Homework 3

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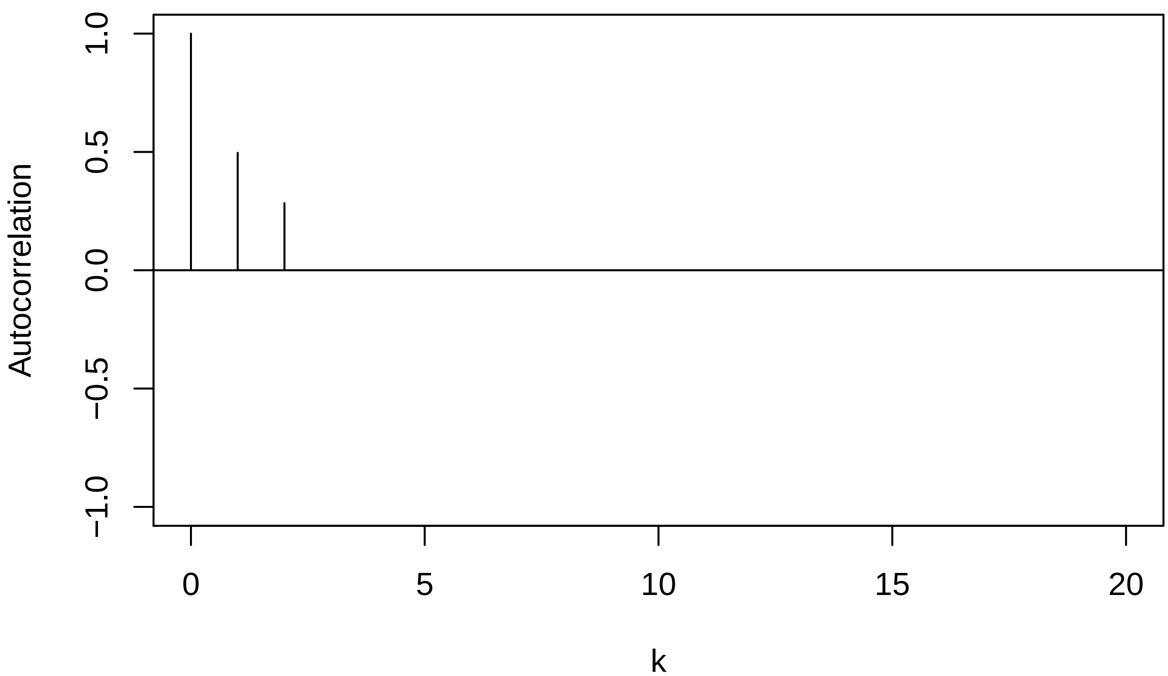
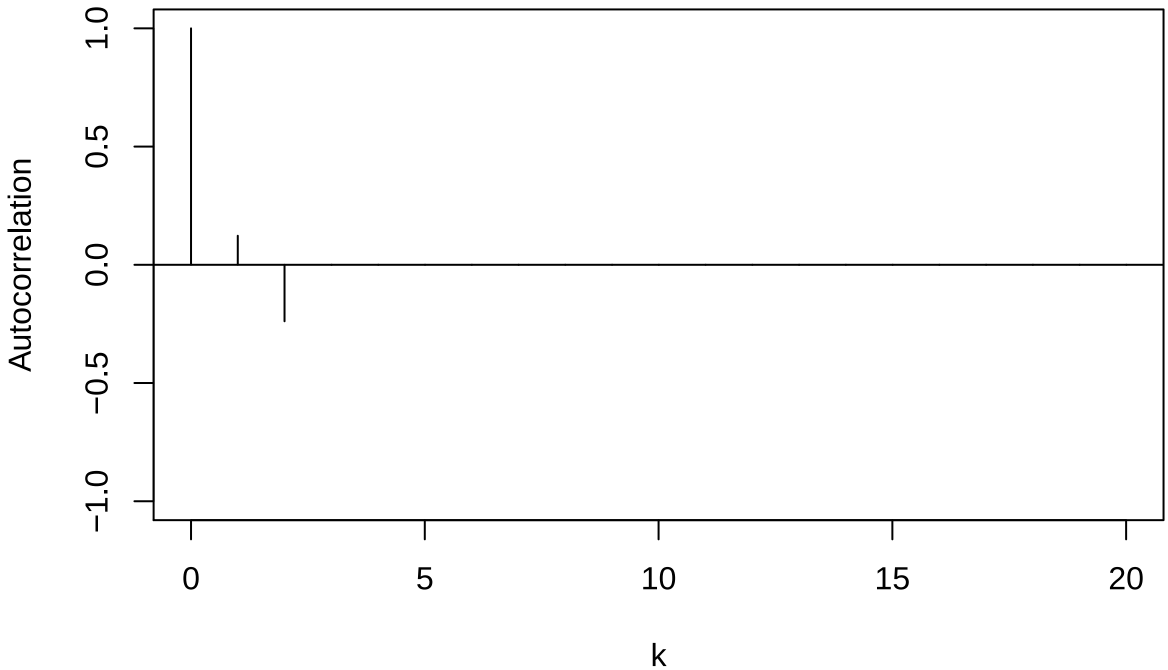
Problems

