Term Structure and Roll Yield: Not Your Father's Backwardation

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Abstract:

This paper investigates potential sources of return to speculators in the commodity futures market. Initially, we focus on "classic commodity theory" based on the ideas of Keynes (1930), Hicks (1939, 1946), Kaldor (1939), Working (1948, 1949) and Brennan (1958). Next our study examines backwardation and contango in line with current convention which references the term structure of the futures price curve. This "simplified commodity theory" provides rationale for a structural risk premium commonly referred to as the 'roll yield' (or 'roll return'). We then introduce our theory of "roll yield permutations" which is derived from integrating the term structure of the futures price curve with the expected future spot price variable. Discussion includes implication of rational expectations, equilibrium theory, reflexivity, the Sonnenschein-Mantel-Debreu theorem and microfoundations theory upon commodity pricing models.

Our research indicates that these models have inherent shortcomings in being able to pinpoint a definitive source of structural risk premium within the complexity of the commodity futures markets. We surmise that the roll yield is a flawed concept derived from an accounting fiction which mixes past and present prices, and skews performance attributions either on a levered or delevered basis. We conclude that classic commodity theory encompasses reflexivity, and as a consequence, reflects the potential for disequilibrium as well as equilibrium. We extend this disequilibrium/equilibrium hypothesis and argue that the term structure of the futures price curve, while suggestive of a potential roll yield benefit or detriment, in fact implies a complex series of roll yield permutations. We are not saying, however, that commodity pricing theory is erroneous. Models as reflections of reality do not operate to the exclusion of the other; rather, these models are inter-related and each reflect certain aspects and dynamics within the futures market paradigm. Consequently, we assume market dynamics reverberate between models, as well as feedback within models resulting in a constant disequilibrium/equilibrium tension.

Keywords: Arbitrage, Backwardation, Contango, Convenience Yield, Equilibrium, Futures Price Curve, Hedging Pressure, Rational Expectations, Roll Yield, Reflexivity, Term Structure, Theory of Storage

JEL Classification: B23, C53, C68, D41, D84, E12, G13, Q11

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Introduction: The 'Source of Return' Conundrum

If something is stated repeatedly as fact, does that make it necessarily true? We no longer believe the sun circles the earth, but that idea was once conventional wisdom. A similar issue faces investors who use passive commodity indexes. In a Financial Times story, "Steep 'contango' forces traders to adapt commodities plans", we are told that investors in commodity index products "obtain a separate return, known as the roll yield, as they shift their positions each month from the expiring futures contract into the following month."[1] This idea of the 'roll yield' (also referred to as the 'roll return') is so commonly asserted by practitioners and purveyors of "commodity investments", as well as by the financial press, that it is now accepted as fact—but is it?[2]

The now mainstream view regarding source(s) of returns from *investing in commodities* is succinctly summarized by Vanguard in their white paper, "The Role of Commodities in a Portfolio". Vanguard's paper states, "Commodity futures index returns may be broken down into collateral return (U.S. Treasury bills), spot return (the return from changes in commodity prices) and roll return (the return associated with rolling a futures contract forward). Over long periods, the spot return is on average not much higher than inflation, so the roll return is an important contributor to the equity-like returns achieved by some commodity investments."[3]

Interestingly, a review of literature prior to the millennium documents that the prevailing wisdom regarding investment in commodities was generally negative. Schneeweis and Spurgin (1996) stated at the time, "One reason for the relatively low level of investment in managed

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[1] Chris Flood, "Steep 'contango' forces traders to adapt commodities plans" *Financial Times*, March 31, 2009.

[2] "The roll return is inherent in the strategy of a total return commodity index and is depicted Figure 8. Erb and Harvey (2005) states that over reasonably long time periods, "roll returns, and not spot returns, have been the driver of investment success."" [Source: Thomas M. Idzorek (2006). "Strategic Asset Allocation and Commodities" Ibbotson Associates, Commissioned by PIMCO, March 27, 2006, p 26.] "This component of the total return realized by investors is referred to as roll yield. As the left-hand panel of Exhibit 1 shows, in backwardated markets, roll yields are positive. In contangoed markets, roll yields are negative since, as the right-hand panel of Exhibit 1 illustrates, replacing contracts results in locking in a loss." [Source: Paul D. Kaplan (2007). "The Next Generation of Commodity Indexes–economic rationale for momentum-based long/short commodity investing" *Morningstar Indexes 2007*, p 2.]

[3] Kimberly A. Stockton (2007). "Understanding Alternative Investments: The Role of Commodities in a Portfolio" The Vanguard Group, Inc., Vanguard Investment Counseling & Research, p 1.

futures[4]… is that, as for traditional investments such as stocks and bond funds, investors require both a theoretical basis for their investment in nontraditional investments as well as supporting empirical results."[5] Paraphrasing Spurgin (2000), if there were excess returns to speculative capital in futures trading, assuming there are participants such as risk averse hedgers willing to lose money over time, then since barriers to entry for trading futures is low, so much capital would flow to this industry that returns would be driven to zero over time, and as a result returns would be spread so thinly that economic profits would not be possible.[6]

Nevertheless, it is assumed that organized futures markets provide important economic benefits. This premise, that properly functioning futures markets serve a valuable economic purpose, is validated by government policy.[7] The secondary benefit provided by the futures market is that it functions as a mechanism for transparent price discovery and liquidity, which therefore mitigates price volatility. The primary benefit provided by these markets, however, is that it allows commercial producers, distributors and consumers of an underlying cash commodity to hedge.[8] This reduces the risk of adverse price fluctuations that may impact business operations, which in turn theoretically results in increased capacity utilization.[9] Hence, it follows that the reallocation of risk affords a reduction in prices of commodity inputs and outputs because businesses need not offset adverse price change risk with increased margins on products or services.

Our investigation starts with established precepts that form the basis of academic studies which attempt to model the sources of return in the futures market. That is, the *source of return* in commodities emanate from capturing 'risk premia' hedgers supposedly offer speculators for assuming the risk that these aforementioned businesses are trying to offset. Till (2007) notes that there are

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[4] Until the development of securitized commodity products in the mid-noughties, speculation in commodity interests was primarily facilitated vis-à-vis the trading of futures contracts on commodity interests, rather than in the physical markets directly. The delegation of such trading to professional speculators is referred to as "managed futures".

[5] Thomas Schneeweis and Richard Spurgin (1996). "Multi-Factor Models in Managed Futures, Hedge Fund and Mutual Fund Return Estimation" University of Massachusetts, School of Management, p 3.

[6] Richard Spurgin (2000). "Some Thoughts on the Source of Return to Managed Futures" Clark University and CISDM, Draft Article, p 1.

[7] In testimony on November 2, 2005 before the Committee on Energy and Commerce United States House of Representatives, Reuben Jeffery III, Chairman U.S. Commodity Futures Trading Commission stated at the time that "Futures markets play a critically important role in the U.S. economy."

[8] By using futures or forward contracts to hedge, a producer, distributor or consumer of an underlying asset can establish a temporary substitute for a cash market transaction that will be realized at a future date.

[9] Capacity utilization is a metric used to measure the rate at which potential output levels are being met or used. Capacity utilization rates can also be used to determine the level at which unit costs will rise.

seven such themes influencing commodity pricing theory: (1) the insurance role of commodity futures contracts, which emphasizes the role of the speculator; (2) the theory of storage, which emphasizes the behavior of the inventory holder and commercial hedger; (3) the net-hedging-pressure hypothesis, which encompasses the behavior of both classes of participant; (4) the statistical behavior of commodity futures prices; (5) the attempt to reconcile commodity futures returns with the CAPM; (6) the role of commodities in a strategic asset allocation; and (7) the importance of yields as a long-term driver of commodity returns.[10]

The insurance-like context was first proposed by Keynes (1923, 1930) in his *theory of normal backwardation*. Essentially, Keynes believed that hedgers have to pay speculators a risk premium to convince them to accept their risk. A key attribute of this theory is the concept of 'congenital weakness' on the demand side for commodities.[11] As expounded by Hicks (1939, 1946), consumers are generally better positioned to choose amongst delivery alternatives as well as time their purchases; whereas producers have operational constraints, are more exposed to commodity price fluctuations, and for that reason are under more pressure to hedge.[12] Hicks' ideas relate to the *theory of storage* and Working's (1948) research on 'carry' versus 'inverse-carry' markets, with the latter involving a producer's commitment to deliver a commodity in the future superseding the increased reward that could result from selling that commodity in the present. As a result, the quoted futures price is theoretically driven below 'expected future spot price'[13] (which is "not observable") because the commodity is held back from the market and kept in storage. As described by Kaldor (1939), holding back a commodity in storage is referred to as 'convenience yield', and together with congenital weakness forms the basis of the phenomenon known as 'backwardation'. These concepts are now part of mainstream thinking, but are they validated by empirical analysis?

Unfortunately for proponents of the roll yield, the legacy of empirical investigations using a variety of asset pricing models including CAPM/C-CAPM, Hedging-Pressure Hypothesis or Arbitrage Pricing Theory have produced contradictory results as to whether there are, in fact,

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[10] Hilary Till (2007). "Part I of A Long-Term Perspective on Commodity Futures Returns: Review of the Historical Literature" *Intelligent Commodity Investing*, Till and Eagleeye, Ed., London: Risk Books, p 39.

[11] Mark Rubenstein (2006). *A History of the Theory of Investments*. Publisher: Wiley Finance, pp 52-54.

[12] Jodie Gunzberg and Hilary Till (2005). "Absolute Returns in Commodity (Natural Resource) Futures Investments" EDHEC Risk and Asset Management Research Centre, p 7.

[13] "The quoted forward price, though above the present spot price, must fall below the anticipated future spot price by at least the amount of the normal backwardation." [Source: John Maynard Keynes (1930). *A Treatise on Money, Volume II: The Applied Theory of Money*. London: Macmillan, 1930, p 144.]

positive expected returns from speculating in the commodity futures market, or whether such markets provide only 'zero systematic risk'.[14] In the past, intellectual curiosity may have been the motivation behind research investigating sources of return in commodities; but contemporaneous with increased activity and volatility in commodities over the past decade, and corresponding to the securitization of commodity-linked investment products and institutionalization of bank-sponsored commodity trading, there has been a rash of papers supportive, if not presumptive, of the idea of a 'structural risk premium' in the commodity futures markets. Forgotten is the counter-argument that trading futures is a *zero-sum game* with the cumulative profit-loss outcome being symmetric, while allowing for the potential of asymmetric winners and losers.

As it stands, the legacy of empirical research is unable to bridge the disparity in results, which only adds to the debate. Dusak (1973) was the first to investigate the beta of the futures market using the CAPM and found zero systematic risk. However, Bodie and Rosansky (1980), Fama and French (1987) found positive returns supporting normal backwardation, as well as Carter, Rausser and Schmitz (1983) who combined both the CAPM and the Hedging-Pressure Hypothesis.[15] Ehrhardt, Jordan and Walking (1987), on the other hand, found no risk premium using the Arbitrage Pricing Theory, while Miffre (2000, 2003), using a combination of asset pricing models, more recently concluded in favor of normal backwardation.[16] Yet only a decade earlier, Kolb (1992) found that only seven out of twenty-nine commodity and financial futures support normal backwardation.[17] As documented by Allen, Cruickshank, Morkel-Kingsbury and Souness (1999), "There is no consistent evidence about the existence of normal backwardation despite a long tradition of research which dates back to Keynes (1930), Hardy (1940), Working (1948, 1949), Houthakker (1957), Telser (1958, 1967), Cootner (1960, 1967), Rockwell (1967) and Dusak (1973)."[18]

In response to this conflict between findings, a study by Erb and Harvey (2006) raises the following question, "for investors considering a long-only investment in commodity futures: how can a commodity futures portfolio have 'equity-like' returns when the average returns of the portfolio's constituents have been close to zero?" At the very least, the catalogue of academic

[14] Shahid M. Ebrahim and Shafiqur Rahman (2004). "The Futures Pricing Puzzle" *Computing in Economics and Finance 2005*, No 35, Society for Computational Economics, Draft: November 8, 2004, pp 2-4.

