Path-dependence of Leveraged ETF Returns

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CIMS

Summary

- Review of exchange-traded funds (ETFs)
- Leveraged ETFs, 3X, -3X,…
- Empirical facts about LETFs
- Path-dependence explained
- Empirical validation of the theoretical formula on 54 LETFs
- Rebalancing: replicating leveraged returns over long-term horizons

Exchange-traded funds

ETF: Investment vehicles similar to mutual funds but “look like stocks”

-- traded on an exchange

-- trading is similar to stocks (long, short, margin)

ETF: can be viewed as a holding company or a fund

-- started as index trackers

-- actively managed ETFs since mid 2000’s

Arbitrage: authorized participants can **create or redeem** ETFs in ``creation units’’

-- creation units: 25K to 100K shares

-- APs often act as market makers, providing liquidity

Milestones

1993: first US ETF
1998: first European ETFs
2006: first actively managed ETFs

More factoids…

1989: Index Participation Shares, stopped by Chicago Mercantile Exchange
1993: SPY Tracking S&P 500 (a.k.a. Spiders or SPDRS, issuer: State Street)
1996: BGI creates WEBS (World Equity Benchmark Shares), later called I-Shares
1998: Sector SPDRS track 9 sectors of the S&P 500
2008: 680 ETFs in US with 610B in assets, increase of 125B in 12 months
2009: more than 800 ETFs

20 most active US ETFs

TICKER	DESCRIPTION	AUM	AVG VOL (3m)	EX RATIO (bps)
SPY	0.1 SP500	93B	366M	8
EFA	MSCI Intl Eq	31B	36M	34
GLD	Gold 0.1 oz	21B	13M	NA
EEM	MSCI Emerging Mkts.	19B	100M	72
IVV	SP 500 I-Share	15B	7M	9
QQQQ	Nasdaq 100 Index 1/10	12B	150M	20
IWM	Russel 2000	10B	77M	20
IWF	Russel 1000 Growth	10B	8M	20
AGG	Bond ETF	9.5B	0.7 M	20
IWD	Russel 1000 Value	9.2B	7M	20
VTI	Vanguard MSCI 1300 Stocks	9.2B	11M	71
DIA	Dow 30	8.9B	32M	14
TIP	Inflation-protected bonds	8.6B	0.7M	20
XLF	Financial SPDR	7.7B	200M	23
SHY	1-3 yr Treasurys	7.7B	1.3M	15
LQD	IG Corporates	6.9B	1.4M	15
MDY	Mid-Cap SPDR	6.8B	10M	25
FXI	China Xinghua 25	5.9B	43M	74
IWB	Russel 1000	5.7B	9M	15
EWJ	Japan	5.6B	32M	52

US Sector ETFs

TICKER	DESC	AUM ($B)	VOL ($M)	LAST ($)
XLF	Financial SPDR	7.7	177	10
XLE	Energy SPDR	4.4	40	49.8
OIH	Oil Services HLDR	5	11	83.99
XLK	Tech	1.8	8	16.5
IGE	Natural Resources	1.2	0.8	25.96
PHO	Water Resources	1.3	0.5	13.25
XLP	Consumer Staples	2.23	6.7	22
XLV	Health Care	2.06	6	27
XLU	Utilities	1.9	8	30.12
MOO	Agribusiness	0.6	0.5	31.19
GDX	Gold Miners	2.7	7.5	35.49
PPH	Pharmaceuticals	1.5	0.6	61.78
PBW	Clean Energy	0.6	0.8	8.38
IYR	Real Estate	1.66	37	32
BBH	Biotech HOLDRS	1.38	0.1	175.69
XLI	Industrials	1.1	12.1	21.1
SMH	Semiconductors	0.75	12	18.88
IYE	Energy Ishare	0.6	1.5	28.38
IBB	Biotechnology	na	0.9	73.99
IYW	Technology	0.6	0.3	36.27
XME	Metals & Mining	0.2	1.8	30.5
RTH	Retail	0.35	5	72.5
RKH	Regional Banks	0.3	2.5	50.36
IYT	Transportation	0.37	0.8	57.5
XLY	Consumer Discretionary	0.6	7	20.2

Leveraged ETFs

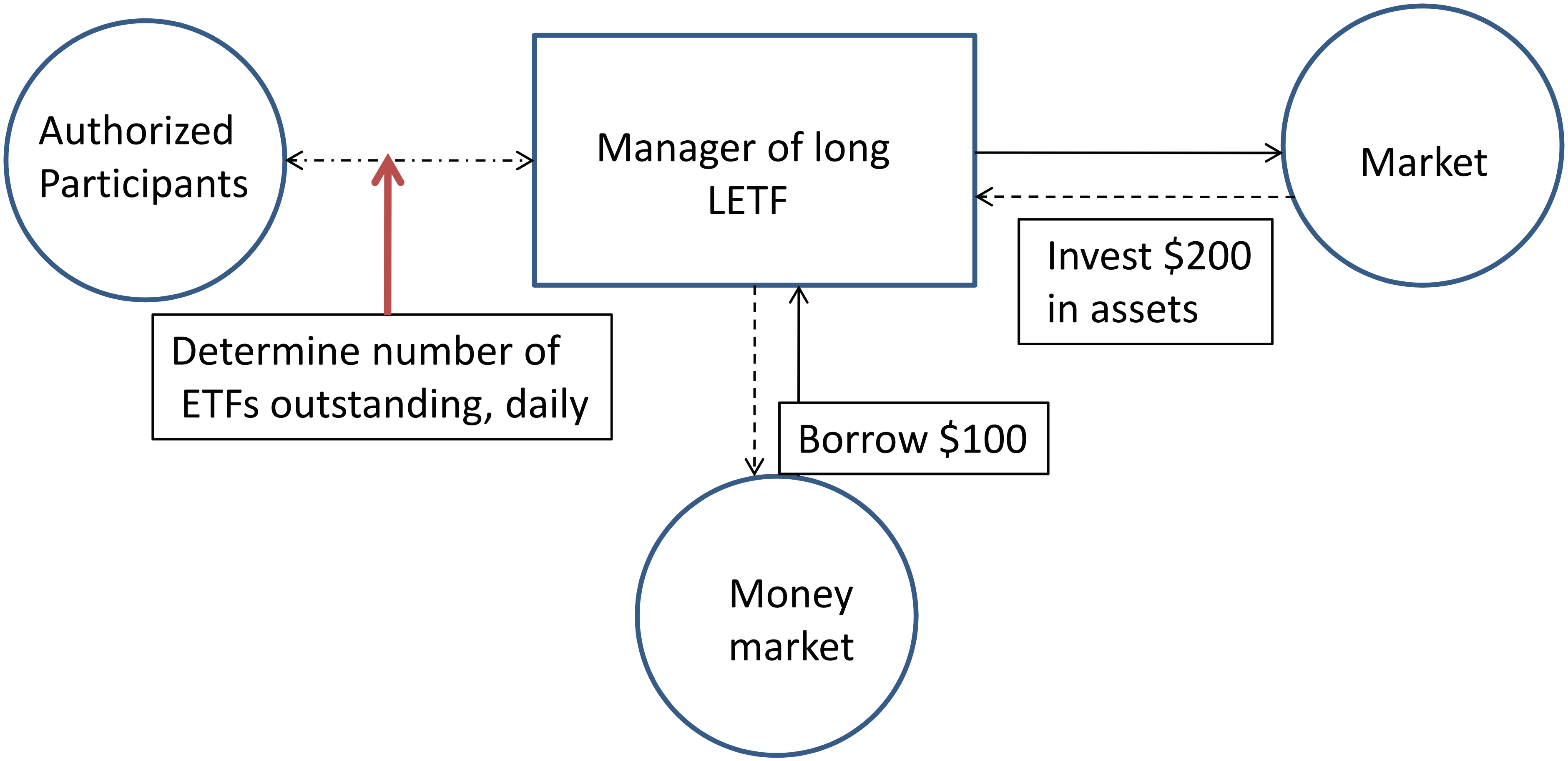
Products offer a multiple of the *daily* return of a reference index

Examples:

