Swiss Finance Institute
Research Paper Series N°14-59

Asymmetric Beta Comovement and Systematic Downside Risk

Eric JONDEAU
University of Lausanne and Swiss Finance Institute

Qunzi ZHANG
Shandong University

Asymmetric Beta Comovement and Systematic Downside Risk*

Eric Jondeau† and Qunzi Zhang‡

October 2014

Abstract

In this paper, we document evidence that downside betas tend to comove more than upside betas during a financial crisis, but upside betas tend to comove more than the downside betas during financial booms. We find that the asymmetry between Downside-Beta Comovement and Upside-Beta Comovement is the main driving force for market level skewness. An indicator called "Systematic Downside Risk" (SDR) is defined to characterize this asymmetry in the comovement of betas. This indicator negatively predicts future market returns. The SDR effectively forecasts future monthly stock market movements with an out-of-sample R-square above 2.27% relative to a strategy based on historical mean. An investor who timed the market using SDR would have obtained a Sharpe ratio gain of 0.206.

Keywords: Systematic Risk, Skewness, Predictability, Trading Strategies

JEL Classification: G11, G12, G14, G17

*The authors would like to thank Jin-Chuan Duan, Loriano Mancini, Michael Rockinger, René M. Stulz, Fabio Trojani, and Yuhang Xing for their valuable comments. We also like to acknowledge conference participants at the Swiss Doctoral Workshop in Finance 2012, China International Conference in Finance 2013, International Risk Management Conference 2014, for valuable discussions. We also benefit from discussions with Michael Hasler, Daniel Andrei, Ngoc Giang Hoang and Stefanie Schraeder. All errors and omissions are the sole responsibility of the authors. A Technical Appendix is available on the websites of the authors.

†Eric Jondeau, Swiss Finance Institute and University of Lausanne. eric.jondeau@unil.ch; http://www.hec.unil.ch/ejondeau/

‡Qunzi Zhang, Shandong University. Corresponding author. qunzi.zhang@sdu.edu.cn; http://www.econ.sdu.edu.cn/tree/content.php?id=52869

1 Introduction

An asset return co-varies with the aggregate market return to a greater extent in a downside market than in an upside market. This asymmetric relationship, which strongly influences risk management strategies and optimal portfolio choices, has received great attention in academic research. For instance, Longin and Solnik (2001) model the multivariate distribution tails based on the extreme value theory and show that the correlation between international equity indices increases in bear markets, but not in bull markets. Ang and Chen (2002) find that the correlation between the equity portfolio and the market is much greater for downside moves than for upside moves. Ang, Chen, and Xing (2006) define downside (upside) beta to characterize stocks with high sensitivity to downside (upside) market movements. In the cross section of firms, they show that downside risk will *positively* predict a firm's future returns because investors who are sensitive to downside losses require a premium for holding assets with higher downside beta. Despite this appealing asset pricing property of downside risk at cross-sectional level, there has been little empirical research into how downside risk or asymmetry between downside and upside risk are priced in the aggregate market return. In this paper, our objective is to address the following questions: How do the downside (upside) betas move together over time? Is there a relationship between beta comovements and market movements, both contemporaneously and temporally? How can we quantify this relation empirically?

We provide evidence that downside betas tend to comove more than upside betas during financial crises but upside betas tend to comove more than downside betas during financial booms. On average, downside betas comove to a greater extent than upside betas. This result has two important implications that we investigate in this paper: (1) the asymmetric comovement of downside risks and upside risks is the main driving force for the asymmetry in market movements, as measured by market skewness; (2) the asymmetric comovement of downside risks and upside risks *negatively* predicts future market returns. Ang, Chen, and Xing (2006) have documented a *positive* relation at the cross-sectional level. Therefore, downside risk appears to play a different role at the cross-sectional level and at the aggregate market level.

We start by describing market skewness in terms of comovements in the sensitivities (or betas) of firms to market news. This decomposition is obtained in two steps. First, at the firm level, we describe returns using a market model conditional on market performance, defined relative to the market return's quantiles.[1] For example, periods when the market return is below its 20% quantile correspond to a downside market, and periods when the market return is above its 80% quantile correspond to a upside market. Accordingly, we obtain downside and upside betas for each firm. Second, given this conditional market model, aggregate skewness is written as a function of aggregate skewness prevailing under each market condition, the comovements in firm's betas, and the individual firm's idiosyncratic return. This decomposition of market skewness serves as the backbone model to study the contemporaneous and temporal relationships between beta comovements and market movements. The conditional market model allows us to obtain several important results: (1) firms are more sensitive to market news (betas are higher) in the downside than in the upside; (2) downside betas are much less dispersed than upside betas; (3) betas comove more in the downside than in the upside; and (4) this asymmetry in beta comovements is the main driver of negative market skewness.

Our results show that the main driving force of negative market skewness is the cross-sectional heterogeneity in the sensitivity of firms' returns to market news. This finding allows us to reconcile the following conflicting empirical evidence: aggregating firm-level returns, which have positive skewness on average (Duffee, 1995, 2002), generates market return, which is negatively skewed (Christie, 1982; Pindyck, 1984; Hong and Stein, 2003). Several papers have interpreted this negative market skewness as resulting from a "leverage effect" (Christie, 1982) or "volatility feedback effect" (Pindyck, 1984; French, Schwert, and Stambaugh, 1987). Hong and Stein (2003) also claim that the negative market skewness arises from investors' heterogeneous beliefs and short-sale constraints. However, these studies do not explain where positive firm skewness comes from. In contrast, studies regarding positive firm skewness (e.g., Duffee, 2002; Grullon, Lyandres, and Zhdanov, 2012) do not explain what generates negative market skewness. As far as we know, Albuquerque (2012) is the only study to propose a theoretical explanation for the different signs of skewness at firm and market levels. In a model with positive individual skewness, the study is able to generate a negative market skewness based on the cross-section heterogeneity in firms' earnings announcement events.[2] We describe a similar

[1] We follow Bawa and Lindenberg (1977) and Ang, Chen, and Xing (2006), and extend the market model to take the asymmetric treatment of risk into account, by specifying asymmetric downside and upside betas.

[2] In his work, positive skewness in individual stock returns is due to the positive skewness in expected returns and to the positive correlation between expected returns and volatility (risk-return trade-off). Then, assuming that stock returns are independent, he shows that negative market skewness arises from cross-sectional

phenomenon but in a market model conditional on market performance. Firms have higher sensitivities to market news in a downside market than in an upside market, for instance due to the well-documented leverage effect (Black, 1976). In addition, sensitivities are more correlated in a market downside than in a market upside. Intuitively, if we interpret shocks on the market return as macro news, we observe that bad macro news often affects most firms negatively. In contrast, good macro news may affect firms differently in a much less correlated way. Therefore, a negative market shock will imply a stronger comovement in firms' returns than a positive market shock of the same magnitude. This observation, in turn, implies that the decrease in market skewness after a negative shock will be larger than the increase in market skewness after a positive shock of the same magnitude. To quantify this empirical evidence, we construct an indicator, which we call "Systematic Downside Risk" (hereafter SDR), defined as the difference between downside-beta and upside-beta comovements. We verify that SDR is the main driving force for the market skewness after controlling for all other possible sources driving market skewness.

We then study the relation between the asymmetry in beta comovements and future market returns. We evaluate this relation with two complementary testing procedures. In the first approach, we use the event study methodology to investigate the ability to predict extreme market movements. The recent subprime crisis has triggered important research related to predicting systemic risk. Kritzman, Li, Page, and Rigobon (2011) and Billio, Getmansky, Lo, and Pelizzon (2012) have proposed the Absorption Ratio, which measures the fraction of the total variance of a system explained by the first principal components. They show that the Absorption Ratio performs well in predicting systemic risk among financial institutions. We adopt a similar approach to compare the two indicators. More specifically, we pick the events when the market has the lowest daily returns, the largest daily returns, or the most turbulent days from the whole sample period and examine the average SDR movement during specific days around these event days. We find that, on average, SDR begins to increase at least 100 days in advance the dates of events with the 1% lowest returns; it begins to increase 18 days in advance of the dates of events with the 1% most turbulent days; and it begins to decrease 22 days ahead the events with the 1% largest returns. The SDR has similar performance as the Absorption

heterogeneity in firms' earnings announcement events. In his model, portfolio return is negatively skewed because a low return for a firm is associated with high volatility in the other firms. This result is obtained because stock return correlation is asymmetric and increases in market downturn.

Ratio in predicting market crashes. However, the SDR outperforms the Absorption Ratio in predicting large market increases. We design a trading strategy based on the signal provided by these indicators and show that SDR generates an outstanding performance, particularly during and after the 2008 subprime crisis. This over-performance can be explained by the success of the SDR in predicting the 2008 market crash as well as the subsequent market rebound. Furthermore, we find that the returns from implementing the SDR-based strategy are minimally explained by the standard risk factors; therefore, the strategy still delivers economically significant alpha after controlling for the usual risk factors.

The second empirical test relies on forecasting regressions. In a recent contribution, Goyal and Welch (2008) examine the out-of-sample performance of a set of potential predictors to predict future market returns. They find that the historical mean has better out-of-sample performance than the traditional predictive regressors. Ferreira and Santa-Clara (2011) find that, once the stock market return is decomposed into components with different time series characteristics (sum-of-parts, or SOP, method), the market return is predictable. The best predictor is found to be the Net Equity Expansion, i.e., the net equity issuing activity. We apply the same regression approach as Goyal and Welch (2008) and Ferreira and Santa-Clara (2011) and test the ability of the SDR and other competitors to predict stock market returns for the 1986-2011 period. Competitors include the Net Equity Expansion but also the Downside-Beta Comovement, the Downside Beta, and the Downside Beta minus Upside Beta, in accordance with Ang and Chen (2002) and Ang, Chen, and Xing (2006) research. The SDR performs better than both the historical mean and the other competitors. We obtain an out-of-sample R-square (relative to the historical mean) of 0.10% with daily data and 2.27% with monthly data; this value is higher than the (negative) out-of-sample R-square of the alternative variables to predict the market return. There are also substantial economic gains from a trading strategy based on the SDR relative to a trading strategy based on the historical mean. The Sharpe ratio gain is approximately 0.2 per year and the certainty equivalent gain is approximately 10% per year; these values can be compared to the much smaller (or even negative) gains from implementing a trading strategy based on the Net Equity Expansion.

The rest of this paper is organized as follows. Section 2 presents the conditional market model and shows how the market skewness is related to the beta comovements. Section 3 introduces the data used in this work and analyzes the empirical decomposition of the market skewness. It provides evidence that the main driver of market skewness is the asymmetry in beta comovements. Section 4 evaluates the ability of the SDR to predict future extreme changes in market price (crashes and booms). It also presents a trading strategy based on the prediction of the sign of the future market return to quantify the gain relative to alternative predictors of the market return. Section 5 evaluates the predictive power of the SDR in the context of predictive regressions. Implementing a trading strategy based on the prediction of future market return shows that SDR again outperforms the other competitors. Section 6 concludes. Appendix A provides technical details.

2 The Conditional Market Model

We consider N firms and designate $R_{i,t}$ to represent the stock return of firm i at date t. We assume that a firm's return is driven by a variant of the market model, in which firm's sensitivity to the market return (its beta) depends on market performance (Bawa and Lindenberg, 1977; Ang, Chen, and Xing, 2006):

$$R_{i,t} - \mu_i = \beta_i^- (R_{m,t} - \mu_m \mid R_{m,t} < 0) + \beta_i^+ (R_{m,t} - \mu_m \mid R_{m,t} \geq 0) + \epsilon_{i,t}, \tag{1}$$

where $R_{m,t} = \sum_{i=1}^{N} w_i R_{i,t}$ is the return of the market portfolio with weights $\{w_1, \cdots, w_N\}$, $\mu_i = E[R_{i,t}]$, and $\epsilon_{i,t} \sim iid(0, \sigma_i)$ is the error term.[3] Parameters β_i^- and β_i^+ measure the firm's sensitivity to the market return under downside and upside markets, respectively. Expected return is then given by the following:

$$E[R_{i,t} - \mu_i] = \beta_i^- E\left[R_{m,t} - \mu_m \,|R_{m,t} < 0\right] + \beta_i^+ E\left[R_{m,t} - \mu_m |\, R_{m,t} \geq 0)\right]. \tag{2}$$

We now define the standardized market skewness as $sk_m = E\left[(R_{m,t} - \mu_m))^3/\sigma_m^3\right]$, where $\mu_m = E[R_{m,t}]$ and $\sigma_m^2 = E[(R_{m,t} - \mu_m)^2]$ denote the expected market return and market variance, respectively. The following proposition provides the expression of the market skewness under the conditional market model.

[3] In the prediction part of the paper (Sections 4 and 5), we will consider an equally-weighted market portfolio, with $w_i = 1/N$, for all i.

Proposition 1. *In the conditional market model defined in Equation (1), market skewness can be decomposed as follows:*

$$
\begin{aligned}
sk_m \;=\;& sk_m^- \left(\frac{\sigma_m^-}{\sigma_m}\right)^3 \sum_{i=1}^{N} \omega_{iii} (\beta_i^-)^3 + sk_m^+ \left(\frac{\sigma_m^+}{\sigma_m}\right)^3 \sum_{i=1}^{N} \omega_{iii} (\beta_i^+)^3 \\
+\;& 3\ sk_m^- \left(\frac{\sigma_m^-}{\sigma_m}\right)^3 \sum_{i=1}^{N} \sum_{j \neq i}^{N} \omega_{ijj} \beta_i^- (\beta_j^-)^2 + 3\ sk_m^+ \left(\frac{\sigma_m^+}{\sigma_m}\right)^3 \sum_{i=1}^{N} \sum_{j \neq i}^{N} \omega_{ijj} \beta_i^+ (\beta_j^+)^2 \\
+\;& 2\ sk_m^- \left(\frac{\sigma_m^-}{\sigma_m}\right)^3 \sum_{i=1}^{N} \sum_{j \neq i}^{N} \sum_{k \neq \{i,j\}}^{N} \omega_{ijk} \beta_i^- \beta_j^- \beta_k^- + 2\ sk_m^+ \left(\frac{\sigma_m^+}{\sigma_m}\right)^3 \sum_{i=1}^{N} \sum_{j \neq i}^{N} \sum_{k \neq \{i,j\}}^{N} \omega_{ijk} \beta_i^+ \beta_j^+ \beta_k^+ \\
+\;& \frac{F(\epsilon_t)}{\sigma_m^3}.
\end{aligned}
\tag{3}
$$

where ω_{ijk} is a generic notation for $(w_i w_j w_k)$ and the functional form $F(\epsilon_t)$ defined in Appendix A involves idiosyncratic terms that can be neglected as N approaches ∞. β_i^- and β_i^+ denote firm i's downside and upside betas, respectively; σ_m^- and σ_m^+ denote the downside and upside market volatilities, respectively; sk_m^- and sk_m^+ represent the downside and upside market skewness, respectively.

Proof. See Appendix A. □

Equation (3) shows that the market skewness is the sum of the beta comovement terms multiplied by the market skewness conditional on the market return performance plus a remainder term. More specifically, to define the beta related terms, the first term (first line), denoted by ***beta*^{3}**, corresponds solely to individual betas. The second term (second line), denoted by ***bco-vol***, describes the comovement of one asset's beta with another asset's squared beta. The third term (third line), denoted by ***bco-cov***, represents one asset beta's comovement with any other two assets' betas.[4] The three terms are multiplied by the market skewness based on the market condition (sk_m^- and sk_m^+) and the relative volatilities prevailing in downside and upside markets ($(\sigma_m^-/\sigma_m)^3$ and $(\sigma_m^+/\sigma_m)^3$).

A natural extension for the individual firm's return based on the conditional market model is to allow firm's beta to depend on more detailed market conditions, such as the market return

[4]We follow Albuquerque (2012) in naming the beta-comovement terms. As he defines non-standardized market skewness, the terms in relative volatility $((\sigma_m^-/\sigma_m)^3$ and $(\sigma_m^+/\sigma_m)^3$ in Equation (3)) do not appear in his expression of the market skewness.

due to being below or above a given quantile. Expected returns are then given by the following:

$$
\begin{aligned}
R_{i,t} - \mu_i &= \sum_{u \in U} \beta_i^{(u)} (R_{m,t} - \mu_m |\ R_{m,t} < R_m^{(u)}) \\
&+ \sum_{u \in U} \beta_i^{(1-u)} (R_{m,t} - \mu_m |\ R_{m,t} \geq R_m^{(1-u)}) + \epsilon_{i,t},
\end{aligned}
\tag{4}
$$

where u denotes the days when market returns are below the u quantile and $1-u$ denotes the days when market returns are above (or equal to) the $1-u$ quantile, with $u \in U$. Therefore, $R_m^{(u)}$ and $R_m^{(1-u)}$ are the left and right market return's quantiles, respectively. $\beta_i^{(u)}$ represents the firm's sensitivity to the market conditional on the market return being below its u quantile and $\beta_i^{(1-u)}$ represents the firm's sensitivity to the market, conditional on the market return being above or equal to its $1-u$ quantile.

