Pairs Trading via Three-Regime Threshold Autoregressive GARCH Models

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Pairs Trading via Three-Regime Threshold Autoregressive GARCH Models

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Abstract. Pairs trading is a popular strategy on Wall Street. Most pairs trading strategies are based on a minimum distance approach or cointegration method. In this paper, we propose an alternative model to the process of pair return spread. Specifically, we model the return spread of potential stock pairs as a three-regime threshold autoregressive model with GARCH effects (TAR-GARCH), and the upper and lower regimes in the model are used as trading entry and exit signals. An application to the Dow Jones Industrial Average Index stocks is presented.

1 Introduction

Pairs trading is a popular strategy on Wall Street. It became well known by the Quant team led by Nunzio Tartagli from Morgan Stanley in the mid-1980s. The main principle underlying pairs trading is the simple idea of reversion, which is the process of identifying two stocks whose prices move together closely. When the spread between them widens, short the high price one and long the low price one. If the past is a good mirror of the future, then prices will converge and the pairs trading will result in profit. There are many ways to find stocks which are moving together. The simplest one is the Minimum Squared Distance method (MSD), which involves calculating the sum of squared deviations between two normalized stock prices and choosing the one which has the minimum value. Gatev, Goetzmann and Rouwenhorst (2006) give detailed results using US CRSP stock prices. Do and Faff (2010) examine the validity of MSD in more recent datasets. Another way to find stocks which are moving together utilizes the equilibrium relation among stocks,

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referred to as the cointegration approach. A group of nonstationary stock prices can have a common stochastic trend (cf. Engel and Granger, 1987). Vidyamurthy (2004) describes how to apply this method to pairs trading. Kawasaki, Tachiki, Udaka, and Hirano (2003) use this cointegration method to find stock pairs in the Tokyo Stock Exchange. Perlin (2009) applies it to the Brazilian stock market. The third way models the spread as a mean-reverting a Gaussian Markov chain and trading is triggered when the forecasting spread is different from the subsequent spread in a significant level. Elliott, van der Hoek, and Malcolm (2005) show how to estimate this model in detail. To obtain appropriate investment decisions, observations of the spread are compared with predictions from calibrated model.

In previous literature, the pair spread is assumed to be a single regime process. However, practitioners often find that the pair spread seems to switch between different regimes, and the usual pairs trading methods fail to identify potential arbitrage opportunities. A recent empirical study by Bock and Mestel (2009) proposes a two-state, first-order Markov-switching process to model the spread and apply it to their trading rules. Since financial time series often exhibit some stylized facts such as volatility clustering, asymmetry in conditional mean and variance, mean reversion, and fat-tailed distributions, it is important to develop an appropriate model which can capture these stylized facts.

To capture the dynamic features of volatility, the popular choices are the autoregressive conditional heteroscedastic (ARCH) and generalized ARCH (GARCH) models of Engle (1982) and Bollerslev (1986), which allow the conditional volatility to be predicted from its lagged terms and past news. Both ARCH and GARCH models are widely employed for describing dynamic volatility in financial time series. Bollerslev, Chou, and Kroner (1992) advocate that a GARCH(1,1) model would usually be sufficient for most financial time series.

In this paper, we propose an alternative: a three-regime threshold nonlinear GARCH model, with a fat-tailed error distribution (TAR-GARCH), to capture mean and volatility asymmetries in financial markets. The salient feature of this model is that it can capture asymmetries in the average return, volatility level, mean reversion, and volatility persistence. For a brief review of the TAR model in finance, refer to Chen, So, and Liu (2011). We employ a Bayesian method, based on Markov chain Monte Carlo (MCMC) methods, allowing simultaneous inference for all unknown parameters in a TAR-GARCH model.

The remainder of this study proceeds as follows. Section 2 introduces the three-regime TAR-GARCH model with a fat-tailed error distribution which is applied to identify pairs trading signals. Bayesian estimation is also briefly discussed in this section. Section 3 presents some results for stocks from the Dow Jones 30 index. These stocks are the most liquid stocks in the US market that traders can buy and sell at any time. Conclusions are presented in Section 4.

2 Methodology

We would like to model the return spread of potential stock pairs as a three-regime threshold autoregressive model with GARCH effects (TAR-GARCH), and the upper and lower regimes in the model are used as trading entry and exit signals.

