Common Factors in Foreign Exchange Returns

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Abstract

Returns to foreign exchange speculation are more predictable than previously thought. In particular, expected returns at all time horizons depend on both the cross section and term structure of foreign currency forward exchange rates relative to the US Dollar. With either set of variables, this dependence works in essentially the same way across time horizons for any single currency, and can be well approximated by a simple linear function of log forward rates. Combining both types of information together, a principal components decomposition of all foreign exchange forward rate data reveals that very few factors are needed to capture even more of the variation in expected returns than is possible with either scheme described above, and that the same factors give expected returns for all currencies and time horizons. Using these common factors, I find that any currency's expected returns over different time horizons are equal to the same function of the data up to a scalar multiplication. I also examine the restrictions that my empirical work implies for multi-country macroeconomic/asset pricing models.

1 Introduction

International finance and macroeconomics offer a variety of puzzles that challenge existing equilibrium asset pricing models. The most prominent of these puzzles is the fact that returns to foreign exchange speculation are predictable: cross-country interest rate differences do not forecast a depreciation of the higher-interest rate currency as one might expect, but instead signal a subsequent appreciation of the higher-interest rate currency. This paper adds to the existing literature by showing that foreign exchange returns are in fact more predictable than previous work would indicate. In particular, I demonstrate that regressions of currency returns relative to the US Dollar on a single interest rate differential alone ignore two important sources of additional data - the cross section of other currencies' interest rate differences relative to the US, and the forward foreign exchange rate term structure of the particular currency for which we wish to predict returns. To my knowledge, this is the first paper to examine the significance of the term structure of forward foreign exchange rates. A number of existing papers already document the importance of the cross section of currency interest rate differentials, see for example [13] and [18], but here the cross section is used in a novel way. That is, I show that for either set of variables, the information which helps predict returns can be effectively collapsed into a single factor which describes expected currency returns at all time horizons I consider. I then pool both sources of information to show that one can find a *smaller* set of variables that uses the same cross section and term structure information from all currencies to describe expected returns to each currency across all time horizons. Moreover, expected returns to a given currency over any time horizon are indistinguishable from different scalar multiples of the same linear function of these common expected return factors. This empirical finding implies tight restrictions on the joint dynamics of pricing kernels for multi-country equilibrium asset pricing models.

Let us review the definition of currency returns in a simple, one-period example. To fix ideas, imagine the home currency as Dollars and the foreign currency as Pounds. The level of the spot rate at time t, written as S_t, is the number of pounds required to purchase 1 Dollar. Equivalently, $1/S_t$ Dollars equals 1 Pound. F_t, the currency forward rate, is the rate at which agents in either country can agree today to trade Pounds for Dollars next period. Consider an agent in the US that takes a long position in a currency forward contract for the Pound: the agent commits today to pay $1/F_t$ Dollars next period to receive 1 Pound next period. Now let the realized spot exchange rate next period be S_{t+1}. The forward contract is fulfilled, and the agent immediately converts the 1 Pound received back into Dollars. So at time 1, the agent pays $1/F_t$ Dollars for a Pound, and then converts the Pound back at the prevailing spot rate to receive $1/S_{t+1}$ Dollars. The agent's return, $\frac{\$\text{ received}}{\$\text{ paid}}$, is then $\frac{1/S_{t+1}}{1/F_t} = \frac{F_t}{S_{t+1}}$. Using lowercase letters for logarithms, the investor's log return at time $t+1$ is:

$$rx_{t+1} = f_t - s_{t+1} \tag{1}$$

where I have used the symbol rx_{t+1} because this is an excess return - no initial investment is required by the agent to agree to the currency forward contract. The return is positive when $s_{t+1} < f_t$. This would correspond to an unexpected appreciation of the Pound relative to the Dollar (fewer Pounds required to purchase a Dollar) if the forward rate were in fact an unbiased predictor of the future value of the spot rate.

The data, show, however, that forward rates are *not* unbiased predictors of future spot rates. Two of the most prominent studies to reach this conclusion include [12] and [9] . A quick look at (1) shows that if f_t is not the time t expectation of the next period's spot rate, $f_t \neq \mathbb{E}_t[s_{t+1}]$, then expected currency returns must be different from zero - that is, returns to currency speculation are predictable.

A no-arbitrage argument shows that today's spot rate and forward rate must satisfy the covered interest parity relationship: the log of the forward rate must equal the log of the spot rate, plus the difference in risk-free interest rates available in the two countries:

$$f_t = s_t + i_t^* - i_t \tag{2}$$

Here i_t^* is the one-period risk-free interest rate for Pounds available at time t, and i_t is the one-period Dollar-denominated risk-free rate. Then the difference in the logs of the forward rate and the spot rate, the forward-spot spread, is the foreign interest rate minus the Dollar interest rate:

$$f_t - s_t = i_t^* - i_t$$

Consider regressions of each of the percentage change in the exchange rate and the currency return on the forward-spot spread:

$$\begin{aligned} s_{t+1} - s_t = & \quad \alpha_1 + \beta_1 (f_t - s_t) + \varepsilon_{1,t+1} \\ f_t - s_{t+1} = & \quad \alpha_2 + \beta_2 (f_t - s_t) + \varepsilon_{2,t+1} \end{aligned} \tag{3}$$

In the data, one typically sees $\beta_2 > 1$. This means that high interest rate currencies appreciate, or that interest rate differences translate into expected returns, contrary to what one might expect. Fama [9] shows that a value of $\beta_2 > 1$ implies that i) expected returns, or risk premiums, are more volatile than expected exchange rate fluctuations, and ii) that foreign currency returns must exhibit negative covariance with expected foreign currency appreciation. This anomalous result is known as the forward premium puzzle; it has launched a wide literature of potential explanations.

The first contribution of this paper is to show that regressions such as the second part of (3) omit variables that provide important information for understanding expected returns. The values of the regression coefficients in (3) present a worthwhile puzzle on their own, but can only paint an incomplete picture of how expected returns to currency speculation vary over time. For a particular example, the one year currency forward rates tells us something about expected returns at a one-month horizon beyond what we can learn from the one-month forward rate alone. Documenting and testing the importance of the term structure will lead us to a consideration of information in the cross section of foreign exchange forward rates, and finally to a parsimonious number of common factors for the expected returns to all currencies across all time horizons.

Section 2 provides an overview of the relevant literature. Section 3 describes the data and variables used throughout the paper. Section 4 documents the increase in return predictability from using the term structure of forward rates. Section 5 looks at the extent to which this information is common across maturities. Section 6 compares the principal components of the term structure for each currency in our set. Section 7 examines the extent to which the cross section of forward-spot spreads for various currencies helps to predict returns at various horizons for each individual currency. Section 8 conducts formal tests of the single factor restrictions. Section 9 identifies a small set of expected return factors common to all currencies and time horizons, and shows that a given currency's expected returns for all time horizons are equal up to a scalar multiplication. Section 10 concludes.

2 Related Literature

Tests of the single-factor restrictions in this paper follow Cochrane and Piazzesi's methodology from their studies of predictability in bond returns. (See [4] and [5].) To my knowledge, this is the first paper to use the entire term structure of foreign exchange forward rates. Hansen and Hodrick [12] and Fama [9] are two foundational papers in the modern study of foreign exchange returns. For bonds, Fama and Bliss find that forward rates on Treasury bonds move together with expected returns. [10] See Cochrane [3] for an overview of return predictability across asset classes. A recent paper by Hassan [14] explains differences in expected currency returns as a function of the countries' relative sizes in the world economy.

A 1993 paper by Backus, Gregory, and Telmer [2] documents predictability of returns to speculation in foreign-currency spot and rates as well as the time variation in the predictable component of returns, and provides a survey of earlier literature in the topic as well. They show that the failure of standard representative agent models with time-separable utility to address either of these empirical facts is ameliorated by allowing for habit formation in the utility of the representative agent, but that this addition still does not produce a satisfactory explanation for the strong autocorrelation of forward premia.

A 2001 paper by Backus, Foresi, and Telmer [1] uses an affine model (of the type familiar from the term structure of interest rates literature) to explore the forward premium puzzle in a two-country world, and finds that affine models require either a positive probability of nominal interest rates becoming negative, or that state variables affect currencies in a non-symmetric fashion.

A recent paper by Hassan and Mano [15] analyzes the covariance of currency returns with forward premia to argue that the forward premium puzzle and the carry trade are distinct phenomena that require distinct explanations. They posit that the carry trade is driven by long-lasting differences in country risk profiles, while variation over time of all other currencies against the US dollar drives the forward premium puzzle. They conclude with the observation that the present stock of international macro and finance models that feature symmetric countries cannot explain either puzzle.

Work by Maggiori [19] and Martin [20] has shown some promise for international macroeconomics and finance models that allow for variations in country size and financial development. Maggiori's model features two countries that are otherwise identical, except that the representative financier in the foreign/ less financially developed country faces a credit constraint on the net worth of the intermediary that they manage, so that the net worth must remain positive at all times. The home/ more financially developed country does not face such a constraint. The better ability of home country intermediaries to cope with funding disruptions allows the home county to consume more, and run a trade deficit, when times are good and on average. This is compensation for assuming higher levels of risk, as the home country fares worse during global downturns. The home currency appreciates during such global crises, and becomes the reserve currency for that reason. Martin considers two countries of different size, where each country features a class of consumers that care only about domestic output, and another class that enjoys both domestic and foreign output, but as imperfect substitutes. The relative price of the small country's output falls when the large country experiences a downturn. This makes the bonds of the small country risky, and causes uncovered interest parity to fail.

Colacito and Croce [6] propose a model that allows for long-run risk in consumption and dividend growth rates to be correlated across countries in a world where agents have Epstein-Zin preferences. They offer this as a resolution of the fact that international stock markets are highly correlated although underlying fundamentals do not appear to be, and show that their model works to reconcile US and UK data. They go on to suggest that exchange rate movements may be driven by linkages among countries' long-run growth prospects. Another paper of theirs [7] reinforces the ability of Epstein-Zin preferences to resolve the weak correlation of consumption growth rates and exchange rate movements, and also develops the ability of these preferences to explain the forward premium puzzle. Allowing for time variations in capital mobility, they arrive at the conclusion that the weak correlation of consumption growth and exchange rate movements as well as the forward premium puzzle, are not anomalies but an equilibrium result in a world where agents have recursive preferences.

Ilut [16] has shown that introducing ambiguity aversion, in which investors behave in such a way as to maximize their utility under the worst of a possible set of probability distributions, implies ex-post departures from uncovered interest parity due to systematic errors in agent's expectations, and also momentum in the returns to currency speculation.

See Lewis [17] for an overview of the current state of research, including a summary of the main puzzles in international finance and macroeconomics, and commentary on possible resolutions.

3 Data, Terms, and Definitions

I consider returns to speculation in currency forward contracts for currencies belonging to the set: AUD, CAD, CHF, GBP, JPY, NOK, NZD, and SEK, for forward contracts of length $n = 1$, 2, 3, 6, 9, or 12 months. This set was selected to achieve the longest possible balanced time series with the broadest set of currencies for which relatively detailed information on the term structure of currency forward rates was available. Throughout I use end-of-month values for daily observations from Thomson Reuters Datastream. Data runs from May 1990 through August 2013.

Let $s_{t,j}$ be the log spot rate at time t for currency j, measured in units of currency j relative to the US Dollar. Similarly, let $f_{t,j}^{(n)}$ be the log currency forward rate at time t for j, n months forward, also measured in units of currency j per US Dollar, where I will occasionally write the current prevailing spot rate as $f_{t,j}^{(0)} \equiv s_{t,j}$. I use a tilde to denote the (log) forward-spot spread at time t for currency j, n months forward, $\widetilde{f}_{t,j}^{(n)} = f_{t,j}^{(n)} - s_{t,j}$. If covered interest parity holds, this is approximately equal to the interest rate differential between currency J and the US. Any use of the phrases *spot rate*, *forward rate*, *forward-spot spread*, or similar should always be understood to mean *with reference to the Dollar*, in the units described above.

I use boldface letters for vectors that group multiple time t observations together on the basis of some common characteristic. In particular, $\mathbf{f}_{t,j}$ is the time t vector of forward rates at all maturities/horizons for currency j, $\mathbf{f}_t^{(n)}$ is the vector of all forward rates at time t with maturity n, across all currencies, and $\mathbf{f}_t$ is the vector of all forward rates prevailing at time t, for all currencies j and all maturities n. Each of these vectors contains the appropriate spot rate(s), and also a constant.

Vectors of forward-spot spreads are defined in the same manner. $\widetilde{\mathbf{f}}_{t,j}$ is the vector of all forward-spot spreads prevailing at time t for currency j, $\widetilde{\mathbf{f}}_t^{(n)}$ is the vector of all forward-spot spreads at time t with maturity n, across all currencies, and $\widetilde{\mathbf{f}}_t$ is the vector of all forward-spot spreads prevailing at time t, for all available maturities of all currencies in the set I consider. Each of these includes a constant as well.

The return to a long position in an n-month forward contract for currency j, initiated at time t and held until maturity, is realized at time $t+n$, and is written as:

$$rx_{t \to t+n,j}^{(n)} \equiv f_{t,j}^{(n)} - s_{t+n} \tag{4}$$

Much of the paper is concerned with returns across different time horizons for currency forward positions initiated at the same time. When discussing returns that are realized at different times but that are linked by a common initiation date, I use a square bracket around the time subscript to indicate that the returns are being grouped together by the time the speculative position was opened, rather than when the returns actually occur. For example, we could group the returns on all forward contracts for a certain currency initiated at the same time together by writing:

$$rx_{[t],j} = \left[\begin{array}{c} rx_{t \to t+1,j}^{(1)} rx_{t \to t+2,j}^{(2)} rx_{t \to t+3,j}^{(3)} rx_{t \to t+6,j}^{(6)} rx_{t \to t+9,j}^{(9)} rx_{t \to t+12,j}^{(12)} \end{array} \right]$$

It will be often useful to quickly make a distinction between the class of vectors $\mathbf{f}$ that contain spot rates, and the class of vectors $\widetilde{\mathbf{f}}$ which do not contain spot rates as individual elements, such as when discussing the results of two otherwise similar regressions that differ only in their use of a member of one or the other case as a variable. In those cases, I will often refer to the vector of type $\mathbf{f}$ as the *rates vector* and the one of type $\widetilde{\mathbf{f}}$ as the *forward-spot spreads vector* or sometimes simply as the *spreads vector*.

4 Importance of the forward foreign exchange term structure

I show first that one can predict currency returns with substantially more accuracy by using foreign exchange term structure information than is possible with the use of the appropriate currency forward-spot spread alone.

I begin by examining a series of three nested regression specifications. For each currency j, I regress both one-month $\left(rx^{(1)}_{t\to t+1,j}\right)$ and one-year $\left(rx^{(12)}_{t\to t+12,j}\right)$ returns on increasing subsets of that currency's foreign exchange term structure information available at time t.

