A Dynamic Analysis of Stock Price Ratios

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Abstract

Stock price ratios have long been used by finance practitioners as a relative value metric. A popular argument for this widespread use is that stock price ratios would tend to revert to their long-run mean so that substantial deviations from historical averages could successfully be arbitraged away. In this work, we lay out the theoretical conditions for the ratio of stock prices to be a trend stationary process. In particular, we theoretically relate statistical price ratio stationarity to economic mean reversion in profitability (as measured by dividends or earnings price ratios) across securities. We further test our theoretical predictions using standard unit root tests and cointegration analysis on a popular example of "close" stocks.

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1. Introduction

Stock price ratio use has long been advocated by finance practitioners as an effective relative valuation metric. Typically, this use is closely associated with the implementation of the so-called pairs trading strategy: "The commonly cited example of this strategy is the coupling of Ford and General Motors (GM) – both American automakers with apparently similar risks. The strategy calls for an investor to look at the ratio of prices, in this case Ford to GM, and when the ratio increases, sell one and buy the other." Vanguard Group (2006).

In the empirical academic literature, Gatev, Goetzmann and Rouwenhorst (2006) provide strong empirical evidence of significant risk-adjusted returns for pairs trading strategies. In their study, they introduce a price distance metric that allows them to match stocks by minimizing the sum of squared deviations between normalized stock prices (i.e. prices including reinvested dividends). Once the pairing process is achieved, they implement dollar neutral (i.e. equal dollar amounts are simultaneously bought and sold) self-financing strategies. They further rebalance their portfolios when normalized stock prices drift apart from each other by more than two standard deviations, as estimated during the matching period. While their methodology does not directly exploit stock price ratios it heavily relies on the economic intuition that paired stock prices are expected to remain "close" to each other. As they profusely point out this directly translates into the statistical concept of cointegrated prices[1] which, as we shall demonstrate, is equivalent to stock price ratio stationarity.

In this work, we first develop a dual stocks model and explore the theoretical requirements for cointegration of the (log) stock prices to obtain. More specifically we

[1] Bossaerts (1988) first documents evidence of price co-integration in the US stock market.

assume that a pair of stocks whose prices follow time-varying diffusion processes is continuously traded in a frictionless economy. Time variation in paired stocks expected returns *spread* is function of a state variable assumed to follow a stationary mean reverting process. In the empirical work, we propose to use the difference between contemporaneous profitability ratios (dividends or earnings price ratios) as a proxy for this state variable. In this regard, we call upon the standard economic argument in Fama and French (2000) that profitability of competitive industries should be mean reverting. This economic intuition is especially relevant for stocks of companies selling substitute products or services. Consequently we tie cointegration, a purely statistical concept, to mean reversion in profitability across stocks, a testable economic argument. This is indeed an important issue since the underlying economic motivation behind pairs trading success remains elusive[2] even among practitioners: "The relative price strategy, however, remains a mystery. What fundamental reason exists for the ratio of stock prices to be 1:2? When we think about it, there is no fundamental reason" Vanguard Group (2006).

The main theoretical result of the paper is that, in a complete economy whereby risk exclusively results from state variable fluctuations, (log) stock prices are cointegrated or alternatively, the (log) stock price ratio is a stationary process. Consequently, under these conditions, investors may expect (log) stock price ratios to revert to their long-term mean. Furthermore, we relate statistical stationarity in stock prices to economic mean reversion in profitability across paired stocks.

We illustrate the theoretical predictions of the paper through an empirical exercise that uses market information for two "close" stocks: Coke and Pepsi. The empirical

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[2] Gatev, Goetzmann and Rouwenhorst (2006) empirically rule out short-term reversals, unrealized bankruptcy risk or short-sales constraints as valid explanations for pairs trading risk-adjusted profitability and instead document that profits would result from exposure to an "unidentified latent" risk factor.

evidence from the unit root tests and cointegration analysis, supports the cointegration and stationarity hypotheses for the price ratio of "close" securities (see e.g., Bossaerts (1988), Lamont (1998), and Gatev, Goetzmann, and Rouwenhorst (2006)).

