

# **Trading Volatility Spreads: A Test of Index Option Market Efficiency**

Ser-Huang Poon and Peter F. Pope

Lancaster University

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# **Trading Volatility Spreads: A Test of Index Option Market Efficiency**

## **Abstract**

If returns on two assets share common volatility components, the prices of options on the assets should be interdependent and the implied volatility spread should mean revert. We first demonstrate, using the canonical correlation method, that there is a common component among the volatilities of the returns on S&P 100 and S&P 500 indexes. We then exploit this commonality by trading on the volatility spread between tick-by-tick OEX and SPX call options listed on the CBOE. Our vega-delta-neutral strategies generated significant profits, even after transaction costs are taken into account. The results suggest that the two options markets are not jointly efficient.

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# Trading Volatility Spreads: A Test of Index Option Market Efficiency

## 1. Introduction

Most prior research on the efficiency of traded option markets usually involves testing whether options are fairly priced, conditional on a pricing model and a volatility forecast.<sup>1</sup> The consensus view emerging from such research is that stock option prices are efficient. Simulated trading of mispriced options generates insignificant abnormal profits, after controlling for transaction costs (Phillips and Smith, 1980). An alternative approach to testing option market efficiency is adopted by Harvey and Whaley (1992b), who test the profitability of a trading rule based on the time series predictability of S&P 100 index option *implied* volatility, estimated from a binomial option pricing model. Their approach has the advantage of not requiring exogenous volatility forecasts of the underlying asset returns. Harvey and Whaley (1992b) also conclude in favor of option market efficiency, after allowing for transaction costs. Thus, the evidence in favor of option market efficiency appears robust to different testing approaches.

The results from prior research do not rule out the possibility that option prices are inefficient with respect to other information excluded from the information set conditioning volatility forecasts. In particular, prior research does not consider information on common volatility components in the returns of other similar assets and their option prices. Our research design focuses on the *relative* pricing of options on “closely related” assets, defined as assets having a large common volatility component. Options on closely related assets will be (imperfect) hedges for one another, and we exploit this property to develop an efficiency test based on trading the volatility spread between the two option series. By concentrating on the relative pricing of options, our approach avoids the need to forecast the absolute level of volatility of the underlying asset. We examine the relative pricing of options written

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<sup>1</sup> Examples include Whaley (1982), Noh, Engel, and Kane (1994) and Kroner (1996).

on the S&P 100 and S&P 500 (i.e. the OEX and the SPX options respectively). We find evidence that trading strategies based on the OEX-SPX implied volatility spread are profitable on an ex ante basis, after transaction costs are taken into account.

The remainder of the paper is organized as follows: in Section 2 we discuss the source of and the test for common volatility component. Section 3 describes the data. Section 4 describes the estimation of contract and index composite implied volatilities. Section 5 describes the trading rule and reports results. Finally, we conclude in Section 6.

## **2. Common volatility components**

### *2.1. Common volatility in stock index returns*

If returns on two assets are related to one or more common factors, the respective volatilities of the two assets will also be related. For example, under the Capital Asset Pricing Model, the volatility of returns on a portfolio of stocks is equal to a systematic component (market volatility multiplied by the portfolio beta squared), plus an unsystematic component (the portfolio's residual risk). Changes in price levels due to common return factors alone is a sufficient condition for the positive association between the returns volatilities of the two assets. Thus, if expected returns on two optionable assets can be described adequately by a common factor model, we would expect prices of options on the two assets to be interdependent. If arbitrage<sup>2</sup> between the two option series is not perfect, implied volatility (or prices) of one option series will predict relative movements of implied volatility (or prices) of the other series. Specifically, if a pair of assets satisfies our definition of being closely related, we expect short-term mean reversion in their implied volatility spread.

In our empirical tests we seek to exploit this intuition by examining options on two U.S. stock indexes: the S&P 100 (for the OEX options) and the S&P 500 index (for the SPX options). The S&P 100 is a value-weighted index comprising one hundred blue-chip stocks listed on U.S. exchanges and the S&P 500 is a value-

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<sup>2</sup> We use the term "arbitrage" in the risky arbitrage sense employed by market practitioners. Spread trading across different option series cannot be totally risk free.

weighted index comprising five hundred widely held U.S.-listed common stocks. Although the S&P 100 index comprises only 20% of the S&P 500 index by number of stocks, it is a very large subset of S&P 500 by market value. For example, as of July 1, 1999, the total market value of the stocks in the S&P 500 index comprised \$11.2 trillion, while market capitalization of the S&P 100 index constituents was \$5.5 trillion, some 49% of the value of the larger index. Thus, even in the highly unlikely event that returns on the four hundred stocks omitted from the S&P 100 index were uncorrelated with those included in the S&P 100 index, their volatilities would contain common volatility component(s).

- Insert Figure 1 about here -

Figure 1 plots the daily index levels and returns on S&P 100 and S&P 500 for the period March 6, 1984 to June 15, 1998. Figures 1(a) to 1(c) suggest that the two indexes have very similar features and are driven by the same factor(s). Indeed, the correlation coefficient between the two index returns is 0.972. A regression of S&P 100 returns on S&P 500 returns produced a regression coefficient of 1.040 (with a  $t$ -ratio of 247.02) and a zero constant term.

## 2.2. *Long range dependence in volatility*

To substantiate the basis of our trading rule, we need to formally test for common factor(s) between S&P 100 and S&P 500 volatilities. Whichever test adopted here must allow for the high degree of persistence in individual stock index volatility documented in prior research (e.g. Bollerslev, Chou, and Kroner, 1992). It must also accommodate the possibility of short-range dependent idiosyncratic noise in index volatility, which has the effect of making long-range dependence harder to detect (see Ray and Tsay, 1997).

- Insert Table 1 about here -

The strong persistence or long memory in volatility is confirmed by the integrating parameter,  $d$ , estimated using the Geweke and Porter-Hudak (1983) method<sup>3</sup> and reported in Table 1. The first two rows of each panel in Table 1 report

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<sup>3</sup> We refer to Geweke and Porter-Hudak (1983) for details and Robinson (1995) for a refinement of the GPH method.

the values of the integrating parameter,  $d$ , for absolute returns, squared returns and their logarithmic counterparts. We report results for the S&P 100 and S&P 500 indexes over the full 1984-1998 sample period and four subperiods. The first two subperiods (in Panels B and C) aim to isolate the effect of the stock market crash of October 19, 1987. The next two sub-periods (in Panels D and E) reflect the volatility relationships prior to and during the period covered by our option trading simulation.

Generally, the results in Table 1 confirm the finding of prior research that volatility is persistent. Except for the shorter pre-crash period in Panel B, the integrating parameters,  $d_{S\&P\ 100}$  and  $d_{S\&P\ 500}$ , are always statistically positive, reflecting the strong persistence in the volatility of the two return series. When  $d > 0.5$ , volatility is non-stationary. This is the case in the post-crash period for absolute returns reported in Panel C, and in the whole sample period for the logarithmic version of both absolute and squared returns reported in Panel A.<sup>4</sup>

### 2.3 *Testing for a common volatility component*

Two main approaches to identifying common volatility components may be adopted. The first is to model the common factor in the first moment of returns (e.g., Engle and Kozicki, 1993).<sup>5</sup> The second approach, adopted here, is to follow Ray and Tsay (1997) and directly model the common component in volatility. This approach captures all co-movements in volatility including those that are driven by common return factors. Ray and Tsay (1997) propose a canonical method that takes advantage of different decay rates of the correlation function for long- and short-range dependent processes. The task of identifying long-term volatility components is then turned into one of finding a linear combination of volatilities that has zero canonical correlation. Details of the Ray and Tsay (1997) estimation method are provided in the Appendix.

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<sup>4</sup> The Ray and Tsay (1997) model, used in the following subsection, is derived for stationary long memory series, but they claim that their derivations can be extended to include cases that involve non-stationary series.

<sup>5</sup> In Engle and Kozicki (1993) a linear combination of stock returns was constructed by minimising the ARCH effect of the combined series. Such a model was tested in Engle and Susmel (1993) on the returns of 18 international stock markets in a pair-wise manner.

The results of estimating the canonical model are reported in rows 3 to 5 in each panel of Table 1. The hypothesis of zero canonical correlation is confirmed in all cases. The  $p$ -value of Ray and Tsay's robust test statistic,  $T^*$ , indicates that there is no residual long memory in the canonical variate,  $\omega = y_{S\&P\ 100} - \beta y_{S\&P\ 500}$ . The lack of long memory is confirmed by the estimated values of the integrating parameter,  $d_\omega$ , all of which are statistically indistinguishable from zero. The estimates of  $\beta$  for  $|r|$  and  $\sqrt{\beta}$  for  $r^2$  suggest an equilibrium ratio, governing the volatility of the S&P 100 to that of the S&P 500, fluctuates in the region of 1.056 to 1.121.

