Pairs Trading: Different Weights, Methods and Markets

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Abstract

The profitability of self-financing portfolios using pairs trading is compared for the American, European and Brazilian stock markets with two different pairs selection methodologies: the distance method proposed by Gatev *et al.* (2006) and the cointegration method suggested by Alexander and Dimitriu (2002). For the United States, the portfolios formed through the distance method have a superior performance, with excess returns of up to 13,9% superior than the 3,9% return obtained by the cointegration method, with a statistically superior sharpe ratio. However, when we look at the european and brazilian markets, the portfolios formed through the cointegration tests perform better. For Europe, they exhibit an average annual excess return of up to 15,92%, with close to zero correlation with the market, and statistically superior to the distance method also in terms of Sharpe Ratio. When we consider the brazilian database the portfolios based on the cointegration method exhibits average annual excess return of up to 19%, while the distance method has an average excess return of up to 6%. They also perform better out of sample with higher net return and Sharpe Ratio. When we look at its betas and alphas we see that both strategies can be considered market neutral for all databasesand their alphas are significant, as seen for the distance method in the US and the cointegration approach in Europe and Brazil. Overall, the results show that different market structures favor different pairs trading strategies, with more simplified market structure such as the US favoring the distance method, while a more segmented and with different share classes favoring the cointegration method.

1 Introduction

The computational advances of the past decades have stimulated the development of trading via computer programs and the rise of algorithmic trading. These systems are designed to search for patterns in financial markets, detect deviation of market prices from these patterns, and profit from detected anomalies. Algorithmic trading is now responsible for more than 70 percent of the trading volume in the the US markets Hendershott *et al.* (2011). On the other hand, events like the Flash Crash of May 6 2010, when the Dow Jones Industrial Average dropped 600 points in less than 5 minutes, revealed the lack of knowledge about the consequences and robustness of algorithms used in practice Nuti *et al.* (2011).

Advances in data storage and access have also opened interesting research possibilities. The availability of "big" data sets allows a more robust detection of statistical arbitrage opportunities within and across markets, allowing a more comprehensive evaluation of the effectiveness of trading algorithms. This paper proposes the use of large datasets from stock markets in the USA, Europe and Brazil to test two popular pairs trading algorithms out-of-sample, namely: the sum of squared deviations approach of Gatev *et al.* (2006), and the cointegration approach suggested by Alexander and Dimitriu (2002). The markets selected allow the analysis of both strategies under different market conditions in an attempt to uncover differences between trading algorithms.

Pairs trading is an algorithmic trading strategy designed to exploit short-term deviations from an existing long-run equilibrium between two stocks. However, different methods have been proposed in the literature to identify pairs to be traded Vidyamurthy (2004); Alexander and Dimitriu

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(2005); Elliott *et al.* (2005); Gatev *et al.* (2006); Caldeira and Moura (2013). The motivation for trading pairs has its roots in works that preach the existence of long term relation between stocks. If there exists indeed a long term equilibrium, deviation from this relation are expected to revert. Since future observations of a mean-reverting time series can potentially be forecasted using historical data, this literature challenges the notion that stock prices cannot be predicted Lo and MacKinlay (1988, 1997); Guidolin *et al.* (2009). Active asset allocation strategies based on mean-reverting portfolios, which generally fall under the umbrella of statistical arbitrage, have been used by investment banks and hedge funds for several years Gatev *et al.* (2006). The word statistical in context of an investment approach is an indication of the speculative character of investment strategy. It is based on the assumption that the patterns observed in the past are going to be repeated in the future. This is in opposition to the fundamental investment strategy that both explores and predicts the behaviour of economic forces that influence the share prices. Pairs trading is possibly one of the simplest statistical arbitrage strategy, since it consists of a portfolio of only two assets. In this approach, we are not interested about trends for particular assets but with a common trend among a pair of stocks, which defines a long-run equilibrium between them. The idea behind pairs trading is that when prices of two shares move together there could be short term deviations to be arbitraged. Thus, this trading strategy consists in detecting pairs of stocks that historically move together, waiting for the spread between them to widen, longing the underpriced stock and shorting the overpriced one to profit when prices revert back to their long-run equilibrium.

Thus, pairs trading is a purely statistical approach designed to exploit equity market inefficiencies defined as the deviation from a long-term equilibrium across stock prices observed in the past. As argued by Do and Faff (2010), pairs trading falls under the big umbrella of the long-short investing approach. According to Avellaneda and Lee (2010) the term statistical arbitrage includes investment strategies that have certain characteristics in common: (i) trading signals follow a systematic rule, in opposition to fundamentals based strategies; (ii) strategies seek to be market-neutral, in the sense that they are not exposed to broad market risk, i.e, they have a zero beta; (iii) the mechanism used to obtain abnormal returns is based on statistical analysis. The success of pairs trading, especially statistical arbitrage strategies, depends heavily on the modeling and forecasting of the spread time series although fundamental insights can aid in the pre-selection step. Pairs trading needs not be market neutral although some say it is a particular implementation of market neutral investing Jacobs *et al.* (1999).

Pairs trading strategies speculate on future convergence of spread between similar securities. Similarity concerns industry, sector, market capitalization, and other common exposures that might imply a comovement between stocks. However a profitable strategy might also be constructed with stocks covering different sectors based purely on statistical properties of the time series. Gatev *et al.* (2006) test a simple non-parametric pairs trading algorithm on the US market between 1963 and 2002, finding average annualized returns of up to 11% for portfolios of pairs. They suggested that the abnormal returns to pairs strategies were a compensation to arbitrageurs for enforcing the law of one price. Another popular algorithm to select pairs is based on the presence of a cointegration relation between stock prices (see Vidyamurthy (2004) for a textbook treament of the subject. The use of the cointegration technique to asset allocation was pioneered by Lucas (1997) and Alexander (1999) and in the previous decade it was increasingly applied in financial econometrics (see, among other, Alexander and Dimitriu (2002); Bessler and Yang (2003); Yang *et al.* (2004); Caldeira and Moura (2013); Galenko *et al.* (2012); Gatarek *et al.* (2014).

Many studies attempted to test the profitability of pairs trading strategies. However, all of them focus on either a single market studies or on a single methodology to select pairs. Do and Faff (2012) examined the impact of trading costs on pairs trading profitability in the U.S. equity market and documented that after 2002 pairs trading strategies were largely unprofitable. Bowen *et al.* (2010) back-test a pairs trading algorithm using intraday data over a twelve month period in 2007, and conclude that returns are highly sensitive to the speed of execution. Moreover, accounting for transaction costs and enforcing a "wait one period" restriction, excess returns are complete eliminated. Broussard and Vaihekoski (2012) tested the profitability of pairs trading under different weighting structures and trade initiation conditions using data from the Finnish stock market. Although the proposed strategy is profitable, the authors note that returns have declined in recent years possible due to increased competition among hedge funds, and/or a reduction in the importance of an underlying common factor that drives the returns in a pairs trading strategy.

The datasets used in these analysis can be divided into two groups: First, the data comes from developed countries which have plenty of historical financial information available, as is the case of the United States. The articles by Gatev *et al.* (2006); Engelberg *et al.* (2009); Huck (2010); Do and Faff (2010); Bowen *et al.* (2010); Do and Faff (2012) are examples that use data from the United States. The second group includes datasets from developing countries. These studies analyse shorter time periods and a smaller number of assets in the database. Yuksel *et al.* (2010) analyse pairs trading in Turkey, Broussard and Vaihekoski (2012) in Finland and Perlin (2009); Caldeira and Moura (2013) in Brazil.

Although many papers have been written about pairs trading, the literature lacks a comprehensive study of the performance of different methodology across developed and emerging markets. Moreover, most studies use different trading periods, different criteria to select assets to be included in the sample and different formation period, rendering a cross study comparison impossible. Since pairs trading performance is influenced by the methodology chosen, it is important to compare them under different circumstances to understand if there is a overall winner, or if some strategies are better suited to specific market conditions. We, on the other hand, use the database of an emerging country Brazil, a developed monetary union, the Euro Area, and an important world financial center, the USA, along with equal parameters for each methodology in order to compare them and test to see which is more suitable to which environment. The cointegration method has shown results statistically superior for the Sharpe Ratio against the distance method for the Euro Area and Brazilian markets studied for all 3 portfolios formed, while the distance method had a superior performance for the American market, signaling that the different market structures affect the performance of each methodology.

The article is organized as follows. In the next section we described both pairs trading methodologies analyzed and some. Section 3 presents some implementation details common to both methods, as well as the evaluation strategy. Section 4 describes the three large data sets used and discusses the results of our comparison. Section 5 evaluates the performance of both methodologies and the last section concludes with some final remarks.

2 Pairs Trading Methodologies

Broadly defined, there are three different approaches to pairs trading: the distance approach, the stochastic approach and the cointegration approach. These methods all vary with regard to how the spread of the stock pairs is defined. This paper compares two most popular methods of selecting pairs of stocks between practitioners and researchers: the distance method proposed by Gatev *et al.* (2006) and the cointegration approach used in Lucas (1997), Alexander and Dimitriu (2005), Do *et al.* (2006), and Caldeira and Moura (2013). The two approaches used in this paper will be discussed in the next subsection.

