Intraday Statistical Properties of Eurofutures

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Abstract

This paper studies the statistical properties of the price, volatility and tick dynamics of the intraday Eurofutures markets by utilizing the transactions and quote data. We build two different types of price series, by position and by contract. The findings indicate numerous sources of intraday and intraweek seasonality. First, volatility tends to decrease towards the maturity date. Second, intraday price changes and tick activity display U-shaped seasonalities with peaks occuring at opening and closing hours. Third, there is evidence of intraweek seasonalities where the level of activity displays a minimum on Monday and a maximum on the last two working days of the weeks. The findings of this paper suggest that a model of volatility for futures markets should correct for the seasonality originating from the maturity effect. In addition, U-shaped intraday seasonalities and intraweek seasonalities should be properly taken into account in the conditional mean and conditional volatility models of futures markets.

In addition, the return series exhibit serial correlation which provides evidence for predictability and timing ability. This predictability does not translate itself into net profitability in the short horizons such as three minutes data. However, the net profitability opportunities exist at the lower frequencies such as the thirty minutes horizon after taking trading costs into account.

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1. Introduction

The explosive growth of interest rate trading together with the deregulation and globalization of the financial industry have fueled the market for interest rate derivatives since the late seventies and early eighties. Although there has been extensive academic research on US Treasury Bill futures, Eurofutures markets have not been studied extensively. In 1997, the daily volume for the Chicago Mercantile Exchange (CME) Eurodollar was about 450 times larger than for CME US Trasurury Bill futures. This takes into account neither the Eurodollar futures traded in other markets, nor Eurofutures for many other currencies while CME accounts for the entire US treasury Bill volume. The bid-ask spread on the CME Eurodollar can be as small as half a basis point¹ compared with quoted spreads on Eurodollar deposits which are at least 2-25 times higher. Due to the size of the small bid-ask spreads even tiny moves are tradable in the Eurofutures markets, whereas they would not be in the Eurodeposit market. This means that the Eurofutures markets act as a very sensitive measuring device for interest rate expectations. They are also transparent markets because transaction prices are public knowledge which is almost never the case for over-the-counter (OTC) markets. The Eurofutures markets are typically the most liquid markets for interest rate instruments, playing a crucial role in the price discovery mechanism. They yield high quality intraday data consisting of transaction prices, firm bid and ask quotes and sometimes volume information. Because of their liquidity and data availability, they are one of the best laboratories in which to investigate market microstructure.

Eurofutures are exchange-traded contracts and entail several differences with respect to over-the-counter instruments. First, Eurofutures are linked to a specific exchange, except when a fungibility agreement is in effect². Second, the trading is typically geographically localized and there is no 24-hour trading unlike the foreign exchange market, although the trend is to effectively lengthen trading hours with after-hours sessions. Given that futures contracts are exchange traded and each transaction is recorded centrally, futures markets offer a high price transparency. The historical data always includes tick-by-tick transaction prices and, depending on the data source, bid and ask quotes, bid and ask sizes and actual volumes for each transaction are also included. This allows us to conduct the first full

 $^{^{1}\}mathrm{A}$ basis point corresponds to 1/100 of a percent and its monetary value (in the case of the Eurodollar futures) is 25 dollars. The minimum price movement for the CME Eurodollar is half a basis point.

²One example of a fungibility agreement is the mutual offset system between the Chicago Mercantile Exchange (CME) and the Singapore International Monetary Exchange (SIMEX), through which contracts opened in one exchange can be liquidated on the other.

investigation of transaction prices³, a comparison between the transaction prices and the bid-ask quotes for the outcry sessions.

Each futures contract has a specific expiry date and normally begins to be traded one year or more in advance. Eurofutures for an underlying three month deposit have four expiry months in a year. These are March, June, September and December contracts known as quarterly expiries. There are also serial expiry contracts such as contracts expiring in months that do not correspond to the quarterly sequence. The serial expiry contracts are not considered here because of the fact that they typically exhibit lower liquidity. We define the first position at a given time as the contract which expires next in the quarterly sequence, the second position as the second contract to expire in the same quarterly sequence. The third and the fourth positions are constructed similarly. For each Eurofutures, we build two different types of price time series which are time series by position and by contract. Position time series is not interrupted by contract expiries and consists of different contracts; at each quarterly expiry it switches to a new contract. The contract time series starts on the opening date of the contract and stops when the contract expires.

In the recent literature, Harvey and Huang (1991) indicate that interest rate and foreign exchange futures prices are much more volatile during the first sixty to seventy minutes of trading on Thursdays and Fridays than at any other hour over the trading week. Harvey and Huang (1991) postulate that this pattern is due to the fact that many macroeconomic announcements occur during the first hour of the trading on these two days and it is not due to the opening itself. These findings are supported by the work of Ederington and Lee (1993, 1995) who analyse the impact of scheduled macroeconomic announcements on interest rate and foreign exchange futures markets. They find that these announcements are responsible for most of the observed time-of-day and day-of-the-week effects. In addition, Ederington and Lee (1993) investigate the speed at which markets adjusts to the news releases. They find that major price adjustments occur within one minute of the release and the direction of the subsequent price adjustments is independent of the first minute's price change. The price continues to be more volatile than normal for fifteen minutes and slightly more volatile for several hours. This adjustment rate is more than which was observed in the securities markets by Patell and Wolfson (1984).

³In the intraday studies of the foreign exchange markets, in Müller et al. (1990), Dacorogna et al. (1993) and Guillaume et al. (1997) studied bid-ask quotes.

Our study of CME and London International Financial Futures Exchange (LIFFE) focuses on Eurodollar (traded at CME), Euroswiss, Euromark, Short Sterling and Three Month ECU (all traded at LIFFE). The results of this paper indicate the following stylized facts.

Eurodollar, Euromark and Short Sterling display a high liquidity⁴ in all positions such that the number of intraday ticks⁵ reaches a maximum in the second position and then decreases. Euroswiss and Three Month ECU, on the contrary, display a decreasing level of liquidity from position one to position four. For Euroswiss, this reduction in the level of liquidity is seven times lower. For all positions, three Month ECU shows a limited tick activity compared to all the other Eurofutures.

For all contracts traded on LIFFE the hourly tick activity displays a U-shape with its minimum between 11am-1pm (GMT) and a clustering of activity around the beginning and the end of the day.

Intraweek tick activity exhibits evidence of a day-of-the-week effect. The level of activity displays a minimum on Monday and a maximum on the last two working days of the week. The difference is significant for the Eurodollar such that for positions one and two the tick activity on Friday is almost twice as much as Monday's and it becomes more significant in positions three and four.

Intraday volatility follows similar patterns to that presented by intraday tick activity. In general, opening hours show the highest price variation and only in some cases the largest price change occur towards closing time which is usually in the last position. Short Sterling and Three Month ECU display the minimum of the U-curve one hour later than in the tick-activity.

For most contracts, the correlation between hourly tick activity and hourly volatility is above 96 percent. Euromark for the first two positions and Three Month ECU for the fourth position show a lower correlation around 90 percent.

In general, Eurodollar, Euromark and Short Sterling hourly volatility tend to increase

⁴Although liquidity has been used in many contexts in the literature, we use tick activity as a measure of liquidity as it is a proxy for the market activity.

⁵Differences in tick activity across different futures positions is also studied in Ballocchi et al. (1998).

from position one to position four. For Euroswiss and Three Month ECU, volatility shows an increase from position one to position two and decreases afterwards.

Eurodollar, Euroswiss and Three Month ECU display bid-ask tick activity higher than transaction activity. In particular, Eurodollar tick activity for bid-ask quotes is about five times higher than for tick activity of transaction prices for all positions. For the Euromark and Short Sterling tick activity, transaction prices are on the average double of that for bid-ask quotes.

During business days, bid-ask price volatility is usually higher than transaction price volatility and the difference is often less than one basis point. Over weekends, with very few exceptions, bid-ask price changes are higher than transaction prices, and even higher on Sunday than on Saturday.

The return series exhibit serial correlation which provides evidence for predictability and timing ability. This predictability does not translate itself into net profitability in the short horizons such as three minutes data. However, the net profitability opportunities exist at the lower frequencies such as the thirty minutes horizon after taking trading costs into account.

In section two, the analysis per position is examined. For all Eurofutures and all positions, the studied sample covers the period of January 1, 1994 to April 15, 1997, more than three years of tick-by-tick data. In section three, the analysis per contract is carried out on Eurodollar (traded at CME), Euromark and Short Sterling (all traded at LIFFE). For each Eurofutures, nine contracts associated with the following expiries are considered: March 1995, June 1995, September 1995, December 1995, March 1996, June 1996, September 1996, December 1996 and March 1997. Between different contracts, sample lengths differ but at least one year of data for each contract is available. Scaling law analysis is presented in section four. Persistence properties of the return, volume and tick dynamics are presented in section five. We conclude afterwards.