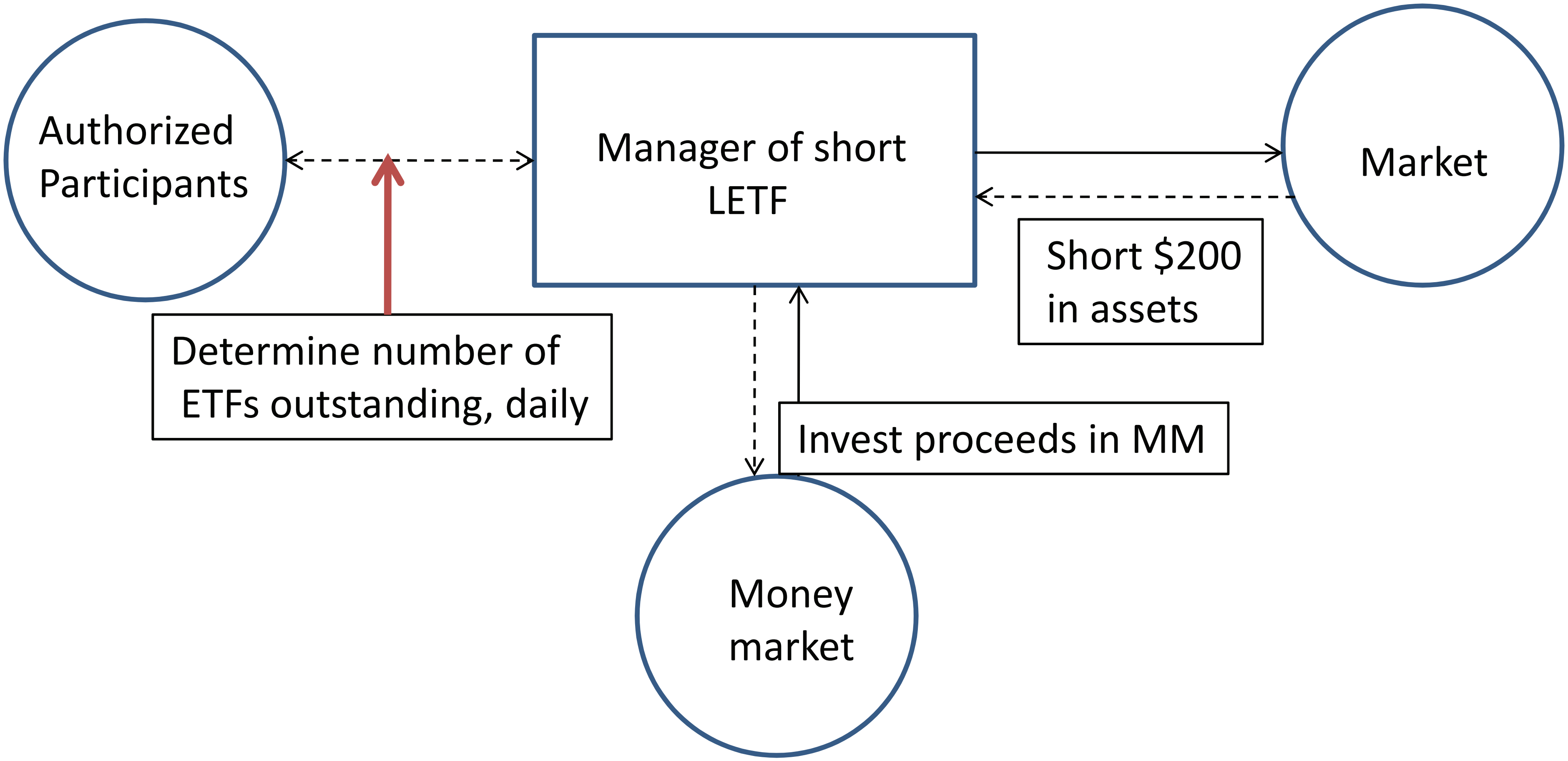
Proshares Ultra Financials ETF (UYG)
Offers a daily exposure to 2 times the Dow Jones Financial Index
(long 200% of underlying undex)

Proshares UltraShort Financials ETF (SKF)
Offers a daily exposure to -2 times the Dow Jones Financial Index
(short 200% of underlying index)

“Bullish” leveraged ETF

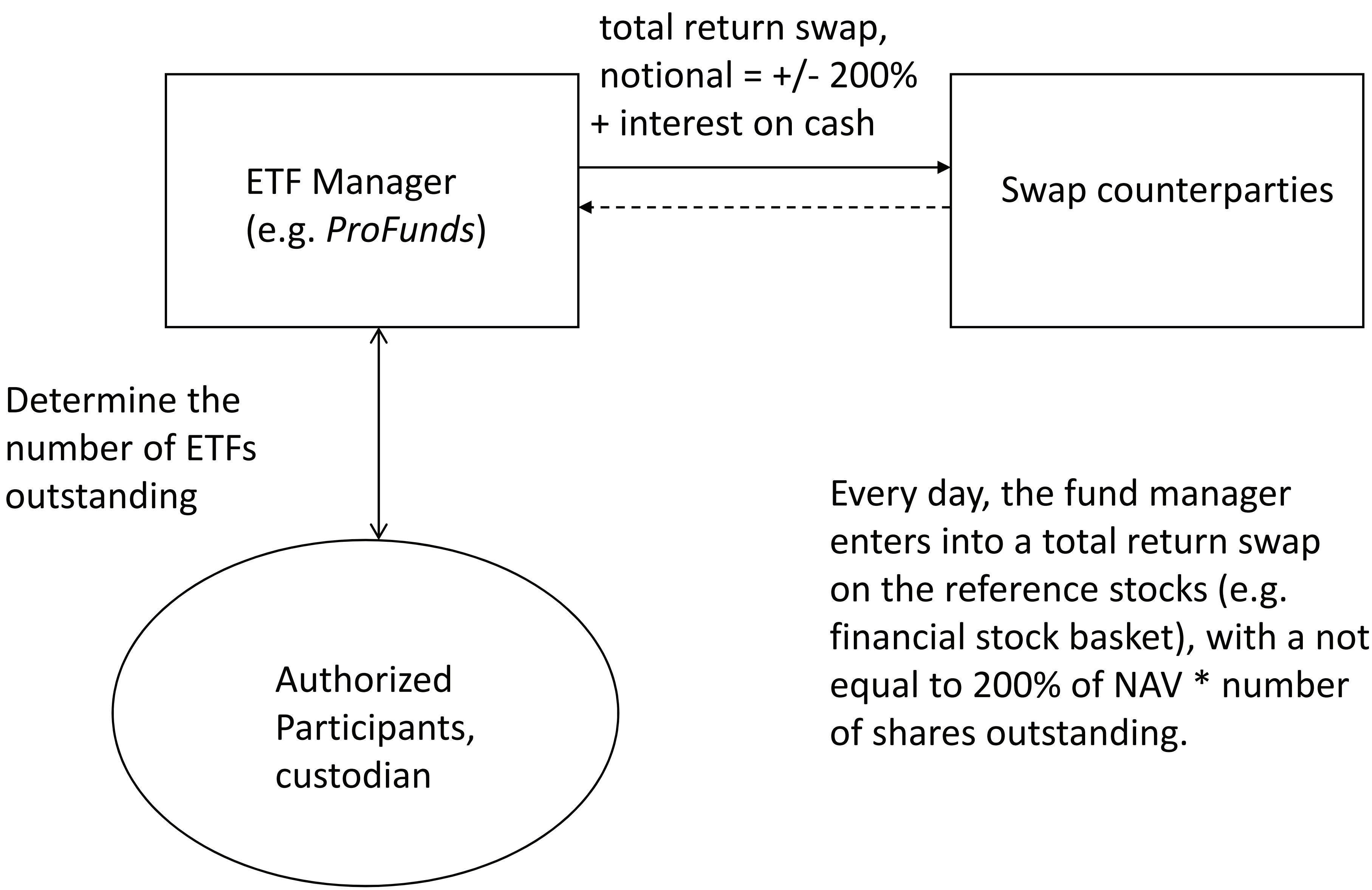


“Bearish” leveraged ETF



In reality this can be more complicated (use of swap counterparties).

Hedging with a Total Return Swap



Every day, the fund manager enters into a total return swap on the reference stocks (e.g. financial stock basket), with a notional equal to 200% of NAV * number of shares outstanding.

