The Estimation of Financial Markets by Means of a Regime-Switching Model

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Zürich, May 2010

Alvin Schwendener

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Abstract

A number of economic and financial time series show abrupt changes in their behavior. This fact has increased the interest in modeling economic and financial time series with periodic switches in parameters. The work at hand develops a Markovian regime-switching model based on a multiple regression framework to describe the dynamics of financial markets. It aims to extend common approaches by considering not just one type of economic or financial time series, but a whole asset universe within one regime-switching model.

The challenging international environment of portfolio management is characterized by the uncertainty of the financial markets, which are in a constant motion. Continuously changing risk/return-structures influence the potential of diversification of international asset allocation. The regime-switching model illustrated allows, by means of two different regimes, the computation of the dynamic risk/return-structure of any international asset universe. The extension to include different asset categories like stocks, bonds and alternative assets in one single model allows to capture the established structures of financial markets, such as fat tails, contagion, decoupling, volatility clustering and other empirical properties of asset returns. By emphasizing the risk/return-structures in order to interpret the potential of diversification for the asset allocation, this work aims to point out the international diversification constituted by the regime switches. Numerical results are presented, which correspond to the last three decades, inclusively to the latest crisis of the financial markets. The Markovian regime-switching model that is introduced estimates the market dynamics using a maximum-likelihood approach.

Finally, an application of the regime-switching model will be introduced to quantify the added value and the performance of a regime-switching framework used in a dynamic asset allocation approach. This introduced asset allocation model is characterized by a dynamic expectation variance analysis and is based on a multistage, stochastic programming. The resulting allocation strategies are optimal in a multiperiod context. They are efficient under consideration of transaction costs, rebalancing activities, stochastic correlations and volatile financial markets. The timing information obtained

from the Markovian characteristics is optimally used by the multistage approach over the whole planning horizon, which is divided into three time periods.

The framework is designed to include liabilities of financial institutes that have to fulfill a pension or insurance mandate. This extension leads to 'liability-adjusted' diversification and integrated portfolio management. Alternatively, one can include a benchmark index in the optimization. This enables to assess his own portfolio result against the background of the general market development, represented by the benchmark index. It is shown that with the help of the timing information, obtained from the Markovian process, benchmarks are not just successfully tracked but even outperformed.

Further the benefit of international investment is being quantified. Despite globalization and increasing contagion effect the potential of international diversification is being revealed and suitable hedging strategies are introduced.

Zusammenfassung

Viele verschiedene Finanz- und makroökonomische Zeitreihen weisen in ihrer stochastischen Entwicklung plötzliche Verhaltensänderungen auf. Diese Tatsache hat das Interesse geweckt, Finanz- und ökonomische Zeitreihen mittels periodisch wechselnder Parameterwerte zu modellieren. In der vorliegenden Dissertation wird ein Markov Regime-Switching Modell entwickelt, das auf einem multiplen Regressionsansatz aufbaut, und das der flexiblen Modellierung der Finanzmarktdynamik dienen soll.

Finanzinvestoren befinden sich in einem anspruchsvollen Umfeld, das unter anderem durch die Unsicherheit an den Finanzmärkten, die sich in ständiger Bewegung befinden, charakterisiert ist. Die sich verändernden Risiko/Rendite-Strukturen nehmen einen starken Einfluss auf das Diversifikationspotenzial internationaler Asset Allokationen. Das Regime-Switching Modell ist in der Lage, mittels zwei verschiedener Regimes die dynamische Entwicklung dieser Risiko/Rendite-Strukturen zu erfassen. Die Berücksichtigung verschiedener Anlageklassen wie Aktien, Obligationen und alternative Anlagen ermöglicht es, die wichtigsten Beobachtungen aus der empirischen Finanzmarktforschung im Modell abzubilden: 'Fat tails', Contagion, Decoupling, Volatilitäts-Clustering sowie weitere empirische Finanzmarkterkenntnisse. Es werden numerische Ergebnisse der letzten drei Jahrzehnte präsentiert, wobei auch auf die jüngste Finanzkrise eingegangen wird. Das eingeführte Markov Regime-Switching Modell schätzt die Finanzmarktentwicklung anhand des Maximum-Likelihood Ansatzes.

Zum Schluss wird eine Anwendung des Regime-Switching Modells vorgestellt. Anhand eines dynamischen Asset Allokations Modells, das auf mehrstufiger stochastische Programmierung basiert, wird der Mehrwert und die Performance des Regime-Switching Modells aufgezeigt. Das Asset Allokations Modell ist charakterisiert durch eine Dynamische Erwartungswert-Varianz-Analyse und basiert auf der mehrstufigen, stochastischen Programmierung. Die daraus resultierenden Allokationsstrategien sind auch im Sinne der Mehrstufigkeit optimal. Sie sind effizient unter der Berücksichtigung der Transaktionskosten, der Rebalancierungsaktivitäten, der stochastischen Korrelationen und der volatilen Finanzmärkte. Die Timing-Information, die der Markov-Charakteristik zu entnehmen ist, wird im mehrstufigen Ansatz über den ganzen Planungshorizont, der in drei Zeitperioden unterteilt ist, optimal eingesetzt.

Die Struktur des Modells ist so konzipiert, dass Liability-Indizes von Pensionskassen oder Versicherungen in der Optimierung berücksichtigt werden können. Dies führt

zu 'liability-adjustierter' Diversifikation und zu einem integrierten Portfoliomanagement. Alternativ können Benchmark-Indizes in der Optimierung integriert werden. Dies ermöglicht die Performance des eigenen Portfolios vor dem Hintegrund der allgemeinen Marktentwicklung zu beurteilen. Es wird aufgezeigt, dass dank der Timing-Information, die aus dem Markov-Prozess gewonnen wird, Benchmark-Indizes nicht nur nachgebildet sondern outperformt werden.

Des Weiteren wird der Mehrwert einer internationalen Allokation quantifiziert. Trotz Globalisierung und dem immer stärkeren Contagion-Effekt werden das Potential der internationalen Diversifikation aufgezeigt und geeignete Hedging-Strategien besprochen.

Chapter 1

Introduction

1.1 Motivation

Financial investors are situated in a challenging environment, which is characterized by the uncertainty of financial markets. The risk/return-structures of the financial markets are in constant motion and present risks but also opportunities to investors. It is important for the investor to be informed about these dynamic processes in order to adequately model and forecast the markets and to be able to compose efficient portfolios. Many financial time series show certain patterns in their behavior, which are characterized by periodic, temporary and dramatic breaks.

For that reason regime-switching models - represented by the influential paper by Hamilton, 1994, [52] - have become more and more popular for appropriately modeling the abrupt changes of the financial markets. In an application of regime-switching models Ang & Bekaert, 1999, [5] point out that stock markets are characterized by two regimes. The stock markets in the first regime are more volatile than in the second one and, compared to the latter, contain different correlations. Based on existing models found in literature, the present work implements an extended regime-switching model to estimate and forecast the financial market dynamics. Besides stock markets, it will also include bond markets as well as alternative assets in order to model established patterns of asset risk/return-structures and to get an overall picture of financial market dynamics.

An application of the regime-switching model will be introduced to quantify the added value and the performance of a regime-switching framework used in a dynamic asset allocation approach. The properties of the risk/return-structure modeled by the regime-switching framework is being consequently used for the scenario generation in this stochastic multistage program. In analogy to the well-known mean-variance model introduced by Harry Markowitz, the application is named DEVA (dynamic expectation variance analysis). It expands the single period approach to a multistage stochastic optimization approach for the identification of tactical and strategic asset allocations. The identified allocation strategies are efficient in a multi-period context, i.e. under consideration of rebalancing activities, transaction costs, stochastic correlations and volatile financial markets. The dynamic asset allocation approach has been extended and adapted for financial institutes, which have to fulfill a pension or insurance mandate (DEVA + L, where L stands for liability), and for investors, who want to track with their own investments a specific benchmark (DEVA + B, where B stands for benchmark) (see Frauendorfer, 2008, [39]).

1.2 Research Question

The dissertation aims to make a contribution towards the development of the risk/return-structure of financial markets over the last three decades, whereby the consolidated findings in empirical finance will be questioned in the light of the latest dynamics. It will point out the different risk return structures of an asset universe due to dissimilar home currencies, in particular the Swiss franc (CHF), euro (EUR) and U.S. dollar (USD), and due to different time periods.

Based on the implementation of the regime-switching model, this work aims to contribute a solution to the challenges of the optimization of high-dimensional non-convex likelihood functions. The optimal start-up values and the boundaries of the estimated parameters are verified. Reliable start-up values and well-placed bounds for the parameters have to be defined in order to avoid singularities and to find satisfying local maxima.

The ongoing globalization, respectively the increasing contagion effect, might raise the concern that international investing is not worth the trouble, since its benefits are not forthcoming at volatile times, where investors need them most. The benefit of international diversification might be neglectable within a static one-period mean-variance framework. However, Ang & Bekaert, 1999, [5] gave proof of the benefit of international investing by analyzing the impact of time-varying correlations on asset allocation in a dynamic portfolio allocation problem. In combination with an introduced multistage dynamic asset allocation approach, the work at hand will prove that despite globalization and an increasing contagion effect, the diversification benefit of international investing still exists. In combination with the multistage dynamic asset allocation, the information of the Markov process, calibrated from the regime-switching model, is being tested on its timing abilities.

Further, this multistage dynamic asset allocation approach aims to prove that the explicit consideration of the liability- or of the benchmark-dynamics optimizes the asset allocation in order to guarantee on the active side the incurred liabilities at all times, respectively to well track the underlying benchmark.

1.3 Structure of the Thesis

Chapter 2 analyzes the most important empirical observations discussed in financial literature which are essential for the generation of a model in line with the market. Chapter 3 provides a classical data analysis of financial time series of the last three decades. Before introducing the regime-switching model in chapter 5, chapter 4 contains the literature research of the chronological development of the regime-switching approach and introduces two important theoretical inputs used for the regime-switching model. In chapter 5 the regime-switching model will then be introduced in great detail. The fundamental ideas and the mathematical derivation of the regime-switching approach will be declared, and the optimization of a non-convex likelihood function will be thematized. Chapter 6 contains an illustrative example based on real market data and discusses the empirical results. Before concluding, chapter 7 introduces a dynamic asset allocation approach to determine the added value and the performance of the regime-switching model.

Chapter 2

Patterns of Financial Market Dynamics

Consolidated findings of empirical finance have revealed some patterns in the dynamic international environment of financial markets. Many empirical observations that are essential for asset allocation management are found in financial literature. In order to appropriately model the financial market dynamics, a certain understanding of these empirical findings is needed. The most important stylized facts of financial market dynamics - like *fat tails, volatility clustering, contagion, co-movement, decoupling,* as well as the *safe haven* function of the Swiss franc - will be explicitly introduced before the regime-switching model is presented. On the one hand the stylized facts reveal the patterns and to a certain point the risk-structure of the observed markets that a suitable model needs to satisfy. On the other hand they give a certain guideline to verify the performance of the chosen model. Although the most stylized facts are elaborately discussed in literature, in the work at hand they are introduced in a short manner.

2.1 Stylized Facts

2.1.1 Fat Tails & Return Asymmetry

Crashes and booms of financial markets lead to a high probability of occurrence of violent market movements, which can not just be negligible by labeling them as outliers. The studies of the asymptotic behavior of the extreme observations, is an important relevance for risk management purposes and a requirement to correctly calculate certain risk measures (Cont, 2001, [26]).

The assumption to consider the empirical distributions of economic and financial time series as normal 'iid' distributed is often not confirmed. In contrast, many economic and financial time series exhibit fat tails, meaning non-Gaussian, sharp-peaked and heavy-tailed distributions (see e.g. Cont, 2001, [26]). This phenomenon was first pointed out by Mandelbrot, 1963, [72] and has been well documented in literature (see e.g. Bidarkota & Dupoyet, 2004, [6] or Blanchard & Watson, 1986, [8]). A distribution with fat tails enables the accommodation of the likelihood of large positive and negative shocks impacting the economy. The normal distribution is inappropriate, since it undervalues the rare events continuously. This might have serious consequences for the asset allocation and its risk management. It may lead on the one hand to payment shortage or even insolvency and on the other hand to arbitrage due to mispricing. Rachev, 2005, [79] shows that fat tails have a significant effect on the optimal portfolio selection. Costs accrue by ignoring them and the calculated risk premiums are significantly underestimated. Hence, he points out how important it is to use a model that satisfies the fat tail assumption of asset returns. However, Bidarkota & Dupoyet, 2004, [6] describe the Gaussian distribution as well understood and analytically tractable, explaining its dominant position throughout macroeconomics and finance.

A popular way to quantify how strong a single marginal distribution differs from the normal distribution is by calculating its kurtosis. The kurtosis (κ), measures the fatness of the tails and is determined such that $\kappa = 3$ for a normal distribution. Distributions with a kurtosis $\kappa > 3$ are called leptokurtic and are characterized by sharp peaks and heavy tails. Cont, 2001, [26] summarizes the empirical results by saying that financial time series tend to have a kurtosis of $\kappa > 3$. He further emphasizes that especially intraday exchange rates exhibit high kurtosis.

However, the kurtosis says nothing about the asymmetry of a distribution. The skewness though, the fourth moment of a distribution, describes the different behavior and different fatness of the left and the right tails. Most financial time series exhibit negative skewness, increasing the probability of big negative realizations and therefore fortifying the fat tail on the left (negative) side. Longin & Solnik, 2001, [69] reject in their study - using extreme value theory[1] to model the multivariate distribution tail - the null hypothesis of multivariate normality for the negative tail, but not for the

[1]The *extreme value theory* provides a framework to estimate the level of 'fatness', by measuring the density in the tail. This allows a more precise evaluation of one's risk exposure.

positive tail. These findings confirm the excess kurtosis and the negative skewness, leading to asymmetric and fat-tailed distributed asset returns. Therefore, at least the first four moments of a distribution are required, in order to successfully fit various functional forms to the distribution of asset returns. According to CONT, 2001, [26] the normal inverse Gaussian distributions, the generalized hyperbolic distributions and the exponentially truncated stable distributions meet these requirements.

As a direct consequence of the result of these findings commonly used risk measurements like the variance or standard deviation and the Sharpe ratio have been criticized (see e.g. ROCKAFELLAR & URYASEV, 2002, [83]), because they do not account for the phenomenon fat tails in loss distributions and even penalize ups and downs equally. Consequently, risk measures like the value at risk (VaR) or lower partial moments (LPM) set the new standards. Many researchers have explored the best measures of risk to be applied in portfolio allocation problems. For example BODA & FILAR, 2006, [10] introduced an alternative *target percentile risk measure* that is applicable to multi-stage investment problems. Based on a study that proves the capability of explaining the negative fat tail of a distribution, RACHEV ET AL., 2004, [80] propose new measures of risk, the so-called *Rachev ratios* (R-ratio), which are based on the value at risk (VaR).

A further risk measurement approach is given by calculating and displaying the *profit & loss distribution* (P&L). The P&L-distribution, equivalent to the cumulative distribution function, captures the first four moments of a stochastic wealth. Next to the mean it exposes the distribution's volatility by its slope as well as the kurtosis and skewness by its shape. Notice, that the P&L-distribution also contains fat tails. Therefore, it is suited for the calculation of additional risk measures. For example, for the values at risk (VaR) that are, due to the P&L-distribution, known at any desired percentage level. A further advantage of this risk measure is that it takes not only the risks but also the chances into account. The model presented in the work at hand models the fat tails and skewness through mixtures of normal distributions and uses the P&L-distribution as the key figure of risk management.

2.1.2 Volatility Clustering

It is well known that the volatility of financial time series varies in the cause of time. As early as the 1960s, MANDELBROT, 1963, [72] observed a certain pattern in this volatility variation, which he summarized as follows: "Large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes" ([72],

p. 418). Hereinafter, many researchers observed this characteristic, which became the well-known phenomenon called *volatility-clustering*.

Many studies have shown that short-term price movements of liquid markets, like e.g. equity or exchange rate markets do not possess any significant linear autocorrelation. CONT, 2001, [26] states that for a time lag of greater than 15 minutes it can be assumed to be zero. He adds that price movements on weekly or monthly bases, however, do exhibit some autocorrelation, although the statistical proof is less conclusive and varies strongly from sample to sample. While returns themselves are non-significantly autocorrelated, nonlinear function of returns, such as absolute or squared returns reveal significant positive and slowly decaying autocorrelation of financial time series (see figure 2.1). Such nonlinear functions are quantitative arguments of the *volatility clustering* phenomenon. Therefore, high or, respectively low volatility events tend to cluster over time, indicating volatility clusterings (see e.g. CONT, 2005, [25]).

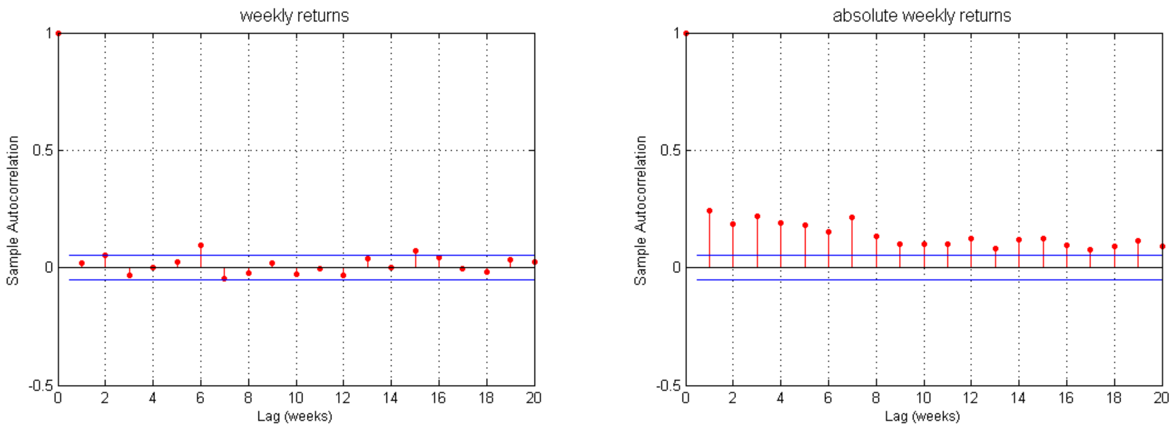

Figure 2.1: **a)** Non-significant autocorrelation of weekly returns of MSCI World Index January 1, 1980 to March 19, 2009. **b)** Significant positive autocorrelation of absolute weekly returns of MSCI World Index January 1, 1980 to March 19, 2009.

One of the first contributions towards modeling heteroscedasticity (time-varying volatility) was made by ENGLE, 1982, [35], who introduced the so-called ARCH-Models. This approach distinguishes between a constant, unconditional variance, modeled by a 'white noise', and between a variable variance, conditional on past residuals. The second variance term enables the modeling of time-dependent volatilities. These new classes of stochastic processes - generalized (GARCH-Models) by BOLLERSLEV, 1986, [11], allowing longer asset history and a more flexible lag-structure - compute the autocorrelation and indicate the volatility structure of a time series. Next to the (G)ARCH-models also stochastic volatility models are often intended to model this phenomenon. As for instance, in a regime-switching model the volatility clusterings are generated exclusively by Markov chain processes hidden in the regime-switching approach. As the probability of a regime change is small, a high- (low-) volatile state is more often followed by a high- (low-) volatile state. Thus, the Markov structure of the model yields volatility clusters. Gulko, 2002, [49] showed through an event-study analysis that clusters of volatile times occur during and after crashes in the financial markets (see also Gulko, 2002, [49]).

According to Gray, 1996, [47] basic structure of existing models, like GARCH, provide a good forecast of the volatility process of financial markets, but structural change in the variance process might lead to bad results. Stating more precisely, Gray adds that the source of this miss-specification of existing models is found in the relatively inflexible structural form of the conditional variance, which is held fixed throughout the sample period. In this sense one can speak of single-regime models. The phenomenon of the volatility clustering however, characterized by the well-known high persistence of financial shocks, leads to structural changes in the variance process. Single-regime approaches, like GARCH can not capture persistence of such periods. Particularly, it underestimates the variance in times of crises, while overestimates them in low-volatility times. The inflexibility of the GARCH parameters leads to poor forecasting results for financial time series that have structural changes over time in their variances. Therefore, models like the regime-switching model, in which the parameters are allowed to switch over time lead to a better fit in modeling volatility of financial markets. For each regime there is a set of parameters, that is fixed over the entire sample period and that has assigned a relative likelihood of its occurrence. Therefore, regime-switching models are a more flexible but compact structure, having the ability to capture structural breaks of time series within a single, unified model. It even allows to combine the regime-switching model with the GARCH approach in order to improve the volatility forecast. These compact regime-switching approach remains tractable and easy to estimate.

2.1.3 Volatility Co-Movement

Changes in volatility tend to occur at the same time in different national markets. History showed that turbulences in important financial centers have an effect on the volatility of worldwide markets, which is called the co-movement effect. Many studies provide evidence of equity market co-movement across national borders. For example, Connolly & Wang, 2003, [24] reveal strong co-movement in international equity markets. In particular they emphasize the coincident correlation among foreign intra-day returns and domestic overnight returns in the U.S., UK and Japanese markets.

Edwards, & Susmel, 2003, [32] found some evidence of interest-rate volatility co-movement. These joint volatility movements influence the distribution of portfolio returns and therefore the portfolio selection as well as the investor's risk management.

2.1.4 Contagion

In the past, little thoughts were given to the option that financial crises could be contagious, due to the fact that the causes of the crises in the early 1980s were mostly attributed to the poor domestic policies and to the high real interest rates in the United States. However, further widespread crises in the 1990s induced the economist to do more research on this 'new' phenomenon of contagion (see Kaminsky & Reinhart, 2000, [60]). Not even good fundamentals were able to insulate a country from the effects of financial contagion. In October 1997 even the Hong Kong Exchange crashed, although there was little evidence of the type of corruption and cronyism that often have been linked to the Asia crisis (see Bordo & Murshid, 2000, [14]).

Different studies have shown that the intensity of causal relationships between nations varies over time. The variance-covariance matrices of international investments are therefore not stable in the cause of time (see e.g. Diebold, F. X., & Yilmaz, K., 2008, [30]; Gebka, B., & Serwa D., 2006, [44] or Longin & Solnik, 1995, [70]). They even change significantly across regimes. The correlations among international stock markets increase in volatile times (e.g. market crashes) and obtain extreme highly positive values. Contagion drives market correlations to unity, and it reduces the potential of portfolio diversification at the time investors need it most (see Gulko, 2002, [49]).

The definitions of contagion vary considerably in literature. They reach from broad to very restrictive definitions. According to the definition, different reasons are considered to be the cause of these two phenomena. Three possible explanatory models might initiate contagion and co-movement among national markets. These are the monsoonal effects, spillover effects and true contagion (see e.g. Masson, 1998, [73] or Tschabold, 2002, [98]).

Monsoonal effect The *monsoonal effect* describes the coherence of the financial markets with an exogenous event that triggers several countries at the same time into a crises. Therefore, the simultaneous occurrence of financial crises is not a result of a chain reaction started by one troubled economy, but a result of a declining fundamental factor of the global business environment. One example of such a exogenous effect is the increase of the world interest rate in the beginning of the eighties, leading into a worldwide debt crisis (see Lutz, 2000, [71]).

Spillover effect The *spillover effects* are defined by the interdependence among national economies. A crisis in a national market might bring other nations to a crisis, by exerting negative influence on these nations' macroeconomic fundamentals. Literature is inconsistent about the transition channels of this interdependence. For Masson, 1998, [73] the commercial and financial relations play no significant role in explaining this interdependence. However, Kaminsky & Reinhart, 1999, [59] put this interdependence down to commercial and financial trade relations and define this phenomenon as fundamental-based contagion.

True contagion Masson, 1998, [73] reserves the term *true contagion* for the transmission of crises that cannot be interpreted by changes in macroeconomic fundamentals. True contagion has its origin in the reaction of financial markets. A crisis in a country causes market sentiment in other countries. It might lead to changes in the risk attitude and in the interpretation given to existing information. If asymmetric information is present it triggers also herd behavior.

This work at hand does not distinguish between these various reasons and applies the term contagion to all three explanatory models, since we are rather interested in the statistical result, than in the cause of the phenomenon. That is to say in the resulting correlations and the derived diversification potential.

2.1.5 Decoupling

At the time stock markets crash, one can observe that bond markets tend to rally, cause investors flee to the safety fixed-interest securities. Due to speculations, risk aversions, trend of interest rates and monetary policy the historical normal situation of positive correlations between stocks and bonds changes in volatile times. The correlation even switches signs from barely positive to significant negative correlations (see e.g. Spremann & Gantenbein, 2005, [91]). Such a situation is maintained days and weeks after a crash and is called decoupling. The significance of this sign switch was given by Connolly, Stivers, & Sun, 2005, [23]. They documented that - for most countries - the correlation of daily stock and bond returns switch signs from significantly positive in low-volatile periods to significantly negative in high-volatile periods. Chou, &

LIAO, [22] confirm in their study of the year 2008 - using a Modified Dynamic Conditional Correlation, called DCCX model - the negative stock-bond returns relation with two measures of market uncertainty: stock volatility and unexpected stock turnover. This phenomenon denotes that bonds offer an effective portfolio diversification during turbulent stock market times (i.e. during a financial crisis), the time when diversification is needed most. Since whenever an internationally important stock market crashes, other national equity markets tumble as well in market contagion (see e.g. GULKO, 2002, [49])[2].

2.1.6 Safe Haven

A safe haven currency is characterized by its reaction in times of crises. It is a misconception to think that a safe haven currency is strong at all times. It is neither immune to the economic developments of the nations surrounding it nor resistant to the markets in which it interacts (see SUESS, 1999, [94]). In times of equity market downturns a safe haven currency appreciates against other currencies, while in upturn markets it depreciates.

The so-called carry trades act as opponents to the safe haven currencies. Their interaction is characterized by a mutually fortifying mechanism. In a carry trade the investor borrows low-yielding currencies in order to reinvest it in high-yielding currencies. Next to the exchange rate risk the investor also takes the interest rate risk. The return of a carry trade consists of two components: On the one hand of the difference between credit and debit interest rates, on the other hand of the gain/loss of the exchange rate development. It is known that since spring 1994 the two common safe haven currencies (Swiss franc and Japan yen (JPY)) have possessed rather lower interest rates compared to the other three world currencies (USD, EUR, British Pound (GBP)). In low-volatile times, when safe haven currencies lose value, carry trades force this depreciation inasmuch as investors sell the safe haven currency for a higher-yield currency. As long as the exchange rates are stable and move within a certain bandwidth the deal is profitable. However, when market volatility becomes more intense, the refuge currency appreciates, the credit, held in this currency, becomes more expensive. Investors then tend to back out of the carry trade deal, boosting the appreciation of the safe haven currencies.

[2]Please note that it basically concerns the treasury bonds, for the simple reason that during financial crises, corporate bonds and mortgage securities often decouple from treasuries and rather move with the equities.

It is a matter of common knowledge among foreign investors that the Swiss franc usually appreciates, even against other strong currencies, when markets become volatile. According to the study of Ranaldo & Söderlind, 2007, [81], the Swiss franc has the strongest safe haven attributes. It is followed by the JPY and the EUR, who also have been used as refuge currencies, although to a smaller extent. In contrast the USD has behaved pro-cyclically with the stock markets. Consequently, a safe haven currency offers a diversification alternative to foreign investors in times of high volatility. However, for Swiss investors the benefit of international diversification is reduced in high-volatile market situations.

2.2 Summary

Two different states of markets (regimes) derived from these stylized facts just elucidated. The relevant determination criteria in the model is given by the volatility of the stock markets. While the first regime is characterized by high volatility, regime 2 reveals lower volatility and by reason of the observed facts also a different risk-structure.

The risk structure of the volatile regime 1 is characterized on the one hand by highly positive correlations among stock markets and on the other hand by significant negative correlations between stock and bond markets. Regime 2, denoting lower volatility markets reveals another risk structure and therefore, a different potential of diversification. The correlation among stock markets turn out to be lower compared to regime 1, even though they are still significantly positive. By contrast with regime 1, the relation between stock and bond markets in regime 2 appears to be significantly different. The correlation even switched signs and proves to be positive. Spremann & Gantenbein, 2007, [90] locate different behavior of equity markets within the regimes and define their historical volatilities. They concluded that in regime 1 the equity markets wave in a wild 'zigzag-curve' (volatility of around 30% to 40%), whereas in regime 2 trends can be identified (volatility of around 15% to 25%). Roughly speaking, regime 1 shows typical features of the *bear* market and regime 2 - with its opposite structure - bears resemblance to the *bull* market (see table 2.1).

Figure 2.2a) illustrates the anti-cyclical pattern of the correlation $\rho_{BD,EQ}$ and the market volatility. It is conspicuous how low the correlation between bond and equity remained in the low-volatile time period from 2004 to 2007. They barely reached a positive correlation, but by no means the high values of earlier non-volatile time periods. However, it fell sharply in the present turbulent market situation, offering

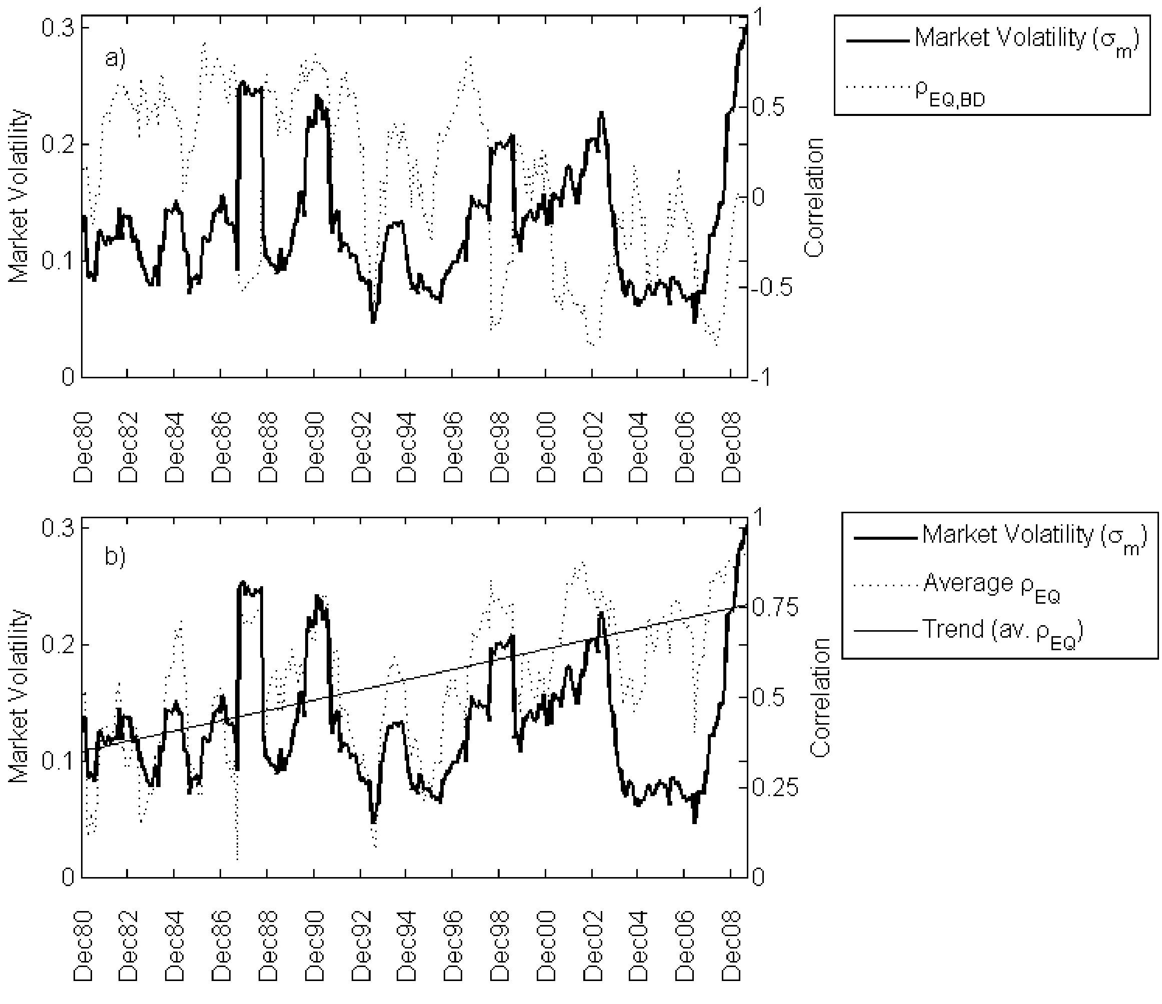

Figure 2.2: a) Market volatility and correlations between stock markets and bond markets over time (moving average: 12 months). **b)** Market volatility and average correlation among stock markets over time (moving average: 12 months).

	Regime 1	**Regime 2**
Volatility:	high	low
Correlation among stocks:	highly positive	low, but positive
Correlation (stock, bond):	negative	positive

Table 2.1: Risk structure of the stock and bond markets depending on the regimes (states of market).

a certain diversification potential in volatile times. Nevertheless, the two regimes are still recognizable.

Figure 2.2b) illustrates the parallel running behavior between the average correlations among national stock markets $\bar{\rho}_{EQ}$ and the market volatility. It is observable that the average correlation among stock markets in the last three years did not fall as low as in the nineties, even though the volatility of the markets was rather low. This might be a result of the age of globalization. Nevertheless, it fell under 0.5 in the beginning of the year 2007. This development of growing correlations of international equity returns was already proved to be significant by Longin & Solnik, 1995, [70]. They indicated that the international correlation between the markets increased over the years 1960 to 1990, reflecting the 'global finance' phenomenon (see also Koch & Koch, 1991, [65] or Furstenberg & Jeon, 1989, [99]). The two regimes are though still recognizable.

The work at hand shows that the regime-switching models are able to capture the changing risk/return-structures of asset returns as well as their higher moments by introducing a state variable, the regime, which can take on two values.

Chapter 3

Data Analysis

The latest trends of financial markets call for a new examination of the stylized facts. It is of interest how the patterns, characterized by the stylized facts, evolved over time. The ongoing globalization, the launch of the European unit currency euro as well as other impacts might have an influence on these common beliefs. These influences will be elaborated more detailed in this chapter and will provide, along with a classical data analysis of financial time series, a first impression of the market dynamics over time. The financial time series refer to monthly excess returns covering period January 1980 to December 2008[1] (348 monthly observations per time series). All time series are held in Swiss franc in order to simulate a portfolio from a Swiss investor's point of view.

3.1 Stock Markets

Based on a data set containing six indices from the three financial world regions Europe, North America and Pacific, the stock behavior of the last three decades will be explored. Next to the European countries Switzerland (CHs), United Kingdom (UKs) and Germany (GEs), stock indices of the U.S.A. (USs), Japan (JPs) and the Pacific area[2] (JPs) are considered. All stock indices stem from MSCI[3] and are total rate of return indices, meaning dividends are reinvested. The excess returns are calculated using the corresponding 3-month (BBA) Libor[4] as risk-free interest rate.

[1] This time period was consciously chosen in order to involve the latest worldwide financial crisis in its peak.

[2] The index of the Pacific area does not include the country Japan, which is considered as an individual market.

[3] Source: Bloomberg

[4] Source: Bloomberg, BBA: British Bankers Association

3.1.1 Descriptive Analysis

Table 3.1 presents the descriptive statistics of the stock market excess returns. The annualized mean over the last three decades lies between 1.9% for the JPs and 5.3% for the CHs. Due to the latest financial crisis, provoked by the U.S. subprime market, the annual returns fell within the last 2 years on average by around 2.5%. Whereas the Indices of the United Kingdom (-3.13%) and the U.S.A. (-2.56%) incur the highest losses during this current crisis. The standard deviation of the annualized excess returns ranges between 16.9% for the CHs[5] and 26.7% for the PAs. The biggest one-month collapse of this time period occurred most often in October 1987, except GEs (September 2002) and JPs (March 1990). It is the 19th of October 1987 that went down in history as 'Black Monday'. To that time it was the biggest one-day stock market fall of history. The index of the Pacific area lost e.g. more than half of its value within one month.

All distributions of the markets exhibit an excess kurtosis, indicating leptokurtic distributions which are characterized by an acute peak and fat tails. The Jarque-Bera test supports the results of the higher moments. On a significance level of 5% the null hypothesis of the normal distributions must be rejected for the excess returns of all indices, whereas the stock market of Japan comes nearest to a normal distribution. The excess returns are all negatively skewed, accentuating the fat tail on the left side.

	CHs	GEs	UKs	USs	JPs	PAs
Mean (annual.)						
- Dec. 2008	5.3%	4.1%	4.8%	5.1%	1.9%	3.7%
- Dec. 2006	7.5%	6.2%	8.0%	7.6%	4.1%	6%
Std.D. (annual.)	16.9%	23.2%	20.2%	20.4%	23.1%	26.7%
Max.	12.4%	21.6%	16.3%	13.2%	23.1%	19.2%
Min.	-26.7%	-29.7%	-31.6%	-31.2%	-21.9%	-64.6%
Skew.	-1.045	-0.9171	-1.082	-0.824	-0.151	-2.039
Kurt.	6.561	5.707	6.887	5.072	3.576	17.26
JB-stat.	247.208	155.066	287.003	101.608	6.142	3190.864
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.044)*	*(0.000)*

Table 3.1: Moment statistics of the stock markets, covering period January 1980 to December 2008 on a monthly basis.

The so-called quantile plots show a normality test for the excess returns. The quantiles of the historical distribution of the excess returns are plotted against the quantiles of a normal distribution. If the historical data were normally distributed, they would lie approximately on the dashed reference line. All stock markets possess similar structures, characterized by large deviations from the normal distributions at the left tail (see figure 3.1). The empirical study of Longin & Solnik, 2001, [69] sustains this graphical

[5]Since all indices are declared in Swiss francs, the Swiss stock market contains no currency risk.

insights. They reject, using 'extreme value theory', the null hypothesis of multivariate normality for the negative tail, but not for the positive tail. These findings confirm the excess kurtosis and the negative skewness documented in table 3.1.

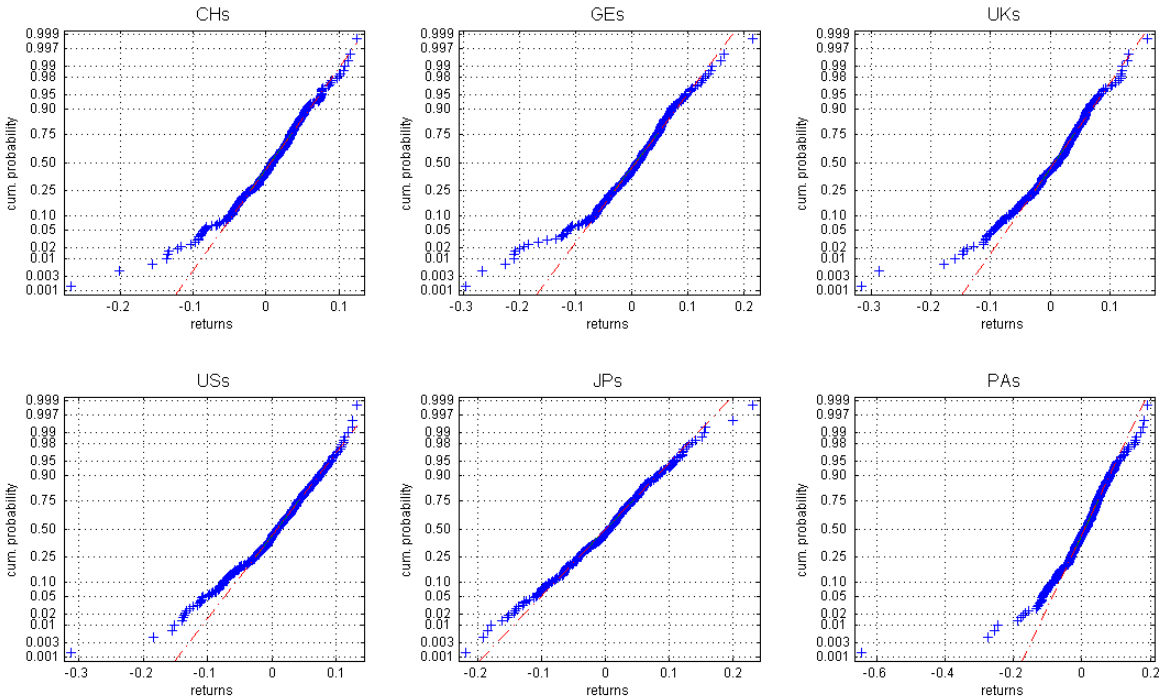

Figure 3.1: Normal probability test for the stock market excess returns covering period January 1980 to December 2008 on a monthly basis. If the historical data were normal distributed, they should lie approximately on the dashed reference line. Please note the different scaling of the x-axes.

