Calendar spreads, risk premium and the convenience yield

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This version: July 2009.

Abstract

This paper studies optimal calendar spreads in commodity futures markets while taking into account a stochastic convenience yield. We show that a convenience yield imperfectly correlated with the spot commmodity price results in an optimal strategy composed of two commodity futures contracts. These strategies reveal a calendar spread effect through the positive correlation between the two futures contracts. These strategies can easily be computed and analyzed under the Samuelson hypothesis.

JEL Classification: G11; G12; G13

Keywords: Commodity futures markets, convenience yield, calendar spread, investment, Samuelson hypothesis, commodity futures prices correlation, market prices of risk.

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1. Introduction

This paper deals with calendar spread in commodity futures markets. Indeed, investors employ different strategies to invest in and to hedge on futures markets. One of these strategies is to take positions in two futures contracts written on the same underlying but having different maturities. This strategy is called a calendar (or time) spread because investors take (partially) offsetting positions in the two different futures contracts. More precisely, a (short) long position in the nearby futures contract, i.e. the futures contract with the shortest maturity, and a (long) short position in the distant futures contract, i.e. the futures contract with the longest maturity, is called a (reverse) calendar spread. This article aims at providing a theoretical insight about calendar spreads in the case of (storable) commodity futures markets.

Since the original works of Keynes (1930) and Hicks (1939) on normal backwardation, and Working (1948, 1949), Brennan (1958), and Telser (1958), on the theory of storage, a lot of theoretical articles have been devoted to hedging on and investing in futures markets. More recently, in the context of continuous time, Ho (1984), Adler and Detemple (1988 a,b), Duffie and Jackson (1990), Briys et al. (1990), Duffie and Richardson (1991), Lioui et al. (1996), and Lioui and Poncet (2001) have studied optimal demands in futures contracts for investors endowed with a non traded cash position. In contrast, Breeden (1984) has examined the case of unconstrained investors who can freely trade on the primitive assets, namely the underlying spot asset and, if need be, other risky assets.

While some of the above articles consider trading in multiple futures contracts, the relationship between the demands for various futures contracts is not emphasized. It follows that the conditions leading to the determination of optimal futures contracts spreads are absent.

Recently, Lioui and Eldor (1998) addressed the issue of optimal spread positions for the Bernoulli investor and characterized the optimal short and long positions in interest rates futures contracts. The key element in their model is the correlation of the futures contracts with the optimal growth portfolio.[1] They

[1] The optimal growth portfolio is a financial asset such that any investment strategy expressed in terms of this numéraire is a martingale.

find that the optimal portfolio consists in a short position with the contract which is the most negatively correlated with the growth portfolio and a long position in the other contract.[2]

Examining futures spreads in commodity markets under a continuous-time setting has not yet been carried out in the literature.[3] Only few papers (see Schrok, 1971; Poitras, 1989) tackled this issue in a static one period analysis under a mean variance framework. Moreover, these works do not highlight the crucial role of the convenience yield[4] in their analysis. In contrast, market participants rely heavily on such a strategy. Indeed, as observed by the Commodity Futures Trading Commission (CFTC, 2008), the number of Calendar spreads of non commercial traders[5] on the NYMEX crude oil market has risen from 10% of trades in March (1999) to 40% in August (2008).[6] From a practical point of view, this rise may be accounted for by the fact that many futures contracts are traded for various, still liquid, maturities ranging from one month to at least 24 months, in the case of copper, and to as long as 48 months in the case of oil, gold and silver.[7]

From a theoretical point of view, Hong (2001) explained the spread open interest observed in commodity futures markets by the presence of a stochastic mean reverting convenience yield. In addition,

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[2] Lioui and Eldor (1998) consider an investor endowed with a non traded cash position so their optimal portfolio also include an additional term linked to this non traded position. However, since our study restrains to investors who only trade in futures markets, our review of their article does not deal with these terms.

[3] To the best of our knowledge.

[4] Brennan (1991) defines the convenience yield as "the flow of services accruing to the owner of the physical inventory, but not to the owner of a contract for future delivery". Indeed, physical inventory provides some services such as the possibility of avoiding shortage of the spot commodity and thus to maintain the production process or even to benefit from a (anticipated) future price increase.

[5] The Commodity Futures and Trading Commission (CFTC, 2008) defines non commercial traders "as any traders not involved in the production, processing, merchandising or any other commercial activity in the commodity it is trading".