[15] Ibid., pp 2-3.

[16] Ibid., p 3.

[17] Ibid., p 3.

[18] D. E. Allen; S. Cruickshank; N. Morkel-Kingsbury; and N. Souness (1999). "Backward to the Future: A Test of Three Futures Markets" Edith Cowan University, School of Finance and Business Economics, p 4.

studies in this area is extensive and continues to grow.[19] Greer (1997) observed that the inherent problem with investing in commodities as an "asset class" may be that these 'real assets' are not 'capital assets' but instead *consumable*, *transformable* [and *perishable*] assets with unique attributes. As noted by Ebrahim and Rahman (2004), who "echo" Malliaris and Stein (1999) as well as Bray (1992) and Sheffrin (1996), "This discrepancy between theoretical assertions and empirical behavior is a puzzle… Is there something missing in the theory?"[20] We think the answer is *yes*.

In line with Jagannathan and Wang's (1993) premise that "the lack of empirical support for the CAPM may be due to inappropriateness of some assumptions",[21] we propose that the assumptions underlying empirical studies which test for sources of return in the commodity markets, specifically those which advocate the idea of a roll yield, has perpetuated deficient theories into the investor mindset.[22] To support our view, this paper probes what we hypothesize is a flawed thought process inherent with current convention which defines 'backwardation' and 'contango' as related to the 'term structure' of the 'futures price curve', as opposed to the concept proposed by Keynes (1930) which relates 'normal backwardation' to the 'expected futures spot price'.

Our research begins by investigating several models which describes potential sources of return to commodity speculators, including one of our own design that exemplifies the complexity of these markets.[23] Initially we focus on what we term "classic commodity theory" rooted in the

[19] The following is an abbreviated list of post-Dusak (1973) studies identified in this area of research: Bodie and Rosansky (1980); Carter, Rausser and Schmitz (1983); Jagannathan (1985); Fama and French (1987); Erhardt, Jordan and Walking (1987); Hirshleifer (1988); Bray (1992); Kolb (1992); Shimko and Masters (1994); Greer (1997); Allen, Cruickshank, Morkel-Kingsbury and Souness (1999); Miffre (2000, 2003); Ebrahim and Rahman (2004); de Roon, van den Goorbergh and Nijman (2004); Erb and Harvey (2006); Gorton and Rouwenhorst (2006).

[20] Ebrahim and Rahman (2004). "The Futures Pricing Puzzle" p 4.

[21] Ravi Jagannathan and Ellen R. McGrattan (1995). "The CAPM Debate" *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 19, No. 4, Fall 1995, p 8.

[22] Empirical studies are not conclusive as to the persistence of a roll yield, and "spirited debate" on this subject continues. Feldman and Till (2007) use a geometric variation of the "roll yield calculation" [see Till (2007) and Shimko and Masters (1994)] and conclude that, "Annual average backwardation can… be considered the average discount on the expected future price and a predictor of the risk premium to passive long speculators." However, they also caution investors should "evaluate what the structural shape of a market's futures curve is expected to be… in determining futures investment returns." In the March/April 2006 *Financial Analysts Journal*, Gorton and Rouwenhorst (2005/2006) provide "evidence of a positive risk premium to a long position in commodity futures". Whereas Erb and Harvey (2005/2006) provide historical evidence suggesting that "tactical asset allocation" strategies within a portfolio of commodity futures "have achieved attractive returns", but "there is no way to guarantee that the historically observed pay-off to momentum or term structure signals will persist in the future."

[23] To minimize confusion we narrowed our analysis to futures contracts based on commodity interests, and excluded futures based on financials assets (e.g., futures contracts based on equity indexes or fixed income) which, because they are based on capital assets, embody intrinsically different economic dynamics and return characteristics.

writings of Keynes (1930), Hicks (1939, 1946), Kaldor (1939), Working (1948, 1949) and Brennan (1958). Next we examine the concepts of backwardation and contango in line with current convention, which is based on the relationship between the nearby futures contract and subsequent contract months. This characteristic, which Till (2007) refers to as the "term structure of the futures price curve", and we label as "simplified commodity theory", provides rationale for a structural risk premium commonly referred to as the roll yield (or roll return). We then introduce our "theory of roll yield permutations" which is derived from integrating the term structure of the futures price curve with the expected future spot price variable. Our investigation includes discussion of rational expectations, equilibrium theory, reflexivity, the Sonnenschein-Mantel-Debreu theorem and the concept of microfoundations in relation to commodity pricing theory.

It is noted that ongoing debate and evolving semantics on the subject has a tendency to create confusion, and that the definition of backwardation and contango as currently adopted by many academics and practitioners is often in line with Till's (2007) term structure of the futures price curve. Alternatively, we have taken the effort in this paper to distinguish backwardation and contango within the context of classic commodity theory as opposed to simplified commodity theory in order to preserve conceptual subtleties. This in turn has allowed us to develop and present our theory of roll yield permutations. In addition, we note that while classic commodity theory is derived from Keynes' writings, where (i) the spot price of a commodity normally exceeds its future price, and (ii) the expected future spot price exceeds the futures price,[24] Keynes' (1930) theory of normal backwardation is considered for the purpose of our investigation to be a variation of classic commodity theory encompassing the bias [constraints] of congenital weakness [risk aversion].

Pricing Theory: Classic, Simplified and Permutations

Keynes' commodity hypothesis has been the subject of continuous controversy since it was first published. As noted by Till (2007), not all of Keyne's (1930) original assertions or Kaldor's (1939) ideas have been accepted, however, both Keynes and Kaldor have "remained relevant" because their "ideas have been continuously reinterpreted". Take, for instance, Keynes' contention that, "The quoted forward price, though above the present spot price, must fall below the anticipated future spot price by at least the amount of the normal backwardation." Within the context of simplified commodity theory, this statement is nonsensical; yet, within classic commodity theory this

[24]Till (2007). "Part I of A Long-Term Perspective on Commodity Futures Returns" p 40.

is a perfectly logical construct illustrating insightful nuance. These two interpretations of the backwardation and contango concepts are reconciled by our theory of roll yield permutations.

Another area of confusion requiring mention here is the topic of spot prices. Commodity pricing theory has a tendency to discuss spot prices as a single representative price. Reality is far different, and producers and end-users who are involved in the cash commodities markets mostly operate "without any formal guidelines on location, time or size of trading unit and only informal requirements for reporting transactions (Anderson, Shafer and Haberer, 1996)."[25] For example, the reported spot prices for cotton "represent an average price for various qualities at multiple levels of production, i.e., they may include producer sales, inter-merchant trading, sales to mills and cooperative pooling."[26] Likewise, the cash markets in the "fed cattle and beef, hog and pork, and lamb and lamb meat industries" are often dispersed geographically with "on the spot" transactions occurring vis-à-vis "auction barn sales; video or electronic auction sales; sales through order buyers, dealers and brokers; and direct trades".[27] Alternatively, the physical markets for coffee and cocoa have increasingly used "price to be fixed" (PTBF) contracts where a relevant delivery month of the futures market is chosen as a reference to determine or fix the price of the physicals contract. This mechanism allows use of the futures market to hedge price risk, while ensuring delivery of a specific grade at a specific time and location vis-à-vis the PTBF agreement. The futures position, on the other hand, is never held to delivery, but offset in the market prior to contract delivery period. Given that grade, quality and location are material factors influencing cash prices, PTBF is a means for commercial producers and end-users of coffee and cocoa to mitigate basis risk.[28]

The above discussion on cash markets raises an interesting question as to what extent spot prices influence the pricing of futures contracts as suggested by classic commodity theory; or, as the coffee and cocoa PTBF agreement implies, are spot prices *now* a derivative of the futures price discovery? This is the conundrum that lies at the heart of the "excessive speculation" debate.[29]

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[25] Joan Evans and James M. Mahoney (1996). "The Effects of Daily Price Limits on Cotton Futures and Options Trading" Federal Reserve Bank of New York, Research Paper No. 9627, August 1996, pp 4-5. [Ref: Anderson, Carl G.; Shafer, C. and Haberer, M. (1996). "Producer price for cotton qualities vague" 1996 Beltwide Cotton Conference.]

[26] Ibid., p 5.

[27] Grain Inspection, Pacers and Stockyard Administration, U.S. Department of Agriculture, *GIPSA Livestock and Meat Marketing Study; Volume 1: Executive Summary and Overview, Final Report*. Prepared by RTI International, Health, Social, and Economics Research, RTI Project Number 0209230; p ES-1.

[28] International Trade Centre, *The Coffee Guide*, 2002.

[29] While the term "excessive speculation" is statutorily defined by the Commodity Exchange Act, its interpretation in *real world* practice is more a function of "political will" than "black-letter law".

Classic Commodity Theory

"Classic commodity theory" focuses on equilibrium relations between the present state and future expectations of three variables: (i) the current spot price of an asset; (ii) the current futures or forward contract price of an underlying asset; and (iii) the expected spot price of an underlying asset on delivery at some future date and location (referred to herein as the 'expected future spot price'). Let S_0 be the current spot price of an asset; F_t be the current price for future delivery of an underlying asset, and $E(S_t)$ be the expected spot price of an underlying asset on the delivery date. It is noted that S_0 is a known variable equal to a price currently obtainable in the spot market for the underlying asset; F_t is also a known variable equal to the current futures or forward contract price quoted on a futures exchange or over-the-counter market; but that $E(S_t)$ is an unobservable variable which converts into S_0 at some future point in time.

There are two underlying speculative strategies (I) and (II), which in combination act to form a third strategy (III) that arbitrages for 'carrying charge parity'. These strategies are reviewed below, but before continuing a description of the "theory of storage" is helpful. Carrying charge or the 'cost-of-carry' is based on the theory of storage first developed by Kaldor (1939) and Working (1949). The theory assumes that holders of commodities incur a storage cost for financing and storing inventories (including insurance and transportation), as well as a convenience benefit of being able to use inventories the moment they are commercially needed. Storage cost, if dominant, is stated to be responsible for producing contango; while convenience yield, if dominant, is said to be the reason for backwardation. In combination, storage cost and convenience yield is expressed as the cost-of-carry which is derived from Kaldor's equation $F_t - S_0 = o + i - y$, where o is storage cost; i is interest costs; and y is the convenience yield; with the variables given based on Kaldor's writings. Using simple algebra, we can then extrapolate that convenience yield or $y = S_0 - F_t + o + i$; spot price or $S_0 = F_t - o - i + y$; and that the futures price or $F_t = S_0 + o + i - y$.[30]

Speculative strategy (I). The first strategy capitalizes on the implied relation between S_0 and $E(S_t)$. This relation is interposed by financing physical delivery of the commodity today at S_0, and selling the same commodity in the future at the *then* prevailing spot price $E(S_t)$. In this

[30] Working (1948, 1949), who examined futures spreads versus prevailing inventories, found that "carrying charges behave like prices of storage as regards their relation to the quantity of stocks held in storage." Accordingly, he defined the cost-of-carry as the "difference at a given time between prices of a commodity for two different dates of delivery", which relates to the 'term structure of the futures price curve' characteristic discussed in this paper.

scenario, an equilibrium state is achieved when S_0 plus cost-of-carry is equal to $E(S_t)$. Hence, if $S_0 + o + i - y$ is theoretically $< E(S_t)$, then speculators would make a profit by taking physical delivery of the commodity today (driving up the current spot price), with the intention of selling the commodity in the future (driving down the expected future spot price).[31] Assuming perfect markets and rational expectations (i.e, market participants are risk neutral, know perfectly the cost-of-carry, and transaction costs are zero), this is the *raison d'etre* that supposedly enforces equilibrium between S_0 and $E(S_t)$. However, since $E(S_t)$ is in the future and therefore an unknown, this strategy is technically speculation, and for that reason determining the theoretical price of $E(S_t)$, if feasible, ought to necessitate interpolation using at least one or more other references.

