The Intraday Properties of the VIX and the VXO

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Abstract

This paper investigates daily and intraday properties of the VIX and its predecessor the VXO. Sampling data at a one-minute frequency, we document that both the VIX and VXO display a negative drift intraday. While this finding is expected in the VXO, given its constant 30-day maturity at a daily frequency, it is surprising to observe the same pattern in the VIX, which maintains the constant maturity at a one-minute frequency. In addition, we document that the VIX has a distinct intraweek pattern, declining during the week and surging over the weekend. We further observe that there is intraday and intraweek variation in the relation between the VIX and the S&P500 (the leverage/feedback effect) which appears to be most negative during the middle of the trading day. Similarly, there is a U-shape pattern in this relation during the week, with the leverage effect being most negative in the middle of the week.

Keywords: VIX; Seasonality; Intraday data.

JEL Codes: G13; G14.

1. Introduction

This paper examines the daily and intraday properties of the volatility index (VIX) with an aim to identify any deterministic and seasonal patterns. Certain observed patterns in the VIX could point towards potential anomalies and should be of interest to (VIX) derivatives traders, as persistent patterns in the VIX could affect the pricing of these derivatives. In addition, persistent patterns in the VIX may have implications for the Exchange Traded Products (ETPs) providers or VIX futures traders who need to hedge their positions by taking off-setting positions in the VIX (e.g., through options on the S&P500). Furthermore, patterns in the VIX could suggest some underlying patterns in option prices on the S&P500, which may be used to develop a trading strategy aimed at exploiting these patterns. We document strong intraweek and intraday patterns in the VIX and its negative relation with the S&P500. To our knowledge, many of these patterns have not been documented previously.

In 1993, the Chicago Board Options Exchange (CBOE) introduced the VIX, an index based on implied volatilities of options traded on the S&P100 index that are computed based on the Black and Scholes (1973) option pricing formula (see Whaley, 1993). The VIX, as a measure of forward-looking stock market volatility, rapidly became known as the 'investor fear gauge'. In 2003, the CBOE changed the computation of the VIX from a model-based approach to a model-free approach (based on the research by Demeterfi et al. (1999) and Carr and Madan (1998)) and also changed the underlying index to the S&P500 (see Whaley, 2009; and Gonzalez-Perez, 2015, for a more detailed overview of the VIX). With the introduction of the new VIX, the old VIX was renamed the VXO. In addition to these changes, the VIX is now based on a much larger universe of out-of-the-money call and put options than the original one (which was based on eight nearthe-money put and call options). Furthermore, while the VXO maintains a constant maturity of 30 days on a daily basis, the VIX maintains its constant 30-day maturity at a one minute frequency. This leads to some distinctive differences in the dynamics of both the VXO and the VIX, on a daily frequency and especially in terms of their intraday behaviour. While many studies have focused on the relation between the VIX and the S&P500, and the forecasting power of the VIX, surprisingly little research exists comparing and contrasting properties of the VIX vis-à-vis the VXO. One exception is Carr and Wu (2006) who study both indices at a daily frequency. These properties, and especially the intraday properties of the VIX, are increasingly important as the market for VIX derivatives (VIX futures and options) and VIX ETPs has grown tremendously, and trading in these products nowadays occurs at very high frequencies. This paper aims to fill this gap by investigating these intraday properties. Given that practitioners and academics still have some interest in the VXO (see some recent examples are Mijatović and Schneider, 2014; Carreiro et al., 2015; Brogaard and Detzel, 2015), we also compare the intraday properties of the VIX to those of the VXO.

We observe that there is a weekly pattern in the VIX, highest on Mondays and declining during the week. This pattern is different from the VXO, which reaches its highest level on Fridays. At an intraday level, we document that the VXO declines from the start to the close of the trading day. This finding is not surprising as the 30-day maturity of the VXO is only rebalanced daily. However, for the VIX, which is rebalanced every minute, we also document a decline throughout the trading day. Additionally, we observe that there is intraday and intraweek variation in the so-called leverage/feedback effect (the relation between VIX and the S&P500) which appears to have a U-shape during the trading day, being most negative in the middle of the trading day. Likewise, there is a U-shape in this relation during the week, with the leverage effect being weakest on Mondays and Fridays.

Although there is limited literature focusing on the intraday properties of the implied volatility indices, there are several studies that use high-frequency data on the VIX. Frijns, Tourani-Rad, and Webb (2016) examine the relation between the VIX and its futures, at a 15-second frequency, over the period 2008 to 2014.[1] Their study shows that while in the early part of their sample Granger causality was mostly from VIX to VIX futures, it reverted in the later part of the sample with Granger causality predominantly running from the VIX futures to the VIX. This observation implies that VIX futures lead the VIX in terms of revealing information about future volatility. Frijns et al. (2016) further find that the VIX futures have essentially replaced the VIX as *the investor fear gauge*; in the sense that the well-documented negative relation between implied volatility and the S&P500 is actually stronger for the VIX futures and the S&P500 than the VIX and the S&P500.

In a related study, Fernandez-Perez, Frijns, and Tourani-Rad (2016) examine the intraday reaction of the VIX and VIX futures to Federal Open Market Committee (FOMC) announcements. Their study documents that, at the time of the FOMC announcement, the VIX declines by about 3%. This decline in VIX is not instantaneous, but lasts for about 45 minutes after the announcement. The VIX futures also show a

[1]Although the VIX is rebalanced every minute in order to maintain a 30-day maturity, the value of the VIX intraday is computed every 15 seconds, thus this is the highest possible frequency one can employ.

decline after the FOMC announcement, with the front-end contract declining by, on average, 1.5%. Given the usual 8 FOMC announcement per year, the study demonstrates that a strategy that takes a short position in the front-end VIX futures contract at the start of the days with a FOMC announcement, and closes this out at the end of the day, could generate an average return of about 10% p.a. after transaction costs.

Bailey, Zheng and Zhou (2014) examine the intraday dynamics of the VIX sampling at a one-minute frequency, focusing on the drivers of the VIX and the variance risk premium with a special interest in understanding their relationships with micro- and macroeconomic factors. Their study documents that various factors, such as Eurodollar futures, gold futures and order imbalances in the S&P500 ETF can explain the dynamics of the VIX.

The few studies mentioned above all focus on whether the intraday dynamics of the VIX is related to the dynamics of derivative products, other assets or news. However, we are not aware of any study to date that has explicitly looked at the intraday properties of the VIX and compared these with the VXO. A recent study by Gonzalez-Perez and Guerrero (2013) focuses on daily properties, specifically whether there are day-of-the-week patterns in the VIX. The authors document that there is indeed a seasonal pattern during the week, where the VIX tends to be higher on Mondays than on other days (a finding which we corroborate). However, Gonzalez-Perez and Guerrero (2013) do not examine the properties of the VIX at the intraday level.

The remainder of this paper is structured as follows. In the next section, we discuss the data employed in this study. Section 3 presents the results of our analysis, which includes the documentation of several intraday properties of the VIX vis-à-vis the VXO and the relation between the implied volatility and the S&P500. Section 4 concludes.