The analyses here are based on the proxies of historical (i.e. realized) volatility. Christensen and Prabhala (1998) find implied volatility contains all information in realized volatility and that implied volatility is an unbiased estimate of realized volatility, after controlling for error-in-variables problems. Hence, we would expect common variation in historical volatilities to be reflected in implied volatilities of options on the two indexes. This commonality in volatilities provides the basis for our trading strategies described in Section 5.

### 3. The option markets and data

The options data used in this study are the OEX (S&P 100) and SPX (S&P 500) options listed on the Chicago Board Options Exchange (CBOE). OEX options are American style, maturing in four consecutive nearby months. Strike prices are set at 5 point intervals for options maturing in near months, and 10 point intervals for options maturing in far months. SPX options have been European style since trading was reintroduced in 1986. Contracts are set to mature in two nearby months and up to four months from the March quarterly cycle.<sup>6</sup> Strike prices are set at 5 point intervals for options maturing in the near month, and 25 point intervals for options maturing in far months. Both OEX and SPX options are traded from 8:30 to 15:15 EST. (Note that the stock market closes at 15:00.) Both options series expire on the Saturday following the third Friday of the expiry month and are settled on a cash basis. The contract size is \$100 times the index level. The tick size is 1/16 (or \$6.25)

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<sup>6</sup> Variants of SPX options (viz. NSX and SPL) can have maturity up to two years. The SPX options stored on the tapes that we received from the CBOE have maturity up to 270 days.

for option premia up to \$3 and 1/8 (or \$12.50) for option premia above \$3. The common expiry date, settlement method, trading time and contract size together with the strong and fundamental volatility linkage between S&P 100 and S&P 500 make OEX and SPX options ideal candidates for our trading rule.

Our options data comprises tick data on all OEX and SPX call options for the period June 1, 1989 to December 31, 1993. All option prices are screened for violations of boundary conditions. 0.5% of the OEX options and 1.4% of the SPX options failed the boundary tests. We have also excluded all options with less than 5 days to maturity. Table 2 below presents a summary on the number of transactions and the trading volume of options that passed the initial screening tests.

- Insert Table 2 about here -

We note from Table 2 that over the four and a half year period, there are 170,450 SPX transactions that passed our exclusion criterion. The equivalent number of OEX transactions is 21 times larger, reaching 3.5 million. The total trading volume is also larger for OEX options, but trading volume per contract is smaller in comparison with that for SPX options.<sup>7</sup>

Table 2 also shows that most transactions and trading volume are in contracts with 5 to 30 days to maturity. In our trading simulations, the maximum holding period is 5 days. Hence, our trading program selects only options with 15 to 35 days to maturity, to allow sufficient time to unwind contracts at the end of the holding period. Restricting the maturity to within a 20-day window has the effect of ensuring that any selected pair of OEX and SPX options comes from the same expiry cycle. This will help to reduce “theta” exposure.<sup>8</sup>

The distribution of moneyness in the last panel of Table 2 shows that the most heavily traded options are the at-the-money options and the out-of-, but still near-to-, the money options. To ensure that our results are not susceptible to problems arising

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<sup>7</sup> This may indicate that participants in these two markets are different with different motivations to trade in options. It is possible that there are more speculators who trade in the highly liquid OEX market with smaller trading volume per contract. Participants in the SPX market could be largely fund managers who rely on the S&P 500 options to hedge their portfolios values, or write deep out-of-the money options to supplement their income.

<sup>8</sup> “Theta” measures the sensitivity of option prices with respect to option time to maturity. Value of option will decrease with the passage of time even if all the other parameters remain the same.



from illiquidity, we have therefore restricted our trading program to select options with moneyness,  $S/X$  (where  $S$  and  $X$  represent the stock index and the strike price respectively), lying within the 0.90 and 1.05 region.<sup>9</sup> The imbalance between the numbers of transactions for the two options series leads us to introduce two features to our trading design. First, our trading program is centered on SPX options during execution. The program matches a SPX transaction with the next immediate OEX transaction, but discards the SPX information if no matching OEX transaction becomes available within the next 10 minutes. Second, the interval for intraday trading and observation is set at 30 minutes to give a maximum chance of observing at least one SPX price in each interval over the entire sample period.

## 4. Estimating option implied standard deviation (*isd*)

### 4.1. Option valuation models and model parameters

The CBOE data tapes provide information on trade date, transaction time stamp, option transaction price, the simultaneous value of the underlying index, exercise price, trading volume and contract expiry month. From the expiry month and the trade date, we can derive the time to maturity, measured in days, excluding weekends and holidays. The time to maturity is then annualised assuming that there are 260 days in a year. To extract *isd*, from the option price, we also need the risk free interest rate and dividend yield of the underlying asset. For the risk free interest rate,  $r$ , we use the rate on the U.S. T-bill maturing closest to the relevant options maturity date. Following Harvey and Whaley (1992a), we assume the dividend yield to be constant, and that expected dividends are equal to ex post realizations. The dividend yield,  $q_t$ , on day  $t$  is annualized and calculated based on all  $n$  dividends distributed during the life of the option as follow:

$$q_t = \frac{1}{T} \ln \left( \frac{S_t + \sum_{i=1}^n D_i e^{r_i T (T-T_i)}}{S_t} \right), \quad (1)$$

<sup>9</sup> Using discounted values of  $S$  and  $X$  in the definition of moneyness did not make any noticeable difference to the distributions reported in Table 2.

where  $D_i$  and  $T_i$  are the amount and the timing of the  $i$ th dividend on the index,  $S_t$  is the stock index level and  $T$  is the option time to maturity.

Given all the above information, one is able to invert the value of implied standard deviation from the relevant option-pricing model. In the case of the OEX options, we use the Barone-Adesi and Whaley (1987) (BAW) model<sup>10</sup> for American style options. In the case of the SPX options, the model used is the Merton-Black-Scholes (1973) (MBS) model for European style options.<sup>11</sup>

#### 4.2. *Creating index composite isd series*

To analyse the relationship between OEX and SPX implied volatilities and identify signals indicating relative option mispricing; we need to create a time series of composite *isd* for each of the indexes. The index composite *isd* at any time point should be based on all implied *isd* derived from individual index option prices observed at that particular point in time. A simplistic approach is to calculate the index volatility as a simple average of all these *isd*. However, this ignores the well-known “smile” and term structure effects of option implied volatility documented in prior research.<sup>12</sup>

- Insert Figure 2 about here -

Figure 2 contains a collection of graphical presentations of the relationship of *isd* with moneyness, time to maturity, vega and trading volume, based on our sample data.<sup>13</sup> In Figure 2(a), we do not observe a complete “smile”, but it is clear that deep in-the-money options tend to be overpriced, relative to the models. Figure 2(b) shows that *isd* of very short maturity options tend to be very erratic. Figures 2(c) and 2(d) jointly suggest that options with high vega (related to the proximity of the exercise

<sup>10</sup> The BAW model does not cater for the embedded wildcard option in OEX contracts. Hence, the implied *isd* produced from the BAW model will include a slight upward bias, which can be counterbalanced, if necessary, by adjusting upward also the pre-set level of volatility spread in the trading program.

<sup>11</sup> Details of the two option models, estimation procedures and the specifications of option’s delta and vega are available for the authors.

<sup>12</sup> See Rubinstein (1994), Heynen, Kemna, and Vorst (1994), and Taylor and Xu (1994).

<sup>13</sup> We use a subsample of data to plot these diagrams. The entire sample is too large to be read into any software.

price to the underlying index level) and large trading volume tend to be more reasonably priced. Based on these observations, we calculate index composite  $isd$  as a weighted average of individual option contract  $isd$  using the product of vega and trading volume as weights. Two measurement intervals are used: 30-minute and daily intervals. In estimating the current level of volatility, the 30-minute series uses all information from the previous half an hour, whereas the daily series uses all information from 8:30 to 15:00.

## 5. Properties of $isd$ and volatility spread

Figure 3 plots the daily index composite  $isd$  compiled following procedures described in Section 4.2. above. The general movements of  $isd_{OEX}$  in Figure 3(a) closely resemble those of  $isd_{SPX}$  in Figure 3(b), although the time series pattern of  $isd_{OEX}$  is somewhat smoother than that of  $isd_{SPX}$ . The volatility spread, defined as the ratio of  $isd_{OEX}$  to  $isd_{SPX}$  and plotted in Figure 3(c), shows greater variations and less persistence. The horizontal line in Figure 3(c) (at 1.010 level) represents the mean level of the volatility spread for the entire sample period. Comparison of the volatility spread from year to year reveals some intertemporal shifts in the mean.

