Chapter 2

The Capital Asset Pricing Model: An Application of Bivariate Regression Analysis

"In investing money, the amount of interest you want should depend on whether you want to eat well or sleep well."

J. KENFIELD MORLEY, "Some Things I Believe," *The Rotarian*, February 1937

"October. This is one of the peculiarly dangerous months to speculate in stocks in. The others are July, January, September, April, November, May, March, June, December, August and February."

MARK TWAIN, *Pudd'nhead Wilson's Calendar* (1899), p. 108

"Nature is the realization of the simplest conceivable mathematical ideas."

ALBERT EINSTEIN, *Ideas and Opinions* (1954), p. 274

"Financial forecasting appears to be a science that makes astrology look respectable."

BURTON MALKIEL, *A Random Walk Down Wall Street* (1985), p. 152

One of the most sought-after possessions for a typical private investor or securities market analyst is a reliable equation predicting the return on alternative securities. A first step in developing and empirically implementing such an equation involves gaining an understanding of why a particular stock has a low or high rate of return. In this chapter we focus attention on the capital asset pricing model (CAPM), a model that helps considerably in developing such understanding. As we shall see, a remarkable feature of CAPM is that its most important parameters can be estimated by using the very simplest of econometric techniques, namely, a bivariate linear model in which a dependent variable is regressed on a constant and a single independent variable. The simple CAPM framework therefore provides a useful introduction to empirical econometrics.

The empirical analysis of stock markets has had a very important role in the development of econometrics. In 1932, Alfred Cowles III, a quantitatively oriented investment analyst, provided leadership to found the Econometric Society.[1] Cowles also initiated funding support to establish the Cowles Commission for Research in Economics. Some of the most important developments in econometric theory, including the theory underlying simultaneous equations estimation and identification, were conceived by researchers at the Cowles Commission, first at the University of Chicago and now at Yale University. In this context it is interesting to note that in the first volume of *Econometrica*, the official publication of the Econometric Society, Cowles published an article purporting to show that the most successful records of stock market forecasters "are little, if any, better than what might be expected from pure chance. There is some evidence, on the other hand, to indicate that the least successful records are worse than what could reasonably be attributed to chance."[2]

It is therefore appropriate that our first application in this hands-on econometrics text involves an empirical examination of stock markets. We begin this chapter with a summary discussion of the financial theory underlying the CAPM, consider the role of diversification, derive the principal estimating equations, interpret them, and then consider issues in empirical implementation.[3] Finally, in the hands-on applications we examine ten years of monthly returns data for a variety of companies and for the market as a whole, estimate company-specific betas using bivariate regression procedures, assess why gold is a special asset, interpret the R^2 measure in terms of the proportion of total risk that is nondiversifiable, evaluate properties of certain portfolios, conduct event studies, estimate a generalized version of CAPM, and then test assumptions concerning stochastic specification.

2.1 DEFINITIONS AND BASIC FINANCE CONCEPTS

Assume that when investors act in the securities markets, their behavior is perfectly rational in the sense that their only concern is the myopic one of assessing returns from their own investments. Define the rate of return on an investment as

$$r = \frac{p_1 + d - p_0}{p_0} \tag{2.1}$$

where

p_1 = price of security at the end of the time period,
d = dividends (if any) paid during the time period,
p_0 = price of security at the beginning of the time period.

Although the return r can be easily calculated ex post (once the investment has been made), r is of course uncertain ex ante (before the investment decision has been made). Hereafter, we interpret r as the expected or ex ante rate of return.

Typically, investors (other than those who enjoy gambling for its own sake) are not only interested in the most likely or *expected return* on an investment; they are also concerned with the possible *distribution* of r, where r is taken as a random variable. The *risk* accompanying a possible investment is typically characterized by the distribution of such possible returns. Returns are often assumed to be distributed normally, and in such cases the distribution can be completely described by two measures, the expected value and the variance σ^2 (or the square root of the variance, σ, called the standard deviation). Under the normality assumption, in the empirical finance literature risk is typically measured by the standard deviation σ.[4]

While investors are virtually unanimous in preferring higher returns to lower ones, other things equal, it is also the case that most investors are risk-averse, that is, investors prefer a lower standard deviation to a higher one, given the same expected return. This implies that if the risk on an investment or portfolio of investments appears to be large, investors are likely to accept such high risk only if it is accompanied by a high expected return; similarly, an investment with a low expected return will be acceptable only if it has a small risk.[5] But how much of a premium will investors require in order to assume greater risk?

If investors were to purchase an asset having zero risk, they would still demand a return as an inducement to postpone current consumption. Such a return is called the *risk-free rate of return*, and we denote it here by r_f; empirical analysts of the U.S. securities market often employ as a measure of r_f the 30-day U.S. Treasury bill rate held until maturity, apparently because investors believe that the probability of default on such a security is virtually zero.[6] We can use these concepts to define compensation for risk, or the risk premium on the jth asset, as the excess return over the risk-free rate r_f, that is,

$$\text{Risk premium}_j \equiv r_j - r_f \tag{2.2}$$

With these definitions in mind we now turn to a consideration of diversification and risk management.

2.2 DIVERSIFICATION AND PORTFOLIO OPTIMALITY

How do intelligent investors manage risk on their investments? To examine the risk management process, it is useful to introduce the notion of *diversification*. Although mathematical discussions of the diversification process can easily become very involved, here we summarize the principal results, using a combination of relatively simple analysis and intuition, based in large part on the pioneering work of Harry M. Markowitz.[7]

If an investor holds two securities, the expected return on the total portfolio r_p is simply the weighted average of the expected returns on each of the assets, the weights being the relative shares invested in each of the two securities,

$$r_p = w_1 r_1 + w_2 r_2 \tag{2.3}$$

where w_j is the proportion of total funds invested in asset j, $j = 1, 2$, and $w_1 + w_2 = 1.0$. Further, the total variance of the portfolio, σ_p^2, is

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2 \cdot w_1 w_2 \sigma_{12} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2 \cdot w_1 w_2 \rho_{12}\sigma_1\sigma_2 \tag{2.4}$$

where

$\sigma_j^2 \equiv$ variance of return on security j, $j = 1, 2$,
$\sigma_j \equiv$ standard deviation of return on security j, $j = 1, 2$,
$\sigma_{12} \equiv$ covariance of returns on securities 1 and 2, and
$\rho_{12} \equiv$ simple correlation between returns on securities 1 and 2.

The second equality in Eq. (2.4) holds, since by definition, $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$.

We now want to show that for a given amount of funds to be invested, diversification generally reduces risk. To see this, assume first the unlikely situation in which returns on securities 1 and 2 are perfectly correlated, that is, assume that ρ_{12}, the simple correlation coefficient between returns on assets 1 and 2, equals 1.0. In this case, $\sigma_{12} = \sigma_1\sigma_2$, which is the largest possible value that σ_{12} can take. But as can be seen by inspecting Eq. (2.4), whenever σ_{12} is at a maximum, given σ_1 and σ_2, so too is the variance on the total portfolio σ_p^2. As the covariance and therefore the correlation between returns on assets 1 and 2 decreases and becomes less than perfect, the final term in Eq. (2.4) becomes smaller, and so too does the total portfolio variance σ_p^2. This is intuitively appealing: By holding two assets whose returns do not move together in perfect harmony, the lower return on one asset can be partially offset by a relatively higher return on the other asset, resulting in a reasonable overall portfolio return yet a reduced total portfolio risk.

It is instructive to demonstrate this with several examples of portfolio risk and return under alternative diversification behavior. Consider the simple case in which the expected returns on securities 1 and 2 both equal 10%, where the standard deviation of returns σ for each is equal to 2.0 and where it is initially assumed that returns on the two securities are perfectly correlated, that is, $\rho_{12} = 1.0$, which implies that $\sigma_{12} = 4.0$.

HARRY M. MARKOWITZ

Father of Modern Portfolio Theory

The basic finance theory underlying modern portfolio analysis is due in large part to the pioneering work of Harry M. Markowitz. Markowitz is a product of Chicago. He was born there in 1927, attended its elementary and secondary schools, and received three degrees from the University of Chicago—a bachelor's, master's, and doctorate in economics. In his Ph.D. dissertation, Markowitz developed the basic portfolio model summarized in this chapter. His seminal analysis appeared in a 1952 issue of the *Journal of Finance*, and a more complete treatment was published in 1959 as a Cowles Foundation report by John Wiley and Sons.

Markowitz's professional career has been in and out of academia and in and out of finance. Leaving Chicago in 1952 with all but his dissertation completed, Markowitz joined the Rand Corporation in Santa Monica, California. In 1960 he took a position at the General Electric Corporation, and then in 1961 he returned to Rand. At Rand and at GE, Markowitz wrote computer code for programs that simulated logistic and/or manufacturing processes. His interests in simulation programming grew, and in 1963 he became Chairman of the Board and Technical Director of Consolidated Analysis Centers, Inc., a computer software firm. One year after assuming an academic position at UCLA in 1968, Markowitz became portfolio manager and later president of an investment firm called Arbitrage Management Company. From 1974 to 1983 he served as a Research Staff Member at IBM's Thomas J. Watson Research Center, focusing on the design of data base computer languages.

Markowitz is a Fellow of the Econometric Society and of the American Academy of Arts and Sciences and has served as President of the American Finance Association. In 1989 he was awarded the John von Neumann Theory Prize by the Operations Research Society of America and The Institute of Management Science, in part for his research on portfolio theory and the SIMSCRIPT programming language. Although he is still very actively involved in real-world financial matters, Harry M. Markowitz currently holds the Marvin Speiser Distinguished Professor of Finance and Economics chair at Baruch College at the City University of New York.

One possible investment strategy is to place funds entirely into security 1, implying that $w_1 = 1.0$ and $w_2 = 0.0$; we name this Case A, and its consequences are presented in the first row of Table 2.1. In Case A the expected portfolio return, based on Eq. (2.3), is calculated as $r_p = 1.0\ (10\%) + 0.0\ (10\%) = 10\%$. Substituting the Case A values into Eq. (2.4) yields a portfolio variance of 4.0, or a standard deviation of 2.0.

A second investment strategy, called Case B, involves putting all funds into security 2 and none into security 1, implying that $w_1 = 0.0$ and $w_2 = 1.0$; this case is presented in the second row of entries in Table 2.1. Use of Eqs. (2.3) and (2.4) again implies that $r_p = 10\%$, while $\sigma_p^2 = 4.0$ and risk σ equals 2.0. Since the risk and return consequences of following Case A or Case B are identical, investors will be indifferent between these two cases.

A third alternative investment strategy, Case C, is to diversify the portfolio by purchasing equal amounts of securities 1 and 2, implying that $w_1 = w_2 = 0.5$. If one substitutes Case C entries in Table 2.1 into Eqs. (2.3) and (2.4), one again obtains the same portfolio return of 10% and standard deviation of 2.0. Notice that in each of these three Cases A, B, and C, because of the perfect correlation assumption, the portfolio risk and return are the same whether the investor holds only security 1, only security 2, or a combination of these assets. If returns on these two securities had not been perfectly correlated, however, the portfolio variance would have been smaller.

To see this, first consider Case D in Table 2.1. Here the correlation between returns on assets 1 and 2 is positive but less than perfect, that is, $\rho_{12} = 0.5$. All other features of this case are the same as in Case C. Notice that owing to diversification (equal amounts of securities 1 and 2 are purchased), the investor is able to exploit the less than perfect correlation between asset returns and obtain the same portfolio return of 10%, yet at a reduced variance of 3.0 and standard deviation of about 1.7. On the basis of Eqs. (2.3) and (2.4) it is simple to show that, in the presence of less than perfect correlation between asset returns, had the investor not diversified and purchased only security 1 (Case E) or only security 2 (Case F), the same 10% return would have been attained but at a higher variance of 4.0 and standard deviation of

Table 2.1 Examples of Risk and Return Under Alternative Portfolio Diversifications

	$r_1 = r_2$	w_1	w_2	σ_1	σ_2	ρ_{12}	σ_{12}	r_p	σ_p^2	σ_p Risk
Case A	10%	1.0	0.0	2.0	2.0	1.0	4.0	10%	4.0	2.0
Case B	10%	0.0	1.0	2.0	2.0	1.0	4.0	10%	4.0	2.0
Case C	10%	0.5	0.5	2.0	2.0	1.0	4.0	10%	4.0	2.0
Case D	10%	0.5	0.5	2.0	2.0	0.5	2.0	10%	3.0	1.7
Case E	10%	1.0	0.0	2.0	2.0	0.5	2.0	10%	4.0	2.0
Case F	10%	0.0	1.0	2.0	2.0	0.5	2.0	10%	4.0	2.0
Case G	10%	0.5	0.5	2.0	2.0	−1.0	−4.0	10%	0.0	0.0

2.0, as in Cases A, B, and C. Cases D, E, and F therefore clearly demonstrate the benefits of diversification in reducing risk.

