The relationship between VIX futures term structure and S&P500 returns

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Abstract

The current paper tests and documents the relationship between the term structure of VIX futures and the underlying equity returns. Furthermore, it investigates the signaling effects of VIX futures term structure in respect to future stock index movements. The objective of this empirical analysis is to verify if a steep upward-sloping term structure indicates a late phase of a bullish trend and conversely if an extreme negative term structure suggests an over-sold market, as certain market participants believe. The proposed term structure measure is estimated as the slope of the term structure of six VIX future contracts and the spot VIX index.

The empirical findings of this study suggest that there is a strong statistical significant positive contemporaneous relationship between the changes in the VIX futures term structure and the returns of the underlying equity index. Finally, the econometric analysis lends some support to the hypothesis that the term structure of VIX futures can be used as a contrarian indicator for investing in the equity market.

Keywords: *VIX futures, volatility term structure, future equity returns*

JEL Classification Codes: G10, G11

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Literature on the CBOE Volatility Index (VIX) and its derivatives is fast growing, but the current study is the first academic attempt to examine directly the relationship between the term structure of VIX futures and the underlying equity returns. In particular, both the contemporaneous relationship between the changes in the term structure of VIX futures and the S&P500 returns and the signaling ability of the VIX futures term structure in respect to future S&P500 returns are tested and documented. This paper is motivated by earlier studies of the potential relationship between VIX and contemporaneous and future stock returns (e.g. Fleming et al 1995; Giot 2005) and extends the VIX futures literature by testing the predicting ability of VIX futures term structure in respect to future equity movements.

In the recent years, several models devoted to VIX futures pricing have been developed in the academic literature. Zhang et al (2010) provide a comprehensive analysis of the VIX futures market. There are also several attempts that investigate the factors that drive the variations in the volatility surface. The empirical findings confirm that about 70-90% of the total variations in the implied volatility surface can be attributed mainly to three factors: parallel shift, twist and curvature[1]. A large part of the existing implied-volatility term structure literature has focused on either options or OTC variance swaps data (indicatively Byoun et al 2003; Mixon 2007; Egloff et al 2010). More recently, exchange listed volatility derivatives data have been used to analyze the volatility term structure. The study by Lu and Zhu (2010) is the first attempt to empirically investigate the whole term structure of VIX futures market.

Nevertheless, since volatility term structure is essentially directly analogous to the interest rates term structure, previous academic research has investigated volatility term structure's information content regarding future short-term volatility (indicatively, Fassas and

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[1] For further details on the dynamics of the factors that drive implied volatility surface, indicatively, see Skiadopoulos et al (1999) and Mixon (2002).

Siriopoulos, 2012 and Zhang et al, 2010). But, academic literature also suggests a link between high expected returns and high conditional volatility (Merton, 1973 was the first to highlight this). Hence, given the abundant empirical evidence that the volatility of the risk-neutral distribution, reflected by the VIX index, tracks and forecasts well the volatility of the physical distribution (e.g. Jiang and Tian, 2005), it is reasonable to assume that VIX will also be related to forward looking returns. Whaley (2000) suggests that the level of VIX shows how much are the market participants willing to pay in volatility terms, either for hedging their equity portfolio (by buying put options), or speculating with limited pre-determined maximum loss (by buying call options). VIX can also be used as a market-timing indicator, since extreme high values of VIX are often concurrent with undervalued level for the underlying S&P500 index (Giot, 2005). His empirical findings suggest that there is certain evidence that positive (negative) forward-looking returns are recorded for long positions initiated when VIX reaches extremely high (low) levels.

It is plausible to assume that the term structure of VIX futures conveys more information regarding the equity market than spot VIX, which is a single constant thirty-day estimate of future realized volatility. With respect to the above framework, this study focuses on two closely related topics, which deal with the empirical link between VIX futures term structure and equity returns. More precisely, it investigates the contemporaneous relationship between VIX futures term and S&P500 index returns and also the possible linkage between changes in the term structure of VIX futures and future stock market returns. This is the first academic attempt to empirically study the whole term structure of VIX futures market and its relationship with equity returns. The empirical analysis of this study has important practical implications for financial market practitioners, as it shows that they can also use the VIX futures term structure as a stock market predictive tool and not only as a proxy of market expectations on forward volatility.