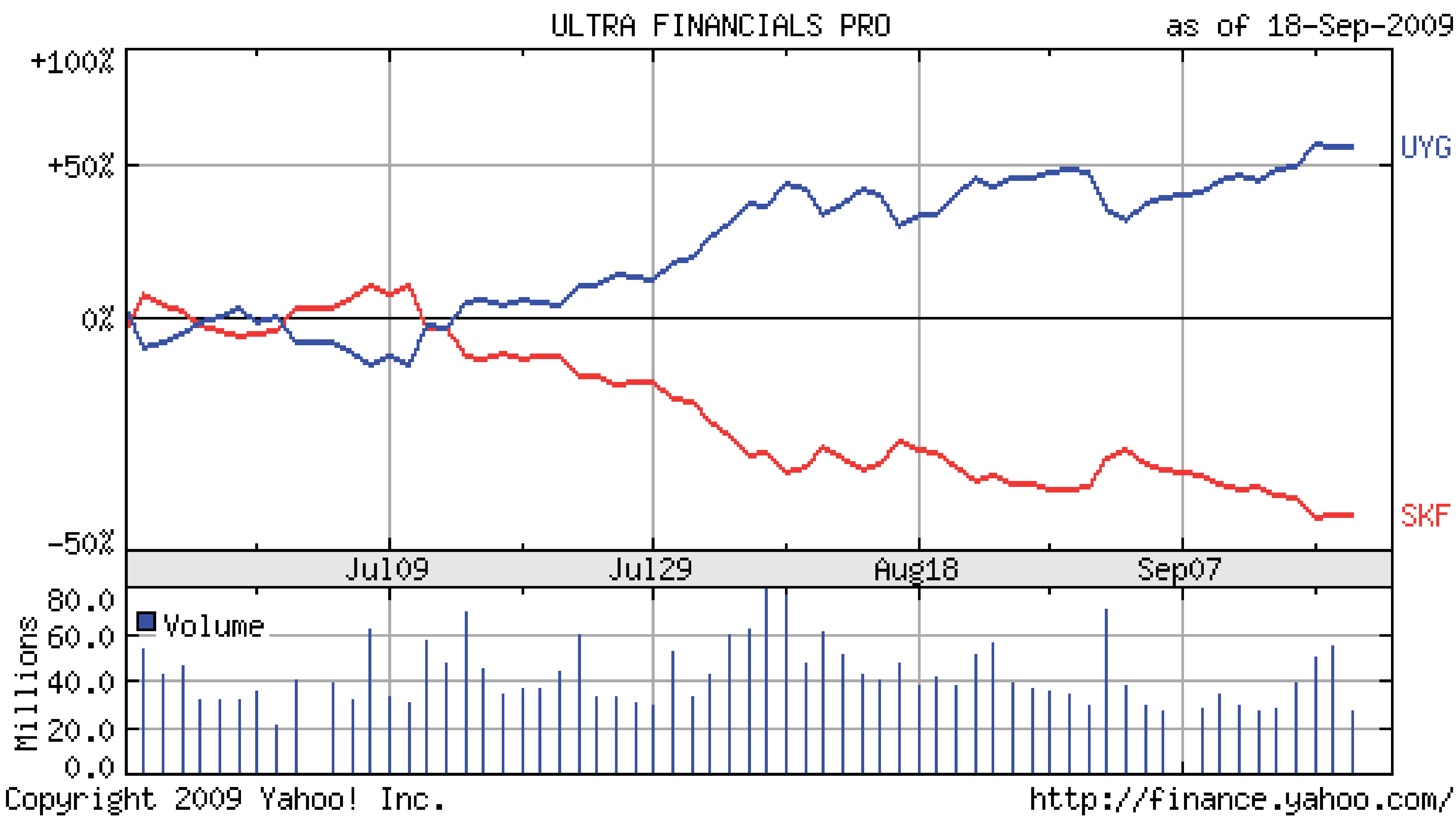
LETFs & buy-and hold investors: *Caveat Emptor*

- Issues have been raised in the marketplace pertaining to the suitability of leveraged ETFs for long-term investors seeking to replicate a multiple of an index performance

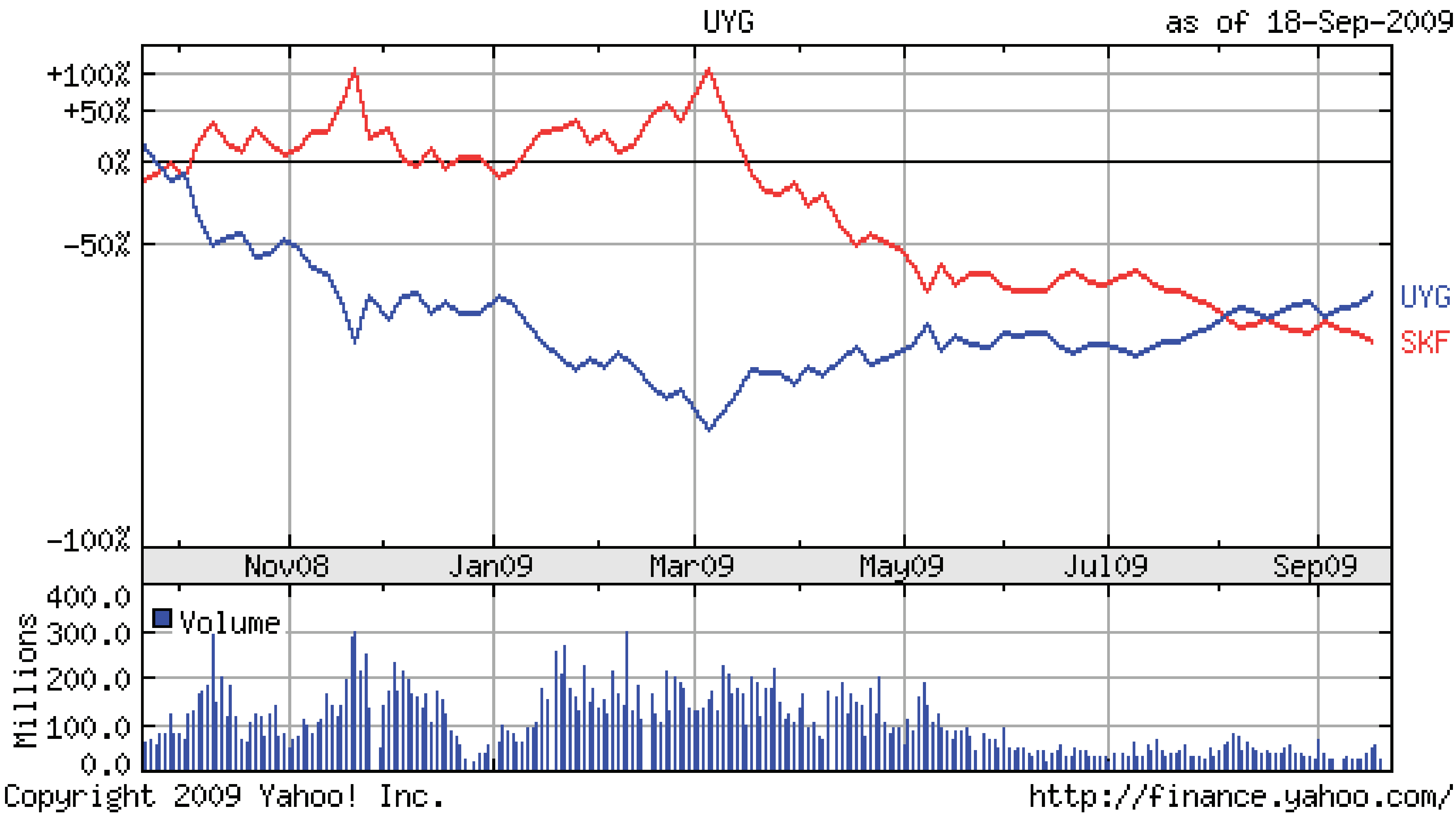
- ``UBS AG U.S. brokerage business stopped selling ETFs that use leverage because such products do not conform to its emphasis on long-term investing'' *Bloomberg News, July 27, 2009*

- `` Due to the effects of compounding, their performance over longer periods of time can differ significantly from their stated daily objective. Therefore, inverse and leveraged ETFs that are reset daily typically are unsuitable for retail investors who plan to hold them longer than one trading session, particularly in volatile markets'' *FINRA Regulatory Notice, June 31, 2009*

