Algorithmic Decision Making Framework

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Abstract:

The emergence of algorithmic trading as a standard and often preferred execution platform has created the need for enhanced trading analytics to compare, evaluate, and select appropriate algorithms. The lack of transparency of many algorithms (due to undisclosed execution methodologies) limits investors' ability to measure the associated cost, risk, and efficiency of execution. In this paper, we outline a dynamic decision making framework to select appropriate algorithms based on pre-trade goals and objectives. The approach employs a three step methodology requiring 1) selection of price benchmark, 2) specification of trading style – passive to aggressive, and 3) determination of adaptation tactic. The framework makes use of the efficient trading frontier introduced by Almgren & Chriss (1999, 2000).

Algorithmic Decision Making Framework, Kissell & Malamut, Forthcoming, Journal of Trading.

I. Introduction

Algorithmic trading is the computerized execution of financial instruments following pre-specified rules and guidelines (e.g., trading strategy). It provides many benefits including anonymity, efficiency, lower commissions, and most importantly, reduced market impact and timing risk. However, utilization of algorithmic trading alone does not guarantee better results. It is essential investors become more proactive in the decision making process.

One of the more unfortunate aspects of algorithmic trading is that most of the existing algorithms provide very little transparency regarding the underlying strategy, expected costs and corresponding risks, and how will the algorithm adapt to changing market conditions such as prices and liquidity. Furthermore, many algorithms contain clever or silly names that have little to do with the underlying objective. This, in turn, often keeps investors in the dark regarding expected algorithmic performance.

Our proposed algorithmic decision making framework provides the necessary transparency and flexibility to develop a customized algorithmic strategy to ensure the strategy and algorithmic parameters are consistent the with overall investment goal (e.g., targeted return and risk distribution). The framework is based on a three step process. First, users choose their desired benchmark price (e.g., the price they are trying to achieve). This can be the previous close, open, arrival, user specified strike price, or day's closing price. Second, users select their desired trading style (e.g., aggressive, normal, moderate, and patient). Trading style is also often described as risk tolerance, risk aversion, or urgency, etc. Third, users specify their preferred adaptation tactic. That is how they want the algorithm to adapt to changing market conditions and manage outliers. For example, should the algorithm become more aggressive in times of favorable prices and more passive in times of adverse price movement or vice versa.

In order to specify the most appropriate algorithm and algorithmic parameters, it is essential that investors have a complete understanding of the cost and risk consequences associated with the trading decision.

A description of our transaction cost modeling methodology and algorithmic decision making framework for a single order is presented next. The proper methodology for a basket of stock (e.g., portfolio, program, trade list, etc.) is the subject of a subsequent article.

II. Description of Transaction Cost Models

An underlying goal of any type of trading is to reduce the hidden costs of trading (e.g., Treynor, 1981). Therefore, algorithms need to consider the effect of transaction costs and balance the tradeoff between market impact and timing risk over the trading horizon. Additionally, it is most beneficial if the prescribed implementation rules are continuously updated in real-time via a dynamic process to take advantage of available liquidity and prices when appropriate. For simplicity, we only consider the effect of market impact and timing risk in this paper. We do, however, point out that price appreciation (e.g., drift, momentum, etc.) is another component of transaction cost and should be included in situations where investors have a directional view on the price evolution of the stock.

Market Impact

Market impact is the movement in the stock price caused by the trade or order. It consists of permanent impact cost due to the information leakage of the order and temporary impact cost due to the liquidity and immediacy needs of the investor. The proposed market impact model is based on a cost allocation process (denoted as I-Star). A complete description of this model can be found in Kissell & Glantz (2003) or in Kissell, Glantz, & Malamut (2004). The most appealing aspect of this model is that it is equipped to handle cost estimation at the single stock and portfolio levels, and can be easily applied to various price benchmarks. The model is as follows:

$$I^* = a_1 \cdot \left(\frac{X}{ADV}\right)^{a2} \cdot \sigma^{a3} \cdot X \cdot P_0 \qquad \textbf{(1)}$$

where, I^* is the instantaneous impact cost expressed in dollars, (e.g., the expected impact cost if the entire order was released to the market immediately), X is the number of shares in the order, ADV is the average daily trading volume, σ is annualized volatility expressed as a decimal (e.g., 0.20 = 20%), P_0 is the current price, and a_1, a_2, a_3 are model parameters.

Then, market impact cost expressed in dollars for a specified trading strategy x_k (how the order is to be transacted over the trading horizon) is:

$$MI_{\$}(x_k) = \underbrace{\sum_{j=1}^{n} \frac{b_1 I^* x_j^2}{X\left(x_j + 0.5 v_j\right)}}_{Temporary} + \underbrace{(1-b_1)\cdot I^*}_{Permanent} \qquad \textbf{(2)}$$

where x_j is the number of shares to transact in period j, v_j is the expected volume (excluding the order) in period j, and b_1 is the percentage of temporary market impact cost where $0 \le b_1 \le 1$. The first part of the expression is temporary impact and is dependent upon the strategy x_k and the second part of the expression is permanent impact and is an unavoidable cost of transacting (e.g., it is not dependent upon the strategy).