The outline of the paper is as follows. Section 1 introduces a frictionless two stocks economy whereby the time varying expected returns *spread* is specified as a mean reverting state variable. Section 2 explores the economic restrictions needed for stock's log-price ratio stationarity to obtain. Section 3 introduces the empirical testing methodology. Section 4 contains the empirical evidence supporting theoretical developments. Section 5 provides a robustness check of the empirical results. Section 6 concludes the paper. Computational details are provided in the Appendix.

2. The Model

Consider a frictionless economy where a pair of stocks is continuously traded. The price $S_{i,t}$ ($i = 1,2$, $0 \le t \le T$) of each stock follows the diffusion process:

$$\frac{dS_{i,t}}{S_{i,t}} = \mu_{i,t}dt + \sigma_i dw_{i,t} \tag{1}$$

where $dw_{i,t}$ denotes a standard arithmetic Brownian motion process**.** The process $\mu_{i,t}$ represents stock's i time-varying expected return while the volatility coefficients σ_i are assumed to be positive and constant with $\sigma_1 \neq \sigma_2$. In addition, ρ denotes the constant instantaneous correlation coefficient between stock return processes. Since we develop here a relative pricing model we define the stock price ratio:

$$R_t = \frac{S_{1,t}}{S_{2,t}} \tag{2}$$

Straightforward application of Ito's product rule to equation (2) yields the following Stochastic Differential Equation (SDE) for the stock price ratio process:

$$\frac{dR_t}{R_t} = \left(\mu_{1,t} - \mu_{2,t} + \sigma_2^2 - \rho\sigma_1\sigma_2\right)dt + \sigma_1 dw_{1,t} - \sigma_2 dw_{2,t} \tag{3}$$

Applying Ito's lemma once more to (3) we verify that the natural logarithm (log-price ratio) of the stock price ratio is given by:

$$\ln(R_t) = Ln(R_0) + \left(\mu_{1,t} - \mu_{2,t} - \tfrac{1}{2}\left(\sigma_1^2 - \sigma_2^2\right)\right)t + \sigma_R\sqrt{t}w_t \tag{4}$$

where $\sigma_R = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$ and $dw_t = \dfrac{\sigma_1 dw_{1,t} - \sigma_2 dw_{2,t}}{\sigma_R}$ is a standard Brownian motion.

A. Constant Expected Returns.

First assume that $\mu_{i,t} = \mu_i$ (i.e. constant stock expected returns). Then, from (4) we deduce that the log-price ratio would be normally distributed with mean $Ln(R_0) + \left(\mu_1 - \mu_2 - ½\left(\sigma_1^2 - \sigma_2^2\right)\right)t$ and variance $\sigma_R^2 t$. Since the variance of the log-price ratio would grow infinite as $t \to \infty$, $\ln(R_t)$ would follow a non-stationary process also referred to as a unit root process in the time series literature. Given that $\ln(R_t) = \ln(S_{1,t}) - \ln(S_{2,t})$, we also conclude that, if stock expected returns were constant, log-prices $\ln(S_{1,t})$ and $\ln(S_{2,t})$ could not be cointegrated since their difference would constitute a non-stationary process. Such a conclusion however is indirectly contrary to the empirical evidence presented by Gatev, Goetzmann and Rouwenhorst (2006) who devise a “distance minimizing” metric between normalized historical stock prices and successfully exploit divergence in pairs trading.

