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Mean reversion in the US stock market

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Abstract

This paper revisits the evidence for the weaker form of the efficient market hypothesis, building on recent work by Serletis and Shintani [Serletis A, Shintani M. No evidence of chaos but some evidence of dependence in the US stock market. Chaos, Solitons & Fractals 2003;17:449–54], Elder and Serletis [Elder J, Serletis A. On fractional integrating dynamics in the US stock market. Chaos, Solitons & Fractals 2007;34;777–81], Koustas et al. [Koustas Z, Lamarche J.-F, Serletis A. Threshold random walks in the US stock market. Chaos, Solitons & Fractals, forthcoming], Hinich and Serletis [Hinich M, Serletis A. Randomly modulated periodicity in the US stock market. Chaos, Solitons & Fractals, forthcoming], and Serletis et al. [Serletis A, Uritskaya OY, Uritsky VM. Detrended Fluctuation analysis of the US stock market. Int J Bifurc Chaos, forthcoming]. In doing so, we use daily data, over the period from 5 February 1971 to 1 December 2006 (a total of 9045 observations) on four US stock market indexes – the Dow Jones Industrial Average, the Standard and Poor's 500 Index, the NASDAQ Composite Index, and the NYSE Composite Index – and a new statistical physics approach – namely the 'detrending moving average (DMA)' technique, recently introduced by Alessio et al. [Alessio E, Carbone A, Castelli G, Frappietro V. Second-order moving average and scaling of stochastic time series. Euro Phys J B 2002;27;197–200.] and further developed by Carbone et al. [Carbone A, Castelli G, Stanley HE. Time dependent hurst exponent in financial time series. Physica A 2004;344;267–71, Carbone A, Castelli G, Stanley HE. Analysis of clusters formed by the moving average of a long-range correlated time series. Phys Rev E 2004;69;026105.]. The robustness of the results to the use of alternative testing methodologies is also investigated, by using Lo's [Lo AW. Long-term memory in stock market prices. Econometrica 1991;59:1279–313.] modified rescaled range analysis. We conclude that US stock market returns display anti-persistence (mean reversion).

1. Introduction

Standard asset pricing models typically imply the 'martingale model,' according to which tomorrow's price is expected to be the same as today's price. The hypothesis that prices fully reflect available information has come to be known as the 'efficient market hypothesis.' The efficient market hypothesis has its roots in the work of Bachelier [3] and Cootner [6], and was explicitly formulated by Samuelson [24] and Fama [8]. In fact Fama [8] defined three types

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of (informational) capital market efficiency, each of which is based on a different notion of exactly what type of information is understood to be relevant. In particular, markets are weak-form, semistrong-form, and strong-form efficient if the information set includes past prices alone, all public information, and any information public as well as private, respectively. Clearly, strong-form efficiency implies semistrong-form efficiency, which in turn implies weak-form efficiency, but the reverse implications do not follow.

In recent years the efficient market hypothesis has been investigated in a large number of studies – see, for example, Fama [10]. As Mantegna and Stanley [20] put it,

> "in the great majority of the empirical studies, the time correlation between price changes has been found to be negligibly small, supporting the efficient market hypothesis. However, it was shown in the 1980s that by using the information present in additional time series such as earnings/price ratios, dividend yields, and term-structure variables, it is possible to make prediction of the rate of return of a given asset on a long time scale, much longer than a month. Thus empirical observations have challenged the stricter form of the efficient market hypothesis."

In this paper we revisit the evidence for the weaker form of the efficient market hypothesis, building on recent work by Serletis and Shintani [26], Elder and Serletis [7], Koustas et al. [13], Hinich and Serletis [11], and Serletis et al. [27]. In doing so, we use daily data, over the period from 5 February 1971 to 1 December 2006 (a total of 9045 observations) on four US stock market indexes – the Dow Jones Industrial Average, the Standard and Poor's 500 Index, the NASDAQ Composite Index, and the NYSE Composite Index – and a new statistical physics approach – namely the 'detrending moving average (DMA)' technique, recently introduced by Alessio et al. [1] and further developed by Carbone et al. [4,5].

The paper is organized as follows. In Section 2 we discuss the martingale hypothesis while in Section 3 we discuss the DMA method for testing for long-range correlations. In Section 4 we discuss the data and use the DMA technique to calculate the Hurst exponent in each of the four stock exchanges and test for random walk type behavior in the US stock markets. In Section 5 we investigate the robustness of our results to alternative testing methodologies. The final section concludes.