Market impact expressed in \$/share is:

$$MI_{\$/sh}(x_k) = \sum_{j=1}^{n} \frac{b_1 I^*}{X} \cdot \frac{x_j}{(x_j + 0.5v_j)} + \frac{(1-b_1) \cdot I^*}{X} \quad \textbf{(3)}$$

For a constant trading rate where $x_j = \alpha \cdot v_j$, or equivalently, $\alpha = x_j / v_j$, and trading intervals expressed in volume-time where each interval contains the same quantity of volume, e.g., $v_1 = ... = v_j = ... = v_n$, we can rewrite (3) as follows:

$$MI_{\$/sh}(\alpha) = \frac{b_1 I^*}{X} \cdot \frac{\alpha}{\alpha + 1/2} + \frac{(1-b_1) \cdot I^*}{X} \quad \textbf{(4)}$$

Finally, for reasonable trading rates (e.g., $5\% \le \alpha \le 40\%$) $\alpha/(\alpha + 1/2) \approx 2/3\alpha^{1/2}$. Thus, equation (4) can be approximated as:

$$MI_{\$/sh}(\alpha) \cong \frac{b_1 I^*}{X} \cdot 2/3\alpha^{1/2} + \frac{(1-b_1) I^*}{X} \quad \textbf{(5)}$$

For simplicity, let $I_1 = 2/3 \cdot b_1 I^* / X$ and $I_2 = (1-b_1) I^* / X$. Then we can rewrite (5) as follows:

$$MI_{\$/sh}(\alpha) \cong I_1 \alpha^{1/2} + I_2 \quad \textbf{(6)}$$

Execution Prices

The expected execution price $\hat{P}(\alpha)$ for a strategy defined in terms of trading rate α is determined from the market impact expression (6) as follows:

$$E_0\left[\hat{P}(\alpha)\right] = P_0 + sign(X) \cdot \left(I_1 \alpha^{1/2} + I_2\right) \quad \textbf{(7)}$$

where, $sign(X) = 1$ for buy orders and $sign(X) = -1$ for sell orders. Notice that the market impact cost causes the price for buys to increase and the price for sells to decrease.

The expected execution price can also be computed at any point in time (e.g., time expectation). This process is to take a weighted average of the realized price at time t and the expected price for the residual shares which according to our cost allocation methodology is simply the current price plus market impact cost for those unexecuted shares. This is as follows:

$$E_t\left[\hat{P}(\alpha)\right] = \theta_t \overline{P_t} + (1-\theta_t)\left\{P_t + sign(X) \cdot \left(I_1 \alpha^{1/2} + I_2\right)\right\} \quad \textbf{(8)}$$

where, θ_t represents the percentage of the order that has executed at time t and $1-\theta_t$ is the unexecuted percentage of the order at time t, $\overline{P_t}$ is the average execution price at time t for $\theta_t \cdot X$ shares, and P_t is the current market price.

Future Price

The market impact expression (6) can also be used to estimate expected future price P_N. Recall that since only permanent (not temporary) impact persists in the price of the stock the intrinsic value of any future price needs to include the permanent impact cost (but not the temporary impact cost). In absence of price appreciation (e.g., drift, momentum, price alpha, etc.) this is as follows:

$$E_0(P_N) = P_0 + sign(X) \cdot I_2 \tag{9}$$

At any point in time P_N is estimated as follows:

$$E_t(P_N) = P_t + sign(X) \cdot (1-\theta) \cdot I_2 \tag{10}$$

Expected Cost

The expected benchmark cost $\phi(\alpha)$ corresponding to a specified trading rate α is defined as follows:

$$E_0[\phi(\alpha)] = sign(X) \cdot \left(E_0\left[\hat{P}(\alpha)\right] - P_b\right) \tag{11}$$

For a historical decision price such as the previous closing price (11) is equivalent to the implementation shortfall measure introduced by Perold (1988).

The expected trading cost can be computed at any point in time over the trading horizon by incorporating actual realized cost for those executed shares and the expected cost for those unexecuted shares. This is as follows:

$$\begin{aligned} E_t[\phi(\alpha)] &= sign(X) \cdot \left(E_t\left[\hat{P}(\alpha)\right] - P_b\right) \\ &= sign(X) \cdot \left(\theta_t \overline{P_t} + (1-\theta_t)\left(P_t + I_1\alpha^{1/2} + I_2\right) - P_b\right) \end{aligned} \tag{12}$$

These cost calculations for a single stock buy order for various price benchmarks are computed next. Costs for a single stock sell order can be determined in a similar manner.

Arrival Price Cost

The expected cost compared to the arrival price (e.g., $P_b = P_0$) is calculated as follows:

$$
\begin{aligned}
E_0[\text{Cost}] &= +1 \cdot \left(E_0\left[\hat{P}(\alpha)\right] - P_0\right) \\
&= P_0 + I_1\alpha^{1/2} + I_2 - P_0 \\
&= I_1\alpha^{1/2} + I_2
\end{aligned} \tag{13}
$$

Previous Close Cost

The expected cost compared to the previous closing price or other decision price that has occurred sometime in the past needs to incorporate the price change between the time of the investment decision t_d and the current time t_0 into the estimated price. In these situations, investors have already incurred a sunk cost (or realized a savings) equal to the price movement between t_d and t_0. Sometimes this sunk cost is due to the overnight price movement where investors do not have opportunity to transact and other times it is due to a delay in trading (Wagner, 1990). For example, suppose the previous day's closing price is P_d so we have $P_b = P_d$. Then the expected cost compared to the previous close price benchmark is:

$$
\begin{aligned}
E_0[\text{Cost}] &= +1 \cdot \left[E_0\left[\hat{P}(\alpha)\right] - P_d\right] \\
&= P_0 + 1I_1\alpha^{1/2} + I_2 - P_d \\
&= (P_0 - P_d) + I_1\alpha^{1/2} + I_2
\end{aligned} \tag{14}
$$

Notice that this cost (14) is equal to (13) plus price change $P_0 - P_d$.

