Jump Spillover in Energy Futures Markets: The Bayesian Viewpoint

Qingfu Liu
Institute for Financial Studies
Fudan University

Anthony H. Tu[*]
Department of Finance
College of Commerce
National Chengchi University

Abstract

In this paper, we investigate jump spillover effects between five energy (petroleum) futures. In order to identify the latent historical jumps of each energy futures, we use a Bayesian MCMC approach to estimate a jump-diffusion model on each energy futures. We examine the simultaneous jump intensities of pairs of energy futures and the probabilities that jumps in crude oil (and natural gas) cause jumps or usually large returns in other energy futures. In all cases, we find significant evidence of jump spillover.

Keywords: Bayesian factor; energy futures; jump-diffusion model; MCMC; spillover; stochastic volatility

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[*] Corresponding author. Department of Finance, National Cheng-Chi University, Taipei, Taiwan.
Tel: 886-2-29387423; Fax: 886-2-29393394; E-mail: atu1106@yahoo.com.tw

I. Introduction

There is strong evidence that jumps (price spikes) play an important role in the continuous diffusion process for asset price. Such models, which allow for the presence of jumps, are often referred to as event risk models. A number of recent theoretical studies analyze the impact of event risk on strategic asset allocation (Liu et al. (2003), Wu (2003)), on option pricing and its ability to explain the observed volatility smiles (Pan (2002), Eraker et al. (2003)), on calculations of risk measures such as value-at-risk (VaR) (Duffie and Pan (2001), Gibson(2001)). Recently, Asgharian and Bengtsson (2006) employ such an event risk model to study jump spillover effects between a number of country equity indexes. They use a Bayesian approach to estimate a jump-diffusion model on each index and find significant evidence of jump spillover.

The recent dramatic spikes in energy prices (in particular oil) that peaked in summer 2008 (see Figure 1) has greatly magnified the importance of understanding and managing risk in these markets. Observers of energy futures markets have long noted that energy futures prices are very volatile and often exhibit jumps during news event periods. The main purpose of this paper is to estimate an event risk model for a number of energy (petroleum) futures contracts (crude oil, natural gas, heating oil, gasoline and fuel oil) in order to identify the latent historical jump times of each energy futures which we then use to quantify the degree of jump spillover between the different futures contracts. We focus on two forms of jump spillover. First, we calculate the simultaneous jump intensities for pairs of energy futures contracts and we test whether or not these simultaneous jump intensities are significant. Second, we perform an analysis of conditional jump spillover to examine to what extent jumps in a specific energy futures increase the probability of jumps in other energy futures, or in a weaker form, cause unusually large negative returns in other energy futures.

To identify the historical jump times, we estimate a univariate jump-diffusion model with stochastic volatility on each energy futures contract. The model, which falls into the class of affine jump-diffusion models proposed by Duffie, Pan, and Singleton (2000), is referred to as the stochastic volatility with correlated jumps (SVCJ) model and it assumes that jumps in returns and volatility arrive simultaneously and that the jump sizes are correlated. The primary reason for estimating such a relatively complex model, instead of simply looking at, say the historically largest price movement of each index, is that in this way we can separate extreme returns that are actual jumps from large diffusive returns simply caused by periods of high volatility[2]. Including jumps in volatility allows for the rapid changes in volatility empirically found by, for instance, Bates (2000), Duffie, Pan, and Singleton (2000), Pan (2002), and Eraker, Johannes, and Polson (2003), and prevent estimated jump times from clustering. One reason for the relatively limited amount of

[2] Our approach does not require us to specify a priori or arbitrary what qualifies a return as extreme, since this is determined in the estimation.

empirical research on event risk is that the complexity of the models makes estimation comparatively difficult. Standard methods such as unconstrained maximum likelihood (ML) and the generalized method of moments (GMM) are, if applicable at all, intractable.

A relatively new approach based on Markov chain Monte Carlo (MCM) methods is used for the estimation of event risk models. The MCMC method to estimate stochastic volatility models was proposed by Jacquier, Polson, and Rossi (1994) and the method was extended to models with jumps in returns and volatility by Eraker, Johannes, and Polson (2003)[3]. MCMC methods are particularly attractive for application in our model for several reasons. First, MCMC is a unified estimation procedure which simultaneously estimates both parameters and state variables. It also estimates the latent processes of the model – the jump times, jump sizes and the future volatility path[4]. Second, MCMC methods account for estimation and model risk. It also allows the researcher to quantify model risk, the uncertainty over the choice of model through the computation of Bayes factor.

Our empirical results show strong evidence for the existence of jump spillover. The estimated simultaneous jump intensities are in general significantly larger than the corresponding intensities under the null hypothesis that the different energy futures' jump processes are independent of each other. Most interestingly, however, we find that the

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3 Other methods that have been used to estimate models with stochastic volatility and jumps include, for instance, the efficient method of moments (EMM) of Gallant and Tauchen (1996), simulated maximum likelihood (SML), the Spectral GMM (SGMM) of Chacko and Viceira (2003), and the implied-state GMM (ISGMM) of Pan (2002).

4 Jacquier, Polson, and Rossi (1994) find in simulations that MCMC outperforms GMM and QMLE in estimation of stochastic volatility models, and Anderson, Chung and Sorensen (1999) find that MCMC outperforms EMM.

historical sample correlations between the energy futures are not good measure to capture the jump spillover effects. This implies that the dependence between the jump processes of the different energy futures returns is quite different from the dependence between returns that are not jumps, implying for instance the mean-variance investors who use these correlations to form portfolios may have little protection against event risk. We also look at the sizes of the simultaneous jumps and find that they can be positive or negative and, in general, larger (in absolute terms) than the sizes of the energy-futures-specifics jumps. This implies that both good and bad news (or events) cause jump spillover.

In our analysis of conditional jump spillover from crude oil futures (and natural gas futures) to other energy futures, we also find strong evidence of jump spillover. A large majority of the estimated conditional jump spillover probabilities are significantly larger than the corresponding probabilities under the null hypothesis of independent jump processes.

The structure of the rest of the article is organized as follows: Section II presents the event risk model and the estimation method. Section III contains the empirical results, which, in addition to the analysis of jump spillover, includes a discussion on the estimated parameters and latent variables. The Appendix interprets the model accuracy through the comparison of Bayes factors. Section IV concludes.

II. Event Risk and Econometrics Model

The SVCJ model assumes that the logarithm of five energy (petroleum) futures price *i*, $F_{i,t}$, $i = 1,2,\ldots, 5$, solves the stochastic differential[5]

$$\begin{pmatrix} d\ln(F_{i,t}) \\ dV_{i,t} \end{pmatrix} = \begin{pmatrix} \mu_i \\ \kappa_i(\theta_i - V_{i,t-}) \end{pmatrix} dt + \sqrt{V_{i,t-}} \begin{pmatrix} dW_{i,t}^{\Upsilon} \\ \sigma_{V,i} dW_{i,t}^{V} \end{pmatrix} + \begin{pmatrix} \xi_{i,t}^{\Upsilon} \\ \xi_{i,t}^{V} \end{pmatrix} dN_{i,t} \qquad (1)$$

Where t- is the point in time that closest precedes time t; $dW_{i,t}^{\Upsilon}$ and $dW_{i,t}^{V}$ are standard one-dimensional wiener processes with instantaneous correlation ρ_i ; $N_{i,t}$ is a one-dimensional Poisson process with constant intensity λ_i; and $\xi_{i,t}^{\Upsilon}$ and $\xi_{i,t}^{V}$ are jump sizes. The jump size of futures volatility, $\xi_{i,t}^{V}$, is assumed to be exponentially distributed with $\mu_{V,i}$, and to allow for the return and volatility jump sizes to be correlated, $\xi_{i,t}^{\Upsilon}$ is assumed to be conditionally normally distributed with conditional mean $\mu_{\Upsilon,i} + \rho_{J,i}\, \xi_{i,t}^{V}$ and standard deviation $\sigma_{\Upsilon,i}$. The correlation between the diffusive terms is allowed for in order to capture the important leverage effect between return and volatility. Typically this correlation is expected to be negative, which induces negative skewness in returns [see Das and Sundaram (1999)]. In what remains of this section, we will drop the subscript *i*.

