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Pairs trading: The performance of a stochastic spread model with regime switching-evidence from the S&P 500☆

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ABSTRACT

There remains a lack of literature on a pairs-trading model that is able to capture the mean reversion and two different states of spreads. The purpose of this study is to combine the Markov regime-switching model and the Vasicek model to implement a pairs-trading strategy that utilizes the S&P 500 stock components from January 1, 2006, through September 28, 2012. We compare our model's performance with the performance of previous methods based on a variety of portfolios and trading periods. The empirical results show that the trading rule of the Markov regime-switching model with mean reversion has the best performance with a simple portfolio. Furthermore, the results show that shorter trading periods produce better performance than longer trading periods and that the trading rule performs strongly during the global financial crisis of 2008 to 2009.

1. Introduction

The statistical arbitrage strategy, which has been utilized for decades, is a market-neutral strategy that is widely used by many hedge funds and that trades on the basis of the relative mispricing between a pair of stocks. Pairs trading is a trading strategy for a portfolio that contains only two stocks; it is generally believed that the pairs-trading strategy is a special type of statistical-arbitrage strategy.

Pairs trading was developed and exploited by the Wall Street quantitative trader Nunzio Tartaglia and the team of mathematicians, physicists, and computer scientists he assembled in the mid-1980s. The deviations that are caused by mispricing between two stocks are expected to disappear in the future; thus, we can short-sell a position on the relatively overvalued stock and buy a long position on the undervalued stock to make a profit. Market neutrality is a concept that is related to statistical arbitrage, and mean reversion is an important feature of statistical arbitrage. Market neutrality means that the returns from the portfolio strategy are unrelated to the performance of the financial markets. Each stock has its own systemic risk and idiosyncratic risk. Systemic risk may be eliminated by employing a pairs-trading strategy because risk will be offset when we buy the undervalued stock and sell the overvalued stock simultaneously. Therefore, there is only idiosyncratic risk in the pairs-trading strategy. The performance of the market-neutral strategy is typically better when trading many portfolios. The pairs-trading strategy assumes that the spread of paired stocks will have mean-reverting characteristics that allow the deviation in the short-term prices of the

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paired stocks to be exploited to earn a profit. However, mean reversion will disappear in long-term spreads if market investors act irrationally over the long term.

Do, Faff, and Hamza (2006) analyze existing pairs-trading methods, including the distance method (Do and Faff (2010); Gatev et al. (1999 and 2006) and Nath (2003)), the cointegration method (Vidyamurthy (2004)), and the stochastic spread method (Elliott, Van Der Hoek, and Malcolm (2005)). The distance method exploits the statistical relationship between “the prices of paired stocks and avoids the risks of model misspecification and misestimation.” Gatev et al. (1999) utilize a simple standard deviation strategy to show that pairs trading can be profitable after costs are accounted for. Gatev, Goetzmann, and Rouwenhorst (2006) apply pairs trading to a sample of U.S. stocks covering the period from 1962 through 2002 and offer a comprehensive analysis. The authors obtain a statistically significant excess return of 11% and rule out certain explanations for the profits of pairs trading, including mean reversion, unrealized bankruptcy risk, and short-sale constraints. Do and Faff (2010) use the sum of squared differences (which was proposed by Gatev et al. (2006)), industry homogeneity, and the reversal frequency of the stock price spread to find that the pairs-trading strategy is not as profitable as shown in the past, although it continues to generate good profits during market downturns, including the period following the global financial crisis that erupted in 2007. Do and Faff also put forward a possible explanation for these dramatically declining returns. They indicate that the main drivers of the declining trend in pairs-trading profitability are the increased rates and magnitude of divergence caused by worsening risks of arbitrage rather than an increase in market efficiency. They find that the worsening risks of arbitrage in various pair portfolios have accounted for as much as 70% of the declining trend in pairs-trading profitability, with the remaining 30% attributable to increased efficiency. However, Do et al. (2006) indicate that the distance method cannot forecast the convergence time or expected holding period, which means that the assumptions of this method are only suitable for paired stocks whose risk-return profiles are similar.

The cointegration approach outlined by Vidyamurthy (2004) attempts to parameterize pairs trading by exploring the possibility of cointegration (Engle & Granger 1987). Vidyamurthy (2004) observes that because the logarithms of two stock prices are assumed to follow a random walk, there is a strong likelihood that they will be cointegrated. If that is the case, the results related to cointegration may be used to determine how far the spread is from its equilibrium, such that long/short positions may be used to find the opportunity to earn profits. However, these econometric techniques may have problems. Engle and Granger's two-step cointegration procedure (Engle & Granger 1987) may cause the residuals to have different sets of statistical properties and lead to the un-cointegration of the two series, which may mean that the mean reversions of the residuals are not reliable. The stochastic spread method developed by Elliott et al. (2005) explicitly models the mean-reverting behavior of the spread between the paired stocks in a continuous time setting in which the spread is assumed to follow the Vasicek model. There are three main advantages of the stochastic spread model: first, the model can capture the mean reversion of the spreads of the paired stocks; second, the random spread model is a continuous time model, which should make it easier to predict the speeds; and third, the parameters of this model can be estimated with a Kalman filter. To overcome the defects of the stochastic spread method, Do et al. (2006) propose the stochastic residual spread method. This model is similar to the stochastic spread method proposed by Elliott et al. (2005); however, to overcome the problem related to phases of imbalance in which the deviation of the price stock spread may temporarily or persistently endure, Bock and Mestel (2009) bridge the literature on Markov regime switching (two-state) and the scientific work on statistical arbitrage to develop useful trading rules for pairs trading. Employing the data from DJ STOXX600, this rule for pairs trading generates positive returns.

Many previous papers have supported the premise that a Markov-switching model provides a better fitting performance to the stock returns than linear models without switching. Turner, Startz, et al. (1989) use S&P monthly index data for the 1946–1989 period and find a Markov-switching model in which the mean, the variance or both may differ between two regimes. Hamilton and Susmel (1994) propose a model with sudden discrete changes in the process that governs volatility. Schaller and Norden (1997) find significant evidence of switching behavior in stock market returns. The switching phenomenon existed in means and variances during the 1929 crash, the Great Depression, and World War II. Not only the return of the stocks but also the spread, defined as the difference between the stock process, exhibit switching behavior. Some studies use Markov-switching framework to investigate the detecting hot and cold IPO cycles (Guo, Brooks, & Shami 2010), the contagion effects among the other assets (Guo, Chen, & Huang 2011), the volatility transmissions (Khalifa, Hammoudeh, & Otranto 2014), and the price–dividend ratio stationarity (Londonoa, Regúlezb, & Vázquezc 2015). Fig. 1 shows that the means (volatilities) of the spreads during 2008–2010 appear to be lower (higher) than the spreads during 2006–2008 and 2010–2012. In other words, it appears that the switching model would be more appropriate for the spreads than the linear model.