[6] "The type of trading conducted by noncommercial traders has also changed significantly over time. For example, in the NYMEX crude oil market, a vast majority of noncommercial traders do not take direct long or short positions in the market where they would benefit directly from prices rising or falling. Rather, most noncommercial traders place spread positions, which amounts to simultaneously buying and selling in different months to trade on pricing relationships over certain time horizons (i.e. calendar spread). Spread trading for NYMEX Crude Oil with equal and offsetting long and short positions has grown from roughly 10 percent (i.e., march 1999) of the market to over 40 percent of the market today (i.e. august 2008)", (CFTC, 2008).

[7] The existence of such different maturities on various commodities explains empirical investigations carried in important works such as those of Casassus and Collin-Dufresne (2005) and Schwartz (1997).

as explained by Brennan (1991), the convenience yield, which constitutes the main determinant of the basis[8], is positively and imperfectly correlated with commodity spot prices. So, to set up their strategies rational traders not only consider a (favorable) change in the spot price but also a (favorable) change in the convenience yield.

The intuition behind a (reverse) calendar spread is as follows. An investor who anticipates an increase (decrease) in the spot price will buy (sell) the contract that is the most correlated with the spot price, i.e. the nearby contract. However, since the correlation between the spot price and the convenience yield is positive, an increase (decrease) in the spot price implies an increase (decrease) in the convenience yield. In addition, due to arbitrage, the negative relationship between futures prices and the convenience yield implies that the impact of the latter is more pronounced in the distant contract than in the nearby one.[9] Hence, a rational investor will sell (buy) the distant contract. Nevertheless, this explanation suffers from two limitations. First, it does not take into account the expected change of futures prices linked to the risk of the convenience yield uncorrelated with the spot price.[10] Second, it does not consider the impact of the convenience yield on the nearby contract.

The goal of this paper is twofold. First, it highlights the interaction between the term structure of futures prices correlation and the Samuelson hypothesis[11] to justify a calendar spread strategy. In particular, we examine the sign and level of the correlation term structure. This is achieved through a simple setting that relies on the widely used model of Gibson and Schwartz (1990). Second, we derive for an investor endowed with a CRRA (Constant Relative Risk Aversion) utility function optimal spread proportions. Once again, we show the crucial role of correlations in an optimal spread strategy.

The remainder of the paper is organized as follows. Section 2 describes the economic framework.

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[8] The basis is defined in this article as the difference between the price of the distant contract and the price of the nearby contract. By (reverse) cash and carry arbitrage, the basis is a decreasing function of the convenience yield.

[9] Indeed, the opportunity cost foregone by not holding the commodity, i.e. the net convenience yield, when owning the futures contract, is proportional to the time to delivery of this commodity.

[10] The risk will be referred to, in the remaining of the paper, as the orthogonal risk of the convenience yield.

[11] The Samuelson hypothesis (Samuelson, 1965) argues that futures price volatility increases when time to maturity decreases; see Bessembinder et al (1996) for empirical evidence.

Section 3 is devoted to the study of the correlation between the various assets. Section 4 analyzes optimal futures contracts proportions. Section 5 provides a numerical illustration of our results. Section 6 concludes. All proofs are available from the authors upon request.

2. Model setting

We rely on the seminal model of Gibson and Schwartz (1990). Let $(S(t))_{t\geq 0}$ designate the process of the spot price of a commodity whose dynamics is given by:

$$\frac{dS(t)}{S(t)} + \delta(t)dt = (r + \sigma_S \lambda_S)dt + \sigma_S dz_S(t), \qquad S(0) = S > 0, \tag{1}$$

where r is the constant positive risk free interest rate, σ_S denotes the constant positive volatility of *S(t)*, λ_S is the constant market price of risk associated to the spot price and $z_S(t)$ is the Brownian motion that governs the spot price evolution. Furthermore, $\delta(t)$ denotes the time-t instantaneous convenience yield whose dynamics obeys the following mean reverting process[12]:

$$d\delta(t) = \kappa(\bar{\delta} - \delta(t))dt + \sigma_\delta dz_\delta(t), \qquad \delta(0) = \delta, \tag{2}$$

where κ and $\bar{\delta}$ are the positive constant adjustment speed and long term mean, respectively.[13] σ_δ denotes the volatility of the convenience yield and $z_\delta(t)$ is the Brownian motion that describes the risk of the convenience yield. The spot price and the convenience yield are supposed to be observable to the investor whose time t information is then given by $F_t \equiv \sigma(S(u), \delta(u); 0 \leq u \leq t)$. Following Brennan (1991), we consider a positive and imperfect correlation between the convenience yield and the spot price. The correlation coefficient, ρ, is such that $\rho dt \equiv d\langle z_S, z_\delta \rangle_t$. Due to this imperfect correlation, a market price of risk related to the convenience yield, λ_δ, exists and is supposed to be constant (see Gibson and

[12] Empirical evidence supports this assumption see for example Bessembinder et al. (1995), Casassus and Collin-Dufresne (2005).