2. The martingale hypothesis

As already noted in the introduction, standard asset pricing models imply the 'martingale model,' according to which tomorrow's price is expected to be the same as today's price. Symbolically, a stochastic process $x(t)$ follows a martingale if

$$E_t[x(t+1)|\Omega(t)] = x(t), \tag{1}$$

where $\Omega(t)$ is the time t information set – assumed to include $x(t)$. Alternatively, the martingale model implies that $x(t+1) - x(t)$ is a 'fair game' (a game which is neither in your favor nor your opponent's)

$$E_t[(x(t+1) - x(t))|\Omega(t)] = 0. \tag{2}$$

Clearly, $x(t)$ is a martingale if and only if $x(t+1) - x(t)$ is a fair game; it is for this reason that fair games are sometimes called 'martingale differences'.

Clearly, the martingale model, Eq. (1), says that if $x(t)$ follows a martingale the best forecast of $x(t+1)$ that could be constructed based on current information $\Omega(t)$ would just equal $x(t)$. The fair game model, Eq. (2), says that increments in value are unpredictable, conditional on the information set $\Omega(t)$. In this sense, information $\Omega(t)$ is fully reflected in prices and hence useless in predicting rates of return.

The martingale model given by (1) can be written equivalently as

$$x(t+1) = x(t) + \varepsilon(t), \tag{3}$$

where $\varepsilon(t)$ is the martingale difference. When written in this form, the martingale looks identical to the random walk model – the forerunner of the theory of efficient capital markets. The martingale, however, is less restrictive than the random walk. In particular, the martingale difference requires only independence of the conditional expectation of price changes from the available information, as risk neutrality implies, whereas the (more restrictive) random walk model requires this and also independence involving the higher conditional moments (i.e., variance, skewness, and kurtosis) of the probability distribution of price changes. By not requiring probabilistic independence between successive price changes, the martingale difference model is entirely consistent with the fact that price changes, although uncorrelated, tend not to be independent over time but to have clusters of volatility and tranquility (i.e., dependence in the higher conditional moments) – a phenomenon originally noted for stock market prices by Mandelbrot [16] and Fama [9].

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3. Detrending moving average analysis

The detrending moving average (DMA) method is an improvement over the detrended fluctuation analysis, introduced by Peng et al. [23]. In particular, the DMA method detrends the series by subtracting a continuous function, the moving average, thereby being more accurate since the moving average is a better low-pass filter when compared to the polynomial filter used for DFA – for more details, see Alessio et al. [1] and Carbone et al. [4,5], and Serletis and Rosenberg [25] for a recent application to energy futures prices (using NYMEX data).

To illustrate the DMA algorithm, consider the time series $x(t)$ with $t = 1, \ldots, N$. Denote the nth order moving average of $x(t)$ by

$$\bar{x}_n(t) = \frac{1}{n} \sum_{k=0}^{n-1} x(t-k).$$

The series $x(t)$ is detrended by subtracting $\bar{x}_n(t)$, and the standard deviation of $x(t)$ around the moving average, $\bar{x}_n(t)$, is calculated as follows

$$\sigma_{\mathrm{DMA}} = \sqrt{\frac{1}{N - n_{\max}} \sum_{t=n_{\max}}^{N} [x(t) - \bar{x}_n(t)]^2},$$

where $n_{\max}$ refers to the maximum value of n.

The Hurst exponent is obtained by graphing σ_{DMA} and n on a log–log plot and computing the slope. In fact, Arianos and Carbone [2] show that

$$\sigma_{\mathrm{DMA}} \propto n^H,$$

where $0 < H < 1$ is the Hurst exponent. A linear relationship between σ_{DMA} and n on a log–log plot indicates the presence of power law (fractal) scaling. Power laws indicate that there is 'scale invariance' (or 'self-similarity') in the sense that fluctuations over small time scales are related to fluctuations at larger time scales. In particular, if n is rescaled (multiplied by a constant), then σ_{DMA} is still proportional to n^H, although with a different factor of proportionality – see, for example, Peitgen et al. [22].

