

# The Cross-Section of Commodity Futures Returns\*

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## Abstract

In this paper we study consumption risk pricing in commodity futures markets. We find that, like stock returns, the conditional Consumption CAPM explains up to 60% of the cross sectional variation in mean futures returns. However, unlike stock returns, using contemporaneous plus future consumption growth reduces the performance of the model. We attribute this result to the fact that for commodities supply changes impact prices and therefore consumption. Consistent with this notion we find that production- and inventory-based factors are significant determinants of the long run risk in commodities markets, which may explain the poor performance of ultimate consumption risk model.

JEL classification: G12 and G13

Keywords: futures, consumption-based model, ultimate consumption risk, production-based model

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# Introduction

This paper investigates the risk premiums in the commodity futures markets in the context of the consumption-based model. Our aim is to determine if commodity futures returns vary cross-sectionally in a systematic way due to differences in consumption risk. So far, the evidence in empirical studies indicates that although commodity returns do not appear to be related to movements in stock market returns (see, e.g., Dusak (1973), Bessembinder (1992), de Roon, Nijman, and Veld (2000)),<sup>1</sup> they do seem to be related to changes in aggregate real consumption (see, e.g., Breeden (1980), Jagannathan (1985)). Nevertheless, an attempt to reconcile the commodity risk premiums with the asset pricing theory have yielded disappointing results (see, e.g., Jagannathan (1985), Erb and Harvey (2006)).

We find that at the quarterly horizon, the (unconditional) CCAPM explains about 50% of the cross-sectional variation in mean futures returns, but there is almost no explained variance at the monthly level, and an intermediate result at the yearly level. This pattern is consistent with the results found by Jagannathan and Wang (2007) based on stock portfolios. However, we find somewhat lower implied consumption risk premiums for our futures contracts. When we assume that the model holds in a conditional sense (as in Jagannathan and Wang (1996)), allowing for time-varying betas and risk premiums, the CCAPM explains up to 60% of the cross-sectional variation in futures returns and again shows the best performance at the quarterly and annual frequency.

Using cumulative consumption growth over the several quarters, which Parker and Julliard (2005) define as ultimate consumption risk, we find that the performance of the CCAPM is best when we use the consumption growth of the contemporaneous quarter of the returns, but that the performance deteriorates over longer horizons. Although this contradicts the findings of Parker and Julliard for stock returns, it is consistent with the finding that the CCAPM performs best at the quarterly frequency.

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<sup>1</sup>This is based on the unconditional version of the CAPM. However, futures contracts do exhibit systematic risk once betas are allowed to vary (e.g., Carter, Rausser, and Schmitz (1983)) and when, additionally, SMB and HML factors are included (e.g., Erb and Harvey (2006)).

We attribute this difference in the performance of the ultimate consumption risk model for stocks and futures data to the offsetting adjustment in the supply side of the economy. We consider a model of consumption, production and storage (e.g., French 1986) and we find that production and inventory based factors are significantly priced in the commodity futures markets. Moreover, the performance of this model improves over longer horizon, which may explain the poor performance of ultimate consumption risk model.

There are two strands of literature that we build on. The first strand focuses on the explanations of the cross-sectional differences in average returns on financial assets. We study whether recent advances to the consumption-based model that have been shown to be successful in explaining the cross-section of stock returns, can also explain a broad cross-section of commodity futures contracts. The consumption-based framework seems to be a good choice for analyzing futures returns, since part of the underlying commodities are strongly related to aggregate consumption itself and may be used for hedging consumption risk.

In particular, we focus on Jagannathan and Wang (2007) who propose that consumption and investment decisions are made infrequently and show that the CCAPM explains more than 70% of the cross-sectional variation in expected stock returns when consumption growth is measured from the 4th quarter of one year to the next. Also, they find that lowering the frequency of consumption growth and returns from monthly to quarterly and annually, significantly improves the performance of the CCAPM, which is likely to result from the smaller measurement error in consumption growth at lower frequencies. Parker and Julliard (2005) conjecture that consumption may be slow to respond to stock returns. They find that ultimate consumption risk, defined as the covariance of a stock return and consumption growth over the quarter of the return and many following quarters, explains between 44% and 73% of the cross-sectional variation in stock returns.

We contribute to this stream of literature by studying commodity futures returns representing a separate asset class, which allows us to draw conclusions on the generalizability of the discussed

advances. This is important especially in view of Lewellen, Nagel, and Shanken (2006) who show that tests of the asset pricing models based on the Fama-French size and book-to-market portfolios are often misleading, as these portfolios are known to have a strong factor structure, i.e. high time-series and cross-sectional predictability based on the Fama-French factors.

It is not known a priori that the same advances should be equally successful in futures markets. There are important differences between the consumption betas in stocks and in futures markets. First, for (commodity) futures it is common to observe positive as well as negative consumption betas, a feature less common in equity markets. Second, the contemporaneous consumption beta for longer maturity futures is usually lower than for shorter maturity futures. This relation suggests that the time period over which consumption risk is measured may play an important role in determining the consumption risk in futures markets, unlike in stock markets.

The second strand of literature that we build on focuses on the existence and determination of commodity futures risk premiums. There are two views of commodity futures price determination. The first approach relates futures price to the expected spot price and futures risk premium. The latter approach, relates futures price to the spot price and the cost of carrying the commodity to the future. Previous research has shown that both these channels are affected by storage (e.g., Gorton, Hayashi, and Rouwenhorst (2007), and Acharya, Lochstoer, and Ramadorai (2009)). So the same primitive shock that affects the convenience yield will also affect the futures risk premium. Given that storage affects the relative present and future scarcity of the commodity we may expect that the third leg in a triad between futures price, spot price and the expected spot price will also be affected by storage. Our finding contributes to this stream of literature by providing an empirical support of this conjecture. We show that the relation between the spot price and expected spot price is affected by the change in storage and production growth. This is important result as it reconciles the the two views of the determination of futures price.

The rest of this paper is structured as follows. The next section gives a brief outline of the

consumption-based models. Section 2 discusses some estimation issues and Section 3 describes the data. The empirical results are discussed in Section 4. Finally, Section 5 concludes.

# 1 Consumption-based models for expected returns

According to finance theory, expected returns on securities are determined by their exposure to systematic economy wide risk. Rubinstein (1976) and Breeden (1979) show that the risk of a security is determined by its covariance with consumption growth (CCAPM). In this framework, a representative agent allocates her resources among consumption and different investment opportunities in order to maximize her utility over lifetime consumption:

$$\max E_t \left[ \sum_{\tau=t}^T \delta^\tau U(C_\tau, \tau) \right], \quad (1)$$

subject to the standard budget constraints, where we assume a time and state separable Von Neumann-Morgenstern utility function  $U(\cdot)$ ,  $C_\tau$  denotes consumption expenditures in period  $\tau$ ,  $\delta$  is the time discount factor, and  $E_t[\cdot]$  denotes the expectation conditioned on the information available at time  $t - 1$ . The first order conditions of the agent's maximization problem imply the following relation that is satisfied by all financial securities:

$$E_{t-1} \left[ \left( \delta \frac{U_c(C_t, t)}{U_c(C_{t-1}, t-1)} \right) r_{i,t} \right] = 0 \quad (2)$$

where  $r_{i,t}$  is the excess return on any security  $i$ , from date  $t - 1$  to  $t$ ,  $U_c(\cdot)$  denotes first derivative of the period utility function.

## 1.1 Contemporaneous Consumption Risk

In the empirical analysis we work with both the unconditional and conditional versions of Equation (2).

We start with the unconditional model. Defining the stochastic discount factor (SDF) as

$$m_t \equiv \delta \frac{U_c(C_t, t)}{U_c(C_{t-1}, t-1)}$$

gives:

$$\begin{aligned} E[m_t r_{i,t}] &= 0 \\ \iff E[r_{i,t}] &= -\frac{Cov[r_{i,t}, m_t]}{E[m_t]} \end{aligned}$$

where the second equality follows from using the definition of covariance and by applying the law of iterated expectations. Defining the sensitivity of excess returns  $r_{i,t}$  to changes in the stochastic discount factor as  $\beta_{ic,j} = \frac{Cov[r_{i,t}, m_t]}{Var[m_t]}$  and the market price of risk  $\lambda_c = -\frac{Var[m_t]}{E[m_t]}$  we get

$$E[r_{i,t}] = \lambda_c \beta_{ic,j}. \quad (3)$$

This is the standard beta representation of the unconditional Consumption CAPM. Expected excess returns on different securities are determined by their covariances with the stochastic discount factor, and thus by their covariances with consumption. A security with greater consumption risk has a higher expected return, since consumption and marginal utility are inversely related.

To model the implications of the conditional version of Equation (2):

$$E_{t-1}[r_{i,t}] = \lambda_{0c,t-1} + \lambda_{1c,t-1} \beta_{ic,t-1}, \quad (4)$$

for the unconditional expected returns, we follow Jagannathan and Wang (1996). First, take unconditional expectations of Equation (4):

$$E[r_{i,t}] = \lambda_{0c} + \lambda_{1c} \bar{\beta}_{ic} + Cov[\lambda_{1c,t-1}, \beta_{ic,t-1}] \quad (5)$$

where  $\bar{\beta}_{ic} = E[\beta_{ic,t-1}]$  and  $\lambda_{1c} = E[\lambda_{1c,t-1}]$ . Then, projecting  $\beta_{ic,t-1}$  on the conditional market risk premium  $\lambda_{1c,t-1}$ , gives:

$$\beta_{ic,t-1} = \bar{\beta}_{ic} + \varphi_{ic} (\lambda_{1c,t-1} - \lambda_{1c}) + \eta_{ic,t-1} \quad (6)$$

with  $E[\eta_{ic,t-1}] = E[\eta_{ic,t-1} \lambda_{1c,t-1}] = 0$ . Finally, substituting Equation (6) into Equation (5) gives for

the unconditional expected returns:

$$\begin{aligned} E[r_{i,t}] &= \lambda_{0c} + \lambda_{1c}\bar{\beta}_{ic} + Var[\lambda_{1c,t-1}]\varphi_{ic}, \\ \varphi_{ic} &= \frac{Cov[\lambda_{1c,t-1}, \beta_{ic,t-1}]}{Var[\lambda_{1c,t-1}]}. \end{aligned} \tag{7}$$

Thus, the conditional CCAPM leads to a two-factor unconditional model, in which the second factor is a risk premium induced by the covariance between the conditional beta  $\beta_{ic,t-1}$  and the conditional market risk premium for consumption risk  $\lambda_{1c,t-1}$ . Not only securities with higher expected betas have higher unconditional expected returns, but also securities with betas that vary more with the risk premium have higher unconditional expected returns, i.e. a positive covariance implies that if  $\beta_{ic,t-1}$  is high when  $\lambda_{1c,t-1}$  is high, which will result in higher unconditional expected returns.

