Option-implied Betas and Cross Section of Stock Returns

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ABSTRACT

On the basis of Buss and Vilkov (2012), we further compare different option-implied beta measures. We confirm that the implied beta method proposed by Buss and Vilkov (2012) (BV) outperforms other beta approaches. We also propose the implied downside beta methods and find that the BV implied downside beta performs best and offers an improvement over the BV implied beta. However, the implied (downside) beta-return relation is not robustness to firm-level variables, like firm size, book-to-market ratio, option-implied moments. These variables are correlated with option-implied betas, which can obscure the relation between implied betas/implied downside betas and stock returns.

Keywords: Option-implied Betas, Option-implied Downside Betas, Cross Section of Stock Returns

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I. Introduction

The Capital Asset Pricing Model (CAPM), introduced by Sharpe (1964), Lintner (1965) and Mossin (1966), independently, indicates that the relationship between market beta and stock return is positive, linear and significant and market betas are the only factor describing the cross-section of expected returns. It has been extensively tested throughout the world equity markets and its validity has often been questioned. For example, the famous study of Fama and French (1992) proves that the relation between market beta and average return disappears during the more recent 1963-1990 period of US stock return data even when beta is the only explanatory variable to average returns. Since there exist serious problems when historical stock return is used to model market betas, like sensitivity to minor changes in the time period used (see McNulty et al., 2002), two commonly used ways arise to improve the measures of traditional market betas: using option-implied information and introducing market downside measures.

The first way is to consider option-implied information in improving traditional betas. Many studies have demonstrated that option-implied volatility is a strong predictor of future volatility in equity markets (see Poon and Granger, 2003). Recent researchers also find that option-implied high moments (skewness and kurtosis) and correlation contain predictive information in the stock market (see Christoffersen et al., 2011). Based on option-implied information, French et al. (1983) (FGK) first introduce a hybrid estimation method to compute market betas using correlations from historical return and the ratio of stock-to-market implied volatilities. Chang et al. (2012) (CCJV) use both option-implied skewness and volatility to estimate market betas. The stock-to-market implied skewness ratio serves as a poxy for expected correlation. They find that the CCJV beta estimates perform relatively well and can explain a sizeable amount of cross-sectional variation in expected returns. Buss and Vilkov (2012) (BV) construct option-implied betas using option-implied correlation and volatility. They report that the BV betas confirm a monotonically increasing risk-return relation consistent with the indication of the CAPM model. Besides the option-implied beta estimation above, some other researchers, e.g. McNulty et al. (2002), Husmann and Stephan (2007), Fouque and Kollman (2011), find other ways to improve market betas using options and find that options can indeed improve the performance of historical betas. For example, Fouque and Kollman (2011) use a calibration technique for the beta parameter and it is calibrated from skews of implied volatilities.

The second common and efficient way to improve the general market betas is modelling downside market betas. Early study of Roy (1952) debates that investors care for more downside risk than upside gains, or simply, safety from disaster as foremost goal. Markowitz (1959) advocates to replace variance by semi-variance as a measure of risk, because semi-variance measures downside losses rather than upside gains. Price et al. (1982) show that the historical downside betas of U.S. stocks systematically differ from the regular betas. Specifically, the regular beta underestimates the risk for low-beta stocks and overestimates the risk for high-beta stocks. This finding may help explain why low-beta stocks appear systematically underpriced and high-beta stocks appear systematically overpriced in empirical tests of the mean-variance CAPM (see, for example, Jensen et al.,1972; Fama and MacBeth, 1973; and Fama and French, 1992). Researchers propose different ways to measure downside betas. For example, Hogan and Warren (1972) propose the semi-variance beta by replacing variance with semi-variance. Bawa (1975) and Bawa and Lindenberg (1977) develop and extend a proxy for downside beta as the Lower Partial Moment (LPM). Ang et al. (2006a) estimate downside betas based on the conditional downside covariance.

In this article, we use option prices on S&P 500 Index and its constituents from OptionMetrics to construct option-implied moments and then option-implied betas. We try to test whether option prices contain important information for the underlying equities to improve traditional betas. We compare the four beta methods—the Historical, FGK, CCJV and BV beta measures by performing portfolio analysis. We sort stocks into quintiles on the basis of ranked betas for each beta method at the end of each month and then calculate both value-weighted and equally-weighted portfolio returns in the next month. Based on the implied beta methods and downside correlations of Ang et al. (2002), this paper first proposes to model implied downside betas, including FGK and BV implied downside betas. We investigate whether the combination of option-implied information and downside measures can improve the general implied beta methods. We also test whether beta-return relation is affected by firm-level factors using portfolio analysis and Fama and MacBeth (1973) (FM) regression methods. These factors include firm size, book-to-market ratio, momentum, option-implied volatility, skewness, kurtosis, variance risk premium and illiquidity. To our knowledge, it is the first paper to study whether the option-implied betas or implied downside betas can be affected by firm-level variables systematically.

The reason to explore whether firm-level factors can help implied betas explain cross-section of stock returns is two folds. First, previous research has found that firm-level factors can help predict stock returns. It is well known that there exist firm size effect of Banz (1981), the book-to-market effect of Basu (1983), the momentum effect of Jegadeesh and Titman (1993). For option-implied moments, Yan (2011), Bali and Murray (2012), Conrad et al. (2013) find that there exists a negative relation between model-free implied skewness and stock returns. In contrary, Xing et al. (2010), Cremers and Weinbaum (2010), Rehman and Vilkov (2012) favour that option-implied ex ante skewness is positively related to future stock returns. In terms of variance risk premium, Bali and Hovakimian (2009) find a significantly negative relation between expected returns and the realized-implied volatility spread, while Bollerslev et al. (2009) find that the post-1990 aggregate stock market returns with high values of the variance risk premium associated with subsequent high returns. Second, these firm characteristics may explain option-implied betas. This is because option-implied betas are constructed by option-implied moments and recent research in risk-neutral moments demonstrates that some firm characteristics are related to option-implied moments. For example, Dennis and Mayhew (2002) investigate the relative importance of various firm characteristics (e.g. implied volatility, firm size, trading volume, leverage, and beta) in explaining the risk-neutral skewness implied from option prices. Hansis et al. (2010) find that option-implied risk-neutral moments (variance, skewness, and kurtosis) are well explained cross-sectionally by a number of firm characteristics. Buss and Vilkov (2012) just provide evidence that the relation between option-implied betas, especially the BV beta and returns is robustness to variance risk premium and option-implied skewness, but they do not study the impact of other firm characteristics, e.g. firm size, book-to-market ratio.

The main contributions of this paper are summarized as follows. First, we compare the Historical, FGK, CCJV and BV beta methods and find that the BV beta measure works best. A portfolio trading strategy that sells stocks ranked in the bottom quintile by BV implied betas and buys stocks in the top quintile by BV implied betas earns positive profit, which is about 6.241% per year for value-weighted return without dividend and 6.669% per year for equally-weighted return without dividend. The monotonicity relation (MR) test method proposed by Patton and Timmermann (2010) shows that only the BV beta has a monotonically increasing relation with equally-weighted return without dividend. This is consistent with the findings of Buss and Vilkov (2012), who find that the BV beta has a monotonically increasing relation with value-weighted returns. Second, we first develop implied downside betas based on the downside correlation of Ang et al. (2006a) and show that the BV implied downside beta performs best, and offers an improvement over the BV implied beta. This can be proved by the biggest return differential between the top and the bottom quintile portfolios sorted by the BV implied downside beta, which is 7.260% for value-weighted return without dividend and 7.551% for equally-weighted return without dividend. The MR test shows that only the BV implied downside beta has a monotonically increasing relation with equally-weighted return without dividend. The positive beta-return relation becomes more pronounced when using downside measures for the BV downside beta. However, the beta-return relation for BV implied betas and BV implied downside betas is not robust to firm-level factors. Once firm-level control variables are included in FM regression, the explanatory power of BV implied betas and downside betas disappears. It means that implied betas and implied downside betas are correlated with these firm-level control variables and this obscures the beta-return relation.

Our paper contributes to the literature that examines the relation between market betas and stock returns. First, our study complements the paper of Buss and Vilkov (2012), who have compared these four beta methods using portfolio analysis based on options on S&P 500 Index and its constituents from January 1996 to December 2009. Second, we contribute to literature on downside betas. Some researchers on downside betas, e.g. Post and Van Vliet (2005), Ang et al. (2006a), Tahir et al. (2013), report that downside risk based CAPM outperforms variance based CAPM. For instance, Post and Van Vliet (2005) support that the mean-semivariance CAPM strongly outperforms the traditional mean-variance CAPM in terms of its ability to explain the cross-section of US stock returns. Tahir et al. (2013) empirically test beta and downside beta based CAPM and find that downside based CAPM comes out to be strong contender compared to CAPM for risk-return relationship.

The rest of this paper is organized as follows: In Section II, we present the calculation of the four different beta methods (Historical, FGK, CCJV and BV betas), as well as the downside beta methods (including Historical, FGK and BV implied downside betas). Section III provides an overview of the data and methodology. Section IV discusses the empirical result for testing the risk-return relation with different implied beta methods. Section V presents the empirical result for the relation between different implied downside beta methods and returns. Section VI summarizes the main findings of this paper.

II. Models

A. Option-Implied Betas

There are four beta models used in this paper, which are historical betas, FGK, CCJV and BV implied beta methods. The calculation of the four different beta methods is shown as follows.

A.1. Historical Beta

Sharpe (1964), Lintner (1965) and Mossin (1966) independently propose the Capital Asset Pricing Model (CAPM), which asserts that the expected return for any individual asset is a positive function of only three variables: beta (the covariance of asset return and market return), the risk-free rate, and the expected market return. The relationship is shown below:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{1}$$

where R_i denotes return on stock i, R_f is risk-free interest rate and R_m is market return.

Let P denote the probability distribution function under the physical measure. The historical beta is usually calculated using the formula below:

$$\beta_{iM}^{His} = \rho_{i,M} \frac{\sigma_{i,t}^P}{\sigma_{M,t}^P} \tag{2}$$

where $\sigma_{i,t}^P$ and $\sigma_{M,t}^P$ are stock and index return standard deviations from historical data, respectively. $\rho_{i,M}$ is the correlation between stock and index returns. Traditionally, the historical beta is calculated using the historical rolling-window method.

A.2. FGK Beta

French et al. (1983) (FGK) first introduce a hybrid estimation method using option-implied volatility to improve the performance of beta forecasts. Let Q denote the probability function under the risk-neutral measure.

$$\beta_{iM}^{FGK} = \rho_{i,M} \frac{\sigma_{i,t}^{Q}}{\sigma_{M,t}^{Q}} \tag{3}$$

where $\sigma_{i,t}^{Q}$ and $\sigma_{M,t}^{Q}$ are option-implied volatility for stock i and index, respectively. $\rho_{i,M}$ is the correlation between historical stock and index returns.

A.3. CCJV Beta

Chang et al. (2012) (CCJV) suppose a one-factor model and assume zero skewness of the market return residual to propose a new market beta method by using both option-implied volatility and option-implied skewness.

$$\beta_{iM}^{CCJV} = \left(\frac{SKEW_{i,t}^{Q}}{SKEW_{M,t}^{Q}}\right)^{1/3} \frac{\sigma_{i,t}^{Q}}{\sigma_{M,t}^{Q}} \tag{4}$$

where $\sigma_{i,t}^{Q}$ and $\sigma_{M,t}^{Q}$ are option-implied volatility for stock i and index, respectively. $SKEW_{i,t}^{Q}$ and $SKEW_{M,t}^{Q}$ are option-implied skewness on stock i and index, respectively. $(\frac{SKEW_{i,t}^{Q}}{SKEW_{M,t}^{Q}})^{1/3}$ serves as a proxy for the risk neutral correlation.

A.4. BV Beta

Buss and Vilkov (2012) (BV) propose a new way to model implied beta by combining option-implied correlation with option-implied volatility.

First, we have one identifying restriction: the observed implied variance of the market index

$(\sigma^Q_{M,t})^2$ equals the implied variance of a portfolio of all market index constituents $i = 1, ..., N$:

$$(\sigma^Q_{M,t})^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma^Q_{i,t} \sigma^Q_{j,t} \rho^Q_{ij,t}, \tag{5}$$

where $\sigma^Q_{i,t}$ denotes the implied volatility of stock i in the index and ω_i represents the index weights.

Empirically, we use stock returns in the market index constituents to identify $N \times (N-1)/2$ physical correlations $\rho^P_{ij,t}$ and then transfer these into implied correlations $\rho^Q_{ij,t}$ [1].

$$\rho^Q_{ij,t} = \rho^P_{ij,t} - \alpha_t (1 - \rho^P_{ij,t}), \tag{6}$$

where $\rho^P_{ij,t}$ is the expected correlation under the physical measure, and α_t denotes the parameter to be identified.

Substituting the implied correlation in equation (5) by the implied correlation in equation (6), we can get the formula to compute α_t, which is shown below:

$$\alpha_t = -\frac{(\sigma^Q_{M,t})^2 - \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma^Q_{i,t} \sigma^Q_{j,t} \rho^P_{ij,t}}{\sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma^Q_{i,t} \sigma^Q_{j,t} (1 - \rho^P_{ij,t})}, \tag{7}$$

After estimating implied volatility and correlation, we can then compute option-implied BV

[1] Buss and Vilkov (2012) identify that the transfer must satisfy two technical conditions and two empirical observations. Two technical conditions are: (i) all correlations $\rho^Q_{ij,t}$ do not exceed one, and (ii) the correlation matrix is positive definite. Furthermore, our implied correlations are consistent with two empirical observations: (i) the implied correlation $\rho^Q_{ij,t}$ is higher than the correlation under the physical measure $\rho^P_{ij,t}$, (ii) the correlation risk premium is larger in magnitude for pairs of stocks that provide higher diversification benefits (i.e., low or negatively correlated stocks), and hence are exposed to a higher risk of losing diversification in bad times characterized by increasing correlations. The second empirical observation is supported by the negative correlation between the correlation under the objective measure and the correlation risk premium in Stathopoulos et al. (2012)

beta as:

$$\beta_{iM,t}^{BV} = \frac{\sigma_{i,t}^{Q} \sum_{j=1}^{N} \omega_j \sigma_{j,t}^{Q} \rho_{ij,t}^{Q}}{(\sigma_{M,t}^{Q})^2}, \tag{8}$$

B. Implied Downside Betas

In Section II.A, we have shown four ways to model betas. In this subsection, we show how to model downside betas. We find three ways to model downside betas, which are Historical, FGK and BV downside betas. We do not find an approach to construct CCJV downside betas. The CCJV beta is constructed from option-implied volatility and skewness (see equation (4)). Moedelling option-implied volatility or skewness does not need to use historical stock returns. However, historical stock returns are needed in modelling downside betas or correlations. Therefore, it is impossible to construct a downside analog of the CCJV beta due to its estimation nature.