In Equation (3), the resulting market skewness decomposition was based on the sign of the market return, with negative and positive signs corresponding to downside and upside markets, respectively. Now, using Equation (4), the market skewness can be written as follows:

$$
\begin{aligned}
sk_m &= \sum_{u \in U} \Bigg[sk_m^{(u)} \left(\frac{\sigma_m^{(u)}}{\sigma_m} \right)^3 \sum_{i=1}^{N} \omega_{iii} (\beta_i^{(u)})^3 + sk_m^{(1-u)} \left(\frac{\sigma_m^{(1-u)}}{\sigma_m} \right)^3 \sum_{i=1}^{N} \omega_{iii} (\beta_i^{(1-u)})^3 \\
&+ 3\ sk_m^{(u)} \left(\frac{\sigma_m^{(u)}}{\sigma_m} \right)^3 \sum_{i=1}^{N} \sum_{j \neq i}^{N} \omega_{ijj} \beta_i^{(u)} (\beta_j^{(u)})^2 + 3\ sk_m^{(1-u)} \left(\frac{\sigma_m^{(1-u)}}{\sigma_m} \right)^3 \sum_{i=1}^{N} \sum_{j \neq i}^{N} \omega_{ijj} \beta_i^{(1-u)} (\beta_j^{(1-u)})^2 \\
&+ 2\ sk_m^{(u)} \left(\frac{\sigma_m^{(u)}}{\sigma_m} \right)^3 \sum_{i=1}^{N} \sum_{j \neq i}^{N} \sum_{k \neq \{i,j\}}^{N} \omega_{ijk} \beta_i^{(u)} \beta_j^{(u)} \beta_k^{(u)} \\
&+ 2\ sk_m^{(1-u)} \left(\frac{\sigma_m^{(1-u)}}{\sigma_m} \right)^3 \sum_{i=1}^{N} \sum_{j \neq i}^{N} \sum_{k \neq \{i,j\}}^{N} \omega_{ijk} \beta_i^{(1-u)} \beta_j^{(1-u)} \beta_k^{(1-u)} \Bigg] + \frac{F(\epsilon_t)}{\sigma_m^3},
\end{aligned}
\tag{5}
$$

where $\sigma_m^{(u)}$ and $\sigma_m^{(1-u)}$ denote the market volatilities conditional on the market return quantiles u and $1-u$, respectively, $sk_m^{(u)}$ and $sk_m^{(1-u)}$ denote the market skewness prevailing when the market return is below quantile u and above quantile $1-u$, respectively. The functional form $F(\epsilon_t)$ is again negligible as $N \to \infty$.

There are three main reasons to extend our decomposition to quantiles. First, including more quantiles enables us to investigate the properties of betas estimated under extreme market conditions. In particular, we want to capture the higher asymmetry between the downside and upside ($\beta_i^{(u)}$ vs $\beta_i^{(1-u)}$). Second, the asymmetry between the comovement of downside betas and the comovement of upside betas is more pronounced at extreme market conditions ($\boldsymbol{bco\text{-}cov}^{(u)}$ vs $\boldsymbol{bco\text{-}cov}^{(1-u)}$). Third, this specification also helps to reproduce market skewness, as we show in the following section.

3 Reconciling Firm-Level and Market-Level Skewness

3.1 Data

We obtain prices for common stocks listed on NYSE/AMEX/NASDAQ from CRSP. The sample period ranges from January 3rd, 1983 to December 30th, 2011. We rule out firms with the following properties: the stock price is less than 5 dollars; the firm has existed for less than 3 months; the number of days when the stock price is zero exceeds 10% of the firm's life, i.e., shares are not actively traded. Finally, we have, in total, $N = 4,962$ firms in our data set.

As we are interested in portfolio aggregation, we use simple (or arithmetic) returns, denoted by $R_{i,t}$, and we construct the market return as the equally-weighted ($R_{m,t}^{EW}$) or value-weighted ($R_{m,t}^{VW}$) average of individual returns. We also consider the equally-weighted and value-weighted CRSP index returns as other market return proxies. However, as they include all firms, they may be more sensitive to outliers and therefore less comparable to the firm's data.

We now analyze the drivers of market skewness and explore the ability of the conditional market model to reproduce the dynamics of the market skewness.

3.2 Main Contributors to Market Skewness

We start with the disconnection between firm and market skewness, as illustrated in Table 1. At firm level (Panel A), we define the sample skewness of firm i by $sk_i = \frac{1}{T}\sum_{t=1}^{T}(R_{i,t} - \mu_i)^3/\sigma_i^3$. The average skewness $\overline{sk} = \sum_{i=1}^{N} w_i sk_i$ is positive whatever the weighting scheme we use to define market returns. It is equal to 0.184 when firms are equally weighted ($w_i = 1/N$) and 0.177 when they are value weighted (w_i = relative market capitalization). The median value is equal to 0.183.

We also report an alternative measure of firm-level asymmetry, namely the sample mean of cross-section skewness. It is defined by $cssk = \frac{1}{T}\sum_{t=1}^{T} cssk_t$, where $cssk_t = \sum_{i=1}^{N} w_i (R_{i,t} - R_{m,t})^3/\sigma_{m,t}^3$ and $\sigma_{m,t}^2 = \sum_{i=1}^{N} w_i \left(R_{i,t} - R_{m,t}\right)^2$. As the table reveals, $cssk$ is also always positive: it is equal to 0.367 when firms are equally weighted and 0.338 when they are value weighted. Both measures of firm-level skewness, whether considered individually or in the cross-section, are consistently positive. The 25% and 75% quantile intervals of firm's skewness are [0.072;0.296] for $\overline{sk}$ and [0.120;0.624] for $cssk$, indicating that most of the individual skewness are indeed positive.

The table also reports the sample market skewness (Panel B). When we consider the market return based on the firms in our sample, we find a negative skewness, equal to -0.111 when firms are equally weighted and -0.367 when they are value weighted. When all the firms in the CRSP database are considered, the sample market skewness is even more negative and equal to -0.689 and -0.675 when firms are equally weighted and value weighted, respectively.

Insert Table 1 here

The decomposition based on Equation (5) provides a model-implied market skewness equal to -0.110, a value close to the corresponding equally-weighted market skewness value for $R_{m,t}^{EW}$ (-0.111) (Panel C). We then compute the contribution of the various constituents (***beta*³**, ***bco-vol***, and ***bco-cov***) appearing in Equation (5) to the negative market skewness. The asymmetry in beta comovement, ***bco-cov***, is the main contributor to market skewness (-0.133). The asymmetry in comovement between two firms, ***bco-vol***, has a smaller contribution (0.024), whereas the asymmetry in betas, ***beta*³**, does not contribute at all. The decomposition of the value-weighted market skewness confirms that the model generates an aggregate skewness (-0.353) close to the actual measure (-0.367) and that the main contributor is by far the ***bco-cov*** term.

We now proceed with in the decomposition and consider the main quantiles for market skewness, the relative market variance, ***beta*³**, ***bco-vol***, and ***bco-cov***, for quantiles $u \in (0.2, 0.3, 0.4, 0.5)$. The first factor that potentially explains the opposite sign of individual and market skewness, is the asymmetry in skewness under downside and upside markets. As Table 2 reveals skewness in a downside market ($sk_m^{(u)}$) is always negative (between -2.4 and -2.6), whereas skewness in an upside market ($sk_m^{(1-u)}$) is always positive (close to 2.7). Moreover, in absolute terms, upside-market skewness for a given quantile $sk_m^{(1-u)}$ is always larger than its downside-market counterpart $sk_m^{(u)}$. Therefore, the negative sign of market return skewness cannot originate from the asymmetry in individual skewness under downside and upside markets.

Second, we consider the betas estimated using the conditional market model, as in Equation (4). The table reports mean values of betas across firms within each market return quantile. The average betas from lower quantiles are always higher than the ones from the upper quantiles, i.e., $\bar{\beta}^{(u)} > \bar{\beta}^{(1-u)}$. This asymmetry between downside and upside betas is similar to the evidence reported by Ang, Chen, and Xing (2006) for downside and upside markets. Our more detailed decomposition shows that downside and upside market conditions have different implications for beta estimates. In downside market, betas are on average relatively high (close to 0.99) and do not vary significantly across market return quantiles. On the opposite, in a market upside, a higher market return corresponds to a lower conditional beta. This result suggests that, in a market boom, a firm's return becomes less sensitive to market fluctuations. The next column indicates that, additionally, the dispersion across betas is much smaller in the downside than in the upside. This result suggests that betas are, in fact, much more concentrated around 1 in the downside market.

Finally, the table reports that the $\boldsymbol{beta^3}$, ***bco-vol***, and ***bco-cov*** terms introduced in the market skewness decomposition. The cubic terms, $\boldsymbol{beta}^3$, are clearly very close to 0 and, therefore, play a negligible role. The ***bco-vol*** terms contribute slightly more than the individual terms, but their role is still very limited. We notice that there is an asymmetric pattern in ***bco-vol***: the dependence between the betas of two firms is lower in the downside than in the upside. The beta comovement between three different assets, ***bco-cov***, clearly contributes the most to market skewness, as its contribution is approximately 1000 times greater than the contribution of ***bco-vol***. We observe that ***bco-cov*** also has a pronounced asymmetry between the lower and upper quantiles, but with the opposite direction. ***bco-cov*** is as large as 0.95 for a downside market but decreases to 0.82 for an upside market.

Insert Table 2 here

In summary, the asymmetric beta comovement, specifically the asymmetry in ***bco-cov***, is the dominant factor for generating negative market skewness. We name this driving factor

"Systematic Downside Risk" (SDR). A possible explanation for the asymmetry embedded in SDR is the following: in a financial downturn, all firms are affected in a similar way, as confirmed by the higher average beta and lower beta dispersion in a downside market. As a consequence, firms' returns tend to comove more in a downside market. This commonality has a feedback effect on market conditions by increasing the likelihood of a new negative shock on the market. Negative market skewness therefore results from an interaction between market shocks and the aggregate reaction of individual firms' returns.

3.3 The Dynamics of Market Skewness

We now evaluate the ability of the conditional market model to reproduce negative market skewness consistently over time, using a 3-year rolling window. Within each of the $6,565$ overlapping rolling windows (with one day as the rolling step), we ensure that each firm has at least two years of data for estimation. The number of firms included in this analysis ranges between $1,892$ and $3,715$.

Figure 1 (Panel A) documents that the mean and median values of firm-level skewness are positive over the entire period. It starts at an average level of 0.3 before 1987, then stabilizes at approximately 0.2 until 2001, and decreases to approximately 0.1 at the end of the sample. Statistics on firm-level skewness are essentially unaltered when we switch from whole-sample data to rolling-window data. We find that the (equally-weighted) average of the firm-level skewness ($\overline{sk}$) is equal to 0.187 and the mean of the cross-sectional skewness ($cssk$) is equal to 0.32. Similarly, the value-weighted average of firm-level skewness is 0.142 and the mean of the cross-sectional skewness is 0.366. The weighting scheme does not play a key role for firm-level statistics.

Figure 1 (Panel B) also displays the evolution of the (equally-weighted) market skewness ($R_{m,t}^{EW}$). On average, market skewness is significantly lower when estimated in rolling windows. It decreases from a sample estimate of -0.111 to a rolling-window average of -0.666. The figure also reveals that the market skewness has been negative over most of the sample. Finally, we find that the model-implied skewness tends to over-estimate the actual market skewness (-0.513 instead of -0.666, on average). However, the dynamics of the actual market skewness is well reproduced by the model. The significant events in stock markets, such as the biggest one-day stock market decline in October 1987, the Asian Financial Crisis in 1997 and the Russian default in 1998, the Dot-com climax and crash in 2000, and the subprime crisis in 2008-09, are all well captured by our model.

Insert Figure 1 here

Figures 2 and 3 display the dynamics of the main constituents of market skewness.[5] Figure 2 shows that market skewness in downside and upside market retain the same property as in the whole sample: $sk_m^{(u)}$ remains negative most of the time, while $sk_m^{(1-u)}$ is always positive. On average, they sum to a positive number over the sampling period. We observe that downside and upside market skewness both converge toward each other over the recent period: they are close to -2 and 2, respectively, at the beginning of the sample and decrease to approximately -0.1 and 0.1 at the end of the sample. This suggests that market skewness alone is not a good indicator of the likelihood of a market downturn.

As Figure 3 (Panel A) reveals, the average beta in the lower quantile is larger than the average beta in the upper quantile most of the time. Downside betas are significantly larger than upside betas in nearly all financial market downturns (including the Asian crisis and Russia default, the Dot-com burst, and the subprime crisis). We also see in Panel B that, in market downturns, the gap between the downside and upside ***bco-cov*** becomes wider, implying that asymmetry becomes stronger, i.e., SDR increases.

Insert Figures 2 and 3 here

4 Forecasting Large Movements of Market Return

4.1 Definition and Evolution of the Alternative Indicators

The empirical evidence reported above demonstrates that the asymmetry in ***bco-cov*** is the main contributing factor to market skewness. We define the SDR index representing the asymmetry

[5] To effectively convey the information generated from the quantile specification, for simplicity, we only report the results obtained from the most representative quantile, i.e., $u = 0.2$, as well as the mean values of results within all the quantiles.

in the beta comovements as follows:

$$
\begin{aligned}
\text{SDR} &\equiv \text{cross-quantile-mean}\left(\textit{bco-cov}^{(u)} - \textit{bco-cov}^{(1-u)}\right) \\
&= \frac{1}{N_U N_\beta} \sum_{u \in U} \sum_{i,j,k}^{N_\beta} \left(\beta_i^{(u)} \beta_j^{(u)} \beta_k^{(u)} - \beta_i^{(1-u)} \beta_j^{(1-u)} \beta_k^{(1-u)} \right),
\end{aligned}
\tag{6}
$$

where $U = (0.2, 0.25, 0.3, \cdots, 0.5)$, N_U is the number of quantiles on downside (upside) market conditions ($N_U = 7$), and N_β is the number of triple products for betas ($N_\beta = N(N-1)(N-2)$). The SDR index sums the differences between $\boldsymbol{bco\text{-}cov}^{(u)}$ and $\boldsymbol{bco\text{-}cov}^{(1-u)}$ for the various quantiles. We also define the Downside-Beta Comovement as follows:

$$
\text{Bcov}_{Down} \equiv \text{mean}(\boldsymbol{bco\text{-}cov}^{(u)}) = \tfrac{1}{N_U N_\beta} \textstyle\sum_{u \in U} \sum_{i,j,k}^{N_\beta} \beta_i^{(u)} \beta_j^{(u)} \beta_k^{(u)}.
$$

In our analysis, these indicators have been found to be the most informative for predicting future market returns. They are computed using exponential weighting, which emphasizes more recent observations.[6]

Figure 4 (Panel A) shows the evolution of the SDR index estimated over 3-year rolling windows. The SDR increased during all the financial crisis episodes. The evolution of the SDR during the subprime crisis markedly differs from the evolution of market quantile skewness, as illustrated in Figure 2. The SDR reached a historically high level (close to 0.15), whereas the market skewness was close to 0 due to the small difference between downside and upside market quantile skewness. An explanation for the contrasting evolution of SDR and market skewness may be as follows: during the subprime crisis, the stock market was very reactive given the high level of uncertainty; both negative and positive shocks propagated more quickly and broadly, which is reflected in the convergence of the low-quantile and high-quantile skewness ($sk_m^{(u)}$ and $sk_m^{(1-u)}$, respectively) and through the increase in both low-quantile and high-quantile average betas. The figure also shows that the evolution of Bcov_{Down} over the recent period has been similar to the evolution of SDR, suggesting that it may be a good predictor for financial crises. However, we notice that Bcov_{Down} has a different evolution during other periods, such as the

[6]In each estimation window, there are $T = 750$ daily return observations. We apply an exponential weighting scheme, with memory parameter ϕ, so that we replace return $R_{i,t}$ by weighted return $\omega_t R_{i,t}$, where $\omega_t = \phi^{(T-t+1)} / \sum_t^T \phi^{(T-t+1)}$. Betas are estimated using these exponentially weighted returns, with $\phi = 0.995$.

dot-com bubble burst in 2000.