This specification nests many of the popular models used for option pricing and portfolio allocation applications. Without jumps, λ=0, (1) reduces to Heston's (1993) square-root

[5] An alternative model specification would be the stochastic volatility with independently arriving jumps (SVIJ) model, which also falls into the general class of models proposed by Duffie, Pan, and Singleton (2002). The SVIJ model assumes different processes for returns and volatility. We choose to work with the SVCJ specification in this article since in an event risk study it simplifies the analysis, and if there are indeed jumps in volatility, it is in some sense more intuitive to assume that major events affect both return and volatility rather than to assume that some events affect only returns and some events affect only volatility.

stochastic volatility model, the SV model. Bates' (1996) SVJ model has normally distributed jumps in returns, $\xi^{\Upsilon} \sim N(\mu_{\Upsilon}, \sigma_{\Upsilon}^2)$, but no jumps in volatility. Duffie, Pan, and singleton (2000) introduced the models with jumps in volatility. The SVIJ model has independently arriving jumps in volatility, $\xi^{V} \sim \exp(\mu_V)$, and jumps in returns, $\xi^{\Upsilon} \sim N(\mu_{\Upsilon}, \sigma_{\Upsilon}^2)$. The SVCJ model has contemporaneous arrivals, $N_t^{\Upsilon} = N_t^{V} = N_t$, and correlated jump sizes, $\xi^{V} \sim \exp(\mu_V)$ and $\xi^{\Upsilon} \big| \xi^{V} \sim N(\mu_{\Upsilon} + \rho_J \xi^{V}, \sigma_{\Upsilon}^2)$. In the Appendix, we interpret the computation of Bayes factors, which offer a summary of the evidence provided by our data in favour of our SVCJ model accuracy.

To estimate the SVCJ model with MCMC, Equation (1) is discretized over a time interval Δ using an Euler discretization[6]. The time discretization generates a much simpler conditional distribution structure and allows the use of standard MCMC techniques. The discretization interval is, since the data in our empirical study are daily, equals to one day (Δ=1) and the discretized version of the model is[7]

$$\begin{pmatrix} \Upsilon_{t+1} \\ V_{t+1} \end{pmatrix} = \begin{pmatrix} \mu \\ \alpha + (1+\beta) V_t \end{pmatrix} + \sqrt{V_t} \begin{pmatrix} \varepsilon_{t+1}^{\Upsilon} \\ \sigma_V \varepsilon_{t+1}^{V} \end{pmatrix} + \begin{pmatrix} \xi_{i,t}^{\Upsilon} \\ \xi_{i,t}^{V} \end{pmatrix} J_{t+1}, \tag{2}$$

6 An advantage of our approach is the ability to formally incorporate prior information. The need for this is not unique to our approach, but is common in estimating models with jumps. Honore (1998) shows that without prior parameter restrictions, a time discretization of Merton's (1976) jump-diffusion model generates an unbounded likelihood function. Moreover, the prior contains information about both the parameters and the structure of the latent process: the stochastic specifications of the jump size, jump times, and volatility. This reinforces the link between parameters and model specification that is often heuristically used to motivate the presence of jumps. Typically, jumps are described as large, but infrequent movements in returns. This is a form of prior information as the parameters are assumed to induce in frequently but relatively large movements, as opposed to frequent but small jumps.

7 The need for continuous-time process to be discretized is a drawback of the MCMC method in the sense that it can potentially introduce discretization biases when low-frequency data are used, However, in a simulation study, Eraker, Johannes, and Polson (2003), show that the biasesin MCMC estimates are very small for daily returns.

Where $\Upsilon_{t+1} = \ln(F_{t+1} / F_t)$ is the log return; J=1 indicates a jump arrival which occurs with probability λ; the drift parameters of the volatility process have been rewritten so that $\alpha = k\theta$ and $\beta = -k$; and $\varepsilon_{t+1}^{\Upsilon}$ and ε_{t+1}^{V} are standard normal stochastic variables with correlation coefficient ρ.

The MCM method for inference and parameter estimation is a Bayesian and simulation-based estimation method. This approach has at least four advantages over other estimation methods: (1) MCMC provides estimates of the latent volatility, jump times, and jump sizes; (2) MCMC accounts for estimation risk; (3) MCMC methods have been shown in related settings to have superior sampling properties to competing methods; and (4) MCMC methods are computationally efficient so that we can check the accuracy of the method using Bayes factors.

While traditional methods treat parameters and latent variables as unknown constants, the Bayesian approach is to treat them as random variables. The foundation of Bayesian analysis is the joint distribution of the parameters and the latent variables conditional on the data. This joint conditional distribution, referred to as the posterior distribution, is derived via Bayes' rule as

$$p(\Theta, V, J, \xi^V, \xi^\Upsilon \,|\, \Upsilon) \propto p(\Upsilon \,|\, \Theta, V, J, \xi^V, \xi^\Upsilon) p(V, J, \xi^V, \xi^\Upsilon \,|\, \Theta) p(\Theta), \tag{3}$$

Where Υ is a T×1 vector of observations; V, J, ξ^V, and ξ^Υ are vectors of latent futures volatilities, jump times, return jumps sizes, and volatility jump sizes, respectively; and Θ is a vector of parameters. The first term on the right-hand side of Equation (3), $p(\Upsilon|\Theta, V, J, \xi^V, \xi^\Upsilon)$, is the likelihood of the data; the second term, $p(V, J, \xi^V, \xi^\Upsilon|\Theta)$, is the (prior) distribution of the latent variables conditional on the parameters; and the last term, $p(\Theta)$, is the prior distribution of the parameters. The Bayesian parameter point estimates of the parameters and the latent variables are typically taken as their posterior means. While knowledge about the normalizing constant is not required, the prior distribution of the parameters has to be specified independently of the data by the researcher (the prior distribution of the latent variables, conditional on Θ, is specified by the model assumptions.). It can be thought as a natural way to impose nonsample information, if there is any, and to impose stationarity and nonnegativity where it is needed. If there is no nonsample information to be imposed, the prior is usually choosen so that it is as uninformative as possible—typically with a very large variance over the relevant parameter space, which is what we do in this article.

The posterior distribution of Equation (3) is extremely complex and nonstandard, with no existing closed-form solution. Consequently simulation-based methods have to be used to explore it. MCMC methods generate a sequence of draws $\{\Theta^{(j)}, V^{(j)}, \xi^{V(j)}, \xi^{\Upsilon(j)}\}_{j=1}^{M}$, which is a Markov chain with equilibrium distribution equal to the posterior distribution. Using this generated sample from the posterior distribution, the point estimates of Θ, V, J, ξ^V, and

ξ^{Υ} are then simply given by their posterior sample means.[8]

In this article we are particularly interested in estimating the latent historical jump times J of each index. The point estimate of J is

$$\hat{J} = \frac{1}{M} \sum_{j=1}^{M} J^{(j)} .$$

It is important to note that this estimate will, unlike the "true" vector of jump times, not be a vector of ones and zeros. Rather, element $t, \hat{J}_t$, is the posterior probability that there was a jump at time t. Following Johannes, Kumar, and Polson (1999), a natural and simple approach to construct from $\hat{J}$, the vector of jump times, is to assert that a jump has occurred if the estimated jump probability is sufficiently large; that is, greater than an appropriately chosen threshold value l, so that

$$\hat{J}_t^* = \begin{cases} 1 & if \quad \hat{J}_t > l, \\ 0 & if \quad \hat{J}_t \le l. \end{cases} \tag{4}$$

In our empirical study we chose l so that the number of inferred jump times divided by the number of observations—the implied jump intensity—is consistent with the estimated jump intensity λ. For simplicity and for consistently, we follow Asgharian and Bengtsson (2006) and use the same value of l for all futures and we choose l=0.1702, since this is the value that turns out to minimize the average distance between the implied jump intensities and the

[8] The details on the MCMC algorithm used to estimate the SVCJ model Equation (2) (3) can be found in Asgharian and Bengtsson (2006) and further details on the theory behind MCMC methods can be found in Johannes and Polson (2004). They are omitted to save space, but are available upon request.

actual estimated jump intensities.

III. Empirical Results

In this section we present and analyze our empirical results. After a presentation of the data, we provide a short discussion on the parameter and the latent variable estimates. Then follows the analysis of jump spillover between pairs of energy futures contracts.