B. Time-Varying Expected Returns.

In order to obtain a more flexible model, assume from now on that $\mu_{i,t}$, stock i time-varying expected return is function of state variable X_t with:

$$\mu_{i,t} = r + \sigma_i X_t \tag{6}$$

where r denotes the constant instantaneous risk-free interest rate of return. State variable X_t is assumed to follow a mean-reverting Ornstein-Uhlenbeck diffusion process:

$$dX_t = \lambda\left(\overline{X} - X_t\right)dt + \sigma_X dz_{X,t} \tag{7}$$

where $dz_{X,t}$ denotes a standard Brownian motion, $\overline{X}$ the constant long run mean, λ the speed of mean reversion coefficient and σ_X the volatility both respectively strictly positive constants. Also let $\rho_{i,X}$ denote the constant instantaneous coefficient of correlation between innovations $dw_{i,t}$ and $dz_{X,t}$. Similar theoretical time-varying expected return specifications have been proposed by previous researchers (see Kim and Omberg (1996), Wachter (2002)). Rewriting (6) as a system of two linear equations (one per stock), it is easily verified that:

$$X_t = \frac{\mu_{1,t} - \mu_{2,t}}{\sigma_1 - \sigma_2} \tag{8}$$

Using a standard orthogonalization argument[3], we notice that stock prices dynamics in expression (1) may be rewritten as:

$$\frac{dS_{i,t}}{S_{i,t}} = \mu_{i,t}dt + \sigma_i \rho_{i,X} dz_{X,t} + \sigma_i \sqrt{1-\rho_{i,X}^2}\, dz_{i,t} \tag{9}$$

where $dz_{X,t}.\, dz_{i,t}$ are independent standard Brownian motions. From expression (9) we observe that each stock's risk may be decomposed into two components. First, there is a systematic source of risk related to the covariance of stock i with the common state variable X_t. Second, there is a residual idiosyncratic risk uncorrelated with either the state variable or the other stock idiosyncratic risk. In empirical implementations of time varying returns models, profitability ratios like the dividends-price ratio (earnings-price ratio) have routinely been used as proxies for time varying expected returns (see Campbell and Viceira (1999), Barberis (2000), Wachter (2002) among others). A salient empirical feature of those ratios is that they tend to be highly persistent and strongly negatively correlated with their associated stock returns (see Barberis (2000)). The latter

[3] See Cvitanić and Zapatero (2004) pp.248 for details.

simply results from the fact that, everything else being equal, higher (lower) returns contemporaneously result in lower (higher) profitability ratios.

In this work however, it is the difference between each stock contemporaneous dividends-price ratio (earnings-price ratio) that is being considered as the empirical proxy for the time varying expected stock returns *spread* X_t. We further conjecture that mean reversion in this difference time series (as opposed to the unit-root type behavior of single stock profitability ratio series) is to be expected as long as profitability across paired securities is expected to mean-revert[4]. This conjecture will be further examined in the empirical work.

2. The Stationary Economy

A. The Joint Distribution of the Log-Price Ratio and the State Variable.

We first derive the joint distribution of the log-price ratio and the state variable as the solution of a linear SDE. Using (7), (8) and (9) the joint process for $\ln(R_t)$ and X_t can be written in vector form as:

$$\begin{pmatrix} d\ln(R_t) \\ dX_t \end{pmatrix} = \left(\begin{pmatrix} 0 & \sigma_1 - \sigma_2 \\ 0 & -\lambda \end{pmatrix} \begin{pmatrix} \ln(R_t) \\ X_t \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}\left(\sigma_1^2 - \sigma_2^2\right) \\ \lambda \overline{X} \end{pmatrix} \right) dt + \begin{pmatrix} \sigma_1\rho_{1,X} - \sigma_2\rho_{2,X} & \sigma_1\sqrt{1-\rho_{1,X}^2} & -\sigma_2\sqrt{1-\rho_{2,X}^2} \\ \sigma_X & 0 & 0 \end{pmatrix} \begin{pmatrix} dz_{X,t} \\ dz_{1,t} \\ dz_{2,t} \end{pmatrix} \tag{10}$$

Equation (10) constitutes an important special case of SDE usually referred to as a linear stochastic differential equation. As shown in Duffie (1996), Appendix E, the

