Correlation Trading Strategies – Opportunities and Limitations

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Gunter Meissner
University of Hawai'i System

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Gunter Meissner[1]

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Abstract: This paper gives an overview and analyzes the most popular correlation trading strategies in financial practice. Six correlation strategies are discussed: 1) Empirical Correlation Trading, 2) Pairs Trading, 3) Multi-asset Options, 4) Structured Products, 5) Correlation Swaps, and 6) Dispersion trading. This paper focuses on trading correlation, however, briefly in point 7, the risk managing properties of correlation products are outlined.

1) Empirical Correlation Trading

Empirical Correlation Trading attempts to exploit historically significant correlations within or between financial markets. Numerous financial correlations can be investigated. One area of interest is the autocorrelation between stocks or indices. Figure 1 shows the autocorrelation of the Dow Jones Industrial Index (Dow) from 1920 to 2014:

[1] Gunter Meissner is President of Derivatives Software, www.dersoft.com, CEO of Cassandra Capital Management, www.cassandracm.com and Adjunct Professor of MathFinance at NYU-Courant. He can be reached at meissner@hawaii.edu

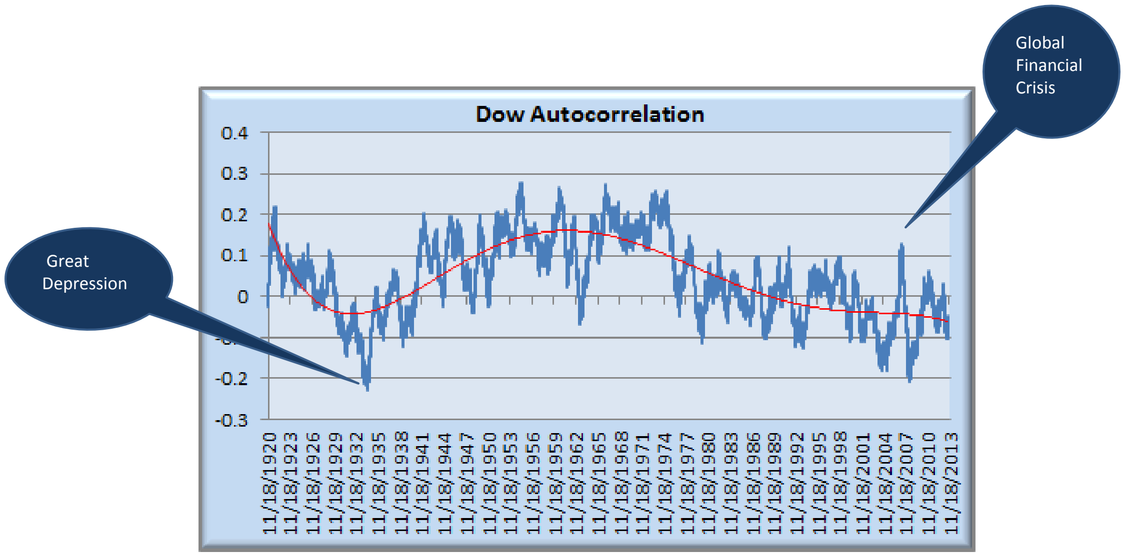

Figure 1: One-day autocorrelation of the Dow Jones Industrial Index (Dow). A positive autocorrelation means that an up-day is followed by an up-day or a down-day is followed by a down-day. A negative autocorrelation means that an up-day is followed by a down-day, or a down-day is followed by an up-day. Figure 1 shows the one-year moving autocorrelation average. The polynomial trendline is of order 5.

From Figure 1 we observe that autocorrelation since the start of World War II in 1939 until the mid-1970's was mostly positive. However, since the mid 1970's autocorrelation has been declining and has mostly been in range with a mean of zero until 2014. An exception was the global financial crisis, in which numerous stocks in the Dow declined, resulting in a positive autocorrelation. Altogether Figure 1 verifies that the Dow is trending less in recent times. This can be interpreted as an increase in the efficiency of the Dow and a demise of technical analysis trend-following strategies.

A further interesting association is the correlation between international equity markets. Numerous studies on this topic exist such as Hilliard (1979), Ibbotson (1982), Schollhammer and Sand (1985), Eun and Shim (1989), Koch (1991), Martens and Poon (2001), Johnson and Soenen (2009), and Vega and Smolarski (2012). Most studies find a positive correlation between international equity markets. This is confirmed by Meissner and Villarreal (2003), whose results are displayed in Table 1:

		Lagging Market						
		Success US	Change US	Success Europe	Change Europe	Success Asia	Change Asia	Limit
Leading Market	US			51.25	0.71	60.50	1.12	> 0.5
				52.45	0.75	65.86	1.31	> 1.0
				51.78	0.78	69.66	1.22	> 1.5
				47.26	0.81	75.74	1.60	> 2.0
				56.30	0.87	**87.21**	**2.18**	> 2.5
				53.61	1.02	74.65	2.86	> 3.0
	Europe	64.10	0.74			57.07	1.07	> 0.5
		67.84	0.84			58.24	1.14	> 1.0
		70.75	0.92			61.55	1.24	> 1.5
		76.03	0.91			58.83	1.24	> 2.0
		64.01	1.15			68.29	1.49	> 2.5
		84.62	1.33			69.90	1.56	> 3.0
	Asia	52.33	0.67	54.95	0.73			> 0.5
		53.74	0.70	56.66	0.75			> 1.0
		54.86	0.68	56.86	0.79			> 1.5
		56.72	0.71	58.62	0.83			> 2.0
		61.38	0.73	61.83	0.92			> 2.5
		60.11	0.71	59.94	0.95			> 3.0

Table 1: Relationship between the US equity market (the Dow Jones Industrial Average), Europe (an average of the DAX, FTSE and CAC), and Asia (an average of the Nikkei, Straits Times and Hang Seng) from 1991 to 2000. For example, the bold number 87.21% means: If the US market had changed (up or down) by more than 2.5%, in 87.21% of these cases the Asian market had the same change the following day. The number 2.18% represents the degree of the percentage change.

From table 1 we observe that the US market follows the European market quite closely. For example, if the European market was up or down more than 2%, the US market had the same directional change in 76.08% of all cases the following day. The degree of the change was 0.91% on average.

We also observe from Table 1 that except for one case (the European market following the US market if the US market has changed by more than 2%), all dependencies are higher than 50%. This confirms the high interdependences between international equity markets.

A word of caution: The Pearson correlation model, which underlies empirical trading strategies suffers from a variety of limitations. Most critically, the Pearson model only measures *linear* associations. As a consequence, the Pearson outcomes can only be meaningfully interpreted if the joint distribution of the variables is elliptical, which comprises the Normal, Student-t, Laplace, Cauchy and the Logistic Distribution. In addition, the correlation coefficient is notoriously volatile, i.e. different time frames can result in very different correlation parameters, i.e. correlations display different regimes. Furthermore, single outliers can distort a regression, and spurious correlations, i.e. correlation without causality can occur. For more on the Pearson model limitations, see point 2.1 below.

2. Pairs Trading

A popular correlation trading strategy in the financial markets is pairs trading. Pairs trading, a type of statistical arbitrage or convergence arbitrage, was pioneered in the quant group of Morgan Stanley in the 1980s. The idea is to find two stocks, which are highly correlated. Once the correlation weakens, the stock that has increased is shorted and the stock which has declined is bought. Presumably the spread will narrow again and a profit is realized. In today's market, pairs trading is often combined with Algorithmic and High Frequency trading. Preprogrammed mathematical algorithms find the pairs and execute the trade in the fastest time possible.