2.1 The Distance Approach

The distance approach is proposed by Gatev *et al.* (2006) and is used among others by Andrade *et al.* (2005), Engelberg *et al.* (2009), Do and Faff (2010), Bowen *et al.* (2010), and Broussard and Vaihekoski (2012). By this approach the co-movement in the pair is measured by the *distance*, which is defined as the sum of squared deviations (SSD) between the two normalized price series. Normalized price series are defined to start from one, and then evolve using the return series. The normalized price series for a stock is given by its cumulative total returns index over the moving formation period of 252 days. Formally, we compute

$$\tilde{P}_{ti} = \prod_{\tau=1}^{t} (1 + r_{\tau i}) \tag{1}$$

where $\tilde{P_{it}}$ is the normalized price of stock i at time t, $r_{i\tau}$ is the dividend-adjusted return of stock i at time τ, and τ is the index for all trading days between $t - 252$ and t. The normalized series begin the observation period with a value equal to one, and increases or decreases each day given its return. For each stock i, we find the stock j that minimizes the sum of square deviations between the two normalized price series. The distance is thus defined as

$$\Delta_t^{ij} = \sum_{t=1}^{252} \left(\tilde{P}_{ti} - \tilde{P}_{tj} \right)^2 \tag{2}$$

where Δ_t^{ij} is the distance between the normalized prices of stock i and j over the formation period. This means that pairs are formed by exhaustive matching in normalized price space, where price is the daily closing price adjusted for dividends and splits. We rank all possible pairs by distance, identify the combinations with the highest measure of co-movement and monitor these pairs for the duration of the trading period. Similar to Gatev *et al.* (2006), we set the periodicity of pair updates to 6 months.

In order to select a pair for a given stock, we search on the database for an asset whose normalized price has the smallest squared distance to the normalized price of the chosen stock up to time t. A long-short position is opened when the distance exceeds a pre-specified threshold[1]based on a standard deviation metric. Following Gatev *et al.* (2006), the signal to start trading occurs when the distance between the normalized price diverges by more than two standard deviations. An open long-short position is closed either upon convergence in normalized prices or at the end of the trading period. The latter imposes a restriction on the investment horizon and works as an automatic risk control mechanism.

The distance approach is a model free approach and non-parametrically exploits a statistical relationship among two stocks prices. From a practical point of view, the distance method is easy to implement and independent of economic models, which avoids misspecification problems. On the other hand, non-parametric strategies have lower prediction ability compared to well-specified parametric models. The fundamental assumption of this approach is that pair spreads exhibit mean-reversion. Accordingly, a price-level divergence is an indication of disequilibrium and price distance is the measure of mispricing.

2.2 The Cointegration Approach

The use of the cointegration technique to asset allocation was pioneered by Lucas (1997) and Alexander (1999) and in the previous decade it was increasingly applied in financial econometrics (see, among others, Alexander and Dimitriu (2002); Bessler and Yang (2003); Yang *et al.* (2004). Cointegration is an extremely powerful technique, which allows dynamic modelling of non-stationary time-series sharing a common stochastic trend. The fundamental observation that justifies the application of the concept of cointegration to the analysis of stock prices is that a system involving non-stationary stock prices in levels can display a common stochastic trend Gatev *et al.* (2006). When compared to the concept of correlation, the main advantage of cointegration is that it enables the use of the information contained in the levels of financial variables.

Similar to the previous trading strategy, the main concern of the cointegration approach is the mean reversion of the spread. However, instead of defining the spread as the distance between standardized prices of a pair of stocks, the spread is defined with respect to the long-run equilibrium of a cointegrated system; that is, the long-run mean of the linear combination of two time series Vidyamurthy (2004). Deviations from the equilibrium should revert to the long-run mean, implying that one or both time series should adjust in order to restore the equilibrium.

Using cointegration as a theoretical basis, the spread is generated based on the actual error term of the long-run relation:

$$log(P_{it}) - \gamma log(P_{jt}) = \mu + \varepsilon_t \tag{3}$$

where γ is the cointegration coefficient, the constant term μ captures a possible premium in stock i versus stock j, and ε_t is the estimated error term. Thus, it is not needed to predict P_t^i

[1]The threshold can be constructed in a variety of ways, but the most common method is to select some proportion of the historical standard deviation of the spread:

$$q = \delta\sigma_{\mathsf{spread}}$$

Gatev *et al.* (2006),Andrade *et al.* (2005) and Do and Faff (2010) set $\delta = 2$, whereas Bowen *et al.* (2010) and Broussard and Vaihekoski (2012) experiment with a range of values. It is also possible to let q be a variable by defining δ as a rolling parameter with window size n; this may allow us to better capture the profit potential of periods with higher volatility in the spread.

and P_t^j, but only their difference $\log\left(P_t^i\right) - \log(P_t^j)$. If we assume that $\left\{\log\left(P_t^i\right), \log(P_t^j)\right\}$ in 3 is a non-stationary VAR(p) process, and there exists a value γ such that $\log\left(P_t^i\right) - \gamma\log(P_t^j)$ is stationary, we will have a cointegrated pair.

For detected cointegrating relations, the algorithm creates trading signals based on predefined investment decision rules. In order to implement the strategy we need to determine when to open and when to close a position. First, we calculate the spread between the shares. The spread is calculated as

$$\varepsilon_t = \log\left(P_t^i\right) - \gamma\log\left(P_t^j\right) - \mu, \tag{4}$$

where ε_t is the value of the spread at time t. Accordingly, we compute the dimensionless z-score defined as

$$z_t = \frac{\varepsilon_t - \mu_\varepsilon}{\sigma_\varepsilon}, \tag{5}$$

the z-score measures the distance to the long-term mean in units of long-term standard deviation. After selecting the most appropriate pairs, the same trading strategy used under the distance approach is executed using the z-score series instead. This method is based on Vidyamurthy (2004); Avellaneda and Lee (2010) and Caldeira and Moura (2013). It is an attempt to parametrize the long-term relationship between two assets and explore price-deviations from their historical relationship using cointegration. Even if two time series are non-stationary, cointegration implies the possibility that a linear combination of both series could be stationary. If this is indeed the case, both series move "closely together" as if they were connected to each other.

The quality of estimation of the correction error model depends on the econometric technique applied. The first method for testing cointegration by Engle and Granger (1987) is a two step procedure in which the first step, stationarity test of the residuals errors, renders results sensitive to the ordering of the variables, and such mispecification error is carried to the second step, the error correction model estimation. The way found to reduce this error is to use two cointegration tests. Besides the Engle and Granger (1987) we also used the Johansen (1988) test, and use only the pairs that are considered cointegrated by both tests. Nonetheless Engle and Granger (1991) well-known limitations (small sample problems, maximum of one cointegrating vector, treating the variables assymetricaly) are not an issue in this work, due to our samples having 252 observations, only two variables are included in the estimation procedure, and it is only possible to find one cointegrating vector.

3 Implementation Details

In this work we follow the methodology by Gatev *et al.* (2006) and Broussard and Vaihekoski (2012) to implement the distance method and the methodology used by Caldeira and Moura (2013) and Vidyamurthy (2004) in the implementation of cointegration methodology. The formation period for the pairs is 12 months long, and the trading period comprises the following 6 months. The pairs of assets are selected by minimizing the sum of squared deviations in the portfolios formed from the distance method and ranked beginning from the smallest sum of squared deviations. In portfolios formed from the cointegration method, the pairs are selected if they are found cointegrated with both tests, Engle and Granger (1987) and Johansen (1988), and later ranked by their Sharpe index within the sample as in Caldeira and Moura (2013).

Next, portfolios are formed with 5, 10, and 20 pairs for each methodology, and are used in the trading period in the 6 months following the formation of pairs. At the end of each period of trading all positions are closed. A new 12 month period for the pairs formation is created and ends on the last observation of the previous trading period, when all cointegration tests and pairing are redone. The assets to be used must be traded in the 12 month formation period, but not necessarily they will be listed during the 6 month trading period.

In order to generate trading signals, it is necessary to calculate the distance between the asset prices in the pair, measured by the spread $\varepsilon_t = P_t^l - \gamma P_t^s$, where ε_t is the spread value at time t.

From the spread, the distance measure is given by the formula $z_t = \frac{\varepsilon_t - \mu_\varepsilon}{\sigma_\varepsilon}$. The goal is to identify when z_t departs from the long term average, given by the error correction model, measured in terms of standard deviation. Initially, the position opens when $|z_t| > 2$ and closes when $z_t = 0$.

Let P_t^l be the long asset price and P_t^s the price of the asset sold short, then the net return in t of pair i is given by:

$$r_{it}^{raw} = ln\left[\frac{P_t^l}{P_{t-1}^l}\right] - \gamma\left[\frac{P_t^s}{P_{t-1}^s}\right] + 2ln\left(\frac{1-C}{1+C}\right) \tag{6}$$

[2]This equation already includes transaction costs in its second term. To calculate the net return of a portfolio with N pairs, we do the weighted average net returns of each pair, with the weight defined by the percentage of the amount invested in each pair with respect to the value of the portfolio in time t. Let p be a portfolio with N pairs, where w_i is the weight for each pair i. Thus, the net return of the portfolio in t is $R_t^p = \sum_{i=1}^N w_{it}R_{it}$. As explained in the Caldeira and Moura (2013), the calculation of compound return (log returns) of a portfolio of assets is, for small values, close to the weighted average of the continuously compounded returns for each asset i.e, $R_t^p \cong \sum_{i=1}^N w_{it}r_{it}$. However, to calculate the return accurately, log-returns are transformed back to simple return, with the monthly compound rate of return, r_{it} given by:

$$r_{it} = ln(1 + R_{it}) = ln\left(\frac{P_t}{P_{t-1}}\right), \tag{7}$$

to transform back we just multiply by e to remove the logarithm and obtain the net return R_{it}. $e^{r_{it}} = 1 + R_{it} \Longrightarrow R_{it} = e^{r_{it}} - 1$.

