Mean-reverting Statistical Arbitrage in Crude Oil Markets

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Abstract

In this paper, we introduce the concept of statistical arbitrage through the definition of a trading strategy that captures persistent anomalies in long-run relationships among assets. We devise a methodology to identify and test mean-reverting statistical arbitrage, and to develop trading strategies. We empirically investigate the existence of statistical arbitrage opportunities in crude oil markets. In particular, we focus on long-term pricing relationships between the West Texas Intermediate crude oil futures and a so-called statistical portfolio, composed by other two crude oils, Brent and Dubai. Firstly, the cointegration regression is used to track the persistent pricing equilibrium, and mispricings arise when West Texas Intermediate crude oil price diverges from the statistical portfolio value. Secondly, we verify

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that mispricing dynamics revert back to equilibrium with a predictable behaviour, and we exploit this stylized fact by applying the trading rules commonly used in equity markets to the crude oil market. The trading performance is measured by three specific profit indicators on out-of-sample data. Lastly, we use a Monte Carlo simulation approach to develop a model for forecasting the expected Value at Risk of the adopted trading strategy over an established holding period.

Keywords: Statistical arbitrage; trading strategy; Value at Risk; commodity markets.

1 Introduction

For years, academics and professionals have been interested in developing quantitative methods and trading strategies to exploit arbitrage opportunities arising in financial markets. One of the first automated trading strategies was developed by a Morgan Stanley trading group under the tutelage of the quant Nunzio Tartaglia in the 1980s and it was called 'Pairs Trading'. It tracked short-term mispricings in a pair of similar securities and exploited it through arbitrage trading by using graphical analysis of trends and their reversion. Asset pairs were selected on the basis of intuition, economic fundamentals, long-term correlations, or simply past experience. By the end of the last millennium, the growing demand for models that could properly describe more sophisticated trading strategies led to the development of the so-called statistical arbitrage strategies. Roughly speaking, a statistical arbitrage is a trading strategy that generates profit in the long-run by pricing inefficiencies identified through mathematical and statistical models. The term statistical arbitrage was used for the first time in the 1990s and it remained widely employed by traders ever since. By 2002, the weak performance of the models caused a loss of confidence in statistical arbitrage methods, primarily due to the changes in market dynamics after the crisis of 2000. Pole (2007) suggests that a relived interest in statistical arbitrage models came about in 2006 when the use of more accurate and advanced algorithms spread.

According to Burgess (1999), a statistical arbitrage may be regarded as a generalization of the traditional zero-risk or pure arbitrage. In the latter case, fair-price relationships between asset pairs with identical cash-flows are constructed and pure arbitrage opportunities are identified when prices deviate from these relationships. Burgess (1999) suggests that zero-risk opportunities cannot exist in the market, due to several uncertain factors such as uncertain future dividend rates, failure to "fill" all legs of the trade, market volatility during the short time required to carry out the lock-in trades. He reports the "basis risk" as an example of source of risk in statistical arbitrage. Basis risk is due to the fluctuations in the differences between spot and futures prices prior to the expiry date. It is a source of uncertainty when positions in securities have to be marked-to-market at current prices by operators due to exchange regulations and companies' internal requirements. On the contrary, it also represents a source of opportunity because an arbitrageur can assume positions in a security and revert the trades before the expiry date, when profits are realized. Thus, the so-called statistical arbitrage opportunities rely on the statistical properties of the security mispricings whose dynamics fluctuate around a stable level. Then, for Burgess (1999) statistical arbitrage attempts to exploit small but consistent regularities in asset price dynamics through suitable strategies.

Bondarenko (2003) derives the definition of statistical arbitrage opportunity from the concept of pure arbitrage opportunity. He defines the pure arbitrage opportunity as a zero-cost trading strategy by which gains are received with no possibility of losses. Instead, the statistical arbitrage opportunity is a zero-cost trading strategy for which the expected payoff is positive and the conditional expected payoff in each state of the economy is nonnegative. This means that the strategy value can be negative in some elementary states, as long as the average payoff in each final state is nonnegative.

For Hogan et al. (2004) and Jarrow et al. (2005), a statistical arbitrage is a long horizon trading opportunity that generates riskless profits. It is a natural extension of the trading strategies utilized in the existing empirical literature on anomalies. Indeed, it is well known that arbitrage opportunities are incompatible with an efficient market (see Jensen (1978)), but tests of market efficiency should refer to an equilibrium model. On the contrary, the existence of a statistical arbitrage rejects market efficiency without invoking the joint hypothesis of an equilibrium model.

Among others, statistical arbitrage approaches based on quantitative methods are proposed by many authors, like Burgess (1999), Vidyamurthy

(2004), Elliot et al. (2005), Do et al. (2006), Bertram (2009), and Cummins and Bucca (2012). Their studies aim to exploit the rising arbitrage opportunities on stock markets.

In this paper, we focus on the specific area of mean-reverting statistical arbitrage. This means that we consider an arbitrage portfolio strategy which is associated with a mean-reverting process. Our purpose is twofold. Firstly, we introduce the concept of statistical arbitrage over a finite time horizon through the definition of a trading strategy that captures persistent anomalies in long-run relationships among asset prices. We provide a computational methodology to develop and test statistical arbitrage strategies. Secondly, we apply the proposed methodology to the crude oil markets. Although in the literature there exist papers that deal with the empirical investigation of statistical arbitrage in several financial markets, to the best of our knowledge and to date, there is no published paper that focuses on modelling statistical arbitrage strategies in crude oil markets. Furthermore, we demonstrate that the proposed methodology does not only extend academic literature on statistical arbitrage, but it is also suitable for practical purposes from a risk management perspective. Indeed, we use a Monte Carlo simulation approach to develop a model for forecasting the expected Value at Risk (VaR) of strategies over an established holding period.

This paper proceeds as follows. In Section 2, we define the mean-reverting statistical arbitrage strategy and we devise a computational methodology to identify and test it. Section 3 deals with the empirical application of the statistical arbitrage methodology to crude oil markets. In particular, Section 3.1 contains the description of the dataset used for the case study. In Section 3.2, we carry out statistical analyses of time series in order to build statistical arbitrage strategies and, in Section 3.3, we verify that strategy dynamics are mean-reverting. In Section 3.4, we outline the used trading rules and we discuss their performance in Section 3.5. In Section 3.6, we test the statistical arbitrage strategies through an out-of-sample analysis. Section 3.7 deals with the development of a model for forecasting the strategies VaR over an established holding period. Finally, Section 4 concludes.

2 Statistical Arbitrage

2.1 Definition and key relationships

We assume a finite time horizon $\overline{T}$, where the uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. $\mathcal{F} = \mathcal{F}_{\overline{T}}$ is the σ-algebra at time $\overline{T}$. All statements and definitions are understood to be valid until the time horizon $\overline{T}$. Furthermore, we assume there are a finite number of trading dates, indexed by $t = 0, 1..., \overline{T}$. Trading strategies are central to the notion of statistical arbitrage. They are formulated using only available information, such as rules based on past returns, firm size, earning announcements, market versus book values, sales growth, or macroeconomic conditions.