2. Data

This study employs intraday data on the VIX, VXO, and the S&P500 index. The CBOE calculates the VIX every 15 seconds, similar to the S&P500 index. The VXO is also computed intraday, but is computed only at a one minute frequency. The constant 30-day maturity of the VIX is maintained at a one minute frequency, i.e. every minute the weight on the options with maturity greater than 30 calendar days increases, while the weight on the options with maturity less than 30 days decreases. For this reason, and to match the sampling frequency with that of the VXO, we also sample the VIX and the S&P500 at a one-minute frequency. We obtain data for the VIX, VXO and S&P500 at one minute frequency from the Thomson Reuters Tick History (TRTH) database maintained by SIRCA (Securities Industry Research Centre of Asia-Pacific) covering the period from September 22, 2003 (which marks the introduction of the current VIX) to December 31, 2013. The calculation of the VIX is from 9:30 until 16:15 Eastern Standard Time (EST), the calculation of the VXO starts at 9:45 and closes at 16:15[2] EST, and finally, the S&P500 is computed from 9:30 until 16:00 EST.

INSERT FIGURE 1 HERE

[2]VXO closed calculations at 16:00 EST before March 2009.

In Figure 1, we plot the daily VIX and S&P500 prices over the sample period (the VXO is omitted because it overlaps with VIX at the daily frequency). We can clearly observe the well-known negative relation between VIX levels and S&P500 prices, where the VIX increases at times when the underlying decreases, and vice versa (see, e.g., Fleming et al., 1995; and Whaley, 2000).

INSERT TABLE 1 HERE

In Table 1, we report daily (close-to-close) summary statistics for the VIX, VXO and the S&P500 index. We observe some well-known documented properties of the VIX, which in levels (Panel A) has an average value of about 20, displays positive skewness and is very persistent as can be seen from the first order autocorrelation. The VXO has very similar characteristics to the VIX. As expected, its average value is similar to that of the VIX at about 20, but this series seems to have a slightly higher standard deviation than the VIX. This higher volatility in the VXO may be due to the options being based on a smaller index and/or the fact that the VXO is based on only 8 options, and thus may be affected more by microstructure noise than the options used in the calculation of the VIX. Considering returns of the series (Panel B), we observe that both VIX and VXO have a negative drift. This is not a common feature of these volatility indices, but is simply a consequence of the index being higher at the start of our sample period and lower at the end, as shown in Figure 1. We also note that the VIX and VXO have similar degrees of skewness and kurtosis. Lastly, Panel C reports the correlations between the series, where we observe the VIX and VXO are highly correlated at the daily frequency, with a correlation of about 0.93. This high correlation is further confirmed when we consider the relation between each implied volatility and the S&P500. We note that the close-to-close correlation between the VIX and the S&P500 and the close-to-close correlation between the VXO and the S&P500 are the same at -0.75.

3. Results

3.1 Descriptive statistics

We first report the intraday patterns in the VIX and VXO (and S&P500 as the main US equity index and underlying of VIX).[3] Figure 2 shows the intraday (open-to-close) cumulative changes in the log value of the VIX, VXO and the S&P500. There is a pronounced decline in the VIX intraday that is most prominent in the first 30 minutes of the trading day, but generally persists over most of the trading day. On average, the decline in the VIX is around -0.80% per day from open to close. Theoretically, the VIX should not decline over the trading day, as the constant maturity of 30 days is maintained throughout the day by adjusting the weights on the nearby and second-nearby options on a one-minute frequency. The decline in the first 30 minutes of the trading day may be related to the resolution of uncertainty that takes place at the start of the trading day (see, e.g., Fernandez-Perez et al., 2016). We also observe a somewhat accelerated decline in VIX towards the end of the trading day. This feature was originally observed by Fleming et al. (1995), and could be related to option traders

[3] To control for the impact of outliers, we delete those minute returns whose value was 7 times higher than the intraday standard deviation.

marking down option prices towards the end of the day, as they have no control over the time value decay that occurs overnight.

Figure 2 also shows the decline in the VXO during the trading day which declines more than the VIX. On average, this decline in the VXO is around -1.36% per day from open to close. This decline is expected as the VXO is rebalanced on a daily basis and thus as time to maturity decreases during the day (see Fleming et al., 1995). While there is an accelerated decline towards the end of the day, we do not observe such a decline at the start of the trading day, as was the case for the VIX. Finally, for the S&P500 index, we do not observe any discernible intraday pattern.

INSERT FIGURE 2 HERE

In Table 2, we report some of the intraday and day-of-the-week properties of the three indices. The mean levels of the VIX, VXO and the S&P500 are given in the first row of each Panel. Panel A confirms the decline in the VIX during the trading day, which is most pronounced during the first hour of its trading. In Panel B, we observe that the VXO declines systematically over the trading day, in line with the observation in Figure 2. The decay is almost linear over the trading day, with an increased decay towards the end of the trading day which has been observed by Fleming et al. (1995), and is due to traders marking down option prices. Finally, for the S&P500 in Panel C, we do not observe any significant intraday pattern, in line with Figure 2.

INSERT TABLE 2 HERE

The last columns of Panels A through C report the average levels of the various indices s for different days of the week. Panel A reveals a day-of-the-week effect in the VIX, where the VIX tends to be highest on Monday, declines during the week up to Friday, where it increases slightly. This weekly pattern is similar to that observed in the VXO, which also declines during the week, but increases slightly on Thursdays and Fridays. The existence of an intraweek pattern in the VXO has also been documented by Fleming et al. (1995), who show that the VXO declines in the intraweek. Likewise, Gonzalez-Perez and Guerrero (2013) document an inverted Monday effect for VIX where it is generally higher on Mondays than previous Fridays and has a U-shaped weekly pattern. We finally note that there is also an intraweek pattern in the S&P500, having a pronounced U-shaped pattern, where the negative Monday or Weekend effect (see, e.g., French, 1980; Gibbons and Hess, 1981; Keim and Stambaugh, 1984; and Rogalski, 1984, among others) and Tuesday effect are very pronounced (Aggarwal and Rivoli, 1989).[4]

Table 2 also reports the summary statistics for the returns (i.e. log changes) in the VIX, VXO, and S&P500. Panel A shows the log changes in the VIX are lowest in the first hour of the trading day (-0.0061% per minute) and the second lowest at the last 45 minutes of the trading day (-0.0025% per minute). Intraday volatility has a pronounced U-shape, the beginning and close of the trading day are characterized by a high volatility, mainly in the first hour of the trading day (0.3329% per minute). Intraday

[4]We should also highlight the work of Connolly (1989, 1991) who argues that many of the previous studies which report anomalies fail to adjust for the power of the test in large samples and use standard significance levels.

skewness decreases during the day, while kurtosis is constant except for the first trading hour. Finally, we observe that autocorrelations of the returns are close to zero during the trading day.

We complement the results of Gonzalez-Perez and Guerrero (2013), who find an inverted Monday effect in VIX index, with the intraday (open-to-close) log changes for every day of the week in the far right columns of Panel A. On average, the intraday log changes in VIX are negative for all weekdays, strongest on Mondays and Tuesdays. Table 2 sheds light on the positive return finding by Gonzalez-Perez and Guerrero (2013), which, as our results clearly show is mostly due to a weekend effect, given that the intraday log changes on Mondays in VIX are the most negative during the whole week. Intraday volatility is almost constant throughout the week with a slight increase only on Fridays. When considering skewness and kurtosis during the week, we note that Mondays and Fridays present the most extreme cases, with Monday displaying negative skewness and high kurtosis in VIX changes and Friday displaying positive skewness and high kurtosis.

