Chapter 6

Explaining and Forecasting Aggregate Investment Expenditures: Distributed Lags and Autocorrelation

". . . much of the investment occurring this year is the result of decisions made last year, the year before, and even the year before that. The decisions were governed by the expectations prevailing in those years about economic conditions this year. New information about this year's conditions that became available after the launching of the projects cannot affect this year's investment in those projects. Much of this year's investment was predetermined by earlier decisions."

ROBERT HALL and JOHN TAYLOR (1988), p. 237

"The important point is that lags in the determination of the level of business investment are long."

RUDIGER DORNBUSCH and STANLEY FISCHER (1987), p. 315

". . . any time series regression containing more than four independent variables results in garbage."

ZVI GRILICHES (1974), p. 335

In this chapter we focus attention on aggregate business investment. The econometric tools that we emphasize include multiple regression estimation and forecasting with distributed lags and various forms of autocorrelation.

Over the last 40 years in the United States, business expenditures on plant and equipment gross investment have averaged approximately 10% of GNP. While such investment spending is typically small in comparison to consumption and government expenditures, for a number of reasons many economists believe that investment is the most important component of GNP.

First, plant and equipment are long-lived, durable goods. Investment outlays that renew and expand the stock of plant and equipment therefore increase potential capacity output supply, not only in the current time period but also into the future. Further, to the extent that new investment goods embody the most recent technical advances, the potential benefits of such technical progress can be realized only as investment occurs. Hence variations in investment expenditures have long-term consequences on a country's productive capacity.

Second, investment expenditures affect demands for the products of the construction and producers' durable good industries. These industries typically alter employment levels in response to variations in the demands for their products; such changes tend to spill over into other industries. This implies that changes in investment expenditures induce shifts in the aggregate levels of employment and personal income through both direct and indirect effects.

Third, the sensitivity of aggregate supply and demand to changes in investment is very important empirically, since investment is the most volatile major component of GNP, varying from about 8.5% of GNP in 1952 and 1958 to above 12.5% in 1984. This volatility of investment expenditures has important "whiplash" consequences. For example, it has been estimated that variations in business expenditures for plant and equipment investment accounted for approximately 25% of the first-year increase in GNP in the United States during the recoveries that began in the third quarter of 1980 and in the fourth quarter of 1982.[1]

Since investment expenditure is so volatile and its movements have important consequences for productive capacity, employment demand, personal income, and the balance of payments, it is critical that the fundamental causes of variations in aggregate investment be understood. If these underlying determinants of changes in investment spending were properly perceived, for example, it might be possible to predict better and accommodate such expected variations. Also, if the volatility in investment expenditures were viewed as being excessive or undesirable, it might be possible to implement government fiscal or monetary policy to control their movements in a more desirable manner.

Unfortunately, however, as we shall see in this chapter, econometricians are still not able to explain and forecast changes in aggregate investment expenditures to the desired degree of precision. Investment models and equations that work exceedingly well in explaining historical variations over one time period often turn out to be less than satisfactory in their forecasting performance into another time period. Moreover, the choice of a preferred model based on historical data often varies considerably with the time period chosen. This state of affairs is humbling, yet it also provides great challenges and opportunities for empirical econometricians.

There are several types of investment. In most industrialized countries, national income and product accountants distinguish among at least three components of investment. *Residential construction* is investment ultimately for use by homeowners and is known to be very sensitive to even minor changes in market interest rates, often written into mortgage contracts. The second major portion of aggregate investment is *changes in business inventories*, which is by far the most volatile component. Firms tend to use inventory changes as buffers against variations in the sales of goods and services, and for this reason, inventory investment is known to be very sensitive to the overall level of economic activity and especially to short-term fluctuations in sales. Inventories may also be held for speculative purposes. The third and largest component of aggregate investment is *fixed business investment*; it incorporates expenditures on nonresidential structures (plant) and producers' durable equipment.

In this chapter we focus on explaining and forecasting variations in the largest component of aggregate investment, namely, fixed business investment.[2] The econometric tools employed in our analysis of fixed business investment include the estimation of distributed lag specifications with autoregressive and moving average error structures, as well as the analysis of static and dynamic forecasting properties of alternative models having autocorrelated errors.[3] We will compare both theoretically and empirically the historical and forecasting performance of five alternative models of aggregate investment, using quarterly data from the U.S. private business sector beginning in 1954. The five alternative theoretical and empirical frameworks considered here are the accelerator, cash flow, neoclassical, Tobin's q, and time series/autoregressive models.

The outline of this chapter is as follows. In Section 6.1 a number of important definitions are presented, a general framework for representing the various models is developed, and issues of stochastic specification are briefly discussed. Then in Sections 6.2 through 6.6 a theoretical and empirical discussion of issues involved in the implementation of each of the five alternative models is presented. Additional econometric specifications and issues are discussed in Section 6.7. The five models are assessed and then compared empirically using a common data base in Section 6.8, and in Section 6.9 other important current research issues are noted. A set of interesting and engaging empirical exercises follows, grounded in a data base of quarterly investment in the U.S. private business sector, from 1952:1 to 1986:4. This data base was constructed by Richard W. Kopcke with assistance from George Houlihan and includes updates from Kopcke's [1985] study.

6.1 INVESTMENT AND CAPITAL STOCK: DEFINITIONS AND GENERAL FRAMEWORK

6.1.A Definitions

In order to understand and implement empirically the various econometric models of investment behavior, it is useful to develop definitions and to establish a general framework for assessing the alternative models. We begin with definitions of the capital stock, whose empirical measurement, as Nobel Laureate Sir John Hicks has noted, "is one of the nastiest jobs that economists have set to statisticians."[4]

Suppose that in time period $t-\tau$ the firm expends $I_{t-\tau}$ dollars for new plant or equipment. Let this $I_{t-\tau}$ value be in constant dollars, that is, define $I_{t-\tau}$ as current dollar investment divided by an asset price index, where the deflator (price index) is normalized to unity in some base year, such as 1977. It is worth noting that in most countries, separate deflators are published for distinct capital assets, such as various types of producers' durable equipment and nonresidential structures.[5]

Because investment goods are durable, they provide services over a multiperiod lifetime. The amount of real investment put in place at time $t-\tau$ and surviving to time t is denoted as $K_{t,t-\tau}$:

$$K_{t,t-\tau} \equiv s_{t,\tau} I_{t-\tau} \tag{6.1}$$

where $s_{t,\tau}$ is the survival rate for age τ investment to time period t. The aggregate of the vintages surviving to the end of time period t—the aggregate capital stock at the end of time period t, denoted K_t—is typically calculated as the sum over vintages

$$K_t \equiv \sum_{\tau=0}^{T} K_{t,t-\tau} = \sum_{\tau=0}^{T} s_{t,\tau} I_{t-\tau} \tag{6.2}$$

where T is the life of the durable good. Computations using Eq. (6.2) can be done separately for various types of capital goods. Further, for each type of capital good it is commonly assumed that the flow of available capital services is a constant proportion of the (constant dollar) capital stock.

One very important empirical issue concerns the life pattern of the survival rates $s_{t,\tau}$. A number of alternative physical deterioration age profiles have been developed. One common alternative, called "one-hoss shay" deterioration, is based on the assumption that once an asset is put into place, it provides the same amount of services during each time period until it "expires" or is scrapped.[6] Recognizing that the time at which durable goods expire is stochastic, analysts have estimated the mean and shape of mortality distributions governing the service lives of various durable assets.

One of the most famous studies of mortality distributions is that by Robert Winfrey, who in 1935 postulated and estimated various bell-shaped mortality distributions centered on the average service lives of a variety of durable goods.[7] Remarkably, until very recently, these Winfrey distributions

Table 6.1 Modified Winfrey S-3 Retirement Patterns, U.S. Department of Commerce

Percent of Mean Service Life	Cumulative Percent of Original Expenditures Discarded	Percent of Mean Service Life	Cumulative Percent of Original Expenditures Discarded
<45	0.0	105	61.6
45	1.2	110	68.8
50	2.4	115	75.4
55	4.1	120	81.3
60	6.5	125	86.3
65	9.7	130	90.3
70	13.7	135	93.5
75	18.7	140	95.9
80	24.6	145	97.6
85	31.2	150	98.8
90	38.4	155	100.0
95	46.1	>155	100.0
100	53.9		

Source: John A. Gorman, John C. Musgrave, Gerald Silverstein, and Kathy A. Comins, "Fixed Private Capital in the United States," *Survey of Current Business*, Vol. 67, No. 7, 1985, Table D, p. 43.

based on pre-1935 data remained essentially unchanged and formed the basis of U.S. Department of Commerce gross capital stock calculations.

To see how these Winfrey mortality distributions have been employed in calculating gross capital stocks based on Eqs. (6.1) and (6.2), let us take a slightly modified Winfrey S-3 retirement pattern published by the U.S. Department of Commerce, reproduced in Table 6.1.[8] For a number of different types of nonresidential capital (except autos), the mean service life is first estimated.[9] The distribution of mortality or retirement patterns $s_{t,\tau}$ is then assumed to be time-invariant, that is, $s_{t,\tau} = s_\tau$, according to the distribution given in Table 6.1.

Given a sufficiently lengthy historical series on real investment expenditures by type of asset, mean service lives, and the above Winfrey mortality distribution, national income accountants calculate the *gross capital stock* using Eqs. (6.1) and (6.2). Note that such gross capital stock series depend on the assumption of one-hoss shay deterioration, fixed expected service lives, and the modified Winfrey mortality distribution.

Although the one-hoss shay decay assumption has some plausibility, other decay patterns are also attractive. A very convenient alternative time path of physical deterioration is the constant exponential decay specification, which is based on the assumption that the rate of physical deterioration as an asset ages is constant and equal to, say, δ% per time period. With constant exponential deterioration the physical survival rate s_τ for an asset of age τ is simply $s_\tau = (1 - \delta)^\tau$. When this survival rate expression is substituted into

Eq. (6.2), the aggregate capital stock at the end of time period t, based on constant exponential deterioration, is computed simply as

$$K_t = \sum_{\tau=0}^{T} K_{t,t-\tau} = \sum_{\tau=0}^{T} (1-\delta)^{\tau} I_{t-\tau} \tag{6.3}$$

National income accountants refer to the capital stock based on the assumption of constant exponential deterioration and calculated by using Eq. (6.3) as the *net capital stock*. Note that the net capital stock is based on the assumption of constant exponential deterioration, while the gross capital stock presumes one-hoss shay decay.[10]

A convenient way of rewriting the (end of time period) net capital stock calculation implicit in Eq. (6.3) is called the *perpetual inventory method*. It is specified as

$$K_t = (1-\delta)K_{t-1} + I_t \tag{6.4}$$

Evidently, the word perpetual is employed because with constant exponential deterioration the quantity of services yielded by an asset as it ages approaches but never actually attains zero, and so an asset is perpetually part of the inventory of capital goods.

One could of course also use an equation like the above perpetual inventory relationship for the case of one-hoss shay decay, but since in that case the rate of scrapping over all vintages would depend on the vintage composition of the surviving capital stock, the overall rate of deterioration δ would vary over time. In such a case, Eq. (6.4) would be rewritten as

$$K_t = (1-\delta_t)K_{t-1} + I_t \tag{6.5}$$

Another important distinction made by analysts of investment behavior is that between replacement and net investment. In each time period a certain amount of investment is designed merely to replace the amount of capital that has deteriorated or been scrapped. With exponential decay and using Eq. (6.4), *replacement investment* equals δK_{t-1}, while for one-hoss shay decay and using Eq. (6.5), replacement investment equals $\delta_t K_{t-1}$.[11] The net increment to the capital stock since the last time period, $K_t - K_{t-1}$, equals total investment minus replacement investment and is called *net investment*. With constant exponential decay, net investment therefore equals $I_t - \delta K_{t-1}$, while for one-hoss shay decay, net investment equals $I_t - \delta_t K_{t-1}$. Finally, regardless of the form of decay, gross investment, replacement investment, and net investment are related by the identity

$$\text{Gross investment} \equiv \text{replacement investment} + \text{net investment.}$$

Incidentally, in the United States, on average, replacement and net investment are roughly equal in magnitude.

The one-hoss shay and constant exponential decay assumptions generally result in different replacement investment, net investment, and therefore net or gross capital stock calculations. Moreover, other forms of decay could also be employed, such as straight-line decay, yielding yet another set of capital stock and investment calculations.[12] This raises the important issue of how one might choose among alternative physical decay assumptions. Fortunately, it is here that economic theory provides very important insights for discriminating among assumptions.

Specifically, the one-hoss shay, straight-line, and constant exponential physical *deterioration* patterns for durable goods imply explicit patterns of price *depreciation* over time as the assets age, that is, each specification of quantity deterioration implies a unique pattern of price depreciation. This relationship is based on the economic reasoning underlying the decision of how much a firm is willing to pay to purchase a used asset.

Suppose that firms seek to minimize the present value of production costs. In equilibrium the price of a used asset must just equal the discounted present value of its expected services over its remaining expected life. If the used price were higher than the present value of future services, the firm would be overinvesting in such assets, receiving services of less value than the amount paid, and would not be minimizing present value costs. On the other hand, if the used price were lower than the present value of services, the firm would be underinvesting in the asset and would be receiving services valued more highly than the price it paid. In equilibrium the present value of future services must just equal the purchase price of the used asset. This implies a relationship between expected physical deterioration and economic depreciation over the lifetime of the asset.

To understand this relationship better, assume initially that the service life of a particular asset were known with certainty and that its physical decay followed a one-hoss shay pattern. With a positive discount rate the price that firms would be willing to pay for such an asset initially would decline gradually in the early years of its service life, and then the price decline would accelerate rapidly as the date of retirement approached.

In Fig. 6.1 the price of a used asset is graphed on the vertical axis, while its age appears on the horizontal axis. The age-price profile for an asset characterized by one-hoss shay decay can be shown to be concave to the origin, as illustrated by the outward-bowed top curve in Fig. 6.1. By contrast, it can be shown that if the quantity decay of a durable good follows a constant exponential decay pattern at rate δ, then the age-price profile also follows the same pattern, that is, the rate of price depreciation is also δ% per time period. Thus with exponential decay, the age-price profile is convex to the origin, as is shown in the bottom curve of Fig. 6.1.[13]

This relationship between quantity decay assumptions and age-price depreciation profiles has stimulated a number of researchers to choose among alternative decay patterns by estimating econometrically the age-price depreciation profiles for used assets and thereby inferring the shape of the underlying s_τ distribution. Although some of the early evidence indicated little sup-

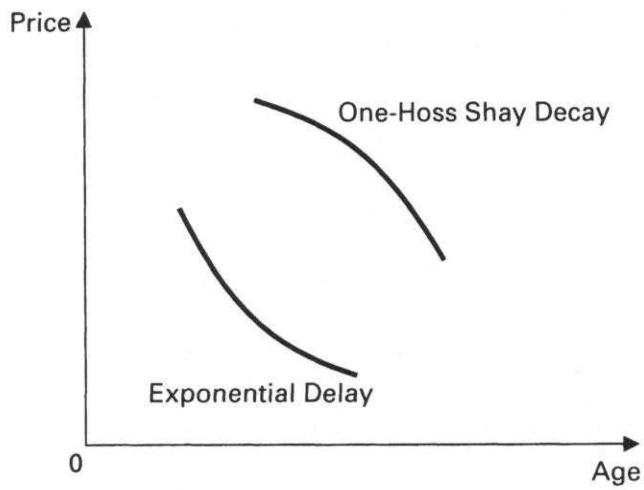


FIGURE 6.1 Age-price profiles under one-hoss shay and constant geometric decay.

port for the exponential decay assumption,[14] more recent studies have examined a larger number of alternative physical deterioration patterns using highly general Box-Cox transformations and have provided greater support for the exponential decay assumption. For example, in their examination of structures, Charles Hulten and Frank Wykoff concluded that

> depreciation patterns are accelerated vis-a-vis straight line, and perhaps also vis-a-vis the geometric [exponential] form. Certainly, the pattern of depreciation does not appear to be linear or decelerated, and obviously, our results, if correct, rule out one-horse shay depreciation as well.[15]

Hulten and Wykoff have conducted similar econometric analyses for a wide variety of differing types of equipment and structures. On the basis of these studies they conclude that while the assumption of exponential or geometric decay tends to be rejected for most assets using conventional statistical criteria, the empirical significance of these rejections is modest, since the age-price profile corresponding with the exponential decay hypothesis very closely approximates that of the more general specifications, such as the hyperbolic.[16] Other simple decay assumptions, such as straight-line or one-hoss shay, are rejected more decisively and do not approximate the more general representations as well.

Today the exponential decay assumption is by far the most widely used deterioration specification in current studies of investment expenditures, most likely owing to its convenient simplicity and its ability to track very closely the age-price profiles of a variety of used assets. It follows also that since there is more empirical support for exponential than one-hoss shay decay, most analysts currently employ as their measure of the capital stock the net rather than the gross construct. A word of caution is in order, however:

Although we will also employ the exponential decay assumption and net capital stocks in the remainder of this chapter, in fact the exponential decay assumption is very restrictive, and results from future empirical research might bring about changes in this common procedure.

6.1.B A General Framework for Representing Various Models

Most theories of investment behavior relate the demand for new plant and equipment to the gap between the desired or optimal amount of capital stock of plant and equipment, denoted K^*, and the actual amount of capital, K. In comparing alternative theories of investment, it will be useful to focus on two aspects concerning K and K^*: (1) What are the factors affecting K^*, and how can such factors be modeled and measured? (2) Why doesn't $K = K^*$, how does K adjust to K^*, and what factors affect the speed of adjustment?

These two aspects of investment behavior can be combined as follows. Let the net capital stock at the end of time period $t - 1$ be K_{t-1}, let K_t^* be the desired capital stock at the end of the current time period, and let the speed of adjustment between K_t^* and K_{t-1} be λ_t. If λ_t were zero, K would be perfectly fixed, and there would be no net investment reducing the gap between K^* and K, while if λ_t were 1, this gap would be closed entirely within one time period, that is, adjustment would be instantaneous. By definition, net investment during time period t equals $\lambda_t(K_t^* - K_{t-1})$, while under the geometric decay assumption, replacement investment equals δK_{t-1}. Since gross investment I_t is the sum of net and replacement investment, gross investment can be written as

$$I_t = \lambda_t(K_t^* - K_{t-1}) + \delta K_{t-1} = \lambda_t K_t^* + (\delta - \lambda_t)K_{t-1} \qquad \textbf{(6.6)}$$

Note that, whenever $\delta < \lambda_t$, the $(\delta - \lambda_t)$ term on the lagged capital stock variable in Eq. (6.6) will be negative.[17]

6.1.C Stochastic Specification

To estimate parameters of an equation like Eq. (6.6), a random disturbance term must be appended, and its distribution properties need to be specified. Unfortunately, relatively little attention has been focused on the sources of these disturbances. One could of course simply assert that investment is inherently stochastic, owing to the fact that it depends on individuals' expectations, which in turn can be heterogeneous and random.

Some analysts also cite measurement error as a source of the disturbance term. Note that if the dependent variable in Eq. (6.6), gross investment, is measured with random error, then so too is the lagged capital stock, a right-hand variable, which by Eq. (6.3) is a weighted sum of previous gross investment expenditures. Hence if one specifies this type of random measurement error as the source of the disturbance term, it is necessary to employ estimation procedures other than ordinary least squares—methods that account for measurement error of a regressor—in order to obtain consistent parameter estimates.[18]

Alternatively, one could specify that at the firm level there are additional parameters affecting firms' investment, parameters that are known to the firm but unknown to the econometrician examining aggregate data. Such parameters could be randomly distributed over the population of firms and could be specified in the aggregate to be identically and normally distributed with mean α (thereby resulting in a constant term being added to Eq. (6.6)) and variance σ^2.

Moreover, since the investment spending decision is a complex one, it is likely that important variables are omitted from simple equations such as Eq. (6.6). If such omitted variables are uncorrelated with the included regressors but have a systematic pattern over time, then their impact could be incorporated in part by specifying a random disturbance term with autoregressive features.

A common procedure in empirical aggregate investment analysis is to append to Eq. (6.6) a first-order autoregressive disturbance,

$$u_t = \rho u_{t-1} + \epsilon_t \qquad t = 2, \ldots, T$$

where $|\rho| < 1$ and ϵ_t is assumed to be independently and identically normally distributed with mean zero and variance σ^2. In some cases, depending in part on the periodicity of the data (annual, quarterly, monthly, etc.), higher-order autoregressive processes may be specified instead.

Equation (6.6), with an autoregressive disturbance term appended, will form the basis of comparison among the five alternative models of investment behavior, beginning with the accelerator model, to which we now turn our attention.

