Extracting market expectations from option prices: an application to over-the-counter New Zealand dollar options

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April 2002

Discussion Paper Series

DP2002/04

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Abstract[1]

What are the odds of a large shift in the exchange rate? Is a large depreciation more likely than a large appreciation? This paper uses over-the-counter New Zealand dollar/US dollar option prices to quantify market expectations of exchange rate uncertainty through measures based on risk-neutral probability distribution functions. Results suggest that the estimated probability distributions can provide important insights into market perceptions about exchange rate risk in the future. Econometric evidence indicates that the higher moments calculated from risk-neutral probability density functions can be used to explain the dynamic behaviour of the forward bias measured in the New Zealand dollar/US dollar exchange rate.

1 Introduction

Derivative markets provide a rich source of information for both market participants and central banks. Due to their forward-looking nature, forward, futures, and option prices encapsulate market expectations about the future price development of the underlying assets. Forwards and futures can be useful for tracing the mean of market expectations. However, in some cases, it is also useful to know both the level and the nature of the uncertainty that the market assigns to the future evolution of different asset prices. Option prices are particularly useful in the latter respect, as they can be used to estimate the higher moments of market expectations, and these higher moments can, in turn, be used to characterise the uncertainty that the market assigns to the price of a particular underlying asset.

In this paper over-the-counter New Zealand dollar/US dollar option prices are used to quantify market expectations of exchange rate uncertainty through risk-neutral probability distribution functions. Results suggest that despite the relative small size of New Zealand dollar currency option market, the estimated probability distributions can provide important insights into market perceptions about exchange rate risk in the future. The relationship between the options-based measures of exchange rate uncertainty and the forward bias has also been examined. We found some econometric support for the hypothesis that changes in the market's perception of risk, measured by the higher moments of the implied PDFs, are reflected in the observed foreign exchange risk premium.

The paper is organised as follows. Section 2 provides an overview of the theoretical underpinnings of extracting expectations from option prices and reviews the recent empirical literature. Section 3 discusses some particular features of the over-the-counter currency option market and shows how implied probability density functions (PDFs) can be derived from the standard option quotes. Section 4 presents the results of estimating implied PDFs for the New Zealand dollar/US dollar exchange rate and discusses how several measures calculated from these distributions behaved over the sample period; special emphasis is given the shock caused by the attack against the World Trade Center on 11 September 2001. Section 5 presents

econometric results that were carried out to test whether option-implied PDFs might shed some light on the forward bias-risk premium puzzle in the case of New Zealand. Section 6 concludes the paper.

2   How to extract expectations from option prices?

The most widely used formula for pricing European-style option contracts is by Black and Scholes (1973).[2,3] The formula assumes that the price of the underlying asset follows a geometric Brownian motion, implying that the logarithmic returns on the asset price are normally distributed with a constant variance. The Black-Scholes formula gives the price of an option as a function of 5 factors:

- the current spot price of the underlying asset (S_t);
- the option's exercise price (X);
- the risk-free rate of interest (r);
- the option's maturity (T-t); and
- the (annualised) volatility of the underlying asset's logarithmic returns (σ).

Of the above parameters, the last one is unobservable in practice. However, it is possible to calculate it from the market prices of traded options by "reversing" the Black-Scholes formula. This estimate is often referred to as implied volatility.

A consequence of the assumptions that lie behind the Black-Scholes pricing function is that implied volatility should be the same for all options on the same underlying asset. However, market data generally does not support this assumption. Firstly, market participants seem to price options with exercise prices close to the current spot price using a smaller volatility than options with more

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[2]   In this paper the analysis is restricted to European-style option contracts. Similar, although more computationally intensive methods have been developed for American-style options (see, for example, Melick and Thomas (1997)).

[3]   The original formula was developed for options on stocks that do not pay dividends. Garman and Kohlhagen (1983) developed the modification of the formula for options on currencies.

distant strikes. As a result, observed implied volatility is usually a convex function of exercise prices. This phenomenon is often referred to as the 'volatility smile'. Secondly, options with the same strike price but with different maturities show differences in implied volatility, implying that the 'term structure' of implied volatility is not horizontal, as it would be under the Black-Scholes assumptions.

