Journal of Money, Investment and Banking
ISSN 1450-288X Issue 4 (2008)

http://www.eurojournals.com/finance.htm

Forecasting VIX

Stavros A. Degiannakis
Department of Statistics, Athens University of Economics and Business
Department of Economic and Regional Development, University of Central Greece
76 Patision street, Athens GR-104 34, Greece
Tel: +30-210-8203-120
E-mail: sdegia@aueb.gr

Abstract

Implied volatility index of the S&P500 is considered as a dependent variable in a fractionally integrated ARMA model, whereas volatility measures based on interday and intraday datasets are considered as explanatory variables. The next trading day's implied volatility forecasts provide positive average daily profits. All the forecasting information is provided by the VIX index itself. There is no incremental predictability from both realized volatility computed from intraday data and conditional volatility extracted from an Arch model. Hence, neither the interday volatility nor the use of intraday data yield any added value in forecasting the S&P500 implied volatility index. However, an agent cannot utilize VIX predictions in creating abnormal returns in implied volatility futures market.

Keywords: ARCH, ARFIMAX, Fractional Integration, Volatility Forecasting, VIX Index.
JEL Classification Codes: C32, C52, C53, G15.

1. Introduction

Financial literature is full of evidence that short-term volatility is predictable. Since Engle (1982) introduced the autoregressive conditional heteroskedasticity (Arch) model, numerous methods have been proposed for predicting future volatility of assets returns. Presently, in a forecast-based evaluation framework, extended versions of Arch volatility specifications have been applied providing added predictive ability in various areas such as option pricing, risk management, portfolio analysis, etc. In the past years, based on Andersen and Bollerslev's (1998) seminal paper, the use of intraday datasets has rekindled the interest of academics to forecast variability of asset returns. The realized volatility, which is defined as the sum of the squared intraday returns, is mainly modeled by autoregressive fractionally integrated moving average with exogenous variables (ARFIMAX) models.

On the other hand, implied volatility[1], first noted by Latane and Rendleman (1976), is considered by many studies as an accurate predictor of future volatility. However, as very well documented by Blair et al. (2001), a number of studies characterize implied volatility measures as less informative than volatility estimated from asset returns, because they induce important biases and contain mis-specification problems. In the recent past, the implied volatility index (VIX) of the Chicago Board of Options Exchange (CBOE) eliminated such measurement errors. As a result, market

[1] Implied volatility is the standard deviation of the return on the asset, which would have to be inputted into a theoretical option pricing formula to yield a theoretical value identical to the price of the option in the marketplace, assuming all other inputs are known.

participants consider the VIX index as the world's premier barometer of investor sentiment and market volatility.

Blair et al. (2001) estimated an Arch model with Glosten's et al. (1993) conditional variance specification. They considered the VIX index of S&P100 and the realized volatility based on five-minute S&P100 returns as explanatory variables. They concluded that the implied volatility index provides more accurate forecasts than either the interday volatility extracted from daily return series or the intraday realized volatility. Koopman et al. (2005) compared the forecasts of various classes of volatility models with realized volatility. They confirmed that volatility forecasts extracted from models of daily returns (such as Arch and Stochastic Volatility models) are less accurate than forecasts based on VIX index. However, models based on realized volatility (such as ARFIMAX and Unobserved Components ARMA models) outperform models with implied volatility.

The aforementioned studies consider the implied volatility index as an explanatory variable in forecasting either the interday conditional volatility or the intraday realized volatility. The present study is the first that models an implied volatility index as a dependent variable. The main purpose of the study is to investigate whether the use of intraday datasets or conditional volatility extracted from an Arch model provides any incremental predictive ability in forecasting the next day's implied volatility. We present evidence that all the forecasting information is provided by the implied volatility index. Neither interday nor intraday volatility measures supply any statistically significant incremental information. The results of the paper point to a fairly important conclusion, that the VIX index is hard to forecast and does not seem to be very closely connected to the volatility of the underlying index. The VIX index is supposed to measure the market's volatility forecast over the future, thus it seems to have little connection to observable behavior of the actual S&P500 volatility. Finally, we conclude that in the case of trading VIX futures instead of VIX itself, there is no economic gain from forecasting the VIX index.

The structure of the paper is as follows. The second section presents the dataset, while the third one describes the estimation procedure of volatility forecasts. The proposed method to evaluate the forecasting performance is presented in section four, whereas section five explores the forecasting ability of the models under investigation. Section six investigates whether an agent can utilize volatility index forecasts to create abnormal returns in the implied volatility futures market. Section seven concludes the paper and provides some ideas for further research.

2. Intraday and Interday Datasets

The S&P500 and VIX indices were obtained from the CBOE for the period 3rd January, 1990 to 24th December, 2003. Index VIX measures the market's expectation of 30-day volatility implicit in the prices of near-term S&P500 options. The result forms a composite hypothetical option that is at-the-money and has 30 calendar (22 trading) days to expiration. Index VIX represents the implied volatility for this hypothetical option. On September 22, 2003, the CBOE announced a new computation of its volatility index. The *old* VIX changed to VXO. The *new* VIX is based on S&P500 index options instead of S&P100 options and uses nearly all of the available S&P500 index options in its calculation, as opposed to just eight options of VXO[2]. In the present study as VIX_{close} the *new* VIX index is considered.

The intraday dataset was obtained from Olsen and Associates for the period ranging from 2nd January, 1997 to 24th December, 2003. The realized volatility on day t is computed as:

$$RV_t = \sqrt{\frac{\hat{\sigma}_{oc}^2 + \hat{\sigma}_{co}^2}{\hat{\sigma}_{oc}^2} \sum_{j=1}^{m-1} \left(100\left(\ln\left(P_{(j+1/m),t}\right) - \ln\left(P_{(j/m),t}\right)\right)\right)^2}\ , \tag{1}$$

[2] For details about the construction of CBOE volatility indices refer to Whaley (1993), Fleming et al. (1995), as well as to CBOE VIX whitepaper in http://www.cboe.com/micro/vix/vixwhite.pdf.

where $P_{(m),t}$ are the S&P500 prices on day t with m observations per day, $\hat{\sigma}_{oc}^2 = T^{-1}\sum_{t=1}^{T}\left(\ln\left(P_{(1),t}\right) - \ln\left(P_{(1/m),t}\right)\right)^2$ is the open-to-close sample variance, and $\hat{\sigma}_{co}^2 = T^{-1}\sum_{t=1}^{T}\left(\ln\left(P_{(1/m),t}\right) - \ln\left(P_{(1),t-1}\right)\right)^2$ is the close-to-open sample variance. Five-minute linearly interpolated prices, from 08:30 CST until 15:00 CST, or $m = 79$, are considered for avoiding market microstructure frictions without lessening the accuracy of the continuous record asymptotics. The scaling factor $\hat{\sigma}_{oc}^{-2}\left(\hat{\sigma}_{oc}^2 + \hat{\sigma}_{co}^2\right)$ accounts for the overnight returns without inserting the *noisy* effect of daily returns[3].