From this net return of the portfolio equation, we used two weighting scheme of returns as in Gatev *et al.* (2006); Broussard and Vaihekoski (2012). The first scheme used is the weighting of the returns to the capital previously committed (committed capital scheme), in which an amount of capital is distributed evenly across the entire universe of pairs for the period. Even if the pair does not open or if it closes before the trading period finishes, capital remains committed to that pair. This scheme divides the payoff in pairs for all pairs that were selected for the period of trading. This method considers the opportunity cost of hedge funds when they commit resources on a pair that ends up not being used during trading. The second scheme used divides the returns by the number of pairs that are open at th moment the return was obtained (fully invested scheme). In other words, this is a less conservative scheme wgere the portfolios resources are are all invested in the pairs that are currently open. We are conservative and assume a rate of return of zero for capital in pairs that are not open, as in Broussard and Vaihekoski (2012), and unlike Gatev *et al.* (2006), which assumes a risk-free rate of return.

The change in the weights of the pairs within the portfolio follows the method of equal weights (Equally weighted approach), defined as in Broussard and Vaihekoski (2012), although we do use the value weighted method and find similar results, hence, we do not report them, but are readily available upon request. The sum of returns of each pair is divided by the number of pairs that were selected for the period of trading, in the committed capital scheme. In practice, the use of stop-loss is critical to minimize losses. However, most academic works on pairs trading don't use them. Exceptions are Nath (2006), and Caldeira and Moura (2013), and in this work we follow the method of Caldeira and Moura (2013) and the stop-loss is triggered and the position in the pair is closed when losses reach 10% and we also include a stop gain of 20% with other values being tested.[3]

Transaction costs considered follow Dunis *et al.* (2010)Dunis et al. (2010) and Caldeira and Moura (2013) and total 0.4% for each change of position in the pair (opening and closing): 0.1% brokerage in total for each action (buying and selling), totaling 0.2% for each pair in brokerage costs. Slippage of 0.05% for each stock in the pair, and 0.2% for the lease of the asset to be sold

[2]This formula can be explained intuitively. Suppose we buy stock ξ at price P_{t-1}^ξ at time $t-1$ and sell it at time t at price P_t^ξ. Including transaction costs, the cost of buying is $P_{t-1}^\xi(1+C)$ and the profit of selling is $P_t^\xi(1-C)$. This corresponds to the decomposed net return:

$ln\left[\frac{P_t^\xi(1-C)}{P_{t-1}^\xi(1+C)}\right] = ln\left[\frac{P_t^\xi}{P_{t-1}^\xi}\right] + ln\left[\frac{(1-C)}{(1+C)}\right] = r_t^\xi + ln\left[\frac{(1-C)}{(1+C)}\right]$

[3]We considered values of 5%, 7%, 10%, 15%, 20% and no trigger for the stop-loss. For the stop-gain we use 5%, 10%, 15% and 20% trigger, as well as no trigger.

short (divided in 0.1% for opening and 0.1% when closing the position). The performance of the pairs portfolios is measured from 4 statistics:

$$Cumulative\ Returns:\ R^A = 252 \times \left(\frac{1}{T} \sum_{t=1}^{T} R_t \right) \tag{8}$$

$$Variance\ of\ Returns:\ \hat{\sigma}^A = \sqrt{252} \times \left(\frac{1}{T} \sum_{t=1}^{T} (R_t - \hat{\mu})^2 \right) \tag{9}$$

$$Sharpe\ Index:\ SR = \frac{\hat{\mu}}{\hat{\sigma}},\ where\ \hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} w_{it} R_{wit} \tag{10}$$

$$Maximun\ Drawdown:\ MDD = \sup_{t \epsilon [0,T]} \left[\sup_{t \epsilon [0,t]} R_s - R_t \right] \tag{11}$$

4 Database

Previous pairs trading studies aimed at testing a specific methodology for a given stock market. However, the availability of "big" financial datasets across the globe allows the researcher to expand the analysis to markets with diferent characteristics, allowing a more robust evaluation of the strategies. Our data comprises three highly liquid financial markets described below: United States Stock Market, Euro Area Stock Markets and the Brazilian Stock Market.

The North American database was obtained at CRSP and contains the most liquid stocks for every 10 year period, totalling 4,471 stocks. The period analyzed goes from 1962 to 2012 comprising a total of 12,586 observations and the market index used is the S&P 500, that is available for the whole sample period. We did not limit the universe with which each stock could pair up. The North American database is the most homogenous one with stocks being traded in stock exchanges close to each other. Also, all stocks considered for trade are ordinary stocks and consequently, unlike for the brazilian and the euro area markets, where pairs can be from the same company due to diferent class shares, in the U.S all stock pairs will necessarily be from different companies, allowing for the comparison of each methodology on diferent institutional environments and types of stocks selected. Finally, from an academic point of view, despite considerable theory about market eciency, not enough empirical information is known regarding how eficiency arises in practice.

The European Union dataset has stocks from companies based in Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain and data from the main stock exchanges based on all those countries, hence we use the MSCI Europe excluding UK and Switzerland Index. This dataset is relevant because the monetary union through the Euro currency eliminates the exchange rate variation. Also, it allows direct arbitrage between diferent countries and diferent stock exchanges within countries. The EU dataset contains daily data from the 1,000 most liquid stocks from 1973 to 2012. All the data is quoted in Euro converted by the conversion rates of the moment the country entered the monetary Union, directly done by the data provider Datastream at it's source. The data was obtained from Datastream comprising 10,435 observations. All countries in the euro area were considered for the sample, however not all countries have stocks in the 1000 most liquid for the sample period.

There are conjunctural and structural reasons to analyze pairs trading in the brazilian financial markets. A considerable amount of international crisis caught Brazil off guard during the 90's and the beginning of the 2000's decade. We use database between 1995 and 2012, which had an amount of 4 international crisis that hit Brazil,[4] and also 2 internal crisis[5] that directly affected the brazilian financial markets. To evaluate pairs trading in such stressful financial conditions provides valuable insight to how this strategy can act as a risk management alternative to other trading strategies, not to mention in a reduced liquidity environment compared to the U.S. Also, brazilian listed companies tend to have preferred and common stocks listed, which is an uncommon characteristic allowing

[4] 1Asian financial crisis in 1997. Russian moratorium and rublo devaluation in 1998. Argentinian moratorium and peso devaluation in 2001 and the U.S. financial crisis in 2008.

[5] Brazilian real devaluation in 1999 and 2002 election of former president Lula.

Table 1: Descriptive statistics of the datasets

Datasets	US data	European Data	Brazilian Data
Start date	Jan-1962	Jan-1973	Jan-1995
End date	Dec-2012	Dec-2012	Dec-2012
Number of stocks	4,471	1,000	450
Number of observations in the sample	12,586	10,435	4,087
Number of training periods	100	80	34
Average number of days in formation period	252	260	247
Average number of days in trading period	126	130	123

closer examination of causes of potential price deviation between assets that have rights to the same cash flow source. The database used in this study consists of all daily closing prices for stocks that have daily trading during the 12 month formation period and are listed in the Bolsa de Valores de Sï¿œo Paulo (Bovespa). The data were obtained from Economï¿œtica for the period between 1995 and 2012, and are adjusted for dividends and splits, in order to avoid false trading signals. The market index considered is the Ibovespa for the whole sample period. Table 4 summarizes the descriptive statistics of the database.

5 Estimation and Results

5.1 Results for the US stock market

5.1.1 Descriptive Statistics

Table 2 shows the results found for the American data set for the whole sample period between january 1962 and december 2012. The average number of pairs opened increases with the number of pairs in the portfolio due to the fact that there are more available pairs that can open at any given moment. However, the distance methodology is much more "trigger happy" than the cointegration method, opening almost double the number of pairs for each different size of portfolio. Nonetheless, it's very interesting to note that the average time the pairs are open is very similar for both strategies, which indicates that they react similarly, but the distance method identifies more trading opportunities. That also reflects on the share of negative excess returns, which is higher for the distance method on the 5 and 10 pairs portfolios. Since the distance method opens more pairs, the situation where the returns are 0, due to the lack of any open pairs, occur less frequent. On the other hand, the cointegration approach opens pairs less frequently, and has more zero return days that does not count as negative returns, which reduces the share of negative excess returns. Hence, even though the distance method performs better, as seen on the next section, it has a tendency of having a higher share of returns below zero.

The average number of round-trip trades per pairs, i. e., the average of pairs that manage to open and close before the closing of the 6 month period is higher for the distance method, for all portfolios sizes. Each pair opens and closes on average between 4,2 and 3,7 times for the distance portfolios while the cointegration method has an average of round trips per pair of 2,3. It's interesting to note that the average time pairs are open is very similar, but the standard deviation is higher for the distance approach, suggesting that this method is less selective on when to open the pairs and that they may take a longer time to converge or they close too rapidly.