Insert Figure 4 here

Because the increase in the SDR is a common pattern observed during all the financial crises, we conjecture that the SDR may be a good monitoring tool for large financial market movements. In our evaluation of the ability of the SDR to predict market crises, we consider another, widely discussed, measure of systemic risk, the Absorption Ratio (AR) proposed by Kritzman, Li, Page, and Rigobon (2011). This measure is based on principal component analysis and has compelling forecasting power for systemic risk in financial market. It is defined as the fraction of the total variance of a set of M asset returns explained (or "absorbed") by a fixed number m of eigenvectors:[7]

$$\mathrm{AR}_m = \frac{\sum\limits_{h=1}^{m} \sigma^2_{E,h}}{\sum\limits_{k=1}^{M} \sigma^2_{A,k}}, \tag{7}$$

where $\sigma^2_{E,h}$ denotes the variance of the h-th eigenvector and $\sigma^2_{A,k}$ the variance of the k-th asset. This measure captures the extent to which markets are unified or tightly coupled. It was initially introduced by Kritzman, Li, Page, and Rigobon (2011) to study systemic risk in portfolio and market indices. Billio, Getmansky, Lo, and Pelizzon (2012) use the AR to measure the degree of connectivity among financial institutions and show how systemic risk can be explained by the risk profiles of the institutions. In accordance with these authors, we estimate AR using data on 49 industry portfolios using the first 10 factors (1/5 of the total assets number).[8]

Figure 4 (Panel B) plots the evolution of SDR and AR over time. The SDR begins to increase at the end of 2006 from a low level of -0.05 and reaches a level of 0.15 in September 2008, i.e., the bankruptcy of Lehman Brothers. Subsequently, the SDR goes back to a level of 0.01 during the rebound period following the subprime crisis. Finally, the SDR strongly

[7]The first eigenvector is the linear combination of asset weights that explains the greatest fraction of the asset total variance. The second eigenvector is the linear combination of asset weights orthogonal to the first eigenvector that explains the greatest fraction of leftover asset variance, i.e., the variance not yet explained (or absorbed) by the first eigenvector. The third eigenvector and beyond are identified the same way.

[8]The data on industry portfolios is from Kenneth French's webpage. Stocks in NYSE/AMEX/NASDAQ are grouped into 49 industry portfolios. Some additional details about the construction of the AR are provided in the Technical Appendix.

increases just after the rating agency Fitch downgrades the Greek sovereign debt at the end of 2009, which can be seen as the starting point of the debt crisis in the Eurozone (Arezki, Candelon, and Sy (2011)). The AR behaves differently in the recent period. After a long period of stability from 1998 to 2006, the AR increases regularly over the subsequent two years (from 0.75 to 0.82). Then, the bankruptcy of Lehman Brothers triggers a dramatic jump of the AR (up to 0.90), which remains at this level until the end of the sample. This evolution suggests that the AR is able to capture market crashes, but does not properly reflect rebounds or booms.

4.2 Event Study: Large Movements of Market Return

To evaluate the ability of these alternative measures to predict large market movements, in accordance with Kritzman, Li, Page, and Rigobon (2011), we standardize our indicators (SDR, Bcov_{Down}, and AR) as follows:

$$X_{Std} = \frac{\text{mean}(X_{3\text{ months}}) - \text{mean}(X_{2\text{ years}})}{\sigma(X_{2\text{ years}})}, \tag{8}$$

where X_{Std} represents the standardized indicator. We select the 1% (5%) of days, in the entire sample period covering 26 years (from 1986 to 2011), with the lowest return value, the largest return increase, or the most turbulent days.[9] Figures 5 and 6 plot the mean values of SDR_{Std}, $\text{Bcov}_{Down,Std}$, and AR_{Std} before and after the 1% and 5% extreme event dates, respectively. SDR_{Std} begins to increase at least 100 days before the event date for the 1% and 5% lowest market returns. AR_{Std} begins to increase 86 (83) days before the event date with the 1% (5%) lowest market return. SDR_{Std} begins to increase 18 (100) days before the event date with the 1% (5%) largest turbulence. AR_{Std} begins to increase 22 (28) days before the event date with the 1% (5%) largest turbulence. Additionally, SDR_{Std} begins to decrease approximately 15 (2) days after these extreme 1% (5%) events occur, and AR_{Std} begins to decrease approximately 40 (35) days after these extreme 1% (5%) events occur.

[9] We measure financial turbulence in the same way as in Kritzman, Li, Page, and Rigobon (2011) (Equation (2), page 17): $d_t = (y_t - \mu)\,\Sigma^{-1}\,(y_t - \mu)'$, where d_t is the turbulence for a particular time period t, y_t the vector of asset returns for period t, μ the sample average vector of historical returns, and Σ the sample covariance matrix of historical returns. This turbulence index captures the extent to which one or more of the returns was unusually high or low. In our work, d_t is calculated at daily frequency within each 3-year rolling window. The 1% (5%) highest d_ts are used to select the 1% (5%) most turbulent dates in the whole sample period.

Insert Figures 5 and 6 here

When predicting the date with maximum market return increase, AR_{Std} behaves in a similar way as it does for predicting the events with the most turbulence and maximum decrease, i.e., it increases. In contrast, SDR_{Std} starts to decrease 22 (8) days before the 1% (5%) maximum market return is realized. Therefore, SDR_{Std} does not only predict turbulent or falling financial market but also forecasts future market increases.

As also documented in these figures, $\mathrm{Bcov}_{Down,Std}$ performs similar to SDR_{Std} and AR_{Std} in predicting the events with high turbulence and low returns. In predicting the largest return increases, $\mathrm{Bcov}_{Down,Std}$ fails to compete with SDR_{Std}: $\mathrm{Bcov}_{Down,Std}$ only predicts the 5% extreme market increases (8 days ahead) but does not predict the 1% extreme market increases.

4.3 Trading Strategy Based on Sign Prediction

An intuitive way to assess the out-of-sample predictive ability of the various indicators is to consider an experiment in which the trading strategy is based on the ability to predict the sign of future returns. We adopt a strategy similar to the one proposed by Kritzman, Li, Page, and Rigobon (2011). The investor can invest a fraction w of her wealth into the S&P 500 and $1-w$ into the risk-free asset. The trading strategy is defined as follows:

Trading Signal	**Portfolio weights**	
	S&P 500	Risk-Free Asset
$X_{Std} < -\theta\ \sigma_{2Y}$	$w = 2$	$1 - w = -1$
$X_{Std} > +\theta\ \sigma_{2Y}$	$w = -1$	$1 - w = 2$
else	$w = 0.5$	$1 - w = 0.5$

where X_{Std} is the standardized indicator representing any of the indicators previously defined. This trading rule means that, when the difference between the 3-month average of the indicator and the 2-year average of the indicator is higher than θ times the indicator's 2-year standard deviation, i.e., X_{Std} is higher than $\theta\ \sigma_{2Y}$, the investor short sells one dollar of the risky asset and buys two dollars of the risk-free asset. When X_{Std} is lower than $-\theta\ \sigma_{2Y}$, the investor buys two dollars of the risky asset and short sells one dollar of the risk-free asset. When $-\theta\ \sigma_{2Y} \leq X_{Std} \leq \theta\ \sigma_{2Y}$, the investor simply implements a naive trading strategy, in which she invests 50% in the risky asset and 50% in risk-free asset. We assume a 1% transaction fee for all trading strategies.

The trading signal and the corresponding investment are plotted in Figures 7 and 8 when $\theta = 1$ and 0.5, respectively. Two results are worth emphasizing. First, the number of trades is relatively low for $\theta = 1$ (in 26 years, 13 trades with SDR, 26 trades with AR). It remains below two trades per year with $\theta = 0.5$ (in 26 years, 47 trades with SDR, 52 trades with AR). Second, SDR_{Std} successfully forecasts, 1 month ahead, the Russian default in August 1998 and signals that the risky asset should be sold; SDR_{Std} and AR_{Std} both predict the subprime crisis. However, in contrast to AR_{Std}, SDR_{Std} offers an efficient signal to buy the risky asset when there is a stock market rebound after the 2007-2008 market crash.

Insert Figures 7 and 8 here

Figure 9 documents the cumulative wealth generated by the four trading strategies. SDR_{Std} outperforms the other three, particularly during the recent subprime crisis. The strategy based on SDR_{Std} generates the highest annualized Sharpe ratio: 0.473 for a signal threshold of $\theta = 1$ and 0.365 for a threshold of $\theta = 0.5$. The ratio is 81% and 179% larger than the Sharpe ratio obtained by AR_{Std} (0.262 and 0.131) and 31% and 1.1% larger than the Sharpe ratio obtained by the simple buy-and-hold strategy (0.361). The final cumulative return (net of transaction fees) obtained with SDR_{Std} strategy is 468% for $\theta = 1$ and 707% for $\theta = 0.5$, which is much higher than the cumulative return obtained with AR_{Std} (254% and 202%, respectively) and with the 50/50 buy-and-hold strategy (249%). The annual return is 5.9% with $\theta = 1$ and 7.5% with $\theta = 0.5$ with the SDR strategy, but 3.5% and 2.6% with the AR strategy and 3.4% with the 50/50 buy-and-hold strategy.

It is noteworthy that, before the subprime crisis, the strategy based on $\mathrm{Bcov}_{Down,Std}$ (with $\theta = 1$) outperforms the others by generating the highest cumulative wealth. However, after 2008, the strategy based on SDR_{Std} significantly outperforms $\mathrm{Bcov}_{Down,Std}$. The cumulative wealth from strategies based on SDR_{Std} and $\mathrm{Bcov}_{Down,Std}$ experience a significant positive increase when the others suffer a loss in their cumulative wealth. When the trading strategy is implemented with $\theta = 0.5$, the strategy based on SDR_{Std} outperforms $\mathrm{Bcov}_{Down,Std}$ most of the time during the entire sample period. During the 1997 Asian crisis and 1998 Russian default period, and after July 2008, the strategy based on SDR_{Std} significantly outperforms $\mathrm{Bcov}_{Down,Std}$.

Insert Figure 9 here

In terms of the risk-adjusted performance, the strategy based on SDR_{Std} generates the highest Sharpe ratio (0.473 with $\theta = 1$ and 0.365 with $\theta = 0.5$), while the strategy based on the Downside-Beta Comovement generates the second largest Sharpe ratio (0.459 and 0.355, respectively).

To summarize, SDR_{Std} outperforms the other indicators in forecasting large market movements, particularly during the recent subprime crisis. We also considered a 3% transaction cost and the result still supported our conclusion because the trading strategies we consider do not generate a large number of trades, and the strategy based on SDR_{Std} triggers the least portfolio rebalances.

4.4 Risk-Adjusted Performance Evaluation of the SDR Strategy

To investigate whether the performance of the SDR_{Std} strategy is driven by the exposure to some risk factors, we apply the approach frequently used in the literature on performance attribution (see, e.g., Carhart, 1997). We regress the return generated by the SDR_{Std} strategy on several risk factors as follows:

$$R_{SDR,t} = \alpha + \beta_F' F_t + \epsilon_t,$$

where $R_{SDR,t}$ represents the daily return generated from implementing the SDR-based trading strategy. In addition, F_t represent the five following factors: the three Fama and French (1993) factors, i.e., the MRP (the return on the value-weighted market portfolio in excess of the 1-month T-bill rate), the SMB (the difference in returns on a portfolio of small-capitalization and large-capitalization stocks), the HML (the difference in returns on a portfolio of high and low book-equity-to-market-equity stocks), the momentum factor UMD proposed by Jegadeesh and Titman (1993) (the difference in returns on a portfolio of winning and losing stocks), and the correlation risk premium (CR) from Buraschi, Kosowski, and Trojani (2014).[10] A positive alpha from this regression means that applying the SDR_{Std} strategy will generate investment profits in addition to the compensation received from exposure to systematic risks.

The results in Table 3 indicate that the SDR_{Std} strategy exhibits statistically significant alpha during the full sample period from 1986 to 2011, even after controlling for various risk factors. The estimates of α show that annual excess returns generated from implementing SDR_{Std} strategy range from 3.7% to 4.5% while considering various sets of standard risk factors. Furthermore, the adjusted R-square statistics range from 0.21% to 0.24%, suggesting that the return of the SDR_{Std} strategy bears little factor risk.

Admittedly, SDR index shares similar property with the correlation risk measure because they both provide information regarding the interconnection between firms. However, the SDR index measures the asymmetric beta comovements while considering downside and upside markets, whereas the CR factor measures the correlation risk among S&P 500 firms while considering physical and risk-neutral distributions. Therefore, we expect that the return generated from SDR_{Std} strategy will not be fully explained by the correlation risk. To investigate this issue, we add the CR factor to the regression. As the series is available from 1996 to 2011, Panel B reports the corresponding regressions over this sample. We observe that there is little amount of SDR_{Std} return that can be explained by the correlation risk factor. As the table shows, after controlling the correlation risk, the alpha increases to the largest value 5.2%; also, the β coefficient on correlation risk is not significantly different from 0 (with a t-statistic equal to 0.309). In fact, the correlation between the return of the SDR_{Std} strategy and correlation risk premium is only -0.0018.

Insert Table 3 here

[10] In Buraschi, Kosowski, and Trojani (2014), the correlation risk is measured using a time-series of market prices for the average correlation of S&P 500 stocks. The correlation risk is measured using a unique data set of actual correlation swap quotes. The correlation risk premium is equal to the difference between the realized and the implied swap rates. Empirically, the correlation risk premium is negative. The reason is that when investors' disagreement increases, the risk-neutral correlation is more likely to increase, as proposed by Buraschi, Trojani, and Vedolin (2014). The data on correlation risk can be found at http://www.alphacruncher.com/data.

5 Forecasting Market Return in Predictive Regressions

This section investigates the ability of the SDR index to predict market return, using a forecasting regression analysis as proposed by Goyal and Welch (2008) and Ferreira and Santa-Clara (2011). In addition to the SDR, Bcov_{Down}, and AR indicators, we also consider the Net Equity Expansion, which was found to be a good predictor of market return in previous literature by Goyal and Welch (2008) and Ferreira and Santa-Clara (2011). It is defined here as the ratio of the 3-year moving sum of net issues by NYSE-listed stocks divided by the total end-of-3-year market capitalization of NYSE stocks. This is the dollar amount (less dividends) of net equity issuing activity, such as IPOs, for NYSE listed stocks, computed from CRSP data as follows:[11]

$$\text{Net Equity Expansion}_t = \text{Mcap}_t - \text{Mcap}_{t-1}(1 + \text{vwretx}_t), \tag{9}$$

where Mcap is the total market capitalization and vwretx is the value weighted return (excluding dividends) on the NYSE index.[12]

We also investigate the predictability of Beta Asymmetry (Beta_{Asy}) and Downside Beta (Beta_{Down}), which are, respectively, defined as follows:

$$\text{Beta}_{Asy} \equiv \text{mean}(\beta^{(u)} - \beta^{(1-u)}) = \frac{1}{N_U N} \sum_{u \in U} \sum_{i=1}^{N} \left(\beta_i^{(u)} - \beta_i^{(1-u)} \right),$$

$$\text{Beta}_{Down} \equiv \text{mean}(\beta^{(u)}) = \frac{1}{N_U N} \sum_{u \in U} \sum_{i=1}^{N} \beta_i^{(u)}.$$

These indicators are inspired by Ang and Chen (2002) and Ang, Chen, and Xing (2006), who show that asymmetry in betas is an important factor in the cross-section of firms' returns. Standardized versions of Beta_{Asy} and Beta_{Down} are defined as before using Equation (8).[13]

[11] Our measure of Net Equity Expansion is similar as in Goyal and Welch (2008) except that they use 1-year moving sums and the total end-of-year market capitalization to obtain the Net Equity Expansion. We take 3 years to be consistent with the length of the window used for estimating the conditional betas.

[12] The Technical Appendix provides additional details on the Net Equity Expansion.

[13] We did not report in Section 4 the estimates based on these indicators for the prediction of large movements of market return, because their performance was very low. The Technical Appendix reports details on this analysis.

5.1 Predictive Regressions

5.1.1 Full Sample Analysis

We regress the market return on the lagged predictor as follows:

$$R_{m,t+1} = \gamma + \delta \, \mathbf{X}_t + \epsilon_{t+1}, \qquad t = 1, \cdots, T, \tag{10}$$

where $R_{m,t+1}$ is the daily (monthly) market return at day (month) $t+1$ and ϵ_{t+1} is the error term. $\mathbf{X}_t$ represents the standardized version of the SDR, Downside-Beta Comovement (Bcov_{Down}), Beta Asymmetry (Beta_{Asy}), Downside Beta (Beta_{Down}), and Absorption Ratio, and the level of the Net Equity Expansion. Based on our previous results from large market movements (Section 4.2), we suppose that a high level of SDR_{Std}, $\text{Bcov}_{Down,Std}$, and AR_{Std} predict decreasing market returns ($\delta < 0$).