[13] In our setting, the convenience yield is defined as the net cost of storage and therefore it can be negative. However, empirical studies (Schwartz, 1997; Casassus et Collin-Dufresne, 2005) suggest that the long term mean is usually positive.

Schwartz, 1990).

To facilitate the exposition of our model, we disentangle the specific risk of the convenience yield that is not correlated with the spot price risk. In order to do so, we introduce a Browian motion $z_u(t)$ uncorrelated with $z_S(t)$ defined as follows:

$$dz_\delta(t) = \rho dz_S(t) + \sqrt{1-\rho^2}\, dz_u(t). \tag{3}$$

In terms of prices of risk, the relation writes:

$$\lambda_\delta = \rho\lambda_S + \sqrt{1-\rho^2}\,\lambda_u, \tag{4}$$

where λ_u is the specific risk price associated with the convenience yield.

Let $H(\tau_H)$, with $\tau_H \equiv T_H - t$, denote the time t futures price maturing on T_H. Its dynamics is given by (Schwartz, 1997):

$$\frac{dH(\tau_H)}{H(\tau_H)} = \left(\sigma_{HS}(\tau_H)\lambda_S - \sigma_{Hu}(\tau_H)\lambda_u\right)dt + \sigma_{HS}(\tau_H)dz_S(t) - \sigma_{Hu}(\tau_H)dz_u(t), \tag{5}$$

where $\sigma_{HS}(\tau_H) \equiv \sigma_S - \rho\sigma_\delta D_\kappa(\tau_H)$ and $\sigma_{Hu}(\tau_H) \equiv \sqrt{1-\rho^2}\,\sigma_\delta D_\kappa(\tau_H)$, with $D_\kappa(\tau_H) \equiv \frac{1}{\kappa}\left(1-e^{-k\tau_H}\right)$, represent the sensitivities of the futures price to the shocks stemming from z_S and z_u, respectively. Therefore, the volatility of the futures price, $\sigma_H(\tau_H)$, is given by: $\sigma_H(\tau_H) = \sqrt{\sigma_{HS}(\tau_H)^2 + \sigma_{Hu}(\tau_H)^2}$.

In what follows, we assume that the commodity futures market is well developed so that trading in futures contracts with different maturities is possible. Let $H_1(\tau_{H_1}) \equiv H_1$ and $H_2(\tau_{H_2}) \equiv H_2$ be two futures contracts maturing at T_{H_1} and T_{H_2}, respectively, with $T_{H_1} < T_{H_2}$, and $\tau_{H_1} \equiv T_{H_1} - t < \tau_{H_2} \equiv T_{H_2} - t$. Thus, H_1 is refered to as the nearby contract and H_2 to as the distant contract.

3. Prices correlation and the Samuelson hypothesis

Having described the economic framework, we now turn to the study of the correlation between futures and spot prices as well as the correlation between the nearby and distant futures prices. First, we analyse the term structure of futures prices correlation (Proposition 1 and Corollary 1). Second, we investigate its sign when the Samuelson hypothesis holds (Proposition 2 and Corollary 2).

Proposition 1 *Let* $\rho_{S,H}$ *denotes the correlation between the spot price and any futures price.*

$\rho_{S,H}(\tau_H)$ *is a decreasing function of* τ_H.

When the time-to-maturity approches 0, arbitrage-free conditions imply that $\rho_{S,H}(0)=1$ *, since* $H \to S$ *as* $\tau_H \to 0$.

Proof. Available from the authors upon request.

As explained in the introduction, distant futures prices are more impacted, in absolute terms, by the convenience yield. So, the more distant is the futures contract, the more is influenced by the orthogonal risk of the convenience yield, and the less is correlated with the spot price.

Corollary 1 *Let* $\rho_{H_1,H_2}(\tau_{H_1};\tau_{H_2})$ *be the correlation between the nearby and distant futures prices. It is:*

- *a decreasing function of* τ_{H_2}.
- *a increasing function of* τ_{H_1}.

The correlation between the spot price and the distant contract is always greater than the correlation between the nearby and the distant futures prices:

$\rho_{S,H_2}(\tau_{H_2}) < \rho_{H_1,H_2}(\tau_{H_1};\tau_{H_2})$

When the nearby time-to-maturity is close to 0, arbitrage-free conditions imply that $\rho_{S,H_2}(\tau_{H_2}) = \rho_{H_1,H_2}(0;\tau_{H_2})$ *, since* $H_1 \to S$ *as* $\tau_{H_1} \to 0$.