2. Analysis per position on LIFFE and CME data

Studying futures by position as proposed in this paper can be justified on the basis of how this market works. Traders (both strategic traders and hedgers) holding close-to-expiry contracts

need to roll their positions forward into the next expiry contract in order to stay in the market. By doing so they are in fact constructing a time series by position, extending beyond the expiry of each contract. The study by contract, on the other hand, provides evidence that properties that depend on the contract lifetime (e.g. the deterministic volatility pattern) are a function of the time left before expiry. From the tick-by-tick data the logarithmic price, $p(t_i)$, is calculated by a simple linear interpolation such that

$$p(t_i) = p(t_p) + (p(t_s) - p(t_p)) \frac{t_i - t_p}{t_s - t_p}$$
(1)

where t_p and t_s are the most recent previous and subsequent tick relative to t_i . The *change* of price or return at time t_i , $r(t_i)$, is defined as

$$r(t_i) \equiv r(\Delta t; t_i) \equiv [p(t_i) - p(t_i - \Delta t)]$$
 (2)

where $p(t_i)$ is the sequence of equally spaced in time logarithmic price, and Δt is the fixed time-interval such as ten minutes, one hour, one day. The *volatility* at time t_i , $v(t_i)$, is defined as

$$v(t_i) \equiv v(\Delta t, S; t_i) \equiv \frac{1}{n} \sum_{k=1}^{n} |r(\Delta t; t_{i-k})|$$
(3)

where S is the sample period on which the volatility is computed. In equation (3), the absolute value of the returns is preferred to the more usual squared value. This is because the former quantity better captures the autocorrelation and the seasonality of the data (Taylor (1988), Müller et al. (1990) and Granger and Ding (1993)). The *tick frequency* at time t_i , $f(t_i)$, is defined as

$$f(t_i) \equiv f(S; t_i) \equiv \frac{1}{S} N(\{x(t_j) \mid t_j \in [t_i - S, t_i]\}).$$
 (4)

The log tick frequency at time t_i , log $f(t_i)$, is defined as

$$\log f(t_i) \equiv \log f(S; t_i) \tag{5}$$

where $N(\lbrace x(t_j)\rbrace)$ is the counting function and S is the sample period on which the counting is computed.

Time series behavior of the Eurofutures indicates at least two different kinds of volatility. One is the spread existing between minimum and maximum prices occurring during the considered period (long-term volatility) and the other is the daily or weekly change in prices (short-term volatility). From both points of view, all Eurofutures display an increase in volatility going from position one to position four. In Figure 1, the time series behavior of Eurodollar and Euromark prices for position one and four are plotted. Both Eurodollar and Euromark display an increase in volatility from position one to four, both on short and long periods of time. This is representative of all the other Eurofutures series.

2.1. Intraday analysis of tick activity and volatility for transaction prices

The intraday statistics uses a uniform time grid with 24-hourly intervals while intraweek statistics uses a uniform 7-day interval grid (from Monday to Sunday) in Greenwich Mean Time (GMT). Both of them are used to measure the volatility and the tick activity which can be considered as an approximation of the level of market activity. The intraday tick activity per position characterizing CME and LIFFE futures prices are summarized in Table 1 which indicates that Eurodollar, Euromark and Short Sterling display a high liquidity in all positions. The number of ticks reaches a maximum in the second position and then decreases. The fourth position has at most as many ticks as the first. Euroswiss and Three Month ECU, on the contrary, display a decreasing level of liquidity from position one to position four. For Euroswiss, this reduction in the level of liquidity is seven times lower. For all positions, three Month ECU shows a limited tick activity compared to all the other Eurofutures. Figure 2 indicates the intraday tick activity and intraday price changes for Short Sterling in position two. The intraday tick activity displays the average number of ticks occurring in each hour of the day while intraday volatility shows the mean absolute change in the logarithm of the price. Both plots display similar U-shapes, the only difference being that the minimum appears one hour later for intraday price changes. The sampling period starts on January 1, 1994 and ends on April 15, 1997. The total number of ticks in Figure 2 is 184,360.

For all contracts traded on LIFFE the hourly tick activity displays a U-shape with its minimum around 11am-1pm (GMT) and a clustering of activity at the beginning and the end of the day⁶. Intraday volatility follows similar patterns to that presented by intraday

⁶Similar forms of intraday seasonality are also reported in Andersen and Bollerslev (1997) and Ghysels and Jasiak (1995) with different data sets of exchange traded instruments.

tick activity. In general, opening hours show the highest price variation⁷ and only in some cases the largest price change occurs towards the closing time which are usually in the last positions. Short Sterling and Three Month ECU display the minimum of the U-curve one hour later than in the tick-activity.

The hourly and daily volatility are defined⁸ as the mean absolute change of logarithmic prices. For most contracts, the correlation between hourly tick activity and hourly volatility is above 96 percent; only Euromark for the first two positions and Three Month ECU for the fourth position show a lower correlation around 90 percent. In general, Eurodollar, Euromark and Short Sterling hourly volatility tend to increase from position one to position four. For Euroswiss and Three Month ECU, volatility shows an increment from position one to position two and then they decrease again.

Values of daily and weekly volatility for each Eurofutures and each position are reported in Table 2. Results on daily volatility confirm the results from the intraweek volatility analysis. For Eurodollar, Euromark and Short Sterling, the level of variation tends to increase from position one to position four. Euroswiss shows an increment from position one to position two and then starts to decrease. Three Month ECU displays a slight tendency to increase. On a weekly basis, for all Eurofutures, price variations tend to increase passing from position one to position four. It is interesting to note that the ratio between the daily and weekly volatility almost always exceeds $\sqrt{7}$, the latter being the ratio predicted by the aggregation properties of the Gaussian random walk.

Intraweek tick activity exhibits evidence of a day-of-the-week effect. The level of activity displays a minimum on Monday and a maximum on Thursdays for LIFFE contracts and on Fridays for CME contracts. The difference is definitely significant for the Eurodollar; in fact, for positions one and two the tick activity on Friday is almost double that on Monday and it becomes more than double for positions three and four. Figure 3 illustrates the intraweek tick activity for Eurodollar and Euromark in position one and position four. They display considerable day-of-the-week differences.

The correlation between intraweek tick activity and intraweek price changes is high, but only Eurodollar intraweek volatility follows a pattern similar to that displayed by intraweek

⁷The difference with respect to the other hours is an average of one basis point.

⁸For completeness, the other definition of the volatility as the square root of the variance computed is also calculated. There are no significant differences between the two definitions.

tick activity. In fact, except for position one, the maximum price variation for Eurodollar is reached on Friday and the volatility between Monday's and Friday's price changes tends to increase from positions two to four. Position one shows the highest level of variation on Monday and presents the lowest value of the correlation coefficient among all the Eurofutures. In addition, the spread of price changes on Friday is on average about two basis points higher than on other days. Three Month ECU, Euroswiss and Euromark reach the maximum spread on Mondays and Thursdays (usually higher on Mondays) and Short Sterling on Wednesday. In Short Sterling's case, it is worth noting that, at least for the first three positions, price changes on Wednesday are about two basis points higher than during the other days (see Figure 4 (a)) in which position two results are displayed. This could be explained by the fact that the relevant news releases in the United Kingdom occur on Tuesdays and Wednesdays.

2.2. Intraday analysis of transaction prices and bid-ask quotes

In this section, the results from the transaction prices are compared with bid-ask quotes. The objective of this further investigation is to examine differences between transaction prices and bid-ask quotes and in-hours and after-hours trading sessions. This section is carried out on the same Eurofutures but on a shorter period from December 2, 1996 to April 15, 1997. This comparison indicates that Eurodollar, Euroswiss and Three Month ECU display bid-ask tick activity higher than transaction activity. In particular, Eurodollar tick activity for bid-ask quotes is about five times higher than tick activity for transaction prices for all positions.

For the Euromark and Short Sterling tick activity, transaction prices are on the average double that of bid-ask quotes. During business days, bid-ask price changes are usually higher than transaction ones and the difference is often less than one basis point. Over weekends, with very few exceptions, bid-ask price changes are higher than transaction prices, and even higher on Sunday than on Saturday. There are practically no transaction price changes on Saturday. In Figures 4 (b) and 4 (c), intraweek tick activity for transaction prices and intraweek tick activity for bid-ask quotes are presented. Both are computed for Short Sterling in position two. Activity on transaction prices appears slighly higher than activity on bid-ask.

For all Eurofutures traded on LIFFE, intraday tick activity displays a U-shape for both transaction prices and bid-ask quotes. In general, both transaction and bid-ask tick activities

are concentrated between 6-7 am and 5-6 pm (GMT time). The hourly tick activity for the Eurodollar bid-ask is on average four times higher than for transaction data, in addition tick activity for transaction data is mostly concentrated between 11 am and 9 pm (GMT) while for bid-ask series there is a significant number of ticks for the entire 24 hours.

For both transaction prices and bid-ask quotes, all Eurofutures traded on LIFFE display significant intraday volatility between 6-7 am and 5-6 pm (GMT) similar to the intraday tick activity behavior. In general, hourly volatility is higher for bid-ask quotes than for transaction prices of all positions. For Eurodollar, transaction price volatility is mostly concentrated between 6 am and 1 am (GMT), while bid-ask volatility is spread over the whole day with important peaks between 12 pm and 8 pm. In Figure 5, hourly volatility for transaction prices and hourly volatility for bid-ask quotes are illustrated. Both are computed for Eurodollar and Euromark in position one. In both cases, the difference in price variation between bid-ask quotes and transaction prices is higher during open market hours than when the market is closed.