The first of the three specifications features a constant and the time t forward-spot spread corresponding to the return horizon, e.g. $f^{(1)}_{t,j} - s_{t,j}$ for one month returns. The second includes a constant, the spot rate, and the n-month forward rate, where the last two variables now enter the regression separately. The final specification includes the entire vector of currency forward rates prevailing at time t for which I have data, $\mathbf{f}_{t,j}$, including the spot rate and a constant. In equations, these are, for return horizons $n = 1$ and $n = 12$:

$$rx^{(n)}_{t\to t+n,j} = a + b(f^{(n)}_{t,j} - s_{t,j}) + \varepsilon_{t+n} \tag{5}$$

$$rx^{(n)}_{t\to t+n,j} = a + b \cdot f^{(n)}_{t} + c \cdot s_t + \varepsilon_{t+n} \tag{6}$$

$$\begin{aligned} rx^{(n)}_{t\to t+n,j} = \quad & \beta_{-1} + \beta_0 s_{t,j} + \beta_1 f^{(1)}_{t,j} + \beta_2 f^{(2)}_{t,j} + \beta_3 f^{(3)}_{t,j} \\ & + \beta_4 f^{(6)}_{t,j} + \beta_5 f^{(9)}_{t,j} + \beta_6 f^{(12)}_{t,j} + \varepsilon_{t+n} \\ & = \beta' \mathbf{f}_{t,j} + \varepsilon_{t+n} \end{aligned} \tag{7}$$

(I write a -1 subscript on the regression constant in (7) to avoid confusion from having a mismatch between regression coefficient subscript and maturity for the shorter-maturity forward rates $n = 0, 1, 2, 3$.) Table 4.1 reports regression R^2 values for $n = 1$ and $n = 12$. In each case, one observes economically significant increases in the magnitude of the R^2 values when moving from the first column to the third column. The second column shows that while the R^2 value does increase when treating the spot rate and the n-period forward rate separately, this alone cannot account for the entirety of the change. One will notice that the entries of the table for one-year returns are larger than their counterparts in the table for one-month returns, and within the table for one-year returns the difference between the first and third columns is often quite large, with the R^2 value increasing by 15% - 20% or more for over half of the currencies considered. The difference is most pronounced for SEK, as the R^2 moves from a value that rounds to 0 to two decimal places to 24%. While the values for one month returns do not show as much of an absolute increase moving from columns one to three, the relative increases are larger in many cases. In particular, half of the currencies considered move from 1% or less in the first column to 5% or more in the third.

To examine the statistical significance of the extra variables, I conduct Wald tests with GMM standard errors that correct for possible heteroskedasticity and serial correlation of the error terms. I explored a variety of estimation methods for the covariance matrix of the regression disturbance terms S. The estimator given by Hansen and Hodrick [12] is not necessarily positive definite in finite samples, and this turns out to be the case in this data for a small subset of returns. For this reason, most statistical tests in the paper will be conducted using the method introduced by Hansen, Eichenbaum, and Singleton (HES) in their study of consumption and leisure choice. [8] I will also occasionally report standard errors calculated using the Newey-West (NW) method. Although this estimator is also constructed to be positive definite in finite samples, this is achieved at the cost of an a priori weighting scheme for the moment covariances. For this reason I typically favor the Hansen-Eichenbaum-Singleton errors for testing throughout the paper. I now briefly review their estimation procedure.

Let u_{t+q} be the vector of moment conditions set to zero in expectation by the regression of concern. For example, in the regression described by (7) for 12-month returns $n = 12$, we would have $q = 12$ and

$$u_{t+12} = \mathbf{f}_{t,j} \otimes \varepsilon_{t+12}$$

Now consider the following (first step) vector autoregression (VAR) representation for the disturbance terms:

$$u_t = A_1 u_{t-1} + \ldots + A_{LAG} u_{t-LAG} + \widetilde{e}_t \tag{8}$$

Here the error term $\widetilde{e}_t$ is the component of the time t vector of moment conditions which is not forecastable as a linear combination of LAG past moment conditions, where number of lags chosen, LAG, should be an increasing function of the sample size. Now perform a (second step) VAR of the moment vector u_t on q lags of the error terms from (8):

$$u_t = B_1 \widetilde{e}_{t-1} + \ldots + B_q \widetilde{e}_{t-q} + v_t \tag{9}$$

Take the error terms v_t from the second step VAR, and use them to define

$$\Omega_T = \frac{1}{T - LAG - q} \sum_{t=LAG+q+1}^{T} v_t v_t' \tag{10}$$

Then a consistent estimator for the variance-covariance matrix of the moment terms u_t, which is constrained to be positive definite in finite samples is given by:

$$S_T = (I + B_1 + \ldots + B_q)\, \Omega_T\, (I + B_1 + \ldots B_q)' \tag{11}$$

For both $n = 1$ and $n = 12$, I estimate Equation (7) for each currency, and consider the following three restrictions: (i) that the coefficients on forward rates of maturities other than n are jointly equal to zero, (ii) that the coefficient on the spot rate is equal to the negative of the coefficient on the corresponding n-month forward rate, and (iii) that both restrictions hold simultaneously.

Restriction (iii) corresponds to the first specification of Table 4.1, restriction (i) corresponds to the second specification. Restriction (ii) does not have a parallel in the table, since it is silent regarding coefficients on forward rates of maturities other than n, but functions as an intermediate step between specifications 2 and 3. In equations, these are:

$$H_0 : \beta_i = 0,\ i \neq n,\ \ 1 \leq i \leq 6 \tag{12}$$

$$H_0 : \beta_0 = -\beta_n \tag{13}$$

$$H_0 : \beta_i = 0,\ i \neq n,\ \ 1 \leq i \leq 6,\ \text{ and } H_0 : \beta_0 = -\beta_n \tag{14}$$

Restriction (ii) is more general than a hypothesis for a particular value of the coefficient on the forward premium. Instead of checking the value or the sign of a regression coefficient on the difference between the forward rate and the spot rate, we are testing that it is appropriate to collapse the information contained in both variables into their difference, at least for the purpose of understanding expected returns.

Table 4.2 reports p-values for tests of the null hypothesis (12) - (14) for the regression (7). The left subtable gives the results for one month returns, $n = 1$, while the right subtable gives results for one year returns $n = 12$. In cases where the moment covariance matrix is ill-conditioned (it is always nonsingular), I compute the inverse using a singular value decomposition. For practically all currencies, one sees strong evidence against the null, with p-values less than one percent. The exceptions to this are JPY, for which one can rather comfortably fail to reject the null of restrictions I and III for both types of return, and the first restriction on 12 month returns for NOK.

To check that the results given in Table 4.2 are robust to the particular regression specification, I considered a variety of related choices designed to make sure that additional term structure information does in fact grant statistically significant improvements in the ability to predict returns. A primary concern is the possibility of a level effect: do the results carry through for regressions on $\widetilde{\mathbf{f}}_{t,j}$ instead of $\mathbf{f}_{t,j}$? Moreover, if the results do survive this change in the regression specification, one might still worry that the presence of the time t spot rate in each entry of the vector $\widetilde{\mathbf{f}}_{t,j}$ is driving the result, rather than forward rates $f_{t,j}^{(n)}$ of longer of shorter maturity than the return horizon of concern. I will now show that neither of these possible concerns present a problem, by considering a collection of alternative regressions and hypothesis tests.

The next few paragraphs describe a total of six regression/hypothesis test combinations. The first four tests use 1-month returns as the left hand side (LHS) variable in the regression, the last two use 12-month returns. Our first concern is the possibility of a level effect driving the result. To check this, let's examine a regression of 1-month returns on the vector of forward spot spreads:

Test 1 Regression:

$$
\begin{aligned}
rx^{(1)}_{t\to t+1,j} = \quad & \beta_{-1} + \beta_1\left(f^{(1)}_{t,j} - s_{t,j}\right) + \beta_2\left(f^{(2)}_{t,j} - s_{t,j}\right) + \beta_3\left(f^{(3)}_{t,j} - s_{t,j}\right) \\
& +\beta_4\left(f^{(6)}_{t,j} - s_{t,j}\right) + \beta_5\left(f^{(9)}_{t,j} - s_{t,j}\right) + \beta_6\left(f^{(12)}_{t,j} - s_{t,j}\right) + \varepsilon_{t+1} \\
& = \beta'\widetilde{\mathbf{f}}_{t,j} + \varepsilon_{t+1}
\end{aligned}
$$

The null hypothesis is that regression coefficients on forward-spot spreads $\widetilde{f}^{(n)}_{t,j} = f^{(n)}_{t,j} - s_{t,j}$ are jointly equal to 0 for $n \neq 1$:

Test 1 H_0: $\beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$

Test 2 is similar, but instead of subtracting the spot rate from each forward rate on the RHS, I instead subtract the time t forward rate with the next shortest time until maturity. Then, the spot rate is part of only one RHS variable:

Test 2 regression:

$$
\begin{aligned}
rx^{(1)}_{t\to t+1,j} = \quad & \beta_{-1} + \beta_1\left(f^{(1)}_{t,j} - s_{t,j}\right) + \beta_2\left(f^{(2)}_{t,j} - f^{(1)}_{t,j}\right) + \beta_3\left(f^{(3)}_{t,j} - f^{(2)}_{t,j}\right) \\
& +\beta_4\left(f^{(6)}_{t,j} - f^{(3)}_{t,j}\right) + \beta_5\left(f^{(9)}_{t,j} - f^{(6)}_{t,j}\right) + \beta_6\left(f^{(12)}_{t,j} - f^{(9)}_{t,j}\right) + \varepsilon_{t+1}
\end{aligned}
$$

I use the same labeling system for Test 1 and Test 2, so the two null hypothesis have identical symbols:

Test 2 H_0: $\beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$

However, the *meaning* of the two restrictions is different. The first restriction says that the one-year forward-spot spread does not provide new information about one-month returns if the one-month forward spot-spread is known, and similarly for the other spreads, while the second restriction says that the difference in one-year and nine-month forward-spot spreads does not provide additional information about expected returns in one month if the one-month forward-spot spread is already known.

Tests 3 and 4 repeat tests 1 and 2 with a smaller set of RHS variables. In particular, the regressions and nulls are:

Test 3 Regression:

$$rx^{(1)}_{t\to t+1,j} = \beta_{-1} + \beta_1\left(f^{(1)}_{t,j} - s_{t,j}\right) + \beta_4\left(f^{(6)}_{t,j} - s_{t,j}\right) + \beta_6\left(f^{(12)}_{t,j} - s_{t,j}\right) + \varepsilon_{t+1} \tag{15}$$

$$\textbf{Test 3 } H_0: \quad \beta_4 = \beta_6 = 0$$

and

Test 4 regression:

$$rx^{(1)}_{t\to t+1,j} = \beta_{-1} + \beta_1\left(f^{(1)}_{t,j} - s_{t,j}\right) + \beta_6\left(f^{(12)}_{t,j} - f^{(9)}_{t,j}\right) + \varepsilon_{t+1}$$

$$\textbf{Test 4 } H_0: \quad \beta_6 = 0$$

The next two tests concern one-year returns. Test 5 simply repeats the substitution of $\widetilde{\mathbf{f}}_{t,j}$ for $\mathbf{f}_{t,j}$:

Test 5 regression:

$$\begin{aligned} rx^{(12)}_{t\to t+12,j} = \; & \beta_{-1} + \beta_1\left(f^{(1)}_{t,j} - s_{t,j}\right) + \beta_2\left(f^{(2)}_{t,j} - s_{t,j}\right) + \beta_3\left(f^{(3)}_{t,j} - s_{t,j}\right) \\ & + \beta_4\left(f^{(6)}_{t,j} - s_{t,j}\right) + \beta_5\left(f^{(9)}_{t,j} - s_{t,j}\right) + \beta_6\left(f^{(12)}_{t,j} - s_{t,j}\right) + \varepsilon_{t+12} \end{aligned}$$

$$\textbf{Test 5 } H_0: \quad \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$$

Finally, the choice of RHS variables for one-year returns must be approached with a bit more care than one-month returns. We want to include the one-year forward-spot spread as a variable, but also other forward rates of shorter maturity, differenced with respect to something other than the spot rate. Including the entire (telescoping) selection of forward rate differences as in the second test will lead to collinear RHS variables, so I only choose a few. The regression is:

Test 6 regression:

$$\begin{aligned} rx^{(12)}_{t\to t+12,j} = & \quad \beta_{-1} + \beta_1 \left(f^{(12)}_{t,j} - s_{t,j}\right) + \beta_2 \left(f^{(1)}_{t,j} - s_{t,j}\right) \\ & + \beta_3 \left(f^{(6)}_{t,j} - f^{(3)}_{t,j}\right) + \beta_4 \left(f^{(12)}_{t,j} - f^{(9)}_{t,j}\right) + \varepsilon_{t+12} \end{aligned}$$

And the null is that regression coefficients on term structure elements other than the forward-spot spread should be jointly equal to 0:

$$\textbf{Test 6 } H_0: \quad \beta_2 = \beta_3 = \beta_4 = 0$$

Table 4.3 reports p-values for the above collection of tests, with standard errors computed using the method of Hansen, Eichenbaum, and Singleton. All p-values are less than one tenth of one percent, and this result is not sensitive to the choice of the number of lagged terms used to estimate the vector auto regressions. For the particular choices used to generate the table, I used six lags in the first step and one in the second step for one month returns, and 18 lags in the first step and 12 lags in the second step for one-year returns.

It seems then that expected returns at both long and short time horizons depend on the forward foreign exchange rate term structure, instead of the relevant forward-spot spread alone. In the next section I show that expected returns to currency forward contracts at various time horizons use essentially the same foreign exchange term structure information.

Table 4.1: R^2 values from regressions (5) - (7). The left hand side variable is the n month currency return relative to the USD. For the first subtable $n = 1$, for the second $n = 12$. The first column corresponds to a regression on a constant and the n-month forward-spot spread. Then second column corresponds to a regression on a constant, the spot rate, and the n-month forward rate. The third column reports the R^2 value for a regression on a constant, the spot rate, and all forward rates for the given currency.

RHS Variable:	$f_{t,j}^{(n)} - s_{t,j}$	$f_{t,j}^{(n)}$, $s_{t,j}$ (separate variables)	term structure: $\mathbf{f}_{t,j}$
R^2		1-month returns	
AUD	0.01	0.03	0.08
CAD	0.01	0.01	0.05
CHF	0.01	0.02	0.04
GBP	0.00	0.02	0.02
JPY	0.03	0.04	0.06
NOK	0.00	0.01	0.05
NZD	0.00	0.01	0.04
SEK	0.00	0.01	0.06
R^2		12 -month returns	
AUD	0.08	0.23	0.26
CAD	0.01	0.04	0.06
CHF	0.12	0.15	0.22
GBP	0.02	0.22	0.27
JPY	0.26	0.43	0.46
NOK	0.02	0.15	0.19
NZD	0.03	0.15	0.18
SEK	0.00	0.21	0.24

Table 4.2: p-values for hypothesis tests (12) - (14) of regression (7) for returns with time horizons of $n = 1$ and $n = 12$ months. Test I examines the restriction that the coefficient on $f_{t,j}^{(i)} = 0$ for $i \neq n$, Test II examines the restriction that the regression coefficient on the spot rate is the negative of the regression coefficient on the n-month forward rate. Test III combines both restrictions.

p-values for tests of term structure coefficient restrictions						
Test	I	II	III	I	II	III
return horizon		$n = 1$			$n = 12$	
AUD	0.00	0.00	0.00	0.00	0.00	0.00
CAD	0.00	0.00	0.00	0.00	0.00	0.00
CHF	0.00	0.00	0.00	0.00	0.00	0.00
GBP	0.00	0.00	0.00	0.00	0.00	0.00
JPY	0.93	0.00	0.97	1.00	0.00	1.00
NOK	0.00	0.00	0.00	0.58	0.00	0.00
NZD	0.00	0.00	0.00	0.00	0.00	0.00
SEK	0.00	0.00	0.00	0.00	0.00	0.00

Table 4.3: p-values for extra hypothesis tests of the significance of FX term structure information. In all cases, one sees strong evidence against the null that coefficients on variables other than $f_{t,j}^{(n)} - s_{t,j}$ are jointly equal to 0. Test 1 is a test that coefficients from the regression of one month returns on $\widetilde{\mathbf{f}}_{t,j}$

p-values for additional robustness tests						
Test	1	2	3	4	5	6
AUD	0.00	0.00	0.00	0.00	0.00	0.00
CAD	0.00	0.00	0.00	0.00	0.00	0.00
CHF	0.00	0.00	0.00	0.00	0.00	0.00
GBP	0.00	0.00	0.00	0.00	0.00	0.00
JPY	0.00	0.00	0.00	0.00	0.00	0.00
NOK	0.00	0.00	0.00	0.00	0.00	0.00
NZD	0.00	0.00	0.00	0.00	0.00	0.00
SEK	0.00	0.00	0.00	0.00	0.00	0.00

5 A common factor in the term structure?

In the last section we saw that expected currency returns depend on the entire term structure of currency forward rates. So far, though, we have only considered individual return horizons in isolation. I now show that one can take the information in the term structure and use it to forecast returns across time horizons. To an excellent approximation, for each individual currency, expected returns at all time horizons are formed from the same linear combination of term structure elements. Following this approximation, almost all of explainable variation in expected returns that is captured by a regression of each return horizon on the term structure of forward rates can be captured by a restricted regressions that allows expected returns over different horizons to vary only in their scalar loadings on the *same* linear function of term structure information.

