Dispersion Trading in South Africa*

An analysis of profitability and a strategy comparison

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Abstract

A dispersion trade is entered into when a trader believes that the constituents of an index will be more volatile than the index itself. The South African derivatives market is fairly advanced, however it still experiences inefficiencies and dispersion trades have been known to perform well in inefficient markets. This paper tests the South African market for dispersion opportunities and explores various methods of executing these trades. The South African market shows positive results for dispersion trading; namely short-term reverse dispersion trading. Call options and Cross-Sectional Volatility (CSV) swaps are also tested. CSV swaps performed poorly whereas call options experienced annual returns well above the market.

Keywords: dispersion trading, volatility arbitrage, cross-sectional volatility, reverse dispersion trade.

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1 Introduction

Dispersion amongst shares can be understood as the share prices not moving in the same direction. If the constituents of an index are highly dispersed, then they will vary a lot individually during the period of the trade. However, the average of all of these movements (that is, the index) will change very little. In statistics, dispersion of a variable measures the spread or variability of the variable (Underhill and Bradfield, 2008).

The most traditional dispersion trade is when vanilla call options are used to trade both the index and its constituents. The index options are shorted and options on the constituents of the index are longed in order to balance out the trade. In a reverse trade the index options are longed and options on the constituents are shorted. Both traditional and reverse trades will be considered in this paper.

When stocks are highly dispersed, the traditional trade for example, will make a profit when the short call on the index expires out-the-money and some of the long calls on the individual stocks expire in-the-money, regardless of which stocks performed well and which performed poorly.

The South African market is interesting because the JSE is an advanced stock exchange, whereas the economic environment is one of an emerging country and should therefore contain informational inefficiencies. This allows an investor to trade options that offer direct exposure to dispersion, in a market that may offer superior returns in dispersion trading.

This paper will test whether or not there are dispersion trading opportunities in the South African market. Tests conducted by Marshall (2009), and Lozovaia and Hizhniakova (2005) will be run on historical data from the South African market for the period 16 May 2006 to 16 May 2012.

If it is established that there are dispersion trading opportunities (traditional or reverse), specific trading rules will be tested under various market conditions. Trades using both call options and swaps on Cross-Sectional Volatility will be tested, as well as different bid-ask spreads.[1]

The remainder of this paper is organised as follows. Section 2 introduces the methods that will be used to test the market for opportunities. Section 3 describes the data collected as well as the methodology employed and the results of running these tests. Section 4 then explores the two vehicles that will be used when conducting dispersion trades. Section 5 provides the method used when backtesting the trades, a sensitivity analysis and a discussion of the results obtained. The paper is concluded in Section 6.

2 Implied, Historic and Theoretical Measures

This section reviews the previous literature, with a focus on identifying good market conditions for dispersion trading. A measure of volatility is taken from Marshall (2009) and a measure of correlation, as well as the "three coefficients of volatility", is taken from Lozovaia and Hizhniakova (2005).

2.1 Volatility Measures

Marshall (2009) conducted an analysis on the S&P 500 as to whether dispersion trading was possible in US markets during the period 31 October 2005 to 1 November 2007. It was discovered that, throughout a 505-day period, there were 84 days on which profitable trading opportunities arose.

Dispersion trading opportunities arise when the day-end index-option-implied volatility (IOIV) exceeds the "Markowitz-implied volatility" (MIV) (Marshall, 2009). IOIV is calculated by taking the market data for calls and

[1]Cross-Sectional Volatility is defined in Section 4.2.

puts on the index, inverting the Black-Scholes option pricing formula, and solving for volatility (Black and Scholes, 1973). Vonhoff (2006) has shown that implied volatility is a function of K, the strike price of the option, and T, the time of maturity of the option.

An index option is a basket option by definition, and thus the theoretical volatility of the index should be a weighted average of the volatility of its components. Equation (1) allows us to calculate the variance of a portfolio's returns (σ_p^2), as shown by Markowitz (1952):

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{i,j} \tag{1}$$

where n is the number of components in the portfolio, w_i is the weight of stock i within the portfolio, and $\sigma_{i,j}$ is the covariance of the returns of stock i with the returns of stock j. Calculating this theoretical portfolio variance with *implied* single stock volatilities, *historic* return correlations and the index weights produces the MIV (Marshall, 2009).

Lozovaia and Hizhniakova (2005) take a similar approach in order to ascertain when a dispersion trade is possible, by expanding Equation (1) as follows:

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j>i} w_i w_j \sigma_i \sigma_j \rho_{i,j} \tag{2}$$

where σ_i is the volatility of stock i, $\rho_{i,j}$ is the correlation of returns between stock i and stock j, and the remaining notation is as defined above. The next four measures are based on Equation (2). The equation is used as is, but the values for the different σ's change.

Lozovaia and Hizhniakova (2005) define three different types of values: *implied*, *theoretical* and *realised*. *Implied* values are values that are not directly observable in the market, and are found using current market data and a corresponding formula. *Realised* values are values computed directly from past data, such as the 30-day moving average volatility on a stock.[2] Alternatively, a value that is calculated based on a theory is called a *theoretical* value (Lozovaia and Hizhniakova, 2005). The MIV, as discussed above, is an example of a *theoretical* value; whereas the IOIV is an *implied* value, which is implied directly from current market data.

2.2 Correlation Measures

Linear correlation between two stocks is a measure of how much their returns move together. In statistics, correlation is a measure of how closely related two sets of data are, that is, how much of the movements in set B are explained by movements in set A (Underhill and Bradfield, 2008). A high correlation between stock i and stock j, or between stock i and the index is bad for a dispersion trade.

There is an inverse relationship between dispersion and correlation. A dispersion trader would want the stocks to disperse and have large up- or downward movements: these large movements will generate profits through the long options held in them. If there is a high correlation amongst the stocks, profits will be generated when stock prices rise. However, the index will move in the same direction as all of the stocks and, because a traditional dispersion trade is short index options, it will make losses on these options.

If there is little correlation amongst the stocks, then the options on outperforming stocks will make leveraged profits, and the options on stocks which perform poorly will expire valueless. The index, being the average of up and down movements, will change little relative to the stocks, and thus a small loss or gain will be experienced on this side of the trade. Therefore,

[2] *Realised* values are also called *historic* values. We will refer to these values as *historic* in this paper.

less correlation is preferred.