5.1.2 Committed Capital for the US

This section uses the Committed Capital weighting scheme, where the returns of the pairs in the portfolio are weighted against all pairs, not only the open pairs at the moment the return was realized. It's cleary a more conservative measure and it takes into account the opportunity cost of committing capital to a given strategy even if there are no trades. Table 3 summarizes the excess

Table 2: Summary statistics of unrestricted pairs trading for the USA dataset

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1962 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

Methodology	Distance Approach			Cointegration Approach		
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Total number of pairs opened	2147	3941	7432	1184	2263	4670
Total number of 6 month trading periods	100	100	100	100	100	100
Mean price deviation for opening pairs	0,026	0,028	0,030	0,406	0,409	0,409
Mean Nï¿œ of pairs opened each 6 month period	21,47	39,41	74,32	11,84	22,63	46,7
Mean Nï¿œ of pairs traded when at least one pair opened	4,302	3,944	3,717	2,368	2,265	2,339
Average Nï¿œ of round-trip trades per pair	4,294	3,941	3,716	2,368	2,263	2,335
Standard deviation of round-trips per pair	2,455	2,319	2,22	1,5458	1,479	1,549
Average time pairs are open in days	16,886	19,405	21,441	16,945	17,283	17,24
Median time pairs are open in days	7	8	10	11	11	11
Average time pairs are open in months	0,804	0,924	1,021	0,806	0,823	0,82
Standard deviation of time open per pair in days	24,239	26,479	28,521	18,110	18,331	18,469
Standard deviation of time open per pair in months	1,154	1,261	1,358	0,862	0,872	0,879
Share of negative excess returns	0,446	0,459	0,441	0,379	0,442	0,469

Table 3: Comparison of strategies for unrestricted pairs trading for the USA dataset

Note: This table reports a summary statistics of the excess returns on portfolios of pairs between January 1962 and December 2012 for the Committed Capital weighting scheme. Pairs are formed over a 12-month period according to a distance and a cointegration criterion and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted weighting scheme.

Methodology	Distance Approach			Cointegration Approach		
Weighting Scheme	Committed Capital					
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	13,937	12,913	12,472	3,956	1,911	3,708
Average Annualized Volatility (in %)	7,247	5,5	4,238	11,527	8,084	5,946
Total Sample Sharpe Ratio	1,800	2,208	2,773	0,336	0,234	0,612
Largest Daily Return (in %)	6,403	6,470	4,327	7,64	3,841	2,657
Lowest Daily Return (in %)	-3,033	-3,224	-2,781	-7,57	-4,323	-2,879
Cumulative Profit (in %)	59.257,77	39.853,71	33.787,045	398,28	118,66	464,39
Spearman Rho	0,016	0,028	0,031	0,012	0,003	0,015
CAPM beta (β-market)	0,024	0,0187	0,016	0,033	0,021	0,023
Jensen's Alpha	0,0005	0,0004	0,0004	0,00014	0,0006	0,0001
Jense's Alpha p.value	0,00	0,00	0,00	0,0263	0,129	0,0003
Annual Skewness	1,297	0,553	0,391	0,680	0,298	0,459
Annual Kurtosis	5,298	2,775	2,364	3,388	2,822	2,756
Total Sample MDD (in %)	24,269	22,30	20,63	34,23	37,380	14,77

return of the pairs portfolios with opening positions at the end of the day that ocurrs the price divergence and closing positions at the end of the day that prices converge.

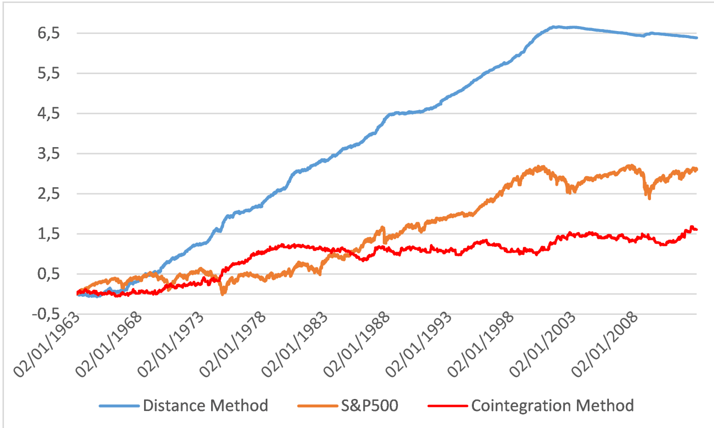
The average annualized return for all portfolios created through the distance method have over 10% yearly return, similar to what Gatev *et al.* (2006) found. The cointegration method performance is lower, with a sharpe ratio ranging between 0,2 and 0,6, while the distance method has a sharpe ratio of up to 2,7. The superior performance of the latter is a direct consequence of not only its higher average return, but also a lower volatility. Both strategies correlation with the market, as seen through the CAPM beta and the Spearman Rho, are not significantly different from zero, exactly what we want and would expect from a self-financed market neutral strategy. Although the beta and rho are important measures of risk, we also considered the maximun drawdown for all porfolios. This is a simple measure thay indicates the largest cumulative loss after a given maximun of a cumulative positive rallying of the returns, signaling how fast the leverage can increase. In the 50 years of the data spam the biggest cumulative loss ranges between 20% and 24% for the distance method and between 14% to 27% for the cointegration approach. Both are considerably inferior to the S&P 500 maximun drawdown of 56%.

It is interesting to see the daily returns as measured by the Jensen's alpha. The method of measuring the excess returns is solely through the returns obtained by the pairs, without being necessary to account for the risk free return, since, in theory, pairs trading are not supposed to have any positive performance. However, in order to be more conservative, we measure Jensen's alpha using 3 months U.S. Treasury Bills as the risk free rate and the market beta using the S&P500 hundred. Even so, the alphas produced by this strategy are statistically significant for all portfolio sizes for the distance methods and for the portfolios with 5 and 20 pairs for the cointegration approach.

Figure 5.1.2 shows the cumulative excess returns of both methodologies using the 5 pairs portfolios. Its very interesting to note that the performance of the distance method is consistente throughout the whole period, in contrast with the volatility of the S&P500 and the cointegration

Figure 1: Cumulative Returns for the USA

Note: This table reports a summary statistics of the cumulative excess returns for the USA in log for the 5 pairs portfolios between January 1963 and December 2012 using the distance method and the cointegration approach using the committed capital weighting scheme.



method. The distance method performs well during difficult times in the U.S and its performance is stable until 2003, then after it stops having a positive performance until the end of the sample in 2012. On the other hand the cointegration approach has a relatively good period during the 70's but its performance is relatively flat for the rest of the period with small cycles of ups and downs that tend to cancel each other.

5.1.3 Fully Invested for the US

The fully invested portfolios are presented on table 4 and show a slightly different picture than the committed capital portfolios. These portfolios have necessarily an equal or higher average return than the committed capital for all portfolio sizes due to its construction methodology. This weighting scheme scales the payoffs by the number of pairs that actually open, implying that all the money is distributed between all open pairs. The distance method is still superior to the cointegration method, with average annualized returns between 22% and 26% while the cointegration method returns are between 9 and 18%. However, its volatility is proportionally higher for both methods and portfolio sizes, and in turn, the sharpe ratios are very similar to the ones found previously. Another consequence of the higher volaility is its impact on the maximun drawdown. All distance method portfolios MDD increased and reached up to 38%, while for the cointegration method the MDD is up to 75%, superior than the S&P500 56% MDD.

These portfolios have a higher average return and volatility, but its betas and rhos are still negligible, and these portfolios can be considered market neutral. This indicates that these portfolios volatility are not based on its market risk, but rather on its own decision on when to open and close pairs and consequently can be included with an index tracking strategy without increasing the portfolios risk. The alphas obtained by the distance approach are higher than the cointegration method generates, but they all are statistically significant, being possible to conclude that both strategies can generate positive performance.

5.1.4 Returns on the long and short side

The inspection of the performance of both sides of the strategy in a separate manner is potentially interesting and is depicted in table 5. Since one of the aspects that makes pairs trading profitable is its mean reversion characteristic, we should expect the excess returns of the long and short side to be equal. Another reason is that if the short side of the strategy is the one driving the results, the short selling barriers and market depth for such instrument should be considered in order to assess why there still exist a profit to be obtained. The results show that for both methods and weighting schemes the long side has a superior return, which are the stocks that have lost value relative to their pairs before the strategy is started. For the cointegration method this difference is even more pronounced, while for the distance approach both sides have more similar performances, although

Table 4: Comparison of strategies for unrestricted pairs trading for the USA dataset
Note: This table reports a summary statistics of the excess returns on portfolios of pairs between January 1962 and December 2012 for the Fully Invested weighting scheme. Pairs are formed over a 12-month period according to a distance and a cointegration criterion and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted weighting scheme.

Methodology	Distance Approach			Cointegration Approach		
Weighting Scheme	Fully invested					
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	26,357	23,197	22,233	14,138	9,602	18,12
Average Annualized Volatility (in %)	13,341	9,805	7,369	31,560	27,954	23,476
Total Sample Sharpe Ratio	1,754	2,128	2,725	0,419	0,328	0,709
Largest Daily Return (in %)	10,673	8,088	6,658	32,14	21,626	34,648
Lowest Daily Return (in %)	-6,434	-6,449	-4,038	-18,92	-17,381	-23,427
Cumulative Profit (in %)	7.624.605,90	2.636.926,02	1.976.508,63	6.128,26	1.285,16	10.4530,87
Spearman Rho	0,015	0,027	0,03	0,011	0,001	0,012
CAPM beta (β-market)	0,038	0,028	0,028	0,083	0,052	0,068
Jensen's Alpha	0,0009	0,0008	0,0007	0,0004	0,0003	0,0006
Jense's Alpha p.value	0,00	0,00	0,00	0,0048	0,026	0,00001
Annual Skewness	1,506	0,947	0,628	2,049	1,162	7,423
Annual Kurtosis	5,363	3,476	2,55	10,072	4,596	65,540
Total Sample MDD (in %)	38,408	33,420	28,295	74,21	75,071	42,81

the short side still performs slightly worse. This provides evidence that the returns re nor driven by non realized profit opportunities created by short selling barriers.

