# Reinforcement Learning for Supply Chains

**Dhruv Madeka*, Dean Foster*, Sham M. Kakade^***

*Amazon ^Harvard University

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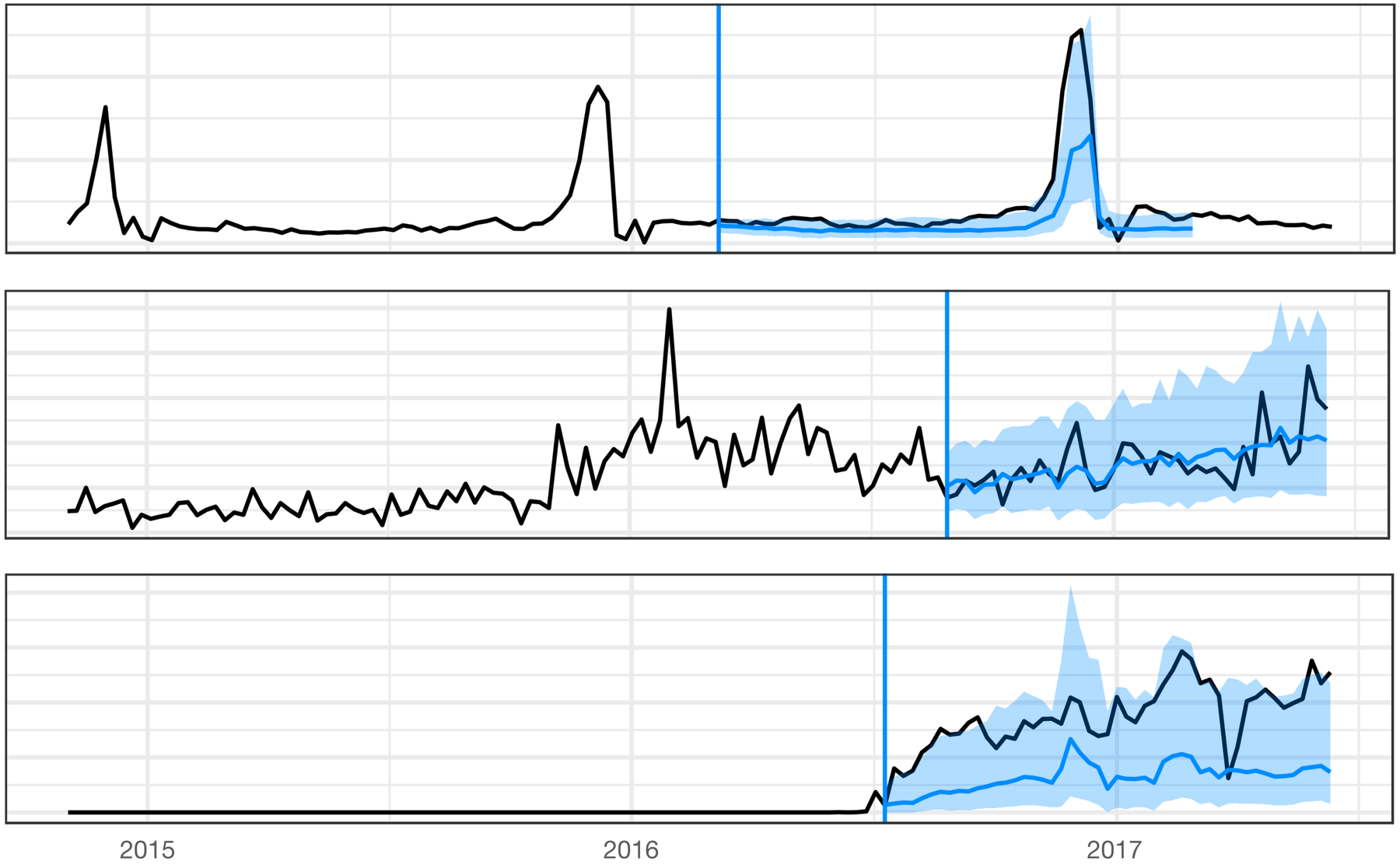

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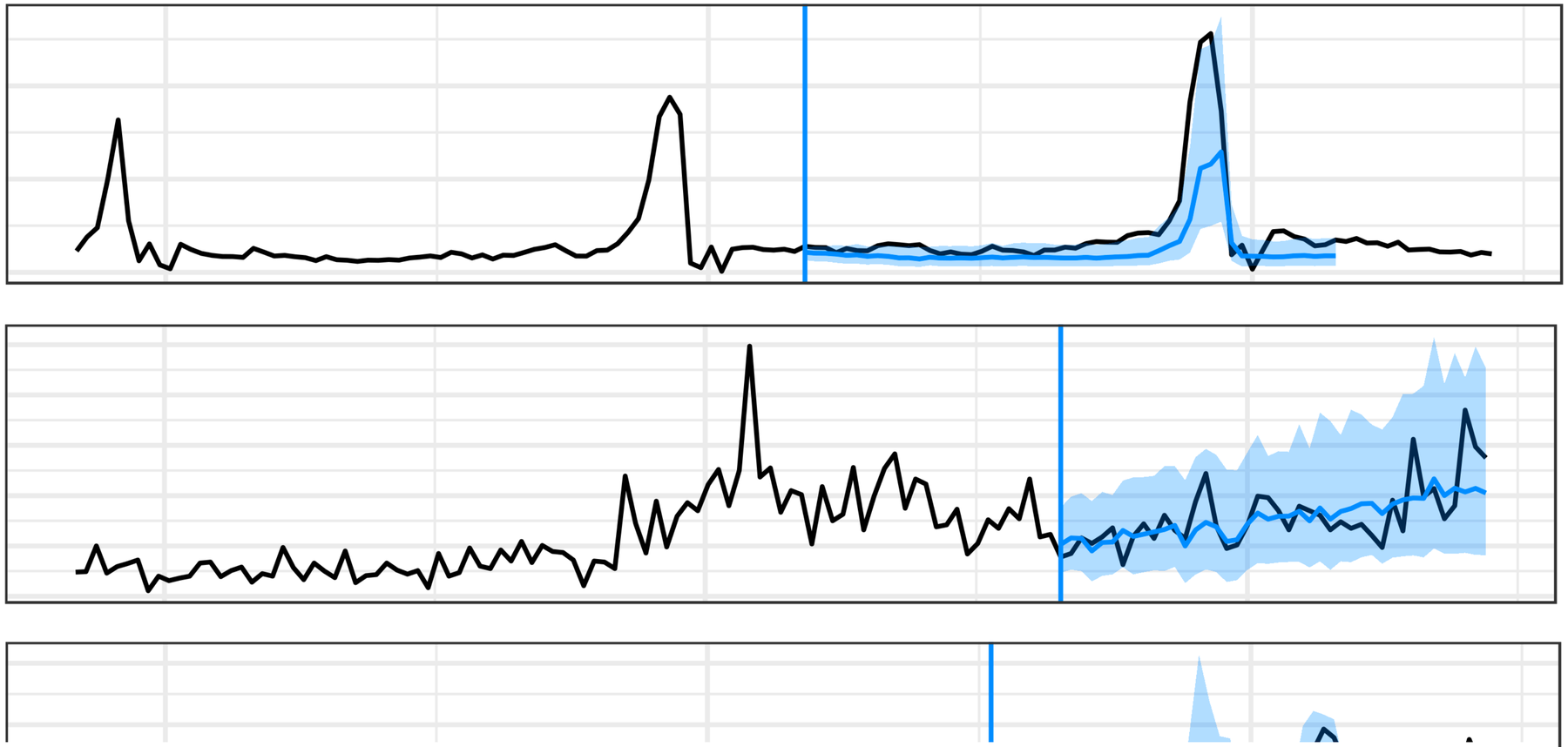

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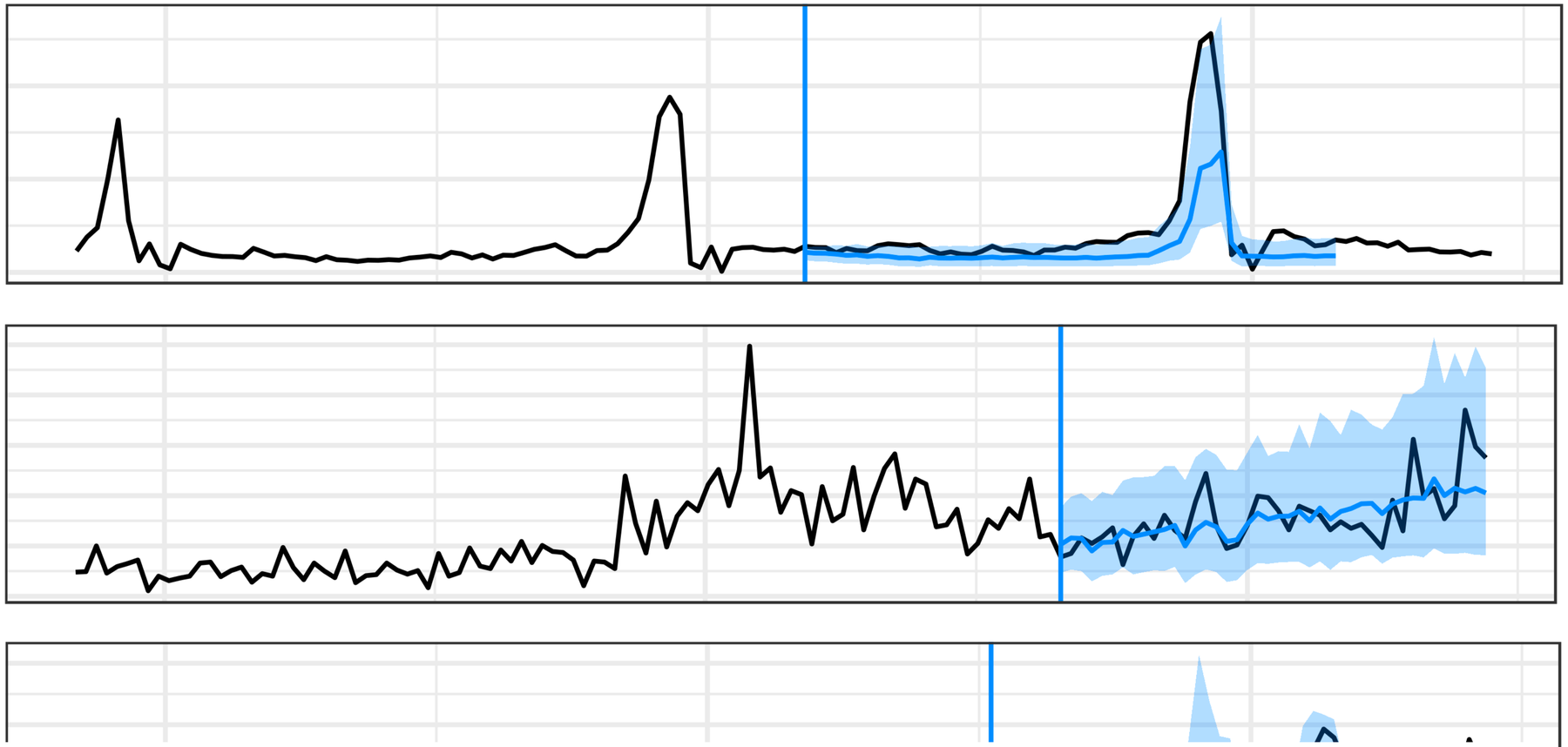

