VIX Futures Trading Activity and Volatility

Bujar Huskaj*

This Draft: September 2015

Abstract

This study extends the literature on the relation between trading activity and volatility by looking at a new asset class in the form of VIX futures, and by decomposing each side of the relation into two components. The results confirm several findings documented in prior studies: The number of transactions is the dominant factor behind the relation; volume covaries positively (negatively) with continuous (jump) volatility; surprises in volume have the largest effect on prices; and market depth reduces volatility. However, the study also finds, among other things, that the dominance of the number of transactions behind the relation with jump volatility is not robust, that the negative relation between market depth and volatility holds only for the continuous volatility component, and that increased VIX futures trading volume is associated with higher VIX options prices.

JEL classification: C13, C22, C24
Keywords: Futures, Options, Realized volatility, VIX, Volume

1 Introduction

The relation between trading activity and volatility has been studied extensively over the past 40 years for various asset classes, including stocks, bonds, currencies and commodities. Early empirical studies of this relation find it to be positive, using volume as a proxy for trading activity and measuring volatility as the absolute price change (see Karpoff (1987) for a survey of early research). More recent empirical studies investigate the impact of numerous trading activity variables on volatility. For example, Jones et al. (1994), Chan and Fong (2000, 2006), Huang and Masulis (2003), and Giot et al. (2010), all of whom examine the relation in stock markets, decompose volume into number of trades and average trade size; Chan and Fong (2000, 2006) and Giot et al. (2010) investigate in addition the effect of order imbalance (buyer- versus seller-initiated trades) and

*Bujar Huskaj is grateful to the Marianne and Marcus Wallenberg foundation for research support.
Contact: Bujar Huskaj, Department of Economics and Knut Wicksell Centre for Financial Studies, Lund University, E-mail: Bujar.Huskaj@nek.lu.se.
A previous version of the working paper was circulated under the title "VIX Futures Volume and Volatility".

Huang and Masulis (2003) examine that of small and large trades. The general finding of these studies is that the number of trades is the main factor behind the positive relation.

Focusing in futures markets, Bessembinder and Seguin (1993) separate volume and open interest into expected and unexpected components to examine if surprises in trading activity convey more information and thus have a larger impact on prices than forecastable activity. They find that surprises in volume have a larger effect on volatility, while forecastable open interest mitigates it. Daigler and Wiley (1999) also separate volume and open interest into two components but use volume data categorized by type of trader. They document that the general public, who they define as a group of traders who are distant from the trading floor, drive the positive trading activity and volatility relation, whereas trades by clearing members and floor traders often decrease volatility.

Less focus has been put on examining the implications of different measures of volatility for the relation, but two studies that address this are those of Chan and Fong (2006) and Giot et al. (2010). Chan and Fong (2006) measure the volatility using the model-free realized volatility estimate, which is relatively less noisy than the absolute price change used in the majority of studies, and find that the general results of the relation still hold; and Giot et al. (2010) examine the impact of trading activity on different components of volatility by decomposing the realized volatility measure into continuous and jump components. These authors document that the relation is positive for the continuous component and negative for the jump component, which they argue supports the consensus among market participants that a low trading volume leads to more erratic returns.

Given the empirical results that expected and unexpected components of trading activity affect total volatility differently (Bessembinder and Seguin, 1993) and that total trading activity has a different impact on continuous and jump volatility (Giot et al., 2010), a natural and interesting extension to previous research is to combine the two methodologies and check how the four trading activity and volatility components relate to each other. This paper examines these relations in the VIX futures market. In doing so, it extends the literature by looking at a new asset class. Focusing on this market also allows to investigate if factors that drive VIX futures volatility have an effect on the prices of VIX options, which are the second most actively traded index options at the Chicago Board Options Exchange (CBOE).[1,2]

Our results support the finding by Bessembinder and Seguin (1993) that shocks in trading volume have the largest impact on total volatility, but show that this heterogeneity is much more pronounced for continuous volatility than for jump volatility. The results are also consistent with those of Giot et al. (2010) that low volume leads to more erratic returns, but show in addition that a decrease in trade size or market depth leads to less erratic price changes. Our options analysis also provides new and interesting results; increases in VIX futures trading volume are found to be associated with higher

[1] In a July 16, 2015 article in Reuters, VIX options were reported to be second only to S&P 500 options, having an average daily volume of about 679,000 according to Trade Alert (http://www.reuters.com/article/2015/07/16/usa-options-vix-idUSL3N0ZQ5E120150716).

[2] The VIX futures and options markets are closely linked as market practice is to use VIX futures as the underlying for VIX options.

VIX options prices, especially for out-of-the-money calls.

The remainder of this study is organized as follows. Some facts about VIX futures are given in Section 2. The trading variables and the realized volatility measure are presented in Section 3, together with the procedure of decomposing them into different components and investigating their relations. The link between VIX futures trading activity and VIX options prices is examined in Section 4. Section 5 discusses the robustness of the results, and Section 6 concludes.

2 A few facts about VIX futures

VIX futures are contracts on forward 30-day stock volatility. They were introduced in March 2004 and are based on the VIX index, which is the leading forward-looking indicator of uncertainty in the U.S. stock market.[3] Given a VIX futures contract that expires at time T, a price of, say, 20 represents an expected annualized volatility of 20% of the S&P 500 over the coming $T + 30$ calendar day period.

