VIX Exchange Traded Products: Price Discovery, Hedging and Trading Strategy

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Christoffer Bordonado, Peter Molnár and Sven R. Samdal* March 22, 2016

Abstract

This paper investigates the most traded VIX exchange traded products (ETPs) with focus on their performance, price discovery, hedging ability and trading strategy. The VIX ETPs track their benchmark indices well. They are therefore exposed to the same time-decay (high negative expected returns) as these indices. This makes them unsuitable for buy-and-hold investments, but it gives rise to a highly profitable trading strategy. Despite being negatively correlated with the S&P 500, the ETPs perform poorly as a hedging tool; their inclusion in a portfolio based on S&P 500 will decrease the risk-adjusted performance of the portfolio.

key words: VIX, exchange traded products, volatility, trading strategy

^{*}All authors are at Norwegian University of Science and Technology, Trondheim, Norway. Please send correspondence to Peter Molnár: peto.molnar@gmail.com. We would like to thank Carol Alexander for the inspiration to investigate this topic.

1. Introduction

Financial research has made great strides in understanding risk and volatility. In recent years, the trading of volatility has also become widespread. A variety of volatility-related exchange traded products (ETPs) listed on several exchanges around the world exist to facilitate volatility trading. For instance, one can trade volatility derivatives on the S&P 500, the Euro Stoxx 50, Nikkei, emerging markets, oil, gold and many other. These volatility ETPs allow investors to trade volatility without using options or futures, and therefore provide them access to seemingly attractive hedging and diversification opportunities. However, many of these products are structured in such a way that their long term expected value is zero. Despite this fact, the popularity and trading volume in these products continue to rise.

Whaley (2013) explains how and why many VIX ETPs are virtually certain to lose money through time due to a "contango trap" in which VIX futures prices fall to the level of the VIX index. Deng et al. (2012) find that VIX ETPs are generally not effective hedges for stock portfolios because of a negative roll yield. However, they propose that medium term ETPs appear to both reduce the volatility and increase the return of stock portfolios. Furthermore, Alexander and Korovilas (2012b) point out that individual positions in VIX futures ETNs, including midterm and longer-term trackers, offer no opportunities for diversification of equity exposure, except during the onset of a major crisis. However, they indicate that certain portfolios of VIX futures, or their ETNs, referred to as 'roll-yield arbitrage' portfolios, can offer unique risk and return characteristics and diversification opportunities. Simon and Campasano (2014), demonstrate that selling (buying) VIX futures contracts when the basis is in contango (backwardation) and hedging market exposure with short (long) S&P futures positions is highly profitable and robust to both conservative assumptions about transaction costs and the use of out of sample forecasts to set up hedge ratios.

Eraker and Wu (2013) propose an equilibrium model to explain the negative return premium for both VIX ETNs and futures. In this model, increases in volatility endogenously lead to decreasing stock prices. The negative return premium is an equilibrium outcome because long VIX futures positions hedge against low returns and high volatility states (i.e, financial crisis).

This paper provides a comprehensive analysis of VIX ETPs with focus on the performance, hedging, price discovery and trading strategy. The finding that the hedging ability of VIX ETPs is rather poor is in line with Alexander and Korovilas (2012b) and Deng et al. (2012).

To our best knowledge, this is the first study of price discovery between different VIX ETPs. Since there exist different categories of VIX ETPs, we study price discovery between the two most traded ETPs within three categories: direct, levered and inverse. We find that more traded and older ETP leads the price discovery process in case of direct and levered ETPs, but less traded and younger product is the information leader in case of inverse ETPs.

Finally, we propose a trading strategy based on buying VIX direct and inverse ETPs and hedging the position with S&P 500 ETF. This part of our work is related to Alexander and Korovilas (2012b) and Simon and Campasano (2014). However, even though Alexander and Korovilas (2012b) study VIX ETPs, the trading strategy they study is different than ours. Our strategy is most related to Simon and Campasano (2014), but they use VIX futures as traded instruments. Our result is different then theirs, we find that an unhedged version of this strategy is the most profitable.

This paper is organized as follows: Section 2 explains the VIX ETPS and presents the data. In Section 3 the price discovery relationship between different pairs of ETPs is studied. Section 4 investigates the hedging ability of the VIX ETPs when included in an equity portfolio tracking the S&P 500. In Section 5 a trading strategy using direct and inverse VIX ETPs is proposed. Section 6 concludes.

2. Data

The S&P 500 index is often considered as a representation of the U.S. stock market, because it covers more than 80% of the US stock market. The VIX index measures the implied volatility of S&P 500 index options. The VIX is often considered not only as an expectation of market volatility, but also as an indication of investor sentiment. The time developement of the VIX and the S&P 500 is plotted in Figure 1. The spikes in the VIX quite clearly correspond to price drawdowns in the S&P 500. From 1990 to 2014, there has been a correlation between the daily returns of the VIX and the S&P 500 of approximately -0.71. The overall tendency of volatility to increase when prices fall is evident in Figure 1.

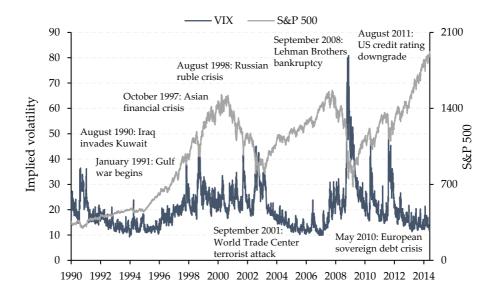


Figure 1: VIX, S&P 500 and geopolitical events from 1990 to 2014.

While the VIX is not a traded entity itself, there is a market in VIX futures and options. Trading of VIX futures began in March 2004 and VIX options in February 2006. VIX exchange traded products (ETPs), the second generation of volatility products, were introduced in 2009. ETPs are traded on a securities exchanges. Exchange traded products include exchange traded funds (ETFs), exchange traded vehicles (ETVs), exchange traded notes (ETNs) and certificates. The VIX ETPs used in this paper are either ETNs or ETFs. ETFs are transparent and specify exactly what instruments are used to generate the benchmark index return. ETNs, on the other hand, are a type of unsecured, unsubordinated debt security that promise the benchmark return over their stated maturity. Hence, the value of an ETN can be affected by the credit rating of the issuer and not just changes in the underlying index.