Written questions are attached at the end

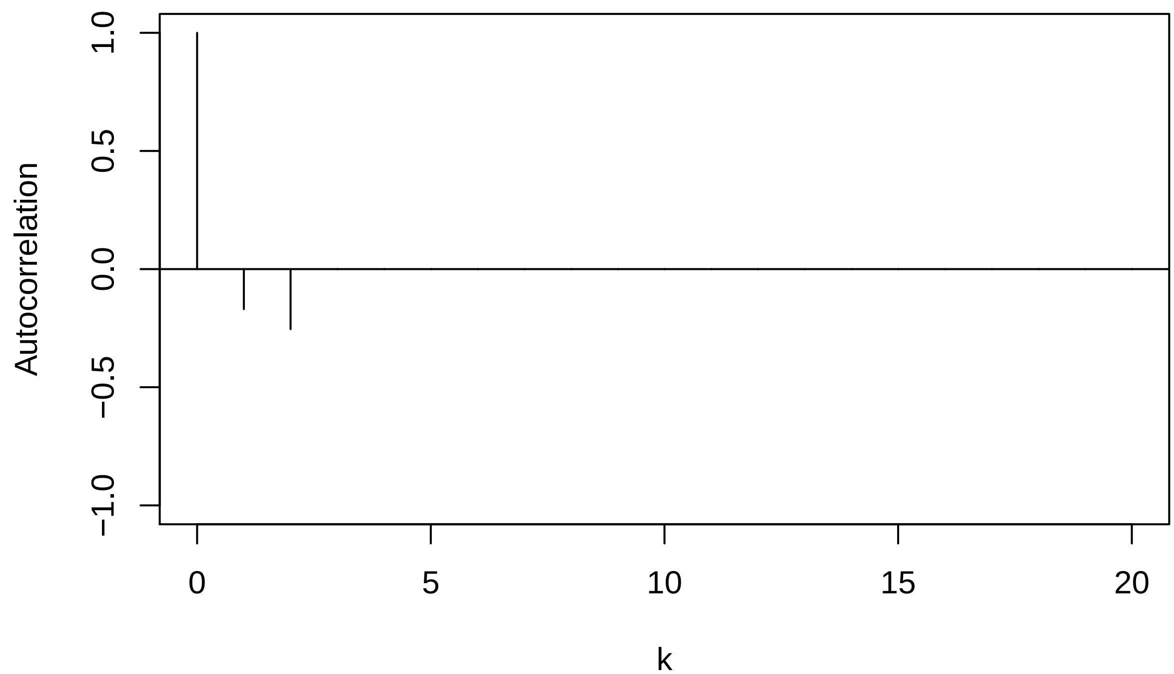
4.2



a)

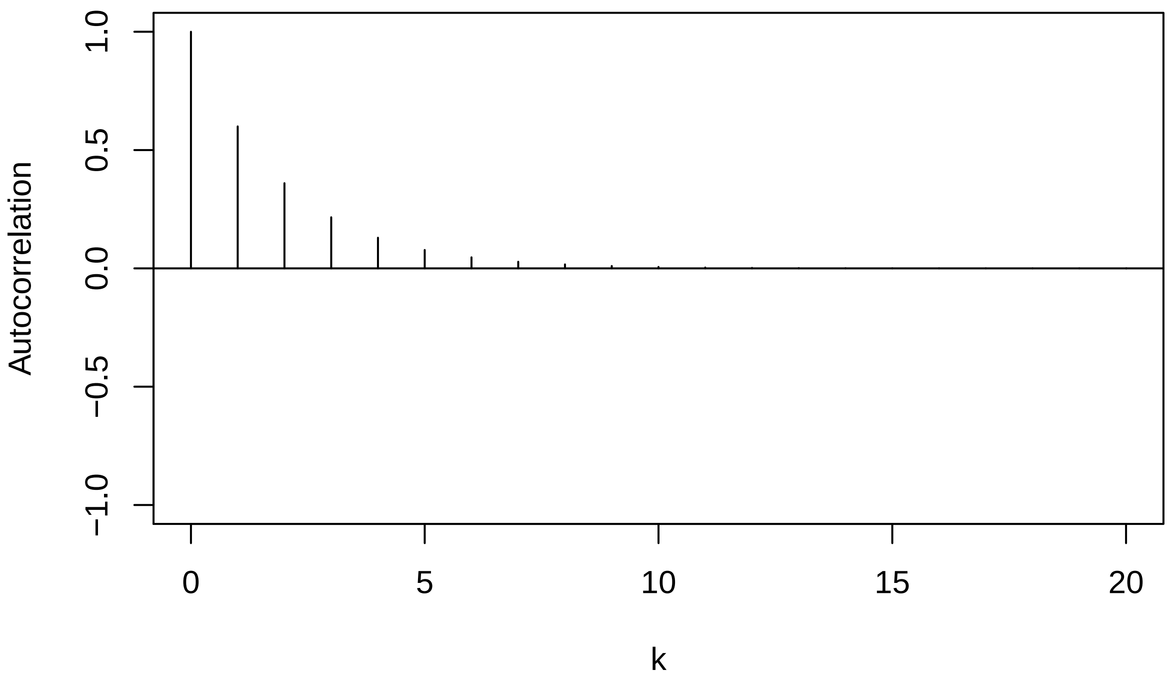


b)

