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Does historical volatility term structure contain valuable information for predicting volatility index futures?

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Abstract

We suggest that the term structure of volatility futures (e.g. VIX futures) shows a clear pattern of dependence on the current level of VIX index. At the low level of VIX (below 20) the term structure is highly upward sloping; at the high VIX level (over 30) it is strongly downward sloping. We use those features to better predict future volatility and index futures. We begin by introducing some quantitative measures of volatility term structure (VTS) and volatility risk premium (VRP). We use them further to estimate the distance between the actual value and the fair (model) value of the VTS. We find that this distance has significant predictive power for volatility futures and index futures and we use this feature to design a simple strategy to invest in VIX index futures and S&P500.

Keywords:
volatility term structure, volatility risk premium, volatility and index futures, realized volatility, implied volatility, investment strategies, returns forecasting, efficient risk and return measures

JEL:
G11, G14, G15, G23, C61, C22

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1. Introduction

We have found that volatility term structure calculated from volatility index futures (VIX index) shows a clear pattern of dependence on the current level of the VIX index. Figure 1 shows that the term structure is highly upward sloping when the level of VIX is relatively low (below 20) and it is significantly downward sloping when the level of VIX is high (over 30). We assume that this is mostly due to the market perception of risk in the short and the long-term. That perception reflects strong mean reversion process and long memory process - both reflected in volatility time series. In order to investigate the volatility term structure phenomenon more deeply, we introduce two quantitative characteristics of volatility term structure derived directly from the levels of the consecutive volatility futures maturities. These measures are calculated independently for different quintile groups[1] of VIX index and the various maturities of volatility index futures. Further, we use these measures in order to estimate the distance between the actual and the fair (model) value of volatility term structure for the given maturity of volatility futures. Subsequently, we include this information in the process of forecasting future levels of volatility and index futures and we find that this distance has significant predictive power. In the last section, we propose a simple investment strategy that uses the volatility term structure in order to predict volatility and equity index futures.

Volatility research and, in particular, volatility forecasting seems to be one of the most active and successful areas in financial econometrics in recent decades (cf. Andersen et al. 2005). The literature on the VIX and its derivatives is growing very fast, but the number of studies testing the predictability of VIX futures prices is – according to our best knowledge – still very low. Konstantinidi and Skiadopoulos (2011) show only weak evidence of statistically predictable patterns in the evolution of volatility futures prices. They also cannot find a trading strategy with economically significant profits.

Some papers suggest strongly that the use of the information content of the volatility term structure may improve this situation, but the number of studies attempting to examine directly the relationship between the term structure of VIX futures and their future returns or the underlying equity returns is still very limited (cf. Fassas 2012, Asensio 2013, Huskaj and Nossman 2013 and the references therein).

The paper is thus organized as follows. The next section describes the data. Methodology of this research is presented in the third section. The fourth section presents the description of simple measures of volatility term structure. Next section describes the dependence between VIX index and S&P500 index and measures of VTS (Volatility Term Structure). Forecasting properties of

[1] As those dependence patterns we mention earlier are not homogenous in the whole sample we divide our data into quintiles and the rest of the study is conducted consistently on this basis.

volatility term structure are presented in the same section. The investment strategies based on these results are presented in the sixth section. The last section concludes and presents possible extensions of this research.

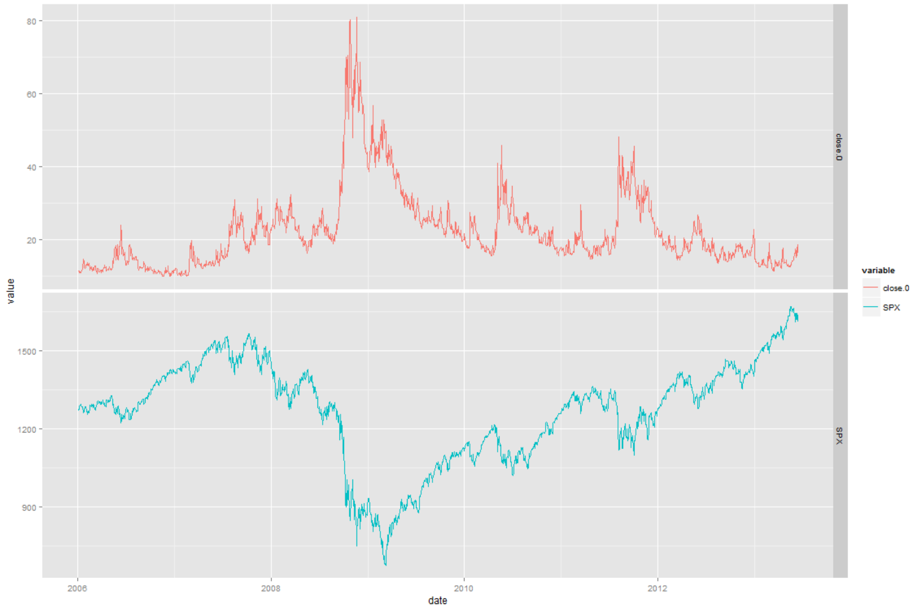
2. Data description.

For each trading day we gather close prices for VIX index, VIX index futures and S&P500 index[2].

Initially, we have up to 24 expiration months (from January, 2006 until July, 2013), but we have to limit their number to 7 because of liquidity problems for longer maturities. Data preparation for VIX index returns has also included the process of gap correction in order to omit the problem of very high positive returns at the moment of series change.

Figure 1 and Table 1 present time series for VIX and S&P500. It is worth noticing that S&P500 returns are leptokurtic and negatively skewed while VIX returns are also leptokurtic and positively skewed. Additionally, both returns series are negatively correlated.

Figure 1. VIX and S&P500 index.



Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013.*

[2] Data are from the following sources: VIX index - http://cfe.cboe.com/Products/historicalVIX.aspx; VIX index futures - http://cfe.cboe.com/Products/historicalVIX.aspx; S&P500 index - www.stooq.pl

Table 1. The descriptive statistics for daily returns of VIX and S&P500 index.

	VIX.r	r
nobs	1872	1872
Minimum	-0.350588	-0.094695
Maximum	0.496008	0.109572
1. Quartile	-0.040153	-0.004962
3. Quartile	0.032535	0.006169
Mean	0.000182	0.000141
Median	-0.005290	0.000813
SE Mean	0.001662	0.000333
LCL Mean	-0.003077	-0.000512
UCL Mean	0.003441	0.000794
Stdev	0.071901	0.014403
Skewness	0.702229	-0.299459
Kurtosis	4.149946	8.992085
Correlation	-0.755	

Note: A*ll calculations were made on the data from 01/01/2006 until 01/07/2013.*

3. Methodology

In order to answer research questions and verify hypotheses we decided to undertake the following steps in this paper. As some earlier studies (i.e. Giot 2005, Simon and Wiggins 2001) suggest we investigate the relationship we are interested in after separating our daily data on VIX index into five quintile groups[3]. We put each day from the sample to 5 quintile groups based on the level of VIX index at the close of the market session. Based on the very low liquidity for longer time to maturity (less than 25% observation), we withdraw from the sample all the information about the volatility index future which has more than seven months to expiration. Then, we estimate simple regression lines independently based on closing prices of VIX index and VIX index futures for each quintile group in order to define average shape of volatility term structure.

$$close_i = \beta_0 + \beta_1 * ttm_i + \beta_2 * ttm_i^2 + \epsilon_i \qquad (1)$$

where:

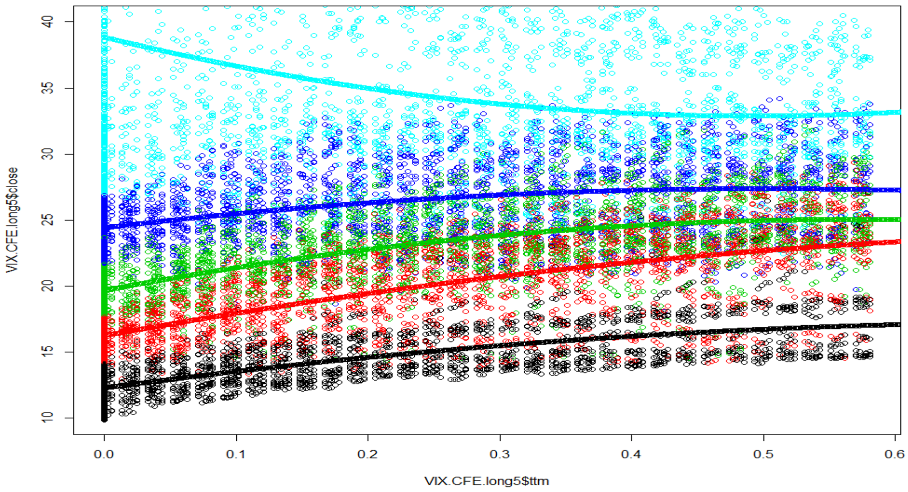
$close_i$ – daily closing price of VIX and VIX futures

ttm_i – time to maturity of i-th VIX futures (for VIX $ttm_i = 0$), $i = 1,\ldots,n_j$

n_j – size of the VIX j-th quintile group, $j = 1,\ldots, 5$.

[3] The selection of quintile group was dictated by very heterogenous shapes of volatility term structure of VTS in different market conditions. It is important to add that it really does not matter if we choose quintile or quartile groups but we had to choose the way of division.

Figure 2. Quadratic functions for volatility term structure in five different quintiles groups.



Note: A*ll calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.VIX index futures are quoted on CFE.*

Then, we draw all the observations and five shapes of term structure estimated in this point on Figure 2. It confirms our initial assumption concerning the patterns of volatility term structure. It is dependent on VIX index level and upward sloping for initial four quintile groups where β_2 parameter has negative value and it is downward sloping for the fifth quintile group where β_2 parameter has positive value.

Based on this initial intuition that volatility term structure is dependent on VIX index level and that the slope of VTS depends on the current level of VIX index we propose two different measures of VTS and three different measures of VRP (Volatility Risk Premium) in order to quantify VTS and risk associated with it. We use the results of regression (1) to construct a reference (theoretical) price of volatility futures as a function of the VIX level (quintile) and time to expiration. The distance between the actual price and the reference price allows us to estimate the VRP. Then, we show that future returns of VIX index and S&P500 index are dependent on the actual level of VIX and the shape of volatility term structure. Thus is confirmed with simple regressions trying to find some robust patterns which then are used in the final investment strategy, where the investment algorithms are based on VTS.

4. Measures of volatility term structure

We propose two measures of VTS and two measures of VRP. The detailed formulas are presented below. *Slope1* is a sum of differences of VIX futures prices with consecutive maturities divided by actual VIX level, calculated separately within each VIX quintile group.

$$slope1_i = \frac{F_j^{(7)} - F_j^{(1)}}{VIX} \quad (2)$$

where:

$F_j^{(7)}$ – close price of 7-th VIX futures within j-th VIX quintile group.

Actual values of *Slope1* confirm our observations from the previous section concerning volatility term structure. We see substantial positive differences between last and first contract levels for first four quintile groups and significant negative differences for the last quintile group (Figure 3).