Under such conditions, the scaling exponent H can be used to identify long-range dependence in the data. In particular, if $0.5 < H < 1$, then $x(t)$ is a persistent process (a process that maintains a trend). That is, if $x(t)$ increased

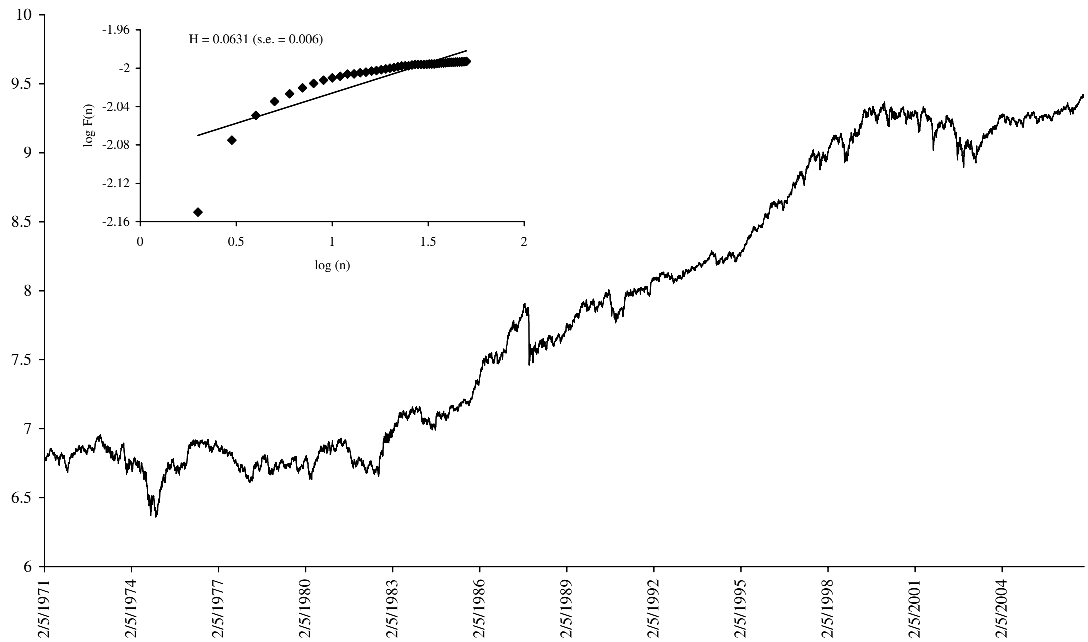

Fig. 1. Dow Jones industrial average (in logs).

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(decreased) in the past, it will most likely increase (decrease) in the future. If $0 < H < 0.5$, then $x(t)$ is an anti-persistent (or mean reverting) process. In this case, if $x(t)$ increased (decreased) in the past, it will most likely decrease (increase) in the future. Finally, $H = 0.5$ corresponds to uncorrelated Brownian motion and we treat it as a necessary condition for the efficient market hypothesis.

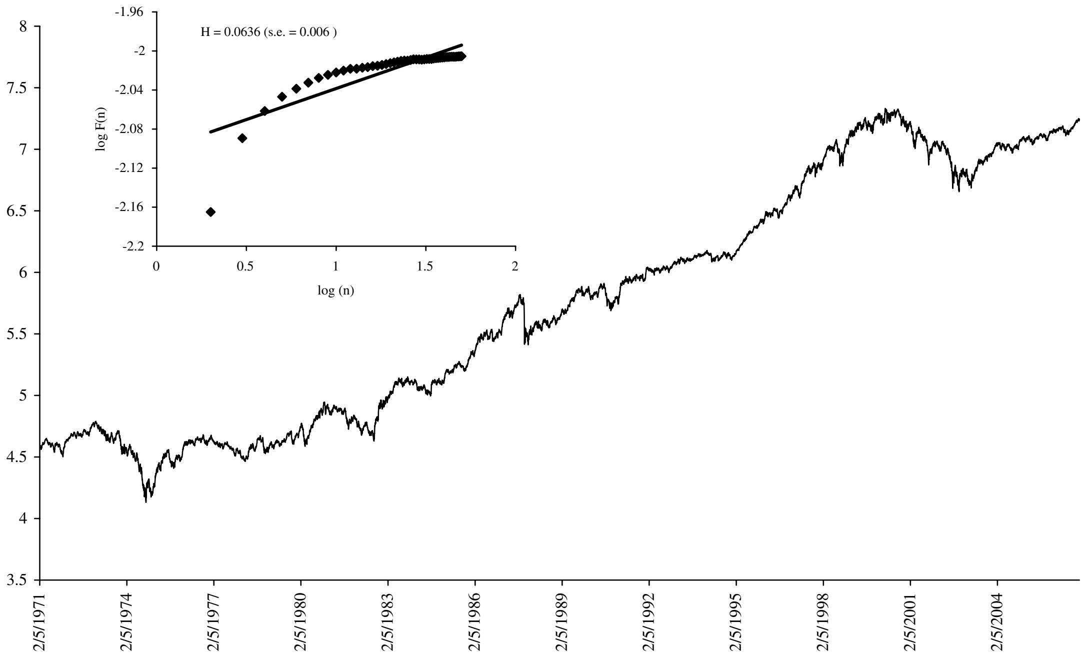

Fig. 2. S&P 500 index (in logs).