The method in this section is borrowed from Cochrane and Piazessi's [4] study of bond returns. However, whereas they examined excess returns for various Treasury maturities realized at the same time period, I instead consider the total returns (when held from initiation to maturity) of all currency forward contracts for which data is available, intimated at a given time. For each maturity, I compare the performance of an unrestricted regression ($rx^{(n)}_{t,\to t+n,j}$ on the vector $\mathbf{f}_{t,j}$ and a constant) with that of a *restricted regression* that uses the data from $\mathbf{f}_{t,j}$ in such a fashion as to force a common relationship on the regression coefficients of the forward rate vector across return maturities. The procedure for the restricted regression is described in detail below.

Consider the simple average over all time horizons of returns to currency j forward contracts initiated at time t. This is:

$$\overline{rx}_{[t],j} \equiv \tfrac{1}{|N|} \textstyle\sum_{n \in N} rx^{(n)}_{t \to t+n,j} =$$
$$\tfrac{1}{6}\left(rx^{(1)}_{t\to t+1,j} + rx^{(2)}_{t\to t+2,j} + rx^{(3)}_{t\to t+3,j} + rx^{(6)}_{t\to t+6,j} + rx^{(9)}_{t\to t+9,j} + rx^{(12)}_{t\to t+12,j} \right)$$
$$N = \{1, 2, 3, 6, 9, 12\}$$

The first information set $\mathcal{F}_{t+k}$ to which this belongs is the largest $k \in N$. I then regress the cross-maturity average return for currency j on a constant and currency j's vector of forward rates prevailing at time t:

$$\overline{rx}_{[t],j} = \gamma' \mathbf{f}_{t,j} + \varepsilon_{[t],j} \tag{16}$$

From this, form for each currency the time t average expected return across time horizons:

$$\mathbb{E}_t \left[\overline{rx}_{[t],j} \right] = \gamma' \mathbf{f}_{t,j}$$

For simplicity, I often call this the expected average return at time t. Now, for each $n \in N$, I examine the *restricted regression* of realized returns on expected average returns:

$$rx^{(n)}_{t \to t+n,j} = b_n \left(\gamma' \mathbf{f}_{t,j} \right) + \varepsilon_{t+n,j} \tag{17}$$

Since the γ terms above are those I obtained in Equation 16, the only term to be estimated in (17) is the horizon-specific scalar loading coefficient b_n. I then compare the estimation in (17) with an unrestricted regression in which the estimated constant and coefficients on the set of forward rates are allowed to vary freely:

$$rx^{(n)}_{t \to t+n,j} = \beta'_n \mathbf{f}_{t,j} + \varepsilon_{t+n,j} \tag{18}$$

The first subtable of Table 5.1 reports R^2 values from the unrestricted regression (18) . The second subtable reports R^2 values from the restricted regression (17), and the last subtable calculates the difference between the two subtables. In both cases, values typically increase with the return horizon, and one-year horizon unrestricted R^2 values are larger than 20% for more than half of the currencies considered. Examining the differences between the unrestricted and restricted values in the third subtable suggests that this single-factor model is a good fit for describing the data. The two single largest discrepancies round to 4% each. The next three largest differences each round to 3%. The remaining 45 currency-return horizon pairs show a difference of 2% or less.

Accurate standard error calculations for the restricted regression coefficients must take into account the dependence of the scalar b_n regression coefficients on the vector of average return regression coefficients γ'. These can be calculated using GMM as in [4], details are provided in the appendix.

Each of the restricted regression coefficients is many standard errors from zero, using either calculation method. One might notice the tables show that some unrestricted regression coefficients are not estimated very precisely, with a few within two standard errors of zero. This is not necessarily a problem for our purposes, at least not yet. All that we have claimed so far is that the term structure as a whole contains useful information beyond the simple forward-spot spread.

Does the restricted regression on the vector of rates above work as well when using the vector of spreads instead? Both the restricted and unrestricted regressions appear to suffer when using spreads instead of rates, but the restricted regression setup still appears moderately successful. Let us now repeat the procedure above, with regressions on the currency-specific vector of forward-spot spreads $\widetilde{\mathbf{f}}_{t,j}$. That is, the unrestricted regression is now:

$$rx^{(n)}_{t\to t+n,j} = \beta'_n \widetilde{\mathbf{f}}_{t,j} + \varepsilon_{t+n,j} \tag{19}$$

while the restricted regressions are formed from

$$\overline{rx}_{[t],j} = \frac{1}{N}\sum_{n\in N} rx^{(n)}_{t\to t+n,j} = \widetilde{\gamma}'\widetilde{\mathbf{f}}_{t,j} + \varepsilon_{[t],j} \tag{20}$$

$$rx^{(n)}_{t\to t+n,j} = \widetilde{b}_n \left(\widetilde{\gamma}'\widetilde{\mathbf{f}}_{t,j}\right) + \varepsilon_{t+n,j} \tag{21}$$

Table 5.2 reports the resulting R^2 values. The first subtable reports values from the unrestricted regression corresponding to Equation 19. One observes a loss of fit comparing these results to the unrestricted regression on levels in Table 5.1. Values in excess of 10 or even 20 percent were relatively common before, while under the spreads specification such values only occur in a small number of cases. Examining the R^2 values for the restricted regressions in the second subtable, and looking at the differences between the two values computed in the third subtable, one sees that the absolute loss of fit caused by imposing the single-factor restriction is small in magnitude, with the largest difference being about 4 percent.

To understand why the vector of rates $\mathbf{f}_{t,j}$ can explain more of the variation in returns than the vector of spreads $\widetilde{\mathbf{f}}_{t,j}$, it is necessary to take a closer look at the composition of the forward foreign exchange term structure. This is the focus of the next section.

Table 5.1: R^2 values from regressions of returns $rx^{(n)}_{t\to t+n,j}$ on the vector of foreign exchange forward rates (and a constant) $\mathbf{f}_{t,j}$. The first subtable gives reports values for the unrestricted regression. The second subtable reports values for the restricted regression. The third subtable reports the difference of the previous two.

$n =$	1	2	3	4	5	6
	Unrestricted Regression R^2					
AUD	0.08	0.11	0.12	0.18	0.21	0.26
CAD	0.05	0.05	0.04	0.05	0.04	0.06
CHF	0.04	0.08	0.06	0.09	0.18	0.22
GBP	0.02	0.05	0.07	0.17	0.23	0.27
JPY	0.06	0.11	0.14	0.25	0.38	0.46
NOK	0.05	0.06	0.07	0.09	0.16	0.19
NZD	0.04	0.08	0.07	0.09	0.14	0.18
SEK	0.06	0.15	0.15	0.15	0.19	0.24
	Restricted Regression R^2					
AUD	0.06	0.08	0.10	0.17	0.20	0.24
CAD	0.03	0.03	0.03	0.05	0.04	0.05
CHF	0.03	0.07	0.06	0.09	0.17	0.22
GBP	0.02	0.04	0.07	0.16	0.23	0.26
JPY	0.05	0.10	0.13	0.24	0.38	0.45
NOK	0.03	0.05	0.07	0.09	0.16	0.19
NZD	0.02	0.04	0.04	0.09	0.13	0.16
SEK	0.05	0.10	0.13	0.15	0.18	0.23
	Difference of R^2s					
AUD	0.03	0.03	0.02	0.01	0.01	0.02
CAD	0.02	0.02	0.01	0.00	0.00	0.01
CHF	0.01	0.01	0.00	0.00	0.00	0.00
GBP	0.01	0.00	0.00	0.01	0.01	0.01
JPY	0.01	0.01	0.01	0.01	0.00	0.01
NOK	0.02	0.01	0.01	0.00	0.00	0.00
NZD	0.02	0.04	0.02	0.01	0.01	0.01
SEK	0.01	0.04	0.03	0.00	0.01	0.01

Table 5.2: R^2 values from regressions of returns $rx^{(n)}_{t\to t+n,j}$ on the vector of foreign exchange forward rates (and a constant) $\widetilde{\mathbf{f}}_{t,j}$. The first subtable gives reports values for the unrestricted regression. The second subtable reports values for the restricted regression. The third subtable reports the difference of the previous two.

$n =$	1	2	3	4	5	6
	Unrestricted Regression R^2					
AUD	0.06	0.07	0.07	0.08	0.08	0.09
CAD	0.05	0.05	0.04	0.04	0.02	0.04
CHF	0.03	0.05	0.04	0.06	0.12	0.15
GBP	0.01	0.01	0.02	0.06	0.08	0.09
JPY	0.04	0.07	0.09	0.16	0.25	0.27
NOK	0.04	0.03	0.03	0.02	0.03	0.04
NZD	0.03	0.06	0.04	0.03	0.04	0.04
SEK	0.04	0.11	0.11	0.04	0.02	0.03
	Restricted Regression R^2					
AUD	0.03	0.04	0.05	0.07	0.07	0.07
CAD	0.03	0.03	0.02	0.04	0.02	0.03
CHF	0.02	0.05	0.03	0.06	0.11	0.15
GBP	0.00	0.01	0.02	0.05	0.08	0.09
JPY	0.03	0.06	0.08	0.15	0.25	0.26
NOK	0.02	0.02	0.03	0.01	0.03	0.03
NZD	0.01	0.02	0.02	0.02	0.03	0.03
SEK	0.04	0.11	0.10	0.04	0.02	0.03
	Difference of R^2s					
AUD	0.03	0.03	0.02	0.01	0.01	0.02
CAD	0.02	0.02	0.01	0.00	0.00	0.01
CHF	0.01	0.01	0.00	0.00	0.00	0.00
GBP	0.00	0.00	0.00	0.01	0.01	0.01
JPY	0.01	0.01	0.01	0.01	0.00	0.01
NOK	0.02	0.01	0.00	0.00	0.00	0.00
NZD	0.02	0.04	0.02	0.01	0.01	0.01
SEK	0.00	0.01	0.00	0.00	0.00	0.00

6 Analysis and comparison of currency forward term structures

We now use principal component analysis (PCA) to better understand why a regression on the term structure of currency spreads explains less of the variation in returns than an analogous regression on the vector of rates.

Principal component decompositions have the advantage of transforming the data into an orthogonal set of variables, where these are ordered by the fraction of the variance of the original data that they are able to explain. The process involves performing an eigenvalue decomposition of the covariance matrix of the variable of interest, say, $\mathbf{f}_{tj}$:

$$Q\Lambda Q' = cov\left(\mathbf{f}_{tj}\right)$$

Here Q is a matrix of eigenvectors, and Λ is the corresponding diagonal matrix of eigenvalues with typical element $\Lambda_{ii} = \lambda_i$. Sort Q and Λ in decreasing order of variance of the original data that they are able to explain, so that the largest eigenvalue is first, and so on. The goal is to explain as much of the variance of the data as possible with a parsimonious (and conveniently linearly independent) number of factors. To recover the ith principal component, multiply the original data by the ith largest (ordered by the size of the associated eigenvalue) eigenvector of Q, which will be its ith column. Moreover, if the original data is D-dimensional and λ_i is the eigenvalue associated with the eigenvector q_i, then $\frac{\lambda_i}{\sum_{k=1}^{D}\lambda_k}$ gives the fraction of variance of the original data explained by the ith component, while $\frac{\sum_{k=1}^{i}\lambda_k}{\sum_{k=1}^{D}\lambda_k}$ gives the cumulative fraction of variance explained by the first i principal components.

Table 6.1 reports cumulative fractions of variance explained for each of the factors determined by principal component analysis on $\mathbf{f}_{t,j}$ and $\widetilde{\mathbf{f}}_{t,j}$. Results for $\mathbf{f}_{t,j}$ are reported in the top subtable, while results for $\widetilde{\mathbf{f}}_{t,j}$ are reported in the bottom subtable. In both cases, the relative importance of the different components is largely the same across currencies. The first factor accounts for over 99% of the variation for all currencies, for either rates or spreads. For forward rates, the cumulative fraction of variance rounds to 100% (to six decimal places) for most currencies by $n = 4$, and for all currencies but one by $n = 5$. (All round to one for $n \geq 6$. Looking at the bottom subtable shows

Table 6.1: Cumulative fraction of variance of each of $\mathbf{f}_{t,j}$ and $\widetilde{\mathbf{f}}_{t,j}$ explained by each of the first i principal components.

j	AUD	CAD	CHG	GBP	JPY	NOK	NZD	SEK
i	Cumulative fraction of variance of $\mathbf{f}_{t,j}$ explained by first i factors							
1	0.999263	0.999152	0.998716	0.996945	0.997726	0.997253	0.999427	0.997162
2	0.999996	0.999996	0.999996	0.999985	0.999994	0.999987	0.999993	0.999977
3	0.999999	1.000000	1.000000	0.999999	0.999999	0.999997	0.999999	0.999991
4	1.000000	1.000000	1.000000	0.999999	1.000000	0.999999	1.000000	0.999998
5	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	0.999999
6	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
7	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
i	Cumulative fraction of variance of $\widetilde{\mathbf{f}}_{t,j}$ explained by first i factors							
1	0.996441	0.996512	0.997563	0.996565	0.997986	0.995634	0.992346	0.995719
2	0.999687	0.999713	0.999839	0.999791	0.999818	0.999535	0.999005	0.998639
3	0.999886	0.999884	0.999939	0.999928	0.999910	0.999831	0.999640	0.999732
4	0.999939	0.999939	0.999969	0.999968	0.999953	0.999959	0.999885	0.999920
5	0.999976	0.999977	0.999989	0.999989	0.999979	0.999991	0.999959	0.999985
6	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

the cumulative fraction of variance explained reaches one more slowly for spreads than for rates.

Table 6.2 does the same for returns and expected returns. The first part of the table reports the cumulative fraction of variance obtained by PCA on the vector of currency returns initiated at time t. The second subtable reports the fraction of variance in *expected returns* to currency forward contracts initiated at time t, where expected returns are calculated using $\mathbf{f}_{t,j}$, as in (18). The last subtable does the same for expected returns calculated using $\widetilde{\mathbf{f}}_{t,j}$, as in (19)

Principal components analysis of returns yields a more complicated structure than obtained for forward rates. Looking at Table 6.2 In this case, the largest component accounts for between 70% and 80% of the variation in total returns across maturities for a given country. The next component adds between 10 and 20 percent, the next about 5 percent. To be able to obtain 99% of the variation, we need to include 5 of the 6 components. This is true for each of the currencies considered.

