The Impact of Overnight Periods on Option Pricing

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Abstract

This paper investigates the effect of overnight trading halts on option prices. We model overnight returns by a pure jump process. Intraday returns follow the literature's standard models by allowing for stochastic volatility and a random jump component. We find that neither the intraday random jumps nor the overnight jumps are able to empirically describe all features of asset prices. We therefore conclude that both random jumps during the day and overnight jumps are important in explaining option prices, where the latter account for about one-third of total jump risk.

KEYWORDS: Derivative pricing, Jump diffusion, Stochastic volatility.

JEL CLASSIFICATION: G11, G13

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1 Introduction

As a result of the shortcomings in the Black-Scholes model for option pricing, two streams of literature can be identified. The first stream extends the Black-Scholes framework by allowing for time varying volatility and the random occurrence of jumps in the underlying stock price process. In Hull and White (1987), option prices in a stochastic volatility model are derived under the assumption that volatility risk is idiosyncratic. Heston (1993) gives closed form option pricing formulas using a mean reverting volatility process and allowing for a volatility risk premium. Merton (1976) motivates that the occurrence of abnormal events can be modelled by a jump component in the specification of the underlying stock price process. That paper discusses the implications for option pricing in case jumps are modelled as a compound Poisson process and under the assumption that jump risk is not priced in options markets. The models derived in Heston (1993) and Merton (1976) can be merged in the affine jump diffusion framework of Duffie, Pan, and Singleton (2000), where asset returns and variances are driven by a finite number of state variables. In this framework, the jump intensity can be made linearly dependent on the state variables which implies a perfect correlation between the state variables and the arrival rate of jumps.

The second stream of literature uses more general Lévy processes instead of Brownian Motion and the compound Poisson process as driving factors for asset returns. Madan, Carr, and Chang (1998) show that, if the parsimonious variance gamma process is assumed to be the stochastic process for underlying stock returns, closed form expressions for the density of asset returns and option prices can be derived. Stochastic volatility models which are driven by Lévy processes are (among others) studied in Carr, Geman, Madan, and Yor (2003).

From the empirical results on the previously described models in, e.g., Bakshi, Cao, and Chen (1997); Pan (2002); Andersen, Benzoni, and Lund (2002); and Madan, Carr, and Chang (1998), it is evident that jumps are important in explaining characteristics of asset returns and option prices. Using a parametrically specified pricing kernel, Pan (2002) provides evidence that both volatility risk and jump risk are priced in the SPX options market. Coval and Shumway (2001) end up with a similar conclusion using returns on option positions that are constructed in such a way that the value of the position is only sensitive for changes in these risk factors. The Lévy literature also provides support for priced volatility and jump risk since the parameter estimates under the objective and the risk neutral measure usually differ. For instance, Madan, Carr, and Chang (1998) find no significant skewness under the objective measure but their risk neutral estimates imply negative skewness. The differences between the objective and the risk neutral distributions are indicative of the presence of a price for crash risk in the options markets. However, it is not always obvious how to infer market prices of risk from the estimation results because a parametric pricing kernel which defines risk prices, is usually not specified in this literature. On the whole, it is clear from both streams of literature that jumps, next to stochastic volatility, are important in explaining some observed patterns in asset returns and option prices.

The present paper considers the jump process in more detail by focusing on jumps in asset prices that are inherent to overnight market closure. Most of the empirical research, cited above, uses daily returns. These returns are calculated using the last tick price on the exchange for each trading day. However, the exchange is closed a large part of the day and information that becomes available during the closing time can not be incorporated in stock prices immediately. More importantly, this information cannot be used to hedge option positions. For instance, European investors use information that is revealed in US stock market changes by submitting orders to their exchange before it opens. This means that all overnight information is reflected in the opening price of the exchange. While the effect of market closure on stock prices has been considered to some extent (see French and Roll (1986)), we are not aware of any paper that focuses on the implications for option pricing. In this paper we stress the difference in information processing intraday and overnight by using different processes driving intraday and overnight returns. In particular, during the day we assume, in the same spirit as Andersen, Benzoni, and Lund (2002), a continuous part with stochastic volatility reflecting the normal vibrations in the stock price and a jump part that models the arrival of important new information leading to a significant change in the stock price. Furthermore, we model the overnight change in the stock price by means of one single jump. We investigate the theoretical and empirical implications of this added factor on option prices. An interesting issue is the proportion of variation in stock prices that can be explained by overnight changes and, subsequently, how much of the jump risk that is present in option markets can be attributed to the fact that investors can't trade overnight. Of course, traders could use exchanges open around the world to hedge their positions. As we use observed option prices, we in fact identify the non-hedgeable overnight shock in stock prices (in our case the S&P-500 index).

The empirical results show that a model including overnight jumps outperforms the above-mentioned models in a significant way. The fact that this overnight jump component accounts for approximately one-third of total jump variation, shows that the overnight local trading halt has a non-trivial impact on option prices. Hence, we show that jump risk consists of two separate significant components. Although overnight jumps are significant, we also show that intraday jumps remain relevant albeit to a somewhat lesser extent. This follows, e.g., from the fact that the estimated volatility of volatility when excluding intraday jumps is too large in comparison to the estimate from volatility series.

The organisation of the paper is as follows. Section 2 provides the theoretical formulation and motivation of the model under both the objective and risk neutral measure. We also give a closed-form option pricing formula int the spirit of Heston (1993). Section 3 describes the data we use and discusses the estimation procedure employed. In Section 4 the empirical results are discussed. Section 5 concludes.

2 The Overnight Jump Model

Financial markets all over the world do not allow for continuous trading on the same exchange. Trading in stocks, interest rate products, and all kind of derivatives usually starts in the morning hours local time and ends in the late afternoon or in the evening (Germany). Of course, it is possible for individual and institutional investors to do 24 hours trading all over the world: by the time London closes, Wall Street is already open and when the US markets stop trading, Asian exchanges have already opened their doors. Due to increasing globalisation and financial market integration, different economies and firms from various countries are related. As a consequence changes in the value of financial instruments on different exchanges are not independent. For instance, a high opening for stocks traded on the Dow Jones usually has a positive effect on stock prices on the exchanges in Europe. This effect is not only present if exchanges are open simultaneously but also if the European market is closed. Relevant news cannot be incorporated immediately in case the exchange is closed. All news that is important for the value of a particular stock should ideally be processed in the opening price of the stock. The difference between the closing price one day and the opening price the next can be seen as a measure of revealed information all over the world during the overnight period.

Up to now, this feature of financial markets has not been considered in the context of derivative pricing, while it seems clear that the overnight period does influence derivatives. This paper tries to fill this gap by adding a jump process to a standard stock price process. The jump in the stock price process exactly models the observed overnight return in asset returns time series.