6.2 THE ACCELERATOR MODEL

One of the earliest empirical models of aggregate investment behavior is the accelerator model, which was put forward by J. M. Clark in 1917 as a possible reason to rationalize the volatility of investment expenditures.[19] The distinguishing feature of the accelerator model is that it is based on the assumption of a fixed capital/output ratio. This implies that prices, wages, taxes, and interest rates have no direct impact on capital spending but may have indirect impacts. We now examine several well-known versions of the accelerator model.

6.2.A Theory

Denote real output during time period t as Y_t, and let the fixed capital/output ratio equal μ. According to the *naive accelerator model*, not only does the optimal capital stock K_t^* bear a fixed factor of proportionality to output,

$$K_t^* = \mu \cdot Y_t \tag{6.7}$$

but the capital stock is always optimally adjusted in each time period, implying that $K_t^* = K_t$ and therefore that net investment I_{nt} equals

$$I_{nt} = K_t - K_{t-1} = \mu(Y_t - Y_{t-1}) \tag{6.8}$$

This naive accelerator model has not fared well in empirical analyses, due in part to the restrictive instantaneous adjustment assumption; one common econometric finding based on estimation of Eq. (6.8) is that the least squares estimate of μ is much smaller than the observed average capital/output ratio.

6.2.B Dealing with Alternative Distributed Lag Forms

A slightly generalized version of the original accelerator model is called the *flexible accelerator* and was put forward by Leendert M. Koyck [1954]. In this specification the adjustment of capital stock to its optimal level is no longer instantaneous, but instead is assumed to be a constant proportion λ of the gap between K^* and K. Denote the partial adjustment coefficient as λ_t, set $\lambda_t = \lambda$ for all t, specify that

$$I_{nt} = \lambda(K_t^* - K_{t-1}) \tag{6.9}$$

and then substitute Eq. (6.7) into Eq. (6.9), which yields

$$I_{nt} = K_t - K_{t-1} = \lambda\mu Y_t - \lambda K_{t-1} \tag{6.10}$$

or

$$K_t = \mu\lambda Y_t + (1 - \lambda)K_{t-1} \tag{6.11}$$

An interesting aspect of Eq. (6.11) is that one can write it for different time periods such as $t - 1$, $t - 2$, $t - 3$, etc. and then repeatedly substitute each such equation back into Eq. (6.11), which then yields a distributed lag formulation with geometrically declining weights

$$K_t = \mu[\lambda Y_t + \lambda(1 - \lambda)Y_{t-1} + \lambda(1 - \lambda)^2 Y_{t-2} + \cdots]$$

or

$$\begin{aligned} K_t - K_{t-1} = \mu[&\lambda(Y_t - Y_{t-1}) + \lambda(1 - \lambda)(Y_{t-1} - Y_{t-2}) \\ &+ \lambda(1 - \lambda)^2(Y_{t-2} - Y_{t-3}) + \cdots] \end{aligned} \tag{6.12}$$

Two aspects of Eqs. (6.12) merit special comment. First, in the top equation of (6.12) the level of capital depends on the levels of current and lagged output, while in the second equation the change in capital, that is, net investment, depends on current and lagged changes in output. Because *levels* of investment depend on *changes* in output, this model of investment is called the accelerator model.

Second, a change in output in period t affects investment not only in period t, but also in future time periods, that is, the effect of output changes on investment is distributed over an infinite number of future time periods.

Conversely, investment in time period t is therefore the result of current and previous output changes. However, the more distant changes are not as important, since in Eq. (6.12) each successive lagged time period is weighted by a geometrically declining factor. For these reasons the partial adjustment specification (6.9) is often called a *geometric distributed lag* formulation.

The investment equation (6.10) is in terms of net investment. Assuming constant geometric decay at rate δ, one can add replacement investment δK_{t-1} to both sides of Eq. (6.10) and obtain the gross investment formulation

$$I_t = K_t - (1 - \delta)K_{t-1} = \lambda\mu Y_t + (\delta - \lambda)K_{t-1} \qquad \textbf{(6.13)}$$

Notice that Eq. (6.13) does not have an intercept term (although in practice this equation is typically estimated with a constant term included) and that, provided that one knew the value of δ (needed to construct the K_t series), estimation by least squares would yield implicit estimates of μ and λ.

Because of the difficulties in obtaining reliable capital stock measures, some analysts prefer to estimate an alternative form of the accelerator model. Specifically, lag Eq. (6.13) by one time period, multiply both sides by $(1 - \delta)$, and then subtract the product from Eq. (6.13). This Koyck transformation yields

$$\begin{aligned} I_t - (1 - \delta)I_{t-1} = {} & \mu\lambda Y_t - (1 - \delta)\mu\lambda Y_{t-1} \\ & + (\delta - \lambda)K_{t-1} - (1 - \delta)(\delta - \lambda)K_{t-2} \end{aligned}$$

which can be rewritten as

$$I_t - (1 - \delta)I_{t-1} = \mu\lambda Y_t - (1 - \delta)\mu\lambda Y_{t-1} + (\delta - \lambda)I_{t-1}$$

since $I_{t-1} = K_{t-1} - (1 - \delta)K_{t-2}$. Collecting terms, we finally obtain

$$I_t = \mu\lambda Y_t - (1 - \delta)\mu\lambda Y_{t-1} + (1 - \lambda)I_{t-1} \qquad \textbf{(6.14)}$$

Equation (6.14) can be estimated without employing any data on capital stock. Moreover, an interesting feature of Eq. (6.14) is that one can also estimate rather than assume the rate of deterioration. Specifically, the least squares estimated parameter on the lagged dependent variable identifies λ, this estimate of λ can be used along with the coefficient estimate on Y_t to identify μ, and these estimates of μ and λ can then be employed along with the estimated coefficient on Y_{t-1} to identify δ.

Note, however, that since Eq. (6.14) contains a lagged dependent variable as a regressor, if the disturbance term appended to Eq. (6.14) follows a first-order autoregressive process, then estimation by ordinary least squares yields inconsistent and biased estimates of the true parameters. Further, because the lagged dependent variable appears as a regressor, one cannot use the traditional Durbin-Watson test statistic to test for autocorrelation. Instead, one can use something like Durbin's m- or h-test statistic procedure.[20]

An attractive feature of the accelerator model (6.13) is its simplicity: Investment is a function only of current and lagged output and the lagged capital stock. The underlying rationale is also intuitively simple. Since investment intentions must pass through the various stages of planning, contracting, and ordering before becoming expenditures and then might still be subject to delivery and gestation lags, the lagged output terms represent the gradual response of investment to changes in final demand. This sequence of lagged output terms can also be interpreted as encompassing output projections, since such projections are often based in part on extrapolated past sales patterns.

The lagged capital stock term in the accelerator model (6.13) can be interpreted as serving two purposes. First, since by assumption the capital/output ratio is fixed, the recent course of output is compared to the lagged capital stock, thereby providing a calibration on whether new investment expenditures are warranted. Second, the amount of replacement investment is typically assumed to be proportional to the capital stock, and so the amount of gross investment should be affected by the level of the lagged capital stock.

While the above accelerator model involves partial adjustment with a geometric distributed lag, there is of course no reason why coefficients on the successive lagged output terms must be constrained to follow a declining geometric pattern. Some empirical researchers have therefore simply inserted a number of lagged output terms as regressors into an equation such as Eq. (6.12) without imposing any restrictions on the coefficients.

Specifically, a maximum length of the lag period is chosen, say, m periods (where $m + 2 < T$, T being the number of observations), and then an equation like

$$I_t = a_0 + \sum_{i=0}^{m-1} b_i Y_{t-i} + b_K K_{t-1} + u_t \qquad \textbf{(6.15)}$$

is estimated with no further coefficient restrictions imposed on the m output variables. In practice, m is typically chosen on the basis of a combination of experimentation, hypothesis testing, and judgment.

One problem with unrestricted distributed lag formulations such as Eq. (6.15) is that the lagged output terms typically tend to be highly correlated, and the resulting multicollinearity often generates large standard errors for the imprecisely estimated distributed lag coefficients, leading to the "garbage" results noted by Zvi Griliches in the quote at the beginning of this chapter.

The effects of such multicollinearity can be reduced by imposing restrictions on the time pattern of distributed lag coefficients (but not as many as are found in the highly restrictive declining geometric weight specifications, Eqs. (6.12) and (6.13)). Such restricted specifications include (1) the inverted-V lag distribution introduced by Frank de Leeuw [1962] (in which the coefficients increase linearly from $b_0 = 0$ to a peak lag at $m/2$ periods, m being an even number, and then decrease symmetrically to zero where $b_m = 0$), (2) the double-lag specification of Michael K. Evans [1967], resulting in an inverted-W distribution lag pattern, and (3) forcing the coefficients to lie along a finite polynomial, the analyst specifying the degree of the polynomial and the maximal lag m. This is often called the Almon lag and is due to Shirley

Almon [1965, 1968]. The choice of the degree of the polynomial and the length of the maximal lag of course leave considerable room for experimentation and judgment.

With each of these distributed lag representations, the choice of preferred specification is to some extent arbitrary and ad hoc. To reduce such arbitrariness and to highlight the important role of hypothesis testing in choosing a preferred specification, Dale W. Jorgenson [1966] introduced the *rational lag* function. Its essential feature is that it approximates a general infinite distributed lag formulation by a ratio of two finite polynomials. An attractive feature of the rational lag distribution is that it takes on as testable special cases the Almon, geometric, and other distributed lag specifications.

Notice that with the generalization of the geometric lag specification (6.15), no restrictions are placed on the m distributed lag parameters. This implies that the rational, Almon, inverted-V, geometric lag and other distributed lag formulations can be tested as special cases of Eq. (6.15), using traditional hypothesis-testing techniques, typically in the context of autoregressive disturbances. Hypothesis testing and the estimation of alternative distributed lag formulations have been discussed extensively in the literature; useful surveys include those by Zvi Griliches [1967] and by Marc Nerlove [1972].[21]

Regardless of the distributed lag formulation chosen, most economists today criticize the accelerator model for being too simplistic.[22] On the other hand, proponents argue that while other models may be preferable on theoretical criteria, the more complicated models (such as those permitting interest rates and taxes to affect optimal capital/output ratios) tend to generate unstable parameter estimates and cannot be estimated with acceptable precision.

6.2.C Empirical Implementations

We now come to an examination of empirical results based on the accelerator model of aggregate investment, a model that has been implemented by a very large number of researchers.[23] For our purposes it will be useful to examine a series of three empirical studies by Richard W. Kopcke [1977, 1982, 1985], who estimated this and other models using quarterly U.S. data, separately for nonresidential structures and producers' durable equipment, drawn from three time periods: (i) 1954:1–1977:4, (ii) 1958:1–1973:3, and (iii) 1956:1–1979:4. In each study, Kopcke allowed for autocorrelation and reported results based on a preferred specification. For studies (ii) and (iii), Kopcke employed the Almon lag and constrained parameters to lie along a third-degree polynomial, whereas in study (i) he left the lagged coefficients unconstrained. Kopcke's results, based on the notation in Eq. (6.15), are reproduced in Table 6.2 (standard error estimates were not reported).

A number of results in Table 6.2 merit special comment. First, regarding the distributed lag coefficients, the length of the preferred lag specification

Table 6.2 Parameter Estimates of Accelerator Investment Equation (6.15) Reported by Kopcke in Three Quarterly U.S. Studies

	Nonresidential Structures			Producers' Durable Equipment		
Parameter	Study (i)	Study (ii)	Study (iii)	Study (i)	Study (ii)	Study (iii)
a_0	3.11	6.74	33.04	−38.03	−60.00	−161.11
b_0	0.025	0.023	0.028	0.095	0.083	0.090
b_1	0.014	0.022	0.021	0.022	0.048	0.067
b_2	0.004	0.015	0.015	0.054	0.036	0.049
b_3	0.028	0.007	0.011	0.028	0.035	0.036
b_4	0.018	0.002	0.007		0.032	0.027
b_5		0.006	0.005		0.015	0.021
b_6			0.003			0.017
b_7			0.003			0.014
b_8			0.002			0.012
b_9			0.002			0.008
b_{10}			0.002			
b_{11}			0.002			
b_K	−0.094	−0.092	−0.177	−0.207	−0.227	−0.272
ρ	0.91	0.995	0.966	0.91	0.997	0.966

Notes: Study (i) covers 1958:1–1973:3; study (ii) covers 1954:1–1977:4; study (iii) covers 1956:1–1979:4. Lagged variables employ earlier observations.

changes depending on the time period estimated, the shortest lags being in the first study and the longest lags being in the last study, for both plant and equipment. In the first study, for structures, $m = 5$, while in the last study, $m = 12$. Also, for equal $t-i$ time displacements the b_i distributed lag coefficients tend to be larger for equipment than for plant. This is plausible, since the time lag for equipment investment is likely to be shorter than that involved in designing and constructing nonresidential structures. In the last two studies the b_i distributed lag coefficients tend to decrease as the time lag increases, but in the first study, for both plant and equipment the distributed lag coefficients follow a reclining-S pattern, initially falling, then increasing and again falling.

Second, in all three studies and for both types of investment goods, the estimated coefficient on the lagged capital stock term is negative, which from Eq. (6.13) suggests that the one-period partial adjustment parameter is larger than the rate of depreciation; incidentally, Kopcke reports that in his net capital stock calculations he employed an annual depreciation rate of about 5% for structures and about 15% for equipment.

Third, although specific test results are not reported, in all cases, first-order autocorrelation appears to be substantial; estimates of ρ range from a low of 0.91 to a high of 0.997. As we shall see, first-order autocorrelation appears to be pervasive in studies of investment demand, no matter what model is used.

This concludes our discussion of the accelerator model. Additional remarks concerning the performance of the accelerator model in estimation and forecasting will be given in Section 6.8, where the accelerator formulation is compared with other aggregate investment models. We now turn to a second model, the cash flow model of investment.

6.3 THE CASH FLOW MODEL

It has long been postulated that the availability of funds has a significant impact on investment behavior. In turn, it has also been argued that internal cash flow is the preeminent source of funds and, in particular, is more important than the availability of external debt or equity financing. The cash flow model posits investment spending as a variable proportion of internal cash flow. Since the supply of internal funds is obviously affected by the current level of profits, it has been suggested that the optimal capital stock K^* should be made to depend not on the level of output, as in the accelerator framework, but instead on variables capturing the level of profits or expected profits.[24]

Consider the specification by Yehuda Grunfeld [1960], who assumed that the optimal capital stock is a linear function of expected profits, as proxied by the market value of the firm, V_t:

$$K_t^* = \alpha + \beta V_t \qquad \textbf{(6.16)}$$

With Eq. (6.16) substituted into Eq. (6.6), Grunfeld obtained an investment equation with an intercept term and with V_t replacing Y_t in Eq. (6.13):

$$I_t = \lambda\alpha + \lambda\beta V_t + (\delta - \lambda)K_{t-1} \qquad \textbf{(6.17)}$$

Equation (6.17) therefore suggests that investment is very much affected by the external market value of the firm.

Among others, John R. Meyer and Edwin E. Kuh [1957] and James Duesenberry [1958][25] have argued that there are important imperfections in capital markets. If the risks associated with firms' increasing the ratio of their debt to earnings lead them to have strong preferences for the internal cash flow financing of investment, then one might want instead to replace V_t in Eq. (6.16) with a liquidity-type variable such as profits or retained earnings after taxes.

A common variable used to measure available funds is cash flow, defined as profits after taxes plus depreciation allowances less dividend payments to shareholders. Cash flows have historically accounted for a substantial proportion of firms' sources of funding for investments in plant and equipment.

Cash flow is not, however, the sole source of available funds. After cash flow the principal source of funds for investors is debt financing. Although debt financing may allow a firm to expand its capital budget, such financing often becomes considerably more expensive than its yield would suggest. For example, debt obligations may place constraints on capital budgeting options, they may increase the risk that is inherent in owning shares of the firm, and they might even eventually increase the risk of managers and owners losing control of their investments. Most empirical analysts also believe that the cost of debt financing exceeds its yield by an increasing margin as the firm's reliance on borrowed funds increases.[26]

A third source of funds for firms is the sale of equity. This type of financing is particularly important for firms whose current or prospective investment opportunities far exceed their cash flow. New equity financing can be very expensive for firms, however, since new equityholders are entitled to their share of any dividends paid by the corporation and because under U.S. tax law these dividend payments are not tax-deductible to the firm (unlike the interest payments on debt). This cost premium can be substantial. For example, Kopcke [1985, p. 25] notes that in 1984 the real cost of investment-grade bonds was about 3%, while the real cost of equity financing exceeded 6.5%.

In summary, according to the cash flow model a firm first commits its retained earnings to financing its capital budget. Only after internal cash flow is exhausted does the firm seek external debt or equity financing. Since internal cash flow serves as a measure of profitability and as an index of the firm's capacity to attract external financing, the amount of the firm's investment is postulated to depend on its available cash flow.

One ambiguity that emerges from the above interpretation on the importance of cash flow is whether cash flow affects the desired capital stock K^* or whether it instead operates by affecting the speed of adjustment λ from K to K^*. The literature is somewhat vague on this, but it is plausible to argue that both channels are potentially significant.[27] Note, however, that if cash flow affects the speed of adjustment, then λ is time-varying and endogenous, rather than fixed and exogenous as in Eqs. (6.13), (6.14), and (6.17). Specifications in which the firm chooses the time path of λ_t as part of its optimal investment process will be considered further in Section 6.7.

The cash flow model has been implemented in a variety of ways by a large number of researchers. In most applications a distributed lag of cash flow is specified, with cash flow in nominal dollars being deflated by the price index for new investment goods. To capture replacement investment and lagged adjustment, the lagged capital stock is also added as a regressor. Note that since the cash flow model (6.18) is very similar to the accelerator model (6.15) with cash flow variables merely replacing output variables, issues of estimation and hypothesis testing tend to be very similar in the two frameworks. For that reason a discussion of these issues is not repeated here.

For our purposes it is useful again to examine the empirical studies based on quarterly U.S. data by Richard Kopcke [1977, 1982, 1985], who estimated separate equations for nonresidential structures and producers' durable equipment. In the 1977 and 1985 studies, Kopcke estimated a cash flow model having the general form

$$I_t = a + \sum_{i=0}^{m-1} b_i(F/J)_{t-1} + cK_{t-1} + u_t \tag{6.18}$$

where the b_i, a, and c are unknown parameters to be estimated, F is internal cash flow in current dollars, and J is a price index for new capital. In his 1982 study, however, Kopcke estimated a somewhat different equation in which I_t/K_{t-1} was the dependent variable and a market value variable was added as a regressor. We defer further discussion on the use of market value and Tobin's q as regressors to Section 6.5, where the Tobin's q model is treated in greater detail. For studies (ii) and (iii), Kopcke employed the Almon lag and constrained parameters to lie along a third-degree polynomial, whereas in study (i) a fourth-degree polynomial was employed.

In Table 6.3, parameter estimates of preferred models from Kopcke's first (1958:1–1977:3) and third (1956:1–1979:4) studies are presented; he did not report standard errors. A number of points merit discussion.

First, regarding the distributed lag specification, the length and shape of the estimated lag distribution vary considerably among studies and assets. In the first study, for both equipment and structures the current period cash flow does not affect current investment at all, whereas in the third study the b_0 coefficient is the largest of the estimated distributed lag parameters. The estimated b_0 through b_4 distributed lag coefficients are always larger for equipment than for plant, implying that cash flow has a more substantial effect on equipment investment than on that for longer-lived plant. In study (i), some-

Table 6.3 Parameter Estimates of Cash Flow Investment Equation (6.18) Reported by Kopcke in Two Quarterly U.S. Studies

	Nonresidential Structures		Producers' Durable Equipment	
Parameter	Study (i)	Study (iii)	Study (i)	Study (iii)
a	0.59	12.71	−19.90	12.71
b_0	—	0.0836	—	0.3702
b_1	0.0681	0.0755	0.2069	0.1677
b_2	0.0222	0.0619	0.0968	0.0963
b_3	0.0421	0.0465	0.1288	0.0898
b_4	0.0546	0.0328	0.1576	0.0818
b_5	0.0381	0.0245	0.1211	0.0062
b_6	0.0224	—	0.0406	—
b_7	—	—	0.0203	—
b_K	0.051	—	0.104	—
ρ	0.83	0.956	0.81	0.936

Notes: Study (i) covers 1958:1–1973:3, while study (iii) covers 1956:1–1979:4. For study (i), Kopcke's [1977] estimated slope coefficients have been divided by 100 to make them comparable to Kopcke's [1985] study (iii). Lagged variables employ earlier observations.

what surprisingly, the length of the lag is 6 for structures and 7 for equipment, while in study (iii) the length is 5 for both assets. Note also that while the distributed lag coefficients in the last study decline as the length of time displacement increases, in the first study for both equipment and structures the estimated distributed lag pattern is saw-toothed, falling from $t-1$ to $t-2$, then gradually increasing to $t-4$, and finally falling again. Together, these estimates indicate a lack of stability over time in the estimated distributed lag coefficients for the cash flow model.

