(6)(200 \times 10^9)(22.5 \times 10^{-9})} = 0.9375 \text{ J}$$

Member BC (Torsion)

$$T = M_B = 225 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 10 \text{ mm} \quad J = \frac{\pi}{2} c^4 = 15.708 \times 10^3 \text{ mm}^4 = 15.708 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(225)^2 L}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 20.874 L$$

$$\frac{1}{2} P S_A = U_{AB} + U_{BC}$$

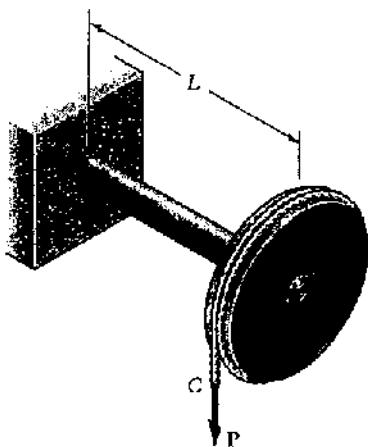
$$\frac{1}{2}(450)(40 \times 10^{-3}) = 0.9375 + 20.874 L$$

$$L = 0.386 \text{ m} = 386 \text{ mm}$$

Problem 11.67

11.67 A disk of radius a has been welded to end B of the solid steel shaft AB . A cable is wrapped around the disk and a vertical force P is applied to end C of the cable. Knowing that the radius of the shaft is r and neglecting the deformations of the disk and of the cable, show that the deflection of point C caused by the application of P is

$$\delta_C = \frac{PL^3}{3EI} \left(1 + 1.5 \frac{Er^2}{GL^2} \right)$$



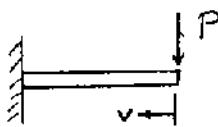
Torsion: $T = Pa$

$$U_t = \frac{T^2 L}{2GJ} = \frac{P^2 a^2 L}{2GJ}$$

Bending: $M = Pv$

$$U_b = \int_0^L \frac{M^2 dv}{2EI} = \int_0^L \frac{P^2 v^2 dv}{2EI}$$

$$= \frac{P^2 L^3}{6EI}$$



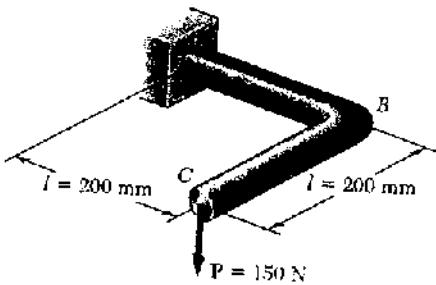
Total $U = \frac{P^2 a^2 L}{2GJ} + \frac{P^2 L^3}{6EI} = \frac{1}{2} P S_c$

$$S_c = \frac{Pa^2 L}{GJ} + \frac{PL^3}{3EI} = \frac{PL^3}{3EI} \left(1 + \frac{3EIa^2}{GJL^2} \right)$$

Since $J = 2I$ $S_c = \frac{PL^3}{3EI} \left(1 + \frac{3}{2} \frac{Ea^2}{GL^2} \right)$

Problem 11.68

11.68 The 12-mm-diameter steel rod ABC has been bent into the shape shown. Knowing that $E = 200 \text{ GPa}$ and $G = 77.2 \text{ GPa}$, determine the deflection of end C caused by the 150-N force.

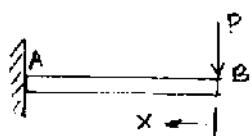


$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{12}{2}\right)^4 = 2.0358 \times 10^{-3} \text{ mm}^4 = 2.0358 \times 10^{-9} \text{ m}^4$$

$$I = \frac{1}{2} J = 1.0179 \times 10^{-9} \text{ m}^4$$

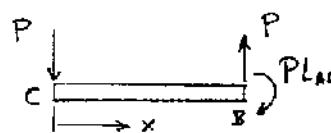
Portion AB: bending $M = -Px$

$$U_{AB,b} = \int_0^{L_{AB}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L_{AB}} x^2 dx \\ = \frac{P^2 L_{AB}^3}{6EI} = \frac{(150)^2 (200 \times 10^{-3})^3}{(6)(200 \times 10^9)(1.0179 \times 10^{-9})} \\ = 0.14736 \text{ J}$$



torsion $T = PL_{BC}$

$$U_{AB,t} = \frac{T^2 L_{AB}}{2GJ} = \frac{P^2 L_{BC}^2 L_{AB}}{2GJ} \\ = \frac{(150)^2 (200 \times 10^{-3})^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(2.0358 \times 10^{-9})} \\ = 0.57265 \text{ J}$$



Portion BC: $M = -Px$

$$U_{BC} = \int_0^{L_{BC}} \frac{M^2}{2EI} dx = \frac{P^2}{3EI} \int_0^{L_{BC}} x^2 dx = \frac{P^2 L_{BC}^3}{6EI} \\ = \frac{(150)^2 (200 \times 10^{-3})^3}{(6)(200 \times 10^9)(1.0179 \times 10^{-9})} = 0.14736 \text{ J}$$

$$\text{Total: } U = U_{AB,b} + U_{AB,t} + U_{BC} = 0.86737 \text{ J}$$

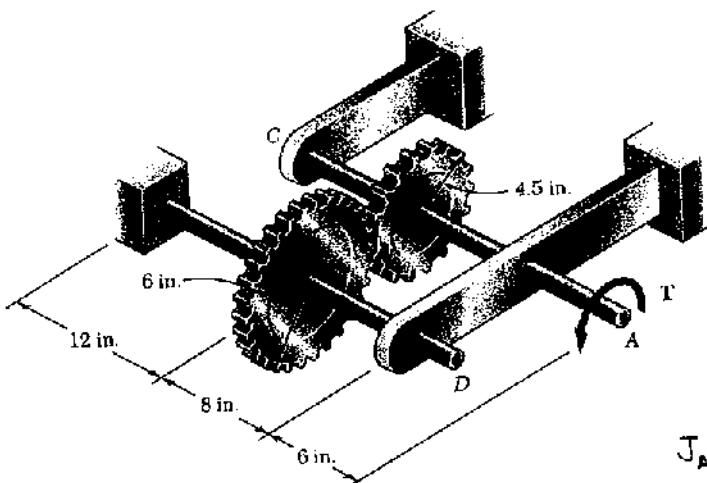
$$\text{Work-energy: } \frac{1}{2} PS = U \quad S = \frac{2U}{P} = \frac{(2)(0.86737)}{150}$$

$$= 11.57 \times 10^{-3} \text{ m} = 11.57 \text{ mm} \blacktriangleleft$$

Problem 11.69

11.69 Two steel shafts, each of $\frac{7}{8}$ -in diameter, are connected by the gears shown.

Knowing that $G = 11.2 \times 10^6$ psi and that shaft DF is fixed at F, determine the angle through which end A rotates when a 1.2 kip · in. torque is applied at A. Ignore the strain energy due to the bending of the shafts.



Work-energy equation:

$$\frac{1}{2}T_A\phi_A = TJ$$

$$\phi_A = \frac{2U}{T_A}$$

Portion AB of shaft ABC:

$$T_{AB} = T_A = 1200 \text{ lb-in}$$

$$L_{AB} = 8 + 6 = 14 \text{ in.}$$

$$J_{AB} = \frac{\pi(\frac{d}{2})^4}{2(2)} = \frac{\pi(2)^4}{2(2)} = 57.548 \times 10^{-5} \text{ in}^4$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(1200)^2 (14)}{(2)(11.2 \times 10^6)(57.548 \times 10^{-5})} = 15.639 \text{ in-lb}$$

Portion BC of shaft ABC: $U_{BC} = 0$

$$\text{Gear B: } F_{BE} = \frac{T_B}{r_B} = \frac{T_{AB}}{r_B} = \frac{1200}{4.5} = 266.67 \text{ lb}$$

$$\text{Gear E: } T_E = r_E F_{BE} = (6)(266.67) = 1600 \text{ lb-in}$$

Portion DE of shaft DEF: $U_{DE} = 0$

Portion EF of shaft DEF: $T_{EF} = T_E = 1600 \text{ lb-in}$

$$L_{EF} = 12 \text{ in. } J_{EF} = J_{AB} = 57.548 \times 10^{-5} \text{ in}^4$$

$$U_{EF} = \frac{T_{EF}^2 L_{EF}}{2GJ_{EF}} = \frac{(1600)^2 (12)}{(2)(11.2 \times 10^6)(57.548 \times 10^{-5})} = 23.831 \text{ in-lb}$$

$$\text{Total: } U = U_{AB} + U_{BC} + U_{DE} + U_{EF} = 39.47 \text{ in-lb.}$$

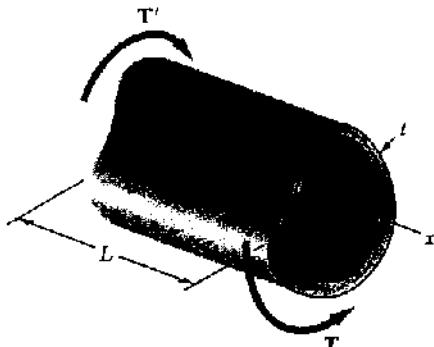
$$\phi_A = \frac{2U}{T_A} = \frac{(2)(39.47)}{1200} = 65.78 \times 10^{-3} \text{ rad} = 3.77^\circ$$

Problem 11.70

11.70 The thin-walled hollow cylindrical member AB has a noncircular cross section of nonuniform thickness. Using the expression given in Eq. (3.53) of Sec. 3.13, and the expression for the strain-energy density given in Eq. (11.19), show that the angle of twist of member AB is

$$\phi = \frac{TL}{4Q^2G} \oint \frac{ds}{t}$$

where ds is an element of the center line of the wall cross section and Q is the area enclosed by that center line.



$$\text{From equation (3.53), } Z' = \frac{T}{2ta}$$

Strain energy density:

$$U = \frac{\chi^2}{2G} = \frac{T^2}{8Gt^2a^2}$$

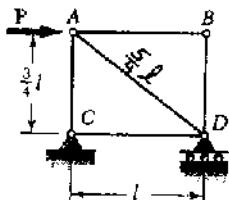
$$U = \int_0^L \oint U t ds dx$$

$$= \int_0^L \frac{T^2}{8Ga^2} \oint \frac{ds}{t} dx = \frac{T^2 L}{8Ga^2} \oint \frac{ds}{t}$$

$$\text{Work of torque} = \frac{1}{2} T \phi = \frac{T^2 L}{8Ga^2} \oint \frac{ds}{t} \quad \phi = \frac{TL}{4Ga^2} \oint \frac{ds}{t}$$

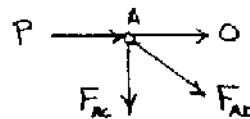
Problem 11.71

11.71 and 11.72 Each member of the truss shown has a uniform cross-sectional area A . Using the method of work and energy, determine the horizontal deflection of the point of application of the load P .



Members AB and BD are zero force members.

Joint A



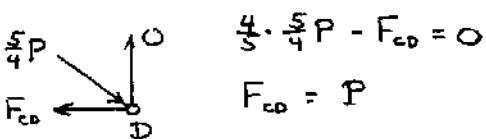
$$\rightarrow \sum F_x = 0$$

$$\frac{4}{5} F_{AD} + P = 0 \quad F_{AD} = -\frac{5}{4} P$$

$$\uparrow \sum F_y = 0$$

$$-F_{AC} - \frac{3}{5} F_{AD} = 0 \quad F_{AC} = \frac{3}{4} P$$

Joint D



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$= \frac{27}{16} \frac{P^2 l}{EA}$$

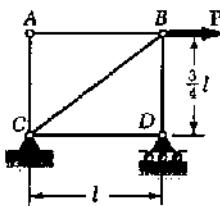
Member	F	L	$F^2 L$
AB	0	l	0
BD	0	$\frac{3}{4}l$	0
AD	$-\frac{5}{4}P$	$\frac{5}{4}l$	$\frac{125}{64}P^2 l$
CD	P	l	$P^2 l$
AC	$\frac{3}{4}P$	$\frac{3}{4}l$	$\frac{27}{64}P^2 l$
			$\frac{27}{8}P^2 l$

$$\text{Work of } P = \frac{1}{2} P \Delta = U$$

$$\Delta = \frac{2U}{P} = \frac{27}{8} \frac{Pl}{EA} = 3.375 \frac{Pl}{EA} \rightarrow \rightarrow$$

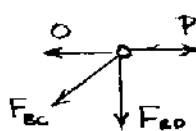
Problem 11.72

11.71 and 11.72 Each member of the truss shown has a uniform cross-sectional area A . Using the method of work and energy, determine the horizontal deflection of the point of application of the load P .



Members AB, AC, and CD are zero force members.

Joint B



$$\rightarrow \sum F_x = 0$$

$$P - \frac{4}{5} F_{BC} = 0 \quad F_{BC} = \frac{5}{4} P$$

$$\uparrow \sum F_y = 0$$

$$-F_{BD} - \frac{3}{5} F_{BC} = 0 \quad F_{BD} = -\frac{3}{4} P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$= \frac{19}{16} \frac{P^2 l}{EA}$$

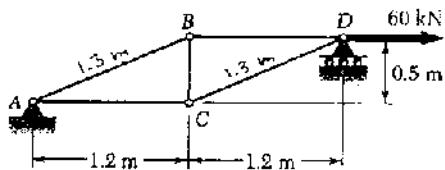
$$\text{Work of } P = \frac{1}{2} P \Delta = U$$

$$\Delta = \frac{2U}{P} = \frac{19}{8} \frac{Pl}{EA} = 2.375 \frac{Pl}{EA} \rightarrow \rightarrow$$

Member	F	L	$F^2 L$
AB	0	l	0
AC	0	$\frac{3}{4}l$	0
CD	0	l	0
BC	$\frac{5}{4}P$	$\frac{5}{4}l$	$\frac{125}{64}P^2 l$
BD	$-\frac{3}{4}P$	$\frac{3}{4}l$	$\frac{27}{64}P^2 l$
			$\frac{19}{8}P^2 l$

Problem 11.73

II.73 Each member of the truss shown is made of steel; the cross-sectional area of member BC is 800 mm^2 and for all other members the cross-sectional area is 400 mm^2 . Using $E = 200 \text{ GPa}$, determine the deflection of point D caused by the 60-kN load.



$$\text{Entire truss } \sum M_A = 0$$

$$2.4 R_D - (0.5)(60) = 0 \quad R_D = 12.5 \text{ kN}$$

Joint D $+ \uparrow \sum F_y = 0 \quad 12.5 - \frac{0.5}{1.3} F_{CD} = 0 \quad F_{CD} = 32.5 \text{ kN}$

$$F_{BD} \quad 60 \quad \rightarrow + \sum F_x = 0 \quad 60 - F_{BD} - \frac{1.2}{1.3} F_{CD} = 0 \quad F_{BD} = 30 \text{ kN}$$

Joint B $+ \rightarrow \sum F_x = 0 \quad 30 - \frac{1.2}{1.3} F_{AB} = 0 \quad F_{AB} = 32.5 \text{ kN}$

$$F_{AB} \quad 30 \quad + \uparrow \sum F_y = 0 \quad - \frac{0.5}{1.3} F_{AC} + F_{BD} = 0 \quad F_{AC} = 12.5 \text{ kN}$$

Joint C $+ \rightarrow \sum F_x = 0 \quad - F_{AC} + \frac{1.2}{1.3}(32.5) = 0 \quad F_{AC} = 30 \text{ kN}$

$$F_{AC} \quad 32.5 \quad U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	$F(\text{kN})$	$L(\text{m})$	$A(10^6 \text{ m}^2)$	$F^2 L/A (\text{N}^2/\text{m})$
CD	32.5	1.3	400	3.4328×10^{12}
BD	30	1.2	400	2.7×10^{12}
AB	32.5	1.3	400	3.4328×10^{12}
BC	12.5	0.5	800	0.0977×10^{12}
AC	30	1.2	400	2.7×10^{12}
Σ				12.3633×10^{12}

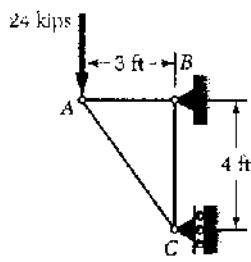
$$U = \frac{12.3633 \times 10^{12}}{(2)(200 \times 10^9)} = 30.908 \text{ J}$$

$$\frac{1}{2} P \Delta = U \quad \Delta = \frac{2U}{P} = \frac{(2)(30.908)}{60 \times 10^3} = 1.030 \times 10^{-3} \text{ m}$$

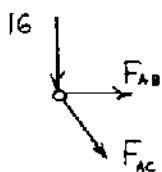
$$= 1.030 \text{ mm} \rightarrow$$

Problem 11.74

11.74 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 3 in². Using $E = 29 \times 10^6$ psi, determine the vertical deflection of joint 4 caused by the application of the 24-kip load.



Joint A



$$+\uparrow \sum F_y = 0$$

$$-16 - \frac{4}{3} F_{AC} = 0$$

$$F_{AC} = -20 \text{ kips}$$

$$+\rightarrow \sum F_x = 0$$

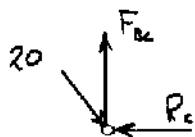
$$\frac{3}{5} F_{AC} + F_{AB} = 0$$

$$F_{AB} = 12 \text{ kips}$$

Joint C

$$+\uparrow \sum F_y = 0 \quad F_B - \frac{4}{5}(20) = 0$$

$$F_B = 16 \text{ kips}$$



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$E = 29 \times 10^3 \text{ ksi}; \\ A = 3 \text{ in}^2$$

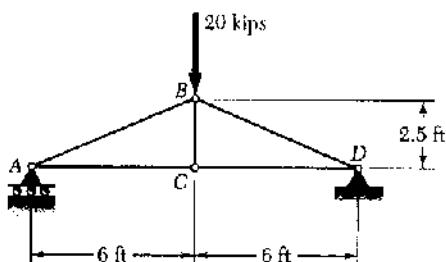
Member	F(kips)	L (in)	$F^2 L$ (kip ² ·in)
AB	18	36	11664
AC	-30	60	54000
BC	24	48	27648
Σ			93312

$$U = \frac{93312}{(2)(29 \times 10^3)(3)} = 0.53628 \text{ kip-in.}$$

$$\frac{1}{2} P \Delta = U \quad \Delta = \frac{2U}{P} = \frac{(2)(0.53628)}{24} = 0.0447 \text{ in. } \downarrow$$

Problem 11.75

11.75 Each member of the truss shown is made of steel and has a cross-sectional area of 5 in². Using $E = 29 \times 10^6$ psi, determine the vertical deflection of point B caused by the 20-kip load.



$$R_A = R_B = 10 \text{ kips} \uparrow$$

$$L_{AC} = L_{CD} = 6 \text{ ft} = 72 \text{ in.}$$

$$L_{BC} = 2.5 \text{ ft} = 30 \text{ in.}$$

$$L_{AB} = L_{BD} = \sqrt{6^2 + 2.5^2} = 6.5 \text{ ft} = 78 \text{ in.}$$

Joint A

$$\uparrow + \sum F_y = 0: \frac{2.5}{6.5} F_{AB} + 10 = 0$$

$$F_{AB} = -26 \text{ kips}$$

$$\uparrow + \sum F_x = 0: \frac{6}{6.5} F_{AB} + F_{AC} = 0$$

$$F_{AC} = 24 \text{ kips}$$

Joint C: $F_{BC} = 0, F_{CD} = 24 \text{ kips}$

By symmetry, $F_{BD} = F_{AB} = -26 \text{ kips}$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

Member	F (kips)	L (in.)	$F^2 L$ (kip ² ·in.)
AB	-26	78	52728
AC	24	72	41472
BC	0	30	0
CD	24	72	41472
BD	-26	78	52728
Σ			188400

$$U = \frac{188400}{(2)(29 \times 10^3)(5)}$$

$$= 0.64966 \text{ kip-in}$$

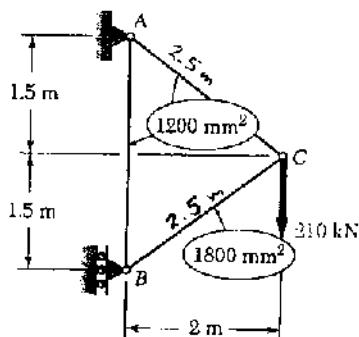
$$\frac{1}{2} P S_B = U$$

$$S_B = \frac{2U}{P} = \frac{(2)(0.64966)}{20}$$

$$= 0.0650 \text{ in.} \downarrow$$

Problem 11.76

11.76 Members of the truss shown are made of steel and have the cross-sectional areas shown. Using $E = 200 \text{ GPa}$, determine the vertical deflection of joint C caused by the application of the 210-kN load.



Joint C

$$+\rightarrow \sum F_x = 0 : \quad -\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} = 0$$

$$+\uparrow \sum F_y = 0 : \quad \frac{3}{5}F_{AC} - \frac{3}{5}F_{BC} - 210 = 0$$

Solving simultaneously,

$$F_{AC} = 175 \text{ kN}$$

$$F_{BC} = -175 \text{ kN}$$

Joint B

$$\begin{array}{l} F_{AB} \\ R_B \\ \hline \end{array} \quad +\uparrow \sum F_y = 0 \quad F_{AB} - \left(\frac{3}{5}\right)(175) = 0 \quad F_{AB} = 105 \text{ kN}$$

$$U_m = \sum \frac{F^2 L}{2EA}$$

Member	F (kN)	L (m)	A (10^{-6} m^2)	$F^2 L / A (\text{N}^2/\text{m})$
AB	105	3.0	1200	27.5625×10^{12}
AC	175	2.5	1200	63.8021×10^{12}
BC	-175	2.5	1800	42.5347×10^{12}
Σ				133.8993×10^{12}

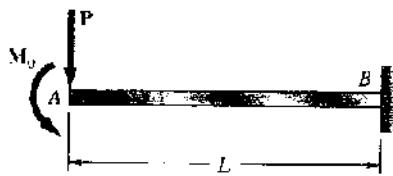
$$U_m = \frac{1}{2E} \sum \frac{F^2 L}{A} = \frac{133.8993 \times 10^{12}}{(2)(200 \times 10^9)} = 334.75 \text{ J}$$

$$\frac{1}{2} P_m \Delta_m = U_m$$

$$\Delta_m = \frac{2 U_m}{P_m} = \frac{(2)(334.75)}{210 \times 10^3} = 3.19 \times 10^{-3} \text{ m} = 3.19 \text{ mm} \downarrow$$

Problem 11.77

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M_0 is applied first.