Proof. Available from the authors upon request.

Since the net convenience yield (or net carrying charge) is an increasing function of the time spread between the nearby and distant futures contracts' time to maturity, the same arguments used for Proposition 1 apply. Note that this result is also in line with empirical evidence, see Manoliu and Tompaidis (2000). Having described the term structure of correlation, we proceed now to investigate its sign.

Proposition 2 *Define* $u \equiv \sqrt{\dfrac{\rho\sigma_\delta}{\kappa\sigma_S}}$. *Then, the following statement holds*:

When the Samuelson effect holds, whatever the maturity, the correlation between the spot price and the futures price is always positive. Moreover, the minimum value of this correlation, $\rho_{S,H}^{min}$, *is equal to :*

$$0 < \rho_{S,H}^{min} = \left(1 + \frac{1-\rho^2}{\rho^2}\left[\frac{u^2}{1-u^2}\right]^2\right)^{-\frac{1}{2}} \tag{6}$$

Proof. Available from the authors upon request.

Proposition 2 states that the smoothing effect of mean reverting prices garantees that the innovation linked to the convenience yield will never be greater, in absolute value, than the innovation related to the spot price.[14] Since, as shown by Corollary 1, the correlation between the spot price and the distant contract is always superior to that between the nearby and the distant futures prices, Corollary 2 obtains immediately.

Corollary 2

When the Samuelson hypothesis holds, the correlation between the nearby and the distant futures contracts is always positive.

Proof. Available from the authors upon request.

Corollary 2 will be shown to be of a great importance for the study of optimal proportions, see Section 4.

[14] As stated by Routledge, Seppi and Spatt (2000), the "Samuelson effect" (or Samuelson hypothesis) is sometimes violated when volatility is low. However, Proposition 2 does not state that when the Samuelson effect is violated, then the correlation is negative. Our simulations for the copper futures market, Section 5, and empirical evidence for the energy futures market (Manoliu and Tompaidis, 2000) suggest that this correlation is always very positive.

4. Optimal spread proportions

Consider an investor endowed with a CRRA utility function for wealth $U(W)=\frac{W^{1-\gamma}}{1-\gamma}$, with $\gamma>0$ [15] denoting the constant risk aversion coefficient. The investor's objective is to maximize the expected utility for his/her terminal wealth over a finite investment horizon T_I when his initial endowment is $W(0)=W_0>0$.

In order to justify optimal spreads, we consider two distinct investment programs, (I) and (II), in the commodity futures market. In program (I), our agent invests his/her initial wealth in two assets: a riskless asset which price is $\beta(t)\equiv\exp(rt)$, and a risky futures contract maturing at T_H ($T_I<T_H$) which price is $H(\tau_H)$. In this case, although the investor faces two imperfectly correlated sources of risk, only one risky asset is used so the market is dynamically incomplete. On the contrary, in program (II), the investor allocates his/her initial wealth between one riskless assets and two futures contracts having short and long maturity T_{H_1} ($T_I<T_{H_1}$)[16] and T_{H_2}, respectively. Hence, in this case the market is complete. The two investment programs (I) and (II) write, respectively:

- $$J_{inc}(\gamma;T_I;W_0)\equiv \sup_{\pi_H(t),0\le t\le T_I} E\left(u\left(W_{inc}(T_I)\right)\right) \tag{7a}$$

  $$s.t.\frac{dW_{inc}(t)}{W_{inc}(t)}=rdt+\pi_H(\tau_H)\frac{dH(\tau_H)}{H(\tau_H)} \tag{7b}$$

- $$J(\gamma;T_I;W_0)\equiv \sup_{\pi_{H_1}(t),\pi_{H_2}(t)0\le t\le T_I} E\left(u\left(W(T_I)\right)\right) \tag{8a}$$

  $$s.t.\frac{dW(t)}{W(t)}=rdt+\pi_{H_1}(\tau_{H_1};\tau_{H_2})\frac{dH(\tau_{H_1})}{H(\tau_{H_1})}+\pi_{H_2}(\tau_{H_1};\tau_{H_2})\frac{dH(\tau_{H_2})}{H(\tau_{H_2})} \tag{8b}$$

[15] The Bernoulli investor optimal proportions are obtained when $\gamma\to 1$.