The paper proceeds as follows: the next section includes a preliminary analysis of the VIX futures market, while section three describes the proposed estimation of the VIX futures term structure and its statistical attributes. Section four includes the empirical analysis of the contemporaneous relationship between VIX futures term structure and S&P500 returns, while section five explains the relevant analysis of the relation between VIX futures term structure and future equity returns. Finally, section six includes the concluding remarks.

I. DESCRIPTIVE STATISTICS AND PRELIMINARY ANALYSIS

VIX futures started to trade in CBOE Futures Exchange (CFE) on March 26, 2004. A detailed description of the VIX futures market, its trading application and the futures settlement procedure can be found in Fassas and Siriopoulos (2012). According to the official specifications of the futures contract, theoretically CBOE "may list for trading up to nine near-term serial months and five months on the February quarterly cycle for the VIX futures contract." In practice, in the period from the contract initiation until March 8, 2006 four futures contracts were listed for each day. Subsequently, on March 9, 2006 six futures contracts were listed. The number of listed contracts gradually increased to seven on April 24, 2006, to nine on October 23, 2006 and to ten on April 22, 2008. Currently there are ten contracts traded on each day.

The data set of the current analysis consists of the daily closing prices of spot VIX and the six nearest VIX futures for the period from March 26, 2004 to July 30, 2010. The daily prices were retrieved from the website of the CBOE Futures Exchange (CFE). Panel A in Exhibit 1 reports the descriptive statistics of the seven VIX-related time-series. The longer-maturity VIX contract tends to be less volatile than the shorter-maturity one, while spot VIX has the largest standard deviation. The contracts time to expiration varies considerably, with the contracts of the February quarterly cycle being traded one year before they expire, while the near-term contracts are usually traded for a notably shorter period. Therefore, not all six time series are continuous: from October 23, 2006 prices exist for at least five contracts, while from April 22, 2008 prices for all six contracts are available. At any case, the sample set includes prices for at least three contracts at any trading day. From the second Monday immediately preceding the expiration day (usually a Wednesday) of the contract month, the next-to-deliver contract is considered. The timing of the rollover in the dataset is set in that way such as no contract under review has a remaining life less than two weeks. Panel B in Exhibit 1 includes the average, maximum and minimum number of days until expiration for the six nearest contracts.

II. THE VIX FUTURES TERM STRUCTURE

The VIX futures chain forms a curve that relates the current value of VIX with its long-term expectation, but unlike the VIX index does not reflect constant-maturity. However, the proposed methodology of estimating the VIX futures term structure does not require an interpolation scheme in order to construct futures contracts with fixed maturities (as Lu and Zhu (2010), who linearly interpolate the VIX futures price on each trading date, together with VIX data, to obtain the prices for fixed maturities). The present estimation of the VIX futures term structure – following Äijö (2008) and Krylova et al (2009) – is conducted on each trading day by fitting a linear model of the available contracts and cash VIX as a function of time to maturity based on least squares criterion (spot VIX index is considered as the price for VIX futures with zero maturity).

In order to test whether the estimated slope measure is a sufficient gauge of the VIX futures term structure, a principal component analysis is conducted. The daily changes of spot VIX and the six futures contracts are used, since the actual levels of all volatility measures are non-stationary. If non-stationary data are used, the portion of the variance captured by the first principal component depends on the sample size and tends to unity as it (the sample size) grows (Gerlach 2003). The empirical findings (Panel A in Exhibit 2) show that the first principal component explains 88.40% of the variance of the data considered. The second principal component explains an additional 4.87% of the variation of the data and the third largest component accounts for less than 3.5% of the total risk. Thus, it is evident that three factors explain an overwhelming proportion of the variation in VIX futures term structure.

