Dependence Structure and Extreme Comovements in International Equity and Bond Markets

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March, 2006

Abstract

Equity returns are more dependent in bear markets than in bull markets. This phenomenon known as asymmetric dependence is well documented. We show analytically that a multivariate GARCH model or a regime switching (RS) model based on normal innovations cannot reproduce this asymmetric dependence. We propose an alternative model which allows tail dependence for lower returns and keeps tail independence for upper returns. This model is applied to international equity and bond markets to investigate their dependence structure. It includes one normal regime in which dependence is symmetric and a second regime characterized by asymmetric dependence. Empirical results show that the dependence between equities and bonds is low even in the same country, while the dependence between international assets of the same type is large in both regimes. The cross-country dependence is specially large in the asymmetric regime.

Keywords: asymmetric correlation, asymmetric dependence, copula, tail dependence measures, GARCH, regime switching model, International finance.

JEL classification: C32, C51, G15

*I specially thank my advisor René Garcia. I also thank Éric Renault, Jean Marie Dufour, Éric Jacquier, Silvia Gonçalves, Nour Meddahi, Benoit Perron and Participants at CIREQ workshop and Financial Management Association Doctoral Consortium at Chicago for their constructive comments and suggestions. I am grateful to Lorenzo Cappiello, Robert Engle and Kevin Sheppard for providing me with their dataset. Financial Support by Centre interuniversitaire de recherche en économie quantitative (CIREQ) and Centre interuniversitaire de recherche en analyse des organisations (CIRANO) is gratefully acknowledged. The usual caveats apply.

1 Introduction

Negative returns are more dependent than positive returns in financial markets, especially in international asset markets. This phenomenon known as asymmetric dependence has been reported by many previous studies including Erb et al (1994), Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002), Das and Uppal (2003), Patton (2004) and references therein. This asymmetric dependence has important implications for portfolio choice[1] and risk management. However, measuring and modeling asymmetric dependence remains a challenge.

Previous studies commonly use simple correlation, dynamic or exceedance[2] correlation to investigate the dependence structure between financial returns. These measures are adequate only when the dependence remains linear and especially when the returns are jointly normal or conditionally normal, a property which is not verified empirically. Boyer et al (1999) and Forbes and Rigobon (2002) remark that correlations estimated conditional on high or low returns or volatility suffer from some conditioning bias. Correlation asymmetry may therefore appear spuriously if these biases are not accounted for. To avoid these problems, Longin and Solnik (2001) use extreme value theory (EVT)[3] by focusing on the asymptotic value of exceedance correlation. The benefit of EVT resides in the fact that the asymptotic result holds regardless of the whole distribution of returns. However as mentioned by Longin and Solnik (2001), EVT cannot help to determine if a given return-generating process is able to reproduce the extreme asymmetric exceedance correlation observed in the data.

This paper provides a first solution to this shortcoming. By reformulating the extreme

[1] Patton (2004) finds that the knowledge of asymmetric dependence leads to gains that are economically significant, while Ang and Bekaert (2002), in a regime switching setup, argue that the costs of ignoring the difference between regimes of high and low dependence are small, but increase with the possibility to invest in a risk-free asset.

[2] The exceedance correlation between two series of returns is defined as the correlation for a sub-sample in which the returns of both series are simultaneously lower (or greater) than the corresponding thresholds θ_1 and θ_2. Formally, exceedance correlation of variables X and Y at thresholds θ_1 and θ_2 is expressed by $Ex\_corr\,(Y, X; \theta_1, \theta_2) = \begin{cases} corr\,(X, Y\,|X \leq \theta_1, Y \leq \theta_2)\,, & \text{for } \theta_1 \leq 0 \text{ and } \theta_2 \leq 0 \\ corr\,(X, Y\,|X \geq \theta_1, Y \geq \theta_2)\,, & \text{for } \theta_1 \geq 0 \text{ and } \theta_2 \geq 0 \end{cases}$. Longin and Solnik (2001) use $\theta_1 = \theta_2 = \theta$, while Ang and Chen (2002) use $\theta_1 = (1+\theta)\,\overline{X}$ and $\theta_2 = (1+\theta)\,\overline{Y}$, where $\overline{X}$ and $\overline{Y}$ are the means of Y and X respectively. We use here thresholds specified in term of quantiles: $\theta_1 = F_X^{-1}(\alpha)$ and $\theta_2 = F_Y^{-1}(\alpha)$ where F_X and F_Y are the cumulative distribution functions of Y and X respectively. All these three definitions are equivalent when X and Y are identically distributed. In Longin and Solnik (2001) and Ang and Chen (2002) exceedance correlations are symmetric if $Ex\_corr\,(Y, X; \theta) = Ex\_corr\,(Y, X; \theta)$ and in our formulation it is the case if $Ex\_corr\,(Y, X; \alpha) = Ex\_corr\,(Y, X; 1-\alpha)\,; \alpha \in (0, 1)$

[3] Extreme Value Theory (EVT) is used to characterize the distribution of variable conditionally to the fact that its values are beyond a threshold, and the asymptotic distribution is obtained when this threshold tends to infinity.

exceedance correlation result of Longin and Solnik (2001) in an equivalent fashion as tail dependence, we can investigate which model can reproduce the empirical facts. First, we provide an analytical result. We show that the multivariate GARCH or regime switching (RS) models with Gaussian innovations that have been used to address asymmetric dependence issues (see Ang and Bekaert, 2002, Ang and Chen, 2002, and Patton, 2004) cannot reproduce an asymptotic exceedance correlation. The key point is that these classes of models can be seen as mixtures of symmetric distributions and cannot produce asymptotically an asymmetric dependence. Of course this does not mean that at finite distance a mixture of these classes cannot produce some asymmetric dependence. The RS model of Ang and Chen (2002) is a good example. However, the asymmetry put forward disappears asymptotically. Moreover, the asymmetry in this RS model comes from the asymmetry created in the marginal distributions with regime switching in the mean and hence it is not separable from the marginal asymmetry (skewness).

We propose an alternative model based on copulas that allows tail dependence for lower returns and keeps tail independence for upper returns as suggested by the findings of Longin and Solnik (2001). We apply this model to international equity and bond markets to investigate their dependence structure. It includes one normal regime in which dependence is symmetric and a second regime characterized by asymmetric dependence. We separately analyze dependence between the two leading markets in North-America (US and Canada) and two major markets of the Euro zone (France and Germany).

Copulas are functions that build multivariate distribution functions from their unidimensional marginal distributions. The theory of this useful tool dates back to Sklar (1959) and a clear presentation can be found in Nelsen (1999). Well designed to analyze nonlinear dependence, they were firstly used by statisticians for nonparametric estimation and measure of dependence of random variables (see Genest and Rivest, 1993 and references therein). Their application to financial and economic problems is a new and fast-growing field of interest. Here, the use of this concept is essentially motivated by two facts. First, it allows to separate the features due to each marginal distribution from the dependence effect between all variables. Second, it extends the linear concept of correlation and is better suited to capture nonlinear dependence.

The empirical investigation shows that the dependence between equities and bonds is low even in the same country, while the dependence between international assets of the same type is large in both regimes. Extreme dependence appears across countries in both the bond and the equity markets, but it is nonexistent across the bond and the equity markets, even in the same country. Another fact is the difference between the unconditional correlation and the correlation in the normal regime. This phenomenon possibly is due to the nonlinearity in dependence of international returns characterized by the presence of extreme dependence that is absent in the tail of a multivariate normal distribution. Exchange rate volatility seems to be a factor contributing to asymmetric dependence. With the introduction of a fixed exchange rate the dependence between France and Germany becomes less asymmetric and more normal than before. High exchange rate volatility is associated with a high level of asymmetry.

The rest of this paper is organized as follows. Section 2 formalizes two empirical facts and shows how classical models fail to capture these facts. In section 3 we develop a model with two regimes that clearly disentangle dependence from marginal features and allows asymmetry in extreme dependence. It results in a four-variate dependence model with asymmetry and a flexible dependence structure. Empirical evidence is examined in section 4 and conclusions are given in section 5.

2 Extreme Asymmetric Dependence and Modeling Issues

In this section we present empirical facts about exceedance correlation in international equity market returns put forward by Longin and Solnik (2001) and the related literature. We next argue that these facts can be equivalently reformulated in terms of tail dependence. The latter formulation will allow us to explain why classical return-generating processes such as GARCH and regime switching models based on a multivariate normal distribution fail to reproduce these empirical facts.

2.1 Empirical Facts

Longin and Solnik (2001) investigate the structure of correlation between various equity markets in extreme situations. Their main finding is that equity markets exhibit a much higher correlation in extreme bear periods and zero correlation for asymptotic upper returns. They arrive at this conclusion by testing the equality of exceedance correlations, one obtained under a joint normality assumption and the other one computed using EVT. For the latter distribution, they model the marginal distributions of equity index returns with a generalized Pareto distribution (GPD) and capture dependence through a logistic function. Their analysis brings forward two important facts.

Fact 1: *There exists asymmetry in exceedance correlation: large negative returns are more correlated than large positive returns.*

This first result has also been obtained by Ang and Chen (2002) who develop a test statistic based on the difference between exceedance correlations computed from data and those obtained from a given model[4]. They find as in Ang and Bekaert (2002) that regime switching models can reproduce the above fact. However, in their regime switching model, it is difficult to separate asymmetric dependence from marginal asymmetries (skewness). A second fact is about asymptotic dependence.

Fact 2: *Asymptotically, exceedance correlation is zero for very large negative returns and strictly positive for very large positive returns.*

Since asymptotic exceedance correlation is zero for both sides of a bivariate normal distribution, Longin and Solnik (2001) interpreted these findings as rejection of normality for large negative returns and non-rejection for large positive returns. In the conclusion of their article, Longin and Solnik stress that their approach has the disavantage of not explicitly specifying the class of return-generating processes that fail to reproduce these two facts.

We provide a first answer to this concern by characterizing some classes of models which cannot reproduce these asymmetries in extreme dependence. The difficulty in telling which model can reproduce these facts is the lack of analytical expressions for the asymptotic exceedance correlation and its intractability even for classical models such as Gaussian GARCH or regime switching model. In order to investigate this issue, we introduce the concept of tail dependence.

2.2 Tail Dependence

To measure the dependence between an extreme event on one market and a similar event on another market, we define two dependence functions one for the lower tail and one for the upper tail and their corresponding asymptotic tail dependence coefficients. For two random variables X and Y with cumulative distribution functions F_X and F_Y respec-

[4] Ang and Chen (2002) define a test statistic $H = \left[\sum_{i=1}^{N} \frac{1}{N} \left(\rho(\vartheta_i) - \widehat{\rho}(\vartheta_i)\right)^2\right]^{1/2}$ which is the distance between exceedance correlations obtained from the normal distribution $(\rho(\vartheta_1), ..., \rho(\vartheta_N))$ and exceedance correlations estimated from the data $(\widehat{\rho}(\vartheta_1), ..., \widehat{\rho}(\vartheta_N))$ for a set of N selected thresholds $\{\vartheta_1, ..., \vartheta_N\}$. In the same way they define H^- and H^+ by considering negative points for H^- and nonnegative points for H^+ such that $H^2 = (H^-)^2 + (H^+)^2$. They can therefore conclude to asymmetry if H^- differs from H^+. Their results depend on the choice of the set of thresholds and can only account for asymmetry at finite distance but not asymptotically.

tively, we call lower tail dependence function[5] (TDF) the conditional probability $\tau^L(\alpha) \equiv \Pr\left[X \leq F_X^{-1}(\alpha) \,\middle|\, Y \leq F_Y^{-1}(\alpha)\right]$ for $\alpha \in (0, 1/2]$ and similarly, upper tail dependence function is $\tau^U(\alpha) \equiv \Pr\left[X \geq F_X^{-1}(1-\alpha) \,\middle|\, Y \geq F_Y^{-1}(1-\alpha)\right]$. The tail dependence coefficient (TDC) is simply the limit (when it exists) of this function when α tends to zero. More precisely *lower TDC* is $\tau^L = \lim_{\alpha \to 0} \tau^L(\alpha)$ and *upper TDC* is $\tau^U = \lim_{\alpha \to 0} \tau^U(\alpha)$. As in the case of joint normality, we have lower tail-independence when $\tau^L = 0$ and upper tail-independence for $\tau^U = 0$.