- Insert Figure 3 and Table 3 about here -

Table 3 reports intraday averages of index  $isd$  and volatility spread compiled at the 30-minute interval and the trading volume calibrated at the 15-minute interval. We note from Table 3 the very low trading volume during the two 15-minute intervals at market open and close. For the time interval from 8:45 to 15:00, trading volume shows a familiar U-shape pattern,<sup>14</sup> with the trough occurring at about 12:00 to 12:30. In contrast, the intraday volatility seems to show a slow decline after 9:00. The

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<sup>14</sup> Chan, Chung and Johnson (1995) find bid-ask spread for actively traded CBOE stock call options to be high only during the opening hour and is the lowest at market close. The patterns of trading volume and bid-ask spread have a direct impact on our trading design. In the trading simulation in Section 6, we target for transactions to take place near (but not at) market open and close. With a more liquid market condition and a narrower bid-ask spread, there is a better chance of finding profitable matching options for our trading strategies.

differences between intraday volatilities are not statistically different from zero, however, given the magnitudes of the standard errors of these volatility estimates.<sup>15</sup>

Table 4 provides some summary statistics and autocorrelation coefficients for implied volatilities and implied volatility spread. There are 1,160 trading days in our four and a half year sample period, which translates into 15,080 30-minute intervals for a daily duration from 8:30 to 15:00. There are 671 30-minute intervals where there was not a single SPX transaction. 33 of these 671 30-minute intervals also do not have any OEX transaction. Omitting these 671 30-minute intervals produced 14,409 observations for our 30-minute sample.

- Insert Table 4 about here -

First part of Table 4 reports descriptive statistics for the 30-minute  $isd$  series and the corresponding volatility spread for the entire sample period and each year in the sample period. We note that the mean  $isd_{OEX}$  is generally higher than the mean  $isd_{SPX}$ , as would be expected due to the greater portfolio diversification of the S&P 500 index. Although the volatility levels of the two indexes display kurtosis, the volatility spread has “thinner” tails, suggesting that extreme values in both volatility series occur simultaneously, reinforcing our conjecture that volatilities of the two indexes are fundamentally related to each other.

The autocorrelation coefficients reported in the second part of Table 4 confirm the observations we made about Figures 3(a) and 3(b), i.e.  $isd_{OEX}$  is more persistent than  $isd_{SPX}$ . The autocorrelation of  $isd_{OEX}$  is stronger than that of  $isd_{SPX}$  at both the 30-minute and the daily intervals. The volatility spread,  $isd_{OEX}/isd_{SPX}$ , is serially correlated, but not to the same extent as the volatility levels. It is interesting to note that increasing the sampling frequency substantially reduces the persistence of volatility spread, producing a lower autocorrelation coefficient at the 30-minute interval.<sup>16</sup> This reduction in autocorrelation suggests that the intraday aggregation may have smoothed out variations within the day. If some of these intraday variations

<sup>15</sup> Interestingly, there is no apparent U-shape pattern in (implied) volatility, similar to the volatility pattern reported in Stoll and Whaley (1990) for stocks traded on the NYSE.

<sup>16</sup> The half-life of daily  $isd$  is 0.89 (measured in day), whereas the half-life of the 30-minute  $isd$  is 0.56 (measured in 30-minute unit), or 16.7 minutes.

are indications of market disequilibrium, the use of daily aggregates may result in fewer arbitrage opportunities being identified.

## 6. Simulated Trading and Results

### 6.1 The trading rule

Our approach to testing the joint efficiency of the two option markets is based on simulating a simple trading rule involving vega-neutral hedge positions, designed to exploit disequilibriums in the relative implied volatilities between the two option markets. At any instance when the volatility spread between OEX and SPX options is too high (too low), we sell (buy) OEX calls and buy (sell) SPX calls. The size of the position in each leg of a trade is determined to ensure overall vega-neutrality. A variation of this trading strategy involves taking addition position in S&P 500 index futures making the portfolio delta neutral as well. Such trading strategies are expected to generate positive gross profits if volatility spread mean reverts, irrespective of which option series reflects the price adjustment causing the spread to return to the equilibrium level. Based on this rule, each simulated trade, generated by our trading program, follows a three-stage cycle.

First the program compares, in the observation period, index composite  $isd$  compiled using individual contract  $isd$  derived from prices of options with 15 to 35 days to maturity and moneyness,  $S/X$ , within the range of 0.90 to 1.05. We also derive the index composite vega,  $\Lambda_{OEX}$  and  $\Lambda_{SPX}$ , using the same groups of options. Denoting the equilibrium spread as  $\bar{\lambda}$ , the program then calculates the predicted minimum option price movement as follow:

$$Abs \left( isd_{OEX} - \bar{\lambda} \times isd_{SPX} \right) \times Min \left( \Lambda_{OEX}, (\bar{\lambda})^{-1} \Lambda_{SPX} \right) \quad . \quad (2)$$

To trigger a new trade, the predicted minimum option price movement must be greater than a pre-set level,  $\delta$ , which we called “filter” and is designed to cover transaction costs. If minimum price change in (2) is greater than  $\delta$ , the program then generates a trade signal to sell OEX option and buy SPX option in the next period

provided that  $isd_{OEX} > \bar{\lambda} isd_{SPX}$ , and vice versa. Otherwise, no new positions will be initiated, although existing portfolios could still be unwound if one of the two the pre-set conditions for unwinding is met. Conditions for unwinding are described later in the third stage.

If a trade signal is generated in the first stage, the second stage involves matching SPX and OEX options and creating a vega-neutral or a vega-delta-neutral hedge portfolio. As the SPX market is not as liquid as the OEX market, the matching process starts with identifying a ‘suitable’ SPX transaction first. A ‘suitable’ SPX transaction is one with 15 to 35 days to maturity and with moneyness lying within the 0.90 and 1.05 region. When a suitable SPX transaction is identified, the program then searches for a matching OEX transaction, meeting similar conditions. If no matching OEX transaction occurs within the next 10 minutes, the initial SPX transaction is abandoned, and the matching process starts again with the next available SPX transaction.

If a matched pair of SPX and OEX transactions are successfully identified, the program next examines if the minimum price change in (2) is still larger than  $\delta$ , substituting all index information with contract information related to the two transactions the program identified. If the minimum price change in (2), calculated for the individual contract pair, is too small, the matching transactions are abandoned and the search process resumes. Otherwise, a new trade consists of two transactions is initiated:  $\eta_{OEX}$  number of OEX options will be sold and  $\eta_{SPX}$  number of SPX options purchased if  $isd_{OEX} > \bar{\lambda} isd_{SPX}$ , and vice versa. The numbers of contracts,  $\eta_{OEX}$  and  $\eta_{SPX}$ , are set following (3a) and (3b) below such that the option portfolio is vega-neutral:

$$\eta_{SPX} \Lambda_{SPX} = -b_1 \eta_{OEX} \Lambda_{OEX} \quad , \quad (3a)$$

$$(1-B)isd_{OEX,t} = a_1 + b_1(1-B)isd_{SPX,t} + e_{1t} \quad , \quad (3b)$$

where  $B$  represents the back-shift operator (for first difference). The coefficient  $b_1$  measures the responsiveness of changes in  $isd_{OEX,t}$  with respect to changes in  $isd_{SPX,t}$ .  $a_1$  and  $e_{1t}$  are, respectively, a constant term and a zero-mean white noise.

Relationships (3a) and (3b) make the option portfolio vega-neutral with respect to small changes in volatility, but leave the portfolio subject to delta risk, i.e. the risk that option prices will change disproportionately when there are changes in the underlying index levels. The portfolio net delta exposure can be estimated as

$$P_{\Delta} = b_2 \eta_{OEX} \Delta_{OEX} + \eta_{SPX} \Delta_{SPX} , \quad (4)$$

where  $\Delta_{OEX}$  and  $\Delta_{SPX}$  are the deltas of the OEX and the SPX options in the portfolio and  $b_2$  is derived from the following regression:

$$(1 - B) \ln S_{S\&P100, t} = a_2 + b_2 (1 - B) \ln S_{S\&P500, t} + e_{2t} , \quad (5)$$

where  $S_{S\&P100}$  and  $S_{S\&P500}$  are, respectively, the S&P 100 and S&P 500 index levels,  $a_2$  and  $e_{2t}$  are, respectively, a constant term and a zero-mean white noise.

A simple and low cost method to hedge delta exposure is to take an opposite position ( $-P_{\Delta}$ ) in S&P 500 futures at the time when the option portfolio is created at time  $t_1$ . The payoff of the futures position, when the option portfolio is liquidated at time  $t_2$ , is

$$-P_{\Delta} (S_{S\&P500, t_2} - S_{S\&P500, t_1}) \times \exp \left[ \frac{r}{360} \times (t_2 - t_1) \right] , \quad (6)$$

where  $r$  is the risk free interest rate, assuming that the S&P 500 futures is fairly priced. Since option positions are expected to be unwound relatively quickly, hedge ratios for the committed contracts are not adjusted dynamically.

The third stage in the trading programme involves unwinding existing positions. We have two strategies for unwinding open positions, viz. “compulsory” unwind and “intelligent” unwind. “Compulsory” unwind takes place at or after a specific time depending on the trading style described in the following subsection. “Intelligent” unwind means contracts are unwound when one of the two conditions are met: (i) the *isd* spread changes direction (e.g. the existing position is “long OEX, short SPX” but the current spread suggests one should short OEX and long SPX instead), and (ii) the portfolio holding period reaches the pre-set limit. The program stops generating new trades one month before our sample period ends, and

concentrates on unwinding all open option positions in the last month of the sample period.