It is important to emphasize that the ETPs are not designed to replicate the VIX itself, but rather futures indices on the VIX. In order to replicate the VIX, one would need to continuously rebalance a basket of options in such a way that the maturity remains constant at 30 days. This means continuously selling options with shorter maturity and buying options with longer maturity. Hence, no ETP sponsor is willing to do this because it would be too costly. VIX ETPs are instead designed to track the value of different S&P 500 VIX Futures Indices.²

¹See http://www.cboe.com/Strategies/VIXProducts.aspx

²http://us.spindices.com/index-family/strategy/vix

For example, the S&P 500 VIX Short-Term Futures Index utilizes prices of the next two near-term VIX futures contracts to replicate a position that rolls the nearest month VIX futures to the next month on a daily basis in equal fractional amounts. This results in a constant one-month rolling long position in first and second month VIX futures contracts. The S&P 500 VIX Mid-Term Futures Index measures the return of a daily rolling long position in the fourth, fifth, sixth and seventh month VIX futures contracts. For an in-depth explanation of how VIX ETFs and ETNs work, see for instance Whaley (2013) or Alexander and Korovilas (2012b).

ETPs have gained great popularity in recent years. Deutsche Bank estimates that the value of ETFs worldwide amount to over \$2.3 trillion and have more than doubled since 2011, exceeding the \$2.2 trillion pool of funds invested in private equity. Fueled by the growth of market participants who do not want or cannot trade in derivatives markets, at least 35 VIX ETPs have been introduced in recent years.³ Figure 2 illustrates that trading of VIX ETPs is growing faster than trading of VIX futures and options.

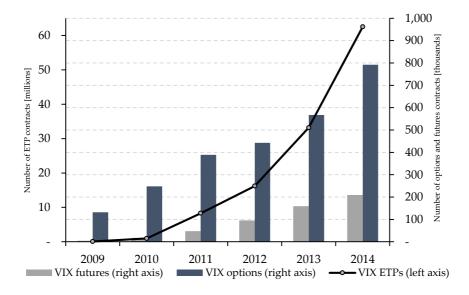


Figure 2: The average number of contracts traded daily in VIX options, futures and ETPs from 2009 to 2014. The number of ETP contracts traded is calculated by summing the average daily volume in seven of the most popular ETPs for the years 2009 to 2014 (see Table 1).

In order to evaluate the different aspects of the VIX ETPs, seven of the most traded ETPs are chosen. Table 1 presents an overview of these.

³http://www.macroption.com/vix-etf-etn-list/

Table 1: An overview of the VIX ETPs used in this paper. The ETPs are sorted by the average number of units traded daily, given in the third column. Assets under management, date of inception and yearly fee are given in the three following columns. ST and MT indicate if the index tracked is based on short-term or mid-term futures. The column denoted TR/ER, indicates whether the ETPs are based on the Excess Return or the Total Return version of the index being tracked. The multiplier denotes the leverage of the ETP, with 1 indicating a direct ETP, -1 indicating an inverse ETP, and 2 indicating leverage with returns twice that of the underlying index. A direct ETP provides long exposure, while an inverse ETP provides short exposure to the underlying. All of these VIX ETPs are traded on NYSE Arca. This information is obtained from http://etfdb.com/etfdb-category/volatility/

| Symbol | Name | Average volume [10 ³] | Asset [mUSD] | Date of inception | Yearly fee [%] | Time horizon | Return type | Multiplier |
|--------|--|-----------------------------------|-----------------|-------------------|-------------------|-----------------|----------------|------------|
| VXX | iPath S&P 500 VIX Short-Term Futures ETN | 20,900 | 919 | 29 Jan 2009 | 0.89 | ST | TR | 1 |
| XIV | VelocityShares Daily Inverse VIX Short-Term ETN | 13,100 | 591 | 29 Nov 2010 | 1.35 | ST | ER | -1 |
| TVIX | VelocityShares Daily 2x VIX Short-Term ETN | 4,600 | 276 | 29 Nov 2010 | 1.65 | ST | ER | 2 |
| UVXY | Proshares Ultra VIX Short-Term Futures ETF | 2,400 | 238 | 04 Oct 2011 | 1.56 | ST | ER | 2 |
| SVXY | Proshares Short VIX Short-Term Futures ETF | 1,500 | 223 | 04 Oct 2011 | 1.32 | ST | ER | -1 |
| VIXY | ProShares VIX Short-Term Futures ETF | 789 | 115 | 03 Jan 2011 | 0.83 | ST | ER | 1 |
| VXZ | iPath S&P 500 VIX Mid-Term Futures ETN | 708 | 77 | 29 Jan 2009 | 0.89 | MT | TR | 1 |

Since the inception of the first VIX ETPs in 2009, the S&P 500 has been trending upward, only interrupted by two periods with high volatility. The first was triggered by the European sovereign debt crisis in 2010 (see Figure 1) and caused the S&P 500 to fall about 15% over a two month period. The second high volatility period followed the downgrading of the US credit rating from AAA to AA+ in August 2011, causing the S&P 500 to fall nearly 11% in six days. What these two periods have in common is that the price fall, although dramatic, was relatively small and short lived. When backtesting different hedging and trading strategies it is desirable to investigate their performance during different market regimes, such as the market crash in 2008. Since the prices of the VIX ETPs are closely linked to the VIX futures indices they are designed to track, it is possible to derive indicative closing values, i.e. *artificial prices*, on the ETPs in the time period before their inception. The prospectus for the VXX explains how the indicative value of the ETN is calculated.⁴

⁴The closing indicative value for a series of ETNs on any calendar day will be calculated in the following manner. The closing indicative value on the inception date will equal \$100. On each subsequent calendar day until maturity or early redemption, the closing indicative value will equal (1) the closing indicative value for that series on the immediately preceding calendar day times (2) the daily index factor for that series on such calendar day (...) The daily index factor for any series of ETNs on any index business day will equal (1) the closing level of the Index for that series

The daily ETP time series has been downloaded from Yahoo Finance. The artificial price data for the pre-inception period used in the analysis has been purchased from sixfigureinvesting.com, and is constructed in the manner described in the prospectus of each VIX ETP. Note however, that the closing market price of an ETP can deviate from its indicative closing price, and bias the analysis. To evaluate the bias, for the overlapping period in the time series, the artificial prices are regressed on the observed market prices. The regression is on the form

$$P_t^{obs} = \alpha + \beta P_t^{art} + e_t, \tag{1}$$

where P_t^{obs} is the observed ETP price at time t, P_t^{art} is the artificial ETP price at time t, and e_t is the residual at time t. The results are reported in Table 2, and it is clear that the artificial prices are very close to the observed market prices. It is therefore concluded that the artificial prices are suitable to expand the time period used to evaluate the trading and hedging strategies.