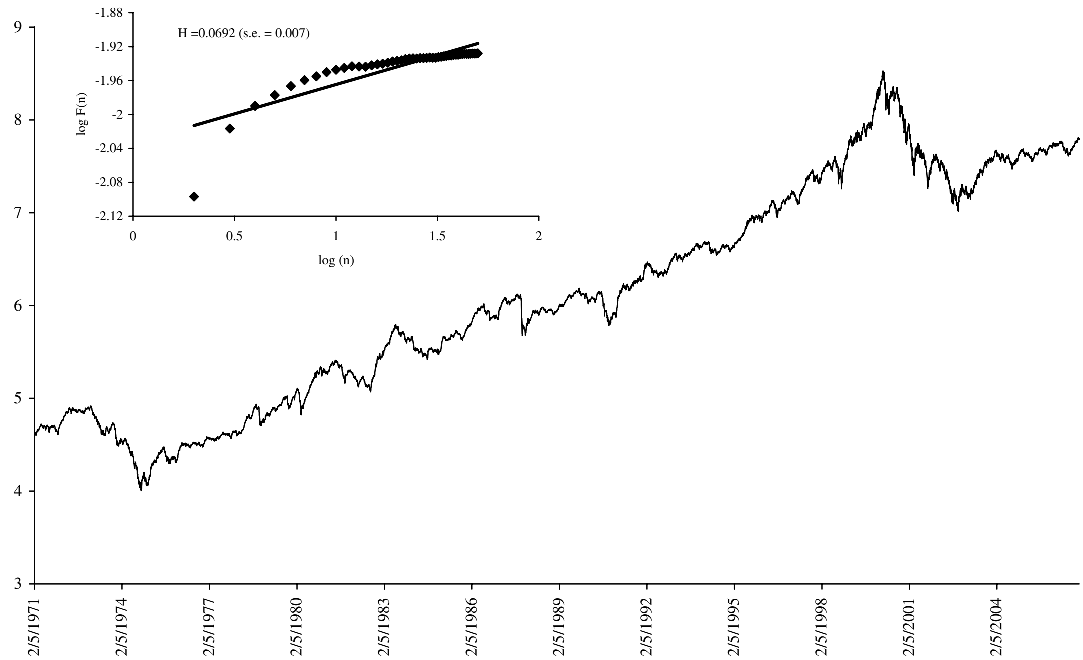

Fig. 3. NASDAQ composite index (in logs).

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4. Data and results

The data that we use in this paper consist of daily observations on four US stock market indexes over the period from 5 February 1971 to 1 December 2006, a total of 9045 observations. The indexes used are the Dow Jones Industrial Average (the most commonly quoted index, composed of 30 blue chip industrial firms), the Standard and Poor's 500 (a capitalization-weighted index of 500 stocks, considered to be the most accurate representation of the US equity securities market), the NASDAQ composite (a market-value weighted index used mainly to track technology stocks of approximately 3200 companies), and the NYSE composite (a capitalization-weighted index of all common stocks listed on the New York Stock Exchange).

Figs. 1–4 show the logarithms of each of the indexes since 1971 (when the NASDAQ stock exchange was founded). The detrending moving average analysis functions, σ_{DMA}, denoted in the figures by $F(n)$, are shown in each of the figures for $n = 50$, using logarithmic first differences, $\Delta \log x(t)$, of the data. In all four cases, $F(n)$ has a linear form in double logarithmic coordinates indicating its power law temporal scaling. The regression slope of this line is fairly close to zero, but statistically significant at conventional significance levels. We interpret this as evidence that the US stock market is strongly anti-persistent or mean reverting.

In addition to the global Hurst exponent, we also calculate the local Hurst exponent. This is accomplished by applying the DMA procedure to subsets of each series. Each subset is created by sliding a window of length L over the data with step θ. Non overlapping subsets are created by setting the size of the window equal to the size of the step. The local Hurst exponent is then calculated for each subset using the method outlined above. A sequence of Hurst exponents for each series is displayed in Figs. 5–8, where $L = \theta = 50$, implying that each subset is slightly more than two months long.