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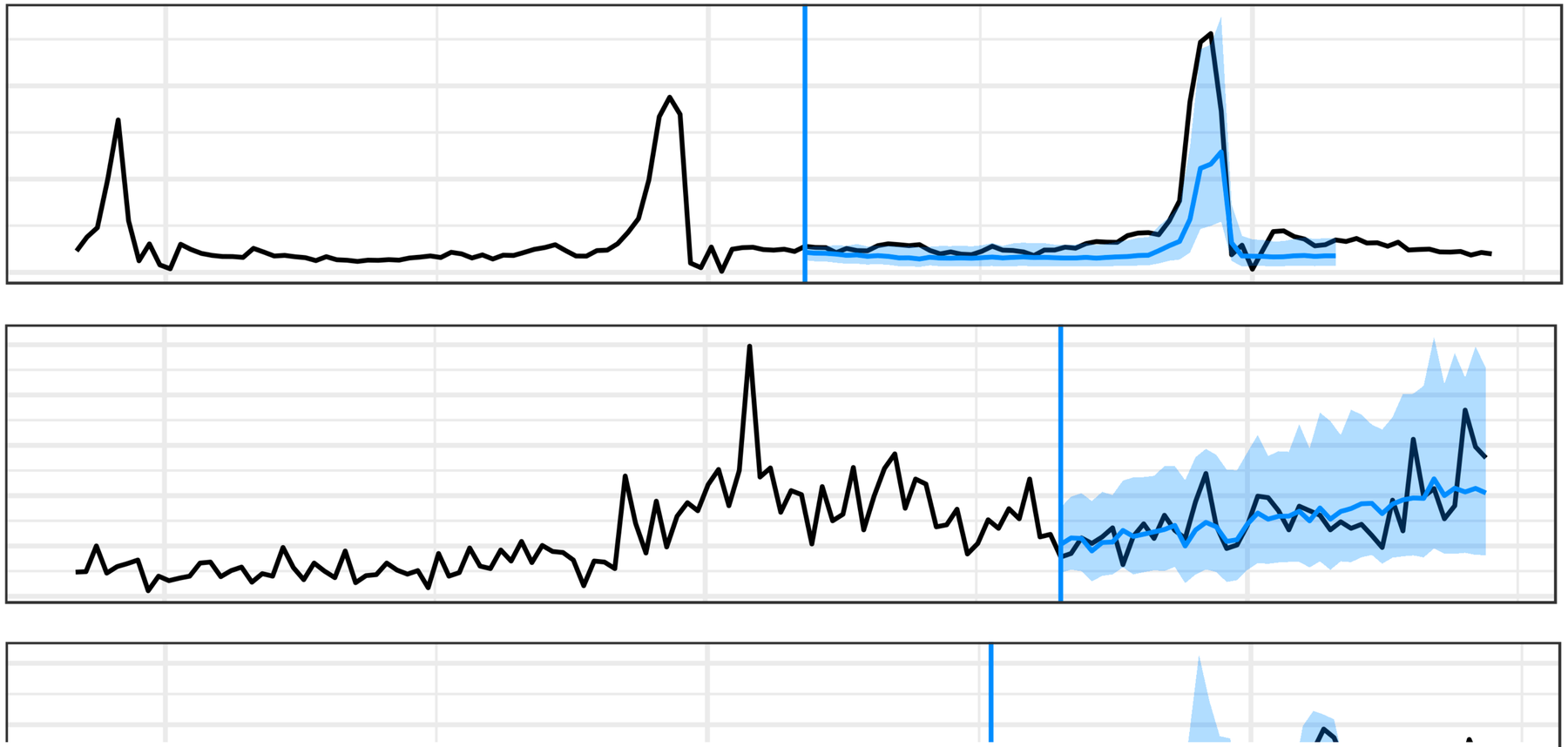

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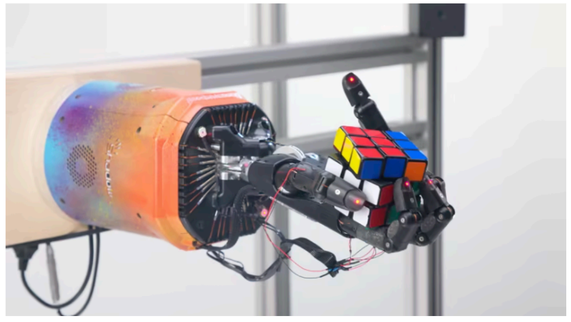
Data reuse: We can compute the loss of every function in a hypothesis class

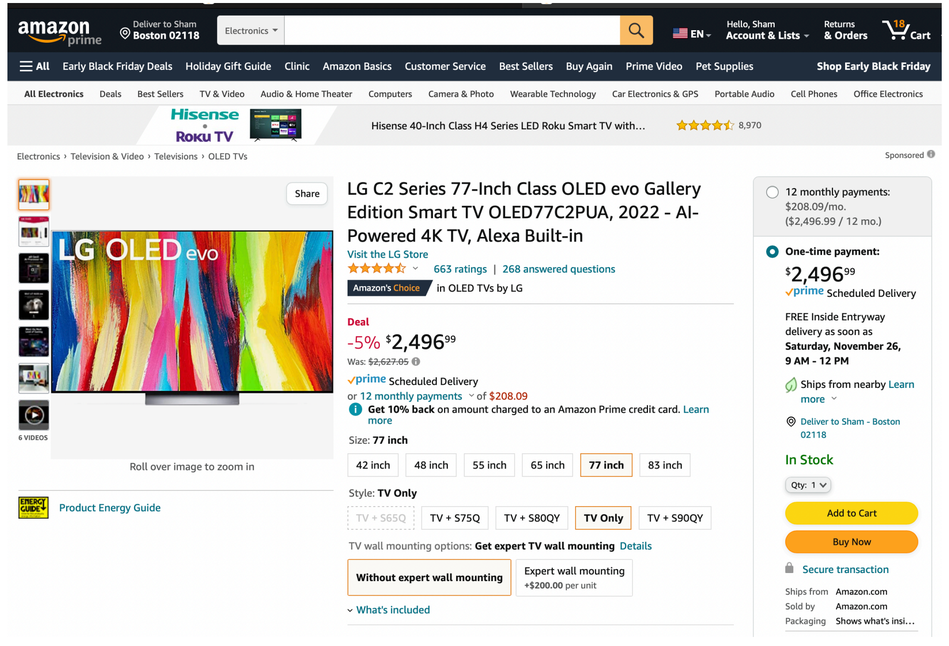
# Real-world RL is hard.



The core challenges Amazon faces are sequential decision making problems.

Can RL help in this space?



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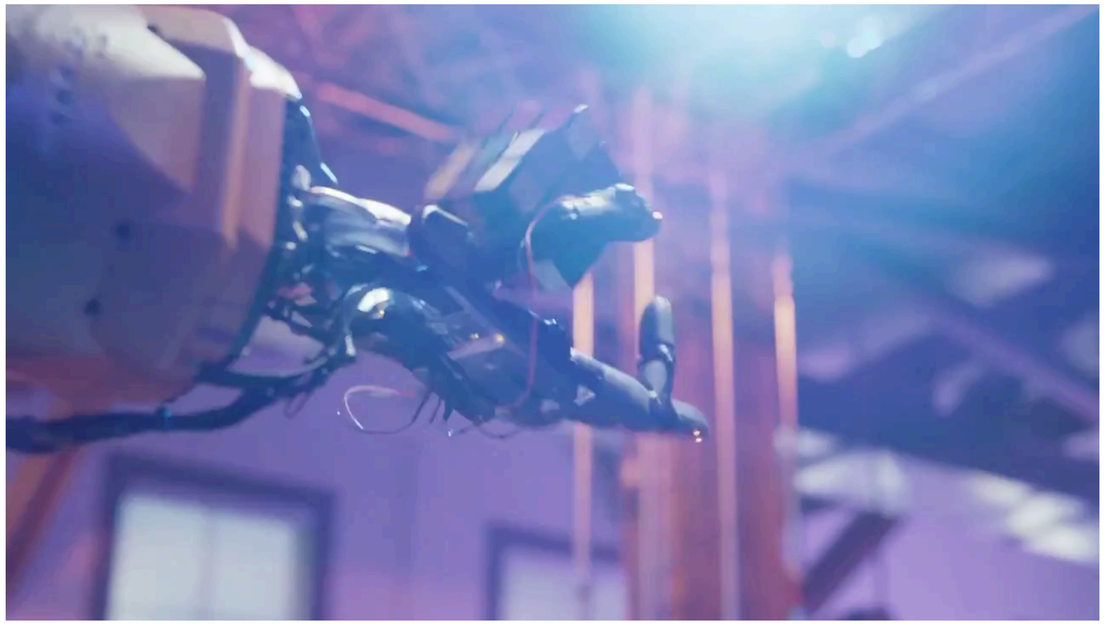
OpenAI, ‘19



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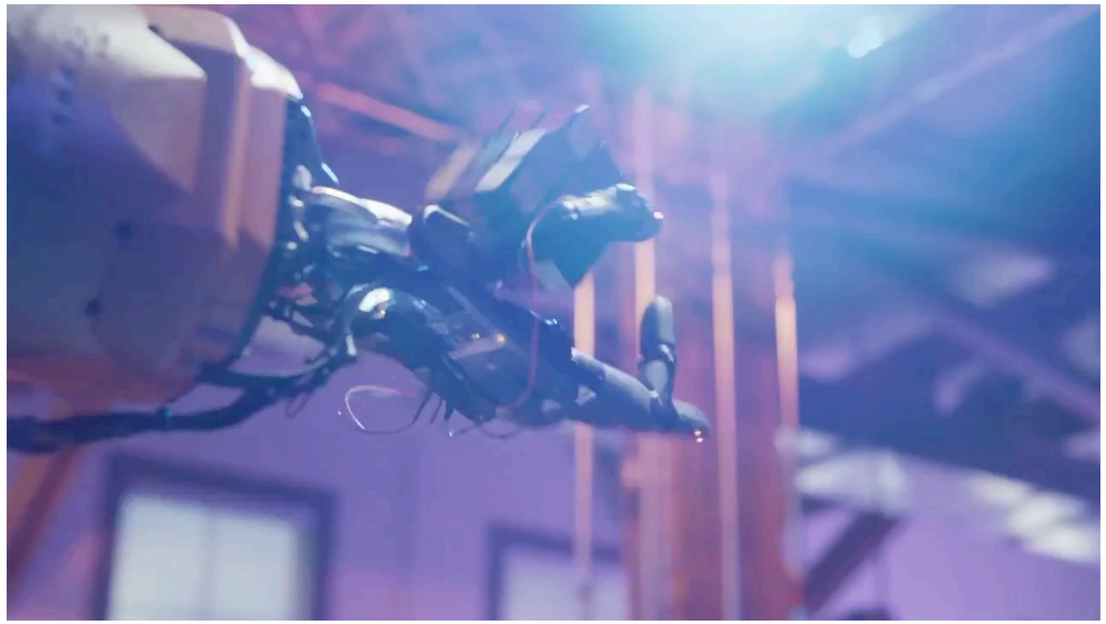

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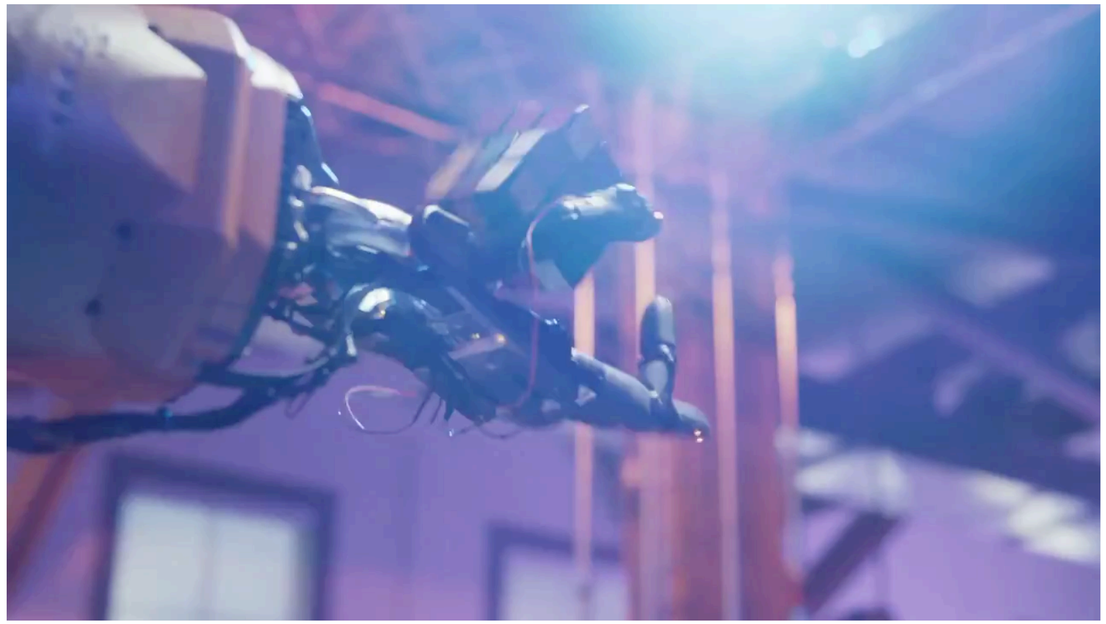