Implied Index Correlation (IIC), as defined by Lozovaia and Hizhniakova (2005), is a measure of the correlation between the index and the constituents' *implied* volatilities. It is calculated by rearranging Equation (2) and substituting the *implied* volatility values for the σ's, resulting in:

$$\text{IIC} \;=\frac{\sigma_p^2 - \sum_{i=1}^{n} w_i^2\sigma_i^2}{2\sum_{i=1}^{n}\sum_{j>i} w_i w_j \sigma_i \sigma_j}. \tag{3}$$

High IIC indicates a better situation for traditional dispersion trading (Ganatra, 2004). This is because IIC is a measure of the correlation of the implied volatilities, not the share returns. A high IIC indicates that index options are trading rich relative to constituent options (Ganatra, 2004). Thus, one would want to sell index options; that is, enter a traditional dispersion trade. It is, however, best to compare IIC to a historic value. The Historic Index Correlation (HIC) can be computed from the same equation but using the *historic* volatility values for the σ's (Lozovaia and Hizhniakova, 2005).[3]

2.3 Volatility Coefficients

Equation (2) can be used with either *historic* or *implied* data on the right-hand side, each giving different results. This gives rise to *theoretical* volatility values for the index. Lozovaia and Hizhniakova (2005) have developed three volatility coefficients, which are indicators of a good time to enter into a dispersion strategy. Table 1 provides a summary of these coefficients.

The first volatility coefficient (CF1) is the ratio of the "weighted components implied volatility" to the implied volatility of the index (Lozovaia and Hizhniakova, 2005:7). The weighted components implied volatility is a weighted sum of all of the components' *implied* volatilities, that is, $\sum_{i=1}^{n} w_i\sigma_i$.

[3]HIC is simply the historic weighted average pairwise correlation (Ganatra, 2004).

Table 1: Summary of the coefficients of volatility.

	Equation[a]	Data used	If relatively low
CF1	$\frac{\sum_{i=1}^{n} w_i \sigma_i^I}{\sigma_m^I}$	Implied	Traditional trading is favoured
CF2	$\frac{\sqrt{\sum_{i=1}^{n} w_i^2 (\sigma_i^I)^2 + 2\sum_{i=1}^{n}\sum_{j>i} w_i w_j \sigma_i^I \sigma_j^I \rho_{i,j}^I}}{\sigma_m^I}$	Implied	Index options are over-priced
CF3	$\frac{\sqrt{\sum_{i=1}^{n} w_i^2 (\sigma_i^H)^2 + 2\sum_{i=1}^{n}\sum_{j>i} w_i w_j \sigma_i^H \sigma_j^H \rho_{i,j}^H}}{\sigma_m^I}$	Historic[b]	Index options are over-performing

[a] Let σ_m^I denote the index *implied* volatility, σ_i^I denote the *implied* volatility of stock i, σ_i^H denote the *historic* volatility of stock i, $\rho_{i,j}^I$ denote the correlation between the *implied* volatility of stock i and stock j and $\rho_{i,j}^H$ denote the correlation between the *historic prices* of stock i and stock j.

[b] CF3 requires historic price correlation, as per Lozovaia and Hizhniakova (2005)

There is an inverse relationship between CF1 and the IIC.[4] We know that a high IIC is good for dispersion, thus a low weighted components implied volatility is also good for dispersion. Hence, a CF1 value close to 1, or relatively low (compared to its historic value), is an indicator of good market conditions for dispersion trading.

The second coefficient of volatility (CF2) is the ratio of "theoretical correlated implied volatility" of the index, to the actual implied volatility of the index (Lozovaia and Hizhniakova, 2005:9). Theoretical correlated implied volatility is calculated using Equation (2), except all of the volatilities used are *implied* volatilities and the correlation is between the *implied* volatilities of the two stocks in question. CF2 is a measure of how theoretical prices differ from the actual index option prices.

If CF2 is less than 1 and relatively low, then theoretical prices are lower

[4] See Appendix A for details.

than actual prices: in other words, the options are over-priced and it is a good time to sell index options. Lozovaia and Hizhniakova (2005) have noted that, in practice, CF2 is a good measure for a Vega trade because it is based on correlations amongst implied volatilities.[5] It performs better when used as an indicator for longer-termed trades, which have a high Vega as their main exposure, as well as a relatively low Gamma exposure.[6]

The third coefficient of volatility (CF3) is similar to CF2, except it is the ratio of "theoretical historic volatility" and actual implied volatility of the index (Lozovaia and Hizhniakova, 2005:10). Theoretical historic volatility is calculated from Equation (2), except all of the volatilities used are *historic* volatilities and the correlation is between the *historic prices* of the two stocks in question.[7] CF3 indicates to what extent the index option's historic volatility differs from its actual implied volatility.

A CF3 that is less than 1 and relatively low, indicates that index options are over-performing based on historic values. This indicates a good time to sell index options as they are trading relatively rich. Lozovaia and Hizhniakova (2005) found that, since CF3 is based on historic values and historic price correlation, it performs better as a measure for Gamma trades.[8] A shorter-term trade is exposed mainly to Gamma risk, thus CF3 is better for

[5]Vega is the sensitivity of the option's price to its volatility.

[6]Gamma is the sensitivity of the option's price to second order changes in the underlying's price.

[7]Unlike most correlation calculations, which are done on log returns, CF3 requires correlation between historic stock prices to be calculated (Lozovaia and Hizhniakova, 2005). Stock prices are normally more correlated than returns because most stock prices trend upwards. Thus, CF3 may be biased upwards (Lozovaia and Hizhniakova, 2005).

[8]The benefits from using historic price correlations (namely the relationship with Gamma) outweigh the upwards bias from historic price correlations (Lozovaia and Hizhniakova, 2005).

shorter-termed trades.

3 Assessing the South African Market

This section explains the data and methodology employed when looking for dispersion trading opportunities in the South African market. The results of these tests are then presented.

3.1 Data Collection and Cleaning

The FTSE/JSE Top 40 index (TOPI) was chosen as the subject of this study. There were 68 stocks in the index over the 6 year study period (16 May 2006 to 16 May 2012), which consists of 1502 trading days. Daily closing prices, dividend yields and SAFEX futures prices for each stock and the index were extracted from Bloomberg for the period.[9]

Historic volatility and correlation was calculated on a 91-day rolling period of daily returns. This results in a loss of data: there are therefore only 1412 days for which dispersion measures can be calculated.

The daily TOPI weights and weekly volatility skew data were supplied by Peregrine Securities. Weightings data were missing for 10 days: the weights were assumed constant for these days. The volatility skew was assumed to remain constant during each week.

At-the-money SAFEX futures were used to extract the implied volatility of the index from the volatility skew data. When time to expiry was below 1 month, futures expiring in 4 months were chosen. This resulted in futures expiring in 1 to 4 months being selected. This excludes contracts expiring in

[9] There was an anomaly in the dividend yield data for stock AGL: Datastream dividend yield data were substituted for this stock.

less than a month, thus avoiding expiry and delivery of a call option.

The constituents implied volatilities needed to be estimated as the Single Stock Futures (SSF) volatility skew was unobtainable. Two estimation methods were used, namely IVolM1 and IVolM2, as defined by:

$$\sigma_{Implied} = \begin{cases} \sigma_{Hist} + q, & \text{under IVolM1,} \\ \sigma_{Hist} \times \frac{\sigma_{IndexImp}}{\sigma_{IndexHist}} \times (1+q), & \text{under IVolM2,} \end{cases}$$

where q ranges from -5 per cent to 10 per cent.