We model the underlying value process as,

$$\begin{aligned} \frac{dS_t}{S_{t_-}} &= c\mu dt + \sqrt{c}\sigma_t dW_t^S + d\left(\sum_{i=1}^{N_t}(Y_i - 1)\right) - \lambda\mu_{RJ}dt + d\left(\sum_{i=1}^{\lfloor 252t \rfloor}(V_i - 1)\right), \qquad (2.1) \\ \log Y_i &\sim N\left(\log(1+\mu_{RJ}) - \frac{1}{2}\sigma_{RJ}^2, \sigma_{RJ}^2\right), \\ \log V_i &\sim N\left((1-c)\frac{\mu_{FJ}}{252} - \frac{1}{2}(1-c)\frac{\sigma_{FJ}^2}{252}, (1-c)\frac{\sigma_{FJ}^2}{252}\right), \end{aligned}$$

where W^S is a standard Brownian Motion and N a standard Poisson process (with intensity λ) under the real world probability measure $\mathbb{P}$. The processes W^S and N as well as the random variables $\{Y_i\}$ and $\{V_i\}$ are independent. Furthermore, μ is the expected rate of return on the stock, σ_t is the stochastically changing volatility (see below), and the parameters μ_{RJ}, and σ_{RJ}^2 determine the random jump size distribution. In comparison to the standard jump model an extra jump term is added to the stock price process. If time in this model is denoted in (trading) years then at time $t = 1$ we should have 252 overnight periods (counting the weekend as one night). In this notation, the extra jump factor represents exactly the overnight jump in the stock price. Finally, the constant c reflects the proportion of the day for which the exchange is open. Note that if $c = 1$, the standard random jump models of Bakshi, Cao, and Chen (1997) and Andersen, Benzoni, and Lund (2002) appear. For our application to S&P-500 options, we take c equal to the proportion of the day that the NYSE is open. The parameter $c\mu$ can be interpreted as the expected yearly return during trading hours while the expected yearly calendar return on asset S is given by,

$$E\left(\log S_{t+1} - \log S_t\right) = c\mu + (1-c)\mu_{FJ}.$$

Note that all parameters should be interpreted on a yearly frequency. The random jumps do not appear in this formula, since the compound Poisson process in (2.1) is compensated. The specification of the stochastic variance process is taken from Heston (1993),

$$\begin{aligned} dc\sigma_t^2 &= -\kappa\left(c\sigma_t^2 - c\sigma^2\right)dt + c\sigma_\sigma\sigma_t dW_t^V \\ \mathrm{Corr}_t(dW_t^V, dW_t^S) &= \rho \end{aligned} \tag{2.2}$$

where κ is the spead of mean reversion, σ^2 is the long run mean of the variance, and σ_σ the volatility of volatility. It is often observed that a large decline in the stock price is followed by a positive shock in volatility levels. The model allows for the possible dependence between stock price movements and volatility movements by means of the parameter ρ.

Finally, the money market process is given by,

$$\frac{dB_t}{B_{t_-}} = crdt + (1-c)d\sum_{i=1}^{\lfloor 252t \rfloor}\frac{r^o}{252} \tag{2.3}$$

with r the annualized continuously compounded risk free rate during the day and r^o the annualized overnight risk free rate. Note that we implicitly assume that investors transfer their money at the end of the trading day to an overnight account. Our model does not take into account time-varying interest rates as we focus on derivatives whose value is known to be relatively robust to interest rate changes. Moreover, the mathematics are simplified a lot.

2.1 Implications for option pricing

In this paper we use the equivalent martingale method for pricing options. In comparison to the standard Black and Scholes (1973) framework there are some added risk factors which make the market described by (2.1)-(2.3) incomplete with respect to the traded financial securities. A consequence is the non-uniqueness of the equivalent martingale measure. Applying

Girsanov's theorem to accomplish the change of measure in this setting results in the following specification of the risk neutral processes

$$
\begin{aligned}
\frac{dS_t}{S_{t_-}} &= cr dt + \sqrt{c}\sigma_t d\tilde{W}_t^S + d\left(\sum_{i=1}^{\tilde{N}_t}\left(\tilde{Y}_i - 1\right)\right) - \tilde{\lambda}\tilde{\mu}_{RJ} dt + d\left(\sum_{i=1}^{\lfloor 252t \rfloor}\left(\tilde{V}_i - 1\right)\right), \\
dc\sigma_t^2 &= -c\tilde{\kappa}\left(\sigma_t^2 - \tilde{\sigma}^2\right) dt + c\sigma_\sigma \sigma_t d\tilde{W}_t^V, \\
\log \tilde{Y}_i &\sim N\left(\log\left(1+\tilde{\mu}_{RJ}\right) - \frac{1}{2}\tilde{\sigma}_{RJ}^2, \tilde{\sigma}_{RJ}^2\right), \\
\log \tilde{V}_i &\sim N\left((1-c)\frac{r^o}{252} - \frac{1}{2}(1-c)\frac{\tilde{\sigma}_{FJ}^2}{252}, (1-c)\frac{\tilde{\sigma}_{FJ}^2}{252}\right), \\
\mathrm{Corr}_t(d\tilde{W}_t^V, d\tilde{W}_t^S) &= \rho,
\end{aligned}
\tag{2.4}
$$

where $\tilde{W}^S$ and $\tilde{W}^V$ are Brownian Motions under the equivalent martingale measure $\mathbb{Q}$ and $\tilde{N}$ is a standard Poisson process under this measure.

Remark 2.1 The variance process under the risk neutral measure can be expressed in terms $\mathbb{P}$-parameters if additional assumptions on the underlying equilibrium model are made. Under certain integrability conditions Girsanov's theorem states that if W_t^V is a $\mathbb{P}$-Brownian Motion then,

$$d\tilde{W}_t^V = dW_t^V + \zeta_t^V dt,$$

is a Brownian motion under a measure $\mathbb{Q}$ that is absolutely continuous with respect to the real world measure $\mathbb{P}$. In this formulation the process ζ_t^V is called the market price of volatility risk. From (2.4), we find that we may choose

$$\zeta_t^V = \frac{\eta^V}{\sigma_\sigma}\sigma_t,$$

where η^V is a constant. This particular choice for the volatility risk premium is, e.g., motivated by the Breeden (1979) consumption based model. If the general volatility risk premium is denoted by $\eta^V\left(S_t, v_t, t\right) \equiv \zeta_t^V \frac{\sigma_\sigma}{\sigma_t}$ then in the Breeden (1979) model,

$$\eta^V\left(S_t, \sigma_t^2, t\right) = \gamma \mathrm{Cov}\left(d\sigma_t^2, \frac{dC_t}{C}\right),$$

with C_t the consumption rate and γ the relative risk aversion of an investor. The additional assumption of a geometric Brownian motion process for consumption with a constant correlation between consumption growth and the asset return, generates a volatility risk premium that is proportional to σ_t^2. As a result the variance process can be rewritten in terms of $\mathbb{P}$-parameters and the volatility risk premium $\eta^V\sigma_t^2$,

$$dc\sigma_t^2 = -\left(\kappa + \eta^V\right)\left(c\sigma_t^2 - \frac{c\kappa\sigma^2}{\kappa + \eta^V}\right) dt + c\sigma_\sigma\sigma_t d\tilde{W}_t^V.$$

	close-close	close-open	open-close
average	0.1322	0.0768	0.0554
std.dev	0.1047	0.0267	0.0993
2.5 perc	-0.0138	-2.20E-03	-0.0139
97.5 perc	0.0141	3.01E-03	0.0128
skewness	-0.0275	-0.0254	-0.0265
kurtosis	4.83	40.77	4.68

Table 1: Summary statistics S&P-500 returns January 1992-August 1997.

This specification allows for a negative premium for volatility risk for which there is both theoretical and empirical evidence, see e.g., Bakshi and Kapadia (2003).

