GARCH MODELS FOR FORECASTING VOLATILITY AND DETERMINING ARBITRAGE IN OPTIONS

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ABSTRACT

Derivatives have become widely accepted as tools for hedging and risk-management, and also to some extent for speculation. A more recent trend has been gaining some ground, that of arbitrage in derivatives.

The critical parameter in derivatives pricing is the volatility of the underlying asset. Exchanges often overestimate volatility in order to cover for any sudden changes in market behavior, leading to systematic overpricing of derivatives. Accurate forecasting of volatility would expose this systematic overpricing.

Unfortunately, volatility is not an easy phenomenon to predict or forecast. One class of models which have proved successful in forecasting volatility in many situations is the GARCH family of models.

The objective of the present study is to analyze systematic mispricing of options derivatives. In order to perform the analysis, data was collected for a sample of NSE-traded stock options and for their underlying stocks for the period of one year prior to the contract. The study uses GARCH models to forecast underlying stock volatility, and uses this forecasted volatility in the Black-Scholes model in order to determine whether the corresponding options are fairly priced. The motivation behind the research was to find systematic mispricing that would provide evidence for arbitrage opportunities.

KEYWORDS: derivatives, hedging, speculation, arbitrage, volatility, overpricing, GARCH, Black-Scholes model.

INTRODUCTION

The volatility of financial markets has been the object of numerous developments and applications over the past two decades, both theoretically and empirically. Financial economists are increasingly concerned with modeling volatility in asset returns. This is important as volatility is considered as a measure of risk, and investors want a premium for investing in risky assets. Banks and other financial institutions apply value-at-risk models to assess their risks. Modeling and forecasting volatility or, in other words, the covariance structure of asset returns, is therefore important.

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Unfortunately, volatility is not an easy phenomenon to predict or forecast. One class of models which have proved successful in forecasting volatility in many situations is the GARCH family of models. The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models were propounded by Engle (1982) and Bollerslev (1986). The distinctive feature of these models is that they recognize that volatilities and correlations are not constant. During some periods, a particular volatility or correlation may be relatively low, whereas during other periods it may be relatively high. The model attempts to keep track of the variations in the volatility or correlations through time. GARCH models are discrete time models that have been used to estimate a variety of financial time series such as stock returns, interest rates and foreign exchange rates. GARCH modeling builds on advances in the understanding and modeling of volatility. It takes into account excess kurtosis (i.e., fat tail behavior) and volatility clustering, two important characteristics of financial time series. It provides accurate forecasts of variances and covariances of asset returns through its ability to model time-varying conditional variances. GARCH models have been applied in such diverse fields as risk management, portfolio management and asset allocation, option pricing, and foreign exchange.

The GARCH (p, q) model is formulated as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2,$$

where p is the order of the GARCH (lagged volatility) terms, and q is the order of the ARC H (lagged squared-error) terms. In the academic literature, the GARCH (1, 1) process seems to be perceived as a realistic data generating process for financial returns. An intuitively appealing interpretation of the GARCH (1, l) model is easy to understand. The GARCH forecast variance is a weighted average of three different variance forecasts. One is a constant variance that corresponds to the long-run average. The second is the forecast that was made in the previous period. The third is the new information that was not available when the previous forecast was made. This could be viewed as a variance forecast based on one period of information. The weights on these three forecasts determine how fast the variance changes with new information and how fast it reverts to its long-run mean.

Volatility and risk both terms are used interchangeably today. If one decides to approach the difficult problem of forecast evaluation, the first consideration is: which volatility is being forecast? For option pricing, portfolio optimization and risk management one needs a forecast of the volatility that governs the underlying price process until some future risk horizon. Future volatility is an extremely difficult thing to forecast because the actual realization of the future process volatility will be influenced by events that happen in the future, e.g. large market movements at any time before the risk horizon. Thus the real problem is that of prediction of volatility. The predicted volatility can be used to determine future prices of the stock or the stock option, and thus an investor can use arbitrage strategies accordingly to benefit from the model.