Table 4 reports the results of the time-series regressions. The coefficients of the SDR_{Std} and $\text{Bcov}_{Down,Std}$ exhibit the expected negative sign and are statistically significant at a 5% significance level with daily market return and at a 1% significance level with monthly market return. Parameter estimates indicate that a one-unit increase in the SDR_{Std} ($\text{Bcov}_{Down,Std}$) is associated with a 0.12% (0.11%) decrease in the future daily market return and a 1.52% (1.15%) decrease in the future monthly market return. Coefficients of AR_{Std} also exhibit the expected negative sign but it is only statistically significant at a 10% significance level while predicting monthly market returns. The coefficient of the Net Equity Expansion has the expected positive sign, although it is not statistically significant. We also notice that the Beta Asymmetry and the Downside Beta do not help at predicting future daily or monthly market return.

Insert Table 4 here

In terms of predictive power, SDR_{Std} and $\text{Bcov}_{Down,Std}$ have by far the highest adjusted R-square. For instance, the values are 2.21% and 1.69%, respectively, at monthly frequency. AR_{Std} and Net Equity Expansion have lower predictive ability, with adjusted R-square values of 0.39% and 0.36%, respectively. The performance of the Net Equity Expansion is weaker than reported in Ferreira and Santa-Clara (2011), maybe because our results are based on a shorter, more recent, sample (1986–2011 instead of 1947–2007). It is plausible that the post subprime crisis period has led to a significant change in predictive regressions.

We also divided the entire period into two subsamples: 1986–1999 and 2000–2011, and ran the same regression tests. Results reported in the Technical Appendix confirm that SDR_{Std} performs consistently better in predicting market returns in both sample periods. Interestingly, SDR_{Std} also performs better in the more recent sample period (2000–2011) with a large gain for predicting monthly returns (the Adjusted R-square increases from 0.68% to 4.51%). To summarize, time-series regressions using the whole sample and subsamples both indicate that SDR_{Std} outperforms the other variables in forecasting daily and monthly market returns.[14]

5.1.2 Out-of-sample Analysis

In-sample analysis does not provide an evaluation of the performance of the various variables in terms of out-of-sample prediction and asset allocation. In accordance with Ferreira and Santa-Clara (2011), we use a sequence of expanding windows to obtain out-of-sample forecasts of the stock market return. The window length is denoted by $s = s_0, \cdots, T-1$, where T is the total number of observations in the whole sample. For the first regression, we take a subsample of the first s_0 observations $t = 1, ..., s_0$ which is called the burning period:

$$R_{m,t+1} = \gamma + \delta\ \mathbf{X}_t + \epsilon_{t+1}, \qquad t = 1, \cdots, s_0.$$

By increasing the sample size s from s_0 to $T-1$, we generate a sequence of $(T-s_0)$ out-of-sample return forecasts based on the information available up to time s:

$$\hat{\mu}_s = E_s[R_{m,s+1}] = \hat{\gamma} + \hat{\delta} R_s.$$

This process mimics the way a forecast would have been achieved in practice.

We evaluate the performance of the competing indicators in the forecasting exercise using an out-of-sample R-square (Goyal and Welch, 2008; Ferreira and Santa-Clara, 2011). This measure compares the predictability of the regression with the historical sample mean return:

$$R^2 = 1 - \frac{MSE_p}{MSE_M}, \tag{11}$$

[14] Empirical results for subsample regression analysis are available in the Technical Appendix.

where $MSE_p = \frac{1}{T-s_0}\sum_{s=s_0}^{T-1}(R_{m,s+1} - \hat{\mu}_s)^2$ is the mean square error of the out-of-sample predictions from the model and $MSE_M = \frac{1}{T-s_0}\sum_{s=s_0}^{T-1}(R_{m,s+1} - \bar{R}_s)^2$ is the mean square error of the historical sample mean, where $\bar{R}_s$ is the historical mean of market returns up to time s.[15] The out-of-sample R-square takes positive (negative) values when the model predicts returns better (worse) than the historical mean. As reported in Table 5, the standardized SDR (SDR$_{Std}$) always generates a positive out-of-sample R-square, implying that the SDR$_{Std}$ can effectively outperforms the historical mean for forecasting future market movements. The out-of-sample R-square (compared with a strategy using historical mean) is higher than 0.101% with daily data and 2.272% with monthly data (for the case with an initial sample of $s_0 = 10$ years).

We notice that Bcov$_{Down,Std}$ performs better than SDR$_{Std}$ at daily frequency but worse than SDR$_{Std}$ at monthly frequency. For all other indicators, we find a negative R-square for daily and monthly market returns.

Insert Table 5 here

5.2 Strategy Based on Return Prediction

As in Ferreira and Santa-Clara (2011), we consider simple out-of-sample trading strategies based on predictive regressions, which combine the stock market with the risk-free asset (1-month Treasury bill). Each period, predictions of market returns are used to calculate the Markowitz optimal weight on stock market:

$$w_s = \frac{\hat{\mu}_s - R_{f,s}}{\gamma\hat{\sigma}_s^2}, \tag{12}$$

where $\hat{\mu}_s$ is the expected market return obtained from the predictive regressions, $R_{f,s}$ is the risk-free rate from s to $s+1$, γ is the risk aversion (assumed to be equal to 2), and $\hat{\sigma}_s^2$ is the variance of stock market returns, estimated using all the available data up to time s. Portfolio decisions could have been made in real time with data available at the time of the decision. The ex-post portfolio return is calculated at the end of each period as follows:

$$R_{p,s+1} = w_s R_{m,s+1} + (1 - w_s)R_{f,s}. \tag{13}$$

[15] The specification and description follow Ferreira and Santa-Clara (2011).

After iterating this process until the end of the sample ($T-1$), we obtain a time series of ex-post returns for each optimal portfolio. We use two measures to evaluate the performance of the trading strategies, the Sharpe ratio (SR), and the certainty equivalent (CE) return, which are defined as follows:

$$\text{SR} = \frac{\bar{R}_p - \bar{R}_f}{\sigma_p} \qquad \text{and} \qquad CE = \bar{R}_p - \frac{\gamma}{2}\sigma_p^2, \tag{14}$$

where $\bar{R}_p$ is the sample mean of the portfolio return and σ_p^2 is the sample variance of the portfolio return. The Sharpe ratio measures the risk-adjusted performance of the strategy, whereas the certainty equivalent return is the risk-free return that a mean-variance investor (with risk aversion γ) would consider equivalent to investing in the strategy.

Table 5 also reports the (annualized) Sharpe ratio gains and the (annualized) certainty equivalent gains relative to investing in the optimal portfolio based on the historical market return mean. We find that forecasting with SDR_{Std} always leads to Sharpe ratio gains relative to using historical mean (0.193 per year at daily frequency and 0.206 at monthly frequency, using a burning period of 10 years). It also generates certainty equivalent gains (10.5% per year at a daily frequency and 9.6% at a monthly frequency). Compared with the standardized Absorption Ratio (AR_{Std}), SDR_{Std} always generates higher certainty equivalent gains and Sharpe ratio gains.

The indicator $\text{Bcov}_{Down,Std}$ also performs well in the out-of-sample analysis, as when using a burning period of 10 years it even dominates the SDR_{Std} in predicting daily market returns by offering a higher out-of-sample R-square (0.151%), a higher Sharpe ratio gain (0.300), and a higher certainty equivalent gain (0.171). In contrast, in predicting monthly market return, SDR_{Std} outperforms $\text{Bcov}_{Down,Std}$ and all the other predictors.

5.3 Risk-Adjusted Performance Evaluation of the SDR Strategy

As in Section 4.4, we now perform an evaluation of the risk-adjusted performance of the various strategies. In Tables 6 and 7, we only report the results of the regressions based on a burning period of $s_0 = 10$ years. The main results below are not altered for $s_0 = 5$ years. We start with the strategy based on the prediction of the next-day market return (Table 6). We notice that the estimate of the alpha, i.e., the risk-adjusted performance of the strategy, is approximately 20% per year over the 1986–2011 period, and 15% per year over the 1996-2011 period. The SDR strategy has some exposure to the market risk: the parameter β_{MRP} is highly significant, although its magnitude is relatively limited (between 0.17 and 0.22). The strategy is also positively exposed to the HML factor. Over the more recent period, for which the correlation risk factor is available, we find that the SDR strategy is negatively exposed to this factor.

This result suggests the following interpretation. Both SDR and CR increase during market downturns. Table 4 documents a significant negative relation between SDR and future market return. Then, the predicted market return is equivalent to the return generated from shorting the SDR. In Table 6, the negative CR risk exposure implies that the SDR predicted market return is *positively* related to a strategy of shorting correlation risk. However, after introducing CR, the alpha from the controlled SDR predicted return remain significantly positive, which supports that most of the SDR predicted return cannot be explained by correlation risk.

If we now turn to the strategy based on the prediction of the next-month market return (Table 7), we find that the estimated α is lower (approximately 12% per year over the 1986-2011 period and 8.7% per year over the 1996-2011 period), although it is still highly significant. As for daily market return forecasts, the SDR strategy is exposed to market risk and HML factors, positively in both cases. It also has a significant negative exposure to the correlation risk factor.

Insert Tables 6 and 7 here

6 Conclusion

This paper builds a conditional market model that reconciles the conflicting evidence of firm-level and market-level skewness. Even when individual skewness is positive on average, cross-sectional firm information, i.e., comovements among betas, successfully explains the negative market skewness. As a main contribution to the literature, we provide a direct channel for the major impact on negative market skewness. In addition we quantify this effect through a new measure, named "Systematic Downside Risk" (SDR), which measures the asymmetry between Downside-Beta Comovement and Upside-Beta Comovement. Both static and dynamic analyses strongly support the ability of our model to reproduce market skewness. To our best knowledge, we are the first to explain negative market skewness through downside risk modeling.

Based on the empirical analysis, we further investigate the ability of the SDR to predict the crashes and booms included in our sample. We conclude that, in addition to being a good proxy to explain the negatively skewed market return, the SDR can be used to predict future stock market movements. This predictive ability is confirmed by the design of profitable trading strategies based on the SDR. After applying a performance evaluation on the returns generated from the SDR strategy, we confirm that the SDR strategy takes on minimal factor risks. We also show, using predictive regressions that SDR is a good predictor of future market returns, not only for crashes but also for booms. It compares favorably with the Net Equity Expansion, which has been identified as a good predictor of market return.

The SDR index can be used at least in two different contexts. First, it is an interesting tool for regulators and financial market authorities, as it is able to predict extreme changes in market prices, such as crashes and booms. Our estimates indicate that the SDR begins to increase approximately half year before extreme negative returns and to decrease approximately three weeks before extreme positive returns (1% quantile). Second, SDR is also a good predictor of future market returns, with an out-of-sample R-square above 2% for next-month market return. A market timing strategy based on the SDR would improve the Sharpe ratio by approximately 0.2 relative to the historical mean and 0.3 relative to the Net Equity Expansion.

References

Albuquerque, Rui, 2012, Skewness in stock returns: Reconciling the evidence on firm versus aggregate returns, *Review of Financial Studies* 25, 1630–1673.

Ang, Andrew, and Joseph Chen, 2002, Asymmetric correlations of equity portfolios, *Journal of Financial Economics* 63, 443–494.

——— , and Yuhang Xing, 2006, Downside risk, *Review of Financial Studies* 19, 1191–1239.

Arezki, Rabah, Bertrand Candelon, and N.R. Amadou Sy, 2011, Sovereign rating news and financial markets spillovers: Evidence from the european debt crisis, *IMF working paper* 1–27.

Bawa, Vijay S., and Eric B. Lindenberg, 1977, Capital market equilibrium in a mean-lower partial moment framework, *Journal of Financial Economics* 5, 189–200.

Billio, Monica, Mila Getmansky, Andrew W. Lo, and Loriana Pelizzon, 2012, Econometric measures of connectedness and systemic risk in the finance and insurance sectors, *Journal of Financial Economics* 104, 535–559.

Black, Fischer, 1976, Studies of stock price volatility changes, *Proceedings of the 1976 Meetings of the American Statistical Association,* 171–181.

Buraschi, Andrea, Robert Kosowski, and Fabio Trojani, 2014, When there is no place to hide: Correlation risk and the cross-section of hedge fund returns, *Review of Financial Studies* 27, 581–616.

Buraschi, Andrea, Fabio Trojani, and Andrea Vedolin, 2014, When uncertainty blows in the orchard: Comovement and equilibrium volatility risk premia, *Journal of Finance* 69, 101–137.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of finance* 52, 57–82.

Christie, Andrew A., 1982, The stochastic behavior of common stock variances: Value, leverage and interest rate effects, *Journal of Financial Economics* 10, 407–432.

Duffee, Gregory R., 1995, Stock returns and volatility a firm-level analysis, *Journal of Financial Economics* 37, 399–420.

——— , 2002, Balance sheet explanations for asymmetric volatility, *University of California at Berkeley working paper.*

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Ferreira, Miguel A., and Pedro Santa-Clara, 2011, Forecasting stock market returns: The sum of the parts is more than the whole, *Journal of Financial Economics* 100, 514–537.

French, Kenneth R., G. William Schwert, and Robert F. Stambaugh, 1987, Expected stock returns and volatility, *Journal of Financial Economics* 19, 3–29.

Goyal, Amit, and Ivo Welch, 2008, A comprehensive look at the empirical performance of equity premium prediction, *Review of Financial Studies* 21, 1455–1508.

Grullon, Gustavo, Evgeny Lyandres, and Alexei Zhdanov, 2012, Real options, volatility, and stock returns, *Journal of Finance* 67, 1499–1537.

Hong, Harrison, and Jeremy C. Stein, 2003, Differences of opinion, short-sales constraints, and market crashes, *Review of Financial Studies* 16, 487–525.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Kritzman, Mark, Yuanzhen Li, Sébastien Page, and Roberto Rigobon, 2011, Principal components as a measure of systemic risk, *Journal of Portfolio Management* 37, 112–126.

Longin, Francois, and Bruno Solnik, 2001, Extreme correlation of international equity markets, *Journal of Finance* 56, 649–676.

Pindyck, Robert S., 1984, Risk, inflation, and the stock market, *American Economic Review* 74, 335–351.

A Appendix: Proof of Proposition 1

In the conditional market model, firm i's excess return is:

$$R_{i,t} - \mu_i = \beta_i^- (R_{m,t} \ - \mu_m \ \mid R_{m,t} < 0) + \beta_i^+ (R_{m,t} \ - \mu_m \ \mid R_{m,t} \geq 0) + \epsilon_{i,t}.$$

Using obvious notations, we rewrite this formula into:

$$R_{i,t} - \mu_i = \beta_i^- (R_{m,t}^- - \mu_m^-) + \beta_i^+ (R_{m,t}^+ - \mu_m^+) + \epsilon_{i,t},$$

with $E[\epsilon_{i,t}] = E[\epsilon_{i,t}(R_{m,t}^- - \mu_m^-)] = E[\epsilon_{i,t}(R_{m,t}^+ - \mu_m^+)] = 0$.