VIX futures are ideal instruments for dealing with stock volatility because they allow to trade it independent of the direction and level of stock prices, on an all-electronic exchange (the CBOE Futures Exchange, CFE) over nearly 24 hours a day, five days a week.[4] There are two types of trading hours: Regular and extended. Regular trading hours are on weekdays between 8:30 a.m. to 3:15 p.m., and extended hours are from 3:30 p.m. on the previous day (5:00 p.m. on Sunday) to 8:30 a.m. The CFE may list the contract for trading up to nine consecutive months and five months on the February quarterly cycle (February, May, August and November). The final settlement day is the Wednesday 30 days before the third Friday of the calendar month that follows the maturity month, and all contracts are settled in cash (the contract multiplier is $1,000 and the minimum price movement for a trade is $50 per contract).[5]

Since its inception, the VIX futures market has grown at a remarkable pace in response to the need for trading volatility.[6] Now with hundreds of thousands contracts traded per day, it has solidified its place as the home of the industry standard contract for trading U.S. stock volatility.

3 The Trading Activity and Volatility Relation

Several theoretical models predict a positive relation between trading activity and volatility. They roughly fall into two categories: Those based on the mixture of distributions

[3] For more information about the VIX, see the *VIX White paper* at http://www.cboe.com/micro/vix/vixwhite.pdf (retrieved on June 16, 2015).

[4] See the VIX options Quick Reference Guide at http://www.cboe.com/micro/vix/VIXoptionsQRG.pdf (retrieved on June 16, 2015).

[5] For more details, see the VIX futures contract specifications at http://cfe.cboe.com/Products/Spec VIX.aspx (retrieved on June 19, 2015).

[6] According to a March 27, 2014 post on the CBOE Options Hub, the average daily volume of VIX futures rose from 4,543 in 2009 to 213,024 in the first two months of 2014 (http://www.cboeoptionshub.com/2014/03/27/panel-10th-anniversary-vix-futures-launch/.)

hypothesis (Clark, 1973; Epps and Epps, 1976; Tauchen and Pitts, 1983; and Harris, 1986), and those based on traders' differences of opinion (Varian, 1985, 1989; Harris and Raviv, 1993; and Shalen, 1993). In short, in the mixture of distributions models, trading activity and volatility are positively related because of joint dependence on a common variable, typically the number of information arrivals; and in the differences of opinion models, they are positively related due to differences in trader beliefs, i.e., when new information arrives, traders' beliefs regarding the true price of the contract are diverse, which results in trading of the contract and consequently in price volatility.

3.1 VIX futures data

Our data set consists of all VIX futures trades and quotes between November 10, 2010 and June 5, 2015. It is obtained from the Thomson Reuters Tick History database (maintained by the Securities Research Centre of Asia-Pacific, SIRCA), and includes daily open interest and microsecond accurate timing information on transaction price, trading volume and order book depth (the best bid and ask quotes and the associated sizes). We exclude records outside the regular trading hours and records that lack either the closing bid or ask quote, as well as non-business and other non-trading days.[7]

In our analysis, we follow the same procedure as in Daigler and Wiley (1999) and use the futures maturity month with the highest open interest. As they argue, this allows us to concentrate on the most active contract month without having to jump back and forth between different contract months (as would be the case if we were to use volume as the selection criterion). Furthermore, all trading variables are for the contracts being analysed, which is advantageous over using aggregated trading variables because it enables us to measure the relation more accurately.

3.2 Trading variables

3.2.1 Descriptive statistics

We use volume, number of trades, average trade size, absolute order imbalance, market depth and open interest as trading variables, all of which have been used in previous studies. The descriptive statistics on the trading series are reported in Table 1, which also contains the autocorrelations at various lags, their statistical significances (market with asterisks) calculated using the Ljung and Box (1978) test for no serial dependence, and the augmented Dickey and Fuller (1979) (ADF) test statistics for nonstationarity with their associated significance levels.

All series exhibit a significant degree of serial correlation, indicating that they are highly forecastable. The strongest dynamic dependence is observed for the market depth and open interest series, while the absolute order imbalance shows the weakest. The ADF test indicates that the open interest series contains a unit root and that all other series are trend stationary. Consequently, the forthcoming analyses are performed on the first

[7]The start of the sample period is selected on the basis of the data availability; order book depth appears in the Thomson Reuters Tick History database only from November 10, 2010.

differenced open interest series and the detrended series of the other trading variables.

[INSERT TABLE 1 HERE]

3.2.2 Decomposing trading variables

We follow Bessembinder and Seguin (1993) and detrend the activity series by removing the 100-day moving average from the original series, and then separate the detrended series into expected and unexpected components using an autoregressive (AR) model of lag order ten. The estimated AR(10) process represents the expected component and the unexpected component is calculated as the detrended series minus the expected component. Unlike them, however, we use an expanding window approach to separate the series. That is, we detrend the series and estimate the model recursively, starting with the first 100 observations and then repeating the procedure after expanding the sample by adding one observation at a time. So the trading components on day t are estimated using only information up to time $t-1$, which is a more realistic way to measure expected activity than using the entire sample as it implies that we have information about the future. For each day, the sum of the two trading activity components and the trend is the original activity series. All detrended/first differenced trading variables and their components are found to be stationary (not reported).