2.1 Threshold AR Model with GARCH Effect

Li and Li (1996) model both mean and volatility asymmetry in a double threshold (DT-)ARCH model; Brooks (2001) further generalizes this to a double threshold GARCH model. Chen, Chiang, and So (2003) further allow an exogenous threshold variable (U.S. market news) and nonlinear mean spill-over effects. Chen and So (2006) propose a threshold heteroskedastic model to capture the mean and variance asymmetries which allows the threshold variable to be formulated with auxiliary variables. This avoids subjectively choosing the threshold variable and enables the relative importance of the auxiliary variables to be examined after model fitting. Most of these studies focus only on two-regime models. The model used here is a three-regime threshold nonlinear GARCH model, with a fat-tailed error distribution, to capture mean and volatility asymmetries in financial markets, which has been studied by Chen, Gerlach, and Lin (2010). This model is characterized by several non-linear factors commonly observed in practice, such as asymmetry in declining and rising patterns of a process. In fact, all mean and volatility parameters are allowed to change between regimes. Due to its complexity, Bayesian estimation and inference for this class of model is considered.

The three-regime model is specified as:

$$
\begin{aligned}
y_t &= \begin{cases} \phi_0^{(1)} + \phi_1^{(1)} y_{t-1} + a_t, & y_{t-d} < c_1 \\ \phi_0^{(2)} + \phi_1^{(2)} y_{t-1} + a_t, & c_1 \le y_{t-d} < c_2 \\ \phi_0^{(3)} + \phi_1^{(3)} y_{t-1} + a_t, & y_{t-d} \ge c_2 \end{cases} \\
a_t &= \sqrt{h_t}\varepsilon_t, \qquad \varepsilon_t \overset{\text{i.i.d.}}{\sim} t_\nu^*, \\
h_t &= \begin{cases} \alpha_0^{(1)} + \alpha_1^{(1)} a_{t-1}^2 + \beta_1^{(1)} h_{t-1}, & y_{t-d} < c_1 \\ \alpha_0^{(2)} + \alpha_1^{(2)} a_{t-1}^2 + \beta_1^{(2)} h_{t-1}, & c_1 \le y_{t-d} < c_2 \\ \alpha_0^{(3)} + \alpha_1^{(3)} a_{t-1}^2 + \beta_1^{(3)} h_{t-1}, & y_{t-d} \ge c_2, \end{cases}
\end{aligned} \tag{1}
$$

where c_1 and c_2 are the threshold values that satisfy $-\infty = c_0 < c_1 < c_2 < c_3 = \infty$; h_t is $\mathrm{Var}(y_t|y_1,\ldots,y_{t-1})$; the integer d is the threshold lag; t_ν^* is a standardized Student-t error distribution with a mean of zero and a variance of one. Some standard restrictions on the variance parameters are given.

$$
\alpha_0^{(j)} > 0,\, \alpha_1^{(j)},\, \beta_1^{(j)} \ge 0 \qquad \text{and} \qquad \alpha_1^{(j)} + \beta_1^{(j)} < 1, \tag{2}
$$

The lagged return y_{t-1} is included in the model (1) in order to test zero serial correlations. We would like to know whether the series has a statistically significant lag-1 autocorrelation which indicates the lagged returns might be useful in predicting y_t. When the lag-one autocorrelation is not statistically significant, it indicates that potential pair arbitrage opportunities may not exist.

Bayesian estimation requires the specification of a likelihood and prior distributions on the model parameters. We select prior distributions that are mostly uninformative, so that the data dominates inference via the likelihood.

2.2 Priors and Likelihood

Let the full parameter vector be denoted as $\theta = (\phi_1, \phi_2, \alpha_1, \alpha_2, c, \nu, d)'$, where ϕ_j = $(\phi_0^{(j)}, \phi_1^{(j)})$, α_j = $(\alpha_0^{(j)}, \alpha_1^{(j)}, \beta_1^{(j)})'$, and $c = (c_1, c_2)'$ and d_0 denotes the maximum delay lag. The conditional likelihood function of the model is:

$$L(\theta \mid y) = \prod_{t=2}^{n} \left\{ \sum_{j=1}^{2} \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{(\nu-2)\pi}} \frac{1}{\sqrt{h_t}} \left[1 + \frac{(y_t - \mu_t)^2}{(\nu-2)h_t} \right]^{-\frac{\nu+1}{2}} I_{jt} \right\}, \quad (3)$$

where $\mu_t = \phi_0^{(j)} + \phi_1^{(j)} y_{t-1}$ and I_{jt} is an indicator variable of $I(c_{j-1} \leq y_{t-d} < c_j)$.