The smile-shaped function of implied volatility indicates that the market assigns a probability distribution of the potential (log) returns on the underlying asset at expiry that is different from the normal distribution. The convex shape of the smile reveals that the market generally attaches greater probability mass to large price changes. In other words, the market weights the tails of the distribution more heavily. As a result, the implied distributions have fatter tails than the normal. Moreover, the volatility smile is generally not symmetric, and this asymmetry implies that the underlying probability distribution, unlike the normal distribution, is skewed.

Several methods have been developed to recover these probability density functions (PDFs) of market expectations from the observed option prices. In this paper we will apply one of these methods, developed originally by Malz (1997), to estimate the implied PDFs of the New Zealand dollar/US dollar exchange rate.

The theoretical underpinnings for deriving implied PDFs from option prices begin with Cox and Ross (1976), who show that the price of a European call option is equal to the discounted value of its expected (risk-neutral) payoffs:

$$c(t,T,X) = e^{-r(T-t)}E\left[\max\left(0, S_T - X\right)\right] =$$
$$= e^{-r(T-t)}\int_X^\infty f(S_T)(S_T - X)dS_T \qquad , \qquad (1)$$

where c is the call price and $f(S_T)$ is the (risk-neutral) PDF of the expectations for the value of the underlying asset. A natural step forward is the result of Breeden and Litzenberger (1978), who show that twice-differentiating the call pricing function C with respect to

the strike price X equals the (discounted value of the) risk-neutral PDF:

$$\frac{\partial^2 c(t,T,X)}{\partial X^2} = e^{-r(T-t)} f(X).$$ (2)

The empirical literature follows two different lines. *Approximating function* methods assume a particular parametric functional form for $f(S_T)$, and then use equation (1) to estimate these parameters while minimising the pricing error (the residual of (1)) across simultaneous observations of c for different strike prices. A popular technique is to use a mixture of two or three lognormal distributions to approximate $f(S_T)$. This technique was initiated by Melick and Thomas (1997) who used this method to analyse expectations about oil prices during the Gulf war.

The second approach adopted in the literature is generally referred to as *smoothed volatility smile* method. It was originally developed by developed by Shimko (1993). Smoothed smile techniques convert option prices into implied volatilities using the Black-Scholes formula, and fit a continuous function – generally a low-order polynomial – on the volatility smile. The fitted volatility function is then converted back to a call price function, and finally equation (2) is used to obtain the implied PDFs by twice-differentiating the call price function. Malz (1997) applies this method to currency options. Campa, Chang and Reider (1997) and Bliss and Panigirtzoglou (2000) use smoothing spline functions instead of polynomials to estimate the volatility smile.

Several central banks use these methods to assess monetary conditions, times of financial stress, the presence of peso problems, central bank credibility, and the effectiveness of monetary policy measures. A thorough overview of the techniques and of the potential use of the options-implied probability distributions is available in Bahra (1997). Examples of applications on Nordic currency options are in Aguilar and Hördahl (1999) and Eitrheim, Frøyland and Røisland (1999).

An important point to note is that the PDFs obtained by the above-mentioned techniques are risk-neutral, meaning that the PDFs we calculate are not the 'true' distributions of the market expectations. Rather, they can be considered as distributions that are consistent with observed market prices under the assumption that market agents are risk neutral. However, as investors are generally considered to be risk-averse – and as a consequence options prices contain information about both expectations and preferences for risk-taking – the option-implied PDFs may deviate from the 'true' probability distribution that market participants attach to different outcomes of the underlying asset's price.

Nevertheless, the empirical literature generally considers the difference between the two distributions as negligible. Firstly, evidence suggests that although the presence of risk aversion may alter the mean (first moment) of the distribution, it alters neither its shape nor its higher moments significantly (Rubinstein (1994)). Secondly, if we assume that preferences for risk taking are constant over time, changes in the risk-neutral distributions account only for changes in expectations, therefore the dynamics of these distributions and their moments can be used to characterise expectational dynamics.

3   Standard quotes on currency options market and their use in estimating implied PDFs

In the following discussion we will review in more detail the method developed by Malz (1997) to estimate implied PDFs from over-the-counter (OTC) currency options prices.