Another VTS measure (*Slope2*) is the slope coefficient of the simple linear regression, calculated separately for every quintile group of VIX levels:

$$close_i = \beta_0 + \beta_1 * ttm_i + \epsilon_i \quad (3)$$

where:

$close_i$ – daily closing price of VIX and VIX futures,

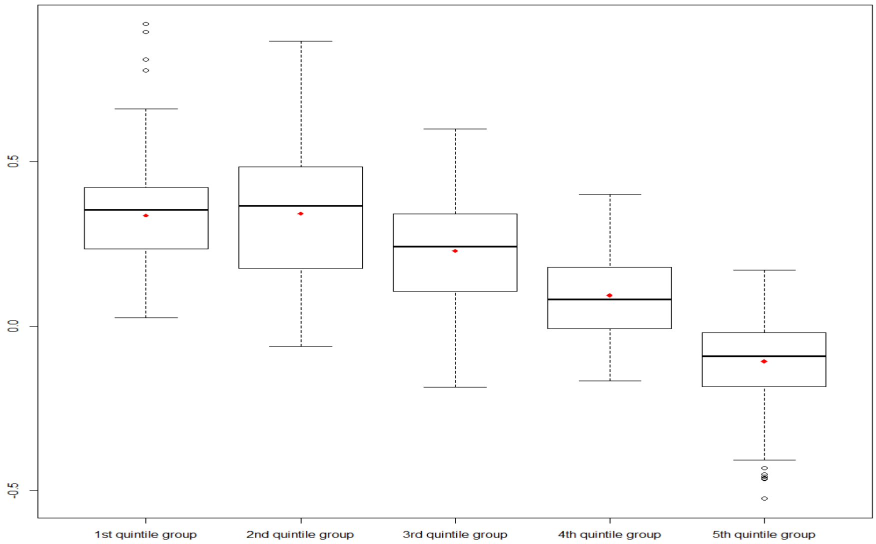
ttm_i – time to maturity of i-th VIX futures, for VIX ttm_i is assumed to be 0;

$i = 1,\dots,n_j$

n_j – size of the VIX j-th quintile group, j = 1,…,5

Slope2 results give support to very similar conclusions as those inferred from *Slope1* values (**Table 2**).

Figure 3. Box plot for *Slope1* with respect to VIX index quintile groups.



Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration. Red dots denote group mean values, while black circles denote outliers which are outside 150% of interquartile range.*

Table 2. The descriptive statistics of *Slope2* for five quintile groups

Group number (i)	Size	parameters VIX level	Min	Max	Avg	Med	SD
1	374	(0;13.14]	0.9	20.3	5.7	5.4	3.1
2	374	(13.14;16.03]	-5.4	20.1	9.4	10.6	5.6
3	375	(16.03;19.55]	-2.2	19.0	8.0	8.7	4.9
4	373	(19.55:25.41]	-10.5	16.1	4.3	4.6	5.1
5	374	(25.41;80.86]	-71.8	8.4	-9.9	-5.6	13.9
ALL	**1870**	**(0:80.86]**	**-71.8**	**20.3**	**3.5**	**4.8**	**10.3**

Note: A*ll calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

As the next step we calculate *individual volatility risk premium* (VRP) by comparing actual VIX futures prices with their „theoretical" values given by formula (1), for each quintile group separately. $VRP_{j,i}^{(k)}$ is defined as a percentage deviation of k-th futures current price from its „theoretical" price:

$$VRP_{j,i}^{(k)} = \frac{F_{j,i}^{(k)} - \hat{F}_{j,i}^{(k)}}{VIX} \tag{4}$$

where:

j – the number of the quintile group, $j \in \{1,2,3,4,5\}$,

k – reference to consecutive VIX futures with ascending time to maturity,

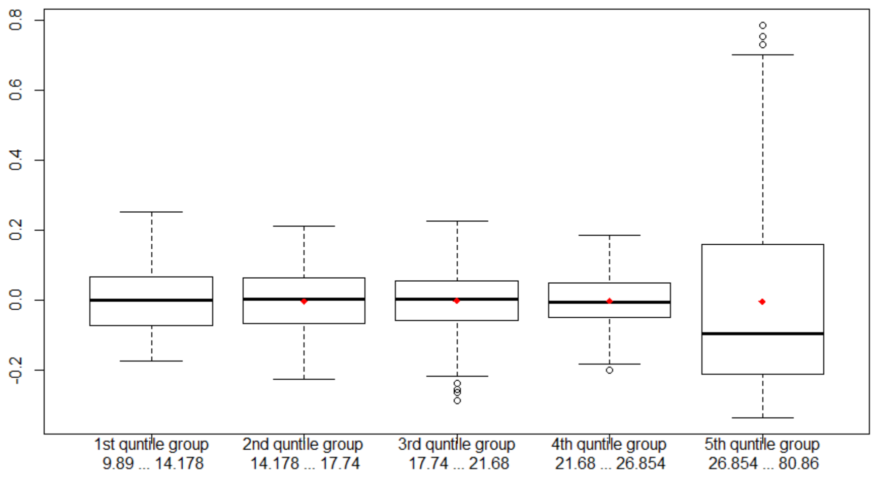
n_j – size of the VIX j-th quintile group, j = 1,…,5

$F_{j,i}^{(k)}$ – closing price of k-th VIX futures contract, within j-th quintile group,

$\hat{F}_{j,i}^{(k)}$ – theoretical value of k-th VIX futures index future, calculated from eq. (1), within VIX j-th quintile group,

VRP_j^I shows us that inside four first quintile groups on average we observe quite substantial departures from „theoretical" volatility term structure ranging between -20% and 20%. What is more important these deviations are neither skewed towards positive nor negative direction what informs us that on average they are equal to zero. The fifth quintile group shows much different picture. Deviations here are much more volatile and they are heavily skewed towards positive values (Figure 4). At the same time, much more than half of observations from this group have values below zero (median is on the level of -10%).

Figure 4. Box plot for VRP_j^I with respect to VIX index quintile groups.



Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration. Red dots denote group mean values, while black circles denote outliers which are outside 150% of interquartile range.*

In the next step, we estimate the *aggregated volatility risk premium* for all maturities and for each quintile group. This measure is proposed in two versions in order to correctly present the direction and the magnitude of deviations from the "theoretical" shape:

- *Aggregated volatility risk premium* is the sum of individual volatility risk premiums for all VIX futures maturities, separately for each VIX quintile group:

$$VRP_{j,i}^{(agg)} = \sum_{k=1}^{7} VRP_{j,i}^{(k)} \tag{5}$$

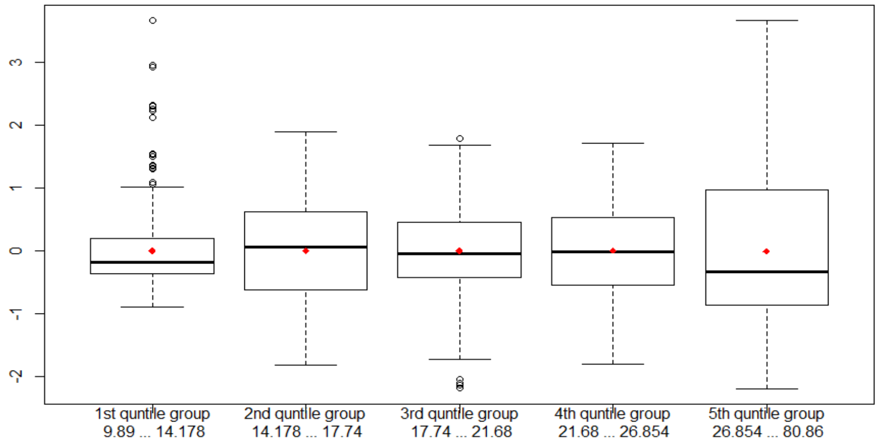
- *Absolute aggregated volatility risk premium* is the sum of absolute individual volatility risk premium for all VIX futures maturities separately for each VIX quintile group:

$$VRP_{j,i}^{|agg|} = \sum_{k=1}^{7} \left| VRP_{j,i}^{(k)} \right| \tag{6}$$

where:
j – the number of the quintile group, $j \in \{1,2,3,4,5\}$,
k – reference to consecutive VIX futures with ascending time to maturity,
$k \in \{1,2,3,4,5,6,7\}$,

$VRP_{j,i}^{(agg)}$ contains information about the magnitude of departures for all maturities taken together, what can be an important factor estimating the value of overall shift of volatility term structure. $VRP_{j,i}^{|agg|}$ contains information about the direction of departures for all maturities together, what can be an important factor in order to estimate the direction of overall shift of volatility term structure. The following two figures show the direction of deviation (Figure 5 - $VRP_{j,i}^{(agg)}$) and their aggregated values (Figure 6 - $VRP_{j,i}^{|agg|}$).

Figure 5. Box plot for $VRP_{j,i}^{(agg)}$ with respect to VIX index quintile groups.

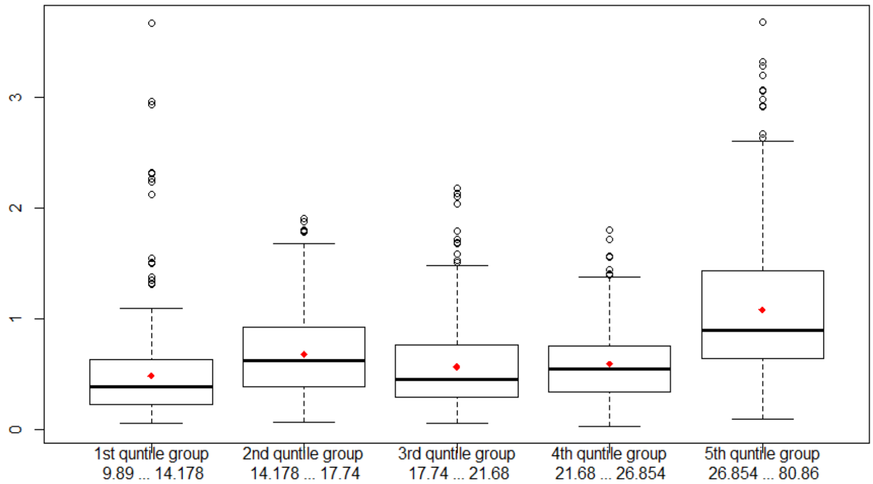


Note: A*ll calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration. Red dots denote group mean values, while black circles denote outliers which are outside 150% of interquartile range.*

Figure 5 allows for similar conclusions as Figure 4 but the range of fluctuations inside each quintile group is now much wider - the reason for that being quite trivial because this is the sum of individual volatility risk premiums. Moreover, we observe a large number of outliers inside the first quintile group. They signal quite substantial deviations of the volatility index futures from their theoretical shape, most often for maturities from the second one up - the result thereof is the highly upward sloping VTS for days with very high values of $VRP_{j,i}^{(agg)}$ or $VRP_{j,i}^{|agg|}$. Furthermore, we observe a positive skewness for the fifth quintile group as is the case for VRP_j^I.

The situation is quite similar when we analyze Figure 6. Once again we observe much wider fluctuations in each quintile group than in the case of Figure 4. Moreover, we identify much more outliers in each quintile group than in Figure 5. Observations in each quintile group are more or less positively skewed with the highest skewness in the fifth quintile group. Additionally, median and mean in fifth quintile group are much higher than in other four quintile groups what is partly the result of the highest fluctuations in case of this group on Figure 5.

Figure 6. Box plot for $VRP_{j,i}^{|agg|}$ with respect to VIX index quintile groups.



Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

5. Forecasting properties of volatility term structure.

In order to check predictive powers of proposed measures of VTS and VRP we present below tables with 1- month returns of S&P500 index and VIX index conditional on five quintile groups of VIX index and:

- Five quintile groups for *Slope1* (Table 3 and Table 4)
- Five quintile groups for *Slope2* (Table 5 and Table 6)
- Four quintile groups[4] for VRP_j^I (Table 7 and Table 8)
- Four quintile groups for $VRP_{j,i}^{(agg)}$ (Table 9 and Table 10)
- Four quintile groups for $VRP_{j,i}^{|agg|}$(Table 11 and Table 12)

We try further to find some patterns in S&P500 index and VIX index returns relationships with various VTS and VRP measures. Table 3 shows that we cannot find any clear dependence between S&P500 return and VIX quintile groups and this observation does not change for all other tables describing S&P500

[4] We divide VRP measures into quartile groups because their fluctuations are not homogenous. We choose quartile groups for VRP instead of quintile groups selected for VIX index in order to make the results and then definitions of investment strategies more transparent.

returns (Table 3, Table 5, Table 7, Table 9, Table 11). Additionally, we can see that S&P500 returns do not depend on *Slope1* levels.

Table 3.Median 1-month S&P500 returns (in %) conditional on VIX &*Slope1* quintile groups.

Group number (i)	*Slope1* / VIX level	I quintile [-0,53: -0,02]	II quintile (-0,02: 0,1]	III quintile (0,1: 0,25]	IV quintile (0,25: 0,39]	V quintile (0,39: 0,92]	**ALL [-0,53: 0,92]**
1	[9,89:14,18]	NA	3,03	1,22	1,74	1,14	**1,27**
2	(14,18:17,74]	5,46	0,81	2,17	-0,18	0,72	**0,86**
3	(17,74:21,68]	0,31	-3,13	0,11	2,34	3,14	**1,19**
4	(21,68:26,85]	1,98	1,39	2,84	3,2	4,12	**2,48**
5	(26,85:80,86]	0,52	2,94	8	NA	NA	**1,23**
ALL	**[9,89:80,86]**	**0,91**	**0,87**	**1,55**	**1,74**	**1,4**	**1,33**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

On the other hand, we can see – in all relevant tables describing VIX returns (Table 4, Table 6, Table 8, Table 10, and Table 12) - that VIX index returns decrease almost monotonically while VIX index level moves from the first to the fifth quintile group. At the same table, we can see that VIX index returns increase almost monotonically while *Slope1* moves from the first to the fifth quintile group. Generally, Table 4 informs us about very high positive VIX index returns for the first and the second VIX index quintile groups and for the fourth and the fifth *Slope1* quintile groups. On the other hand, we observe very high negative VIX index returns for the fourth and the fifth VIX index quintile groups and for the first and for the second *Slope1* quintile groups.

Table 4. Median1-month VIX returns (in %) conditional on VIX &*Slope1* quintile groups.

Group number (i)	*Slope1* / VIX level	I quintile [-0,53: -0,02]	II quintile (-0,02: 0,1]	III quintile (0,1: 0,25]	IV quintile (0,25: 0,39]	V quintile (0,39: 0,92]	**ALL [-0,53: 0,92]**
1	[9,89:14,18]	NA	-0,04	0,91	1,43	5,01	**2,52**
2	(14,18:17,74]	-27,7	-2,88	-8,46	9,84	6,54	**5,36**
3	(17,74:21,68]	1,27	14,95	-0,92	-7,27	-8,53	**-3,39**
4	(21,68:26,85]	-14,99	-1,34	-7,7	-8,5	-11,94	**-7,62**
5	(26,85:80,86]	-11,44	-12,78	-16,01	NA	NA	**-12,06**
ALL	**[9,89:80,86]**	**-11,95**	**-1,9**	**-3,07**	**-1,12**	**2,92**	**-2,68**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration. Red font denotes significantly negative returns while green font denotes significantly positive returns.*

Slope2 discloses additional information Table 5 shows that S&P500 index returns increase monotonically while *Slope2* moves from the first to the fifth quintile group. Table 6, describing the dependence of VIX index returns on the *Slope2* values, allows for almost the same conclusions as in the case of Table 4.

The general conclusion concerning the part of the table with high positive and high negative returns is once again the same as in the case of Table 4.

Table 5. Median1-month S&P500 returns (in %) conditional on VIX &*Slope2* quintile groups.

Group number (i)	*Slope2* / VIX level	I quintile [-71,83: -0,72]	II quintile (-0,72: 2,87]	III quintile (2,87: 6,88]	IV quintile (6,88: 10,54]	V quintile (10,54: 20,36]	**ALL [-71,83: 20,36]**
1	[9,89:14,18]	NA	1,07	1,34	1,23	2,61	**1,27**
2	(14,18:17,74]	5,46	0,81	1,76	1,67	0,07	**0,86**
3	(17,74:21,68]	1,04	-2,57	-2,29	1,63	2,99	**1,19**
4	(21,68:26,85]	1,98	-2,72	2,79	3,27	3,21	**2,48**
5	(26,85:80,86]	0,59	2,88	1,97	9,1	NA	**1,23**
ALL	**[9,89:80,86]**	**0,98**	**0,55**	**1,44**	**1,75**	**1,83**	**1,33**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

Table 6. Median1-month VIX returns (in %) conditional on VIX &*Slope2* quintile groups.