Figure 3 reports the evolution of the different VIX ETPs and the S&P 500 index. The prices are rebased to 100 at the beginning of the period. The ETPs which track the same underlying VIX futures index have been merged, since their price series are identical in the period leading up to their inception, and more or less identical after inception. For instance, VXX/VIXY in Figure 3 depicts the rebased prices of both VXX and VIXY in one, which track the short term VIX futures index. When the market crashed in 2008, the direct ETPs doubled many times over, while the inverse ETPs were almost wiped out. In addition, the constant rolling of the VIX futures contracts used to construct the ETPs represent a massive drag on the performance of the direct ETPs, causing them to decline rapidly in stable periods. On the other hand, this contributes to the outperformance of the inverse VIX ETPs.

Table 2: Results when artificial VIX ETP prices are regressed on observed VIX ETP prices.

| Ticker | Start date | No. of obs. | R^2 | Adj. R ² | α | t-stat α | β | t-stat β |
|--------|------------|-------------|--------|---------------------|---------|----------|------|----------|
| VXX | 30 Jan 09 | 1256 | 0.9999 | 0.9999 | -0.191 | -0.3956 | 1.00 | 4502 |
| VXZ | 20 Feb 09 | 1242 | 0.9999 | 0.9999 | 0.010 | 0.5490 | 0.99 | 3404 |
| TVIX | 30 Nov 10 | 794 | 0.9991 | 0.9991 | 23.732 | 7.7996 | 0.96 | 932 |
| XIV | 30 Nov 10 | 794 | 0.9995 | 0.9995 | -0.002 | -0.1572 | 0.99 | 1322 |
| UVXY | 04 Oct 11 | 581 | 0.9996 | 0.9996 | -25.376 | -1.8432 | 1.02 | 1224 |
| VIXY | 04 Jan 11 | 770 | 0.9997 | 0.9997 | 0.107 | 0.7614 | 1.00 | 1695 |
| SVXY | 04 Oct 11 | 581 | 0.9995 | 0.9995 | -0.014 | -0.3864 | 0.99 | 1093 |

on such index business day divided by (2) the closing level of the Index for that series on the immediately preceding index business day. (...) The **investor fee** for any series of ETNs is 0.89% per year times the applicable closing indicative value times the applicable daily index factor, calculated on a daily basis in the following manner. On each subsequent calendar day until maturity or early redemption, the investor fee will equal to (1) 0.89% times (2) the closing indicative value for that series on the immediately preceding calendar day times (3) the daily index factor for that series on that day divided by (4) 365.

Tables 3, 4 and 5 report descriptive statistics for three versions of the data used in this paper. Table 3 shows the descriptive statistics for daily returns of the VIX ETPs for the period 21 June 2006 to 25 April 2014. In the time series for this period, returns on the ETPs are calculated based on their artificial prices until their date of inception. From their date of inception, and until the end of the period, the observed closing prices downloaded from Yahoo Finance are used. This data set is used for testing the trading and hedging strategies presented in Sections 5 and 4 respectively. Table 4 reports descriptive statistics for the returns of the observed ETP market prices. Table 5 reports descriptive statistics for the one-minute prices that will be used to examine price discovery in Section 3. The one-minute data set has been downloaded from the WRDS TAQ database covering the period fron the inception of VIX ETP until December 31, 2012. To reduce noise that might occur during opening and closing trading minutes, the observations from the first and last fifteen minutes of each day have been removed from this data set.

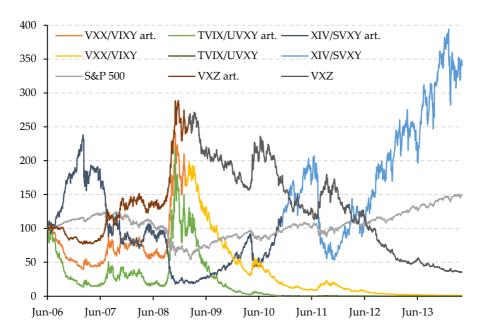


Figure 3: Rebased prices of VIX ETPs and the S&P 500 index. ETPs tracking the same index are more or less identical in price, so their series have been merged. The time before and after inception is shown by dividing the series for each ETP into a series for the artificial (art.) prices, and the observed closing prices.

Table 3: Descriptive statistics for daily VIX and S&P 500 ETP returns from June 21, 2006 to April 25, 2014. The VIX ETP series are based on artificial and observed VIX ETP returns.

| | Mean | Median | Std. Dev. | Variance | Kurtosis | Skewness | Min | Max | No. of obs. |
|------|--------|--------|-----------|----------|----------|----------|--------|-------|-------------|
| VXX | -0.16% | -0.54% | 3.83% | 0.0015 | 3.02 | 0.85 | -14.3% | 24.6% | 1974 |
| VIXY | -0.16% | -0.54% | 3.84% | 0.0015 | 3.08 | 0.87 | -14.1% | 24.6% | 1974 |
| VXZ | -0.03% | -0.18% | 1.99% | 0.0004 | 3.19 | 0.68 | -8.2% | 11.1% | 1974 |
| TVIX | -0.35% | -1.11% | 7.44% | 0.0055 | 3.60 | 0.86 | -29.8% | 49.1% | 1974 |
| UVXY | -0.33% | -1.14% | 7.75% | 0.0060 | 3.29 | 0.87 | -32.2% | 49.1% | 1974 |
| XIV | 0.14% | 0.54% | 3.83% | 0.0015 | 3.05 | -0.88 | -24.5% | 14.1% | 1974 |
| SVXY | 0.14% | 0.56% | 3.88% | 0.0015 | 3.31 | -0.88 | -24.5% | 16.1% | 1974 |
| SPY | 0.04% | 0.07% | 1.40% | 0.0002 | 13.48 | 0.23 | -9.8% | 14.5% | 1974 |
| SH | -0.03% | -0.08% | 1.39% | 0.0002 | 9.42 | 0.10 | -11.0% | 9.4% | 1974 |

Table 4: Descriptive statistics for daily VIX ETP returns from the date of their inception until 25 Apr. 2014.

| | Start date* | Mean | Median | Std. Dev. | Variance | Kurtosis | Skewness | Min | Max | No. of obs. |
|------|-------------|--------|--------|-----------|----------|----------|----------|--------|-------|-------------|
| VXX | 02 Feb 2009 | -0.32% | -0.71% | 3.77% | 0.0014 | 2.92 | 0.81 | -14.3% | 20.7% | 1338 |
| VIXY | 05 Jan 2011 | -0.24% | -0.52% | 4.01% | 0.0016 | 2.88 | 0.80 | -13.1% | 20.7% | 831 |
| VXZ | 23 Feb 2009 | -0.13% | -0.25% | 1.91% | 0.0004 | 2.89 | 0.57 | -8.2% | 10.3% | 1303 |
| TVIX | 01 Dec 2010 | -0.60% | -1.11% | 7.43% | 0.0055 | 4.12 | 0.77 | -29.8% | 40.8% | 855 |
| UVXY | 05 Oct 2011 | -0.83% | -1.11% | 7.74% | 0.0060 | 2.02 | 0.47 | -26.7% | 37.0% | 642 |
| XIV | 01 Dec 2010 | 0.22% | 0.52% | 3.96% | 0.0016 | 2.86 | -0.82 | -20.0% | 13.2% | 855 |
| SVXY | 05 Oct 2011 | 0.35% | 0.55% | 3.88% | 0.0015 | 2.09 | -0.50 | -19.2% | 13.2% | 642 |

^{*}Time series downloaded from finance.yahoo.com. Note that start date might differ from inception date in Table 1 because of unavailable data.