[4] Fama and French (2000) suggest that:"… in a competitive environment, profitability is mean reverting within as well as across industries. Other firms eventually mimic innovative products and technologies that produce above normal profitability for a firm".

solution of this SDE can be expressed in explicit form as a joint Gaussian process denoted as follows:

$$\begin{pmatrix} \ln(R_t) \\ X_t \end{pmatrix} \sim N\left(\begin{pmatrix} m_R(t) \\ m_X(t) \end{pmatrix}, \begin{pmatrix} V_{RR}(t) & V_{RX}(t) \\ V_{RX}(t) & V_{XX}(t) \end{pmatrix} \right) \tag{11}$$

where the mean vector $m(t) = \begin{pmatrix} m_R(t) \\ m_X(t) \end{pmatrix}$ and the variance-covariance matrix $V(t) = \begin{pmatrix} V_{RR}(t) & V_{RX}(t) \\ V_{RX}(t) & V_{XX}(t) \end{pmatrix}$ satisfy a specific system of ordinary differential equations. A similar system is solved by Wachter (2002). The details of the solution are provided in the Appendix.

B. A Special Case: Completeness.

We now consider the special case where the economy developed in Section1 is complete that is $\rho_{i,X} = -1$. Market completeness here refers to the fact that idiosyncratic risk in expression (9) is zero or in other words stock returns are perfectly negatively correlated with the state variable process. This assumption has been previously proposed by Wachter (2002) in a single stock dynamic allocation model. We now state and prove the main theoretical proposition of this research.

<u>Proposition:</u>

If the economy is complete (i.e. $\rho_{i,X} = -1$) and $\sigma_X = \lambda$ then:

i) In the limit (as $t \to \infty$), the SDE solution in expression (11) is a joint stationary process. In particular, the log-price ratio process $\ln(R_t)$ is a trend stationary process.

ii) In the limit (as $t \to \infty$), the log-price ratio process $\ln(R_t)$ and the state variable process X_t are perfectly negatively correlated.

Proof:

First observe that the completeness assumption (i.e. $\rho_{i,X} = -1$) implies that the instantaneous covariance between $\ln(R_t)$ and X_t equals $-\sigma_X\sigma_1 + \sigma_X\sigma_2$. The negative term $-\sigma_X\sigma_1$ is consistent with the intuition that changes in stock 1 price are negatively correlated with the spread profitability ratio variable X_t while the positive term $+\sigma_X\sigma_2$ reflects that changes in stock2 price instead are positively correlated with X_t.

To prove i) we examine the limiting distribution of the joint process in expression (11). From (A.1) and (A.2) in the Appendix, we observe that $m_R(t)$ is indeed a linear function of t while, as $t \to \infty$ while $m_X(t) = \overline{X}$. For weak stationarity of the joint process, it is required that all the components of variance-covariance matrix $V(t)$ be bounded as $t \to \infty$. From (A.3) and (A.4) in the Appendix it is easily seen that, as $t \to \infty$:

$$V_{XX}(\infty) = \frac{\sigma_X^2}{2\lambda} \tag{12}$$

$$V_{RX}(\infty) = \frac{(\sigma_1 - \sigma_2)\sigma_X^2}{2\lambda^2} - \frac{(\sigma_1 - \sigma_2)\sigma_X}{\lambda} \tag{13}$$