For the historical downside beta, we follow the semi-variance beta approach of Hogan and Warren (1972). The computation of historical downside betas is shown as follows:

$$\beta_{\theta}^{D-His} = \frac{E[r_i r_M | r_M < \theta]}{E[r_M^2 | r_M < \theta]} \tag{9}$$

where the numerator is the second lower partial co-movement between the stock excess return r_i and the market excess return r_M. It measures the co-movements with the market during market downturns. θ is the threshold to define the downside market.

The principle to model implied downside betas is mainly based on modelling downside correlations. Ang et al. (2002) decompose the downside betas into a conditional correlation term and a ratio of conditional total volatility to conditional market volatility. The downside correlation is:

$$\rho_\theta^- = corr\{r_i, r_M | r_M < \theta\} = \frac{E[r_i r_M | r_M < \theta]}{\sqrt{E[r_i^2 | r_M < \theta] E[r_M^2 | r_M < \theta]}} \tag{10}$$

Following their way, we combine downside correlations and option-implied volatility to obtain option-implied downside betas. We substitute the historical correlation of the FGK beta in equation (3) by the downside correlation in equation (10) to obtain FGK implied downside betas.

$$\beta_\theta^{D-FGK} = \rho_\theta^- \frac{\sigma_{i,t}^Q}{\sigma_{M,t}^Q} \tag{11}$$

For the BV beta method, we use individual stock returns satisfying $[r_i | r_{M,t} < \theta]$ to calculate physical downside correlations ρ_θ^- and then get the BV option-implied downside betas following equations (5)-(8) in Section II.A.

III. Data and Methodology

A. *Data*

We use the daily data of the S&P 500 Index, and its constituents for a sample period from January 4, 1996 to December 31, 2012, a total of 4,278 trading days. The S&P 500 Index serves as a proxy for the US market.

The constituents of the S&P 500 Index are obtained from COMPUSTAT year by year. The total number of companies (including addition and deletion) in the index from 1996 to 2012 is reported in the second column of Table I. The financial statement data—like book value of common equity and balance-sheet deferred taxes used in this paper is also from COMPUSTAT. They are quarterly data, so we fill in the other months for each quarter. The daily S&P 500 Index and stock prices on its constituents are obtained from the Center for Research in Security Prices (CRSP). The index weights on each day are computed using closing market capitalization of all current index components from the previous day. It is the percentage of market value of security used, which is downloaded from index data. For stocks, firm size is measured by market capitalization, which is equal to equity price multiplying shares outstanding. Daily option data on the S&P 500 Index and its constituents with all maturities are obtained from OptionMetrics. This database contains data for all US exchange-listed equities and market indices, as well as for all listed US index and equity options. We extract secid, date, exdate, last date, call or put flag, strike price$\times$1000, best bid, best offer, volume, open interest rate, implied volatility, delta. For European options, implied volatilities are calculated using mid-quotes and the Black-Scholes formula. For American options, a binomial tree approach that takes into account the early exercise premium is employed. Treasury bills, as a proxy for risk-free interest rates are obtained from CRSP Treasuries database.

After obtaining the constituents of the S&P 500 Index from COMPUSTAT, stock data from CRSP and option data from OptionMetrics, we merge these three databases. First, we merge stock data with index constituents (COMPUSTAT) and delete stock data which are not in the index period. Second, we merge the combined stock data with option data. After merging these three databases, we get the number of companies in the index with both option and stock data, which is reported in the fourth column of Table I. The percentage of companies with both option and stock data is provided in the last column of Table I.

Sorted by secid or PERMNO, we have a total number of 922 companies with both option and stock data in our data sample from 1996 to 2012. From Table I, the companies in the S&P 500 Index with both option and stock data compose above 90% of all constituents. This percentage generally increases from 92.68% in 1996 to 100% in 2012. We find that for every year the number of companies in the S&P 500 Index exceeds 500 because of index additions and deletions. From the difference between Column 2 and 3 in Table I, we find that some firms are added and deleted in the S&P 500 Index in the same year.

[Insert Table I here]

After obtaining and merging data, we need to do data dealing in order to delete data that does not meet some standards. As in Bakshi et al. (1997), Bakshi et al. (2003) and Jiang and Tian (2005), Chang et al. (2012), we use the average of the bid and ask quotes for each option contract. We filter out average quotes less than \$3/8. We also filter out quotes that do not satisfy standard no-arbitrage conditions. We eliminate in-the-money options because they are less liquid than out-of-the-money and at-the-money options. We diminish the influence of an early exercise premium on our estimations. We eliminate put options with $K/S \geq 1.03$ and eliminate call options with $K/S \leq 0.97$.

B. Option-Implied Moments

Option-implied moments (including variance, skewness and kurtosis) are extracted from option data with the model-free approach. We follow the formula of Bakshi et al. (2003), who present a direct approach for the estimation of forward-looking risk-neutral variance, skewness, and kurtosis from option prices. Their method relies on a continuum of strikes and does not incorporate specific assumptions on an underlying model. The three moments can be expressed as functions of payoffs on a quadratic, a cubic, and a quartic contract. The detail for the three option-implied moments is provided in Appendix A. In order to calculate the integrals in the formulas precisely, we need a continuum of option prices. In practice, we do not have a continuum of option prices across moneyness, and we therefore have to make a number of choices regarding implementations. We approximate them from available option data: As in Carr and Wu (2009) and Jiang and Tian (2005), for each maturity, we interpolate implied volatilities using a cubic interpolation across moneyness levels (K/S) to obtain a continuum of implied volatilities. The cubic interpolation is only effective for interpolating between the maximum and minimum available strike price. For moneyness levels below or above the available moneyness level in the market, we simply extrapolate the implied volatility of the lowest or highest available strike price. For the risk-free interest rate, we also do the interpolation to get interest rates with different maturities.

After implementing the interpolation-extrapolation technique, we extract a fine grid of 1000 implied volatilities for moneyness levels between 1/3 and 3. Then we convert these implied volatilities into call and put option prices based on the following rule: moneyness levels less than 100% ($K/S \leq 1$) are used to generate put prices and moneyness levels greater than 100% ($K/S \geq 1$) are used to generate call prices. This fine grid of option prices is then used to compute option-implied moments by approximating the Quad, Cubic and Quartic contracts using numerical integration. It is important to note that this procedure does not assume that the Black-Scholes model correctly prices options. It merely provides a translation between option prices and implied volatilities.

For each day, we calculate risk-neutral moments using options with different maturities for one stock. In each calculation, we require that a minimum of two OTM calls and two OTM puts have valid prices. If not enough data are available, the observation is discarded. When using options with all maturities on each day for one stock, we may get option-implied volatility, skewness and kurtosis with various maturities for one stock on one particular day. We then linearly interpolate to get the 180-day VAR, SKEW and KURTOSIS, using both contracts with maturity more than 180 days and contracts with maturity less than 180 days. If there is only one maturity in a particular day, we do not do interpolation and just use this to represent the 180-day VAR, SKEW and KURTOSIS on that day. The choice of a 180-day horizon is to some extent based on a trade-off between option liquidity that is largest for options with 30-90 days to maturity and the relevant horizon for firm risk, which is arguably considerably longer (see Chang et al., 2012).

[Insert Table II here]

Table II presents summary statistics for option-implied volatility, skewness and kurtosis for the sample period from January 1996 to December 2012. It reports the number of observations, average, standard deviation, median as well as 25th, 75th percentiles of implied moments for both S&P 500 Index and its constituents. The average of S&P 500 Index volatility is 0.2422 and the average of stock volatility is 0.3934. It is obvious to find that the average of the S&P 500 Index volatility is less than the average of stock volatility. This is because stocks in the S&P 500 Index are not perfectly correlated. The average of S&P 500 Index skewness is -1.5342, more negative than the average of stock skewness, which is -0.4417. It shows that the distribution of both index and stock returns is negatively skewed. The average of S&P 500 Index kurtosis is 7.1139 and the average of stock kurtosis is 3.5738. The average of the S&P 500 Index kurtosis is much greater than the average of stock kurtosis. The averages of kurtosis for both index and stock are greater than 3, which indicates that the distribution of both index and stock returns has high peaks. Overall, the risk-neutral distribution of index return is more skewed and fat-tailed than stock risk-neutral distribution.

[Insert Figure 1 here]

Figure 1 displays some properties of the S&P 500 implied moments from January 1996 to December 2012. It is obvious that the S&P 500 Index option-implied volatility fluctuates between 0 and 0.6. The market becomes more fluctuant in the recent few years. Volatility peaks at 0.6 around the year 2008. The S&P 500 Index option-implied skewness is always negative from 1996 to 2012. It becomes more negative since 2008. The S&P 500 Index option-implied kurtosis always fluctuates above 3. The index kurtosis becomes even higher in the recent few years since 2008.

C. Option-Implied Betas and Downside Betas

In this subsection, we compute the different beta methods and downside beta methods discussed in Section II by applying daily data on S&P 500 Index and its constituents from January 1996 to December 2012.

Traditionally, historical betas are calculated using historical rolling windows. In our test, the historical beta is calculated using daily stock and index returns with the respective rolling-window length of 180 days (previous 126 trading days). For the FGK beta, the historical correlation is calculated using stock and index returns with the rolling-window length of 180 days (previous 126 trading days). Option-implied volatility is calculated following the BKM method. We put historical correlations and option-implied stock-to-market volatility ratio in equation (3) to calculate FGK market betas for each stock. For the CCJV beta, we compute model-free implied volatility and skewness of the BKM method and put them in equation (4) to get the CCJV betas. For the BV beta, we first estimate implied volatility following the BKM method. We then compute correlations under the physical measure using daily returns with the respective rolling-window length of 180 days. The correlations under the risk-neutral measure are computed following equation (6) after calibrating the only unknown parameter α_t from equation (7). Then the BV beta is computed from equation (8). The calculation of downside betas is very similar to the general betas. The only difference is that when measuring downside betas, we use stock or index returns in market downturns. Following the computation procedure of different beta methods, we obtain daily historical, FGK and BV betas, as well as downside betas for most stocks from the first trading day of July in 1996 to the last trading day of December in 2012. We get daily CCJV betas from the first trading day of January in 1996 to the last trading day of December in 2012.

[Insert Table III here]

Table III reports the descriptive statistics for different beta methods. Panel A presents the summary statistics for general beta methods. We find that for all these four beta methods, the average and value-weighted average of betas are around unit. Both the mean and value-weighted mean of historical betas are about 0.5% higher than 1. Both the mean and value-weighted mean of FGK betas are about 15% less than 1. For the CCJV beta, the mean and value-weighted mean are about 3% greater than 1. The mean and value-weighted mean of BV betas are about 7% greater than 1. The Historical, FGK and CCJV betas have median less than 1 and the BV beta has median around 1. Panel B reports the summary statistics for downside betas. The average of Historical and BV downside betas is slightly greater than unit. The mean of historical downside betas is 1.0081 and the mean of BV downside betas is 1.0976. For FGK downside betas, the average is less than 1, which is 0.6506.

IV. Option-Implied Betas and Stock Returns

As discussed in Section III.C, the calculation of the four beta methods exists difference and the summary statistics confirm that there is a variety among these four beta methods. We investigate whether the different beta methods can cause difference for the risk-return relationship. Alternatively, we find out which beta method gives better performance in terms of the positive, linear and significant beta-return relationship. In order to investigate this, we sort stocks into quintile portfolios based on the values of market betas discussed in Section IV.A. To see whether betas are related to firm-level variables, we adopt the portfolio analysis shown in Section IV.B. In Section IV.C, we conduct the Fama and MacBeth (1973) regression to further examine whether market betas can still predict stock returns, when controlling for different firm-level variables.

A. *Portfolio Analysis of Beta-Return Relation*

Before doing our analysis, we replicate the paper of Buss and Vilkov (2012) using the same sample and sample period. The replication result is shown in Appendix B. Our replication result is very close to the result shown in BV's original paper and it demonstrates that the results of our paper are reasonable.

In order to study the risk-return relation, we perform the portfolio sorting methodology similar to the early study of Jensen et al. (1972) and the recent study of Buss and Vilkov (2012). We sort the individual securities in S&P 500 Index into five groups at the end of each month, and separately for each beta method, according to their pre-ranked betas. The pre-ranked betas are estimated using previous 180-day (126-trading day) daily returns at the end of month t. Jensen et al. (1972), Fama and French (1992) use at least 24 months of 5-year monthly returns to calculate pre-ranked betas. Similar to them, we use 180-day rolling-window returns to calculate Historical, FGK and BV betas. For example, we begin by estimating the coefficient beta for a half-year period from January, 1996 to June, 1996 for all equities listed on the S&P 500 Index at the beginning of July 1996. These stocks are then ranked from low to high on the basis of the estimated pre-ranked betas and are assigned to five portfolios with equal number of securities — the 20% of stocks with the smallest betas to the first portfolio, the 20% of stocks with the biggest betas to the fifth portfolio and so on. For the CCJV beta, the pre-ranked beta is the implied beta calculated at the end of the month. After constructing the portfolios based on the pre-ranked betas, we calculate the value-weighted and equally-weighted averages of betas, the annualized value-weighted and equally-weighted average of return without dividend, holding period return, for each beta methodology, for each portfolio in the next month $t+1$. The entire process is repeated for each month after July of 1996 until December of 2012. Finally, we calculate the time-series means of betas and realized returns.

The CAPM implies a monotonically increasing pattern in the stock return ranked by their market beta. In order to test the monotonically increasing beta-return relation, we adopt the monotonicity relation (MR) test method proposed by Patton and Timmermann (2010). The null hypothesis is based on the sign of the return spread between the fifth portfolio (highest beta) and the first portfolio (lowest beta), named High-Low return spread. If the High-Low return spread is negative, we will test whether the beta-return relation is monotonically decreasing. If the High-Low return spread is positive, we will test whether the beta-return relation is monotonically increasing. The MR test result is decided by p-values. If the MR p-value is less than 5% and High-Low return spread is positive (negative), it means that there is a monotonically increasing (decreasing) risk-return relation.