The IVolM1 was chosen as it is tractable and allows one to explore the factors that influence the results, without adding the further complication of changing implied volatilities. IVolM2 was used to generate more realistic results.[10] The SSF skew was then found by linearly shifting the index skew to the SSF at-the-money volatility, assuming both skews have the same shape.

When exploring the following results, it is important to note that q is a flat premium (based on the summary statistics of the index implied volatility premium) when used in IVolM1, but in IVolM2, q is the additional premium over and above the index premium for that day. The mean index premium for the period was 2.61 per cent with a standard deviation of 5.1 per cent. IVolM1 generates a mean SSF at-the-money volatility (across all stocks and days) of 33.45 per cent (q=0), whereas IVolM2 generates 36.96 per cent.[11]

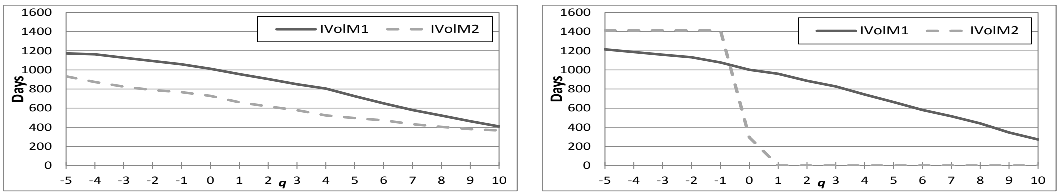
3.2 Volatility and Correlation

Marshall (2009) used the "modified Markowitz equation", which avoids calculating all 124 750 constituent correlations for each day.[12] The TOPI has

[10]It is important to note that the premium applied to SSF historic volatility under IVolM2 varies with time as the premium on the index implied volatility changes daily.

[11]See Appendix B for further details.

[12]See Marshall (2009) for further details.



(a) IOIV>MIV

(b) IIC>HIC

Figure 1: The number of days, out of 1412, that IOIV exceeds MIV and IIC exceeds HIC under IVolM1 and IVolM2.

a maximum of 42 constituents at any given time, which made it feasible to calculate a time-series including all 861 correlations each day. Therefore, the following more accurate formula for theoretical volatility was used:

$$\sigma^2_{MIV} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_i \sigma_j \rho_{i,j}.$$

It was hypothesised that MIV is cheaper than IOIV, and thus shorting the index and longing the constituents would result in profits. However, when comparing the two data sets for South Africa, results show that the opposite is true. Under IVolM1, IOIV was greater than MIV on 726 of 1412 days, when q is set to 5 per cent, as seen in Figure 1(a). The mean difference between IOIV and MIV was -1.17 per cent. There were periods during the 2008 recession when this difference was larger than -10 per cent, as can be seen in Figure 2.[13]

In the US, there were 315 of 505 days where IOIV exceeded MIV with a mean of 1.21 per cent (Marshall, 2009). This is opposite to the South African results, where a negative mean difference was recorded. When comparing the number of days that IOIV exceeded MIV, South African results are similar to the US results, but as q increases there is a noted decrease in this measure. This inverse relationship between IOIV$-$MIV and q holds true under IVolM2

[13] A larger version of Figure 2 is available in Appendix C.

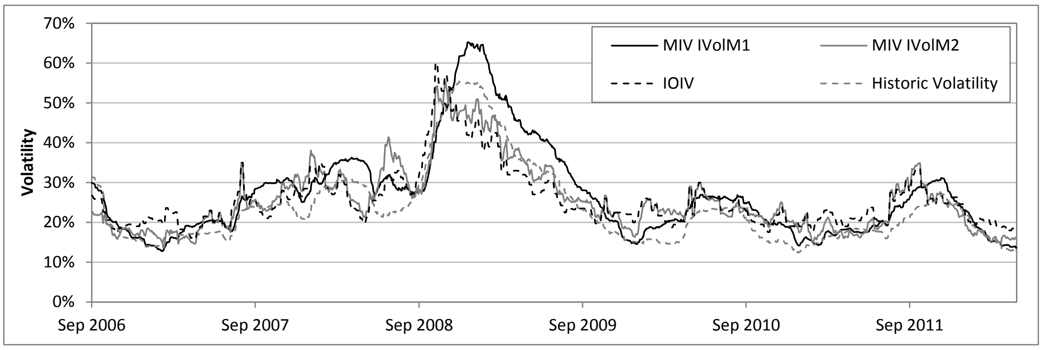


Figure 2: The MIV, under IVolM1 (q=5) and IVolM2 (q=0); IOIV; and historic index volatility over time.

as well. This is due to MIV having a linear relationship with q, and IOIV being independent of q.

Figure 2 shows that there is no systematic difference between the periods before and after the 2008 market crash. All three measures seem to track each other relatively well and are mostly above historic volatility. However, during the recovery period (December 2008 to December 2009), MIV (under IVolM2) and IOIV are below historic volatility. Under IVolM1, MIV is significantly above IOIV for this period; this implies that the market was unfavourable towards dispersion trades.

There are some dispersion trading opportunities when q is small, however, it is more likely that q is larger than 0 under IVolM1. Figure 1(a) shows that, for $q > 4$ per cent, the number of days that IOIV is greater than MIV is less than half the number of days under consideration and drops as q gets large. This is indicative that reverse dispersion trades may be more profitable than traditional dispersion trades in the South African market.

The correlation measures present similar results to the variance measures. Under both IVolM1 and IVolM2, IIC and q have an inverse relationship. As q increases, IIC falls, and thus the number of days that IIC is greater than HIC (which is independent of q) also falls, as seen in Figure 1(b). Under

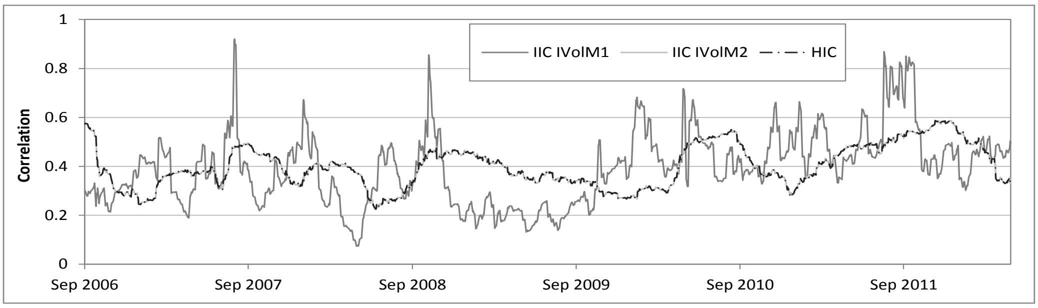


Figure 3: The IIC, under IVolM1 (q=5) and IVolM2 (q=0), and the HIC.

IVolM1, IIC was greater than HIC for less than half the number of days in the period, when q was set to 5 per cent.[14] Under IVolM2, with q at 1 or more per cent, IIC does not exceed HIC.[15]

The number of dispersion trading opportunities drop off as q increases. HIC is higher than IIC for most cases: this is indicative that reverse dispersion trades will be more profitable than traditional dispersion trades in the South African market. This is due to reverse dispersion trades making profits when *historic* correlation is above *implied* correlation.