The limiting behavior of $V_{RR}(t)$ is trickier. First observe that $\rho_{i,X} = -1$ implies $\sigma_1^2(1-\rho_{1,X}^2)t + \sigma_2^2(1-\rho_{2,X}^2)t = 0$ for all t. Second, computing (A.5) yields:

$$V_{RR}(t) = (\sigma_1 - \sigma_2)^2\left(\frac{\sigma_X^2}{\lambda^2} - 2\frac{\sigma_X}{\lambda} + 1\right)t + K \tag{14}$$

where K is a constant term as $t \to \infty$. Consequently, for $V_{RR}(t)$ to be bounded as $t \to \infty$ we need $\frac{\sigma_X^2}{\lambda^2} - 2\frac{\sigma_X}{\lambda} + 1 = 0$ for all t. This is achieved if and only if $\sigma_X = \lambda$. Hence after integration, we obtain $V_{RR}(t) = K = \frac{(\sigma_1 - \sigma_2)^2}{2\lambda}\left(1 - e^{-2\lambda t}\right)$ and finally:

$$V_{RR}(\infty) = \frac{(\sigma_1 - \sigma_2)^2}{2\lambda} \tag{15}$$

For ii) we simply verify that if $\sigma_X = \lambda$:

$$\lim_{t\to\infty} Correl(\ln(R_t), X_t) = \frac{V_{RS}(\infty)}{\sqrt{V_{RR}(\infty)}\sqrt{V_{XX}(\infty)}} = -1 \tag{16}$$

This proposition has important implications for practitioners who seek to use a pairs-trading investment strategy in the stock market as it gives a clear economic intuition to the pairs trading strategy with straightforward empirical content to be tested. In this respect, as long of the proposition holds, any investment strategy that exploits short-term "error" deviations of stock prices of close firms apart from their long run (cointegrated) relation, e.g., matching stocks by minimizing the sum of squared deviations between normalized stock prices as in Gatev, Goetzmann, and Rouwenhorst (2006) should produce significant risk-adjusted returns.

To illustrate the implications of the proposition, in the next section we provide a simple empirical exercise where we analyze the time series behavior of the Coca Cola and Pepsi stock price ratio. These two stocks provide us with a straightforward example of relative pricing between close substitutes. We give empirical support to part i) of the proposition running several unit root tests on $\ln(R_t)$. The empirical support to part ii) of the proposition consists in performing a standard cointegration analysis and show the existence of one cointegrating vector between $\ln(R_t)$ and X_t.

3. Empirical Implementation

A. Data description

The dataset used in the empirical analysis includes as dependent variables quarterly log prices of Coca Cola Co. stock (Ticker: *KO*), and Pepsico Inc. stock (Ticker: *PEP*) from 1973Q1 to 2007Q4. We include as state variables proxying for stock profitability the earnings-price ratio and the dividend-price ratio (the last one for robustness check purposes)[5] of both stocks.

There is a large empirical financial literature that shows the predictive power of both earnings and dividend-price ratios on future stock returns (see e.g., Shiller (1984), Fama and French (1988), Campbell and Shiller (1988), Campbell (1991), Lamont (1998), and Campbell, Chan and Viceira (2003)). All data has been obtained from the Datastream series published by Thomson Financial Ltd. Table I provides the correlation matrix and the summary statistics of all the variables included in the empirical analysis.

B. The Discrete-Time Vector Error Correction Model (VECM)

We assume that the dynamics of the continuous-time state space model defined in equations 10 and 11 can be approximated by a discrete-time first-order vector autoregression VAR (1) in levels. This parsimonious specification is not too restrictive because any high-order VAR can be re-expressed as a first-order (its companion) VAR (see e.g., Campbell, Chan, and Viceira (2003)). In particular:

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[5] According to some authors, the E/P ratio is more variable than the D/P ratio (see e.g., Fama and French (1988), and Lamont (1998)).