5. Robustness

As a robustness check, we use the 'range over standard deviation' or R/S statistic to detect long-range or 'strong' dependence in economic time series – see Mandelbrot [16], Hurst [12], Mandelbrot [17], and Mandelbrot and Taqqu [19]. Through the use of stock returns data, it has been recently shown that the Husrt–Mandelbrot (classical) R/S statistic is sensitive to short-range dependence – see Mandelbrot [15]. In fact rejections of the null hypothesis (of no long-range dependence versus the alternative of long-range dependence) based on long time scales can be erroneous and can instead be due to bias induced by short term dependencies – Lo [14].

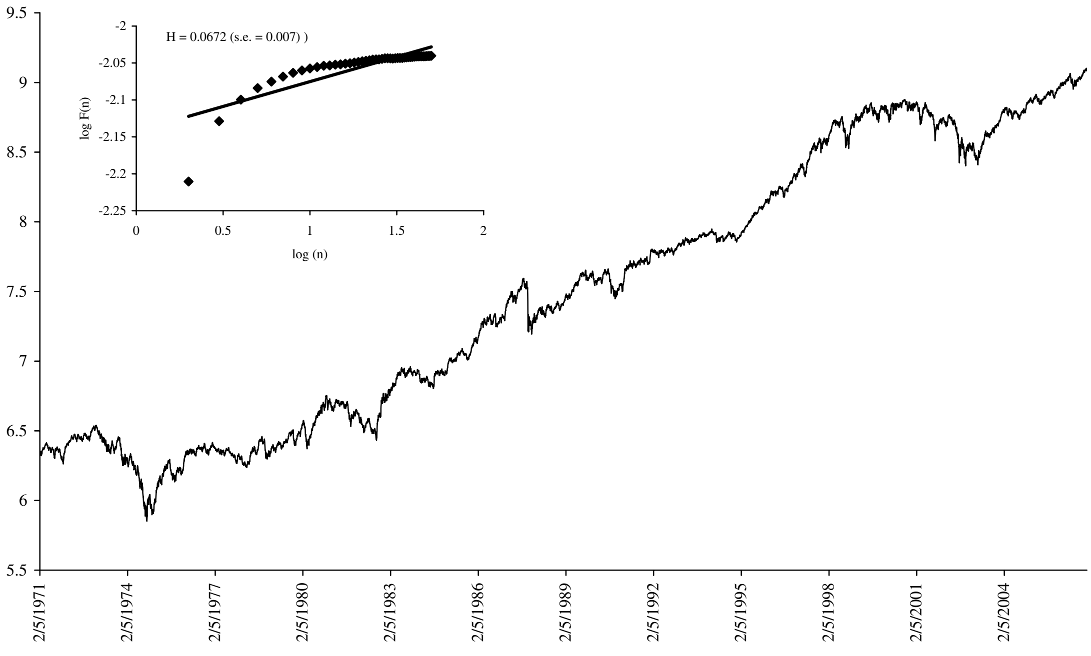

Fig. 4. NYSE composite index (in logs).