SKF/UYG Past 3 months

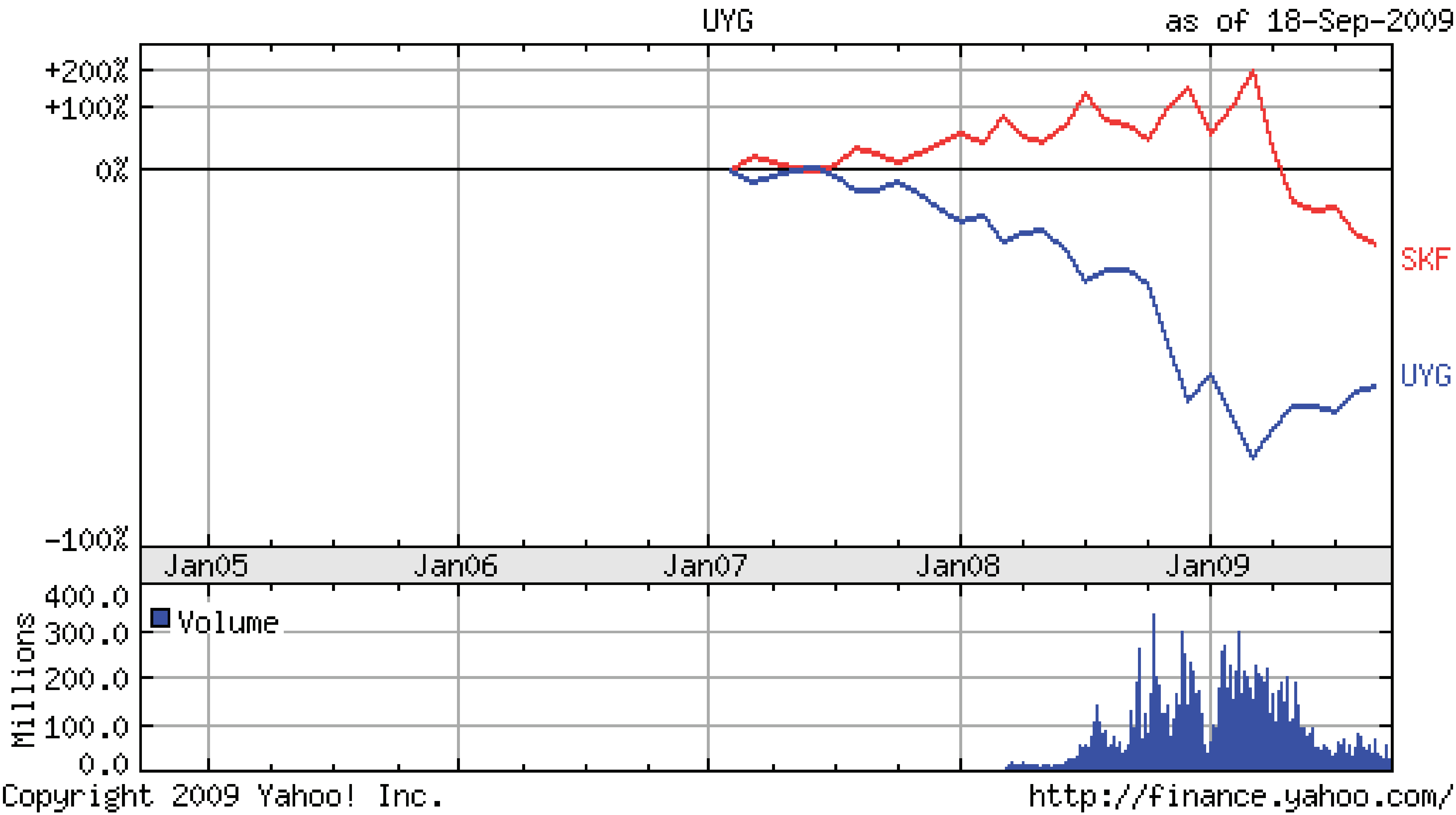


Past year

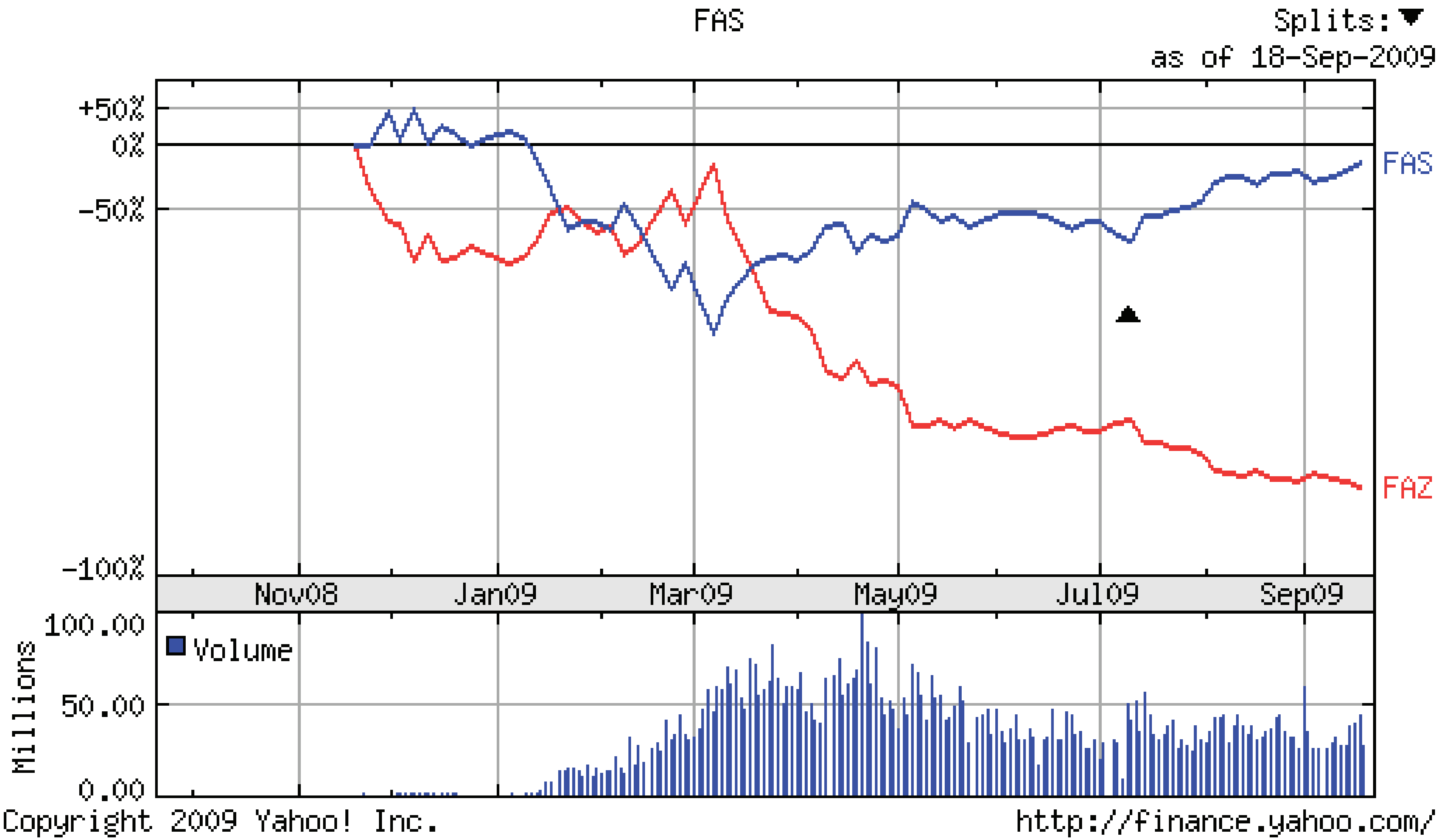


Notice that both returns are negative (big) over 1 year

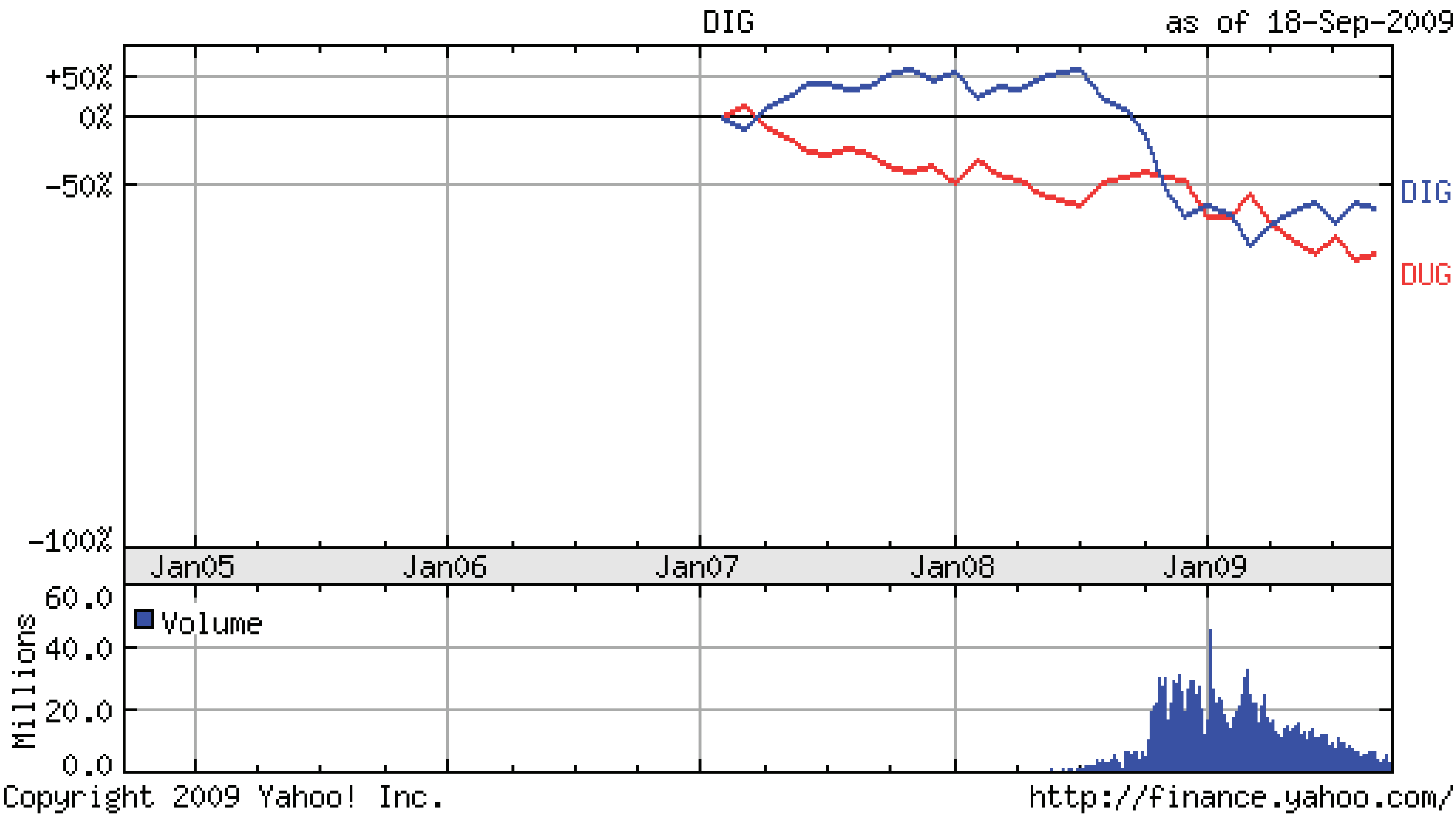
Since inception



Another example: FAS/FAZ
Direxion 3X and -3X Financial ETF

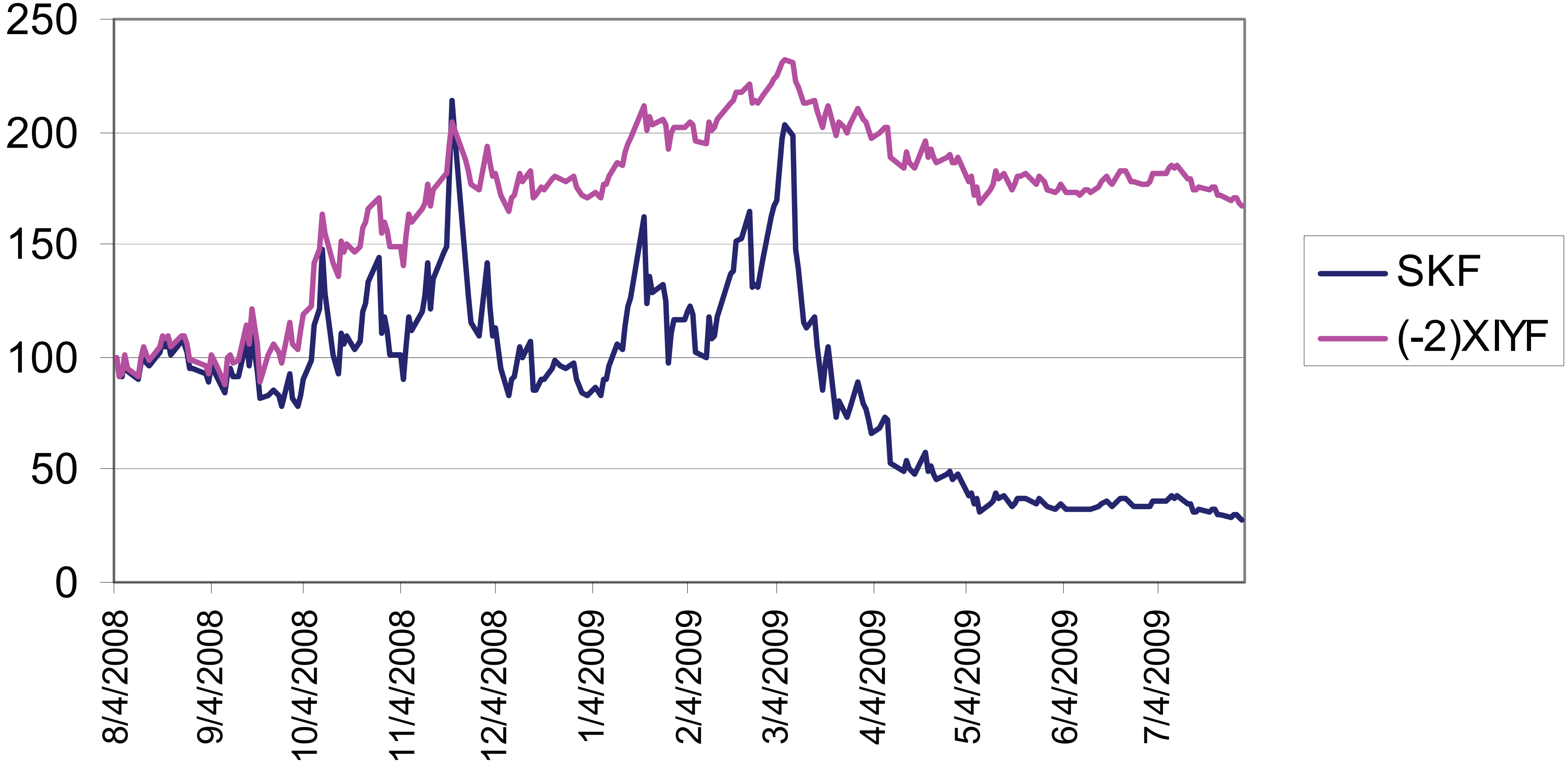


Oil & Gas Proshares DIG (long) DUG (short)



UYG vs. 2X IYF, 1 year

SKF vs. -2X IYF



250
200
150
100
50
0
8/4/2008
9/4/2008
10/4/2008
11/4/2008
12/4/2008
1/4/2009
2/4/2009
3/4/2009
4/4/2009
5/4/2009
6/4/2009
7/4/2009
SKF
(-2)XIYF

LETFs: The discrete model

$R_{S,n}$ = return of the underlying index over the nth period

$R_{L,n}$ = return of the leveraged ETF over the nth period

S_t = price of the underlying index or ETF

L_t = price of the leveraged ETF

f = expense ratio for leveraged ETF

$$R_{L,n} = \beta R_{S,n} + (1-\beta) r\Delta t - f\Delta t$$

$$\begin{aligned} L_t &= \prod_{n=1}^{N}\left(1 + R_{L,n}\right) \\ &= \prod_{n=1}^{N}\left(1 + \beta R_{S,n} + (1-\beta) r\Delta t - f\Delta t\right) \end{aligned}$$

$$\ln\left(1+\beta R_{S,n}+(1-\beta)r\Delta t-f\Delta t\right)\approx \beta R_{S,n}+(1-\beta)r\Delta t-f\Delta t-\frac{1}{2}\beta^2 R_{S,n}^2$$

$$\beta\ln\left(1+R_{S,n}\right)\approx \beta R_{S,n}-\frac{1}{2}\beta R_{S,n}^2 \qquad\qquad \therefore$$