Table 1 presents the descriptive statistics of daily log-returns $\left(100\ln\left(P_{(1),t}/P_{(1),t-1}\right)\right)$, implied volatility index closing prices (VIX_{close}), annualized realized standard deviation $\left(\sqrt{252RV}\right)$, and annualized conditional standard deviation $\sqrt{252}\sigma$, extracted from TARCH model in Equation (3), whereas Figure 1 depicts the relative line graphs. Figure 2 plots the distributions of daily log-returns, logarithmic implied volatility index, and logarithmic intraday standard deviation. The density estimates are based on the normal Kernel with bandwidths method calculated according to Equation 3.31 of Silverman (1986). The daily log-returns indicate nonzero skewness and excess kurtosis relative to that of the normal distribution. The annualized unconditional volatility of daily log-returns is 16.62%, (1.0468* $\sqrt{252}$). The mean value of the implied volatility index is 20.19%. The average annualized realized volatility is 15.86%, whereas during the same period the annualized unconditional (conditional) volatility of daily returns is 20.55% (19.34%). The intraday volatility is much less as in Blair et al. (2001), who noted that this is a consequence of positive correlation between consecutive intraday returns. The intraday standard deviation is leptokurtic and skewed to the right. Our findings are in line with the previous studies (i.e., Ebens 1999, Andersen et al. 2001, Thomakos and Wang 2003, Giot and Laurent 2004) as the intraday logarithmic standard deviation is close to the normal distribution but statistically distinguishable from it. The Jarque-Bera (56.40), Anderson-Darling (3.94), and Crámer-Von Misses (0.63) statistics reject the hypothesis of normality at any level of significance.

Table 1: Descriptive statistics of S&P500 daily log-returns, implied volatility index (VIX_{close}), annualized realized standard deviation and annualized conditional standard deviation (extracted from TARCH model).

	Daily Returns (3rd Jan.90-24th Dec. 03)	**VIX index (3rd Jan.90-24th Dec. 03)**	**Realized Volatility (2nd Jan.97-24th Dec. 03)**	**Conditional Volatility (2nd Jan.97-24th Dec. 03)**
Mean	0.031375	20.19891	15.86304	19.33992
Median	0.032147	19.52000	14.58000	17.92984
Maximum	5.575686	45.74000	57.87000	46.67359
Minimum	-7.115025	9.310000	4.590000	9.269228
Std. Dev.	1.046804	6.454584	6.527193	6.097755
Skewness	-0.023829	0.824203	1.849690	1.199369
Kurtosis	6.099974	3.569710	8.626524	4.423520
Jarque-Bera	1408.174	445.6255	3302.495	566.6693
Probability	0.000000	0.000000	0.000000	0.000000

[3] The method we follow to account for the effects of overnight returns and intraday noise is similar to Martens (2002) and Koopman et al. (2005).

Figure 1: Figures of daily log-returns, implied volatility index, annualized realized standard deviation and annualized daily conditional standard deviation (extracted from TARCH model).

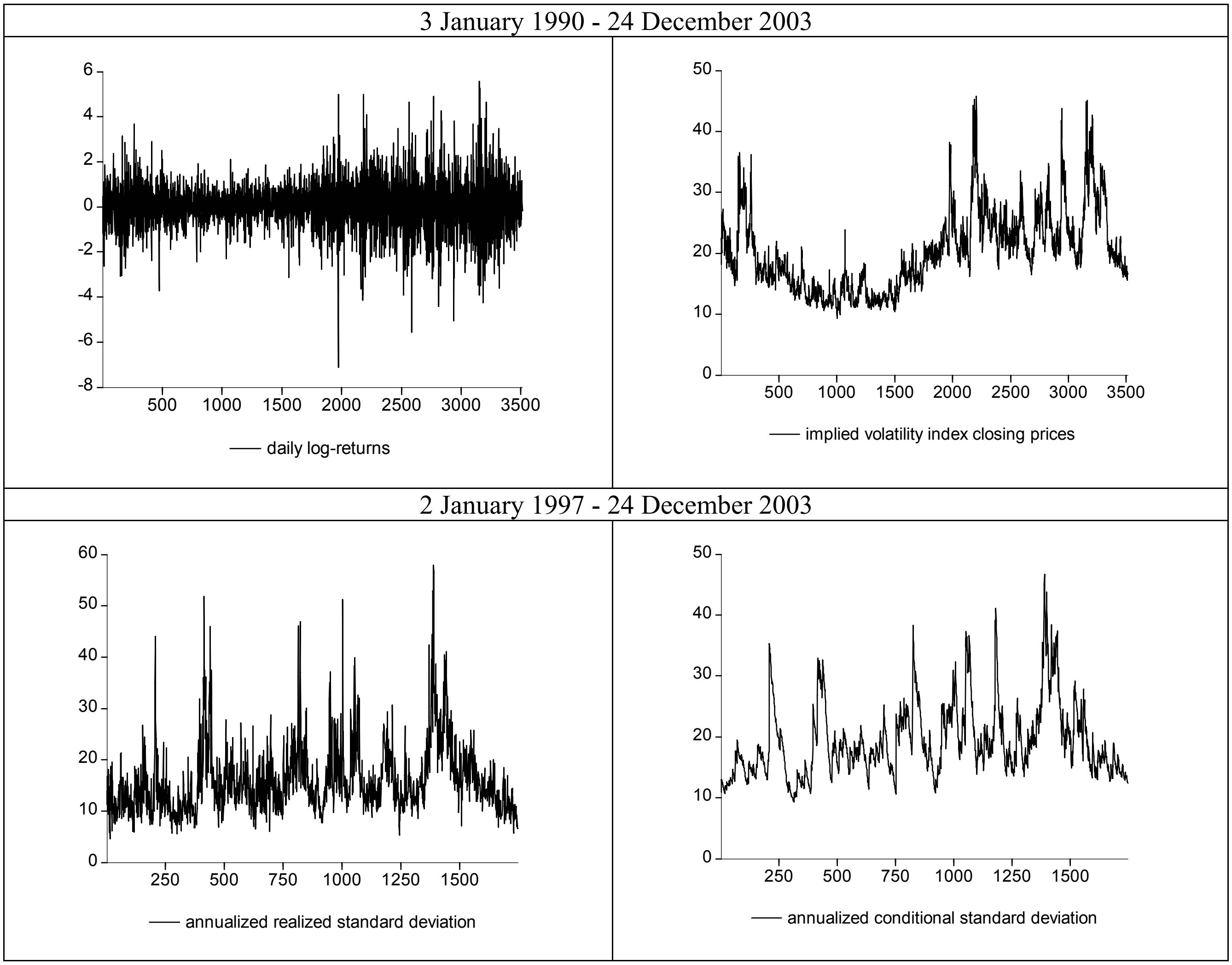
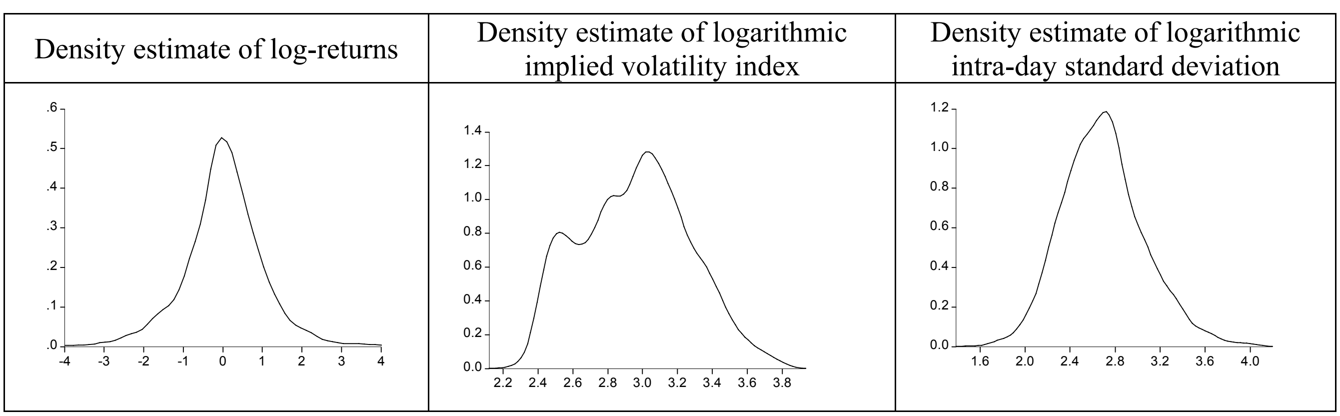


