Trading Strategy With Local Volatility Model

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Abstract: As the local volatility plays an important part in volatility modeling, we have built up two volatility trading strategies based on the local volatility model derived from Dupire. First strategy is based on the smoothed local volatility surface and the second strategy is based on the local and implied volatility. After the backtesting with options of S&P 500 Index, we prove that the strategy based on the smoothed local volatility surface is profitable with the best trading signals we got from the backtest.
Keywords: Local volatility; Surface; Trading strategy.

1 Introduction

As the financial market develops, the volatility plays an important role for expecting the price in the option market. So, to evaluate the volatility is crucial to meaningful financial decision-making. We have learned the Black-Scholes model (Black and Scholes (1973)) to price options. By inverting the Black-Scholes function, we can get the implied Black-Scholes volatility. And if the model is perfect, the implied volatility would be the same for option market prices. But the reality shows this is not the case. It strongly depends on the maturity and the strike of the European option. To solve this paradox, the study of implied volatility has become a central preoccupation for both academics and practitioners. The local volatility model is one of popular models to calculate the volatility. The advantage of pricing the option with this model is that it is fast and exact calibration to the volatility surface,and it is also suitable for products which only depend on terminal distributions of the underlying.

In this paper, we are aiming to compare the local volatility model and the implied volatility model using the real data from the market. Based on the results, we smoothed the local volatility surface and the implied volatility surface. Then we designed two volatility trading strategies which have realistic meaning.

2 Literature Review

Option pricing is a developing process. One of the most popular assumptions is geometric Brownian motion for the underlying asset price which is firstly raised by Osborne (1959). The famous Black–Scholes model derived a partial differential equation which is called Black-Scholes formula now which was first published by Black and Scholes (1973). The volatility is the only parameter in the Black-Scholes formula which is not observed directly in the market. It assumes the volatility is constant which can lead to a significant error. To avoid the lack of the implied volatility, there are two main assumption of the volatility. One is stochastic volatility model which is the variance of a stochastic process is itself randomly distributed, such as Hull and White (1987), Stein and Stein (1991), Heston (1993) and so on; another is local volatility model such as Dupire et al. (1994), Derman and Kani (1994), Dumas et al. (1998) and so on. Gatheral (2011) published a book which introduced the volatility systematically.

The stochastic volatility approach taken by Heston (1993) involved defining a stochastic process for the instantaneous volatility with a number of free parameters. These parameters can be chosen so that the model can fit the vanilla option market price.

The original model of Dupire and Derman and Kani assumes that the asset value undergoes a random walk with the returns being normally distributed but where the instantaneous local volatility is a deterministic function of asset value and time $\sigma = \sigma(S_t, t)$. Dupire shows that, under some regularity assumptions, this local volatility function can be uniquely determined if the prices of European options of all strikes and maturities are available.

3 Methodology

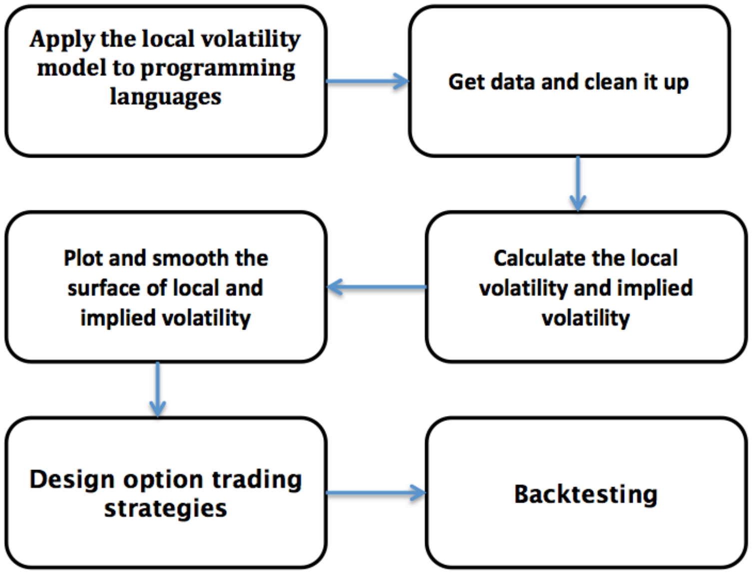

Figure 1: The flow chart

Firgure 1 presents that how we conduct our project. Firstly, we understand the local volatility model. Then we use R and Matlab to construct the model. After that, we get data and clean it up. And then we calculate the local volatility and implied volatility. In the end by constructing the smoothed surface, we design opiton trading strategies and do the backtest.