Finally, in the most unlikely case that returns on assets 1 and 2 were perfectly negatively correlated, diversification could eliminate risk entirely. For example, in Case G, $\rho_{12} = -1.0$, but if $w_1 = w_2 = 0.5$, $\sigma_p^2 = \sigma_p = 0$.

Now consider the case of an investor diversifying by holding n securities, where n can be larger than 2. As before, the expected return on the total portfolio is a weighted average of security-specific expected returns r_j, where the w_j weights are shares of total funds invested in each asset, that is,

$$r_p = \sum_{j=1}^{n} w_j r_j \tag{2.5}$$

Once again, with n securities the total variance of the portfolio depends not only on the variances of the n individual securities, but also on their covariances. Specifically, the portfolio variance is calculated as

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \cdot \sum_{i=1}^{n}\sum_{j=i+1}^{n} w_i w_j \sigma_{ij} \tag{2.6}$$

where σ_{ij} is the covariance between returns on securities i and j and σ_i^2 is the variance. Note that the total portfolio variance in Eq. (2.6) has n variance terms and $n(n-1)$ covariances, with $n(n-1)/2$ of them being different. Therefore the larger is n, other things equal, the greater is the relative importance of asset covariances to total portfolio variance. For example, when n is 5, there are five variances and 20 covariances; when n is doubled to 10, the number of variances doubles to 10, but the number of covariance terms in Eq. (2.6) increases to 90! As n becomes very large, the portfolio variance approaches a (weighted) average of the covariances. Hence covariances are extremely important in the diversification process.

The above discussion focused on the average return and variance of a diversified portfolio. For portfolio decision-making purposes, marginal returns and variances are also important. Suppose that in the initial portfolio of n assets held by an investor, there were zero holdings of security k, implying that initially $w_k = 0$. Next, assume that the investor decided to acquire a very small positive amount of security k, but that other holdings remained unchanged. Define the *marginal return* of the kth asset on r_p as the change in r_p, given a small change in w_k. From Eq. (2.5) this marginal return is simply equal to r_k:

$$\text{Marginal return}_k \equiv \partial r_p/\partial w_k = r_k \tag{2.7}$$

This small change in asset holdings also affects the portfolio variance. Define the *marginal variance* of the kth asset as the change in σ_p^2 given a small change in w_k. From Eq. (2.6) and using the fact that a weighted sum of individual covariances with security k equals the covariance of security k with the portfolio—itself a weighted sum of other securities—it follows that the marginal variance is simply

$$\text{Marginal variance}_k \equiv \frac{\partial \sigma_p^2}{\partial w_k} = 2 \cdot \sum_{i=1}^{n} w_i \sigma_{ik} = 2\sigma_{kp} \tag{2.8}$$

where σ_{kp} is the covariance between security k and the portfolio p.[8] Hence the marginal variance—the change in total portfolio variance as a result of a small change in the holdings of asset k—depends simply on the covariance between returns on asset k and the portfolio.

Given these definitions, we can now present an important principle of portfolio optimality derived in finance theory. If two securities in a portfolio have the same marginal variance but different expected returns, then that portfolio cannot be optimal in the sense of providing a maximum return for given risk. The reason such a portfolio could not be optimal is that it would be possible to obtain a higher return without increasing risk by holding more of the asset with the higher return (the marginal variances of the two assets are assumed to be identical). Therefore if a portfolio is *optimal*, all securities with the same marginal variance must have identical expected returns.

The marginal variance and the variances and covariances in Eqs. (2.6) and (2.8) all depend on units of measurement. Like the economists' notion of elasticity, financial economists have found it convenient to adopt relative measures that are independent of units of measurement. Perhaps the best-known relative measure is the *beta value* for security k, computed simply as

$$\text{Beta}_k \equiv \sigma_{kp}/\sigma_p^2 \tag{2.9}$$

Since a security's beta value depends on its covariance, which in turn is closely related to its marginal variance, one can combine Eqs. (2.8) and (2.9) to derive a factor of proportionality between beta and marginal variance:

$$\text{Marginal variance}_k = 2\sigma_{kp} = 2\sigma_p^2 \cdot \text{beta}_k$$

Given this relationship, the previous discussion on portfolio optimality can be expressed equivalently in terms of beta values rather than marginal variances. Specifically, if a portfolio is optimal, then all securities with the same beta value relative to the portfolio must have identical expected returns.

2.3 DERIVATION OF A LINEAR RELATIONSHIP BETWEEN RISK AND RETURN

To this point we have related variances, covariances, marginal variances, and beta values, and we have presented an important principle of portfolio optimality. But how might one move from these insights to portfolio choice and an empirically implementable relationship between risk and return? In the next few pages we summarize the very important contribution of the CAPM to facilitating relatively simple empirical analysis, and we show that the relationship between risk and return is a linear one.[9]

Suppose an investor has a portfolio, called a, consisting of a mix of two assets. The blend of these two assets generates an expected portfolio return r_a and has a variance of σ_a^2. Now let there be a risk-free asset whose return is r_f, and let the investor be able to borrow or lend indefinitely at the risk-free rate of return r_f. One possibility facing this investor is to combine portfolio a with the risk-free asset into a new portfolio. In such a case the expected return on the new portfolio would equal

$$r_p = (1 - w_a)r_f + w_a r_a \tag{2.10}$$

where w_a is the proportion of total funds invested in portfolio a. The variance of this new portfolio would be

$$\sigma_p^2 = w_a^2\sigma_a^2 + (1 - w_a)^2\sigma_f^2 + 2w_a(1 - w_a)\sigma_{af} \tag{2.11}$$

where σ_{af} is the covariance between the expected return of portfolio a and that on the risk-free asset. However, since by definition the risk-free asset has a zero variance return, this risk-free return is also uncorrelated with that on any other security, implying that $\sigma_f^2 = \sigma_{af} = 0$. Thus Eq. (2.11) reduces to

$$\sigma_p^2 = w_a^2\sigma_a^2 \qquad \text{or} \qquad \sigma_p = w_a\sigma_a \tag{2.12}$$

Rearranging the second expression in Eq. (2.12) yields $w_a = \sigma_p/\sigma_a$ and $(1 - w_a) = 1 - \sigma_p/\sigma_a$, which, after substituting back into Eq. (2.10) and collecting terms, gives us

$$r_p = r_f + \left[\frac{r_a - r_f}{\sigma_a}\right]\sigma_p \tag{2.13}$$

In Eq. (2.13) we have a simple linear relationship between portfolio return r_p and portfolio risk σ_p, one that even Albert Einstein would admire (recall the Einstein quote at the beginning of this chapter). Specifically, the total portfolio return r_p is the sum of two terms: the risk-free rate of return r_f and $(r_a - r_f)/\sigma_a$ times the portfolio risk σ_p. This linearity is shown in Fig. 2.1 in which the expected return is measured on the vertical axis, risk is on the horizontal axis, the intercept term is r_f, and the slope term is $(r_a - r_f)/\sigma_a$.

Several features of Fig. 2.1 are worth noting. First, if the investor chose to invest only in the risk-free asset such that $w_a = 0$, then by Eq. (2.10), r_p would equal r_f, and by (2.11), σ_p would equal zero. Second, if the investor instead chose to invest only in the a portfolio and entirely avoided the risk-free asset (say, at point a in Fig. 2.1), then $w_a = 1$, $r_p = r_a$, and $\sigma_p = \sigma_a$. Third, the slope of the line in Fig. 2.1 represents the reward to the investor of accepting increased risk, that is, of increasing the proportion of funds invested in the risky portfolio a.

Portfolio a is, of course, but one of many possible risky portfolios constructed by our investor; assets 1 and 2 could have been combined in numerous alternative combinations. This raises the interesting issue of what the risk-

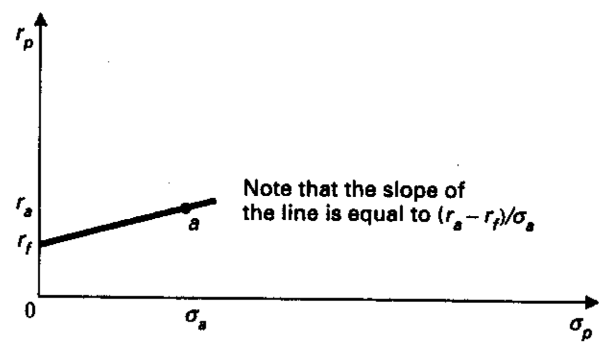


FIGURE 2.1 The linear relationship between risk and return

return frontier would look like for an investor who is considering other alternative possibilities of a portfolio blended from these two risky assets.

Suppose we place two securities on a risk-return diagram, as in Fig. 2.2. Let asset 1 be low risk–low return, while asset 2 is high risk–high return. Further, let the correlation between returns on assets 1 and 2 be less than perfect. As was pointed out in Eq. (2.3), a combination of the two stocks will yield an expected return of $r_p = w_1 r_1 + w_2 r_2$, a weighted average of returns on the two assets. However, because of diversification, the total portfolio risk σ_p will be smaller than the weighted average of the standard deviations, since $(w_1\sigma_1 + w_2\sigma_2)^2$ is less than the right-hand side of Eq. (2.4) whenever $\rho_{12} < 1$. As a result, provided that $\rho_{12} < 1$, the risk-return frontier for various combinations of assets 1 and 2 will look like the concave curve depicted in Fig. 2.3. Incidentally, it is worth noting that as $\rho_{12} \to 1$, the concave curve converges to a straight line.

Our investor is now faced with the following problem: Among all the possible risky asset portfolio combinations in Fig. 2.3, which blend of a portfolio of risky assets with the risk-free asset will yield the maximum return for

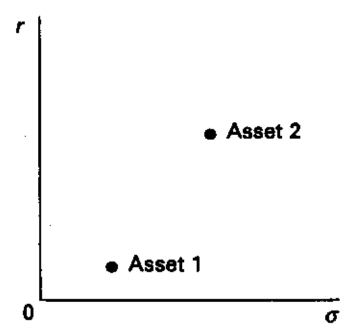


FIGURE 2.2

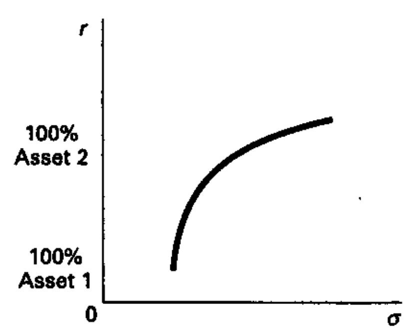


FIGURE 2.3

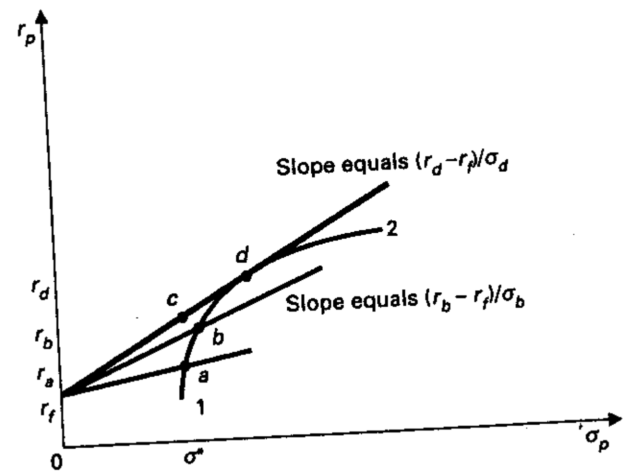


FIGURE 2.4

any given portfolio risk? The CAPM solution to this optimality problem is in fact quite simple.