Panel B reports the results for the VXO. In line with Figure 2, the intraday decline in the log changes in VXO is stronger than in VIX and relatively constant at about -0.0043% per minute, although this is less pronounced between 12:30 and 14:30. Otherwise, the intraday volatility of the VXO shows a U-shape, and is about twice the volatility of the VIX in every hour of the day. This finding could be due to the composition of both volatility indices, the VIX being more diversified than the VXO. The intraday skewness seems to be more negative in the VXO but the kurtosis shows a similar pattern to that of VIX (except that the spike in kurtosis is observed in the 10:30-11:30 window, whereas for the VIX it is in the opening hour). Interestingly, there is strong intraday autocorrelation in the log changes of VXO of about -20% which could be due to market microstructure noise, such as the bid-ask bounce. The intraday (open-to-close) statistics for every day of the week are reported in the last columns of Panel B. Similar to the VIX, there is a daily drop in the VXO. However, this drop is slightly stronger on Mondays and Tuesdays. The volatility in VXO changes seems to be constant during the week. In addition, there is no clear pattern in the skewness and kurtosis which seems constant on every day but Tuesdays, where kurtosis is the highest. Finally, the autocorrelation is significant and constant at about -20% in every day of the week.

The intraday statistics for the log returns of S&P500 are reported in the Panel C. As expected, the intraday returns are close to zero throughout the day, and there is the well-known U-shape in the intraday volatility. Intraday skewness and kurtosis also display interesting patterns, and are relatively low at the start of the day and increase towards the end of the trading day. Finally, autocorrelations in the returns are positive throughout the day about 5%, but in the last 45-minutes of the trading day it jumps to 10%. In the last column of Panel C, we report the intraday (open-to-close) statistics for every day of the week, where we observe no clear differences in the intraday returns of S&P500 for any day of the week.

In the Panel D, the intraday correlations between the log changes in the VIX and the returns in the S&P500 at one-minute frequency are reported.[5,6] First, we observe that the

[5]TRTH provides the VXO data at one-minute frequency with 4 seconds of difference with the clock used for VIX and S&P500. Due to this mismatch, the correlations between VXO and VIX (or S&P500) at 1-minute frequency are not statistically significant. Therefore, we decided to skip the correlations with VXO at this high frequency.

well-known negative correlation between VIX and S&P500 is also present at the one-minute frequency.[7] Second, we note that the intraday correlation is broadly constant at about -53%, but it is less negative during the first and last hour of the trading day. In unreported results, we obtain the intraday correlations year by year and find that the same U-shape in the intraday correlations is persistent over years. Regarding the weekly pattern, the intraday correlations show also a U-shape, decreasing during the week with a sudden increase on Fridays and reaching a similar magnitude on Mondays.

To visualize the intraday patterns in VIX and VXO over the week, we plot the cumulative log changes for the whole week, overnight and weekend log changes.

INSERT FIGURES 3 AND 4 HERE

In Figure 3, we plot the cumulative (average) log changes in the VIX for the whole week. There is a clear pattern in the weekly evolution of the log changes in VIX. The VIX drops, on average, by 3% per week and then jumps on the weekend to reach a value of zero.[8] The biggest drop is on Mondays, Tuesdays and also on Thursday's

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[6]We match the trading sessions of both VIX and S&P500 for calculating their correlations so these are from 9:30 until 16:00 EST.

[7]This negative relation has already been documented in the literature and has generally two main explanations. The first one is the so-called leverage effect (e.g. Black, 1976; and Christie, 1982) which postulates that the negative relation exists because a decline in stock prices results in an increase in the leverage of the firm, which makes the firm more risky and thus volatility increases. The second explanation is related to volatility feedback effect (French et al., 1987; Campbell and Hentschel, 1992; and Bekaert and Wu, 2000), which postulates that the negative relation exists because as volatility increases investors require higher expected returns, and thus stock prices must decline to offer these higher expected returns.

[8]The cumulative log changes at the end of the weekend is not exactly zero, which is due to the slightly negative trend in the VIX over the sample period as documented in Table 1 and Figure 1.

overnight (Thursday close to Friday open) which coincides with rollover day in the VIX options portfolio.[9] Figure 4 shows a completely different pattern in the weekly log cumulative changes in VXO. As expected, the daily rebalancing to a fixed 30-calendar day has a clear effect on the weekly evolution of the VXO, where it declines intraday but jumps at overnight/weekend and finishes the week at zero.

3.2 Regression Analysis

In this section, we conduct several regressions to estimate the intraday pattern in implied volatility as well as the lagged, contemporaneous and future relations with the S&P500 index. To do so, we split each day into 8 periods and calculate the log changes in VIX (VXO) and S&P500 for each period: I) 9:30-10:30, II) 10:30-11:30, III) 11:30-12:30, IV) 12:30-13:30, V) 13:30-14:30, VI) 14:30-15:30, VII) 15:30-16:15 and VIII) overnight (from 16:15 until the next trading day open at 9:30).

We run the following three regressions:

$$\Delta\log(VIX_t) = \alpha + \beta R_t + \beta_{lev}|R_t| + \varepsilon_t^1 \tag{1}$$

$$\Delta\log(VIX_t) = \alpha + \beta_{-8}R_{t-8:t-1} + \beta R_t + \beta_{lev}|R_t| + \beta_{+8}R_{t+1:t+8} + \varepsilon_t^2 \tag{2}$$

$$\Delta\log(VIX_t) = \alpha + \beta_{-8}R_{t-8:t-1} + \beta R_t + \beta_{lev}|R_t| + \beta_{+8}R_{t+1:t+8} + \sum_{i=1}^{12}\theta_i D_{i,t} + \varepsilon_t^3 \tag{3}$$

[9]CBOE rolls over into second and third maturity options to compute the VIX when the first maturity is below eight calendar days, to reduce the impact of market microstructure noise in the volatility index.

where $\Delta\log(VIX_t)$ is the hourly log changes in the VIX (or VXO), R_t is the hourly return in the S&P500, $R_{t-8:t-1}$ ($R_{t+1:t+8}$) is the cumulative returns in S&P500 over the previous (next) 24 hours, and finally, $D_{i,t}$ are time dummy variables equal 1: 1) 9:30-10:30, 2) 10:30-11:30, 3) 11:30-12:30, 4) 12:30-13:30, 5) 13:30-14:30, 6) 14:30-15:30, 7) 15:30-16:15, 8) Monday, 9) Tuesday, 10) Wednesday, 11) Thursday and 12) Roll days, and zero otherwise. These dummies control for all the intraday and day of the week patterns that we observed in the previous sections. The parameters of interest are β_{-8} (β_{+8}) which measure the lag (lead) effect of the stock return on VIX; β which measures the impact of contemporaneous changes in stock return on VIX; β_{lev} which measures the impact of contemporaneous changes in stock return on VIX, independently of their direction, so $\beta + \beta_{lev}$ measures the asymmetric or leverage effect, and finally, θ_i measure all the intraday and day of the week effects.

INSERT TABLE 3 HERE

In the Table 3, we report the results of the regressions for log changes in VIX (Panel A) and log changes in VXO (Panel B). In line with the literature, we observe that β is significantly negative and β_{lev} is significantly positive. This finding confirms the negative relation between the VIX and the S&P500 index at the intraday level, and also provides evidence on the leverage/feedback effect, i.e. the VIX increases more when the S&P500 declines, at the intraday level. Adding the lead/lag stock returns in Model 2, we observe that only the lagged effects are significant and positive. This finding is in line with Fleming et al. (1995) and is due to the mean reverting property of VIX (VXO), i.e.

the strong negative contemporaneous relation between stock returns and VIX induces a slightly positive association between log changes in VIX and past stock returns.