Our prior settings are similar to those used by Chen, Gerlach, and Lin (2010). A Gaussian prior distribution is assumed for $\phi_j \sim N(\phi_{j0}, V_j)$, constrained for mean stationarity, where $\phi_{j0} = 0$ and V_j^{-1} is a matrix with 'large' numbers on the diagonal. With the maximum delay d_0, we assume a discrete uniform prior $p(d)$ = $\frac{1}{d_0}$ for d. To ensure the required constraint equation (2) on $p(\alpha_j)$, we adopt a uniform prior $p(\alpha_j)$ over the region which is the indicator $I(S_j)$, $j = 1, 2$, where S_j is the set of α_j that satisfies the restriction in (2). The prior for the threshold parameters c, as proposed by Chen, Gerlach, and Lin (2010), for three regimes is:

$$c_1 \sim \text{Unif}(lb_1, ub_1) \ ; \ c_2|c_1 \sim \text{Unif}(lb_2, ub_2),$$

where we can set $lb_1 = h$ and $ub_1 = (1-2h)$. If we choose $h = 0.1$, then $c_1 \in (0.1, 0.8)$. Further, set $ub_2 = (1-h)$ and $lb_2 = c_1 + h$. The prior for (c_1, c_2) is flat over the region ensuring $c_1 + h \leq c_2$ and at least $100h\%$ of observations are contained in each regime. Finally, the degrees of freedom ν is re-parametrised to $\nu^* = \nu^{-1}$ with uniform prior $I(\nu^* \in [0, 0.25])$, this restricts $\nu > 4$, so that the first four moments of the error distribution are finite.

We assume a prior independence among the groupings $\phi_1, \phi_2, \alpha_1, \alpha_2, c$, ν, and d. Multiplication of each prior followed by the conditional likelihood function in (3) leads to the conditional posterior density for each parameter group. Detailed conditional posteriors can be found in Chen, Gerlach, and Lin (2010). Except for parameter d, the conditional posterior distributions for each remaining parameter group are non-standard. We thus incorporate the Metropolis and Metropolis-Hastings (MH) methods to draw the MCMC iterates for the other parameter groups, see Chen and So (2006) for the discussion on the MH method. To speed up convergence and allow optimal mixing, we employ an adaptive MH-MCMC algorithm that combines a random walk Metropolis and an independent kernel MH algorithm. We extensively examine trace plots and autocorrelation function (ACF) plots from multiple runs of the MCMC sampler for each parameter to confirm convergence and to infer adequate coverage. We set the MCMC sample size N sufficiently large, discarding the burn-in iterates, and keep the last $N-M$ iterates for inference.

2.3 Pairs Selection

The series of returns are calculated by taking differences of the logarithms of the daily closing price, $r_t^j = \ln(P_t^j/P_{t-1}^j)$, where P_t^j is the closing price index of asset j on day t. Our implementation of pairs trading has two stages. The procedure is given as follows:

Stage 1: We calculate the MSD between the two normalized price series among the pairs. The formula of MSD is given as follows.

$$MSD = \sum_{t=1}^{n}(P_t^A - P_t^B)^2, \tag{4}$$

where P_t^i is normalized price of asset i at time t. Five pairs are selected with the smallest MSD.

Stage 2: We calculate the return spread between the selected pairs, $y_t = r_t^A - r_t^B$, and fit a three-regime TAR model with GARCH effect to the return spread. Once the model is fitted, the upper and lower threshold values in the model are used as trading entry and exit signals.

We open a position in a pair when the pair return spread (y_t) is larger (smaller) than the high (low) threshold value, as estimated by the TAR-GARCH model, that is, we short (long) A and long (short) B. We unwind the position when the pair return spread crosses over the same threshold value again. If the spread doesn't cross before the end of the last trading day of the trading period, gains or losses are calculated at the end of the last trade of the trading period.

The average trading return on the short stock A and long stock B position is calculated as follows:

$$r_1 = \frac{1}{D}\left[-\ln\frac{P^A_{sold}}{P^A_{bought}} + \ln\frac{P^B_{sold}}{P^B_{bought}}\right], \tag{5}$$

where D stands for the number of holding days. On the other hand, the average trading return on the long stock A and short stock B position is given as follows.

$$r_2 = \frac{1}{D}\left[\ln\frac{P^A_{sold}}{P^A_{bought}} - \ln\frac{P^B_{sold}}{P^B_{bought}}\right], \tag{6}$$

where D stands for the number of holding days.

3 Empirical Results

The daily close prices (adjusted for dividends and splits) of constituents of the Dow Jones Industrial Average Index (DJIA) are used as an illustration. The data are obtained from Yahoo Finance US over a 7-year time period, from January 2, 2006 to May 31, 2013. The in-sample period of this study is from January 2, 2006 to February 28, 2013 and the out-of-sample period is from March 1, 2013 to May 31, 2013.

