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Regime-Switching Models in the Foreign Exchange Market

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Abstract

This paper compares three regime-switching models in estimating and forecasting behavioral heterogeneity in the AUD/USD foreign exchange market. The three heterogeneous agent models (HAMs) allow different elements in the models to be regime-dependent in different mechanisms. The first model pioneered by Boswijk et al. (2007) models the fraction of each type of agents as a function of its relative past performance. The second model developed by Lof (2012) allows agents to switch their strategies based on macroeconomic fundamentals. The third model proposed by Chiarella et al. (2012) sets agents beliefs to be dependent on a markov-switching process. Based on AUD/USD monthly bilateral exchange rate and money market rate from 2000:1 to 2013:6, our empirical results show that (i) the model by Lof (2012) provides the best in-sample estimation efficiency and (ii) the model by Boswijk et al. (2007) significantly outperforms the model by Lof (2012) in terms of out-of-sample forecasting accuracy, but not that by Chiarella et al. (2012) in the medium to long run.

Keywords: Regime Switching, Heterogeneity, Out-of-Sample Forecasting, Trading Behavior

JEL classification: G12, G15

1 Introduction

A large number of studies have confirmed the application of heterogeneous bounded-rational strategies based on technical and fundamental analysis in the trading of financial assets by investment professionals. For instance, Boswijk et al. (2007), Chiarella et al. (2012) and Yamamoto and Hirata (2013) have documented the evidence of behavioral heterogeneity and the simultaneous presence of both fundamentalists and chartists in the stock markets. Similar observation is also found in the options market (Frijns et el., 2010) and the foreign exchange markets (Gilli and Winker, 2003; Westerhoff and Reitz, 2005; Mankhoff et al., 2009; de Jong et al., 2010). Such heterogeneity in investment behavior is also supported by the experimental evidence of Hommes (2011) and the questionnaire survey of Allen and Taylor (1990), Cheung and Chinn (2011) and Gehrig and Menkhoff (2004), among others. A more comprehensive summary of these surveys can be found in the study of Menkhoff and Taylor (2007).

Heterogeneity can either be cross-sectional where different types of investors adopt different strategies or time-variant where the same investor can switch among heterogeneous strategies over time (Boswijk et al., 2007; Chiarella et al., 2012; Yamamoto and Hirata, 2013). Accounting for such a switch in trading behavior is important in both theoretical and empirical work. Theoretically, Huang and Zheng (2012) and Huang et al. (2010, 2012) show that, besides the cross-sectional heterogeneity, accounting for the regime-switching beliefs itself can improve the models capability to generate financial market stylized facts that match well with actual financial market data. Empirically, Manzan and Westerhoff (2005), De Grauwe and Grimaldi (2006), de Jong et al. (2010) and Chiarella et al. (2012) show that models with the regime-switching behavior tend to outperform those without in terms of better in-sample explanatory power and out-of-sample forecasting ability. Besides, these models also demonstrate better predictive power than a simple foreign exchange rate random walk model that outperforms many structural exchange rate models (Meese and Rogoff, 1983).

Existing regime-switching models that are used to estimate behavioral heterogeneity can be broadly classified into three categories according to their switching mechanisms. The first category is pioneered by Boswijk et al. (2007, BHM hereafter) where investors switch their strategies based on past performance. In the original work of Boswijk et al. (2007), investors cluster evolutionarily to the strategy that generate higher past realized profits according to some discrete choice probability, with root tracing back to Brock and Hommes (1998). Many empirical studies later follow similar switching rule but refer to various switching criteria based on backward looking indicators. For example, in Manzan and Westerhoff (2007), investment decision is updated according to the deviation between actual and fundamental prices whereas studies by de Jong et al. (2010), Jongen et al. (2012) and ter Ellen et al. (2013) assume that investors switch to strategies with relatively accurate past forecasting. This strand of regime-switching models is by far the most commonly applied type of models in the current literature. The second strand of empirical models follows closely the work of Lof (2012) where investors update their trading strategies according to business cycles. The switching rule is governed by a smooth-transition function of some macroeconomic variables such as GDP growth rate, industrial production and consumer price index. In this class of models, investors follow a chartist strategy during economic expansion and switch to a fundamentalist strategy during economic contraction. The third strand of models follows the work of Chiarella et al. (2012) where the switching originates from the structural change in expectations based on some unobserved conditions in the financial markets that governed by a Markov process. In this set up, investors initially play the same trading strategy but subsequently deviate from their original forecasting formula and switch completely to a different strategy depending on the market conditions. Putting these together, it is noted that behavioral heterogeneity can be cross-sectional and time-variant. It can also arise within the same trading strategy. In the first and second types of models, investors switch evolutionarily between trading strategies that are government by some rule. In the third type of models, even though investors do not switch between strategies but strategy may switch according to different market conditions that are governed by a Markov switching process.

Although there is clear evidence that the regime-switching models provide a good empirical specification to estimate behavioral heterogeneity, it remains unclear as in which type of switching mechanism performs better. In view of the lack of performance comparison, in this paper, we aim to compare different switching mechanisms and evaluate their performance in estimating the foreign exchange market.[1] Specifically, we identify the comparative advantage of each switching mechanism in terms of its goodness-of-fit, estimation efficiency and predictive power, using a framework that is as simple as possible. In doing so, we first develop a benchmark model that is sufficiently general to incorporate all the three types of switching mechanisms while keeping other factors unchanged. We then estimate the three models separately.

We choose to compare the three switching mechanisms in the foreign exchange market with the context of AUD/USD for several reasons. First, AUD/USD is the fourth-most-traded currency pair, which accounts for 7% of the global foreign-exchange market (Economist, 2013). The common practices of various types of traders in such a market is relatively general and representative. Second, the Federal Reserve of United States has kept the interest rate close to zero for more than five years while the interest rate in Australia has been high (its current interest rate is still 2.75% after several round of rate cuts), which makes this currency pair one of the most important vehicle for carry trade. Third, to the best of our knowledge, no empirical literature on behavioral heterogeneity has studied AUD/USD despite its importance in the foreign exchange market. Our studies complement existing literature with the trading behavior on AUD/USD.

Using monthly data from January 2000 to June 2013, our empirical results suggest that (1) behavioral heterogeneity can arise not only due to cross-sectional heterogeneity with investors taking different strategies or time-variant heterogeneity with investors switching their strategies over time but also within-group heterogeneity when investors who stick to the same strategy shift their expectations over time; (2) while strategy switching based on a smooth-transition function of various macroeconomic variables proposed by Lof (2012) provides the best in-sample explanatory power and strategy switching based on past performance as in Boswijk et al. (2007) exhibits better out-of-sample forecasting power, there is no significant evidence to confirm any of these

[1] Existing papers typically apply BHM to estimate HAMs, using various foreign exchange data. ter Ellen et al. (2013) apply weekly survey data to estimate HAM with the BHM method. They find existence of value traders and momentum traders as well as the switching between them. Perhaps due to the monthly data that we have applied, we find no evidence of switching behavior in the AUD/USD exchange market with the BHM method.

two switching mechanisms outperforms that of Chiarella et al. (2012) in terms of forecasting accuracy.

The remaining of the paper is organized as follows. Section 2 describes the model. Section 3 presents the data and methodology. Section 4 discusses the estimation results of the three models. Sections 5 and 6 compare the three models in terms of their estimation efficiency and out-of-sample forecasting power, respectively. Section 7 concludes.

2 Model Specifications

Like many other asset markets, the foreign exchange market is also believed to consist of investment professionals who use both fundamental and technical analysis in their trading activities (Menkhoff and Taylor, 2007). Many existing studies have indeed confirmed the simultaneous presence of fundamentalists and chartists in the foreign exchange market. These include the work of Cheung and Chinn (2011) and Gehrig and Menkhoff (2004). Additionally, many researchers who incorporate fundamentalist and chartist strategies in their asset pricing models show that simulated data in these models match with almost all stylized facts (De Grauwe and Grimaldi, 2006; Huang et al., 2012). These models also provide empirical specifications that outperform the commonly used random walk model (Chiarella et al., 2012; de Jong et al, 2010). Motivated by these findings, we consider a foreign exchange market that consists of both fundamentalists and chartists who submit their orders to a market maker. The market maker then adjusts the exchange rate up or down according to the size of aggregate order.

2.1 Benchmark Model

2.1.1 Fundamentalists

In the benchmark model, the fundamentalists perceive the deviation between spot exchange rate S_t and fundamental exchange rate u_t as a trading opportunity. According to them, expected price movement is caused by the mispricing of either an undervaluation or an overvaluation of a currency.

While some of them believe in the persistence of the mispricing in the short term, others may expect a reversion back to its fundamental value. Together, their aggregate demand function is given by:

$$D_{f,t} = \alpha_{ft} \left(u_t - S_{t-1} \right), \tag{1}$$

where α_{ft} measures the extent to which the fundamentalists act on their belief. With $\alpha_{ft} > 0$, the majority of the fundamentalists believe in the mean-reverting of price, that is, they expect the currency to appreciate (depreciate) in the future if S_{t-1} is below (above) u_t. In this case, the aggregate actions of the fundamentalists stabilize the currency. With $\alpha_{ft} < 0$, the majority of the fundamentalists believe that the spot rate will continue to deviate from its fundamental, at least for a while. In this case, the aggregate actions of the fundamentalists destabilize the currency and worsen the deviation. Our specification of the fundamentalists is slightly different from the traditional definition, where they always trade to drive prices towards its fundamentals. The traditional definition is a special case in our setup.

There is a lack of consensus in estimating the fundamental value u_t in the foreign exchange market (Taylor, 1995). While many theoretical works propose to estimate the fundamental exchange rate using monetary models based on money stock and real income (Mark, 1995), recent works suggest an extended framework that includes money supply, output, interest rate, expected inflation and trade balance (Neely and Sarno, 2002).[2] There is, however, no convention on estimating the corresponding coefficients, such as the money-demand income elasticity. Therefore, these methods that work well in the existing theoretical literature are not sufficient for our purposes. Most of the foreign exchange rate determination models draw upon purchasing power parity (PPP) and uncovered interest rate parity (UIP). However, the calculation of true PPP value of an exchange rate is ambiguous and not straightforward (ter Ellen et al., 2013). Therefore, in our studies, we choose to proxy the fundamental exchange rate based on UIP, which requires only directly observable data and is relatively easy to estimate. The application of UIP is also justified by the observation that foreign exchange traders watch closely the interest rates movement underlying the

[2]See Taylor (1995) for a survey on various models of foreign exchange rate determination.

currency pairs (in our case, the federal funds rate in the United States and the cash rate in Australia), the two most important indicators underlying UIP. Specifically, we measure the fundamental foreign exchange rate for AUD/USD based on the following equation:

$$u_{t+1} = S_t(1 + r_t^{AU}/12)/(1 + r_t^{US}/12), \tag{2}$$

where r_t^{US} is the annualized US money market rate and r_t^{AU} is the annualized Australia money market rate.