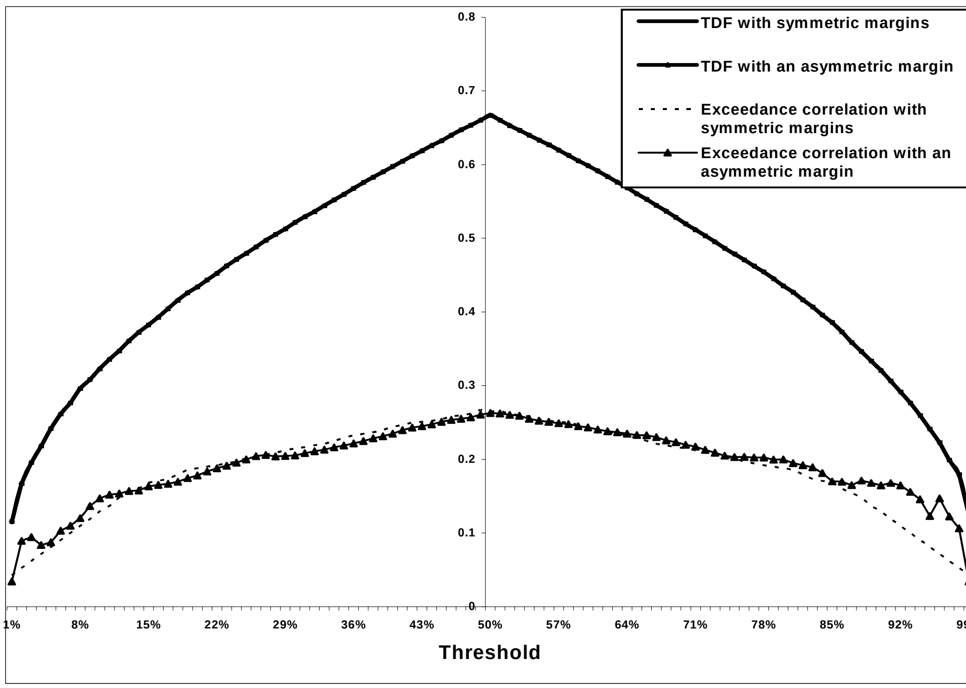

Figure 1: Compared effect of skewness on the Tail Dependence Function (TDF) and the Exceedance correlation: First we simulate standard bivariate Gaussian distribution with correlation 0.5 and compute TDF and Exceedance correlation. Secondly, we create asymmetry in one marginal distribution by replacing the $N(0,1)$ by a mixture of $N(0,1)$ and $N(4,4)$ with equal weight. The symmetric TDF remains the same while the symmetric exceedance correlation is affected by the change in the marginal distribution.

Compared to exceedance correlation used by Longin and Solnik (2001), Ang and Chen (2002), Ang and Bekaert (2002), and Patton (2004), one of the advantages of TDF and corresponding TDCs is their invariance to modifications of marginal distributions that do not affect the dependence structure. Figure 1 gives an illustration of this invariance. We simulate bivariate Gaussian distribution $N(0, I_\rho)$, where I_ρ is the two dimensional matrix with standard deviation one in all elements of the diagonal and $\rho = 0.5$ is the correlation coefficient outside the diagonal elements. Both exceedance correlation and tail dependence

[5] In the literature, the important thing is the limit of this function, but here, it is important to keep it in order to make the comparison with conditional correlation which is also a function of a threshold.

measures show a symmetric behavior of dependence in extreme returns. However, when we replace one of the marginal distributions $N(0,1)$ by a mixture of normals one $N(0,1)$ and one $N(4,4)$ with equal weights and let the other marginal distribution and the dependence structure unchanged, the TDF remains the same while the exceedance correlation is affected. In fact, the correlation coefficient and the exceedance correlation are a function of the dependence structure and of marginal distributions while the tail dependence is a sole function of the dependence structure, regardless of the marginal distributions.

By observing that for the logistic function used by Longin and Solnik (2001), the zero value for the asymptotic correlation coefficient is exactly equivalent[6] to tail independence, we deduce from fact 2 the equivalent following fact.

Fact 2': *Lower extreme returns are tail-dependent, while upper extreme returns are tail-independent.*

The advantages to write fact 2 in its equivalent form fact 2' are many. Compared to exceedance correlation, the tail dependence coefficient is generally easier to compute. Moreover we can easily derive the tail dependence of a mixture from the tail dependence of the different components of the mixture. This last property will be used below to investigate the issue pointed out at the end of Longin and Solnik (2001) paper about which model can or cannot reproduce their results.

2.3 Why classical multivariate GARCH and RS model cannot reproduce asymptotic asymmetries?

Ang and Chen (2002) and Ang and Bekaert (2002) try to reproduce the facts with classical models such as GARCH and RS based on a multivariate normal distribution: After examining a number of models, they found that GARCH with constant correlation and fairly asymmetric GARCH cannot reproduce the asymmetric correlations documented by Longin and Solnik. However, they found that a RS model with Gaussian innovations reproduces better asymmetries in exceedance correlation. However, what they reproduce is an asymmetric correlation at finite distance, but not the asymptotic asymmetric dependence put forward in Longin and Solnik (2001). Their finite distance asymmetric correlation comes from the asymmetries produced in the marginal distributions with a regime switching in means[7]. Therefore it becomes difficult to distinguish asymmetries in dependence from

[6] For the logistic function with parameter α, the correlation coefficient of extremes is $1-\alpha^2$ (see Longin and Solnik, 2001). We find that the upper tail dependence coefficient is $2-2^{\alpha}$. Then, both coefficients are zero when α equals 1 and different from zero when α is different from 1.

[7] Ang and Bekaert (2002) note that the ability of RS model (compared to GARCH model) to reproduce asymmetries, derives from the fact that it accounts the persistence in both first and second moments while

asymmetry in marginal distributions.

By reinterpreting Longin and Solnik (2001) results in term of TDC instead of asymptotic exceedance correlation, we show analytically that all these models cannot reproduce asymptotic asymmetry even if some can reproduce finite distance asymmetry. These results are extended to the rejection of more general classes of return-generating processes. By doing so we provide the beginning of an answer to Longin and Solnik (2001) concerns about the ability of some models to reproduce their findings.

The key point of this result is the fact that many classes of models including Gaussian(or Student) GARCH and RS can be seen as mixtures of symmetric distributions. We establish the following result.

Proposition 2.1:

(i) *Any GARCH model with constant mean and symmetric conditional distribution has a symmetric unconditional distribution and hence has a symmetric TDC.*

(ii) *If the conditional distribution of a RS model has a zero TDC, then the unconditional distribution also has a zero TDC.*

(iii) *From a multivariate distribution with symmetric TDC, it is impossible to construct an asymmetric TDC with a mixture procedure (as GARCH, RS or any other) by keeping all marginal distributions unchanged across mixture components.*

Proof: see Appendix A.

This proposition allows us to argue that the classical GARCH or RS models cannot reproduce asymmetries in asymptotic tail dependence. Therefore, the classical GARCH models (BEKK, CCC or DCC)[8] with constant mean can be seen as a mixture of symmetric distributions with the same first moments and therefore exhibit a symmetric tail dependence function as well as a symmetric TDC. When the mean becomes time-varying as in the GARCH-M model the unconditional distribution can allow asymmetry in dependence (Ang and Chen, 2002), but this asymmetry comes from the mixture of the marginal distributions. The resulting skewness cannot be completely disentangled from the asymmetric dependence. Similarly, the classical RS model with Gaussian innovations is a discrete mixture of normal distributions which has a TDC equal to zero on both sides. Therefore, by

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the GARCH accounts this persistence only in second moments. We give analytical arguments to their intuitions.

[8] The BEKK proposed by Engle and Kroner (1995) is the straightforward generalization of the GARCH model to a multivariate case which guarantees positive definiteness of the conditional variance-covariance matrix. In the CCC model proposed by Bollerslev (1990) the conditional variance-covariance matrix is assumed constant, while in the DCC of Engle (2002) this matrix is dynamic.

(ii) we argue that both its TDCs are zero. However, at finite distance, when the mean changes with regimes, the dependence function is not symmetric. This asymmetry is found by Ang and Chen (2002) and Ang and Bekaert (2002) in their RS model, but it disappears asymptotically and it comes from the asymmetry created in the marginal distributions by regime switching in means. Hence, the asymmetries in dependence is not separable from the marginal asymmetry, exactly like in the GARCH-M case. The latter criticism can be generalized to GARCH models with regime switching or to regime switching models with symmetric distributions for the innovations. The part (iii) of proposition2.1 extends this intuition in terms of more general multivariate mixture models based on symmetric innovations. Actually when the marginal distributions are the same across all symmetric TDC components of a mixture, it is impossible to create asymmetry in TDCs.

Two clear questions arise from the discussion above. First, how can we separate the marginal asymmetries from the asymmetry in dependence? Second, how can we account not only for asymmetries at finite distance but also for asymptotic dependence? In the next section, we propose a flexible model based on copulas that addresses these two issues.

3 A Copula Model for asymmetric dependence

Our model aims at capturing the asymmetric dependence found in international equity returns. Our discussion in the last section showed that it is important to control the marginal distributions and the dependence structure separately, by disentangling the two components. Therefore, we need to allow for asymmetry in tail dependence, regardless of the possible marginal asymmetry (skewness). Copulas, also known as dependence functions or uniform representations, are adequate tools to achieve this aim.

3.1 Disentangling the marginal distributions from dependence with copulas

Estimation of multivariate models is difficult because of the large number of parameters involved. Multivariate GARCH models are a good example since the estimation becomes intractable when the number of series being modeled is high. The CCC of Bollerslev (1990) and the DCC of Engle (2002) deal with this problem by separating the variance-covariance matrix in two parts, one part for the univariate variances of the different marginal distributions, a second part for the correlation coefficients. This separation allows them to estimate the model in two steps. In the first step, they estimate the marginal parameters and use them in the estimation of the correlation parameters in a second step. Copulas offer a tool to generalize this separation while extending the linear concept of correlation to nonlinear dependence.

Copulas are functions that build multivariate distribution functions from their unidimensional margins. Let $X \equiv (X_1, ..., X_n)$ be a vector of n univariate variables. Denoting F the joint n-dimensional distribution function and $F_1, ..., F_n$ the respective margins of $X_1, ..., X_n$. Then the Sklar theorem[9] states that there exists a function C called copula which joins F to $F_1, ..., F_n$ as follows.

$$F(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)) \tag{3.1}$$

This relation can be expressed in term of densities by differentiating with respect to all arguments. We can therefore write (3.1) equivalently as

$$f(x_1, ..., x_n) = c(F_1(x_1), ..., F_n(x_n)) \times \prod_{i=1}^{n} f_i(x_i) \tag{3.2}$$

where f represents the joint density function of the n-dimensional variable X and f_i the density function of the variable X_i for $i = 1, ..., n$. The copula density function is naturally defined by $c(u_1, ..., u_n) \equiv \frac{\partial^n}{\partial u_1 ... \partial u_n} C(u_1, ..., u_n)$. Writing the joint distribution density in the above form, we understand why it can be said that copula contains all information about the dependence structure[10].

We now suppose that our joint distribution function is parametric and we separate the marginal parameters from the copula parameters. So the relation (3.2) can be expressed as:

$$\begin{aligned} f(x_1, ..., x_n; \delta, \theta) &= c(u_1, ..., u_n; \theta) \times \prod_{i=1}^{n} f_i(x_i; \delta_i); \\ u_i &= F_i(x_i; \delta_i) \qquad \text{for} \quad i = 1, ..., n \end{aligned} \tag{3.3}$$

where $\delta = (\delta_1, ..., \delta_n)$ are the parameters of the different margins and θ denotes the vectors of all parameters that describe dependence through the copula. Therefore, copulas offer a very nice tool to separate margins from the dependence structure and to build more flexible multivariate distributions.

More recent work allow some dynamics in dependence. In a bivariate context, Rodriguez (2004) introduces regime switching in both theparameters of marginal distributions and the

[9] See Nelsen (1999) for a general presentation. Note that if F_i is continuous for any $i = 1, ..., n$ then the copula C is unique.