The companies that comprise the DJIA are 3M (MMM), Alcoa (AA), American Express (AXP), AT&T (T), Bank of America (BAC), Boeing (BA), Caterpillar (CAT), Chevron Corporation (CVX), Cisco Systems (CSCO), Coca-Cola (KO), Dupont (DD), ExxonMobil (XOM), General Electric (GE), Hewlett-Packard (HPQ), The Home Depot (HD), Intel (INTC), IBM (IBM), Johnson & Johnson (JNJ), JPMorgan Chase (JPM), McDonald's (MCD), Merck (MRK), Microsoft (MSFT), Pfizer (PFE), Procter & Gamble (PG), Travelers (TRV), UnitedHealth Group Incorporated (UNH), United Technologies Corporation (UTX), Verizon Communications (VZ), Wal-Mart (WMT), and Walt Disney (DIS).

Table 1 gives the descriptive statistics of the DJIA stock prices. Table 1 shows that 13 companies have a standard deviation of greater than 10, and IBM has the highest standard deviation, 41.67. A volatile stock will have a high standard deviation. Figure 1 shows the time series plots of 30 DJIA company's daily closing prices.

We calculate the MSD between the two normalized price series, and the number of possible pairs is 435 (i.e. C_2^{30}). The five pair trading candidates are

Pair 1: Caterpillar (CAT) vs Chevron Corporation (CVX)
Pair 2: IBM (IBM) vs Johnson & Johnson (JNJ)
Pair 3: Merck (MRK) vs Microsoft (MSFT)
Pair 4: Verizon (VZ) vs Wal-Mart Stores Inc.(WMT)
Pair 5: Wal-Mart Stores Inc. (WMT) vs The Walt Disney Company (WAL).

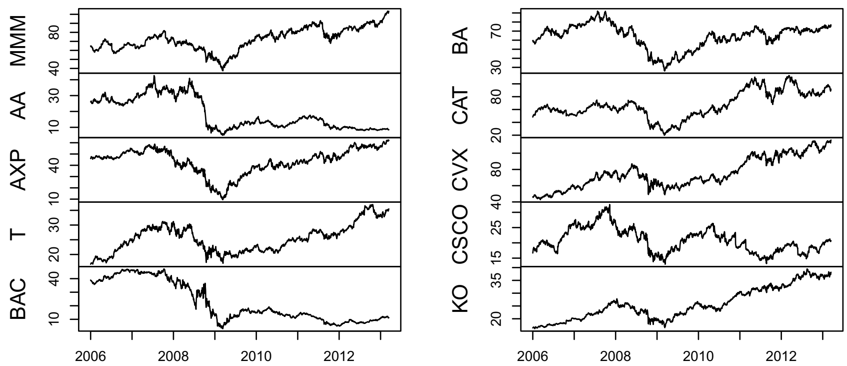

Fig. 1 The times series plots of 30 DJ company's prices

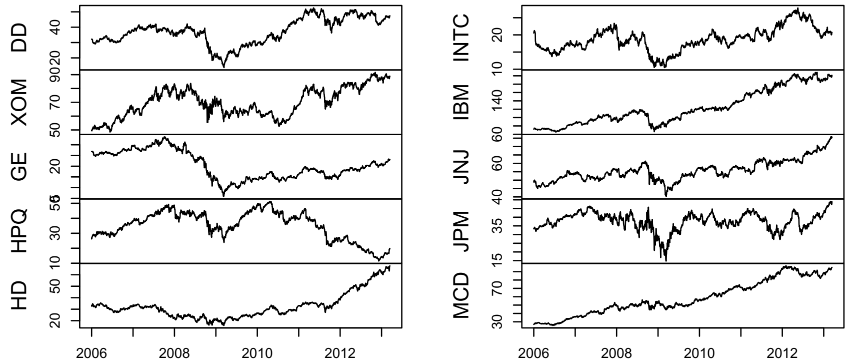

Fig. 1 (*continued*)

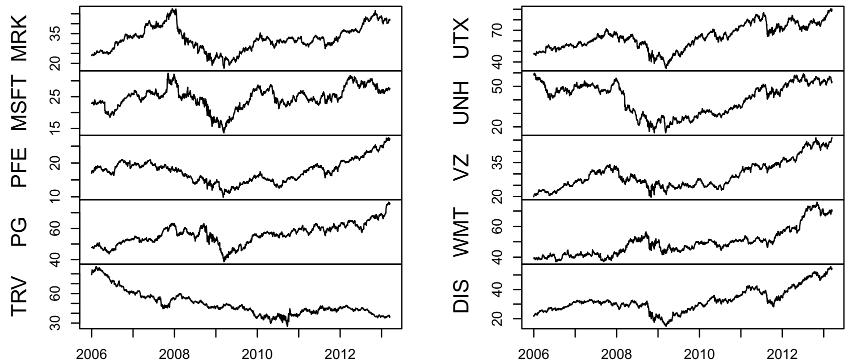

Fig. 1 (*continued*)