Standardized market skewness can be decomposed as follows:

$$\begin{aligned}
sk_m \ &\equiv \ E\left[\left(\frac{R_{m,t} - \mu_m}{\sigma_m}\right)^3\right] = \left(\frac{1}{\sigma_m}\right)^3 E\left[\left(\sum_{i=1}^{N} \omega_i R_{i,t} - E\left[\sum_{i=1}^{N} \omega_i R_{i,t}\right]\right)^3\right] \\
&= \ \left(\frac{1}{\sigma_m}\right)^3 E\left[\sum_{i=1}^{N} \omega_{iii} \left(R_{i,t} - \mu_i\right)^3 + 3 \ \sum_{i=1}^{N} \sum_{j \neq i}^{N} \omega_{ijj} \left(R_{i,t} - \mu_i\right) \times \left(R_{j,t} - \mu_j\right)^2 \right. \qquad \text{(A.1)} \\
&+ \ 2 \ \left. \sum_{i=1}^{N} \sum_{j \neq i}^{N} \sum_{k \neq i, k \neq j}^{N} \omega_{ijk} \left(R_{i,t} - \mu_i\right) \times \left(R_{j,t} - \mu_j\right) \times \left(R_{k,t} - \mu_k\right)\right].
\end{aligned}$$

The individual terms in Equation (A.1) are given by:

$$\begin{aligned}
R_{i,t} - \mu_i \ &= \ \beta_i^- \left(R_{m,t}^- - \mu_m^-\right) + \beta_i^+ \left(R_{m,t}^+ - \mu_m^+\right) + \epsilon_{i,t}, \\
\left(R_{i,t} - \mu_i\right)^2 \ &= \ (\beta_i^-)^2 \left(R_{m,t}^- - \mu_m^-\right)^2 + 2\beta_i^- \left(R_{m,t}^- - \mu_m^-\right) \epsilon_{i,t} \\
&+ \ (\beta_i^+)^2 \left(R_{m,t}^+ - \mu_m^+\right)^2 + 2\beta_i^+ \left(R_{m,t}^+ - \mu_m^+\right) \epsilon_{i,t}, \\
\left(R_{i,t} - \mu_i\right)^3 \ &= \ (\beta_i^-)^3 \left(R_{m,t}^- - \mu_m^-\right)^3 + 3(\beta_i^-)^2 \left(R_{m,t}^- - \mu_m^-\right)^2 \epsilon_{i,t} + 3\beta_i^- \left(R_{m,t}^- - \mu_m^-\right) \epsilon_{i,t}^2 \\
&+ \ (\beta_i^+)^3 \left(R_{m,t}^+ - \mu_m^+\right)^3 + 3(\beta_i^+)^2 \left(R_{m,t}^+ - \mu_m^+\right)^2 \epsilon_{i,t} + 3\beta_i^+ \left(R_{m,t}^+ - \mu_m^+\right) \epsilon_{i,t}^2 + \epsilon_{i,t}^3.
\end{aligned}$$

After taking the expectation of $\left(R_{i,t} - \mu_i\right)^3$ and using the conditions on the error terms, we have:

$$E\left[\left(R_{i,t}-\mu_i\right)^3\right] = (\beta_i^-)^3 E\left[\left(R_{m,t}^- - \mu_m^-\right)^3\right] + (\beta_i^+)^3 E\left[\left(R_{m,t}^+ - \mu_m^+\right)^3\right] + \epsilon_{i,t}^3. \tag{A.2}$$

Similarly, we obtain the following expressions for the cross-product expectations:

$$\begin{aligned} &E\left[\left(R_{i,t}-\mu_i\right)\times\left(R_{j,t}-\mu_j\right)^2\right] \\ &\qquad = \beta_i^-(\beta_j^-)^2 E\left[\left(R_{m,t}^- - \mu_m^-\right)^3\right] + \beta_i^+(\beta_j^+)^2 E\left[\left(R_{m,t}^+ - \mu_m^+\right)^3\right] + \epsilon_{i,t}\epsilon_{j,t}^2, \end{aligned} \tag{A.3}$$

and

$$\begin{aligned} &E\left[\left(R_{i,t}-\mu_i\right)\times\left(R_{j,t}-\mu_j\right)\times\left(R_{k,t}-\mu_k\right)\right] \\ &\qquad = \beta_i^-\beta_j^-\beta_k^- E\left[\left(R_{m,t}^- - \mu_m^-\right)^3\right] + \beta_i^+\beta_j^+\beta_k^+ E\left[\left(R_{m,t}^+ - \mu_m^+\right)^3\right] + \epsilon_{i,t}\epsilon_{j,t}\epsilon_{k,t}. \end{aligned} \tag{A.4}$$

After combining Equations (A.2)–(A.4) and simplifying some terms, we find:

$$\begin{aligned} sk_m &= \left(\frac{1}{\sigma_m}\right)^3 E\left[\sum_{i=1}^N \omega_{iii}(\beta_i^-)^3\left(R_{m,t}^- - \mu_m^-\right)^3 + \sum_{i=1}^N \omega_{iii}(\beta_i^+)^3\left(R_{m,t}^+ - \mu_m^+\right)^3\right. \\ &+ 3\sum_{i=1}^N\sum_{j\neq i}^N \omega_{ijj}\beta_i^-(\beta_j^-)^2\left(R_{m,t}^- - \mu_m^-\right)^3 + 3\sum_{i=1}^N\sum_{j\neq i}^N \omega_{ijj}\beta_i^+(\beta_j^+)^2\left(R_{m,t}^+ - \mu_m^+\right)^3 \\ &+ \left. 2\sum_{i=1}^N\sum_{j\neq i}^N\sum_{k\neq\{i,j\}}^N \omega_{ijk}\beta_i^-\beta_j^-\beta_k^-\left(R_{m,t}^- - \mu_m^-\right)^3 + 2\sum_{i=1}^N\sum_{j\neq i}^N\sum_{k\neq\{i,j\}}^N \omega_{ijk}\beta_i^+\beta_j^+\beta_k^+\left(R_{m,t}^+ - \mu_m^+\right)^3\right] \\ &+ \frac{F\left(\epsilon_t\right)}{\sigma_m^3}, \end{aligned}$$

where $F\left(\epsilon_t\right)$ is defined as:

$$F\left(\epsilon_t\right) = E\left[\sum_{i=1}^N \omega_{iii}\epsilon_{i,t}^3 + 3\sum_{i=1}^N\sum_{j\neq i}^N \omega_{ijj}\epsilon_{i,t}\epsilon_{j,t}^2 + 2\sum_{i=1}^N\sum_{j\neq i}^N\sum_{k\neq\{i,j\}}^N \omega_{ijk}\epsilon_{i,t}\epsilon_{j,t}\epsilon_{k,t}\right].$$

Finally, if we normalize centered returns by the appropriate volatility to define downside and upside market skewness, we find the following expression for the market skewness:

$$
\begin{aligned}
sk_m &= E\left[\sum_{i=1}^{N}\omega_{iii}(\beta_i^-)^3\left(\frac{\sigma_m^-}{\sigma_m}\right)^3 sk_m^- + \sum_{i=1}^{N}\omega_{iii}(\beta_i^+)^3\left(\frac{\sigma_m^+}{\sigma_m}\right)^3 sk_m^+\right. \qquad \text{(A.5)}\\
&+ 3\sum_{i=1}^{N}\sum_{j\neq i}^{N}\omega_{ijj}\beta_i^-(\beta_j^-)^2\left(\frac{\sigma_m^-}{\sigma_m}\right)^3 sk_m^- + 3\sum_{i=1}^{N}\sum_{j\neq i}^{N}\omega_{ijj}\beta_i^+(\beta_j^+)^2\left(\frac{\sigma_m^+}{\sigma_m}\right)^3 sk_m^+\\
&+ \left. 2\sum_{i=1}^{N}\sum_{j\neq i}^{N}\sum_{k\neq\{i,j\}}^{N}\omega_{ijk}\beta_i^-\beta_j^-\beta_k^-\left(\frac{\sigma_m^-}{\sigma_m}\right)^3 sk_m^- + 2\sum_{i=1}^{N}\sum_{j\neq i}^{N}\sum_{k\neq\{i,j\}}^{N}\omega_{ijk}\beta_i^+\beta_j^+\beta_k^+\left(\frac{\sigma_m^+}{\sigma_m}\right)^3 sk_m^+\right]\\
&+ \frac{F\left(\epsilon_t\right)}{\sigma_m^3}.
\end{aligned}
$$

Table 1: Individual Firm-level and Market Skewness

This table provides summary statistics for firm-level and market skewness based on the whole sample. Panel A reports measures of firm-level skewness. Cross-sectional statistics include the cross-firm average of firm skewness (sk_i) with equal weights (Mean (EW)) and with value weights (Mean (VW)), the 25th percentile, the median, and the 75th percentile. Time-series statistics include the time-series average of cross sectional skewness ($cssk_t$) with equal weights (Mean (EW)) and with value weights (Mean (VW)), the 25th percentile, median, and 75th percentile. Panel B reports measures of market-level skewness based on equally- and value-weighted sample market return ($sk(R_{m,t})$) and equally- and value-weighted CRSP return ($sk(R_{m,t}^{CRSP})$). Panel C reports the contributions of the various terms described in Equation (5). Sample: 3rd January 1983 through 30th December 2011.

	Mean (EW)	**Mean (VW)**	**25th percentile**	**Median**	**75th percentile**
Panel A: Statistics on firm-level skewness					
(i) Cross-section statistics on firm skewness:					
sk_i	0.184	0.177	0.072	0.183	0.296
(ii) Time-series statistics on cross-section skewness:					
$cssk_t$	0.367	0.338	0.120	0.372	0.624
Panel B: Statistics on market skewness					
$sk(R_{m,t})$	-0.111	-0.367			
$sk(R_{m,t}^{CRSP})$	-0.689	-0.675			
Panel C: Contributors to model-implied skewness					
bcov-cov part	-0.133	-0.349			
+ *bco-vol* part	0.024	-0.0036			
+ $beta^3$ part	1.058×10^{-7}	1.557×10^{-6}			
= Model Implied skewness	-0.110	-0.353			

Table 2: Static Decomposition for Market Skewness at Quantile Level

This table presents the quantile-based constituents of market skewness defined in Equation (5). These constituents are the market skewness, sk_m, the relative market variance, $(\sigma_m^{(u)})^2/\sigma_m^2$, the cross-sectional mean and variance of the conditional betas, $\bar{\beta}^{(u)}$ and $(\sigma_\beta^{(u)})^2$, respectively, and the three components of the model-implied market skewness: *beta*^{3} denotes $\sum_{i=1}^{N}(\beta_i^{(u)})^3/N^3$, *bco-vol* denotes $\sum_{i=1}^{N}\sum_{j\neq i}^{N}\beta_i^{(u)}(\beta_j^{(u)})^2/N^3$, and *bco-cov* denotes $\sum_{i=1}^{N}\sum_{j\neq i}^{N}\sum_{k\neq\{i,j\}}^{N}\beta_i^{(u)}\beta_j^{(u)}\beta_k^{(u)}/N^3$. u denotes the days when market returns are below u quantile, and $1-u$ when market returns are above (or equal to) $1-u$ quantile where $u \in (0.2, 0.3, 0.4, 0.5)$. Sample: 3rd January 1983 through 30th December 2011.

Percentile	sk_m	$(\sigma_m^{(u)}/\sigma_m)^2$	$\bar{\beta}^{(u)}$	$(\sigma_\beta^{(u)})^2$	*beta*^{3} ($\times 10^{-8}$)	*bco-vol* ($\times 10^{-3}$)	*bco-cov*
20th	−2.552	0.934	0.985	0.249	0.001	0.121	0.955
30th	−2.556	0.774	0.985	0.234	0.001	0.120	0.954
40th	−2.438	0.688	0.986	0.229	0.001	0.120	0.957
50th	−2.405	0.631	0.988	0.228	0.001	0.120	0.961
50th	2.644	0.527	0.958	0.397	4.044	1.639	0.865
60th	2.686	0.580	0.954	0.423	5.510	1.948	0.851
70th	2.660	0.671	0.951	0.451	7.951	2.434	0.838
80th	2.721	0.891	0.947	0.495	12.904	3.261	0.823

Table 3: Exposure of the SDR Strategy to Risk Factors (Signal Strategy)

The table presents the results of time series regressions. The dependent variable is the excess return of the SDR_{Std} trading strategy (in excess of the 1-month T-bill rate), when the trading signal threshold $\theta = 1$. The independent variables include the three Fama and French (1993) factors, i.e., MRP (the return on the value-weighted market portfolio in excess of a 1-month T-bill rate), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), HML (the difference in returns on a portfolio of high and low book equity to market equity stocks), the momentum factor UMD (the difference in returns on a portfolio of winning and loosing stocks) proposed by Jegadeesh and Titman (1993), and the correlation risk premium CR from Buraschi, Kosowski, and Trojani (2014). ***, **, and * indicate significance at the 1%, 5%, and 10% level respectively. Values for α are annualized. The adjusted R-square is in percentage. The first set of regressions uses data sample from 1st January 1986 through 30th December 2011. Since data on CR start from February 1996, we run the second set of regressions using data sample from 2nd February 1996 to 30th December 2011.

Panel A: Sample 1986–2011

	I	II	III	IV
α	0.037*	0.038*	0.045**	0.042**
	(1.721)	(1.755)	(2.133)	(2.004)
β_{MRP}	0.316***	0.309***	0.287***	0.291***
	(42.979)	(41.757)	(38.220)	(37.665)
β_{SMB}		-0.117***	-0.139***	-0.139***
		(-8.065)	(-9.674)	(-9.659)
β_{HML}			-0.195***	-0.183***
			(-12.750)	(-11.308)
β_{UMD}				0.027**
				(2.473)
$Adj.R^2(\%)$	0.219	0.227	0.245	0.246
$Fstat$	1847.164	965.113	713.430	537.019

Panel B: Sample 1996–2011

	I	II	III	IV	V
α	0.040	0.039	0.046	0.049	0.052
	(1.296)	(1.270)	(1.493)	(1.595)	(1.495)
β_{MRP}	0.217***	0.217***	0.206***	0.197***	0.197***
	(23.214)	(23.201)	(21.972)	(19.918)	(19.910)
β_{SMB}		0.038*	0.017	0.020	0.020
		(1.938)	(0.880)	(1.039)	(1.040)
β_{HML}			-0.154***	-0.170***	-0.170***
			(-8.278)	(-8.693)	(-8.687)
β_{UMD}				-0.035***	-0.034***
				(-2.632)	(-2.629)
β_{CR}					0.0002
					(0.209)
$Adj.R^2(\%)$	0.119	0.120	0.134	0.136	0.136
$Fstat$	538.903	271.516	206.924	157.157	125.704

Table 4: Predictive Regressions of Market Return (In-Sample Analysis)

Panel A provides results of the daily time-series regressions of market return on the SDR_{Std}, Downside-Beta Comovement ($Bcov_{Down,Std}$), Absorption Ratio (AR_{Std}), Beta Asymmetry ($Beta_{Asy,Std}$), Downside Beta ($Beta_{Down,Std}$) (all in standardized version) as well as Net Equity Expansion. Panel B provides results of the monthly time-series regressions. The market return is the simple return of the S&P 500 index. Newey-West robust t-statistics are in parentheses (10 lags for daily regression and 6 lags for monthly regression). ***, **, and * indicate significance at the 1%, 5%, and 10% level respectively. The adjusted R-square is in percentage. Sample: 1st January 1986 through 30th December 2011.

Panel A: Forecasting Daily Market Return

	I	II	III	IV	V	VI
Intercept	0.0007***	0.0007***	0.0040	0.0007***	0.0007***	0.0006**
	(2.8095)	(2.7246)	(1.3618)	(2.6113)	(2.6615)	(2.1717)
SDR_{Std}	-0.0012**					
	(-2.3437)					
$Bcov_{Down,Std}$		-0.0011**				
		(-2.5575)				
AR_{Std}			-0.0043			
			(-1.0963)			
$Beta_{Asy,Std}$				-0.0005		
				(-1.0265)		
$Beta_{Down,Std}$					-0.0002	
					(-0.5734)	
Net Equity Expansion						0.0045
						(0.6844)
$Adj.R^2(\%)$	0.131	0.135	0.013	0.003	-0.010	0.001
$Fstat$	9.626	9.860	1.832	1.211	0.373	1.078

Panel B: Forecasting Monthly Market Return

	I	II	III	IV	V	VI
Intercept	0.0085***	0.0084***	0.0488**	0.0082***	0.0083***	0.0076***
	(4.4338)	(4.3179)	(2.2153)	(4.1359)	(4.206)	(3.284)
SDR_{Std}	-0.0152***					
	(-3.6571)					
$Bcov_{Down,Std}$		-0.0115***				
		(-3.5888)				
AR_{Std}			-0.0517*			
			(-1.7837)			
$Beta_{Asy,Std}$				-0.0056		
				(-1.3198)		
$Beta_{Down,Std}$					-0.0016	
					(-0.6268)	
Net Equity Expansion						0.0665
						(1.2765)
$Adj.R^2(\%)$	2.213	1.695	0.391	0.187	0.026	0.355
$Fstat$	148.413	113.319	26.596	13.173	2.662	24.236

Table 5: Predictive Regressions of Market Return (Out-of-Sample Performance)

This table reports out-of-sample R-square (in percentage), annualized Sharpe ratio gains, and annualized certainty equivalence gains for stock market return forecasts at daily and monthly frequencies from predictive regressions with expanding window. The explanatory variables are SDR_{Std}, Downside-Beta Comovement ($\text{Bcov}_{Down,Std}$), Absorption Ratio (AR_{Std}), Beta Asymmetry ($\text{Beta}_{Asy,Std}$), Downside Beta ($\text{Beta}_{Down,Std}$) (all in standardized version) as well as Net Equity Expansion. The endogenous variable is the future market return (simple return of S&P 500 index). The out-of-sample R-square (in percentage) compares the forecast error of the model with the forecast error of the historical mean. The Sharpe ratio gains and the certainty equivalent gains are portfolio gains of a trading strategy based on different return forecasts relative to the one with the historical mean return. Forecasts begin s_0 (5 and 10) years after the sample starts. Sample: 1st January 1986 through 30th December 2011.