[Insert Table IV here]

Table IV provides a summary of the mean expected betas and the mean realized return for the beta-sorted quintile portfolios from January 1996 to December 2012. Panel A reports the time-series average of portfolio betas and returns sorted on the Historical beta. For the value-weighted portfolios, the High-Low return spread is 3.545% for return without dividend and 2.174% for holding period return. For the equally-weighted portfolios, the High-Low return spread is 6.014% for return without dividend, 4.574% for holding period return. This is consistent with Fama and MacBeth (1973) and Jensen et al. (1972), who find evidence to support the positive risk-return relationship as predicted by the CAPM. However, the t-statistics of High-Low return spread do not exceed the 10% level threshold, which means that the positive beta-return relation is not significant. Panel B presents the quintile portfolio performance sorted on the FGK beta measure. For both the value-weighted and the equally-weighted portfolios, a long-short portfolio buying stocks in the highest beta quintile and shorting stocks in the lowest beta quintile produces positive average returns. For the value-weighted portfolios, the average is 2.376% per year for return without dividend and 1.097% per year for holding period return. For the equally-weighted portfolios, the average return is 5.577% for return without dividend and 4.422% for holding period return. The t-statistics of return difference between the fifth and first portfolio are in the range of 0.15 to 0.65 for both the value-weighted and equally-weighted portfolios, which are less than the 10% significance level. Panel C reports quintile portfolios sorted on the CCJV beta method. We find that the High-Low return spread is negative for both the value-weighted and the equally-weighted portfolios. The most negative return spread is -3.012% for equally-weighted holding period return and the least negative return spread is -0.518% for value-weighted return without dividend. The t-statistics of High-Low return spread for the CCJV beta indicate that the negative return difference is not significant. Chang et al. (2012) run a cross-section regression of stock returns on CCJV betas year by year and find that for some years the slopes of CCJV beta are negative. The results for portfolio analysis sorted on the basis of the BV beta is shown in Panel D. On a value-weighted basis, the portfolio return without dividend increases by 6.241% per year from 4.644% in the first portfolio to 10.884% in the fifth portfolio and the portfolio holding period return increases by 4.470% per year. On a equally-weighted basis, the portfolio return spread is 6.669% for return without dividend and 5.014% for holding period return. The t-statistics vary between 0.6 and 1, which are less than the 10% significance level.

We perform a formal monotonicity test of the risk-return relation, applying the non-parametric technique of Patton and Timmermann (2010). The result for the monotonicity relation test with p-values obtained from time-series block bootstrapping is shown in the last column of Table IV. From the MR test, we find that all MR p-values are greater than 10% except for equally-weighted return without dividend for the BV beta method, whose MR p-value is 0.093. There is no significant evidence to support that there is a monotonically increasing relation between all betas and returns. Buss and Vilkov (2012) find that there is a monotonically increasing relation between the BV beta and value-weighted returns. Here, we find a monotonically increasing relation between the BV beta and equally-weighted returns without dividend at the 10% significance level. Although our finding is slightly different from Buss and Vilkov (2012) in the weighting scheme, it still confirms the result of Buss and Vilkov (2012) that the BV implied beta performs best.

To summarize, Table IV shows that the relationship between Historical, FGK, BV betas and returns is positive, but it is not significant indicated from the p-values of High-Low spread (greater than 10%). More importantly, the BV beta method gives the biggest value-weighted and equally-weighted return spread across High and Low portfolios (including return without dividend and holding period return). For example, the return spread of the BV beta is about 2.7% greater than that of the Historical beta and 3.9% greater than that of the FGK beta for value-weighed return without dividend. Our findings are consistent with Buss and Vilkov (2012), who find that the BV beta has the biggest High-Low return spread.

[Insert Figure 2 here]

Figure 2 shows that all beta methods display a noisy beta-return relation across different quintiles for value-weighted return without dividend. For the value-weighted portfolios, the return difference between the extreme quintile portfolios is more pronounced for the BV and Historical betas rather than for the FGK and CCJV beta methods. The High-Low return spread for FGK and CCJV betas, is 2.376% and -0.518%, respectively. For Historical and BV betas, the average High-Low return difference is 3.545% and 6.241%, respectively. The return spread of the BV beta is more pronounced than that for the Historical beta. The return difference for the BV beta is about 2.70% greater than that of the Historical beta. The plot of the BV beta and value-weighted returns shows that the pattern is closest to linear compared with Historical, FGK and CCJV betas. The equally-weighted return for both Historical and BV beta methods displays a monotonically increasing risk-return relation, but the MR test in the last column of Table IV proves that only the BV beta has a monotonically increasing risk-return relation at the 10% significance level.

Overall, the results in Table IV and Figure 2 indicate that the positive relation between market beta risk and returns is much more pronounced for the BV beta, consistent with the findings of Buss and Vilkov (2012).

B. *Firm-Level Factors Affecting Beta-Return Relation*

In Section IV.A, we use portfolio analysis to examine the risk-return relation for historical and option-implied betas. In this subsection, we do the robustness test to see whether the beta-return relationship can be affected by some other factors related to firm characteristics or option-implied moments. If there is a difference in the distribution of these characteristics across beta-sorted portfolios for the different beta methods, then the risk-return relation may be not robustness[2].

[2] Buss and Vilkov (2012) identify three conditions to see whether beta-return relation is robustness to these factors. The three conditions are shown as follows: (i) there must be a sizable difference and clear pattern in the average characteristic value of the beta-sorted portfolios. (ii) there must be strong differences in the average characteristic value of the portfolios across beta methodologies. (iii) the return pattern arising from the predictive power of a given characteristic and the beta-return relation for our betas must work in the same direction. Importantly, all three conditions must be satisfied to give rise to a spurious risk-return relation

Buss and Vilkov (2012) show that the risk-return relation for the BV beta cannot be explained by variance risk premium and option-implied skewness. In this subsection, we use portfolio analysis to test whether betas have a clear pattern with firm size, book-to-market ratio, option-implied volatility, skewness and kurtosis, and variance risk premium.

Banz (1981) may be the first empirical paper to present evidence of a size effect in US stock returns that there is a negative relation between firm size and stock returns. Similar findings of the size effect are found by Levis (1985) in the UK market and Ho et al. (2000) in the Hong Kong market. Basu (1983) documents the book to market effect that book-to-market ratio has a positive relation with returns. Studies of the Japanese, UK and Hong Kong markets by Chan et al. (1991), Chan and Chui (1996) and Ho et al. (2000), respectively, have found similar results for book to market effect. Fama and French (1992) confirm that there exist the size effect and book-to-market effect. The reason for including volatility is that Ang et al. (2006b) show that high historical volatility strongly predicts low subsequent returns. The evidence for the explanatory power of implied skewness to stock return is provided by recent studies (e.g. Xing et al., 2010; Cremers and Weinbaum, 2010; Rehman and Vilkov, 2012; Conrad et al., 2013). Some few researchers find the effect of implied kurtosis on returns (e.g. Diavatopoulos et al.,2012; Bali et al., 2014). In terms of variance risk premium, Bali and Hovakimian (2009) find a significantly negative relation between expected returns and the realized-implied volatility spread, while Bollerslev et al. (2009) find that the post-1990 aggregate stock market returns with high values of the variance risk premium associated with subsequent high returns.

The calculation of firm size and book-to-market ratio follows Fama and French (1992). *Firm* *size* - ln(ME) is the log of the market capitalization from the previous day. Market capitalization - ME is equal to stock price multiplying shares outstanding. *Book-to-market ratio* is equal to ln(BE/ME), where BE is the book value of common equity plus balance-sheet deferred taxes. *Implied volatility*, *skewness* and *kurtosis* are calculated following the BKM method. *Variance risk premium*(VRP) is the difference between the realized and implied variance, similar to Carr and Wu (2009).

As in Merton (1980) and Andersen et al. (2003), the realized physical (annualized) variance is computed in a model-free manner, using daily stock returns.

$$\sigma_P^2(t) = \frac{252}{21}\sum_{l=1}^{21}(R(t+l)-\overline{R})^2 \tag{12}$$

The variance risk premium is defined as the difference between the realized and implied variance.

$$VRP(t) = \sigma_P^2(t) - \sigma_Q^2(t) \tag{13}$$

where $\sigma_P^2(t)$ and $\sigma_Q^2(t)$ denote the realized and implied variances on day t, respectively.

[Insert Table V here]

Table V shows the average of firm characteristics and option-implied moments from 1996 to 2012 for portfolios formed on different beta methods. Panel A presents the portfolio result based on Historical betas. The High-Low spread for option-implied volatility, skewness and variance risk premium is positive and significant at the 5% significance level. However, the relation between historical betas and option-implied skewness is significantly negative. We also find that the historical beta has a monotonically increasing relationship with option-implied volatility and a monotonically decreasing relationship with implied kurtosis. From the results in Panel B, we find that the FGK beta is negatively correlated with firm size, option-implied kurtosis, and the decreasing relation is monotonic. The FGK beta has a positive relation with book-to-market ratio, option-implied volatility, skewness and variance risk premium. The findings in Panel C report that the CCJV beta is significantly and negatively related to option-implied firm size and option-implied skewness. It has a monotonically decreasing relationship with option-implied skewness. However, the CCJV beta has a monotonically increasing relation with option-implied volatility. Panel D presents that there is a significantly positive relation between the BV beta and book-to-market ratio, option-implied volatility and skewness, while there is a negative relation between the BV beta and firm size and option-implied kurtosis. The relationship between the BV beta and firm size, book-to-market ratio, option-implied volatility and kurtosis is monotonic. Overall, we see clear patterns of betas with some of these factors. Thus the beta-return relationship may be blurred by these factors.

Since we can see clear patterns between betas and some of these factors when portfolios are sorted by the different beta methods in Table V, next we check whether these factors have a significant relationship with returns. The procedure is similar to the portfolio analysis for the beta-return relation in Section IV.A. At the end of each month, we calculate the firm-level variables, including firm size, book-to-market ratio, option-implied volatility, skewness, kurtosis, and variance risk premium. We sort the individual securities in S&P 500 Index into five groups at the end of each month, and separately for each variable, according to the value of these factors. After constructing the portfolios based on these variables, we calculate annualized value-weighted and equally-weighted return without dividend, holding period return for each variable, for each portfolio in the next month *t+1*. The entire process is repeated for each month in the whole sample period. Finally, we calculate the time-series average of realized returns.

[Insert Table VI here]

Table VI reports the quintile portfolios formed on firm-level factors from January 1996 to December 2012. Panel A reports that the relation between firm size and returns is negative and significant at the 10% significance level. The MR p-value shows that the size-return relation is monotonically decreasing at the 10% significance level. This is consistent with the size effect by Banz (1981) and Fama and French (1992). However, the negative relation between firm size and return is only significant at the10% level. This is because the firms in our sample are the constituents of S&P 500 Index, which are relatively big companies. Panel B presents the results for portfolio analysis based on book-to-market ratio. The relation between book-to-market ratio and returns is positive. For both value-weighted and equally-weighted returns, the High-Low return spread is not significant at the 10% significance level. The MR p-value shows that the relation between book-to-market ratio and return is not monotonically increasing. The spread t-statistics and MR p-value in Panel C show that model-free implied volatility has no significant and positive relation with returns. In Panel D, we find that there exists a significant and positive relation between model-free implied skewness and stock returns, consistent with Xing et al. (2010), Cremers and Weinbaum (2010), Rehman and Vilkov (2012). However, our findings are in contrast to Conrad et al. (2013), Bali and Murray (2012), who find that option-implied ex ante skewness is negatively related to future stock returns. Panel E shows that model-free implied kurtosis is negatively related to both value-weighted and equally-weighted returns, but the High-Low return spread is not significant at the 10% significance level. The MR p-value shows that the kurtosis-return relation is not monotonic. For the portfolios sorted by variance risk premium shown in Panel F, the High-Low return spread is negative and significant for both value-weighted and equally-weighted returns at the 10% significance level. The MR test shows that there is a monotonically decreasing pattern between variance risk premium and value-weighted returns at the 1% significance level. The findings are in line with the result of Bali and Hovakimian (2009), who find a significantly negative relation between expected returns and the realized-implied volatility spread.

We have a close look at the explanation of the negative relation between the CCJV beta and returns shown in Table IV. The CCJV beta is composed by implied skewness and volatility. From Table V, we find a monotonically decreasing relation between the CCJV beta and option-implied skewness. From Panel C of Table VI, we find that there exists a positive relation between model-free implied skewness and stock returns. When the skewness is more negative (smaller), the CCJV beta becomes bigger; the portfolio returns become smaller. Therefore, the negative beta-return relation can be explained by the skewness-return relationship.

Overall, the portfolio analysis in Table VI shows that firm size, option-implied skewness and variance risk premium have a significant relationship with returns. Firm size and variance risk premium are negatively related to returns, while option-implied skewness is positively related to returns. From Table V, we find that all beta methods are linked to some of the firm factors.

C. Fama-MacBeth (1973) Regression

Through portfolio analysis, we can see a simple picture of how average portfolio returns vary across the spectrum of betas and other variables. However, portfolio analysis has its own potential pitfalls to test the risk-return relation (see Fama and French, 2008). For example, portfolio sorts are clumsy for examining the functional form of the relation between average returns and the variables. Therefore, we adopt the Fama and MacBeth (1973) (FM) regression to further test whether the risk-return relationship is robustness to firm-level variables. We perform a cross-sectional regression of value-weighted returns on one or more explanatory variables monthly. We then calculate the time-series average of the cross-sectional regression coefficients. In addition to enabling us to control for multicollinearity among the explanatory variables, the slope coefficients from the regression analysis can also be interpreted as the risk premium associated with taking one unit of risk associated with each of the risk variables.

Except betas, we include additional control variables in the month-by-month FM regression. Some previous literature supports there exist firm size effect of Banz (1981), the book-to-market effect of Basu (1983), the momentum effect of Jegadeesh and Titman (1993), the volatility effect of Ang et al. (2006b), exposure to implied skewness and kurtosis risk (see Cremers and Weinbaum, 2010), exposure to illiquidity risk of Amihud (2002), and exposure to the variance risk premium (see Bali and Hovakimian, 2009). All these factors imply different patterns for the cross section of stock returns. To control for these effects, we define firm size and book-to-market ratio variables following Fama and French (1992). We calculate option-implied moments—volatility, skewness, kurtosis following the method of Bakshi et al. (2003). *Variance risk premium*(VRP) is the difference between the realized and implied variance. *Momentum* is the cumulative daily return over the previous six months. *Illiquidity* is defined as the average ratio of the daily absolute return to the (dollar) trading volume on that day, $|R_{iyd}|/VOLD_{iyd}$. R_{iyd} is the return on stock i on day d of year y and $VOLD_{iyd}$ is the respective daily volume in dollars. It is followed by Amihud (2002), who finds that expected market illiquidity has a positive and significant effect on ex ante stock excess return, and unexpected illiquidity has a negative and significant effect on contemporaneous stock return.

The standard FM regression has two stages. Before the two stages, we first sort stocks into quintile portfolios according to the pre-ranked betas at the end of month t, for each beta methodology. We then calculate the annualized value-weighted average of stock returns in the next month $t+1$. We also calculate the average of beta, firm size, book-to-market ratio, option-implied volatility, skewness, kurtosis, variance risk premium, momentum and illiquidity at the end of month t. In the first stage, we estimate the following regression in cross section for each month t:

$$r_{p,t+1} = \gamma_{0.t} + \gamma_{2,t}\beta_{p,t} + \phi_t' Z_{p,t} + \varepsilon_{p,t} \tag{14}$$

where $r_{p,t+1}$ is the portfolio return for portfolio p in month $t+1$. $\beta_{p,t}$ is portfolio betas for portfolio p in month t. $Z_{p,t}$ is the other explanatory variables for portfolio p in month t.