[16] The conditions, $T_I<T_H$ and $T_I<T_{H_1}$, avoid the strategy of rolling over future contracts (Neuberger 1999). However, since in the Gibson and Schwartz (1990) model market prices of risks are constant over time, investors do not take into account the intertemporal substitution effect linked to future values of these market prices of risks (see Kim and Omberg, 1996; Wachter, 2002). As a consequence, the investment horizon is not necessary to compute optimal investment and this condition is purely formal in our setting.

where the two budget constrains (7b) and (8b) are obtained from the self financing property. $\pi_H(\tau_H)$ denotes the wealth proportion[17] invested in the futures contract in program (I), and $\pi_{H_1}\left(\tau_{H_1};\tau_{H_2}\right)$ and $\pi_{H_2}\left(\tau_{H_1};\tau_{H_2}\right)$ are the proportions invested in the nearby and distant futures contracts in program (II), respectively.[18] Moreover, $J_{inc}(\gamma,T_I,W_0)$ and $J(\gamma,T_I,W_0)$ measure the satisfaction gained by the investor for the best feasible strategy in programs (I) and (II), respectively. Since, any strategy of (I) is feasible by a strategy of (II), we have $J_{inc}(\gamma,T_I,W_0)\le J(\gamma,T_I,W_0)$. Proposition 3 below states that the latter inequality is actually strict.

Proposition 3 *An agent will achieve a better strategy investing in two futures contracts than investing in only one contract whatever his/her investment horizon, risk aversion or initial wealth. Formally, we have:*

$$J_{inc}(\gamma,T_I,W_0)<J(\gamma,T_I,W_0) \tag{9}$$

Proof. Available from the authors upon request.

Since, in our setting, it is never optimal to invest in one futures contract only, we focus on the strategy in the nearby and distant futures contract of investment program (II).

Proposition 4 *Under a CRRA framework, optimal spread proportions are given by:*

$$\pi_{H_1}\left(\tau_{H_1};\tau_{H_2}\right)=\frac{\lambda^{\perp}_{H_2}(\tau_{H_2})}{\gamma\sigma_{H_1}(\tau_{H_1})\sqrt{1-\rho_{H_1,H_2}\left(\tau_{H_1};\tau_{H_2}\right)^2}} \tag{10}$$

$$\pi_{H_2}\left(\tau_{H_1};\tau_{H_2}\right)=\frac{\lambda_{H_2}(\tau_{H_2})}{\gamma\sigma_{H_2}(\tau_{H_2})}-\frac{\sigma_{H_1}(\tau_{H_1})}{\sigma_{H_2}(\tau_{H_2})}\rho_{H_1,H_2}\left(\tau_{H_1};\tau_{H_2}\right)\pi_{H_1}\left(\tau_{H_1};\tau_{H_2}\right) \tag{11}$$

with $\lambda^{\perp}_{H_2}(\tau_{H_2})$ and $\lambda_{H_2}(\tau_{H_2})$ the market prices of risks of any strategy uncorrelated with the distant futures contract and of any strategy perfectly correlated with the distant futures contract,

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17 $\pi_H(\tau_H)\equiv\dfrac{\theta_H(\tau_H)H(\tau_H)}{W_{inc}(t)}$, where $\theta_H(\tau_H)$ is the number of futures contracts held at time t.

18 $\pi_{H_1}\left(\tau_{H_1};\tau_{H_2}\right)\equiv\dfrac{\theta_{H_1}(\tau_{H_1};\tau_{H_2})H(\tau_{H_1})}{W(t)}$ and $\pi_{H_2}\left(\tau_{H_1};\tau_{H_2}\right)\equiv\dfrac{\theta_{H_2}(\tau_{H_1};\tau_{H_2})H(\tau_{H_1})}{W(t)}$, where $\theta_{H_1}(\tau_{H_1};\tau_{H_2})$ and $\theta_{H_1}(\tau_{H_1};\tau_{H_2})$ are the number of nearby and distant futures contracts held at time t, respectively.

respectively. These market prices of risks write:

$$\lambda_{H_2}^{\perp}(\tau_{H_2}) = \sqrt{1-\rho_{S,H_2}(\tau_{H_2})^2}\,\lambda_S + \rho_{S,H_2}(\tau_{H_2})\lambda_u \tag{12a}$$

$$\lambda_{H_2}(\tau_{H_2}) = \rho_{S,H_2}(\tau_{H_2})\lambda_S - \sqrt{1-\rho_{S,H_2}(\tau_{H_2})^2}\,\lambda_u \tag{12b}$$

Proof. Available from the authors upon request.