1. Data Description

In this study, we examine jump spillover for five contracts from the European energy futures markets. The contracts are on crude oil futures, natural gas futures, heating oil futures, gasoline futures and fuel oil futures, all of which are traded on the ICE futures Europe. The data are between September 29, 2006 and October 29, 2008 and all data are from Datastream. The daily log return of each futures contract is

$$Y_t = \log(F_t / F_{t-1}),$$

where F_t is the settled futures price at date *t.* Descriptive statistics for the data can be found in Table 1.[9] Figure 1 shows price trends for five energy futures corresponding to the analyzed period and indicates that the oil price have mainly fluctuated relative to other four energy prices. From these prices we calculated log-returns and volatilities and plot them in

[9] It can be argued that the effects of differences in market opening hours can be avoided, for example, weekly data. However, lower frequency data would smooth out the effects of jumps and invalidate the assumption that at most one jump can occur per discretization interval. A better alternative would instead be to use higher frequency data, such as hourly returns, in order to really find out when in the day the jumps have occurred and how fast they are transmitted across borders. Unfortunately, high-frequency datasets that go back sufficiently long in time are hard to obtain for more than perhaps international equity indexes. Consequently, our best alternative is to use daily data, taking into account in our analysis that differences in market opening hours are present.

Figure 2. Figure 2 also shows the probabilities of jumps, downside jumps and upside jumps.

2. Estimated Parameters and Latent Variables

The MCMC estimates for the parameters of SVCJ model and for the different energy futures contracts are presented in Table 2.

Since this article is focused on the analysis of jumps, we begin by examining the estimated unconditional average sizes of jumps in returns, which for each energy futures equals $\mu_{\Upsilon} + \rho_J \mu_V$. The values for crude oil, natural gas, heating oil, gasoline and fuel oil are, respectively, 2.3263, 4.8152, 2.0413, 2.4158, 2.6474. Thus, the futures that has the largest (in absolute terms) unconditional average size of jumps in returns is natural gas, closely followed by fuel oil and gasoline. The futures that has the smallest unconditional average size of jumps in returns is heating oil, closely followed by crude oil. The estimate of ρ_J, which measures the dependency between the size of jumps in returns and the size of jumps in volatility, is positive (negative) for natural gas, heating oil and gasoline (crude oil and fuel oil).

The parameter λ, which is the arrival intensity of jumps, is estimated at values between 0.0853 for natural gas and 0.1118 for heating oil. So jumps in the heating oil market are approximately 1.5 times more frequent than jumps in the natural gas market.

Although it has no direct connection to jumps, it might be interesting to look at the estimated correlation coefficient ρ between diffusive shocks to return and volatility for each futures. This parameter is, negative for all futures and significantly. Unlike the equity market evidence, the energy futures prices exhibit very little leverage effect. As expected, the jump correlation coefficients ρ_J are positive for natural gas, heating oil and gasoline.

Figure 2 shows, together with the historical log returns and volatilities, the estimated jump probabilities for up jumps and down jumps for the crude oil, the natural gas, the heating oil, the gasoline and the fuel oil. The estimated jump probabilities appear to be clustered during periods of high volatility. The relatively high jump intensity of heating oil compared to that of, for example, the natural gas is illustrated by the relatively large number of days with high jump probabilities. Looking at the estimated futures volatility paths in Figure 2, it is interesting to see that the futures volatilities appear to be quite correlated.

The lower triangular part of Table 3 shows the sample correlation matrix of the estimated futures volatility changes, $\sqrt{\hat{V}_{i,t}} - \sqrt{\hat{V}_{i,t-1}}$, $t = 1, 2, ..., T$, for all future contracts $i = 1, 2, ..., 5$. The correlation coefficients are all positive and the largest correlation is between the crude oil and the heating oil, while the smallest correlation is between crude oil and natural gas. A potentially important issue is whether or not the presence of jumps make biased estimates of correlation coefficients between futures volatility changes as well as between returns. The upper triangular part of Table 3 shows the correlation matrix of the futures volatility changes filtered from the estimated jump components $\widehat{\xi^V_{1,t} J_{1,t}}$, $i = 1, 2, ..., 5$ and $t = 1, 2, ..., T$. In all ten possible cases we find out that the correlation decreases when the futures volatilities are filtered from jumps which implies that models that do not include jumps in volatility may overestimate correlation between futures volatilities. A similar pattern is present also in returns.

Table 4 shows, in the lower triangular part, the correlation matrix of returns and, in the upper triangular part, the correlation matrix of returns filtered from the estimated jump components $\widehat{\xi^{Y}_{1,t} J_{1,t}}$, $i = 1, 2, ..., 5$ and $t = 1, 2, ..., T$. In all ten possible cases, filtering the returns from jumps decreases the sample correlation.

3. Jump Spillover

In this section we use our estimates of the latent historical jump times to analyze jump spillover effects. We first look at the simultaneous jump intensities between pairs of energy futures. Then we examine the conditional jump spillover probabilities that jump in a specific futures cause jumps or unusually large returns in other futures. Finally, we take a brief look at the average latent jumps sizes of the simultaneous jumps that from the basis of our analysis of jump spillover.

3.1 Simultaneous jump intensities

We start our study of jump spillover by taking a look at the simultaneous jump intensities between pairs of energy futures. We calculate the simultaneous jumps intensity of two energy futures in a straightforward manner simply as the number of identified simultaneous jumps divided by the number of overlapping observations. The simultaneous jump intensity of a specific futures with itself is what we refer to as the “implied jump intensity”; that is, the number of jump times divided by the number of observations—a number that should be roughly consistent with the respective estimate of λ in Table 2.

Table 5 reports the estimated simultaneous jump intensities and whether the estimated values are significant at the 95% (*) or 99% (**) level. The significance levels are obtained by testing if the estimated intensities are greater than what they would be under the null hypothesis that the different futures’ jump processes are completely independent. Under the null hypothesis, the simultaneous jump intensity of two futures is estimated simply as the product of the futures’ implied jump intensities. For example, the simultaneous downside jump intensity of the crude oil and the natural gas under the null hypothesis is equal to 0.1031 multiplied by 0.1289.

An initial observation is that the estimated simultaneous jump intensities are generally quite significant, which is evidence of the existence of jump spillover.

3.2 Conditional jump spillover probabilities

Next we look at the conditional spillover probability that, given a jump in a chosen benchmark futures, other futures jump on the same or on the following day—denoted by same-day (conditional) jump spillover and next-day (conditional) jump spillover. We examine two cases for the choice of benchmark futures: (1) crude oil, (2) natural gas.

Table 6 shows the estimated same-day and next-day (conditional) jump spillover probabilities. The same-day jump spillover probability for a specific futures contract is estimated as the number of simultaneous jump times with the benchmark futures divided by the number of jump times of the benchmark futures. The next-day jump spillover probability is calculated in the same way, but by using instead the number of jumps that occur on the day after jumps in the benchmark futures. The table also shows which of these estimated probabilities are significant at 95% or 99% level. The significance levels are obtained by testing for quality of the estimated spillover probabilities with the corresponding probabilities under the null hypothesis of independent jump process. Under the null hypothesis, the same-day and next-day jump spillover probabilities of a futures are both estimated as the number of identified jumps divided by the number of observations. The results show that all the estimated probabilities are significant.

To analyze a weaker form of jump spillover, we look at the conditional probabilities that jumps in the benchmark futures merely result in unusually large returns in other futures and not necessarily jumps. For simplicity, we define an unusually large return as a return belonging to the lower decile of the historical returns of each energy futures. These probabilities are estimated in the same fashion as above, and we again test if they are greater than the corresponding probabilities under the null hypothesis of independence between the benchmark jumps and the unusually large returns.

The result for this type of jump spillover is shown in Table 7. It is easy to see that almost all values for the same-day jumps are considerably larger than the corresponding values in Table 6 and, at the same time, almost the same pattern as above in terms of spillover and significance is present.

