Hw 1 R Code

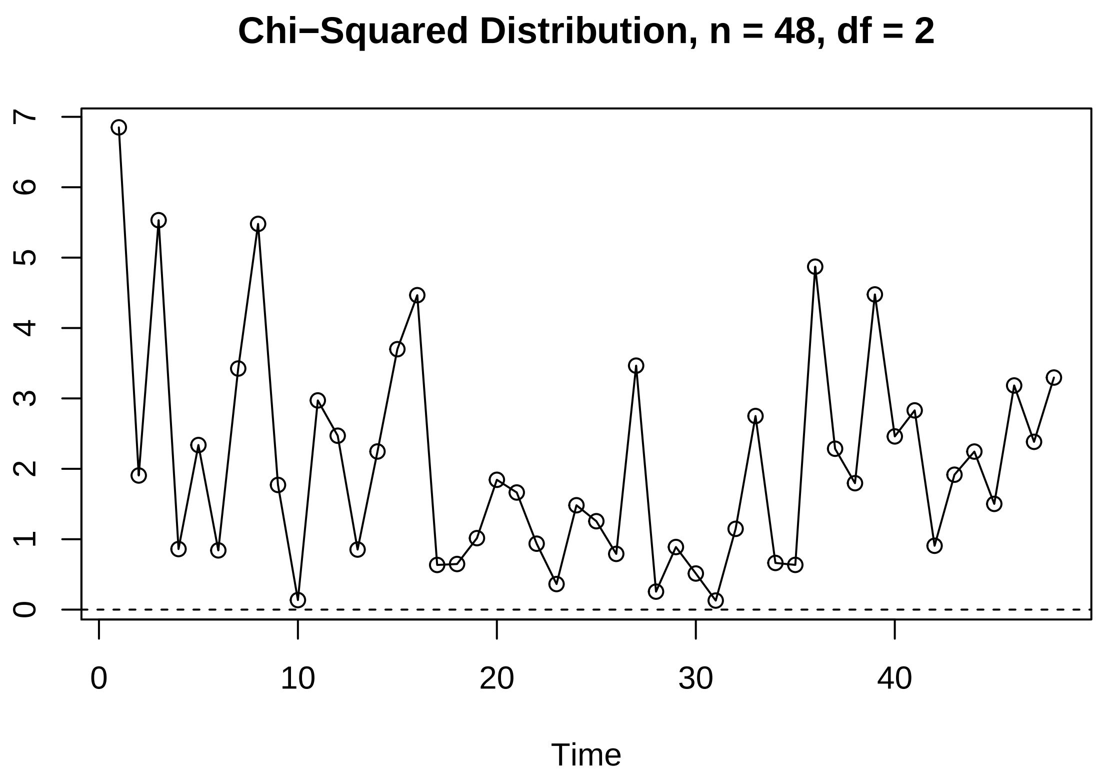
Deanna Springgay

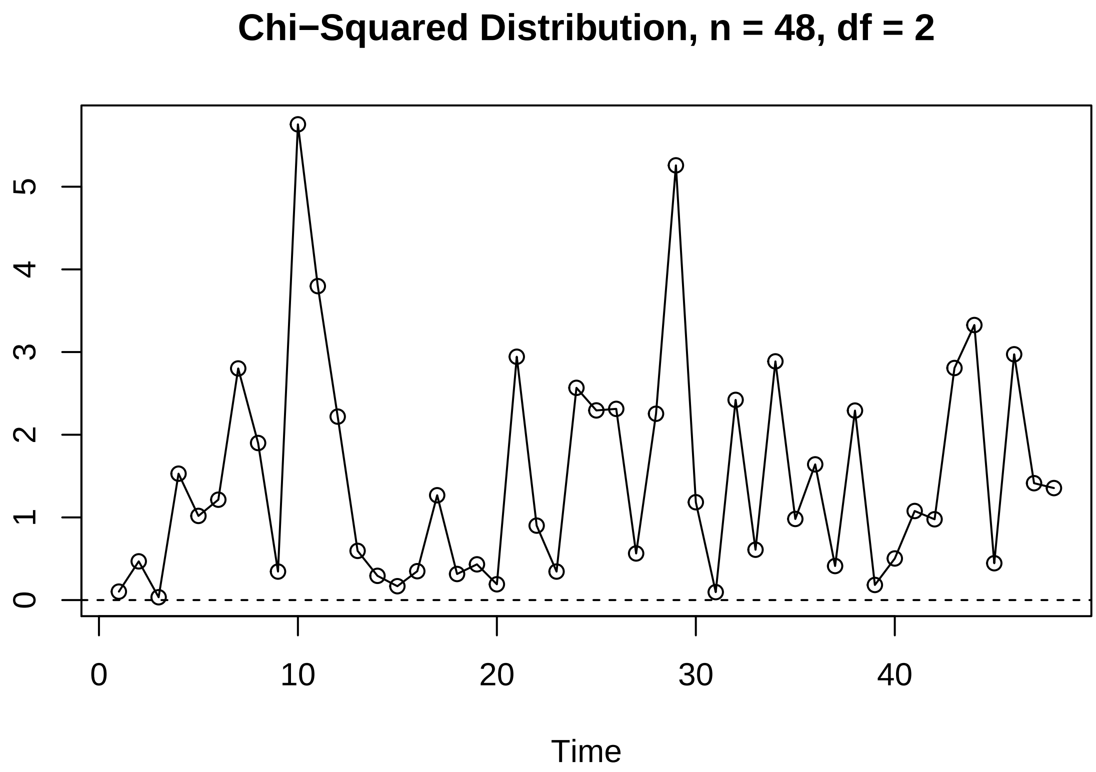
1/31/2021

1.4

Below are two plots of different simulations of the chi-squared distribution in following code block. Both appear to follow non-normal distributions but they are not random - they follow a chi-squared distribution.

```
set.seed(7)
chiSquare <- ts(rchisq(n = 48, df = 2))
plot(chiSquare, type = "o", ylab = "", main = "Chi-Squared Distribution, n = 48, df = 2")
abline(h = 0, lty = 2)
```