The MSD values of the five pairs are shown in Table 2. All five return spread series exhibit the standard property: they have fat-tailed distributions, as indicated by the highly significant Jarque-Bera normality test statistics, a joint test for the absence of skewness and kurtosis which are given in Table 2. Descriptive statistics of the DJIA stock returns are given in Table 3. Table 3 shows that the returns of all companies are close to zero, and this result coincides with the mean reversion theory. Furthermore, there are 6 companies' stock index returns that are negative.

We then fit a three-regime TAR model with GARCH effect to these five selected pairs' return spreads. Allowing y_{t-1} in the conditional mean helps account for possible autocorrelations in the pairs' return spreads. Once the model is fitted, the upper and lower threshold values in the model are used as trading entry and exit signals. When the return spread is above (below) the upper (lower) threshold value, we then short (long) one share A stock and long (short) one share B stock. Once the position is open and the spread falls back to the threshold, the position is closed.

Table 1 The descriptive statistics of the DJIA stock prices from January 2, 2006 to February 28, 2013

Company	Symbol	Mean	Maximun	Minum	Std
3M	MMM	72.30	103.59	37.40	12.41
Alcoa	AA	18.50	42.63	4.99	9.84
American Express	AXP	43.59	62.38	9.47	11.35
AT&T	T	25.37	36.98	16.69	4.75
Bank of America	BAC	22.51	47.13	3.09	14.58
Boeing	BA	64.12	92.06	26.24	13.33
Caterpillar	CAT	67.05	112.91	19.87	20.77
Chevron Corporation	CVX	75.19	116.21	42.71	18.95
Cisco Systems	CSCO	20.78	32.55	13.01	3.96
Coca -Cola	KO	25.76	39.45	16.13	6.27
DuPont	DD	37.33	52.57	13.68	8.51
ExxonMobil	XOM	69.90	91.67	48.45	10.49
General Electric	GE	20.38	33.71	5.83	6.42
Technology	HPQ	35.06	51.30	11.46	9.51
The Home Depot	HD	32.36	67.79	15.80	11.10
Intel	INTC	18.73	27.86	10.47	3.47
IBM	IBM	123.84	208.22	65.28	41.68
Johnson&Johnson	JNJ	55.72	75.78	40.23	6.19
JPMorgan Chase	JPM	37.59	49.14	14.78	5.15
MeDonald's	MCD	58.55	97.03	25.57	20.45
Merck	MRK	32.28	47.56	17.45	6.30
Microsoft	MSFT	24.62	32.39	13.62	3.45
Pfizer	PFE	17.66	27.48	9.85	3.35
Procter & Gamble	PG	56.10	76.82	38.60	6.51
Travlers	TRV	49.24	87.37	26.74	12.17
United Technologies Corp.	UTX	63.43	90.51	33.84	11.76
UnitedHealth Group Incorporated	UNH	40.95	59.75	15.51	11.55
Verizon Communications Inc.	VZ	29.54	46.05	19.63	6.26
Wal-Mart Stores Inc.	WMT	49.34	75.78	36.96	8.91
The Walt Disney Company	DIS	32.52	55.73	14.77	8.19

Table 2 Stock pairs with the smallest MSD and Jarque-Bera test for pair return spreads

Pairs	Company A	Company B	MSD	Jarque-Bera test	
				Statistic	p-value
1	Caterpillar(CAT)	Chevron Corporation(CVX)	454.94	2363.27	0.00
2	IBM (IBM)	Johnson & Johnson(JNJ)	649.18	5515.45	0.00
3	Merck (MRK)	Microsoft(MSFT)	661.11	7845.45	0.00
4	Verizon(VZ)	Wal-Mart Stores Inc.(WMT)	681.79	4031.88	0.00
5	Wal-Mart Stores Inc.(WMT)	The Walt Disney Company(WAL)	701.61	1755.15	0.00