The unconditional correlation coefficients of the stock market pairs reach from 0.39 between Japan and Germany stock markets to 0.75 between the United Kingdom and the U.S. stock markets. The smallest average correlation to all the other considered markets shows the JPs with an average of 0.46.

	CHs	GEs	UKs	USs	JPs	PAs
CHs	**1**					
GEs	0.72	**1**				
UKs	0.66	0.63	**1**			
USs	0.67	0.63	0.75	**1**		
JPs	0.44	0.39	0.50	0.48	**1**	
PAs	0.60	0.57	0.73	0.72	0.50	**1**

Table 3.2: Unconditional correlation coefficients of the stock market pairs, covering period January 1980 to December 2008 on a monthly basis.

By the examination of the correlation over time, one recognizes on the one hand an up-moving trend and on the other hand a certain mean reverting process around this trend (see figure 3.2). The figure shows the monthly average correlation of stock market excess returns of the respective market against the other five remaining indices (equally weighted). By opposing the correlations to the market volatility - measured by the standard deviation of the corresponding national stock market - the pattern of the correlation around the trend turns out to be strongly related to the market uncertainty. When the market volatility increases, the correlation among stock markets rises as well and vice versa (contagion, see section 2.1.4). The linear regression line, with its positive slope, as well as the corresponding 95%-confidence interval point out that the average stock market correlations were rising within the last three decades. In average it rose by a value of 0.35 from 0.40 to 0.75, being located more and more within the declared interval. At the end of the year 2008 the correlations almost tend to unity. The most independent market among these financial world regions from the Swiss franc perspective is the JPs, the most dependent is the UKs followed by the USs. The biggest slope discovers the German index (GEs). Despite of this strong trend the contagion phenomenon is still recognizable, even though not that concisely anymore. In the last low-volatile period from the year 2005 to 2007 the correlation did not fall as low as history would have expected. The two lines still move align with each other, however, the gap between them gets larger. The difference of the two mapped volatilities - graphically distinguished by the thin solid line and the dotted line - uncovers the currency risks from a Swiss investor. A large percentage of the uncertainty is driven by the stock market movement itself, only a small percentage, which does not change the volatility pattern significantly, is due to the currency risks.

	CHs	GEs	UKs	USs	JPs	PAs
CHs		0.74	0.73	0.70	0.51	0.65
GEs	0.57		0.70	0.67	0.45	0.64
UKs	0.54	0.56		0.79	0.56	0.76
USs	0.56	0.54	0.67		0.56	0.77
JPs	0.30	0.31	0.41	0.39		0.59
PAs	0.50	0.53	0.69	0.66	0.42	
ø(high volatility)	0.66	0.64	0.71	0.70	0.53	0.68
ø(ø(high volatility))	0.65					
ø(low volatility)	0.50	0.50	0.57	0.57	0.37	0.56
ø(ø(low volatility))	0.51					

Table 3.3: Stock market semi-correlations in volatile and non-volatile time periods. Semi-correlations between any two markets are calculated using, out of the whole sample, only the data points when the volatility of a world index lies above its median (high volatility) or below its median (low volatility). The semi-correlations of the high volatility times are displayed in the upper right side of the matrix while the semi-correlations of the low volatile times are displayed in the lower left side. The excess returns are calculated in Swiss francs, meaning currency risk is not hedged.

The semi-correlations found in table 3.3 sustain the patterns found in the graphical presentation (see figure 3.2). In all markets the average correlation against the other five indices is higher in high-volatile than in low-volatile periods. The lowest correlations in both time periods are the ones with the JPs.

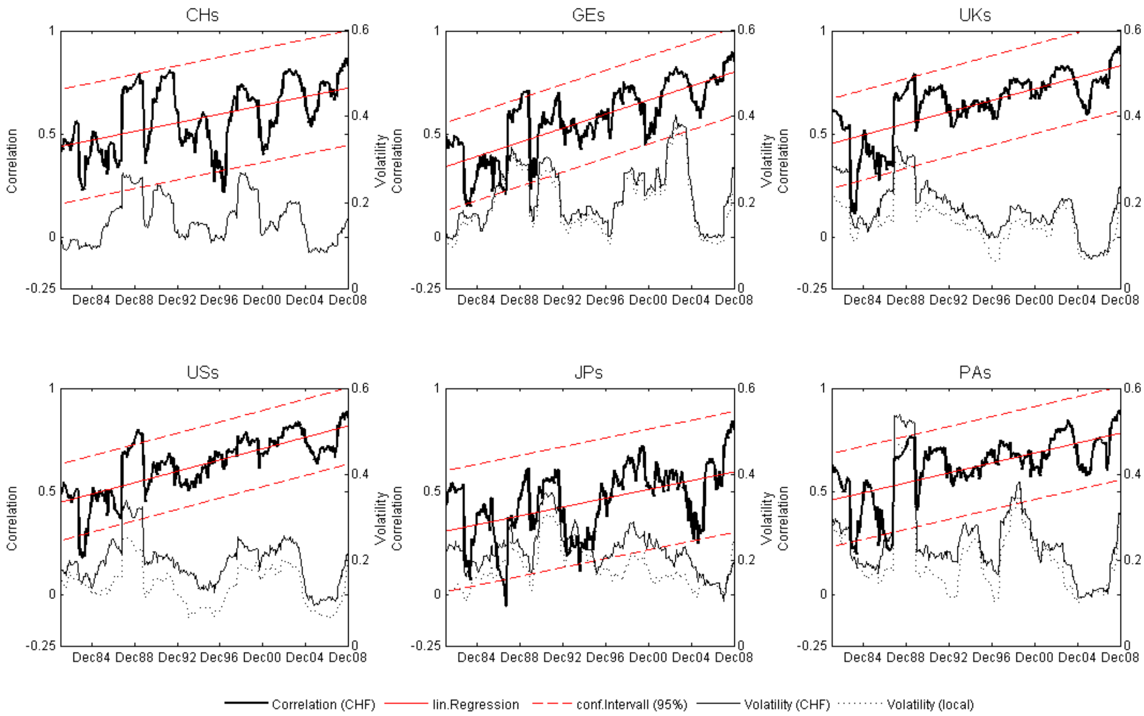

Figure 3.2: The figure shows the monthly average correlations of stock market excess returns of the respective market against the other five remaining indices (equally weighted). The correlations are defined as rolling windows of two years for the period of January 1980 to December 2008. The market volatility of the six regions is measured as the yearly standard deviation of the corresponding stock index.

3.2 Bond Markets

The bond indices are based on total all lives government indices found on Datastream. The excess returns are calculated using the corresponding 3-month (BBA) Libor[6] as risk-free interest rates. Six bond indices, covering the main world currencies are compared. Next to the local Swiss franc bonds (CHb), euro bonds (Eub), sterling bonds (UKb), USD bonds (USs) and Japan Yen bonds (JPb) are considered.

3.2.1 Descriptive Analysis

The descriptive statistics of the bond market excess returns is summarized in table 3.4. The annualized mean over the last three decades ranges from 0.7% for the CHb and

[6]Source: Bloomberg, BBA: British Bankers Association

4.0% for the JPb. Their standard deviation lies between 3.6% for CHb and 11.8% for the JPb, however, it is worth mentioning that the volatility of CHb contains no currency risk. The biggest one-month loss of the bond returns, since January 1980, has appeared in all markets (except Japan) in the early nineties, to a time where the bond markets used to be more volatile than in the following two decades. The highest bond market loss within a month was observed in the United Kingdom (-14.5%), while Switzerland only records -3.7%. The minimum and maximum losses reflect the volatilities of the different bond markets, which are strongly influenced by the risk of the exchange rates of the particular currency to the defined home currency Swiss franc.

The distributions of bond market excess returns show an excess kurtosis, if not necessarily as high as the stock markets. The Jarque-Bera test confirms that the distributions are significantly different from a normal distribution. Except the USb and to a certain significance level ($\alpha = 2.2\%$) also CHb could be considered as normally distributed. Two out of five regarded bond markets exhibit a negative skewness, headed by the United Kingdom (-0.485). Japan has a positive skewness of 0.492, while the bond market of the U.S.A. seems to be approximately symmetric.

	CHb	EUb	UKb	USb	JPb
Mean (annual.)	0.7%	2.5%	3.8%	3.8%	4.0%
Std.D. (annual.)	3.6%	5.9%	11.7%	11.8%	11.9%
Max.	3.5%	8.0%	12.1%	10.2%	14.0%
Min.	-3.7%	-5.5%	-14.5%	-11.0%	-10.8%
Skew.	-0.172	0.116	-0.490	0.063	0.491
Kurt.	3.672	5.150	4.670	3.075	4.304
JB-stat.	8.228	67.572	54.168	0.309	38.556
	(0.022)	*(0.000)*	*(0.000)*	*(0.500)*	*(0.000)*

Table 3.4: Moment statistics of the bond markets, covering period January 1980 to October 2008 on a monthly basis.

Figure 3.3 sustains the findings in table 3.4. The quantile plots elucidate graphically the assumption of the normal distributed excess returns for the USb and the positive skewness for the JPb.

The unconditional correlation coefficients of the bond market pairs disclose two groups of dependency. In the first group UKb (0.48) and JPb (0.41) gear to the USb, in the second group UKb (0.40) and CHb (0.42) gear to the EUb. The correlation of all other bond market pairs are below 0.32, nevertheless ever positive. The Swiss bond has conspicuously low correlations ($\leq$0.16) towards the other bond markets.

Since the year 1980 the correlations among the bond markets have moved mostly between 0.0 and 0.5. However, there are time periods in which they broke the level of 0.5 and even times were they showed negative relations.

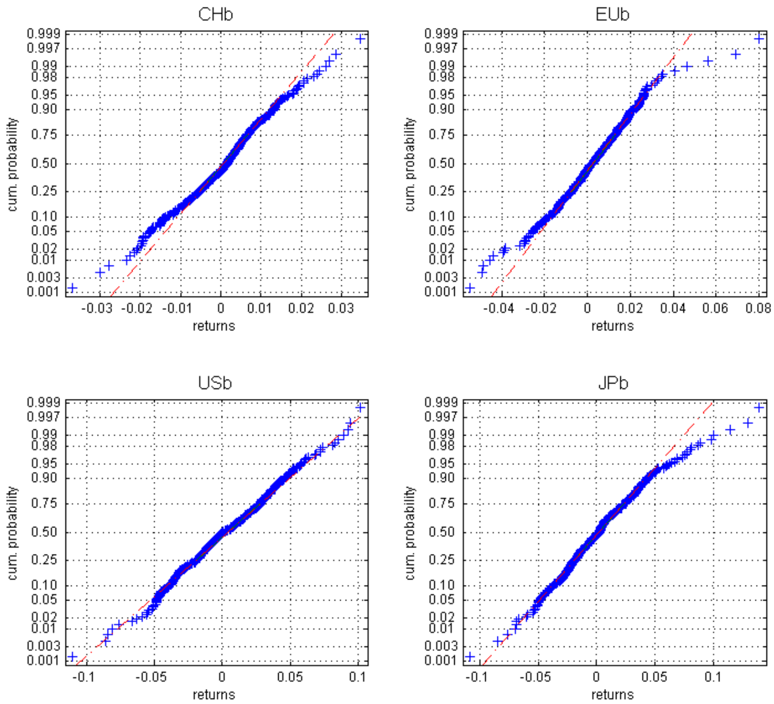

Figure 3.3: Normal probability test for the bond market excess returns covering period January 1980 to October 2008 on a monthly basis. If the historical data were normal distributed, they should lie approximately on the dashed reference line. Please note the different scaling of the x-axes.

	CHb	EUb	UKb	USb	JPb
CHb	**1**				
EUb	0.42	**1**			
UKb	0.16	0.40	**1**		
USb	0.13	0.31	0.48	**1**	
JPb	0.14	0.19	0.30	0.41	**1**

Table 3.5: Unconditional correlation coefficients of the bond market pairs, covering period January 1980 to November 2008 on a monthly basis.

Figure 3.4 exposes the two different risk components of foreign bond market investments. The correlation over time displays the average correlation of the respective market to the other four indices (equally weighted). Next to the national bond market uncertainty (dotted line) a great deal of the volatility is due to the foreign exchange exposure (difference to the thin solid line). Especially the foreign bonds USb and JPb contain an over-average amount of currency risk.

3.3 Exchange Markets

In an international environment an investor has on the one hand greater diversification potential, however, on the other hand he has to cope with volatile developing exchange

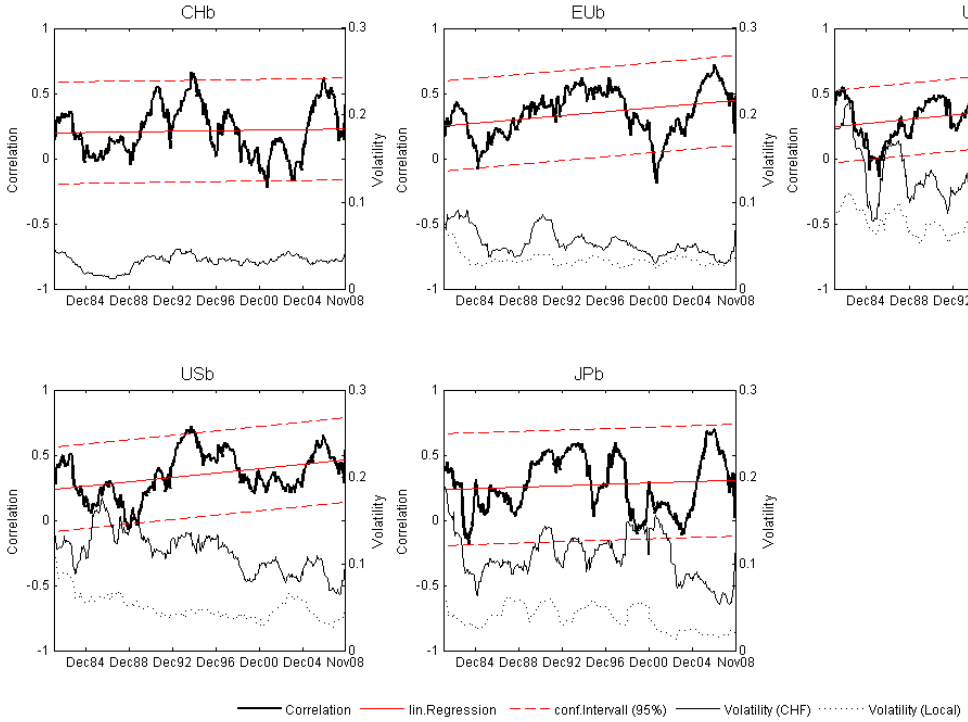

Figure 3.4: The correlation over time displays the average correlation of the respective market to the other four indices (equally weighted). Correlation over time opposed to the market volatility from a Swiss franc point of view (24-months rolling moving average). The market volatility of the five areas is measured as the yearly standard deviation of the corresponding bond index.

rate markets. The currency exposure risk is one of the key sources of the overall risk of an international investment. Even though, on average, the market risk might be larger than the currency risk it is well-advised to consider the exchange rate fluctuations in order to improve the performance of an international diversified portfolio. Studies have shown how strategies that include foreign currencies significantly outperform strategies that exclude foreign currencies (see Santis, Gerard, & Hillion, 1999, [85]).

The market risk and the currency risk are not just supplementary but they also affect each other. A popular belief found in the financial literature says that there is a statistical interaction between stock prices and exchange rates. On the one hand stock market fluctuations might affect exchange rates, on the other hand changes in the exchange rates influences the stock market prices. The exchange rate dynamics influence the competitiveness of international firms. It affects the trade balance, by directly manipulating the real income and output. Thereby, one has to distinguish between exporting and importing firms. A depreciation of the local currency leads to cheaper exporting goods, which may result in an increase of foreign demand and sales. Therefore, an exporting firm's value increases during a depreciation of its local currency. An appreciation has exactly a contrary effect on the firm's stock prices. An importing firm however benefits from a appreciation and sustains a loss during depreciation of its local currency.

3.3.1 Descriptive Analysis

In the last three decades the Swiss franc experienced an appreciation towards three of the four world currencies in a long term trend. The GBP fell after its peak rate of 5.47 in January 1981 gradually down to 1.56. The USD increased in the early nineties from a rate of around 1.5 up to 2.86. After this peak in February 1985 the USD rapidly depreciated and has moved since the end of the nineties around a rate of 1.5. In March 2008 the USD rate even fell under 1.00 and leveled out at this low rate. The German Mark (DEM) as well as the EUR had lower volatility compared to the USD or GBP. However, a closer look at the Euro, launched in January 1999, points out that the EUR rate experienced already two depreciations within its young history. Around the turn of the millennium as well as after mid 2007 the EUR lost value compared to the Swiss franc. In both cases the rate fell under 1.5. In October 2008 one EUR costed only 1.43 CHF, as cheap as never before in the history of the European single currency. One year before, the rate reached with 1.68 its highest value ever. The yen appreciated in the last three decades compared to the base currency Swiss franc. The latest peaks are visible around the year 2000 and at the end of 2008. In comparison to other base currencies like USD or GBP this appreciation is even more pronounced.

The short term movements of the exchange rates around these long term trends, reveal a certain linkage between the stock prices and the exchange rates. Figure 3.6 illustrates this relation by plotting the exchange rates against the market volatility: When market volatility is intense, the Swiss franc strengthens. The latest financial market crisis reverified this phenomenon. In the last 10 years this pro-cyclical movements were visible after the 9/11 terror attack in the year 2001 and with the beginning of the subprime crises at midsummer of 2007.

Table 3.6 contains the descriptive statistics of the considered exchange rates. All currencies except the other safe haven currency JPY depreciated towards the CHF. The EUR/CHF (resp. DEM/CHF) has compared to the other exchange rates lower volatility. All other exchange rates demonstrate high volatilities and a respectively high currency risk. The negative skewness between high-interest rate (USD, GBP and EUR (DEM)) and low-interest rate currencies (CHF and JPY) might be because of abrupt dealings of carry trades, who boost the appreciation of the safe haven currency (see section 2.1.6, also e.g. Brunnermeier, Nagel & Pedersen, (2008), [17]). Also

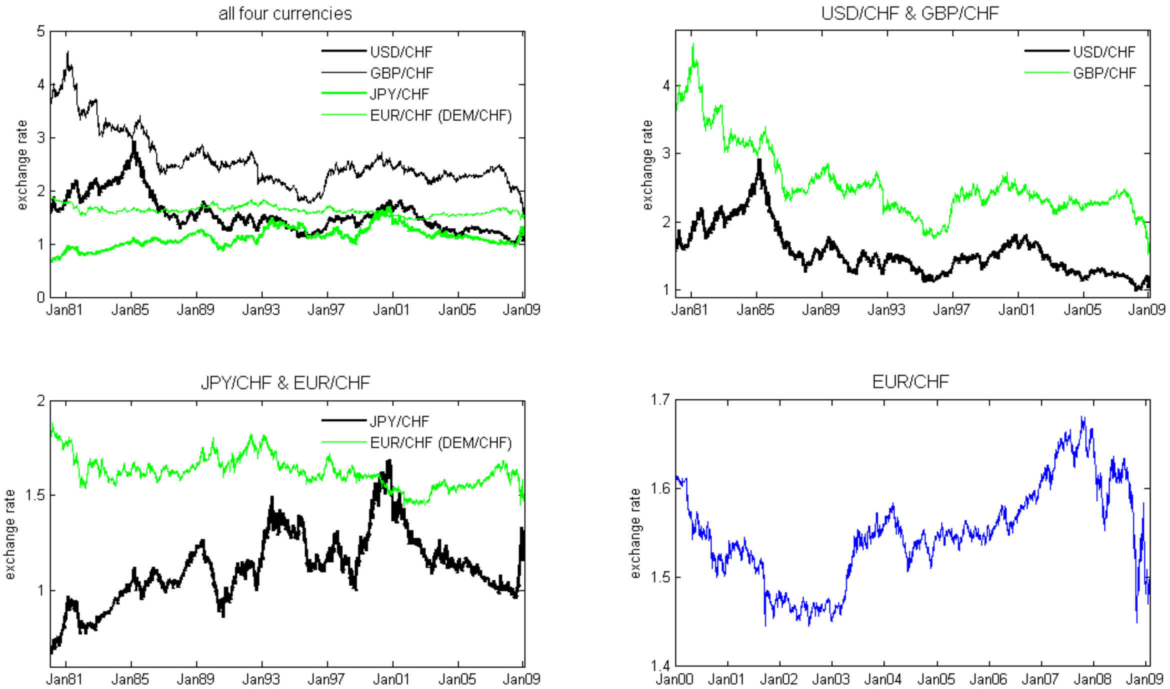

Figure 3.5: Swiss franc exchange rates over time. The EUR series is a synthetic series obtained by multiplying the whole DEM series (1980-1998) with the exchange rate EUR/DEM of the reference date January 29, 1999 (=1.9542). Please note the different scaling on the y-axes.

the exchange rates have a excess kurtosis, indicating leptokurtic distributions with acute peaks and fat tails. Therefore, the extreme events occur more often than the normal distribution would assume. Due to the negative skewness (except JPY/CHF) especially the rare events on the left side appear more often. This goes along with the appreciation of the Swiss franc as safe haven currency in turbulent times.

This paragraph describes how the Swiss franc reacted during the latest financial crisis. It answers the question whether the Swiss franc still acts as a safe haven currency or whether it has lost this function. Since the beginning of the subprime mortgage crises (defined reference date: June 1, 2007) the Swiss franc has appreciated against almost all of the 150 currencies of the world and reinforced its safe haven characteristics. Also in comparison with four of the world's leading currencies the Swiss franc gained ground (see figure 3.7). These currencies all depreciated against the Swiss franc. The USD lost 19.64% within 9.5 months (March 17, 2008) towards the Swiss franc. After a short rally its value fell drastically again around new year 2008/2009. The British pound depreciated in the first year along with the USD before it decoupled from it and continued to fall in value, hitting the lowest level of history end of December 2008 with -37.58% against the Swiss franc. The unit currency euro experienced no depreciation until the beginning of the year 2008. After a short recovery over the summer 2008 it depreciated rapidly against the Swiss franc by even -12.35% on October 28, 2008.

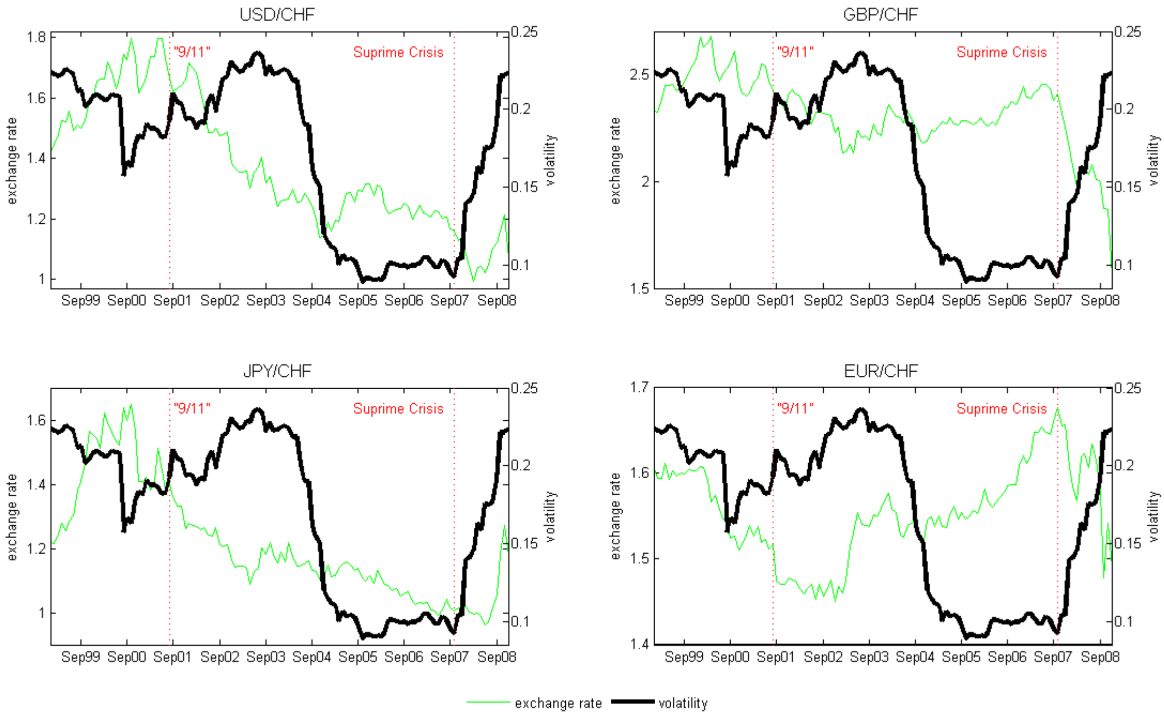

Figure 3.6: Swiss franc exchange rates (USD/CHF, GBP/CHF, JPY/CHF, EUR/CHF) and the market volatility over time.

	USD/CHF	GBP/CHF	JPY/CHF	EUR/CHF (from Jan.99)	DEM/CHF (to Dec.98)
Mean (annual.)	-1.4%	-2.8%	2.0%	-0.8%	-0.6%
Std.D. (annual.)	12.0%	9.7%	11.0%	4.3%	4.7%
Max.	10.5%	6.8%	14.3%	4.3%	4.6%
Min.	-12.8%	-18.1%	-10.6%	-6.6%	-4.0%
Skew.	-0.275	-1.240	0.454	-0.951	-0.068
Kurt.	3.443	8.637	4.486	9.480	3.600
JB-stat.	7.221	549.839	53.924	227.967	3.547
	(0.030)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.132)*

Table 3.6: Moment statistics of the exchange rates, covering period January 1980 to December 2008 on a monthly basis

The Swiss franc also revaluated in a certain time period towards the other low-yield rate currency yen (-5.06% on July 21, 2008). These facts sustain that investors still consider the Swiss franc as a safe haven currency in volatile times.

3.4 Summary

In summary it can be said that stocks are leptokurtic, fat-tailed distributed with negative skewness. Over the years they became more and more contagious among themselves. Over time their correlations follow align with the market volatility. One can

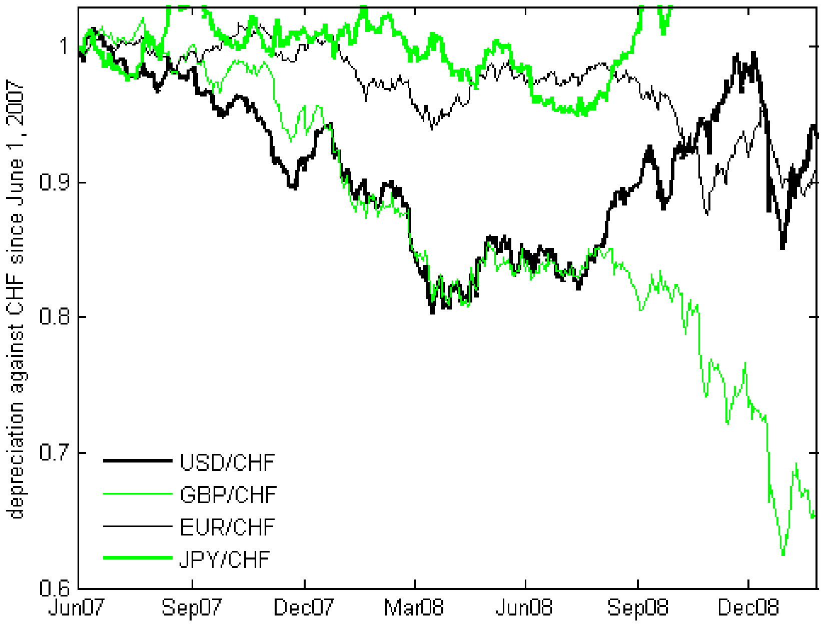

Figure 3.7: Safe haven function of the Swiss franc in the latest crisis.

still distinguish the high-volatile and the low-volatile times, even though not that pronounced anymore and on a high level close to unity.

The bond markets correspond more likely to a normal distribution. However, according to the statistical significance they also follow, with exception, a leptokurtic, fat-tailed distribution. The skewness varies from market to market and reaches from significantly negative to significantly positive values. The correlation among themselves lies mostly between 0.0 and 0.5. They vary over time, however, in the relation to the market volatility there is no clear pattern recognizable.

The relation between stocks and bonds is visible and described in section 2.2 under figure 2.2a). The important findings are that the correlation among stock and bonds declined over time. But the anti-cyclical pattern toward the market volatility is still given.

The Swiss franc exchange rates follow also a leptokurtic, fat-tailed distribution with negative skewness. Only the exchange rate to the other safe haven currency JPY has a positive skewness. As a result of the safe haven characteristics, the Swiss franc exchange rate moves contrary to the market volatility.

Chapter 4

Regime-Switching Models

The stylized facts just mentioned lead to rigorous changes in the behavior of the financial markets. The challenge is to model these changes in the process followed by particular time series. A simple idea is to change the constant terms of the stochastic process after such a structure-changing event. But if the process has changed in the past, it could as well change in the future. When one considers business cycles or financial crises, the effect might be dramatic but not permanent. This prospect should be taken into account in modeling a forecast. A plausible approach is constituted by a regime-switching model. A regime-switching model includes a law that describes the regime changes. The change in a regime should certainly not be regarded as the outcome of perfectly foreseeable, deterministic events. Because the change in the regimes itself is a stochastic process. Hence, the process of such a regime-switching model depends on an unobserved random variable s_t, denoting the state or regime the process is in at time t. A common time series model for such a discrete-valued random number is a Markov chain, making it possible to design a model that allows given time series to follow a stochastic process with changing parameters. Within a given regime, the dynamics of the observed data are determined by a conventional stochastic process with a different set of regime-dependent parameters (see Hamilton, 1994, [51]).

Before introducing the regime-switching model developed in this work at hand, three further introductory sections are discussed in advance. First, the literature research section summarizes the chronological development of the regime-switching approach and its applications. Secondly, two theoretical input sections introduce important modules of a the holistic regime-switching model.

4.1 Literature Research

In the last decades the interest in time-varying parameter models has been increasing. Especially for macroeconomic and financial time series, since they might contain dramatic changes in their behavior. These changes are mostly associated with events like financial crises (see CERRA & SAXENA, 2005, [21] or HAMILTON, 2005, [50]) or with sharp changes in government policies (see HAMILTON, 1988, [53] or DAVIG, 2004, [28]). Conventional models with fixed density functions or only one single set of parameters were to some extent no longer suitable. The possible structural changes are to be considered in the model (KIM, PIGER, & STARTZ, 2005, [63]). One major set of such sophisticated frameworks are regime-switching models.

Regime-switching models have a long history. They go at the minimum 50 years back to QUANDT, 1958, [77]. Quandt introduced a linear regression model obeying two different regimes. He estimated the proper time of the switching point by examining the appropriate likelihood function. However, his model depends on the a priori knowledge of the exact number of switches. Further models contain this simple type of structure, consisting the assumption that there is at most one switch in the data series (see e.g. BROWN & DURBIN, 1968, [15]; QUANDT, 1960, [76] or FARLEY & HINICH, 1970, [36]). Based on the former, rather simple models, GOLDFELD & QUANDT, 1972, [46] and QUANDT, 1972, [75] presented two methods for a multiswitch problem that allows numerous switches. The so-called *D-method* introduces an unknown function, possibly linear that contains an exogenous variable which partially explains which regime generates an observation. An alternative method, called λ-*method*, assumes that nature chooses between regimes 1 and 2 with probabilities λ and $1 - \lambda$. The log likelihood function is being maximized with respect to λ, which is a priori unknown to the investigator. Although the probabilities are independent of what the system was in on the previous trial, both multiswitch methods have acceptable sampling properties. A more sophisticated multiswitch model, referring to in the following as Markov-switching models or hidden Markov models, was introduced by GOLDFELD & QUANDT, 1973, [45]. This particular useful version redefines the latent state variable - controlling the regime switches - as a Markov process. Hence, the characteristics of such a Markov chain lead to serially dependent state variables and set the new standard to describe the regime-switching. HAMILTON, 1989, [52] extended in an influential paper the Markov-switching models and introduced the notable Hamilton-filter, in which the objects of interest are estimated as a by-product of an iterative algorithm related to a Kalman-Filter.

Critics on given regime-switching models led to even more sophisticated models. Diebold, Lee & Weinbach, 1994, [31] criticized that Hamilton's approach treats the transition probabilities as constant over time. In their research they suggest to allow time-varying transition probabilities for explaining the switching processes of exchange rates. They might vary with fundamentals like relative money supplies, relative real outputs or interest rate differentials. Further, most literature typically assumes that the regime switches are exogenous with respect to all observations of the regression disturbance. Kim, Piger & Startz, 2005, [63] relax this requirement by introducing an endogenous Markov regime-switching that defines the latent regimes with a probit specification. Nonetheless, the popularity of Hamilton's model is well deserved. Its general characteristics of moments and stationary conditions were published by Tjøstheim, 1986, [97], Yang, 2000, [101], Timmermann, 2000, [96] and Francq & Zakoïan, 2000, [38].

Substantive applications of regime-switching models mostly inspired by Hamilton, 1989, [52] include Hamilton, 1988, [53] and Gray, 1996, [47] on interest rates, Hamilton, 1989, [52] on aggregate output, Engel & Hamilton, 1990, [34] and Diebold, Lee & Weinbach, 1994, [31] on exchange rate as well as Cecchetti, Lam & Mark, 1990, [19], Abel, 1994, [1] and Ang & Berkaert, 1999, [5], 2002, [4] on stock returns. Many researchers use the Markov characteristics to describe abrupt changes of different stochastic processes, i.e. no autoregressive processes (see e.g. Lindgren, 1978, [67]), autoregressive processes (see e.g. sc Rabiner, 1989, [78]) or regressions (see e.g. Goldfeld & Quandt, 1973, [45]).

Regime-switching models are not only common in financial markets but also in energy markets, since they allow a more appropriate modeling of the spiking behavior of spot prices than conventional jump-diffusion models (see e.g. Huisman & Mahieu, 2003, [56] and Culot, Goffin, Lawford, De Merten & Smeers, 2006, [27]). According to Blöchlinger, 2007, [9] regime-switching models for electricity prices can mainly be divided into two branches. On the one hand the spot prices are determined by dividing the time series into separate regimes with different underlying process. The process in the normal regimes is modeled by a mean-reverting process, while the spike regimes follow different (mean-reverting) jump processes. The change in regime is typically governed by an unobserved discrete-valued random number that follows a Markov chain (see e.g. Huisman & Mahieu, 2003, [56]). On the other hand there are regime-switching approaches that model the spot prices as a product of two independent Markov processes. The first Markov process describes the normal regime, the second is used for the spikes (see e.g. Kholodnyi, 2005, [61]).

4.2 Markov Chains

The Markov chain is a special type of discrete-time stochastic processes. In most regime-switching models it is used to describe the probability law governing the change from one regime to another. This chapter briefly introduces the main characteristics of this type of stochastic processes as well as the requirements in order to be used in a regime-switching model (see HAMILTON, 1994, [51] or WINSTON, 2004, [100])[1].

A discrete-time stochastic process is called Markov chain if the probability distribution of the state s_t at time t depends only through the most recent state s_{t-1} on the past and does not depend on the states the chain passed through on the way to s_{t-1}. Let s_t be one of the finite numbers of states $1, 2, \ldots, K$:

$$P\{s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots\} = P\{s_t = j | s_{t-1} = i\} \tag{4.2.1}$$

A further assumption is made that at all times and for all states i and j the expression $P\{s_t = j | s_{t-1} = i\}$ is time-independent. This assumption results in the following expression:

$$P\{s_t = j | s_{t-1} = i\} = p_{ij}, \tag{4.2.2}$$

where p_{ij} derives the probability that state i will be followed by state j. Therefore, we get an K-state Markov chain with transition probabilities p_{ij} $[i, j = 1, 2, \ldots, K]$. The transition probabilities are often collected in an $(K \times K)$-transition matrix P:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{22} & \cdots & p_{2K} \\ \vdots & \vdots & & \vdots \\ p_{K1} & p_{K2} & \cdots & p_{KK} \end{pmatrix} \tag{4.2.3}$$

Note that each row must sum up to unity $\sum_{j=1}^{K} p_{ij} = 1$, and each entry in the transition matrix has to be nonnegative. Equation 4.2.2 implies that the transition probabilities remain stable over time. If this assumption is satisfied one speaks of a stationary or homogeneous Markov chain[2].

In order to compute forecasts HAMILTON, 1994, [51] introduces, next to the transition matrix P, a $(1 \times K)$-vector ξ_t as the probability distribution of the states at time t. The probability distribution of the states at time $t+1$ is then given by multiplying the

[1]The following remarks are mainly based on HAMILTON, 1994, [51] and WINSTON, 2004, [100].

[2]There are cases where the economic considerations suggest time-varying transition probabilities (see e.g. DIEBOLD, LEE & WEINBACH, 1994, [31]).

known probability distribution ξ_t with the constant transition matrix P. Due to the Markov property (see equation 4.2.1), it follows

$$E(\xi_{t+1}|\xi_t, \xi_{t-1}, \ldots) = \xi_t P \tag{4.2.4}$$

This result implies that it is possible to express a Markov chain in the form of a first-order vector autoregression for ξ_t

$$\xi_{t+1} = \xi_t P + v_{t+1}, \tag{4.2.5}$$

and to calculate m-period-ahead forecasts

$$\xi_{t+m} = v_{t+m} + v_{t+m-1}P + v_{t+m-2}P^2 + v_{t+1}P^{m-1} + \ldots + \xi_t P^m, \tag{4.2.6}$$

where the expected value is given by

$$E(\xi_{t+m}) = \xi_t P^m. \tag{4.2.7}$$

Depending on the classification of the states, a Markov chain is either called reducible or ergodic (irreducible). Whenever in an K-state Markov chain at least one state i $[i = 1, 2, \ldots, K]$ is absorbing, meaning that once the process enters state i there is no possibility of ever leaving this state ($p_{ii} = 1$), the Markov chain is said to be reducible. Such reducible Markov chains are interesting whenever the change in regime is a permanent event. The advantage of using a reducible Markov chain instead of a deterministic specification for such a process is that it allows to generate meaningful forecasts prior to the change that consider the possibility of the change from a current regime to the absorbing regime (see Hamilton, 1994, [51]). The permanent regime change could be implemented by defining the permanent state in which the model will switch any time in the future as the absorbing state.