After obtaining a time series of slope coefficients, the second stage of the standard FM regression is to calculate the time-series average of these coefficients. With the FM regression, we can easily examine the significance of the predictability of betas, as well as we can control for several firm characteristics at the same time.

[Insert Table VII here]

Table VII presents the results for FM regressions of value-weighted portfolio returns on different betas and firm-level variables. In the first regression of each panel, betas are the only explanatory variable. We find that the coefficients of all beta methods are positive but not always significant. The coefficient of the FGK beta in Regression 1 of Panel B is 0.126 with t-statistic of 1.73. The coefficient of the BV beta in Regression 1 of Panel D is 0.183 with t-statistic of 2.28. The coefficients of the FGK, BV betas are significant at the 10% and 5% significance levels, respectively.

In order to separate the predictive power of betas from other control variables, we consider these control variables in the FM regression. Panel A shows that when the historical beta is the only dependent variable in Regression 1, the coefficient of the historical beta is 0.083, which is not significant. When adding all other control variables in the FM regression (Regression 2-4), the historical beta still has no explanatory power to returns. The coefficients of historical betas even become negative. This result is consistent with Fama and French (1992), who find that the beta-return relation is very flat even when beta is the only explanatory variable. In Panel B, the FGK beta coefficient is 0.126 in Regression 1 and significant at the 10% level. We find that the significant coefficient of the FGK beta disappears when other control variables are included in the FM regression (Regression 2-4). In Regression 1 of Panel C, we find that the coefficient of the CCJV beta is 0.101 (but insignificant), when only beta is included in the FM regression. But when other control variables are added in Regression 2-4, the coefficient of the CCJV beta even becomes significantly negative. From Regression 1 of Panel D, we find that when the BV beta is the only explanatory variable in the FM regression, it gives the positive coefficient, which is 0.183 and significant at the 5% significance level. The BV beta outperforms the other beta methods. This is consistent with the portfolio analysis result in Table IV, which gives the biggest High-Low spread. However, this positive relation for BV beta does not persist when other explanatory variables are allowed in the FM regression indicated from Regression 2-4 in Panel D. When including option-implied volatility, skewness, kurtosis and variance risk premium, the coefficient of the BV beta becomes significantly negative (see Regression 3 of Panel D). When including all control variables, the coefficient of the BV beta becomes insignificantly negative (see Regression 4 of Panel D).

We now make an interpretation to the FM regression result in Table VII. When beta is the only explanatory variable to returns, the relation between the BV beta and returns is significantly positive and it outperforms the other beta methods. However, the positive BV beta and return relation disappears when other control variables are included in the FM regression. From Table V, we find that all beta methods have clear patterns with some of the control variables included in the FM regression in Table VII. Especially for the BV beta, it has a relationship with firm size, book-to-market ratio, implied volatility and kurtosis. When we add option-implied moments and variance risk premium in Regression 3 of Panel D, the coefficient of option-implied volatility becomes significant. Only one possibility to explain this is that betas are correlated with other explanatory variables, and this obscures the relation between betas and stock return. This can also be demonstrated by previous research in cross-sectional analysis of risk-neutral moments who finds option-implied moments can be explained by firm characteristics. More specifically, Hansis et al. (2010) find that the option-implied risk-neutral moments (variance, skewness , and kurtosis) of the BKM method are well explained cross-sectionally by a number of firm characteristics (like size and market-to-book ratio of equity). Dennis and Mayhew (2002) investigate the relative importance of various firm characteristics (e.g., implied volatility, firm size, trading volume, leverage, and beta) in explaining the risk-neutral skewness implied from option prices. Taylor et al. (2009) find that risk-neutral skewness of individual firms varies significantly and negatively with firm size, firm systematic risk and market volatility. Overall, the positive beta and return relation is not robustness to the firm-level variables.

V. Implied Downside Betas and Stock Returns

In this section, we examine whether the risk-return relation is improved when using implied downside betas. We include the Historical, FGK and BV downside beta methods in this section. The construction of implied downside betas follows Ang et al. (2002), who decompose the downside betas into a conditional correlation term and a ratio of conditional total volatility to conditional market volatility. The computation of downside betas follows the formulas in Section II.B. In order to demonstrate the predictability of downside betas, we conduct the portfolio analysis in Section V.A and the FM regression in Section V.B.

A. Portfolio Analysis

In order to study the downside risk-return relation, we perform the portfolio sorting methodology similar to the procedure illustrated in Section IV.A. We sort the individual securities in S&P 500 Index into five groups at the end of each month, and separately for each downside beta method, according to their pre-ranked downside betas. Portfolio 1 includes firms with the lowest downside betas, and portfolio 5 contains firms with the highest downside betas. We then calculate the value-weighted and equally-weighted downside betas, annualized value-weighted and equally-weighted return without dividend, holding period return for each beta methods, for each portfolio in the next month. The procedure is repeated for all months. Finally, we calculate the time-series average of downside betas and realized returns.

[Insert Table VIII here]

Table VIII provides a summary of the mean downside betas and the mean realized returns for the downside beta sorted quintile portfolios. The table shows that the High-Low return spread is positive for Historical, FGK and BV downside beta methods in most cases. Taking the value-weighted return without dividend for example, the High-Low return spread is 3.401% for the Historical downside beta in Panel A, 0.443% for the FGK downside beta in Panel B and 7.260% for the BV downside beta in Panel C. The portfolio sorting method shows that there is a positive relationship between Historical, FGK, BV downside betas and returns. From the t-statistics of these High-Low return spreads, we find that no one exceeds the 10% significance level. More importantly, we find that the BV implied downside beta gives the biggest value-weighted and equally-weighted return spread across high and low portfolios (including return without dividend and holding period return), consistent with the result for general beta methods in Table IV.

From the monotonicity relation test in the last column of Table VIII, we find that all MR p-values are greater than 10% for value-weighted returns, which means that there is no monotonically increasing relation between all downside betas and value-weighted returns. However, there exists some monotonically increasing relations for the equally-weighted portfolios. For example, the historical downside beta gives MR p-value equal to 0.085 (less than 10%) for returns without dividend in Panel A. It means that there exists a monotonically increasing relation between the historical downside beta and equally-weighted return without dividend at the 10% significance level. The BV downside beta gives the MR p-value equal to 0.042 (less than 5%) for return without dividend and 0.066 (less than 10%) for holding period returns. There exist a monotonically increasing relation between the BV downside beta and equally-weighted return without dividend at the 5% significance level and a monotonically increasing relation between the BV downside beta and equally-weighted holding period return at the 10% significance level. Compared with the Historical and FGK downside methods, the monotonically increasing relation between the BV downside beta and return is more pronounced.

We now have a close look at the comparison of BV implied downside betas with BV implied betas. The BV implied downside beta performs better than the BV beta in terms of the positive, significant and linear beta-return relation. More specifically, the BV downside beta gives the average High-Low return spread of 7.260% for value-weighted return without dividend in Panel C of Table VIII, which is 1.02% greater than that of the BV implied beta (It is 6.241%) shown in Panel D of Table IV. The monotonicity relation test for the BV implied beta gives the MR p-value of 0.093 for equally-weighted return without dividend in Panel D of Table IV. Panel C of Table VIII shows the MR p-value of 0.042 for the BV implied downside beta. The monotonically increasing relation between the BV downside beta and the equally-weighted returns becomes more significant. The result that BV downside beta outperforms BV beta is consistent with the research in downside stock markets (e.g. Ang et al.,2006a; Post and Van Vliet,2005). For instance, Post and Van Vliet (2005) find that the Mean Semivariance CAPM strongly outperforms the traditional Mean Variance CAPM in terms of its ability to explain the cross-section of US stock returns.

[Insert Table 3 here]

Figure 3 shows that all downside beta methods display a noisy beta-return relation across different quintile portfolios for value-weighted returns. For the value-weighted portfolios, the return difference between the extreme quintile portfolios is more pronounced for the BV downside beta rather than for the Historical and FGK downside beta methods. The FGK downside beta method gives the most flat beta-return relation with 0.443% return spread, while the BV downside beta displays a relatively increasing risk-return relation to some extent with 7.260% return spread. The equally-weighted quintile portfolio returns for the historical and BV downside beta methods display a monotonically increasing risk-return relation, and the MR test proves that the Historical and BV downside betas indeed have a monotonically increasing risk-return relation at the10% significance level.

Overall, the results in Table VIII and Figure 3 indicate that the BV downside beta outperforms the Historical, FGK downside beta methods. Additionally, the BV downside beta offers an improvement over the BV beta. The positive relation between market beta risk and returns is much stronger and more pronounced for the BV downside beta than the general BV beta method.

B. Fama-MacBeth Regression

After the portfolio analysis sorted on the basis of downside betas in Section V.A, we run the month-by-month FM regression in this subsection. Except different downside betas, it includes the control variables—ln(firm size), ln(book-to-market ratio), implied volatility, skewness, kurtosis, variance risk premium, momentum, and illiquidity.

[Insert Table IX here]

Table IX presents the results for FM regressions of value-weighted returns on different downside betas and other control variables. The first regression in each panel shows the result for the FM regression on only downside betas. When doing regressions on only downside betas, we find that the coefficients of all downside beta methods are positive but not always significant. Only the coefficient of the BV downside beta in Regression 1 of Panel C is 0.219 with t-statistics of 2.31, significant at the 5% significance level. This is consistent with the portfolio analysis result in Table VIII, which gives the biggest High-Low spread—7.260% per year. However, this positive relation for BV downside is not robust when other explanatory variables are allowed in Regression 2-4 in Panel C. When including option-implied volatility, skewness, kurtosis and variance risk premium, the coefficient of the BV downside beta becomes negative and insignificant (see Regression 3 of Panel C). When including all control variables, the slope of the BV downside beta becomes significantly negative (see regression 4 of Panel C). Only one possibility to explain this is that BV downside betas are correlated with other explanatory variables, and this obscures the relation between BV downside betas and returns.

Therefore, the FM regression confirms that the BV downside beta outperforms the Historical, and FGK downside beta methods in terms of the positive beta-return relation. However, this kind of positive relationship disappears when other control variables are included in the cross-section FM regression, which is similar to the findings of the BV beta in Table VII.

VI. Conclusion

On the basis of previous research of Buss and Vilkov (2012), this paper further investigates the relation between option-implied betas and returns using portfolio analysis and FM regression. We employ options on S&P 500 Index and its constituents from January 1996 to December 2012.

Consistent with Buss and Vilkov (2012), we find that the BV beta outperforms the other beta methods, which gives the biggest positive High-Low return spread through the portfolio analysis. A long-short portfolio buying stocks in the highest BV beta quintile and shorting stocks in the lowest BV beta quintile produces positive average returns, which is 6.241% per year for value-weighted return without dividend. The BV beta has a monotonically increasing relation with equally-weighted return without dividend at the 10% significance level, while there is no monotonically increasing or decreasing relation between other betas and returns for both the value-weighted and the equally-weighted portfolios.

This paper first proposes to combine downside correlations of Ang et al. (2002) and option-implied moments of Bakshi et al. (2003) to measure implied downside betas following previous implied beta methods of FGK and BV. When sorting stocks on downside betas, we find that the BV downside beta performs best, which gives the biggest High-Low return spread compared with the Historical and FGK downside betas. The monotonicity test (MR) shows that the BV downside beta is more pronounced in terms of the monotonically increasing beta-return relation than the Historical and FGK downside betas. The BV downside beta has a monotonically increasing relation with equally-weighted return without dividend and holding period returns at the 5% and 10% significance levels, respectively.

Additionally, the BV downside beta can improve the performance of the BV beta in terms of the beta-risk relation. The BV downside beta gives1.02% bigger High-Low return spread than that of the general BV beta for value-weighted return without dividend. The MR test shows that the monotonically increasing beta-return relation becomes more significant when using the downside measures. The BV downside beta has a monotonically increasing relation with equally-weighted return without dividend and holding period returns at the 5% and 10% significance levels, respectively. The BV beta only has a monotonically increasing relation with equally-weighted return without dividend at the 10% significance level.

However, we find that the implied (downside) beta-return relation is not robust to firm characteristics. The positive beta-return relation for BV betas and BV downside betas disappears when other control variables are included in the FM regression. For example, when implied betas or implied downside betas are the only explanatory variable in the cross-sectional FM regression, the coefficients of BV betas and BV downside betas are significant and positive. When considering other control variables, we find that the coefficients of BV betas and BV downside betas become insignificant or even negative. Overall, we find that BV betas and BV downside betas are correlated with other explanatory variables, and this obscures the relation between betas and returns.

Appendix

A. *Estimation of Risk-Neutral Moments–Details*

In this section, we show how to compute risk-neutral model-free variance, skewness and kurtosis from individual or index options. We follow the formulas in Bakshi and Madan (2000) and Bakshi et al. (2003). Bakshi and Madan (2000) show that the continuum of characteristic functions of risk-neutral return density and the continuum of options are equivalent classes of spanning securities. Any payoff function with bounded expectation can be spanned by out-of-the-money (OTM) European calls and puts. Based on this insight, Bakshi et al. (2003) formalize a mechanism to extract the variance, skewness and kurtosis of the risk-neutral return density from a contemporaneous collection of OTM calls and puts.

Let the $\tau-$period return be given by the log price relative:

$$R(t,\tau) = ln[S(t+\tau)] - ln[S(t)]. \tag{15}$$

Define the quadratic, cubic and quartic contracts with the following payoffs:

$$H[S] = \begin{cases} R(t,\tau)^2, & \text{quadratic contract} \\ R(t,\tau)^3, & \text{cubic contract} \\ R(t,\tau)^4, & \text{quartic contract} \end{cases} \tag{16}$$

Let E^Q denote the expected value operator under the risk neutral measure. The variance, skewness and kurtosis under the risk-neutral measure are defined as

$$\begin{aligned}
VAR =& E^Q[(R - E^Q[R])^2], \\
SKEW =& \frac{E^Q[(R - E^Q[R])^3]}{VAR^{3/2}}, \\
KURTOSIS =& \frac{E^Q[(R - E^Q[R])^4]}{VAR^2},
\end{aligned} \tag{17}$$

Following BKM, we define the 'Quad', 'Cubic' and 'Quartic' contracts as having a payoff function equal to the squared return and cubic return, respectively, for a give maturity τ. The fair value of these contracts are

$$\begin{aligned}
Quad =& e^{-r\tau}E^Q[R^2], \\
Cubic =& e^{-r\tau}E^Q[R^3], \\
Quartic =& e^{-r\tau}E^Q[R^4]
\end{aligned} \tag{18}$$

The price of the Quadratic, Cubic and Quartic contracts is

$$\begin{aligned}
Quad &= \int_S^\infty \frac{2(1 - ln(K/S))}{K^2} C(\tau, K) dK + \int_0^S \frac{2(1 + ln(S/K))}{K^2} P(\tau, K) dK \\
Cubic &= \int_S^\infty \frac{6ln(K/S) - 3(ln(K/S))^2}{K^2} C(\tau, K) dK - \int_0^S \frac{6ln(S/K) + 3(ln(S/K))^2}{K^2} P(\tau, K) dK \\
Quartic &= \int_S^\infty \frac{12(ln(K/S))^2 - 4(ln(K/S))^3}{K^2} C(\tau, K) dK + \int_0^S \frac{12(ln(K/S))^2 + 4(ln(K/S))^3}{K^2} P(\tau, K) dK
\end{aligned} \tag{19}$$

where S, K is the underlying stock price and strike price, respectively. C and P are call and put price, respectively.