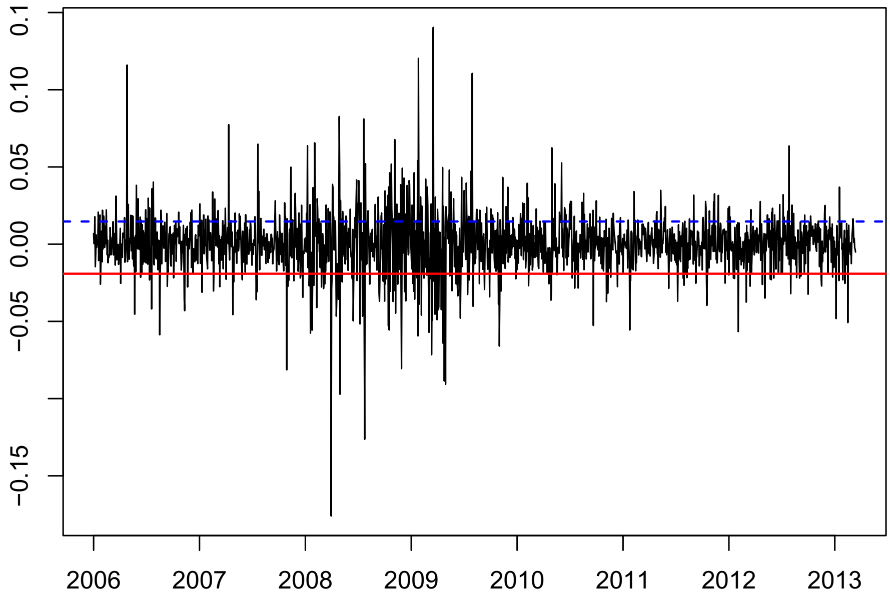

Fig. 2 MRK-MSFT pair return spread from January 2, 2006 to February 28, 2013

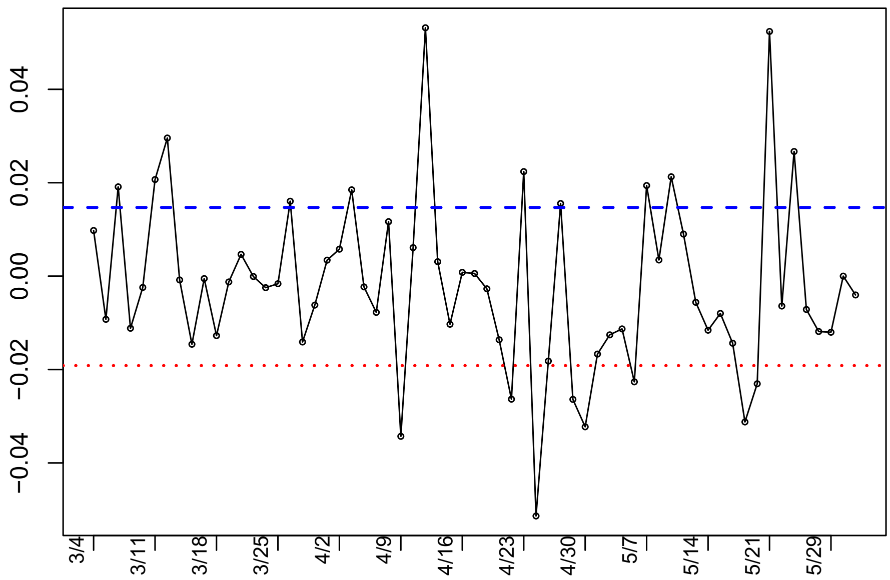

Fig. 3 MRK-MSFT pair return spread from March 1, 2013 to May 31, 2013

Table 3 The descriptive statistics of the DJIA stock log returns from January 2, 2006 to February 28, 2013