In the attempt to identify these three factors, the components loadings are examined (Panel B in Exhibit 2). It is evident that the loadings for the first component are all positive and rather similar. Hence, the first principal component can be considered as representing level shifts of the futures term structure. The factor loadings for the second principal component are positive for the spot and the one-, two- and three-month contracts, but negative for the four-, five-, and six-month futures contracts. Therefore, the second component captures changes in the slope of the VIX futures term structure. Finally, the third factor is interpreted as "curvature", since it is positive for short and long maturities and negative for intermediate maturities. Consequently, the estimated slope measure should provide a sufficient amount of information of the VIX term structure, since the principal component analysis shows that a small portion of the variance of the VIX futures term structure can be explained by the twisting of the term structure.

Panel A in Exhibit 3 includes the descriptive statistics of the goodness of fit of the daily least-square estimation as measured by the respective R-squared. The results show that the estimated slope measure captures well the shape of the term structure, as the median value of the R-squared is above 80%. Looking closer into the data, we notice that the values for the R-squared converge to zero, when the slope of the VIX futures term is flat. Panel B in Exhibit 3 reports the descriptive statistics of the estimated VIX futures term structure time-series. The estimated term structure is in units of volatility percentage points per year. Thus, the mean VIX term structure estimate of 1.935 indicates that the level of VIX futures price increases, on average, by 1.935 volatility percentage points per one-year horizon, or equivalently, about

0.161 percentage points per month. A simple t-test for means and the Wilcoxon signed ranks test for medians show that the upward sloping term structure is statistically significant (at 1% significance level). Therefore, on average, during the period under examination (March 2004 – July 2010) the VIX futures term structure is upward sloping, i.e. short-term VIX futures trade at lower price than long-term contracts.

However, the range of observations is relatively large, suggesting that the VIX futures term structure is considerably time varying. In fact, as shown in Exhibit 4, VIX futures term structure may change considerably and turn from upward sloping to downward sloping and vice versa in a limited period of time. For example, for the larger part of 2007 the slope changed sign several times. Furthermore, it can be noted from Exhibit 4 that although the estimated term structure tends to be upward sloping, there are also periods of downward sloping term during the time under review. The VIX futures term structure turned negative for the first time since the inception of VIX futures (in 2004) during the global stock market correction in the second quarter of 2006. Nevertheless, it was not until the global credit and financial crisis of 2008 that we experienced extreme negative slopes. In the fall of 2008, VIX reached exceptional high prices and remained in extremely elevated levels for an extended period of time. VIX achieved its intraday high in October, but closed at a new all-time high on November 20, 2008. With VIX at such high levels, it is not surprising that VIX futures traded at a discount. Theoretically, when spot VIX is very low compared to its long-term average level, as it was true during the 2005-2007 period, VIX futures trade at a premium because market participants expect that VIX will rise in the future. Conversely, when VIX is very high, VIX futures trade at a discount, implying that spot VIX will eventually decline to its normal levels. In practice, during the fall of 2008, the discounts on VIX futures reached unprecedented levels.

Finally, regarding the other preliminary statistics, it can be noted that if VIX futures term structure were calculated as the difference between the short-term and the long-term contracts, the observed negative skewness would suggest that the short-term contract exhibits occasional jumps, while the longer-term contract is more stable. Then again, as Krylova et al (2009) suggest, since the definition of the VIX futures term structure is the slope of seven different VIX time series, this interpretation is not necessarily valid. In order to determine whether the VIX futures slope time-series is stationary, the augmented Dickey-Fuller unit root test is conducted[2]. The lag length used is decided based on the Schwartz information criterion. The results of the unit root test soundly reject the null hypothesis of a unit root. Last, the autocorrelations (for up to five lags) of VIX futures term structure are reported in Panel C in Exhibit 3. All five orders exhibit statistically significant positive autocorrelation. Their significance is tested with the Ljung-Box Q-statistic. The first order autocorrelation is 0.970 suggesting that VIX futures term structure is mean reverting. A conventional measure of persistence of shocks (Krylova et al 2009), the ratio $\ln 0.5 / \ln \rho_1$, implies a mean half-life of shocks to the VIX futures term structure process of approximately 23 trading days.