3.3 Size of simultaneous jumps

Finally, it might be interesting to look also at the estimated latent jump sizes of the identified simultaneous jumps in returns on which we base our simultaneous jump intensities. The element in row *i* and column *j* of Table 8 shows the average size of the jumps in futures *i* that are simultaneous with jumps in futures *j*, calculated as

$$\frac{1}{\sum_{t=1}^{T}\hat{J}^{*}_{i,t}\hat{J}^{*}_{i,t}}\sum_{t=1}^{T}\hat{J}^{*}_{i,t}\hat{J}^{*}_{i,t}\hat{\xi}^{\Upsilon}_{i,t},$$

where $\hat{\xi}^{\Upsilon}_{i,t}$,t=1,2,…,T are the estimated latent sizes of jumps in returns for futures *i*. Consequently, the diagonal elements of the table are equal to the average latent jump sizes of the different futures. For example, the average latent size of all the jumps identified in the crude oil is equal to -1.3876% for downside jump and 1.7791% for upside jump, whereas the average size of the subset of the jumps in the crude oil that are simultaneous with jumps in the natural gas is equal to -1.1117% for downside jump and 1.5449% for upside jump.

IV. Conclusion

In this paper we examine jump spillover effects between a number of energy (petroleum) futures. Our contribution is twofold: First, we fit a stochastic volatility jump-diffusion model to each individual futures and compare features of the different futures such as jump frequencies and jump magnitudes. This analysis is motivated by the fact that the most previous empirical research on event risk models have focused mostly on the equity markets and little is known about the impact of jumps in energy markets. Second, we look at jump spillover effects between energy futures. This is the central issue of the article, and to our knowledge, it has not been analyzed by previous studies.

To identify the historical jump times of different energy futures, we use the SVCJ model. This model helps us to separate out returns that are related to sudden unexpected events (jumps) from large diffusive returns caused by periods of high volatility without the need to make any a priori assumptions on what is an extreme return and what is not, as this determined in the estimation. We estimate the model with the MCMC method of Eraker, Johannes, and Polson (2003) and Asgharian and Bengtsson (2006).The advantage of this method compared to most other methods is that it makes it possible to estimates the latent processes of the model－in particular the jump times, jump sizes and futures volatility paths.

Our study of jump spillover begins with an analysis of the simultaneous jump intensities of pairs of energy futures. We find that these intensities are generally quite significant, which is evidence of the existence of jump spillover.

We end our study of jump spillover by looking at the conditional probabilities that jumps in crude oil and natural gas cause jumps or large price movements in other energy futures. The results show that the estimated spillover probabilities are all significant. We also find that these estimated conditional jump spillover probabilities are also generally significantly larger than what they would be under the null hypothesis of no jump spillover.

Appendix: Bayes Factor and Model Comparison

Model risk arises from the uncertainty over selecting a model specification. As mentioned above, the SVCJ nests other popular continuous-time model, such as SV, SVJ and SVIJ. Consistent with our Bayesian approach, a natural statistical criterion for resolving this uncertainty is computing Bayes factors. Bayes factors offer a summary of the evidence provided by the data in favour of one statistical model. More importantly, they account for parameter uncertainty and impose a penalty for lack of parsimony (higher dimension)[10]. Consider two competing models M_1 and M_2. Using Bayes theorem, it is straight-forward to show that the Bayes factor B_{12} (in favour of model M_1) is the ratio of posterior to prior odds, which is equal to the ratio of the marginal likelihoods:

$$B_{12} = \frac{p(r|M_1)}{p(r|M_2)} \quad ,$$

where the marginal likelihood of model M_1 is defined as:

$$\begin{aligned} p(r|M_1) &= \int_\theta p(r,\theta|M_1)d\theta \\ &= \int_\theta p(r|\theta,M_1)\pi(\theta|M_1)d\theta \, . \end{aligned}$$

It is important to note that the marginal likelihood is an averaged (not a maximized) likelihood. This implies that the Bayes factor is an automatic "Occam's Razor" in that it

[10] For a review and computation of Bayes factors, see Kass and Raftery (1995) and Johannes and Polson (2002).

integrates out parameter uncertainty[11]. Furthermore, the marginal likelihood is simply the normalizing constant of the posterior density and (suppressing the model index for simplicity) it can be written as:

$$p(r) = \frac{f(r|\theta)\pi(\theta)}{\pi(\theta|r)},$$

Where $f(r|\theta)$ is the likelihood, $\pi(\theta)$ the prior density, $\pi(\theta|r)$ the posterior density, θ is evaluated at the posterior mean estimate. Since θ is drawn in context of MCMC sampling, the posterior density $\pi(\theta|r)$ is computed using the technique of reduced conditional MCMC runs of Chib (1995) and Chib and Jeliazkov (2001).

To assess the information provided by a Bayes factor, it is useful to consider twice its natural logarithm so as to be on the same scale as likelihood ratio statistics. To make the interpretation more familiar, Panel A of Table A1 presents the range of the values of $2\ln(B_{12})$ that constitute evidence in favour of model M_1. Finally, note that model comparisons based on Bayes factors are asymptotically equivalent to evaluations based on the Schwartz (or equivalently the BIC) criterion.

In Table A1 we rank the in-sample performance of the models according to the Bayes factors. The key input to this criterion is the calculation of the marginal likelihood. Therefore, Table A1 gives us a distinct statistical perspective on performance because the

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[11] *Occam's Razor* is the principle of parsimony, which states that among two competing theories making the exact same prediction, the simpler one is best.

marginal likelihood is computed in a way that integrates out parameter uncertainty and imposes a penalty for lack of parsimony (higher dimension). Panel A of Table A1 presents the range of Bayes factors that constitute evidence in favor (or against) a given model. The results reported in Panel B of Table A1 indicate two clear patterns in ranking the models: (i) SVCJ and SVJ are always better than SV for all energy futures; and (ii) SVCJ is better than SVJ for all energy futures. The former result emphasizes the importance of the jump element in model specification. The latter result supports our assertation that jumps between different energy futures are actually correlated.

Table A1 The model comparison by Bayes Factors

Panel A: Interpreting Bayes factors

2ln(B12)	B_{12}	Evidence against model M2
0-2	1-3	Not worth more than a bare mention
2-6	3-20	Positive
6-10	20-150	Strong
>10	>150	Very strong

Panel B: Bayesi factors 2ln(B12)

	Crude oil	Natural gas	Heating oil	Gasoline	Fuel oil
SVCJ vs. SV	123.61	108.74	97.08	170.77	145.59
SVJ vs. SV	107.22	88.04	74.38	119.26	101.60
SVCJ vs. SVJ					

Reference

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Table 1 Descriptive statistics of returns

Description statistics of the energy futures returns included in this study. The data consist of daily percentage log returns for each energy futures contract from September 29, 2006 to October 29, 2008. The mean returns and standard deviations have been annualized through multiplication by 252 and $\sqrt{252}$, respectively.

	Crude oil	Natural gas	Heating oil	gasoline	Fuel oil
Mean	0.008609	0.018049	0.024316	-0.007428	0.011620
Median	0.164406	-0.30808	0.000000	0.192988	0.000000
Maximum	8.242577	34.40974	7.664302	6.747177	8.004271
Minimum	-10.94552	-13.62917	-9.019634	-11.50368	-12.21918
Std. Dev.	2.173236	4.531974	2.132602	2.482446	2.148959
Skewness	-0.459732	2.042574	-0.252806	-0.631543	-0.873541
Kurtosis	5.495041	14.20782	4.108804	4.670400	8.062012
Observations	543	543	543	543	543

Table 2 Parameter estimates for five energy futures

The MCMC estimates of SVCJ model parameters for the futures of crude oil, natural gas, heating oil, gasoline and fuel oil. Also reported are the posterior standard errors and the posterior confidence intervals for each parameter.