The three critical elements of pairs trading are

a) Selection of the pairs

b) Timing of trade execution

c) Timing of trade closing

In this paper we will concentrate on point a), the selection of the pairs. For an empirical paper on timing and closing of pairs see Gatev, Goetzman, and Rouwenhorst (2006).

Several statistical concept can be applied to identify potentially interesting pairs.

2.1 Applying Correlations to determine the Pairs

A simple Pearson correlation model could be used to identify the pairs. First we screen for pairs, which are highly correlated, i.e. have a high correlation coefficient. If the correlation weakens, the pair's trade is executed, i.e. the stock which has increased is sold and the stock which has decreased is purchased. However, the Pearson correlation model suffers from a variety of limitations:

a) The Pearson model evaluates the strength of the *linear* association between two variables. However, most dependencies in Finance, in particular stock price movements, are non-linear.

b) In particular, linear correlation measures are only natural dependence measures if the joint distribution of the variables is elliptical[2]. However, only few distributions such as the multivariate normal distribution and the multivariate student-t distribution are special cases of elliptical distributions, for which a linear correlation measure can be meaningfully interpreted. See Embrechts, McNeil, and Straumann (1999) and Binghma and Kiesel (2001) for details.

c) As a consequence of point a), zero correlation derived by the Pearson model does not necessarily mean independence. For example, the parabola $Y = X^2$ will lead to a correlation coefficient of 0, which is arguably misleading.

d) Pearson correlations are non-robust, i.e. highly time-frame sensitive. Shorter time frames can lead to a highly positive (negative) correlation, whereas longer time frames can display a negative (positive) correlation. See Wilmott (2009) and Meissner (2015) for details.

e) Pearson himself mentioned a limitation of his model with respect to 'Spurious Correlations'. Spurious correlations occur when the absolute values of variables show no pairwise correlation, even though the relative values show a non-zero correlation.

f) Correlation analysis can also result in 'Spurious Relationships'. A Spurious Relationship (also termed 'Nonsense correlation' or 'Correlation does not imply Causation') refers to the fact that two variables may be highly correlated without causation. This may occur if

1) The two variables both change together in time. For example an increase in organic food consumption will be highly correlated with an increase in Autism even though the two are not causally related. In finance stocks often trend upwards. Hence two upward trending stocks can be correlated without causation simply because they both increase in time.
2) A third (lurking) factor impacts the two variables. For example, a heat wave increases ice cream consumption and death in older people. The correlation between ice cream consumption and death in older people will be highly correlated without direct causation.

For a list and detailed discussion of ten limitations of the Pearson model, see Meissner (2015). We conclude that due to the severe limitations of the Pearson correlation model, it is not

[2] An elliptical distribution is a generalization of multivariate normal distributions.

well suited for the application in finance. In particular it is not well suited to identify potentially interesting pairs.

2.2 Mean Reversion Techniques

Pairs trading assumes that a spread which has widened, will revert to its long term mean. Therefore, mean reverting techniques can be applied to find potentially interesting pairs. Formally, mean reversion exists if

$$\frac{\partial(S_t - S_{t-1})}{\partial S_{t-1}} < 0 \qquad (1)$$

where

S_t, S_{t-1}: Spread at time t and time t-1 respectively

We can apply the Ornstein-Uhlenbeck 1930 model -also known as the Vasicek 1977 model- to quantify the degree of mean reversion, which a spread exhibits. The discrete version of the Ornstein-Uhlenbeck process is

$$S_t - S_{t-1} = a\,(\mu_S - S_{t-1})\,\Delta t + \sigma_S\,\varepsilon_t \sqrt{\Delta t} \qquad (2)$$

where

S_t, S_{t-1}: Spread at time t and time t-1 respectively
a : Degree of mean reversion, also called mean reversion rate or gravity, $0 \le a \le 1$
μ_S : Long term mean of S
σ_S : Volatility of S
ε : White noise, i.e. ε is iid and n(0,1).

We are currently only interested in mean reversion, so we will ignore the stochasticity part in equation (2) $\sigma_S\,\varepsilon\sqrt{\Delta t}$. For ease of exposition let's also assume Δt=1. Hence equation (2) simplifies to

$$S_t - S_{t-1} = a\,\mu_S - a\,S_{t-1} \qquad (3)$$

To find the degree of mean reversion 'a', we can run a standard regression analysis of equation (3) of the form Y = α + β X, where Y corresponds to S_t-S_{t-1}, α corresponds to $a\,\mu_S$ and importantly, the regression coefficient β corresponds to the inverse of the mean reversion rate 'a'.

The approach of equations (1) to (3) is a reasonable approach to quantify mean reversion of the spread between two stocks. The higher the spread mean reversion rate -a = β, the more promising a spread trade is once the spread has diverted from its long term mean μ_S. However, the approach (1) to (3) applies the Pearson regression model to quantify the mean reversion rate –a. Therefore, the limitations a) to e) mentioned in point 2.1 also apply to this approach.

2.3 Cointegration

The 2003 Nobel-Prize rewarded Cointegration approach goes back to Robert Engle and Steve Granger (1987). Cointegration is a natural and mathematically rigorous model to find potentially interesting pairs. The idea is to identify a linear combination of two stocks which is cointegrated. A linear combination, i.e. the spread of two stocks is S_1 – a S_2, where S_1 and S_2 are stocks and 'a' is a constant. Formally, this spread is cointegrated if S_1 and S_2 are individually integrated but the spread S_1 – a S_2 has a lower order of integration[3]. In particular, we are looking for a spread which is integrated to the order 0, I(0). In this case the spread is stationary.[4] A stationary process is defined by three criteria

1) A constant drift,
2) A constant variance, and
3) Constant autocorrelation.

If we can verify that our spread S_1 – a S_2 is stationary, this means that the spread will never divert too far from its mean. Once it has diverted from its mean, it can be expected to revert to its mean due to the constant mean, variance, and autocorrelation. Hence stationary spreads are promising candidates for Pairs trading!

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[3] Here the domain of integration is in a time series sense, i.e. the domain of integration is one-dimensional. This means that we are summing up incremental units of a times series, i.e. a real line (contrary to the often applied two-dimensional integration concept, which calculates surfaces under a function).
[4] To be precise, being I(0) is a necessary but not sufficient condition for being stationary process. So all stationary processes are I(0), but not all I(0) processes are stationary.

Typically we apply the Dickey-Fuller test to find critical t-values for the degree of stationarity of our spread. Dickey and Fuller tabulated the asymptotic distribution of the t-statistic of our null-hypothesis of a unit root process to determine the degree of stationarity in a time series.

So why can Cointegration be seen as superior to the Correlation when identifying Pairs? The answer is that Cointegration, besides being mathematically more rigorous (see point 2.1) is a more 'natural fit' for financial markets. Most stocks trend upwards, i.e. they are not stationary, but integrated to the order 1, I(1). Correlation analysis often leads to 'Spurious regressions' if two times series are I(1) and detrending is often not possible. In addition, Granger and Newbold (1974) showed that even for detrended time series, spurious relationships can occur.

