Cointegration Portfolios of European Equities for Index Tracking and Market Neutral Strategies

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Abstract

Traditional quantitative porfolio construction relies on the analysis of correlations between assets. Over the last 10 years, following the generalised use of JP Morgan (1994) RiskMetrics approach, quantitative portfolio managers have made a growing use of conditional correlations.

If correlations are indeed time-varying, unfortunately their many changes make them in practise a difficult tool to use when managing quantitative portfolios, as the frequent rebalancing they imply may be very costly.

In this paper, we use the concept of cointegration which relies on the long-term relationship between time series, and thus assets, to devise quantitative European equities portfolios in the context of two applications: a classic index tracking strategy and a long-short equity market neutral strategy.

We use data from the Dow Jones EUROStoxx50 index and its constituent stocks from 4 January 1999 to 30 June 2003. Our results show that the designed portfolios are strongly cointegrated with the benchmark and indeed demonstrate good tracking performance. In the same vein, the long-short market neutral strategy generates steady returns under adverse market circumstances but, contrary to expectations, does not minimise volatility.

Keywords: cointegration, index tracking, market neutral strategy, portfolio optimisation, vector autoregression models.

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1. Introduction

Financial markets are highly interdependent and, for many decades, portfolio managers have scrutinised the co-movements between markets. It is however regrettable that traditional quantitative porfolio construction still heavily relies on the analysis of correlations for modelling the complex interdependencies between financial assets. Admittedly, the application of the concept of correlation has been improved and, over the last 10 years, following the generalised use of JP Morgan (1994) RiskMetrics approach, quantitative portfolio managers have made a growing use of conditional correlations.

Yet, if correlations are indeed time-varying, their many changes across time make them in practise a difficult tool to use when managing quantitative portfolios, as the frequent rebalancing they imply may be very costly. Correlation and cointegration are somewhat related concepts but the key distinction between them is that correlation reflects short run comovements in returns while cointegration measures long run comovements in prices.

Accordingly, the main motivation for this paper is to gauge the benefits of less frequent portfolio rebalancing through the use of the concept of cointegration which relies on the long term relationship between time series, and thus assets, to devise quantitative European equities portfolios in the context of two applications: a classic index tracking strategy and a long-short equity market neutral strategy.

When index tracking portfolios are constructed on the basis of returns analysis, i.e. correlation, it is necessary to rebalance them frequently to keep them in line with the benchmark index to be tracked. Yet if the allocations in a portfolio are designed such that the portfolio tracks an index then the portfolio should be cointegrated with the index: in the short run the portfolio might deviate from the index but they should be tied together in the longer run. Optimal cointegration portfolios, as they rely on the long run trends between asset prices, should therefore not require as much rebalancing.

Market neutral strategies have become popular among investment managers, particularly since the end of the stock market bull run in 2000, as their key characteristic is that, if constructed and implemented properly, the underlying stock market behaviour does not impact the results of the portfolio. In other words, returns generated by an equity market neutral portfolio should be independent of the general stock market returns. A long-short equity market neutral strategy consists in buying a portfolio of attractive stocks, the long portion of the portfolio, and selling a portfolio of unattractive stocks, the short portion of the portfolio. The spread between the performance of the longs and the shorts provides the value added of this investment strategy and, here, again, the frequency of rebalancing is a key element in final performance.

We use data from the Dow Jones EUROStoxx50 index and its constituent stocks from 4 January 1999 to 30 June 2003 to construct cointegration portfolios of European equities, implementing in turn index tracking and long-short equity market neutral strategies: our results show that the designed portfolios are strongly cointegrated with the benchmark and indeed demonstrate good tracking performance; in the same vein, the long-short market neutral strategy generates steady returns under adverse market circumstances but, contrary to expectations, does not minimise volatility.

The rest of the paper is organised as follows. Section 2 briefly reviews the literature on common trends in equity markets and cointegration-based trading strategies. Section 3 describes the techniques and investment strategies retained for this study while section 4 documents the data used and the construction of the cointegration portfolios. Section 5 presents our estimation results. Finally, section 6 closes this article with a summary of our conclusions.

2. Literature Review

Since the seminal work of Engle and Granger (1987), cointegration has emerged as a powerful technique for investigating common trends in multivariate time series, providing a sound methodology for modelling both long run and short run dynamics in a system.

Although models of cointegrated financial time series are now relatively common, their importance for quantitative porfolio optimisation has remained until now very limited because the traditional starting point for portfolio construction since Markowitz (1952, 1959) is a correlation analysis of returns, whereas cointegration is based on the raw price, rate or yield data: any decision based on long term common trends in the price data is excluded in standard risk-return modelling.

Recent research on stock market linkages has emphasised finding common stochastic trends for a group of stock markets through testing for cointegrating relationships. Using monthly and quarterly data for the period January 1974 to August 1990 and the Johansen (1988) test for multiple cointegration, Kasa (1992) investigates whether there are any common stochastic trends in the equity markets of the US, Japan, UK, Germany and Canada. The results indicate the presence of a single common trend driving these countries stock markets. Corhay *et al.* (1993) study whether the stock markets of different European countries display a common long-run trend. They use static regression models and a VAR-based maximum likelihood framework, which provides empirical evidence of common stochastic trends among five important European stock markets over the period 1975-1991. Masih and Masih (1997) underline the growing leading role of the US market following the 1987 crash.

Meanwhile, Choudhury (1997) analyses the long-run relationships between six Latin American stock markets and the US market using weekly data for the period January 1989 to December 1993. The cointegration tests indicate the presence of a long-run relationship between the six Latin American indices with and without the US index. Other studies looking at linkages across developing countries include Cheung and Mak (1992), Chowdhury (1994), Garrett *et al*. (1994), Ng (2002) and Dunis and Shannon (2004).

Yet these papers focus primarily on stock market linkages. Closer to our preoccupation of optimal portfolio construction, Cerchi and Havenner (1988) and Pindyck and Rothemberg (1992) underline that an equity index is by definition a weighted sum of its constituents so that there should a sufficiently large basket of component equities which is cointegrated with the index, provided index weights are reasonably stable across time. Alexander and Dimitriu (2002) build index tracking and market neutral cointegration portfolios for domestic US equities based on the Dow Jones Industrial Average index with daily data from January 1990 to December 2001, whereas, using 12 years of daily data from January 1990 to March 2002, Qiu (2002) devises a cointegration-based portfolio of international bonds from 8 different countries to replicate the 13-country JP Morgan global government bond index. Finally, using the same EUROStoxx50 index and constituent series as we do, but with daily data from September 1998 to July 2002, Burgess (2003) develops cointegration-based strategies for hedging a given equity position or implementing statistical arbitrage trading opportunities.

3. Methodology and Investment Strategies

3.1 Cointegration Models

The issues of common trends and the interdependency of financial markets have come under increased scrutiny in recent years, following Engle and Granger (1987) who point out that a linear combination of two or more non-stationary series may be stationary: if such a stationary linear combination exists, the non-stationary time series are said to be cointegrated. The stationary linear combination is called the cointegrating equation and may be interpreted as a long-run equilibrium relationship among the variables. Thus, cointegration of stock markets means there is a long-run relationship between them: if Y and X are I(1) time series and are cointegrated so that $u = Y - \alpha - \beta x$ is I(0), then, in the long run, Y and X do not drift apart, since u has a constant mean, which is zero. Hence $Y = \alpha + \beta X$ can be interpreted as an equilibrium or long-run relationship between these markets and u is referred to as the error-correction term (ECT) since it gives the 'error' value in $Y = \alpha + \beta X$ and so is the deviation from equilibrium which, in the long run, is zero.