Thus, there is no example in the literature of a pairs-trading model that can capture both mean reversions and the two different states of spreads. The purpose of this study is to develop a pairs-trading model that combines the Markov regime-switching model with the Vasicek model with mean reversion and compare this new model with previous models, including the distance method model and the stochastic spread method model (the Vasicek model). The empirical results show that rule I (the stochastic spread method with Markov regime switching) has the best performance with a simple portfolio (TOP 1) and show that a shorter trading period leads to better performance. However, rule III (the distance method) is the least effective of the three rules from a risk-management point of view. Hence, rule I is effective for investors who attempt to generate substantial profits by utilizing pairs trading. Our results also provide supportive evidence in accordance with Do and Faff (2010), confirming the continuing downward trend in the profitability of pairs trading; the strategy performs strongly during periods of the recent global financial crisis.

This paper is organized as follows: Section 1 offers the introduction. The Markov regime-switching model with mean reversion that is examined in this study and its maximum likelihood estimation are presented in Section 2. In Section 3, we discuss the criteria for pairs and trade rules. The empirical results are shown in Section 4. Finally, Section 5 presents the conclusions.

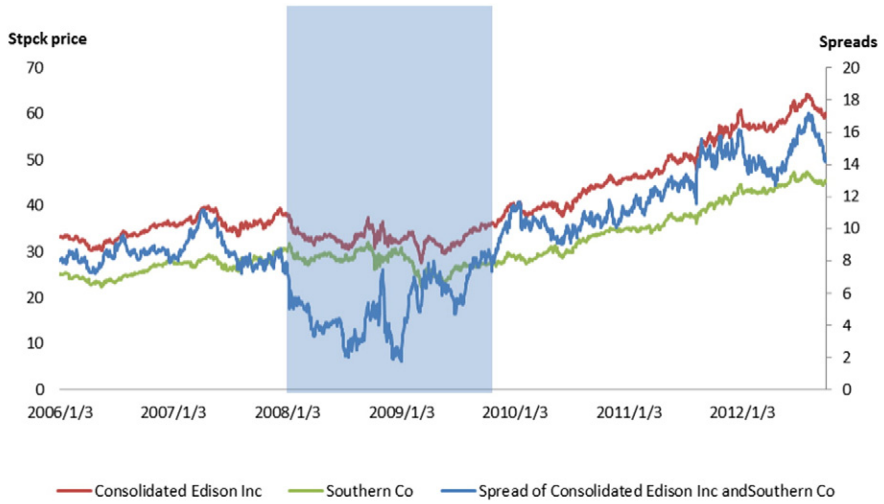

Fig. 1. Spread of stock prices (TOP 1 portfolio of unrestricted pairs). The price spreads of Consolidated Edison Inc. and Southern Co. are calculated from January 3, 2006, to September 28, 2012. The shaded area indicates the low mean state regime.

2. Stochastic spread method with regime switching

This section first describes the framework of the stochastic spread method with regime switching. Furthermore, a maximum likelihood estimation of the stochastic spread method with regime switching is illustrated.

2.1. Model

Roughly speaking, our model consists of the Markov regime-switching model proposed by Hamilton (1989) and the mean reversion model introduced by Vasicek (1977). We define a spread,Spread_t, at time t as

$$\text{Spread}_\text{t} = \text{price}_t^A - \text{price}_t^B, \tag{1}$$

the difference between price_t^A and price_t^B that are paired with one another. Moreover, this spread is assumed to follow a mean reversion stochastic process with regime-switching states. We write

$$d(\text{ Spread}_t) = \kappa_{S_t}\left(\theta_{S_t} + \text{Spread}_{t-1}\right)dt + \varepsilon_t, \text{where } \varepsilon_t \sim N\left(0, \sigma_{S_t}^2\right), \tag{2}$$

where S_t follows the two states of the Markov chain process with states $S_t = \text{high}$ and $S_t = \text{low}$. For each state, the $d(\text{ Spread}_t)$ follows a mean reversion stochastic process with a constant speed of reversion,k_{S_t}, and long-term mean level, κ_{S_t}; the residual ε_t is assumed to follow a normal distribution with a zero mean and variance, $\sigma_{S_t}^2$. The transition probability matrix of the Markov chain process is given by the following.

$$\Sigma = \begin{bmatrix} P_\text{high,high} & P_\text{low,high} \\ P_\text{high,low} & P_\text{low,low} \end{bmatrix}, \tag{3}$$

which captures the probabilities of a state change from time $t-1$ to time t. For example, the term $= P(S_t = \text{low} | S_{t-1} = \text{high})$ indicates the probability of switching from a low state at time $t-1$ to a high state at time t. Note that each column in Σ sums to one. Our model defined in Eqs. (1)–(3) can capture the mean reversion of the spread between the paired stocks among different states, which solves the problems of the linear model that cannot measure the switching states and those of the traditional pairs-trading model that does not imply mean reversion. Note that if the state S_t is the same for all time t, this nonlinear model is reduced to the traditional Vasicek model. In the next section, we will show the ways to estimate our model by the maximum likelihood estimation method.

2.2. Maximum likelihood estimation

We will estimate our model in Eqs. (1)–(3) by the maximum likelihood estimation method. The estimation process is similar to, but more complicated than, the process used by Hamilton (1989). We write the logarithm of the likelihood function of Eqs. (1)–(3) as

$$\ln \mathcal{L} = \sum_{t=1}^n lnf(d(\text{ Spread}_t)| \varphi_{t-1}) \tag{4}$$

The probability density function of $d(\text{Spread}_t)$,$f(d(\text{ Spread}_t)|\ \varphi_{t-1})$, can be derived as

$$\begin{aligned} f(d(\text{ Spread}_t)|\ \varphi_{t-1}) &= f(d(\text{ Spread}_t)|S_t, \varphi_{t-1})P(S_t|\ \varphi_{t-1}) \\ &= f(d(\text{ Spread}_t)|S_t = \text{high}, \varphi_{t-1})P(S_t = \text{high}|\ \varphi_{t-1}) \\ &\quad + f(d(\text{ Spread}_t)|\mathrm{S_t} = \text{low}, \varphi_{t-1})P(S_t = \text{low}|\ \varphi_{t-1}) \end{aligned} \tag{5}$$

Under normal assumptions for residual ε_t, $f(d(\text{ Spread}_t)|\mathrm{S_t}{=}i,\varphi_{t-1})$ will be

$$\begin{aligned} &f(d(\text{Spread}_t)|S_t = i, \varphi_{t-1}) \\ &= \frac{1}{\sqrt{2\pi\sigma^2_{S_t=i}}} \exp\left\{ -\frac{\left[d(\text{ Spread}_t) - \kappa_{S_t=i}\left(\theta_{S_t=i} + \text{Spread}_{t-1}\right)\right]^2}{2\sigma^2_{S_t=i}} \right\} \end{aligned} \tag{6}$$

where $i{=}$high and low.