Fundamentalists who expect mean reversion are those who believe in UIP. They will buy the currency of low-interest country and sell the currency of high-interest country and anticipate the exchange rate of low-interest country to appreciate. These are known as the UIP traders. On the other hand, for fundamentalists who expect the price deviation to persist will sell the currency of low-interest country and buy the currency of high-interest country. They will profit from the interest rate differential as the exchange rate remains constant or depreciates. They are known as the carry traders. Therefore, when the majority of the fundamentalists are UIP traders, $\alpha_{ft} > 0$. When carry traders form the majority, $\alpha_{ft} < 0$. We distinguish the fundamentalists as carry traders and UIP traders, beyond the archetypical specifications of fundamentalists in HAMs. This set up is in accordance with the importance of interest differential as a determinant of the foreign exchange expectations, in addition to the conventional fundamental and momentum considerations in other financial markets (Jorgen et al., 2012).[3]

2.1.2 Chartists

The chartists in the model are those who conduct technical analysis to form their expectation of future exchange rate. While there are many technical rules, the most commonly applied rule is the momentum rule. Empirical studies have indeed confirmed the presence of such a trading rule. Besides, the momentum rule is found to be relatively more profitable as compared to the other

[3] One recent study which combines the carry trade observation with the heterogeneous literature is Spronk et al. (2013), where they use simulations to provide an explanation of the exchange rate puzzles.

technical rules, such as moving average (Jongen et al., 2012). Motivated by these findings, we assume that the chartists form their expectation based on the most basic form of momentum rule AR(1):

$$E_{c,t}(S_t) = S_{t-1} + \beta_t (S_{t-1} - S_{t-2}),$$

where β_t is the extrapolation rate of the chartists and it measures the degree of expected autocorrelation. When $\beta_t > 0$, the chartists expect the price trend to persist (bandwagon expectation). On the other hand, when $\beta_t < 0$, the chartists expect the past price trend to reverse (contrarian expectation).

The aggregate demand of the chartists is given by:

$$\begin{aligned} D_{c,t} &= \eta[E_{c,t-1}(S_t) - S_{t-1}] \\ &= \lambda_t (S_{t-1} - S_{t-2}), \end{aligned} \tag{3}$$

where $\eta > 0$ is a parameter that measures the extent to which the chartists act on their beliefs and the second line is obtained by letting $\lambda_t = \eta\beta_t$. Note that $\lambda_t > 0$ ($\lambda_t < 0$) if and only if $\beta_t > 0$ ($\beta_t < 0$).

2.1.3 The Market Maker

Within a market marker framework, the market maker collects orders from all traders and subsequently quotes the spot exchange rate according to the aggregate demand with a speed of $\gamma > 0$. We use $\omega_{f,t}$ and $\omega_{c,t}$ to denote the market weights (or fractions) of fundamentalists and chartists, respectively. The exchange rate is updated according to the aggregate demand and a noise term ε_t, which can be written as:

$$\begin{aligned} \Delta S_t &= S_t - S_{t-1} \\ &= \gamma\left(\omega_{f,t} D_{f,t} + \omega_{c,t} D_{c,t}\right) + \varepsilon_t \\ &= \gamma\omega_{f,t}\alpha_{ft}(u_t - S_{t-1}) + \gamma\omega_{c,t}\lambda_t (S_{t-1} - S_{t-2}) + \varepsilon_t. \end{aligned} \tag{4}$$

where γ is the speed of exchange rate adjustment and the third line is obtained by substituting $D_{f,t}$ and $D_{c,t}$ from Eqs.(1) and (3).

In the following, we specify the three types of switching mechanisms separately. In particular, the proportion of fundamentalists, $\omega_{f,t}$, and chartists, $\omega_{c,t}$, are modeled to evolve according to different rules — switching based on past performance (Boswijk et al., 2007) and switching based on macroeconomic fundamentals (Lof, 2012), while the intensity of actions of fundamentalists and chartists, α_t and λ_t, are modeled to be contingent on the foreign exchange market state for the Markov-switching beliefs (Chiarella et al., 2012).

2.2 Strategy Switching Based on Past Performance (BHM)

Following BHM, we assume that traders update their market weight every period according to the most recent realized profits. The proportion of fundamentalists, $\omega_{f,t}$, and chartists, $\omega_{c,t}$, evolve according to a discrete choice model with multinomial logit probabilities:

$$\begin{aligned} \omega_{f,t} &= \frac{\exp\left(\rho \pi_{f,t-1}\right)}{\exp\left(\rho \pi_{f,t-1}\right) + \exp\left(\rho \pi_{c,t-1}\right)} \\ \omega_{c,t} &= \frac{\exp\left(\rho \pi_{c,t-1}\right)}{\exp\left(\rho \pi_{f,t-1}\right) + \exp\left(\rho \pi_{c,t-1}\right)} = 1 - \omega_{f,t} \end{aligned}, \tag{5}$$

where ρ is a scaled intensity of choice that measures the sensitivity to the relative profitability of the trading rules, and $\pi_{f,t-1}$ and $\pi_{c,t-1}$ represent the most recent realized return of fundamentalists and chartists, respectively. Specifically, the profits of each fundamentalist and chartist can be described by:

$$\begin{aligned} \pi_{f,t} &= D_{f,t}\left(S_t - S_{t-1}\right) \\ \pi_{c,t} &= D_{c,t}\left(S_t - S_{t-1}\right) \end{aligned},$$

To comply with BHM, we further assume that $\alpha_{ft} \equiv \alpha_f$ and $\lambda_t \equiv \lambda$. This implies that both fundamentalists and chartists maintain the same degree of acting on their believes. In the BHM style of switching, Eq. (4) can be written as:

$$
\begin{aligned}
\Delta S_t &= S_t - S_{t-1} \\
&= \frac{\gamma\alpha_f \exp\left(\rho\pi_{f,t-1}\right)(u_t - S_{t-1})}{\exp\left(\rho\pi_{f,t-1}\right)+\exp\left(\rho\pi_{c,t-1}\right)} + \frac{\gamma\lambda \exp\left(\rho\pi_{c,t-1}\right)(S_{t-1} - S_{t-2})}{\exp\left(\rho\pi_{f,t-1}\right)+\exp\left(\rho\pi_{c,t-1}\right)} + \varepsilon_t \\
&= \frac{\gamma\alpha_f(u_t - S_{t-1})}{1+\exp\left[-\rho\Delta S_{t-1}\left(\pi_{f,t-1}-\pi_{c,t-1}\right)\right]} + \frac{\gamma\lambda(S_{t-1}-S_{t-2})}{1+\exp\left[\rho\Delta S_{t-1}\left(\pi_{f,t-1}-\pi_{c,t-1}\right)\right]} + \varepsilon_t \\
&= \frac{\gamma\alpha_f(u_t - S_{t-1})}{1+\exp\left[-\rho\Delta S_{t-1}\left(\alpha_f(u_{t-1}-S_{t-2})-\lambda(S_{t-2}-S_{t-3})\right)\right]} + \frac{\gamma\lambda(S_{t-1}-S_{t-2})}{1+\exp\left[\rho\Delta S_{t-1}\left(\alpha_f(u_{t-1}-S_{t-2})-\lambda(S_{t-2}-S_{t-3})\right)\right]} + \varepsilon_t \\
&= \frac{\gamma\alpha_f(u_t - S_{t-1})}{1+\exp\left[-\frac{\rho}{\gamma}\Delta S_{t-1}\left(\gamma\alpha_f(u_{t-1}-S_{t-2})-\gamma\lambda(S_{t-2}-S_{t-3})\right)\right]} + \frac{\gamma\lambda(S_{t-1}-S_{t-2})}{1+\exp\left[\frac{\rho}{\gamma}\Delta S_{t-1}\left(\gamma\alpha_f(u_{t-1}-S_{t-2})-\gamma\lambda(S_{t-2}-S_{t-3})\right)\right]} + \varepsilon_t.
\end{aligned} \tag{6}
$$

While the time series of spot exchange rate S_t is directly observable, the fundamental exchange rate u_t is calculated based on Eq. (2). Other parameters to be estimated are γ, α_f, λ and ρ. To prevent under-specification, we estimate $\gamma\alpha_f$, $\gamma\lambda$ and ρ/γ instead. As $\gamma > 0$ is only a scaling factor, it will not affect the statistical significance of the estimation coefficients.

2.3 Strategy Switching Based on Macro Fundamentals (LSTR)

Following Lof (2012), traders are assumed to update their strategy according to macroeconomic fundamentals. The weight of fundamentalists, $\omega_{f,t}$, follows a logistic smooth-transition regressive function (LSTR) of a lagged matrix of macroeconomic fundamental variables X_{t-1}:

$$
\omega_{f,t} = \frac{1}{1+\exp\left[\tau\left(X_{t-1}A - c\right)\right]},
$$

where A is a column vector denoting the weight of the fundamental variables, the parameter $\tau > 0$ measures the sensitivity to the fundamental matrix, and c is the threshold exceeding which the foreign exchange market will be dominated by chartists. The model implies transitions between two regimes: the regime dominated completely by fundamentalists, $\omega_{f,t} = 1$, which tends to occur when $X_{t-1}A < c$ and the regime dominated completely by chartists, which tends to occur when $X_{t-1}A > c$. When $X_{t-1}A = c$, fundamentalists and chartists share the market equally. The selection of macroeconomic fundamental variables X_{t-1} will be based on a series of linearity tests. More details are discussed in the methodology section.

Following Lof (2012), we let $\alpha_{ft} \equiv \alpha_f$ and $\lambda_t \equiv \lambda$. In Lof's LSTR switching model, Eq. (4) can be written as:

$$\Delta S_t = \frac{\gamma\alpha_f\left(u_t - S_{t-1}\right)}{1+\exp\left[\tau\left(X_{t-1}A - c\right)\right]} + \frac{\gamma\lambda\exp\left[\tau\left(X_{t-1}A - c\right)\right]\left(S_{t-1} - S_{t-2}\right)}{1+\exp\left[\tau\left(X_{t-1}A - c\right)\right]} + \varepsilon_t. \tag{7}$$

As the time series of u_t, S_t and X_t are directly observable, we can estimate the LSTR for $\gamma\alpha_f$, $\gamma\lambda$, A, c and τ.