5.1.5 Without Transaction Costs

The analysis of the performance without transaction costs can be very elucidating. If there are no costs to open or close any position, we can measure through the difference between the returns of the portfolios with and without transaction costs what is the size of the possible trading gains. The ineficiency generated by transaction costs can be measured in terms of excess return that could be obtained. Table 6 shows that the returns without transaction costs are almost double the ones calculated with a 0.8% transaction cost. This suggests that, since transaction costs account for a percentage of the price, the higher the return obtained in a given portfolio, the higher it could have been in absolute terms. Also, since the excess return is higher and the volatility remains the same, the sharpe ratio is superior for all portfolios types and sizes.

5.1.6 Returns with one day waiting period

We report the one day waiting period results on table 7. The reason for analyzing the one day waiting results is for the possibility of a delay with the implementation of the trade after the signal has been observed. Also, it allows for a more conservative view of the strategy since there may be a bid-ask bounce effect. The results indicate that both methodologies have a significant loss of performance when a one day lag is introduced contrary to what Gatev *et al.* (2006) found. For the distance approach, the five pairs portfolio has an average annualized return of -0,038, while the cointegration approach has negative average returns for its 5 and 10 pairs portfolios. This suggests that pairs trading profits may not survive in a competitive environment, when problems with executing a trade because of a thin market may happen or due to the bid-ask bounce vanishing

Table 5: Comparison of long and short returns for unrestricted pairs trading for the USA dataset

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs for the long side and the short side between January 1962 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully invested weighting scheme.

Methodology		Distance Approach				Cointegration Approach			
Weighting Scheme		Committed Capital		Fully Invested		Committed Capital		Fully Invested	
Pairs Portfolio		Long	Short	Long	Short	Long	Short	Long	Short
5 Pairs	Excess Return	10,042	7,139	16,705	15,239	5,796	0,143	16,491	2,759
	Std. Dev.	10,119	9,845	17,687	17,174	10,096	9,460	26,316	24,684
	S.R	0,945	0,700	0,873	0,826	0,558	0,015	0,58	0,11
10 Pairs	Excess Return	9,163	6,733	16,549	11,978	5,676	-1,799	18,357	-2,242
	Std. Dev.	9,389	9,261	14,468	14,218	7,666	7,300	24,576	22,966
	S.R	0,934	0,703	1,058	0,795	0,720	-0,248	0,686	-0,098
20 Pairs	Excess Return	8,804	6,478	15,019	12,11	5,948	-0,264	23,609	1,561
	Std. Dev.	8,446	8,302	12,709	12,411	6,348	5,954	22,455	20,784
	S.R	0,999	0,756	1,101	0,921	0,91	-0,044	0,944	0,074

Table 6: Excess return of unrestricted pairs trading for the USA dataset without transaction costs

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1962 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully Invested weighting scheme. 0% Transaction Costs.

Methodology		Distance Approach		Cointegration Approach	
Pairs Portfolio		CC	FI	CC	FI
5 Pairs	Excess Return	21,9321	42,362	8,218	25,563
	Std. Dev.	7,4264	13,6243	11,685	31,878
	S.R	2,6713	2,5944	0,676	0,714
10 Pairs	Excess Return	20,2064	37,1624	6,044	22,479
	Std. Dev.	5,6369	9,9676	8,236	27,916
	S.R	3,2662	3,1723	0,712	0,726
20 Pairs	Excess Return	19,3413	35,0806	7,902	31,619
	Std. Dev.	4,3248	7,4984	6,048	22,914
	S.R	4,09	4,0127	1,257	1,199

Table 7: Excess return of unrestricted pairs trading for the USA dataset with one day waiting

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1962 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. One day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

Methodology	Distance Approach			Cointegration Approach		
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	-0,038	0,153	1,072	-0,664	-0,598	0,326
Average Annualized Volatility (in %)	7,575	5,827	4,509	11,444	8,319	6,039
Total Sample Sharpe Ratio	-0,005	0,026	0,236	-0,058	-0,072	0,054
Largest Daily Return (in %)	6,314	6,456	3,058	7,855	4,442	2,856
Lowest Daily Return (in %)	-5,713	-3,109	-3,974	-7,598	-4,27	-2,895
Cumulative Profit (in %)	-15,004	-0,822	61,961	-48,331	-37,65	7,446
Spearman Rho	0,009	0,021	0,028	0,012	0,004	0,005
CAPM beta (β-market)	0,015	0,013	0,014	0,025	0,015	0,014
Jensen's Alpha	-0,00006	0,00001	0,0003	-0,0003	-0,0002	0,00008
Jensen's Alpha p-value	0,883	0,951	0,132	0,596	0,541	0,79
Annual Skewness	1,565	0,94	0,632	0,792	0,795	0,345
Annual Kurtosis	7,627	3,708	3,352	3,454	3,966	3,37
Total Sample MDD (in %)	78,026	68,497	55,853	73,056	55,158	35,527
Share of negative excess returns (in %)	0,503	0,522	0,518	0,394	0,455	0,488

when a one day waiting period is inserted. Hence, the profitability of pairs trading is tightly linked with the speed of execution, which although it raises questions on its profitability, the fact that we could still find considerable excess return with transaction costs on a daily basis suggests that pairs trading is still profitable. The tendency in terms of profitability regarding market efficiency is for the intra-day execution or pairs trading to take away the profits that could be obtained by a daily execution, with its limits on return being set by the transaction costs that incur and consenquently the necessity for the spread to widen enough in order to create a profitable opportunity.

5.2 Results for the Europeans Markets

5.2.1 Descriptive Statistics

Table 8 shows the summary statistics for the european data set for the whole sample period between january 1973 and december 2012. The average number of pairs opened, also increases with the number of pairs in the portfolio. However, when compared to the USA, the difference between both strategies is smaller. Similar to the USA results, the cointegration approach opens less pairs on average than the distance method. It is interesting to note that the median time pairs are open in days is very similar for both strategies. This suggests that both strategies may be closing too soon, and its performance could be improved by either identifiying an opening opportunity earlier, or by taking longer to close. The distance method has a higher average time pairs are open as well as a standard deviation, which is a consequence of this method identification procedure, not requiring a long term relationship between the stocks. Also, the average and median time pairs remain open increases with the portfolio size, specially for the cointegration method, indicating on this approach that the convergence takes longer as we increase the number of pairs, suggesting that the stocks may not move so close together as we widen our portfolio, but end up converging after a while. As a matter of fact, the distance method does not directly considers the convergence towards a long

Table 8: Summary statistics of unrestricted pairs trading for the European Union dataset
Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1973 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

Methodology	Distance Approach			Cointegration Approach		
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Total number of pairs opened	2107	4261	8254	1733	3404	6470
Total number of 6 month trading periods	78	78	78	78	78	78
Mean price deviation for opening pairs	0,017	0,019	0,021	0,225	0,25	0,286
Mean Nï¿œ of pairs opened each 6 month period	27,01	54,62	105,8	22,217	43,641	82,948
Mean Nï¿œ of pairs traded in months when at least one pair opened	5,472	5,533	5,359	4,524	4,438	4,264
Average Nï¿œ of round-trip trades per pair	5,402	5,462	5,291	4,443	4,364	4,147
Standard deviation of round-trips per pair	3,404	3,531	3,585	3,288	3,211	3,236
Average time pairs are open in days	10,12	9,801	10,21	7,403	8,324	8,874
Median time pairs are open in days	3	3	3	3	3	3
Average time pairs are open in months	0,481	0,466	0,486	0,352	0,3963	0,422
Standard deviation of time open per pair in days	20,70	19,65	19,77	12,91	14,858	15,472
Standard deviation of time open per pair in months	0,985	0,936	0,941	0,614	0,707	0,736
Share of negative excess returns	0,366	0,404	0,411	0,284	0,365	0,394

term relationship, unlike the cointegration method does. The distance method selects stocks that are very close to each other, but not based on convergence, hence explaining why the average time pairs remain open is higher for the distance method.

We can see that the share of negative excess returns is higher for the distance method, similar to the result obtained for the USA. As the share of negative returns shows, the distance method tends to have more open pairs at a given moment, due to its higher average time pairs are open, which consequently generates the opportunit for more non-zero returns. Hence, unlike the cointegration approach that tends to have pairs open for a shorter period, and less orpportunities to have negative returns days, the distance method has a higher share of negative excess returns.

5.2.2 Committed Capital for the European Union

Table 9 shows the results for the committed capital weighting scheme for the European Union, for both investment strategies. Both strategies have a very good performance, and an average annualized return superior to 6% a year. Also, their volatility is fairly low, and falls with the size of the pairs portfolios. The result of such facts is that both methods have very high sharpe ratios. The distance method has a SR ranging between 1,2 and 2,1 while the cointegration approach varies between 1,5 and 2,4. Also, when we look at the Jensen's alpha, we see that they are statistically significant, which adds to the conclusion that both strategies have a positive performance.