In Model 3, we add the dummy variables. Unlike Table 2, the regression approach is appealing because it allows us to uncover each intraday and day of the week effect after controlling for all the other time effects and co-movements with the underlying. We find several interesting results. First, adding the dummy variables reduces the asymmetric or leverage effect in both VIX and VXO by 26% and 31%, respectively, compared to Model 2. This result may suggest that part of the asymmetric effect observed in Models 1 and 2 are actually deterministic and induced by the intraday and day-of-the-week effects. Second, the lagged effects of stock returns are reduced by about 3.5% in both VIX and VXO. Third, the intraday effects on VXO are stronger than VIX, but the day-of-the-week effect is stronger in the VIX than in the VXO. The intraday effect is expected as the VXO is rebalanced daily. Interestingly, the day of the week effect in VXO disappears once the intraday effects are taken into account (only the Wednesday dummy is statistically significant). With regards to the intraday pattern, we again observe the inverse U-shape pattern with a strong drop in both VIX and VXO at the beginning of the trading day which is lessened throughout the day, but they drop strongly again at the end of the trading day. This intraday inverse U-shape pattern is exacerbated on Mondays and lessened along the week in VIX. Fourth, the roll day has an important impact on both VIX and VXO, leading to an additional decline in both indexes. Fifth, the weekend effect (captured by the constant in Model 3) is positive and highly significant. Finally, the total variability in the log changes in VIX and VXO explained by the time dummy variables, measured by the increase in the adj-R^2 with regard to Models 1 and 2, is as high as 2% and 4%, respectively.[10]

The results presented in Table 3 assume that the observed patterns in VIX and VXO remain constant over time. However, many changes that have occurred during our sample period may have affected the dynamics of these volatility indices, e.g. the introduction of VIX futures (2004), VIX options (2006) and VIX ETPs (2009), as well as important market events, such as the Global Financial Crisis (2007-2009) and the European Debt crisis (2010). All these events may have influenced the intraday pattern in the VIX and VXO. Therefore, we perform a regression analysis per year and report the results in Table 4.

INSERT TABLE 4 HERE

The results in Table 4 show how the intraday and day of the week patterns observed in Table 3 are relatively stable over time. Namely, there is an inverse U-shape in the intraday and day of the week pattern in VIX, though this is more pronounced in certain years. The drop in the intraday of VXO is stronger than VIX but VXO does not show any clear day of the week effect. The roll day effect is important in most of the years. Otherwise, this yearly analysis puts forward a change in the asymmetric or leverage

[10]Additionally, we have conducted several Wald tests with Newey-West h.a.c. standard errors whose null hypothesis are $H_0^1: \theta_1 = \theta_2 = \cdots = \theta_{12} = 0$; $H_0^2: \theta_1 = \theta_2 = \cdots = \theta_7 = 0$; and $H_0^3: \theta_8 = \theta_9 = \theta_{10} = \theta_{11} = 0$ for both VIX and VXO. In all the cases, the null hypothesis is rejected at the 1% level, showing the intraday calendar effects are statistically significant. In order to test if the day of the week effect in VXO, once we drop the significant Wednesdays dummy, is statistically significant, we have run the following Wald test for VXO, $H_0^4: \theta_8 = \theta_9 = \theta_{11} = 0$, which does not reject the null hypothesis.

effect which started to be stronger from 2009 onwards (coincidently this is the year when the first VIX ETPs were introduced)[11]. This may be due to the strong impact of investors in volatility products that have exacerbated this leverage/feedback effects between VIX and its underlying (see, e.g., Fernandez-Perez, Frijns, Tourani-Rad and Webb (2016), who find that the properties of price process in VIX futures have changed since the introduction of the VIX ETPs in 2009). However, studying which factors explain these changes in the leverage/feedback effects is beyond the scope of this paper so we leave this question open as an interesting topic for further research.

4. Conclusion

The aim of this paper is to provide an overview of the intraday properties of the VIX vis-à-vis the VXO. Sampling data at a one-minute frequency, we document that both VIX and VXO display a negative drift intraday. While this finding is expected in the VXO (as it maintains its constant 30-day maturity at a daily frequency), it is not expected in the VIX, which maintains the constant maturity at a one-minute frequency. In addition, we document that the VIX has a distinct intraweek pattern, declining during the week and having a jump over the weekend. We further observe that there is intraday and intraweek variation in the leverage/feedback effects which appears to be most negative during the middle of the trading day. Similarly, there is a U-shape pattern in this relation during the week, with the leverage effect being weakest on Mondays and Fridays.

[11]The VIX ETP market is large with over $US 2 billion invested in these ETPs. The market for VIX futures is large as well, with over 300,000 contracts traded per day in June of 2016. In fact, VIX derivatives are the second-largest contributor to the profits of the Chicago Board Options Exchange, after options on the S&P500 stock index.

Overall, the findings we document are relevant to traders in VIX derivatives, as patterns in the VIX could affect the pricing of these derivatives due to arbitrage links between their prices. In addition, the observed patterns in the VIX may have implications for ETP providers or VIX futures providers who need to hedge their positions by taking off-setting positions in VIX (options on the S&P500). Finally, the patterns observed in the VIX could suggest some underlying patterns in option prices on the S&P500, which may be used to develop a trading strategy aimed at exploiting these patterns.

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Figure 1: Daily VIX (y-axis) and S&P500 (opposite y-axis) from 22/September/2003 to 31/December/2013.