The conditional model expressed in this way requires estimation of the expected beta,  $\bar{\beta}_{ic}$  and the sensitivity of the conditional beta to the risk premium,  $\varphi_{ic}$ , which cannot be done directly. Alternatively, we can directly estimate the average reaction of the returns to the changes of the stochastic discount factor and the average reaction to the changes of the risk premium. This leads to the following unconditional betas:

$$\begin{aligned} \beta_{ic} &\equiv \frac{Cov[r_{i,t}, m_t]}{Var[m_t]}, \\ \beta_{i\varphi} &\equiv \frac{Cov[r_{i,t}, \lambda_{1c,t-1}]}{Var[\lambda_{1c,t-1}]}. \end{aligned} \tag{8}$$

Jagannathan and Wang (1996) show that if  $\beta_{i\varphi}$  is not a linear function of  $\beta_{ic}$ , then there exist some constants  $a_0, a_1, a_2$  such that for every security  $i$  the unconditional expected return is a linear function of the above two unconditional betas:

$$E[r_{i,t}] = a_0 + a_1\beta_{ic} + a_2\beta_{i\varphi}. \tag{9}$$

We only summarize the idea of the proof here, for details see Appendix A or the proof of Theorem 1 in Jagannathan and Wang (1996). First it is shown that when betas vary over time, then  $(\beta_{ic}, \beta_{i\varphi})$  is a linear function of  $(\bar{\beta}_{ic}, \varphi_{ic})$ , which follows from additional assumptions about the residual term from

projection equation  $\eta_{ic,t-1}$ . Second, when  $\beta_{i\varphi}$  is not linear in  $\beta_{ic}$  (i.e. when the single beta CCAPM does not hold unconditionally, even though it holds conditionally), then  $(\beta_{ic}, \beta_{i\varphi})$  will contain all necessary information contained in  $(\bar{\beta}_{ic}, \varphi_{ic})$ . Hence, expected returns will be linear in  $(\bar{\beta}_{ic}, \varphi_{ic})$  as well as in  $(\beta_{ic}, \beta_{i\varphi})$ .

## 1.2 Ultimate Consumption Risk

Recently, Parker and Julliard (2005) find that contemporaneous consumption risk, as in the models discussed in the previous sections, is not sufficient to explain the cross-section of stock returns. They propose to extend the contemporaneous measure with the subsequent time periods to account for possible slow consumption adjustment. To see this, let us define the stochastic discount factor as

$$m_t \equiv \delta \frac{U_c(C_t)}{U_c(C_{t-1})}$$

and rearrange the terms in Equation (2) in the following way:

$$E_{t-1} [U_c(C_t) r_{i,t}] = 0.$$

Combining the above with the Euler equation for the risk-free rate between time  $t$  and  $t + S$ :

$$E_t [\delta U_c(C_{t+S}) R_{t,t+S}^f] = U_c(C_t),$$

gives the following representation for expected returns:

$$\begin{aligned} E[r_{i,t}] &= \lambda_{c,S} \beta_{ic,S}, \\ \beta_{ic,S} &= \frac{Cov[r_{i,t}, m_t^S]}{Var[m_t^S]}, \\ m_t^S &= \delta R_{t,t+S}^f \frac{U_c(C_{t+S})}{U_c(C_{t-1})}, \end{aligned} \tag{10}$$

where  $Cov[m_t^S, r_{i,t}]$  for large  $S$  is referred to as ultimate consumption risk and the market price of this risk is  $\lambda_{c,S} = -\frac{Var[m_t^S]}{E[m_t^S]}$ .



## 2 Estimation issues

We use Fama and MacBeth (1973) two stage cross-sectional regression approach with the first passage time-series slope coefficients estimated on the full sample. All standard errors are corrected for the estimation error in the dependent variable and for the fact that  $\beta'$ s are pre-estimated as suggested by Fama and MacBeth (1973), Shanken (1992), and Jagannathan and Wang (1996). Moreover, since our sample consists of return histories that differ in length, we apply the procedure of Stambaugh (1997) to compute the multivariate moments without discarding any observations. The validity of the models is examined by testing whether Jensen's alphas are zero (see Appendix B for details).

To parameterize the consumption-based model we assume that the period utility function has a constant relative risk aversion  $\gamma$ . This implies the following for the stochastic discount factor:

$$m_t = \delta \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma},$$

where  $\left( \frac{C_t}{C_{t-1}} \right)^{-\gamma}$  is the growth in per capita consumption from time  $t - 1$  to time  $t$ . Given the above representation of the SDF, expected returns are a non-linear function of consumption growth. In the following, we assume that consumption growth and security returns are jointly log-normally distributed, which implies that the expected excess returns are linear in log-consumption growth:

$$E[r_{i,t}] + 0.5Var[r_{i,t}] = \gamma Cov[r_{i,t}, \Delta c_t], \quad (11)$$

where  $\Delta c_t \equiv \log\left(\frac{C_t}{C_{t-1}}\right)$ . The above can be also expressed in the following beta representation:

$$\begin{aligned} E[r_{i,t}] &= \lambda_0 + \lambda_c \beta_{ic,j}, \\ \beta_{ic,j} &= \frac{Cov[r_{i,t}, \Delta c_t]}{Var[\Delta c_t]}. \end{aligned} \quad (12)$$

where the implied coefficient of relative risk aversion is  $\gamma = \frac{\lambda_c}{Var[\Delta c_t]}$  and the intercept is  $\lambda_0 = -0.5Var[r_{i,t}]$ .

A similar beta representation can be obtained, without the need to assume log normality but by using a Taylor series approximation of the stochastic discount factor around expected consumption growth.

In order to account for possibly slow consumption adjustment the log-consumption growth is measured over an extended horizon:

$$\Delta c_t^S = \log \left( \frac{C_{t+S}}{C_{t-1}} \right). \quad (13)$$

See Appendix C for details on the derivations. In addition, the conditional model requires observations on the conditional market risk premium  $\lambda_{1c,t-1}$ . We follow the approach of Jagannathan and Wang (1996) utilizing the fact that a variable that helps predict the business cycle can also forecast the market risk premium. The logic behind this is based on the presumption that if prices vary over the business cycle, so might the market risk premiums. The empirical research on predictability has identified several potential variables, from which the most widely used are: a dummy for the January effect; a credit risk premium defined as the difference in yields between Moody's Baa rank bonds and Moody's Aaa rank bonds; a term structure premium defined as the difference between 90 days and 30 days Treasury Bill rate; a dividend yield on the S&P 500 index; and the return on the market index (see, e.g., Kirby (1998), Pesaran and Timmermann (1995), and Ferson and Harvey (1991)). Based on previous studies we use a term structure variable  $(r_{t-1}^{term})$ .<sup>2</sup> Assuming that the market risk premium is linear in the conditioning variable, i.e.  $\lambda_{1c,t-1} = b_0 + b_1 r_{t-1}^{term}$ , we can estimate the following conditional model:

$$\begin{aligned} E[r_{i,t}] &= \lambda_0 + \lambda_1 \beta_{ic} + \lambda_{term} \beta_{i,term} \\ \beta_{ic} &\equiv \frac{Cov[r_{i,t}, \Delta c_t]}{Var[\Delta c_t]}, \\ \beta_{i,term} &\equiv \frac{Cov[r_{i,t}, r_{t-1}^{term}]}{Var[r_{t-1}^{term}]}. \end{aligned} \quad (14)$$

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<sup>2</sup>We also experiment with other predictive instruments, but our results are robust with respect to the choice of the instrument.

## 3 Data

### 3.1 Futures data

We use data on 20 futures contracts that are obtained from the Futures Industry Institute (FII) Data Center. The starting date of our sample period varies between contracts, as we use all available information for each futures contract. The earliest starting date is February 1968. The end date, December 2004, is common for all series. Hence the number of observations varies between futures contracts in our sample.

The data can be divided<sup>3</sup> into grains (3), oil and meals (3), meats (4), energy (3), precious metals (4), and food and fiber (3). These markets have relatively large trading volumes and provide a broad cross-section of commodity futures contracts. Details about the delivery months, the exchanges where these futures contracts are traded and the starting dates for each contract are given in Table 1.

Futures returns are calculated using a rollover strategy of nearest- and second-nearest-to-maturity futures contracts. Until the delivery month, we assume a position in the nearest-to-maturity contract. At the start of the delivery month, the position is changed to the contract with the following delivery month, which then becomes the nearest-to-maturity contract. Prices of futures observed in the delivery month are excluded from the analysis to avoid irregular price behavior that is common during the delivery month. We compute the returns for the second-nearest-to-maturity futures contracts in a similar way. Depending on the delivery dates during the year, the different series are for delivery one to three months apart. We obtain a minimum of 194 and a maximum of 442 time-series observations.

We use first- and second-nearest-to-maturity futures contract as these are the most actively traded contracts. This gives us 39 cross-sectional observations. Since these two contracts are highly correlated in the cross-sectional tests we use the returns on the first nearest-to-maturity futures contracts,  $r_{S,t+1}$ , (hereafter referred to as spot returns) and a spreading returns that combine a long position in the second

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<sup>3</sup>The classification we use is similar to the one used by the Institute for Financial Markets (IFM).

nearest-to-maturity futures contract with the short position in the first nearest-to-maturity contract,  $r_{F,t+1} - r_{S,t+1}$  (hereafter referred to as spreading returns).

Descriptive statistics are presented in Table 2. Panel A describes the returns on 20 futures markets: for both spot and spreading returns. We report annualized mean returns computed for different frequencies. Consistent with previous studies<sup>4</sup> we find that except for few futures contracts (crude oil, unleaded gasoline and live cattle futures) the estimated mean returns are statistically indistinguishable from zero at the 5% significance level. The highest average returns - more than 10% on an annual basis - are earned by energy futures. For some futures, e.g., soybean oil, cotton, coffee, and copper, we observe large differences in mean returns across the different frequencies of the data.<sup>5</sup>

The estimated average spreading returns usually have the opposite signs of the corresponding average of the spot returns. This may follow from a common risk factor that affects both the spot prices and futures yields for commodity futures. For example, a positive demand shock leads to an immediate increase in the spot prices. However, it will also lead inventories to fall and the convenience yields to rise. Hence a positive demand shock decreases futures yields and thus influences both the spot returns and spreading returns but in the opposite way. The volatility of the short-term futures contract is always higher than the volatility of the spreading returns for the same underlying asset, implying that spot price risk is larger than yield risk.

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<sup>4</sup>See. e.g., Bessembinder (1992), Bessembinder and Chan (1992), and de Roon, Nijman, and Veld (2000).