A Markov chain is called ergodic if at any time it is possible to go from every state to every state, however, not necessarily in one move. If the Markov chain is ergodic and homogeneous, meaning that the process is defined by a time-independent transition matrix P, then there exists a vector π that denotes the steady-state probabilities:

$$\pi = \pi P \tag{4.2.8}$$

subject to $\sum_{j=1}^{K} p_{ij} = 1$. The initial state probabilities ξ_0 have no influence on the steady-state distribution. They depend solely on the structure of the transition matrix.

The long-run forecast of an ergodic Markov chain is therefore independent of the current state. Ergodic Markov chains are suitable to model time series that have no permanent, but only temporary regime changes.

4.3 i.i.d. Mixture Distributions

In the following regime-switching model, presented in chapter 5, the process of the risk/return-structure of the financial markets is described as a conditional factor model with regime-switching. The parameters of this factor model change according to the results of a hidden Markov chain that controls the regime-switches. Such a framework may lead to identical independent distributed (i.i.d.) mixture distributions where the real observations can be addressed more flexible[3].

Many financial market models assume an unimodal, normal distribution in order to estimate the observed data. However, if the observations can be distinguished into several groups, i.e. volatile and non-volatile regimes, they are more adequately addressed by the assumption of i.i.d. mixture distributions. Due to the fact that the shape of a mixture density has the striking property of being extremely flexible.

Lets presume that the process of our observed variable Y is being described by a finite number of normally distributed regimes ($s_t = 1, 2, ..., K$), where the random variable s_t denotes in which regime the process is in at time t. Depending on s_t the variable y_t is generated from the corresponding regime and therefore from its normal distribution $\sim N(\mu_{s_t}, \sigma_{s_t})$. The conditional density of y_t for each regime k ($k = 1, 2, ..., K$) is defined as

$$f(y_t \mid s_t = k; \theta) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{\left(\frac{-(y_t-\mu_k)^2}{2\sigma_k^2}\right)}, \tag{4.3.1}$$

where θ contains the population parameters, including the first two moments of the regimes' normal distributions ($\mu_1, \mu_2, ..., \mu_K$ and $\sigma_1^2, \sigma_2^2, ..., \sigma_K^2$). The latent regime s_t, which follows a hidden Markov process, is drawn from a probability distribution itself. The unconditional probability of the regime k is defined as:

$$P\{s_t = k; \theta\} = \pi_k \qquad \text{for } k = 1, 2, ..., K \tag{4.3.2}$$

and is also included in the vector of the population parameters θ:

$$\theta = (\mu_1, \mu_2, ..., \mu_K, \sigma_1^2, \sigma_2^2, ..., \sigma_K^2, \pi_1, \pi_2, ..., \pi_K)'.$$

[3]The following remarks refer closely to Hamilton, 1994, [51].

According to the Bayes' theorem the joint density-distribution function of y_t and s_t is given by:

$$p(y_t, s_t = k; \theta) = f(y_t \mid s_t = k; \theta) \cdot P\{s_t = k; \theta\}. \tag{4.3.3}$$

Under the terms of equation 4.3.1 in association with equation 4.3.2 the joint density-distribution can also be written as:

$$p(y_t, s_t = k; \theta) = \frac{\pi_k}{\sqrt{2\pi}\sigma_k} e^{\left(\frac{-(y_t-\mu_k)^2}{2\sigma_k^2}\right)}. \tag{4.3.4}$$

By summing these different conditional densities over all K regimes one obtains the unconditional density of observing y_t:

$$\begin{aligned} f(y_t, \theta) &= \textstyle\sum_{k=1}^{K} p(y_t, s_t = k; \theta) \\ &= \tfrac{\pi_1}{\sqrt{2\pi}\sigma_1} e^{\left(\frac{-(y_t-\mu_1)^2}{2\sigma_1^2}\right)} \\ &\quad + \tfrac{\pi_2}{\sqrt{2\pi}\sigma_2} e^{\left(\frac{-(y_t-\mu_2)^2}{2\sigma_2^2}\right)} + \ldots \\ &\quad + \tfrac{\pi_K}{\sqrt{2\pi}\sigma_K} e^{\left(\frac{-(y_t-\mu_K)^2}{2\sigma_K^2}\right)}. \end{aligned} \tag{4.3.5}$$

As a result of Markovian characteristics the regime variable s_t is i.i.d. distributed across time. Therefore, the aggregation over time of equation 4.3.5 yields the log likelihood function:

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t; \theta). \tag{4.3.6}$$

By maximizing the log likelihood function subject to two constraints,

$$\sum_{k=1}^{K} \pi_{k,1} = 1 \quad \pi_{k,1} \geq 0,$$

one obtains the maximum likelihood estimate of θ. Once the population parameter vector has been estimated, following equation determines the probability that the observation at time t was drawn from regime k:

$$P\{s_t = k \mid y_t; \theta\} = \frac{p(y_t, s_t = k; \theta)}{f(y_t; \theta)} = \frac{\pi_k \cdot f(y_t \mid s_t = k; \theta)}{f(y_t; \theta)}. \tag{4.3.7}$$

The distribution of our observed variable y_t is a mixture of a finite amount of normal distributions. Meaning that each sample y_t was drawn with the probability of π_k from regime k, and therefore statistically belongs to regime k. Such mixture distributions cover a large spectrum of different densities and allow both excess kurtosis and skewness. They can be far from being Gaussian.

The following examples for $K = 2$ give a short insight of different mixture distributions. Lets assume that the variable X follows a process that is best explained by two regimes, while in both regimes the variable is normally distributed.

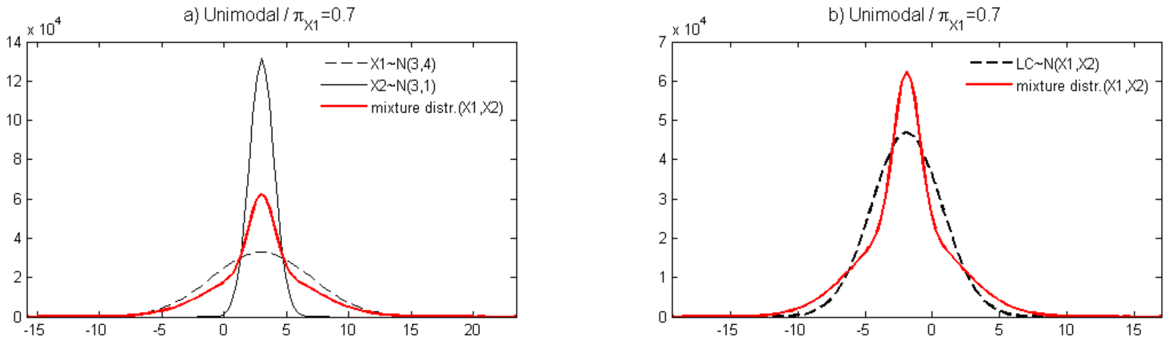

Figure 4.1: Unimodal mixture distribution: Density of mixture of two normal distributions with equal mean values: $X1 \sim N(3,4)$, $X2 \sim N(3,1)$ and $\pi_{X1} = 0.7$.

Figure 4.1 shows that the mixture of two normally distributed variables $X1$ and $X2$ with equal mean values but different standard deviations, lead to a unimodal non-Gaussian, fat-tailed distribution. Each observed sample of the mixture distribution is drawn from regime 1 $[X1 \sim N(3,4)]$ with a probability of $\pi_{X1} = 0.7$ or from regime 2 $[X2 \sim N(3,1)]$ with a probability of $\pi_{X2} = 1 - \pi_{X1} = 0.3$. Compared to the normal distribution $LC \sim N(3, 2.18)$, each sample being the result of the linear combination (LC) of $X1$ and $X2$, the mixture distribution contains an excess kurtosis ($\kappa = 4.1$) and fatter tails. In this case the density of $X2$ mainly characterizes the body of the mixture distribution, while the density of $X1$ primarily calls forth the fat tails.

If in addition to the standard deviation also the means of the regimes differ from each other, the mixture distribution might become bimodal. Figure 4.2 gives an example for $X1 \sim N(5,1)$, $X2 \sim N(0,4)$ and $\pi_{X1} = 0.5$. The density of the mixture of these two normal distributions is far away from Gaussian. It contains two local maxima and assigns higher probability to the rare events, especially to the left side due to a conspicuous negative skewness of -0.991. For the observation $x_t = -5$ one can be pretty sure that it was rather drawn from regime 2 then from regime 1 $(P\{s_t = 1 \mid y_t; \theta\} \gg P\{s_t = 2 \mid y_t; \theta\})$.

Proceeding on the assumption of a 2-dimensional asset universe, figure 4.3 illustrates how sensitive the density of a multidimensional mixture distribution reacts on the state

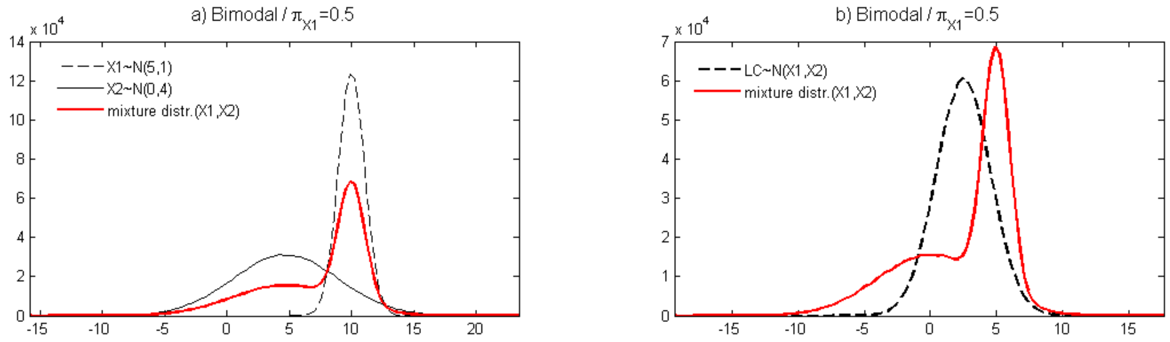

Figure 4.2: Bimodal mixture distribution: Density of mixture of two normal distributions with different mean values: $X1 \sim N(5,1)$, $X2 \sim N(0,4)$ and $\pi_{X1} = 0.5$.

variable s_t. The two regimes were defined as:

$$X1 \sim N\left[\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 25 & 16 \\ 16 & 16 \end{pmatrix}\right]$$

$$X2 \sim N\left[\begin{pmatrix} 8 \\ 6 \end{pmatrix}, \begin{pmatrix} 16 & -6 \\ -6 & 9 \end{pmatrix}\right].$$

There is a structural break between regime 1 and regime 2 in the dependency of the two variables. In times of regime 1 the two variables have a positive correlation of $\rho_1 = 0.8$. In regime 2 their dependency is negative with $\rho_2 = -0.5$. The risk that two negative returns appear simultaneously is much more probable in the case where regime 1 dominates ($\pi_1 = 0.7$) than in the case where regime 2 is more likely ($\pi_2 = 0.8$). This is comprehensible, since the characteristics in regime 1, with lower means, higher volatilities and a highly positive correlation of $\rho = 0.8$, offers minor diversification than regime 2. Therefore, next to the moments of the conditional normal distributions, the density of the mixture distribution strongly depends on the state variable s_t.

Ray & Lindsay, 1995, [82] illustrated in their study on the topography of mixtures that a mixture density can be analyzed in lower dimensions $(K-1)$ by use of the so-called ridgeline surface or manifold, containing all critical points of the mixed density (modes, antimodes and saddlepoints). The ridgeline manifold of a multivariate density with D dimensions and a mixture of K components, is defined by a surface with (K-1)-dimensions:

$$\begin{aligned} x^*(\alpha) &= [\alpha_2\Sigma_1^{-1} + \alpha_1\Sigma_2^{-1} + \ldots + \alpha_K\Sigma_K^{-1}]^{-1} \\ &\quad \times[\alpha_1\Sigma_1^{-1}\mu_1 + \alpha_2\Sigma_2^{-1}\mu_2 + \ldots + \alpha_K\Sigma_K^{-1}\mu_K], \end{aligned} \tag{4.3.8}$$

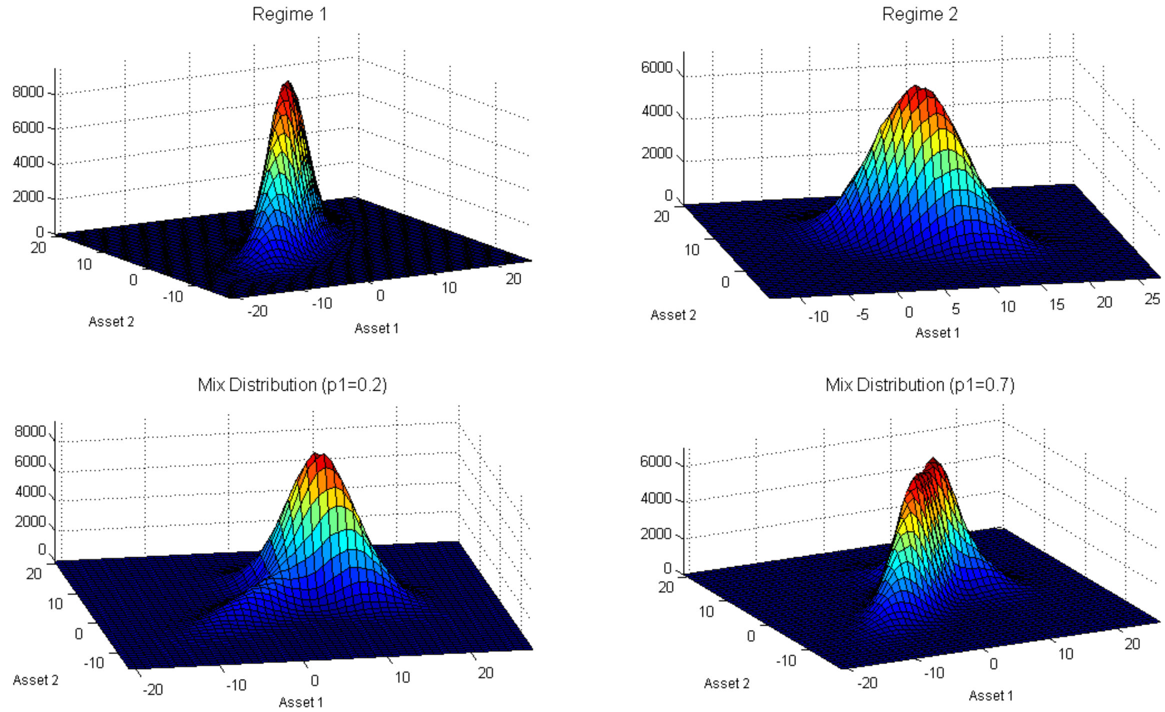

Figure 4.3: Bivariate mixture distribution.

where $\alpha \quad \epsilon \quad [0,1]$ and Σ_k $(k = 1, 2, \ldots, K)$ denotes the variance-covariance matrices of the K different normal densities. In the case of a mixture of two normals ($K = 2$), it is called the ridgeline, since it is a curve, represented as

$$x^*(\alpha) = [(1-\alpha)\Sigma_1^{-1} + \alpha\Sigma_2^{-1}]^{-1}[(1-\alpha)\Sigma_1^{-1}\mu_1 + \alpha\Sigma_2^{-1}\mu_2]. \tag{4.3.9}$$

The ridgeline defines a curve between μ_1 and μ_2, as α varies from 0 to 1. Hence, all the critical points are located on one line.

Now in the cases where the means μ are regime-independent, the ridgeline (surface) reduces to one point, defining the only mode and therefore the global maximum of the mixture distribution.

Chapter 5

A Factor Model with Regime-Switching

In this chapter a regime-switching framework, which depicts several of the empirical facts just mentioned will be introduced. This sophisticated framework is based on a multivariate factor model and is being supplemented by some latent regimes, which follow a discrete Markov chain. The latent Markov-switching approach was first studied by Goldfeld & Quandt, 1973, [45] and later extended by Hamilton, 1989, [52]. Since then it has been used in many branches of econometrics and finance. The factor model includes the single capital asset pricing model (CAPM) of Sharpe, 1964, [86] and Lintner, 1965, [68] and its generalization contains the arbitrage pricing theory of capital asset pricing (APT) of Ross, 1976, [84]. In the international context, the APT is also often denoted as the international asset pricing theory (IAPT, see Adler & Dumas, 1983, [3]). The excess return-generating function is defined by an APT solution with multiple factors. The expected excess returns, respectively the risk premiums of the securities and the covariance between them are described by the factors, dividing the total risk into the systematic and idiosyncratic part. It is assumed that the chosen factors capture all the sources of covariance between the asset categories. This approach is still consistent with the form of the CAPM, since the simple CAPM does not assume that the market is the only source of covariance between the returns (see Elton, Gruber, Brown & Goetzman, 2003, [33]). The risk/return-structure is then determined by the estimated relative systematic risks (betas) and by the variance-decomposition of the factors' variance-covariance matrix (see equation 5.1.4, Spremann, 2007, [88] or Spremann, 2007, [89]).

The paper written by Ang & Bekaert, 1999, [5]) deals with a similar model. Even though they consider only equity markets in their model and are therefore not able to capture the phenomenon of decoupling, the model presented in the paper at hand is inspired by their work.

5.1 General Regime-Switching Framework

To estimate the excess returns of assets y_n for $n = 1, ..., N$ and their risk-structure Σ_y (variance-covariance matrix of the assets), it is supposed that the excess returns of all assets depend on a small number of factors x_m for $m = 1, ..., M$ and on an idiosyncratic part ε_n. In order to capture the properties presented above, the parameters are allowed to change according to some latent regimes $s(t)$, which follow a discrete Markov process. The continuous excess returns of the factors are assumed to have the following distribution

$$x(t) \sim N\left(\bar{x}\left(s(t)\right), \Sigma_x\left(s(t)\right)\right) \tag{5.1.1}$$

for all $t = 1, \ldots, T$. The excess return of asset n at time t is then described by

$$y_n(t) = \sum_{m=1}^{M} \beta_{m,n}\left(s(t)\right) x_m(t) + \varepsilon_n\left(t, s(t)\right) \tag{5.1.2}$$

where the state variable $s(t)$, denoting the regime at time t, follows a K-state homogeneous Markov chain, hence stationary transition probabilities as defined in equation 4.2.3. The lth row and the kth column of the transition matrix P define the transition matrix p_{lk}:

$$p_{lk} := \Pr\left[s(t) = k | s(t-1) = l\right].$$

The joint distribution of the factor and of the asset excess returns conditioned on the regime is

$$\left(\begin{array}{c} x(t) \\ y(t) \end{array} \middle| s(t) = k \right) \sim \tag{5.1.3}$$

$$N\left(\left(\begin{array}{c} \bar{x}(k) \\ \beta(k)\bar{x}(k) \end{array}\right), \left(\begin{array}{cc} \Sigma_x(k) & \Sigma_x(k)\beta(k) \\ \beta'(k)\Sigma_x(k) & \beta'(k)\Sigma_x(k)\beta(k) + \Sigma_\varepsilon(k) \end{array}\right)\right),$$

where $\beta(k) \in \mathbb{R}^{N \times M}$ denotes the regression coefficients, representing the sensitivity of asset n to the factor m. $\Sigma_x(k) \in \mathbb{R}^{M \times M}$ is the covariance matrix of the factors in regime k. $\Sigma_\varepsilon(k) \in \mathbb{R}^{N \times N}$ is assumed to be a diagonal, respectively a sparse matrix with elements indicating the variances of the idiosyncratic risks.

Note that it is assumed that the parameters only depend on the current regime $s(t)$ and not on past regimes. Theoretically, all variables are allowed to vary with the regimes. However, as we shall see later on in the empirical implementation, it might be sensible to hold certain parameters identical in both regimes.

Due to the regime-switching the flexibility of the parameters of the factor model increases, allowing a representative modeling of the real financial markets. It shall avoid a dilution of different financial regimes. Gray, 1996, [47] states that a one-regime model, averaging the two financial regimes, delivers poor results. In the regime-switching model in each period, the excess returns $y_n(t)$ are drawn from a different distribution, depending on the prevailing regime $s(t)$:

$$\begin{aligned} y_t(s_t) &= \mu(s_t) + \Sigma_y^{\frac{1}{2}}(s_t)\varepsilon_t \\ &= \beta(s_t)\bar{x}(s_t) + \left[\beta(s_t)\Sigma_x(s_t)\beta^{'}(s_t) + \Sigma_\varepsilon(s_t)\right]^{\frac{1}{2}} \cdot \varepsilon_t \end{aligned} \tag{5.1.4}$$

Since the excess returns follow a regime-dependent normal distribution the implicit distribution over all regimes is thus a mixture of normal distributions. Hence, the regime-switching framework allows us to capture fat tails as well as other empirical properties of asset returns like e.g. contagion, decoupling and stochastic volatilities.

5.2 Maximum Likelihood Estimation

The above regime-switching factor model is estimated using a maximum likelihood approach. The determination of the likelihood function is based on the following notation (the following remarks refer closely to Hamilton, 1994, [51] and Gray, 1996, [47]):

$$\begin{aligned} z_t &= \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \\ \Phi_t &= \{z_t, z_{t-1}, \ldots, z_1\}, \\ f_k(z_t|\Phi_{t-1};\theta) &= f(z_t, s(t) = k|\Phi_{t-1};\theta), \\ \xi_{k,t} &= \Pr[s(t) = k|\Phi_{t-1};\theta], \\ \theta &= \{\beta(k), \bar{x}(k), \Sigma_x(k), \Sigma_\varepsilon(k), P\} \quad \text{for } k, l = 1, \ldots, K. \end{aligned}$$

The unconditional density function of the multivariate distribution of z_t is expressed by $f(z_t)$. According to this, $f_k(z_t)$ stands for the conditional density function given

$s(t) = k$, Φ_t defines the history, including all available data up to date t. Further - as introduced in previous sections - $\xi_{k,t}$ denotes the $(K \times 1)$ vector containing the probability distributions of the states (section 4.2) and the vector θ contains the population parameters, which are to be estimated (section 4.3).

The maximum likelihood function is derived by aggregating over the regimes and time. The regime-independent probability distribution function of z_t is obtained by the product of the conditional density of z_t $(f_k(z_t))$ and the marginal probability of the regime $s(t)$

$$f(z_t|\Phi_{t-1}) = f(z_t) = \sum_{k=1}^{K} f_k(z_t) \Pr\left[s(t) = k|\Phi_{t-1}\right].$$

Aggregation over time yields then the likelihood function

$$L(\theta, \Phi_T) = \prod_{t=1}^{T} f(z_t; \theta),$$

with its respective log likelihood function

$$\begin{aligned} \log L(\theta, \Phi_T) &= \sum_{t=1}^{T} \log f(z_t; \theta) \\ &= \sum_{t=1}^{T} \log \left(\sum_{k=1}^{K} f_k(z_t; \theta) \Pr[s(t) = k|\Phi_{t-1}] \right). \end{aligned} \tag{5.2.1}$$

The conditional densities $f_k(z_t; \theta)$ in equation (5.2.1) are given by equation (5.1.3). In order to derive the state probabilities conditioned on the past observations, the Hamilton filter is applied (see Hamilton, 1989, [52] and Hamilton, 1994, [51]). The filter is an iterative procedure, which involves two phases, an updating phase (see equation 5.2.2) and a prediction phase (see equation 5.2.3). By iterating the two following equations the inference and forecast for each date t is optimized:

$$\xi_{t|t} = \frac{(\xi_{t|t-1} \bigodot \eta_t)}{1'(\xi_{t|t-1} \bigodot \eta_t)} \tag{5.2.2}$$

$$\xi_{t+1|t} = P \cdot \xi_{t|t} \tag{5.2.3}$$

Here η_t represents the $(K \times 1)$ vector containing all K conditional densities. The kth element of the vector represents the conditional density $f_k(z_t|\Phi_{t-1}; \theta)$ defined in the introduced notation. P depicts the $(K \times K)$ transition matrix introduced in equation 4.2.3. 1 denotes an $(K \times 1)$ vector of ones, and the sign $\bigodot$ stands for an element-by-element multiplication. Given the starting values for the population parameter vector

θ (see section 5.2.3) and a start-up value for $\xi_{1,0}$ (see section 5.2.4), the iteration on equation 5.2.2 and 5.2.3 for $t = 1, 2, \ldots, T$ leads to the values of $\xi_{t|t}$ and $\xi_{t+1|t}$ for each date t. This iterative algorithm is used as a by-product to calculate the log likelihood function $L(\theta)$:

$$L(\theta) = \sum_{t=1}^{T} logf(z_t|\Phi_{t-1};\theta), \tag{5.2.4}$$

where

$$logf(z_t|\Phi_{t-1};\theta) = 1'(\xi_{t|t-1} \odot \eta_t). \tag{5.2.5}$$

The derivation of equation 5.2.2 is given through equation 5.2.5. First calculate the conditional joint density function of z_t and $s_t = k$, by multiplying the kth element of $\xi_{t|t-1}$, described as $P[s_t = k|\Phi_{t-1};\theta]$ and the kth element of η_t, described as $f(z_t|s_t = k, \Phi_{t-1};\theta)$:

$$P[s_t = k|\Phi_{t-1};\theta] \cdot f(z_t|s_t = k, \Phi_{t-1};\theta) = p(z_t, s_t = k|\Phi_{t-1};\theta). \tag{5.2.6}$$

The sum of the K conditional densities in equation 5.2.6 for $k = 1, 2, \ldots, K$ defines the density of the observed vector z_t based on the past observations:

$$f(z_t|\Phi_{t-1};\theta) = 1'(\xi_{t|t-1} \odot \eta_t), \tag{5.2.7}$$

as claimed in equation 5.2.5. By dividing the joint density distribution in equation 5.2.6 by the density of z_t given in equation 5.2.7, one obtains the posterior probability of state k:

$$\begin{aligned} \frac{p(z_t, s_t = k|\Phi_{t-1};\theta)}{f(z_t|\Phi_{t-1};\theta)} &= P[s_t = k|z_t, \Phi_{t-1};\theta] \\ &= P[s_t = k|\Phi_t;\theta]. \end{aligned}$$

According to equation 5.2.5:

$$P[s_t = k|\Phi_t;\theta] = \frac{p(z_t, s_t = k|\Phi_{t-1};\theta)}{1'(\xi_{t|t-1} \odot \eta_t)}. \tag{5.2.8}$$

Note that the numerator on the right side of equation 5.2.8 is the kth element of the vector $(\xi_{t|t-1} \bigodot \eta_t)$ and the left side of equation 5.2.8 corresponds to the kth element of the vector $\xi_{t|t}$. Therefore, writing equation 5.2.8 in vectorial notation results in:

$$\xi_{t|t} = \frac{(\xi_{t|t-1} \bigodot \eta_t)}{1'(\xi_{t|t-1} \bigodot \eta_t)}, \tag{5.2.9}$$

as claimed in equation 5.2.2.

A regime-switching framework is a more complex, flexible structure, able to model data, generated by different economic mechanisms - all within a single, unified model. This recursive nature simplifies the construction of the likelihood function, allowing simple estimation of relatively difficult models (see Gray, 1996, [47]).

Former studies have pointed out that, by reasons of the observed stylized facts mentioned above, two regimes capture most of the financial market dynamics and get satisfactory results.

5.2.1 Convergence of the Likelihood Function

Due to the statistical advantages of maximum likelihood estimators, many researchers applied the method of maximum likelihood to estimate normal mixture models, respectively a mixture of normal regressions. This method, however, bears two main challenges. On the one hand the density function $f(z_t)$, given by the mixture of several normal distributions, leads to a likelihood surface with several local maxima, where the global maximum does not exist. On the other hand, next to the missing global maximum, the likelihood function might contain singularities. Such a singularity occurs, if one of the standard deviations of any component converges to zero. In such a case the value of the likelihood function of the mixture converges to the major mode of likelihood, since it becomes infinite.

In the case of a mixture of normal regressions a singularity arises whenever $\beta(s_t)$ is chosen so that any observation $y_t(s_t)$ is exactly equal to $\beta(s_t)\bar{x}(s_t)$. Subsequently, as the variances of the error terms tend to zero, the likelihood function $L = \sum_{t=1}^{T}(f_t)$ increases to infinity:

$$f(y_t|s_t) = 2\pi^{-\frac{N}{2}}|\Sigma_{y(s_t)}|^{-\frac{1}{2}} e^{\left(-\frac{1}{2}[y(s_t)-\beta(s_t)\bar{x}(s_t)]'\ \Sigma_{y(s_t)}^{-1}\ [y(s_t)-\beta(s_t)\bar{x}(s_t)]\right)}. \tag{5.2.10}$$

Researchers came up with two different solution to solve this inconvenience. On one side Quandt & Ramsey, 1978, [74] introduced with the moment generating function estimator an alternative method. On the other side the maximum likelihood approach was extended by constraints for the variances of the components of the mixture. Prior to the estimation the variances have to be bound away from zero, in order to avoid any singularities of the likelihood function. This has a certain 'smoothing' effect on the likelihood surface. For univariate mixtures, respectively the mixture of two normal regressions, feasible restrictions are of the form $\sigma_1 = \sigma_2$, $\sigma_1 = k\sigma_2$ or $\min(\sigma_1, \sigma_2) > c$ (see CAUDILL & ACHARYA, 1998, [18]).

By developing and using sophisticated methods for the maximum likelihood estimation of mixture models, researchers were successful in reducing the computational difficulty and in weakening the singularity issue (e.g. the Expectation Maximization (EM) method by DEMPSTER, LAIRD & RUBIN, 1977, [29]). Many stated that the singularity of the likelihood surface has been exaggerated in the past (see FOWLKES, 1979, [37]). HOSMER, 1978, [55], however, points out that for small sample sizes (less than 300) and for mixtures where the components are close together, singularity still poses a problem. LEYTHMAN, 1984, [66] examined this statement by conducting a Monte Carlo experiment to test the incidence of singularities occurring in the maximum likelihood estimation of a mixture of two univariate normals. His findings indicate that singularities might be a problem. However, he relativizes it by stating that even in the worst case the number of singularities is relatively small and that the incidence of these singularities declines with an increasing sample size and increasing degree of separation between the components. CAUDILL & ACHARYA, 1998, [18] extended this experiment to the regression case. They found similar results and came to the conclusion that a convergence to singularities in maximum likelihood of mixtures of regressions might in fact be lower than in the case of univariate normal mixtures.

To put it simply, due to the likelihood surface with several local maxima the singularities do not cause a main problem. The optimization rather converge to one of these reasonable local maxima than to a singularity (see e.g. HAMILTON, 1994, [51] or KIEFER, 1978, [62]). Thus, the choice of the start-up values has an important relevance in the maximum likelihood estimation (see ABIAD, 2007, [2]). The model introduced in section 5.2 avoids the incidence of singularities by setting boundaries to the estimated parameters, mainly to the variance parameters. How narrow the interval of these boundaries needs to be is discussed in section 5.2.3.

The maximum likelihood algorithm increases very quickly close to the local maximum, but then needs more iterations to reach the convergence. A phenomenon which has often been observed in other studies (see e.g. DIEBOLD, WEINBACH & LEE, 1994,

[31]). The successful convergence and its speed can be controlled by the dimensions of the model (see section 5.2.2) and by the chosen starting values (see section 5.2.3).

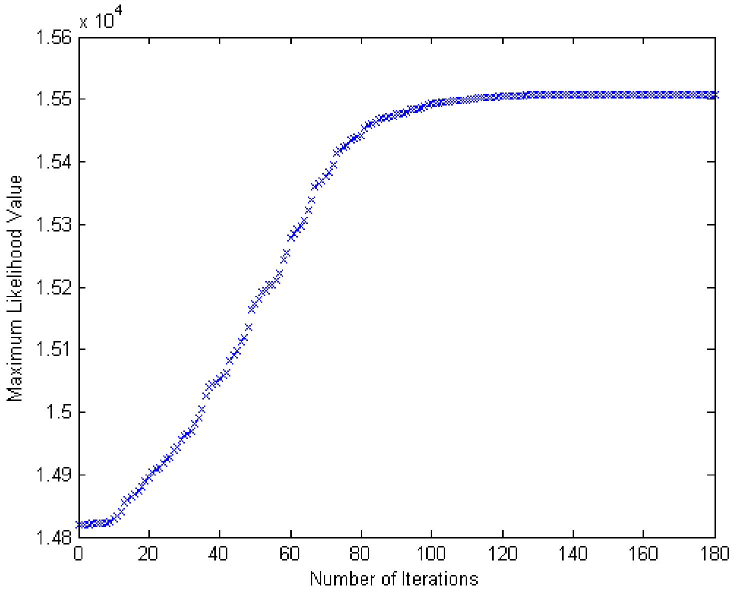

Figure 5.1: Log likelihood function converging to a local maximum. Based on a model with 145 dimension: 6 factors, 10 assets and time series of 353 observations.

5.2.2 Dimensions of the model

Calibrations have shown that the model might fail to converge when too many explanatory variables are included (see also Abiad, 2007, [2]). For that reason, to avoid a fail of convergence especially for big asset universes, a reduction of dimensions is inevitable. This section as well as section 6.2.1 'Restrictions and Significance Tests' deal with this issue.

To perform portfolio analyses, the expected returns and the variances for each asset have to be computed. In addition, numerous correlation coefficients ρ_{ij} for all pairs of assets i and j ($i \neq j$) have to be estimated. An institution following N assets would end up estimating $N(N-1)/2$ correlation coefficients. This results in a total of $2N+N(N-1)/2$ estimations. Due to the exponential growth of this term an institution would quickly hit several capacity limits[1].

This issue has led researchers to develop models who describe and predict more compactly the correlation structures between securities. They came up with different mod-

[1] According to Elton, Gruber, Brown & Goetzmann, 2003, [33] most financial institutions follow between 150 and 250 stocks, which would lead to the estimation of 11′715 and 31′125 correlation coefficients.

els that can be classified into two categories: index models and averaging techniques. The conditional factor model introduced in chapter 5, belonging to the class of index models, lies in an intermediate position between the full historical correlation matrix and the single-index model (e.g. CAPM model). Due to such a factor model the estimation parameters for portfolio analyses can drastically be reduced. According to the factor model described in equation 5.1.4, the number of estimates is diminished to $N + NL + 2L + L(L-1)/2$, where L denotes the number of factors. Figure 5.2a) demonstrates how the number of estimations for the full historical correlation matrix, due to the exponential effect, pulls away. Figure 5.2b), magnifying a part of figure 5.2a), illustrates the break-even points towards factor models with different numbers of factors ($L = 1, 2, \dots, 10$).

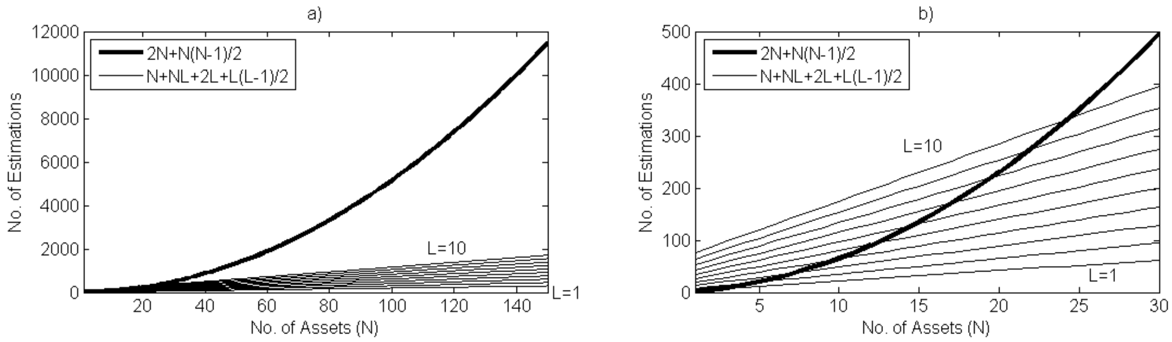

Figure 5.2: Dimension of the model subject to the amount of factors (L) and assets (N).

In order to cover a significant part of the variance-covariance matrix of the given asset universe the factors need to be well chosen. The more factors are included in the model, the better the historical correlation matrix will is estimated, however, the more complex the model will become. The amount of the included factors needs to be well-balanced, all the more, given the fact that added factors might contribute more random noise than further useful information into the forecasting process (see Elton et al., 2003, [33]). The assumption of the multi-factor model is that the covariance of the assets' residuals are zero ($E(\epsilon_i \epsilon_j) = 0$), where $i = 1, \dots, N$ and $j = 1, \dots, N$ ($i \neq j$). Meaning that the dependency structure of the assets is fully explained by the vairance-covariance matrix of the chosen factors (Σ_x). The challenge lies in finding factors who complementarily describe the whole risk-structure of the underlying asset universe (Σ_y).

A well-chosen factor model is able to drastically reduce the amount of estimating parameters. Such a reduction is essential to a regime-switching model with k regimes, in which all estimating parameters are to be estimated for each regime and also for the

Markov parameters. In section 6.2.1, the different parameters will be tested for their regime dependency. On the one hand this procedure further reduces the dimension of the model, on the other hand it will provide more information about the dynamics of the financial markets.

5.2.3 Determination of Optimal Start-Up Values and Bounds

The likelihood function of a regime-switching model to be optimized is non-convex and therefore many different local maxima exist. It is thus essential to define reliable start-up values. Caudill & Acharya, 1998, [18] mention three reasons why starting values for mixtures of normal distributions (resp. regressions) are of important matter. At first, good starting values reduce the frequency of the occurrence of singularities. Secondly, feasible starting values accelerate the estimation procedures. Finally, they argue that a simple method for generating feasible starting values would help make the estimation of mixtures of normal regressions faster and more practical, since mixture normal distributions models can require a high number of starting values for regression parameters. Since the starting values are inferred from the historical data, the next paragraph explores the history in the light of data clustering (regime pattern). Based on these findings a feasible starting value generater is introduced.

The historical data of the financial markets mainly follow two regimes. A three dimensional histogram explains the separation of these two regimes. The x-axis denotes the correlation between the equity factor (EQ) and the bond factor (BD) $\rho_{EQ,BD}$, while the y-axis defines the market volatility σ_m. The z-axis contains the frequency of each combined bin. Figures 5.3 a) to c) illustrate how the observations of the past were distributed, depending on different historical time periods. The graph can be divided into four regions: Region 1 is characterized by low volatility and positive correlation. Data in region 2 contain low volatility and negative correlations. Region 3 consists of high volatility and negative correlated data and region 4 is characterized by high volatility and positive correlation. The observations are mainly gathered in two of these four regions, namely in region 1 and region 3. They represent the two regimes, defined in section 2.1.

Region 3 represents regime 1 with high volatility and negative correlation between EQ and BD ($\rho_{Eq,BD}$). Region 1 represents regime 2 with lower volatility and positive correlation ($\rho_{Eq,BD}$). The aggregated heights of the beams within a regime determine that over the long term (see figures b and c) the financial market had more often the risk/return-structure represented by region 1. Regime 2 occurs at around two thirds of time, while regime 1 (region 3) occurs to one third. This proportion reflects the business cycles of the past and is in line with the steady state probabilities of the regime-switching probabilities.

The observations of the histogram in figure 5.3a), covering period September 1992 to August 2004 are almost exclusively either located in region 1 or in region 3. Observations of the years before and after this time period skewed this pattern to some degree. Around the year 1990 there were some observations with high volatility and positive correlation (region 4). During these days the decoupling phenomenon did not hold and lead to extremely low possibility to diversify the high risks (see figure 5.3b). During the time period September 2004 and August 2008 the observations were characterized by low volatility and negative correlation, visualized in figure 5.3c) by the positive frequencies (higher beams) in region 2[2].