III. THE CONTEMPORANEOUS RELATIONSHIP BETWEEN VIX FUTURES TERM STRUCTURE AND S&P500 RETURNS

The contemporaneous relationship of stock market returns and VIX futures term structure changes is tested with a regression analysis of the equity index returns (R_t) against the changes of the estimated slope measure ($\Delta slope_t$). Furthermore, this relationship is tested over several frequencies (for 1 day, 1 week, 2 weeks, 1 month and 2 months) in order to test the potential different dynamics of this relationship. In order to deal with the overlapping

[2] It is plausible to assume that there is no time trend and intercept in the slope series in the long run. Nonetheless, the ADF test was also performed with a time trend and intercept yielding identical results; furthermore, the findings are not sensitive to the number of lags used.

observations problem, the sample is reduced in a way in which none of the observations overlap.

Additionally, since there is clear evidence of a strong asymmetric contemporaneous relationship between the changes of VIX and the underlying index returns[3], the changes of the VIX futures term structure are separated into positive ($\Delta slope_t^+$) and negative ($\Delta slope_t^-$), in order to test for a similar asymmetry effect. Last, existing empirical evidence shows that the stock market returns are time varying and conditionally heteroskedastic; hence, the use of Newey-West standard errors is appropriate for this empirical investigation.

As a result, the testing of the contemporaneous relationship between S&P500 returns and VIX futures term structure changes is conducted using the following specification:

$$R_{t,K} = \alpha + \beta \Delta slope_{t,K}^{+} + \gamma \Delta slope_{t,K}^{-} + \varepsilon_t \qquad (1)$$

in which, $R_{t,K}$ represents percentage returns in S&P500 in the non-overlapping K period (where K = 1 day, 1 week, 2 weeks, 1 month and 2 months), $\Delta slope_{t,K}^{+}$ is the change in Slope over the same K period, if positive, and $\Delta slope_{t,K}^{-}$ is the change in Slope over the same K period, if negative. If my proposition is true, the two coefficients should be significantly different from zero and from each other.

The usual regression statistics for Equation 1 for all five time horizons are given in Exhibit 5. In particular, it includes the estimators and their Newey-West standard errors, the adjusted R-square, the F-statistic and the Durbin-Watson statistic. The findings suggest that there is a statistical significant positive relationship between the changes in the VIX futures term structure and the returns of the underlying equity index in all five time frames. The R-

[3] Indicatively, Fleming et al (1995), Whaley (2000), Giot (2005) and Carr and Wu (2006) all identify a statistically significant negative and asymmetric contemporaneous relationship between VIX and S&P100 (or S&P500) returns. Additionally, a similar relationship has also been documented empirically in other equity markets.

squared is ranging from 63% (for daily observations) to 47% (for bi-weekly observations). Furthermore, a negative change in the VIX futures slope is associated with a larger negative return of the stock index, than an equivalent positive change. The coefficients of the negative slope changes are in all instances higher (in absolute values) than the respective coefficients of the positive changes, but, in order to test the significance of their difference, the Wald restriction test is employed (Exhibit 7). The reported p-values indicate that the null hypothesis that the difference is not statistically relevant can be rejected (at the 90% confidence level) in four out of five different timeframes.

The presumed underlying rationale of the recorded positive relationship is that market participants are willing to pay hefty premiums to buy protection in the form of out-of-the-money S&P500 options (with several expirations in the future). Under this reasoning, it is rather safe to assume that when equity markets are advancing, investors are buying puts in order to hedge their existing stock portfolio.