First, Proposition 4 obtains optimal spread proportions invested in the nearby and distant futures contracts. Apart from the two market prices of risk (λ_S and λ_u), proportions are expressed in terms of easily computable quantities $\sigma_{H_1}(\tau_{H_1})$, $\sigma_{H_2}(\tau_{H_2})$, $\rho_{S,H_2}(\tau_{H_2})$, $\rho_{H_1,H_2}(\tau_{H_1};\tau_{H_2})$. Moreover, optimal proportion in the distant futures contract can be decomposed into two parts. The first is an autonomous part, $\pi_{H_2\_aut}(\tau_{H_2}) \equiv \dfrac{\lambda_{H_2}(\tau_{H_2})}{\gamma\sigma_{H_2}(\tau_{H_2})}$, since it is not linked to the nearby contract. This part is, actually, the traditional mean variance term associated with an investment strategy in the distant futures contract. The second part, $\pi_{H_2\_cor}(\tau_{H_2}) \equiv -\dfrac{\sigma_{H_1}(\tau_{H_1})}{\sigma_{H_2}(\tau_{H_2})}\rho_{H_1,H_2}(\tau_{H_1};\tau_{H_2})\pi_{H_1}(\tau_{H_1};\tau_{H_2})$, stems from the correlation between the nearby and distant futures contract.[19] Using Proposition 2 as well as Corollary 2, and since there is an empirical support for a (high) positive correlation, (Manoliu and Tompaidis, 2000)[20] , the term $\pi_{H_2\_cor}(\tau_{H_2})$ justifies the use of calendar spreads in commodity markets. Indeed, $\dfrac{\partial\pi_{H_2}}{\partial\pi_{H_1}}(\tau_{H_1};\tau_{H_2}) = -\dfrac{\sigma_{H_1}(\tau_{H_1})}{\sigma_{H_2}(\tau_{H_2})}\rho_{H_1,H_2}(\tau_{H_1};\tau_{H_2})$. As for the optimal proportion in the nearby contract, it is affected by i) the market prices of risk associated with the spot price and the specific risk of the convenience yield, ii) the investor's risk aversion and iii) the correlation between the distant futures contract and the spot price on one hand and the nearby contract on the other hand. Indeed, the nearby proportion is, in absolute value, an increasing function of this correlation.

[19] Indeed, if the two contracts are not correlated (which is quite unlikely) then this part vanishes out.

[20] See Table 6 page 45 in the paper.

5. Numerical illustration

This section provides an illustration of the various propositions and corollaries of the article in the case of the NYMEX copper futures markets. Our base case parameters are adapted from the empirical study in Casassus and Collin-Dufresne (2005) and are given in Table 1. In addition, we choose a relative risk aversion equal to 3 (Meyer and Meyer, 2005).

Table 1: model parameters for copper

λ_u	λ_S	κ	σ_S	ρ	σ_δ
0.13	0.26	1.1	0.23	0,7	0,20

Figure 1 plots the results provided by Propositions 1, 2 and Corollaries 1 and 2. It illustrates the increasing and decreasing shape of the correlation with respect to the nearby and distant time to maturity, respectively. In addition, whereas the pattern in the distant time to maturity is almost linear, the correlation function of the nearby time to maturity is highly concave.

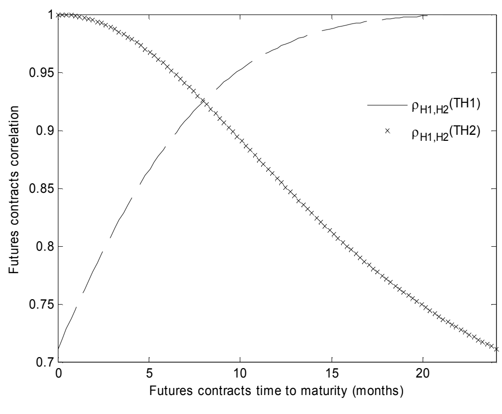

Fig. 1. *Futures contracts correlation as a function of the nearby and distant times to maturity.* This graph plots $\rho_{H_1,H_2}(\tau_{H_1};\tau_{H_2})$ as a function of τ_{H_1} (dash) from 0 to 24 months while $\tau_{H2} = 24$ months, as a function of τ_{H2} (cross) from 0 to 24 months while $\tau_{H1} = 0$ month. Other parameters are given in table 1.

Figures 2 and 3 are devoted to the study of optimal proportions as a function of the residual maturity of the nearby and distant futures, respectively. First, the two figures clearly illustrate the spread positions. For our base-case parameters, the investor is always long in the nearby and short in the distant. However, a close inspection shows that this spread is never one to one.