One possible strategy is one that we considered earlier when we constructed portfolio *a*: holding assets 1 and 2 in the proportions corresponding to, say, point *a* in Fig. 2.4. Note that at point *a* on the $(r_f - a)$ bottom line in Fig. 2.4, the total portfolio risk is σ^* and the return is r_a. We now show that while such a strategy would be feasible, it would not be optimal in the sense that the investor could attain an even higher return given total risk σ^* by lending a portion of funds, say, $1 - w_p$, at the risk-free rate r_f and then investing the remaining portion of funds, w_p, according to the portfolio proportions represented by some other point on the concave risk-return frontier in Fig. 2.4 (as we shall soon see, portfolio *d*). An investor who followed this mixed strategy would be able to reach point *c*, where the return is larger than at point *a*, yet the σ^* risk is the same as at point *a*.

To see this, consider another portfolio (called *b*) and the risk-return possibilities available when this risky portfolio is combined with the risk-free asset. Specifically, if we repeated our analysis that began just above Eq. (2.10) and derived a linear relationship between risk and return for various mixes of portfolio *b* with the risk-free asset, we would obtain a linear equation analogous to Eq. (2.13), having intercept r_f and slope equal to $(r_b - r_f)/\sigma_b$. Such a line is the middle one drawn in Fig. 2.4; its slope is larger than that obtained with various mixes of the earlier portfolio *a* and the risk-free asset, implying that the rewards to risk are higher when portfolio *b* is mixed with the risk-free asset instead of portfolio *a*. An important result here is that with this *b* portfolio, as is shown in Fig. 2.4, one could hold a mixture of it and the risk-free asset to attain any desired return on the straight line through points r_f and b; except at the intercept point where $\sigma_p = 0$, for any given risk the portfolio return r_p is larger on the portfolio b line than on the portfolio a line. In this sense, portfolio b dominates portfolio a.

But why stop with portfolio b? Using the very same reasoning as with portfolios a and b, consider yet another portfolio combination of assets 1 and 2, say, that at point d on the concave risk-return frontier in Fig. 2.4. Analogous to Eq. (2.13), one could now blend portfolio d with a risk-free asset in numerous ways to obtain a risk-return linear equation relating r_p to σ_p, having intercept r_f and slope now equal to $(r_d - r_f)/\sigma_d$. Such a straight line is the top one in Fig. 2.4; note that its slope is larger than that based on portfolio b. Points on this $r_f - d$ line indicate the expected return r_p corresponding with alternative levels of risk σ_p.

Note that with a strategy based on a mix of portfolio d with the risk-free asset, the investor could always attain a higher return for the same risk than with the b mixture, since the straight line connecting r_f through point d is always above the $r_f - b$ line except at the point where $\sigma_p = 0$. Furthermore, since this $r_f - d$ line is tangent to the concave risk-return frontier, there is no other portfolio superior to d, for any other line emanating from r_f and having a larger slope would not touch the concave risk-return frontier and therefore would not be feasible. Portfolio d is said to be *efficient*.

The implication of this analysis is startling: Each investor should hold portfolio d, regardless of his or her risk-return preferences, and then achieve the desired amount of risk by borrowing or lending at the risk-free rate r_f. In particular, if σ^* is the desired maximum amount of risk given the investor's preferences, the optimal procedure for the investor is to mix assets 1 and 2 according to portfolio d and then blend by borrowing or (in this case) lending funds at the risk-free rate until point c is attained. Note that each investor is therefore involved in only two investments—a risky portfolio d and borrowing or lending on a risk-free loan.[10]

This same line of reasoning can easily be generalized to more realistic situations in which the number of risky securities available to investors is greater than two. In the case of n securities the CAPM best strategy for each investor is to hold the n assets in optimal proportions on the concave risk-return frontier and then adjust for the person-specific desired risk level by borrowing or lending at the risk-free rate.[11]

The thoughtful reader might now ask, "But just where is this optimal portfolio d really located?" Although in general it might in fact be very hard to plot out the entire concave frontier of attainable risk-return combinations and its tangency with the risk-free lending line, under CAPM assumptions this calculation is simple, perhaps even unnecessary. Specifically, if one makes the assumption that all investors have the same information and opportunities and that there are no taxes or transactions costs, then, even if people's attitudes toward risk differ, everyone will process information in the same way and will view their investment prospects identically. In such a case, everyone will hold exactly the same portfolio d mix of securities 1 and 2, but each person will then blend portfolio d and the risk-free asset to suit his or her own risk-return tastes. In this case the total market portfolio for all investors will simply be a blowup of portfolio d. Alternatively, each investor's portfolio will be a microscopic clone of the market as a whole. Therefore, according to the CAPM, the optimal strategy is to invest in securities in the same proportion as they are in the overall market, since these are the same as those of the best attainable portfolio, and then adjust for person-specific risk preferences by borrowing or lending at the risk-free rate.

2.4 FURTHER INTERPRETATION OF THE CAPM RISK-RETURN LINEAR RELATIONSHIP

We have shown that diversification is effective in reducing risk because prices of different stocks are imperfectly correlated. We now examine risk in greater detail. In a classic experimental study by Wayne Wagner and Sheila Lau [1971] it was shown that diversification reduces risk very rapidly at first, but that after some point, additional diversification has little effect on risk or variability. Specifically, using portfolios of different size drawn from an historical sample of stocks, Wagner and Lau demonstrated that diversification can almost halve the variability of returns but that most of this benefit can be obtained by holding relatively few stocks; the improvement becomes rather small when the number of securities is increased beyond, say, ten.[12]

Diversification cannot, of course, eliminate risk entirely. The risk that can be potentially eliminated by diversification is called *specific*, unique, or unsystematic risk. Specific risk derives from the fact that many of the dangers or opportunities that surround an individual company are peculiar to that company and, perhaps, its immediate competitors; specific risk can therefore be eliminated by holding a well-diversified portfolio. But there is also some risk that cannot be avoided no matter how much one diversifies. This risk is generally known as *market* or systematic risk. Market risk derives from the fact that there are other economywide and global dangers and opportunities that are faced by all businesses. The fact that stocks have a tendency to "move together" reflects the presence of market risk, risk that cannot be eliminated through diversification. Note that this risk will remain even when an optimal portfolio is attained.

To explore further this dependence of stock returns on market risk, note that one implication of the CAPM is that the risk of a well-diversified portfolio depends only on the market risk of the securities included in the portfolio. Suppose, therefore, that you had a well-diversified portfolio (say, a microscopic clone of the entire stock market portfolio) and that you wanted to measure the dependence of stock returns on risk further by computing the sensitivity of a particular company's stocks in your portfolio, say, those of company j, to variations in the overall market return. Obviously, you would not want to compute this by looking at company j's returns in isolation. Rather, you would want to use the covariance information from Eqs. (2.8) and (2.9) relative to the market as a whole.

Specifically, recall that we noted earlier that one measure of the relative marginal variance of a security, say, the kth asset, is called its beta value relative to the portfolio; in Eq. (2.9) this was defined as $\text{beta}_k = \sigma_{kp}/\sigma_p^2$. One interpretation of this relative variance notion is that if the return on the portfolio is expected to increase by, say, 1%, then the return on the kth asset is expected to increase by beta_k times 1%. The investment beta is therefore a measure of the sensitivity of the return on the kth asset to variation in returns on the portfolio; beta_k summarizes the dependence on portfolio risk.

It will be useful to think of the portfolio as the entire market's portfolio. Let us therefore define an investment beta for, say, company j relative to the entire market portfolio as

$$\text{Beta}_j = \sigma_{jm}/\sigma_m^2 \tag{2.14}$$

where σ_{jm} is the covariance between company j's return and that of the market as a whole and σ_m^2 is the variance of the market's returns.

There is one problem, however, in relating beta_j to the CAPM framework. The covariance and variance terms for beta_j in Eq. (2.14) refer to the *total* returns on assets, whereas by contrast in the CAPM development to this point we have dealt with variations in risk *premiums*, that is, the excess return over the risk-free rate, such as $r_m - r_f$, where r_m is the return on the entire market's assets. Can one rewrite Eq. (2.14) in terms of risk premiums rather than total returns? As we now show, we can, since beta_j is unaffected by this change.

To see this, note that since the ratio of the covariance term σ_{jm} to the variance term σ_m^2 is unaffected by subtracting the risk-free return from the total return, the investment beta ratio Eq. (2.14) holds even when it is defined in terms of risk premiums rather than in terms of total returns. This has an important implication in the CAPM context, where we deal with risk premiums rather than with total returns. Specifically, since Eq. (2.14) is invariant to rescaling in terms of risk premiums rather than total returns, the value of beta for a particular company equals the covariance of its risk premium with the market portfolio risk premium, divided by the variance of the market's risk premium. This suggests that the beta_j summary measure of dependence on market risk has wide applicability.

Securities vary considerably in the value of their investment betas; some, for example, have values as large as 2, indicating that a 1% rise or fall in the market results in a 2% rise or fall in the value of that security. Such securities are relatively risky. Other truly "blue chip" stocks, on the other hand, are not as sensitive to market movements and have much smaller betas of, say, 0.5, implying that a 1% rise or fall in the market results in a ½% rise or fall in their value. Traditionally, holding stocks with betas greater than 1 is called an "aggressive" stance, while holding stocks with betas less than unity is called "defensive." As we shall see in Exercise 3 of this chapter, for some assets, beta can even be negative—these are "superdefensive" assets!

Investment betas can also be defined for portfolios of assets (rather than individual assets) relative to the market as a whole. For example, consider a portfolio q consisting of n assets, and define its beta value relative to the market as a whole as $\text{beta}_{qm} \equiv \sigma_{qm}/\sigma_m^2$. Given the definition of covariances, one can rewrite beta_{qm} as

$$\text{Beta}_{qm} = \sum_{i=1}^{n} w_{iq} \cdot \text{beta}_{im} \tag{2.15}$$

where w_{iq} is the proportion of portfolio q invested in asset i and beta_{im} is asset i's beta value relative to the market's portfolio. Hence the beta value of a portfolio is simply a weighted average of the beta values of the component securities, the weights being the portfolio investment shares.

Obviously, for the stock market as a whole the covariation with itself is the same as its variance, implying that the beta ratio for the stock market as a whole is 1.0. Moreover, since by Eq. (2.15) the beta of a well-diversified entire market portfolio depends on the weighted average betas of the securities included in the portfolio, it follows that, on average, individual stocks have a beta of 1.0. Finally, it is worth noting that since the covariance of a riskless asset with the market portfolio is zero, beta_{im} equals zero whenever asset i is risk-free.

2.5 ECONOMETRIC ISSUES INVOLVED IN IMPLEMENTING THE CAPM

We now want to move toward econometric implementation of the CAPM framework. Our first objective is to derive an estimable equation. Consider a small portfolio p whose sole security is j and a large, well-diversified portfolio m, which is the entire market's portfolio. Substituting j for p and m for a into Eq. (2.13), we can rewrite the CAPM linear relationship (2.13) as

$$r_j - r_f = (\sigma_j/\sigma_m) \cdot (r_m - r_f) \tag{2.16}$$

where r_j and r_f are returns to security j and to a risk-free asset, respectively; r_m is the return on the overall securities market portfolio; and σ_j/σ_m is the ratio of the standard deviations of returns on asset j and the market portfolio m. The term $r_j - r_f$ is the risk premium for security j, while $r_m - r_f$ represents the overall market's risk premium. By the way, over the last 60 or so years in the United States the average market risk premium has been about 8.4% per year.[13]

According to Eq. (2.16), the risk premium on the jth asset is simply a factor of proportionality σ_j/σ_m times the market risk premium; this factor of proportionality expresses the dependence of security j's return on the market return, a dependence highlighted by the CAPM. Such reasoning suggests that the proportionality factor σ_j/σ_m must be related in some way to the investment beta_j discussed in the previous section.