Engle and Granger (1987) and Engle and Yoo (1987) propose a 2-step estimation method, where the first step consists of estimating a long-run equilibrium relationship and the second is the estimation of the dynamic error-correction relationship using lagged residuals. Holden and Thompson (1992) claim that this two-step approach has the advantage that the estimation of the two steps is quite separate so that changes in the dynamic model do not enforce re-estimation of the static model obtained in the first step. As such it offers a tractable modelling procedure.

Alexander (1999) suggests nevertheless that the problem of uniqueness arises when there are more than two variables included in the model, i.e. the possibility of more than one cointegrating vector between the selected variables according to the choice of dependent variable. In the circumstances, the well documented Johansen (1988) method for multiple cointegration allows testing for a number of cointegrating vectors at the same time. It relies on estimating a vector autoregression (VAR) model in differences, such as:

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{p-1} \Delta X_{t-p-1} + \Pi X_{t-p-1} + BZ_t = u_t \qquad (1)$$

where X is a $(m \times 1)$ matrix of I(1) variables, Z is a $(s \times 1)$ matrix of I(0) variables, the Γ_j and Π are $(m \times m)$ matrices of unknown parameters and B is a $(m \times s)$ matrix of unknown parameters. M is the number of variables in X, and p is the maximum lag in the equation, which is a VAR model. If Π has zero rank, no stationary linear combination can be identified and the variables in X_t are not cointegrated. The number of lags to be included within the model is determined by minimising Akaike's error criterion.

In our applications however, the choice of the dependent variable is completely obvious, i.e. the EUROStoxx50 index for the index tracking application and the *ad hoc* artificial 'long' and 'short' benchmarks for the long/short equity market neutral strategy. There is therefore no doubt as to what the endogenous variable in the cointegration equation should be and which cointegrating vector we should be looking for, so the original Engle and Granger (1987) approach can also be applied to estimate cointegration equations such as:

$$Y_t = \alpha + \beta X_t + u_t \qquad (2)$$

where Y_t and X_t are cointegrated time series and therefore the residual series and tracking error u_t is stationary.

It is worth noting that, with a large amount of stocks, there may be no alternative to using equation (2) for technical reasons: indeed, multicollinearity may occur, in which case least squares estimates are unbiased, but their variances are large and may be far from the true value. This can be solved by using ridge regression (Hoerl and Kennard 1970a, 1970b), where, by adding a degree of bias to the regression estimates, it is hoped that the net effect will be to give more reliable ones.

3.2 Index Tracking

The first investment strategy selected in this paper is a classic index tracking strategy which aims at replicating the benchmark in terms of returns and volatility using cointegration rather than correlation. This allows us to make use of the full information contained in stock prices and base our portfolio weights on the long run behaviour of stocks.

As with traditional correlation-based portfolio construction, the selection of the stocks to be included in the cointegration portfolio is 'exogenous', so to speak. Obviously, the quality of the index tracking will highly depend on the stock selection and several alternative combinations should be tried out before choosing the final tracking portfolio.

Then portfolio weights are determined over the chosen in-sample period by the coefficients of the cointegration equation between the log price of the market index and the portfolio stocks log prices as exogenous variables.

$$\log(STOXX_t) = a_0 + \sum_{k=1}^{n} a_k * \log(P_{k,t}) + \varepsilon_t \qquad (3)$$

where $STOXX_t$ is the EUROStoxx50 index and $P_{k,t}$ is the price of the constituent stock P_k at time *t*, the series $STOXX_t$ and $P_{k,t}$ are cointegrated and therefore the residual series, i.e. the tracking error, ε_t is stationary.

Using log prices has the advantage that the tracking error ε_t is in return format and the a_k coefficients are portfolio weights: they need however to be normalized to sum up to one to give the percentage weight of each selected stock in the index tracking portfolio. The index tracking portfolio daily returns are computed as the weighted sum of the daily returns of its constituent stocks.

3.3 Long/Short Equity Market Neutral Strategy

As underlined by Lederman (1996) and Jelicic and Munro (1999), market neutral strategies are often considered by fund managers as state-of-the-art investment strategies. They actually include many different complex trading strategies in the bond and equity markets and it is beyond the scope of this paper to review them all. We concentrate here exclusively on long/short equity market neutral strategies.

Long/short equity investment can be traced back to the late 1940s and the A.W. Jones investment partnership that bought and shorted stocks. It was later refined by N. Tartaglia at Morgan Stanley in the late 1980s. However, it was not before recently that long/short equity strategies gained any real institutional appeal. In fact, these strategies have really become popular among investment managers since the stock market downturn in 2000, because their key characteristic is that, if constructed and implemented properly, the underlying stock market behaviour should not impact the results of the portfolio. In other words, returns generated by an equity market neutral portfolio should be independent of the general stock market returns.

A long-short equity market neutral strategy consists in buying a portfolio of attractive stocks, the long portion of the portfolio, and selling a portfolio of unattractive stocks, the short portion of the portfolio. The spread between the performance of the longs and the shorts provides the value added of this investment strategy which seeks to provide a return in excess of the risk-free rate. The strategy is not a pure enhanced cash strategy because of the significantly higher risk and return expectations of the strategy, but it is an absolute return investment approach, hence its frequent description as a 'double alpha' strategy.

Indeed, there are two primary sources of return to a long/short equity neutral strategy. The first component is the 'long' portfolio where the investor is a buyer of stocks: in this 'long' portfolio, the investor profits when the stocks in the portfolio rise in price, on average, and loses when the stock prices fall[1]. The second component is the 'short' portfolio where the long/short equity investor borrows stocks from another investor and then sells the stocks to generate the short portfolio (note the self-financing aspect of the long/short strategy): in this 'short' portfolio, the investor profits when the prices of the constituent stocks fall, on average, and loses when these stocks rise in price.

In practise, the construction of both 'long' and 'short' portfolios derives from the index tracking strategy: only this time we aim to devise 2 cointegrating portfolios tracking 2 benchmarks, a benchmark 'plus' and a benchmark 'minus' constructed by adding to (respectively subtracting from) the main benchmark daily returns an annual excess return of x% (equally distributed on the daily returns). The 2 cointegration equations tested are:

$$\log(STOXX^{+}_{t}) = a_0 + \sum_{k=1}^{n} a_k * \log(P^{+}_{k,t}) + \varepsilon^{+}_{t} \qquad (4)$$

[1] Note that, if there are no such constraints imposed on the 'long' and 'short' portfolios, both are likely to include some short equity positions.

where $STOXX^{+}_{t}$ is the EUROStoxx50 'plus' index devised as a benchmark for the 'long' portfolio and $P^{+}_{k,t}$ is the price of the constituent stock P^{+}_{k} at time *t*, the series $STOXX^{+}_{t}$ and $P^{+}_{k,t}$ are cointegrated and therefore the residual series ε^{+}_{t} is stationary.

$$\log(STOXX^{-}_{t}) = a_0 + \sum_{k=1}^{n} a_k * \log(P^{-}_{k,t}) + \varepsilon^{-}_{t} \qquad (5)$$

where $STOXX^{-}_{t}$ is the EUROStoxx50 'minus' index devised as a benchmark for the 'short' portfolio and $P^{-}_{k,t}$ is the price of the constituent stock P^{-}_{k} at time *t*, the series $STOXX^{-}_{t}$ and $P^{-}_{k,t}$ are cointegrated and therefore the residual series ε^{-}_{t} is stationary.

Clearly the choice of the annual excess return to construct the 2 'long' and 'short' cointegrated portfolios is critical. If, as mentioned before, there is a good reason to expect *a prioiri* that a sufficiently large basket of component equities will be cointegrated with the reference market index, this may not be true in the case of *ad hoc* benchmarks, such as those created for the 'long' and 'short' portfolios. The satisfaction of the cointegration tests in (4) and (5) is therefore essential, but it can be reasonably expected that the larger the annual excess return chosen, the more difficult it will be to satisfy these tests.