$$\ln\left(1+\beta R_{S,n}+(1-\beta)r\Delta t-f\Delta t\right)\approx \beta\ln\left(1+R_{S,n}\right)+(1-\beta)r\Delta t-f\Delta t-\frac{1}{2}\left(\beta^2-\beta\right)R_{S,n}^2$$

$$\frac{L_t}{L_0}\approx\left(\frac{S_t}{S_0}\right)^{\beta}\exp\left[(1-\beta)rt-ft-\frac{1}{2}\left(\beta^2-\beta\right)\sum_{n=1}^{N}R_{S,n}^2\right]$$

$$\boxed{\frac{L_t}{L_0}\approx\left(\frac{S_t}{S_0}\right)^{\beta}\exp\left[(1-\beta)rt-ft-\frac{1}{2}\left(\beta^2-\beta\right)\int_0^t\sigma_s^2\,ds\right]}$$

Continuous-time model

$$\frac{dS_t}{S_t} = \sigma_t dZ_t + \mu_t dt$$

$$\frac{dL_t}{L_t} = \beta \frac{dS_t}{S_t} + (1-\beta) r dt - f dt$$

$$d\ln L_t = \frac{dL_t}{L_t} - \frac{1}{2}\left(\frac{dL_t}{L_t}\right)^2 = \frac{dL_t}{L_t} - \frac{1}{2}\beta^2 \sigma_t^2 dt$$

$$d\ln S_t = \frac{dS_t}{S_t} - \frac{1}{2}\left(\frac{dS_t}{S_t}\right)^2 = \frac{dS_t}{S_t} - \frac{1}{2}\sigma_t^2 dt$$

$$d\ln L_t - \beta d\ln S_t = (1-\beta) r dt - f dt - \frac{1}{2}\left(\beta^2 - \beta\right)\sigma_t^2 dt$$

$$\boxed{\frac{L_t}{L_0} = \left(\frac{S_t}{S_0}\right)^{\beta} \exp\left[(1-\beta) r t - f t - \frac{1}{2}\left(\beta^2 - \beta\right)\int_0^t \sigma_s^2 ds\right]}$$

Path-dependence of LETF returns has to do with the volatility

$$\frac{L_t}{L_0} = \left(\frac{S_t}{S_0}\right)^{\beta} \exp\left[(1-\beta)rt - ft - \frac{1}{2}\left(\beta^2 - \beta\right)\int_0^t \sigma_s^2 ds\right]$$