Second, in terms of the effect of the lagged capital stock on current gross investment, only in the first study is this variable included as a right-hand variable, and here its estimated value is positive in both the plant and equipment investment equations. In the last study this variable is omitted from the preferred equation, perhaps because its estimated value was statistically insignificantly different from zero. Note that such insignificance is not entirely unexpected; our previous discussion of the accelerator model implied that the expected coefficient on lagged capital stock cannot be signed a priori, since by Eq. (6.13) it represents $(\delta - \lambda)$. Hence if $b_K \approx (\delta - \lambda)$ is insignificantly different from zero, this may simply reflect a finding that with the cash flow model, the estimate of λ is approximately equal to that of δ.

Finally, although statistical tests of significance are not reported, for both equipment and structures and in both time periods, considerable first-order autocorrelation of residuals is present. In the first study the estimated ρ are 0.83 and 0.81, while in the last study, estimates of ρ are 0.956 and 0.936.

This concludes our discussion of the cash flow model, although market value as a determinant of investment will be addressed further in the Tobin's q model of Section 6.5. Moreover, the performance of the cash flow model in estimation and forecasting will be considered again in Section 6.8, where the cash flow formulation will be compared with other aggregate investment models.

6.4 THE NEOCLASSICAL MODEL

Earlier, it was noted that one highly restrictive assumption embodied in the accelerator model of investment is that the capital/output ratio is fixed, an assumption that implies that substitution possibilities among capital, labor, and other inputs are constrained to be zero. Similarly, in the cash flow model, only internal cash flow affects the optimal capital stock, and again there is no role for input substitution. By contrast, economic theory textbooks have long emphasized the role of input substitution as a critical element in the economic theory of cost and production. This inconsistency has been highlighted by Dale Jorgenson [1963, p. 247]: "There is no greater gap between economic theory and econometric practice than that which characterizes the literature on business investment in fixed capital." In the decade that followed, Jorgenson and his associates worked at closing this gap, and their pioneering studies resulted in a model that is widely used to this day, namely, the neoclassical model of investment. Since the neoclassical model currently receives the greatest attention from econometricians, we consider it in some detail in this section.

The distinguishing feature of the neoclassical model is that it is based on an explicit model of optimization behavior that relates the desired capital stock to interest rates, output, capital prices, and tax policies. As we shall see, however, the major pitfall of the neoclassical model is that while it provides a clear framework for understanding factors affecting the firm's optimal demand for capital, it does not rationalize investment or movements toward the optimal capital stock. More specifically, as Nobel Laureate Trygve Haavelmo had explained already in 1960, "Demand for a finite addition to the stock of capital can lead to any rate of investment, from almost zero to infinity, depending on the additional hypotheses we introduce regarding the speed of reaction of capital users."[28] As a result, although econometric models of investment based on the neoclassical paradigm have explicit theoretical foundations concerning the optimal capital stock, their empirical implementation has until very recently required appending to this demand model an ad hoc specification of the adjustment process of K to K^*. Later in this chapter, however, we will discuss recent developments that allow λ_t, the speed of adjustment, to be a choice variable in the firm's overall optimization process.

6.4.A Theory

Define profits π at time t as revenue minus costs. In the case of a firm using two inputs, capital and labor, profits can be written as

$$\pi_t \equiv P_tY_t - w_tL_t - c_tK_t \tag{6.19}$$

where P is the price of output, Y is value-added output quantity, w is the wage rate, L is the hours of labor services, c is the cost of capital services, and K is the quantity of capital services. (Recall that an assumption made in virtually all investment models is that the quantity of capital services is a constant proportion of the capital stock.) The measurement of c, often called the *user cost of capital*, will be discussed in further detail later in this section.

Jorgenson specified that the firm chooses time paths of inputs and output so as to maximize the present value of profits, subject to a neoclassical production function constraint

$$Y_t = f(K_t, L_t) \tag{6.20}$$

Since capital goods are durable, however, by purchasing long-lived plant and equipment, firms could potentially lock themselves into a situation in which they might not be able to dispose of unwanted capital goods. This implies that the present value optimization problem facing a firm is a very complex one involving uncertainties concerning lifetimes of capital goods, future input prices, and future output demands. How can such a complex problem be made more manageable and amenable to empirical implementation? To attain empirical tractability, Jorgenson and his associates made a number of important simplifying assumptions.

First, Jorgenson assumed the existence of a perfect market for used or secondhand capital goods, as well as perfect markets for all inputs and output. The existence of a perfect used market for capital goods implied that firms did not need to worry about locking themselves in by purchasing long-lived investment goods, since such goods could always be sold on the secondhand market at prices just equal to the present value of their expected services over their expected remaining lifetimes. Further, the existence of a perfect market for secondhand capital goods allowed Jorgenson to view firms as renting capital goods to themselves during each time period, charging themselves an implicit rental price for capital, a price that is called the *user cost of capital.*

Second, in his theoretical framework, Jorgenson assumed that the adjustment of K to K^* was costless. Like others before him, he assumed that the physical decay of capital goods followed an exponential pattern. Finally, Jorgenson calculated the capital stock using the perpetual inventory relation (6.4).

The above assumptions brought about a major simplification of the optimization problem facing firms. Specifically, Jorgenson showed that under these conditions the very complex present value optimization problem reduced to a sequence of one-period profit maximization problems for which the firm chooses optimal values of K_t, L_t, and Y_t so as to maximize one-period profits (6.19) subject to the production function constraint (6.20).

We now examine this one-period optimization problem in further detail. Under the above profit maximization conditions, use of the traditional Lagrangian multiplier procedure yields the familiar necessary conditions for optimality, namely, for capital,

$$P_t \cdot \frac{\partial Y_t}{\partial K_t} = c_t \rightarrow \frac{\partial Y_t}{\partial K_t} \equiv MPP_{K,t} = \frac{c_t}{P_t} \tag{6.21}$$

and for labor,

$$P_t \cdot \frac{\partial Y_t}{\partial L_t} = w_t \rightarrow \frac{\partial Y_t}{\partial L_t} \equiv MPP_{L,t} = \frac{w_t}{P_t} \tag{6.22}$$

where $MPP_{K,t}$ and $MPP_{L,t}$ denote the marginal physical products of K and L, respectively. These equations simply restate the familiar result that profit-maximizing firms will choose that set of inputs such that for each input, the marginal benefit of employing another unit of the input (the additional real output) just equals the marginal cost of employing another unit of the input (the additional real wage or real user cost of capital).

To implement an equation like Eq. (6.21) empirically, one needs to specify an explicit form of the production function f in Eq. (6.20), derive the corresponding expression for the marginal product of capital, and then solve for that level of K^* such that the marginal physical product of capital just equals the real user cost of capital, as in Eq. (6.21). In turn, this requires a measure of the user cost of capital c_t and a mathematical form for the production function f in Eq. (6.20).

6.4.B On Measuring the User Cost of Capital

Let us first examine the user cost of capital. Unlike labor input, for which wage rate data are typically readily available, the one-period user cost of capital is seldom directly observed. Although some types of capital have active rental markets (e.g., airplanes), in most cases, firms purchase capital inputs and consume them entirely by themselves. An implication of this is that because of the lack of available data, one must typically infer indirectly the user cost of capital that firms implicitly charge themselves to use their own capital inputs. If the secondhand market is assumed to be perfect and firms are indifferent between renting and owning capital, the implicit user cost of capital that firms charge themselves must just equal the price that firms could fetch were they to rent their capital to others.

In their pioneering study on the effects of tax policy on investment, Robert E. Hall and Dale W. Jorgenson [1967] emphasized that the rental price of capital must incorporate at least four effects. First, there is the opportunity cost of having funds tied up in plant and equipment. As before, let the asset price for new capital goods be J_t, and let the current one-period interest rate yield be r_t. In such a case the opportunity cost of capital equals $r_t \cdot J_t$. Second, assuming that capital decays at a constant one-period rate of δ%, the renter would need to compensate the owner for depreciation, and this depreciation would equal $\delta \cdot J_t$. Third, in addition to effects due to depreciation, durable capital goods experience price changes over time that result in capital gains or losses to their owners. Let the expected percentage change in the asset price be $\Delta J_t/J_t$. In the Hall-Jorgenson formulation the user cost of capital is the sum of the above three effects, that is,

$$c_t = J_t(r_t + \delta - \Delta J_t/J_t) \tag{6.23}$$

Finally, Hall and Jorgenson noted that Eq. (6.23) should be adjusted to take into account the effects of various taxes. The introduction of taxes into the user cost formula involves an implicit assumption that firms are unable to shift taxes forward to consumers and that firms' user costs are therefore affected by taxes.[29] Through their effects on the user cost of capital, various tax policies can then be related to investment spending.

As Hall and Jorgenson emphasized, since various types of taxes affect after-tax returns, the incorporation of tax effects into the user cost of capital formula depends on the particular statutory provisions of the relevant tax laws. In the United States, such relevant taxes include the federal and state corporate income taxes, accelerated depreciation provisions, investment tax

ROBERT E. HALL

Computing the Effects of Tax Policy on Investment

As his undergraduate thesis in economics at the University of California–Berkeley in Spring 1964, Robert E. Hall elaborated on the neoclassical model of investment developed by his thesis supervisor, Dale W. Jorgenson. The following September, Hall enrolled as a graduate student at MIT, and during his first year he took a public finance course from E. Cary Brown. Public finance interested Hall, and the following summer, Hall and Jorgenson resumed working together in Berkeley, jointly deriving the famous rental price formula that incorporated the effects of tax policy. Their resulting article in the *American Economic Review*, "Tax Policy and Investment Behavior," was the first extensive examination of tax policy in a neoclassical model of investment behavior.

Hall's contributions to computation in econometrics go far beyond his work in investment modeling. When Dale Jorgenson asked him to organize software to implement empirically the computation of the effects of tax policy on investment, Hall found the available software woefully inadequate. Working originally on an IBM 1620 machine that IBM had nick-named CADET (*C*ouldn't *A*dd, *D*idn't *E*ven *T*ry—it used a table look-up to add rather than doing addition internally), Hall set about to writing the computer code for a user-friendly regression program. MIT graduate students began using the program in 1966, and after naming the program TSP (*T*ime *S*eries *P*rocessor), Hall revamped it completely in 1967, making it more modular and adding nonlinear estimation features. Hall credits Mark Eisner with helping him enormously, particularly in learning how to program analytic differentiation.

When Hall assumed an academic position at Berkeley in 1967, he took TSP with him; and with funding support to Jorgenson and Hall from David Wood of the U.S. Executive Office of the President, Office of Emergency Preparedness, TSP was essentially completed by 1968. Interestingly, much of the TSP programming took place on a remote terminal to a UNIVAC 1108 (complete with a printer, a card reader, and a 2400-baud modem that hardly ever worked), on the ground floor of a Baptist seminary, within easy walking distance of the famous 1960s People's Park student antiwar demonstrations at Berkeley.

Robert E. Hall was born in Palo Alto in 1943, received his B.A. degree from Berkeley in 1964, and received his Ph.D. from MIT in 1967. After three years at Berkeley, Hall returned to MIT, where he stayed until 1978, when he accepted an offer from Stanford University. Currently, Hall is Professor of Economics at Stanford, is a Senior Fellow at the Hoover Institution, and is Director of the Research Program on Economic Fluctuations of the National Bureau of Economic Research. Hall's research is primarily in macroeconomics, focusing on issues of inflation, unemployment, taxation, and monetary policy. With John Taylor he is coauthor of *Macroeconomics: Theory, Performance and Policy*. He has been elected a Fellow of the Econometric Society and of the American Academy of Arts and Sciences.

credits, capital gains taxes, and federal, state, and local property taxes. One user cost formula that accounts for the presence of such taxes and is commonly used in investment studies is a slightly revised version of the Hall-Jorgenson equation, derived by Laurits R. Christensen and Dale W. Jorgenson [1969]. This modified user cost formula is written as

$$c_t = TX_t \cdot [J_t r_{t-1} + \delta \cdot J_t - \Delta J_t] + b_t J_t \tag{6.24}$$

where b_t is the effective rate of property taxes and TX_t is the effective rate of taxation on capital income given by

$$TX_t \equiv \frac{1 - \Upsilon_t \cdot z_t - k_t}{1 - \Upsilon_t} \tag{6.25}$$

where Υ_t is the effective corporate income tax rate, z_t is the present value of depreciation deductions for tax purposes on a dollar's investment over the lifetime of the good, and k_t is the effective rate of the investment tax credit.

A rather serious practical problem with empirically implementing the neoclassical model of investment is that there are a number of difficult and unsettled issues in how one obtains measures of the various components in the tax-adjusted user cost of capital formulae (6.24) and (6.25). Several of these problems merit special attention.

First, to be consistent with the underlying theory, researchers usually seek data on marginal effective tax rates, not average rates. However, since tax law interpretation is complex, it is often very difficult to obtain reliable data on effective marginal tax rates. Note also that in the United States, corporate taxes (and, for that matter, subsidies) are levied at the federal, state, and local levels of government. Effective tax rate calculations should take into account these various levels and interactions of taxes and subsidies. Because of the complexity in undertaking such calculations, however, some analysts use as their measure of tax impacts the average effective tax rate or, in certain cases, marginal statutory tax rates.[30] Further, a number of U.S. and international studies have concluded that these effective and statutory tax rates vary considerably among differing types of assets, although the 1986 Tax Reform Act in the United States was designed in part to reduce such variations.[31] This suggests that in studying the effects of taxes on investment it is important to distinguish among the various types of investment goods.

A second set of issues in implementing the tax-adjusted user cost of capital empirically involves choosing "the" interest rate yield r_t. Since firms have available various possible sources of funding, including internal cash flow and external debt or equity, and since each of these sources can have distinct costs, it is not entirely clear how one should compress these various costs into a single r_t measure. In practice, researchers have used a variety of variables as their measure of r_t, including a government risk-free bond yield, a Moody's corporate bond yield such as the Baa yield, weighted averages of debt and equity costs, and ex post average internal rates of return. Econometric estimates of the neoclassical investment model parameters will depend, of course, on what measure of r_t is incorporated into the user cost formula (6.24).[32]

A third issue involved in implementing empirically the user cost of capital formula (6.24) concerns measurement of the capital gains term $\Delta J_t \equiv J_t - J_{t-1}$. This capital gains term is typically envisaged as incorporating *expected* capital gains, and so the empirical researcher must deal with the issue of measuring unobserved expectations. In practice, Christensen and Jorgenson assume perfect foresight and replace expected with realized new investment goods prices, while others have assumed static expectations, either in levels or in growth rates. Numerous alternative approaches have also been examined empirically, and it is clear that estimates of investment equations are somewhat sensitive to how one incorporates nonstatic expectations.[33] We shall return to a discussion of expectations later in this chapter (as well as in Chapter 10).

Each of these measurement issues is in principle important. One clear implication is that with the neoclassical investment model, considerable care must be taken in empirically assessing the sensitivity of one's results to alternative measures of the tax rate variables, the interest rate, and the capital gains term in user cost formulae such as Eqs. (6.24) and (6.25).

6.4.C Toward Empirical Implementation

Let us now return to the estimating equation (6.21), which serves as the basis of the neoclassical investment model. In this equation the optimal capital stock K^* is obtained by finding that level of capital at which its marginal physical product equals the real user cost. To implement this equation econometrically, one must therefore assume a specific form of the production function and obtain the marginal physical product by taking the partial derivative with respect to capital.

In the investment studies by Hall and Jorgenson and by Jorgenson a simple Cobb-Douglas form of the production function was assumed (with the implied capital-labor substitution elasticity equaling unity):

$$Y_t = A \cdot K_t^{\alpha} \cdot L_t^{\beta} \tag{6.26}$$

where, under the assumption of constant returns to scale, $\alpha + \beta = 1$. Differentiating Eq. (6.26) with respect to K_t, rearranging, and substituting into Eq. (6.21) yields

$$\alpha(Y_t/K_t) = c_t/P_t$$

This equation is then solved for the optimal capital stock K^*:

$$K_t^* = \alpha \cdot (P_t/c_t) \cdot Y_t \tag{6.27}$$

Notice that if the firm is assumed to maximize profits, then Y_t in Eq. (6.27) is an endogenous choice variable for the firm, while if it is instead assumed only that the firm minimizes costs, then Y_t is exogenous.

In his empirical research, Jorgenson assumed that Y_t was exogenous. Moreover, while his theoretical model was based on instantaneous adjustment (zero adjustment costs were assumed), in his empirical implementation, Jorgenson specified partial adjustment in the form of a distributed lag specification. This apparent inconsistency between theory and practice was rationalized by arguing that while the firm always attempted instantaneous adjustment, such attempts were perpetually frustrated by unanticipated delivery delays.

More specifically, in each period the firm was assumed to place orders for new net investment so that, had these orders been filled, it would have been the case that $K_t^* = K_t$, $K_{t-1}^* = K_{t-1}$, and so on. It therefore follows that

$$IO_t = K_t^* - K_{t-1}^* \equiv \Delta K_t^* \tag{6.28}$$

where IO_t is investment orders in time period t. Letting ϕ_j represent the proportion of all orders that take j periods to be delivered, and assuming that these ϕ_j are constant over time, Jorgenson related current net investment spending I_{nt} to a distributed lag function of current and previous investment orders $IO_{t-\tau}$, then, via Eq. (6.28), to current and previous changes in the desired or optimal capital stock $K_{t-\tau}^*$, and finally, using Eq. (6.27), to current and lagged output and real user cost, thereby ultimately obtaining

$$I_{nt} = \sum_{j=0}^{\infty} \phi_j IO_{t-j} = \sum_{j=0}^{\infty} \phi_j \, \Delta K_{t-j}^* = \sum_{j=0}^{\infty} \alpha\phi_j(P \cdot Y/c)_{t-j} \tag{6.29}$$

Provided that no orders were cancelled, the distributed lag parameters ϕ_j should sum to unity.

Equation (6.29) is in terms of net investment. Since replacement investment is proportional to the capital stock, Jorgenson added $\delta \cdot K_{t-1}$ to both sides of Eq. (6.29), thereby obtaining the estimating equation

$$I_t = \sum_{j=0}^{\infty} \alpha\phi_j(P \cdot Y/c)_{t-j} + \delta \cdot K_{t-1} \tag{6.30}$$

Jorgenson [1963] appended an additive "white noise" disturbance term to Eq. (6.30) and then approximated the infinite lag by using the rational lag specification discussed in Section 6.3.

Working with a number of associates over an extended time period, Jorgenson has reported estimates of the gross investment equation (6.30) using several differing measures of the r_t, tax variables, and expected capital gains components in the user cost formula (6.24). Some of this work is with data at an aggregated sectoral level (e.g., total manufacturing), while other studies are based on data from individual firms.[34]

One important empirical implication of virtually all of Jorgenson's empirical research on investment is that investment spending is very much influenced by tax policy that affects the user cost of capital. Notice that in the Hall-Jorgenson neoclassical framework, tax policy directly affects investment via changing the user cost of capital, whereas with the accelerator model, tax policy has only an indirect effect through induced changes in output. With the cash flow model, tax policy can have an impact, depending on whether cash flow is calculated by using before- or after-tax net revenues.

A rather common and troubling result based on Jorgenson's research, however, is that the estimate of α is very small, often less than 0.05, whereas on the basis of the Cobb-Douglas production function (6.26), one would expect a larger estimate of around 0.25.[35] While some analysts attribute this small estimate to measurement error, the source is still a matter of some controversy.

Jorgenson's neoclassical model has been generalized in a number of ways. First, if one takes a logarithmic transformation of Eq. (6.27), then differentiates with respect to the $\ln c_t$ and compares this with the partial derivative with respect to $\ln Y_t$, it is clear that the elasticity of optimal capital with respect to output is $+1$, while that with respect to the user cost of capital is -1, that is, the elasticities are equal in magnitude but differ in sign. When one adds the distributed lag specification (6.29) to the optimal capital stock relation (6.27), the short-run, period-by-period responses of investment to changes in output and to changes in the user cost are still equal to each other (differing in sign), but now of course they are not necessarily equal to $+1$ or -1; rather, provided that the ϕ_i sum to unity, the short-run user cost responses sum to -1, while those for output sum to $+1$.

