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Momentum and Contrarian Stock-Market Indices

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Abstract:
We propose a new class of investable momentum and contrarian stock-market indices
that partition a benchmark index, such as the Russell 1000. Our momentum indices
overweight stocks that have recently outperformed, while our contrarian indices under-
weight these same stocks. Our index construction methodology is extremely flexible, and
allows the index provider to trade-off the distinctiveness of the momentum/contrarian
strategies with portfolio turnover. Momentum investment styles in particular typically
entail a high level of turnover, and hence high associated transaction costs. The creation
of momentum and contrarian indices and exchange traded funds (ETFs) based on our
methodology would allow investors to access these styles at lower cost than is currently
possible. Our indices also provide performance benchmarks for momentum/contrarian
investment managers, and good proxies for a momentum factor. Over the period 1995-
2007 we find that short term momentum and long term contrarian indices outperform
the reference Russell 1000 index. We also document the changing interaction between
the momentum/contrarian and value/growth styles. (*JEL* C43, G11, G23)

KEY WORDS: Momentum index; Contrarian index; Performance measurement; Turnover;
Momentum factor; Behavioral finance

1. Introduction

In recent years, the effectiveness of momentum and contrarian investment strategies has been the subject of much discussion. A contrarian investment strategy earns risk-adjusted excess returns when investors overreact to news, while a momentum strategy earns excess risk-adjusted returns when investors underreact. DeBondt and Thaler (1985, 1987) find evidence of overreaction over periods of a few years, while Jegadeesh and Titman (1993, 2001) find evidence of underreaction over periods of a few months. These findings are consistent with the experimental results of Kahneman and Tversky (1982), who observed systematic violations of Bayes Rule amongst subjects when revising their beliefs in the presence of new information [see also Barberis, Shleifer and Vishny (1998)].

When attempting to explain stock returns and investment manager performance, researchers generally turn to the 3 factor Fama and French (1992, 1993, 1996) or 4 factor Carhart (1997) model. This is because a significant body of research evidence exists demonstrating that the market return, firm size, valuation and momentum are important drivers of stock returns. Each of these factors, excluding momentum, has a standard market proxy in the form of a stock market index (e.g., a market cap weighted index, large and small cap indices, value and growth indices). The missing ingredient in the market is a momentum index.

This is particularly important given that large numbers of investors follow contrarian and momentum strategies [see for example Goetzmann and Massa (2002)]. The same is true of mutual funds. Grinblatt, Titman and Wermers (1995) find that 77 percent of the 155 mutual funds in their sample engage in momentum investing. Similarly, Menkhoff and Schmidt (2005) use survey data to show that momentum and contrarian strategies are common among German fund managers. Given the extensive use of these strategies, it is surprising that no explicitly momentum and contrarian indices are currently computed by any of the major index providers.

There is therefore a clear need for momentum and contrarian indices. Such indices would provide useful performance benchmarks for active managers following these

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strategies. Active managers should not be unduly rewarded or punished for performance attributable to their innate factor tilts [Mulvey and Kim (2008) find that the best performing managers tend to have a momentum tilt]. The development of momentum and contrarian indices could therefore enhance the performance evaluation of active managers following these styles. Furthermore, these indices could facilitate the development of momentum and contrarian exchange traded funds (ETFs), providing investors with a lower cost option for gaining access to the momentum and contrarian modes of investing (we return to this issue shortly). Given the findings of deBondt-Thaler and Jegadeesh-Titman, for example, there might be particular demand for momentum funds with horizons of less than a year and contrarian funds with longer horizons.

The vast majority of stock-market indices are market-cap weighted. This is because market-cap weighting is conceptually straightforward, consistent with a passive buy-and-hold strategy, has relatively low transaction costs (due to low turnover rates and the focus on larger cap and hence more liquid stocks), and when followed by all investors does not violate market clearing [see Bailey (1992), Siegel (2003) and Arnott, Hsu and Moore (2005)]. In recent years there has been a proliferation of stock-market indices, for example value and growth indices, small and mid-cap indices, and sector (e.g., banks, health, information technology) indices. Small, mid-cap and sector indices are usually also market-cap weighted over their chosen domain, while value and growth indices are usually constructed by splitting a market-cap-weighted portfolio into two more or less equal halves (in terms of market capitalization).

Momentum indices are conspicuous in their absence. The only example of a contrarian index currently available is an equal-weighted index that is rebalanced periodically. The act of rebalancing back to equal weights requires buying past losers and selling past winners. In recent years, equal-weighted versions of the S&P 500, Dow-Jones Wilshire 5000 and Nasdaq 100 indices have appeared [see for example Standard and Poor's and Rydex Global Advisors (2003)]. Value Line also produces an equal-weighted index (VLA) defined on about 1620 stocks. It should be noted, however, that these indices are all advertised as equal-weighted indices. They are contrarian by

accident rather than by design.

One reason for the lack of momentum and contrarian indices in the public domain is the way momentum portfolios are typically constructed. The literature generally mimics the momentum factor by constructing portfolios that go long the best performing stocks and short the worst performers [see for example Jegadeesh and Titman (1993)]. Such investment strategies are not practical for investors since they require shorting of assets, which many investors are either not able or not permitted to do. Also, by excluding most stocks in a benchmark, such indices will tend to have high turnover (and hence high transaction costs) and low diversification. Further, they do not provide a measure of momentum relative to the market return (which is available currently in the value and growth dimension but not in the momentum and contrarian dimension) and their high levels of concentration make them less able to adequately evaluate the performance of investment managers. See Bailey (1992) for a list of properties that a performance benchmark should ideally possess.

In this article we propose a new class of flexible momentum/contrarian indices that does not require shorting and allow the index provider to trade-off turnover against the distinctiveness of the momentum/contrarian strategy. Like many value/growth indices, our momentum/contrarian indices are benchmarked to existing indices – usually of the market-cap-weighted variety although this is not necessary. For example, momentum and contrarian versions of equal-weighted indices or of fundamental indices [Arnott, Hsu and Moore (2005)] are also possible. Here we use the Russell 1000 index as our benchmark.

We calculate the performance, risk and turnover characteristics of our momentum and contrarian indices over a 12 year period. Our momentum and contrarian indices partition the benchmark index into momentum and contrarian portfolios that when combined yield the original index portfolio. This approach should be familiar to investors since value/growth indices are typically constructed in a similar way. Also, this approach has a distinct advantage over the traditional long minus short momentum factors in that it better approximates the actual behavior of active investment

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managers.

We begin by constructing a partition in the momentum/contrarian domain based on the past price performance of stocks relative to the benchmark index over a specified time horizon. We then propose a new and flexible approach for transforming this base partition that makes use of the beta function. By varying the parameter β in the beta function the index provider has considerable flexibility with regard to the design of the final partition actually used to construct the momentum and contrarian portfolios. This beta function transformation is a useful innovation in its own right, and represents an attractive method for constructing passive factor portfolios. Although here we focus on the price momentum factor, it could equally well be applied to other factors such as value and growth, earnings momentum, dividend yield or price acceleration (the rate of change of momentum).

During the "tech" boom we find a close affinity between momentum indices and growth indices, and between contrarian indices and value indices. This relationship is consistent with the findings of Assness (1997), but it reverses after 2001. Thereafter the momentum indices move more in line with value indices, and contrarian indices with growth indices. In the more recent history we find little relationship between momentum/contrarian and growth/value styles. These findings highlight the fact that momentum/contrarian strategies are not simply a proxy for growth and value, and represent unique investment strategies in their own right.

We also compute the turnover of our momentum and contrarian indices. Although momentum strategies inevitably lead to indices with higher turnover than the reference Russell 1000 index and corresponding value/growth indices, this turnover can be controlled to an acceptable level without sacrificing the distinctiveness of the momentum and contrarian styles. Furthermore, the more relevant comparisons of turnover for these types of indices are with traditional (long minus short) momentum factors and investment managers following momentum investment strategies. Our indices compare favorably on both counts, and hence could potentially allow investors to access the momentum style of investing at lower cost than is currently possible.

Momentum/contrarian ETFs based on our indices therefore are viable and could be of interest to investors aiming to tactically or strategically tilt towards or away from momentum at various horizons, or as a passive factor exposure in a multi-manager structure (e.g. if the fund has an unwanted bias away from momentum or if a manager wants to tilt towards it). These strategies are not currently implementable at low cost.