[10] The tail dependence coefficients are easily defined through copula as $\tau^L = \lim_{\alpha \to 0} \frac{C(\alpha, \alpha)}{\alpha}$ and $\tau^U = \lim_{\alpha \to 0} \frac{2\alpha - 1 + C(1-\alpha, 1-\alpha)}{\alpha}$

copula function[11]. Ang and Bekaert (2002; 2004) allow all parameters of the multivariate normal distribution to change with the regime. The extension of these models to a large number of series faces the above mentioned curse of dimensionality. Since the switching variable is present in both the margins and the dependence function, separation of the likelihood function into two part is not possible and the two-step estimation cannot be performed. Pelletier (2004) uses the same separation as in the CCC or DCC and introduce the regime switching variable only into the correlation coefficients. By doing so, he can proceed with the two-step procedure to estimate the model while limiting the number of parameters to be estimated[12]. We carry out a similar idea but for nonlinear dependence.

Therefore, we separate the modeling of marginal distributions from the modeling of dependence by using univariate GARCH models for the marginal distributions and introducing regime changing in the copula dependence structure. The pattern of the model with four variables (two countries, two markets in our following application) is illustrated in Figure 2.

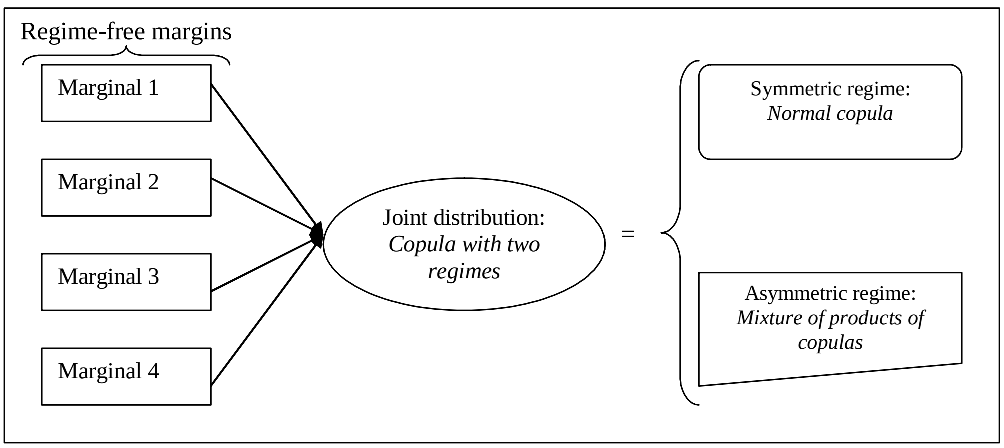

Figure 2. Model structure: Disentangling marginal distribution from the dependence structure with a two-regime copula, one symmetric regime and one asymmetric regime. The marginal distributions are regime-free.

[11] The models proposed by Rodriguez (2004) in his analysis of contagion can reproduce asymmetric dependence but it cannot distinguish between skewness and asymmetry in the dependence structure. In fact, a change in regime produces both skewness and asymmetric dependence, two different features that must be characterized separately.

[12] Since Pelletier (2004) uses the normal distribution with constant mean, the resulting unconditional distribution is symmetric and cannot reproduce asymmetric dependence.

3.2 Specification of the Marginal Distribution

For marginal distributions, we use a M-GARCH (1,1) model similar to Heston-Nandi (2000):

$$
\begin{aligned}
x_{i,t} &= \mu_i + \lambda_i \sigma_{i,t}^2 + \sigma_{i,t} z_{i,t}; \qquad z_{i,t} \sim N(0,1); \quad i = 1, \cdots, 4 & (3.4)\\
\sigma_{i,t}^2 &= \omega_i + \beta_i \sigma_{i,t-1}^2 + \alpha_i \left(z_{i,t-1} - \gamma_i \sigma_{i,t-1}\right)^2. & (3.5)
\end{aligned}
$$

The variables $x_{1,t}$ and $x_{2,t}$ represent the log returns of equities and bonds respectively for the first country while $x_{3,t}$ and $x_{4,t}$ are the corresponding series for the second country; $\sigma_{i,t}^2$ denotes the conditional variance[13] of $x_{i,t}$, λ_i can be interpreted as the price of risk and γ_i captures potential asymmetries in the volatility effect. The parameters of the marginal distributions are grouped into one vector $\delta \equiv (\delta_1, \cdots, \delta_4)$, with $\delta_i = (\mu_i, \lambda_i, \omega_i, \beta_i, \alpha_i, \gamma_i)$. In the Heston-Nandi (2000) interpretation, μ_i represents the interest rate. Here we keep μ_i as a free parameter to give more flexibility to our model and will verify if its estimates correspond to a reasonable value for the interest rate[14].

3.3 Specification of the Dependence Structure

Our dependence model is characterized by two regimes[15], one Gaussian regime in which the dependence is symmetric (C_N) and a second regime that can capture the asymmetry in extreme dependence (C_A). The conditional copula is given by:

$$
C\left(u_{1,t}, ..., u_{4,t}; \rho^N, \rho^A \left| s_t\right.\right) = s_t C_N \left(u_{1,t}, ..., u_{4,t}; \rho^N\right) + (1 - s_t) C_A \left(u_{1,t}, ..., u_{4,t}; \rho^A\right), \quad (3.6)
$$

where $u_{i,t} = F_{it}(x_{i,t}; \delta_i)$, with F_{it} denoting the conditional cumulative distribution function of $x_{i,t}$ given the past observations. The variable s_t follows a Markov chain with a time-invaryling transitional probability matrix

$$
M = \begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix}; P = \Pr\left(s_t = 1 \left| s_{t-1} = 1\right.\right) \text{ and } Q = \Pr\left(s_t = 0 \left| s_{t-1} = 0\right.\right) \quad (3.7)
$$

[13]The condition $\beta_i + \alpha_i \gamma_i^2 < 1$ is sufficient to have the stationarity of the process $x_{i,t}$ with finite unconditional mean and variance (see Heston and Nandi, 2000).

[14]Since we have to estimate the same model for bonds and equities in the same country, we would have to test the natural hypothesis.

$$
H_0 : \mu_i^{bond} = \mu_i^{Equity}.
$$

[15]This regime change does not affect the marginal distributions to allow separation between the marginal dynamics and changes in dependence structure. The fact that the margins are regime-free will also allow us to decompose the log likelihood function in order to perform a two-step estimation.

The normal regime ($s_t = 1$) corresponds to the symmetric regime where the conditional joint normality can be supported and the asymmetric regime ($s_t = 0$) corresponds to the asymmetric regime in which markets are strongly more dependent for large asymptotic negative returns than for large positive returns.

The Gaussian copula C_N is defined straightforwardly by (3.1) where the joint distribution $F = \Phi_{\rho^N}$ is the 4-dimensional normal cumulative distribution function with all diagonal elements of the covariance matrix equal to one, i.e. $C_N\left(u_1, ..., u_4; \rho^N\right) = \Phi_{\rho^N}\left(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_4)\right)$, where Φ is the univariate standard normal cumulative distribution function.

The asymmetric components of the copula are illustrated in figure 3. The first one is characterized by independence between the two countries, but possibly extreme dependence between equities and bonds for each country. The second one is characterized by independence between equity and bond markets but allows for extreme dependence between equity returns and bond returns separately. The third one allows for possible extreme dependence between bonds in one country and equities in another country but supposes independence for the rest.

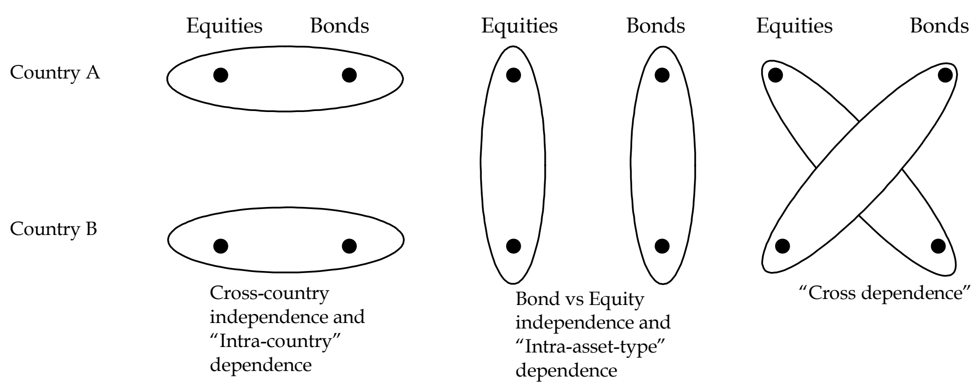

Figure 3. Illustration of the three components of asymmetric copula. Each component is the product of the two bivariate copulas representing the dependence between the two corresponding asset returns.

Formally the asymmetric copula is the mixture of these three components and is expressed as follows

$$\begin{aligned} C_A\left(u_1, ..., u_4; \rho^A\right) \equiv \; & \pi_1 C_{GS}\left(u_1, u_2; \tau_1^L\right) \times C_{GS}\left(u_3, u_4; \tau_2^L\right) \\ & + \pi_2 C_{GS}\left(u_1, u_3; \tau_3^L\right) \times C_{GS}\left(u_2, u_4; \tau_4^L\right) \\ & + \left(1 - \pi_1 - \pi_2\right) C_{GS}\left(u_1, u_4; \tau_5^L\right) \times C_{GS}\left(u_2, u_3; \tau_6^L\right) \end{aligned} \tag{3.8}$$

with $\rho^A = \left(\pi_1, \pi_2, \tau_1^L, \tau_2^L, \tau_3^L, \tau_4^L, \tau_5^L, \tau_6^L\right)$, and the bivariate component is the Gumbel survival copula given by

$$C_{GS}\left(u, v; \tau^L\right) = u + v - 1 + \exp\left[-\left(\left(-\log\left(1-u\right)\right)^{\theta\left(\tau^L\right)} + \left(-\log\left(1-v\right)\right)^{\theta\left(\tau^L\right)}\right)^{1/\theta\left(\tau^L\right)}\right], \tag{3.9}$$

where $\theta\left(\tau^L\right) = \dfrac{\log\left(2\right)}{\log\left(2-\tau^L\right)}$, $\tau^L \in [0,1)$ is the lower TDC and the upper TDC is zero[16].

One can notice that our asymmetric copula specification assumes some constraints in the dependence structure. For three different couples from different components of this copula, the sum of their TDC is lower than one[17]. Without any constraints this sum may reach 3. Such constraints are dictated by some copula limitations[18]. A major problem in multivariate distributions' construction today and perhaps the most important open question concerning copulas as mentioned by Nelsen (1999, page 86) is how to construct multivariate copulas with specific bivariate marginal distributions. An "impossibility" theorem by Genest et al (1995) states that it is not always possible to construct multivariate copulas with specific bivariate margins. Therefore even if in the bivariate case we can have a nice asymmetric copula with lower tail dependence and upper tail independence as Longin and Solnik (2001) suggest, some problems remain when we contemplate more than two series. Most existing asymmetric tail dependent copulas are in the family of archimedian copulas and the usual straightforward generalization in multivariate copula constraints all bivariate marginal copulas to be the same. This is clearly unadmissible in the context of our analysis. In the above model, we allow each of the six couples of interest to have different levels of lower TDC. As C_A is constructed, it is easy to check that it is a copula since each component of the mixture is a copula[19] and the mixture of copulas is a copula.

It is important to notice that, in this model, the labeling of each regime is defined ex-ante. The normal regime ($s_t = 1$) corresponds to the symmetric regime where the conditional joint normality can be supported and the asymmetric regime ($s_t = 0$) corresponds to the asymmetric regime in which markets are strongly more dependent for large negative returns

[16] The Longin and Solnik (2001) result implies that lower tails are dependent while upper tails are independent. Hence, the Gumbel survival copula is designed to model this feature since it has this tail dependence structure.

[17] For example, the TDC between bonds and equities in the first country is $\pi_1\tau_1^L$, between equities of two countries $\pi_2\tau_3^L$, and between equities in the first country and bonds in the second country $\left(1-\pi_1-\pi_2\right)\tau_5^L$. Therefore, the sum is $\pi_1\tau_1^L + \pi_2\tau_3^L + \left(1-\pi_1-\pi_2\right)\tau_5^L \leq 1$, since $\tau_1^L \leq 1$, $\tau_3^L \leq 1$, and $\tau_5^L \leq 1$.

[18] This model can be generalized in the same way to a copula of any dimension. The same type of restrictions are applied, but we obtain a copula with a more flexible dependence structure.

[19] A copula can be seen as a cdf of multidimensional variable with uniform $[0,1]$ margins, so by considering two bivariate independent variables with uniform margins the copula of 4-variate variable including both variables is simply the product of their bivariate copulas. Hence, such a product is always a copula.

than for large positive returns.