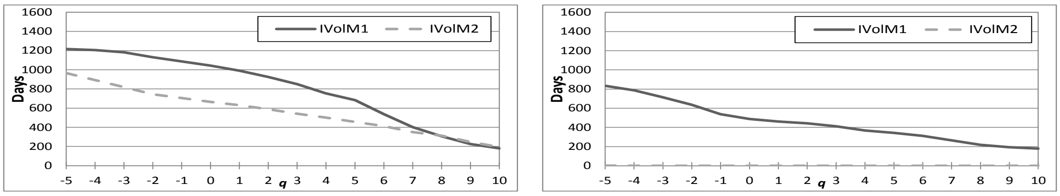
The correlation of the market pre- and post-crash is similar, as seen in Figure 3. However, during the recovery period, IIC (under IVolM1) was well below HIC, indicating poor market conditions for dispersion trades.

3.3 Coefficients of Volatility

CF1 is related to IIC, so we expect to see similar dispersion trading opportunities to those we saw with IIC. CF1 increases as q is increased; this results in decreasing trading opportunities, as seen in Figure 4(a). CF1 in South

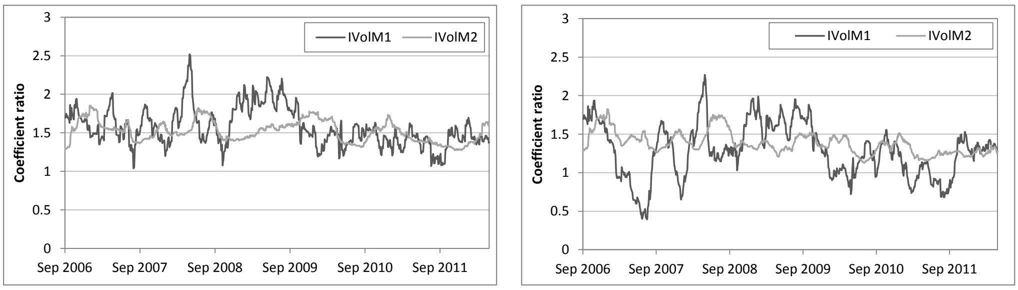
[14]More results are available in Table 6 in Appendix C.

[15]This is because with q at 0 per cent (under IVolM2), IIC almost perfectly tracks HIC as seen in Figure 3 and increasing q has the effect of linearly shifting IIC downwards, thus eliminating trading opportunities.



(a) CF1<1.5 (b) CF2<1

Figure 4: The number of days, out of 1412, that specific dispersion trading measures were reached under IVolM1 and IVolM2.



(a) CF1 (b) CF2

Figure 5: CF1 and CF2 over time, under IVolM1 (q=5) and IVolM2 (q=0).

Africa is relatively high: under IVolM1 the mean CF1 across the 1412 days was 1.57 (q=5). Under IVolM2, the mean CF1 was 1.59 (q=5). Figure 4(a) shows that for $q > 5$ per cent, reverse dispersion trading opportunities start to become more prevalent than traditional dispersion trading opportunities.

There is no significant difference between the levels of CF1 and CF2 pre- and post-crash. However, there does appear to be a slight downward trend after the recovery period in Figure 5, suggesting that the market may be becoming more favourable towards dispersion trading.

Figure 4(b) shows that there are very few dispersion trading opportunities when using CF2 as a proxy (CF2 is never less than 1 under IVolM2). Thus, CF2 yields unsatisfactory results for dispersion trading opportunities. CF2 is an indicator of long-termed dispersion trading opportunities, as explained in Section 2.3. Therefore, the South African market is not a good market

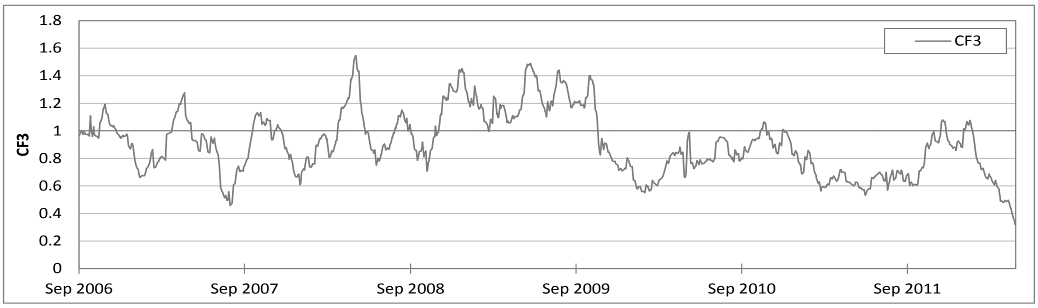


Figure 6: The progression of CF3 over time.

for longer-termed dispersion trades. CF2 being relatively high indicates that index options are relatively under-priced, or that single stock options are relatively over-priced. This is most likely due to the fact that single stock options are thinly traded in South Africa, causing them to be priced higher. It is hypothesised that index options are trading close to their true price, and that this apparent relative mispricing is due to the much higher prices of single stock options.

CF3 is independent of both q and the single stock implied volatility, as it uses purely historical values. CF3 was below 1 on 973 of the 1412 days within the study period, as can be seen in Figure 6. The average CF3 over the period was 0.91, with a standard deviation of 0.23. This indicates that the market displayed good conditions for short-term dispersion trading over the period. It is clear that there were more trading opportunities (CF3 < 1) in the last 3 years than in the first 3 years. This period of good short-term dispersion trading conditions began as the recovery period ended (that is, once volatilities had returned to a normal level).

Dispersion trading is therefore not advised during and immediately after a market crash. However, once the dispersion measures have returned to their pre-crash levels, the market is favourable for dispersion trading once again. After the 2008 market crash (and its recovery period), there were more dispersion trading opportunities in South Africa than before.

4 Dispersion Vehicles

Different vehicles can be used when constructing dispersion trades. This section introduces the two vehicles considered in this paper: call options and Cross-Sectional Volatility (CSV) swaps.

4.1 Vanilla Call Options

When using call options, Vonhoff (2006) states that the options used on both sides of the portfolio should have the same time to maturity and should be trading at-the-money. At-the-money options are more liquid and have an approximate Delta of 0.5, irrespective of the underlying, leading to an overall portfolio Delta that is relatively low (Nelken, 2006).[16]

A suitable weighting strategy for the representative set needs to be chosen before a dispersion trade can be conducted.[17] The simplest strategy is an index-weighted one, where the weights of the positions taken in the options on the constituents are in proportion to the weights of the constituents in the index (Lisauskas, 2010). This is simple and crude, but useful when conducting a theoretical analysis, due to a high level of tractability.

4.2 Cross-Sectional Volatility

Senechal (2004:1) describes CSV as a measure of "the dispersion of stock returns at one point in time". As CSV increases from zero, stocks begin to

[16]Delta is the sensitivity of the option's price to changes in the underlying's price.

