Smile in motion: An intraday analysis of asymmetric implied volatility

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Abstract. We present a new method to measure the intraday relationship between movements of implied volatility smiles and stock index returns. It exploits a specific characteristic of the smile profile in high-frequency data. Using transaction data for EuroStoxx 50 options from 2000 to 2011 and DAX options from 1995 to 2011 (14 million transactions), we find that the intraday evolution of volatility smiles is generally not consistent with traders' rules of thumb such as the sticky strike or sticky delta rule. On average, the impact of index return on implied volatility is 1.3 to 1.5 times stronger than the sticky strike rule predicts. The main factor driving variations of the adjustment factor is the index return. Our results have implications for option valuation, hedging and the understanding of the leverage effect.

1. Introduction

When studying options with different strike prices, it is common practice to translate option prices into implied volatilities. The strike price structure of implied volatilities directly reveals deviations from the flat line prediction of the Black-Scholes model. The patterns typically found in option markets are subsumed under the term "smile", because they often show an increase in implied volatility for high and low strike prices. For stock index options, implied volatility tends to decrease monotonically with strike price, which is why the pattern is better known as a "skew". In line with a part of the literature, we use "smile" as a general term for the strike price profile of implied volatility, which also includes a skew pattern.

Figure 1A shows a typical scatterplot of implied volatility against moneyness, where moneyness is defined as a scaled ratio of strike price and underlying index value. Each point represents a transaction on 21st January 2009 in DAX options with a time to maturity of 30 days. Clearly, there is an overall downward sloping skew profile, which is consistent with the findings of many previous studies. But the graph also reveals a secondary pattern where implied volatility *increases* with moneyness. In Fig. 1B this structure is highlighted by plotting neighboring strike prices in different grey shades (same data as in Fig. 1A). For fixed strike, the moneyness variation is caused by index changes alone. Thus, when the index falls (moneyness increases), implied volatility tends to increase, and vice versa. The whole smile structure appears to move systematically in relation to index changes. The main idea of this paper is to exploit these intraday movements to analyze the index-volatility relationship. We apply this analysis to options on the EuroStoxx 50 and the German DAX index, which both belong to the most actively traded stock index options in the world. Our study includes 4.6 million transactions of EuroStoxx 50 options over the period from 2000 to 2011 and 9.3 million transactions of DAX options between 1995 and 2011.

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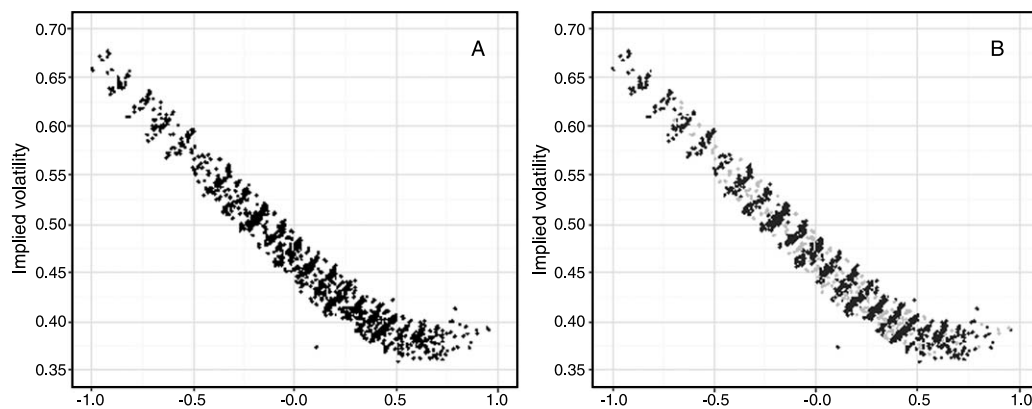
Fig. 1. Smile profile of the DAX option with a time to maturity of 30 calendar days on September 21, 2009. Left: all trades in black. Right: same data, but different grey shades for neighboring strikes.

Our study is related to three streams of literature. The first deals with the dynamics of the surface of implied volatilities. The surface is typically estimated on a daily basis using parametric or non-parametric methods. In the parametric approach, which goes back to Shimko (1993), implied volatilities are modeled by polynomial functions of moneyness and time to maturity. Non-parametric methods include kernel regressions (e.g. Fengler (2005)) and grouping techniques (e.g. Pena et al. (1999)). The dynamics of implied volatilities is then analyzed by Principal Components Analysis (PCA) or related methods. Several studies show that a small number of two to four factors explains much of the daily variation of the smile surface. These factors are related to shocks to (1) the overall level of the smile, (2) its steepness, (3) curvature and (4) the term structure of implied volatilities (see Skiadopoulos et al. (1999), Cont and Fonseca (2002), Hafner (2004), Fengler et al. (2003), Fengler et al. (2007)). The studies also find that the level-related factor has a strongly negative correlation to the return of the underlying index. Goncalves and Guidolin (2006) take a more direct approach than PCA by using VAR models for the parameters of polynomial smile regressions. They find that the movements of the surface are highly predictable, but it remains an open question if this predictability can be exploited by profitable trading strategies.

The negative return-volatility correlation, which is at the heart of the second stream of literature, is as strong as about −0.6 to −0.8 in daily data, which is why it is regarded as an important stylized fact (see, e.g., Christoffersen (2012), p. 11). It is often called asymmetric volatility or leverage effect, because a decrease of stock prices brings about higher leverage ratios and therefore higher equity risk. However, this leverage argument is insufficient to explain the size of the observed correlation (see Figlewski and Wang (2000)). Recent evidence from high-frequency data suggests that the relation is initiated by index return followed by a volatility reaction (see Masset and Wallmeier (2010)), but the economic causes of the effect are still unclear. In line with other studies, we still use the term "leverage effect" although the leverage argument cannot fully account for the effect.

The third stream of literature considers the asymmetric volatility reaction from a trader's point of view. Traders need to know the return-volatility relation for hedging plain-vanilla options and pricing and hedging exotic options. They often rely on rules of thumb instead of sophisticated but possibly less robust theoretical models (see Kim (2009)). Derman (1999) analyzes three rules of thumb known as "sticky moneyness", "sticky strike" and "sticky implied tree". The first two rules suggest that implied volatility remains constant for given moneyness or given strike. The third rule assumes that there is a deterministic relation between asset price and local volatility, so that volatility is not a risk factor of its own. In this case, the familiar binomial and trinomial trees can be modified to reflect the node-dependent local volatility. However, empirical studies do not support the deterministic volatility approach (see Dumas et al. (1998)). It implies exaggerated shifts of the smile profile when the asset price changes. Ultimately, the return-volatility relation is overstrained if it is considered as the sole cause of the observed skew. Nevertheless, Crépey (2004) concludes from numerical and empirical tests that, in negatively

skewed equity index markets, the mean hedging performance of the model is better than the Black-Scholes implied delta. Thus, the local volatility model might be useful in practice despite its known weaknesses. The results for the other rules of thumb are mixed. For a one year time period, Derman (1999) identifies seven different regimes in which different rules prevail. Daglish et al. (2007) analyze monthly S&P500 option data from 1998 to 2002 and find support for the sticky moneyness rule in a relative form, which means that the excess of implied volatility over the ATM level is a function of moneyness. Gatheral and Kamal (2010) report that the ATM implied volatility of S&P500 options reacts 1.5 times stronger than expected according to the sticky strike rule. This estimate reflects the average relationship from daily data over a time period of eight years.

Compared to these studies, the first contribution of this paper is to introduce the stylized fact illustrated in Fig. 1, and to show how it can be exploited to estimate the index-implied volatility relationship. For this purpose, we extend the commonly applied smile model. One advantage of our approach is that it is based on *implied* volatility as opposed to *realized* volatility. In a high-frequency environment, implied volatilities can be measured with great precision, while estimates of realized volatility typically suffer from large standard errors. A further advantage comes from using a regression-based estimation. The common approach to measure the leverage effect is to compare volatility changes and index returns for a given option series over high-frequency intervals. The shorter the return period, the more important microstructure effects (such as the bid-ask bounce) become. The paired return observations are then aggregated in some way to obtain an overall measure of the leverage effect. In our approach, we do not have to compute and aggregate returns explicitly. The regression approach measures the overall effect more directly, and it provides a simple and powerful way to deal with microstructure noise which is part of the error term of the regression.

Our second contribution is to check the validity of traders' rules of thumb using the enhanced smile model. We find that none of the three commonly proposed rules (sticky moneyness, sticky strike, sticky implied tree) is valid over any extended time period. "Sticky moneyness" has to be rejected, because it ignores the observed systematic movements of the skew pattern altogether. "Sticky strike" goes in the right direction, but the empirical intraday movements are, on average, about 1.3 times stronger than the rule predicts. In contrast, the smile movements implied by the "implied tree rule" are clearly exaggerated compared to empirical observations. These findings are remarkably stable over time and across time to maturity classes. They are practically relevant for hedging index options, testing option pricing models and modeling and trading volatility (see Gatheral and Kamal (2010), 930).