<sup>5</sup>We sometimes observe rather extreme returns, usually related to specific events. One example is the silver bubble in the turn of 1979. By the early 1970s silver began to rise in price, along with gold, platinum, oil, inflations and U.S. interest rates. The Commodities Futures Trading Commission falsely attributed the rise in silver prices to the market manipulations and changed the trading rules (i.e., margin requirements were raised to 100%, only futures sell orders were allowed), which lead to a collapse of the silver bubble. This is reflected in our data, where we observe almost 300% annual return for silver in 1979. In general we observe more volatility in the futures data, which is reflected in rather high standard deviations reported earlier. Since, these are inherent features of futures data we do not exclude any observations.

## 3.2 Consumption data

It is a well known fact that reported consumption data are subject to measurement problems. Theory implies that consumption risk is measured with respect to aggregate consumption growth between two points in time. In practice, however, we observe total expenditures on goods and services over a period of time. This creates a so called "summation (or time-aggregation) bias" (e.g., Breeden, Gibbons, and Litzenberger (1989)).

One way to avoid this problem is to use higher frequency consumption data. On the other hand, high frequency consumption data are measured less precisely, which may lead to less reliable (less stable) estimates. The higher frequency data may exhibit seasonal patterns, which might be especially important among the returns on commodity futures.<sup>6</sup> Moreover, recent work by Jagannathan and Wang (2007) shows that consumption-risk measured with lower frequency data, can better explain the cross-section of the 25 Fama-French portfolios. Since establishing which of the aforementioned biases dominate in futures markets remains an empirical issue, we use monthly, quarterly and yearly consumption data in our empirical tests.

Following the literature, we measure consumption growth as the percentage change in the seasonally adjusted, aggregate, real per capita consumption expenditures on nondurable goods and services. We use monthly, quarterly and annual consumption and population data from the National Income and Product Accounts (NIPA) tables in Section 2 on Personal Income and Outlays. The sample period is dictated by the availability of futures prices, as the consumption data at all frequencies are observed at a longer time interval.

Panel B of Table 2 gives the descriptive statistics for the log consumption growth. The consumption growth during our sample period is slightly above 2% per annum for all frequencies. Monthly consumption exhibits the highest growth and the highest volatility.

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<sup>6</sup>For example, futures contracts on grains may exhibit seasonality around (due to) the harvest times.

## 4 Empirical analysis

### 4.1 The Consumption CAPM

We start our empirical analysis with the unconditional version of the CCAPM in Equation (3). The contemporaneous cross-sectional estimates of  $\lambda_0$  and  $\lambda_c$  are reported in Table 3 in the first rows (i.e.,  $S=0$ ) of each panel corresponding to different frequencies: monthly, quarterly and yearly. Estimating beta at a monthly frequency leads to a very poor performance of the consumption model. The  $R^2$  of the cross-sectional regression is 0.5%, and the market price of consumption risk is not significantly different from zero.

At lower frequencies, the model fares much better: for quarterly estimates, the  $R^2$  increases to 56% and for yearly estimates it is 24%. To test the hypothesis that the Jensen's alphas are zero, we test whether the estimated intercept is equal to half of the variance of futures returns (see Appendix B for details). Again, the model performs better at lower frequencies, as it generates insignificant Jensen's alphas for quarterly and yearly returns.

The implied consumption risk premium,  $\lambda_c$ , is about one percent per year based on quarterly estimates and 74 basis points based on yearly estimates. These estimates are somewhat lower than the estimates found by Jagannathan and Wang (2007) based on stock portfolios, but the order of magnitude and the patterns that we find are comparable, except for the fact that for our futures data quarterly estimates provide the best results while they find yearly returns to give the best fit of the CCAPM for stock returns.

Table 4 provides the estimates based on the conditional CCAPM in Equation (14). Again, the contemporaneous case is reported in the first rows ( $S=0$ ) of each panel.

The results indicate that for all frequencies the conditional models show a much better performance than the unconditional models in Table 3. It is only for the quarterly estimates that the  $R^2$  is less than ten percentage points higher for the conditional model (64% versus 56%). In other cases the  $R^2$

improves by at least 20 percentage points. In this case we are not able to test if the Jensens's alphas are zero, because the intercept is partially unobservable as a result of the transformation from (7), in which expected returns are linear in two betas: one that is induced by the covariance between returns and the pricing kernel and the other induced by the covariance between the conditional beta and the conditional consumption price of risk, to the linear relation between expected returns and the two unconditional betas given in (14).

Again, the explanatory power of the CCAPM is the highest for the quarterly estimates (64%), but the difference with the yearly estimates (60%) is small. The estimated consumption risk premium and the implied risk aversions are close to the ones in Table 3.

The ability of the CCAPM to explain the cross section of futures returns well on the one hand, gives very high estimates for the risk aversion of the representative investor on the other hand. However, this result is consistent with other empirical studies on stock market returns (e.g., the evidence varies from risk aversion between 20 and 40 in Parker and Julliard (2005) and Jagannathan and Wang (2007) to 160 in Duffee (2005)). This result is also consistent with numerous theoretical explanations for the equity premium (e.g., heterogenous consumers in Constantinides and Duffie (1996), habit formation in Campbell and Cochrane (1999), or infrequent revision of consumption and investment decisions in Lynch (1996)), which makes the linear relation between expected returns and the covariance with consumption growth hold only approximately resulting in a high implied coefficient of risk aversion.

## 4.2 Ultimate Consumption Risk

As the previous results indicate, the horizon over which consumption is measured clearly matters. Table 3 reports the performance of the (unconditional) CCAPM based on ultimate consumption risk (Parker and Julliard (2005)) for different horizons  $S$  as defined in (13). This separates out the frequency effects of consumption from futures returns, as the latter are measured at a constant frequency in the ultimate consumption risk model.

First, the results based on monthly mean returns indicate that the  $R^2$  first increases until the horizon is about seven months, and then starts to decrease. However, the monthly mean returns almost invariably yield negative market prices of consumption risk. For the quarterly and annual returns the price of consumption risk is always positive, but here our results are contradictory to those of Parker and Julliard (2005). Where Parker and Julliard find an increasing performance as the number of quarters increases, we find the best performance for the contemporaneous first quarter, after which the performance of the ultimate risk measure deteriorates. We also find that the best fit for monthly data at the horizon of seven months, does not correspond to the results for quarterly data, where the best fit appears for the contemporaneous quarter. This is likely to be a result of the differences in the measurement errors in consumption data across frequencies.

The results for the conditional CCAPM with ultimate consumption risk (Table 4) basically show the same pattern. As for the contemporaneous case, the performance of the conditional model improves significantly on the unconditional one. However, the main finding for the ultimate risk is similar to the one in Table 3: after the first contemporaneous quarter (year), the performance of the model actually decreases by increasing the ultimate risk horizon, rather than increasing as in Parker and Julliard (2005).

### 4.3 What is going on?

The difference in the performance of the ultimate risk for our futures data relative to stock market data needs further analysis.<sup>7</sup> Table 5 shows the estimation results separately for the short term futures contracts and the longer term contracts. The results show that consumption betas explains the cross-sectional variation in short term futures returns contemporaneously and that this relation weakens as we extend the horizon for the consumption growth. For the longer term contracts we find insignificant consumption risk premiums at all horizons. Say differently, consumption adjusts immediately to the

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<sup>7</sup>Note that our results appear to be robust with respect to the sample period, the sample size, the consumption measure, the use of real or nominal series (these auxiliary results are available from the authors on request).



shock in the short term futures returns but is not related to the shock in the longer term returns.

This result is consistent with the Theory of Storage which indicates that any shock in the economy produces larger changes in near term expected spot prices than in more distant ones because these shocks are progressively offset by demand and supply responses (Fama and French, 1988). Furthermore, French (1986) pointed out that demand and supply shocks to the current output are transmitted to future periods through inventories. The change in inventories is spread between consumption and storage, hence only part of the shock will affect future consumption.

Breeden (1980) show in a theoretical model that commodity betas depend on their supply and demand elasticities and the covariances of goods' production with aggregate consumption. For instance, a positive demand shock will for many commodities lead to higher prices, but will also be associated with higher consumption, implying a positive short term beta. Following the demand shock however, demand may lower (because of a negative price elasticity) and supply may gradually increase, which both have off-setting effects on the relation between commodity prices on the one hand and longer term consumption on the other hand. Similarly, following a positive supply shock, prices will decrease and consumption will increase, leading to a negative short-term beta. Again, changes in demand and supply for the commodity following the price change will have off-setting effects in the longer run.

Figure 1 provides some support for this, based on the consumption betas of our futures contracts. For each futures contract, the figure plots the beta with respect to the contemporaneous quarter consumption growth, as well as with respect to longer horizon consumption growth. For almost every futures contract the betas decrease in absolute value as the horizon increases, basically fading out to zero. This is consistent with the hypothesis formulated above that for commodities - which are part of aggregate consumption - supply and demand changes induce short term consumption betas, but basically zero longer term consumption betas. This may also explain the fact that in Tables 3 and 4 we find the strongest relation between consumption growth and futures returns at the quarterly horizon and not at the annual horizon: decreasing the frequency of returns and consumption growth from monthly

to quarterly reduces the estimation error in consumption data, but a further decrease in the frequency actually decreases the performance because of the changing consumption betas.

#### 4.4 Production- and inventory-based factors

In general, the smoothing of the return shock that hits the economy can be done via the demand side (adjusting the consumption) but also via the supply side (adjusting production and/or inventories). If the poor performance of the long run consumption risk model is offset by the supply changes, we can expect that the supply-based model will outperform the consumption-based model in the long run. To see this, we assume a model of consumption, production and storage (e.g., Brennan (1958), French (1986)). The model relates the quantity consumed in a given period to the amount produced and the change in storage

$$\begin{aligned} C_t &= \sum w_i C_{it} = \sum w_i Q_{it} - \sum w_i \Delta I_{it}. \\ &= Q_t - \Delta I_t \end{aligned} \tag{15}$$

where  $Q_t$  is the aggregate production, and  $\Delta I_t$  is the change in the aggregate inventories, i.e.,  $\Delta I_t = \sum w_i \Delta I_{it}$ . The production data are observed on the aggregate level, but we need to aggregate inventory data from individual contracts. We do this in two ways, first by assuming weights  $w_i$  are constant across commodities and second by using open interest of each commodity futures contract to determine these weights. Plugging equation (15) into the Consumption-CAPM leads to the following cross-sectional relation

$$\begin{aligned} E[r_{i,t}] &= \lambda_0 + \lambda_{QI} \beta_{QI} \\ \beta_{QI} &= \frac{Cov\left[r_{i,t}, \lg\left(\frac{Q_t - \Delta I_t}{Q_{t-1} - \Delta I_{t-1}}\right)\right]}{Var\left[\lg\left(\frac{Q_t - \Delta I_t}{Q_{t-1} - \Delta I_{t-1}}\right)\right]} \end{aligned} \tag{16}$$

The model is stated in terms of the observable data, however there are several reasons why we might

want to avoid using the inventory data and instead focus on the price-based proxy. The most important is that the proxy does not suffer from the common data revisions. Second, the price information is readily available to all investors instantaneously while the inventory data are often delayed. Finally, several papers document a strong relation between the price-based proxy and inventory data (e.g., Gorton, Hayashi, and Rouwenhorst (2007), and Acharya, Lochstoer, and Ramadorai (2009)).