Speculative strategy (II). The second strategy capitalizes on the implied relation between F_t and $E(S_t)$. This relation is interposed by buying the futures contract today at F_t, taking physical delivery of the commodity when the contract expires, and *at such time* simultaneously selling it in the spot market at $E(S_t)$. In this scenario, an equilibrium state is achieved when F_t is equal to $E(S_t)$. Hence, if F_t is theoretically $< E(S_t)$, then speculators would make a profit by purchasing the futures contract (driving up the futures contract price), with the intention of taking delivery and selling the commodity at the *then* prevailing spot price (driving down the expected future spot price). In addition, since it is easy to short futures contracts, if F_t is theoretically $> E(S_t)$, then speculators could also make a profit by shorting the future contract (driving down the futures contract price), with the intention of making delivery in the future by purchasing the commodity at the *then* prevailing spot price (driving up the expected future spot price). Assuming perfect markets and rational expectations, this is the *raison d'etre* that imposes equilibrium between F_t and $E(S_t)$. In fact, F_t functions as the *price discovery mechanism* and current indication of $E(S_t)$ in both theory and practice.

Arbitrage strategy (III). As a result of speculative strategies (I) and (II), assuming perfect markets, rational expectations and arbitrage convergence, then F_t should equal $S_0 + (o_t + r_t - y_t)$, where o_t is the outlay on physical storage including facilities, insurance and interest; r_t is a marginal risk-aversion factor (applied to commodities without futures contracts); y_t is the convenience yield which is expected to increase as inventories decrease; and t is the time to delivery of the underlying asset. This third strategy arbitrages for ‘carrying charge parity’, with variables given based on Brennan’s (1958) ideas, who reasoned that outlays are generally constant until facilities reach

[31] The reason why the opposite trade is less likely to occur, besides it being difficult to borrow and short S_0, relates to Keynes theory of normal backwardation and congenital weakness. This is discussed further on page 16.

capacity, at which point costs would increase as storage becomes limited.[32] Brennan, who applied his research to actual market data, referred to $E(S_t) - S_0$ as the price spread, and inferred that $r_t - y_t$ was a result of $E(S_t) - S_0 - o_t$, where $E(S_t)$ is equal to forward contracts in lieu of commodities without futures. Evidence of convenience yields at low inventory levels was also confirmed by Telser (1958) in his empirical study on futures price spreads and inventories from 1926 to 1954.[33]

Taking the speculative strategies (I) and (II) together with arbitrage strategy (III), assuming perfect markets and rational expectations, an equilibrium state is achieved when S_0 plus cost-of-carry equals $E(S_t)$, F_t is equal to $E(S_t)$, and S_0 plus cost-of-carry equals F_t. More simply, we can state $F_t = S_0 + (o_t + r_t - y_t) = E(S_t)$. However, if as Keynes (1930) suggests scenarios exist where (I) $E(S_t)$ may be > or < $S_0 + (o_t + r_t - y_t)$, or (II) $E(S_t)$ may be > or < F_t, then by inference either: (i) the assumptions underlying cost-of-carry are incorrect and should be modified such that $F_t = S_0 + (o_t + r_t - y_t) = E(S_t)$; (ii) markets are imperfect and 'rational expectations equilibrium' is flawed, which allows for arbitrage opportunities; or (iii) there is an unknown factor causing an unexplained anomaly.

For reasons which shall become apparent, since o_t may be ascertained (i.e., estimated from observable fundamentals) whereas r_t and y_t are inferred, we propose three alternative equations to solve for cost-of-carry based on the difference between F_t and S_0. First, the equation $F_t = S_0 + (o_t + r_t - y_t)$ can be simplified to $F_t = S_0 + (o_t - y_t \pm \varepsilon_t)$, where $\pm\varepsilon_t$ is an *error term* over/under y_t, and where y_t is determinable in keeping with Kaldor's (1939) equation and assumed to include Brennan's (1958) marginal risk aversion factor r_t. It should be noted, however, that Kaldor qualified his storage theory equation and the convenience benefit as not applicable when hedgers are forward buyers. We therefore took the liberty of dubbing such conditions an "inconvenience yield" as a counterpoint to the convenience yield factor.[34] Since convenience yield $-y_t$ and inconvenience yield $+y_t$ is inferred by netting o from $E(S_t) - S_0$ or more readily from $F_t - S_0$, we can rewrite carrying charge parity as $F_t = S_0 + (o_t \pm y_t \pm \varepsilon_t)$. This second variation of the formula still presumes that $\pm y_t$ is distinguishable from other factors such as storage outlay and/or financing costs. Accordingly, our third variant of the cost-of-carry equation, in keeping with classical convention, rewrites ε as a second derivative, such that $F_t = S_0 + (o_t - y_t \cdot \varepsilon_t)$, where ε_t is an *error factor* from which $\pm y_t$ can be derived, but only

[32] This is an interesting observation in light of traders leasing oil tankers as a means to speculate on the physical product. See: "Tanker Glut Signals 25% Drop on 26-Mile Line of Ships" *Bloomberg*, December 28, 2009.

[33] Till (2007). "Part I of A Long-Term Perspective on Commodity Futures Returns" p 48.

[34] Given the growth of China and other emerging economies, as well as increased allocations to long-only commodity funds as an inflation hedge, we anticipate that Kaldor's observation of conditions when hedgers are forward buyers (i.e., "inconvenience yield") will be subject to additional study in the coming years if this factor becomes persistent.

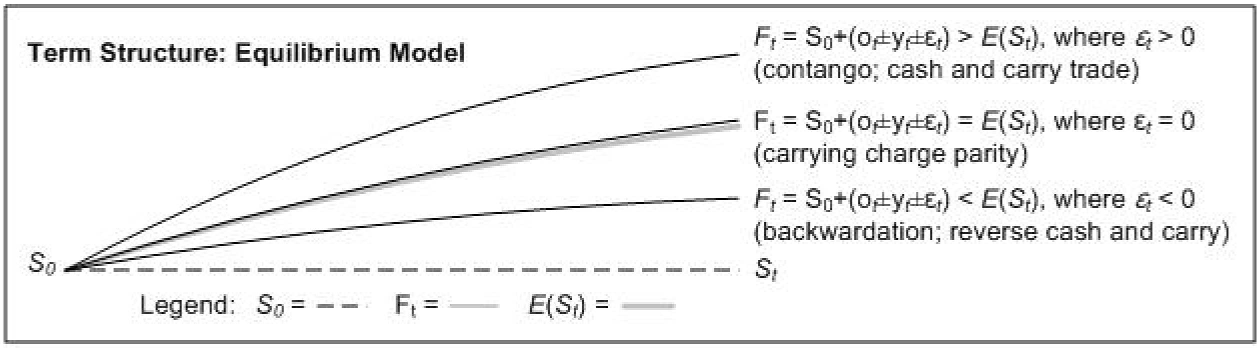
as a function of whether ε_t is either = 1, ≥ 1, or ≤ 1 including scenarios where ε_t equals 0, in which case the cost-of-carry consists only of storage outlay.[35]

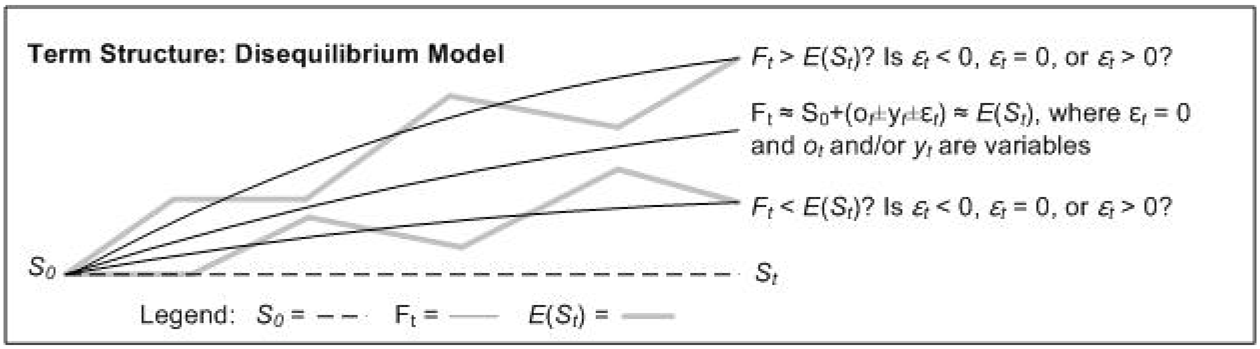
We now reprise Keynes' (1930) statement that, "The quoted forward price, though above the present spot price, must fall below the anticipated future spot price by at least the amount of the normal backwardation." How is this possible? We interpret Keynes to mean the following: classic commodity theory assumes that if arbitrage convergence is imperfect, and there exists economic stimuli which causes normal backwardation, it is possible to have $E(S_t)$, which in the real world is unobservable, valued above F_t, even if the term structure of the futures price curve exhibits contango as conventionally defined (i.e., $F_t = S_0 + (o_t + r_t - y_t)$, where r_t and y_t are null and thus $F_t > S_0$).

In effect, classic commodity theory allows for and generates circular logic which is inherent in the classical literature. We tackle such implied arbitrage by including an error term [or factor] in our $S_0 + (o_t \pm y_t \pm \varepsilon_t)$ [or $S_0 + (o_t - y_t \cdot \varepsilon_t)$] equations, which can either be: (a) negative [or a factor >1], if convenience yield dominates (backwardation), in which case $F_t < E(S_t)$; or (b) positive [or a factor <1], if storage costs dominates (contango), in which case $F_t > E(S_t)$. However, for either of these scenarios, *causal relativity* (i.e., the outcome of a cause must always be determined/evaluated relative to a control condition) between S_0 and/or F_t and $E(S_t)$, while intrinsic to the price discovery process, is not conventionally addressed because rational expectations is assumed. Yet, if one assumes imperfect markets and the potential for variability in either F_t, $E(S_t)$ or S_0, the consequential effect is *price reflexivity* in the absence of a rigorous control condition.

Hence, if there are conditions where F_t is backwardated or contango relative to $E(S_t)$, which we acknowledge is not observable, equilibrium (i.e., $F_t = S_0 + (o_t + r_t - y_t) = E(S_t)$) implies that either F_t is mispriced or alternatively S_0 is mispriced. This in turn raises concerns about the viability of F_t and/or S_0 serving as a control condition when cost-of-carry (i.e., $(o_t \pm y_t \pm \varepsilon_t)$ [or $(o_t - y_t \cdot \varepsilon_t)$]) is also indeterminable based on a potential error term [or factor]. As a result, if fundamentals underlying cost-of-carry assumptions change, such divergence should subsequently influence the price of either F_t, S_0, $E(S_t)$, or two, or all three of these variables simultaneously. The following graphs help illustrate this conundrum using our more simplistic $S_0 + (o_t \pm y_t \pm \varepsilon_t)$ variation of the cost-of-carry equation. In this particular example, we illustrate what Keynes intended by his quote above.