Table 5: Descriptive statistics for the log returns of 1-minute price data used in the price discovery section.

| | Startdate | Mean | Median | Stdev | Variance | Kurtosis | Skewness | Min | Max | Nobs |
|------|-------------|----------|---------|-------|----------|----------|----------|--------|-------|--------|
| VXX | 30 Jan 2009 | -0.0010% | 0.0000% | 0.20% | 0.000004 | 519 | -0.68 | -19.3% | 12.8% | 409646 |
| VIXY | 04 Jan 2011 | -0.0008% | 0.0000% | 0.22% | 0.000005 | 556 | 1.49 | -14.0% | 13.6% | 201768 |
| TVIX | 30 Nov 2010 | -0.0023% | 0.0000% | 0.43% | 0.000018 | 290 | 0.42 | -22.7% | 24.4% | 216636 |
| UVXY | 04 Oct 2011 | -0.0037% | 0.0000% | 0.42% | 0.000017 | 342 | -0.06 | -22.2% | 23.3% | 128756 |
| XIV | 30 Nov 2010 | 0.0002% | 0.0000% | 0.22% | 0.000005 | 341 | -3.86 | -14.5% | 9.7% | 215511 |
| SVXY | 03 Jan 2012 | 0.0008% | 0.0000% | 0.20% | 0.000004 | 228 | -3.33 | -9.3% | 5.5% | 100816 |

Figure 4 shows that the average term structure was steeply upward sloping during 2012. From January 3 to December 31 2012, the return of UVXY and VXX was -96.80% and -76.38%, respectively. In 2008 however, the average term structure was steeply downward sloping. The artificial ETP prices indicate that the return of holding UVXY and VXX from January 3 to December 31 2008 would have been 214% and 121%, respectively. This indicates that it might be profitable to buy direct VIX ETPs when the term structure is in backwardation (downward sloping), and buy inverse VIX ETPs when the term structure is in

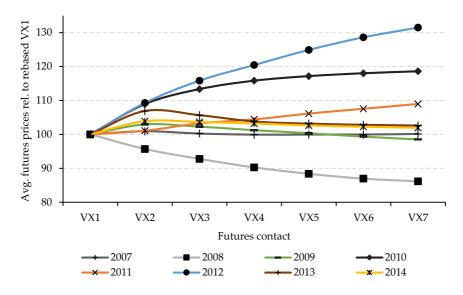


Figure 4: Average VIX futures term structure per year, 2007 - 2014.

contango (upward sloping). We use this insight in a trading strategy presented In Section 5.

3. Price Discovery

We analyze pairs of similar VIX ETPs in order to decide which of the ETPs within each category is the most efficient at absorbing new information. The reason why we study price discovery only withing each ETP category is that ETPs across categories differ significantly⁵ and are therefore not suitable for the price discovery analysis designed for almost identical assets.

The VIX ETPs have been divided into three categories:

direct: VXX and VIXYleveraged: TVIX and UVXYinverse: XIV and SVXY

⁵ Even though intuition suggests that one VIX ETPs in one category can be replicated using another VIX ETP in another category, it is not really the case. First of all, VIX ETPs that have as an underlying short-term VIX futures index are very different from VIX ETP which has as an underlying medium-term VIX futures index. However, even the VIX ETPs with the same underlying (short-term VIX futures index), but different multiplier, are not simple multiples of each other, as can be observed in Table 4 and Table 5. The question why the actual returns of a leveraged exchange-traded fund deviate significantly from the leveraged multiple of the underlying index return is in detail studied and explained in Tang and Xu (2013).

and the price discovery relationship between the two ETPs within each category is analyzed. Notice that the VXZ presented in Section 2 is not among the ETPs mentioned above. Since VXZ is the only ETP in this study which tracks the S&P 500 VIX Mid-Term Futures Index, it is left out of this part of the paper. Although some ETPs are more popular than others, all ETPs considered in this paper are very liquid.

In the price discovery analysis we use 1-minute data. For the descriptive statistics of 1-minute returns see Table 5 in Section 2. The corrections between the prices of different ETPs occur very quickly and daily prices are therefore almost perfectly correlated. On the other hand, 1-minute data are not perfectly correlated (see Table 6) and allow the price discovery analysis.

Table 6: This table presents correlations between studied VIX ETP pairs are daily and 1-minute frequency. The data span the period from the date of inception of the last ETP in each pair until 31 December 2012.

| | Daily data | 1-minute data |
|-----------|------------|---------------|
| VXX-VIXY | 0.999 | 0.723 |
| SVXY-XIV | 0.997 | 0.704 |
| TVIX-UVXY | 0.948 | 0.619 |

Several measures of price discovery exist. The most widely used are Granger causality of Granger (1986), the common factor weights of Schwarz and Szakmary (1994) and Hasbrouck (1995) information share (IS). Gonzalo and Granger (1995) extended the causality framework to a common factor model (CF). Baillie et al. (2002) show that these two models are directly related and provide similar results if the residuals are uncorrelated between markets. Comparing the three measures, Theissen (2002) shows that the Schwarz and Szakmary measure can be derived from the Gonzalo-Granger framework and conclusions are qualitatively similar to those based on information shares. In this paper we use Common Factor Weights introduced by Schwarz and Szakmary (1994). In next few paragraphs we explain how are these weights calculated.

First, the Engle-Granger two-step procedure is applied to the different VIX ETP pairs. First the log price of one ETP is regressed on the log price of the other ETP:

$$Y_t = z + \gamma X_t + \varepsilon_t \tag{2}$$

Once the cointegrating regression in Equation (2) is run and estimates of γ and z are obtained, the Engle-Granger test is performed. This is a unit root test on the residuals:

$$\Delta \varepsilon_t = \varphi \varepsilon_{t-1} + \varepsilon_t. \tag{3}$$

If the unit root test indicates that the residuals are stationary then the prices are cointegrated.