This equality of lagged investment responses to output and user costs has been questioned by Charles W. Bischoff [1971a], who argues that the response of investment to output changes should be more rapid than that to changes in user cost. More specifically, assume that capital is substitutable with labor before capital is put into place (ex ante substitutability, often called "putty") but cannot be substituted for labor once it is put into place (ex post zero substitutability, often called "clay"). Now suppose that the user cost of capital unexpectedly fell and that it was expected to remain at this lower level. This would bring about an increase in the optimal capital/labor ratio, but because of the putty-clay nature of the capital stock, the existing capital stock cannot be remolded to adapt to the new capital/labor ratio. Rather, the effects of the capital user cost change can be realized only as the older capital stock is retired or as total capacity is increased. By contrast, when output increases unexpectedly, new capital equipment is required immediately, having the same capital/labor operating attributes as the existing capital stock, and its installation is not impeded by the need to retire existing equipment first.

An important implication of this putty-clay framework is therefore that the response of investment to an increase in output should be shorter and more rapid than the response to a decrease in the user cost of capital. On the basis of this reasoning, Bischoff generalized the change in optimal capital relation (6.29), allowing for the effect of the real user cost to differ from that of output. This resulted in an investment order equation of the general form

$$IO_t = \Delta K_t^* = \alpha(P/c)_t(Y_t - Y_{t-1}) = \alpha(P/c)_t Y_t - \alpha(P/c)_t Y_{t-1} \qquad \textbf{(6.31)}$$

Current net investment was then specified again to be the sum of current and lagged investment orders, where, however, the lag coefficients on output differed from those on user cost as follows:

$$I_{nt} = \sum_{j=0}^{\infty} IO_{t-j} = \alpha \sum_{j=0}^{\infty} \phi_j (P/c)_{t-j} Y_{t-j} - \alpha \sum_{j=0}^{\infty} \omega_j (P/c)_{t-j} Y_{t-j-1} \qquad \textbf{(6.32)}$$

To capture the effects of replacement investment, δK_{t-1} is added to both sides of Eq. (6.32). Further, in most empirical implementations the infinite lag is approximated by using a finite lag, resulting in an equation of the form

$$I_t = a_0 + \sum_{j=0}^{m-1} b_j (P/c)_{t-j} Y_{t-j} - \sum_{j=0}^{m-1} c_j (P/c)_{t-j} Y_{t-j-1} + \delta K_{t-1} \qquad \textbf{(6.33)}$$

where the b_j and c_j parameters are functions of the underlying α, ϕ_j, and ω_j parameters in Eq. (6.32). Notice that if one takes and subtracts the derivative $\partial I_t / \partial \ln (c/P)_{t-j}$ from $\partial I_t / \partial \ln Y_{t-j}$, this difference equals $c_j (P/c)_{t-j} Y_{t-j-1}$, which is positive if c_j is positive. Thus a 1% increase in output at time $t - j$ has a larger effect on I_t than a 1% decrease in the user cost of capital at time $t - j$, provided that $c_j > 0$.

In his empirical research, Bischoff found that the c_j were primarily positive and statistically significant. The null hypothesis that the price and output variables act with the same lag distribution was decisively rejected; that is, the joint null hypothesis that $c_j = 0, j = 0, \ldots, m - 1$ was rejected.[36]

A different generalization of the Jorgenson specification involved the use of a less restrictive production function underlying the marginal productivity relation (6.21). As was noted earlier, with the Cobb-Douglas specification (6.26) the elasticity of the optimal capital stock with respect to the real user cost of capital is -1; this result is obtained by taking a logarithmic transformation of Eq. (6.27) and then differentiating with respect to $\ln c_t$. A more general production function is called the CES (constant elasticity of substitution) function, and while it constrains the elasticity of substitution to be constant, it does not constrain it to equal unity.

Although a detailed discussion of properties of the CES function is outside our current focus (see Chapter 9 for further analysis), for our purposes it is sufficient to note that with the CES function, the marginal productivity relation (6.21) yields an expression for the optimal capital stock,

$$K_t^* = \alpha' \cdot (P_t/c_t)^\sigma \cdot Y_t \tag{6.34}$$

where σ is the elasticity of substitution between capital and labor. Note that when $\sigma = 1$, Eq. (6.34) collapses to the Cobb-Douglas optimal capital stock relation (6.27). Equation (6.34) provides the basis of an investment equation in which the long-run price response of investment to the user cost of capital is not constrained a priori to equal unity, but instead is estimated.

Earlier, it was noted that with the optimal capital stock relation (6.27) based on the Cobb-Douglas production function with constant returns to scale, the elasticity of capital with respect to output was also constrained to equal unity. Following Robert Eisner and M. Ishaq Nadiri [1968], one can allow returns to scale to differ from unity. With the CES function this implies an optimal capital relation based on the marginal productivity expression (6.21) of the form

$$K_t^* = \alpha' \cdot (P_t/c_t)^\sigma \cdot Y_t^{[\sigma+(1-\sigma)/v]} = \alpha' \cdot (P_t/c_t)^\sigma \cdot Y_t^\eta \tag{6.35}$$

where v is the returns to scale parameter and $\eta \equiv \sigma + (1 - \sigma)/v$ is the elasticity of optimal capital stock with respect to output.

Instead of employing linear specifications, Eisner and Nadiri used a logarithmic regression equation based on Eq. (6.35). Specifically, taking first differences, they began with an equation of the form

$$\Delta \ln K^* = \sigma \, \Delta \ln (P/c) + \eta \, \Delta \ln Y$$

and then permitted different lagged responses to (P/c) and Y, thereby obtaining the equation

$$\begin{aligned} \Delta \ln K_t = & \sum_{j=0}^{m-1} [\phi_{cj} \Delta \ln (P/c)_{t-j} + \phi_{Yj} \Delta \ln Y_{t-j}] \\ & - \sum_{i=1}^{s} \omega_i \Sigma \, \Delta \ln K_{t-i} \end{aligned} \tag{6.36}$$

Estimates of the long-run price and output responses were shown to equal

$$\sigma = \sum_{j=0}^{m-1} \phi_{cj} \Bigg/ \left(1 + \sum_{i=1}^{s} \omega_i\right), \qquad \eta = \sum_{j=0}^{m-1} \phi_{Yj} \Bigg/ \left(1 + \sum_{i=1}^{s} \omega_i\right) \tag{6.37}$$

In implementing estimation of Eq. (6.36) empirically, Eisner and Nadiri experimented with different lag lengths for m and s and found that, regardless of the lag specification, the estimate of σ was very small (certainly less than unity and occasionally insignificantly different from zero).

Note that if one constrains the price responses to be zero ($\sigma = 0$) and the output elasticity $\eta = 1$, then the generalized neoclassical model in Eq. (6.36) reduces to the accelerator model considered earlier. The Eisner-Nadiri results imply some support for this simple accelerator specification and suggest that even when tax policy affects the user cost of capital, its direct effect on investment is nil, since investment is not sensitive to price changes (although it appears to be affected considerably by output changes).

An important empirical controversy on the effectiveness of tax policy in stimulating investment has focused in large part on the estimated value of σ, which Jorgenson and his co-workers assumed to equal unity and which others estimated, obtaining results implying that σ was close to zero.

Neither Jorgenson nor Eisner and Nadiri worried much at all about stochastic specification, simply adding disturbance terms to their equations and then estimating parameters using ordinary least squares. Recall that in Section 6.1 a number of reasons were given suggesting why the stochastic specification might follow an autoregressive process. Such considerations led Bischoff [1969] to specify a levels form of Eq. (6.36):

$$\ln K_t = a + \sum_{j=0}^{m-1} [\phi_{cj} \ln (P/c)_{t-j} + \phi_{Yj} \ln Y_{t-j}] - \sum_{i=1}^{s} \omega_i \ln K_{t-i} + u_t \tag{6.38}$$

where u_t followed a first-order autoregressive process, $u_t = \rho u_{t-1} + \epsilon_t$.

If $\rho = 0$, then OLS estimates of Eq. (6.38) would be consistent, but if $\rho \neq 0$, then because the lagged dependent variable is a regressor, estimation by OLS will yield inconsistent estimates of the parameters. On the other hand, if $\rho = 1$, then the usual procedure for dealing with first-order autocorrelation by taking generalized first differences would yield the first difference equation (6.36) estimated by Eisner and Nadiri, whose parameters in such a case could be estimated consistently by OLS. However, if ρ is between 0 and 1, regardless of whether one estimates Eq. (6.36) or Eq. (6.38), since each specification contains a lagged dependent variable as a regressor, consistent and efficient estimates of the parameters can be obtained only by using a generalized least squares estimator that allows for first-order autocorrelation.

Using the same data as that of Eisner and Nadiri (also used earlier by Jorgenson) and the Hildreth-Lu estimator, Bischoff obtained results that differed dramatically from those of Eisner and Nadiri. Bischoff's conditional maximum likelihood estimate of ρ was 0.20 (although he discovered an inferior local maximum at $\rho = 0.92$). However, the estimated likelihood function was quite flat, with a 95% confidence interval for ρ ranging from -0.02 to 0.97 for one set of data and from -0.2 to 1.0 for another set. Therefore Bischoff could not reject the null hypothesis that $\rho = 0$, and whether $\rho = 1$ was rejected was marginal.

Bischoff then proceeded to estimate σ and η, assuming a number of values for ρ. The only situation in which Bischoff was able to reject the null hypothesis that $\sigma = 1$ was when $\rho = 1$ (the Eisner-Nadiri specification), whereas in all other cases for $0 < \rho < 1$, the Jorgenson-maintained assumption that $\sigma = 1$ could not be rejected. Hence, although Bischoff's point estimates of σ were less than unity, the Eisner-Nadiri result that σ was significantly different from (less than) unity appeared to be a direct consequence of their implicit assumption that $\rho = 1$. In essence, therefore, Bischoff established the empirical significance of the autoregressive stochastic specification; unfortunately, the data were not rich enough to enable him to discriminate effectively among competing hypotheses concerning the value of σ. Note, however, that Bischoff did not examine higher-order autoregressive or other moving average processes that might have been suggested by the underlying theory.

The controversy over how large the response of investment is to changes in the user cost of capital has filled many pages of professional economic journals, and for reasons of brevity we will not consider this literature further here.[37] For our purposes, however, it is useful to examine briefly a series of three recent empirical estimates of the neoclassical investment model reported by Kopcke. Recall that Kopcke's studies cover differing time periods, study (i) encompassing the pre-OPEC oil embargo period 1958:1–1973:3, study (ii) including earlier as well as post-OPEC responses during 1954:1–1977:4, and study (iii) ranging from 1956:1 to 1979:4.

The investment equation specified and estimated by Kopcke is Eq. (6.33) and is based on the constant-returns-to-scale Cobb-Douglas production function, which assumes $\sigma = \eta = 1$. It permits distinct distributed lag responses of investment to changes in the user cost of capital and to changes in output, and it allows for first-order autocorrelation. In studies (ii) and (iii), Kopcke employs the Almon technique and constrains the distributed lag parameters to lie along a third-degree polynomial, while in study (i) a fourth-degree polynomial is specified. Although Kopcke does not report standard errors for his estimated parameters, he states [1982, p. 30] that the length of the lag was determined "by considering the sensibility of the estimated coefficients and their standard errors." Kopcke's parameter estimates for equipment are reproduced in Table 6.4, while those for structures appear in Table 6.5.

As is seen in Table 6.4, for equipment the preferred estimated neoclassical model has a very large number of estimated parameters: in studies (ii) and (iii) a total of 26 distributed lag parameters are estimated, 13 each for the output (the b's) and the user cost (the c's) lagged variables. In all three studies, for equipment, both the estimated b_j and the estimated c_j parameters follow an inverted-V pattern, first increasing to $j = 2$ (study (i)) or $j = 4$ or 5 (studies (ii) and (iii)) and then falling. Also, the estimated c_j are often as large as the b_j

Table 6.4 Parameter Estimates of Neoclassical Investment Equation (6.33) for Producers' Durable Equipment Reported by Kopcke in Three Quarterly U.S. Studies

Parameter	Study (i)	Study (ii)	Study (iii)	Parameter	Study (i)	Study (ii)	Study (iii)
a_0	−18.24	−11.60	8.41	c_0	0.0060	0.019	0.0190
b_0	0.0063	0.020	0.0202	c_1	0.0080	0.029	0.0277
b_1	0.0079	0.030	0.0281	c_2	0.0086	0.037	0.0337
b_2	0.0087	0.037	0.0337	c_3	0.0081	0.042	0.0373
b_3	0.0085	0.042	0.0371	c_4	0.0068	0.044	0.0389
b_4	0.0074	0.044	0.0387	c_5	0.0050	0.045	0.0386
b_5	0.0056	0.044	0.0386	c_6	0.0030	0.043	0.0369
b_6	0.0035	0.043	0.0371	c_7	0.0013	0.040	0.0339
b_7	0.0015	0.040	0.0344	c_8	0.0001	0.036	0.0301
b_8	0.0003	0.036	0.0308	c_9		0.031	0.0257
b_9		0.031	0.0265	c_{10}		0.025	0.0210
b_{10}		0.026	0.0217	c_{11}		0.018	0.0162
b_{11}		0.020	0.0166	c_{12}		0.011	0.0118
b_{12}		0.013	0.0116	ρ	0.60	0.839	0.785
b_K	0.141	0.141	0.135				

Notes: Study (i) covers 1958:1–1973:3; study (ii) covers 1954:1–1977:4; study (iii) covers 1956:1–1979:4. In studies (ii) and (iii) the trailing term on the Almon lag is constrained to equal zero. Lagged variables employ earlier observations.

for equal time displacements j. Test results on the joint null hypothesis that the $c_j = 0, j = 1, \ldots, m - 1$, are not reported; however, the fact that Kopcke retains them suggests that they are statistically significant.

A second feature of the neoclassical investment model for equipment is that the estimated coefficient on the lagged capital stock is rather stable over the three studies, ranging only from 0.135 to 0.141. Further, in terms of sta-

Table 6.5 Parameter Estimates of Neoclassical Investment Equation (6.33) for Nonresidential Structures Reported by Kopcke in Three Quarterly U.S. Studies

Parameter	Study (i)	Study (ii)	Study (iii)	Parameter	Study (i)	Study (ii)	Study (iii)
a_0	16.84	−1.78	−50.71	b_6	0.00005	0.00045	0.0006
b_0	—	0.00074	0.0014	b_7	0.00009	0.00035	0.0005
b_1	0.00029	0.00076	0.0011	b_8	0.00015	0.00026	0.0004
b_2	0.00029	0.00074	0.0010	b_9	0.00016	0.00017	0.0004
b_3	0.00021	0.00069	0.0008	b_{10}	0.00003	0.00009	0.0003
b_4	0.00012	0.00063	0.0007	b_{11}	−0.00035	0.00003	0.0002
b_5	0.00006	0.00054	0.0006	b_K	0.048	0.025	0.096
				ρ	0.92	0.925	0.976

Notes: Study (i) covers 1958:1–1973:3; study (ii) covers 1954:1–1977:4; study (iii) covers 1956:1–1979:4. Lagged variables employ earlier observations.

bility, parameter estimates on each of the distributed lag variables appear to be very similar in studies (ii) and (iii) but are generally three or more times larger than those in study (i). Whether this difference is due to different scalings of the data in the first study is not clear. One substantial difference between the three studies, however, is the extent of first-order autocorrelation. In studies (ii) and (iii) the estimate of ρ is 0.839 and 0.785, respectively, but for study (i) the estimate of ρ is but 0.60.

For nonresidential structures, point estimates of the parameters appear to suggest less stability than those for equipment. As is seen in Table 6.5, while the length of the lag is 12 quarters in each of the three studies, for equal time displacements the estimated b_j and c_j from study (iii) are larger than those from study (ii), which in turn are larger than those from the earliest study. Moreover, unlike the distributed lag pattern for equipment, which displayed an inverted-V shape, with structures the pattern in studies (i) and (iii) is one of estimated coefficients continuously falling as the time displacement increases; in study (ii) the coefficients fall after $j = 1$.

Estimates of the effect of the lagged capital stock on current gross investment in structures also vary considerably among the three studies. In the first study, $b_K = 0.048$, and with the second study this estimate falls to about half of its value, $b_K = 0.025$. Then in the last study the b_K estimate jumps dramatically to 0.096. Finally, autocorrelation appears to be important in all three studies; estimates of ρ range from 0.92 to 0.976.

In summary, the neoclassical model is attractive on theoretical criteria because it provides a rigorous framework for specifying the optimal capital stock, which depends on prices, tax policy, and output. Like the accelerator and cash flow models, however, the neoclassical model is one of optimal capital, not optimal investment, and so the choice of distributed lag specification for investment tends to be ad hoc rather than based on optimization theory. Finally, while the neoclassical model is attractive on theoretical grounds, numerous practical problems emerge in implementing it empirically, including in particular the measurement of tax, interest rate, and capital gains variables.

This concludes our discussion of the neoclassical model. Further remarks concerning the empirical performance of the neoclassical model in estimation and forecasting will be given in Section 6.8, where its properties will be compared with those of other models. We now turn to a fourth model of aggregate investment, namely, the Tobin's q specification.

6.5 TOBIN'S q MODEL

Earlier, in discussing the cash flow model of investment we noted that the optimal capital stock was postulated to be a function of expected profits, which in turn might be measured by the market value of the firm. James Tobin [1969] has generalized the cash flow model and has provided a rigorous framework for an investment model in which net investment depends on the ratio of the market value of business capital assets to their replacement value, a ratio known as "q." The theory underlying Tobin's q model is relatively straightforward and in fact is closely related to the neoclassical investment model considered in Section 6.4.

6.5.A Theory

On the basis of the expected profitability of an investment project, managers reckon the price they are willing to pay for it; call this the demand price for an asset. The demand price for an entire firm is the market value of all its securities, that is, the market value of all its debt and equity in securities markets. The cost of producing all new capital goods is the supply price and is typically measured by assessing the replacement cost of a firm's assets. In equilibrium, the demand and supply prices for plant and equipment must be equal. If the ratio of the market value of the firm to the replacement value of its assets were unity, then there would be no incentives for the firm to invest.

Suppose, however, that a firm was operating in a relatively profitable environment and that if it added \$1 to its capital stock of plant and equipment, its expected profitability would increase sufficiently that its market value would increase by more than \$1. In this case the value of the marginal q ratio would be greater than unity, and the firm should invest in the plant and equipment in order to maximize the return to its shareholders. According to Tobin, such investment should continue until the incremental market value just equaled the incremental cost of the plant and equipment, that is, investment should continue until marginal q equals unity.

A similar argument could be made for a firm operating in a relatively unprofitable environment. Suppose that the firm were already overly capital-intensive and that if it added \$1 to its capital plant and equipment stock, its expected profitability would increase negligibly, so that its market value would increase by less than \$1. In this case the value of the marginal q ratio would be less than unity, and the firm should not invest in the plant and equipment; its shareholders could earn a higher return elsewhere. Indeed, the firm might work better on behalf of its shareholders if it sold off part of its capital plant and equipment.

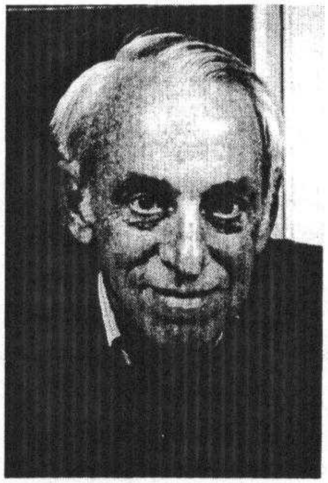
The above considerations suggest that in its naive form the Tobin's q model of investment implies that whenever marginal q is greater (less) than unity, there are incentives for net investment (disinvestment) in capital plant and equipment. Such reasoning has led to the specification of investment equations of the form

$$I_t = a + \sum_{j=0}^{m-1} b_j \cdot (q-1)_{t-j} K_{t-j-1} + b_K \cdot K_{t-1} + u_t \tag{6.39}$$

where the b_j are expected to be positive.

JAMES TOBIN

Expectations and the q Theory of Investment



Econometricians have long recognized that a major problem in modeling investment behavior involves dealing satisfactorily with the unobserved expectations of firms and investors. In a series of papers, some written with colleague William Brainard, James Tobin suggested using the securities market evaluation of a firm as a summary statistic of the market's expectations concerning the firm's future profitability as discounted for interest and risk. Tobin defined q as the ratio of the market value of the firm divided by the replacement value of its capital assets. He then showed that whenever the marginal value of q was greater (less) than unity, the firm should undertake investment (disinvestment); in long-run equilibrium the q of a competitive firm should equal unity. Tobin and Brainard also applied the q theory to macroeconomic models.