## 6.2. *Implementation*

To implement the trading rule, we set  $\bar{\lambda} = b_1 = b_2 = 1$ , and let  $\delta$  take one of the three values; 0.125, 0.25 and 0.375 representing one, two and three ticks respectively. For highly liquid index option markets, the bid-ask spread often reduces to just one tick.<sup>17</sup> The maximum holding period for a position is set equal to 5 days. We have two trading intervals, viz. 30-minute and daily, and five trading styles.

The first trading style is the most aggressive. It is designed to mimic a “Die-hard” speculator who trades all day, revising his/her decision to buy/sell OEX/SPX options every half and hour, but trades only once in any 30-minute interval.<sup>18</sup> Since the “Die-hard” speculator is observing the market movements all day, he/she will unwind the portfolio when the volatility spread changes direction or when the portfolio is held for more than 5 days. The second trading style mimics the “Day-punter” who enters the trading place in the morning, absorbs information at the market open, takes action after 10:00 and unwinds all open positions on the same day at market close.

The last three types of trading style adopt a slower pace for trading, hence, the style description “Go-leisurely”. This group of traders will observe the markets movements for the large part of the day and take position after 14:30 before market close. Style 3 trader will unwind the portfolio in the next morning. Style 4 trader holds the portfolio for at least one whole day and liquidates it near market close on the next day, while Style 5 trader is more sophisticated. Style 5 trader will unwind his/her position only if the volatility spread changes direction or when the position holding period exceeds 5 days. Styles 2, 3 and 4 are the same as those tested in Harvey and Whaley (1992b).

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<sup>17</sup> We follow Harvey and Whaley (1992b) by assuming that a round-trip transaction in index option is one tick.

<sup>18</sup> We have to constraint the number of trades taking place within any time interval to avoid the possibility that all profits generated at the end of the simulation are attributed to only a few specific time intervals or contracts.



### 6.3. Trading results

Table 5 provides a summary of trading profits (in index points per trade<sup>19</sup>) for vega-neutral strategies. The trading profits in Table 5 do not include transaction costs. Profits, net of transaction costs, together with results for vega-delta-neutral trading strategies are reported later in Table 6.

- Insert Table 5 about here -

Table 5 reports, for each trading style, the number of trades over the entire sample period. All five trading styles produce gross profits but some of these profits have large time series variations. Gross profits that are statistically greater than zero at the 5% level are those produced by Trading Style 1 and 5; and with a low filter ( $\delta = 0.125$ ), Trading Style 2 and 3 also.

The percentage figure, reported underneath the number of trades, shows the proportion of trades that involved selling OEX options and buying SPX options. These percentages showed that the majority of the simulated trades (about 69% to 87% depending on the trading style) involved selling OEX options and buying SPX options. Reducing the number of trades, either by increasing the size of the filter or by restricting the number of trades to one trade per day, has the effect of making this type of strategy more dominant. The average time lapse between the executions of the matching contracts is 27 seconds. For the most aggressive Trading Style 1 and the least stringent filter ( $\delta = 0.125$ ), the average number of trades per day is 5 trades with a maximum of 12 trades per day. Moreover, 13 trades, on average, are left open in the book each day, and the maximum number of trades left open on any one day is 39 trades.

Table 6 reports net trading profits for both vega-neutral and vega-delta-neutral trading strategies. Option round-trip transaction costs are assumed to be  $\frac{1}{8}$  (i.e. one tick). So the transaction cost for each trade is  $(\eta_{OEX} + \eta_{SPX}) \times 0.125$  in index points.

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<sup>19</sup> Each of these trades consists of two transactions; one in OEX option and the other in SPX option.

Delta hedging is achieved by taking position in S&P 500 futures. Assuming that each futures position costs 0.05 in index points,<sup>20</sup> the hedging cost amounts to  $P_{\Delta} \times 0.05$ .

- Insert Table 6 about here -

All five trading styles produced gross profits as previously reported in Table 5, but only two of them remain profitable after taking into account transaction costs as shown in Table 6. The profitable styles are those with “intelligent” unwind, viz. Trading Style 1 and 5. The  $t$ -ratios for the profits generated from vega-delta-neutral strategy confirms that delta hedging has the effects of enhancing the profitability and reducing the variations in performance.

Tables 5 and 6 show that the application of a more stringent filter produced larger profits per trade, but also resulted in fewer trades. The total profits (net of transaction costs from Table 6) for the vega-delta-neutral portfolios are about the same at different levels of  $\delta$ . With higher  $t$ -ratios associated with high  $\delta$ , the results in Table 6 are in favor of a bigger filter. Note that there are very few opportunities to trade if one restricts oneself to maximum one trade per day as in the cases of Trading Style 2 to 5. For example, Trading Style 5 produces two trades in every 15 days compared with two trades per day for the more aggressive Trading Style 1. To produce the same level of total profit, Style 5 trader would have to become more aggressive by taking a much larger position for each trade.

It is worth noting from Tables 5 and 6 that the average number of days positions are held increases as the size of the filter increases. With a larger filter, the observed volatility spread is further away from the equilibrium level and will take a longer time to revert to equilibrium. The portfolio hedge ratios are unlikely to remain the same as time passes, hence the rule requiring unwinding of a position after day 5, even if the volatility spread has not yet returned to the equilibrium level. In practice, one could set the portfolio holding period to be unconstrained, but dynamically update the hedge ratio and buy/sell the correct amount of options/futures for the net position where necessary. The dynamic approach is likely to enhance the profitability of our trading rule.

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<sup>20</sup> Both estimates of transaction costs for trading in index options and in index futures followed those adopted in Harvey and Whaley (1992b).

- Insert Table 7 about here -

Finally, table 7 provides detailed analysis of the vega-delta hedged profits net of all transaction costs, and separates the profits according to the number of days the portfolio is held and the buy-sell strategy. The table includes results for Trading Style 1 and 5 only; the two more promising trading strategies. Table 7 shows that delta hedging via S&P 500 futures has the effect of reducing performance variations by about a quarter. Large numbers of trades were unwound on day 1, and for Trading Style 1, day 0 as well. This early unwinding pattern is consistent with the lag 1 autocorrelations of *isd* spread (0.287 for 30-minute *isd* and 0.460 for daily *isd*). Over 40% of the trades were unwound on day 5, the pre-set portfolio holding period. It is possible that profitability may be enhanced if we were to relax this constraint and hedge the option portfolios dynamically. Overall, profitability does not appear to be concentrated on a particular holding period or to be dependent on the buy-sell strategy. Trades involving buying OEX options and selling SPX options are equally profitable, although market conditions leading to such trades occurred less frequently.

## 7. Conclusion

In this paper we suggest a new way to test the joint efficiency of markets in options written on financial assets that are close substitutes. The advantage of this approach to testing option market efficiency is that we do not need to condition the efficiency tests on forecasts of volatility of the underlying asset returns. Instead, we effectively condition our tests on forecasts of relative volatility movements. When assets are close substitutes we expect their volatilities to display a long-run equilibrium relationship and the volatility spread to be mean reverting. In this case, predictions of volatility spread changes are expected to be more precise than predictions in the absolute levels of volatility.

Our trading rule results suggest that the OEX and SPX options markets are not jointly efficient. The strategy of trading the volatility spread appears to generate consistent profits even after transaction costs are taken into account. We expect that relaxing the constraints on the portfolio holding period introduced in our trading rule, and switching to a dynamic hedging approach will increase profitability still further.

Our results indicate the existence of short-run relative mispricing between the OEX and SPX options markets. Hence, the two option markets are not jointly efficient. This is consistent with information discovery, relating to common volatility components, occurring at different rates in the two option markets. Future work might seek to analyze the dynamics of joint information discovery across the two markets. It is likely that other classes of options will lend themselves to this type of analysis. Obvious examples of optionable financial assets likely to display significant common volatility components include foreign currencies and individual stocks belonging to the same industrial sector.

