9.1 a) $\hat{Y}_t(1) = \mu + \phi(Y_t - \mu) = 10.8 + (-0.5)(12.2 - 10.8) = 10.1$

b) $\hat{Y}_t(2) = \mu + \phi(\hat{Y}_t(1) - \mu) = 10.8 + (-0.5)(10.1 - 10.8) = 11.15$

$\hat{Y}_t(2) = \mu + \phi^2(Y_t - \mu) = 10.8 + (-0.5)^2(12.2 - 10.8) = 11.15$

c) $\hat{Y}_t(10) = \mu + \phi^{10}(Y_t - \mu) = 10.8 + (-0.5)^{10}(12.2 - 10.8) = 10.801$

9.2 a) $Y_t = 5 + 1.1Y_{t-1} - 0.5Y_{t-2} + e_t \quad \sigma_e^2 = 2$

$\hat{Y}_t(l) = \phi_1 \hat{Y}_t(e-1) + \phi_2 \hat{Y}_t(e-2) + \dots$

$\hat{Y}_{2008} = \hat{Y}_{2007}(1) = 5 + 1.1Y_{2007} - 0.5Y_{2006} = 5 + 1.1(10) - 0.5(11) = 10.5$ MILLION

$\hat{Y}_{2009} = \hat{Y}_{2007}(2) = 5 + 1.1\hat{Y}_{2008} - 0.5Y_{2007} = 5 + 1.1(10.5) - 0.5(10) = 11.55$ MILLION

b) $\psi_0 = 1$

$\psi_1 - \phi_1\psi_0 = 0$

$\psi_j - \phi_1\psi_{j-1} - \phi_2\psi_{j-2} \dots = 0$

For AR(2)

$\psi_1 - \phi_1\psi_0 = 0$

$\psi_1 - \phi_1(1) = 0$

$\psi_1 = \phi_1 = 1.1$

c) $\hat{Y}_t(l) \pm z_{1-\alpha/2}\sqrt{Var(e_t(l))}$

$\alpha = 0.05$

$\hat{Y}_{2007}(1) \pm z_{0.95}\sqrt{\sigma_e^2}$

$10.5 \pm 2.575\sqrt{2}$

10.5 ± 3.6416

$(6.8584, 14.1416)$ MILLION

$Var(e_t(l)) = \sigma_e^2 \sum_{j}^{e-1} \psi_j^2$

$Var(e_t(1)) = \sigma_e^2$

d) $Y_{2008} = 12$

$\hat{Y}_{2009} = \hat{Y}_{2008}(1) = 5 + 1.1Y_{2008} - 0.5Y_{2007} = 5 + 1.1(12) - 0.5(10) = 13.2$ MILLION

10.2 a) $(1 - 1.6x + 0.7x^2)(1 - 0.8x^{12})$

$\phi_1 = 1.6 \quad \phi_2 = -0.7 \quad \theta_1 = 0.8$

$\phi_1 + \phi_2 = 1.6 - 0.7 = 0.9 < 1 \quad \phi_2 - \phi_1 = -0.7 - 1.6 = -2.3 < 1 \quad |\phi_2| = 0.7 < 1$

$\therefore$ STATIONARY

b) $ARIMA(2,0,0) \times (1,0,0)_{12}$

10.3 $Y_t = a + bt + S_t + X_t$ ARIMA$(p,0,q) \times (P,1,Q)_s$

$W_t = Y_t - Y_{t-s} = (a + bt + S_t + X_t) - (a + b(t-s) + S_{t-s} + X_{t-s})$

$= bs + S_t - S_{t-s} + X_t - X_{t-s} = bs + \nabla_s X_t$

ARIMA$(p,0,q) \times (P,0,Q)_s$

10.5 a) $Y_t = 0.5Y_{t-1} + Y_{t-4} - 0.5Y_{t-5} + e_t - 0.3e_{t-1}$

$Y_t - Y_{t-4} = 0.5Y_{t-1} - 0.5Y_{t-5} + e_t - 0.3e_{t-1}$

$= 0.5(Y_{t-1} - Y_{t-5}) + e_t - 0.3e_{t-1}$

$\Phi = 0.5$ $\theta_1 = 0.3$ ARIMA$(1,0,1) \times (0,1,0)_4$

b) $Y_t = Y_{t-1} + Y_{t-12} - Y_{t-13} + e_t - 0.5e_{t-1} - 0.5e_{t-12} + 0.25e_{t-13}$

$(Y_t - Y_{t-1}) - (Y_{t-12} - Y_{t-13}) = e_t - 0.5e_{t-1} - 0.5e_{t-12} + 0.25e_{t-13}$

$= e_t - 0.5(e_{t-1} + e_{t-12}) + (0.5)(0.5)e_{t-13}$

$\theta_1 = 0.5$ $\Theta_1 = 0.5$ ARIMA$(0,1,1) \times (0,1,1)_{12}$