Figure 2: Distribution of daily log-returns, logarithmic implied volatility index, and logarithmic intraday standard deviation.



3. Volatility Models

The logarithm of the VIX index is regarded as the dependent variable in an ARFIMAX specification with normally distributed innovations. The interday conditional volatility and the intraday realized volatility are considered as exogenous variables to investigate their contribution in forecasting the next day's VIX value. The ARFIMAX model is defined as:

$$(1-aL)(1-L)^d\left(\ln VIX_t - w_0 - w_1 y_{t-1} - \delta_1 f(RV_{t-1}) - \delta_2 f(\sigma_{t-1})\right) = (1+bL)u_t \tag{2}$$
$$u_t \overset{i.i.d.}{\sim} N\left(0,\sigma_u^2\right),$$

where $VIX_t = VIX_{close,t}\left(\sqrt{252}\right)^{-1}$, $y_t = 100\ln\left(P_{(1),t}/P_{(1),t-1}\right)$ is the return series from day $t-1$ to t, $P_{(1)t}$ is the S&P500 closing price at day t, and L is the lag operator. The AFRIMAX model was introduced by Granger (1980) and Granger and Joyeux (1980) and was applied first in modeling realized volatility by Ebens (1999). The fractional integration parameter, d, captures the slow hyperbolic decay of the response of the implied volatility index to past shocks, whereas the parameter w_1 takes into account the response of daily S&P500 returns to the implied volatility index. The realized volatility specification is modeled in the forms $f(RV_{t-1}) = RV_{t-1}, RV_{t-1}^2, \ln\left(RV_{t-1}^2\right)$. The conditional volatility extracted from an Arch model is considered either as the in-sample volatility estimated at day $t-1$ given the information set that is available at the same day, $f\left(\sigma_{t-1|t-1}\right)$, or as the out-of-sample volatility of day t given the information set that is available at day $t-1$, $f\left(\sigma_{t|t-1}\right)$. Thus, $f\left(\sigma_{t-1}\right) = \sigma_{t-1|t-1}, \sigma_{t-1|t-1}^2, \ln\sigma_{t-1|t-1}^2, \sigma_{t|t-1}, \sigma_{t|t-1}^2, \ln\sigma_{t|t-1}^2$. The parameters δ_1 and δ_2 represent the added contribution of realized intraday volatility and conditional interday volatility, respectively. Since $u_t \sim N\left(0,\sigma_u^2\right)$, the one-day-ahead VIX is $VIX_{t+1|t}^* = \exp\left(\ln VIX_{t+1|t} + 0.5\sigma_u^2\right)$.

In the sequel, we propose an Arch model for computing the conditional volatility on the S&P500 index daily returns. The TARCH model, which was introduced by Glosten et al. (1993), with skewed Student t distributed standardized innovations, represents a parsimonious Arch model that accounts for the asymmetric response of innovations to volatility:

$$\begin{aligned} y_t &= c_0' + \left(1-c_1'L\right)^{-1}\varepsilon_t \\ \varepsilon_t &= z_t\sigma_t \\ z_t &\overset{i.i.d.}{\sim} skT(0,1;v,g) \\ \sigma_t^2 &= a_0' + a'\left(z_{t-1}\sigma_{t-1}\right)^2 + \gamma' d_{t-1}'\left(z_{t-1}\sigma_{t-1}\right)^2 + b'\sigma_{t-1}^2, \end{aligned} \tag{3}$$

where $skT(z_t;v,g) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}}\left(\frac{2s}{g+g^{-1}}\right)\left(1+\frac{sz_t+m}{v-2}g^{-d_t}\right)^{-\frac{v+1}{2}}$, g is the asymmetry parameter, $v>2$ denotes the degrees of freedom of the distribution, $\Gamma(.)$ is the gamma function, $d_t' = 1$ if $z_t > 0$ and $d_t' = 0$ otherwise, $m = \Gamma((v-1)/2)\sqrt{(v-2)}\left(\Gamma(v/2)\sqrt{\pi}\right)^{-1}\left(g-g^{-1}\right)$ and $s = \sqrt{g^2+g^{-2}-m^2-1}$. The autoregressive component of the conditional mean is considered to account for the nonsynchronous trading effect. The in-sample conditional variance is estimated as $\sigma_{t|t}^2 = a_0'^{(t)} + \left(a'^{(t)} + \gamma'^{(t)}d_{t-1}'\right)\varepsilon_{t-1|t}^2 + b'^{(t)}\sigma_{t-1|t}^2$. The one-day-ahead conditional variance forecast is computed as $\sigma_{t+1|t}^2 = a_0'^{(t)} + \left(a'^{(t)} + \gamma'^{(t)}d_t'\right)\varepsilon_{t|t}^2 + b'^{(t)}\sigma_{t|t}^2$.

4. Evaluation Methodology

The common method to evaluate the forecasting performance is through defining a statistical loss function that measures the distance between predictions and observations. In our study, we create an economic loss function that calculates the cumulative returns from trading VIX index on a daily basis. If the VIX price forecast is greater than the VIX closing price, the VIX index is bought. If the VIX price forecast is less than the VIX closing price, the index is sold. For each transaction, traders should pay a transaction cost. Moreover, trades will be executed only when profits are expected to exceed the transaction cost. Thus, a filter is applied, so as to proceed in a trade only when the difference between forecast and observed VIX prices exceeds the amount of filter F. The model's i average daily return after a transaction cost, X, is

$$\overline{R}^{(i)} = s^{*-1}\left(\sum_{t=1}^{s^*}\left(\frac{\left(VIX_{close,t+1} - VIX_{close,t}\right)d_t^{(i)}}{VIX_{close,t}}\right) - \sum_{t=1}^{n^{(i)}}\frac{X}{VIX_{close,t}}\right), \tag{4}$$

where s^* is the number of trading days, $n^{(i)}$ is the number of transactions, $d_t^{(i)} = 1$ if $VIX_{t+1|t}^{*(i)} > VIX_t + F$, $d_t^{(i)} = -1$ if $VIX_{t+1|t}^{*(i)} \leq VIX_t - F$ and $d_t^{(i)} = 0$ otherwise [4].