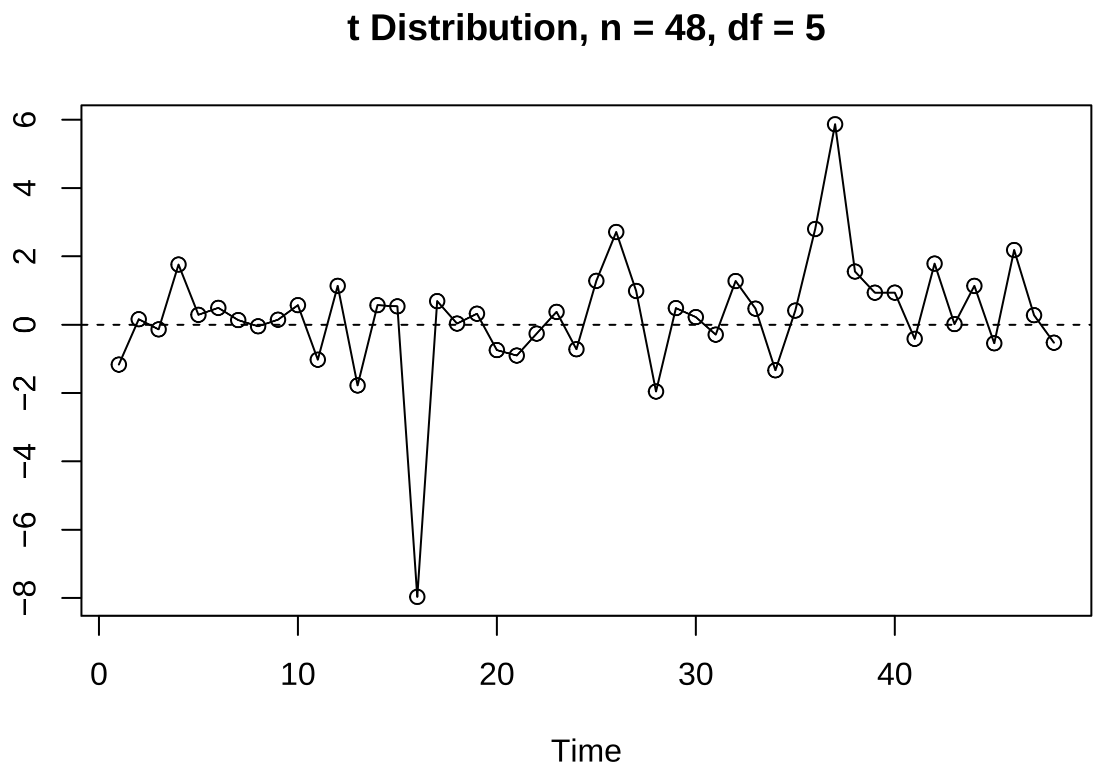




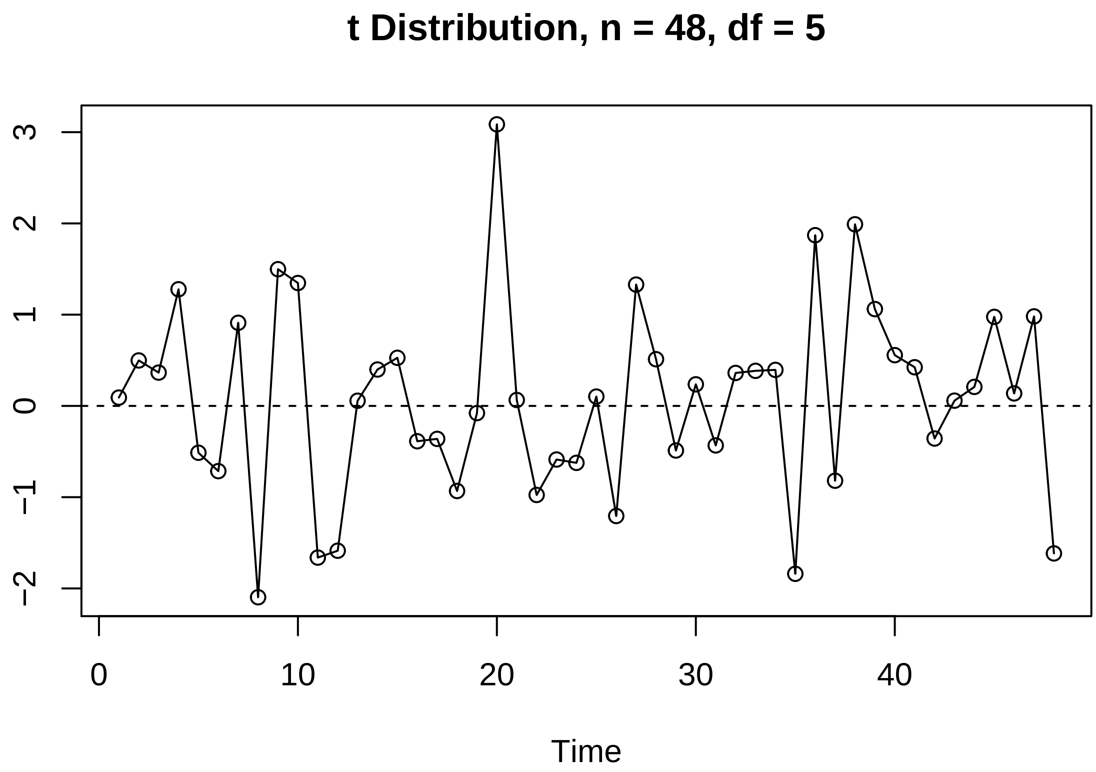
1.5

Below are two plots of different simulations of the t distribution in following code block. Both appear to follow normal and do not appear to be random.

```
set.seed(17)
chiSquare <- ts(rt(n = 48, df = 5))
plot(chiSquare, type = "o", ylab = "", main = "t Distribution, n = 48, df = 5")
abline(h = 0, lty = 2)
```