The Granger Representation Theorem states that the cointegrated series *X* and *Y* have an error correction representation of order *k*, given by

$$\Delta Y_t = \alpha_Y (Y_{t-1} - \gamma X_{t-1} - z) + \sum_{i=1}^k \beta_{11}^i \Delta Y_{t-i} + \sum_{i=1}^k \beta_{12}^i \Delta X_{t-i} + \varepsilon_{1t}$$
 (4)

$$\Delta X_t = \alpha_X (Y_{t-1} - \gamma X_{t-1} - z) + \sum_{i=1}^k \beta_{21}^i \Delta Y_{t-i} + \sum_{i=1}^k \beta_{22}^i \Delta X_{t-i} + \varepsilon_{2t}$$
 (5)

where the coefficients of Equations (4) and (5) can be estimated using OLS. The appropriate number of lags to use in each model is determined by using the Bayesian information criterion. Hence, the value of k might be different in Equations (4) and (5). Numbers of lags are reported in Table 7. The alphas (α) describe the speed of adjustment back to equilibrium. For this to be true, the alphas should have opposite signs.

The common factor weights are calculated using the estimated coefficients of each of the error correction models. The magnitude of each weight (θ_i) then indicates which market mainly initiates the price discovery process, and which market mainly follows. The weights are calculated as follows:

$$\theta_X = \frac{|\alpha_Y|}{|\alpha_Y| + |\alpha_X|} \tag{6}$$

$$\theta_{Y} = 1 - \theta_{X} = \frac{|\alpha_{X}|}{|\alpha_{X}| + |\alpha_{Y}|} \tag{7}$$

where α_Y and α_X are the error correction coefficients from equation (4) and (5) respectively.

The intuition behind the coefficients θ_X and θ_Y is the following. When two time series are cointegrated and they deviate from each other, then some force (usually arbitrage) pushes them together. This force is captured by the coefficients α_X and α_Y . If it is mostly X pushed towards Y (α_X is larger than α_Y), then Y is the price leader and vice versa.

Table 7 reports the results of the estimated error correction model⁶ for each VIX ETP, as well as its common factor weight (CFW). In case of direct ETPs (VXX and VIXY), VXX is the price leader with a common factor weight of 75%. With VXX being one of the most traded VIX ETPs, this result is in line with what one

 $^{^6}$ For sake of brevity, we report only the estimated α coefficients, because only these coefficients enter the calculation of information share.

might expect. In case of inverse ETP's (XIV and SVXY) is XIV the price leader. Of the two, XIV is older, it has a larger average daily volume and measured in assets under management it is about 2.5 times larger than the SVXY. For levered ETP's (TVIX and UVXY), UVXY is the leader. This is somehow surprising, given that IVIX is older and has higher trading volume.

Notice that there are a few other signs indicating the price leadership. In each pair, the leading ETP has fewer significant lagged returns (k) than the following, and its adjusted R-squared is lower than for the following ETP.

| ETP | lags | R^2 | adj.R ² | α | SE | t-stat | p-value | CFW |
|------|------|-------|--------------------|---------|--------|--------|---------|------|
| VXX | 1 | 0.002 | 0.002 | -0.0015 | 0.0006 | -2.54 | 0.01 | 0.75 |
| VIXY | 15 | 0.053 | 0.053 | 0.0045 | 0.0006 | 7.38 | 0.01 | 0.25 |
| XIV | 2 | 0.003 | 0.003 | -0.0006 | 0.0004 | -1.70 | 0.08 | 0.54 |
| SVXY | 5 | 0.062 | 0.062 | 0.0007 | 0.0004 | 2.07 | 0.03 | 0.46 |
| TVIX | 9 | 0.039 | 0.039 | -0.0003 | 0.0001 | -1.72 | 0.05 | 0.43 |
| UVXY | 5 | 0.010 | 0.010 | 0.0002 | 0.0001 | 1.31 | 0.19 | 0.57 |

Table 7: Results of the estimated error correction models and Common Factor Weights.

4. VIX ETP's as hedging tools

This section investigates the effect of including a VIX ETP in a portfolio tracking the S&P 500 index. As discussed in Section 2, the VIX usually spikes during market drawdowns. All the direct VIX ETPs are strongly negatively correlated with the S&P 500, and therefore they might be suitable instruments for hedging or diversification to an equity portfolio. We therefore study performance of the portfolio based on S&P 500 and VIX ETPs.

In order to select which ETP to use as the hedge, regressions are performed with the ETP returns on the returns of the S&P 500. The preferred ETP is chosen by looking at the estimated coefficients. To minimize negative drag on the portfolio, the ETP with the largest constant term is chosen with a hedge ratio indicated by the estimated β . The results are reported in Table 8. The alphas are negative for all the ETPs except for that of the VXZ, which is the ETP based on the mid-term VIX futures index. However, by looking at the t-statistics of the reported alphas, it cannot be concluded that none of them are significantly different from zero. However, because the VXZ is based on longer term futures (where the term structure is flatter), it suffers less from time-decay. This is consistent with the findings reported in Section 2. Also, Figure 3 shows that the VXZ is the direct ETP with the lowest decay.

The hedging ability of the chosen VIX ETPs is evaluated from the viewpoint of an investor holding a portfolio tracking the S&P 500 index. Based on these

⁷Correlation coefficients are between -0.78 and -.076.

results we can easily calculate that an investor seeking a full hedging should allocate approximately 35% of his holdings to VXZ and 65% to S&P 500. Based on this, three simple strategies are proposed and compared to holding an unhedged portfolio of S&P 500.

Table 8: Regression results indicating that VXZ constitues the best hedging instrument among the VIX ETPs considered. $rSP500_t = \alpha + \beta \times rETP_t + u_t$.

| ETP | R^2 | Adj.R ² | α | t-stat α | β | t-stat β |
|------|-------|--------------------|---------|----------|--------|----------|
| VXX | 0.60 | 0.60 | -0.015% | -0.74 | -0.286 | -54.8 |
| VIXY | 0.60 | 0.60 | -0.015% | -0.73 | -0.286 | -54.9 |
| VXZ | 0.58 | 0.58 | 0.012% | 0.61 | -0.540 | -52.2 |
| UVXY | 0.61 | 0.61 | -0.016% | -0.82 | -0.142 | -55.1 |
| TVIX | 0.61 | 0.61 | -0.022% | -1.11 | -0.148 | -55.4 |

The first strategy is called *The Passive Investor*. With this strategy, a portfolio of S&P 500 and VXZ is set up at the appropriate weights, and never rebalanced. This implies that the amount of portfolio protection will vary as time passes, depending on the performance of the VXZ and the S&P 500. With the poor performance of the VXZ in times with low volatility, it is expected that the amount of portfolio protection will decline as the price of VXZ declines, and spike when volatility rises. The second strategy is called *The Semi-Passive*. This strategy differs from the first one in that the portfolio weights are rebalanced every 90 trading days. This number is chosen arbitrarily and implies that the portfolio will be rebalanced roughly every four and a half months. It is expected that the weight of VXZ in the portfolio will diverge from the initial 35% between each rebalancing point. The final strategy is called *The Active Investor*. In this strategy, the portfolio is rebalanced at the end of every day, to keep the weight of VXZ close to 35%. Note that costs of rebalancing the portfolios have been neglected. However, these costs would in practice affect only The Active Investor. Later in this section it is shown that this hedging strategy performs quite poorly, even without transaction costs.