Substituting these expressions into the variance, skewness and kurtosis formulas, we then get the risk-neutral three moments:

$$\begin{aligned} VAR =& e^{r\tau} Quad - \mu^2 \\ SKEW =& \frac{e^{r\tau} Cubic - 3\mu e^{r\tau} Quad + 2\mu^3}{VAR^{3/2}} \\ KURTOSIS =& \frac{e^{r\tau} Quartic - 4\mu e^{r\tau} Cubic + 6\mu e^{r\tau} Quad - 3\mu^3}{VAR^2} \end{aligned} \tag{20}$$

where

$$\mu = e^{r\tau} - 1 - \frac{e^{r\tau} Quad}{2} - \frac{e^{r\tau} Cubic}{6} - \frac{e^{r\tau} Quartic}{24} \tag{21}$$

B. *Results for Replicating Buss and Vilkov (2012)*

B.1. Data Comparison

In order to compare our result with Buss and Vilkov (2012) efficiently, we provide the result of replication of Buss and Vilkov (2012). It is necessary to point out the same and different points in our analysis and in the original paper. For the same aspects, both papers use the S&P 500 Index and its constituents. The sample period is ranged from January 4, 1996 to December 31, 2009. We use the same data dealing standard. We select out-of-the-money options (puts with deltas strictly larger than -0.50 and calls with deltas smaller than 0.5. We estimate option-implied moments with the same time horizon of one year. Risk-neutral moments are computed following the same method of Bakshi et al. (2003).

There are some aspects that Buss and Vilkov (2012) do not describe in their article. First, Buss and Vilkov (2012) do not identify the risk-free interest rate used to estimate the option-implied moments. In our replication, we use treasury bills from CRSP as a proxy for risk-free interest rate. Second, Buss and Vilkov (2012) do not clarify how to get option-implied volatility with one-year to maturity. We compute option-implied moments with various maturities on each day. We then use linear interpolation to get the 365-day VAR, SKEW and KURTOSIS, using both contracts with maturity more than 365 days and contracts with maturity less than 365 days. If there is just one maturity in one day, we do not do interpolation and just use this to represent the 365-day VAR, SKEW and KURTOSIS in that day.

There exist some data difference between our replication and Buss and Vilkov (2012). The number of firms with both stock and option data available in the S&P 500 Index we collect is different from the description by Buss and Vilkov (2012). Sorted by PERMNO, Buss and Vilkov (2012) have a total of 950 firms in their data, which exceeds 500 because of index additions and deletions. We obtain the constituents of S&P 500 Index from COMPUSTAT. Sorted by gvkey, we get 908 companies from January 4, 1996 to December 31, 2009. Then we use the firms from COMPUSTAT to collect stock data from CRSP and option data from OptionMetrics. For firms with stock data available, we have a total of 897 names sorted by PERMNO. When applying the same data filtering role as in Buss and Vilkov (2012)—selecting out-of-the-money options (puts with deltas strictly larger than -0.5 and calls with deltas smaller than 0.5), Buss and Vilkov (2012) got 373 in 1996 to 483 in 2009 out of 500 stocks in the S&P 500 Index. We obtain more than 450 in 1996 and 496 stocks at the beginning of 2009. This may be because the database has updated in recent years.

[Insert Table X here]

Table X presents the descriptive statistics of implied volatility and skewness for the sample period from January 1996 to December 2009. It reports the number of observations, average, standard deviation, median as well as 25th, 75th percentiles of implied volatility and skewness for both S&P 500 Index and stocks. We Winsorize the variables—stock implied volatility and skewness at the 1% and 99% levels following Ang et al. (2006a). For example, if an observation for stock option-implied volatility is extremely large and above the 99th percentile of all the firms' implied volatility, we replace the firm's option-implied volatility with the implied volatility corresponding to the 99th percentile. The same procedure applies to option-implied skewness. Form Table X, we find that the average of S&P 500 Index volatility is 0.2378 and the average of stock volatility is 0.3708. It is obvious to find that the average of the S&P 500 Index volatility is less than the average of stock volatility. The average of S&P 500 Index skewness is -0.9608, more negative than the average of stock skewness, which is -0.3804. It shows that the distribution of both index and stock return is negatively skewed.

B.2. Portfolio Analysis

[Insert Table XI here]

Table XI provides a summary of the mean expected betas and the mean realized returns for the beta-sorted quintile portfolios from the paper of Buss and Vilkov (2012). From the High-Low return spread and the monotonicity test, we can confirm there is a monotonically increasing relation between the BV beta and returns.

Based on our collected option data from January1996 to December 2009, we do the portfolio analysis. We sort the individual securities in S&P 500 Index into five groups at the end of each month, and separately for each beta method, according to their pre-ranked betas. The re-ranked betas are estimated using previous one-year daily stock returns at the end of month *t* (at least 40% of 365-day daily returns). For example, we begin by estimating the coefficient beta for one-year period from January, 1996 to December, 1996 for all equities listed on the S&P 500 Index at the beginning of January 1997. These stocks are then ranked from low to high on the basis of the estimated pre-ranked beta and are assigned to five portfolios with equal number of securities — the 20% with the smallest to the first portfolio, the 20% of stocks with the biggest betas to the fifth portfolio and so on. After constructing the portfolios based on the pre-ranked betas, we calculate the value-weighted betas, annualized value-weighted return without dividend for each beta methods, for each portfolio in the next month *t+1*. The procedure is repeated for the whole sample. Then we calculate the time-series average of value-weighted betas, value-weighted return without dividend in the next month *t+1*.

For some trading days and firms, option and stock data are available but the risk-neutral moments may not be available after a series of data dealing and calculation. We use the available risk-neutral moments to calculate implied betas. For the portfolio analysis, we can just use the avaliable implied betas at the end of the month, or we can fill in the missing implied betas (stock data available at this day) at the end of the month for maximum 10 times. The portfolio sorting result with beta filling and without beta filling is shown in XII.

[Insert Table XII here]

Table XII provides a summary of the mean expected betas and the mean realized returns for the beta-sorted quintile portfolios for our replication from 1996 to 2009. We find that for all methods, the difference between our replicated quintile betas and betas from Buss and Vilkov (2012) is very small. For quintile portfolio returns, the biggest difference for the BV beta is 1.79%. The biggest difference for the Historical beta is 1%. The biggest difference for the FGK beta is 1.03%. We conclude two main reasons for the difference of quintile portfolio return. The first reason is data difference. We have described the data difference above. The number of companies we collect is slightly different from Buss and Vilkov (2012) because of database updating. The interpolation method to get one-year option-implied moments may exist difference. The risk-free rate proxy may be different. The second reason is the sensitivity of portfolio sort method. We use daily returns in the portfolio sort, while we report annualized returns. Daily returns are very small and sensitive. Once annualized, a little difference can cause big changes. We need to multiply daily returns by 252 to get annualized returns.

[Insert Figure 4 here]

Figure 4 compares the portfolio result of replication with the result in Buss and Vilkov (2012). We find that the linear shapes for all beta methods in our replication are quite similar to the shapes from the original paper. This indicates that our replication is very close to the paper and it demonstrates that the results of this paper are reasonable.

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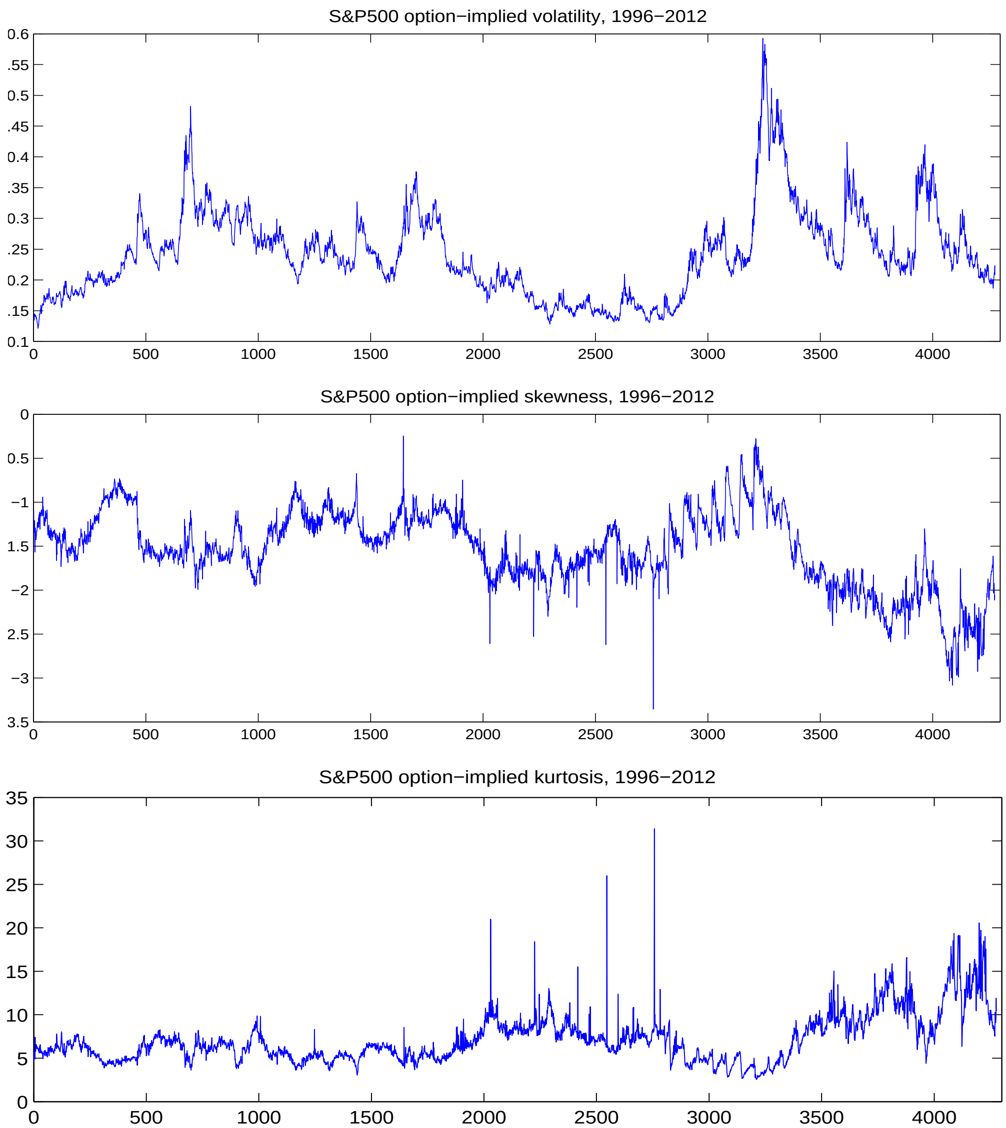
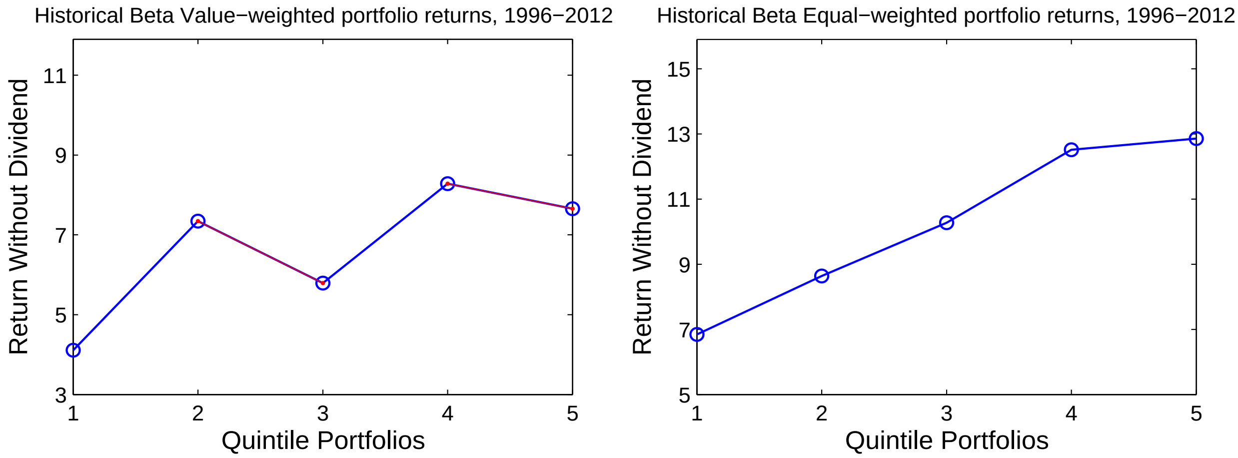
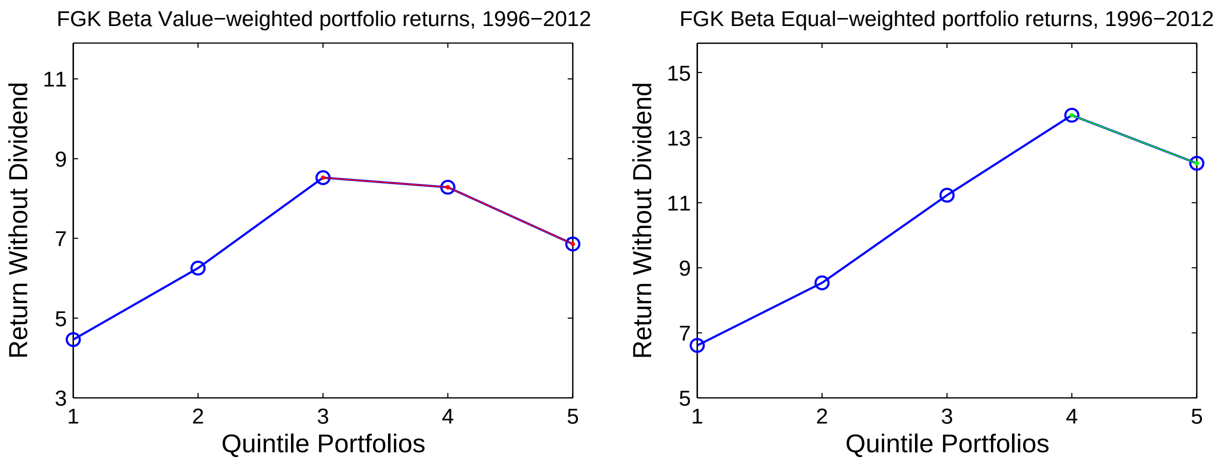