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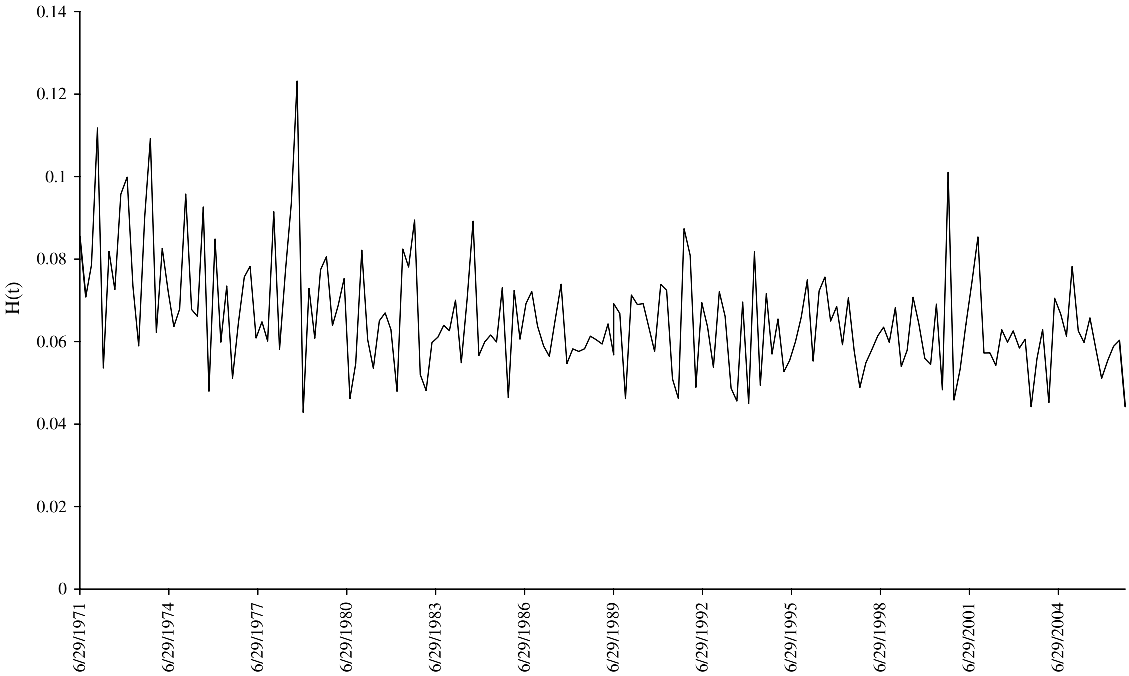

Fig. 5. Time dependent hurst exponent for the dow jones industrial average.

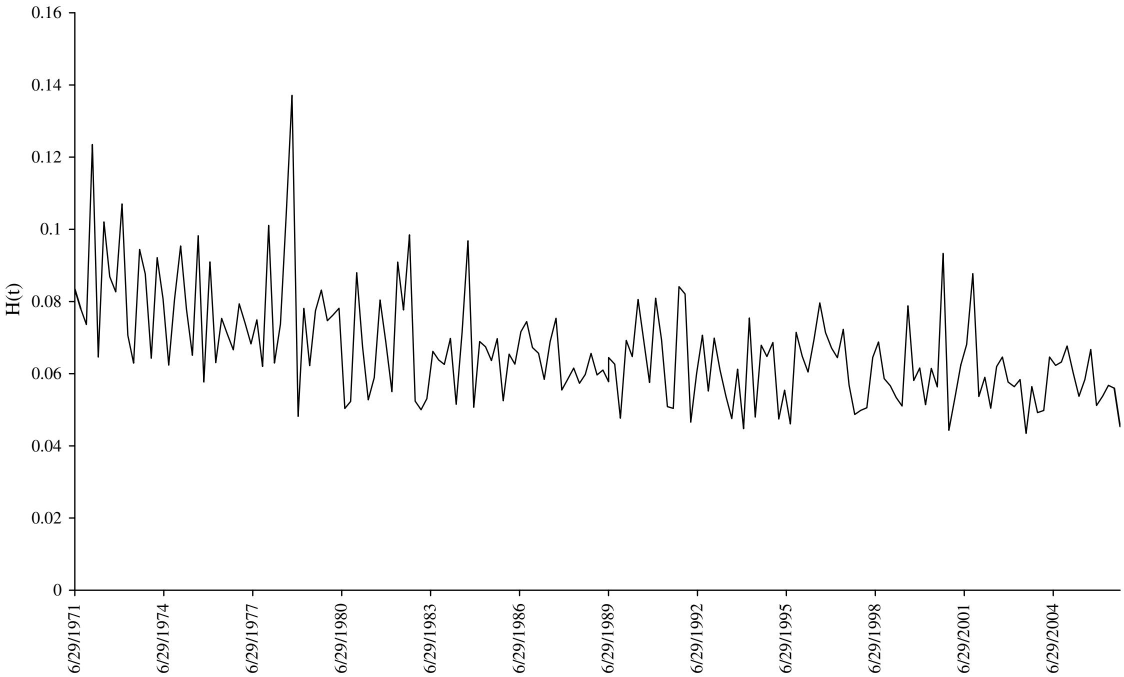

Fig. 6. Time dependent hurst exponent for the S&P 500 index.