3.1 Model

3.1.1 The Black-Scholes implied volatility model

The Black-Scholes formula model is derived from a geometric Brownian motion:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

S_t is the asset value, μ is the risk-neutral drift and dW_t is a Wiener process with a mean of zero and variance of dt.

The Black-Scholes can be used to calculate the call and put option price. The call option for a non-dividend-paying underlying stock in terms of the Black–Scholes parameters is:

$$C(S,T) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}[ln(\frac{S}{K}) + \frac{(r+\sigma^2)}{2}(T-t)]$$

$$d_2 = \frac{1}{\sigma\sqrt{T-t}}[ln(\frac{S}{K}) + \frac{(r-\sigma^2)}{2}(T-t)] = d_1 - \sigma\sqrt{T-t}$$

For put option is:

$$P(S,T) = Ke^{-r(T-t)} - S + C(S,t) = N(-d_2)Ke^{-r(T-t)} - N(-d_1)S$$

T-t is the time to maturity. S is the asset value. N is the cumulative distribution function of the standard normal distribution. K is the strike price. r is the risk free rate. σ is the volatility. Let us assume that:

$$f(\sigma) \equiv= N(d_1)S - N(d_2)Ke^{-r(T-t)} - C_{real}$$

C_{real} is the option price in the market. We should find the $\sigma = \sigma_{implied}$ let the $f(\sigma) = 0$. There are two popular method to calculate the σ. One is Newton-Raphson method. Another is bisection method. We use bisection method. We can calculate the σ_i using σ_{i-1} as below:

$$\sigma_i = \begin{cases} \sigma_{i-1} + \frac{\sigma_0}{2^i} & if \quad f(\sigma_{i-1}) > 0 \\ \sigma_{i-1} + \frac{\sigma_0}{2^i} & if \quad f(\sigma_{i-1}) < 0 \end{cases}$$

The estimates of the $\sigma_{implied}$ are getting closer and closer when the number of steps increase.

3.1.2 The local volatility model

The Dupire's equation is:

$$\sigma^2(K,T) = \frac{2(\frac{\partial C}{\partial T} - (r_T - q_T)(C - K\frac{\partial C}{\partial K}))}{K^2\frac{\partial^2 C}{\partial K^2}}$$

Where C is short for C(S_0,K,T), r_t is the risk-free rate, D_t is the dividend yield. If the risk-free rate and the dividend yield equal to zero, the volatility equation will be:

$$\sigma^2(K,T) = \frac{2\frac{\partial C}{\partial T}}{K^2\frac{\partial^2 C}{\partial K^2}} \tag{1}$$

The local volatility in terms of implied volatility equation is:

$$v_l = \frac{\frac{\partial \omega}{\partial T}}{1 - \frac{y}{w}\frac{\partial w}{\partial T} + \frac{1}{2}\frac{\partial^2 w}{\partial y^2} + \frac{1}{4}(-\frac{1}{4} - \frac{1}{w} + \frac{y^2}{w})(\frac{\partial w}{\partial y})^2} \tag{2}$$

where $w = \sum(K,T)^2 T$ is the Black-Scholes total implied variance. $y = ln\frac{K}{T}$ where $F_T = e^{\int_0^T u_t dt}$ is the forward price with $u_t = r_t - q_t$.

3.2 Surface

Using R and Bloomberg, we collect historical S&P 500 Index call and put options with 69 trading days and choose the at the money strikes from $0.8S_0$ to $1.2S_0$. S_0 is current stock price.

Firstly, we use one trading day call option data to calculate the value of the local volatility and the local volatility in terms of implied volatility. Then we plot the surface as below.