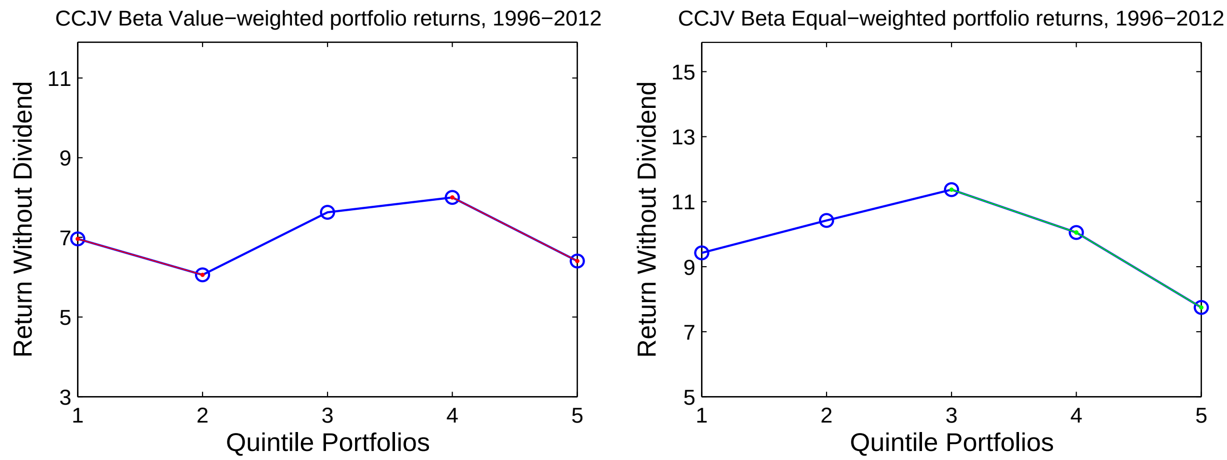
Figure 1. Plot of S&P 500 option-implied moments This figure plots the option-implied volatility, skewness and kurtosis for the S&P 500 Index from January 1996 to December 2012. The computation follows the model-free method of Bakshi et al. (2003).



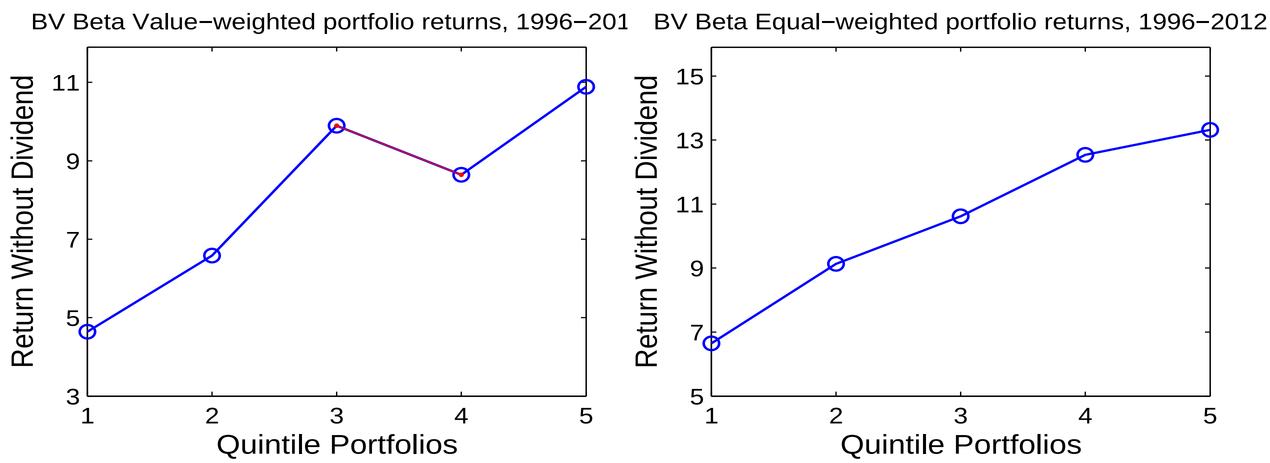
(a) Panel A: Historical Beta



(b) Panel B: FGK Beta

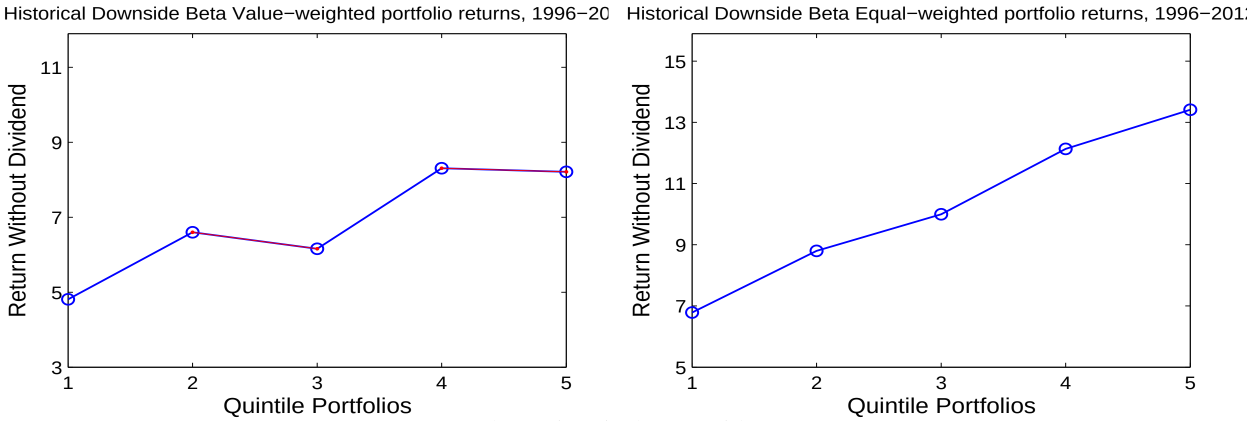


(c) Panel C: CCJV Beta

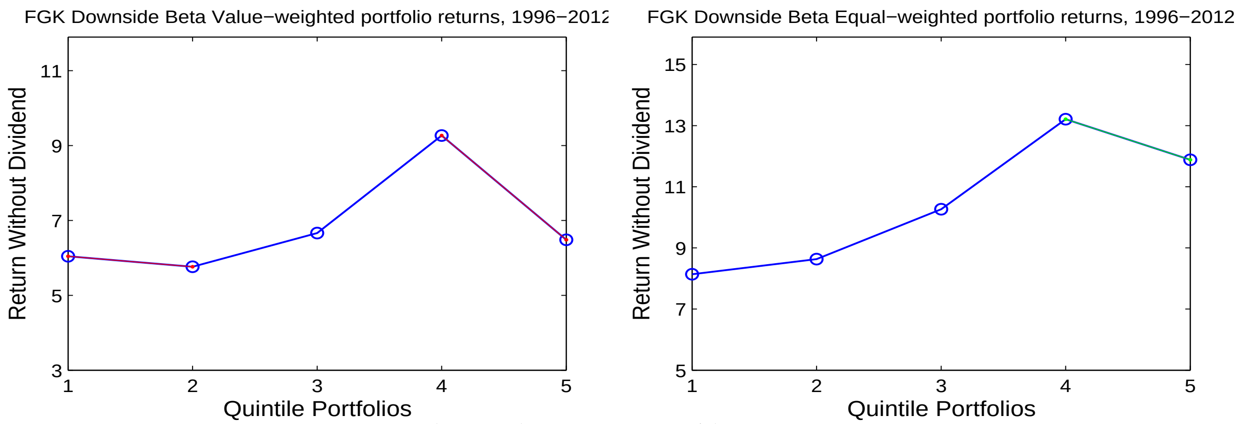


(d) Panel D: BV Beta

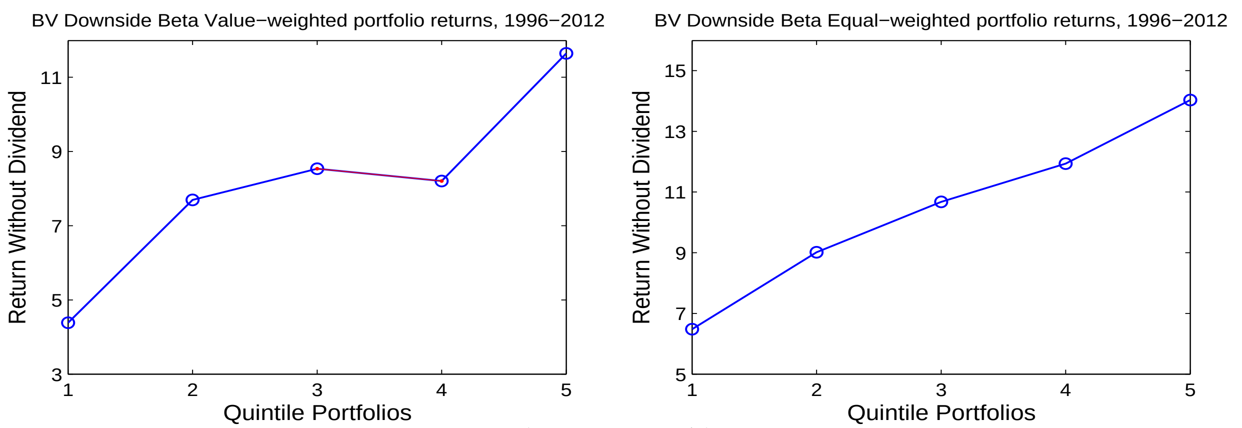
Figure 2. The plot of betas and returns The figure shows the annualized mean return without dividend of the five quintile portfolios sorted by pre-ranked betas over the sample period from January 1996 to December 2012. At the end of each month, we sort the stocks into quintiles based on their pre-ranked betas calculated at the end of the month. The first portfolio then contains the stocks with the lowest beta, while the last portfolio contains the stocks with the highest beta. We then compute the annualized value-weighted and equally-weighted realized return without dividend over the next month for each quintile portfolio, month and beta methodology. Exact numerical values for the betas and returns of each portfolio are shown in Table IV. The four panels present the results for four different beta methods. The returns are expressed in percent.



(a) Panel A: Historical Downside Beta

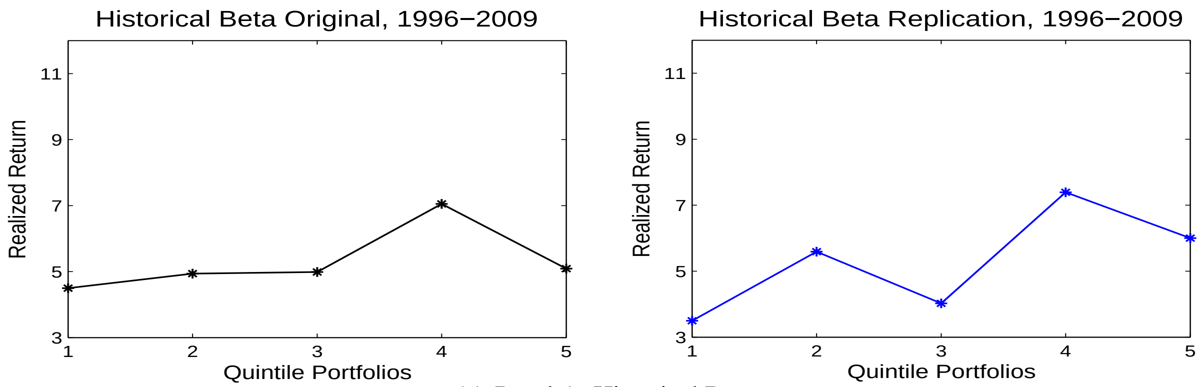


(b) Panel B: FGK Downside Beta

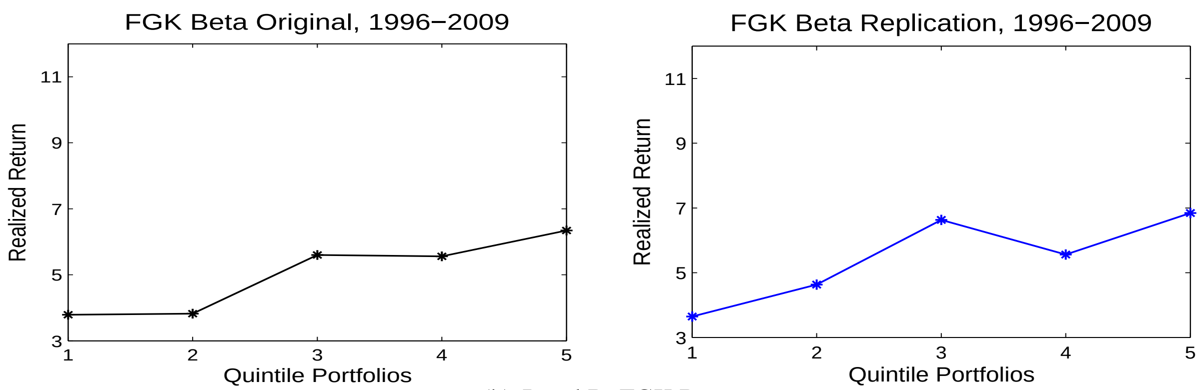


(c) Panel C: BV Downside Beta

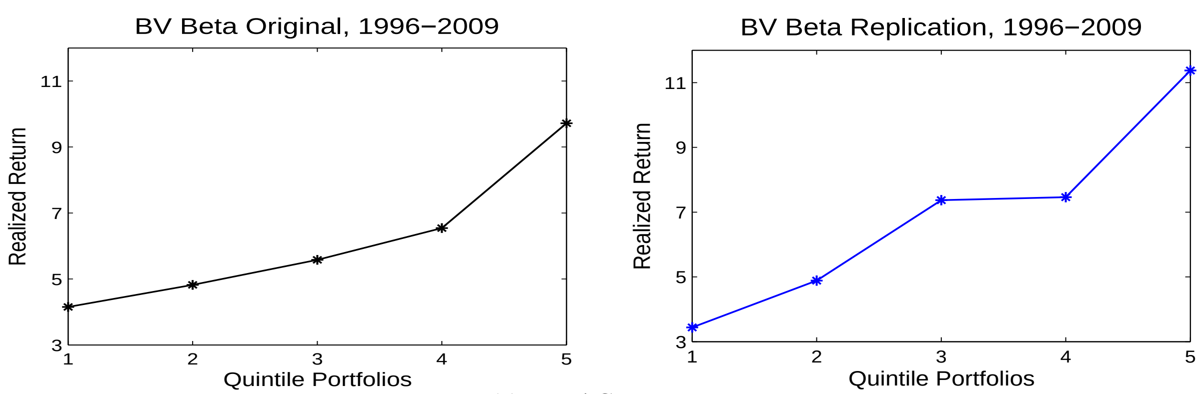
Figure 3. The plot of downside betas and returns The figure shows the annualized mean return of the five quintile portfolios sorted by pre-ranked downside betas over the sample period from January 1996 to December 2012. At the end of each month, we sort the stocks into quintiles based on their pre-ranked downside betas calculated at the end of the month. The first portfolio then contains the stocks with the lowest pre-ranked downside market beta, while the last portfolio contains the stocks with the highest pre-ranked downside market beta. We then compute the annualized value-weighted and equally-weighted realized returns without dividend over the next month for each quintile portfolio, month and downside beta methodology. Exact numerical values for the returns and betas of each portfolio are shown in Table VIII. The three panels present the results for different downside beta methods. The returns are expressed in percent.