Tobin's q theory has had an enormous impact on the econometric modeling of investment decisions. Although the theory is simple and persuasive, econometric implementation has not been quite as straightforward. In particular, as is discussed in detail in this chapter, measurement problems occur for both the numerator and the denominator of q, and it is difficult to distinguish marginal from average q empirically.

James Tobin was born in Champaign, Illinois in 1918. His father was Athletic Publicity Director at the University of Illinois, and his mother was Executive Secretary of the local Family Service agency. After attending public high school in Urbana, Tobin won a Harvard National Scholarship and graduated from Harvard summa cum laude in economics in 1939. World War II interrupted his graduate studies at Harvard. After a brief stint in Washington as an economist in 1941–1942, he enlisted for Naval Reserve officer training and served as a line officer on a destroyer from 1942 to 1945. Tobin returned to Harvard in 1946, finished his Ph.D. in 1947, spent several years at Harvard as a Junior Fellow, and then studied in the Department of Applied Economics at Cambridge University. Tobin took an academic position at Yale University in 1950 and, aside from occasional visits, has been there ever since. He retired from teaching duties in 1988 and is now Sterling Professor Emeritus of Economics.

Tobin's contributions to economics are enormous. Tobin is the author or editor of 13 books and more than 300 articles; his main professional interests have been in macroeconomics, monetary theory and policy, fiscal policy and public finance, consumption and saving, unemployment and inflation, portfolio theory and asset markets, and econometrics (in Chapter 11 of this text, readers will implement his Tobit estimator). Also active in the public policy arena, Tobin was a member of President John F. Kennedy's Council of Economic Advisers and has frequently testified before Congress. In 1981, Tobin was awarded the Prize in Economic Science in Memory of Alfred Nobel, in part for his work on the q theory of investment.

An attractive feature of the q investment model is that because the market's expectations concerning future profitability are summarized completely by the securities market evaluation of the firm, with q as a regressor the lag distribution excludes delays due to expectational lags. Rather, lagged values of q represent only order, delivery, and gestation delays. Note also that in Eq. (6.39), $q_t \cdot K_t$ represents the market value of the firm; in this sense, Eq. (6.39) is very similar to Grunfeld's cash flow equation (6.17).

As we shall see, although the theory underlying q is seemingly persuasive, in fact, aggregate investment does not always respond to changes in market value consistent with the simple q theory.

6.5.B Issues in Empirical Implementation

In practice, there are some serious problems in empirically implementing the q model. Recall that the denominator of q is the replacement value of the firm's assets, typically measured as the sum of the replacement values of its plant, equipment, land, working capital, and inventories. A denominator computed in this way omits other assets of the firm, including intangibles such as brand recognition and goodwill. This underestimation of the denominator can lead to a situation in which measured q is greater than the firm's true q.

The measurement of the numerator of q also presents problems in empirical implementation, particularly with respect to valuing untraded outstanding and unretired debt obligations, as well as accounting properly for tax factors that drive wedges between before- and after-tax bond yields and dividends.[38]

More generally, a major problem with empirical implementation of the Tobin's q model is that a measure of q based on the market value of the firm divided by the replacement value of its assets represents an *average*, not a *marginal* value of q. For example, suppose that an unexpected energy price increase rendered obsolete a considerable portion of a firm's existing plant and equipment, yet simultaneously created substantial opportunities for profitable new investment in more energy-efficient equipment. In such a case, average q might be less than unity, while marginal q could exceed unity.

For reasons such as these, empirical researchers have tended to be rather cautious in arguing that whenever average q is greater than 1, incentives for positive net investment exist. Rather, most now simply postulate that at best, net investment should be an increasing function of measured average q. Further, since the underlying theory is vague on functional form, the q investment equation is often estimated in variants of Eq. (6.39) including, for example, having I_t/K_{t-1} as the left-hand variable. Note also that if one sets $a = 0$ in Eq. (6.39) and then divides both sides by K_{t-1}, the new intercept term is δ, and so this intercept can be interpreted as an estimate of the rate of depreciation.

The Tobin's q model of investment can be related to the neoclassical framework, as has been emphasized by Andrew Abel, Hiroshi Yoshikawa, and Fumio Hayashi.[39] More specifically, define the one-period shadow price of capital π_t as the additional profits expected in time period t if one more unit of capital were in place, and then set the one-period tax-adjusted price of uninstalled capital goods equal to the user cost of capital c_t—see Eq. (6.24). Now define an amended q as the ratio of these one-period prices, that is,

$$q_t \equiv \pi_t/c_t \tag{6.40}$$

Equation (6.40) differs from the earlier definition of q in that here q is the ratio of one-period prices (often called flow prices), whereas the earlier definition of q was in terms of asset prices—the ratio of the market value of the firm to the replacement value of its assets. An advantage of the formulation in Eq. (6.40), however, is that it highlights the expectational and marginal (rather than average) nature of q. One further result explored by Hayashi in this context is that if the firm operates in an imperfectly competitive output market, in equilibrium the firm's marginal q can be greater than unity.[40]

Although the q model is attractive because of its theoretical foundations and its ability to distinguish order, delivery, and gestation from expectational lags, its empirical performance has been less than impressive to date. A number of studies have regressed investment on q, and a common finding is that variations in q are unable to explain a large part of the variation in investment; further, as with other empirical investment models, the residuals or unexplained movements in investment tend to be highly serially correlated, suggesting that important explanatory variables are omitted.[41]

The relatively poor performance of q is troubling and could be due to a number of reasons. One obvious possibility is the use of average in place of marginal q. Abel and Blanchard [1986] have reported results in which they first compute a measure of marginal q based on the expectation of a present value of a stream of marginal profits. Such a calculation involves expectations of both future profitability and the discount rate (cost of capital).

Using a number of alternative measures of marginal q, Abel and Blanchard find that, as in previous studies, q is generally a statistically significant regressor, but it leaves unexplained a large and serially correlated fraction of investment. They conclude that "Since our findings are so similar to the results obtained relating investment to average q, we find little support for the view that the low explanatory power of average q is due to the fact that average q is simply a poor proxy for the theoretically more appealing marginal q."

The incompleteness of the q model in explaining investment is also evident in that output and profit variables still enter significantly when added to the q investment equations. Finally, Abel and Blanchard examine specifications in which the two components of marginal q—the marginal profit component and the cost of capital—are specified as separate regressors. They find that the marginal profit component has a larger and more significant effect on investment than does the cost of capital component, thus lending further support to the common empirical finding in neoclassical investment studies that investment responds more to output changes than to variations in the user cost of capital.

Let us now briefly examine results from a series of three empirical studies of investment in the United States by Kopcke in which estimates of the q model are reported. Recall that Kopcke's three studies cover differing time periods, study (i) encompassing the pre-OPEC oil embargo period 1958:1–1973:3, study (ii) including earlier as well as post-OPEC responses during 1954:1–1977:4, and study (iii) ranging from 1956:1 to 1979:4.

The investment equation specified and estimated by Kopcke is Eq. (6.39) with several modifications. In study (i) and reflecting roots from the internal cash flow literature, the q_{t-j} variable is multiplied by real cash flow $(F/P)_{t-j}$ rather than by the capital stock variable K_{t-j} as in Eq. (6.39). In study (ii), Kopcke divides both sides of Eq. (6.39) by K_{t-1}, whereas in study (iii) the dependent variable is respecified as I_t/U_t, where U_t is the rate of capacity utilization.[42] The estimation of these equations allows for first-order autocorrelation and employs the Almon polynomial distributed lag technique; in studies (ii) and (iii) the coefficients are constrained to lie along a third-degree polynomial, while in study (i) a fourth-degree polynomial is specified. Kopcke specifies separate investment equations for equipment and structures. While this facilitates comparison with other models estimated by Kopcke, it is worth noting that the measure of q employed as a regressor is not specific to equipment or structures, but rather reflects their combined market value.[43]

Kopcke's results are reproduced in Table 6.6. Since the dependent variables differ in the three studies, it is not meaningful directly to compare magnitudes of the estimated coefficients across studies. Some patterns among the coefficients are, however, evident in Table 6.6.

First, the length of the estimated distributed lag estimated based on the q model varies from five to nine quarters, which is about the same length as that for the accelerator and cash flow models but is shorter than for the neoclassical model estimated by Kopcke. Second, the shape of the lag distribution varies considerably among the three studies. For structures the effect of the lag on investment peaks at zero, two, and five quarters for studies (i), (ii), and (iii), respectively, while for equipment the associated peaks are at zero, one, and four quarters. For both equipment and structures, in study (iii) the lag coefficients gradually increase with time and then taper off, while in study (i) the pattern of the lag distribution coefficients is much more complex. A curious feature of the lag coefficients from the most recent study is that simulta-

Table 6.6 Parameter Estimates in Three Versions of Tobin's q Investment Equation (6.39) Reported by Kopcke in Three Quarterly U.S. Studies

	Nonresidential Structures			Producers' Durable Equipment		
Parameter	Study (i)	Study (ii)	Study (iii)	Study (i)	Study (ii)	Study (iii)
a	2.86	0.044	12.92	−19.21	0.056	−18.52
b_0	3.338	0.004	−0.0059	11.439	0.036	−0.0260
b_1	2.278	0.012	−0.0016	4.472	0.037	−0.0020
b_2	2.333	0.015	0.0019	5.618	0.034	0.0123
b_3	2.018	0.014	0.0044	9.523	0.028	0.0188
b_4	1.015	0.011	0.0060	4.248	0.021	0.0193
b_5	0.179	0.007	0.0067		0.014	0.0159
b_6	1.532	0.003	0.0064		0.008	0.0102
b_7		0.00001	0.0053		0.002	0.0043
b_8			0.0031			
b_K	0.063	—	0.082	0.177	—	0.293
ρ	0.82	1.000	0.872	0.82	0.994	0.866

Notes: Study (i) covers 1958:1–1973:3; study (ii) covers 1954:1–1977:4; study (iii) covers 1956:1–1979:4. In studies (ii) and (iii) for both equipment and structures the trailing term in the Almon lag is constrained to equal zero. Lagged variables employ earlier observations.

neous and one-period lagged increases in q tend to decrease investment, that is, for both equipment and structures, estimates of b_0 and b_1 are negative.

Since both sides of Eq. (6.39) are divided by K_{t-1} in study (ii), the lagged capital stock is no longer included as a regressor. The intercepts therefore represent estimates of δ, which are 0.044 (structures) and 0.056 (equipment). For equipment this estimate appears to be somewhat low.

Finally, as in all of the investment equation estimates reported by Kopcke, the unexplained residuals display a marked serial correlation. For structures, estimates of ρ range from 0.82 to 1.00, while for equipment they vary between 0.82 and 0.994.

In summary, the q model is attractive on theoretical criteria because it provides a rigorous framework for specifying the effect of market value on investment. It therefore provides a theoretical foundation to an investment equation that is similar in form to some of the earlier, more ad hoc cash flow models. Although use of market value incorporates the effects of expectational lags, like the accelerator, cash flow, and neoclassical models, the underlying theory of the q model does not provide guidance on factors governing the shape and length of the distributed lag specification. Therefore in practice its form tends to be chosen on an ad hoc basis, rather than being based on optimization theory.

Further, while the q model is attractive on theoretical grounds, numerous problems emerge in implementing it empirically, including in particular measuring marginal rather than average q, accounting for intangibles that affect market value, and properly incorporating tax factors that interact among market value and the tax treatment of dividends and yields. Estimates of the q model tend to vary considerably, depending on the data time period chosen, and unexplained residuals display a substantial amount of serial correlation. Moreover, since forecasting stock market trends is known to be difficult, the q model would not seem to be very useful in the forecasting context. Finally, there is a serious question as to the imperical validity of q. If q were correct, then following a dramatic decline in stock market prices, one would expect investment plans to decline as well. As the investment banking firm Deloitte, Haskins, and Sells [1988, p. 3] has noted, this was not the case following the 1987 stock market crash: "Preliminary results from a Dun & Bradstreet (D&B) survey conducted in the wake of the [1987] stock market crash support the findings that the outlook for 1988 remains healthy. In the D&B 5,000 Survey, roughly 75% of the companies said their capital spending plans for 1988 would not be negatively affected by the stock market slump."

This concludes our discussion of the Tobin's q model, although additional remarks concerning its empirical performance in estimation and forecasting will be given in Section 6.8, where its properties will be compared with those of other models. We now turn to a fifth model of aggregate investment, namely, the time series/autoregressive formulation.

6.6 TIME SERIES/AUTOREGRESSIVE MODELS OF AGGREGATE INVESTMENT

In contrast to the four competing theories of investment discussed in the previous sections, the time series/autoregressive approach does not directly use output, cash flow, market value, prices, or taxes as determinants of investment expenditures. Rather, in its simplest form, investment is merely regressed on a series of previous investment expenditures. Such a model with m lagged investment terms could of course be interpreted as resulting from a specification in which $I_t = a + u_t$, where u_t followed an autoregressive process of degree m.

6.6.A Measurement without Theory?

Viewed in such a manner, the autoregressive model of investment would seem to be a classic example of measurement without theory, and indeed practitioners of time series modeling have encountered many such criticisms. Proponents of the time series approach have argued, however, that despite the superficial elegance of some of the economic theory-based competing models of investment, their empirical implementation requires making a number of arbitrary statistical assumptions. Rather than employing such ad hoc stochastic specification assumptions, time series practitioners argue that for a class of macroeconomic models linear in the variables, the reduced form for the investment equation can be shown to have the form

$$I_t = a + \sum_{j=1}^{m} b_j I_{t-j} + u_t \tag{6.41}$$

provided that the exogenous variables are covariance stationary. For macro models that are "nearly linear" the time series model may still approximate rather closely the unconstrained reduced form equation for investment. Hence the simple representation (6.41) can be viewed as being the "final form" derived from a more structural macro model of the economy.[44]

Critics of the time series approach contend that equations such as Eq. (6.41) are not very useful in that they do not permit forecasters or policymakers to assess directly the effects of changes in business conditions or economic policy on investment. Such criticisms merely highlight the fact that unless Eq. (6.41) is specified within a more explicit structural model, and unless appropriate cross-equation restrictions are imposed, the interpretation of parameter estimates in Eq. (6.41) is essentially unclear. Considerations such as these once led Zvi Griliches [1974, p. 335] to venture a "law" stating that "any time series regression containing more than four independent variables results in garbage."

Defenders of the more structural models of investment, however, are also somewhat vulnerable to specification issues. For example, in the accelerator and neoclassical models, output affects investment. But causality can plausibly be argued to run in the other direction from investment to output. Similarly, interest rates and market value might be influenced by investment and so might be endogenous rather than exogenous. These are issues to which we will turn our attention in the next section. However, time series practitioners take the view that the potential for misspecification is very real in the more structural models and that difficult pitfalls in model building might best be mitigated by analyzing the underlying dynamics embedded in investment outlays alone.

6.6.B Empirical Implementation

Empirical implementations of time series investment models have not been confined to purely autoregressive specifications, such as those in Eq. (6.41). Rather, as in Zellner and Palm [1974], it is common to estimate investment equations that allow both for autoregressive (AR) and moving average (MA) error structures. MA specifications are attractive in that they can be shown to emerge from expectations formations processes that are adaptive or rational, as will be discussed further in Section 6.7.

Let us now briefly examine results from a series of three empirical studies of investment in the United States by Kopcke in which estimates of the time series model are given. In studies (i) and (iii), Kopcke estimates parameters of Eq. (6.41), while in study (ii), I_t/K_{t-1} is the dependent variable, rather than I_t. Kopcke uses OLS estimation procedures in all three studies; results are reproduced in Table 6.7.

Table 6.7 Parameter Estimates of Time Series/Autoregressive Investment Equation (6.41) Reported by Kopcke in Three Quarterly U.S. Studies

	Nonresidential Structures			Producers' Durable Equipment		
Parameter	Study (i)	Study (ii)	Study (iii)	Study (i)	Study (ii)	Study (iii)
a	0.29	0.0021	0.565	0.48	0.012	0.215
b_1	1.00	1.21	1.24	1.41	1.40	1.35
b_2	—	−0.043	−0.085	—	−0.464	−0.061
b_3	—	−0.191	0.079	—	—	−0.306
b_4	—	—	−0.311	—	—	−0.143
b_5	—	—	−0.017	—	—	0.245
b_6	—	—	−0.015	—	—	−0.078
b_7	—	—	0.071	—	—	—
b_8	—	—	0.029	—	—	—

Notes: Study (i) covers 1958:1–1973:3; study (ii) covers 1954:1–1977:4; study (iii) covers 1956:1–1979:4. In Study (ii), I_t/K_{t-1} is the dependent variable, not I_t. Lagged variables employ earlier observations.

A striking result from Table 6.7 is that the preferred specification is highly unstable in the series of three studies, both for structures and for equipment. In particular, for structures the preferred number of lags is one in study (i), three in study (ii), and eight in study (iii); for equipment the number of lagged terms in the three studies is one, two, and six, respectively. Critics of the time series approach would likely interpret such instability as being due to the fact that during the 1970s a number of major economic disruptions occurred and that by failing to account properly for the effects of such disruptions the unconstrained distributed lag time series approach represents a serious misspecification. It should be noted, however, that the time series model is not alone in manifesting instability; each of the other four investment models has also displayed substantial unstable tendencies.

In Section 6.8 of this chapter we will consider in greater detail the comparative performance of each of the five models in estimation and forecasting. Before doing that, however, we digress briefly to discuss several other important econometric specifications and issues.

6.7 ADDITIONAL ECONOMETRIC SPECIFICATIONS AND ISSUES

In this section we briefly discuss a number of econometric issues that emerge from our overview of the five alternative models of investment behavior. These issues include allowing for possible simultaneity of certain regressors, incorporating moving average error structures, dealing with nonstatic expectations (including the rational expectations hypothesis), and introducing adjustment costs so that speeds of adjustment are endogenous and time-varying.

6.7.A Simultaneity Issues

In the original derivation of the neoclassical investment model, profit-maximizing firms were assumed to choose levels of K_t, L_t, and Y_t so as to maximize one-period profits (6.19). Even though Y_t is therefore an endogenous variable, Jorgenson and his co-workers typically ignored possible simultaneity issues and simply assumed that there was no correlation between Y_t and the disturbance term u_t.[45] If simultaneity (nonzero correlation between Y_t and u_t) is present, however, estimation by OLS yields biased and inconsistent parameter estimates.

The simultaneity of output and investment could be handled by assuming cost minimization rather than profit maximization, in which case, by assumption, output would be exogenous. However, under cost minimization the basic optimal capital stock equation changes from Eq. (6.27) (assuming a Cobb-Douglas production function with constant returns to scale) to

$$K_t^* = \{[\alpha/(1-\alpha)] \cdot (w_t/c_t)\}^{1-\alpha} \cdot (Y_t/A) \qquad \textbf{(6.42)}$$

Hence if one wants to assume cost minimization rather than profit maximization, an equation that is different from Eq. (6.27) must be estimated.

Potential simultaneous equations problems are not necessarily confined to the output variable, however. Suppose, for example, that government tax policies changed, creating investment incentives. The increased demand for new investment could increase interest rates and/or the supply price of new investment goods, thereby affecting the r_t and ΔJ_t components of the user cost of capital (6.24). The market value might also be affected, implying that both the numerator and the denominator of Tobin's q could be simultaneously determined. Finally, depending on the structure of the tax rate schedule, marginal tax rates might be jointly determined with investment in response to the government investment incentives. Note that in some of these cases, simultaneity occurs at the level of the firm, while in other cases it is only at a more macro level.