3.3 Realized volatility

3.3.1 Decomposing realized volatility

To decompose realized VIX futures volatility into one continuous and one jump component, the constructed VIX futures logarithmic price series is assumed to obey the following continuous-time jump-diffusion process,

$$\ln(F(t+1)) - \ln(F(t)) = r(t+1) = \int_t^{t+1} \mu(s)ds + \int_t^{t+1} \sigma(s)dW(s) + \sum_{t<s\leq t+1} \kappa(s),$$

where $\ln(F(t+1))$ is the logarithmic price at time $t+1$, $r(t+1)$ is the logarithmic return, $\mu(t+1)$ and $\sigma(t+1)$ are the instantaneous drift and volatility functions, $W(t+1)$ is a standard Brownian motion, and $\kappa(t+1)$ is the jump size.

Using largely the same notations as in Andersen et al. (2007), the daily realized volatility (RV) is defined as the sum of $1/\Delta$ intraday squared returns, and it is a consistent estimator of the continuous integrated volatility and the jump volatility for increasingly finer sampled returns:

$$RV_{t+1}(\Delta) = \sum_{j=1}^{1/\Delta} r^2_{t+j\Delta,\Delta} \xrightarrow{p} \int_t^{t+1} \sigma^2(s)ds + \sum_{t<s\leq t+1} \kappa^2(s)$$

for $\Delta \to 0$, where $r_{t,\Delta} = \ln(F(t)) - \ln(F(t-\Delta))$ is the within-day discretely sampled Δ-period logarithmic return.

The diffusion and jump components can be disentangled by first using the realized bi-power variation (BV) of Barndorff-Nielsen and Shephard (2004, 2005), which is a consistent estimator of the continuous integrated volatility even in the presence of jumps:

$$BV_{t+1}(\Delta) = \frac{\pi}{2}\sum_{j=2}^{1/\Delta} |r_{t+j\Delta,\Delta}||r_{t+(j-1)\Delta,\Delta}| \xrightarrow{p} \int_t^{t+1} \sigma^2(s)ds.$$

Thereafter, a consistent estimator of the jump component (J) is simply the difference between the realized volatility and the bi-power variation,

$$J_{t+1}(\Delta) = RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \xrightarrow{p} \sum_{t<s\leq t+1} \kappa^2(s).$$

Having separated the two components, one can then identify statistically significant jumps via the jump ratio statistic used in Huang and Tauchen (2005) and Andersen et al. (2007),[8]

$$Z_{t+1}(\Delta) = \Delta^{-1/2} \frac{[RV_{t+1}(\Delta) - BV_{t+1}(\Delta)]RV_{t+1}(\Delta)^{-1}}{\left[\left(\mu_1^{-4} + 2\mu_1^{-2} - 5\right)\max\left\{1, TQ_{t+1}(\Delta)BV_{t+1}(\Delta)^{-2}\right\}\right]^{1/2}} \xrightarrow{p} N(0,1). \tag{1}$$

The jump ratio statistic is thus approximately normally distributed under the null hypothesis of no jump. A series of statistically significant jumps is then given by

$$J_{t+1,\alpha}(\Delta) = I[Z_{t+1}(\Delta) > \Phi_\alpha^{-1}] \times [RV_{t+1}(\Delta) - BV_{t+1}(\Delta)],$$

where α is the significance level, Φ is the cumulative density function of the standard normal distribution, and $I(.)$ is the indicator function of a significant jump over $[t, t+1]$.[9]

3.3.2 Correcting for microstructure noise

As discussed in Andersen et al. (2007), practical microstructure frictions such as nonsynchronous trading, price discreteness and bid-ask spreads might cause the high frequency returns to be serially correlated and make the realized volatility measure an inconsistent estimator. Huang and Tauchen (2005) also report that such frictions will generally make

[8] $\mu_1 = \sqrt{2/\pi}$ and $TQ_{t+1}(\Delta)$ is the standardized realized tri-power quarticity, defined as

$$TQ_{t+1}(\Delta) = \Delta^{-1}\mu_{4/3}^{-3}\sum_{j=3}^{1/\Delta} |r_{t+j\Delta,\Delta}|^{4/3}|r_{t+(j-1)\Delta,\Delta}|^{4/3}|r_{t+(j-2)\Delta,\Delta}|^{4/3},$$

where $\mu_{4/3} = 2^{2/3}\Gamma(7/6)\Gamma(1/2)^{-1}$ and where Γ is the Gamma function, $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$.