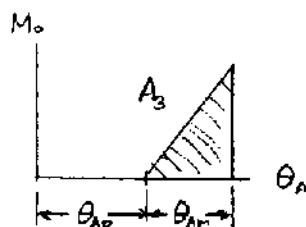
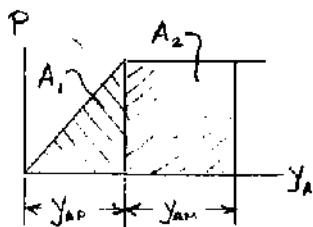
From Appendix D, Case 1

$$y_{AP} = \frac{PL^3}{3EI} \quad \theta_{AP} = \frac{PL^2}{2EI}$$

From Appendix D, Case 3

$$y_{AM} = \frac{M_0 L^2}{2EI} \quad \theta_{AM} = \frac{M_0 L}{EI}$$

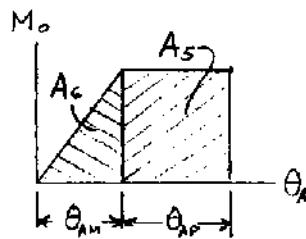
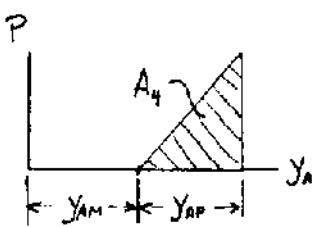
(a) First P , then M_0 .



$$U = A_1 + A_2 + A_3$$

$$\begin{aligned} &= \frac{1}{2}P y_{AP} + P y_{AM} + \frac{1}{2}M_0 \theta_{AM} \\ &= \frac{P^2 L^3}{6EI} + \frac{PM_0 L^2}{2EI} + \frac{M_0^2 L}{2EI} \end{aligned}$$

(b) First M_0 , then P .

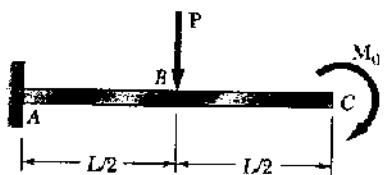


$$U = A_4 + A_5 + A_6$$

$$\begin{aligned} &= \frac{1}{2}P y_{AP} + M_0 \theta_{AP} + \frac{1}{2}M_0 \theta_{AM} \\ &= \frac{P^2 L^3}{6EI} + \frac{M_0 PL^2}{2EI} + \frac{M_0^2 L}{2EI} \end{aligned}$$

Problem 11.78

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M_0 is applied first.



Appendix D Cases 1 and 3

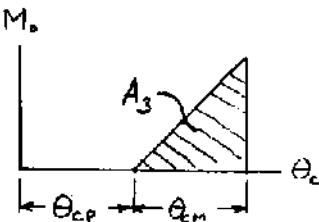
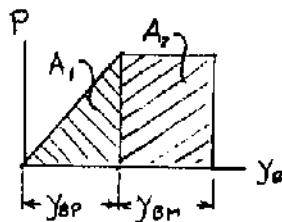
$$y_{BP} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\theta_{CP} = \frac{P(L/2)^2}{2EI} = \frac{PL^2}{8EI}$$

$$y_{BM} = \frac{M_0(L/2)^2}{2EI} = \frac{M_0L^2}{8EI}$$

$$\theta_{BM} = \frac{M_0L}{EI}$$

(a) First P , then M_0 .

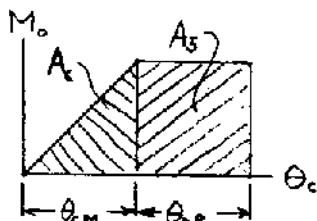
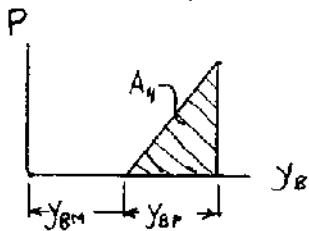


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2}Py_{BP} + Py_{BM} + \frac{1}{2}M_0\theta_{CM}$$

$$= \frac{P^2L^3}{48EI} + \frac{PM_0L^2}{8EI} + \frac{M_0^2L}{2EI}$$

(b) First M_0 , then P .



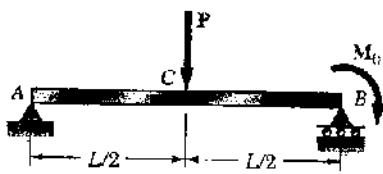
$$U = A_4 + A_5 + A_6$$

$$= \frac{1}{2}Py_{BP} + M_0\theta_{CP} + \frac{1}{2}M_0\theta_{CM}$$

$$= \frac{P^2L^3}{48EI} + \frac{PM_0L^2}{8EI} + \frac{M_0^2L}{2EI}$$

Problem 11.79

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M_o is applied first.



From Appendix D, Case 4

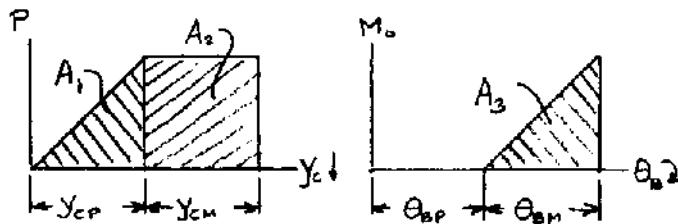
$$\downarrow y_c = \frac{PL^3}{48EI} \quad C\theta_B = -\frac{PL^2}{16EI}$$

From Appendix D, Case 7

$$\downarrow y_c = \frac{M_o}{6EI} \left((L/2)^3 - L^2(L/2) \right) = -\frac{M_o L^2}{16EI}$$

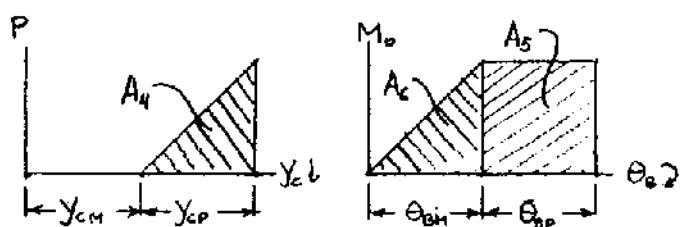
$$C\theta_B = \frac{M_o L}{3EI}$$

(a) First P , then M_o .



$$\begin{aligned} U &= A_1 + A_2 + A_3 \\ &= \frac{1}{2}P y_{cp} + P y_{cm} + \frac{1}{2}M_o \theta_{bm} \\ &= \frac{P^2 L^3}{96EI} - \frac{PM_o L^2}{16EI} + \frac{M_o^2 L}{6EI} \end{aligned}$$

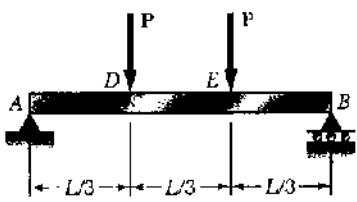
(b) First M_o , then P .



$$\begin{aligned} U &= A_4 + A_5 + A_6 \\ &= \frac{1}{2}P y_{cp} + M_o \theta_{bp} + \frac{1}{2}M_o \theta_{bm} \\ &= \frac{P^2 L^3}{96EI} - \frac{M_o PL^2}{16EI} + \frac{M_o^2 L}{6EI} \end{aligned}$$

Problem 11.80

11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.



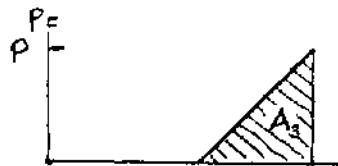
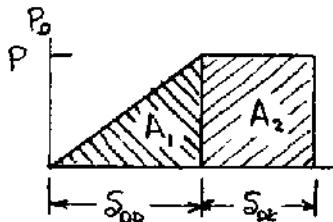
(a) Label the forces P_D and P_E .

Using Appendix D, Case 5

$$S_{EE} = \frac{P_E a^2 b^2}{3EI L} = \frac{P_E (\frac{2}{3}L)^2 (\frac{1}{3}L)^2}{3EI L} = \frac{4}{243} \frac{P_E L^3}{EI}$$

$$S_{DE} = \frac{P_E b}{6EI L} [(L^2 - b^2)x - x^3] = \frac{P_E (\frac{1}{3}L)}{6EI L} [(L^2 - (\frac{1}{3}L)^2)(\frac{1}{3}) - (\frac{L}{3})^3] = \frac{7}{486} \frac{P_E L^3}{EI}$$

Likewise, $S_{DD} = \frac{4}{243} \frac{P_D L^3}{EI}$ and $S_{ED} = \frac{7}{486} \frac{P_D L^3}{EI}$



Let P_D be applied first.

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} P_D S_{DD} + P_D S_{DE} + \frac{1}{2} P_E S_{EE} = \frac{2}{243} \frac{P_D^2 L^3}{EI} + \frac{7}{486} \frac{P_D P_E L^3}{EI} + \frac{2}{243} \frac{P_E^2 L^3}{EI}$$

With $P_D = P_E = P$ $U = \frac{5}{162} \frac{P^2 L^3}{EI}$

(b) Reactions: $R_A = R_B = P \uparrow$

Over portion AD. ($0 \leq x \leq \frac{L}{3}$) $M = Px$

$$U_{AD} = \int_0^{L/3} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L/3} x^2 dx = \frac{P^2}{6EI} (\frac{L}{3})^3 = \frac{1}{162} \frac{P^2 L^3}{EI}$$

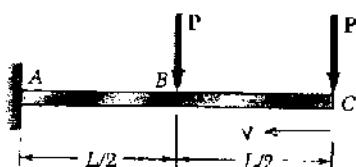
Over portion DE. ($\frac{L}{3} \leq x \leq \frac{2L}{3}$) $M = \frac{PL}{3}$

$$U_{DE} = \int_{L/3}^{2L/3} \frac{M^2}{2EI} dx = \frac{(PL/3)^2}{2EI} (L/3) = \frac{1}{54} \frac{P^2 L^3}{EI}$$

Over portion ED. By symmetry, $U_{EB} = U_{AD} = \frac{1}{162} \frac{P^2 L^3}{EI}$

Total. $U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{162} \frac{P^2 L^3}{EI}$

Problem 11.81



11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.

(a) Label the forces P_B and P_C

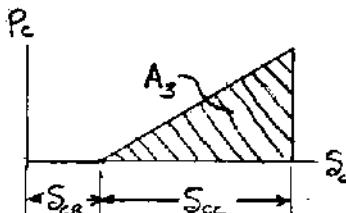
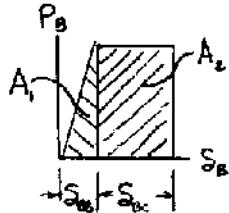
Using Appendix D Case 1

$$\epsilon_{BB} = \frac{P_B(L/2)^3}{3EI} = \frac{1}{24} \frac{P_B L^3}{EI}$$

$$S_{CB} = S_{BB} + \frac{L}{2} \Theta_B = \frac{1}{24} \frac{P_B L^3}{3EI} + \frac{L}{2} \frac{P_B (L/2)^2}{EI} = \left(\frac{1}{24} + \frac{1}{16} \right) \frac{P_B L^3}{EI} = \frac{5}{48} \frac{P_B L^3}{EI}$$

$$S_{CC} = \frac{1}{3} \frac{P_C L^3}{EI}$$

$$S_{BC} = \frac{P_C}{GEI} (3Lx^2 - x^3) = \frac{P_C}{GEI} \left(3L\left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^3 \right) = \frac{5}{48} \frac{P_C L^3}{EI}$$



Apply P_B first, then P_C

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} P_B S_{BB} + P_B S_{BC} + \frac{1}{2} P_C S_{CC} = \frac{1}{24} \frac{P_B L^3}{EI} + \frac{5}{48} \frac{P_B P_C L^3}{EI} + \frac{1}{6} \frac{P_C^2 L^3}{EI}$$

$$\text{With } P_B = P_C = P \quad U = \left(\frac{1}{24} + \frac{5}{48} + \frac{1}{6} \right) \frac{P^2 L^3}{EI} = \frac{7}{24} \frac{P^2 L^3}{EI}$$

$$\text{Over AB: } M = Pv + P(v - \frac{L}{2}) = P(2v - \frac{L}{2})$$

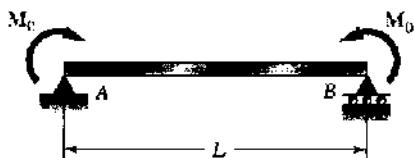
$$\begin{aligned} U_{AB} &= \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_{\frac{L}{2}}^L (4v^2 - 2Lv + \frac{1}{4}L^2) dv \\ &= \frac{P^2}{2EI} \left\{ \frac{4}{3} \left[L^3 - \left(\frac{L}{2}\right)^3 \right] - 2L \cdot \frac{1}{2} \left[L^3 - \left(\frac{L}{2}\right)^3 \right] + \frac{1}{8} L^4 \left[L - \frac{L}{2} \right] \right\} \\ &= \frac{P^2}{2EI} \left\{ \frac{7}{6} L^3 - \frac{3}{4} L^3 + \frac{1}{8} L^3 \right\} = \frac{13}{48} \frac{P^2 L^3}{EI} \end{aligned}$$

$$\text{Over BC: } M = Pv \quad U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_0^{\frac{L}{2}} v^2 dv = \frac{P^2}{2EI} \cdot \frac{1}{3} \left(\frac{L}{2}\right)^3 = \frac{P^2 L^3}{48 EI}$$

$$\text{Total } U = U_{AB} + U_{BC} = \left(\frac{13}{48} + \frac{1}{48} \right) \frac{P^2 L^3}{EI} = \frac{7}{24} \frac{P^2 L^3}{EI}$$

Problem 11.82

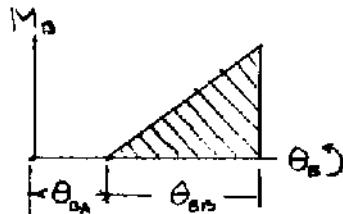
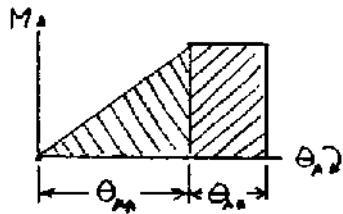
11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.



(a) Label the couples M_A and M_B .

Using Appendix D, Case 7

$$C\theta_{AA} = \frac{M_A L}{3EI} \quad C\theta_{BA} = \frac{M_A L}{GEI} \quad C\theta_{BB} = \frac{M_B L}{3EI} \quad C\theta_{AB} = \frac{M_B L}{GEI}$$



Apply M_A First, then M_B .

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} M_A \theta_{AA} + M_A \theta_{AB} + \frac{1}{2} M_B \theta_{BB} = \frac{1}{6} \frac{M_A^3 L}{EI} + \frac{1}{6} \frac{M_A M_B L}{EI} + \frac{1}{6} \frac{M_B^3 L}{EI}$$

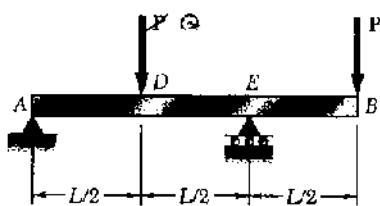
$$\text{With } M_A = M_B = M_0 \quad U = \frac{1}{2} \frac{M_0^3 L}{EI}$$

(b) Bending moment $M = M_0$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2 L}{2EI}$$

Problem 11.83

11.83 and 11.85 For the prismatic beam shown, determine the deflection of point D.

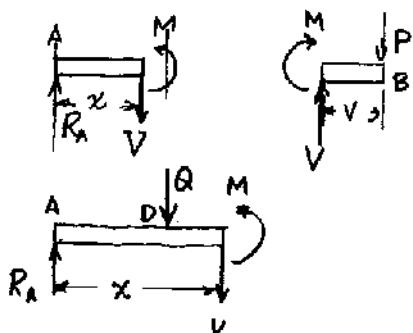


Change force at D from P to Q.

Reactions. $R_A = \frac{1}{2}(Q-P) \uparrow$, $R_B = \frac{1}{2}(3P+Q) \uparrow$

Strain energy $\bar{U} = U_{AD} + U_{DE} + U_{EB}$

Deflection of point D.



$$S_D = \frac{\partial U}{\partial Q} = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q}$$

$$\text{Portion AD. } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx$$

$$M = R_A x = \frac{1}{2}(Q-P)x$$

$$\frac{\partial M}{\partial Q} = \frac{1}{2}x \quad \text{With } Q = P, M = 0$$

$$\frac{\partial U_{AD}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dx = 0$$

$$\text{Portion DE: } U_{DE} = \int_{L/2}^L \frac{M^2}{2EI} dx$$

$$M = R_A x - Q(x - \frac{L}{2}) = \frac{1}{2}(Q-P)x - Q(x - \frac{L}{2}) = \frac{1}{2}Q(L-x) - \frac{1}{2}Px$$

$$\frac{\partial M}{\partial Q} = \frac{1}{2}(L-x)$$

$$\text{With } Q = P, M = -\frac{1}{2}R(2x-L)$$

$$\begin{aligned} \frac{\partial U_{DE}}{\partial Q} &= \int_{L/2}^L \frac{M}{EI} \frac{\partial M}{\partial Q} dx = -\frac{P}{4EI} \int_{L/2}^L (2x-L)(L-x) dx \\ &= -\frac{P}{4EI} \int_{L/2}^L (3Lx - 2x^2 - L^2) dx \\ &= -\frac{P}{4EI} \left[\left(\frac{3}{2}Lx^2 - \frac{2}{3}x^3 - L^2x \right) \right]_{L/2}^L \\ &= -\frac{PL^3}{4EI} \left[\frac{3}{2} - \frac{2}{3} - 1 - \left(\frac{3}{2} \right) \left(\frac{1}{4} \right) + \left(\frac{2}{3} \right) \left(\frac{1}{8} \right) + \frac{1}{2} \right] = -\frac{PL^3}{96EI} \end{aligned}$$

$$\text{Portion EB: } U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$M = -Pv$$

$$\frac{\partial M}{\partial Q} = 0$$

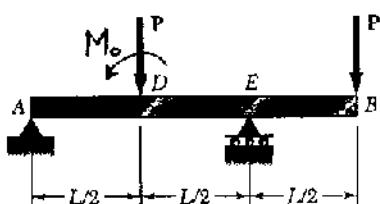
$$\frac{\partial U_{EB}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dv = 0$$

$$\text{Deflection of Point D. } S_D = 0 - \frac{PL^3}{96EI} + 0 = -\frac{PL^3}{96EI} \downarrow$$

$$\delta_D = \frac{PL^3}{96EI} \uparrow$$

Problem 11.84

11.84 and 11.86 For the prismatic beam shown, determine the slope at point D.



Add couple M_o at point D as shown.

$$\text{Reactions. } R_A = \frac{M_o}{L} f, \quad R_E = 2P - \frac{M_o}{L} \uparrow$$

$$\text{Strain energy. } U = U_{AD} + U_{DE} + U_{EB}$$

Slope at point D.

$$\theta_D = \frac{\partial U}{\partial M_o} = \frac{\partial U_{AD}}{\partial M_o} + \frac{\partial U_{DE}}{\partial M_o} + \frac{\partial U_{EB}}{\partial M_o}$$

$$\text{Portion AD. } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx$$

$$M = R_A x = \frac{M_o}{L} x$$

$$\frac{\partial M}{\partial M_o} = \frac{x}{L} \quad \text{With } M_o = 0, \quad M = 0$$

$$\frac{\partial U_{AD}}{\partial M_o} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial M_o} dx = 0$$

$$\text{Portion DE: } U_{DE} = \int_{L/2}^L \frac{M^2}{2EI} dx$$

$$M = R_A x - P(x - \frac{L}{2}) - M_o = \frac{M_o}{L} x - P(x - \frac{L}{2}) - M_o$$

$$\frac{\partial M}{\partial M_o} = (\frac{x}{L} - 1) \quad \text{With } M_o = 0, \quad M = -P(x - \frac{L}{2})$$

$$\frac{\partial U_{DE}}{\partial M_o} = \int_{L/2}^L \frac{M}{EI} \frac{\partial M}{\partial M_o} dx = -\frac{P}{EI} \int_{L/2}^L (\frac{x}{L} - 1)(x - \frac{L}{2}) dx$$

$$= -\frac{P}{2EI} \int_{L/2}^L (L-x)(2x-L) dx = \frac{P}{2EI} \int_{L/2}^L (3Lx - 2x^2 - L^2) dx$$

$$= \frac{P}{2EI} \left(\frac{3}{2}Lx^2 + \frac{2}{3}x^3 - L^2x \right) \Big|_{L/2}^L$$

$$= \frac{PL^3}{2EI} \left[\frac{3}{2} - \frac{2}{3} - 1 - (\frac{3}{2})(\frac{1}{4}) + (\frac{2}{3})(\frac{1}{8}) + \frac{1}{2} \right] = \frac{PL^2}{48EI}$$

$$\text{Portion EB: } U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$M = -Pv$$

$$\frac{\partial M}{\partial M_o} = 0$$

$$\frac{\partial U_{EB}}{\partial M_o} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial M_o} dv = 0$$

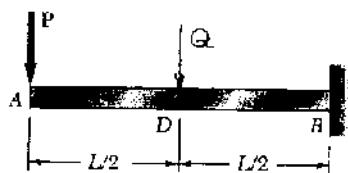
Slope at point D.

$$\theta_D = 0 + \frac{PL^2}{4EI} + 0$$

$$= \frac{PL^2}{48EI}$$

Problem 11.85

11.83 and 11.85 For the prismatic beam shown, determine the deflection of point D.



Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_D = \frac{\partial U}{\partial Q} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx$$

Over portion AD $0 < x < \frac{L}{2}$ $M = -Px$, $\frac{\partial M}{\partial Q} = 0$

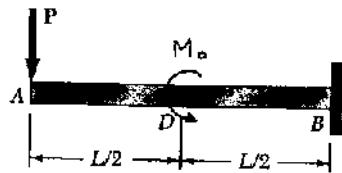
Over portion DB $\frac{L}{2} < x < L$ $M = -Px - Q(x - \frac{L}{2})$, $\frac{\partial M}{\partial Q} = -(x - \frac{L}{2})$

Set $Q = 0$,

$$\begin{aligned} S_D &= \frac{1}{EI} \int_0^{\frac{L}{2}} (-Px)(0) dx + \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px)[-(x - \frac{L}{2})] dx \\ &= \frac{P}{EI} \int_{\frac{L}{2}}^L (x^2 - \frac{L}{2}x) dx = \frac{P}{EI} \left\{ \frac{1}{3}L^3 - \frac{1}{3}(\frac{L}{2})^3 - (\frac{L}{2})\frac{1}{2}L^2 + \frac{L}{2}\frac{1}{2}(\frac{L}{2})^2 \right\} \\ &= \left(\frac{1}{3} - \frac{1}{24} - \frac{1}{4} + \frac{1}{16} \right) \frac{PL^3}{EI} = \frac{5}{48} \frac{PL^3}{EI} \end{aligned}$$

Problem 11.86

11.84 and 11.86 For the prismatic beam shown, determine the slope at point D.