2.4 Markov-Switching Beliefs (MS)

As in the financial market, the foreign exchange market is characterized by currency appreciation with low volatility in a boom state, and currency depreciation with high volatility in a bust state. Investors' trading behavior is likely to vary in different states. Motivated by these observations, the beliefs of fundamentalists and chartists as well as the noise term are modeled to be state-dependent.

Following Chiarella et al. (2012), λ_t (to be exact β_t) is assumed to be contingent on the foreign exchange market state m_t, which takes a discrete value of 0 or 1 so that $m_t \in M = \{0,1\}$. The dynamics of the state aim at capturing the changes in the market conditions through the observed prices. The state m_t is modeled as a stationary ergodic two-state Markov chain on M with transition probabilities given by:

$$P\left(m_t = j | m_{t-1} = i, m_{t-2} = k, \ldots\right) = P\left(m_t = j | m_{t-1} = i\right) = P_{j,i} \tag{8}$$

for $i, j, k \in M$, where $P_{j,i}$ indicates the probability that state (regime) i transits to state j for $i, j \in \{0,1\}$. The transition probabilities are constants and satisfy the conditions of $\sum_{j=0}^{1} P_{j,i} = 1$ and $0 \leq P_{j,i} \leq 1$ for $i = 0,1$. The state m_t is a random variable that is not directly observable. However, a filter estimate can be computed from the time series of the exchange rate. Some filters, such as sequential filter, are capable of performing accurate inferences of m_t. It is therefore reasonable to assume that investment professionals can estimate the state with high precision. The regime-dependent α_{ft} and λ_t are then given by:

$$\alpha_{ft} = \begin{cases} \alpha_{f0}, & m_t = 0, \\ \alpha_{f1}, & m_t = 1. \end{cases} \tag{9}$$

and

$$\lambda_t = \begin{cases} \lambda_0, & m_t = 0, \\ \lambda_1, & m_t = 1. \end{cases} \tag{10}$$

The noise term ε_t is assumed to be drawn from an $N(0, \sigma_t^2)$ distribution and σ_t^2 is regime-dependent, that is:

$$\varepsilon_t \sim \begin{cases} N(0, \sigma_0^2), & m_t = 0, \\ N(0, \sigma_1^2), & m_t = 1. \end{cases} \tag{11}$$

In Chiarella et al. (2012), the expectation formation process is regime-dependent but the fraction of fundamentalists and chartists are constant such that $\omega_{f,t} \equiv \omega_f$ and $\omega_{c,t} \equiv \omega_c$. Under the Markov regime-switching (MS) process, the price dynamic function is given by:

$$\Delta S_t = \gamma \omega_f \alpha_{ft} (u_t - S_{t-1}) + \gamma \omega_c \lambda_t (S_{t-1} - S_{t-2}) + \varepsilon_t. \tag{12}$$

The parameters estimated are $\gamma\omega_f\alpha_{f0}$, $\gamma\omega_f\alpha_{f1}$, $\gamma\omega_c\lambda_0$, $\gamma\omega_c\lambda_1$, σ_0, σ_1, $P_{0,0}$ and $P_{0,1}$.

3 Methodology and Data Description

3.1 Data

We use period average AUD/USD monthly exchange rate, Australia and US money market rates from 2000:1 to 2013:6. The use of monthly data is consistent with most literature in HAM for the foreign exchange market, for example, de Jong et al. (2010), Jongen et al. (2012) and Spronk et al. (2013). Using data with lower frequency may smooth out much of the behavioral changes of the traders.[4] Even though foreign exchange rate data are available at high frequency, we use monthly data for two reasons. The first is to avoid directly addressing issues arising from short-term noise such as the day-of-the-week effect in exchange rate volatility (see Hsieh, 1988). The second is to link the decision making process to macroeconomic data that are typically available at a relatively low frequency.

Other data collected are potential transition variables which are used in the LSTR model. These include real effective exchange rate ($REER$), real GDP (GDP)[5], unemployment rate (UNE), real industrial production (IND), consumer price index (CPI), money supply ($M1$), short-term (STY) and long-term (LTY) interest rates measured by government bond yield. Among these variables, $REER$, GDP, IND, CPI and $M1$ are measured in month-on-month growth rates. Since nominal exchange rate is a relative measure, we take country-on-country differences (Australian value minus US value) of each potential transition variable.

The statistics of all these variables are summarized in Table 1. Figure 1 shows the nominal exchange rate and its deviation from the fundamental value. Over the sample period, the Australian dollar has in general appreciated strongly against the US dollar and AUD/USD exchange rate has been around or below the fundamental value, with two notable exceptions of continuing depreciation. Between 2000 and 2001, due to falling market confidence and rising concerns over political instability, the Australian dollar fell to its lowest level since the currency was floated in 1983, with large deviations from its fundamental value. The second notable depreciation episode happened between August 2008 and March 2009 during the Global Financial Crisis, where the nominal exchange rate exhibits unprecedent deviations from its fundamental value.

The summary statistics suggest a large fluctuation in the AUD/USD exchange rate with a minimum value of 0.929, and a maximum value of 1.996, and a standard deviation of 0.315. Its maximum deviation from the fundamental exchange rate is 0.218 and on average it has been be-

[4] Some empirical studies with HAM also use annual data of the financial market indices. For example, Boswijk et al. (2007) use annual S&P500 index since the index can be traced back to 1871 and thus have a large sample even with annual data. However, Australia adopted a fixed exchange rate regime prior to 1984. Therefore, using annual exchange rate data would largely reduce our sample size and negatively affect the asymptotic properties of the estimates.

[5] Since real GDP data is available at quarterly frequency only, we first calculate the quarterly growth rate and then calculate the average monthly growth rate assuming a geometric growth pattern in each quarter.

Table 1: Summary Statistics

Variable	Mean	Std. dev.	Min	Max
S_t	1.350	0.315	0.929	1.996
$S_t - u_t$	-0.006	0.044	-0.104	0.218
r_t^{AU}	4.990	1.086	2.750	7.250
r_t^{US}	2.225	2.119	0.070	6.540
$REER$	0.297	3.098	-18.753	8.003
GDP	0.098	0.230	-0.372	0.775
UNE	-0.941	2.096	-4.692	2.610
IND	1.405	2.949	-7.346	9.054
CPI	0.046	0.322	-0.805	1.494
$M1$	-0.003	2.142	-15.885	5.491
STY	2.217	1.185	-0.400	4.555
LTY	1.368	0.602	0.026	2.618

Notes: Sample period is from 2000:1 to 2013:6. All numbers are expressed in percentage, except S_t and $S_t - u_t$.

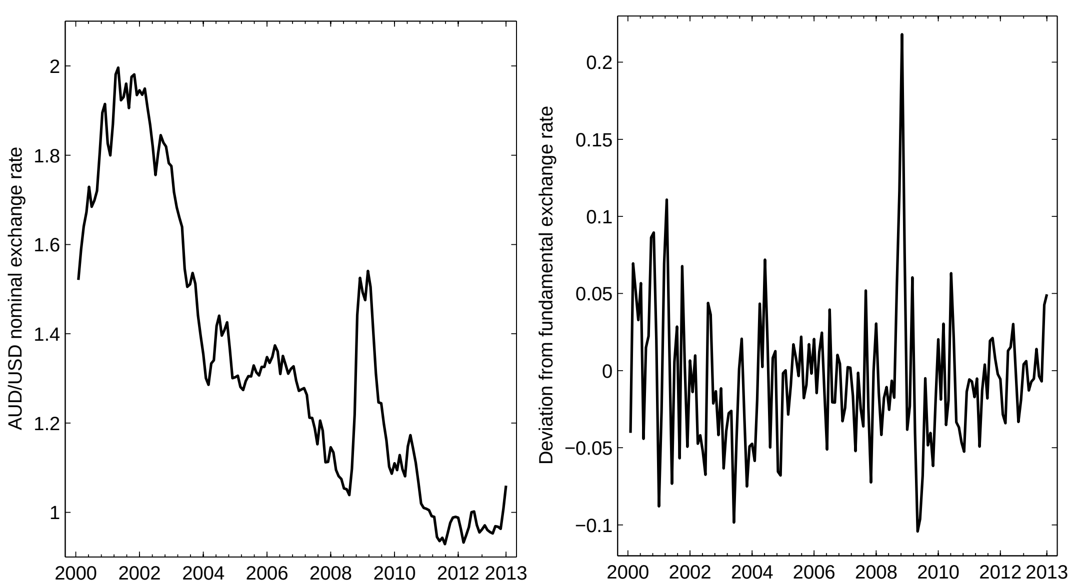

Figure 1: Nominal Exchange Rate and Its Deviation from Fundamental Exchange Rate

low its fundamental value. This deviation arises from the interest rate differences between the two economies. The Australia average money market rate of 4.990% has been more than twice of that of the US 2.225%. The real effective exchange rate growth rates in Australia and the US exhibit significant differences, with the maximum of 8% and the minimum of -18%. Such differences are also observed in other variables, with industrial production and money supply growth rates being more significant. All potential transition variables are then standardized to accommodate the numerical estimation of the nonlinear model.

3.2 Linearity Tests

To determine which set of variables are valid transition variables in the LSTR model, following Luukkonen et al. (1988), linearity tests based on Taylor approximations of the model are performed. The potential transition variables are divided into four groups: (1) foreign exchange market indicators ($REER$); (2) business cycle indicators (GDP, UNE, IND); (3) money supply ($M1$) and inflation rate (CPI); and (4) interest rates (STY, LTY). First, we consider the univariate transition function and select the variable that yields the strongest rejection of linearity. Next, we consider multivariate transition functions with two, three and four transition variables, separately. Linearity tests are performed on each possible set of two to four transition variables. We never include more than one variable from each of the four groups. This approach allows us to avoid multicollinearity within the transition function since variables within each group are likely to be highly correlated. Then, for each of the two-, three- and four-transition-variable case, we choose the set of variables that yields the strongest linearity rejection as the optimal set of transition variables. In total, we select four sets of transition variables from the linearity tests, which are $\{REER\}$, $\{REER, M1\}$, $\{REER, CPI, STY\}$ and $\{REER, CPI, STY, UNE\}$. The linearity test results for three-transition-variable are presented in Table 2 since its estimation results exhibit the best goodness-of-fit. The remaining linearity tests along with their estimation results are summarized in the appendices.

