Microphone Calibration

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1 Acoustic sensing basics

1.1 Octave bands:

Let n be the band number. Then the center of octave band n (in Hz) is,

$$f_c(N) = 2^{n-5} \times 10^3.$$

The band limits, in Hz, are given by

$$f_{\text{upper}}(n) = \sqrt{2} f_c(n)$$
$$f_{\text{lower}}(n) = \frac{1}{\sqrt{2}} f_c(n)$$

1.2 Equivalent sound pressure level:

This is a measurement of average sound pressure. Let $p(t)$ be the instantaneous sound pressure at time t and let T be the time duration of a measurement. Then,

$$L_{eq}(T) = 10 \log_{10} \left[\frac{1}{T} \int_0^T \left(\frac{p(t)}{p_0} \right)^2 dt \right]$$

where p_0 is the reference sound pressure (usually 20μPa). For discrete time measurements,

$$L_{eq}(N) = 10 \log_{10} \left[\frac{1}{N} \sum_{k=0}^{N-1} \left(\frac{p(k)}{p_0} \right)^2 \right]$$

where N is the number of measurements.
We can also compute the equivalent sound pressure level in an octave band using a band pass filter, H_n, which pass frequencies in octave band n. Then the equivalent sound pressure level in octave band n is

$$L_{eq}^n(N) = 10 \log_{10} \left[\frac{1}{N} \sum_{t=0}^{N-1} \left(\frac{H_n[p(t)]}{p_0} \right)^2 \right]$$

Here we assume that H_n has unit gain (0dB) at $f_c(n)$. I use 6^{th} order Butterworth filters for H_n.

To get the total sound pressure level across a set $\mathcal{N}$ of octave bands, the following calculation is performed:

$$L_{eq}(N) = 10\log_{10}\left(\sum_{n\in\mathcal{N}} 10^{\frac{L^n_{eq}(N)}{10}}\right)$$

On the other hand, the average L_{eq} of a set of sound pressure measurements taken over distinct time intervals, $\{L_{eq,i}\}_{i=0}^{M-1}$, is given by

$$L_{eq} = 10\log_{10}\left(\frac{1}{M}\sum_{i=0}^{M-1} 10^{\frac{L_{eq,i}}{10}}\right)$$

1.3 A-weighted L_{eq}:

The human ear is less sensitive to low frequencies; the perceived loudness of a low-frequency signal is less than the perceived loudness of a high-frequency signal of equal L_{eq}. The A-weighting of a sound pressure level accounts for this by weighting high-frequency components in a signal more than the low-frequency components. The A-weighted L_{eq} in octave band n is defined as

$$LA^n_{eq}(N) = L^n_{eq}(N) + A(f_c(n))$$

where

$$A(f) = 20\log_{10} R_A(f) + 2$$
$$R_A(f) = \frac{12194^2 f^4}{(f^2+20.6^2)\sqrt{(f^2+107.7^2)(f^2+737.9^2)}(f^2+12194^2)}$$

The units of LA_{eq} are dBA. The total A-weighted sound pressure level for a signal can be computed as

$$LA_{eq}(N) = 10\log_{10}\left(\sum_{n\in\mathcal{N}} 10^{\frac{LA^n_{eq}(N)}{10}}\right)$$

2 Calibration

For a sound pressure signal $p(k)$ in octave band n, we assume that the microphone measurement is given by

$$y(k) = K(n)p(k) + y_{DC}$$

Note that there is a frequency dependence on the sensor gain $K(n)$ which is consistent with the data sheet.

2.1 DC bias:

To compute the DC bias we take advantage of the zero-mean property of $p(k)$. Taking the average over many measurements,

$$\frac{1}{N}\sum_{k=0}^{N-1} y(k) = \frac{K(n)}{N}\sum_{k=0}^{N-1} p(k) + y_{DC}$$
$$y_{DC} = \frac{1}{N}\sum_{k=0}^{N-1} y(k)$$

2.2 Sensor gain:

Suppose that $p(k)$ is a calibration tone at frequency $f_c(n)$ at x dB. Then,

$$x = 10\log_{10}\left[\frac{1}{N}\sum_{k=0}^{N-1}\left(\frac{H_n(p(k))}{p_0}\right)^2\right]$$
$$x = 10\log_{10}\left[\frac{1}{N}\sum_{k=0}^{N-1}\left(\frac{H_n((y(k)-y_{DC})/(K(n))}{p_0}\right)^2\right]$$
$$x = 10\log_{10}\left[\frac{1}{(K(n)p_0)^2}\frac{1}{N}\sum_{k=0}^{N-1}\Big(H_n(y(k)-y_{DC})\Big)^2\right]$$
$$K_{cal}(n) := \frac{1}{(K(n)p_0)^2} = \frac{10^{x/10}}{\frac{1}{N}\sum_{k=0}^{N-1}\big(H_n(y(k)-y_{DC})\big)^2}$$

The calibration routine is then given by:

1. Generate a sound signal at $f_c(n)$ at known x dB,
2. For each new measurement:
    - Subtract the computed y_{DC} from the raw measurement
    - Update the filter states with input $y(k) - y_{DC}$
    - Square the filter output and add it to the previous squared output
3. After taking sufficient measurements, compute $K_{cal}(n)$

Note that we do not solve for $K(n)$ explicitly since $K_{cal}(n)$ is sufficient to compute the equivalent sound pressure level:

$$L_{eq}^n(N) = 10\log_{10}\left[\frac{K_{cal}(n)}{N}\sum_{k=0}^{N-1}\big(H_n(y(k)-y_{DC})\big)^2\right]$$

Note that if H_n does not have unit gain at $f_c(n)$ this is absorbed into $K_{cal}(n)$. Thus, we do not actually design the band pass filters to have unit gain at the octave frequency and rather compensate for this through calibration.