3.4 An adapted parsimonious model

Given our application, we impose an additional constraint: $\pi_1 + \pi_2 = 1$. This means that we neglect the asymmetric cross-dependence between equities in one country and bonds in another country, which seems like an economically reasonable assumption given that we maintain cross-dependence through the normal regime. The mixed copula becomes.

$$\begin{aligned} C_A\left(u_1,...,u_4;\rho^A\right) \equiv \;& \pi C_{GS}\left(u_1,u_2;\tau_1^L\right) \times C_{GS}\left(u_3,u_4;\tau_2^L\right) \\ & +(1-\pi) C_{GS}\left(u_1,u_3;\tau_3^L\right) \times C_{GS}\left(u_2,u_4;\tau_4^L\right) \end{aligned} \tag{3.10}$$

Therefore, the asymmetry copula is now characterized by just five parameters $\rho^A = \left(\pi,\tau_1^L,\tau_2^L,\tau_3^L,\tau_4^L\right)$.

3.5 Estimation

As already mentioned, our structure allows for a two-step estimation procedure. The likelihood function must be evaluated unconditionally to the unobservable regime variable s_t and decomposed in two parts. Let us denote the sample of observed data by $\underline{X}_T = \{X_1,\cdots,X_T\}$ where $X_t \equiv \{x_{1,t},\cdots,x_{4,t}\}$. The log likelihood function is given by:

$$L\left(\delta,\theta;\underline{X}_T\right) = \sum_{t=1}^{T} \log f\left(X_t;\delta,\theta \middle| \underline{X}_{t-1}\right) \tag{3.11}$$

where $\underline{X}_{t-1} = \{X_1,...X_{t-1}\}$ and θ is a vector including the parameters of the copula and the transition matrix. Hamilton (1989) describes a procedure to perform this type of evaluation[20]. With $\xi_t = (s_t, 1-s_t)'$ and denoting

$$\eta_t = \left[\begin{array}{c} f\left(X_t;\delta,\theta \middle| \underline{X}_{t-1}, s_t=1\right) \\ f\left(X_t;\delta,\theta \middle| \underline{X}_{t-1}, s_t=0\right) \end{array}\right] \tag{3.12}$$

the density function conditionally to the regime variable s_t and the past returns can be written as:

$$f\left(X_t;\delta,\theta \middle| \underline{X}_{t-1}, s_t\right) = \xi_t' \eta_t \tag{3.13}$$

Since s_t (or ξ_t) is unobservable, we integrate on s_t and obtain the unconditional density function:

[20] A general presentation can be found in Hamilton (1994, chapter 22).

$$
\begin{aligned}
f\left(X_t;\delta,\theta|\,\underline{X}_{t-1}\right) = \quad & \Pr\left[s_t=1|\underline{X}_{t-1};\delta,\theta\right]\times f\left(X_t;\delta,\theta|\,\underline{X}_{t-1},s_t=1\right) \\
& +\Pr\left[s_t=0|\underline{X}_{t-1};\delta,\theta\right]\times f\left(X_t;\delta,\theta|\,\underline{X}_{t-1},s_t=0\right)
\end{aligned} \tag{3.14}
$$

The conditional probabilities of being in different regimes at time t conditional on observations up to time $t-1$, denoted by $\widehat{\xi}_{t|t-1}\equiv(\Pr\left[s_t=1|\underline{X}_{t-1};\delta,\theta\right],\Pr\left[s_t=0|\underline{X}_{t-1};\delta,\theta\right])'$, are computed through the Hamilton filter. Starting with the initial value $\widehat{\xi}_{1|0}$, the optimal inference and forecast for each date in the sample is given by the iterative equations:

$$
\widehat{\xi}_{t/t} = \left[\widehat{\xi}'_{t|t-1}\eta_t\right]^{-1}\left(\widehat{\xi}_{t|t-1}\odot\eta_t\right) \tag{3.15}
$$

$$
\widehat{\xi}_{t+1/t} = M'.\widehat{\xi}_{t|t} \tag{3.16}
$$

where $\odot$ denotes element-by-element multiplication. Finally, the unconditional density can be evaluated with the observed data as $f\left(X_t;\delta,\theta|\,\underline{X}_{t-1}\right)=\widehat{\xi}'_{t|t-1}\eta_t$ and the log likelihood becomes:

$$
L\left(\delta,\theta;\underline{X}_T\right)=\sum_{t=1}^{T}\log\left(\widehat{\xi}'_{t|t-1}\eta_t\right) \tag{3.17}
$$

To perform the two-step procedure, we decompose the log likelihood function into two parts: the first part includes the likelihood functions of all margins, while the second part represents the likelihood function of copula.

Proposition 3.2 (Decomposition of the log likelihood function) *The log likelihood function can be decomposed into two parts including the margins and the copula*

$$
L\left(\delta,\theta;\underline{X}_T\right)=\sum_{i=1}^{4}L_i\left(\delta_i;\underline{X}_{i,T}\right)+L_C\left(\delta,\theta;\underline{X}_T\right) \tag{3.18}
$$

where

$\underline{X}_{i,t}=\{x_{i,1},...,x_{i,t}\}$;

$L_i\left(\delta_i;\underline{X}_{i,T}\right)=\sum_{t=1}^{T}\log f_i\left(x_{i,t};\delta_i|\,\underline{X}_{i,t-1}\right)$

$L_C\left(\delta,\theta;X\right)=\sum_{t=1}^{T}\log\left(\widehat{\xi}'_{t|t-1}\eta_{ct}\right)$

with

$\eta_{ct}=\left[\begin{array}{c} c\left(u_{1,t}(\delta_1),...,u_{n,t}(\delta_n);\theta|\,s_t=1\right) \\ c\left(u_{1,t}(\delta_1),...,u_{n,t}(\delta_n);\theta|\,s_t=0\right)\end{array}\right];\quad u_{i,t}(\delta_i)=F_i\left(x_{i,t};\delta_i|\,\underline{X}_{i,t-1}\right)$

and $\widehat{\xi}'_{t|t-1}$ *filtered from* η_{ct} *as*

$$\widehat{\xi}_{t/t} = \left[\widehat{\xi}'_{t|t-1}\eta_{ct}\right]^{-1}\left(\widehat{\xi}_{t|t-1}\odot\eta_{ct}\right)$$
$$\widehat{\xi}_{t+1/t} = M'.\widehat{\xi}_{t|t}$$

Proof*:* see Appendix A

Several options are available for the estimation of the initial value $\widehat{\xi}_{1|0}$. One approach is to set it equal to the vector of unconditional probabilities, which is the stationary transitional probability of the Markov chain. Another simple option is to set $\widehat{\xi}_{1|0} = N^{-1}1_N$. Alternatively it could be considered as another parameter, which will be estimated subject to the constraint that $1'_N\widehat{\xi}_{1|0} = 1$. We use the first option here, which seems to be the more appropriate one, since it is the probability to be in different regimes given that we do not know the previous regime.

Through the above decomposition, we notice that each marginal log likelihood function is separable from the others. Therefore, even if the estimation of all margins is performed in a first step, we can estimate each set of marginal parameters separately into this step. The first step is then equivalent to n single estimations of univariate distributions. The two-step estimation is formally written as follows:

$$\widehat{\delta} = \underset{\delta=(\delta_1,\ldots,\delta_4)\in\Delta}{\arg\max}\sum_{i=1}^{4}L_i\left(\delta_i; X_{i,.}\right) \tag{3.19}$$

$$\widehat{\theta} = \underset{\theta\in\Theta}{\arg\max} L_C\left(\widehat{\delta},\theta; X\right) \tag{3.20}$$

The estimator for the parameters of the marginal distributions is then $\widehat{\delta} = \left(\widehat{\delta}_1, ..., \widehat{\delta}_4\right)$, with $\widehat{\delta}_i = \left(\widehat{\mu}_i, \widehat{\lambda}_i, \widehat{\omega}_i, \widehat{\beta}_i, \widehat{\alpha}_i, \widehat{\gamma}_i\right)'$; and $\widehat{\theta} = \left(\widehat{\rho}^N; \widehat{\rho}^A; \widehat{P}; \widehat{Q}\right)$ includes all the estimators of the parameters involved in the dependence structure. Δ and Θ represent all possible values of δ and θ respectively.

3.6 Test of asymmetry in dependence

The natural way to evaluate this model, if dependence is asymmetric, is to test the null hypothesis of one normal copula regime against the alternative hypothesis of two copula regimes including the normal one and the asymmetric one. This test faces many irregularity problems. Under the null hypothesis, some nuisance parameters are unidentified and the scores are identically zero. These are the general problems of testing in RS models. Hansen (1996) describes the asymptotic distributions of standard test statistics in the context of regression models with additive nonlinearity. Garcia (1998) and Hansen (1996) provide the asymptotic null distribution of the likelihood ratio test. Andrews and Ploberger (1993) address the first problem in a general context and derive an optimal test. The above procedures solve the problem of unidentified nuisance parameters under the null and the identically zero scores. However, there is an additional problem of testing parameter on the boundary. Andrews (2001) deals with this boundary problem but in the absence of the two first problems.

Maximized Monte Carlo (MMC) tests of Dufour (2005), which are a generalization of classical Monte Carlo (MC) tests of Dwass (1957) and Barnard (1963), are adapted for tests facing all these problems. The MC tests of Dwass (1957) and Barnard (1963) are performed by doing many replications (with the same size as the sample data) under the null hypothesis, and compute the test statistic for each replication. The distribution of the test statistic is therefore approximated by the distribution of the obtained values. One can therefore compute the value of the test statistic with the data and deduce from the MC distribution the p-value of the test. The classical MC test does not deal with the presence of nuisance parameters under the null hypothesis. The MMC of Dufour (2005) addresses the problem of nuisance parameters under the null. When the tests statistic involve the nuisance parameters as in the case of the likelihood ratio test under the alternative, the values of these parameters are needed to compute the test statistic on simulated data. The MMC technique is the maximization of the p-values given all the possible values of the nuisance parameters. This test is computationally very demanding. However, Dufour (2005) proposes a simplified version which focus on the estimated values of the nuisance parameters and shows that it works under the assumptions of uniform continuity, and convergence over the nuisance parameter space. Our model satisfied these assumptions of uniform continuity and convergence. Therefore, we will apply this simpler version also known as bootstrap tests. Results confirm statistically the presence of asymmetric dependence in both pairs of countries.

4 Dependence structure in international bond and equity markets: An empirical Investigation

4.1 Data

We will consider the same model for two pairs of two countries. First, we model the equity and bond markets in US and Canada. The series for the US equity returns is the SP 500, while the Canadian equity returns is the Datastream index. The bond series are indices of five-year Government bonds. We also consider France and Germany as a pair of countries. The interest here will be to see how the introduction of the European common currency changed the dependence structure between the asset markets in these two countries. The bond indices are the five-year Government indices, while the equity indices are the MSCI series. All returns are Datastream total returns and are expressed in US dollars on a weekly basis from January 01, 1985 to December 21, 2004, which corresponds to a sample of 1044 observations. Table 1 gives descriptive statistics for these bond and equity series.

The Sharpe ratios seem to be comparable between countries. However this ratio is low for equities (60% in average) compared to bonds (115% in average). All returns present negative skewness except the France bond index. Asset returns and return volatility in France and Germany seem to be high compared to US and Canada. The volatility of returns in France and Germany is more than 23%, while it is only 18% for the US and Canada. This is the effect of the increasing value of the Euro relative to the US dollar after its introduction in January 01, 1999.

4.2 Marginal distributions

The estimates of the marginal parameters are reported in table 3. The large values for the β_i parameters (arround 90%) capture the high persistence in volatility. The values of the parameters α are not significant at the level 5%. It means that ARCH effects are not present. However, the high degree of significance for the parameter λ indicates that asset return are skewed.

Notice that the mean parameter μ_i is very different for the bond and the equity markets in all considered countries except for France with a t-statistic of 1.8114 for testing if the difference between the values of this parameter for bond and equity returns is zero. Therefore the interpretation of this parameter as an interest rate by Heston and Nandi (2000) does not seem to be supported by the data[21].

4.3 Dependence structure in bond and equity markets

We apply our two-country, two-market model to two set of countries: US and Canada for North-America, France and Germany for Europe. Three main conclusions emerge from the results. First, there appears to be a large extreme cross-country dependence in both markets, while there is little dependence between equities and bonds in the same country. Second, the dependence structure exhibits a strong nonlinearity. Third, there seems to be

[21] The null hypothesis tested is $H_{0i} : \mu_i^{Equity} = \mu_i^{Bond}$, i representing the country.

a link between exchange rate volatility and asymmetry of dependence.