The term $P(S_t{=}i|\ \varphi_{t-1})$ is known as the prediction probability of the state $S_t{=}i$ at time t, which is obtained based on the available information set φ_{t-1}. That is,

$$\begin{aligned} &P(S_t = i|\varphi_{t-1}) \\ &= \sum_{j=\text{high}}^{\text{low}} P(S_t = i|S_{t-1} = j, \varphi_{t-1})\mathrm{P}(S_{t-1} = j\ |\varphi_{t-1}) \\ &= \sum\nolimits_{j=\text{high}}^{\text{low}} P(S_t = i\ |S_{t-1} = j)\mathrm{P}(S_{t-1} = j|\varphi_{t-1}), i = \text{high}, \text{low}. \end{aligned} \tag{7}$$

Note that $P(S_t{=}i|S_{t-1}{=}j\ )$ represents the transition probabilities defined in Eq. (3). The term $\mathrm{P}(S_{t-1}{=}j\ |\varphi_{t-1})$ is known as the filtered probability, which may be calculated using the following equation. Using the Bayes rule, we can rewrite $\mathrm{P}(S_{t-1}{=}j\ |\varphi_{t-1})$ as

$$\begin{aligned} \mathrm{P}(S_{t-1} = j|\varphi_{t-1}) &= \mathrm{P}(S_{t-1} = j\ |d(\text{ Spread}_{t-1}), \varphi_{t-2}) \\ &= \frac{f(S_{t-1} = \mathrm{j}, d(\text{Spread}_t)\ |\ \varphi_{t-2})}{f(d(\text{Spread}_t)\ |\ \varphi_{t-2})} \\ &= \frac{f(d(\text{Spread}_{t-1})|S_{t-1} = \mathrm{j}, \varphi_{t-2})P(S_{t-1} = \mathrm{j}|\ \varphi_{t-2})}{\sum_{j=\text{high}}^{\text{low}} f(d(\text{Spread}_{t-1})|S_{t-1} = \mathrm{j}, \varphi_{t-2})P(S_{t-1} = \mathrm{j}|\ \varphi_{t-2})}, j = \text{high}, \text{low} \end{aligned} \tag{8}$$

Thus far, we have provided the most important estimating formulas, Eqs. (4)–(8), for this model. Thereafter, given an initial value of the filtered probability, the filtered probability and the prediction probability are computed and iterated by Eqs. (7) and (8). Finally, we can compute the logarithmic likelihood function of each state, and the log-likelihood function defined in Eq. (4) can be obtained.

3. Arbitrage rule

3.1. Pair formation and trading period

Every pair in the pairs-trading strategy has two periods: the formation period and the trading period. In the formation period, which is set at 1 year, the model parameters of this study are estimated by moving windows (1-day move). Next, we choose the smallest sum of the squared differences (SSD) of pairs to form a portfolio by calculating the SSD in the normalized prices of each pair over 1 year, and we rank the pairs of stocks from smallest to largest. For example, the two stocks with the smallest SSD pair among the 20 small SSD pairs are called the "first pair of TOP 20," and the next two stocks are called the "second pair of TOP 20." We repeat the strategy every month during our sample by overlapping. We refer to the two stocks in a pair as stock A and stock B.

$$\begin{aligned} \text{normalized price } \mathrm{A_{i,t}} &= \prod_{\tau=1}^{t}\left(1 + \mathrm{r}_\tau^{\mathrm{A_i}}\right) \\ \text{normalized price } \mathrm{B_{i,t}} &= \prod_{\tau=1}^{t}\left(1 + \mathrm{r}_\tau^{\mathrm{B_i}}\right) \\ \mathrm{SSD} = \sum\nolimits_{t=1}^{n} &\left(\textit{normalized price } A_{i,t} - \text{normalized price } B_{i,t}\right)^2, \end{aligned} \tag{15}$$

where $r_\tau^{A_i}$ is the normalized stock price's daily return on each day of the ith stock A of the portfolio at time t, and n is the number days of the formation period.

In the trading period, we trade the portfolio under three trading rules and comply with the trading rules by moving 1 day in the subsequent 1-, 2-, 3-, 6-, and 12-month periods.

3.2. Arbitrage rule

3.2.1. Trading rule I: stochastic spread model with regime switching

Trading rule I is adapted to this primary model that contains the Vasicek model and the Markov regime-switching model. This study uses the Vasicek model to capture the mean reversion of spreads. We estimate the Vasicek model with the Markov regime-switching model to detect the phases of imbalances in which the spreads have two different means and variances, such as in Fig. 1. The paired stocks, Consolidated Edison Inc. and Southern Co., are the pair with the smallest SSD during the formation period from 1/3/2006 to 12/29/2006. The shaded area is the low mean state regime, and the unshaded area is the high mean state regime. We apply a Markov regime-switching model with switching means and switching variances to develop the trade rules as follows:

If the predicted state is the low mean regime,

$$Z_t = \begin{cases} -1, \text{if Spread} \geq \widetilde{\kappa_{\text{low}}}\left(\widetilde{\theta_{\text{low}}} + \text{Spread}_{-1}\right) + \delta \times \widetilde{\sigma_{\text{low}}} \\ +1, \text{if} \leq \widetilde{\kappa_{\text{low}}}\left(\widetilde{\theta_{\text{low}}} + \text{Spread}_{-1}\right) - \delta \times \widetilde{\sigma_{\text{low}}} \end{cases} \tag{16}$$

If the predicted state is the high mean regime,

$$Z_t = \begin{cases} -1, \text{if Spread} \geq \widetilde{\kappa_{\text{high}}}\left(\widetilde{\theta_{\text{high}}} + \text{Spread}_{-1}\right) + \delta \times \widetilde{\sigma_{\text{high}}} \\ +1, \text{if} \leq \widetilde{\kappa_{\text{high}}}\left(\widetilde{\theta_{\text{high}}} + \text{Spread}_{-1}\right) - \delta \times \widetilde{\sigma_{\text{high}}} \end{cases} \tag{17}$$

Otherwise, $Z_t = 0$, and the critical value δ is set at 1.96, according to the normal distribution of the residual at the 5% significance level, where $\widetilde{\sigma_{\text{low}}}$ is the variance of the innovation in the low state, $\widetilde{\kappa_{\text{low}}}$ is the speed of reversion in the low state, $\widetilde{\theta_{\text{low}}}$ is the long-term mean level in the low state, $\widetilde{\sigma_{\text{high}}}$ is the variance of the innovation in the high state, $\widetilde{\kappa_{\text{high}}}$ is the speed of reversion in the high state, and $\widetilde{\theta_{\text{high}}}$ is the long-term mean level in the high state.