In terms of neutrality, both strategies have a close to zero beta and rho, another relevant characteristic, given that both strategies perform well. This shows that the performance is not a consequence of a high beta combined with a good market performance in the period. When we look at the strategies maximun drawdown we see that they perform very well and have a small MDD. The distance method has an MDD that can be as low as 13% while the cointegration approach loses up to only 16,9% of its value, much smaller than the MSCI Europe's MDD of 48%. All results indicate that both strategies have a very positive average performance through the sample period, and a high return stability, as seen by the low MDD, as well as a negligible beta.

Table 9: Comparison of strategies for unrestricted pairs trading for the European Union dataset

Note: This table reports a summary statistics of the excess returns on portfolios of pairs between January 1973 and December 2012 for the Commited Capital weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion or cointegration approach and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted.

Methodology	Distance Approach			Cointegration Approach		
Weighting Scheme	Committed Capital					
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	6,560	7,477	7,753	14,338	15,092	13,221
Average Annualized Volatility (in %)	5,231	4,208	3,417	8,77	6,623	5,00
Total Sample Sharpe Ratio	1,214	1,713	2,185	1,528	2,122	2,483
Largest Daily Return (in %)	4,173	2,227	1,964	7,078	4,392	3,714
Lowest Daily Return (in %)	-5,23	-1,97	-1,83	-7,309	-3,746	-3,306
Cumulative Profit (in %)	1.092,01	1.610,35	1.817,65	17.815,55	24.772,35	13.333,54
Spearman Rho with	0,015	0,003	0,015	0,009	0,012	0,029
CAPM beta (β-market)	0,001	0,001	0,002	0,003	0,006	0,008
Jensen's Alpha	0,00025	0,00028	0,00029	0,0005	0,0005	0,0005
Jense's Alpha p.value	0,00	0,00	0,00	0,00	0,00	0,00
Annual Skewness	1,513	1,386	1,346	0,507	0,751	0,627
Annual Kurtosis	7,161	5,671	5,095	2,622	3,116	2,419
Total Sample MDD (in %)	23,66	13,86	30,35	40,554	23,515	16,936

Table 10: Comparison of strategies for unrestricted pairs trading for the European Union dataset
Note: This table reports a summary statistics of the excess returns on portfolios of pairs between January 1973 and December 2012 for the Fully Invested weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion or cointegration approach and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted.

Methodology	Distance Approach			Cointegration Approach		
Weighting Scheme	Fully Invested					
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	19,45	27,79	27,94	56,016	82,69	81,849
Average Annualized Volatility (in %)	10,37	9,785	8,198	25,567	23,092	19,025
Total Sample Sharpe Ratio	1,714	2,507	3,007	1,741	2,612	3,147
Largest Daily Return (in %)	6,956	6,115	5,245	15,20	13,54	10,697
Lowest Daily Return (in %)	-12,8	-4,82	-6,55	-20,725	-11,631	-14,371
Spearman Rho	0,0152	14571	16146	0,008	0,0105	0,028
CAPM beta (β-market)	0,015	0,005	0,017	0,015	0,028	0,039
Jensen's Alpha	0,009	0,005	0,009	0,001	0,002	0,002
Jensen's Alpha p-value	0,000	0,000	0,000	0,000	0,000	0,00
Annual Skewness	6,666	1,557	1,651	1,853	2,539	2,767
Annual Kurtosis	3,403	3,680	2,379	6,652	9,940	11,309
Total Sample MDD (in %)	37,05	39,24	48,67	80,593	49,179	37,579

5.2.3 Fully Invested in the European Union

Table 10 shows that the results for the fully invested portfolios are a little different than the committed capital ones. The performances for the cointegration method are superior in terms of average excess return when compared with the distance approach but when we look at the sharpe ratios, the results are very similar. This emanates from the higher volatility the cointegration portfolios have, hence, not being able to generated a better portfolio.

The fully invested scheme has a higher sharpe ratio thna the committed capital scheme possibly due to the short length of time pairs remain open at a single time. Hence, although the volatility is higher, the higher returns more than compensates the gains, resulting in a higher sharpe ratio. For the cointegration method, it has a higher excess return for all portfolio sizes as well as a higher sharpe ratio than when we use the committed capital scheme. This happens because the cointegration method has a median time of open pairs of only three days and a smaller standard deviation of the average time pairs remain open, causing the fully invested scheme to weight the returns by a much smaller number of pairs than the committed capital or even than the distance method that uses the fully invested scheme. Hence the returns increase much more than for the distance method. However, the Jensen's alpha is higher for the distance method a result of the high risk that the cointegration approach has.[6].

Even though the volatility is higher for all portfolio sizes and both strategies, the beta is still not statistically different than zero, as well as the Spearman Rho. Finally, the MDD for both strategies is higher for all portfolio sizes, something that is expected, given the weighting methodology, but the MDD for the 5 pairs cointegration approach is higher than 80%, a result that signals that even though the strategy is market neutral, its downside risk is still very high.

[6] 0,002 for the cointegration method using fully invested scheme, compared to the 0,0005 using the committed capital weighting scheme.

Table 11: Comparison of long and short returns for unrestricted pairs trading for the European Union dataset
Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs for the long side and the short side between January 1973 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully invested weighting scheme.

Methodology		Distance Approach				Cointegration Approach			
Weighting Scheme		Committed Capital		Fully Invested		Committed Capital		Fully Invested	
Pairs Portfolio		Long	Short	Long	Short	Long	Short	Long	Short
5 Pairs	Excess Return	7,836	3,058	24,52	6,626	11,868	5,797	42,772	21,578
	Std. Dev.	8,012	7,908	19,95	19,73	8,377	7,996	25,142	23,565
	S.R	0,941	0,381	1,099	0,325	1,339	0,704	1,417	0,829
10 Pairs	Excess Return	9,395	2,514	30,64	11,38	11,699	6,599	51,832	37,172
	Std. Dev.	6,737	6,621	21,01	20,59	6,612	6,486	24,64	23,676
	S.R	1,333	0,375	1,272	0,523	1,673	0,985	1,696	1,335
20 Pairs	Excess Return	9,863	2,202	30,30	11,27	11,567	4,826	57,908	31,667
	Std. Dev.	5,973	5,903	18,25	17,86	5,601	5,435	22,200	21,630
	S.R	1,575	0,368	1,451	0,598	1,954	0,867	2,059	1,272

5.2.4 Returns on the long and short side

When we look at table 11 we see clearly see that both approaches have a tendency for the long side to perform better than the short side. The long side returns with committed capital for both strategies range between 7,8% and 11,8% while the short side is between 2,2% and 6,5%. For the fully invested scheme the difference is a bit bigger, but nonetheless the strategy still benefits strongly from both sides of the pair. Another important characteristic is that the sharpe ratio on both sides are smaller than the pairs sharpe ratio, which indicates that the strategy fullfills its objective of portfolio building and benefiting from the diversification, not only based on the selection of different stocks but also on simmetrically opposite investments.

The results for the cointegration approach are not very different but, nonetheless, have some interesting characteristics. The long side's return is more than double the short side for all pairs size and weighting schemes. This can be explained by the existance of a bull market through the most part of the analyzed period, and the fact that the strategy selects different stocks and pairs than the distance method. This also reflects on the sharpe ratio with the long side having a superior performance than the short side. On the other hand, the distance method has a long side performance that os around 3 to four times superior to the short side, hence, with a higher dependence on the long side.

5.2.5 Without Transaction Costs

The performances without transaction costs are presented on table 12. For both strategies the excess return increases when compared both weighting schemes with their counterparts with transaction costs. The main difference is that the cointegration method has a higher absolute and percentage change on its excess return when using the fully invested scheme. in general, the results indicate that the transaction costs reduces the profitability of pairs trading.

5.2.6 Returns with one day waiting period

When we include the one day waiting period we see a shift in performance for both strategies. Table 13 depicts the statistics for both strategies using a committed capital weighting scheme and including a one day waiting period for the opening and closing of pairs in order to account for

Table 12: Excess return of pairs trading for the European Union dataset without transaction costs
Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1973 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 0% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully Invested weighting scheme.

Methodology		Distance Approach		Cointegration Approach	
Pairs Portfolio		CC	FI	CC	FI
5 Pairs	Excess Return	16,122	48,202	22,285	92,949
	Std. Dev.	5,588	11,353	9,059	26,353
	S.R	2,675	3,467	2,221	2,497
10 Pairs	Excess Return	27,565	52,307	23,099	137,449
	Std. Dev.	6,248	11,238	6,878	24,005
	S.R	3,898	3,746	3,022	3,608
20 Pairs	Excess Return	24,951	46,15	20,737	135,668
	Std. Dev.	4,916	8,427	5,224	19,84
	S.R	4,533	4,506	3,608	4,328

unexpected situations, for example, the difficulty in executing the order as well as bid-ask bounce. Both strategies have a negative average excess return for the sample period which translates in a negative sharpe and alpha. The MDD is very high, as much as 97,1% which indicates that pairs trading strategy, if the orders execution is delayed, can be very unprofitable. These results indicate that in a competitive environment, although daily pairs trading can still be profitable given a sufficient speed of execution, it is no longer profitable to trade with a one day delay, be it due to low processing capabilities, thin trading or any other reason that would impose a delay between the moment the price is discovered, and the order executed. As seen for the USA, and now for the european union, the speed of execution is a paramount necessity in pairs trading.