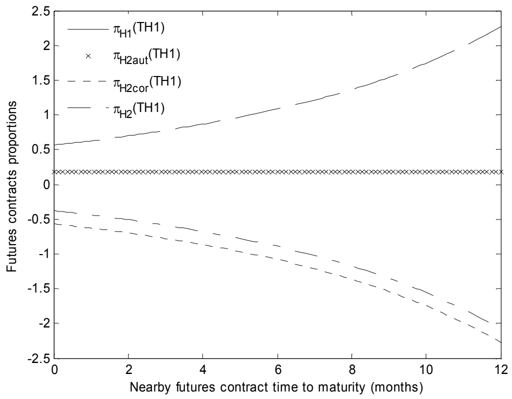

Fig. 2. *Proportions in nearby and distant futures contract as a function of the nearby time to maturity. This graph plots* $\pi_{H_1}(\tau_{H_1};\tau_{H_2})$ (dash), $\pi_{H_2\_aut}(\tau_{H_1};\tau_{H_2})$ (cross), $\pi_{H_2\_corr}(\tau_{H_1};\tau_{H_2})$ (dot) , $\pi_{H_2}(\tau_{H_1};\tau_{H_2})$ (dash-dot) as a function of τ_{H_1} from 0 to 12 months while τ_{H_2} = 24 months. Other parameters are given in table 1.

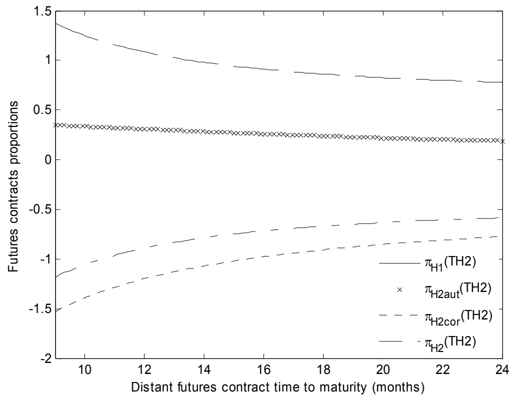

Fig. 3. *Proportions in nearby and distant futures contract as a function of the distant time to maturity. This graph plots* $\pi_{H_1}(\tau_{H_1};\tau_{H_2})$ (dash), $\pi_{H_2\_aut}(\tau_{H_1};\tau_{H_2})$ (cross), $\pi_{H_2\_corr}(\tau_{H_1};\tau_{H_2})$ (dot) , $\pi_{H_2}(\tau_{H_1};\tau_{H_2})$ (dash-dot) as a function of τ_{H_2} from 9 to 24 months while τ_{H_1} = 3 months. Other parameters are given in table 1.

Moreover, these two figures display the main properties of Proposition 4. When the nearby (distant) time to maturity increases, the correlation between the two contracts increases (decreases). Therefore, the proportion invested in the nearby contract increases (decreases). Also, we point out the convex shape of the curves related to the nearby proportion as the time spread between the nearby and distant contracts reduces. Furthermore, the correlated component of the distant proportion and the nearby proportion exhibit similar characteristics, that is a non linear pattern as well as a constant, although opposite, sign. In addition, we notice that the autonomous part of the distant contract proportion has the same sign as the nearby proportion. The former has much smaller magnitude, though. This difference in magnitude is due to the significant impact of the correlation function on the proportion allocated to the distant contract.

6. Conclusion

This paper focuses on optimal calendar spreads for an investor with a constant relative risk aversion. We highlight the crucial role of the correlation between the distant and nearby futures contracts. First, this correlation affects positively and significantly the proportion invested in the nearby contract, and second, it implies a negative relationship between the distant and the nearby proportions. Moreover, we derive optimal spread proportions and show that they depend on futures contracts' volatilities and correlations.

Our analysis can be carried out under a more achieved setting where a stochastic feature of the market price of risk is introduced (Casassus and Collin-Dufresne, 2005; Richter and Sorensen, 2006). This stochastic market price of risk creates an intertemporal substitution effect that can potentially affect the spread analysis.

References

Adler, M., Detemple, J., 1988a. On the optimal hedge of a non-traded cash position. *The Journal of Finance* 43, 143-153.

Adler, M., Detemple, J., 1988b. Hedging with futures in an intertemporal portfolio context. *The Journal of Futures Markets* 8, 249-269.

Bessembinder, H., J. F. Coughenour, P. J. Seguin, & M. M. Smoller, 1995. Mean reversion in equilibrium asset prices: evidence from the futures term structure. *Journal of Finance* 50, 205-238.