(a) Panel A: Historical Beta



(b) Panel B: FGK Beta



(c) Panel C: BV Beta

Figure 4. The plot of betas and returns comparing our replication with the original paper The figure shows the annualized realized return of the five quintile portfolios sorted by pre-ranked betas over the sample period from January 1996 to December 2009. At the end of each month, we sort the stocks into quintiles based on their pre-ranked betas calculated at the end of the month. The first portfolio then contains the stocks with the lowest pre-ranked market beta, while the last portfolio contains the stocks with the highest pre-ranked market beta. We then compute the annualized value-weighted realized returns over the next month for each quintile portfolio, month and beta methodology. Exact numerical values for the returns and betas of each portfolio can be found in Table XI and XII. The three panels present the results for different beta methods. The returns are expressed in percent. The figures in the left hand side are the original figures from Buss and Vilkov (2012) and the figures in the right hand side are the figures from our replication. Note, the figure plots the quintile portfolios with beta filling.

Table I
Data Descriptive for the Number of Companies in the S&P 500 Index
Table I presents the data descriptive for the number of companies in the S&P 500 Index year by year from 1996 to 2012. Column 2 reports the number of firms in the S&P 500 Index, obtained from Compustat for each year. For each year, there are some companies appeared in the S&P 500 Index more than once. Sorted by 'gvkey', the number of unique firms in the S&P 500 Index is shown in Column 3. The number of firms with both option and equity data available sorted by PERMNO or secid is reported in Column 4. The percentage of firms with both option and equity data available is shown in Column 5. The percentage in Column 5 is obtained by using the data in Column 4 divided by Column 3.

Year	NO. of firms	NO. of unique firms	NO. of firms with both option and stock prices	Percentage of firms with both option and stock prices available
1996	539	519	481	92.68%
1997	531	526	495	94.11%
1998	537	533	512	96.06%
1999	545	540	515	95.37%
2000	555	552	524	94.93%
2001	532	529	505	95.46%
2002	526	522	507	97.13%
2003	520	508	497	97.83%
2004	526	519	508	97.88%
2005	536	516	505	98.45%
2006	530	529	519	98.11%
2007	536	536	528	98.51%
2008	534	534	529	99.06%
2009	525	523	522	99.81%
2010	523	516	514	99.61%
2011	521	518	517	99.81%
2012	517	516	516	100.00%

Table II
Descriptive Statistics for Option-implied Moments

Table II reports the data descriptive statistics on risk-neutral moments—volatility, skewness and kurtosis for S&P 500 Index and its constituents from January 1996 to December 2012. Risk-neutral moments are calculated following the model-free procedure of Bakshi et al. (2003). The table reports the number of observations, mean, median, standard deviation and 25th, 75th percentiles of the risk-neutral moments.

	Observation	Mean	Standard Deviation	25th Percentile	Median	75th Percentile
S&P 500 Volatility	4,277	0.2422	0.0713	0.1934	0.2338	0.2810
S&P 500 Skew	4,276	-1.5342	0.4333	-1.7882	-1.5165	-1.2222
S&P 500 Kurtosis	4,276	7.1139	2.7082	5.2367	6.5108	8.3001
Stock Volatility	1,683,646	0.3934	0.1760	0.2760	0.3526	0.4600
Stock Skew	1,683,646	-0.4417	0.4035	-0.6586	-0.4318	-0.1975
Stock Kurtosis	1,683,646	3.5738	1.1391	3.0026	3.2660	3.7450

Table III
Descriptive Statistics for Different Beta Methods

Table III provides summary statistics for general beta methods and downside beta methods. The sample period is from January 1996 to December 2012. For each day, we compute, for each methodology separately, the daily betas for all stocks in the S&P 500 Index. The table reports the number of observations, mean, value-weighted mean, standard deviation and the 25th, 50th, 75th percentiles. Panel A reports the summary descriptives for general beta methods. Panel B presents the summary statistics for downside beta methods.

	Observation	Mean	Weighted Mean	Standard Deviation	25th Percentile	Median	75th Percentile
Panel A: General Betas							
Historical	1,964,655	1.0046	1.0050	0.5029	0.6739	0.9373	1.2555
FGK	1,612,617	0.8476	0.8483	0.4082	0.5779	0.8045	1.0586
CCJV	1,531,197	1.0323	1.0329	0.4086	0.7643	0.9770	1.2314
BV	1,612,617	1.0732	1.0734	0.3891	0.8207	1.0162	1.2655
Panel B: Downside Betas							
Historical	1,964,655	1.0081	1.0086	0.4990	0.6782	0.9425	1.2600
FGK	1,612,949	0.6506	0.6511	0.3566	0.4089	0.6224	0.8601
BV	1,612,949	1.0974	1.0976	0.3708	0.8456	1.0310	1.2800

Table IV
Portfolios Sorted by Different Beta Methods and MR Test

The five quintile portfolios are sorted by pre-ranked betas over the sample period from January 1996 to December 2012. At the end of each month, we sort the stocks into quintiles based on their pre-ranked betas calculated at the end of each month. The first portfolio then contains the stocks with the lowest expected market beta, while the last portfolio contains the stocks with the highest pre-ranked market beta. We then compute the annualized value-weighted and equally-weighted realized returns over the next month for each quintile portfolio, month and beta methodology. The table reports the time-series average of value-weighted and equally-weighted betas and portfolio returns, as well as the High-Low portfolio return spread, separately for each methodology. In addition, the table provides t-statistics and p-values for the High-Low spread to test whether the spread is significant or not. It also provides p-values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonicity relation test. *, **, *** denote the 10%, 5%, and 1% significance levels, respectively. In this table, v-beta, e-beta denote the value-weighted, equally-weighted betas, respectively. v-rwd, v-hpr represent the value-weighted return without dividend, value-weighted holding period return, respectively. e-rwd, e-hpr denote the equally-weighted return without dividend, equally-weighted holding period return, respectively. The returns are expressed in percent.

	Low	2	3	4	High	High-Low	t-stat	t-pval	MR-p
				Panel A: Historical Beta					
v-beta	0.470	0.732	0.928	1.169	1.643	1.172	-	-	-
e-beta	0.464	0.732	0.929	1.165	1.713	1.249	-	-	-
v-rwd	4.110	7.342	5.792	8.284	7.656	3.545	0.567	0.285	0.490
v-hpr	6.680	9.598	7.664	9.913	8.854	2.174	0.349	0.363	0.608
e-rwd	6.844	8.639	10.273	12.510	12.858	6.014	0.895	0.185	0.131
e-hpr	9.403	10.711	12.042	14.113	13.978	4.574	0.682	0.248	0.170
				Panel B: FGK Beta					
v-beta	0.411	0.624	0.783	0.967	1.283	0.872	-	-	-
e-beta	0.408	0.634	0.794	0.969	1.343	0.934	-	-	-
v-rwd	4.483	6.238	8.499	8.309	6.859	2.376	0.349	0.363	0.251
v-hpr	6.935	8.305	10.333	9.869	8.032	1.097	0.164	0.435	0.301
e-rwd	6.629	8.513	11.220	13.699	12.207	5.577	0.817	0.207	0.288
e-hpr	8.874	10.299	12.887	15.207	13.297	4.422	0.648	0.258	0.337
				Panel C: CCJV Beta					
v-beta	0.617	0.787	0.919	1.090	1.419	0.801	-	-	-
e-beta	0.695	0.834	0.956	1.114	1.457	0.762	-	-	-
v-rwd	6.969	6.060	7.589	7.952	6.450	-0.518	0.089	0.465	0.498
v-hpr	9.576	8.067	9.295	9.259	7.502	-2.074	0.358	0.360	0.406
e-rwd	9.443	10.405	11.379	9.970	7.804	-1.640	0.301	0.382	0.424
e-hpr	11.824	12.229	12.972	11.288	8.811	-3.012	0.554	0.290	0.344
				Panel D: BV Beta					
v-beta	0.644	0.848	1.000	1.189	1.537	0.893	-	-	-
e-beta	0.658	0.862	1.010	1.195	1.620	0.963	-	-	-
v-rwd	4.644	6.584	9.893	8.645	10.884	6.241	0.838	0.201	0.325
v-hpr	7.315	8.529	11.525	10.066	11.785	4.470	0.602	0.274	0.366
e-rwd	6.647	9.131	10.616	12.537	13.316	6.669	0.940	0.173	0.093*
e-hpr	9.171	10.976	12.238	13.973	14.185	5.014	0.708	0.240	0.129

Table V
Properties of Portfolios Formed on Different Betas

Table V provides the average characteristics related to firms and option-implied moments for quintile portfolios sorted on the pre-ranked betas, over the sample period from January 1996 to December 2012. At the end of each month t, five portfolios are formed based on the pre-ranked different beta methods. For each quintile portfolios, month, and methodology, we compute the average of ln(ME), ln(BE/ME), model-free implied volatility, skewness, kurtosis and variance risk premium for all stocks at the time when portfolios are sorted. The table reports the time-series means of these statistics for all quintile portfolios, as well as the High-Low portfolio spread, separately for each methodology. In addition, the table provides t-statistics and p-values for the High-Low spread to test whether the spread is significant or not. It also provides p-values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonicity relation test. Note: *, **, *** denote the 10%, 5%, and 1% significance levels, respectively.

	Low	2	3	4	High	High-Low	t-stat	t-pval	MR-pval
				Panel A: Historical Beta					
ln(ME)	2.327	2.271	2.246	2.297	2.285	-0.042	0.337	0.368	0.594
ln(BE/ME)	-7.931	-7.924	-7.909	-7.885	-7.901	0.030	0.333	0.369	0.311
MFIV	0.305	0.328	0.354	0.397	0.517	0.212	11.300	0.000***	0.000***
MFIS	-0.410	-0.422	-0.430	-0.422	-0.359	0.051	1.912	0.028**	0.566
MFIK	4.014	3.837	3.627	3.481	3.232	-0.781	9.842	0.000***	0.000***
VRP	-0.008	-0.007	-0.003	0.005	0.046	0.054	1.749	0.040**	0.228
				Panel B: FGK Beta					
ln(ME)	2.560	2.516	2.498	2.473	2.372	-0.188	1.795	0.036**	0.063*
ln(BE/ME)	-8.089	-8.006	-7.965	-7.924	-7.930	0.159	2.081	0.019**	0.210
MFIV	0.301	0.327	0.357	0.405	0.536	0.234	11.926	0.000***	0.000***
MFIS	-0.402	-0.421	-0.427	-0.428	-0.372	0.030	1.283	0.100*	0.946
MFIK	3.970	3.723	3.644	3.508	3.261	-0.709	9.419	0.000***	0.000***
VRP	-0.002	-0.002	0.004	0.004	0.035	0.038	1.316	0.094*	0.237
				Panel C: CCJV Beta					
ln(ME)	2.644	2.639	2.610	2.501	2.307	-0.338	4.776	0.000***	0.153
ln(BE/ME)	-8.007	-8.037	-8.032	-8.021	-8.013	-0.005	0.109	0.457	0.403
MFIV	0.287	0.319	0.356	0.407	0.515	0.228	18.283	0.000***	0.000***
MFIS	-0.329	-0.433	-0.480	-0.520	-0.596	-0.267	17.947	0.000***	0.000***
MFIK	3.485	3.387	3.422	3.443	3.494	0.009	0.157	0.438	0.995
VRP	0.006	0.004	0.002	0.001	0.005	-0.001	0.080	0.468	0.306
				Panel D: BV Beta					
ln(ME)	2.796	2.619	2.530	2.393	2.082	-0.714	12.268	0.000***	0.000***
ln(BE/ME)	-8.124	-8.009	-7.959	-7.921	-7.905	0.219	3.511	0.000***	0.046**
MFIV	0.269	0.317	0.357	0.416	0.568	0.298	20.423	0.000***	0.000***
MFIS	-0.429	-0.439	-0.434	-0.417	-0.330	0.100	4.886	0.000***	0.423
MFIK	3.995	3.738	3.627	3.524	3.222	-0.773	8.231	0.000***	0.000***
VRP	0.001	-0.001	0.000	0.005	0.033	0.032	1.190	0.117	0.620

Table VI
Portfolio Returns Sorted by Firm Characteristics

The five quintile portfolios are sorted by firm-level factors over the sample period from January 1996 to December 2012. At the end of each month, we sort the stocks into quintiles based on these factors calculated at the end of the month, respectively. We then compute the value-weighted and equally-weighted realized returns over the next month. The time-series average of value-weighted and equally-weighted portfolio returns is reported in Table VI. In addition, the table provides t-statistics and p-values for the High-Low spread to test whether the spread is significant or not. It also provides p-values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonicity relation test. *, **, *** denote the 10%, 5%, and 1% significance levels, respectively. v-rwd, v-hpr represent the value-weighted return without dividend, value-weighted holding period return, respectively. e-rwd, e-hpr denote the equally-weighted return without dividend, equally-weighted holding period return, respectively. The returns are expressed in percent.