Add couple M_o at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\Theta_D = \frac{\partial U}{\partial M_o} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_o} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_o} dx$$

Over portion AD: $0 < x < \frac{L}{2}$ $M = -Px$ $\frac{\partial M}{\partial M_o} = 0$

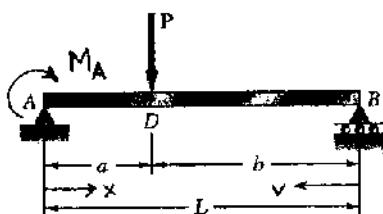
Over portion DB: $\frac{L}{2} < x < L$ $M = -Px - M_o$ $\frac{\partial M}{\partial M_o} = -1$

Set $M_o = 0$,

$$\begin{aligned} \Theta_D &= \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px)(0) dx + \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px)(-1) dx \\ &= \frac{P}{EI} \int_{\frac{L}{2}}^L x dx = \frac{P}{EI} \left[\frac{1}{2}L^2 - \frac{1}{2}(\frac{L}{2})^2 \right] \\ &= \left(\frac{1}{2} - \frac{1}{8} \right) \frac{PL^2}{EI} = \frac{3}{8} \frac{PL^2}{EI} \end{aligned}$$

Problem 11.87

11.87 For the prismatic beam shown, determine the slope at point A.



Add couple M_A at point A.

$$\text{Reactions: } R_A = \frac{Pb}{L} - \frac{M_A}{L}, \quad R_B = \frac{Pa}{L} + \frac{M_A}{L}$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a M^2 dx + \frac{1}{2EI} \int_a^b M^2 dv$$

$$C\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI} \int_0^a M \frac{\partial M}{\partial M_A} dx + \frac{1}{EI} \int_a^b M \frac{\partial M}{\partial M_A} dv$$

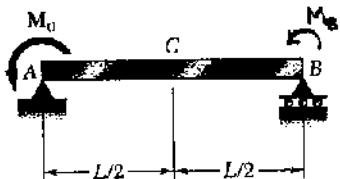
$$\text{Over portion AD } (0 < x < a) \quad M = M_A + R_A x = M_A(1 - \frac{x}{L}) + \frac{Pbx}{L}, \quad \frac{\partial M}{\partial M_A} = 1 - \frac{x}{L}$$

$$\text{Over portion DB } (0 < v < b) \quad M = R_B v = \frac{Pav}{L} + \frac{M_A v}{L}, \quad \frac{\partial M}{\partial M_A} = \frac{v}{L}$$

$$\begin{aligned} \text{Set } M_A = 0 \quad \theta_A &= \frac{1}{EI} \int_0^a \left(\frac{Pbx}{L} \right) \left(1 - \frac{x}{L} \right) dx + \frac{1}{EI} \int_a^b \frac{Pav}{L} \frac{v}{L} dv \\ &= \frac{P}{EI L^2} \left(\frac{1}{2} b L a^2 - \frac{1}{3} b a^3 + \frac{1}{6} a b^3 \right) \\ &= \frac{Pab}{6EI L^2} (3La - 2a^2 + 2b^2) \end{aligned}$$

Problem 11.88

11.88 For the prismatic beam shown, determine the slope at point B.



Add couple M_B at point B as shown.

$$\text{Reactions: } R_A = \frac{1}{L} (M_o + M_B) \uparrow$$

$$\text{Strain energy: } U = \int_0^L \frac{M^2}{2EI} dx$$

$$\text{Slope at point B: } \theta_B = \frac{\partial U}{\partial M_B}$$

$$M = R_A x - M_o = (M_o + M_B) \frac{x}{L} - M_o$$

$$\frac{\partial M}{\partial M_B} = \frac{x}{L} \quad \text{With } M_B = 0 \quad M = M_o \left(\frac{x}{L} - 1 \right)$$

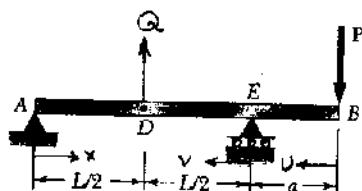
$$\frac{\partial U}{\partial M_B} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_B} dx = \frac{M_o}{EI} \int_0^L \left(\frac{x}{L} - 1 \right) \frac{x}{L} dx = \frac{M_o}{EI L^2} \int_0^L (x - L) x dx$$

$$= \frac{M_o}{EI L^2} \int_0^L (x^2 - Lx) dx = \frac{M_o}{EI} \left(\frac{x^3}{3} - \frac{Lx^2}{2} \right) \Big|_0^L = - \frac{M_o L}{6EI}$$

$$\theta_B = - \frac{M_o L}{6EI} = \frac{M_o L}{6EI}$$

Problem 11.89

11.89 and 11.90 For the prismatic beam shown, determine the deflection at point D.



Add force Q at point D.

$$\text{Reactions } R_A = -\frac{Pa}{L} - \frac{1}{2}Q, \quad R_E = \frac{P(a+L)}{L} - \frac{1}{2}Q$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad S_D = \frac{\partial U}{\partial Q}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx, \quad M = R_A x = -\frac{Pa}{L}x - \frac{1}{2}Qx, \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}x$$

$$\text{Set } Q = 0. \quad \frac{\partial U_{AD}}{\partial Q} = \frac{1}{EI} \int_0^{\frac{L}{2}} (-\frac{Pa}{L}x)(-\frac{1}{2}x) dx = \frac{Pa}{2EI L} \int_0^{\frac{L}{2}} x^2 dx \\ = \frac{Pa}{2EI L} \frac{1}{3} (\frac{L}{2})^3 = \frac{Pa L^2}{48 EI}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$$

$$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{1}{2}Qv - P(a+v) = \frac{Pa}{L}v - Pa - \frac{1}{2}Qv$$

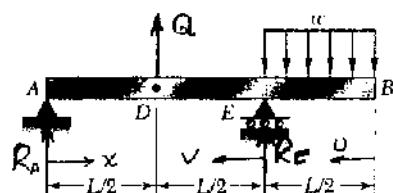
$$\text{Set } Q = 0. \quad \frac{\partial U_{DE}}{\partial Q} = \frac{1}{EI} \int_0^{\frac{L}{2}} (\frac{Pa}{L}v - Pa)(-\frac{1}{2}v) dv = \frac{Pa}{2EI L} \int_0^{\frac{L}{2}} (-v^2 + Lv) dv \\ = \frac{Pa}{2EI L} \left[-\frac{1}{3}(\frac{L}{2})^3 + (L)\frac{1}{2}(\frac{L}{2})^2 \right] = \frac{Pa}{2EI L} \left[-\frac{L^3}{24} + \frac{L^3}{8} \right] \\ = \frac{1}{24} \frac{Pa L^2}{EI}$$

$$\text{Over portion EB: } M = -Pu \quad \frac{\partial M}{\partial Q} = 0 \quad \frac{\partial U_{EB}}{\partial Q} = 0$$

$$S_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{Pa L^2}{48 EI} + \frac{Pa L^2}{24 EI} + 0 = \frac{L}{16} \frac{Pa L^2}{EI} \uparrow$$

Problem 11.90

11.89 and 11.90 For the prismatic beam shown, determine the deflection at point D.



Add force Q at point D.

$$\text{Reactions: } R_A = -\frac{1}{8}wL - \frac{1}{2}Q, R_B = \frac{5}{8}wL - \frac{1}{2}Q$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad S_Q = \frac{\partial U}{\partial Q}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = -2wLx - \frac{1}{2}Qx, \quad \frac{\partial M}{\partial x} = -\frac{1}{2}Q$$

$$\text{Set } Q = 0. \quad M = -\frac{1}{8}wLx$$

$$\frac{\partial U_{AD}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dx = \int_0^{L/2} \left(-\frac{wLx}{8EI} \right) \left(-\frac{1}{2}Q \right) dx = \frac{wL}{16EI} \int_0^{L/2} x^2 dx = \frac{wL^4}{384EI}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$M = R_E v - \frac{wL}{2}(v + \frac{L}{4}) = \frac{1}{8}wL(v - L) - \frac{1}{2}Qv \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$$

$$\text{Set } Q = 0 \quad M = -\frac{wL}{8}(L-v)$$

$$\begin{aligned} \frac{\partial U}{\partial Q} &= \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dv = \frac{wL}{16EI} \int_0^{L/2} (L-v)v dv = \frac{wL}{16EI} \left(\frac{Lv^2}{2} - \frac{v^3}{3} \right) \Big|_0^{L/2} \\ &= \frac{wL^4}{192EI} \left(\frac{1}{8} - \frac{1}{24} \right) = \frac{wL^4}{192EI} \end{aligned}$$

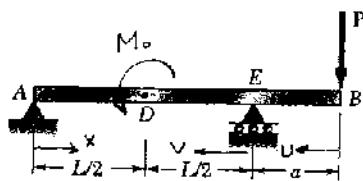
$$\text{Over portion EB: } M = -\frac{1}{2}wv^2 \quad \frac{\partial M}{\partial Q} = 0 \quad U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} du$$

$$\frac{\partial U_{EB}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} du = 0$$

$$S_Q = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{wL^4}{384EI} + \frac{wL^4}{192EI} + 0 = \frac{wL^4}{192EI} \uparrow$$

Problem 11.91

11.91 and 11.92 For the prismatic beam shown, determine the slope at point D.



Add couple M_o at point D.

$$\text{Reactions: } R_A = -\frac{Pa}{L} + \frac{M_o}{L}, \quad R_B = \frac{P(a+L)}{L} - \frac{M_o}{L}$$

$$U = U_{AD} + U_{DE} + U_{EB} \quad \Theta_D = \frac{\partial U}{\partial M_o}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx, \quad M = R_A x = -\frac{Pa}{L} x + \frac{M_o}{L} x, \quad \frac{\partial M}{\partial M_o} = \frac{1}{L} x$$

$$\text{Set } M_o = 0. \quad \frac{\partial U_{AD}}{\partial M_o} = \frac{1}{EI} \int_0^{\frac{L}{2}} \left(-\frac{Pa}{L} x \right) \left(\frac{1}{L} x \right) dx = -\frac{Pa}{EIL^2} \int_0^{\frac{L}{2}} x^2 dx \\ = -\frac{Pa}{EIL^2} \cdot \frac{1}{3} \left(\frac{L}{2} \right)^3 = -\frac{PaL}{24EI}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv \quad \frac{\partial M}{\partial M_o} = -\frac{1}{L} v$$

$$M = R_E v - P(a+v) = \frac{P(a+L)}{L} v - \frac{M_o}{L} v - P(a+v) = \frac{Pa}{L} v - Pa - \frac{M_o}{L} v$$

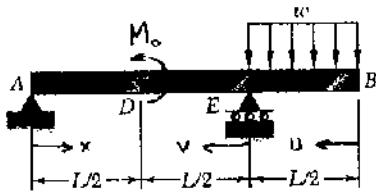
$$\text{Set } M_o = 0. \quad \frac{\partial U_{DE}}{\partial M_o} = \frac{1}{EI} \int_0^{\frac{L}{2}} \left(\frac{Pa}{L} v - Pa \right) \left(-\frac{1}{L} v \right) dv = -\frac{Pa}{EIL^2} \int_0^{\frac{L}{2}} (v^2 - Lv) dv \\ = -\frac{Pa}{EIL^2} \left[\frac{1}{3} \left(\frac{L}{2} \right)^3 - L \cdot \frac{1}{2} \left(\frac{L}{2} \right)^2 \right] = -\frac{Pa}{EIL^2} \left[\frac{1}{24} L^3 - \frac{1}{8} L^3 \right] \\ = \frac{1}{12} \frac{PaL}{EI}$$

$$\text{Over portion EB: } M = -P_v \quad \frac{\partial M}{\partial M_o} = 0 \quad \frac{\partial U_{EB}}{\partial M_o} = 0$$

$$\text{Total } \Theta_D = \frac{\partial U_{AD}}{\partial M_o} + \frac{\partial U_{DE}}{\partial M_o} + \frac{\partial U_{EB}}{\partial M_o} = -\frac{1}{24} \frac{PaL}{EI} + \frac{1}{12} \frac{PaL}{EI} + 0 = \frac{1}{24} \frac{PaL}{EI} \quad \square$$

Problem 11.92

11.91 and 11.92 For the prismatic beam shown, determine the slope at point D.



Add couple M_0 at point D.

$$\text{Reactions. } R_A = \frac{M_0}{L} - \frac{wL}{8}, \quad R_E = -\frac{M_0}{L} + \frac{5wL}{8}$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad \theta_D = \frac{\partial U}{\partial M_0}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = \frac{M_0 x}{L} - \frac{wLx}{8} \quad \frac{\partial M}{\partial M_0} = \frac{x}{L}$$

$$\text{Set } M_0 = 0, \quad M = -\frac{wLx}{8}$$

$$\frac{\partial U_{AD}}{\partial M_0} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \int_0^{L/2} \left(-\frac{wLx}{8EI} \right) \left(\frac{x}{L} \right) dx = -\frac{w}{8EI} \int_0^{L/2} x^2 dx = -\frac{wL^3}{192EI}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$M = R_E v - \frac{wL}{2} \left(v + \frac{L}{4} \right) = -\frac{M_0 v}{L} + \frac{wL}{8} (v - L) \quad \frac{\partial M}{\partial M_0} = -\frac{v}{L}$$

$$\text{Set } M_0 = 0, \quad M = -\frac{1}{8} wL(L-v)$$

$$\frac{\partial U_{DE}}{\partial M_0} = \frac{w}{8EI} \int_0^{L/2} (L-v)v dv = \frac{w}{8EI} \left(\frac{Lv^2}{2} - \frac{v^3}{3} \right) \Big|_0^{L/2} = \frac{wL^3}{96EI}$$

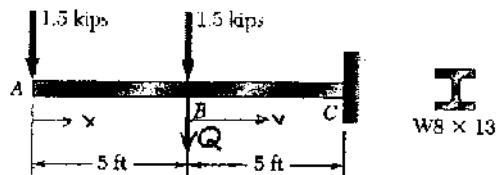
$$\text{Over portion EB: } U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} du \quad M = -\frac{1}{2} wu^2 \quad \frac{\partial M}{\partial M_0} = 0$$

$$\frac{\partial U_{EB}}{\partial M_0} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial M_0} du = 0$$

$$\theta_D = \frac{\partial U_{AD}}{\partial M_0} + \frac{\partial U_{DE}}{\partial M_0} + \frac{\partial U_{EB}}{\partial M_0} = -\frac{wL^3}{192EI} + \frac{wL^3}{96EI} + 0 = \frac{wL^3}{192EI}$$

Problem 11.93

11.93 For the beam and loading shown, determine the deflection of point B. Use $E = 29 \times 10^6$ psi.



W8 x 13

Add force Q at point B

Units: forces in kips, lengths in ft.

$$E = 29 \times 10^3 \text{ ksi} \quad I = 39.6 \text{ in}^4$$

$$EI = (29 \times 10^3)(39.6) = 1.148 \times 10^6 \text{ kip-in}^2 = 7975 \text{ kip-ft}^2$$

$$U = \int_0^5 \frac{M^2}{2EI} dx + \int_0^5 \frac{M^2}{2EI} dv \quad S_B = \frac{\partial U}{\partial Q} = \frac{1}{EI} \left\{ \int_0^5 M \frac{\partial M}{\partial Q} dx + \int_0^5 M \frac{\partial M}{\partial Q} dv \right\}$$

$$\text{Over AB: } M = -1.5x, \quad \frac{\partial M}{\partial Q} = 0 \quad \int_0^5 M \frac{\partial M}{\partial Q} dx = 0$$

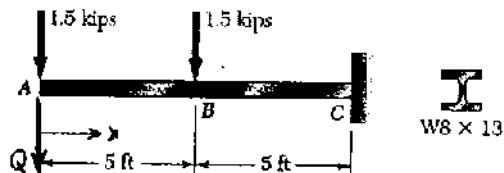
$$\text{Over BC: } M = -1.5(v+5) - 1.5v - Qv = -3v - 7.5 - Qv, \quad \frac{\partial M}{\partial Q} = -v$$

$$\int_0^5 M \frac{\partial M}{\partial Q} dv = \int_0^5 (-3v^2 - 7.5v) dv = (3(\frac{1}{3})(5)^3) + (7.5)(\frac{1}{2})(5)^2 = 218.75$$

$$S_B = \frac{1}{EI} \left\{ 0 + 218.75 \right\} = \frac{218.75}{7975} = 27.43 \times 10^{-3} \text{ ft} = 0.329 \text{ in.} \quad \blacksquare$$

Problem 11.94

11.94 For the beam and loading shown, determine the deflection of point A. Use $E = 29 \times 10^6$ psi.



W8 x 13

Add force Q at point A.

Units: forces in kips, lengths in ft.

$$E = 29 \times 10^3 \text{ ksi}, \quad I = 39.6 \text{ in}^4$$

$$EI = (29 \times 10^3)(39.6) = 1.148 \times 10^6 \text{ kip-in}^2 = 7975 \text{ kip-ft}^2$$

$$U = \int_0^{10} \frac{M^2}{2EI} dx \quad S_A = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^{10} M \frac{\partial M}{\partial Q} dx$$

$$\text{Over portion AB} \quad 0 < x < 5, \quad M = -1.5x - Qx \quad \frac{\partial M}{\partial Q} = -x$$

$$\int_0^5 M \frac{\partial M}{\partial Q} dx = \int_0^5 (-1.5x)(x) dx = 1.5 \int_0^5 x^2 dx = (1.5)(\frac{1}{3})(5)^3 = 62.5$$

$$\text{Over portion BC} \quad 5 < x < 10 \quad M = -1.5x - 1.5(x-5) - Qx$$

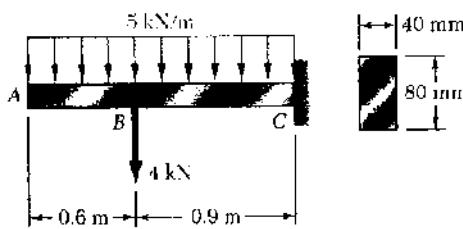
$$M = -3x + 7.5 - Qx \quad \frac{\partial M}{\partial Q} = -x$$

$$\int_5^{10} M \frac{\partial M}{\partial Q} dx = \int_5^{10} (-3x^2 + 7.5x) dx = (3)(\frac{1}{3})(10^3 - 5^3) - (7.5)(\frac{1}{2})(10^2 - 5^2) = 593.75$$

$$S_A = \frac{1}{EI} \left\{ 62.5 + 593.75 \right\} = \frac{656.25}{7975} = 82.29 \times 10^{-3} \text{ ft} = 0.987 \text{ in.} \quad \blacksquare$$

Problem 11.95

11.95 For the beam and loading shown, determine the deflection at point B. Use $E = 200 \text{ GPa}$.



$$U = \int_0^a \frac{M^2}{2EI} dx + \int_a^L \frac{M^2}{2EI} dx$$

$$S_B = \frac{\partial U}{\partial P} = \int_0^a \frac{M}{EI} \frac{\partial M}{\partial P} dx + \int_a^L \frac{M}{EI} \frac{\partial M}{\partial P} dx$$

Partion AB ($0 \leq x \leq a$)

$$M = -\frac{1}{2}wx^2 \quad \frac{\partial M}{\partial P} = 0$$

$$\int_0^a \frac{M}{EI} \frac{\partial M}{\partial P} dx = 0$$

Partion BC ($a < x \leq L$)

$$M = -\frac{1}{2}wx^2 - P(x-a)$$

$$\frac{\partial M}{\partial P} = -(x-a)$$

$$\begin{aligned} \int_a^L \frac{M}{EI} \frac{\partial M}{\partial P} dx &= \frac{w}{2EI} \int_a^L x^2(x-a) dx + \frac{P}{EI} \int_a^L (x-a)^2 dx \\ &= \frac{w}{2EI} \int_a^L (x^3 - ax^2) dx + \frac{P}{EI} \int_a^b v^2 dv \\ &= \frac{w}{2EI} \left(\frac{L^4}{4} - \frac{aL^3}{3} - \frac{a^4}{4} + \frac{a^4}{3} \right) + \frac{Pb^3}{3EI} \end{aligned}$$

$$S_B = 0 + \frac{w}{2EI} \left(\frac{L^4}{4} - \frac{aL^3}{3} + \frac{a^4}{12} \right) + \frac{Pb^3}{3EI}$$

Data: $a = 0.6 \text{ m}$, $b = 0.9 \text{ m}$, $L = a+b = 1.5 \text{ m}$

$$w = 5 \times 10^3 \text{ N/m} \quad P = 4 \times 10^3 \text{ N}$$

$$I = \frac{1}{12}(40)(80)^3 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(1.70667 \times 10^{-6}) = 341333 \text{ N}\cdot\text{m}^2$$

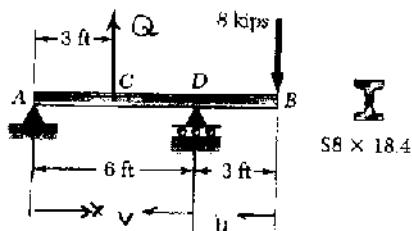
$$S_B = 0 + \frac{5 \times 10^3}{(2)(341333)} \left[\frac{(1.5)^4}{4} - \frac{(0.6)(1.5)^3}{3} + \frac{(0.6)^4}{12} \right] + \frac{(4 \times 10^3)(0.9)^3}{(3)(341333)}$$

$$= 7.25 \times 10^{-3} \text{ m}$$

$$= 7.25 \text{ mm} \downarrow$$

Problem 11.96

11.96 For the beam and loading shown, determine the deflection at point C. Use $E = 29 \times 10^6$ psi.



Units: Forces in kip, lengths in ft.

$$E = 29 \times 10^6 \text{ ksi} \quad I = 57.6 \text{ in}^4$$

$$EI = (29 \times 10^6)(57.6) = 1.6704 \times 10^9 \text{ kip-in}^2 = 11600 \text{ kip-ft}^2$$

Add dummy force Q at point C. Reactions $R_A = 4 + \frac{1}{2}Q \downarrow$, $R_B = 12 - \frac{1}{2}Q \uparrow$

$$U = U_{AC} + U_{CD} + U_{DB} \quad \delta_c = \frac{\partial U}{\partial Q} = \frac{\partial U_{AC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} + \frac{\partial U_{DB}}{\partial Q}$$

$$\text{Over AC } 0 < x < 3 \quad M = -(4 + \frac{1}{2}Q)x \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}x \quad \text{Set } Q = 0$$

$$\frac{\partial U_{AC}}{\partial Q} = \frac{1}{EI} \int_0^3 (4x)(\frac{1}{2}x) dx = \frac{2}{EI} \int_0^3 x^2 dx = \frac{(2)(3)^3}{3EI} = \frac{18}{EI}$$

$$\text{Over CD } 0 < v < 3 \quad M = R_B v - 8(v+3) = 12v - \frac{1}{2}Q - 8v - 24 = 4v - 24 - \frac{1}{2}Qv$$

$$\frac{\partial M}{\partial Q} = -\frac{1}{2}v \quad \text{Set } Q = 0$$

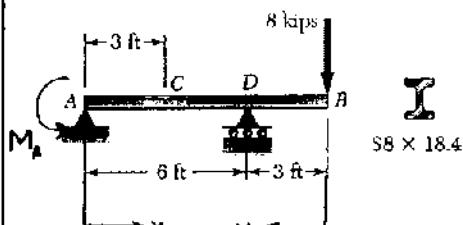
$$\frac{\partial U_{CD}}{\partial Q} = \frac{1}{EI} \int_0^3 (24 - 4v)(-\frac{1}{2}v) dv = \frac{1}{EI} \int_0^3 (12v - 2v^2) dv = \frac{1}{EI} \left\{ (12) \frac{(3)^2}{2} - (2) \frac{(3)^3}{3} \right\} \\ = \frac{36}{EI}$$

$$\text{Over DB } 0 < v < 3 \quad M = -8v \quad \frac{\partial M}{\partial Q} = 0 \quad \frac{\partial U_{DB}}{\partial Q} = 0$$

$$\delta_c = \frac{18}{EI} + \frac{36}{EI} + 0 = \frac{54}{11600} = 4.655 \times 10^{-5} \text{ ft} = 0.0559 \text{ in.}$$

Problem 11.97

11.97 For the beam and loading shown, determine the slope at end A.
Use $E = 29 \times 10^3 \text{ ksi}$



Units: Forces in kips, lengths in ft.

$$E = 29 \times 10^3 \text{ ksi}, I = 57.6 \text{ in}^4$$

$$EI = (29 \times 10^3)(57.6) = 1.6704 \times 10^6 \text{ kip-in}^2 = 11600 \text{ kip-ft}^2$$

Add dummy couple M_A at end A. Reactions: $R_A = -4 + \frac{M_A}{6}$, $R_B = 12 - \frac{M_A}{6}$

$$U = U_{AD} + U_{DB} \quad \delta\theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AD}}{\partial M_A} + \frac{\partial U_{DB}}{\partial M_A}$$

Over AD $0 < x < 6$ $M = -M_A + R_A x = -M_A - 4x + \frac{M_A}{6}x$

$$\frac{\partial M}{\partial M_A} = -\left(1 - \frac{x}{6}\right) \quad \text{Set } M_A = 0.$$

$$\frac{\partial U_{AD}}{\partial M_A} = \frac{1}{EI} \int_0^6 (4x)\left(1 - \frac{x}{6}\right) dx = \frac{1}{EI} \int_0^6 \left(4x - \frac{2}{3}x^2\right) dx = \frac{1}{EI} \left\{ \left(4 \cdot \frac{6^2}{2} - \frac{2}{3} \cdot \frac{6^3}{3} \right) \right\}$$

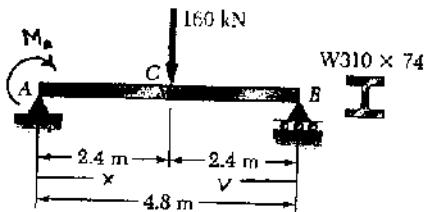
$$= \frac{24}{EI}$$

Over DB $0 < v < 3$ $M = -8v \quad \frac{\partial M}{\partial M_A} = 0 \quad \frac{\partial U_{DB}}{\partial M_A} = 0$

$$\delta\theta_A = \frac{24}{EI} + 0 = \frac{24}{11600} = 2.07 \times 10^{-3} \text{ rad}$$

Problem 11.98

11.98 For the beam and loading shown, determine the slope at end A.
Use $E = 200 \text{ GPa}$.



Add couple M_A at point A.