3.2.2. Trading rule II: stochastic spread model

Trading rule II is adapted for the Vasicek model for trading, and the following parameters are estimated by the Vasicek model:

$$Z_t = \begin{cases} -1, & \text{if Spread}_t \geq k(\theta + \text{Spread}_{t-1}) + \delta \times \sigma \\ +1, & \text{if Spread}_t \leq k(\theta + \text{Spread}_{t-1}) - \delta \times \sigma \end{cases} \tag{18}$$

Otherwise, $Z_t = 0$. δ is set as 1.96.σ is the variance of the innovation, κ is the speed of reversion, and θ is the long-term mean level.

3.2.3. Trading rule III: distance method

Trading rule III is proposed by Gatev et al. (2006). The position will open when the spread of the paired stocks is larger than 1.96 historical standard deviations.

$$Z_t = \begin{cases} -1, & \text{if Spread}_t \geq \mu + \delta \times \sigma \\ +1, & \text{if Spread}_t \leq \mu - \delta \times \sigma \end{cases} \tag{19}$$

Otherwise, $Z_t = 0$. δ is set as 1.96.μ and σ are the average and the standard deviation of past stock spreads during the formation period, respectively.

We will short-sell stock A (the overvalued stock) and buy stock B long (the undervalued stock) in a pair when the observed spread is larger than the predicted value that was estimated during the pair-formation period of this model ($Z_t = -1$). By contrast, we will short-sell stock B and buy stock A long in a pair when the spread is smaller than the predicted value ($Z_t = +1$). Positions are unwound to make a profit when the spread reverts. If the positions do not clear at the end of the trading period, we will clear and calculate the return as of the last day of the trading period.

3.3. Excess return computation

In accordance with Gatev et al. (2006), the performance return during the trading period is computed by way of reinvestment.

$$\begin{aligned} R_{p,t} &= \frac{\sum_{i=1}^{n} W_{\text{i,t}} R_{\text{i,t}}}{\sum_{i=1}^{n} W_{\text{i,t}}}, \\ W_{\text{i,t}} &= \left(1 + R_{i,1}\right) \times \left(1 + R_{i,2}\right) \times \ldots \times \left(1 + R_{\text{i},t\text{-}1}\right). \end{aligned} \tag{20}$$

In Eq. (20), R, W, and n are the returns, weight, and number of pairs of the portfolios, respectively. We trade the portfolios by overlapping to reduce the influence of choosing the starting point of the formation period of the pairs-trading returns. To isolate

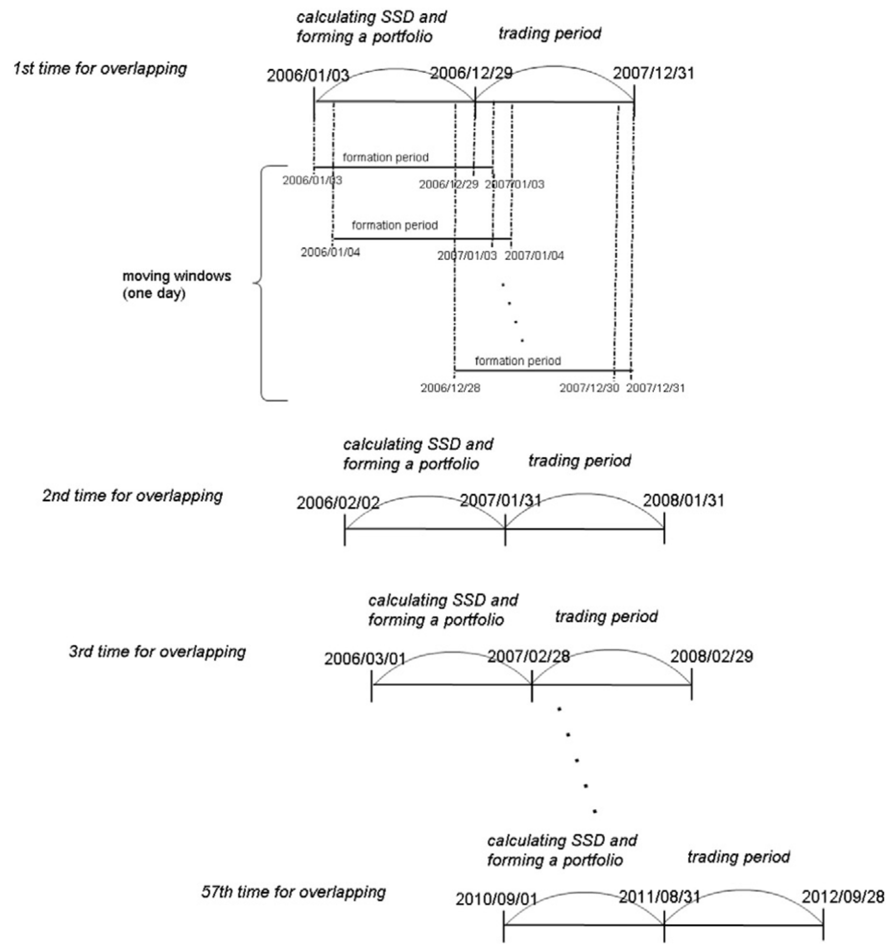

Fig. 2. Overlapping.

Table 1
Parameter estimates for trading rule I and trading rule II.

Transition probabilities	Trading rule I			Trading rule II	
			k	0.0452	(0.01)*
$P_{\mathrm{low,low}}$	0.909	(0.439)*	θ	8.3635	(0.14)*
$P_{\mathrm{low,high}}$	0.091	(0.35)	σ	0.0102	(0.00)*
$P_{\mathrm{high},low}$	0.919	(0.24)*			
$P_{high,\mathrm{high}}$	0.081	(0.25)			
Low regime					
$\widetilde{\kappa_{\mathrm{low}}}$	0.0920	(0.38)			
$\widetilde{\theta_{\mathrm{low}}}$	8.3853	(4.20)*			
$\widetilde{\sigma_{\mathrm{low}}}$	0.0183	(0.03)			
High regime					
$\widetilde{\kappa_{\mathrm{high}}}$	−0.0001	(0.00)			
$\widetilde{\theta_{\mathrm{high}}}$	15.9263	(1719.66)			
$\widetilde{\sigma_{\mathrm{high}}}$	0.0014	(0.00)			
Log likelihood	660.35			437.75	
LR test	445.20				
p-value	0.00				
Critical value	3.84				

The parameters are reported and estimated by maximum likelihood estimation in Section 2.2. The numerals in parentheses are standard errors. The likelihood ratio test (LR test) is used to compare the fit of trading rule I and trading rule II.

the potential effects of different industries, we form different industrial portfolios in addition to the full sample. Therefore, we classify these stocks by sector, require that the two stocks belong to the same industry, and compare them with the non-classified pairs. Standard and Poor's defines these sectors as the Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Telecommunications Services, and Utilities sectors.