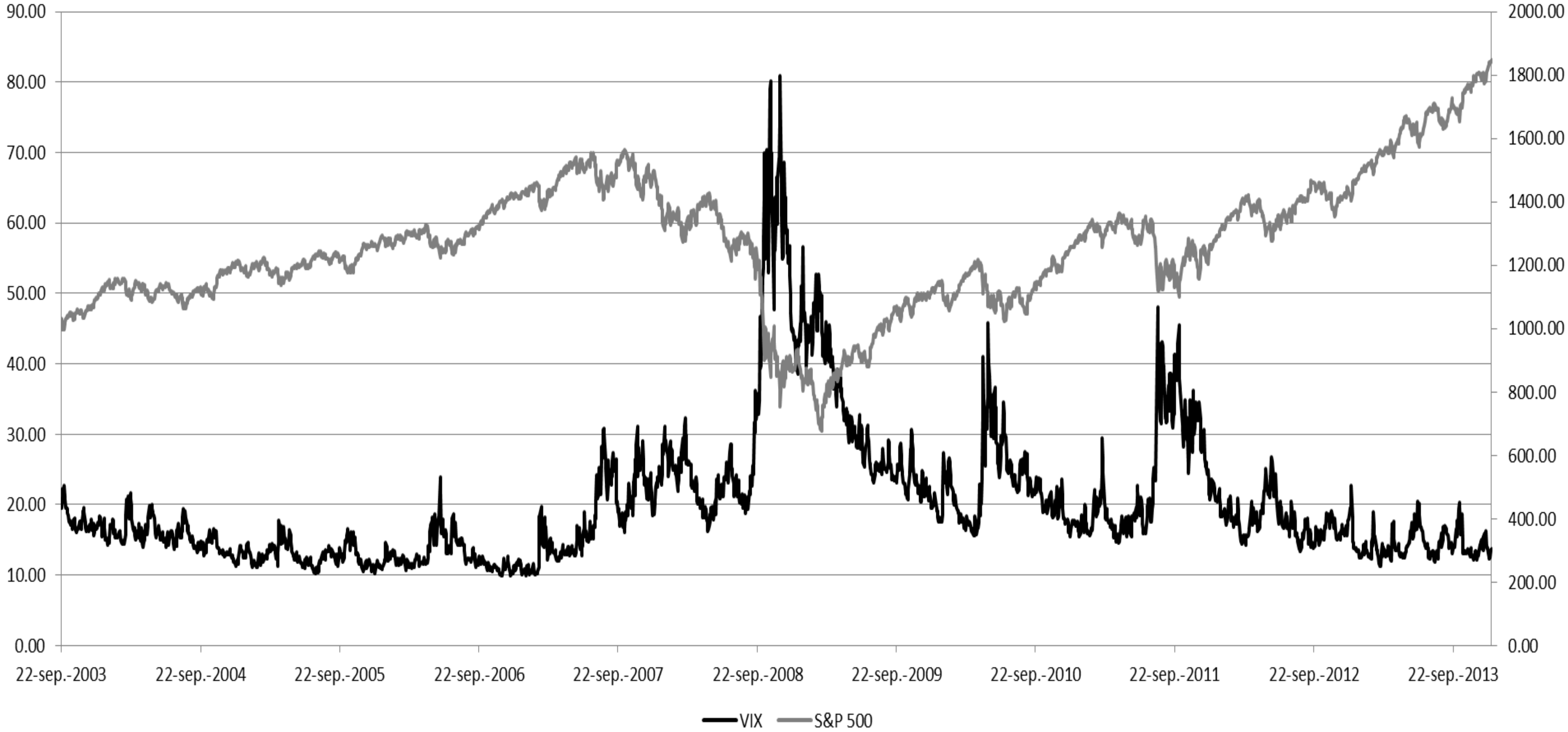

Figure 2: Cumulative log changes in VIX, VXO and cumulative returns in S&P500 from 9:30 to 16:15 EST, from 22/September/2003 to 31/December/2013.