Besides the average daily return, the Sharpe ratio is computed as the ratio of the annualized average returns with annualized standard deviation of daily returns:

$$SR = \frac{\overline{R}^{(i)}\sqrt{252}}{\sqrt{V\left(R^{(i)}\right)}}. \tag{5}$$

To investigate whether the model achieves the highest performance and is significantly different from its competitors, we apply the Diebold and Mariano (1995) test. The null hypothesis of equivalent predictive ability of models i and i^* against the alternative hypothesis that the benchmark model i is superior to model i^* is tested. Let $r_t^{(i)} = \dfrac{\left(VIX_{\text{close},t+1} - VIX_{\text{close},t}\right)d_t^{(i)} - Xn_t}{VIX_{\text{close},t}}$ denote the return on day t based on model i, where $n_t = 2$ if $d_t^{(i)}d_{t-1}^{(i)} = -1$, $n_t = 0$ if $d_t^{(i)} = d_{t-1}^{(i)}$, $n_t = 1$ otherwise. For $z_t^{(i,i^*)} = \left(r_t^{(i)} - r_t^{(i^*)}\right)$, the Diebold–Mariano (DM) statistic is the t-statistic derived by the regression of $z_t^{(i,i^*)}$ on a constant with HAC standard errors.

5. Empirical Results

The models were estimated in the G@RCH and ARFIMA packages of Ox. The first VIX forecast is generated for January 2nd, 2002. For each trading day, the models are re-estimated based on the rolling sample of constant size equal to $s = 1249$ trading days, hence, $s^* = 499$ one-day-ahead volatility forecasts are estimated. We take into consideration a transaction cost of \$0.2, which reflects 20 times the minimum price interval of VIX point, and consider various values for the filter and, in particular, $F = 0(0.1)0.4$.[5]

The orders of the autoregressive and the moving average components of the ARFIMAX framework are not determined based on in-sample model selection criteria, such the Akaike's and Schwarz's information criteria, as a good in-sample performance of a model is not a prerequisite for its good out-of-sample precision[6]. We have estimated various versions of the framework given by Equation (2), that is, higher orders of the autoregressive and the moving average components, and different sets of exogenous variables, but we exhibit the models that are useful for the presentation of the results[7]. The ARIMAX model for zero autoregressive order and moving average order of one achieves the highest forecasting performance.

In the sequel, we name the model ARFIMAX, but the autoregressive component is omitted. The specification given by Equation (2) is estimated for $\delta_1 = \delta_2 = 0$ as well as for the various functional forms of the realized intraday and the conditional interday volatility, providing 12 models in

---

4 Two transactions costs are charged because we have to unwind yesterday's position and put a new one on today. Both transaction costs are applied to today's returns. Yesterday's transactions cost could be applied to yesterday's return, which, however, reaches to almost identical average daily returns.

5 $F = a(b)c$ denotes $T = a, a+b, a+2b, \ldots, c-b, c$.

6 For more details see Brooks and Burke (2003), Angelidis et al. (2004), and Degiannakis and Xekalaki (2007).

7 However, all the estimated models are available upon request.

total. Table 2 presents the 12 models, numbered from 0 to 11. For example, model 1 denotes the ARFIMAX model in Equation (2) with the RV_{t-1}^2 as exogenous variable. According to Table 2, which presents the average daily returns and the Sharpe ratios of these models, the daily returns, without assuming any trading cost, are between 0.141% and 0.251% with Sharpe ratios ranging from 0.453 to 0.809. The ARFIMAX model, without any functional form of RV_{t-1} or σ_{t-1} as the exogenous variable, achieves the highest profit. However, after a transaction cost of \$0.2, the average returns are negative in all the cases. We proceed on a trade only when profits are predicted to exceed the assumed trading cost. Hence, trades are executed only when the absolute difference between forecast and today's VIX price exceeds the amount of the filter F. For \$0.2 trading cost and filter, the ARFIMAX model that takes into consideration volatility information solely from the lag values of the VIX index is still the model with the highest returns. After a trading cost of \$0.2 and a filter rule of \$0.4, the ARFIMAX model with the realized variance RV_{t-1}^2 as the exogenous variable is the best performing model. In general, all the forecasting information is provided by the VIX index. There is no substantial improvement in the forecasting ability of the models that take into consideration information from interday or intraday S&P500 volatility. Models 1 (with RV_{t-1}^2 as exogenous variable) and 8 (with $\sigma_{t|t-1}^2$ as exogenous variable) as well as the model without any exogenous volatility information achieve the highest rate of returns.

Table 2: The average daily returns and the Sharpe ratios of the 12 models, after a trading cost of \$0.0 and \$0.2 and various values for filter $F = 0(0.1)0.4$.

		Average Daily Return				Sharpe Ratio			
		Cost 0.0	**Cost 0.2**			**Cost 0.0**	**Cost 0.2**		
ARFIMAX (0,d,1) with exogenous	**Model**	**Filter 0,0**	**Filter 0.0**	**Filter 0.2**	**Filter 0.4**	**Filter 0.0**	**Filter 0.0**	**Filter 0.2**	**Filter 0.4**
-	0	**0.251%**	-0.015%	**0.118%**	0.005%	**0.809**	-0.049	**0.420**	0.022
RV_{t-1}^2	1	0.223%	-0.024%	0.089%	**0.070%**	0.717	-0.077	0.318	**0.290**
RV_{t-1}	2	0.231%	-0.047%	0.060%	-0.024%	0.743	-0.151	0.214	-0.101
$\ln RV_{t-1}^2$	3	0.204%	-0.097%	0.017%	-0.044%	0.655	-0.307	0.059	-0.181
$\sigma_{t-1\|t-1}^2$	4	0.150%	-0.061%	-0.010%	-0.112%	0.482	-0.194	-0.036	-0.469
$\sigma_{t-1\|t-1}$	5	0.141%	-0.065%	-0.043%	-0.048%	0.453	-0.207	-0.157	-0.210
$\ln \sigma_{t-1\|t-1}^2$	6	0.209%	-0.024%	-0.122%	-0.165%	0.673	-0.077	-0.441	-0.745
$\ln RV_{t-1}^2, \ln \sigma_{t-1\|t-1}^2$	7	0.208%	-0.076%	0.015%	-0.031%	0.670	-0.242	0.052	-0.128
$\sigma_{t\|t-1}^2$	8	0.238%	**-0.004%**	0.071%	0.063%	0.764	**-0.013**	0.256	0.260
$\sigma_{t\|t-1}$	9	0.187%	-0.048%	0.037%	0.046%	0.601	-0.152	0.133	0.188
$\ln \sigma_{t\|t-1}^2$	10	0.214%	-0.038%	-0.001%	-0.072%	0.688	-0.119	-0.005	-0.300
$\ln RV_{t-1}^2, \ln \sigma_{t\|t-1}^2$	11	0.179%	-0.092%	-0.016%	-0.082%	0.576	-0.292	-0.055	-0.349

Table 3 presents the percentage of long and short trading positions for trading VIX index based on the signals generated by the predictions of the models. Without transaction costs, the long and short trading positions are suggested, on average, in 65% and 35% of the trading days, respectively. In the cases of a \$0.2 and \$0.4 filters, the long positions are also more often suggested than the short ones. The average daily profit from always taking long trading positions is 0.06% without assuming any trading cost and negative in the case of adding a trading cost and any filter.