Our proxy is based on the predictions from the Theory of Storage that a negative shock to current inventories widens the basis, i.e. the difference between the spot and futures price of a commodity. The fall in inventories raises the scarcity of the commodity and hence the spot price. Futures price also increases but by less, as the shocks are offset by demand and supply responses (Fama and French (1988)). Hence, we define the following proxy

$$\begin{aligned}\sum w_i I_{it} &\simeq \sum w_i S_{it} - \sum w_i F_{it} \\ I_t &\simeq S_t - F_t\end{aligned}$$

where  $S_{it}$  is the spot price and  $F_{it}$  is the futures price of a commodity and we use the same weights as above. For aggregate production we use Industrial Production Index (derived from the Federal Reserve Board).

Using the above definition of inventories allows us to write the following multifactor model

$$E[r_{i,t}] = \lambda_0 + \lambda_Q \beta_Q + \lambda_{\Delta FMS_t} \beta_{\Delta FMS_t} + \lambda_{\Delta FMS_{t-1}} \beta_{\Delta FMS_{t-1}} \quad (17)$$

where

$$\begin{aligned}\beta_Q &= \frac{Cov\left[r_{i,t}, \lg\left(\frac{Q_t}{Q_{t-1}}\right)\right]}{Var\left[\lg\left(\frac{Q_t}{Q_{t-1}}\right)\right]} \\ \beta_{\Delta FMS_t} &= \frac{Cov\left[r_{i,t}, r_{F,t} - r_{S,t}\right]}{Var\left[r_{F,t} - r_{S,t}\right]}\end{aligned}$$

and  $FMS$  stands for futures minus spot return,  $r_{S,t} = \lg\left(\frac{S_t}{S_{t-1}}\right)$  is the aggregate log spot return, and

$r_{F,t} = \lg\left(\frac{F_t}{F_{t-1}}\right)$  is the aggregate log futures return<sup>8</sup>.  $\beta_{\Delta FMS_{t-1}}$  uses one period lagged values of the aggregate log spot and futures returns. See Appendix D for details.

The results are in Table 6. We find strong relation between production, storage and futures risk premia. First, the results in Panel A show that the covariation between futures returns and the lagged value of our inventory proxy is never significant. That is why in Panel B we also report results without this factor. In both panels, it is apparent that the production and inventory factors explain slightly less of the contemporaneous variations in expected futures returns than consumption growth. However, once we allow supply side to adjust to the return shock the performance of the model increases and exceeds that of the ultimate consumption growth.

Moreover, as seen from Table 7 the results are equally strong for short term and the longer term contracts and in both cases they do increase with the horizon. This further supports the notion that production and inventories are more strongly related to long run risk in commodities markets than consumption-based factors.

This result may explain the poor performance of the ultimate consumption growth model for commodity futures returns. Since the supply side typically reacts with a delay to the shock in the economy, we observe contemporaneously stronger relation between commodity futures returns and consumption growth. Once we give more time to the consumers and producers to react to the initial shock, the slower adjustment on the supply side starts to offset the consumption reaction with a delay. Hence, we observe the strongest link between contemporaneous consumption growth and commodity futures returns but not in the long run.

To further verify whether it is supply that off-sets the consumption adjustments, we estimate the supply and demand elasticity directly from the aggregate demand and supply data. We assume a constant elasticity function and estimate elasticity using regressions of log demand (supply) on log

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<sup>8</sup>In the empirical estimation we use the first nearest-to-maturity contracts as a proxy for spot price, and second nearest-to-maturity contracts for the futures price.

prices:

$$\Delta \log D_{i,t}^S = \alpha_D + \beta_{D,i} \Delta \log P_{i,t} + u_{i,t},$$

$$\Delta \log S_{i,t}^S = \alpha_S + \beta_{S,i} \Delta \log P_{i,t} + \varepsilon_{i,t},$$

where change in demand and supply is measured over different horizons  $S$ :  $\Delta \log I_{i,t}^S = \log(\frac{I_{i,t+S}}{I_{i,t-1}})$  with  $I_{i,t}^S = \{D_{i,t}^S, S_{i,t}^S\}$ . We report the estimates of the slope coefficients and their standard errors in Table 8. For demand we use our consumption data, and for supply we use the Industrial Production Index (derived from the Federal Reserve Board). The demand elasticity remains small and insignificant across all considered horizons (i.e., on average it decreases from 0.15% to -0.15% during 4 quarters), while the supply elasticities are large and significant and they increase sharply with the horizon (i.e., on average by ten percentage points per year).<sup>9</sup>

This is in line with the conjecture that for commodities supply changes have a direct impact on commodity prices and consumption adjustments, since part of the commodities are (strongly related to) consumption goods. Moreover, since this is an inherent feature of commodities we expect that in general it will not be true for the stock market, where supply changes are not directly related to the consumption decisions. To see this, we have added futures contracts on S&P 500 Index to our analysis. We find the highest and actually increasing demand elasticity for futures on S&P 500 Index, in comparison to our commodity futures, which is in line with slow consumption adjustment to the shocks in stock returns as found in Parker and Julliard (2005).

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<sup>9</sup>We have additionally looked at futures contracts on S&P 500 Index to compare our results with those of Parker and Julliard (2005). We find the highest and actually increasing demand elasticity for futures on S&P 500 Index, in comparison to our commodity futures, which is in line with slow consumption adjustment to the shocks in stock returns as found in Parker and Julliard (2005).

## 5 Summary and Conclusions

Recent studies on consumption based models show that measuring consumption growth and/or returns over longer horizon improves the performance of the CCAPM in explaining the cross-sectional variation of expected stock returns. Drawing on these results, we study whether excess returns on futures contracts vary in a systematic way due to differences in consumption risk similar to the returns on stocks. Our aim is to contribute to the discussion whether futures markets are subject to the same sources of priced risk as stock markets.

In this paper, we show that, similarly to stock returns, the (unconditional) CCAPM explains about 50% of the cross-sectional variation in mean futures returns at the quarterly frequency, while there is almost no explained variance at the monthly level, and an intermediate result at the yearly level. The conditional model yields even better performance than the unconditional model (i.e., the  $R^2$  is about 60%) and, again, it is best at the quarterly and annual frequency. This pattern is consistent with the results found by Jagannathan and Wang (2007) based on stock portfolios. However, we find somewhat lower implied consumption risk premiums for our futures contracts.

Using ultimate consumption risk, we find that the performance of the CCAPM is best using consumption growth of the contemporaneous quarter of the returns, but then deteriorates for the longer horizons. Although this contradicts the findings of Parker and Julliard (2005) for stock returns, it is consistent with the finding that the CCAPM performs best at the quarterly frequency and may be the result of supply and demand elasticities of many of the commodities that underlie our futures contracts, inducing time-varying consumption betas.

We attribute this difference in the performance of the ultimate consumption risk model for stocks and futures data to the offsetting adjustment in the supply side of the economy. We consider a model of consumption, production and storage (e.g., French 1986) and we find that production and inventory based factors are significantly priced in the commodity futures markets. Moreover, the performance

of this model improves over longer horizon, which may explain the poor performance of ultimate consumption risk model.

## Appendix A The conditional Consumption CAPM

This section repeats the proof of Theorem 1 from Jagannathan and Wang (1996) but in terms of the conditional CCAPM instead of their conditional CAPM. The proof proceeds in two steps:

1. If betas vary over time, then  $(\beta_{ic}, \beta_{i\varphi})$  is a linear function of  $(\bar{\beta}_{ic}, \varphi_{ic})$ .
2. When  $\beta_{i\varphi}$  is not linear in  $\beta_{ic}$  (i.e. when single beta CCAPM does not hold unconditionally, even though it holds conditionally), then expected returns are linear in  $(\bar{\beta}_{ic}, \varphi_{ic})$  as well as in  $(\beta_{ic}, \beta_{i\varphi})$ .

**Ad. 1** Define the return on a portfolio that is perfectly correlated with consumption as  $r_{c,t}$ . Then, note that for this portfolio the conditional CCAPM implies:

$$\begin{aligned} E_{t-1}[r_{c,t}] &= \lambda_{0c,t-1} + \lambda_{1c,t-1} \\ \lambda_{1c,t-1} &= E_{t-1}[r_{c,t} - \lambda_{0c,t-1}]. \end{aligned}$$

Define  $\varepsilon_{i,t}$  as

$$\varepsilon_{i,t} = r_{i,t} - \lambda_{0c,t-1} - (r_{c,t} - \lambda_{0c,t-1})\beta_{ic,t-1}. \quad (18)$$

This implies the following orthogonality conditions:

$$\begin{aligned} E[\varepsilon_{i,t}] &= 0 \\ E[\varepsilon_{i,t}r_{c,t}] &= 0 \\ E[\varepsilon_{i,t}\lambda_{1c,t-1}] &= 0. \end{aligned}$$

We can substitute equation (6) into (18) to obtain

$$r_{i,t} = \lambda_{0c,t-1} + (r_{c,t} - \lambda_{0c,t-1})\bar{\beta}_{ic} + (r_{c,t} - \lambda_{0c,t-1})(\lambda_{1c,t-1} - \lambda_{1c})\varphi_{ic} + (r_{c,t} - \lambda_{0c,t-1})\eta_{ic,t-1} + \varepsilon_{i,t}$$

From the definition of covariance and the orthogonality conditions given above we obtain

$$Cov(r_{i,t}, \Delta c_t) = Cov(r_{i,t}, r_{c,t}) \quad (19)$$

$$\begin{aligned} &= Cov(\lambda_{0c,t-1}, r_{c,t}) + Cov(r_{c,t} - \lambda_{0c,t-1}, r_{c,t})\bar{\beta}_{ic} \\ &+ Cov((r_{c,t} - \lambda_{0c,t-1})(\lambda_{1c,t-1} - \lambda_{1c}), r_{c,t})\varphi_{ic} + Cov((r_{c,t} - \lambda_{0c,t-1})\eta_{ic,t-1}, r_{c,t}), \end{aligned}$$

$$Cov(r_{i,t}, \lambda_{1c,t-1}) = Cov(\lambda_{0c,t-1}, \lambda_{1c,t-1}) + Cov(r_{c,t} - \lambda_{0c,t-1}, \lambda_{1c,t-1})\bar{\beta}_{ic} \quad (20)$$

$$+ Cov((r_{c,t} - \lambda_{0c,t-1})(\lambda_{1c,t-1} - \lambda_{1c}), \lambda_{1c,t-1})\varphi_{ic} + Cov((r_{c,t} - \lambda_{0c,t-1})\eta_{ic,t-1}, \lambda_{1c,t-1}).$$

Since, for the CCAPM  $\beta_{ic} = \frac{Cov(r_{i,t}, \Delta c_t)}{Var(\Delta c_t)}$  and  $\beta_{i\varphi} = \frac{Cov(r_{i,t}, \lambda_{1c,t-1})}{Var(\lambda_{1c,t-1})}$ , these equations imply that  $(\beta_{ic}, \beta_{i\varphi})$  will be a linear function of  $(\bar{\beta}_{ic}, \varphi_{ic})$ , if the last terms in both equations (19) and (20) are zero. Hence, we assume that the residual  $\eta_{ic,t-1}$  from the projection equation (6) satisfies the following orthogonal conditions:

$$E[\eta_{ic,t-1} E_{t-1}[r_{c,t}]] = 0$$

$$E[\eta_{ic,t-1} \lambda_{1c,t-1}^2] = 0$$

$$E[\eta_{ic,t-1} \lambda_{1c,t-1} \lambda_{0c,t-1}] = 0.$$

This completes step 1.