We consider N assets[1] whose prices at time t are the row vector $[v_t^1, ..., v_t^N]$

[1]One of the assets may represent a risk-free bond.

and an additional asset whose value is Z_t. We define a portfolio of $N+1$ assets as a $N+1$-dimensional row vector $[h^0, h^1, ..., h^N]$ and each h^i, $i = 0, ..., N$, represents the weight of the i-th constituent asset in the portfolio that is bought at time $t = 0$ and held until time T. In particular, h^0 is the weight of the additional asset Z_t. The coefficients h^i, $i = 0, ..., N$, can be either positive or negative, so that we may have, respectively, both long or short positions in the assets. The value process of the $N + 1$-asset portfolio is $(X_t)_{t\geq 0}$, defined as

$$X_t = h^0 Z_t + \sum_{i=1}^{N} h^i v_t^i, \quad t \geq 0. \tag{1}$$

Definition 1 *The portfolio process $(X_t)_{t\geq 0}$ generates a statistical arbitrage if there exists a time T such that the following conditions are satisfied:*

1. $X_0 = 0$,
2. $E[X_T|\mathcal{F}_0] \geq 0$,
3. *The variance $Var[X_T|\mathcal{F}_t]$ decreases monotonically through time*[2] .

where $E[\cdot|\mathcal{F}_0]$ is the expected value under the objective probability measure $\mathbb{P}$ and $Var[\cdot|\mathcal{F}_t]$ is the variance, conditional to the information available at time t.

Then, the portfolio $(X_t)_{t\geq 0}$ is called statistical arbitrage strategy[3].

Remark 1 *The statistical arbitrage strategy of Definition 1 satisfies three conditions:* 1. *It is a zero initial cost strategy;* 2. *The expected payoff at the*

[2] Conditional variance is a decreasing function of time t, that is $\frac{\partial}{\partial t} Var[X_T|\mathcal{F}_t] \leq 0$
[3] The portfolio $(X_t)_{t\geq 0}$ is also called simply "statistical arbitrage".

trading day T as seen at time 0 is positive; and 3. The strategy reduces its variance over time by adjusting magnitude of its long and short positions. Condition 3. is essential to generate a statistical arbitrage[4].

Remark 2 *In the literature, the statistical arbitrage is opposed to the pure arbitrage (see for example Bondarenko (2003)). Given the portfolio with process $(X_t)_{t\geq 0}$, a pure arbitrage has $X_0 = 0$ and there exists a time T such that $X_T \geq 0$ with probability 1 and $X_T > 0$ with positive probability. This means that a pure arbitrage portfolio is basically a deterministic money making machine that exploits mispricings on the market. On the contrary, in statistical arbitrage the mispricings on the market are based on the expected value of the assets, that is the mispricing of price relationships are true in expectation, in the long-run.*

Let $(Z_t)_{t\geq 0}$ be the price process of a particular asset, called *target asset*. We consider the portfolio vector $\mathbf{h} = [h^1, ..., h^N]$ that consists of the N assets of prices vector $\mathbf{v}_t = [v_t^1, ..., v_t^N]$ and such that the portfolio value is

$$V_t = \mathbf{h}\mathbf{v}_t = \sum_{i=1}^{N} h^i v_t^i, \quad t \geq 0. \tag{2}$$

Definition 2 *The portfolio $\boldsymbol{h}$ is a statistical portfolio for the target asset,*

[4]In economic terms, Condition *3.* implies that the Sharpe ratio associated to the strategy increases monotonically through time. This is consistent with the policy adopted by hedge funds that profit by exploiting mean-reverting dynamics of a portfolio driven by a continuously evolving Sharpe Ratio (see Lo (2010)).

$(Z_t)_{t\geq 0}$, *if the following fair-price relationship holds:*

$$E[Z_t|\mathcal{F}_s] = E[V_t|\mathcal{F}_s], \quad 0 \leq s \leq t, \tag{3}$$

where $E[\cdot|\mathcal{F}_s]$ *is the expected value under the objective probability measure* $\mathbb{P}$ *conditional to the information available at time* s, $\mathcal{F}_s$, *and* V_t *is the portfolio value in equation (2).*

Equation (3) gives a long-run equilibrium relationship between the target asset and the portfolio **h**.

The definition of the mispricing portfolio at a generic time t follows:

Definition 3 *The mispricing portfolio is a trading strategy,* $(M_t)_{t\geq 0}$, *which has mean-reverting dynamics described by the following equation:*

$$dM_t = \alpha(\Theta - M_t)dt + \sigma dW_t, \quad M_0 = 0, \tag{4}$$

where $\alpha > 0$ *is the speed of mean reversion,* $\Theta > 0$ *is the long-run mean,* σ *is the return volatility and* $(W_t)_{t\geq 0}$ *is a Brownian motion.*

Definition 3 implies that M_t is normally distributed and the conditional mean and variance between any two instants s and t, $0 \leq s < t$, given M_s, are:

$$E[M_t|\mathcal{F}_s] = \Theta + (M_s - \Theta)e^{-\alpha(t-s)}, \quad Var[M_t|\mathcal{F}_s] = \frac{\sigma^2}{2\alpha}\left(1 - e^{-2\alpha(t-s)}\right), \tag{5}$$

where we again consider the conditional expected value and variance.

In the following proposition we link the concept of mispricing portfolio to statistical arbitrage.

Proposition 1 *The mispricing portfolio $(M_t)_{t\geq 0}$ whose dynamics is given by formula (4) is a statistical arbitrage strategy. Indeed, all conditions in Definition 1 are fulfilled:*

1. $M_0 = 0$,
2. $E[M_T|\mathcal{F}_0] = \Theta\left(1 - e^{-\alpha T}\right) > 0$,
3. $\frac{\partial}{\partial t} Var[M_T|\mathcal{F}_t] = -\sigma^2 e^{-2\alpha(T-t)} \leq 0$.

Proposition 2 *The portfolio given by a long position on the target asset and a short position on the statistical portfolio and whose process follows the mean-reverting dynamics (4) is a mispricing portfolio $(M_t)_{t\geq 0}$.*

Therefore, given the target asset $(Z_t)_{t\geq 0}$ and the statistical portfolio $(V_t)_{t\geq 0}$, consisting of N assets and obtained according to formula (3), the mispricing portfolio $(M_t)_{t\geq 0}$ is a $N+1$-dimensional vector $\widehat{\mathbf{h}}= [1, -h^1, ..., -h^N]$, such that

$$M_t = Z_t - V_t = Z_t - \sum_{i=1}^{N} h^i v_t^i, \quad t \geq 0. \tag{6}$$

.

Remark 3 *Formula (6) is equivalent to formula (1), where $h^0 = 1$ and h^i, $i = 1, ..., N$, are negative.*

2.2 The Statistical Arbitrage Methodology

In this Subsection, we develop a computational methodology to identify mean-reverting statistical arbitrages according to Subsection 2.1. Our methodology consists of three steps:

1. Statistical mispricing dynamics investigation: a portfolio of selected assets is defined such that the assets prices are in a long-run equilibrium; the deviations from this equilibrium determine the portfolio values and they are called statistical mispricings.