Given the risk neutral processes in (2.4), a standard plain vanilla call option can be priced using the standard valution formula,

$$C_t = B_t E_t^{\mathbb{Q}} \left(\frac{\max(S_T - X, 0)}{B_T} \right),$$

where T is the maturity, X the strike price of the option. Following Heston (1993) we show in the appendix that the pricing formulae for the value of a call option C and a put option P at time t can be simplified as,

$$\begin{aligned} C_t &= S_t P_1 - X e^{-cr(T-t)-n(1-c)r^o} P_2, \\ P_t &= X e^{-cr(T-t)-n(1-c)r^o} (1 - P_2) - S_t (1 - P_1). \end{aligned}$$

where probabilities P_1 and P_2 are given by $(A.1)$ and $(A.2)$, and $n = \lfloor 252T \rfloor - \lfloor 252t \rfloor / 252$. The appendix utilizes the fact that the trading day part of the model is an affine jump diffusion in the spirit of Duffie, Pan, and Singleton (2000) and that the overnight process is independent of the intraday process.

3 Data and Estimation Issues

In the previous section we motivated that different processes describe the intraday and overnight returns. In this paper we focus on the S&P-500 index. A first kind of evidence for modelling the intraday return in a different way can be found in Table 1 which shows the sample statistics of the close-to-close, close-to-open, and open-to-close return series in the sample period January 1992 to August 1997.

From Table 1 it is clear that the overnight return is an important part of the total daily return. For instance, looking at the average return learns that more than 50% of the total daily return is composed of the overnight return in our sample. The sample standard deviation of the overnight returns is much lower (but not negligible) than the standard devation of the intraday returns, indicating that information important for S&P stocks generally arrives during trading hours. Information of significant importance during the night often leads to a high absolute return on the S&P-500 which determines the high kurtosis of overnight returns in Table 1. Return data between July 1999 and November 2003, summarized in Table 8 (see appendix), confirm these results.

In addition to S&P-500 return data, we have intraday option data of European options that are written on the S&P-500 index. The S&P-500 data set includes all reported trades and quotes covering S&P-500 European index options traded on the Chicago Board Options Exchange and intraday S&P-500 index levels from January 1992 through June 1997. Following Bakshi, Cao, and Chen (1997) for each day in the sample, only the midprice based on the last reported bid-ask quote (prior to 3:00 pm Central Standard Time) of each option contract is used for estimation. This means that the recorded S&P-500 index levels are the corresponding index levels at the moment the option bid-ask quote is recorded. Since SPX options are European, early exercise is not possible. Unfortunately, interest rates and dividend yields have to be extracted from different sources since these data are not provided together with the option quote data. Jackwerth and Rubinstein (1996) assume that the dividend amount and timing expected by the market is identical to the dividends actually paid on the index. Also, they settled, after experimenting with a variety of possible interest rates including T-bill rates and CD-rates, on using implied interest rates imbedded in the European put-call parity. In contrast, Bakshi, Cao, and Chen (1997) extracted daily dividend distributions for the S&P-500 index from the S&P-500 Information Bulletin and they used daily Treasury-bill bid and ask discounts to obtain interest rates. We have employed interpolated LIBOR rates as a proxy for the risk free rate. Information on overnight interest rates is taken from Bloomberg. The challenge of calculating dividend rates remains. This research resolves this problem by using the dividend rates that are calculated from the actual dividends paid out by the SPX stocks, following Jackwerth and Rubinstein (1996). Finally, we have the closing option quotes of SPX options and corresponding index levels from July 1999 to November 2003. These data are extracted from the ABN-Amro Asset Management option database. Summary statistics and empirical results corresponding to this dataset are given in the appendix.

Table 2 provides descriptive statistics on call option prices (stated in terms of Black-Scholes implied volatilities) that (a) have time-to-expiration of greater than or equal to six calendar days, (b) have a bid price of greater than or equal to 3/8$ and a bid-ask spread of less than or equal to 1$, (c) have a Black-Scholes implied volatility greater than zero and less than or equal to 0.70, and (d) have quotes that satisfy the arbitrage restriction

$$C(t,T) \geq \max\left(0, S_t e^{-q(T-t)} - X e^{-r(T-t)}\right). \tag{3.1}$$

		Days to Expiration			
	Moneyness	<60	60-180	>180	**Subtotal**
ITM	< 0.97	0.210	0.171	0.140	
		14753	14802	6821	36376
ATM	0.97-1.03	0.136	0.138	0.152	
		14611	13693	5571	33875
OTM	> 1.03	0.124	0.118	0.172	
		4768	9380	5836	19984
Subtotal		34132	37875	18228	90235

Table 2: Summary statistics SPX call option implied volatilities. The reported numbers are implied volatilities of call options on the S&P-500 index corresponding to the average last tick before 3:00 PM and the total number of observations for each maturity category. The sample period ranges from January 1, 1992 to June 30, 1997, respectively.

In this formula X is the option exercise price, q the dividend rate, and r the continuously compounded intraday riskfree rate.

From the numbers in Table 2 the well-known patterns of implied volatilities across strikes and maturities are recognized. The volatility skew is clearly present for options with maturity smaller than 180 calendar days. For options with maturity longer than 6 months the standard patterns are not recognized since these options are less frequently traded. Table 3 provides the implied volatilities for put option contracts where (3.1) is replaced by its put option equivalent.

From the return data given in Table 1 it is clear that the 1992-1997 sample period can be characterized as a 'low volatility' period. This observation is confirmed by the implied volatility data from Tables 2 and 3. From a theoretical point of view there is a strong link between the realized volatility and the Black-Scholes ATM implied volatility. This means that in a low realized volatility period the Black-Scholes ATM volatility is also low on average. The final indication for a low volatility period is given by the steepness of the volatility skew. Low volatility markets usually have skews that are less steep than skews in high volatility markets. Using the same reasoning, the numbers in Table 8, Table 9, and Table 10 show that the sample period July 1999 to November 2003 can be seen as a 'high volatility' period.

The goal of the present paper is to study the effect of overnight trading halts on option pricing. The models presented in the previous section contain a number of parameters that need to be estimated. Numerous estimation strategies have been proposed in the literature. Examples are, the Efficient Method of Moments which is applied by, e.g., Chernov and Ghysels (2000) in the Heston stochastic volatility model and by Andersen, Benzoni, and Lund (2002) in a jump diffusion model; the Simulated Method of Moments approach in Duffie and Singleton

		Days to Expiration			
	Moneyness	<60	60-180	>180	**Subtotal**
OTM	< 0.97	0.191	0.173	0.172	
		12912	14729	7065	34706
ATM	0.97-1.03	0.137	0.139	0.151	
		14690	13771	5709	34170
ITM	> 1.03	0.163	0.125	0.130	
		8513	11259	6122	25894
Subtotal		36115	39759	18896	94770

Table 3: Summary statistics SPX put option implied volatilities. The reported numbers are implied volatilities of put options on the S&P-500 index corresponding to the average last tick before 3:00 PM and the total number of observations for each maturity category. The sample period ranges from January 1, 1992 to June 30, 1997, respectively.

(1993); the Markov Chain Monte Carlo methods applied by Eraker, Johannes, and Polson (2003) in a jump diffusion setting; and the spectral GMM estimator utilizing the empirical characteristic function in Chacko and Viceira (2003) who apply the method to stochastic volatility and jump diffusion models. The empirical applications in these papers focus on parameter estimation under the real world measure $\mathbb{P}$ using the information contained in returns series. An exception is Chernov and Ghysels (2000) who use both returns and at-the-money options in order to identify the market price of volatility risk. From a practical point of view these estimation techniques are not always convenient because long historical data series are required or computer intensive simulation techniques are used.