[35] The reason we insert an error term [or factor] into the classical equations of Kaldor (1939) and Brennan (1958), when r and y have conventionally sufficed in accounting for risk aversion factor and convenience yield based on the relationship between F_t and S_0, is because it permits modeling backwardation and contango relative to $E(S_t)$.





As discussed previously, Keynes (1930) formulated his commodity hypothesis and the theory of normal backwardation arguing that F_t is typically less than $E(S_t)$. This is based on two assumptions. First, since market participants are risk averse (this is not universally agreed upon by academics), it implies that $E(S_t) > S_0 + (o_t + r_t - y_t)$ and therefore $F_t < E(S_t)$;[36] hence, futures markets are *normally* backwardated. Second, commodities that are used for consumption or production purposes may not be easily shorted (i.e., S_0 borrowed and sold). That said, Keynes allows for the possibility of $E(S_t) < S_0 + (o_t + r_t - y_t)$, which then implies that F_t can also be $> E(S_t)$ undermining the "rule" that $F_t < E(S_t)$.[37] Thus, there are situations when Keynes acknowledges that the futures market may exhibit contango, although the term was never specifically used by Keynes when describing his theory. Nevertheless, the theory of normal backwardation does not detract from the observation that classic commodity theory encompasses circular logic. Rather, the *Keynes-Hicks hypothesis* involving congenital weakness just places certain constraints on the model.

With respect to our variations of the cost-of-carry equations, $(o_t \pm y_t \pm \varepsilon_t)$ [or $(o_t - y_t \cdot \varepsilon_t)$], the problem facing arbitrageurs seeking to effect carrying charge parity (III) is determining whether: (i) $\varepsilon_t = 0$ [or ε_t = factor of 1] and the fundamentals underlying o_t and/or y_t assumptions have shifted; or

[36] Rubenstein (2006). *A History of the Theory of Investments*, p 52.

[37] Ibid., p 52.

(ii) ε_t > or < 0 [or ε_t > or < factor of 1] and there exists an arbitrage opportunity. Since the model's logic is circular, the relationships between (I) S_0 and $E(S_t)$ versus o_t, y_t and/or ε_t; and between (II) F_t and $E(S_t)$ versus o_t, y_t and/or ε_t; as well as between (III) F_t and S_0 versus o_t, y_t and/or ε_t is complex and iterative. Therefore, when arbitraging for carrying charge parity, in order to determine if any of these variables are correctly valued or mispriced an arbitrageur must rely on fundamental analysis in estimating o_t. Conversely, the question facing speculators with respect to speculative scenarios (I) or (II) is estimating whether the market's current implied cost-of-carry is accurate, which allows interpolation of $E(S_t)$; or whether the market's implied cost-of-carry is inaccurate and there exists a speculative opportunity. If the latter, then a speculator has the choice to: (i) purchase the physical commodity at S_0, assume the risk of storage outlays, and then sell the physical in the future at the then prevailing price; or (ii) enter into a long or short futures position at F_t, and then take or make delivery upon contract settlement when $E(S_t)$ is converted into S_0.[38]

Hence, for practitioners, applicability of classic commodity theory is largely dependent on whether rational expectations equilibrium is presumed to accurately reflect how the *real world* works, or whether one may assume that markets are imperfect and rational expectations is untenable. Siding with the latter argument, classic commodity theory while rendered less useful for instigating trade decisions (or, for that matter, less capable as a methodology for empirically testing for backwardation or contango), intrinsically generates price reflexivity. As a result, constant and proportional change to variable(s) in the model have the potential to cause S_0, F_t and/or $E(S_t)$ to skew in a directional bias (rather than converge toward equilibrium) resulting in price exponentiation. If, however, rational expectations is tenable, then the definition of backwardation or contango within the context of classic commodity theory is either: (i) discordant—because $E(S_t)$ is according to both the model's definition and the real world, an *unknown*; or (ii) harmonious—yet engendering reflexivity too, but where variable(s) are by default assumed to already be in equilibrium as a result of arbitrage convergence. This is where economic modeling becomes philosophical.[39]

With respect to point (i) above, Keynes (1937) states, "The orthodox theory assumes that we have a knowledge of the future of a kind quite different from that which we actually possess."[40]

[38] In actual practice, the majority of speculators offset the futures contract before expiry.

[39] "Of course, all models are wrong. The only model that is not wrong is reality and reality is not, by definition, a model." [Source: Andrew G. Haldane (2009). "Why Banks Failed the Stress Test" Executive Director for Financial Stability, Bank of England, speech given at Marcus-Evans Conference on Stress-Testing, February 13, 2009.]

[40] John M. Keynes (1937). "General Theory of Employment" *Quarterly Journal of Economics*, CWK, vol. XIV, p 122.

Keynes also noted that fundamental uncertainty about the future is such that "We simply do not know",[41] although proponents of *ergodic systems* dismiss Keynes in this respect.[42] Davidson (1991) retorts with a Post-Keynesian perspective by making the case that "no expenditure of current resources on analyzing past data or current market signals can provide reliable statistical or intuitive clues regarding future prospects."[43] As demonstrated by Calvo and Mendoza (2000), "in the presence of short-selling constraints, the gains of gathering information at a fixed cost diminish as markets grow… [thereby] weakening incentives for gathering costly information and by strengthening incentives for imitating arbitrary market portfolios."[44] As to point (ii) in the prior paragraph, Muth (1961) states, "The way expectations are formed depends specifically on the structure of the relevant system describing the economy."[45] In other words, models based on rational expectations hypothesis assume a predetermined equilibrium around which expectations are formed, which in effect reverses the model's line of causation.[46] According to Davidson, "For the rational expectations hypothesis to provide a theory of expectational formation without persistent errors, not only must the subjective and objective distribution functions be equal at any given point of time, but these functions must be derived from what are called ergodic stochastic processes. By definition, an ergodic stochastic process simply means that averages calculated from past observations cannot be persistently different from the time average of future outcomes… Indeed, Samuleson (1969, p. 184) has made the acceptance of the 'ergodic hypothesis' the sine qua non of the scientific method in economics."[47]

[41] Keynes (1937). "General Theory of Employment" p 114.

[42] "General theory is an obscure book. I am not sure that even Keynes himself knew completely what he really meant… We are in a much better position than Keynes was to figure out how the economy works… Classical economics is right in the long run… Moreover, economists today are more interested in the long-run equilibrium." [Source: N. Gregory Mankiw (1992). "The Reincarnation of Keynesian Economics" *European Economic Review*, 36(2-3), April 1992, pp 560-561.]

[43] "From this Post Keynesian perspective, decision makers either avoid choosing between 'real' alternatives because they 'haven't got a clue' about the future, or follow their 'animal spirits' for positive investment action in a 'damn the torpedoes, full speed ahead' approach." Paul Davidson (1991). "Is Probability Theory Relevant for Uncertainty? A Post Keynesian Perspective" *The Journal of Economic Perspectives*, Vol. 5, No. 1, Winter 1991, p 130.

[44] Guillermo Calvo and Enrique Mendoza (2000). "A Rational Contagion and the Globalization of Securities Markets" *Journal of International Economics*, Vol. 51, p 79.

[45] "In order to explain fairly simply how expectations are formed, we advance the hypothesis that they are essentially the same as the predictions of the relevant economic theory." John F. Muth (1961). "Rational Expectations and the Theory of Price Movements" *Econometrica*, Vol. 29, No. 3, July 1961, p 316.

[46] "One must assume that the people in one's models do not know what is going to happen, and know that they do not know just what is going to happen." [Source: John R. Hicks (1979). *Causality in Economics*. New York. p vii.]

[47] Davidson (1991). "Is Probability Theory Relevant for Uncertainty? A Post Keynesian Perspective" p 130; [Ref: Paul A. Samuelson (1969). "Classical and Neoclassical Theory" *Monetary Theory*. London: Penguin, 1969.]

Another problem, as premised by the Sonnenschein-Mantel-Debreu theorem,[48] revolves around the application of rational expectations to aggregate behavior. Microfoundations literature has depicted microeconomics as concerned only with individual behavior and has regarded macroeconomic propositions as consequences of the interaction of rational individuals. Colander et al. (2008) notes, "What makes macroeconomics a separate field of study is the complex properties of aggregate behavior that emerges from the interaction among subjects. Since in a complex system aggregate behavior cannot be deduced from an analysis of individuals alone, representative-agent models fail to address the most basic questions of macroeconomics."[49] As argued by Guesnerie (1992), "It is rational for individual players to have rational expectations if other players have these very same rational expectations, but not necessarily otherwise."[50] Janssen (1993) concludes, "The term 'rational expectations' is thus rather misleading… [rational expectations] is an aggregate hypothesis that cannot unconditionally be regarded as being based on [methodological individualism]."[51] This conclusion is consistent with the problem we previously noted regarding cost-of-carry, and determining if o_t or y_t has changed as a result of aggregate microeconomic factors or whether rational equilibrium systems are misleading. Thus the combination of the Sonnenschein-Mantel-Debreu theorem and microfoundations theory raise the specter that generalized macroeconomic assumptions about cost-of-carry may be inconsistent with the specific operating context, microeconomics and subsequent "rational" actions of an individual bona fide hedger.

It is not surprising then that Keynes (1923, 1930) offered little in terms of empirical evidence for the theory of normal backwardation. As Rubenstein (2006) notes, "Since the expected future spot price is not observable, the signature of normal backwardation will be the tendency of the forward price to rise (more than the opportunity costs of holding the commodity would suggest) as the delivery date approaches."[52] Nevertheless, mainstream thinking endures that backwardation is the *normal* state for futures on commodities as a result of "natural" arbitrage pressures despite

[48] Hugo F. Sonnenschein (1973). "Do Walras Identity and Continuity Characterize the Class of Community Excess Demand Functions?" *Journal of Economic Theory*, 6, p 345-54; Rudolf Mantel (1974). "On the Characterization of Excess Demand" *Journal of Economic Theory*, 7, pp 348-353; Gerard Debreu (1974) "Excess Demand Functions" *Journal of Mathematical Economics*, 1, pp 15-21.

[49] David Colander; Peter Howitt; Alan Kirman; Axel Leijonhufvud and Perry Mehrling (2008). "Beyond DSGE Models: Toward an Empirically Based Macroeconomics" *American Economic Review*, vol. 98(2), May 2008, p 236.

[50] Maarten Janssen (2006). "Microfoundations" Tinbergen Institute Discussion Paper, TI 2006-041/1, p 6. [Ref: Roger Guesnerie (1992). "An Exploration of the Eductive Justifications of the Rational Expectations Hypothesis" *American Economic Review*, Vol. 82, No. 85, pp 1254-1278.]

[51] Maarten Janssen (1993). "Microfoundations: A Critical Inquiry" London: Routledge, p 142.

[52] Rubenstein (2006). *A History of the Theory of Investments*, p 54.

studies such as Allen, Cruickshank, Morkel-Kingsbury and Souness (1999) who concluded that "normal backwardation does not prevail for a majority of the contracts examined" and that the "remaining contracts can however clearly be classified as being in contango."[53] Perhaps key to the ongoing confusion may be adherence to a conventional definition of backwardation and contango as related to the term structure of the futures price curve, versus Keynes perception that the futures/ forward price is less than the expected future spot price—an unobservable variable.

We therefore offer the following hypothesis as to why research continues to produce inconsistent results with respect to empirical research on futures market risk premia.[54] Despite that rational expectations equilibrium is *the* underlying assumption, modeling the relation between S_0 and/or F_t in order to determine $E(S_t)$ intrinsically encompasses some form of circular logic, and accordingly, is subject to the problem of reflexivity. To resolve this otherwise, requires a quixotic control condition to be assumed as a fixed variable within the model. Therefore, since classic commodity theory is actually reflexive, its *natural* state is arguably disequilibrium not equilibrium, and constant and proportional change to variable(s) in the model has the potential to skew prices in a directional bias rather than converge toward equilibrium.