$$\mathbf{y}_{i,t+1} \equiv \begin{bmatrix} s_{i,t+1} \\ x_{i,t+1} \end{bmatrix} \; \forall i, \tag{17}$$

where $s_{i,t+1} \equiv \log(S_{i,t+1})$, and $x_{i,t+1}$ is the state variable $\forall i = KO, PEP$, with non-stationary dynamics:

$$\mathbf{y}_{i,t+1} = \mathbf{\Phi}_{i,0} + \mathbf{\Phi}_1 \mathbf{y}_{i,t} + \boldsymbol{\varepsilon}_{i,t+1} \; \forall i\,, \tag{18}$$

where $\mathbf{\Phi}_{i,0}$ is the vector of intercepts, $\mathbf{\Phi}_{i,1}$ is the matrix of regressors, and $\boldsymbol{\varepsilon}_{i,t+1}$ is the matrix of innovations in the state variables $\forall i$, such that $\boldsymbol{\varepsilon}_{i,t+1} \overset{i.i.d.}{\sim} N(0, V_\varepsilon)$ and possibly cross-sectionaly correlated. In error correction form equation (18) can be written as (Hamilton (1994), page 580):

$$\Delta \mathbf{y}_{t+1} = -\boldsymbol{\gamma}\boldsymbol{\alpha}^T \mathbf{y}_t + \sum_{j=1}^{\infty} \mathbf{\Phi}_j^* \Delta \mathbf{y}_{j,t-j} + \boldsymbol{\xi}_{t+1}\,, \tag{19}$$

where $\mathbf{\Phi}(1) = \boldsymbol{\gamma} \times \boldsymbol{\alpha}^T$, $\boldsymbol{\gamma} = \begin{bmatrix} b \\ -c \end{bmatrix}$, and $\boldsymbol{\alpha}^T = \begin{bmatrix} 1 & a_2 & a_3 & a_4 \end{bmatrix}$ is the (normalized) cointegration vector between the four variables under analysis in levels. Thus, the discrete-time error correction representation of the state-space model defined in equations 10 and 11 is:

$$\begin{aligned} \ln(R_t) &= -b\mathbf{e}_{t-1} + \phi_R X_{t-1} + \delta_t \\ X_t &= c\mathbf{e}_{t-1} + \phi_X \ln(R_t) + v_t \end{aligned}, \qquad \text{with } E\left(\xi\xi^T\right) = \begin{bmatrix} V_{\delta\delta} & 0 \\ 0 & V_{vv} \end{bmatrix} \tag{20}$$

where $\ln(R_t)$, X_t are defined as in section 2, and $\mathbf{e}_{t-1} = \boldsymbol{\alpha}^T \times \mathbf{y}_{t-1}$ is the error correction term that restores the long-run (cointegration) relation between the variables.

4. Empirical Results

A. Preliminary Unit Root Tests

The first step in any cointegration analysis consists in performing unit root tests on the variables included in the VAR in levels. We run different unit root tests in order to mitigate the well-known problem of standard tests with low power due to size distortions. We also improve the power of the standard tests running 1,000 simulations of the *p*-values using an AR specification, and a wild bootstrapping procedure.[6] Table II, shows the test results for all the variables in levels. As expected from an eye inspection of the time-series behavior of these variables (Figures 1a and 1b), we cannot reject the null hypothesis of the presence of a unit root (reject the null of stationarity for the KPSS test) in all the cases at the 5%-10% level of significance. These results are similar to those found elsewhere (see e.g., Lamont (1998)).

B. VAR in Levels, Rank Tests, and Cointegrating Vector - $x_t = E/Ps$

The next step following Johansen's methodology to cointegration analysis is to estimate the VAR in levels, and determine the rank of $\mathbf{\Phi}(1)$. Table III reports the estimation results of the VAR (1) in levels. The VAR (1) system is estimated using a conditional full information maximum likelihood (FIML) procedure, which basically consists of repeating Zellner's seemingly unrelated regression (SUR) estimation procedure until the lower triangle matrix converges to the FIML solution.

The top panel of the table presents the coefficient estimates with their respective *t*-statistics and *p*-values, the log-likelihood value, sum of squares, adjusted-R^2s, and residual standard errors for each equation in the VAR using E/Ps as state variables. The lower panel shows the correlation matrix of the innovations from each equation in the VAR (1) with its respective *t*-statistics and *p*-values.