4.3.1 US-Canada Dependence Structure

The cross-country extreme dependence is large in the two different markets, but the dependence across the two markets is relatively low. In the asymmetric regime, the TDCs are larger than 54 % in both bond-bond and equity-equity markets, while both equity-bond TDCs in US and Canada are lower than 2%. This observation has an important implication for international diversification. The fact that extreme dependence in international equity and bond markets is larger than national bond-equity dependence reduces the gain of international diversification and encourage the switching from equity to the domestic bond or risk-free asset in case of bear markets. The average absolute value of correlation in the normal regime is larger than 60% for cross-country dependence and lower than 20% for equity-bond dependence. The results underline the differences between unconditional correlation and the correlation in the normal regime. In fact, the presence of extreme dependence in the negative returns explains this difference since the multivariate Gaussian distribution has independence in the tails of returns regardless of the level of correlation.

The separation of the distribution into two parts, including the normal regime and the asymmetric regime, allows to capture the strong nonlinear pattern in the dependence structure. Moreover, it is interesting to see that for a high unconditional correlated couple such as the US and Canada equity markets, this separation gives not only an extreme dependence for the asymmetric regime, but also a correlation in the normal regime (87 %) that appears larger than the unconditional correlation (72 %). This result may seem counter-intuitive if we take the unconditional correlation as a “mean” of the correlations in the two regimes. Of course, one must realize that the asymmetric regime can be characterized by a low correlation but by a large TDC. This demonstrates the importance of distinguishing between correlation and extreme dependence. The mixture model is better able to capture this distinction in fitting the data. A normal distribution may be a good approximation for measuring finite distance dependence, but an appropriate copula structure is necessary for characterizing extreme dependence.

4.3.2 France-Germany Dependence Structure

Due to a high cross-country unconditional correlation in both markets, the results for France and Germany are more eloquent. The dependence between equities and bonds is low, while the dependence between assets of the same type is large in both regimes. For France and

Germany, equity-equity correlation and bond-bond correlation are larger than 90% while bond-equity correlations are lower than 21% in the same country as well as between the two countries. In the asymmetric regime, the TDC are larger than 66% between assets of the same type and lower than 2% between bond and equities in both France and Germany.

The introduction of the Euro increases the dependence between France and Germany markets. Before the introduction of the Euro, in the normal regime, the cross-country correlation between assets of the same type is in average 80%, against more than 96% after the introduction of this currency. The cross-asset correlations exhibit a similar pattern. This result is consistent with those of Cappiello, Engle and Sheppard (2003)[22] who find that the introduction of fixed exchange rate leads to a structural break characterized by a high correlation. For the asymmetric regime, except for the fact that the extreme dependence between the France and Germany equity markets drastically decreases from 87% to 26%, all extreme dependences increase. This change in the level of dependence suggests a relationship between the dependence structure and the exchange rate.

4.3.3 Link between asymmetric dependence and exchange rate

The graph of filtered probabilities for France and Germany shows that after the introduction of the Euro the dependence is more likely Gaussian than asymmetric. To confirm this graphical observation, we perform a logistic[23] regression of the conditional probabilities to be in the asymmetric regime on the volatility of exchange rate.

For US and Canada, we obtain:

$$\widehat{P}_t = \underset{\substack{\text{-7.71e-1} \\ (1.76\text{e-}1)}}{a} + \underset{\substack{9.30\text{e+}1 \\ (2.36\text{e+}1)}}{b} \times Vol_t + e_t$$

For France and Germany, we have:

$$\widehat{P}_t = \underset{\substack{\text{-1.26e+0} \\ (6.81\text{e-}2)}}{a} + \underset{\substack{5.06\text{e+}2 \\ (2.29\text{e+}1)}}{b} \times Vol_t + e_t$$

[22] The goal of Cappiello, Engle and Sheppard (2003) was to investigate the asymmetric effect of past news on the correlation. Since it is well documented that the negative shocks have a larger effect on volatility than the positive shocks of same magnitude, they try to see if the result is similar for correlation.

[23] Since the probability P_t to be in a regime is between 0 and 1, the logistic regression allows us to keep this constraint by proceeding as follows $P_t = \exp(a + Vol_t + \varepsilon_t)/(1 + \exp(a + Vol_t + \varepsilon_t))$ or equivalently $\log(P_t/(1 - P_t)) = a + bVol_t + \varepsilon_t$ and we can perform the usual regression.

The $R - square$ of these regressions are respectively 0.75, and 0.86. The explained variable $\widehat{P_t} = \log(P_t/(1-P_t))$, P_t is the conditional probability to be in the asymmetric regime given the available information, and Vol_t is the exchange rate volatility between the two countries obtained by a M-GARCH(1,1) filter.

These results suggest that high exchange rate volatility is associated with asymmetric dependence. With the introduction of the European currency the dependence between France and Germany becomes more normal. This result is coherent with the literature, which finds asymmetric dependence mainly in the international markets (see Longin and Solnik, 2001). This evidence is simply the fact that in the normal regime the correlation is higher than the unconditional correlation. Moreover, since the introduction of the Euro reduces the volatility of the exchange rate, it increases the correlation due to the link between a fixed exchange rate and the normal distribution regime. Monte Carlo test results reveal the presence of asymmetry in dependence structure.

5 Conclusion

In this paper, we show the limitations of some classical models to reproduce asymmetric dependence and the need to disentangle marginal asymmetry from dependence asymmetry. Using copulas we provide a flexible model to achieve this aim. We applied this model to international bond and equity markets and put forward some interesting facts about the dependence structure.

The dependence between the equity markets on one hand and the bond markets on other were found to be much larger than the dependence between equities and bonds even in the same country. It is especially the extreme dependence that appears to be large in cross-country bond markets and equity markets taken separately. This result may explains the lack of international diversification known as the home bias puzzle. International investors face high extreme dependence in bear markets and therefore lose the diversification gain when they most need it. We have suggested that the exchange rate volatility may be a factor behind the asymmetric behavior of international markets' dependence. Therefore, it will be interesting to use a model similar to the model explored in this paper, possibly incorporating exchange rates, to study the portfolio of an international investor in the spirit of Ang and Bekaert (2002).

Appendix A. Proofs

Proof of Proposition 2.1

To prove this proposition, we need the two following lemmas

Lemma 1: *(a) Let* $\left\{f^{(s)}\right\}_{s=1}^{n}$ *be a family of symmetric multivariate density functions of* $n\,(\leq\infty)$ *variables with same mean. The mixture* $f=\sum_{s=1}^{n}\pi_s f^{(s)}$*, where* $\sum_{s=1}^{n}\pi_s=1$*, and* $\pi_s\geq 0$ *for any* s*, is a symmetric multivariate density function. (b) Moreover for a continuum of symmetric multivariate density function* $\left\{f^{(\sigma)}\right\}_{\sigma\in A\subseteq\mathbb{R}}$ *with same mean, the mixture* $f=\int_A \pi_\sigma f^{(\sigma)}d\sigma$*, where* $\int_A \pi_\sigma d\sigma=1$*, is a symmetric multivariate density function.*

Proof: Let μ be the mean of all $f^{(s)}$ (and all $f^{(\sigma)}$)

$$f\left(\mu-x\right)=\sum_{s=1}^{n}\pi_s f^{(s)}\left(\mu-x\right)$$

by symmetry of all $f^{(s)}$, we have, $\sum_{s=1}^{n}\pi_s f^{(s)}\left(\mu-x\right)=\sum_{s=1}^{n}\pi_s f^{(s)}\left(\mu+x\right)=f\left(\mu+x\right)$

i.e. $f\left(\mu-x\right)=f\left(\mu+x\right)$ and the part (a) follows. Similarly for mixture of continuum,

$f\left(\mu-x\right)=\int_A \pi_\sigma f^{(\sigma)}\left(\mu-x\right)d\sigma=\int_A \pi_\sigma f^{(\sigma)}\left(\mu+x\right)d\sigma=f\left(\mu+x\right)$ and we have (b).

Lemma 2: *Let* $\left\{F^{(s)}\right\}_{s=1}^{n}$ *be a family of bivariate cdf with zero lower (upper) TDC. The mixture* $F=\sum_{s=1}^{n}\pi_s F^{(s)}$*, where* $\sum_{s=1}^{n}\pi_s=1$*, and* $\pi_s\geq 0$*, for any* s*, is a bivariate density function with lower (upper) TDC.*

Proof: we do the proof for lower tail since by "rotation" we have the same result for upper tail.

Let τ_L^F be the lower TDC of F, we have

$$\begin{aligned}\tau_L^F &= \lim_{\alpha\to 0}\Pr\left[X\leq F_x^{-1}\left(\alpha\right)\middle|Y\leq F_y^{-1}\left(\alpha\right)\right]\\ &= \lim_{\alpha\to 0}\frac{\Pr\left[X\leq F_x^{-1}\left(\alpha\right),Y\leq F_y^{-1}\left(\alpha\right)\right]}{\Pr\left[Y\leq F_y^{-1}\left(\alpha\right)\right]}\\ &= \lim_{\alpha\to 0}\frac{F\left(F_x^{-1}\left(\alpha\right),F_y^{-1}\left(\alpha\right)\right)}{F_y\left(F_y^{-1}\left(\alpha\right)\right)}\end{aligned}$$

and since $F=\sum_{s=1}^{n}\pi_s F^{(s)}$, we have

$$
\begin{aligned}
\tau_L^F &= \lim_{\alpha\to 0}\frac{\sum\limits_{s=1}^{n}\pi_s F^{(s)}\left(F_x^{-1}(\alpha),F_y^{-1}(\alpha)\right)}{\alpha}\\
&= \lim_{\alpha\to 0}\sum_{s=1}^{n}\pi_s\frac{F^{(s)}\left(F_x^{-1}(\alpha),F_y^{-1}(\alpha)\right)}{\alpha}\\
&= \sum_{s=1}^{n}\pi_s\lim_{\alpha\to 0}\frac{F^{(s)}\left(F_x^{-1}(\alpha),F_y^{-1}(\alpha)\right)}{\alpha}
\end{aligned}
$$

by definition

$$
F^{(s)}\left(F_x^{-1}(\alpha),F_y^{-1}(\alpha)\right)=C^{(s)}\left(F_x^{(s)}\left(F_x^{-1}(\alpha)\right),F_y^{(s)}\left(F_y^{-1}(\alpha)\right)\right)
$$

where $C^{(s)}$ is the copula and $F_x^{(s)}$, $F_y^{(s)}$ the marginal cdf corresponding to $F^{(s)}$, we have

$$
\alpha=F_x\left(F_x^{-1}(\alpha)\right)=\sum_{s=1}^{n}\pi_s F_x^{(s)}\left(F_x^{-1}(\alpha)\right)
$$

so
$F_x^{(s)}\left(F_x^{-1}(\alpha)\right)\leq\alpha/\pi_s$ for all s and similarly $F_y^{(s)}\left(F_y^{-1}(\alpha)\right)\leq\alpha/\pi_s$,
hence

$$
\begin{aligned}
\lim_{\alpha\to 0}\tfrac{F^{(s)}\left(F_x^{-1}(\alpha),F_y^{-1}(\alpha)\right)}{\alpha} &= \lim_{\alpha\to 0}\frac{C^{(s)}\left(F_x^{(s)}\left(F_x^{-1}(\alpha)\right),F_y^{(s)}\left(F_y^{-1}(\alpha)\right)\right)}{\alpha}\\
&\leq \lim_{\alpha\to 0}\frac{C^{(s)}\left(\alpha/\pi_s,\alpha/\pi_s\right)}{\alpha}, \text{ since copula is increasing function}\\
&= 1/\pi_s\lim_{\alpha'\to 0}\frac{C^{(s)}\left(\alpha',\alpha'\right)}{\alpha'} \text{ by setting } \alpha'=\alpha/\pi_s\\
&= 0, \text{ since } F^{(s)} \text{ and hence } C^{(s)} \text{ is zero lower TDC}
\end{aligned}
$$

we therefore have $\tau_L^F=0$

The part (i) and (ii) of the proposition is the straightforward application of above lemma

- For GARCH with constant mean and symmetric conditional distribution

$$
X_t=\mu+\Sigma_{t-1}^{1/2}\varepsilon_t
$$
$$
(+ \text{ any GARCH dynamic equation of } \Sigma_{t-1})
$$

where ε_t is stationary with symmetric distribution such that $E(\varepsilon_t)=0$. The unconditional distribution of X_t is a mixture of distribution of symmetric variable with same mean μ but possibly different variance covariance matrix. By applying the lemma 1, we conclude that the unconditional distribution of X_t is symmetric and (i) follows.