Panel A: Forecasting Daily Market Return

	R^2 (%)	**SR Gain**	**CE Gain**
Burning period $s_0 = 5$ **years**			
SDR_{Std}	0.095	0.207	0.101
$\text{Bcov}_{Down,Std}$	0.115	0.234	0.117
AR_{Std}	−0.044	0.054	0.034
$\text{Beta}_{Asy,Std}$	−0.027	−0.037	−0.016
$\text{Beta}_{Down,Std}$	−0.067	−0.089	−0.037
Net Equity Expansion	−0.101	−0.178	−0.080
Burning period $s_0 = 10$ **years**			
SDR_{Std}	0.101	0.193	0.105
$\text{Bcov}_{Down,Std}$	0.151	0.300	0.171
AR_{Std}	−0.018	−0.010	0.025
$\text{Beta}_{Asy,Std}$	−0.006	−0.002	0.002
$\text{Beta}_{Down,Std}$	−0.041	−0.060	−0.018
Net Equity Expansion	−0.062	−0.094	−0.045

Panel B: Forecasting Monthly Market Return

	R^2 (%)	**SR Gain**	**CE Gain**
Burning period $s_0 = 5$ **years**			
SDR_{Std}	1.787	0.180	0.077
$\text{Bcov}_{Down,Std}$	1.158	0.129	0.061
AR_{Std}	−1.807	−0.003	−0.010
$\text{Beta}_{Asy,Std}$	−0.140	−0.013	−0.004
$\text{Beta}_{Down,Std}$	−1.660	−0.137	−0.071
Net Equity Expansion	−1.609	−0.159	−0.081
Burning period $s_0 = 10$ **years**			
SDR_{Std}	2.272	0.206	0.096
$\text{Bcov}_{Down,Std}$	1.998	0.198	0.106
AR_{Std}	−1.285	−0.081	−0.037
$\text{Beta}_{Asy,Std}$	−0.021	0.004	0.004
$\text{Beta}_{Down,Std}$	−1.261	−0.119	−0.066
Net Equity Expansion	−1.134	−0.077	−0.052

Table 6: Exposure of the SDR Strategy to Risk Factors (Daily Predictive Regressions)

The table presents the results of time series regressions. The dependent variable is the predicted daily market return (using SDR_{Std}) in excess of the T-bill rate. Forecasts begin $s_0 = 10$ years after the sample starts. The independent variables include the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a 1-month T-bill rate), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), HML (the difference in returns on a portfolio of high and low book equity to market equity stocks), the momentum factor UMD (the difference in returns on a portfolio of winning and loosing stocks) proposed by Jegadeesh and Titman (1993), and the correlation risk premium CR from Buraschi, Kosowski, and Trojani (2014). ***, **, and * indicate significance at the 1%, 5%, and 10% level respectively. Values for α are annualized. The adjusted R-square is in percentage. The first set of regressions uses data sample from 1st January 1986 through 30th December 2011. Since data on CR is available from February 1996, we run the second set of regressions using data sample from 2nd February 1996 to 30th December 2011.

Panel A: Sampe 1986–2011

	I	II	III	IV
α	0.205***	0.205***	0.205***	0.205***
	(105.444)	(105.421)	(105.412)	(105.332)
β_{MRP}	0.149**	0.149**	0.175***	0.166***
	(2.506)	(2.502)	(2.912)	(2.622)
β_{SMB}		0.085	0.131	0.134
		(0.682)	(1.053)	(1.075)
β_{HML}			0.351***	0.335***
			(2.956)	(2.684)
β_{UMD}				-0.034
				(-0.404)
$Adj.R^2(\%)$	0.154	0.166	0.380	0.384
$Fstat$	6.282	3.373	5.166	3.915

Panel B: Sample 1996–2011

	I	II	III	IV	V
α	0.150***	0.150***	0.150***	0.150***	0.140***
	(39.669)	(39.653)	(39.642)	(39.661)	(36.301)
β_{MRP}	0.222**	0.216**	0.198*	0.141	0.132
	(2.296)	(2.211)	(1.777)	(1.223)	(1.168)
β_{SMB}		0.092	0.100	0.137	0.063
		(0.386)	(0.416)	(0.571)	(0.268)
β_{HML}			0.084	-0.194	-0.209
			(0.337)	(-0.678)	(-0.747)
β_{UMD}				-0.319**	-0.319**
				(-2.026)	(-2.065)
β_{CR}					-0.086***
					(-7.470)
$Adj.R^2(\%)$	0.355	0.365	0.372	0.648	4.265
$Fstat$	5.272	2.709	1.843	2.411	13.160

Table 7: Exposure of the SDR Strategy to Risk Factors (Monthly Predictive Regressions)

The table presents the results of time series regressions. The dependent variable is the predicted monthly market return (using SDR_{Std}) in excess of the T-bill rate. Forecasts begin s_0 (5 and 10) years after the sample starts. The independent variables include the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a 1-month T-bill rate), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), HML (the difference in returns on a portfolio of high and low book equity to market equity stocks), the momentum factor UMD (the difference in returns on a portfolio of winning and loosing stocks) proposed by Jegadeesh and Titman (1993), and the correlation risk premium CR from Buraschi, Kosowski, and Trojani (2014). ***, **, and * indicate significance at the 1%, 5%, and 10% level respectively. Values for α are annualized. The adjusted R-square is in percentage. The first set of regressions uses data sample from 1st January 1986 through 30th December 2011. Since data on CR is available from February 1996, we run the second set of regressions using data sample from 2nd February 1996 to 30th December 2011.

Panel A: Sample: 1986–2011

	I	II	III	IV
α	0.119***	0.119***	0.119***	0.119***
	(98.368)	(98.347)	(98.319)	(98.259)
β_{MRP}	1.966**	1.970**	2.293***	2.142**
	(2.403)	(2.407)	(2.771)	(2.456)
β_{SMB}		1.250	1.820	1.879
		(0.735)	(1.062)	(1.094)
β_{HML}			4.132**	3.844**
			(2.540)	(2.248)
β_{UMD}				-0.628
				(-0.546)
$Adj.R^2(\%)$	0.144	0.157	0.317	0.325
$Fstat$	5.773	3.157	4.258	3.268

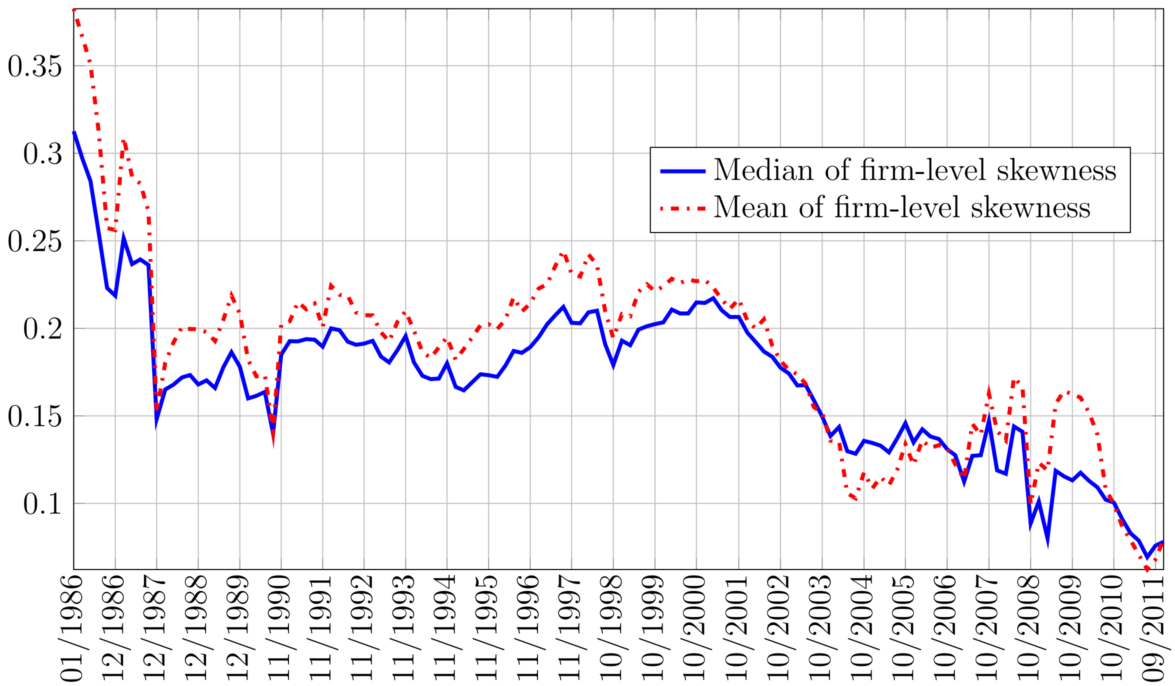
Panel B: Sample: 1996–2011

	I	II	III	IV	V
α	0.087***	0.087***	0.087***	0.087***	0.083***
	(36.398)	(36.384)	(36.370)	(36.386)	(33.584)
β_{MRP}	2.842**	2.764**	2.783*	1.931	1.830
	(2.106)	(2.030)	(1.786)	(1.209)	(1.156)
β_{SMB}		1.490	1.483	2.193	1.541
		(0.449)	(0.445)	(0.657)	(0.465)
β_{HML}			-0.086	-4.656	-4.777
			(-0.025)	(-1.172)	(-1.212)
β_{UMD}				-5.128**	-5.144**
				(-2.358)	(-2.385)
β_{CR}					-0.791***
					(-4.953)
$Adj.R^2(\%)$	0.309	0.323	0.323	0.709	2.384
$Fstat$	4.435	2.317	1.544	2.552	6.981

Figure 1: Dynamics of Firm-Level and Market-Level Skewness

Panel A presents cross-firm median (solid line) and mean (dash line) values of individual firm-level skewness. Panel B reports the actual market skewness (solid line) and the model-implied market skewness (dash line). Market skewness is calculated using 3-year rolling windows. Sample: 1st January 1986 through 30th December 2011.

Panel A: Firm-level skewness



Panel B: Model implied vs real market skewness

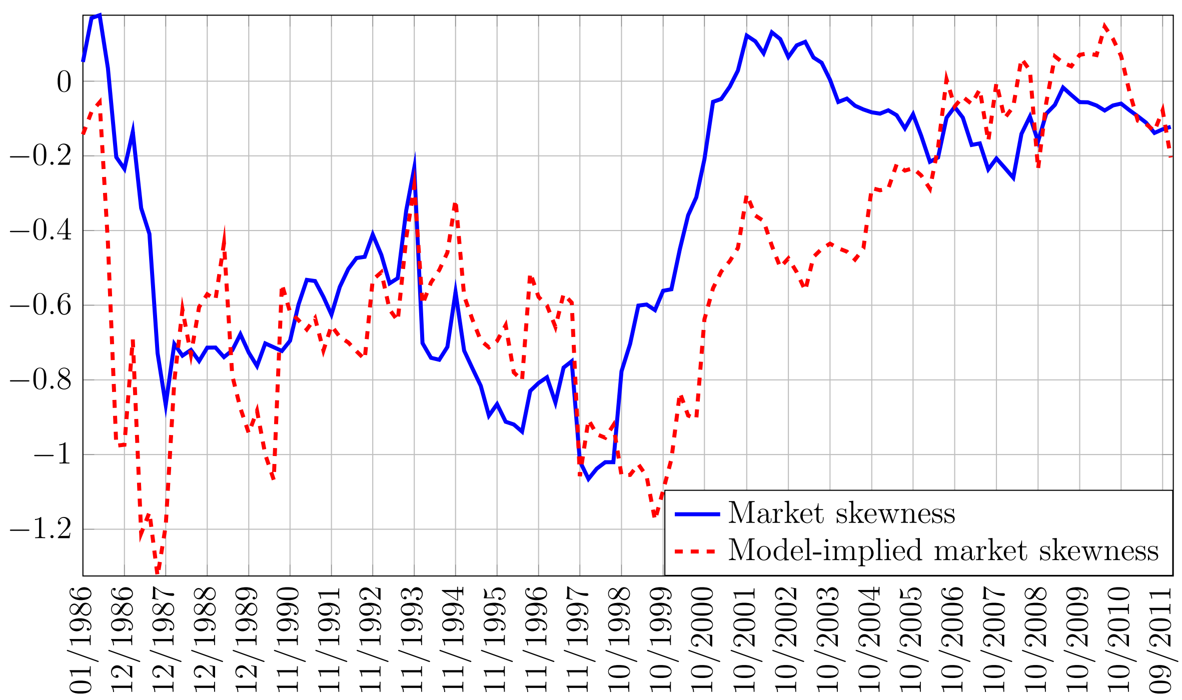


Figure 2: Dynamics of Market Skewness across Quantiles

This figure presents the behavior of market skewness for downside market and upside market: $sk_m^{(u)}$ and $sk_m^{(1-u)}$, where u denotes the days when market returns are below u quantile, and $1-u$ when market returns are above (or equal to) $1-u$ quantile. $sk_m^{(u)}$ and $sk_m^{(1-u)}$ represent market skewness prevailing when market return is below quantile u (solid line with markers) and above (or equal to) quantile $1-u$ (dash line with markers), respectively. Here, $u = 0.2$. The plotted values for the conditional market skewness are calculated using 3-year rolling windows. Sample: 1st January 1986 through 30th December 2011.

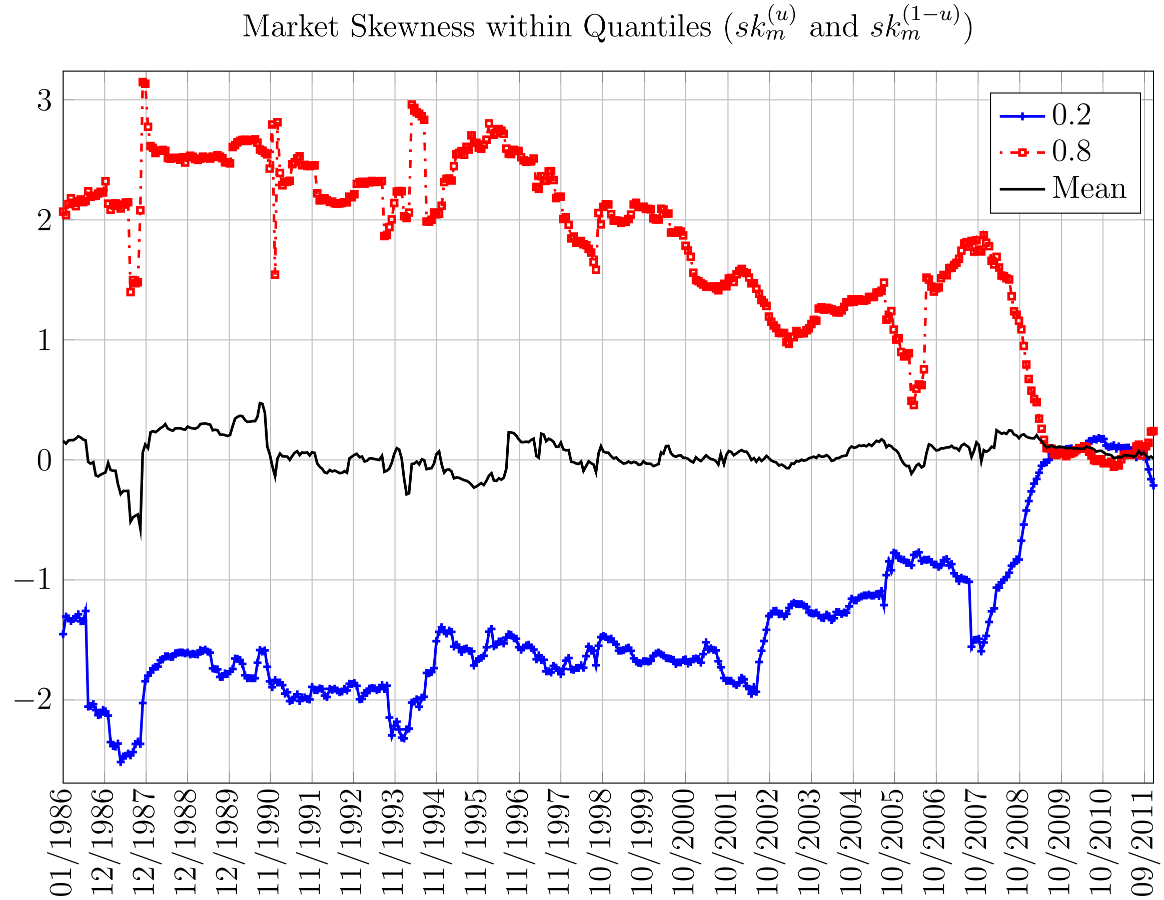
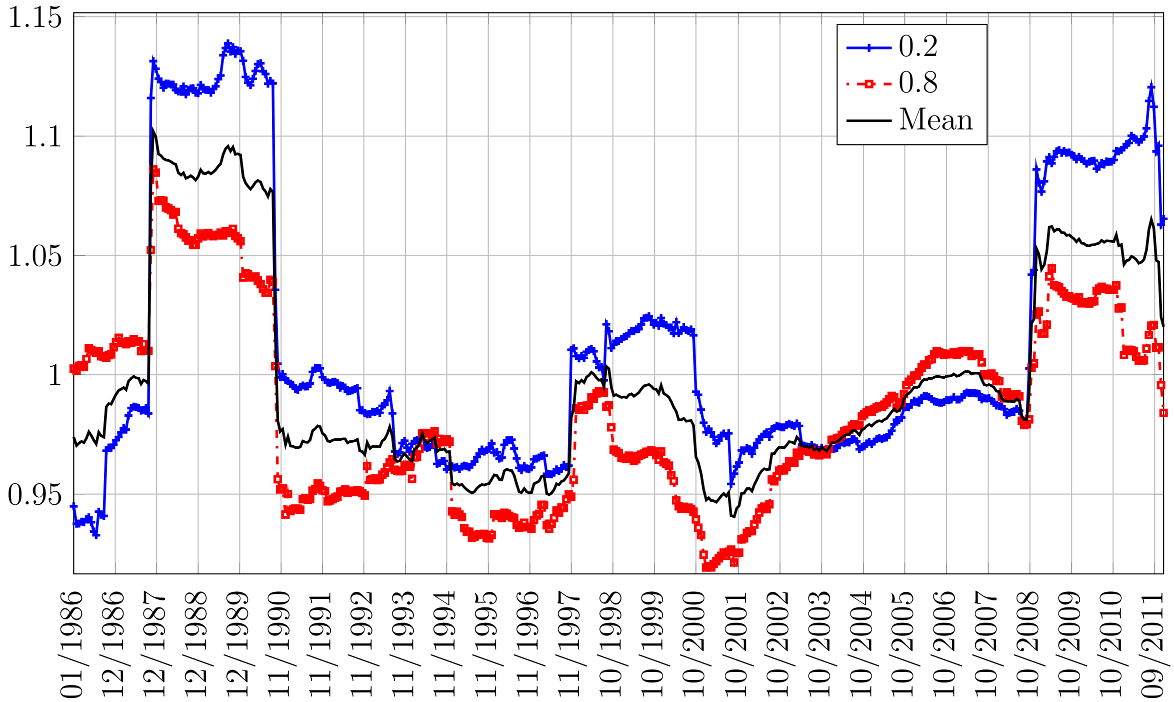