## Appendix

### Estimating Common Volatility Component(s) using Canonical Correlation

This appendix describes the canonical correlation method proposed in Ray and Tsay (1997) for estimating common volatility components. Denote daily volatility as  $y_{it}$  with dimension  $k \times 1$ , and  $Y_{h,j,t} = (y'_{t-j}, y'_{t-j-1}, \dots, y'_{t-j-h+1})'$  as the set of  $h$  past values of the system after lagging  $j$  times for  $h > 0$ ,  $j > 0$ . If there are  $r$  persistent common components, there will be  $s = k - r$  zero canonical correlations between  $y_t$  and  $Y_{h,j,t}$ . For given  $h$  and  $j$ , the squared canonical correlations between  $y_t$  and  $Y_{h,j,t}$  are the eigenvalues of the matrix

$$A(h, j) = [Var(y_t)]^{-1} Cov(y_t, Y_{h,j,t}) [Var(Y_{h,j,t})]^{-1} Cov(Y_{h,j,t}, y_t) . \quad (A.1)$$

Denote the eigenvalues of  $A(h, j)$  by  $0 \leq \rho_1^2 \leq \dots \leq \rho_k^2 \leq 1$ . We need to test the hypothesis  $H_0 : \rho_s^2 = 0$  against the alternative  $H_1 : \rho_s^2 > 0$  where  $s = 1, \dots, k$  and a robust test statistic for  $H_0$ , that allows for serially correlated innovations, is

$$T_s^* = -(n - h) \sum_{i=1}^s \ln \left( 1 - \frac{\hat{\rho}_i^2}{g_i} \right), \quad (A.2)$$

where  $n$  is the sample size,  $g_i = \left( 1 + 2 \sum_{u=1}^{j-1} \hat{\rho}_{r,u} \hat{\rho}_{l,u} \right)$ ,  $\hat{\rho}_{r,u}$  is the lag- $u$  sample autocorrelation of the canonical variate  $y_{it}^* = c_i' y_t$ , and  $\hat{\rho}_{l,u}$  is the lag- $u$  sample autocorrelation of another canonical variate  $x_{it}^* = C_i' Y_{h,j,t}$ . For each eigenvalue  $\hat{\rho}_i^2$ ,  $c_i$  is the eigenvector of  $A(h, j)$ , whereas  $C_i$  is the eigenvector of  $\hat{\Sigma}_{YY}^{-1} \hat{\Sigma}_{Yy} \hat{\Sigma}_{yy}^{-1} \hat{\Sigma}_{yY}$ .

The robust test statistic,  $T^*$ , has an asymptotic  $\chi^2$  distribution with  $s[(h-1)k + s]$  degrees of freedom, where  $k$  denotes the dimension of vector of volatilities (i.e., the number of assets). In this paper,  $k = 2$  as we are modeling the common volatility in two indexes. We test the hypothesis that there is one common component, i.e.  $s = r = 1$ , among the volatilities of the two indexes. We set  $h = j = 10$  following Ray and Tsay (1997).<sup>21</sup>

<sup>21</sup> See Ray and Tsay (1997) for a discussion on appropriate choice of  $h$  and  $j$  in practice.

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Table 1  
Estimated integrating parameters,  $d$ , for returns on S&P 100 ( $y_{S\&P100}$ ), S&P 500 ( $y_{S\&P500}$ ),  
and their canonical variate,  $\omega = y_{S\&P100} - \beta y_{S\&P500}$

		$ r $	$\log r $	$r^2$	$\log r^2$
A. Whole sample period (N=3,599) March 6, 1984 to June 15, 1998	$d_{S\&P100}$	.486 (6.84)	.571 (5.62)	.148 (7.11)	.581 (6.12)
	$d_{S\&P500}$	.479 (6.74)	.554 (6.78)	.139 (7.16)	.551 (6.92)
	$d_\omega$	.173 (1.77)	.116 (1.51)	-.062 (-.72)	.138 (1.73)
	$\beta$	1.075	.986	1.115	.986
	$T^*$	21.77 [0.30]	17.34 [0.57]	10.78 [0.93]	18.79 [0.47]
B. Pre-crash period (N=903) March 6, 1984 to October 9, 1987	$d_{S\&P100}$	.269 (2.24)	.369 (2.28)	.075 (0.65)	.348 (2.43)
	$d_{S\&P500}$	.233 (2.07)	.209 (1.40)	.137 (1.44)	.245 (1.74)
	$d_\omega$	.023 (0.21)	-.058 (-.49)	-.122 (-1.57)	-.016 (-.15)
	$\beta$	1.121	.976	1.221	.976
	$T^*$	11.99 [0.89]	10.02 [0.95]	3.91 [1.00]	8.78 [0.98]
C. Post-crash period (N=2,686) October 26, 1987 to June 15, 1998	$d_{S\&P100}$	.562 (6.81)	.579 (4.70)	.468 (5.82)	.557 (5.08)
	$d_{S\&P500}$	.556 (6.05)	.590 (5.17)	.440 (4.91)	.600 (5.25)
	$d_\omega$	.222 (1.85)	-.024 (-.21)	.008 (0.10)	.054 (0.48)
	$\beta$	1.067	.989	1.144	.989
	$T^*$	24.83 [0.17]	25.99 [0.13]	12.27 [0.87]	26.81 [0.11]
D. Pre-option period (N=1,317) March 6, 1984 to May 31, 1988	$d_{S\&P100}$	.345 (4.44)	.418 (3.10)	.108 (6.22)	.419 (3.00)
	$d_{S\&P500}$	.341 (5.79)	.471 (4.27)	.108 (7.24)	.485 (4.47)
	$d_\omega$	.044 (.415)	-.110 (-1.08)	.083 (1.01)	.127 (1.31)
	$\beta$	1.099	.977	1.197	.977
	$T^*$	12.63 [0.86]	14.81 [0.74]	3.92 [1.00]	12.08 [0.88]
E. Option sample period (N=1,160) June 1, 1989 December 30, 1993	$d_{S\&P100}$	.445 (3.83)	.379 (2.41)	.287 (2.55)	.351 (2.35)
	$d_{S\&P500}$	.432 (3.57)	.338 (2.81)	.241 (2.17)	.347 (3.00)
	$d_\omega$	-.034 (-.22)	.012 (0.08)	.028 (0.18)	.043 (0.28)
	$\beta$	1.056	.990	1.132	.990
	$T^*$	16.28 [0.64]	23.13 [0.23]	28.84 [0.07]	22.62 [0.25]

Notes:  $t$ -ratios are reported in parentheses ( ),  $p$ -values are reported in brackets [ ].  $d$  is the long-range dependence integrating parameter,  $T^*$  is the robust test statistic for zero canonical correlation in the cross product of lagged transformed returns.  $T^*$  has an asymptotic  $\chi^2$  distribution with 19 degrees of freedom.



Table 2  
Summary of S&P 100 and S&P 500 call option transactions and trading volume  
(June 1, 1989 through December 31, 1993)

	S&P 100		S&P 500	
Option ticker symbol	OEX		SPX	
Total number of contracts	3,585,534		170,490	
Average number of contracts per day	3,091		147	
Total trading volume	62,803,101		8,508,856	
Average volume per contract	17.52		49.91	
Average volume per day	54,141		7,335	

	Number of transactions		Trading volume	
	S&P100	S&P500	S&P100	S&P500
Maturity (in days)				
5-10	921,803	30,310	15,959,571	1,099,743
11-20	1,690,842	57,648	28,120,468	2,145,809
21-30	672,841	36,190	12,265,557	1,799,400
31-40	192,777	15,649	3,765,872	936,398
41-60	91,026	14,922	2,145,611	1,178,772
61-90	16,245	8,876	546,022	757,354
91-180	-	5,788	-	495,443
181-270	-	1,107	-	95,937
	3,585,534	170,490	62,803,101	8,508,856

Moneyness $S/X$				
0.70-0.85	541	365	37,170	56,153
0.85-0.90	8,011	2,414	386,634	268,682
0.90-0.95	165,231	15,556	4,956,064	1,385,933
0.95-1.00	2,274,133	100,132	39,724,761	4,638,187
1.00-1.05	1,119,124	45,550	17,243,757	1,813,841
1.05-1.10	16,018	4,320	372,332	244,777
1.10-1.15	2,005	1,083	63,481	53,950
1.15-1.20	425	550	16,918	27,937
>1.20	46	520	1,984	19,396
	3,585,534	170,490	62,803,101	8,508,856

The total number of trading days in the sample period is 1,160. 0.46% (1.35%) of the S&P 100 (S&P 500) violated the boundary condition  $c > S \exp(-qt) - X \exp(-rt)$ . Defining moneyness as  $S \exp(-qt) / X \exp(-rt)$  instead of  $S/X$  has not changed the distribution noticeably. Options with less than 5 days to maturity are omitted.