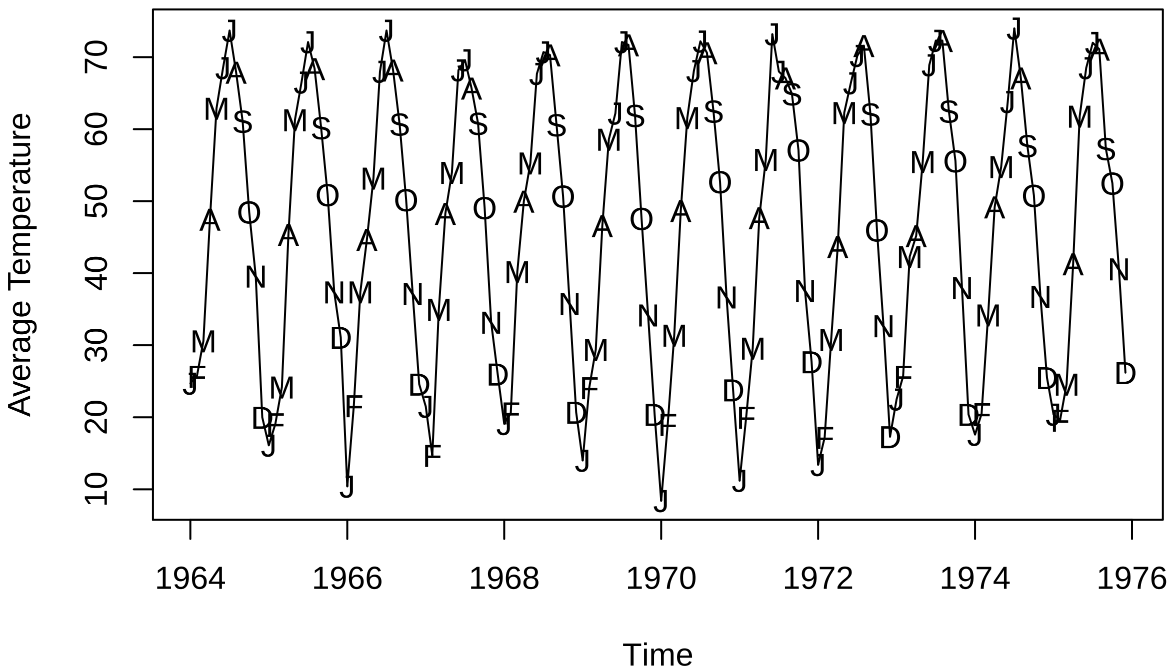


t Distribution, n = 48, df = 5
6
4
2
0
−2
−4
−6
−8
0
10
20
30
40
Time



1.6

```r
data("tempdub")
plot(tempdub, type = "l", ylab = "Average Temperature")
points(y = tempdub, x = time(tempdub), pch = as.vector(season(tempdub)))
```


Average Temperature
10
20
30
40
50
60
70
1964
1966
1968
1970
1972
1974
1976
Time

2.9 a) $Y_t = \beta_0 + \beta_1 t + X_t$

$E[Y_t] = \beta_0 + \beta_1 t + E[X_t] = \beta_0 + \beta_1 t + \mu_{tx}$

↳ DEPENDS ON t, $\therefore$ NOT STATIONARY

$W_t = \nabla Y_t = Y_t - Y_{t-1}$

$E[W_t] = E[Y_t - Y_{t-1}] = E[\beta_0 + \beta_1 t + X_t - (\beta_0 + \beta_1(t-1) + X_{t-1})]$

$= \beta_0 + \beta_1 t + E[X_t] - \beta_0 - \beta_1(t-1) - E[X_{t-1}]$

$= \beta_0 + \beta_1 t - \beta_0 - \beta_1 t + \beta_1 = \beta_1 \rightarrow$ NOT DEPENDENT ON t

$cov(W_t) = cov(Y_t, Y_{t-1}) = cov(\beta_0 + \beta_1 t + X_t, \beta_0 + \beta(t-1) + X_{t-1})$

$= cov(X_t, X_{t-1}) = \gamma_k \rightarrow$ NOT DEPENDENT ON t

$\therefore W_t$ IS STATIONARY

2.10 a) $Y_t = \mu_t + \sigma_t X_t$

$E[Y_t] = E[\mu_t + \sigma_t X_t] = \mu_t + \sigma_t E[X_t] = \mu_t + \sigma_t(0) = \mu_t$

$cov(Y_t, Y_{t-k}) = cov(\mu_t + \sigma_t X_t, \mu_{t-k} + \sigma_{t-k} X_{t-k}) = \sigma_t \sigma_{t-k} cov(X_t, X_{t-k})$

$= \sigma_t \sigma_{t-k} \rho_k$

b) $Var[Y_t] = Var[\mu_t + \sigma_t X_t] = \sigma_t^2 Var[X_t] = \sigma_t^2(1) = \sigma_t^2$

$corr[Y_t, Y_{t-k}] = \dfrac{\sigma_t \sigma_{t-1} \rho_k}{\sqrt{Var[Y_t] Var[Y_{t-k}]}} = \dfrac{\sigma_t \sigma_{t-k} \rho_k}{\sigma_t \sigma_{t-k}} = \rho_k$

Y_t COULD BE STATIONARY AS LONG AS μ_t DOESN'T DEPEND ON t, BUT WE DON'T HAVE THAT INFORMATION

c) ρ_k IS FREE OF t BUT AGAIN, WE DON'T KNOW IF σ_t DEPENDS ON t.