**Ad. 2** From step 1 we know that

$$\begin{pmatrix} \beta_{ic} \\ \beta_{i\varphi} \end{pmatrix} = \begin{pmatrix} b_0 \\ c_0 \end{pmatrix} + \begin{pmatrix} b_1 & b_2 \\ c_1 & c_2 \end{pmatrix} \begin{pmatrix} \bar{\beta}_{ic} \\ \varphi_{ic} \end{pmatrix}. \quad (21)$$

To show that  $(\bar{\beta}_{ic}, \varphi_{ic})$  are linear in  $(\beta_{ic}, \beta_{i\varphi})$  we need to show that equation (21) is invertible. Suppose that the 2 by 2 matrix in this equation is singular, then there is a nonzero vector  $(x, y)$  such that

$$(x, y) \begin{pmatrix} b_1 & b_2 \\ c_1 & c_2 \end{pmatrix} = 0,$$

which implies that  $x\beta_{ic} + y\beta_{i\varphi}$  is a constant across securities. Since  $\beta_{ic}$  is not a constant across assets, we must have  $y \neq 0$  for singularity to hold. In this case  $\bar{\beta}_{ic}$  will be linear in  $\varphi_{ic}$  and hence both  $\beta_{ic}$  and



$\beta_{i\varphi}$  will be linear in  $\bar{\beta}_{ic}$  only. But this contradicts the initial assumption of non-linearity between  $\beta_{i\varphi}$  and  $\beta_{ic}$  and hence this 2 by 2 matrix must be invertible. Now we can invert equation (21) such that  $(\bar{\beta}_{ic}, \varphi_{ic})$  are linear in  $(\beta_{ic}, \beta_{i\varphi})$  and substitute them into (7) to obtain (9), which completes step 2 and the proof.

## Appendix B Estimation error in the intercept

Recall that when we assume that returns and consumption growth are jointly log-normally distributed then the unconditional model in (12) implies that the expected log (excess) returns are linear in consumption betas:

$$E[r_{f,i,t}] = \lambda_0 + \lambda_c \beta_{ic,j},$$

where  $\lambda_0 = -0.5 \text{Var}(r_{f,i,t})$ . Thus, in order to test if Jensen's alpha is zero we need to incorporate the estimation error in  $\text{Var}(r_{f,i,t})$ . Note that for futures returns we can rewrite the LHS in the following way:

$$E[r_{f,i,t}] + 0.5 \text{Var}[r_{f,i,t}] = \log \{E[R_{f,i,t}]\},$$

where  $R_{f,i,t} = \exp(r_{f,i,t}) = \frac{P_t}{F_{t-1}}$ . Hence, to test if Jensen's alpha is zero in the unconditional models we can estimate the cross-sectional regression using  $\log \{E[R_{f,i,t}]\}$  as a dependent variable and test the hypothesis that the intercept is zero.

## Appendix C Log-linearization for ultimate risk horizon

The assumption on joint log-normality of consumption growth and returns implies the following for expected returns:

$$E[r_{i,t}] = -\log \delta + \gamma E \left[ \log \left( \frac{C_t}{C_{t-1}} \right) \right] + 0.5 (\sigma_i^2 + \gamma^2 \sigma_c^2 - 2\gamma \sigma_{ic}).$$

Next, consider the excess returns on the securities of the following form:

$$E \left[ r_{i,t} - r_{t-1}^f \right] = -0.5 \text{Var} [r_{i,t}] + \gamma \text{Cov} \left[ r_{i,t}, \log \left( \frac{C_t}{C_{t-1}} \right) \right]. \quad (22)$$

Assuming that the risk free rate of borrowing between time  $t$  and  $t + S$  is constant so that the consumption-CAPM holds in the following way:

$$E \left[ r_{t,t+S}^f \right] = -\log \delta + \gamma_S E \left[ \log \left( \frac{C_{t+S}}{C_t} \right) \right] + 0.5 \gamma_S^2 \sigma_{c,S}^2,$$

allows us to substitute out  $C_t$  in (22) and obtain:

$$E \left[ r_{i,t} - r_{t-1}^f \right] + 0.5 \text{Var} [r_{i,t}] = \gamma \text{Cov} \left[ r_{i,t}, \log \left( \frac{C_{t+S}}{C_{t-1}} \right) \right] - \theta,$$

where  $\theta = \frac{\gamma}{\gamma_S} \text{Cov} \left[ r_{i,t}, r_{t,t+S}^f + \log \delta - 0.5 \gamma_S^2 \sigma_{c,S}^2 \right]$  which is assumed to be zero. It follows immediately, that we can use the beta representation of the form:

$$\begin{aligned} E [r_{i,t}] &= \lambda_0 + \lambda_c \beta_{ic,j}, \\ \beta_{ic,j} &= \frac{\text{Cov} [r_{i,t}, \Delta c_t^S]}{\text{Var} [\Delta c_t^S]}. \end{aligned}$$

where  $\Delta c_t^S = \log \left( \frac{C_{t+S}}{C_{t-1}} \right)$ .

## Appendix D Production- and inventory-based model

We start by assuming a model of consumption, production and storage (Brennan, 1958, French 1986).

The model relates the quantity consumed in a given period to the amount produced and the change in the amount stored

$$\begin{aligned} C_t &= \sum w_i C_{it} = \sum w_i Q_{it} - \sum w_i \Delta I_{it}. \\ &= Q_t - \Delta I_t \end{aligned}$$

Drawing on previous studies (e.g. Gorton, Hayashi, and Rouwenhorst 2007) we avoid the inventory data and we use a basis (i.e., the difference between the spot and futures price of the same commodity) as a proxy

$$I_{it} \simeq S_{it} - F_{it}$$

Using the above relations we can rewrite the consumption model

$$\begin{aligned} \lg\left(\frac{C_t}{C_{t-1}}\right) &= \lg\left(\frac{Q_t - \Delta I_t}{Q_{t-1} - \Delta I_{t-1}}\right) \\ &\simeq \lg\left(\frac{Q_t - (S_t - F_t) + (S_{t-1} - F_{t-1})}{Q_{t-1} - (S_{t-1} - F_{t-1}) + (S_{t-2} - F_{t-2})}\right) \\ &= \lg\left(\frac{Q_t - S_t + F_t + S_{t-1} - F_{t-1}}{Q_{t-1} - S_{t-1} + F_{t-1} + S_{t-2} - F_{t-2}}\right) \\ &\simeq \alpha_0 \lg\left(\frac{Q_t}{Q_{t-1}}\right) - \alpha_1 \lg\left(\frac{S_t}{S_{t-1}}\right) + \alpha_2 \lg\left(\frac{F_t}{F_{t-1}}\right) + \alpha_3 \lg\left(\frac{S_{t-1}}{S_{t-2}}\right) - \alpha_4 \lg\left(\frac{F_{t-1}}{F_{t-2}}\right) \end{aligned}$$

where the last equation follows by iterating the following approximation (Campbell and Shiller (1988))

$$\begin{aligned} \Delta(y_{t+1} + x_t) &\simeq \frac{Y_{t+1} + X_{t+1} - Y_t - X_t}{Y_t + X_t} \\ &= \frac{Y_{t+1} - Y_t}{Y_t + X_t} + \frac{X_{t+1} - X_t}{Y_t + X_t} \\ &\simeq a\Delta y_{t+1} + (1-a)\Delta x_t \\ a &= \frac{Y_t}{X_t + Y_t}. \end{aligned}$$

Now using the definitions of log returns and assuming that  $\alpha_1 = \alpha_2$  and  $\alpha_3 = \alpha_4$ , we can write

$$\lg\left(\frac{Q_t - \Delta I_t}{Q_{t-1} - \Delta I_{t-1}}\right) \approx \alpha_0 \Delta q_t - \alpha_1 (r_{S,t} - r_{F,t}) + \alpha_3 (r_{S,t-1} - r_{F,t-1}) \quad (23)$$

where  $r_{S,t}$  is the log spot return, and  $r_{F,t}$  is the log futures return. Hence we can write a single factor consumption-based model as a multifactor production- and inventory-based model (as in equation (17)).

We assess the degree of this approximation in simulations. We randomly generate positive numbers for  $Q_t$ ,  $S_t$ , and  $F_t$ . The series are constructed in such a way that the numerator and denominator in  $\lg\left(\frac{Q_t - S_t + F_t + S_{t-1} - F_{t-1}}{Q_{t-1} - S_{t-1} + F_{t-1} + S_{t-2} - F_{t-2}}\right)$  are positive. We, then, regress  $\lg\left(\frac{Q_t - S_t + F_t + S_{t-1} - F_{t-1}}{Q_{t-1} - S_{t-1} + F_{t-1} + S_{t-2} - F_{t-2}}\right)$  on three

factors as given in (23). The estimated fit varies between 96% and 99%. These results are available from the authors on request.