2. Mean reverting analysis: mispricing dynamics are analyzed in order to find predictable components and verify mean reversion.

3. Trading strategy implementation: appropriate trading strategies are implemented to profit and performance indicators are calculated to evaluate their reliability.

In order to build the mispricing portfolio of step *1.*, and according to Subsection 2.1, we need first to choose a target asset, then to identify N assets such that condition (3) holds. These N assets are selected on the basis of a subjective analysis of investors, based on information coming from price behaviour, market rumors, asset physical or financial characteristics, etc. Usually, the target asset and the N assets have common characteristics, such as similar physical or financial characteristics, the same reference market, or they are assets whose prices are affected by the same external factors. However, the technique adopted to obtain the weights of the statistical portfolio $\mathbf{h} = [h^1, ..., h^N]$ of Definition 2 is the cointegration regression. The concept of cointegration has a financial meaning, indeed it represents a long-term relationship among assets. On one hand, the cointegration approach allows us to obtain the coefficients of the constituent asset prices v_t^i, $i = 1, ..., N$, in order to form portfolio (6). On the other hand, it allows us to verify that the chosen constituent assets are appropriate in the sense that their prices are positive correlated with the target asset price, namely they share the same common trend, long-run equilibrium (3). The coefficients h^i, $i = 1, ..., N$, in (2) are elements of the cointegration vector. They are estimated by regressing a set of historical prices v_t^i, $i = 1, ..., N$, over historical target asset prices, such that

$$\mathbf{h} = \arg\min \sum_t \left(Z_t - \sum_{i=1}^{N} h^i v_t^i \right)^2 . \tag{7}$$

It is important to state that the linear cointegration assumes coefficient stability, in particular in the long-run equilibrium between oil prices. However, it is recognized that such stability may not reflect empirical data, particularly large sample data. In the literature, some authors consider the possibility of cointegration even if there are structural breaks in time series, such as Gregory and Hansen (1996). Therefore, the Quandt likelihood ratio (henceforth QLR) test of Stock and Watson (2003) is applied to verify that coefficients h^i, $i = 1, ..., N$, are stable in the long period. The QLR F-statistics test the hypothesis that the intercept and coefficients in formula (7) are constant against the alternative of break in the central 70% of the sample.

In step *2.*, statistical tests are used to verify the mean reversion of mispricing portfolio dynamics. We analyze the autocorrelations across time steps, and apply the Augmented Dickey-Fuller (ADF) Test to search for unit roots and to study time series stationarity. However, a theoretical problem about the low power of classical Dickey-Fuller tests (see Dickey and Fuller (1979)) to clearly identify the stationarity and so the predictability of a price process is well known in econometrics field. Therefore, we use the more robust test of Variance Ratio (see Cochrane (1988) and Lo and MacKinlay (1988), among others), in order to verify if the dynamics of the mispricing portfolio deviate from the random walk behaviour. If we calculate the variance ratio over consecutive time periods $\tau > 0$, we obtain the variance ratio function. Its analysis allows us to find out a mean-reverting nature of the mispricing portfolio. The variance ratio statistic is defined as the normalized ratio of the long-term variance calculated over a period τ to single-period variance. Values of variance ratio bigger than one for any τ suggest that the historical prices are positively serially correlated and the mispricing portfolio has a trending behaviour. On the contrary, values of variance ratio less than one for any τ suggest that the historical prices are negatively serially correlated and the mispricing portfolio has a mean-reverting behaviour.

In step *3.*, appropriate trading rules may be developed in order to take advantage of the mean-reverting behaviour and to open or close positions to profit.

3 Application to the crude oil market

In this Section, we aim at applying the theory discussed in Section 2 to real market data. Although many statistical arbitrage opportunities have been empirically identified in stock markets, commodity markets can be explored. Some forms of arbitrage may be identified in these markets, as reviewed by Fanelli (2015). In this article, we focus on crude oils traded on different markets, that is the West Texas Intermediate, Brent and Dubai. We applied the methodology of Subsection 2.2. We show their prices are related each others and we build a mispricing portfolio by assuming a long position on the West Texas Intermediate crude oil futures and a short position on the statistical portfolio composed by futures on Brent and Dubai crude oils. Furthermore, we develop three basic trading strategies that rely on the mean-reverting behaviour of the mispricing portfolio and we measure their profitability through performance indicators. We carry out a backtest of the strategies on an out-of-sample data and, finally, we propose a model for forecasting strategy losses from a risk management perspective.

3.1 Description of the data

We consider three crude oils. The three largest crude benchmarks in the world are the West Texas Intermediate (henceforth WTI), Brent and Dubai crude oils. The first two are the most important global crude benchmarks for the light and sweet crude. Instead, the Dubai is the most important benchmark for the sour and heavy crude. The WTI crude oil is traded on the New York Mercantile Exchange and was launched in March, 1983. Nowdays, it is the most liquid futures contract in crude oil markets. The WTI is deliverable to Cushing, Oklahoma, which is accessible to the spot market via pipeline. The Brent crude oil, which is traded on the Intercontinental Exchange, was launched in July, 1989. The Dubai crude oil is quoted by Platt's. The dataset consists of weekly futures prices for the first month, spanning from 10/25/2000 to 10/19/2009, resulting in 461 observations.

3.2 Time series analysis

We choose the WTI crude oil as the target commodity and we consider the statistical portfolio (3) composed by Brent and Dubai crude oils, according to Definition 2. Therefore, the weight vector $\mathbf{h}$ is obtained by applying the cointegration regression according to (7). The results of the cointegration regression on the 461 observations are summarized in Table I. Furthermore, the ADF test statistic for residuals is -3.75029 with a p-value of 0.04869 and this implies a cointegration relationship is evidenced. Consequently, mispricing portfolio value time series are obtained through formula (6) and the values are plotted in Figure 1.

Table I:

Cointegration regression for crude oils

We consider equation $Z_t = c - h^1 v_t^1 - h^2 v_t^2$. Z_t is the price of the WTI futures, whereas v_t^1 is the price of the Brent futures and v_t^2 is the price of the Dubai futures. h^1 and h^2 are the weights of v_t^1 and v_t^2 in the statistical portfolio. c is the constant of regression.