In addition to moving windows, the pairs are also traded by overlapping to reduce the influence of different pairs that are selected in different formation periods with excessively high or low returns. Fig. 2 shows the portfolio of the TOP 1 portfolio traded for 1 year by overlapping.

4. Empirical results

4.1. Data description

This paper uses the daily S&P 500 Index from January 1, 2006, to September 28, 2012, to conduct an empirical study of the performance of the pairs-trading strategy using our model. Thus, for the sample period, we choose to analyze and compare the

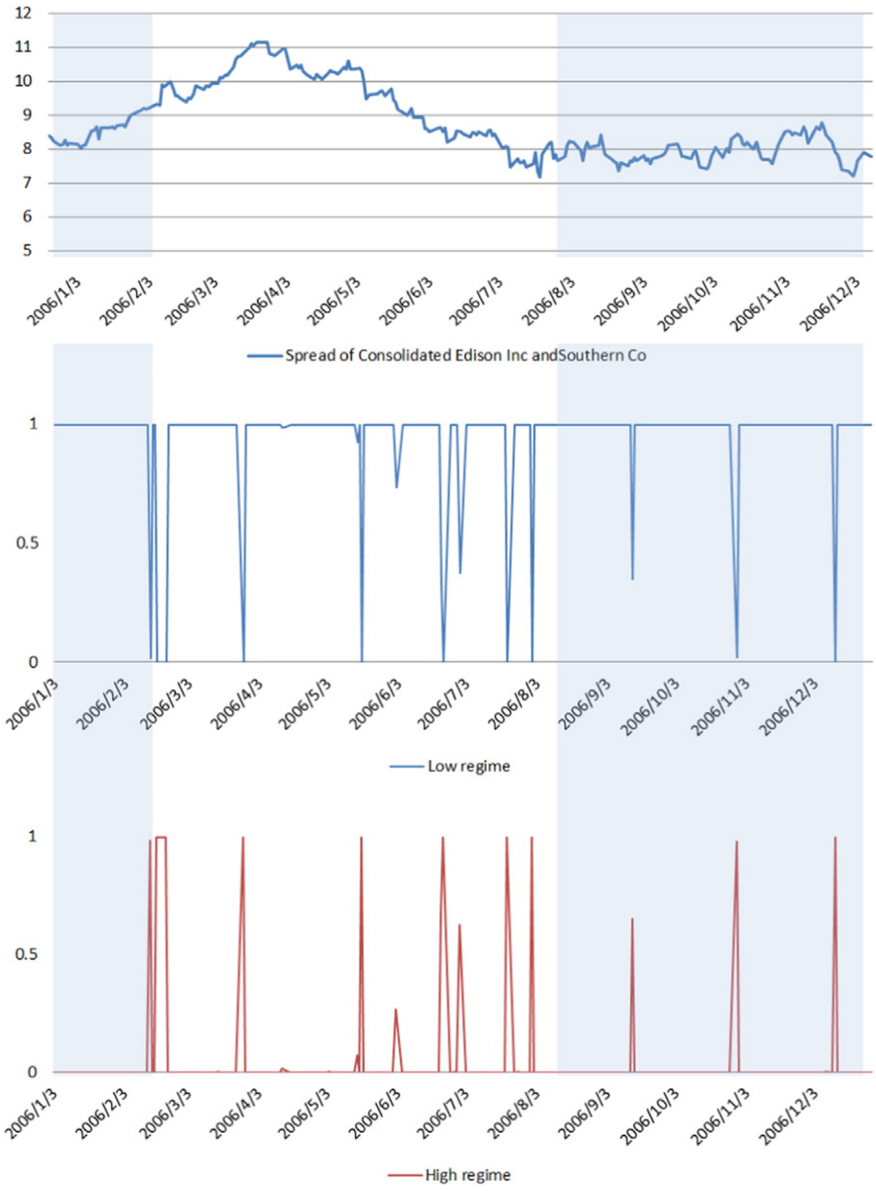

Fig. 3. The smoothed probabilities. The smoothed probabilities of the TOP 1 portfolio of unrestricted pairs (Consolidated Edison Inc. and Southern Co.) from 2006/1/3 to 2006/12/29.

Table 2
TOP 1 portfolio. Average excess returns of unrestricted pairs.

Trading Period (months)	1			2			3			6			12		
Trading rule	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
Average excess return (%)	0.34	−0.60	0.02	0.12	−0.13	−0.11	−0.03	−0.08	−0.15	0.02	−0.09	0.00	−0.04	−0.01	−0.06
Standard error	0.20	0.14	0.17	0.09	0.06	0.11	0.07	0.05	0.10	0.03	0.07	0.09	0.03	0.02	0.05
T-statistic	1.73	−4.21	0.10	1.31	−2.14	−1.07	−0.47	−1.68	−1.53	0.53	−1.32	−0.05	−1.05	−0.73	−1.15
Sharpe ratio	0.21	−0.51	0.01	0.16	−0.26	−0.13	−0.06	−0.21	−0.19	0.07	−0.17	−0.01	−0.14	−0.10	−0.15
Standard deviation	1.62	1.18	1.40	0.73	0.50	0.87	0.60	0.40	0.77	0.24	0.56	0.75	0.26	0.14	0.37
Kurtosis	3.11	7.07	4.96	1.22	2.40	3.90	2.46	5.10	0.50	3.30	46.61	12.17	14.73	4.99	1.47
Skewness	1.40	−2.11	0.07	0.73	−0.69	−1.35	0.28	−1.70	−0.81	1.31	−6.44	−2.51	−2.99	−0.06	−0.43
Minimum	−2.91	−6.05	−5.35	−1.65	−1.98	−3.67	−1.78	−1.82	−2.36	−0.49	−4.17	−3.98	−1.43	−0.52	−1.09
Maximum	6.05	1.31	4.46	2.33	1.08	1.43	1.72	0.56	1.27	0.91	0.52	1.31	0.48	0.51	0.91
Range	8.97	7.36	9.81	3.98	3.06	5.10	3.50	2.38	3.63	1.40	4.69	5.29	1.91	1.03	2.01
Observations	68	68	68	67	67	67	66	66	66	63	63	63	57	57	57

Paired-samples *t* test

	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
I		3.8785	1.2434		2.2946	1.6618		0.5424	0.9191		1.4240	0.2048		−0.5805	0.3170
		(0.00)*	(0.22)		(0.02)*	(0.10)		(0.59)	(0.36)		(0.16)	(0.84)		(0.56)	(0.75)
II			−2.7948			−0.1362			0.5842			−0.7573			0.8067
			(0.01)*			(0.89)			(0.56)			(0.45)			(0.42)

Summary statistics of the monthly excess returns of the TOP 1 portfolio of unrestricted pairs. Trading rule I is adapted to this main model, which incorporates the Vasicek model and the Markov regime-switching model. Trading rule II is adapted to the Vasicek model. Trading rule III is proposed by Gatev et al. (2006). The position will open when the spread of the paired stocks is larger than 1.96 historical standard deviations. The paired-samples *t* test is reported. In the panel "paired-samples *t* test," the first numeral is the *t*-statistic and the second numeral is the *p*-value in the paired-samples *t* test.

performance of the pairs-trading strategy during the global financial crisis of 2007–2009 and during the period following the global financial crisis.