Table 3  
Intraday seasonality in options implied standard deviation and trading volume  
(June 1, 1989 through December 31, 1993)

30-minute interval	Option <i>isd</i> and volatility spread				Option trading volume (in %)		
	No Obs	<i>isd</i> <sub>OEX</sub>	<i>isd</i> <sub>SPX</sub>	<i>isd</i> <sub>OEX</sub> / <i>isd</i> <sub>SPX</sub>	15-minute interval	OEX	SPX
8:30-9:00	1140	.1493	.1471	1.0152	8:30-8:45	3.51	1.94
		(.0011)	(.0010)	(.0022)	8:45-9:00	6.28	5.36
9:00-9:30	1144	.1501	.1471	1.0205	9:00-9:15	6.26	5.96
		(.0011)	(.0010)	(.0024)	9:15-9:30	5.61	5.60
9:30-10:00	1133	.1498	.1468	1.0211	9:30-9:45	5.04	5.16
		(.0011)	(.0010)	(.0025)	9:45-10:00	4.68	4.82
10:00-10:30	1127	.1502	.1467	1.0255	10:00-10:15	4.26	4.19
		(.0011)	(.0010)	(.0026)	10:15-10:30	3.89	4.20
10:30-11:00	1102	.1501	.1463	1.0269	10:30-10:45	3.68	3.61
		(.0011)	(.0010)	(.0026)	10:45-11:00	3.40	3.70
11:00-11:30	1095	.1498	.1461	1.0277	11:00-11:15	3.28	3.40
		(.0011)	(.0011)	(.0027)	11:15-11:30	2.90	2.96
11:30-12:00	1079	.1500	.1461	1.0298	11:30-11:45	2.73	2.68
		(.0011)	(.0011)	(.0028)	11:45-12:00	2.52	2.58
12:00-12:30	1073	.1499	.1461	1.0288	12:00-12:15	2.47	2.51
		(.0011)	(.0011)	(.0028)	12:15-12:30	2.46	2.41
12:30-1:00	1069	.1496	.1460	1.0269	12:30-12:45	2.63	2.63
		(.0011)	(.0011)	(.0026)	12:45-13:00	2.80	2.94
1:00-1:30	1089	.1486	.1453	1.0262	13:00-13:15	3.09	2.82
		(.0011)	(.0011)	(.0026)	13:15-13:30	3.11	3.23
1:30-2:00	1113	.1488	.1464	1.0211	13:30-13:45	3.50	3.55
		(.0011)	(.0011)	(.0027)	13:45-14:00	3.68	4.21
2:00-2:30	1117	.1483	.1457	1.0207	14:00-14:15	4.00	4.55
		(.0011)	(.0010)	(.0026)	14:15-14:30	4.14	4.36
2:30-3:00	1128	.1480	.1454	1.0199	14:30-14:45	4.52	5.15
		(.0011)	(.0010)	(.0026)	14:45-15:00	5.03	5.47
					15:00-15:15	0.56	0.02

The volatility estimates are compiled as the weighted average of all option *isd* observed within 30-minute intervals from 8:30 to 15:00, using the product of vega and volume as weight. The model used to estimate *isd*<sub>OEX</sub> is the Barones-Adesi and Whaley model, and the model used to estimate *isd*<sub>SPX</sub> is the Merton-Black-Scholes model. Standard errors are in parentheses. There are 671 missing observations in the SPX sample, and 33 in the OEX sample.

Table 4  
Descriptive statistics and autocorrelation for implied volatilities and volatility spread  
(June 1, 1989 through December 31, 1993)

30-minute sample		No Obs	Mean	Std Dev	Kurtosis	Skewness	Min	Max
1989-93	$isd_{OEX}$	14,409	.149	.0367	.695	.904	.081	.313
	$isd_{SPX}$		.146	.0348	.855	.876	.072	.301
	$isd_{OEX} / isd_{SPX}$		1.024	.0868	.386	-.050	.673	1.452
1989	$isd_{OEX}$	1,757	.160	.0204	2.163	1.208	.123	.284
	$isd_{SPX}$		.153	.0216	1.795	1.161	.113	.272
	$isd_{OEX} / isd_{SPX}$		1.048	.0808	.348	-.191	.780	1.358
1990	$isd_{OEX}$	3,156	.190	.0343	-.751	.548	.129	.293
	$isd_{SPX}$		.183	.0338	-.536	.573	.115	.301
	$isd_{OEX} / isd_{SPX}$		1.045	.0903	.464	.126	.746	1.452
1991	$isd_{OEX}$	3,170	.161	.0248	3.920	1.691	.115	.313
	$isd_{SPX}$		.158	.0236	4.269	1.664	.112	.286
	$isd_{OEX} / isd_{SPX}$		1.025	.0800	.694	-.078	.673	1.351
1992	$isd_{OEX}$	3,127	.131	.0139	-.253	.364	.100	.186
	$isd_{SPX}$		.133	.0162	-.179	.339	.090	.196
	$isd_{OEX} / isd_{SPX}$		.995	.0792	.014	-.090	.731	1.265
1993	$isd_{OEX}$	3,199	.109	.0099	-.300	.272	.081	.144
	$isd_{SPX}$		.108	.0118	.361	.517	.072	.160
	$isd_{OEX} / isd_{SPX}$		1.015	.0907	.166	-.239	.697	1.354

Autocorrelation	30-minute $isd$ (14,409 observations)			Daily $isd$ (1,160 observations)		
	$isd_{OEX}$	$isd_{SPX}$	$isd_{OEX} / isd_{SPX}$	$isd_{OEX}$	$isd_{SPX}$	$isd_{OEX} / isd_{SPX}$
Lag 1	.987	.910	.287	.977	.950	.460
Lag 2	.984	.899	.215	.954	.931	.390
Lag 3	.981	.895	.198	.935	.914	.325
Lag 4	.978	.892	.184	.920	.901	.334
Lag 5	.976	.890	.172	.907	.887	.275
Lag 20	.947	.869	.140	.782	.772	.215
Lag 50	.906	.834	.115	.694	.681	.125
Lag 100	.862	.796	.079	.497	.467	.063

The volatility estimates are calculated as the weighted average of all  $isd$  observed within selected intervals, viz. 30 minutes or daily from 8:30 to 15:00, using the product of vega and volume as weight. The Barones-Adesi and Whaley model is used to estimate  $isd_{OEX}$ , and the Merton-Black-Scholes model is used to estimate  $isd_{SPX}$ . Standard errors are in parentheses. There are 671 (33) missing observations in the SPX (OEX) 30-minute sample. There is no missing value in the daily sample.

Table 5  
Summary of gross trading profits (per trade and in index points) for vega-neutral trading strategies  
(Simulation period: June 1, 1989 to December 31, 1993)

Trading Styles	Filter $\delta = .125$			Filter $\delta = .25$			Filter $\delta = .375$		
	No of trades	Profit (t-ratio)	Ave day	No of trades	Profit (t-ratio)	Ave day	No of trades	Profit (t-ratio)	Ave day
<i>The “Die-hard”</i>									
1. Trade once every 30 minutes from 9:00 to 15:00, and unwind when the ratio of volatilities goes back to 1, or on day 5 whichever is earlier.	6,254 68.6%	10.68 (17.66)	2.34	4,032 69.8%	12.45 (14.66)	2.70	2,283 70.0%	14.58 (11.22)	3.00
<i>The “Day-punter”</i>									
2. Trade once at 10:00 and start to unwind at 14:30 on the same day.	764 73.2%	2.13 (2.01)	0.68	520 74.2%	3.10 (2.12)	0.75	294 72.8%	2.28 (1.03)	0.81
<i>The “Go-leisurely”</i>									
3. Trade once at 14:30 and start to unwind at 9:00 on the following day.	590 78.8%	1.70 (1.97)	1.12	341 81.8%	2.00 (1.65)	1.12	158 86.7%	3.00 (1.72)	1.16
4. Trade once at 14:30 and start to unwind at 14:30 on the following day.	590 78.8%	1.84 (1.66)	1.58	341 81.8%	2.58 (1.64)	1.64	158 86.7%	3.91 (1.68)	1.71
5. Trade once at 14:30, and start to unwind when the ratio of volatilities goes back to 1, or on day 5 whichever is earlier.	590 78.8%	9.39 (5.05)	2.67	341 81.8%	8.97 (3.36)	3.00	158 86.7%	16.17 (3.61)	3.42

Notes: Each trade consists of a long position in OEX option and a short position in SPX option (or vice versa) with a zero vega exposure for the portfolio. “Ave Day” is the average number of days lapsed before the portfolio is liquidated.  $\delta$  is the minimum expected change in the value of the option portfolio for any new trade to be initiated. The percentage figure reported below the number of trades is the percentage of trades involved selling OEX options and buying SPX options. The rests of the trades involved selling SPX options and buying OEX options.