To explore this relationship further, let us generalize Eq. (2.16) by adding to it an intercept term α_j and a stochastic disturbance term ϵ_j, and then define a new parameter β_j as being equal to the proportionality factor, that is, $\beta_j \equiv \sigma_j/\sigma_m$. This gives us an estimable equation relating the total risk premium of security j to the market risk premium and to the stochastic disturbance term:

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f) + \epsilon_j \tag{2.17}$$

In Eq. (2.17) the stochastic disturbance term ϵ_j reflects the effects of specific (unsystematic) and diversifiable risk. We will assume that ϵ_j has an expected value of zero and a variance of σ_ϵ^2 and that it is also independently and identically normally distributed.

Now comes the clincher: The least squares estimate of β_j in Eq. (2.17) is in fact identical to the investment beta defined in Eq. (2.14)! To see this, consider the bivariate regression model $y = \alpha + \beta x + \epsilon$. The least squares estimate of β is $\text{COV}(x, y)/\text{VAR}(x)$. Now let $y \equiv r_j - r_f$ from Eq. (2.17), and let $x \equiv r_m - r_f$. Then the least squares estimate of β_j is simply $\hat{\beta}_j = \text{COV}(r_j - r_f, r_m - r_f)/\text{VAR}(r_m - r_f)$. But this is precisely equal to σ_{jm}/σ_m^2, the investment beta defined in Eq. (2.14). Intuitively, the least squares estimate of the factor of proportionality β_j is simply the ratio of the standard deviations of the risk premiums for asset j and for the market m.

These results imply that, for any security j, one can estimate β_j using bivariate least squares procedures on Eq. (2.17). Graphically, the covariance-variance ratio is the least squares estimate of the slope of a regression line relating the risk premium of a particular security j on the vertical axis to the market risk premium on the horizontal axis; the linearity of this relationship derives from Eq. (2.13). Although one can use least squares procedures, in the bivariate regression framework, estimating β_j is in fact trivial: Once one has values of the appropriate covariances and variances, all one need do is simply compute β_j as their ratio.

To estimate the β_j parameter based on time series data of individual companies, it must of course be assumed that, for a particular company, β_j is relatively stable over time. Quite frequently, monthly data are employed that are based on returns from the New York Stock Exchange. Econometric studies based on such data have found that in many studies (but with some notable exceptions), β_j has tended to be relatively stable over a five-year (60-month) time span.[14] There are cases, however, in which the conditions in an industry or a firm abruptly change, implying that the relevant β_j might also vary. Oil company stocks, for example, had a beta below unity before the 1973 OPEC oil embargo, but this soon changed after 1973, and since then oil company betas have typically increased. Similarly, when the airline industry in the United States was deregulated in 1978, the betas of most major U.S.

airline companies rose; analogous changes in betas occurred for electric utilities, particularly for those with substantial nuclear-generating capacity, after the Chernobyl nuclear power accident in 1986. This stability of beta remains a problematic issue, however, and therefore in Exercises 6 and 7 of this chapter we will encourage you to examine the variability of estimated betas over time, using statistical techniques known as Chow tests and event study methodologies.

While the parameter β_j is of obvious interest and importance, in our estimable Eq. (2.17) another parameter appears that was added in an ad hoc manner, namely, α_j. Specifically, recall that α_j does not appear in Eq. (2.16), the linear equation analogous to Eq. (2.13). On the basis of the finance theory underlying the CAPM and summarized in Eq. (2.16), one should therefore expect estimates of α_j to be close to zero on average (more on this later, however). In fact, a typical empirical result obtained when one estimates parameters of Eq. (2.17) is that the least squares estimate of α_j is insignificantly different from zero. The null hypothesis that $\alpha_j = 0$ can be tested of course simply by determining whether the t-statistic corresponding to the estimate of α_j is greater than the critical value at some predetermined significance level. Moreover, if one wants to impose the restriction that $\alpha_j = 0$ in (2.17), most computer regression programs provide options that allow the user to omit an intercept term.

It is worth noting that in some investment houses, portfolio managers' performances have occasionally been evaluated by computing the predicted returns the manager would have earned given the betas of the companies in his or her portfolio and subtracting this predicted return from realized returns, thereby obtaining an implicit estimate of the manager's α; if the realized return were larger than that predicted by the overall portfolio β, then the portfolio manager was said to have produced a positive α, and he or she was compensated accordingly. Such simple compensation schemes are seldom used today, but they suggest an interesting interpretation of α for portfolio managers.[15]

Suppose that a securities market analyst employed monthly time series data on returns of a particular company over the preceding five years and estimated the parameters α and β using conventional linear regression techniques. Perhaps the analyst obtained the empirical finding that for a particular company the estimate of α was positive and significantly different from zero. This would imply that even if the market as a whole were expected to earn nothing (that is, if $r_m - r_f$ in Eq. (2.17) equalled zero), investors in this company expected a positive rate of price appreciation. (Note that, for example, an estimate of 0.67 for α based on monthly data is interpreted as an annual rate of price appreciation of approximately 12 times 0.67, or about 8% per year.) For some other company the analyst might find that the estimated α was negative and significantly different from zero. True adherents to the CAPM would therefore argue that, on average, one should expect estimates of α to be zero.

This raises the important issue of whether one can test the CAPM. Five remarks are worth making in this context. First, the finance theory underlying the CAPM explicitly employs *expected* or ex ante returns, whereas all we can observe is *realized* or ex post returns. This makes rigorous testing of the CAPM more difficult, although not impossible if more sophisticated econometric techniques are employed.[16]

Second, according to the CAPM, the market portfolio should include *all* risky investments, whereas most market indexes and estimates of r_m contain only a sample of stocks, say, those traded on the New York Stock Exchange (thereby excluding, for example, all risky assets traded in the rest of the world—even human capital, private businesses, and private real estate). Collecting sufficient data on risky assets to estimate r_m reliably could be a formidable and prohibitively expensive task. In this context it is worth noting that early empirical studies based on the CAPM discovered that estimates of β depended critically on the choice of r_m; specifically, r_m measures based on the Dow Jones 30 Industrials Index provided different results from those based on the Standard & Poor 500 Index or the Wilshire 5000.[17] Today, funded in large part by Merrill, Lynch, Pierce, Fenner & Smith, Inc., the Center for Research on Securities Prices (CRSP) at the University of Chicago makes available to researchers estimates of r_m based on the value-weighted transactions of all stocks listed on the New York and American Stock Exchanges. Fortunately, such data series on r_m have been made available to us, and in the hands-on applications of this chapter we will employ the CRSP measures of r_m.

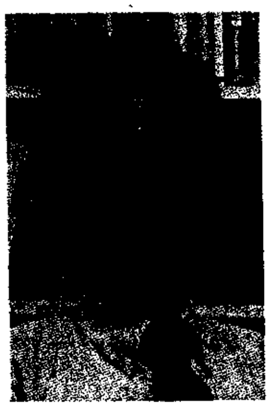
Third, the typical measure of a risk-free asset is something like the 30-day U.S. Treasury bill rate, which really is risk-free only if it is held to maturity. Further, it is risk-free only in a nominal sense, for even if it is held to maturity, the uncertainty of inflation makes the real rate of return uncertain. Therefore it is difficult, if not impossible, to obtain a good measure of the risk-free return. This makes testing of the CAPM model even more troublesome.

Fourth, there is a strong tradition within statistics that argues that it is not possible to test a specific model unless there is an alternative viable model against which it can be compared. In the case of the CAPM, one alternative that might be used is the ingenious arbitrage pricing model (APM) developed by Stephen Ross, which in its simplest interpretation essentially involves adding right-hand variables to Eq. (2.17), such as the rate of unexpected price inflation.[18] Provided that the above issues were properly taken into account, one could in principle test the CAPM against the APM merely by testing the null hypothesis that the parameters on the additional right-hand variables were simultaneously equal to zero against the alternative hypothesis that these parameters were simultaneously not equal to zero.

Fifth and finally, a more informal method of testing the CAPM involves checking whether its predictions are consistent with what is observed. In this context, two aspects of the CAPM model appear to be inconsistent with observation.

STEPHEN A. ROSS

Generalizing the CAPM



Although the capital asset pricing model of modern finance theory is widely taught and implemented today, the validity of the CAPM hinges on a number of restrictive assumptions. One very useful generalization of the CAPM is called the arbitrage pricing model (APM). The APM allows for factors other than the market risk premium to affect returns on securities. The APM was developed in 1976 by Stephen A. Ross, a financial economist then at the University of Pennsylvania.

Ross's research on the APM has attracted a great deal of interest and controversy and has raised important methodological issues of whether it is possible to test the CAPM as a special case of the APM (see the text for further details and references).

Born in Boston in 1944, Stephen Ross developed a strong interest in applied mathematics. He received his B.S. degree with honors from the California Institute of Technology, majoring in physics. His attention shifted to economics and finance, however, and in 1969 he earned a doctorate in economics from Harvard.

Although Ross is widely recognized as a rigorous theoretical financial researcher, he is also respected as an unusually clear expositor. At the University of Pennsylvania, for example, he was nominated for distinguished teaching awards three years in succession. In 1977, Ross moved to Yale University, and today he holds its Sterling Professor of Economics and Finance chair, a chair previously held by Nobel Laureate James Tobin.

In addition to undertaking academic research in finance theory, Ross regularly travels from Yale to Wall Street and New York City, where he engages in financial consulting and serves as a principal with Richard Roll in Roll and Ross Asset Management Corporation. Ross is on the editorial board of numerous professional journals, is a Fellow of the Econometric Society, and in 1988 was elected President of the American Finance Association.

1. One important implication of the CAPM framework is that the relationship between risk and return is not only linear but is also positive. As has been noted by Fischer Black, Michael Jensen, and Myron Scholes [1972], however, there have been a number of instances in which this relationship has appeared to be negative rather than positive. Specifically, Black, Jensen, and

Scholes found that over the entire nine-year period from April 1957 through December 1965, securities with higher βs produced lower returns than less risky (lower-β) securities. Why this happened is not yet entirely clear and constitutes a puzzling contradiction of the CAPM framework.

2. Another implication of the CAPM framework is that a security with a zero β should give a return exactly equal to the risk-free rate. Black, Jensen, and Scholes exhaustively studied security returns on the New York Stock Exchange over a 35-year period and found instead that the measured zero-β rate of return exceeded the risk-free rate, implying that some unsystematic (or non-β) risk makes the return higher for the zero-β portfolio than is predicted by the CAPM. Moreover, the actual risk-return relationship examined by Black, Jensen, and Scholes appeared to be flatter than that predicted by the CAPM. It is therefore not clear what factors other than the market risk premium are being valued in the marketplace. According to the CAPM, it is only market risk that matters, since unsystematic risk can be diversified away. Much research is currently underway that attempts to examine whether factors other than r_m affect market risk.

In this context it is worth noting that some securities firms such as Merrill, Lynch, Pierce, Fenner and Smith regularly publish a "beta book" in which they report estimates of α and β based on standard OLS regression methods, as well as "adjusted β" estimates that attempt to deal with the zero-β portfolio problem mentioned above, using more complex Bayesian statistical procedures. Although these Bayesian procedures are of considerable interest, they are beyond the scope of this chapter.

Several other econometric issues should be briefly noted. The printed output of computer regression programs typically includes measures of R^2, the standard error of the regression, and t-statistics. These and other standard statistical measures have a particularly interesting interpretation and application within the CAPM. Consider, for example, the simple correlation between the risk premium on the security j $(r_j - r_f)$ and the market risk premium $(r_m - r_f)$—variables that are on the left- and right-hand sides, respectively, of the CAPM regression equation (2.17). The sample correlation coefficient between them can be rewritten as follows:

$$\rho_{jm} = \frac{\hat{\sigma}_{jm}}{\hat{\sigma}_j \hat{\sigma}_m} = \frac{\hat{\sigma}_{jm}}{\hat{\sigma}_m^2} \cdot \frac{\hat{\sigma}_m}{\hat{\sigma}_j} = \hat{\beta}_j \cdot \frac{\hat{\sigma}_m}{\hat{\sigma}_j} \tag{2.18}$$

where $\hat{\sigma}_{jm}$, $\hat{\sigma}_m^2$, $\hat{\sigma}_j^2$ are sample covariances and variances for $r_m - r_f$ and $r_j - r_f$, and $\hat{\beta}_j$ is the least squares estimate of β_j. Hence the sample correlation between the portfolio and market risk premiums is simply the product of the least squares estimate of β_j and the relative sample standard deviations of the market and j-portfolio risk premiums.