The second subtable of 6.2 reports cumulative fraction of variance by number of components for expected returns, conditional on $\mathbf{f}_{t,j}$, the vector of forward rates. The first component accounts for an average of about 95% of the variation, the first and second components together account for

Table 6.2: Fraction of variance explained by each component of $Cov(\mathbf{rx}_{t,j})$

$j =$	AUD	CAD	CHF	JPY	GBP	NOK	NZD	SEK
i	Fraction of variance explained by each component of $Cov(\mathbf{rx}_{t,j})$							
1	0.791517	0.773517	0.756630	0.732956	0.772188	0.751527	0.819218	0.773370
2	0.910811	0.899144	0.878159	0.884867	0.887674	0.888089	0.924900	0.901492
3	0.956983	0.949404	0.942114	0.950183	0.954320	0.946683	0.963229	0.954965
4	0.977386	0.972589	0.973232	0.977100	0.978974	0.976166	0.981666	0.978904
5	0.992582	0.989426	0.990817	0.992293	0.991578	0.991818	0.993278	0.993551
6	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
i	Fraction of variance explained by each component of $Cov(\mathbb{E}\left[\mathbf{rx}\right]_{t,j})$, rates specification							
1	0.938505	0.873883	0.979101	0.973683	0.982283	0.974192	0.925679	0.945440
2	0.983595	0.985523	0.987373	0.991502	0.994998	0.991105	0.971354	0.992537
3	0.996901	0.992343	0.994390	0.996617	0.998798	0.997232	0.994196	0.998146
4	0.999403	0.996712	0.998679	0.998768	0.999747	0.999338	0.998885	0.999297
5	0.999869	0.999797	0.999889	0.999945	0.999983	0.999919	0.999834	0.999779
6	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
i	Fraction of variance explained by each component of $Cov(\mathbb{E}\left[\mathbf{rx}_{t,j}\right])$, spreads specifyication							
1	0.847043	0.824354	0.968949	0.928161	0.973311	0.898690	0.735259	0.946547
2	0.958832	0.981327	0.981443	0.977794	0.993079	0.959473	0.898482	0.983931
3	0.992335	0.991350	0.991764	0.991727	0.999111	0.987793	0.980259	0.993224
4	0.998571	0.997143	0.998024	0.996475	0.999853	0.997016	0.996193	0.998345
5	0.999744	0.999812	0.999842	0.999895	1.000000	0.999699	0.999553	0.999352
6	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

about 98% of the variance.

The third subtable does the same for expected returns conditional on the vector of forward-spot spreads $\widetilde{\mathbf{f}}_{t,j}$. For every currency, the first component of expected returns explains a smaller relative share of the variance when calculated using spreads than when calculating returns. Once the second component is taken into account, both methods of forming expected returns have a comparable cumulative fraction of variance share.

Figure 1 reports loading of the first three principal components of $\mathbf{f}_{t,j}$ for each currency j. The loadings shapes are a match for the level, slope, and curvature factors familiar from the literature on bond pricing, and occur in the same order, so I refer to them by the same names. (See, for example, [5] for a discussion of the role of these factors in bond pricing.) The first factor, level, has practically identical loadings on the across maturities. The second factor, slope, would look like a straight line, except for the fact that the time horizons of the various forward rates are not evenly spaced. The spot rate, 1 month, 2 month, and 3 month rates are each one month apart, while the difference in longer-maturity forward rates is a constant three months. This causes a kink in the plot of the second factor at $n = 3$. To the left of the kink we see the gentler slope associated with a spacing of one between points, to the right the steeper slope corresponding to a difference of three months. Slope factor loadings shapes are also the same across currencies, up to a multiplication by -1 for AUD. The third factor, curvature, has low loadings for both short and long maturities, and high loadings for intermediate maturities for each of CAD, CHF, GBP, JPY, and NOK. Loading shapes for AUD and NZD are the same as the earlier group up to a multiplication by -1. Of the three components, the only currency with a loading shape that is not identical to the others up to a scalar multiplication is SEK's third (curvature) component, which displays too large of a loading on $f^{(2)}_{t.SEK}$, and loadings that are too low on intermediate maturities, to fit in with the others. Additional factors do not explain much of the variance, and display irregular loading patters.

Figure 2 reports loadings on $\widetilde{f}^{(n)}_{t,j}$ for the first two principal components of $\widetilde{f}_{t,j}$. These look like the slope and curvature factors from the decomposition on $\mathbf{f}_{t,j}$. This seems to offer a partial explanation for the loss of R^2 in moving from regressions of returns on $\mathbf{f}_{t,j}$ to $\widetilde{f}_{t,j}$: subtracting the spot rate from each of the forward rates effectively eliminates the level factor. The loss in fit then

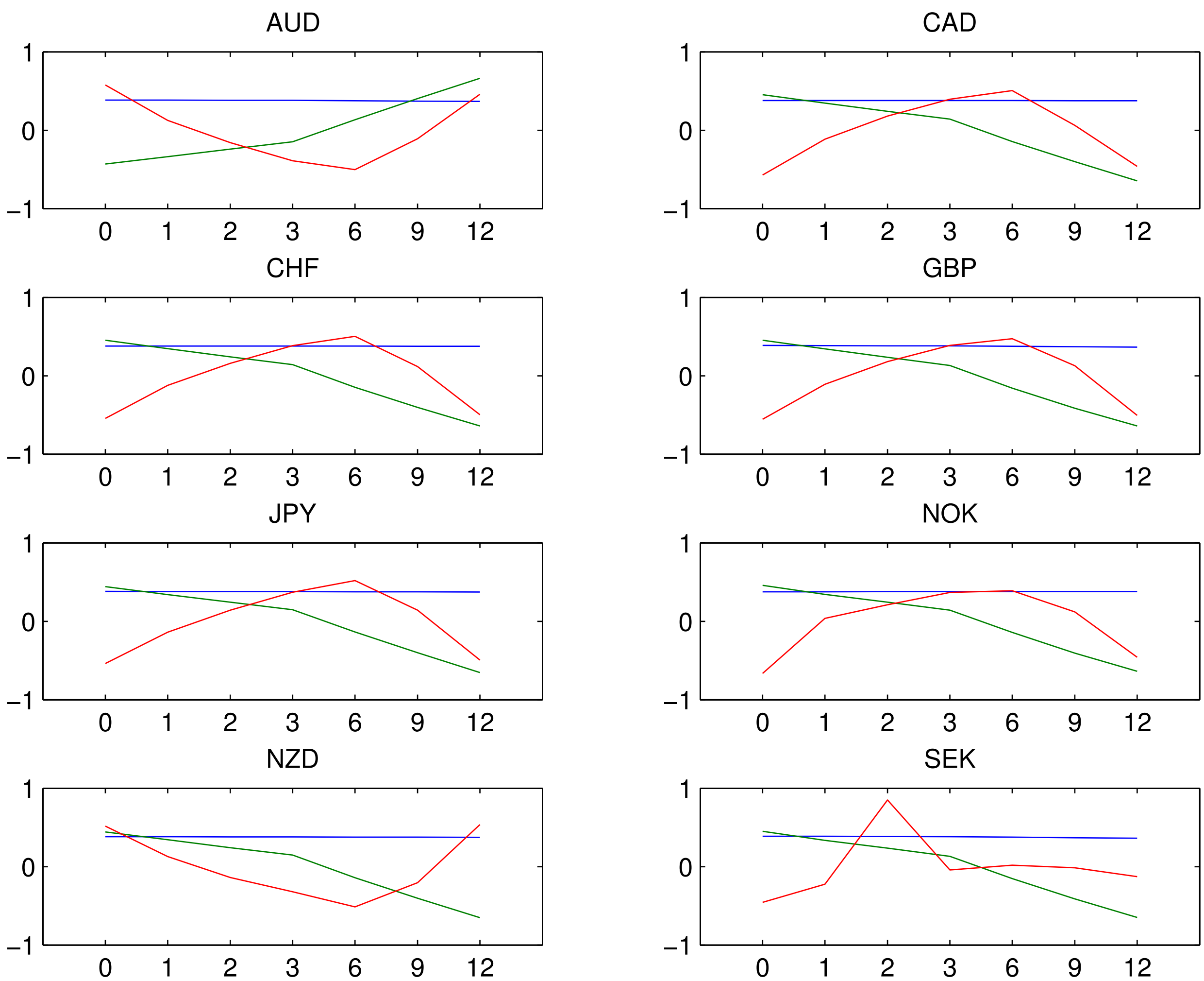

Figure 1: Loadings of the first three principal components of $\mathbf{f}_{t,j}$. Loading shapes are nearly identical across currencies. The level, slope, and curvature factors are the flat line, the kinked line, and the curve, respectively.

suggests that the level factor makes up an important part of expected returns.

So far, we have seen that the first few principal components of both the forward rate and forward-spot spread vectors display rather similar loading shapes across currencies, and explain the about the same share of variance. To what extent do the foreign exchange term structure factors of different currencies move together? Table 6.3 displays the cross-currency correlation matrices for the first three components of the vector of forward rates $\mathbf{f}_{tj}$. For the level factor, most cross-currency correlation pairs are in excess of 60%, with many as large as $80-90\%$. Correlation pairs for the slope factor are roughly similar in magnitude. (The spread factor in AUD is negatively correlated with that of every other currency, all correlations are positive.) The third factor curvature, is more weakly correlated across currencies, and also displays more variability in its sign. Table 6.4 reports correlations for the first two principle components of $\widetilde{f}_{t,j}$, with roughly comparable results. Although some signs are different, correlation magnitudes are qualitatively similar.

Table 6.5 provides correlations for factors of realized returns. Table 6.6 and Table 6.7 provide correlations for factors of expected returns calculated using $\mathbf{f}_{t,j}$ and $\widetilde{\mathbf{f}}_{t,j}$, respectively. In both tables, one sees moderately high cross-currency correlations of the largest component of expected returns, with about half of the values for each about 30% or larger in magnitude.

So any currency's vector of expected returns depends on its forward foreign exchange term structure, and term structure factors display similar loading shapes and high correlations across currencies. Moreover, the largest components of expected currency returns are noticeably correlated across currencies as well. Since the term structure predicts returns across time horizons for a given currency, one might expect that the cross section of foreign exchange forwards would also predict returns across time horizons. We will see in the next section that it does.

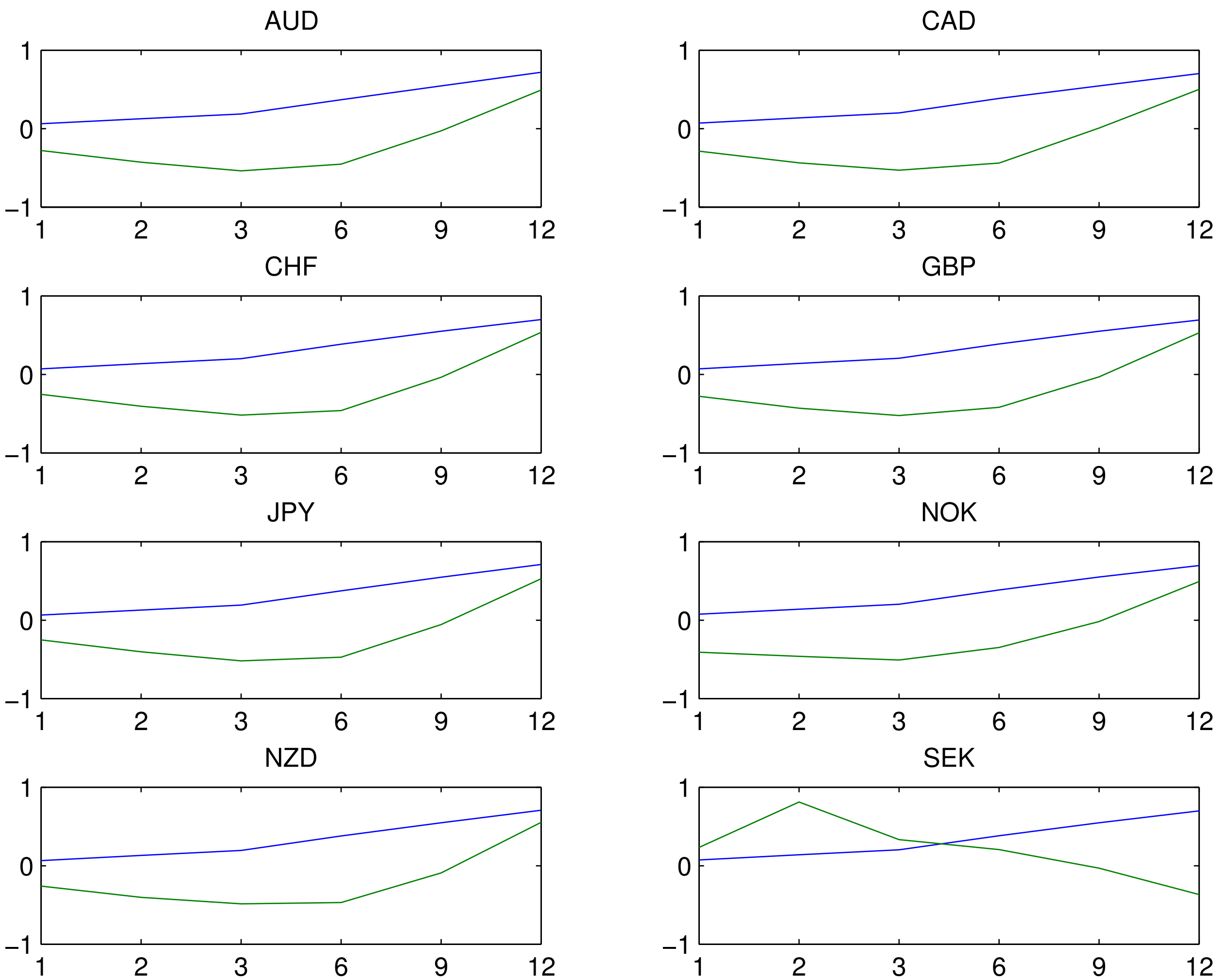

Figure 2: Loadings of the first two principal components of $\widetilde{f}_{t,j}$. Loading shapes are nearly identical across currencies. The level and curvature factors are the kinked line and the curve, respectively.