Table 6  
Summary of net trading profits (per trade and in index point) for vega-neutral and vega-delta-neutral trading strategies  
(Simulation period: June 1, 1989 to December 31, 1993)

Trading Styles	Vega-neutral strategies			Vega-delta-neutral strategies		
	$\delta = .125$	$\delta = .25$	$\delta = .375$	$\delta = .125$	$\delta = .25$	$\delta = .375$
<i>The “Die-hard”</i>						
1. Trade once every 30 minutes from 9:00 to 15:00, and unwind when the ratio of volatilities goes back to 1, or on day 5 whichever is earlier.	1.02 (1.69)	2.77 (3.27)	4.97 (3.82)	2.78 (6.80)	5.24 (9.35)	7.77 (9.32)
<i>The “Day-punter”</i>						
2. Trade once at 10:00 and start to unwind at 14:30 on the same day.	-7.70 (-7.26)	-6.76 (-4.63)	-7.60 (-3.41)	-7.80 (-14.37)	-6.42 (-8.91)	-6.75 (-6.42)
<i>The “Go-leisurely”</i>						
3. Trade once at 14:30 and start to unwind at 9:00 on the following day.	-7.98 (-9.24)	-7.63 (-6.31)	-6.50 (-3.74)	-7.58 (-13.12)	-7.31 (-10.03)	-5.60 (-5.23)
4. Trade once at 14:30 and start to unwind at 14:30 on the following day.	-7.84 (-7.06)	-7.06 (-4.50)	-5.59 (-2.42)	-6.52 (-9.83)	-6.00 (-7.00)	-3.23 (-2.38)
5. Trade once at 14:30, and start to unwind when the ratio of volatilities goes back to 1, or on day 5 whichever is earlier.	-0.29 (-0.16)	-0.67 (-0.25)	6.67 (1.49)	2.10 (1.67)	2.30 (1.33)	7.98 (2.54)

Notes:

Numbers reported in the table are profits net of all transaction costs with  $t$ -ratios in parentheses.  $\delta$  is the minimum expected change in the value of the option portfolio for any new trade to be initiated. Each trade consists of a long position in OEX option and a short position in SPX option (or vice versa) with a zero vega exposure for the portfolio. Delta hedging is done using S&P 500 futures assuming that futures contract is fairly priced. We assume non-integer contracts exist throughout. The option bid-ask spread is assumed to be 1/8 for a round-trip transaction. The cost of hedging via S&P 500 futures is assumed to be 0.05.

Table 7

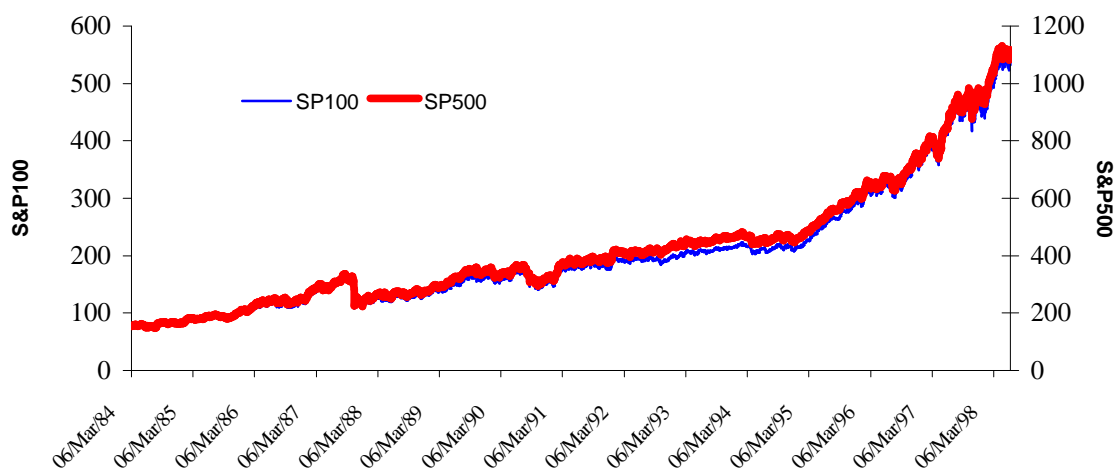
Detailed analysis of net profits (per contract and in index points) from vega-delta-neutral strategies under Trading Style 1 and 5

		No of trades	Mean	Standard Error	Median	Kurtosis	Skewness	Min	Max			
Trading Style 1: $\delta = 0.125$												
Gross profits from vega-neutral strategies		2,283	14.58	1.298	15.56	4.208	0.009	-294.02	350.03			
Net profits from vega-neutral strategies		2,283	4.97	1.296	6.08	4.239	-0.008	-301.81	339.75			
Net profits from vega-delta-neutral strategies		2,283	7.77	0.834	6.67	5.117	0.596	-258.14	243.06			
Vega-delta-neutral	Strategy: Sell OEX Option & Buy SPX Option						Strategy: Buy OEX Option & Sell SPX Option					
No of days held	No	Mean	Std Err	Med	Min	Max	No	Mean	Std Err	Med	Min	Max
0	223	8.61	2.096	6.26	-146.45	160.25	75	18.20	3.662	11.09	-40.23	139.27
1	333	11.02	2.004	7.12	-108.68	191.20	116	22.56	4.140	12.58	-73.30	210.11
2	189	17.23	3.080	10.08	-88.14	191.13	66	11.76	4.601	9.735	-89.28	195.05
3	153	10.92	2.615	8.89	-94.62	124.86	46	24.99	7.818	13.14	-58.21	243.06
4	108	18.15	4.745	11.67	-113.92	169.03	35	19.56	6.417	13.17	-44.47	116.23
5	589	2.05	1.693	3.73	-258.14	137.24	347	-7.44	1.936	-4.63	-140.19	104.6
6	1	-64.61	0	-64.61	-64.61	-64.61	1	41.27	0	41.27	41.27	41.27
8	1	78.58	0	78.58	78.58	78.58	0	-	-	-	-	-
		No of trades	Mean	Standard Error	Median	Kurtosis	Skewness	Min	Max			
Trading Style 5: $\delta = 0.125$												
Gross profits from vega-neutral strategies		158	16.17	4.484	11.88	1.376	0.379	-150.07	185.80			
Net profits from vega-neutral strategies		158	6.67	4.465	3.75	1.430	0.365	-159.17	178.05			
Net profits from vega-delta-neutral strategies		158	7.98	3.135	7.29	4.014	0.409	-146.41	182.57			
Vega-delta-neutral	Strategy: Sell OEX Option & Buy SPX Option						Strategy: Buy OEX Option & Sell SPX Option					
No of days held	No	Mean	Std Err	Med	Min	Max	No	Mean	Std Err	Med	Min	Max
0	4	0.77	5.006	1.66	-12.26	12.01	0	-	-	-	-	-
1	26	13.68	6.205	4.60	-26.80	131.51	5	21.45	5.347	21.29	8.50	37.12
2	19	5.57	8.254	12.16	-70.28	62.52	0	-	-	-	-	-
3	12	13.76	7.167	18.04	-41.53	48.52	3	75.72	54.161	37.69	6.89	182.57
4	17	17.67	10.952	14.10	-79.34	102.34	1	-16.09	0.000	-16.09	-16.09	-16.09
5	59	0.23	5.554	4.00	-146.41	116.15	12	-0.07	8.414	6.94	-57.10	38.53

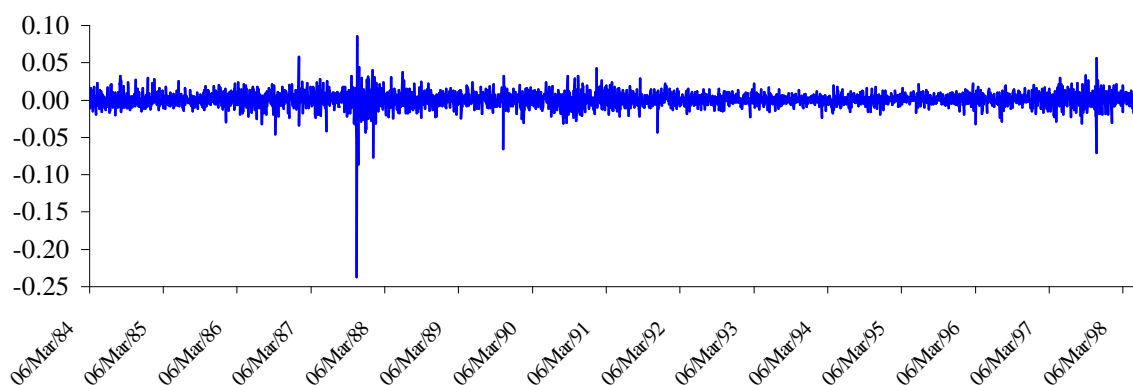
Trading Style 1 trader trades once every 30 minutes from 9:00 to 15:00, and unwinds when the volatility spread returns to 1, or on day 5 whichever is earlier. Trading Style 5 trader trades once at 14:30, and starts to unwind when volatility spread returns to 1, or on day 5 whichever is earlier. Cost for a round-trip transaction in index option is assumed to be 0.125 or 1 tick. Hedging cost via S&P 500 index futures is assumed to be 0.05.  $\delta$  is the threshold for minimum expected change in option portfolio value before a new trade can be executed.