The standard error of the residual in the regression equation (2.17) also has a useful interpretation. Specifically, while the left-hand side of Eq. (2.17) reflects the effects of both specific (unsystematic) and market (systematic) risk on the portfolio in company j, the $\beta_j(r_m - r_f)$ term on the right-hand side reflects only the impact of market risk. It therefore follows that the estimated residual in Eq. (2.17) incorporates only the effects of specific (unsystematic) risk. The standard error of the residual (also often called the standard error of the regression), computed as the square root of s^2, defined as

$$s^2 = \sum_{t=1}^{T} e_t^2/(n - 2) \tag{2.19}$$

where e_t is the least squares residual for the tth observation, therefore measures the standard deviation of the specific (unsystematic) risk—portfolio risk that is not responsive to market fluctuations. A large standard error of the residual, say, $s = 15\%$ per month, would indicate that a substantial amount of change in the portfolio j risk premium could not be explained by changes in the market risk premium.

Further, since the R^2 value from regression computer output indicates what proportion of the variation in the dependent variable is explained by variation in the right-hand or independent variables, in the CAPM context of Eq. (2.17), R^2 measures the market (systematic) portion of total risk. On the other hand, $1 - R^2$ is the proportion of total risk that is specific (unsystematic). William F. Sharpe [1985, p. 167] notes that for an individual company a typical R^2 measure from a CAPM equation is about .30 but that as one diversifies across companies' assets into a larger portfolio, the R^2 measure increases, owing to the reduction of specific risk through diversification.

It is important to note that, since in the bivariate regression model $R^2 = \rho_{jm}^2$, high R^2 values do not necessarily correspond with large estimates of β_j. To see this, note that from Eq. (2.15),

$$R^2 = \rho_{jm}^2 = \hat{\beta}_j^2 \cdot \frac{\hat{\sigma}_m^2}{\hat{\sigma}_j^2} \tag{2.20}$$

It follows that for some stocks with very large variance $\hat{\sigma}_j^2$, R^2 can be low even while the estimate of β_j is high; in such cases the reaction of the particular stock (or portfolio) to market variations is very sharp, yet market variation explains only a small portion of the stock's large variability. The regression equation for other stocks might have a high R^2 but a low β_j estimate; this can occur when variation in the stock's (or portfolio's) risk premium is small in relation to variation in the market risk premium, that is, the ratio of sample variances in Eq. (2.20) is large. Moreover, note that a very low R^2 does not invalidate the CAPM framework; rather, it simply indicates that the total risk of a particular company's assets is almost entirely company-specific, unrelated to the market as a whole.

The final typical regression output of particular interest to us here is the t-statistic. Earlier, it was noted that the t-statistic on the estimate of α can be used to test directly the null hypothesis that $\alpha = 0$ against the alternative hypothesis that $\alpha \neq 0$. Failure to reject this null hypothesis might be viewed as evidence in support of the CAPM.

The t-statistic on the estimate of β corresponds to an analogous null hypothesis, namely, that $\beta = 0$, against the alternative hypothesis that $\beta \neq 0$. Quite frequently, it is of interest to test a different hypothesis, namely, that the movement of asset prices of a particular company is the same as that of the market as a whole; this corresponds to a test of the null hypothesis that $\beta = 1$ against the alternative hypothesis that $\beta \neq 1$. To conduct such an hypothesis test, simply take the estimated standard error of the β estimate (usually provided on computer regression outputs), construct a confidence interval given some reasonable level of confidence (e.g., 95% or 99%), and then determine whether $\beta = 1$ falls within this confidence interval. If it does, the null hypothesis is not rejected; if $\beta = 1$ does not fall within this confidence interval, the null hypothesis is rejected.

One other remark is worth making before we go on to hands-on empirical implementations. Since within the CAPM framework, β enables one to calculate what the required return on a particular stock might be, it also indirectly allows one to compute the all-equity firm's cost of capital. For a well-managed firm that is considering a particular investment project, the combination of expected return and its β should, of course, place the project above its cost of capital if the project is to be accepted. However, if the company is considering a new project that is more risky than the company's average projects, a larger expected return should be required by the firm before investing, since such a project would increase the average risk of the company and, according to the CAPM model, would result in investors' requiring a higher return. This implies that, analogous to Eq. (2.15), projects also have their own betas, and if the β of a particular project is higher than the company's average β, so too should be the required expected rate of return.[19]

This completes our discussion of the theory underlying the CAPM and our brief review of econometric issues involved in estimating its parameters. We are now ready to become directly involved in implementing the CAPM empirically.

2.6 HANDS ON WITH THE CAPM

The purpose of the exercises in this chapter is to help you gain an empirical hands-on experience and understanding of the CAPM using bivariate least squares regression techniques. Exercise 1 is particularly useful, since in it you become acquainted with data used in subsequent exercises. In Exercise 2 you obtain and interpret least squares estimates of β, while in Exercise 3 you encounter the β of a very interesting asset—gold. In Exercise 4 you are given the opportunity to learn how you can recover from mistakes—running a regression of X on Y rather than of Y on X for Delta Airlines. In Exercise 5 you gain a better understanding of the benefits of diversification by employing the CAPM to construct a portfolio.

The last five exercises of this chapter employ slightly more sophisticated econometric techniques; therefore you might want to return to these exercises later after becoming more familiar with the requisite econometric tools. In particular, in Exercise 6 you test for the stability of β using the Chow test procedure. In Exercise 7 you employ dummy variables in "event studies" for the Three Mile Island nuclear power accident and for the bidding war between DuPont and Dow Chemical in their takeover attempt of Conoco. In Exercise 8 you employ Chow tests and dummy variables to assess whether January returns are different from those of other months, as has apparently been observed. Then in Exercise 9 you assess an alternative to the CAPM framework, the arbitrage pricing model, by comparing results from multivariate and bivariate regressions. In the final application, Exercise 10, you conduct a number of tests to determine the empirical validity of the stochastic specification.

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On the data diskette provided, you will find a subdirectory called CHAP2.DAT with numerous data files. In these data files a number of data series are provided, including data on ten years of monthly returns for 17 companies over the time period January 1978 to December 1987 (values for r_p). In another data file named MARKET, you will find a data series taken from the Center for Research on Securities Prices (often called CRSP) that measures r_m; specifically, MARKET is a value-weighted composite monthly market return based on transactions from the New York Stock Exchange and the American Exchange over the same ten-year time span. Note that the CRSP measure of r_m includes data on all stocks listed at the New York and American Stock Exchanges, not just the 30 used for the Dow Jones index or the Standard & Poor 500. Another data file is called RKFREE, and it provides you with a measure of r_f, namely, the return on 30-day U.S. Treasury bills. The 17 companies whose returns are in the master file of CHAP2.DAT operate primarily in eight industries:

Industry	*Company*	*File Name*
Oil	Mobil	MOBIL
	Texaco	TEXACO
Computers	International Business Machines	IBM
	Digital Equipment Company	DEC
	Data General	DATGEN
Electric utilities	Consolidated Edison	CONED
	Public Service of New Hampshire	PSNH
Forest products	Weyerhauser	WEYER
	Boise	BOISE
Electronic components	Motorola	MOTOR
	Tandy	TANDY
Airlines	Pan American Airways	PANAM
	Delta	DELTA

Banks	Continental Illinois	CONTIL
	Citicorp	CITCRP
Foods	Gerber	GERBER
	General Mills	GENMIL

These firms and industries display considerable variation in risk and returns. Several other data files are in the CHAP2.DAT subdirectory, such as GOLD, EVENTS, and APM. The contents of those data files are discussed in the exercises below.

At numerous times in the following exercises we will ask you to test an hypothesis using a "reasonable level of significance." Since what is "reasonable" is to some extent discretionary, you are asked to follow the convention of stating precisely what percent significance level is being used to test hypotheses; in that way, others with different preferences can still draw their own inferences.

Note: Remember that before the data diskette can be used, the data files must be properly formatted. For further information, refer back to Chapter 1, Section 1.3. MAKE SURE THAT ALL DATA FILES TO BE USED BELOW ARE FORMATTED.

EXERCISES

EXERCISE 1: Getting Started with the Data

The purpose of this exercise is to help you become more familiar with the data series in your subdirectory CHAP2.DAT and to have you speculate on what values company-specific betas might take.

(a) Using data in the file named MARKET, print and plot returns for the last 36 months in the data series, from January 1985 through December 1987. (The output of most computer regression programs looks rather messy if the full time period of 120 months is plotted.)

(b) Next, using data from the MARKET and RKFREE files covering the entire 120-month time span, construct the risk premium measure ($r_p - r_f$) for any company of your choice and the overall market risk premium ($r_m - r_f$). (*Note:* It is possible that for some months the risk premium of a particular company's assets, or that for the market as a whole, is negative. Does this occur in your sample? In particular, check for October 1987. What happened in that month?) Compute and print out sample means for r_p, r_m, r_f, $r_p - r_f$, and $r_m - r_f$. Note that these returns are monthly, not annual. One way of converting the mean monthly returns to annual equivalents is based on the formula

$$r_{\text{annual}} = (1 + \bar{r})^{12} - 1$$

where r_{annual} is the annual return and $\bar{r}$ is the mean monthly return. A resulting value of .060, for example, corresponds to a 6.0% annual return. Compute annual returns for r_p, r_m, r_f, $r_p - r_f$, and $r_m - r_f$. Do these values appear plausible? Why or why not?

(c) Next plot both the company's and the market's risk premium for the last 36 months in the data series, from January 1985 through December 1987 (make sure months coincide for the two series). Are there any noteworthy aspects of these plots? Do you think that this company's β for this time period will be greater than or less than unity? Why? Does this make intuitive sense?

(d) Finally, calculate the variance and the standard deviation of the company's and market's risk premiums over the same 36-month time period as in part (c), as well as the simple correlation coefficient between them. Using Eq. (2.18) and the above values, calculate the implied β for this company. Is this β roughly equal to what you expected?

EXERCISE 2: Least Squares Estimates of β

From the list of industries on pages 42–43, choose one industry that you think is highly risky and another industry that you think is relatively "safe." Divide your sample into the first half (January 1978–December 1982) and the second half (January 1983–December 1987) and choose the half with which you will work.

(a) Using your computer regression software, the 60 observations you have chosen, and Eq. (2.17), estimate by ordinary least squares the parameters α and β for one firm in each of these two industries. Do the estimates of β correspond well with your prior intuition or beliefs? Why or why not?

(b) For one of these companies, make a time plot of the historical company risk premium, the company risk premium predicted by the regression model, and the associated residuals. Are there any episodes or dates that appear to correspond with unusually large residuals? If so, attempt to interpret them.

(c) For each of the companies, test the null hypothesis that $\alpha = 0$ against the alternative hypothesis that $\alpha \neq 0$, using a significance level of 95%. Would rejection of this null hypothesis imply that the CAPM has been invalidated? Why or why not?

(d) For each company, construct a 95% confidence interval for β. Then test the null hypothesis that the company's risk is the same as the average risk over the entire market, that is, test that $\beta = 1$ against the alternative hypothesis that $\beta \neq 1$. Did you find any surprises?

(e) For each of the two companies, compute the proportion of total risk that is market risk, also called systematic and nondiversifiable. William F.

Sharpe [1985, p. 167] states that "Uncertainty about the overall market . . . accounts for only 30% of the uncertainty about the prospects for a typical stock." Does evidence from the two companies you have chosen correspond to Sharpe's typical stock? Why or why not? What is the proportion of total risk that is specific and diversifiable? Do these proportions surprise you? Why?

(f) In your sample, do large estimates of β correspond with higher R^2 values? Would you expect this always to be the case? Why or why not?