Figure 3: Dynamics of Beta and Bco-cov across Quantiles

Panel A presents the cross-firm average of the downside $\beta^{(u)}$ and upside $\beta^{(1-u)}$, respectively. Panel B presents the cross-firm average of the downside *bco-cov*$^{(u)}$ and upside *bco-cov*$^{(1-u)}$, respectively. u denotes the days when market returns are below u quantile (solid line with markers), and $1-u$ when market returns are above (or equal to) $1-u$ quantile (dash line with markers), where $u = 0.2$. The solid line presents the mean values of *beta* and *bco-cov* at each quantile. The values for the conditional *beta* and *bco-cov* are calculated using 3-year rolling windows. Sample: 1st January 1986 through 30th December 2011.

Panel A: Beta



Panel B: Bco-cov

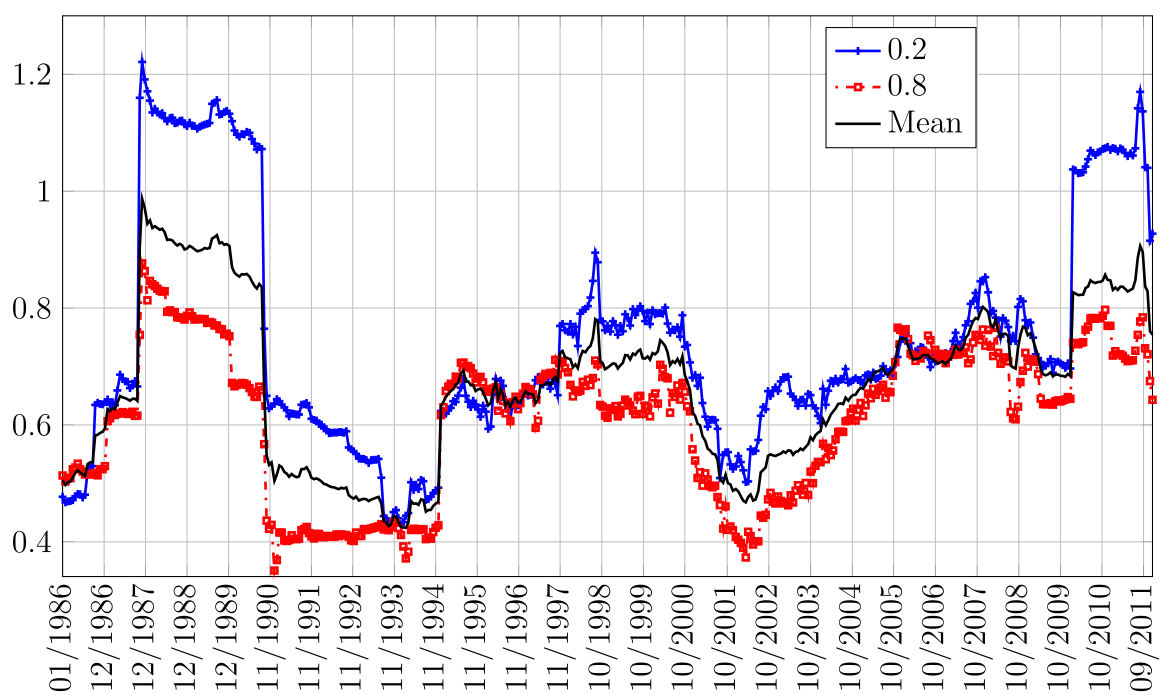


Figure 4: Dynamics of SDR, Downside-Beta Comovement, and Absorption Ratio

In Panel A, solid line represents SDR Index (on the left axis) and dot line represents Downside-Beta Comovement (Bcov_{Down}) (on the right axis). In Panel B, solid line represents SDR Index (on the left axis) and dot line represents Absorption Ratio (on the right axis). Results are calculated using 3-year rolling windows. Sample: 1st January 1986 through 30th December 2011.

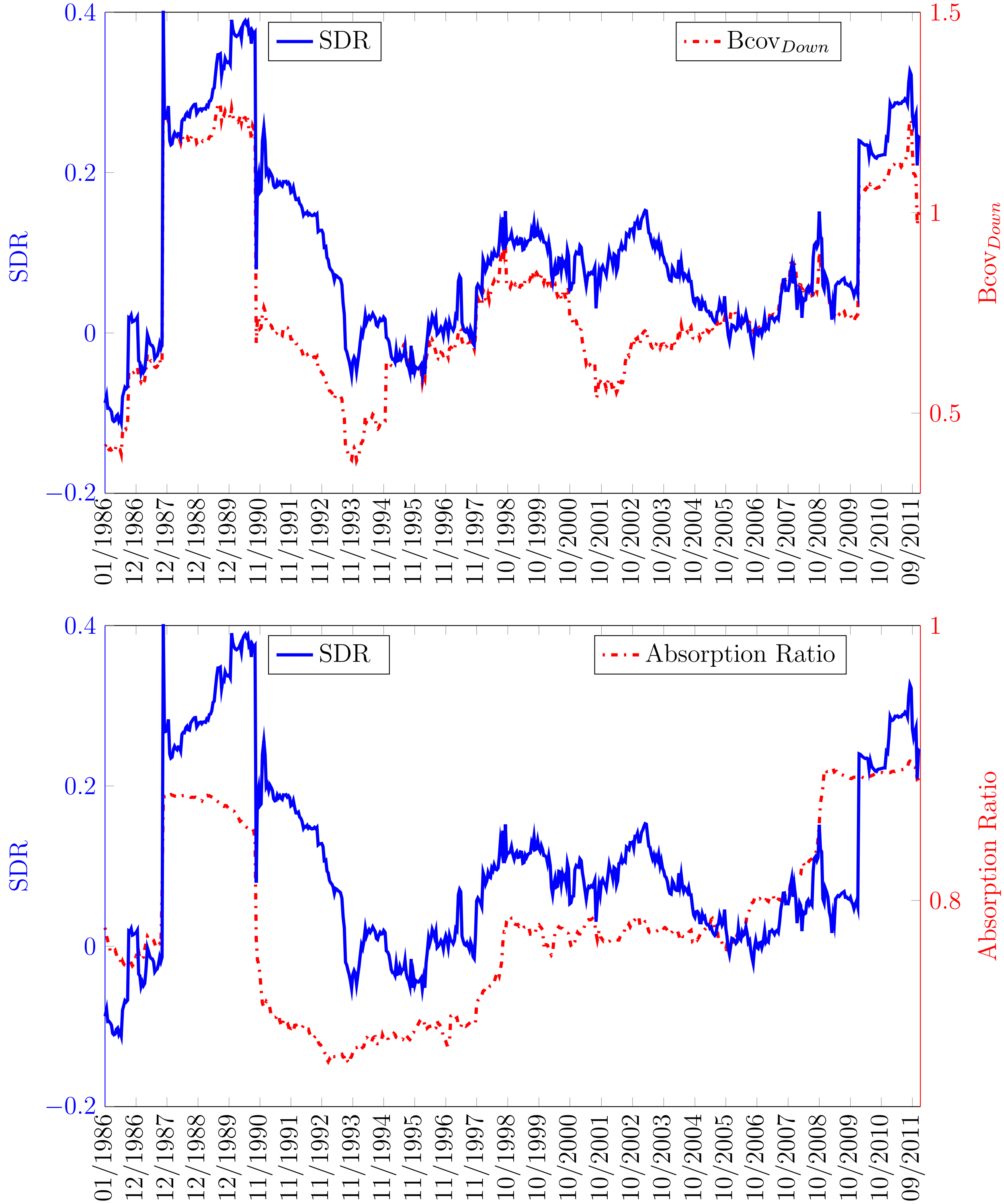


Figure 5: Standardized SDR, Downside-Beta Comovement, and Absorption Ratio around 1% Extreme Market Movements

This figure presents the predictability of standardized SDR, Downside-Beta Comovement (Bcov_{Down}), and Absorption Ratio (AR) based on event study analysis. Plotted values are mean values in percentage. The extreme market movements are the 1% days with the lowest market return, largest market return, and most turbulent days over the whole sample period. Sample: 1st January 1986 through 30th December 2011.

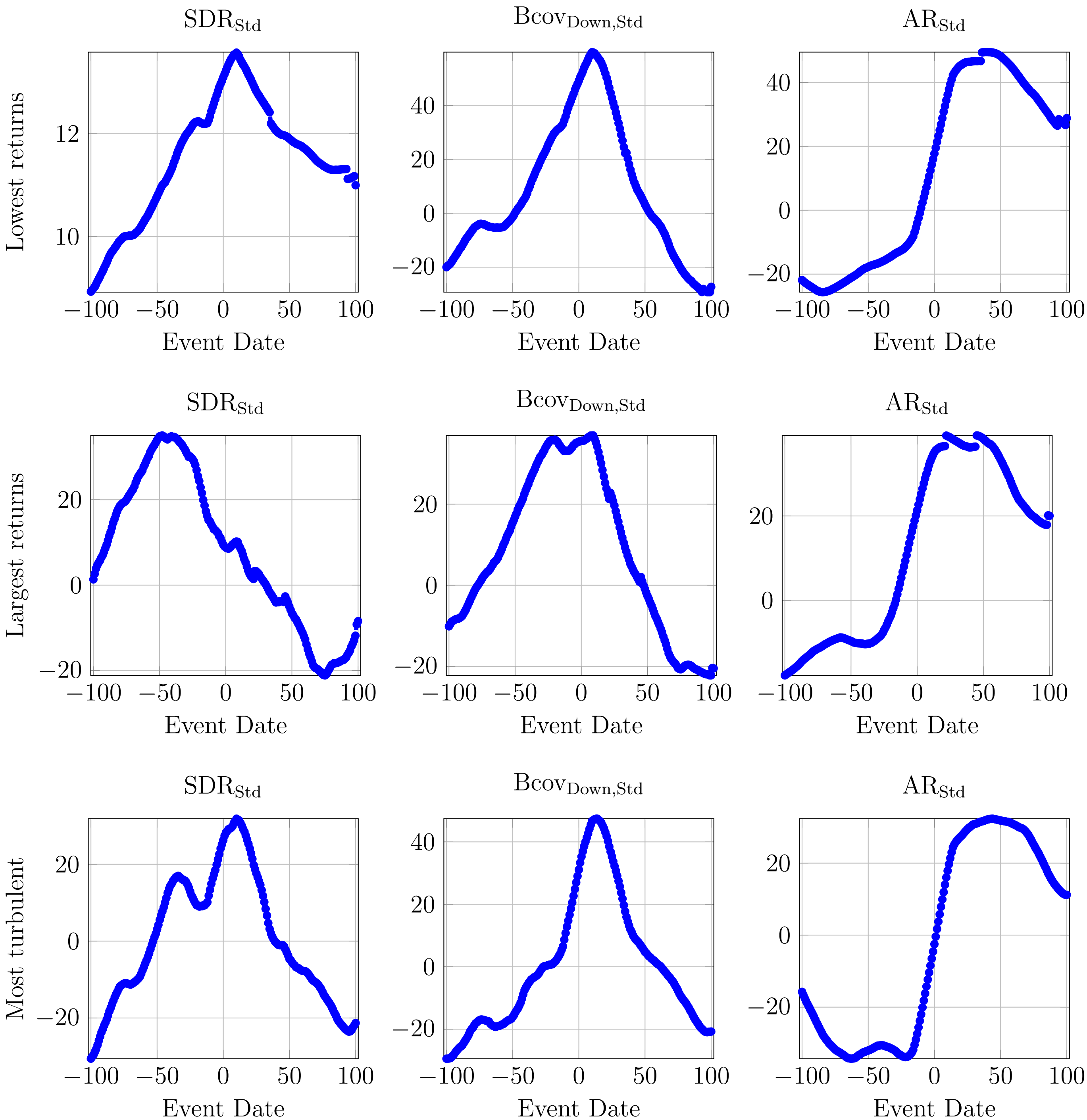


Figure 6: Standardized SDR, Downside-Beta Comovement, and Absorption Ratio around 5% Extreme Market Movements

This figure presents the predictability of standardized SDR, Downside-Beta Comovement ($Bcov_{Down}$), and Absorption Ratio (AR) based on event study analysis. Plotted values are mean values in percentage. The extreme market movements are the 5% days with the lowest market return, largest market return, and most turbulent days over the whole sample period. Sample: 1st January 1986 through 30th December 2011.

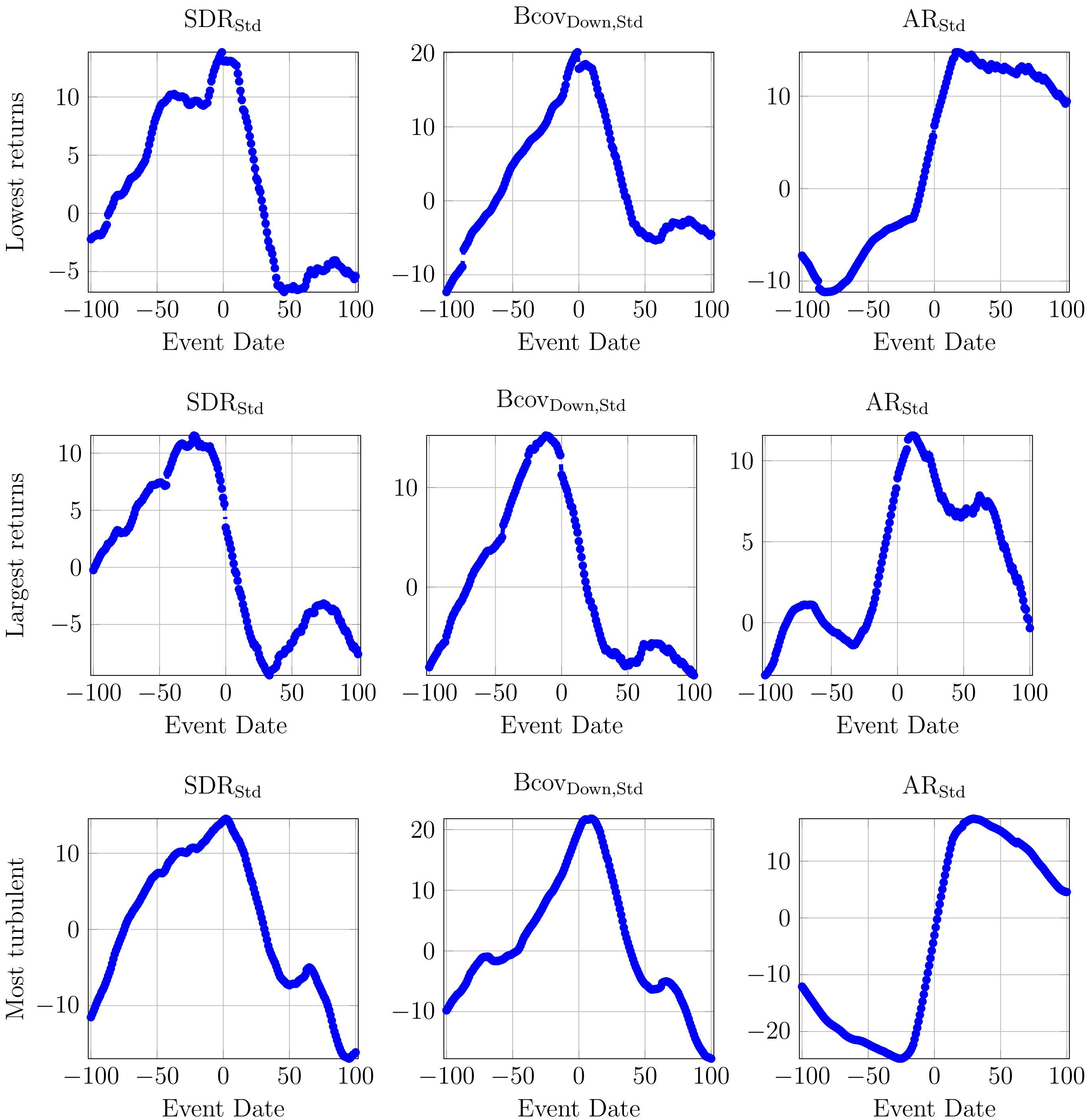


Figure 7: Trading Signal, Asset Allocation, and Market Index (for $\theta = 1$)

This figure presents trading signals (solid line) based on standardized SDR, standardized Downside-Beta Comovement (Bcov_{Down}), and standardized Absorption Ratio (AR), respectively. It also presents investments in risky asset: S&P500 Index (circle-marked line) and risk-free asset (dash line). Trading signals and investment holdings are plotted on the left axis; S&P500 Index level is plotted in dot line on the right axis. Portfolios are rebalanced once the 3-month signal's absolute value (demeaned by its 2-year mean) is larger than its 2-year standard deviation. Sample: 1st January 1986 through 30th December 2011.