Cointegration naturally applies I(1) stock processes and evaluates if a combination, i.e. a spread of the I(1) processes is stationary I(0). In addition, the Granger causality concept, which includes an autoregressive process augmented by independent variables, can determine the direction and degree of the causal relationships Y(X) and X(Y).

In summary, the benefits of Pairs trading are a high degree of market neutrality (β close to zero) and largely self-funding since one asset is shorted and the other purchased. Pairs can best be identified using cointegration techniques. As with all risk-arbitrage strategies, the limitations are that profits are 'arbed away' (arbitraged away), i.e. the more the strategy is applied, the less pairs exist which can be exploited. For example, the originators of pairs trading at Morgan Stanley were very successful at first, but after a few years abandoned the strategy. In today's market traders try to generate profits from pairs trading using efficient mathematical algorithms combined with high frequency trading.

3. Multi-asset Options

A further way to trade correlation are Multi-asset options, also called Correlation Options or Rainbow Options. Multi-asset options are options, whose payoff depends at least partially on the correlation between two or more underlying assets in the option.

The following list displays popular multi-asset options, which started trading in the 1990s.

	Payoff at option maturity
Option on the better of two	$\max(S_1, S_2)$
Option on the worse of two[5]	$\min(S_1, S_2)$
Call on the maximum of two	$\max[0, (S_1, S_2) - K]$
Exchange option	$\max(0, \max(S_2 - S_1))$
Spread option	$\max[0, (S_2 - S_1) - K]$
Option on better of two or cash	$\max(S_1, S_2, \text{Cash})$
Dual strike option	$\max(0, S_1 - K_1, S_2 - K_2)$
Basket option[6]	$\max(\sum_{i=1}^{n} n_i S_i - K, 0)$ n_i : weight of asset S

Table 2: List of popular Multi-asset Options

Let V be the value of a multi-asset option and ρ the Pearson correlation coefficient between the prices of the underlying assets in the option. Interestingly, for all of the options except two in Table 1 we have $\frac{\partial V}{\partial \rho} < 0$, i.e. the more negative the correlation, the higher the options price. The two options for which $\frac{\partial V}{\partial \rho} > 0$ applies are Options on the worse of two, and Basket options.

In an Option on the worse of two, the options buyer will receive the underlying with the lower price. Hence if the correlation between the underlying assets is positive, they will both on average go up or down together, minimizing the chance of a high S_1 and a low S_2 and the change of a low S_1 and a high S_2, which are both negative for the option buyer.

[5] In 1998 Societe General marketed an extension of the worst-of-two, termed Everest Option. The payoff is on the worst performer of typically 10 to 15 asset at maturity T: $\min_{i=1 \ldots n} \frac{S_i(T)}{S_i(0)}$, where S_i is the price of the i^{th} asset and n is the number of assets.

[6] A variation of the Basket option is Societe General's Himalayan option. At multiple points in time t_i, the payoff of the best performing asset S_b in a basket $\max\left(\frac{S_b(t_i) - S_b(t_0)}{S_b(t_0)}, 0\right)$ is paid out and this asset is then removed until the basket is empty.

For a Basket option, also termed Portfolio option, the higher the correlation between the assets in the basket, the higher the probability of a high payoff, since the assets have a high probability of increasing together. For a high correlation, the probability of the assets in the portfolio decreasing together is also higher, however, the loss of the (any) option for the option buyer is floored at the typically low option premium.

Investment banks, also referred to as the dealer, are typically sellers of multi-asset options. While from a seller's perspective only two of the eight options mentioned are short correlation -the Option on the worse of two and the Basket option- these two options comprise most of the multi-asset option market. Therefore, the equity portfolio of investment banks typically has a short correlation position.

The Quanto option is another popular option which depends critically on correlation. However it is technically not a multi-asset option since it does not include two or more assets. A Quanto option allows a domestic investor to exchange her potential payoff in a foreign currency back into his home currency at a fixed exchange rate. A quanto option therefore protects an investor against currency risk: E.g. an American believes the Nikkei will increase, but is worried about a decreasing yen. The investor can buy a quanto call on the Nikkei with the yen payoff being converted into dollars at a fixed (usually the spot) exchange rate. The term quanto comes from quantity, meaning the amount that is re-exchanged to the home currency is unknown because it depends on the payoff of the option.

Let S' be the price of the foreign underlying (e.g. the Nikkei), and let the investor be American, i.e. the investor wants to exchange her potential payoff in yen into US$ at the rate X = $/Yen. The payoff of the quanto call then is X max [S' - K', 0]. The value of a Quanto call option is highly sensitive to the correlation between S' and X, ρ(S',X). We have $\frac{\partial Q}{\partial \rho(S', X)} < 0$, where Q is the value of the Quanto call. This is intuitive since a negative correlation ρ(S',X) implies a hedge:

If S' increases as X decreases, the Quanto call seller (typically the investment bank) faces a high payoff but has to convert less Yen to US$ to satisfy the payoff. Conversely, if S' decreases and X increases, the Quanto call seller has to convert more Yen into US$, but the amount of Yen is low since S' is low.[7]

Since most investment banks are sellers of Quanto options and the Quanto option value has a negative relationship to correlation, $\frac{\partial Q}{\partial \rho(S',X)} < 0$, investment banks in a Quanto option are short correlation, adding to the already short correlation position of multi-asset options.

Interestingly, the sensitivity of the Quanto option value, Q, to the volatility of the exchange rate, σ(X), depends on the absolute value of the correlation ρ(S',X). We have

$$\frac{\partial Q}{\partial \sigma(X)} < 0 \text{ if } \rho(S',X) > 0 \quad (4)$$

and

$$\frac{\partial Q}{\partial \sigma(X)} > 0 \text{ if } \rho(S',X) < 0 \quad (5)$$

Typically an increase in volatility leads to an increase in an option value. However, equation (4) shows that an increases in the volatility of the exchange rate σ(X) lowers the value of the Quanto call Q if the correlation coefficient ρ(S',X) is positive. The reason is that the negative impact of the positive correlation ρ(S',X) on Q is reduced by the higher volatility of the correlation σ(X), hence the option value is lowered. However, if the correlation ρ(S',X) is negative as in equation (5), a higher volatility of the exchange rate σ(X) increases the option price Q, since the built-in hedge of the negative correlation is reduced by the higher volatility of σ(X), hence increasing the option value Q. This effect is similar to a binary option, which has a positive Vega if it is out-of-the-money and a negative Vega if it is in-the-money.

Generally, the sensitivity of an option, a structured product such as a CDO or CMO, or any financial value as VaR (Value at Risk) or ES (Expected Shortfall) to correlation can be quantified

[7] If the underlying in a quanto is a basket as the Nikkei, another correlation exposure exists: The volatility of the Nikkei depends on the correlation of its components. The higher the correlation of the components, the higher the volatility, see point 6, dispersion trading, for details.

with mathematical derivatives, the first order termed Cora and the second order termed Gora. For an option value V, we have

$$\mathrm{Cora} = \frac{\partial V}{\partial \rho(x_{i=1,\dots,n})} \tag{6}$$

where $x_{i=1,\dots,n}$ are independent variables, in the case of the Quanto option $x_1 \equiv S'$ and $x_2 \equiv X$.