- For RS model with zero TDC

$$X_t = \mu_{s_t} + \Sigma_{s_t}^{1/2}\varepsilon_t$$

where s_t takes a discrete value. Without loss of generality assume that X_t is bivariate and that $s_t = s$, $\mu + \Sigma^{1/2}\varepsilon_t$ is zero TDC such as in the normal case, therefore the unconditional distribution of X_t is a mixture of distribution with zero TDC. By applying the lemma 2, we conclude that the unconditional distribution of X_t has zero TDC. and (ii) follows

For (iii), with the same notations as lemma 1, keeping marginal distribution unchanged across mixture components means that. For discrete case
$f^{(s)}(x_1, ..., x_n; \delta, \rho) = c^{(s)}(u_1, ..., u_n; \theta) \times \prod_{i=1}^{n} f_i(x_i; \delta_i)$, with $u_i = F_i(x_i; \delta_i)$, hence

$$\begin{aligned}
f(x_1, ..., x_n; \delta, \rho) &= \sum_{s=1}^{n} \pi_s f^{(s)}(x_1, ..., x_n; \delta, \rho) \\
&= \sum_{s=1}^{n} \pi_s c^{(s)}(u_1, ..., u_n; \theta) \times \prod_{i=1}^{n} f_i(x_i; \delta_i) \\
&= c(u_1, ..., u_n; \theta) \times \prod_{i=1}^{n} f_i(x_i; \delta_i)
\end{aligned}$$

with $c(u_1, ..., u_n; \theta) = \sum_{s=1}^{n} \pi_s c^{(s)}(u_1, ..., u_n; \theta)$ is the copula of f and we can see that c is a mixture of copula with symmetric TDC and hence is a copula with symmetric TDC.

for the continuum case

$$\begin{aligned}
f(x_1, ..., x_n; \delta, \rho) &= \int_A \pi_\sigma f^{(\sigma)}(x_1, ..., x_n; \delta, \rho)\, d\sigma \\
&= \int_A \pi_\sigma c^{(\sigma)}(u_1, ..., u_n; \theta)\, d\sigma \times \prod_{i=1}^{n} f_i(x_i; \delta_i) \\
&= c(u_1, ..., u_n; \theta) \times \prod_{i=1}^{n} f_i(x_i; \delta_i)
\end{aligned}$$

with $c(u_1, ..., u_n; \theta) = \int_A \pi_\sigma c^{(\sigma)}(u_1, ..., u_n; \theta)\, d\sigma$ which is a copula with symmetric TDC for same the reasons mentioned above.

Q.E.D

Proof of Proposition 3.2.

By copula definition, we have

$$
\begin{aligned}
\eta_t &= \begin{bmatrix} f\left(X_t;\delta,\theta\right|\underline{X}_{t-1},s_t=1) \\ f\left(X_t;\delta,\theta\right|\underline{X}_{t-1},s_t=0) \end{bmatrix} \\
&= \begin{bmatrix} c\left(u_{1,t}\left(\delta_1\right),...,u_{4,t}\left(\delta_4\right);\theta\left|s_t=1\right.\right)\times\prod_{i=1}^{4}f_i\left(x_{i,t};\delta_i\right) \\ c\left(u_{1,t}\left(\delta_1\right),...,u_{4,t}\left(\delta_4\right);\theta\left|s_t=0\right.\right)\times\prod_{i=1}^{4}f_i\left(x_{i,t};\delta_i\right) \end{bmatrix}
\end{aligned}
$$

with $u_{i,t}\left(\delta_i\right)=F_i\left(x_{i,t};\delta_i\right)$

By denoting $\widehat{\xi}_{t|t-1}=\left(\widehat{\xi}^{(1)}_{t|t-1},\widehat{\xi}^{(0)}_{t|t-1}\right)'$, the likelihood can be rewritten

$$
\begin{aligned}
&L\left(\delta,\theta;\underline{X}_T\right) \\
&=\sum_{t=1}^{T}\log\left(\widehat{\xi}'_{t|t-1}\eta_t\right) \\
&=\sum_{t=1}^{T}\log\left(\sum_{k=0}^{1}\widehat{\xi}^{(k)}_{t|t-1}c\left(u_{1,t}\left(\delta_1\right),...,u_{4,t}\left(\delta_4\right);\theta\left|s_t=k\right.\right)\times\prod_{i=1}^{4}f_i\left(x_{i,t};\delta_i\right)\right) \\
&=\sum_{t=1}^{T}\left[\sum_{i=1}^{4}\log\left(f_i\left(x_{i,t};\delta_i\right)\right)+\log\left(\sum_{k=0}^{1}\widehat{\xi}^{(k)}_{t|t-1}c\left(u_{1,t}\left(\delta_1\right),...,u_{4,t}\left(\delta_4\right);\theta\left|s_t=k\right.\right)\right)\right]
\end{aligned}
$$

it follows that

$$
L\left(\delta,\theta;\underline{X}_T\right)=\sum_{i=1}^{4}L_i\left(\delta_i;\underline{X}_T\right)+L_C\left(\delta,\theta;\underline{X}_T\right)
$$

where

$$
\begin{aligned}
L_i\left(\delta_i;\underline{X}_{i,T}\right) &= \sum_{t=1}^{T}\log f_i\left(x_{i,t};\delta_i\right|\underline{X}_{i,t-1}) \\
L_C\left(\delta,\theta;X\right) &= \sum_{t=1}^{T}\log\left(\widehat{\xi}'_{t|t-1}\eta_{ct}\right)
\end{aligned}
$$

with

$$
\eta_{ct}=\begin{bmatrix} c\left(u_{1,t}(\delta_1),...,u_{n,t}(\delta_n);\theta\right|s_t=1) \\ c\left(u_{1,t}(\delta_1),...,u_{n,t}(\delta_n);\theta\right|s_t=0) \end{bmatrix}
$$

by noticing that $\eta_t=\eta_{ct}\times\prod_{i=1}^{4}f_i\left(x_{i,t};\delta_i\right)$ we have that

$$
\widehat{\xi}_{t/t}=\left[\widehat{\xi}'_{t|t-1}\eta_t\right]^{-1}\left(\widehat{\xi}_{t|t-1}\odot\eta_t\right)=\left[\widehat{\xi}'_{t|t-1}\eta_{ct}\right]^{-1}\left(\widehat{\xi}_{t|t-1}\odot\eta_{ct}\right)
$$

Q.E.D

Appendix B. Copulas' expressions

Normal copula

$$C_N(u_1,\dots,u_n;\rho)=\Phi_\rho\left(\Phi^{-1}(u_1),\dots,\Phi^{-1}(u_n)\right)$$

$$C_N(u_1,\dots,u_n;\rho)=\int_{-\infty}^{\Phi^{-1}(u_1)}\dots\int_{-\infty}^{\Phi^{-1}(u_n)}\left[(2\pi)^n\det(\rho)\right]^{-\frac{1}{2}}\exp\left[-\tfrac{1}{2}\left(z'\rho^{-1}z\right)\right]dz_1\dots dz_n$$

where $z=(z_1,\dots,z_n)'$, $\rho=\left(\rho_{ij}\right)_{i,j=1}^n$, with $\left|\rho_{ij}\right|\le 1$, $\rho_{ii}=1$ and ρ positive defined matrix

$$c_N(u_1,\dots,u_n;\rho)=\left(\det(\rho)\exp\left[x'\rho^{-1}x-x'x\right]\right)^{-1/2}$$

with $x=\left(\Phi^{-1}(u_1),\dots,\Phi^{-1}(u_n)\right)'$,

Φ is *cdf* of standard normal distribution and Φ_ρ is *cdf* of multivariate normal distribution with correlation matrix ρ.

Tail dependence coefficients are

$\tau^L=\tau^U=0$

Bivariate Gumbel copula

$$C_G(u,v;\theta)=\exp\left[-\left((-\log(u))^\theta+(-\log(v))^\theta\right)^{1/\theta}\right]$$

$$c_G(u,v;\theta)=\frac{C_G(u,v;\theta)\left(\log(u)\,.\,\log(v)\right)^{\theta-1}}{uv\left((-\log(u))^\theta+(-\log(v))^\theta\right)^{2-1/\theta}}\left(\left((-\log(u))^\theta+(-\log(v))^\theta\right)^{1/\theta}+\theta-1\right)$$

Bivariate Gumbel Survival copula

$$C_{GS}(u,v;\theta)=u+v-1+C_G(1-u,1-v;\theta)$$

$$c_{GS}(u,v;\theta)=c_G(1-u,1-v;\theta)$$

The tail dependence coefficients of C_{GS} are

$\tau^L=2-2^{\frac{1}{\theta}}$ and $\tau^U=0$

so $\theta=\theta\left(\tau^L\right)=\dfrac{\log(2)}{\log\left(2-\tau^L\right)}$

and we can re-parameterize the Copula $C_{GS}(u,v;\theta)$ with τ^L as $C_{GS}\left(u,v;\tau^L\right)=C_{GS}\left(u,v;\theta\left(\tau^L\right)\right)$

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6 Tables and Graphs

Table 0: Two letters country code

United States	Canada	France	Germany
US	CA	FR	DE

Table 1: Summary statistics of weekly bond and equity index returns for the four countries. All returns are expressed in US dollars on a weekly base from January 01, 1985 to December 21, 2004, what corresponds to a sample of 1044 observations. ($^{\delta}$ Denotes annualized percent). Sharpe ratio represents the ratio of the mean over the standard deviation of return.

	Mean$^{\delta}$	Std$^{\delta}$	Kurtosis	Skewness	Min$^{\delta}$	Max$^{\delta}$	Sharpe ratio
US Equity	13.67	17.51	17.00	-1.55	-680.36	311.10	0.78
US Bond	7.57	4.69	0.67	-0.06	-66.91	58.81	1.61
CA Equity	11.24	16.72	13.62	-1.67	-610.87	225.15	0.67
CA Bond	8.81	8.15	1.13	-0.24	-130.55	118.07	1.08
FR Equity	14.72	23.43	7.18	-0.09	-582.12	512.16	0.63
FR Bond	11.52	11.16	0.92	0.04	-142.02	166.68	1.03
DE Equity	12.57	24.97	8.01	-0.46	-574.96	463.08	0.50
DE Bond	10.44	11.56	0.82	-0.01	-142.54	171.39	0.90

Table 2: Unconditional correlations between different assets (bond and equity) of four considered countries.

	US Equity	US Bond	CA Equity	CA Bond	FR Equity	FR Bond	DE Equity
US Bond	0.0576						
CA Equity	0.7182	0.0116					
CA Bond	0.1783	0.4706	0.4392				
FR Equity	0.1957	-0.0182	0.1974	0.1065			
FR Bond	-0.0499	0.3386	-0.0080	0.2433	0.3066		
DE Equity	0.2089	-0.0536	0.1995	0.1009	0.8099	0.2625	
DE Bond	-0.0832	0.3081	-0.0234	0.2143	0.3084	0.9403	0.2847

Table 3: Estimates of M-GARCH (1, 1) parameters for all bond and equity returns of four countries. The value into the brackets represents the standard deviation of parameter estimator. L is the Log likelihood function of variable. t represent the student t statistic of the difference between parameter μ for Equity and μ for Bond in the same country. This statistic allow to test if the parameter μ can be interpreted as the interest rate of the country.