Table 6.3: Correlations of principal components of $\mathbf{f}_{t,j}$

First factor: Level								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.90	0.87	0.36	0.58	0.90	0.93	0.79
CAD		1.00	0.81	0.43	0.47	0.86	0.80	0.73
CHF			1.00	0.25	0.74	0.85	0.88	0.60
GBP				1.00	-0.21	0.56	0.43	0.60
JPY					1.00	0.43	0.58	0.14
NOK						1.00	0.88	0.87
NZD							1.00	0.70
SEK								1.00
Second factor: Slope								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	-0.65	-0.61	-0.55	-0.68	-0.65	-0.80	-0.57
CAD		1.00	0.74	0.77	0.70	0.74	0.50	0.84
CHF			1.00	0.74	0.88	0.87	0.46	0.79
GBP				1.00	0.61	0.71	0.56	0.82
JPY					1.00	0.74	0.53	0.61
NOK						1.00	0.47	0.81
NZD							1.00	0.44
SEK								1.00
Third factor: Curvature								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	-0.40	-0.46	-0.46	-0.36	-0.32	0.49	-0.19
CAD		1.00	0.18	0.46	0.24	0.27	-0.19	0.07
CHF			1.00	0.52	0.65	0.49	-0.49	0.19
GBP				1.00	0.34	0.45	-0.23	0.22
JPY					1.00	0.35	-0.38	0.14
NOK						1.00	-0.36	0.30
NZD							1.00	-0.09
SEK								1.00

Table 6.4: Correlations of principal components of $\widetilde{\mathbf{f}}_{t,j}$

First factor: Slope								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.62	0.61	0.41	0.78	0.48	0.77	0.52
CAD		1.00	0.78	0.69	0.74	0.74	0.45	0.85
CHF			1.00	0.68	0.88	0.85	0.40	0.84
GBP				1.00	0.52	0.63	0.51	0.77
JPY					1.00	0.69	0.51	0.70
NOK						1.00	0.30	0.78
NZD							1.00	0.42
SEK								1.00
Second factor: Curvature								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.42	0.46	0.48	0.39	0.31	0.53	-0.33
CAD		1.00	0.21	0.41	0.28	0.31	0.24	-0.18
CHF			1.00	0.59	0.73	0.52	0.52	-0.37
GBP				1.00	0.44	0.46	0.27	-0.41
JPY					1.00	0.42	0.42	-0.28
NOK						1.00	0.37	-0.52
NZD							1.00	-0.24
SEK								1.00

Table 6.5: Correlations of principal components of realized returns $\mathbf{rx}_{[t],j}$

First factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.81	0.51	0.61	0.26	0.69	0.91	0.72
CAD		1.00	0.31	0.52	0.13	0.57	0.71	0.60
CHF			1.00	0.49	0.44	0.81	0.57	0.73
GBP				1.00	0.04	0.73	0.62	0.76
JPY					1.00	0.22	0.25	0.14
NOK						1.00	0.73	0.85
NZD							1.00	0.68
SEK								1.00
Second factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.73	0.40	0.59	0.05	0.64	0.87	0.65
CAD		1.00	0.27	0.43	-0.01	0.53	0.64	0.51
CHF			1.00	0.65	0.42	0.81	0.47	0.78
GBP				1.00	0.19	0.80	0.60	0.79
JPY					1.00	0.21	0.12	0.12
NOK						1.00	0.63	0.87
NZD							1.00	0.61
SEK								1.00
Third factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.70	0.31	0.52	-0.02	0.54	0.77	0.55
CAD		1.00	0.24	0.51	-0.05	0.50	0.57	0.51
CHF			1.00	0.62	0.51	0.77	0.45	0.76
GBP				1.00	0.17	0.79	0.52	0.79
JPY					1.00	0.22	0.10	0.21
NOK						1.00	0.50	0.84
NZD							1.00	0.54
SEK								1.00

Table 6.6: Correlations of principal components of expected returns calculated from $\mathbf{f}_{t,j}$

First factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.49	0.51	0.19	0.57	0.33	0.75	0.43
CAD		1.00	0.36	0.25	0.28	0.45	0.45	0.37
CHF			1.00	0.18	0.67	0.34	0.49	0.27
GBP				1.00	-0.11	0.45	0.14	0.49
JPY					1.00	0.04	0.52	-0.01
NOK						1.00	0.50	0.83
NZD							1.00	0.47
SEK								1.00
Second factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	-0.25	0.17	-0.10	0.00	0.15	0.32	-0.11
CAD		1.00	-0.00	0.06	0.00	-0.06	-0.37	0.13
CHF			1.00	-0.07	-0.42	0.24	0.28	0.05
GBP				1.00	0.00	-0.10	-0.08	-0.01
JPY					1.00	-0.08	-0.25	0.09
NOK						1.00	0.15	-0.58
NZD							1.00	-0.20
SEK								1.00
Third factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.22	0.18	-0.04	-0.23	0.35	0.30	-0.19
CAD		1.00	0.05	0.02	-0.19	0.30	0.12	-0.02
CHF			1.00	-0.12	0.11	0.37	0.01	-0.09
GBP				1.00	-0.12	-0.15	0.19	-0.02
JPY					1.00	-0.07	-0.13	0.14
NOK						1.00	0.08	-0.21
NZD							1.00	0.01
SEK								1.00

Table 6.7: Correlations of principal components of expected returns calculated from $\widetilde{\mathbf{f}}_{t,j}$

First factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.36	0.56	-0.09	0.75	0.28	0.63	0.20
CAD		1.00	0.33	-0.07	0.33	0.27	0.10	0.10
CHF			1.00	-0.09	0.84	0.40	0.32	-0.07
GBP				1.00	-0.18	-0.12	-0.19	-0.28
JPY					1.00	0.45	0.48	0.08
NOK						1.00	0.09	0.45
NZD							1.00	0.06
SEK								1.00
Second factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	-0.23	0.15	-0.00	0.02	0.24	0.30	0.09
CAD		1.00	0.01	0.08	0.02	-0.08	-0.35	0.09
CHF			1.00	-0.01	-0.44	0.33	0.26	-0.11
GBP				1.00	0.04	-0.03	-0.07	-0.08
JPY					1.00	-0.11	-0.25	0.20
NOK						1.00	0.17	0.11
NZD							1.00	-0.10
SEK								1.00
Third factor:								
j	AUD	CAD	CHF	GBP	JPY	NOK	NZD	SEK
AUD	1.00	-0.23	-0.17	-0.12	-0.21	0.36	0.30	0.06
CAD		1.00	0.04	0.09	0.18	-0.28	-0.12	0.17
CHF			1.00	0.15	-0.12	-0.35	-0.01	0.09
GBP				1.00	-0.06	-0.29	0.11	0.20
JPY					1.00	-0.02	-0.14	-0.01
NOK						1.00	0.07	-0.26
NZD							1.00	0.19
SEK								1.00

7 Expected returns at all time horizons depend on the cross section of foreign exchange forward rates

To see the contribution of the cross section of currency forward rates to return predictability, I begin by comparing regression R^2s of all returns on the appropriate currency and time horizon-specific forward-spot spread alone with the vector of a constant and all currency forward-spot spreads for the same time horizon, $\widetilde{\mathbf{f}}_t^{(n)}$. Values are reported in Table 7.1. The first subtable provides the R^2 from a regression of the return $rx_{t\to t+n,j}^{(n)}$ on a constant and $\widetilde{f}_{t,j}^{(n)}$ alone. The second subtable reports R^2 values from the regression:

$$rx_{t\to t+n,j}^{(n)} = \beta' \widetilde{\mathbf{f}}_t^{(n)} + \varepsilon_{t+n} \tag{22}$$

The bottom subtable reports the differences between the two tables. The values in the second subtable are noticeably larger, and the differences increase with the time horizon. Many of the differences are about twenty percentage points or more. For an example, the R^2s on 12-month returns on CAD and SEK rise from about 1% and 0% when only the particular currency's forward-spot spread is used, to 21% and 22%, respectively, when the cross section is used. Moreover, comparing the R^2 values from the cross section regressions in the second subtable of Table 7.1 with those of the (unrestricted) regressions on the term structure of forward-spot spreads for the currency of interest, one sees that the cross sectional regressions have an R^2 value that is often 10% higher or more. The same comparison with the unrestricted regressions on the vectors of currency-specific forward rates provides a mixed result. Some currency - horizon pairs have higher R^2 values when using the cross section, while others have higher values in regressions using the term structure.

Table 7.2 reports p-values for a test of the null hypothesis for each currency and time horizon, the regression coefficients in (22) on forward-spot spreads of currencies other than currency j (the currency corresponding to the return on the right hand side) are jointly equal to zero:

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_{j-1} = \beta_{j+1} \ldots = \beta_n = 0 \tag{23}$$

where β_k is the regression coefficient corresponding to $\widetilde{f}_{t,k}^{(n)}$, the n-month forward-spot spread for currency k, observed at time t, and currencies are listed in alphabetic order. The pattern of the results is interesting in that while we reject the null of no joint significance for other forward-spot spreads in the cross section for at least some subset of time horizons for every currency, and for half or more of the currencies at all but one time horizon, there is no clear pattern of reject/fail to reject decisions across currencies, or even *within* a given currency across time horizons. At conventional significance levels, one rejects the null at short time horizons but not long time horizons for NZD, and at long time horizons but not short time horizons for SEK. For each of AUD, CAD, and JPY, we see strong evidence against the null at both short and long time horizons, but not at intermediate time horizons. A relatively uniform trend across time horizons of reject/fail to reject decisions might suggest a story about cross-country consumption insurance against high frequency or low frequency risks though not both, but it is not immediately obvious that a simple model could explain the pattern of results seen here.

In each subplot Figure 3, I plot for a particular currency the regression betas for each return horizon $rx_{t\to t+n,j}^{(n)}$ on the cross section of one month forward rates $\widetilde{\mathbf{f}}_t^{(1)}$. For every currency, we see very similar regression coefficients across return time horizons. The pattern is robust to the choice of the currency forward rate maturity date of the right-hand side variable $\widetilde{\mathbf{f}}_t^{(n)}$, all produce the same picture. This suggests that despite the odd pattern of test results in Table 7.2, a version of the single factor restriction from Section 5 might work well to describe currency expected returns across time horizons as a function of the foreign exchange forward rate cross section.

Consider again each currency's average return across time horizons to currency forward contracts initiated at time t, and regress this on the time t cross section of currency forward rates for one month in the future, $\widetilde{\mathbf{f}}_t^{(1)}$:

$$\overline{rx}_{[t],j} = \gamma' \widetilde{\mathbf{f}}_t^{(1)} + \varepsilon_{t+\bar{n}} \tag{24}$$

and a restricted regression for each maturity:

$$rx_{t\to t+n,j}^{(n)} = b_n \left(\gamma' \widetilde{\mathbf{f}}_t^{(n)}\right) + \varepsilon_{t+n} \tag{25}$$

A comparison of the unrestricted and restricted regression R^2 values gives the somewhat surprising result that for a few currencies one sees a slightly *higher* R^2 value in the restricted regression than in the unrestricted regression. When the restricted regression has a lower R^2 value, the difference is typically about two percentage points or less. Differences (unrestricted R^2 minus restricted R^2) remain generally small when positive for alternative choices of the maturity of the cross section vector on the right hand side of (24), but the negative differences basically disappear when the 9 month or 12 month cross section is used in (24).

Table 7.1: R^2 values from regressions on the cross section of currency forward rates

$n =$	1	2	3	6	9	12
R^2 from reg on own f-s spread						
AUD	0.01	0.02	0.03	0.03	0.05	0.08
CAD	0.01	0.01	0.01	0.01	0.01	0.01
CHF	0.01	0.03	0.03	0.05	0.09	0.12
GBP	0.00	0.00	0.00	0.01	0.02	0.02
JPY	0.03	0.05	0.08	0.15	0.24	0.26
NOK	0.00	0.00	0.00	0.00	0.01	0.02
NZD	0.00	0.00	0.01	0.01	0.01	0.03
SEK	0.00	0.00	0.00	0.00	0.00	0.00
R^2 from regression on the cross section						
AUD	0.05	0.09	0.14	0.25	0.28	0.27
CAD	0.05	0.11	0.15	0.20	0.21	0.21
CHF	0.07	0.13	0.15	0.16	0.24	0.30
GBP	0.10	0.14	0.17	0.19	0.20	0.19
JPY	0.05	0.10	0.13	0.22	0.35	0.42
NOK	0.06	0.10	0.12	0.13	0.16	0.17
NZD	0.04	0.07	0.12	0.22	0.26	0.26
SEK	0.03	0.08	0.13	0.20	0.21	0.22
difference in values						
AUD	0.03	0.07	0.11	0.21	0.23	0.20
CAD	0.04	0.10	0.14	0.19	0.20	0.20
CHF	0.05	0.10	0.12	0.11	0.15	0.19
GBP	0.10	0.14	0.17	0.18	0.17	0.17
JPY	0.02	0.04	0.05	0.07	0.11	0.16
NOK	0.06	0.10	0.12	0.13	0.15	0.16
NZD	0.04	0.07	0.12	0.21	0.24	0.23
SEK	0.03	0.08	0.13	0.19	0.21	0.22

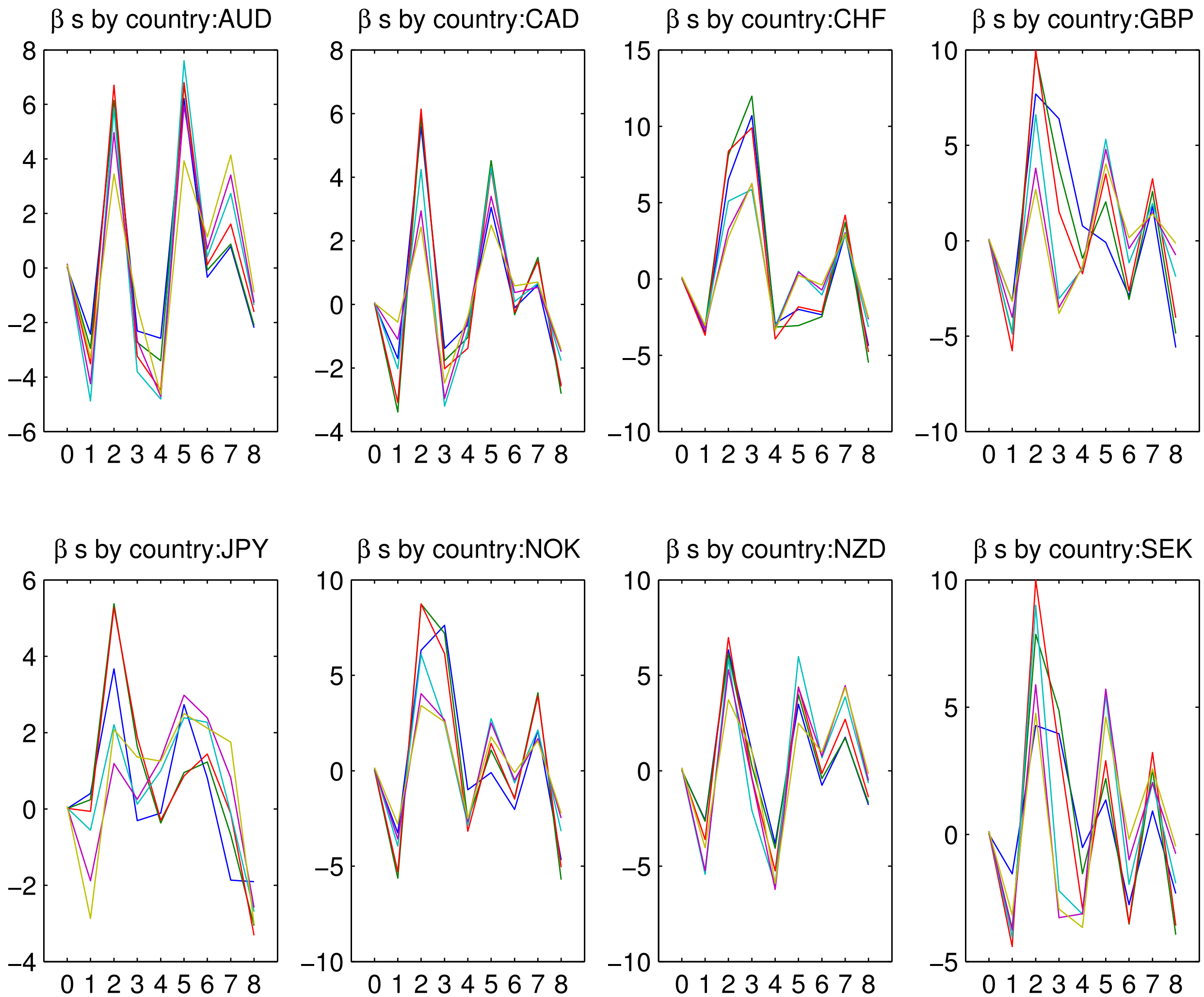

Figure 3: βs from the regression $rx^{(n)}_{t\to t+n,j} = \beta_{-1} + \beta' \widetilde{\mathbf{f}}^{(1)}_t + \varepsilon_{t+n}$. The regression constant is the first point on the horizontal axis, other points on the horizontal axis are the betas on the various currencies, listed in alphabetical order by currency. Different lines within a plot correspond to different return horizons n.