[17]A representative set of stocks is chosen such that it adequately describes the index and has a relatively low implied volatility (Vonhoff, 2006).

disperse from their respective means (Ankrim and Ding, 2002).

CSV as a vehicle for a dispersion trade is a relatively new idea. However, Morgan Stanley (2010:1) have introduced the idea of a "Lock-In Dispersion Swap" (LIDS), which is a swap on the highest dispersion value of the period. A LIDS is similar to a CSV swap, and therefore, the following profit and loss function for a CSV swap can be derived:

$$\text{P\&L}_{CSV} = \max_{t=1...T} \left(\sqrt{\sum_{i}^{n} w_{i,t}(r_{i,t} - \bar{r}_t)^2} \right) - K \tag{4}$$

where T is the time of maturity in days, $r_{i,t}$ is the return on stock i at time t, $\bar{r}_t$ is the arithmetic average return of the n stocks at time t, and K is the strike price of the swap.[18]

5 Conducting Dispersion Trades

This section describes the methodology used when backtesting dispersion trades and presents the results in the form of a sensitivity analysis.

5.1 Trading Rules and Method

Table 2 describes the 16 trading rules. There are 8 trading rules for traditional dispersion trades. Of these 8 there are 4 simple rules, and 4 stricter versions of the simple rules. These 8 rules are then inverted to give 8 rules for reverse dispersion trades. CF2 was not used, because longer-termed dispersion trades were deemed less profitable in Section 3.3.[19]

[18] See Appendix D for the derivation.

[19] This rule was tested once and did not yield above-average returns. Due to the numerous covariance computations required to calculate CF2 for every q, no further investigation into this measure was conducted.

Table 2: The 16 dispersion trading rules.

Traditional Rules		Reverse Rules	
Simple	Stricter	Simple	Stricter
IIC > HIC	IIC > 1.1(HIC)	IIC < HIC	1.1(IIC) < HIC
CF1 < 1	CF1 < 0.9	CF1 > 1	CF1 > 1.1
CF3 < $\overline{\text{CF3}}$ [a]	CF3 < 0.9($\overline{\text{CF3}}$)	CF3 > $\overline{\text{CF3}}$	CF3 > 1.1($\overline{\text{CF3}}$)
IOIV > MIV	IOIV > 1.1(MIV)	IOIV < MIV	IOIV < 0.9(MIV)

[a] $\overline{\text{CF3}}$ is the historic arithmetic average of the CF3.

Stricter versions of these rules were introduced in an attempt to stop trading on dispersion trades that have just become profitable, but may move back to the unprofitable region. These stricter rules should take the volatile nature of the measures of dispersion into account, and may result in superior profits.

Each dispersion trading rule was tested both with a portfolio that had an initial value of R1 000 000, and with each trade having a value of R1 000 000. The second test involved running trades of the same value each time, and recording the total mean profit for each trade conducted over the period in question.

The trading rules were first run without any trading costs nor a bid-ask spread. Various bid-ask spreads were then added and the tests were rerun. Finally trading costs, in the form of a further 2 per cent, were added to the largest bid-ask spread. This was to account for a worst-case trading cost and bid-ask spread scenario.

5.2 Call options

The call options offered in South Africa are on futures on the underlying, not on the underlying itself (West, 2011). Therefore, a modified version of the Black-Scholes formula is used to value the options; the exact formula used was taken from West (2011:70). Each trade is held for 21 working days.

5.2.1 Weighting Methods

Two weighting methods were tested.[20] The first, simpler method involved selecting all the stocks in the index with a weighting larger than 2 per cent. The second method sorted all of the stocks by their weight in the index, and chose stocks until the total selected weight was greater than 75 per cent.[21]

Once the constituents to be traded had been chosen through a weighting method, the futures data for each stock was considered. If a futures contract for a stock could not be found, that stock was then removed from the trade. This resulted in an overall average of 37.8 stocks being used in a trade, if the entire constituent list was chosen. Once the list of stocks had been finalised, it was arithmetically reweighted to a total of 1.

Both weighting methods had a higher return than if all of the constituents were chosen. This confirms the assumption that selecting a representative stock set is important. The more complex method performed the best, using the least number of stocks on average (11.22), which accounted for 71.05 per cent of the index weight. The mean annual return across all trading rules was 39.29 per cent.[22]

[20]There was no bid-ask spread and IVolM1 was used for these tests.

[21]Less stringent versions of each rule (allowing more stocks to be chosen) performed worse and were therefore excluded from analysis.

[22]All returns are Nominal Annual rates Compounded Continuously (NACC), unless stated otherwise.

Table 3: Annual percentage portfolio and trade returns, (standard deviation) and [average trades per year] under traditional and reverse trading rules.

	Traditional	Reverse
All trading rules:		
Portfolio return	41.52 (45.38) [5]	28.08 (24.85) [6.88]
Trade returns annualised	66.55 (47.44) [5]	51.58 (15.98) [6.88]
Specific trading rules:		
CF1 $<$ 1 trade return	113.31 (99.8) [2.07]	-
1.1(IIC) $<$ HIC trade return	-	84.95 (13.84) [4.93]

The simpler method performed similarly, with a return of 38.71 per cent (and using 11.43 stocks on average). This method was used for the other tests below, as it has similar results to the best method, and is more tractable.[23]

5.2.2 Trading Rules

Table 3 shows that, although reverse dispersion trades underperformed traditional trades, they were less volatile over the period.[24] This difference is more pronounced when looking at the annualised trade returns. It should also be noted that there were more reverse trades than traditional trades per year; this is because the South African market favours reverse trades.

In all but one case (CF1 $<$ 1), the stricter rules had a higher mean profit per trade, and traded less often. The stricter rules also had a lower standard deviation on average. The most profitable rule was CF1 $<$ 1, a traditional trading rule that only traded 2.07 times a year, as shown in Table 3.

The third most profitable rule (and most profitable reverse trading rule)

[23] An extract of the trade results is available in Appendix E.

[24] The 16 trading rules were tested using the simple ($w_i > 0.02$) weighting rule, without a bid-ask spread and using IVolM1.

was 1.1(IIC) $<$ HIC, which had a standard deviation 6.1 times smaller than its mean profit per trade. Thus, it takes an extreme deviation from the mean to result in losses when using this trading rule. This rule has a 99 per cent confidence interval of $68.26 \leq \mu \leq 103.15$ per cent around the (annualised) mean return per trade. Thus, the rule 1.1(IIC) $<$ HIC is chosen to be the safest and most profitable rule.

5.2.3 Implied Volatility Estimation Methods and Bid-Ask Spreads

IVolM2 performed poorly and had a return well below that of IVolM1.[25] IVolM1 and IVolM2 had mean returns (across all trading rules) of 38.71 and 6.90 per cent respectively. When looking at reverse dispersion trades only, the methods performed in a similar manner, with returns of 32.09 and 12.72 per cent respectively.