Table 2: Linearity Tests: Three-variable Transition Function

X_{t-1}	p-value	Model	X_{t-1}	p-value	Model
$(GDP, M1, LTY)$	0.0368	LSTR	$(GDP, M1, STY)$	0.0358	LSTR
(IND, CPI, LTY)	0.1768	Linear	(IND, CPI, STY)	0.3047	Linear
$(IND, M1, LTY)$	0.3825	Linear	$(IND, M1, STY)$	0.1831	Linear
$(REER, GDP, CPI)$	4×10^{-9}	LSTR	$(REER, GDP, LTY)$	5×10^{-11}	LSTR
$(REER, GDP, M1)$	6×10^{-7}	LSTR	$(REER, GDP, STY)$	3×10^{-10}	LSTR
$(REER, IND, LTY)$	3×10^{-8}	LSTR	$(REER, IND, CPI)$	7×10^{-4}	LSTR
$(REER, IND, M1)$	8×10^{-4}	LSTR	$(REER, IND, STY)$	5×10^{-8}	LSTR
$(REER, CPI, LTY)$	4×10^{-14}	LSTR	$(REER, CPI, STY)$*	1×10^{-14}	LSTR
$(REER, M1, LTY)$	1×10^{-11}	LSTR	$(REER, M1, STY)$	1×10^{-11}	LSTR
$(REER, UNE, CPI)$	8×10^{-9}	LSTR	$(REER, UNE, LTY)$	5×10^{-9}	LSTR
$(REER, UNE, M1)$	6×10^{-7}	LSTR	$(REER, UNE, STY)$	3×10^{-11}	LSTR
(UNE, CPI, LTY)	0.1060	Linear	(UNE, CPI, STY)	0.1420	Linear
$(UNE, M1, LTY)$	0.0594	Linear	$(UNE, M1, STY)$	0.0594	Linear
(GDP, CPI, STY)	0.1589	Linear	(GDP, CPI, LTY)	0.3509	Linear

Notes: p-values of F-tests for linearity are presented. Column 'Model' indicates the suggested model by the tests. The set of variables that yields the strongest rejection of linearity is marked with *, which is $(REER, CPI, STY)$.

4 Estimation Results

In this section, we present the estimation results from the proposed model with the three types of regime switching, including (1) switching between strategies based on past performance (BHM), (2) switching between strategies based on macroeconomic fundamentals (LSTR) and (3) switching between regime-dependent beliefs (MS).

4.1 BHM Estimation Results

The parameter estimates for the BHM-type switching, defined by Eq. (6), are presented in Table 3. The model is estimated by nonlinear least squares following Boswijk et al. (2007). The logit switching rule in the BHM model essentially represents a special case of the generalized logistic smooth transition where the transition variables are differences in the realized profits of fundamentalists and chartists. The estimate of $\gamma\alpha_f$ is negative (-2.225). This implies that among the fundamentalists, carry traders dominate the foreign exchange market. The estimate of $\gamma\lambda$ is positive (0.723). This suggests that among the chartists bandwagon expectations dominate. The intensity of choice parameter ρ is not identified in the estimation procedure, but is captured by ρ/γ (17.571). This is not statistically significant, suggesting that there is no direct evidence of switching between the fundamentalists and the chartists based on past performance. This implies that the relative profitability of the two trading rules is not a significant strategy switch in the AUD/USD foreign exchange market. However, this result does not exclude the possible switch driven by other mechanisms (for example, LSTR and/or MS).[6]

Table 3: Estimation Results for the BHM Model

Parameter	$\gamma\alpha_f$	$\gamma\lambda$	ρ/γ
Coefficient	-2.225	0.723	17.571
p-Value	0.270	0.000	0.738

The upper panel of Figure 2 plots the estimated evolution of the fraction of fundamentalists and the AUD/USD exchange rate. The lower panel of Figure 2 is a scatter plot of the fraction of fundamentalists $\omega_{f,t}$ against the relative profitability of fundamentalism and chartism trading rules captured by $\gamma\Delta S_{t-1}[\lambda_t(S_{t-2}-S_{t-3})-\alpha_f(u_{t-1}-S_{t-2})]$. As suggested by the estimated coefficients in Table 3, there is no significant fluctuation in the fraction of fundamentalists, staying around 0.5. The scatter plot is a relatively flat line, indicating that agents respond sluggishly to differences in performance. The negative slope of the scatter plot, albeit small in absolute magnitude, somehow indicates that a positive difference in profits between chartism and fundamentalism strategy results in a smaller fraction of fundamentalists. Around the period of September 2008, the fraction of fundamentalists deviated from its mean level, suggesting some switching from fundamentalists to chartists with the relative appreciating US dollar.

[6] No switching cannot be taken as a criteria to exclude a model since the existence of strategy switching in the AUD/USD foreign exchange market is not a stylized fact. The relatively poor performance of BHM in the AUD/USD foreign exchange market however cannot and should not be generalized to the other financial markets. There is significant difference between equity market and foreign exchange market. For example, investors may hold the stock position for years while currency traders typically close their positions daily, which may result in that BHM can capture the switching behavior in stock market with yearly data while fail to capture the switching behavior in foreign exchange market even with monthly data.

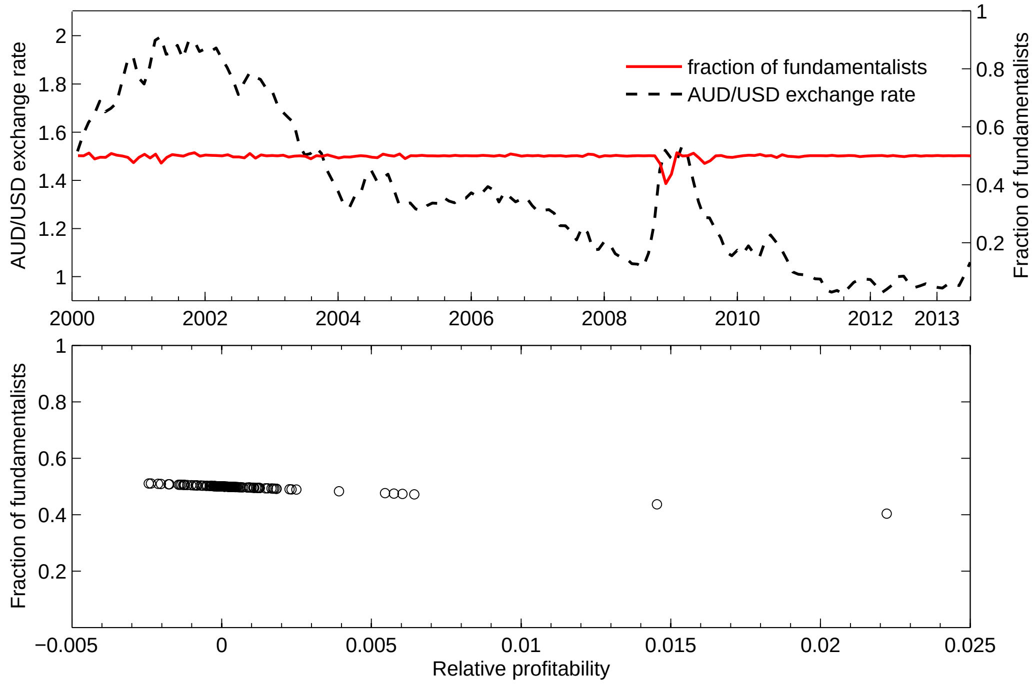

Figure 2: Fraction of Fundamentalists Estimated from BHM

4.2 LSTR Estimation Results

Following Lof (2012), we estimate the switching between strategies based on macroeconomic fundamentals, defined by Eq. (7), using nonlinear least squares, for each set of transition variables separately. In the LSTR model with a multivariate transition function, τ, A and c cannot be all identified at the same time without further restrictions. Therefore, we restrict the summation of the elements in A to be one such that $X_{t-1}A$ represents the weighted sum of multiple transition variables. The estimation results with three transition variables $\{REER, CPI, STY\}$ are presented since it has the best goodness-of-fit. The other linearity test results and estimation results are discussed in the appendices.

Table 4 summarizes the estimation results. The results identify two distinct regimes with statistically significant $\gamma\alpha_f$ and $\gamma\lambda$. Since $\gamma\alpha_f < 0$ (-19.553), among the fundamentalists carry traders dominate. On the other hand, $\gamma\lambda > 0$ (0.370) suggests that among the chartists bandwagon expectations dominate. These results are found to be consistent with those estimated using the BHM model. The significance of $\gamma\alpha_f$ and $\gamma\lambda$ also implies the presence of between-group behaviorial heterogeneity. The intensity of choice parameter τ (-0.933) supports a smooth transition between regimes but this is found to be not significant. However, the insignificance of the intensity of choice parameter with a large standard deviation is a common result in switching-type regression models since large changes in τ only cause a small variation of the fraction of fundamentalists $\omega_{f,t}$ (see for example, Boswijk et al., 2007 and de Jong et al., 2010). Teräsvirta (1994) suggests that this effect is not relevant as long as there is significant heterogeneity in the estimated regimes.

The interpretation of $A = (a_1, a_2, a_3)^T$ reveals that fundamentalists dominate during the periods of relative lower growth in Australia compared the US. Real effective exchange rate growth ($REER$) has a positive coefficient a_1 (3.558). This implies that a relative economic downturn in Australia depreciates the Australia real effective exchange rate and appreciates the US real effective exchange rate, and thus increases $REER$, causing an increase in the fraction of fundamentalists. Inflation rate (CPI) has a negative but not significant coefficient a_2 (-0.283),. A lower inflation rate in Australia indicates a relative economic downturn in Australia, and a larger fraction of fundamentalists in this model. Also the short-term government bond yield (STY) has a negative and significant coefficient a_3 (-2.275). A relative low yield on Australian government bonds (low-risk assets) implies high levels risk aversion, and in this model a high fraction of fundamentalists. In summary, fundamentalism is the dominant strategy during low economic growth periods in Australia.

Table 4: Estimation Results for the LSTR model

Parameter	$\gamma\alpha_f$	$\gamma\lambda$	τ	a_1	a_2	a_3	c
Coefficient	-19.553	0.370	-0.933	3.558	-0.283	-2.275	3.365
p-Value	0.000	0.000	0.605	0.000	0.190	0.000	0.000

Notes: a_1, a_2 and a_3 are the coefficients of the transitions variables $REER$, CPI and STY, respectively.

The upper panel of Figure 3 shows the plot of the fraction of fundamentalists and the AUD/USD nominal exchange rate overtime. The lower panel of Figure 3 is the scatter plot of the fraction of fundamentalists $\omega_{f,t}$ against the weighted sum of the transition variables $X_{t-1}A$. Most of the time, the economy is represented by both fundamentalists and chartists, with $\omega_{f,t}$ ranges between 0 and 1. Overtime, chartists dominate since the fraction of fundamentalists are mostly below 0.5. The fraction of fundamentalists increases when S_t goes down, that is, the US dollar depreciates against the Australian dollar. In 2008, the market was dominated mostly by the chartists for a prolonged period, when the US dollar appreciated substantially against the Australian dollar. The scatter plot clearly shows a logistic curve, suggesting a smooth transition between fundamentalists and chartists. The curve is more dense in the lower part (below 0.5), indicating a relative smaller portion of fundamentalists in the market.