LITERATURE

The ARCH model was developed by Robert Engle to assess the validity of a conjecture of Friedman (1977) that the unpredictability of inflation was a primary cause of business cycles. He hypothesized that the level of inflation was not a problem; it was the uncertainty about future costs and prices that would prevent entrepreneurs from investing and lead to a recession, and that this could only be plausible if the uncertainty were changing over time, i.e. in the presence of heteroscedasticity. The ARCH model described the forecast variance in terms of current observable. Instead of using short or long sample standard deviations, the ARCH model proposed taking weighted averages of past squared forecast errors, a type of weighted variance. These weights could give more influence to recent information and less to the distant past. Clearly the ARCH model was a simple generalization of the sample variance. The big advance was that the weights could be estimated from historical data even though the true volatility was never observed. Forecasts can be calculated every day or every period. By examining these forecasts for different weights, the set of weights can be found that make the forecasts closest to the variance of the next return. This procedure, based on the maximum likelihood method, gives a systematic approach to the estimation of the optimal weights. Once the weights are determined, this dynamic model of time-varying volatility can be used to measure the volatility at any time and to forecast it into the near and distant future.

There are many benefits to formulating an explicit dynamic model of volatility. Tests of the adequacy and accuracy of a volatility model can be used to verify the procedure. One-step and multi-step forecasts can be constructed using these parameters. Inserting the relevant variables into the model can test economic models that seek to determine the causes of volatility. Incorporating additional endogenous variables and equations can similarly test economic models about the consequences of volatility.

The application that appeared in Engle (1982) was to inflation in the United Kingdom since this was Friedman's conjecture. While there was plenty of evidence that the uncertainty in inflation forecasts was time varying, it did not correspond to the U.K. business cycle. Similar tests for U.S. inflation data, reported in Engle (1983), confirmed the finding of ARCH but found no business-cycle effect. While the trade-off between risk and return is an important part of macroeconomic theory, the empirical implications are often difficult to detect as they are disguised by other dominating effects, and obscured by the reliance on relatively low-frequency data. In finance, the risk/return effects are of primary importance and data on daily or even intra daily frequencies are readily available to form accurate volatility forecasts. Thus finance is the field in which the great richness and variety of ARCH models developed.

In the academic literature, the GARCH (1, 1) process seems to be perceived as a realistic data generating process for financial returns. As a result, a large number of econometric and statistical papers that develop estimation and testing techniques based on the assumption of a GARCH type data generating mechanism, use the GARCH(1, 1) model to actually implement their results, (Catalin Starica, 2003). GARCH models can be used for valuing options across strike prices and times-to-maturity, while estimating parameters using information on option prices, (P.Christofferson and K.Jacobs, 2002). While the autoregressive conditional heteroskedastic approach to modeling time-varying volatility is currently prevalent, alternative methodologies for volatility modeling exist in the econometric literature.

Important papers that apply GARCH Models include Bollerslev, Engle and Woolbridge (1988), French, Schwert and Stambaugh (1987), Glosten, Jagannathan and Runkle (1993), Pagan and Schwert (1990) and Schwert (1990). Motivated equally by the success of GARCH Models in fitting asset returns and by failure of deterministic volatility models in fitting option prices, researchers have extended the GARCH Model into domain of option valuation. A theoretical aspect of hedging in the GARCH option pricing model is further discussed in Garcia and Renault (1998).

A few papers have investigated certain aspects of empirical performance of specific GARCH option pricing models like Amin (1993), Engle and Mustafa (1992). Several authors have suggested approaches that attempt to take advantage of price discrepancies by taking proper transformations of financial time-series like N. Burgess (2002) and N. Towers (2002). The most important work though is by Engle and Bollerslev in 1988. Since then many researchers have contributed to the GARCH models.

Generalizations to different weighting schemes can be estimated and tested. The very important development by Tim Bollerslev (1986), called Generalized Autoregressive Conditional Heteroscedasticity, or GARCH, is today the most widely used model. This essentially generalizes the purely autoregressive ARCH model to an autoregressive moving average model. The weights on past squared residuals are assumed to decline geometrically at a rate to be estimated from the data.