In this paper we consider information about the parameters as revealed by option prices. We mainly focus our research on the influence of overnight jumps on option prices. This is important since, from Table 1, it is clear that, under $\mathbb{P}$, these jumps have a significant impact on expected returns and, especially, volatilities. The practical implementation of the estimation procedure is straightforward and follows Bakshi, Cao, and Chen (1997). For a particular day t, a set of N options is chosen for which the closing price is observed. Henceforth, the price of the i-th option in this set will be denoted by O_{it}^{obs}. For all of these options the strike price, the remaining time to maturity, the risk free interest rate, and the (dividend discounted) value of the underlying are observed as well. Subsequently, we have a model price O_{it}^{model} for option i at time t that is a function of the structural $\mathbb{Q}$ parameter vector $\Theta = (\kappa + \eta^V, \kappa\sigma^2, \sigma_\sigma, \rho, \tilde{\mu}_J, \tilde{\sigma}_{RJ}^2, \tilde{\lambda}, \tilde{\sigma}_{FJ}^2)$ and the unobservable instantaneous variance σ_t^2. For a particular time t the estimated parameter vector is given by,

$$\left[\hat{\Theta}_t, \hat{\sigma}_t^2\right] = \text{argmin}_{\Theta,\sigma_t^2} \sum_{i=1}^{N} \left(\frac{O_{it}^{\text{model}} - O_{it}^{\text{obs}}}{O_{it}^{\text{obs}}} \right)^2 .$$

This procedure is repeated for each day in both samples resulting in two time series of estimators. Similar procedures are applied in Bakshi, Cao, and Chen (1997) to stochastic volatility, stochastic interest rates and random jump option pricing models and in Madan, Carr, and Chang (1998) in a Lévy based option pricing model.

In the implementation of the procedure above we only use the out-of-the money options (for low strikes put options and for high strikes call options), since out-of-the-money options are more liquid than in-the-money-options. Together with the objective function this implies that we are particularly focused on fitting the steepness of the observed (Black-Scholes) implied volatility skews or otherwise stated the tails of the market implied risk neutral distribution.

4 Empirical Results

This section gives an overview of the estimation results that are obtained by applying the data and estimation techniques as described in Section 3 to the model formulation of Section 2. First, as a benchmark, the results are presented for the standard stochastic volatility model (SV) and the standard stochastic volatility with random jumps (SVRJ) followed by a treatment of the results of the extended model including the fixed jump specficiation, both in a setting with only stochastic volatility during the day (SVFJ) as well as a setting where also random jumps may occur intraday (SVFJRJ). For the SV and $SVRJ$ models the constant c is set equal to 1 in (2.4). The choice for c in the extended model will be discussed in Section 4.2.

4.1 Standard option pricing models

Presenting the results for the stochastic volatility model and the stochastic volatility model with random jumps makes our results comparable those of Bakshi, Cao, and Chen (1997), since their specification of these models and their estimation technique is similar to that in the present paper. These results will serve as a benchmark in Section 4.2. Table 4 and Table 5 give an overview of the estimation results for these models using the data of the 1992-1997 sample period. Note that in the stochastic volatility model the parameters κ, σ^2, and η^V are not separately identified from option data alone (in a statistical sense).

Table 4 confirms, as we have seen in the previous section, that the average instantaneous volatility of 14.4% in our sample is low in comparison to, for example, the estimated values in Bakshi, Cao, and Chen (1997) over the period June 1988 to May 1991. Besides the instantaneous volatility there are also differences in estimates for the parameters $\sigma_\sigma, \kappa + \eta^V$, and

	mean	st.dev.	2.5 perc	97.5 perc
$\kappa\sigma^2$	0.0415	0.0142	0.0397	0.0495
$\kappa + \eta^V$	1.6679	0.9610	1.2536	1.6873
$\sqrt{\kappa\sigma^2/(\kappa + \eta^V)}$	0.1602	0.0429	0.1468	0.1601
σ_σ	0.6105	0.1872	0.4243	1.0491
ρ	-0.6893	0.1507	-0.9939	-0.4283
σ_t	0.1441	0.0306	0.1012	0.2111
SSE	0.7039	0.5817	0.0717	2.3471

Table 4: Summary statistics parameters SV model using option data from 1992-1997.

$\kappa\sigma^2$. Besides the different sample period, one explanation could be that Bakshi, Cao, and Chen (1997) only use call prices to fit the model while in this research both puts and calls are employed. Furthermore, the results could be driven by model misspecification which becomes more apparent in our results since we focus on relative pricing errors and not on absolute pricing errors. The reason for this choice is that we want, in addition to at-the-money options, to fit options with high and low strike prices and since we only use cheap out-of-the-money option for parameter estimation, we should choose relative errors as objective.

To address the latter issue in more detail, we consider a situation where the option implied volatility curve is downward sloping in the strike price for a given maturity of the option. The steepness of the implied volatility curve provides information about the risk neutral distribution of the underlying index at the maturity date. The steeper the implied volatility curve for a certain strike price region, the more probability mass in the tails of the implied risk neutral distribution, since the risk neutral probability of a certain future state of the stock price is the second derivative of the option with respect to the strike price evaluated in this future state. There is an enormous literature on methodologies that extract information about the risk neutral distribution from option prices, see for example Britten-Jones and Neuberger (2000). Because we minimize squared relative errors, the fit of cheaper options (short term OTM puts and OTM calls) is relatively more important as compared to that of the more expensive options in the sample (long term ATM puts and calls). Stated differently, the focus is on fitting the tails of the market implied risk neutral distribution. Since the implied volatility curve for short term options is often downward sloping, the algorithm generally chooses on average parameters that are able to generate negative skewness in the risk neutral distribution. Both ρ and σ_σ can generate the desired skewness. Table 4 shows at which levels these parameters are estimated. The parmater estimate for σ_σ implies a volatility of volatility (9%) that is too high in comparison to empirically found levels in low volatility periods (5%) which indicates that the stochastic volatility model without jumps is misspecified.

	mean	st.dev.	2.5 perc	97.5 perc
$\tilde{\mu}_{RJ}$	-0.0634	0.0392	-0.1377	0.0305
$10^{-3} * \tilde{\sigma}^2_{RJ}$	9.121	8.256	0.001	28.91
$\tilde{\lambda}$	0.6001	0.0480	0.5450	0.6308
$\kappa\sigma^2$	0.0438	0.0097	0.0243	0.0655
$\kappa + \eta^V$	3.5527	1.0003	1.4731	5.4069
$\sqrt{\kappa\sigma^2/(\kappa + \eta^V)}$	0.1161	0.0349	0.0662	0.2062
σ_σ	0.3997	0.0026	0.3973	0.4015
ρ	-0.5885	0.1996	-0.9293	-0.1613
σ_t	0.1174	0.0318	0.0744	0.1962
SSE	0.1625	0.1504	0.0210	0.5526

Table 5: Summary statistics parameters SVRJ model using option data from 1992-1997.

This reasoning is confirmed by the estimation results in Table 5 where we include random jumps. Compared to the estimates in Table 4, σ_σ is estimated much smaller and this is due to the appearance of (on average) negative jumps that capture (part of) the negative skewness in the implied risk neutral distribution. The three parameter random jump size process combined with stochastic volatility is superior to the stochastic volatility model in describing the tails of the market implied risk neutral distribution. Table 11 and Table 12 show similar kind of patterns in the second sample period. The $SVRJ$ model also gives a much better fit of the option data and the parameter estimates of ρ and σ_σ in this model are estimated lower than in the SV model. Furthermore, comparing the estimates of σ_t, $\tilde{\lambda}, \tilde{\mu}_{RJ}$, and $\tilde{\sigma}_{RJ}$ in Table 5 and Table 12, it is seen that the volatility in the 1999-2003 period is higher than in the 1992-1997 period.