3. Analysis per contract

The purpose of this section is to analyze the volatility behavior of futures prices per contract. As shown before, futures contracts exhibit volatility seasonalities like those reported in the literature for foreign exchange data (Müller et al. (1990)) and also for equity markets (Andersen and Bollerslev (1997); Ghysels and Jasiak (1995)). Those seasonalities were attributed to dealing patterns, such as market presence (Dacorogna et al. (1993)). However, the characteristic feature of the futures markets when compared to foreign exchange or equity markets is that each contract has an expiry. We show that this leads to another seasonality, depending on the time left to expiry. On a daily or weekly basis, there are indications that Eurodollar, Euromark and Short Sterling display a decreasing volatility towards the expiry. We define deterministic volatility pattern as the relation between volatility and time left to expiry. In order to probe the existence of a seasonality related to contract expiry, a sample consisting of many futures contracts is needed.

For each Eurofutures (Eurodollar, Euromark, Short Sterling) and for each contract, we built a series of hourly price differences determined by linear interpolation. Then we computed daily volatilities taking the mean absolute value of hourly price differences from 00:00

to 24:00 (GMT) of each working day (weekends and holidays were excluded). The daily volatilities are then plotted against time to expiry. The result is shown in Figure 6. The vertical axis represents the deterministic volatility computed on all Eurofutures and all contracts together. The horizontal axis represents the time left to expiry and as we move towards the origin the number of days before expiry decreases⁹. Figure 6 spans a period of about 300 days because only within that period were we able to compute our deterministic volatility on at least 35 contracts.

After 300 days, only a small number of contracts displays any real activity hence the average is affected by the volatility characterizing only few Eurofutures and it becomes less significant. The results indicate that there is a downward trend in volatility as the time left before expiry decreases. There is also an unexpected behavior consisting in oscillatory movements with peaks every 90 days corresponding to rollover activities near the ending of contracts.

These results are also confirmed by a deterministic volatility study on each single Eurofutures. Eurodollar, Euromark and Short Sterling show a decreasing volatility at least for the 300 days before expiry. All Eurofutures display oscillatory movements with peaks around expiry dates.

4. Scaling law and drift exponent

Another test of a random walk process is given by the existence of an empirical scaling law which relates the mean absolute price changes as to the time interval on which the change is measured. This test was proposed in Müller et al. (1990) and empirically studied for the foreign exchange rates. The idea behind the scaling law is straightforward. The random walk model is

$$x_t = x_{t-1} + \varepsilon_t \tag{6}$$

where x_t is the logarithm of a financial price series and ε_t are independently and identically distributed following a normal distribution with mean zero and variance σ^2 . Let r_t^n denote

⁹The smoothing in Figure 6 is calculated by the maximum overlap discrete wavelet transformation which is a time series multi-resolution technique and it is applicable to non-stationary time series (Percival and Mofjeld (1997)).

the *n*-period return at time t, i. e. $r_t^n = x_t - x_{t-n}$ so that

$$r_t^n = \sum_{i=0}^{n-1} \varepsilon_{t-i} . (7)$$

Due to the independence of the ε_t , the variance of r_t^n is equal to $\sigma_{rw}^2 = n\sigma^2$. The plot of the logarithm of the variance of the *n*-period random walk return $(\log(n\sigma^2))$ against the logarithm of the horizon n $(\log(n))$, leads to a linear function with slope equal to one (or 0.5 when $\log(\sigma_{rw})$ is plotted against $\log(n)$). This simple calculation provides an intuitive basis for a test of the random walk hypothesis. The idea is to plot the logarithm of the *n*-period return variance – or some other measure of volatility (here the mean absolute price change) – against the return period and to test whether the relationship is linear and the slope coefficient is the one implied by the random walk process. The important aspect of the scaling law is that it provides a simultaneous test of many different time horizons (Guillaume et al. (1997)) for a particular process.

An empirical scaling law¹⁰ examines the relationship between intervals Δt and the average absolute price changes,

$$\overline{|\Delta x|} = c \Delta t^{\frac{1}{E}}, \qquad (8)$$

where the bar over the $|\Delta x|$ indicates the average over the whole sample period, c is a constant and 1/E is defined to be the drift exponent. The drift exponent is 0.5 for the Gaussian random walk process. Results for drift exponents of different Eurofutures contracts, using overlapping observations are shown in Table 3. The drift exponents are all significantly above the value 0.5 which is the value expected for a Gaussian random walk. The time intervals we considered for the absolute returns varied from below one day to half a year.

In a second step, we have repeated the scaling law analysis on an average of contracts. We averaged the mean absolute values (associated with each time interval) on the number of contracts. When the analysis referred to single Eurofutures the average was computed on 9 contracts; when it referred to all Eurofutures and all contracts together, the average was computed on 36 contracts. Then we performed a linear regression for the logarithm of the computed averages against the corresponding logarithm of time intervals, taking the following time intervals: 1 day, 2 days, 1 week, 2 weeks, 4 weeks, 8 weeks and half a year.

¹⁰An application of the empirical scaling law within the context of foreign exchange markets is studied in Müller et al. (1990).

The scaling law analysis and the regression results are also presented in Figure 7. The confidence interval around the Ordinary Least Squares (OLS) represents two standard deviations from the regression fit. The results indicate that the OLS model fits fairly well to the data implied by the empirical scaling law. Figure 7 also demonstrates that the scaling law implied by the random walk model stays outside of the confidence intervals implying the rejection of the Gaussion random walk model as the data generation process. The results in Table 4 for single Eurofutures indicate that Short Sterling on short periods of time (1 and 2 days) displays a slightly higher volatility than Eurodollar, while on the remaining intervals the opposite is true. In fact, Short Sterling's drift exponent is lower than that of Eurodollar. Eurodollar and Short Sterling show higher volatility than Euromark for all intervals except the longest interval corresponding to 26 weeks or more. Euromark's drift exponent is the highest among all Eurofutures. The drift exponent for all Eurofutures is remarkably similar to those obtained on foreign exchange rates (Guillaume et al. (1997)) and on inter-bank money market rates (Ballocchi and Dacorogna (1996)).

It is a further indication that, in addition to being a test of the random walk hypothesis, the drift exponent is measuring something related to the nature of the market rather than the behavior of a particular asset. In Guillaume et al. (1997), it was shown that the drift exponent for regulated markets like the European Monetary System (EMS) was smaller than 0.5 and that it went back to values larger than 0.5 when the EMS bands were relaxed. Here we have the confirmation (after the interbank interest rate and the stock market (Mantegna and Stanley (1995)) that, for liquid markets, the exponent is around 0.6, independent of the asset which is traded.

5. Persistence properties of return, volume and tick dynamics

5.1. By position

In this section, the time series properties of the returns, volatility and the tick dynamics are investigated for the 3 and 30 minutes series by position. In Tables 5-8, the 3 minutes USD series are investigated from Position 1 to Position 4. The return series exhibit a significant negative skewness with a large excess kurtosis. The skewness for the returns are -28.28,

-13.72, -2.66 and -7.61 for Positions 1-4. The size of the skewness decrease as it is moved from Position 1 to Position 4. The kurtosis are 3455.90, 1147.54, 1985.30 and 476.13 for Position 1 to 4, respectively. There is a gradual decrease of excess kurtosis as it is moved from Position 1 to Position 4 although the kurtosis of Position 3 is slightly higher than that of Position 2. Similar to the returns, the volatility exhibits significant skewness and excess kurtosis relative to the Gaussian distribution. Tick series are also highly skewed and have excess kurtosis although their skewness and kurtosis relative to the volatility dynamics are much less pronounced.

For the returns, the first four autocorrelations of Position 1, the first three autocorrelations of Position 2, the first two autocorrelations of Position 3 and first six autocorrelations of Position 4 are statistically significant and negative. The statistical significance of the autocorrelations are measured by the Bartlett standard errors and the Ljung-Box-Pierce statistic. The size of an average movement of the Eurofutures in basis points can be measured from the mean absolute volatility. For Position 1 in Table 5, the average movement in basis points for the USD is about .247 basis points. An average 12 minutes movement is .494 for Position 1. Given that the trading costs for Eurofutures is about .5 basis point for transaction costs (this is roundturn execution cost; for one way the execution cost is .25 basis point.) and .5 basis point for the spread, the size of the movement in basis points for a 12 minutes interval stays below the level of profitability. The average movement in the basis points gets larger from Position 1 towards Position 4. For Position 4 in Table 8, the average movement for the 3 minute data is .329 basis points. The persistence of the return dynamics lasts six consequtive 3 minutes and the average movement in basis points is .806 for 18 minutes. The size of the trading costs is one basis point for a roundtrip trade and .75 for one way trade. For Position 4, the predictability gets close in terms of translating it into profitability if the trade is one way but not roundtrip. One interpretation of these results is that conditional mean predictability can be interpreted as a timing advantage. For those who plan to buy, they may wait a bit longer given that the autocorrelations are negative. This may give price advantage when the market is moving down. For those who plan to sell, it indicates that they have to move quickly with their trade.