Overall, the long/short equity market neutral strategy consists of buying the 'long' portfolio and selling the 'short' portfolio. The global portfolio daily returns are computed as the sum of the daily returns of the 'long' and 'short' portfolios (multiplied by –1 for the 'short' portfolio), where the daily returns of each of these portfolios is the weighted sum of the daily returns of their constituent stocks. In other words, the strategy returns depend on the spread between the benchmarks tracked.

Finally, as the 'long' and 'short' portfolios are both highly correlated with the reference stock market benchmark, and assuming that each tracking error is not correlated with the market, one would expect a low correlation of their difference with the market benchmark, a key characteristic of a market neutral strategy.

4. Data and Portfolio Construction

4.1 Data

The data used in this paper is the Dow Jones EUROStoxx50 index and its constituent stocks as at 30 June 2003. The databank spans from 4 January 1999 to 30 June 2003, four and a half years of data with 1084 readings in total. It was obtained from the Yahoo financial website (www.finance.yahoo.co.uk). The advantage of taking this stock index is that it covers a panel of international stocks from different European countries, all denominated in a common currency, the euro. Yet, as rightly mentioned by Burgess (2003), the slightly nonsynchronous closing times of the different European stock markets would induce distortions in a true trading environment, but, for this paper, it is deemed that these closing prices are good enough and serve well our purpose of demonstrating the use of cointegration portfolios.

The 50 stocks listed in the EUROStoxx50 index, their ticker symbols and their weights in the index as at 30 June 2003 are given in Appendix 1.

A log transformation is applied to both the benchmark and the underlying stocks, as this ensures that the cointegration equation coefficients can be interpreted as portfolio weights and because, if the level variables are cointegrated, so will be their logarithms. Traditional ADF tests are performed for the EUROStoxx50 index and its constituent time series to confirm that they all are nonstationary[2].

4.2 Portfolio Construction

For both applications, an initial in-sample portfolio is constructed initially for the period from January 1999 to December 2001, and it is progressively expanded monthly until June 2003: the initial portfolio (P0) is constructed over the period from January 1999 to December 2001 and simulated out-of-sample in January 2002 as the first tracking portfolio (P1), then the second tracking portfolio is constructed over the period from January 1999 to January 2002 and simulated out-of-sample in February 2002 (P2), the third tracking portfolio is constructed using data from January 1999 to February 2002 and simulated out-of-sample in March 2002 (P3), and so on. We therefore obtain 18 out-of-sample portfolios (P1-P18).

The initial portfolio P0 is based on 3 years of daily data and the coefficients of the cointegration regression are subsequently reestimated monthly using the Johansen (1988) test procedure (see Appendix 2 for an example). The first cointegration tracking portfolio (P1) is simulated from 2 through 31 January 2002, using estimation data from 4 January 1999 to 28 December 2001 to determine portfolio weights. The last tracking portfolio (P18) is simulated from 2 through 30 June 2003, using data from 4 January 1999 to 30 May 2003 to estimate portfolio weights.

To build our index tracking portfolio, we first need to apply a *stock selection* procedure: for the purpose of diversification, we initially apply the simplest stock selection criterion available, i.e. the weight of the stocks in the index at the moment of the portfolio construction to construct P0 portfolios containing 5, 10, 15 and 20 constituent stocks that are most highly cointegrated with the EUROStoxx50 index as at 28 December 2001. Only relative weights are subsequently modified.

As mentioned before, the *cointegration equation* then allows to determine portfolio weights, using the regression coefficients and normalizing their sum to 1.There is no specific constraint, both long and short positions are allowed.

The *stationarity of the tracking error* in each regression is then tested with a traditional ADF test, the more stationary the tracking error, the greater the cointegration between the benchmark and the constructed portfolio.

The final stage is the *computation and analysis of portfolio results*. To gauge portfolio performance, for each tracking portfolio, annualised returns (using portfolio returns estimated as the first difference in portfolio log prices), annualised volatility, excess returns, information ratio[3], Sharpe ratio[4] and correlation of the tracking portfolio returns with the index returns are calculated.

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[2] These results and descriptive statistics are not reproduced here to conserve space. They are available from the authors upon request.

[3] The information ratio is simply the average annualised return of an investment strategy divided by its average annualised volatility.

In this paper, we devise cointegration portfolios as decribed above for three different applications, (i) a simple index tracking, (ii) the same, but with different rebalancing frequencies and (iii) a long/short market neutral strategy.

4.2.1 Simple index tracking methods

We construct cointegrated portfolios tracking the EUROStoxx50 index which contain respectively 5 stocks, 10 stocks, 15 stocks and 20 stocks.

4.2.2 Different rebalancing frequencies

To investigate whether the stock selection method is responsible for potential weight instability, we use alternative stock selection methods, also based on price ranking criteria. To reduce turnover, each portfolio is kept constant for 3-month, 6-month and 1-year investment periods. The initial strategy based on monthly rebalancing is subsequently referred to as RM, while the quarterly, semi-annually and annually rebalancing strategies are denoted respectively RQ, RSA and RA. Note that an important difference between the initial stock selection method and the alternative ones proposed here will be associated transaction costs.

4.2.3 Long/short equity market neutral

As mentioned before, an extension for exploiting the tracking potential of cointegrated portfolios is to replicate ‘plus’ and ‘minus’ benchmarks by creating ‘long’ and ‘short’ portfolios. Yet many different ‘plus’ and ‘minus’ benchmarks can be devised on the back of the EuroStoxx50 index leading to alternative tracking portfolios.

Concerning the 'constrained' long/short strategy, we need to construct 2 new artificial benchmarks by adding/subtracting an annualised return of x% uniformly from the daily returns of the EuroStoxx50 index (for instance, to construct the ‘EuroStoxx50 - 5%’ artificial benchmark, we need to subtract 0.01984%, i.e. 5%/252 assuming a 252-day trading year, from the EuroStoxx50 daily returns and then find a cointegration relationship between this new benchmark and some of the stocks available). The methodology for an artificial 'EuroStoxx50 plus' benchmark is obviously similar.

Having ensured that the portfolios pass the cointegration test, we then compute the weights exactly as with the simple index tracking strategy. The long/short portfolio manager gets the sum of the return of the ‘long’ portfolio and the return (multiplied by –1) of the ‘short’ portfolio (in fact, less the borrowing cost of the ‘short’ portfolio as he needs to borrow to ‘buy’ the stocks of the ‘short’ portfolio before selling them and we therefore subtract 4% p.a. from the ‘short’ portfolio return to reflect borrowing costs).

We use 9 combinations of artificial benchmarks in order to implement different long/short equity market neutral portfolios: (1) ‘plus’ 2.5% vs ’minus’ 2.5%; (2) ‘plus’ 2.5% vs ’minus’ 5%; (3) ‘plus’ 2.5% vs ’minus’ 10%; (4) ‘plus’ 5% vs ’minus’ 2.5%; (5) ‘plus’ 5% vs ’minus’ 5%; (6) ‘plus’ 5% vs ’minus’ 10%; (7) ‘plus’ 10% vs ’minus’ 2.5%; (8) ‘plus’ 10% vs ’minus’ 5% ; and (9) ‘plus’ 10% vs ’minus’ 10%.

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[4] The Sharpe ratio was computed as the average annualised return of an investment strategy minus the risk free rate (assumed at 4% p.a.) divided by the average annualised volatility.