Table 7.2: GMM Wald test p-values for the hypothesis that coefficients of other currency's forward-spot spreads are jointly equal to 0 in regressions of individual returns on the cross section of currency forward-spot spreads with the same forward maturity as the return horizon. Standard errors are calculated using the HES method with 15 lags in the first VAR, and $q = n$ lags in the second VAR.

p-values for tests that $\beta_k = 0$, $k \neq j$, in $rx^{(n)}_{t \to t+n,j} = \beta' \widetilde{\mathbf{f}}^{(n)}_t + \varepsilon_{t+n}$						
$n =$	1	2	3	6	9	12
AUD	0.03	0.02	0.07	0.14	0.18	0.03
CAD	0.00	0.00	0.00	0.18	0.24	0.00
CHF	0.00	0.02	0.06	0.06	0.00	0.00
GBP	0.07	0.00	0.00	0.05	0.02	0.01
JPY	0.04	0.01	0.01	0.32	0.00	0.00
NOK	0.00	0.00	0.01	0.00	0.07	0.00
NZD	0.05	0.04	0.02	0.24	0.25	0.44
SEK	0.11	0.14	0.06	0.12	0.00	0.00

Table 7.3: For all currency and return horizon combinations, this table reports the differences in the R^2 values of the unrestricted regression regression (22) and the restricted (24), for various choices of the cross section variable used in the average return regression.

$n =$	1	2	3	6	9	12
Unrestricted R^2 - restricted R^2, $\widetilde{\mathbf{f}}^{(1)}_t$ used in (24)						
AUD	0.01	0.02	0.02	0.04	0.04	0.03
CAD	0.01	0.03	0.04	0.04	0.02	-0.00
CHF	0.02	0.03	0.02	-0.04	-0.05	-0.06
GBP	0.06	0.04	0.04	0.03	0.07	0.10
JPY	0.01	0.02	0.02	-0.02	-0.03	0.02
NOK	0.02	0.02	0.01	-0.01	-0.01	-0.02
NZD	0.01	0.02	0.03	0.03	0.01	-0.01
SEK	0.02	0.02	0.03	0.01	-0.00	-0.01
Unrestricted R^2 - restricted R^2, $\widetilde{\mathbf{f}}^{(12)}_t$ used in (24)						
AUD	0.00	0.00	0.01	0.01	0.01	0.02
CAD	-0.00	0.01	0.01	0.02	0.01	0.01
CHF	0.04	0.05	0.04	0.02	0.02	0.02
GBP	0.07	0.05	0.03	-0.01	0.01	0.03
JPY	0.01	0.02	0.02	0.01	0.01	0.02
NOK	0.04	0.05	0.04	0.02	0.02	0.01
NZD	0.01	0.01	0.02	0.02	0.02	0.01
SEK	0.02	0.03	0.03	0.01	0.02	0.02

8 Formal tests of single-factor restrictions

The evidence so far suggests that the single factor restrictions for both term structure and cross section information work reasonably well, but do they pass a statistical test? In particular, we want to make sure that the moment conditions (pricing errors) ignored by the two-step procedure still provide an adequate fit to the data, and also that the all return horizons' collections of unrestricted regression coefficients are jointly equal to the same vector, up to multiplication by a scalar. Fortunately, GMM makes this easy to do, and the work has been done for us. In particular, the testing procedure and calculations are identical to those outlined in [4] and the related appendix, up to corrections for the numbers of variables and parameters to be estimated. They are briefly reproduced here for completeness.

We still need to determine formally whether the restricted regression models provide an adequate fit to the data. This is accomplished by nesting the restricted regression as a special case of the unrestricted restricted regression where certain combinations of moments are set to zero. The original (unrestricted) vector of moment conditions is

$$g_T^{unrest}(\varepsilon) = \mathbb{E}\left[\vec{\varepsilon}_{[t]} \otimes f_t\right] = \vec{0}$$

Here I use the general expression f_t for the right-hand-side variable of the regression, which is the appropriate vector of forward rates or forward-spot spreads, plus a constant, and the symbol $\otimes$ denotes the Kronecker product of the two column vectors. $\vec{\varepsilon}_{[t]}$ is the vector of errors for the unrestricted regression on f_t for returns at all horizons initiated at time t, as, for example, by collecting the errors in (18) for all horizons n. The restricted regression sets the following collection of moments to 0:

$$g_T^{rest}(\varepsilon) = \mathbb{E}\begin{bmatrix} \overline{\varepsilon}_{[t]} \times f_t \\ \varepsilon_{[t]}^{rest} \times (\gamma' f_t) \end{bmatrix} = 0$$

The scalar $\overline{\varepsilon}_{[t]}$ is the error from the regression on averaged returns, as in (16), and $\varepsilon_{[t]}^{rest}$ is the vector of all restricted regression errors as in (17). This is equivalent to multiplying the vector of unrestricted moment conditions $g_T(\varepsilon)$ by the following selection matrix a_T:

$$a_T = \begin{bmatrix} \vec{1} \otimes I_b & \\ I_c \otimes \gamma' & 0_{b\times c} \end{bmatrix}$$

Here $\vec{1}^T$ is a row vector of 1s, with one entry for each restricted regression equation, the dimension c of the top block row of identity matrices is equal to the number of right hand side variables in each regression, and the size of the identity matrix I_c in the bottom left block is equal to the number of restricted regression equations minus 1, since the full set of moment restrictions is linearly dependent. The restricted regression sets $a_T g_T^{rest}(\varepsilon) = 0$ We also need to compute the matrix

$$d \equiv \frac{\partial g_T\left(\varepsilon^{unrest}\right)}{\partial \begin{bmatrix} b' & \gamma' \end{bmatrix}} =$$

$$= \left[\begin{array}{c|c} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \otimes \mathbb{E}\left[ff'\right]\gamma & -b \otimes \mathbb{E}\left[ff'\right] \end{array} \right]$$

The covariance matrix for the vector of restricted moments is then given by

$$cov\left(g_T\left(\varepsilon^{rest}\right)\right) = \frac{1}{T}\left(I - d\left(ad\right)^{-1}a\right) S \left(I - d\left(ad\right)^{-1}a\right)'$$

The test statistic is

$$g_T' cov\left(g_T\right)^{+} g_T \sim \chi^2$$

where the degrees of freedom are equal to the number of unrestricted moments minus the sum of the number of entries in γ and the number of linearly independent restricted regression coefficients. The $+$ symbol indicates pseudo-inversion by singular value decomposition, since the covariance matrix is singular. (See [4]) I ran tests for the fit of the restricted regression for each of the single factor regressions on rates, spreads, and the cross section. I calculated the long-run covariance matrix of the moment disturbances using the method of Hansen, Eichenbaum, and Singleton. For each f vector and currency, test statistic p-values are larger than 99% with $NLAG = q = 12$, and each of the corresponding χ^2 values is less than one. I explored a variety of other combinations of lag lengths and obtained the same result. It would appear, then, that each of the two-step regression models fits the data well.

A direct comparison of the unrestricted regression against the single factor model involves a Wald test of the parameter regression $\beta = \mathbf{b}\gamma'$. Here the i^{th} row of β is the set of unrestricted regression coefficients on the i^{th} entry of the vector of expected returns, and the i^{th} entry of the column vector $\mathbf{b}$ is the restricted regression scalar coefficient corresponding to the i^{th} expected return. Denote the row-wise vectorization (forming a column vector) of the unrestricted and restricted regression coefficient matrices as $vec(\beta)$ and $vec\left(\mathbf{b}\gamma'\right)$. Then we find the Generalized Method of Moments distribution for the covariance matrix of the entries of the unrestricted regression coefficients by computing:

$$cov\left(vec\left(\beta\right)\right) = \frac{1}{T} d^{-1} S \left(d^{-1}\right)'$$

$$d \equiv \frac{\partial g_T}{\partial vec\left(\beta\right)'} = I_6 \otimes \mathbb{E}\left[f f'\right]$$

$$g_T \equiv \mathbb{E}\left[\vec{\varepsilon}_{[t]} \otimes f_t\right]$$

The test statistic is

$$\left[vec\left(\mathbf{b}\gamma'\right) - vec\left(\beta\right)\right]' cov\left(vec\left(\beta\right)\right)^{-1} \left[vec\left(\mathbf{b}\gamma'\right) - vec\left(\beta\right)\right] \sim \chi^2 \tag{26}$$

Where the degrees of freedom are the number of moments to be fit in the unrestricted model, minus the sum of the independent parameters in the restricted model. (The last restricted regression coefficient is constrained to be a linear combination of the others.) I again estimate the long-run covariance matrix of the moment discrepancy terms using the 2-step VAR method of Hansen, Eichenbaum, and Singleton. Table 8.1 reports p-values from tests of the various restrictions for different choices of the first and second stage regression lag parameters. The cross-section tests are done using the cross section of one-year forward rates as the right-hand side variable, results are similar for the other choices of forward contract maturity. The initial choice of 12 lags for each regression decisively rejects the restrictions. I also examine the results with when more lags are used to estimate standard errors than are strictly necessary from the regression model's assumption that errors are uncorrelated past twelve months. With 24 lags used in each step of the regression, I again reject the null in all cases. However, expanding the number of lags to 36, one fails to reject the single factor restriction for about half of the currencies for each specification. In general, increasing either the first step or second step lag VAR length continues to reduce evidence against the null.

One should exercise caution in drawing conclusions from these results. In particular, if we believe the regression model, covariances of lags longer than 12 months are zero, and there is no reason to include them in the estimation of parameter standard errors. Moreover, the sensitivity of the test results implies that the regression model might be missing one or more important variables. Keeping all of this in mind, if one does take the failure to reject the various single-factor models under the use of extra lags to estimate the autocovariance matrix as a vindication of its success, one obtains a rather strong restriction on the dynamics of the volatilities of currency pricing kernels. In particular, we have the following:

Proposition 1. *Suppose that the n-period product of stochastic discount factors $M^i_{t+1}M^i_{t+2}\cdots M^i_{t+n}$ is log-normally distributed for $i \in \{H, *\}$, that markets are complete and in particular that covered interest parity holds, and also that expected log returns to a long position in currency $*$ from the perspective of an investor in H are a separable linear function of some possibly time-varying scalar state variable X_t and a constant that depends only on the return horizon $b(n)$, so that $\mathbb{E}_t\left[f^{(n)}_{t,j} - s_{t+n}\right] = \mathbb{E}_t\left[rx^{(n)}_{t\to t+n,j}\right] = b(n) \times X_t$. Then one obtains the following equation governing the relationship of the variances of the respective currency pricing kernels, where $m_{t+p} \equiv \log M_{t+p}$:*

$$\begin{aligned}\varphi_t(k) \equiv \textstyle\sum_{p=1}^{k} \frac{1}{2}\left\{Var_t\left(m_{t+p}\right) - Var_t\left(m^*_{t+p}\right)\right\} \\ + \textstyle\sum_{p,q\neq p}^{p,q\leq k}\left\{Cov_t\left(m_{t+p}, m_{t+q}\right) - Cov_t\left(m^*_{t+p}, m^*_{t+q}\right)\right\} \propto \\ \varphi_t(n) = \textstyle\sum_{p=1}^{n} \frac{1}{2}\left\{Var_t\left(m_{t+p}\right) - Var_t\left(m^*_{t+p}\right)\right\} \\ + \textstyle\sum_{p,q\neq p}^{p,q\leq n}\left\{Cov_t\left(m_{t+p}, m_{t+q}\right) - Cov_t\left(m^*_{t+p}, m^*_{t+q}\right)\right\}, \quad \forall k, n\end{aligned} \tag{27}$$

In words, this says that the difference in the variances of the k-step pricing kernels is proportional to the difference in the variances of the n-step pricing kernels, for any time horizons k and n.

Proof. Write the yield on an n-period bond nominally risk-free bond for currency j as $\left(1 + y^{(n)}_{t,j}\right)^n$. The price of such a bond in levels is

$$P^{(n)}_{t,j} = \mathbb{E}_t\left[M_{t+1}M_{t+2}\cdots M_{t+n}\right]$$

Write the log price as

$$p^{(n)}_{t,j} = \log \mathbb{E}_t\left[M_{t+1}M_{t+2}\cdots M_{t+n}\right] \approx -ny^{(n)}_{t,j}$$

If covered interest parity holds, then $f^{(n)}_{t,*} = s_t + ny^{(n)}_{t,*} - ny^{(n)}_{t,H}$, which, using the approximation above, is the difference in the log prices of the two bonds: $f^{(n)}_{t,*} = s_t + p^{(n)}_{t,H} - p^{(n)}_{t,*}$. Then the expected return to an investor in country H holding an n-period forward contract for currency $*$ is:

$$\mathbb{E}_t\left[f^{(n)}_{t,*} - s_{t+n}\right] = \mathbb{E}_t\left[s_t + p^{(n)}_{t,H} - p^{(n)}_{t,*} - s_{t+n}\right] = p^{(n)}_{t,H} - p^{(n)}_{t,*} - \mathbb{E}_t\left[s_{t+n} - s_t\right] \tag{28}$$

Write the expectation term in the last expression as the telescoping sum:

$$\mathbb{E}_t\left[s_{t+n} - s_t\right] = \mathbb{E}_t\left[\sum_{p=1}^{n}\left(s_{t+p} - s_{t+p-1}\right)\right]$$

Backus, Foresi, and Telmer have shown in [1] that the absence of arbitrage opportunies requires that the change in the log exchange rate is equal to the difference of the log pricing kernels:

$$s_{t+p} - s_{t+p-1} = m^H_{t+p} - m^*_{t+p}$$

Making this substitution in the telescoping sum above, we obtain:

$$\mathbb{E}_t\left[s_{t+n} - s_t\right] = \mathbb{E}_t\left[\sum_{p=1}^{n} m_{t+p}^{H}\right] - \mathbb{E}_t\left[\sum_{p=1}^{n} m_{t+p}^{*}\right] \tag{29}$$

Now use the assumption that each currency's pricing kernel is lognormal:

$$\begin{aligned} p_{t,j}^{(n)} = \log \mathbb{E}_t\left[M_{t+1}^{j} \cdots M_{t+1}^{j}\right] &= \mathbb{E}_t\left[\log\left(M_{t+1}^{j} \cdots M_{t+1}^{j}\right)\right] + \tfrac{1}{2} Var\left(\log\left(M_{t+1}^{j} \cdots M_{t+1}^{j}\right)\right) \\ &= \mathbb{E}_t\left[\textstyle\sum_{p=1}^{n} m_{t+p}^{j}\right] + \tfrac{1}{2} Var\left(m_{t+1}^{j} + \ldots + m_{t+n}^{j}\right) \end{aligned} \tag{30}$$

Substituting (29) and (30) in (28), we obtain that

$$\begin{gathered} \mathbb{E}_t\left[f_{t,*}^{(n)} - s_{t+n}\right] \\ = \textstyle\sum_{p=1}^{n} \tfrac{1}{2}\left\{Var_t\left(m_{t+p}\right) - Var_t\left(m_{t+p}^{*}\right)\right\} \\ + \textstyle\sum_{p,q\neq p}^{p,q\leq n}\left\{Cov_t\left(m_{t+p}, m_{t+q}\right) - Cov_t\left(m_{t+p}^{*}, m_{t+q}^{*}\right)\right\} \\ \equiv \varphi_t(n) \end{gathered}$$

Now use the assumption that $\mathbb{E}_t\left[rx_{t\to t+n,*}^{(n)}\right] = b(n)X_t$, where X_t is a scalar. Comparing expected returns at horizons n and k periods ahead, we see that

$$\varphi_t(n) = \frac{b(n)}{b(k)}\varphi_t(k)$$

and we have established the proportionality restriction on the behavior of the two SDF processes.

□

The result above provides a fairly strong restriction on the relationship between the conditional distributions of pricing kernels, and the evidence I find in favor of its most important (and also, perhaps, least credible - or in any case most unusual) hypothesis is limited at best. Moreover, there is an inherent tension in the little support available, in that by failing to reject the null in the single factor restriction Wald test for both the term structure and cross section regressions (albeit only for certain currencies and with a somewhat concerning number of estimated moment autocovariance terms) it would seem that neither single factor is alone sufficient to describe returns.

To make further progress in understanding return predictability, a natural next step is to combine both sources of information, preferably without drowning in a sea of variables. This is the focus of the next section.