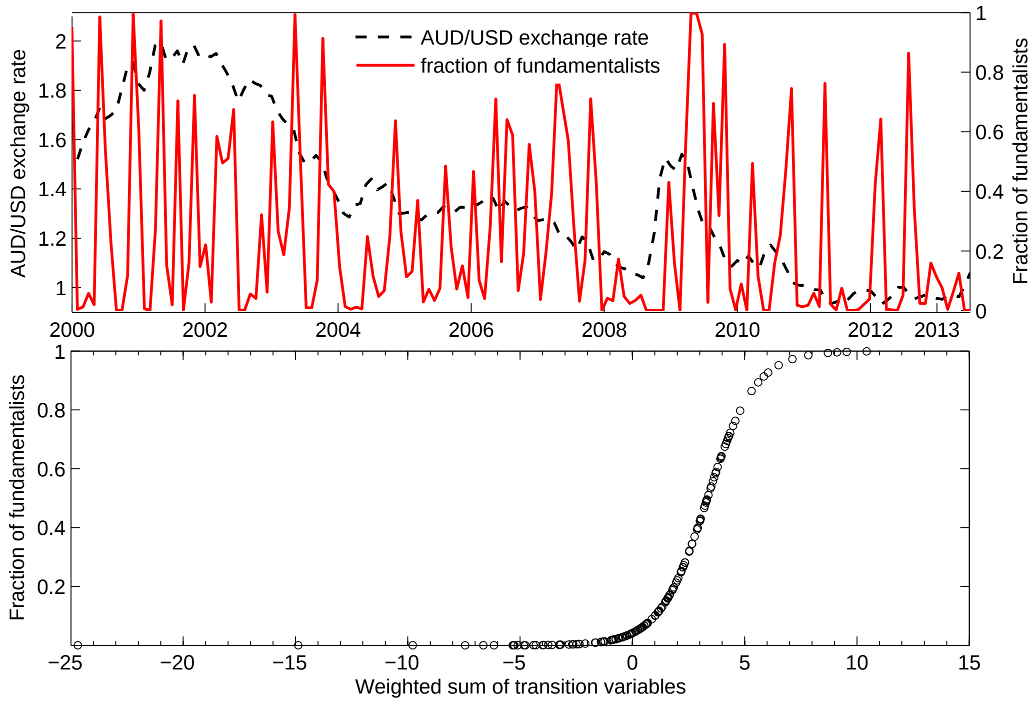

Figure 3: Fraction of Fundamentalists Estimated from LSTR

4.3 MS Estimation Results

The Markov regime switching model, defined by Eqs. (9) - (12), is estimated using maximum likelihood (Hamilton, 1994, Chap. 22). The estimation results are summarized in Table 5. We use state 0 to denote a boom state where the Australia dollar is appreciating (decreasing S_t) against the US dollar with a relatively low volatility (low risk) and state 1 to denote a bust state where the Australia dollar is depreciating (increasing S_t) against the US dollar with a relatively high volatility (high risk).

Since both γ and ω_f are positive, $\gamma\omega_f\alpha_{f0} < 0$ and $\gamma\omega_f\alpha_{f1} > 0$ suggest that $\alpha_{f0} < 0$ and $\alpha_{f1} > 0$. This implies the dominance of carry traders in the boom state (state 0) and UIP traders dominate in the bust state (state 1). In bust state (state 1), the large deviation of AUD/USD nominal exchange rate from its fundamental value as shown in Figure 1 fades away quickly, as the UIP traders who are dominating are mean-reverting. As most of the sample period is covered by the boom state (state 0), we should expect an overall dominance of carry traders from 2000:1 to 2013:6. This is consistent with the observations from the estimation results of the BHM and LSTR models. The positive estimates of $\gamma\omega_c\lambda_0$ and $\gamma\omega_c\lambda_1$ suggest that the chartists expect the exchange rate movement to persist even though the intensity of the bandwagon effect may be different. This finding is also consistent with the results estimated based on the BHM and LSTR models. In the bust state, the degree of bandwagon expectation is larger than that in the boom state, suggesting a more pessimistic sentiment when the Australian dollar is depreciating against the US dollar. Both the different behavior among the fundamentalists and the different behavior among the chartists in different states provide evidence of within-group heterogeneity over time. The regime-dependent standard deviations are estimated to be $\sigma_0 = 0.029$ and $\sigma_1 = 0.062$ in states 0 and 1, respectively. The volatility in bust state is twice as much as that in the boom state, suggesting that the market is more sensitive to external news/shocks and thus exhibits higher volatility in the bust state than in the boom state.

The switching of the beliefs of the fundamentalists and chartists and the changing market volatilities between the two states are indicated by the transition probabilities. The fundamentalists do not fix their strategies over time. Instead, they switch from UIP traders to carry traders or vice versa with a time-varying probability. Similarly, the chartists adjust their degree of bandwagon expectation according to the market condition which can be differentiated by the state variable m_t. In addition, the overall market sensitivity to external news/shocks are conditioned on the state variable. The results in Table 5 show that the probability of remaining in the boom state is 0.986 ($P_{0,0}$), suggesting that the boom state is persistent on an average of $1/(1-P_{0,0}) \approx 71$ months. It means that the probability for the fundamentalists to stay in the carry trading strategy, the chartists to have a smaller degree of bandwagon expectation and the market to be less volatile is 0.986. Given the two states identified, this also means that the probability for the fundamentalists and chartists to switch their beliefs from the boom state to the bust state is 0.014 ($P_{1,0}$). Similarly, the fundamentalists maintain their UIP trading strategies and the chartists maintain higher degree of bandwagon expectation in the bust state with a probability 0.931 ($P_{1,1}$), and switch to the boom state with a probability of 0.069 ($P_{0,1}$).

Table 5: Estimation Results for the MS Model

Parameter	$\gamma\omega_f\alpha_{f0}$	$\gamma\omega_f\alpha_{f1}$	$\gamma\omega_c\lambda_0$	$\gamma\omega_c\lambda_1$	σ_0	σ_1	$P_{0,0}$	$P_{0,1}$
Coefficient	-2.231	2.043	0.223	0.439	0.029	0.062	0.986	0.069
p-Value	0.010	0.592	0.021	0.007	0.000	0.000	0.000	0.167

Figure 4 demonstrates that the fitted value based on the MS model matches well with the AUD/USD exchange rate series and the 1-step ahead prediction captures the ups and downs of the exchange rate movements even when they are at extreme. The shaded areas are the periods classified as state 1 or the bust state by the estimated model.

With the estimates of the transition probabilities, we can further calculate the smoothed probability $P(m_t = j|p_1,...,p_N)$ for each period, conditioning on the whole price series $\{p_1,...,p_N\}$ (for details of the algorithm, see Kim and Nelson, 1999). Figure 5 shows the smoothed probability for α_{ft}, λ_t and σ_t to fall into the two states over the sample period of January 2000 to June 2013. The bust periods corresponding to state 1 cover the 2000 - 2001 falling market confidence in Australia and the 2008 - 2009 global financial crisis. The boom periods corresponding to state 0 include the time from late 2001 to early 2008, when the financial market was prospering in Australia. Recently, after the Australian dollar depreciates to the bottom in the late 2008, fundamentalists and chartists became optimistic with the global recovery and switched their estimates from regime 1 to regime 0.

Table 6 further classifies the sample periods according to the regime switching probability.

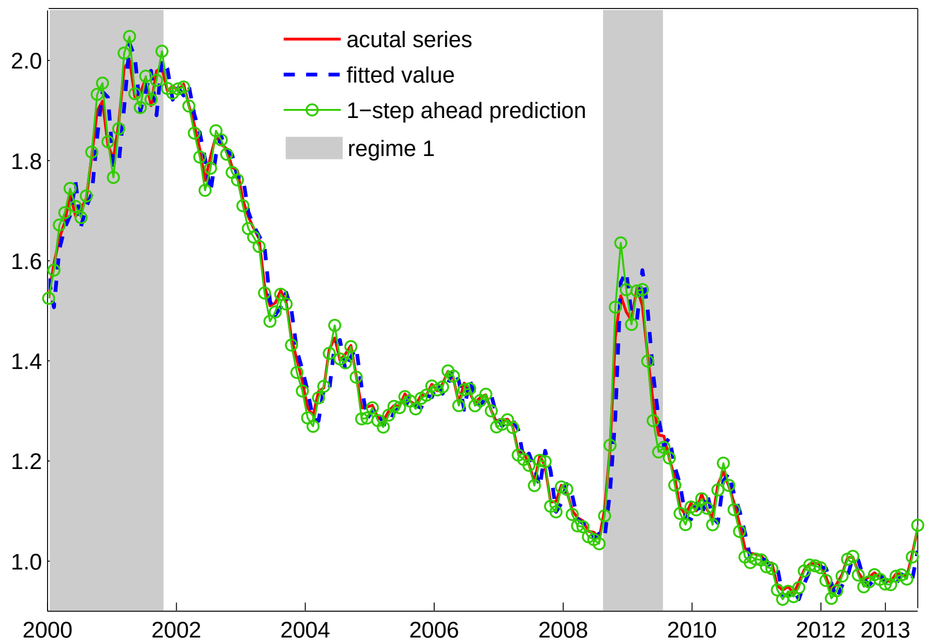

Figure 4: Fitted Value and 1-step-ahead Prediction from the MS Model

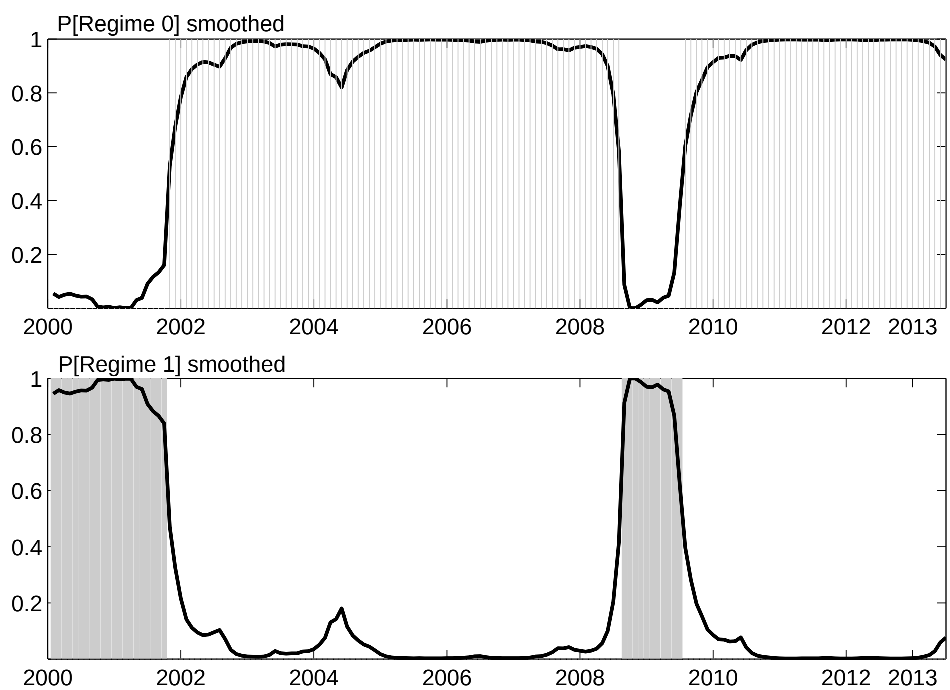

Figure 5: Smoothed Transition Probability.