	US		CA		FR		DE	
	Equity	Bond	Equity	Bond	Equity	Bond	Equity	Bond
β	7.94e-1	7.82e-1	8.09e-1	9.07e-1	9.68e-1	9.36e-1	9.24e-1	9.56e-1
	(3.49e-1)	(1.62e-1)	(4.06e-1)	(1.79e-1)	(3.61e-1)	(4.21e-1)	(1.54e-1)	(2.45e-1)
α	5.46e-5	2.63e-6	6.40e-5	7.30e-6	2.28e-5	1.51e-5	2.22e-5	1.08e-5
	(4.04e-5)	(6.36e-5)	(8.16e-5)	(2.94e-5)	(9.35e-6)	(2.17e-5)	(2.14e-4)	(1.88e-5)
γ	4.45e+1	3.84e+1	2.73e+1	3.28e+1	1.91e+1	6.53e+0	1.19e+1	3.26e+0
	(1.70e-2)	(6.11e-3)	(1.14e-2)	(1.22e-2)	(1.61e-2)	(1.85e-1)	(8.07e-2)	(2.45e-2)
λ	1.72e+0	1.37e+1	3.13e+0	1.01e+1	1.61e+0	5.61e+0	1.78e+0	6.13e+0
	(1.39e-2)	(1.05e-2)	(2.09e-2)	(7.59e-3)	(7.22e-3)	(1.96e-1)	(6.33e-2)	(7.86e-3)
ω	7.57e-6	6.49e-6	1.21e-5	3.49e-6	1.99e-6	1.51e-7	6.46e-5	4.79e-7
	(9.64e-5)	(1.90e-5)	(1.74e-5)	(2.52e-5)	(6.53e-5)	(4.33e-5)	(1.92e-4)	(3.25e-5)
μ	1.07e-3	7.18e-4	1.32e-3	4.73e-4	1.48e-3	5.37e-4	6.51e-4	1.35e-4
	(1.29e-4)	(6.74e-5)	(3.76e-5)	(5.26e-5)	(5.00e-4)	(1.45e-4)	(1.32e-4)	(3.38e-5)
L	2.49e+3	3.77e+3	2.50e+3	3.20e+3	2.10e+3	2.88e+3	2.04e+3	2.84e+3
t	2.4185		13.0999		1.8114		3.7869	

Table 4: Dependence structure between United States and Canada of equity and bond markets. Correlation coefficients are reported for normal regime, while Tail dependence coefficients described the asymmetric regime. The tail dependence coefficient is obtained as the product of parameter τ and the respective weight π for cross-asset dependence and 1-π for cross-country dependence. Standard Deviation in the parenthesis is reported just for all parameters directly estimated from the model. Last raw reports the diagonal elements of transitional probability matrix

Cross-Country (US-CA) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
				τ	TDC((1-π)τ)
US Equity - CA Equity		0.8739		0.9100	0.7917
		(0.1560)		(0.0185)	
US Bond - CA Bond		0.3870		0.6234	0.5424
		(0.0831)		(0.0124)	
			1-π	0.6897	
Cross-Asset (Equity-Bond) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
	US Bond	CA Bond		τ	TDC($\pi\tau$)
US Equity	-0.1101	0.1234	US Equity - US Bond	0.1300	0.0169
	(0.0416)	(0.0312)		(0.041)	
CA Equity	-0.0812	0.4085	CA Equity - CA Bond	0.1385	0.0180
	(0.0207)	(0.0103)		(0.0145)	
			π	0.3102	
				(0.0207)	
Parameters of transitional probability matrix					
	P	0.9020	Q	0.9586	
		(0.0207)		(0.0206)	

Table 5: Dependence structure between France and Germany of equity and bond markets. Correlation coefficients are reported for normal regime, while Tail dependence coefficients described the asymmetric regime. The tail dependence coefficient is obtained as the product of parameter τ and the respective weight π for cross-asset dependence and 1-π for cross-country dependence. Standard Deviation in the parenthesis is reported just for all parameters directly estimated from the model. Last raw reports the diagonal elements of transitional probability matrix

Cross-Country (FR-DE) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
				τ	TDC((1-π)τ)
FR Equity - DE Equity		0.9083		0.9554	0.7787
		(0.0267)		(0.0603)	
FR Bond - DE Bond		0.9901		0.8261	0.6733
		(0.058)		(0.027)	
			1-π	0.8151	
Cross-Asset (Equity-Bond) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
	FR Bond	DE Bond		τ	TDC($\pi\tau$)
FR Equity	0.1893	0.2023	FR Equity - FR Bond	0.0923	0.0171
	(0.0170)	(0.0129)		(0.028)	
DE Equity	0.1175	0.1294	DE Equity - DE Bond	0.0969	0.0179
	(0.0214)	(0.030)		(0.029)	
			π	0.1849	
				(0.0294)	
Parameters of transitional probability matrix					
	P	0.8381	Q	0.9373	
		(0.0270)		(0.0373)	

Table 6: Subperiod I (period before the introduction of Euro money: from January 01, 1985 to December 29, 1998 what corresponds to a sample of 731 observations). Dependence structure between France and Germany of equity and bond markets. Correlation coefficients are reported for normal regime, while Tail dependence coefficients described the asymmetric regime. The tail dependence coefficient is obtained as the product of parameter τ and the respective weight π for cross-asset dependence and 1-π for cross-country dependence. Standard Deviation in the parenthesis is reported just for all parameters directly estimated from the model. Last raw reports the diagonal elements of transitional probability matrix

Cross-Country (FR-DE) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
				τ	TDC((1-π)τ)
FR Equity - DE Equity		0.6924		0.9554	0.8663
		(0.0760)		(0.035)	
FR Bond - DE Bond		0.9082		0.8388	0.7606
		(0.038)		(0.061)	
			1-π	0.9067	
Cross-Asset (Equity-Bond) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
	FR Bond	DE Bond		τ	TDC($\pi\tau$)
FR Equity	0.2091	0.1641	FR Equity - FR Bond	0.1130	0.0105
	(0.0123)	(0.0151)		(0.021)	
DE Equity	0.1205	0.1519	DE Equity - DE Bond	0.0067	0.0006
	(0.0106)	(0.049)		(0.072)	
			π	0.0933	
				(0.010)	
Parameters of transitional probability matrix					
	P	0.0651	Q	0.9438	
		(0.0103)		(0.0102)	

Table 7: Subperiod II (period after the introduction of Euro money: from January 05, 1999 to December 21, 2004 what corresponds to a sample of 313 observations). Dependence structure between France and Germany of equity and bond markets. Correlation coefficients are reported for normal regime, while Tail dependence coefficients described the asymmetric regime. The tail dependence coefficient is obtained as the product of parameter τ and the respective weight π for cross-asset dependence and 1-π for cross-country dependence. Standard Deviation in the parenthesis is reported just for all parameters directly estimated from the model. Last raw reports the diagonal elements of transitional probability matrix

Cross-Country (FR-DE) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
				τ	TDC((1-π)τ)
FR Equity - DE Equity		0.9426		0.2598	0.2582
		(0.0950)		(0.0106)	
FR Bond - DE Bond		0.9937		0.8946	0.8892
		(0.0382)		(0.071)	
			1-π	0.9940	
Cross-Asset (Equity-Bond) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
	FR Bond	DE Bond		τ	TDC($\pi\tau$)
FR Equity	0.2272	0.2350	FR Equity - FR Bond	0.2249	0.0013
	(0.0241)	(0.0177)		(0.024)	
DE Equity	0.1516	0.1573	DE Equity - DE Bond	0.9760	0.0059
	(0.0118)	(0.059)		(0.082)	
			π	0.0060	
				(0.012)	
Parameters of transitional probability matrix					
	P	0.9212	Q	0.2274	
		(0.0118)		(0.0117)	

Table 8: Monte Carlo Test of Asymmetry Dependence. LR is the likelihood ratio statistic compute from data. $p-value$ is obtained from 1000 Monte Carlo trails with size 1043 (equal to the sample size) each.

	US-Canada	France-Germany
LR	0.0731	0.7889
$p-value$	0.0090	0.0000

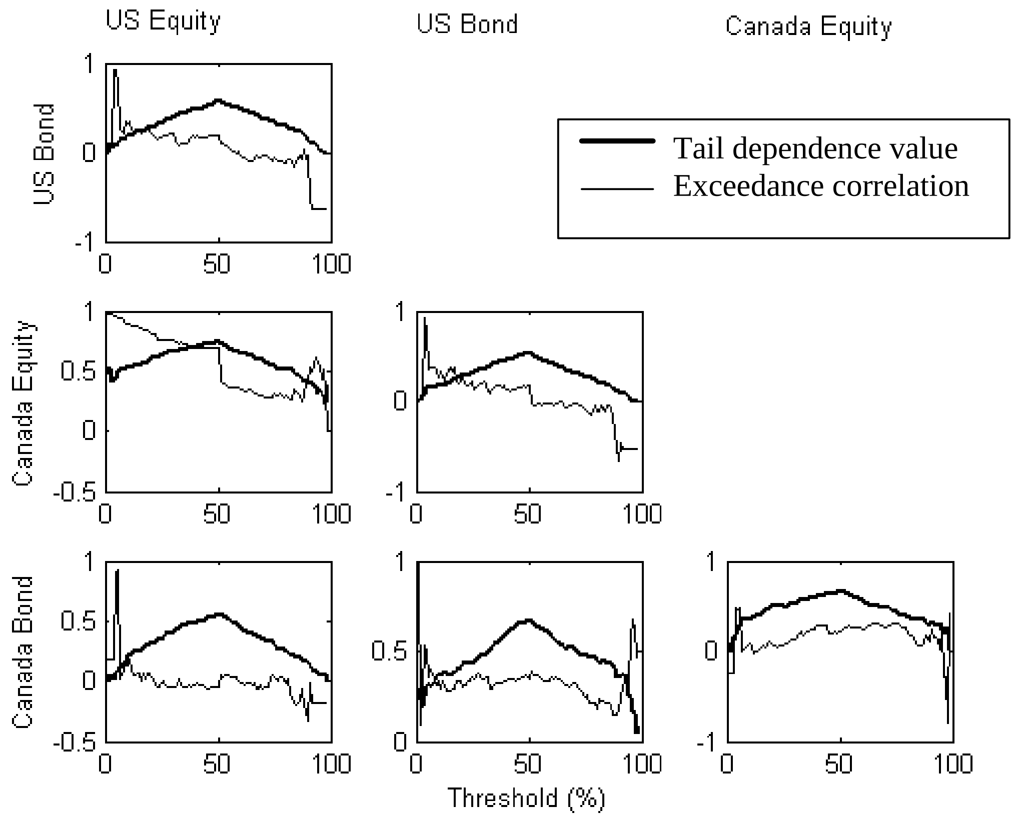

Figure 4.
Tail dependence function and Exceedance correlation between US and Canada asset (bond and equity) returns.

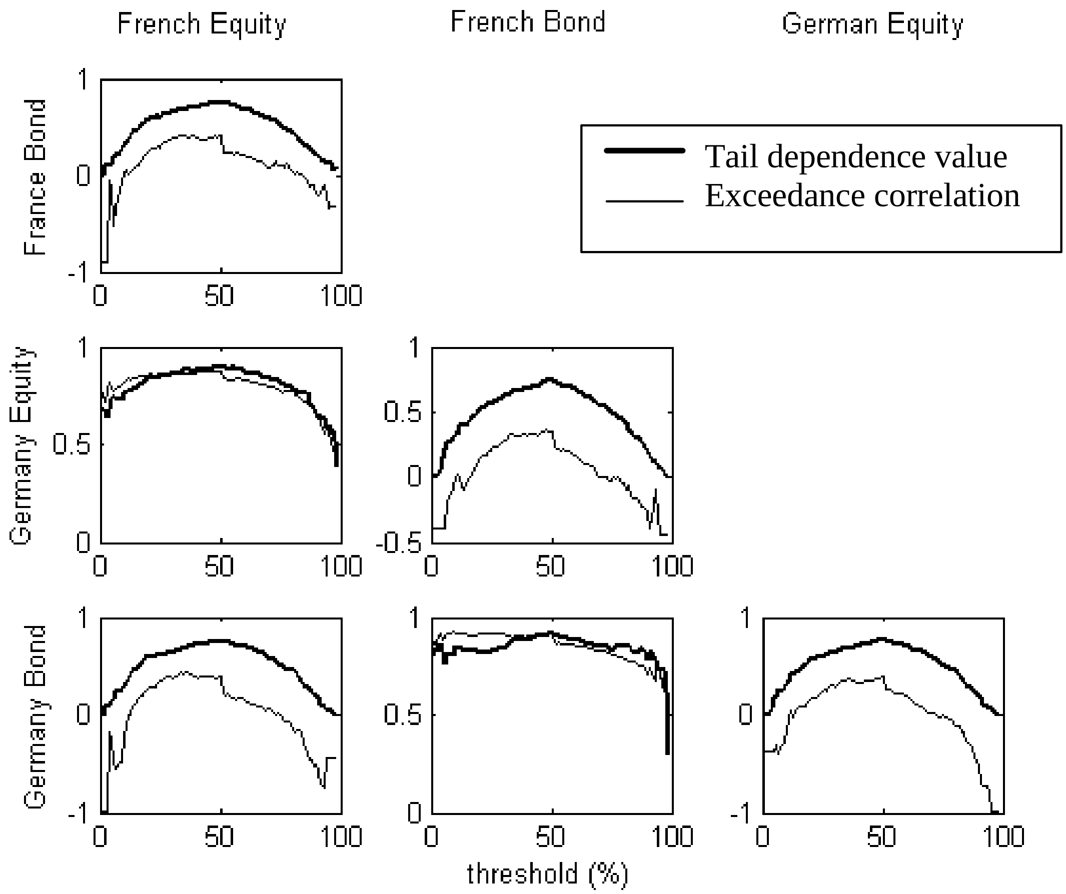

Figure 5.
Tail dependence function and Exceedance correlation between France and Germany asset (bond and equity) returns