[6] The errors are drawn from a Normal distribution with zero mean and variances equal to the squared OLS residuals.

Next, we proceed formally to test for the presence of any cointegration(s) vector(s). For this purpose, we run Johansen's trace and lambda-max tests of the cointegration rank assuming no drift in the cointegration vector. The test results shown in Table IV lead us to conclude that there is one cointegration vector between the four variables under analysis. Table V presents the ordinary least squares (OLS) estimates of the cointegration vector without drift. Note that OLS estimates are super-consistent even with the right hand side variables correlated with the errors, with a fast convergence rate towards their true values of *T*. Figure 2a plots the residual "error" correction term from the cointegrating regression. These results give empirical support to part ii) of the proposition in section 2.

C. Error Correction Representation – State Variable: X_t using E/P

We now proceed to test for the presence of unit roots on both $\ln(R_t)$ and X_t. Once more, we improve the power of the standard tests running 1,000 simulations of the *p*-values using an AR specification, and a wild bootstrapping procedure. Although results for the log price ratio are somewhat mixed, those from the most robust tests: Breitung's non-parametric and the KPSS tests, lead us to reject the null hypothesis of the presence of a unit root (accept the null of stationarity around some constant in the KPSS test) for both variables at the 5%-10% level of significance (see Table VI). These results give empirical support to part i) of the proposition in section 2.

The final step in the cointegration analysis entails the estimation of the VECM as defined in equation (20). For this purpose, we use a FIML estimation procedure imposing the following restrictions: 1) only X_{t-1} and the error-correction term from the long-run equilibrium relation Granger causes $\ln(R_t)$; and 2) in the state variable regression we allow for a feedback effect from $\ln(R_{t-1})$ besides the corresponding error correction term. The results are presented in Table VII and Figure 3a. The top panel of the table presents the coefficient estimates with their respective *t*-statistics and *p*-values, the log-likelihood value, sum of squares, adjusted-R^2s, and residual standard errors for each equation in the VECM using E/Ps as state variables. The lower panel shows the diagonal correlation matrix of the innovations from each equation in the VECM with its respective *t*-statistics and *p*-values. Except for the intercepts all the regressors are statistically significant at the 5%-10% level with statistically significant $\overline{R}^2$s.

The positive signs of the error correction terms can be explained from the way the cointegrating relation is specified given the estimation results shown in Table V. Hence, any transitory "error" deviations from the long run equilibrium relation between the log-price ratio of *KO* and *PEP* and the difference in E/P of *PEP* and *KO* will pull both variables back to their long run cointegrating relation. Importantly, the speed of this adjustment process for the log-price ratio is very high, with a low speed in the adjustment process for the state variable.

5. Robustness check

We now proceed to repeat the cointegration analysis using D/P ratios as state variables. Hence, we start estimating the VAR in levels, and then determine the rank of $\mathbf{\Phi}(1)$. Table VIII reports the estimation results of the VAR (1) in levels using a conditional full information maximum likelihood (FIML) procedure as before.

Once more, the top panel of each table presents the coefficient estimates with their respective *t*-statistics and *p*-values, the log-likelihood value, sum of squares, adjusted-R^2s, and residual standard errors for each equation in the VAR system. The lower panel shows the correlation matrix of the innovations from each equation in the VAR (1) with its respective *t*-statistics and *p*-values. Note, that now the correlations are higher than when we use E/P ratios.

Next, we proceed to test formally for the presence of any cointegration(s) vector(s) between the four variables. For that purpose, we run Johansen's trace and lambda-max tests of the cointegration rank assuming no drift in the cointegration vector. The test results shown in Table IX, lead us to conclude that there is one cointegration vector between the four variables under analysis at the 5% level of significance. Table X presents the ordinary least squares (OLS) estimates of the cointegration vector without drift. The results are qualitative and quantitatively similar to the ones obtained when we use E/Ps as state variables, hence giving additional empirical support to part ii) of the proposition in section 2.