Our analysis requires very precise estimates of implied volatility. Hentschel (2003) shows that the impact of estimation errors is potentially large, which makes it difficult to identify the secondary smile pattern this study is about. Therefore, we make every effort to reduce estimation errors. For example, we perfectly synchronize index levels and option prices and make use of put call parity to obtain an implicit market estimate of expected dividends including tax effects. With this clean database, it turns out that the relationship between index and option markets is closer than previous literature suggests. Almost all of the variation of implied volatility across transactions in one option series on one day can be explained by moneyness and index return (average daily R^2 of the enhanced smile model of 99%). This finding might be of interest on its own.

The rest of the paper is structured as follows. Section 2 describes our data and the matching of index and option prices. Section 3 presents our enhanced smile model and Section 4 the results of its empirical estimation. Section 5 concludes.

2. Data

We analyze options on the European stock index EuroStoxx 50 (OESX) and on the German stock index DAX (ODAX). They are traded at the Eurex and belong to the most liquid index options in the world. The options are European style. At any point in time during the sample period, at least eight option maturities were available. However, trading is heavily concentrated on the nearby maturities. Trading hours changed several times during our sample period, but both products were traded at least from 09:30 to 16:00. Our sample period extends from 1995 to 2011 for DAX options and from 2000 to 2011 for ESX options (which were launched in 2000).

For this study of intraday smile movements, it is crucially important to measure implied volatilities with

great precision. Hentschel (2003) describes the main sources of measurement error as follows (p. 788): "For the index level, a large error typically comes from using closing prices for the options and index that are measured 15 minutes apart. This time difference can be reduced by using transaction prices, but such careful alignment of prices is not typical. Even when option prices and published index levels are perfectly synchronous, large indexes often contain stale component prices." We address these concerns in the following ways. To overcome stale prices in the index, we derive the appropriate index level from transaction prices of the corresponding index future, which is the most common index trading instrument. We match each option trade with the previous future trade and require that the time difference does not exceed 30 seconds. In fact, the median time span between matched future and option trades in 2011 is 240 milliseconds. Even with perfect matching, the index level might still be flawed since it is not adjusted for dividends during the option's lifetime. This is particularly relevant for the ESX which is a price index, while the DAX is a performance index. An adjustment might still be relevant for DAX options, because the assumption about taxation of dividends underlying computation of the index does not necessarily correspond to the actual taxation of marginal investors (see Hafner and Wallmeier (2000)). The necessary adjustment is not straightforward since dividend expectations of option traders are not directly observable. Instead, following Han (2008) and, for European markets, Hafner and Wallmeier (2007), we use put-call parity to derive a market estimate of the appropriate index adjustment. Put-call parity is directly applicable because our index options are of European type and transaction costs are small.

Our procedure to measure implied volatilities can be summarized as follows. To obtain the index level S_{mt} corresponding to an observed futures market price F_{mt} at time m on day t, we solve the futures pricing model $F_{mt} = S_{mt} e^{r(T-t)}$ for S_{mt}, where r is the risk-free rate of return and T the futures contract maturity date. We only consider the contract most actively traded on that day, which is normally the nearest available. The futures implied index level S_{mt} is then adjusted such that transaction prices of pairs of at-the-money (ATM) puts and calls traded within 30 seconds are consistent with put-call-parity. The adjusted index level is $S_{mt}^{adj} = S_{mt} + A_t$, where A_t is the same adjustment value for all index levels observed intraday. Empirically, the adjustment is usually negligible with

the exception of short-term ESX options traded in March (after the third Friday) and April. The reason is that for these options, the maturity months (April and May) are different from the next maturity date of the future (June). Between the two maturity dates, most ESX firms pay out dividends, which are therefore considered differently in options and futures prices.

3. Estimation method

In line with Natenberg (1994) and Goncalves and Guidolin (2006), among others, we define time to maturity adjusted moneyness as:

$$M(m, t, T, X) = \frac{\ln\left(\frac{Xe^{-r(T-t)}}{S_{mt}^{adj}}\right)}{\sqrt{T-t}},$$

where m is the intraday time (down to the level of seconds), t denotes the trading day, T is the option's maturity date and X the exercise price. The typical smile regression based on transaction data considers all trades on one day in options with different strike prices but the same time to maturity. Figure 2 shows typical scatterplots of implied volatility across moneyness for different times to maturity (trading date 14th December 2011).

A common way to model these patterns is to use the cubic regression function:

$$IV(M) = \beta_0 + \beta_1 M + \beta_2 M^2 + \beta_3 D \cdot M^3 + \varepsilon, \quad (1)$$

where IV is the implied volatility, β_i, $i = 0, 1, 2, 3$ are regression coefficients, ε is a random error, and D a dummy variable defined as:

$$D = \begin{cases} 0, & M \leq 0 \\ 1, & M > 0 \end{cases}.$$

The dummy variable accounts for an asymmetry of the pattern of implied volatilities around the ATM strike ($M = 0$). The cubic smile function is twice differentiable so that the corresponding risk-neutral probability density is smooth.

A weakness of regression model (1) is the underlying assumption that the smile pattern is constant during each trading day. By contrast, empirical observations suggest that implied volatilities change in accordance with intraday index returns (see Fig. 1). Therefore, we propose an enhanced regression model which considers the index level as an additional determinant
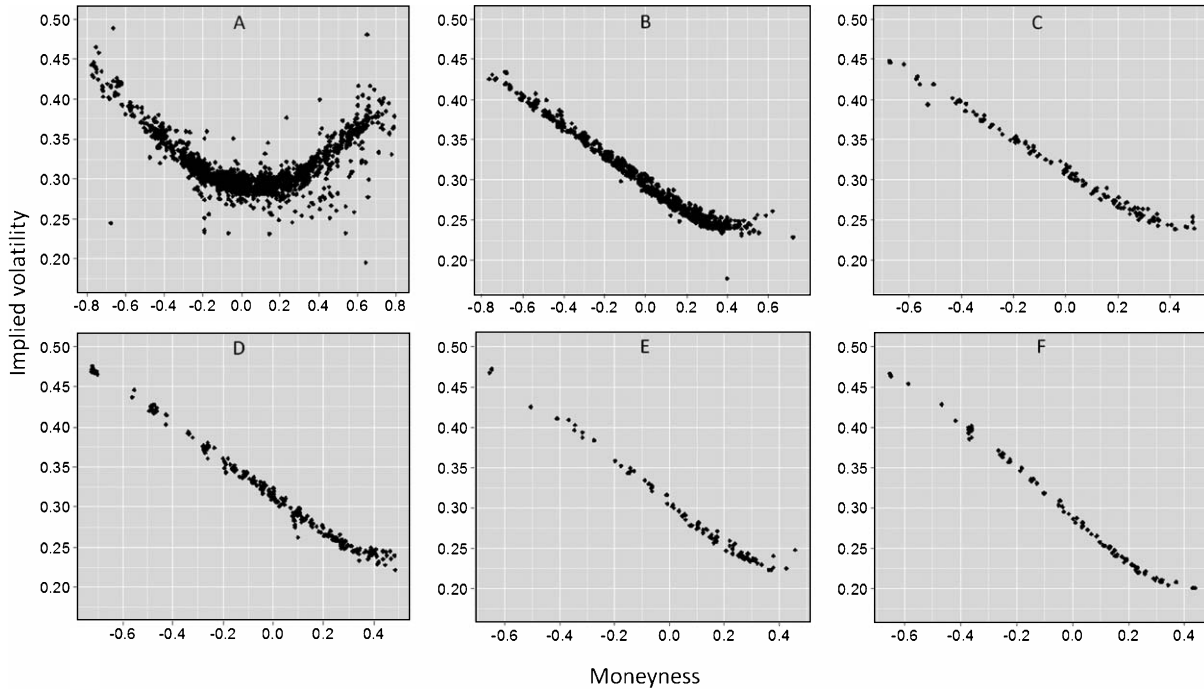
Fig. 2. Implied volatilities of ODAX trades on 14th December 2011. Times to maturity: 2 (A), 37 (B), 65 (C), 93 (D), 184 (E), 373 (F) calendar days.
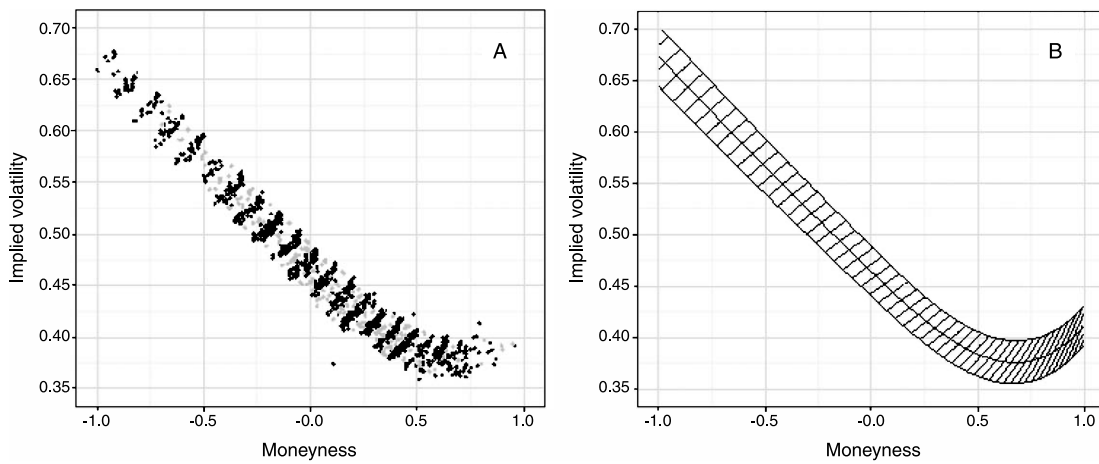
Fig. 3. Smile profile and regression function of the ODAX option with time to maturity of 30 calendar days on 21st September 2009. Graph A: all trades; different grey shades for neighboring strikes (distance 50). Graph B: regression function for minimum, maximum and mean index level. In addition, for each strike between 3050 to 6000 (at intervals of 50), the regression function is shown for index levels between the maximum and minimum on this day.