Units: forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}, I = 165 \times 10^8 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(165 \times 10^{-6}) = 33 \times 10^6 \text{ N} \cdot \text{m}^2 = 33000 \text{ kN} \cdot \text{m}^2$$

$$\text{Reactions: } R_A = 80 - \frac{M_A}{4.8} \quad R_B = 80 + \frac{M_A}{L}$$

$$U = U_{AB} + U_{BC} = \int_0^{2.4} \frac{M^2}{2EI} dx + \int_0^{2.4} \frac{M^2}{2EI} dv \quad \partial \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_A}$$

$$\text{Over AB: } M = M_A + R_A x = M_A + 80x - \frac{M_A}{4.8}x \quad \frac{\partial M}{\partial M_A} = (1 - \frac{x}{4.8})$$

$$\text{Set } M_A = 0. \quad \frac{\partial U_{AB}}{\partial M_A} = \frac{1}{EI} \int_0^{2.4} (80x)(1 - \frac{x}{4.8}) dx = \frac{1}{EI} \int_0^{2.4} (80x - 16.6667x^2) dx \\ = \frac{1}{EI} \left\{ (80)(\frac{1}{2})(2.4)^2 - (16.6667)(\frac{1}{3})(2.4)^3 \right\} = \frac{153.6}{EI}$$

$$\text{Over BC: } M = R_B v = 80v + \frac{M_A}{4.8}v, \quad \frac{\partial M}{\partial M_A} = \frac{1}{4.8}v$$

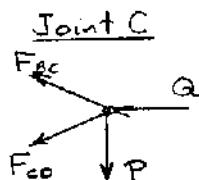
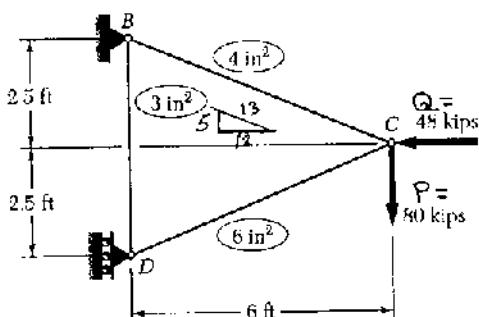
$$\text{Set } M_A = 0. \quad \frac{\partial U_{BC}}{\partial M_A} = \frac{1}{EI} \int_0^{2.4} (80v)(\frac{1}{4.8}v) dv = \frac{16.6667}{EI} \int_0^{2.4} v^2 dv \\ = \frac{(16.6667)(2.4)^3}{3EI} = \frac{76.8}{EI}$$

$$\partial \theta_A = \frac{1}{EI} \left\{ 153.6 + 76.8 \right\} = \frac{230.4}{33000} = 6.98 \times 10^{-3} \text{ rad.} \quad \text{Ans}$$

Problem 11.99

11.99 and 11.100 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.99 Vertical deflection of joint C.



$$\begin{aligned} & \uparrow \sum F_x = 0 : \\ & -\frac{12}{13} F_{Bc} - \frac{12}{13} \frac{5}{12} F_{cd} - Q = 0 \\ & F_{Bc} + F_{cd} = -\frac{13}{12} Q \quad (1) \\ & +\uparrow \sum F_y = 0 : \\ & \frac{5}{13} F_{Bc} - \frac{5}{13} F_{cd} - P = 0 \\ & F_{Bc} - F_{cd} = -\frac{13}{5} P \quad (2) \end{aligned}$$

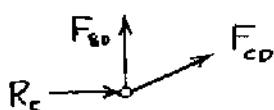
Solving (1) and (2) simultaneously,

$$F_{Bc} = \frac{13}{10} P - \frac{13}{24} Q$$

$$F_{cd} = -\frac{13}{10} P - \frac{13}{24} Q$$

Joint D

$$+\uparrow \sum F_y = 0 : \frac{5}{13} F_{cd} + F_{BD} = 0$$



$$F_{BD} = -\frac{5}{13} F_{cd} = \frac{1}{2} P + \frac{5}{24} Q$$

Lengths of members: $L_{BC} = 78 \text{ in.}$, $L_{CD} = 78 \text{ in.}$, $L_{BD} = 60 \text{ in.}$

$$U = \sum \frac{F^2 L}{2EA}$$

$$S_p = \frac{\partial U}{\partial P} = \sum \frac{FL}{EA} \frac{\partial F}{\partial P} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

Member	F	L (in.)	$\frac{\partial F}{\partial P}$	A (in^2)	$\frac{FL}{A} \frac{\partial F}{\partial P}$
BC	$\frac{13}{10} P - \frac{13}{24} Q$	78	$\frac{13}{10}$	4	$32.955 P - 13.73125 Q$
CD	$-\frac{13}{10} P - \frac{13}{24} Q$	78	$-\frac{13}{10}$	6	$21.97 P + 9.45417 Q$
BD	$\frac{1}{2} P + \frac{5}{24} Q$	60	$\frac{1}{2}$	3	$5.00 P + 2.08333 Q$
Σ					$59.975 P - 2.49375 Q$

Further data: $E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$

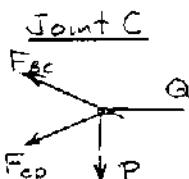
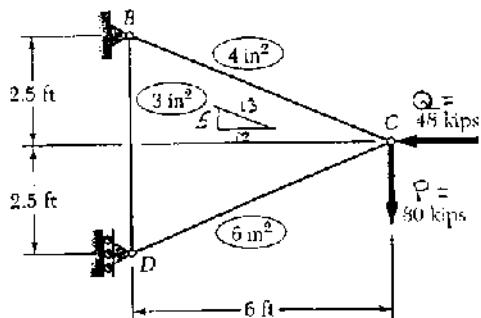
$P = 80 \text{ kips}$, $Q = 48 \text{ kips}$

$$S_p = \frac{(59.975)(80) + (-2.49375)(48)}{29000} = 0.1613 \text{ in.}$$

Problem 11.100

11.99 and 11.100 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.100 Horizontal deflection of point C



$$\begin{aligned} \text{Horizontal equilibrium: } & \sum F_x = 0 : -\frac{13}{12} F_{Bc} - \frac{13}{12} \frac{5}{13} F_{cd} - Q = 0 \\ & F_{Bc} + F_{cd} = -\frac{13}{12} Q \quad (1) \\ \text{Vertical equilibrium: } & \sum F_y = 0 : \frac{5}{13} F_{cd} - \frac{5}{13} F_{Bc} - P = 0 \\ & F_{cd} - F_{Bc} = -\frac{13}{5} P \quad (2) \end{aligned}$$

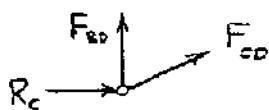
Solving (1) and (2) simultaneously,

$$F_{Bc} = \frac{13}{10} P - \frac{13}{24} Q$$

$$F_{cd} = -\frac{13}{10} P - \frac{13}{24} Q$$

Joint D

$$+\uparrow \sum F_y = 0 : \frac{5}{13} F_{cd} + F_B = 0$$



$$F_{BD} = -\frac{5}{13} F_{cd} = \frac{1}{2} P + \frac{5}{24} Q$$

Lengths of members: $L_{BC} = 78 \text{ in}$, $L_{CD} = 78 \text{ in}$, $L_{BD} = 60 \text{ in}$.

$$U = \sum \frac{F^2 L}{2EA}$$

$$S_Q = \frac{\partial U}{\partial Q} = \sum \frac{FL}{EA} \frac{\partial F}{\partial Q} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

Member	F	L(in.)	$\frac{\partial F}{\partial Q}$	A(in²)	$\frac{FL}{A} \frac{\partial F}{\partial Q}$
BC	$\frac{13}{10} P - \frac{13}{24} Q$	78	$-\frac{13}{24}$	4	$-13.73125 P + 5.72135 Q$
CD	$-\frac{13}{10} P - \frac{13}{24} Q$	78	$-\frac{13}{24}$	6	$9.15467 P + 3.81424 Q$
BD	$\frac{1}{2} P + \frac{5}{24} Q$	60	$\frac{5}{24}$	3	$2.08333 P + 0.86806 Q$
Σ					$-2.49325 P + 10.40365 Q$

Further data: $E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$

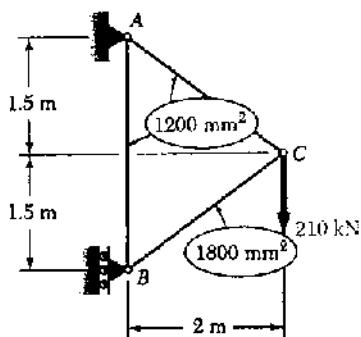
$$P = 80 \text{ kips}, \quad Q = 48 \text{ kips}$$

$$S_Q = \frac{(-2.49325)(80) + (10.40365)(48)}{29000} = 0.01034 \text{ in.} \leftarrow$$

Problem 11.101

11.101 and 11.102 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

11.101 Vertical deflection of joint C.



Call the vertical load P . The vertical deflection of joint C is δ_P .

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

$$\text{Joint C: } +\sum F_x = 0: -\frac{4}{5} F_{Ax} - \frac{4}{5} F_{Ac} = 0$$

$$F_{Ac} \nearrow \quad \uparrow \sum F_y \quad \frac{3}{5} F_{Ac} + \frac{3}{5} F_{Bc} - P = 0$$

F_{Ac}

$$\text{Solving simultaneously, } F_{Ac} = \frac{5}{6} P \quad F_{Bc} = -\frac{5}{6} P$$

Joint B $+f \sum F_y = 0:$

$$F_{Bc} \uparrow \quad \frac{5}{6} P \quad F_{AB} - \frac{3}{5} \cdot \frac{5}{6} P = 0$$

$$F_{AB} = \frac{1}{2} P$$

Member	F	$\frac{\partial F}{\partial P}$	$L (\text{m})$	$A (10^{-6} \text{ m}^2)$	$F(\frac{\partial F}{\partial P})L/A$
AB	$\frac{1}{2} P$	$\frac{1}{2}$	3	1200	$625 P$
AC	$\frac{5}{6} P$	$\frac{5}{6}$	2.5	1200	$1446.76 P$
BC	$-\frac{5}{6} P$	$-\frac{5}{6}$	2.5	1800	$964.51 P$
Σ				$3036.27 P$	

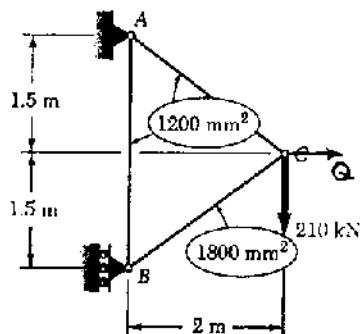
$$\delta_P = \frac{1}{E} (3036.27 P) = \frac{(3036.27)(210 \times 10^3)}{200 \times 10^9} = 3.19 \times 10^{-3} \text{ m}$$

$$= 3.19 \text{ mm} \downarrow$$

Problem 11.102

11.101 and 11.102 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

11.102 Horizontal deflection of point C.



Call the vertical force P . Add a dummy horizontal force Q (positive \rightarrow) at joint C. The horizontal deflection of joint C is S_Q .

$$S_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EI} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

Joint C $\rightarrow \sum F_x = 0:$

$$F_{AC} - \frac{4}{5} F_{BC} + Q = 0$$

+f $\sum F_y = 0:$

$$\frac{3}{5} F_{AC} - \frac{3}{5} F_{BC} - P = 0$$

Solving simultaneously

$$F_{AC} = \frac{5}{6} P + \frac{5}{8} Q \quad F_{BC} = -\frac{5}{6} P + \frac{5}{8} Q$$

Joint B $+f \sum F_y = 0:$

$$F_{AB} + F_{BC} = 0$$

$$F_{AB} = -\frac{3}{5} F_{BC} = \frac{1}{2} P - \frac{3}{8} Q$$

Member	$\int F$	$\partial F / \partial Q$	$L(\text{m})$	$A (10^6 \text{ m}^2)$	$F (\partial F / \partial Q) L / A$ with $Q = 0$
AB	$\frac{1}{2} P - \frac{3}{8} Q$	$-\frac{3}{8}$	3	1200	$-468.75 P$
AC	$\frac{5}{6} P + \frac{5}{8} Q$	$+\frac{5}{8}$	2.5	1200	$1085.07 P$
BC	$-\frac{5}{6} P + \frac{5}{8} Q$	$+\frac{5}{8}$	2.5	1800	$-723.38 P$
					$-107.06 P$

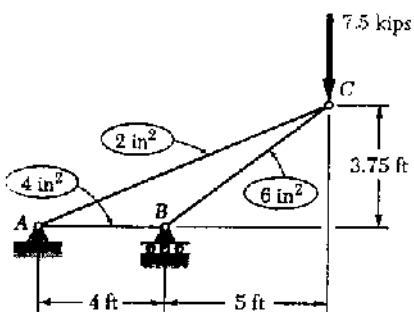
$$S_Q = \frac{1}{E} (-107.06 P) = -\frac{(107.06)(210 \times 10^3)}{200 \times 10^9} = -0.1124 \times 10^{-3} \text{ m}$$

$$= 0.1124 \text{ mm} \leftarrow$$

Problem 11.103

11.103 and 11.104 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.103 Vertical deflection of joint C.



Call the vertical load P . The vertical deflection of joint C is δ_P .

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

$$\text{Geometry} \quad \overline{AC} = \sqrt{9^2 + 3.75^2} = 9.75 \text{ ft} = 117 \text{ in}$$

$$\overline{BC} = \sqrt{5^2 + 3.75^2} = 6.25 \text{ ft} = 75 \text{ in}$$

$$4 \text{ ft} = 48 \text{ in.}, \quad 5 \text{ ft} = 60 \text{ in.}, \quad 3.75 \text{ ft} = 45 \text{ in.}$$

Joint C

$$+ \rightarrow \sum F_x = 0: \quad - \frac{108}{117} F_{Ac} - \frac{60}{75} F_{Bc} = 0$$

$$+ \uparrow \sum F_y = 0: \quad - \frac{45}{117} F_{Ac} - \frac{45}{75} F_{Bc} - P = 0$$

$$\text{Solving simultaneously, } F_{Ac} = 3.25 P, \quad F_{Bc} = -3.75 P$$

Joint B

$$3.75 P \quad + \rightarrow \sum F_x = 0: \quad -F_{AB} - \frac{60}{75} F_{Ac} = 0$$

$$F_{AB} = -3.00 P.$$

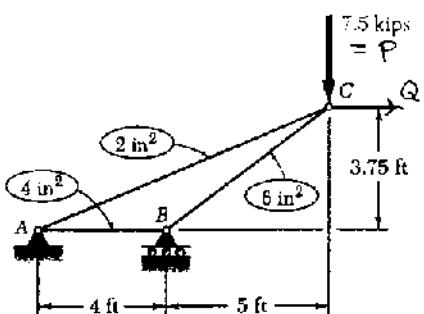
Member	F	$\Delta F / \Delta P$	L (in)	A (in^2)	$F(\Delta F / \Delta P)L / A$
AB	-3.00 P	-3.00	48	4	108.00 P
AC	3.25 P	3.25	117	2	617.91 P
BC	-3.75 P	-3.75	75	6	175.78 P
Σ			901.69 P		

$$\delta_P = \frac{901.69 P}{E} = \frac{(901.69)(7.5 \times 10^3)}{29 \times 10^6} = 0.233 \text{ in.} \downarrow$$

Problem 11.104

11.103 and 11.104 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.104 Horizontal deflection of point C.



Call the vertical load P . Add horizontal dummy load Q at joint C. The horizontal deflection of joint C is s_Q .

$$s_Q = \frac{\partial U}{\partial Q} = \frac{1}{2Q} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

$$\text{Geometry: } \bar{AC} = \sqrt{9^2 + 3.75^2} = 9.75 \text{ ft} = 117 \text{ in.}$$

$$\bar{BC} = \sqrt{5^2 + 3.75^2} = 6.25 \text{ ft} = 75 \text{ in.}$$

$$4 \text{ ft} = 48 \text{ in.}, 5 \text{ ft} = 60 \text{ in.}, 3.75 \text{ ft} = 45 \text{ in.}$$

Joint C

$$\begin{aligned} & \rightarrow \sum F_x = 0: -\frac{108}{117} F_{AC} - \frac{60}{75} F_{BC} + Q = 0 \\ & + \uparrow \sum F_y = 0: -\frac{45}{117} F_{AC} - \frac{45}{75} F_{BC} - P = 0 \end{aligned}$$

Solving simultaneously, $F_{AC} = 3.25P + 2.4375Q$

$$F_{BC} = -3.75P - 1.5625Q$$

Joint B

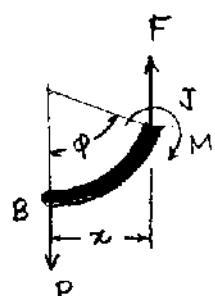
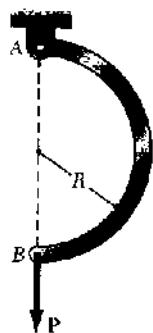
$$\begin{aligned} & \rightarrow \sum F_x = 0: \frac{4}{5} F_{AC} - F_{AB} = 0 \\ & F_{AB} = \frac{4}{5} F_{AC} = -3.00P - 1.25Q \end{aligned}$$

Member	F	$\partial F / \partial Q$	$L (\text{in.})$	$A (\text{in}^2)$	$F(\partial F / \partial Q)L/A$	$Q = 0$
AB	$-3.00P - 1.25Q$	-1.25	48	4	$45.00P$	
AC	$3.25P + 2.4375Q$	2.4375	117	2	$463.43P$	
BC	$-3.75P - 1.5625Q$	-1.5625	75	6	$73.24P$	
Z					$581.67P$	

$$s_Q = \frac{581.67P}{E} = \frac{(581.67)(7.5 \times 10^3)}{29 \times 10^6} = 0.1504 \text{ in.} \rightarrow$$

Problem 11.105

11.105 For the uniform rod and loading shown and using Castigiano's theorem, determine the deflection of point B.



Use polar coordinate ϕ .

Calculate the bending moment $M(\phi)$ using free body BJ.

$$\rightarrow \sum M_J = 0 : Px - M = 0$$

$$M = Px = PR \sin \phi$$

Strain energy: $U = \int_0^L \frac{M^2}{2EI} ds$

$$U = \int_0^{\pi} \frac{(PR \sin \phi)^2}{2EI} (R d\phi) = \frac{P^2 R^3}{2EI} \int_0^{\pi} \sin^2 \phi d\phi$$

$$= \frac{P^2 R^3}{2EI} \int_0^{\pi} \frac{1 - \cos 2\phi}{2} d\phi$$

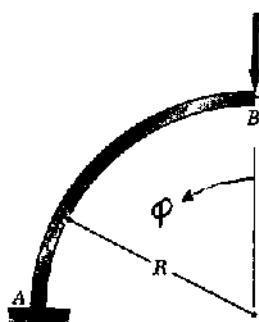
$$= \frac{P^2 R^2}{2EI} \left(\frac{1}{2}\phi \Big|_0^{\pi} - \frac{1}{4} \sin 2\phi \Big|_0^{\pi} \right) = \frac{\pi P^2 R}{4EI}$$

By Castigiano's theorem,

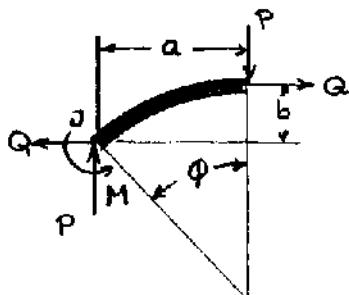
$$S = \frac{\partial U}{\partial P} = \frac{\pi P R^3}{2EI} \downarrow$$

Problem 11.106

11.106 For the beam and loading shown and using Castigiano's theorem, determine (a) the horizontal deflection of point B, (b) the vertical deflection of point B.