1. Crude Oil

	Mixed jumps			Downside jumps			Upside jumps		
	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI
μ	-0.3745	1.5838	(-3.4790, 2.7299)	-3.5652	2.2975	(-8.0712, 0.9420)	2.831	2.7037	(-2.4699, 8.1304)
θ	3.8856	1.2721	(1.3921, 6.3789)	6.3834	1.9551	(-2.5518, 10.2183)	3.1924	1.0981	(-1.0402,5.3448)
κ	0.1404	0.1374	(-0.1289, 0.4097)	0.0984	0.1551	(-0.2056, 0.4024)	0.1768	0.1748	(-0.1659, 0.5188)
σ_V	0.2245	0.0586	(0.1096, 0.3394)	0.2097	0.0673	(0.0778, 0.3416)	0.2304	0.0735	(0.0863, 0.3745)
ρ	-0.0057	0.3802	(-0.7510, 0.7389)	-0.0019	0.5743	(-1.1240, 1.1281)	0.0131	0.5784	(-1.1212, 1.1468)
μ_V	0.3180	0.1127	(0.0971, 0.5389)	0.2745	0.5454	(-0.7941, 1.3432)	0.3487	0.6853	(-0.9940, 1.6922)
μ_Y	2.3289	2.1018	(-1.7902, 6.4487)	2.5385	2.6933	(-2.7395,7.8204)	2.0261	2.3512	(-2.5807, 6.6319)
σ_Y	1.1288	1.2032	(-1.2289, 3.4870)	1.1212	1.8515	(-2.5082, 4.7486)	1.0913	1.5164	(-1.8809, 4.0631)
ρ_J	-0.0081	0.1517	(-0.3054, 0.2892)	-0.1093	0.178	(-0.2402, 0.4581)	0.0934	0.1863	(-0.2721, 0.4590)
λ	0.1079	0.1586	(-0.2030, 0.4188)	0.0999	0.1929	(-0.2782, 0.4790)	0.1115	0.1878	(-0.2570, 0.4801)

2. Natural Gas

	Mixed jumps			Downside jumps			Upside jumps		
	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI
μ	2.3274	2.2255	(-2.0308, 6.6891)	-4.7755	2.8056	(-10.2708, 0.7217)	9.4744	2.5358	(4.5018, 14.4402)
θ	2.4085	1.2492	(-0.0402, 4.8570)	2.934	1.7848	(-0.5607, 6.4319)	1.7867	1.2135	(-0.5921, 4.1602)
κ	0.0649	0.1379	(-0.2054, 0.3352)	0.0730	0.1359	(-0.1934, 0.3394)	0.0542	0.1951	(-0.3280, 0.4372)
σ_V	0.1115	0.0496	(0.0143, 0.2087)	0.1084	0.0546	(0.0014, 0.2154)	0.1102	0.0645	(-0.0162, 0.2366)
ρ	-0.0052	0.3830	(-0.7560, 0.7448)	-0.0057	0.5758	(-1.1232, 1.1341)	-0.0045	0.5836	(-1.1482, 1.1390)
μ_V	0.1373	0.0234	(0.0914, 0.1832)	0.1512	0.0271	(0.0981, 0.2043)	0.1180	0.0291	(0.0610, 0.1750)
μ_Y	4.8117	1.7935	(1.2870, 8.3291)	3.0247	2.0994	(-1.0904, 7.1402)	6.4063	2.2052	(2.0807, 10.7273)
σ_Y	0.9988	1.0630	(-1.0850, 3.0821)	0.9608	1.7691	(-2.5095, 4.4297)	0.9969	2.2221	(-3.3607, 5.3518)
ρ_J	0.0257	0.0560	(-0.0841, 0.1355)	-0.0802	0.0299	(-0.0216, 0.1388)	0.1305	0.1046	(-0.0745, 0.3355)
λ	0.0853	0.1525	(-0.2136, 0.3842)	0.0931	0.1762	(-0.2520, 0.4381)	0.074	0.1898	(-0.2981, 0..4460)

3. Heating Oil

	Mixed jumps			Downside jumps			Upside jumps		
	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI
μ	-0.1621	1.2206	(-2.5540, 2.2301)	-3.0562	2.3589	(-7.6820, 1.5692)	2.7384	2.2506	(-1.6704, 7.1520)
θ	3.8170	1.2774	(1.3131, 6.3209)	4.9132	2.1607	(0.6827, 9.1520)	2.5683	0.9052	(0.7942, 4.3421)
κ	0.1451	0.1253	(-0.1005, 0.3907)	0.1288	0.1791	(-0.2220, 0.4801)	0.1557	0.1697	(-0.1769, 0.4883)
σ_V	0.2382	0.0614	(0.1179, 0.3585)	0.2325	0.0730	(0.0894, 0.3756)	0.2344	0.0745	(0.0884, 0.3804)
ρ	-0.0026	0.2813	(-0.5538, 0.5487)	-0.0026	0.5778	(-1.1302, 1.1352)	0.0078	0.5775	(-1.1240, 1.1402)
μ_V	0.3461	0.4134	(-0.4640, 1.1559)	0.3371	0.5808	(-0.8012, 1.4750)	0.3412	0.6514	(-0.9361, 1.6182)
μ_Y	2.0375	1.3770	(-0.6611, 4.7359)	2.1328	2.4926	(-2.7483,7.0192)	1.8607	1.7723	(-1.6128, 5.3309)
σ_Y	1.1918	1.1301	(-1.0232, 3.4068)	1.1288	1.6944	(-2.1902, 4.4518)	1.2072	1.738	(-2.1956, 4.5830)
ρ_J	0.0110	0.3645	(-00.7031, 0.7250)	-0.0754	0.6846	(-1.2660,1.4172)	0.097	0.4302	(-0.74608, 0.9402)
λ	0.1118	0.1172	(-0.1179, 0.3415)	0.1055	0.1894	(-0.2657, 0.4769)	0.1136	0.1879	(-0.2551, 0.4820)

4. Gasoline

	Mixed jumps			Downside jumps			Upside jumps		
	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI
μ	-0.6708	1.0202	(-2.6703 1.3289),)	-4.1004	2.2866	(-8.5801,0.3792)	2.7855	2.082	(-1.3009, 6.8718)
θ	-2.3268	2.0945	(-6.4301, 1.7797)	-9.2494	2.9225	(-14.9806,-3.5210)	4.6889	2.1045	(0.5607, 8.8091)
κ	0.1332	0.1168	(-0.0957, 0.3621)	0.0950	0.1446	(-0.1884, 0.3784)	0.1661	0.1838	(-0.1940, 0.5260)
σ_V	0.2223	0.0569	(0.1108, 0.3338)	0.1881	0.0595	(0.0715, 0.3047)	0.2476	0.0771	(0.0965, 0.3987)
ρ	-0.0158	0.2805	(-0.5660, 0.5339)	-0.0194	0.5762	(-1.1101,1.1490)	-0.0116	0.577	(-1.1420,1.1192)
μ_V	0.3151	0.4397	(-0.5471, 1.1768)	0.2344	0.1277	(-0.0159, 0.4847)	0.3832	0.9277	(-1.4350, 2.2014)
μ_Y	2.4122	2.0537	(-1.6086, 6.437)	2.9346	2.8794	(-2.7102, 8.5806)	1.7933	2.0495	(-2.221, 5.8108)
σ_Y	1.1512	1.2803	(-1.3582, 3.6608)	1.0668	1.7814	(-2.4206, 4.5586)	1.1895	1.7714	(-2.2794, 4.6590)
ρ_J	0.0116	0.2767	(-0.5309, 0.5538)	-0.1116	0.1603	(-0.2026, 0.4258)	0.1344	0.5039	(-0.8533, 0.1220)
λ	0.1078	0.1268	(-0.1407, 0.3563)	0.1004	0.1965	(-0.2848, 0.4860)	0.1109	0.1799	(-0.2421, 0.4629)