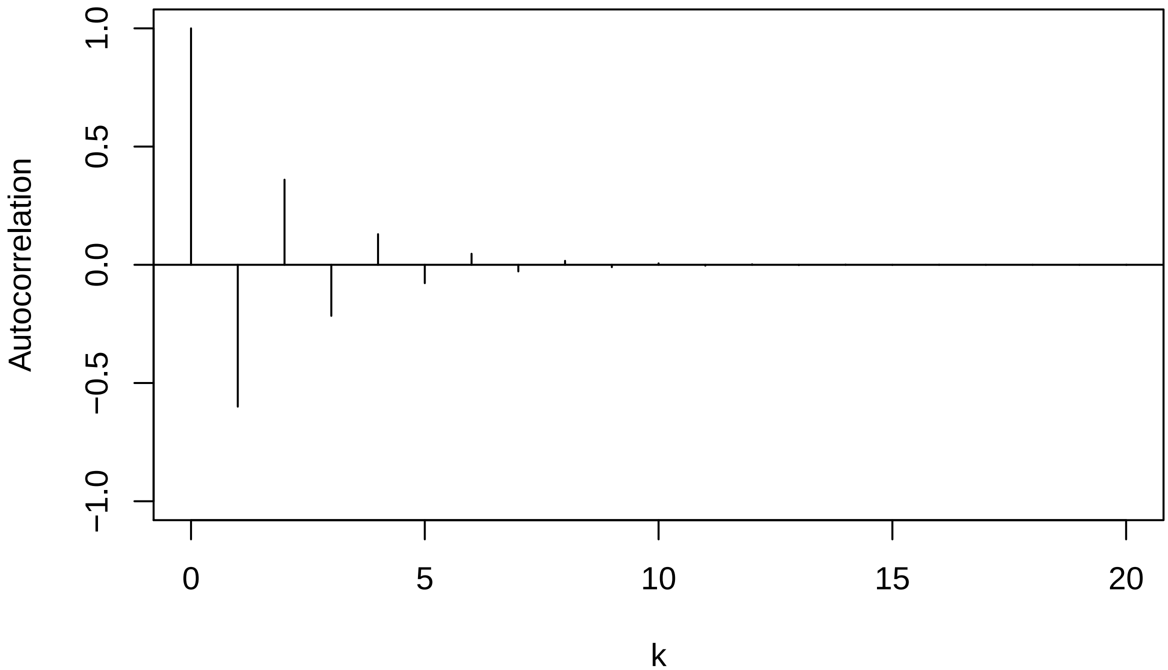


c)