Simultaneous equations problems can be addressed in a number of ways. One possibility is to solve explicitly for the reduced form and then estimate it by OLS. Alternatively, one might employ an instrumental variable estimator such as two-stage least squares and estimate the investment structural equation directly, the choice of instruments being guided by observing what exogenous variables would appear in a reduced form equation. Care must be taken, however, in choosing instruments if one has lagged dependent variables and allows for an autoregressive error structure in the structural equation. In particular, Ray C. Fair [1970] has shown that if the disturbance term follows an mth-order autoregressive process, then to purge the disturbance term of correlations with all m components, the instrument list for any $X_{t-\tau}$ regressor must include not only those from time period $t - \tau$, but also all those back to time period $t - \tau - m$, as well as the Y_{t-m}. In particular, if one includes only the $t - \tau$ current-valued instruments and excludes Y_{t-m}, estimation by instrumental variable techniques will yield biased and inconsistent estimates of the parameters.[46]

6.7.B Moving Average Errors

One very important aspect of the investment equation that has not yet been discussed concerns error structures that could be generated by alternative expectations formations. This can be illustrated by modifying slightly Tobin's q model (6.39), which we now write as

$$I_{nt} = a + b \cdot Q^*_{t+1} + \epsilon_t \tag{6.43}$$

where $Q^*_{t+1} \equiv q^*_{t+1} \cdot K_t$ is the *expected* market value of the firm at the beginning of the next time period and ϵ_t is an independently and identically normally distributed random disturbance term. Since expected market value is not observed, let us specify an *adaptive expectations* mechanism in which

$$Q^*_{t+1} - Q^*_t = (1 - \ell) \cdot (Q_t - Q^*_t) \Rightarrow Q^*_{t+1} = (1 - \ell)Q_t + \ell Q^*_t \tag{6.44}$$

where the adaptive expectations coefficient ℓ is in the range $0 \leqslant \ell < 1$. Note that ℓ is interpreted as representing the speed with which expectations adapt and not the proportion of the gap between K^* and K adjusted within one time period. In particular, according to Eq. (6.44), expected market value at time $t + 1$ is a weighted average of current market value and the market value expected in the current time period. Hence current expectations are derived by modifying previous expectations in light of current experience.

To implement the adaptive expectations empirically, repeatedly substitute $Q^*_{t-\tau}$ in $Q^*_{t-\tau+1}$ in Eq. (6.44), and then substitute the result into Eq. (6.43). This yields

$$I_{nt} = a + b \cdot (1 - \ell) \cdot (Q_t + \ell Q_{t-1} + \ell^2 Q_{t-2} + \cdot \cdot \cdot) + \epsilon_t \tag{6.45}$$

Since Eq. (6.45) contains an infinite number of regressors, it is not practical for estimation. This problem is solved by employing the Koyck transformation in which Eq. (6.45) is lagged once and multiplied by ℓ, the product then being subtracted from Eq. (6.45), yielding

$$I_{nt} = a \cdot (1 - \ell) + b \cdot (1 - \ell) \cdot Q_t + \ell \cdot I_{n,t-1} + \xi_t \tag{6.46}$$

where ξ_t is a first-order moving average process

$$\xi_t \equiv \epsilon_t - \ell \epsilon_{t-1} \tag{6.47}$$

It can also be shown that if one has additional explanatory variables in the net investment equation (6.43), each involving an adaptive expectations process, then the Koyck transformation can be applied successively, yielding a composite disturbance term involving a higher-order moving average process. In a neoclassical model, for example, one might have both expected output and expected user cost variables following adaptive expectations. More generally, considerations such as these suggest that moving average errors could be very important in the stochastic specifications of investment models.

An important econometric implication of the moving average error process in Eq. (6.47) is that since ξ_t is correlated with the lagged dependent variable $I_{n,t-1}$ (owing to the presence of ϵ_{t-1} in ξ_t), estimation by OLS yields biased and inconsistent estimates of the parameters. However, consistent estimates can be obtained by use of instrumental variables procedures or by the maximum likelihood method. Many econometric theory textbooks provide further details regarding estimation and inference of Box-Jenkins models with moving average or integrated autoregressive–moving average error (ARIMA) specifications.[47]

6.7.C Nonstatic and Rational Expectations

Expectations of future events play a very important role in economic behavior. Since anticipations are important, it is often necessary to separate out the effects of changed expectations from other explanatory variables in analyzing investment behavior. Failure to do so can result in, for example, erroneous predictions concerning the effects of changes in tax policy.[48]

In the last few decades, economists have made substantial advances in incorporating the effects of nonstatic expectations into econometric models. According to the rational expectations hypothesis (REH), economic agents efficiently use all the information at their disposal to form expectations. Indeed, omniscient economic agents are assumed to construct optimal predictors based on sophisticated economic theory and available information and data.[49] One important implication of the REH is that expectational errors should be uncorrelated with any of the variables entering into the information set used by economic agents, since if errors were correlated with these variables, the information contained in the variables could not have been employed rationally.

Optimal predictors of variables are predictors that are uncorrelated with expectational errors. Optimal predictors can be constructed in a number of ways. Suppose that in a regression equation the practicing econometrician seeks to employ as a regressor the optimal or rational predictor of X_{t+1}. John F. Muth [1960] has derived the optimal predictor in one special case. Specifically, if one assumes that X is stochastic and that its realizations are governed by a first-order moving average process of the form

$$X_t = X_{t-1} + u_t - \ell u_{t-1} \tag{6.48}$$

where u_t is an independently and identically distributed disturbance term with mean zero, then the optimal or rational predictor of X_{t+1}, denoted X^*_{t+1}, equals

$$X^*_{t+1} = (1 - \ell) \cdot (X_t + \ell X_{t-1} + \ell^2 X_{t-2} + \cdots) \tag{6.49}$$

But Eq. (6.49) is simply the adaptive expectations specification considered in the previous section, where $Q^*_{t+1} = X^*_{t+1}$. This implies that if values of X are generated as specified in Eq. (6.48)—admittedly a special case—then adaptive expectations are rational, that is, under these conditions, estimation of Eq. (6.46) incorporates the REH. Other stochastic processes can be specified that lead to different optimal predictors; such predictors often involve rather complex distributed lag representations.[50] Moreover, with many nonstatic expectations formulations, K^* is no longer a fixed target, but instead becomes a moving one.

Empirical studies of investment have incorporated nonstatic expectations in a number of ways. Eisner [1967], for example, replaced expectations variables directly with estimates based on survey data. A more common procedure, however, is to replace an expectations variable with the fitted value of a separately estimated equation. Examples of this include a Canadian study by Helliwell and Glorieux [1970] in which separate extrapolative, regressive, and trend growth elements of the expectations formation process are specified and estimated, and a British study by Feldstein and Flemming [1971]. Both studies find that this predicted variable performs well in the investment equation. Another example is provided by Ando, Modigliani, Rasche, and Turnovsky [1974], who compare empirically the importance of alternative forecasts of price changes in investment equations.

It is worth remarking that replacing an expectations variable with the fitted value from an auxiliary regression equation and then doing OLS on the new equation is of course an instrumental variables technique that in certain cases provides parameter estimates that are numerically equivalent to the two-stage least squares (2SLS) procedure.[51] Moreover, since by construction the fitted value in the 2SLS estimation procedure is orthogonal to the residual, use of 2SLS in this context ensures that elements of the information set (the variables employed in the first stage of 2SLS) are uncorrelated with the sample residuals, implying that this 2SLS procedure is consistent with the REH.[52]

6.7.D Adjustment Costs

Several times in this chapter it has been stressed that the economic theory underlying the investment models is in fact a theory of optimal capital stock, not of investment. In the last two decades, however, developments have occurred that now make it possible to implement empirically models of optimal investment, not just optimal capital. Such models are typically called adjustment cost models; we now briefly outline their salient features.

Recall that the investment process entails a number of time-consuming stages, including changing expectations, making decisions and appropriating orders, experiencing delivery delays, installing the new plant and equipment, and getting it to function. As originally presented by Robert Eisner and Robert H. Strotz [1963], the distinctive feature of the adjustment cost approach is that it is based on the assumption that when a firm expends effort to install new plant and equipment, it forgoes current output production. Internal costs of adjustment are therefore measured by the cost of the gestation lag, that is, by the value of current output forgone by devoting resources to the installation of new plant and equipment. If marginal adjustment costs are an increasing function of net or gross investment, then the firm might not wish to close the gap between K^* and K entirely within one time period and instead might find it optimal in a present value sense to reduce current adjustment costs at the expense of having a less efficient capital stock, thereby optimally spreading the adjustment process out over several time periods.[53]

While early adjustment cost specifications resulted in constant speeds of adjustment, in a series of papers Lucas and Treadway introduced important innovations that enormously facilitated empirical implementations of models with endogenous, time-varying adjustment paths.[54] On the basis of expected output demands, expected input prices, and increasing marginal costs of adjustment for capital, Lucas and Treadway employed dynamic optimization techniques and solved for the time path of investment that maximized present value. This resulted in a flexible accelerator model in which the optimal speed of adjustment λ_t is a function of substitution possibilities involving capital inputs, parameters of the adjustment cost function, and the discount rate. An interesting implication of this flexible accelerator model is that increases in interest rates have two separate but reinforcing effects on current investment. First, increases in r raise the user cost of capital and thereby reduce the optimal capital stock. Second, increases in r enhance the relative importance of current adjustment costs and thereby reduce the optimal speed of adjustment λ_t. Hence investment is a decreasing function of r.

Empirical implementation of the dynamic flexible accelerator model requires specification of cost or production functions, assumptions concerning the shape of the adjustment cost function, and expectations formation specifications. Since such models are reasonably complex and have typically been estimated in the context of many inputs—not just investment in capital plant and equipment—we omit further discussion here. For further details, see Chapter 9, "Modeling the Interrelated Demands for Factors of Production," especially Section 9.7.

6.8 EMPIRICAL COMPARISONS OF FIVE INVESTMENT MODELS

The theoretical investment literature is voluminous, and so it is not surprising that researchers have frequently compared the empirical estimation and forecasting properties of alternative investment models.[55] Rather than surveying that literature, however, we now compare results of a series of three "horse races" reported by Richard Kopcke [1977, 1982, 1985]. It would be comforting if this series of comparisons produced a consistent set of winners or losers, but as we shall see, this is unfortunately not the case, since the relative rankings of the models change dramatically from one study to the next.

Kopcke compares the five models on the basis of three aspects of performance: estimation, static forecast properties, and dynamic forecast properties. We have already discussed Kopcke's estimation of each of the models, but several other points should now be made. First, the length of the estimated lag distribution varies considerably among models, as is indicated in the second and sixth columns of Table 6.8. The time series model typically has the shortest lag (sometimes as small as 1), the neoclassical model usually has the longest (up to 13 lags), and the accelerator, cash flow, and Tobin's q models have lags of intermediate length. While increasing the length of the lag improves fit during estimation, it does not necessarily enhance forecasting properties.

Second, in each of the five models estimated by Kopcke, residuals were found to be highly autoregressive. Since allowing for first-order autoregressive errors essentially amounts to adding the lagged residual as a regressor, in esti-

Table 6.8 Selected Statistics of Five Investment Models during Estimation Period Reported by Kopcke in Three Quarterly U.S. Studies

	Nonresidential Structures				Producers' Durable Equipment			
Model	Length of Lag	Percent RMSE	% |Errors| >$1B	% |Errors| >$2B	Length of Lag	Percent RMSE	% |Errors| >$1B	% |Errors| >$2B
			Study (i) 1958:1–1973:3					
Accelerator	4	0.78	17.5	3.2	3	0.97	28.6	4.8
Cash flow	6	0.80	17.5	3.2	6	1.21	38.1	7.9
Neoclassical	11	0.78	12.7	1.6	9	0.94	28.6	1.6
Tobin's q	7	0.76	15.9	0.0	4	1.17	39.7	6.3
Time series	1	0.91	22.2	3.2	2	1.53	42.9	14.3
			Study (ii) 1954:1–1977:4					
Accelerator	6	0.82	19.8	2.1	6	1.22	42.7	10.4
Cash flow	10	0.74	15.6	1.0	9	1.58	51.0	15.6
Neoclassical	12	0.78	16.7	3.1	13	1.11	37.5	8.3
Tobin's q	8	0.78	16.7	1.0	8	1.58	42.7	17.7
Time series	3	0.89	28.1	2.1	2	1.73	41.7	18.8
			Study (iii) 1956:1–1979:4					
Accelerator	11	0.9	20	2	9	1.2	34	13
Cash flow	5	0.9	20	0	5	1.4	45	13
Neoclassical	11	0.9	25	4	12	1.2	42	9
Tobin's q	8	1.0	28	7	7	1.3	46	11
Time series	8	0.9	25	3	6	1.8	46	22

Notes: % |Errors| >$1B and >$2B refer to the percentage of absolute errors exceeding one and two billion dollars, respectively. Percent RMSE is the percent root mean squared error.

mation the goodness of fit benefits from such autocorrelation. As we shall soon see, however, this benefit for estimation does not always carry over to forecasting.

Third, goodness of fit can be measured by R^2 or by a number of other measures. In columns three and seven of Table 6.8, Kopcke reports the percent root mean squared error (RMSE) measure of fit, computed as

$$\%\,\text{RMSE} \equiv \sqrt{\sum_{t=1}^{T} e_t^2/T} \qquad e_t \equiv \frac{I_t - \hat{I}_t}{I_t} \tag{6.50}$$

where $\hat{I}_t$ is the fitted value of investment in time t based on the regression estimates and T is the sample size from estimation. Note that the residual here is in percent form. The RMSE for structures never exceeds 1.0%; and although that for equipment is typically larger, it is always 1.8% or less. Differences among the models are very small, indicating that all equations can be made to track the data equally well.

In the other four columns of Table 6.8, Kopcke reports a different measure of fit, namely, the percentage of the (absolute value of) residuals that exceed one and two billion (constant 1972) U.S. dollars. Since equipment investment is on average at least twice as large as that for plant (roughly, \$120 versus \$50 billion), we expect this percentage to be larger for equipment than for plant, and indeed this is the case. The neoclassical model displays the smallest percentage of very large (>\$2 billion) residuals for equipment, while Tobin's q generally performs very well for structures; in most cases, for both equipment and structures the time series model has the largest proportion of very large residuals. This relative ranking of the neoclassical and time series models in estimation is not unexpected, since the neoclassical model generally has the longest lag structure, while the time series specification has the shortest.

Kopcke assesses forecasting properties of the five investment models using ex post static and dynamic forecasts. In ex post static forecasts, each quarterly forecast of investment spending is computed with full knowledge of the previous quarter's actual investment outlays, the actual lagged capital stock, and the corresponding lagged forecast error.[56] While such a static forecast yields little understanding of how well a forecaster would actually have fared, the error performance of the static forecast can be compared to the model's error performance during estimation. If, for example, the model is unable to track the data as well during the forecast interval as it does during the estimation period, then there is evidence indicating instability of the model.

It is therefore instructive to compare % RMSE values from estimation in Table 6.8 to the static forecast % RMSE entries in Table 6.9. On average, for both equipment and structures the % RMSE measures from the static forecast are about twice as large as those from estimation, regardless of the model estimated. Further, in most cases the percentage of large (>\$1 billion) and

Table 6.9 Selected Statistics of Five Investment Models for Static Forecasts Reported by Kopcke in Three Quarterly U.S. Studies

	Nonresidential Structures				Producers' Durable Equipment			
Model	% Mean Error	Percent RMSE	% |Errors| >$1B	>$2B Length	% Mean Error	Percent RMSE	% |Errors| >$1B	>$2B
Study (i):	*Estimation 1958:1–1973:3*				*Forecasting 1973:4–1976:4*			
Accelerator	−0.9	1.4	61.5	7.7	−0.3	1.8	53.8	15.4
Cash flow	−1.3	1.6	61.5	38.5	−1.6	2.8	84.6	69.2
Neoclassical	−0.6	1.1	30.8	7.7	−1.6	2.9	92.3	61.5
Tobin's q	−1.3	1.5	61.5	23.1	−0.7	2.1	76.9	23.1
Time series	−0.9	1.5	38.5	15.4	−0.8	2.0	61.5	23.1
Study (ii):	*Estimation 1954:1–1977:4*				*Forecasting 1978:1–1981:4*			
Accelerator	0.57	1.1	50	0	0.97	2.3	81	44
Cash flow	0.78	1.5	50	19	0.33	2.9	81	56
Neoclassical	1.00	1.6	56	25	0.82	2.1	75	44
Tobin's q	0.53	1.5	44	19	−0.16	3.2	69	56
Time series	0.52	1.3	56	19	4.70	6.3	94	88
Study (iii):	*Estimation 1956:1–1979:4*				*Forecasting 1980:1–1984:4*			
Accelerator	0.4	1.3	60	10	1.3	2.8	75	55
Cash flow	0.5	1.6	55	20	0.2	2.3	95	35
Neoclassical	−0.4	1.7	60	20	1.3	2.5	75	45
Tobin's q	1.1	2.1	70	50	0.9	2.7	75	50
Time series	0.2	1.7	70	20	0.3	3.4	85	70

Notes: % |Errors| >$1B and >$2B refer to the percentage of absolute errors exceeding one and two billion dollars, respectively. Percent RMSE is the percent root mean squared error.

very large (>$2 billion) absolute residuals also increases severalfold in the static forecast. While some deterioration in precision might be expected, this universal, substantial increase in the dispersion of errors is disconcerting and suggests that none of the equations is stable.

Additional evidence on the forecasting properties of the various models can be obtained by examining signs of the residuals. In columns two and six of Table 6.9, mean values of the percent residuals in the static forecasts are presented. In the first study, for both equipment and structures the means are all negative, indicating that each of the models systematically overpredicted investment during the 1973:4–1976:4 interval. By contrast, in both the second and third studies the % mean error values are positive in nine of ten cases, implying that over the 1978:1–1981:4 and 1980:1–1984:4 forecast intervals, the models generally underpredicted investment. Together these results suggest that the explanation of investment spending is much more complicated than any of the five models suggests or that the link between investment spending and its determinants changed from the estimation to the forecast time period.

The final set of forecast performance measures reported by Kopcke is based on ex post dynamic forecasts. With dynamic forecasts, quarterly investment forecasts do not benefit by knowing investment in the previous time period or by knowing previous forecast errors. Unlike the static forecasts, the dynamic forecast is not prevented from straying far from the trend of actual investment, since in the dynamic forecast the previous error is not checked, nor are corrections for errors incorporated into subsequent forecasts. Possibilities for substantial forecast error are therefore more likely with dynamic forecasts.

In one important sense, however, these dynamic forecasts do not reflect the actual experiences of forecasters: Values of the exogenous variable (such as output, market value, user cost of capital, internal cash flow) are based on historical data, not on forecasted values. For this reason, such forecasts are called ex post dynamic forecasts; they differ from ex ante forecasts in which all exogenous variables must be forecasted.

To obtain best linear unbiased forecasts, Kopcke computes predicted values based on the estimated regression equation for each model, and then adds to this prediction the effect of the autocorrelated residual. With first-order autocorrelation the additive effect of the last residual from the estimation period, denoted e_T, on the forecast in period $T + \tau$ equals $\rho^{T+\tau} \cdot e_T$, where ρ is the estimated first-order autocorrelation coefficient.[57] Notice that since $0 < \rho < 1$, this effect declines as τ increases, that is, the effect of the residual on the forecast diminishes as the distance between the forecast period and the estimation period increases.

While differences among the models are small in the estimation and only modest in the static forecasts, with the dynamic forecasts, Kopcke finds substantial variations among models. This highlights the fact that models that work well in estimation do not necessarily perform similarly well in forecasting. Performance measures of the ex post dynamic forecasts employed by Kopcke include % mean error, % RMSE, and % absolute errors exceeding \$4 billion and \$8 billion; these are reproduced in Table 6.10.[58] The results are very revealing.

In the first study, in nine of ten cases the % mean errors are negative, implying that almost all models failed to gauge sufficiently the weakness of investment during the immediate post-OPEC oil embargo time period. While such errors are relatively small for the accelerator model, with the time series model, average errors are huge: -8.9% for structures and -11.9% for equipment. The accelerator and Tobin's q models do not have any very large errors (% absolute errors > \$8 billion), but for equipment investment this measure varies between 30% and 70% for the neoclassical, cash flow, and time series models. The % RMSE measures range from 5.2 (accelerator) to 9.7 (time series) for structures, but for equipment investment (which is more than twice

Table 6.10 Selected Statistics of Five Investment Models for Dynamic Forecasts Reported by Kopcke in Three Quarterly U.S. Studies

	Nonresidential Structures				Producers' Durable Equipment			
Model	% Mean Error	Percent RMSE	% |Errors| >$4B	% |Errors| >$8B	% Mean Error	Percent RMSE	% |Errors| >$4B	% |Errors| >$8B
Study (i):	*Estimation 1958:1–1973:3*				*Forecasting 1973:4–1976:4*			
Accelerator	−5.1	5.2	76.9	0.0	1.8	4.2	46.2	0.0
Cash flow	−4.7	6.2	69.2	23.1	−4.1	9.2	76.9	46.1
Neoclassical	−4.6	4.9	76.9	0.0	−3.3	6.2	46.2	30.8
Tobin's q	−5.5	5.9	76.9	0.0	−1.9	4.9	53.8	0.0
Time series	−8.9	9.7	76.9	61.5	−11.9	14.3	69.2	69.2
Study (ii):	*Estimation 1954:1–1977:4*				*Forecasting 1978:1–1981:4*			
Accelerator	3.7	4.4	44	6	6.8	8.3	63	31
Cash flow	7.6	8.2	88	50	8.4	9.0	94	50
Neoclassical	9.1	10.0	88	63	6.8	8.1	75	38
Tobin's q	7.0	7.4	88	38	7.8	9.6	63	50
Time series	2.0	2.7	19	0	2.5	4.4	38	13
Study (iii):	*Estimation 1956:1–1979:4*				*Forecasting 1980:1–1984:4*			
Accelerator	2.8	3.9	30	0	8.6	10.1	85	45
Cash flow	4.8	5.9	50	30	2.0	3.9	35	0
Neoclassical	−3.0	5.3	40	15	6.9	7.6	75	40
Tobin's q	6.6	7.5	70	35	3.5	5.7	35	15
Time series	0.5	3.6	30	0	−4.8	10.7	70	45

Notes: % |Errors| >$4B and >$8B refer to the percentage of absolute errors exceeding four and eight billion dollars, respectively. Percent RMSE is the percent root mean squared error.

as large as structures) the difference is even larger, from 4.2 (accelerator) to 14.3 (time series).