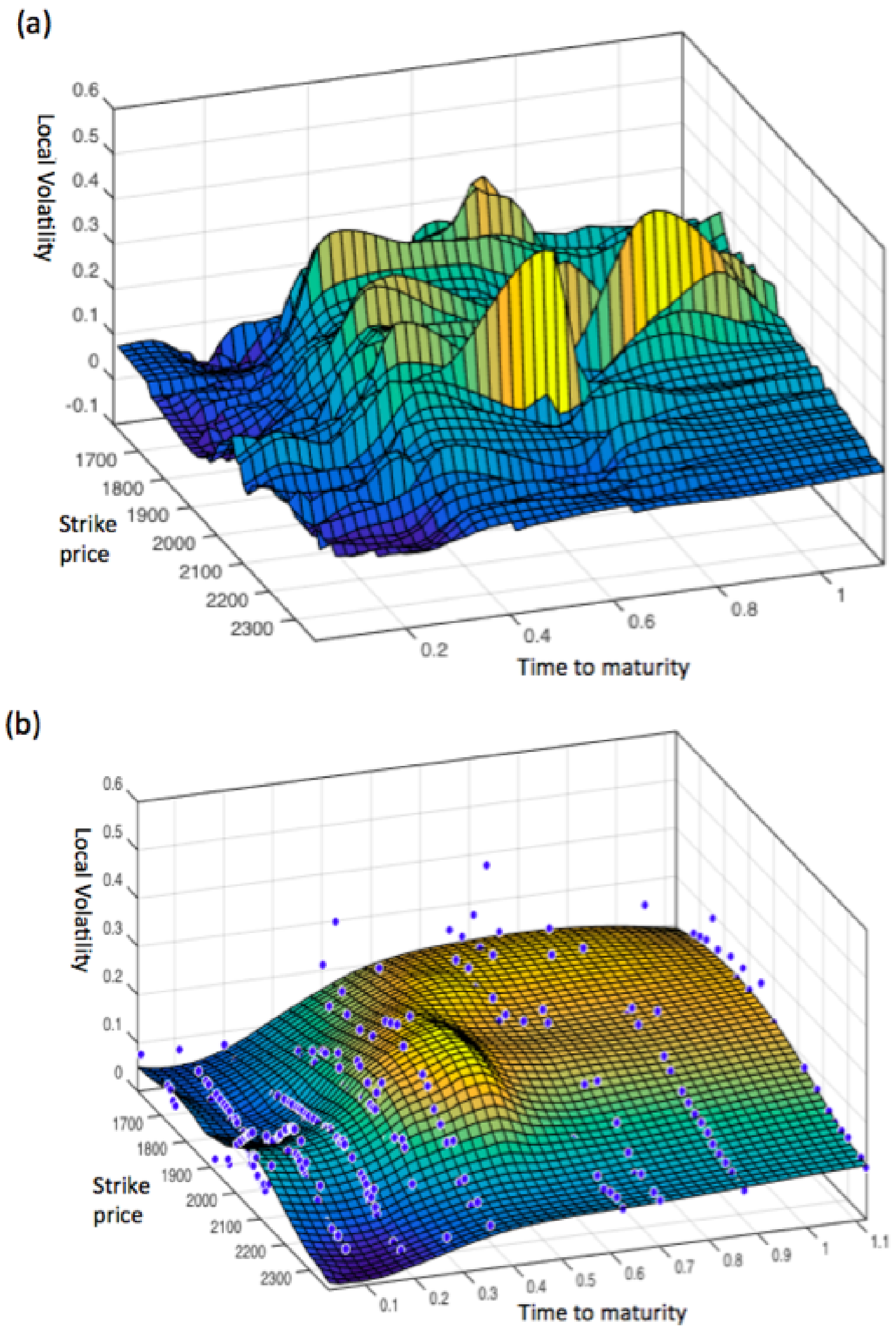

Figure 2: The local volatility surface

Figure 2 (a) presents the volatility surface before smoothing. Its R-square is 1. We choose the Lowess method to smooth the surface. We can get the result in Figure 2 (b). Its R-square is 0.7364. Lowess is derived from the term "locally weighted scatter plot smooth". It is a method which uses locally weighted linear regression to smooth data.