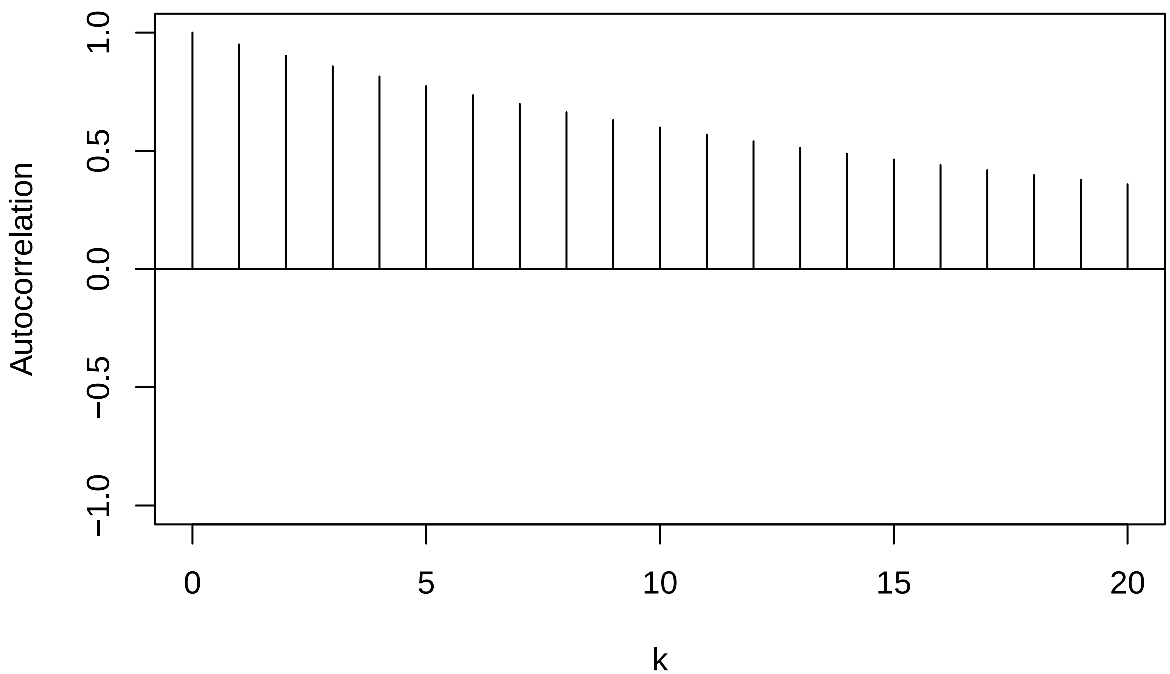
4.5



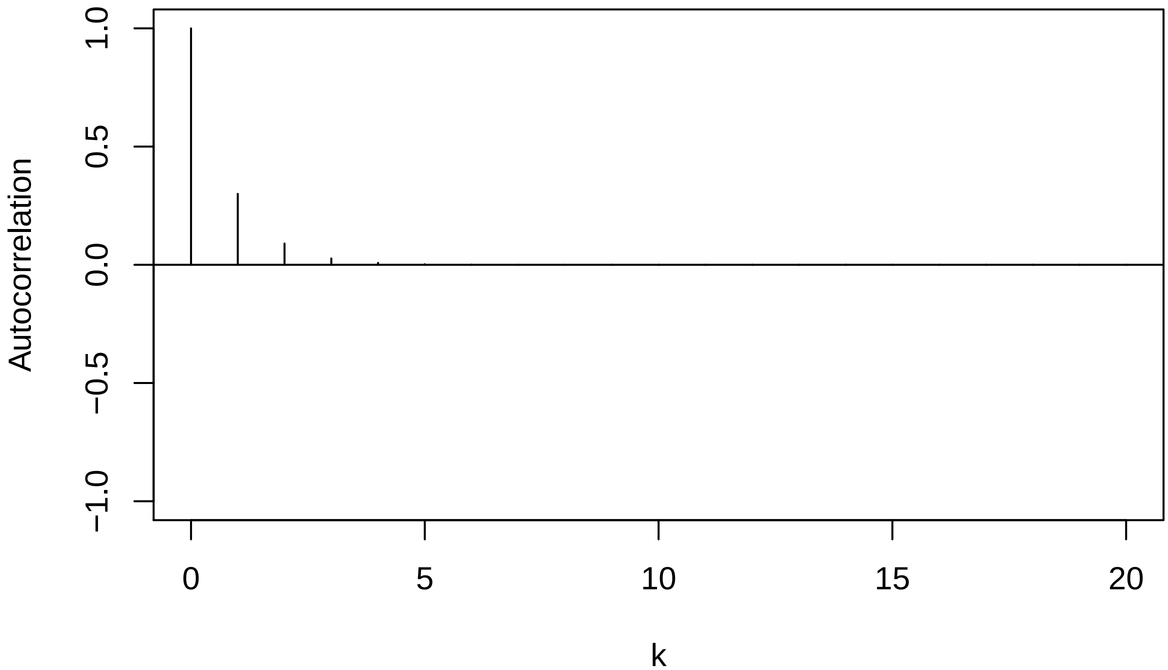
a)



b)



c)

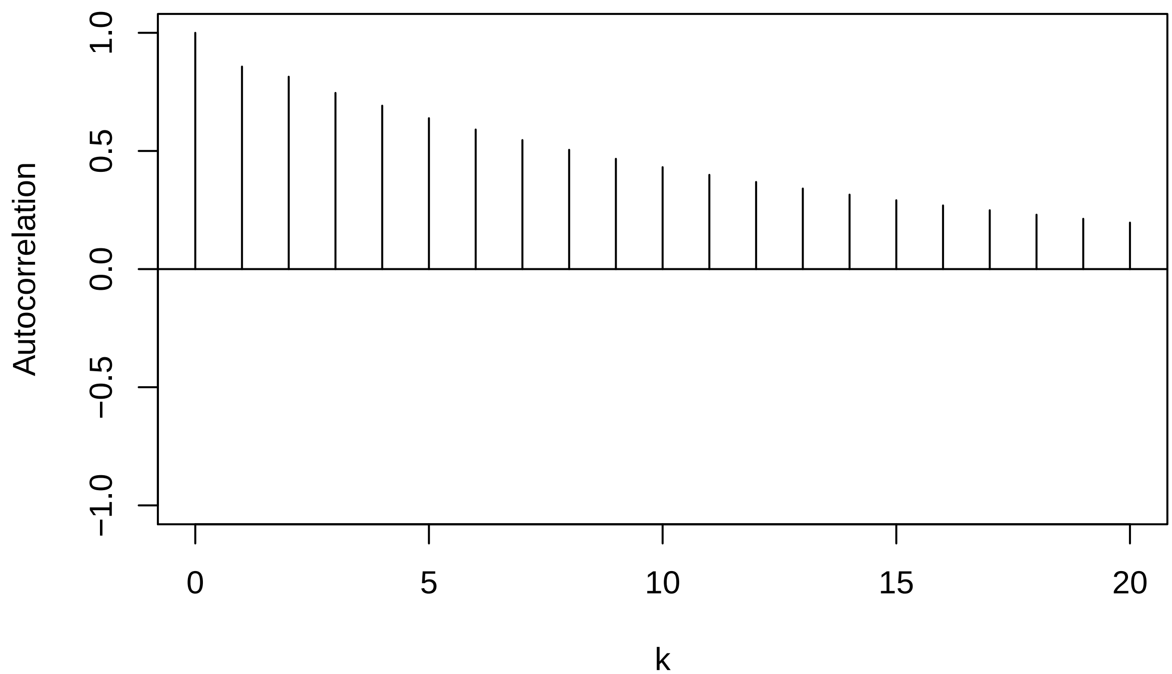


d)