Add dummy load Q at point B.
Use polar coordinate ϕ .



$$U = \int_0^{\frac{\pi}{2}} \frac{M^2}{2EI} R d\phi$$

Bending moment

$$\rightarrow \sum M_J = 0: M - Pa - Qb = 0$$

$$\begin{aligned} M &= Pa + Qb \\ &= PR \sin \phi + QR(1 - \cos \phi) \end{aligned}$$

$$\frac{\partial M}{\partial P} = R \sin \phi$$

$$\frac{\partial M}{\partial Q} = R(1 - \cos \phi)$$

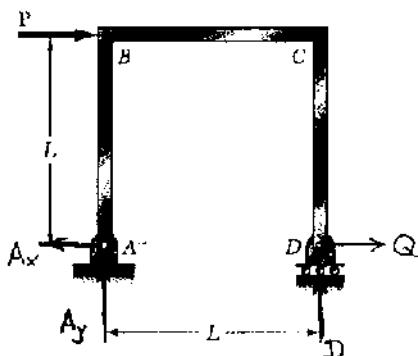
Set $Q = 0$

$$\begin{aligned} (a) S_Q &= \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial Q} R d\phi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} PR \sin \phi R(1 - \cos \phi) R d\phi \\ &= \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} (\sin \phi - \sin \phi \cos \phi) d\phi = \frac{PR^3}{EI} \left(-\cos \phi - \frac{1}{2} \sin^2 \phi \right) \Big|_0^{\frac{\pi}{2}} \\ &= \frac{PR^3}{EI} \left(-\cos \frac{\pi}{2} + \cos 0 - \frac{1}{2} \sin^2 \frac{\pi}{2} + \frac{1}{2} \sin^2 0 \right) \\ &= \frac{PR^3}{EI} (0 + 1 - \frac{1}{2} + 0) = \frac{1}{2} \frac{PR^3}{EI} \end{aligned}$$

$$\begin{aligned} (b) S_P &= \frac{\partial U}{\partial P} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial P} R d\phi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} PR \sin \phi R \sin \phi R d\phi \\ &= \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} \sin^2 \phi d\phi = \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 2\phi) d\phi \\ &= \frac{PR^3}{EI} \left(\frac{1}{2}\phi - \frac{1}{4} \sin 2\phi \right) \Big|_0^{\frac{\pi}{2}} = \frac{PR^3}{EI} \left(\frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{4} \cdot 0 - \frac{1}{4} \sin \pi + \frac{1}{4} \sin 0 \right) \\ &= \frac{PR^3}{EI} \left(\frac{\pi}{4} - 0 - 0 + 0 \right) = \frac{\pi}{4} \frac{PR^3}{EI} \end{aligned}$$

Problem 11.107

11.107 Three rods, each of the same flexural rigidity EI , are welded to form the frame ABCD. For the loading shown, determine the deflection at point D.



Add dummy force Q at point D as shown.

$$\begin{aligned} \text{Statics: } & \sum M_A = 0 : DL - PL = 0 \quad D = P \uparrow \\ & \sum F_x = 0 : -A_x + P + Q = 0 \quad A_x = (P+Q) \leftarrow \\ & +\uparrow \sum F_y = 0 : A_y + D = 0 \quad A_y = P \downarrow \end{aligned}$$

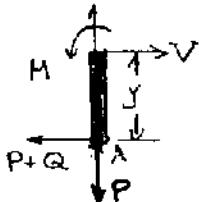
$$U = U_{AB} + U_{BC} + U_{CD}$$

By Castiglione's theorem, $S_d = \frac{\partial U}{\partial Q}$

$$S_d = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q}$$

Member AB

$$M = (P + Q)y \quad \frac{\partial M}{\partial Q} = y \quad \text{Set } Q = 0 \quad M = Py$$

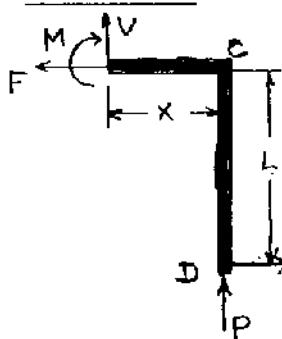


$$U_{AB} = \int_0^L \frac{M^2}{2EI} dy$$

$$\frac{\partial U_{AB}}{\partial Q} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial Q} dy = \frac{P}{EI} \int_0^L y^2 dy = \frac{PL^3}{3EI}$$

Member BC

$$M = Px + QL \quad \frac{\partial M}{\partial Q} = L \quad \text{Set } Q = 0 \quad M = Px$$



$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx$$

$$\frac{\partial U_{BC}}{\partial Q} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial Q} dx = \frac{PL}{EI} \int_0^L x dx = \frac{PL^3}{2EI}$$

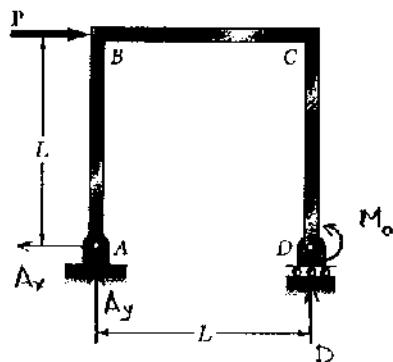
Member CD $M = Qy \quad \frac{\partial M}{\partial Q} = y \quad \text{Set } Q = 0 \quad M = 0$

$$U_{CD} = \int_0^L \frac{M^2}{2EI} dy \quad \frac{\partial U_{CD}}{\partial Q} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial Q} dy = 0$$

$$S_d = \frac{PL^3}{3EI} + \frac{PL^3}{2EI} + 0 = \frac{5PL^3}{6EI} \rightarrow$$

Problem 11.108

11.108 Three rods, each of the same flexural rigidity EI , are welded to form the frame ABCD. For the loading shown, determine the angle formed by the frame at point D.



Add dummy couple M_o at point D.

$$\text{Statics} \quad +\circlearrowleft M_A = 0: \quad M_o + DL - PL = 0$$

$$D = P - \frac{M_o}{L} \uparrow$$

$$\pm \sum F_x = 0: \quad -A_x + P = 0 \quad A_x = P \leftarrow$$

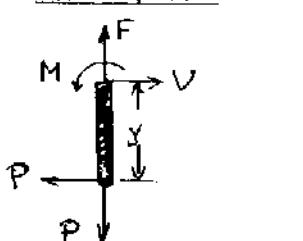
$$+\uparrow \sum F_y = 0: \quad A_y + D = 0 \quad A_y = P - \frac{M_o}{L} \downarrow$$

$$U = U_{AB} + U_{BC} + U_{CD}$$

$$\text{By Castiglione's theorem, } \theta_D = \frac{\partial U}{\partial M_o}$$

$$\theta_D = \frac{\partial U_{AB}}{\partial M_o} + \frac{\partial U_{BC}}{\partial M_o} + \frac{\partial U_{CD}}{\partial M_o}$$

Member AB



$$M = P y \quad \frac{\partial M}{\partial M_o} = 0 \quad U_{AB} = \int_0^L \frac{M^2}{2EI} dy$$

$$\frac{\partial U_{AB}}{\partial M_o} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_o} dy = 0$$

Member BC

$$M = M_o + Dx = M_o + P x - \frac{M x}{L}$$

$$\text{Set } M_o = 0 \quad M = P x$$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx$$

$$\frac{\partial U_{BC}}{\partial M_o} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_o} dx = \frac{P}{EI} \int_0^L x(1 - \frac{x}{L}) dy = \frac{PL^2}{6EI}$$

$$\text{Member CD} \quad M = M_o \quad \frac{\partial M}{\partial M_o} = 0$$

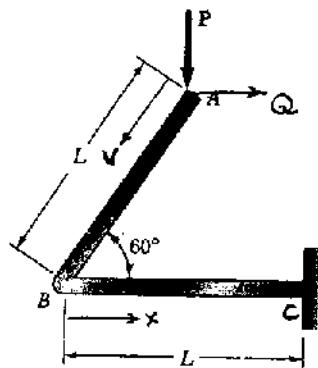
$$\text{Set } M_o = 0 \quad M = 0$$

$$U = \int_0^L \frac{M^2}{2EI} dy \quad \frac{\partial U}{\partial M_o} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_o} dy = 0$$

$$\theta_D = 0 + \frac{PL^2}{6EI} + 0 = \frac{PL^2}{6EI} \angle$$

Problem 11.109

II.109 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the vertical deflection of point A, (b) the horizontal deflection of point A



Add dummy horizontal force Q at point A.

$$\text{Over AB} \quad M = \frac{1}{2} Pv + \frac{\sqrt{3}}{2} Qv$$

$$\frac{\partial M}{\partial P} = \frac{1}{2} v \quad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2} v$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dx \quad \text{Set } Q = 0$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dv = \frac{1}{EI} \int_0^L (\frac{1}{2} Pv)(\frac{1}{2} v) dv \\ = \frac{1}{12} \frac{PL^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dv = \frac{1}{EI} \int_0^L (\frac{1}{2} Pv) \frac{\sqrt{3}}{2} dv \\ = \frac{\sqrt{3}}{12} \frac{PL^3}{EI}$$

$$\text{Over BC} \quad M = -P(x - \frac{L}{2}) + \frac{\sqrt{3}}{2} QL, \quad \frac{\partial M}{\partial P} = (x - \frac{L}{2}), \quad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2} L$$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx \quad \text{Set } Q = 0.$$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dx = \frac{1}{EI} \int_0^L P(x - \frac{L}{2})^2 dx = \frac{P}{3EI} (x - \frac{L}{2})^3 \Big|_0^L = \frac{1}{12} \frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx = -\frac{1}{EI} \int_0^L P(x - \frac{L}{2})(\frac{\sqrt{3}}{2})L dx = -\frac{\sqrt{3}P}{4EI} (x - \frac{L}{2})^2 \Big|_0^L = 0$$

(a) vertical deflection of point A.

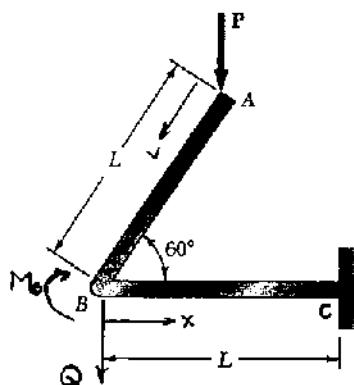
$$S_p = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} = \frac{1}{6} \frac{PL^3}{EI}$$

(b) horizontal deflection of point A.

$$S_q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{\sqrt{3}}{12} \frac{PL^3}{EI} = 0.1443 \frac{PL^3}{EI}$$

Problem 11.110

11.110 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the vertical deflection of point B, (b) the slope of BC at point B.



Add dummy vertical Q and dummy couple M_0 at B.

$$\text{Over AB} \quad M = \frac{1}{2}Pv, \quad \frac{\partial M}{\partial Q} = 0, \quad \frac{\partial M}{\partial M_0} = 0$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv$$

$$\frac{\partial U_{AB}}{\partial Q} = 0$$

$$\frac{\partial U_{AB}}{\partial M_0} = 0$$

$$\text{Over BC} \quad M = -P(x - \frac{L}{2}) - Qx + M_0, \quad \frac{\partial M}{\partial Q} = -x, \quad \frac{\partial M}{\partial M_0} = 1$$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx \quad \text{Set } Q = 0 \text{ and } M_0 = 0.$$

$$\begin{aligned} \frac{\partial U_{BC}}{\partial Q} &= \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L P(x - \frac{L}{2})x dx = \frac{P}{EI} \left[\frac{L^3}{3} - (\frac{L}{2}) \frac{L^2}{2} \right] \\ &= \frac{1}{12} \frac{PL^3}{EI} \end{aligned}$$

$$\frac{\partial U_{BC}}{\partial M_0} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_0} dx = \frac{1}{EI} \int_0^L P(x - \frac{L}{2}) dx = 0$$

(a) vertical deflection of point B.

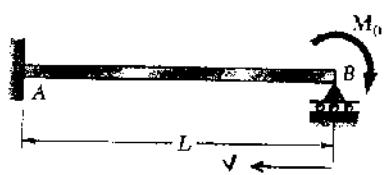
$$S_0 = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{1}{12} \frac{PL^3}{EI} \downarrow$$

(b) slope of BC at point B.

$$\theta_B = \frac{\partial U_{AB}}{\partial M_0} + \frac{\partial U_{BC}}{\partial M_0} = 0$$

Problem 11.111

11.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.

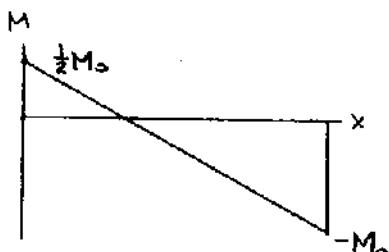


Remove support B and add reaction R_B as a load.

$$U = \int_0^L \frac{M^2}{2EI} dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_B} dv = 0$$

$$M = R_B v - M_0 \quad \frac{\partial M}{\partial R_B} = v$$

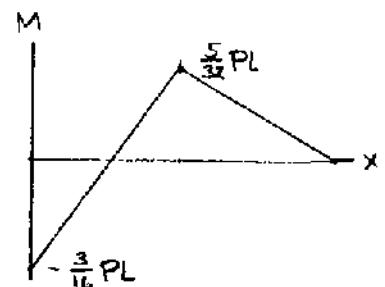
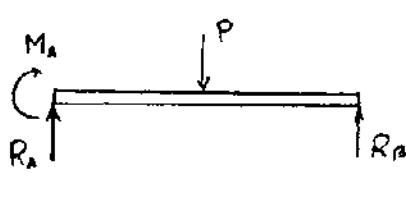
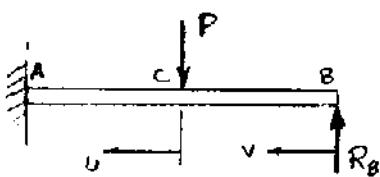
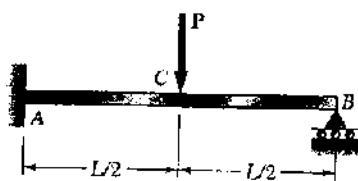


$$y_B = \frac{1}{EI} \int_0^L (R_B v - M_0) v dv \\ = \frac{R_B}{EI} \int_0^L v^2 dv - \frac{M_0}{EI} \int_0^L v dv \\ = \frac{R_B L^3}{3EI} - \frac{M_0 L^2}{2EI} = 0 \quad R_B = \frac{3}{2} \frac{M_0}{L} \uparrow$$

$$M_A = R_B - M_0 = \frac{3}{2} M_0 - M_0 = \frac{1}{2} M_0$$

Problem 11.112

11.111 through 11.114 Determine the reaction at the roller support and the draw the bending-moment diagram for the beam and loading shown.



Remove support B and add reaction R_B as a load.

$$U = U_{AC} + U_{CB} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} du + \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dv$$

$$Y_B = \frac{\partial U}{\partial R_B} = \frac{\partial U_{AC}}{\partial R_B} + \frac{\partial U_{CB}}{\partial R_B} = 0$$

$$\text{Over } AC: M = R_B(u + \frac{L}{2}) - Pv, \quad \frac{\partial M}{\partial R_B} = (u + \frac{L}{2})$$

$$\begin{aligned} \frac{\partial U_{AC}}{\partial R_B} &= \frac{1}{EI} \int_0^{\frac{L}{2}} [R_B(u + \frac{L}{2}) - Pv](u + \frac{L}{2}) du \\ &= \frac{R_B}{EI} \int_0^{\frac{L}{2}} (u + \frac{L}{2})^2 du - \frac{P}{EI} \int_0^{\frac{L}{2}} u(u + \frac{L}{2}) du \\ &= \frac{R_B}{3EI} [L^3 - (\frac{L}{2})^3] - \frac{P}{EI} [\frac{1}{8}(\frac{L}{2})^3 + \frac{L}{2} \cdot \frac{1}{2}(\frac{L}{2})^2] \\ &= \frac{7}{24} \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI} \end{aligned}$$

$$\text{Over } CB: M = R_B v \quad \frac{\partial M}{\partial R_B} = v$$

$$\frac{\partial U_{CB}}{\partial R_B} = \frac{1}{EI} \int_0^L (R_B v) v dv = \frac{R_B}{3EI} (\frac{L}{2})^3 = \frac{1}{24} \frac{R_B L^3}{EI}$$

$$Y_B = (\frac{7}{24} + \frac{1}{24}) \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI} = 0$$

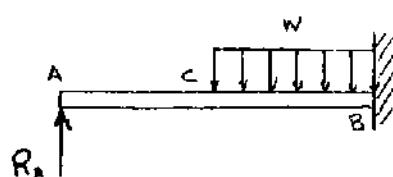
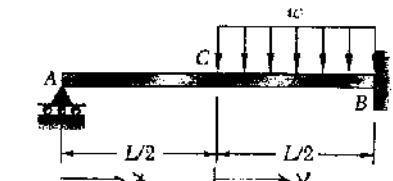
$$R_B = \frac{5}{16} P \uparrow$$

$$M_C = R_B \frac{L}{2} = \frac{5}{32} PL$$

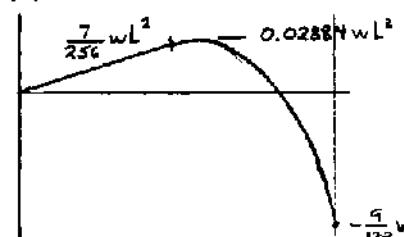
$$M_A = R_B L - P \frac{L}{2} = (\frac{5}{16} - \frac{1}{2}) PL = -\frac{3}{16} PL$$

Problem 11.113

11.111 through 11.114 Determine the reaction at the roller support and the draw the bending-moment diagram for the beam and loading shown.



M



Remove support A and add reaction R_A as a load.

$$U = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx + \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dx + \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dv = 0$$

Portion AC: $0 < x < \frac{L}{2}$ $M = R_A x$ $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AC}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} (R_A x)(x) dx = \frac{R_A L^3}{24 EI}$$

Portion CB: $0 < v < \frac{L}{2}$

$$M = R_A(v + \frac{L}{2}) - \frac{1}{2} w v^2 \quad \frac{\partial M}{\partial R_A} = (v + \frac{L}{2})$$

$$\frac{\partial U_{CB}}{\partial R_A} = \frac{1}{EI} \int_0^L [R_A(v + \frac{L}{2}) - \frac{1}{2} w v^2](v + \frac{L}{2}) dv$$

$$= \frac{1}{EI} \left\{ R_A \int_0^{\frac{L}{2}} (v + \frac{L}{2})^2 dv - \frac{1}{2} w \int_0^{\frac{L}{2}} (v^3 + \frac{L}{2} v^2) dv \right\}$$

$$= \frac{R_A}{EI} \left[\frac{1}{3} L^3 - \frac{1}{3} (\frac{L}{2})^3 \right] - \frac{w}{2EI} \left[\frac{1}{4} (\frac{L}{2})^4 + \frac{L}{2} \cdot \frac{1}{3} (\frac{L}{2})^3 \right]$$

$$= (\frac{1}{3} - \frac{1}{24}) \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI}$$

$$S_A = \frac{\partial U_{AC}}{\partial R_A} + \frac{\partial U_{CB}}{\partial R_A} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} = 0$$

$$R_A = \frac{7}{128} w L \uparrow$$

Over AC: $M = \frac{7}{128} w L x$

$$M_c = \frac{7}{256} w L^2 = 0.02734 w L^2$$

Over CB: $M = \frac{7}{128} w L(v + \frac{L}{2}) - \frac{1}{2} w v^2$

$$M_B = \frac{7}{128} w L^2 - \frac{1}{2} w (\frac{L}{2})^2 = -\frac{9}{128} w L^2 = -0.07031 w L^2$$

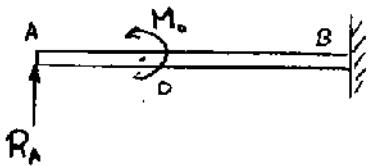
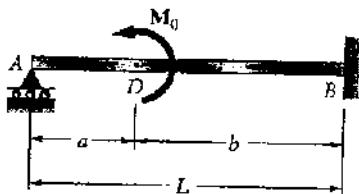
$$\frac{dM}{dv} = \frac{7}{128} w L - w v_m = 0 \quad v_m = \frac{7}{128} L$$

$$M_m = \frac{7}{128} w L \left(\frac{7}{128} L + \frac{L}{2} \right) - \frac{1}{2} w \left(\frac{7}{128} L \right)^2$$

$$= \frac{945}{32768} w L^2 = 0.02884 w L^2$$

Problem 11.114

11.111 through 11.114 Determine the reaction at the roller support and the draw the bending-moment diagram for the beam and loading shown.



Remove support A and add reaction R_A as a load.

$$U = \int_a^L \frac{M^2}{2EI} dx$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_a^L M \frac{\partial M}{\partial R_A} dx = 0$$

$$\text{Portion AD: } 0 < x < a \quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI} \int_0^a (R_A x)(x) dx = \frac{R_A a^3}{3EI}$$

$$\text{Portion DB: } (a < x < L) \quad M = R_A x - M_0 \quad \frac{\partial M}{\partial R_A} = x$$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI} \int_a^L (R_A x - M_0)(x) dx = \frac{1}{EI} \left\{ \frac{1}{3} R_A (L^3 - a^3) - \frac{1}{2} M_0 (L^2 - a^2) \right\}$$

$$S_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI} \left\{ R_A \left(\frac{1}{3} a^3 + \frac{1}{3} L^3 - \frac{1}{3} a^3 \right) - \frac{1}{2} M_0 (L^2 - a^2) \right\} = 0$$

$$R_A = \frac{3}{2} \frac{M_0 (L^2 - a^2)}{L^3} = \frac{3}{2} \frac{M_0 b (L + a)}{L^3}$$

$$M_A = 0$$

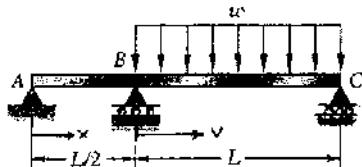
$$M_{D-} = R_A a = \frac{3}{2} \frac{M_0 ab (L + a)}{L^3}$$

$$M_{D+} = M_{D-} - M_0 = \frac{3}{2} \frac{M_0 ab (L + a)}{L^3} - M_0$$

$$M_B = R_A L - M_0 = \frac{3}{2} \frac{M_0 b (L + a)}{L^2} - M_0$$

Problem 11.115

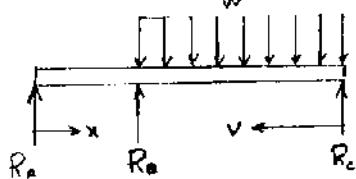
11.115 For the uniform beam and loading shown, determine the reaction at each support.