The results of the strategies are reported in Table 9 and compared to the performance of an unhedged investment in the S&P 500 index. Figure 5 reports the evolution of the portfolio with different strategies compared with S&P 500. Figure 6 show how the weight of VXZ in the portfolio changes through time.

In terms of hedging against tail events, all strategies perform as intended. For instance, when the S&P 500 fell 37.6% in 2008, the portfolios of the Passive Investor, Semi-Passive and Active Investor hedge strategies returned 8%, 3.8% and -3%, respectively. However, during the course of 2013 the market gained 26.4% while the strategies returned 0%, -0.2% and -1.6%. So the VXZ does a good job in removing the large negative returns during market drawdowns, but the cost of being hedged in upward moving markets removes the potential positive returns of the portfolio.

Table 9: Results and returns statistics of three simple hedging strategies, as well as an unhedged postion.

| | The Passive | The Semi-Passive | The Active | Unhedged |
|----------------------|-------------|------------------|------------|----------|
| Initial ETP holdings | 35% | 35% | 35% | 0% |
| Rebalancing | Never | Every 90 days | Every day | Never |
| Std. dev. | 0.58% | 0.53% | 0.59% | 1.41% |
| Avg. Rtn. | 0.006% | 0.012% | 0.008% | 0.030% |
| Max | 3.85% | 3.88% | 5.92% | 11.58% |
| Min | -3.19% | -2.84% | -3.47% | -9.04% |
| Median | 0.000% | -0.005% | -0.010% | 0.082% |
| Sharpe Ratio | 0.011 | 0.023 | 0.014 | 0.021 |
| Sortino Ratio | 0.015 | 0.034 | 0.021 | 0.030 |
| Corr. with S&P500 | -0.06 | 0.42 | 0.65 | 1.00 |
| Total return | 9% | 29% | 13% | 49% |
| Annualized return | 1.1% | 3.3% | 1.6% | 5.1% |

Figure 6 shows that the weight of VXZ for the Passive Investor strategy change significantly over time. The weight of VXZ rises rapidly during the onset of the financial crisis. It then decreases steadily from mid 2008 until the end of the period, only interrupted by the volatile periods in 2010 and 2011. It is clear that, given enough time before a major market crisis, the value of the VXZ holdings will eventually decline towards zero. By then, the protection it offers to the portfolio will be negligible.

The weight of VXZ in the portfolio *The Semi-Passive* is corrected back to 35% of the total portfolio value every 90 trading days. During the fall of 2008, the value of the holdings in VXZ rose above 60% of total portfolio value. This drastic shift in portfolio weights is caused by two factors. First, as the S&P 500 index falls rapidly, so does its value in the portfolio. In turn, this decreases its portfolio weight. The decreasing weight of the S&P 500 in the portfolio implies that the weight of VXZ increases. However, since the value of the VXZ holdings rises due to the rising price of VXZ, its weight in the portfolio increases even further.



Figure 5: The performance of the Passive, Semi-Passive and Active hedging strategies compared with the S&P 500.

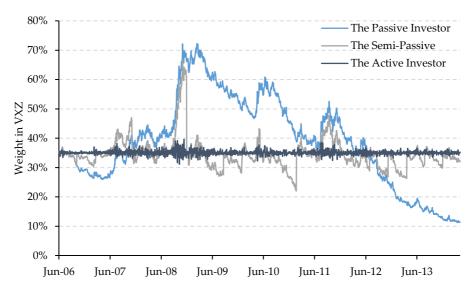


Figure 6: The evolution of weights in VXZ for the three hedging strategies.

Interestingly, the Semi-Passive hedging strategy indicates some degree of increased risk-adjusted performance. Table 9 reports that this strategy has better Sharpe- and Sortino-ratios than the unhedged position. The maximum draw-

down is also quite high for the unhedged position relative to the Semi-Passive (-9.04% vs. -2.83%).

However, the performance of the Semi-Passive⁸ portfolio could be influenced by particular rebalancement dates. In order to check whether this is an issue, we investigated several semi-passive investment strategies which differ in the length of rebalancement period, in particular 25, 50, 75, ..., 250 trading days. We find that even though performance of these strategies varies with the length of rebalancing period, the average performance (average daily return 0.012% and average Sharpe ratio 0.22%) is very similar to the performance of the previously studied 90-day rebalancing (average daily return 0.012% and average Sharpe ratio 0.23%). We therefore conclude that the choice of 90 days is representative and use it throughout this paper.

To further evaluate the performance of the Semi-Passive hedging strategy, a sensitivity analysis is performed to see how the risk-adjusted return varies when the portion of VXZ is changed. The risk-adjusted returns are measured by calculating the Sharpe Ratio and the average return per tail risk (average daily return divided by 1% daily VaR) for different holdings. Then the same is done for the VXX, also using the Semi-Passive strategy. Strictly for illustrative purposes, the same analysis is performed when pretending as if the VIX itself could actually have been bought and sold just like any other VIX ETP.

Since the financial crisis following the Lehman Brothers bankruptcy was particularly severe, this may skew the results in favor of ETP hedging. Because of this, the same calculations are done with the real prices starting 20 Feb 2009 for the VXZ and 20 Jan 2009 for the VXX and spot VIX. This way the major VIX spike in September 2008 is filtered out, but the period still covers the big market drawdowns during the European sovereign debt crisis in May 2010 and the US credit downgrade in August 2011. The results can be seen in Figure 7.

When evaluating the whole period, including a small portion of VXZ in the portfolio would have marginally improved its risk adjusted returns when measured both with the Sharpe Ratio and the average return per 1% daily VaR. As for including the VXX in the portfolio, it would not have improved its Sharpe Ratio. However, its average return per 1% daily VaR would have improved slightly. When looking at the period after the inception of each ETP, after the 2008 crisis, it is clear that the inclusion of VIX ETPs in the portfolio would have offered no improvement in the risk adjusted returns.