For the volatility and the tick dynamics, the first ten autocorrelations are statistically significant and positive. In particular, the serial persistence in the tick dynamics is highly significant. For instance, the Ljung-Box-Pierce statistics for volatility and the tick dynamics

for Position 1 are 4230 and 71,200 where the $\chi^2(10)$ at the 5 percent level is 18.307. The days of the week effects are reported at the lower part of Tables 5-8. The *p*-values of *t*-statistics for the days-of-the week dummies indicate that the days of the week effects are statistically insignificant for the returns. For the volatility and the tick dynamics, there are significant days of the week effects and this effect is highly significant for the tick dynamics.

The same analysis is also presented for the DEM series by position in Tables 9-12 for the 3 minutes data. Similar to the USD Eurofutures, the returns, volatility and the tick series are highly skewed and have excess kurtosis. The return series have negative skewness for all positions and this was also the case for the USD Eurofutures. There is also a gradual decline in the excess kurtosis as it is moved from Position 1 to 4. Although we do not report the analysis with the GBP and JPY Eurofutures here, similar findings also prevail for these series as well. Overall, all Eurofutures studied here are highly skewed where the skewness is negative on the return series and positive for the volatility and tick activity. All positions have large excess kurtosis and the size of the kurtosis decline from Position 1 to Position 4.

For the DEM Eurofutures in Tables 9-12, the returns series have statistically significant negative serial correlations. For Positions 1 and 2, the length of the persistence prevails up to the 9th lag which accounts for 27 minutes of persistence. For Position 3 and 4, the serial persistence lasts up to the 6th lags. For Position 1, the mean movement is .156 basis point and it is .468 basis point for 27 minutes. For Position 2, the mean movement for the basis points is .212 and it is .636 basis points in a 27 minutes window.

The volatility and the tick activity of the DEM position series are highly serially correlated and the serial correlation is more prominent in the tick activity. The days of the week effect indicates that the returns do not have significant days of the week seasonalities. The volatility and the tick activity however exhibit statistically significant days of the week effects. This is similar to the USD Eurofutures analysis above. Also, similar results hold for the GBP and the JPY Eurofutures although they are not reported here. The qualitative results here support the histogram analysis for the days of the weeks effects earlier.

We further analysed the USD and the DEM Eurofutures with the 30 minutes data. The skewness and the excess kurtosis properties are similar to that of the 3 minutes data although the size of the kurtosis is relatively smaller. The kurtosis for the returns for USD (Tables 13-16) are 756.53, 271.52, 269.63 and 89.98 for Positions 1 to 4. The serial correlations indicate that the 1st order serial correlations are negative and statistically significant. For Position 2,

the serial correlations for the 1st, 3rd and 4th lags are statistically significant. For Position 2, the first two autocorrelations are statistically significant. For Position 4, the first four lags have statistically significant serial correlations. In the case of Position 4, the average movement is 1.181 basis points in 30 minutes. This is approximately 2.362 basis points move in a two hour interval and indicates the existence of net profit opportunities. Similar findings also prevail for the DEM series with the 30 minutes data. For all four positions, Tables 17-20 indicate that the first two autocorrelations for all four positions are negative and statistically significant. The mean movement in the basis points increases from Position 1 to Position 4. This was also the case for the USD series. In Table 20, the mean movement for DEM in Position 4 is .959 in basis points. For 1 hour horizon, this is approximately 1.36 basis points. The GBP and the JPY series with the 30 minutes data also have similar characteristics but are not reported here.

5.2. By contract

The time series properties of the USD and DEM Eurofutures for the March and September 1995 contracts are studied here with the 3 and 30 minutes data. The return series for contract data exhibit rather small skewness but the excess kurtosis remains to be large. The volatility and the tick activity are highly skewed and have excess kurtosis for 3 minutes as well as the 30 minutes series. The serial correlation analysis indicates that USD futures have statistically significant autocorrelations at the first, fourth, fifth, seventh to tenth lags with the 3 minutes data in Table 21. The LBP test statistic on the returns is 1210 where the 5 percent level for the χ^2 is 18.307. For the 30 minutes series, Table 22 indicates that the first two and fourth to nineth lags of the autocorrelations are statistically significant. The mean volatility for 30 minutes series is 0.0001219 which indicates that the 30 minutes average movement for these series is 1.2 basis points. For a two hour move this is approximately a 2.258 basis point move and it may be profitable after the trading costs.

The DEM Eurofutures by contract are analysed in Tables 23 and 24 for the 3 and 30 minutes series. For the 3 minutes data, the first six autocorrelations are statistically significant for the return series. Given that the size of the average move is .23 basis points, this persistence may not be profitable at the 3 minutes frequency. The volatility and the tick activity display strong persistence and this persistence is more prominent in the tick activity. For the 30 minutes series, the first two autocorrelations are statistically significant.

Given that the mean volatility is .87 basis points, this is approximately 1.23 basis points movements in an hour. This may be at the margin for net profitability. We further analysed other contracts inluding the 1996 and 1997 contracts on USD, DEM, GBP and JPY Eurofutures. The results are similar such that the persistence properties of returns can be used as a timing indicator and may lead to pockets of predictability after taking the trading costs into account.

6. Conclusions

This paper presents a statistical study of fundamental properties characterizing Eurofutures markets. Intraday price changes and tick activity present a high positive correlation and display a U-shape confirming the existence of an intraday seasonality. The activity and volatility peak at the opening and closing times. There is evidence of intraweek seasonalities such that the level of activity displays a minimum on Monday and a maximum on the last two working days of the week (usually on Thursday for LIFFE contracts and on Friday for CME contracts). There is practically no activity during weekends.

A more detailed analysis of bid-ask quotes and transaction prices has displayed two additional features. The first is that Eurodollar, Euroswiss and Three Month ECU intraday tick activity is higher for bid-ask quotes than for transaction prices; except over weekends bid-ask tick activity prevails for all Eurofutures. The second is that the difference between hourly volatility of the transaction prices and hourly volatility of the bid-ask quotes is very small (below one basis point). For Eurodollar, the difference is much less than a basis point when the market is fully active. This difference increases when the market is not active. For futures traded on LIFFE the difference is more important during active hours and it reaches a maximum during the first working hours of the day.

Our analysis on an average of contracts has shown the existence of a scaling law which relates the volatility to the corresponding time intervals. The scaling law displays a drift exponent significantly larger than that expected for a Gaussian random walk and very close to the values obtained for foreign exchange rates. For data based on per contract we have shown that price volatility displays a dependence on the time left to expiry. On average, volatility tends to decrease as we move towards expiry and peaks approximately every 90 days near quarterly expiries.

This study shows that on one hand, the Eurofutures markets present remarkable similarities with the other markets studied so far with high frequency data (Guillaume et al. (1997); Andersen and Bollerlev (1997); Ghysels and Jasiak (1995), in terms of intraday/intra-week seasonalities and scaling law for absolute price changes. On the other hand, we have found new properties that depend on the fact that futures contracts have predefined expiry dates. A striking example of such a property is the deterministic volatility which varies with contract expiry.

The results of this paper also indicate that the return series exhibit serial correlation which provides evidence for predictability and timing ability. This predictability does not translate itself into net profitability in the short horizons such as three minutes data. However, the net profitability opportunities exist at the lower frequencies such as the thirty minutes horizon after taking trading costs into account.

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 ${\bf Table~1} \\$ Average number of ticks per business day for each Eurofutures and each position

	Eurodollar	Euromark	Sterling	Euroswiss	ECU
Position 1	104	161	151	84	41
Position 2	180	227	214	81	30
Position 3	151	215	158	29	14
Position 4	117	175	110	12	9

Notes: The intraday tick activity per position characterizing CME and LIFFE futures prices. Eurodollar, Euromark and Short Sterling display a high liquidity in all positions. The number of ticks reaches a maximum in the second position and then decreases. The fourth position has at most as many ticks as the first.

Table 2
Daily and weekly volatility in basis points for different Eurofutures positions

	Eur	odollar	Eur	oswiss	Eur	omark	Ste	rling	Е	CU
	day	week	day	week	day	week	day	week	day	week
Position 1	5.8	16.3	4.6	12.1	3.3	8.2	6.5	17.8	5.2	14.1
Position 2	6.7	18.9	5.4	13.7	4.6	11.2	7.1	19.7	5.2	13.5
Position 3	9.5	24.2	5.4	14.5	5.7	14.0	7.4	21.2	5.2	14.5
Position 4	7.1	21.0	5.2	14.9	6.1	16.2	7.6	23.0	5.4	15.8

Notes: Values of daily and weekly volatility for each Eurofutures and each position. Results on daily volatility confirm the results from the intraweek volatility analysis. For Eurodollar, Euromark and Short Sterling the level of variation tends to increase from position one to position four. Euroswiss shows an increment from position one to position two and then starts to decrease. Three Month ECU displays a slight tendency to increase. The basis points are 10^{-4} .