5. Fuel Oil

	Mixed jumps			Downside jumps			Upside jumps		
	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI	Posterior mean	Posterior std. dev.	Posterior 95% CI
μ	-0.8019	1.0330	(-2.8270, 1.2231)	-4.451	2.8063	(-9.9489,1.0487)	2.8793	2.0730	(-1.1792, 6.9428)
θ	-0.3325	1.0365	(-2.3639, 1.6988)	6.4848	2.1003	(2.3728, 10.6019)	-7.1366	2.0674	(-11.1908,-3.0802)
κ	0.1083	0.1206	(-1.1281, 0.3447)	0.0864	0.1261	(-0.1608, 0.3336)	0.1259	0.1874	(-0.2409, 0.4928)
σ_V	0.2117	0.0586	(0.0968, 0.3266)	0.1766	0.0633	(0.0525, 0.3007)	0.2384	0.0774	(0.0867, 0.3901)
ρ	-0.0048	0.2759	(-0.5458, 0.5360)	-0.0046	0.5720	(-1.1160, 1.1262)	0.0141	0.5703	(-1.1043, 1.1320)
μ_V	0.2852	0.1991	(-0.1052, 0.6748)	0.2027	0.1835	(0.1569, 0.5622)	0.3562	0.5345	(-0.6910, 1.4042)
μ_Y	2.6524	1.1895	(0.3212, 4.9837)	3.1954	1.0185	(1.1992 ,5.1921)	2.0034	1.8364	(-1.6010, 5.6017)
σ_Y	1.1762	1.0819	(-0.9443, 3.2966)	1.0701	1.7845	(-2.4324, 4.5703)	1.2353	1.7721	(-2.2432,4.7107)
ρ_J	-0.0176	0.1268	(-0.2661, 0.2309)	-0.1252	0.6769	(-1.2010, 1.4522)	0.0907	0.2035	(-0.3082, 0.4901)
λ	0.0981	0.1074	(-0.1124, 0.3086)	0.086	0.1876	(-0.2824, 0.4539)	0.1062	0.1901	(-0.2661, 0.4790)

Table 3 Sample correlation matrix for futures volatility changes and for futures volatility changes filtered from jump components

Below the diagonal is the sample correlation matrix of the estimated futures volatility changes, $\sqrt{\hat{V}_{i,t}} - \sqrt{\hat{V}_{i,t-1}}, i = 1.2....5, t = 1,2,...,T$. Above the diagonal is the sample correlation matrix of the estimated futures volatility changes filtered from the estimated jump components

$$\widehat{\xi^{V}_{1,t} J_{1,t}} = (M-m)^{-1} \sum_{j=m+1}^{M} \xi^{V(j)}_{i,t} J^{(j)}_{i,t}, i = 1,2,...,5, t = 1,2,...,T \cdot$$

	Crude oil	Natural gas	Heating oil	gasoline	Fuel oil
Crude oil		0.0019	0.813	0.7055	0.7288
Natural gas	0.0115		0.0648	0.0819	0.0419
Heating oil	0.8412	0.0667		0.6804	0.7047
gasoline	0.72491	0.0857	0.7145		0.6528
Fuel oil	0.7581	0.0446	0.7224	0.67016	

Table 4 Sample correlation matrix for returns and for returns filtered from jump components

Below the diagonal is the sample correlation matrix of log returns; $\Upsilon_{i,t}, i = 1,2,...,5, t = 1,2,...,T$. Above the diagonal is the sample correlation matrix of log returns filtered from the estimated jump components

$$\widehat{\xi^{\Upsilon}_{1,t} J_{1,t}} = (M-m)^{-1} \sum_{j=m+1}^{M} \xi^{\Upsilon(j)}_{i,t} J^{(j)}_{i,t}, i = 1,2,...,5, t = 1,2,...,T \cdot$$

	Crude oil	Natural gas	Heating oil	gasoline	Fuel oil
Crude oil		0.1017	0.8987	0.8416	0.6819
Natural gas	0.1207		0.1	0.0087	0.0913
Heating oil	0.9135	0.1134		0.8022	0.6501
gasoline	0.8543	0.0178	0.8208		0.5733
Fuel oil	0.7174	0.1011	0.6757	0.6215	

Table 5 Estimated simultaneous jump intensities

Estimated simultaneous jump intensities between pairs of energy futures. For two given futures, the intensity is estimated as the number of simultaneous jumps divided by the number of overlapping observations. For a single futures, the intensity is simply the number of jump times divided by the number of observations. (**) denotes a one-side significance at 99% level and (*) denotes a one-side significance at 95% level. The left (right) of (.,.) denotes the simultaneous downside (upside) jump intensities.

	Crude oil	Natural gas	Heating oil	Gasoline	Fuel oil
Crude oil	(0.1031 , 0.1178)				
Natural gas	(0.0203** , 0.0037**)	(0.1289 , 0.035)			
Heating oil	(0.0902** , 0.0847**)	(0.0331** , 0.0018**)	(0.1657 , 0.1031)		
Gasoline	(0.0737* , 0.0718*)	(0.0147** , 0.0000*)	(0.0828** , 0.0700**)	(0.1123 , 0.1344)	
Fuel oil	(0.0442** , 0.0645**)	(0.0092* , 0.0036*)	(0.0445** , 0.0497**)	(0.0450** , 0.0516**)	(0.0534 , 0.1271)

Table 6 Conditional jump spillover probabilities

Estimated jump spillover probabilities from chosen benchmark energy futures to other energy futures. For a given nonbenchmark energy futures, the jump spillover probability is estimated as the number of the futures jumps that occur, depending on which is meaningful, either simultaneously as or on the day following a benchmark futures jump, divided by the number of jump times of the benchmark futures. (**) denotes a one-side significance at 99% level and (*) denotes a one-side significance at 95% level.

	Benchmark											
	Crude oil						Natural gas					
	Same-day			Next-day			Same-day			Next-day		
	jump	Downside jump	Upside jump	jump	Downside jump	Upside jump	jump	Downside jump	Upside jump	jump	Downside jump	Upside jump
Crude oil	—	—	—	—	—	—	0.2360**	0.1571**	0.1579**	0.2247*	0.1429**	0.1053**
Natural gas	0.1917**	0.1964**	0.0313**	0.1667**	0.1262**	0.0156**	—	—	—	—	—	—
Heating oil	0.7917**	0.875**	0.7188**	0.3583**	0.3928**	0.2813**	0.2697**	0.2571**	0.1053**	0.2360**	0.2143*	0.0526**
Gasoline	0.6583**	0.7143**	0.6094**	0.3250**	0.2857**	0.2500**	0.2921**	0.1714**	0.0526**	0.2809**	0.1571**	0.0526*
Fuel oil	0.4917**	0.4286**	0.5469**	0.2833**	0.2321**	0.3125**	0.1685**	0.0857**	0.2105**	0.1685**	0.0714*	0.1053**

Table 7 Conditional probabilities of Spillover from Jumps to Return

Estimated probabilities that jumps in the chosen benchmark energy futures translate into unusually large negative returns in other energy futures. We define an unusually large negative return of given futures as a historical return that belongs to the lowest decile of that futures dataset. For a given nonbenchmark futures, the probability is estimated as the number of the futures unusually large negative returns that occur, depending on which is meaningful, either simultaneously as or on the day following a benchmark futures jump, divided by the number of jump times of the benchmark futures. (**) denotes a one-side significance at 99% level and (*) denotes a one-side significance at 95% level.

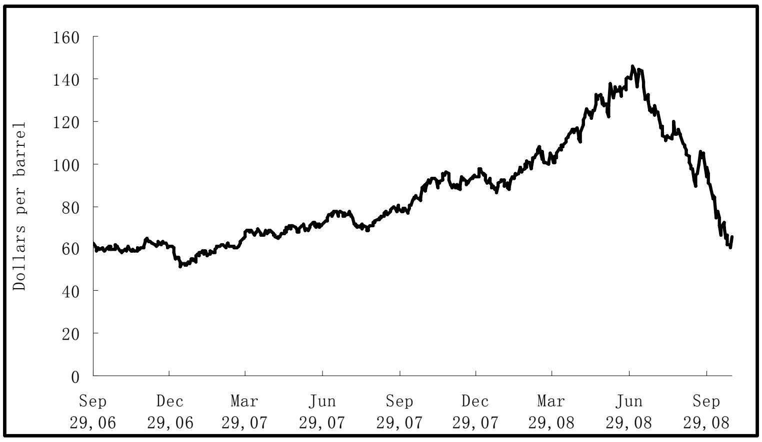
	Benchmark											
	Crude oil						Natural gas					
	Same-day			Next-day			Same-day			Next-day		
	jump	Downside jump	Upside jump	jump	Downside jump	Upside jump	jump	Downside jump	Upside jump	jump	Downside jump	Upside jump
Crude oil	—	—	—	—	—	—	0.2809**	0.2286**	0.3684**	0.2135**	0.2000**	0.2105**
Natural gas	0.2167**	0.2632**	0.1852**	0.1583**	0.0313**	0.1563**	—	—	—	—	—	—
Heating oil	0.8250**	0.8947**	0.8148**	0.6583**	0.2969**	0.3125**	0.2697**	0.3714**	0.2632**	0.2360**	0.1857**	0.1053**
Gasoline	0.7000**	0.7368**	0.7778**	0.5250**	0.2969**	0.2813**	0.3258**	0.2286**	0.1579**	0.3034**	0.1286**	0.1053*
Fuel oil	0.4917**	0.4737**	0.5926**	0.3000**	0.1563**	0.1563**	0.2135**	0.1857**	0.3158**	0.1685**	0.1000**	0.1579**