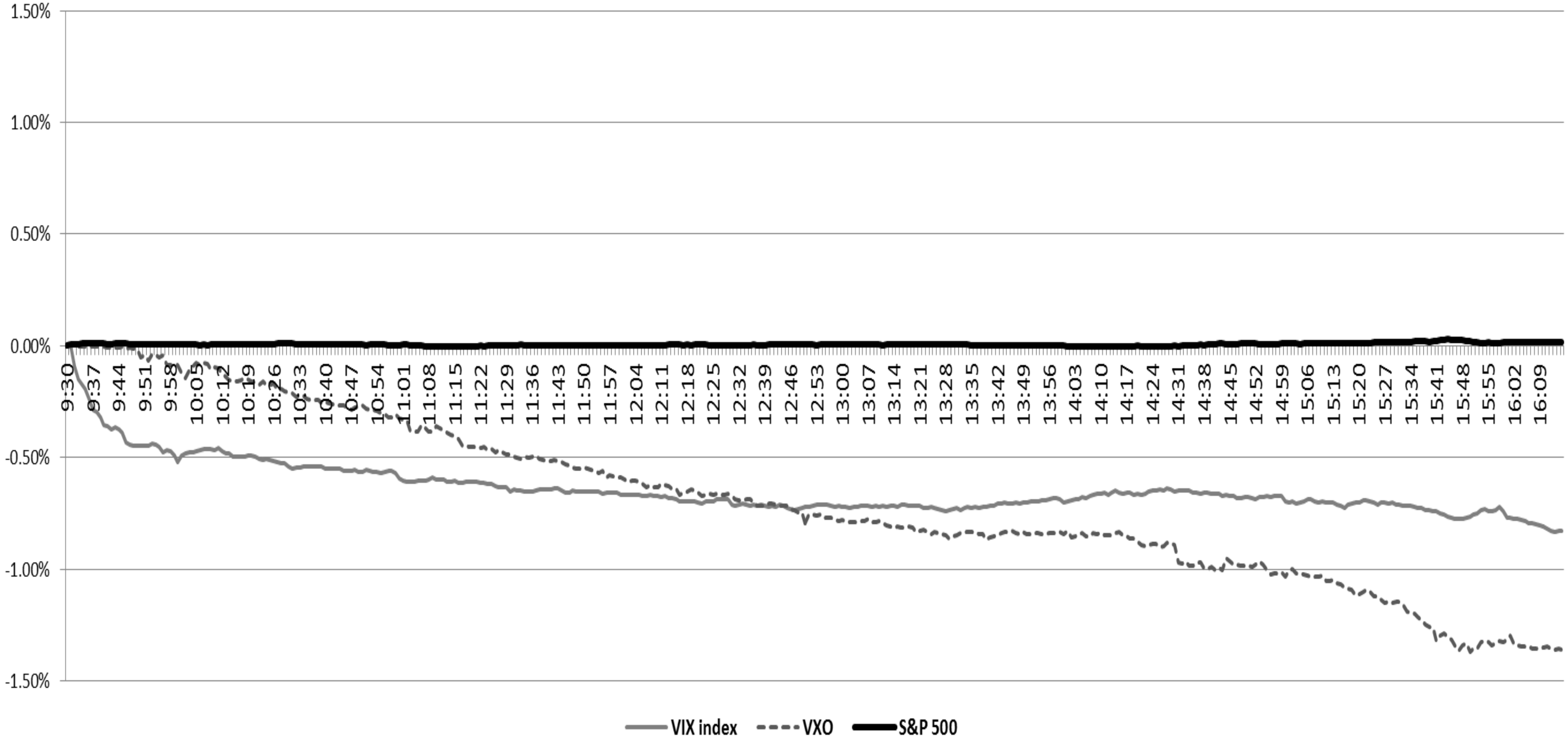

Figure 3: Weekly cumulative log changes in VIX index

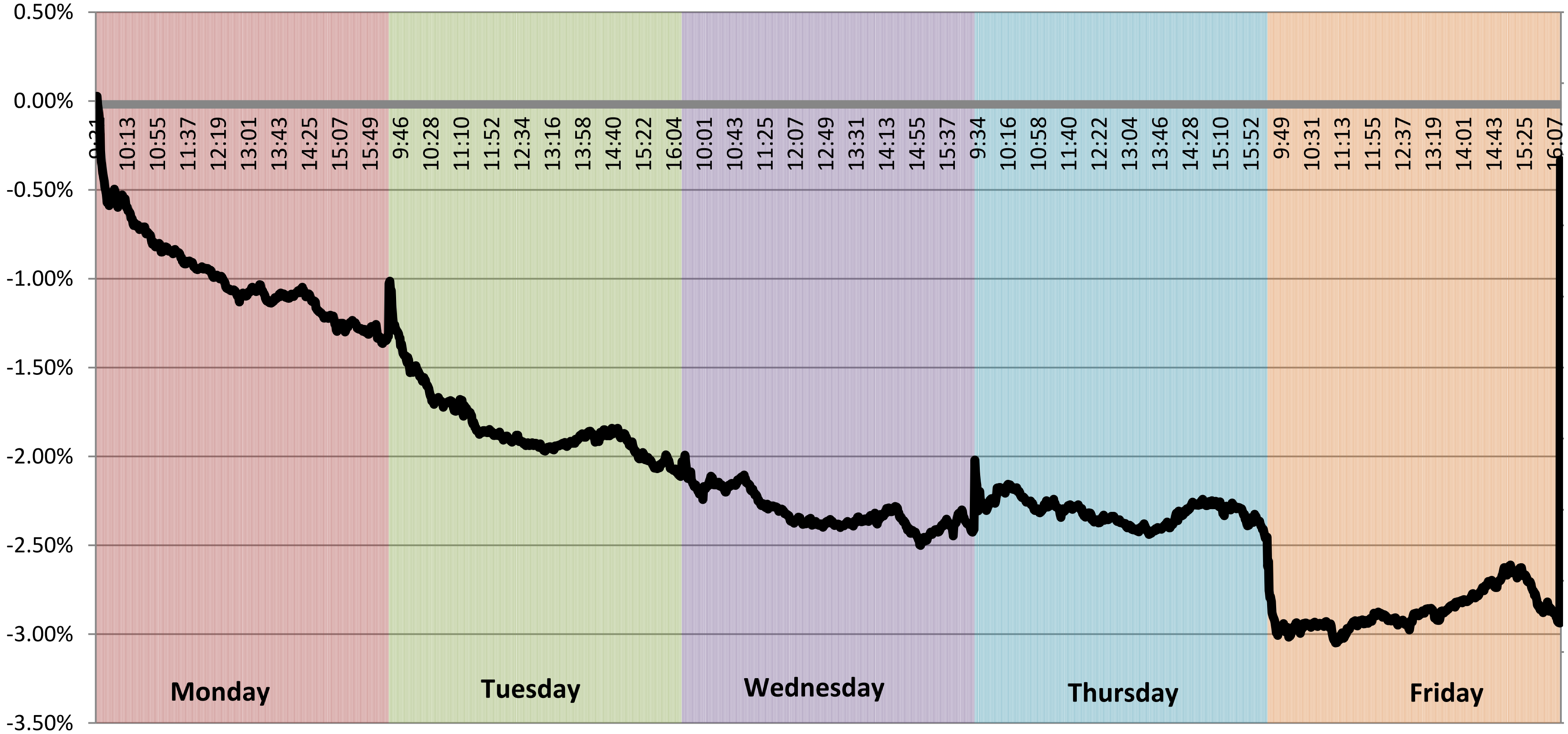

Figure 4: Weekly cumulative log changes in VXO index

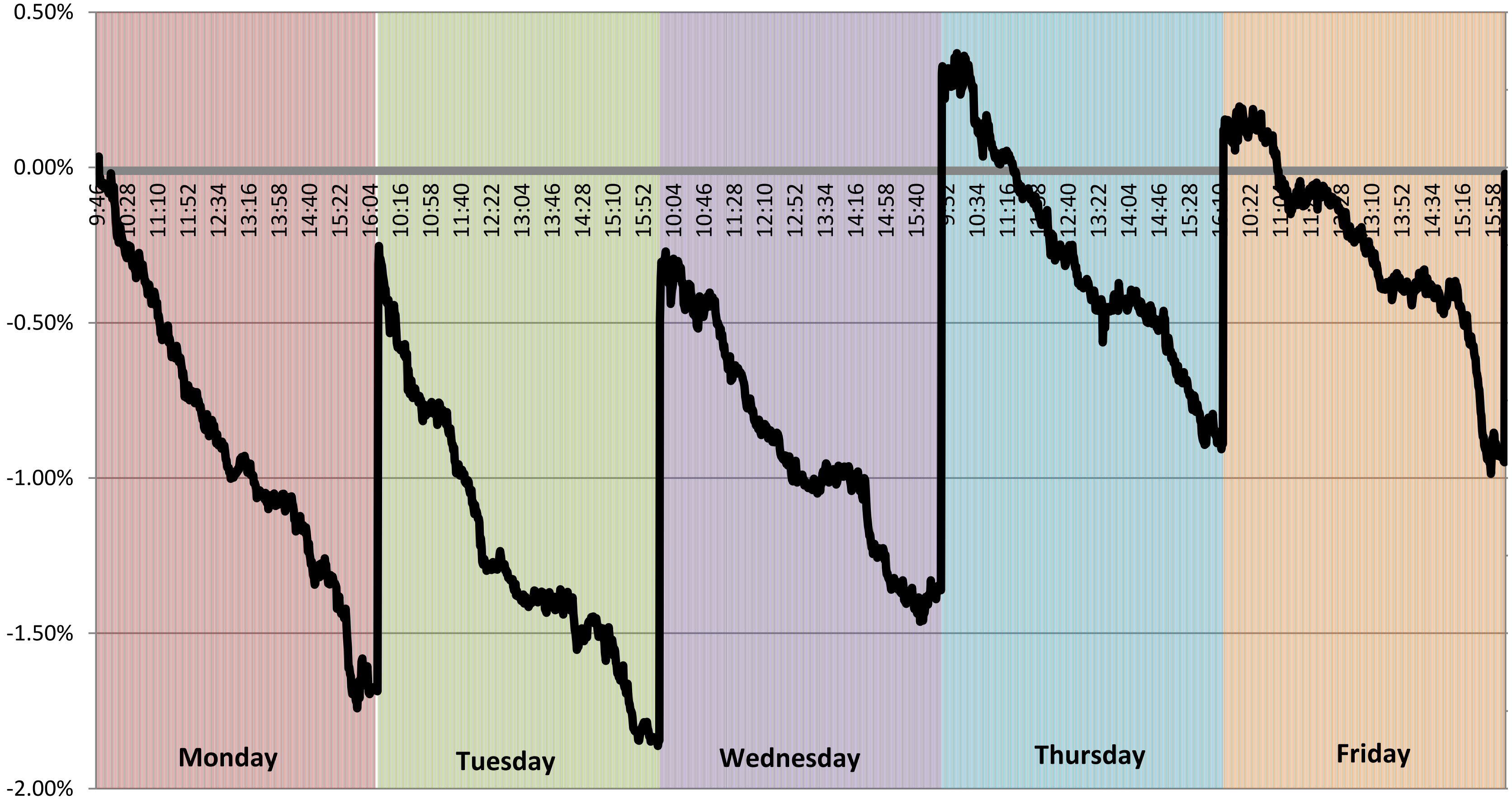

Table 1: Daily (close-to-close) summary statistics.

	VIX	**VXO**	**S&P500**
Panel A: Levels			
Mean	20.11	19.88	1266.35
Std	9.78	10.35	205.02
Skewness	2.37	2.48	0.13
Kurtosis	10.26	11.08	3.18
ρ(1)	0.98***	0.98***	1.00***
ADF	-1.334	-1.372	1.162
JB test	8095.58***	9688.50***	10.84***
Panel B: Returns			
Mean (annual)	-3.50%	-5.89%	5.77%
Std (annual)	106.03%	118.70%	20.04%
Skewness	0.67	0.56	-0.29
Kurtosis	7.41	7.14	13.72
ρ(1)	-0.13***	-0.16***	-0.11***
ADF	-17.718***	-18.156***	-40.416***
JB test	2292.16***	1982.40***	12401.62***
Panel C: Correlations			
	VIX	VXO	S&P500
VIX	1.00		
VXO	0.93***	1.00	
S&P500	-0.75***	-0.75***	1.00

Table 2: Statistics for the level and log changes in VIX (Panel A), VXO (Panel B), returns in S&P500 (Panel C) and cross correlation between VIX and S&P500 (Panel D) for each hour from 9:30 to market closes, and day of the week,, from 22/September/2003 to 31/December/2013. I) 9:30-10:30, II) 10:30-11:30, III) 11:30-12:30, IV) 12:30-13:30, V) 13:30-14:30, VI) 14:30-15:30, VII) 15:30-16:15