Coefficient	Estimate	Std. Error	t-Statistic	Prob.
c	1.617633	0.194177	8.330716	0.0000
h^1	1.193785	0.0406112	9.39531	0.0000
h^2	-0.217020	0.042228	-5.139301	0.0000

R^2	0.995153
Adjusted R^2	0.995131
S.E. of regression	1.832739
Akaike info criterion	4.055987
Schwarz criterion	4.082885
F-statistic	47012.19
Prob(F-stat)	0.000000
Durbin-Watson statistic	0.256805

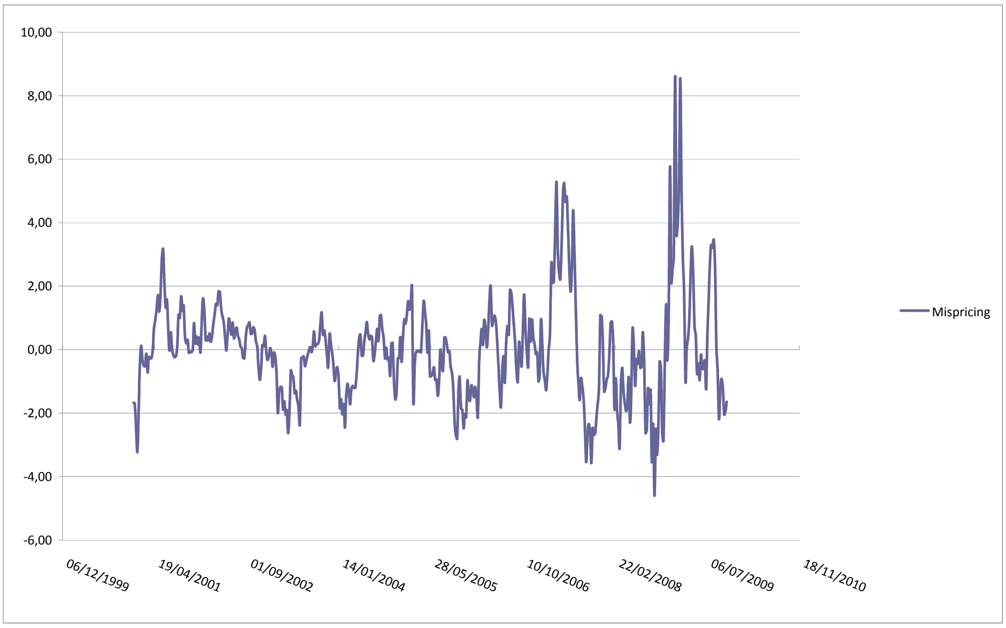

Figure 1: Crude oil mispricing portfolio

In Figure 1 we observe that up to the end of 2006 the mispricing (in $) fluctuates in the range [-2,2]. This is due to the fact that most of the time WTI was higher than Brent crude oil price and the three crude oil prices behave in the same way. The exception is in June 2001, when a weakened US economy and an increased non-OPEC production put downward pressure on WTI prices with respect to Brent oil, and made markets more volatile. Since 2006 and even more so in 2007, the gap switched and Brent oil price was higher than WTI crude oil price, in which the peak came in February 2009 with an average gap of 4.23$/barrel. There could be some macroeconomic changes affecting this spread and recent years, such as the changes in

Euro/USD. This evidence caused more volatility in crude oil markets that is reflected in a wider mispricing oscillation.

Then, the QLR test is applied to check whether the long-run relationship between the WTI, Brent and Dubai crude oils is stable. In particular, here the QLR F-statistics tests the hypothesis that the intercept and coefficients in equation

$$Z_t = c - h^1 v_t^1 - h^2 v_t^2,$$

(see Table I), are constant against the alternative of the break in the central 70% of the sample. We find the critical value F is 61.2695, which means that the null hypothesis that these coefficients are stable is rejected at the 1% significant level. This is the result of the structural break in the sample. The breakpoint data is taken on February 14, 2005. We consequently divide our data in two sub-sets, one for the pre-breakpoint dates and the other for the post-breakpoint dates, leaving the data of 2008 to 2009 to test the model by an out-of-sample analysis. We estimate the mispricing coefficients by OLS regression. Tables II and III provide the details of the regression estimates obtained for each sub-sample.

Table II:

Pre-Breakpoint regression results

The dataset spans from 10/25/2000 to 02/14/2005. We consider equation $Z_t = c - h^1 v_t^1 - h^2 v_t^2$. Z_t is the price of the WTI futures, whereas v_t^1 is the price of the Brent futures and v_t^2 is the price of the Dubai futures. h^1 and h^2 are the weights of v_t^1 and v_t^2 in the statistical portfolio. c is the constant of regression.

Coefficient	Estimate	Std. Error	t-Statistic	Prob.
c	-2.22903	0.403895	-5.519	0.0000
h^1	0.975588	0.0398486	24.482	$<$0.0000
h^2	0.184552	0.0518756	3.558	0.0000

R^2	0.982966
Adjusted R^2	0.982807
Durbin-Watson Statistic	0.251507
Akaike info criterion	622.284
Schwarz criterion	632.41

Table III:

Post-Breakpoint regression results

The dataset spans from 02/14/2005 to 12/31/2007. We consider equation $Z_t = c - h^1 v_t^1 - h^2 v_t^2$. Z_t is the price of the WTI futures, whereas v_t^1 is the price of the Brent futures and v_t^2 is the price of the Dubai futures. h^1 and h^2 are the weights of v_t^1 and v_t^2 in the statistical portfolio. c is the constant of regression.

Coefficient	Estimate	Std. Error	t-Statistic	Prob.
c	-1.07760	0.992658	-1.086	0.27943
h^1	1.31559	0.102020	12.895	0.0000
h^2	-0.317238	0.102883	-3.083	0.00244

R^2	0.982966
Adjusted R^2	0.972844
Durbin-Watson Statistic	0.972477
Akaike info criterion	605.086
Schwarz criterion	614,138

Comparing Table II and Table III, we observe that there is a change of Dubai position in the mispricing portfolio after February 2005. Indeed, before the structural break the mispricing portfolio consists in a long position on the target commodity WTI crude oil and short positions on Brent and Dubai oils. Instead, after the break the long position on WTI crude oil is balanced by a short position on Brent oil and a long position on Dubai oil.

By applying the Johansen (1991) test we verify that the cointegration relation holds also in the presence of a structural break. The results of the test are shown in Tables IV and V.

Table IV: Pre-Breakpoint Johansen test

Rank	Eigenvalue	Trace Test	Lmax Test
0	0.073294	28.206 [0.0766]	16.366 [0.2123]
1	0.052025	11.840 [0.1666]	11.487 [0.1324]
2	0.001640	0.353 [0.5524]	0.35307 [0.5524]

Table V: Post-Breakpoint Johansen test

Rank	Eigenvalue	Trace Test	Lmax Test
0	0.17463	37.851 [0.0042]	28.789 [0.0024]
1	0.058590	9.0619 [0.3664]	9.0565 [0.2880]
2	3.59e-005	0.005388 [0.9415]	0.005388 [0.9415]

3.3 Mean-reversion analysis

In this Subsection, we analyze the mispricing portfolio time series obtained in the previous Subsection, in order to find predictable components and verify the mean-reverting behaviour.

The autocorrelation function of the mispricing time series is used to examine the short-term effects. As we can see from Figure 2, an autocorrelation coefficient with a value different from zero means that a mispricing value is related to the past value and hence the presence of a predictable component is expected.