Among the 162 periods, 130 months (80.25%) are associated to regime 0 when the market is in the boom state and the remaining 32 months (19.75%) are in regime 1 when the market is in the bust state. The average duration to remain in regime 0 consecutively is 65 months, which is much longer than that in regime 1 (16 months). However, given the incomplete episodes at the beginning and at the end of our sample, it does not necessary mean that tranquil episodes are more persistent than turbulent episodes.

Table 6: Regime Classification from the MS Model

Regime	Start	End	Avg. Prob.	Months	Total months
Regime 0	2001(10)	2008(07)	0.947	82	130
	2009(07)	2013(06)	0.960	48	
Regime 1	2000(01)	2001(09)	0.954	21	32
	2008(08)	2009(06)	0.929	11	

Overall, the MS estimation results provide evidence on the co-existence of time-varying within-group heterogeneity and between-group heterogeneity. Most interestingly, although the dates of domestic economic instabilities and financial crises are not used in any way to estimate the parameters or form inference about transition probabilities, the classified regimes match well with the market booms and busts in actual episodes.

5 Efficiency Tests

In this section, we apply several measures to evaluate the goodness-of-fit of the three regime-switching models and aim to identify the most well-specified model. Table 7 presents the log-likelihood, Akaike Information Criterion (AIC)[7], the p-values of the linearity likelihood ratio test and the three misspecification diagnostics computed from the model residuals. Following Deschamps (2008), the three misspecification diagnostic tests are:

1. The Jarque-Bera statistic, used as an indicator of error non-normality;

[7] In comparing the in-sample fit, we use the AIC in addition to the log-likelihood. The advantage of AIC is that it can deal with the trade-off between the goodness of fit of the model and the complexity of the model. AIC not only rewards goodness of fit, but also includes a penalty that is an increasing function of the number of estimated parameters. This penalty discourages over-fitting.

2. A χ^2-statistic for the nullity of the autoregression coefficients in an AR(12) model of the residuals. This is used as an indicator of error autocorrelation, and is denoted by AC(12).

3. An F-statistic for the nullity of the autoregression coefficients in an AR(12) models of the squared residuals. This is used as an indicator of error conditional heteroscedasticity, and is denoted by ARCH(12).

Table 7: Goodness-of-fit and Misspecification Tests

Model	log-likelihood	AIC	Linearity LR	Jarque-Bera	AC(12)	ARCH(12)
BHM	291.939	-3.567	0.729	0.000	0.129	0.214
LSTR	318.272	-3.855	0.000	0.000	0.570	0.010
MS	305.850	-3.677	0.000	0.691	0.315	0.334

In terms of in-sample estimation, the log-likelihood and AIC suggest that the LSTR model has a better fit than the BHM and MS models. The p-values for the likelihood ratio tests strongly reject the null hypothesis of linearity in favor of the LSTR and MS models. However, when comparing the BHM model with the linear model, the results fails to reject the linearity. This finding is consistent with the BHM estimation results that there is no significant switching between the chartists and fundamentalists. The estimated p-values of the Jarque-Bera statistics indicate non-normality of residuals for the BHM and LSTR models. This result could be attributed to the different volatilities of the foreign exchange market in different states, which are modeled explicitly in the MS model but not the BHM or the LSTR models. The estimated p-values for error autocorrelation (AC(12)) and error conditional heteroscedasticity (ARCH(12)) are all larger than 0.01, with the exception of that for ARCH(12) in the LSTR model. Therefore, we conclude that there is weak evidence of conditional heteroscedasticity in this model.

6 Predictive Power

Besides the in-sample efficiency, another important criterion to evaluate the different model specifications is the out-of-sample predictability. We compare the out-of-sample forecasting accuracy of the three regime-switching models, by dividing the data sample into two segments: one for the in-sample estimation and the other for the out-of-sample comparison. Specifically, we use data from 2000:1 to 2012:5 to estimate the models, on the basis of which we forecast the exchange rate from 2012:6 to 2013:6. The forecast series are then compared with the actual exchange rate as shown in Figure 6. The model with switching between strategies based on macroeconomic fundamentals (LSTR) exhibits better forecasting accuracy in the short run (3-month horizon). However, the other two regime-switching models (BHM and MS) tend to outperform in a longer horizon. The regime-switching based on past performance (BHM) gives the best out-of-sample prediction accuracy in the long run.

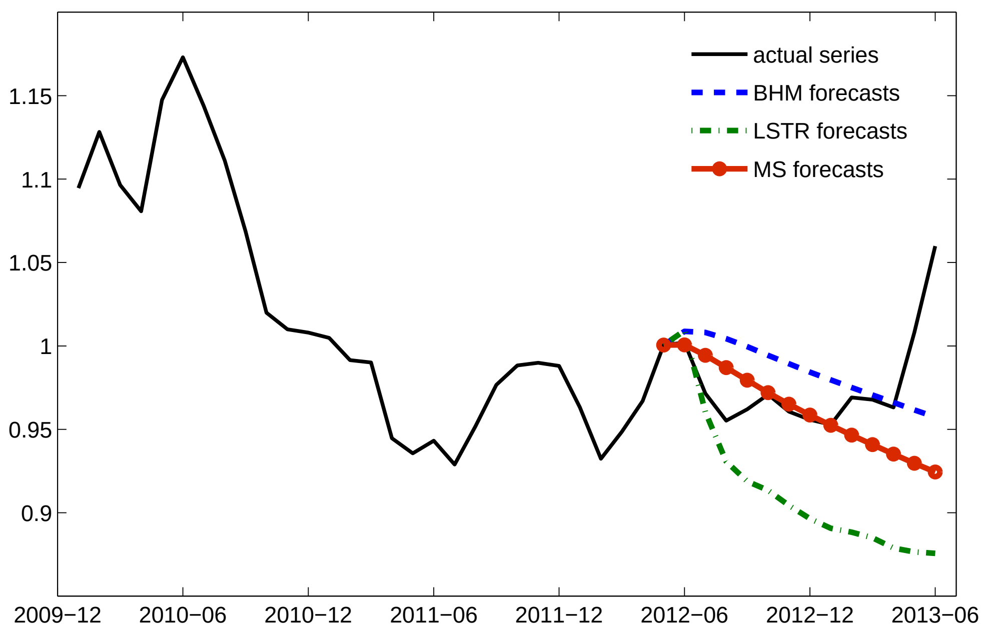

Figure 6: Comparison of Out-of-sample Predictability

In the above exercises, the separation of the sample is somewhat arbitrary, a natural question is whether the results is sensitive to the sample selection. As a robustness check, we apply a rolling forecasting technique with a fixed sample size and then compare the forecasting performance using the root-mean-squared forecast error (RMSE) and mean absolute error (MAE) for different forecasting horizons. The significance of the difference in forecasting differences is tested by the Diebold and Mariano (1995) test statistic.

Specifically, for each regime-switching model, we calculate the h-months-ahead forecasting error, for $h = 1, 2, \ldots, 12$. The forecasting recursion is based on a rolling estimation window with a fixed sample size, which is 108 (9 years of data) in our practice. First, the model is estimated for the period 2000:1 to 2008:12, that is, the first 108 observations $(S_1, S_2, \ldots, S_{108})$, and then obtain the first h-month-ahead forecasts $(\hat{S}_{108+1}, \ldots, \hat{S}_{108+h})$. Similarly, we obtain the second h-month-ahead forecasts $(\hat{S}_{109+1}, \ldots, \hat{S}_{109+h})$ from observations $(S_2, S_3, \ldots, S_{109})$. Such a process is repeated until we obtain the last h-month ahead forecasts $(\hat{S}_{162-h+1}, \ldots, \hat{S}_{162})$ from observations $(S_{55}, S_{56}, \ldots, S_{162-h})$ (162 is the total number of observations in the full sample). This recursion leads to a sequence of $k_h = 162 - 108 + 1 - h = 55 - h$ density forecasts. Based on recursive forecasting, we calculate h-step-ahead RMSE and MAE as:

$$RMSE_h = \sqrt{\frac{\sum_{t=108}^{t=162-h} (\hat{S}_{t+h} - S_{t+h})}{k_h}}, \qquad MAE = \frac{\sum_{t=108}^{t=162-h} |\hat{S}_{t+h} - S_{t+h}|}{k_h}$$

Table 8 presents the RMSE's (Panel A) and MAE's (Panel B)of the three models and the Diebold-Mariano test statistics (Panel C) for the 12 forecast horizons. Note that the smaller the value of the RMSE and MAE, the better the forecasting accuracy. Panel *A* contains the ratio of the RMSE of any two models. They are BHM vs. LSTR, BHM vs. MS and LSTR vs. MS. A ratio < 1 (> 1) indicates a better forecasting performance for the model mentioned first (second). Panel *C* reports the Diebold-Mariano test statistics of equality of forecasting performance. We use the RMSE as loss function with a rectangular lag window with $h - 1$ sample autocovariances for the h-step-ahead forecast error. A negative (positive) number indicates better performance for the model mentioned first (second).