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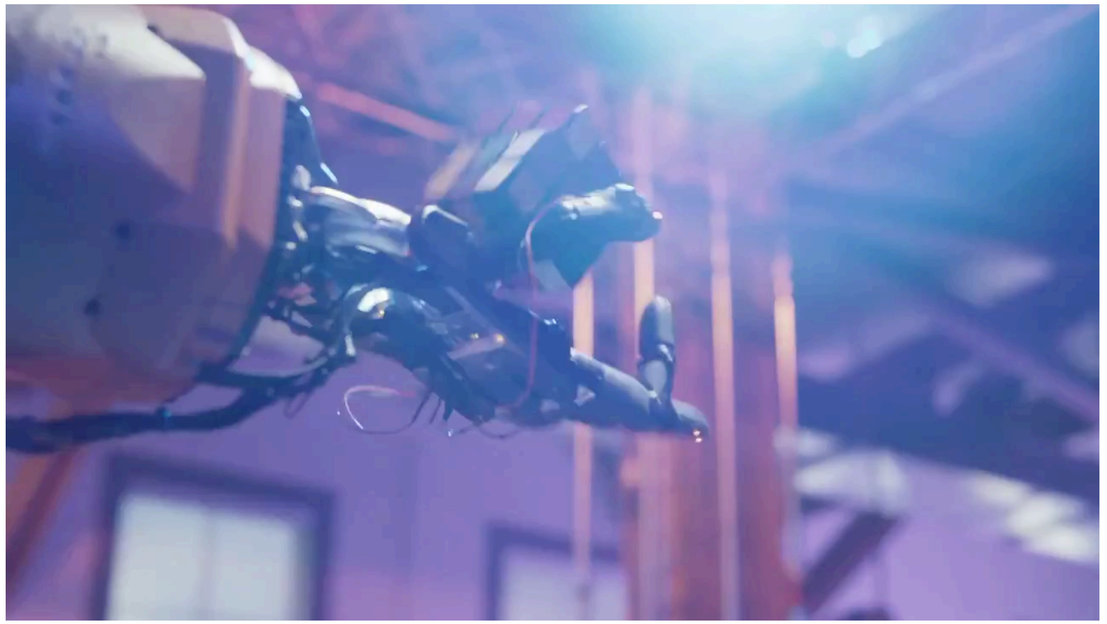

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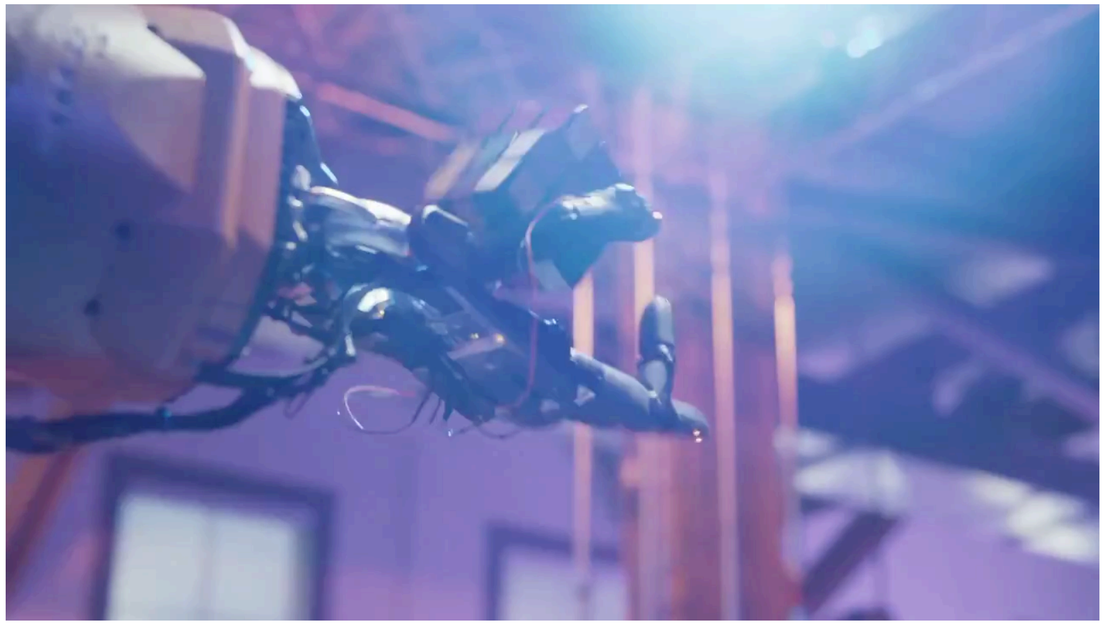

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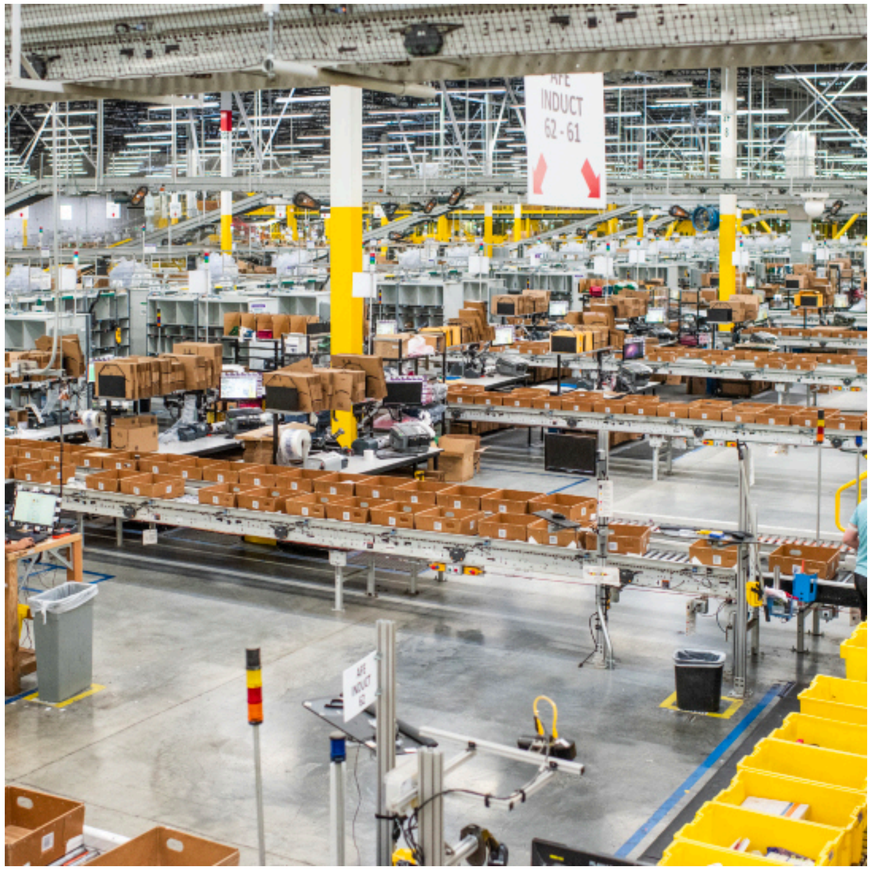

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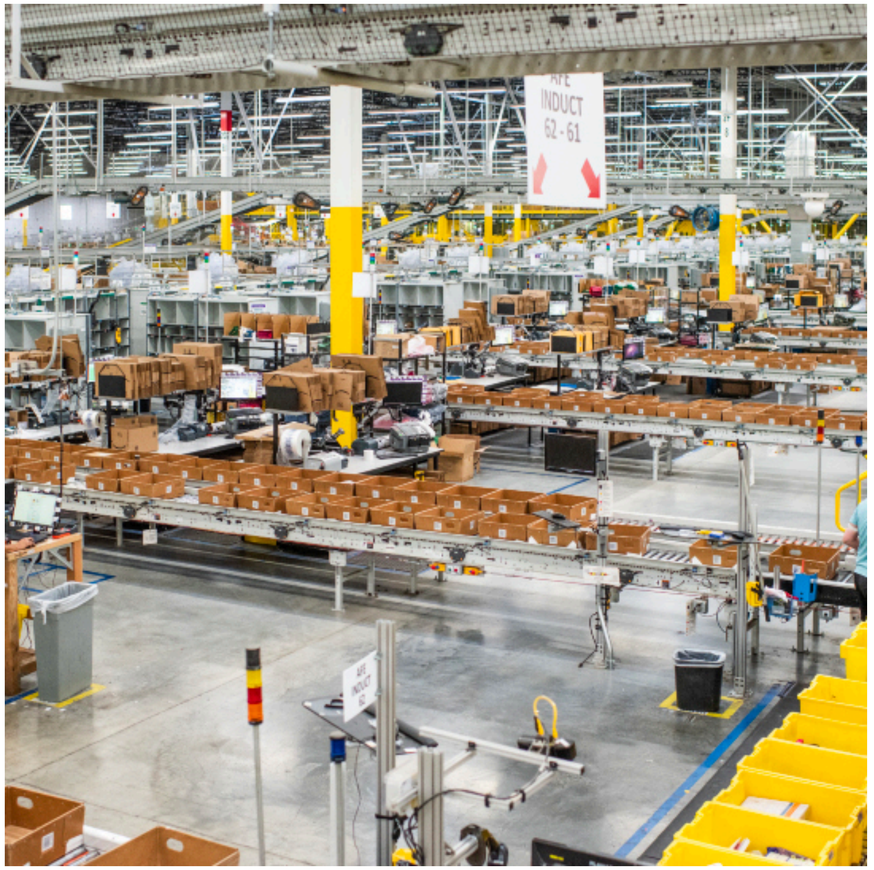

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# Outline

Can we use historical data to solve inventory management problems in supply chain?

- Part I: Utilizing Historical Data
- Part II: Moving to real-world inventory management problems
- Part III: Real World Results