3.3 Strategy

We now have come up with two strategies both based on the local volatility. In both of our strategies, we use the bid and ask prices separately to calculate the local volatility and the implied volatility. It is because that we use only the bid price volatility to make decision of shorting options and use only the ask price volatility to make decision of longing options.

3.3.1 Strategy 1

Strategy 1 is based on the smoothed local volatility surface (Figure 2 (b)):

If the real local volatility is above the smoothed local volatility surface and reaches the open position signal, we will short those options.

If the real local volatility is below the smoothed local volatility surface and reaches the open position signal, we will short those options.

If the option bid (ask) prices is higher (lower) than the ask (bid) option price + (-) closing signals, we will close those position and calculate the real profit&loss.

If the unrealized P&L is smaller than M, we will close all the positions and then exit the strategy. M is decided by the level of risk the investor is willing to take.

The basic idea of strategy 1 is that the smoothed surface of local volatility is the local volatility that we believe it should be. So we believe that the local volatility should tend to come closer to the smoothed surface and then we can make profit from the predicted volatility changes.

The signals are chosen by the backtesting with the best P&L. It is obviously that the goodness-of-fit will significantly affect the result. So we test surfaces with different goodness-of-fit, and then pick out the surface with the best total P&L.

In strategy 1 we test both the local volatility with Dupire's equation (1) and the local volatility in terms of the implied volatility with Dupire's equation (2). In the code we reconstruct the local volatility equation (1) into:

$$\sigma_{local} = \frac{\frac{C_{close,j} - C_{i,j}}{\Delta_T}}{\frac{1}{2}K^2 \frac{\frac{C_{i,j+1} - C_{i,j}}{\Delta_{K_1}} - \frac{C_{i,j} - C_{i,j-1}}{\Delta_{K_2}}}{\frac{\Delta_{K_1} + \Delta_{K_2}}{2}}} \tag{3}$$

Where $C_{close,j}$ is the closest option price with the same strike price, $C_{i,j+1}$ and $C_{i,j-1}$ are the two closest option price with the same maturity, Δ_T and Δ_K are interval between each Maturity and Strike. And we also reconstruct the local volatility in terms of the implied volatility which is shown below:

$$a = \sigma_{imp}^2 + 2\sigma_{imp}K(\frac{\sigma_{imp_{close,j}} - \sigma_{imp_{i,j}}}{\Delta_T} + (r_T - q_T)K\frac{\sigma_{imp_{i,close}} - \sigma_{imp_{i,j}}}{\Delta_K})$$

$$b = (1 + \frac{Kln(\frac{K}{S_0})}{\sigma_{imp}} \frac{\sigma_{imp_{i,close}} - \sigma_{imp_{i,j}}}{\Delta_K})^2$$

$$c = KT\sigma_{imp}[\frac{\sigma_{imp_{i,close}} - \sigma_{imp_{i,j}}}{\Delta_K} - \frac{1}{4}KT\sigma_{imp}(\frac{\sigma_{imp_{i,close}} - \sigma_{imp_{i,j}}}{\Delta_K})^2 + \frac{\frac{\sigma_{imp_{i,j+1}} - \sigma_{imp_{i,j}}}{\Delta_{K_1}} - \frac{\sigma_{imp_{i,j}} - \sigma_{imp_{i,j-1}}}{\Delta_{K_2}}}{\frac{\Delta_{K_1} + \Delta_{K_2}}{2}}]$$

$$\sigma_{local\_imp} = \frac{a}{b+c} \tag{4}$$

Where σ_{imp} is the implied volatility from Black-Scholes formula. By using equation (3) and (4), we calculated the local volatility and the local volatility in terms of the implied volatility for all options in 69 days and then do the backtesting.

3.3.2 Strategy 2

Strategy 2 is a strategy we used to make a comparison with strategy 1, the basic idea and the opening closing strategy are basically the same with strategy 1, except that we are not using the smoothed volatility surface anymore. Instead we use the implied volatility calculated from the Black-Scholes formula, as the local volatility should show the exact volatility faster and more accurately than the implied volatility. We may use the differences between local and implied volatility to make profit. Below is the basic logic of strategy 2:

If the implied volatility is higher than the real local volatility and reaches the open position signal, we will short those options.

If the implied volatility is higher than the real local volatility and reaches the open position signal, we will long those options.