We explore the impact of varying the time horizon (formation and holdings periods) on the performance and turnover of momentum and contrarian indices. The time horizons considered range from six months to three years. Our findings are consistent with those of the behavioral finance literature. Momentum strategies tend to work well over shorter horizons, while contrarian strategies outperform over longer horizons. We also show how our momentum and contrarian indices can be used to construct proxies that correlate very closely with the momentum factor in a Carhart (1997) factor model.

The rest of the article is structured as follows. In the next section we describe our momentum/contrarian index methodology. Section 3 provides an empirical demonstration of our momentum/contrarian indices benchmarked to the Russell 1000 index over the period 1995-2007. Our main findings are summarized in the conclusion.

2. Constructing momentum/contrarian stock-market indices

We propose a class of momentum/contrarian indices that partitions a benchmark portfolio. That is, a momentum and contrarian allocation is allotted to each stock in the benchmark index, such that the allocations sum to one. For example, it could be decided that stock X is 80 percent momentum and 20 percent contrarian. These allocations are then used to construct momentum and contrarian index portfolios, which when combined exactly equal the portfolio underlying the benchmark index.

The momentum/contrarian allocations are constructed in two stages. In the first stage, the momentum and contrarian allocations for each stock are determined as follows:

Stage 1 Allocations:

$$\text{Momentum}: \quad \mu_{t,n}^M = \frac{p_{t,n}/p_{t-k,n}}{(p_{t,n}/p_{t-k,n}) + (I_t/I_{t-k})},$$

$$\text{Contrarian}: \quad \mu_{t,n}^C = \frac{I_t/I_{t-k}}{(p_{t,n}/p_{t-k,n}) + (I_t/I_{t-k})}, \qquad (1)$$

where $\mu_{t,n}^M$ and $\mu_{t,n}^C$ denote the fraction of total shares from the reference index of stock n allocated to the momentum and contrarian portfolios, $p_{t,n}$ denotes the price of stock n and I_t denotes the level of the benchmark index. The allocations $\mu_{t,n}^M$ and $\mu_{t,n}^C$ are functions of the price relatives $p_{t,n}/p_{t-k,n}$. When a stock n in the benchmark portfolio in period t is not listed in period $t-k$ (i.e., $p_{t-k,n}$ is not available), we simply set $\mu_{t,n}^M = \mu_{t,n}^C = 0.5$. That is, this stock is split equally between the momentum and contrarian portfolios.

The benchmark index I_t here excludes dividends. This is not a requirement. The stage 1 formulas could be reformulated to include dividend payments. Also, other performance measures such as earnings per share (EPS) growth could be used in place of price to construct a different factor portfolio/index that may be useful as a benchmark and investment vehicle for say an EPS growth strategy.

Both $\mu_{t,n}^M$ and $\mu_{t,n}^C$ are bounded between 0 and 1. Also, by construction, $\mu_{t,n}^M + \mu_{t,n}^C = 1$. When a stock n rises by the same proportion as the index itself (i.e., $p_{t,n}/p_{t-k,n} = I_t/I_{t-k}$), it follows that the stock is allocated equally between the momentum and contrarian portfolios (i.e. $\mu_{t,n}^M = \mu_{t,n}^C = 0.5$). When a stock rises by more than the reference index, $\mu_{t,n}^M > 0.5$ and $\mu_{t,n}^C < 0.5$.

A distinction must be drawn between the formation and holding periods of a momentum/contrarian strategy. The formation period is the period over which prices are observed to determine the momentum/contrarian allocations for each stock. The formation period here is from period $t-k$ to t, and hence the formation horizon is k periods. The holding period is the period over which a momentum/contrarian portfolio is held, i.e., it measures the frequency with which the index is rebalanced.

Rebalancing of the momentum/contrarian indices should not be confused with reconstitution of the benchmark index itself. For example, the Russell 1000 index is

reconstituted (i.e., stocks are added and deleted) on an annual basis. Using these definitions, note that equal-weighted, fundamental, value/growth and momentum/contrarian indices all need to be both rebalanced and reconstituted periodically, whereas market-cap weighted indices generally only require reconstitution.

To provide an easy way to distinguish the various momentum/contrarian indices presented in the empirical section, we use the notation (f,h), where f=formation period and h=holding period. For example, a (6,12) index would determine the momentum/contrarian allocations based on 6 months of price performance, and then hold the resulting portfolios for 12 months, at which time the process is repeated.

To give the index provider some flexibility with regard to the distinctiveness of the momentum/contrarian strategies, the stage 1 allocations are then transformed using the regularized incomplete beta function with its two parameters α and β set equal to each other.[1] The stage 2 transformed momentum and contrarian allocations $\theta_{t,n}^M$ and $\theta_{t,n}^C$ are determined as follows:[2]

Stage 2 Allocations

$$\theta_{t,n}^M = \frac{B_{\mu_{t,n}^M}(\beta)}{B(\beta)}, \tag{2}$$

where

$$B_{\mu_{t,n}^M}(\beta) = \int_0^{\mu_{t,n}^M} x^{\beta-1}(1-x)^{\beta-1}dx,$$

$$\theta_{t,n}^C = \frac{B_{\mu_{t,n}^C}(\beta)}{B(\beta)} = 1 - \theta_{t,n}^M,$$

and

$$B_{\mu_{t,n}^C}(\beta) = \int_0^{\mu_{t,n}^C} x^{\beta-1}(1-x)^{\beta-1}dx,$$

$$B(\beta) = \int_0^1 x^{\beta-1}(1-x)^{\beta-1}dx.$$

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[1]Tables of the beta function can be easily accessed via the internet, and are also available in most statistical software packages such as Matlab and Ox. The properties of the beta function are discussed at the following website: ≪http://functions.wolfram.com≫.

[2]The parameter β used in the Stage 2 allocations refers to the beta function only, and is unrelated to the β typically associated with the CAPM. Unless otherwise stated, throughout this paper β refers to the parameter from the beta function, which is selected by the index compiler to gain the desired trade-off between the turnover and distinctiveness of the momentum/contrarian indices.

The objective here is to provide the index provider with a means of varying the allocations in a way that preserves the ranking across stocks. For example, for stocks m and n, suppose $\mu_{t,m}^M > \mu_{t,n}^M$. This means that stock m is more momentum than stock n. Hence a greater proportion of the total holding of m is allocated to the momentum portfolio than is the case for n in period t. The transformation of allocations using the regularized incomplete beta function will preserve this ranking. That is, irrespective of the choice of β (as long as it is positive and finite), when in stage 1 $\mu_{t,m}^M > \mu_{t,n}^M$ it will also be the case for the stage 2 transformed allocations that $\theta_{t,m}^M > \theta_{t,n}^M$.

Another important feature of the beta function as defined above is that it has a fixed point at 0.5. It follows from this and the fact that it is a monotonically increasing function that $\mu_{t,n}^M > 0.5$ implies that $\theta_{t,n}^M > 0.5$, and that $\mu_{t,n}^M < 0.5$ implies that $\theta_{t,n}^M < 0.5$. That is, if the original μ values identify an asset as more (less) momentum than contrarian, the transformed θ allocations will do likewise. Also, the transformed momentum and contrarian allocations will still sum to one for each asset. That is, $\mu_{t,n}^M + \mu_{t,n}^C = 1$ and $\theta_{t,n}^M + \theta_{t,n}^C = 1$ for all assets n in the portfolio.

Once a positive value for β has been chosen, the transformed allocations $\theta_{t,n}^M$ and $\theta_{t,n}^C$ can be derived from $\mu_{t,n}^M$ and $\mu_{t,n}^C$, respectively, using tables of the beta function. The impact of changes in β on the relationship between $\mu_{t,n}^M$ and $\theta_{t,n}^M$ is graphed in Figure 1. Figure 1 shows that increasing the value of β leads to more distinct momentum/contrarian allocations.

Insert Figure 1 Here

The momentum/contrarian portfolios are obtained by multiplying the benchmark portfolio $Q_{t,n}$ (for example that of the Russell 1000 index) by the stage 2 momentum/contrarian allocations:

$$\text{Momentum portfolio}: \quad q_{t,n}^M = \theta_{t,n}^M \times Q_{t,n},$$

$$\text{Contrarian portfolio}: \quad q_{t,n}^C = \theta_{t,n}^C \times Q_{t,n}, \tag{3}$$

where $Q_{t,n}$ denotes the number of units of stock n in the benchmark index in period t, and $q_{t,n}^M$ and $q_{t,n}^C$ denote the number of units of stock n in the momentum and contrarian

index, respectively, in period t.

