The Information Content in Implied Idiosyncratic Volatility and the Cross-Section of Stock Returns: Evidence from the Option Markets

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Abstract

Current literature is inconclusive as to whether idiosyncratic risk influences future stock returns and the direction of the impact. Prior studies are based on historical realized volatility. Implied volatilities from option prices represent the market's assessment of future risk and are likely a superior measure to historical realized volatility. We use implied idiosyncratic volatilities on firms with traded options to examine the relation between idiosyncratic volatility and future returns. We find a strong positive link between implied idiosyncratic risk and future returns. After considering the impact of implied idiosyncratic volatility, historical realized idiosyncratic volatility is unimportant. This performance is strongly tied to small size and high book-to-market equity firms.

INTRODUCTION

The relation between risk and return in the stock market is a long standing fundamental issue in finance. Standard asset pricing models such as the Sharpe-Lintner (1964, 1965) capital asset pricing model (CAPM) and the Ross (1976) arbitrage pricing theorem (APT) suggest a positive relation between systematic risk and return, and many studies test this theoretical prediction. Theoretically, systematic risk is the aspect of total risk linked to returns. A more recent strand of literature, however, suggests that unsystematic, or idiosyncratic, risk may be what is actually driving this risk-return relation.

Many examine the cross-sectional relation between equity returns and idiosyncratic risk. Douglas (1969), Lintner (1965), and Lehmann (1990) find that returns are positively related to market model residuals. Merton (1987), Barberis and Huang (2001), Jones and Rhodes-Kropf (2003), and Malkiel and Xu (2006) develop asset pricing models establishing that returns are a positive function of idiosyncratic risk. The arguments of these models generally center on the failure by investors to hold diversified portfolios and these investors requiring compensation for the additional risk. Malkiel and Xu empirically find a positive link between returns and idiosyncratic risk, even in the presence of variables representing size, book-to-market equity, and liquidity. Chua, Goh, and Zhang (2005) use an autoregressive model and Fu (2007) and Spiegel and Wang (2007) use EGARCH models to estimate expected idiosyncratic volatility. They all find that expected returns are positively related to expected idiosyncratic volatility.

In contrast, Ang, Hodrick, Xing, and Zhang (2006) find a negative cross-sectional relation between returns and idiosyncratic risk. These results are robust to the inclusion of other independent variables and different market conditions. Their results do not seem attributable to exposure to aggregate volatility risk. Ang, Hodrick, Xing, and Zhang note that their findings are counter to the theoretical argument that investors require additional returns for bearing unsystematic risk and prior empirical findings of a positive relation between returns and idiosyncratic risk. The authors attribute this discrepancy with other studies to their focusing and sorting on firm-level idiosyncratic volatility, something other studies fail to do. Bali and Cakici (2007), however, demonstrate there is no robust significant relation between idiosyncratic volatility and the cross-section of expected stock returns.

It is possible that the puzzling results of Ang, Hodrick, Xing, and Zhang (2006) are due to limits to arbitrage or short sale constraints. Miller (1977) posits that large constraints on short selling may lead to lower future returns. Boehme, Danielsen, Kumar, and Sorescu (2005) and Duan, Hu, and McLean (2007) find short interest ratios are linked to the relation between returns and idiosyncratic risk. Duan, Hu, and McLean find a negative relation for the 5% of stocks with the highest short interest, but no relation for the remaining stocks. Boehme, Danielsen, Kumar, and Sorescu use option listing and short interest as measures of short sale constraints. Stocks with (without) traded options and with low (high) short interest have low (high) short sale constraints. They find a positive (negative) relation between returns and idiosyncratic risk for stocks with low (high) short sale constraints. There is no relation between returns and idiosyncratic risk for the entire option sample and the entire non-option sample.

Battalio and Schultz (2006) do not find any evidence of short sales restrictions for Internet stocks early in the year 2000. This is important because it provides evidence of an absence of short sales restrictions for stocks underlying actively traded options. By examining a broad spectrum of stocks from the NYSE, AMEX, and Nasdaq, Ang Hodrick, Xing, and Zhang (2006) are likely mixing many securities with limits to arbitrage and short sale constraints along with other securities without these limits and constraints. This may affect their finding of a negative relation between stock returns and idiosyncratic risk. Therefore, in the spirit of Boehme, Danielsen, Kumar, and Sorescu (2005) and Battalio and Schultz, we examine the effect of short sale constraints and idiosyncratic volatility on stock returns.

The nature of the cross-sectional relation between idiosyncratic risk and returns is currently unclear. One weakness with prior studies is the use of historical risk measures. Since ex-ante market expectations likely provide better assessments for future volatility than past volatility, we examine the relation between expected volatility and future returns. Thus, our analysis is similar in spirit to that in Chua, Goh, and Zhang (2005), Fu (2007), and Spiegel and Wang (2007). We do not, however, employ realized idiosyncratic volatility and a statistical model to estimate expected idiosyncratic volatility. Instead, we directly assess expected volatility as the implied volatility from observed option prices. From this measure we then derive implied idiosyncratic volatility.[1]

[1] As shown in the next section, we require an estimate of a firm's beta to derive an estimate of implied idiosyncratic volatility. Although we explore alternative beta estimation methods and obtain robust results, our estimates of implied idiosyncratic volatility are subject to possible errors from beta estimation.

By definition, implied volatility is the market's best guess of the future volatility over the term of the option. Consequently, the predictive power of implied volatility is examined in multiple studies. For example, Christensen and Prabhala (1998) find that the volatility implied by S&P 100 index option prices outperforms past volatility in forecasting future index volatility. Doran and Ronn (2006) note that this estimate is biased and may be related to the volatility risk premium. Other studies examine the predictive ability of the CBOE volatility index, VIX, for stock returns.[2] Giot (2005) finds that VIX is useful for predicting returns on the S&P 100. Copeland and Copeland (1999) determine that VIX levels predict returns on various indices formed on size and growth versus value characteristics. Banerjee, Doran, and Peterson (2007) find that VIX levels and innovations predict the returns of characteristic based portfolios.

We use a forward looking measure of idiosyncratic volatility and examine its cross-sectional relation to firm-level future returns. Thus, the objective of our study is to investigate whether expected idiosyncratic risk, as measured from implied volatility, is related to future returns. We also examine if the predictive ability of implied idiosyncratic risk is stronger than that of realized idiosyncratic volatility and forecasts of volatility from statistical models.

Using stock and option prices for all firms with traded options, we calculate measures of implied and realized idiosyncratic volatility. We also calculate idiosyncratic volatility forecasts from EGARCH and autoregressive models. We find that implied idiosyncratic volatility strongly predicts realized volatility at the firm level, and the effect is greater than that from past realized idiosyncratic volatility or volatility forecasts from statistical models. Next, using a Fama and MacBeth (1973) cross-sectional analysis and controlling for firm-specific effects, we show that implied idiosyncratic volatility positively predicts future stock returns, but past realized idiosyncratic volatility is unrelated to future returns. Idiosyncratic volatility forecasted from statistical models is not significantly related to returns when implied idiosyncratic volatility is included as an explanatory variable.

Finally, we sort our sample by implied and realized idiosyncratic volatilities and examine portfolio returns. We both equal and value weight the portfolios. We consistently find that implied idiosyncratic volatility is more closely linked to future returns than realized idiosyncratic volatility. We directly compare the future return forecasting power for implied idiosyncratic and

[2] VIX is a measure of market expectations of stock index return volatility over the next 30 calendar days. Beginning in 2003, VIX is calculated from the S&P 500 index option prices. The calculation is based on a wide range of strike prices and is independent of any option pricing model.

past realized volatilities. Implied idiosyncratic volatility has a strong, positive forecasting power for future returns when realized idiosyncratic volatility is controlled for. However, realized idiosyncratic volatility has no significant forecasting power when implied idiosyncratic volatility is controlled for. The relation is strongest for small firms and high book-to-market equity firms. While we present high alphas, we suggest an alternative explanation that idiosyncratic risk should be a priced factor in asset pricing models.

The rest of the paper is organized as follows. In the next section the data is described and we explain how the measure of implied idiosyncratic volatility is calculated. In the third section we report the methodology and results for the predictive power of implied total and idiosyncratic volatility for future realized volatility, and the Fama and MacBeth (1973) cross-sectional analysis of future firm returns. Then in the fourth section we present and analyze portfolio returns formed on implied idiosyncratic volatility. A conclusion is provided in the final section.

DATA

Data Description and Measures of Volatility

We employ individual company daily implied return volatility data, from January 1996 through June 2005, made available from OptionMetrics.[3] Option open interest is also provided by OptionMetrics. We obtain stock returns, share prices, and number of shares outstanding from CRSP and book value of equity from Compustsat.[4] The CRSP and Compustat data we obtain is not restricted to solely firms with options or to the period 1996-2005. Daily returns for the three Fama and French (1993) factors (MKT, SMB, HML) and the Carhart (1997) momentum factor (UMD) are obtained from Kenneth French's website.