Regardless, classic commodity theory, while an abstraction from reality, provides significant insight into various aspects of how the futures market operates. Further, we do not claim that backwardated or contango conditions cannot exist in the classic sense; that is, $F_t < E(S_t)$ or $F_t > E(S_t)$. In accordance with the Sonnenschein-Mantel-Debreu theorem and microfoundations theory, we posit that individual bona fide hedgers, assuming carrying charge parity $F_t = S_0 + (o_t \pm y_t \pm \varepsilon_t)$ [or $(o_t - y_t \cdot \varepsilon_t)$] $= E(S_t)$, can determine backwardation or contango in relation to their specific business context at a particular point in time. Rather, it is impossible for the broad mass of market participants, specifically a "crowd" of speculators, to know perfectly whether $\varepsilon_t = 0$ [or ε_t = factor of 1] and o_t or y_t has increased, decreased or not due to a change in micro- or macro-economic factors, or whether $\varepsilon_t >$ or < 0 [or $\varepsilon_t >$ or $<$ factor of 1] and an arbitrage opportunity exists. Furthermore, analysis of market fundamentals (e.g., storage outlays) on an aggregate basis is prone to interpretation and thus error—this is well understood by professional traders who rely on money management techniques to control risk and the incongruities of the market.

[53] Allen, et al. (1999). "Backward to the Future: A Test of Three Futures Markets" p 13.

[54] As an aside, we note that there may also be biases built into the construction of continuous series of futures data, which also instigates contradictory results from empirical research.

Simplified Commodity Theory

"Simplified commodity theory" is based on a conventional and mainstreamed interpretation of backwardation and contango. The model looks at the term structure of the futures price curve, and compares the 'nearby futures' contract with such futures' subsequent contract months. If the nearby futures contract price is trading at a premium (higher) than the 'second nearby' or following successive delivery contracts, the market is said to be backwardated or in backwardation. If the nearby futures contract price is trading at a discount (lower) than the 'second nearby' or following successive delivery contracts, the market is said to be in contango. It is noted here that the term structure of the futures price curve provides the underlying rationale for an alleged structural risk premium known as the 'roll yield' or 'roll return' (term used depends on the literature source).

The model focuses on the relation between present state variables only. These variables are the (i) current spot price of an asset; (ii) current nearby futures contract price; and (iii) the current second nearby and/or successive futures contract prices. Let S_0 be the current spot price of the asset; F_1 be the current futures price for nearby contract delivery of the underlying asset, and F_2, F_3, F_t be the current futures price for second nearby, third nearby and subsequent contract deliveries of the underlying asset. By not accounting for the expected future spot price $E(S_t)$, examination of relative pricing is made uncomplicated. Based on Kaldor's (1939) ideas about 'supply-of-storage', Working (1948, 1949) observed that since storage costs are normally higher the longer a commodity is stored, the futures price at increasingly distant delivery dates will naturally be higher than at earlier dates, and that the difference will be the 'cost of storage'. As a consequence, the *natural term structure* of the futures price curve is contango such that $S_0 < F_1 < F_2 < F_3 < F_t$.

This seeming contradiction between normal backwardation and the theory of storage is reconciled by classic commodity theory in that the term structure of the futures price curve can be contango, but the underlying expected future spot price can also be backwardated in relation to each of its corresponding futures contract. Therefore, if $S_0 < F_1 < F_2 < F_3 < F_t$ is true and indicates contango, then $F_1 < E(S_1)$, $F_2 < E(S_2)$, $F_3 < E(S_3)$, and $F_t < E(S_t)$ can also be true and is indicative of backwardation, where $E(S_1)$ is the expected future spot price for F_1 future delivery, $E(S_2)$ is the expected future spot price for F_2 future delivery, etc. Likewise the reverse scenario is possible. However, while this conjecture is based on inference and reconciles the divergence between normal backwardation and the impact of time on storage outlays, theorists generally prefer to reference Working's (1948, 1949) research on the relationship between inventories and carrying charges in order to provide an economic rationale for resolving this seeming contradiction. Working concluded that (a) ‘inverse carrying charges’ are a reliable indicator of scarcity and that (b) during times of scarcity, Kaldor’s (1939) convenience yield becomes sufficiently large as to overwhelm a commodity’s storage and financing costs, leading to a negative price of storage.

As initially discussed, simplified commodity theory utilizes the term structure of the futures price curve characteristic from which a supposed structural risk premium known as the roll return is derived. As indicated by our exposé of classic commodity theory, for the economic relationship between a futures contract and the underlying cash commodity to exist, it is necessary that the underlying be deliverable against that futures position (either in the form of physical delivery or cash settlement). Interestingly, across all futures markets, less than one percent of open futures positions are settled by delivery.[55] However, the fact that delivery may be made means that the price of a futures contract must relate realistically to the price of the underlying commodity. This relationship is facilitated through arbitrage, ensuring convergence during delivery month, as theorized by classic commodity theory. However, since speculators rarely take or make delivery of the underlying commodity, and thus rarely hold a futures contract through delivery settlement, a speculator’s exposure to the commodity futures price eventually involves closing out the held contract and *rolling* into a successive futures position with a later expiry date. This process, known as “rolling the futures contract forward”, is said to indicate an ‘excess return’ which is either positive or negative depending on whether the speculator is long or short, and whether the futures price curve is backwardated or contango within the definition of simplified commodity theory.

Such is the rationale from which the premise developed that commodity futures investment encompasses returns from three sources: (i) ‘collateral return’ which is the risk-free return from investment in collateral set aside for *de minimis* margin (such good faith deposits are often placed in US Treasury bills); (ii) the so-called ‘spot return’ which is based on the change in futures prices from the point at which a position is initiated until it is either liquidated (in the case of longs) or covered (in the case of shorts); and (iii) the roll yield, which can be either positive or negative, and results from replacing an expiring contract with a second nearby or subsequent contract in order to avoid physical delivery, yet at same time maintain a long or short exposure to the underlying commodity in the futures market.

[55] Source of statistic is testimony by Reuben Jeffery III, Chairman U.S. Commodity Futures Trading Commission on November 2, 2005 before the Committee on Energy and Commerce United States House of Representatives.

The convention for calculating spot returns, excess returns and roll returns, as enumerated by Till (2007) whose formulae is based on Shimko and Masters (1994), is as follows: let F_1^t equal the nearby futures price at the current point in time *t*, let F_1^{t+1} equal the nearby futures price at a subsequent point in time *t+1*, let F_2^t equal the second nearby futures price at the current point in time *t*, and let F_2^{t+1} equal the second nearby futures price at a subsequent point in time *t+1*. Given these variables, the conventional formula for roll return is as follows:

Excess return = $(F_1^{t+1} – F_2^t) / F_2^t$

Spot return = $(F_1^{t+1} – F_1^t) / F_1^t$

Roll return = $((F_1^{t+1} – F_2^t) / F_2^t) – ((F_1^{t+1} – F_1^t) / F_1^t)$.

Example/Data Table A[56]			
Point in time	*t*	*t+1*	*t+2*
First nearby	F_1^t = \$100	F_1^{t+1} = \$120	N/A
Second nearby	F_2^t = \$90	F_2^{t+1} = \$100	N/A

Using the data from Table A, excess return equals (\$120 – \$90) / \$90 or 33%, spot return equals (\$120 – \$100) / \$100 or 20%, and the "arithmetically derived" roll return is equal to 33% – 20% = 13%. As can be inferred based on this example and our discussion of classic commodity theory, the roll return essentially propositions a speculative attempt at capturing convenience yield without paying for the carry associated with cash commodities. However, outright long (or short) futures positions are not without speculative risk, regardless of any supposed roll yield benefit. Accordingly, from this exposé we provide four observations on simplified commodity theory.

<u>First</u>, we must point out an issue with the above math in regards to the convention for calculating excess returns, and as a consequence the formula for roll return. While the equations above are computatively possible and calculate a yield that may approximate what a speculator might *hypothetically* capture when rolling the contract forward, it is *in reality* impossible to roll F_1^{t+1} into F_2^t, since the former is a price from point in time *t+1*, the latter is a price from point in time *t*, and more importantly they are two different/distinct contracts. Accordingly, the calculation for excess return is an *accounting fiction*, and it should be obvious that this mathematical "trick" mixes up past and present prices creating roll yield out of an imaginary transaction that is impossible to duplicate

[56] Source of roll yield formulae and example calculations from: Hilary Till (2007). "Part I of A Long-Term Perspective on Commodity Futures Returns: Review of the Historical Literature" *Intelligent Commodity Investing*, Till and Eagleeye, Ed., London: Risk Books, pp 73-78. Note that while the formulas and example data is exactly the same as used by Till (2007), variables are labeled using our conventions not those in Till's exposé.

in the *real world*. Further, the use of F_2^t as the denominator in the formula for excess return, $(F_1^{t+1} - F_2^t) / F_2^t$, is source for a performance skew over time. Specifically, using F_2^t (e.g., $90) as the denominator rather than F_1^t (e.g., $100) is akin to collateralizing the trade at 90 cents on the dollar rather than 100 cents on the dollar. And while leverage is possible in futures trading due to *de minimis* margin, in backwardated conditions the excess return formula precipitates levered results when using such equation. We therefore surmise that empirical studies which calculate the roll return using Shimko and Masters' (1994) convention or Till's (2007) variation reflect flawed conclusions.

What actually happens in reality is as follows. To begin, we shall introduce an additional variable F_2^{t+2}, where the second nearby is also priced at point in time $t+2$. In our revised *real world* example, a speculator goes long the contract at F_1^t and liquidates the contract at F_1^{t+1}, at which time he/she rolls into the second nearby futures contract at F_2^{t+1}, and then subsequently closes out the trade by liquidating the contract at F_2^{t+2}. Let's assume that F_2^{t+2} = $110 but that all other variables are the same as in Till's (2007) original example given above. For further extrapolation, we shall also introduce the variable $E(S_2)$ into the equation.

Example/Data Table B			
Point in time	t	$t+1$	$t+2$
First nearby	F_1^t = $100	F_1^{t+1} = $120	N/A
Second nearby	F_2^t = $90	F_2^{t+1} = $100	F_2^{t+2} = $110
Expected spot price	N/A	$E(S_2)^{t+1}$ = $102	N/A

The formula we propose, $[\{1 + ((F_1^{t+1} - F_1^t) / F_1^t)\} \cdot \{1 + ((F_2^{t+2} - F_2^{t+1}) / F_2^{t+1})\} - 1]$, calculates a profit of $30 and a compounded return of 32% to the speculator. It should be noted that as a result of this *real life* transaction the roll yield is not observable. For arguments sake, assuming that $F_2^{t+1} < E(S_2)$ at point in time $t+1$, one could hypothetically argue that a $2 roll yield is a component of $F_2^{t+2} - F_2^{t+1}$. This contention, that a roll yield benefit resulted from rolling the contract forward is rooted on the assumption that the F_2^{t+2} liquidation price of $110 includes a $2 excess premia attributable to difference between $E(S_2)$ and F_2 at point in time $t+1$. This presents a conundrum because the $10 difference between F_2^{t+2} and F_2^{t+1} is simultaneously attributable to a change in the spot price. Regardless, here again we must revisit classic commodity theory and introduce the variable $E(S_t)$ in order to determine a supposed roll yield.