Company	Symbol	Mean	Maximun	Minum	Std
3M	MMM	0.0003	0.0941	−0.0938	0.0156
Alcoa	AA	−0.0006	0.2077	−0.1742	0.0315
American Express	AXP	0.0002	0.1877	−0.1933	0.0283
AT&T	T	0.0004	0.1506	−0.0799	0.0154
Bank of America	BAC	−0.0007	0.3020	−0.3422	0.0422
Boeing	BA	0.0001	0.1437	−0.0805	0.0198
Caterpillar	CAT	0.0004	0.1375	−0.1569	0.0233
Chevron Corporation	CVX	0.0005	0.1895	−0.1333	0.0188
Cisco Systems	CSCO	0.0001	0.1482	−0.1768	0.0212
Coca-Cola	KO	0.0005	0.1298	−0.0907	0.0125
DuPont	DD	0.0002	0.1088	−0.1205	0.0197
ExxonMobil	XOM	0.0003	0.1587	−0.1504	0.0174
General Electric	GE	−0.0001	0.1800	−0.1365	0.0221
Technology	HPQ	−0.0002	0.1353	−0.2238	0.0217
The Home Depot	HD	0.0004	0.1316	−0.0858	0.0193
Intel	INTC	0.0000	0.1119	−0.1318	0.0204
IBM	IBM	0.0006	0.1089	−0.0611	0.0145
Johnson&Johnson	JNJ	0.0002	0.1154	−0.0798	0.0106
JPMorgan Chase	JPM	0.0002	0.2240	−0.2325	0.0315
MeDonald's	MCD	0.0007	0.0897	−0.0830	0.0130
Merck	MRK	0.0003	0.1192	−0.1595	0.0180
Microsoft	MSFT	0.0001	0.1707	−0.1247	0.0185
Pfizer	PFE	0.0003	0.0975	−0.1121	0.0157
Procter & Gamble	PG	0.0003	0.0972	−0.0823	0.0118
Travlers	TRV	−0.0004	0.2005	−0.2274	0.0213
United Technologies Corp.	UTX	0.0004	0.1281	−0.0917	0.0169
UnitedHealth Group Incorporated	UNH	−0.0001	0.2984	−0.2059	0.0244
Verizon Communications Inc.	VZ	0.0005	0.1369	−0.0842	0.0150
Wal-Mart Stores Inc.	WMT	0.0003	0.1051	−0.0840	0.0130
The Walt Disney Company	DIS	0.0005	0.1484	−0.1026	0.0192

We performed 20,000 MCMC iterations and discarded the first 8,000 iterates as a burn-in sample for each data series. The parameter estimates for the model in each selected pairs' return spreads are summarized in Table 4, which include posterior medians and standard deviations (Std.) for each parameter. Note that the c_1 and c_2 are the threshold values. All five pairs return spreads clearly display no series correlation across the three regimes, in response to the lag-one spread return. Since in-sample fitting may contain information for out-of-sample trading, AR lag-one coefficient remains in our proposed model. Regarding volatility persistence ($\alpha_1^{(j)} + \beta_1^{(j)}$), the second regime displays the lowest level of persistence across spreads. We can see from Table 4 that most of the high and low threshold values are opposite signs, indicating possible different regime processes.

Table 5 shows the mean returns of companies in five pairs from March 1, 2013 to March 31, 2013, and the mean return of five pairs. It is a one-month out-of-sample result. In a similar way, we also calculate a three-month out-of-sample result. Table 6 shows the mean returns of companies in five pairs from March 1, 2013 to May 28,

Table 4 Bayesian inference of three-regime TAR model with GARCH effect for the five pair return spreads

Pairs	CAT-CVX		IBM-JNJ		MRK-MSFT		VZ-WMT		WMT-WAL	
Par.	Med	Std	Med	Std	Med	Std	Med	Std	Med	Std
$\phi_0^{(1)}$	0.0000	0.0031	-0.0001	0.0012	-0.0012	0.0028	0.0017	0.0011	-0.0041	0.0028
$\phi_1^{(1)}$	0.0591	0.0971	-0.0297	0.0710	0.0855	0.0811	0.1242	0.0644	-0.0942	0.0921
$\phi_0^{(2)}$	0.0000	0.0005	0.0002	0.0003	0.0008	0.0004	0.0003	0.0004	-0.0002	0.0004
$\phi_1^{(2)}$	0.0558	0.0685	0.0068	0.0680	0.0229	0.0469	0.0029	0.0712	-0.0633	0.0595
$\phi_0^{(3)}$	-0.0006	0.0020	-0.0007	0.0017	0.0023	0.0024	0.0005	0.0022	-0.0014	0.0021
$\phi_1^{(3)}$	0.0200	0.0789	0.0855	0.0790	-0.0470	0.0772	-0.0088	0.0916	0.0943	0.0863
$\alpha_0^{(1)}$	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
$\alpha_1^{(1)}$	0.0281	0.0223	0.0308	0.0169	0.0182	0.0154	0.0575	0.0192	0.0379	0.0178
$\beta_1^{(1)}$	0.9289	0.0459	0.9474	0.0283	0.9674	0.0232	0.9124	0.0276	0.9279	0.0323
$\alpha_0^{(2)}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\alpha_1^{(2)}$	0.0393	0.0194	0.0698	0.0219	0.0758	0.0228	0.0542	0.0191	0.0365	0.0174
$\beta_1^{(2)}$	0.9159	0.0229	0.8890	0.0190	0.8696	0.0222	0.8993	0.0245	0.9104	0.0156
$\alpha_0^{(3)}$	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
$\alpha_1^{(3)}$	0.0870	0.0260	0.0287	0.0156	0.0291	0.0211	0.0359	0.0170	0.0870	0.0255
$\beta_1^{(3)}$	0.7847	0.0675	0.9313	0.0353	0.9400	0.0387	0.9039	0.0460	0.8880	0.0360
c_1	-0.0184	0.0015	-0.0062	0.0018	-0.0192	0.0020	-0.0052	0.0020	-0.0172	0.0027
c_2	0.0105	0.0046	0.0103	0.0018	0.0147	0.0032	0.0139	0.0021	0.0113	0.0032
ν	5.5336	0.7293	4.2734	0.2758	4.2981	0.2910	5.1025	0.5995	5.1695	0.6468