For the option sample we use all firms with traded options with the condition that there is a least five years of prior stock return data. This is necessary for the calculation of the firm's beta and the calculation of idiosyncratic implied and realized volatility. To calculate firm *j*'s beta, monthly firm returns, r, are regressed on market returns using the prior 60 months:

$$r_{j,t} = \alpha_j + \beta_j MRET_t + e_{j,t} \tag{1}$$

[3] OptionMetrics is a financial research and consulting firm specializing in econometric analysis of options markets.
[4] We exclude all ETFs and foreign, financial, and utility stocks. We include only firms with a 10 or 11 share code.

where MRET is the return on the CRSP value-weighted index. Each subsequent month the sample is updated to use only the prior 60 months, giving a rolling beta estimate for each firm.[5]

For each firm within the option sample, the option implied volatilities are calculated by OptionMetrics using American or European models where appropriate. Since there are a variety of strike prices and maturities for each firm on a given day, a standardized implied volatility is calculated by employing the most weight on implied volatilities with at-the-money options closest to 30 days to maturity for both calls and puts. Averaging across all options reduces the measurement error associated with inverting option prices to obtain implied volatilities.[6]

To calculate the idiosyncratic portion of implied volatility, we express implied market volatility as a function of market volatility, in a fashion similar to Dennis, Mayhew, and Stivers (2006), such that:

$$\sigma^2_{IV,j,t} = \beta^2_j \sigma^2_{IV\_M,t} + \sigma^2_{IV\_idio,j,t} \tag{2}$$

where $\sigma^2_{IV\_M,t}$ is the implied market variance from VIX on day *t*, $\sigma^2_{IV,j,t}$ is the implied total variance for firm *j* at time *t*, β^2_j is the squared market beta from the estimation of equation (1), and $\sigma^2_{IV\_idio,j,t}$ is the idiosyncratic portion of implied variance for firm *j* at time *t*. Our measure of implied idiosyncratic volatility is the square root of the idiosyncratic portion of implied variance. Theoretically, this value should not be less than or equal to zero, but empirically it is possible. A small number of them have non-positive values and we set these equal to zero.

We create one month annualized realized volatility as the annualized standard deviation of daily returns within the given month for each firm. To extract the realized idiosyncratic portion, returns of the individual firms are regressed on market returns using equation (1), but with a daily frequency. To create the realized idiosyncratic risk measure, $\sigma_{RV\_idio,j,t}$, the standard deviations of the daily residuals are calculated for the given month *t* as:

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[5] To check for the robustness of our beta calculation, we use the Fama and French (1993) three-factor model market beta and a portfolio beta. The portfolio beta is calculated in a similar fashion to Fama and French (1992) and Fu (2007) by forming a rolling monthly estimation of equal-weighted returns for 10x10 portfolios based on size and firm betas. These portfolio returns are then regressed on the current and one-month lagged value-weighted index returns to generate portfolio betas, which are assigned to the individual firms depending on their size and beta decile. The subsequent calculation of $\sigma^2_{IV\_idio,j,t}$ using either the Fama and French (1993) three-factor model market beta or the portfolio beta have correlations with the market model beta of .83 and .72, respectively. There is no qualitative impact on the predictive power of implied idiosyncratic volatility to forecast future idiosyncratic realized volatility and returns using either alternative beta calculation. The results are available upon request.

[6] See Hentschel (2003) for details.

$$\sigma_{RV\_idio,j,t} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(e_{j,t,n} - \bar{e}_{j,t}\right)^2} \tag{3}$$

where N equals the number of days in the given month, $e_{j,t,n}$ is the residual for firm j on day n in month t, and $\bar{e}_{j,t}$ is the mean residual for firm j over the N days in month t. This standard deviation measure is then annualized.

To compare the forecast power of implied idiosyncratic volatility relative to realized idiosyncratic volatility, we also construct two statistical forecasts of realized idiosyncratic volatility. The first is from the Nelson (1991) EGARCH (p,q) model, as used in Fu (2007) and Spiegel and Wang (2007). The benefit of the EGARCH versus the GARCH model is that is does not require restricting the parameters to insure a non-negative variance. The function form is:

$$R_{j,t} = \alpha_j + \beta_{1,j} MKT_{M,t} + \beta_{2,j} SMB_t + \beta_{3,j} HML_t + \varepsilon_{j,t}, \quad \varepsilon_{j,t} \sim N(0, \sigma_{j,t}^2) \tag{4}$$

$$\ln \sigma_{EG\_idio,j,t}^2 = a_i + \sum_{\phi=1}^{p} b_{j,\phi} \ln \sigma_{j,t-1}^2 + \sum_{\Phi=1}^{q} c_{j,\Phi} \left\{ \theta \frac{\varepsilon_{j,t-k}}{\sigma_{j,t-k}} + \gamma \left[\left| \frac{\varepsilon_{j,t-k}}{\sigma_{j,t-k}} \right| - \sqrt{2/\pi} \right] \right\} \tag{5}$$

where the monthly returns are described in the three-factor model in equation (4), and the conditional variance for firm j, $\sigma_{EG\_idio,i,t}^2$, is a function of the past p residual variances and q-period return shocks. Equations (4) and (5) are estimated for each stock using at least 60 monthly returns. The square root of the conditional variance is the measure of idiosyncratic volatility. The second statistical forecast uses a 2nd order autoregressive model, AR(2), to estimate idiosyncratic volatility, similar to that of Chua, Goh, and Zhang (2005). Using the squared residual from equation (4), idiosyncratic variance for firm j is expressed as:

$$\sigma_{AR\_idio,j,t}^2 = \vartheta_{1,j} + \vartheta_{2,j}\varepsilon_{j,t-1}^2 + \vartheta_{3,j}\varepsilon_{j,t-2}^2 + \eta_{j,t} \tag{6}$$

An AR(2) is preferred to an AR(1) process since the latter tends to have high serial correlation. Our measure of idiosyncratic volatility is the square root of the idiosyncratic variance.

Two additional controls are included to account for possible liquidity issues or short-sale constraints. They reflect our focus on firms with traded options. Highly liquid stocks are less likely to have market frictions. Our measure of liquidity, OI, is defined as the log of option open interest plus one, where open interest is aggregated across all options for a firm. We employ the short-sale constraint measure defined in Ofek, Richardson, and Whitelaw (2004) as:

$$\text{ORW\_Ratio} = 100 * \ln\left(\frac{S}{S^*}\right) \tag{7}$$

where S is the current stock price, and S^* is the theoretical stock price calculated from the put-call parity relation which includes the early exercise premium on the put.[7] If there is a short-sale constraint, the ORW_Ratio should exceed zero. The inclusion of these controls is designed to test the hypothesis that firms that have higher liquidity and/or lower short-sale constraints may have different price responses to volatility than those firms that are more constrained.[8]

Descriptive Statistics

In Table I we provide descriptive statistics for the option sample over the period 1996 through June 2005. Data is averaged across time for an individual firm, and then descriptive statistics across firms are presented. The total number of monthly firm observations is 132,634, with a total of 2,253 unique firms. On average there are 1704 firms in any given month and the average firm within the sample has 75 observations. Market value of equity (SIZE) is measured at the end of each month. The cross-sectional average, median, and 5th and 95th percentile values for firm size correspond very closely to the Fama and French (1993) percentiles. Book-to-market equity (B/M), calculated for a given month in calendar year *t*, is computed using the end of prior month market value of equity and book equity from fiscal year *t-2*. This insures that equity values are known at the time they are used. The B/M values are slightly lower than Fama and French values in each category, with the median and 95th percentile values equal to the 40th percentile and 85th percentile values in Fama and French. Beta is calculated from equation (1) using the prior 60 months of returns. The median beta is 1.03, and the 5th and 95th percentile values are 0.27 and 2.66, respectively. The distribution of values for the short-sale constraint (ORW_Ratio) is similar to that in the Ofek, Richardson, and Whitelaw (2004) sample. So while the number of firms in our sample is significantly less than the full universe of available stocks, based on the option firm characteristics the sample is very representative.

The cross-sectional average and median firm implied volatility is higher than the firm realized volatility counterparts. This is similar to the market relation between implied volatility

[7] Refer to equation (3) in Ofek, Richardson, and Whitelaw (2004).
[8] Although option listing is, itself, often considered a mitigation of short-sale constraints, there may still be cross-sectional differences in these constraints across firms with options.

and realized volatility, where the VIX index average volatility over the period is 23.6% and the realized volatility on the S&P 500 is 16.8%. These differences are significant, at the 1% level, for both the market and firm levels. As Doran and Ronn (2006) point out, differences in implied and realized market volatility may be a direct result of a volatility risk premium, which has a significant impact on the value of the underlying options. Given that this pattern persists at the firm level, it is possible that the volatility premium influences firm-level option prices as well.