4.9

a) $0.6 + 0.3 = 0.9 < 1$, $0.3 - 0.6 = -0.3 < 1$, $|0.3| < 1$

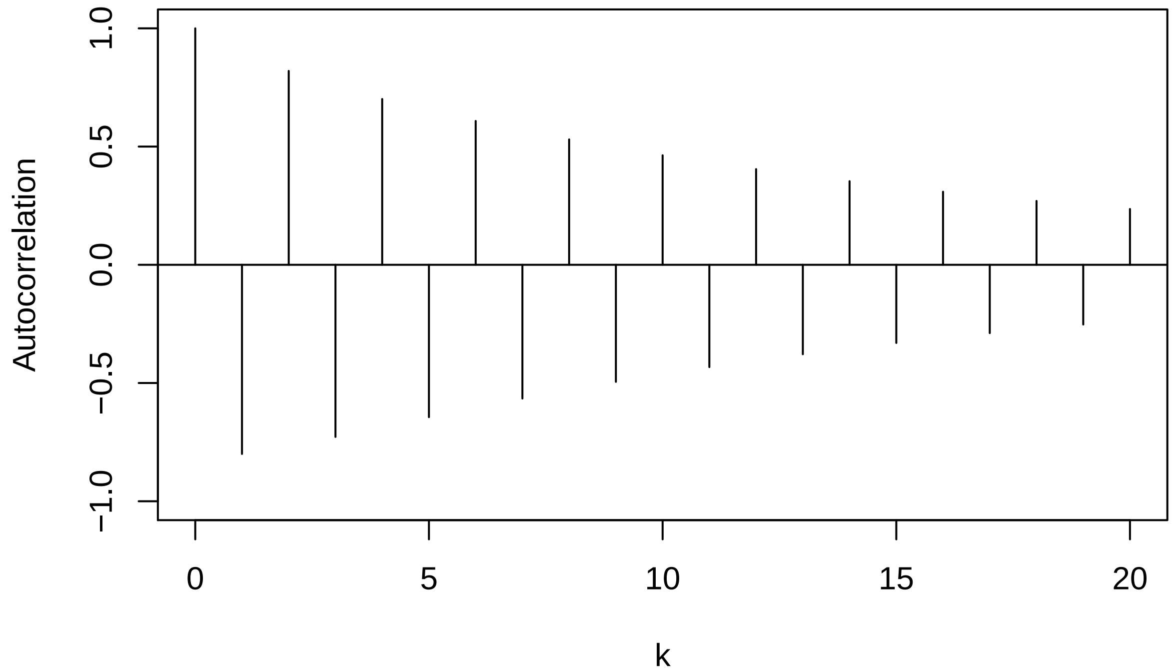
Therefore, this process is stationary and causal. The roots of the characteristic equation are real.




```
## [1]  1.081666-0i -3.081666+0i
```

b) -0.4 + 0.5 = 0.1 < 1, 0.5 - -0.4 = 0.9 < 1, |0.5| < 1

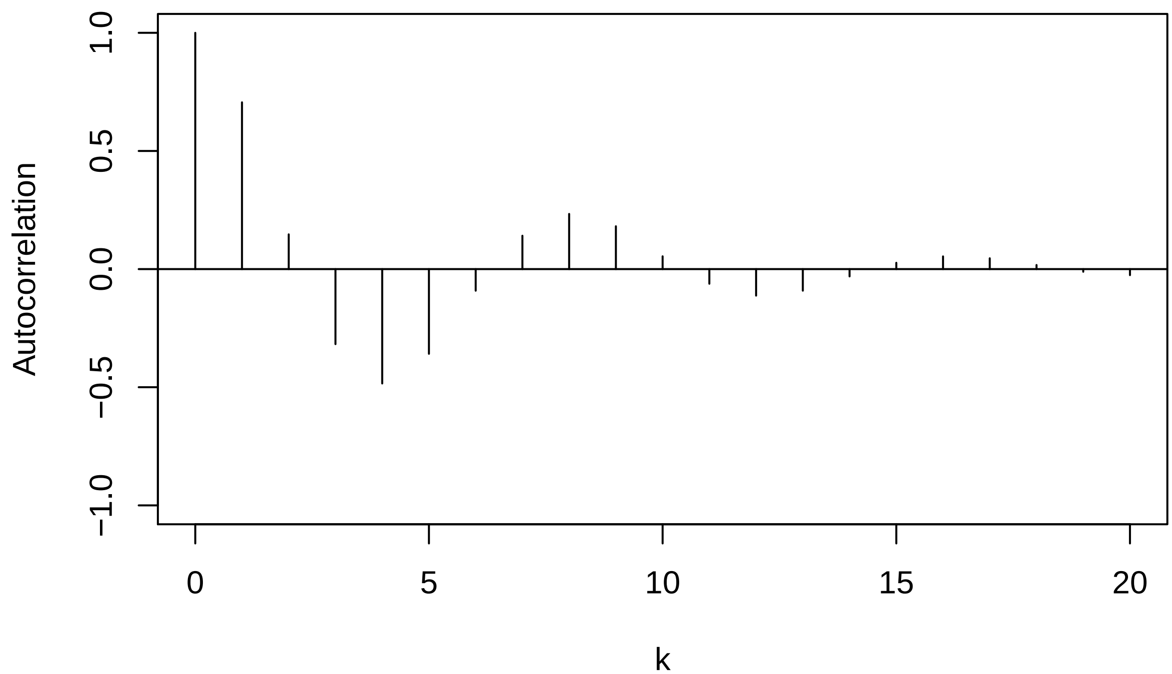
Therefore, this process is stationary and causal. The roots of the characteristic equation are real.




```
## [1] -1.069694+0i  1.869694-0i
```