 Table 3

 Drift exponents for all Eurofutures

Expiry	Eurodollar	Euromark	Sterling	
March 1995	$.60 \pm .02$	$.60 \pm .01$	$.61 \pm .02$	
$\mathrm{June}\ 1995$	$.66\pm.02$	$.65\pm.01$	$.62\pm.02$	
${\bf September} \ 1995$	$.68\pm.02$	$.66\pm.01$	$.62\pm.02$	
December 1995	$.64\pm.02$	$.66\pm.01$	$.64\pm.02$	
March 1996	$.57\pm.03$	$.66\pm.01$	$.63\pm.02$	
$\mathrm{June}\ 1996$	$.70\pm.01$	$.62\pm.01$	$.62\pm.02$	
${\bf September} 1996$	$.70\pm.01$	$.65\pm.01$	$.62\pm.01$	
December 1996	$.69\pm.01$	$.63\pm.02$	$.60\pm.02$	
March 1997	$.66 \pm .02$	$.62 \pm .02$	$.63 \pm .02$	

Notes: An empirical scaling law examines the relationship between intervals Δt and the average absolute price changes, $\overline{|\Delta x|} = c \Delta t^{\frac{1}{E}}$ where the bar over the $|\Delta x|$ indicates the average over the whole sample period, c is a constant and 1/E is defined to be the drift exponent. The drift exponent is 0.5 for the Gaussian process. The drift exponents are all significantly above the value 0.5 which indicates deviation from a Gaussian random walk.

Table 4
Drift exponents for scaling law on Eurodollar, Euromark and Short Sterling

	All-Eurofutures	Eurodollar	Euromark	Sterling
drift exponent	.599	.634	.664	.599
standard error	.007	.008	.023	.013

Notes: This table reports the drift exponents for all contracts and single contracts for Eurodollar, Euromark and Short Sterling. For single Eurofutures, the average was computed on 9 contracts. For All-Eurofutures, the average was computed on 36 contracts.

	T		
Description	Return	Volatility	Tick
Sample size	124838	124838	124838
Mean	0000002	.0000247	.63
Std.	.00007	.00007	1.30
${\bf Skewness}$	-28.28	43.66	3.99
Kurtosis	3455.90	4333.64	32.69
Max	.0047	.0087	40
Min	0087	.0000	0
ρ_1	1416	.1212	.4522
$ ho_2$	0245	.0792	.3591
$ ho_3$	0109	.0682	.3199
$ ho_4$	0126	.0712	.2778
$ ho_5$.0027	.0563	.2400
$ ho_6$	0028	.0571	.2157
$ ho_7$	0027	.0464	.1968
$ ho_8$	0034	.0405	.1757
$ ho_9$	0008	.0406	.1615
$ ho_{10}$	0038	.0376	.1457
Bartlett std. errors	.0028	.0028	.0028
LBP	2610	4230	71200
$\chi^2_{0.05}(10)$	18.307		
$\beta_{Constant}$.059	.000	.000
$eta_{Tuesday}$.053	.023	.000
$eta_{Wednesday}$.041	.036	.000
$eta_{Thursday}$.057	.028	.000
eta_{Friday}	.125	.000	.000
$F\ (zero\ slopes)$.056	.000	.000
R^2	.0004	.008	.064

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	-	7		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Sample size	147257	147257	147257
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Mean	0000002	.0000319	1.03
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Std.	.00008	.00007	1.85
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Skewness	-13.72	21.30	3.29
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Kurtosis	1147.54	1623.98	18.10
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Max	.0022	.0075	34
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Min	0075	.0000	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\overline{\rho_1}$	1635	.1910	.5264
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_2$	0217	.1348	.4338
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_3$	0135	.1189	.3893
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_4$	0013	.1239	.3471
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_5$.0007	.1009	.3044
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_6$	0003	.0965	.2768
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_7$.0001	.0929	.2578
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_8$	0077	.0819	.2339
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_9$.0010	.0756	.2147
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ ho_{10}$.0035	.0721	.2007
$\begin{array}{c cccc} \chi^2_{0.05}(10) & 18.307 \\ \hline \beta_{Constant} & .009 & .000 & .000 \\ \beta_{Tuesday} & .013 & .000 & .000 \\ \beta_{Wednesday} & .014 & .000 & .000 \\ \beta_{Thursday} & .019 & .000 & .000 \\ \beta_{Friday} & .056 & .000 & .000 \\ F\left(zero\ slopes\right) & .037 & .000 & .000 \\ \hline \end{array}$	Bartlett std. errors	.0026	.0026	.0026
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	LBP	4040	19100	164000
$\begin{array}{c ccccc} \beta_{Tuesday} & .013 & .000 & .000 \\ \beta_{Wednesday} & .014 & .000 & .000 \\ \beta_{Thursday} & .019 & .000 & .000 \\ \beta_{Friday} & .056 & .000 & .000 \\ F\left(zero\ slopes\right) & .037 & .000 & .000 \end{array}$	$\chi^2_{0.05}(10)$	18.307		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\beta_{Constant}$.009	.000	.000
$eta_{Thursday}$.019 .000 .000 eta_{Friday} .056 .000 .000 $F(zero\ slopes)$.037 .000 .000	$eta_{Tuesday}$.013	.000	.000
β_{Friday} .056 .000 .000 $F(zero\ slopes)$.037 .000 .000	$eta_{Wednesday}$.014	.000	.000
$F(zero\ slopes)$.037 .000 .000	$eta_{Thursday}$.019	.000	.000
_ ` _ /		.056	.000	.000
R^2 .0007 .003 .012	- /	.037	.000	.000
	R^2	.0007	.003	.012

Table 7 Summary Statistics for the 3 minutes USD Eurofutures (Position 3) January 1, 1994 - April 15, 1997

Description	Return	Volatility	Tick
Sample size	146052	146052.	146052
Mean	0000002	.0000328	.87
Std.	.00011	.00011	1.66
Skewness	-2.66	18.18	4.13
Kurtosis	1985.30	2349.12	36.64
Max	.0220	.0042	50
Min	0222	.0000	0
$\overline{\rho_1}$	0608	.0956	.5171
$ ho_2$	0116	.0728	.4313
$ ho_3$	0022	.0631	.3866
$ ho_4$	0022	.0670	.3533
$ ho_5$.0002	.0534	.3036
$ ho_6$.0009	.0478	.2793
$ ho_7$	0008	.0441	.2584
$ ho_8$	0022	.0418	.2361
$ ho_9$	0006	.0375	.2253
$ ho_{10}$.0024	.0361	.2061
Bartlett std. errors	.0026	.0026	.0026
LBP	562	5030	163000
$\chi^2_{0.05}(10)$	18.307		
$\beta_{Constant}$.579	.000	.000
$eta_{Tuesday}$.493	.000	.000
$eta_{Wednesday}$.482	.000	.000
$eta_{Thursday}$.619	.000	.000
eta_{Friday}	.804	.000	.000
$F\ (zero\ slopes)$.764	.000	.000
R^2	.0001	.003	.015

Description	Return	Volatility	Tick
Sample size	126605	126605	126605
Mean	0000002	.0000329	.78
Std.	.00008	.00008	1.50
Skewness	-7.61	13.93	3.66
Kurtosis	476.13	648.90	25.55
Max	.0022	.0056	39
Min	0056	.0000	0
$\overline{\rho_1}$	0910	.2115	.4924
$ ho_2$	0127	.1632	.4059
$ ho_3$	0031	.1476	.3665
$ ho_4$	0028	.1546	.3338
$ ho_5$.0076	.1259	.2863
$ ho_6$	0044	.1117	.2662
$ ho_7$.0004	.0958	.2423
$ ho_8$.0019	.0922	.2213
$ ho_9$.0058	.0900	.2087
$ ho_{10}$.0036	.0858	.1941
Bartlett std. errors	.0028	.0028	.0028
LBP	1094	22600	126000
$\chi^2_{0.05}(10)$	18.307		
$\beta_{Constant}$.008	.000	.000
$eta_{Tuesday}$.016	.000	.000
$eta_{Wednesday}$.035	.000	.000
$eta_{Thursday}$.022	.000	.000
eta_{Friday}	.334	.000	.000
$F\ (zero\ slopes)$.146	.000	.000
R^2	.0005	.007	.018

Description	Return	Volatility	Tick
Sample size	165684	165684	165684
Mean	.0000001	.0000156	.83
Std.	.00005	.00004	2.06
${\bf Skewness}$	6.92	14.48	10.75
Kurtosis	800.66	996.82	244.03
Max	.0046	.0046	96
Min	0023	.0000	0
$\overline{\rho_1}$	1343	.1132	.2843
$ ho_2$	0604	.0844	.1787
$ ho_3$	0337	.0675	.1437
$ ho_4$	0181	.0577	.1120
$ ho_5$	0171	.0542	.1144
$ ho_6$	0070	.0483	.1010
$ ho_7$	0098	.0521	.0912
$ ho_8$	0073	.0462	.0861
$ ho_9$	0035	.0383	.0731
$ ho_{10}$	0019	.0410	.0685
Bartlett std. errors	.0025	.0025	.0025
LBP	3920	6810	32300
$\chi^2_{0.05}(10)$	18.307		
$\beta_{Constant}$.874	.000	.000
$eta_{Tuesday}$.988	.003	.000
$eta_{Wednesday}$.615	.000	.000
$eta_{Thursday}$.813	.000	.000
eta_{Friday}	.408	.000	.000
$F\ (zero\ slopes)$.766	.000	.000
R^2	.0001	.001	.003