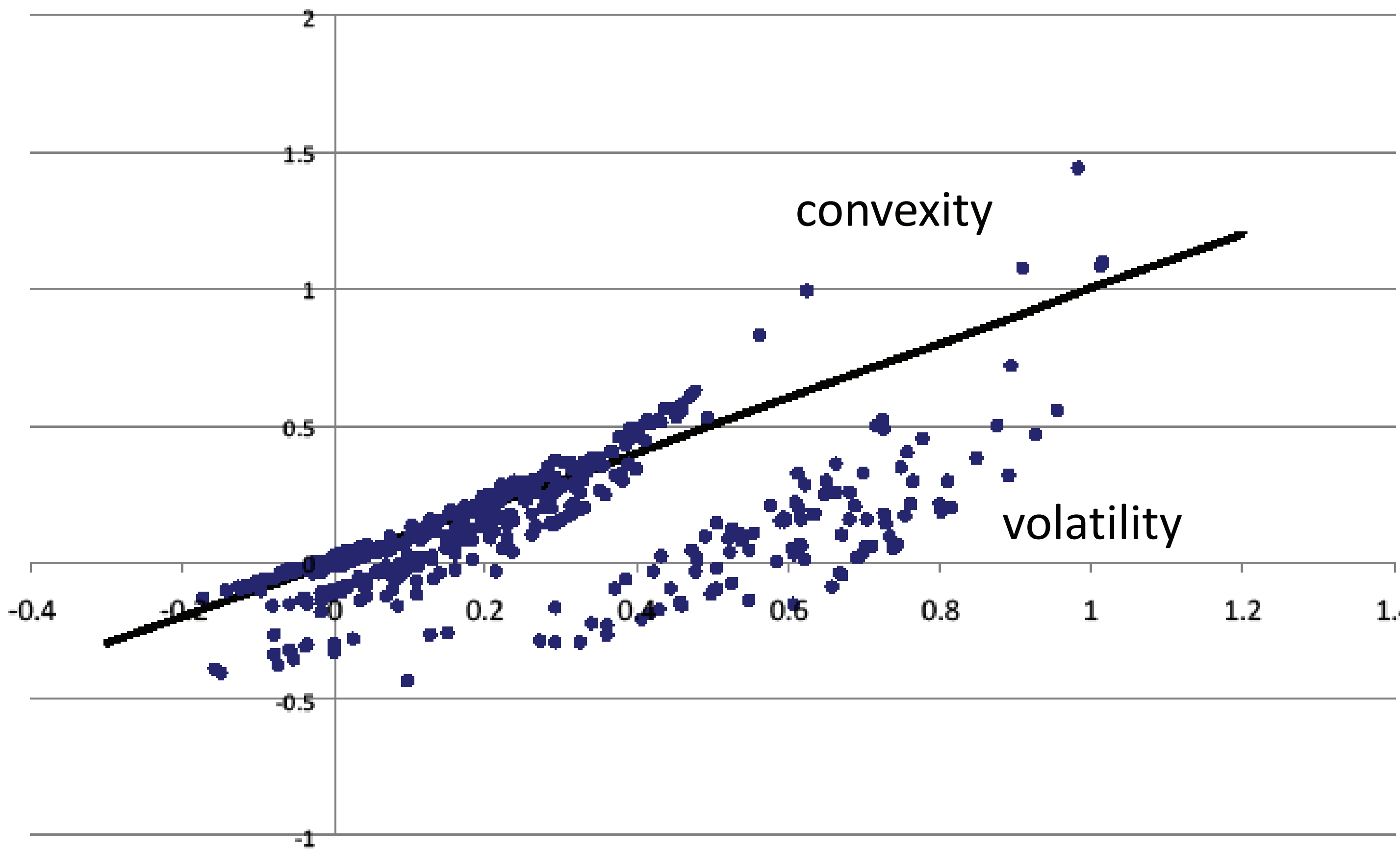
convexity due to daily compounding

financing & fees

volatility

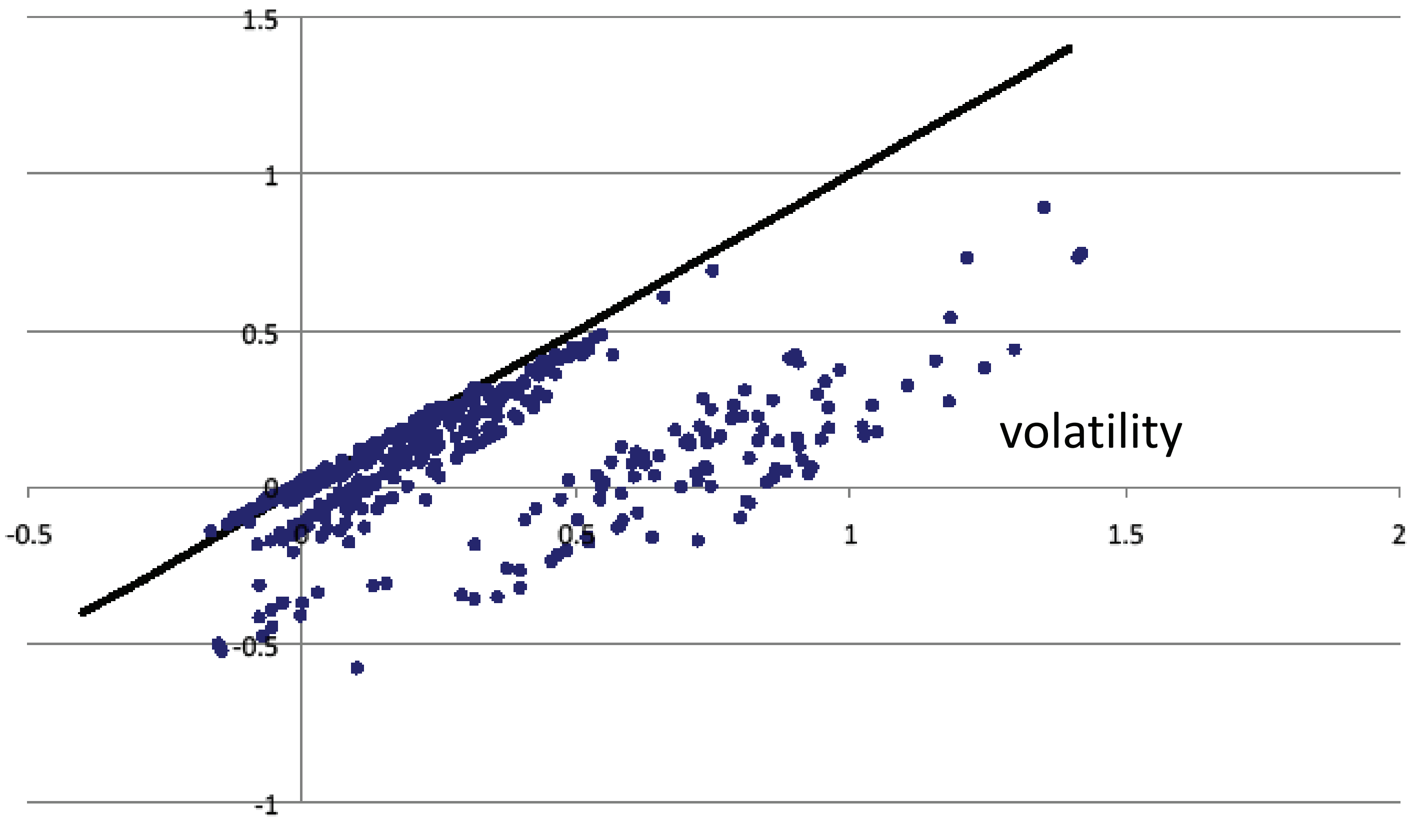
In practice, we will estimate the stochastic volatility as the 10-day standard deviation of the underlying ETF returns.

SKF vs. -2*IYF 3-month returns



Feb 2, 2008 to Mar 2, 2009, overlapping 3M returns

Ln SKF vs. -2*Ln IYF, 3 month returns



Feb 2, 2008 to Mar 2, 2009, overlapping 3M returns

Double Leveraged ETFs analyzed in the study

Underlying ETF	Proshares Ultra	Proshares Ultra Short	Index/Sector
QQQQ	QLD	QID	Nasdaq 100
DIA	DDM	DXD	Dow 30
SPY	SSO	SDS	S&P500 Index
IJH	MVV	MZZ	S&P MidCap 400
IJR	SAA	SDD	S&P Small Cap 600
IWM	UWM	TWM	Russell 2000
IWD	UVG	SJF	Russell 1000
IWF	UKF	SFK	Russell 1000 Growth
IWS	UVU	SJL	Russell MidCap Value
IWP	UKW	SDK	Russell MidCap Growth
IWN	UVT	SJH	Russell 2000 Value
IWO	UKK	SKK	Russell 2000 Growth
IYM	UYM	SMN	Basic Materials
IYK	UGE	SZK	Consumer Goods
IYC	UCC	SCC	Consumer Services
IYF	UYG	SKF	Financials
IYH	RXL	RXD	Health Care
IYJ	UXI	SIJ	Industrials
IYE	DIG	DUG	Oil & Gas
IYR	URE	SRS	Real Estate
IYW	ROM	REW	Technology
IDU	UPW	SDP	Utilities

Triple Leveraged ETFs used in this study

Underlying ETF	Direxion 3X Bull	Direxion 3X Bear	Index/Sector
IWB	BGU	BGZ	Russell 1000
IWM	TNA	TZA	Russell 2000
RIFIN	FAS	FAZ	Russell 1000 Financial Services
RGS	ERX	ERY	Russell 1000 Energy
EFA	DZK	DPK	MSCI EAFE Index
EEM	EDC	EDZ	MSCI Emerging Markets Index