of implied volatilities. The new regression function is:

$$IV(M, S) = \beta_0 + \beta_1 M + \beta_2 M^2 + \beta_3 D \cdot M^3$$
$$+ \gamma \ln S + \delta M \ln S + \varepsilon, \qquad (2)$$

where S is the short-form of S_{mt}^{adj}. For $\delta = 0$, this model implies parallel shifts of the smile pattern in response to changes in the index level. The interaction term with moneyness is included to also allow for twists of the smile pattern ($\delta \neq 0$). Figure 3B shows the fitted

regression function for the previous example of Fig. 1. On this day (21st January 2009), the index varied between 4140 and 4310. For a given strike price, index changes are directly reflected in inverse moneyness changes. These, in turn, are systematically related to changes in implied volatilities, as can be seen from the positive relation of implied volatilities and moneyness for constant X (Fig. 3A). This structure overlying the general smile pattern is well captured in the enhanced smile model (Fig. 3B).

If S in Equation (2) takes on the value $\overline{S}$ defined as the average stock price on that day, $IV(M, \overline{S})$ represents the "average" daily smile:

$$IV(M, \overline{S}) = IV^*(M) = \beta_0^* + \beta_1^* M$$
$$+ \beta_2 M^2 + \beta_3 D \cdot M^3 + \varepsilon, \qquad (3)$$

where β_0^* and β_1^* are defined as $\beta_0^* = \beta_0 + \gamma \ln \overline{S}$ and $\beta_1^* = \beta_1 + \delta \ln \overline{S}$. The "average" smile function $IV(M, \overline{S}) = IV^*(M)$ of Equation (3) is expected to be very similar to the simple smile function $IV(M)$ of Equation (1), because both estimate the overall smile pattern without considering intraday movements of the pattern.

Based on the enhanced regression model (2), we propose two simple measures for the index dependency of the smile. The first measure is defined as the partial derivative of ATM implied volatility with respect to the log index level and is therefore equal to the coefficient γ:

$$\gamma = \frac{\partial IV(0, S)}{\partial \ln S}. \qquad (4)$$

A γ-coefficient of -1 means that ATM volatility decreases by one percentage point when the log index rises by one percent. The second measure is closely related to well-known traders' rules of thumb for the smile dynamics. Following Gatheral and Kamal (2010), this measure is the parameter c so that the following relation holds:

$$\frac{dIV}{dS}(0, \overline{S}) \cdot \frac{1}{dM(\overline{S})/dS} = (1 - c)\frac{\partial IV(0, \overline{S})}{\partial M}. \qquad (5)$$

The left hand side of Equation (5) describes the total change of the implied volatility of an ATM option per unit of change in moneyness, where the changes in moneyness and implied volatility are induced by index return. The right hand side relates this change in implied volatility to the change we would observe with a constant smile pattern. Under the assumption

of a constant smile, we would just have to update moneyness in accordance with index changes and read off the new implied volatility from the initial smile function ("sticky moneyness"). The smile dynamics corresponds to this sticky moneyness rule if $c = 0$. In case of $c = 1$, the index-induced change in implied volatility of an option is zero, which corresponds to the "sticky strike" rule. As a third rule of thumb, Derman (1999) introduced the "sticky implied tree" rule which is characterized by an inverse movement of implied volatilities compared to "sticky moneyness". In our formulation, the sticky implied tree rule approximately corresponds to $c = 2$.

Inserting the derivatives

$$\frac{dIV(0, S)}{dS} = \frac{1}{S}\left(\gamma - \frac{\beta_1 + \delta \ln S}{\sqrt{T - t}}\right)$$

$$\frac{dM(S)}{dS} = -\frac{1}{S\sqrt{T - t}}$$

$$\frac{\partial IV(0, S)}{\partial M} = \beta_1 + \delta \ln S$$

into Equation (5) and solving for c, we obtain:

$$c = \frac{\gamma}{\beta_1 + \delta \ln \overline{S}}\sqrt{T - t}. \qquad (6)$$

Thus, the second measure of the intraday leverage effect, c, expresses the coefficient γ as a multiple of the slope of the smile function. In case of $c = 1$, the parallel shift of the smile function just offsets the effect of "riding" on the initial smile function (sticky strike). The sticky implied tree rule postulates that the parallel shift more than offsets the effect of a movement along the smile, while sticky moneyness implies that shifts of the smile are non-existent. Therefore, our enhanced smile regression model provides the opportunity to test these rules of thumb on a daily basis.

The error terms in Equations (1) and (2) are supposed to be heteroscedastic, because the sensitivity of the implied volatility estimates with respect to the index level is larger for deep in-the-money options. Therefore, we apply a weighted least squares (WLS) estimation assuming that the error variance is proportional to the (positive) ratio of the option's delta and vega. This ratio indicates how a small increase in the index level affects the implied volatility. We note that the impact of the WLS estimation (as opposed to OLS) is negligible in all but very few cases. We exclude an observation as outlier if the absolute value of the regression residual exceeds five standard deviations of the

residuals. Such outliers can be due to mistrades which are unwound but still contained in the database. Less than 0.3% of all observations are identified as outliers according to the 5-sigma rule.

4. Empirical results

4.1. Smile pattern over time

Our sample consists of short-term options, which are typically more actively traded than long-term options. We classify the options into three maturity groups. The last two weeks before the maturity date (which is a third Friday) are excluded to leave out expiration-day effects. The weeks 3 to 6, 7 to 10, and 11 to 14 before maturity each constitute one group, so that the time to maturity ranges from 14 to 39 days (TtM1), 42 to 67 days (TtM2), and 70 to 95 days (TtM3). The days in-between these intervals are Saturdays and Sundays. Table 1 shows summary statistics for the estimated parameters of simple smile regressions (1) in Panel A and the enhanced regressions (3) in Panel B over the time period from January 2000 to December 2011. Results for ESX and DAX options are very similar. The ATM implied volatility (β_0) is about 24% on average. The negative β_1- and positive β_2-coefficients reflect the typical skew pattern. The smile function is curved for the shortest maturity and almost linear for longer maturities. Note that differences of smile profiles across maturity classes depend on the definition of moneyness. The differences would be stronger when using a simple moneyness measure without maturity adjustment. The estimated coefficient β_3 is mostly positive, because implied volatility often increases near the upper boundary of available moneyness. In most cases, the highest moneyness actively traded lies between 0.5 and 1.0. Of course, the estimated regression function cannot necessarily be extrapolated beyond the maximum of traded moneyness. Panel B reveals that the β-parameters of the extended regression model are almost identical to those in Panel A. This is not surprising, because the inclusion of the index level as an additional explanatory variable does not modify the average moneyness profile of implied volatilities.

Figure 4 illustrates the evolution of the daily estimates of smile characteristics for ODAX over the time period from January 1995 to December 2011. Graph 4A shows the ATM implied volatility, which corresponds to β_0^* in Equation (2). Figures 4B and 4C show

spreads between implied volatilities at different moneyness levels. The spreads are defined as:

$$Spread1 = IV^*(M_1 = -0.3) - IV^*(M_0 = 0),$$

$$Spread2 = IV^*(M_2 = 0.139) - IV^*(M_0 = 0).$$

Levels $M_1 = -0.3$ and $M_2 = 0.139$ are chosen such that an option with a time to maturity of 45 days has a strike price of 90% or 105% of the index level.