We follow the same criterion of pairs trading as described by Gatev et al. (2006). In each study, S&P 500 stocks are paired by using the SSD criterion; we have 124,750 pairs in total. This study uses 12 months of S&P 500 Index daily price data to form and trade pairs over the subsequent 1-, 2-, 3-, 6-, and 12-month periods.

4.2. Parameter estimation

Table 1 represents the estimation parameter results of the TOP 1 portfolio of unrestricted pairs (Consolidated Edison Inc. and Southern Co.) under trading rule I and trading rule II during the first overlapping period. Trading rule I is adapted to this primary model, which incorporates the Vasicek model and the Markov regime-switching model. Trading rule II is adapted to the Vasicek model. The expected length of stay is $\frac{1}{1-P_{\text{low,low}}} = 10.989$ days in the low mean state regime and $\frac{1}{1-P_{\text{high,high}}} = 1.088$ days in the high mean state regime. All parameters of trading rule II are significant variables, and $\widetilde{\theta}_{\text{low}}$ of trading rule I is significant at the 5% significance level. The results indicate that the spread has two different states and that trading rule I has a good fit (445.20 > 3.84) according to the likelihood ratio test (LR Test), which is used to compare the fits of trading rule I and trading rule II.

The smoothed probability is estimated by using all of the information from the period. Fig. 3 demonstrates that the expected length of stay in the low mean state regime, which is represented by the shaded area, is longer than the stay in the high mean state regime; the results are identical to the results shown in Table 1.

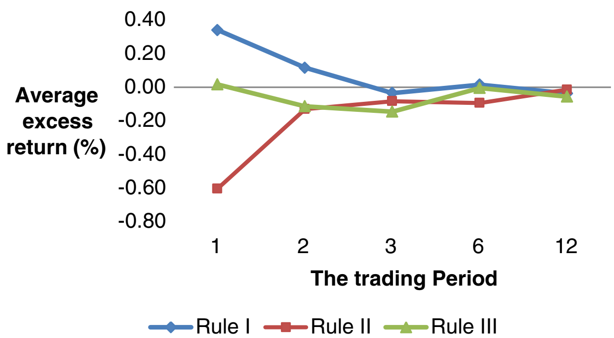

Fig. 4. TOP 1 portfolio. Average excess returns of unrestricted pairs.

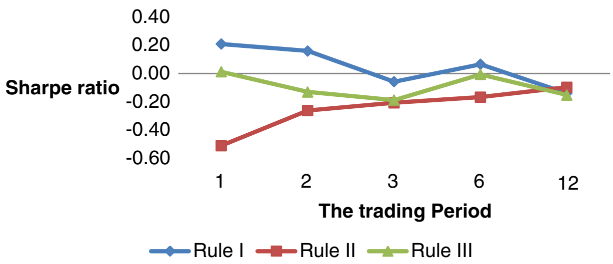

Fig. 5. TOP 1 portfolio. Sharpe ratio of unrestricted pairs.

4.3. Profits and risk of trading rules

The results shown in Table 2 and Figs. 4 and 5 are the returns and Sharpe ratio of the TOP 1 portfolio of unrestricted pairs when transaction costs are considered. Table 2 shows that trading rule I's original returns for trading periods of 1, 2, and 6 months are positive, and trading rule I has the best performance of the three trading rules. Table 2 and Fig. 5 show that the Sharpe ratios of trading rule I are positive for trading periods of 1, 2 and 6 months. Trading rule III is the least effective of the three rules from a risk-management perspective. Because the standard deviations of trading rule III's returns are the highest for trading periods of 2, 3, 6 and 12 months, the minimum returns are the lowest for trading periods of 2, 3, and 6 months, and the maximum returns are the highest for trading periods of 6 and 12 months, i.e., the range of returns is the largest for trading periods of 1, 2, 3, 6 and 12 months. These results show that the risk of trading rule III is the highest among the three trading rules and that the risk of trading rule I is the lowest among the three rules.

The results in Table 2 show that according to the paired-samples *t* test, trading rule I is better than trading rule II at a 5% significance level for trading periods of 1 and 3 months, and trading rule I is better than trading rule III at a 5% significance level for a trading period of 1 month. In Fig. 4, the observed trends of the returns are that a shorter trading period will yield a higher performance for trading rule I; in addition, the returns of trading rule II improve as the trading period grows. Fig. 4 also shows that the returns of trading rule I decrease as the trading period increases.

Fig. 6 shows the TOP 1 portfolio overlapped monthly returns of unrestricted pairs for a trading period of 1 month. We observe that most of the returns of trading rule II are negative, and rule II contains the largest negative returns. The remarkable performance of trading rule I is repeated during the global financial crisis of 2008 to 2009, with the portfolio reporting a mean monthly excess return of 4.56%; this is decidedly superior to the performance of the preceding period (2006–2007) and the subsequent period (2010–2012). Associated with this higher level of returns, however, is an elevated volatility in monthly performance. The Sharpe ratio is 0.32 during the bear market (2008–2009); however, it remains superior to those of the adjacent periods. This result also confirms the empirical evidence of Do and Faff (2010). Furthermore, the defensive characteristic of trading rule I also enables it to perform better than the other two rules during the global financial crisis. After the financial crisis (2010–2012), although trading rule I earns almost negative returns, it remains superior to trading rule II and trading rule III. The possible explanation of negative returns may originate from the worsening arbitrage risks found by Do and Faff (2010). That is, the increased rates and magnitude of divergence caused by worsening arbitrage risks leads to the declining trend in the profitability of pairs trading.

4.4. Pair portfolios and trading period

Tables 3 and 4 show the overlapped TOP 3 portfolios' and overlapped TOP 5 portfolios' monthly returns of unrestricted pairs, respectively. Similar to the results for TOP 1 portfolios, we find that the risk of rule I is the smallest among the three rules, whereas rule III is not the best trading rule from a risk-management perspective. Furthermore, we compare the average excess returns

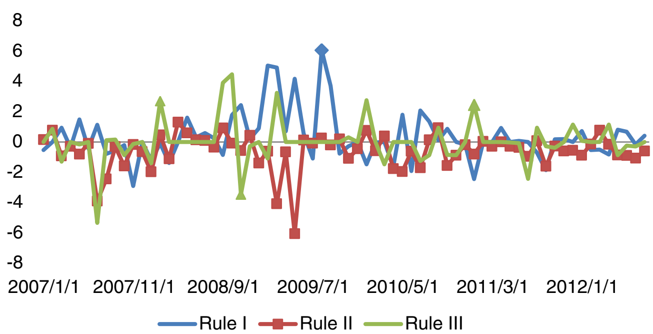

Fig. 6. TOP 1 portfolio. Monthly excess return of unrestricted pairs for a trading period of 1 month.