IV. THE RELATIONSHIP BETWEEN VIX FUTURES TERM STRUCTURE AND FUTURE S&P500 RETURNS

Investors very often use extreme levels of the VIX as a contrary trading signal for the equity market. In particular, high levels of VIX may indicate capitulation that leads to extreme under-valuation of stock prices and thus, buying opportunities. Conversely, abnormal low VIX levels suggest complacency by investors and thus, increased probability for market correction (Simon 2003). Simon and Wiggins (2001) show that, during the period from January 1989 through June 1999, when the VIX is in its top decile, S&P500 futures are on average 2.8 percent higher over the next 30 days than they otherwise would be. This paper extends the use of VIX as a trading tool by investigating the predictive power of the VIX futures term structure in respect to future equity market returns. Since the VIX futures premium/ discount can be related to investor risk aversion, it may be informative about the equity returns.

The model used to examine the relationship between VIX futures term structure changes and future S&P500 returns is similar to the specification used in the previous section. Specifically, the following regression is estimated:

$$R_{t+K} = \alpha + \beta \Delta slope^{+}_{t,K} + \gamma \Delta slope^{-}_{t,K} + \varepsilon_t \tag{2}$$

in which, R_{t+K} represents percentage returns in S&P500 in the subsequent non-overlapping K period (where K = 1 day, 1 week, 2 weeks, 1 month and 2 months), $\Delta slope^{+}_{t,K}$ is the change in Slope over the previous K period, if positive, and $\Delta slope^{-}_{t,K}$ is the change in Slope over the previous K period, if negative. The significance of their difference is investigated with the use of the Wald restriction tests (Exhibit 7). Positive slope coefficients of Equation 2 suggest that the change in the slope of VIX futures is a straight buying or selling indicator, while negative slope coefficients imply that the change in the slope is a contrarian indicator.

The empirical findings (shown in Exhibit 6) are not as robust as in the contemporaneous relationship, but show that VIX futures term structure can be used as a stock market predictive tool. In particular, there is a negative relationship between changes in the estimated VIX futures slope and forward-looking S&P500 returns. The positive changes prove statistically more important, as they are significant (at least at the 10% level) for the weekly, the biweekly and bimonthly future returns. This finding seems consistent with market practitioners' view, who consider a steep upward sloping term structure as an indication of an overbought market. The VIX futures term structure will ultimately flatten out and according to the empirical history this will happen when the stock market declines. The upward sloping average VIX futures term structure suggests that short-term volatility is relatively low compared with its long-term average and that market participants expect an increase in volatility in the future (Zhang et al 2010). This may prove bearish for the stock market as usually when VIX advances, S&P500 declines.

A potential trading strategy for benefiting from an upward sloping term structure involves buying both VIX and S&P 500 Index puts for the same expiration month. VIX puts pricing is based on the respective futures price, which trades at a premium relatively to spot VIX. At expiration, by definition, the prices of spot and future VIX should converge. Therefore, if market participants prove wrong and spot VIX doesn’t increase, the VIX put will become profitable. Conversely, if VIX price advances in order to meet the futures prices, the VIX puts will expire worthless, but, most probably, the S&P500 put options will make money (since when VIX advances, usually S&P500 declines). Hence, the one leg or the other of this trading strategy should prove profitable. Still, if the combined position expires worthless, the risk for the investor is limited, since the strategy involves buying options.

An alternative strategy to speculate on the VIX futures term structure is to directly buy/sell futures contracts. If the term structure is considered too steep and is expected to flatten, then an investor should consider buying a shorter-term futures contract and simultaneously go short on a longer-term contract. One of the main attributes of the futures spread strategy is its high leverage. Then again another way to trade the term structure is by using options with different maturities instead of futures spreads. For example, in an upward sloping term structure, an investor can simultaneously buy a call option with a near-term expiration and a put option with a longer-term expiration. In this case, the riskiness of the position (compared to the futures position) is considerably lower, as the maximum loss is the paid premiums. On the other hand though, the leverage factor will shrink and the time value of the options will be an additional possible risk.