	I	II	III	IV	V	VI	VII	Monday	Tuesday	Wednesday	Thursday	Friday
Panel A: VIX Index [9:30 to 16:15]												
Mean (level)	20.20	20.15	20.14	20.13	20.14	20.14	20.13	20.45	20.19	20.06	20.01	20.11
Mean	-0.0061%	-0.0019%	-0.0013%	-0.0003%	0.0014%	-0.0007%	-0.0025%	-0.0035%	-0.0023%	-0.0010%	-0.0006%	-0.0005%
Std	0.3329%	0.2005%	0.1631%	0.1532%	0.1772%	0.1979%	0.2072%	0.1992%	0.2077%	0.2159%	0.2104%	0.2230%
Skewness	0.5565	0.0965	0.0970	0.1425	0.0662	-0.0061	0.0110	-0.2220	0.0498	0.0098	0.0659	0.1122
Kurtosis	58.26	14.20	12.60	14.64	14.73	15.67	14.84	30.80	25.92	26.86	16.39	34.22
ρ(1)	-0.03**	0.02**	0.01*	0.00	0.02**	0.02**	0.04**	-0.01**	0.00	0.00	0.02**	0.00
Panel B: VXO Index [9:30 to 16:00]												
Mean (level)	20.22	20.07	20.03	20.00	19.99	19.95	19.90	20.09	19.89	19.94	20.00	20.29
Mean	-0.0048%	-0.0043%	-0.0034%	-0.0028%	-0.0006%	-0.0043%	-0.0043%	-0.0044%	-0.0042%	-0.0031%	-0.0028%	-0.0033%
Std	0.5350%	0.4432%	0.3796%	0.3618%	0.3971%	0.4275%	0.4156%	0.4194%	0.4268%	0.4340%	0.4260%	0.4241%
Skewness	-0.0404	-0.0873	-0.0437	-0.0425	-0.0186	-0.1094	0.0431	-0.0201	-0.0245	-0.1083	-0.0726	0.0186
Kurtosis	14.18	57.91	9.58	9.92	9.41	10.79	17.08	12.24	21.72	11.49	10.88	9.22
ρ(1)	-0.19***	-0.20***	-0.20***	-0.19***	-0.19***	-0.19***	-0.17***	-0.20***	-0.19***	-0.20***	-0.19***	-0.19***
Panel C: S&P500 [9:30 to 16:00]												
Mean (level)	1266.38	1266.26	1266.28	1266.20	1265.74	1266.10	1266.31	1266.23	1264.47	1264.72	1267.79	1268.59
Mean	0.0001%	-0.0001%	0.0000%	0.0000%	-0.0002%	0.0002%	-0.0007%	-0.0001%	0.0001%	0.0000%	0.0000%	0.0000%
Std	0.0579%	0.0430%	0.0359%	0.0345%	0.0407%	0.0478%	0.0510%	0.0409%	0.0439%	0.0448%	0.0464%	0.0463%
Skewness	-0.0076	-0.0068	-0.0367	-0.0243	-0.0730	0.0682	0.0962	0.0052	0.0528	-0.0066	-0.0403	0.0559
Kurtosis	11.54	11.83	13.45	14.39	14.78	15.62	16.16	13.12	13.67	13.25	14.58	15.42
ρ(1)	0.04**	0.05**	0.06**	0.06**	0.05**	0.03**	0.10**	0.04**	0.05**	0.04**	0.05**	0.05**
Panel D: Corr(Δlog(VIX), S&P500) [9:30 to 16:00]												
Correlation	-0.43***	-0.52***	-0.52***	-0.52***	-0.55***	-0.55***	-0.50***	-0.48***	-0.50***	-0.51***	-0.52***	-0.49***

Table 3: OLS regressions where the dependent variable is the log changes in VIX (Panel A) and VXO (Panel B), Rt is the returns of S&P500, R(t-8:t-1) (R(t+1:t+8)) are the cumulative returns in S&P500 over the previous (next) 24 hours, and the rest are dummy variables for each hour (e.g. 9:30-10:30 = dummy equal to 1 in 9:30-10:30, zero otherwise), day of the week (e.g. Monday = dummy equal to 1 on Mondays, zero otherwise) and roll day (Roll day = dummy equal to 1 on overnight period from open of third Friday of the month to previous close); constant represents the weekend returns (i.e. all hours equal to zero means overnight, all days equal to zero means Friday and no roll day). t-test adjusted by Newey-West in the parenthesis. In bold the significant t-statistics at the 1% level or better.

	Panel A: VIX						**Panel B: VXO**					
	Model 1		**Model 2**		**Model 3**		**Model 1**		**Model 2**		**Model 3**	
Constant	**-0.0007**	(-3.22)	**-0.0008**	(-3.80)	**0.0090**	(13.91)	**-0.0008**	(-3.48)	**-0.0009**	(-4.33)	**0.0130**	(18.38)
R(t-8:t-1)			**0.0824**	(6.21)	**0.0795**	(6.08)			**0.1248**	(8.77)	**0.1202**	(8.51)
Rt	**-4.3754**	(-33.16)	**-4.3790**	(-33.18)	**-4.3792**	(-33.23)	**-4.7133**	(-35.73)	**-4.7237**	(-35.87)	**-4.7235**	(-35.74)
|Rt|	**0.2990**	(3.06)	**0.3403**	(3.55)	**0.2514**	(2.62)	**0.3489**	(3.39)	**0.4138**	(4.12)	**0.2847**	(2.81)
R(t+1:t+8)			-0.0027	(-0.19)	-0.0025	(-0.18)			-0.0216	(-1.33)	-0.0215	(-1.34)
9:30-10:30					**-0.0098**	(-12.20)					**-0.0159**	(-18.10)
10:30-11:30					**-0.0079**	(-13.00)					**-0.0170**	(-23.99)
11:30-12:30					**-0.0071**	(-11.97)					**-0.0161**	(-23.39)
12:30-13:30					**-0.0064**	(-11.13)					**-0.0156**	(-22.93)
13:30-14:30					**-0.0060**	(-10.00)					**-0.0147**	(-20.78)
14:30-15:30					**-0.0066**	(-10.75)					**-0.0161**	(-21.95)
15:30-16:15					**-0.0077**	(-12.01)					**-0.0162**	(-21.88)
Monday					**-0.0047**	(-11.74)					-0.0003	(-0.76)
Tuesday					**-0.0042**	(-11.30)					0.0005	(1.46)
Wednesday					**-0.0029**	(-7.71)					**0.0011**	(2.86)
Thursday					**-0.0037**	(-9.30)					0.0006	(1.45)
Roll day					**-0.0110**	(-4.31)					**-0.0145**	(-5.56)
adj-R^2	49.47%		49.63%		51.30%		43.31%		43.62%		47.45%	

Table 4: Regressions for VIX (Panel A) and VXO (Panel B) per year. t-test adjusted by Newey-West in the parenthesis. In bold the significant t-statistics at the 1% level or better.