Table 8.1: GMM Wald test p-values for the test of the single factor restriction associated with the test statistic (26)

Rates			
N,Q	12.00	24.00	36.00
AUD	0.00	0.00	0.00
CAD	0.00	0.00	0.40
CHF	0.00	0.00	0.00
GBP	0.00	0.00	1.00
JPY	0.00	0.00	0.00
NOK	0.00	0.00	0.00
NZD	0.00	0.00	0.22
SEK	0.00	0.00	0.00
Spreads			
N,Q	12.00	24.00	36.00
AUD	0.00	0.00	1.00
CAD	0.00	0.00	0.00
CHF	0.00	0.00	1.00
GBP	0.00	0.00	1.00
JPY	0.00	0.00	0.00
NOK	0.00	0.00	0.00
NZD	0.00	0.00	0.00
SEK	0.00	0.00	1.00
The cross section			
N,Q	12.00	24.00	36.00
AUD	0.00	0.00	0.00
CAD	0.00	0.00	0.00
CHF	0.00	0.00	1.00
GBP	0.00	0.00	0.00
JPY	0.00	0.00	0.67
NOK	0.00	0.00	0.12
NZD	0.00	0.00	0.00
SEK	0.00	0.00	0.59

9 Identification of a common factor in expected returns across currencies and time horizons

We have already seen that expected returns depend on both the term structure and cross section of currency forward rates. A next step is to see how expected returns for various currencies and time horizons depend on both variables together. In order to make progress toward this goal, there is a problem we need to address: it is not clear how to do this in a way that uses the same variables for each currency, especially if one wants to use only a few variables to understand expected returns across all currencies and time horizons. The right hand side of term structure regressions is necessarily different for each currency, and currencies also differ in how their expected returns vary relate to the cross section. Moreover, for any particular currency, the term structure of forward rates $\mathbf{f}_{t,j}$ contributes 7 variables, and the cross section of forward rates for any particular time horizon i contributes another 7 variables. One then has 14 variables and a constant on the right hand side of any proposed regression, for the returns of just one currency.

With this in mind, I follow a procedure from [5] to pare down the number of variables. We will soon see that one can achieve better results than in either section above, using both types of data in the same way for all currencies and maturities, and with fewer variables than either section considered on its own.

Begin by regressing the entire vector of returns to forward contracts at all horizons initiated at time t on the entire vector of forward rates $\mathbf{f}_t$ (or the entire vector of forward-spot spreads $\widetilde{\mathbf{f}}_t$) prevailing at time t. (This includes all time horizons, for all currencies.)

$$\mathbf{rx}_{[t]} = \beta \mathbf{f}_t + \varepsilon_{[t]} \tag{31}$$

From this, form the vector of expected returns:

$$\mathbb{E}_t \left[\mathbf{rx}_{[t]} \right] = \beta \mathbf{f}_t \tag{32}$$

Now we perform an eigenvalue decomposition of the covariance matrix of the vector of expected returns:

$$Q\Lambda Q' = Cov(\mathbb{E}_t\left[\mathbf{rx}_{[t]}\right]) \tag{33}$$

Consider the truncated matrix of eigenvectors Q'_k, the transpose of the first k columns of Q corresponding to the k largest entries of Λ. Then

$$X_t^k \equiv Q'_k\left(\beta \mathbf{f}_t\right) \tag{34}$$

is a vector of size $k \times 1$, made up of the k most important (largest amount of variance explained) orthogonal factors of the vector of expected returns for all currencies and time horizons. These are the first k principal components of the vector of all expected returns. To see how well just a small number k of factors can do for predicting returns, I begin by simple regressions of returns on X_t^k:

$$rx_{t\to t+n,j}^{(n)} = a_{n,j} + \mathbf{b}_{n,j}^T X_t^k + \varepsilon_{t\to t+n,j}^{(n)} \tag{35}$$

for $1 \leq k \leq 4$ expected return factors, for each currency/time horizon pair. Throughout, results for expected returns formed from the vector of forward rates are presented in parallel with those formed from the vector of forward-spot spreads. I will denote the former as factors formed from $\mathbb{E}\left[\mathbf{rx}_{[t]}|\mathbf{f}_t\right]$, and the latter as $\mathbb{E}\left[\mathbf{rx}_{[t]}|\widetilde{\mathbf{f}}_t\right]$.

Table 9.1 reports R^2 values for expected return factors formed from $\mathbb{E}\left[\mathbf{rx}_{[t]}|\mathbf{f}_t\right]$, Table 9.2 does this for expected return factors formed from $\mathbb{E}\left[\mathbf{rx}_{[t]}|\widetilde{\mathbf{f}}_t\right]$. For either method of computing expected returns, a single factor gives a higher R^2 than either term structure vector (rates or spreads) or the cross section. (The only widespread exceptions to this for $k = 1$ are JPY in almost every case, and $\mathbb{E}\left[\mathbf{rx}_{[t]}|\widetilde{\mathbf{f}}_t\right]$ against $\widetilde{\mathbf{f}}_t^{(n)}$.) A series of tables in the appendix report the differences in R^2 values relative to the currency-specific (term structure) or time horizon specific (cross section) variables that we have seen in previous sections.

For $k \geq 3$, we can explain more of returns with either common factor than any of the three variables above, for practically all currency and time horizons. (There is no longer a pattern to the very few cases where idiosyncratic information gives a higher R^2 than X_t. So one can explain substantially more variation in returns, with less than half as many variables, using the same information for all return horizons and currencies, than with either of the previous schemes that use currency and/or horizon specific information. Moreover, the increase in R^2 relative to the early variables is often quite large, especially for $\mathbf{f}_t$ at longer time horizons, where one sees increases of as many as 50 percentage points.

Regression coefficients for expected return factors formed from the vector of rates are plotted in Figure 4, regression coefficients for expected return factors formed from the vector of spreads are plotted in Figure 5. These are the coefficients on non constant variables X_t^k for $k = 3$. Each subtable plots the coefficients for a given currency, and coefficients on returns of the same time horizon are grouped together. Examining Figure 4 and Figure 5, one sees a similar pattern of coefficients for most currencies for either type of expected return factor. In general, coefficients on shorter horizon returns are relatively flat, while coefficients on longer horizon returns display greater amplitude but the same basic shape. The clearest violations of this occur in factors formed from spreads for GBP and SEK.

Are the regression coefficients statistically distinct from zero? For each currency and time horizon combination, I consider the null:

$$b_1 = b_2 = b_3 = 0 \tag{36}$$

and the related series of individual regressions:

$$b_i = 0, \quad i \in \{1, 2, 3\} \tag{37}$$

Where the b coefficients are obtained in the regression (35) with $k = 3$. p-values are calculated using the method of Hansen, Eichenbaum, and Singleton, with the number of lags used to estimate both regressions set to 12. I obtain a strong rejection of (36) for all currencies and time horizons, with all p-values much less than one percent. For individual restrictions, I see the same trend of uniform rejection for all coefficients. Since p-values from this series of tests are nearly all zero to any appreciable level of precision, I report χ^2 test statistics instead in Table 9.4. I also explored a variety of other combinations of lag choices for the first and second step regressions, and found these results to be robust with respect to the choice of lag parameters.

The coefficient graphs in Figure 4 and Figure 5 suggest we might be able to describe the dependence of expected currency returns across time horizons with the same type of single-factor model explored in previous sections. Let the vector $\widetilde{X}_t$ contain a constant and the first three factors of the vector of all expected returns, $X_t^{k=3}$, formed from either $\mathbf{f}_t$ or $\widetilde{\mathbf{f}}_t$. For either choice of conditioning information, regress each currency's average return across time horizons on $\widetilde{X}_t$:

$$\overline{rx}_{[t],j} = \gamma' \widetilde{X}_t + \varepsilon_{[t],j} \tag{38}$$

Now use the γ coefficients from (38) to estimate the restricted regression:

$$rx_{t \to t+n,j} = b_n \left(\gamma' \widetilde{X}_t \right) + \varepsilon_{t+n,j}^{(n)} \tag{39}$$

We want to compare these to the unrestricted regression:

$$\mathbf{rx}_{[t]} = \beta \widetilde{X}_t + \varepsilon_{[t],j} \tag{40}$$

As before, I consider the GMM tests that i) the moments neglected by the two-step estimate procedure are not too large, and ii) that the single-factor restriction holds for the unrestricted regression (40) coefficients: $\mathbf{b}\gamma' = \beta$. Table 9.6 reports p-values for both tests, for both choices ($\mathbf{f}_t$ and $\widetilde{\mathbf{f}}_t$) of the variable used to form the expected return factors X_t. As in the other restricted regression tests, the data gives practically no evidence against the two-step regression procedure, and all p-values round to one. More surprising is that for X_t formed from $\mathbb{E}\left[\mathbf{rx}_{[t]} | \mathbf{f}_t\right]$, we see practically no evidence against the null of the second test - the GMM Wald test that the unrestricted regression coefficients are jointly equal up to a scaling factor. The was rather decisively rejected (at 12 HES standard error lags) for all variables for all currencies in the previous section. Moreover, we see a similar result in the bottom-right subtable of Table 9.6, where I report results of the analogous test for coefficients on X_t, where X_t is derived using $\widetilde{\mathbf{f}}_t$. For six of the 8 currencies, we fail to reject the null at the 5% level with 12 Hansen-Eichenbaum-Singleton standard error lags (for each step). The two currencies for which we do reject the null, GBP and SEK, are those that we would expect to fail the coefficient restriction test just by looking at Figure 5. Results are qualitatively similar when 18 lags are used to estimate the long-run covariance matrix of the moment disturbances, except for the fact that one now also rejects the null in the GMM-Wald test for CAD as well. It appears then that one can approximate expected currency returns across currencies and time horizons by using only a small number of principal components. Moreover, in most cases the currency-specific linear combinations of the X_t components are the same across time horizons, up to a scaling factor.

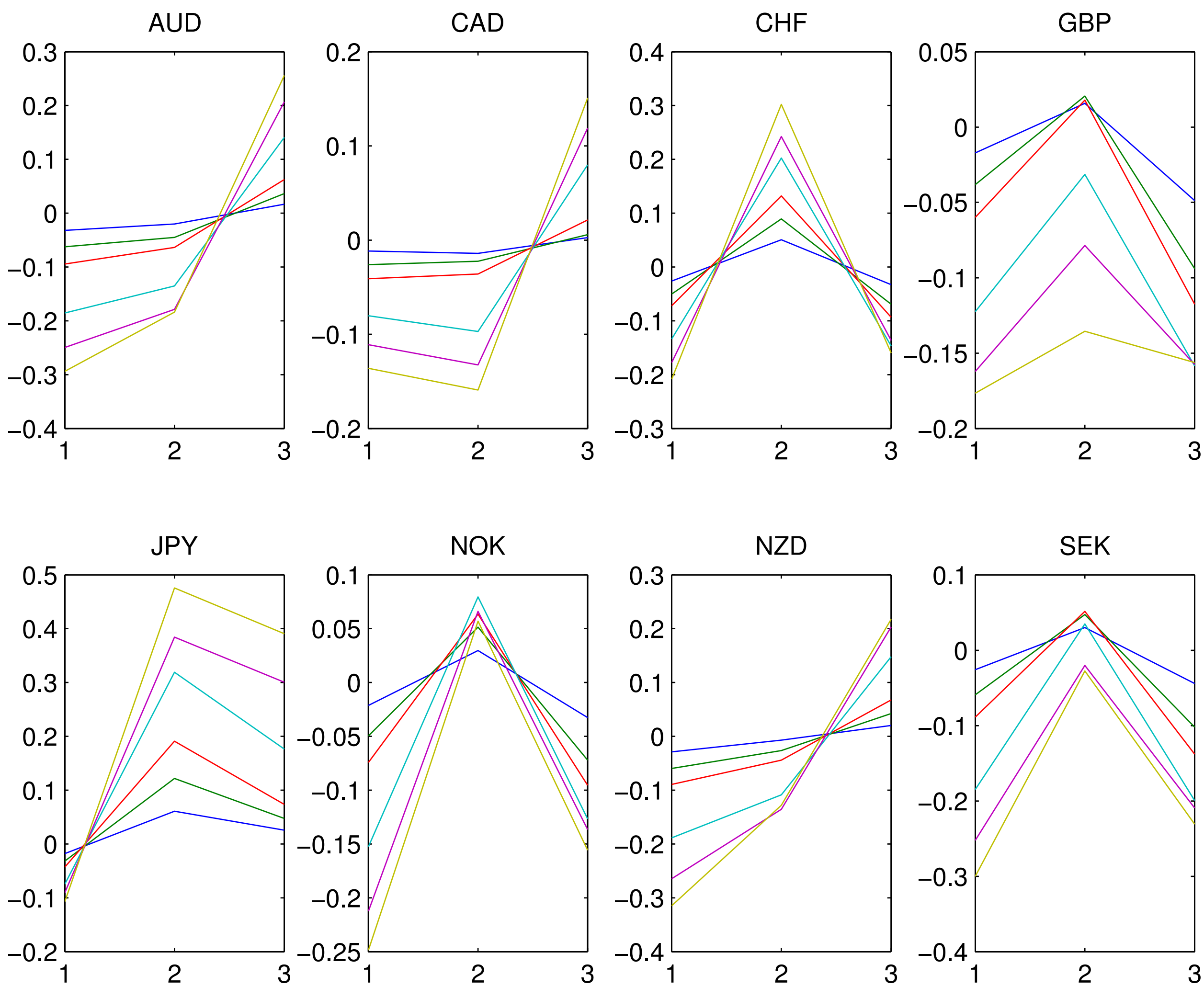

Figure 4: Coefficient loadings on the common expected return factors formed from the covariance matrix of expected returns conditional on $\mathbf{f}_t$, $k = 3$. The dark blue line corresponds to one month returns, the green line to two month returns, the red line to three month returns, the light blue line to 6 month returns, the purple line to 9 month returns, and the gold line to one year returns.

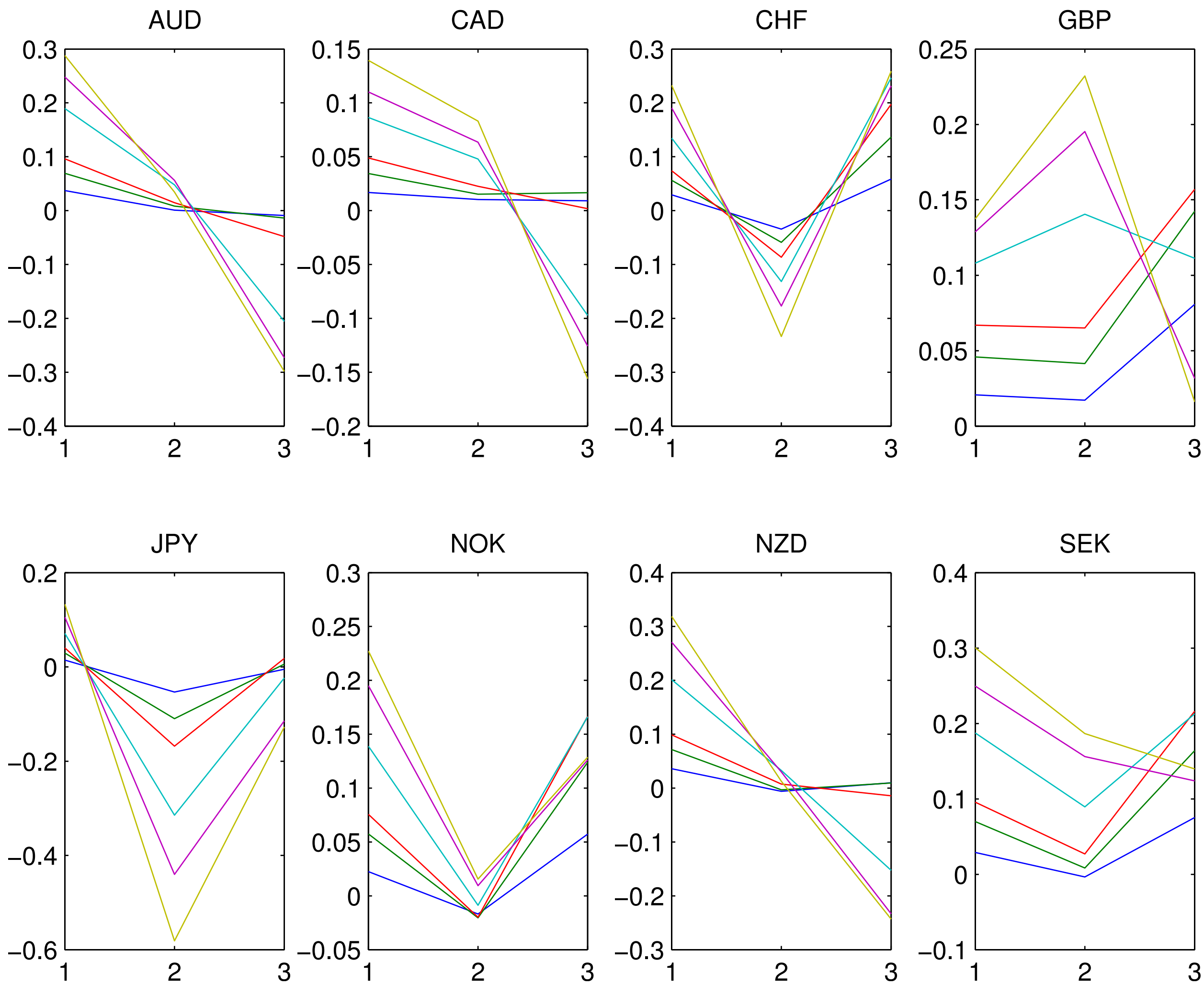

Figure 5: Coefficient loadings on the common expected return factors in the regression (35) with $k = 3$.