ATM in Graph 4A basically replicates the volatility index VDAX. The largest peaks are related to the Russian crisis in 1998, the September 2001 attack on the World Trade Center, the market turmoil of 2002, the Iraq war in 2003 and the subprime crisis with the bankruptcy of Lehman Brothers in September 2008. *Spread*1 in Graph B is always positive and varies mostly between 2 and 8 percentage points. The fluctuations are significant and do not seem to be related to the overall level of the smile as measured by ATM. The spread appears to follow an upward trend during the sample period. *Spread*2 (Graph C) is negative over the whole period, which means that the negatively sloped skew extends well beyond a moneyness of 0.

Figure 5 shows the same graphs for OESX over the shorter period since 2000. The main observations are the same as for ODAX.

4.2. Intraday movements of the smile

We now turn to the empirical results on intraday movements of the smile in relation to intraday index changes. Our first measure of the index-smile relation, which is the γ-coefficient of the enhanced smile regression (2), is on average significantly negative (see Table 1, Panel B). Between 80% (OESX, TtM3) and 96% (ODAX, TtM1) of the daily γ-estimates are negative. Thus, the intraday index level turns out to be an important explanatory variable. The negative sign means that the smile shifts upwards when the index value decreases. The shift tends to be stronger the shorter the time to maturity. The estimated δ-coefficient for the interaction of moneyness and index level is negative on average, but significantly negative only for OESX with TtM3. The adjusted R^2-coefficients of the extended smile model are significantly higher than those of the simple smile model. On average, the adjusted R^2 is about 98% in the first moneyness class and 99% in the second and third classes. Therefore, the intraday index level explains about 50% of the variation of the remaining errors of the simple smile regression.

Table 1

Descriptive statistics of smile regression parameters from January 2000 to December 2011 (both for ODAX and OESX). The time to maturity ranges from 14 to 39 days (TtM1), 42 to 67 days (TtM2), and 70 to 95 days (TtM3). Asterisk * indicates mean values that are significant at the 1% level. Perc. sig. is the percentage of significant coefficients in daily regressions (1% level). Perc10 is the 10% percentile of the distribution of daily R^2-values. Perc. LT0, Perc. 0-2, and Perc. HT2 denote the percentage of c-values lower than 0, between 0 and 2, and higher than 2, respectively

		ODAX			OESX		
		TtM1 ($N = 2770$)	TtM2 ($N = 2789$)	TtM3 ($N = 2219$)	TtM1 ($N = 2694$)	TtM2 ($N = 2609$)	TtM3 ($N = 1805$)
		Panel A: Simple smile regression					
β_0	Mean	0.2383*	0.2387*	0.2355*	0.2416*	0.2407*	0.2416
	Std.	0.1027	0.0925	0.0844	0.1029	0.0945	0.0897
	Perc. sig.	99.3%	99.3%	99.4%	99.3%	99.2%	99.4%
β_1	Mean	−0.1494*	−0.1777*	−0.1941*	−0.1592*	−0.1910*	−0.2145
	Std.	0.0465	0.0475	0.0464	0.0445	0.0438	0.0444
	Perc. sig.	99.3%	99.3%	99.4%	99.2%	99.2%	99.4%
β_2	Mean	0.0594*	0.0257*	0.0085*	0.0659*	0.0293*	−0.0017
	Std.	0.0695	0.0614	0.0627	0.0719	0.0743	0.0810
	Perc. sig.	94.4%	87.9%	77.7%	91.6%	80.5%	71.6%
β_3	Mean	0.6576*	0.7870*	0.9414*	0.7919*	1.0003*	1.1921
	Std.	0.7204	0.7216	0.8367	1.0351	1.1415	1.3735
	Perc. sig.	98.6%	97.5%	88.1%	92.5%	89.5%	87.4%
Adj. R^2	Mean	96.6%	98.3%	98.5%	96.5%	97.8%	97.9%
	Std.	8.6%	2.7%	3.1%	5.8%	4.0%	3.7%
	Perc10	93.3%	96.3%	96.9%	92.9%	95.9%	95.7%
		Panel B: Enhanced smile regression					
β_0^*	Mean	0.2381*	0.2385*	0.2353*	0.2394*	0.2374*	0.2372
	Std.	0.1024	0.0922	0.0841	0.1026	0.0943	0.0893
	Perc. sig.	99.3%	99.2%	99.4%	99.3%	99.0%	99.4%
β_1^*	Mean	−0.1490*	−0.1772*	−0.1935*	−0.1584*	−0.1905*	−0.2148
	Std.	0.0469	0.0474	0.0461	0.0452	0.0446	0.0454
	Perc. sig.	99.3%	99.3%	99.3%	99.2%	99.1%	99.4%
β_2	Mean	0.0632*	0.0277*	0.0116*	0.0684*	0.0306*	−0.0009
	Std.	0.0755	0.0629	0.0606	0.0729	0.0728	0.0768
	Perc. sig.	96.1%	92.3%	84.8%	94.2%	84.6%	74.7%
β_3	Mean	0.6460*	0.7671*	0.9018*	0.7821*	0.9831*	1.1707
	Std.	0.7269	0.6985	0.7924	1.0387	1.1167	1.3169
	Perc. sig.	98.8%	97.9%	90.9%	92.7%	90.8%	90.4%
γ	Mean	−0.7603*	−0.6094*	−0.5331*	−0.7938*	−0.6529*	−0.5834
	Std.	0.4617	0.3350	0.2927	0.4878	0.3704	0.3482
	Perc. sig.	96.0%	95.1%	90.0%	88.8%	85.2%	80.0%
δ	Mean	−0.0069	0.0003	−0.0210	0.0705*	−0.0401	−0.0809
	Std.	0.7281	0.6142	0.7919	1.1912	1.0804	1.1969
	Perc. sig.	67.0%	45.5%	32.3%	54.3%	34.8%	27.7%
c	Mean	1.3407*	1.3105*	1.2790*	1.3130*	1.3072*	1.2723
	Std.	0.7493	0.6571	0.6533	0.7648	0.7013	0.7371
	Perc. LT0	3.8%	2.6%	2.8%	4.3%	3.9%	4.7%
	Perc. 0–2	81.6%	86.3%	87.3%	80.4%	83.0%	84.5%
	Perc. HT2	14.6%	11.2%	9.8%	15.4%	13.0%	10.9%
Adj. R^2	Mean	98.4%	99.2%	99.2%	97.9%	98.7%	98.6%
	Std.	2.4%	1.6%	1.9%	4.1%	3.2%	3.2%
	Perc10	96.7%	98.3%	98.4%	96.2%	97.5%	97.1%

The c-coefficient as our second measure of the relation between index level and smile profile is about 1.3 on average (Table 1, Panel B). The mean value is significantly larger than 1 for ODAX and OESX in all maturity classes. On general grounds, we expect values of c between 0 and 2, although values below 0 or above 2 are not excluded by theory. Between 80% and 87% of the daily c-estimates lie in this range (see Perc.0-2, Table 1, Panel B). Values above 2 are more frequently observed than values below 0. As will be shown in Section 4.3, the c-values higher than 2 mainly occur on days with a strongly negative index return.