## Deep Inventory Management

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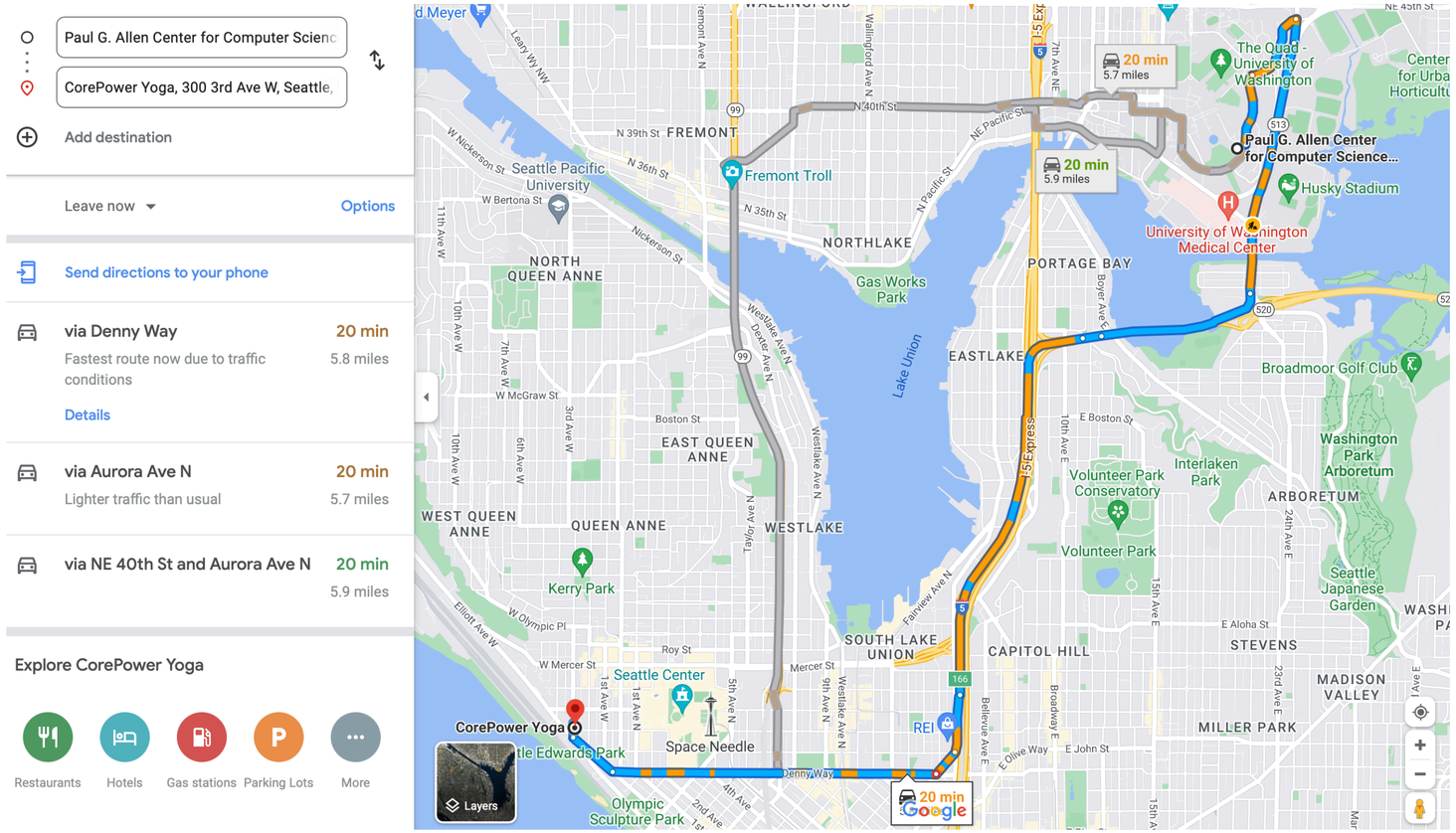
Amazon, Harvard University, shamisme@amazon.com

Largely based on this paper: arxiv/2210.03137

# I: Utilizing historical data

# Warm up: Vehicle Routing

(when using historical data might be ok)




- We want a good policy for routing a single car.

- Policy $\pi$: features -> directions

  features:

  time of day, holiday indicators, current traffic, sports games, accidents, location, weather,

- Historical Data:

  suppose we have logged historical data of features

- Backtesting policies:
  - Key idea: a single route minimally affects traffic
  - Counterfactual: with the historical data, we can see what would have happened with another policy.

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- Backtesting policies:
  - Key idea: a small fleet route may have small affects on traffic.
  - Counterfactual: with the historical data, we can see what would have happened with another policy.

# Supply Chain Data

# Supply Chain Data

Price= $2
Cost= $1

| Time | Inventory | Demand | Order | Revenue |
|---|---|---|---|---|
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| | | | | |

# Supply Chain Data

Price= $2
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| Time | Inventory | Demand | Order | Revenue |
|---|---|---|---|---|
| **0** | 100 | 20 | - | 40 |
| | | | | |
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# Supply Chain Data

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| **0** | 100 | 20 | - | 40 |
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| **1** | 90 | 20 | - | 40 |
| | | | | |
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| **2** | 120 | 60 | - | 120 |
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  - Empirically, backlog due to unmet demand does not look significant.[1]

1. See Verhoef et al (2006)

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  - Exogenous random variables: evolving under $\Pr$ and not dependent on our actions

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$$V_T(\pi) = E_\pi\left[\sum_{t=1}^{T} \gamma^t r(s_t, I_t, a_t)\right]$$

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- We can explore (linear risk in every product)

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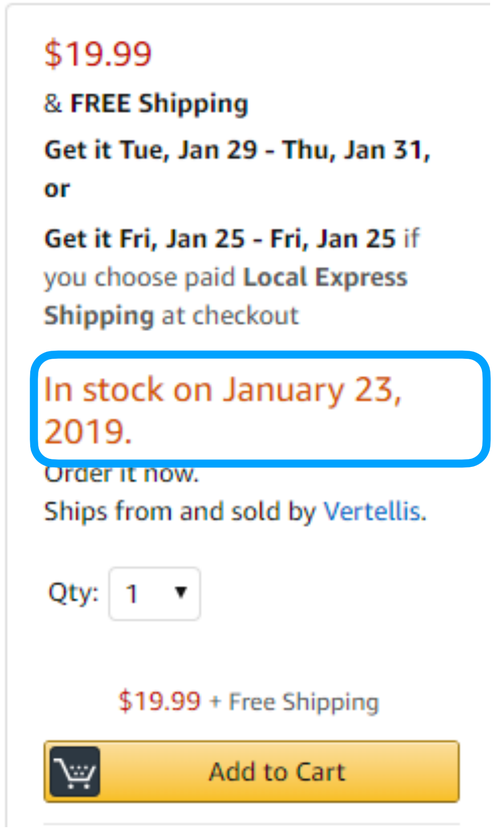
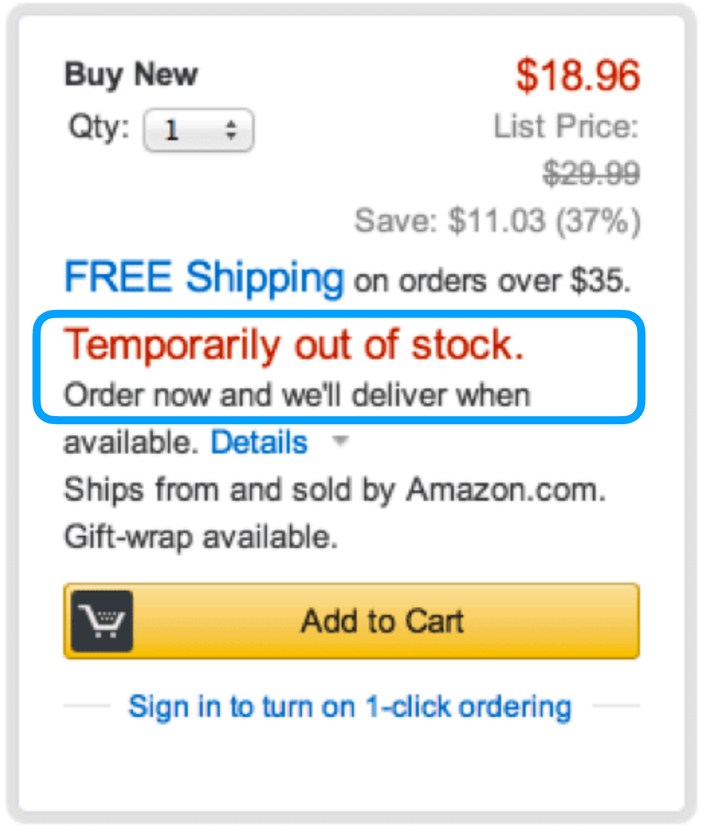
  - We can optimize a neural policy on the past data.
  - In the usual RL setting (not exogenous), we would have an amplification factor of (at least) $\min\{2^T, K\}$, using historical data due to the counterfactual issue.

# II: Real World Inventory Management Problems

# Real-world Issue: Censored Demand

- When demand $\geq$ inventory, what customers see:



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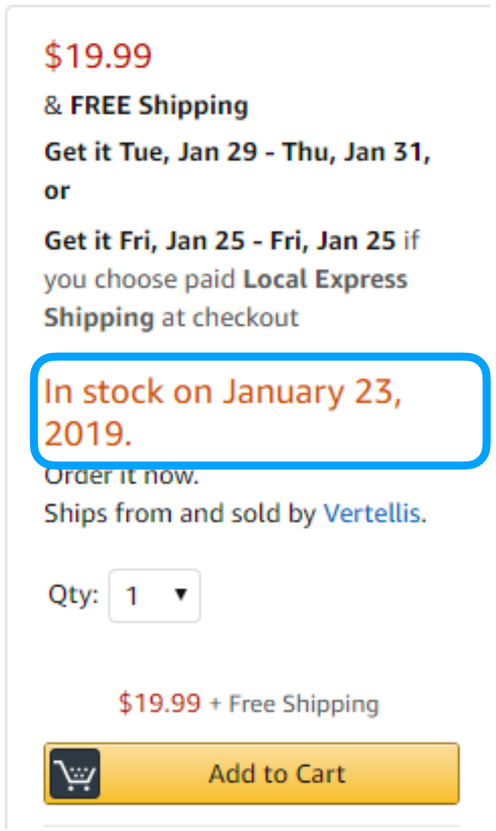


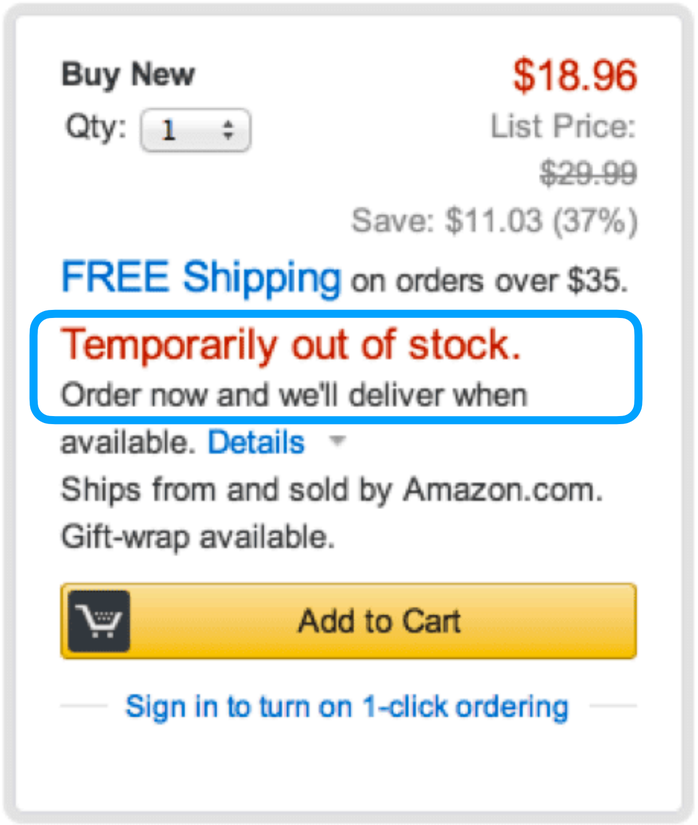


We only observe **sales** not the **demand:**

**Sales := min(Demand, Inventory)**

**Can we still backtest?**

# Our historical data is then censored….

$$\text{Sales} := \min(\text{Demand}, \text{Inventory})$$

Price= $2

Cost= $1

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Sales := min(Demand, Inventory)

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| Time | Inventory | True Demand | Sales | Order | Revenue |
|---|---|---|---|---|---|
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| ⋮ | ⋮ | | | | |
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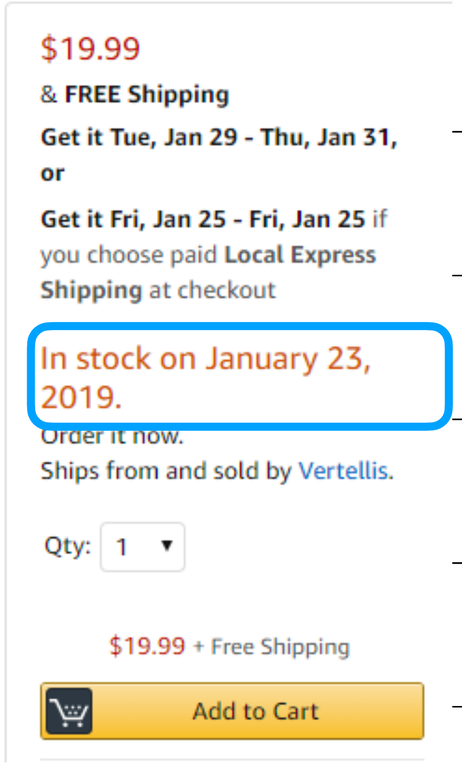