Table 9.1: R^2 values of returns on expected return factors formed from all rates $\mathbf{f}_t$

	One factor					
AUD	0.10	0.18	0.28	0.49	0.58	0.61
CAD	0.03	0.08	0.13	0.24	0.32	0.37
CHF	0.07	0.12	0.17	0.31	0.41	0.41
GBP	0.04	0.10	0.16	0.33	0.46	0.45
JPY	0.04	0.06	0.06	0.09	0.10	0.10
NOK	0.05	0.12	0.18	0.38	0.53	0.58
NZD	0.08	0.17	0.26	0.48	0.59	0.62
SEK	0.06	0.15	0.21	0.43	0.56	0.62
	Two factors					
AUD	0.11	0.20	0.31	0.54	0.64	0.66
CAD	0.04	0.09	0.15	0.31	0.41	0.47
CHF	0.12	0.20	0.29	0.45	0.57	0.59
GBP	0.05	0.10	0.16	0.33	0.48	0.50
JPY	0.12	0.22	0.32	0.44	0.47	0.48
NOK	0.07	0.15	0.21	0.40	0.54	0.59
NZD	0.09	0.18	0.27	0.51	0.62	0.64
SEK	0.08	0.16	0.23	0.43	0.56	0.62
	Three factors					
AUD	0.11	0.21	0.33	0.59	0.71	0.74
CAD	0.04	0.09	0.16	0.35	0.47	0.55
CHF	0.14	0.24	0.34	0.52	0.61	0.63
GBP	0.11	0.21	0.27	0.43	0.56	0.56
JPY	0.13	0.24	0.35	0.53	0.66	0.71
NOK	0.09	0.20	0.26	0.45	0.58	0.63
NZD	0.09	0.20	0.30	0.57	0.68	0.69
SEK	0.11	0.24	0.32	0.52	0.63	0.68
	Four factors					
AUD	0.14	0.26	0.38	0.61	0.71	0.75
CAD	0.09	0.17	0.25	0.41	0.49	0.55
CHF	0.14	0.25	0.34	0.52	0.66	0.72
GBP	0.17	0.33	0.42	0.59	0.63	0.60
JPY	0.15	0.27	0.39	0.58	0.68	0.72
NOK	0.10	0.22	0.28	0.46	0.58	0.66
NZD	0.11	0.22	0.32	0.57	0.69	0.74
SEK	0.12	0.26	0.34	0.56	0.63	0.69

Table 9.2: R^2 values of returns on expected return factors formed from all spreads $\widetilde{\mathbf{f}}_t$

	One factor					
AUD	0.08	0.13	0.17	0.30	0.34	0.34
CAD	0.04	0.08	0.11	0.16	0.18	0.23
CHF	0.05	0.09	0.11	0.19	0.28	0.30
GBP	0.04	0.08	0.12	0.15	0.17	0.16
JPY	0.01	0.03	0.03	0.05	0.08	0.09
NOK	0.03	0.10	0.11	0.19	0.26	0.29
NZD	0.08	0.15	0.19	0.32	0.37	0.37
SEK	0.05	0.12	0.15	0.26	0.32	0.36
	Two factors					
AUD	0.08	0.13	0.17	0.31	0.34	0.35
CAD	0.04	0.09	0.12	0.18	0.20	0.25
CHF	0.07	0.12	0.15	0.23	0.35	0.39
GBP	0.05	0.10	0.15	0.22	0.28	0.29
JPY	0.07	0.14	0.19	0.32	0.47	0.56
NOK	0.04	0.10	0.11	0.19	0.26	0.29
NZD	0.08	0.15	0.19	0.33	0.37	0.37
SEK	0.05	0.12	0.15	0.28	0.36	0.40
	Three factors					
AUD	0.08	0.13	0.18	0.36	0.40	0.40
CAD	0.04	0.09	0.12	0.21	0.24	0.29
CHF	0.10	0.20	0.27	0.33	0.41	0.45
GBP	0.13	0.23	0.25	0.25	0.28	0.29
JPY	0.07	0.14	0.19	0.32	0.49	0.57
NOK	0.07	0.17	0.19	0.23	0.28	0.30
NZD	0.08	0.15	0.19	0.35	0.41	0.41
SEK	0.09	0.22	0.26	0.33	0.37	0.42
	Four factors					
AUD	0.13	0.20	0.24	0.39	0.41	0.41
CAD	0.11	0.19	0.22	0.30	0.28	0.30
CHF	0.10	0.21	0.27	0.33	0.45	0.53
GBP	0.16	0.29	0.32	0.31	0.29	0.29
JPY	0.08	0.16	0.22	0.36	0.51	0.58
NOK	0.08	0.20	0.21	0.23	0.29	0.34
NZD	0.11	0.20	0.22	0.36	0.41	0.45
SEK	0.11	0.25	0.29	0.37	0.37	0.42

Table 9.3: χ^2 test statistics from the regressions (36) and (37), expected returns formed from spreads vector

$n =$	1	2	3	6	9	12
			$b_1 = b_2 = b_3 = 0$			
AUD	663	419	1021	6044	3361	2051
CAD	2391	1947	686	1261	774	937
CHF	85373	65021	47378	23281	8279	6226
GBP	57140	54114	46470	15762	4889	3922
JPY	18963	19136	19607	27688	30572	24602
NOK	29988	40837	39734	10565	2453	1718
NZD	1469	911	288	1844	1767	1029
SEK	22642	34604	37385	20572	6167	5391
			$b_1 = 0$			
AUD	448	414	378	383	200	125
CAD	86	102	101	89	53	67
CHF	1079	574	353	341	230	184
GBP	124	168	214	154	75	55
JPY	55	62	52	56	56	43
NOK	228	393	355	266	153	127
NZD	293	308	260	253	165	111
SEK	140	236	253	370	235	218
			$b_2 = 0$			
AUD	2	71	107	298	126	22
CAD	400	242	263	334	215	293
CHF	18215	7671	5883	4012	2426	2273
GBP	1050	1672	2487	3194	2119	1918
JPY	8967	10432	11265	13374	12071	10168
NOK	1579	609	301	13	4	7
NZD	97	8	18	82	25	2
SEK	24	41	246	1027	1125	1030
			$b_3 = 0$			
AUD	247	146	850	4064	2186	1205
CAD	225	212	1	1014	621	760
CHF	38321	30318	22480	10322	3041	2046
GBP	16955	14507	10648	1473	41	7
JPY	56	21	97	55	600	359
NOK	13422	16400	15470	3425	577	366
NZD	205	47	49	1313	1096	583
SEK	8470	11667	11653	4285	521	424

Table 9.4: χ^2 test statistics from the regressions (36) and (37). expected returns formed from rates vector

$n =$	1	2	3	6	9	12
			$b_1 = b_2 = b_3 = 0$			
AUD	1097	1440	1764	4070	7898	9864
CAD	436	286	379	1554	1667	2554
CHF	15703	9252	8373	7726	5608	5337
GBP	8727	9550	8818	6684	4683	4975
JPY	10065	11558	13154	21280	32933	31724
NOK	12692	10259	11248	7425	4813	4779
NZD	2339	2576	2607	4195	4704	2914
SEK	3448	4769	5317	5455	5414	7412
			$b_1 = 0$			
AUD	166	172	181	317	525	594
CAD	22	32	40	61	59	87
CHF	223	135	124	160	164	144
GBP	38	52	72	96	93	89
JPY	25	23	19	27	31	26
NOK	186	172	238	343	342	348
NZD	196	230	208	314	364	273
SEK	43	57	75	140	195	302
			$b_2 = 0$			
AUD	726	989	914	1890	3018	2602
CAD	364	264	342	992	942	1332
CHF	9448	4745	4700	4115	3426	3388
GBP	353	169	72	71	245	589
JPY	3144	3792	4354	5503	6214	5686
NOK	4037	2069	1941	1044	370	204
NZD	136	505	577	1171	1069	504
SEK	649	407	283	56	14	28
			$b_3 = 0$			
AUD	983	1252	1690	3931	7737	9671
CAD	22	34	232	1292	1459	2293
CHF	7712	5483	4507	4140	2090	1817
GBP	6548	6764	5941	3459	1891	1502
JPY	1069	1106	1244	3224	7287	7363
NOK	9283	7789	8410	5080	3034	2930
NZD	2007	2502	2551	4144	4607	2801
SEK	2677	3629	3898	3517	2877	3836

Table 9.5: R^2 values of $\widetilde{\mathbf{f}}_{t+n}$ formed from factors of $\mathbb{E}\left[rx_{t+n}|\widetilde{\mathbf{f}}_t\right]$

	One factor					
AUD	0.13	0.15	0.17	0.25	0.31	0.30
CAD	0.07	0.07	0.08	0.11	0.14	0.13
CHF	0.10	0.10	0.09	0.07	0.06	0.05
GBP	0.00	0.00	0.00	0.00	0.00	0.00
JPY	0.21	0.22	0.21	0.18	0.15	0.12
NOK	0.02	0.02	0.02	0.02	0.01	0.01
NZD	0.03	0.03	0.03	0.05	0.08	0.12
SEK	0.01	0.02	0.03	0.03	0.05	0.05
	Two factors					
AUD	0.19	0.23	0.26	0.35	0.39	0.38
CAD	0.24	0.26	0.29	0.36	0.37	0.36
CHF	0.47	0.48	0.49	0.44	0.37	0.30
GBP	0.24	0.22	0.20	0.16	0.08	0.05
JPY	0.45	0.46	0.46	0.43	0.38	0.33
NOK	0.39	0.43	0.46	0.40	0.28	0.18
NZD	0.05	0.06	0.06	0.08	0.09	0.12
SEK	0.22	0.23	0.25	0.26	0.26	0.26
	Three factors					
AUD	0.23	0.26	0.29	0.37	0.40	0.40
CAD	0.28	0.27	0.30	0.36	0.38	0.36
CHF	0.47	0.48	0.49	0.44	0.37	0.30
GBP	0.24	0.22	0.21	0.17	0.10	0.05
JPY	0.47	0.48	0.49	0.48	0.42	0.36
NOK	0.47	0.48	0.49	0.40	0.28	0.18
NZD	0.07	0.07	0.07	0.09	0.09	0.12
SEK	0.25	0.24	0.25	0.27	0.27	0.27
	Four factors					
AUD	0.24	0.27	0.30	0.37	0.41	0.44
CAD	0.30	0.29	0.31	0.36	0.38	0.39
CHF	0.49	0.49	0.50	0.45	0.38	0.31
GBP	0.27	0.25	0.24	0.20	0.11	0.06
JPY	0.47	0.48	0.49	0.48	0.43	0.38
NOK	0.48	0.50	0.51	0.42	0.31	0.22
NZD	0.07	0.07	0.07	0.09	0.10	0.18
SEK	0.26	0.24	0.25	0.27	0.28	0.28

Table 9.6: The top two subtables report p-values for the null hypothesis that the two-step regression procedure still fits the data. The bottom two subtables report p-values for the GMM Wald test that the unrestricted regression coefficients are jointly equal to different scalar loadings on the average regression coefficients. Left subtables correspond to factors X_t formed from $\mathbf{f}_t$, right subtables correspond to factors formed from $\widetilde{\mathbf{f}}_t$. Standard errors are calculated using the method of Hansen, Eichenbaum, and Singleton, with the same number of lags used in the estimation of both the first and second step VARs. The number of lanes is reported in the row labelled L.

p-values for single-factor model tests				
X_t	$\mathbb{E}_t\left[\mathbf{rx}_{[t]}\middle\vert\mathbf{f}_t\right]$		$\mathbb{E}_t\left[\mathbf{rx}_{[t]}\middle\vert\widetilde{\mathbf{f}}_t\right]$	
L	12	18	12	18
Test of two-step regression fit				
AUD	1.00	1.00	1.00	1.00
CAD	1.00	1.00	1.00	1.00
CHF	1.00	1.00	1.00	1.00
GBP	1.00	1.00	1.00	1.00
JPY	1.00	1.00	1.00	1.00
NOK	1.00	1.00	1.00	1.00
NZD	1.00	1.00	1.00	1.00
SEK	1.00	1.00	1.00	1.00
GMM-Wald Single-Factor Test				
AUD	1.00	1.00	1.00	0.21
CAD	1.00	1.00	0.09	0.00
CHF	1.00	1.00	0.79	0.30
GBP	1.00	1.00	0.00	0.00
JPY	1.00	1.00	1.00	1.00
NOK	1.00	1.00	0.68	0.26
NZD	1.00	1.00	1.00	1.00
SEK	1.00	1.00	0.00	0.00

10 Conclusion

We have seen over the previous sections that foreign currency returns are more predictable than indicated elsewhere in the literature. In particular, expected currency returns at all time horizons depend on the entire term structure of foreign currency forward rates. The particular form of this dependence is similar across time horizons, as GMM tests find practically no evidence against the two-step regression procedure for returns as a function of the term structure. We find the same in regressions of returns at different time horizons on the cross section of foreign currency forward rates. However, with either variable one sees strong evidence against the null that unrestricted regression coefficients are scalings of the same average regression coefficients, at least for most methods of calculating the hypothesis test standard errors.

Our search for common factors in expected returns across both currencies and time horizons appears to have been successful, in that a principal components analysis of expected returns to all currencies at all time horizons shows that the three largest components better predict returns than either the term structure or cross section individually. Moreover, these variables are the same for all currencies and all return horizons, and we find very little evidence against the null that a given currency's expected returns at different time horizons are just different scalings of the same linear function of the common factors described above. This stands in contrast to our findings with respect to either currency-specific term structure information of maturity-specific cross section information.

Further work on this topic could include the identification of cross-currency analogues to the currency-specific level, slope, and curvature factors documented in section 6, and comparison of these with the common expected return components X_t. A related paper (currently in progress) explores this area. Another extension of the work here would be a multi-country equilibrium asset pricing model that rationalizes the dependence of expected returns on the term structure.

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