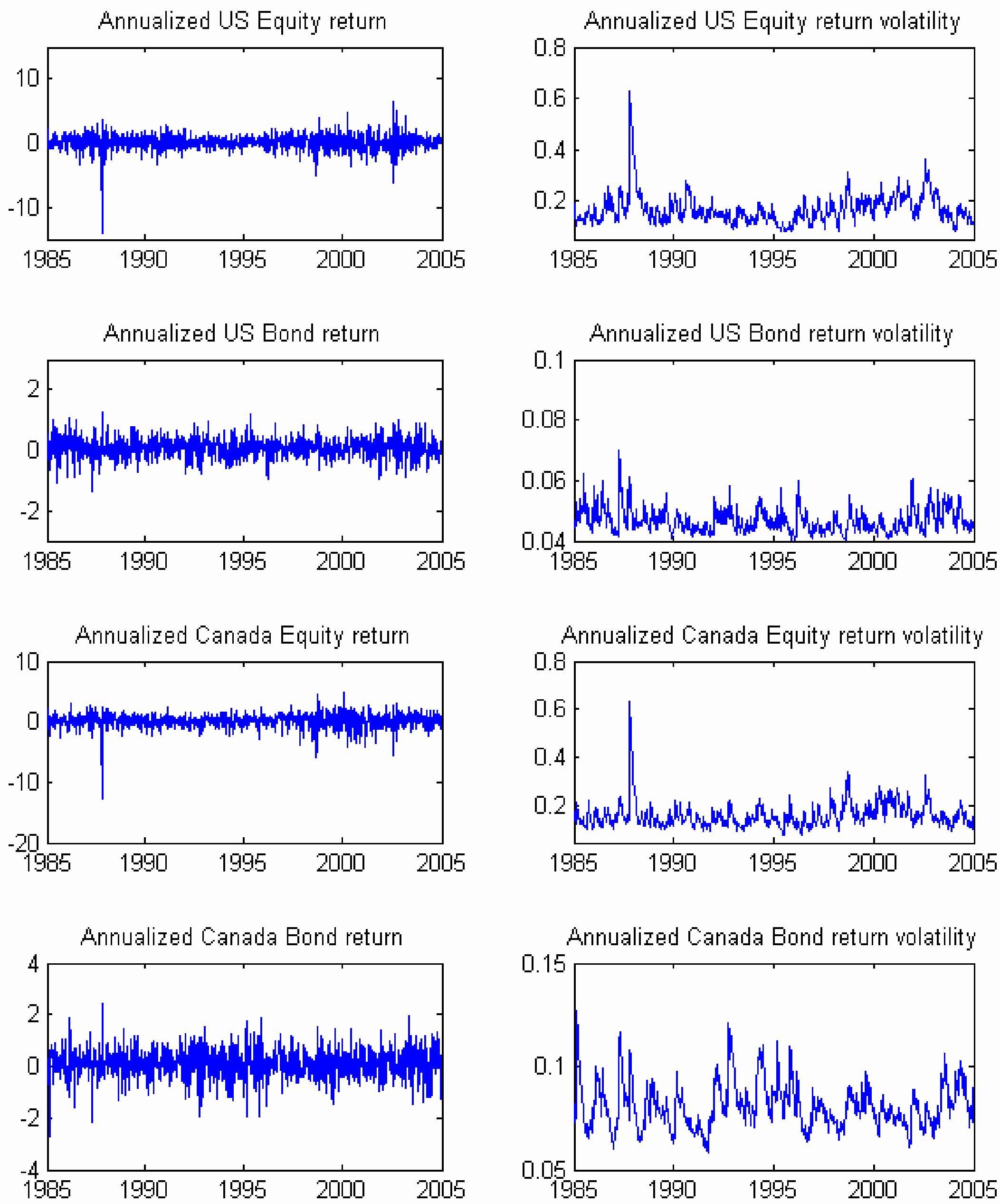

Figure 6.
Annualized bond and equity returns time series for US and Canada, with their conditional volatilities obtained using the M-GARCH (1,1).

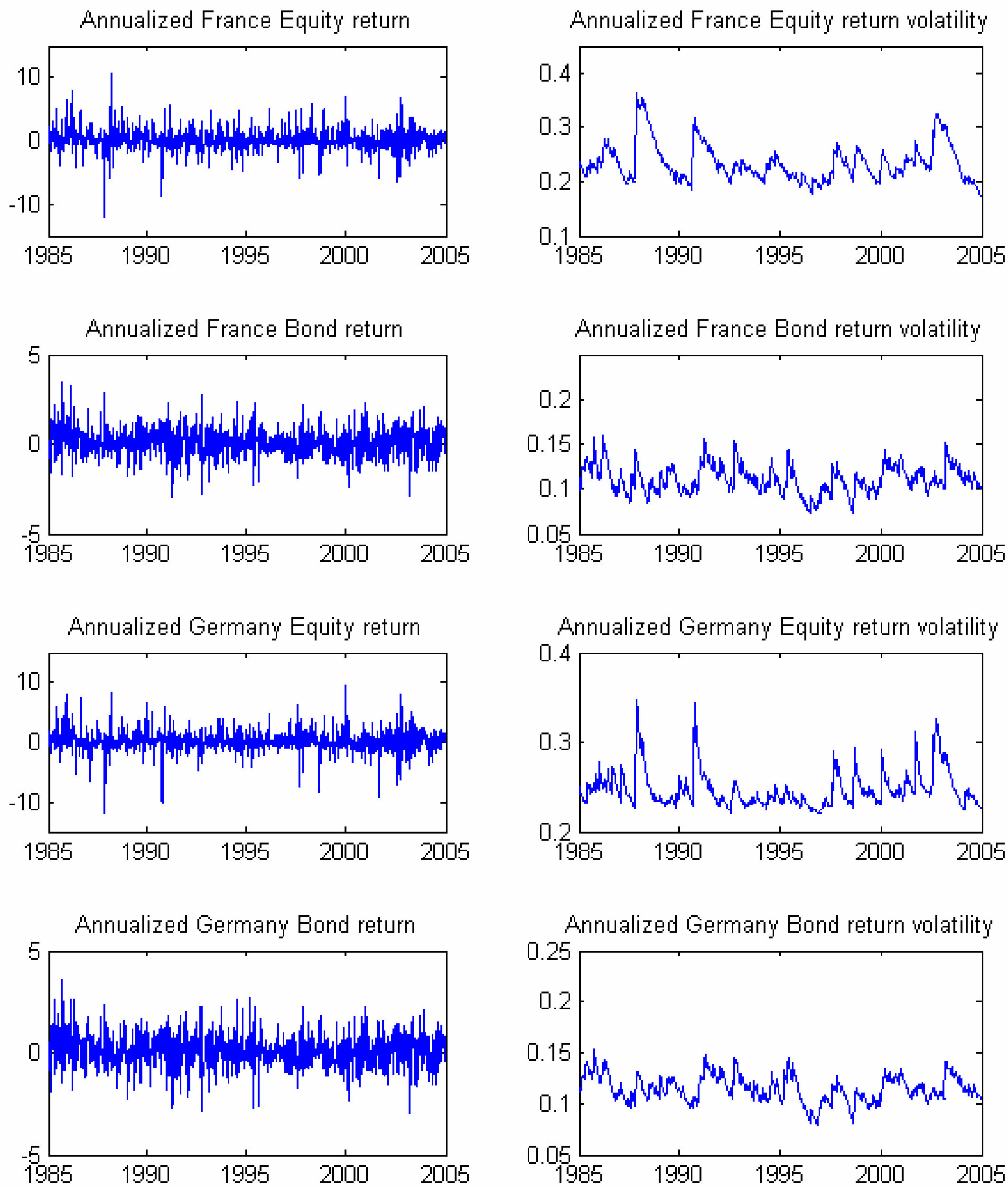

Figure 7.
Annualized bond and equity returns time series for France and Germany, with their conditional volatilities obtained using the M-GARCH (1,1).

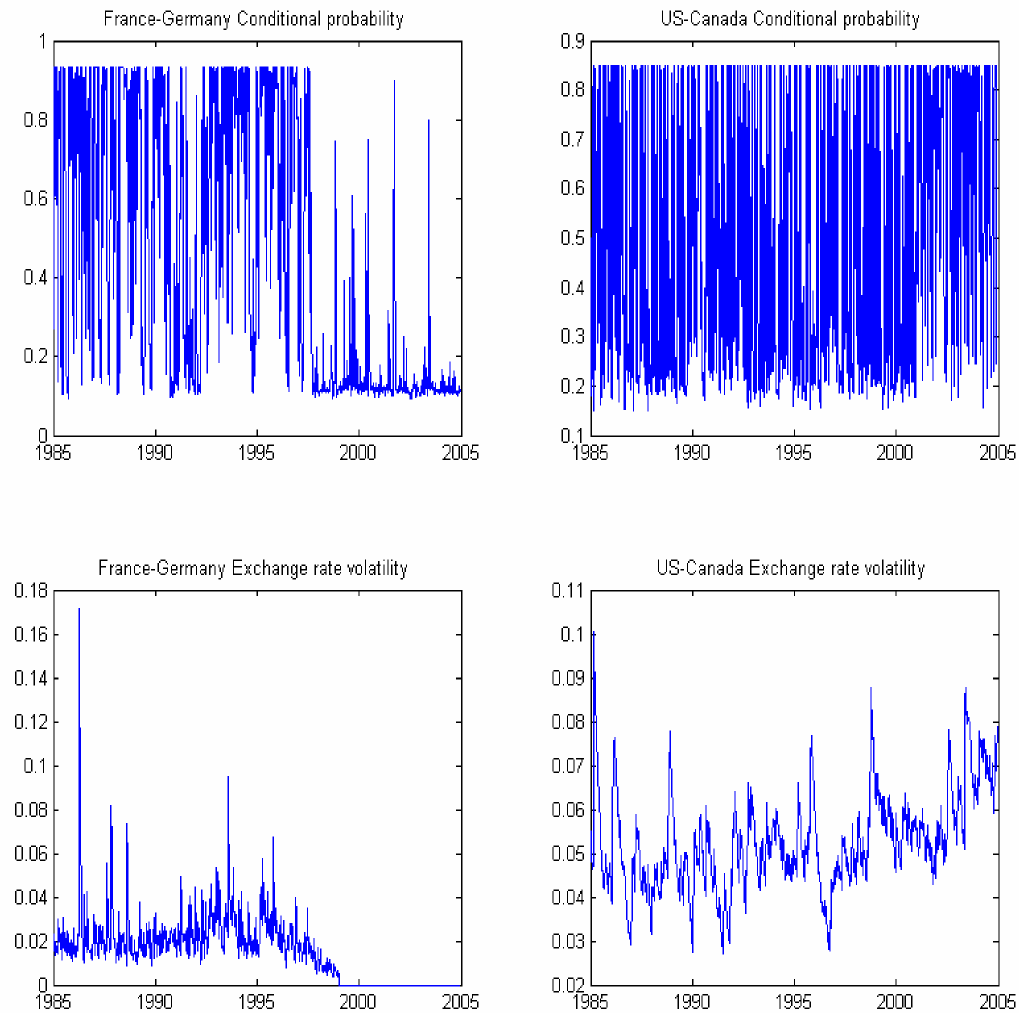

Figure 8.
Conditional probability denotes the probability to be in asymmetric regime conditional to available information. Exchange rate volatility is the conditional volatility filtered with the M-GARCH (1,1) model.