Figure 1 plots the strike price of put options against the Black-Scholes implied volatility on January 9, 1992. The closing value of the S&P-500 this day (discounted for dividends) equals 418.155 and the time to maturity for the option series under consideration is 8 calendar days. Figure 1 exactly confirms the reasoning above. The high implied volatilities are better matched by the random jump model. In this picture it appears that this comes at the cost of a worse implied volatility fit for near the money options. However, since our criterion is relative pricing errors, the pricing error for OTM put options has a much larger weight than the pricing errors for ATM put options. The sum of the relative pricing errors for the stochastic volatility model is given by 0.21 while for the random jump model the pricing error equals 0.12. The pricing error of the stochastic volatility model is mainly driven by the mispricing of the put option that is farthest out-of-the-money. Figure 1 looks similar on other dates. Furthermore, we see that for longer dated options the added value of the random jumps is

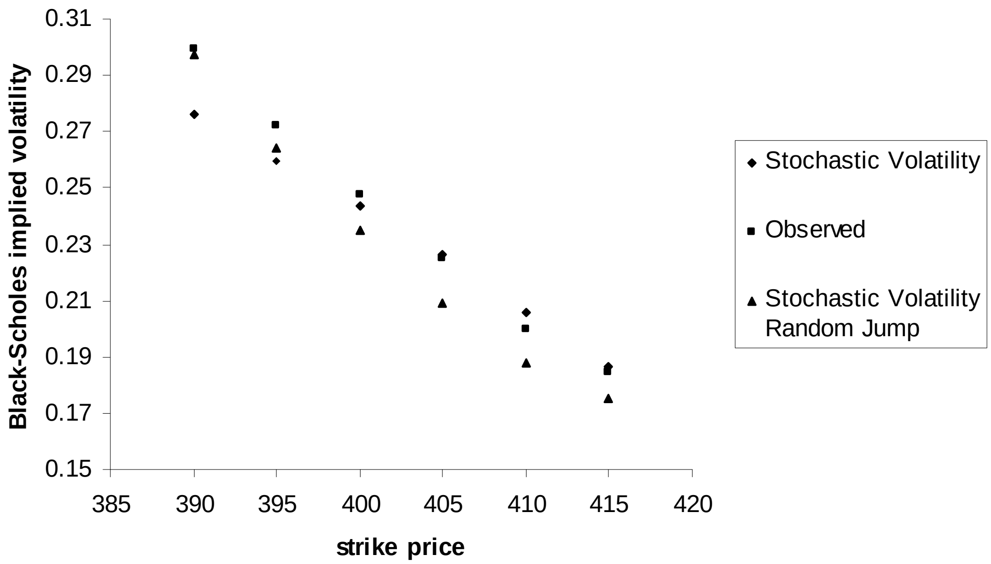

Figure 1: Black-Scholes implied volatilities for out-of-the-money puts on January 9, 1992.

negligible: stochastic volatility explains long term option prices quite well. If we consider, for instance, the relative pricing error of both models on January 13, 1992, where the selected options all have a maturity longer than 39 calendar days, then we notice that there is just a small difference between these pricing errors. More precisely, the stochastic volatility model has a relative pricing error of 0.045 while the random jump model has a pricing error of 0.049.

Comparing Table 5 with Table 4, we see that the instantaneous volatility has decreased on average. This makes sense since the total variation in the underlying value is now divided in the variation of a jump component and the variation that stems from the stochastic volatility part of the model. For instance, the average yearly instantaneous volatility of the S&P-500 index in the SV model is 14.4% and in the $SVRJ$ model it is estimated as 11.7%. A back of the envelope calculation gives a volatility of approximately 8.40% for the jump part. Adding the variance of the continuous part and the discrete jump part of the $SVRJ$ model gives approximately the average instantaneous variance in the SV model. The same observation can be made from Table 11 and Table 12. The average instantaneous volatility drops from 24.8% in the SV model to 20.2% in the $SVRJ$ model but adding the variance implied by the jump parameters to the average instantaneous variance of the $SVRJ$ model approximately gives the average instantaneous variance in the SV model.

The fact that the jump component in the 1992-1997 sample period looks less important than in the Bakshi, Cao, and Chen (1997) sample period is explained by the fact that this sample period is a low volatility period that is generally characterized by less pronounced skews. The implied risk neutral distributation in those markets is less negatively skewed than in high volatility markets hence, less frequent and smaller jumps (in absolute value) are required to generate the skewness implied by option prices.

4.2 Option pricing models with overnight jumps

To goal of the present paper is to assess importance of overnight trading halts for derivative pricing. In this section we discuss the estimation results for both the $SVFJ$ and the $SVFJRJ$ model.

In this subsection we set the constant $c = \frac{1}{3}$. Setting the parameter c unequal to 1 changes the interpretation of the parameters. For example, the model under the objective probability measure $\mathbb{P}$ can be reformulated with parameter $\mu^* = c\mu$ where μ^* is the expected yearly return during trading periods. To make this return comparable to the standard close to close yearly returns, we have to correct for the return during the periods in which trading is impossible. The expected overnight return μ_{FJ} can be estimated using overnight returns. The total expected yearly return in the $SVFJ$ and the $SVFJRJ$ model using the information in both intraday and overnight returns is then given by $\mu^* + (1-c)\mu_{FJ}$, whilst the total expected yearly return in the standard SV and $SVRJ$ is summarized in one single parameter μ. In the same spirit the variance of the yearly return can be decomposed in a variance part concerning trading periods and a part due to the non trading overnight periods. The trading periods variance consists of the variance of the continuous component (stochastic volatility) and the random jump component. The non trading overnight period variance is modelled by the fixed jump part. In contrast with the random jump part, these fixed jumps take place at fixed points in time, namely: at the end of each trading day. The results in Table 1 suggest that the trading period part of the variance is estimated much higher than the variance of the non trading overnight part (represented by parameter $\tilde{\sigma}^2_{FJ}$). Furthermore, the estimate of the parameter $\tilde{\sigma}^2_{FJ}$ gives an indication of how total jump risk, that is priced in option markets, is divided in the jump risk during trading periods and the risk that comes from the possible arrival of important new information during non trading overnight periods. Note that, since our focus is on risk neutral parameters, we basically treat the non systematic part of the jump risk that can't be hedged away.

For illustrative purposes, we first focus on the statistical results of the $SVFJ$ model where fixed jumps are added to the stochastic volatility model and where we set $c = \frac{1}{3}$. In this model there are no discontinuities in the asset price process during trading days but at the end of each trading day the stock price jumps to the opening price of the next trading day. This particular setting results in the following specification of the asset price process under the risk

	mean	**st.dev.**	**2.5 perc**	**97.5 perc**
$10^{-3} * \tilde{\sigma}^2_{FJ}$	9.712	6.623	0.001	28.91
$c\kappa\sigma^2$	0.0414	0.0061	0.0388	0.0441
$\kappa + \eta^V$	1.6185	0.0081	1.6128	1.6243
$\sqrt{c\kappa\sigma^2/(\kappa + \eta^V)}$	0.1597	0.0101	0.1545	0.1654
$\sqrt{c}\sigma_\sigma$	0.9243	0.3222	0.3079	1.5000
ρ	-0.8849	0.1298	-1.0000	-0.5555
$\sqrt{c}\sigma_t$	0.1186	0.0342	0.0723	0.2049
SSE	0.3956	0.3477	0.0455	1.2568