Figure 5.3d) illustrates the cumulated frequency of the four regions over time[3]. It is obvious that region 1 and 3 are dominating and usually alternate between each other. Region 2 and 4 are either transition regions or rather one-time-only occurrences, not yet established in the market. Therefore, in the last three decades, the financial markets followed mainly either the characteristic of region 1 or region 3, captured by regime 2 and regime 1. These insights about the historical evolvement of the regimes, help to model an appropriate starting value generator.

Since the likelihood surface contains several local maxima, the starting values for the two regimes already need to possess the structure of the stylized facts observed at the financial markets. Otherwise the maximum likelihood estimation might stuck in a suboptimal local maximum. According to figure 5.3 and in order to gain the structure described by the stylized facts, the market volatility is used as key differentiator to distinguish the given historical data into two data pools (representing the regimes). The median of the historical market volatility $\sigma_{med.}$ as a reference, assigns each point of time t into one of the data pools, depending on the market volatility $\sigma_{m,t}$ at time t (rolling window with a lag of l). If the market volatility at time t is higher than the median ($\sigma_{m,t} > \sigma_{med.}$) the data t to $t + l$ are assigned to the high-volatile pool, otherwise ($\sigma_{m,t} \leq \sigma_{med.}$) they are put into the low-volatile pool. These two data pools are then used to calculate the regime-dependent starting values. They contain[4] either the observations of the regions 1 and 2 or of the regions 3 and 4.

[2]For more details to this phenomenon see section 2.2.

[3]Note: The annualized market volatility $\sigma_{m,t}$ and the correlation $\rho_t(EQ, BD)$ are calculated by a 24-months rolling window. Therefore, one has to take a lag into account concerning the market development.

[4]By ignoring any differences between the mean and the median.

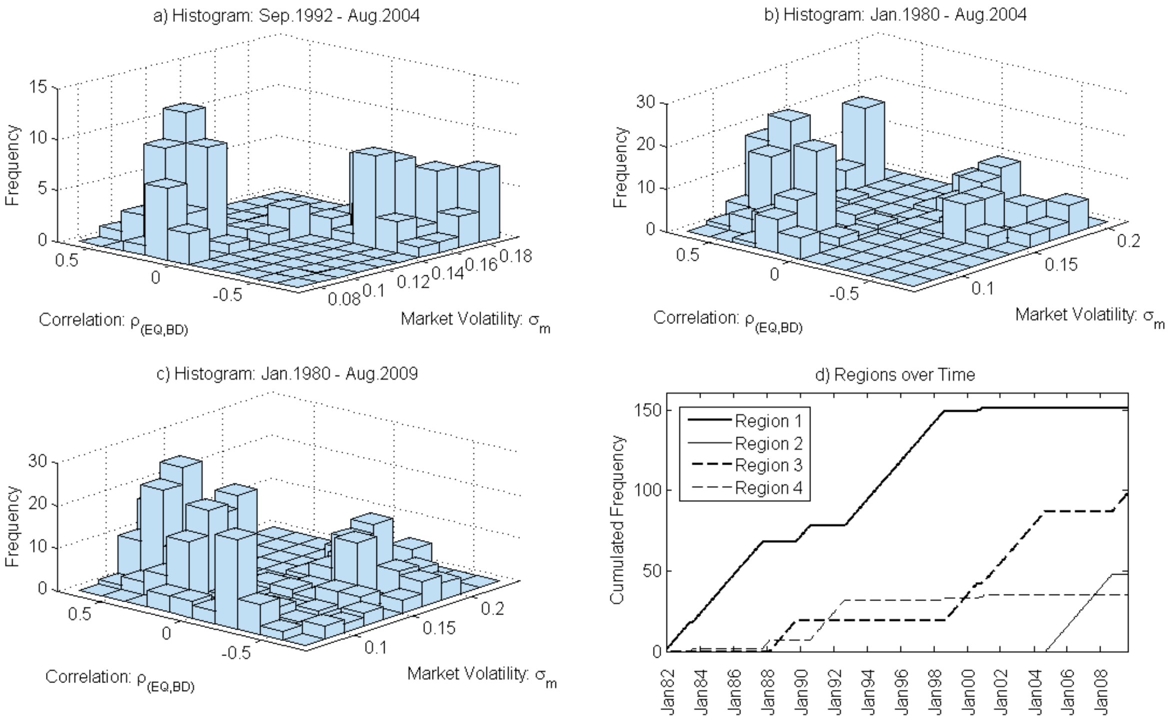

Figure 5.3: Historical data risk structure visualized within a three dimensional histogram.

The feasible starting values are then obtained by calculating the descriptive statistics for these two data pools. This results in two mean-vectors ($\tilde{\mu}_{s(t)}$) and two variance-covariance matrices ($\tilde{\Sigma}_{x,s(t)}$) of the chosen factors. In this manner the starting values, describing the two multivariate normal distributions, already capture the two different risk/return-structures of regime 1 and regime 2 and are close to the historically[5] expected local maximum.

One of the most influential parameters on singularities are the volatilities of the factors. Their starting values ($\tilde{\sigma}_X$) need to be on a feasible level and should be kept within specified limits, in order to avoid volatilities too close to zero and therefore to avoid singularities. One solution to bound the volatilities is by using a simple interval. Adding or subtracting a certain percentage α from the starting value, leads to the upper $(1-\alpha)\tilde{\sigma}_X$ and lower bound $(1+\alpha)\tilde{\sigma}_X$ of the volatilities. On the one hand this approach assures that each volatility is bound away from zero with an amount of $(1-\alpha)\tilde{\sigma}_X$. On the other hand it allows an optimization margin of $2\alpha \cdot \tilde{\sigma}_X$. The maximum likelihood estimation finds a local maximum within these ranges without exhausting the restrictions and by strictly avoiding any singularity. However, when neglecting any boundaries the estimation overshoots and might end in singularity. The

[5]Since the model at hand includes no fundamental analysis, the history is the only information and the best estimator for the future.

correlations among the factors have as an absolute measure defined boundaries of -1 and 1. These boundaries are sufficient and need not to be more restricted to find a satisfying local maximum.

The starting values of the (L x N)-matrix of the coefficient parameters β, denoting the factor loadings of each asset, are best chosen by setting them equal to the estimation given by the ordinary least square method. The convergence to the global, respectively to a feasible local maximum is independent of the starting values of the betas. Even for extreme and absolutely unrealistic starting values $\tilde{\beta}$ the maximum likelihood estimation finds the optimal beta close to the value given by the estimation of the ordinary least square method. Therefore, the starting values of the betas are irrelevant when it comes to find a global, respectively feasible local maximum. However, good starting values improve the time of convergence. Since the starting values are irrelevant in order to find the optimum, also the boundaries of these betas are negligible. Without losing a lot of flexibility one could also fix the betas beforehand by predefining them with the OLS-method. This would reduce the dimensions of the model.

Based on the starting values of $\tilde{\Sigma}_{x,s(t)}$ and of $\tilde{\beta}$, the starting values of the volatilities of the residual terms ($\tilde{\sigma}_\epsilon$) are being calculated. The rules for the boundaries of $\tilde{\sigma}_\epsilon$ are equal to the rules of $\tilde{\sigma}_X$. For the correlations of the residuals the starting value is zero, the value of the expected correlations. The correlations among the residual terms of the single assets have as an absolute measure defined boundaries of -1 and 1. These boundaries are sufficient and need not to be more restricted to find a satisfying local maximum.

Next to the starting values of the parameters that describe the conditional normal distributions of the regimes, also the starting values of the parameters of the Markov chain need to be defined.

The mixture of two normal regressions is characterized by the probability distribution of the states i at time zero $\xi_{i,0}$ $(i = 1, \ldots, N)$ and by the parameters of the transition matrix P. Therefore, in the present 2-regime-switching model only three parameters are needed in order to fully define the Markov chain: $\xi_{1,0}$, p_{12} and p_{21}[6]. Carefully selected starting values for these Markov chain parameters are based on historical observations. The determination of the starting values for the initial probabilities $\xi_{i,0}$ has many options and is discussed separately in section 5.2.4. The boundaries of $\xi_{i,0}$ are given by definition with 0 and 1. This boundaries are sufficient and need not to be more restrictive to find a satisfying local maximum. The ratio given by the potential observations of regime 1 and potential observations of regime 2 (see figure 5.3) broadly

[6] Alternative: $\xi_{2,0}$, p_{11} and p_{22}

defines the starting values of the transition probabilities. The starting values for the transition matrix P, modeled as a logistic function, should contain the information of the volatility clustering phenomenon. The bigger the clusters, thus the longer the financial markets are nonstop located in the same regime i, the smaller the starting values of the transition probabilities p_{ij} $(i \neq j)$ are to be set. In the 2-regime-switching model at hand, the starting value for the less-volatile regime 2 is lower than for the volatile regime 1. Cause according to the steady-state probabilities of the historical data and due to the results found in figure 5.3 the market is more often and for longer time periods located in the low-volatile markets (regime 2). The boundaries for the parameters p_{ij} $(i \neq j)$ of the transition matrix P are set by 1% and 20%. The lower boundary guarantees that the Markov chain is ergodic (irreducible), and has no absorbing state. The upper bound assures that the Markov chain model clusters and does not switch too often. In most cases these boundaries are negligible, since they are not exhausted in the optimal result of the maximum likelihood estimation. This again sustains the stylized facts of the financial markets and speaks for the regime-switching approach.

By computing the starting values with the procedure just discussed, the mixture of the two multivariate normal regressions seems to be located on the likelihood surface close to the satisfying and optimal local maximum. However, when the returns for each factor, respectively assets are defined equal in both regimes, keeping the volatilities regime-dependent, the likelihood surface will be by definition a mixed multivariate fat-tailed unimodal distribution (see section 4.3). When the returns μ are regime-independent, the likelihood surface contains only one mode. In this case, the starting values do not need to possess the structure of the stylized facts in order to reach the global maximum, since each local maximum is also the global one. Optimal starting values are a lot more arbitrary. Nevertheless, on the one hand good starting values reduce the time, respectively the iterations to convergence and on the other hand, even more importantly, the ever present singularity problematic still needs to be bound.

In the case where the returns are regime-independent, it does not matter whether the starting values already possess the two-regime structure, generated according to the above-mentioned pool-approach or whether the starting values are randomly chosen, with reasonable values. The maximum likelihood estimation finds the same optimal global maximum, with exactly the same maximum likelihood value. In a specific example, with an asset universe of 6 assets and with optimal starting values the model needs 68 iterations to reach the optimal maximum likelihood value. When one uses suboptimal starting values (randomly chosen) it needs e.g. 75 iterations to attain the same optimal maximum likelihood value.

In the case where the returns are regime-dependent optimal starting values are crucial, due to bimodal (multimodal) mixture distributions. The calibration with optimal starting values leads to a risk/return-structure that is close to the one of the case where the returns are regime-dependent. According to the four further parameters the maximum likelihood value is, however, slightly higher. The same calibration with suboptimal starting values, gets stocked in a suboptimal local maximum, without hitting any boundaries. On the one hand the likelihood function value is significantly lower than for the optimal local maximum and on the other hand the risk/return-structure does not fully match the given stylized facts of the financial markets. Figure 5.4 illustrates this issue by plotting the smoothed state variable $\xi_{1,t|T}$ over time.

5.2.4 Initial Probabilities $\xi_{k,0}$

The treatment of the start-up values of $\xi_{k,0}$ depend, according to Diebold, Weinbach & Lee, 1994, [31], on whether the applied time series are stationary or not. If they are stationary, then $\xi_{k,0}$ is to be set at the level of the steady-state probabilities. Whenever the time series are non-stationary, then the start-up probabilities of the states at time 0 is an additional parameter that must be estimated. In practice, however, the start-up probabilities have an insignificant effect on the likelihood function, on the basis that the start-up values of the probabilities become irrelevant in a large sample. For a long enough data set it gets therefore insignificant whether one estimates $\xi_{k,0}$ as a separate parameter, sets it at the level of the steady-state probabilities or even at an arbitrary chosen value in the interval [0,1] (see also Gray, 1996, [47] or Abiad, 2007, [2]).

Significance tests by means of the likelihood ratio test (Hamilton, 1994, [51]) sustained the negligible effect of the start-up values for the initial probability $\xi_{k,0}$ in a large sample. The sensitivity analysis of the value of the log likelihood function $\log L(\theta)$, to changes in the initial probability $\xi_{k,0}$ and to changes in the sample size, are registered in table 5.1. Four different sample sizes with 50, 150, 250 and 350 observations were selected, covering different time periods: July 2007 - August 2009 (50 observations), March 1997 - August 2009 (150 obs.), November 1988 - August 2009 (250 obs.) and July 1980 - August 2009 (350 obs.). The predefined initial probabilities range over the whole feasible spectrum, thus from 0 to 1 with a step size of 0.1. It is in evidence that the log likelihood values $\log L(\theta)$ of longer sample sizes are less sensitive to changes in $\xi_{k,0}$ than shorter sample sizes. The resulting parameter vector θ is distinguished into the unrestricted maximum likelihood estimate $\hat{\theta}$ and into the restricted maximum likelihood estimate $\tilde{\theta}$. The values of the log likelihood function for the unrestricted case $\log L(\hat{\theta})$, in which the initial probabilities $\xi_{k,0}$ are not predefined but estimated, are by

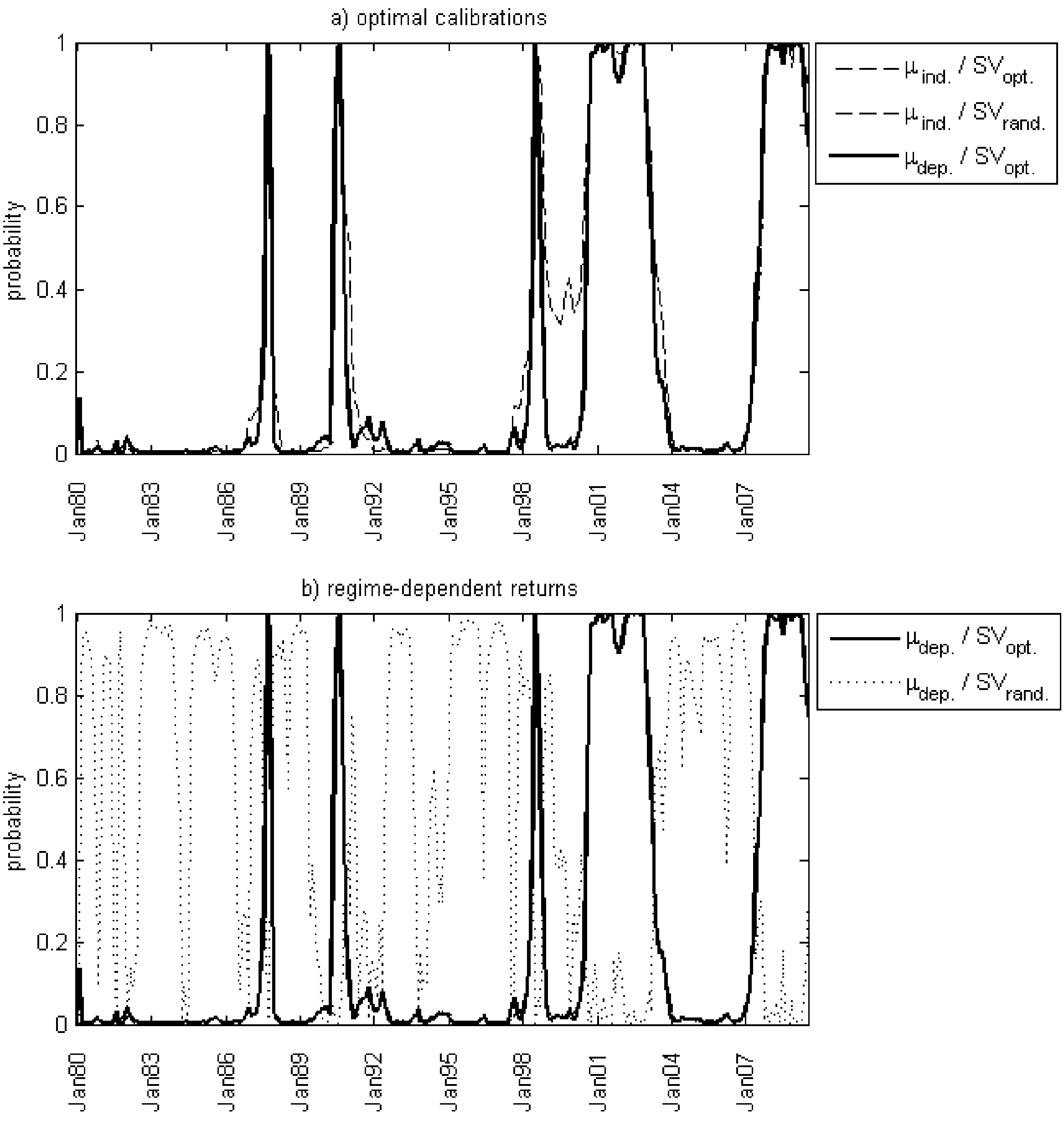

Figure 5.4: Smoothed state variables $\xi_{1,t|T}$ over time. Legend: $SV_{opt.}$ = optimal starting value, $SV_{rand.}$ = randomly chosen starting values, $\mu_{ind.}$ = regime-independent means, $\mu_{dep.}$ = regime-dependent means.

definition greater or equal than for the restricted case $\log L(\tilde{\theta})$. $\log L(\tilde{\theta})$ denotes the possible maximum value attained for the log likelihood function while still satisfying the restrictions. In the 2-regime case at hand, the optimal values of the initial probabilities $\xi^*_{1,0}$ are all found in the boundaries. Depending on the starting date of the time series, the four sample sizes have an optimal initial probability of either $\xi^*_{1,0} = 0$ (for 50, 250 and 350 obs.) or $\xi^*_{1,0} = 1$ (for 150 obs.). On the one hand the restricted log likelihood values $\log L(\tilde{\theta})$ diminish the further the predefined start-up probability $\xi_{1,0}$ deviates from the optimal value $\xi^*_{1,0}$. On the other hand the shorter the time series, the bigger the impact on the log likelihood value.

	50 Obs.	**150 Obs.**	**250 Obs.**	**350 Obs.**
$\boldsymbol{\log L(\hat{\theta})}$	1'556.15	4'490.29	7'111.23	9'678.30
$\boldsymbol{\xi_{1,0}}$		$\boldsymbol{\log L(\tilde{\theta})}$		
0.0	(*)1'556.15	4'482.61	(*)7'111.23	(*)9'678.30
0.1	1'556.04	4'488.38	7'111.13	9'678.23
0.2	1'555.93	4'488.86	7'111.02	9'678.16
0.3	1'555.79	4'489.19	7'110.90	9'678.08
0.4	1'555.64	4'489.44	7'110.76	9'678.00
0.5	1'555.46	4'489.64	7'110.60	9'677.91
0.6	1'555.23	4'489.81	7'110.41	9'677.82
0.7	1'554.95	4'489.95	7'110.17	9'677.72
0.8	1'554.54	4'490.08	7'109.87	9'677.63
0.9	1'553.85	4'490.19	7'109.44	9'677.55
1.0	1'552.81	(*)4'490.29	7'108.77	9'677.47

$\log L(\hat{\theta})$ = max. likelihood value, $\xi_{1,0}$ estimated

$\log L(\tilde{\theta})$ = max. likelihood value, $\xi_{1,0}$ predefined

(*): $logL(\hat{\theta}) = logL(\tilde{\theta})$, optimal $\xi^*_{1,0}$

Table 5.1: Sensitivity analysis of the value of the log likelihood function $\log L(\theta)$ to changes in the initial probability $\xi_{k,0}$ and to changes in the sample sizes. Column: Four different sample sizes with 50, 150, 250 and 350 observations, covering different time periods: Jul.2007 - Aug.2009 (50 obs.), Mar.1997 - Aug.2009 (150 obs.), Nov.1988 - Aug.2009 (250 obs.), Jul.1980 - Aug.2009 (350 obs.). Row: The predefined initial probabilities, ranging from 0 to 1 with a step size of 0.1.

One way to determine if these differences are statistically significant or not is the likelihood ratio test (see HAMILTON, 1994, [51]). This significance test proceeds on the assumption that the twofold difference between $L(\hat{\theta})$ and $L(\tilde{\theta})$ is approximately χ^2-distributed. Where the degree of freedom m denotes the number of restrictions:

$$2[L(\hat{\theta}) - L(\tilde{\theta})] \approx \chi^2(m). \tag{5.2.11}$$

When the size of the unrestricted parameter vector $\hat{\theta}$, containing all parameters that are to be estimated, is defined as a $(a \times 1)$-vector, then the restricted parameter vector $\tilde{\theta}$ has the size of $[(a - m) \times 1]$. The likelihood ratio $\chi^2(m)$, obtained by the term on the left-hand side of the equation 5.2.11 is summarized in table 5.2. The critical value of the test $\chi^2_{c,m=1}$, involving a single restriction $(m = 1)$, is 2.706 at the 10% significance level, 3.841 at the 5% significance level and 6.635 at the 1% significance level. For the calibration with a sample size of 350 observations, the starting value of the initial probability $\xi_{1,0}$ is statistically insignificant, since $\chi^2 \leq \chi^2_c$ holds for all possible $\xi_{1,0}$. The null hypothesis $L(\hat{\theta}) \neq L(\tilde{\theta})$ can not be rejected at the 10% significance level. For the three smaller sample sizes not every predefined $\xi_{1,0}$ has an insignificant impact on the likelihood function. Certain discrepancies between $\xi_{1,0}$ and $\xi^*_{1,0}$ lead to a significantly smaller log likelihood value. If $|\xi_{1,0} - \xi^*_{1,0}| \geq 0.8$ then the null hypothesis has to be rejected at the 10% significance level for all three sample sizes. For the extreme discrepancy of $|\xi_{1,0} - \xi^*_{1,0}| = 1.0$ the null hypothesis has to be rejected at the 5% significance level for the sample size of 250 observations and even at the 1% significance level for the two smallest sample sizes[7].

$\xi_{1,0}$	$\chi^2(m=1)$			
	50 Obs.	150 Obs.	250 Obs.	350 Obs.
0.0 (*1.0*)	0.00	*0.00*	0.00	0.00
0.1 (*0.9*)	0.21	*0.20*	0.20	0.14
0.2 (*0.8*)	0.45	*0.42*	0.41	0.29
0.3 (*0.7*)	0.71	*0.67*	0.66	0.44
0.4 (*0.6*)	1.02	*0.96*	0.93	0.61
0.5 (*0.5*)	1.39	*1.30*	1.26	0.78
0.6 (*0.4*)	1.83	*1.70*	1.64	0.96
0.7 (*0.3*)	2.41	*2.20*	2.11	1.15
0.8 (*0.2*)	3.22	*2.86*	2.72	1.34
0.9 (*0.1*)	4.60	*3.82*	3.58	1.51
1.0 (*0.0*)	6.67	*15.36*	4.92	1.66

$\chi^2(m) = 2[\log L(\hat{\theta}) - \log L(\tilde{\theta})]$

Table 5.2: The likelihood ratio $\chi^2_{(m=1)} = 2[\log L(\hat{\theta}) - \log L(\tilde{\theta})]$. Column: Four different sample sizes with 50, 150, 250 and 350 observations, covering different time periods: (50) Jul.2007 - Aug.2009, (150) Mar.1997 - Aug.2009, (250) Nov.1988 - Aug.2009, (350) Jul.1980 - Aug.2009. Row: The predefined initial probabilities, ranging from 0 to 1 with a step size of 0.1. Please note the reverse order for the second column (highlighted by cursive script).

The high likelihood ratio of $\xi_{1,0}(m = 1) = 15.36$ for the sample size with 150 observations demonstrates the danger of poorly predefined initial probabilities for short

[7]Please note the reverse order of the second column (highlighted by cursive script).

time series. The given severer restrictions, forces the inference of the regime probabilities $\xi_{1,t|t}$ to find a suboptimal path. While after three time steps the state inferences for $\xi_{1,0} \leq 0.9$ match the optimal state inference (see figure 5.5a), the inference for the predefined $\xi_{1,0} = 1$ runs another and significantly worse path (see figure 5.5b), even though optimal for the given restrictions. This does not only lead to suboptimal Markov structures but also to suboptimal risk/return-structures of the two observed regimes.

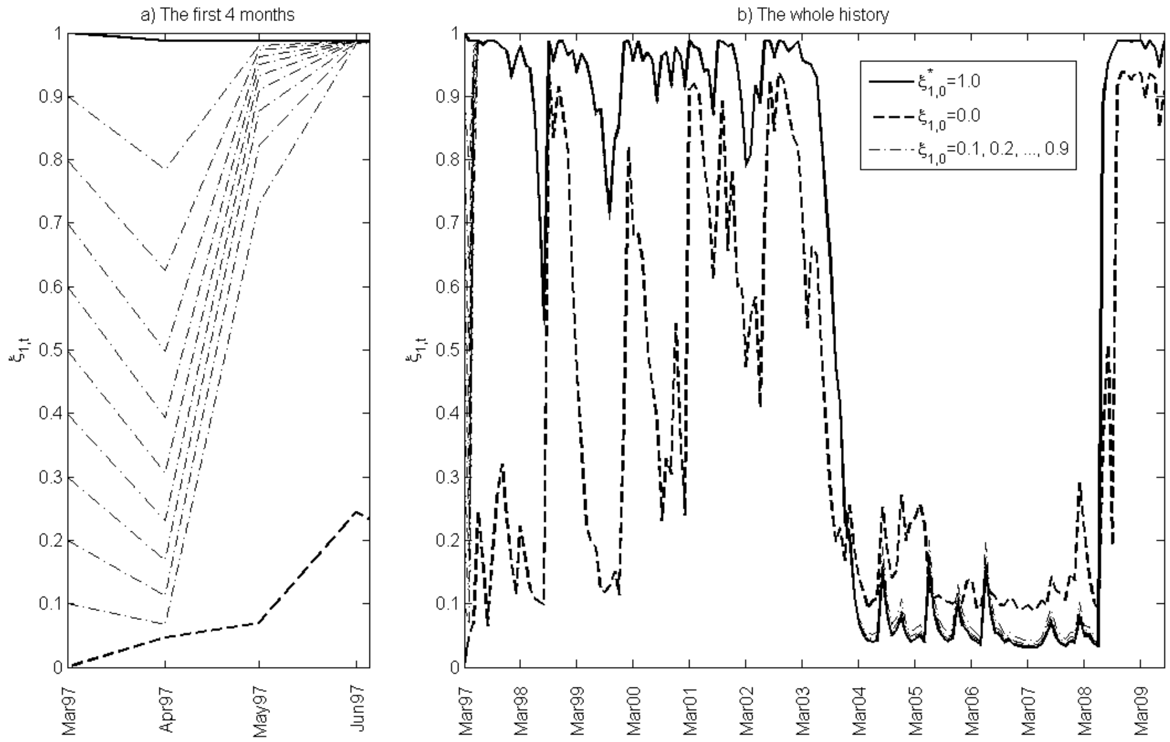

Figure 5.5: Actual and smoothed inference depending on the initial probability $\xi_{1,0}$.

In summary the fact can be confirmed that for long enough sample sizes the initial probability $\xi_{1,0}$ becomes insignificant concerning the maximization of the likelihood function. Therefore, it does not matter for big sample sizes wether one estimates it as a separate parameter, sets it at the level of the steady-state probabilities or even at an arbitrary chosen value within the interval [0,1]. For small sample sizes its recommended to either estimate it as a separate parameter or to use the steady state probabilities as predefined values.

5.2.5 State Variable $\xi_{k,t}$ and Smoothing Algorithm

In order to calculating the smoothed inferences for the regime probabilities the Kim filter is suggested as an extension of the Kalman filter (see KIM, 1994, [64]). It is a combination of the Kalman filter and the Hamilton filter, specially-designed for Markov-switching models (HAMILTON, 1994, [51]; HAMILTON, 1989, [52] or HAMILTON, 1988, [53]).

The smoothed inference of the state variable $\xi_{t|\tau}$ does not only consider the data up to time t ($\xi_{t|t}$) but also the data obtained up to the date T ($\xi_{t|T}$). Whenever $\tau < t$ holds, it is a matter of forecasts about the regimes for some future date, however, when $\tau > t$ holds, one speaks of the smoothed probabilities for the regime at time t. KIM, 1994, [64] shows that the smoothed inferences can be estimated by using following iterative algorithm:

$$\xi_{t|T} = \xi_{t|t} \odot \{P' \cdot [\xi_{t+1|T}(\div)\xi_{t+1|t}]\} \tag{5.2.12}$$

where the signs $\odot$ and $(\div)$ stand for element-by-element multiplication respectively division. This iterative process starts with the probability $\xi_{T,T}$, which is obtained from the equation [5.2.2] for $t = T$ and iterates backwards on equation [5.2.12]. It is, however, only valid when s_t follows a first-order Markov chain as explained in section 4.2.

Therefore, once the parameters are estimated the smoothed inference about s_t can be calculated based on the full information in the sample. Figure 5.6 shows the actual $\xi_{1,t|t}$ and smoothed inferences $\xi_{1,t|T}$ for the probability of regime 1 over time. It is obvious that the inferences, based on the full information proceeds smoother over time, since it does not react on short term deviations but only on significant changes that are sustained by future and known data. These probabilities are also more extreme, since more information reduces uncertainty. Since January 1980 the smoothed probabilities of the regimes have been lower than 5% or higher than 95% in 77.3% of all months. Further, the full information makes it possible to react earlier on significant changes in risk/return-structures (regime changes). Consequently, the process of $\xi_{1,t|T}$ moves a few time-periods ahead of the process $\xi_{1,t|t}$. However, one has to keep in mind that the smoothed inference results from a backward iteration on the full information. After the maximum likelihood estimation the calculated smoothed inference could be used to recalibrate the parameters. This would lead to smoothed explanatory parameters that are based on the full information ($\Sigma_{x_{t|T}}, \beta_{t|T}, \ldots$).

The actual inference $\xi_{1,t|t}$, based on the information given up to time t, fluctuates more and has not as extreme probabilities as the smoothed inference, since it does not know the true values of the future observations and therefore already reacts on short term trends. However, volatility clusterings and hence the changing risk/return-structures are good visible. The model is therefore able to distinguish the regimes over time.

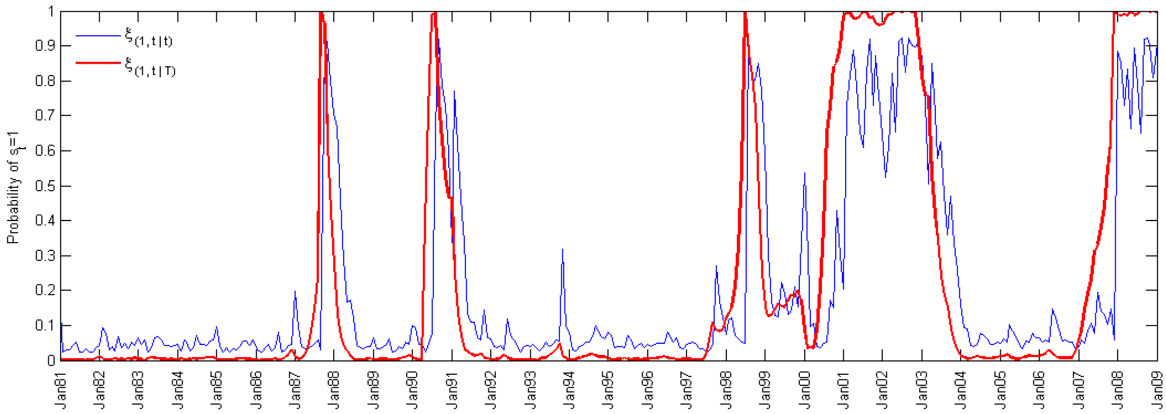

Figure 5.6: Actual and smoothed Inferences for the regime probabilities.

5.2.6 Summary

The used starting value generator and the chosen boundaries have been proved to be suitable. This is mainly shown in the fact that the optimal local maximum of the model with regime-dependent returns closely matches the global maximum of the model with regime-independent returns. However, only if one starts with starting values obtained by the pool-approach.

The interaction between the regarded data set, the reliable start-up values and the required boundaries of the estimated parameters needs to be understood in order to handle the non-convex likelihood function and to find satisfying local maxima.

The regime-switching approach introduced in the work at hand, aims to model the dramatic breaks of the financial time series, to capture the stylized facts of financial markets. Therefore, it makes little sense to use a two regime-switching model if the available data set does not cover the data of at least one business cycle. It would still lead to correct results, but the resulting regimes would not differ significantly from the historical risk/return-structure.

Chapter 6

Illustrative Example

6.1 Data

The illustrative example, containing monthly data and covering period January 1980 to August 2009 (356 monthly observations per time series), is based on seven factors and considers an international portfolio with eleven asset categories. The financial time series used for empirical analysis refer to monthly, continuous excess returns from a Swiss franc point of view. To obtain useful information concerning international asset allocation we proceed on an asset universe of eleven worldwide assets: Next to assets from Switzerland, assets of the following three important financial world regions were considered: Europe, U.S.A. and Japan.

Token	Description	Details
CHb	Swiss Bond	Swiss Total All Lives *DS Govt. Index - Total Return
EUb	Euro Bond	German Total All Lives DS Govt. Index - Total Return
UKb	United Kingdom Bond	UK Total All Lives DS Govt. Index - Total Return
USb	U.S. Bond	U.S. Total All Lives DS Govt. Index - Total Return
JPb	Japan Bond	Japan Total All Lives DS Govt. Index - Total Return
CHs	Swiss Stock	Swiss Performance Index, Total Return
EUs	European Stock	MSCI Europe Standard Index, Gross Return
USs	U.S. Stock	MSCI U.S. Standard Index, Gross Return
JPs	Japan Stock	MSCI Japan Standard Index, Gross Return
EMs	Emerging Market Stock	MSCI EM Standard Index, Gross Return
ReE	Swiss Real Estate	Schroder ImmoPlus, Close-End Fund, prime Swiss locations. The fund distributes its earnings.
Sources	Bond Indices: Datastream (*DS), Stocks and ReE Indices: Bloomberg.	

Table 6.1: Asset universe of the illustrative example and the corresponding indices.

The seven considered factors to describe the risk/return-structure of this asset universe are two market indices (EQ: equity-factor, BD: bond-factor), four currency indices

(EUR: euro-factor, GBP: British pound-factor, USD: U.S. dollar-factor and JPY: Japan yen-factor) and a real estate factor (RE). The last factor (RE) is needed to properly describe the dynamics of the real estate asset (ReE), which is rarely defined by the other 6 assets.

The EQ-factor is composed of several - market capitalization weighted - MSCI Indices for stock markets in local currency. The bond factor contains several - GDP weighted - Data-Stream indices for government bonds, also in local currency. Worldwide national indices of the following countries were taken into account: Canada, France, Germany, Great Britain, Italy, Japan, Netherlands, Switzerland and U.S.A. The currency-factors show the change of the hedging value of the foreign currency to the local Swiss franc.

$$\ln\left(\frac{CHF}{USD}\right)_t - \ln\left(\frac{CHF}{USD}\right)_{t-1} + \left[\left(\frac{\ln(1+3mL_t^{(USD)}) - \ln(1+3mL_t^{(CHF)})}{12}\right)\right]$$

These currency factors are essential in order to capture the foreign currency risk exposure of an international investment strategy, having the Swiss franc as home currency. They do not only affect the volatility of the foreign assets but also the correlation structure of the whole international asset universe. Therefore, they are an important source of the covariance structure of the asset categories. Each asset return ($y_{n,t}$) is therefore described by following multivariate factor model[1]:

$$\begin{aligned} y_{n,t} &= \beta_n^{EQ} x_t^{EQ} + \beta_n^{BD} x_t^{BD} + \beta_n^{EUR} x_t^{EUR} + \beta_n^{USD} x_t^{USD} \\ &\quad + \beta_n^{GBP} x_t^{GBP} + \beta_n^{JPY} x_t^{JPY} + \beta_n^{RE} x_t^{RE} + \varepsilon_{nt} \end{aligned} \tag{6.1.1}$$

The model works consciously with macroeconomic indicators. On the one hand the economic interpretation of the factors - and therefore of the risk exposure - is more obvious compared to principal components. On the other hand the macroeconomic factor loadings are more stable over time than the ones of the principal components (see BOOS, 2004, [12]).

6.2 Empirical Results

6.2.1 Restrictions and Significance Tests

Based on the mixed multivariate distribution introduced in section 5.2, where all parameters are denoted as regime-dependent, and with help of significance analyses, the

[1] Please note that for each regime the parameters can be different.

model is being specified in greater detail. On the one hand it might reduce the dimensions of the model, on the other hand it provides further information of the dynamics of the historical financial markets.

Under the assumption of normal distributed assets the factor loadings are given through the OLS-method (Ordinary Least Squares). A particular analysis of the factor loadings brings new insights into the dynamics and risk exposures of the assets (see table 6.2). At first it is to mention that none of the regarded assets depend on all seven factors. The chosen factors describe only a specific selection of the assets. According to the t-test and as expected the bond indices are mainly described by the BD-factor and the stock indices mainly by the EQ-factor. The exchange rate dynamics found in the foreign assets are further covered by the corresponding currency factors. Therefore, the European asset categories (EUb and EUs) depend on the EUR-factor, likewise the JPY-factor describes the dynamics of the Japanese asset categories (JPb and JPs). Besides the UKb also the EUs depends on the GBP-factor, since the UKs play a major role in the European stock index. The other two positive factor loadings of the GBP-factor, namely JPs and ReE are admittedly significant, however, close to zero. The USD-factor depicts first and foremost the USb and the USs but also to a great part the EMs. Further, the CHs and the EUs depend with a small but significant positive factor loading on the USD-factor. If the GBP-factor and the JPY-factor would be excluded, the USD-factor would fill in and play a significant role in the explanation of British and Japanese asset categories. The Swiss real estate asset is almost exclusively driven by its factor. This confirms its value for diversification. All other factor loadings are set to zero, since these coefficients are according to the t-test not significantly different from zero. This brings about a large reduction of dimensions, without any significant loss of information. In the case at hand the dimension of the factor loading got more than halved, from 77 to 35 parameters.

The current factor loadings are a measure for the systematic risk (market risk) of a single asset. The market risk itself is modeled by the volatility of the EQ- and BD-factors. The USb ($\beta_{BD,USb} = 1.20$) is the only aggressive bond in the regarded asset universe ($\beta > 1$). The other five are defensive assets ($\beta < 1$), with the CHb ($\beta_{BD,CHb} = 0.44$) as the most defensive one. The influence of the bond factor to the stock markets is widely different. Some factor loadings are significantly positive (USs), others like the JPs and the EMs are significantly negative. The emerging markets are conspicuous, with a beta of $\beta_{BD,EMs} = -1.01$. The factor loadings of the stock markets to the equity factor are currently not that spread out, all close to one. The only aggressive stock markets are the emerging markets with a beta of $\beta_{EQ,EMs} = 1.11$. The other markets are defensive ones, if only weak. The Swiss market is the most defensive stock market($\beta_{BD,CHs} = 0.86$).[2]

	CHb	EUb	UKb	USb	JPb	CHs	EUs	USs	JPs	EMs	ReE
EQ	0	0	0	0	0	**0.86**	**0.97**	**0.98**	**0.97**	**1.11**	0
BD	**0.44**	**0.65**	**0.95**	**1.20**	**0.50**	0.08	-0.01	0.16	-0.26	-1.01	-0.01
EUR	0	1.02	0	0	0	0	0.53	0	0	0	0
USD	0	0	0	0.99	0	0.21	0.22	0.95	0	0.78	0
GBP	0	0	1.07	0	0	0	0.34	0	0.06	0	-0.01
JPY	0	0	0	0	1.00	0	0	0	0.92	0	0
RE	0.06	0	0	0	0	0	0	0	0	0	0.99

Table 6.2: Factor loadings of the single assets to the six risk factors.