Groupnumber (i)	*Slope2* / VIX level	I quintile [-71,83: -0,72]	II quintile (-0,72: 2,87]	III quintile (2,87: 6,88]	IV quintile (6,88: 10,54]	V quintile (10,54: 20,36]	**ALL [-71,83: 20,36]**
1	[9,89:14,18]	NA	8,63	0,85	3,04	-0,84	**2,52**
2	(14,18:17,74]	-27,7	-2,88	-11,78	2,53	10,79	**5,36**
3	(17,74:21,68]	-14,21	13,89	13,99	-5,05	-8,47	**-3,39**
4	(21,68:26,85]	-14,99	0,63	-3,38	-11,93	-8,33	**-7,62**
5	(26,85:80,86]	-11,91	-11,24	-14,69	-16,1	NA	**-12,06**
ALL	**[9,89:80,86]**	**-12,64**	**0,87**	**-1,71**	**-0,99**	**-0,57**	**-2,68**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration. Red font denotes significantly negative returns while green font denotes significantly positive returns..*

Table 7. Median1-month S&P500 returns (in %) conditional on VIX quintile groups &VRP_j^I quartile groups.

Group number (i)	VRP_j^I / VIX level	I quartile [-0,34: -0,08]	II quartile (-0,08: -0,01]	III quartile (-0,01: 0,06]	IV quartile (0,06: 0,79]	**ALL [-0,34: 0,79]**
1	[9,89:14,18]	0,68	0,95	1,83	1,56	**1,27**
2	(14,18:17,74]	1,77	0,44	-1,74	2,18	**0,86**
3	(17,74:21,68]	1,84	1,19	0,19	1,78	**1,19**
4	(21,68:26,85]	0,56	1,23	2,6	4,78	**2,48**
5	(26,85:80,86]	1,08	2,99	5,68	-1,6	**1,23**
ALL	**[9,89:80,86]**	**1,15**	**1,1**	**1,28**	**1,92**	**1,33**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

Table 7 shows that S&P500 returns increase monotonically with the increase of $\boldsymbol{VRP_j^I}$ values. On the other hand, we cannot see any clear dependence between VIX returns and $\boldsymbol{VRP_j^I}$ values (Błąd! Nieprawidłowy odsyłacz do zakładki: wskazuje na nią samą.). Contrary to the latter, the general conclusion concerning the part of the table with high positive and high negative returns still holds.

Table 8. Median1-month VIX returns (in %) conditional on VIX quintile groups &VRP_j^Iquartile groups.

Group number (i)	VRP_j^I / VIX level	I quartile [-0,34: -0,08]	II quartile (-0,08: -0,01]	III quartile (-0,01: 0,06]	IV quartile (0,06: 0,79]	ALL [-0,34: 0,79]
1	[9,89:14,18]	5,74	0,26	0,49	5,43	**2,52**
2	(14,18:17,74]	-6,32	2,1	15,27	1,69	**5,36**
3	(17,74:21,68]	-12	0,31	3,52	-6,55	**-3,39**
4	(21,68:26,85]	-3,67	-8,01	-8,59	-6,31	**-7,62**
5	(26,85:80,86]	-11,79	-15,9	-16,55	-9,76	**-12,06**
ALL	**[9,89:80,86]**	**-5,37**	**-2,28**	**0,15**	**-3,03**	**-2,68**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration. Red font denotes significantly negative returns while green font denotes significantly positive returns.*

In Table 9 we find the most ideal dependence of S&P500 returns. They increase monotonically with the increase of $\boldsymbol{VRP_{j,i}^{(agg)}}$values. There is no clear dependence between VIX returns and $\boldsymbol{VRP_{j,i}^{(agg)}}$values (Table 10). Once again, the general conclusion concerning the part of the table with high positive and high negative returns still holds.

Table 9. Median1-month S&P500 returns (in %) conditional on VIX quintile groups &$VRP_{j,i}^{(agg)}$quartile groups.

Group number (i)	$VRP_{j,i}^{(agg)}$ / VIX level	I quartile [-2,19: -0,55]	II quartile (-0,55: -0,11]	III quartile (-0,11: 0,58]	IV quartile (0,58: 3,68]	ALL [-2,19: 3,68]
1	[9,89:14,18]	0,77	0,82	2,11	2,02	**1,27**
2	(14,18:17,74]	1,41	1,4	-1,93	2,16	**0,86**
3	(17,74:21,68]	0,28	-0,4	-0,17	2,75	**1,19**
4	(21,68:26,85]	-0,56	1,88	2,6	4,21	**2,48**
5	(26,85:80,86]	0,42	2,9	5,5	0,28	**1,23**
ALL	**[9,89:80,86]**	**0,6**	**1,05**	**1,4**	**2,58**	**1,33**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

Table 11 and Table 12 do not add any new information. There is no any clear dependence between S&P500 returns and $\boldsymbol{VRP_{j,i}^{|agg|}}$. The same conclusion we can drawn in the case of VIX returns and $\boldsymbol{VRP_{j,i}^{(agg)}}$ values (Table 12). The general conclusion concerning the part of the table with high positive and high negative VIX returns holds only in case of the negative returns.

Table 10. Median1-month VIX returns (in %) conditional on VIX quintile groups &$VRP_{j,i}^{(agg)}$quartile groups.

Group number (i)	$VRP_{j,i}^{(agg)}$ / VIX level	I quartile [-2,19: -0,55]	II quartile (-0,55: -0,11]	III quartile (-0,11: 0,58]	IV quartile (0,58: 3,68]	**ALL** **[-2,19: 3,68]**
1	[9,89:14,18]	7,59	3,01	-2,64	2,96	**2,52**
2	(14,18:17,74]	-7,72	0,06	12,13	2,17	**5,36**
3	(17,74:21,68]	-10,23	5,36	-1,48	-9,96	**-3,39**
4	(21,68:26,85]	-5,87	-12,34	-5,21	-7,42	**-7,62**
5	(26,85:80,86]	-8,72	-16,39	-18,52	-11,68	**-12,06**
ALL	**[9,89:80,86]**	**-5,13**	**-0,59**	**-0,56**	**-4,63**	**-2,68**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration. Red font denotes significantly negative returns while green font denotes significantly positive returns.*

Table 11. Median1-month S&P500 returns (in %) conditional on VIX quintile groups &$VRP_{j,i}^{|agg|}$quartile groups.