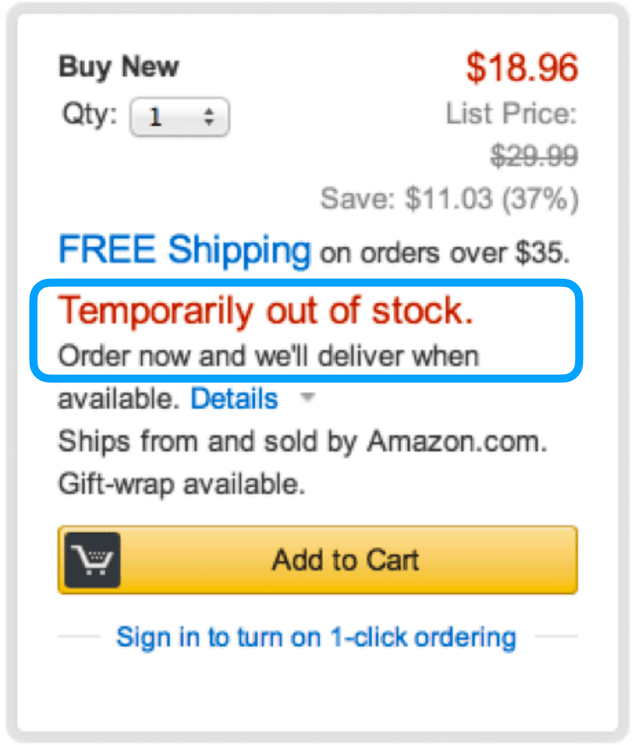

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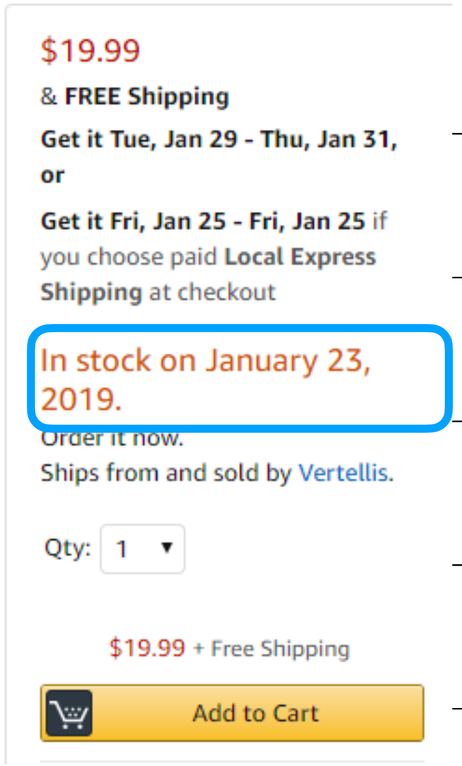
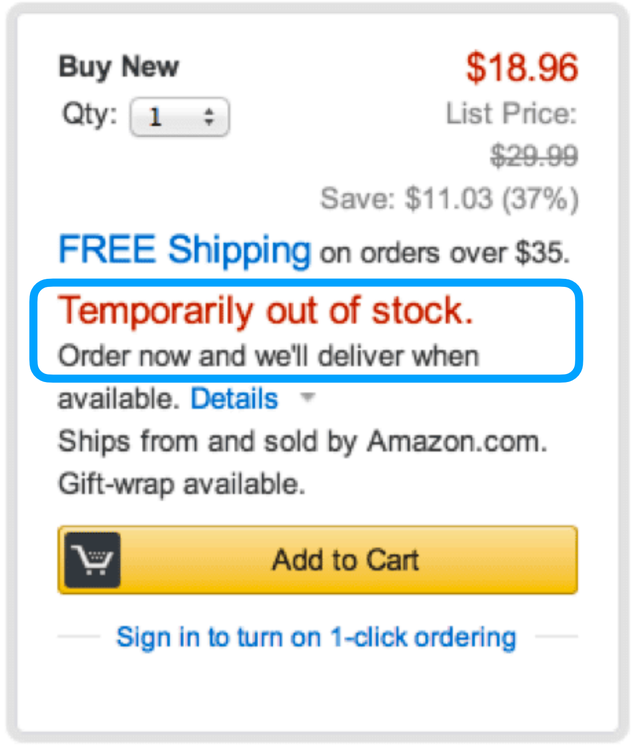


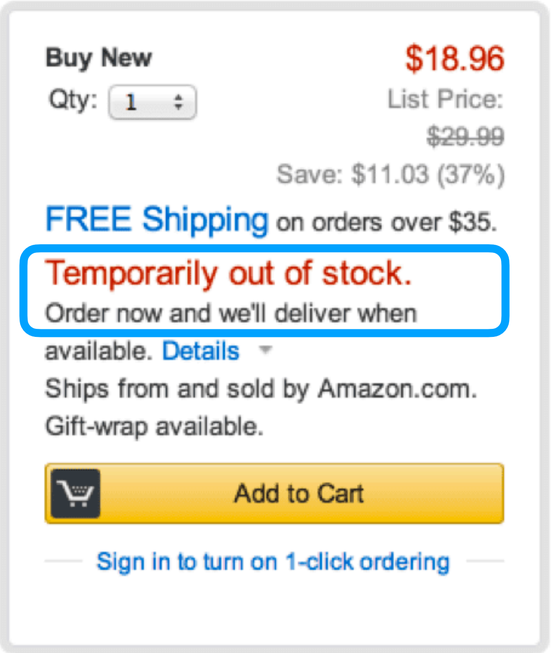

Price= $2

Cost= $1

**If we could fill in the missing demand, then we could still backtest!**

# We have many observed historical covariates

- Covariates:
  Sales, Web Site, Glance Views, Product Text, Reviews
- Example: the #times customers look at an item gives us info about the unobserved demand.




- Let's forecast the missing variables from the observed covariates!
  $\hat{\mathbb{P}}(\text{Missing Data} \mid \text{Observed Data})$

# Uncensoring the data….

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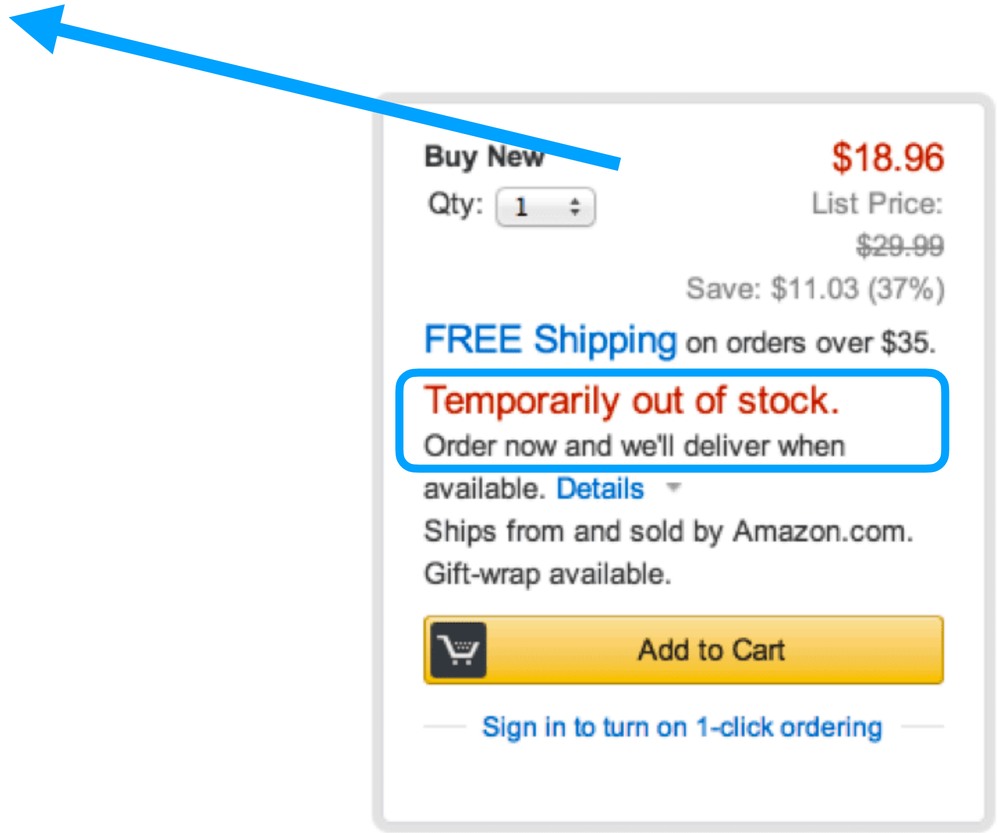

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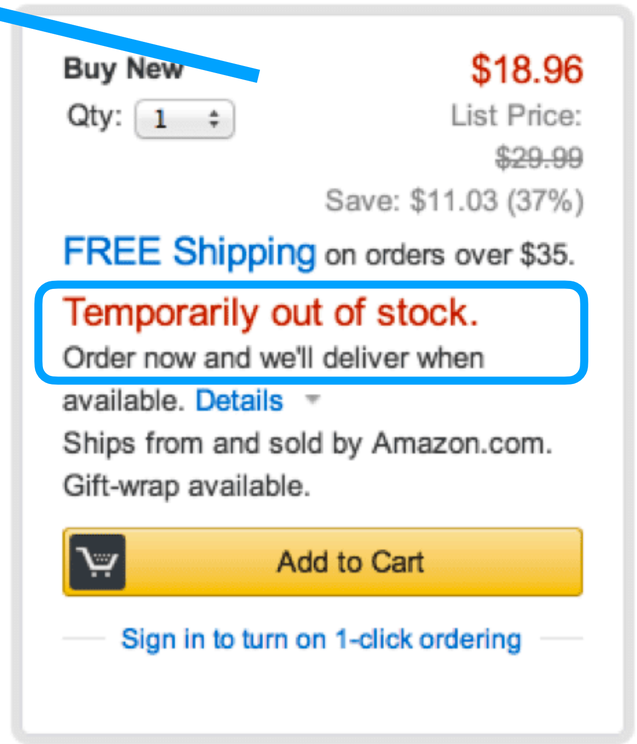

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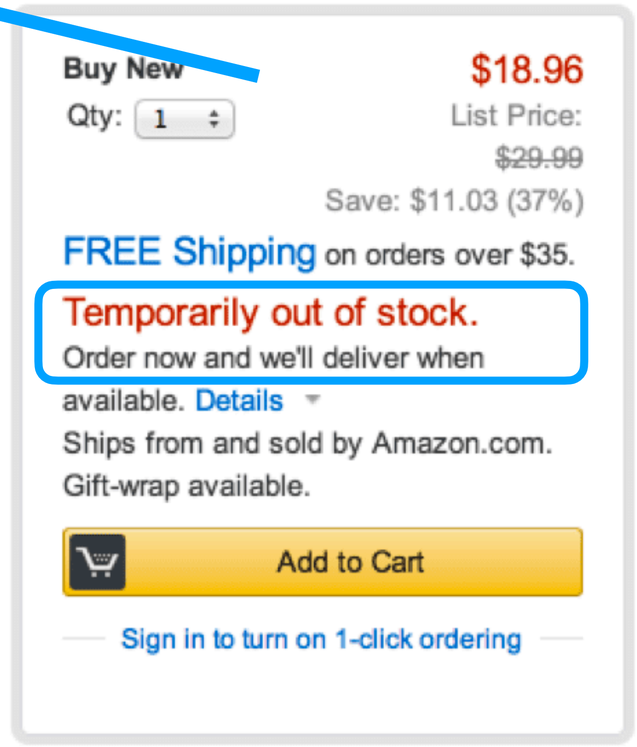


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**Key idea:**
**Use covariates (e.g. glance views) to forecast missing demand, vendor lead times, etc**

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If we can forecast the missing variables accurately (in a total variation sense),
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Setting: we have $N$ sampled sequences $\{s_1^i, s_2^i, \ldots s_T^i\}_{i=1}^N$,
where $M_i$ and $O_i$ are the missing and observed exogenous variables in sequence $i$.

Forecast: $\widehat{\mathbb{P}}^i = \widehat{\Pr}(M_i \mid O_i)$ is our forecast of $\mathbb{P}^i = \Pr(M_i \mid O_i)$.