[9] To ensure that the estimated continuous and jump volatilities sum to the realized volatility, one can put the insignificant jumps in the continuous volatility process as

$$BV^*_{t+1,\alpha}(\Delta) = I[Z_{t+1}(\Delta) < \Phi_\alpha^{-1}] \times RV_{t+1}(\Delta) + I[Z_{t+1}(\Delta) > \Phi_\alpha^{-1}] \times BV_{t+1}(\Delta).$$

the jump test statistic biased against finding jumps. Therefore, we calculate returns from average bid-ask quotes and follow Andersen et al. (2007)'s suggestion of calculating the bi-power variation and the tri-power quariticity using staggered returns to break the erroneous serial correlation in the returns.[10]

3.3.3 Resulting realized volatilities

The realized VIX futures volatility is calculated using $1/\Delta = 81$ logarithmic returns per day and statistically significant jumps are identified for α equal to 0.999, corresponding to a significance level of 0.1%. In contrast to Giot et al. (2010), the insignificant jumps are not put in the continuous volatility process (see footnote 9) but are excluded from the sample. This is because these jumps, although identified as small and insignificant by the jump ratio statistic [Equation (1)], are still large in magnitude relative to the continuous volatility.

In Table 2, we present the summary statistics on the volatility series, together with their autocorrelations, ADF test statistics and correlations. Jumps are identified as significant in 251 days out of the total 1,029 in the sample, corresponding to 24%. This is comparable to the 26% reported in Giot et al. (2010). The realized volatility and continuous series display statistically significant autocorrelation at any reasonable level, but not the jump series. Comparing the magnitudes of the first two series' autocorrelation coefficients also shows that removing jumps enhances the dynamic dependence of the resulting series. Based on the ADF test, we conclude that all series are stationary. Realized volatility is strongly related with its continuous component, but its correlation with the jump component is of the opposite sign, considerably weaker and insignificant. The correlation between the two components, however, is negative, relatively stronger and highly significant. These results are in line with those reported by Giot et al. (2010).

[INSERT TABLE 2 HERE]

[10] These versions are given by

$$BV_{t+1}(\Delta) = \frac{\pi}{2}\,(1-2\Delta)^{-1}\sum_{j=3}^{1/\Delta} |r_{t+j\Delta,\Delta}||r_{t+(j-2)\Delta,\Delta}|$$

and

$$TQ_{t+1}(\Delta) = \Delta^{-1}\mu_{4/3}^{-3}\,(1-4\Delta)^{-1}\sum_{j=5}^{1/\Delta} |r_{t+j\Delta,\Delta}|^{4/3}|r_{t+(j-2)\Delta,\Delta}|^{4/3}|r_{t+(j-4)\Delta,\Delta}|^{4/3}.$$

3.4 Regressions of realized volatilities on trading variables

We investigate the trading activity and volatility relation by estimating the following regressions:

$$y_t = c_y + \sum_{i=1}^{4} \delta_{yi} D_{it} + \sum_{j=1}^{12} \rho_j y_{t-j} + \theta_y TTM_t + \sum_{k=1}^{m} \alpha_k A_{kt} + u_{yt}, \quad (2)$$

$$J_t = c_J + \sum_{i=1}^{4} \delta_{Ji} D_{it} + \theta_J TTM_t + \sum_{k=1}^{m} \beta_k A_{kt} + u_{Jt}. \quad (3)$$

Equation (2) is a linear regression model and Equation (3) is a tobit model. y_t is either realized volatility or the continuous component, D_{it} are dummies that capture day-of-the-week differences in volatilities, ρ_j measures the persistence in the volatility process (which Table 2 provided evidence of), TTM_t is the number of days to maturity of the futures contract, included as a control variable, and α and β measure the impact of m trading activity variables.[11] Regressions similar to Equation (2) are also used in Jones et al. (1994) and Chan and Fong (2006), and models similar to Equation (3) are used in Giot et al. (2010).

The linear regression model is estimated by ordinary least squares and the tobit model is estimated by maximum likelihood. Each activity series is standardized by subtracting and dividing it by its associated mean and standard deviation to allow for direct comparison of the slope coefficients, and Newey and West (1987) standard errors are used. We initially regress on only one trading variable, and then we include more to check if any of them are dominant. The estimation results are presented in Table 3, where we report the estimated trading variable coefficients, their statistical significances (marked with asterisks), and the adjusted (pseudo) R-squared for the linear (tobit) regression. For brevity, we do not report the estimates of the other coefficients, and we only report the results of the simple (one trading variable) and complete models (all trading variables).[12]

The estimated coefficients relating volume and number of trades to continuous volatility are all positive and significant at the 1% level, and their unexpected component parameters are larger in magnitude than the parameters associated with their expected component. A heterogeneous effect of the volume components on total volatility is also found by Bessembinder and Seguin (1993). Similar to volume and trade frequency, the estimated parameters on absolute order imbalance are all positive. However, of the two components, only the parameter associated with the unexpected one is significant at a reasonable level, and its size is about six times smaller than that of volume and number of trades. The parameters associated with the unexpected component of average trade size and market depth are also larger in magnitude and more important than those of the expected one. Their signs, however, are negative, indicating that large transactions and a deep market both reduce continuous volatility. Using a proxy for market depth,

[11] VIX futures volatility is also found to be long-term dependent in Huskaj (2013).

[12] All results are available upon request.

Bessembinder and Seguin (1993) also document a negative relation between depth and volatility, but the effect of average trade size contrasts previous results.[13] Furthermore, none of the estimated coefficients associated with open interest are significant, which is in line with the results of Daigler and Wiley (1999).