The momentum/contrarian index is momentum/contrarian relative to the benchmark index. This distinction is important since if the benchmark index itself is contrarian (for example, the equal-weighted S&P 500 index), it is possible that a momentum index defined relative to the benchmark index may still be contrarian (if its composition is tracked from one period to the next), albeit less contrarian than the benchmark index itself.

Three special cases of particular interest are when $\beta = 0, 1$ or infinity. When $\beta = 0$, the momentum and contrarian indices are identical to each other and the benchmark index. This is because, in this case, $\theta_{t,n}^M = \theta_{t,n}^C = 0.5$ for all n.

From Figure 1 it can be seen that as the parameter β increases the momentum/contrarian portfolios diverge more and more. When $\beta = 1$, the stage 2 allocations are the same as those from stage 1. That is $\theta_{t,n}^M = \mu_{t,n}^M$ and $\theta_{t,n}^C = \mu_{t,n}^C$ for all n. In the limiting case as β tends to infinity, there are no partial allocations, except when $\mu_{t,n}^M = \mu_{t,n}^C = 0.5$. In all other cases, each stock is placed in either the momentum portfolio or the contrarian portfolio, depending on whether it outperformed or underperformed the benchmark index in the formation period.

A higher value of β will tend to make the momentum portfolio more momentum and the contrarian portfolio more contrarian. At the same time this will also tend to increase the turnover in these portfolios when they are rebalanced, which may undermine their usefulness as investable benchmarks. The index provider must balance these two issues when choosing a value of β. Our empirical analysis suggests that a value of β of about 15 achieves an attractive balance for a (12,12) momentum/contrarian index benchmarked to the Russell 1000 index. However, higher values of β may also be justified when compared to traditional momentum factors or active managers employing a similar style. The preferred value of β may also vary with the formation and holding periods and the choice of reference index. The important point is that the proposed methodology is flexible and hence can accommodate different investor preferences.

It is possible to further reduce turnover by setting thresholds for θ. For example,

values of $\theta_{t,n}M$ and $\theta_{t,n}^C$ greater than 0.9 could be rounded up to 1, and values below 0.1 rounded down to zero. Such a rule has the advantage of eliminating small positions from the indices, thus reducing turnover and making passive management easier and more cost effective. Some index providers currently apply this type of trading rule when constructing value/growth indices [see Russell Indexes (2007)].

Insert Table 1 Here

Table 1 shows how the stage 1 and stage 2 momentum/contrarian allocations work in practice for the 20 largest stocks in the Russell 1000 index as of June 30, 2006. For example, Exxon Mobile is the largest cap stock in the Russell 1000 index with a weight of approximately 2.9 percent. In the year to June 30, 2006 the price return for Exxon Mobile was 6.75 percent, while the price return for the Russell 1000 index was 6.60 percent. Using the notation $\theta^M(\beta)$ for the momentum allocation for a particular value of β, the stage 1 allocation is given by $\theta^M(1)$. For Exxon Mobile, $\theta^M(1) > 0.5$. Since Exxon Mobile only marginally outperformed the benchmark index, small rises in β increase $\theta^M(\beta)$ only slightly. A more extreme example is provided by Intel. Intel fell 27.3 percent in the year to June 30, 2006, which makes it an ideal candidate for the contrarian index. Its stage 1 allocation is $\theta^M(1) = 0.4055$. Given a β of 15, the stage 2 momentum allocation falls to 0.1495. Table 1 also highlights that when $\beta =$ infinity there are no partial allocations.

A further issue of interest is the market capitalization shares of the momentum and contrarian portfolios defined as follows:

$$s_t^M = \frac{\sum_{n=1}^N \theta_{t,n}^M p_{t,n} Q_{t,n}}{\sum_{n=1}^N p_{t,n} Q_{t,n}}, \quad s_t^C = \frac{\sum_{n=1}^N \theta_{t,n}^C p_{t,n} Q_{t,n}}{\sum_{n=1}^N p_{t,n} Q_{t,n}}. \tag{4}$$

For example, if $s_t^M = 0.6$ this means that the momentum index has a market capitalization equal to 60 percent of that of the benchmark index. By construction $s_t^M + s_t^C = 1$. In the value/growth context, approximately equal shares of the original benchmark index are generally allocated to the value and growth indices. It follows that an investor allocating half her funds to index funds benchmarked to value and growth indices would approximately replicate an index fund on the original benchmark portfolio. In our context it will generally be the case that $s_t^M > s_t^C$ (see the appendix for a more detailed

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explanation). Intuitively this result arises because all of the stocks with a momentum allocation greater than 0.5 have had their market capitalization share in the reference index increase (due to above-market price performance) over the formation period.

The magnitude of the gap between s_t^M and s_t^C is explored in a later section. For now, however, we note that at times this gap can grow quite large. In cases where the split is considered too uneven, the original momentum and contrarian allocations $\mu_{t,n}^M$ and $\mu_{t,n}^C$ can be modified as follows:

Modified Stage 1 Allocations

$$\text{Momentum}: \quad \mu_{t,n}^M = \frac{p_{t,n}/p_{t-k,n}}{(p_{t,n}/p_{t-k,n}) + (I_t/I_{t-k}) + z_t},$$

$$\text{Contrarian}: \quad \mu_{t,n}^C = \frac{I_t/I_{t-k} + z_t}{(p_{t,n}/p_{t-k,n}) + (I_t/I_{t-k}) + z_t}, \tag{5}$$

where z_t denotes a parameter that the index provider can adjust each period to get the desired market capitalization split for the momentum and contrarian portfolios. By construction, it will still be the case that $\mu_{t,n}^M + \mu_{t,n}^C = 1$. Also, in stage 2 $\mu_{t,n}^M$ and $\mu_{t,n}^C$ are still transformed using the beta function. To obtain a value of $\theta_{t,n}^M$ greater than 0.5 it is now necessary for $p_{t,n}/p_{t-k,n} > I_t/I_{t-k} + z_t$. As can be seen, when $z_t > 0$, this will cause a reallocation of market capitalization away from the momentum portfolio towards the contrarian portfolio. By adjusting z_t, the market capitalizations of the momentum and contrarian portfolios can be made to be as close to each other as desired.

3. Empirical Results

(i) Formation and holding periods

Our empirical comparisons cover the period June 1995 to June 2007. The benchmark index is the US large cap Russell 1000 index, which is reconstituted on an annual basis. Before we can compute our momentum/contrarian indices it is first necessary to specify the formation and holding periods and the value of β. We initially consider formation and holding periods of 12 months for various levels of β. We refer to these as (12,12) strategies. We then consider (6,6), (24,24) and (36,36) strategies.

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(ii) Comparing the market capitalization shares of momentum and contrarian indices

The market capitalization shares of the momentum portfolio s_t^M and contrarian portfolio s_t^C as defined in (4) are graphed in Figure 2 over the period 1996-2007 for $\beta = 15$. For almost the whole sample $s_t^M > s_t^C$, and in 2001 s_t^M rises as high as 0.65. This spike in s_t^M can be attributed to the tech boom which caused large changes in the relative market capitalizations of stocks in the Russell 1000 index.

Insert Figure 2 Here

Also shown in Figure 2 is the impact on market capitalization shares of the inclusion of the adjustment z_t as defined in (5). The inclusion of z_t ensures that $s_t^M = s_t^C = 0.5$ whenever the momentum/contrarian indices are rebalanced (here on 30 June each year). Over the course of the next twelve months it can be seen from Figure 2 that the market capitalization shares can depart from 0.5, before reverting to 0.5 at the next rebalance. This means that the benchmark index – here the Russell 1000 – can be approximately replicated by investing an equal amount of money in its corresponding momentum and contrarian indices on the rebalance date. This will not be the case when the z_t adjustment is excluded.

The inclusion of z_t, however, entails a trade-off. Its inclusion muddies the waters with regard to the classification of stocks as either momentum or contrarian. With z_t set to zero, a stock that rises in price more than the benchmark index will be allocated predominantly to the momentum portfolio. When z_t is included, this is no longer necessarily the case. In practice, including z_t has only a minor impact on realized outcomes (eg. returns and turnover). The correlation in returns obtained with and without z_t is in excess of 0.999, with a low associated tracking error.[3] For the remainder of this article, therefore, we exclude z_t.

(iii) Turnover and the choice of the β parameter

The choice of a value of β requires a trade-off between two considerations. Lower values of β imply reduced turnover. Figure 3 plots turnover as a function of β for the

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[3]These results are not presented here but are available on request.