EXERCISE 3: Why Gold Is Special

The purpose of this exercise is to acquaint you with features of a rather remarkable asset whose peculiar covariance of returns with the market as a whole often makes it attractive to investors.

(a) There is one asset in the data directory whose data file is named GOLD. The GOLD data file contains series on monthly returns for GOLD, as well as data series for the market (MARK76) and risk-free (RKFR76) variables, all for the January 1976–December 1985 time period. Using the January 1976–December 1979 four-year time period and the CAPM, generate variables measuring the GOLD-specific and market risk premiums, and then estimate the β for GOLD. Compute a 95% confidence interval for β. Do your estimates make sense? Why might such an asset be particularly desirable to an investor who is attempting to reduce risk through diversification? What does this imply concerning the expected return on such an asset?

(b) Now estimate the β for GOLD using data from January 1980–December 1985. Construct a 95% confidence interval for β. Has anything changed? Comment on supply and demand shift factors possibly altering the β of GOLD.

EXERCISE 4: Consequences of Running the Regression Backward

The purpose of this exercise is to explore the consequences of running a regression backwards, that is, of regressing X on Y rather than Y on X, and to discover how one can recover the "correct" estimates from the computer output of the "incorrect" regression.

(a) Using data from January 1983 through December 1987 for Delta Airlines in the data file DELTA, as well as data on r_m in MARKET and on r_f in RKFREE, construct the risk premium for Delta Airlines and for the market as a whole. To simplify notation, now define $Y_t = r_{pt} - r_{ft}$ (the risk premium for Delta Airlines) and $X_t = r_{mt} - r_{ft}$ (the market risk premium), $t = 1, \ldots, 120$.

Suppose that instead of specifying the "correct" CAPM regression equation

$$Y_t = \alpha + \beta X_t + \epsilon_t \tag{2.21}$$

you inadvertently specified the "incorrect" reciprocal regression equation

$$X_t = \delta + \gamma Y_t + \upsilon_t \tag{2.22}$$

Show that in the incorrect equation, one can still find the original CAPM parameters and disturbances, that is, solve Eq. (2.21) for X_t in terms of Y_t and ϵ_t and show that $\delta = -\alpha/\beta$, $\gamma = 1/\beta$, and $\upsilon_t = (-1/\beta)\epsilon_t$.

(b) Now estimate by OLS the parameters in the incorrect equation (2.22), and denote these estimates of δ and γ by d and g, respectively. What is the R^2 from this incorrect regression? At a reasonable level of significance, test the null hypothesis that $\gamma = 0$ against the alternative hypothesis that $\gamma \neq 0$. Next, construct implicit estimates of the CAPM β and α parameters from this incorrect regression as $b_x = 1/g$ and $a_x = -d/g$ (the subscript x indicates that the estimate is obtained from a regression of X on Y).

(c) Having suddenly discovered your mistake, now run the correct regression and estimate by OLS the α and β parameters in Eq. (2.21), and denote these parameter estimates as a_y and b_y (the subscript y indicates that the estimate is obtained from a regression of Y on X). What is the R^2 from this regression? At a reasonable level of significance, test the null hypothesis that $\beta = 0$ against the alternative hypothesis that $\beta \neq 0$.

(d) Notice that the R^2 from the correct equation (2.21) in part (c) is identical to the R^2 from the incorrect equation (2.22) in part (b) and that so too are the t-statistics computed in parts (b) and (c). Why does this occur? Using the formula for R^2 from your econometric theory textbook and for the OLS estimates of β in Eq. (2.21) and γ in Eq. (2.22), show that

$$R^2 = b_y \cdot g = b_y/b_x \tag{2.23}$$

Notice that what Eq. (2.23) implies is that if you run the wrong regression (Eq. 2.22) and obtain g rather than b_y, you can simply use your R^2 from this incorrect regression, your value of g and Eq. (2.23) to solve for b_x—the estimated value of β you would have obtained had you run the correct regression! Verify numerically that this relationship among R^2, b_y, g, and b_x occurs with your data for Delta Airlines.

(e) Finally, notice that since R^2 is always less than unity, Eq. (2.23) implies that $b_y < b_x$. Which estimate of β do you prefer—the smaller b_y or the larger b_x? Why?

EXERCISE 5: Using the CAPM to Construct Portfolios

The purpose of this exercise is to provide you with a greater appreciation of the benefits of diversification. Imagine that in December 1982 your rich uncle died and bequeathed to you one million dollars, which he specified that you must invest immediately in risky assets. Further, according to the terms of the estate, you could not draw on any funds until you graduated. Therefore in late 1982 you needed to decide how to invest the proceeds of this estate so that upon graduation you would have ample funds for the future. You will now examine the effects of diversification upon risk and return, given three alternative portfolios that you might have chosen in January 1983.

Choose one industry that you think is relatively risky and another industry that is relatively safe, from the list on pages 42–43. Choose two companies in the safe industry and two in the relatively riskier industry.

(a) Calculate the means and standard deviations of the returns for each of the four companies over the January 1983–December 1987 time period. Do the risk-return patterns for these companies correspond with your prior expectations? Why or why not? In which might your uncle have invested? Why?

(b) Construct three alternative (one million dollar total) portfolios as follows. Portfolio I: 50% in a company in the safe industry and 50% in a company from the risky industry. Portfolio II: 50% in each of the two companies in the safe industry. Portfolio III: 50% in each of the two companies in the risky industry. Calculate the sample correlation coefficient between the two company returns in each of the three portfolios. Comment on the size and interpretation of these correlations. For each of the three portfolios, calculate the means and standard deviations of returns over the January 1983–December 1987 time period. Are there any surprises?

(c) Which of the three alternative portfolio diversifications would have been most justifiable in terms of reducing the unsystematic risk of investment? Why? (Choose your words carefully.)

(d) Next estimate the CAPM equation (2.17) for each of these three portfolios over the same January 1983–December 1987 time period. For each portfolio, using a reasonable level of significance, test the null hypothesis that $\beta = 1$ against the alternative hypothesis that $\beta \neq 1$. Which portfolio had the smallest proportion of unsystematic risk?

(e) For Portfolio I, compare the R^2 from the portfolio regression in part (d) with the R^2 from the separate regressions for the two companies. Would you expect the R^2 from the portfolio equation to be higher than that from the individual equations? Why or why not? Interpret your findings.

EXERCISE 6: Assessing the Stability of β over Time and Among Companies

An important maintained assumption of much empirical work in the CAPM framework is that the β parameter is stable over time. To the extent that companies within the same industry are relatively similar, one might also expect that the βs are equal across companies in the same industry. In this exercise, such assumptions are tested, using the Chow test for parameter stability.[20]

(a) From your data file, choose two industries. To allow for possible structural change, for each company in the sample, divide the historical data series into two halves, one for January 1978–December 1982 and the other for January 1983–December 1987. Using Eq. (2.17) and a reasonable level of significance, test the null hypothesis that for each company the parameters α and β are equal over the two half-samples, that is, that for each company these parameters are constant over time.

(b) Now, for each industry, using a reasonable level of significance, test the null hypothesis that the parameters α and β are identical for all firms in the industry over the entire January 1978–December 1987 time period. (Be particularly careful in how you compute degrees of freedom.)

(c) Finally, for each industry, using a reasonable level of significance, test the null hypothesis that the parameters α and β are identical for all companies in the industry *and* are equal over *both* the January 1978–December 1982 and January 1983–December 1987 time intervals. (Be particularly careful in calculating degrees of freedom.) What do you conclude concerning the stability of parameters over companies and time?

EXERCISE 7: Three Mile Island and the Conoco Takeover: Event Studies

An important behavioral assumption underlying the CAPM is that all investors efficiently use whatever information is available to them. It is also typically assumed that all investors have equal access to information. One implication of this is that if an unexpected event occurs that generates information to all investors concerning changes in the expected future profitability of a particular company, such a release of information should immediately raise or lower the price of that company's securities. This sharp change in the value of a company's assets will often, but not always, tend to be a "once and for all" shift, and thereafter movements in the company's securities are often again determined by its β and the overall market risk premium.

In this exercise we examine and quantify the effects of certain events that generated information relevant to investors. We consider two cases:

(1) the effect of the Three Mile Island nuclear power plant accident on March 28, 1979, on the returns earned by General Public Utilities, the parent holding company of the Three Mile Island generating plant[21]; and (2) the effect of the bidding war between, among others, DuPont and Dow Chemical in their attempt to take over Conoco in June–August 1981.[22] Incidentally, in both cases it will be implicitly assumed that the companies' subsequent betas were unaffected by the new information.

In your subdirectory CHAP2.DAT is a data file named EVENTS, containing 120 months' data on monthly returns for GPU, DUPONT, and DOW as well as the RKFREE return and the overall MARKET return; these data cover the time period from January 1976 through December 1985. This file also includes data for CONOCO, but the data cover only the time period of January 1978 through September 1981, since Conoco was taken over at that time.

(a) First, generate values of the company-specific and overall market risk premiums for GPU, DUPONT, and DOW over the January 1976–December 1985 time period and for CONOCO covering the January 1976–September 1981 time interval. Compute sample means and print them out.

(b) Then, using the CAPM equation (2.17), estimate the α and β of General Public Utilities over the January 1976–December 1985 time period, but exclude from your sample the month of April 1979. On the basis of your regression model, compute the fitted value for April 1979 and then the residual for April 1979, the period just after the Three Mile Island nuclear power accident occurred. (*Note:* The monthly data are daily averages, and since the accident occurred in the last days of March 1979, it does not substantially affect the monthly March 1979 data.) How do you interpret this residual?

(c) Now form a dummy variable (denoted TMIDUM) whose value in April 1979 is unity and whose value is zero in all other months. Using least squares regression methods, estimate an expanded CAPM model in which Eq. (2.17) is extended to include the dummy variable TMIDUM. Compare the value of the estimated coefficient on the TMIDUM variable with the value of the residual in part (b) for April 1979. Also compare the estimated slope coefficients in the two regressions. Why are these equal? Finally, using a reasonable level of significance, test the null hypothesis that the coefficient on the TMIDUM variable is equal to zero against the alternative hypothesis that it is not equal to zero. Was the Three Mile Island accident a significant event?

(d) Using the CAPM equation (2.17) and as lengthy a time period sample as possible, estimate the α and β of the three companies involved in the Conoco takeover event: DuPont (the eventually successful takeover bidder), Dow (an unsuccessful bidder), and Conoco (the takeover target); note that for Conoco, the data series terminates in September 1981, since at that time the takeover was completed. For each company, construct the average of the regression residuals for the months of June, July, August, and September 1981.

(e) Next, construct for DuPont, Dow, and Conoco a dummy variable whose value is 1 during the four months of June, July, August, and September 1981 and whose value is 0 in all other months. Using least squares regression methods, estimate an expanded CAPM model over the January 1976–December 1985 time period for DuPont and Dow (and for January 1976–September 1981 for Conoco), in which Eq. (2.17) is extended to include the June–September 1981 dummy variable. Compare the value of the estimated coefficient on this dummy variable for each company with the arithmetic average value of its residuals over the June–September 1981 time period from part (d). Then compare the estimated betas. Why does the equality relationship of part (d) no longer hold here? How do you interpret the signs of these estimated coefficients; in particular, which shareholders were the winners and which were the losers during the Conoco takeover efforts? Is this plausible? Finally, using a reasonable level of significance, for each of the three companies test the null hypothesis that the coefficient on the June–September 1981 dummy variable is equal to zero against the alternative hypothesis that it is not equal to zero.

EXERCISE 8: Is January Different?

There is some tentative evidence supporting the notion that stock returns in the month of January are, other things being equal, higher than in other months, especially for smaller companies.[23] Why this might be the case is not clear, since even if investors sold losing stocks during December for tax reasons, the expectation that January returns would be higher would shift supply and demand curves and thereby would tend to equilibrate returns over the year via the possibility of intertemporal arbitrage.[24] However, the "January is different" hypothesis does seem worth checking out empirically. In this exercise you investigate how this hypothesis might be tested.

(a) First, if the "January premium" affected the overall market return r_m and the risk-free return r_f by the same amount, say, j_m, show that the market risk premium would be unaffected. In this case, could the "January is different" hypothesis be tested within the CAPM framework of Eq. (2.17)? Would it not make more sense, however, to assume that the "January is different" hypothesis referred only to risky assets? Why or why not?