Bessembinder, H., J. F. Coughenour, P. J. Seguin, & M. M. Smoller, 1996. Is there a term structure of futures volatilities? Reevaluating the Samuelson hypothesis. *Journal of Derivatives* 4. 45–58.

Breeden, D., 1984. Futures markets and commodity options – hedging and optimality in incomplete markets. *Journal of Economic Theory* 32, 275-300.

Brennan, M., 1958. The supply of storage. *American Economic Review* 48, 50-72.

Brennan, M., 1991. The pricing of convenience and the valuation of commodity contingent claims, in:

Lund, D., Oskendal, B. (Eds), Stochastic models and option values, Elsevier Science Publishers, North Holland, Amsterdam, The Netherlands, 33-71.

Briys, E., Crouhy, M., Schlesinger, H., 1990. Optimal hedging under intertemporally dependent preferences. *Journal of Finance* 45, 1315-1324.

Casassus, J. P., Collin-Dufresne, J-P, 2005. Stochastic convenience yield implied from commodity futures and interest rates. *The Journal of Finance* 60(5), 2283-2331.

Commodity Trading Future Commission (CFTC), 2008. Staff Report on Commodity Swap Dealers & Index Traders with Commission Recommendations.

Duffie, D., Jackson, O., 1990. Optimal hedging and equilibrium in a dynamic futures market. *Journal of Economic Dynamics and Control* 14, 21-33.

Duffie, D., Richardson, H., 1991. Mean-Variance hedging in continuous time. *Annals of Applied Probability* 1, 1-15.

Gibson, R., Schwartz, E., 1990. Stochastic conveniences yield and the pricing of oil contingent claims. *Journal of Finance* 45, 959–976.

Hicks, J., 1939. *Value and Capital*, Oxford University Press. Oxford.

Ho, T., 1984. Intertemporal commodity futures hedging and the production decisions. *Journal of Finance* 39, 351-376.

Hong, H., 2001. Stochastic convenience yield, optimal hedging and the term structure of open interest and futures prices. Working Paper, Graduate School of Business, Stanford University.

Keynes, J., 1930. A Treatise on Money: the Applied Theory of Money. Macmillan, Vol. 2, Londres.

Kim, Omberg, 1996. Dynamic non myopic Portfolio behaviour. *Review of Financial Studies*, 9(1), p141-161.

Lioui, A., Eldor, R., 1998. Optimal spreading when spreading is optimal. *Journal of Economic Dynamics and Control* 23, 277-301.

Lioui, A., Nguyen Duc Trong, P., Poncet, P., 1996. Optimal dynamic hedging in incomplete futures markets. *The Geneva Papers on Risk and Insurance Theory* 21-1, 103-122.

Lioui, A., Poncet, P., 2001. On the optimal portfolio choice under stochastic interest rates. *Journal of Economic Dynamics and Control* 25, 1841-1865.

Manoliu and Tompaidis, 2000. Energy futures prices: term structure model with Kalman filter estimation. Working paper, University of Texas.

Meyer and Meyer, 2005. Relative Risk Aversion: What Do We Know? *Journal of Risk and Uncertainty*, 31 (3), 243-262.

Neuberger, A. 1999. Hedging Long-Term Exposures with Multiple Short-Term Futures Contracts. *Review of Financial Studies*, 9(1),p141-161.

Poitras, G., 1989. Optimal futures spreads positions. *Journal of Futures Markets* 9, 123-133.

Richter, M., Sorensen, C., 2006. Stochastic volatility and seasonality in commodity futures and optionsthe case of soybeans. Working paper. Copenhagen Business School.

Routledge, B., Seppi, D., Spatt, C., 2000. Equilibrium forward curves for commodities. *Journal of Finance* 55, 1297-1338.

Samuelson, P., 1965. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review* 6, , 41-49.

Schrock, N., 1971. The theory of asset choice: simultaneous holding of short and long positions in the futures market. *Journal of Political Economy* 79, 270-293.

Schwartz, E., 1997. The stochastic behavior of commodity prices: Implications for valuation and hedging. *Journal of Finance* 52, 923-973.

Telser, L. G. 1958. Futures trading and the storage of cotton and wheat. *Journal of Political Economy*, 66, 233–255.

Wachter J., 2002. Portfolio and consumption decisions under mean-reverting returns: and exact solution for complete markets. *Journal of Financial and Quantitative Analysis* 37, 63-91.

Working, H., 1948. Theory of the inverse carrying charge in futures markets, *Journal of Farm Economics* 30, 128.

Working, H., 1949. The theory of the price of storage. *American Economic Review* 39, 1254-62.