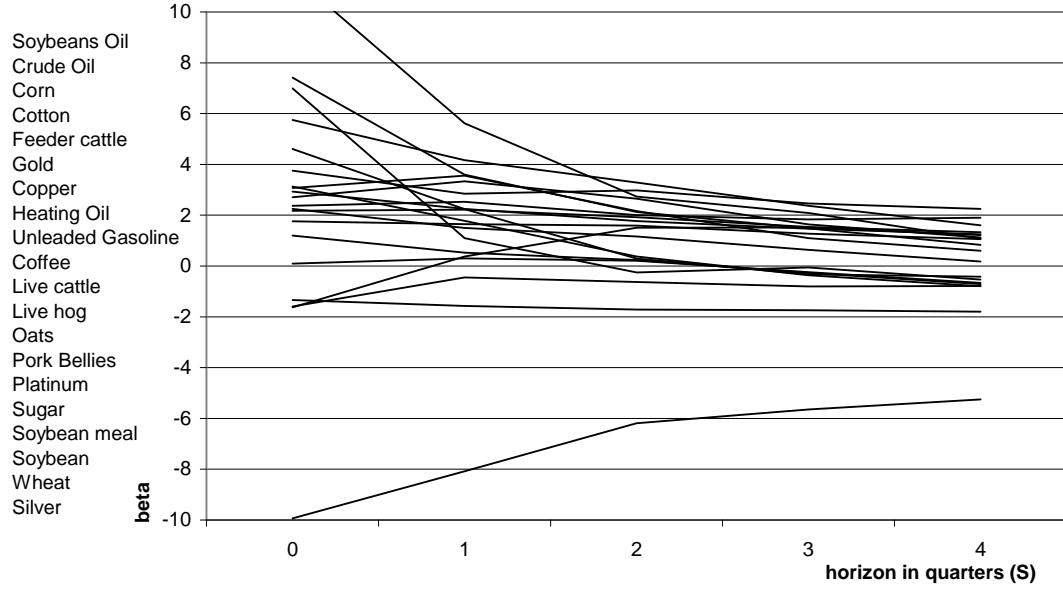
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Figure 1: Consumption betas at different horizon.



The figure depicts consumption betas estimated on 20 commodity futures contracts with varying sample period (see Table 1) using the following time-series regression:

$$r_{i,t} = \alpha_i + \beta_{i,cS} \Delta c_t^S + \varepsilon_{i,t}$$

based on ultimate consumption risk for different horizons S:

$$\Delta c_t^S = \log \left( \frac{C_{t+S}}{C_{t-1}} \right).$$



Table 1: **Futures contracts.**

The table reports the futures exchange, the delivery months, and the beginning date of the sample period for the 20 futures contracts in our sample. The end date of the sample period, December 2004, is common for all contracts.

Futures contract	Exchange	Delivery months	Start date
Commodities			
Grains			
Wheat	Chicago Board of Trade	3,5,7,9,12	1968 Dec
Corn	Chicago Board of Trade	3,5,7,9,12	1968 Dec
Oats	Chicago Board of Trade	3,5,7,9,12	1974 Dec
Oil & Meal			
Soybean	Chicago Board of Trade	1,3,5,7,8,9,11	1968 Nov
Soybeans Oil	Chicago Board of Trade	1,3,5,7,8,9,10,12	1968 Nov
Soybean meal	Chicago Board of Trade	1,3,5,7,8,9,10,12	1968 Nov
Meats			
Live cattle	Chicago Mercantile Exchange	2,4,6,8,10,12	1976 Dec
Feeder cattle	Chicago Mercantile Exchange	1,3,4,5,8,9,10,11	1977 Oct
Live (lean) hog	Chicago Mercantile Exchange	2,4,6,7,8,10,12	1969 Dec
Pork Bellies	Chicago Mercantile Exchange	2,3,5,7,8	1969 Aug
Energy			
Crude Oil	New York Mercantile Exchange	All	1983 Dec
Heating Oil	New York Mercantile Exchange	All	1979 Dec
Unleaded Gas	New York Mercantile Exchange	All	1985 Apr
Metals			
Gold	Commodity Exchange, Inc.	1,2,4,6,8,10,12	1975 Jan
Silver	Commodity Exchange, Inc.	3,5,7,9,12	1968 Feb
Platinum	New York Mercantile Exchange	1,4,7,10	1972 Sep
Copper	Commodity Exchange, Inc.	1,3,5,7,9,12	1988 Oct
Food/Fiber			
Coffee	New York Board of Trade	3,5,7,9,12	1973 Dec
Sugar	New York Board of Trade	3,5,7,10	1974 Oct
Cotton	New York Board of Trade	3,5,7,10,12	1972 Dec

Table 2: **Descriptive statistics.**

The table gives the descriptive statistics for the commodity futures contracts, and consumption growth. The sample period varies across futures (see Table 1). Panel A describes the statistics for returns estimated from data for different frequency: monthly, quarterly and yearly; and for the nearest-to-maturity futures contracts (spot returns), and the second nearest-to-maturity futures contracts (spreading returns). Panels B gives the same statistics for consumption growth rates at different frequencies.

Futures name	Annualized spot returns						Annualized spreading returns					
	Monthly		Quarterly		Yearly		Monthly		Quarterly		Yearly	
	Mean	St. Dev	Mean	St. Dev	Mean	St. Dev	Mean	St. Dev	Mean	St. Dev	Mean	St. Dev
Panel A: expected returns												
Wheat	-2.38	(23.82)	-1.96	(26.33)	0.16	(34.94)	0.78	(4.64)	0.77	(5.14)		(4.59)
Corn	-4.58	(23.56)	-4.08	(25.30)	-3.28	(28.52)	0.93	(4.42)	0.92	(3.67)		(6.39)
Oats	-8.17	(29.42)	-7.70	(31.83)	-7.09	(32.57)	1.04	(6.96)	1.14	(7.00)		(3.03)
Soybean	0.57	(27.08)	0.61	(28.56)	0.81	(28.00)	1.27	(4.23)	1.30	(4.22)		(6.53)
Soybeans Oil	3.97	(31.14)	3.52	(31.31)	6.77	(45.02)	-0.34	(3.80)	-0.30	(4.16)		(3.26)
Soybean meal	2.06	(29.83)	2.43	(31.38)	2.85	(32.44)	1.11	(5.53)	1.12	(6.02)		(0.54)
Live cattle	5.20	(15.50)	5.36	(14.95)	4.96	(14.94)	-1.66	(5.51)	-1.74	(5.99)		(3.06)
Feeder cattle	3.96	(14.93)	4.14	(15.27)	3.33	(19.75)	0.17	(3.07)	0.14	(2.97)		(5.30)
Live hog	3.12	(26.91)	2.76	(26.98)	1.66	(26.29)	2.47	(8.52)	2.43	(9.31)		(7.14)
Porl Bellies	-4.04	(35.10)	-5.22	(33.58)	-8.85	(25.94)	2.81	(10.33)	2.82	(10.09)		(7.35)
Crude Oil	10.67	(32.85)	13.03	(38.48)	11.70	(44.43)	-0.43	(4.69)	-0.48	(5.15)		(6.20)
Heating Oil	4.15	(30.33)	6.24	(34.26)	4.42	(35.20)	-0.71	(6.24)	-0.86	(6.28)		(11.40)
Unleaded Gasoline	13.85	(33.74)	14.53	(34.52)	14.09	(38.17)	-2.80	(7.75)	-2.91	(7.19)		(7.54)
Gold	-3.16	(19.42)	-3.61	(19.42)	-2.71	(28.76)	-0.07	(0.54)	-0.08	(0.64)		(9.96)
Silver	-3.59	(31.67)	-4.82	(32.14)	-1.04	(58.37)	0.17	(1.11)	0.17	(0.98)		(1.11)
Platinum	2.44	(28.87)	1.37	(25.32)	1.92	(31.99)						
Copper	6.14	(24.12)	6.07	(25.05)	2.68	(29.05)	0.02	(3.16)	-0.17	(2.69)		(7.93)
Coffee	0.85	(38.25)	1.49	(44.49)	3.52	(53.07)	-1.37	(7.76)	-1.42	(8.13)		(3.87)
Sugar	-10.59	(39.77)	-9.22	(43.10)	-11.94	(37.73)	2.85	(8.43)	2.90	(8.54)		(6.61)
Cotton	-0.01	(24.57)	0.23	(26.77)	4.26	(38.59)	0.70	(5.76)	0.91	(6.44)		(0.95)
Panel B: consumption growth												
Consumption	2.08	(1.21)	2.07	(0.84)	2.04	(1.11)						

Table 3: **Ultimate risk for unconditional consumption model.**

The table reports the cross-sectional regression estimation results for the Consumption CAPM:

$$E[r_{i,t+1}] = \lambda_0 + \lambda_1 \beta_i,$$

where  $\beta_i$  is the time-series futures returns loadings on consumption growth based on different horizons S:  $\Delta c_t^S = \log \left( \frac{C_{t+S}}{C_{t-1}} \right)$ . We use 39 commodity futures contracts (spot and spreading returns) with varying sample period (see Table 1).  $\gamma$  is the implied coefficient of risk aversion defined as  $\frac{\lambda_c}{Var[\Delta c_t^S]}$ . We report Fama-MacBeth standard errors, and next to them Shanken corrected standard errors.

S	$\lambda_0$	St.err	Corr err	$\lambda_1$	St.err	Corr err	$R^2$	$R_{adj}^2$	$\gamma$
Panel A: monthly									
0	0.80	(0.81)	(0.82)	-0.43	(1.31)	(1.33)	0.55%	-0.02	-345
1	0.78	(0.81)	(0.81)	-0.34	(1.39)	(1.39)	0.33%	-0.02	-269
2	0.96	(0.93)	(0.97)	-1.10	(1.25)	(1.28)	5.40%	0.03	-775
3	0.88	(0.98)	(1.01)	-1.11	(1.32)	(1.35)	4.43%	0.02	-639
4	0.84	(1.02)	(1.06)	-1.33	(1.39)	(1.42)	5.94%	0.03	-646
5	0.74	(1.06)	(1.11)	-1.72	(1.44)	(1.50)	9.44%	0.07	-709
6	0.69	(1.07)	(1.13)	-2.00	(1.56)	(1.64)	10.93%	0.09	-718
7	0.62	(1.08)	(1.14)	-2.08	(1.63)	(1.71)	10.15%	0.08	-657
8	0.60	(1.08)	(1.13)	-2.02	(1.73)	(1.81)	7.78%	0.05	-564
9	0.58	(1.08)	(1.13)	-1.94	(1.82)	(1.90)	6.02%	0.03	-482
10	0.53	(1.09)	(1.12)	-1.86	(1.88)	(1.96)	5.06%	0.02	-414
11	0.56	(1.08)	(1.11)	-1.55	(1.96)	(2.03)	3.14%	0.01	-310
12	0.57	(1.08)	(1.10)	-1.38	(2.01)	(2.07)	2.32%	0.00	-249
Panel B: quarterly									
0	-0.34	(1.11)	(1.29)	0.98	(0.38)	(0.45)	55.54%	0.54	545
1	-0.05	(1.10)	(1.26)	1.15	(0.49)	(0.57)	32.78%	0.31	393
2	0.27	(1.12)	(1.23)	1.14	(0.60)	(0.67)	17.93%	0.16	270
3	0.45	(1.15)	(1.27)	1.38	(0.75)	(0.84)	18.63%	0.16	236
4	0.67	(1.16)	(1.25)	1.36	(0.85)	(0.94)	14.20%	0.12	180
Panel C: yearly									
0	-0.40	(0.92)	(1.02)	0.54	(0.25)	(0.33)	21.21%	0.19	44
1	0.30	(1.15)	(1.25)	0.60	(0.34)	(0.43)	14.50%	0.12	31

Table 4: **Ultimate risk for conditional consumption model.**

The table reports the cross-sectional regression estimation results for the conditional Consumption CAPM:

$$E[r_{i,t+1}] = \lambda_0 + \lambda_1 \beta_i + \lambda_{term} \beta_{i,term},$$

where  $\beta_i$  is the time-series futures returns loadings on consumption growth based on different horizons S:  $\Delta c_t^S = \log \left( \frac{C_{t+S}}{C_{t-1}} \right)$ .  $\beta_{i,term}$  is the slope coefficient from a time-series regression of futures returns on the term structure variable that proxies for the time varying risk premiums. We use 39 commodity futures contracts (spot and spreading returns) with varying sample period (see Table 1).  $\gamma$  is the implied coefficient of risk aversion defined as  $\frac{\lambda_c}{Var[\Delta c_t^S]}$ . We report Fama-MacBeth standard errors, and next to them Shanken corrected standard errors.