(b) Suppose instead, therefore, that the "January premium" did not affect risk-free assets. In this case, one might hypothesize that in January the overall market return would change from r_m to r'_m, where $r'_m = r_m + j_m$

and j_m is the January premium. Would the market risk premium be affected? Why or why not? Further, if the CAPM model were true and the α and β parameters were constant, show that in January the expected portfolio return should increase to r_p', where $r_p' = r_p + \beta \cdot j_m$. Rewrite Eq. (2.17) using the right-hand sides of the above r_m' and r_p' expressions in place of r_m and r_p, respectively; then, noting that $\beta \cdot j_m$ is unobservable, subtract $\beta \cdot j_m$ from both sides. With what are you left compared to Eq. (2.17)? What does this indicate concerning the possibility of testing the "January is different" hypothesis within the CAPM framework?

(c) Given the conclusions that emerged from parts (a) and (b), it would seem to make sense to abandon the CAPM framework and instead to examine alternative ways of testing the "January is different" hypothesis. To do that, choose any three industries in your January 1978–December 1987 sample. Form a dummy variable called DUMJ that takes the value of unity if the month is January and is zero for all other months; also ensure that for each company the variable r_p is accessible from your data files (*not* the $r_p - r_f$ risk premium variable used in the CAPM regression). Now for each company run the regression of r_p on an intercept term and on the dummy variable DUMJ. Using a reasonable level of significance, test the null hypothesis that the coefficient on the DUMJ variable is zero against the alternative hypothesis that it is not equal to zero. Is January different?

(d) Now suppose that in spite of the reservations developed in parts (a) and (b) concerning the possibility of testing the "January is different" hypothesis within the CAPM framework, sheer perseverance compels you still to estimate Eq. (2.17). More specifically, for each company in the same sample as in part (c), form a subsample that consists only of the ten January observations (one from each year). Then form the complementary subsample consisting of the remaining 11 months of each year for each company. Next, using the Chow test procedure,[25] test the null hypothesis that the parameters of the CAPM in January are equal to those of the remaining months of the year. Interpret your results carefully. What is the alternative hypothesis in this case?

(e) Yet one other way of examining this "January is different" hypothesis within the CAPM framework is to assume that the β parameter is constant over all months but that the January intercept term might differ from that for the remaining 11 months of the year. Set up and estimate a CAPM model, using the same companies as in parts (c) and (d), in which the slope coefficient β is the same for all months, while the intercept term for January and the common intercept term for all other months of the year are permitted to differ. Using a reasonable level of significance, test the null hypothesis that "January is different." Then test the null hypothesis that "January is better."

(f) Comment on the various tests and test results in parts (a) through (e). What do you conclude? Is January different?

EXERCISE 9: Comparing the Capital Asset and Arbitrage Pricing Models

As was noted in the text, an alternative model of asset pricing has been developed by Stephen A. Ross and has been called the arbitrage pricing model (APM).[26] In our context the APM is of interest in that the CAPM might be viewed as a testable special case of a more general model. More specifically, in the APM, securities are allowed to respond differentially to economywide "surprises," such as unexpected oil price shocks, or unexpected changes in the overall rate of inflation. In this exercise, the CAPM is treated as a special case of the APM using the multivariate linear regression model. It is worth noting here that while typical applications of the APM use quite sophisticated empirical methods, such as factor analysis, for pedagogical purposes here we will simply treat the CAPM and the bivariate regression model as a special case of the APM in the multiple regression context.

A data file named APM in the subdirectory CHAP2.DAT contains monthly time series for three data series: the consumer price index (CPI), the price of domestic crude oil (POIL), and the Federal Reserve Board index of industrial production (FRBIND). Make sure that all these data series are properly formatted (see Chapter 1, Section 1.3 for further details).

(a) Choose two industries whose companies, in your judgment, might be particularly sensitive to aggregate economic conditions. For each firm in this sample, using the entire January 1978–December 1987 time span, first construct a data series called RINF reflecting the rate of inflation (CPI in month t minus CPI in month $t-1$ divided by CPI in month $t-1$); another data series called ROIL—the growth rate in the real price of oil—computed as $(\text{POIL/CPI})_t - (\text{POIL/CPI})_{t-1}$, all divided by $(\text{POIL/CPI})_{t-1}$; and finally, a data series GIND reflecting growth in industrial production, computed as FRBIND in month t minus FRBIND in month $t-1$, divided by FRBIND in month $t-1$. (*Note:* The data series for CPI, POIL, and FRBIND begin in December 1977, not January 1978; this permits you to compute changes in these variables for the full January 1978–December 1987 time period.) Compute the sample means of RINF, ROIL, and GIND and then generate simple "surprise" variables SURINF, SUROIL, and SURIND as RINF minus its sample mean, ROIL minus its sample mean, and GIND minus its sample mean, respectively. Print out the data series for SURINF, SUROIL, and SURIND.

(b) Next, estimate the standard CAPM model for your chosen company and then estimate a multiple regression model in which the right-hand variables include a constant, the market risk premium of the CAPM model, and the "surprise" variables SURINF, SUROIL, and SURIND.

Using a reasonable level of significance, test the joint null hypothesis that the coefficients of these three additional variables are simultaneously equal to zero. Interpret the results.

(c) On the basis of your results in part (b), develop and defend a model specification for each company that best reflects the factors affecting company returns. Do you conclude that the CAPM model is supported, or do you reject it? Is market risk entirely captured by the market risk premium, or do other variables affect market risk? Are any of these results surprising? Why?

EXERCISE 10: What about Our Assumptions Concerning Disturbances? Did We Mess Up?

In order for the ordinary least squares estimator to have the optimal properties, it has been assumed that the ϵ_j disturbances are independently and identically normally distributed with mean zero and variance σ^2. In this exercise we test these assumptions. Note that parts (a), (b), (c), and (d) are independent questions and, in particular, are not sequential; hence any subset of them can be performed once the requisite econometric procedures have been learned.

(a) Testing for homoskedasticity: A number of tests have been proposed to test the null hypothesis of homoskedasticity against an alternative hypothesis consisting of either a specific or some unspecified form of heteroskedasticity. Here we employ a very simple test, due to Halbert J. White [1980].

From your master data file in CHAP2.DAT, choose any two firms. For the first firm in the sample, on the basis of the entire January 1978–December 1987 data series, estimate parameters in the CAPM equation (2.17) using ordinary least squares. Retrieve the residuals from this OLS regression and then generate a new variable defined as the square of the residual at each observation. (*Note:* Take a look at the residual for October 1987. Is it unusual? Why might this be the case?) Next, run an auxiliary OLS regression equation in which the dependent variable is the squared residual series from the first regression and the right-hand variables include a constant term, the market risk premium r_{mt}, and the square of the market risk premium, that is, r_{mt}^2. If one assumes that the original disturbances are homokurtic (that is, the expected value of ϵ_t^4 is a constant), then under the null hypothesis, T (the sample size, here, 120) times the R^2 from this auxiliary regression is distributed asymptotically as a chi-square random variable with two degrees of freedom. Compute this chi-square test statistic for homoskedasticity and compare it to the 95% critical value. Are your data from the first firm consistent with the homoskedasticity assumption? Perform the same White test for tributed reasonably symmetrically and approximately normally; see, for example, Franco Modigliani and Gerald A. Pogue [1974a, b].

5. There is a substantial body of literature demonstrating that, on average, investors in the United States have in fact received higher rates of return for bearing greater risk. See, for example, the empirical study by Roger Ibbotson and Rex Sinquefield [1986] covering the 60-year time span from 1926 to 1985.
6. Note, however, that the price of this asset is guaranteed only if it is held to maturity. Further, although the *nominal* return is known with certainty if the asset is held to maturity, the extent of inflation is uncertain, and thus its *real* rate of return is not risk-free.
7. For a more complete treatment of diversification, see the seminal article by Harry M. Markowitz [1952].
8. This exposition is admittedly very simple and ignores complications of shares adding to unity, as well as the source of funding for the additional purchases of security k. For a more detailed and rigorous discussion, see Harry M. Markowitz [1952, 1959]. An intuitive exposition using discrete changes is found in William F. Sharpe [1985], pp. 154–156.
9. Two seminal articles on the CAPM are those by William F. Sharpe [1964] and John Lintner [1965]. A third pioneering article on CAPM by Jack L. Treynor [1961] has never been published.
10. This two-step procedure has been called the separation theorem and was first derived by James Tobin [1958].
11. Obviously, the concave risk-return frontier in this case represents alternative optimal portfolios composed of n assets. To ensure that the portfolio is on the frontier rather than under it, each portfolio mix must fulfill the portfolio optimality conditions discussed near the end of Section 2.2.
12. Other useful studies demonstrating the rapid benefits of diversification are by Franco Modigliani and Gerald Pogue [1974a, b] and, in an international context, Bruno Solnik [1974].
13. Richard A. Brealey and Stewart C. Myers [1988], p. 136, taken from Roger Ibbotson and Rex Sinquefield [1986].
14. Studies examining the historical stability of β include Marshall E. Blume [1971]; Robert A. Levy [1971]; William F. Sharpe and Guy M. Cooper [1972]; and Fischer Black, Michael C. Jensen, and Myron Scholes [1972].
15. For further discussion, see Burton Malkiel [1985], Chapter 9.
16. For one attempt to circumvent the ex ante, ex post problem, see Eugene F. Fama and James D. MacBeth [1973].
17. See, for example, Richard Roll [1983].
18. For further discussion of the arbitrage pricing model, see modern finance textbooks such as Richard A. Brealey and Stewart C. Myers [1988], pp. 163–164, or William F. Sharpe [1985], Chapter 8, pp. 182–201. The classic article on the arbitrage pricing theory is that by Stephen A. Ross [1976]. Empirical implementations of the APM typically involve rather sophisticated econometric techniques, including various types of factor models. Some examples of APM empirical implementations include Richard Roll and Stephen A. Ross [1980]; Edwin Burmeister and Kent D. Wall [1986]; Nai-Fu Chen, Richard Roll, and Stephen A. Ross [1986]; and Marjorie B. McElroy and Edwin Burmeister [1988a, b].
19. Most modern finance texts have discussions on procedures for estimating project-specific betas and incorporating them properly into cost of capital calculations.

See, for example, Richard A. Brealey and Stewart C. Myers [1988], Chapter 9, pp. 173–203. An analytical framework for envisaging such project-specific betas has been developed by William F. Sharpe [1977].

20. The Chow test is described in most standard econometrics textbooks. The original reference is Gregory C. Chow [1960]; an expository treatment is provided in Franklin M. Fisher [1970], and a further discussion is found in Damodar N. Gujarati [1970].
21. For further discussion on the effects of the Three Mile Island accident on securities prices of GPU and other electric utility companies, see Carl R. Chen [1984] and Thomas J. Laslavic [1981]; for an analysis of the effects of the Chernobyl nuclear power accident on U.S. electric utility security prices, see Juanita M. Haydel [1988].
22. For a description and analysis of the Conoco takeover, see Richard S. Ruback [1982].
23. See, for example, Richard H. Thaler [1987a]; Philip Brown, Allan W. Kleidon, and Terry A. Marsh [1983]; Donald B. Keim [1983]; Josef Lakonishok and Seymour Smidt [1984]; and Richard Roll [1983]. Incidentally, empirical evidence has also been reported supporting the notion of Monday, end of weekend, holiday, and even intraday effects; see, for example, Kenneth R. French [1980] and, for additional discussion and references, Richard Thaler [1987b]. Finally, the May/June 1978 (Vol. 6, No. 2/3) issue of the *Journal of Financial Economics* is devoted entirely to articles discussing a variety of anomalous returns.
24. However, see Burton G. Malkiel [1985], pp. 179–180, for a discussion of the role of relatively large transactions costs for securities of small companies in reducing possible intertemporal arbitrage.
25. For references on the Chow test, see footnote 20.
26. For references, see footnote 18.
27. The Durbin-Watson test statistic and the Hildreth-Lu and Cochrane-Orcutt estimation procedures are described in considerable detail in most standard econometrics textbooks. Your instructor can provide you with further details.
28. Most graduate econometrics textbooks now include discussions of estimation and inference in models with moving average disturbances. A useful and readable specialized reference text on time series estimation and inference is Charles Nelson [1973].
29. Chi-square goodness-of-fit tests are described in most statistics textbooks, as is the Kolmogorov-Smirnov test; in the context of econometrics textbooks a brief review of tests for normality is found in Judge et al. [1985], pp. 826–827. An alternative test procedure is the studentized range test procedure. For an application in the CAPM context and for tables, see Eugene F. Fama [1976], especially pp. 8–11 and Table 1.9, p. 40.