Table 3: Percentage of long and short trading positions for trading VIX index.

		Filter 0.0		Filter 0.2		Filter 0.4	
ARFIMAX (0,d,1) with exogenous	**Model**	**Long**	**Short**	**Long**	**Short**	**Long**	**Short**
-	0	64%	36%	53%	27%	35%	21%
RV_{t-1}^2	1	66%	34%	52%	28%	35%	22%
RV_{t-1}	2	66%	34%	52%	28%	36%	22%
$\ln RV_{t-1}^2$	3	66%	34%	52%	28%	36%	22%
$\sigma_{t-1\|t-1}^2$	4	65%	35%	51%	29%	32%	20%
$\sigma_{t-1\|t-1}$	5	64%	36%	50%	27%	30%	21%
$\ln\sigma_{t-1\|t-1}^2$	6	65%	35%	51%	27%	28%	20%
$\ln RV_{t-1}^2, \ln\sigma_{t-1\|t-1}^2$	7	66%	34%	52%	28%	36%	21%
$\sigma_{t\|t-1}^2$	8	64%	36%	51%	29%	36%	21%
$\sigma_{t\|t-1}$	9	65%	35%	51%	28%	34%	22%
$\ln\sigma_{t\|t-1}^2$	10	64%	36%	51%	28%	33%	22%
$\ln RV_{t-1}^2,\ \ln\sigma_{t\|t-1}^2$	11	65%	35%	53%	28%	33%	22%

The first two columns denote the percentage of long and short trading positions for no transaction costs. The last four columns denote the percentage of long and short trading positions for \$0.2 and \$0.4 filters.

Table 4 presents the corresponding p-values of the DM test for the null hypothesis that model i has statistically equal loss function as model i^*. The null hypothesis is not rejected at any rational level of significance, indicating that realized volatility and the extracted volatility from the Arch model do not provide any significant incremental predictive ability in forecasting VIX index. The model without any functional form of RV_{t-1} or σ_{t-1} as exogenous variable has statistically equal loss function as its competing models[8].

Table 4: The p-values of the DM statistic for the null hypothesis that model i has equal predictive ability as model i^* (after a trading cost and filter of 0.2).

ARFIMAX (0,d,1) with exogenous	**-**	RV_{t-1}	$\ln RV_{t-1}^2$	$\sigma_{t\|t-1}^2$	$\sigma_{t\|t-1}$
-	-	0.344	0.152	0.695	0.457
RV_{t-1}^2		0.464	0.162	0.842	0.610
RV_{t-1}		-	0.086	0.912	0.818
$\ln RV_{t-1}^2$			-	0.585	0.844
$\sigma_{t\|t-1}^2$				-	0.604
$\sigma_{t\|t-1}$					-

Figure 3 depicts, indicatively, the cumulative daily returns of Models 1 and 8. Figure 4 plots the VIX index and the corresponding one-day-ahead forecasts of Models 1 and 8, whereas Figure 5 presents the scatter plot of VIX index and the one-day-ahead VIX forecasts. In both cases, the one-day-

[8] The evaluation was also conducted based on statistical loss functions that measure the distance between predicted and actual VIX values. The minimum values of MSE and MAE criteria were achieved by Model 5. The hypothesis that Model 5 has superior predictive ability is not rejected at any reasonable level of significance. Therefore, if the evaluation were based on the distance between observed and predicted values, we would have concluded that the conditional volatility extracted from the TARCH model provides added predictive ability in forecasting the VIX index.

ahead prediction line graphs are almost indistinguishable from the index itself. Table 5 presents the estimated parameters of the Models 1 and 8 by using the entire dataset. All the parameters except the constant coefficients are statistically significant at least at 5% level of significance. The variance is characterized by slowly mean-reverting fractionally integrated process as the degree of integration is lower than 0.5 (although statistically equal to 0.5 for the intraday model) and therefore, there are indications that the intraday volatility is covariance stationary. Cases in which the logarithmic realized volatility of stock indices is the dependent variable, parameter d was estimated at 0.46 and 0.48 for the CAC40 and S&P500 indices, respectively, by Giot and Laurent (2004), less than 0.4 (Dow Jones Industrial Average) by Ebens (1999), and around 0.45 (S&P500) by Thomakos and Wang (2003). In regard to the TARCH model, the asymmetry between past bad or good news and volatility is statistically significant and the estimated parameters of the skewed Student-t distribution indicate that the innovations are asymmetric and leptokurtic.

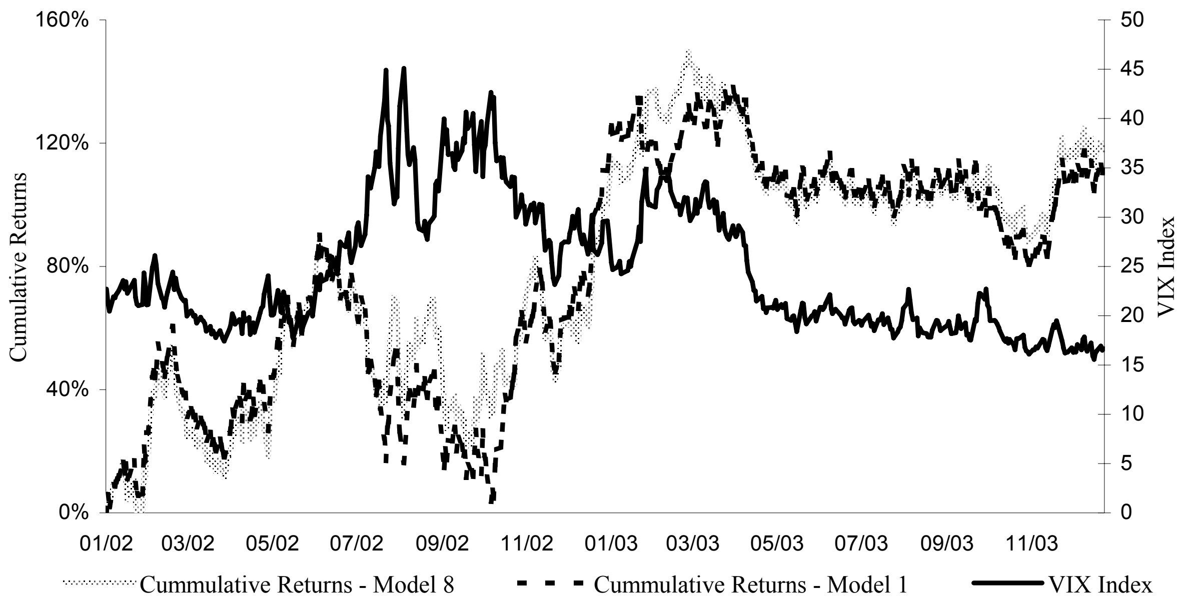
Table 5: Parameter estimates for Models 1 and 8 (2nd January 1997 – 24th December 2003).