The factor loadings β are by no means constant over time (see figure 6.1). There are many different events leading to changes in the beta of an index. While a companies' beta is influenced by firm-specific and microeconomic aspects a national economy and its indices are rather depending on macroeconomic indicators. Among other indicators the real exchange rate, defining the international competitiveness of the companies and the interest rates of the world leading currencies are to be mentioned.

The factor loadings vary in the course of time. However, in the model they are treated as regime-independent, since in their dynamic is neither a pattern nor a regime-dependency found. The bond markets reveal over time different sensitivities to the BD-factor. Over the last ten years the USb used to be the most sensitive bond market out of the regarded asset universe. The European bond markets (CHb, EUb, UKb) had a rather moderate sensitivity. The JPb with a low but positive beta was especially in the years 2003 to 2006 poorly described by the bond factor. The equity markets CHs, EUs and USs seem to have a stable beta towards the EQ-factor of around 1. Whereas the CHs always turned out to be defensive. The emerging market stocks showed, especially in the years 2007 and 2008 high sensitivity, and the JPs possessed on average the lowest beta of all equity markets over the last ten years.

Statistical tests, in particular likelihood ratio tests (see Hamilton, 1994, [51]) illustrate that the variance-covariance matrix of the factor (Σ_x) is regime-dependent. However, table 6.3 including the results of the likelihood ratio tests, reveals differences among the single factors.

Table 6.3 distinguishes two different time periods, in order to measure the impact of the latest financial crisis. The first calibration covers time period January 1981 to December 2006. The second calibration is based on the data from January 1981 to August 2009, which include the latest financial crisis.

[2]Note that the factor loadings are mainly determined by the factor composition.

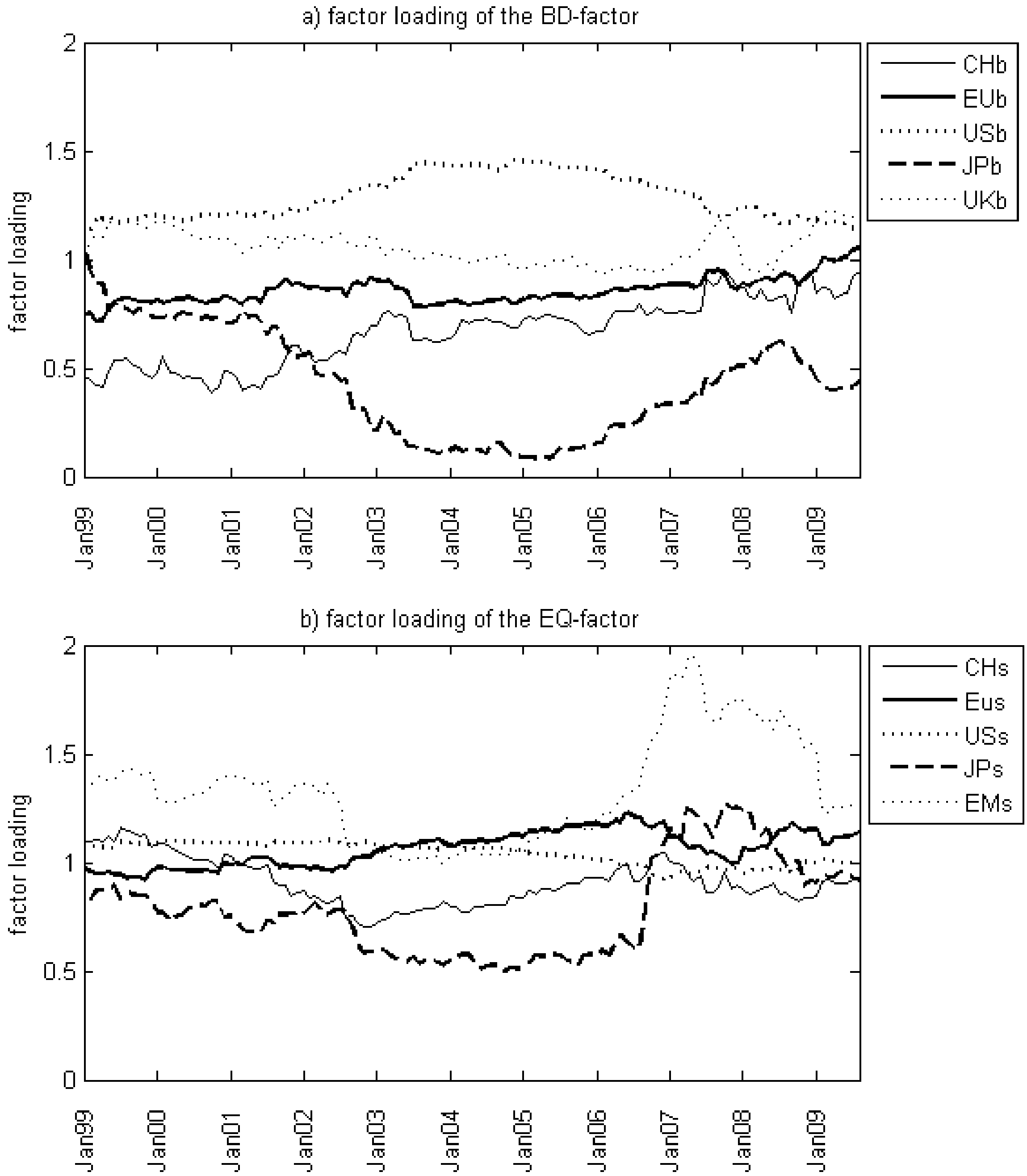

Figure 6.1: Factor loadings in the course of time. 48-months rolling windows. **a)** Factor loadings of the stock markets towards the EQ-factor. **b)** Factor loadings of the bond markets towards the BD-factor.

Regime-dep. factor	Jan.1981 - Dec.2006 Dim.	$logL(\theta)$	χ^2	p-value	Jan.1981 - Aug.2009 Dim.	$logL(\theta)$	χ^2	p-value
none	83	14'510.91			83	15'803.95		
EQ	90	14'549.32	76.83	0.0000	90	15'844.08	80.26	0.0000
BD	96	14'556.08	13.51	0.0357	96	15'856.39	24.62	0.0004
EUR	96	14'553.08	7.50	0.2766	96	15'850.93	13.70	0.0331
USD	96	14'555.72	12.79	0.0464	96	15'848.81	9.46	0.1494
GBP	96	14'559.14	19.62	0.0032	96	15'851.90	15.64	0.0159
JPY	96	14'556.59	14.54	0.0242	101	15'866.47	44.78	0.0000

Table 6.3: Results of the likelihood ratio test, examining the regime-independency of the variance-covariance matrix of the factors (Σ_x).

The likelihood ratio test indicates the EQ-factor in both calibrations as significantly regime-dependent. Before the crises the BD-factor, with a p-value of 0.0357, can be regarded as regime-independent. After the crisis and according to the likelihood ratio test the BD-factor behaves regime-dependent. A closer look reveals that this regime-dependency is mainly due to the UKb and JPb. The regime-dependency of these two asset categories can also be achieved by the corresponding currency factors. Thereby, for robustness and dimensional reasons and without mentionable loss of information, only the significantly affected bond markets are modeled as regime-dependent through their currency-factors.

The currency factors are therefore divided into two groups. While the EUR- and the USD-factor are significantly regime-independent in both calibrations, the GBP- and the JPY-factor require a closer elaboration. After the likelihood ratio test the GBP-factor behaves regime-dependent in both calibrations. Since the crises the JPb-factor became significantly dependent on the regimes as well.

Further tests kept on reducing the complexity and dimensions of the optimization problem and revealed additional information on the financial market dynamics.

The variance-covariance matrix of the residuals (Σ_ε) is reduced to a sparse matrix by testing the coefficient of correlation. The analysis of this matrix is not just done to reduce the dimensions but also to validate the whole factor model by proving if the model assumptions are correct and by indicating potential missing factors. Most correlations are not significantly different from zero, however, a few correlations are. The maximum likelihood estimation contains these non-zero correlations and therefore the model allows a certain correction due to the correlating error terms.

Most research studies on asset allocation mention the problem of the extreme sensitivity of the composition of the asset allocation to small changes of asset means. Many papers claim that the noisiness in asset returns leads to noisy and unrealistic asset allocations. In order to diminish this sensitivity, Ang & Bekaert, 1999, [5] consider to restrict the regime-switching model by setting the means equal across regimes. They argue that the restricted model is statistically not rejectable and offers in most cases even a better regime classification. This constrain also puts the focus more on the effect of the time-varying risk structure (correlations). Based on the results of the likelihood ratio test, the regime-switching model presented in the work at hand defines the asset returns also as regime-independent. This statistically insignificant restriction has several advantages. First, it reduces the dimension of the estimated model. Secondly, it also puts the focus on the risk-structure of the regimes. Finally, a rather technical but not lesser argument. When the returns μ are regime-independent, the likelihood surface contains only one mode (see section 4.3). This valuable attribute makes the maximum likelihood estimation much easier.

The expected asset returns are estimated based on the theory of the dividend discount model. Under the assumption of constant dividend growth and flat interest rate curve, the expected asset returns are described by the interest rate r, by the risk premium λ of the global factors (EQ- and BD-factor) and by the exposure to the risk premium defined by the coefficient parameter β. The variation of the excess returns of the single assets are therefore explained by the variation of the global price of risk and according to their exposure to it, defined by β. Further, the variation of the riskless interest rate r equally affects the asset returns.

In summary it can be said that statistical tests, in particular likelihood ratio tests (see Hamilton, 1994, [51]) illustrate that besides the variance-covariance matrix of the factors - and therefore of the asset categories - all other parameters are regime-independent. The betas, however, vary over time, but these fluctuations are independent of the regimes. Thus equation 5.1.4 diminishes to:

$$y_t(s_t) = \beta\bar{x} + [\underbrace{\beta\Sigma_x(s_t)\beta' + \Sigma_\varepsilon}_{\Sigma_y(s_t)}]^{\frac{1}{2}} \cdot \varepsilon_t \tag{6.2.1}$$

Overall the financial market dynamics are modeled by two variance-covariance matrices:

$$\begin{aligned} \Sigma_y(s_t = 1) &= \beta'\Sigma_x(s_t = 1)\beta + \Sigma_\varepsilon && (6.2.2) \\ \Sigma_y^{(}s_t = 2) &= \beta'\Sigma_x(s_t = 2)\beta + \Sigma_\varepsilon && (6.2.3) \end{aligned}$$

6.2.2 Risk/Return-Structure of the Regimes

This subsection discusses the risk structures of the two regimes. A closer look at the variance-covariance matrices of regime 1 and regime 2 aims to identify the stylized facts mentioned above.

Figure 6.2 draws the comparison between the realized market volatility and the volatility implied by the regime switching model. The implied volatility is measured based on the annualized volatilities of the equity factor at time T for regime 1 ($\sigma_{1,T(EQ)}$) and regime 2 ($\sigma_{2,T(EQ)}$) and is either weighted with the actual inference of the regime probabilities $\xi_{t|t}$ or with the smoothed inferences $\xi_{t|T}$:

$$\begin{aligned} \sigma_{t|t(implied)} &= \sqrt{\xi_{1,t|t}\sigma^2_{1,T(EQ)} + \xi_{2,t|t}\sigma^2_{2,T(EQ)}} \\ \sigma_{t|T(implied)} &= \sqrt{\xi_{1,t|T}\sigma^2_{1,T(EQ)} + \xi_{2,t|T}\sigma^2_{2,T(EQ)}}, \end{aligned} \tag{6.2.4}$$

where $t = 0, \ldots, T$ and $\xi_{1,t} = 1 - \xi_{2,t}$. Hence, the implied volatility has by definition its lower bound at $\sigma_{2,T(EQ)}$ and its upper bound at $\sigma_{1,T(EQ)}$. Currently the annualized implied volatility ranges between 10.86% and 22.14%.

The volatility implied by the regime-switching model moves along with the market volatility, defined as 12-month rolling window standard deviation of the equity factor. The model captures the volatility clusters and therefore identifies the dramatic breaks of the the financial market's structure. The volatility clusters are modeled exclusively by the hidden Markov chain process, driven by low transition probabilities - currently $p_{12} = 11.52\%$ and $p_{21} = 3.7\%$. The transition probability p_{12} with a value of 11.52% is compared to the historical average of transition probabilities relatively high. Along with the decreasing initial probability $\xi_{1,t}$ of currently 73.6% it increases the possibility of an early regime switch and indicates a possible sustainable market upturn.

The risk structures of the two regimes $\Sigma_y(s_t)$ (tables 6.4 and 6.5), reveal the above-mentioned features of empirical finance and confirm the two regimes of the financial markets.

Table 6.4 itemizes the volatilities of the asset for regime 1 and 2. The volatilities of the stock indices identify regime 1 as the volatile regime, where the annualized volatilities of the national stock markets are situated on a high level of around 30%. Whereas in regime 2 the volatility decreases in all stock markets - almost exclusively under 20% - emphasizing the co-movement effect. A phenomenon, where a national increase of the market volatility triggers the increase of other linked financial markets.

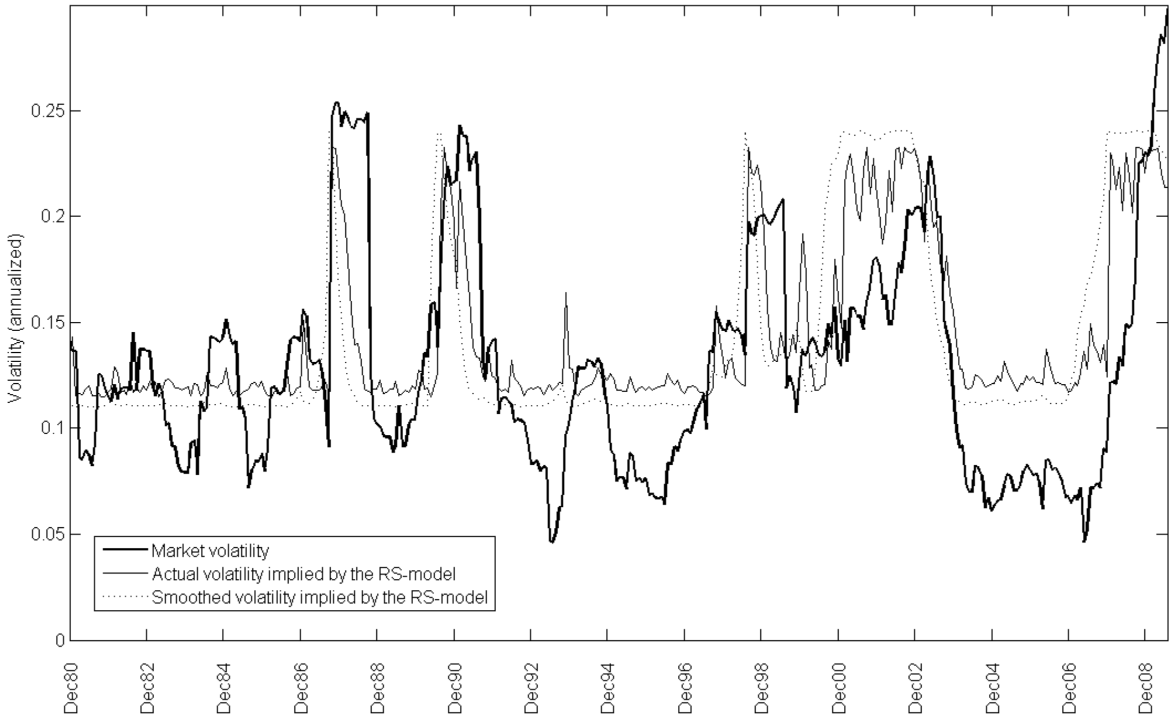

Figure 6.2: Market volatility (moving average of EQ-factor, 18 months) and volatility implied by the regime-switching model over time.

The Swiss stock index 'CHs', containing no foreign currency exposures, has of all observed stock indices the lowest volatility. The foreign stock indices are more volatile than the 'CHs', with the emerging market stock index 'EMs' leading the way. Next to the price and to the exchange rate fluctuations, the 'EMs' contains, compared to industrial countries, higher risk exposure resulting from lower political stability and higher potential liquidity problems.

The volatility difference between the Swiss bond index and the foreign bond indices is even more pronounced. The exchange rate volatility covers a major part of the overall risk exposure in this asset category. The Euro bond index has, compared to the other three considered foreign bond indices, a moderate volatility, tracing back to the low volatility of the exchange rate EUR/CHF. The volatilities are regime-independent for CHb, EUb and USb; therefore equal in both regimes. The two regime-dependent bond markets (UKb and JPb) have higher volatilities in regime 1 than in regime 2. This applies especially to the JPb.

Significant higher correlations ($\rho \geq 0.65$) among stock indices in regime 1 compared to regime 2 indicate the contagion phenomenon, which reduces the benefits of portfolio diversification at the time investors need them most. The correlations are on average 0.26 higher in volatile times and reach impressive high correlation up to 0.90 (CHs vs.

σ(*)	Regime 1	Regime 2	μ(*)	regime-indep.
CHb	3.53%	3.53%		1.68%
EUb	5.84%	5.84%		1.49%
UKb	12.42%	11.45%		1.74%
USb	11.86%	11.86%		2.12%
JPb	14.63%	10.67%		1.06%
CHs	**21.98%**	14.28%		3.86%
EUs	**25.71%**	15.36%		4.19%
USs	**26.78%**	17.65%		4.50%
JPs	**29.22%**	20.18%		3.78%
EMs	**34.88%**	24.20%		3.46%
ReE	3.64%	3.64%		1.80%

(*): annualized

Table 6.4: Annualized volatility of the assets. The co-movement effect is highlighted by bold characters. Annualized assets returns (regime-independent). Cash position: 0.32%.

EUs) or 0.89 (EUs vs. USs). In the low-volatile regime 2 the stock indices show on average a correlation of 0.51, leaving some opportunities of diversification.

The three bond indices CHb, EUb and USb, have according to definition, regime-independent correlations. The correlations among the bond markets lie somewhat between 0 and 0.5, leaving in both regimes a certain potential of diversification. The CHb and the UKb gear to the EUb. The JPb and UKb gear to the USb.

The correlations among stocks and Swiss bonds are negative in regime 1 but positive in regime 2 (except towards EMs), confirming the decoupling of financial markets. This phenomenon denotes that bonds offer an effective portfolio diversification potential during turbulent stock markets (i.e. financial crisis), at the time diversification is needed most. The decoupling phenomenon is also attenuated cognizable for the correlations among stocks and foreign bonds. Due to the safe haven role of the Swiss franc, the currency exposure of the foreign bond dilutes the decoupling effect to a certain degree. Under the assumption that the Swiss franc depreciates in volatile times, the rally of the foreign bonds is relativized due to foreign currency losses. On the one hand the foreign bond price increases on the other hand the won foreign currency losses its value. As a result of these two stylized facts (decoupling, safe haven), the correlations between foreign bonds and stocks are lower in regime 1 than regime 2, however, they do not switch signs.

The real estates, as a further traditional asset category, provide with low volatility of 3.64% and low correlations towards stocks and bonds, also in the volatile regime 1, a significant diversification investment.

Even though the emerging markets stocks contain higher volatility in comparison to other foreign stocks, due to the high influence of domestic issues, they are attractive to invest in. On the one hand, the higher risk brings higher expected returns. And indeed,

Regime 1:

ρ	CHb	EUb	UKb	USb	JPb	CHs	EUs	USs	JPs	EMs	ReE
CHb	1	0.40	-0.02	0.07	-0.02	*-0.12*	*-0.21*	*-0.21*	*-0.21*	*-0.25*	0.10
EUb		1	0.34	0.33	-0.01	*0.18*	*0.27*	*0.23*	*0.08*	*0.12*	0.14
UKb			1	0.56	0.25	*0.05*	*0.23*	*0.24*	*0.10*	*0.05*	0.15
USb				1	0.43	*0.11*	*0.20*	*0.40*	*0.19*	*0.22*	-0.06
JPb					1	0.02	0.05	0.17	0.41	0.10	-0.12
CHs						1	**0.90**	**0.81**	**0.65**	**0.73**	0.10
EUs							1	**0.89**	**0.69**	**0.80**	0.12
USs								1	**0.68**	**0.82**	0.07
JPs									1	**0.69**	0.03
EMs										1	0.06
ReE											1

Regime 2:

ρ	CHb	EUb	UKb	USb	JPb	CHs	EUs	USs	JPs	EMs	ReE
CHb	1	0.40	0.17	0.07	0.12	*0.18*	*0.08*	*0.03*	*0.06*	*-0.07*	0.10
EUb		1	0.44	0.33	0.28	*0.19*	*0.37*	*0.26*	*0.19*	*0.10*	0.13
UKb			1	0.42	0.31	*0.25*	*0.50*	*0.39*	*0.25*	*0.21*	0.07
USb				1	0.44	*0.30*	*0.44*	*0.72*	*0.31*	*0.42*	-0.05
JPb					1	0.25	0.34	0.42	0.54	0.24	0.11
CHs						1	**0.76**	**0.56**	**0.35**	**0.39**	0.11
EUs							1	**0.71**	**0.43**	**0.50**	0.15
USs								1	**0.37**	**0.60**	0.06
JPs									1	**0.39**	0.14
EMs										1	0.05
ReE											1

Table 6.5: Risk structure of the asset categories for the two regimes.

the emerging markets, taking the globalization for their benefit[3], outperformed the developed world markets in the last two decades (see figure 6.3). Further, there is the strategy to invest in many domestic driven emerging markets to diversify away much of the risk. On the other hand they have lower correlations to developed stock markets (USs, EUs, CHs), than the developed markets have among themselves. Although the emerging market stocks are also affected by the co-movement and the contagion phenomenon, the issue of lower correlation yields a further potential diversification investment. In volatile times (regime 1) the average correlation among the three selected developed markets[4] (USs, EUs, CHs) is $\o\rho_{R1}(DMs) = 0.87$. Their average correlation to the emerging markets, however, is only $\o\rho_{R1}(DMs, EMs) = 0.78$. In regime 2, during low-volatile markets, the differences are even bigger: $\o\rho_{R2}(DMs) = 0.68$, respectively $\o\rho_{R2}(DMs, EMs) = 0.50$.

This risk structure of the asset categories ($\Sigma_y(s_t)$) derives from the regression coefficients β and the risk structure of the seven factors $\Sigma_x(s_t)$[5], which contain the same observable features (see equation 6.2.1 and table 6.6). The EQ-factor x^{EQ} has a higher

[3] The globalization opened the trade and capital flows and reduced the communication and transporting costs (see HAMMOND, KAMP & FORE, 2006, [54])

[4] DMs = Developed market stocks.

[5] For completeness: Σ_ε, in a perfect model a diagonal matrix but mostly a sparse matrix, handles the potential rest correlations of the asset's residuals.

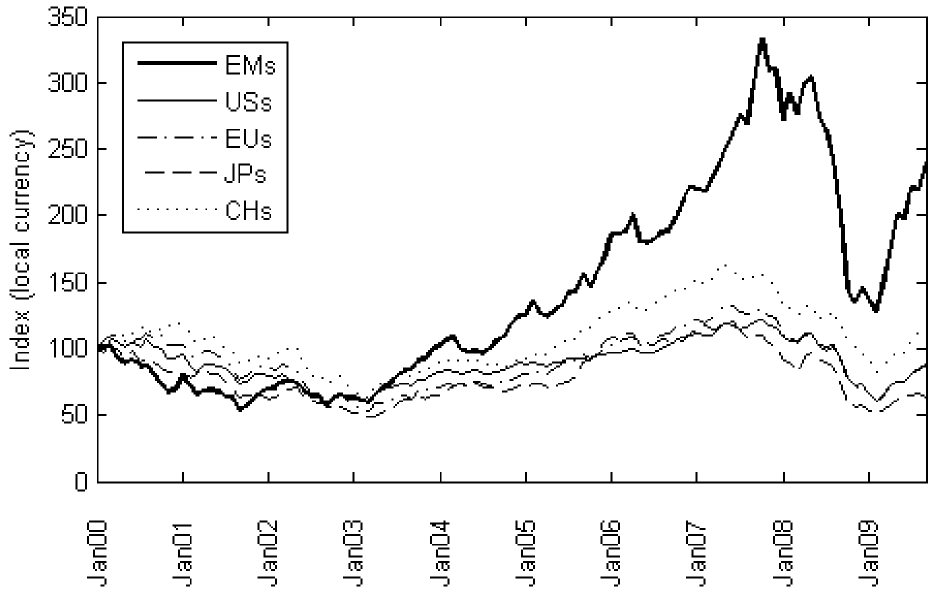

Figure 6.3: Performance of the emerging market stocks (EMs), compared to four developed market stocks: USs, EUs, JPs and CHs. The indices cover time period January 2000 to September 2009. The indices are in local currency and the base value is set to 100.

volatility in regime 1 (22.14%) than in regime 2 (10.86%), proving regime 1 as the volatile regime. The risk structure of the factors Σ_x also provides evidence of the decoupling phenomenon. While the correlations between the EQ- and the BD-factor is positive in regime 2 (0.29), the BD-factor decouples in the volatile regime 1 from the EQ-factor indicated by the significantly negative correlation of -0.36. The two risk/return-structures of the factors Σ_x reveal a further stylized fact of empirical finance. The high correlations in regime 1 between the EQ- and the two currency-factors (x^{EUR}, x^{USD}) verify the safe haven phenomenon of the Swiss franc. Because of significant positive correlations with the stocks, foreign investors avoid the currencies euro ($\rho = 0.45$) and U.S. dollar ($\rho = 0.16$) and invest in volatile times in Swiss francs. Whereas in the low-volatile times there are non-significant correlations seen among EQ- and the two currency-factors.

The high volatility of the U.S. dollar-factor (see table 6.6) as well as the positive correlation of the U.S. dollar-factor with the EQ-factor in turbulent times signalizes the considerable currency risk of the U.S. dollar. Also the foreign currencies GPB and JPY show according to their factors high currency risks. Therefore, the foreign currency positions need to be well observed. Note, that the volatility of bonds and the correlations among them (exclusively UKb and JPb) are regime-independent by definition (see section 5.2.1).

Regime 1:

Vola	(annual.)	ρ	EQ	BD	EUR	USD	GBP	JPY	RE
EQ	**22.14%**	EQ	1	*-0.36*	0.45	0.16	0.10	0.01	0.14
BD	4.40%	BD		1	-0.08	-0.27	-0.39	-0.17	0.06
EUR	4.56%	EUR			1	0.36	0.46	0.02	0.15
USD	12.13%	USD				1	0.69	0.51	-0.06
GBP	11.52%	GBP					1	0.32	0.08
JPY	14.4%	JPY						1	-0.12
RE	3.68%	RE							1

Regime 2:

Vola	(annual.)	ρ	EQ	BD	EUR	USD	GBP	JPY	RE
EQ	**10.86%**	EQ	1	*0.29*	0.10	0.07	0.21	0.22	0.20
BD	4.40%	BD		1	-0.08	-0.27	-0.08	0.00	0.06
EUR	4.56%	EUR			1	0.36	0.44	0.26	0.15
USD	12.13%	USD				1	0.48	0.47	-0.06
GBP	9.15%	GBP					1	0.32	0.09
JPY	9.79%	JPY						1	0.12
RE	3.68%	RE							1

Table 6.6: Risk structure of the factors for the tow regimes.

Graphical outputs of the regime-switching model (figure 6.4, 6.5 and 6.6) point out, by visualizing the evolvement of the financial markets over time that the risk/return-structure of the actual financial situation results from the underlying data (history). Figure 6.4 visualizes the anti-cyclical movement of the correlation $\rho_{EQ,BD}$ towards the market volatility, emphasizing the decoupling effect.

The next figure (figure 6.5) underlines on the one hand the increasing contagion among stock markets and on the other hand it shows the cyclical behavior of the correlations among stock markets, moving align with the market volatility.

A further graphical output (see figure 6.6) of the regime-switching model depicts the logical conclusion of the figures 6.4 and 6.5 that the correlations among stocks move contrary to the correlations between stocks and bonds $\rho_{EQ,BD}$.

6.2.3 Initial and Transition Probabilities

The probability of being currently located in the volatile regime 1 ($\xi_{1,t|T}$) turns out to be 73.6% ($\xi_{1,t|t} = 61.29\%$), reaching its latest peak in October 2008 at the level of 100.0% ($\xi_{1,t|t} = 88.48\%$). This affirms the current *bearish* markets as well as the intuitive assumption. Figure 6.7 illustrates how the probabilities $\xi_{1,t|t}$ (actual) and $\xi_{1,t|T}$ (smoothed) changed over time. Since regime 1 denotes the volatile *bearish* market situations, the probabilities $\xi_{1,t}$ unsurprisingly move align with the market volatility.

A closer look at the development over the last 13 years discloses the recent financial market crises and their influence on the risk/return-structures of the international

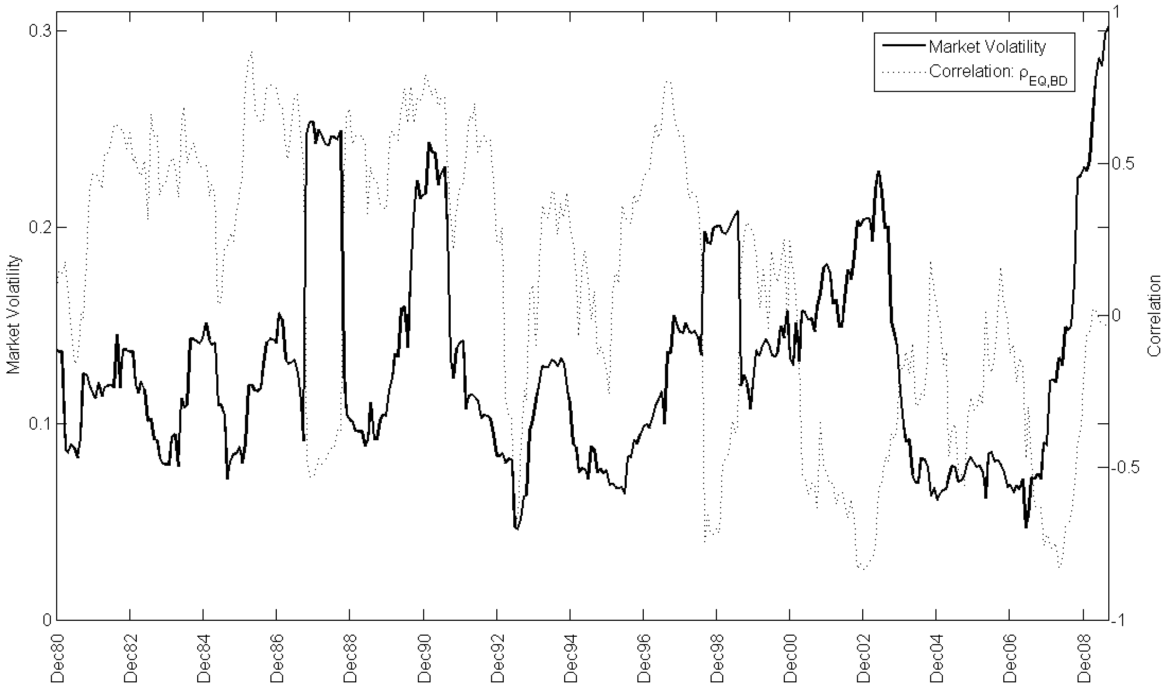

Figure 6.4: Anti-cyclical movement of the correlation $\rho_{EQ,BD}$ and the market volatility (12-month rolling windows).

financial markets. The rising emerging market crisis (Asia Crisis) of the year 1997 and the following Russia crisis in August 1998 brought the historical low volatility market period of the mid-nineties to an end. After a short calm down, it is conspicuous how long the probability of the volatile market regime at time t ($\xi_{1,t}$) has been high after the the new economy crisis 2001/02 boosted by the terror attacks *9/11* in the year 2001 and the following incidents. The current market situation is dominated by the latest financial crisis, ending the booming financial market period of the last three years. The "subprime"-mortgage crisis, which started in the United States in late 2006 led to an slowdown of the U.S. economy and had an effect on the global financial markets during 2007 and the beginning of 2008. The future will show if the current situation is only a *bear market rally* or an early indicator of a regime switch into a period of low-volatile financial markets.

These crises have a strong influence on the state probabilities $\xi_{1,t}$ ($\xi_{2,t} = 1 - \xi_{1,t}$) and therefore on the risk/return-structure of the international financial markets. Since the risk/return-structures of the regimes stay more or less stable[6] over time, the relevant mixed distribution, containing fat tails depends strongly on these state probabilities.

[6]This robustness of the risk/return-structures of the regimes sustain the evidence of the patterns given in section 2.1 and of the regime-switching approach.

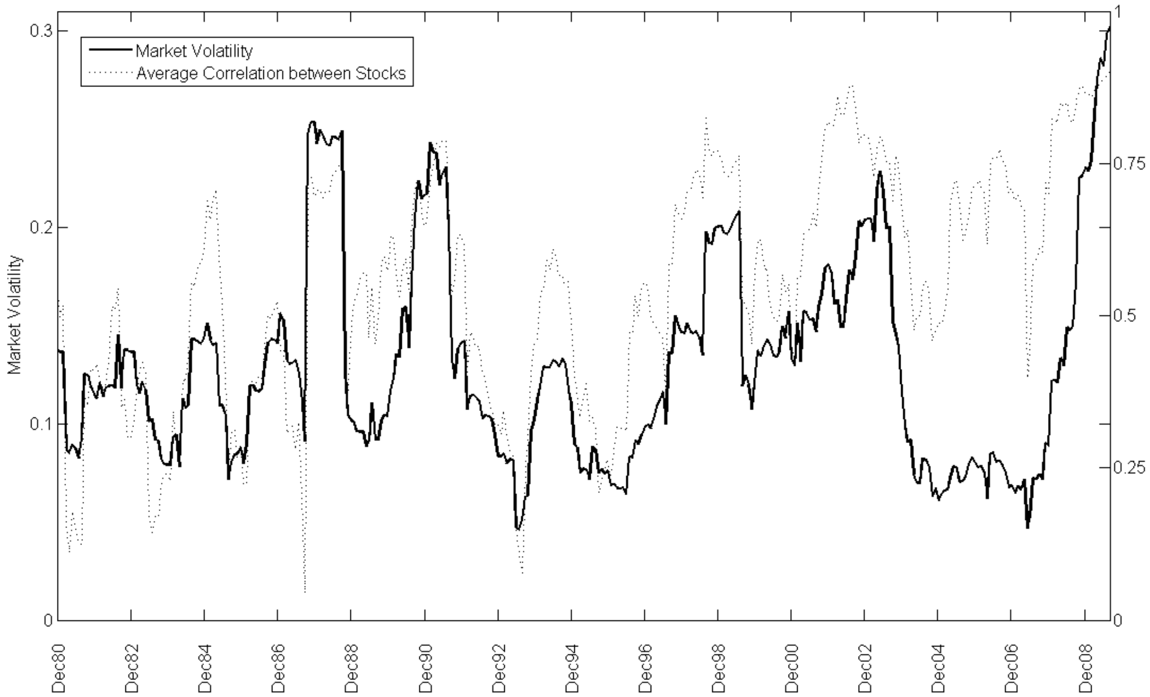

Figure 6.5: The correlations among stocks become more and more contagious, but still move align with the market volatility (12-month rolling windows).

The long-run forecasts or steady-state probabilities of the two regimes (π) depend on the transition probabilities. The transition matrix P, based on monthly time steps is:

$$P = \begin{pmatrix} 0.88 & 0.12 \\ 0.04 & 0.96 \end{pmatrix}$$

The high probabilities (> 0.88) of staying in the same regime lead to volatility clustering, meaning that high- (low-) volatile observations are more often followed by high- (low-) volatile observations. The steady-state distribution, regardless of the current value of $\xi_{1,t}$ is

$$\pi = \begin{pmatrix} 0.24 \\ 0.76 \end{pmatrix}$$

Over the long run the model is approximately in three-quarter of the time located in regime 2 and in one-quarter of time in regime 1. This is a result of the ratio between p_{11} and p_{22}. Figure 6.8 illustrates that it takes at least 5 years to reach the steady state. These findings are align with the empirical observation on business cycles, which last approximately 5 years, whereof a little more than one year the economy tumbles in a recession. The regime probabilities $\xi_{1,t}$ and $\xi_{2,t}$ cross within the first year, since in the long run the financial markets are more likely stated in regime 2.

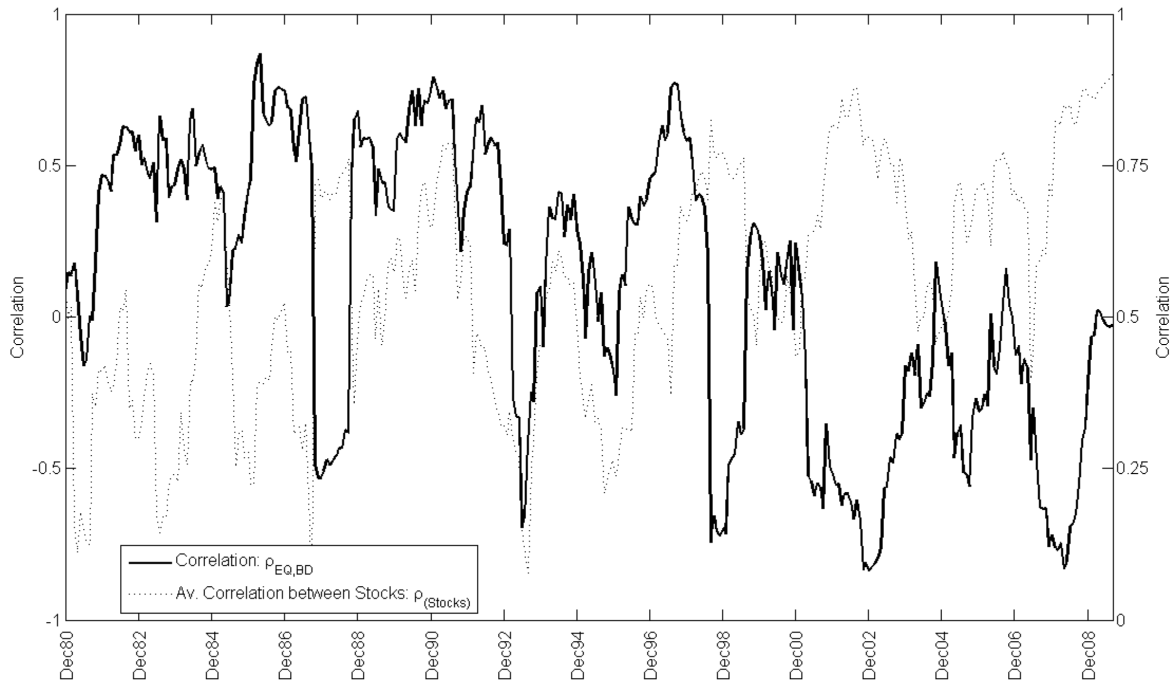

Figure 6.6: Anti-cyclical movement of the correlation $\rho_{EQ,BD}$ and the correlations among stocks (12-month rolling windows).