Day's Close Cost

For a future price benchmark such as the day's closing price we need to estimate that expected price since it has not yet been recorded. Unlike a historical price benchmark such as the previous close or the current price a future price benchmark is not know in advance. In absence of price appreciation, the future price will be the current price plus permanent market impact cost. In situations with expected price appreciation this quantity will also need to be incorporated into the future price. Following, we compute the expected cost for the day's closing price following via (7), (9) and (11). For $P_b = E_0[P_N]$ we have:

$$E_0[\text{Cost}] = +1 \cdot \left(E_0\left[\hat{P}(\alpha)\right] - E_0[P_N]\right) \quad \textbf{(15)}$$
$$= P_0 + I_1\alpha^{1/2} + I_2 - (P_0 + I_2)$$
$$= I_1\alpha^{1/2}$$

An interesting aspect of using a future price benchmark is that expected cost only consists of the temporary market impact cost. This is because the permanent market impact cost is projected into the future prices and investors are not penalized for permanent impact. Therefore, execution cost is look more favorable on average using a future price benchmark than a previous or current price benchmark. Notice that the expected cost for the day's closing price (15) the expected cost is less than the expected cost for the arrival price benchmark (13) by the permanent impact cost quantity. Since the permanent impact cost is projected into all future prices, investors who measure cost compared to a future price benchmark such as the day's close are expected to do better since they will in effect save the permanent impact cost than if they measured cost against a previous or current benchmark price.

Timing Risk

The timing risk of a trade strategy x_k is the uncertainty surrounding expected market impact cost. It consists of price volatility, uncertainty in daily volume, and uncertainty in intraday volume patterns. For simplicity we only consider the price volatility component of timing risk. Readers interested in the full calculation are referred to Kissell & Glantz (2003).

For a discrete trade strategy x_k the timing risk expressed in \$/share is:

$$\Re(x_k) = P_0 \cdot \sqrt{\sum_{j=1}^{n} r_j^2 \cdot t\sigma^2 / n} \quad \textbf{(16)}$$

where $t\sigma^2$ is annualized variance scaled to the length of the trading horizon, $t\sigma^2/n$ is annualized variance scaled to the trading interval, and r_j is the number of unexecuted shares in period j where with $r_j = \sum_{l=j}^{n} x_l$.

For a constant trade rate $x_j = \alpha \cdot v_j$ and trading intervals expressed in volume-time, timing risk computed for discrete time intervals is:

$$\Re(x_k) = P_0 \cdot \sqrt{\frac{(2n+1)(n+1)n}{6n^3} \cdot t\sigma^2} \quad \textbf{(17)}$$

where t is the expected time to complete the order. Under continuous trading we have:

$$\Re(\alpha) = P_0 \cdot \sqrt{\sigma^2 \cdot \frac{1}{3} \cdot \frac{1}{250} \cdot \frac{X}{V^*} \cdot \frac{1}{\alpha}} \qquad \textbf{(18)}$$

since $\lim_{n \to \infty} (2n+1)(n+1)n \big/ 6n^3 = 1/3$ and
$t = 1/250 \cdot V_t \big/ V^* = 1/250 \cdot (V_t / X) \big/ \left(V^* / X\right) = 1/250 \cdot X \big/ V^* \cdot 1/\alpha$. Here V^* represents the expected daily volume and V_t represents the expected volume over the trading horizon.

The expected timing risk can also be computed at any point in time over the trading horizon. It is the incremental risk for the remainder of the order (unexecuted shares). This is as follows:

$$E_t\left[\Re(\alpha)\right] = \theta_t \cdot P_0 \cdot \sqrt{\sigma^2 \cdot \frac{1}{3} \cdot \frac{1}{250} \cdot \frac{X}{V^*} \cdot \frac{1}{\alpha}} \qquad \textbf{(19)}$$

III. Quantitative Decision Making Framework

The proposed algorithmic decision making framework is based on a three-step process:

1. Choose Price Benchmark
2. Select Trading Style (Risk Aversion)
3. Specify Adaptation Tactic

Investors choose these parameters based on their overall investment objectives and trading goals. Other algorithmic parameters such as start & end time, volume limits, price limits, maximum percentage of volume, minimum percentage of volume, participation with market on close (for day's close algorithm), etc., also need to be specified prior to trading. The quantitative decision making framework is described next.

Step 1 - Choose the Price Benchmark

The first step in the algorithmic decision making framework is to choose the desired benchmark price P_b. For example, previous close, open, arrival, day's close, user specified strike, etc. The benchmark price is then used to construct the efficient trading frontier (ETF) introduced by Almgren & Chriss (1999, 2000). Recall, the ETF consists of the set of all optimal trading strategies, e.g., those strategies with the least cost for a given level of timing risk and the lowest timing risk for a specified cost. An efficient or rational algorithmic strategy thus will reside on the ETF.

Figure 1 illustrates the ETF for three benchmark prices P_b : previous close, arrival price (current price), and the day's close. The expected cost for the arrival price benchmark is computed following equation (13) and consists of both temporary and permanent impact cost. The expected cost for the day's closing price benchmark is computed following equation (15) and consists of only temporary impact cost since permanent impact cost is projected into the future closing price and does not constitute an additional cost. This results in the day's closing price ETF being shifted below the arrival price ETF to reflect the lower expected benchmark cost. For the previous day's closing price benchmark the ETF will be shifted by $sign(X)\cdot(P_0 - P_d)$. Depending upon the side of the order price movement will constitute either a sunk cost or a savings. The assumption in **figure 1** is that there has been adverse price movement between t_d and t_0, e.g., $sign(X)\cdot(P_0 - P_d) > 0$, resulting in the previous close ETF shifting above the arrival price ETF. It is important to note that the previous close ETF could also have shifted downwards if there was favorable price movement, e.g., $sign(X)\cdot(P_0 - P_d) < 0$.

Now suppose that the market has just closed and we are at time t_d. Therefore, there has not been any observed price movement (e.g., no incurred cost or realized savings). In this case, however, there is incremental timing risk due to the uncertainty in overnight price movement. This causes the previous close ETF to shift to the right to account for the incremental overnight risk (**figure 2**). Notice in this situation for the same quantity of expected cost "C" there is increased timing risk (R2 > R1) accounting for the additional uncertainty in overnight price movement. But as soon as the market opens the next trading day, incremental overnight risk converts into the delay component which could be an increment cost if adverse price movement or an incremental savings if favorable price movement. This results in the ETF shifting either upwards or downwards as shown in **figure 1**.