Further generalizations have been proposed by many researchers, including AARCH, APARCH, FIGARCH, FIEGARCH, STARCH, SWARCH, GJR-GARCH, TARCH, MARCH, NARCH, SNPARCH, SPARCH, SQGARCH, CESGARCH, Component ARCH, Asymmetric Component ARCH, Taylor-Schwert, Student-t-ARCH, GED-ARCH, and several others. Many of these models were surveyed in Bollerslev et al. (1992), Bollerslev et al. (1994), Engle (2002b), and Engle and Isao Ishida (2002). These models recognize that there may be important nonlinearity, asymmetry, and long memory properties of volatility and that returns can be non-normal with a variety of parametric and nonparametric distributions. A closely related but econometrically distinct class of volatility models called stochastic volatility (SV) models have also seen dramatic development (see, for example, Peter K. Clark (1973), Stephen Taylor (1986), Andrew C. Harvey et al. (1994), and Taylor (1994)). These models have a different data-generating process which makes them more convenient for some purposes, but more difficult to estimate.

Much research has been published on the accuracy of different volatility forecasts for financial markets: see, for example, Andersen and Bollerslev (1998) Alexander and Leigh (1997), Brailsford and Faff (1996), Cumby et al.(1993), Dimson and Marsh (1990), Figlewski (1997), Frennberg and Hansson (1996), and West and Cho (1995). Given the remarks just made about the difficulties of this task it should come as no surprise that the results are inconclusive. However, there is one finding that seems to be common to much of this research, and that is that 'historical' volatility is just about the worst predictor of a constant volatility process. A performance criterion for volatility or correlation forecasts should be based on hedging performance (Engle and Rosenberg, 1995) or on trading a volatility- or correlation-dependent product.

Alexander and Leigh (1997) perform a statistical evaluation of the three types of statistical volatility forecasts that are in standard use: 'historical' (equally weighted moving averages), EWMAs and GARCH. Given the remarks just made, it is impossible to draw any firm conclusions about the relative effectiveness of any volatility forecasting method for an arbitrary portfolio. However, using data from the major equity indices and foreign exchange rates, some broad conclusions do appear. While EWMA methods perform well for predicting the centre of a normal distribution, the VaR model back-testing indicates that GARCH and equally weighted moving average methods are more accurate for the tails prediction required by VaR models. These results seem relatively independent of the data period used. GARCH forecasts are designed to capture the fat tails in return distributions, so VaR measures from GARCH models tend to be larger than those that assume normality.

DATA & METHODOLOGY

The sample consisted of forty-one stocks listed on National Stock Exchange (NSE) and which have options actively traded on the NSE. The sample stocks were chosen is from fourteen sectors, as follows: aviation (Air Deccan, Jet Airways), auto and auto components (Amtek Auto, Hero Honda, Maruti), banking and financial services (Allahabad Bank, Canara Bank, Corporation Bank, Reliance Capital), capital goods (ABB, Aditya Birla Nuvo, AIA Engineering), cement (ACC, Ambuja, Shree Cement), chemicals (Chambal Fertilizers, Orchid Chemicals), FMCG (Colgate, Dabur, Hindustan Unilever), IT (3i Infotech, CMC, Infosys Technologies, Wipro), media (Adlabs, Zee Ltd, Sun TV), oil & gas (Aban Lloyd, GAIL, HPCL), pharmaceuticals (Cipla, Dr. Reddy's Ltd), power (CESC, Reliance Energy, Suzlon, Tata Power), textiles (Century Textiles, Skumars, Welspun Gujarat), and real estate (DLF, Unitech).

The data for the study consisted of the closing prices and volumes of the stocks in the period Jan'07 - Dec.'07 and the closing prices and volumes of the corresponding stock options in the period Jan.'08 - Mar.'08. These were directly collected from the NSE website.

The objective of the present study is to analyze systematic mispricing of options derivatives. In order to perform the analysis, data was collected for a sample of NSE-traded stock options and for their underlying stocks for the period of one year prior to the contract. The study uses a GARCH (1, 1) model to forecast underlying stock volatility, and uses this forecasted volatility in the Black-Scholes model in order to determine whether the corresponding options are fairly priced. The motivation behind the research is to find systematic mispricing that would provide evidence for arbitrage opportunities.

