
(name)

Write your name both on the exam and on the blue book. Circle the one best answer for the multiple choice below. Tear off the multiple choice and turn them in with your blue book. (5pts each)

1. The set $(A \cup C)(B \cup C)$ is best represented as

- (a) C .
- (b) $C \cup AB$.
- (c) $AB \cup BC \cup CB \cup CC$.
- (d) $(A^c B \cup B^c A)^c$
- (e) none of the above.

2. The sum $\sum_{i=1}^n \binom{n}{i}$ is

- (a) 2^n
- (b) $2^n + 1$
- (c) 1^n
- (d) $\sum_{i=0}^{n-1} 2^i$
- (e) more than one of the above

3. The identity $(1 - 1)^n = 0$ can be used to evaluate which of the following

- (a) $\sum_{j=1}^n \binom{n}{j} \binom{j}{i}$
- (b) $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} \cdots \binom{n}{n}$
- (c) $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} \cdots \binom{n}{n}$
- (d) $\sum_{i=0}^n 2^i$
- (e) none of the above

4. Suppose a person chooses one letter from MISSISSIPPI and one letter from TENNESSEE, what is the probability that they spell IT:

- (a) $4/99$
- (b) 0
- (c) $4/11$

- (d) $4/11 + 1/9$
- (e) none of the above

5. If $A \subset B$ then

- (a) $P(A \cup B) > P(B)$
- (b) $P(B|A) < P(B)$
- (c) $P(A) < 1$
- (d) $P(A|B) > P(A)$
- (e) $P(B|A) = 1$
- (f) More than one of the above

6. Which one of the following is always true

- (a) $P(E|E \cup F) \leq P(E|F)$
- (b) $P(E|F) \geq P(E|E)$
- (c) $P(E|E \cup F) \geq P(E|F)$
- (d) $P(E|EF) \leq P(E|F)$

7. In a clinical trial we are looking for patients which satisfy our rules for entry. Suppose that the probability of a patient doing so is $2/3$'s. What is the probability that we get our 2nd patient when the 4th candidate walks in the door:

- (a) $\binom{4}{2}(1/3)^2(2/3)^2$
- (b) $4/27$
- (c) $4/81$
- (d) $ppqp + pqpp + qppp$
- (e) none of the above

8. For the probability space (Ω, \mathcal{A}, P) the function $P()$

- (a) maps Ω to the real line
- (b) maps subset of Ω to the real line
- (c) maps elements of Ω to the real line
- (d) maps subsets of \mathcal{A} to the real line
- (e) maps elements of \mathcal{A} to the real line

Answer the rest in your blue book (20 pts each)

9. We will say that a sequence of events E_1, E_2, E_3, \dots converges quickly to zero if $P(E_i) \leq p^i$ for some $p < 1$.

(A) Prove that there exists some m such that $P(\cup_{i=m}^{\infty} E_i) < .05$.

(B) Let $X = \sum_i I_{E_i}$. Prove that $E(X) < \infty$.

10. Let X be a random variable with density function $f(x) = .5^x$ for $x = 1, 2, 3, \dots$. Let Y be a random variable defined by the equation $Y = a^X$.

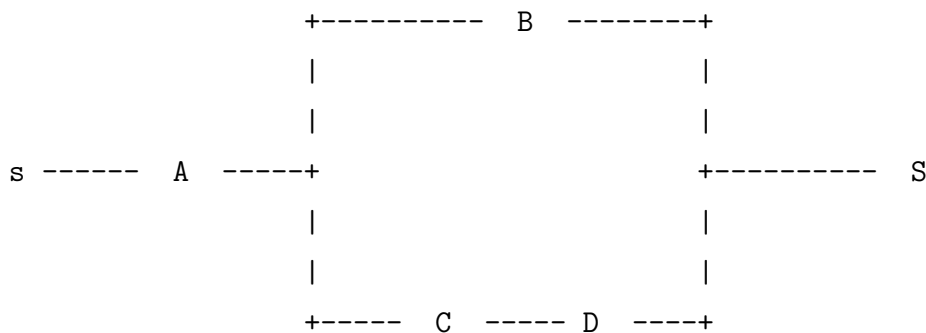
(A) What is $E(Y)$?

(B) For what values of a is $E(Y) < \infty$?

(C) If $a = 1 + \epsilon$, for $\epsilon \approx 0$, show $X \approx \frac{Y-1}{\epsilon}$.

(D) From this, compute $E(X)$.

11. Let $\mathcal{A} \equiv \{\text{the event A works}\}$, and likewise for \mathcal{B}, \mathcal{C} and \mathcal{D} . Assume that each part fails independently of the other pieces.



Let $\mathcal{T} \equiv \{\text{the event a path from s to S works}\}$.

(A) Write \mathcal{T} in terms of $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and \mathcal{D} .

(B) If $P(\mathcal{A}) = P(\mathcal{B}) = P(\mathcal{C}) = P(\mathcal{D}) = 3/5$, what is the $P(\mathcal{T})$?

(Bonus) If you could pay a dollar to increase one of the probabilities by a small amount say ϵ , which of $P(\mathcal{A}), P(\mathcal{B}), P(\mathcal{C})$ or $P(\mathcal{D})$ would you choose? Why?