	Low	2	3	4	High	High-Low	t-stat	t-pval	MR-p
				Panel A: Firm Size					
v-rwd	13.016	9.751	8.999	7.188	6.007	-7.009	1.459	0.072*	0.045**
v-hpr	14.722	11.572	10.927	8.994	7.931	-6.791	1.413	0.079*	0.056*
e-rwd	14.839	10.412	9.881	8.324	7.701	-7.139	1.559	0.060*	0.057*
e-hpr	16.526	12.227	11.808	10.141	9.537	-6.988	1.526	0.064*	0.068*
				Panel B: Book-to-Market Ratio					
v-rwd	6.449	5.713	6.837	8.145	6.777	0.328	0.067	0.473	0.279
v-hpr	7.945	7.401	8.958	10.563	9.322	1.377	0.286	0.388	0.251
e-rwd	8.863	7.639	8.661	10.893	13.608	4.745	1.095	0.137	0.300
e-hpr	10.286	9.071	10.392	12.853	16.092	5.806	1.336	0.091*	0.301
				Panel C: Option-Implied Volatility					
v-rwd	4.919	8.093	9.290	8.843	10.082	5.163	0.677	0.249	0.185
v-hpr	7.590	9.994	10.758	9.988	10.795	3.205	0.421	0.337	0.230
e-rwd	6.587	9.934	10.795	12.623	12.147	5.560	0.777	0.218	0.206
e-hpr	9.257	11.861	12.357	13.904	12.904	3.647	0.511	0.305	0.256
				Panel D: Option-Implied Skewness					
v-rwd	4.584	7.644	7.820	9.219	10.213	5.630	1.46	0.07*	0.05**
v-hpr	6.385	9.473	9.690	10.846	12.104	5.720	1.49	0.07*	0.04**
e-rwd	5.434	9.046	10.035	13.295	14.257	8.823	2.20	0.01***	0.02**
e-hpr	7.246	10.776	11.683	14.743	15.818	8.573	2.14	0.02**	0.01***
				Panel E: Option-Implied Kurtosis					
v-rwd	8.465	9.198	10.812	5.155	5.665	-2.800	0.547	0.292	0.379
v-hpr	9.714	10.730	12.507	7.176	7.701	-2.013	0.394	0.347	0.426
e-rwd	12.918	15.002	10.943	6.171	7.032	-5.886	1.233	0.109	0.301
e-hpr	14.056	16.417	12.575	8.171	9.043	-5.013	1.052	0.146	0.298
				Panel F: Variance Risk Premium					
v-rwd	19.201	15.224	9.096	5.617	-9.573	-28.774	4.306	0.000***	0.000***
v-hpr	20.249	16.758	10.981	7.730	-7.527	-27.776	4.137	0.000***	0.001***
e-rwd	13.170	13.552	12.301	10.342	3.108	-10.062	1.646	0.050**	0.149
e-hpr	14.234	15.085	14.094	12.324	4.871	-9.363	1.531	0.063*	0.236

Table VII
Results for Fama-MacBeth Regression

The table shows the results for Fama and MacBeth (1973) regression of annualized value-weighted returns on betas and firm characteristics. The sample period is from January 1996 to December 2012. The t-statistics are shown in brackets. We report the average of coefficients and their t-statistics of the dependent variables. Note: *, **, *** denote the 10%, 5%, and 1% significance levels, respectively.

	constant	β	ln(ME)	ln(BE/ME)	MFIV	MFIS	MFIK	VRP	mom	Illiq
					Panel A: Historical Beta					
1	-0.009	0.083								
	(-0.23)	(1.54)								
2	-0.819	-0.091	-0.024	-0.126						
	(-0.43)	(-0.72)	(-0.18)	(-0.58)						
3	-0.749	-0.669***			2.922*	-1.781	-0.093	2.150		
	(-0.47)	(-2.71)			(1.81)	(-1.39)	(-0.26)	(1.46)		
4	-1.924	0.024	-0.070	-0.183	-0.002	-0.088	0.208	0.036	0.151	0.000
	(-1.19)	(0.18)	(-0.50)	(-1.14)	(-0.06)	(-0.75)	(0.93)	(0.37)	(1.52)	(omt)
					Panel B: FGK Beta					
1	-0.020	0.126*								
	(-0.44)	(1.73)								
2	0.614	0.030	0.049	0.072						
	(0.39)	(0.18)	(0.30)	(0.33)						
3	-2.686**	-0.153			-0.065	0.878	-0.802**	2.917**		
	(1.98)	(-0.49)			(-0.07)	(0.70)	(-2.39)	(2.41)		
4	-0.701	-0.082	0.168	-0.183	0.137	-0.158	-0.327*	-0.006	0.207	0.000
	(-0.39)	(-0.59)	(1.33)	(-0.91)	(1.42)	(-0.95)	(-1.79)	(-0.08)	(1.24)	(omt)
					Panel C: CCJV Beta					
1	0.009	0.101								
	(0.17)	(1.42)								
2	-0.186	0.029	-0.083	-0.056						
	(-0.10)	(0.23)	(-0.80)	(-0.24)						
3	1.121	-1.631***			5.080***	-1.892**	-0.647	-0.042		
	(0.91)	(-2.85)			(3.11)	(-2.58)	(-1.57)	(-0.05)		
4	-0.128	-0.336**	-0.103	0.057	0.313	-0.235**	0.284*	0.008	0.078	0.000
	(-0.08)	(-2.09)	(-0.90)	(0.32)	(1.54)	(-2.01)	(1.67)	(0.07)	(0.44)	(omt)
					Panel D: BV Beta					
1	-0.103	0.183**								
	(-1.65)	(2.28)								
2	-0.195	0.019	-0.119	-0.070						
	(-0.15)	(0.14)	(-1.08)	(-0.42)						
3	1.793	-1.178**			3.507**	0.362	-0.470	-0.561		
	(1.62)	(-2.57)			(2.44)	(0.53)	(-1.59)	(-0.37)		
4	1.694	-0.129	-0.146	-0.017	0.124	0.014	-0.390*	0.005	-0.108	0.000
	(0.85)	(-0.61)	(-0.72)	(-0.07)	(1.48)	(0.09)	(-1.66)	(0.05)	(-1.24)	(omt)

Table VIII
Portfolio Sorts on Downside Betas and MR Test

The five quintile portfolios are sorted by pre-ranked downside betas over the sample period from January 1996 to December 2012. At the end of each month, we sort the stocks into quintiles based on their pre-ranked downside betas calculated at the end of the month. The first portfolio then contains the stocks with the lowest pre-ranked market downside beta, while the last portfolio contains the stocks with the highest pre-ranked market downside beta. We then compute the value-weighted and equally-weighted realized returns over the next month. The time-series average of value-weighted and equally-weighted portfolio returns is reported in Table VIII. Note: we do not find ways to model the CCJV downside beta. In addition, the table provides t-statistics and p-values for the High-Low spread to test whether the spread is significant or not. It also provides p-values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonicity relation test. Note: *, **, *** denote the 10%, 5%, and 1% significance levels, respectively. v-rwd, v-hpr represent the value-weighted return without dividend, value-weighted holding period return, respectively. e-rwd, e-hpr denote the equally-weighted return without dividend, equally-weighted holding period return, respectively. The returns are expressed in percent.

	Low	2	3	4	High	High-Low	t-stat	t-pval	MR-p
				Panel A: Historical Downside Beta					
v-beta	0.473	0.738	0.934	1.174	1.641	1.169	-	-	-
e-beta	0.468	0.739	0.936	1.167	1.712	1.244	-	-	-
v-rwd	4.812	6.601	6.158	8.310	8.213	3.401	0.549	0.291	0.232
v-hpr	7.383	8.809	8.057	9.940	9.407	2.025	0.328	0.371	0.333
e-rwd	6.788	8.801	9.996	12.132	13.410	6.622	0.979	0.164	0.085*
e-hpr	9.325	10.897	11.777	13.767	14.485	5.160	0.763	0.223	0.118
				Panel B: FGK Downside Beta					
v-beta	0.266	0.465	0.603	0.764	1.026	0.760	-	-	-
e-beta	0.258	0.472	0.610	0.766	1.066	0.807	-	-	-
v-rwd	6.043	5.762	6.665	9.268	6.485	0.443	0.076	0.470	0.448
v-hpr	8.322	7.757	8.506	10.909	7.786	-0.537	-0.092	0.537	0.513
e-rwd	8.139	8.636	10.268	13.210	11.886	3.747	0.622	0.267	0.259
e-hpr	10.125	10.452	11.973	14.831	13.051	2.926	0.486	0.314	0.323
				Panel C: BV Downside Beta					
v-beta	0.692	0.878	1.016	1.200	1.540	0.848	-	-	-
e-beta	0.710	0.891	1.029	1.212	1.632	0.923	-	-	-
v-rwd	4.384	7.695	8.536	8.206	11.643	7.260	0.939	0.174	0.145
v-hpr	7.065	9.620	10.060	9.572	12.461	5.395	0.700	0.242	0.166
e-rwd	6.479	9.013	10.675	11.933	14.030	7.551	1.068	0.143	0.042**
e-hpr	9.067	10.898	12.262	13.340	14.858	5.791	0.820	0.206	0.066*

Table IX
Results for Fama-MacBeth Regression for Downside Beta

The table shows the results for Fama and MacBeth (1973) regression of annualized value-weighted returns on downside betas and firm characteristics. The sample period is from January 1996 to December 2012. The t-statistics are shown in brackets. We report the coefficients and t-statistics of the dependent variables. Note: *, **, *** denote the 10%, 5%, and 1% significance levels, respectively.

	constant	β	ln(ME)	ln(BE/ME)	MFIV	MFIS	MFIK	VRP	mom	Illiq
					Panel A: Historical Downside Beta					
1	-0.006	0.081								
	(-0.16)	(1.50)								
2	-1.134	-0.130	0.014	-0.161						
	(-0.77)	(-1.11)	(0.13)	(-0.88)						
3	-1.783	-0.173			1.729	-0.846	0.350	-1.320		
	(-1.38)	(-0.70)			(1.35)	(-1.09)	(1.03)	(-1.37)		
4	-2.113*	-0.029	-0.258	-0.403**	-0.006	0.048	-0.089	0.182**	0.005	0.000
	(-1.66)	(-0.17)	(-1.37)	(-2.49)	(0.13)	(0.41)	(-0.59)	(2.33)	(0.05)	(omt)
					Panel B: FGK Downside Beta					
1	0.023	0.091								
	(0.63)	(1.26)								
2	0.058	-0.018	-0.379	-0.113						
	(0.04)	(-0.08)	(-1.37)	(-0.52)						
3	0.687	-0.486			0.403	-1.500	-0.528*	-1.367		
	(0.50)	(-1.25)			(0.35)	(-0.94)	(-1.74)	(-0.66)		
4	0.241	-0.064	-0.029	0.015	0.132	-0.045	-0.009	-0.040	0.099	0.000
	(0.16)	(-0.42)	(-0.22)	(0.09)	(0.50)	(-0.28)	(-0.06)	(-0.87)	(0.79)	(omt)
					Panel C: BV Downside Beta					
1	-0.140*	0.219**								
	(-1.80)	(2.31)								
2	-1.371	-0.040	-0.164	-0.240						
	(-0.98)	(-0.19)	(-0.93)	(-1.32)						
3	-0.805	-1.539***			4.937***	0.096	-0.177	-0.805		
	(0.08)	(-3.86)			(4.11)	(0.13)	(-0.58)	(-0.46)		
4	2.872*	-0.412**	-0.174	0.098	0.098	-0.039	-0.360**	-0.113	0.029	0.000
	(1.88)	(-2.23)	(-1.03)	(0.56)	(1.40)	(-0.24)	(-2.00)	(-0.78)	(0.17)	(omt)

Table X
Descriptive Statistics for Option-implied Volatility and Skewness (1996-2009)

Table X reports the data descriptive statistics on risk-neutral moments—volatility and skewness for S&P 500 Index and its constituents from January 1996 to December 2009. Risk-neutral moments are calculated using the model-free procedure in Bakshi et al. (2003). The table reports the number of observations, average, median standard deviation and 25th, 75th percentiles of the risk-neutral moments.

	Observation	Mean	Standard Deviation	25th Percentile	Median	75th Percentile
S&P 500 Volatility	3,520	0.2378	0.0744	0.1812	0.2311	0.2691
S&P 500 Skew	3,520	-0.9608	0.3307	-1.1301	-0.9691	-0.8286
Stock Volatility	1,243,943	0.3708	0.1410	0.2691	0.3394	0.4369
Stock Skew	1,243,943	-0.3804	0.2736	-0.5587	-0.3784	-0.1854

Table XI
Quintile Portfolio Betas and Returns from Original Paper

Table XI provides the mean expected beta and the annualized mean realized return for the five quintile portfolios sorted on expected market beta, over the sample period from January 1996 to December 2009. At the end of each month, we sort, for each beta methodology, the stocks into quintiles based on their expected market beta. The first portfolio thereby contains the stocks with the lowest expected market betas, and the last portfolio contains the stocks with the highest expected market betas. For each quintile portfolio, month, and methodology, we then compute the value-weighted expected portfolio market beta and the annualized value-weighted realized return over the next month. We only include a stock in our sorting procedure if its expected beta is available for all approaches within a certain group (Daily, Monthly, CCJV). The table reports the time-series means of the expected betas and the realized returns for each methodology. In addition, the table provides p-values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonicity test of the hypotheses for monotonically increasing and monotonically decreasing relations between expected betas and returns.

	Low	2	3	4	High	High-Low	H_0: increasing	H_0: decreasing
Panel A: Historical Daily								
Expected Beta	0.48	0.71	0.88	1.09	1.52	1.04	-	-
Realized Return	4.50	4.94	4.99	7.05	5.09	0.60	0.40	0.31
Panel B: BV Option-Implied Daily								
Expected Beta	0.66	0.83	0.96	1.12	1.45	0.79	-	-
Realized Return	4.15	4.82	5.58	6.54	9.72	5.57	0.71	0.05
Panel C: FGK Daily								
Expected Beta	0.40	0.59	0.72	0.88	1.19	0.78	-	-
Realized Return	3.79	3.82	5.60	5.56	6.34	2.55	0.40	0.07

Table XII
Quintile Portfolio Betas and Returns from Replication

Table XII provides the mean expected beta and the annualized mean realized return for the five quintile portfolios sorted on different beta measures, over the sample period from January 1996 to December 2009. At the end of each month, we sort, for each beta methodology, the stocks into quintiles based on their expected market beta. The first portfolio thereby contains the stocks with the lowest expected market betas, and the last portfolio contains the stocks with the highest expected market betas. For each quintile portfolio, month, and methodology, we then compute the value-weighted expected portfolio market beta and the annualized value-weighted realized return over the next month. The table reports the time-series means of the expected betas and the realized returns for each methodology. In addition, the table provides p-values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonicity relation test.

	Low	2	3	4	High	High-Low	MR p-val
Panel A: Historical Daily							
Expected Beta	0.49	0.72	0.89	1.13	1.59	1.11	-
Realized Return	3.50	5.59	4.03	7.39	6.00	2.50	0.43
Panel B: BV Option-Implied Daily							
(No data filling)							
Expected Beta	0.69	0.84	0.97	1.14	1.43	0.74	-
Realized Return	3.17	4.66	7.84	6.74	8.95	5.79	0.27
(Dta filling)							
Expected Beta	0.70	0.86	0.99	1.16	1.47	0.78	-
Realized Return	3.44	4.88	7.37	7.46	11.37	7.94	0.12
Panel C: FGK Daily							
(No data filling)							
Expected Beta	0.41	0.59	0.73	0.89	1.16	0.75	-
Realized Return	3.18	5.02	6.02	4.81	5.46	2.28	0.30
(Dta filling)							
Expected Beta	0.42	0.60	0.74	0.90	1.18	0.77	-
Realized Return	3.65	4.63	6.63	5.56	6.85	3.20	0.32