Assume: With pr. 1, forecasting has low error: $\frac{1}{N}\sum_{i=1}^{N} \text{TotalVar}\left(\mathbb{P}^i, \widehat{\mathbb{P}}^i\right) \leq \epsilon_{\text{sup}}$.

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- Key idea: We can backtest even in the censored setting!

# III: Training Policies & Empirical Results

# The Simulator

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- Collection of historical trajectories:
  - 1 million products
  - 104 weeks of data per product

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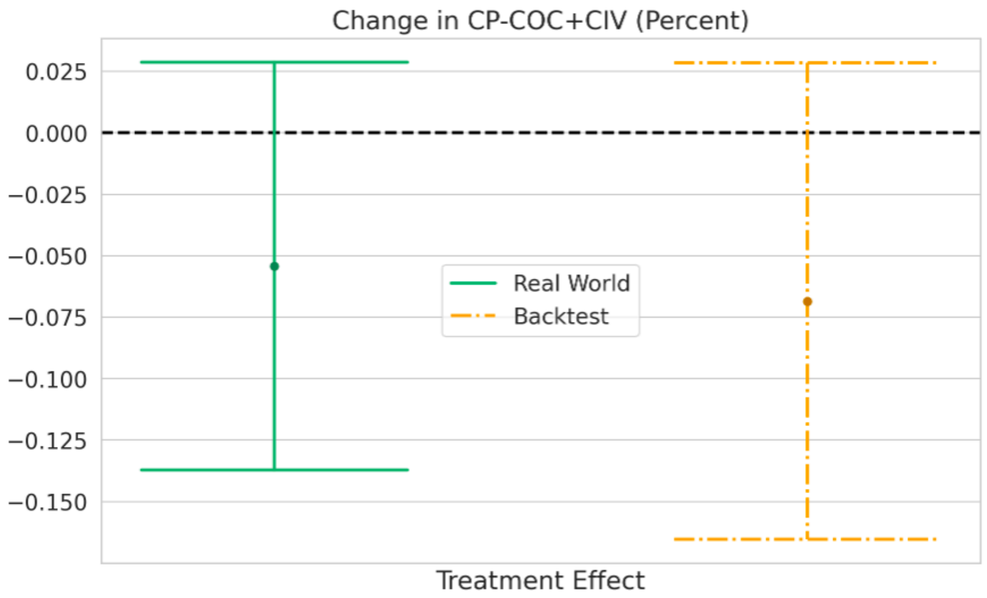
- Policy gradient methods in a “gym”:
  - “gym” ↔ backtesting ↔ simulator
    (note the “simulator” isn’t a good world model).
  - The policy can depend on many features.
    (seasonality, holiday indicators, demand history, ASIN, text features)

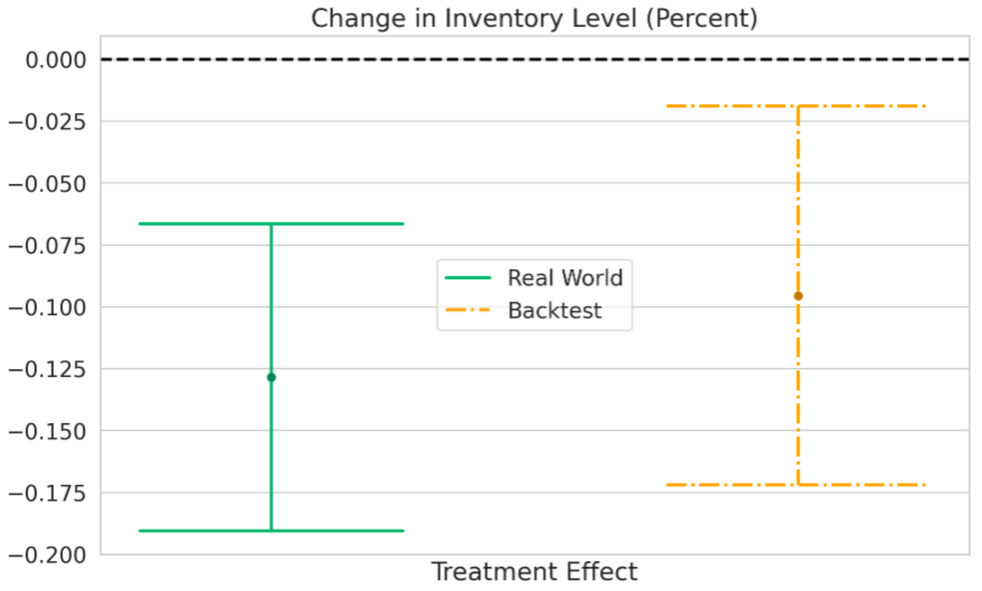
# Differentiable Control Problem

- Note that each term of our state evolution is a differentiable function of previous actions
- So, we can take gradients directly from our Reward through our policy
- This is our current production policy, called *DirectBackprop*
- Similar in spirit to Perturbation Analysis (Glasserman et al 1995), except it uses a neural policy

# Sim to Real Transfer

Checking the calibration on a small experiment



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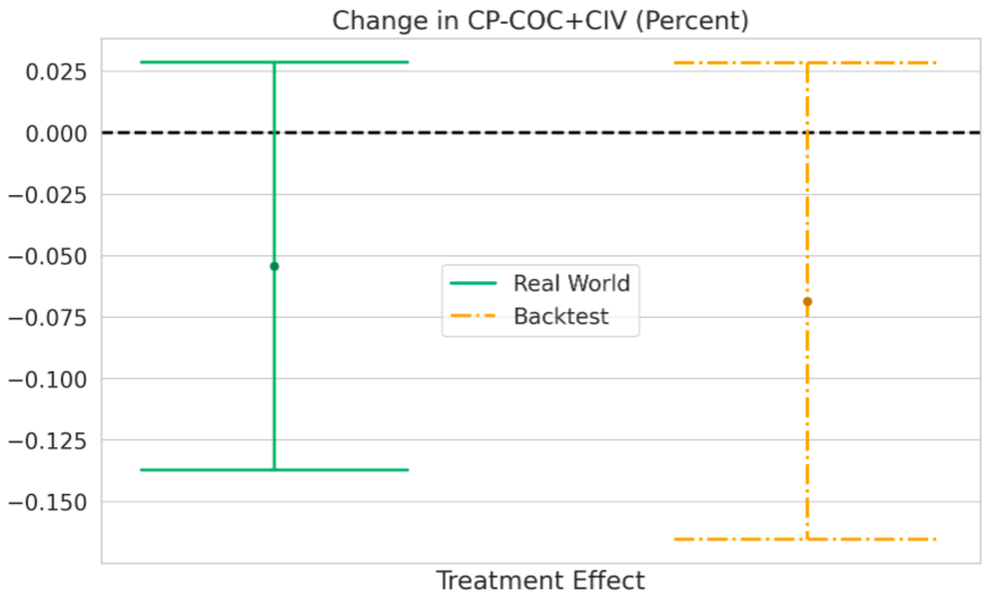


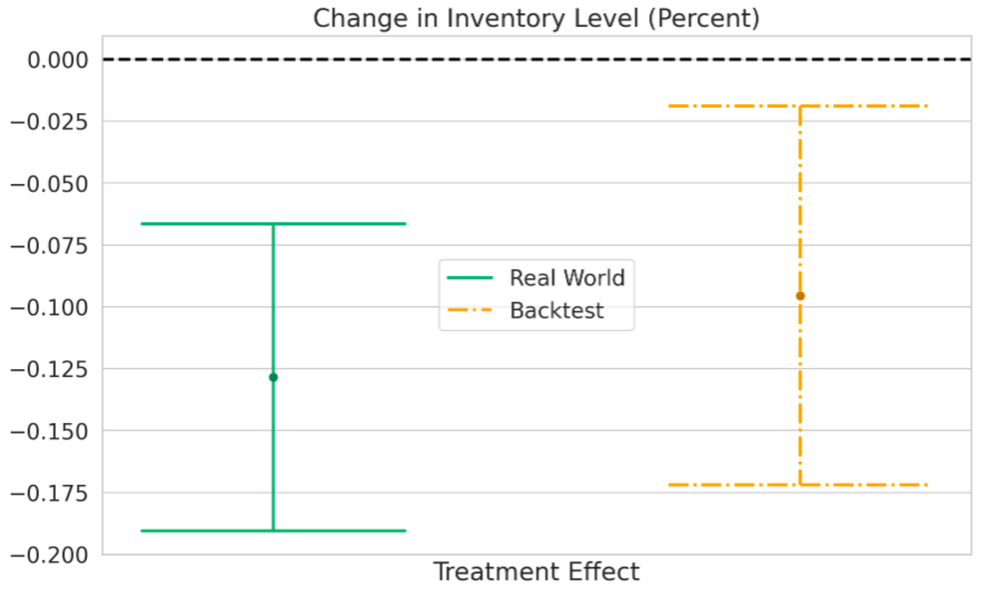

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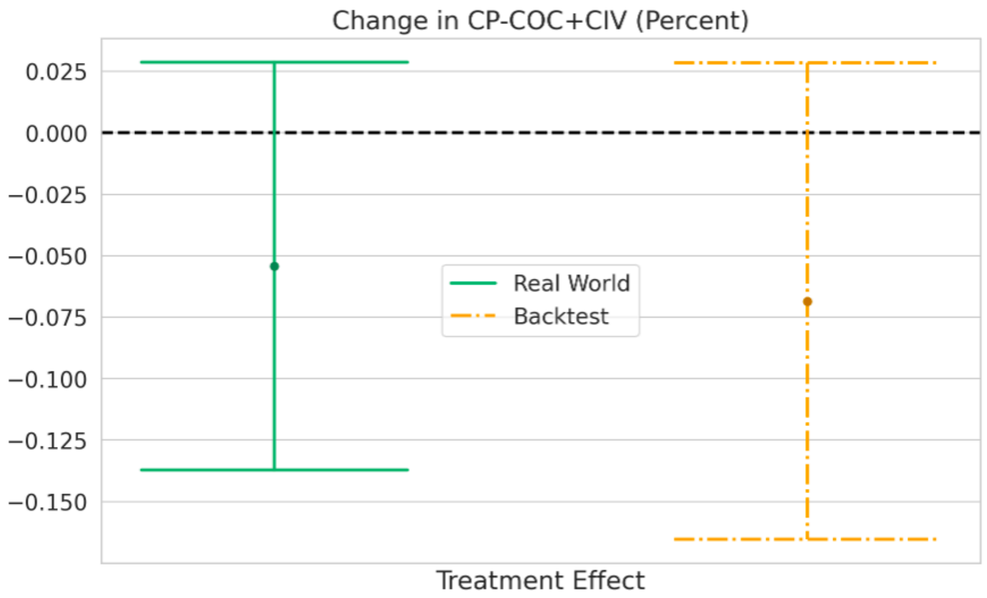