Description	Return	Volatility	Tick
Sample size	170716.	170716.	170716.
Mean	.0000001	.0000212	1.14
Std.	.00006	.00005	2.25
${\bf Skewness}$	-6.26	13.27	7.48
Kurtosis	581.75	772.47	124.35
Max	.0028	.0046	84
Min	0046	.0000	0
$\overline{\rho_1}$	1348	.1283	.3122
$ ho_2$	0479	.0984	.2100
$ ho_3$	0298	.0876	.1736
$ ho_4$	0196	.0731	.1454
$ ho_5$	0079	.0686	.1408
$ ho_6$	0096	.0692	.1265
$ ho_7$	0076	.0663	.1160
$ ho_8$	0046	.0611	.1040
$ ho_9$	0029	.0576	.0935
$ ho_{10}$	0008	.0514	.0925
Bartlett std. errors	.0024	.0024	.0024
LBP	3750	10700	746100
$\chi^2_{0.05}(10)$	18.307		
$\beta_{Constant}$.067	.000	.000
$eta_{Tuesday}$.035	.023	.000
$eta_{Wednesday}$.027	.036	.000
$eta_{Thursday}$.138	.028	.000
eta_{Friday}	.020	.000	.000
$F(zero\ slopes)$.076	.000	.000
R^2	.0004	.002	.004

Description	Return	Volatility	Tick
Sample size	164702	164702	164702
Mean	.0000001	.0000233	1.07
Std.	.00006	.00006	1.86
${\bf Skewness}$	-6.48	14.38	5.30
Kurtosis	594.73	792.28	69.06
Max	.0037	.0046	60
Min	0046	.0000	0
$\overline{\rho_1}$	1054	.1350	.3261
$ ho_2$	0369	.1090	.2351
$ ho_3$	0220	.0950	.2018
$ ho_4$	0174	.0834	.1798
$ ho_5$	0069	.0785	.1638
$ ho_6$	0097	.0769	.1479
$ ho_7$	0013	.0690	.1370
$ ho_8$	0018	.0645	.1284
$ ho_9$.0035	.0603	.1174
$ ho_{10}$	0015	.0583	.1201
Bartlett std. errors	.0025	.0025	.0025
LBP	2210	12200	57100
$\chi^2_{0.05}(10)$	18.307		
$\beta_{Constant}$.022	.000	.000
$eta_{Tuesday}$.009	.000	.000
$eta_{Wednesday}$.017	.000	.000
$eta_{Thursday}$.081	.000	.000
eta_{Friday}	.005	.000	.000
$F(zero\ slopes)$.016	.000	.000
R^2	.0007	.002	.005

Description	Return	Volatility	Tick
Sample size	169605	169605	169605
Mean	.0000001	.0000230	1.07
Std.	.00006	.00006	1.89
${\bf Skewness}$	-6.56	14.39	5.62
Kurtosis	596.63	791.60	75.91
Max	.0037	.0046	60
Min	0046	.0000	0
$\overline{\rho_1}$	1054	.1345	.3234
$ ho_2$	0366	.1086	.2320
$ ho_3$	0226	.0949	.1969
$ ho_4$	0172	.0837	.1749
$ ho_5$	0070	.0783	.1619
$ ho_6$	0095	.0768	.1438
$ ho_7$	0014	.0688	.1317
$ ho_8$	0017	.0643	.1267
$ ho_9$.0033	.0606	.1179
$ ho_{10}$	0015	.0580	.1159
Bartlett std. errors	.0024	.0024	.0024
LBP	2280	12500	56900
$\chi^2_{0.05}(10)$	18.307		
$\beta_{Constant}$.016	.000	.000
$eta_{Tuesday}$.006	.023	.000
$eta_{Wednesday}$.013	.036	.000
$eta_{Thursday}$.058	.028	.000
eta_{Friday}	.003	.000	.000
$F\ (zero\ slopes)$.010	.000	.000
R^2	.0008	.002	.004

Description	Return	Volatility	Tick
Sample size	12481	12481	12481
Mean	0000017	.0000718	.62
Std.	.00019	.00018	1.31
Skewness	-15.74	25.43	3.80
Kurtosis	756.53	987.53	23.86
Max	.0048	.0087	19
Min	0087	.0000	0
$\overline{\rho_1}$	0504	.0337	.1327
$ ho_2$.0064	.0144	.0768
$ ho_3$	0069	.0210	.1241
$ ho_4$.0099	.0301	.0830
$ ho_5$	0103	.0190	.0552
$ ho_6$	0027	.0087	.0456
$ ho_7$	0084	.0136	.0266
$ ho_8$	0005	.0081	.0193
$ ho_9$	0130	.0052	.0432
$ ho_{10}$.0005	.0075	.0293
Bartlett std. errors	.0090	.0090	.0090
LBP	38.5	43.2	683
$\chi^2_{0.05}(10)$	18.307		

	1		
Description	Return	Volatility	Tick
Sample size	14723	14723	14723
Mean	0000018	.0000932	1.01
Std.	.00020	.00017	1.81
${\bf Skewness}$	-8.05	13.99	3.10
Kurtosis	271.52	424.58	15.00
Max	.0028	.0074	25
Min	0074	.0000	0
$\overline{\rho_1}$	0359	.0949	.1987
$ ho_2$.0052	.0474	.1066
$ ho_3$.0081	.0592	.1181
$ ho_4$.0146	.0500	.0776
$ ho_5$.0010	.0487	.0556
$ ho_6$.0002	.0207	.0608
$ ho_7$.0032	.0280	.0457
$ ho_8$	0149	.0270	.0200
$ ho_9$.0033	.0297	.0385
$ ho_{10}$.0095	.0222	.0372
Bartlett std. errors	.0082	.0082	.0082
LBP	28.4	338	1220
$\chi^2_{0.05}(10)$	18.307		

Description	Return	Volatility	Tick
Sample size	14602	14602	14602
Mean	0000018	.0001113	.88
Std.	.00033	.00031	1.66
Skewness	74	48.81	3.74
Kurtosis	269.63	3353.05	25.48
Max	.0221	.0222	30
Min	0222	.0000	0
$\overline{\rho_1}$	0088	.0399	.2165
$ ho_2$	3010	.3560	.1442
$ ho_3$.0018	.0264	.1392
$ ho_4$.0053	.0257	.1089
$ ho_5$.0009	.0180	.0936
$ ho_6$.0080	.0118	.0673
$ ho_7$	0053	.0113	.0592
$ ho_8$.0048	.0132	.0551
$ ho_9$	0008	.0125	.0551
$ ho_{10}$.0022	.0106	.0546
Bartlett std. errors	.0083	.0083	.0083
LBP	1330	1910	1820
$\chi^2_{0.05}(10)$	18.307		

Table 16 Summary Statistics for the 30 minutes USD Eurofutures (Position 4) January 1, 1994 - April 15, 1997

Description	Return	Volatility	Tick
Sample size	12658	12658	12658
Mean	0000019	.0001181	.78
Std.	.00023	.00020	1.52
Skewness	-3.53	8.43	3.76
Kurtosis	89.98	143.53	25.60
Max	.0031	.0056	28
Min	0056	.0000	0
$\overline{\rho_1}$.0117	.1220	.1747
$ ho_2$.0181	.0598	.0996
$ ho_3$.0096	.0803	.1140
$ ho_4$.0128	.0689	.0807
$ ho_5$.0044	.0551	.0575
$ ho_6$.0195	.0482	.0566
$ ho_7$.0021	.0248	.0565
$ ho_8$.0034	.0261	.0503
$ ho_9$	0048	.0506	.0585
$ ho_{10}$.0122	.0393	.0558
Bartlett std. errors	.0089	.0089	.0089
LBP	16.5	512	997
$\chi^2_{0.05}(10)$	18.307		

	•		
Description	Return	Volatility	Tick
Sample size	16566	16566	16566
Mean	.0000014	.0000571	.81
Std.	.00011	.00009	1.96
Skewness	5.15	13.39	9.77
Kurtosis	259.64	453.24	190.67
Max	.0046	.0046	68
Min	0025	.0000	0
$\overline{\rho_1}$	1483	.0627	.0716
$ ho_2$	0308	.0382	.1100
$ ho_3$	0087	.0254	.0342
$ ho_4$.0031	.0290	.0205
$ ho_5$	0054	.0307	.0428
$ ho_6$	0019	.0325	.0208
$ ho_7$	0055	.0248	.0117
$ ho_8$	0097	.0205	.0153
$ ho_9$	0002	.0125	.0108
$ ho_{10}$.0063	.0220	.0176
Bartlett std. errors	.0078	.0078	.0078
LBP	385	175	363
$\chi^2_{0.05}(10)$	18.307		