Group number (i)	$VRP_{j,i}^{|agg|}$ / VIX level	I quartile [0,03: 0,33]	II quartile (0,33: 0,58]	III quartile (0,58: 0,87]	IV quartile (0,87: 3,68]	**ALL** **[0,03: 3,68]**
1	[9,89:14,18]	1,69	0,75	1,31	2,24	**1,27**
2	(14,18:17,74]	-2,28	0,87	1,63	2,16	**0,86**
3	(17,74:21,68]	0,95	-0,76	1,66	2,01	**1,19**
4	(21,68:26,85]	2,74	1,12	2,22	3,83	**2,48**
5	(26,85:80,86]	4,25	3,27	0,38	0,22	**1,23**
ALL	**[9,89:80,86]**	**1,47**	**0,86**	**1,36**	**1,82**	**1,33**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

Table 12. Median1-month VIX returns (in %) conditional on VIX quintile groups &$VRP_{j,i}^{|agg|}$quartile groups.

Group number (i)	$VRP_{j,i}^{|agg|}$ / VIX level	I quartile [0,03: 0,33]	II quartile (0,33: 0,58]	III quartile (0,58: 0,87]	IV quartile (0,87: 3,68]	**ALL** **[0,03: 3,68]**
1	[9,89:14,18]	-2,49	7,54	6,39	-0,44	**2,52**
2	(14,18:17,74]	13,68	5,89	-3,32	-0,57	**5,36**
3	(17,74:21,68]	0,35	1,56	-10,17	-8,34	**-3,39**
4	(21,68:26,85]	-13,8	-3,5	-3,67	-9,24	**-7,62**
5	(26,85:80,86]	-26,61	-15,39	-8,24	-10,27	**-12,06**
ALL	**[9,89:80,86]**	**-3,24**	**1,45**	**-2,73**	**-6,42**	**-2,68**

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration. Red font denotes significantly negative returns while green font denotes significantly positive returns.*

Having found strong dependence patterns we will try to use them to design a simple investment strategy which is supposed to beat the market (S&P500 buy & hold).

6. Investment model

The main task of this section is to design investment strategies which will implement the dependence of S&P500 and VIX index returns on various VTS and VRP measures. Our investment algorithms use the idea of mean reversion process of VIX index fluctuations and additionally, the information hidden in VIX index futures term structure shape and VRP values. Tables and figures presented in this paper confirmed our initial intuition that VIX Index returns (and partly S&P500 returns[5]) depend on the current level of VIX Index, VTS shape and VRP values. Higher slope of VTS (or high VRP value) together with lower VIX index quintile group generally implicates high VIX index returns while lower slope of VTS (or low VRP value) together with higher VIX index quintile group generally implicates very low VIX index returns. Additionally, we observed that S&P returns rises almost monotonically with the increase of VRP values and slope 2 revealing strong dependence between index future returns and the current level of risk perceived by market participants.

We propose five simple strategies which invests in VIX index futures or in S&P500 index[6]. The signal for equity index is always opposite than for VIX index futures. For comparison purposes we use S&P500 buy& hold strategy results. The general assumptions – valid for all strategies - are as follows:

- transaction costs = 0.1%,
- data gathering window: minimum one year,
- first signals: 2008-01-01, then the current closing price is used to generate a new signal for each consecutive day ,
- trade price: closing price of VIX futures with nearest maturity,
- switch to the 2nd contract on the last trading day (rolling yields included),
- two different basis instruments in one strategy: VIX and S&P500 index.
- leverage: 100%
- margin: we do not receive any additional interests from cash above margin,

The detailed assumptions for each strategy and theirs results are presented below.

6.1. I strategy.

buy: if VRP_j^I is in 1st quartile group

sell: if VRP_j^I in 4th quartile group

close: if VRP_j^I in 2nd quartile group after sell signal or in 3rd after buy signal

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5 We admit that dependence in case of S&P500 returns is much weaker than in case of VIX index returns.

6 Roll yield is much less important in case of S&P500 futures than in case of VIX index futures, so we decided to use S&P500 index data because it does not influence final results.

hold: if VRP_j^I in 2nd quartile group after buy signal or in 3rd after sell signal

The first strategy utilize the assumption that VRP is mean reverting process and every departure from „theoretical" volatility term structure should be reverted and should go back to zero. The problem is that while it is true inside VIX quintile groups the logic of the signal can be weakened by switches between VIX quintile groups. This could be the reason of rather poor results of this strategy what is presented in detail on Figure 7 and in Table 13.

6.2. II strategy.

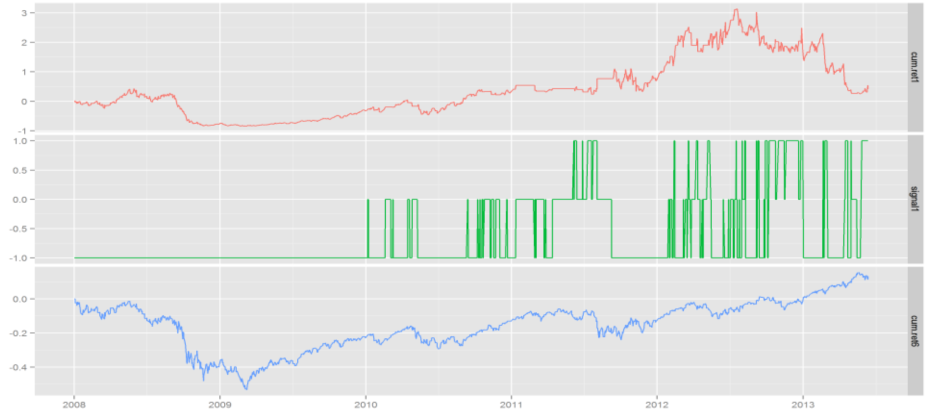
buy: if VRP_j^I in 1st quartile group

sell: if VRP_j^I in 4th quartile group

hold: if VRP_j^I in 2nd or 3rd quartile group

Second strategy is very similar to the first one but it has less strict rules concerning the moment when we close the position. The problem with switches between VIX quintile groups still exists but thanks to less frequent close signals the results of this strategy are much better and among the best ones within all strategies (Figure 8 and Table 13).