Table 5 Company returns in five pairs and pairs returns from March 1, 2013 to March 28, 2013

Pairs	Company A	Mean Return	Company B	Mean Return	No. of Trading	Pairs Return
1	CAT	-0.260 %	CVX	0.086 %	4	-1.173 %
2	IBM	0.263 %	JNJ	0.321 %	4	3.494 %
3	MRK	0.241 %	MSFT	0.122 %	3	3.849 %
4	VZ	0.267 %	WMT	0.255 %	7	4.758 %
5	WMT	0.255 %	WAL	0.138 %	5	1.019 %

Table 6 Company returns in five pairs and pairs returns from March 1, 2013 to May 31, 2013

Pairs	Company A	Mean Return	Company B	Mean Return	No. of Trading	Pairs Return
1	CAT	-0.090 %	CVX	0.090 %	13	10.448 %
2	IBM	0.047 %	JNJ	0.160 %	17	3.225 %
3	MRK	0.160 %	MSFT	0.363 %	17	15.780 %
4	VZ	0.075 %	WMT	0.087 %	24	8.082 %
5	WMT	0.087 %	WAL	0.208 %	11	1.600 %

2013, and the mean return of five pairs. Figure 2 is a time series plot of the third pair's return spread (MRK-MSFT) during in-sample period. The red and blue lines are estimated threshold values, c_1 and c_2, respectively. Figure 3 shows the pairs return spread of asset MRK and its pair, MSFT, during the out-of-sample period. Again, red and blue lines locate at threshold values which are employed as trading entry and exit signals.

From Tables 5 and 6, we find that there are 4.6 round trips trading on average in the one-month period and 16.4 round trips trading on average in the three-month period. The average 5 pairs profits are 2.389% and 7.827%, respectively. The pair returns increase with the trading horizons in most pairs. This indicates that the longer the trading horizon, the better the mean reversion process works.

For a comparison, we would like to consider the cointegration approach. In the cointegration pairs trading literature, there is a potential problem, that is, we can't find enough cointegration pairs in the sample. Hakkio and Rush (1991) had observed that "cointegration is a long-run concept and hence requires long spans of data to give tests for cointegration much power rather than merely large numbers of observations." Indeed when we apply the cointegration test to the five selected pairs, only the third pair (MRK/ MSFT) is found to be cointegrated in the in-sample period.

$$P_{MRK} = -4.13 + 1.47 P_{MSFT} + \varepsilon_{IN}.$$

We then define the residual out-of-sample as

$$\varepsilon_{OUT} = P_{MRK,OUT} + 4.13 - 1.47 P_{MSFT,OUT}.$$

When the indicator (defined as $\varepsilon_{OUT}/\varepsilon_{IN}$) is larger (smaller) than one (negative one), then we short (long) one share of the first stock (MRK) and long (short) 1.47 shares of the second stock (MSFT). In this case, we short 1 share MRK, and long 1.47 shares of MSFT. The final pairs average return is 0.85% ($\frac{1}{41} \times (-\ln(\frac{41.84}{47.38}) + 1.47 \times \ln(\frac{32.38}{27.76})$) in the three-month out of sample period. The profit is significantly less than that of the proposed method which yields an average return 15.780%.

4 Conclusions

In this study, we model the daily return spread of stock pairs as a three-regime TAR-GARCH process, and the upper and lower regimes in the model are used as trading entry and exit signals. We apply the trading rules to the Dow Jones Industrial Average Index stocks. The in-sample period of this study is from January 2, 2006 to February 28, 2013 and the out-of-sample period is from March 1, 2013 to May 31, 2013.

The empirical results suggest that the combination of MSD and TAR-GARCH trading rules generate positive excess returns, relative to the underlying stocks. The average pairs trading profits are 2.389% and 7.827% in the one-month and three-month trading periods, respectively. With the proposed three-regime TAR-GARCH pairs trading strategy, traders can reap adequate profits from the Dow Jones 30 stocks. Transaction costs and rolling in-sample fitting and out-of-sample trading will be analyzed in future studies.

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