By comparing the means and medians, the idiosyncratic portion of realized total volatility is about 80% percent. This is slightly lower than the 85% found by Goyal and Santa-Clara (2003), but the idiosyncratic portion still makes up the majority of realized volatility. The mean and median implied idiosyncratic volatility are similar to the realized and two statistical forecast idiosyncratic volatility measures, and make up about 70% percent of implied total volatility. So while implied idiosyncratic volatility accounts for a lower portion of implied total volatility than realized idiosyncratic volatility does for realized total volatility, it is clearly the significant component of implied total volatility. This is important given the relation of implied and future realized volatility and returns. If most of the implied volatility is idiosyncratic, at the firm level the idiosyncratic portion may be a strong predictive component.

METHODOLOGY AND RESULTS FOR SECURITY-LEVEL ANALYSIS

Predictive Power of Implied Volatility

To test for the information content in implied volatility in the option sample, we first examine the predictive power in forecasting future realized volatility. We use monthly data, observing the implied and realized volatility on the last day of each month, where realized volatility is measured over the month. This is done at the market and individual firm levels as:

$$\sigma_{RV\_M,t+1} = \alpha + \xi_1 \sigma_{IV\_M,t} + \xi_2 \sigma_{RV\_M,t} + \varepsilon_{t+1} \tag{8}$$

$$\sigma_{RV,j,t+1} = \alpha_j + \xi_{1,j} \sigma_{IV,j,t} + \xi_{2,j} \sigma_{RV,j,t} + \varepsilon_{j,t+1} \tag{9}$$

where $\sigma_{RV\_M,t}$ is the annualized realized monthly volatility on the S&P 500 in month *t* and $\sigma_{IV\_M,t}$ is the VIX index in month *t*. ε denotes a residual, and α and ξ represent coefficients to be estimated. Equation (8) is the regression specification for the market. Equation (9) is the regression specification for the individual firm *j*. In each case this is a test of the information content in total implied volatility, and the samples have non-overlapping observations. For the firm level regressions, each firm's coefficients are estimated separately and then mean and median coefficients across firms are presented, along with the proportion significant at the 5% level. In a reverse Fama and MacBeth (1973) methodology, t-statistics are formed from the cross-sectional distribution of the firm coefficients. We require at least 60 observations for the firm to be included, reducing the sample to 1310 firms. We present the results in Table II.

Panel A reports the coefficient estimates and t-statistics (in parentheses) for the market regression, with and without the restriction that $\xi_2 = 0$. The coefficients on implied market volatility are positive and significant at the 1% level, while the coefficient on realized market volatility is insignificant. Consistent with Christiansen and Prabhala (1998) and others, implied volatility is the efficient predictor of future realized volatility, even in the presence of past realized volatility. From column one, a test for $\xi_1 = 1$ reveals a t-statistic of 3.49 (not shown), suggesting that ξ_1 is significantly different from one. Thus, implied volatility is an upward biased predictor of future realized volatility. This suggests a strong volatility risk premium.

Panel B reports the firm-level results. Mean and median (across firm) coefficients are presented, in brackets are the 25th and 75th percentile values, and in parentheses are the reverse Fama and MacBeth (1973) t-statistics. Firm-level results are similar to the market-level results. When past realized volatility is included, firm implied volatility is still the efficient predictor of future realized volatility. This is different from prior findings, such as those by Bakshi and Kapadia (2003) and Dennis, Mayhew, and Stivers (2006), showing that total implied volatility is an unbiased and efficient estimator of total realized volatility. These studies use data from a small sample of firms and employ the 1988-1995 time period, which had very low volatility. The conclusions in Bakshi and Kapadia and Dennis, Mayhew, and Stivers at the firm level are consistent with the conclusion in Christensen and Prabhala (1998) for the market level. However, we now know that the Christensen and Prabhala results are a function of the time period, and do not hold generally. Our findings at the firm level are consistent with new evidence, as in Doran and Ronn (2006), showing implied volatility is an upward biased predictor of future realized volatility at the market level.

Predictive Power of Implied Idiosyncratic Volatility

Next, for the option sample we compare the forecasting ability of implied idiosyncratic volatility, statistical forecasts from the EGARCH and AR(2) models, and historical realized volatility, as predictors for future realized idiosyncratic volatility. We observe historical realized idiosyncratic volatility for month *t*, implied idiosyncratic volatility at the end of month *t* that is a forecast for month *t+1*, and forecasts of month *t+1* idiosyncratic volatility from EGARCH and AR(2) models using data through month *t*. For each firm *j* we regress future realized idiosyncratic volatility on the forecasted measures as:

$$\sigma_{RV\_idio,j,t+1} = \alpha_j + \psi_{1,j}\sigma_{IV\_idio,j,t} + \psi_{2,j}\sigma_{EG\_idio,j,t} + \psi_{3,j}\sigma_{AR\_idio,j,t} + \psi_{4,j}\sigma_{RV\_idio,j,t} + \varepsilon_{j,t+1} \qquad (10)$$

where α and ψ s are estimated coefficients. Similar to the regression results reported in Panel B of Table II, coefficients for each firm are estimated separately, with mean and median coefficients presented along with their 25th and 75 th percentile values. We also present t-statistics from the reverse Fama-MacBeth procedure. We show the results in Table III with the same format as Panel B of Table II. They are presented for the full model in equation (10) as well as restricted subsets. The sample size drops from Table II because we lack sufficient data to calculate beta for some firms; beta is needed so we can calculate implied idiosyncratic volatility.

The first column in Table III shows that both implied and historical realized idiosyncratic volatility are positively related to future idiosyncratic volatility, but the relation is stronger for implied volatility with a significant t-statistic from the Fama-MacBeth regressions. The second and third columns show that historical realized idiosyncratic volatility is a stronger predictor of future volatility than each of the statistical models; only historical volatility has a significant t-statistic. Finally, the last column includes all four idiosyncratic volatility forecast measures. Implied idiosyncratic volatility has the strongest relation with future idiosyncratic volatility and it is the only measure with a significant Fama-MacBeth t-statistic. Thus, implied idiosyncratic volatility dominates the other measures as a predictor of future idiosyncratic volatility.[9]

Fama-MacBeth Future Return Estimation

[9] Our results join others demonstrating the importance of volatility forecasts for predicting future realized volatility.

To test the relation between firm future returns and idiosyncratic risk, we first revisit the sample and results presented in Ang, Hodrick, Xing, and Zhang (2006). They find a negative correlation between future returns and historical realized idiosyncratic volatility. They do not, however, examine relations at the firm level or control for firm characteristics. We estimate at the firm level and on a monthly basis the relation between future returns and historical idiosyncratic volatility, using Fama and MacBeth (1973) regressions and firm-specific controls:

$$r_{j,t+1} = \alpha + \lambda_1 \sigma_{RV\_idio,j,t} + \lambda_2 LSIZE_{j,t} + \lambda_3 LBM_{j,t} + \lambda_4 r_{j,t}$$

$$+ \lambda_5 r_{j,t-11:t-1} + \lambda_6 r_{j,t-35:t-12} + \lambda_7 \beta_{j,t} + \varepsilon_{j,t+1} \tag{11}$$

where r_j is the return for stock *j*, LSIZE is the log of market equity, and LBM is the log of book-to-market equity. Both are measured at the end of the month and book-to-market equity is calculated as defined previously. The three returns that are independent variables precede the dependent variable return by one month, the eleven months prior to the first return independent variable, and the 24 months prior to the second independent return variable, respectively. α and λ s are coefficients to be estimated. The cross-sectional regressions are estimated with and without the firm characteristic controls, and over two separate time periods. The first is the same time period used in Ang et al., 1963-2000. The second is from 1996 through June 2005 and corresponds to the period of our options data. For both periods we use all available firms and do not confine the analysis to firms with traded options. We present the results in Table IV.[10]

The results for the Ang et al. (2006) time period show a negative but insignificant coefficient on realized idiosyncratic volatility. This holds regardless of whether the firm controls are included or not. The direction of the coefficient sign is consistent with their findings, but the lack of statistical significance is troubling. The results for the period 1996 through June 2005 also show no important role for realized idiosyncratic volatility. Without the firm controls, the coefficient on realized idiosyncratic volatility has a positive, insignificant sign. With firm controls, the coefficient sign is negative but it is still insignificant. From the results in Table IV there is little we can conclude about the relation between idiosyncratic realized volatility and future returns. This is consistent with the findings in Bali and Cakici (2007).

[10] In an earlier version of this paper we included the macro factors dividend-price ratio, relative Treasury bill rate, term spread, and default spread, along with implied idiosyncratic volatility, to examine the time-series properties of each individual firm. Implied idiosyncratic volatility retained a positive and significant effect on future returns.