Step 2 - Select Trading Style (Risk Aversion)

The second step in the algorithmic decision making framework requires investors to specify their trading style. This is often referred to as level of risk aversion or urgency and dictates the preferred tradeoff between market impact cost and timing risk. The more aggressive strategies incur higher market impact cost and lower the timing risk. The more passive strategies incur less market impact cost and higher timing risk. Alternatively, investors may specify a strategy that minimizes risk for a specified level of cost, minimizes risk or tracking error for a specified cost, based upon a fair value calculation or value-at-risk value. Proceeding, we will discuss the process for investors seeking to balance the trade-off between cost and risk via a risk aversion parameter λ following the specification of Almgren & Chriss (1999, 2000). Kissell, Glantz, and Malamut (2004) provide detailed guidelines for specifying trading style for alternative trading goals.

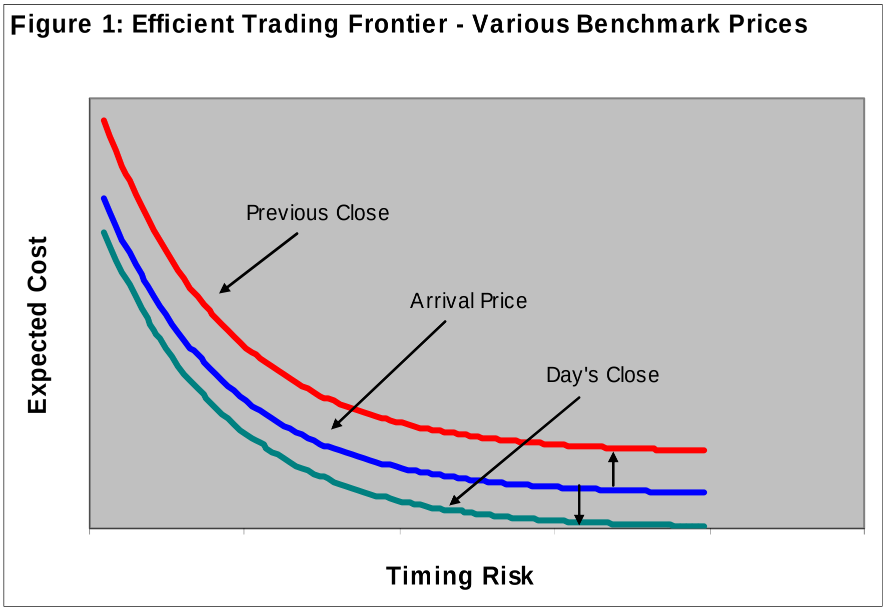

Figure 1: Efficient Trading Frontier (ETF). The arrival price ETF is constructed with temporary and permanent market impact cost. The day's closing price ETF will be lower (shifted down) than the arrival price ETF because it consists of only temporary market impact cost. Permanent impact is projected into the day's close and does not constitute an incremental cost. The previous close price ETF can be either above or below the arrival price ETF because it includes permanent and temporary impact, as well as price movement between the time of the investment decision and the time trading begins.

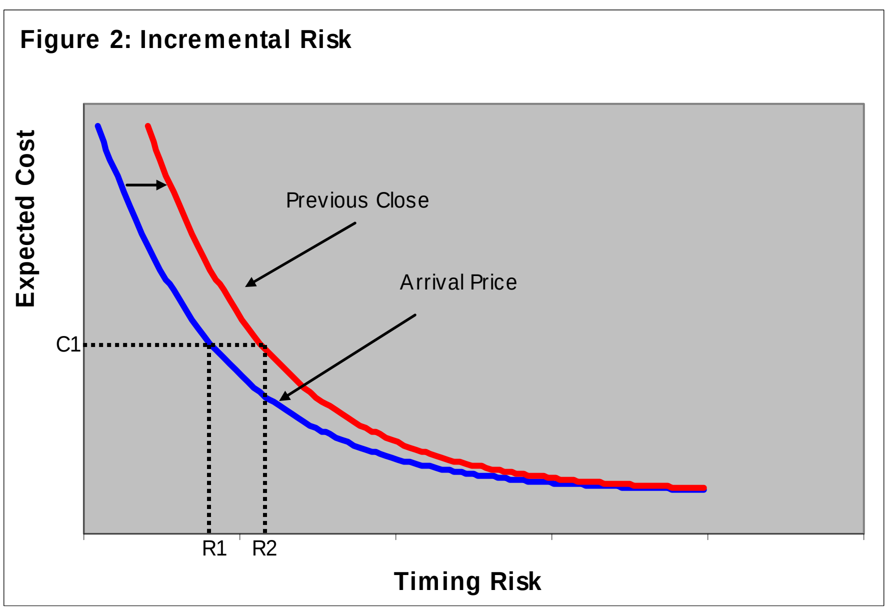

Figure 2: Incremental Risk. In situations where trading will begin at some future point in time (such as the next day) the ETF will shift to the right to account for the incremental price risk. This is most often observed when the investment decision is determined after the market close but trading can not begin until the next trading day. However, as soon as we reach that future point in time the incremental risk translates to either a cost or savings.

Figure 3 depicts strategies corresponding to different values of λ. Notice those strategies with higher values of lambda correspond to higher cost and less risk and those strategies with lower values of lambda correspond to lower cost and higher risk. Lambda also correspond to trading time where the higher the lambda the shorter the execution time and the lower the lambda the longer the execution time.