Let us start with some special features and conventions of the OTC currency option market. The first peculiarity is that option prices on the market are quoted in terms of deltas and volatilities, instead of strikes and money prices.[4] At the time of the settlement of a given

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[4]   The delta is the rate of change of the Black-Scholes call pricing function with respect to the spot exchange rate. It gives us the change in the (Black-Scholes) value of a call option induced by a unit change in the underlying asset's spot value. The delta is close to zero for out-of-the-money options and is close to 1 for

deal, the volatility quotes are translated to money prices by the Garman-Kohlhagen option pricing formula, the equivalent of the Black-Scholes formula for currency options. As a result, dealers do not need to change their quotes every time the spot exchange rate moves, just if their view about future volatility changes. It is important to note that using the Black-Scholes/Garman-Kohlhagen formula to translate the volatilities into money prices does not imply that the dealers accept the assumptions behind the Black-Scholes model. They only use the formula as a one-to-one non-linear mapping between the volatility-delta space (where the quotes are made) and the strike price-option price space (in which the final specification of the deal is expressed for the settlement).

The second important convention to notice is that a large part of the trading involves option combinations. The most popular standard combinations are straddles, risk reversals, and strangles.

Figure 1:
Payoff diagrams of standard option combinations
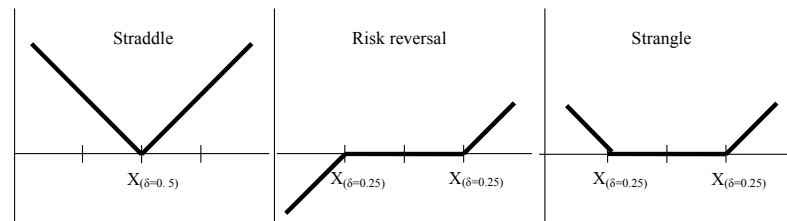
A *straddle* is a combination of a call and a put option with the same strike price. This combination has a V-shaped payoff function: the further the spot price is away from the strike price at the time of expiry, the higher is the payoff. The price of a straddle yields information about the expected variance of the exchange rate: the higher the variance is expected to be, the higher is the profit expected from holding a straddle, and as a result, the higher is its price. The standard quoted OTC straddle contract is the 'at-the-

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deeply in-the-money options, therefore it is often used as a measure of the options' "moneyness".

money' straddle, where the strike price for both options is equal to the current forward price. This also means that the delta for both options that make a standard straddle is approximately equal to 0.5. Like other options, straddles are quoted in terms of volatility, therefore the quoted price of a straddle gives us the implied volatility of an option with an approximate delta of 0.5. The straddle price therefore is often referred to as the at-the-money implied volatility.

A *risk reversal* is a combination of buying an out-of-money call option and selling an out-of-money put option. The price of the risk reversal is equal to the price of the call minus the price of the put. The call option turns profitable at high values of the exchange rate, while the put option turns profitable at low values of the exchange rate. As a consequence, risk reversals bring information about the skewness of market expectations on future price changes. If market expects a large appreciation of the exchange rate, the call option is more likely to become profitable, therefore its price will exceed the price of the put, and the risk reversal will have a positive value. Similarly, if the risk reversal price is negative, then the market expects the put option being more likely to yield profits, signifying a skew towards depreciation. The standard risk reversal contract is the 25-delta, where both options have a delta value of 0.25.

A *strangle*, also consists of an out-of-money put and an out-of-money call, however, the payoff is different as both options have the same 'direction'. When buying a strangle, the investor is betting on the tails of the distribution, as the strangle turns profitable with large exchange rate moves. Strangle prices, therefore, convey information about the kurtosis of the distribution: the higher the probability assigned to large exchange rate movements, the higher the price of a strangle. As for the risk reversals, the standard strangle contract is the 25-delta.

Quotes on these combinations are available on a real-time basis from investment banks. Therefore a data-set of at least three time series: at-the-money implied volatility (straddle quotes), 25-delta risk reversal quotes, and 25-delta strangle quotes is available for several currencies and several maturities on a daily basis.

The Malz method uses these series to estimate the implied probability density function for each trading day. Using the three quote observations, a second-order Taylor approximation of the volatility smile (ie a quadratic function in delta) is estimated. This volatility smile function gives us the implied volatility for each delta. Then the volatility smile function is converted into a call pricing function using the Black-Scholes/Garman-Kohlhagen pricing formula. As a result, we get a continuous approximating function that gives the call option prices for each possible strike price. Then we can use the Breeden-Litzenberger formula, equation (2), to obtain the probability density function by differentiating the call pricing function twice. For a technical presentation of the method, see the appendix.

Once the probability density functions are in hand, it is possible to calculate the higher moments of the distributions in order to obtain measures for the variance, skewness, and kurtosis coefficients.