ANALYSIS & INTERPRETATION

The results of the volatility and pricing analysis of the sample stock options are shown in Table 1. It was found that the implied volatility based on call prices was undervalued as compared to the forecast volatility from the GARCH model for 26.83% of the sample stock options, and was overvalued for 73.17% of the sample stock options. It was also found that the implied volatility based on put prices was undervalued as compared to the forecast volatility from the GARCH model for 12.195% of the sample stock options, and was overvalued for 87.805% of the sample stock options. On the other hand, it was found that the call price was undervalued as compared to the forecast call price based on the forecast volatility from the GARCH model applied in the Black-Scholes model for 36.585% of the sample stock options, and was overvalued for 63.415% of the sample stock options. It was also found that the put price was undervalued as compared to the forecast put price based on the forecast volatility from the GARCH model applied in the Black-Scholes model for 14.63% of the sample stock options, and was overvalued for 85.37% of the sample stock options.

The results of the paired-samples t-test for the implied volatilities (for both calls and puts) and the forecast volatility (based on the GARCH model) are shown in Table 2. It was found that the mean implied volatilities (for both calls and puts) were significantly lower than the mean forecast volatility. Also, it was found that the implied volatility based on call prices was significantly lower than the implied volatility based on put prices.

The results of the paired-samples t-test for the option prices and the forecast option prices (based on the GARCH and Black-Scholes models) are shown in Table 3. It was found that the mean call price was significantly higher than the mean forecast call price. It was also found that mean put price was significantly higher than the mean forecast put price.

The results of the paired-samples t-test for the overpricing of call and put option prices (as compared to forecasts based on the GARCH and Black-Scholes models) are shown in Table 4. It was found that the mean overpricing of call options is significantly lower than the mean overpricing of put options.

DISCUSSION

Volatility permeates modern financial theories and decision making processes. The GARCH model provides accurate measures and good forecasts of future volatility which is critical for evaluation of option pricing.

The results of the study indicate that the implied volatilities (for both calls and puts) were predominantly overvalued, as compared to the forecast volatility (based on the GARCH model), and that call and put option prices were predominantly overvalued, as compared to the forecast call and put options prices (based on the GARCH and Black-Scholes models). Further, put options were more overpriced than call options, as compared to the forecasts based on the GARCH and Black-Scholes models.

These observed discrepancies in the market make it necessary for investors to forecast volatility. Options are priced on basis of the volatility, but there is a difference between the theoretical pricing of options and their actual pricing. To take the advantage of such mispricing in the market, arbitrage can be done.

Table 4 below provides a framework for strategic development with respect to options arbitrage.

Table 5: strategies for options arbitrage due to mispricing of options

Situation	Description	Strategy Description	Investor's Decision
Theoretical Call Price > Quoted Call Price	In this case the call prices quoted by exchange for trading are lower than the theoretical call price calculated	As the quoted call prices are lower than the theoretical price, an investor should buy a call option	Buy Call Option
Theoretical Call Price < Quoted Call Price	In this case the call prices quoted by exchange for trading are higher than the theoretical call price calculated	As the quoted call prices are higher than the theoretical price, an investor should write a call option	Sell Call Option
Theoretical Put Price < Quoted Put Price	In this case the put prices quoted by exchange for trading are higher than the theoretical call price calculated	As the quoted call prices are higher than the theoretical price, an investor should sell a put option	Sell Put Option
Theoretical Put Price > Quoted Put Price	In this case the put prices quoted by exchange for trading are lower than the theoretical call price calculated	As the quoted call prices are lower than the theoretical price, an investor should buy a put option	Buy Put Option

Unfortunately, the results of the study are not indicative only, not conclusive. The major limitations of the study are the relatively small sample size, and the limited research period considered for the study. The research period is in particular a drawback because of the global crisis that surfaced towards the tail end of the time period. In order to generalize the results of the study, similar analysis would need to be carried out for different periods, and taking into account all macroeconomic factors that may affect the pricing of options.

The study provides understanding of option prices and systematic mispricing by the exchanges. Further research can assess the arbitrage profits for each of the stocks-option strategies. Further investigation may also consider the estimation of the prices for index options and currency options using the GARCH model as in the present study.