The sensitivity of Cora to correlation can be quantified with Gora,

$$\mathrm{Gora} = \frac{\partial \mathrm{Cora}}{\partial \rho(x_{i=1,\dots,n})} = \frac{\partial^2 V}{\partial \rho^2 (x_{i=1,\dots,n})} \tag{7}$$

For an exchange option E with a payoff max (0, max(S_2 - S_1)), the pricing equation in the Black-Scholes-Merton environment is

$$E = S_2 e^{-q_2 T} N\left(\frac{\ln\left(\frac{S_2 e^{-q_2 T}}{S_1 e^{-q_1 T}}\right) + \frac{1}{2}(\sigma_1^2 + \sigma_2^2 - 2\rho\,\sigma_1\sigma_2)T}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\,\sigma_1\sigma_2}\sqrt{T}} \right) - S_1 e^{-q_1 T} N\left(\frac{\ln\left(\frac{S_2 e^{-q_2 T}}{S_1 e^{-q_1 T}}\right) - \frac{1}{2}(\sigma_1^2 + \sigma_2^2 - 2\rho\,\sigma_1\sigma_2)T}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\,\sigma_1\sigma_2}\sqrt{T}} \right) \tag{8}$$

where

S_1: asset to be given away
S_2: asset to be received
q_2 : return of asset 2
q_1 : return of asset 1
σ_1 : volatility of asset S_1
σ_2 : volatility of asset S_2
ρ : correlation coefficient for assets S_1 and S_2
T : option maturity in years
N(x) : the cumulative standard normal distribution of x.

Differentiating equation (8) partially with respect to ρ, we derive the Cora of an exchange option E

$$\mathrm{Cora_E} = \frac{\partial E}{\partial \rho} = -\frac{e^{-q_2 T}\sqrt{T} S_2 \sigma_1 \sigma_2 \, n\left[\frac{\ln\left[\frac{S_2 e^{-q_2 T}}{S_1 e^{-q_1 T}}\right] + \frac{1}{2} T(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}{\sqrt{T}\sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}}\right]}{\sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}} \tag{9}$$

Differentiating equation (9) partially with respect to ρ, we derive the Gora

$$\mathrm{Gora_E} = \frac{\partial \mathrm{Cora_E}}{\partial \rho}$$
$$= -\left(e^{-q_2 T} S_2 \sigma_1^2 \sigma_2^2 \left(-4 \ln\left[\frac{S_2 e^{-q_2 T}}{S_1 e^{-q_1 T}}\right]^2 + T(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)(4 + T\sigma_1^2 - 2T\rho\sigma_1\sigma_2 + T\sigma_2^2) \right) n\left[\frac{\ln\left[\frac{S_2 e^{-q_2 T}}{S_1 e^{-q_1 T}}\right] + \frac{1}{2} T(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}{\sqrt{T}\sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}}\right] \right) \Big/ \left(4\sqrt{T}(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)^{5/2}\right) \tag{10}$$

In summary, multi-asset option allow an investor to trade correlation between desired assets. Multi-asset options which contain only two assets are typically priced in the Black-Scholes-Merton environment, i.e., have a closed form solution. As a consequence, conveniently, the correlation risk parameters Cora and Gora can also be derived closed form. The pricing of multi-asset options, which contain more than two assets require Monte Carlo simulations. Typically the assets follow correlated geometric Brownian motions, see Zhang (1997) and Linders and Schoutens (2014) for details.

4. Structured Products

Structured products are customized instruments, designed to provide the investor with a relatively high return and -due to diversification- relatively low risk. Typically, a structured product

a) Contains multiple assets
b) Often includes a derivative
c) Is sometimes tranched

Structured products comprise a wide range of instruments. CDOs (Collateralized Debt Obligations) and a CMOs (Collateralized Mortgage Obligations) comprise all criteria above. The multi-asset options, which we discussed in section 3, are simple structured products, most just containing two assets. Pension funds, Mutual Funds and cost-efficient ETFs (Exchange Traded Funds) can also be considered structured products, only satisfying criteria a) above however.

Especially tranched structured products are highly sensitive to correlation between the assets in the structure and the correlation between the tranches. We will show this with the example of a cash CDO.

A cash CDO is a structured product, referencing typically 125 bonds. The default risk of these bonds is tranched. The equity tranche holder is exposed to the first 3% of defaults, the mezzanine tranche holder is exposed to the 3% - 7% of defaults and so on. Figure 1 shows the relationship of the tranche spread with respect to the degree of correlation between the assets in the CDO, when the Gaussian copula correlation model is applied.[8]

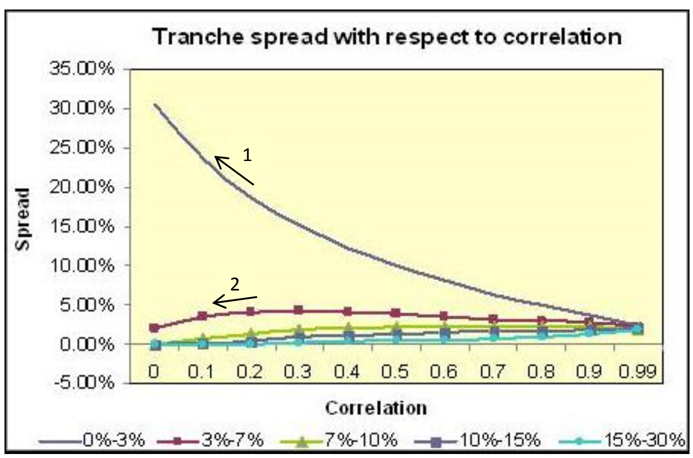

Figure 1: Tranche Spread with respect to correlation between the assets in the CDO. The equity tranche investor, (0-3% tranche), is 'long correlation', since when the correlation between the assets in the CDO increases, the equity tranche spread decreases, and the investor now receives an above market spread.

[8] For details on the Copula model see Cherubini, Luciano and Vecchiato (2004) and Nelsen (2006).

The correlation properties of a CDO, displayed in Figure 1 led to huge Hedge Fund losses in 2005. Hedge funds had shorted the equity tranche (0%-3% in Figure 1) to collect the high equity tranche spread. They had then presumably hedged the risk by going long the mezzanine tranche (3% to 7% in Figure 1). However, as we can see from Figure 1, this 'hedge' is flawed.

When the correlations of the assets in the CDO decreased in 2005 due to the downgrade of Ford and General Motors, the hedge funds lost on both positions: 1) The equity tranche spread (0%-3%) increased sharply, see arrow 1. Hence the fixed spread that the hedge fund received in the original transaction was now significantly lower than the current market spread, resulting in a paper loss. 2) In addition, the hedge funds lost on their long mezzanine tranche position, since a lower correlation lowers the mezzanine tranche spread, see arrow 2. Hence the spread that the hedge fund paid in the original transactions was now higher than the market spread, resulting in another paper loss. As a result of the huge losses, several hedge funds such as Marin Capital, Aman Capital and Baily Coates Cromwell filed for bankruptcy.

Correlation properties of a CDO had a critical impact on the global financial crisis 2007 - 2009. When default probabilities and with it default correlations increased, the correlation between the tranches also increased, providing less protection of the lower tranches for the higher tranches. Especially the default probability of AAA rated super-senior tranches increased sharply due to the decreased protection of the lower tranches. Investors had to buy back super-senior tranches at significantly higher spreads, realizing big losses.[9] The issuers of the CDOs containing super-senior tranches realized large gains.