4 Application to over-the-counter New Zealand dollar options

In this section we present implied probability density functions estimated from options on the New Zealand dollar/US dollar exchange rate using the Malz method, and discuss the interpretation of the information content of these measures. The implied PDFs are derived from a daily dataset that covers the period 3 January 2000 to 11 November 2001. The estimations were made for options with 1, 2, 3, 6, 9 and 12 month maturity. Historical option price data is provided by UBS Warburg, while daily exchange rate and interest rate data are from Datastream.

One potential concern about the data is the liquidity of the options contracts. If the daily quotes on these options are illiquid, the information content of the implied PDFs may become noisy. According to the latest BIS survey on currency market liquidity, the monthly turnover of the NZD/USD option contracts amounts to 340

million USD (BIS (2001)).[5] We assumed that this quantity is sufficient to enable market prices to represent expectations. However, graphical inspection of the time series suggests that at the beginning of the data set some short periods of illiquidity are present.

Throughout this section special attention is given to the period surrounding 11 September 2001, the day of the attack on the World Trade Center, as it provides an opportunity to analyse the behaviour of options-based indicators during times of market stress and to demonstrate the use of the analytical techniques.

Figure 2 displays the path of the spot exchange rate and the calculated higher moments of the implied PDFs.[6] It can be seen that the significant episodes in the exchange rate history of the last two years are relatively well reflected in the higher moments. We highlight two particular episodes. The first is the sharp decline of the exchange rate during August-September 2000. During this particular period, the implied volatility and kurtosis were increasing strongly, while the skewness measure went deeply into the negative zone, indicating bearish expectations on the exchange rate. The second highlighted episode is the aftermath of the terrorist attack against the WTC, which again showed similar, although rather temporary, fluctuations in the higher moments. In the rest of the section we will analyse this period more thoroughly.

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[5]   This figure is likely to underestimate the global turnover of options written on the NZD/USD as the BIS report does not provide data on the New Zealand dollar trades that take place on the London market.

[6]   The moments are calculated from the implied distributions of the expected percentage (logarithmic) returns. The standard deviation is annualised.

Figure 2:
The spot NZD/USD exchange rate and the higher moments of implied PDFs
(*calculated from 3-month NZD/USD options*)