(12,12), (24,24) and (36,36) strategies. In all cases turnover is an increasing function of β. These findings suggest that lower values of β should be preferred. However, this conclusion must be balanced against the fact that higher values of β increase the distinctiveness of the momentum and contrarian portfolios, and hence better encapsulate the momentum and contrarian styles. In particular, in the limiting case when β equals zero, the momentum and contrarian portfolios become identical. Hence one must trade-off turnover against distinctiveness.

Insert Figure 3 Here

We define turnover as the minimum of aggregate purchases or sales of securities, divided by average Total Net Assets. This definition is consistent with both the CRSP and U.S. Securities and Exchange Commission Form N-SAR definitions of turnover (see ≪http://www.sec.gov/about/forms/formn-sar.pdf≫). This measure allows for useful comparisons between notional indices (such as our momentum and contrarian indices, the Russell 1000, S&P 500 and S&P 500 Equal Weighted Index) and live funds (e.g., index and exchange traded funds based on notional indices, and actively managed equity funds), since it excludes the impact of investor cash flows into and out of the live funds.

Depending on the value of β, our (12,12) momentum/contrarian indices vary in terms of turnover from as low as 10.0 percent per annum to as high as 49.8 percent per annum (see Table 2). Table 3 shows that increasing (decreasing) the formation and holding periods leads to lower (higher) levels of turnover. For example, the (6,6) indices have higher turnover than the (12,12) indices, while the (24,24) and (36,36) indices have lower turnover.

A number of issues need to be considered when evaluating the turnover of momentum and contrarian indices. First, it must be acknowledged that momentum/contrarian strategies, particularly those focusing on shorter horizons, will inevitably have higher turnover than standard market-cap weighted indices. Further, we should expect (and do indeed find) that momentum and contrarian indices generate higher turnover than value and growth indices, since valuation characteristics tend to be more stable over time than relative price performance.

It should be remembered, however, that the main goals of our momentum/contrarian indices are, first, to capture the relevant momentum/contrarian factor, and, second, to do so in a transparent way at lower cost than existing momentum/contrarian factors and active managers employing these same styles. The more relevant comparisons, therefore, for our momentum/contrarian indices are with traditional long minus short momentum factors and active managers following momentum/contrarian investment strategies. Our indices compare favorably on both accounts. For example, Agyei-Ampomah (2007) shows that the turnover for traditional long-minus short momentum factors can be prohibitively high; over 150 percent per annum for each side (long and short) of a (6,6) strategy and around 80 percent per annum for each side of a (12,12) strategy. By comparison, our highest turnover (12,12) momentum index (where $\beta =$ infinity) has annual turnover of just 40.5 percent, while the contrarian equivalent has turnover of 49.8 percent. Our indices also have the advantage of being more diversified and more consistent with how active managers construct momentum/contrarian portfolios than the traditional momentum factors.

According to Cremers and Petajisto (2007), the average US mutual fund has turnover of approximately 95 percent per annum, which is around double the level of our highest turnover momentum and contrarian indices. Grinblatt, Titman and Wermers (1995) present evidence that managers following a momentum style tend to have higher than average turnover, which suggest that active momentum managers likely have turnover levels greater than 100 percent. This makes even our highest turnover indices appear attractive from a turnover and transaction costs perspective.

Another useful comparison is with more specialized existing indices, such as the equal-weighted S&P 500 index, which we have previously noted is in fact an example of a contrarian index (although with rather different properties to those proposed in this paper). For example, in historical back-tests similar to those performed in this paper, the equal-weighted S&P 500 index has a turnover of about 30 percent per annum [see Standard and Poor's and Rydex Global Advisors (2003)]. This is relevant since the fact that successful index and exchange traded funds have been launched to

track this index implies that turnover of around 30 percent per annum is acceptable to investors for certain types of passive investments. Further, the ETFs based on this index are designed explicitly for active trading investors, similar to how we envisage momentum/contrarian indices being used. As of 31 October 2007, the equal-weighted S&P 500 index has a $2.2 billion ETF tracking it (see the Rydex ETF Annual Report at ≪http://www.rydexfunds.com/ETFCenter/products/overview.rails?rydex_symbol=RSP≫).

We focus below on indices with $\beta = 15$ for each of the (f,h) momentum/contrarian strategies, which over the sample period results in annual turnover of about 25 percent for the (12,12) strategy. A key feature of our methodology, however, is that if 25 percent turnover is considered too high (or even too low), then β can be simply adjusted to obtain the desired trade-off between turnover and distinctiveness.

(iv) Diversification

The diversification of a portfolio can be measured by its concentration index. This is equal to the inverse of the Herfindahl-Hirschmann index, and is defined as follows [see Strongin, Petsch and Sharenow (2000)]:

$$CC_t^i = \left[\sum_{n=1}^{N} (w_{t,n}^i)^2 \right]^{-1},$$

where

$$w_{t,n}^M = \frac{p_{t,n} q_n^M}{\sum_{m=1}^{N} p_{t,m} q_m^M}, \quad w_{t,n}^C = \frac{p_{t,n} q_n^C}{\sum_{m=1}^{N} p_{t,m} q_m^C},$$

denote the portfolio shares of stock n in the momentum and contrarian portfolios, respectively. The concentration indices here must lie between 0 and 1000, since $N = 1000$. A value of 100 would imply that the portfolio has the same level of diversification as an equal-weighted portfolio of 100 stocks. Concentration ratios for the Russell 1000 index and (12,12) momentum/contrarian indices when $\beta = 15$ and 30 are graphed in Figure 4. It can be seen from Figure 4 that the concentration of the momentum/contrarian indices are not particularly sensitive to variations in β over the specified range. Also, between 1996 and 2001, the momentum index is more concentrated than the contrarian index. After 2001, this pattern is reversed. Furthermore, before 2001, the Russell 1000 has about the same concentration as the contrarian index. After 2001, it has about the

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same concentration as the momentum index. These results illustrate that our momentum/contrarian indices differ little in terms of concentration compared to the reference Russell 1000 index.

Insert Figure 4 Here

(v) Performance and Risk of Momentum and Contrarian Indices

Tables 2 and 3 present summary performance and risk results for the momentum/contrarian indices over the 11 year period to June 2007. Table 2 focuses solely on the (12,12) strategy and shows how the results vary with β. Table 3 fixes $\beta = 15$ and investigates the impact of changing the formation and holding periods from (6,6), (12,12), (24,24) to (36,36). The time period for Table 3 is shorter than in Table 2 since the longer formation periods of the (24,24) and (36,36) strategies require two and three years of historical performance respectively. To facilitate comparisons, we therefore look at the period June 1998 to June 2007 in Table 3.

Insert Table 2 Here

Insert Table 3 Here

Table 2 shows that increasing β leads to more divergent performance from the benchmark index. Momentum outperforms contrarian, and this outperformance increases as β rises. The volatility (standard deviation) of total returns increases as the parameter β rises, while tracking error against the Russell 1000 index and turnover also increase with β. Correlation with the reference index falls as β rises. Comparisons of momentum/contrarian pairs (i.e., momentum and contrarian indices with the same values of β) show that tracking error between the pairs rises and correlations fall as β rises.

From Table 3 it can be seen that the momentum style outperforms contrarian over shorter horizons but this pattern reverses over longer horizons. These findings are consistent with DeBondt and Thaler (1985, 1987) and Jegadeesh and Titman (1993, 2001).

While Tables 2 and 3 provide a useful summary of the performance characteristics over the whole period, they give no indication of how the results have changed over time.

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Figure 5 plots the performance of (12,12) momentum and contrarian indices defined on the Russell 1000 index from June 1996 to June 2007 for $\beta = 15$ and infinity. Figure 5(a) charts the increase in the value of \$100 invested in June 1996. Figure 5(b) measures performance relative to the Russell 1000 index. From both graphs it can be seen that the momentum strategy outperforms the contrarian strategy, and that the divergence between the two strategies rises with β. The outperformance of momentum strategies over a 12 month horizon is again consistent with the literature on momentum strategies [see Jegadeesh and Titman (1993, 2001)].

Insert Figure 5 Here

(vi) Style interactions

Fama and French (1996), Lakonishok, Shleifer and Vishny (1994) and Asness (1997) document a negative relationship between the performance of momentum and value stocks. Scowcroft and Sefton (2005) find that this relationship has weakened since 2000. Our results go further and find a reversal in this relationship after 2001, and no significant relationship in the more recent data.