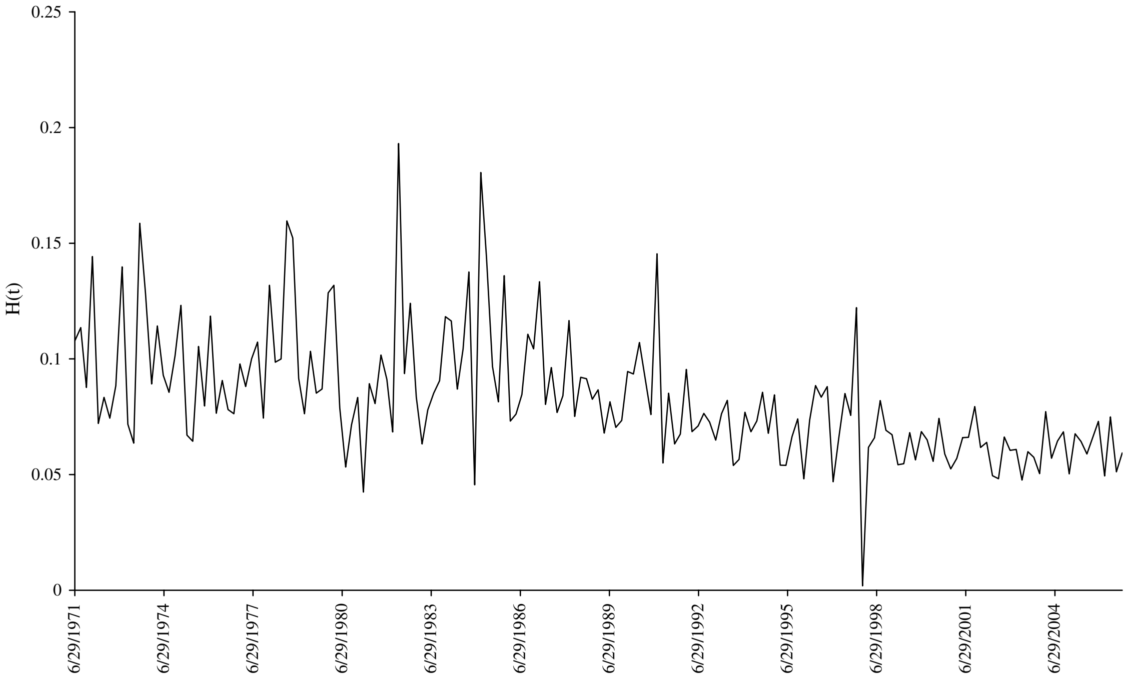

Fig. 7. Time dependent hurst exponent for the NASDAQ composite index.

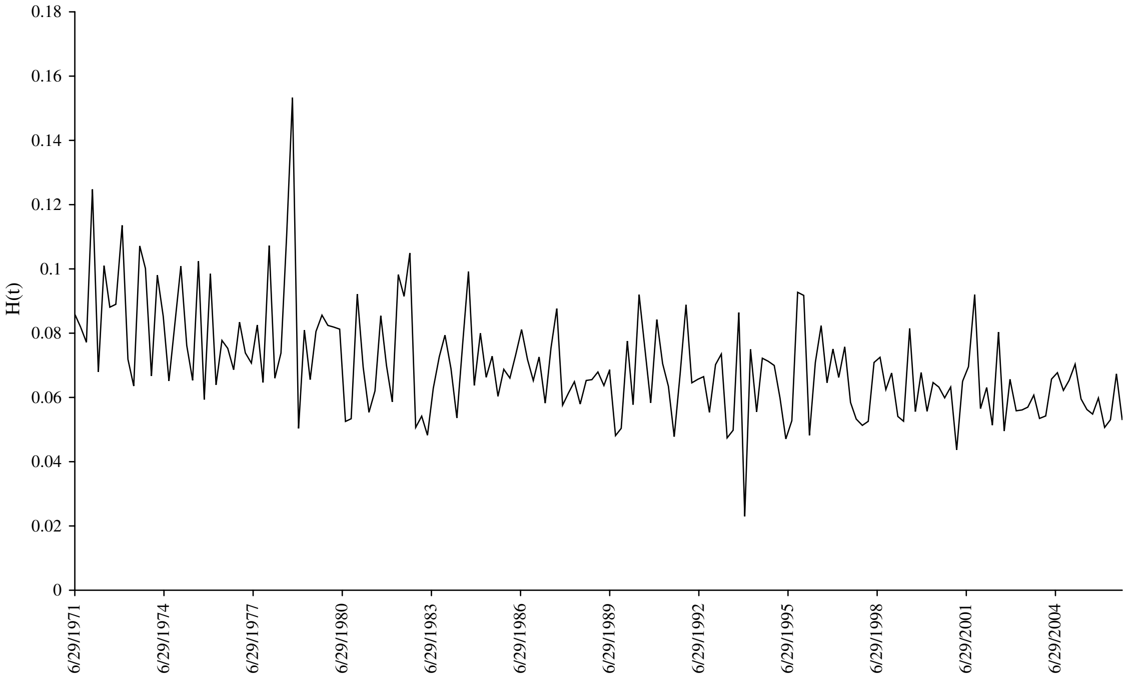

Fig. 8. Time dependent hurst exponent for the NYSE composite index.