Spot rate
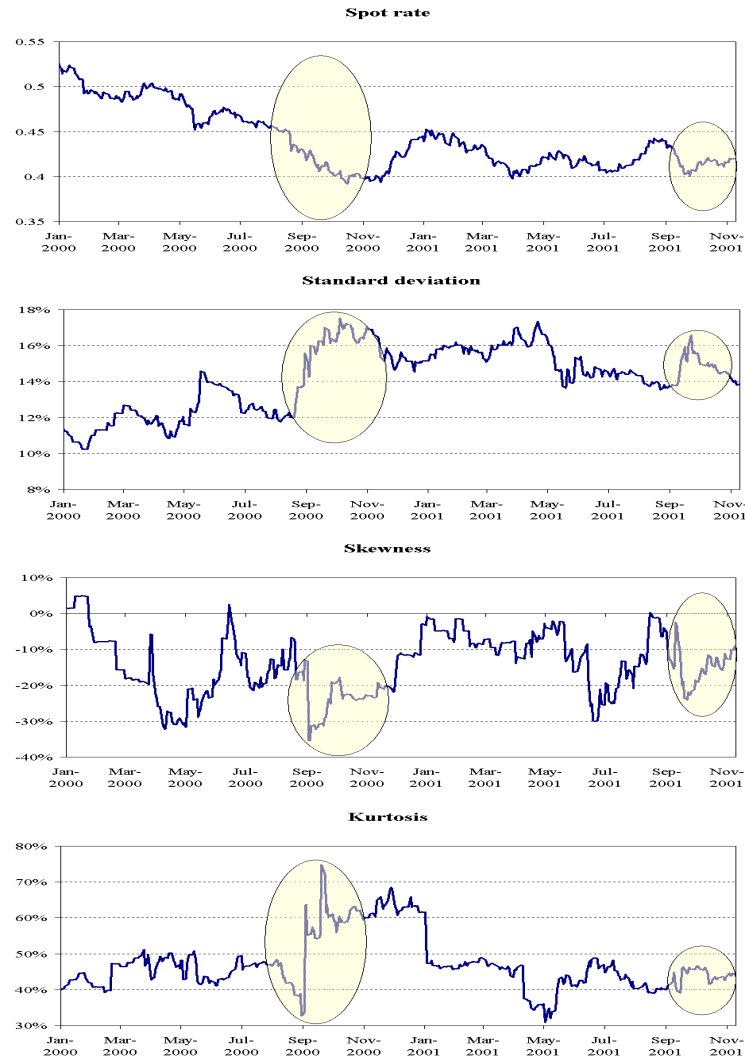
Standard deviation

Skewness

Kurtosis

Figure 3 shows the shape of the options-implied probability density functions of the NZD/USD before and after the WTC attack. The PDFs were calculated from options with a one-month maturity. It can be observed that the market turmoil caused by the attacks resulted in a more dispersed distribution of the one-month-ahead expected returns. Moreover, the increase in the dispersion was more marked at the lower tail, implying an increase in the skewness.

Figure 3:
Options-implied probability density functions for the NZD/USD 1 month ahead
(*expected percentage change relative to the spot rate*)
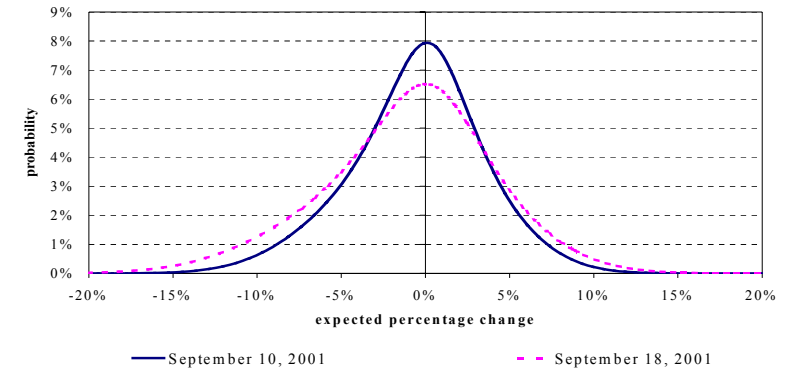
——— September 10, 2001   – – – September 18, 2001

Figure 4 displays the evolution of the standard deviation and the skewness measure of the implied probability densities in the aftermath of the crisis. It can be observed that although initially the standard deviation increased quickly and the skewness turned into highly negative values indicating depreciation expectations, the market views gradually started to ease from their extremes by the last week of September. By the end of October both the standard deviation and the skewness had practically returned to their pre-crisis level.
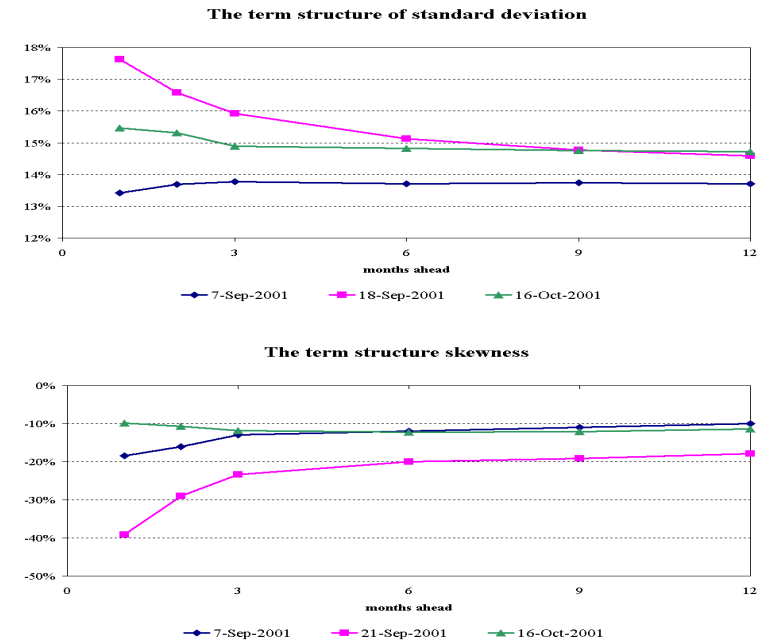
Figure 4:
Standard deviation and skewness during the WTC crisis
(*calculated from 3-month NZD/USD options*)