Remove support A and add reaction R_A as a load.

$$\sum M_B = 0 \quad -R_A \frac{L}{2} - \frac{1}{2}wL^2 + R_c L = 0$$

$$R_c = \frac{1}{2}R_A + \frac{1}{2}wL$$



$$U = U_{AB} + U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx + \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dv$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = 0$$

Portion AB: $M = R_A x, \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AB}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_0^{\frac{L}{2}} (R_A x)(x) dx = \frac{R_A}{3EI} (\frac{L}{2})^3 = \frac{1}{24} \frac{R_A L^3}{EI}$$

Portion BC: $M = R_c v - \frac{1}{2}wv^2 = \frac{1}{2}R_A v + \frac{1}{2}wLv - \frac{1}{2}wLv^2$

$$\frac{\partial M}{\partial R_A} = \frac{1}{2}v$$

$$\begin{aligned} \frac{\partial U_{BC}}{\partial R_A} &= \frac{1}{EI} \int_0^L [\frac{1}{2}R_A v + \frac{1}{2}w(Lv - v^2)](\frac{1}{2}v) dv = \frac{1}{4EI} \int_0^L [R_A v^2 + w(Lv^2 - v^3)] dv \\ &= \frac{1}{4EI} \left[R_A \frac{L^3}{3} + w \left(\frac{L^4}{3} - \frac{L^4}{4} \right) \right] = \frac{R_A L^3}{12EI} + \frac{wL^4}{48EI} \end{aligned}$$

$$S_A = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = \left(\frac{1}{24} + \frac{1}{12} \right) \frac{R_A L^3}{EI} + \frac{wL^4}{48EI} = 0$$

$$R_A = -\frac{1}{6}wL = \frac{1}{6}wL \downarrow$$

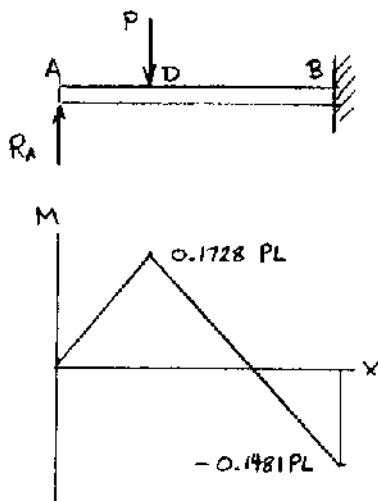
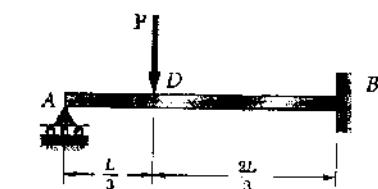
$$R_c = \frac{1}{2}(-\frac{1}{6}wL) + \frac{1}{2}wL = \frac{5}{12}wL \uparrow$$

$$+\uparrow \sum F_y = 0: R_A + R_B + R_c - wL = 0$$

$$-\frac{1}{6}wL + R_B + \frac{5}{12}wL - wL = 0 \quad R_B = \frac{3}{4}wL \uparrow$$

Problem 11.116

11.116 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and load shown.



Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{EI} dx$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_A} dx$$

$$\text{Portion AD: } 0 < x < \frac{L}{3} \quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$$

$$\begin{aligned} \frac{\partial U_{AD}}{\partial R_A} &= \frac{1}{EI} \int_0^{\frac{L}{3}} M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_0^{\frac{L}{3}} (R_A x)(x) dx \\ &= \frac{R_A}{3EI} \left(\frac{L}{3}\right)^3 = \frac{1}{81} \frac{R_A L^3}{EI} \end{aligned}$$

$$\text{Portion DB: } \frac{L}{3} < x < L \quad M = R_A x - P(x - \frac{L}{3})$$

$$\frac{\partial M}{\partial R_A} = x$$

$$\begin{aligned} \frac{\partial U_{DB}}{\partial R_A} &= \frac{1}{EI} \int_{\frac{L}{3}}^L M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_{\frac{L}{3}}^L [R_A x - P(x - \frac{L}{3})] x dx \\ &= \frac{R_A}{EI} \int_{\frac{L}{3}}^L x^2 dx - \frac{P}{EI} \int_{\frac{L}{3}}^L (x^2 - \frac{L}{3}x) dx \\ &= \frac{R_A}{3EI} \left[L^3 - \left(\frac{L}{3}\right)^3\right] - \frac{P}{EI} \left[\frac{1}{3}(L^3 - (\frac{L}{3})^3) - \frac{L}{6}(L^2 - (\frac{L}{3})^2)\right] \\ &= \left(\frac{1}{3} - \frac{1}{81}\right) \frac{R_A L^3}{EI} - \left(\frac{1}{3} - \frac{1}{81} - \frac{1}{6} + \frac{1}{54}\right) \frac{PL^3}{EI} \end{aligned}$$

$$S_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \left(\frac{1}{81} + \frac{1}{3} - \frac{1}{81}\right) \frac{R_A L^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = 0$$

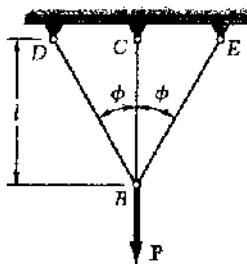
$$R_A = \frac{14}{27} P \uparrow$$

$$\text{Bending moments} \quad M_D = R_A \left(\frac{L}{3}\right) = \frac{14}{81} PL = 0.1728 PL$$

$$M_B = R_A L - P \left(\frac{2L}{3}\right) = -\frac{4}{27} PL = -0.1481 PL$$

Problem 11.117

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC.

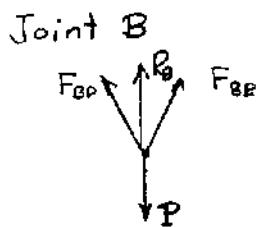


Detach member BC at support C.

Add reaction R_c as a load.

$$U = \sum \frac{F^2 L}{2EA} \quad y_c = \frac{\partial U}{\partial R_c} = \sum \frac{FL}{EA} \frac{\partial F}{\partial R_c} = 0$$

$$\text{Joint C} \quad F_{bc} = R_c$$



$$+\rightarrow \sum F_x = 0: \quad F_{BE} \sin \phi - F_{BD} \sin \phi = 0 \quad F_{BE} = F_{BD}$$

$$+\uparrow \sum F_y = 0: \quad F_{BD} \cos \phi + F_{BE} \cos \phi + R_B - P = 0$$

$$F_{BD} = F_{BE} = \frac{P - R_B}{2 \cos \phi}$$

Member	F	$\frac{\partial F}{\partial R_B}$	L	$(FL/EA) \frac{\partial F}{\partial R_B}$
BD	$(P - R_B)/2 \cos \phi$	$-1/2 \cos \phi$	$l/\cos \phi$	$(R_B - P)l/4EA \cos^3 \phi$
BE	$(P - R_B)/2 \cos \phi$	$-1/2 \cos \phi$	$l/\cos \phi$	$(R_B - P)l/4EA \cos^3 \phi$
BC	R_B	1	l	$R_B l/EA$

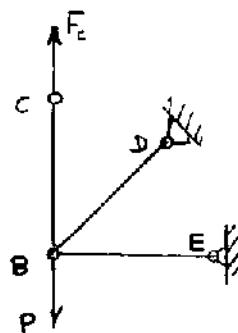
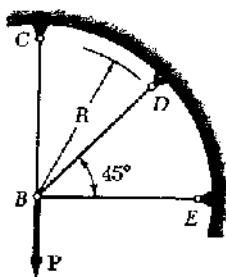
$$y_B = -Pl/2EA \cos^3 \phi + R_B l/2EA \cos^3 \phi + R_B l/EA = 0$$

$$R_B = \frac{P}{1 + 2 \cos^3 \phi}$$

$$F_{BE} = R_B = \frac{P}{1 + 2 \cos^3 \phi}$$

Problem 11.118

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .



$$U = \frac{\sum F^2 R}{2EA} = \frac{R}{2EA} \sum F^2$$

$$S_c = \frac{\partial U}{\partial F_c} = \frac{R}{EA} \sum F \frac{\partial F}{\partial F_c} = 0$$

Member	F	$\partial F / \partial F_c$	$F(\partial F / \partial F_c)$
BC	F_c	1	F_c
BD	$\sqrt{2}P - \sqrt{2}F_c$	$-\sqrt{2}$	$-2P + 2F_c$
BE	$-P + F_c$	1	$-P + F_c$
Σ			$-3P + 4F_c$

Detach member BC from its support at point C . Add reaction F_c as a load.

Joint B.

$$+\uparrow \sum F_y = 0:$$

$$\frac{\sqrt{2}}{2} F_{BD} + F_c - P = 0$$

$$F_{BD} = \sqrt{2}P - \sqrt{2}F_c$$

$$+\rightarrow \sum F_x = 0:$$

$$\frac{\sqrt{2}}{2} F_{BD} + F_{BE} = 0$$

$$F_{BE} = -P + F_c$$

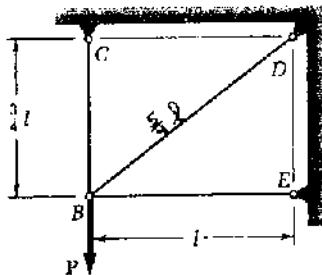
$$S_c = \frac{R}{EA} (-3P + 4F_c) = 0$$

$$F_c = \frac{3}{4}P$$

$$F_{BC} = F_c = \frac{3}{4}P$$

Problem 11.119

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .

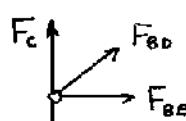


Detach member BC from support C . Add reaction F_c as a load.

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$S_c = \frac{\partial U}{\partial F_c} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_c} L$$

Joint B



$$+\uparrow \sum F_y = 0 : F_c - P + \frac{3}{5} F_{BD} = 0 \quad F_{BD} = \frac{5}{3} P - \frac{5}{3} F_c$$

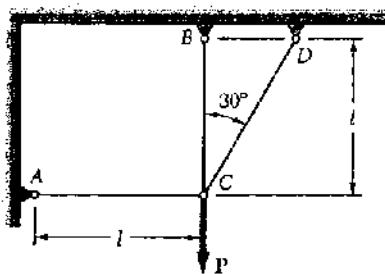
$$\pm \sum F_x = 0 : F_{BE} + \frac{4}{3} F_{BD} = 0 \quad F_{BE} = -\frac{4}{3} P + \frac{4}{3} F_c$$

Member	F	$\frac{\partial F}{\partial F_c}$	L	$F(\frac{\partial F}{\partial F_c})L$
BC	F_c	1	$\frac{3}{4}l$	$\frac{3}{4}F_c l$
BD	$\frac{5}{3}P - \frac{5}{3}F_c$	$-\frac{5}{3}$	$\frac{5}{4}l$	$-\frac{125}{36}Pl + \frac{125}{36}F_c l$
BE	$-\frac{4}{3}P + \frac{4}{3}F_c$	$\frac{4}{3}$	l	$-\frac{16}{9}Pl + \frac{16}{9}F_c l$
Σ				$-\frac{21}{4}Pl + 6F_c l$

$$S_c = \frac{1}{EA} \left(-\frac{21}{4} Pl + 6 F_c l \right) = 0 \quad F_c = \frac{7}{8} P \quad F_{ac} = F_c = \frac{7}{8} P$$

Problem 11.120

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC.



Cut member BC at end B and replace member force F_{BC} by load F_B acting on member BC at B.

$$S_B = \frac{\partial U}{\partial F_B} = \frac{\partial}{\partial F_B} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_B} L$$

Joint C $\uparrow \sum F_y = 0: \quad \frac{\sqrt{3}}{2} F_{CD} + F_{AC} - P = 0$

$F_{CD} = \frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$

$\pm \sum F_x = 0: \quad F_{AC} - \frac{1}{2} F_{CD} = 0$

$F_{AC} = \frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$



Member	F	$2F/\partial F_B$	L	$F(\partial F/\partial F_B)L$
AC	F_B	1	l	$F_B l$
BC	$\frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$	$-\frac{1}{\sqrt{3}}$	l	$-\frac{1}{3} Pl + \frac{1}{3} F_B l$
CD	$\frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}} l$	$-\frac{4}{3} Pl + \frac{8}{\sqrt{3}} F_B l$
Σ				$-(\frac{1}{3} + \frac{8}{\sqrt{3}}) Pl + (\frac{4}{3} + \frac{8}{\sqrt{3}}) F_B l$

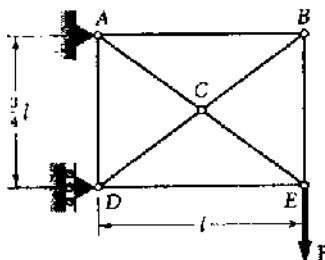
$$S_B = -(\frac{1}{3} + \frac{8}{\sqrt{3}}) \frac{Pl}{EA} + (\frac{4}{3} + \frac{8}{\sqrt{3}}) \frac{F_B l}{EA}$$

$$F_B = \frac{\frac{1}{3} + \frac{8}{\sqrt{3}}}{\frac{4}{3} + \frac{8}{\sqrt{3}}} P = \frac{8 + \sqrt{3}}{8 + 4\sqrt{3}} P = 0.652 P$$

$$F_{AC} = F_B = 0.652 P$$

Problem 11.121

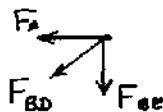
11.121 and 11.122 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.



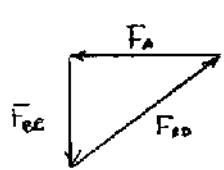
Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$

Joint B



$$F_{OP} = -\frac{5}{4} F_A$$



$$F_{BE} = \frac{3}{4} F_A$$

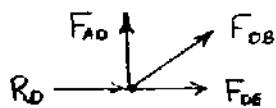
Joint E

$$+\uparrow \sum F_y = 0:$$

$$F_{BE} - P + \frac{3}{5} F_{AE}$$

$$\begin{aligned} F_{AE} &= \frac{5}{3} P - \frac{5}{3} F_{BE} \\ &= \frac{5}{3} P - \frac{5}{4} F_A \end{aligned}$$

Joint D



$$+\uparrow \sum F_y = 0: \quad F_{AD} + \frac{3}{5} F_{DB} = 0$$

$$F_{AD} = -\frac{3}{5} F_{DB} = -\frac{3}{4} F_A$$

$$+\rightarrow \sum F_x = 0: \quad -\frac{4}{5} F_{AE} - F_{DE} = 0$$

$$F_{DE} = -\frac{4}{5} F_{AE} = -\frac{4}{3} P + F_A$$

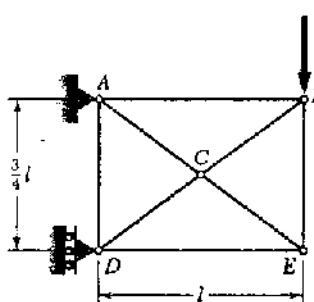
Member	F	$\partial F / \partial F_A$	L	$F(\partial F / \partial F_A) L$
AB	F_A	1	l	$F_A l$
AD	$-\frac{3}{4} F_A$	$-\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$-\frac{125}{48} Pl + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$\frac{125}{64} F_A l$
BE	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	l	$-\frac{4}{3} Pl + F_A l$
Σ				$-\frac{63}{16} Pl + \frac{27}{4} F_A l$

$$S_A = \frac{1}{EA} \left(-\frac{63}{16} Pl + \frac{27}{4} F_A l \right) = 0 \quad F_A = \frac{7}{12} P$$

$$F_{AB} = F_A = \frac{7}{12} P = 0.583 P$$

Problem 11.122

11.121 and 11.122 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.



Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$

Joint B $\rightarrow \sum F_x = 0 \quad -F_A - \frac{4}{3} F_{BD} = 0 \quad F_{BD} = -\frac{5}{4} F_A$

$$F_A \leftarrow +\uparrow \sum F_y = 0 \quad -P - F_{BE} - \frac{3}{5} F_{BD} = 0 \quad F_{BE} = -P + \frac{3}{5} F_A$$

Joint E $+ \uparrow \sum F_y = 0 \quad F_{CE} + \frac{3}{5} F_{AE} = 0 \quad F_{AE} = \frac{5}{3} P - \frac{5}{4} F_A$

$$F_{AE} \leftarrow +\rightarrow \sum F_x = 0 \quad -\frac{4}{5} F_{AE} + F_{DE} = 0 \quad F_{DE} = -\frac{4}{3} P + F_A$$

Joint D $+ \uparrow \sum F_y = 0 \quad F_{AD} + \frac{3}{5} F_{CD} = 0 \quad F_{AD} = \frac{3}{4} F_A$

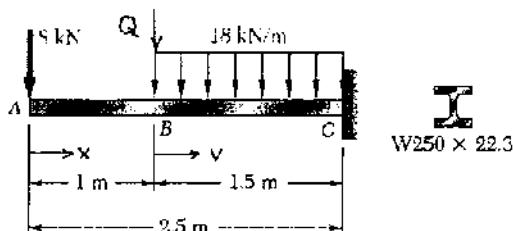
Member	F	$\partial F / \partial F_A$	L	$F(\partial F / \partial F_A)L$
AB	F_A	1	l	$F_A l$
AD	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{64} P l + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$\frac{125}{64} F_A l$
BE	$-P + \frac{3}{5} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$-\frac{9}{16} P l + \frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	l	$-\frac{4}{3} P l + F_A l$
Σ				$-\frac{9}{2} P l + \frac{27}{4} F_A l$

$$S_A = \frac{1}{EA} \left(-\frac{9}{2} P l + \frac{27}{4} F_A l \right) = 0 \quad F_A = \frac{2}{3} P$$

$$F_{AB} = F_A = \frac{2}{3} P = 0.667 P$$

Problem 11.123

11.123 For the beam and loading shown, determine the deflection at point B. Use $E = 200 \text{ GPa}$.



Add force Q at point B.

Units: Forces in kN, lengths in m.

$$\text{Over AB} \quad M = -8x \quad \frac{\partial M}{\partial Q} = 0$$

$$\text{Over BC} \quad M = -8(v+1) - \frac{1}{2}(18)v^2 - Qv \quad \frac{\partial M}{\partial Q} = -v$$

$$E = 200 \times 10^9 \text{ Pa}, \quad I = 28.9 \times 10^6 \text{ mm}^4 = 28.9 \times 10^{-6} \text{ m}^4$$

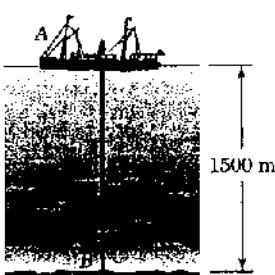
$$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^4 \text{ N}\cdot\text{m}^2 = 5780 \text{ kN}\cdot\text{m}^2$$

$$U = \int_0^1 \frac{M^2}{2EI} dx + \int_0^{1.5} \frac{M^2}{2EI} dv$$

$$\begin{aligned} S_B &= \frac{\partial U}{\partial Q} = \frac{1}{EI} \left\{ \int_0^1 M \frac{\partial M}{\partial Q} dx + \int_0^{1.5} M \frac{\partial M}{\partial Q} dv \right\} \\ &= \frac{1}{EI} \left\{ 0 + \int_0^{1.5} [-8(v+1) - \frac{1}{2}(18)v^2](-v) dv \right\} = \frac{1}{EI} \int_0^{1.5} (9v^3 + 8v^2 + 8v) dv \\ &= \frac{1}{EI} \left\{ \frac{9}{4}(1.5)^4 + \frac{8}{3}(1.5)^3 + \frac{8}{2}(1.5)^2 \right\} = \frac{29.391}{EI} = \frac{29.391}{5780} \\ &= 5.08 \times 10^{-3} \text{ m} = 5.08 \text{ mm} \end{aligned}$$

Problem 11.124

11.124 The ship at A has just started to drill for oil on the ocean floor at a depth of 1500 m. The steel drill pipe has an outer diameter of 200 mm and a uniform wall thickness of 12 mm. Knowing that the top of the drill pipe rotates through two complete revolutions before the drill bit at B starts to operate and using $G = 77.2 \text{ GPa}$, determine the maximum strain energy acquired by the drill pipe.



$$\phi = 2 \text{ rev} = (2)(2\pi) = 4\pi \text{ radians}$$

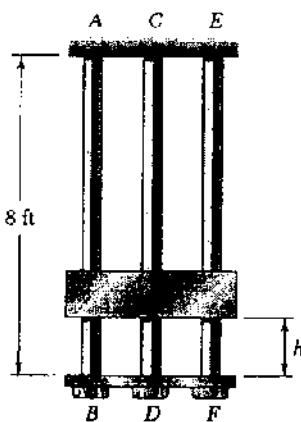
$$L = 1500 \text{ m}, \quad c_o = \frac{1}{2}d_o = 100 \text{ mm}, \quad c_i = c_o - t = 88 \text{ mm}$$

$$J = \frac{\pi}{2} (c_o^4 - c_i^4) = \frac{\pi}{2} (100^4 - 88^4) = 62.88 \times 10^6 \text{ mm}^4 = 62.88 \times 10^{-6} \text{ m}^4$$

$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ\phi}{L}$$

$$\begin{aligned} U &= \frac{T^2 L}{2GJ} = \frac{(GJ\phi)^2}{L} = \frac{GJ\phi^2}{2L} \\ &= \frac{(77.2 \times 10^9)(62.88 \times 10^{-6})(4\pi)^2}{(2)(1500)} = 256 \times 10^3 \text{ J} = 256 \text{ kJ} \end{aligned}$$

Problem 11.125



11.125 The 100-lb collar G is released from rest in the position shown and is stopped by plate BDF that is attached to the $\frac{7}{8}$ -in.-diameter steel rod CD and to the $\frac{5}{8}$ -in.-diameter steel rods AB and EF . Knowing that for the grade of steel used $\sigma_{\text{all}} = 24 \text{ ksi}$ and $E = 29 \times 10^6 \text{ psi}$, determine the largest allowable distance h .

Let Δ_m be the elongation.

$$\Delta_m = \frac{\sigma_{AB} L}{E} = \frac{\sigma_{CD} L}{E} = \frac{\sigma_{EF} L}{E}$$

$$\sigma_{AB} = \sigma_{CD} = \sigma_{EF} = 24 \times 10^3 \text{ psi}$$

$$L = 8 \text{ ft} = 96 \text{ in.}$$

$$\Delta_m = \frac{(24 \times 10^3)(96)}{29 \times 10^6} = 79.448 \times 10^{-3} \text{ in.}$$

$$\text{For each rod } U = \frac{F^2 L}{2EA} = \frac{(EA\Delta_m/L)^2 L}{2EA} = \frac{EA \Delta_m^2}{2L}$$

$$\text{Rod } CD: A_{CD} = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$

$$U_{CD} = \frac{(29 \times 10^6)(0.60132)(79.448 \times 10^{-3})^2}{(2)(96)} = 573.28 \text{ in-lb.}$$

$$\text{Rods } AB \text{ and } EF: A_{AB} = A_{EF} = \frac{\pi}{4} \left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$$

$$U_{AB} = U_{EF} = \frac{(29 \times 10^6)(0.30680)(79.448 \times 10^{-3})^2}{(2)(96)} = 292.49 \text{ in-lb}$$

$$\text{Total } U_m = U_{AB} + U_{CD} + U_{EF} = 1158.27 \text{ in-lb}$$

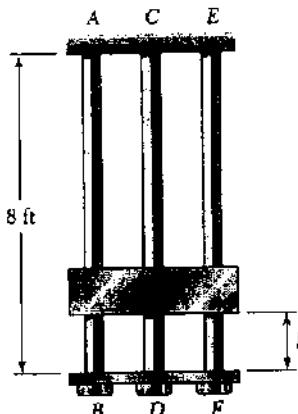
$$\text{Falling distance is } h + \Delta_m. \quad W = 100 \text{ lb}$$

$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{1158.27}{100} = 11.583 \text{ in.}$$

$$h = 11.583 - 79.448 \times 10^{-3} = 11.50 \text{ in.}$$

Problem 11.126



11.126 Solve Prob. 11.125, assuming that the $\frac{7}{8}$ -in.-diameter steel rod CD is replaced by a $\frac{7}{8}$ -in.-diameter rod made of a grade of aluminum for which $\sigma_u = 20$ ksi and $E = 10.6 \times 10^6$ psi.

11.125 The 100-lb collar G is released from rest in the position shown and is stopped by plate BDF that is attached to the $\frac{7}{8}$ -in.-diameter steel rod CD and to the $\frac{5}{8}$ -in.-diameter steel rods AB and EF. Knowing that for the grade of steel used $\sigma_u = 24$ ksi and $E = 29 \times 10^6$ psi, determine the largest allowable distance h .