Table 6: Summary statistics SVFJ model using option data from 1992-1997.

neutral probability measure:

$$\begin{aligned}
\frac{dS_t}{S_{t_}} &= \frac{1}{3} r dt + \tilde{\sigma}_t d\tilde{W}_t^S + d\left(\sum_{i=1}^{\lfloor 252t \rfloor}\left(\tilde{V}_i - 1\right)\right) \qquad (4.1)\\
d\tilde{\sigma}_t^2 &= -\left(\kappa + \eta^V\right)\left(\tilde{\sigma}_t^2 - \frac{\kappa\tilde{\sigma}^2}{\kappa + \eta^V}\right) dt + \tilde{\sigma}_\sigma \tilde{\sigma}_t d\tilde{W}_t^V\\
\log \tilde{V}_i &\sim N\left(\frac{2}{3}\frac{r^o}{252} - \frac{1}{3}\frac{\tilde{\sigma}^2_{FJ}}{252}, \frac{2}{3}\frac{\tilde{\sigma}^2_{FJ}}{252}\right)\\
\mathrm{Corr}_t(d\tilde{W}_t^V, d\tilde{W}_t^S) &= \rho
\end{aligned}$$

where $\tilde{\sigma}_t^2 = \frac{1}{3}\sigma_t^2$, $\tilde{\sigma}^2 = \frac{1}{3}\sigma^2$, and $\tilde{\sigma}_\sigma = \sqrt{\frac{1}{3}}\sigma_\sigma$. This reparameterized specification allows us to evaluate the standard Heston (1993) option pricing formula in $\left(\frac{1}{3}r, \kappa + \eta^V, \kappa\tilde{\sigma}^2, \tilde{\sigma}_\sigma, \tilde{\sigma}_t\right)$. The resulting sample statistics, using the 1992-1997 dataset, for the estimators are given in Table 6.

The average trading period volatility in the $SVFJ$ model is given by the estimate of $\sqrt{c}\sigma_t$ in Table 6 and therefore equals 11.9%. The estimator of parameter σ_t reflects the total yearly asset return volatility, if the overnight return would have the same variance as the trading day return. However, the estimate of $\tilde{\sigma}^2_{FJ}$ shows that the volatility in overnight periods is smaller and approximately equals 9.8%. The sum of the intraday variance and the overnight variance roughly gives the total variance of the yearly return implied by the estimates in the SV and $SVRJ$ of the previous section.

Although we have seen that the fixed jump component of the $SVFJ$ model captures a significant amount of the total variance in the yearly return and hence provides evidence that the impact of overnight jumps on option prices is non trivial, we also see that the statistical fit of the model measured by the average of the sum of squared (relative) errors does not improve much in comparison to the SV model. If we continue to compare the results of Table 4 to the

	mean	std.dev	2.5 perc	97.5 perc
$\tilde{\mu}_{RJ}$	-0.072	0.036	-0.133	0.003
$10^{-3} * \tilde{\sigma}^2_{RJ}$	9.812	5.81	0.402	20.57
$\tilde{\lambda}$	0.541	0.415	0.016	1.448
$10^{-3} * \tilde{\sigma}^2_{FJ}$	4.447	5.706	0.001	22.341
$c\kappa\sigma^2$	0.039	0.015	0.002	0.065
$\kappa + \eta^V$	3.324	1.084	1.555	5.016
$\sqrt{c\kappa\sigma^2/(\kappa+\eta^V)}$	0.112	0.048	0.022	0.207
$\sqrt{c}\sigma_\sigma$	0.506	0.289	0.173	1.451
ρ	-0.706	0.228	-0.976	-0.142
$\sqrt{c}\sigma_t$	0.095	0.034	0.020	0.148
σ_t	0.164	0.058	0.035	0.256
SSE	0.123	0.102	0.019	0.416

Table 7: Summary statistics parameters SVFJRJ model using option data from 1992-1997.

results of Table 6, we see the biggest difference in the estimate for parameter ρ. The addition of the fixed jump component to the model shifts the risk neutral distribution of the underlying index at a future point in time to the right. In order to fit the left tail of this distribution a more negative ρ is required. However, if we compare the estimates of the $SVFJ$ model to the estimates of the $SVRJ$ model of the previous subsection, we have reason to believe that the $SVFJ$ suffers from the same problem as the pure SV model: the estimated volatility of volatility in the $SVFJ$ is too large in comparison to the empirically observed levels.

The most appealing model, however, is clearly the $SVFJRJ$ model, since it allows for difference in intraday asset return variance and overnight asset return variance. The estimation results of the $SVFJRJ$ model for the 1992-1997 sample period are given in Table 7. From a statistical point of view, we see, that in comparison to the SV, $SVRJ$, and $SVFJ$, the $SVFJRJ$ improves the fit of option prices considerably. This makes sense since the possible arrival of unexpected news to the stock market is accounted for in option prices. The estimates of $\sqrt{c}\sigma_\sigma$ and ρ in the $SVFJRJ$ model differ from the estimates σ_σ and ρ in the $SVRJ$ model. The estimated variance of overnight returns $\tilde{\sigma}^2_{FJ}$ implies that the risk neutral distribution shifts to the right in comparison to the risk neutral distributions implied by the $SVRJ$ model. The lower skewness in the left tail has to be compensated for by a higher σ_σ and a lower ρ. Furthermore, the appearance of random jumps makes the estimates less extreme than the estimates in the $SVFJ$ models since the random jump parameters generate the desired negative skewness (in this case by means of a lower value of the parameter μ_{RJ}). Comparing the $SVFJRJ$ model to the $SVFJ$ model, we see that the addition of random jumps to the

$SVFJ$ model has the same effect on parameters σ_σ and ρ as the addition of random jumps to the SV model. The reasoning is also the same: the random jump part captures (part of) the negative skewness in the risk neutral distribution required to fit option prices that otherwise could only partly be captured by large changes in parameters σ_σ and ρ.

Given the estimates of the $SVFJRJ$ model in Table 7, the total yearly return variance can be separated in three components. The intraday volatility consists of the continuous part equal to 9.5% and the random jump part volatility of approximately 8.61%. Given that the volatility of the fixed jump is 6.67%, the estimated total yearly variance is roughly the same as we have seen before. These numbers show that the fixed jump part covers approximately one-third of total jump variation.

We showed that the overnight jump component is non trivial in empirical option prices and that $SVFJRJ$ model gives a significantly better fit of option prices in the 1992-1997 sample period. We also applied the $SVFJ$ model and the $SVFJRJ$ model to the 1999-2003 sample period to see if the conclusions are robust for the input data that are used. Table 13 and Table 14 in the appendix show the estimation results of the $SVFJ$ model and $SVFJRJ$ model using the data of the second sample period. Table 13 confirms the conclusions of Table 6: the volatility of volatility parameters is high compared to reasonable levels which indicates misspecification of the $SVFJ$ model. Using the estimates in Table 14 the variance is decomposed in three parts. The average instantaneous volatility is estimated as 17.4% and the volatility that comes from the random jump part equals 16.12%. The fixed jump part of total volatility is 11.0% and hence the risk neutral fixed jump variance (again) captures approximately one third of total jump variation.