Figure 7. Equity line and signal of I strategy for VIX index futures and S&P500 index.



Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

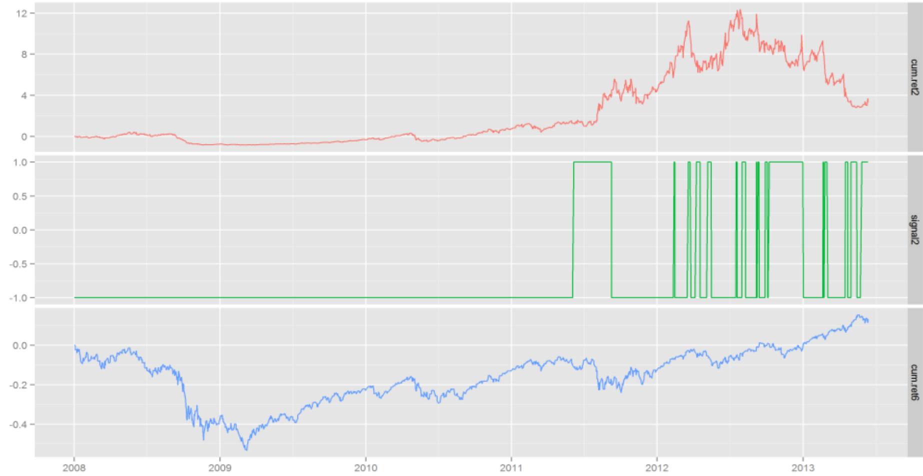
6.3. III strategy.

buy: if VRP_j^I in 1st or 2nd quartile group

sell: if VRP_j^I in 3rd or 4th quartile group

The next strategy characterizes the most often switches between short and long positions but as we can see this modification does not enhance the results (Figure 9 and Table 13)

Figure 8.Equity line and signal of II strategy for VIX index futures and S&P500 index.



Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

Figure 9.Equity line and signal of III strategy for VIX index futures and S&P500 index.

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

6.4. IV strategy.

buy: if VIX in 1st or 2nd quintile group
sell: if VIX in 4th or 5th quintile group
hold: otherwise

Two consecutive strategies base their logic on VIX index levels at the moment when signal is generated. The fourth strategy uses extreme quintiles of VIX in order to generate buy and sell signals for VIX index futures. The position is hold until the opposite signal is generated. The results of this approach are rather poor (Figure 10and Table 13).

Figure 10. Equity line and signal of IV strategy for VIX index futures and S&P500 index.

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

Figure 11. Equity line and signal of V strategy for VIX index futures and S&P500 index.

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

6.5. V strategy.

buy: if VIX in 1st or 2nd quintile group
sell: if VIX in 4th or 5th quintile group
close: otherwise

The fifth strategy uses the same logic as the fourth one but closes positions more often what in fact is once again not the best option (Figure 11 and Table 13).

6.6. Comparison of all strategies.

After detailed analysis of presented results we can notice that we can beat the market represented by S&P500 buy&hold (VI strategy) only in case of one investment strategy (II strategy). Return and Return&Risk statistics are much better for this strategy but at the same time risk statistics informs us that II strategy is characterized by much higher level of aSD and MaxD.

Table 13. Return and risk statistics for all strategies under investigations.

	#1	#2	#3	#4	#5	#6 – S&P500 buy&hold
ARC(%)	6,56	30,81	-36,84	-17,80	-29,24	2,28
aSD(%)	71,48	81,02	81,17	54,97	81,09	25,43
IR	0,09	0,38	-0,45	-0,32	-0,36	0,09
Sharpe	0,08	0,37	-0,46	-0,34	-0,37	0,06
Treynor	0,03	0,18	-0,21	-0,14	-0,20	0,02
Beta	1,67	1,67	1,80	1,32	1,47	1,00
MaxD(%)	-69,58	-71,57	-95,04	-89,55	-90,82	-53,25

Note: *All calculations were made on the data from 01/01/2006 until 01/07/2013 on the basis of VIX index futures with up to 7 months to expiration.*

7. Conclusions and extensions of this research

Based on the presented results we can conclude that:

1. VTS shape and VRP values are important in order to predict VIX and S&P500 index futures .
2. We observe very high positive VIX index returns for the first and the second VIX index quintile groups and for the fourth and the fifth *Slope1*(*Slope2*, VRP_j^I, $\boldsymbol{VRP_{j,i}^{(agg)}}$, $\boldsymbol{VRP_{j,i}^{|agg|}}$) quintile groups. On the other hand, we observe very high negative VIX index returns for the fourth and the fifth VIX index quintile groups.
3. It is possible to use information from VIX index futures term structure in order to construct profitable strategies (strategy II) which enhance our Return&Risk ratio in comparison to S&P500 buy&hold.

We think that it would be important to extend the conclusions of this research on other volatility and equity index futures (e.g. Vstoxx and

EuroStoxx50, VNKY and Nikkei225 and other volatility futures quoted on CBOE/CFE). Additionally, we would like to test investment strategies where each characteristics will be calculated on rolling two years window instead of anchored window used in this study. Further research on various definition of volatility term structure are even more important because more adequate reference to so called "normal" or equilibrium level of VTS is the crucial point in defining diverse volatility arbitrage strategies.

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