S	$\lambda_0$	St.err	Corr err	$\lambda_1$	St.err	Corr err	$\lambda_{term}$	St.err	Corr err	$R^2$	$R_{adj}^2$	$\gamma$
Panel A: monthly												
0	1.83	(0.73)	(0.78)	0.62	(1.13)	(1.22)	11.51	(6.51)	(6.96)	18.13%	0.14	496
1	1.84	(0.76)	(0.84)	1.26	(1.11)	(1.24)	13.22	(6.32)	(6.99)	20.51%	0.16	1001
2	1.87	(0.77)	(0.81)	0.00	(1.13)	(1.19)	10.43	(6.41)	(6.75)	17.17%	0.13	-3
3	1.87	(0.76)	(0.80)	-0.03	(1.19)	(1.26)	10.37	(6.54)	(6.89)	17.17%	0.13	-18
4	1.82	(0.75)	(0.79)	-0.40	(1.28)	(1.35)	9.62	(6.59)	(6.94)	17.60%	0.13	-195
5	1.70	(0.72)	(0.77)	-0.93	(1.36)	(1.44)	8.74	(6.64)	(7.04)	19.51%	0.15	-386
6	1.65	(0.72)	(0.76)	-1.20	(1.48)	(1.58)	8.47	(6.69)	(7.13)	20.46%	0.16	-429
7	1.63	(0.71)	(0.76)	-1.29	(1.56)	(1.67)	8.70	(6.77)	(7.23)	20.60%	0.16	-408
8	1.67	(0.72)	(0.77)	-1.16	(1.67)	(1.78)	9.12	(6.83)	(7.28)	19.45%	0.15	-324
9	1.69	(0.72)	(0.77)	-1.07	(1.77)	(1.89)	9.44	(6.87)	(7.32)	18.83%	0.14	-265
10	1.69	(0.72)	(0.77)	-1.07	(1.84)	(1.97)	9.63	(6.90)	(7.35)	18.75%	0.14	-239
11	1.75	(0.73)	(0.77)	-0.78	(1.93)	(2.06)	9.94	(6.92)	(7.34)	17.92%	0.13	-156
12	1.77	(0.73)	(0.78)	-0.69	(1.99)	(2.11)	10.08	(6.92)	(7.34)	17.72%	0.13	-124
Panel B: quarterly												
0	0.68	(0.57)	(0.65)	0.87	(0.37)	(0.42)	10.61	(8.36)	(9.49)	67.31%	0.65	487
1	1.19	(0.72)	(0.83)	1.09	(0.48)	(0.55)	14.08	(8.49)	(9.68)	54.55%	0.52	373
2	1.57	(0.81)	(0.92)	1.16	(0.61)	(0.68)	15.31	(8.61)	(9.67)	43.73%	0.41	275
3	1.70	(0.84)	(0.94)	1.29	(0.74)	(0.82)	14.38	(8.49)	(9.40)	41.31%	0.38	220
4	1.89	(0.89)	(0.97)	1.21	(0.83)	(0.91)	14.23	(8.44)	(9.18)	36.33%	0.33	160
Panel C: yearly												
0	0.68	(0.70)	(0.86)	0.59	(0.25)	(0.31)	35.73	(17.31)	(21.00)	57.59%	0.55	48
1	1.42	(0.96)	(1.12)	0.63	(0.34)	(0.40)	34.14	(17.24)	(20.03)	47.89%	0.45	33

Table 5: **Ultimate consumption risk on spot and spreading returns separately.**  
The table reports the cross-sectional regression estimation results for the Consumption CAPM:

$$E[r_{i,t+1}] = \lambda_0 + \lambda_1 \beta_i,$$

where  $\beta_i$  is the time-series futures returns loadings on consumption growth based on different horizons S:  $\Delta c_t^S = \log\left(\frac{C_{t+S}}{C_{t-1}}\right)$ . Panel A gives results for 20 spot returns and Panel B for 19 spreading returns with varying sample period (see Table 1).  $\gamma$  is the implied coefficient of risk aversion defined as  $\frac{\lambda_c}{Var[\Delta c_t^S]}$ . We report Fama-MacBeth standard errors, and next to them Shanken corrected standard errors.

S	$\lambda_0$	St.err	Corr err	$\lambda_1$	St.err	Corr err	$R^2$	$R_{adj}^2$	$\gamma$
Panel A: spot returns									
quarterly									
0	-1.76	(2.46)	(2.99)	1.15	(0.44)	(0.54)	66.22%	0.64	645
1	-1.02	(2.38)	(2.82)	1.34	(0.56)	(0.66)	38.07%	0.35	458
2	-0.19	(2.30)	(2.58)	1.27	(0.67)	(0.76)	19.41%	0.15	299
3	0.23	(2.24)	(2.49)	1.46	(0.81)	(0.92)	19.54%	0.15	249
4	0.72	(2.19)	(2.36)	1.40	(0.89)	(0.99)	14.71%	0.10	184
yearly									
0	-2.65	(2.70)	(3.40)	0.83	(0.30)	(0.41)	30.06%	0.26	68
1	-0.10	(2.37)	(2.64)	0.66	(0.38)	(0.47)	15.01%	0.10	34
Panel B: spreading returns									
quarterly									
0	0.49	(0.26)	(0.25)	0.27	(0.35)	(0.37)	2.65%	-0.03	149
1	0.49	(0.26)	(0.25)	0.35	(0.49)	(0.52)	2.34%	-0.03	119
2	0.51	(0.26)	(0.25)	0.52	(0.61)	(0.64)	3.29%	-0.02	123
3	0.51	(0.26)	(0.25)	0.59	(0.72)	(0.75)	3.07%	-0.03	101
4	0.48	(0.25)	(0.25)	0.49	(0.81)	(0.85)	1.62%	-0.04	65
yearly									
0	0.63	(0.34)	(0.40)	0.56	(0.30)	(0.37)	22.34%	0.18	46
1	0.60	(0.39)	(0.41)	0.52	(0.28)	(0.46)	10.93%	0.06	27

Table 6: Ultimate production and inventory risk  
The table reports the cross-sectional regression estimation results for the production- and inventory-based model:

$$E[r_{i,t+1}] = \lambda_0 + \lambda_Q \beta_{i,Q} + \lambda_{\Delta FMS_t} \beta_{i,\Delta FMS_t} + \lambda_{\Delta FMS_{t-1}} \beta_{i,\Delta FMS_{t-1}},$$

where  $\beta_Q$  is the time-series futures returns loadings on aggregate production growth based on different horizons  $S$ .  $\beta_{i,\Delta FMS_t}$  is the slope coefficient from a time-series regression of futures returns on the aggregate futures minus spot return, which is a proxy for inventory-based risk factor.  $\beta_{i,\Delta FMS_{t-1}}$  uses one period lagged values of the aggregate spot and futures returns. We use 39 commodity futures contracts (spot and spreading returns) with varying sample period (see Table 1). We report Fama-MacBeth standard errors, and next to them Shanken corrected standard errors.

S	$\lambda_0$	St.err	Corr.err	$\lambda_Q$	St.err	Corr.err	$\lambda_{\Delta FMS_t}$	St.err	Corr.err	$\lambda_{\Delta FMS_{t-1}}$	St.err	Corr.err	$R^2$	$R^2_{adj}$
Panel A														
quarterly														
0	-0.26	(0.68)	(0.93)	1.67	(0.77)	(1.02)	0.49	(0.29)	(0.51)	0.31	(0.50)	(0.59)	33.92%	28.26%
1	0.14	(0.73)	(0.90)	2.70	(0.85)	(1.20)	1.54	(0.62)	(1.22)	-0.77	(0.55)	(0.97)	42.72%	37.81%
2	0.11	(0.72)	(0.85)	2.97	(0.85)	(1.04)	1.81	(0.76)	(1.50)	-0.92	(1.10)	(1.60)	47.09%	42.55%
3	0.43	(0.74)	(0.79)	3.13	(1.06)	(1.18)	2.77	(0.83)	(1.82)	-2.38	(1.05)	(1.97)	46.07%	41.44%
4	0.55	(0.72)	(0.78)	2.83	(1.16)	(1.20)	3.62	(1.02)	(2.39)	-2.14	(1.47)	(2.84)	52.88%	48.84%
yearly														
0	1.57	(0.79)	(1.25)	1.17	(0.84)	(0.86)	0.79	(0.36)	(0.39)	0.12	(0.27)	(0.34)	15.57%	8.33%
1	2.21	(0.83)	(0.96)	0.09	(0.68)	(0.99)	1.05	(0.43)	(0.55)	-0.17	(0.41)	(0.48)	21.38%	14.65%
Panel B														
quarterly														
0	-0.27	(0.67)	(0.92)	1.50	(0.72)	(0.97)	0.56	(0.26)	(0.51)				33.52%	29.83%
1	0.14	(0.71)	(0.92)	2.61	(0.67)	(0.91)	1.57	(0.61)	(1.22)				42.72%	39.54%
2	0.16	(0.69)	(0.76)	2.99	(0.83)	(1.04)	1.96	(0.61)	(1.32)				47.33%	44.41%
3	0.37	(0.73)	(0.74)	3.19	(1.04)	(1.17)	2.54	(0.74)	(1.60)				45.34%	42.30%
4	0.67	(0.70)	(0.72)	2.76	(1.15)	(1.22)	3.96	(0.91)	(2.02)				51.62%	48.93%
yearly														
0	1.25	(0.74)	(1.19)	1.47	(0.80)	(0.84)	0.93	(0.34)	(0.40)				17.23%	12.63%
1	1.98	(0.80)	(1.01)	0.23	(0.65)	(1.01)	1.07	(0.40)	(0.51)				18.77%	14.26%

Table 7: **Ultimate production and inventory risk on spot and spreading returns separately.** The table reports the cross-sectional regression estimation results for the production- and inventory-based model:

$$E[r_{i,t+1}] = \lambda_0 + \lambda_Q \beta_{i,Q} + \lambda_{\Delta FMS_t} \beta_{i,\Delta FMS_t},$$

where  $\beta_Q$  is the time-series futures returns loadings on aggregate production growth based on different horizons S.  $\beta_{i,\Delta FMS_t}$  is the slope coefficient from a time-series regression of futures returns on the aggregate futures minus spot return, which is a proxy for inventory-based risk factor. Panel A gives results for 20 spot returns and Panel B for 19 spreading returns with varying sample period (see Table 1). We report Fama-MacBeth standard errors, and next to them Shanken corrected standard errors.