V. CONCLUSION

The CBOE Volatility Index (VIX) has long been a popular measure of investors' forecast of future volatility, as well as a measure of "fear" in the stock market (Whaley 2000). However, VIX futures are a relatively new instrument that allows market participants to trade volatility directly, instead of indirectly trying to gain volatility exposure through options or over-the-counter products. The Chicago Board Options Exchange (CBOE) introduced the first exchange-traded product, VIX futures, on March 26, 2004. Investors that want to have exposure into VIX are better off by investing in the near-term futures contracts (and roll-over from month to month) instead of buying a longer-term contract.

The major contribution of this paper is that it extends the VIX-related literature, as it is the first attempt to investigate the relationship between VIX futures term structure changes and equity market returns. The first part of the current analysis is the estimation of the VIX futures term structure and the description of its statistical attributes. VIX futures term structure varies heavily over time. Although the term structure tends to be upward sloping, sustained periods of downward sloping term structure can also be observed. The principal component analysis suggests that the evolution of innovations in the VIX futures term structure over their entire life (March 2004 to July 2010) can be mainly captured by three factors. In particular, these components can be interpreted as the level factor (parallel shifts of the term structure), the twist factor (shifts in the slope of the term structure) and the curvature factor (changes in the steepness of term skew).

The proposed estimation of the VIX futures term structure has two estimation advantages, as it allows the underlying future contracts to have non-constant maturity and in addition it doesn't require a continuous dataset. The empirical results show a strong contemporaneous relationship between VIX futures term structure changes and S&P500 returns. Furthermore, they confirm that VIX futures market does predict forward returns, but the predictability as measured by adjusted R-squared is in the low single digits. We should of course take into consideration that VIX futures are relatively new instruments and maybe there are behavioral characteristics that we have not yet experienced.

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APPENDIX

Exhibit 1 –Properties of spot VIX and VIX futures (March 2004 – July 2010)

	VIX	FVIX1	FVIX2	FVIX3	FVIX4	FVIX5	FVIX6
Panel A: Descriptive statistics							
Mean	20.91	21.47	22.81	23.69	23.94	23.80	24.60
Median	16.94	18.79	21.65	23.13	23.36	23.43	23.95
Maximum	80.86	66.23	59.77	54.67	50.58	47.02	45.00
Minimum	9.89	11.15	12.21	12.90	13.38	13.73	14.12
Std. Dev.	11.48	9.80	9.22	8.67	8.11	7.72	7.56
Skewness	2.06	1.57	1.18	0.87	0.71	0.65	0.48
Kurtosis	7.91	5.46	4.14	3.28	2.91	2.71	2.49
Observations	1598	1598	1374	1224	1190	1196	1033
Panel B: Days until maturity for futures contracts							
Mean		34.6	65.2	97.7	128.3	160.3	189.0
Max		79	121	170	198	261	268
Min		18	47	75	103	138	166

Note: The data set consists of the daily closing prices of spot VIX and the six nearest VIX futures (FVIX) for the period from March 26, 2004 to July 30, 2010. Panel A includes the descriptive statistics of the seven time series. Panel B includes the average, maximum and minimum number of days until expiration for the six futures contracts. From the second Monday immediately preceding the expiration day of the contract month, the next-to-deliver contract is considered. The timing of the rollover in the dataset is set in that way such as no contract under review has a remaining life less than two weeks.

Exhibit 2 – Principal Components Analysis

The principal component analysis is conducted using the daily changes of spot VIX and the six futures contracts.

Panel A: Principal Components

Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	6.1883	5.8474	0.8840	6.1883	0.8840
2	0.3409	0.1088	0.0487	6.5292	0.9327
3	0.2321	0.1196	0.0332	6.7613	0.9659
4	0.1124	0.0450	0.0161	6.8737	0.9820
5	0.0674	0.0337	0.0096	6.9411	0.9916
6	0.0337	0.0084	0.0048	6.9748	0.9964