Fig. 4. Estimated smile characteristics of ODAX options with a time to maturity between 42 and 67 calendar days from 1995 to 2011. ATM is the ATM implied volatility according to the enhanced smile model of Equation (2). Spread1 (Spread2) is the difference between the implied volatility of OTM puts (calls) and ATM implied volatility.
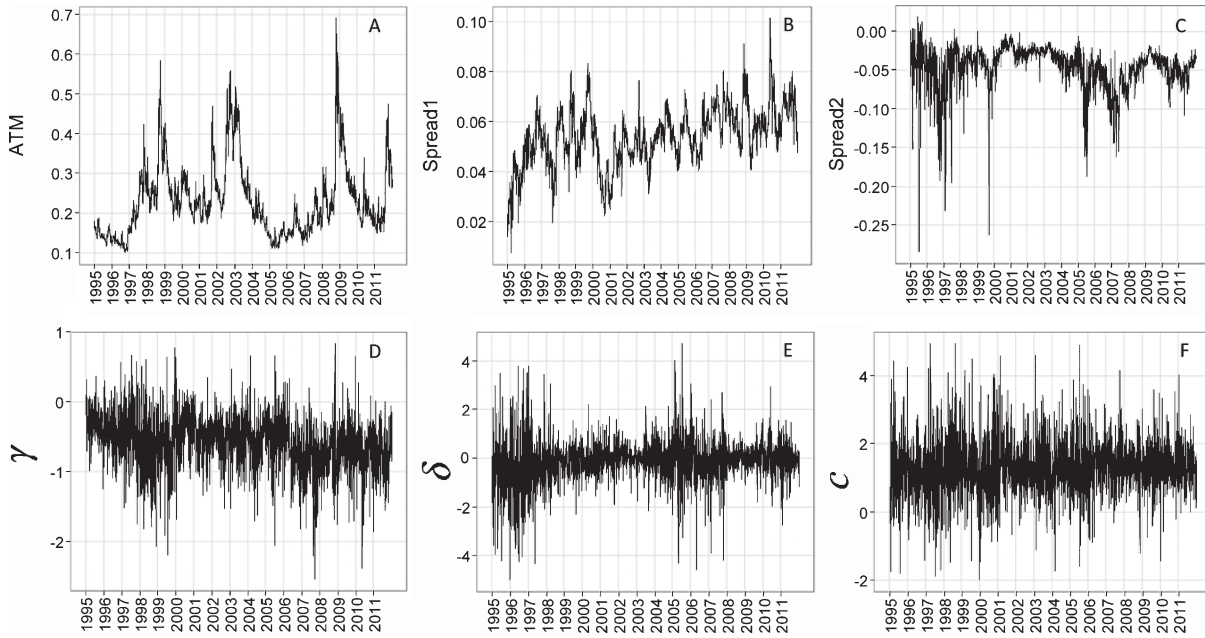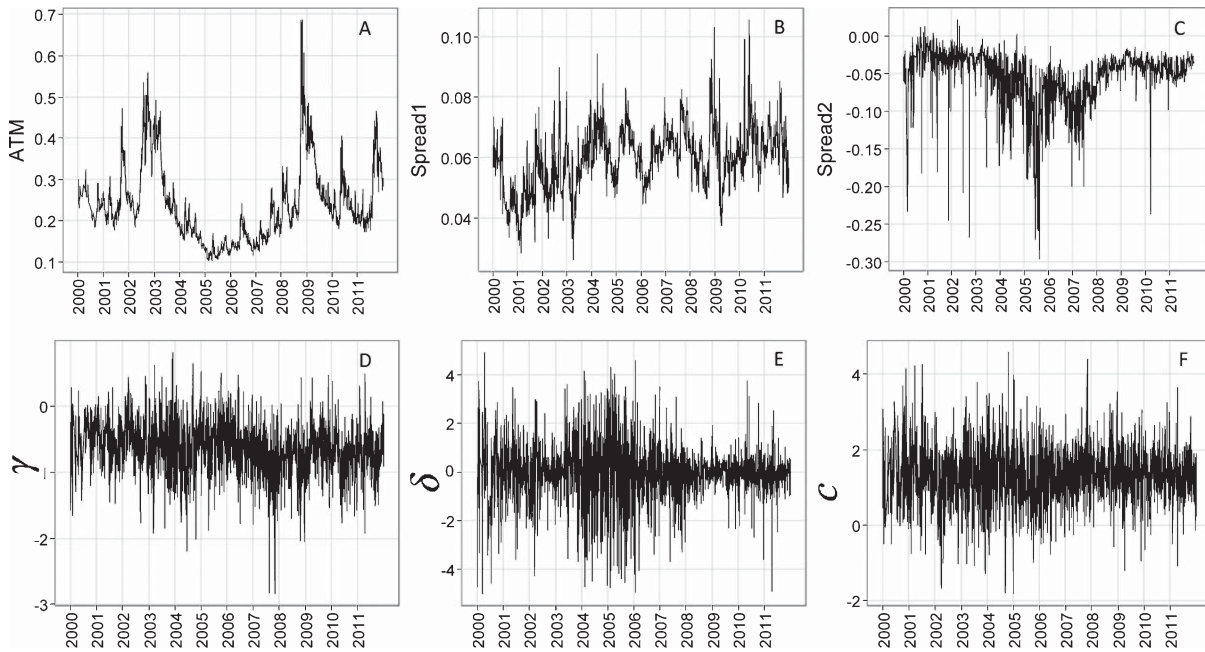

Fig. 5. Estimated smile characteristics of OESX options with a time to maturity between 42 and 67 calendar days from 2000 to 2011. ATM is the ATM implied volatility according to the enhanced smile model of Equation (2). Spread1 (Spread2) is the difference between the implied volatility of OTM puts (calls) and ATM implied volatility.

Table 2

Correlation coefficients of daily parameter estimates of the enhanced smile model between ODAX and OESX (January 2000 to December 2011). The time to maturity ranges from 14 to 39 days (TtM1), 42 to 67 days (TtM2), and 70 to 95 days (TtM3)

	TtM1 $N = 2664$	TtM2 $N = 2598$	TtM3 $N = 1770$
β_0^*	0.979	0.978	0.976
β_1^*	0.829	0.845	0.793
β_2	0.784	0.781	0.687
β_3	0.652	0.675	0.611
γ	0.691	0.603	0.436
δ	0.136	0.066	0.052
c	0.518	0.458	0.364

One possible explanation is that a particularly high c reflects the fear of a major market downturn. Taking market sentiment into account, values of c above 2 are not theoretically inconsistent. They tend to occur simultaneously in ODAX and OESX options.

The average coefficients of ODAX and OESX options are strikingly similar in Table 1. As additional evidence, Table 2 shows the correlation coefficients of daily parameter estimates. The β_0-parameters of ODAX and OESX are almost perfectly correlated. For β_1, β_2, β_3 and γ, the correlations are smaller but still substantial. The interaction terms δ, however, are not highly correlated, which is in line with expectations. In all, the smile patterns of ODAX and OESX are found to move largely in parallel and to react in a similar way to changes in the underlying index level.

For a more detailed analysis, we split our sample into years and compute yearly averages of the γ- and c-coefficients. The results in Table 3 indicate that mean γ is always negative and mean c always greater than 1. Coefficient γ varies more strongly than c. One reason is that γ is positively related to the slope of the skew. Such a relation does not exist for the c-coefficient. Thus, the steeper the smile, the larger is the parallel shift of the smile with respect to changes in the index level. In a world with a constant smile, when the index level decreases, implied volatility falls along the initial smile pattern. To exactly offset this decrease of implied volatility, the shift in the smile pattern would need to be directly related to the slope of the skew. We find that the actual shift is typically about 1.3 times bigger. The yearly averages of the c-coefficients are remarkably stable across time to maturity classes and for the two options (ODAX, OESX).

Graphs D and F in Figs. 4 and 5 show the *daily* estimates of γ and c. The estimates are noisy, which is not surprising given the fact that an accurate estimation is only possible if considerable index changes occur during the day. For the c-coefficient, trends or cyclical patterns do not seem to exist. The graphs confirm the finding that the mean value of c remains rather stable at a level of about 1.3. Thus, we conclude that none of the three traders' rules of thumb is consistent with empirical evidence for ESX and DAX options. The sticky moneyness rule is rejected by clear evidence of systematic, index related shifts of the smile pattern. The sticky strike rule, postulating a c-coefficient of 1, correctly captures the direction of index-related smile movements, but the empirically observed shifts are more pronounced. Conversely, the predictions of the implied tree rule are too extreme. As Dumas et al. (1998) and others have shown, the reason is that implied tree models assume a deterministic relation between volatility changes and asset returns. This negative relation is regarded as the sole cause of the skew in option prices. However, to explain the strong skew observed in index option markets in this way, the relation between volatility and return would have to be even stronger than it actually is.

4.3. Determinants of intraday movements

The last section revealed that the c-coefficient does not appear to follow a trend or cyclical pattern. Yet, it might be systematically related to other economic variables. One obvious candidate is the index return, because negative index returns are often found to have a stronger impact on volatility than positive returns.

We define daily log return as $r_t = \ln(S_{t,close}/S_{t,open})$, where $S_{t,close}$ and $S_{t,open}$ denote the index levels at closing and opening of option trading on day t. Days are grouped into eight daily return (DR) groups. Group 1 includes all days with $r_t \leq -3\%$, Group 8 all days with $r_t > 3\%$. The groups in-between cover a return interval of one percentage point each, in ascending order. For example, Groups 4 and 5 are characterized by $-1\% < r_t \leq 0$ and $0 < r_t \leq 1\%$, respectively. As a second return variable, we define previous overnight return (OR) as $r_t^o = \ln(S_{t,open}/S_{t-1,close})$. We build four OR classes in ascending order with thresholds for r_t^o of -0.5%, 0%, and 0.5%. Thus, Group OR1 includes all days t with $r_t^o \leq -0.5\%$, Group MR4 all days t with $r_t^o > 0.5\%$. Sorting is done in one dimension, either by DR or OR, based on all days of the sample period (1995 to 2011 for ODAX and 2000 to 2011 for OESX).