CHAPTER REFERENCES

Black, Fisher, Michael C. Jensen, and Myron Scholes [1972], "The Capital Asset Pricing Model: Some Empirical Tests," in Michael C. Jensen, ed., *Studies in the Theory of Capital Markets*, New York: Praeger, pp. 79–121.

Blume, Marshall E. [1971], "On the Assessment of Risk," *Journal of Finance*, 26:1, March, 1–10.

the second firm. If the null hypothesis of homoskedasticity is rejected for any firm, make the appropriate adjustments and reestimate the CAPM equation using an appropriate weighted least squares procedure. In such cases, does adjusting for heteroskedasticity affect the parameter estimates significantly? The estimated standard errors? The t-statistics of significance? Is this what you expected? Why or why not?

(b) Testing for first-order autoregressive disturbances: From your master data file in CHAP2.DAT, choose two industries. For each firm in the January 1978–December 1987 sample of these two industries, use the Durbin-Watson statistic and test for the absence of a first-order autoregressive stochastic process employing an appropriate level of significance. If the Durbin-Watson statistic for any firm reveals that the null hypothesis of no autocorrelation either is rejected or is in the range of "inconclusive" inference, reestimate the CAPM equation for those firms using either the Hildreth-Lu or Cochrane-Orcutt estimation procedure.[27] Are the estimated parameters, standard errors, and/or t-statistics affected in any important way? Are any of the above results surprising? Why?

(c) Testing for first-order moving average disturbances: From your master data file in CHAP2.DAT, choose two industries. For each firm in the January 1978–December 1987 sample of these two industries, estimate the CAPM model allowing for a first-order moving average disturbance process. Using either an asymptotic t-test or a likelihood ratio test statistic and a reasonable level of significance, test the null hypothesis that the moving average parameter is equal to zero against the alternative hypothesis that it is not equal to zero.[28]

(d) Testing for normality: From your data file, choose two industries. For each firm in the January 1978–December 1987 sample of these industries, use residuals from CAPM regression equations and the chi-square goodness-of-fit test (or the Kolmogorov-Smirnov test procedure) to test for normality of the residuals.[29]

CHAPTER NOTES

1. For an historical account of the founding of the Econometric Society, see Carl F. Christ [1983].
2. Alfred Cowles III [1933], p. 324.
3. For a more complete discussion of the CAPM, see any textbook in modern corporate finance, e.g., Richard Brealey and Stewart Myers [1988], Chapters 7–9, or William Sharpe [1985], Chapters 6–8, as well as the references cited therein.
4. It might be argued that risk should be measured only by downside surprises. However, if the distribution of returns is symmetric, as is the case with the normal distribution, then use of either the variance or the standard deviation as a measure of risk is appropriate. Incidentally, although the issue remains somewhat controversial, there is a substantial amount of evidence suggesting that returns are dis-

Brealey, Richard A. and Stewart C. Myers [1988], *Principles of Corporate Finance*, Third Edition, New York: McGraw-Hill.

Brown, Philip, Allan W. Kleidon, and Terry A. Marsh [1983], "Stock Return Seasonalities and the Tax-Loss Selling Hypothesis: Analysis of the Arguments and Australian Evidence," *Journal of Financial Economics*, 12:1, June, 105–127.

Burmeister, Edwin and Kent D. Wall [1986], "The Arbitrage Pricing Theory and Macroeconomic Factor Measures," *The Financial Review*, 21:1, February, 1–20.

Chen, Carl R. [1984], "The Structural Stability of the Market Model After the Three Mile Island Accident," *Journal of Economics and Business*, 36:1, February, 133–140.

Chen, Nai-Fu, Richard Roll, and Stephen A. Ross [1986], "Economic Forces and the Stock Market," *Journal of Business*, 59:3, July, 383–403.

Chow, Gregory C. [1960], "Tests of Equality between Sets of Coefficients in Two Linear Regressions," *Econometrica*, 28:3, July, 591–605.

Christ, Carl F. [1983], "The Founding of the Econometric Society and Econometrics," *Econometrica*, 51:1, January, 3–6.

Cowles, Alfred III [1933], "Can Stock Market Forecasters Forecast?" *Econometrica*, 1:3, July, 309–324.

Einstein, Albert [1954], *Ideas and Opinions*, New York: Crown Publishers.

Fama, Eugene F. [1970], "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance*, 25:2, May, 383–417.

Fama, Eugene F. [1976], *Foundations of Finance*, New York: Basic Books.

Fama, Eugene F. and James D. MacBeth [1973], "Risk, Return and Equilibrium: Empirical Tests," *Journal of Political Economy*, 81:3, May, 607–636.

Fisher, Franklin M. [1970], "Tests of Equality between Sets of Coefficients in Two Linear Regressions: An Expository Note," *Econometrica*, 38:2, March, 361–366.

French, Kenneth R. [1980], "Stock Returns and the Weekend Effect," *Journal of Financial Economics*, 8:1, March, 55–70.

Gujarati, Damodar N. [1970], "Use of Dummy Variables in Testing for Equality between Sets of Coefficients in Two Linear Regressions—A Note," *American Statistician*, 24:1, February, 50–52.

Haydel, Juanita M. [1988], "The Effect of the Chernobyl Nuclear Accident on Electric Utility Security Prices," unpublished M.S. thesis, Massachusetts Institute of Technology, A.P. Sloan School of Management, June.

Heck, Jean Louis [1988], *Finance Literature Index*, New York: McGraw-Hill.

Ibbotson, Roger G. and Rex A. Sinquefield [1986], *Stocks, Bonds, Bills, and Inflation: 1986 Yearbook*, Chicago: Ibbotson Associates.

Judge, George G., William E. Griffiths, R. Carter Hall, Helmut Lutkepohl, and Tsoung-Chao Lee [1985], *The Theory and Practice of Econometrics*, Second Edition, New York: John Wiley and Sons.

Keim, Donald B. [1983], "Size Related Anomalies and Stock Return Seasonability: Further Empirical Evidence," *Journal of Financial Economics*, 12:1, June, 13–32.

Lakonishok, Josef and Seymour Smidt [1984], "Volume and Turn of the Year Behavior," *Journal of Financial Economics*, 13:3, September, 435–455.

Laslavic, Thomas J. [1981], "A Market Shock: The Effect of the Nuclear Accident at Three Mile Island upon the Prices of Electric Utility Securities," unpublished M.S. in Management thesis, Massachusetts Institute of Technology, A.P. Sloan School of Management, June.

Levy, Robert A. [1971], "On the Short-Term Stationarity of Beta Coefficients," *Financial Analysts Journal*, 27:6, November/December, 55–62.

Lintner, John [1965], "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics*, 47:1, February, 13–37.

Malkiel, Burton G. [1985], *A Random Walk down Wall Street*, New York: W.W. Norton.

Markowitz, Harry M. [1952], "Portfolio Selection," *Journal of Finance*, 7:1, March, 77–91.

Markowitz, Harry M. [1959], *Portfolio Selection: Efficient Diversification of Investments*, New York: John Wiley & Sons.

McElroy, Marjorie B. and Edwin Burmeister [1988a], "Arbitrage Pricing Theory as a Restricted Nonlinear Multivariate Regression Model," *Journal of Business and Economic Statistics*, 6:1, January, 29–42.

McElroy, Marjorie B. and Edwin Burmeister [1988b], "Joint Estimation of Factor Sensitivities and Risk Premia for the Arbitrage Pricing Theory," *Journal of Finance*, 43:3, July, 721–735.

Modigliani, Franco and Gerald A. Pogue [1974a], "An Introduction to Risk and Return: I," *Financial Analysts Journal*, 30:2, March/April, 68–80.

Modigliani, Franco and Gerald A. Pogue [1974b], "An Introduction to Risk and Return: II," *Financial Analysts Journal*, 30:3, May/June, 69–86.

Nelson, Charles R. [1973], *Applied Time Series Analysis for Managerial Forecasting*, San Francisco: Holden-Day.

Reinganum, Marc R. [1983], "The Anomalous Stock Market Behavior of Small Firms in January: Empirical Tests for Tax-Loss Selling Effects," *Journal of Financial Economics*, 12:1, June, 89–104.

Roll, Richard [1983], "Vas Ist Das?" *Journal of Portfolio Management*, 9:2, Winter, 18–28.

Roll, Richard and Stephen A. Ross [1980], "An Empirical Investigation of the Arbitrage Pricing Theory," *Journal of Finance*, 35:5, December, 1073–1103.

Ross, Stephen A. [1976], "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory*, 13:4, December, 341–360.

Ruback, Richard S. [1982], "The Conoco Takeover and Stockholder Returns," *Sloan Management Review*, Cambridge, Mass.: MIT Sloan School of Management, 23:2, Winter, 13–32.

Sharpe, William F. [1964], "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance*, 19:3, September, 425–442.

Sharpe, William F. [1977], "The Capital Asset Pricing Model: A 'Multi-Beta' Interpretation," in Haim Levy and Marshall Sarnat, eds., *Financial Decision Making Under Uncertainty*, New York: Academic Press.

Sharpe, William F. [1985], *Investments*, Third Edition, Englewood Cliffs, N.J.: Prentice-Hall.

Sharpe, William F. and Guy M. Cooper [1972], "Risk-Return Classes of New York Stock Exchange Common Stocks, 1931–1967," *Financial Analysts Journal*, 28:2, March/April, 46–54.

Solnik, Bruno [1974], "Why Not Diversify Internationally Rather Than Domestically?" *Financial Analysts Journal*, 30:4, July/August, 48–54.

Thaler, Richard H. [1987a], "Anomalies: The January Effect," *Journal of Economic Perspectives*, 1:1, Summer, 197–201.

Thaler, Richard H. [1987b], "Anomalies—Seasonal Movements in Security Prices: II. Weekend, Holiday, Turn of the Month, and Intraday Effects," *Journal of Economic Perspectives*, 1:2, Fall, 169–177.

Tobin, James M. [1958], "Liquidity Preference as Behavior toward Risk," *Review of Economics and Statistics*, 25:1, February, 65–86.

Treynor, Jack L. [1961], "Toward a Theory of Market Value of Risky Assets," unpublished manuscript.

Twain, Mark (Samuel Clemens) [1899], *Pudd'nhead Wilson*, New York: Harper and Row.

Wagner, Wayne H. and Sheila C. Lau [1971], "The Effect of Diversification on Risk," *Financial Analysts Journal*, 27:6, November/December, 48–53.

White, Halbert J. [1980], "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica*, 48:4, May, 817–838.

FURTHER READINGS

Cragg, John G. and Burton G. Malkiel [1982], *Expectations and the Structure of Share Prices*, Chicago: University of Chicago Press. A rigorous empirical analysis of stock market returns.

Fama, Eugene F. [1976], *Foundations of Finance*, New York: Basic Books. Especially Chapters 1–4. A readable and useful classic text.

Huang, Chi-fu and Robert H. Litzenberger [1988], *Foundations for Financial Economics*, Amsterdam: North-Holland. Especially Chapter 10, "Econometric Issues in Testing the Capital Asset Pricing Model." Contains a useful summary of recent research issues.

Malkiel, Burton G. [1985], *A Random Walk down Wall Street*, New York: W.W. Norton. Especially Chapters 8 and 9. Enjoyable and stimulating reading.

Markowitz, Harry M. [1959], *Portfolio Selection: Efficient Diversification of Investments*, New York: John Wiley & Sons. A classic treatment of portfolio theory.