Dean Foster **Stat 430: First Midterm Exam** November 4, 2011

1. (10 pts) This problem has you show deMorgan's law:

   - Draw a Venn diagram of $(A \cup B)^c$.
   - Draw a Venn diagram of $A^c \cap B^c$.

   Are they the same sets?

2. (10 points) Take $\Omega = \{1, 2, 3, 4\}$, and suppose we know that $\{1, 3\}$ and $\{3, 4\}$ are both in the algebra $\mathcal{A}$. Show that $\{1, 4\}$ is in $\mathcal{A}$.

3. (10 points) Suppose $\Omega = \{1, 2, 3\}$, Give an example of an algebra of sets $\mathcal{A}$ over Ω such that $\{1\} \notin \mathcal{A}$.

4. (10 points) Suppose $\Omega = \{1, 2\}$, then clearly it is impossible to get the set $\{3\}$. But, argue why it doesn't make sense to say $P(\{3\}) = 0$.

5. (20 points) Suppose Tom either takes a taxi or the bus. To save money, he only takes the taxi 1/10 of the time. When he takes a taxi, he is late 20% of the time, but when he takes the bus he is late 80% of the time. One morning, Tom, Dick and Harry are scheduled to have a meeting. When, Tom arrives on time Dick says, "I see you sprung for a cab this morning!" But before Tom can reply, Harry jumps in with "Nah, he's too cheap! I bet he was just lucky!"

   (a) What is the probability that Tom took a taxi?

   (b) What amounts should Dick and Harry bet to make this fair?

   (c) Who is more likely to be right?

6. (20 points) Suppose the generating function for the non-negative integer valued random variable X is $E(s^X) = .5 + .2s + .3s^2$.

   (a) What is $P(X = 1)$?

   (b) What is $E(X)$?

   (c) Suppose $Y = \sum_{i=1}^{n} X_i$ where each X_i is IID with the same generating function. What is the generating function for Y, and from this find what $P(Y = 0)$ is?

7. (15 points) Suppose we have an asset that returns a random amount each year. We will define R_t to be the ratio of the price at the end of year t to the price at the beginning of the year. So after T years, the total value when you start with a single dollar is:

$$W_T = \prod_{t=1}^{T} R_t = R_1 \times R_2 \times R_3 \times \cdots \times R_T$$

   Assume that the returns are independent and identically distributed. If $E(R_t) = 1.7$ what is the formula for $E(W_T)$? What will $E(W_{20})$ equal?

8. (5 points) Let X_i be a Bernulli trial, namely, $P(X_i = 1) = p = 1 - P(X_i = 0)$, where $\{X_i\}_{i=1,n}$ are an IID sequence. Let $Y = \sum_{i=1}^{n} X_i$. Then Y is a Binomial with parameters p and n, and hopefully you have written on your cheat sheet that $Var(Y) = np(1-p)$. Let's check that you wrote this down correctly:

   (a) Warmup: What is the $var(X_i)$?

   (b) Trivia: What is the $cov(X_i, X_j)$ if $i \neq j$?

   (c) Real question: What is $var(\sum_{i=1}^{n} X_i)$?

   (d) Bonus: Starting from $P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}$ compute the variance of Y.