Panel (a) of Figure 6 shows the performance of (12,12) momentum/contrarian indices with $\beta = 15$ and Russell 1000 value/growth indices relative to the benchmark Russell 1000 index. The indices are all normalized to 1 in June 1996. Thereafter, a value greater than 1 implies the index has outperformed the Russell 1000 index, while a value less than 1 implies the index has underperformed. Consistent with Fama and French (1996), Lakonishok, Shleifer and Vishny (1994) and Asness (1997), we see in the early part of the sample (during the "tech" boom) that value and contrarian indices move together, as do growth and momentum indices. Contrary to these earlier studies, however, after 2001 we find a reversal of these relationships. In the later part of the sample there has been no noticeable relationship between value/growth and momentum/contrarian styles.

Panel (b) shows rolling three year excess return (relative to the benchmark Russell 1000 index) correlations between the momentum/contrarian indices and the Russell 1000 value index. Panel (c) shows correlations between the momentum/contrarian in-

dices and the Russell 1000 Growth index. Again we see positive correlations between value and contrarian indices on the one hand and between growth and momentum indices on the other in the early years, before these correlations reverse and then neutralize.

An important conclusion from this is that value/growth and momentum/contrarian strategies do not measure the same thing; that is, momentum/contrarian is not simply a proxy for growth/value, and these styles represent unique investment strategies in their own right.[4]

Insert Figure 6 Here

(vii) Momentum as a factor

The three-factor model of Fama and French (1992, 1993, 1996) is useful for evaluating the performance and risk exposures of a portfolio or index. The model attributes the expected return on a portfolio in excess of the risk-free rate to three factors: the return on a broad market portfolio in excess of the risk-free rate; the difference between the return on a portfolio of small stocks and a portfolio of large stocks (SMB - small minus big); the difference between the return on a portfolio of high-book-to-market stocks and a portfolio of low-book-to-market stocks (HML - high minus low). Faff (2003) argues that useful proxies for SMB and HML factors can be constructed from publicly available stock market indices. For example, a SMB factor proxy can be obtained by taking the difference between the returns on the Russell small cap and Russell 1000 indices. Similarly, a HML factor proxy can be obtained by taking the difference between the returns on the Russell 1000 value and growth indices.

Carhart (1997) extends the Fama-French model to include the momentum factor of Jegadeesh and Titman in addition to SMB and HML. Most academics and practitioners use the traditional long minus short approach as in Jegadeesh and Titman to compute this momentum factor. A momentum factor proxy can be obtained, however, by simply

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[4]Results not included here also depict a changing relationship between value/growth and momentum/contrarian styles for the (6,6), (24,24) and (36,36) indices. Consistent with Fama and French, the (36,36) indices show a generally positive relationship between the contrarian and value styles.

taking the difference between the returns on our momentum and contrarian indices.

The correlation between our momentum factor proxy and Carhart's momentum factor is high as shown in Figure 7 below. Figure 7 shows the rolling three year correlation between momentum factors constructed from our momentum/contrarian (6,6) and (12,12) indices and the traditional (long minus short) momentum factors. Here we use $\beta = 15$, however, changing β has little impact on the results. In fact, increasing β generally leads to slightly higher correlations. These results show that the momentum/contrarian indices proposed here are measuring what we want them to measure, but with the advantage of being investable quasi-passive portfolios rather than hypothetical long-short portfolios. These indices could therefore be useful for manager performance evaluation (e.g., comparing a manager with a momentum style to a momentum index) or as a convenient off-the-shelf proxy for the returns to the momentum factor, which will make a Carhart (1997) model simpler to implement for practitioners. Finally, these results suggest that investors interested in investing in momentum/contrarian ETFs can be confident they will be gaining exposure to the momentum factor they desire.

Insert Figure 7 Here

4. Conclusion

The momentum/contrarian styles are common among investors. This is in spite of the fact that they are expensive to access and lack clear performance benchmarks. In this article we have shown how momentum/contrarian indices and ETFs can be created that would allow investors to access these styles at lower cost, provide performance benchmarks for momentum/contrarian oriented funds, and provide a simple proxy for the traditional and computationally intensive momentum factor typically used in the literature. Our two-stage index construction methodology is very flexible and can also be usefully applied in other style dimensions.

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Appendix

To see why we would normally expect $s_t^M > s_t^C$, it is useful to define variants on s_t^M and s_t^C that use the prices and portfolio holdings of period $t - k$:

$$\hat{s}_t^M = \frac{\sum_{n=1}^{N} \theta_{t,n}^M p_{t-k,n} Q_{t-k,n}}{\sum_{n=1}^{N} p_{t-k,n} Q_{t-k,n}}, \quad \hat{s}_t^C = \frac{\sum_{n=1}^{N} \theta_{t,n}^C p_{t-k,n} Q_{t-k,n}}{\sum_{n=1}^{N} p_{t-k,n} Q_{t-k,n}}.$$

The momentum market shares can be reexpressed as follows:

$$s_t^M = \sum_{n=1}^{N} w_{t,n} \theta_{t,n}^M, \quad \hat{s}_t^M = \sum_{n=1}^{N} w_{t-k,n} \theta_{t,n}^M,$$

where

$$w_{t,n} = \frac{p_{t,n} Q_{t,n}}{\sum_{n=1}^{N} p_{t,m} Q_{t,m}}, \quad w_{t-k,n} = \frac{p_{t-k,n} Q_{t-k,n}}{\sum_{n=1}^{N} p_{t-k,m} Q_{t-k,m}},$$

and by construction $\sum_{n=1}^{N} w_{t,n} = \sum_{n=1}^{N} w_{t-k,n} = 1$. Now, abstracting from changes in the benchmark portfolio (i.e., assuming $Q_{t-k,n} = Q_{t,n} = Q_n$) it follows that

$$w_{t,n} = \frac{p_{t,n} Q_n}{\sum_{m=1}^{N} p_{t,m} Q_m} > \frac{p_{t-k,n} Q_n}{\sum_{m=1}^{N} p_{t-k,m} Q_m} = w_{t-k,n} \Rightarrow \frac{p_{t,n}}{p_{t-k,n}} > \frac{\sum_{m=1}^{N} p_{t,m} Q_m}{\sum_{m=1}^{N} p_{t-k,m} Q_m} = \frac{I_t}{I_{t-k}}.$$

This means that

$$w_{t,n} > w_{t-k,n} \Rightarrow \theta_{t,n}^M > 0.5,$$

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$$w_{t,n} < w_{t-k,n} \Rightarrow \theta_{t,n}^M < 0.5.$$

These inequalities in turn imply that

$$\sum_{n=1}^{N} w_{t-k,n}\theta_{t,n}^M < 0.5 < \sum_{n=1}^{N} w_{t,n}\theta_{t,n}^M. \tag{6}$$

That is, $s_t^M > 0.5$. Given that $s_t^M + s_t^C = 1$, it follows that $s_t^C < 0.5$.

Table 1. Momentum/Contrarian Allocations on June 30, 2006
for 20 Largest Stocks in Russell 1000

	$(P_t/P_{t-k})-1$	Stage 1 $\theta(\beta=1)$	Stage 2 $\theta(\beta=15)$	Stage 2 $\theta(\beta=100)$	Stage 2 $\theta(\beta=Inf)$
Exxon Mobil Corp.	6.75	0.50035	0.50039	0.50101	1.00000
General Electric Co.	-4.88	0.47155	0.37745	0.20846	0.00000
Citigroup Inc.	4.35	0.49466	0.47573	0.43711	0.00000
Bank of America Corp.	5.46	0.49731	0.48720	0.46674	0.00000
Microsoft Corp.	-6.20	0.46806	0.36311	0.18127	0.00000
Procter & Gamble Co.	5.40	0.49717	0.48662	0.46523	0.00000
Johnson & Johnson	-7.82	0.46374	0.34557	0.15078	0.00000
Pfizer Inc.	-14.90	0.44391	0.26958	0.05516	0.00000
Altria Group Inc.	13.56	0.51581	0.56708	0.66979	1.00000
JPMorgan Chase & Co.	18.91	0.52730	0.61559	0.77765	1.00000
Chevron Corp.	10.98	0.51006	0.54240	0.60906	1.00000
American International Group Inc.	1.64	0.48807	0.44733	0.36531	0.00000
Cisco Systems Inc.	2.20	0.48945	0.45327	0.38006	0.00000
International Business Machines	3.53	0.49269	0.46724	0.41535	0.00000
Wal-Mart Stores Inc.	-0.06	0.48387	0.42931	0.32160	0.00000
Wells Fargo & Co.	8.93	0.50540	0.52228	0.55776	1.00000
Intel Corp.	-27.28	0.40552	0.14950	0.00346	0.00000
AT&T Inc.	17.43	0.52417	0.60253	0.75042	1.00000
PepsiCo Inc.	11.33	0.51085	0.54578	0.61756	1.00000
Russell 1000 Index	6.60				

Notes: This table illustrates how changing the parameter β impacts on the Stage 2 momentum/contrarian allocations. The price return over the formation period for each of the 20 largest stocks in the Russell 1000 is displayed in the second column, along with the price return of the Russell 1000 index itself. Columns three to six show the Stage 2 allocations for β = 1, 15, 100 and infinity respectively.