5 Conclusion

We present an option pricing model that explicitly models the influence of non trading overnight periods on option prices. We conclude that both random jumps during trading periods and the overnight jump are important in explaining observed option prices. We found that in two sample periods, of which the first can be characterized as a period of low volatility and the second as a period of high volatility, the added jump component covers a significant amount of the variation in the underlying value (risk neutral) process. In more detail, we found that the fixed jump part covers approximately one-third of total jump variation. Furthermore, the empirical results reveal that model including the overnight jump component gieves a better fit of empirical option prices than the traditional asset pricing model. Finally, we show that a model containing the overnight jump in combination with stochastic volatility has the same problem as the standard stochastic volatility model: the estimated volatility of volatility is too large in comparison to the volatility of volatility extracted from volatility series.

Hence, this paper concludes that total jump risk should be separated in random jump risk during the trading day and overnight jump risk for option pricing purposes. How the market prices these separate risk factors, is a topic that we leave for future research.

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A Option Pricing

We derive the theoretical formula for a plain vanilla call option given the risk neutral process in (2.4). Using Ito's Lemma, the stochastic differential for $\log S_t$ is,

$$d\log S_t = \left(cr - \frac{1}{2}\sigma_t^2\right)dt + \sigma_t d\tilde{W}_t^S + d\left(\sum_{i=1}^{\tilde{N}_t}\log\tilde{Y}_i\right) - \tilde{\lambda}\tilde{\mu}_{RJ}dt + d\left(\sum_{i=1}^{\lfloor 252t\rfloor}\log\tilde{V}_i\right).$$

Following Scott (1997), the call option value formula is given by,

$$\begin{aligned} C_t &= B_t E_t^{\mathbb{Q}}\left(\frac{\max\left(S_T - X, 0\right)}{B_T}\right) \\ &= e^{-cr(T-t)-n(1-c)r^o}\left\{E_t^{\mathbb{Q}}(S_T)P_1 - XP_2\right\}, \end{aligned}$$

where

$$P_1 = \int_X^\infty \frac{S_T}{E^{\mathbb{Q}}(S_T)} p^{\mathbb{Q}}(S_T) dS_T, \tag{A.1}$$

$$P_2 = P^{\mathbb{Q}}(S_T > X). \tag{A.2}$$

Since the probability density function is unknown under our assumptions regarding the evolution of stock and money market, Fourier inversion techniques are used to derive expressions for P_1 and P_2 (see for, Bakshi and Madan (2000)),

$$\begin{aligned}
P_1 &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left(\frac{\exp\left(-i\alpha\log X\right)\varphi\left(\alpha - i\right)}{i\alpha\varphi\left(-i\right)}\right) d\alpha, \\
P_2 &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left(\frac{\exp\left(-i\alpha\log X\right)\varphi\left(\alpha\right)}{i\alpha}\right) d\alpha,
\end{aligned}$$

where $\varphi(\alpha)$ denotes the characteristic function of the random variable $\log S_T$. Given the process for $\log S_t$ above, $\varphi(\alpha)$can be written as

$$\begin{aligned}
\varphi(\alpha) &= E_t^{\mathbb{Q}}\left\{\exp\left(i\alpha\log S_T\right)\right\}, \\
&= E_t^{\mathbb{Q}}\left\{\exp\left(i\alpha\left[\log S_t + cr\tau - \frac{1}{2}\int_t^T c\sigma_u^2 du + \int_t^T \sqrt{c}\sigma_u d\tilde{W}_u^S + \right.\right.\right. \\
&\quad \left.\left.\left. + \sum_{i=\tilde{N}_t}^{\tilde{N}_T} \log\tilde{Y}_i - \tilde{\lambda}\tilde{\mu}_{RJ}\tau + \sum_{i=\lfloor 252t\rfloor}^{\lfloor 252T\rfloor} \log\tilde{V}_i\right]\right)\right\}, \\
&= E_t^{\mathbb{Q}}\left\{\exp\left(i\alpha\left[\log S_t + cr\tau - \frac{1}{2}\int_t^T c\sigma_u^2 du + \int_t^T \sqrt{c}\sigma_u d\tilde{W}_u^S\right]\right)\right\} \times \\
&\quad E_t^{\mathbb{Q}}\left\{\exp\left(i\alpha\left[\sum_{i=\tilde{N}_t}^{\tilde{N}_T} \log\tilde{Y}_i - \tilde{\lambda}\tilde{\mu}_{RJ}\tau\right]\right)\right\} E_t^{\mathbb{Q}}\left\{\exp\left(i\alpha\sum_{i=\lfloor 252t\rfloor}^{\lfloor 252T\rfloor} \log\tilde{V}_i\right)\right\}.
\end{aligned}$$

The characteristic functions for the various parts will be derived separately. The first part is (apart from c) equal to formula (17) in Heston (1993), i.e,

$$\begin{aligned}
& E_t^{\mathbb{Q}}\left\{\exp\left(i\alpha\left(\log S_t + cr\tau - \frac{1}{2}\int_t^T c\sigma_u^2 du + \int_t^T \sqrt{c}\sigma_u d\tilde{W}_u^S\right)\right)\right\} \\
=\ & \exp\left(C\left(\tau;\alpha\right) + D\left(\tau;\alpha\right)c\sigma_t^2 + i\alpha\log S_t\right),
\end{aligned}$$

where

$$\begin{aligned} C(\tau;\alpha) &= cri\alpha\tau + \frac{c\kappa\sigma^2}{c\sigma_\sigma^2}\left\{\left(\kappa+\eta^V-\rho\sqrt{c}\sigma_\sigma i\alpha+d\right)\tau - 2\log\left(\frac{1-ge^{d\tau}}{1-g}\right)\right\}, \\ D(\tau;\alpha) &= \frac{\kappa+\eta^V-\rho\sqrt{c}\sigma_\sigma i\alpha+d}{c\sigma_\sigma^2}\frac{1-e^{d\tau}}{1-ge^{d\tau}}, \end{aligned}$$

and,

$$\begin{aligned} g &= \frac{\kappa+\eta^V-\rho\sqrt{c}\sigma_\sigma i\alpha+d}{\kappa+\eta^V-\rho\sqrt{c}\sigma_\sigma i\alpha-d}, \\ d &= \sqrt{\left(\rho\sqrt{c}\sigma_\sigma i\alpha-(\kappa+\eta^V)\right)^2+c\sigma_\sigma^2\left(i\alpha+\alpha^2\right)}. \end{aligned}$$

The random jump part of the model is described by means of a compensated compound Poisson process. The lognormal distribution of the jump sizes $\tilde{Y}_i$ determines the characteristic function as,

$$\begin{aligned} &E_t^{\mathbb{Q}}\left\{\exp\left(i\alpha\left[\sum_{i=\tilde{N}_t}^{\tilde{N}_T}\log\tilde{Y}_i-\tilde{\lambda}\tilde{\mu}_{RJ}\tau\right]\right)\right\}= \\ &\tilde{\lambda}\tau\left[(1+\tilde{\mu}_{RJ})^{i\alpha}\exp\left(\left(\frac{\alpha i}{2}\right)(i\alpha-1)\,\tilde{\sigma}_{RJ}^2\right)-1\right]-i\alpha\tilde{\lambda}\tilde{\mu}_{RJ}\tau. \end{aligned}$$

The expression for the characteristic function of the fixed jump part is more tractable since (in comparison to the random jump part) one source of randomness disappears. The characteristic function then can be calculated, using the lognormal jump sizes $\tilde{V}_i$, as

$$E_t^{\mathbb{Q}}\left\{\exp\left(i\alpha\sum_{i=\lfloor 252t\rfloor}^{\lfloor 252T\rfloor}\log\tilde{V}_i\right)\right\}=\exp\left(i\alpha(1-c)nr^o-\frac{1}{2}\alpha\left(\alpha+i\right)n\tilde{\sigma}_{FJ}^2\right).$$

where $n=\frac{\lfloor 252T\rfloor-\lfloor 252t\rfloor}{252}$. The characteristic function for the terminal stock price is determined and can be used to obtain P_2 in the option pricing formula.