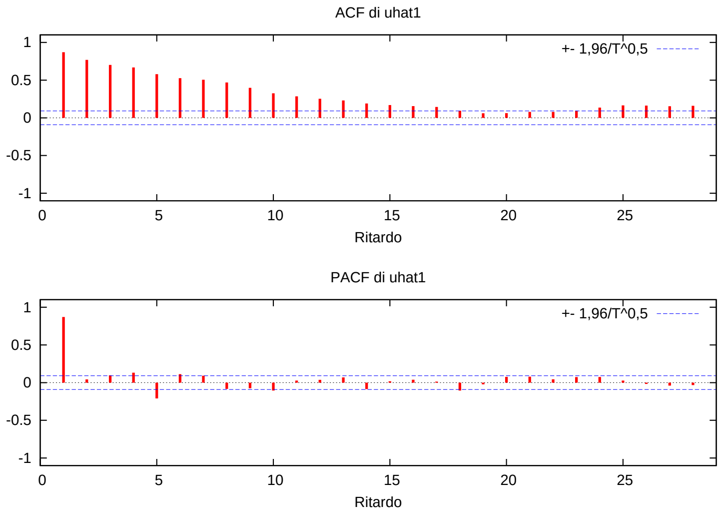

Figure 2: Autocorrelation Function

If we look at the results for unit root tests shown in Table I, we verify the stationarity of the time series. The stationarity is asserted by the value -3.75 of the ADF statistic test, even if acceptable but high value of the p-value 0.04869 could mean an absence of mean reversion. Therefore, we calculate Variance Ratio statistics according to different time lags and we plot them in Figure 3. The Variance Ratio function assumes values lower than one and it is also a decreasing function. We can conclude that the mispricing dynamics follow a mean-reverting behaviour, confirming the existence of predictable components.

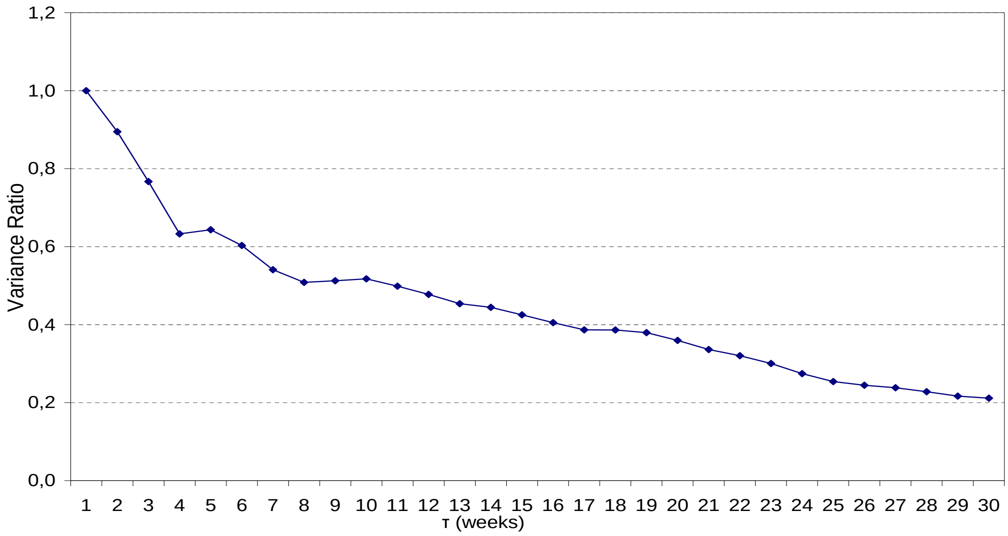

Figure 3: Variance Ratio Function

3.4 Trading rules implementation

In this subsection, we aim at investigating suitable trading rules that identify trading signals for opening and closing positions in the mispricing portfolio. We review three basic trading rules described in Burgess (1999) and we test them on our commodity data.

The adopted trading rules rely implicitly on the mean-reverting behaviour of the mispricing time series. In fact, if in the long run the misprising reduces as prices change, a trader, who has previously opened a position in the mispricing portfolio, can realize profits. The trader should only optimize the tradeoff between transaction costs and trading gains.

These trading rules define the sign and the magnitude of the mispricing portfolio components $\widehat{\mathbf{h}}$ in formula (6). Although in the following we will define three trading rules as functions of the time, we do not need to verify that they fulfill the statistical arbitrage conditions of Definition 1 because the rule functions acquire the mean-reverting characteristics of the mispricing portfolio, and they can be considered strategies along the same line of Proposition 1. Hereafter, we will use indifferently the term trading rule and trading strategy.

The characteristics of the three adopted trading rules are summarized in Table 3.4. For each strategy we give a short description.

Table VI: Trading Strategies

Name	**Simbol**	**Description**
Plain Vanilla strategy	S_t^k	The mispricing is traded according to the investor risk profile. Very risky and aggressive positions can be taken.
Moving-average strategy	S_t^h	The mispricing is traded according to a prudential consideration of the investor risk profile.
Smooth strategy	S_t^O	The mispricing is traded according to the investor risk profile and by considering the transaction costs.

We recall that we suppose a finite number of trading dates, $t = 1, ..., \overline{T}$. Let S_t^k be the plain vanilla strategy, which is the basic trading rule at date t that depends on the sign and the level of the mispricing at the previous time and on the value of a sensitivity parameter $k \in \mathbb{R}$ according to the following formula:

$$S_t^k = -sign(M_{t-1})|M_{t-1}|^k. \tag{8}$$

The mispricing portfolio must be sold when S_t^k is negative and bought when it is positive. An example of trading rule as a function of the time is displayed in Figure 4.

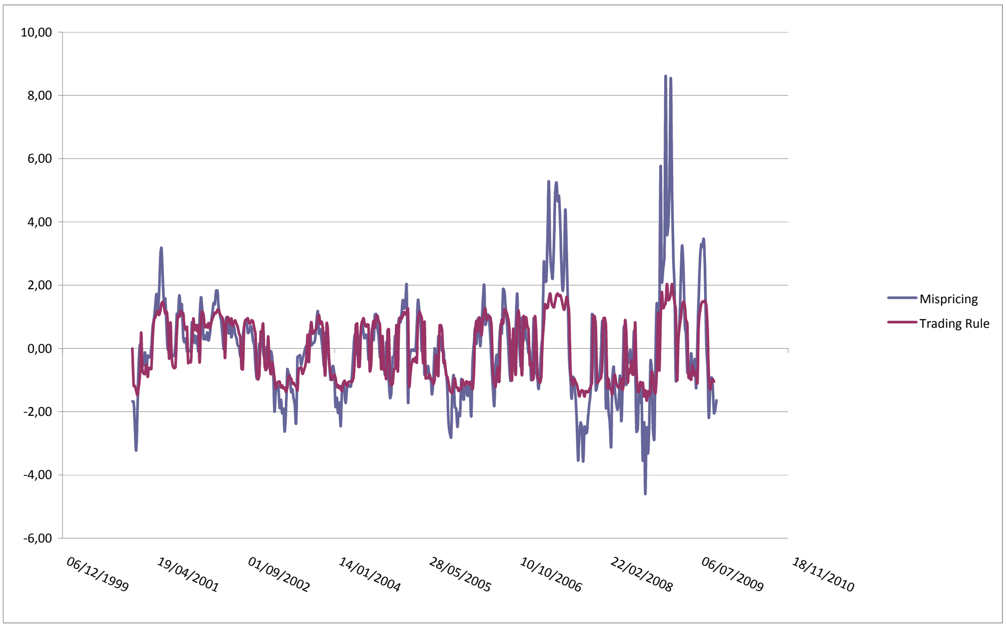

Figure 4: Trading rule with $K = 0.33$

The holding magnitude varies as a function of the size of the previous mispricing through the sensitivity parameter k. We implement this rule according to different values of k on the mispricing time series. We can summarize our results as follows. When $k = 0$, we have a step function meaning that the entire holding is always invested in the mispricing portfolio. If $k > 0$, the size of portfolio increases as the magnitude of the mispricing enlarges and, in particular, a $k > 1$ corresponds to more aggressive strategies.