	Model 1		**Model 8**	
Parameters	**Coefficient**	**Standard error**	**Coefficient**	**Standard error**
		ARFIMAX Model		
b	0.4874[a]	(0.0242)	0.4398[a]	(0.0276)
d	0.4992[a]	(0.0009)	0.4990[a]	(0.0012)
w_0	0.3458	(1.2630)	0.3129	(1.0690)
w_1	0.0043[a]	(0.0007)	0.0053[a]	(0.0008)
δ_1	0.0026[b]	(0.0014)	-	-
δ_2	-	-	0.0265[a]	(0.0043)
σ_u^2	0.0032	-	0.0032	-
		TARCH Model		
c'_0	-		0.0018	(0.0267)
c'_1	-		-0.0125	(0.0225)
a'_0	-		0.0412[a]	(0.0131)
a'	-		-0.0310[a]	(0.0101)
γ'	-		0.1733[a]	(0.0288)
b'	-		0.9220[a]	(0.0168)
v	-		14.712[a]	(4.8772)
g	-		-0.0755[b]	(0.0372)

[a] Indicates that the coefficient is statistically significant at 1% level of significance.
[b] Indicates that the coefficient is statistically significant at 5% level of significance.

Figure 3: The VIX index and the cumulative rate of returns of Models 1 and 8. (2nd January 2002 – 24th December 2003).

Without any transaction cost



After a \$0.2 transaction cost and a \$0.4 filter

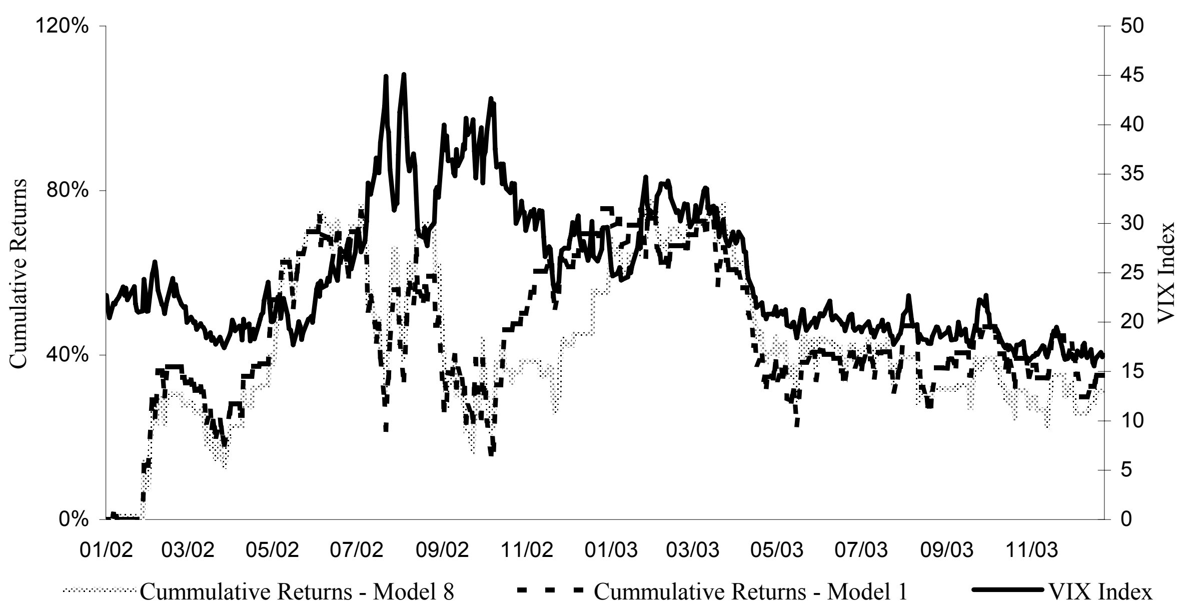
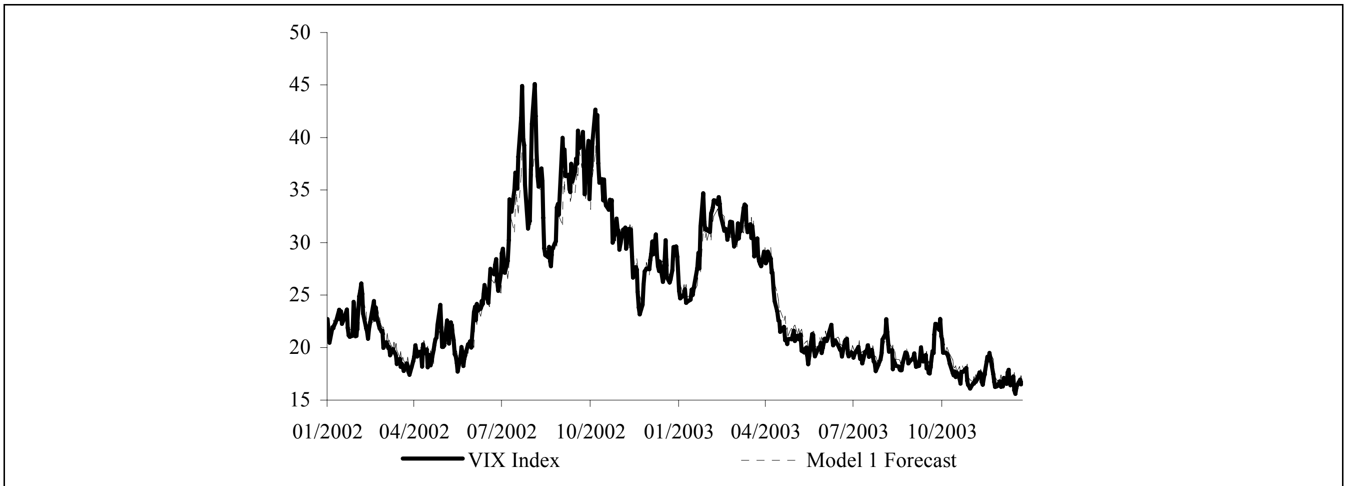


Figure 4: VIX index and its one-day-ahead forecasts, $VIX^{*}_{t|t-1}\sqrt{252}$, of Models 1 and 8 (2nd January 2002 - 24th December 2003, 499 observations).