Table 3
TOP 3 portfolios. Average excess returns of unrestricted pairs.

Trading period (months)	1			2			3			6			12		
Trading rule	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
Average excess return (%)	0.21	−0.37	0.01	−0.09	−0.16	−0.09	0.00	−0.08	−0.06	0.02	−0.04	−0.12	0.00	−0.03	−0.09
Standard error	0.11	0.10	0.10	0.11	0.05	0.07	0.04	0.04	0.06	0.02	0.02	0.06	0.02	0.01	0.04
T-statistic	2.01	−3.73	0.10	−0.81	−2.94	−1.40	0.08	−1.95	−1.05	0.98	−1.85	−2.13	−0.01	−3.02	−2.28
Sharpe ratio	0.24	−0.45	0.01	−0.10	−0.36	−0.17	0.01	−0.24	−0.13	0.12	−0.23	−0.27	0.00	−0.40	−0.30
Standard deviation	0.87	0.83	0.83	0.89	0.44	0.55	0.33	0.32	0.49	0.17	0.19	0.44	0.13	0.08	0.31
Kurtosis	3.07	8.98	3.09	39.91	23.52	5.28	1.88	2.54	1.27	−0.09	15.02	2.61	9.76	1.11	−0.12
Skewness	0.83	−1.92	−0.05	−5.53	−3.93	−1.61	−0.30	0.34	−0.32	0.39	−2.69	−1.16	−2.07	−0.13	−0.53
Minimum	−1.76	−4.48	−2.93	−6.43	−2.90	−2.55	−1.19	−0.84	−1.67	−0.34	−1.10	−1.61	−0.62	−0.24	−0.95
Maximum	3.67	1.48	2.52	1.52	0.52	0.78	0.78	1.11	1.29	0.43	0.35	0.75	0.26	0.19	0.44
Range	5.43	5.96	5.45	7.94	3.42	3.33	1.97	1.95	2.96	0.78	1.45	2.36	0.88	0.43	1.39
Observations	68	68	68	67	67	67	66	66	66	63	63	63	57	57	57

Paired-samples *t* test

	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
I		4.0241	1.3856		0.5707	0.0415		1.4019	0.9204		2.0337	2.3379		1.5418	2.1036
		(0.00)*	(0.17)		(0.57)	(0.97)		(0.16)	(0.36)		(0.04)	(0.02)*		(0.13)	(0.04)
II			−2.7001			−0.7475			−0.1580			1.2337			1.4898
			(0.01)*			(0.46)			(0.87)			(0.22)			(0.14)

Summary statistics of the monthly excess returns of the TOP 3 portfolios of unrestricted pairs. Trading rule I is adapted to this main model, which contains the Vasicek model and the Markov regime-switching model. Trading rule II is adapted to the Vasicek model. Trading rule III is proposed by Gatev et al. (2006). The position will open when the spread of the paired stocks is larger than 1.96 historical standard deviations. The paired-samples *t* test is reported. In the panel "paired-samples *t* test," the first numeral is the *t*-statistic, and the second numeral is the *p*-value in the paired-samples *t* test.

of the trading rules by different pair portfolios and trading periods, as shown in Table 2 to Table 4. We find that most returns for rule I are positive and that the returns for rules I and III are decreasing as the trading period increases, whereas the returns for rule II are increasing as the trading period increases. Moreover, we find that the largest Sharpe ratio is 0.34 with the TOP 1 portfolio for a trading period of 1 month for rule I, and the lowest Sharpe ratio is −0.51 with the TOP 1 portfolio for a trading period of 1 month for rule II. Fig. 7 exhibits the average excess returns for rule I by different pair portfolios and trading periods. Notably, the overlapped TOP 1 portfolios' monthly returns of unrestricted pairs for a trading period of 1 month yield the greatest profits. This result implies that shorter trading periods and fewer pairs produce better trading performance for rule I.

Table 4
TOP 5 portfolios. Average excess returns of unrestricted pairs.

Trading period (months)	1			2			3			6			12		
Trading rule	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
Average excess return (%)	0.18	−0.35	0.01	−0.04	−0.16	−0.10	0.02	−0.06	−0.13	0.02	−0.06	−0.13	0.00	−0.03	−0.14
Standard error	0.07	0.10	0.08	0.07	0.04	0.06	0.04	0.03	0.05	0.01	0.02	0.04	0.01	0.01	0.04
T-statistic	2.47	−3.62	0.14	−0.53	−4.02	−1.77	0.53	−2.18	−2.33	1.24	−3.20	−2.89	0.39	−2.98	−3.55
Sharpe ratio	0.30	−0.44	0.02	−0.06	−0.49	−0.22	0.07	−0.27	−0.29	0.16	−0.40	−0.36	0.05	−0.39	−0.47
Standard deviation	0.59	0.79	0.64	0.57	0.33	0.48	0.30	0.24	0.44	0.12	0.14	0.35	0.09	0.07	0.29
Kurtosis	−0.06	16.31	3.19	28.57	9.46	1.27	3.84	2.79	0.11	0.58	5.31	1.23	18.60	1.25	2.09
Skewness	0.28	−3.09	−0.75	−4.42	−2.20	−0.78	−0.16	0.00	−0.18	0.68	−1.72	−0.80	−3.31	−0.19	−1.34
Minimum	−1.23	−4.94	−2.55	−3.79	−1.83	−1.44	−1.00	−0.77	−1.31	−0.20	−0.67	−1.28	−0.50	−0.25	−1.08
Maximum	1.63	0.97	1.56	0.80	0.37	0.97	1.05	0.69	0.78	0.39	0.18	0.50	0.13	0.16	0.32
Range	2.86	5.90	4.11	4.58	2.20	2.41	2.05	1.46	2.09	0.59	0.85	1.79	0.63	0.41	1.40
Observations	68	68	68	67	67	67	65	65	65	63	63	63	57	57	57

Paired-samples *t* test

	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
I		4.3801	1.5652		1.5423	0.7271		1.7722	2.2248		3.2523	3.1307		2.1868	3.5084
		(0.00)*	(0.12)		(0.13)	(0.47)		(0.08)	(0.03)		(0.00)*	(0.00)*		(0.03)	(0.00)*
II			−2.8992			−0.8194			1.0071			1.4670			2.7165
			(0.00)*			(0.41)			(0.32)			(0.14)			(0.01)*

Summary statistics of the monthly excess returns on the TOP 5 portfolio of unrestricted pairs. Trading rule I is adapted to this main model, which contains the Vasicek model and the Markov regime-switching model. Trading rule II is adapted to the Vasicek model. Trading rule III is proposed by Gatev et al. (2006). The position will open when the spread of paired stocks is larger than 1.96 historical standard deviations. The paired-samples *t* test is reported. In the panel "paired-samples *t* test," the first numeral is the *t*-statistic, and the second numeral is the *p*-value in the paired-samples *t* test.