Using stocks with traded options, we examine the relation between implied idiosyncratic risk and future returns. The estimation period is 1996 through June 2005. Equation (11) is modified to include implied idiosyncratic volatility, the statistical idiosyncratic volatility forecasts, the short sale constraint, ORW_Ratio, and the log of monthly open interest, OI:

$$\begin{aligned} r_{j,t+1} = \alpha + \lambda_1 \sigma_{RV\_idio,j,t} + \lambda_2 LSIZE_{j,t} + \lambda_3 LBM_{j,t} + \lambda_4 r_{j,t} + \lambda_5 r_{j,t-11:t-1} + \lambda_6 r_{j,t-35:t-12} \\ + \lambda_7 \beta_{j,t} + \lambda_8 \sigma_{IV\_idio,j,t} + \lambda_9 \sigma_{EG\_idio,j,t} + \lambda_{10} \sigma_{AR\_idio,j,t} + \lambda_{11} ORW\_Ratio_{j,t} + \lambda_{12} OI_{j,t} + \varepsilon_{j,t+1} \end{aligned} \quad (12)$$

In Table V we present the estimation results from various combinations of independent variables from equation (12), using the Fama and MacBeth (1973) methodology.[11] The results demonstrate a strong positive and significant relation between implied idiosyncratic volatility and future returns; coefficients are significant at the 1% level regardless of the specification or sample. In contrast, the coefficients on realized idiosyncratic volatility are insignificant in all specifications.[12] The coefficients on the statistical forecasts from the EGARCH and AR(2) models are positive and significant when the respective variables are individually combined with realized idiosyncratic volatility. However, when all four idiosyncratic volatility measures are included together, the only significant coefficient is for implied volatility. The coefficients for size, book-to-market equity, and returns for the prior month are consistently significant. Also, the coefficients on size, book-to-market equity, and all three lagged return measures are similar to those reported in Table IV for the period 1996 through June 2005. The coefficient on beta is positive and, when idiosyncratic volatility is included, the relation with returns gets stronger.

The findings suggest that high implied idiosyncratic volatility in month *t* should result in high returns in month *t+1*. Intuitively, the results are pleasing since there is a positive theoretical relation between volatility and returns and because the idiosyncratic portion of implied volatility makes up the majority of total implied volatility. Additionally, implied volatility, by definition, is a forward looking measure, while realized volatility is an ex-post measure. Thus, the fact that realized volatility is insignificant is not surprising.

[11] We also examined results excluding stocks below $5 with little change in results.
[12] Our sample of stocks with options consists of relatively large firms. The insignificant coefficients on realized volatility are consistent with weak or insignificant results for large stocks found by Ang, Hodrick, Xing, and Zhang (2006), Bali and Cakici (2007), and Fu (2007).

The coefficient signs for the short-sale constraint and the log of option open interest are also in the correct direction.[13] The coefficient for the short-sales constraint is significantly negative at the 1% level, implying that the higher the constraint, the lower the next period's return. This suggests that stocks that are overvalued due to difficulty in shorting the stock should have lower future returns. Option open interest is a proxy for liquidity, and the more liquid or active the option markets are, the less likely the stock suffers from short-sale constraints. Open interest always has a positive coefficient that is significant at the l% level, suggesting that firms with more active options suffer less from short-sale constraints. However, this variable is highly correlated with size. When $\lambda_2 = 0$, the coefficient on open interest is insignificantly negative.

Our results so far for idiosyncratic risk are in contrast to those of Ang et al. (2006) and in agreement with economic theories which posit that higher idiosyncratic volatility stocks should earn higher expected returns. Our findings in Table III show that implied idiosyncratic volatility is an efficient but upwardly biased predictor of future realized volatility. Our results in Table V show that higher implied idiosyncratic volatility results in higher future returns. This suggests that there is a premium for bearing implied idiosyncratic volatility, a market price of idiosyncratic volatility risk that may contribute to higher future returns. Additionally, it appears that stocks suffering from high short-sale constraints or low liquidity underperform stocks with limited short-sale constraints or high liquidity.

PORTFOLIO RETURN ANALYSIS

Analysis Based on Single Sorts

If there is a premium for bearing idiosyncratic volatility risk, traditional asset pricing models will not capture this premium, and the results will appear to be low (high) abnormal returns for low (high) idiosyncratic risk stocks. Since our prior results show that implied idiosyncratic volatility forecasts future idiosyncratic volatility and returns better than those forecasts from statistical models, we further analyze the former. We now examine, in a portfolio context, returns as a function of idiosyncratic risk. We consider the equal and value-weighted returns to portfolios formed by sorting individual securities on the basis of implied idiosyncratic

[13] We also estimated equation (12) with option trading volume instead of open interest. Volume and open interest are highly correlated and the results did not change.

volatility, realized idiosyncratic volatility, the ORW_Ratio, and the log of option open interest. Our goal is to see whether returns are more closely associated with implied or realized idiosyncratic volatility and if short-sale constraints affect this relation. To form the portfolios, our stocks with traded options are independently sorted at the end of each month into five implied idiosyncratic volatility, five realized idiosyncratic volatility, five ORW_Ratio, and five log of option open interest quintiles. We then examine subsequent one-month returns.

Mean values of various firm characteristics are presented for each of the portfolios in Table VI. Size and past monthly returns are inversely related to the two idiosyncratic volatility measures, while implied volatility and ORW_Ratio tend to be positively related to the two volatility measures. Size and beta tend to be positively related to option open interest, while book-to-market equity and the ORW_Ratio tend to be negatively related to open interest. Generally, relations in Table VI are as expected.

For each of the four sorted parameters, we compute the time-series mean and standard deviation of the monthly returns over the entire sample period from January 1996 to June 2005. Additionally, we form high minus low, zero-cost portfolios using the top and bottom quintiles of the respective sorts. The four portfolios are formed as: (1) long high implied idiosyncratic volatility and short low implied idiosyncratic volatility stocks, (2) long high realized idiosyncratic volatility and short low realized idiosyncratic volatility stocks, (3) long high ORW_Ratio and short low ORW_Ratio stocks, and (4) long high option open interest and short low option open interest stocks. Abnormal returns are alphas that are estimated for all portfolios by regressing equal and value-weighted monthly returns on the four-factor model including the Fama and French (1993) market, size, and value factors, and the Carhart (1997) momentum factor. The regression equation is:

$$r_{p,t} = \alpha_p + \psi_{1,p} MKT_t + \psi_{2,p} SMB_t + \psi_{3,p} HML_t + \psi_{4,p} UMD_t + \varepsilon_{p,t} \tag{13}$$

where r_p is the return to a given portfolio.[14]

In Table VII we report the equal and value-weighted mean monthly raw returns, standard deviation of returns, and four-factor alphas with t-statistics in parentheses. Across the idiosyncratic volatility portfolios the highest raw returns and standard deviations are for the top

[14] We repeat all the analyses excluding stocks whose prices are less than or equal to five dollars on the portfolio formation date *t*. We do this out of concern that low-priced stocks may distort the results for the broad sample of stocks. The results are similar to those presented. Additionally, CAPM alphas and Fama and French (1993) three-factor alphas were calculated. The results are similar to the four-factor alphas we report.

quintile portfolios. The returns decrease somewhat monotonically for both the equal and value-weighted portfolios, although the lowest volatility quintiles have higher returns then the next to lowest quintiles. For the ORW_Ratio portfolios, the highest raw returns are for the lowest quintile, for both the equal and value-weighted portfolios. For the log of option open interest portfolios, higher returns tend to be associated with lower portfolios.

For the highest implied idiosyncratic volatility quintiles there are significantly positive alphas at the 1% (5%) level for equal (value) weighting. For realized idiosyncratic volatility, the equal-weighted highest volatility portfolio has a significantly positive alpha at the 5% level. The results of high-minus-low portfolios show that the alphas for implied idiosyncratic volatility are positive and significant, while those of realized idiosyncratic volatility are not significant. For the equal (value) weighted zero-cost portfolios, the coefficient is over three (four) times as big for implied volatility then for realized volatility and the t-statistic is significant at the 5% level. This provides preliminary evidence suggesting there is information in the idiosyncratic component of implied volatility beyond that of realized volatility. The general portfolio findings for idiosyncratic volatility are contrary to the Ang et al. (2006) findings, but consistent with our Fama-MacBeth cross-sectional analysis for individual securities.

A question arises as to how to interpret these significant alphas. We have been calling them abnormal returns because they are unexplained by the four-factor model. However, there may be an omitted variable in the model. As given, the four-factor model is structured to price systematic risk, not to explain returns on portfolios sorted on idiosyncratic risk. Thus, an alternative explanation is that idiosyncratic risk should be a priced factor in asset pricing models. In this case, the returns would not be abnormal.

For the ORW_Ratio quintiles, low constrained portfolios outperform high constrained portfolios for both equal and value-weighted portfolios. High-minus-low portfolios have negative and significant alphas at the 1% level. This is consistent with earlier results, implying high short-sale constrained portfolios are overvalued. There is the same pattern in alphas across option open interest quintiles, in this case refuting the notion that more active options are less constrained than those that are less active.[15]

In Figure 1 we show the cumulative dollar values to investing in each of the following value-weighted strategies. The first is buying and holding the S&P 500. The dollar values of the

[15] Further analysis shows that the low open interest firms tend to be small firms, with associated high returns.