In conclusion, the value of structured products depends critically on the correlation between the assets in the structured product. The correlation properties of the assets in a structured product can be fairly complex. Investors should well understand the correlation properties before investing in a structured product.

[9] For details see Gregory 2010

5. Correlation Swaps

Correlation Swaps are pure correlation plays, i.e. contrary to the previously discussed correlation trading strategies in point 1 to 4, no price or volatility components of an underlying instrument are involved. In a correlation swap one party pays a fixed correlation rate in exchange for a realized, stochastic correlation rate. The fixed rate payer is ‘buying correlation’, since she benefits from an increase in correlation, the fixed rate receiver is ‘selling correlation’. Figure 2 displays a Correlation swap.

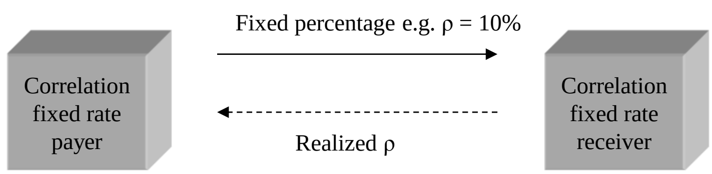

Figure 2: Correlation Swap with 10% fixed rate

The payoff of a correlation swap for the fixed rate payer is N ($\rho_{realized} - \rho_{fixed}$), where N is the notional amount. $\rho_{realized}$ is the average correlation between the assets in the correlation swap, which is realized during the time period of the swap. Formally we have

$$\rho_{realized} = \frac{\sum_{i>j} w_i \, w_j \, \rho_{ij}}{\sum_{i>j} w_i \, w_j} \tag{11}$$

where $\rho_{i,j}$ is the Pearson correlation coefficient between assets i and j. In trading practice, we typically have identical weights $w_i = w_j$. In this case equation (11) reduces to

$$\rho_{realized} = \frac{2}{n^2 - n} \sum_{i>j} \rho_{i,j} \tag{12}$$

The critical question is how to value correlation swaps, i.e. how to derive $\rho_{realized}$. A first thought is to use interest rate swap valuation techniques. In an interest rate swap, forward interest rates are derived by arbitrage arguments from the term structure of spot interest rates.[10]

[10] See Hull 2011 or Meissner 1998 for details.

However, currently in 2015, a correlation term structure does not yet exist. We can derive implied correlations from option prices on indices (see point 6 for details). However, often the implied correlations differ quite strongly from the zero-cost correlation swap fixed rate in the correlation swap market.

One approach to derive the forward correlation rate $\rho_{realized}$ is to model correlation with a stochastic process, just as we model stocks, bonds, interest rates, exchange rates, commodities, volatility and other financial variables with a stochastic process. Quite a bit of research has recently been done in stochastic correlation modeling, see Engle (2002), Emmerich (2006), Düllmann, Küll and Kunisch (2008), Ma (2009a) and (2009b), Da Fonseca, Grasselli, and Ielpo, (2006), Buraschi, Porchia, and Trojani (2010) and Lu, Lobachevskiy, Meissner (2015).

When modeling a financial variable, a critical question is whether to include mean reversion. Except for stocks, which have increased on average annually by about 7.9% (including dividends) each year since 1920, we typically model other financial variables as bonds, interest rates, exchange rates, commodities, volatility with a mean reverting process.

Empirical studies find that correlation exhibits strong mean reversion. The monthly mean reversion of stocks in the Dow Jones industrial average from 1972 to 2012 was 77.51%, (meaning the average monthly Dow correlation reverted each month to its long term mean by 77.51%) see Meissner 2014. Hence we should definitely include a mean reversion component in a stochastic correlation process.

In addition, when modeling Pearson correlations, we should limit the stochastic process between -1 and +1. The bounded Jacobi process includes mean reversion and bounds:

$$d\rho = a\,(m_\rho - \rho_t)\,dt + \sigma_\rho \sqrt{(h-\rho_t)(\rho_t - f)}\,\varepsilon_t \sqrt{dt} \qquad (13)$$

where
ρ : Pearson correlation coefficient
a : degree of mean reversion (gravity), $0 \le a \le 1$.
m_ρ: long term mean of the correlation ρ
σ_ρ : volatility of correlation ρ
h : upper boundary level, f : lower boundary level, i.e. $h \ge \rho \ge f$
ε : Brownian motion ε is iid and $\varepsilon = n(0,1)$

For bounds of h=1 and f=-1, equation (13) reduces to

$$d\rho = a\,(m_\rho - \rho_t)\,dt + \sigma_\rho \sqrt{(1-\rho_t^2)}\;\varepsilon_t \sqrt{dt} \tag{14}$$

Equation (14) is a good candidate for modeling correlations and awaits empirical testing.

A further rigorous model, which avoids the issue of boundaries, since covariances (which take value between -∞ and +∞) are modeled, is the Buraschi, Porchia and Trojani (2010) approach. The core equations are a covariance matrix following a mean reverting stochastic process and a stochastic process of the underlying price matrix. Since the Brownian motions of the Covariance matrix the underlying price matrix are correlated, the Buraschi, Porchia and Trojani (2010) model can be viewed as an extension of the Heston 1993 model.

While correlation swaps can be applied as a speculation tool, their value lies in the convenient hedging of correlation risk. Most clients of investment banks go long correlation in a multi-asset option (see point 3), or a structured product (see point 4). Consequently investment banks find themselves typically with a short correlation portfolio and a correlation swap provides a direct hedge for this short correlation position.

In conclusion, correlation swaps are a direct way to trade correlation and hedge correlation risk. However, currently (2015) no agreed valuation procedure exists. Option implied correlations (see point 6 for details) often differ quite strongly from zero-cost fixed rates in a correlation swap and are therefore not applicable to derive the realized swap rate. Stochastic correlation models are developing, which seem to be a promising approach to model the stochastic correlation in a correlation swap.

6. Dispersion Trading

Dispersion trading emerged in the late 1990s from index arbitrage. In a long index arbitrage trade, the trader buys certain components (e.g. stocks) of an index (e.g. the S&P 500) and shorts the whole index. The index components are expected to outperform the index, so that

$\sum_{i=1}^{n} w_i r_i > r_I$ where w_i are the component weights, r_i is the return of the index components and r_I is the return of the Index.

Dispersion trading applies the same idea, just with respect to component volatility and index volatility. The strategy can be well implemented with options. Three types of dispersion trades can be employed:

a) Directional Dispersion trading. Here call options on index components can be bought and call options on the index[11] sold in the expectation that

$$\sum_{i=1}^{n} w_i \max(S_i - K_i) > \max(S_I - K_I) \tag{15}$$

where S_i are prices of the index components, S_I is the price of the Index and K_i and K_I are the strike prices of the i^{th} component and index respectively.