The optimal trading rate α that balances the tradeoff between cost and risk at the investor's specified level of risk aversion is determined via the following optimization:

$$\underset{\alpha}{Min} \quad Cost(\alpha) + \lambda \cdot Risk(\alpha) \tag{20}$$

where λ is the investor specified risk aversion parameter. Loosely, a value of $\lambda = 1$ indicates an investor equally concerned about risk and, a value of $\lambda = 2$ indicates an investor twice as concerned about risk than cost, and a value of $\lambda = 1/2$ indicates an investor one-half as concerned about risk as cost.

For an arrival price benchmark, the objective function is determined via (6) and (18) as follows:

$$\begin{aligned} \underset{\alpha}{Min} \quad & L = \left(I_1 \alpha^{1/2} + I_2\right) + \lambda \cdot P_0 \cdot \sqrt{\sigma^2 \cdot \frac{1}{3} \cdot \frac{1}{250} \cdot \frac{X}{V^*} \cdot \frac{1}{\alpha}} \\ s.t. \quad & LB \le \alpha \le UB \end{aligned} \tag{21}$$

where LB and UB are user specified lower and upper bounds (respectively) on trading rate with LB>0.

The optimal trading rate α_0 is the solution to $\partial L/\partial \alpha = 0$. A graphical illustration of this optimization is shown in **figure 4**. Solving (21) we obtain:

$$\alpha_0 = \frac{\lambda \cdot P_0}{I_1} \cdot \sqrt{\frac{\sigma^2 \cdot X}{3 \cdot 250 \cdot V^*}} \tag{22}$$

Arrival Price Reference Cost

The expected reference cost ϕ_{ref} corresponding to optimal trading rate α_0 for the arrival price benchmark is by substituting α_0 in (22) into (6) as follows:

$$\phi_{ref} = (\lambda \cdot I_1 \cdot P_0)^{1/2} \cdot \left(\frac{\sigma^2 \cdot X}{3 \cdot 250 \cdot V^*}\right)^{1/4} + I_2 \tag{23}$$

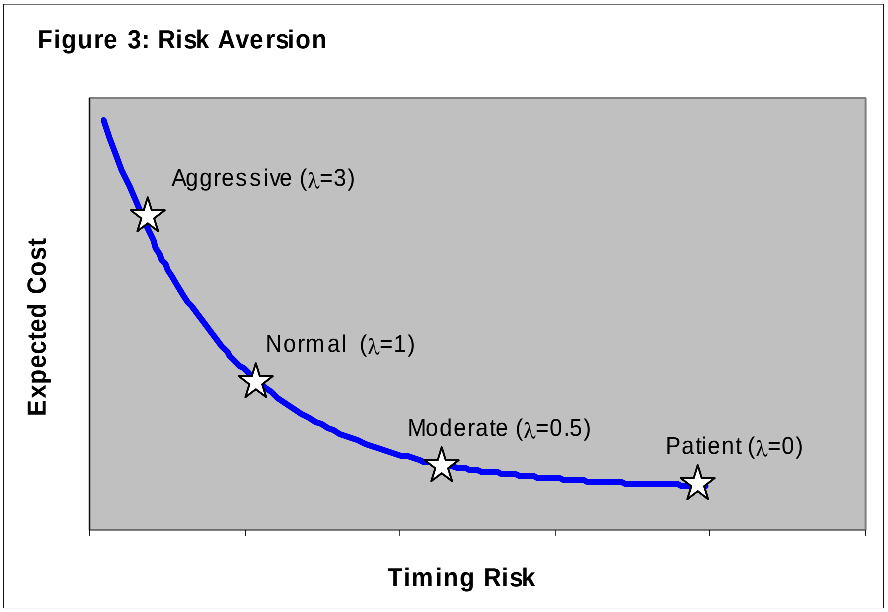

Figure 3: Risk Aversion. This diagram depicts various optimal trading strategies on the ETF. Higher levels of risk aversion (e.g., more aggressive, higher urgency, etc.) are associated with higher expected cost and lower timing risk. Lower levels of risk aversion (e.g., less aggressive, lower urgency, etc.) are associated with lower expected cost and higher the timing risk.

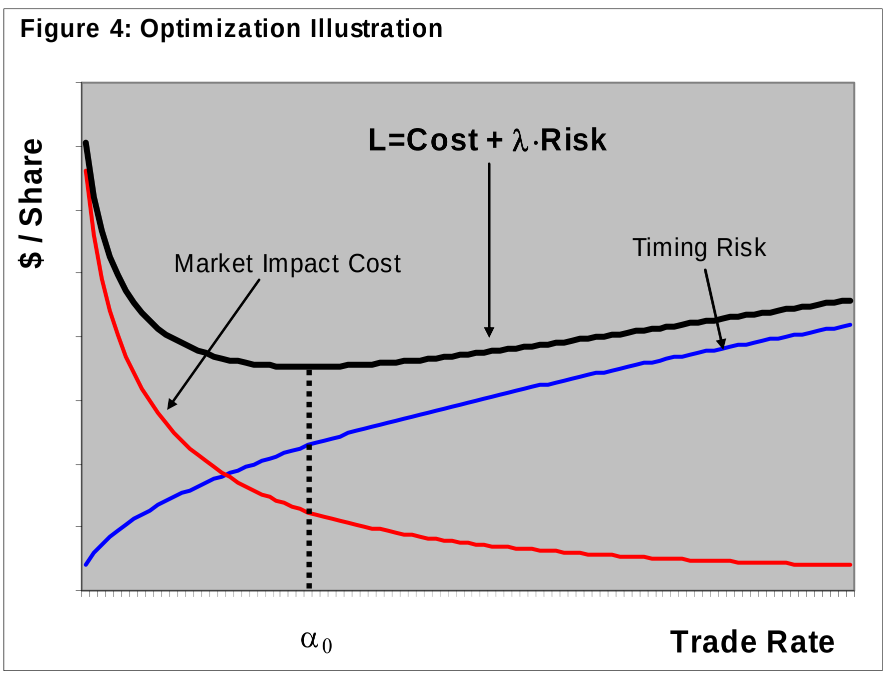

Figure 4: Optimization Illustration. This diagram illustrates the evolution of market impact cost and timing risk corresponding to various trading rates. The optimal trading rate α_0 is determined from the minimum point on the curve $L = Cost + \lambda \cdot Risk$,where λ is the specified level of risk aversion. Higher levels of risk aversion will correspond to higher trading rates and lower levels of risk aversion will correspond to lower trading rates.