5. Results and Performance Analysis

In this section, we present only some of the results obtained for the three strategies followed, i.e. the simple index tracking, the different rebalancing frequency and the long/short equity market neutral strategies. Complete results are available from the authors upon request.

5.1 Simple index tracking

The actual stocks contained in the different tracking portfolios are given in Appendix 3. Table 5.1 below documents the in-sample results of the tracking portfolios compared with the benchmark.

Table 5.1 – In-sample results for EuroStoxx50 and tracking portfolios (January 1999 - December 2001)

Portfolio	Annualised return	Annualised volatility	Correlation with benchmark	Information ratio	Sharpe ratio
Benchmark	*5.33%*	*23.71%*	*-*	*0.23*	*0.06*
5 stocks	*86.58%*	*91.34%*	*0.21*	*0.95*	*0.90*
10 stocks	*13.05%*	*49.02%*	*0.13*	*0.27*	*0.18*
15 stocks	*19.18%*	*34.30%*	*0.48*	*0.56*	*0.44*
20 stocks	*29.71%*	*45.33%*	*0.44*	*0.66*	*0.57*

Table 5.2 below documents the out-of-sample results of the tracking portfolios compared with the benchmark.

Table 5.2 – Out-of-sample results for EuroStoxx50 and tracking portfolios (January 2002 - June 2003)

Portfolio	Annualised return	Annualised volatility	Correlation with benchmark	Information ratio	Sharpe ratio
Benchmark	*-24.62%*	*34.01%*	*-*	*-0.72*	*-0.84*
5 stocks	*0.23%*	*38.33%*	*0.65*	*0.01*	*-0.10*
10 stocks	*41.75%*	*77.37%*	*0.06*	*0.54*	*0.49*
15 stocks	*-6.28%*	*31.23%*	*0.79*	*-0.20*	*-0.33*
20 stocks	*-9.45%*	*37.28%*	*0.75*	*-0.25*	*-0.36*

The overall conclusion is that, over a 18-month period where the benchmark lost 24.62%, all tracking portfolios produced better out-of-sample returns and risk-adjusted returns. The portfolio comprising 10-stocks registers the best performance, but it is also the least correlated with the benchmark.

Figure 5.1 – Sharpe ratio for EuroStoxx50 and 2 tracking portfolios (January 2002 – June 2003)

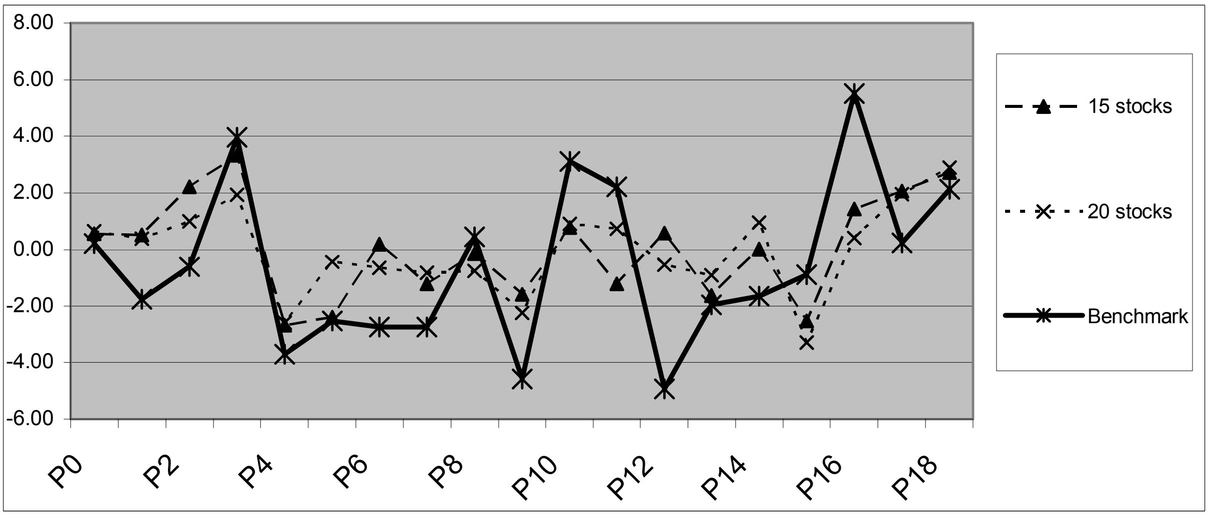

Figure 5.1 above shows that the Sharpe ratios for the 15- and 20-stock tracking portfolios are less volatile compared to the benchmark.

5.2 Different rebalancing frequencies

The results of the simple index tracking show that the 10-stock tracking portfolio has the best performance out-of-sample. As all tracking errors are stationary throughout the whole period, we select this portfolio to compare its results when using different rebalancing strategies, monthly (RM), quarterly (RQ), semi-annually (RSA) and annually (RA).

Table 5.3 - Out-of sample results for EuroStoxx50 and 10-stock tracking portfolios with various rebalancing frequencies (January 2002 - June 2003)

Portfolio	***Annualised return***	***Annualised volatility***	***Correlation with benchmark***	***Information ratio***	***Sharpe ratio***
Benchmark	*-24.62%*	*34.01%*	*-*	*-0.72*	*-0.84*
10 (RM)	*41.75%*	*77.37%*	*0.06*	*0.54*	*0.49*
10 (RQ)	*42.33%*	*57.21%*	*0.18*	*0.74*	*0.67*
10 (RSA)	*9.90%*	*56.58%*	*0.13*	*0.18*	*0.10*
10 (RA)	*-21.98%*	*48.03%*	*0.25*	*-0.46*	*-0.54*

As can be seen from table 5.3, all portfolios with 10 stocks using different rebalancing frequencies have a better performance than the benchmark. In terms of volatility, all tracking portfolios show a higher volatility than the EuroStoxx50 index. Using monthly and quarterly rebalancing produces similar annualised returns of about 42%. The 10-stock tracking portfolio with quarterly rebalancing has the best overall performance with the highest information ratio and a 0.18 correlation to the benchmark.

We conclude that quarterly rebalancing is better than monthly rebalancing, especially if transaction costs are included (see Appendix 4 for the weights profiles): true, an important difference between the rebalancing strategies is transaction costs which will be lower, the lower the rebalancing frequency retained. Still, with estimated round trip transaction costs of 12 basis points (b.p.) for the EuroStoxx50 index, and about twice as much for its component stocks in the cash market, these costs are not as such a major drawback for the more active strategies[5].

5.3 Long/short equity market neutral

Table 5.4 below compares 10-stock tracking portfolios obtained by adding to and subtracting from the benchmark returns an annual excess return of respectively -2.5%, -5%, -10%, +2.5%, +5% and +10%.

Table 5.4 - Average out-of sample results for 10-stock long/short portfolios (January 2002 - June 2003)

	Benchmark	**+2.5% /-2.5%**	**+2.5% /-5%**	**+2.5% /-10%**	**+5% /-2.5%**	**+5% /-5%**	**+5% /-10%**	**+10% /-2.5%**	**+10% /-5%**	**+10% /-10%**
Annualised return	-24.62%	-486.75%	81.15%	37.21%	-384.15%	**183.74%**	139.80%	-368.26%	199.64%	155.69%
Annualised volatility	34.01%	179.63%	96.83%	84.00%	206.63%	**132.92%**	119.56%	285.72%	239.65%	225.66%
Correlation with benchmark	-	-0.17	0.20	0.18	-0.13	**0.20**	0.18	-0.05	0.18	0.17
Information ratio	-0.72	-2.71	0.84	0.44	-1.86	**1.38**	1.17	-1.29	0.83	0.69
Sharpe ratio	-0.84	-2.73	0.80	0.40	-1.88	**1.35**	1.14	-1.30	0.82	0.67

[5] Assuming that, each time, the entire portfolio is reshuffled, which is not the case in this application, monthly rebalancing implies at most 12 round trips per year or 288 b.p., quarterly rebalancing 4 round trips or 96 b.p., semi-annual rebalancing 2 round trips or 48 b.p., whereas annual rebalancing entails only 1 round trip or 24 b.p. For trading costs assumptions, see www.interactive-brokers.com and Bessimbinder (2003).