Table 2. Performance of (12,12) Indices (June 1996-June 2007)

	Return	XS(Ref)	StDev	TE(Ref)	IR	Turnover	Corr(Ref)	XS(MC)	TE(MC)	Corr(MC)
Ref index	9.70	0.00	15.14	0.00	N/A	5.1%	1.0000			
R1000	9.59	-0.11	15.17	0.23	-0.48	9.2%	0.9999			
R1000V	11.81	2.11	14.00	6.77	0.31	16.4%	0.8948	5.21	13.20	0.71
R1000G	6.59	-3.11	18.87	6.51	-0.48	19.2%	0.9501			
Mom(1)	9.75	0.05	15.52	1.70	0.03	10.0%	0.9942	0.13	3.57	0.97
Mom(5)	9.89	0.19	15.98	3.23	0.06	16.8%	0.9799	0.64	7.16	0.90
Mom(10)	9.99	0.29	16.21	3.88	0.08	20.7%	0.9717	1.09	8.80	0.85
Mom(15)	10.04	0.34	16.37	4.24	0.08	23.1%	0.9668	1.43	9.71	0.82
Mom(20)	10.09	0.39	16.47	4.46	0.09	24.9%	0.9636	1.67	10.32	0.80
Mom(30)	10.15	0.45	16.60	4.75	0.09	27.3%	0.9594	2.01	11.10	0.77
Mom(50)	10.22	0.52	16.75	5.04	0.10	30.0%	0.9550	2.41	11.94	0.74
Mom(100)	10.30	0.60	16.88	5.33	0.11	33.2%	0.9504	2.81	12.82	0.71
Mom(Inf)	10.41	0.71	17.34	6.16	0.12	40.5%	0.9369	3.55	14.69	0.65
Cont(1)	9.62	-0.08	14.95	1.88	-0.04	10.0%	0.9923			
Cont(5)	9.26	-0.44	15.18	3.94	-0.11	17.5%	0.9662			
Cont(10)	8.90	-0.80	15.50	4.94	-0.16	22.0%	0.9483			
Cont(15)	8.62	-1.09	15.75	5.51	-0.20	25.0%	0.9371			
Cont(20)	8.42	-1.28	15.94	5.90	-0.22	27.3%	0.9291			
Cont(30)	8.14	-1.56	16.21	6.42	-0.24	30.4%	0.9183			
Cont(50)	7.81	-1.89	16.54	7.00	-0.27	34.4%	0.9061			
Cont(100)	7.50	-2.20	16.92	7.63	-0.29	39.3%	0.8927			
Cont(Inf)	6.86	-2.84	17.57	8.76	-0.32	49.8%	0.8670			

Notes:

The reference index here is the Russell 1000 ignoring any index changes between its annual reconstitutions. It therefore omits some IPOs and corporate actions and any other additions and deletions that take place between reconstitution dates. The reference index has a very low tracking error with respect to the Russell 1000 index.

XS(Ref)/TE(Ref)/Corr(Ref)=Excess return/tracking error/correlation relative to reference index;

XS(MC)/TE(MC)/Corr(Ref)=Excess return/tracking error/correlation of momentum index relative to contrarian index with the same β value.

Tracking error is measured by the annualized standard deviation of monthly excess returns;

IR=XS(Ref)/TE(Ref) is the information ratio.

Table 3. Impact of Varying the Formation and Holding Periods on Index Performance When β=15 (June 1998-June 2007)

	Return	XS	StDev	TE	IR	Turnover	Corr(Ref)
Ref index	5.37	0.00	15.18	0.00	N/A	5.1%	1.0000
(6,6) - Mom(15)	6.42	1.05	15.25	4.04	0.26	36.6%	0.9647
(12,12) - Mom(15)	5.46	0.09	16.49	4.64	0.02	23.1%	0.9605
(24,24) - Mom(15)	4.77	-0.60	16.87	3.95	-0.15	14.3%	0.9751
(36,36) - Mom(15)	4.47	-0.90	16.54	2.61	-0.34	11.0%	0.9901
(6,6) - Cont(15)	4.16	-1.21	16.70	5.08	-0.24	39.8%	0.9536
(12,12) - Cont(15)	4.38	-0.98	16.08	6.04	-0.16	25.0%	0.9268
(24,24) - Cont(15)	5.38	0.01	14.61	4.53	0.00	18.1%	0.9544
(36,36) - Cont(15)	5.90	0.53	14.31	3.48	0.15	15.0%	0.9738

Note: XS=Excess return; TE=Tracking error; IR=XS/TE is the information ratio

Figure 1. Deriving Stage 2 Allocations from Stage 1 Allocations using the Regularized Incomplete Beta Function
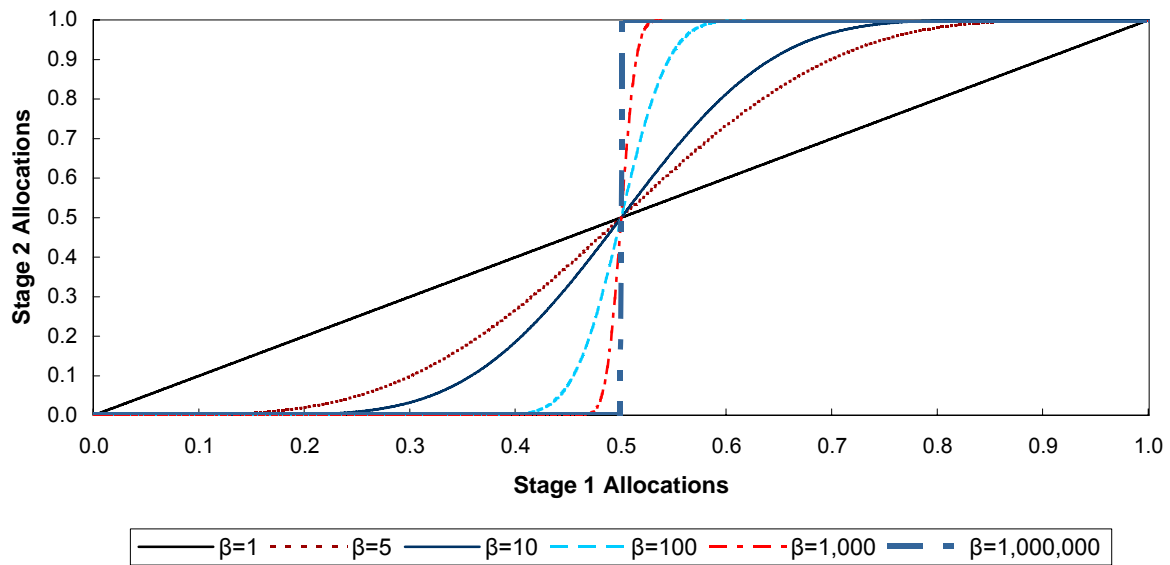

Figure 2. Market Capitalization Share of the (12,12) Momentum Portfolio with Beta=15