Standard deviation
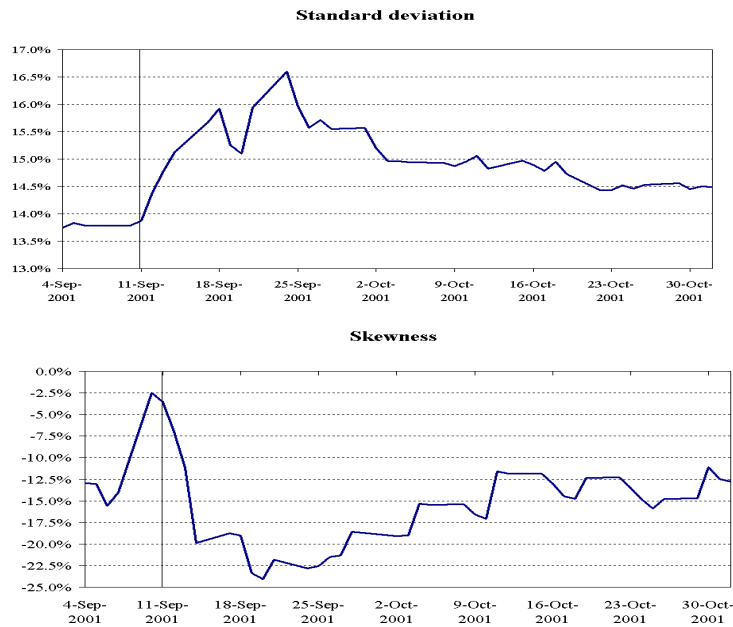
Skewness

The data also reveals that the market considered the vast majority of the shock as a temporary phenomenon.

Figure 5 shows the term structure of standard deviation and skewness. It can be seen that the change in standard deviation and skewness observed immediately after the events was much more marked for the shorter maturities than for the longer ones: the higher moments of the 6- to 12-month-ahead PDFs have changed only marginally. This reflects the fact that market participants expected the exchange rate risk to increase for a 1 to 3 month period, and then to return to the proximity of the pre-crisis level.

Figure 5:
The term structure of higher moments

The term structure of standard deviation

The term structure skewness

Another simple yet interesting measure that can be calculated from the data is the probability of the exchange rate falling below a certain level. Figure 6 shows the probability that a depreciation of 10 per cent or more will occur within a month. It can be seen that the probability of a sharp depreciation increased in the wake of the crisis.

Figure 6:
Implied probability of a more than 10 per cent depreciation of the exchange rate within 1 month
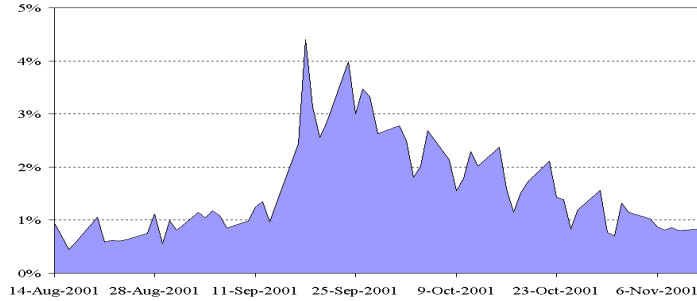
We can conclude that the options-implied PDFs captured the negative sentiment of the New Zealand foreign exchange market after the WTC crisis. However, we have to take into account that the shock was relatively large. For smaller shocks it may be difficult to distinguish between the noise in the data and the shocks themselves. In other words, as the technique does not allow for calculating confidence intervals for the implied PDFs and the other measures derived from them, we cannot quantify whether changes in the PDFs are significant or not. It is therefore recommended to crosscheck the results obtained from the options-implied probability densities by comparing them with other information sources on market expectations.

5 Can the higher moments of options-implied probability measures explain the forward premium?

In this section we will analyse whether the options-implied higher moments of the expectations can explain the existence and the direction of the forward bias observed on the New Zealand dollar market.

The forward bias is defined as the difference between the spot exchange rate at time T and the forward exchange rate at time t for a contract to be delivered at T:

$$bias_T = s_T - f_t^T$$

If the forward exchange rates are unbiased predictors of the future spot rate, it implies that the forward bias has a zero mean and it is uncorrelated with the information set available at time t.

There exists a vast literature in empirical finance on forward rate unbiasedness. International evidence suggests that the unbiasedness hypothesis generally does not hold for currency markets. (For the case of the New Zealand dollar see Ha and Reddell (1998)). Put another way, systematic excess returns can be achieved on forward currency markets. One possible explanation for the existence of the forward bias derives from investors' risk preference: the excess return may be due to a time-varying risk premium. If risk-averse agents invest abroad, they require compensation for the risk that they are running due to the volatility of the exchange rate.

