

1. (10 pts) This problem will walk you through a proof that \cap and \cup don't commute.
- (a) Draw a Venn diagram of $(A \cap B) \cup C$.
 - (b) Draw a Venn diagram of $A \cap (B \cup C)$.
 - (c) Describe a point that is in one set but not the other. Represent this as an intersection of sets.
 - (d) Argue logically that this intersection should be in one but not in the other of the two sets.
2. (30 pts) This question tests your ability to ignore your intuition. Each of the following statement is about a probability space (Ω, \mathcal{A}, P) . Your problem is to say which are correct and which are wrong. For the incorrect statement, rewrite the statement to make it correct. What we know about the probability space is:

$$\begin{aligned}\Omega &= \{1, 2, 3, 4, 5, 6\} & P(\{1, 2, 3\}) &= 1/2 \\ \{1\} &\in \mathcal{A} & P(\{2, 3\}) &= 1/3 \\ \{1, 2, 3\} &\in \mathcal{A} & P(\{1, 4\}) &= 1/3\end{aligned}$$

- (a) $P(\{7\}) = 0$.
 - (b) $P(4) = 1/6$.
 - (c) If $\{2, 4, 6\} \in \mathcal{A}$ then $\{1, 3, 5, 7\} \in \mathcal{A}$.
 - (d) $\{\emptyset\} \in \mathcal{A}$.
 - (e) $\{\emptyset\} \subset \mathcal{A}$.
 - (f) If $A \in \mathcal{A}$ then $A \subset \Omega$.
 - (g) If $A \subset \Omega$ then $A \in \mathcal{A}$.
 - (h) If $\{5\} \in \mathcal{A}$ then $P(\{5\}) \leq 1/4$.
 - (i) $P(\{2, 3\} | \{4, 5, 6\}) = 0$.
3. (25 points) You are a lawyer defending a man who is accused of murder. Unfortunately, the murderer left some of his own blood behind at the scene of the crime. So the prosecution has done a DNA match with your client. The outcome of the test is either “match” or “no match,” which we will call M and M^c respectively. If your client is innocent (which we will call G^c), there is a 1-in-a-million chance of a match occurring. If your client is guilty (G), there is a 90% chance of the test generating a match.
- The above assumed that the lab didn't make a gross error. If the laboratory made a gross error (E), then there is a 50% chance of the test generating a match regardless of whether your client is innocent or guilty.

Assume that errors in the laboratory are independent of the guilt of your client. Assume that there is a 95% chance that your client is innocent. Assume that there is only a 2% chance that the laboratory made a mistake.

- (a) What is the probability of $G \cap E$? Of $G \cap E^c$? And of $G^c \cap E$?
 - (b) What is the probability of $G \cap E^c \cap M$? (You probably should make a table of probabilities.)
 - (c) Given a Match didn't occur, what is the conditional probability of E ? What is the conditional probability of G ?
 - (d) Given a Match did occur, what is the conditional probability of E ? What is the conditional probability of G ?
 - (e) What should your strategy be as a lawyer in this case? (Hint: what will you argue if there is a match and what will you argue if there isn't a match.)
4. (15 pts) At a very slow day 8 hour day on the NJ turnpike, a total of 2 cars arrive at a toll booth which has two lanes. After work, the two toll collectors (Bob and Dean) are comparing how many people came through each of their lanes. Dean had 2 cars, and Bob had 0 cars.
- Dean argues that the sample space is $\Omega = \{(D = 2, B = 0), (D = 1, B = 1), (D = 0, B = 2)\}$. Assuming each of these are equally likely he argues that the probability that he handles 2 cars and Bob handles zero cars is $1/3$.
 - Bob argues that no-one ever goes in a lane with a car in it when a lane is free. Thus, if Dean gets the first car, then Bob will get the second car. Likewise if Bob gets the first car, then Dean will get the second car. Each of these two are equally likely.

Critique each of these arguments. (In other words, if you agree with one of them, say so. If you disagree with both of them then say what you think is the correct distribution. In either case give a sentence as to what is wrong with the thinking of anybody you disagree with.)

5. (15 points) Yesterday, you received two heart shaped boxes of candies for valentines day. (Exactly why you got more than one box, is a question of sociology and not statistics.) The first box has 8 caramels in it and 4 other candies. The second box has 5 caramels and 5 other candies in it. Tonight, while it is dark, you reach into a draw and pull out one of the boxes and offer a candy to the "friend" who gave you the second box. (Exactly why it is dark is left up to your imagination.) Each of you eat one caramel. All of a sudden, you realize that you better have offer a candy from the box that your friend gave you, or you will be out one friend. (The problem of how you will argue your way out losing this friend is the question of a moral philosophy not statistics.) What is the probability that your two candies came from the second box given the two candies drawn were both caramels?