Table 3

Mean estimates of daily measures of asymmetric volatility by year. The time to maturity ranges from 14 to 39 days (TtM1), 42 to 67 days (TtM2), and 70 to 95 days (TtM3)

Year	TtM1			TtM2			TtM3		
	γ	c	R^2_{adj}	γ	c	R^2_{adj}	γ	c	R^2_{adj}
				Panel A: ODAX					
1995	−0.4136	1.3141	90.5%	−0.3160	1.2723	93.9%	−0.2580	1.1807	92.6%
1996	−0.5812	1.0487	96.3%	−0.4450	1.0659	98.1%	−0.4152	1.1532	98.1%
1997	−0.6118	1.1914	96.0%	−0.4307	1.1210	96.9%	−0.3737	1.1429	96.7%
1998	−0.9291	1.4214	98.2%	−0.7011	1.4047	99.2%	−0.5947	1.3427	98.6%
1999	−0.7483	1.2755	98.5%	−0.6366	1.2508	99.5%	−0.5234	1.1053	99.4%
2000	−0.4271	1.2408	97.6%	−0.3557	1.1617	98.7%	−0.3014	1.0821	98.8%
2001	−0.6426	1.4110	97.3%	−0.5127	1.4252	98.4%	−0.4272	1.3321	98.6%
2002	−0.6679	1.2952	97.8%	−0.4970	1.2281	98.8%	−0.4375	1.2269	98.9%
2003	−0.6331	1.3246	97.2%	−0.5027	1.2471	98.9%	−0.4360	1.1940	99.0%
2004	−0.7214	1.4561	98.6%	−0.5968	1.3630	99.4%	−0.5077	1.2724	98.9%
2005	−0.5306	1.1529	98.2%	−0.4738	1.2562	99.2%	−0.4515	1.2798	99.4%
2006	−0.8666	1.3705	99.1%	−0.6648	1.3745	99.5%	−0.5714	1.3618	99.6%
2007	−1.1012	1.3940	99.4%	−0.8612	1.4099	99.6%	−0.7289	1.4189	99.6%
2008	−0.9709	1.4515	98.7%	−0.7756	1.3566	99.2%	−0.6618	1.3039	99.1%
2009	−0.6811	1.2475	99.0%	−0.5883	1.2306	99.5%	−0.5411	1.2435	99.4%
2010	−0.9133	1.4365	99.0%	−0.7403	1.3834	99.7%	−0.6421	1.3347	99.7%
2011	−0.9600	1.3130	98.7%	−0.7392	1.2888	99.3%	−0.6420	1.2699	99.5%
				Panel B: OESX					
2000	−0.4951	1.1989	94.6%	−0.4273	1.1534	98.1%	−0.4086	1.2047	98.1%
2001	−0.6476	1.3439	96.9%	−0.5247	1.3784	97.8%	−0.5183	1.4187	97.2%
2002	−0.7034	1.2726	96.7%	−0.5181	1.1328	98.3%	−0.4891	1.2090	98.2%
2003	−0.6836	1.3326	97.4%	−0.5870	1.2899	97.6%	−0.5233	1.2426	98.7%
2004	−0.7627	1.2594	98.2%	−0.6865	1.3796	98.7%	−0.6580	1.3840	99.0%
2005	−0.6432	1.0746	98.6%	−0.5458	1.2207	98.8%	−0.4074	1.0254	99.0%
2006	−0.8277	1.2918	98.8%	−0.6352	1.2781	98.9%	−0.5418	1.2192	98.7%
2007	−1.0644	1.2864	99.2%	−0.8570	1.3672	99.3%	−0.6888	1.3094	99.0%
2008	−0.9821	1.4980	98.5%	−0.7954	1.4208	98.8%	−0.6777	1.3431	98.6%
2009	−0.7165	1.3218	98.8%	−0.6227	1.3133	98.8%	−0.5671	1.2823	98.7%
2010	−0.9578	1.4408	98.9%	−0.7759	1.3899	99.3%	−0.6774	1.3348	98.9%
2011	−0.9695	1.3902	98.1%	−0.7217	1.2835	99.2%	−0.6292	1.2554	98.6%

For each DR and OR group, Table 4 shows the number of days, N, in this group and the average γ and c coefficients. We focus on coefficient c as it takes the slope of the smile into account. The results show a strongly negative relationship between c and the daily return DR. In line with expectations, c is generally higher for negative compared to positive returns. In the DR8 group ($r_t > 3\%$), smile movements are consistent with the sticky strike rule (c close to 1). For small absolute returns (DR4 and DR5), c corresponds to the sample average of about 1.3. In the DR1 and DR2 groups, the smile movements are stronger, with c-coefficients clearly above 1.5. The c-values tend to be larger for shorter times to maturity, but the differences between the maturity classes are small. The overnight return OR appears to be related to the c-coefficient in a similar way as DR, but with a smaller impact. The mean c value for OR1 ($r_t^o \leq −0.5\%$) is about 1.5, while it is about 1.2 in the OR4 group ($r_t^o > 0.5\%$).

A comparison of Tables 3 and 4 gives rise to the question of whether the yearly averages reported in Table 3 can be reconciled with the findings related to DR, i.e. whether the mean c value for a given year reflects the number of days within each DR category observed in that year. To examine this question, we calculate yearly c-values based on 1) the actual distribution of index returns across the DR categories observed in each specific year; and 2) the average c-values for each DR category reported in Table 4. The results vary between 1.27 and 1.37 for TtM1, between 1.28 and 1.34 for TtM2 and between 1.25 and 1.30 for TtM3. These variations are clearly smaller than those actually observed (see the yearly averages in Table 3). Thus, the yearly variation can only to a small extent be explained by the different number of days within each DR category. This conclusion is consistent with the finding that a decrease of the daily index return by one percentage point tends to increase c by approxi-

Table 4

Mean estimates of daily measures of asymmetric volatility by classes of daily return (DR) and overnight return (OR). The time to maturity ranges from 14 to 39 days (TtM1), 42 to 67 days (TtM2), and 70 to 95 days (TtM3). The days of the sample period are grouped into eight daily return and four overnight return groups. Group DR1 includes all days with a return below −0.03, group DR8 all days with a return above 0.03. The groups in-between cover a return interval of one percentage point each, in ascending order. OR is defined as the log return of the index based on the opening price of the current day and the closing price of the previous trading day. OR groups are built in ascending order with return thresholds of 0.005, 0.0, and 0.005. Sorting is done in one dimension, either by DR or OR

| | | TtM1 | | | TtM2 | | | TtM3 | | |
		N	γ	c	N	γ	c	N	γ	c
					Panel A: ODAX (1995–2011)					
DR	1	98	−1.0480	1.7941	108	−0.7998	1.6984	82	−0.6514	1.5476
	2	149	−0.9617	1.7761	146	−0.7276	1.5962	113	−0.6072	1.5583
	3	432	−0.8726	1.5991	441	−0.6840	1.5137	317	−0.5906	1.4333
	4	1174	−0.7404	1.3593	1161	−0.5797	1.3072	868	−0.5175	1.2939
	5	1410	−0.6747	1.2022	1414	−0.5459	1.2108	1045	−0.4858	1.1966
	6	439	−0.6406	1.1190	433	−0.5309	1.1371	319	−0.4686	1.1018
	7	125	−0.6834	1.1663	128	−0.5404	1.1563	97	−0.4942	1.1497
	8	69	−0.5205	0.9237	75	−0.4192	0.9080	62	−0.3560	0.8706
OR	1	391	−0.9084	1.5305	386	−0.7079	1.4667	285	−0.6240	1.4344
	2	1348	−0.7483	1.3787	1342	−0.5916	1.3330	1013	−0.5143	1.2900
	3	1612	−0.6830	1.2610	1620	−0.5444	1.2328	1245	−0.4860	1.2255
	4	545	−0.6996	1.1621	558	−0.5747	1.2007	360	−0.5050	1.1293
					Panel B: OESX (2000–2011)					
DR	1	85	−1.1507	1.8569	83	−0.9027	1.6923	63	−0.7340	1.5720
	2	132	−1.1221	1.8429	127	−0.8230	1.6629	76	−0.7020	1.5352
	3	335	−0.8993	1.5251	319	−0.7511	1.4948	230	−0.6448	1.4113
	4	793	−0.8141	1.3330	756	−0.6654	1.3315	532	−0.5940	1.3015
	5	839	−0.7372	1.2150	829	−0.6140	1.2278	558	−0.5588	1.2091
	6	345	−0.6457	1.0866	325	−0.5498	1.1352	228	−0.5256	1.1567
	7	95	−0.6733	1.0722	99	−0.5909	1.1783	68	−0.5286	1.1541
	8	70	−0.5769	1.0296	71	−0.4951	1.0142	50	−0.4313	0.9380
OR	1	157	−1.0759	1.6089	149	−0.8641	1.5484	124	−0.7146	1.4346
	2	1110	−0.8231	1.3807	1069	−0.6575	1.3375	721	−0.5962	1.3333
	3	1261	−0.7241	1.2192	1229	−0.6142	1.2554	834	−0.5481	1.2016
	4	166	−0.8605	1.2935	162	−0.7211	1.2791	126	−0.6144	1.2316

mately 0.1 (see Table 4). As the difference of average daily index returns between boom and recession years is much smaller than 1%, the corresponding difference in mean c between these years will be small.