Let Δ_m be the elongation, $L = 8 \text{ ft} = 96 \text{ in.}$

$$\Delta_m = \frac{\sigma_{AB} L}{E_{AB}} = \frac{\sigma_{CD} L}{E_{CD}} = \frac{\sigma_{EF} L}{E_{EF}}$$

$$\text{If } \sigma_{AB} = 24 \times 10^3 \text{ psi}, \quad \Delta_m = \frac{(24 \times 10^3)(96)}{29 \times 10^6} = 79.448 \times 10^{-3} \text{ in.}$$

$$\text{If } \sigma_{CD} = 20 \times 10^3 \text{ psi}, \quad \Delta_m = \frac{(20 \times 10^3)(96)}{10.6 \times 10^6} = 181.13 \times 10^{-3} \text{ in.}$$

Smaller value governs $\Delta_m = 79.448 \times 10^{-3} \text{ in.}$

$$\text{For each rod } U = \frac{F^2 L}{2EA} = \frac{(EA\Delta_m/L)^2 L}{2EA} = \frac{EA \Delta_m^2}{2L}$$

$$\text{Rod CD: } A_{CD} = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2, \quad E_{CD} = 10.6 \times 10^6 \text{ psi}$$

$$U_{CD} = \frac{(10.6 \times 10^6)(0.60132)(79.448 \times 10^{-3})^2}{(2)(96)} = 209.54 \text{ in-lb}$$

$$\text{Rods AB and EF: } A_{AB} = A_{EF} = \frac{\pi}{4} \left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$$

$$U_{AB} = U_{EF} = \frac{(29 \times 10^6)(0.30680)(79.448 \times 10^{-3})^2}{(2)(96)} = 292.49 \text{ in-lb}$$

$$\text{Total } U_m = U_{AB} + U_{CD} + U_{EF} = 794.52 \text{ in-lb.}$$

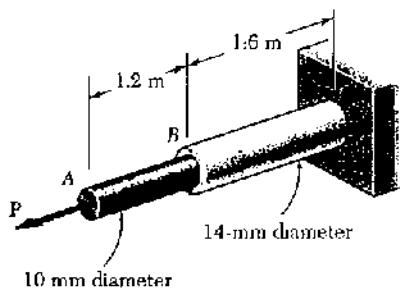
$$\text{ Falling distance is } h + \Delta_m \quad W = 100 \text{ lb.}$$

$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{794.52}{100} = 7.9452 \text{ in.}$$

$$h = 7.9452 - 79.448 \times 10^{-3} = 7.87 \text{ in.}$$

Problem 11.127



11.127 Rod *AB* is made of a steel for which the yield strength is $\sigma_y = 450 \text{ MPa}$ and $E = 200 \text{ GPa}$; rod *BC* is made of an aluminum alloy for which $\sigma_y = 280 \text{ MPa}$ and $E = 73 \text{ GPa}$. Determine the maximum strain energy that can be acquired by the composite rod *ABC* without causing any permanent deformations.

$$A_{AB} = \frac{\pi}{4}(10)^2 = 78.54 \text{ mm}^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(14)^2 = 153.94 \text{ mm}^2 = 153.94 \times 10^{-6} \text{ m}^2$$

$$P_{all} = G_y A \quad \text{for each portion.}$$

$$\text{AB: } P_{all} = (450 \times 10^6)(78.54 \times 10^{-6}) = 35.343 \times 10^3 \text{ N}$$

$$\text{BC: } P_{all} = (280 \times 10^6)(153.94 \times 10^{-6}) = 43.103 \times 10^3 \text{ N}$$

Use the smaller value.

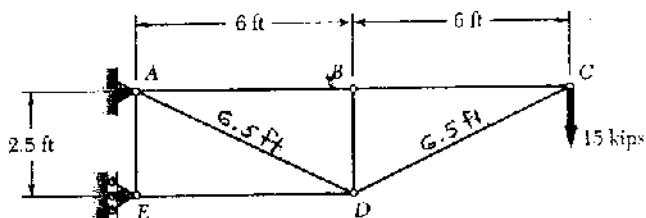
$$P = 35.343 \times 10^3 \text{ N}$$

$$U = \frac{P^2 L_{AB}}{2 E_{AB} A_{AB}} + \frac{P^2 L_{BC}}{2 E_{BC} A_{BC}} = \frac{(35.343 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})} + \frac{(35.343 \times 10^3)^2 (1.6)}{(2)(73 \times 10^9)(153.94 \times 10^{-6})}$$

$$= 136.6 \text{ J}$$

Problem 11.128

11.128 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 5 in^2 . Using $E = 29 \times 10^6 \text{ psi}$, determine the vertical deflection of joint C caused by the application of the 15-kip load.



Members BD and AE are zero force members.

For entire truss $\rightarrow M_A = 0$:

$$2.5 R_D - (12)(15) = 0$$

$$R_D = 72 \text{ kips}$$

For equilibrium of joint E

$$F_{EB} = -R_o = -72 \text{ kips}$$

Joint C

$$+1 \sum F_y = 0$$

$$- \frac{2.5}{6.5} F_{cb} - 15 = 0$$

$$F_{cb} = - 39 \text{ kips}$$

$$\rightarrow \sum F_x = 0:$$

$$-\frac{6}{6.5} F_{cb} - F_{bc} = 0$$

$$F_{ac} = 36 \text{ kips.}$$

Joint D

$$+ \rightarrow \sum F_x = 0 \\ 72 - \frac{6}{2.5} (F_{AD} + 39) = 0 \\ F_{AD} = 39 \text{ kips}$$

$$\underline{\text{Joint B}} \quad \sum F_x = 0:$$

$$-F_{ax} + F_{ay} = 0$$

$$F_{Ae} = 36 \text{ kips}$$

$$\text{Strain energy: } U_s = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \cdot \sum F^2 L$$

Member	F (kips)	L (in)	$F^2 L$ (kip $^2 \cdot$ in)
AB	36	72	93312
BC	36	72	93312
CD	-39	78	118638
DE	-72	72	373248
BD	0	30	0
AE	0	30	0
AD	39	78	118638
Σ			797148

Data: $E = 29 \times 10^3$ ksi

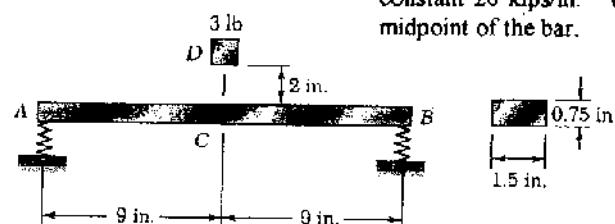
$$A = 5 \text{ in}^2$$

$$U_m = \frac{797148}{(2)(29 \times 10^3)(5)} = 2.7488 \text{ kip-in}$$

$$\frac{1}{2} P_n \Delta_m = U \quad \Delta_m = \frac{2U_m}{P_n} = \frac{(2)(2.7488)}{15} = 0.366 \text{ in. } \downarrow$$

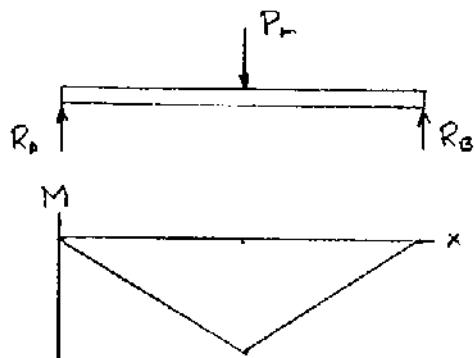
Problem 11.129

11.129 The 3-lb block D is released from rest in the position shown and strikes a steel bar AB having the uniform cross section shown. The bar is supported at each end by springs of constant 20 kips/in. Using $E = 29 \times 10^6$ psi, determine the maximum deflection at the midpoint of the bar.



$$k = 20 \text{ kips/in} = 20 \times 10^3 \text{ lb/in}$$

$$R_A = R_B = \frac{1}{2} P_m$$



$$\text{For spring A, } U_A = \frac{1}{2} R_A y_A = \frac{1}{2} \frac{P_m^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$$

$$\text{For spring B, } U_B = \frac{1}{2} R_B y_B = \frac{1}{2} \frac{P_m^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$$

$$\text{Portion AC of beam ACB: } M = \frac{1}{2} P_m x$$

$$U_{AC} = \int_0^{L_{AC}} \frac{M^2}{2EI} dx = \frac{P_m^2}{8EI} \int_0^{L_{AC}} x^2 dx = \frac{P_m^2 L_{AC}^3}{24EI}$$

$$\text{Portion CB of beam ACB:}$$

$$\text{By symmetry, } U_{CB} = U_{AC} = \frac{P_m^2 L_{AC}^3}{24EI}$$

$$\text{Total } U = U_A + U_B + U_{AC} + U_{CB} = \frac{P_m^2}{4k} + \frac{P_m^2 L_{AC}^3}{12EI}$$

$$I = \frac{1}{12} bd^3 = \frac{1}{12}(1.5)(0.75)^3 = 52.734 \times 10^{-6} \text{ in}^4$$

$$U = \left\{ \frac{1}{(4)(20 \times 10^3)} + \frac{(9)^2}{(12)(29 \times 10^6)(52.734 \times 10^{-6})} \right\} P_m^2 = 52.224 \times 10^{-6} \quad P_m^2 = \frac{1}{2} P_m y_m$$

$$y_m = \frac{2U}{P_m} = 104.448 \times 10^{-6} P_m \quad P_m = 9.5741 \times 10^3 y_m$$

$$U = (52.224 \times 10^{-6})(9.5741 \times 10^3)^2 y_m^2 = 4.7871 \times 10^3 y_m^2$$

$$\text{Work of falling weight: } W(h + y_m) = (3)(2 + y_m) = 6 + 3y_m$$

$$\text{Equating, } 6 + 3y_m = 4.7871 \times 10^3 y_m^2$$

$$y_m^2 - 626.69 \times 10^{-6} y_m - 1.25338 \times 10^{-3} = 0$$

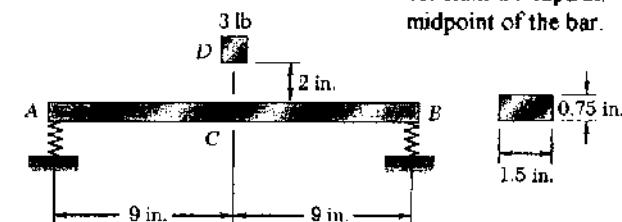
$$y_m = \frac{1}{2} \left\{ 626.69 \times 10^{-6} + \sqrt{(626.69 \times 10^{-6})^2 + (4)(1.25338 \times 10^{-3})} \right\}$$

$$= 35.7 \times 10^{-3} \text{ in.} = 0.0357 \text{ in.} \blacksquare$$

Problem 11.130

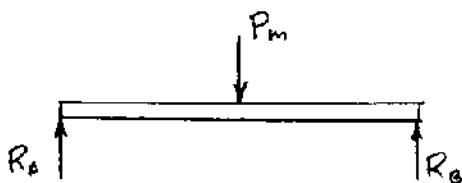
11.130 Solve Prob. 11.129, assuming that the constant of each spring is 40 kips/in.

11.129 The 3-lb block D is released from rest in the position shown and strikes a steel bar AB having the uniform cross section shown. The bar is supported at each end by springs of constant 20 kips/in. Using $E = 29 \times 10^6$ psi, determine the maximum deflection at the midpoint of the bar.



$$k = 40 \text{ kips/in.} = 40 \times 10^3 \text{ lb/in.}$$

$$R_A = R_B = \frac{1}{2} P_m$$



$$\text{For spring A, } U_A = \frac{1}{2} R_A y_A = \frac{1}{2} \frac{P_m^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$$

$$\text{For spring B, } U_B = \frac{1}{2} R_B y_B = \frac{1}{2} \frac{P_m^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$$

$$\text{Portion AC of beam ACB: } M = \frac{1}{2} P_m x$$

$$U_{AC} = \int_0^{L_{AC}} \frac{M^2}{2EI} dx = \frac{P_m^2}{8EI} \int_0^{L_{AC}} x^2 dx = \frac{P_m^2 L_{AC}^3}{24EI}$$

$$\text{Portion CB of beam ACB:}$$

$$\text{By symmetry, } U_{CB} = U_{AC} = \frac{P_m^2 L_{AC}^3}{24EI}$$

$$\text{Total } U = U_A + U_B + U_{AC} + U_{CB} = \frac{P_m^2}{4k} + \frac{P_m^2 L_{AC}^3}{12EI}$$

$$I = \frac{1}{12} bd^3 = \frac{1}{12} (1.5)(0.75)^3 = 52.734 \times 10^{-3} \text{ in.}^4$$

$$U = \left\{ \frac{1}{(4)(40 \times 10^3)} + \frac{(9)^3}{(12)(29 \times 10^6)(52.734 \times 10^{-3})} \right\} P_m^2 = 45.974 \times 10^{-6} P_m^2 = \frac{1}{2} P_m y_m$$

$$y_m = \frac{2U}{P_m} = 91.949 \times 10^{-6} P_m \quad P_m = 10.8756 \times 10^3 \text{ ym}$$

$$U = (45.974 \times 10^{-6})(10.8756 \times 10^3)^2 y_m^2 = 5.4378 \times 10^3 y_m^2$$

$$\text{Work of falling weight: } W(h + y_m) = (3)(2 + y_m) = 6 + 3y_m$$

$$\text{Equating, } 6 + 3y_m = 5.4378 \times 10^3 y_m^2$$

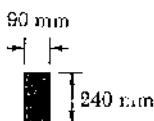
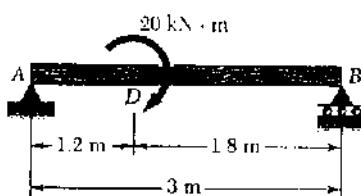
$$y_m^2 - 551.70 \times 10^{-6} y_m - 1.1034 \times 10^{-3} = 0$$

$$y_m = \frac{1}{2} \left\{ 551.70 \times 10^{-6} + \sqrt{(551.70 \times 10^{-6})^2 + (4)(1.1034 \times 10^{-3})} \right\}$$

$$= 33.5 \times 10^{-3} \text{ in.} = 0.0335 \text{ in.} \downarrow$$

Problem 11.131

11.131 Using $E = 12 \text{ GPa}$, determine the strain energy due to bending for the timber beam and loading shown.

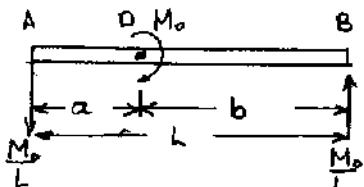


$$+\sum M_A = 0 : -M_o + R_A L = 0$$

$$R_A = \frac{M_o}{L} \downarrow, \quad R_B = \frac{M_o}{L} \uparrow$$

$$\text{Over portion AD: } M = R_A x = \frac{M_o x}{L}$$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_o^2}{2EI L^2} \int_0^a x^2 dx \\ = \frac{M_o^2 a^3}{6EI L^2}$$



$$\text{Over portion DB: } M = R_B v = \frac{M_o v}{L}$$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{M_o^2}{2EI L^2} \int_0^b v^2 dv \\ = \frac{M_o^2 b^3}{6EI L^2}$$

$$\text{Total: } U = U_{AD} + U_{DB} = \frac{M_o^2 (a^3 + b^3)}{6EI L^2}$$

Data: $a = 1.2 \text{ m}$, $b = 1.8 \text{ m}$, $L = 3 \text{ m}$, $M_o = 20 \times 10^3 \text{ N}\cdot\text{m}$

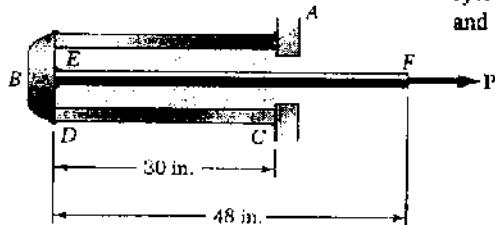
$$I = \frac{1}{12}(90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

$$EI = (12 \times 10^9)(103.68 \times 10^{-6}) = 1.24416 \times 10^6 \text{ N}\cdot\text{m}^2$$

$$U = \frac{(20 \times 10^3)^2 [(1.2)^3 + (1.8)^3]}{(6)(1.24416 \times 10^6)(3)^2} = 45.0 \text{ N}\cdot\text{m} = 45.0 \text{ J}$$

Problem 11.132

11.132 A 30-in. length of aluminum pipe of cross-sectional area 1.85 in^2 is welded to a fixed support *A* and to a rigid cap *B*. The steel rod *EF*, of 0.75 in. diameter, is welded to cap *B*. Knowing that the modulus of elasticity is $29 \times 10^6 \text{ psi}$ for the steel and $10.6 \times 10^6 \text{ psi}$ for the aluminum, determine (a) the total strain energy of the system when $P = 10 \text{ kips}$, (b) the corresponding strain-energy density of the pipe *CD* and in the rod *EF*.



$$\text{For } EF: A = \frac{\pi}{4} d^2 = 0.4418 \text{ in}^2$$

$$CD: U_{cd} = \frac{P^2 L}{2EA} = \frac{(10 \times 10^3)^2 (30)}{(2)(10.6 \times 10^6)(1.85)} = 76.49 \text{ in-lb}$$

$$EF: U_{ef} = \frac{P^2 L}{2EA} = \frac{(10 \times 10^3)^2 (48)}{(2)(29 \times 10^6)(0.4418)} = 187.33 \text{ in-lb}$$

$$(a) \text{ Total: } U = U_{cd} + U_{ef} = 264 \text{ in-lb}$$

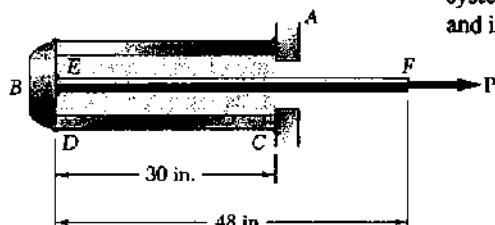
$$(b) CD: \sigma = -\frac{10000}{1.85} = -5405 \text{ psi}, \quad U = \frac{\sigma^2}{2E} = \frac{(-5405)^2}{(2)(10.6 \times 10^6)} = 1.378 \text{ in-lb/in}^3$$

$$EF: \sigma = \frac{10000}{0.4418} = 22635 \text{ psi}, \quad U = \frac{\sigma^2}{2E} = \frac{22635^2}{(2)(29 \times 10^6)} = 8.83 \text{ in-lb/in}^3$$

Problem 11.133

11.133 Solve Prob. 11.132, when $P = 8 \text{ kips}$.

11.132 A 30-in. length of aluminum pipe of cross-sectional area 1.85 in^2 is welded to a fixed support *A* and to a rigid cap *B*. The steel rod *EF*, of 0.75 in. diameter, is welded to cap *B*. Knowing that the modulus of elasticity is $29 \times 10^6 \text{ psi}$ for the steel and $10.6 \times 10^6 \text{ psi}$ for the aluminum, determine (a) the total strain energy of the system when $P = 10 \text{ kips}$, (b) the corresponding strain-energy density of the pipe *CD* and in the rod *EF*.



$$\text{For } EF: A = \frac{\pi}{4} d^2 = 0.4418 \text{ in}^2$$

$$CD: U_{cd} = \frac{P^2 L}{2EA} = \frac{(-8000)^2 (30)}{(2)(10.6 \times 10^6)(1.85)} = 48.95 \text{ in-lb}$$

$$EF: U_{ef} = \frac{P^2 L}{2EA} = \frac{(8000)^2 (48)}{(2)(29 \times 10^6)(0.4418)} = 119.89 \text{ in-lb}$$

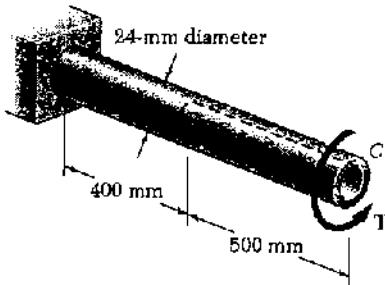
$$(a) \text{ Total: } U = U_{cd} + U_{ef} = 168.8 \text{ in-lb}$$

$$(b) CD: \sigma = -\frac{8000}{1.85} = -4324 \text{ psi}, \quad U = \frac{\sigma^2}{2E} = \frac{(-4324)^2}{(2)(10.6 \times 10^6)} = 0.882 \text{ in-lb/in}^3$$

$$EF: \sigma = \frac{8000}{0.4418} = 18108 \text{ psi}, \quad U = \frac{\sigma^2}{2E} = \frac{(18108)^2}{(2)(29 \times 10^6)} = 5.65 \text{ in-lb/in}^3$$

Problem 11.134

11.134 Rod *AC* is made of aluminum and is subjected to a torque *T* applied at *C*. Knowing that $G = 73 \text{ GPa}$ and that portion *BC* of the rod is hollow and has an inner diameter of 16 mm, determine the strain energy of the rod for a maximum shearing stress of 120 MPa.



$$C_o = \frac{d_o}{2} = 12 \text{ mm}, \quad C_i = \frac{d_i}{2} = 8 \text{ mm}$$

$$J_{AB} = \frac{\pi}{2} C_o^4 = \frac{\pi}{2} (12)^4 = 32.572 \times 10^9 \text{ mm}^4 = 32.572 \times 10^{-9} \text{ m}^4$$

$$\begin{aligned} J_{BC} &= \frac{\pi}{2} (C_o^4 - C_i^4) = \frac{\pi}{2} (12^4 - 8^4) = 26.138 \times 10^8 \text{ mm}^4 \\ &= 26.138 \times 10^{-9} \text{ m}^4 \end{aligned}$$

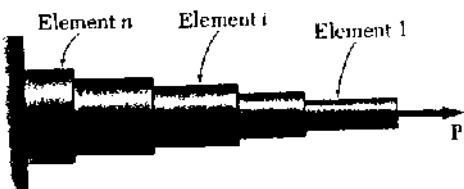
$$\tau_{\text{all}} = \frac{Tc}{J_{\text{min}}} \quad T = \frac{J_{\text{min}} \tau_{\text{all}}}{c} = \frac{(26.138 \times 10^{-9})(120 \times 10^6)}{12 \times 10^{-3}} = 261.38 \text{ N}\cdot\text{m}$$

$$U_{AB} = \frac{T^2 L_{AB}}{2G J_{AB}} = \frac{(261.38)^2 (400 \times 10^{-3})}{(2)(73 \times 10^9)(32.572 \times 10^{-9})} = 5.747 \text{ J}$$

$$U_{BC} = \frac{T^2 L_{BC}}{2G J_{BC}} = \frac{(261.38)^2 (500 \times 10^{-3})}{(2)(73 \times 10^9)(26.138 \times 10^{-9})} = 8.951 \text{ J}$$

$$\text{Total } U = U_{AB} + U_{BC} = 14.70 \text{ J}$$

PROBLEM 11.C1



11.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to a load P applied at its free end. The length of element i is denoted by L_i and its diameter by d_i . (a) Denoting by E the modulus of elasticity of the material used in the rod, write a computer program that can be used to determine the strain energy acquired by the rod and the deformation measured at the free end. (b) Use this program to determine the strain energy and deformation of the rods of Probs. 11.9 and 11.10.