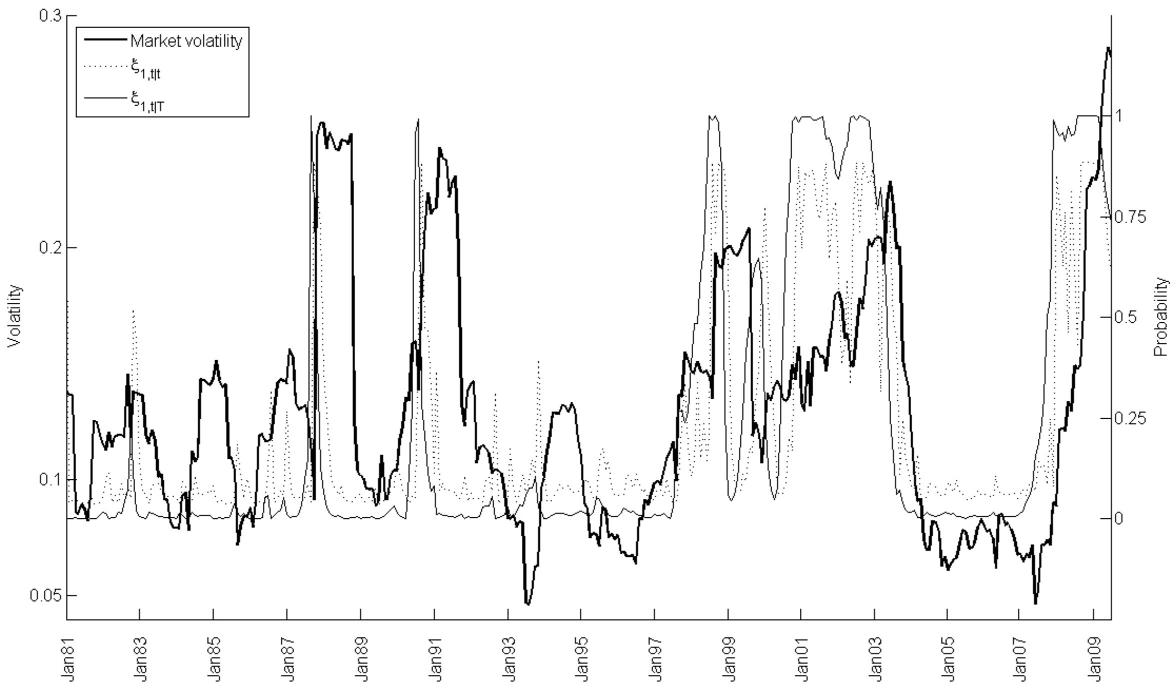

Figure 6.7: State probability $\xi_{1,t}$ moves align with the market volatility (moving average: 12 months), proving regime 1 as the volatile regime.

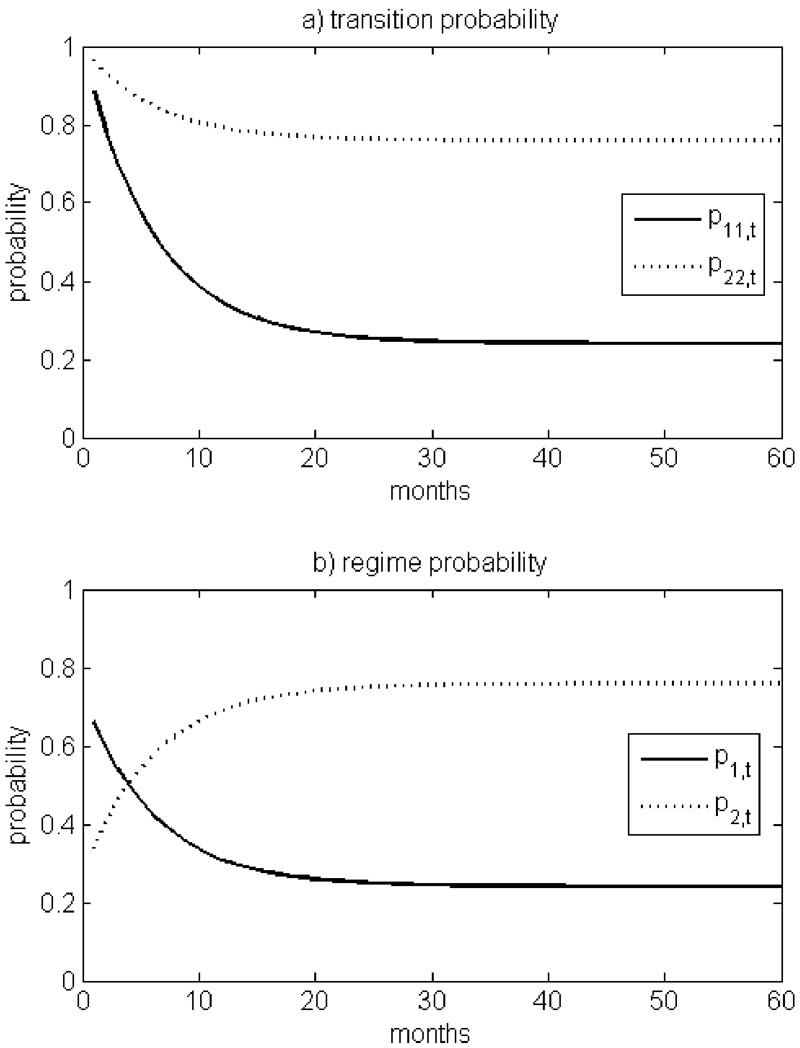

Figure 6.8: a) Transition probabilities p_{ii} over time, reaching steady state distribution π_i after approximately 60 months (time-steps). b) Regime probabilities $p_{i,t}$ over time reach the steady state distribution π_i also within the next 60 months. This illustrates that the steady state probabilities π_i are independent on the current regime probability $\xi_{i,t}$.

6.3 Influence of the Length of Historical Data

Since the regime-switching model is solely based on technical analysis, neglecting any aspects of the fundamental analysis, it is of interest how the length of the historical data set influences the resulting risk/return-structure. The long-term pattern of the financial markets is characterized by business cycles with alternating periods of low-volatile markets (regime 2) and high-volatile markets (regime 1). Depending on the chosen time period the ratio between potential observations of regime 1 and potential observations of regime 2 changes, which might lead to different results. In order to investigate the sensitivity of the size of the time series, 22 models with different time horizons but the same end date were calibrated. The model is based on monthly data and contains a representative asset universe of six assets (CHs, EUs, USs, CHb, EUb, USb). The start dates of the corresponding time series reach with yearly time steps from January 1980 to January 2001. The end date of all time series is May 2009.

Figure 6.9 illustrate the actual ($\xi_{1,t|t}$) and the smoothed ($\xi_{1,t|T}$) inferences of the regime probability, which are based on the historical data from January 1980 to May 2009. These inferences reveal the major global financial crises of the last three decades. The Black Monday on the 19th of October 1987 went down in history as the biggest one-day percentage decline in the stock market. Around the year 1990 on the one hand, the Japanese asset price bubble collapsed and on the other hand the U.S. savings and loan crisis of the eighties led from 1990 to 1991 to an economic recession. The volatile times from mid 1997 to 2004 were characterized by successive crises. The Asian financial crisis starting 1997, raised the fear of a global economic crash. It was followed by the Russian crisis in August 1998. After a short time of recovery the financial markets were hit by the New Economy crisis, which was boosted by the international political turmoil due to the 9/11-attacks. The "subprime"-mortgage crisis, which started in the United States in late 2006 led to a slowdown of the U.S. economy and triggered a worldwide financial market crash (see also section 6.2.3).

Figure 6.10a) contains some selected smoothed inferences of the regime probability ($\xi_{1,t|T}$), while figure 6.10b) presents the corresponding actual inferences of the regime probabilities ($\xi_{1,t|t}$). They point out how the length of the historical data set affects the process of the inferences.

At first view there are no big differences determinable. Each single inference discloses the global crisis, respectively the major volatile time periods. The different sample sizes of the historical data seem to have low influence on the general process of the

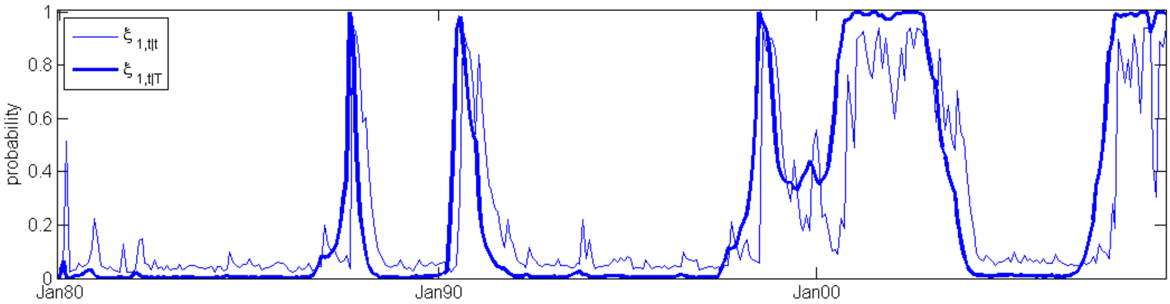

Figure 6.9: a) Actual and smoothed inferences for the regime probability based on time series from Jan. 1980 to May 2009. b) Smoothed inferences for the regime probability based on different sizes of time series.

regime switches. Already a small sample size of around 100 observations perceives the two regimes in the financial markets.

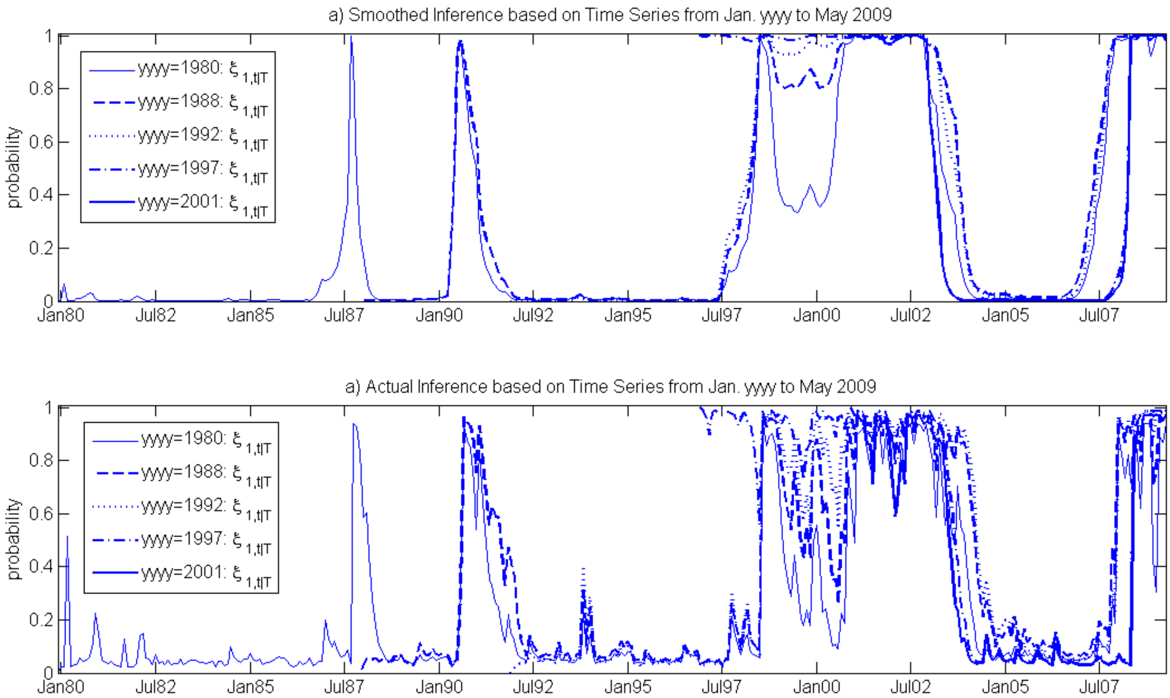

Figure 6.10: a) Smoothed inferences for the regime probability $\xi_{1,t|T}$ based on different sizes of time series. b) The corresponding actual inferences for the regime probability $\xi_{1,t|t}$.

Nevertheless, a closer look reveals some interesting differences among the inferences of various sample sizes. In order to well distinguish the last three decades in different and expressive time periods, each starting date of the consciously selected historical time periods lies either short before or short after a global crisis: January of the years 1980, 1988, 1992, 1997, 2001. The differences are best illustrated with the unequal occurrence of the bear market rally around the turn of the millennium. The longer the time history, the more pronounced is the bear market rally. This difference is traced back to the long time ranges of low-volatile markets (regime 2) in the eighties and nineties.

This phenomenon can be explained by introducing the parameters of the underlying Markov process. Figure 6.11a) shows the transition probabilities (p_{11} and $p_{22} = 1 - p_{11}$) of the corresponding time series. The transition probability of staying in regime one (p_{11}) increases inversely proportional to the length of the time series. Therefore, calibrations which are based on longer time series, containing i.e. the low-volatile market periods of the nineties or even the eighties assign a lower value to p_{11}, meaning that the probability of switching from regime 1 to regime 2 is higher. This is the case because the risk/return-structure of regime 2 dominates the overall historical time series. This reflection becomes more explicit when bringing the steady-state probabilities π_i into this explanation. The long-run forecasts or steady-state probabilities π of the two regimes, depending solely on the transition matrix P, reflect the ratio between potential observations of regime 1 and potential observations of regime 2 enforcedly based on the underlying historical data (see figure 6.11b).

This behavior of the inferences is not only visible around the year 2000, but also in the first switches of the new millennium. The inflection point of the switch from regime 1 to regime 2 at the beginning of 2003 is equal for all smoothed inferences. However, the speed of the switch varies according to the size of the sample. According to the transition probabilities p_{12} it is therefore the time period January 1980 to May 2009 that switches at the beginning of the year 2003 most rapidly back to regime 2. It is not explainable by the estimated Markov parameters alone, why the calibrations of the short time periods starting after the low-volatile time of the nineties (starting date: 1997 and younger) switches even faster than the time period January 1980 to May 2009. However, it brings the focus of the discussion to the structural change of the time series.

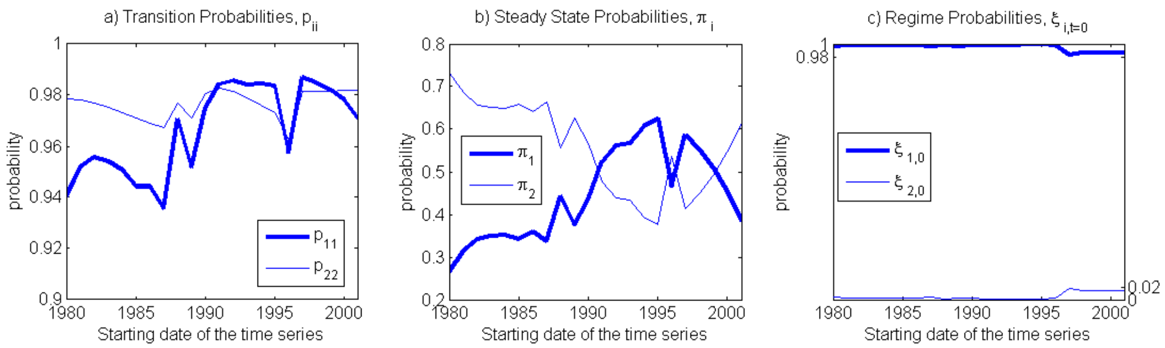

Figure 6.11: Markov information depending on the length of the historical data. Figure c) shows that the actual regime probability of May 2009 is more or less robust towards the length of the historical time series.

Since 1997 the financial market, hence the time series have been dominated by volatile times. Therefore, the risk/return-structures within the regimes, especially in regime 2 experienced significant changes. For example, the decoupling effect, well illustrated by the regime-switching model, is being diluted. The historical correlation between the EQ- and BD-factors $\rho_{hist}(EQ, BD)$ experienced a decline over time and even switch sign from positive to negative (see figure 6.12a), dotted line). While monthly time series covering period from May 2009 back to at least Dec 1989 show a positive correlation, shorter time series show a negative statistical connection. This is mainly due to the changes of $\rho_{R2}(EQ, BD)$ in the risk/return-structure of the second regime. The area between the thin and the thick solid line, representing the decoupling effect, is being dramatically reduced. Time will show, if the change of this stylized fact is only a result of the many financial market crises of the last decades, leaving no time to a real recovery of the markets or if this phenomenon called decoupling is a thing of the past. It is, however, important to mention that the added-value of the decoupling effect is still given to this day. First, there is still a lower correlation in regime 1 than in regime 2 [$\rho_{R1}(EQ, BD) < \rho_{R2}(EQ, BD)$]. Secondly, in times of high-volatile stock markets, bond markets still serve as secure alternative and provide a certain potential of diversification. This structural change is also visible on the level of the single assets (see figure 6.12b). The correlation between CHb and the three stock markets CHs, EUs and USs show the same pattern as the above-mentioned behavior of their global factors.

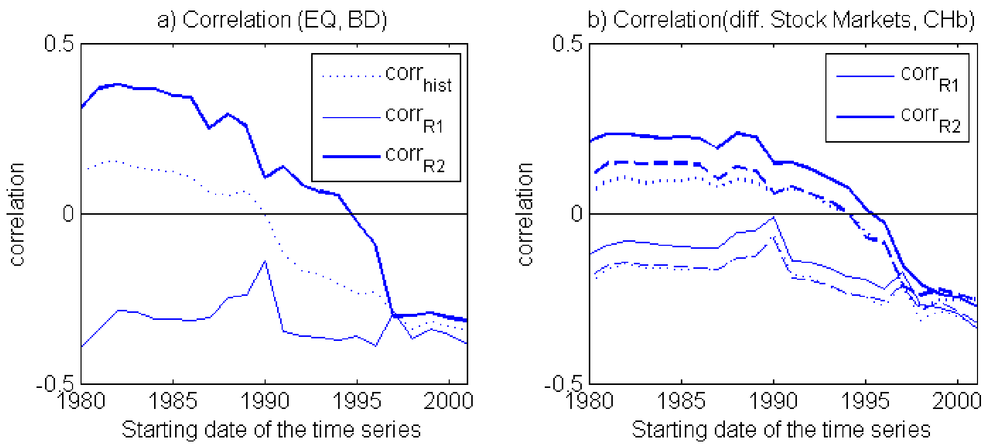

Figure 6.12: Decoupling Effect depending on the length of the underlying time series.

It is obvious that the length of the historical data set influences the output. However, good news is that if the historical data set covers data of both regimes, the model is able to distinguish the low-volatile times from the high-volatile ones. Even for a short history. Therefore, whether one uses a short or a long historical data set depends solely on the subjective expectations and beliefs of the future. Is the future better represented by the latest financial market dynamics or do the long established market dynamics describe the evolvements of the future? Nevertheless, it is to mention that even though the risk/return-structure changes over time the stylized facts are always given.

6.4 Influence of different Base Currencies

An international asset allocation requests a distinction between local currency and base currency. While each foreign asset is exposed to its local currency, the investor measures the performance of the portfolio in his base respectively home currency. Local and base risk/return-structure of the assets differ, depending on the movements of the corresponding exchange rate. As a result, the risk/return-structure of investors with unequal home currencies will be different. Table 6.7 shows that the differences in the risk structures are significant. The annualized volatilities of the bond securities depend strongly on the corresponding home currency. Therefore, the big part of the foreign bond risks is due to currency risks. This affects the correlations as well. Correlation among bonds, and between bonds and stocks are highly sensitive to changes in home currency. In contrast, the currency risks have lower influence on the volatility of stock markets and on their correlations. This might be due to stock markets themselves being highly volatile and rather globalized (contagious) than bond markets. The high volatility of the USD-factor (x^{USD}) lead to bigger differences in the risk/return-structure of the USD-model compared to the CHF- and EUR-model. In each home currency the decoupling effect is visible.

Regime 1 in CHF

Vola	(annual.)
CHb	2.12%
EUb	4.23%
USb	9.39%
CHs	19.75%
EUs	23.52%
USs	24.28%

ρ	CHb	EUb	USb	CHs	EUs	USs
CHb	1	0.33	-0.04	*-0.27*	*-0.30*	*-0.30*
EUb		1	0.27	*0.09*	*0.20*	*0.14*
USb			1	*0.23*	*0.29*	*0.50*
CHs				1	**0.85**	**0.76**
EUs					1	**0.85**
USs						1

Regime 1 in EUR

Vola	(annual.)
CHb	4.55%
EUb	2.60%
USb	8.94%
CHs	18.69%
EUs	22.05%
USs	23.04%

ρ	CHb	EUb	USb	CHs	EUs	USs
CHb	1	0.46	0.07	*-0.26*	*-0.40*	*-0.36*
EUb		1	0.03	*-0.32*	*-0.36*	*-0.36*
USb			1	*0.11*	*0.12*	*0.39*
CHs				1	**0.83**	**0.73**
EUs					1	**0.83**
USs						1

Regime 1 in USD

Vola	(annual.)
CHb	10.94%
EUb	10.58%
USb	3.64%
CHs	18.79%
EUs	21.45%
USs	19.72%

ρ	CHb	EUb	USb	CHs	EUs	USs
CHb	1	0.93	0.41	*0.14*	*-0.01*	*-0.24*
EUb		1	0.46	*0.18*	*0.08*	*-0.16*
USb			1	*-0.19*	*-0.25*	*-0.29*
CHs				1	**0.82**	**0.67**
EUs					1	**0.80**
USs						1

Table 6.7: Risk/return-structures of regime 1 depending on different home currencies. Calibration is based on historical data sets with weekly time series covering period January 2000 to May 2008. The bond securities correspond to 3-5 year government bonds (e.g. EUb = Euro bond). The stocks securities are MSCI Indices.

Chapter 7

Application of the Regime-Switching Model

The properties of the risk/return-structure modeled by the regime-switching framework can consequently be used for the scenario generation in a multistage dynamic asset allocation approach. The results of the regime-switching framework serve as input data for a dynamic asset allocation model (DEVA), which is characterized by a dynamic expectation variance analysis (see Frauendorfer, Jacoby & Schwendener, 2006, [41]; Frauendorfer & Siede, 2000, [42]; Frauendorfer, 1995, [43]; Siede, 2000, [87] or Steiner, 2002, [93]) based on a multistage, stochastic programming. (see e.g. Bire & Louveaux, 1997, [7] or Kall & Wallace, 1994, [58]).

The fundamental idea of this approach, illustrated in figure 7.1, is that the rebalancing activities in each node is optimized under consideration of the initial (actual) portfolio, future rebalancing activities, transaction costs and the uncertain risk/return-structure of the financial markets. The rebalancing activities in the nodes, representing decision points depend on the current regime illustrated by the two paths, leading to a stochastic final wealth. Besides the expected final wealth and its variance, the shortfall probability of a chosen threshold value and other important risk indications are provided by the implied density function. Note that this density function contains, due to the mixture of two normal distributions (regimes) fat tails, which characterize the associated risk measures.

While section 7.1 introduces the multistage dynamic asset allocation approach in a rather technical way, section 7.2 contains some numerical results and performances of this approach.

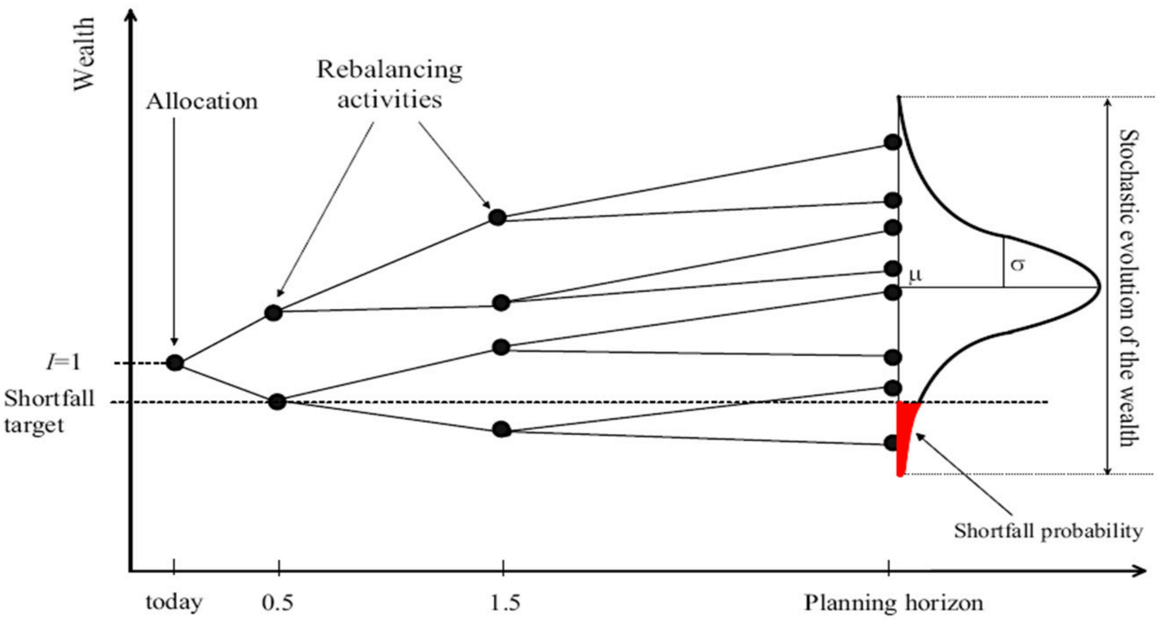

Figure 7.1: Basic concept of DEVA. (Source: FRAUENDORFER, 2008, [39])

7.1 Multistage Dynamic Asset Allocation Approach[1]

The planning horizon of an investor, defined as $\mathcal{T} := \{0, \ldots, T\}$, is divided into several sub-periods $t \epsilon \mathcal{T}$, each of them allowing readjustments of the asset allocation. Such a multi-period approach takes additional in- and outflows, future rebalancing activities and the dynamics of the asset returns into account. In each decision point, not only the actual situation but the future stochastic evolvement, as well as its corresponding optimal decisions are considered.

7.1.1 Evolution of Wealth

In each time step $t = 0, \ldots, T$ a portfolio turnover can be executed by purchasing $\nu^+_{t,n} \geq 0$ or selling $\nu^-_{t,n} \geq 0$ assets $(n = 1, \ldots, N)$. The costs and benefits of the rebalancing activities are compensated with in- and outflows in the cash position $(n = 0)$.

The transaction costs for selling $c_{t,n}$ and for purchasing $d_{t,n}$ are defined to be proportional to the corresponding amounts. By allowing exogenous cashflow $C_t \epsilon \mathbb{R}$, the budget restrictions for the first period, starting with a given initial portfolio in money

[1]The remarks in this sub-section refer closely to BOOS, SCHMID & KOLLER, 2003, [13].

term $x_{ini} \epsilon \mathbb{R}$, is then defined as:

$$x_{0,n} = x_{ini,n} + \nu^+_{0,n} - \nu^-_{0,n} \qquad n = 1, \ldots, N \tag{7.1.1}$$

$$x_{0,0} = x_{ini,0} + C_0 - \sum_{n=1}^{N} [(1 + d_{0,n})\nu^+_{0,n} + (1 - c_{0,n})\nu^-_{0,n}] \tag{7.1.2}$$

After rebalancing the portfolio, the allocation x_0 is hold during the first period. The wealth at the end of this period is stochastic and depends on the realization of the returns $r_{1,n}$. The particular assets of the portfolio at time $t = 1$ are of the value $x_{0,n} \cdot r_{1,n}$ $(n = 1, \ldots, N)$. Analogously to the time $t = 0$ rebalancing activities and exogenous cashflow may occur. The actual portfolio is adjusted to the new market situation. The conditional budget restriction for $t = 1, \ldots, N - 1$ is then given by:

$$x_{t,n} = r_{t,n}x_{t-1,n} + \nu^+_{t,n} - \nu^-_{t,n} \qquad n = 1, \ldots, N \tag{7.1.3}$$

$$x_{t,0} = r_{t,0}x_{t-1,0} + C_t - \sum_{n=1}^{N} [(1 + d_{t,n})\nu^+_{t,n} + (1 - c_{t,n})\nu^-_{t,n}] \tag{7.1.4}$$

The final wealth $W_T := x'_{T-1} \cdot r_T$ accrues inductively from the budget restrictions. At time $t = T$ there are no further rebalancing activities, since they would only yield transaction costs.

7.1.2 Objective Function and Constraints

In analogue to the classical mean-variance model of Markowitz[2] the objective function is defined as a parametric minimization of the variance. The model is an extension of the mean variance criterion in a multi-period environment, i.e.

$$minV[x'_{T-1} \cdot r_T] \tag{7.1.5}$$

subject to the budget constraint

$$E[x'_{T-1} \cdot r_T] = \bar{W}_T, \tag{7.1.6}$$

where $\bar{W}_T$ represents the expected final wealth, calculated from the predefined target return.

[2]For $T = 1$ and by ignoring the transaction costs $c_n = d_n = 0$ the model simplifies to the classical Markowitz approach. This is, however, only the case cause the initial portfolio is obsolete when the transaction costs are neglected.

According to the FRAUENDORFER & SIEDE, 2000, [42] and SIEDE, 2000, [87] the variance can be written as:

$$V[x'_{T-1} \cdot r_T] = E[x'_{T-1}(\Sigma_T + \bar{r}_T \cdot \bar{r}'_T)x_{T-1}] - \left(E[x'_{T-1}\bar{r}_T]\right)^2, \tag{7.1.7}$$

where Σ_T defines the conditional covariance matrix and $\bar{r}_T$ the vector of the conditional expected returns, both contingent on the regime. The decision vector x_{T-1} results, due to the budget restrictions, from the rebalancing activities and from the actual evolvement of the financial markets.

For a robust and expressive international asset allocation certain inequality conditions are crucial. Such restrictions are considered in the above optimization problem to indicate the bounds of certain assets or asset groups ($x_{i,t}$), to define bounds on any decision variable ($v^+_{i,t}, v^-_{i,t}$), or just to constrain short sales.

On the one hand such inequalities are crucial for practical applications but on the other hand they considerably complicate the solving procedure. Namely, an analytical solution is no longer feasible and is being replaced by a numerical approach.

7.1.3 Optimizing by means of Stochastic Programming

By discretizing the continuous distributions of the asset returns by means of a scenario tree, the multistage dynamic asset allocation approach turns into a stochastic multistage optimization problem. The solution of the stochastic program is an approximation to the continuous original problem, with a finite set of scenarios and their corresponding probabilities. For detailed information on the discretization procedure it is being referred to SIEDE, 2000, [87]. Standard solvers for quadratic optimization problems with linear restriction are suited to solve the resulting optimization program. DEVA uses an efficient interior point algorithm developed by STEINBACH, 1998, [92] which considers the specific structure of the approach.

7.1.4 Liability based Asset Allocation and Benchmark Tracking

Financial institutions that have to fulfill a pension or insurance mandate, grant certain guarantees on the passive side. However, no equivalent assurance on the active side, provided by the financial and capital markets does exist. All the more, with considering the interest rate risk on the liability side. The multistage dynamic asset allocation approach allows the explicit consideration of the liability dynamics within the optimization process in order to guarantee the incurred liabilities on the active side at all times. The framework in equation 7.1.5 is designed to include several liabilities[3] as fixed short positions in the optimization process and to consider their correlations to the asset returns accordingly. With this extension an integrated asset & liability system supports investors in analyzing and controlling shortfall-risks as well as in assessing strategic specifications.

It has become fashion to measure the performance of a portfolio manager against a stochastic benchmark. Many professional investors are judged relevant to an index that reflects its investing style. For example, the mutual funds take the Standard and Poors 500 Index as benchmark, the commodity futures try to beat the Goldman Sachs Commodity Index and the bond funds compete with the Barclays Capital Multiverse Index who maintain the Lehman Brothers Bond Index. It does not matter how well a portfolio manager does, as long as he beats the index, even if it just means losing less than the benchmark does. Mostly, he receives an incentive (e.g. bonus) on outperforming a benchmark by a certain percentage, but will be punished if he underperforms the benchmark by some other predetermined amount. Not just professional agents but also ordinary investors implicitly follow this moving target approach, for example by trying to cover inflation costs or exchange rate costs or by trying to beat other indices. Equivalent to the liability based asset allocation, the framework includes several benchmarks, which are not perfectly correlated with the tradeable assets, as fixed short positions in the optimization process. This benchmark tracking enables to objectively assess the portfolio performance against the background of a specific index, representing the general market development. The utility of this approach is cognizable, since it is well known that many professional investors measure their performance relatively to a benchmark (see Browne, 1999, [16]).

The presented multistage dynamic asset allocation approach, optimizes benchmark tracking and liability based asset allocation with a mean variance criterium for the surplus.

7.2 Numerical Results and Performance

How well the results of the multistage dynamic asset allocation approach, combined with a regime-switching model perform is shown in former case studies (see e.g. Centineo, 2004, [20]). Centineo compares the portfolio suggested by DEVA with the

[3] For more information on modeling pension fund liabilities see Jacoby, 2006, [57].

Pictet BVG-40 Index (2000), which provides a suitable benchmark for Swiss pension funds. Based on the initial portfolio, which corresponds to a typical Swiss pension fund, the efficient portfolios calculated by DEVA outperform the benchmark, while BVV2-restrictions and transaction costs are taken into account. It is shown that the efficient portfolios of the DEVA model keep up with the Swiss benchmark index. They reveal risk-adjusted added value and decent Sharpe ratios. Further, it is shown that they possess significant positive Merton-γ pointing out the timing contribution of DEVA.

This section will introduce some further numerical results and will discuss the performance of this approach.

7.2.1 Initial & Transition Probabilities and their Effect on the Asset Allocation

Since the regimes have seen to be quite robust over time, the initial and transition probabilities of the regimes have a significant influence on the optimal asset allocation. The coefficient $\xi_{1,0}$ denotes the initial probability that the market is located in regime 1 at the current date $t = 0$ and therefore determines, in combination with the transition matrix, the weights of the different risk/return-structures of regime 1 and regime 2. The transition matrix P, particularly the ratio between p_{11} and p_{22}, defines the steady state probability and the time of reaching it. The closer p_{11} and p_{22} to unity, the longer is the influence of the initial probability $\xi_{1,0}$ during the planning horizon.

Lets assume an asset universe composed of three bond indices and three stock indices, each from the markets Switzerland (CHb, CHs), Europe (EUb, EUs) and U.S.A. (USb, USs). The risk/return-structures of the different regimes was estimated, based on monthly data from January 1980 to May 2009 in Swiss franc home currency. Different efficient portfolios where optimized by DEVA using the estimated risk structures in combination with various initial probabilities $\xi_{1,0}$, transition matrices P and investment horizons T:

The impact of the parameters $\xi_{1,0}$, P and T on the efficient portfolio compositions is revealed in table 7.1, containing the aggregated equity positions in percentage of the whole portfolio. Beforehand, it can be said that a parameter sensitivity analysis of the portfolio composition to changes of the initial probability $\xi_{1,0}$, of the transition matrix P and of the investment horizon T sustain the hypothesis that stocks are less attractive in high-volatile times (regime 1) than in low-volatile times (regime 2).

The cases C1-1 to C3-1 have an investment horizon of one year, while the cases C1-7 to C3-7 have an investment horizon of 7 years (ceteris paribus). The efficient portfolios of

Criteria	Cases					
	C1-1	C2-1	C3-1	C1-7	C2-7	C3-7
Parameter						
T :	1 year			7 years		
$\xi_{1,0}$:	0.95		0.05	0.95		0.05
p_{11} :	0.95	0.60	0.95	0.95	0.60	0.95
Target Return	aggregated equities in %					
$x^* =$ 2.00%	0.09	0.15	0.11	0.19	0.14	0.24
2.50%	0.13	0.19	0.15	0.24	0.20	0.32
3.00%	0.20	0.29	0.23	0.34	0.32	0.49
3.50%	0.28	0.41	0.33	0.48	0.46	0.71
4.00%	0.35	0.53	0.42	0.60	0.60	0.93
4.50%	0.43	0.63	0.51	0.69	0.73	1.00
5.00%	0.51	0.71	0.61	0.80	0.87	1.00
5.50%	0.60	0.77	0.73	0.91	1.00	1.00
6.00%	0.69	0.86	0.90	1.00	1.00	1.00
6.50%	0.80	1.00	1.00	1.00	1.00	1.00
Steady State						
π_1 :	0.29	0.05	0.29	0.29	0.05	0.29
approx. time (in years):	7	1	7	7	1	7

$p_{22} = 0.98$ for all cases

Table 7.1: Aggregated equity positions (CHs, EUs, USs) in percentage of the whole portfolio. Sensitivity analysis of the portfolio composition to changes in the initial probability $\xi_{1,0}$, the transition matrix P and the investment horizon T.

longer horizons allocate more towards stocks. Since over the long run financial markets used to be situated more often in low-volatile regimes than high-volatile regimes, the probability that the financial markets switch from the high- to the low-volatile regime is higher for longer investment horizons. The allocation to stocks is therefore proportionate to the investment horizon, subject to normal business cycles, where regime 2 dominates regime 1 in the long run. It further shows that time and risk are inseparable.

A similar structure is also found by comparing the cases C1 and C2 within the same investment horizon. The cases start with an initial probability of $\xi_{1,0} = 0.95$, meaning that the current market situation is located in a high-volatile environment. Cause the cases C1 have a p_{11} of 0.95, the probability of staying in this high-volatile market situation is much higher than in the cases C2 with a p_{11} of only 0.60. The steady-state probability of C1 ($\pi_{1(p_{11}=0.95,p_{22}=0.98)} = 0.29$) is bigger than of C2 ($\pi_{1(p_{11}=0.60,p_{22}=0.98)} = 0.05$) and it takes longer, namely approximately 7 years, compared to a little more than 1 year to reach it. Therefore, the efficient portfolio in the cases C2 allocate more towards equity than the cases C1, since the probability is higher and the time is shorter to switch back to normal-volatile periods (see also figure 7.2). This statement, however, is not correct for the efficient portfolios with a target return of $x^* \leq 3.5\%$ and an investment horizon of 7 year. There is an explanation for this exception. The multi-period approach allows to adequately consider rebalancing activities and the dynamics of the asset returns. Hence, the planning horizon, divided into three sub-periods, allows two further rebalancing activities in the future, based on future market dynamics. These dynamics are characterized by the current risk/return-structure (Σ_x) and the Markov process ($\xi_1, 0$ and P). According to figure 7.2 cases C2 have the advantage to keep the portfolio at lower risk in sub-period 1, in order to profit from the less-volatile times in sub-periods 2 and especially 3, where $\xi_{1,t>13} = 0.05$. The effect of this investment tactic diminishes with higher target returns x^*, due to the fact that the target function dominates.

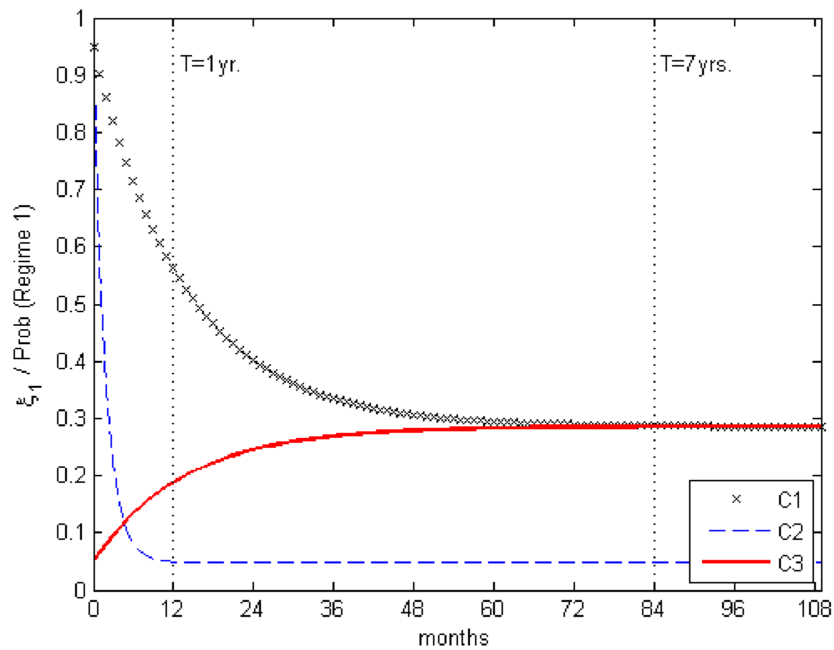

Figure 7.2: Probability of being located in regime 1 over time for the cases C1, C2 and C3 ($\xi_{1,t}$ for $t = 0, 1, 2, \dots, 110$).