S&P 500 are calculated by initially investing $10,000 in 1996 and holding through June 2005. The second strategy is going long high implied idiosyncratic volatility and short low implied idiosyncratic volatility stocks. The third is going long high realized idiosyncratic volatility and short low realized idiosyncratic volatility stocks. The idiosyncratic risk portfolios have $5000 invested long in the high volatility stocks and $5000 invested short in the low volatility stocks. As seen in the figure, returns based on implied idiosyncratic volatility are the best, followed by the S&P 500, and then realized idiosyncratic volatility. $10,000 invested in the implied idiosyncratic volatility portfolio at the beginning of 1996 increases in value to over $35,000 at the end of June, 2005. This is over three times the cumulative return of the equivalent S&P 500. The realized idiosyncratic volatility portfolio has almost no return over the period.

Analysis Based on Double Sorts

The results we report in Table VII suggest that implied idiosyncratic volatility can predict future stock returns better than realized idiosyncratic volatility. However, it is still not clear if these represent two separately useful explanatory variables or if one risk measure subsumes the other. In cross-sectional regressions with individual securities we found that implied idiosyncratic volatility predicted future returns, but realized idiosyncratic volatility does not. We now investigate, in the context of a portfolio analysis similar to Ang et al. (2006), if implied idiosyncratic volatility is a better predictor of future returns than past realized idiosyncratic volatility.

We test this relation using a double-sort procedure. At the end of every month we sort stocks into five portfolios, with equal numbers of securities in each, based on their realized idiosyncratic volatility over the prior 30 days. We then form 25 portfolios, with equal numbers of securities, by sorting the stocks in each of these five portfolios into five sub-portfolios based on the implied idiosyncratic volatility measured at the end of month.[16] For each of the five initial realized idiosyncratic volatility sorted portfolios, we form both equal and value -weighted zero-cost portfolios that are long the high quintile of implied idiosyncratic volatility stocks and short the low quintile of implied idiosyncratic volatility stocks. We measure returns for the next

[16] The double sorting procedure results in approximately 50 stocks within each of the 25 portfolios.

month for each of these ten portfolios. Finally, we regress the returns of these ten zero-cost portfolios on the four-factor model and focus on the alphas (abnormal returns). We report the alphas and their t-statistics in the first row of Panel A in Table VIII. For both the equal and value-weighted portfolios the alphas are positive and significant, at the 1% level, for all high-minus-low implied idiosyncratic volatility portfolios across realized idiosyncratic volatility quintiles.

We then reverse the sorting process, sorting first on implied idiosyncratic volatility and then on realized idiosyncratic volatility, yielding 25 new portfolios. We form zero-cost portfolios for each of the five implied idiosyncratic volatility portfolios as long the high quintile of realized idiosyncratic volatility stocks and short the low quintile of realized idiosyncratic volatility stocks. We regress the returns of these zero-cost portfolios on the four-factor model and report the alphas and t-statistics in the second row of Panel A in Table VIII. All alphas are negative and insignificant. This indicates that realized idiosyncratic volatility does not significantly impact future returns once implied idiosyncratic risk is controlled for. Thus, results indicate that implied idiosyncratic volatility is important for predicting future returns, but realized idiosyncratic volatility is uninformative.

Since implied idiosyncratic volatility is positively related to future stock returns, we next investigate if this effect is pervasive across the firm characteristics of market value of equity (SIZE), book-to-market equity (B/M), short-sale constraints (ORW_Ratio), and liquidity (OI). Each month stocks are independently sorted by SIZE, B/M, ORW_Ratio, and OI, making five portfolios for each containing equal numbers of stocks. We sort the securities in each of these portfolios by implied idiosyncratic volatility and form five sub-portfolios for each. We again place equal numbers of stocks in each portfolio. Equal and value-weighted portfolios are formed for each of the 25 portfolios for the four double sorts and the returns are measured for the month following the portfolio formation date. For each of the ten SIZE, B/M, ORW_Ratio, and OI categories, zero-cost portfolios are formed that are long the high quintile of implied idiosyncratic volatility stocks and short the low quintile of implied idiosyncratic stocks. There are 20 equal and 20 value-weighted zero-cost portfolios. We regress the returns for each of these zero-cost portfolios on the four-factor model.

We report the alphas and their t-statistics from these regressions in Table VIII, Panel B. For the sort on SIZE there are positive and significant alphas, at the 1% level, for the smallest

SIZE quintile for both equal and value-weighted portfolios. All other portfolios have insignificant alphas. For the sort on B/M there are positive and significant alphas, at the 1% level, for the two highest B/M quintiles for both equal and value-weighted portfolios. All other portfolios have insignificant alphas. Thus, small size and high B/M firms that have high implied idiosyncratic volatility outperform small size and high B/M firms with low implied idiosyncratic volatility. These findings suggest that the well-documented size and book-to-market equity effects may be linked to a positive relation between returns and implied idiosyncratic risk.

In Figure 2 we show the cumulative dollar values to the S&P 500, the small SIZE quintile high minus low implied idiosyncratic volatility portfolio, and the high B/M quintile high minus low implied idiosyncratic volatility portfolio. We assume a $10,000 investment in each strategy, starting in 1996. The figure is consistent with the results in Table VIII, Panel B, and show the strong, consistent performance of the high B/M and small size firms relative to the market.

The alphas across the ORW_Ratio show no discernable pattern. For the ten ORW_Ratio portfolios, five of the alphas are significantly positive at the 1% level, and two of the alphas have a negative sign. For the OI portfolios, seven out of the ten have positive and significant alphas at the 5% level or better, with the higher quintiles insignificant. Thus, low open interest firms may be more closely associated with the positive relation between returns and implied idiosyncratic volatility.

CONCLUSION

Prior studies disagree about the type of relation idiosyncratic risk has with future returns. Douglas (1969), Lintner (1965), Lehmann (1990) and Malkiel and Xu (2006) find a positive relation, but Ang, Hodrick, Xing, and Zhang (2006) find a negative relation. All of these studies use historical realized volatility measures. We believe that implied idiosyncratic volatility from option prices represents a better measure of the market's anticipated volatility and provides a better way to examine the relation between future returns and idiosyncratic risk.

We employ stock and option prices from firms with traded options from 1996 through June, 2005. At the firm level implied idiosyncratic volatility is an upward biased predictor of realized idiosyncratic volatility, even in the presence of past realized volatility. Implied idiosyncratic volatility is a stronger predictor of future idiosyncratic volatility than idiosyncratic volatility forecasts from statistical models. We control for firm characteristics in a Fama-MacBeth approach and find that implied idiosyncratic risk positively predicts future stock returns while realized idiosyncratic volatility does not. For predicting returns, implied idiosyncratic volatility dominates idiosyncratic volatility forecasts from statistical models. Also, the Ofek, Richardson, and Whitelaw (2004) measure of short-sale constraints has a strong negative effect on future returns. We use portfolio analysis and four-factor regressions to show that implied idiosyncratic volatility positively predicts future returns. When we directly compare implied and realized idiosyncratic volatility to each other, implied idiosyncratic volatility is positively linked to future returns but realized idiosyncratic risk is not. This suggests that idiosyncratic risk matters as long as it is derived from the forward measure.

Controlling for size and book-to-market equity, we find that the strength of idiosyncratic volatility is strongly tied to small size and high book-to-market equity firms. So, implied idiosyncratic risk predicts future returns and zero-cost portfolios based on implied idiosyncratic risk sorts lead to significant four-factor alphas. This may be indicative of abnormal returns or, alternatively, may mean that idiosyncratic risk is an omitted risk factor.

Our evidence is consistent with prior studies advocating a positive relation between idiosyncratic risk and future returns, but in contrast to the results in Ang, Hodrick, Xing, and Zhang (2006). This may be directly related to the information content in implied volatility, which by definition is a forward looking measure. Realized volatility measures are uninformative in the presence of implied volatility measures both for predicting future volatility and returns. Thus, our main contribution is to provide an appropriate ex-ante measure of risk which demonstrates the positive relation between risk and return.