The estimated parameters relating the trading variables to jump volatility have the opposite sign to those linking them to continuous volatility, and except for a few coefficients on order imbalance and open interest, all are significant at the 1% level. Overall, the trading effect varies again depending on which activity component it is, but the difference is considerably lower than that for continuous volatility, and neither component is dominant across all trading variables. The negative coefficients for volume and number of trades are consistent with the results of Giot et al. (2010), and thus provide additional evidence that low activity in these variables lead to more erratic price changes. Absolute order imbalance has, as before, a much lower impact than volume and trade frequency, and the positive coefficients for average trade size and market depth suggest that a decrease in either one of them leads to less erratic price changes. Giot et al. (2010) also documents a positive average trade size coefficient, but it is not statistically significant. The component parameters on open interest are again not significant at any reasonable level.

The adjusted R-squared values for continuous volatility show that the number of trades explains the most, and when including all trading variables in the regressions, the estimated parameters associated with the number of trades remain roughly the same and are also the ones that are most statistically significant. This clearly shows that trade frequency is the main factor behind the (positive) relation with continuous volatility. Including the same variables in the tobit regression suggests that the number of trades also plays the dominant role behind the (negative) relation with jump volatility. These results are in accordance with those of Giot et al. (2010).

[INSERT TABLE 3 HERE]

4 The Impact on VIX Options Prices

4.1 VIX options data

The VIX options data set is also obtained from SIRCA and it consists of daily observations on VIX call and put options over the same period as the VIX futures trading activity series (April 6, 2011 to June 5, 2015). Again, non-business and other non-trading days are removed, and we use the mid-point of the closing bid-ask quote as the option price.

[13] Bessembinder and Seguin (1993) proxy market depth using expected open interest by arguing (roughly) that if factors which market depth depend on do not change quickly, such as traders' risk aversion, wealth constraints and other underlying determinants, then a variable constructed from lagged open interest should contain information on current depth.

4.2 Implied volatility

We construct daily implied volatility time series of nearest-the-money (NTM) put options and 25% out-of-the-money (OTM) call and put options.[14] The underlying series is the same VIX futures series as in the previous section, and we use the simple Whaley (1993) model to back out the implied volatility since there is no reason to use a more sophisticated model for this analysis. If more than one option is found for a category on any given day, we simply pick the one with the highest open interest. Each resulting implied volatility is squared and converted into a daily basis.

Table 4 provides the summary statistics, autocorrelations and the ADF test statistics on the converted series. Similar to the realized volatility and its continuous component, all three implied volatility series exhibit significant autocorrelation and can be considered stationary. The OTM call and put series are higher respectively lower than the NTM series, reflecting the implied volatility skew, and the serial dependence is the strongest for the OTM put series, followed by the NTM series.

[INSERT TABLE 4 HERE]

4.3 Regressions of implied volatilities on trading variables

Trading activity in VIX futures is related to VIX options implied volatility by a similar model as Equation (2). The regression results are provided in Table 5, where we, like Table 3, report the estimated trading variable parameters, their statistical significances (marked with asterisks) and the adjusted R-squared. Again, we do not report the estimates of the other coefficients and we only report the results of the simple and complete models.

The regression results are in line with those in Table 3 in terms of significance, coefficient sign and relative magnitude: Surprises in trading activity are more important in explaining volatility; volume and trade frequency covary positively with it; the relation is the opposite for trade size and market depth; order imbalance has a very low impact; and open interest is not significant at all. Furthermore, trade frequency has again the greatest effect and explains the most, and its estimated parameters in the complete model remain once again almost unaltered and most significant. These results clearly suggest that factors driving VIX futures trading also have an impact on VIX options prices, where the largest effect is observed for OTM call options. A possible explanation for this is that as increased trading in VIX futures is associated with greater uncertainty about the future, hedgers are more likely to buy cheaper OTM than NTM VIX call options during these times to protect themselves against potential drops in the stock market (as they would with OTM S&P 500 puts), which drives up the prices of these options.

[INSERT TABLE 5 HERE]

[14]NTM put options are used because they are more liquid than NTM call options over our sample period, with a daily average volume (open interest) of 14,155 (110,102) for puts versus 14,089 (84,373) for calls.

5 Robustness check of the empirical results

We evaluate the robustness of our results by splitting the sample in half and repeating the entire analysis for each subperiod. The first subsample is from April 4, 2011 to May 3, 2013 (515 observations), and the second is from May 6, 2013 to June 5, 2015 (514 observations). The regression coefficients linking the trading variables to continuous volatility are very alike their whole-sample counterparts in terms of importance, sign and relative size. This clearly shows that the main results on this relation are robust. However, our findings on the relation between trading activity and jump volatility hold only for the second period. This obviously has to due with the difference in the number of significant jumps between the subsamples (14% in the first versus 35% in the second). The signs on the regression parameters are robust across both periods, but trade frequency is not dominant in the first one, market depth is. This is an important finding because the choice of significance level to identify jumps with is clearly subjective. As for our results on the relation between VIX futures trading activity and VIX options prices, they are robust across the subsamples: Activity in VIX futures in general and trade frequency in particular is associated with changes in VIX options prices, especially for OTM calls. We do not report any results here but they are available upon request.