The key is to find index components S_i, which outperform relative to the index. Naturally directional dispersion trading can also be implemented with put options, if a trader believes she can identify index components which will underperform relative to the index.

b) Non-directional dispersion trading. If the trader primarily wants to trade volatility and not the direction of the components or the index, dispersion trading can be implemented with straddles. In a long dispersion trade, the trader would purchase straddles on individual index components and sell straddles on the index in the expectation that

$$\sum_{i=1}^{n} w_i St_i > St_I \tag{16}$$

[11] Index options are identical with Basket options, which we discussed in section 3.

where St_i is the payoff of the straddle on the i^{th} components and St_I is the payoff of the straddle on the index.[12] Equation (16) can be well approximated with the volatilities

$$\sum_{i=1}^{n} w_i \sigma_i > \sigma_I \tag{17}$$

where σ_i is the volatility of i^{th} price component, w_i is the weighting of the i^{th} component and σ_I is the volatility of the index.

c) Non-directional dispersion trading can also be implemented by buying call or put options on individual components and selling call or put options on the index and delta-hedging both legs. In this case the expectation is that the volatility of the index components is bigger than the volatility of the Index, therefore the gamma – theta difference of the index components is bigger than gamma-theta difference of the index[13]: $\sum_{i=1}^{n} w_i\,(\text{gamma}_i - \text{theta}_i) > \text{gamma}_I - \text{theta}_I$.

The three dispersion trading strategies are also termed 'standard dispersion' or 'vanilla dispersion'.

6.1 Why is dispersion trading a play on correlation?

To derive why dispersion trading is a play on correlation, let's start with the variance equation for two assets i and j:

$$Var_{ij} = Var_i + Var_j + 2\,Cov_{ij} \tag{18}$$

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[12] More precisely, equation (16) is $\sum_{i=1}^{n} \max(S_i^T - K) \cup \max(K - S_i^T) > \max(S_I^T - K) \cup \max(K - S_I^T)$, where S_i^T is the price of the i^{th} component at maturity T, and S_I^T is the price of the index I at maturity T.

[13] In a long option position the trader gains on the delta hedge measured by the gamma, and loses time value, measured by the theta, and vice versa. So in a long dispersion trade, the trader would generate profits from the individual components gamma, lose money on the individual components theta, gain on the index theta and lose on the index gamma.

where Var_{ij} is the variance of the assets i and j and Cov_{ij} is the covariance of i and j.

Generalizing for n = {i,j=1,…,n} assets which comprise the index I, and using financial notation, i.e. $Var \equiv \sigma^2$, equation (18) becomes

$$\sigma_I^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{n-1}\sum_{j>i}^{n} w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{19}$$

where σ_I^2 is the implied variance of the Index, i.e. the variance implied by option prices on the index, and σ_i^2 is the implied variance of an option on the component i, and w_i and w_j are weighting factors. Solving equation (19) for the average pairwise correlation coefficient between assets i and j, ρ_{ij}, we derive

$$\rho_{ij} = \frac{\sigma_I^2 - \sum_{i=1}^{n} w_i^2 \sigma_i^2}{2\sum_{i=1}^{n-1}\sum_{j>i}^{n} w_i w_j \sigma_i \sigma_j} \tag{20}$$

Equation (20) shows the general concept of dispersion trading. The correlation between the components i and j, ρ_{ij}, is not derived by data points in a two-dimensional coordinate system as in the Pearson model, but by the relationship between the index implied volatility σ_I and component implied volatility σ_i.

A trader can now assess the value of ρ_{ij} derived by equation (20) and possibly compare it with the historical values of ρ_{ij} or with her views on future values of ρ_{ij}. From equation (20) we observe that $\frac{\partial \sigma_I^2}{\partial \rho_{ij}} > 0$ and $\frac{\partial \sigma_i^2}{\partial \rho_{ij}} < 0$. So if a trader believes in an increase of correlation, she will buy index volatility (e.g. buy straddles on the index) and sell component volatility (e.g. sell straddles on index components), termed short dispersion. As an example, let's assume the trader sell straddles on index components 1 to 5 and buys a straddle on the index. This would be a successful trade if the components and the index volatility behave as in Figure 3:

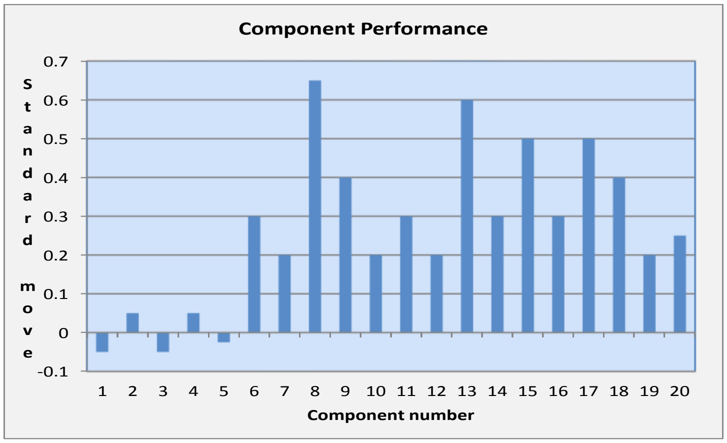

Figure 3: Example of a high positive correlation ρ_{ij} of the index components and consequently high standard move of the index. In the scenario of Figure 3, a short dispersion trade, e.g. selling straddles on the index components 1 to 5, and buying a straddle on the index would have been warranted.

From Figure 3 we observe that the loss of the trader from selling straddles on the components 1 to 5 is low, but the profit from buying an index straddle is high (since the calls produce a high payoff). Conversely, if the components behave as in Figure 4, the strategy of selling straddles on components 1 to 5 and buying a straddle on the index is now a disaster.

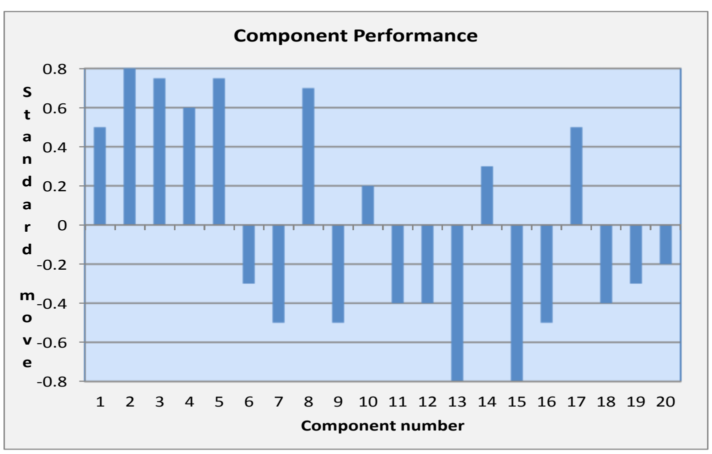

Figure 4: Example of a low correlation ρ_{ij} of index components and consequently a low index standard move; in fact in Figure 4 the sum of the standard moves of the index components = 0, $\sum_{i=1}^{n} \sigma_i = 0$, resulting in a constant index. In the scenario of Figure 4, a long dispersion trade, i.e. buying straddles on the components 1 to 5 and selling a straddle on the index would have been warranted.

From Figure 4 we observe that the loss of the trader from selling an index straddle is zero, since the index has not moved, and the profit from buying straddles on the components 1 to 5 is high, since the calls are in-the-money.

Correlations always increase in distressed markets (John Hull)

The critical questions with respect to dispersion trading are

1) When to go long dispersion, i.e. buying volatility on index components and selling volatility on the index, when to go short dispersion i.e. selling volatility on index components and buying volatility on the index.
2) How many and which components of the index to trade
3) Which type of dispersion trade to implement, see points a) to c) in section 6 above.

With respect to point 1), empirical studies show that correlations levels and correlation volatility are higher in recessions and lower in normal economic periods and expansionary periods, see Meissner 2014. Hence in anticipation of a recession a short dispersion trade is warranted, in anticipation of a normal economic period or an expansion, a long dispersion trade may be implemented. Component selection is a matter of the trader's skill, possibly enhanced by algorithmic and high frequency trading techniques.