There are of course many ways in which one could rank these models. If one uses the % RMSE criterion and then weights the equipment % RMSE twice that for structures (reflecting relative investment magnitudes), then in the first study the ranking is as follows: accelerator, Tobin's q, neoclassical, cash flow, and time series.

The performance of the five models changes dramatically, however, in the second of Kopcke's studies. Here the % mean error values are all positive, indicating that actual investment 1978:1–1981:4 was larger than that predicted by all the models; the underprediction is particularly large for equipment. In a sharp reversal from Kopcke's first study, in which the time series model fared worst, in this study the time series model easily outperforms the other models in all criteria. The accelerator model still does well, but its performance is clearly inferior to that of the time series model. For all models except the accelerator, the percentage of very large errors in study (ii) is larger than those of study (i). If one uses the same % RMSE criteria as noted above, then in the second study the ranking is time series, accelerator, Tobin's q, neoclassical, and cash flow.

In the final of Kopcke's three studies, a sharp change again occurs in the performance of the five models. While % mean error values are again primarily positive, indicating systematic underprediction of investment over the 1980:I–1984:IV time span, the cash flow model, which ranked last in study (ii), now dominates, particularly for equipment investment. Further, the time series model, which dominated all models for both equipment and structures in study (ii), is still dominant in study (iii) in terms of the % RMSE criterion for structures, but for equipment its % RMSE is a substantial 10.7%, and it comes in last, faring only marginally worse than the accelerator model (10.1%). If one uses the same % RMSE criteria as noted above, then in this final study the ranking is cash flow, Tobin's q, neoclassical, accelerator, and time series.

Kopcke's tables merit close attention. But one point is already abundantly clear: No investment model consistently dominates its competitors. In particular, the relative rankings of the models, summarized in Table 6.11, change dramatically from one study to the next. Further, although the accelerator, cash flow, and time series model each put in one very good effort, they also each put forth at least one very bad performance. It is also the case that Tobin's q model is reasonably consistent at second or third place and outperforms the neoclassical model in each of these ex post forecasts. However, the usefulness of Tobin's q for ex ante forecasting is rather limited, since the ability to forecast overall stock market movements accurately is still severely circumscribed.

In summary, this series of empirical comparisons of five investment models indicates that econometricians are still a long way from reaching a

Table 6.11 Rankings of Five Investment Models Based on the % RMSE Dynamic Forecast Criterion* for Kopcke's Series of Three Quarterly U.S. Investment Studies

Study Forecast Period	Model				
	Accelerator	Cash Flow	Neoclassical	Tobin's q	Time Series
Study (i) 1973:4–1976	1	4	3	2	5
Study (ii) 1978:1–1981:4	2	5	4	3	1
Study (iii) 1980:1–1984:4	4	1	3	2	5

*% RMSE for equipment is weighted twice that for structures.

consensus on what form of investment equation is most preferable. There is much—very much—that still needs to be learned.

6.9 CONCLUDING REMARKS

There is an old saying, "We are so close, and yet so far." As we have seen in this chapter, theoretical developments in dynamic demand analysis over the last 50 years have been substantial, a large amount of data has been collected, and sophisticated computational techniques are now widely available for use on increasingly inexpensive computers. But what have we learned? While theoretical and computational developments in expectations formation and in the understanding of gestation lags (adjustment costs) have provided us with necessary tools, successful measurement and forecasting still elude us. As we observed in the previous section, we are still not able to predict investment to a reasonably precise level, nor can we even conclude on the basis of empirical performance what form of the investment equation is preferable and stable. Particularly frustrating is the fact that it has proved to be very difficult to separately identify and estimate reliably the expectational, order, delivery, and gestation lags. Progress has occurred, but it has been slow. It is difficult to reduce with success the very complex investment process to a limited number of variables and parameters.

Fortunately, the analysis of investment continues to attract great attention. Researchers today are attacking investment issues from a variety of vantage points, as has been indicated in the numerous references in this chapter. Some researchers believe that we may be asking too much from our data, whose construction deserves considerably more attention, particularly with respect to timing issues (expenditures versus installation), quality adjustment, tax impacts (marginal and average, effective and statutory), and the measurement of other components of the cost of capital. Others are working on the theory of risk and focusing more attention on the stochastic nature of the investment process. Still others are investigating aggregation problems over firms, the homogeneity of capital, the implications of irreversible investments, and the effects of imperfect competition.

There is still very much to be learned. The challenge is great, the opportunities are enormous, and the pace of ongoing research is furious. For young econometricians it is the greatest of times. So let us now get on with the hands-on exercises.

6.10 HANDS-ON ESTIMATION AND FORECASTING OF INVESTMENT EQUATIONS

The purpose of these exercises is to gain experience in actual estimation of distributed lag models with autoregressive disturbances, and to better understand how one chooses among alternative investment model specifications in estimation. A particularly enlightening experience afforded by these exercises is the encounter with models that perform admirably in estimation, yet achieve only modest success in dynamic forecasting.

Exercise 1 is especially important and useful; it acquaints you with the underlying data and encourages you to think through possible hypotheses to help explain broad trends in the historical investment data. Exercises 2 through 6 then deal successively with the accelerator, time series, cash flow, neoclassical, and Tobin's q investment models. Econometric techniques that are considered in these exercises include testing for autocorrelation with lagged dependent variables, traditional regression analysis of models with autoregressive errors, and estimation of Almon polynomial distributed lags as special cases of more general specifications, with and without endpoint restrictions.

Then in the more difficult set of Exercises 7 through 9, Box-Jenkins ARIMA identification, estimation, and forecasting techniques are implemented empirically, the estimation of simultaneous equations models with autoregressive errors is introduced, and the issue of just how price-responsive is the demand for capital is addressed in the context of a putty-clay neoclassical model of investment based on the CES production function. Finally, although it is not necessarily the most difficult exercise, in Exercise 10 a "horse race" project is outlined, similar to those conducted by Kopcke. This horse race project involves considerable effort and therefore might be most suitable for a team project or term paper, rather than just a normal exercise.[59]

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In the directory of the data diskette provided, you will find a subdirectory called CHAP6.DAT with a data file called KOPCKE. This data file contains quarterly U.S. data, 1952:1–1986:4 on 14 variables, each having 140 observations. There are two panels of data in KOPCKE. In the first panel there are nine variables: a date variable (DATE) indicating the year and quarter (in the form YYYYQ, where YYYY is year and Q is the quarter); implicit price deflators for structures (JS) and equipment (JE), indexed to 1.000 in 1982; cash flow of nonfinancial corporate business (F) in millions of current dollars; gross private domestic investment in nonresidential structures (IS) and producers' durable equipment (IE) in millions of 1982 dollars; once-lagged capital stocks of equipment (KELAG) and structures (KSLAG); and gross domestic business product (Y), all in millions of 1982 dollars.

The five variables in the second panel of data are another date variable (TIME), indicating the year and quarter (in the YYYYQ form); the Federal Reserve Board capacity utilization rate for manufacturing (U); Tobin's q, the ratio of the market value of nonfinancial corporations to the replacement value of their net assets (Q); and the real user cost of capital for nonresidential structures (CS) and producers' durable equipment (CE). Note that the last two variables are in real, not nominal terms, that is, $C \equiv c/P$, where c is defined as in Eq. (6.24). The annual rates of depreciation employed by Kopcke in the user cost formulae (6.24) are 0.15 for equipment and .05 for structures.

Comment: These data have been provided by Richard Kopcke, with assistance from George Houlihan. There are several differences in this data set from those used in the earlier Kopcke studies, and so you will be unable to replicate exactly Kopcke's previously reported results. In particular, the current data set incorporates data revisions that were published after Kopcke's 1977, 1982, and 1985 studies. Further, units of measurement differ, since in the three Kopcke studies, investment was in millions of 1972 dollars, while in the current data set, investment is in millions of 1982 dollars. Additional details concerning these data are found in the appendices of Kopcke [1977, 1982, 1985].

Important note: Although there are 140 observations in the data set, some of these observations will essentially be lost in estimation owing to the presence of distributed lags and autoregressive or moving average error structures. Following Kopcke's last [1985] study, it is recommended that direct estimation begin with the 1956:1 observation and that the previous 16 observations be reserved for use as appropriately lagged variables.

EXERCISES

EXERCISE 1: Examining the Data

The purpose of this exercise is to help you become familiar with important features of the investment data. You examine levels and relative growth rates of investment in equipment and plant, as well as capital/output ratios, and growth rates of price deflators and user costs. You interpret high and low values of Tobin's q and capacity utilization, and you assess the internal consistency of the net capital stock and investment data with the assumed rates of depreciation.

In the data diskette directory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real investment in equipment and structures (IE and IS), real output (Y), once-lagged capital stocks of equipment (KELAG) and structures (KSLAG), implicit price deflators for equipment and structures (JE and JS), cash flow (F), Tobin's q (Q), the rate of capacity utilization (U), and real user costs of capital for equipment and structures (CE and CS).

(a) Print out the entire data series for IE, IS, KELAG, KSLAG, JE, JS, F, Q, U, CE, and CS. Make sure that you have 140 observations on each of these variables and that printed values for the first and last observations are the same as those in the KOPCKE data file when file contents are viewed on a monitor screen. Why is this a useful procedure to follow before embarking on further data analysis?

(b) The quarterly investment data in producers' durable equipment and nonresidential structures are in millions of 1982 dollars, seasonally adjusted and at annual rates. Compare the ratio of investment in equipment to investment in structures at the beginning of the sample and at the end of the sample. Which type of investment has been growing more rapidly, equipment or structures? Do these relative growth rates also hold for capital stocks of equipment and structures? Compare relative growth rates of the asset price deflators for new equipment (JE) and structures (JS) over the entire 1952:1–1986:4 sample and then for the real user costs of capital CE and CS. What hypotheses could help explain why IE grew more rapidly than IS over this sample time period?

(c) Many students of economic development believe that countries with high investment rates and capital stocks relative to output eventually experience greater growth in output and in labor productivity. Compute capital/output ratios separately for equipment and structures in 1952:1 and 1986:4, and then construct an aggregate capital/output ratio as the sum of capital stocks of equipment and structures, divided by output. Is production in the United States becoming more capital-intensive? Comment and interpret the major trends in these capital/output data series.

(d) Tobin's q, the ratio of the market value of nonfinancial corporations to the replacement value of their net assets, has a very interesting time series. What is the lowest sample value of q, and what is the highest? What historical events might help explain these dramatically different levels of q? Do they appear to be related to relative levels of capacity utilization? When do the peaks and troughs occur for capacity utilization? For user costs of capital CE and CS? Notice that average q is less than unity for all but four observations, yet much net investment has occurred. Is it more reasonable to state that net investment is an increasing function of q or that q must be greater than 1 in order for net investment to occur? Why?

(e) The once-lagged capital stock data KELAG and KSLAG are fourth-quarter to fourth-quarter linear interpolations of annual net capital stock data published by the U.S. Department of Commerce, Bureau of Economic Analysis. Earlier, it was noted that the quarterly investment data IE and IS are seasonally adjusted and at annual rates and that in the user cost formula, Kopcke assumed rates of depreciation for equipment and structures to be $\delta_E = 0.15$ and $\delta_S = 0.05$. It is useful to compare rates of depreciation assumed by Kopcke with those embodied in the U.S. Department of Commerce net capital stock calculations. To do this, solve Eq. (6.5) for δ_t; this yields

$$\delta_t = 1 - ((K_t - I_t)/K_{t-1}) \qquad \textbf{(6.51)}$$

For equipment, sum the four quarters of investment in 1953 and in 1986, and then divide by 4 (since the quarterly data are at annual rates); this yields annual investment for 1953 and for 1986. Then substitute this 1953 value of I_t into Eq. (6.51), let K_t, K_{t-1} be the 1953:4 and 1952:4 values, respectively, and compute δ_{1953}. Similarly, substitute the 1986 value of I_t into Eq. (6.51), let K_t, K_{t-1} be the 1986:4 and 1985:4 values, respectively, and compute δ_{1986}. How well do these values of δ_t compare with the $\delta_E = 0.15$ assumed by Kopcke? (*Note:* The Department of Commerce capital stock calculations represent sums of capital stocks for various types of equipment. While each type of equipment could have a constant exponential decay rate, the aggregate capital stock could have a time-varying decay rate, owing to the changing composition within the equipment capital stock.) Perform the same calculations for structures, and compare with Kopcke's assumed $\delta_S = 0.05$. Comment on the internal consistency of the capital stock, investment, and user cost data. Finally, for the entire 1952:1–1986:4 data sample, compute replacement investment as a proportion of total investment for equipment and for structures, using Kopcke's values for δ_E and δ_S. How important is replacement investment? Is net investment ever negative? Interpret these results.

(f) With time series data, severe multicollinearity can often occur; this can result in imprecise parameter estimates and large standard errors. Using data from 1954:1 to 1986:4, construct a simple correlation matrix for IE_t, IS_t, Y_t, Y_{t-1}, Y_{t-2}, . . . , Y_{t-8}. Construct another correlation matrix involving investment and current and lagged values for some other variable that might be used as a regressor. Are the explanatory variables highly correlated with each other? Do you think this might cause problems in estimation? How might the adverse effects of this collinearity be mitigated?

EXERCISE 2: Testing for Autocorrelation with Lagged Dependent Variables

The purpose of this exercise is to enable you to gain experience with procedures designed to test for the presence of first-order autocorrelation in models with lagged dependent variables. You will use Durbin's m and h test statistic procedures in the context of the accelerator model and will undertake estimation using the OLS, Hildreth-Lu, and iterative Cochrane-Orcutt procedures.

In the data diskette directory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real investment in equipment and structures (IE and IS) and real output (Y).

(a) Using OLS and data corresponding to 1956:1–1986:4, estimate parameters of Eq. (6.14) for either equipment or structures investment, with an intercept term added. On the basis of these parameter estimates, construct the implicit estimates of the capital/output coefficient μ, the partial adjustment coefficient λ, and the rate of depreciation δ, as discussed in the text under Eq. (6.14). Does the implied estimate of δ correspond well with the $\delta_E = 0.15$ or $\delta_S = 0.05$ rate assumed by Kopcke? How do you interpret this? Using the reported Durbin-Watson (DW) test statistic and a 0.05 level of significance, test the null hypothesis of no autocorrelation. Since DW is approximately equal to $2(1 - \rho)$, calculate the implied estimate of ρ, the first-order autocorrelation coefficient. Why might this estimate of ρ be biased toward zero?

(b) James Durbin [1970] has developed two test statistics that are strictly valid in large samples of data but that are often employed in small samples as well. In our context, Durbin's h-statistic is calculated as

$$\text{Durbin's } h \equiv \hat{\rho} \sqrt{\frac{T}{1 - T(\text{Var } b)}} \tag{6.52}$$

where $\hat{\rho}$ is the estimated first-order autocorrelation coefficient from part (a), T is the sample size (here, $T = 124$), and Var b is estimated as the square of the standard error of the coefficient of the lagged dependent variable from part (a). In large samples, h is approximately normally distributed with unit variance. Notice that, provided $T(\text{Var } b) < 1$, the ratio in the square root term will be greater than 1, and so $h > \rho$. Using results from part (a), if possible, compute h, and then using the standard normal distribution table and a 0.05 level of significance, test the null hypothesis that $h = 0$.

In some cases, however, $T(\text{Var } b) > 1$, and in such situations one obviously cannot use Durbin's h-statistic, since square roots of negative numbers cause well-known problems! Instead, one can employ Durbin's m-test procedure. Specifically, from part (a), retrieve the 124 residuals, and denote these as e_t. Then, with the first observation now deleted, estimate by OLS an equation with e_t as the dependent variable and with regressors the same variables as in the original equation estimated in part (a) plus the lagged residual e_{t-1}; that is, estimate the equation

$$e_t = a + b_1 Y_t + b_2 Y_{t-1} + b_3 I_{t-1} + \rho^* e_{t-1}$$

over observations 1956:2–1986:4. Compare the magnitude of this estimate of ρ^* with that implied in the OLS equation estimated in part (a) and with Durbin's h-statistic (if you were able to compute it). Now test whether ρ^* here is significantly different from zero by comparing the calculated t-statistic on ρ^* with the 0.05 critical value from the normal distribution table. Does the statistical inference on whether first-order autocorrelation is present vary depending on the procedure employed? Which procedure is preferable, and why?[60]

(c) Now estimate Eq. (6.14), allowing for first-order autocorrelation, using data over the 1956:1–1986:4 time period and the Hildreth-Lu estimation technique, with the grid of values for ρ originally ranging from $\rho = 0.00$ to $\rho = 1.00$ in steps of 0.05. Does any conflict appear between local and global maxima? On the basis of the initial results, refine the grid search to steps of 0.01. Test the null hypothesis that $\rho = 0$. Compare the implicit estimates of μ, λ, and δ to those obtained in part (a). Which estimates are preferable, and why?

(d) With lagged dependent variables, one must be particularly cautious in using the iterative Cochrane-Orcutt estimator, since the first-round estimate of ρ is biased.[61] However, for numerical reasons this estimator often (but not always) coincides with the Hildreth-Lu estimator. Estimate Eq. (6.14), allowing for first-order autocorrelation using the iterative Cochrane-Orcutt estimator, and then compare parameter estimates and statistical inference with those from part (c). In this particular case, do estimates of ρ and the parameters coincide? Interpret your results.

EXERCISE 3: Regression Estimation of the Time Series/Autoregressive Model

The purpose of this exercise is to have you estimate parameters of a time series/autoregressive model and choose a preferred specification based on hypothesis testing and properties of the estimated parameters. Procedures used here involve the classical hypothesis-testing approaches of traditional regression analysis. An alternative time series approach, based on Box-Jenkins procedures, is employed in Exercise 7 of this chapter.

In the data diskette directory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real investment in equipment and structures (IE and IS).

(a) In the three time series/autoregressive studies of Kopcke discussed in Section 6.6, the number of lagged dependent variables is one, three, and eight for structures in studies (i), (ii), and (iii), respectively, and one, two, and six for equipment. Using the quarterly U.S. investment data 1956:1–1979:4, estimate Eq. (6.41) for equipment investment by OLS, where m, the number of lagged terms, equals ~~8~~. Compare these estimates to those reported by Kopcke in his study (iii), reproduced in Table 6.7. How well are you able to replicate Kopcke's results? (*Note:* See the comment just preceding Exercise 1.) Then estimate the same equation using more recent data, that is, estimate Eq. (6.41) with $m =$ ~~8~~ over the 1956:1–1986:4 time period. Do estimates of the parameters change substantially? What can you infer concerning parameter stability? 6 6

(b) Using the estimated residuals from your 1956:1–1979:4 regression equation of part (a), convert these residuals into percent form, and then compute the % root mean squared error, as in Eq. (6.50), with T now equal to 96. Compare your % RMSE to that reported by Kopcke (see Table 6.~~7~~). Then compute % RMSE for the estimated residuals from your 1956:1–1986:4 regression equation of part (a). Does the size of the % RMSE change substantially when the more recent investment data are included? Interpret these results. 8

(c) Repeat parts (a) and (b) but for investment in structures rather than in equipment, with $m = 6$, as in Kopcke [1985], reproduced in Table 6.7. Does the % RMSE for equipment continue to exceed that for structures, as Kopcke reported?