These considerations do not argue against an important impact of DR on c. They rather suggest that the strength of the association between DR and c as well as the overall level of c vary over time. To capture these differences and to examine the determinants of the c-coefficient in more detail, we run a regression analysis. In each year, we estimate the following regression model:

$$c_t = b_0 + \sum_{i=1}^{7} b_i F_{i,t} + \varepsilon_t,$$

where $F_{i,t}$ is the value of determinant i on day t of the respective year, c_t is the estimated c-coefficient according to Equation (6) on day t, b_i are regression coefficients and ε is an error term. Based on the time series of coefficients from these yearly regressions, we compute the Fama and MacBeth (1973) t-statistic to examine if an overall significant influence over the sample period exists. Our explanatory variables are the daily return (DR) and the overnight return (OR) as defined before, the ATM implied volatility (variable ATM), the slope of the smile function (Skew), and two variables related to net buying pressure of OTM puts (see Bollen and Whaley (2004)) and thin trading. Following Masset and Wallmeier (2010), we use trading volume (in Euro per day) as liquidity measure and the ratio of trading volume of puts to the trading volume of calls (per day) as a measure of net buying pressure (Put-Call Ratio). We also include the time to maturity in calendar days (variable TtM).

The regression results in Table 5 confirm that c is strongly related to DR. In each year, the slope coefficient of DR is significantly negative. The results for

Table 5

Regression results for determinants of leverage measures. The table shows estimated regression coefficients of yearly regressions of leverage coefficient c on explanatory variables. DR is daily return, OR overnight return, TtM the time to maturity in days, ATM the at-the-money implied volatility, Skew the slope of the smile function, Trading Volume is trading volume measured in Euro per day, and Put-Call Ratio is the ratio of trading volume of puts to the trading volume of calls (per day). Significance levels of 1%, 5% and 10% are indicated by superscripts a, b and c. F/MB shows t-statistics based on the time series of yearly coefficient estimates according to the Fama/MacBeth (1973) method.

	DR	OR	TtM	ATM	Skew	Trading volume	Call ratio	Adj. R^2
				Panel A: ODAX				
1995	−23.84[a]	−22.03[a]	0.0005	−2.7107	3.8245[c]	2.98E-07[a]	0.1087[b]	8.6%
1996	−31.19[a]	−14.20[b]	0.0013	4.3701[c]	1.3766	−2.42E-08	0.0488	10.2%
1997	−43.40[a]	2.65	−0.0022	1.4728[b]	2.3113[c]	−3.91E-08[b]	−0.0314	10.3%
1998	−14.53[a]	−16.61[a]	−0.0007	−0.2848	1.2761	2.08E-08	0.0452	12.1%
1999	−19.35[a]	−3.67	−0.0050[a]	1.4808[b]	−0.9242	−5.02E-08[a]	0.0406	14.0%
2000	−13.12[a]	57.68[b]	0.0013	3.9148[b]	5.2990[a]	3.37E-08[b]	0.0337[c]	9.5%
2001	−5.98[a]	37.00[b]	0.0024	0.3156	3.4597[a]	5.34E-09	0.0554[a]	5.6%
2002	−5.22[a]	−16.25	−0.0025[c]	0.1515	−0.8987	2.80E-09	−0.0064	5.7%
2003	−5.45[a]	−22.47	0.0004	−0.3329	1.4546	3.24E-08[b]	0.0068	3.1%
2004	−7.50[b]	−7.62	−0.0010	6.1724[a]	2.6584[b]	2.44E-08	−0.0016	5.5%
2005	−37.65[a]	−39.03[a]	0.0000	5.2865[b]	−0.5846	−5.19E-09	0.0029	11.7%
2006	−13.46[a]	−9.30	0.0000	0.2663	−1.7777[a]	1.71E-08	0.0200	5.2%
2007	−17.21[a]	−5.54	0.0010	1.6307[c]	1.6654[b]	−4.17E-09	−0.0094	6.8%
2008	−9.72[a]	−2.73	−0.0006	0.2638	1.2301	9.16E-10	0.0138	15.7%
2009	−9.59[a]	−17.60[a]	−0.0011	0.1004	−0.3687	−8.39E-10	0.0150	9.6%
2010	−9.93[a]	1.95	−0.0012	1.4502	0.2976	8.58E-09[c]	−0.0007	4.0%
2011	−6.64[a]	−9.64[a]	−0.0007	−0.0866	−1.1108	7.77E-09[b]	−0.0073	9.1%
F/MB	−5.73[a]	−0.94	−1.15	2.47[b]	2.36[b]	1.06	2.45[b]	
				Panel B: OESX				
2000	−12.67[a]	20.37	0.0013	2.1648	3.2477[c]	3.78E-08	0.0162	4.2%
2001	−6.69[a]	−17.85	0.0033	0.5016	1.2730	5.03E-08	0.0312[b]	3.3%
2002	−6.25[a]	−26.23	−0.0043[b]	−0.0147	−1.8068[c]	1.08E-08	−0.0055	5.2%
2003	−10.40[a]	−4.88	0.0005	0.7935[c]	1.6152	3.72E-09	0.0070	5.2%
2004	−9.78[b]	−26.54	0.0012	5.6427[a]	0.1915	−8.20E-09	−0.0018	3.9%
2005	−45.91[a]	64.68	0.0020	5.1670[b]	2.5864[b]	−2.46E-08	−0.0044	14.2%
2006	−18.33[a]	2.05	−0.0003	4.3278	1.6346	2.92E-09	−0.0157	7.9%
2007	−15.36[a]	−17.99[a]	0.0021[c]	3.6963[a]	3.6594[a]	3.31E-09	−0.0002	11.8%
2008	−8.31[a]	−15.45[a]	0.0007	0.4159[a]	2.5000	1.91E-09[b]	0.0094	16.4%
2009	−11.35[a]	−15.61[a]	0.0001	0.6609[c]	1.0244	−1.50E-09	0.0178	11.0%
2010	−9.92[a]	−1.82	0.0003	2.8160[a]	1.5125[b]	2.11E-09	−0.0014	10.6%
2011	−7.42[a]	−2.76	−0.0013	0.6793[b]	−0.2500	4.34E-09[a]	−0.0140[c]	7.8%
F/MB	−4.34[a]	−0.48	0.81	3.81[a]	3.24[a]	1.22	0.81	

OR are mixed. In most years, the OR-coefficient is negative, but the Fama/MacBeth *t*-statistic suggests that the mean of the yearly coefficients is not significantly different from 0. The positive coefficients of ATM and Skew show that the characteristics of the smile function contribute to explaining the intraday smile movements. The higher the ATM implied volatility and the slope of the smile function, the stronger the shift in implied volatilities for a given index return. The results for net buying pressure are mixed. The ratio of put and call trading volume is positively related to *c* only for DAX options. Trading volume as such does not seem to be a determinant of smile movements. The adjusted R^2 coefficients are about 8.5% on average and significantly different from zero in each year. This degree of explanatory power is in line with our previous observation that the *c*-coefficient is relatively stable and does not follow clearly discernable patterns over time.

4.4. Day-to-day movements

The *c*-measure of our intraday analysis is based on a marginal analysis. In particular, the slope coefficient in the denominator of Equation (6) is the

differential change of implied volatility. For significant index moves, however, the *discrete* change in volatility also depends on the curvature of the smile function. Therefore, in this section, we examine day-to-day movements of implied volatilities. The enhanced smile regression model is particularly suitable for this analysis, because it provides a mapping from index level to smile profile, so that smile changes can be accurately matched to index returns. According to Equation (2), the index level dependent smile is given by:

$$IV_t(M(X, S), S) = \beta_{0,t} + \beta_{1,t}M(X, S) + \beta_{2,t}M(X, S)^2$$
$$+ \beta_{3,t}D \cdot M(X, S)^3 + \gamma_t \ln S$$
$$+ \delta_t \ln S \cdot M(X, S) + \varepsilon_t,$$

where moneyness is a function of discounted strike price X and index level S. Let $S = \overline{S}_t$ denote the average index level observed on day t. If the smile function remains constant from day $t-1$ to t, the stock price change from $\overline{S}_{t-1}$ to $\overline{S}_t$ induces a change x_t of the implied volatility of an ATM option of

$$x_t = IV_{t-1}(M(\overline{S}_{t-1}, \overline{S}_t), \overline{S}_t) - IV_{t-1}(0, \overline{S}_{t-1}). \quad (7)$$

The additional change y_t due to a shift of the smile function is equal to

$$y_t = IV_{t-1}(M(\overline{S}_{t-1}, \overline{S}_t), \overline{S}_{t-1}) - IV_t(M(\overline{S}_{t-1}, \overline{S}_t), \overline{S}_t). \quad (8)$$

The variables x_t and y_t in Equations (7) and (8) are defined in such a way that a *positive* ratio y_t/x_t indi-

cates that the effect of moving along the initial smile is partly ($0 < y_t/x_t < 1$) or fully ($y_t/x_t \geq 1$) offset by shifts of the smile. For a stock price *decrease*, both x_t and y_t are typically *negative*, and vice versa.