Three different levels of bid-ask spreads were applied. A 2 per cent spread was applied to the index options: this is in line with what is experienced in the market. The spread applied to the single stock options was varied between 2, 5 and 10 per cent.

Table 4 shows the annual returns generated when bid-ask spreads were added. Both methods are affected similarly by the addition of bid-ask spreads. As one would expect, increasing the bid-ask spread reduced returns. IVolM2 had negative returns when high bid-ask spreads were introduced. If this estimation method most accurately describes the South African market, this could mean that dispersion trades are unprofitable in South Africa due to high bid-ask spreads on single stock options.

To simulate trading costs, the trading rules were run at a bid-ask spread of 4:12. Table 4 includes these results for comparison to other bid-ask spreads.

[25] The simple ($w_i > 0.02$) weighting rule was used along with no bid-ask spread.

Table 4: Mean NACC returns (and standard deviation) in per cent, across the trading rules, generated under IVolM1 and IVolM2 when different bid-ask spreads were applied.

	All trading rules				Reverse rules only			
Spread	2:2[a]	2:5	2:10	4:12	2:2	2:5	2:10	4:12
IVolM1	28.39	21.92	12.93	5.55	25.05	21.31	14.41	7.68
	(39.93)	(28.80)	(16.06)	(8.75)	(25.81)	(21.99)	(15.56)	(9.98)
IVolM2	-0.58	-4.47	-9.22	-11.44	2.09	-4.68	-15.13	-20.60
	(6.22)	(7.75)	(7.03)	(6.21)	(7.01)	(11.30)	(11.28)	(10.41)

[a] A bid-ask spread of 2:5 equates to a 2 per cent spread being applied to the index option and a 5 per cent spread being applied to single stock options.

This is a conservative estimate and trading costs are expected to be below this in practice. Therefore, if a trading rule earns a profit under these conditions, it is deemed to be potentially profitable in practice.

Figure 7 shows the performance of the chosen rule under IVolM1 and IVolM2 when the worst-case bid-ask spread was applied. A buy-and-hold index portfolio and a risk-free portfolio are provided for comparison. The performance of the rule under no bid-ask spread and IVolM2 is provided, so the progression of this portfolio can also be noted.

The chosen rule (under IVolM1 and the worst-case bid-ask spread) comfortably outperformed the market, with an annual return of 19.94 per cent compared to the market's 7.68 per cent. This rule had the second highest return, but had the lowest standard deviation (1.42 per cent) per trade. It traded 4.9 times per year and used 11.68 stocks on average.

Under IVolM2 and the worst-case bid-ask spread of 4:12, none of the trading rules produced positive returns. Figure 8 shows the profit per trade of the chosen rule under these conditions. The median profit was positive,

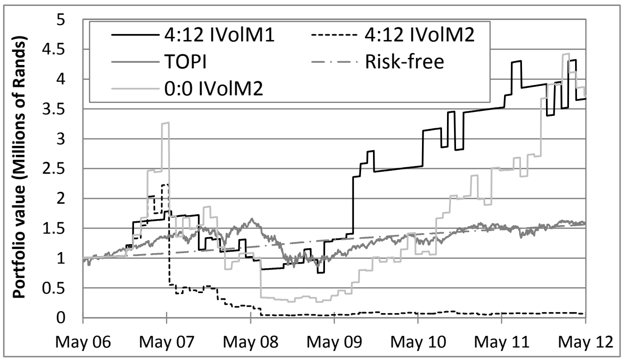


Figure 7: The performance of various portfolios over time. The rule 1.1(IIC) < HIC (q=5) was used where applicable and the risk-free portfolio was invested at the 91-day T-Bill rate.

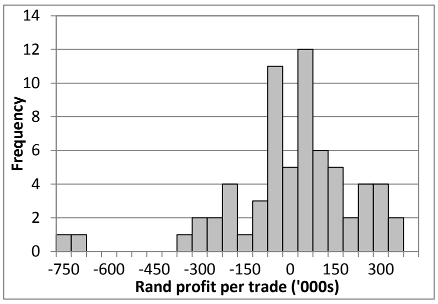


Figure 8: Histogram of the profit per trade of 1.1(IIC) < HIC under IVolM2 (q=5), with 4:12 bid-ask. Each trade was on R1 000 000 and held for a month.

however the mean is pulled left by the negative skew. This histogram shows that dispersion trades were still profitable under these high bid-ask spreads. However, due to their negative skew, a better trading rule is required. There were 11 trades a year: this rule is trading too often. The rule, 1.1(IIC) < HIC, behaves in this way because of the relationship (under IVolM2) between IIC and HIC, as explained in Section 3.2. IIC almost perfectly maps HIC (as seen in Figure 3); therefore, the rule either trades all the time or not at all as q changes.[26]

5.3 Cross-Sectional Volatility

The CSV swap contract traded was set up very similarly to the LIDS contract introduced in Section 4.2.[27] The CSV swap's strike price needed to be set for each trade, depending on current or historical data. CSV derivatives have not been priced before; thus, an approximation of the strike was sought, as

[26]See Figure 1(b), which depicts the number of times this rule would conduct traditional dispersion trades.

[27]See Appendix D for a detailed explanation.

shown in Appendix D. The CSV swap contract was for 21 working days and was held to expiry. Exposure to the amount of 80 per cent of the portfolio's capital was taken in each trade.

Under IVolM1, the entire portfolio was lost under every trading rule tested. The mean return over all trades (not annualised, i.e. over a one month trade) was -105.21 per cent.[28] Traditional dispersion trades were more profitable than reverse dispersion trades, with a mean return per trade of 35.8, as compared to -246 per cent. The returns per trade are negatively skewed, and thus, even though a positive mean is recorded, the portfolio was lost before these profits could be realised.

Under IVolM2, the results were similar, except one case where a profit of R49 billion was recorded.[29] Once again, traditional trades outperformed reverse trades, with a mean return per trade of 34.2 and -168.73 per cent respectively. It is clear that CSV swaps work better when used as a vehicle for traditional dispersion trades. However, in South Africa, the market favours reverse dispersion trading and therefore CSV swaps perform poorly.

CSV swap trading can be highly profitable in theory (as seen in one case out of 512), however, it is not suitable for practical use. The mean profit per traditional trade under IVolM2 was more than triple the mean profit of the call options from Section 5.2. However, the returns were negatively skewed and more volatile than those experienced with call options. This caused large losses: often more than the portfolio value was lost in a single trade. Bid-ask spreads and trading costs were not considered, as CSV swaps were deemed unprofitable in the base scenario.

[28]The nature of a swap allows for more than 100 per cent to be lost.

[29]The mean return per one month trade was 102.37 per cent and 27 trades were run. The final portfolio value was R49 653 343 002.86; this result is accepted and is deemed to not be a calculation error.

6 Conclusion

In this paper, dispersion trading opportunities were found and dispersion trading strategies were tested. The South African market presents opportunities for profits to be made from dispersion trades. The market favoured reverse dispersion trades over traditional dispersion trades. The market also favoured short-term dispersion trades over longer-termed trades.