Second, we posit that the roll yield hypothesis is related to calendar spread trading. Till (2007) in an afterthought suggests that "one cannot invest and receive the spot return separate from the roll return; and, correspondingly, one cannot invest solely to receive the roll return, but not the spot return."[57] Utilizing calendar spreads, however, one can simultaneously sell the nearby futures and buy the second nearby futures if the term structure is considered backwardated; or vice versa, buy the nearby futures and sell the second nearby futures if the term structure is considered contango. Hence, it is theoretically possible to develop a trading approach that isolates the term structure phenomena.[58] In fact, de Roon, van den Goorbergh and Nijman (2004) noted that "passive spreading strategies… capture the term premiums" and "spreading returns are predictable by net hedge demand observed in the past, which can be exploited by active trading, but only if transaction costs are relatively low."[59]

Third, widespread application of calendar spread trading and "next generation" long-short commodity index funds hypothetically alters the term structure.[60] In theory, a *flat* term structure of the future price curve represents equilibrium in simplified commodity theory, which is why such theory is imperfect. Based on commonly-held assumptions derived from this model, speculators should then theoretically sell premium (i.e., contango) futures contracts and purchase discount (i.e., backwardated) futures contracts. With respect to the structural risk premium that is supposedly derived from the roll yield, *real-life* active traders have already developed trading strategies with the idea of getting out in front of the 'flow-of-money' of publicly disclosed roll dates of passive commodity funds. Therefore, in actual practice the so-called roll yield is reduced if not eliminated by certain 'economic agents' seeking to take advantage of *dumb money* in reliance of the "greater fool theory". This explains why institutional sponsors of passive commodity index products have become concerned with term structure. Lewis (2007) writes that "the traditional approach to rolling commodity index futures on a predefined monthly schedule is in need of reform to address the implications that unstable term structures imply for the roll return within a commodity index."[61]

Fourth, it should be pointed out that calendar spreads are a quasi-arbitrage. Arbitrageurs engaged in 'cash and carry' and 'reverse cash and carry' strategies theoretically earn excess risk premia from convergence of the futures contract with spot price upon physical or cash settlement. In practice, many commodity futures contracts specify a physical delivery notice period prior to the contract's settlement date, whereby a holder of a long or short futures contract is exposed to delivery assignment of the underlying commodity (rather than cash settlement). For this reason, only well capitalized arbitrageurs or bona fide hedgers can actually enter into this period without

[57] Till (2007). "Part I of A Long-Term Perspective on Commodity Futures Returns" p 74 (Appendix A).

[58] In practice, this approach is limited given contract specifications and the risk of delivery during delivery periods.

[59] F. de Roon; R. van den Goorbergh; and T. Nijman (2004). "An Anatomy of Futures Returns: Risk Premiums and Trading Strategies" *WO Research Memoranda 747*, Netherlands Central Bank, Research Department, p 26.

[60] Anecdotally, introduction of long-only commodity index products has been responsible for reshaping the term structure of the futures price curve from predominantly backwardated to contango, as predicted by Kaldor (1939).

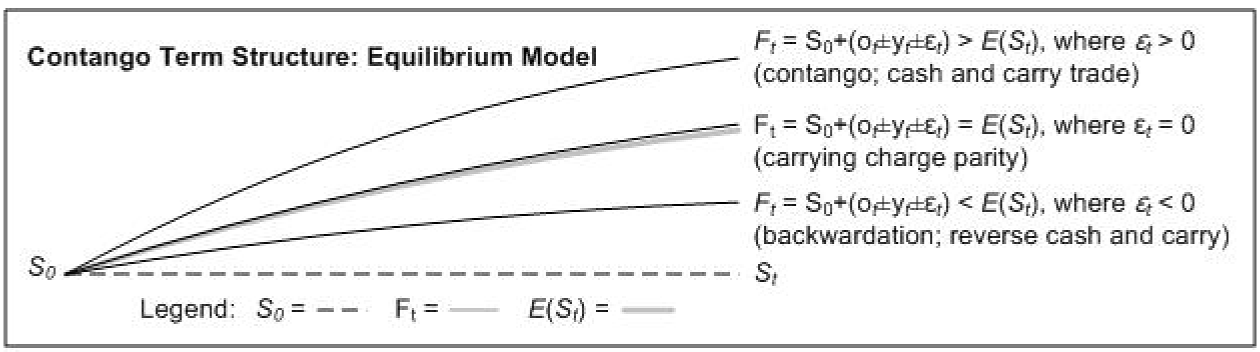
[61] Michael Lewis (2007). "Structural Shifts in Commodity-Index Investing" *Intelligent Commodity Investing*, p 213.

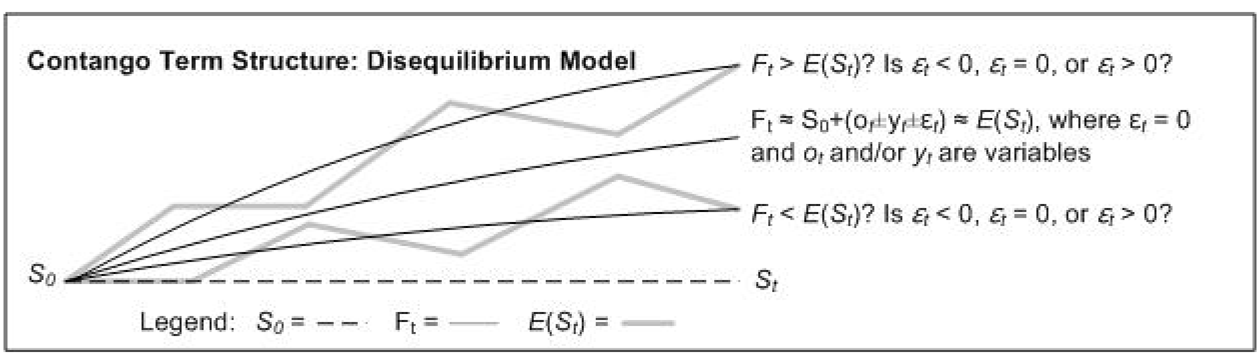
significant risk. Presumptively, this may be the exact period when Keynes' "signature of normal backwardation"[62] is most apparent. As previously posited, the roll yield is essentially a speculative play at trying to capture the convenience yield without paying for the outlay of storage costs. Correspondingly, the establishment of outright positions versus relative positions such as calendar spreads exposes a speculator to significant price risk (i.e., spot return). Even assuming the existence of a roll yield, such intangible return is only beneficial within the context of long-term bull market or flat market cycle assuming markets are backwardated during such period. As reiterated by multiple studies, roll yield as a structural risk premium is arguably only relevant over the very long-term.

<u>Roll Yield Permutations</u>

Our theory of "roll yield permutations" combines characteristics of both classic commodity theory and simplified commodity theory. We alluded to our underlying rationale when we discussed and illustrated how it was theoretically possible for the term structure of the futures price curve to be contango within the context of simplified commodity theory, while at the same time the expected future spot price may be backwardated within the context of classic commodity theory.

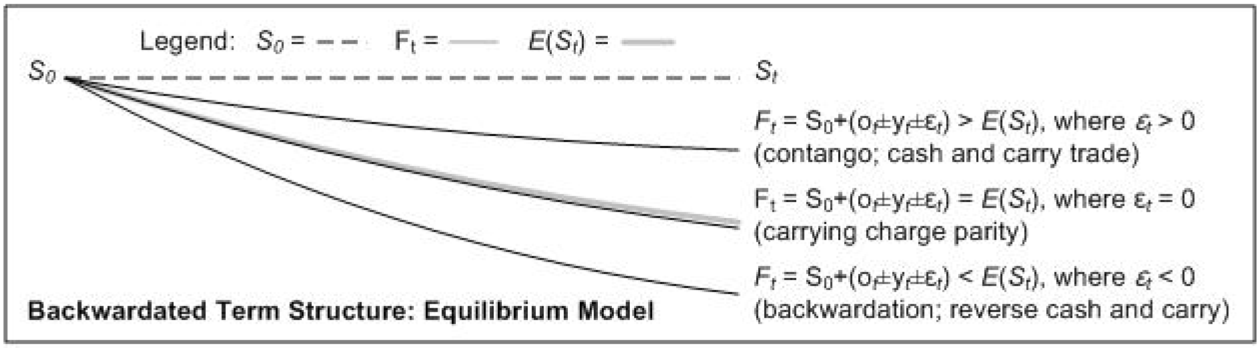
This first graphic was presented within the section on classic commodity theory, and stresses Working's (1948, 1949) observations about relationship between futures prices and storage costs.

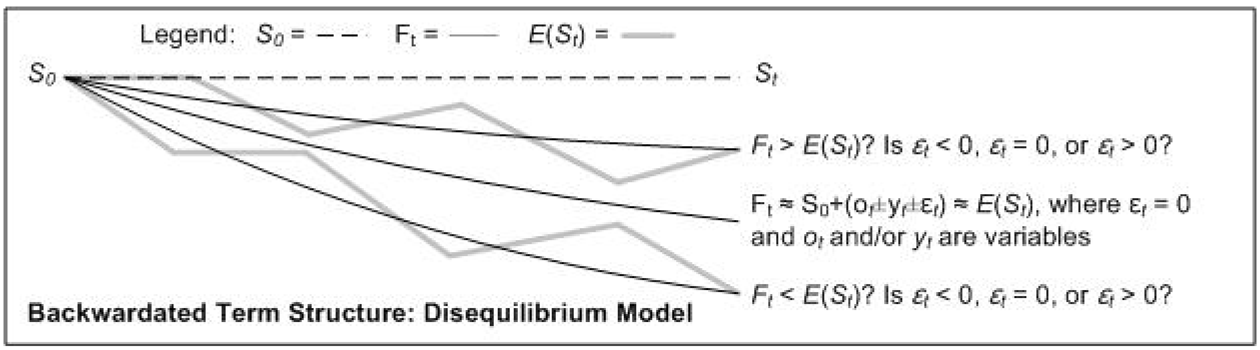




[62] Rubenstein (2006). *A History of the Theory of Investments*, p 54.

The second graphic on the following page, extends this model's qualities by illustrating how it functions when the term structure of the futures price curve reflects backwardation.





<u>Permutation Model</u>: Building upon the assumptions, models, formulae and variables presented in the prior sections, let us assume that there are three types of market conditions: (i) flat market where the spot and futures price remains static, (ii) bull market where the spot and futures price generally trends upwards, and (iii) bear market where the spot and futures price generally trend downwards. Again, let S_0 be the current spot price of the asset, and F_0 be the current futures price for an illiquid futures contract trading after first notice day within the delivery period, but prior to the last physical delivery date and contract expiration; let F_1 be the current futures price for the liquid nearby future delivery of the underlying asset, and F_2, F_3, F_4 be the current futures price for second nearby, third nearby and fourth nearby future delivery of the underlying asset. Additionally, let $E(S_1)$ be the expected future spot price for F_1 future delivery, $E(S_2)$ be the expected future spot price for F_2 future delivery, $E(S_3)$ be the expected future spot price for F_3 future delivery, and $E(S_4)$ be the expected future spot price for F_4 future delivery. Further, we shall assume that each of F_0, F_1, F_2, F_3, and F_4 futures contracts are physically settled in the current year December, and the following year March, June, September and December, respectively.

<u>Permutation Level 1</u>: Assuming one week prior to contract expiration, after first notice day within the delivery period, but prior to the last physical delivery date and contract expiration, there are three likely scenarios with respect to the relationship between S_0 and F_0. Either: $F_0 < S_0$, or $F_0 = S_0$, or $F_0 > S_0$, with convergence occurring at the time of physical settlement, such that $S_0 = F_0$ on the contract settlement date. We can also extend these three basic relationships to encompass bullish, flat and bearish market conditions. Hence, we note that there are three sets of these permutations for each type of assumed market scenarios.