If the option bid (ask) prices is higher (lower) than the ask (bid) option price + (-) closing signals, we will close those position and calculate the real profit&loss.

If the unrealized P&L is smaller than M, we will close all the positions and then exit the strategy. M is decided by the level of risk the investor is willing to take.

4 Result

The backtesting data we used is historical daily option data of S&P 500 Index starting from 2015-01-02. In total there are 69 trading days, we used the first 59 days to find out the best signals and then used the last 10 days to do the paper trading to test the strategy.

After comparing different goodness-of-fit, we choose the local regression method with a span of 5 as the smoothing method for the smoothed surface, because a surface with a span of 5 has the best trading result among different goodness-of-fit surfaces.

The results in Figure 3 is the result of local volatility derived from equation (2), the results of local volatility in terms of the implied volatility are much worse than the results in Figure 3, so we only choose the local volatility results to show here. All full results for strategy 1 and 2 can be found in the attached files.

4.1 Strategy 1

total P&L = last day's unrealized P&L + total realized P&L

		signal.close								
		6			7			8		
		total P&L	first day's investment	transactions	total P&L	first day's investment	transactions	total P&L	first day's investment	transactions
signal. open	0	2926130	4084590	6484	2694490	4084590	6173	2839500	4084590	5940
	0.01	2129780	2245990	5684	2144380	2245990	5449	2224740	2245990	5228
	0.02	1745865	1675140	4937	1884380	1675140	4750	1790655	1675140	4626
	0.03	1204630	1252410	4084	1393365	1252410	3981	1307355	1252410	3855
	0.04	1058090	1043880	3399	1184710	1043880	3284	1248570	1043880	3184
	...									
	0.16	-30075	99980	285	-39485	99980	281	-28315	99980	277
	0.17	-15240	34850	202	-45185	34850	197	-59845	34850	189
	0.18	6620	34850	180	3545	34850	176	-705	34850	171
	0.19	50890	34850	134	74925	34850	133	68035	34850	128
	0.2	69690	31130	115	44320	31130	114	41010	31130	109
	0.21	54580	31130	91	56760	31130	91	59210	31130	89
	0.22	43200	31130	67	45380	31130	67	47150	31130	65

Figure 3: The shortcut signals result of strategy 1

The short cut of the total trading signals's results is shown in Figure 3, the red cells are the best signals we get from the backtesting.

Although the total P&L with low opening signals seems to be high, but after comparing to their first day's investment amount, the total P&L will then be not so good. Since the lower the opening signals are, the higher the risk of losing money with a large opening positions will be. The result also shows that a certain level of closing signals will increase the profit of strategy 1, this may be because the changing of option prices is a continuous process during a couple of days.

After getting the result of best signals, then we apply both the best signals we got and the comparison signals to strategy 1 with the data from day 60 to 69. We have the result shown in Figure 4:

total P&L = last day's unrealized P&L + total realized P&L

		signal.cose					
		6			7		
		total P&L	first day's investment	transactions	total P&L	first day's investment	transactions
signal. open	0.05	69495	1078650	499	72345	1078650	481
	0.1	-80430	224970	147	-78430	224970	142
	0.19	6820	19680	23	7310	19680	23
	0.2	5040	19680	15	5530	19680	15
	0.25	1660	7380	5	1660	7380	5

Figure 4: The paper trading result of strategy 1

From the result we get from the paper trading test, we can see the differences between each pair of trading signals, the red cells show the best trading signals in the paper trading, this is also the best trading signals we got from the result in Figure 3.