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TABLE I

Descriptive Statistics of Option Firm Sample

This table reports the mean, median, 5th percentile, and 95th percentile values for firm characteristics within the option firm sample from January 1996 through June 2005. All available monthly data for each firm is used. SIZE is price times shares outstanding, B/M is book-to-market equity, β is calculated from the market model using the past 60 months of returns, ORW_Ratio is the Ofek, Richardson, and Whitelaw (2004) short-sales constraint ratio, and OI is the log of option open interest. The six volatility measures are: 1) σ_{IV}, the average firm implied volatility, 2) σ_{RV}, the average firm realized volatility, calculated as the annualized standard deviation of the current month's daily returns, 3) σ_{IV_idio}, the idiosyncratic portion of implied volatility coming from equation (2), 4) σ_{RV_idio}, the idiosyncratic portion of realized volatility calculated using the market model, 5) σ_{EG_idio}, the idiosyncratic portion of volatility coming from the EGARCH model in equation (5), and 6) σ_{AR_idio}, the idiosyncratic portion of volatility coming from the AR(2) model in equation (6). Obs by firm is the average number of observations for any given firm in the sample. Firms by month is the average number of firms for any given month in the sample.

	mean	median	5th	95th
SIZE ($ thousands)	6,165	1,283	179	23,633
B/M	0.52	0.45	0.06	1.04
Beta	1.20	1.03	0.27	2.66
ORW_Ratio	0.08	0.06	-1.05	1.21
OI	11.22	11.09	8.25	14.71
σ_{IV}	50.7%	46.6%	27.5%	85.6%
σ_{RV}	48.1%	43.2%	24.4%	85.5%
σ_{IV_idio}	35.3%	32.9%	15.4%	64.8%
σ_{RV_idio}	38.3%	35.1%	19.7%	67.4%
σ_{EG_idio}	39.0%	34.1%	13.9%	79.1%
σ_{AR_idio}	43.1%	37.9%	16.7%	85.9%
Obs by Firm	75	79	17	114
Firms by Month	1,704	1,643	1,421	2,018

TABLE II

Forecasting Future Realized Volatility

This table shows the estimation results of the following models:

$$\sigma_{RV\_M,t+1} = \alpha + \xi_1 \sigma_{IV\_M,t} + \xi_2 \sigma_{RV\_M,t} + \varepsilon_{t+1}$$

$$\sigma_{RV,j,t+1} = \alpha_j + \xi_{1,j} \sigma_{IV,j,t} + \xi_{2,j} \sigma_{RV,j,t} + \varepsilon_{j,t+1}$$

where σ_{RV} is realized volatility, calculated as the annualized standard deviation of month *t* daily returns, σ_{IV} is the implied volatility for month *t*, ε is the residual, and α and ξ represent coefficients to be estimated. The first model is for the market and the second is for individual firms. Panel A presents the market coefficients, with t-statistics in parentheses, where realized volatility is for the S&P 500 index and implied volatility is the VIX index. Panel B presents the firm-level coefficients, with implied volatilities from OptionMetrics. For the firm-level results, we report the mean and median coefficient estimates for the 1310 firms that have at least 60 observations. The numbers in brackets are the 25th and 75th percentile values of the coefficients. In the first column ξ_2 is restricted to zero. In parentheses are t-statistics from reverse Fama and Macbeth (1973) regressions, where the model is estimated for each firm through time and then t-statistics are from the cross-sectional distribution of coefficients across firms. The sample period is from 1996 through June 2005. * is significant at the 5% level. ** is significant at the 1% level.

	Panel A: Market Estimates		Panel B: Firm-Level Estimates	
			Mean/Median	Mean/Median
	$\xi_2 = 0$		$\xi_2 = 0$	
ξ_1	0.728	0.599	0.731 / 0.738	0.611 /0.603
	(10.15)**	(5.43)**	[0.540 / 0.918]	[0.408 / 0.809]
			(51.94)**	(77.62)**
ξ_2		0.160		0.136/0.124
		(1.52)		[0.021 / 0.560]
				(27.66)**
□	-0.003	-0.001	0.099 / 0.087	0.092 / 0.080
	(0.21)	(0.02)	[0.029 / 0.154]	[0.029 / 0.137]
			(18.63)**	(33.35)**

TABLE III

Forecasting Firm Future Realized Idiosyncratic Volatility

This table shows the estimation results of the following model on individual stocks:

$$\sigma_{RV\_idio,j,t+1} = \alpha_j + \psi_{1,j}\sigma_{IV\_idio,j,t} + \psi_{2,j}\sigma_{EG\_idio,j,t} + \psi_{3,j}\sigma_{AR\_idio,j,t} + \psi_{4,j}\sigma_{RV\_idio,j,t} + \varepsilon_{j,t+1}$$

The four volatility measures are: 1) σ_{IV_idio}, the idiosyncratic portion of implied volatility coming from equation (2), 2) σ_{EG_idio}, the forecast of idiosyncratic volatility from the EGARCH model in equation (5), 3) σ_{AR_idio}, the forecast of idiosyncratic volatility from the AR(2) model in equation (6), and 4) σ_{RV_idio}, the idiosyncratic portion of realized volatility calculated from the market model. ε is the residual, and α and ψ represent coefficients to be estimated. We report the mean and median coefficient estimates for the 1255 firms that have at least 60 observations. The numbers in brackets are the 25th and 75th percentile values of the coefficients. In parentheses are t-statistics from reverse Fama and MacBeth (1973) regressions, where the model is estimated for each firm through time and then t-statistics are from the cross-sectional distribution of coefficients across firms. The sample period is from 1996 through June 2005. * is significant at the 5% level. ** is significant at the 1% level.

	Mean/Median	Mean/Median	Mean/Median	Mean/Median
ψ_1	0.365 / 0.354			0.330 / 0.312
	[0.197 / 0.519]			[0.149 / 0.485]
	(2.84)**			(2.43)*
ψ_2		0.182 / 0.185		-0.006 / 0.001
		[0.019 / 0.337]		[-0.194 / 0.188]
		(1.39)		(0.87)
ψ_3			0.296 / 0.277	0.174 / 0.157
			[0.093 / 0.492]	[-0.106 / 0.419]
			(1.74)	(1.04)
ψ_4	0.184 / 0.185	0.323 / 0.326	0.295 / 0.298	0.157 / 0.156
	[0.067 / 0.305]	[0.199 / 0.448]	[0.171 / 0.420]	[0.044 / 0.269]
	(1.95)	(3.48)**	(3.18)**	(1.73)
α	0.169 / 0.150	0.186 / 0.150	0.148 / 0.115	0.133 / 0.104
	[0.106 / 0.210]	[0.098 / 0.239]	[0.055 / 0.206]	[0.049 / 0.184]
	(3.76)**	(2.69)**	(1.96)*	(1.74)

TABLE IV

Fama-MacBeth Estimation with All Available Stocks

This table shows summary results of the Fama-MacBeth monthly cross-sectional regressions with future one-month returns the dependent variable,

$$r_{j,t+1} = \alpha + \lambda_1 \sigma_{RV\_idio,j,t} + \lambda_2 LSIZE_{j,t} + \lambda_3 LBM_{j,t} + \lambda_4 r_{j,t}$$

$$+ \lambda_5 r_{j,t-11:t-1} + \lambda_6 r_{j,t-35:t-12} + \lambda_7 \beta_{j,t} + \varepsilon_{j,t+1}$$

σ_{RV_idio} is the idiosyncratic portion of realized volatility calculated from equation (3). LSIZE is the log of market equity and LBM is the log of the ratio of book-to-market equity. The three returns that are independent variables precede the dependent variable return by one month, the eleven months prior to the first return independent variable, and the 24 months prior to the second independent return variable, respectively. β is calculated from the market model using the past 60 months of returns. α and λ s are the coefficients to be estimated. The cross-sectional regressions are estimated over two separate time periods. The first is the same time period used in Ang et al. (2006), 1963-2000. The second time period is from 1996 through June 2005 and corresponds to the period of our options data. The time-series average of the coefficient estimates are reported with the associated t-statistics in parentheses. * is significant at the 5% level. ** is significant at the 1% level.

	1963 - 2000			1996 - June 2005	
σ_{RV_idio}	-0.0576	-0.0492		0.0182	-0.0575
	(1.19)	(1.42)		(0.19)	(1.05)
LSIZE		-0.0009			-0.0016
		(2.15)*			(1.69)
LBM		0.0031			0.0034
		(4.14)**			(2.21)*
r_t		-0.0725			-0.0504
		(19.01)**			(5.54)**
$r_{t-11:t-1}$		0.0072			0.0023
		(4.15)**			(0.63)
$r_{t-35:t-12}$		-0.0035			-0.0051
		(4.36)**			(2.52)*
β		-0.0054			0.0021
		(0.76)			(0.84)

TABLE V

Fama-MacBeth Estimation on Stocks with Options

This table shows summary results of the Fama-MacBeth monthly cross-sectional regressions with future one-month returns as the dependent variable,

$$r_{j,t+1} = \alpha + \lambda_1 \sigma_{RV\_idio,j,t} + \lambda_2 LSIZE_{j,t} + \lambda_3 LBM_{j,t} + \lambda_4 r_{j,t} + \lambda_5 r_{j,t-11:t-1} + \lambda_6 r_{j,t-35:t-12}$$
$$+ \lambda_7 \beta_{j,t} + \lambda_8 \sigma_{IV\_idio,j,t} + \lambda_9 \sigma_{EG\_idio,j,t} + \lambda_{10} \sigma_{AR\_idio,j,t} + \lambda_{11} ORW\_Ratio_{j,t} + \lambda_{12} OI_{j,t} + \varepsilon_{j,t+1}$$