This set of permutations relates to the small percentage of open interest futures positions that are settled through delivery, either by bona fide hedgers or by institutional arbitrageurs who still hold the contract into this period of the contract's life cycle. At this juncture, most speculators have rolled forward and are trading the next liquid futures contract month.

<u>Permutation Level 2</u>: The next set of permutations revolves around the relationship between S_0 and F_1 and the aforementioned market conditions. Assuming bullish market conditions, either: $F_1 < S_0$, or $F_1 = S_0$, or $F_1 > S_0$; assuming flat market conditions, either: $F_1 < S_0$, or $F_1 = S_0$, or $F_1 > S_0$; and assuming bearish market conditions, either: $F_1 < S_0$, or $F_1 = S_0$, or $F_1 > S_0$. However, for each of these permutations, given the potential for either backwardation, contango or equilibrium market conditions as implied by classic commodity theory, it is necessary to relate the F_1 future delivery to its corresponding $E(S_1)$. As a result, we can extend the model to include nine possible permutations for each type of market scenario. Assuming the value of $E(S_1)$ is known, either:

$F_1 < S_0$, where $F_1 < E(S_1)$
$F_1 < S_0$, where $F_1 = E(S_1)$
$F_1 < S_0$, where $F_1 > E(S_1)$
$F_1 = S_0$, where $F_1 < E(S_1)$
$F_1 = S_0$, where $F_1 = E(S_1)$
$F_1 = S_0$, where $F_1 > E(S_1)$
$F_1 > S_0$, where $F_1 < E(S_1)$
$F_1 > S_0$, where $F_1 = E(S_1)$
$F_1 > S_0$, where $F_1 > E(S_1)$

As noted these nine permutations can be applied to bullish, flat and bearish market conditions. The significance of the underlying market conditions relates to the benefit or detriment the roll yield theoretically provides under either bullish, flat or bearish market scenarios. For example, assuming an established long position, each roll into a forward contract month potentially results in one of the following hypothetical outcomes: (1) roll during a bull market when $F_1 < E(S_1)$ results in a yield benefit and a positive price change; (2) roll during a flat market when $F_1 < E(S_1)$ results in a yield benefit yet no change in price; (3) roll during a bear market when $F_1 < E(S_1)$ results in a yield benefit but a negative price change; (4) roll during a bull market when $F_1 = E(S_1)$ results in no yield benefit yet a positive price change; (5) roll during a flat market when $F_1 = E(S_1)$ results in no yield benefit and no change in price; (6) roll during a bear market when $F_1 = E(S_1)$ results in no yield benefit but negative price change; (7) roll during a bull market when $F_1 > E(S_1)$ results in a yield detriment yet positive price change; (8) roll during a flat market when $F_1 > E(S_1)$ results in a yield detriment but no change in price; (9) roll during a bear market when $F_1 > E(S_1)$ results in a yield detriment and a negative price change. Likewise, assuming an established short position, opposite outcomes of these nine theoretical scenarios could result.

Additionally, markets do not go straight up in bull markets or straight down in bear markets, and flat markets could first go up then down, down then up, sideways, or any combination thereof. Therefore, assuming that each of F_0, F_1, F_2, F_3, and F_4 futures contracts are physically settled in the current year December, and following year March, June, September and December, respectively; and further, we assume four rolls are transacted into the following year: December to March, March to June, June to September, and September to December; then during each quarterly roll (assuming just bull and bear market conditions, not flat market conditions) twelve separate market cycles could theoretically occur: (i) bull, bull, bull, bull; (ii) bull, bear, bull, bull; (iii) bull, bull, bear, bull; (iv) bull, bear, bear, bull; (v) bull, bull, bear, bear; (vi) bull, bear, bull, bear; (vii) bear, bear, bear, bear; (viii) bear, bull, bear, bear; (ix) bear, bear, bull, bear; (x) bear, bull, bull, bear; (xi) bear, bear, bull, bull; and (xii) bear, bull, bear, bull. Hence, in order to appreciate the complexity of possible permutations, one should overlay quarterly beneficial or detrimental roll yield scenarios from the prior paragraph with the various market cycles described in this paragraph.

It should be noted that these permutations can be extended to include the relationships between F_0 and F_2, F_0 and F_3, or F_0 and F_4, etc. resulting in additional permutations assuming the following four roll scenarios in any one year: December to March, December to June, December to September, and December to December. We also note that certain of these permutations may be logistically difficult for the typical real-life speculators to execute since such permutations may potentially involve spot market transactions (i.e., S_0) and require actual outlay for storage costs.

Permutation Level 3: The next set of permutations builds upon ideas presented in Permutation Level 1 and 2, but extends established concepts around the relationship between F_1 and F_2 (and by extrapolation, the relationships between F_1 and F_2, F_3, F_4; and F_2 and F_3, F_4; and F_3 and F_4). Assuming bullish market conditions, then either: $F_1 > F_2$, or $F_1 = F_2$, or $F_1 < F_2$; assuming flat market conditions, then either: $F_1 > F_2$, or $F_1 = F_2$, or $F_1 < F_2$; and assuming bearish market conditions, then either: $F_1 > F_2$, or $F_1 = F_2$, or $F_1 < F_2$. For each of these permutations, given the potential for either backwardation, contango or equilibrium markets as implied by classic commodity theory, in addition to relating F_1 contract delivery to its corresponding $E(S_1)$, we also relate F_2 contract delivery to its corresponding $E(S_2)$. As a result, the model is extrapolated to include twenty-seven possible permutations. Hence, for each type of market scenario, assuming we know the value of $E(S_1)$ as well as $E(S_2)$, then either:

$F_1 > F_2$, where $F_1 > E(S_1)$ and $F_2 > E(S_2)$
$F_1 > F_2$, where $F_1 = E(S_1)$ and $F_2 > E(S_2)$
$F_1 > F_2$, where $F_1 < E(S_1)$ and $F_2 > E(S_2)$
$F_1 > F_2$, where $F_1 > E(S_1)$ and $F_2 = E(S_2)$
$F_1 > F_2$, where $F_1 = E(S_1)$ and $F_2 = E(S_2)$
$F_1 > F_2$, where $F_1 < E(S_1)$ and $F_2 = E(S_2)$
$F_1 > F_2$, where $F_1 > E(S_1)$ and $F_2 < E(S_2)$
$F_1 > F_2$, where $F_1 = E(S_1)$ and $F_2 < E(S_2)$
$F_1 > F_2$, where $F_1 < E(S_1)$ and $F_2 < E(S_2)$

$F_1 = F_2$, where $F_1 > E(S_1)$ and $F_2 > E(S_2)$
$F_1 = F_2$, where $F_1 = E(S_1)$ and $F_2 > E(S_2)$
$F_1 = F_2$, where $F_1 < E(S_1)$ and $F_2 > E(S_2)$
$F_1 = F_2$, where $F_1 > E(S_1)$ and $F_2 = E(S_2)$
$F_1 = F_2$, where $F_1 = E(S_1)$ and $F_2 = E(S_2)$
$F_1 = F_2$, where $F_1 < E(S_1)$ and $F_2 = E(S_2)$
$F_1 = F_2$, where $F_1 > E(S_1)$ and $F_2 < E(S_2)$
$F_1 = F_2$, where $F_1 = E(S_1)$ and $F_2 < E(S_2)$
$F_1 = F_2$, where $F_1 < E(S_1)$ and $F_2 < E(S_2)$

$F_1 < F_2$, where $F_1 > E(S_1)$ and $F_2 > E(S_2)$
$F_1 < F_2$, where $F_1 = E(S_1)$ and $F_2 > E(S_2)$
$F_1 < F_2$, where $F_1 < E(S_1)$ and $F_2 > E(S_2)$
$F_1 < F_2$, where $F_1 > E(S_1)$ and $F_2 = E(S_2)$
$F_1 < F_2$, where $F_1 = E(S_1)$ and $F_2 = E(S_2)$
$F_1 < F_2$, where $F_1 < E(S_1)$ and $F_2 = E(S_2)$
$F_1 < F_2$, where $F_1 > E(S_1)$ and $F_2 < E(S_2)$
$F_1 < F_2$, where $F_1 = E(S_1)$ and $F_2 < E(S_2)$
$F_1 < F_2$, where $F_1 < E(S_1)$ and $F_2 < E(S_2)$

As can be inferred by the variety of roll yield permutations shown, the number of potential manifestations can be countless when one also considers variations as a result of timing rolls.

Economic modeling is premised on the idea of distilling complex situations into axioms that can be robustly applied to the intricacies of the real world. However, the purpose of our roll yield permutations model is based on the opposite reasoning: take conventional precepts about the commodities futures market and reveal how truly complex the trade decision-making process is. We do not portend that these permutations illustrate "natural" economic constraints or elucidate the underlying dynamics of the arbitrageur, hedger, speculator paradigm in the legacy of Keynes, Hicks, Kaldor, Working, Brennan, Cootner, etc. Rather, these permutations demonstrate the variety of scenarios that occur when one clarifies simplified commodity theory by incorporating the pivotal relationship between F_t and its corresponding $E(S_t)$ as described by classic commodity theory. Nevertheless, we admit that if F_2 is backwardated relative to F_1 [i.e., $F_1 > F_2$], then it is a likely indication that $E(S_2)$ is also backwardated relative to $E(S_1)$ [i.e., $E(S_1) > E(S_2)$]; however, this may not always be the case and researchers and practitioners should not presume as much.

<u>Permutation Hypothesis</u>: Our hypothesis for classic commodity theory propositions that the model contains circular logic, and as a consequence is subject to reflexivity. If such model is in fact a reflexive, we propose that it reflects disequilibrium as well as equilibrium. We now extend this hypothesis to suggest that the term structure of the futures price curve, while suggestive of a potential roll yield benefit or detriment, in fact implies a complex and reflexive series of roll yield permutations which amplifies into an exponential number of potential market scenarios. Further, models as reflections of reality do not operate at the exclusion of the other; rather, these models are inter-related and each reflect certain aspects and dynamics within the entire futures market paradigm. Consequently, such market dynamics reverberate between models, as well as feedback within models resulting in constant disequilibrium/equilibrium tension. Hence, we posit that these models in fact support a Post-Keynesian view that the world is messy and uncertain.

Conclusion: Flawed Assumptions Induce Returns

In comparing modern finance with behavioral finance, Frankfurter and McGoun (2002) in their article "Resistance is Futile: The Assimilation of Behavioral Finance", make the following astute observation which can similarly be applied to our analysis of the various commodity asset pricing models we investigated: "What has happened is that we've used these assumptions for so long that we've forgotten that we've merely made assumptions, and we've come to believe that the world is necessarily this way."[63] Likewise, our investigation suggests that the pricing theories we examined have inherent shortcomings when analyzing the commodity futures markets. As a result, such models, which are conventionally regarded as validation for persistent and replicable sources of return in the commodity futures markets, may be widely misunderstood.

We note that the roll yield is derived from a simplified definition of backwardation and contango based on what Till (2007) describes as the "term structure of the futures price curve". Because this assumption "is in line with current market convention",[64] backwardation is now commonly defined as conditions when *the futures price is below the current spot price* and contango as conditions when *the futures price is above the current spot price*. However, this paradigm is not in line with the original ideas of normal backwardation as described by Keynes (1923, 1930), as well as related phenomena identified by Hicks (1939, 1946), Kaldor (1939), Working (1948, 1949) and Brennan (1958). In classic literature, concepts of backwardation and contango correlate the futures price to the expected future spot price, which is an unobservable variable, to be discovered in the future upon settlement when the futures contract converges with the cash market. This difference in assumptions is not insignificant. The 'source of return' conundrum is that for every buyer of a futures contract there is a seller—*sine qua non*, there is no intrinsic value in forward contracts—they are simply agreements which commit delivery of an asset at some place/point in time.