4.2 Strategy 2

total P&L = last day's unrealized P&L + total realized P&L

		signal.close											
		6			7			8			9		
		total P&L	first day's investment	transactions	total P&L	first day's investment	transactions	total P&L	first day's investment	transactions	total P&L	first day's investment	transactions
signal. open	0	1009290	3124520	7619	1119455	3124520	7376	1254805	3124520	7147	1313445	3124520	6865
	0.01	683540	2780610	7364	727575	2780610	7060	565180	2780610	6814	902955	2780610	6732
	0.02	-79975	2421440	6974	-21440	2421440	6742	458340	2421440	6526	304870	2421440	6405
	0.03	-367365	2130190	6667	-579635	2130190	6418	-378010	2130190	6265	-294720	2130190	6117
	0.04	-1257705	1859240	6129	-1161625	1859240	5987	-987375	1859240	5779	-1084795	1859240	5622
	0.05	-2102770	1642960	5727	-2173745	1642960	5537	-1992675	1642960	5401	-1932835	1642960	5271
	...												
	0.19	-3518840	177780	1535	-3491080	177780	1527	-3492560	177780	1520	-3500500	177780	1510
	0.2	-2984125	175640	1376	-2994305	175640	1358	-2938275	175640	1354	-3043955	175640	1338
	0.21	-2570940	139240	1207	-2576340	139240	1196	-2540350	139240	1184	-2461250	139240	1168
	0.22	-2137930	139240	1089	-2091580	139240	1073	-2187510	139240	1068	-2147470	139240	1051
	0.23	-1806645	127260	923	-1770345	127260	914	-1814125	127260	905	-1827915	127260	891
	0.24	-1498675	127260	789	-1562065	127260	777	-1552085	127260	778	-1536735	127260	768

Figure 5: The shortcut signals result of strategy 2

For strategy 2, we do basically the same as strategy 1, one part of trading signals result is shown in Figure 5: we can see that although we can get the positive total P&L with low opening signals, their first day's investment are too large. Also as the low opening signals allows a large amount of opening positions, the risk of losing money with these large opening position is too high to take.

total P&L = last day's unrealized P&L + total realized P&L

		signal.close								
		7			8			9		
		total P&L	first day's investment	transactions	total P&L	first day's investment	transactions	total P&L	first day's investment	transactions
signal. open	0	-819190	6723565	1897	-810040	6723565	1836	-916150	6723565	1785
	0.01	-816310	6032195	1818	-865051	6032195	1767	-957830	6032195	1720
	0.02	-918080	5251310	1748	-936935	5251310	1696	-995970	5251310	1665
	0.1	-1884975	1497415	947	-1920035	1497415	939	-1938695	1497415	923
	0.2	-797805	447855	316	-801235	447855	315	-805685	447855	310
	0.24	-293675	280280	158	-323265	280280	157	-322735	280280	156

Figure 6: The paper trading result of strategy 2

Figure 6 shows the result of paper trading for the last 10 days with the trading signals we choose. We can see that the total P&L are all negative with a large amount of investment on the first day. Also the transactions amount is larger than the strategy 1 with the same trading signals.

5 Conclusion

From the results above, we now have the conclusion:

Strategy 1 should be profitable, the results show us that strategy 1 can make profit with the best trading signals we choose, it has a small amount of opening positions and relatively low risk.

Strategy 2 may be profitable, although the paper trading test we done is not so good, in a long period, strategy 2 can still make some profit as Figure 5 shows to us. But the investor will have to take a relatively high risk, since the low opening signals causes a large opening positions, with a very high risk of losing money with this large opening positions.

Both strategies need to do the backtesting to find out the best trading signals, and both the strategies may not work on options with a low volatility. For what we have done we can only make the conclusions on the S&P 500 Index options, we would like to find out if the strategy also work on other options with high volatility, this is what we should improve in the further study.

References

Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *The journal of political economy*, 637–654.

Derman, E. and I. Kani (1994). Riding on a smile. *Risk Magazine 7*, 32–39.

Dumas, B., J. Fleming, and R. E. Whaley (1998). Implied volatility functions: Empirical tests. *The Journal of Finance 53*(6), 2059–2106.

Dupire, B. et al. (1994). Pricing with a smile. *Risk 7*(1), 18–20.

Gatheral, J. (2011). *The volatility surface: a practitioner's guide*, Volume 357. John Wiley & Sons.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of financial studies 6*(2), 327–343.

Hull, J. and A. White (1987). The pricing of options on assets with stochastic volatilities. *The journal of finance 42*(2), 281–300.

Osborne, M. F. (1959). Brownian motion in the stock market. *Operations research 7*(2), 145–173.

Stein, E. M. and J. C. Stein (1991). Stock price distributions with stochastic volatility: an analytic approach. *Review of financial Studies 4*(4), 727–752.