6 Conclusions

This study has investigated the relation between trading activity and volatility for an asset class that has not been considered in the literature before, and it has also extended the methodology used in previous research by decomposing each side of the relation into two components. Consistent with prior studies for other markets, we find that volume, trade frequency and order imbalance are positively (negatively) related with continuous (jump) volatility, and that total volatility is most affected by the number of transactions. We also confirm that the impact of the unexpected volume component on total volatility is much larger than that of the expected one. However, we find in addition that the relations are the opposite for trade size and market depth, that the dominance of the number of transactions behind the relation with jump volatility is not robust to alternative jump proportions, and that the difference between the effects of the trading components is not relatively large for jump volatility. That an increase in market depth reduces (total) volatility is consistent with previous empirical studies and theory (e.g., Kyle, 1985), but our results also show that depth is positively related to jump volatility. This indicates that while the majority of order flows in a deep market cause small price changes, occasionally, some orders are large enough to cause significant price moves.

Our VIX options study provides evidence that activity in the VIX futures market is associated with price changes for VIX options, especially for OTM call options. The strongest relation is with trade frequency, where a higher frequency is related with higher options prices.

Future work should investigate whether the results found in this study hold for other markets as well, and an interesting extension would be to use trading data that is categorized by type of trader, as in Daigler and Wiley (1999).

References

Andersen, T. G., Bollerslev, T., Diebold, F. X., 2007. Roughing it up: Including jump components in the measurement, modeling and forecasting of return volatility. Review of Economics and Statistics 89, 701–720.

Barndorff-Nielsen, O. E., Shephard, N., 2004. Power and bipower variation with stochastic volatility and jumps. Journal of Financial Econometrics 2, 1–37.

Barndorff-Nielsen, O. E., Shephard, N., 2005. Econometrics of testing for jumps in financial economics using bipower variation. Journal of Financial Economics 4, 1–30.

Bessembinder, H., Seguin, P. J., 1993. Price volatility, trading volume, and market depth: Evidence from futures markets. Journal of Financial and Quantitative Analysis 28, 21–40.

Chan, K., Fong, W. M., 2000. Trade size, order imbalance, and the volatility-volume relation. Journal of Financial Economics 57, 247–273.

Chan, K., Fong, W. M., 2006. Realized volatility and transactions. Journal of Banking and Finance 30, 2063–2085.

Clark, P. K., 1973. A subordinated stochastic process model with finite variance for speculative prices. Econometrica 41, 135–156.

Daigler, R. T., Wiley, M. K., 1999. The impact of trader type on the futures volatility-volume relation. Journal of Finance 54, 2297–2316.

Dickey, D. A., Fuller, W. A., 1979. Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association 74, 427–431.

Epps, T. W., Epps, M. L., 1976. The stochastic dependence of security price changes and transaction volumes: Implication for the mixture-of- distributions hypothesis. Econometrica 44, 305–322.

Giot, P., Laurent, S., Petitjean, M., 2010. Trading activity, realized volatility and jumps. Journal of Empirical Finance 17, 168–175.

Harris, L., 1986. Cross-security tests of the mixture of distributions hypothesis. Journal of Financial and Quantitative Analysis 21, 39–46.

Harris, M., Raviv, A., 1993. Differences of opinion make a horse race. Review of Financial Studies 6, 473–506.

Huang, R., Masulis, R., 2003. Trading activity and stock price volatility: Evidence from the london stock exchange. Journal of Empirical Finance 10, 249–269.

Huang, X., Tauchen, G., 2005. The relative contribution of jumps to total price variance. Journal of Financial Econometrics 3, 456–499.

Huskaj, B., 2013. Long memory in vix futures volatility. Review of Futures Markets 21, 31–49.

Jones, C., Kaul, G., Lipson, M., 1994. Transactions, volume and volatility. Review of Financial Studies 7, 631–651.

Karpoff, J., 1987. The relationship between price changes and trading volume: A survey. Journal of Financial and Quantitative Analysis 22, 109–126.

Kyle, A. S., 1985. Continuous auctions and insider trading. Econometrica 53, 1315–1335.

Ljung, G., Box, G., 1978. On a measure of lack of fit in time series models. Biometrica 65, 297–303.

Newey, W. K., West, K. D., 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703–708.

Schwarz, G., 1978. Estimating the dimension of a model. Annals of Statistics 6, 461–464.

Shalen, C. T., 1993. Volume, volatility, and the dispersion of beliefs. Review of Financial Studies 6, 405–434.

Tauchen, G., Pitts, M., 1983. The price-variability-volume relationship on speculative markets. Econometrica 51, 485–505.

Varian, H., 1985. Divergence of opinion in complete markets: A note. Journal of Finance 40, 309–317.

Varian, H., 1989. Differences of opinion in financial markets. In *Financial risk: theory, evidence and implications, C. Stone, (eds)*, Proceedings of the eleventh annual economic policy conference of the Federal Reserve Bank of St. Louis, Boston: Kluwer.

Whaley, R. E., 1993. Derivatives on market volatility: Hedging tools long overdue. Journal of Derivatives 1, 71–84.

Table 1
Summary statistics on the trading variables.