5.3 Brazilian Results

5.3.1 Descriptive Statistics

The results for the brazilian dataset are shown on table 14 for the whole sample period between january 1996 and december 2012. The results are very similar to the american and european datasets, with the main difference being the higher median and average time pairs remain open. This reflects also on a higher standard deviation of time pairs remain open as well as in a lower average number of round-trip trades per pair. Since pairs remain open longer, they have less opportunites to have more round-trip trades in a given trading window. Another consequence of its longer average time open, is a higher share of negative excess return, which is superior to the ones found for the USA and european datasets.

5.3.2 Committed Capital for Brazil

The performance of both strategies is presented on table 15 using the committed capital weighting scheme. Some periods exhibit less than 20 cointegrated pairs, specially the years between 1996 and 2002, consequently when applicable, we used the maximun number available of pairs to form a portfolio. Unlike for the previous databases, the performance is fairly low for both methods. The top performance in terms of annual excess returns and sharpe ratio is for the 5 pairs cointegration approach reaching, respectively, 8,3% and 0,641. The performance of all other portfolios range between 0,3% and 3,6% annual excess return. The alphas are not statistically significant, except for the 5 pairs cointegration approach which are significant at the 5% level. Although both strategies do not exhibit great results, their MDD are small ranging between 15% and 23%, which suggest

Table 13: Excess return of unrestricted pairs trading for the European Union dataset with one day waiting
Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1973 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. One day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

Methodology	Distance Approach			Cointegration Approach		
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	-6,901	-8,388	-8,22	-4,684	-3,678	-3,571
Average Annualized Volatility (in %)	6,114	5,067	4,136	8,846	6,72	5,008
Total Sample Sharpe Ratio	-1,169	-1,728	-2,073	-0,542	-0,557	-0,726
Largest Daily Return (in %)	4,248	2,671	2,221	7,334	3,652	3,909
Lowest Daily Return (in %)	-5,648	-3,668	-2,552	-8,233	-5,016	-3,520
Cumulative Profit (in %)	-94,636	-97,11	-96,838	-87,366	-79,49	-77,681
Spearman Rho	0,006	0,002	0,014	0,002	0,005	0,007
CAPM beta (β-market)	-0,002	-0,002	-0,001	0,006	0,006	0,005
Jensen's Alpha	-0,0002	-0,0003	-0,0003	-0,0002	-0,0001	-0,0001
Jensen's Alpha p-value	0,00	0,00	0,00	0,0005	0,0003	0,00006
Annual Skewness	1,392	0,766	1,012	1,898	1,922	1,428
Annual Kurtosis	4,827	2,767	3,429	8,144	9,064	6,597
Total Sample MDD(in %)	95,659	97,150	97,124	88,525	81,188	78,48
Share of negative excess returns (in %)	0,498	0,588	0,618	0,391	0,487	0,528

Table 14: Summary statistics of unrestricted pairs trading for the Brazilian dataset

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1996 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

Methodology	Distance Approach			Cointegration Approach		
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Total number of pairs opened	415	734	1407	389	638	1156
Total number of 6 month trading periods	34	34	34	34	34	34
Mean price deviation for opening pairs	0,049	0,063	0,078	0,745	0,775	0,759
Mean Nï¿œ of pairs opened each 6 month period	12,205	21,588	41,382	11,441	18,764	34
Mean Nï¿œ of pairs traded in months when at least one pair opened	2,515	2,224	2,131	2,542	2,294	2,298
Average Nï¿œ of round-trip trades per pair	2,441	2,158	2,069	2,288	1,876	1,7
Standard deviation of round-trips per pair	1,810	1,562	1,45	1,748	1,589	1,576
Average time pairs are open in days	30,828	30,209	27,064	17,051	18,155	18,135
Median time pairs are open in days	13	15	14	11	12	12
Average time pairs are open in months	1,468	1,438	1,288	0,811	0,864	0,863
Standard deviation of time open per pair in days	37,418	35,174	31,712	19,538	19,87	19,383
Standard deviation of time open per pair in months	1,781	1,674	1,510	0,930	0,946	0,923
Share of negative excess returns	0,472	0,492	0,493	0,383	0,426	0,443

Table 15: Comparison of strategies for unrestricted pairs trading for the Brazilian dataset
Note: This table reports a summary statistics of the excess returns on portfolios of pairs between January 1996 and December 2012 for the Commited Capital weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion or cointegration approach and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted.

Methodology	Distance Approach			Cointegration Approach		
Weighting Scheme	Committed Capital					
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	3,188	0,640	0,378	8,370	3,534	3,688
Average Annualized Volatility (in %)	8,252	6,676	5,297	12,53	10,37	9,314
Total Sample Sharpe Ratio	0,380	0,095	0,071	0,641	0,334	0,388
Largest Daily Return (in %)	2,934	3,571	2,036	5,512	5,512	5,512
Lowest Daily Return (in %)	-3,41	-3,94	-2,30	-7,68	-7,68	-7,68
Cumulative Profit (in %)	57,42	6,97	3,92	224,2	60,93	67,70
Spearman Rho with	0,064	0,060	0,071	0,069	0,050	0,042
CAPM beta (β-market)	0,017	0,013	0,013	0,014	0,008	0,005
Jensen's Alpha	0,00014	0,000017	0,000007	0,00031	0,00013	0,00014
Jense's Alpha p.value	0,157	0,785	0,890	0,011	0,192	0,125
Annual Skewness	0,663	-0,37	0,728	1,375	1,280	1,536
Annual Kurtosis	4,099	3,020	4,543	4,185	4,586	5,948
Total Sample MDD (in %)	15,06	23,17	18,68	19,55	18,56	18,56

that this strategies are very stable over time. This can be seen through its beta and rho, which are very close to zero, similar to these statistics obtained for the previous datasets.

5.3.3 Fully Invested for Brazil

Table 16 presents the results for the whole trading out of sample, between july 1996 and december 2012 already discounted for transaction costs and slippage effects. The performance for the cointegration method are superior in terms of average excess return, as well as in sharpe ratio. However, this does not mean much, since its sharpe ratio is fairly low for the cointegratio method, between 0,36 and 0,57, but still higher than the ones for the distance approach. This low performance reflects on the Jensen's alpha, with it not being statistically significant for the distance method, and only the five pairs portfolio through the cointegration method is significant at the 5% level. Both strategies are market neutral, as reflected by its beta and rho, and as expected from this self-financing strategy. The MDD from both strategies is around 50%, indicating a quite risky strategy, but still lower than the 65% MDD of the market index (Ibovespa).

5.3.4 Returns on the long and short side

The results on the long and short side are quite different than the ones found for the american and european dabase as seen on table 17. For example, the distance approach short side has a negative average annual excess return. This may be because the brazilian stock market rallied after 2002, and this method might have selected stocks that were both increasing in value together. Hence, the short side would lose money. Another interesting fact is that the cointegration approach using the fully invested weighting scheme also has a negative performance on its portfolios with 10 and 20 stocks. As a general rule, the long side is responsible for the results of the whole portfolio, different from the american and european datasets. This indicates that the hedging abilities of the short

Table 16: Comparison of strategies for unrestricted pairs trading for the Brazilian dataset

Note: This table reports a summary statistics of the excess returns on portfolios of pairs between January 1996 and December 2012 for the Fully Invested weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion or cointegration approach and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted.

Methodology	Distance Approach			Cointegration Approach		
Weighting Scheme	Fully Invested					
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	6,507	0,091	-0,84	19,50	10,99	12,39
Average Annualized Volatility (in %)	17,65	17,89	16,38	31,01	28,26	24,25
Total Sample Sharpe Ratio	0,357	0,005	-0,05	0,574	0,369	0,481
Largest Daily Return (in %)	13,52	13,52	14,49	13,53	15,27	11,52
Lowest Daily Return (in %)	-15,1	-9,80	-8,01	-18,3	-16,0	-15,3
Cumulative Profit (in %)	116,0	-21,6	-29,8	721,3	183,3	312,4
Spearman Rho	0,058	0,050	0,064	0,069	0,051	0,044
CAPM beta (β-market)	0,033	0,035	0,038	0,052	0,035	0,022
Jensen's Alpha	0,00023	-0,00006	-0,000052	0,00067	0,000039	0,000045
Jensen's Alpha p-value	0,183	0,924	0,731	0,026	0,157	0,059
Annual Skewness	5,010	3,273	2,790	1,832	2,401	2,631
Annual Kurtosis	27,735	17,352	12,878	5,77	9,735	9,737
Total Sample MDD (in %)	46,66	58,73	59,83	46,98	57,73	41,68

Table 17: Comparison of long and short returns for unrestricted pairs trading for the Brazilian dataset
Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs for the long side and the short side between January 1996 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully invested weighting scheme.