If the forward bias is due to a risk premium component, it is logical to assume that its size depends on market expectations about the nature of this risk. Following Lyons (1988), Pagès (1996), and Malz (1997), we test whether the higher moments of market expectations – derived from options prices – can help in explaining the forward bias. We estimate the following equation:

$$s_T - f_t^T = \alpha + \beta std_t + \gamma skew_t + \delta kurt_t + \varepsilon_t$$

where *std*, *skew* and *kurt* are the standard deviation, the skewness and the kurtosis of the option-implied PDFs of the NZD/USD exchange rate, respectively.

We run the regression for the forward bias calculated from the one-month-ahead forward rates. Similarly, the standard deviation, skewness, and kurtosis measures were calculated from options with a maturity of one month, therefore they can be considered to be

estimates of the higher moments of the one-month-ahead market expectations. The original data-set covers the period 3 January 2000 to 11 November 2001. As daily data frequency was used for a one-month-ahead forecasting equation, the data is overlapping. As a result, although ordinary least squares yield to consistent parameter estimates, the OLS variance estimates would be biased. To correct for this, heteroskedasticity- and autocorrelation-consistent standard errors were calculated using the technique of Newey and West (1987). The truncation lag was selected according to the rule suggested by Newey and West (1994). Table 1 displays the regression results.

Table 1: Full-sample regression

Dependent Variable: $s_T\text{-}f_t^T$
Method: Least Squares
Sample(adjusted): 22 486
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.115078	0.037081	-3.103404	0.0020
std_t	0.630991	0.174637	3.613160	0.0003
$skew_t$	-0.038671	0.034914	-1.107607	0.2686
$kurt_t$	0.013897	0.055036	0.252513	0.8008
R-squared	0.134975	Mean dependent var		-0.008380
Adjusted R-squared	0.129346	S.D. dependent var		0.040885
S.E. of regression	0.038149	Akaike info criterion		-3.686059
Sum squared resid	0.670921	Schwarz criterion		-3.650429
Log likelihood	861.0088	F-statistic		23.97748
Durbin-Watson stat	0.086416	Prob(F-statistic)		0.000000

The results suggest that the standard deviation of the option-implied probability densities has a highly significant effect on the forward bias. However, the other higher moments are insignificant. The signs of the coefficients are in the right direction: since the New Zealand dollar is the "weak" currency relative to the U. S. dollar, an increase in standard deviation and kurtosis is likely push the risk premium upwards, resulting in a higher forward bias. Similarly, the negative (but insignificant) sign of the skewness parameter is in line with

skewness towards depreciation (negative sign) pushing risk premium upwards.

Table 2: Short-sample regression

Dependent Variable: $s_T\text{-}f_t^T$
Method: Least Squares
Sample(adjusted): 250 486
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.271186	0.070590	-3.841678	0.0002
std_t	1.059251	0.385770	2.745814	0.0065
$skew_t$	-0.162105	0.036916	-4.391223	0.0000
$kurt_t$	0.204015	0.043583	4.681024	0.0000
R-squared	0.420687	Mean dependent var		0.002880
Adjusted R-squared	0.413228	S.D. dependent var		0.042424
S.E. of regression	0.032498	Akaike info criterion		-3.998574
Sum squared resid	0.246068	Schwarz criterion		-3.940041
Log likelihood	477.8310	F-statistic		56.40016
Durbin-Watson stat	0.149731	Prob(F-statistic)		0.000000

Diagnostic tests revealed that some parameter estimates are quite unstable. If we run the regression on the second half of our sample (by skipping the first 250 observations) the results are much more supportive of our hypothesis: the coefficients of the skewness and the kurtosis terms become highly significant with the signs being in line with the intuition (table 2). Also, the overall explanatory power of the equation is higher.

One possible – although rather speculative – explanation of this may be the development of the market. Data suggest that during the first half of the data-set risk reversals and strangles (the option combinations related to the higher moments) were rather illiquid: during the initial period the quoted prices sometimes show stepwise changes. It is possible therefore that their price cannot be considered as good representation of market expectations, whereas in the second half of the data-set the trade of these option combinations might have been more liquid.