For that reason, we surmise that roll yield is a flawed concept in contrast with Erb and Harvey (2006) and others who "find that the term structure of futures prices is an important determinant of the cross-section of commodity futures returns."[65] To begin with, the "convention" in calculating the roll yield as described by Shimko and Masters (1994) and Till (2007) mixes past and present prices creating an imaginary transaction that is impossible to duplicate in the *real world*. Even Till notes "that the convention of separating out futures-only returns into spot return and roll return is solely for performance-attribution purposes."[66] Second, in addition to the formulae for excess return/roll return

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[63] George M. Frankfurter and Elton G. McGoun (2002). "Resistance is Futile: The Assimilation of Behavioral Finance" Bucknell University, Department of Management, p 13.

[64] "When a commodity's immediately deliverable futures contract is at a premium to the next deferred delivery contract, we will refer to the commodity as being *in backwardation*. When the nearby contract is at a discount to the second nearby contract, we will refer to the commodity as being *in contango*." Source: Till (2007). "Part I of A Long-Term Perspective on Commodity Futures Returns" p 42.

[65] Claude B. Erb and Campbell R. Harvey (2005/2006). "The Tactical and Strategic Value of Commodity Futures" *Financial Analysts Journal*, 62(2), pp 69-97; Unabridged Version: January 12, 2006, p 36.

[66] Till (2007). "Part I of A Long-Term Perspective on Commodity Futures Returns" p 74 (Appendix A).

being an accounting fiction, the equation's denominator theoretically results in levered or delevered performance attributions. In the case of markets that exhibit backwardation (as defined by "current market convention"), excess returns shall skew positively on a levered basis; and in markets that exhibit contango (as defined by "current market convention"), excess returns shall skew negatively on a delevered basis. Third, the contention that a roll yield benefit or detriment is created from rolling contracts is rooted in the assumption that the change in spot price includes an excess premia attributable to the expected futures spot price, which as pointed out is an unobservable variable.[67]

Further, the roll yield hypothesis implies a trade that can be executed vis-à-vis calendar spreads. As previously discussed, de Roon, van den Goorbergh and Nijman (2004) noted that "passive spreading strategies… capture the term premiums" and "spreading returns are predictable by net hedge demand observed in the past, which can be exploited by active trading, but only if transaction costs are relatively low."[68] Accordingly, a flat term structure theoretically represents equilibrium. Based on assumptions derived from simplified commodity theory, speculators should buy futures when the markets exhibit backwardation (as defined by "current market convention"), and sell futures when the markets exhibit contango (as defined by "current market convention"). By inference, the roll yield is essentially a speculative attempt at trying to capture the convenience yield in backwardated markets [or "inconvenience yield" in contango markets] without assuming the risks associated with the cost-of-carry. Nevertheless, outright long [or short] futures positions explicitly exposes speculators to significant price risk (i.e., spot return). Alternatively, if excess risk premia is available as a result of convenience [or inconvenience] yield, then such premia is implicitly captured by arbitrageurs who engage in 'cash and carry' and 'reverse cash and carry' strategies. Hence, even assuming the existence of a roll yield, such supposed return to an investor is both unobservable and therefore unquantifiable. This finding is supported by our theory of roll yield permutations.

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[67] To buttress our argument, we present a comparison to the well-vetted Black-Scholes options pricing model. One way to capture risk premium is by writing naked options. In such a strategy the option's strike price serves as an explicit control condition, and time decay (i.e., theta) erodes the value of the option over time resulting in a return that can be quantitatively measured. The roll yield hypothesis also implies a construct similar to the Black-Scholes options pricing model. The risk premium, in the case of roll returns, is assumed to encompass convenience [or inconvenience] yield, which is earned as the contract approaches physical or cash settlement. The material difference between models, however, is that the expected future spot price, which we note is analogous to an option strike price, is not a explicit control condition. Consequently, the convention for calculating roll yield is a formulae not without inherent issues due to the fact that simplified commodity theory's "strike price," the expected futures spot price, is in actuality a quixotic variable—that is, an unobservable control condition until conversion into spot price.

[68] de Roon; van den Goorbergh; and Nijman (2004). "An Anatomy of Futures Returns: Risk Premiums and Trading Strategies" p 26.

In summary, the legacy of research is inconclusive with respect to modeling the sources of returns in the commodity futures markets largely because these models have inherent shortcomings in being able to pinpoint a definitive source of structural risk premium within the complexity of such markets. We conclude that classic commodity theory encompasses reflexivity, and as a consequence, has the potential to skew variable(s) in either a directional bias resulting in price exponentiation, or induce variable(s) to converge toward price expectations equilibrium. Hence, classic commodity theory is based on asset pricing models which reflect disequilibrium as well as equilibrium. We extend this disequilibrium/equilibrium hypothesis and argue that the term structure of the futures price curve, while suggestive of a potential roll yield benefit or detriment, in fact implies a complex and reflexive series of roll yield permutations.[69] Further, models as reflections of reality do not operate at the exclusion of the other; rather, these models are inter-related and each reflect certain aspects and dynamics within the entire futures market paradigm. Consequently, we assume such market dynamics reverberate between models, as well as feedback within models resulting in a constant disequilibrium/equilibrium tension.

Thoughts on Rational Expectations

Admittedly, models are only an abstraction from reality. Expecting these models to be exactly right is unreasonable, and it is generally understood that neoclassical economic models have inherent limitations related to the analysis of markets within the context of rational expectations equilibrium. Such systems are based on 'perfect competition', assume that the economy is stable, and that markets naturally return to equilibrium after a disturbance. Hence, such models maximize utility and/or profits in a world of constraints based on the choices of "rational" economic agents. By definition then, these models relegate speculators to the role of that very economic agent which maintains equilibrium. Yet a survey of *real-life* traders reveals that many of these practitioners do not as a general rule use academic models in their day-to-day speculative trading decisions.[70] Paradoxically, this same group plays a key influence upon the

[69] Similarly, Spurgin's (2000) 'hedging response functions', as described in his article "Some Thoughts on the Source of Return to Managed Futures", models behavioral risk averse mechanisms, corroborating social reflexivity.

[70] See: Lawrence Harris (1993). "The Winners and Losers of the Zero-Sum Game: The Origins of Trading Profits, Price Efficiency and Market Liquidity" University of Southern California, May 7, 1993. In addition, we note it is dangerous to extrapolate past performance into multi-factor regression analysis as a means to predict future outcomes. It is a well understood tenet of systematic traders that expanding the number of factors, conditions and variables increases the likelihood of curve-fitting to historical data, also referred to as 'over-optimization'. Note: one exception to this assertion is the Black-Scholes option pricing model, which is widely used by the industry and practitioners.

selfsame price data from which theories/models are constructed. So if past data is assumed to represent equilibrium and "the future is merely the statistical reflection of the past,"[71] then one could inversely argue that perfect competition and rational expectations minimizes these models' usefulness as a mechanism from which to make speculative decisions. In other words, rational expectations compel such models to simply validate that current market prices are equal to equilibrium; unless the opposite is true—that markets are in fact imperfect and rational expectations is untenable, which in turn undermines the veracity of these models.

The comeuppance is that formulaic anomalies are often rationalized, as for example with Shimko and Masters (1994), who explain that "the convention in calculating excess returns is to treat the futures investment as being fully collateralized based on the second nearby price."[72] We, on the other hand concur with Lavoie (1992) who warns that, "Axioms are chosen, not for their likelihood, but for their ability to allow the existence of equilibrium or its uniqueness. Neo-Walrasians describe the world as it should be rather than as it is."[73] Stated otherwise, in our collective pursuit to understand the *real world*, we must not fall into the trap of accepting theories and models because it is mathematically possible. Not questioning a models' logic and the formulae upon which widely-held conclusions are perpetuated, simply because it may lead to the whole system crumbling, is intellectually dishonest and not without consequence. Not surprisingly, this conundrum is becoming increasingly recognized by economists, policy-makers and traders given the current financial crisis—see for example, FSA's "The Turner Review".[74]

"Keynes believed that fundamental uncertainty is a crucial element in any economic processes. And that under most circumstances, even if probabilities could be estimated, they [are]

[71] Davidson (1991). "Is Probability Theory Relevant for Uncertainty? A Post Keynesian Perspective" p 132.

[72] Till (2007). "Part I of A Long-Term Perspective on Commodity Futures Returns" p 77.

[73] Marc Lavoie (1992). "Towards a New Research Programme for Post-Keynesianism and Neo-Ricardianism" *Review of Political Economy*, Vol. 4, Issue 1, 1992, p 42.

[74] The Turner Review (March 2009) begins by stating: "Over the last 18 months, and with increasing intensity over the last six, the world's financial system has gone through its greatest crisis for at least half a century, indeed arguably the greatest crisis in the history of finance capitalism." (p. 5). Of the two explanations it advances one includes reliance on "the theory of efficient and rational markets" (p. 39). In this regard, the report notes, "the predominant assumption behind financial market regulation – in the US, the UK and increasingly across the world – has been that financial markets are capable of being both efficient and rational…" (p. 39). "In the face of the worst financial crisis for a century, however, the assumptions of efficient market theory have been subject to increasingly effective criticism… These criticisms include that: Market efficiency does not imply market rationality… Individual rationality does not ensure collective rationality… Individual behaviour is not entirely rational… Allocative efficiency benefits have limits… Empirical evidence illustrates large scale herd effects and market overshoots…" (pp. 40-41). [Source: Financial Services Authority, "The Turner Review: a regulatory response to the global banking crisis", March 2009. http://www.fsa.gov.uk/Pages/Library/Corporate/turner/index.shtml]

meaningless for long period decision making. The nature and power of market forces cannot deal with the unpredictability of the long run, and relying on them to do so will lead to incomplete information."[75] Ric Holt's Post-Keynesian articulation of Keynes's ideas on uncertainty and predictability underscores our thesis. From our perspective as practitioners, it seems obvious that the truth is somewhere in the middle rather than at the ideological extremes of neoclassical ergodic systems. We are not saying, however, that commodity pricing theory is erroneous. Models are not exclusive and each reveals intrinsic *qualities* within the "aggregate wealth portfolio of all agents in the [global] economy."[76] Nevertheless, unto themselves, they do not provide that one universal asset pricing solution which encompasses all cross-sectional variations. *That model* is akin to seeking the holy-grail. What these models do convey is an insightful understanding, provided one accepts that *in the real world* economic agents can act irrationally, that assumptions are by their very nature flawed, and that markets drift from disequilibrium to equilibrium and back in a never ending cycle. Hence, we posit that commodity pricing models in fact support a Post-Keynesian view that the world is messy and uncertain. In the words of Keynes (1937), "But at any given time facts and expectations were assumed to be given in a definite and calculable form… Actually, however, we have, as a rule, only the vaguest idea of any but the most direct consequences of our acts… The hypothesis of a calculable future leads to a wrong interpretation of the principles of behavior which the need for action compels us to adopt, and to an underestimation of the concealed factors of utter doubt, precariousness, hope and fear."[77]

[75] Ric Holt, "What is Post Keynesian Economics?" http://cc.shu.edu.tw/~tsungwu/holt.htm

[76] Jagannathan and McGrattan (1995). "The CAPM Debate" p 8.

[77] Keynes (1937). "General Theory of Employment," p. 112, 113, 122

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