Panel B: Factor Loadings

Change in	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6
VIX	0.3440	0.7657	0.5393	-0.0357	0.0440	0.0266
FVIX1	0.3720	0.2637	-0.5410	0.6399	-0.2912	-0.0399
FVIX2	0.3878	0.0134	-0.3657	-0.2262	0.5628	0.5195
FVIX3	0.3927	-0.0305	-0.1963	-0.3789	0.1981	-0.4686
FVIX4	0.3932	-0.1615	-0.0233	-0.2944	-0.3198	-0.4416
FVIX5	0.3861	-0.3029	0.2037	-0.1737	-0.5589	0.5440
FVIX6	0.3673	-0.4746	0.4496	0.5269	0.3777	-0.1313

Exhibit 3 – Descriptive statistics of VIX futures term structure

Panel A: Goodness of fit	
Mean R-sq	0.7228
Median R-sq	0.8221
Maximum R-sq	0.9997
Minimum R-sq	0.0000
Panel B: Descriptive statistics	
Mean	1.935
Median	4.782
Maximum	16.352
Minimum	-75.884
Std. Dev.	11.147
Skewness	-3.28
Kurtosis	16.85
Observations	1598
Panel C: Autocorrelations	
ρ_1	0.970
ρ_2	0.950
ρ_3	0.935
ρ_4	0.917
ρ_5	0.904

Note: Panel A reports the goodness of fit of the daily least-square estimation, as measured by the R-squared. Panel B shows the descriptive statistics for the estimated VIX futures term structure, while Panel C reports the autocorrelations of the VIX futures term structure.

Exhibit 4 – VIX futures Term Structure

The estimation of the VIX futures Term Structure is conducted on each trading day by fitting a linear model of the available contracts and cash VIX as a function of time to maturity based on least squares criterion (spot VIX index is considered as the price for VIX futures with zero maturity). The estimated term structure is in units of volatility percentage points per year.

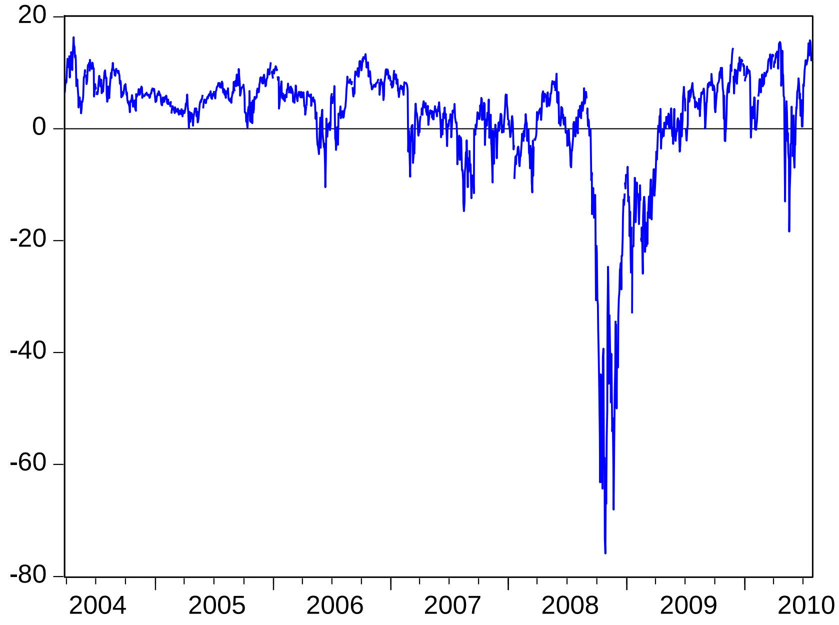

Exhibit 5 – The contemporaneous relationship between VIX futures slope and S&P500 returns

The testing of the contemporaneous relationship between S&P500 returns and VIX futures term structure changes is conducted using the following specification:

$$R_{t,K} = \alpha + \beta \Delta slope^{+}_{t,K} + \gamma \Delta slope^{-}_{t,K} + \varepsilon_t$$

in which, $R_{t,K}$ represents percentage returns in S&P500 in the non-overlapping K period (where K = 1 day, 1 week, 2 weeks, 1 month and 2 months), $\Delta slope^{+}_{t,K}$ is the change in the VIX futures term structure over the same K period, if positive, and $\Delta slope^{-}_{t,K}$ is the change in the VIX futures term structure over the same K period, if negative.