In general, long/short strategies produce better results than the benchmark except for the 3 strategies replicating the benchmark minus 2.5% which produce very negative performance. Yet, contrary to what one would expect, the long/short strategies do not minimise volatility: annualised volatility is generally higher than the benchmark. The long/short combination 'plus 5%/minus 5%' has the best out-of-sample performance with a Sharpe ratio of 1.35 compared with -0.84 for the EuroStoxx50 index during this 18-month period.

Still, table 5.4 shows the out-of-sample results with the benefit of hindsight. In fact, fund managers do not have the benefit of hindsight and would have traded the 'best portfolio' at the end of each calibration period.

Table 5.5 shows that the combination 'plus 5%/minus 5%' has the highest in-sample Sharpe ratio at 0.43 agsainst 0.06 for the EuroStoxx50 index.

Table 5.5 – In-sample results for 10-stock long/short portfolios (January 1999 - December 2001)

	Benchmark	+2.5% /-2.5%	+2.5% /-5%	+2.5% /-10%	+5% /-2.5%	+5% /-5%	+5% /-10%	+10% /-2.5%	+10% /-5%	+10% /-10%
Annualised return	5.33%	4.25%	5.92%	7.35%	5.75%	**7.42%**	8.85%	78.94%	80.61%	82.04%
Annualised volatility	23.71%	5.67%	10.54%	21.87%	4.85%	**7.97%**	18.36%	1761.45%	1762.00%	1763.34%
Correlation with benchmark	-	0.19	0.23	0.26	-0.14	**0.08**	0.21	0.03	0.03	0.03
Information ratio	0.23	0.75	0.56	0.34	1.19	**0.93**	0.48	0.04	0.05	0.05
Sharpe ratio	0.06	0.04	0.18	0.15	0.36	**0.43**	0.26	0.04	0.04	0.04

Table 5.4 shows that, after 18 months, the combination 'plus 5%/minus 5%' was still the best strategy. In real life however, as fund managers do not know the future, they would probably have modified their choice of long/short combination every 3 or 6 months. It is unlikely that they would leave their portfolio using the same long/short portfolio mix for more than a year.

Accordingly, we assume in the following that investment managers manage their long/short portfolios using 3- and 6-month rebalancing frequencies.

5.3.1 Long/short neutral strategies rebalancing every 6 months

Table 5.6 below shows that for the out-of-sample period from January 2002 to June 2002, the combination 'plus 2.5%/minus 10%' produced the best Sharpe ratio at 0.58. Unfortunately, in January 2002, a fund manager would have used the results from table 5.5 to set up his trading strategy using the combination 'plus 5%/minus 5%': six months later, in June 2002, this strategy had produced a Sharpe ratio of -0.58, still superior to the -1.69 achieved by the EuroStoxx50 index.

Table 5.6 - Out-of sample results for 10-stock long/short portfolios (January 2002 - June 2002)

	Benchmark	+2.5% /-2.5%	+2.5% /-5%	+2.5% /-10%	+5% /-2.5%	+5% /-5%	+5% /-10%	+10% /-2.5%	+10% /-5%	+10% /-10%
Annualised return	-34.52%	-22.43%	-2.04%	**19.52%**	-34.59%	-14.21%	7.36%	-700.02	-679.63%	-658.07%
Annualised volatility	22.75%	22.93%	30.22%	**26.75%**	25.15%	31.50%	26.96%	136.57%	141.33%	135.30%
Correlation with benchmark	-	-0.08	-0.14	**-0.16**	-0.07	-0.13	-0.16	-0.21	-0.19	-0.19
Information ratio	-1.52	-0.98	-0.07	**0.73**	-1.38	-0.45	0.27	-5.13	-4.81	-4.86
Sharpe ratio	-1.69	-1.15	-0.20	**0.58**	-1.53	**-0.58**	0.12	-5.16	-4.84	-4.89

If we use the results from table 5.6 with the combination 'plus 2.5%/minus 10%' for the following six months, table 5.7 shows that for the following 6-month out-of-sample period from July 2002 to December 2002, the retained strategy produces a Sharpe ratio of -0.23 (still far superior to the -1.13 of the EuroStoxx50 index), whereas the best Sharpe ratio for that period is provided by the combination ‘plus 10%/minus 5%’ with a Sharpe ratio of 2.36.

Table 5.7 - Out-of sample results for 10-stock long/short portfolios (July 2002 - December 2002)

	Benchmark	+2.5% /-2.5%	+2.5% /-5%	+2.5% /-10%	+5% /-2.5%	+5% /-5%	+5% /-10%	+10% /-2.5%	+10% /-5%	+10% /-10%
Annualised return	-47.67%	-424.38%	14.90%	-28.25%	-116.57%	322.70%	279.55%	514.72%	**954.00%**	910.85%
Annualised volatility	45.88	198.76%	142.41%	143.31%	280.82%	238.67%	238.89%	402.84%	**402.90%**	402.40%
Correlation with benchmark	-	-0.05	0.31	0.28	0.03	0.33	0.31	0.16	**0.32**	0.31
Information ratio	-1.04	-2.14	0.10	-0.20	-0.42	1.35	1.17	1.28	**2.37**	2.26
Sharpe ratio	-1.13	-2.16	0.08	**-0.23**	-0.43	1.34	1.15	1.27	**2.36**	2.25

Using the results from tables 5.5, 5.6 and 5.7, we can simulate the trading performance of a fund manager rebalancing his portfolio every 6 months. Starting from January 2002 to June 2002, he would have traded the combination ‘plus 5%/minus 5%’ (i.e. the best in-sample combination), then, from July 2002 to December 2002, the combination ‘plus 2.5%/minus 10%’ (i.e. the best strategy between January 2002 and June 2002) and, from January 2003 to June 2003, the combination ‘plus 10%/minus 5%’ (i.e. the best strategy between July 2002 and December 2002).

Table 5.8 - Out-of-sample trading simulation of successive optimal long/short portfolio combinations rebalanced every 6 months and EuroStoxx50 (January 2002 - June 2003)

	Long/short strategies	EuroStoxx50
Annualised return	124.07%	-24.62%
Annualised volatility	116.55%	34.01%
Correlation with benchmark	0.19	-
Information ratio	1.06	-0.72
Sharpe ratio	1.03	-0.84

The trading simulation with semi-annual rebalancing yields a Sharpe ratio of 1.03 compared to 1.35 for the best single out-of-sample long/short strategy chosen from the in-sample optimisation (see table 5.4 above). This is still far superior to the -0.84 achieved by the EuroStoxx50 index over the same 18-month period.

5.3.2 Long/short neutral strategies rebalancing every 3 months

We use a similar approach as that adopted for the 6-month rebalancing, but this time we assume a trading strategy whereby the fund manager changes the structure of his portfolio every 3 months (see Appendix 5). Starting with from January 2002 to March 2002, a fund manager would have traded the combination ‘plus 5%/minus 5%’ (i.e. the best in-sample combination), then from April 2002 to June 2002, the combination ‘plus 2.5%/minus 5%’ (i.e. the best strategy between January and March 2002), then from July 2002 to September 2002, the combination ‘plus 2.5%/minus 10%’ (i.e the best strategy between April and June 2002), then from October 2002 to December 2002, the combination ‘plus 10%/minus 5%’ (i.e. the best strategy between July and September 2002), then from January 2003 to March 2003, the combination ‘plus 10%/minus 10%’ (i.e. the best strategy between September and December 2002) and, finally, from April 2003 to June 2003, the combination ‘plus 10%/minus 5%’ (i.e. the best strategy between January and March 2003).