	Mean	Median	Std	Autocorrelation at lag 1	Autocorrelation at lag 20	ADF
V	47,637	41,284	32,394	0.824***	0.478***	-12.383*** [1]
NT	8,151	6,975	5,308	0.829***	0.340***	-10.202*** [1]
ATS	5.817	5.526	1.836	0.792***	0.594***	-6.340*** [3]
Depth	424	312	360	0.946***	0.735***	-5.154*** [3]
|OB|	2,962	2,034	3,014	0.987***	0.838***	-29.354*** [0]
O	123,082	132,242	47,258	0.227***	0.135***	-2.340 [14]

Notes: Summary statistics (daily averages) on the trading variables over the period November 10, 2010 to June 5, 2015. V, NT, ATS, Depth, |OB| and O denote volume, number of trades, average trade size, market depth, absolute order imbalance and open interest, respectively. Depth is obtained as the average volume available on the bid and ask side for the best level in the order book, and absolute order imbalance is defined as the absolute value of the difference between the number of buyer- and seller-initiated trades for the day. All data are obtained during the regular trading hours (8:30 a.m. to 3:15 p.m.), and bid-ask spreads and depths are observed at the end of one-minute intervals. ADF denotes the augmented Dickey and Fuller (1979) test with an intercept and trend, i.e., $\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t$ where $\Delta y_t = y_t - y_{t-1}$, and the lag length p is determined using the Schwarz (1978) information criterion; the values are the test statistics associated with γ from testing the null hypothesis of a unit root, i.e., $\gamma = 0$. *** denotes significance at the 1% level.

Table 2
Summary statistics on the realized volatilities.

	Mean	Median	Std	Autocorrelation at lag 1	Autocorrelation at lag 20	ADF	Correlations RV	Correlations C	Correlations J
RV	0.101	0.076	0.081	0.576***	0.084***	-12.315*** [1]	1		
C	0.095	0.072	0.083	0.579***	0.085***	-12.330*** [1]	0.98***	1	
J	0.006	0.000	0.015	0.042	-0.001**	-30.718*** [0]	-0.04	-0.22***	1

Notes: Summary statistics (daily averages) on the realized volatilities over the period April 6, 2011 to June 5, 2015. RV, C, and J denote, respectively, realized volatility, its continuous component, and its jump component (which consists of statistically significant jumps identified for α equal to 0.999) in percent. The volatility series are calculated using logarithmic five minute returns based on mid-quote prices during the regular trading hours (8:30 a.m. to 3:15 p.m.) ADF denotes the augmented Dickey and Fuller (1979) test with an intercept, i.e., $\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t$ where $\Delta y_t = y_t - y_{t-1}$, and the lag length p is determined using the Schwarz (1978) information criterion; the values are the test statistics associated with γ from testing the null hypothesis of unit root, i.e., $\gamma = 0$. *** and ** denote significance at the 1% and 5% levels.

Table 3

Regressions of daily realized volatility and its two components on the total trading variables and their two components.

	RV					*C*					*J*				
	T*	*adj* R^2	E	U	*adj* R^2	T*	*adj* R^2	E	U	*adj* R^2	T*	*pse* R^2	E	U	*pse* R^2
Panel A: Simple models															
V	0.426***	0.587	0.148***	0.409***	0.621	0.430***	0.585	0.147***	0.415***	0.619	-0.116***	0.007	-0.087***	-0.084***	0.007
NT	0.491***	0.620	0.167***	0.435***	0.670	0.502***	0.621	0.164***	0.447***	0.673	-0.164***	0.011	-0.110***	-0.130***	0.011
ATS	-0.092***	0.369	-0.049*	-0.076***	0.369	-0.103***	0.376	-0.050*	-0.088***	0.376	0.083***	0.005	0.051***	0.065***	0.005
Depth	-0.199***	0.403	-0.078***	-0.247***	0.449	-0.206***	0.409	-0.080***	-0.257***	0.456	0.094***	0.006	0.076***	0.069***	0.007
|OB|	0.080***	0.369	0.027	0.078***	0.368	0.073***	0.372	0.024	0.072***	0.371	0.005	0.001	-0.034**	0.010	0.002
O	-0.019	0.360	-0.006	-0.018	0.359	-0.018	0.364	-0.008	0.016	0.364	0.036*	0.002	0.030	0.024	0.002
Panel B: Complete models															
NT	0.517***	0.626	0.187***	0.434***	0.684	0.525***	0.626	0.185***	0.443***	0.688	-0.137***	0.012	-0.070**	-0.117***	0.014
ATS	0.081***		0.048**	0.058***		0.072***		0.046**	0.048***		0.024		0.022	0.023	
Depth	-0.022		0.009	-0.079***		-0.022		0.010	-0.084***		0.030		0.033	0.028*	
|OB|	-0.023		-0.003	-0.036**		-0.027		0.005	-0.042***		0.016		-0.035*	0.024	
O	-0.035		-0.024	0.012		-0.035		-0.025	-0.009		0.022		0.026	0.010	

Notes:

$$y_t = c + \sum_{i=1}^{4} \delta_{i,y} D_{it} + \sum_{j=1}^{12} \rho_j y_{t-j} + \theta_y TTM_t + \sum_{k=1}^{m} \alpha_k A_{kt} + u_t, \quad \text{(a)}$$

$$J_t = c + \sum_{i=1}^{4} \delta_{i,J} D_{it} + \theta_J TTM_t + \sum_{k=1}^{m} \beta_k A_{kt} + u_t. \quad \text{(b)}$$