6.2 Should we have a bias towards long dispersion?

To analyze whether a long dispersion trade typically leads to a higher payoff, let's start with equation (19), which rearranged, is

$$\sigma_I = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \bar{\rho}_{ij}} \tag{21}$$

where $\overline{\rho}_{ij}$ is the historical (not option implied) average correlation coefficient between the components i and j, and σ_i and σ_j are, as in equation (19), the option implied volatilities of the components i and j. The σ_I of equation (21) is referred to as the MIV, Markowitz implied volatility. The MIV is the theoretical value, based on implied component volatility and historical correlation, at which the index volatility σ_I should trade. However, MIV often differs from the IOIV, the index option implied volatility, at which the index actually trades. Most existing studies find that IOIV > MIV, see Lozovaia and Hizhniuakova (2005), Marshall (2009), Maze (2012), or Bossu (2014). The reasons for the option implied volatility IOIV to be higher than Markov implied volatility MIV may be due to

a) The Risk-aversion of investors. In virtually every market we have a risk-premium:

   - In the bond market the credit spread, which reflects default probability of traded bonds is significantly higher than the historical default probability, which Altman (1989) pointed out.

   - In the equity market, the equity risk premium is $r_S > r_f$, where r_S is the return of a stock and r_f is the risk-free interest rate, accounting for higher stock volatility.

   - In the option market the implied volatility, reflecting option prices is typically higher than historical option volatility.

   Therefore, we can also expect the option implied volatility IOIV to be higher than the Markov implied volatility MIV, due to risk-averse investors buying protection on the whole index.

b) Related to point a) is the perception of systemic risk, i.e. the possibility of a significant downturn of several major markets as in the 2007 to 2009 crisis, which investors want to protect against by buying protection on the whole index, hence increasing IOIV.

c) Liquidity is a further reason for option implied volatility IOIV being higher than Markov implied volatility MIV. Most investors buy protection on the more liquid index option market, driving up option implied volatility IOIV.

d) Risk-taking professionals often sell individual option volatility, driving down market implied volatility MIV.

In conclusion, there is theoretical rational and empirical evidence that the option implied volatility IOIV is typically higher than the market implied volatility MIV. In the long run this can be exploited by traders by going short IOIV and long MIV. Traders should be aware of a systemic downturn though, in which IOIV is expected to increase sharply due to increased correlation.

6.3 Risk-managing Dispersion trades

In trading practice, traders often take a position in the cora (the correlation exposure, introduced in section 3) and hedge the delta, vega, gamma, and theta risk. These risks are typically quite small at inception of the dispersion trade, since options are bought in one leg of the dispersion trade and sold in the other. For example in a long dispersion trade, straddles can be bought on index components, and a straddle is sold on the index, netting some of the initial risks.

However, once the underling components diverge from their initial values, the risk parameters can become quite large. This is especially the case if only some of the index components are traded: The index may have diverged strongly from its initial value, resulting in a change of the index risk parameters, however, the partial components may have a low correlation, resulting in only a modest change of the risk parameters as we displayed in Figure 3. Conversely, the risk parameters of the partial index components can change strongly if they are highly correlated, but the index standard move may be small due to a low correlation of all components. We displayed this situation in Figure 4, where the traded index components are components 1 to 5.

The derivation of an index option value and its risk parameters is occasionally done by treating the basket as a single underlying variable and applying a Black-Scholes-Merton approach. However, this is an overly simplistic approximation for two main reasons

1) The sum of the lognormal distributed components is not lognormally distributed.
2) The critical factor correlation between the components is not incorporated.

To derive index option values and its risk parameters, typically Monte Carlo simulation is applied, assuming the components follow correlated geometric Brownian motions. For more details see

Zhang (1997), Liners and Schoutens (2014) and for nice overview paper Krekel, de Kock, Korn, and Man (2004).

Risk managing dispersion trades is also costly, computationally intensive, and quite complex. If all options of the S&P 500 are traded, the bid-ask spread has to paid for each trade and additionally transaction costs occur. The options have to be risk-managed continuously, especially the pair-wise correlation matrix of the positions can become operationally and computationally intensive: Trading 500 S&P options results in n(n-1)/2, so 500(499)/2 = 124,750 pair-wise correlations, which have to be implied by option prices and their impact on the dispersion value has to be derived. In addition, the risk parameters can be become quite large. Therefore dispersion desks typically demand high risk limits to manage the exposure. The dispersion risk-parameters can also be quite complex. The greeks (delta, gamma, vega, theta) are typically highly interdependent. For example dispersion trades include a 'hidden Vega risk': Changes in the implied component volatility σ_i can have a significant impact on the index volatility - component volatility ratio σ_I/σ_i, but depending on parameter constellations can also have very little impact, see Avellaneda (2002) and Bossu (2014).

In summary, the high cost, high computational intensity, and fairly high mathematical complexity of continuously hedging dispersion trades limits the application of actively hedged dispersion trading in practice.

6.4 Variance Dispersion Trading

If dispersion trading is executed via straddles as discussed in section 6, the trader has to deal with the price and volatility of the underlying components and the index. If the dispersion trade is hedged, the trader has to additionally deal with the risk parameters, delta, vega, gamma, theta, vanna, volga, and rho, which as we explained in section 6.2, are complex and correlated. An easier and more direct way to trade correlation is variance dispersion trading, which is a combination of a standard dispersion trade and variance swaps. The payoff of a short variance dispersion trade is

$$\sigma_I^2 - \sum_{i=1}^{n} w_i \, \sigma_i^2 \tag{22}$$

where σ_I^2 is the realized variance of the index for the time period of the trade, σ_i^2 is the realized variance of the index components for the time period of the trade, and w_i is the weighting of the i^{th} component. Hence in a short variance dispersion trade, the trader is anticipating that the index volatility is higher than the component volatility during the time period of the trade and the payoff is as in equation (22). Naturally, for a long variance dispersion trade, the payoff is equation (22) multiplied with -1.

Note that equation (22) is the square of equation (17), which is an approximation of the standard dispersion trade via straddles. Applying a variance dispersion trade with the payoff of equation (22) enables investment banks to take a position in variances directly without the hassle of having to trade and hedge the underlying options. This is convenient for investment banks, who, as we explained in point 3, typically have short correlation portfolios and can hedge this short correlation position directly by going long correlation via a short variance dispersion trade. The other product, which allows a direct trading and hedging of correlation risk are correlation swaps, which we discussed in point 5.

Mathematically, it can be shown that the payoff of a short variance dispersion trade can be approximated by

$$\sum_{i=1}^{n} w_i \, \sigma_i^2 \, (\rho_{ij} - \bar{\rho}_{ij}) \tag{24}$$

where σ_i^2 is the realized variance of the index components for the time period of the trade, ρ_{ij} is the option implied correlation and $\bar{\rho}_{ij}$ is the realized correlation, see Bossu (2006) and Jacquier and Slaoui (2010). As we discussed in section 6.2, there is a risk-premium in the correlation market. It follows that the implied correlation is typically higher than the realized correlation, i.e.