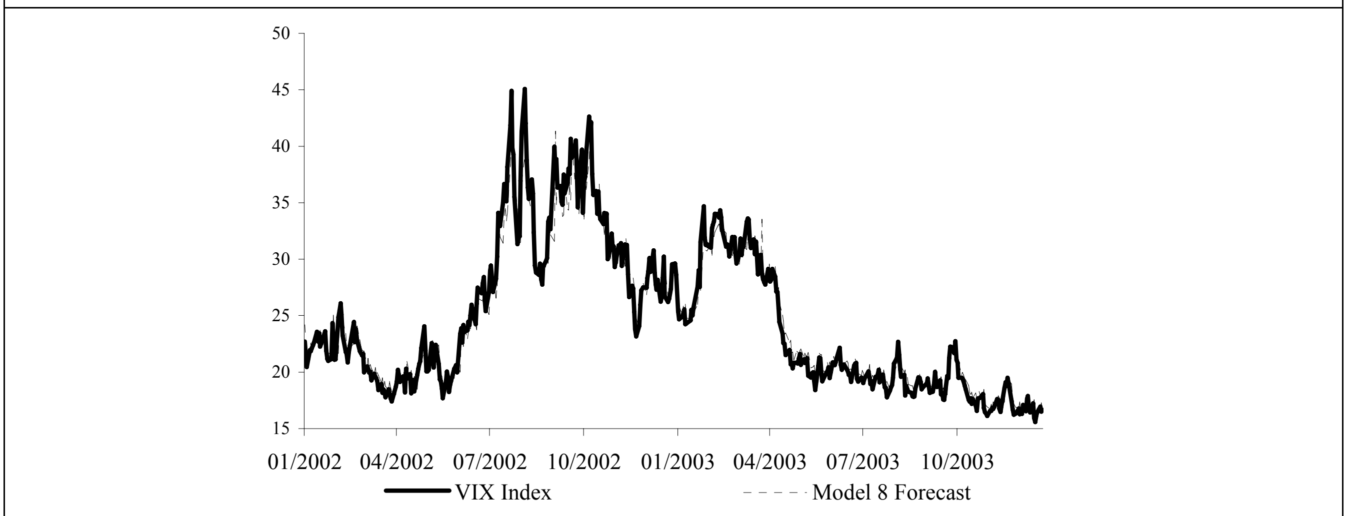
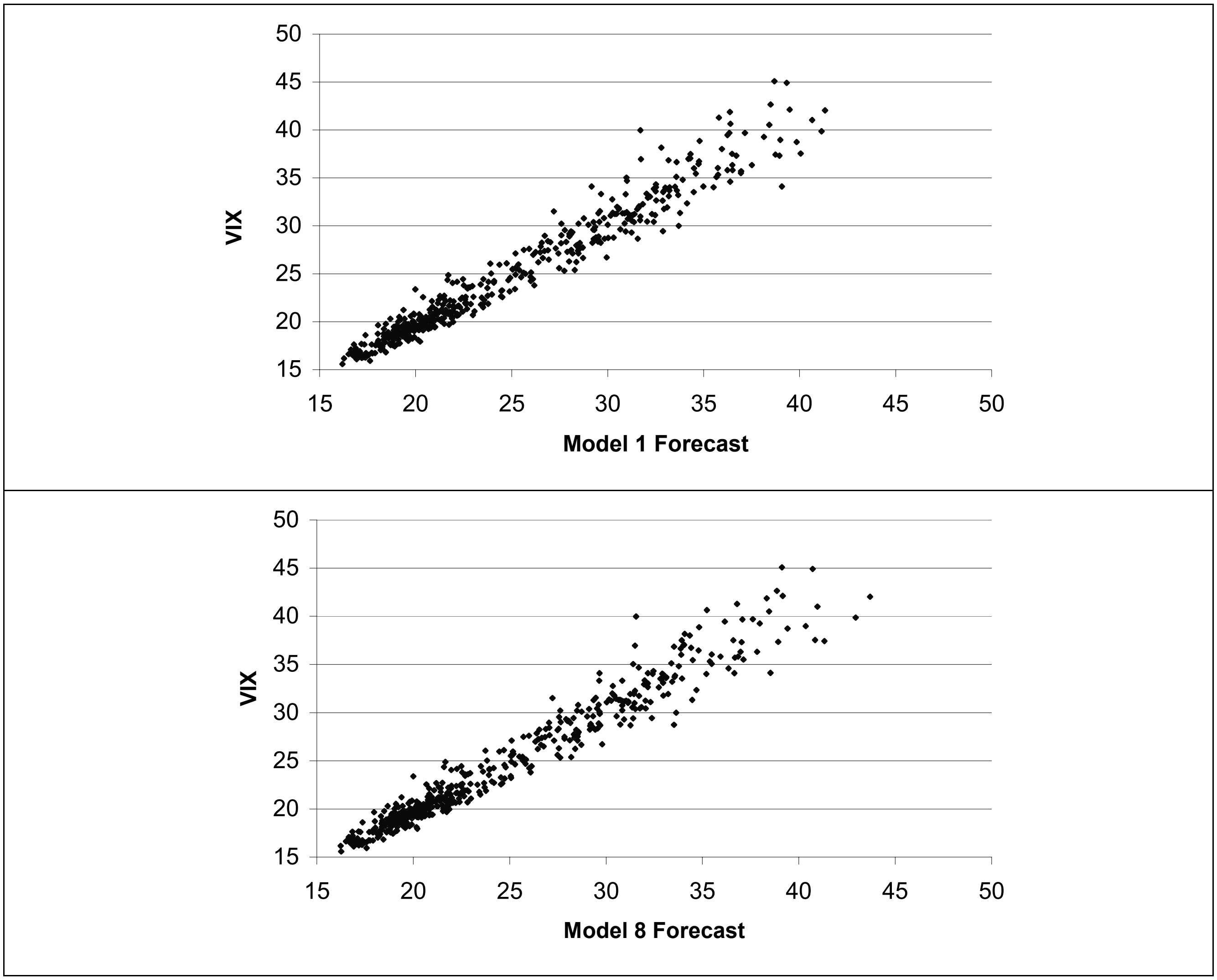


Figure 5: VIX index and one-day-ahead VIX forecasts scatter plots of Models 1 and 8 (2nd January 2002 - 24th December 2003).



6. Trading Game with VIX Futures

VIX index is a volatility forecast, not an asset. Hence, in reality, we cannot create a position by buying or short-selling the index itself. On March 26th, 2004, the CBOE announced the trading of futures on the VIX index. VIX futures are contracts on forward 30-day implied volatilities. They are quoted 10 times the value of VIX and the contract multiplier is \$100. The minimum price interval is 0.01 of VIX point or \$10 per contract. The final settlement date is the Wednesday prior to the third Friday of the expiring month.

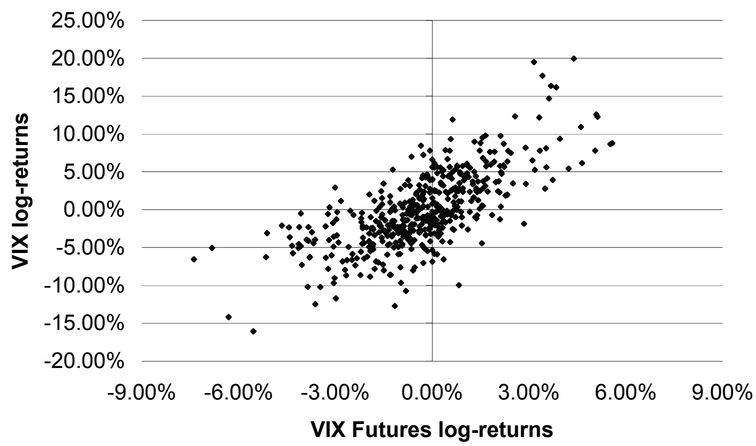
The data of futures on VIX were obtained from the CBOE over the sample period of 26th March, 2004 through 23rd February, 2006 [9]. We repeat the estimation of the model framework given in Equation (2) by expanding the sample period up to 23rd February, 2006. The loss function given in Equation (4) is measured by replacing $VIX_{\text{close},t}$ with one tenth of the VIX futures settlement value. The futures with contract months on the February quarterly cycle and a maturity period of length no shorter than 4 trading days were considered as these are the contracts with the highest trading volume. In the case of trading VIX futures instead of VIX itself, there is no economic gain from forecasting the VIX index. Thus, an agent who applies the proposed forecasting model is unable to create trading strategies

[9] CBOE VIX futures dataset is available at: http://cfe.cboe.com/Products/historicalVIX.aspx.

that yield abnormal returns. Of course, we can reasonably conclude that the VIX index on day t expresses the volatility expected for the period from day t to day $t+30$, while the next month's VIX futures express the volatility expected for the period from the expiration day to 30 days ahead.