Ordinary least squares and maximum likelihood results of the linear and tobit regression models given by Equation (a) and (b), respectively, for the period April 6, 2011 to June 5, 2015. Panel A (B) provides the results of the regressions on a single (all) trading variable(s). See footnote to Tables 1 and 2 for explanations of V, NT, ATS, Depth, |OB| and O, and RV, C and J. All trading series are standardized by subtracting and dividing them by their associated mean and standard deviation, and all volatility series are scaled up by a factor of $1,000$. T* is the detrended/first-differenced series, and E and U are its expected and unexpected components. y_t is RV or C, D_{it} are dummies that capture day-of-the-week differences in volatilities, ρ_j measures the persistence in the volatility process, TTM_t is the number of days to maturity of the futures contract, and α and β measure the impact of m trading activity variables. ***, ** and * denote significance at the 1%, 5% and 10% levels.

Table 4
Summary statistics on the implied volatilities.

	Mean	Median	Std	Autocorrelation at lag 1	Autocorrelation at lag 20	ADF
OTM call	0.336	0.327	0.090	0.773***	0.360***	-9.242*** [1]
NTM put	0.206	0.194	0.081	0.830***	0.376***	-7.570*** [1]
OTM put	0.118	0.105	0.056	0.805***	0.434***	-6.933*** [2]

Notes: Summary statistics (daily averages) on the implied volatilities over the period April 6, 2011 to June 5, 2015. NTM and OTM denote near-the-money and 25% out-of-the-money options. Each implied volatility series is squared and converted into a daily basis in percent. ADF denotes the augmented Dickey and Fuller (1979) test with an intercept, i.e., $\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t$ where $\Delta y_t = y_t - y_{t-1}$, and the lag length p is determined using the Schwarz (1978) information criterion; the values are the test statistics associated with γ from testing the null hypothesis of a unit root, i.e., $\gamma = 0$. *** denotes significance at the 1% level.

Table 5

Regressions of daily VIX options implied volatility on the total VIX futures trading variables and their two components.

	OTM call					NTM put					OTM put				
	T*	*adj* R^2	E	U	*adj* R^2	T*	*adj* R^2	E	U	*adj* R^2	T*	*adj* R^2	E	U	*adj* R^2
Panel A: Simple models															
V	0.200***	0.736	0.104***	0.183***	0.739	0.163***	0.750	0.054***	0.170***	0.759	0.106***	0.732	0.042***	0.107***	0.737
NT	0.228***	0.745	0.121***	0.197***	0.751	0.190***	0.757	0.056***	0.197***	0.774	0.127***	0.741	0.049***	0.127***	0.753
ATS	-0.073***	0.697	-0.033	-0.066***	0.697	-0.050***	0.719	-0.001	-0.062***	0.721	-0.038***	0.704	-0.006	0.044***	0.705
Depth	-0.114***	0.703	-0.052**	-0.138***	0.714	-0.087***	0.724	-0.028*	-0.126***	0.738	-0.051***	0.706	-0.016	-0.078***	0.717
|OB|	0.019	0.692	0.021	0.017	0.692	0.035**	0.717	0.011	0.034**	0.717	0.021*	0.701	0.006	0.020*	0.701
O	0.019	0.691	0.021	0.011	0.691	-0.014	0.716	-0.016	-0.007	0.716	-0.004	0.700	-0.001	-0.003	0.699
Panel B: Complete models															
NT	0.235***	0.745	0.135***	0.181***	0.756	0.191***	0.756	0.067***	0.180***	0.778	0.132***	0.740	0.062***	0.116***	0.755
ATS	0.019		0.027	-0.009		0.015		0.026	-0.009		0.004		0.017	-0.011	
Depth	-0.011		0.009	-0.061***		-0.010		0.001	-0.055***		0.007		0.012	-0.029***	
|OB|	-0.028*		-0.008	-0.033**		-0.002		-0.000	-0.011		-0.004		-0.007	-0.009	
O	0.021		0.021	0.021		-0.013		-0.013	-0.002		-0.002		0.000	0.002	

Notes:

$$y_t = c + \sum_{i=1}^{4} \delta_{i,y} D_{it} + \sum_{j=1}^{12} \rho_j y_{t-j} + \theta_y TTM_t + \sum_{k=1}^{m} \alpha_k A_{kt} + u_t,$$

Ordinary least squares results of the linear regression model above for the period April 6, 2011 to June 5, 2015. Panel A (B) provides the results from the regressions on a single (all) trading variable(s). See footnote to Table 1 for explanations of V, NT, ATS, Depth, |OB| and O. All trading series are standardized by subtracting and dividing them by their associated mean and standard deviation, and the implied volatility series of nearest-the-money (NTM) put options and 25% out-of-the-money (OTM) call and put options are squared and converted into a daily basis in percent. T* is the detrended/first-differenced series, and E and U are its expected and unexpected components. D_{it} are dummies that capture day-of-the-week differences in volatilities, ρ_j measures the persistence in the volatility process, TTM_t is the number of days to maturity of the futures contract, and α and β measure the impact of m trading activity variables. ***, ** and * denote significance at the 1%, 5% and 10% levels.