The two top plots in Figure 7 reports the results when it is pretended that hedging with spot VIX is a viable option. Inclusion of the VIX index (unfortunately not possible) would have offered a considerable improvement in risk adjusted returns, but inclusion of VXZ and VXX mostly deteriorates the performance of the whole portfolio. To justify the use of VIX ETPs as a hedging

⁸ This is not an issue for the Active (Passive) Investor portfolio, because this porfolio is being rebalanced every day (never).

tool, one should possess some ability or tool for prediction. And if one is able to successfully predict extreme tail events, hedging for protection would not be necessary.

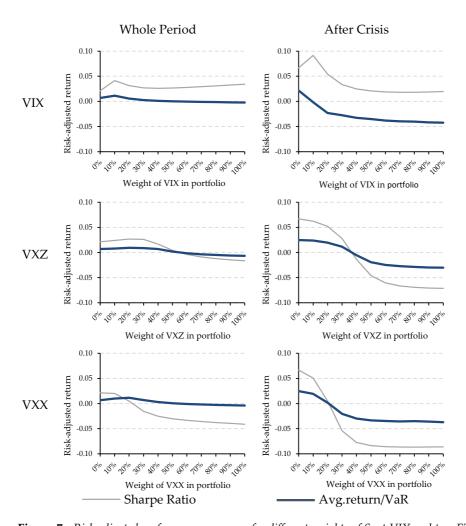


Figure 7: Risk adjusted performance measures for different weights of Spot VIX and two ETPs, during the whole sample period, and the period after inception of each ETP. The X-axis represents the relative amount of VIX ETPs (or spot VIX) included in the portfolio, the Y-axis gives the value of the ratios.

These results are in line with Alexander and Korovilas (2012a) who show that single positions on direct VIX futures ETNs of any maturity could only provide a hedge of equity exposure during the first few months of a great crisis.

and crisis is hard to predict.

5. Trading Strategy - Reaping the Market Fear

In this section a trading strategy intended to extract the roll-yield inherent in the VIX ETPs is presented . A similar strategy using VIX futures was studied by Simon and Campasano (2014), Chan (2013) and Sinclair (2013).

According to the rational expectations hypothesis, futures prices should be unbiased predictors of the future value of the spot prices. This means that the slope of the futures curve today should be concurrent with the direction and magnitude of subsequent changes of the spot itself. However, this arbitrage-free reasoning does not apply to VIX futures, because VIX itself is not a traded instrument. VIX futures are consistently overpriced relative to the subsequent moves in the underlying VIX index. Furthermore, Simon and Campasano (2014) present evidence that the futures prices can be predicted by looking at the difference between the VIX front month futures price and the VIX Index. If the futures are trading over the VIX, the futures tend to fall and if the futures are trading below the VIX they tend to rise.

The term structure can possibly be explained by the fact that most investors are risk-averse. They are therefore willing to pay a premium for VIX exposure because it represents an insurance against losses in the equity market. This means that the expected return when going long a VIX futures is typically negative, which is consistent with the typical contango pattern. In a sense, it is this premium that is being harvested in the trading strategy.

The roll-yield is caused by the rolling of futures contracts necessary when constructing the ETPs. If the total return of the futures is equal to the spot return plus the roll-yield, the roll-yield can be extracted by shorting (buying) the futures and buying (shorting) the underlying spot when the market is in contango (backwardation). However, because it is unfeasible to trade the VIX index directly, the usual futures pricing relationship based on cash-and-carry arbitrage does not hold. However, it is not necessary to use the spot itself, as long as the instrument used has a high correlation with the spot. Since the S&P 500 index has a high negative correlation with the VIX, we will use it for this purpose.

To implement the trading strategy the following ETPs are used. SPDR S&P 500 Trust ETF (SPY) and ProShares Short S&P 500 ETF (SH) provide long and short exposure to the S&P 500, respectively. Path S&P 500 VIX ST Futures ETN (VXX) and VelocityShares Daily Inverse VIX ST ETN (XIV) provide long and short exposure to short-term VIX futures, respectively. In other words, when the market is in contango, XIV and SH are bought, and when the market is in backwardation, VXX and SPY are bought.

⁹Prospectus for these products are available on www.spdr.com and www.proshares.com

First, the relative difference between the front month VIX futures and spot VIX is calculated. This is defined as the *relative basis*, R^{B} . If it is positive and exceeds a certain upper buy-threshold B^{U} set by the trader, a position is opened by buying the XIV (short volatility exposure). The XIV position is hedged by buying SH (short S&P 500 exposure). The size of the hedge is also determined by the trader, depending on her risk aversion. This position will make money as long as the market stays in contango, and the VIX futures term structure does not shift downward. The position is closed when the relative basis falls below an upper sell-threshold, $\hat{S}^{\hat{U}}$, which may be set equal to, or lower than the buy-threshold. A reason why one might want the upper sell-threshold lower than the upper buy-threshold is to avoid too frequent trading. The same strategy applies when the relative difference between the front month VIX futures and spot VIX is negative and falls below a certain lower buy-threshold, B^L . Then a position is opened where the trader buys VXX (long volatility exposure) and hedges part of the position by also buying SPY (long S&P 500 index ETF). The position is closed when the relative difference between the front month VIX futures contract and the spot VIX rises above the lower sell-threshold S^{L} . The upper and lower buy and sell threshold are illustrated in Figure 8.

The time series used to test the trading strategy contain the closing prices of the ETPs and spot VIX, and the settle prices after closing of the front month VIX futures.

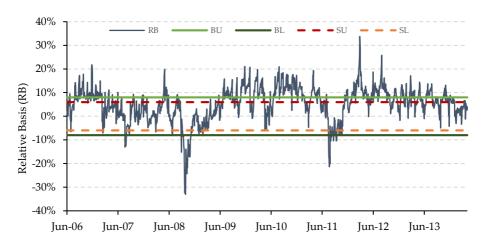


Figure 8: Relative basis throughout the period, and the upper and lower buy and sell thresholds.

In order to take into account trading costs and bid-ask spread, buying and selling prices are calculated as follows:

$$P^{BUY} = P^{close}(1 + spread)(1 + fee)$$

$$P^{SELL} = P^{close}(1 - spread)(1 - fee)$$
(8)

where *fee* represents a percentage paid per trade, and *spread* is expressed as a percentage of the closing price. These two terms are important as they might influence the performance of the trading strategy.

The brokerage fee is set to 0.20%. Several brokerages provide access to US equity markets for fees lower than this. The bid-ask spread is based on historical average and is set to $0.15\%.^{10}$

The flowchart of the algorithm is shown in Figure 9. Right before the end of each trading day, the relative basis R^B , i.e. the percentage difference between the price of the front month VIX futures contract and VIX spot is calculated. If the relative basis is above (below) an upper (lower) buy threshold, B^U (B^L) determined by the trader, it indicates that the market is in contango (backwardation) and that one should hold XIV (VXX) and hedge with SH (SPY).

