Probability Final

You may use your sheet from the midterm and a new sheet of notes. No calculators, cell phones, PDA's, laptops, or HAL 9000's. Show your reasoning. Don't just give the answer.

1. A box contains two gold balls and two clay balls. You are allowed to choose successively balls from the box at random. You win 1 dollar each time you draw a gold ball and lose 1 dollar each time you draw a clay ball. After a draw, the ball is not replaced.
   (a) If you draw exactly one ball, what is your expected earnings?
   (b) What is the moment generating function for the value of the first draw?
   (c) If you draw exactly k balls (for $k = 1, 2, 3, 4, 5$) what is your expected earnings?
   (d) If you draw until you are ahead by 1 dollar or until there are no more gold balls, what is your expected earnings?

2. Suppose you win 1 dollars when an black card is drawn from a deck of cards but you lose 1 dollar when a red card is drawn. (So out of the 52 cars, you win with 4 of them and lose with 4 of them.)
   (a) Let X_1 be the amount you win on the first draw, and X_2 be the amount you win on the second draw. (Assume you don't put the card back.) What is $Cov(X_1, X_2)$?
   (b) What is the mean and variance of $X_1 + X_2$?
   (c) What is the mean and variance of $\sum_{i=1}^{52} X_i$? (Hint: think before you compute.)

3. Consider a non-negative random varible: $X \geq 0$.
   (a) If $E(X) = 1$, what is a good bound on $P(X \geq 100)$?
   (b) If $E(X) = 1$, and $V(X) = 1$ what is a good bound for $P(X \geq 100)$?
   (c) If the generating function $h_X(2) = 4$, (I.e. $E(2^X) = 4$ then what is a good bound for $P(X \geq 100)$?

4. Suppose the moment generating function for X is $g(t) = 1 + t$. In other words, $E(e^{tX}) = 1 + t$. What can you tell me about X?

5. The law of large numbers tells us alot about a sum of random variables. The CLT tells us even more about sums. But what about products? Let X_i be a random variable that takes on either $+1$ or -1 with equal probability. Let $P_n = \prod_{i=1}^{n} X_i$. Will P_n converge to some fixed value? (I.e. law of large numbers?) If it converges, what is this value, if it doesn't converge, what does P_n look like?

6. Statistics is often driven by two things, a prediction and a residual. Define the random variable $Z = E(Y|X)$ and the random variable $W = Y - Z$. Then Z is the prediction and W is the residual of the "regression" of Y on X.

   (a) What is $E(Z)$?

   (b) What is $E(XZ)$

   (c) Let $h()$ be an arbitary function, show $E(h(X)Z) = E(h(X)Y)$.

   (d) What is $E(WZ)$?

7. Let X_i be a random variable with mean 1.01 and standard deviation .2. (For example, $X = 1.21$ or $X = .81$ with equal probability, but that is such an ugly statement, lets pretend I didn't mention it.) Let $W = \prod_{i=1}^{n} X_i$. Suppose all the X_i's are independent, so the whole series is IID.

   (a) What is $E(W)$?

   (b) What is the long run growth rate (i.e. $\lim_{n\to\infty}(\log W_n)/n$)?

   (c) What will W_{1000} look like?

8. Suppose you put 100 mice on a calorie restriction diet. Normal mice on a normal diet live 1000 days with a standard deviation of 150 days.

   (a) If this diet doesn't change the length of life for these mice, what will be the mean, variance and distribution of $\overline{T}$? (Where $\overline{T}$ is the average number of days a mouse in the experiment lives.)

   (b) Find a good estimate the probability that $\overline{T}$ is bigger than 1300.

   (c) If your experiment actually yielded an average of 1300, would you believe that these mice have the same mean as typical mice?

   (d) (bonus) Using generating functions, provide a better bound.

CLASS COMMENTS

You will be asked by a computer to evaluate the class. But since there were a few new things I tried that were new, I wanted to get your reactions to them. If you are feeling time stressed–feel free to wait until after the exam to fill this out, or drop it by my office at some later time. (Use the back for more space if you want.)

Stories: I tried to start most classes with an unrelated factoid / story. The educational concept is that these are easier to remimber. So my challenge is three fold: Can you remimber any of them? Is there one that stuck with you? Did you like them?

Book: We used an open source book this semester. I liked the content, but I'm not the target audience.

- Which format did you use the book in:
  - Only pdf
  - only Book
  - Both hard copy and pdf
- Did you like the book?
- Was it too hard / too easy?
- Which chapter did you like the best?
- Which one the least?

Other: If you like, feel free to comment on anything else in the class (use the other side). These comments will only go to me. If you want to make your comment public to future students–then you have to use the on line system.