Testing the Model: 2X ETFs, 2/2008 to 3/2009

Underlying ETF	Tracking_error(average, %)	standard_deviation (%)	Ultralong Ticker
QQQQ	0.04	0.47	QLD
DIA	0.00	0.78	DDM
SPY	-0.06	0.40	SSO
IJH	-0.06	0.38	MVV
IJR	1.26	0.71	SAA
IWM	1.26	0.88	UWM
IWD	1.00	0.98	UVG
IWF	0.50	0.59	UKF
IWS	-0.33	1.20	UVU
IWP	-0.02	0.61	UKW
IWN	2.15	1.29	UVT
IWO	0.50	0.74	UKK
IYM	1.44	1.21	UYM
IYK	1.20	0.75	UGE
IYC	1.56	1.04	UCC
IYF	-0.22	0.74	UYG
IYH	0.40	0.42	RXL
IYJ	1.05	0.74	UXI
IYE	-0.73	1.71	DIG
IYR	1.64	1.86	URE
IYW	0.51	0.55	ROM
IDU	0.25	0.55	UPW

tracking error = average of target return - model return,
where target return = L_t/L_0 and model return is according to our formula

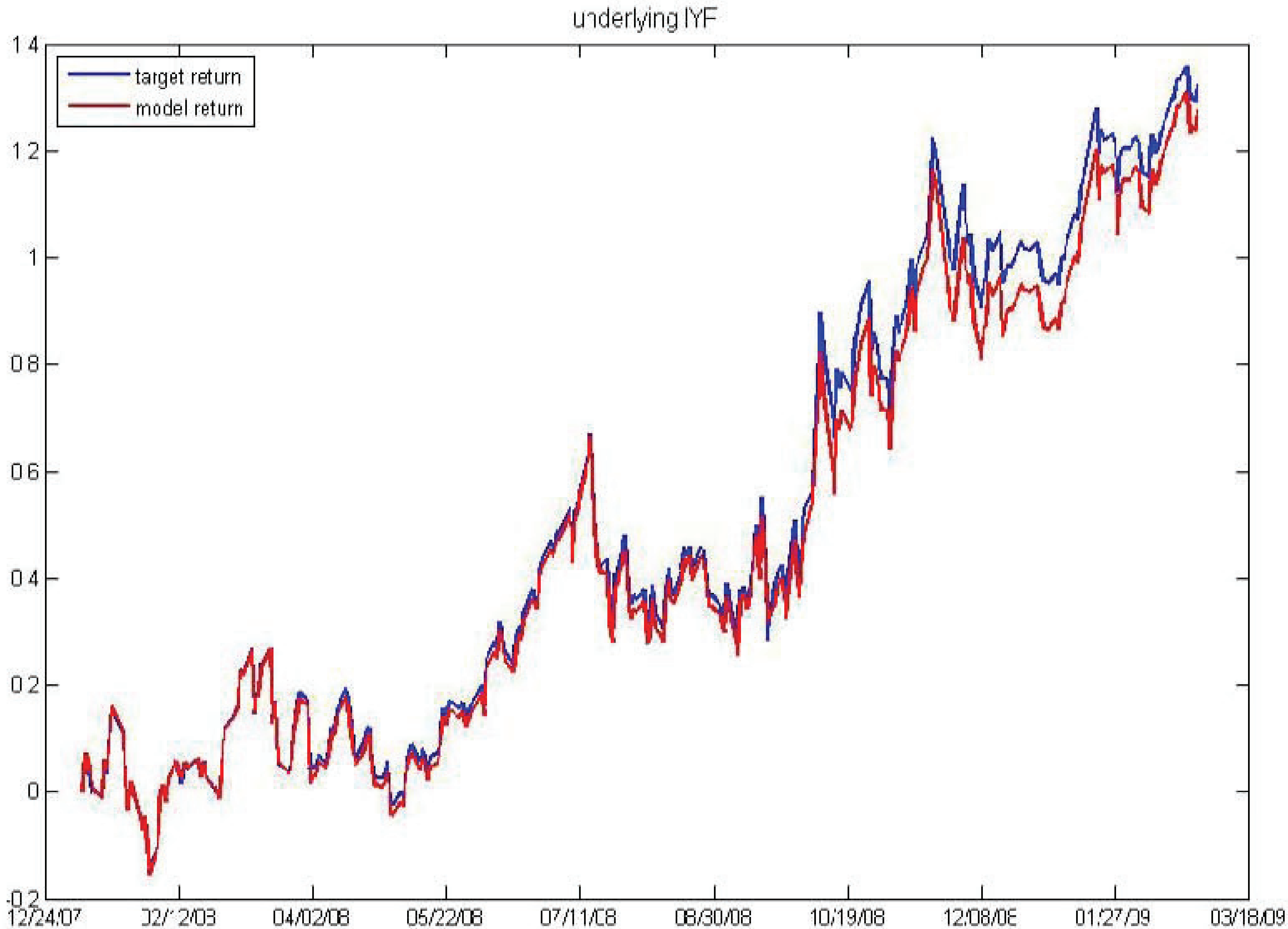
-2X Leveraged ETFs

Underlying ETF	Tracking_error(average)	standard_deviation	ultra short tickers
QQQQ	-0.76%	1.66%	QID
DIA	2.73%	3.12%	DXD
SPY	2.21%	2.74%	SDS
IJH	-1.20%	1.61%	MZZ
IJR	0.81%	1.63%	SDD
IWM	-1.17%	1.30%	TWM
IWD	-0.08%	3.14%	SJF
IWF	1.40%	3.40%	SFK
IWS	-2.06%	3.03%	SJL
IWP	-1.31%	2.36%	SDK
IWN	1.07%	1.91%	SJH
IWO	0.14%	1.47%	SKK
IYM	-2.54%	2.20%	SMN
IYK	1.92%	3.90%	SZK
IYC	-1.44%	1.85%	SCC
IYF	-7.96%	9.42%	SKF
IYH	-1.61%	1.67%	RXD
IYJ	-0.18%	1.46%	SIJ
IYE	0.43%	3.09%	DUG
IYR	-2.04%	3.30%	SRS
IYW	0.32%	1.67%	REW
IDU	-3.17%	2.86%	SDP

Triple Leveraged ETFs, since inception (Nov 2008-Mar 2009)

Underlying	Tracking_error(average)	standard_deviation	3X bull ticker
IWB	0.44%	0.55%	BGU
IWM	0.81%	0.75%	TNA
RIFIN	3.67%	2.08%	FAS
RGS	2.57%	0.70%	ERX
EFA	1.26%	2.32%	DZK
EEM	1.41%	1.21%	EDC

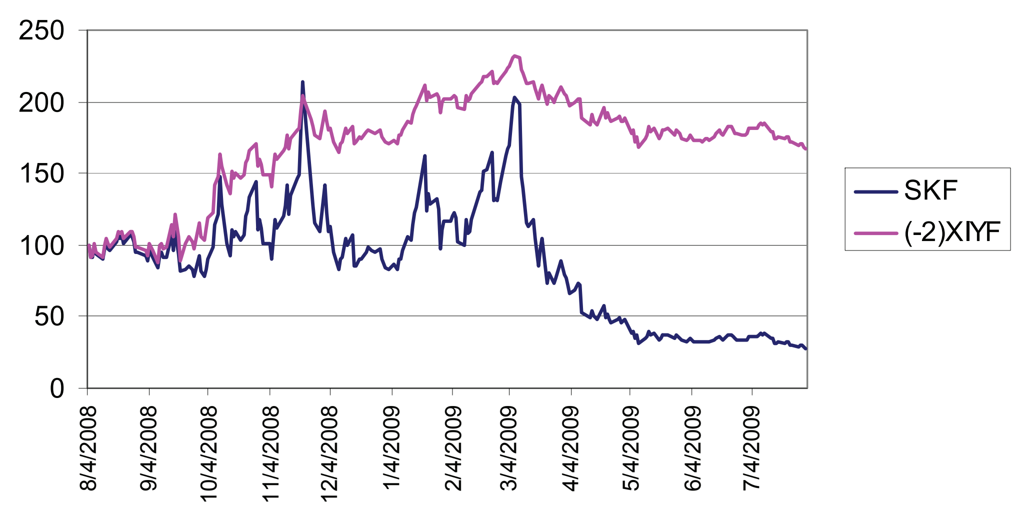
Underlying	Tracking_error(average)	standard_deviation	3X bear ticker
IWB	-0.08%	0.64%	BGZ
IWM	0.65%	0.76%	TZA
RIFIN	-1.63%	4.04%	FAZ
RGS	-1.41%	1.01%	ERY
EFA	-1.54%	1.86%	DPK
EEM	0.49%	1.43%	EDZ



Tracking SKF since December 2007 using the actual prices and the formula

Using LETFs to target long-term leveraged returns

Problem: how can investors replicate 2X the returns of an ETF over a period > 1 day (6m, 1y) with LETFs *without* undergoing tracking error?