Figure 1  
Daily index values and returns for the period March 6, 1984 to June 15, 1998

(a) Daily stock index level of S&P100 and S&P500



(b) Daily stock index returns of S&P100



(c) Daily stock index returns of S&P500

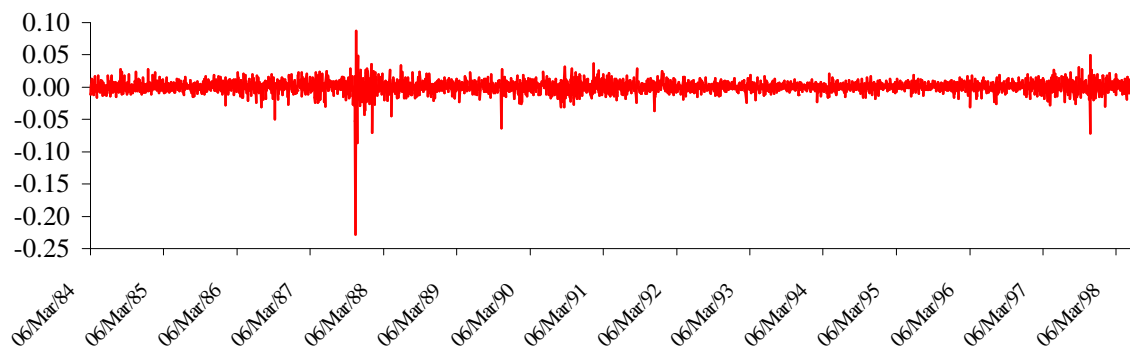
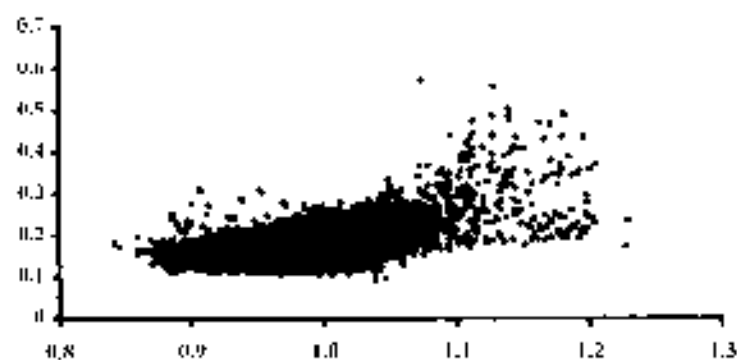
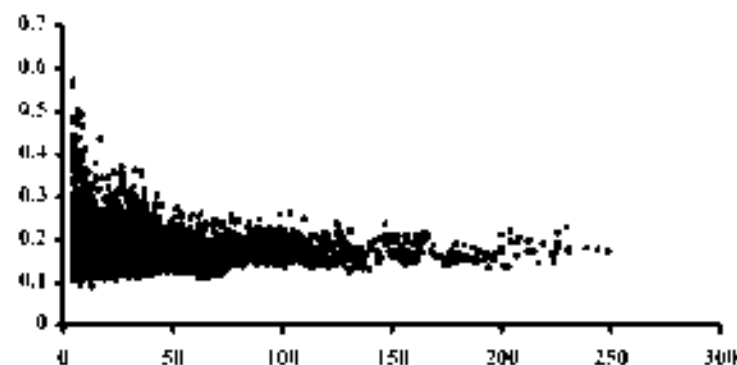


Figure 2  
Relationships among isd, moneyness, time to maturity,  
trading volume and vega in tick-by-tick sample

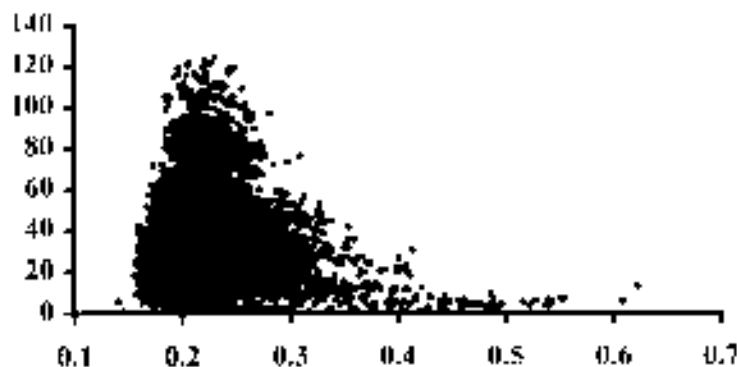
(a) isd vs. moneyness



(b) isd vs. time to maturity



(c) Vega vs. isd



(d) Trading volume vs. isd

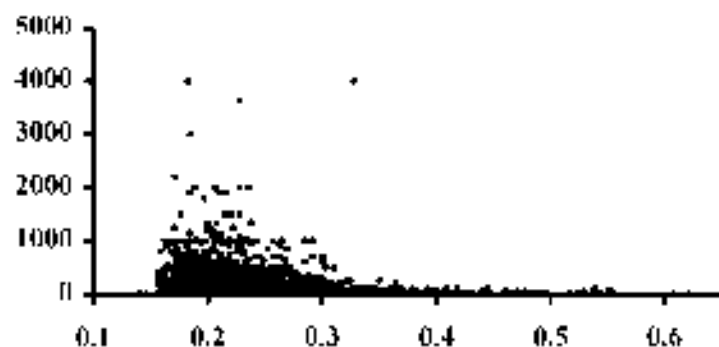
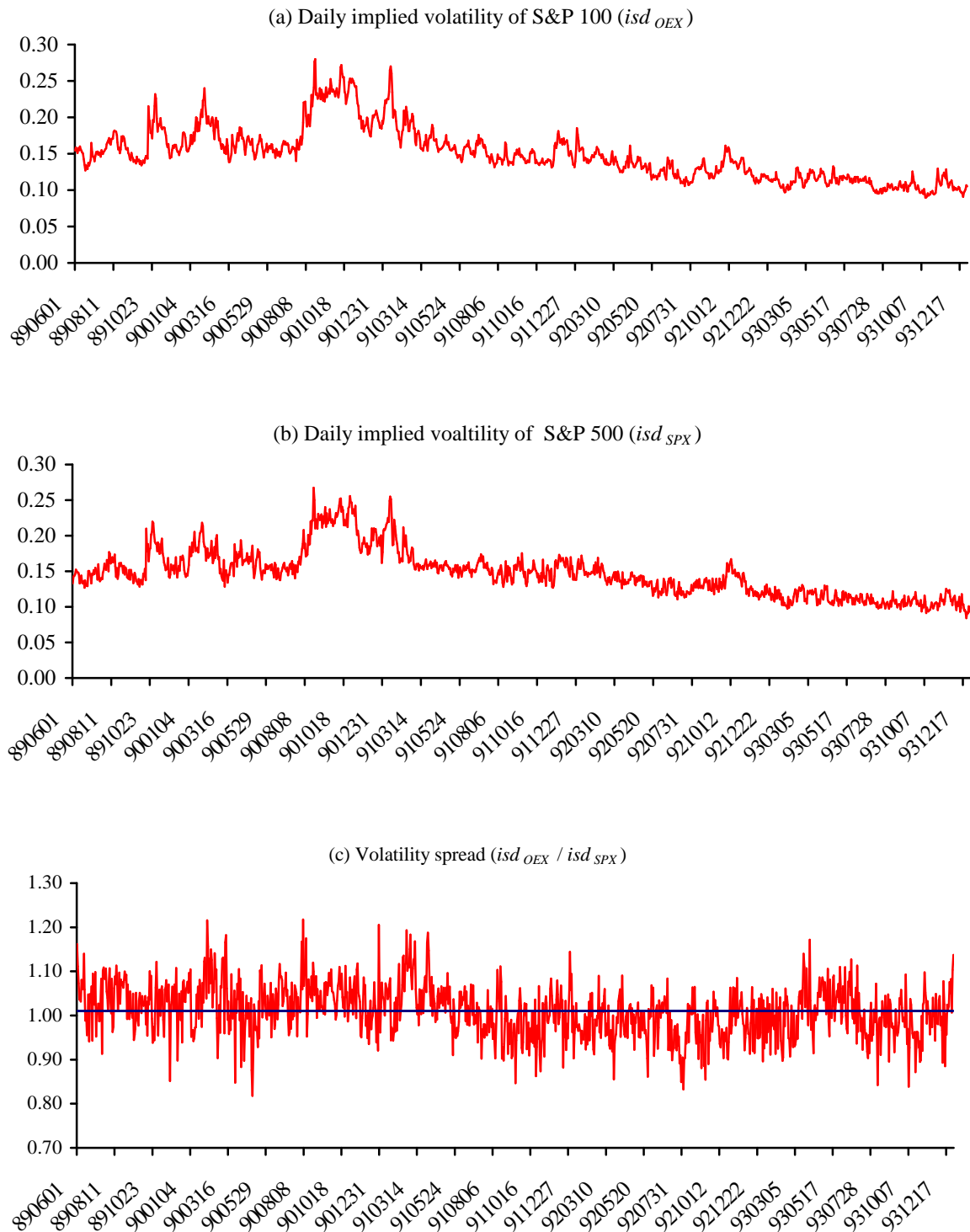




Figure 3  
Daily index option implied volatility and volatility spread  
(June 1, 1989 through December 31, 1993)



Index volatilities are compiled as the weighted average of all options implied volatilities observed from 8:30 to 15:00, using the product of vega and trading volume as weight. The model used to estimate  $isd_{OEX}$  is the Barones-Adesi and Whaley model, and the model used to estimate  $isd_{SPX}$  is the Merton-Black-Scholes model. The horizontal line in Fig.3(c) (at 1.010 level) represents the mean level of the volatility spread for the entire sample period.