SOLUTION ENTER: P AND E

FOR EACH ELEMENT

ENTER A_i AND δ_L

$$\text{COMPUTE: NORMAL STRESS: } \sigma_i = \frac{P}{A_i}$$

$$\text{STRAIN ENERGY: } U_i = \frac{\sigma_i^2 L_i}{2 E}$$

$$\text{STRAIN ENERGY DENSITY: } u = \frac{\sigma_i^2}{2 E}$$

TOTAL STRAIN ENERGY
UPDATE THROUGH n ELEMENTS

$$U = U_1 + U_2 + \dots$$

TOTAL DEFORMATION

$$\frac{1}{2} P \Delta = U : \Delta = \frac{2U}{P}$$

PROGRAM OUTPUT

Problem 11.9

Axial load = 8.000 kips Modulus of elasticity = 29×10^6 psi

Element	Length in.	delta L in.	Stress ksi	Strain Energy in.lb	Strain Energy Density lb.in./in. ³
1	24.000	0.022	26.08	86.32	11.72
2	36.000	0.022	18.11	89.92	5.65

Total Strain Energy = 176.24 in.lb

Total Deformation = 0.0441 in.

Problem 11.10

Axial load = 25.000 kN Modulus of elasticity = 200 GPa

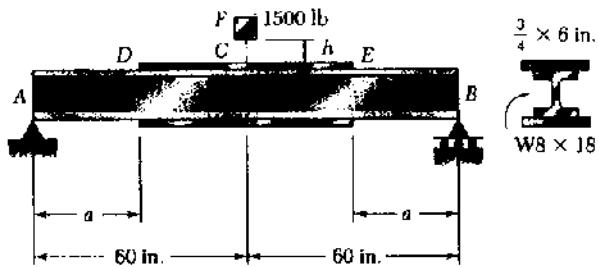
Element	Length m	delta L mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m ³
1	0.80	0.497	124.34	6.22	38.65
2	1.20	0.477	79.58	5.97	15.83

Total Strain Energy = 12.1853 J

Total Deformation = 0.9748 mm

PROBLEM 11.C2

11.C2 Two 0.75 × 6-in. cover plates are welded to a W8 × 18 rolled-steel beam as shown. The 1500-lb block is to be dropped from a height $h = 2$ in. onto the beam. (a) Write a computer program to calculate the maximum normal stress on transverse sections just to the left of D and at the center of the beam for values of a from 0 to 60 in., using 5-in. increments. (b) From the values considered in part a, select the distance a for which the maximum normal stress is as small as possible. Use $E = 29 \times 10^6$ psi.



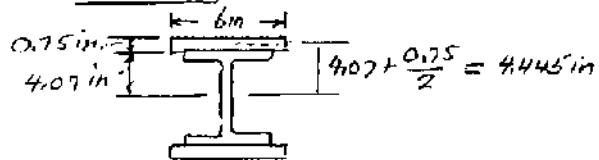
SOLUTION

COMPUTE AND ENTER MOMENTS OF INERTIA AND SECTION MODULI

FOR AD AND EB: W8X18

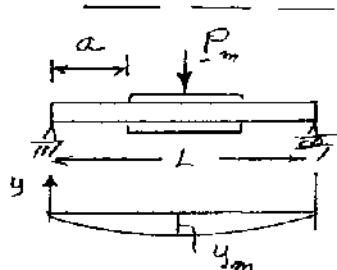
$$I_1 = 61.9 \text{ in}^4 \quad S_1 = 15.2 \text{ in}^3$$

FOR DCE: W8X18 PLUS COVER PLATES



$$I_2 = 61.9 + 2(6 \times 0.75)(4.445)^2 = 239.72 \text{ in}^4$$

$$S_2 = \frac{I_2}{(4.07 + 0.75)} = \frac{239.72}{4.82} = 49.7 \text{ in}^3$$



$$y_m = P_m x$$

WHERE $\alpha = \text{INFLUENCE COEFFICIENT}$
SEE NEXT PAGE FOR DETERMINATION OF α

$P_m = \text{EQUIVALENT STATIC LOAD}$

$$V_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{P_m^2}{\alpha}$$

WORK DONE BY W IS $W(h + y_m)$

$$\frac{1}{2} \frac{P_m^2}{\alpha} = Wh + W y_m$$

$$\text{OR: } y_m^2 - 2W\alpha y_m - 2Wh\alpha = 0 \quad (A)$$

POSITION 1

POSITION 2

PROGRAM SOLUTION OF (A) FOR y_m

ENTER $L = 10 \text{ in.}$, $h = 2 \text{ in.}$, $W = 1500 \text{ lb}$, $E = 29 \times 10^6 \text{ psi}$

FOR $\alpha = 0$ TO 60 in. , STEP 5 in. :

SOLVE (A) FOR y_m , $P_m = y_m / \alpha$, $y_{st} = W\alpha$

$$\bar{T}_D = \bar{T}_1 = \frac{1}{2} P_m a / S_1, \quad ; \quad \bar{T}_C = \bar{T}_2 = \frac{1}{4} P_m L / S_2$$

PRINT: α , y_{st} , y_m , P_m , \bar{T}_1 , \bar{T}_2 , AND $(\bar{T}_1 - \bar{T}_2)$

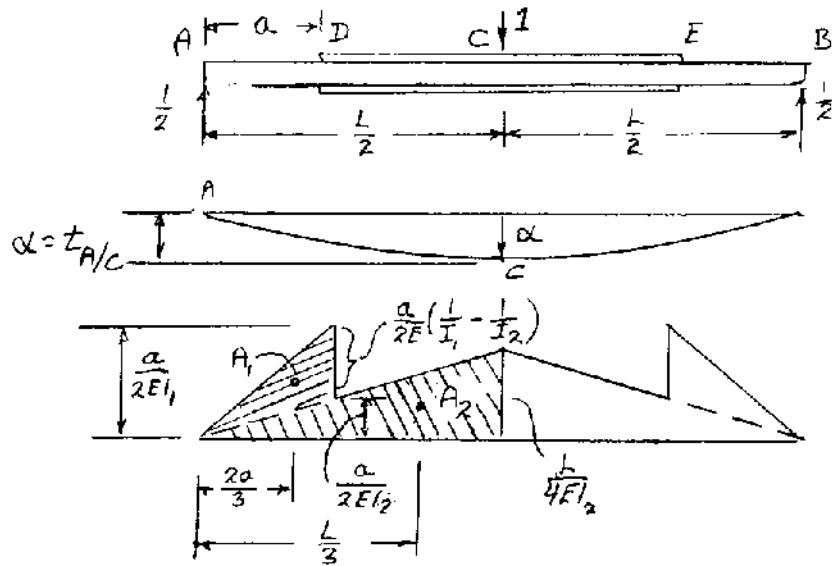
REPEAT WITH SMALLER INTERVALS TO FIND α FOR $(\bar{T}_1 - \bar{T}_2) = 0$

THIS IS THE DISTANCE α FOR T_{max} AS SMALL AS POSSIBLE

CONTINUED

PROBLEM 11.C2 - CONTINUED

DETERMINATION OF α : α IS DEFLECTION AT C FOR A UNIT LOAD AT C.



$$\alpha = t_{A/C} = A_1 \left(\frac{2a}{3} \right) + A_2 \left(\frac{L}{3} \right) = \left[\frac{1}{2} \frac{a}{E I_1} \left(\frac{1}{I_1} - \frac{1}{I_2} \right) \frac{a}{2} \right] \frac{2a}{3} + \left[\frac{1}{2} \cdot \frac{L}{4 E I_2} \cdot \frac{L}{2} \right] \frac{L}{3}$$

$$\alpha = \left[\left(\frac{1}{I_1} - \frac{1}{I_2} \right) \alpha^3 + \frac{1}{8 I_2} L^3 \right] / 6E$$

△

PROGRAM OUTPUT

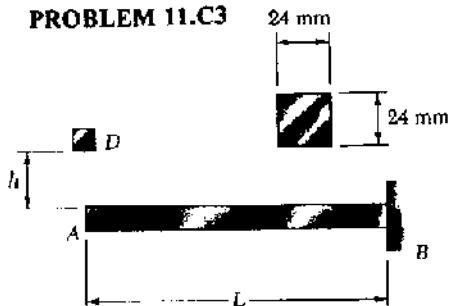
Beam = W 8x18 with two 6 by 0.75-in. cover plates
 $h = 2$ in. $W = 1500$ lb $L = 120$ in.

a in.	y _{stat} in.	y _{max} in.	P _{max} lb	σ_1 ksi	σ_2 ksi	$\sigma_1 - \sigma_2$ ksi
0.00	0.00777	0.1842	35572	0.00	21.46	-21.46
5.00	0.00778	0.1844	35544	5.85	21.44	-15.59
10.00	0.00787	0.1855	35348	11.63	21.32	-9.69
15.00	0.00812	0.1885	34834	17.19	21.01	-3.82
20.00	0.00859	0.1942	33896	22.30	20.45	1.85
25.00	0.00938	0.2033	32509	26.73	19.61	7.13
30.00	0.01056	0.2163	30736	30.33	18.54	11.79
35.00	0.01220	0.2334	28706	33.05	17.32	15.73
40.00	0.01438	0.2546	26563	34.95	16.02	18.93
45.00	0.01718	0.2799	24436	36.17	14.74	21.43
50.00	0.02068	0.3090	22415	36.87	13.52	23.35
55.00	0.02496	0.3419	20550	37.18	12.40	24.78
60.00	0.03008	0.3783	18862	37.23	11.38	25.85

Use smaller increments to seek the smallest maximum normal stress

18.33	0.00840	0.1919	34259	20.657	20.665	-0.01
18.34	0.00840	0.1920	34257	20.667	20.664	0.00
18.35	0.00841	0.1920	34255	20.677	20.663	0.01

Max stress small as possible for $a = 18.34$ in.
Smallest max stress = 20.67 ksi

PROBLEM 11.C3


11.C3 The 16-kg block *D* is dropped from a height *h* onto the free end of the steel bar *AB*. For the steel used $\sigma_{all} = 120 \text{ MPa}$ and $E = 200 \text{ GPa}$. (a) Write a computer program to calculate the maximum allowable height *h* for values of the length *L* from 100 mm to 1.2 m, using 100-mm increments. (b) From the values considered in part *a*, select the length corresponding to the largest allowable height.

SOLUTION

$$\underline{\text{ENTER}} \quad \sigma_{all} = 120 \text{ MPa}, E = 200 \text{ GPa}, d = 0.024 \text{ m}$$

$$m = 16 \text{ kg}; g = 9.81 \text{ m/s}^2$$

$$I = d^4/16 \quad S = \frac{I}{z} = \frac{I}{d/2} = \frac{d^3}{12}$$

For $L = 100 \text{ mm}$ to 1200 mm STEP 100 mm

$$L = L/1000$$

$$y_{st} = mg L^3 / 3EI$$

$$M_{max} = \sigma_{all} S$$

$$P_{max} = M_{max}/L$$

$$y_{max} = P_{max} L^3 / 3EI$$

From Prob. 11.B9, page 325

$$y_{max} = y_{st} \left[1 + \sqrt{1 + \frac{2h}{y_{st}}} \right] \quad \underline{\text{SOLVE FOR}} \quad h = \left[\left(\frac{y_{max}}{y_{st}} - 1 \right)^2 - 1 \right] \frac{y_{st}}{2}$$

Print: L , y_{st} , y_{max} , P_{max} , M_{max} , h

RETURN

PROGRAM OUTPUT
Problem 11.C3

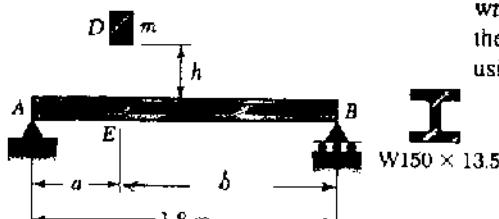
$m = 16.0 \text{ kg}$ $d = 24 \text{ mm}$ $\sigma = 120 \text{ MPa}$ $G = 200 \text{ GPa}$

L mm	y_{stat} mm	y_{max} mm	P_{max} N	M_{max} $N \cdot m$	h mm
100	0.00946	0.167	2764.8	276.48	1.301
200	0.07569	0.667	1382.4	276.48	2.269
300	0.25547	1.500	921.6	276.48	2.904
400	0.60556	2.667	691.2	276.48	3.205
500	1.18273	4.167	553.0	276.48	3.173
600	2.04375	6.000	460.8	276.48	2.807
700	3.24540	8.167	395.0	276.48	2.109
800	4.84445	10.667	345.6	276.48	1.076
900	6.89766	13.500	307.2	276.48	-0.289
1000	9.46181	16.667	276.5	276.48	-1.988
1100	12.59367	20.167	251.3	276.48	-4.020
1200	16.35000	24.000	230.4	276.48	-6.385

Use smaller increments to seek the largest height *h*

435	0.77883	3.154	635.6	276.48	3.2316
440	0.80599	3.227	628.4	276.48	3.2320
445	0.83378	3.300	621.3	276.48	3.2317

PROBLEM 11.C4



11.C4 The block D of mass $m = 8 \text{ kg}$ is dropped from a height $h = 750 \text{ mm}$ onto the rolled-steel beam AB . Knowing that $E = 200 \text{ GPa}$, write a computer program to calculate the maximum deflection of point E and the maximum normal stress in the beam for values of a from 100 to 900 mm, using 100-mm increments.

SOLUTION

$$\underline{\text{ENTER}}: L = 1.8 \text{ m}, E = 200 \text{ GPa}, h = 0.75 \text{ m}$$

$$m = 8 \text{ kg}, g = 9.81 \text{ m/s}^2$$

$$I = 6.87 \times 10^{-6} \text{ m}^4$$

$$S = 91.6 \times 10^{-6} \text{ m}^3$$

FOR $a = 100 \text{ mm}$ TO 900 mm STEP 100 mm

$$a = a/1000$$

$$b = L - a$$

SEE PROB. 11.71, page 705 →

$$y_{st} = mg a^2 b^2 / 3EI L$$

INFLUENCE COEFFICIENT FOR Δ_E →
FOR UNIT LOAD AT E

$$\alpha = a^2 b^2 / 3EI L$$

SEE PROB 11.69, page 705 →

$$y_m = y_{st} \left[1 + \sqrt{1 + \frac{2h}{y_{st}}} \right]$$

$$P_{max} = y_m / \alpha$$

$$M_{max} = P_{max} a b / L$$

$$\tau_{max} = M_{max} / S$$

PRINT: $\alpha, y_{st}, y_m, P_{max}, \tau_{max}$

RETURN

Problem 11.C4

Beam: W 150 x 13.5

$$I = 6.87 \times 10^{-6} \text{ m}^4 \quad S = 91.6 \times 10^{-6} \text{ m}^3$$

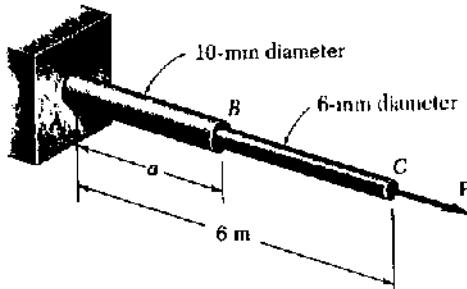
$$L = 1.8 \text{ m} \quad h = 750 \text{ mm} \quad m = 8 \text{ kg} \quad g = 9.81 \text{ m/s}^2$$

a mm	y _{stat} mm	y _{max} mm	P _{max} N	σ_{max} MPa
100	0.0003	0.6775	173.93	179.33
200	0.0011	1.2757	92.43	179.40
300	0.0021	1.7946	65.75	179.46
400	0.0033	2.2339	52.85	179.51
500	0.0045	2.5936	45.55	179.55
600	0.0055	2.8734	41.13	179.59
700	0.0063	3.0734	38.46	179.61
800	0.0068	3.1934	37.02	179.63
900	0.0069	3.2334	36.56	179.63

NOTE: THE SMALL VARIATION IN τ_{max} . THIS IS DUE TO THE ENERGY ACQUIRED BY THE MASS AS IT FALLS THROUGH y_{max} .

SEE PROB. 11.147, page 731, FOR A CASE WHERE ENERGY DELIVERED IS CONSTANT AND τ_{max} IS ALSO CONSTANT.

PROBLEM 11.C5



11.C5 The steel rods AB and BC are made of a steel for which $\sigma_y = 300 \text{ MPa}$ and $E = 200 \text{ GPa}$. (a) Write a computer program to calculate, for values of a from 0 to 6 m, using 1-m increments, the maximum strain energy that can be acquired by the assembly without causing any permanent deformation. (b) For each value of a considered, calculate the diameter of a uniform rod of length 6 m and of the same mass as the original assembly, and the maximum strain energy that could be acquired by this uniform rod without causing permanent deformation.

SOLUTION

$$\underline{\text{ENTER: } \sigma_y = 300 \text{ MPa}, E = 200 \text{ GPa}, L = 6 \text{ m}}$$

$$\text{AREA}_{AB} = \frac{\pi}{4}(0.010 \text{ m})^2, \text{ AREA}_{BC} = \frac{\pi}{4}(0.006 \text{ m})^2$$

$$P_m = \sigma_y \text{ AREA}_{BC}$$

FOR $a = 0 \text{ to } 6 \text{ m STEP } 1 \text{ m}$

$$U = \frac{P_m^2}{2E} \left(\frac{a}{\text{AREA}_{AB}} + \frac{L-a}{\text{AREA}_{BC}} \right)$$

FOR UNIFORM ROD OF SAME VOLUME

$$\text{VOL} = a(\text{AREA}_{AB}) + (L-a)(\text{AREA}_{BC})$$

$$d = \sqrt{\frac{4 \text{ VOL}}{\pi L}}$$

$$\text{AREA}_{\text{NEW}} = \frac{\pi}{4} d^2$$

$$P_{\text{NEW}} = \sigma_y (\text{AREA}_{\text{NEW}})$$

$$U_{\text{NEW}} = \frac{P_{\text{NEW}}^2 L}{2E(\text{AREA}_{\text{NEW}})}$$

PRINT $a, U, \text{VOL}, d, P_{\text{NEW}}, U_{\text{NEW}}$

RETURN

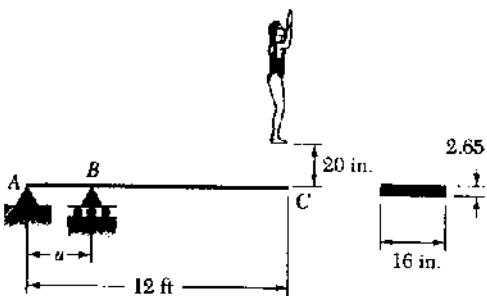
PROGRAM OUTPUT

Problem 11C5

$\sigma_y = 300 \text{ MPa}, P_m = 8482 \text{ N}, L = 6 \text{ m}, E = 200 \text{ GPa}$

a m	U J	Vol m^3	d mm	New P N	newU J
0.00	38.17	169.65	6.00	8482.30	38.17
1.00	34.10	219.91	6.83	10995.58	49.48
2.00	30.03	270.18	7.57	13508.85	60.79
3.00	25.96	320.44	8.25	16022.12	72.10
4.00	21.88	370.71	8.87	18535.40	83.41
5.00	17.81	420.97	9.45	21048.67	94.72
6.00	13.74	471.24	10.00	23561.95	106.03

PROBLEM 11.C6



11.C6 A 160-lb diver jumps from a height of 20 in. onto end C of a diving board having the uniform cross section shown. Write a computer program to calculate for values of a from 10 to 50 in., using 10-in. increments, (a) the maximum deflection of point C, (b) the maximum bending moment in the board, (c) the equivalent static load. Assume that the diver's legs remain rigid and use $E = 1.8 \times 10^6$ psi.

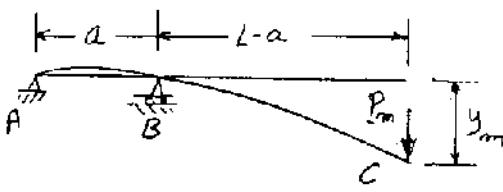
SOLUTION

$$\text{ENTER: } L = 12 \text{ ft}, h = 20 \text{ in.}, W = 160 \text{ lb}$$

$$E = 1.8 \times 10^6 \text{ psi}$$

$$I = (16 \text{ in.})(2.65 \text{ in.})^3 / 12$$

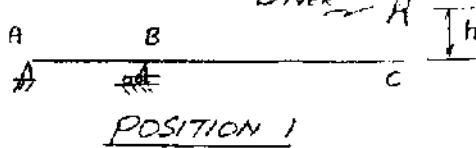
$$S = (16 \text{ in.})(2.65 \text{ in.})^2 / 6$$



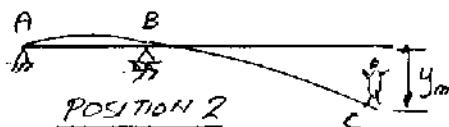
$$y_m = P_m \alpha \quad \text{WHERE } \alpha = \text{INFLUENCE COEFFICIENT}$$

SEE BELOW FOR DETERMINATION OF α

WHERE P_m = EQUIVALENT STATIC LOAD



POSITION 1



POSITION 2

$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

$$\text{WORK} = W(h + y_m)$$

$$\text{WORK} = U_2 - \frac{1}{2} \frac{y_m^2}{\alpha}$$

$$W(h + y_m) = \frac{1}{2} \frac{y_m^2}{\alpha}$$

PROGRAM SOLUTION OF α FOR y_m . ENTER α

FOR $\alpha = 10 \text{ in.}$ TO 50 in. STEP 10 in.

$$\text{SOLVE } \alpha \text{ FOR } y_m, \quad P_m = y_m / \alpha$$

$$M_{max} = M_B = P_m(L-a)$$

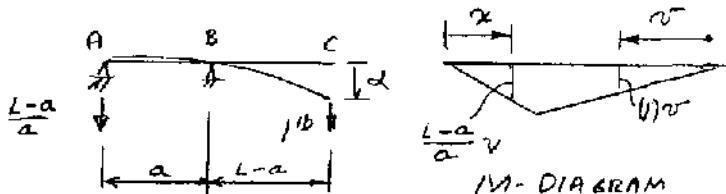
$$J = M_{max} / S$$

PRINT α, y_m, P_m, M_m, J

PROGRAM OUTPUT

α in.	y_m in.	P_m lb	Max M kip-in.	sigma psi
10	14.622	757.7	101.532	5422
20	13.262	802.6	99.519	5314
30	11.950	855.6	97.536	5208
40	10.683	919.1	95.583	5104
50	9.462	996.4	93.661	5001

DETERMINATION OF INFLUENCE COEFFICIENT α



$$U = \frac{1}{2} (1/b) \alpha = \sum \int \frac{M^2}{2EI} dx$$

$$\frac{\alpha}{2} = \frac{1}{2EI} \left[\int_0^{L-a} \left(\frac{L-a}{a} \right)^2 x^2 dx + \int_{L-a}^L v^2 dx \right]$$

$$\alpha = \frac{1}{EI} \left[\frac{(L-a)^2 a^3}{a^2} + \frac{(L-a)^3}{3} \right]$$

$$\alpha = \frac{1}{3EI} \left[(L-a)^2 a + (L-a)^3 \right]$$

△