This trading simulation with quarterly portfolio rebalancing produces a Sharpe ratio of 0.70 compared to 1.03 for the 6-month rebalancing and 1.35 for the best single out-of-sample long/short strategy chosen from the in-sample optimisation. Here again, this trading strategy yields a much better Sharpe ratio than the -0.84 achieved by the EuroStoxx50 index over the same 18-month period.

Table 5.9 - Out-of-sample trading simulation of successive optimal long/short portfolio combinations rebalanced every 3 months and EuroStoxx50 (January 2002 - June 2003)

	Long/short strategies	EuroStoxx50
Annualised return	90.45%	-24.62%
Annualised volatility	122.79%	34.01%
Correlation with benchmark	0.18	-
Information ratio	0.74	-0.72
Sharpe ratio	0.70	-0.84

5.3.3 Transaction costs

As mentioned before when analysing the performance of the index tracking strategy, transaction costs will obviously be lower, the lower the rebalancing frequency retained. This is an even more important issue in the case of long/short market neutral strategies as these entail trading 2 tracking portfolios and the self-financing feature offered by the short sale generally implies a leverage of 2:1 and thus double transaction costs. Still, with at most 8 round trips in total for quarterly rebalancing and 4 for semi-annual rebalancing, the transaction costs involved (respectively 192 b.p. and 96 b.p.) are minimal compared to the annualised returns, before transaction costs, of the long/short strategies achieved in our trading simulations.

6. Concluding Remarks

The main motivation for this paper was to demonstrate the benefits arising from the use of the concept of cointegration which relies on the long term relationship between time series, and thus assets, to devise quantitative European equities portfolios in the context of two applications: a classic index tracking strategy and a long-short equity market neutral strategy. Indeed, its key characteristics, i.e. a mean-reverting tracking error (i.e stationary residuals from the cointegration equation), enhanced portfolio weight stability over time and the full use of the information contained in stock prices allow for a flexible design of various investment strategies in equity markets, from index tracking to long/short market neutral.

Clearly, our results suffer from some of the simplifying assumptions adopted. Firstly, we arbitrarily choose to select at most 20 of the 50 stocks in the EuroStoxx50 index: a larger equity basket would probably have led to better results for the index tracking application. Secondly, we apply the simplest stock selection criterion available, i.e. the weight of the stocks in the index at the moment of the portfolio construction: the quality of the benchmark tracking highly depends on the stock selection procedure and much improvement could be achieved in this respect. Finally, the slightly nonsynchronous closing times of the different European stock markets would induce distortions in a true trading environment, but closing prices serve well our purpose of demonstrating the use of cointegration portfolios.

Yet, our results are quite impressive. Over the 18-month out-of-sample period from January 2002 to June 2003 where the EuroStoxx50 index lost 24.62%, all tracking portfolios produce much better returns and risk-adjusted returns, with less volatile Sharpe ratio profiles than those of the benchmark.

Strategies based on correlation would require rebalancing portfolios frequently. In contrast, cointegration-based portfolios require less frequent turnover: an analysis of alternative rebalancing frequencies shows that a quarterly portfolio update appears preferable to monthly, semi-annual or annual reallocations.

Furthermore, the tracking capabilities offered by cointegration make it possible to track different benchmarks and thus to implement long/short equity market neutral strategies. Most of the long/short combinations analysed in this paper produce better out-of-sample results and risk-adjusted results than the EuroStoxx50 benchmark, albeit at the cost of higher volatility, which may be linked to the smaller number of stocks included in the ‘long’ and ‘short’ porfolios. Two trading simulations with quarterly and semi-annual rebalancing show that, during the adverse market conditions of the January 2002 to June 2003 out-of-sample period, the selected long/short combinations would have attracted Sharpe ratios of 1.03 and 0.70 respectively, against -0.84 for the EuroStoxx50 index. As we have seen, these results are robust to the introduction of transaction costs.

Overall, the main conclusion from this research is that cointegration portfolios add economic value for investors and fund managers. In the circumstances, our results should go some way towards convincing a growing number of quantitative fund managers to experiment beyond the bounds of correlation analysis for portfolio construction.

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Appendix 1 - Stocks comprised in Dow Jones EUROStoxx50 as at 30 June 2003

(Ordered descendingly according to their weight in EUROStoxx50 index)

Company	ISIN	Market Sector	Float Factor[6]	Adjusted Weight (%)
TOTAL FINA ELF	FR0000120271	Energy	1.00	8.03
ROYAL DUTCH PETROLEUM	NL0000009470	Energy	1.00	7.83
NOKIA	FI0009000681	Technology	1.00	6.10
TELEFONICA	ES0178430E18	Telecommunications	0.94	4.12
ENI	IT0003132476	Energy	0.65	3.19
SIEMENS	DE0007236101	Technology	0.93	3.16
UNILEVER NV	NL0000009348	Food & Beverage	1.00	3.14
BNP	FR0000131104	Banks	0.94	3.11
BCO SANTANDER CENTRAL HIS	ES0113900J37	Banks	1.00	2.81
AVENTIS	FR0000130460	Healthcare	0.87	2.80
BCO BILBAO VIZCAYA ARGENT	ES0113211835	Banks	1.00	2.46
DEUTSCHE TELEKOM	DE0005557508	Telecommunications	0.57	2.45
DEUTSCHE BANK R	DE0005140008	Banks	0.95	2.29
E.ON	DE0007614406	Utilities	0.87	2.28
DAIMLERCHRYSLER	DE0007100000	Automobiles	0.81	2.21
ASSICURAZIONI GENERALI	IT0000062072	Insurance	0.86	2.09
GROUPE SOCIETE GENERALE	FR0000130809	Banks	1.00	2.05
CARREFOUR SUPERMARCHE	FR0000120172	Noncyclical Goods & Services	0.80	1.99
ABN AMRO	NL0000301109	Banks	0.89	1.91
SANOFI SYNTHELABO	FR0000120578	Healthcare	0.56	1.90
ING GROEP	NL0000303600	Insurance	0.88	1.87
PHILIPS ELECTRONICS	NL0000009538	Cyclical Goods & Services	1.00	1.85
FRANCE TELECOM	FR0000133308	Telecommunications	0.43	1.82
BASF	DE0005151005	Chemicals	0.91	1.78
L'OREAL	FR0000120321	Noncyclical Goods & Services	0.47	1.78
AXA UAP	FR0000120628	Insurance	0.82	1.58
GROUPE DANONE	FR0000120644	Food & Beverage	0.95	1.52
UNICREDITO ITALIANO	IT0000064854	Banks	0.69	1.51
TELECOM ITALIA	IT0001127429	Telecommunications	0.45	1.51
TIM	IT0001052049	Telecommunications	0.44	1.39
FORTIS	BE0003801181	Financial Services	0.89	1.37
REPSOL YPF	ES0173516115	Energy	0.82	1.33
VIVENDI UNIVERSAL	FR0000127771	Media	1.00	1.31
AIR LIQUIDE	FR0000120073	Chemicals	1.00	1.23
ENDESA	ES0130670112	Utilities	0.95	1.13
ENEL	IT0003128367	Utilities	0.32	1.01
SUEZ	FR0000120529	Utilities	0.93	1.00
ALLIANZ	DE0008404005	Insurance	0.74	0.90
AEGON	NL0000301760	Insurance	0.88	0.87
SAINT GOBAIN	FR0000125007	Construction	1.00	0.87
BAYER	DE0005752000	Chemicals	0.94	0.86
LVMH MOET HENNESSY	FR0000121014	Cyclical Goods & Services	0.46	0.82
RWE	DE0007037129	Utilities	0.76	0.82
SAN PAOLO IMI	IT0001269361	Banks	0.86	0.78
ALCATEL	FR0000130007	Technology	0.93	0.73
LAFARGE	FR0000120537	Construction	1.00	0.69
VOLKSWAGEN	DE0007664005	Automobiles	0.69	0.65
MUENCHENER RUECKVER R	DE0008430026	Insurance	0.62	0.58
AHOLD	NL0000331817	Noncyclical Goods & Services	1.00	0.29
BAYERISCHE HYPO & VEREINS	DE0008022005	Banks	0.63	0.23

[6] The free float factor is the percentage of shares remaining after the block ownership and restricted shares adjustments are applied to the total number of shares. We have:

Strategic shareholding (%) = Number of shares classified as strategic / Total number of shares outstanding

Free float (%) = 100% - Strategic shareholding (%).