In order to reduce the investor risk attitude in mispricing trading, we use the moving-average strategy S_t^h, defined as follows:

$$S_t^h = \frac{1}{h}\sum_{j=1}^{h} S_{t-j}^k, \tag{9}$$

where $h > 0$ is the moving average parameter. Furthermore, any transaction, made according to any trading strategy, implies some costs and, obviously, every operator wants to optimize the tradeoff between costs and gains from the exploitation of trading signals. In order to do this we consider the following smooth strategy:

$$S_t^O = (1 - O)S_t^k + OS_{t-1}^O, \tag{10}$$

where $O \geq 0$ is a smoothing parameter. By increasing the values of h and O, on one hand the number of transactions comes down so that the transaction costs decrease, on the other hand the accuracy of the smoothed trading signal diminishes. This behaviour is illustrated in Figure 5 where we compare the three trading strategies (8), (9) and (10) as functions of the time, assuming $K = 0.33$, $h = 6$ and $O = 0.9$. From the figure we deduce that strategy function (8) changes more frequently with respect to function (9) and has larger amplitude compared with function (10). Function (10) has a very smooth trend in comparison with the other functions meaning that, ceteris paribus, adopting this strategy the number of transactions reduces.

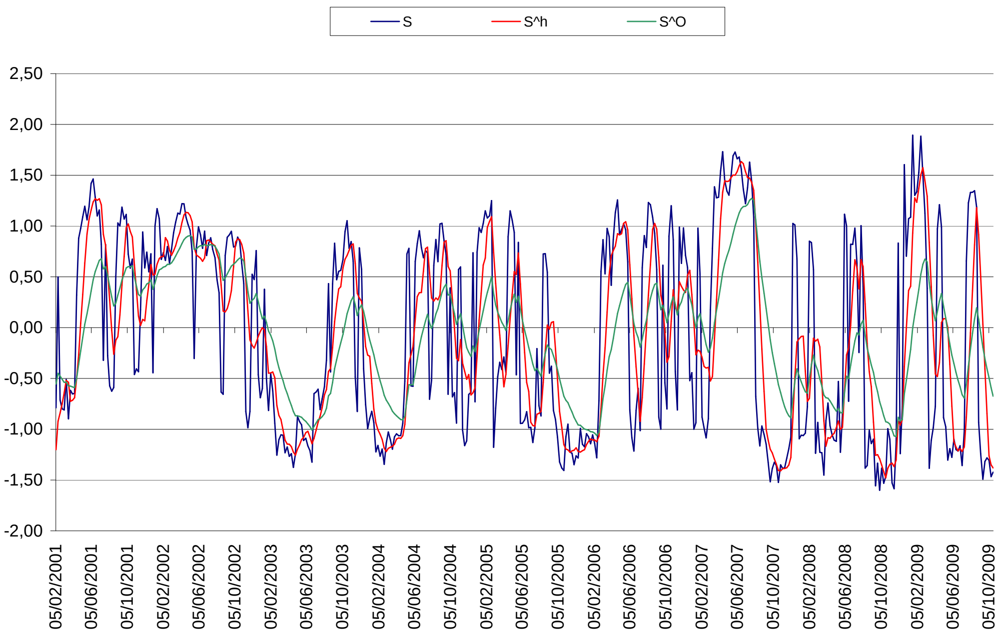

Figure 5: Trading signal comparison: $K = 0.33$; $h = 6$; $O = 0.9$

3.5 Performance analysis

In order to evaluate the performance of the trading rules defined in Section 3.4, we define and estimate the following performance indicators.

The first indicator R_t is the mark-to-market profit & loss, which evaluates the return gotten over a generic trading time period $[t-1, t]$ by applying any trading rule. Let us consider for example the rule (8), the mark-to-market profit & loss at time t is computed by the following formula

$$R_t = S_t^k \frac{\Delta M_t}{S_t + V_t^h} - c|\Delta S_t^k|, \tag{11}$$

where $\Delta M_t = M_t - M_{t-1}$, $\Delta S_t^k = S_t^k - S_{t-1}^k$, c is the percentage transaction costs and $S_t + V_t^h$ is the sum of the mispricing portfolio components. Then, we can substitute S_t^k with the other trading strategies (9) and (10) in order to obtain the return of those strategies.

A strategy profitability indicator is the cumulative mark-to-market profit & loss, ρ_t, that represents the total return or cumulative profit of a strategy from the inception $t = 0$ to the generic trading date t. It is computed as the cumulative sum of the R_s, $s = 0, ..., t$:

$$\rho_t = \sum_{s=0}^{t} R_s. \tag{12}$$

We can use the indicator ρ_t to compare the performances of strategy (8) according to different values of k.

A performance indicator that takes into account not only the level of profit, but also the level of strategy risk, measured by the variability of profits, is the Sharpe Ratio. The Sharpe Ratio calculated at date t is Π_t. As in the traditional sense, it measures the profit per unit of risk. In this context of the statistical arbitrage, it is calculated as the ratio between the annualized mean profitability of the strategy and its annualized standard deviation of the profits:

$$\Pi_t = \frac{\frac{1}{t}\sum_{s=1}^{t} R_s}{\sqrt{\frac{1}{t}\sum_{s=1}^{t}\left[(R_s - \frac{1}{t}\sum_{s=1}^{t} R_s\right]^2}} \tag{13}$$

Figure 6 shows the cumulative profit functions for $k = 0, 0.5, 1$ and confirms the value $k = 1$ ensures the greater profit.

