4.12 a) $\theta_1 = \theta_2 = 1/6$

$$\rho_k = \frac{-1/6 + 1/6 \cdot 1/6}{1 + (1/6)^2 + (1/6)^2} = \frac{1/6(1/6 - 1)}{1 + 2/36} = -5/38$$

$\theta_1 = -1 \quad \theta_2 = 6$

$$\rho_k = \frac{1 - 6}{1 + 1^2 + 36} = -\frac{5}{38}$$

b) $\theta_1 = \theta_2 = 1/6$

$$\frac{1/6 \pm \sqrt{1/36 + 4 \cdot 1/6}}{-2 \cdot 1/6} = -1/2 \pm \frac{\sqrt{25/36}}{-1/3} = 1/2 \pm \frac{5/6}{1/3} = \{-3, -2\}$$

$\theta_1 = -1 \quad \theta_2 = 6$

$$\frac{-1 \pm \sqrt{1 + 4 \cdot 6}}{-2 \cdot 1/6} = \frac{-1 \pm 5}{-12} = \frac{1}{12} \pm \frac{5}{12} = \{-1/3, 1/2\}$$

4.15 $$Var(Y_t) = Var(\phi Y_{t-1} + e_t) = \phi^2 Var(Y_{t-1}) + \sigma_e^2$$
$$= \phi^2 Var(\phi Y_{t-2} + e_t) + \sigma_e^2$$
$$= \phi^4 Var(Y_{t-2}) + 2\sigma_e^2$$
$$= \phi^{2n} Var(Y_{t-n}) + n\sigma_e^2$$

$\lim_{n \to \infty} Var(Y_t) \to \infty$ IF $\phi = 1 \longrightarrow$ IMPOSSIBLE

4.16 a) $Y_t = \phi Y_{t-1} + e_t$

$$-\sum_{j=1}^{\infty} (1/3)^j e_{t+j} = 3\left(-\sum_{j=1}^{\infty} (1/3)^j e_{t-1+j}\right) + e_t$$

$$-\sum_{j=1}^{\infty} (1/3)^{j+1} e_{t+j} = -\sum_{j=1}^{\infty} (1/3)^j e_{t-1+j} + 1/3\, e_t$$

$$-\sum_{j=1}^{\infty} (1/3)^{j+1} e_{t+j} = -\sum_{j=2}^{\infty} (1/3)^j e_{t-1+j}$$

$$-\sum_{j=1}^{\infty} (1/3)^{j+1} e_{t+j} = -\sum_{j+1=2}^{\infty} (1/3)^{j+1} e_{t+j}$$

b) $E(Y_t) = E\left(\sum_{j=1}^{\infty} (1/3)^j e_{t-j}\right) = 0$ SINCE ALL TERMS ARE WHITE NOISE

$$Cov(Y_t, Y_{t-1}) = Cov\left(-\sum_{j=1}^{\infty}(1/3)^j e_{t+j}, \sum_{j=1}^{\infty}(1/3)^j e_{t+j-1}\right)$$

$$= Cov\left(-1/3 e_{t+1} - (1/3)^2 e_{t+2} \cdots - (1/3)^n e_{t+n-1}\right)$$

$$= Cov\left(-1/3 e_{t+1}, -1/3^2 e_{t+1}\right) + Cov\left(-1/3 e_{t+2}, -1/3^2 e_{t+2}\right) \cdots$$

$$= 1/26\, \sigma_e^2 (1 + 1/3 + 1/3^2 + \cdots + 1/3^n)$$

↳ FREE OF t ∴ STATIONARY

c) IT IS UNSATISFACTORY BECAUSE Y_t DEPENDS ON FUTURE OBSERVATIONS

4.18 a) $E(W_t) = E(Y_t + c\phi^t) = E(Y_t) + E(c\phi^2) = 0 + c\phi^2 = c\phi^2$

b) $\phi(Y_{t-1} + c\phi^{t-1}) + e_t = \phi Y_{t-1} + c\phi^t + e_t$

$$= \phi\left(\frac{Y_t - e_t}{\phi}\right) + c\phi^t + e_t = Y_t + c\phi^t$$

c) NO → W_t IS NOT FREE OF t