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Table 1: undervaluation/overvaluation of volatility and price of the sample options

stock option	implied volatility (call option)	implied volatility (put option)	call price	put price
Air Deccan	undervalued	overvalued	undervalued	overvalued
Jet Airways	overvalued	overvalued	undervalued	overvalued
Amtek Auto	undervalued	overvalued	undervalued	overvalued
Hero Honda	overvalued	overvalued	undervalued	undervalued
Maruti	overvalued	overvalued	overvalued	overvalued
Allahabad Bk	overvalued	overvalued	overvalued	overvalued
Canara Bank	overvalued	overvalued	overvalued	overvalued
Corpn Bank	overvalued	overvalued	overvalued	overvalued
Reliance Cap	overvalued	overvalued	overvalued	overvalued
ABB	overvalued	overvalued	overvalued	overvalued
Abirlanuvo	overvalued	overvalued	overvalued	overvalued
Aia Eng	overvalued	overvalued	undervalued	overvalued
ACC	overvalued	overvalued	overvalued	overvalued
Ambuja	overvalued	overvalued	overvalued	overvalued
Shree Cement	overvalued	overvalued	overvalued	overvalued
Chambal Fert	undervalued	overvalued	undervalued	overvalued
Orchid Chem	overvalued	overvalued	overvalued	overvalued
Colgate	overvalued	overvalued	overvalued	overvalued
Dabur	undervalued	undervalued	undervalued	undervalued
HUL	overvalued	overvalued	overvalued	overvalued
3IINFOTECH	undervalued	undervalued	undervalued	undervalued
CMC	undervalued	overvalued	undervalued	overvalued
Infosys	overvalued	undervalued	undervalued	undervalued
Wipro	overvalued	overvalued	overvalued	overvalued
Adlabs	overvalued	overvalued	overvalued	overvalued
SUN TV	undervalued	overvalued	undervalued	overvalued
ZEE	overvalued	overvalued	overvalued	overvalued
Aban	overvalued	overvalued	overvalued	overvalued
Gail	overvalued	overvalued	overvalued	overvalued
HPCL	overvalued	overvalued	overvalued	overvalued
CIPLA	undervalued	overvalued	undervalued	overvalued
DRL	overvalued	overvalued	overvalued	overvalued
Matrix Labs	undervalued	overvalued	undervalued	overvalued
CESC	overvalued	overvalued	overvalued	overvalued
Reliance Energy	overvalued	overvalued	overvalued	overvalued
Suzlon	overvalued	overvalued	overvalued	overvalued
Skumars	undervalued	undervalued	undervalued	undervalued
DLF	overvalued	overvalued	overvalued	overvalued
Unitech	undervalued	undervalued	undervalued	undervalued
Century	overvalued	overvalued	overvalued	overvalued
Welspun guj	overvalued	overvalued	overvalued	overvalued

Table 2: paired-samples t-tests for volatility of the sample options

	mean	std. dev.	correlation	p-value	t-test	p-value
implied volatility (call)	0.4566	0.1294	0.6700	0.0000	-10.5780	0.0000
implied volatility (put)	0.5172	0.1214				
implied volatility (call)	0.4566	0.1294	0.3630	0.0000	-2.5490	0.0110
theoretical volatility (GARCH model)	0.8208	2.5918				
implied volatility (put)	0.5172	0.1214	0.1500	0.0070	-2.1010	0.0360
theoretical volatility (GARCH model)	0.8208	2.5918				

Table 3: paired-samples t-tests for prices of the sample options

	mean	std. dev	correlation	p-value	t-test	p-value
quoted call price	65.4558	69.6281	0.7880	0.0000	2.9750	0.0030
theoretical call price (GARCH model)	58.1934	62.1679				
quoted put price	55.7475	58.8239	0.6650	0.0000	5.2470	0.0000
theoretical put price (GARCH model)	42.3088	51.6172				

Table 4: paired-samples t-tests for prices of the sample call and put options

	mean	stdev	correlation	p-value	t-test	p-value
overpricing of calls	7.2624	43.5275	0.9390	0.0000	-7.0040	0.0000
overpricing of puts	13.4387	45.6704				