Description	Return	Volatility	Tick
Sample size	17069	17069	17069
Mean	.0000010	.0000728	1.13
Std.	.00014	.00011	2.28
${\bf Skewness}$	-4.49	11.53	8.66
Kurtosis	179.55	325.18	169.59
Max	.0027	.0047	75
Min	0047	.0000	0
ρ_1	1037	.0802	.1262
$ ho_2$	0123	.0615	.0572
$ ho_3$.0027	.0355	.0354
$ ho_4$.0004	.0422	.0257
$ ho_5$	0004	.0230	.0455
$ ho_6$	0050	.0511	.0326
$ ho_7$.0021	.0348	.0123
$ ho_8$.0150	.0395	.0097
$ ho_9$.0106	.0360	.0087
$ ho_{10}$.0032	.0385	.0205
Bartlett std. errors	.0077	.0077	.0077
LBP	193	375	426
$\chi^2_{0.05}(10)$	18.307		

Description	Return	Volatility	Tick
Sample size	16957	16957	16957
Mean	.0000006	.0000851	1.05
Std.	.00016	.00013	1.86
Skewness	-3.88	10.09	6.10
Kurtosis	132.05	236.12	98.29
Max	.0035	.0047	57
Min	0047	.0000	0
ρ_1	0619	.0774	.1440
$ ho_2$	0164	.0591	.0586
$ ho_3$	0018	.0441	.0542
$ ho_4$.0113	.0299	.0358
$ ho_5$.0067	.0295	.0473
$ ho_6$	0007	.0503	.0437
$ ho_7$.0039	.0350	.0193
$ ho_8$.0029	.0377	.0183
$ ho_9$.0157	.0460	.0202
$ ho_{10}$.0222	.0503	.0319
Bartlett std. errors	.0077	.0077	.0077
LBP	85.4	390	588
$\chi^2_{0.05}(10)$	18.307		

Description	Return	Volatility	Tick
Sample size	16467	16467	16467
Mean	.0000004	.0000959	.90
Std.	.00018	.00016	1.59
${\bf Skewness}$	-4.23	10.83	6.41
Kurtosis	139.63	239.90	138.20
Max	.0044	.0049	59
Min	0049	.0000	0
$\overline{\rho_1}$	0375	.0784	.1282
$ ho_2$	0249	.0638	.0690
$ ho_3$.0010	.0357	.0489
$ ho_4$.0092	.0379	.0321
$ ho_5$.0066	.0354	.0428
$ ho_6$	0008	.0376	.0317
$ ho_7$.0099	.0404	.0299
$ ho_8$.0177	.0535	.0254
$ ho_9$.0051	.0252	.0272
$ ho_{10}$.0151	.0331	.0369
Bartlett std. errors	.0078	.0078	.0078
LBP	46.5	359	512
$\chi^2_{0.05}(10)$	18.307		

Table 21 Summary Statistics for the 3 minutes USD Eurofutures (March 1995 Contract)

Description	Return	Volatility	Tick
Sample size	37195	37195	37195
Mean	0000002	.0000391	1.02
Std.	.00008	.00007	1.72
${\bf Skewness}$	29	4.04	3.20
Kurtosis	30.48	43.80	18.32
Max	.0013	.0018	34
Min	0018	.0000	0
$\overline{\rho_1}$	1779	.2465	.5017
$ ho_2$	0048	.1673	.4196
$ ho_3$.0049	.1507	.3700
$ ho_4$	0210	.1518	.3306
$ ho_5$.0172	.1189	.2913
$ ho_6$	0051	.1031	.2681
$ ho_7$	0121	.0920	.2449
$ ho_8$	0158	.0861	.2228
$ ho_9$.0054	.0876	.2054
$ ho_{10}$.0168	.0721	.1825
Bartlett std. errors	.0052	.0052	.0052
LBP	1210	7000	37800
$\chi^2_{0.05}(10)$	18.307		

Table 22 Summary Statistics for the 30 minutes USD Eurofutures (March 1995 Contract)

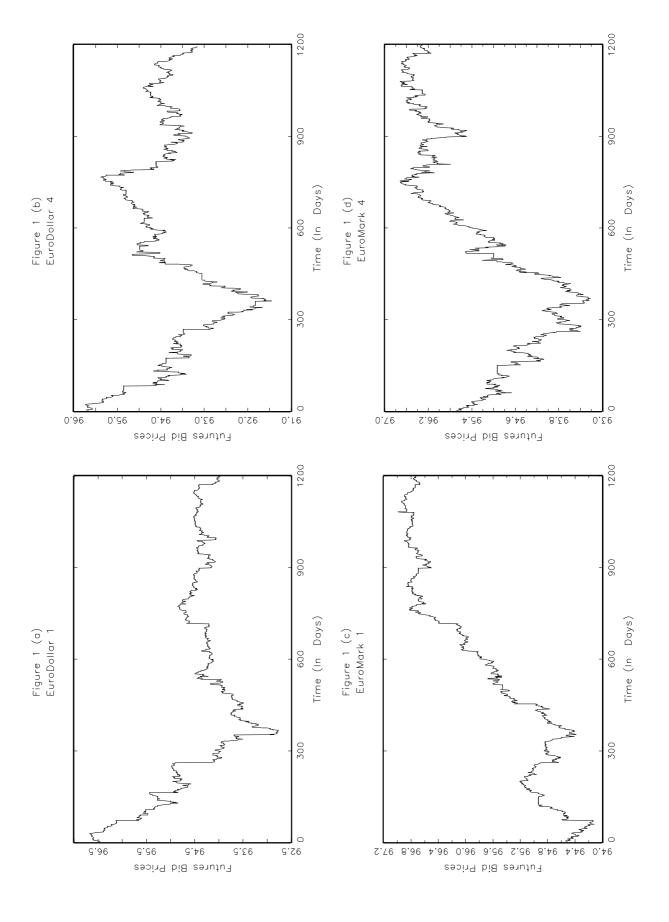
Description	Return	Volatility	Tick
Sample size	3717	3717	3717
Mean	0000014	.0001219	1.00
Std.	.00020	.00016	1.72
Skewness	42	3.26	3.15
Kurtosis	11.37	19.68	16.47
Max	.0017	.0020	22
Min	0020	.0000	0
$\overline{\rho_1}$	0683	.1185	.1841
$ ho_2$.0378	.0646	.0939
$ ho_3$.0094	.1098	.1595
$ ho_4$.0245	.1003	.0820
$ ho_5$	0130	.0879	.0704
$ ho_6$.0167	.0729	.0421
$ ho_7$.0162	.0592	.0448
$ ho_8$.0130	.0537	.0314
$ ho_9$	0116	.0831	.0320
$ ho_{10}$	0070	.0652	.0295
Bartlett std. errors	.0164	.0164	.0164
LBP	29.2	264	322
$\chi^2_{0.05}(10)$	18.307		

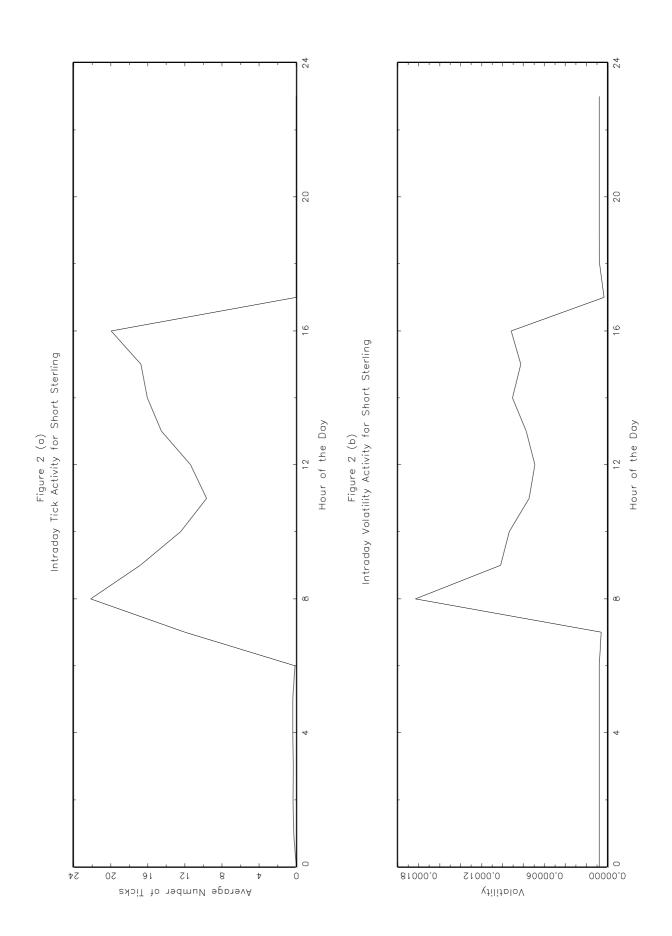
Table 23
Summary Statistics for the 3 minutes DEM Eurofutures (September 1995 Contract)