It is important to keep in mind that for a historical price benchmark (e.g., previous close) and for a future price benchmark (e.g., day's close) the reference cost ϕ_{ref} must include appropriate adjustments. For a historical price benchmark the adjustment is $side(X)\cdot(P_0 - P_d)$ and for a future price benchmark the adjustment is to exclude the permanent impact cost. These are computed as follows:

Previous Close Reference Cost

$$\phi_{ref} = side(X)\cdot(P_0 - P_d) + (\lambda \cdot I_1 \cdot P_0)^{1/2} \cdot \left(\frac{\sigma^2 \cdot X}{3\cdot 250 \cdot V^*}\right)^{1/4} + I_2 \quad \textbf{(24)}$$

Day's Close Reference Cost

$$\phi_{ref} = (\lambda \cdot I_1 \cdot P_0)^{1/2} \cdot \left(\frac{\sigma^2 \cdot X}{3\cdot 250 \cdot V^*}\right)^{1/4} \quad \textbf{(25)}$$

Step 3 - Specify Adaptation Tactic

The third step in the algorithmic decision making process is for investors to specify how the algorithm is to adapt to changing market conditions (e.g., adaptation tactic). For example, liquidity & volume profiles, price movement, volatility, or market index movement such as the S&P500 or perhaps the stock's specific sector index movement. Adaptation tactics can also incorporate news, company earnings, or fed announcements. In specifying the appropriate tactics it is essential investors have a complete understanding of the cost consequences of the decision, that is, how will the decision affect the expected cost, timing risk, risk exposure (e.g., value-at-risk) and varying profit and loss distribution (Kissell & Malamut, 2005). Otherwise, it is unlikely the most appropriate algorithm will be selected.

It is important to note that there are times when investors may not be interested in specifying any adaptation tactic. In these circumstances, the algorithm will transact at the initial trading rate over the entire trading horizon (e.g., constant trade rate). An example where it is not appropriate to specify any adaptation tactic is for the day's close benchmark because any adaptation tactic may cause the algorithm to either finish early causing high benchmark risk or not finish by the end of the day potentially causing large opportunity cost.

The previous sections have defined trading strategy in terms of trading rate which adjusts to market volumes. We next describe a process to determine appropriate dynamic price based scaling rules where trading rates additionally adjust to prices. The more traditional price based scaling rules become more aggressive in times of favorable prices and less aggressive in times of adverse price movement. But the exact change in level of aggressive and/or passiveness is often arbitrary and unless investors are aware of the cost consequence of the decision it is difficult to determine beforehand if this is the most appropriate dynamic strategy. One of the more important goals involving adaptation tactics is to select the tactic that best manages potential outliers, expectation of price trend, and price uncertainty. In effect, this amounts to real-time management of the expected cost distribution (which is based on realized costs and a marginal ETF).

We discuss three types of price-based scaling algorithms: target, aggressive in-the-money (AIM), and passive in-the-money (PIM) type strategies.

Target Cost

The targeted cost adaptation tactic minimizes the squared difference between expected cost and reference cost ϕ_{ref}. It becomes more aggressive in times of favorable price movement and more passive in times of adverse price movement so not to incur unnecessary market impact cost.

Compared to a constant trading rate, the target cost adaptation tactic will incur lower cost on average but increases risk exposure (**figure 5a**).Mathematically, the target cost adaptation tactic is found by minimizing the following equation:

$$\underset{\alpha_t}{Min} \quad L = \left(E_t\left[\phi(\alpha)\right] - \phi_{ref}\right)^2 \tag{26}$$

where $E_t\left[\phi(\alpha)\right]$ is determined according to (12) and ϕ_{ref} is found in step 2 . Solving, we obtain:

$$\alpha_t = \left(\frac{\phi_{ref} - \left(\theta_t \overline{P_t} + (1-\theta_t)(P_t + I_2) - P_b\right)}{(1-\theta_t)I_1}\right)^2 \tag{27}$$

with, $LB \le \alpha_t \le UB$.

Aggressive in the Money

The *Aggressive in the Money (AIM)* adaptation tactic maximizes the probability that the actual cost will be less than the reference cost ϕ_{ref}. This is equivalent to maximizing the Sharpe Ratio where performance (return) is measured as the difference of reference cost minus actual cost. This has also been defined as maximizing the information ratio of the trade (Almgren & Chriss, 2000). The AIM tactic becomes more aggressive in times of favorable prices and less aggressive in times of adverse price movement but will do so in a manner that manages the tradeoff between continued favorable prices and price risk (i.e., the possible of prices continuing to be favorable versus the chance the prices will move away and execution will become more costly). This is similar behavior to the target tactic but additionally incorporates price risk.

Compared to a constant trading rate, AIM will incur lower cost on average but increased risk exposure (**figure 5b**). Compared to the target cost strategy, AIM incurs a slightly lower cost with increased potential for better prices, but this comes with an increased risk exposure (**figure 5c**). Mathematically, the AIM tactic is found by maximizing the following equation:

$$\max_{\alpha_t} \quad L = \frac{\phi_{ref} - E_t[\phi(\alpha)]}{E_t[\Re(\alpha)]} \tag{28}$$

where $E_t[\phi(\alpha)]$ is determined according to (12), $E_t\Re[\phi(\alpha)]$ is determined according to (19), and ϕ_{ref} is found in step 2. Solving we obtain:

$$\alpha_t = \left(\frac{\phi_{ref} - \left(\theta_t \overline{P_t} + (1-\theta_t)(P_t + I_2) - P_b\right)}{2(1-\theta_t)I_1} \right)^2 \tag{29}$$

with, $LB \le \alpha_t \le UB$.