The results in Panels A and B suggest that the LSTR model generally outperforms both the BHM model and the MS model in short run forecasts (up to 3-month horizon), while both the MS and BHM models outperform the LSTR model in the medium to long run. The BHM model

Table 8: Forecasting Performance

Steps	Panel A: Ratio of RMSE's			Panel B: Ratio of MAE's			Panel C: Diebold-Mariano		
	BHM /LSTR	BHM /MS	LSTR /MS	BHM /LSTR	BHM /MS	LSTR /MS	BHM /LSTR	BHM /MS	LSTR /MS
1	1.396	0.928	0.665	1.515	0.937	0.618	3.506***	-2.057***	−3.175***
2	1.267	0.913	0.720	1.222	0.944	0.772	1.181	-1.113	-1.165
3	1.087	0.872	0.802	0.968	0.912	0.942	0.375	-1.220	-0.707
4	0.980	0.833	0.850	0.922	0.881	0.955	-0.086	-1.220	-0.476
5	0.884	0.803	0.909	0.857	0.846	0.987	-0.509	-1.218	-0.265
6	0.855	0.778	0.910	0.777	0.798	1.026	-0.556	-1.218	-0.237
7	0.821	0.769	0.937	0.764	0.784	1.025	-0.739	-1.237	-0.169
8	0.787	0.763	0.969	0.767	0.768	1.001	-1.090	-1.230	-0.094
9	0.721	0.753	1.044	0.721	0.760	1.055	-2.215**	-1.242	0.149
10	0.666	0.743	1.116	0.681	0.767	1.126	-6.296***	-1.259	0.463
11	0.612	0.725	1.184	0.614	0.733	1.192	-3.465***	-1.302	0.668
12	0.559	0.707	1.264	0.566	0.702	1.240	-27.52***	-1.402	1.095

Notes: *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively.

always performs better than the MS model in terms of RMSE and MAE. For both the BHM and MS model, we observe that the forecasting power of the model vis-à-vis the LSTR model generally improves as the forecasting horizon increases. Similar observations are found in the the forecasting power of the model BHM vis-à-vis the MS model, although the improvement is relatively smaller.

In terms of significant improvements as presented in Panel C, we observe a similar pattern. In terms of the 1-month-ahead forecasts, the differences of the three models are highly significant, where the LSTR model has the best performance while the MS has the worst performance. The good performance of the LSTR model dies out in the medium to the long run. The BHM model performs significantly better than the LSTR in 9- to 12-month step ahead forecasts. The BHM model is always doing better than the MS model, however this difference is only significant for the 1-step-ahead forecast.

7 Conclusion

We evaluate the performance of three switching mechanisms, developed by Boswijk et al. (2007, BHM), Lof (2012, LSTR) and Chiarella et al. (2012, MS), in estimating behavioral heterogeneity. The BHM switching mechanism highlights the importance of past performance in determining the dynamic weight of heterogeneous trading rules. The LSTR switching model emphasizes the role of macroeconomic fundamental in shaping the trading behavior and affecting the choice of the trading strategy. The MS model addresses how agents shift their expectations by extrapolating the financial market conditions from the price information and change their trading behavior accordingly. Applying AUD/USD monthly exchange rate data from 2000:1 to 2013:6, we document empirical evidence on the comparative advantage of the three switching models. While the LSTR model provides better in-sample explanatory power than the other two, there is significant evidence to support the performance of the BHM model in terms of its out-of sample forecasting accuracy. There is, however, no significant evidence that either the BHM or the LSTR model outperforms the MS model in terms of predictive power. All the three models have consistently confirmed the presence of behavior heterogeneity. More interestingly, both the LSTR and the MS models highlight a significant dominance of carry traders in the AUD/USD foreign exchange market.[8] The carry traders sell the currency of low-interest country and buy the currency of high-interest country expecting to profit from interest rate difference. This finding is in line with other findings in literature that the influence of carry traders on exchange rates is large and increasing (Galati et al., 2007; Pojarliev and Levich, 2011). The role of carry traders has also been highlighted by Spronk et al. (2013) using simulations to generate stylized facts observed in empirical exchange rates.

In reality, the three different types of regime switching mechanisms can co-exist. In this paper, we isolate the three types and investigate which type can fit the AUD/USD exchange rate data better. But this superiority is not in absolute terms and we cannot rule out the co-existence of other types of switching in the market. The relative good performance of the LSTR model suggests that agents in the AUD/USD foreign exchange market are more responsive to the changes in the macroeconomic fundamentals than to the past profit and to the market states.

Future research in this area may include an investigation of the other financial markets, such as commodity market and stock market, in order to compare the relative performance of the three

[8]The results however cannot rule out the presence of UIP traders. The BHM model also suggests the dominance of carry traders, even though it is not statistically significant. The conclusion, however, is restricted to AUD/USD exchange rate and hence should not be generalized.

switching models in different markets. Another future research direction would be to improve the empirical estimation of the fundamental exchange rate, by taking into account the central bank's monetary policy. Other possible directions include innovating a model which is able to integrate the three types of regime switching and check whether such an integration improves the overall empirical performance.

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A Other LSTR Estimation Results

In the linearity tests for the LSTR model (see Section 3.2), four sets of transition variables are selected, which are $\{REER\}$ (see Table 9), $\{REER, M1\}$ (see Table 10), $\{REER, CPI, STY\}$ (see Table 2) and $\{REER, CPI, STY, UNE\}$ (see Table 11). In the main text, the estimation results with three transition variables $\{REER, CPI, STY\}$ are discussed (referred to as LSTR3 hereafter). In this appendix, we present results for the LSTR model with one, two and four transition variables, denoted as LSTR1, LSTR2 and LSTR4, respectively.

Table 9: Linearity Tests: Univariate Transition Function

X_{t-1}	p-value	Model	X_{t-1}	p-value	Model
UNE	0.0981	Linear	STY	0.0626	Linear
$REER$*	0.0000	LSTR	$M1$	0.1546	Linear
LTY	0.3300	Linear	CPI	0.0865	Linear
IND	0.1585	Linear	GDP	0.3555	Linear

Notes: p-values of F-tests for linearity are presented. Column 'Model' indicates the suggested model by the tests. The variable that yields the strongest rejection of linearity is marked with *, which is $REER$.

Table 10: Linearity Tests: Two-variable Transition Function

X_{t-1}	p-value	Model	X_{t-1}	p-value	Model
(GDP, CPI)	0.3958	Linear	$(GDP, M1)$	0.2119	Linear
(GDP, LTY)	0.3667	Linear	(GDP, STY)	0.1453	Linear
(IND, CPI)	0.0079	LSTR	(IND, LTY)	0.4183	Linear
$(IND, M1)$	0.1653	Linear	(IND, STY)	0.0898	Linear
(CPI, LTY)	0.3888	Linear	(CPI, STY)	0.1733	Linear
$(M1, LTY)$	0.3002	Linear	$(M1, STY)$	0.1698	Linear
$(REER, IND)$	7×10^{-4}	LSTR	$(REER, CPI)$	2×10^{-57}	LSTR
$(REER, LTY)$	6×10^{-14}	LSTR	$(REER, M1)$*	3×10^{-59}	LSTR
$(REER, STY)$	1×10^{-14}	LSTR	$(REER, UNE)$	1×10^{-9}	LSTR
(UNE, CPI)	0.2016	Linear	(UNE, LTY)	0.1320	Linear
$(UNE, M1)$	0.0566	Linear	(UNE, STY)	0.0324	LSTR
$(REER, GDP)$	4×10^{-12}	LSTR			

Notes: p-values of F-tests for linearity are presented. Column 'Model' indicates the suggested model by the tests. The set of variables that yields the strongest rejection of linearity is marked with *, which is $(REER, M1)$*.

Table 12 summarizes the estimation results. Consistent with the precious estimation for LSTR3, the results for LSTR1, LSTR2 and LSTR4 all identify two distinct regimes with statistically significant $\gamma\alpha_f$ and $\gamma\lambda$, implying the presence of between-group behaviorial heterogeneity. For all cases, $\gamma\alpha_f$ is negative and $\gamma\lambda$ is positive, suggesting that carry traders dominate among the fundamentalists while the bandwagon expectations dominate among the chartists. The intensity of choice parameter τ is significant for LSTR1 and LSTR2, but not for LSTR4. As mentioned earlier, this should not be worrying as long as there is significant heterogeneity in the estimated regimes. The estimated A are not significant for LSTR2 while a_1 and a_3 are significant for LSTR4. Despite the insignificance, the interpretations for A still reveal that fundamentalism is the dominant strategy during relative recessive periods in Australia compared to US, signalled by high $REER$, high $M1$, low CPI, low STY and high UNE.

Figure 7, Figure 8 and Figure 9 plot the fraction of fundamentalists $\omega_{f,t}$ and its scatter plot against the weighted sum of transition variables $X_{t-1}A$, for each model respectively. For all these three models, we have similar observation as the LSTR3 results. Both fundamentalists and chartists are represented in the economy in most of the time. The scatter plots clearly suggest smooth transition between fundamentalists and chartists.

In terms of goodness-of-fit, the LSTR1, LSTR2 and LSTR4 models are inferior compared to the LSTR3 model as shown in Table 13. For all the LSTR models, the likelihood ratio tests reject the linear model. However, the p-values for Jarque-Bera statistics imply non-normality of residuals from the estimated models. The estimated p-values for error AC(12) and ARCH(12) suggest weak evidence of conditional heteroscedasticity in the LSTR2 and LSTR3 models, and error autocorrelation in the LSTR2 model.

Figure 10 compare the out-of-sample forecasting performance of all LSTR models together with the BHM and MS models. Among the LSTR models, LSTR1 and LSTR2 exhibit similar forecasting accuracy, which is slightly better than that of LSTR3 and LSTR4 in the medium to long horizons. Overall, the LSTR models show better forecasting power in the short run but poorer performance in the medium to long run.

Table 11: Linearity Tests: Four-variable Transition Function

X_{t-1}	p-value	Model
$(REER, GDP, CPI, LTY)$	1.82×10^{-9}	LSTR
$(REER, GDP, CPI, STY)$	2.22×10^{-8}	LSTR
$(REER, GDP, M1, LTY)$	9.12×10^{-8}	LSTR
$(REER, GDP, M1, STY)$	6.93×10^{-7}	LSTR
$(REER, IND, CPI, LTY)$	1.28×10^{-8}	LSTR
$(REER, IND, CPI, STY)$	2.10×10^{-8}	LSTR
$(REER, IND, M1, LTY)$	8.03×10^{-7}	LSTR
$(REER, IND, M1, STY)$	5.11×10^{-6}	LSTR
$(REER, UNE, CPI, LTY)$	1.83×10^{-9}	LSTR
$(REER, UNE, CPI, STY)$*	2.62×10^{-12}	LSTR
$(REER, UNE, M1, LTY)$	2.53×10^{-9}	LSTR
$(REER, UNE, M1, STY)$	2.25×10^{-9}	LSTR

Notes: p-values of F-tests for linearity are presented. Column 'Model' indicates the suggested model by the tests. The set of variables that yields the strongest rejection of linearity is marked with *, which is $(REER, UNE, CPI, STY)$.