σ_{RV_idio} is the idiosyncratic portion of realized volatility from equation (3). LSIZE is the log of market equity and LBM is the log of book-to-market equity. The three returns that are independent variables precede the dependent variable by one month, the eleven months prior to the first return independent variable, and the 24 months prior to the second independent return variable, respectively. β is calculated from the market model using the past 60 months of returns, σ_{IV_idio} is the idiosyncratic portion of implied volatility from equation, σ_{EG_idio} is the idiosyncratic portion of volatility from the EGARCH model in equation (5), σ_{AR_idio} is the idiosyncratic portion of volatility from the AR(2) model in equation (6), ORW_Ratio is the Ofek, Richardson, and Whitelaw (2004) short sales constraint ratio, and OI is the log of monthly firm option open interest plus one. The time-series average of the coefficient estimates are reported with t-statistics in parentheses. The sample period is 1996 - June 2005. The average number of firms per month is 1467. * is significant at the 5% level. ** is significant at the 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
σ_{RV_idio}			0.0116		-0.0021	0.0100	0.0092	-0.0026	-0.0036	0.0007
			(1.30)		(0.43)	(0.69)	(1.58)	(0.55)	(0.75)	(0.14)
LSIZE	-0.0048	-0.0040	-0.0035	-0.0019	-0.0018	-0.0032	-0.0030	-0.0018	-0.0050	
	(4.54)**	(4.39)**	(4.20)**	(2.31)*	(2.18)*	(3.88)**	(3.59)**	(2.23)*	(5.09)**	
LBM	0.0053	0.0058	0.0059	0.0065	0.0064	0.0049	0.0049	0.0051	0.0049	0.0063
	(4.11)**	(4.00)**	(4.52)**	(5.00)**	(5.05)**	(4.92)**	(5.07)**	(5.43)**	(5.17)**	(6.29)**
r_t	-0.0240	-0.0244	-0.0224	-0.0216	-0.0216	-0.0214	-0.0221	-0.0208	-0.0193	-0.0185
	(2.14)*	(2.46)*	(2.47)*	(2.38)*	(2.41)*	(2.54)**	(2.61)**	(2.56)**	(2.38)*	(2.28)*
$r_{t-11:t-1}$	0.0027	0.0023	0.0022	0.0023	0.0022	0.0009	0.0007	0.0010	0.0019	0.0017
	(1.18)	(1.11)	(1.09)	(1.14)	(1.13)	(0.47)	(0.37)	(0.52)	(0.96)	(0.89)
$r_{t-35:t-12}$	-0.0014	-0.0005	-0.0005	-0.0006	-0.0006	-0.0008	-0.0008	-0.0008	-0.0006	-0.0004
	(1.77)	(0.83)	(0.89)	(1.02)	(1.01)	(1.44)	(1.54)	(1.45)	(1.20)	(0.82)
β		0.0017	0.0031	0.0051	0.0061	0.0025	0.0020	0.0063	0.0040	0.0058
		(0.53)	(0.95)	(1.42)	(1.77)	(0.80)	(0.65)	(1.81)	(1.16)	(1.75)
σ_{IV_idio}				**0.0350**	**0.0371**			**0.0352**	**0.0307**	**0.0426**
				(4.20)**	**(5.67)****			**(5.56)****	**(4.92)****	**(5.77)****
σ_{EG_idio}						0.0124		0.0031		
						(2.20)*		(0.45)		
σ_{AR_idio}							0.0148	0.0034		
							(2.42)*	(0.46)		
ORW_Ratio									-0.0027	-0.0027
									(4.43)**	(4.33)**
OI									0.0026	-0.0001
									(4.17)**	(0.18)
α	0.1276	0.1113	0.1072	0.0777	0.0745	0.093	0.0904	0.0651	0.0854	0.0565
	(8.22)**	(8.07)**	(7.17)**	(4.45)**	(4.41)**	(6.21)**	(5.79)**	(3.85)**	(5.28)**	(4.82)**

TABLE VI

Mean Values of Firm Characteristics for Sorted Portfolios

This table reports mean values of characteristics of portfolios whose securities are sorted into quintiles four different ways. Securities are sorted by σ_{IV_idio} , the idiosyncratic portion of implied volatility from equation (2), σ_{RV_idio} , the idiosyncratic portion of realized volatility from equation (3), ORW_Ratio, the Ofek, Richardson, and Whitelaw (2004) short sales constraint ratio, and OI, the log of monthly firm option open interest plus one. Size is price times shares outstanding, B/M is book-to-market equity, implied volatilities are from OptionMetrics, beta is calculated from the market model using the past 60 months of returns, OI is the log of option open interest, and the past one month return is from the prior month.

		1(Low)	2	3	4	5(High)
σ_{IV_idio}	Size	13,900	11,500	5,990	3,068	1,302
	B/M	0.40	0.40	0.42	0.45	0.50
	Implied Volatility	36.4%	33.3%	40.7%	51.4%	73.0%
	Beta	1.55	0.89	0.94	1.07	1.27
	ORW_Ratio	0.03	0.01	0.02	0.08	0.15
	OI	11.79	11.39	11.18	11.11	11.15
	Past Month Return	1.33%	1.20%	1.29%	1.06%	0.26%
σ_{RV_idio}	Size	10,890	10,700	7,665	4,509	2,264
	B/M	0.42	0.41	0.43	0.45	0.44
	Implied Volatility	36.4%	36.4%	42.9%	52.3%	66.9%
	Beta	1.18	0.87	1.00	1.20	1.50
	ORW_Ratio	0.03	0.02	0.06	0.09	0.09
	OI	11.25	11.30	11.32	11.32	11.44
	Past Month Return	1.45%	1.30%	1.23%	1.02%	0.15%
ORW_Ratio	Size	5,392	7,245	8,613	5,860	3,712
	B/M	0.49	0.41	0.39	0.42	0.49
	Implied Volatility	52.6%	48.6%	47.0%	49.4%	56.0%
	Beta	1.11	1.14	1.11	1.15	1.24
	ORW_Ratio	-0.99	-0.17	0.07	0.31	1.18
	OI	10.97	11.34	11.49	11.30	10.98
	Past Month Return	0.26%	1.28%	1.67%	1.28%	0.61%
OI	Size	1,040	1,503	2,211	3,698	22,400
	B/M	0.55	0.51	0.45	0.41	0.33
	Implied Volatility	47.1%	50.2%	52.6%	53.7%	49.9%
	Beta	0.94	1.08	1.16	1.24	1.31
	ORW_Ratio	0.10	0.10	0.09	0.07	0.04
	OI	8.70	10.23	11.12	12.10	13.94
	Past Month Return	1.17%	1.23%	1.22%	1.00%	0.52%

TABLE VII

Idiosyncratic Volatility Portfolio Percentage Returns

This table shows the raw and abnormal percentage monthly returns to portfolios formed by sorting stocks based on implied idiosyncratic volatility (σ_{IV_idio}) and realized idiosyncratic volatility (σ_{RV_idio}) as defined by equations (2) and (3), respectively. At the end of each month *t*, firms are sorted into one of five idiosyncratic volatility portfolios. Portfolio 1 (5) contains low (high) idiosyncratic volatility firms. Portfolio returns are calculated over the following month *t+1*. This is done separately for both implied and realized idiosyncratic volatility. The raw monthly return mean and standard deviation (sd) are given for both equal and value-weighted portfolios. The alpha is from the four-factor Fama-French and Cahart model, and is reported for both the equal and value-weighted portfolio. The high-low alpha is from a regression of the returns to portfolio 5 minus those of portfolio 1. A similar analysis is conducted for portfolios formed by sorting securities on the basis of ORW_Ratio, which is the Ofek, Richardson, and Whitelaw (2004) short sales constraint ratio, and the log of option open interest plus one, OI. The t-statistics are given in parentheses. * is significant at the 5% level. ** is significant at the 1% level.

		Equal Weighted						Value Weighted					
		1(Low)	2	3	4	5(High)	High-Low	1(Low)	2	3	4	5(High)	High-Low
σ_{IV_idio}	mean	0.76	0.40	0.72	1.15	2.28		0.69	0.36	0.66	1.03	2.10	
	sd	5.69	4.15	4.90	6.23	9.12		5.55	4.05	4.81	6.13	8.95	
	Alpha	0.116	-0.019	0.165	0.532	1.509	1.393	0.061	-0.050	0.120	0.429	1.360	1.299
	t-stat	(0.37)	(0.08)	(0.75)	(1.77)	(2.76)**	(2.05)*	(0.20)	(0.22)	(0.57)	(1.47)	(2.55)*	(1.96)*
σ_{RV_idio}	mean	1.03	0.75	1.09	1.36	0.33		0.93	0.67	0.98	1.02	1.20	
	sd	4.72	4.44	5.09	6.49	9.23		4.58	4.35	4.98	6.33	9.04	
	alpha	0.156	-0.164	0.179	0.262	0.541	0.385	0.093	-0.218	0.097	0.163	0.396	0.303
	t-stat	(0.66)	(1.00)	(0.87)	(1.31)	(2.07)*	(1.11)	(0.41)	(1.39)	(0.49)	(0.84)	(1.54)	(0.89)
ORW_Ratio	mean	2.30	0.90	0.46	0.65	1.02		2.09	0.81	0.38	0.58	0.89	
	sd	6.31	5.98	5.77	6.11	6.63		6.03	5.81	5.61	5.91	6.36	
	alpha	1.448	0.067	-0.290	-0.157	0.129	-1.319	1.267	-0.005	-0.346	-0.211	0.024	-1.243
	t-stat	(6.00)**	(0.47)	(2.01)*	(1.04)	(0.58)	(6.36)**	(5.60)**	(0.04)	(2.52)*	(1.48)	(0.11)	(6.32)**
OI	mean	1.54	1.42	1.14	0.78	0.44		1.42	1.29	1.04	0.70	0.37	
	Sd	5.17	5.78	6.31	6.73	7.05		4.99	5.57	6.04	6.42	6.74	
	alpha	0.605	0.510	0.291	0.035	-0.242	-0.848	0.507	0.402	0.202	-0.034	-0.294	-0.801
	t-stat	(3.06)**	(2.99)**	(1.56)	(0.22)	(1.30)	(3.65)**	(2.66)**	(2.47)*	(1.12)	(0.24)	(1.70)	(3.60)**