Passive in the Money

The *Passive in the Money (PIM)* adaptation tactic is a price based scaling tactic with underlying goal to limit potential losses (in times of adverse price movement) and provide opportunity to share in gains (in times of favorable trends). For example, PIM tactics manages potential adverse outliers. PIM becomes more aggressive in times of adverse price movement to reduce the likelihood that the price will move too much away from the benchmark price. It becomes more passive in times of favorable prices allowing opportunity to participate with the better prices if the favorable trend persists. It behaves in the opposite manner of AIM. The PIM adaptation tactic is most consistent with the Arrow-Pratt constant relative risk aversion (CRRA) formulation (see Pratt, 1964 and Arrow, 1971).

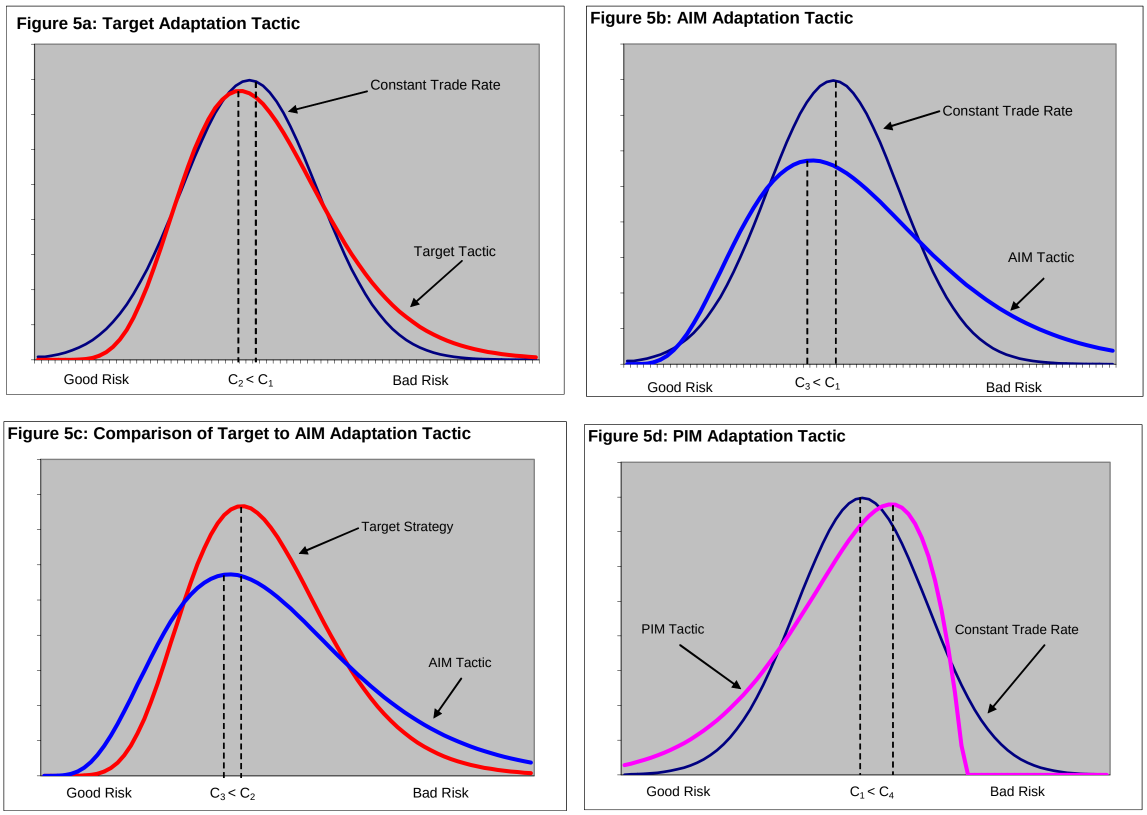

Figure 5: Cost Distribution of Adaptation Tactics. Figure 5a compares the cost distribution of the target cost tactic to a constant trade rate. The target tactic incurs a lower cost on average C_2 compared to C_1 for the constant trade rate, but is associated with increased risk exposure (e.g., bad risk) and less opportunity to achieve better prices. Figure 5b compares the cost distribution of the AIM tactic to a constant trade rate. The AIM tactic is similar to the target cost tactic. It incurs a lower cost on average C_3 compared to C_1 for the constant trade rate, but with increased risk exposure and less opportunity to achieve better prices. Figure 5c compares the cost distribution of AIM to target. The AIM tactic incurs a lower cost on average than the target cost tactic, C_3 compared to C_2. It is also associated with higher risk exposure but more opportunity to achieve favorable prices. Figure 5d compares the cost distribution of the PIM tactic to a constant trade rate. The PIM tactic incurs a slightly higher cost on average C_4 compared to C_1 for the constant trade rate, but is associated with decreased risk exposure (e.g., bad risk) and increased opportunity to realize better prices in times of favorable trends (e.g., good risk).

Compared to a constant trading rate strategy PIM realizes slightly higher costs on average but decreased risk exposure and increased opportunity to participate in better prices (**figure 5d**). Mathematically, the PIM adaptation tactic is found by minimizing the information ratio of the trade or equivalently, maximizing the following equation:

$$\underset{\alpha_t}{Max} \quad L = \frac{E_t[\phi(\alpha)] - \phi_{ref}}{E_t[\Re(\alpha)]} \tag{30}$$

where $E_t[\phi(\alpha)]$ is determined according to (12), $E_t \Re[\phi(\alpha)]$ is determined according to (19), and ϕ_{ref} is found in step 2. Solving we obtain:

$$\alpha_t = \left(\frac{\left(\theta_t \overline{P_t} + (1-\theta_t)(P_t + I_2) - P_b\right) - \phi_{ref}}{2(1-\theta_t) I_1} \right)^2 \tag{31}$$

with, $LB \le \alpha_t \le UB$.