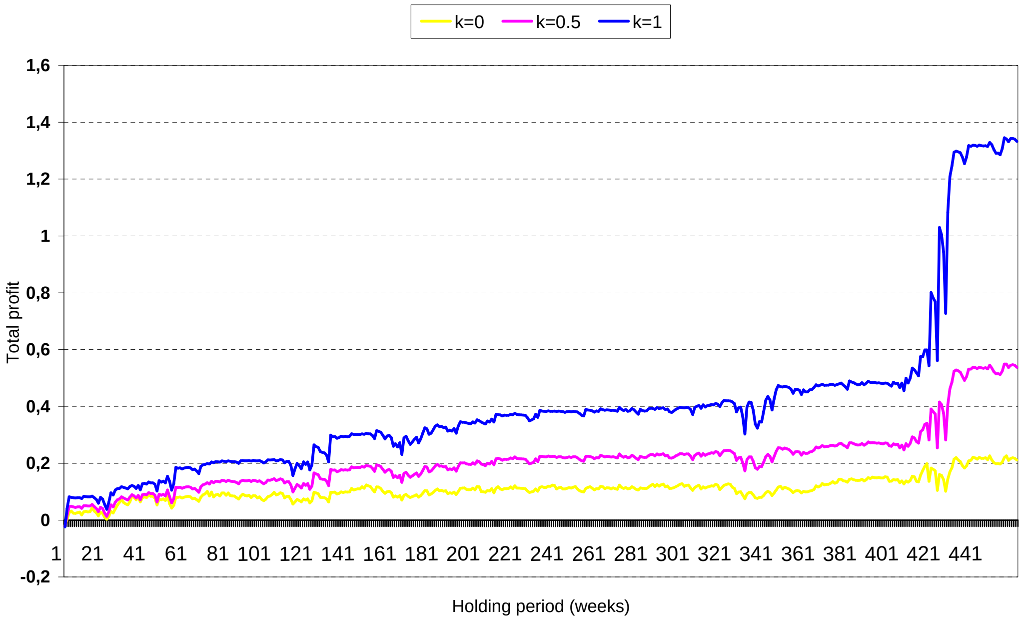

Figure 6: Total return function comparison

Using the performance indicators described above, we compare the trading strategies according to certain parameters and we investigate the most efficient one. We apply the strategy S_t^O, equation (10), on our mispricing data letting k varies between 0 and 1 and O varies between 0 and 0.75. We calculate the values of ρ_t at the last observation for each k and represent them in the Figure 7. From the figure we can deduce that the optimal strategy ensuring the maximum profit is when $k = 1$ and $O = 0.5$ and assuming a cost percentage equal to 0.25%. We use this optimal rule to test the effectiveness of our statistical arbitrage strategy by an out-of-sample analysis as described in the following subsection.

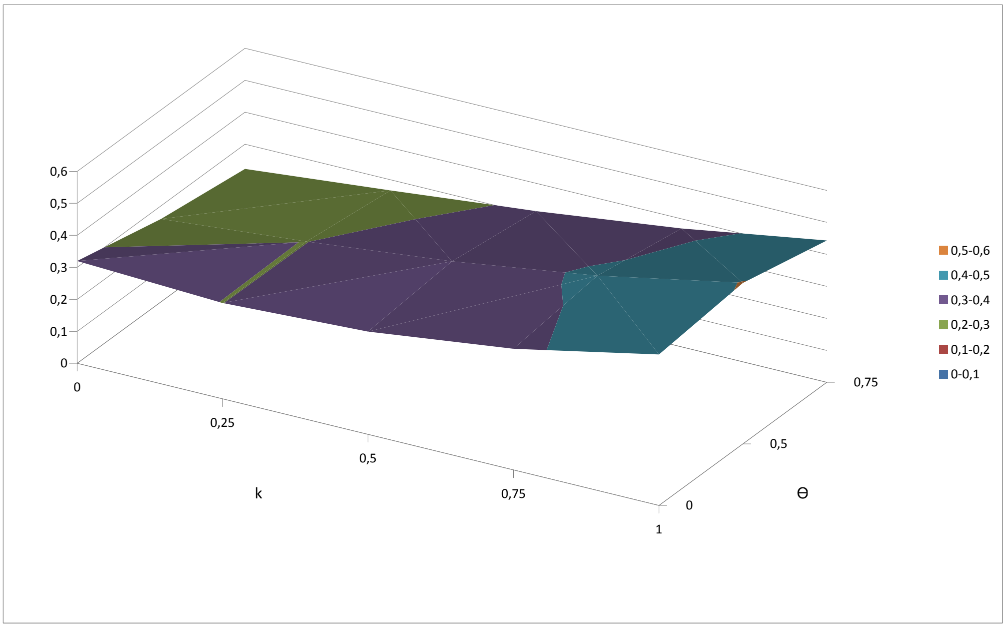

Figure 7: Optimal ISA trading strategy

3.6 Out-of-sample analysis

In order to carry out a backtest of the strategy, we apply the optimal strategy (10) with parameters $k = 1$, $O = 0.5$ and $c = 0.25\%$ to weekly out-of-sample data spanning from 01/07/2008 to 12/23/2010, so that we increase our data set with the data of the year 2010. On the contrary, we consider the time series spanning from 10/25/2000 to 12/31/2007 as in-sample data, and we have used them to estimate the parameters of the mispricing in Section 3.2. We compare the yearly performances of the in-sample data with those of the out-of-sample data. The trading performances are measured using the total return, the annual Sharpe ratio and calculating the percentage profitable weeks as the percentage of periods corresponding to positive returns. The results of the in-sample analysis are illustrated in Table VII, instead, those of the out-of-sample analysis are in Table VIII. We keep into consideration the structural break in 2005, so that we change the cointegration coefficients in the mispricing estimation.

Table VII:

In-sample performance

We apply the optimal strategy in equation (10) with parameters $k = 1$, $O = 0.5$ and $c = 0.25\%$ to weekly in-sample data spanning from 10/25/2000 to 12/31/2007. We calculate yearly performance indicators.

Year	2001	2002	2003	2004
Total Return	7.76%	5.74%	4.76%	2.71%
Sharpe Ratio	1.67	1.42	1.14	0.98
Profitable weeks	45.45%	55.77%	42.31%	44.23%

Year	2005	2006	2007
Total Return	2.09%	4.01%	13.89%
Sharpe Ratio	0.90	0.86	0.83
Profitable weeks	2.69%	50.00%	52.83%

Table VIII:

Out-of-sample performance

We apply the optimal strategy in equation (10) with parameters $k = 1$, $O = 0.5$ and $c = 0.25\%$ to weekly out-of-sample data spanning from 01/07/2008 to 12/23/2010. We calculate yearly performance indicators.

Year	2008	2009	2010
Total Return	4.16%	31.69%	4.12%
Sharpe Ratio	0.84	0.68	0.79
Profitable weeks	42.31%	59.52%	50.00%

From this simple analysis we find out that the strategy performs well in the out-of-sample years, in line with the results obtained on in-sample data. The anomalous value of 2009 Total Return is mainly due to some macroeconomic events already evidenced in Figure 1.

We could state that an optimal strategy may be developed and updated daily taking into consideration the three indices of performance (total return, Sharpe ratio and profitable periods), so that any trading decision would be taken in line with the specific risk profile of the investor.