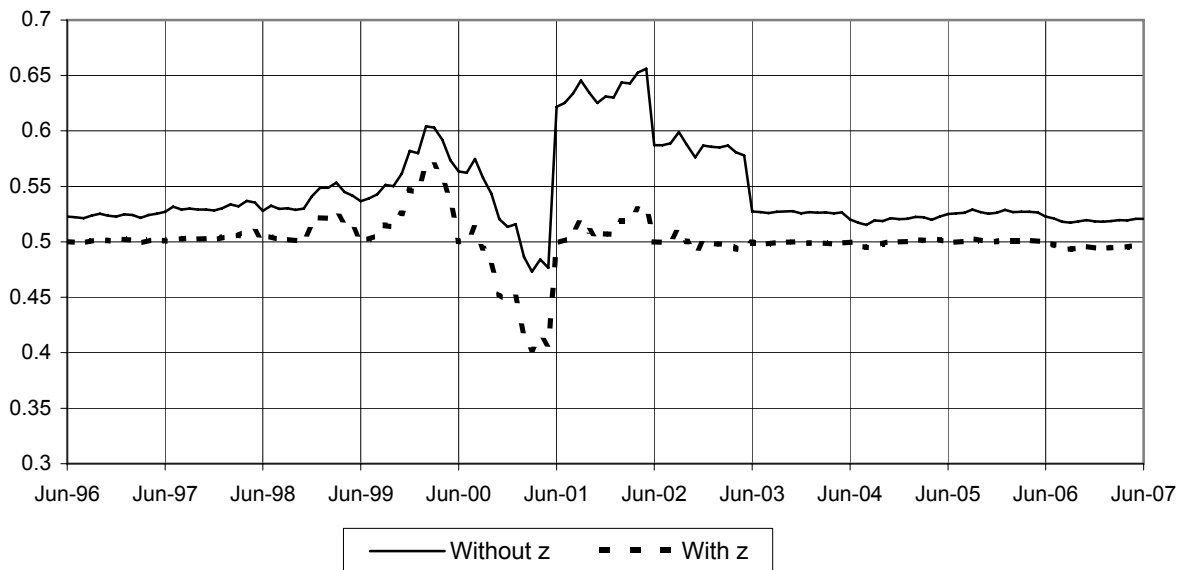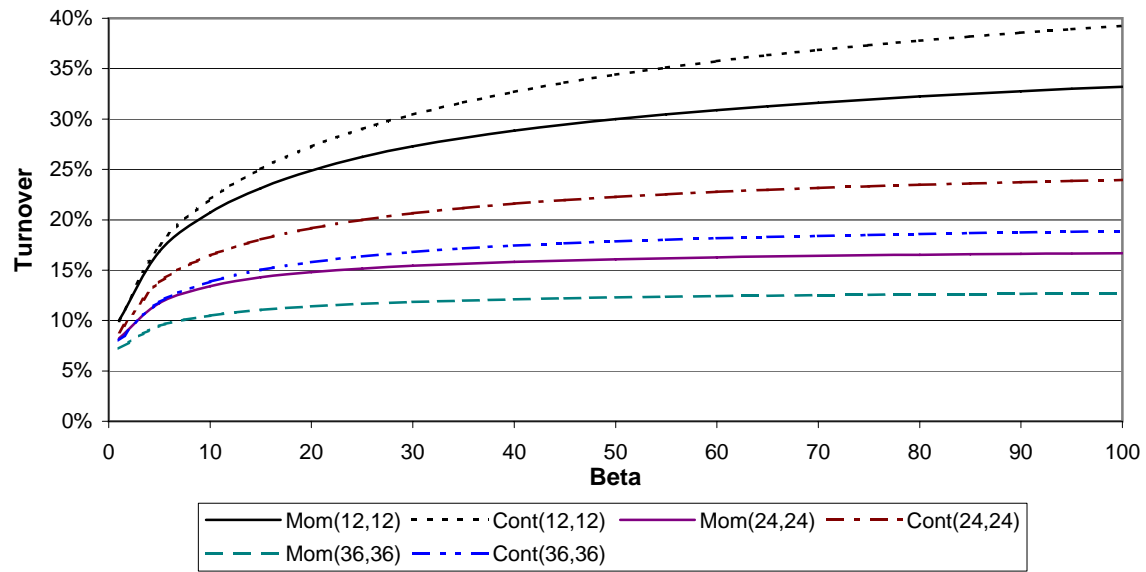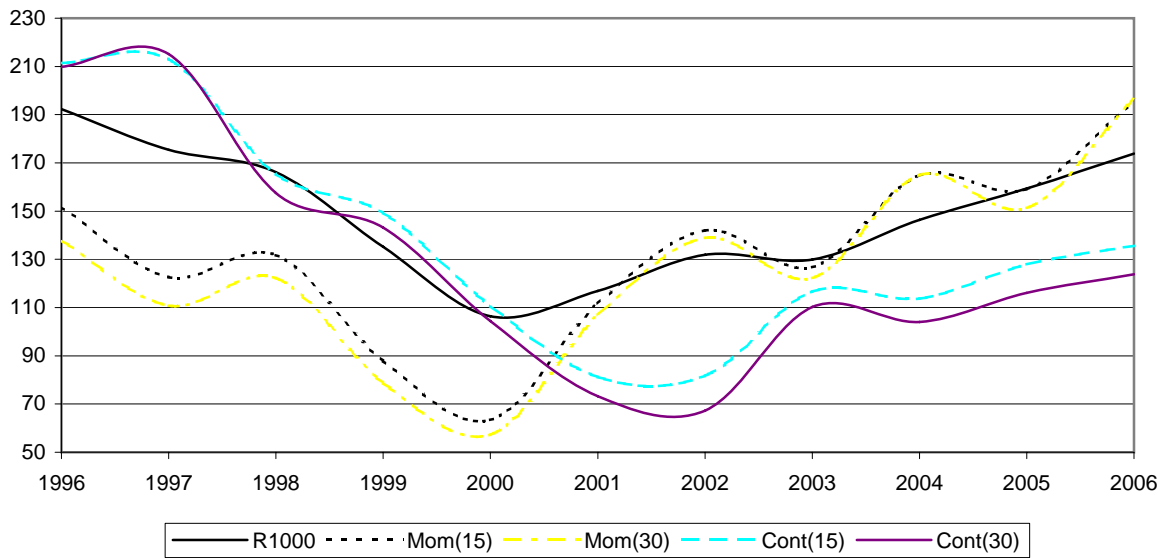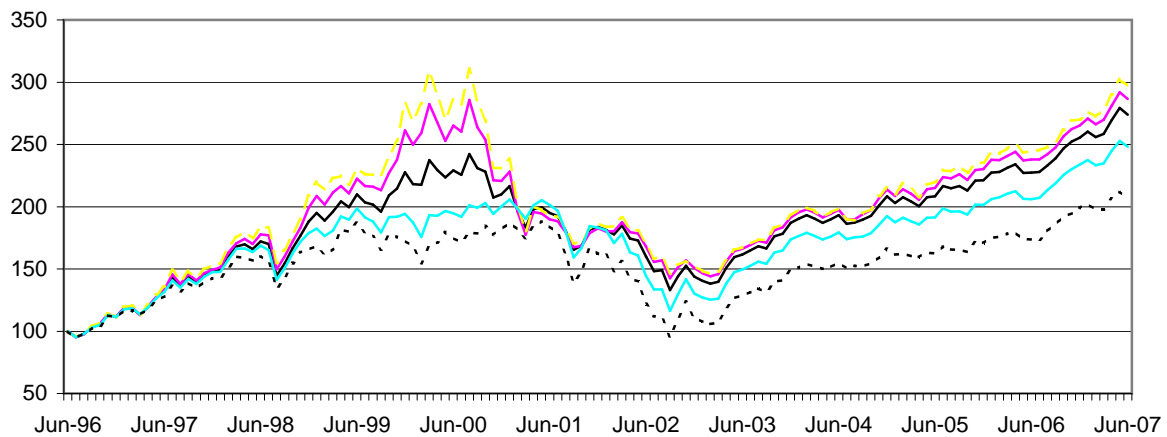**Figure 3. Turnover as a Function of Beta and the Formation and Holding Periods**