Table 8 Average Jump Sizes of Simultaneous Jumps

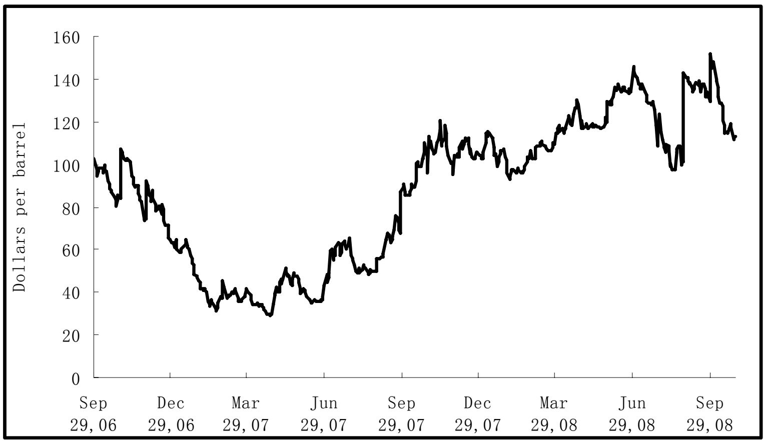
Average estimated latent sizes of simultaneous jumps in returns. Element $i\ j$ i, j=1,2,…,5 is the average size of the jumps in futures i that are simultaneous with jumps in futures j and is calculated as $\sum_{t=1}^{T} \hat{J}^*_{i,t}\hat{J}^*_{j,t}\hat{\xi}^{\Upsilon}_{i,t} / \sum_{t=1}^{T} \hat{J}^*_{i,t}\hat{J}^*_{j,t}$, where $\hat{\xi}^{\Upsilon}_{i,t}$, t=1,2,…,T, are the estimated latent sizes of jumps in returns for futures i. The left (right) of (.,.) denotes the simultaneous downside (upside) jump intensities.

(downside , upside)	Crude oil	Natural gas	Heating oil	gasoline	Fuel oil
Crude oil	(-1.3876 , 1.7791)	(-1.1117 , 1.5449)	(-1.1573 , 0.5677)	(-0.8862 , 0.5183)	(-0.3754 , 0.6932)
Natural gas	(-1.3238 , 1.3617)	(-1.6501 , 3.2665)	(-1.3218 , 2.5912)	(-1.7991 , 0.4085)	(-2.3616 , 0.8670)
Heating oil	(-0.7726 , 0.3206)	(-1.0496 , 0.5787)	(-1.2048 , 0.5408)	(-0.7795 , 0.3482)	(-0.2004 , 0.3126)
gasoline	(-1.2962 , 0.3568)	(-0.7503 , 1.2056)	(-1.4214 , 0.2974)	(-1.6715 , 0.5556)	(-0.7156 , 0.4336)
Fuel oil	(-0.8420 , 0.6568)	(-0.0320 , 1.7562)	(-0.8396 , 0.4950)	(-0.884 ,0.5856)	(-1.1442 , 0.9818)

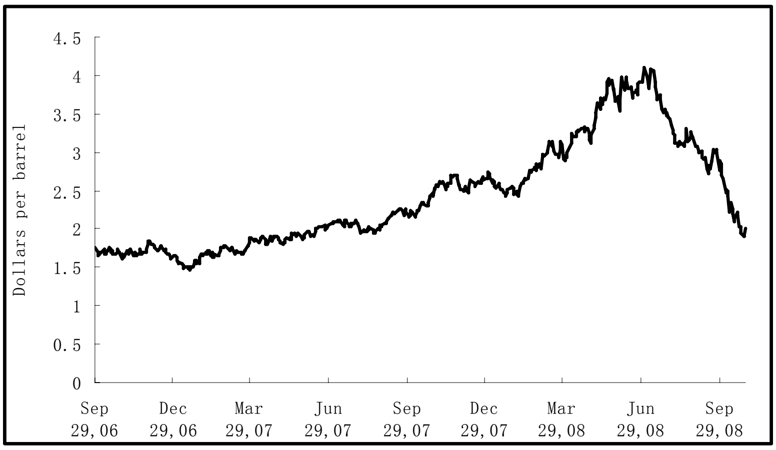
Figure 1 The Price of five energy futures



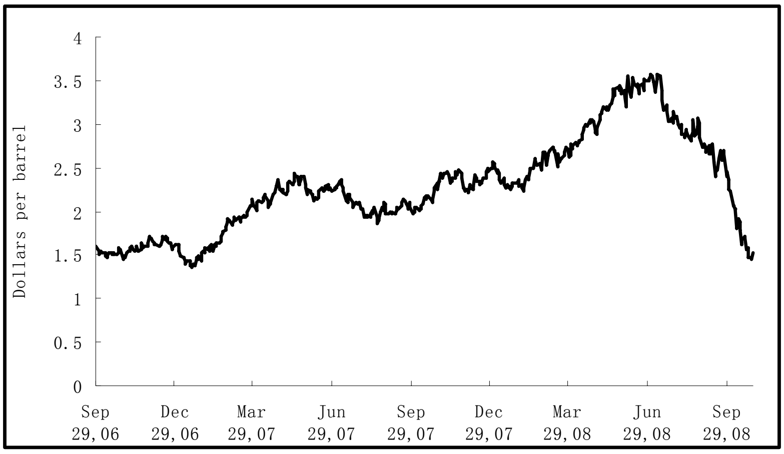
Crude Oil



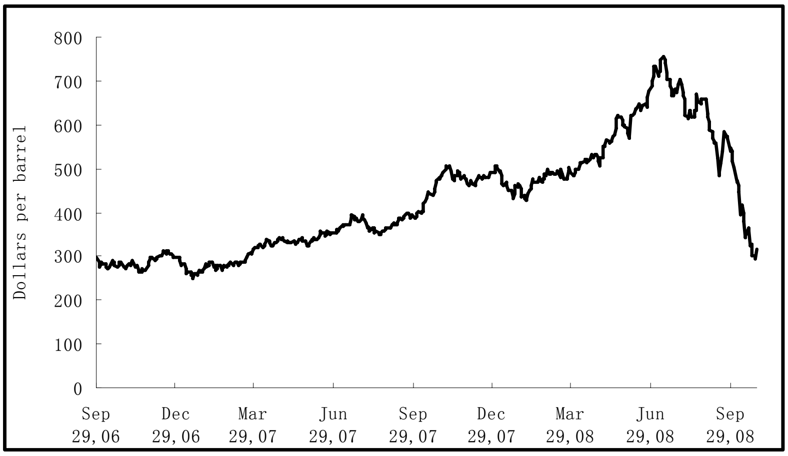
Natural Gas



Heating Oil

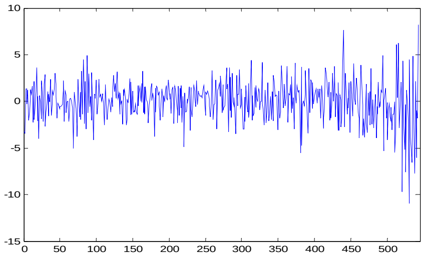


Gasoline

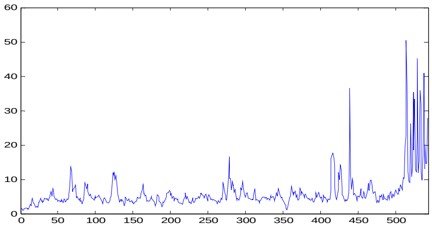


Fuel Oil

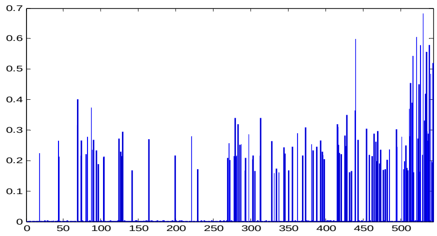
Figure 2 Log-returns, Volatilities, and the Probabilities of Jumps for Five Energy Futures



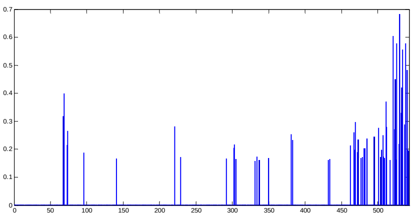
Log return



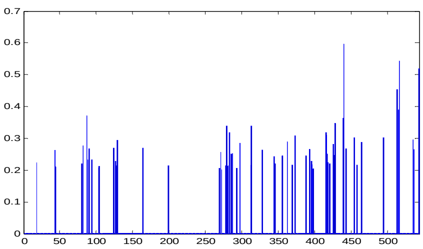
Futures volatility



Jump prob.