In order to obtain P_1 observe the following lemma with $Y=\log S_T$.

Lemma A.1 *Let Y be a random variable whose distribution has density p and characteristic function φ and for which $E\{\exp(\alpha Y)\}<\infty$ for all $\alpha\in\mathbb{R}$. Define the distribution F by its survival function*

$$1-F(z)=\int_z^\infty\frac{\exp(y)}{E\{\exp(Y)\}}p(y)dy.$$

Then, F has characteristic function $\tilde{\varphi}$ with,

$$\tilde{\varphi}(\alpha)=\frac{\varphi(\alpha-i)}{E\{\exp(Y)\}}.$$

Proof. Let Z have distribution function F and density

$$f(z) = \frac{\exp(z)dz}{E\left\{\exp(Y)\right\}}.$$

Now

$$\begin{aligned}
\tilde{\varphi}\left(\alpha\right) &= E\exp\left(i\alpha Z\right) \\
&= \int_{-\infty}^{\infty} \exp\left(i\alpha z\right)\frac{\exp(z)p(z)}{E\left\{\exp(Y)\right\}}dz \\
&= \int_{-\infty}^{\infty} \frac{\exp\left(i\left(\alpha - i\right)z\right)}{E\left\{\exp(Y)\right\}}p(z)dz \\
&= \frac{E\exp\left\{i\left(\alpha - i\right)z\right\}}{E\left\{\exp(Y)\right\}} \\
&= \frac{\varphi\left(\alpha - i\right)}{E\left\{\exp(Y)\right\}}.
\end{aligned}$$

B Statistics and Results 1999-2003 sample period

Table 8 gives the sample statistics of S&P-500 returns between July 1999-November 2003:

	close-close	**close-open**	**open-close**
average	-0.0428	-0.0008	-0.0351
std.dev	0.2057	0.0786	0.1890
2.5 perc	-0.0252	-0.0108	-0.0238
97.5 perc	0.0267	0.0100	0.0242
skewness	0.1338	0.2543	0.2102
kurtosis	4.57	10.37	5.91

Table 8: Summary statistics S&P-500 returns July 1999-November 2003

Table 9 gives an overview of the call option Black-Scholes implied volatilities between July 1999-November 2003:

		Days to Expiration			
	Moneyness	<60	60-180	>180	**Subtotal**
ITM	< 0.97	0.319	0.277	0.252	
		12618	10918	2882	26418
ATM	0.97-1.03	0.222	0.225	0.238	
		7945	6834	2906	17685
OTM	> 1.03	0.302	0.234	0.208	
		13886	13742	2198	29826
Subtotal		34449	31494	7986	73929

Table 9: Summary statistics SPX call option implied volatilities. The sample period ranges from July 9, 1999 to November 27, 2003, respectively.

Table 10 gives an overview of the put option Black-Scholes implied volatilities between July 1999-November 2003:

		Days to Expiration			
	Moneyness	<60	60-180	>180	**Subtotal**
ITM	< 0.97	0.330	0.283	0.249	
		14321	12132	2965	29418
ATM	0.97-1.03	0.220	0.222	0.233	
		7942	6805	2906	17653
OTM	> 1.03	0.256	0.220	0.205	
		10223	10630	2161	23014
Subtotal		32486	29567	8032	70085

Table 10: Summary statistics SPX put option implied volatilities. The sample period ranges from July 9, 1999 to November 27, 2003, respectively.

Table 11 shows the summary statistics of the estimated parameters in the stochastic volatility model using option data from July 1999-November 2003:

	mean	**st.dev**	**2.50%**	**97.50%**
$\kappa\sigma^2$	0.041	0.016	0.040	0.041
$\kappa + \eta^V$	1.598	0.505	1.429	1.621
$\sqrt{\kappa\sigma^2/(\kappa + \eta^V)}$	0.159	0.015	0.157	0.167
σ_σ	0.868	0.187	0.312	0.993
ρ	-0.642	0.178	-1.000	-0.373
σ_t	0.248	0.053	0.175	0.381
SSE	0.695	0.656	0.036	2.592

Table 11: Summary statistics parameters SV model using option data from 1999-2003.

Table 12 shows the summary statistics of the estimated parameters in the stochastic volatility model including random jumps using option data from July 1999-November 2003:

	mean	**st.dev**	**2.5 perc**	**97.5 perc**
$\tilde{\mu}_{RJ}$	-0.115	0.076	-0.238	0.000
$\tilde{\sigma}^2_{RJ}$	0.029	0.038	0.000	0.132
$\tilde{\lambda}$	0.625	0.114	0.543	0.768
$\kappa\sigma^2$	0.053	0.035	0.039	0.173
$\kappa + \eta^V$	3.904	0.111	3.618	3.933
$\sqrt{\kappa\sigma^2/(\kappa + \eta^V)}$	0.113	0.031	0.100	0.220
σ_σ	0.393	0.112	0.085	0.622
ρ	-0.528	0.071	-0.771	-0.386
σ_t	0.202	0.050	0.132	0.317
SSE	0.216	0.312	0.010	1.020

Table 12: Summary statistics parameters SVRJ model using option data from 1999-2003.

Table 13 shows the summary statistics of the estimated parameters in the stochastic volatility model including overnight jumps using option data from July 1999-November 2003:

	mean	**st.dev**	**2.5 perc**	**97.5 perc**
$10^{-3} * \sigma^2_{FJ}$	11.29	13.41	0.004	40.61
$c\kappa\sigma^2$	0.042	0.011	0.040	0.045
$\kappa + \eta^V$	1.616	0.024	1.606	1.623
$\sqrt{c\kappa\sigma^2/(\kappa + \eta^V)}$	0.160	0.017	0.156	0.167
$\sqrt{c}\sigma_\sigma$	0.792	0.389	0.281	1.500
ρ	-0.885	0.147	-1.000	-0.459
$\sqrt{c}\sigma_t$	0.223	0.070	0.111	0.377
SSE	0.493	0.526	0.031	1.954

Table 13: Summary statistics SVFJ model using option data from 1999-2003.

Table 14 shows the summary statistics of the estimated parameters in the stochastic volatility model including random jumps and overnight jumps using option data from July 1999-November 2003:

	mean	**st.dev**	**2.5 perc**	**97.5 perc**
$\tilde{\mu}_{RJ}$	-0.085	0.058	-0.197	0.009
$10^{-2} * \tilde{\sigma}^2_{RJ}$	1.799	2.531	0.001	10.14
$\tilde{\lambda}$	1.178	1.045	0.209	3.622
$10^{-2} * \sigma^2_{FJ}$	1.199	1.144	0.002	3.822
$c\kappa\sigma^2$	0.042	0.035	0.004	0.169
$\kappa + \eta^V$	3.270	3.545	1.611	3.937
$\sqrt{c\kappa\sigma^2/(\kappa + \eta^V)}$	0.119	0.052	0.033	0.215
$\sqrt{c}\sigma_\sigma$	0.567	0.403	0.012	1.498
ρ	-0.696	0.221	-1.000	-0.266
$\sqrt{c}\sigma_t$	0.174	0.074	0.061	0.353
SSE	0.103	0.133	0.006	0.497

Table 14: Summary statistics parameters SVFJRJ model using option data from 1999-2003.