To analyze the net effect of the change in volatility, we run the time-series regression:

$$y_t = a + cx_t + e_t \quad (9)$$

where a and c are regression coefficients and e is an error term. We expect coefficient a to be 0 (no drift). Coefficient c has a similar interpretation as before in the intraday analysis, i.e. $c = 0$ characterizes sticky moneyness and $c = 1$ corresponds to the sticky strike rule.

We additionally run the quadratic regression:

$$y_t = a_0 + a_1 x_t + a_2 x_t^2 + e_t, \quad (10)$$

where a_0, a_1 and a_2 are regression coefficients. The intraday analysis in Section 4.3 revealed that the ratio y_t/x_t depends on the daily return and therefore on x_t. Thus, we expect y_t/x_t to be higher the smaller x_t, which means that coefficient a_2 is supposed to be negative.

To be consistent with our previous intraday analysis, we also compute the mean of the ratio y_t/x_t. The mean is equal to the estimated c' coefficient in a regression of y_t/x_t on a constant:

$$y_t/x_t = c' + \epsilon_t. \quad (11)$$

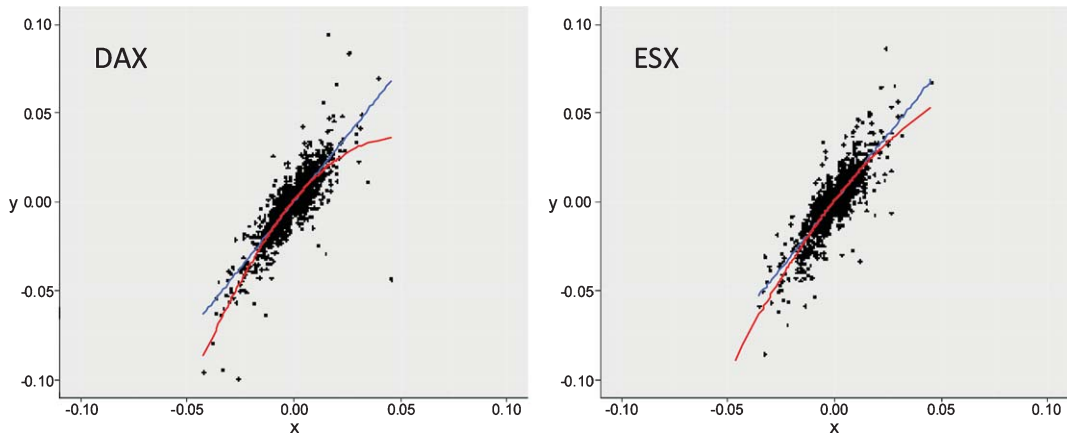We can interpret c' in Equation (11) as an unconditional estimate of the ratio $-y_t/x_t$, while $-y_t/x_t$ in

Fig. 6. Scatterplots of day-to-day movements of the smile. x is the change of the implied volatility from day $t-1$ to day t along the initial smile function (for an option which is at-the-money on day $t-1$). y is the additional change in the implied volatility due to a shift in the smile pattern. The sticky moneyness rule predicts a slope coefficient of 0, the sticky strike rule a slope coefficient of 1. The lines show the estimated linear and quadratic regression functions. The estimated slope coefficients of the linear regressions are 1.52 (DAX) and 1.55 (ESX) with adjusted R^2 coefficients of 72% and 71% (see Table 6 for more detail). In both cases, the quadratic term is significantly negative on the 1% level. Data are from 1995 to 2011 for DAX and from 2000 to 2011 for ESX.

Table 6

Regression results for day-to-day movements. The table shows results of regression models (9), (10) and (11). Median is the median of (y_t/x_t), and $\hat{c}'$ is the mean of (y_t/x_t). TtM is the time-to-maturity class. Superscript a indicates significance on the 1 percent level (two-sided test).

Option, TtM	$\hat{c}'$	Median	Linear regression			Quadratic regression			
			$\hat{a}$	$\hat{c}$	R^2_{adj}	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	R^2_{adj}
ODAX, 1	1.339	1.431	< 0.001	1.506[a]	66.3%	0.001[a]	1.474[a]	−10.300[a]	68.2%
ODAX, 2	1.310	1.398	< 0.001	1.519[a]	71.9%	0.001[a]	1.448[a]	−14.977[a]	74.7%
ODAX, 3	1.258	1.379	< 0.001	1.409[a]	73.3%	0.000	1.423[a]	0.343	73.3%
OESX, 1	1.233	1.361	< 0.001	1.574[a]	71.2%	0.001	1.565[a]	−5.404[a]	71.6%
OESX, 2	1.255	1.376	< 0.001	1.550[a]	70.5%	0.001	1.529[a]	−8.397[a]	71.2%
OESX, 3	1.206	1.330	< 0.001	1.369[a]	58.3%	0.001	1.417[a]	−14.218[a]	61.3%

Equation (9) is conditional on x_t. The regression results of Equation (9) and Equation (11) are shown in Table 6. The sample mean value c' lies between 1.20 and 1.34 while the median is always higher with values of 1.33 to 1.43. Regression (9) provides even higher estimated c-coefficients between 1.36 and 1.57. The latter values are similar to results found by Gatheral and Kama (2010) for daily S&P500 options data.

The difference between the estimated c' and c coefficients are only observed for daily data. In the intraday analysis, the mean values for c reported in Tables 1 to 4 are always close to the median as well as the slope coefficient of a regression analogous to Equation (9). In this regression, $\gamma\sqrt{T-t}$ is the independent variable and $(\beta_1 + \delta\ln\overline{S})$ the dependent variable. The main reason for the difference between daily and intraday results is that the relation between y_t and x_t is nonlinear, as the significantly negative estimates of a_2 indicate. This can also be seen from the scatterplots in Fig. 6 which include linear and quadratic regression lines. Thus, whether the best overall point estimate is 1.3 or rather 1.5, depends on the time horizon and the loss function of an agent.

5. Conclusion

This paper presents a new way to analyze the leverage effect in intraday option data. Contrary to the overall downward sloping skew profile, when the strike price is fixed, implied volatilities tend to *increase* with moneyness. Because the corresponding moneyness variation is caused by index changes alone, this implies that when the index falls (moneyness increases), implied volatility increases, and vice versa. We exploit these intraday movements of the smile structure to estimate the relationship between index return and changes of implied volatilities. For this purpose, we propose an extended smile regression model.

An application to EuroStoxx 50 and DAX options for the sample periods 2000-2011 and 1995-2011, respectively, indicates that almost all of the in-sample daily variation of implied volatility across transactions can be explained by moneyness and index return (average R^2 of about 99%). This data precision allows us to clearly identify the index-volatility relation. We find that the three commonly proposed traders' rules of thumb for describing the movements of the smile pattern (sticky moneyness, sticky strike, sticky implied tree) are not compatible with the empirical data over any extended time period. The sticky moneyness and sticky implied tree rules are clearly rejected. The sticky strike rule goes in the right direction, but systematically underestimates the impact of index changes which are about 1.3 times stronger than predicted. Day-to-day movements of the smile are consistent with a higher overall adjustment factor of about 1.5, depending on the specific assumptions underlying the estimation. More important than the average parameter is the finding that the adjustment factor is strongly associated with the index return on the same day. A decrease of the index return by one percentage point tends to increase the factor by about 0.1. The adjustment factor does not follow clearly discernable patterns over time. These results are very similar for different time to maturity classes.

Our findings can be used to test option pricing models by comparing the model implied leverage coefficient with the empirical estimates. They are also relevant for hedging, because the delta hedge ratio has to be adapted to predictable index-dependent shifts in the smile structure (see Rosenberg (2000)). According to our results, explanations for the leverage effect should be consistent with the fact that it has existed on the intraday level continuously for more than ten years. In general, this study shows that index-related changes of the smile in option prices are highly predictable, and the relationship between index and option markets tends to be closer than previous literature suggests.

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