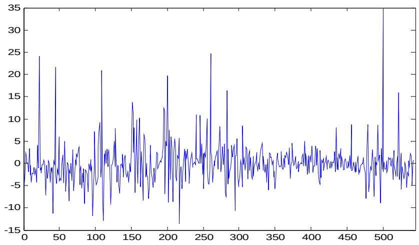


Downside jump prob

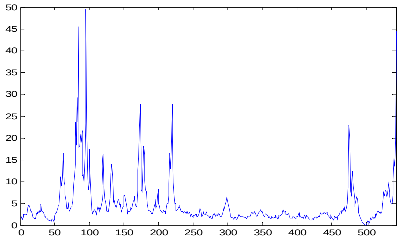


Upside jump prob

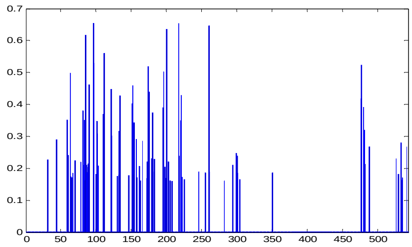
Figure 2(a): Crude Oil



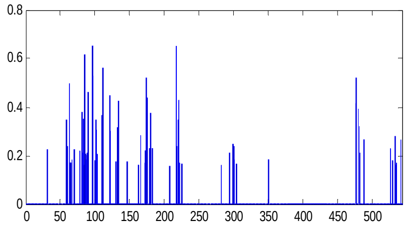
Log return



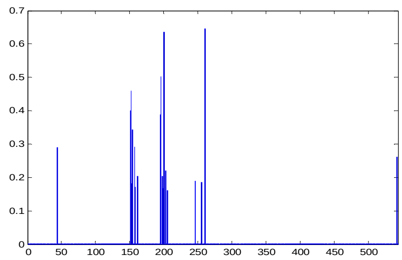
Futures volatility



Jump prob.

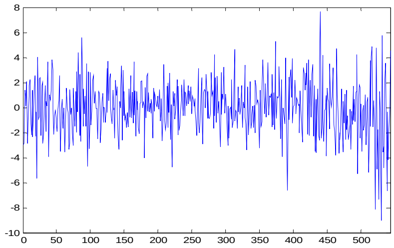


Downside jump prob

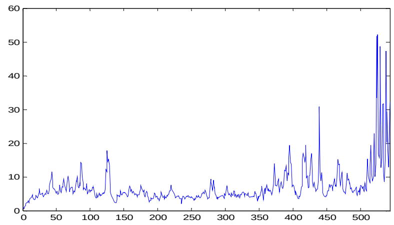


Upside jump prob

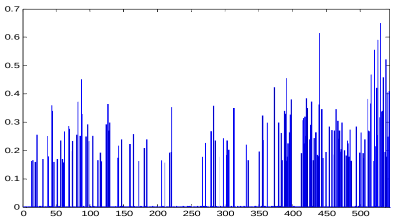
Figure 2(b): Natural Gas



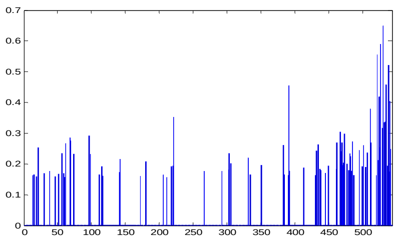
Log return



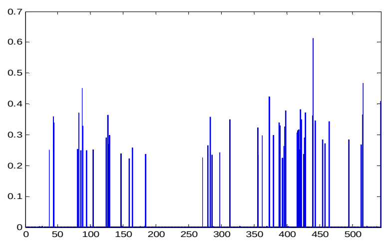
Futures volatility



Jump prob.

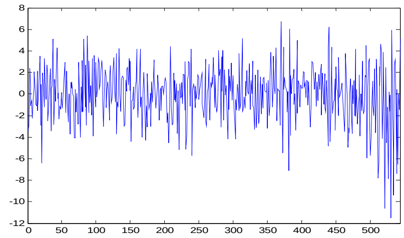


Downside jump prob.

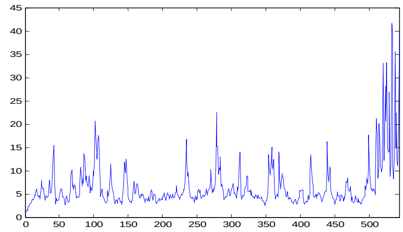


Upside Jump prob.

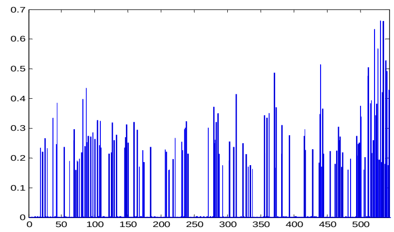
Figure 2(c): Heating Oil



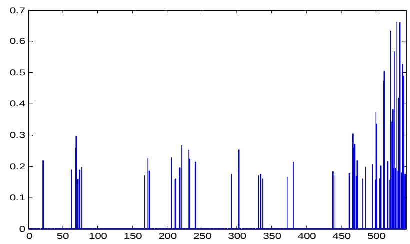
Log return



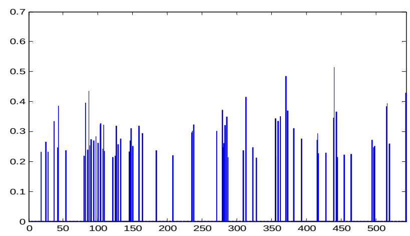
Futures volatility



Jump prob.

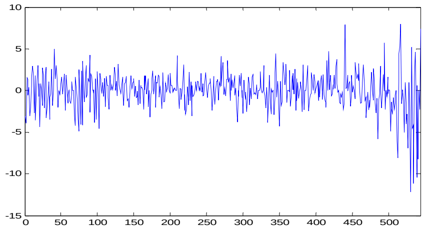


Downside jump prob.

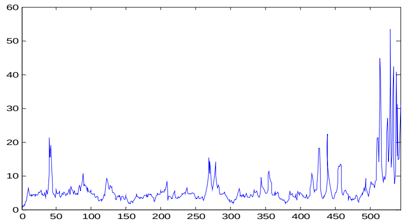


Upside Jump prob.

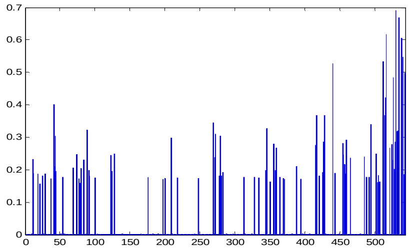
Figure 2(d): Gasoline



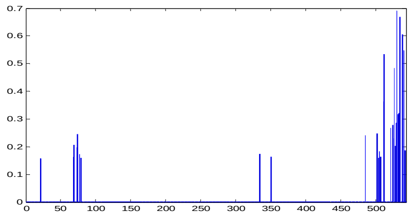
Log return



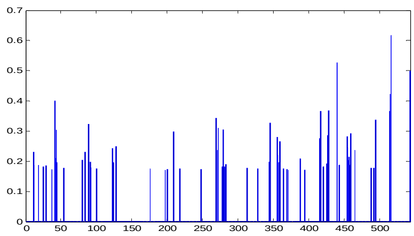
Futures volatility



Jump prob.



Downside Jump prob.



Upside Jump prob.

Figure 2(e): fuel oil