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Table 1
Lo's modified $\widetilde{Q}_n$ analysis

Series	Test statistic	Critical values for H_0		
		90%	95%	99%
DJIA	1.46	[.861, 1.747]	[.809, 1.862]	[.721, 2.098]
S& P 500	1.45	[.861, 1.747]	[.809, 1.862]	[.721, 2.098]
NASDAQ composite	1.81	[.861, 1.747]	[.809, 1.862]	[.721, 2.098]
NYSE composite	1.22	[.861, 1.747]	[.809, 1.862]	[.721, 2.098]

To distinguish between long-range and short-range dependence, Lo's [14] modified R/S statistic is used, denoted here by $\widetilde{Q}_n$, and defined as

$$\widetilde{Q}_n \equiv \frac{1}{\hat{\sigma}_n(q)}\left[\max_{1\leqslant n\leqslant N}\sum_{t=1}^{n}(x(t)-\langle x\rangle)-\min_{1\leqslant n\leqslant N}\sum_{t=1}^{n}(x(t)-\langle x\rangle)\right],$$

with $\hat{\sigma}_n(q)$ estimated by

$$\hat{\sigma}_n^2(q)=\hat{\sigma}_x^2+2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j,$$

where $x(t)$ with $t = 1, \ldots ,N$ is the time series under consideration, $\langle x\rangle$ its sample mean, and $\hat{\sigma}_x^2$ and $\hat{\gamma}_j$ are estimates of the sample variance and autocovariances of x and the $\omega_j(q)$'s are weights suggested by Newey and West [21].

This procedure calculates the range of partial sums of deviations of $x(t)$ from $\langle x\rangle$, rescaled by its standard deviation, $\hat{\sigma}_n(q)$. This (standard deviation) adjustment captures possible short-range dependence yielding a more robust estimate of the long-range dependence properties of $x(t)$. Intuitively, the process involves comparing the dependence characteristics of the entire sample with those of continually shrinking subsamples. A linear relationship between $\widetilde{Q}_n$ and n on a log–log plot indicates the presence of power law (fractal) scaling.

A major shortcoming of rescaled range analysis was the lack of a systematic way of determining significance levels of the estimated Hurst exponent. Using Monte Carlo simulation techniques, Lo [14] calculated the distributional properties of $\widetilde{Q}_n$ in order to obtain the critical values necessary for drawing conclusions regarding statistical significance. We used Lo's [14] modified $\widetilde{Q}_n$ test statistic and tested the null hypothesis of no long-range dependence in each of the four US stock exchanges and report the results in Table 1.

A test of the null hypothesis is performed at the 95% confidence level by accepting or rejecting according to whethet the test statistic is or not contained in the interval [.809, 1.862]. Since the test statistics for the Dow Jones Industrial Average, the S&P 500, the NASDAQ Composite, and the NYSE Composite are 1.46, 1.45, 1.81, and 1.22, respectively, the null hypothesis cannot be rejected for all four stock market indexes.

6. Conclusion

We have calculated the Hurst exponent for four US stock market indexes – the Dow Jones Industrial Average, the Standard and Poor's 500 Index, the NASDAQ Composite Index, and the NYSE Composite Index – using a recently proposed (statistical physics) scaling technique – the detrending moving average (DMA). Our results confirm that US stock market returns display anti-persistence (that is, anti-correlation or mean reversion), meaning that if returns have been up (down) in the previous period, then they are more likely to be down (up) in the next period. We have investigated the robustness of our results to alternative testing methodologies, by using Lo's [14] modified rescaled range analysis.

It is to be noted that the conclusions reached in this paper are different from those in Serletis et al. [27] who use daily data on the Dow Jones Industrial Average, over the period from January 3, 1928 to March 15, 2006, and a different statistical physics approach – detrended fluctuation analysis (DFA). They conclude that the stock market in the United States is consistent with the efficient market hypothesis.

Acknowledgement

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