(d) Kopcke states that his choice of preferred specification is based on a combination of statistical significance, the sensibility of the parameter estimates, and judgment. Begin with a rather general specification, say, $m = 12$, use the full sample 1956:1–1986:4, and then conduct a series of hypothesis tests that leads you to settle on a preferred specification, both for structures and for equipment investment. Comment on the role and interpretation of the Durbin-Watson test statistic in this specification choice process. Note also that if the b_j parameters on the lagged dependent variables sum to a value greater than $+1$, the estimated model is nonstationary (predicted investment will increase indefinitely with time). Does this occur with any of your estimated models? Is it reasonable? Finally, for both equipment and structures, compare your preferred specifications to those reported by Kopcke in his studies (i) and (iii), reproduced in Table 6.7. Does the number of lagged variables in your preferred specification differ from those reported by Kopcke? If possible, test the number of lags reported by Kopcke as special cases of your preferred specification or vice versa. What do you conclude on the basis of these hypothesis tests?

EXERCISE 4: Almon Lag Estimation of the Cash Flow Model

The purpose of this exercise is to give you experience in using the Almon polynomial distributed lag (PDL) procedure in the estimation of a cash flow model of investment spending. You will test coefficient restrictions that are implicit in the PDL procedure and allow for first-order autocorrelation of residuals.

In the data diskette directory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real investment in equipment and structures (IE and IS), cash flow in millions of current dollars (F), price deflators for new equipment (JE) and new structures (JS) capital goods, and the once-lagged capital stocks for equipment and structures (KELAG and KSLAG) in millions of 1982 dollars.

Note: For all parts of this exercise, choose *either* equipment investment *or* structures investment.

(a) Using the same time period (1956:1–1979:4) and specification as in Kopcke's last study (see Table 6.3), construct the real cash flow variable as the ratio of nominal cash flow (F) to the price deflator for new structures (JS) or new equipment (JE). Estimate the cash flow investment equation (6.18) using the Almon PDL procedure: As in Kopcke [1985], for both the equipment and structures equation, set m, the number of distributed lags of cash flow, equal to 6 (including the current time period), and constrain the parameters to lie along a third-degree polynomial, but impose no other constraints on the near or far PDL coefficients. Note that in Kopcke's estimated equations the lagged capital stock is omitted as a regressor, that is, the parameter c in Eq. (6.18) is constrained to zero. Test for first-order autocorrelation using the Durbin-Watson test statistic, and then estimate this PDL model allowing for first-order autocorrelation. How well can you replicate Kopcke's results, reproduced in Table 6.3? Comment on the pattern of the estimated PDL coefficients. How do you interpret the sum of the estimated PDL coefficients?

(b) Now test for restrictions implied by the above PDL estimation. Using the same time period as in part (a), that is, 1956:1–1979:4, estimate parameters of a model in which the PDL constraints are not imposed on the parameters. Specifically, estimate Eq. (6.18) with $m = 6$ and with $c = 0$, first by OLS and then by any of the procedures that allow for first-order autocorrelation (i.e., Hildreth-Lu or iterative Cochrane-Orcutt). Are any adverse effects of multicollinearity now apparent? Does the Almon PDL procedure of part (a) reduce these adverse effects? What is the difference in the number of "free" or independent parameters estimated here, relative to those estimated in the third-degree PDL of part (a)? On the basis of the OLS results here and in part (a), test for the validity of the constraints implied by the third-degree PDL using a reasonable level of significance and an F-test. Then, on the basis of the generalized least squares results (allowing for first-order autocorrelation) here and in part (a), again test for the validity of these third-degree PDL restrictions using a reasonable level of significance and the likelihood ratio test procedure. Comment on the appropriateness of your estimated PDL model.

(c) Next, check on other empirical support underlying the third-degree PDL specification. Specifically, using the PDL procedure and allowing for first-order autocorrelation over the 1956:1–1979:4 time period, estimate a model in which the number of distributed lag terms m is six as in part (a) but the degree of the PDL is 4 rather than 3. Compare results with those obtained in part (a). Are there any major changes in the parameter estimates or standard errors? Then, using a large sample test and a reasonable level of significance, test for a PDL of degree 3 (estimated in part (a)) as a special case of the PDL of degree 4 estimated here.

(d) Finally, choose a preferred cash flow investment specification for a data set including more recent data. Specifically, using the 1956:1–1986:4 data (you will need to construct the real cash flow variable as in part (a)), experiment with alternative degrees of the PDL procedure, allow for differing numbers of distributed lag terms, permit the coefficient on the lagged capital stock to be nonzero, and then estimate a number of alternative PDL equations that allow for first-order autocorrelation. From these various estimated equations, choose a preferred specification. Comment on and defend your choice.

EXERCISE 5: Putty-Clay in the Neoclassical Model of Investment

The purpose of this exercise is to engage you in estimating and interpreting parameters of a neoclassical investment equation, based on the Cobb-Douglas production function, incorporating putty-clay effects by allowing for differential impacts on investment of price and output changes. This specification was originally employed by Bischoff [1971a] and was discussed in Section 6.4.C.

In the data diskette subdirectory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real investment in equipment (IE), the real user cost of capital for equipment (CE), a price deflator for new equipment (JE) capital goods, and the once-lagged equipment capital stock (KELAG).

(a) First you will need to construct the price-output variables that appear in the basic investment equation (6.33). Specifically, since CE is the real user cost of capital equipment defined as $CE \equiv cE/P$, where the lowercase cE is in nominal terms, on the basis of Eq. (6.33) construct new variables X_{1t} and X_{2t} defined as

$$X_{1t} \equiv (P/cE)_t Y_t = Y_t/CE_t \qquad X_{2t} \equiv (P/cE)_t Y_{t-1} = Y_{t-1}/CE_t$$

Note: Make sure that X_{1t} and X_{2t} are constructed for the data period 1952:2 to 1986:4.

(b) Having constructed the price/output variables X_{1t} and X_{2t}, you are now ready for estimation of Bischoff's putty-clay specification. As in Kopcke [1985], on the basis of the 1956:1–1979:4 time period, employ the Almon PDL procedure allowing for first-order autocorrelation, and estimate parameters of the now modified equipment investment equation (6.33):

$$IE_t = a_0 + \sum_{j=0}^{m-1} b_j X_{1,t-j} - \sum_{j=0}^{m-1} c_j X_{2,t-j} + d_K K_{t-1} + u_t \quad \textbf{(6.33')}$$

with $m = 13$ and with the PDL coefficients constrained to lie on a third-degree polynomial. Impose no further restrictions on the PDL parameters. Compare your results to those of Kopcke, reproduced in Table 6.4, noting the negative sign in front of the c_j in (6.33'). How well are you able to replicate Kopcke's results? For small j, are your estimated c_j positive and significantly different from zero? How do you interpret these c_j?

(c) Next test whether the output responses differ from the price responses, that is, test whether Bischoff's putty-clay insight significantly improves the fit of the model. Recall from the discussion following Eq. (6.33) that for any given time displacement j, the effect on current investment I_t will be larger for an increase in Y_{t-j} than for an equal decrease in the user cost of capital $(c/P)_{t-j}$, provided that $c_j > 0$ (again note the negative sign in front of Eq. (6.33′)). This implies that the equal lag response hypothesis for prices and output corresponds to a special case of Bischoff's modified equation (6.33′) in which the c_j are simultaneously constrained to equal zero, $j = 0, \ldots, m - 1$. Using the same data sample as in part (a), $m = 13$, and a third-degree PDL specification allowing for first-order autocorrelation, estimate parameters of Eq. (6.33′) with the $X_{2,t-j}$ variables excluded from the regression equation. Impose no further restrictions on the PDL parameters. What is the number of constraints imposed in this restricted model? Using a reasonable level of significance and the likelihood ratio test procedure, test the joint null hypothesis that $c_j = 0, j = 0, \ldots, 12$, against the alternative hypothesis that these $c_j \neq 0$. Is the putty-clay hypothesis significant empirically?

(d) Finally, generate a preferred neoclassical equipment investment specification for a data set that includes more recent data. Specifically, using the 1956:1–1986:4 data (with the X_{1t} and X_{2t} variables constructed as in part (a)), experiment with alternative degrees of the PDL procedure, allow for Bischoff's putty-clay specification, and then estimate a number of alternative PDL equations that allow for first-order autocorrelation. To keep the number of regressions manageable, limit the number of distributed lag terms to $m = 13$ (including the current time period), as in Kopcke [1985]. From these various estimated equations, choose a preferred specification. Comment on and defend your choice, also comparing your results with those previously reported by Kopcke, reproduced in Table 6.4.

EXERCISE 6: Almon PDL Endpoint Restrictions in Tobin's *q* Model

The purpose of this exercise is to involve you in estimating a Tobin's q investment equation using Almon's polynomial distributed lag (PDL) procedure with endpoint restrictions imposed, as was done by Kopcke [1982, 1985]. Recall that PDL is a method for including a large number of lagged variables in a model but reducing the number of free coefficients to be estimated by requiring the coefficients to lie on a smooth polynomial in the lag. The purpose of endpoint constraints is to force the lag coefficients at either end of the lags over which one is estimating to go to zero. The LEADING constraint forces the coefficient of the hypothetical first lead to zero, while the TRAILING constraint forces the coefficient of the lag that is one past the last included lag to equal zero. See your econometrics theory textbook or the classic Almon [1965, 1968] references for further details.

In the data diskette directory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real investment in equipment and structures (IE and IS), values of the Tobin's q ratio (q), and the once-lagged capital stocks for equipment and structures (KELAG and KSLAG).

Notes: For all parts of this exercise, choose *either* equipment investment *or* structures investment. The same Tobin's q variable is used in the equipment and structures investment equation, since separate measures are not available.

(a) To attempt replication of Kopcke's estimated equation (6.39), in which $(q - 1)_{t-j} \cdot K_{t-j-1}$ appear as regressors, construct a new variable called X_t and defined as $X_t \equiv (q - 1)_t \cdot K_{t-1}$. *Note:* Generate this variable for the entire 1952:1–1986:4 time period.

(b) Having constructed the X_t variable, you are now ready to proceed with estimation. Using the PDL estimation procedure and allowing for first-order autocorrelation, estimate parameters of the equation

$$I_t = a + \sum_{j=0}^{m-1} b_j \cdot X_{t-j} + b_K K_{t-1} + u_t \qquad \textbf{(6.53)}$$

over the 1956:1–1979:4 time period. Following Kopcke [1985], set $m = 9$ for the structures equation or $m = 8$ for the equipment equation, and constrain the PDL coefficients to lie on a third-degree polynomial. Impose no further restrictions on the PDL coefficients. Compare your results with those of Kopcke [1985], reproduced in Table 6.6. *Note:* You will not be able to replicate Kopcke's results, since his left-hand variable is I_t/U_t (where U is the rate of capacity utilization) rather than I_t as in Eq. (6.53). Further, he imposes the restriction that the TRAILING lag term be zero.

(c) To impose and test the restriction that the TRAILING lag term is zero, estimate Eq. (6.53) just as in part (b), but now impose the restriction that the implied coefficient on the b_m lagged variable is zero. (*Note:* Following J. Phillip Cooper [1972], most computer programs impose this endpoint restriction by subtracting the last scrambled variable from each of the other scrambled variables and then dropping the last scrambled variable from the regression.) Test for the empirical validity of this single TRAILING lag restriction by using either a t-test, F-test, or likelihood ratio test statistic and a reasonable significance level.

(d) Kopcke apparently experienced some difficulties in obtaining a desirable q investment equation in his third study, since his preferred specification involved introducing a new capacity utilization variable that did not appear in his previous studies (i) and (ii). An alternative direction on which Kopcke might have focused attention is his regressor X_t, defined in part (a). Recall that for a number of reasons discussed in Section 6.5, researchers currently tend to argue only that net investment is an increasing function of q, rather than stating the stronger hypothesis that if $q > 1$, then net investment will be positive. This suggests that one might want to define a new variable $X_t' \equiv q_t \cdot K_{t-1}$ and then substitute this into Eq. (6.53) in place of X_t. Notice that X_t' is the market value of the firm and that with this specification we are very close to the cash flow–market value specification (6.17) originally used by Grunfeld. Why will parameter estimates change when X_t' is used instead of X_t? Construct this new variable X_t' for the 1952:1–1986:4 time period. Using the same data, PDL degree, and value of m as in part (b), estimate Eq. (6.53) with the Almon PDL estimator, allowing for first-order autocorrelation. Which specification do you prefer, the one here or that from (b)? Why?

(e) Finally, generate a preferred Tobin's q investment specification for a data set that includes more recent data. Specifically, using the 1956:1–1986:4 data and staying with the third-degree PDL, experiment with the X_t or X_t' variables constructed in (a) and (d), allow for alternative TRAILING endpoint restrictions, and estimate a number of different PDL equations with first-order autocorrelation. To keep the number of regressions manageable, limit the number of distributed lag terms to at most 9, as in Kopcke [1985]. From these various estimated equations, choose a preferred specification. Comment on and defend your choice, also comparing results with those previously reported by Kopcke, reproduced in Table 6.6.

EXERCISE 7: Box-Jenkins Identification, Estimation, and Forecasting of an Investment Equation

In Exercise 3 a time series/autoregressive model was estimated using traditional regression techniques. The purpose of this exercise is to give you hands-on experience in modeling the investment time series using an alternative procedure called the Box-Jenkins time series approach. Many econometric theory textbooks now devote discussion to time series analysis. If your econometric theory text does not, see the classic reference, a book by George P. Box and Gwilym M. Jenkins [1976], or the very readable texts by Charles R. Nelson [1973], Andrew C. Harvey [1981, 1990], and Clive W. J. Granger [1989].

In the data diskette directory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real investment in equipment and structures (IE and IS). *Note:* Since these investment series are seasonally adjusted, the series should not contain a seasonal component.

(a) The first task is to generate a transformation of the investment time series that is stationary. Using 1956:1–1979:4 data, plot the raw data series for IE and IS. Do the data appear to follow a trend? What does this imply concerning stationarity? Next construct the sample autocorrelations (and their standard errors) for IE and IS where the maximal lag m is 16. Determine whether each series is stationary. If a series is nonstationary, then difference the data one, two, or more times until the sample autocorrelation function indicates that the differenced data are stationary, that is, until the sample autocorrelations go to zero as the length of the lag increases. Denote this degree of differencing by d. In choosing d, make use of the Box-Pierce portmanteau test, typically called the Box-Pierce Q-statistic:

$$\text{Box-Pierce } Q \equiv T \sum_{k=j+1}^{m} \hat{\rho}_k^2 \tag{6.54}$$

which tests the null hypothesis that all further (longer lagged) autocorrelations are simultaneously equal to zero. At the jth lag the Box-Pierce Q-statistic is distributed independently as a chi-square random variable with degrees of freedom equal to $m - j - 1$.

(b) Having generated a stationary time series for IE and IS, now identify the order of the moving average (MA) process, denoted q, and the order of the autoregressive (AR) process, denoted p. Recall that spikes in the sample autocorrelation function often indicate MA components, while observations that are highly correlated with surrounding ones, resulting in discernible up-and-down patterns, suggest that an AR process may be generating the data. Plot the sample autocorrelation function from your stationary series, and examine its properties. Is there any evidence of an MA component? An AR component? Why or why not? The partial autocorrelation function can be also used for guidance in determining the order of the AR portion of the ARIMA process. Using the stationary data series on IE and IS, calculate partial autocorrelation functions for time displacements up to 12 quarters, and on the basis of these autocorrelation functions and the Box-Pierce Q-statistic, choose reasonable values for p and q; in particular, choose two sets of p, q for IE and two for IS. Defend your choices.

(c) Having identified several possible ARIMA(p, d, q) processes underlying the IE and IS data series, now estimate their parameters. Specifically, for each of your ARIMA(p, d, q) specifications chosen in part (b) for IE and IS, use the Box-Jenkins estimation technique and estimate the ARIMA parameters employing quarterly investment data, 1956:1–1979:4. From your four alternative ARIMA(p, d, q) specifications from part (b), choose one preferred model among these for IE and one for IS. Base your choice on the statistical significance of the estimated parameters, the sample autocorrelations, partial autocorrelations, and the Box-Pierce Q-statistics. Compare your final choices to the ARIMA specifications of Kopcke [1985], whose final (p, d, q) models, reproduced in Table 6.7, are (8, 0, 0) for IS and (6, 0, 0) for IE.

(d) Now employ eight more recent observations, redo the identification and estimation of ARIMA models, and then perform an ex post dynamic forecast. First, using the quarterly IE and IS data, 1956:1–1981:4, redo part (a) and generate a stationary data series. Next, as in part (b), identify several plausible ARIMA(p, d, q) specifications. Estimate parameters of these specifications using Box-Jenkins procedures, and choose a preferred specification for IE and IS. Compare these final specifications to those of part (c). Are the specifications robust? How do you interpret these results? Finally, on the basis of your preferred specifications for IE and IS, use the Box-Jenkins forecasting procedure and forecast for the 20 quarters following the estimation period, that is, construct a dynamic forecast over 1982:1–1986:4, using forecasted rather than actual lagged values of investment as appropriate. Compare the forecasted with the actual investment data series over this time period, compute % RMSE using Eq. (6.50), examine whether any residuals are particularly large, and comment on properties of the forecasts. How well do your ARIMA models perform?

EXERCISE 8: Estimation with Simultaneity and Autocorrelation

One important specification issue in the investment literature concerns whether certain right-hand variables are exogenous or are instead endogenous. Although a variety of variables could be viewed as being jointly determined with investment, as was noted in Section 6.7.A, most attention has been focused on the output variable. The purpose of this exercise is to help you gain firsthand experience with instrumental variable estimation of several models in the presence of simultaneity and first-order autocorrelation.

In the data diskette directory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real investment in equipment and structures (IE and IS) and real output (Y).

Notes: (1) For all parts of this exercise, choose *either* equipment investment *or* structures investment. (2) Although this exercise suggests using the OLS estimation method, following Kopcke [1985], you might instead wish to use the Almon polynomial distributed lag (PDL) estimator. If you employ PDL, set $m = 12$ (structures) or $m = 10$ (equipment), and constrain the PDL parameters to lie on a third-degree polynomial, as in Kopcke.

(a) Using OLS and data covering the 1956:1–1986:4 time span, estimate parameters of the accelerator equation (6.15), where $m = 7$. Then estimate this same equation allowing for first-order autocorrelation. Do the AR(1) estimates differ substantially from OLS estimates? If first-order autocorrelation is present, will the OLS parameter estimates be consistent? Why or why not? (*Hint:* Note that K_{t-1} appears as a regressor and that with the perpetual inventory procedure of Eq. (6.4) the capital stock data construction methods ensure that K_{t-1} depends on lagged investment I_{t-1}.)

(b) Next, allow for simultaneity but not for autocorrelation. Assume that output Y_t is jointly determined with investment I_t but that $Y_{t-\tau}$ are predetermined, $\tau = 1, \ldots, m - 1$. From among the various variables in the KOPCKE data file, choose two variables that could serve as legitimate instruments for I_t; denote these instrumental variables as Z_{1t} and Z_{2t}. Defend your choice of Z_{1t} and Z_{2t}. Then employ the two-stage least squares (2SLS) or instrumental variable (INST) procedure and estimate parameters of Eq. (6.15), where $m = 7$. Compare results with those obtained in part (a). Do the estimates vary substantially?

(c) Now estimate allowing both for simultaneity and for autocorrelation. As discussed in Section 6.7.A, you must take care in specifying the set of instruments in this context. Specifically, Fair [1970] has shown that if the disturbances follow a kth-order autoregressive process, then in addition to Z_{1t} and Z_{2t}, the set of instruments here must include $I_{t-\tau}$, $Z_{1,t-\tau}$, and $Z_{2,t-\tau}$, for $\tau = 1, \ldots, k$. In the current context, if you specify an AR(1) process, then the set of instruments should be Z_{1t}, Z_{2t}, I_{t-1}, $Z_{1,t-1}$, and $Z_{2,t-1}$. Outline the intuition that helps explain why this expanded set of instruments must be employed in order to obtain consistent estimates of the parameters. Then, with $m = 7$ and this enlarged set of instruments, estimate parameters of Eq. (6.15) by 2SLS or INST, allowing for first-order autocorrelation. Is autocorrelation statistically significant? Do the 2SLS or INST estimates of the distributed lag parameters vary significantly when account is taken of autocorrelation? Does allowing for simultaneity matter empirically? Why or why not?

EXERCISE 9: Levels and First Differences of the CES Capital Demand Equation with Autocorrelation

The controversy over how large is the response of investment to changes in the user cost of capital has filled many pages of professional economic journals. Recall that in the investment equations employed by Jorgenson, and more recently by Kopcke, the estimating equation is derived from the Cobb-Douglas production function, and so the magnitude of the long-run price response is constrained a priori. The purpose of this exercise is to have you relax the Cobb-Douglas specification, and instead estimate capital demand equations based on the less restrictive constant elasticity of substitution (CES) production function.

In the data diskette directory CHAP6.DAT there is a data file called KOPCKE that contains quarterly data series 1952:1–1986:4 on real invest-