Of the 16 trading rules that were tested, $1.1(IIC) < HIC$ had the highest and least volatile mean return per trade entered using call options. After the application of a conservative bid-ask spread and trading costs, one of the implied volatility estimation techniques had mean returns well above market returns. The return generated by these trades is, however, highly dependent on the implied volatility of the single stocks, the bid-ask spread and the trading costs. Therefore, care must be taken when implementing these strategies. A further study into the implied volatility of single stocks in the market, and the effect that they have on dispersion opportunities, is suggested.

CSV swaps, as defined in this paper, are too volatile to be used effectively when conducting dispersion trades. These CSV swaps are designed to return positive profits under traditional dispersion trades; shorting the swap contract did not work as a reverse dispersion trading vehicle. A reverse CSV swap could be explored, potentially with a minimum lock-in value.

Backtesting trades generated annual returns in excess of 19 per cent on average, which is relatively high for a period that includes the 2008 market crash. Finally, it is concluded that the South African market does present viable conditions for dispersion trades; even when mean returns were negative, there were trades that made profits.

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Appendices

A IIC and the First Coefficient of Volatility

There is an inverse relationship between IIC and CF1. This can be seen by considering Equation (2):

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2(\mathrm{IIC}) \sum_{i=1}^{n} \sum_{j>i} w_i w_j \sigma_i \sigma_j$$

where the σ's are implied volatilities (Lozovaia and Hizhniakova, 2005). It can be seen that this expression holds by substituting in Equation (3) for IIC and simplifying. By re-arranging this expression, we can get:

$$\sigma_p^2 = \left(\sum_{i=1}^{n} w_i \sigma_i \right)^2 + 2(\mathrm{IIC} - 1) \sum_{i=1}^{n} \sum_{j>i} w_i w_j \sigma_i \sigma_j. \tag{A.1}$$

The right-hand side of Equation (A.1) is a sum of the weighted components implied volatility (part of CF1) and an expression containing IIC. From this it can be seen that the lower the IIC (which ranges from -1 to 1), the higher the weighted components implied volatility, as the right-hand side must equal the left-hand side of the equation.

B IVolM1 and IVolM2

IVolM1 and IVolM2 estimate the at-the-money implied volatilities of the constituents of the index. They use the historic volatilities of these stocks as inputs. Therefore, their distribution is similar to that of the historic volatilities.

IVolM1 just applies a flat premium, q, to the historic volatility. This has the effect of linearly shifting the historic volatility's time series up or down. Table 5 displays the mean implied volatility (over every stock and every day

Table 5: The mean implied volatility (per cent) (over every stock and every day in the study period) that IVolM1 and IVolM2 estimated at various values of q.

q	IVolM1	IVolM2
-5	28.5	35.1
0	33.5	37.0
5	38.5	38.8
10	43.5	40.7

in the study period) that IVolM1 and IVolM2 estimated at various values of q. The mean historic volatility across the SSFs is 33.5 per cent, which is the same as setting q to 0 per cent under IVolM1.

Under IVolM2 the ratio between implied and historic index volatility was multiplied by $(1 + q)$ and then applied to the SSF historic volatility. Firstly, this includes the premium between the index implied volatility and historic volatility in the estimate when q is 0 per cent. Secondly, this results in an implied volatility estimate that is less sensitive to changes in q, as seen in Table 5.

C Volatility, Correlation and Coefficients of Volatility Results

Figure 9 is a larger version of Figure 2. Table 6 contains the results under the two implied volatility estimation methods, for the number of days IOIV exceeded MIV; IIC exceeded HIC; and the mean difference between IOIV and MIV. Table 7 contains the results for the number of days CF1 was below 1.5 and CF2 was below 1.

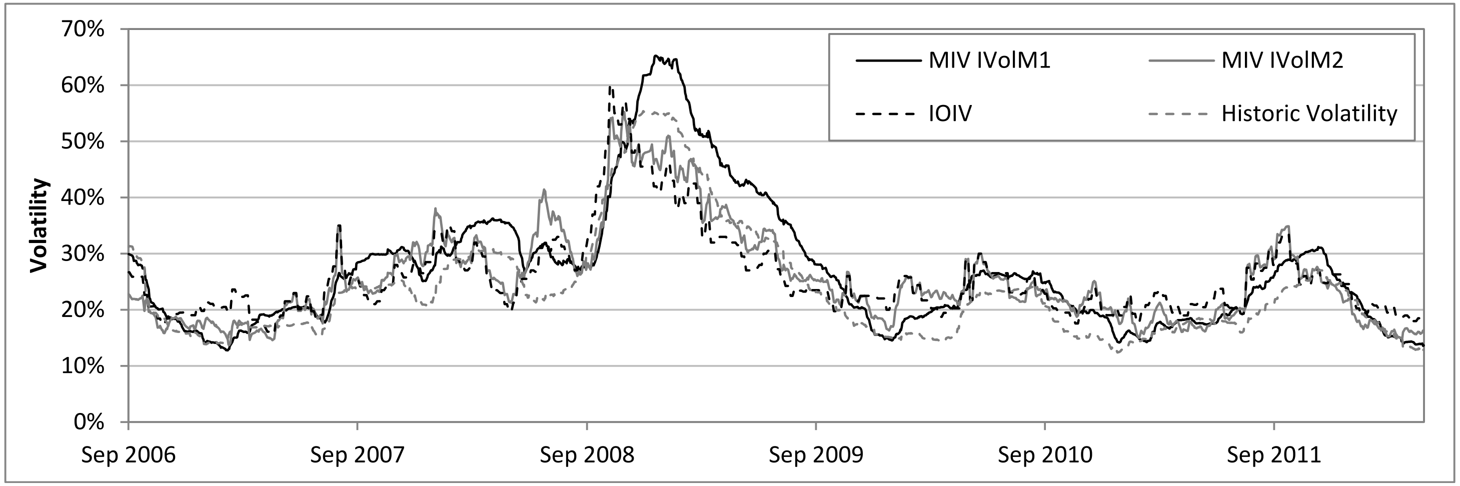

Figure 9: The MIV, under IVolM1 (q=5) and IVolM2 (q=0); IOIV; and historic index volatility over time.

Table 6: Number of days, out of 1412, that IOIV exceeds MIV; the mean difference between IOIV and MIV over the period; and the number of days that IIC exceeds HIC.

	IOIV>MIV		IOIV−MIV		IIC>HIC	
q	IVolM1	IVolM2	IVolM1	IVolM2	IVolM1	IVolM2
−5	1172	933	5.30	1.39	1215	1412
−4	1163	873	4.65	1.13	1187	1412
−3	1127	825	4.01	0.87	1159	1412
−2	1093	789	3.36	0.61	1132	1412
−1	1059	767	2.71	0.36	1077	1412
0	1012	729	2.07	0.10	1001	295
1	956	662	1.42	−0.16	959	0
2	905	617	0.77	−0.41	886	0
3	850	578	0.12	−0.67	827	0
4	804	524	−0.52	−0.93	743	0
5	726	496	−1.17	−1.18	663	0
6	651	473	−1.82	−1.44	581	0
7	580	433	−2.47	−1.70	516	0
8	523	404	−3.11	−1.96	440	0
9	464	381	−3.76	−2.21	346	0
10	409	367	−4.41	−2.47	273	0

Table 7: Number of days, out of 1412, that CF1 is less than 1.5 and that CF2 is less than 1.