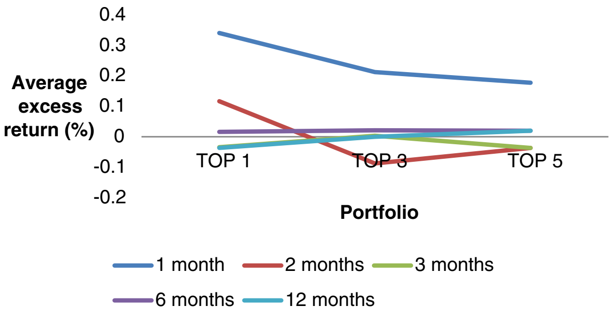

Fig. 7. Average excess returns of rule I by different pair portfolios and trading periods.

4.5. Pairs trading by industry group

To clarify the potential effects of pairs trading from different industries, industrial portfolios have been created in addition to the full sample that is used. Therefore, we classify these stocks by sector; we require that the two stocks belong to the same industry, and we compare them with the unrestricted pairs. Standard and Poor's defines these sectors as the Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Telecommunications Services, and Utilities sectors. Fig. 8 shows TOP 1 portfolio excess returns of restricted and unrestricted pairs. This figure shows that the TOP 1 portfolio excess returns of the unrestricted pairs is the highest for trading periods of 1 and 2 months. The returns of the Information Technology sector are the highest for trading periods of 1 and 2 months. In the Utilities sector, rule I has positive returns for trading periods of 1, 2, 3, 6 and 12 months. However, the other sectors do not necessarily earn positive or negative returns. Overall, the performance of the utilities sector is relatively better than those of other industry sectors because this sector has positive returns in any trading period. These results are practically identical to those of Gatev et al. (2006) and Do and Faff (2010).

5. Conclusions

The purpose of this study is to develop a pairs-trading model that combines the Markov regime-switching and Vasicek models with a mean-reverting strategy and to compare the model with previous models, including the distance method and the Markov regime-switching model. Trading rule I is adapted to this main model, which contains the Vasicek model and the Markov regime-switching model. This study uses the Vasicek model to capture the mean reversion of spreads, and it uses the Markov regime-

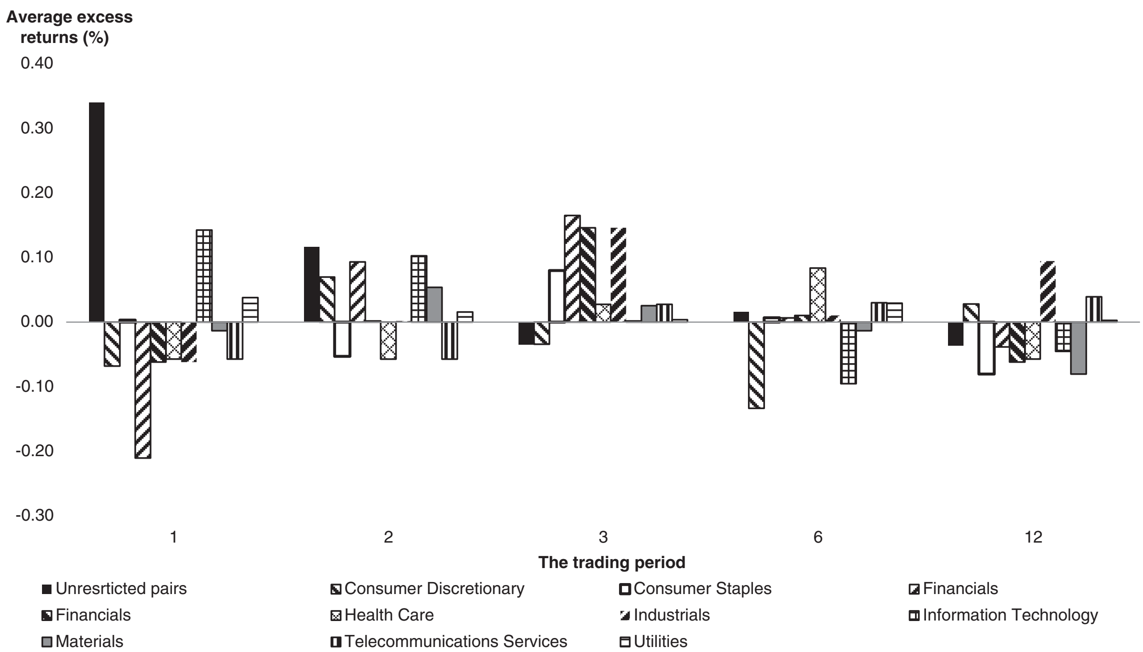

Fig. 8. TOP1 portfolio. Excess returns of restricted and unrestricted pairs.

switching model to detect the phases of imbalances in which the spreads have two different means and variances. Trading rule II is adapted to the Vasicek model, and rule III is proposed by Gatev et al. (2006).

We follow the same criterion of pairs trading described by Gatev et al. (2006). This study uses 12 months of S&P 500 index daily price data to form pairs and trades them in the subsequent 1, 2, 3, 6, and 12 months. The empirical results show that rule I has the best performance in a simple portfolio (TOP 1) and that the shorter the trading period is, the higher the performance is. This result implies that shorter trading periods and fewer pairs produce better trading performance for rule I. In particular, the Sharpe ratio of returns is positive for a trading period of 1 month. This study found that rule III is the least effective of the three rules from a risk management perspective. Because the standard deviations are high, investors must undertake a higher risk. Therefore, rule I is practical for investors who attempt to generate satisfactory profits by pairs trading for short-term trading. Rule II and rule III are suitable for investors who prefer to take on risk and for those who attempt to hold long-term positions and obtain a better return.

Our results also provide supportive evidence of the results obtained by Do and Faff (2010) regarding a continuing downward trend in the profitability of pairs trading; the reason for this conclusion may be the increased rates and magnitude of the divergence caused by worsening arbitrage risks. Furthermore, we also find that trading rule I performs strongly during the recent global financial crisis period and even has a performance superior to those of the adjacent periods. This result supports the empirical evidence of Do and Faff (2010). In addition, we show that trading rule I performs better than the other two rules during the global financial crisis.

Future research should aim to find the suitable critical value of the trading rule and consider alternative portfolio formations that include intra-industry matched pairs from different industries instead of top pairs within a single industry.

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