c) $1.2 + \text{-}0.7 = 0.5 < 1$, $\text{-}0.7 - 1.2 = \text{-}1.9 < 1$, $|\text{-}0.7| < 1$

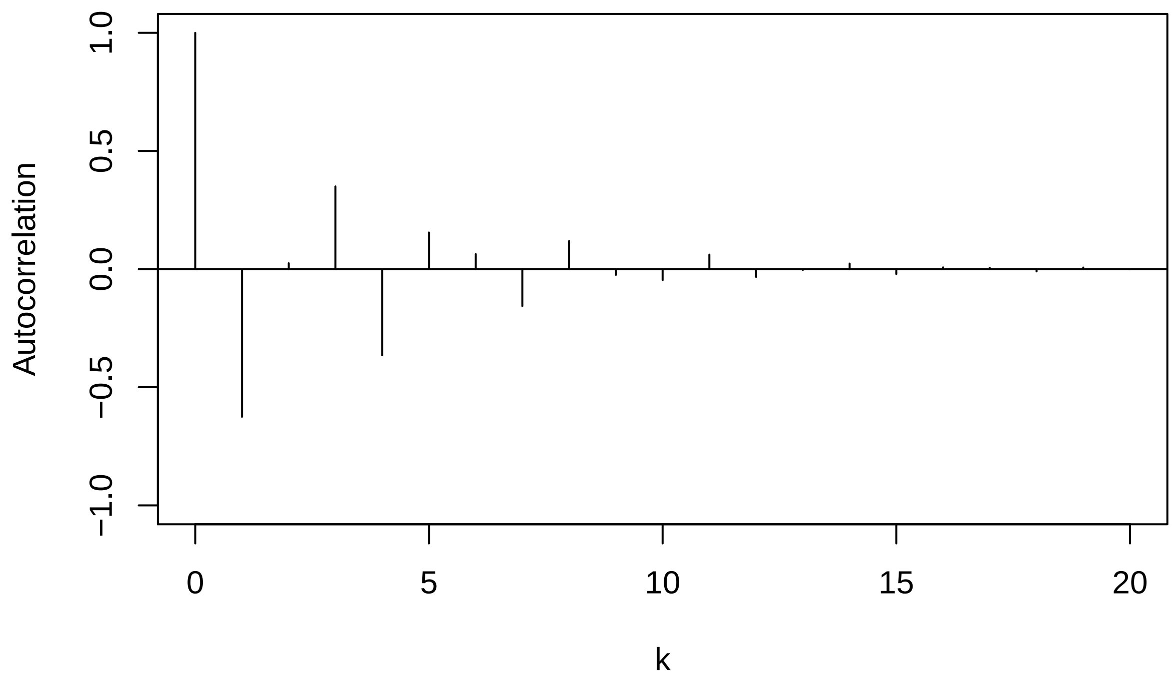
Therefore, this process is stationary and causal. The roots of the characteristic equation are complex.




```
## [1] 0.8571429+0.8329931i 0.8571429-0.8329931i
```

