

3.2  $Y_t = \beta_0 + \beta_1 t + X_t$

$$\bar{Y} = \frac{1}{n} \sum Y_t = \frac{1}{n} \sum (\mu + e_t - e_{t-1}) = \mu + \frac{1}{n} \sum (e_t - e_{t-1}) = \mu + \frac{1}{n} (e_n - e_0)$$

$$\text{Var}(\bar{Y}) = \text{Var}\left(\mu + \frac{1}{n} (e_n - e_0)\right) = \frac{1}{n^2} \text{Var}(e_n - e_0) = \frac{1}{n^2} (\sigma_e^2 + \sigma_e^2) = \frac{2\sigma_e^2}{n^2}$$

IT'S UNUSUAL FOR THE SAMPLE SIZE TO HAVE A BIG IMPACT ON THE VARIANCE.

$$Y_t = \mu + e_t$$

$$\bar{Y} = \frac{1}{n} \sum Y_t = \frac{1}{n} \sum (\mu + e_t) = \mu + \frac{1}{n} \sum e_t = \mu + \bar{e}$$

$$\text{Var}(\bar{Y}) = \text{Var}(\mu + \bar{e}) = \text{Var}(\bar{e}) = \sigma_e^2/n$$

3.3  $Y_t = \mu + e_t + e_{t-1}$

$$\bar{Y} = \frac{1}{n} \sum Y_t = \frac{1}{n} \sum (\mu + e_t + e_{t-1}) = \mu + \frac{1}{n} \sum (e_t + e_{t-1})$$

$$= \mu + \frac{1}{n} (e_n + e_0 + 2 \sum_{t=1}^{n-1} e_t)$$

$$\text{Var}(\bar{Y}) = \text{Var}\left(\mu + \frac{1}{n} (e_n + e_0 + 2 \sum_{t=1}^{n-1} e_t)\right)$$

$$= \frac{1}{n^2} (\sigma_e^2 + \sigma_e^2 + 4(n-1)\sigma_e^2) = \frac{2(2n-1)\sigma_e^2}{n^2}$$

$$Y_t = \mu + e_t$$

FROM 3.2:  $\text{Var}(\bar{Y}) = \sigma_e^2/n$

AS  $n \rightarrow \infty$ , THE VARIANCE FROM THE ORIGINAL  $Y_t$  BECOMES 4 TIMES LARGER THAN THE SECOND  $Y_t$