Methodology		Distance Approach				Cointegration Approach			
Weighting Scheme		Committed Capital		Fully Invested		Committed Capital		Fully Invested	
Pairs Portfolio		Long	Short	Long	Short	Long	Short	Long	Short
5 Pairs	Excess Return	6,844	-1,44	18,30	-7,05	8,333	2,112	22,35	2,491
	Std. Dev.	21,97	21,21	36,81	35,87	16,12	15,81	34,27	33,24
	S.R	0,301	-0,06	0,456	-0,20	0,496	0,132	0,588	0,074
10 Pairs	Excess Return	7,127	-4,34	14,72	-9,71	3,274	2,153	18,62	-1,62
	Std. Dev.	17,64	17,13	35,61	34,46	14,95	14,71	33,68	32,89
	S.R	0,390	-0,25	0,385	-0,29	0,215	0,144	0,507	-0,04
20 Pairs	Excess Return	5,419	-3,12	13,76	-9,49	4,456	1,144	22,60	-3,56
	Std. Dev.	14,34	13,78	33,34	32,25	14,52	14,27	33,63	32,94
	S.R	0,367	-0,23	0,386	-0,30	0,300	0,079	0,606	-0,11

side could be improved in order to increase the portfolios returns. We also see that the short side is not the one driving the results due to short selling barriers or lack of liquidity.

5.3.5 Without Transaction Costs

For the brazilian database the results without transaction costs are presented on table 18. Its clear that all portfolios perform much better without transaction costs, and also have a higher sharpe ratio. However, the 5 and 10 pairs portfolios created through the distance method have an average annual excess return of slightly over 4% (4,3 and 4,08%, respectively). Since their performance with transaction costs is slightly over 0% as seen on section 5.3.2, we can estimate that transaction costs after including bid-ask bounce, slippage effects and liquidity effects, is around 4 percentage points of the final return on these portfolios.

5.3.6 Returns with one day waiting period

As previously seen with the american and european databases, the returns with one day waiting period tend to be negative. As seen on table 19, the brazilian dataset is not very different. The exception is the positive returns obtained by the 5 and 10 pairs portfolios created using the cointegration method, which have an average annualized excess return of 2,4% and 0,8% respectively. Nonetheless, their sharpe ratio are 0,191 and 0,081, and their Jensen's alpha are not statistically significant. The share of negative exess returns is not much higher than without the one day waiting period, indicating that the number of days the strategy loses money is not more frequent, but the loss in itself is higher. As for the MDD, both strategies have a worse maximun drawdown when compared to the committed capital scheme without one day waiting period.

6 Pairs Trading Performance Evaluation

6.1 Boostrap for Assessing Pairs Trading Performance

In order to evaluate the performance of the strategies, we compare it to a naive strategy, i.e., we create bootstrapped return series in which the signal to start the strategy of pairs trading is inserted,and the performance of such a strategy is monitored and compared to the performance

Table 18: Excess return of pairs trading for the Brazilian dataset without transaction costs

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1996 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 0% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully Invested weighting scheme.

Methodology		Distance Approach		Cointegration Approach	
Pairs Portfolio		CC	FI	CC	FI
5 Pairs	Excess Return	7,523	12,96	12,54	30,60
	Std. Dev.	8,492	17,54	12,66	31,60
	S.R	0,854	0,694	0,933	0,845
10 Pairs	Excess Return	4,343	7,187	7,928	23,13
	Std. Dev.	6,806	17,94	10,53	28,15
	S.R	0,624	0,386	0,724	0,739
20 Pairs	Excess Return	4,088	6,764	8,016	24,51
	Std. Dev.	5,470	16,83	9,450	24,30
	S.R	0,732	0,388	0,816	0,902

Table 19: Excess return of unrestricted pairs trading for the Brazilian dataset with one day waiting

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1996 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. One day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

Methodology	Distance Approach			Cointegration Approach		
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	-4,24	-2,33	-1,81	2,449	0,861	-0,03
Average Annualized Volatility (in %)	10,34	9,705	8,537	12,66	10,51	9,371
Total Sample Sharpe Ratio	-0,41	-0,24	-0,21	0,191	0,081	-0,00
Largest Daily Return (in %)	5,919	4,946	4,089	5,205	4,927	4,927
Lowest Daily Return (in %)	-4,17	-3,15	-2,61	-6,97	-6,97	-6,97
Cumulative Profit (in %)	-54,6	-36,8	-29,9	29,97	5,05	-7,37
Spearman Rho	0,038	0,063	0,091	0,061	0,050	0,043
CAPM beta (β-market)	0,012	0,019	0,023	0,018	0,012	0,011
Jensen's Alpha	-0,00	-0,00	-0,000089	-0,000085	0,000026	-0,000075
Jensen's Alpha p-value	0,078	0,271	0,305	0,492	0,795	0,934
Annual Skewness	0,449	0,622	0,848	0,175	0,920	0,800
Annual Kurtosis	2,950	3,756	3,735	2,010	3,800	3,235
Total Sample MDD (in %)	58,24	39,55	38,05	27,37	24,34	23,78
Share of negative excess returns (in %)	0,503	0,505	0,509	0,401	0,437	0,453

of the original series of returns. We follow the method used by Gatev *et al.* (2006) and Caldeira and Moura (2013), in which the bootstrap initiates at the time at which the signal is sent to begin trading pairs. In each bootstrap, the original series is replaced by two series of random assets similar to the assets earlier, similarity being defined as returns in the previous month belonging to the same decile. Thus, the difference in performance of the original assets and simulated give an indication of performance. The net return of the naive strategy is given by:

$$R_t^{naive} = \sum_{i=1}^{N} w_{it} r_{it} + 2N \, ln \left(\frac{1-C}{1+C} \right) \tag{12}$$

The results were calculated in every 6 month trading period and are withheld due to space constraints and can be obtained by contacting the author. We bootstrap each period 2500 for each of the pairs selection methodology and for each portfolio size, and found that both strategies obtain statisticallly significant positive performance when compared to a naive trader for both countries. In other words, the pairs trading strategies based on the selection of pairs through cointegration and through the distance method have a superior performance when compared to the random selection of pairs of stocks to be traded. The average returns on the random pairs is slightly negative for all databases, possibly due to the inclusion of transaction costs, and the standard deviations are large compared to the pairs trading portfolio's standard deviations.

6.2 Hypothesis testing for the difference between the Sharpe Ratios

Given that the objective of this paper is to compare the performance of two pairs selection methods, we must use a metric in order to assess if any of the strategies has a superior performance. In order to test the statistical significance of the difference between the Sharpe ratios of both strategies we use the methodology proposed in Ledoit and Wolf (2008) and obtain the p-values of the stationary bootstrap of Politis and Romano (1994) with B = 1000 bootstrap resamples and block length b= 5.

The whole sample result for the difference between Sharpe ratios through the methodology proposed by Politis and Romano (1994) indicates that for Brazil, the cointegration method is superior during the whole sample period to the distance method. However, for the subperiods the results are not as robust, with most subperiods results, available upon request, indicating that the cointegration method does not deliver a statistically significant higher Sharpe Ratio which hints at the fact that some subperiods might be driving the full sample results, or that the performance is slightly superior in each period, but not statistically higher due to sample limitations, since the size of most subperiods sample is 120 compared to the whole period that comprises 4087 observations. For europe the results also indicate that for the whole sample period the cointegration strategy is superior when using a portfolio consisting of 10 and 20 pairs. However for the 5 pairs portfolio the p-value of the statistic calculated is 0.107, and the cointegration strategy cannot be considered superior. Also, for the 6 month subperiods the cointegration strategy in most subsample periods is not superior, with the results most likely being driving by some subperiods. These findings hint that the cointegration strategy may be superior to the distance method in some periods, while the distance method may be superior in other subperiods, but on average the cointegration method delivers a higher Sharpe Ratio. For the USA the results are the other way around. The distance method Sharpe Ratio is statistically superior than on the cointegration method, for all 3 portfolio sizes. However, again as in Brazil and for Europe, the results in the subperiods are mixed, with the distance method not being superior in most 6 month subperiods due to high volatility and small period (tables available upon request).

7 Conclusion

In this paper we compared two methodologies for the strategy called pairs trading. The distance method presented in Gatev *et al.* (2006) and the cointegration method used by Caldeira and Moura (2013), for the american stock market between 1962 and 2012, for the brazilian stock market between 1996 and 2012 and for the european market between 1973 and 2012. We create portfolios comprising 5, 10 and 20 pairs for each method, and bootstrap the results in order to compare the their performance. The pairs were ranked by their in sample sharpe in the cointegration method and by the smallest to the highest SSD for the distance method in order to form the portfolios. The signal to open the position out of sample was given whenever the distance between the stocks on a given pair crossed the 2 standard deviation threshold. Both methodologies had a good performance when compared to a naive trader that randomly selection pairs to trade on a given period. For Brazil, the cointegration method had a cumulative return between 1996 and 2012 of up to 721%, while the distance method had up to 116% of cumulative return. When compared to each other, the cointegration method had a clear, statistically significant higher average annualized return, with a superior Sharpe Ratio, and, most of the time, a statistically significant inferior volatility. Both strategies can be considered market neutral, with a close to zero spearman correlation with the market.

For Europe, while the results were not so clear cut, they also pointed towards the cointegration method being superior, delivering an average of 15% of excess returns for the committed capitals against 7% for the distance method. The Sharpe Ratio was also considered superior for the whole sample period, although in some subsamples both strategies were very similiar. Both strategies had an excess returns superior than a naive trader. For the United States, the results were very different, indicating that the distance method is superior, delivering up to 13.% of average annual excess return, more than the 3% of the cointegration method. Considering that this strategy is self-financed, since the cash obtained by shortening a stock is used to buy the long stock in the pair, these results are encouraging and indicate a clear path for more research regarding the drivers of such difference in performance, the optimality of the trading thresholds and the stability of the cointegration parameters.

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