d) -1 + -0.6 = -1.6 < 1, -0.6 - -1 = 0.4 < 1, |-0.6| < 1

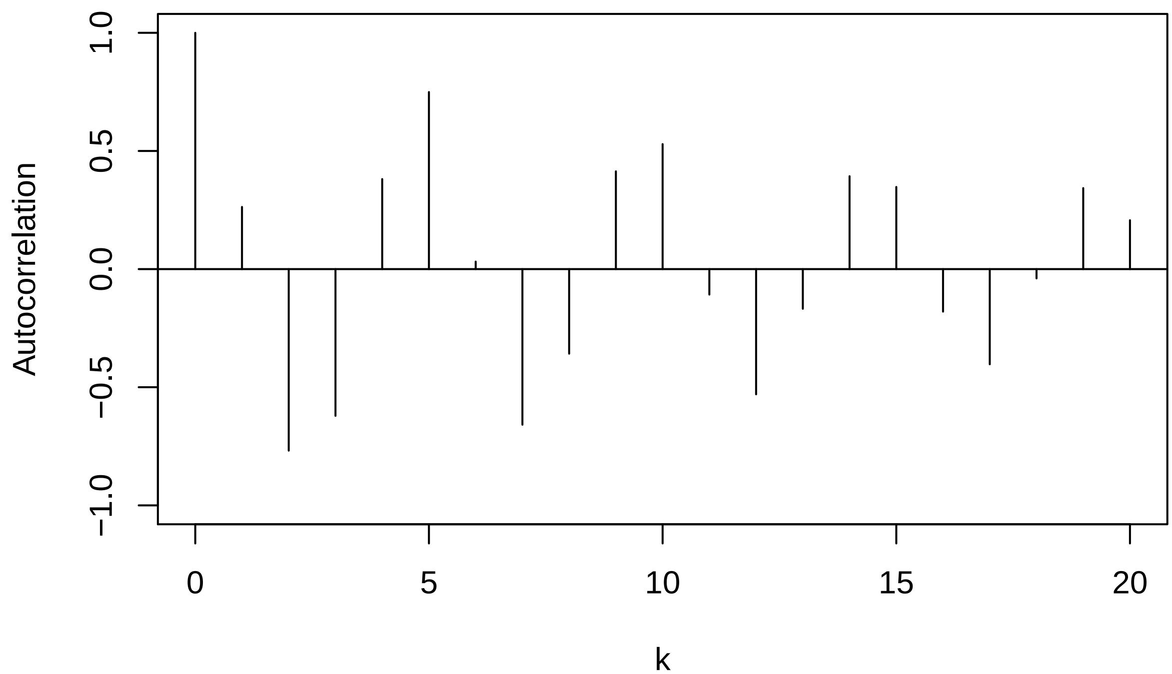
Therefore, this process is stationary and causal. The roots of the characteristic equation are complex.




```
## [1] -0.8333333+0.9860133i -0.8333333-0.9860133i
```

e) 0.5 + -0.9 = -0.4 < 1, -0.9 - 0.5 = -1.4 < 1, |-0.9| < 1

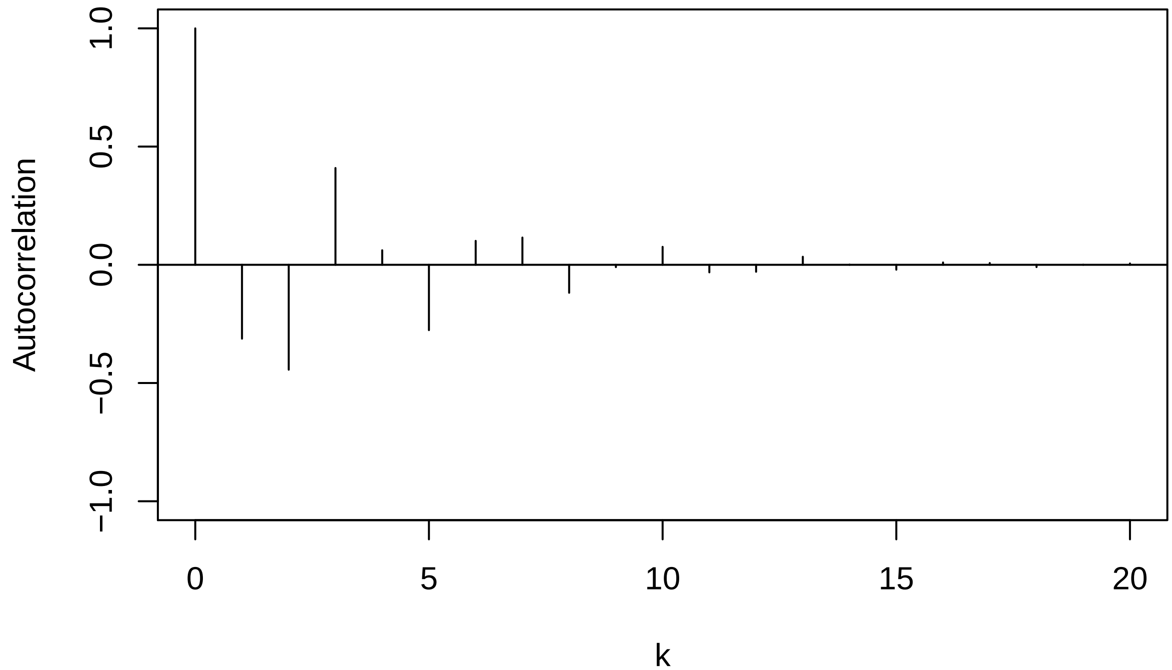
Therefore, this process is stationary and causal. The roots of the characteristic equation are complex.




```
## [1] 0.277778+1.016834i 0.277778-1.016834i
```

f) -0.5 + -0.6 = -1.1 < 1, -0.6 - -0.5 = -0.1 < 1, |-0.6| < 1

Therefore, this process is stationary and causal. The roots of the characteristic equation are complex.




```
## [1] -0.416667+1.221907i -0.416667-1.221907i
```

4.19

This is similar to an AR(1) with pk = -(-0.5)^k

```
ARMAacf(ar=-0.5, lag.max=7)
```

```
##          0          1          2          3          4          5          6
##  1.0000000 -0.5000000  0.2500000 -0.1250000  0.0625000 -0.0312500  0.0156250
##          7
## -0.0078125
```

```
ARMAacf(ma = -c(0.5, -0.25, 0.125, -0.0625, 0.03125, -0.0015625))
```

```
##            0            1            2            3            4            5
##  1.000000000 -0.499669415  0.249157053 -0.123223218  0.058900991 -0.024029260
##            6            7
##  0.001172159  0.000000000
```