Appendix 2 - Johansen (1988) Cointegration Test

Sample(adjusted): 6 716
Included observations: 711 after adjusting endpoints
Trend assumption: Linear deterministic trend
Series: LOG_STOXX LOG_FR_12027 LOG_NL_RD LOG_FI_870737 LOG_IT_ENI LOG_DE_723610 LOG_FR_13110 LOG_ES_SAN LOG_FR_13046 LOG_ES_BBVA LOG_DE_555750
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.104080	**310.8791**	**277.71**	293.44
At most 1	0.086735	**232.7369**	**233.13**	247.18
At most 2	0.052139	168.2288	192.89	204.95
At most 3	0.040202	130.1564	156.00	168.36
At most 4	0.036513	100.9820	124.24	133.57
At most 5	0.032318	74.53538	94.15	103.18
At most 6	0.028475	51.17799	68.52	76.07
At most 7	0.017255	30.63820	47.21	54.46

*(**) denotes rejection of the hypothesis at the 5%(1%) level

1 Cointegrating Equation(s): Log likelihood 20651.38

Normalized cointegrating coefficients (std.err. in parentheses)

LOG_STOXX	LOG_FR_12027	LOG_NL_RD	LOG_FI_870737	LOG_IT_ENI	LOG_DE_723610	LOG_FR_13110
1.000000	0.445615	-0.813099	-0.072151	0.942272	-0.133696	-0.093449
	(0.10463)	(0.11559)	(0.03832)	(0.13883)	(0.04958)	(0.08731)

Appendix 3 - Stocks contained in various tracking portfolios

5 stocks tracking portfolio

Company	ISIN	Market Sector
TOTAL FINA ELF	FR0000120271	Energy
ROYAL DUTCH PETROLEUM	NL0000009470	Energy
NOKIA	FI0009000681	Technology
ENI	IT0003132476	Energy
SIEMENS	DE0007236101	Technology

10 stocks tracking portfolio

Company	ISIN	Market Sector
TOTAL FINA ELF	FR0000120271	Energy
SIEMENS	DE0007236101	Technology
BNP	FR0000131104	Banks
AVENTIS	FR0000130460	Healthcare
BCO BILBAO VIZCAYA ARGENT	ES0113211835	Banks
DEUTSCHE TELEKOM	DE0005557508	Telecommunications
DEUTSCHE BANK R	DE0005140008	Banks
DAIMLERCHRYSLER	DE0007100000	Automobiles
ASSICURAZIONI GENERALI	IT0000062072	Insurance
ABN AMRO	NL0000301109	Banks

15 stocks tracking portfolio

Company	ISIN	Market Sector
TOTAL FINA ELF	FR0000120271	Energy
ROYAL DUTCH PETROLEUM	NL0000009470	Energy
NOKIA	FI0009000681	Technology
ENI	IT0003132476	Energy
SIEMENS	DE0007236101	Technology
BNP	FR0000131104	Banks
BCO SANTANDER CENTRAL HIS	ES0113900J37	Banks
AVENTIS	FR0000130460	Healthcare
BCO BILBAO VIZCAYA ARGENT	ES0113211835	Banks
DEUTSCHE TELEKOM	DE0005557508	Telecommunications
DEUTSCHE BANK R	DE0005140008	Banks
E.ON	DE0007614406	Utilities
DAIMLERCHRYSLER	DE0007100000	Automobiles
ASSICURAZIONI GENERALI	IT0000062072	Insurance
ABN AMRO	NL0000301109	Banks

20 stocks tracking portfolio

Company	ISIN	Market Sector
TOTAL FINA ELF	FR0000120271	Energy
ROYAL DUTCH PETROLEUM	NL0000009470	Energy
NOKIA	FI0009000681	Technology
ENI	IT0003132476	Energy
SIEMENS	DE0007236101	Technology
BNP	FR0000131104	Banks
BCO SANTANDER CENTRAL HIS	ES0113900J37	Banks
AVENTIS	FR0000130460	Healthcare
BCO BILBAO VIZCAYA ARGENT	ES0113211835	Banks
DEUTSCHE TELEKOM	DE0005557508	Telecommunications
DEUTSCHE BANK R	DE0005140008	Banks
E.ON	DE0007614406	Utilities
DAIMLERCHRYSLER	DE0007100000	Automobiles
ASSICURAZIONI GENERALI	IT0000062072	Insurance
ABN AMRO	NL0000301109	Banks
ING GROEP	NL0000303600	Insurance
PHILIPS ELECTRONICS	NL0000009538	Cyclical Goods & Services
BASF	DE0005151005	Chemicals
L'OREAL	FR0000120321	Noncyclical Goods & Services
REPSOL YPF	ES0173516115	Energy

Appendix 4 - Portfolio weights in 10-stock tracking portfolios

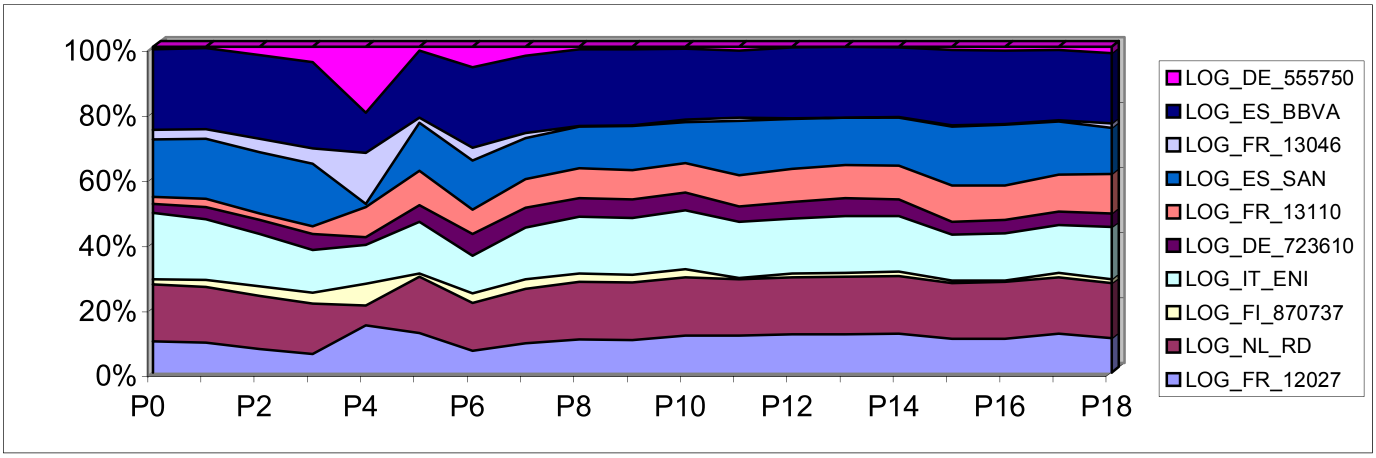

Figure 1 - Portfolio weights in 10-stock tracking portfolio with monthly rebalancing