Cost Consequence

As a strategy deviates from an initially prescribed trade rate it alters the expected cost distribution. But there is no free lunch, as the expected cost decreases overall risk exposure to less favorable prices increases and potential for more favorable prices decreases. It is possible to decrease risk exposure to less favorable prices and increase potential for better prices but this comes at the expense of increased expected cost. For example, both target and AIM tactics are associated with better prices (lower costs) on average but this comes at the expense of increased "bad" risk and truncated "good" risk. There is higher probability that that these strategies will incur less favorable prices than a constant trade rate because they slow down in times of adverse price movement and if the adverse trend persists less favorable prices will be realized. These strategies do not provide opportunity to participate in better prices if the favorable trend continues because the order will likely be completed ahead of schedule. In comparison, target is more aggressive than AIM. The PIM tactic, on the other hand, provides decreased "bad" risk exposure and increased "good" risk exposure, but it is also associated with less favorable prices (higher cost) on average. Since PIM becomes more aggressive in times of adverse price movement it is likely to finish ahead of schedule, so if the adverse trend continues investors are protected from the less favorable prices. Also, since the algorithm slows down in times of favorable prices it provided opportunity to participate in very favorable prices if the trend persists.

The rate of change for these various adaptation strategies is shown in **figure 6** for a buy order. Notice that as prices move higher target and AIM become more passive and as prices move lower they become more aggressive. Also, since AIM simultaneously manages cost and timing risk whereas target only manages cost the AIM adaptation tactic changes at a slower rate than target. PIM, conversely, behaves in the exact opposite manner as AIM where it becomes more aggressive in times of increasing prices to truncate risk and more passive in times of favorable trends to take advantage of better prices.

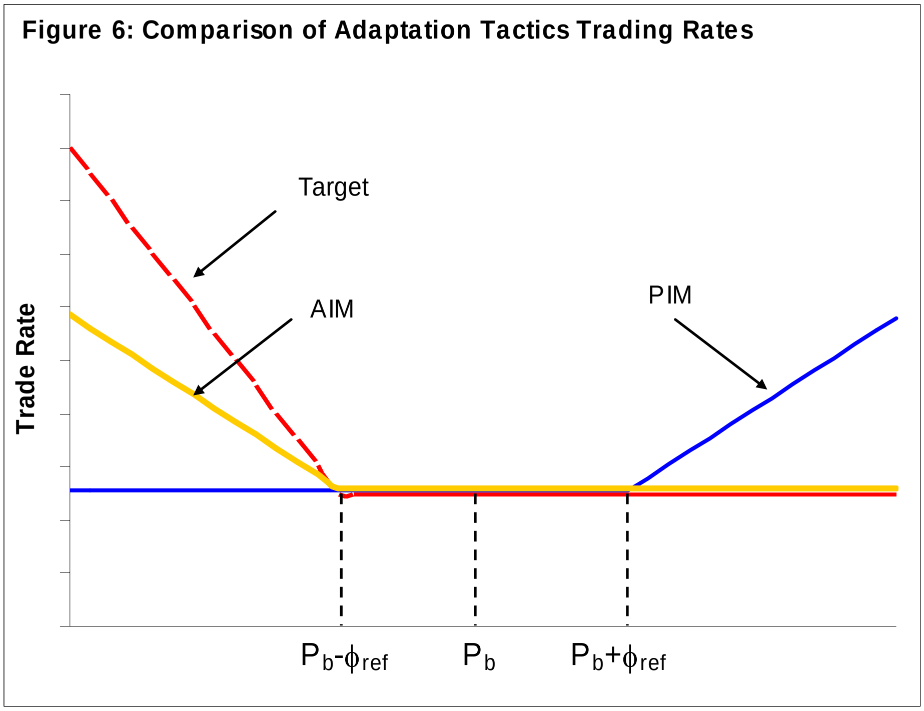

Figure 6: Comparison of Adaptation Tactic Trading Rates. This diagram illustrates the change in trading rate for each of the adaptation tactics for a buy order. The target and AIM tactics become more aggressive in times of lower prices (in-the-money) and less aggressive in times of higher prices (out-of-the-money).The target tactic is more aggressive than the AIM in times of favorable prices. The PIM strategy behaves in the opposite manner of AIM by being more aggressive in times of adverse prices and less aggressive in times of favorable prices.

IV. Conclusion

In conclusion, our algorithmic decision making framework utilizes a cost allocation market impact model (I-Star) that is readily adaptable to any benchmark price and market conditions (liquidity and prices). Furthermore, the underlying modeling methodology provides necessary transparency for investors to determine the trading strategy and adaptation tactics most consistent with overall investment objectives. The framework is based on a three step process where investors i) choose their benchmark price, ii) select their preferred trading style (risk aversion), and iii) specify their preferred adaptation tactic.

It is essential that investors understand the overall cost consequences of these decisions on the trading cost distribution. For example, target and AIM adaptation tactics will decrease expected cost but this comes with increase risk exposure and could result in unfavorable prices if adverse movement persists. Furthermore, they do not provide opportunity to participate in better prices if favorable trends persist. PIM, on the other hand, decrease risk exposure to adverse price trends and provides opportunity to participate in the most favorable prices, but this tactic comes with an increase in expected cost. Finally, it is essential that managers and traders work together to ensure that implementation plans are in sync with the overall investment goals and objectives. Otherwise, it is unlikely funds will achieve maximum levels of return.

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