A little more complicated is the comparison between the cases C2 and C3, since their state probabilities $\xi_{1,t}$ cross each other at approximately $t = 5$. In the cases of long time horizons of 7 years, the efficient portfolios of C3-7 have a larger overall stock position than those of C2-7. This again is a result of the multi-period investment. Investors in C3-7 profit from the non-volatile times at the beginning of the planning horizon, while the investors in C2, currently located in a rather volatile time period, are able to wait for low-volatile market situations during sub-periods 2 an 3, where $\xi_{1,t>13} = 0.05$. It is, however, difficult to find a regularity for the investment horizon of 1 year, where the state probabilities cross each other in midst of the planning horizon.

The last comparison between the cases C1 and C3 is trivial. Their transition matrix is equal, but their initial probability $\xi_{1,0}$ is significantly different. While C1 starts in a highly volatile market, with $\xi_{1,0} = 0.95$, C2 begins in a low-volatile market situation, where $\xi_{1,0}$ is only 0.05. Due to equal transition probabilities the steady state probabilities π of C1 and C3 converge to the same values ($\pi_1 = 0.29$ and $\pi_2 = 0.71$), but they reach it from opposite directions. Independent of the investment horizon T, the cases C1, converging from high-volatile periods, contain consequently lower portfolio weights for the stock positions than the cases C3.

7.2.2 Base Currencies and their Effect on the Asset Allocation

In Section 6.4 the distinction between local currencies and the base currency, the so called home currency of the investor, was introduced. Their risk/return-structures differ according to the dynamics of the exchange rate of the asset's local currency and the investor's base currency. Not only the risk/return-structure of investors with unequal home currencies will differ, but also their efficient asset allocations.

Figure 7.2 shows the asset allocation of three different home currencies: Swiss francs (CHF), Euro (EUR) and U.S. dollar (USD). The target returns in each currency range from 3.0% to 8.5% with steps of 10 base points for which in each case the efficient portfolio was optimized. Transaction costs are intentionally neglected in order to augment the effect of the home currency on the asset allocation and to keep the optimal portfolios independent from the initial portfolio. Therefore, the initial portfolio is simply chosen, by holding 100% of the money at disposal in cash.

The lower target returns are attained with a high proportion of bonds. It is, however, conspicuous, if not necessarily surprising that in each case the foreign currency bonds are hardly considered. Only the Swiss franc government bonds (CHb) have a significant amount in the efficient portfolios of the other two home currencies. In the EUR-allocations the asset category CHb is found throughout the whole range of the calculated target returns, having its peak at $x^* = 6.00\%$. In the USD-allocations the amount of the CHb increases from 0.78% at $x^* = 5.50\%$ to 8.99% at $x^* = 8.50\%$. The other two bond categories EUb and USb are negligible out of their home currency allocations. The explanation of this artefact, to avoid to invest in foreign bond categories, is that a significant part of the risk of foreign bonds is traced to their exchange rate exposure. Now, under the assumption that the market does not compensate this kind of risk, such asset categories have a poor risk/return-ratio and are therefore barely considered in an efficient portfolio. Table 7.3 demonstrates this phenomenon:

It appears that foreign bonds in foreign currencies have high exchange rate risks. In the USD home currency portfolio, three quarter of the whole risk of the foreign bonds

CHF	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%	8.5%
Risk %	0.32	1.02	1.87	3.05	4.75	6.70	8.74	10.84	12.98	15.21	17.56	20.15
Shortfall %	0.00	0.07	0.18	0.63	3.51	7.57	11.14	13.89	16.26	18.75	21.05	23.38
CASH	73.71	2.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHb	23.45	87.93	88.27	83.68	73.06	62.08	50.96	39.78	29.70	20.75	11.89	0.00
EUb	0.06	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
USb	0.63	1.32	0.74	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHs	1.27	4.62	5.68	8.12	13.05	18.08	23.17	28.27	32.43	35.53	38.80	44.09
EUs	0.32	1.44	1.98	3.17	6.16	9.26	12.40	15.55	18.60	21.89	25.10	28.55
USs	0.56	2.27	3.32	4.98	7.73	10.58	13.47	16.40	19.27	21.83	24.21	27.36
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

EUR	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%	8.5%
Risk %	0.34	1.04	1.83	2.85	4.31	6.15	8.16	10.27	12.46	14.76	17.25	20.10
Shortfall %	0.00	0.01	0.11	0.34	1.58	5.65	9.85	13.13	15.74	18.38	21.13	23.74
CASH	79.69	13.83	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHb	3.18	13.52	18.55	20.49	27.91	37.41	46.74	41.18	30.67	21.44	11.47	0.00
EUb	14.22	60.51	66.87	61.37	46.23	25.82	5.15	0.00	0.00	0.00	0.00	0.00
USb	0.67	2.65	1.28	1.66	0.63	0.05	0.03	0.00	0.00	0.00	0.00	0.00
CHs	0.83	3.44	3.77	5.67	9.10	13.40	17.58	22.32	26.93	29.92	34.04	35.68
EUs	0.85	3.68	5.83	6.38	9.11	13.28	17.68	20.99	24.19	28.11	31.56	38.18
USs	0.56	2.37	3.69	4.43	7.02	10.04	12.82	15.51	18.21	20.53	22.93	26.14
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

USD	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%	8.5%
Risk %	0.40	1.31	2.25	3.28	4.51	6.08	7.95	9.99	12.13	14.37	16.79	19.54
Shortfall %	0.00	0.29	1.34	2.39	3.33	5.77	9.36	12.39	15.19	18.08	20.62	23.00
CASH	82.59	36.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHb	0.00	0.00	0.00	0.00	0.00	0.78	2.58	4.54	5.97	7.81	8.98	0.00
EUb	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
USb	14.61	52.87	79.61	75.43	65.86	52.43	38.51	25.32	13.34	1.71	0.00	0.00
CHs	0.83	3.02	4.56	6.00	8.22	11.99	16.67	21.40	25.83	28.19	32.38	36.89
EUs	0.53	1.92	3.47	4.10	5.20	7.76	10.96	14.20	17.43	20.43	22.94	31.62
USs	1.44	5.25	9.10	10.29	11.15	13.61	17.36	21.35	25.45	30.23	33.99	31.50
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

Table 7.2: Efficient portfolios in different home currencies. For the ease of exposition, transaction costs were not considered. The target returns range from 3.0% to 8.5% with steps of 10 base points. The asset allocations are based on the calibration introduced in section 6.4.

CHb and EUb are due to the dynamics of the particular exchange rates. Also in an EUR or CHF home currency portfolio the total risk of foreign bonds contains a high proportion of exchange rate risk (around 50%). In comparison to the foreign bonds, the volatility of the foreign stocks is not that highly influenced by the exchange rate. On the one hand stocks are regardless of any exchange rates riskier than bonds, on the other hand they are more contagious in the international environment than the bonds.

7.2.3 Liability-based Asset Allocation and its Effect on the Asset Allocation

The following numerical example illustrates how liability-based asset allocation helps to meet the incurred liabilities at all times. It shows the effect on the asset allocation that comes from the incorporation of stochastic liabilities in the optimization procedure of the multistage stochastic program.

CHF	Regime 1			Regime 2		
	$\sigma_{incl.}$	$\sigma_{excl.}$	rel.diff.	$\sigma_{incl.}$	$\sigma_{excl.}$	rel.diff.
CHb	2.10%					
EUb	4.23%	2.58%	-38.93%		R1 = R2	
USb	9.39%	3.53%	-62.43%			
CHs	19.02%			14.05%		
EUs	23.52%	21.25%	-9.69%	16.13%	14.13%	-12.37%
USs	24.28%	20.14%	-17.03%	15.51%	12.78%	-17.61%

EUR	Regime 1			Regime 2		
	$\sigma_{incl.}$	$\sigma_{excl.}$	rel.diff.	$\sigma_{incl.}$	$\sigma_{excl.}$	rel.diff.
CHb	4.55%	2.20%	-51.62%			
EUb	2.59%				R1 = R2	
USb	8.94%	3.72%	-58.40%			
CHs	18.69%	18.91%	1.17%	13.39%	13.99%	4.49%
EUs	21.60%			14.32%		
USs	23.04%	20.05%	-12.98%	13.85%	12.69%	-8.40%

USD	Regime 1			Regime 2		
	$\sigma_{incl.}$	$\sigma_{excl.}$	rel.diff.	$\sigma_{incl.}$	$\sigma_{excl.}$	rel.diff.
CHb	10.94%	2.60%	-76.26%			
EUb	10.58%	2.58%	-75.56%		R1 = R2	
USb	3.64%					
CHs	18.79%	18.96%	0.90%	15.98%	14.07%	-11.91%
EUs	21.45%	21.44%	-0.02%	16.52%	14.40%	-12.83%
USs	19.93%			12.77%		

Table 7.3: Quantification of the foreign assets' exchange rate risk. Three considered home currencies (CHF, EUR, USD) are compared. $\sigma_{incl.}$ = total risk inclusively exchange rate risk; $\sigma_{excl.}$ = total risk exclusively exchange rate risk; rel.diff.= relative difference.

The historical data to estimate the parameters of the regime switching model cover period January 1980 to August 2009 on a monthly basis. The model consists of four factors and calibrates an asset universe with six asset categories as well as three liability indices (ConstMix 25, 40 and 60). To better point out the effect of these liabilities, a specific liability evolution has been chosen for them. The evolvement of these liabilities is in fact a synthetic replication based on the considered six asset categories. The weightings of these replications are shown in the following table 7.4 and are inspired, respectively an approximation of the Pictet LPP-Indices 2000[4].

The strategic planning horizon of 4.5 years has been chosen, which was divided into 3 periods of the length of 6 months, 1 year and 3 years. For the ease of exposition, trans-

[4]Pictet LPP-Indices are suitable and well-known benchmarks for Swiss pension funds (see Appendix A.1).

Asset Class	ConstMix25	ConstMix40	ConstMix60
Cash	**0.0%**	**0.0%**	**0.0%**
Bonds	**75.0%**	**60.0%**	**40.0%**
CHb	60.0%	45.0%	25.0%
EUb	10.0%	10.0%	10.0%
USb	5.0%	5.0%	5.0%
Equity	**25%**	**40.0%**	**60.0%**
CHs	10.0%	15.0%	20.0%
EUs	7.5%	12.5%	20.0%
USs	7.5%	12.5%	20.0%
Mean (μ)	**2.24%**	**2.64%**	**3.18%**
Volatility (σ)	**5.33%**	**7.51%**	**10.76%**

Table 7.4: Composition of the three ConstMix liabilities. The evolvement of these liabilities is in fact a synthetic replication with the weightings shown. The weightings refer to the composition of the Pictet LPP-Indices 2000. Whereas the asset USb replaces the LPP 2000 sub-index 'World Bond' and assets USs and EUs are substitutes for the LPP-2000 sub-index 'World Equities'.

action costs as well as restrictions were not considered. The target returns with steps of 5 base points have different ranges, according to the considered liability (ConstMix).

Liability-based asset allocation leads to efficient portfolios that replicate the structure of the underlying liability index quite well. This 'liability-adjusted' diversification is clearly visible for the target returns that are close to the growth of the liability (see table 7.5). For the ConstMix-25 with a liability growth of 2.24% the 'liability-adjusted' diversification is found around the target return $x^* = 2.40\%$. For the ConstMix-40 with a liability growth of 2.64% it is found around $x^* = 2.70\%$ and for the ConstMix-60 with a growth of 3.18% around $x^* = 3.20\%$.

The comparison between the liability-based asset allocation with the non-liability one (see table 7.6) is striking. In the relevant range of $2\% \leq x^* \leq 4\%$ the efficient portfolios without liabilities invest no wealth into foreign bond positions, due to reasons already discussed. Also the stock positions, dominated by the 'USs', are less diversified. Therefore, incorporating the liabilities into the optimization procedure of the multistage stochastic program leads to a 'liability-adjusted' diversification and is indispensable in an integrated portfolio management process.

According to figure 7.3, the efficient frontiers of table 7.5 cross the efficient frontier of table 7.6. The risk of the liability-adjusted efficient portfolios, compared to the non-liability efficient portfolios, are higher for small target returns but lower for target returns above a certain value. It is notorious that for lower target returns, the risk consists mainly of the inherent risk of the liabilities. In the case where the liabilities are not considered, the risk decreases for small target returns towards zero, since a main portion of the wealth is placed in the riskless cash position. The point of intersection

ConstMix25	**Target returns in %**											
	2.25	**2.30**	**2.35**	**2.40**	**2.45**	**2.50**	**2.55**	**2.60**	**2.65**	**2.70**	**2.75**	**2.80**
Risk %	0.59	0.63	0.67	0.74	0.81	0.89	0.99	1.11	1.25	1.39	1.56	1.73
Shortfall %:												
- with Liab.	69.91	58.45	42.75	33.37	29.85	28.00	27.27	26.87	26.37	25.79	25.41	25.17
- w/out Liab.	48.58	47.80	47.11	46.59	45.94	45.30	44.79	44.15	43.61	42.85	42.28	41.72
CASH	13.38	8.51	3.74	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHb	49.10	53.59	57.91	61.18	61.53	61.95	62.28	62.36	62.56	62.82	63.17	63.51
EUb	11.45	11.21	11.02	10.70	9.67	8.79	8.21	7.58	6.60	5.35	3.85	2.32
USb	3.88	3.90	3.93	4.08	4.35	4.43	4.25	4.20	4.22	4.31	4.38	4.48
CHs	9.94	9.97	10.05	10.33	10.56	10.65	10.51	10.38	10.33	10.37	10.45	10.51
EUs	6.35	6.66	6.93	7.04	7.16	7.37	7.72	8.18	8.64	9.14	9.75	10.37
USs	5.90	6.16	6.42	6.64	6.73	6.81	7.03	7.30	7.64	8.01	8.40	8.81
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

ConstMix40	**Target returns in %**											
	2.50	**2.55**	**2.60**	**2.65**	**2.70**	**2.75**	**2.80**	**2.85**	**2.90**	**2.95**	**3.00**	**3.05**
Risk %	0.72	0.72	0.73	0.75	0.79	0.83	0.88	0.95	1.02	1.12	1.23	1.52
Shortfall %:												
- with Liab.	84.08	81.43	75.84	69.48	62.03	50.98	39.37	33.23	30.74	29.50	28.65	29.07
- w/out Liab.	53.82	52.85	51.98	51.25	50.57	49.73	49.05	48.27	47.65	47.05	46.41	45.80
CASH	23.04	21.39	18.32	13.36	8.46	3.64	0.11	0.01	0.01	0.00	0.01	0.03
CHb	31.20	30.55	31.47	36.01	40.49	44.86	48.04	48.45	48.89	49.17	49.20	49.30
EUb	10.09	11.47	12.59	12.34	12.09	11.84	11.37	10.36	9.48	8.92	8.37	6.09
USb	4.17	4.17	4.00	4.05	4.10	4.17	4.36	4.62	4.65	4.49	4.45	4.67
CHs	12.98	13.82	14.53	14.59	14.66	14.79	15.11	15.34	15.42	15.32	15.21	51.31
EUs	9.37	9.54	9.80	10.11	10.42	10.67	10.77	10.88	11.11	11.44	11.85	12.88
USs	9.15	9.06	9.29	9.54	9.78	10.03	10.24	10.34	10.44	10.66	10.91	11.72
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

ConstMix60	**Target returns in %**											
	2.95	**3.00**	**3.05**	**3.10**	**3.15**	**3.20**	**3.25**	**3.30**	**3.35**	**3.40**	**3.45**	**3.50**
Risk %	0.76	0.76	0.77	0.78	0.81	0.85	0.90	0.95	1.02	1.11	1.21	1.34
Shortfall %:												
- with Liab.	88.27	85.46	82.39	78.03	71.76	65.09	56.10	46.31	39.16	35.97	34.27	32.75
- w/out Liab.	54.01	53.33	52.52	51.89	51.24	50.54	49.89	49.31	48.63	48.00	47.59	47.06
CASH	23.11	22.53	20.45	15.48	10.56	5.69	1.05	0.03	0.02	0.00	0.01	0.00
CHb	15.15	15.25	14.13	18.71	23.26	27.73	31.86	32.92	33.38	33.78	33.88	34.05
EUb	10.35	10.60	12.47	12.14	11.79	11.46	11.14	10.35	9.31	8.60	8.06	7.15
USb	4.57	4.52	4.24	4.32	4.41	4.51	4.68	4.90	5.06	4.96	4.89	4.88
CHs	17.14	17.30	18.43	18.48	18.53	18.62	18.86	19.14	19.30	19.32	19.21	19.18
EUs	15.08	15.14	15.41	15.77	16.12	16.43	16.63	16.71	16.90	17.14	17.53	17.99
USs	14.60	14.66	14.87	15.10	15.33	15.56	15.78	15.95	16.03	16.20	16.42	16.75
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

Table 7.5: Liability-based efficient portfolios with different liabilities. For the ease of exposition, transaction costs as well as restrictions were not considered. The target returns with steps of 5 base points have different ranges, according to the considered liability (ConstMix).

between the two efficient frontiers, with and without considering a stochastic liability, depends on the risk of the liabilities as well as on the level of the riskless rate. The higher the liability risk, the higher the target return at the point of intersection (see figure 7.3b). As self-evident conjunction the higher the riskless rate, the higher the target return at the point of intersection. Since the riskless rate in this numerical example is very low (0.32%) the point of intersection is between $0.55\% \leq x^* \leq 0.76\%$, depending on the risk of the different stochastic ConstMix-liabilities.

The integrated asset allocation is better adjusted to the stochastic properties of the liabilities. This leads on the one hand to lower portfolio risks and on the other hand to

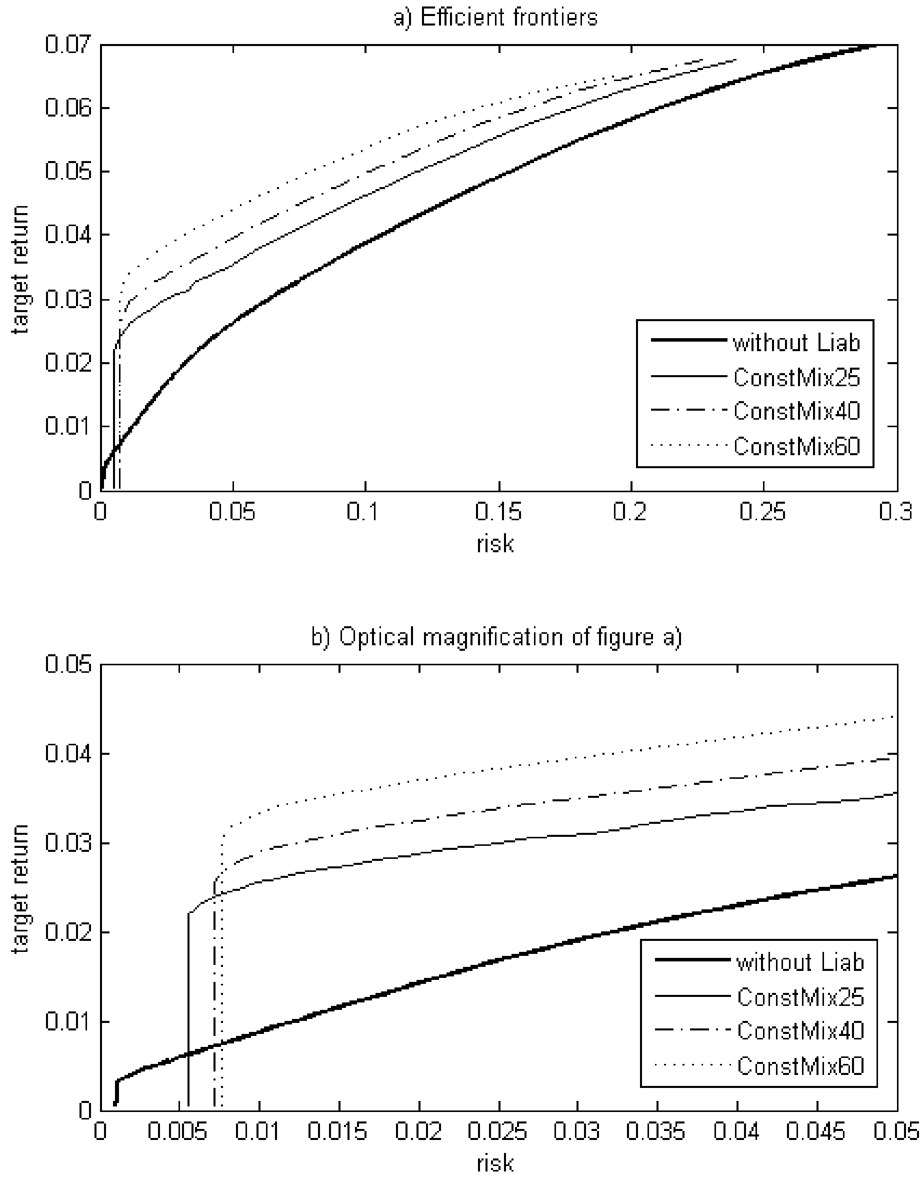

Figure 7.3: a) Efficient frontiers of the 'liability-adjusted' portfolios (ConstMix) versus the non-liability efficient frontier (without Liab). b) Optical magnification of the relevant part.

w/out Liab.	Target returns in %										
	2.00	**2.20**	**2.40**	**2.60**	**2.80**	**3.00**	**3.20**	**3.40**	**3.60**	**3.80**	**4.00**
Risk %	3.23	3.74	4.31	4.95	5.64	6.38	7.16	7.98	8.82	9.69	10.59
Shortfall %	53.57	49.32	46.59	44.15	41.72	39.36	37.29	35.81	34.24	33.20	32.51
CASH	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHb	83.42	82.43	79.78	76.51	72.54	68.24	63.61	58.71	54.05	49.55	45.19
EUb	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
USb	0.71	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHs	3.16	3.02	3.33	3.99	5.10	6.09	7.17	8.43	9.49	10.37	11.15
EUs	5.47	6.30	7.52	8.51	9.59	11.11	12.57	13.96	15.22	16.41	17.45
USs	7.24	8.24	9.37	10.99	12.77	14.56	16.65	18.90	21.24	23.67	26.21
TOTAL	100	100	100	100	100	100	100	100	100	100	100

Table 7.6: Efficient portfolios without liabilities. For the ease of exposition, transaction costs as well as restrictions were not considered. The target returns range from 0.02 to 0.04 with steps of 20 base points.

smaller shortfall probabilities[5] within the liability-relevant range of the target returns (see figure 7.4). Not considering the liabilities (but same threshold) would, from the viewpoint of an integrated portfolio manager, result in a replication with an unfavorable risk/return-pattern and thus in a higher shortfall risk.

To summarize the integration of liabilities in the asset allocation process has a crucial influence on the efficient frontier as well as on the composition of the efficient portfolios and its shortfall probabilities. The consistent modeling and integration of the liabilities is therefore essential in an integrated portfolio management process. It better fulfills the task to secure the funding of the incurred liabilities.

7.2.4 Benchmark Tracking and its Effect on the Asset Allocation

This section generates, using the DEVA approach, three indices, namely Index-25, Index-40 and Index-60 that track, with semi-annual rebalancing activities, the three corresponding Pictet indices LPP-25 plus, LPP-40 plus and LPP-60 plus (see Appendix A.2).

The following numerical example contains an asset universe of twelve assets. Next to the cash position, the asset universe consists of three bond indices, six stock indices and two alternative assets.

The reference date of the benchmark tracking is June 1, 2006. Each index starts with the value of 100. The initial portfolios for the Index-25, Index-40 and Index-60 are generated, based on the resulting risk/return-structure of the regime-switching model,

[5]The corresponding thresholds to the shortfall probabilities are calculated under the assumption that the annual portfolio return matches the annual liability growth. In order to ensure an equal coverage ratio of currently 125% in the future.

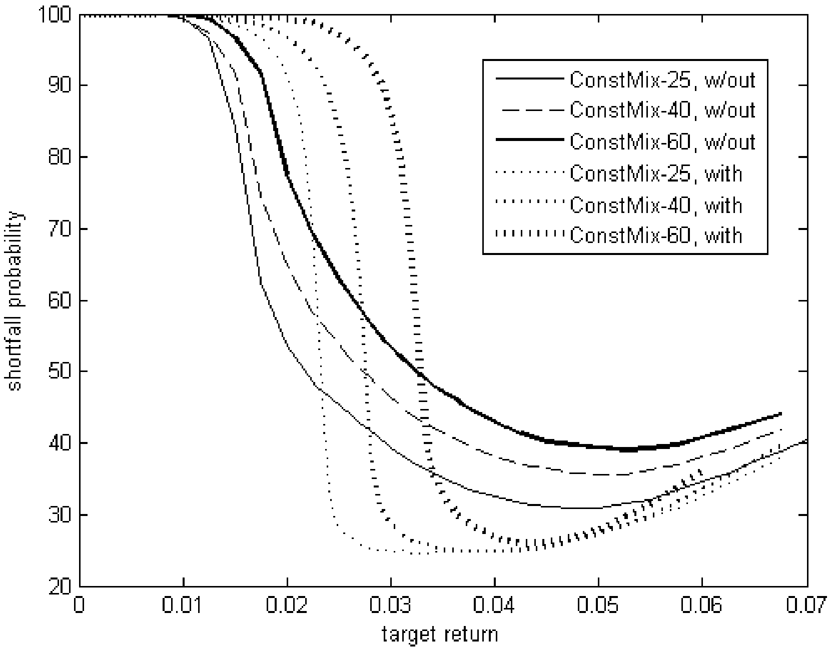

Figure 7.4: Sensitivity analysis of the shortfall probability to changes in the target returns.

using only daily data up to May 2006. The strategic planning horizon of 2.5 years is divided into 3 periods of the length of 6 months, 1 year and 1 year. By holding 100% of the wealth in cash and by neglecting the transaction costs, the resulting efficient portfolio can be used as the initial portfolios at the reference date. The decision criteria to chose one of the efficient portfolios lying on the efficient frontier is given by the smallest shortfall probability. The corresponding thresholds to the shortfall probabilities are calculated under the assumption that the generated index matches the current benchmark value.

At the end of the year 2006 the portfolio weights of the indices are rebalanced for the first time. Based on an updated risk/return-structure, using daily data up to December 2006 the portfolio weights are newly optimized. Meanwhile, transaction costs of 5 basis points (equal for selling and purchasing) are taken into account. Further updates are done on a semi-annual frequency, up to June 2008.

The performances of the indices compared to the benchmarks are visible in figure 7.5. The Index-25 outperforms its benchmark in the the year 2007. At the beginning of the year 2008 the out-performance is rapidly lost. In the downturn of the year 2008, the Index-25 tracks its benchmark quite well. The rebalancing activities of mid 2008 reduced the portfolio exposure by investing more in cash and bond positions (see figure 7.6). Thanks to these rebalancing activities the Index-25 was able to keep the level with its benchmark, even during the collapse at the end of the year 2008. The Index-40 outperforms its benchmark through the entire time period. The risk exposure of the index was already reduced at the end of the year 2007, by emphasizing bond positions. This might be the key reason of the outperformance during the collapse at the end of the year 2008. The Index-60 tracks its benchmark quite well. It outperforms slightly in the second half of the year 2007 and matches its benchmark during the collapse in October 2008.

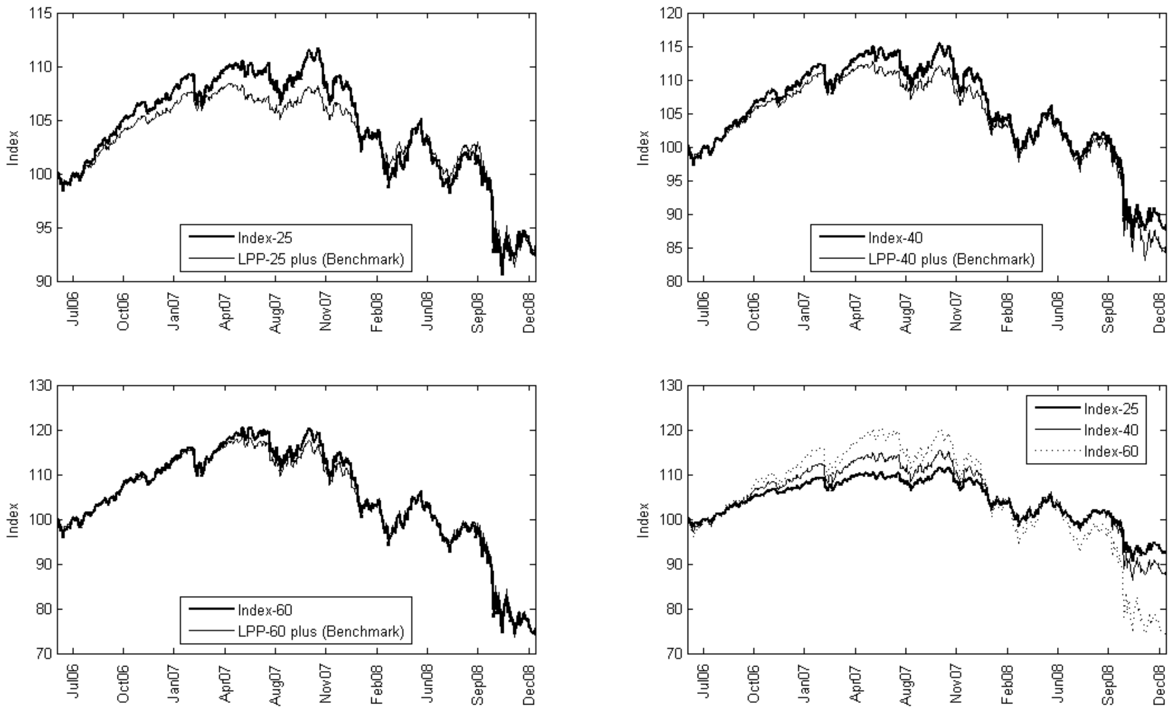

Figure 7.5: The performances of the generated indices over time compared to the benchmarks Pictet LPP plus. Covering time period June 2006 to December 2008.

Figure 7.6 shows the portfolio structures over time. It is reasonable that the Index-25 contains the largest cash and bond position, while Index-60 is dominated by stock positions.

Over the long run benchmark tracking will not, by definition, outperform the general market development. Further information is needed in order to gain an edge over the benchmarks. The regime-switching model does not only contain the information concerning the risk/return-structure but also, due to the actual inference $\xi_{1,t|t}$, a certain timing information. Based on this information not only the risk/return-structure is adapted to the new financial market situation but one can also adapt the benchmark that is being tracked. In volatile time periods one tracks a conservative benchmark

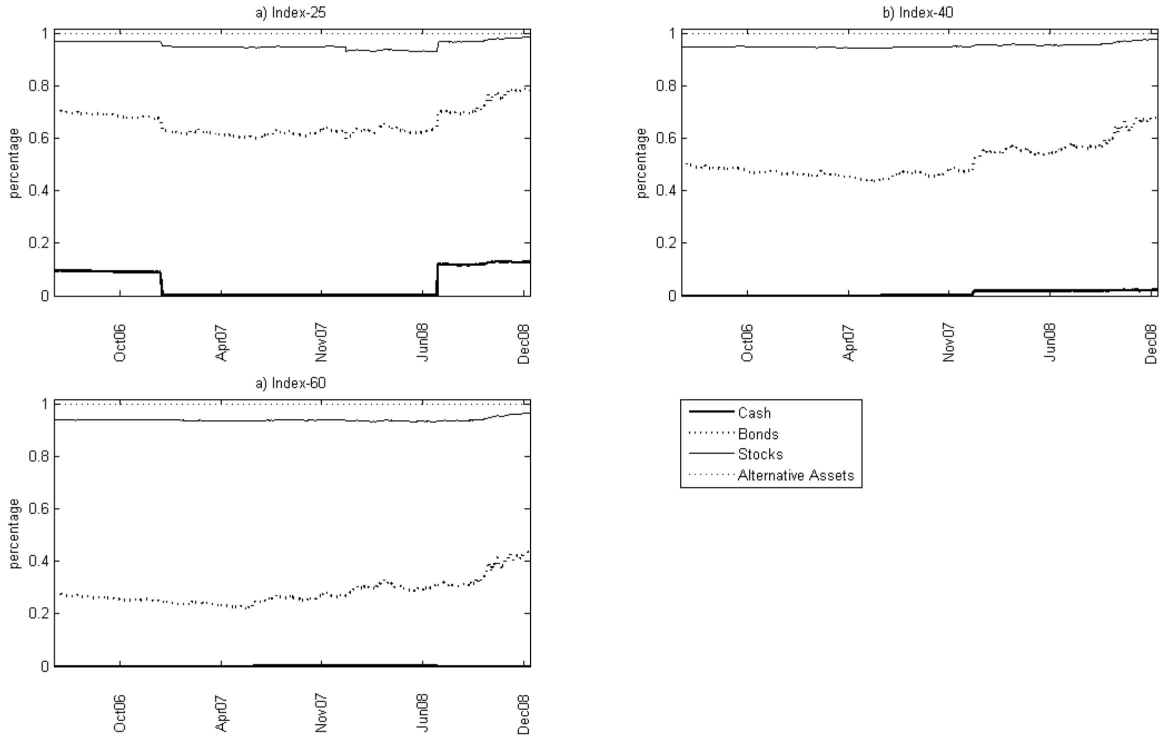

Figure 7.6: The portfolio structures of the generated indices over time. Aggregated in four asset categories: cash, bonds, stocks and alternative assets.

(e.g. the LPP-25 plus) and during low-volatile time periods a more risky benchmark (e.g. the LPP-60 plus).

A optimal rule of switching the benchmark is defined as follows. If $\xi_{1,t|t}$ is higher than 0.5, the model tracks a conservative benchmark and only switches to a riskier one, if $\xi_{1,t|t} < 0.4$ holds. However, if $\xi_{1,t|t}$ is lower or equal to 0.5, the model tracks a risky benchmark and only switches to a conservative one, if $\xi_{1,t|t} > 0.6$ holds.

Adapting this rule to the Index-25 (benchmark: LPP-25 plus) and to the Index-60 (benchmark: LPP-60 plus) leads to an outstanding index, named RS-Index (regime-switching index). The RS-index tracks the LPP-60 plus index from June 2006 to February 2008. Since in march $\xi_{1,t|t} > 0.6$ holds, the RS-index switches to the more conservative benchmark LPP-25 plus index[6]. This procedure assures that the generated index always matches the better benchmark. Therefore, over time it will outperform both benchmarks (see figure 7.7).

[6]In this example the switch neglects the transaction costs.

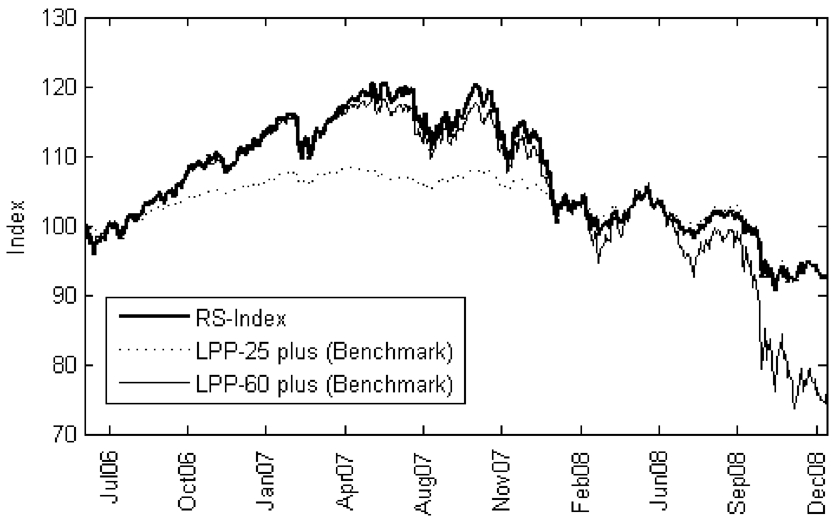

Figure 7.7: Performance of the generated RS-index over time period June 2006 to December 2008 compared to the benchmarks.

7.2.5 Hedging and its Effect on the Asset Allocation

Many investors consider international investing in order to either increase the diversification or to gain a higher return on investments in an economy, where one believes that it grows on a faster pace than the domestic one. These two new options, however, bring along some further risks, mainly in the form of exchange rate exposure. Exchange rate risk is not to be underestimated in international investments. A well-placed foreign exchange hedge, however, provides an effective mechanism for offsetting such exchange rate risks. The following case study illustrates the dangers, but also the benefits of international investments.

The case study is based on monthly data covering period January 1980 to December 2007 and consists of an asset universe of six indices: CHb, EUb, USb and CHs, EUs, USs. On the basis of these data, three different Swiss franc market models were calibrated. Each model representing a certain hedging strategy: no-hedge, part-hedge, total-hedge (see Appendix, table A.3). The first calibration 'no-hedge' regards all foreign investments as unhedged. Each foreign asset is next to the market risk also entirely exposed to the corresponding exchange rate risk. To that fact the volatilities of the two involved foreign regions U.S.A. and Europe are, in both regimes, higher compared to the equivalent assets of the domestic Swiss market. The second calibration 'part-hedge' considers the foreign bonds (EUb, USb) as fully hedged against any exchange rate risks. According to table A.3, the volatility of the foreign bonds, reduced to market risk only, relatively decreases by -32.80% (EUb) and -52.19% (USb). Next to the reduction of the volatility it is noticeable that the correlations among domestic and hedged foreign bonds increases. The dynamics of the exchange risk do no longer dilute the relation among investments of the same asset category. Even the decoupling effect, neutralized by the dynamics of the exchange rates, become evident. The third calibration 'total-hedge' considers not only the foreign bonds as fully hedged against any exchange rate risk but also the foreign stocks (EUs, USs). The hedged foreign stocks experience a relative volatility reduction of -11.15% (regime 1), -7.54% (regime 2) for the EUs and of -23.53% (regime 1), -26.01% (regime 2) for the USs.

Hedged asset categories are calculated by retransferring the corresponding asset index into its local currency. It is to mention that neither interest rate risks, nor hedging costs are considered in this hedge procedure. On the one hand the differences of the interest rates between the Swiss franc and the foreign currencies are neglected, on the other hand no derivatives (e.g. forwards) are used in the portfolio to hedge the currency exposure. The case study is deliberately kept simple and aims to point out the exposure of a investment in foreign currencies due to the volatility of the exchange rates. This exposure is not only given by investing in international financial markets but is also an issue for international or exporting companies with businesses in different currencies.

The efficient portfolios for each of these three hedging strategies are found in table 7.7. A planning horizon of three years has been chosen, which was divided into three periods of length: 6 months, 1 year and $1\frac{1}{2}$ years. Since the riskless cash position at the end of 2007 had a return of 2.76%, the expressive target returns start at 3.5% and range to 8.5% with steps of 50 base points. For each of these target returns the initial portfolio was optimized and reallocated. Transaction costs are again neglected in order to augment the effect of the hedging strategies on the asset allocation and to keep the optimal portfolios independent from the initial portfolio. Therefore, the initial portfolio is simply chosen, by holding 100% of the money in cash at disposal.