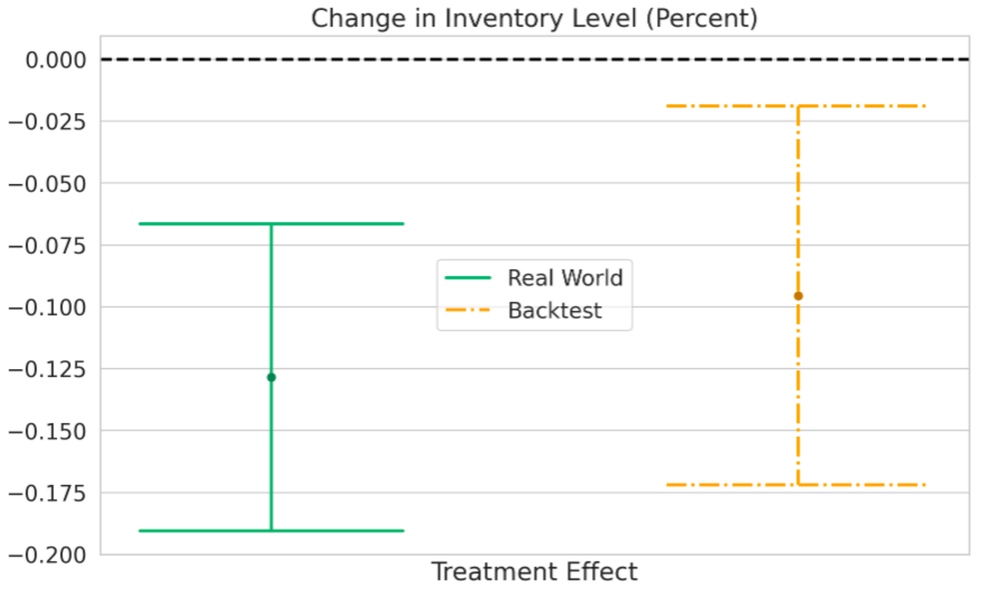

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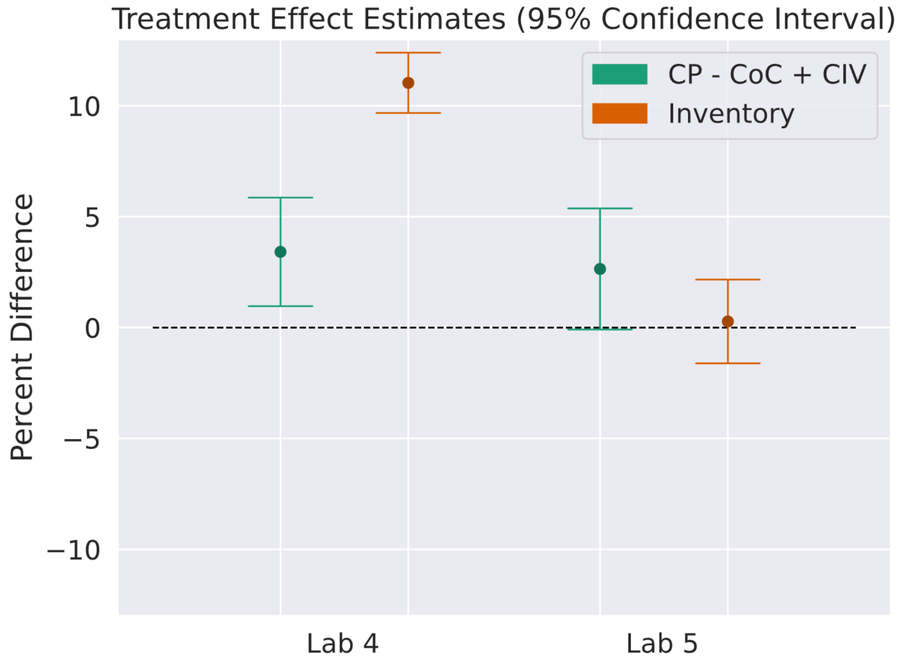

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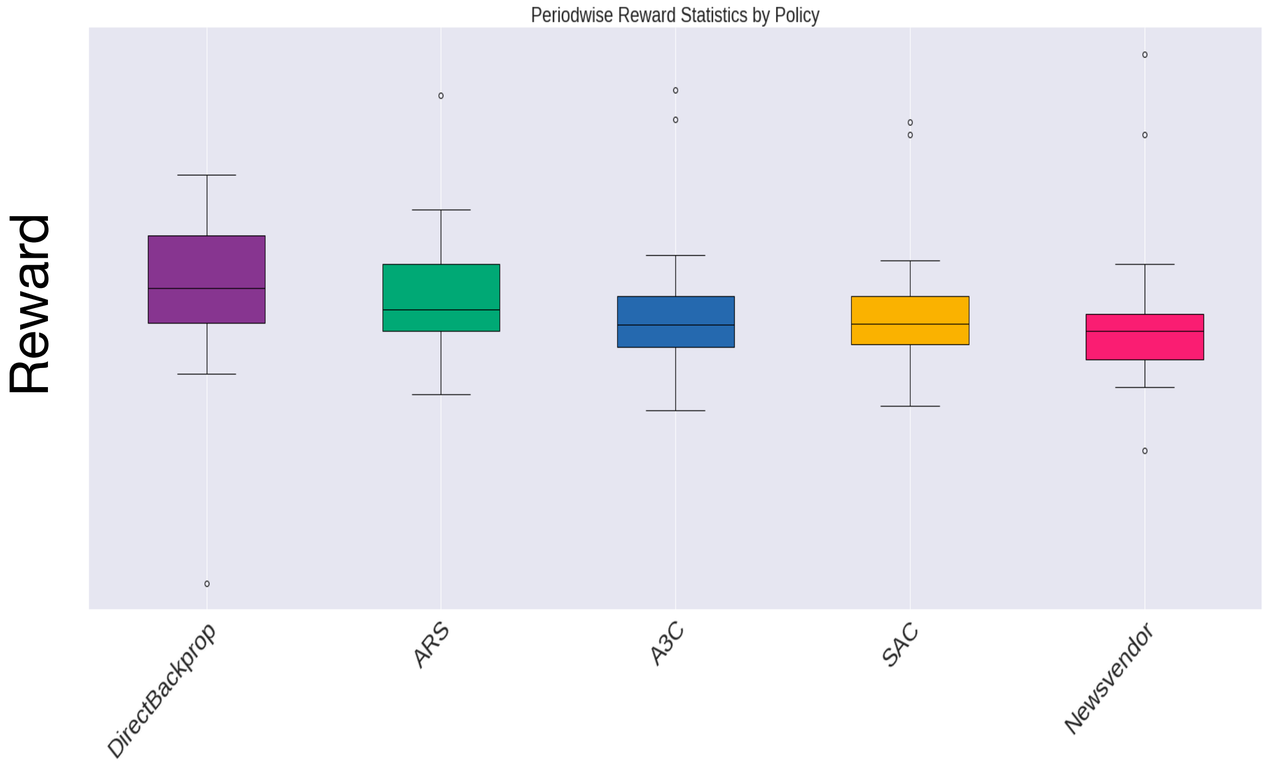


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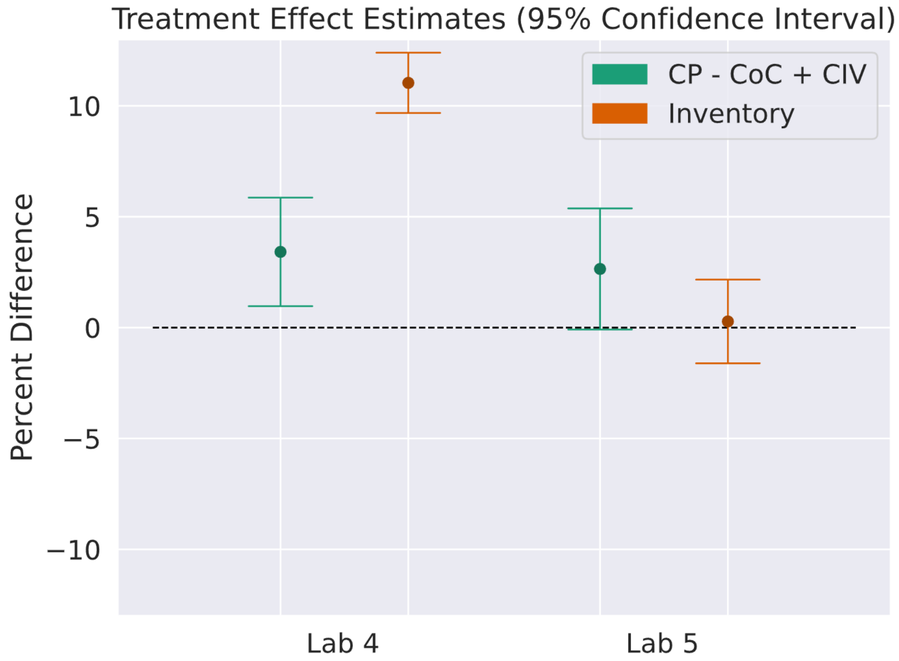

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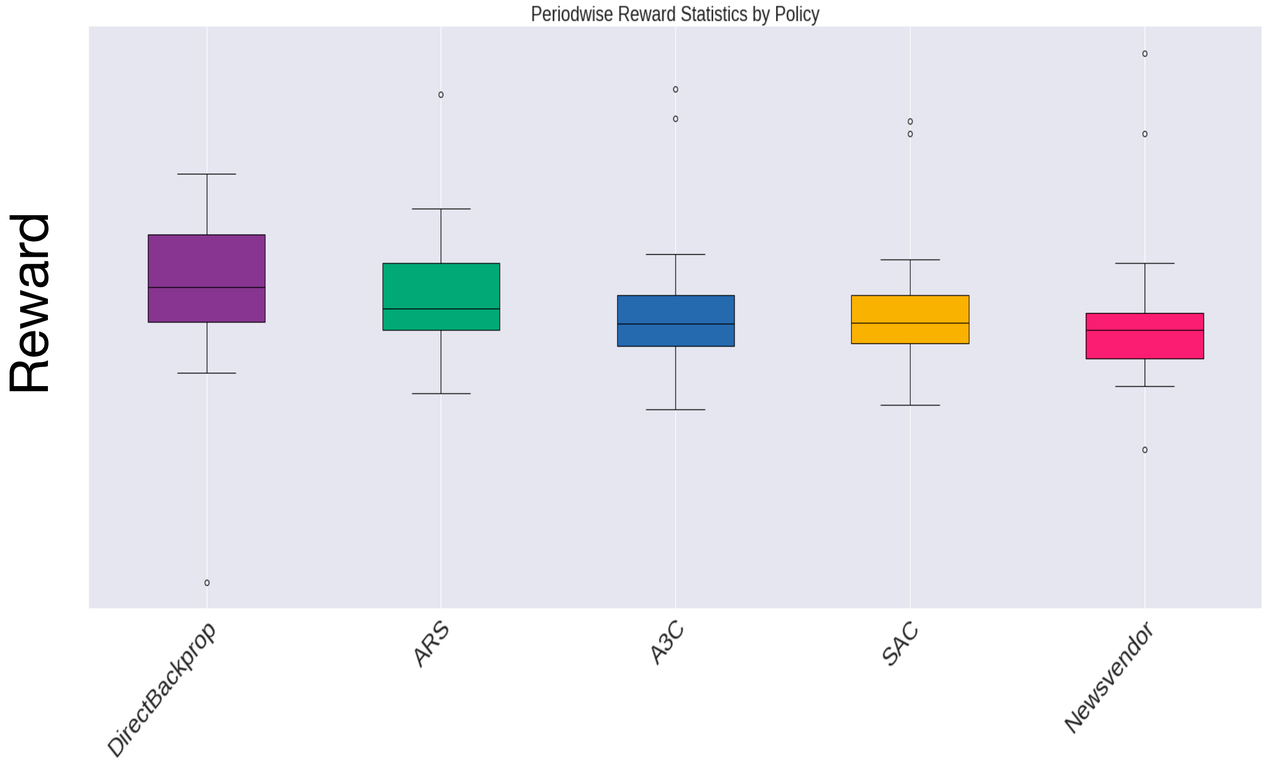


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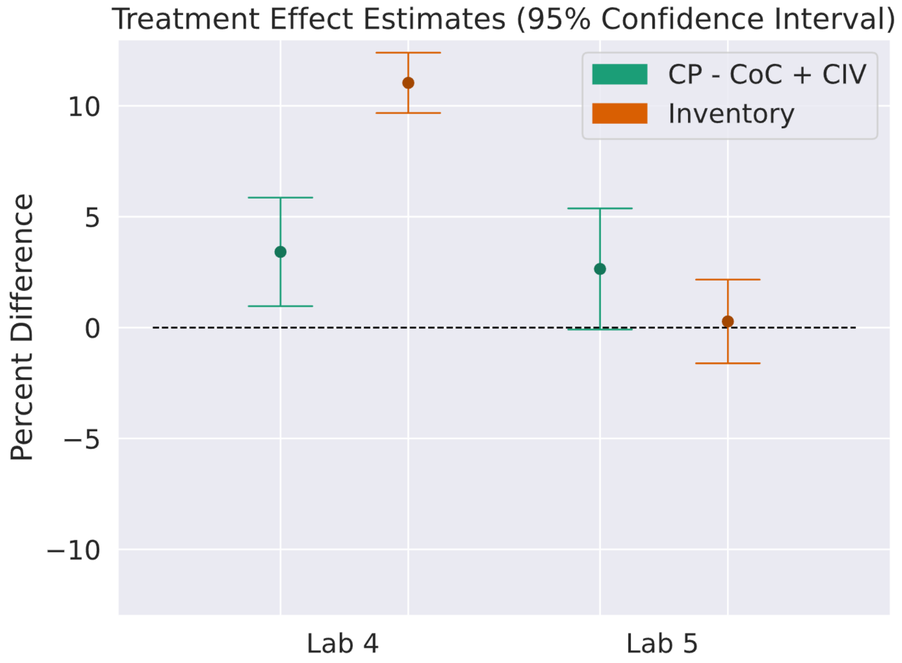