4.20

This is similar to an ARMA(1,1) with phi = -0.5 and theta = 0.5

```
ARMAacf(ar = -0.5, ma = -0.5, lag.max = 8)
```

```
##            0            1            2            3            4            5
##  1.000000000 -0.714285714  0.357142857 -0.178571429  0.089285714 -0.044642857
##            6            7            8
```

```
## 0.022321429 -0.011160714 0.005580357
```

```r
ARMAacf(ma = -c(1, -0.5, 0.25, -0.125, 0.0625, -0.03125, 0.015625))
```

```
##            0            1            2            3            4            5
## 1.000000000 -0.714240871  0.357015800 -0.178298629  0.088730773 -0.043528304
##            6            7            8
## 0.020089986 -0.006696662  0.000000000
```

4.12 a) $\theta_1 = \theta_2 = 1/6$

$$\rho_k = \frac{-1/6 + 1/6 \cdot 1/6}{1+(1/6)^2+(1/6)^2} = \frac{1/6(1/6-1)}{1+2/36} = -5/38$$

$\theta_1 = -1 \quad \theta_2 = 6$

$$\rho_k = \frac{1-6}{1+1^2+36} = -\frac{5}{38}$$

b) $\theta_1 = \theta_2 = 1/6$

$$\frac{1/6 \pm \sqrt{1/36 + 4 \cdot 1/6}}{-2 \cdot 1/6} = -1/2 \pm \frac{\sqrt{25/36}}{-1/3} = 1/2 \pm \frac{5/6}{1/3} = \{-3, -2\}$$

$\theta_1 = -1 \quad \theta_2 = 6$

$$\frac{-1 \pm \sqrt{1+4 \cdot 6}}{-2 \cdot 1/6} = \frac{-1 \pm 5}{-12} = \frac{1}{12} \pm \frac{5}{12} = \{-1/3, 1/2\}$$

4.15
$$Var(Y_t) = Var(\phi Y_{t-1} + e_t) = \phi^2 Var(Y_{t-1}) + \sigma_e^2$$
$$= \phi^2 Var(\phi Y_{t-2} + e_t) + \sigma_e^2$$
$$= \phi^4 Var(Y_{t-2}) + 2\sigma_e^2$$
$$= \phi^{2n} Var(Y_{t-n}) + n\sigma_e^2$$

$\lim_{n \to \infty} Var(Y_t) \longrightarrow \infty$ IF $\phi = 1 \longrightarrow$ IMPOSSIBLE

4.16 a) $Y_t = \phi Y_{t-1} + e_t$

$$-\sum_{j=1}^{\infty} (1/3)^j e_{t+j} = 3\left(-\sum_{j=1}^{\infty} (1/3)^j e_{t-1+j}\right) + e_t$$

$$-\sum_{j=1}^{\infty} (1/3)^{j+1} e_{t+j} = -\sum_{j=1}^{\infty} (1/3)^j e_{t-1+j} + 1/3\, e_t$$

$$-\sum_{j=1}^{\infty} (1/3)^{j+1} e_{t+j} = -\sum_{j=2}^{\infty} (1/3)^j e_{t-1+j}$$

$$-\sum_{j=1}^{\infty} (1/3)^{j+1} e_{t+j} = -\sum_{j+1=2}^{\infty} (1/3)^{j+1} e_{t+j}$$

b) $E(Y_t) = E\left(\sum_{j=1}^{\infty} (1/3)^j e_{t+j}\right) = 0$ SINCE ALL TERMS ARE WHITE NOISE

$$Cov(Y_t, Y_{t-1}) = Cov\left(-\sum_{j=1}^{\infty}(1/3)^j e_{t+j}, \sum_{j=1}^{\infty}(1/3)^j e_{t+j-1}\right)$$

$$= Cov\left(-1/3\, e_{t+1} - (1/3)^2 e_{t+2} \dots - (1/3)^n e_{t+n-1}\right)$$

$$= Cov(-1/3\, e_{t+1}, -1/3^2 e_{t+1}) + Cov(-1/3\, e_{t+2}, -1/3^2 e_{t+2}) \dots$$

$$= 1/26\, \sigma_e^2 (1 + 1/3 + 1/3^2 + \dots + 1/3^n)$$

$\hookrightarrow$ FREE OF t $\therefore$ STATIONARY

c) IT IS UNSATISFACTORY BECAUSE Y_t DEPENDS ON FUTURE OBSERVATIONS

4.18 a) $E(W_t) = E(Y_t + c\phi^t) = E(Y_t) + E(c\phi^2) = 0 + c\phi^2 = c\phi^2$

b) $\phi(Y_{t-1} + c\phi^{t-1}) + e_t = \phi Y_{t-1} + c\phi^t + e_t$

$$= \phi\left(\frac{Y_t - e_t}{\phi}\right) + c\phi^t + e_t = Y_t + c\phi^t$$

c) NO $\rightarrow$ W_t IS NOT FREE OF t