Table 12: Estimation Results for the other LSTR models

Model	Parameter	$\gamma\alpha_f$	$\gamma\lambda$	τ	a_1	a_2	a_3	a_4	c
LSTR1	Coefficient	-15.856	0.351	-3.176	.	.	.	.	0.764
	p-Value	0.000	0.000	0.031	.	.	.	.	0.214
LSTR2	Coefficient	-14.687	0.363	-3.939	0.947	0.053	.	.	0.612
	p-Value	0.000	0.000	0.041	0.340	0.719	.	.	0.237
LSTR4	Coefficient	-19.550	0.370	-0.939	3.535	-0.281	-2.257	0.002	3.340
	p-Value	0.000	0.000	0.637	0.000	0.163	0.000	0.993	0.000

Notes: In LSTR2, a_1 and a_2 are the coefficients for transitions variables $REER$ and $M1$, respectively. In LSTR2, a_1, a_2 and a_3 are the coefficients for transitions variables $REER$, CPI, STY and UNE, respectively.

Table 13: Goodness-of-fit and Misspecification Tests of All LSTR Models

Model	log-likelihood	AIC	Linearity LR	Jarque-Bera	AC(12)	ARCH(12)
LSTR1	315.571	-3.846	0.000	0.000	0.267	0.569
LSTR2	315.821	-3.837	0.000	0.000	0.049	0.008
LSTR3	318.272	-3.855	0.000	0.000	0.570	0.010
LSTR4	318.272	-3.843	0.000	0.000	0.265	0.698

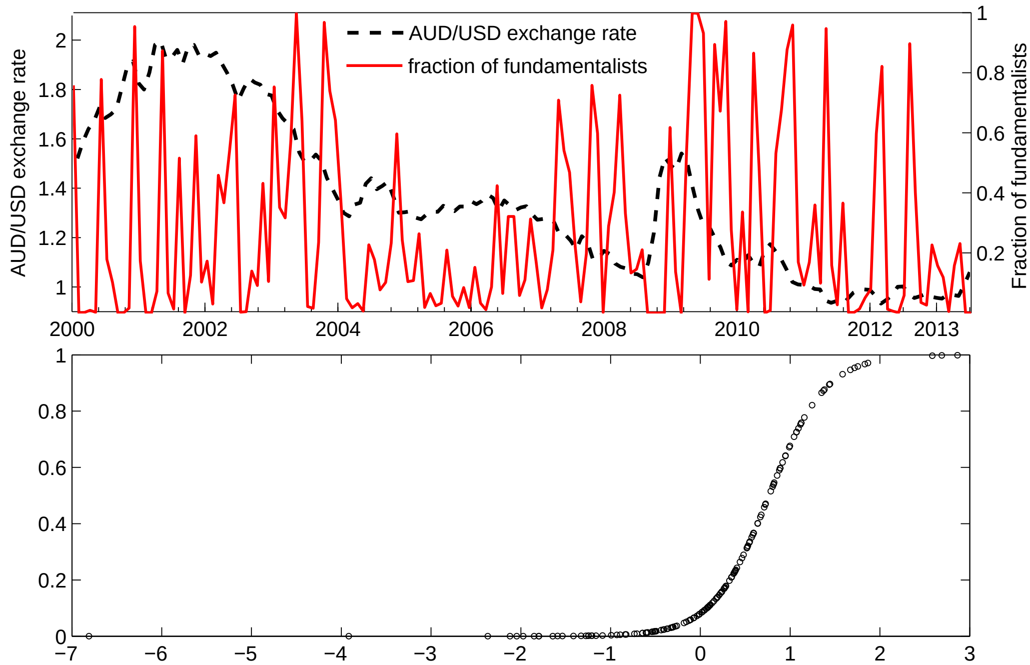

Figure 7: Fraction of Fundamentalists Estimated from LSTR1

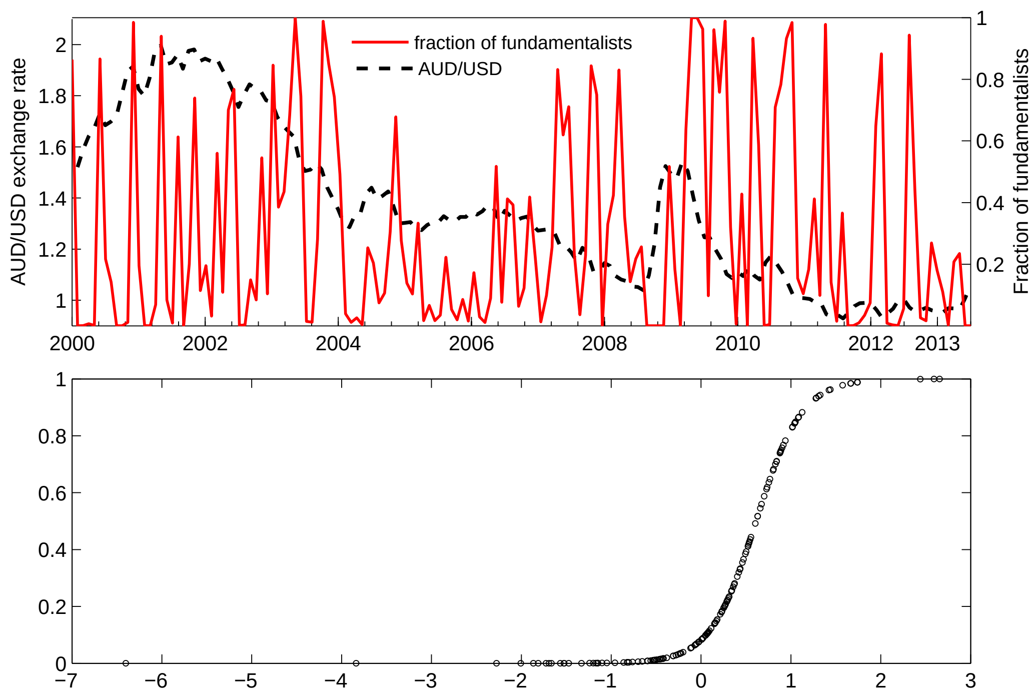

Figure 8: Fraction of Fundamentalists Estimated from LSTR2

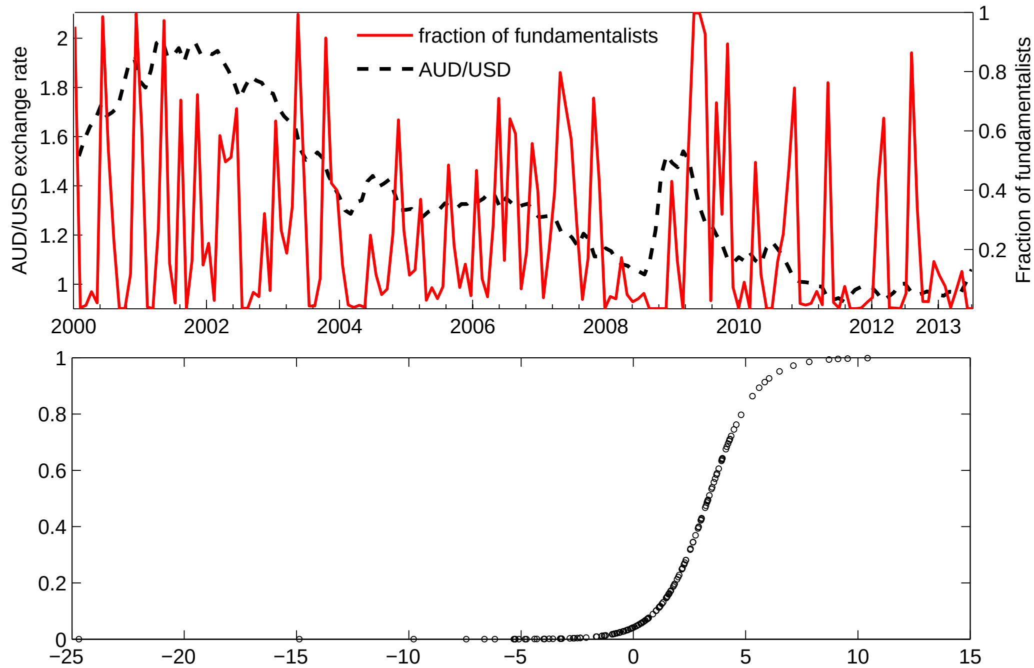

Figure 9: Fraction of Fundamentalists Estimated from LSTR4

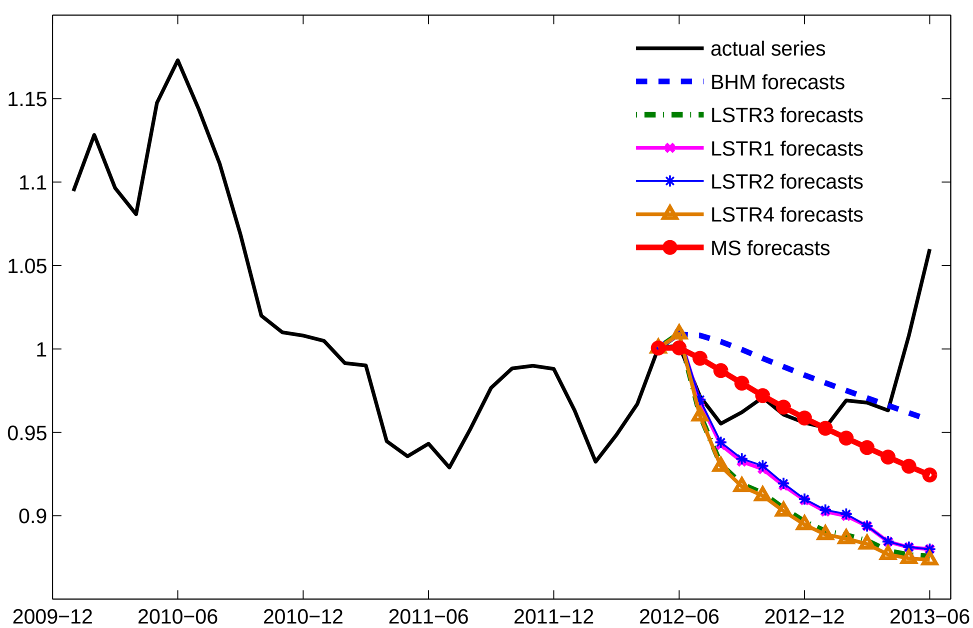

Figure 10: Comparison of Out-of-sample Forecasting Power of All Models