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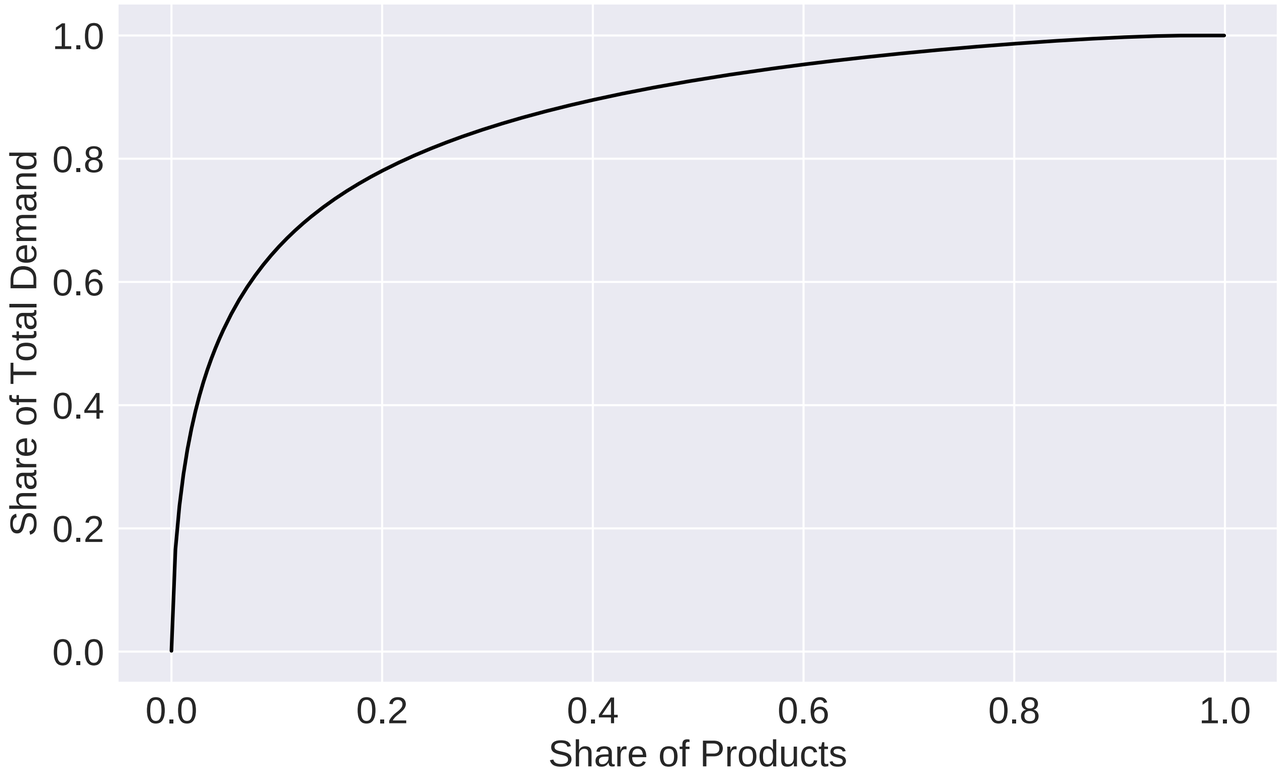
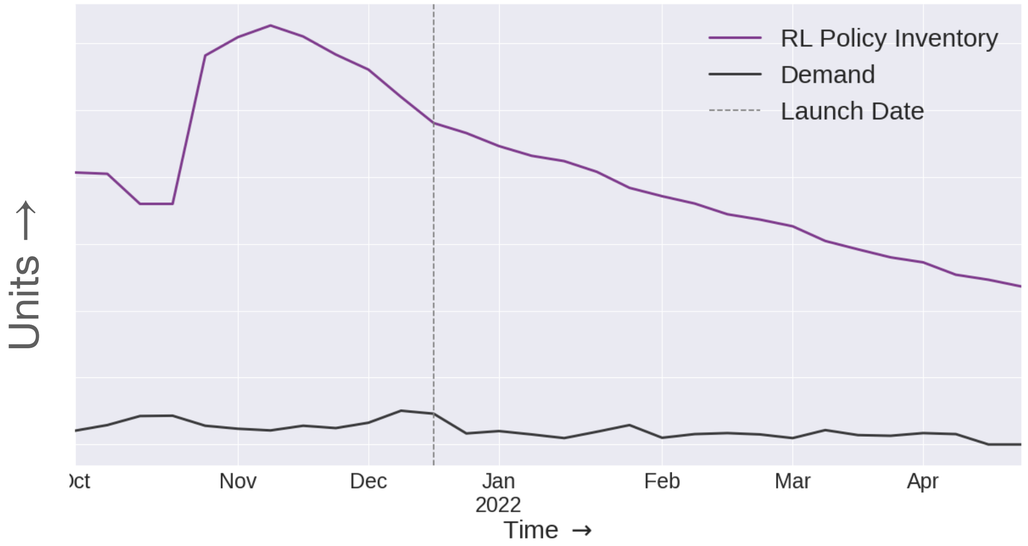
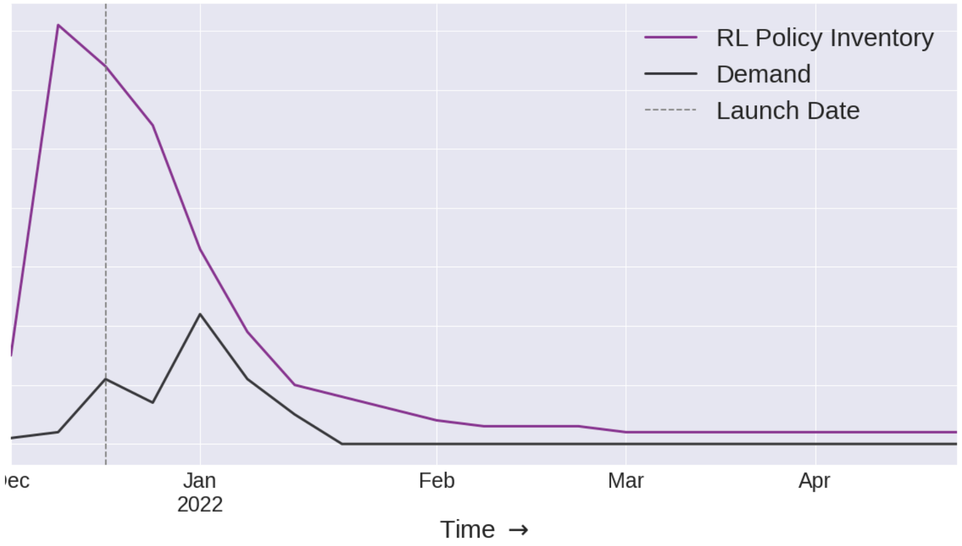
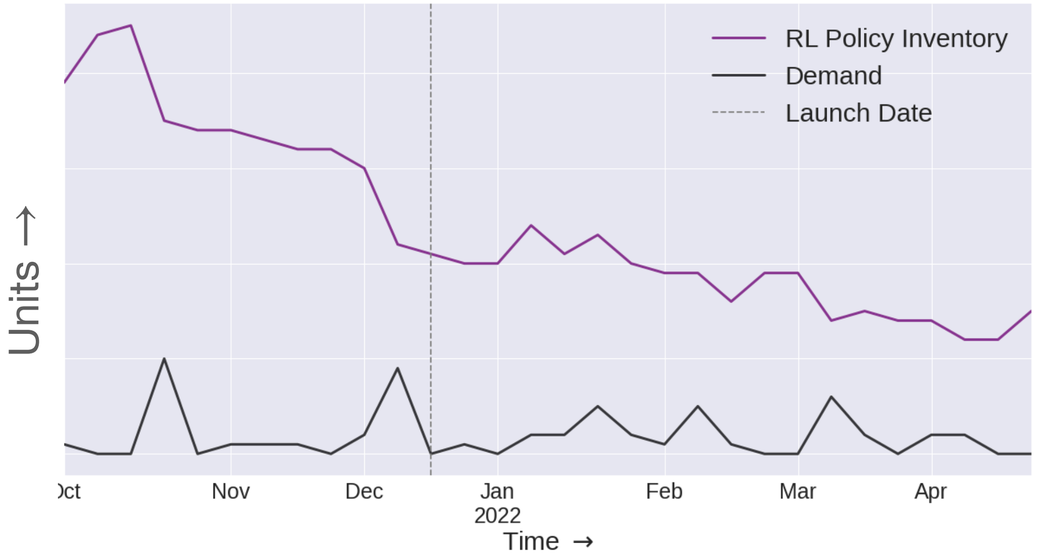
  - A few products contribute to most of the reward

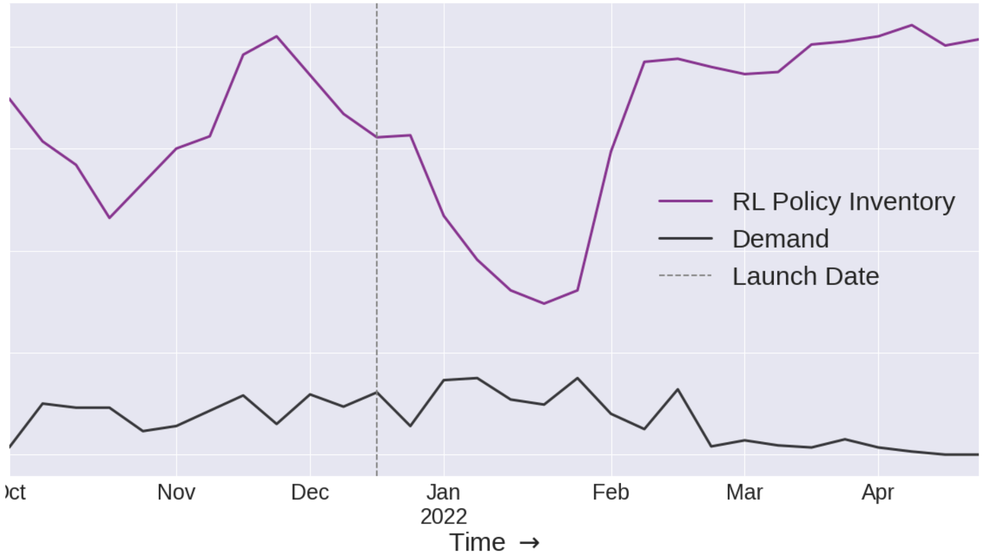


# Anecdotally, RL has reasonable strategies in the real world…



# Real World RL Challenges

- World is not perfectly exogenous (some terms may depend on our actions)
- Cross product constraints are computationally intensive
- Not every Supply Chain problem can be written in this framework

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- A large number of classical Operations Research problems fall into this class of Interactive Decision-Making problems

# The RL Team

Abhi

Angel

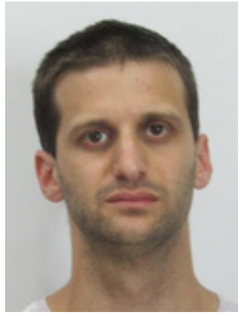
Carson

Dhruv

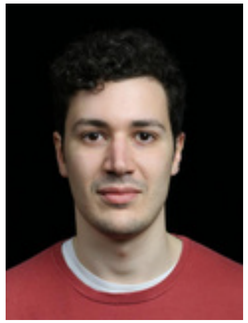
Sham

Dominique

Kari



Omer



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