Treat the target return as the payoff of a ``derivative'' written on the LETF

$$\text{Final payoff} = \frac{S_T}{S_0} - 1$$

$$\forall t, \quad L_t = L_t \left(\frac{S_t}{S_0} \right)^{\beta} e^{-A(0,t)} \qquad A(s,t) = (\beta - 1) r (t-s) + f(t-s) + \frac{\beta^2 - \beta}{2} \int_s^t \sigma_u^2 du$$

$$S_t = S_0 \left(\frac{L_t}{L_0} \right)^{1/\beta} e^{A(0,t)/\beta}$$

$$V(L_t, A_t, t) = e^{-r(T-t)} E\left\{ \frac{S_T}{S_0} - 1 \middle| L_t, A_t \right\}$$

``Fair value'' of a derivative that delivers the target payoff in S

Compute Fair Value and Delta

$$V(L,A,t) = e^{-r(T-t)} E\left\{ \left(\frac{L_T}{L_0}\right)^{1/\beta} e^{A(0.,T)/\beta} \text{-1} \quad | L_t = L, A(0,t) = A \right\}$$

$$= \left(\frac{L}{L_0}\right)^{1/\beta} e^{A/\beta} - e^{-r(T-t)}$$

$$\therefore$$

$$\frac{\partial V}{\partial L}(L,A,t) = \frac{1}{\beta}\left(\frac{L}{L_0}\right)^{1/\beta - 1} e^{A/\beta}\left(\frac{1}{L_0}\right) \qquad \therefore \qquad L\frac{\partial V}{\partial L} = \frac{1}{\beta}\left(\frac{L}{L_0}\right)^{1/\beta} e^{A/\beta}$$

$$\boxed{L\frac{\partial V}{\partial L} = \frac{1}{\beta}\frac{S}{S_0}}$$

The hedge ratio is proportional to the value of the underling index

Verification

Consider a dynamic portfolio that holds $\Delta_t = \Pi_0 \frac{1}{\beta}\frac{S_t}{S_0}$ dollars of LETF at time t, $0 < t < T$.

$$d\Pi_t = r\Pi_t dt + \Delta_t \frac{dL_t}{L_t} - \Delta_t r dt$$

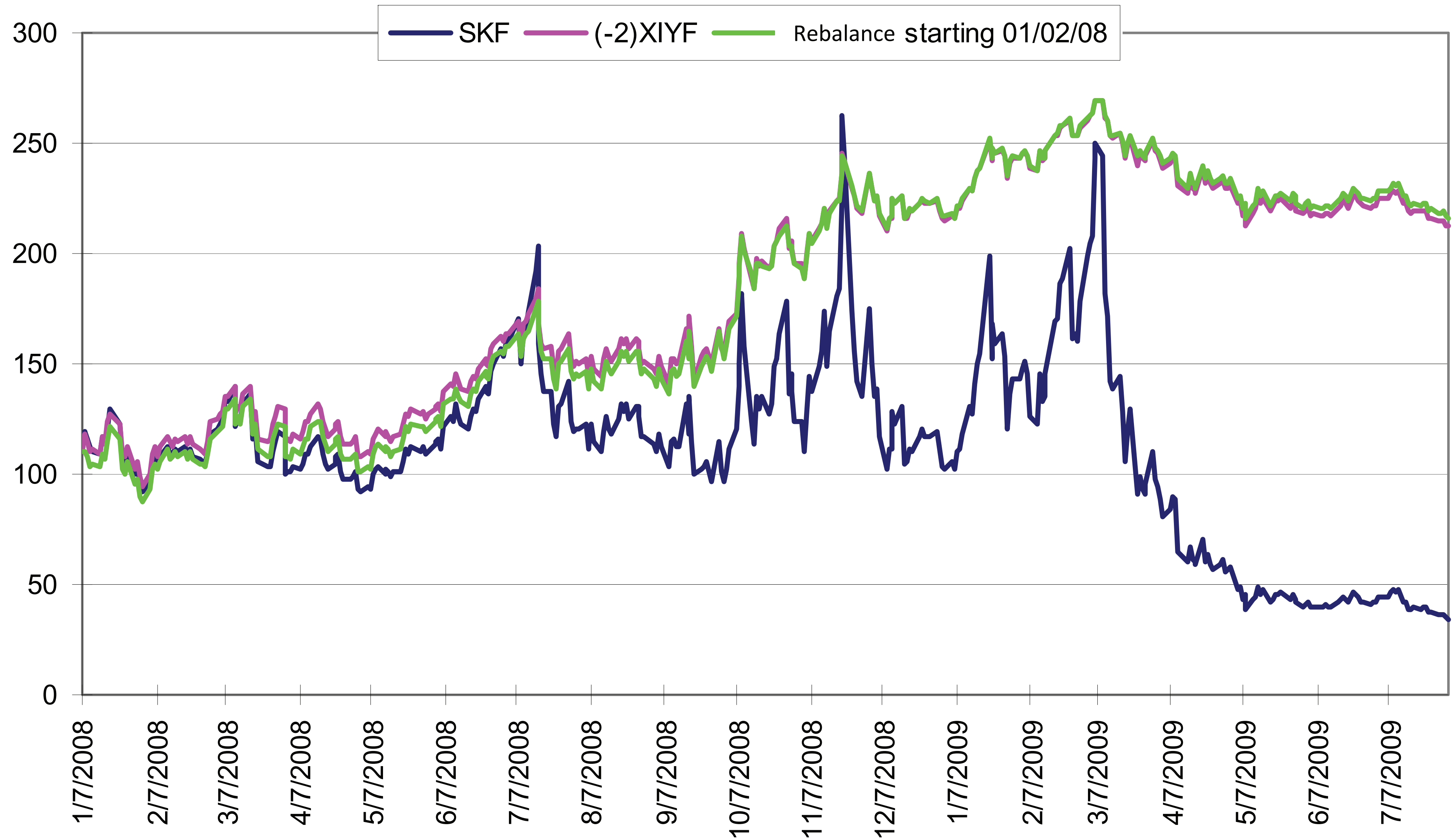
$$= \Delta_t \left(\beta \frac{dS_t}{S_t} + (1-\beta) r dt \right) - \Delta_t r dt$$

$$= r\Pi_t dt + \Pi_0 \frac{1}{\beta}\frac{S_t}{S_0}\left(\beta \frac{dS_t}{S_t} + (1-\beta) r dt \right) - \Delta_t r dt$$

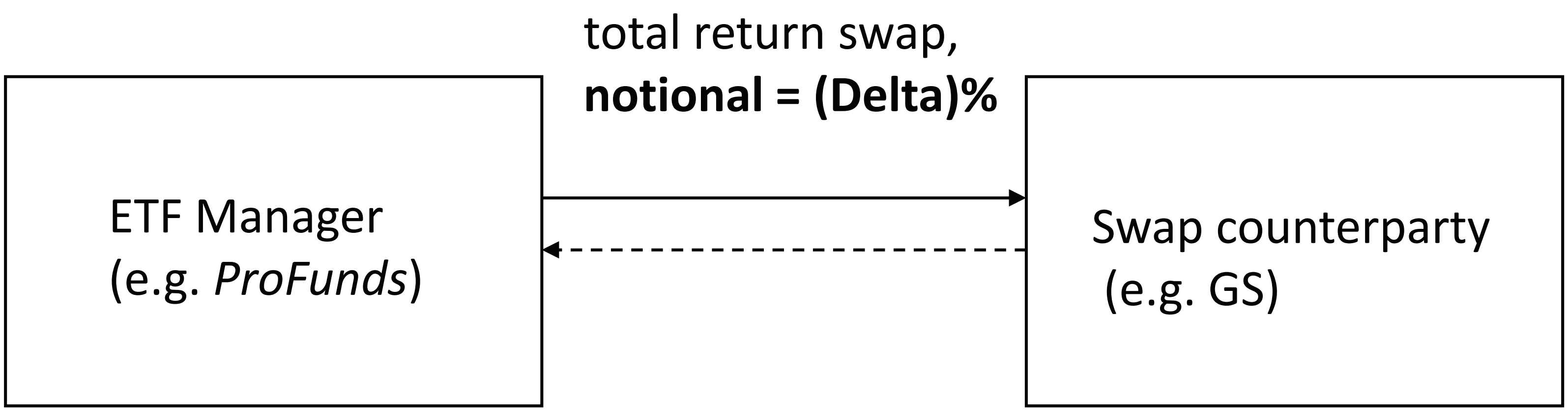
$$= r\Pi_t dt + \Pi_0 \frac{dS_t}{S_0} - \Pi_0 \frac{S_t}{S_0} r dt + \Pi_0 \frac{1}{\beta}\frac{S_t}{S_0} r dt - \Pi_0 \frac{1}{\beta}\frac{S_t}{S_0} r dt$$

$$d\Pi_t = r\Pi_t dt + \Pi_0 \left(\frac{dS_t}{S_0} - \frac{S_t}{S_0} r dt \right) \quad \therefore \quad \frac{\Pi_T e^{-rT} - \Pi_0}{\Pi_0} = \frac{S_T e^{-rT} - S_0}{S_0} \quad \therefore \quad \frac{\Pi_T}{\Pi_0} = \frac{S_T}{S_0}$$

Dynamically hedged SKF investment strategy



Modified Leveraged ETF: A long-term replication of leveraged returns



Delta depends on market levels, volatility, and dividend yield of the index $\Delta_t = \Pi_0 \frac{m}{\beta} \frac{S_t}{S_0}$

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