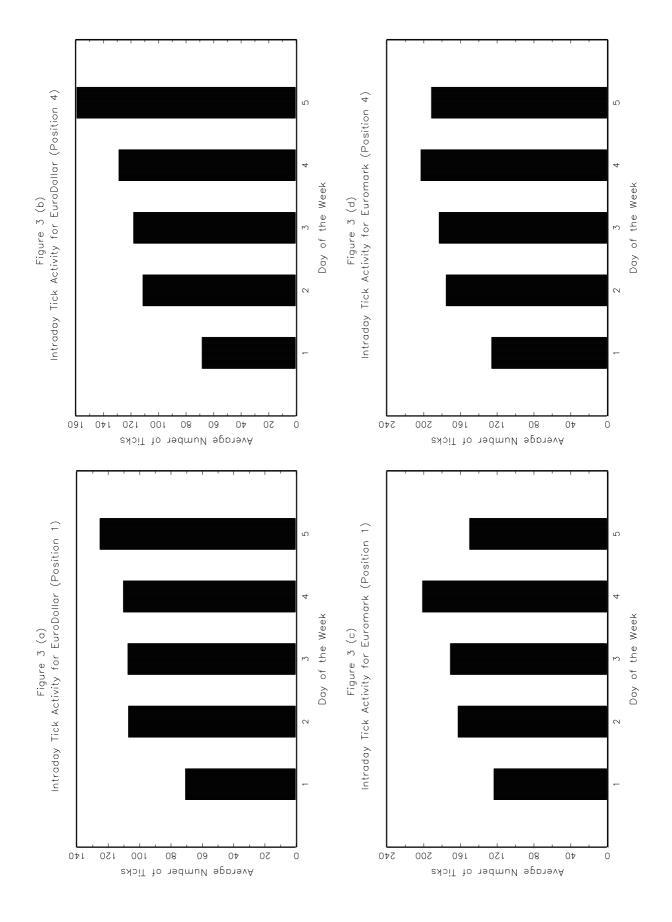
Description	Return	Volatility	Tick
Sample size	50747	50747	50747
Mean	.0000005	.0000233	.92
Std.	.00006	.00005	1.58
${\bf Skewness}$	2.31	5.82	5.33
Kurtosis	128.49	173.00	104.43
Max	.0028	.0028	68
Min	0009	.0000	0
$\overline{\rho_1}$	0962	.1463	.3182
$ ho_2$	0597	.1269	.2320
$ ho_3$	0261	.1146	.2063
$ ho_4$	0188	.0856	.1710
$ ho_5$	0100	.0821	.1487
$ ho_6$	0172	.0724	.1339
$ ho_7$	0028	.0681	.1318
$ ho_8$	0044	.0663	.1140
$ ho_9$.0055	.0669	.1123
$ ho_{10}$	0085	.0575	.1113
Bartlett std. errors	.0044	.0044	.0044
LBP	707	4400	16400
$\chi^2_{0.05}(10)$	18.307		

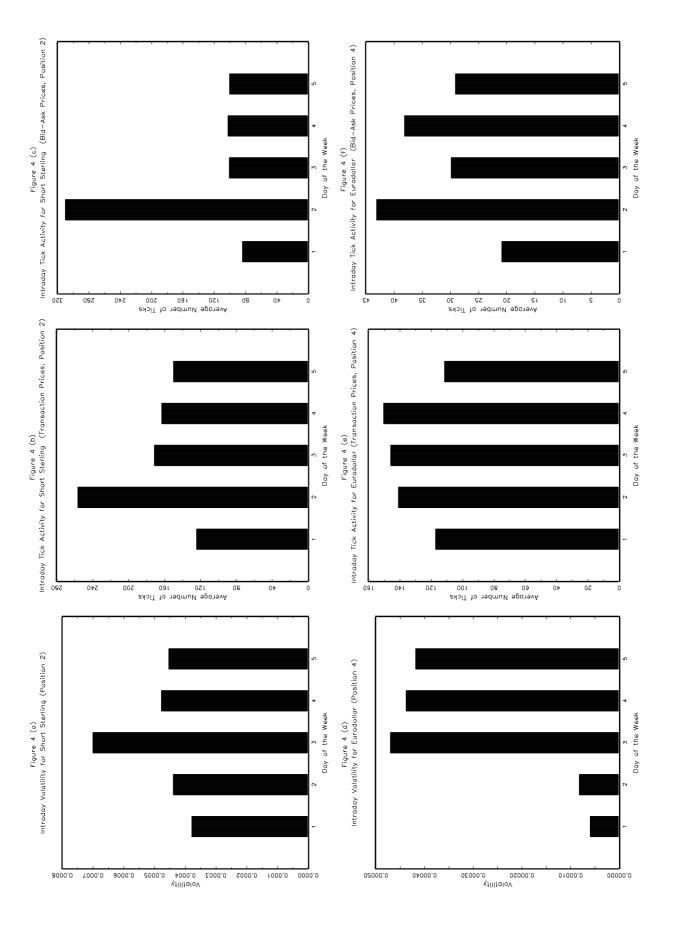
Table 24
Summary Statistics for the 30 minutes DEM Eurofutures (September 1995 Contract)

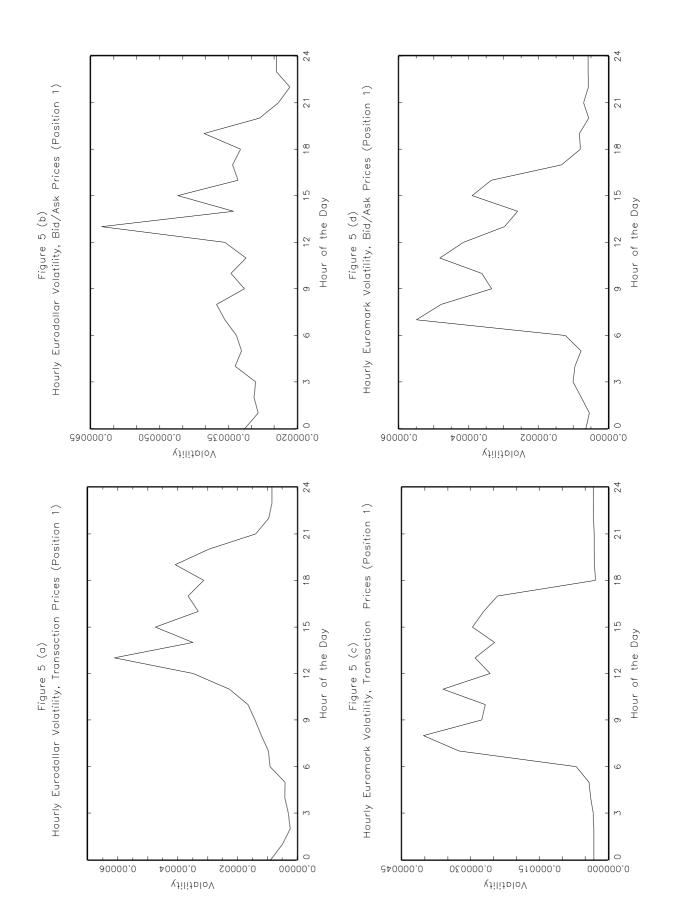
Description	Return	Volatility	Tick
Sample size	5072	5072	5072
Mean	.0000050	.0000872	.91
Std.	.00015	.00012	1.52
Skewness	1.20	4.61	3.28
Kurtosis	30.06	58.98	19.33
Max	.0026	.0026	21
Min	0014	.0000	0
ρ_1	0985	.0929	.1242
$ ho_2$	0275	.0683	.0863
$ ho_3$	0035	.0221	.0464
$ ho_4$	0127	.0294	.0181
$ ho_5$.0205	.0242	.0472
$ ho_6$.0085	.0492	.0235
$ ho_7$	0360	.0349	.0585
$ ho_8$.0303	.0421	.0392
$ ho_9$.0055	.0318	.0587
$ ho_{10}$	0001	.0348	.0518
Bartlett std. errors	.0140	.0140	.0140
LBP	67.9	116	199
$\chi^2_{0.05}(10)$	18.307		

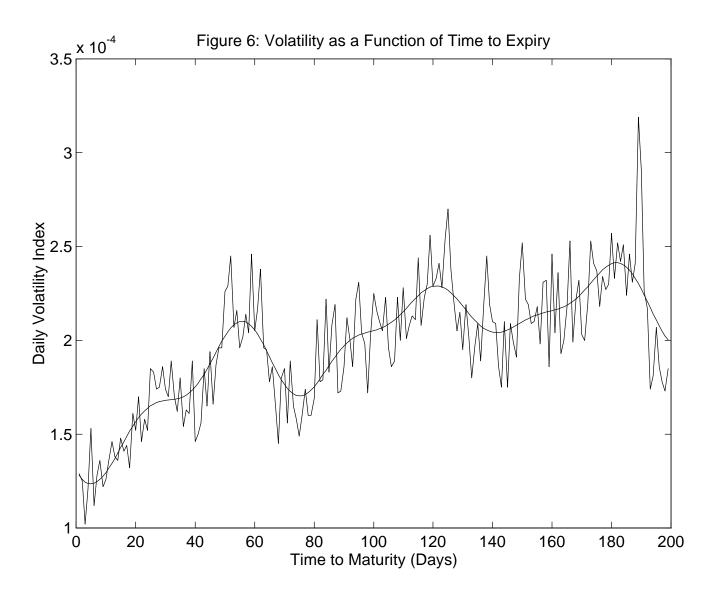












Data Lower Confidence Interval OLS Fit Upper Confidence Interval Random Walk log(Time Interval in Minutes) ∞ Z.8-G.√-G.4− G.8log(Mean Volatility)

Figure 7 Scaling Law on All Contracts