The hedging strategy 'no-hedge' reveals no new insights. It is well known that foreign bonds have, due to the relative high exchange rate exposure, an unprofitable risk/return-ratio. Hence, the foreign bond positions are of negligible quantity. The foreign stock markets demonstrate a far better risk/return-ratio than foreign bonds, due to the fact that the compensated market risk make up the largest part of the total risk. Hence, they serve already in unhedged efficient portfolios for diversification. In the lower target returns, which are in range of the expected bond returns, the foreign stock positions are small and mainly used for diversification reasons. The larger the target return, the larger the foreign stock positions in order to reach the

No-Hedge: Foreign Assets Unhedged

	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%	8.5%
Risk %	0.49	1.39	2.36	3.43	4.74	6.39	8.30	10.36	12.53	14.80	17.23	19.90
Shortfall %	0.00	0.47	1.72	3.01	4.60	7.19	10.23	12.66	15.35	17.74	20.51	22.77
CASH	84.34	47.96	14.61	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHb	13.11	43.65	70.66	81.57	80.21	70.55	59.25	47.76	36.55	25.25	13.51	0.00
EUb	0.52	1.94	4.80	4.77	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
USb	0.18	0.26	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHs	0.28	0.77	0.71	1.30	3.36	7.05	11.03	14.98	18.64	22.22	26.01	29.91
EUs	0.39	1.34	2.35	3.14	4.52	5.93	7.86	9.89	11.98	14.02	15.94	17.55
USs	1.18	4.08	6.86	9.21	11.89	16.47	21.86	27.37	32.83	38.51	44.54	52.54
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

Part-Hedge: Foreign Bonds Hedged (*)

	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%	8.5%
Risk %	0.43	1.25	2.11	3.07	4.22	5.64	7.36	9.41	11.77	14.32	17.03	20.00
Shortfall %	0.00	0.29	1.26	2.13	3.44	5.64	8.46	11.45	15.28	18.97	21.56	23.76
CASH	84.77	48.72	15.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHb	6.61	22.04	33.60	34.99	29.74	16.08	0.00	0.00	0.00	0.00	0.00	0.00
EUb*	2.09	10.19	22.05	26.16	15.31	3.43	0.00	0.00	0.00	0.00	0.00	0.00
USb*	4.08	13.50	20.00	26.50	38.54	57.35	68.49	59.08	47.76	35.94	23.92	9.34
CHs	0.31	0.97	0.87	1.22	3.41	6.17	8.86	11.69	14.65	17.47	19.26	22.54
EUs	0.52	1.80	3.24	4.25	4.99	6.68	8.56	10.06	12.18	14.49	16.90	18.53
USs	0.81	2.78	5.07	6.88	8.01	10.29	14.09	19.17	25.41	32.10	39.92	49.59
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

Total-Hedge: Foreign Assets Hedged (*)

	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%	8.5%
Risk %	0.44	1.26	2.12	3.08	4.21	5.60	7.26	9.20	11.39	13.79	16.49	20.11
Shortfall %	0.00	0.30	1.31	2.29	3.42	5.57	8.33	11.25	14.55	18.57	21.63	23.14
CASH	85.20	48.54	15.64	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CHb	6.79	23.41	35.86	37.87	33.88	21.25	0.01	0.00	0.00	0.00	0.00	0.00
EUb*	2.38	8.71	18.94	23.00	11.15	0.07	0.01	0.00	0.00	0.00	0.00	0.00
USb*	3.71	12.58	18.32	23.94	35.40	51.69	63.48	53.26	40.89	27.42	9.66	0.00
CHs	0.14	0.34	0.04	0.04	1.37	3.28	5.10	6.32	7.16	7.34	8.36	0.00
EUs*	0.90	3.23	5.55	7.28	8.69	11.17	14.26	16.23	19.33	22.82	25.69	31.23
USs*	0.88	3.19	5.65	7.86	9.51	12.54	17.14	24.19	32.62	42.42	56.29	68.77
TOTAL	100	100	100	100	100	100	100	100	100	100	100	100

Table 7.7: Efficient portfolios for different hedging strategies. For the ease of exposition, transaction costs were not considered. The target returns range from 3.0% to 8.5% with steps of 50 base points. The asset allocations are based on the calibration found in table A.3.

desired expected returns. The accumulated foreign stocks even exceed the domestic stock market. On the one hand it sustains the statement about the risk/return-ratio on the other hand it might be a reason of diversification against the cluster risk of the domestic bond position.

In the hedging strategy 'part-hedge' foreign bonds, which are fully currency-hedged, become competitive and valuable for international investments. The efficient portfolios for each target return have a lower risk and lower shortfall probability (threshold = 100; return of 0%), which is - amongst other - referable to the increased potential of diversification and the higher returns on foreign assets. The efficient portfolios of target returns which are in the range of the expected bond returns are well diversified among the three bond markets. For higher target returns the hedged USb replaced the role of the domestic CHb. The slightly higher expected return and volatility of the USb compared to the CHb allows to reach higher target returns with less stock positions, pushing down the portfolio risk.

The hedging strategy 'total-hedge', in which all foreign assets are fully hedged against the exchange rate exposure, improves mainly the efficient frontier above the target return of $x^* \geq 5\%$. The efficient portfolios increase their foreign stock positions at the charge of the bond positions and the CHs.

Figure 7.8 illustrates in a μ/σ-space the added-value of an international investment and of its hedging strategies. While the dotted line denotes the efficient frontier that results in only investing in domestic assets, the other efficient frontier refers to international investments with different hedging strategies. Already unhedged international investments ('no-hedge') generate added-value. Since unhedged foreign bonds are unprofitable, the added value is mainly due to the foreign stock positions. Therefore, the added-value increases along with the level of the target returns. The hedging strategy 'part-hedge' leads to further added-value, especially in the mid-range of the suitable target returns. This hedging strategy opens new investment possibilities, then one mostly enters the foreign bond markets only by hedging them. By reasons already discussed, the additional hedging of the foreign stocks (full-hedging) brings little efficiency improvement and little risk reduction to an efficient frontier. This small added value is visible above a certain target return of around $x^* > 6\%$.

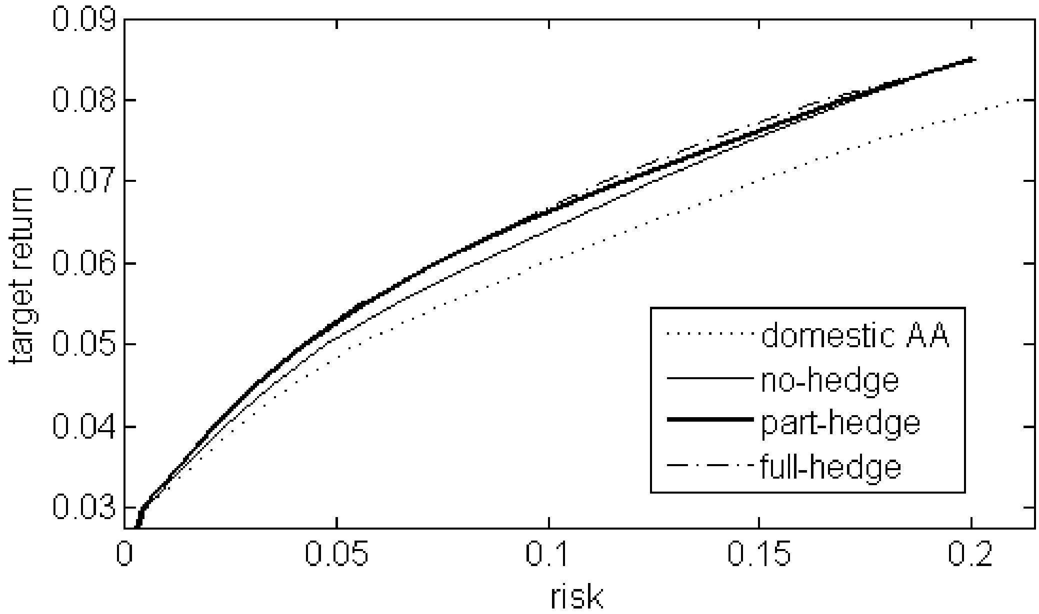

Figure 7.8: Efficient frontiers of different hedging strategies. A planning horizon of 3 years has been chosen, which was divided into 3 periods of 6 months, 1 year and $1\frac{1}{2}$ years.

How these different hedging strategies performed during the year 2008 is visualized in figure 7.9a), assuming to keep the weights of the portfolios equal during the whole year. The year 2008 was intentionally chosen, due to the high-volatile exchange rates during this particular year (see figure 7.9b). The hedging strategy 'no-hedge' is being outperformed by the other two hedging strategies. In the first half of the year the relative appreciation of the Swiss franc towards the two foreign currencies USD and EUR depressed the performance of the 'no-hedge' strategy. Since this strategy contains no foreign bonds, it is traced back to the foreign stock positions. The 'part-hedge' strategy derives profit from the hedged foreign bond positions using their potential of diversification and their higher returns. The importance of hedging foreign bonds is shown by the dotted line, visualizing the performance of an unhedged portfolio containing the same weighting like the efficient portfolio of the 'part-hedge' strategy. During appreciation of the domestic currency, the exchange risk exposure of the foreign bonds pushes the whole performance down. On the other side it has a positive effect on the performance during a depreciation, as seen at the end of the year 2008 (see figure 7.9). The 'full-hedge' strategy has no further apparent effect on the performance, it rather moves around the 'part-hedge' strategy. The index of the unhedged portfolio containing the same weighting as the efficient portfolio of the 'full-hedge' strategy (thin solid line) moves more or less along the index of the corresponding unhedged portfolio of the 'part-time' strategy (dashed line).

In summary, it can be stated that from the Swiss franc point of view it is recommendable to protect open positions against adverse movements in foreign exchange rates. On the one hand it preserves a double loss in volatile times: lower returns as well as appreciation of the safe haven currency Swiss franc. On the other hand it provides international potential of diversification. A currency hedge for foreign stocks seems to be of low benefit, all the more whilst taking into account that the costs of hedging are not considered in this case study. However, the currency hedge for foreign bonds has to be considered even though it might be a complex and costly business (See Swensen, 2005, [95]).

7.2.6 The Potential of Diversification

The financial markets provide a large basket of investment vehicles, which can be well arranged in a risk/return-diagram. Apart from the cash market, government and investment grade bonds tend to be the lowest risk assets, however, they also offer the lowest expected returns. Stocks, the third main asset class, are more risky than bonds, with correspondingly higher expected returns. However, asset managers mostly

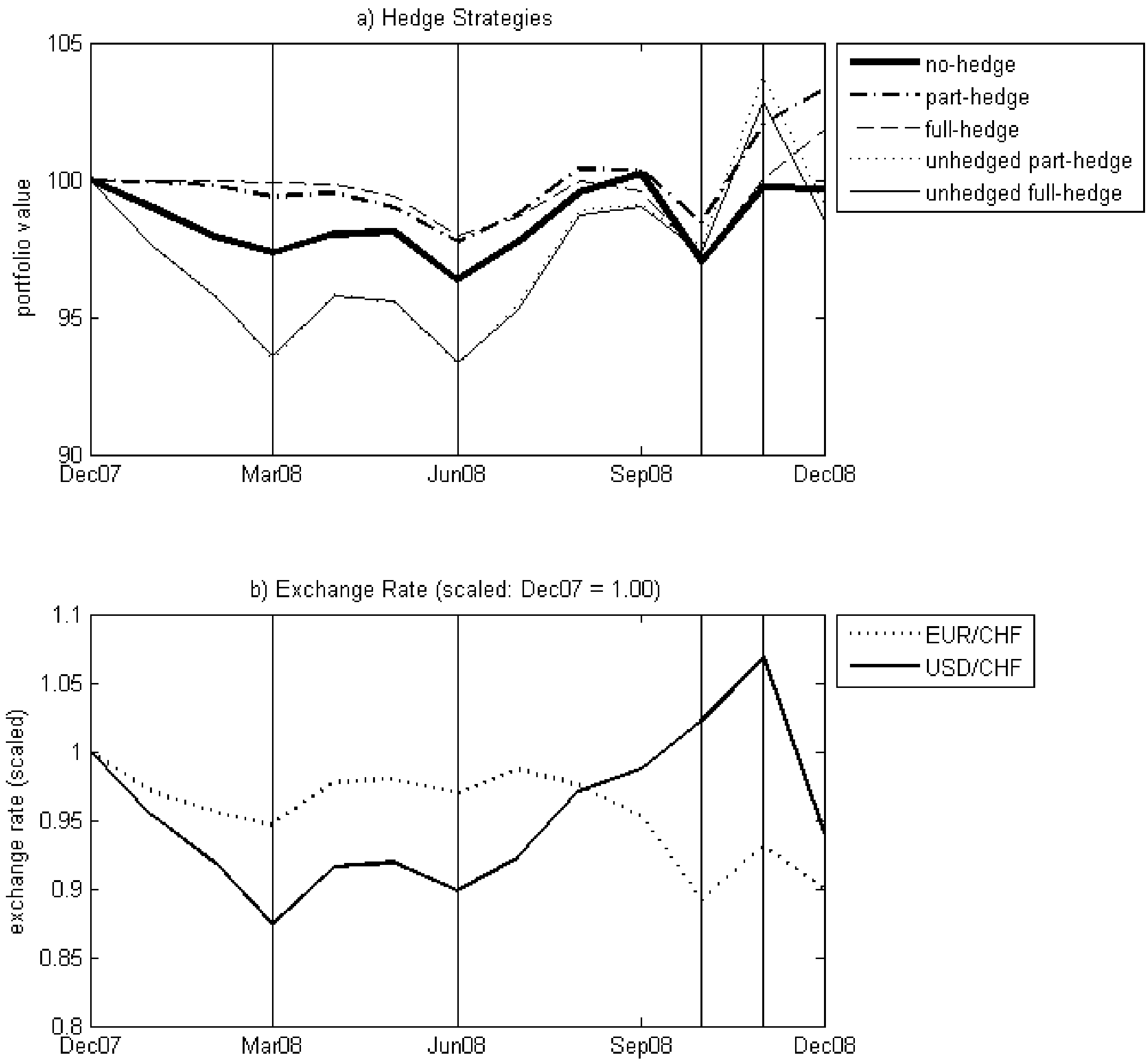

Figure 7.9: Portfolio performance of different hedging strategies of the year 2008. A planning horizon of 3 years has been chosen, which was divided into 3 periods of 6 months, 1 year and $1\frac{1}{2}$ years.

distinguish the stock markets between industrial countries and emerging countries. For the simple reason that the emerging markets are classified as more risky, since next to price fluctuations, also political crises and potential liquidity problems have to be considered. Other asset categories are at a rough estimate classified as shown in figure 7.10.

Private equity and other high-volatile investment instruments can be characterized by a high concentration on single positions (turnarounds) and by applying significant leverage. Beyond that, the unpredictable specific risks (counterparty, liquidity, untransparent investment strategies and potential unlimited losses due to short positions) make these instruments highly volatile. Further, according to their degree of creditworthiness, the bond investments spread over the whole spectrum of risk. There-

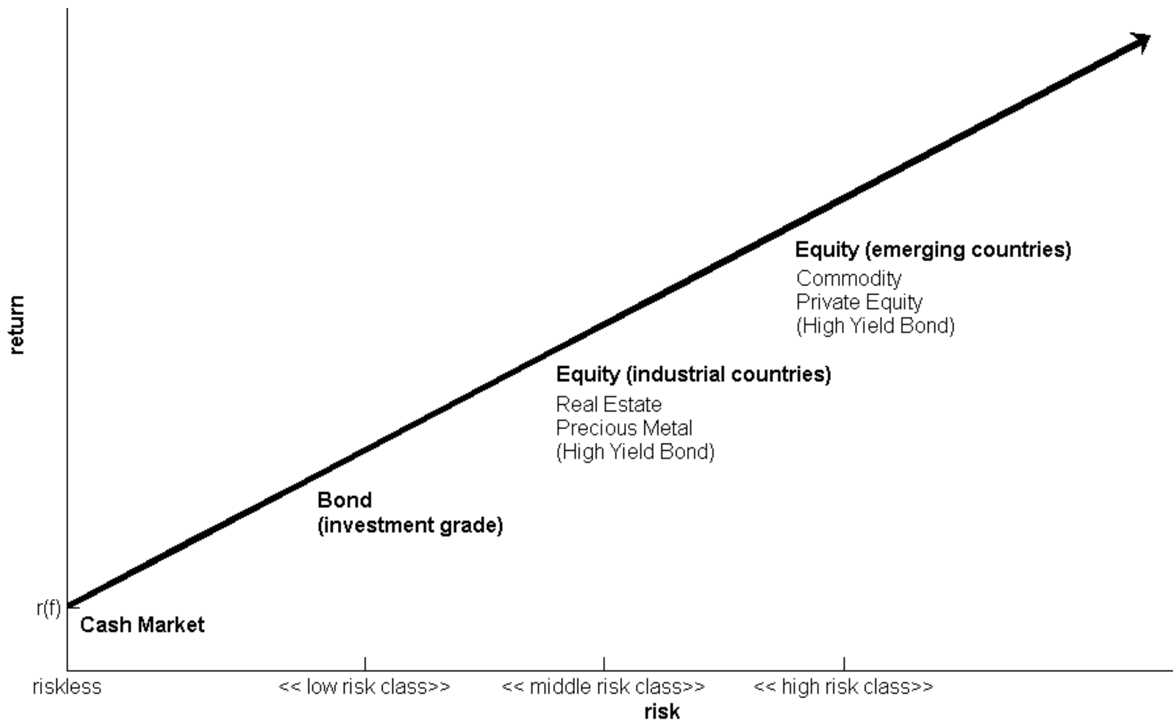

Figure 7.10: Rough classification of different asset categories within a μ/σ-space.

fore, high-yield bonds would be classified as middle-risk assets or even high-risk assets. All these different investment possibilities might be clustered into three risk classes (low, middle and high) as shown on the x-axes of figure 7.10.

The risk aversion, respectively the risk ability of an investor give first insights on his efficient portfolio structure. The more risk-averse an investor is, the more cash and bonds are found in his efficient portfolio. The riskier an investor gets the more he invests in risky assets, on the assumption of higher expected returns. It is, however, a pitfall to only invest in asset categories of the risk class that corresponds to ones risk affinity. Not only the risk/return-diagram but also the diversification among all these investment vehicles have to be considered. Instruments from the same asset classes (such as bond, stock, etc.) or within the same risk classes are not only similar in their risk/return-position but also in their dynamics, while instruments from different asset, respectively risk classes might have lower correlations, leading to higher potential of diversification. Recall, for example, the contagion and decoupling effects mentioned in section 2.1. Therefore, one way to control the risk of a portfolio includes to invest in a number of different types of asset classes and therefore in a number of different risk classes. The asset allocation is based on the fact that different asset categories produce positive returns in different ways and on different times.

The following illustrative example demonstrates the inefficiency that occurs by selecting a restrictive asset universe a priori. An investable universe of 16 assets was divided into three risk/return-classes (see table 7.8).

Investment strategy:		**balanced**			
Investable universe:		(All risk classes, 16 assets)			
moderate (low- & middle-risk classes, 13 assets)					
conservative (low-risk class)		**growth** (middle-risk class)		**risky** (high-risk class)	
<u>*5 assets*</u>		<u>*8 assets*</u>		<u>*3 assets*</u>	
Swiss Bond	CHb	Swiss Stock	CHs	Emerging Markets Stock	EMs
United Kingdom Bond	UKb	United Kingdom Stock	UKs	Commodity Index	Cmdt
Euro Bond	EUb	Euro Stock	EUs	Private Equity Index	PE
United States Bond	USb	United States Stock	USs		
Emerging Market Bond	EMb	Pacific Area Stock	PAs		
		Industr. Countries Stock	ICs		
		Gold	Gold		
		Hedge Fund Index	HF		

Table 7.8: Asset universe of 16 assets divided into three risk/return classes and two mixed strategies (moderate and balanced.

The efficient frontiers for each risk class (low, middle and high), representing a sub-investable universe were optimized. The target returns for each efficient frontier range from 2.8% (equals the riskfree rate) to the highest reachable target return possible, with steps of 10 base points (see figure 7.11). The comparison between the efficient frontiers of these risk classes shows that their efficient portfolios are only optimal within a certain risk range. To invest exclusively in assets out of the low-risk class is only efficient when the target return is $x^* \leq 4.10\%$. The efficient interval for middle-risk class portfolios ranges between the target returns $4.10\% < x^* \leq 7.50\%$. Investments with an expected return of $x^* > 7.50\%$ have the lowest risks by investing only in assets out of the high-risk class.

In the interval up to $x^* = 4.10\%$ the low-risk class investment dominates the two riskier classes. However, the efficient frontier of the low-risk class has a sharp bend at around $x^* = 4.00\%$. This is the point where the investor needs to invest more than 20% in foreign bond assets to reach further excess return. Such an investment consequently involves higher risk. Due to the fact that only the market risk but not the exchange rate risk is compensated, the additional return is relatively low compared to the extra risk, which leads to a sharp bend in the curve.

The inefficiency of the investment strategies that invest only in assets out of the investor's preferred risk class is best illustrated, by taking two further investment strategies into the analysis. The large solid line represents the efficient frontier of balanced investment strategy, allowing to invest in assets out of all three risk classes. The dotted line represents the efficient frontier of the moderate investment strategy, which results from investing in assets solely out of the low- and middle-risk classes. These investment strategies dominate the ones from a single risk class (conservative, growth and risky). After starting from the riskfree rate at $r_f = 2.82\%$ their efficient frontiers contain a steeper slope within the $\mu/\sigma-$ space before it converges from above to the efficient frontiers of the growth, respectively risky investment strategy.

The added value given through higher potential of diversification is also quantified in figure 7.12. The area plot represents the rough portfolio structure of the balanced investment strategy (left y-axis). The lines denote the risk reduction given by comparing each efficient frontier of the single risk classes (low, middle, high) with the balanced efficient frontier (right y-axis). These three lines emphasize the risk ranges found above: Within the range of $2.8\% \leq x^* \leq 4.10\%$ it is the conservative strategy that keeps best up with the balanced strategy. Up to the target return of $x^* = 5.70\%$ it is the growth strategy and beyond that the risky strategy that perform least poorly, compared to the balanced strategy.

While the potential of diversification is small at the tails of the balanced efficient frontier, it reaches its peak at the target return $x^* = 4.80\%$, with a reduction of $\sigma_{(diff.)} = 2.12\%$. In relative terms it is an impressive reduction of risk exposure of 41.25%. On the left tail all efficient frontiers start at the riskless rate, by definition. However, already little increases in the target returns show potential of diversification with the balanced investment strategy. The right tail illustrates that from a certain level of target return x^*, also the balanced investment strategy needs to invest almost inclusively in assets from the high-risk class, in order to reach the ambitious target returns. Inevitably it converges with the efficient frontier of the risky investment strategy, albeit from above. The huge potential for diversification lies in between of the tails. Already a small amount of around 20% of the portfolio invested in higher risk classes is sufficient to avoid the sharp bend of the conservative strategy. In other words, with a great amount of assets out of the low-risk class even high expected returns are reachable with a relative low-risk exposure. It is also interesting that efficient portfolios for low target returns of $x^* < 4\%$ invest around 5% of the principle in assets from the high-risk class, to use the potential of diversification.

From this discussion is to take that any a priori restriction like e.g. asset bounds leads to inefficiency. The restricted efficient frontier will always be dominated by the unrestricted one. The optimal course of action is to first calculate the unbounded efficient frontier. Then the investor will choose his optimal portfolio along the efficient

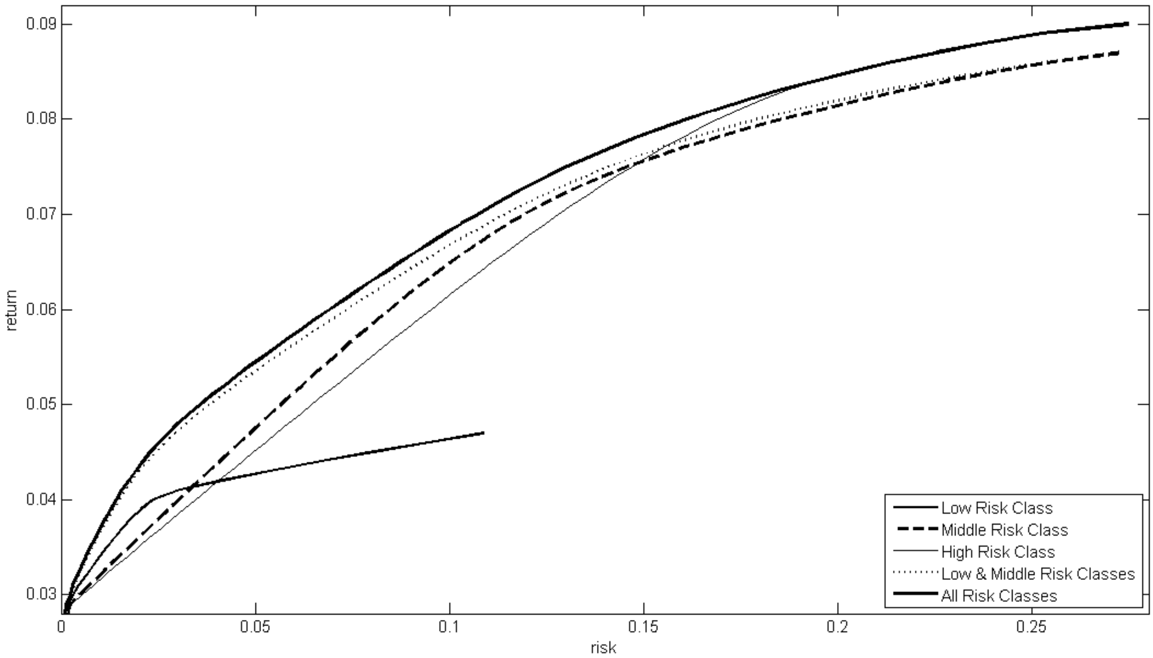

Figure 7.11: The efficient frontiers based on different risk classes respectively combinations of it. All optimizations are based on the calibration in Swiss francs introduced in Section 6.4, yet extended from 6 to 16 assets.

frontier, according to his risk-aversion, respectively risk-ability a posteriori. He will always do better than by choosing a restricted asset universe a priori.

Comparing the profit and loss distributions (P&L) reinforces this conclusion. Figure 7.13a) compares the P&L-distribution of the two efficient portfolios with the same risk exposure of $\sigma = 3.07\%$, taken from the conservative and the balanced efficient frontier. The conservative portfolio (C) has an expected return of $x^* = 4.10\%$, while the balanced portfolio (B) reveals an expected return of $x^* = 4.82\%$. The balanced P&L-distribution dominates the one of the conservative investment strategy with first-order stochastic dominance. Meaning that for any portfolio value y, the relation of the cdf's (F) equals: $F_C(y) \geq F_B(y)$. For any argument y, the probability of obtaining y or less than this value is always higher for the conservative portfolio than for the balanced portfolio. Which does not mean that the balanced strategy will always obtain a higher return, but the chance for higher returns is bigger than for the alternative strategy. In other words, the shortfall probability of any argument y, will always be smaller when investing in the balanced investment strategy than in the conservative one. Any investor who prefers more to less will always invest in the balanced portfolio.

Figure 7.13b) compares the P&L-distributions of the two efficient portfolios with the same expected return of $x^* = 4.10\%$, taken from the conservative and the balanced efficient frontier. The conservative portfolio has a risk of $\sigma = 3.07\%$, while the balanced

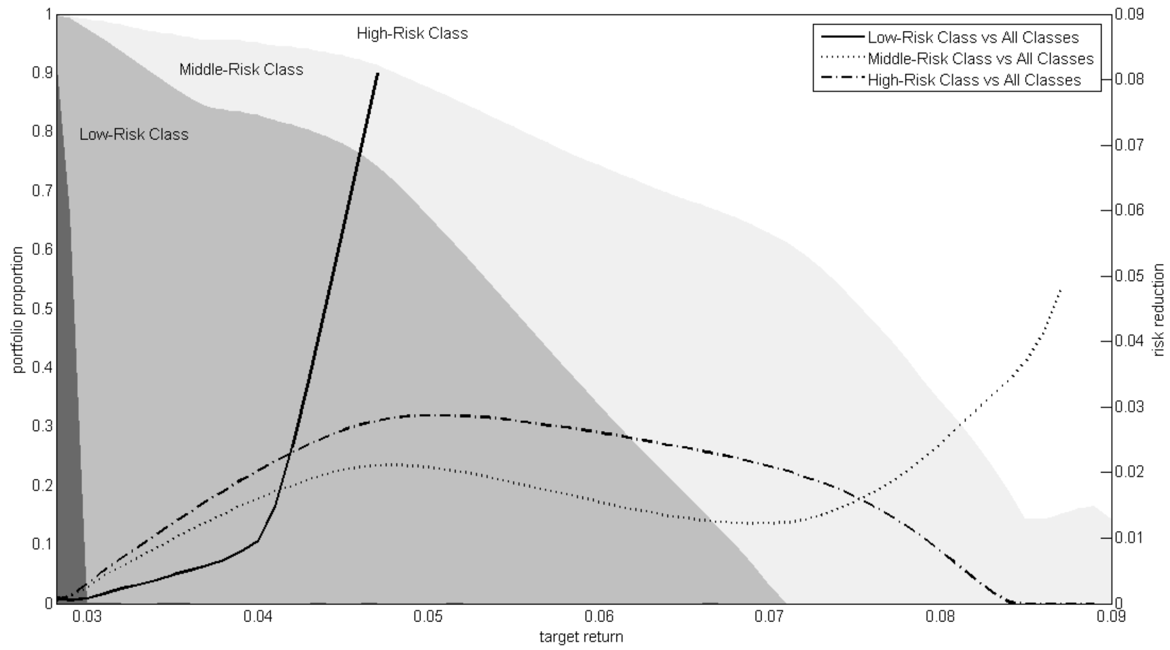

Figure 7.12: Rough portfolio structure of the balance strategy and the added value towards the single risk class strategies.

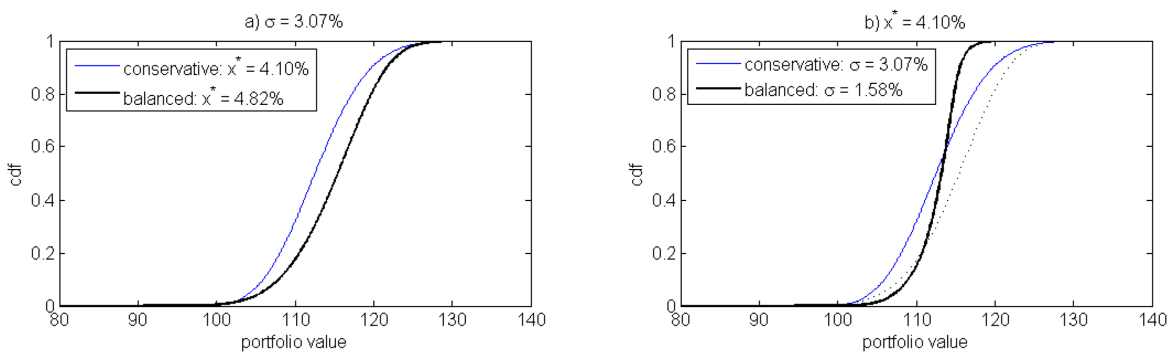

Figure 7.13: First- and second-order stochastic dominance, between the conservative and the balanced strategies.

portfolio reduced the risk exposure by diversification to $\sigma = 1.58\%$. Since these P&L-distributions cross each other at their expected return $x^* = 4.10\%$, the investor cannot select between B and C using first-order stochastic dominance. It depends whether he weights the possible higher losses under the target return of $x^* = 4.10\%$ as more important than the possible higher returns above x^*. A further assumption on the characteristics of the investor's utility function is needed. Under the further premise of risk aversion, in addition to preferring more to less, a rational decision is possible. A risk-averse investor has to be compensated for bearing risk. Therefore, when the expected returns are equal, a risk-averse investor will always decide in favor of the lower-risk strategy. Strategy B is said to dominate strategy C with second-order stochastic dominance, when following inequality applies to all x:

$$\int_{-\infty}^{x} F_C(y) \geq \int_{-\infty}^{x} F_B(y).$$

If an investor would like to take the risk by investing in C, in order to possibly participate on the higher returns above x^*, he would be better advised to invest in the dotted line, representing the efficient portfolio with $x^* = 4.80\%$ and $\sigma = 3.07\%$ on the balanced efficient frontier. Since it dominates C as seen in figure 7.13a). The comparison of the thick solid line and the dotted line reveals no stochastic dominance, since the corresponding portfolios lie on the same efficient frontier. Both are efficient, the decision depends on the investor's risk affinity. He has to determine the target return x^* according to his risk affinity, respectively risk ability.

Chapter 8

Conclusion

There is much evidence in financial literature that the dynamics of the financial markets show some established pattern. This paper documents how well the application of a regime-switching model captures these patterns. On the one hand it considers fat tails, which are essential for risk management. On the other hand it contains several features, like volatility-clustering, co-movement, contagion and decoupling, which are essential to adequately estimate the current potential of diversification. The increased flexibility of the parameters, given by the regime-switching characteristic, allows a representative modeling of the real financial market dynamics, since it avoids the dilution of different financial regimes, even in indisputable high-volatility market situation as in these days. The used starting value generator and the chosen boundaries have been proved to be suitable, in order to face the challenges of the non-convex likelihood function and in order to find satisfying local maxima.

It is also shown that the properties of the risk/return-structures - modeled by the regime-switching approach - serve as input data for a stochastic multistage optimization program. Hence, in the range of dynamic asset allocation, the regime-switching framework provides a suitable added value:

- The regime-switching model does not only provide the risk/return- structure of the current market situation, but - due to the Markov process - also a certain timing information to the dynamic asset allocation. The multistage approach of the dynamic asset allocation is able to optimally use this timing effect over the whole planning horizon, divided into three time periods.

- The explicit consideration of liability-dynamics in the asset allocation process has a crucial influence on the efficient frontier, as well as on the composition of the efficient portfolios and their shortfall probabilities. The consistent modeling and integration of the liabilities is therefore essential in an integrated portfolio management process. It better fulfills the task to secure the funding of the incurred liabilities.

- The explicit consideration of a benchmark in the asset allocation process has a crucial influence on the performance of benchmark tracking. The consistent modeling and integration of the benchmark is therefore essential in order to evaluate the asset allocation against the background of an index, representing the general market development. Further, the timing information of the Markov process turns out to be the key figure to outperform the general market indices.

- Despite the fact of globalization and of the increasing contagion effect, the diversification benefits of international investments still exist. The added value of international investments are quantified and suitable hedging strategies are determined.

Appendix A

A.1 Composition of the Pictet LPP-Indices

Asset Class	LPP-25	LPP-40	LPP-60
Cash	0.0%	0.0%	0.0%
Bonds	**75.0%**	**60.0%**	**40.0%**
CHF	60.0%	45.0%	25.0%
EUR	10.0%	10.0%	10.0%
World	5.0%	5.0%	5.0%
Equity	**25%**	**40.0%**	**60.0%**
Switzerland	10.0%	15.0%	20.0%
World	15.0%	25.0%	40.0%

Source: Pictet, www.pictet.com

Table A.1: Composition of the Pictet 2000 Index Family. The table shows the weightings of the three LPP 2000 indices (LPP: Law on occupational pension schemes).

Asset Class	LPP-25 plus	LPP-40 plus	LPP-60 plus
Cash	0.0%	0.0%	0.0%
Bonds	**65.0%**	**50.0%**	**30.0%**
CHF	40.0%	30.0%	15.0%
World	25.0%	20.0%	15.0%
Equity	**20%**	**30.0%**	**45.0%**
Switzerland	7.5%	10.0%	15.0%
World	12.5%	20.0%	30.0%
Real Estate	**10%**	**10.0%**	**10.0%**
Switzerland	7.5%	5.0%	2.5%
World	2.5%	5.0%	7.5%
Hedge Funds	**2.5%**	**5.0%**	**7.5%**
Private Equity	**2.5%**	**5.0%**	**7.5%**
Commodities	**0.0%**	**0.0%**	**0.0%**

Source: Pictet, www.pictet.com

Table A.2: Composition of the Pictet 2005 Index Family. The table shows the weightings of the three LPP 2005 indices (LPP: Law on occupational pension schemes).

A.2 Risk Structures of different Hedge Strategies

Foreign Assets Unhedged

R1	ann.σ	ρ	CHb	EUb	USb	CHs	EUs	USs
CHb	3.47%	CHb	1	0.41	0.11	*-0.13*	*-0.19*	*-0.21*
EUb	5.58%	EUb		1	0.29	*0.26*	*0.33*	*0.25*
USb	11.67%	USb			1	*0.38*	*0.42*	*0.58*
CHs	24.60%	CHs				1	**0.93**	**0.87**
EUs	28.80%	EUs					1	**0.92**
USs	31.75%	USs						1

R2	ann.σ	ρ	CHb	EUb	USb	CHs	EUs	USs
CHb	3.47%	CHb	1	0.41	0.11	*-0.13*	*-0.19*	*-0.21*
EUb	5.58%	EUb		1	0.29	*0.19*	*0.36*	*0.22*
USb	11.67%	USb			1	*0.27*	*0.43*	*0.71*
CHs	14.44%	CHs				1	**0.75**	**0.56**
EUs	14.85%	EUs					1	**0.70**
USs	17.34%	USs						1

Foreign Bonds Hedged (*)

R1	ann.σ	ρ	CHb	EUb*	USb*	CHs	EUs	USs
CHb	3.47%	CHb	1	0.66	0.44	*-0.13*	*-0.19*	*-0.21*
EUb*	3.75%	EUb*		1	0.61	*-0.26*	*-0.27*	*-0.31*
USb*	5.58%	USb*			1	*-0.36*	*-0.39*	*-0.37*
CHs	24.59%	CHs				1	**0.93**	**0.87**
EUs	28.79%	EUs					1	**0.92**
USs	31.74%	USs						1

R2	ann.σ	ρ	CHb	EUb*	USb*	CHs	EUs	USs
CHb	3.47%	CHb	1	0.66	0.44	*0.21*	*0.10*	*0.03*
EUb*	3.75%	EUb*		1	0.61	*0.15*	*0.12*	*-0.02*
USb*	5.58%	USb*			1	*0.16*	*0.08*	*0.06*
CHs	14.44%	CHs				1	**0.75**	**0.56**
EUs	14.85%	EUs					1	**0.70**
USs	17.34%	USs						1

Foreign Assets Hedged (*)

R1	ann.σ	ρ	CHb	EUb*	USb*	CHs	EUs*	USs*
CHb	3.47%	CHb	1	0.66	0.44	*-0.13*	*-0.20*	*-0.22*
EUb*	3.75%	EUb*		1	0.61	*-0.26*	*-0.28*	*-0.30*
USb*	5.58%	USb*			1	*-0.36*	*-0.41*	*-0.35*
CHs	24.59%	CHs				1	**0.92**	**0.87**
EUs*	25.59%	EUs*					1	**0.89**
USs*	24.28%	USs*						1

R2	ann.σ	ρ	CHb	EUb*	USb*	CHs	EUs*	USs*
CHb	3.47%	CHb	1	0.66	0.44	*0.21*	*0.13*	*0.13*
EUb*	3.75%	EUb*		1	0.61	*0.15*	*0.14*	*0.19*
USb*	5.58%	USb*			1	*0.16*	*0.13*	*0.33*
CHs	14.44%	CHs				1	**0.76**	**0.58**
EUs*	13.73%	EUs*					1	**0.61**
USs*	12.83%	USs*						1

Table A.3: Risk structure of different hedge strategies. The bond securities correspond to 3-5 year government bonds (e.g. EUb = Euro bond). The stocks securities are MSCI Indices.

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