First it is checked whether any positions are open. If not, the cash on hand is used to buy XIV (VXX) and SH (SPY) with the appropriate weights. If a position in XIV (VXX) and SH (SPY) is already open, nothing is done and the position is held through the next trading day. If a position in VXX (XIV) and SPY (SH) is held, this is closed and a position in XIV and SH (VXX and SPY) is opened. If the relative basis lies within the interval of the lower sell and upper sell, S^L and S^U thresholds (also determined by the trader), it is checked whether there are any open positions. If not, the cash on hand is kept until the end of the next trading day, and if there are any open positions these are closed. One then waits until right before the end of the next trading day, and repeats the process.

The thresholds described above must satisfy the following conditions:

$$B^{L} \leq S^{L}$$

$$B^{U} \geq S^{U}$$

$$S^{U} \geq B^{L}$$

$$S^{L} < B^{U}$$
(9)

The first two conditions in (9) imply that the magnitude of the sell-threshold must be less than or equal to the buy-threshold. The last two conditions must be met in order to avoid having open positions that are betting on opposite directions of the futures curve.

Table 10 reports the results of the trading strategies with different fraction of the ETP position hedged with the S&P 500 direct and inverse ETPs. The values in the column *Hedge* report the share of total capital allocated to the hedge (SPY or SH). Surprisingly, the unhedged strategy performs the best. By only

¹⁰See for instance http://www.cboe.com/rmc/2013/Day3-Session-1ACross.pdf

switching the position between XIV and VXX, the trader would have earned an annualized return of 69% with a yearly volatility of 39%, including transaction costs. The average duration between the trades is 18 trading days. This equates to 106 changes in position over the whole period.

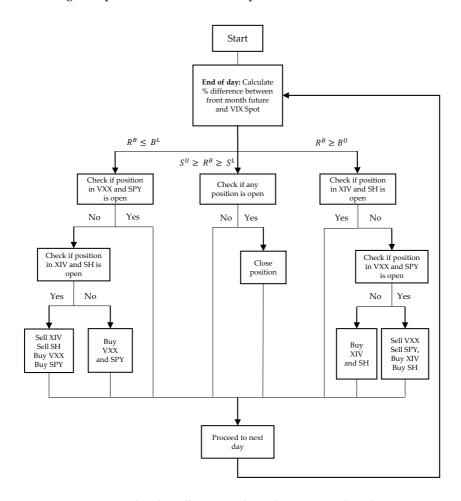


Figure 9: Flowchart illustrating the trading strategy algorithm.

Table 10: Results of the trading strategy when the following buy and sell conditions are set: $B^L = -8\%$, $B^U = 8\%$, $S^L = -6\%$, $S^U = 6\%$, fee = 0.20% and spread = 0.15%. The total number of changes in position is 106.

| Hedge ratio | Avg. return per trade | Winners/ losers | Total return | Annualized return | Volatility | Sharpe ratio | Sortino ratio | Largest drawdown |
|----------------|--------------------------|--------------------|-----------------|-------------------|------------|-----------------|------------------|---------------------|
| 0% | 5.24% | 53/53 | 5919% | 69% | 39% | 2.11 | 3.17 | -24.5% |
| 10% | 4.64% | 53/53 | 4027% | 61% | 35% | 2.02 | 3.04 | -22.3% |
| 20% | 4.04% | 52/54 | 2668% | 53% | 31% | 1.94 | 2.91 | -19.9% |
| 30% | 3.43% | 50/56 | 1711% | 45% | 27% | 1.86 | 2.78 | -17.5% |
| 40% | 2.81% | 52/54 | 1052% | 37% | 23% | 1.77 | 2.64 | -14.9% |
| 50% | 2.19% | 50/56 | 610% | 28% | 18% | 1.66 | 2.48 | -12.2% |
| 60% | 1.56% | 46/60 | 322% | 20% | 14% | 1.53 | 2.27 | -9.3% |
| 70% | 0.93% | 42/64 | 140% | 12% | 10% | 1.26 | 1.89 | -6.3% |
| 80% | 0.28% | 34/72 | 30% | 3% | 7% | 0.51 | 0.77 | -3.8% |
| 90% | -0.37% | 28/78 | -33% | -5% | 9% | -0.51 | -0.73 | -6.0% |
| 100% | -1.03% | 35/71 | -69% | -14% | 16% | -0.82 | -1.14 | -9.8% |

6. Conclusion

This paper investigates VIX exchange traded products with focus on their performance, price discovery and usefulness as hedging and trading instruments.

By measuring the price discovery between the different ETPs it is found that they impound information at different speeds. The VXX impounds information faster than the VIXY, which one might expect since it has higher volume and has existed longer. Similarly for the most popular inverse ETPs, the slightly older and more traded XIV impounds information faster than SVXY. However, for the leveraged ETPs, TVIX and UVXY, the younger and less traded UVXY is leading the price formation process.

The VIX futures term structure makes direct positions in VIX ETPs unsuitable for buy-and-hold strategy due to the negative expected returns. For instance, a position in the most popular direct VIX ETP, the VXX, would have lost over 99.5% of its value if opened in January 2009 and held to April 2014. The returns of direct ETPs are particularly bad when the market is in contango. During periods when the term structure is in backwardation, the direct VIX ETPs perform well. But, backwardation occurs seldom, and when it does, it is short-lived. Hence, ETPs which offer long volatility exposure are too expensive to be treated as buy-and-hold investments.

We study also how VIX ETPs perform as a hedging tools. If VIX index would be a tradable index, investor holding the S&P 500 portfolio would benefit from investing (e.g. 10% of his wealth) in the VIX index, but not in VIX exchange traded products. Our results show that high negative expected returns of these products outweight the diversification benefits and that adding these products to the S&P 500 does not improve, but deteriorate risk-adjusted returns. However, time-decay is not a feature exclusive to VIX ETPs. The presence of volatility risk premiums would likely impose time-decay on investors looking for volatility

exposure using other products too, such as put options, variance swaps, volatility futures, or other related products.

The time-decaying feature of the VIX ETPs gives rise to a trading opportunities. A trading strategy of buying inverse (direct) VIX ETPs when the term structure is sufficiently in contango (backwardation), is highly profitable and robust to transaction costs. With conservative assumptions about trading costs, the strategy yielded annual returns of over 60% and Sharpe Ratio above 2. VIX exchange products are poor hedging tools, but offer great opportunities for investors seeking high profits for willingness to bear some risks.

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