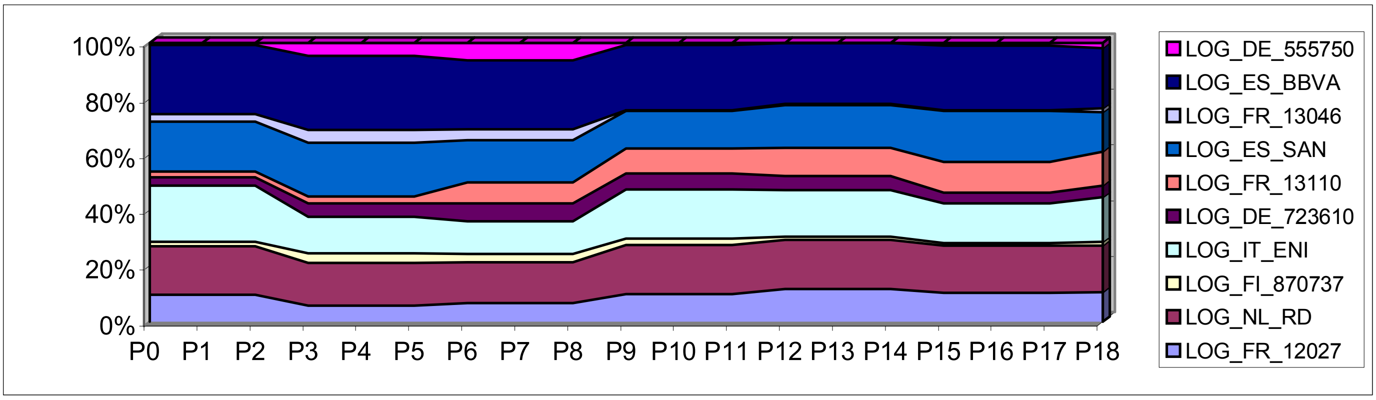

Figure 2 - Portfolio weights in 10-stock tracking portfolio with quarterly rebalancing

Appendix 5 - Out-of-sample results for 10-stock long/short portfolios with quarterly rebalancing

Table A5.1 - Out-of-sample results for 10-stock long/short portfolios (January - March 2002)

	Benchmark	+2.5% /-2.5%	**+2.5% /-5%**	+2.5% /-10%	+5% /-2.5%	+5% /-5%	+5% /-10%	+10% /-2.5%	+10% /-5%	+10% /-10%
Annualised return	3.94%	-31.83%	**-6.58%**	-10.46%	-37.75%	-12.50%	-16.37%	-1348.00%	-1322.75%	-1326.62%
Annualised volatility	20.52%	12.80%	**17.25%**	18.37%	13.50%	17.35%	18.21%	234.22%	236.47%	235.93%
Correlation with benchmark	-	0.13	**0.14**	0.16	0.11	0.12	0.14	-0.23	-0.04	0.04
Information ratio	0.19	-2.49	**-0.38**	-0.57	-2.80	-0.72	-0.90	-5.76	-5.59	-5.62
Sharpe ratio	0.00	-2.80	**-0.61**	-0.79	-3.09	**-0.95**	-1.12	-5.77	-5.61	-5.64

Table A5.2 - Out-of-sample results for 10-stock long/short portfolios (April - June 2002)

	Benchmark	+2.5% /-2.5%	+2.5% /-5%	**+2.5% /-10%**	+5% /-2.5%	+5% /-5%	+5% /-10%	+10% /-2.5%	+10% /-5%	+10% /-10%
Annualised return	-72.98%	-13.02%	2.5%	**49.50%**	-31.44%	-15.91%	31.09%	-52.04%	-36.52%	10.48%
Annualised volatility	24.97%	33.05%	43.19%	**35.14%**	36.81%	45.64%	35.72%	38.91%	46.20%	34.66%
Correlation with benchmark	-	-0.29	-0.41	**-0.47**	-0.24	-0.38	-0.45	-0.19	-0.34	-0.42
Information ratio	-2.92	-0.39	0.06	**1.41**	-0.85	-0.35	0.87	-1.34	-0.79	0.30
Sharpe ratio	-3.08	-0.52	**-0.03**	**1.29**	-0.96	-0.44	0.76	-1.44	-0.88	0.19

Table A5.3 - Out-of-sample results for 10-stock long/short portfolios (July - September 2002)

	Benchmark	+2.5% /-2.5%	+2.5% /-5%	+2.5% /-10%	+5% /-2.5%	+5% /-5%	+5% /-10%	+10% /-2.5%	**+10% /-5%**	+10% /-10%
Annualised return	-121.08%	-834.1%	-104.6%	-187.6%	-233.8%	495.7%	412.7%	710.34%	**1439.81%**	1356.80%
Annualised volatility	53.48%	190.7%	87.76%	94.86%	360.9%	258.2%	263.9%	563.18%	**461.60%**	465.98%
Correlation with benchmark	-	0.03	-0.01	-0.07	0.07	0.03	-0.01	0.09	**0.05**	0.02
Information ratio	-2.26	-4.37	-1.19	-1.98	-0.65	1.92	1.56	1.26	**3.12**	2.91
Sharpe ratio	-2.34	-4.39	-1.24	**-2.02**	-0.66	1.90	1.55	1.25	**3.11**	2.90

Table A5.4 - Out-of-sample results for 10-stock long/short portfolios (October - December 2002)

	Benchmark	+2.5% /-2.5%	+2.5% /-5%	+2.5% /-10%	+5% /-2.5%	+5% /-5%	+5% /-10%	+10% /-2.5%	+10% /-5%	**+10% /-10%**
Annualised return	25.73%	-14.70%	134.39%	131.09%	0.63%	149.7%	146.42%	319.11%	468.19%	**464.90%**
Annualised volatility	38.28%	206.1%	197.06%	191.76%	200.8%	219.1%	213.92%	242.51%	344.20%	**338.82%**
Correlation with benchmark	-	-0.14	0.64	0.63	0.00	0.63	0.63	0.24	0.60	**0.60**
Information ratio	0.67	-0.07	0.68	0.68	0.00	0.68	0.68	1.32	1.36	**1.37**
Sharpe ratio	0.58	-0.09	0.66	0.66	-0.02	0.67	0.67	1.30	**1.35**	**1.36**

Table A5.5 - Out-of-sample results for 10-stock long/short portfolios (January - March 2003)

	Benchmark	+2.5% /-2.5%	+2.5% /-5%	+2.5% /-10%	+5% /-2.5%	+5% /-5%	+5% /-10%	+10% /-2.5%	**+10% /-5%**	+10% /-10%
Annualised return	-58.30%	-1977%	17.74%	1.39%	-1975%	20.06%	3.70%	-1869.2%	**125.39%**	109.03%
Annualised volatility	40.94%	620.80%	129.6%	108.39%	617.0%	146.7%	125.66%	624.16%	**227.79%**	207.20%
Correlation with benchmark	-	-0.26	0.34	0.34	-0.25	0.34	0.33	-0.20	**0.34**	0.33
Information ratio	-1.42%	-3.18	0.14	0.01	-3.20	0.14	0.03	-2.99	**0.55**	0.53
Sharpe ratio	-1.52%	-3.19	0.11	-0.02	-3.21	0.11	0.00	-3.00	**0.53**	**0.51**