Dependent variable: S&P500 returns ($R_{t,K}$)						
	Independent Variables			**Adj. R-sq**	**F-statistic**	**DW**
	Intercept	$\Delta slope^{+}_{t,K}$	$\Delta slope^{-}_{t,K}$			
K = 1-day	0.0006* (0.0003)	1.3667*** (0.1247)	1.6540*** (0.1086)	62.5%	1,332.24	2.15
K = 1 week	0.0033** (0.0013)	1.1101*** (0.1761)	1.9290*** (0.2023)	54.9%	201.43	2.27
K = 2 weeks	0.0038 (0.0023)	1.1296*** (0.4149)	1.9063*** (0.1353)	47.1%	74.02	1.94
K = 1 month	0.0085* (0.0051)	1.2881*** (0.2595)	2.8218*** (0.5590)	53.1%	43.53	1.86
K = 2 months	0.0151 (0.0107)	1.2686 (1.2145)	3.4410*** (0.6509)	51.7%	20.82	1.81

Newey-West standard errors in parenthesis

*** Identifies coefficient significant at the 1% level

** Identifies coefficient significant at the 5% level

* Identifies coefficient significant at the 10% level

Exhibit 6 – The relationship between VIX futures term structure and future S&P500 returns

The testing of the relationship between VIX futures term structure changes and future S&P500 returns is conducted using the following specification:

$$R_{t+K} = \alpha + \beta \Delta slope^{+}_{t,K} + \gamma \Delta slope^{-}_{t,K} + \varepsilon_t$$

in which, R_{t+K} represents percentage returns in S&P500 in the subsequent non-overlapping K period (where K = 1 day, 1 week, 2 weeks, 1 month and 2 months), $\Delta slope^{+}_{t,K}$ is the change in the VIX futures term structure over the previous K period, if positive, and $\Delta slope^{-}_{t,K}$ is the change in the VIX futures term structure over the previous K period, if negative.

Dependent variable: S&P500 returns (R_{t+K})						
	Independent Variables			**Adj. R-sq**	**F-statistic**	**DW**
	Intercept	$\Delta slope^{+}_{t,K}$	$\Delta slope^{-}_{t,K}$			
K = 1-day	-0.0007*	-0.0631	-0.3953***	1.9%	16.48	2.06
	(0.0004)	(0.1269)	(0.1416)			
K = 1 week	0.0029*	-0.5159***	0.2034	1.4%	3.36	2.09
	(0.0015)	(0.1697)	(0.2329)			
K = 2 weeks	0.0028**	-0.8305**	0.5893	3.4%	3.91	2.08
	(0.0028)	(0.3480)	(0.3976)			
K = 1 month	0.0095	-0.5435	1.2293	4.3%	2.68	2.05
	(0.0048)	(0.5154)	(0.8014)			
K = 2 months	0.0103	-2.1236*	-0.5133	10.3%	3.07	1.38
	(0.0111)	(1.2145)	(0.7039)			

Newey-West standard errors in parenthesis
*** Identifies coefficient significant at the 1% level
** Identifies coefficient significant at the 5% level
* Identifies coefficient significant at the 10% level

Exhibit 7 – Wald Coefficient Restriction tests

The hypothesis that the difference between the coefficients of the negative slope changes and the respective coefficients of the positive changes is not statistically relevant is tested using the Wald restriction test.

H_0: Equal coefficients of positive and negative VIX slope changes		
	F-statistic	**Probability**
Panel A: contemporaneous relationship		
K = 1-day	3.80	0.0515
K = 1 week	7.12	0.0080
K = 2 weeks	2.88	0.0917
K = 1 month	5.43	0.0225
K = 2 months	1.75	0.1940
Panel B: future relationship		
K = 1-day	2.56	0.1096
K = 1 week	5.86	0.0160
K = 2 weeks	5.76	0.0175
K = 1 month	1.64	0.2049
K = 2 months	0.84	0.3647