2.13 a) $Y_t = e_t - \theta(e_{t-1})^2$

$E[Y_t] = E[e_t - \theta(e_{t-1})^2] = E[e_t] - \theta E[e_{t-1}^2] = \theta Var[e_{t-1}] = -\theta \sigma_e^2$

$Var[Y_t] = Var[e_t - \theta(e_{t-1})^2] = Var[e_t] + \theta^2 Var[e_{t-1}^2]$

$= \sigma_e^2 + \theta^2(E[e_{t-1}^4] - E[e_{t-1}^2]^2) = \sigma_e^2 + \theta^2(3\sigma_e^4 - \sigma_e^4)$

$= \sigma_e^2 + 2\theta^2 \sigma_e^4$

$$\operatorname{cov}(Y_t, Y_{t-1}) = \operatorname{cov}(e_t - \theta e_{t-1}^2, e_{t-1} - \theta e_{t-2}^2)$$
$$= \operatorname{cov}(e_t, e_{t-1}) - \theta \operatorname{cov}(e_t, e_{t-2}^2) - \theta \operatorname{cov}(e_{t-1}^2, e_{t-1})$$
$$+ \theta^2 \operatorname{cov}(e_{t-1}^2, e_{t-2}^2)$$
$$= \theta \operatorname{cov}(e_{t-1}^2, e_{t-1}) = -\theta(E[e_{t-1}^3] + \mu_{t-1} + \mu_t) = 0$$
$$\gamma_{t,s} = \frac{\operatorname{cov}(Y_t, Y_{t-1})}{\sqrt{Var[Y_t]Var[Y_{t-1}]}} = 0$$

b) AUTOCORRELATION IS 0 AND $E[Y_t]$ IS CONSTANT

$\therefore$ Y_t IS STATIONARY

2.14 a) $Y_t = \theta_0 + t e_t$

$$E[Y_t] = E[\theta_0 + t e_t] = \theta_0 + t E[e_t] = \theta_0 + t(0) = \theta_0$$

↳ NOT DEPENDENT ON t

$$Var[Y_t] = Var[\theta_0 + t e_t] = 0 + t^2 Var[e_t] = t^2 \sigma_e^2$$

↳ DEPENDENT ON t

Y_t IS NOT STATIONARY

b) $W_t = \nabla Y_t = Y_t - Y_{t-1} = t e_t - (t-1) e_{t-1}$

$$E[W_t] = E[\theta_0 + t e_t - \theta_0 - (t-1) e_{t-1}] = t E[e_t] - (t-1) E[e_{t-1}]$$
$$= t E[e_t] - t E[e_{t-1}] + E[e_{t-1}] = 0$$

↳ NOT DEPENDENT ON t

$$Var[W_t] = Var[t e_t] - Var[(t-1) e_{t-1}] = t^2 Var[e_t] - (t-1)^2 Var[e_{t-1}]$$
$$= t^2 \sigma_e^2 - (t-1)^2 \sigma_e^2 = (2t-1)\sigma_e^2$$

↳ DEPENDENT ON t

W_t IS NOT STATIONARY

c) $Y_t = e_t e_{t-1}$

$$E[Y_t] = E[e_t e_{t-1}] = E[e_t] E[e_{t-1}] = 0$$

↳ NOT DEPENDENT ON t

$$\operatorname{cov}(Y_t, Y_{t-1}) = \operatorname{cov}(e_t e_{t-1}, e_{t-1} e_{t-2}) = E[(e_t e_{t-1} - \mu_t^2)(e_{t-1} e_{t-2} - \mu_t^2)]$$
$$= E[e_t] E[e_{t-1}] E[e_{t-1}] E[e_{t-2}] = 0$$

↳ NOT DEPENDENT ON t

Y_t IS STATIONARY

2.19 a) $Y_1 = \theta_0 + e_1$

$t > 1 \quad Y_t = \theta_0 + Y_{t-1} + e_t$

$Y_1 = \theta_0 + e_1$

$Y_2 = \theta_0 + \theta_0 + e_2 + e_1$

$Y_t = \theta_0 + \theta_0 + \ldots + e_t + e_{t-1} \cdots + e_1$

$Y_t = t\theta_0 + e_t + e_{t-1} \cdots + e_1$

b) $E[Y_t] = E[t\theta_0 + e_t + e_{t-1} \cdots e_1] = t\theta_0 + E[e_t] + E[e_{t-1}] \cdots E[e_1]$

$= t\theta_0 + 0 + 0 + \ldots + 0 = t\theta_0$

c) $cov(Y_t, Y_{t-k}) = cov(t\theta_0 + e_t + e_{t-1} + \ldots, (t-k)\theta_0 + e_{t-k} + \ldots)$

$= cov(e_{t-k} + e_{t-k-1} + \ldots, e_{t-k} + e_{t-k-1} + \ldots)$

$= Var[e_{t-k} + e_{t-k-1} + \ldots] = (t-k)\sigma_e^2$