3.7 Loss forecasting Model

In previous Subsections, we have applied our statistical arbitrage methodology to commodity data and we have shown the performance of three basic trading rules. The proposed methodology may be used for practical purposes for trading and risk management. In particular, from a risk management perspective, an investor is interested in forecasting the expected loss deriving from the application of investment strategies. In this context, we develop a loss forecasting model that allows us to calculate the expected potential loss of a portfolio over a given holding period. In particular, we aim at evaluating at time t the expected VaR[5], $\varphi(t,T)$, of the mispricing portfolio (6) as the expectation of the VaR at the end of the holding period T, $VaR(T)$, conditional to the information available in t, $\mathcal{F}_t$:

$$\varphi(t,T) = E[VaR(T)|\mathcal{F}_t]. \tag{14}$$

In order to estimate $\varphi(t,T)$ we apply the Monte Carlo method by implementing the following steps:

1. Consider the mispricing portfolio (6): use historical data to estimate the parameters of dynamics (4) in order to simulate them.

2. Simulate N trajectories of the mispricing portfolio.

3. For each trajectory i in Step 2., $i = 1, ..., N$:

    - Apply one of the three trading strategies (8), (9) and (10) over an established holding period (i.e., one week, two weeks, or eight weeks);

    - Calculate returns according to formula (11);

[5]The Value a Risk measures the potential loss in value of a risky asset or portfolio over a defined period for a given confidence interval. Given the portfolio return distribution it is obtained as the quantile corresponding to a confidence level, that is a probability.

- Assume normal distribution of returns and estimate the expected return and the variance by using simulated returns of the trajectory;
- Calculate the VaR of trajectory i, $VaR^i(T)$, on return distribution, given the confidence level;

4. The expected VaR, $\varphi(t,T)$, is obtained as

$$\varphi(t,T) = \frac{\sum_{i=1}^{N} VaR^i(T)}{N}. \tag{15}$$

According to Step 1., once the mispricing portfolio (6) have been built, its dynamics can be easily simulated according to formula (4). Indeed, given the mispricing time series and in order to obtain estimates of mean reversion parameters in (4), we have to consider the process (4) in discrete time:

$$\Delta M_t = \alpha(\theta - M_t)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_t, \tag{16}$$

where $\Delta M_t = M_{t+1} - M_t$, Δt is the unit time interval, and $\varepsilon_t \sim N(0, \Delta t)$ is a normally distributed random variable. The discrete-time mean reversion model (16) is equivalent to the following simple linear model (first-order autoregressive model):

$$\Delta M_t = \alpha_0 + \alpha_1 M_t + \sigma\varepsilon_t \tag{17}$$

where $\alpha_0 = \alpha\Theta\Delta t$, $\alpha_1 = \alpha\Delta t$ and $\varepsilon_t \sim N(0, \Delta t)$. Parameters in equation (16) are estimated by applying the OLS regression to (17) and the results are summarized in Table IX.

Table IX: Regression results for equation $\Delta M_t = \alpha_0 + \alpha_1 M_t + \sigma\varepsilon_t$

	Coefficient	Std. Error	t-ratio	p-value
α_0	0.6967454	0.0308819	22.5616	0.0000
α_1	- 0.108796074	0.0157461	-6.9094	0.0000

Residual Std Error	0.65401123
R^2	0.094395944
Adjusted R^2	0.092418642

Then, according to Step 2., we consider $N = 10.000$ scenarios assuming different mispricing portfolio holding periods, from one week to eight weeks. For each scenario, after simulating the mispricing trajectory according to dynamics (16), we apply one of the three strategies (8), (9) and (10) described in Section 3.4. For example, in the first graphic of Figure 3.7 we plot the simulated strategy (8) with $k = 0.33$ when T is eight weeks, while in the second graphic of Figure 3.7 we plot the Total Return function relative to the same strategy.

We estimate $\varphi(t, T)$ according to the formula (15) of Step 3. In the third graphic of Figure 3.7 we show the expected VaR function $\varphi(t, T)$ over the holding period measured as weeks. Function $\varphi(t, T)$ is obtained by interpolating the VaR values calculated through formula (15) by assuming $t = 0$, holding weeks $T = 1, 2, ..., 8$ and a confidence level of 5%. As we expected,

$\varphi(t, T)$ increases as the holding period enlarges.

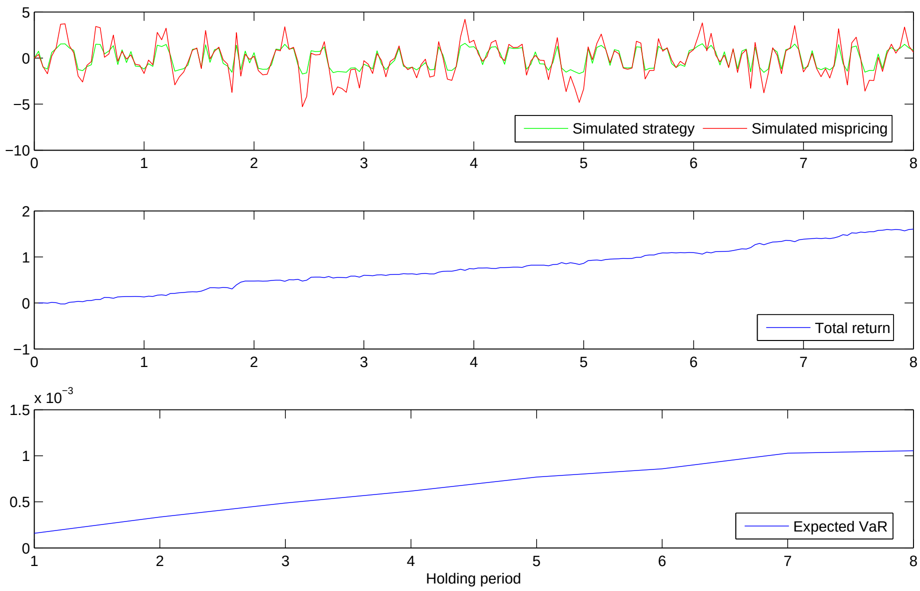

Figure 8: **Loss Forecasting Model.** In the top figure the simulated path of strategy (8) is plotted over eight weeks by considering $k = 0.33$, instead in the middle figure the Total Return function relative to the same strategy is represented. In the bottom figure the expected VaR function $\varphi(t, T)$ is plotted according to different holding weeks T, represented on the x-axis, when $t = 0$. Function $\varphi(t, T)$ is obtained by interpolating the VaR values calculated through formula (15) and assuming holding weeks $T = 1, 2, ..., 8$ and a confidence level of 5%.

4 Conclusions

In this paper, we have introduced the concept of statistical arbitrage through the definition of a trading strategy, called mispricing portfolio. We have focused on mean-reverting strategies in order to capture persistent anomalies in the markets. Furthermore, we have devised a computational methodology to identify statistical arbitrages and apply trading rules adopted from equity markets.

We have shown the empirical evidence of statistical arbitrage in crude oil markets. We have built the mispricing portfolio by using a cointegration regression in order to identify long-term pricing relationships between the WTI crude oil futures and the price of a replication portfolio composed by other two crude oils, Brent and Dubai. We have applied trading rules commonly used in equity markets to profit. Finally, we have proposed a model for forecasting the strategy expected potential loss over an established holding period according to a Monte Carlo simulation approach and we have demonstrated our methodology could be used in research and practice for risk management purposes.

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