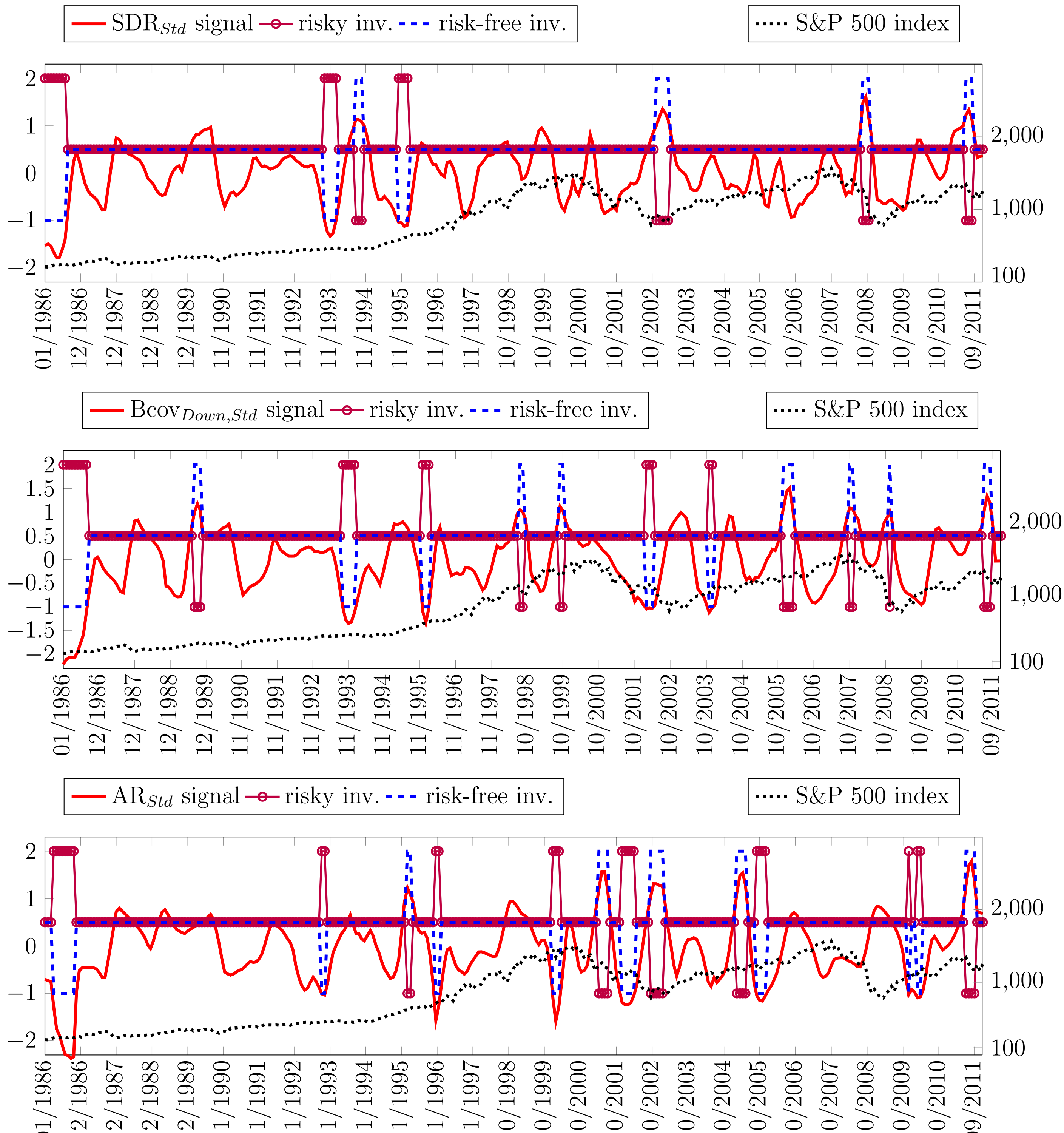


Figure 8: Trading Signal, Asset Allocation, and Market Index (for $\theta = 0.5$)

This figure presents trading signals (solid line) based on standardized SDR, standardized Downside-Beta Comovement (Bcov_{Down}), and standardized Absorption Ratio (AR), respectively. It also presents investments in risky asset: S&P500 Index (circle-marked line) and risk-free asset (dash line). Trading signals and investment holdings are plotted on the left axis; S&P500 Index level is plotted in dot line on the right axis. Portfolios are rebalanced once the 3-month signal's absolute value (demeaned by its 2-year mean) is larger than half of its 2-year standard deviation. Sample: 1st January 1986 through 30th December 2011.

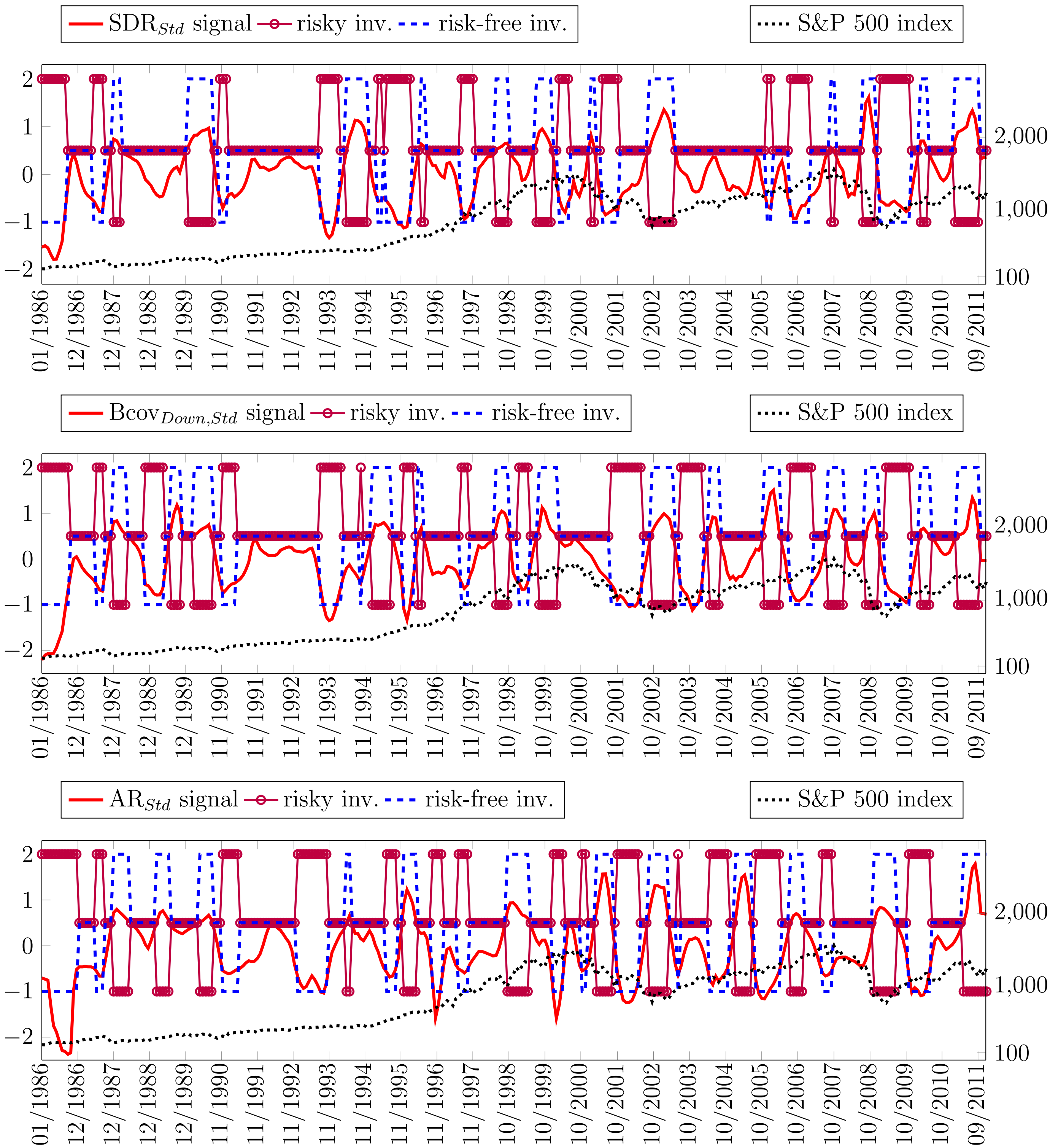
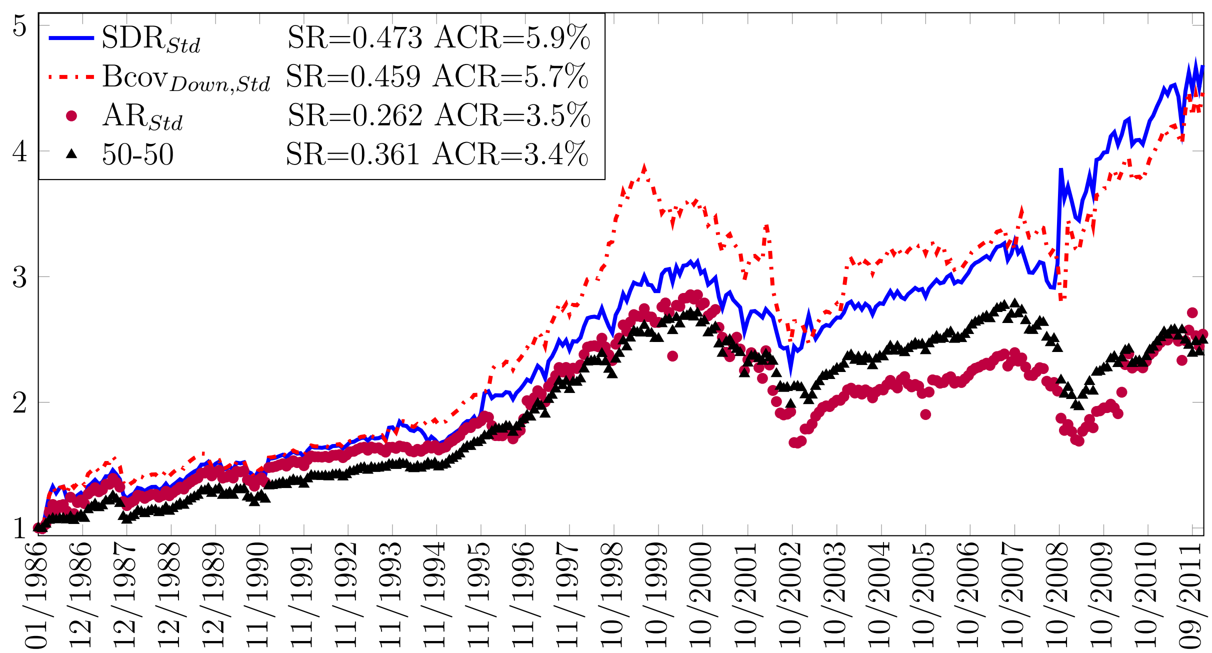


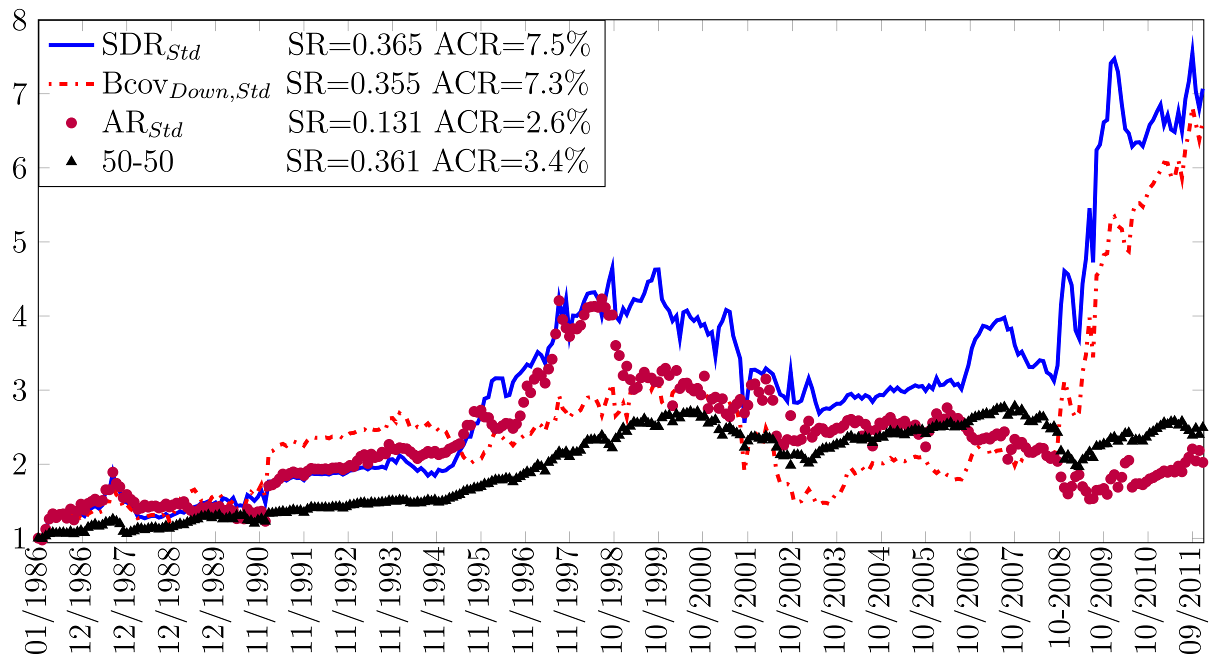
Figure 9: Cumulative Wealth of the Strategies Based on a Trading Signal

The figure presents cumulative wealth generated from trading strategies based on standardized SDR, standardized Downside-Beta Comovement (Bcov_{Down}), standardized Absorption Ratio (AR), and 50/50 buy-and-hold. Panels A and B correspond to a trading signal based on $\theta = 1$ and $\theta = 0.5$, respectively. All strategies start with an initial investment of 1 dollar and have 1% transaction cost proportional to the unit of trading. Annualized Sharpe ratios (SR) and annualized cumulative return (ACR) for trading strategies are reported. Portfolios are rebalanced once the 3-month signal's value (demeaned by its 2-year mean) is larger than its 2-year standard deviation multiplied by θ. Panels A and B correspond to $\theta = 1$ and $\theta = 0.5$, respectively. Sample: 1st January 1986 through 30th December 2011.

Panel A: Cumulative Wealth ($\theta = 1$)



Panel B: Cumulative Wealth ($\theta = 0.5$)



swiss:finance:institute

c/o University of Geneva
40 bd du Pont d'Arve
1211 Geneva 4
Switzerland

T +41 22 379 84 71
F +41 22 379 82 77
RPS@sfi.ch
www.SwissFinanceInstitute.ch