	CF1<1.5		CF2<1	
q	IVolM1	IVolM2	IVolM1	IVolM2
-5	1217	966	834	0
-4	1205	892	787	0
-3	1182	819	713	0
-2	1131	744	636	0
-1	1087	706	538	0
0	1044	665	488	0
1	991	630	461	0
2	925	588	442	0
3	851	542	411	0
4	754	502	368	0
5	683	457	343	0
6	537	412	311	0
7	401	350	264	0
8	306	313	218	0
9	225	248	193	0
10	181	195	179	0

D Cross-Sectional Volatility and Lock-In Dispersion Swaps

Menchero and Morozov (2010) found that volatility is a large driver of CSV, making it a good proxy for dispersion. Most of the volatilities that have been discussed previously are time-series volatilities, as they need a large time-series of data in order to be calculated. CSV however, is not a time-series volatility, as it is a fixed-point cross-sectional measure (Menchero and Morozov, 2010).

Menchero and Morozov (2010), as well as Senechal (2004), define CSV at a specific point in time t, as:

$$\mathrm{CSV}(t) = \sqrt{\sum_{i}^{n} w_{i,t}(r_{i,t} - \bar{r}_t)^2} \tag{A.2}$$

where $\bar{r}_t$ is the weighted average return across all the stocks in the estimation environment at time t, $r_{i,t}$ is the return of stock i at time t, and $w_{i,t}$ is the weight of stock i at time t.

Morgan Stanley (2010) have introduced the idea of a "Lock-In Dispersion Swap" (LIDS), which is a swap on the highest dispersion value over the period, protecting against sharp drops in dispersion.

The LIDS has a payoff to the buyer of:

$$\text{P\&L}_{LIDS} = \max_{t=1\ldots T} [D_{MS}(t)] - K \tag{A.3}$$

where K is the strike price of the swap, T is the time of maturity in months, and $D_{MS}(t)$ is Morgan Stanley dispersion (as defined in Equation (A.4)) at time t.

Morgan Stanley dispersion (D_{MS}) is defined as:

$$D_{MS}(t) = \frac{1}{n} \sum_{i=1}^{n} |r_{i,t} - \bar{r}_t| \tag{A.4}$$

where $r_{i,t}$ is the return on stock i at time t and $\bar{r}_t$ is the arithmetic average return of the n stocks at time t. Stock return in this context is defined as the growth in the price of stock i from time 0 until time t.

Comparing Equation (A.4) to Equation (A.2), it can be seen that D_{MS} and CSV are very similar. A lock-in CSV swap payoff function for the buyer can be found by replacing D_{MS} in Equation (A.3) with Equation (A.2) to get:

$$\text{P\&L}_{CSV} = \max_{t=1\ldots T} \left(\sqrt{\sum_{i}^{n} w_{i,t}(r_{i,t} - \bar{r}_t)^2} \right) - K,$$

which is Equation (4).

A LIDS trade is thus similar to a CSV trade, allowing the assumption that CSV trading results should be similar to those of LIDS. Therefore, the use of CSV as a dispersion trading vehicle can be justified by the use of LIDS for dispersion trading. One of the major differences between CSV and D_{MS} is that CSV uses index weights, whereas D_{MS} uses arithmetic weights. However, the weights under CSV could be set to arithmetic weights, making the two measures even more similar.

Morgan Stanley (2010) justifies the use of LIDS by noting that, based on empirical evidence from the credit crisis, dispersion increases slowly through time, but contracts quickly. This can be understood by looking at correlation, which is inversely related to dispersion; correlation will rise sharply during a market crash and slowly return to a normal level after the crash (as is the case with HIC in Figure 3) (Morgan Stanley, 2010).

Morgan Stanley (2010) have also shown that the return from a LIDS trade is greater than or equal to the return from a call on dispersion, and that LIDS trades are cheaper, because they have an initial value of zero.

As CSV options have not been priced yet, an approximation of the strike price is necessary. CSV has a relationship with historic volatility, and thus the strike of a volatility swap was found using Equation (A.5). Demeterfi *et al.* (1999:23) show that:

$$K_{VOL} \approx \sigma_{ATMF}\sqrt{1+3Tb^2} \tag{A.5}$$

where σ_{ATMF} is the implied at-the-money forward volatility of the underlying and b is the skew (which is set at 0.2) is a good approximation of a volatility swap's strike price. The strike of the CSV swap was then found by manipulating the following relationship:

$$\frac{K_{CSV}}{K_{VOL}} = \frac{CSV}{\sigma_{Historic}}.$$

E Dispersion Trading Results

Table 8 is an extract of the trading results that were generated. These results were from a test conducted under the simple ($w_i > 0.02$) weighting rule; without any bid-ask spread and under IVolM1. [Rev] denotes that reverse dispersion trades were conducted; traditional trades were conducted in all other cases.

Table 8: Extract of the results showing the final portfolio value (in Rands) for some trading rules for q from -5 to 10 per cent, under IVolM1 and no bid-ask spread.

q	IIC > HIC	IIC > 1.1(HIC)	IIC < HIC [Rev]	1.1(IIC) < HIC [Rev]
-5	12 982 528.07	15 862 255.69	1 944 661.39	2 560 103.42
-4	7 430 886.03	14 286 121.55	4 911 547.26	2 926 183.20
-3	3 306 335.27	19 718 441.57	6 965 836.99	4 877 379.38
-2	14 076 403.33	31 633 017.00	10 612 263.11	6 067 650.80
-1	9 639 876.08	23 554 637.18	5 576 089.32	7 122 923.51
0	12 382 001.84	4 079 339.00	5 201 704.76	7 873 925.92
1	5 177 874.69	6 157 907.62	8 702 384.52	7 063 507.06
2	3 862 371.18	10 329 549.38	5 565 405.01	6 335 521.56
3	6 027 800.43	4 796 698.33	11 404 059.17	11 083 118.56
4	7 148 917.55	12 209 855.42	20 006 593.87	11 520 152.08
5	11 427 448.28	2 004 460.65	23 182 384.29	11 494 317.88
6	478 488.33	6 540 769.00	9 286 172.48	26 196 770.33
7	2 581 384.79	7 280 638.37	16 392 566.20	15 238 953.58
8	4 440 759.48	4 179 984.76	8 596 788.00	31 818 934.00
9	2 045 866.16	2 486 039.53	21 361 600.53	26 572 981.18
10	2 707 352.89	2 718 883.63	27 912 765.54	47 235 700.74

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