6 Conclusion

A method of extracting implied risk-neutral PDFs of the exchange rate was applied to New Zealand dollar/US dollar options. The estimated probability density functions provided us with an insight into the dynamics of market expectations over the last two years.

The techniques presented can be useful for both market participants and policymakers. From a central bank's point of view, besides using this information in the monetary policymaking and policy evaluation process, the options-based indicators of market expectations are also useful for analysing the stability of the financial system, as these measures may shed some light on the market's perception of risk.

We also found some – although not entirely robust – econometric evidence that the higher moments calculated from risk-neutral PDFs can be used to explain the dynamic behaviour of the forward bias measured in the New Zealand dollar/US dollar exchange rate. Expected volatility, skewness and high kurtosis seem to affect the foreign exchange risk premium of the New Zealand dollar. Longer data series and further work are necessary, however, to confirm these findings.

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Appendix:
Malz's method for deriving the risk-neutral probability distribution from OTC currency option prices.

This appendix provides a brief description of how the implied risk-neutral distribution can be calculated from the standard OTC currency option quotes. The method described here was originally developed by Malz (1997). For further technical details, see the original article.

The Black-Scholes/Garman-Kohlhagen formula for pricing call options on currencies takes the following form:

$$c = e^{-r^*(T-t)} S_t \Phi\left[\frac{\ln(S_t/X) + (r - r^* + \sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}}\right] - $$
$$- e^{-r(T-t)} X \Phi\left[\frac{\ln(S_t/X) + (r - r^* - \sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}}\right] \quad (A.1)$$

where S_t is the spot exchange rate, X is the exercise price, r and r^* are the domestic and foreign risk free interest rates, respectively, σ is the volatility, and $\Phi(.)$ is the standard normal cumulative distribution function.

The rate of change of the Black-Scholes function with respect to the spot exchange rate is called the delta (δ) and is often used as a measure of options' "moneyness". The delta of a call option is always between 0 and 1 and can be calculated as

$$\delta_c = \frac{\partial c(.)}{\partial S_t} = e^{-r^*(T-t)} \Phi\left[\frac{\ln(S_t/X) + (r - r^* + \sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}}\right] \quad (A.2)$$

As was mentioned in the main text, due to the quoting conventions of the OTC options markets, the quoted prices of straddles (atm_t), risk reversals (rr_t), and strangles (str_t) are quoted directly in terms of volatility and delta. Using $\sigma(\delta_c)_t$ as a notation for the volatility smile

function, the quoting conventions can be written in the following form:

$$atm_t = \sigma(0.5)_t$$
$$rr_t = \sigma(0.25)_t - \sigma(0.75)_t \tag{A.3}$$
$$str_t = \frac{\sigma(0.25)_t + \sigma(0.75)_t}{2} - \sigma(0.5)_t$$

Rearranging the above equations for the volatilities gives us:

$$\sigma(0.25)_t = atm_t + 0.5rr_t + str_t$$
$$\sigma(0.5)_t = atm_t \tag{A.4}$$
$$\sigma(0.75)_t = atm_t - 0.5rr_t + str_t$$

Malz's method assumes that the implied volatility function can be expressed as the following second-order Taylor approximation around the point $\delta_c = 0.5$:

$$\hat{\sigma}(\delta_c)_t = b_0 atm_t + b_1 rr_t(\delta_c - 0.5) + b_2 str_t(\delta_c - 0.5)^2 \tag{A.5}$$

One can use (A.4) to obtain the values for the parameter vector (b_0, b_1, b_2). It can be shown that the solution that is compatible with (A.4) is (1, -2, 16), thus the approximation of the implied volatility function takes the following form:

$$\hat{\sigma}(\delta_c)_t = atm_t - 2rr_t(\delta_c - 0.5) + 16str_t(\delta_c - 0.5)^2 \tag{A.6}$$

As the delta itself is a function of implied volatility, we have to solve (A.2) and (A.6) simultaneously – using numerical optimisation – to obtain the implicit functions $\hat{\sigma}(\delta_c)_t$, which gives the implied volatility for each delta, and $\hat{\sigma}(X)_t$, which then gives us the implied volatility for each exercise price.

Substituting the values of the latter into the Black-Scholes equation (A.1) provides us with the estimated call pricing function $\hat{c}(X)_t$. Finally, the implied risk-neutral probability distribution $\hat{f}(X)_t$ is

derived using the Breeden-Litzenberger result (Equation (2) in the main text).