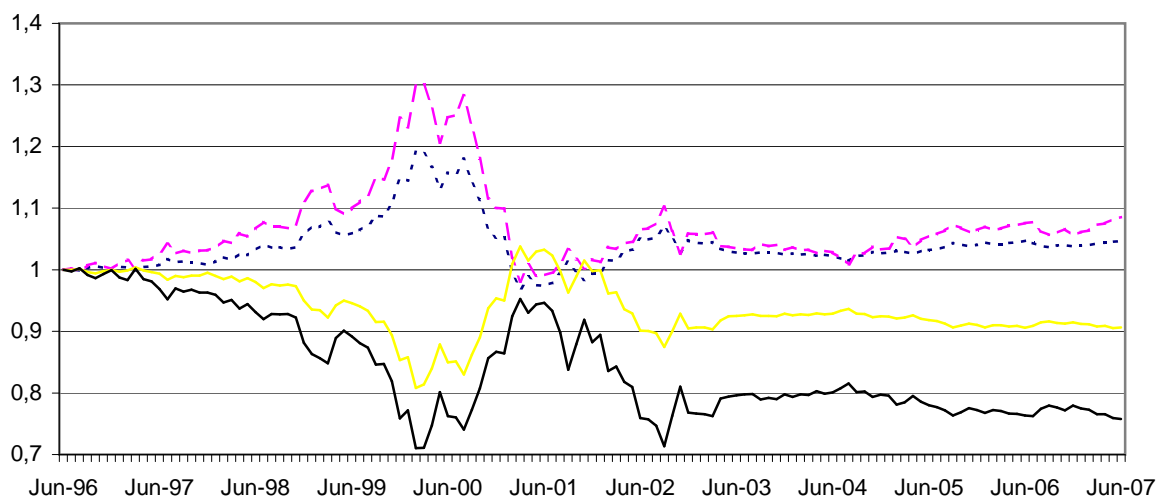**Figure 4. Concentration Coefficients for Russell 1000 and (12,12) Momentum Indices**
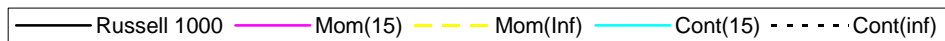

Note: M(15) denotes a momentum index with $\beta=15$.

Figure 5. Performance of (12,12) Momentum and Contrarian Indices

(a) Index Levels (Jun-96=100)

Russell 1000 —— Mom(15) —— Mom(Inf) —— Cont(15) ····· Cont(inf)

(b) Performance Relative to the R1000 (Jun-96=1)
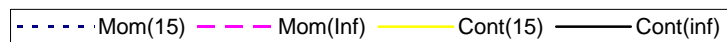
····· Mom(15) —— Mom(Inf) —— Cont(15) —— Cont(inf)

Figure 6. Comparing (12,12) Momentum/Contrarian and Value/Growth Styles

(a) Performance Relative to the R1000 (Jun-96=1)

Legend: Ref — R1000V — R1000G — Mom(15) — Mom(Inf) — Cont(15) — Cont(Inf)

(b) Rolling 3 Year Correlation of Excess Returns (vs. R1000) - Mom/Cont vs. Value

Legend: Mom(15) — Mom(Inf) — Cont(15) — Cont(Inf)

(c) Rolling 3 Year Correlation of Excess Returns (vs. R1000) - Mom/Cont vs. Growth

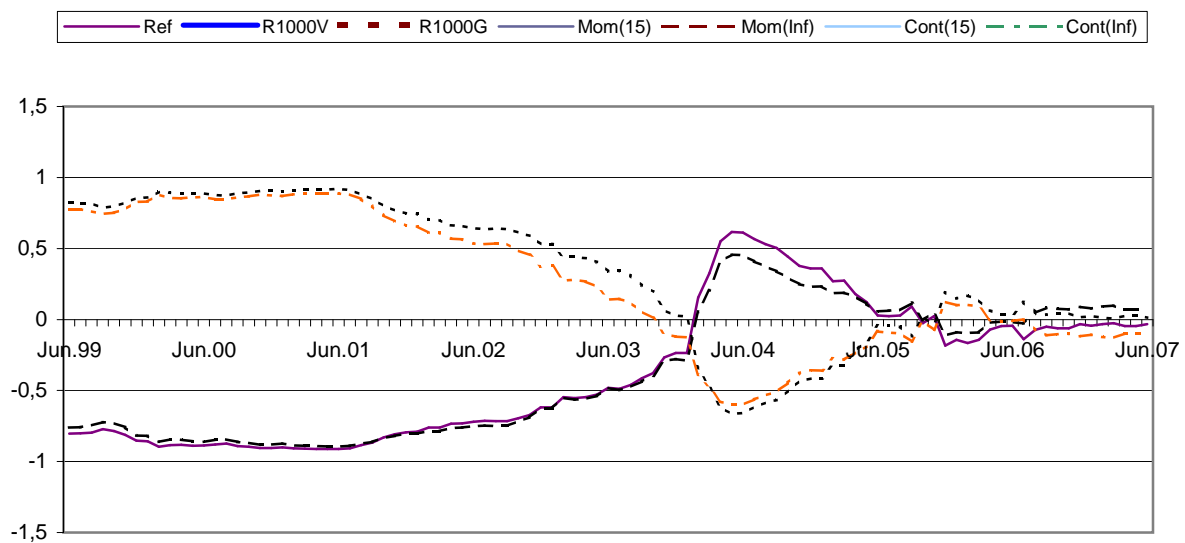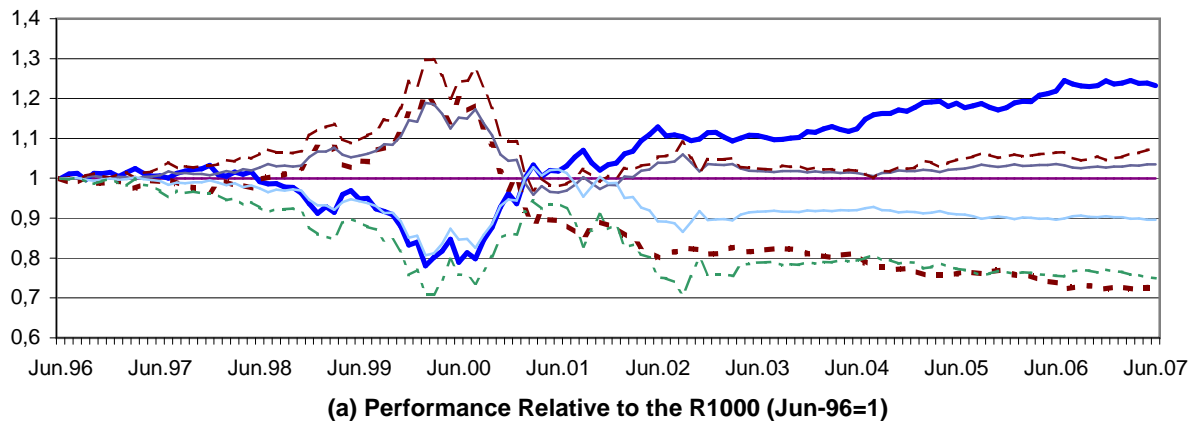Legend: Mom(15) — Mom(Inf) — Cont(15) — Cont(Inf)
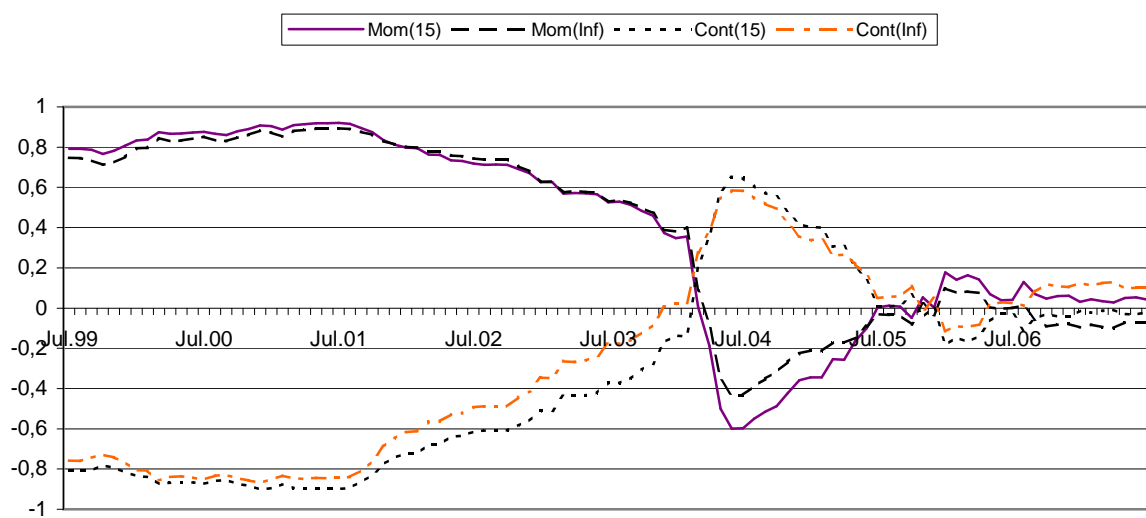
Figure 7. Rolling Three Year Correlations between MCI and Traditional Momentum Factor Returns