TABLE VIII

Double-Sorted High Minus Low Percentage Portfolio Returns

This table shows the abnormal percentage monthly returns from the four-factor Fama-French and Carhart model on high minus low double-sorted equal and value- weighted portfolios. At the end of each month firms are sorted into one of five portfolios, and then each portfolio is sorted into five additional portfolios for a total of 25 portfolios. Each portfolio's returns are calculated over the following month. Panel A shows two double-sorts. The first is a sort of realized idiosyncratic volatility (σ_{RV_idio}) and then implied idiosyncratic volatility (σ_{IV_idio}). The second sorts implied idiosyncratic volatility (σ_{IV_idio}) and then realized idiosyncratic volatility (σ_{RV_idio}). In Panel B, firms are sorted by characteristic, and then by idiosyncratic implied volatility (σ_{IV_idio}). SIZE is calculated as the price times shares outstanding, B/M is book value of equity divided by market equity, ORW_Ratio is the Ofek, Richardson, and Whitelaw (2004) short sales constraint ratio, and OI is the log of option open interest plus one. The t-statistics are given in parentheses. * is significant at the 5% level. ** is significant at the 1% level.

	Equal Weighted					Value Weighted				
Panel A:	1(Low)	2	3	4	5(High)	1(Low)	2	3	4	5(High)
σ_{RV_idio} (σ_{IV_idio})	1.034	0.989	1.984	2.442	2.741	0.895	0.811	1.838	2.286	2.543
	(3.04)**	(2.86)**	(3.71)**	(5.05)**	(3.99)**	(2.99)**	(2.78)**	(3.61)**	(5.01)**	(4.05)**
σ_{IV_idio} (σ_{RV_idio})	-0.381	-0.517	-0.561	-0.510	-0.622	-0.289	-0.414	-0.450	-0.446	-0.562
	(1.03)	(1.75)	(1.84)	(1.44)	(0.84)	(1.05)	(1.72)	(1.79)	(1.55)	(0.91)
Panel B:										
SIZE(σ_{IV_idio})	3.176	0.618	0.365	0.352	-0.296	2.513	0.600	0.254	0.358	-0.041
	(3.78)**	(1.17)	(0.61)	(0.81)	(0.82)	(3.20)**	(1.14)	(0.46)	(0.82)	(0.09)
B/M(σ_{IV_idio})	0.692	0.436	1.839	2.566	3.533	0.755	-0.450	1.665	2.114	2.948
	(1.24)	(0.79)	(1.93)	(4.71)**	(5.78)**	(0.87)	(0.55)	(1.53)	(3.28)**	(4.05)**
ORW_Ratio(σ_{IV_idio})	2.585	0.568	0.526	2.192	1.951	1.938	-0.488	-0.861	2.366	0.916
	(4.32)**	(1.32)	(1.25)	(4.38)**	(3.41)**	(2.78)**	(0.76)	(1.15)	(3.63)**	(1.49)
OI(σ_{IV_idio})	1.885	2.251	2.074	1.327	0.602	1.040	0.959	1.046	-0.029	0.453
	(4.89)**	(5.05)**	(4.00)**	(1.97)*	(1.16)	(2.76)**	(2.26)*	(1.97)*	(0.05)	(0.77)

Figure 1

Cumulative Dollar Values for Implied and Realized Idiosyncratic Volatility

This figure demonstrates the cumulative dollar values to investing in one of the following strategies. First, buy and hold the S&P 500. Second go long high implied idiosyncratic volatility and short low implied idiosyncratic volatility stocks. Third, go long high realized idiosyncratic volatility and short low realized idiosyncratic volatility stocks. The last two portfolios are value weighted and formed every month by ranking the idiosyncratic volatilities at the end of the month and then holding them for one month. The dollar values are calculated by initially investing $10,000 in each strategy in 1996 and holding through June 2005.

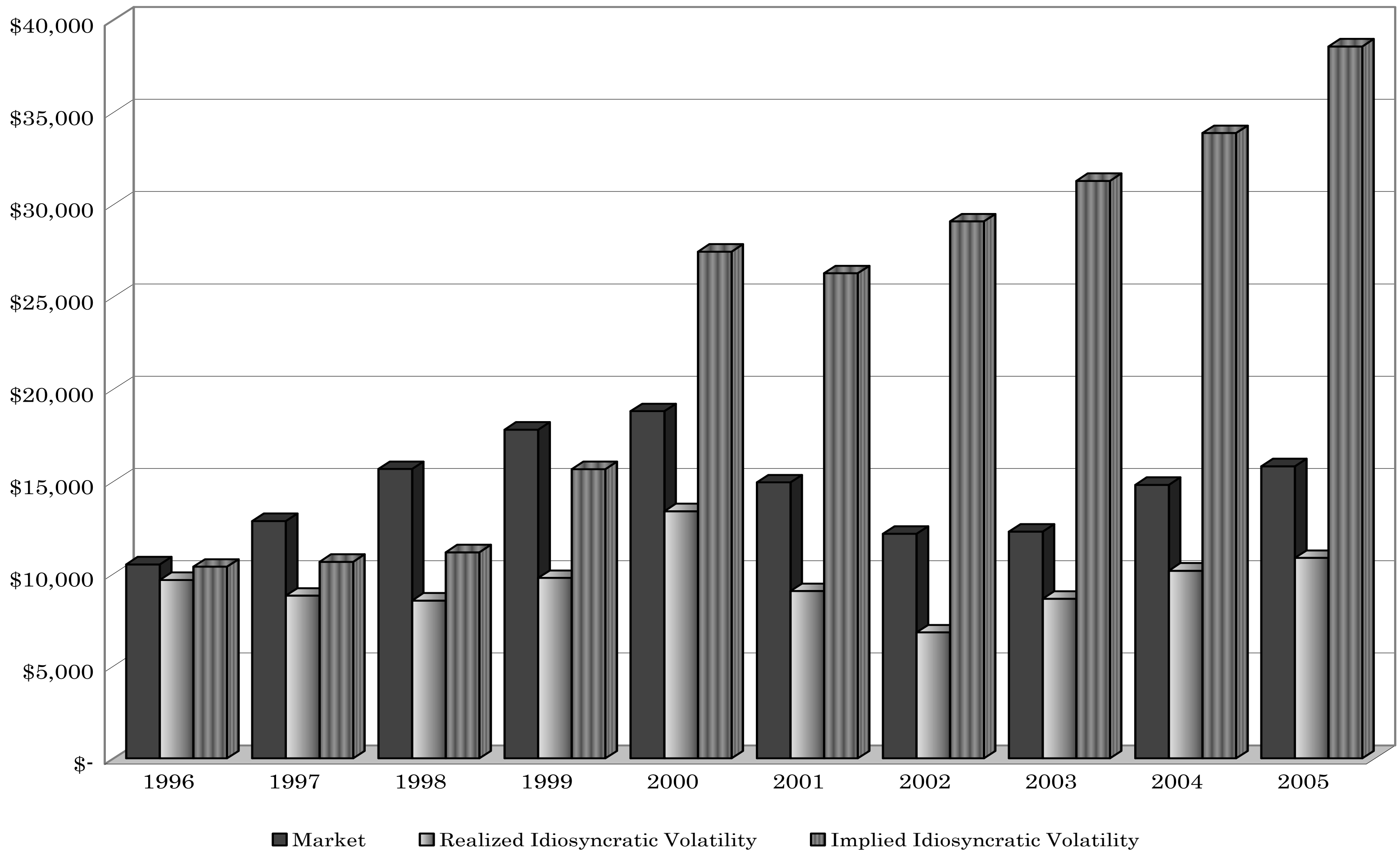

Figure 2

Small Size and High Book-to-Market Equity Cumulative Dollar Values

This figure demonstrates the cumulative dollar values to investing in one of the following strategies. First, buy and hold the S&P 500. Second, go long high idiosyncratic implied volatility and short low idiosyncratic implied volatility stocks in the small size category. Third, go long high idiosyncratic implied volatility and short low idiosyncratic implied volatility stocks in the high B/M category. The portfolios are value-weighted, and formed every month by ranking the implied volatilities at the end of the month and then holding them for one month. The dollar values are calculated by initially investing $10,000 in each strategy in 1996 and holding through June 2005.