	Panel A: VIX										
	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Constant	**0.0132**	**0.0104**	**0.0069**	**0.0121**	**0.0119**	**0.0117**	**0.0059**	**0.0087**	**0.0046**	**0.0050**	**0.0088**
R(t-8:t-1)	0.0245	0.0569	0.0074	0.0165	**0.1524**	0.0142	**0.0556**	**0.1231**	**0.1417**	**0.1574**	**0.1954**
Rt	**-2.6655**	**-4.9688**	**-6.1046**	**-7.3948**	**-7.1226**	**-2.6872**	**-2.8646**	**-6.0445**	**-5.6135**	**-6.4952**	**-7.9834**
|*Rt*|	0.1082	**0.4782**	**0.5305**	0.0872	0.3058	0.0065	**0.3923**	**0.5567**	**0.7549**	**1.0925**	0.5378
R(t+1:t+8)	-0.0026	-0.0211	-0.0374	**-0.0977**	-0.0397	-0.0088	0.0059	-0.0305	0.0313	-0.0225	-0.1012
9:30-10:30	**-0.0152**	**-0.0125**	**-0.0090**	**-0.0125**	**-0.0110**	**-0.0139**	**-0.0078**	**-0.0114**	**-0.0092**	**-0.0042**	**-0.0067**
10:30-11:30	**-0.0144**	**-0.0102**	**-0.0056**	**-0.0106**	**-0.0099**	**-0.0099**	**-0.0054**	**-0.0091**	**-0.0031**	**-0.0054**	**-0.0083**
11:30-12:30	**-0.0136**	**-0.0091**	**-0.0057**	**-0.0094**	**-0.0082**	**-0.0075**	**-0.0050**	**-0.0095**	**-0.0039**	**-0.0040**	**-0.0073**
12:30-13:30	**-0.0139**	**-0.0081**	**-0.0048**	**-0.0084**	**-0.0074**	**-0.0067**	**-0.0052**	**-0.0071**	-0.0028	**-0.0036**	**-0.0077**
13:30-14:30	**-0.0139**	**-0.0078**	**-0.0041**	**-0.0102**	**-0.0069**	**-0.0063**	**-0.0047**	**-0.0069**	**-0.0030**	-0.0028	**-0.0061**
14:30-15:30	**-0.0161**	**-0.0090**	**-0.0056**	**-0.0086**	**-0.0072**	**-0.0087**	**-0.0044**	**-0.0072**	**-0.0031**	**-0.0047**	**-0.0064**
15:30-16:15	**-0.0151**	**-0.0081**	**-0.0060**	**-0.0098**	**-0.0081**	**-0.0079**	**-0.0055**	**-0.0091**	**-0.0047**	**-0.0077**	**-0.0102**
Monday	-0.0011	**-0.0044**	**-0.0037**	**-0.0045**	**-0.0069**	**-0.0075**	**-0.0036**	**-0.0043**	**-0.0045**	**-0.0042**	**-0.0035**
Tuesday	-0.0003	**-0.0044**	**-0.0038**	**-0.0047**	**-0.0059**	**-0.0072**	**-0.0055**	**-0.0023**	**-0.0044**	**-0.0022**	**-0.0018**
Wednesday	-0.0007	**-0.0035**	**-0.0022**	**-0.0026**	**-0.0046**	**-0.0036**	-0.0015	**-0.0021**	**-0.0029**	**-0.0030**	**-0.0038**
Thursday	**-0.0016**	**-0.0028**	**-0.0034**	**-0.0038**	**-0.0062**	**-0.0051**	**-0.0026**	**-0.0023**	**-0.0032**	**-0.0054**	**-0.0023**
Roll day	**-0.0134**	**-0.0115**	-0.0019	-0.0057	**-0.0211**	-0.0154	**-0.0082**	**-0.0067**	**-0.0105**	0.0018	**-0.0212**
adj-R^2	45.39%	48.42%	44.65%	51.97%	64.25%	53.78%	57.45%	68.63%	72.05%	60.24%	65.02%

Table 4 (continued)

	Panel B: VXO										
	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Constant	**0.0160**	**0.0179**	**0.0150**	**0.0170**	**0.0146**	**0.0142**	**0.0109**	**0.0093**	**0.0061**	**0.0113**	**0.0107**
R(t-8:t-1)	0.0695	0.0583	0.0060	0.0351	**0.1896**	0.0347	**0.1013**	**0.2626**	**0.2131**	**0.2317**	**0.2305**
Rt	**-4.3610**	**-5.8118**	**-6.5705**	**-7.9768**	**-7.6519**	**-3.0012**	**-2.8261**	**-6.3686**	**-6.0807**	**-7.1195**	**-8.7125**
|Rt|	0.1539	-0.1732	**0.5656**	-0.1675	0.4489	0.2017	**0.3993**	**0.8417**	**0.7434**	**0.7869**	0.3628
R(t+1:t+8)	-0.0192	-0.0229	**-0.0869**	**-0.1208**	-0.0725	0.0224	**-0.0383**	**-0.0852**	-0.0432	-0.0611	**-0.1597**
9:30-10:30	**-0.0195**	**-0.0193**	**-0.0169**	**-0.0165**	**-0.0175**	**-0.0203**	**-0.0146**	**-0.0152**	**-0.0106**	**-0.0148**	**-0.0118**
10:30-11:30	**-0.0207**	**-0.0213**	**-0.0193**	**-0.0195**	**-0.0196**	**-0.0178**	**-0.0146**	**-0.0154**	**-0.0096**	**-0.0166**	**-0.0156**
11:30-12:30	**-0.0206**	**-0.0193**	**-0.0196**	**-0.0192**	**-0.0183**	**-0.0161**	**-0.0128**	**-0.0164**	**-0.0096**	**-0.0141**	**-0.0141**
12:30-13:30	**-0.0193**	**-0.0186**	**-0.0180**	**-0.0183**	**-0.0152**	**-0.0169**	**-0.0138**	**-0.0127**	**-0.0099**	**-0.0157**	**-0.0149**
13:30-14:30	**-0.0204**	**-0.0183**	**-0.0175**	**-0.0185**	**-0.0162**	**-0.0140**	**-0.0137**	**-0.0138**	**-0.0080**	**-0.0127**	**-0.0130**
14:30-15:30	**-0.0211**	**-0.0214**	**-0.0195**	**-0.0196**	**-0.0176**	**-0.0172**	**-0.0138**	**-0.0135**	**-0.0100**	**-0.0154**	**-0.0120**
15:30-16:15	**-0.0193**	**-0.0204**	**-0.0207**	**-0.0205**	**-0.0167**	**-0.0175**	**-0.0137**	**-0.0149**	**-0.0101**	**-0.0157**	**-0.0133**
Monday	0.0012	-0.0011	0.0006	-0.0002	-0.0010	**-0.0034**	-0.0015	0.0017	0.0006	0.0005	0.0003
Tuesday	0.0013	-0.0001	0.0006	-0.0004	0.0001	-0.0008	-0.0014	0.0017	0.0006	**0.0025**	**0.0032**
Wednesday	**0.0030**	0.0000	0.0008	0.0009	0.0019	0.0002	0.0014	**0.0027**	0.0016	0.0011	0.0012
Thursday	0.0022	0.0008	0.0011	0.0010	-0.0002	0.0009	-0.0004	**0.0022**	-0.0001	-0.0012	**0.0024**
Roll day	**-0.0137**	**-0.0177**	-0.0058	-0.0003	-0.0082	**-0.0359**	**-0.0146**	-0.0014	**-0.0263**	-0.0060	**-0.0200**
adj-R^2	49.77%	48.75%	47.50%	54.08%	56.18%	51.94%	54.66%	60.83%	62.19%	57.36%	52.62%