$\rho_{ij} > \bar{\rho}_{ij}$. Hence a short variance swap trade i.e. receiving σ_I^2 and paying $\sum_{i=1}^{n} w_i \sigma_i^2$ with a payoff as in equation (22) is typically a promising trade. However, the trader should be aware of systemic risk: In a downturn of several markets and its components, the realized correlation $\bar{\rho}_{ij}$ will increase sharply, leading to losses of the short variance dispersion trade.

7. Risk Management with Correlation Products

This paper focusses on correlation trading. However, we will briefly outline the risk management properties of correlation products.

After the global financial crisis 2007 to 2009, risk-reduction became the new paradigm in finance. Correlation risk, a relatively overlooked risk before the crisis, has come to the forefront since correlations within and between markets increased sharply in the systemic crisis and caused devastating losses for investors as well as financial institutions. For details see 'Sunk by correlation' in Risk Magazine 2008, and Meissner 2014.

Naturally all portfolio risk-measures such as VaR (Value at Risk), ES (Expected Shortfall), and ERM (Enterprise Risk Management), and EVT (Extreme Value Theory) depend critically on the input factor correlation between the assets in the portfolio to derive the risk-measure. The well-known model for the parametric VaR is

$$VaR_P = \sigma_P \ \alpha \ \sqrt{x}$$

where VaR_P : Value at Risk of a portfolio P

σ_P : Volatility of the portfolio P

α : Abscise value of a standard normal distribution, corresponding to a certain confidence level c, $\alpha = N^{-1}(c)$, where N^{-1} is the inverse of a standard normal distribution

x : Time horizon for the VaR, typically measured in days

Importantly, the volatility of the portfolio σ_P is derived by the correlation between the assets in the portfolio

$$\sigma_P = \sqrt{\beta_h \, C \, \beta_h^T} \quad (25)$$

where β_h is a horizontal vector of invested amounts (price times quantity) and importantly

C is the covariance matrix of the returns of the assets

Equation (25) applies the same concept as the Dispersion trading equation (19). The higher the correlation, the higher is the variance of the index in equation (19). Here with VaR, the higher the correlation, the higher the volatility of the portfolio. Hence we have $\frac{\partial \text{VaR}}{\partial C} > 0$. This is intuitive since the higher the correlation between the assets, the higher is the probability of many assets declining together, leading to high losses. This is exactly what happened in the global financial crisis: Risk managers had derived VaR numbers with low correlation inputs from benign times 2003 to 2006. When correlations increased dramatically in 2007 to 2009, VaR values and actual losses increased dramatically.

The risk-products, which were discussed in points 3 to 6 can be applied to reduce correlation risk. Especially correlation swaps and variance dispersions are a convenient correlation risk hedging tool, since they are pure correlation plays. As an example, Figure 5 shows the VaR of a 10-asset portfolio, AT&T, Citi, Ford, GE, GM, HPQ, IBM, JPM, MSFT, and P&G from August 1st 2011 to July 31, 2012 (function without squares) with respect to the change in the pair-wise asset correlation. The positive VaR correlation exposure is then hedged with paying fixed in a correlation swap and receiving the average pairwise correlation of the 10 assets, (compare Figure 2). Hence the correlation exposure of VaR is reduced (graph with squares).

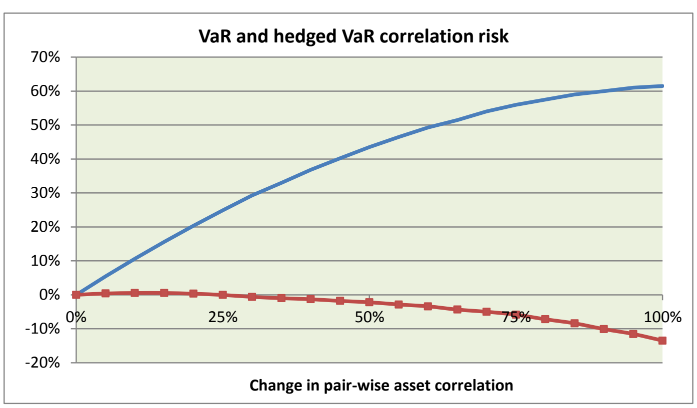

Figure 5: VaR with respect to change in the pair-wise asset correlation of a 10-asset portfolio (straight line) and VaR hedged with a correlation swap (function with squares).

Naturally also individual positions can be hedged with correlation products. As mentioned in section 3 and 4, equity desks often have a short correlation position, since they are sellers of Basket options, Quanto options and Worst-of. This position can be hedged with going long in a correlation swap or short dispersion.

In addition, correlation products can also enhance liquidity. A large mutual fund or insurance company might want to buy single stock options in big size from an investment bank. This illiquid single option exposure can be converted into more liquid index exposure by a dispersion trade.

Generally, equity prices and correlations are negatively correlated as seen in Figure 6.

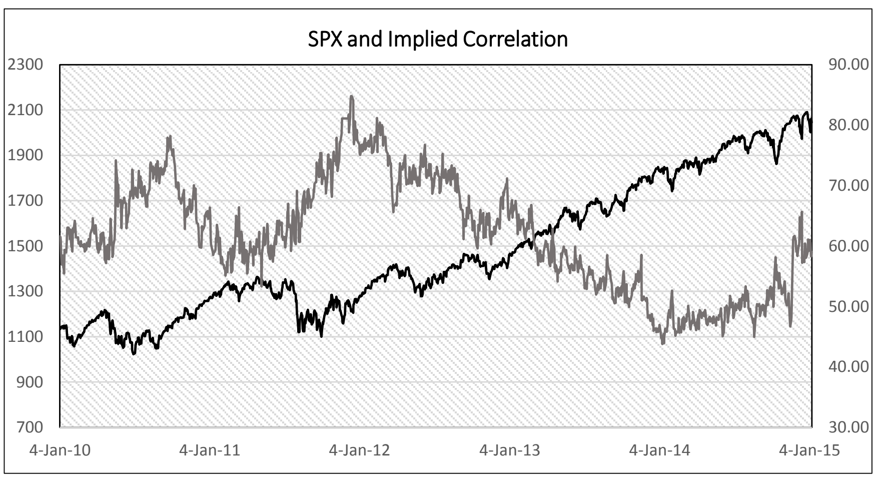

Figure 6: Negative correlation between the SPX, a tracking index of the S&P 500 (black line, left axis), and its implied component correlations (gray line, right axis), which are derived by equation (20). The correlation coefficient is -0.74. Data source: CBOE

Hence going long correlation via one of the correlation products discussed above provides a hedge against systemic equity downside risk. The exact sensitivities of the hedge i.e. Cora, Gora, see equations (7) and (8), as well as other greeks may be approximated by historical data.

8. Concluding Summary

This paper gives an overview of the most popular correlation trading strategies in practice, which comprise 1) Empirical Correlation Trading, 2) Pairs Trading, 3) Multi-asset Options, 4) Structured Products, 5) Correlation Swaps, and 6) Dispersion trading. Most strategies involve trading an underlying asset, i.e. levels and volatilities of the underlying have to be managed. However, pairs trading and variance dispersions are pure correlation plays, which simplifies correlation trading. Correlation levels and volatility display expected properties, i.e. they are low in economic expansions and higher in economic downturns. This facilitates correlation trading.

Correlation products can also be applied in risk management to reduce correlation exposure in portfolio risk measures as VaR, ES, ERM and EVT. In addition, there is a strong negative correlation between equity prices and equity correlation. Therefore, correlation products can also be applied to hedging systemic equity risk.

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