S	$\lambda_0$	St.err	Corr err	$\lambda_Q$	St.err	Corr err	$\lambda_{\Delta FMS_t}$	St.err	Corr err	$R^2$	$R^2_{adj}$
Panel A: spot premiums											
quarterly											
0	-2.64	(2.62)	(2.80)	2.18	(1.20)	(1.35)	0.88	(0.67)	(0.72)	50.05%	44.17%
1	-2.05	(2.62)	(2.87)	4.38	(1.79)	(2.01)	0.87	(0.94)	(1.48)	58.85%	54.00%
2	-2.74	(2.70)	(3.04)	5.06	(2.33)	(2.64)	0.97	(0.99)	(1.65)	62.20%	57.75%
3	-3.10	(2.80)	(3.13)	6.42	(2.87)	(3.21)	0.83	(1.03)	(2.13)	57.04%	51.99%
4	-2.58	(2.74)	(3.26)	6.20	(3.18)	(3.83)	1.94	(1.16)	(2.74)	61.37%	56.82%
yearly											
0	1.25	(1.49)	(2.35)	1.86	(1.34)	(1.20)	1.16	(0.53)	(0.65)	22.32%	13.18%
1	3.63	(1.62)	(2.36)	0.20	(0.89)	(1.06)	1.38	(0.59)	(0.72)	27.66%	19.15%
Panel B: spreading returns											
quarterly											
0	-0.45	(0.38)	(0.44)	5.10	(1.73)	(2.51)	-1.03	(0.39)	(0.55)	39.95%	32.45%
1	0.01	(0.33)	(0.39)	3.66	(1.88)	(2.79)	-1.70	(0.68)	(0.92)	44.41%	37.46%
2	0.44	(0.35)	(0.39)	2.46	(2.64)	(3.70)	-0.26	(1.10)	(1.50)	37.57%	29.77%
3	0.40	(0.32)	(0.37)	2.46	(3.03)	(4.48)	-0.41	(1.28)	(1.79)	38.31%	30.60%
4	0.65	(0.37)	(0.46)	0.55	(4.01)	(5.69)	1.49	(2.21)	(2.52)	33.16%	24.80%
yearly											
0	-0.02	(0.28)	(0.22)	2.53	(0.61)	(1.15)	-0.61	(0.21)	(0.38)	60.71%	55.80%
1	0.28	(0.46)	(0.25)	1.81	(1.05)	(1.40)	-0.18	(0.73)	(0.65)	17.02%	6.65%

Table 8: **Demand and supply elasticity.**

The table reports the estimation of the constant elasticity functions of demand (D) and supply (S) from the following regressions:

$$\begin{aligned}\Delta \log D_{i,t}^S &= \alpha_D + \beta_{D,i} \Delta \log P_{i,t} + u_{i,t}, \\ \Delta \log S_{i,t}^S &= \alpha_S + \beta_{S,i} \Delta \log P_{i,t} + \varepsilon_{i,t},\end{aligned}$$

where change in demand and supply is measured over different horizons S:  $\Delta \log I_{i,t}^S = \log(\frac{I_{i,t+S}}{I_{i,t-1}})$  with  $I_{i,t}^S = \{D_{i,t}^S, S_{i,t}^S\}$ . We use 39 commodity futures contracts (spot and spreading returns) with varying sample period (see Table 1). The estimates are based on the aggregate values of demand (consumption) and supply (Industrial Production Index). We report the estimates of the slope coefficients and their standard errors.

futures \ S	Panel A: Demand elasticity									
	0	St. err	1	St. err	2	St. err	3	St. err	4	St. err
Wheat	0.09	(0.28)	0.11	(0.47)	0.01	(0.63)	-0.43	(0.78)	-1.10	(0.90)
Corn	0.04	(0.29)	-0.14	(0.48)	-0.13	(0.65)	-0.72	(0.81)	-1.65	(0.94)
Oats	0.33	(0.29)	0.16	(0.48)	0.17	(0.64)	-0.11	(0.80)	-0.54	(0.96)
Soybean	0.18	(0.24)	0.10	(0.40)	0.20	(0.54)	-0.03	(0.68)	-0.57	(0.79)
Soybeans Oil	0.04	(0.25)	-0.14	(0.42)	-0.30	(0.56)	-0.67	(0.69)	-1.19	(0.81)
Soybean meal	0.28	(0.22)	0.25	(0.37)	0.42	(0.50)	0.27	(0.63)	-0.18	(0.73)
Live cattle	0.28	(0.42)	0.58	(0.69)	0.48	(0.92)	0.24	(1.17)	0.48	(1.42)
Feeder cattle	0.54	(0.51)	1.24	(0.83)	1.23	(1.10)	1.78	(1.43)	2.42	(1.72)
Live hog	0.17	(0.21)	0.04	(0.35)	0.05	(0.46)	0.33	(0.57)	0.20	(0.67)
Pork Bellies	0.17	(0.20)	0.21	(0.32)	0.07	(0.43)	0.36	(0.54)	0.23	(0.63)
Crude Oil	0.08	(0.22)	-0.26	(0.35)	-0.15	(0.47)	0.07	(0.60)	-0.42	(0.71)
Heating Oil	0.17	(0.24)	0.07	(0.40)	0.03	(0.52)	0.01	(0.65)	-0.30	(0.77)
Unleaded Gasoline	0.23	(0.22)	-0.02	(0.35)	-0.05	(0.46)	0.12	(0.58)	-0.12	(0.71)
Gold	0.27	(0.39)	-0.40	(0.64)	-1.26	(0.84)	-0.97	(1.07)	-1.47	(1.27)
Silver	0.53	(0.22)	0.32	(0.37)	-0.03	(0.49)	0.05	(0.61)	-0.36	(0.71)
Platinum	0.48	(0.27)	0.27	(0.44)	0.06	(0.58)	0.21	(0.73)	-0.24	(0.87)
Copper	0.51	(0.38)	0.52	(0.61)	0.05	(0.80)	0.13	(1.02)	-0.11	(1.35)
Coffee	-0.01	(0.20)	-0.26	(0.33)	-0.09	(0.43)	-0.20	(0.54)	0.02	(0.64)
Sugar	-0.08	(0.17)	-0.20	(0.28)	-0.37	(0.38)	-0.73	(0.47)	-0.91	(0.56)
Cotton	0.48	(0.25)	0.56	(0.41)	-0.16	(0.55)	-0.45	(0.70)	-0.62	(0.83)
S&P 500	0.71	(0.48)	1.35	(0.78)	1.80	(1.04)	2.11	(1.31)	2.81	(1.53)
futures \ S	Panel B: Supply elasticity									
	0	St. err	1	St. err	2	St. err	3	St. err	4	St. err
Wheat	4.17	(1.42)	5.29	(2.50)	9.19	(3.34)	13.06	(4.04)	14.56	(4.67)
Corn	5.06	(1.43)	7.03	(2.53)	10.53	(3.44)	13.16	(4.21)	12.72	(4.95)
Oats	1.64	(1.53)	3.49	(2.52)	5.80	(3.30)	7.34	(4.07)	5.95	(4.78)
Soybean	4.93	(1.18)	7.32	(2.08)	8.55	(2.87)	10.42	(3.53)	11.69	(4.11)
Soybeans Oil	5.28	(1.22)	8.07	(2.16)	9.54	(2.94)	10.41	(3.65)	9.72	(4.29)
Soybean meal	3.85	(1.11)	5.87	(1.94)	6.95	(2.70)	9.16	(3.29)	11.07	(3.81)
Live cattle	3.85	(2.04)	7.58	(3.47)	9.14	(4.67)	7.99	(5.86)	12.37	(6.90)
Feeder cattle	4.65	(2.45)	9.92	(4.16)	11.86	(5.60)	12.06	(7.12)	20.28	(8.19)
Live hog	1.34	(1.07)	2.04	(1.86)	0.69	(2.52)	1.41	(3.09)	4.28	(3.55)
Pork Bellies	0.25	(1.01)	0.28	(1.75)	-0.48	(2.36)	-0.45	(2.90)	2.36	(3.34)
Crude Oil	4.80	(0.95)	7.83	(1.62)	7.62	(2.30)	8.29	(2.86)	8.40	(3.32)
Heating Oil	4.15	(1.04)	7.03	(1.75)	6.65	(2.44)	6.46	(3.03)	6.08	(3.45)
Unleaded Gasoline	4.61	(1.01)	8.18	(1.68)	7.88	(2.43)	7.81	(3.04)	7.78	(3.54)
Gold	2.86	(1.89)	4.88	(3.25)	7.06	(4.40)	12.64	(5.39)	18.81	(6.18)
Silver	3.59	(1.11)	5.80	(1.93)	8.05	(2.58)	11.54	(3.12)	13.15	(3.64)
Platinum	4.07	(1.39)	6.43	(2.40)	8.82	(3.18)	12.15	(3.80)	13.26	(4.38)
Copper	3.08	(1.91)	8.98	(3.08)	11.34	(4.11)	14.63	(4.98)	12.29	(6.55)
Coffee	1.50	(1.04)	2.48	(1.77)	4.08	(2.32)	4.40	(2.77)	3.14	(3.19)
Sugar	2.10	(0.89)	2.81	(1.48)	2.88	(1.97)	2.86	(2.44)	3.17	(2.84)
Cotton	3.08	(1.33)	6.99	(2.23)	10.51	(2.91)	12.75	(3.54)	15.61	(4.03)
S&P 500	-3.44	(2.33)	0.29	(4.06)	2.76	(5.49)	6.90	(6.69)	10.64	(7.57)