The VIX itself, an approximation to the overall implied volatility of the S&P500 options, is a prediction of future volatility. VIX futures on the other hand is a prediction of where the prediction of future volatility will be on the day that the future expires. The futures contract is based on a future version of the VIX index. According to Figure 6, which presents the VIX and VIX futures scatter plot, the correlation between VIX and VIX futures is not high and can be even negative. For example, in 26% of the trading days the VIX and VIX futures log-returns have opposite signs.

Figure 6: VIX index and VIX futures scatter plot, (26th March 2004 to 23rd February 2006).



7. Conclusions

We provided an empirical model that produced adequate one-day-ahead predictions of VIX index. Instead of evaluating forecasts based on statistical loss functions, we measured an economic loss function as the average return from trading VIX index on a daily basis. All the forecasting information is provided by the VIX index itself. Both realized volatility and conditional volatility extracted from an Arch model were considered as exogenous variables in the model, but they did not provide any incremental information in forecasting VIX index. Hence, there is no added value either from the use of the more hard-to-collect intraday datasets or from estimating an Arch model using daily datasets.

Blair et al. (2001) and Koopman et al. (2005) also provided evidence that interday volatility does not provide more accurate forecasts than the implied volatility. We also confirmed Blair's et al. (2001) finding that the intraday volatility measure does not yield significant incremental forecasting information.

An interesting point that is left for further study is the evaluation of the models' predictability in a multiperiod framework. Moreover, there is not yet a standard method of computing realized volatility based on the intraday datasets. Recently, Zhang et al. (2003), Engle and Sun (2005), and Hansen and Lunde (2005) proposed various measures of realized volatility. However, the most precise method to compute the realized variance is still an open area for research. Whether the new methods of computing realized volatility will increase its forecasting ability is also a very interesting topic.

An important issue that is still unanswered and a future study should be focused on is whether the VIX index is efficient or not. The dynamics of the VIX, which is supposed to measure the market's volatility forecast over the next 30 days, seem to have little connection to observable behavior of the actual S&P500 volatility. This could mean that the VIX is highly efficient and fully discounts the information that can be extracted from recent historical data, or it could mean that the VIX itself is inefficient and valuable information from Arch type models (or realized volatility measure) is ignored?

In the previous section we discovered that forecasts of the VIX index are unable to provide any information in forecasting VIX futures. VIX futures were introduced on March 26th, 2004. Hence, adequate sample size is not available to estimate the proposed model with the VIX futures series as dependent variable. When an adequate sample size of VIX futures becomes available, it will be interesting to investigate whether an ARFIMAX model on the VIX futures can produce forecasts that yield positive profits.

Acknowledgement
I would like to thank Pierre Giot and Sébastien Laurent for the availability of their program codes and Stephen Figlewski for his valuable and constructive suggestions.

References
[1] Andersen, T. and Bollerslev, T. (1998). "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts." *International Economic Review*, 39, 885-905.

[2] Andersen, T., Bollerslev, T., Diebold, F.X. and Ebens, H. (2001). "The Distribution of Realized Stock Return Volatility." *Journal of Financial Economics*, 61, 43-76.

[3] Angelidis, T., Benos, A. and Degiannakis, S. (2004). "The Use of GARCH Models in VaR Estimation." *Statistical Methodology*, 1(2), 105-128.

[4] Blair, B.J., Poon, S-H and Taylor S.J. (2001). "Forecasting S&P100 Volatility: The Incremental Information Content of Implied Volatilities and High-Frequency Index Returns." *Journal of Econometrics*, 105, 5-26.

[5] Brooks, C. and Burke, S.P. (2003). "Information Criteria for GARCH Model Selection." *European Journal of Finance*, 9, 557-580.

[6] Degiannakis, S. and Xekalaki, E. (2007). "Assessing the Performance of a Prediction Error Criterion Model Selection Algorithm in the Context of ARCH Models." *Applied Financial Economics*, 17, 149-171.

[7] Diebold, F.X. and Mariano, R. (1995). "Comparing Predictive Accuracy." *Journal of Business and Economic Statistics*, 13(3), 253-263.

[8] Ebens, H. (1999). "Realized Stock Volatility." Department of Economics, *Johns Hopkins University*, Working Paper, 420.

[9] Engle, R.F. (1982). "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation." *Econometrica*, 50, 987-1008.

[10] Engle, R.F. and Sun, Z. (2005). "Forecasting Volatility Using Tick by Tick Data." *European Finance Association*, 32th Annual Meeting, Moscow.

[11] Fleming, J., Ostdiek, B. and Whaley, R.E. (1995). "Predicting Stock Market Volatility: A New Measure." *Journal of Futures Markets*, 15, 265-302

[12] Giot, P. and Laurent, S. (2004). "Modelling Daily Value-at-Risk Using Realized Volatility and ARCH Type Models." *Journal of Empirical Finance*, 11, 379-398.

[13] Glosten, L., Jagannathan, R. and Runkle, D. (1993). "On the Relation between Expected Value and the Volatility of the Nominal Excess Return on Stocks." *Journal of Finance*, 48, 1779-1801.

[14] Granger, C.W.J. (1980). "Long Memory Relationships and the Aggregation of Dynamic Models." *Journal of Econometrics*, 14, 227-238.

[15] Granger, C.W.J. and Joyeux, R. (1980). "An Introduction to Long Memory Time Series Models and Fractional Differencing." *Journal of Time Series Analysis*, 1, 15-39.

[16] Hansen, P.R. and Lunde, A. (2005). "A Realized Variance for the Whole Day Based on Intermittent High-Frequency Data." *Journal of Financial Econometrics*, 3(4), 525-554.

[17] Koopman, S.J., Jungbacker, B. and Hol, E. (2005). "Forecasting Daily Variability of the S&P100 Stock Index Using Historical, Realised and Implied Volatility Measurements." *Journal of Empirical Finance*, 12(3), 445-475.

[18] Latane, H.A. and Rendleman, R.J. (1976). "Standard Deviations of Stock Price Ratios Implied in Option Prices." *Journal of Finance*, 31, 369-381.

[19] Martens, M. (2002). "Measuring and Forecasting S&P500 Index-Futures Volatility Using High-Frequency Data." *Journal of Futures Markets*, 22, 497-518.

[20] Silverman, B.W. (1986). *Density Estimation for Statistics and Data Analysis,* Chapman and Hall.

[21] Thomakos, D.D. and Wang, T. (2003). "Realized Volatility in the Futures Markets." *Journal of Empirical Finance*, 10(3), 321-353.

[22] Whaley, R.E. (1993). "Derivatives on Market Volatility: Hedging Tools Long Overdue." *Journal of Derivatives*, 1, 71-84.

[23] Zhang, L., Mykland, P.A. and Ait-Sahalia, Y. (2003). "A Tale of Two Time Scales: Determining Inttegrated Volatility with Noisy High-Frequency Data." *National Bureau of Economic Research*, Working Paper, 10111.