The final step in the cointegration analysis entails the test for the presence of a unit root on X_t defined on D/Ps (the result doesn't change for $\ln(R_{t-1})$), and estimate the corresponding VECM defined as in equation (20). The top panel of the table presents the coefficient estimates with their respective *t*-statistics and *p*-values, the log-likelihood value, sum of squares, adjusted-R^2s, and residual standard errors for each equation in the VECM using E/Ps as state variables. The lower panel shows the diagonal correlation matrix of the innovations from each equation in the VECM with its respective *t*-statistics and *p*-values. The results are qualitative and quantitatively similar to the ones obtained when we use E/Ps as state variables as shown in Tables VI, XI and Figure 3b. Hence, we can conclude that our empirical results passed the robustness check.

6. Conclusion

If prices are cointegrated then the statistical analysis of stock price ratios time series is well founded. In this research we precisely establish the necessary theoretical restrictions to obtain stock price ratio stationarity. In particular we relate stock prices cointegration to mean reversion in profitability.

To illustrate the main theoretical result of the paper we provide a simple empirical exercise using market data from two "close" stocks: Coke and Pepsi. We run standard unit root tests, and a complete cointegration analysis following Johansen's standard procedure to give empirical support to the main theoretical proposition in the paper. Further robustness checks using a different state variable confirm our initial empirical findings. Overall, the empirical evidence supports the intuition that stochastic shocks to those particular stocks's price ratio should fade away on the long run. An important issue would be the development and testing of an optimal strategy (in the traditional utility maximizing sense) to take advantage of such a mean reverting behavior. This is left for future research.

Appendix

To obtain formulae for the mean vector $m(t)=\begin{pmatrix} m_R(t) \\ m_X(t) \end{pmatrix}$ and the variance-covariance matrix $V(t)=\begin{pmatrix} V_{RR}(t) & V_{R\mu}(t) \\ V_{R\mu}(t) & V_{\mu\mu}(t) \end{pmatrix}$ in equation (11), let $a(t)=\begin{pmatrix} 0 & \sigma_1-\sigma_2 \\ 0 & -\lambda \end{pmatrix}$,

$b(t)=\begin{pmatrix} -\frac{1}{2}\left(\sigma_1^2-\sigma_2^2\right) \\ \lambda\overline{X} \end{pmatrix}$ and $c(t)=\begin{pmatrix} \sigma_1\rho_{1,X}-\sigma_2\rho_{2,X} & \sigma_1\sqrt{1-\rho_{1,X}^2} & -\sigma_2\sqrt{1-\rho_{2,X}^2} \\ \sigma_X & 0 & 0 \end{pmatrix}$

Equation (10) can be rewritten as:

$$\begin{pmatrix} d\ln(R_t) \\ dX_t \end{pmatrix}=\left(a(t)\begin{pmatrix} \ln(R_t) \\ X_t \end{pmatrix}+b(t)\right)dt+c(t)\begin{pmatrix} dz_{X,t} \\ dz_{1,t} \\ dz_{2,t} \end{pmatrix}$$

From Duffie (1996), Appendix E, we know that for any t, the mean vector $m(t)$ and the variance-covariance matrix $V(t)$ are solutions of the following system of ordinary differential equations:

$$\frac{dm(t)}{dt}=a(t)m(t)+b(t)\ ,\ m(0)=\begin{pmatrix} \ln(R_0) \\ X_0 \end{pmatrix}$$

$$\frac{dV(t)}{dt}=a(t)V(t)+V(t)a(t)^T+c(t)c(t)^T\ ,\ V(0)=0$$

Hence we obtain the following system of ordinary differential equations for the mean vector:

$$\frac{dm_R(t)}{dt}=(\sigma_1-\sigma_2)m_X(t)-\frac{1}{2}\left(\sigma_1^2-\sigma_2^2\right)$$

$$\frac{dm_X(t)}{dt}=-\lambda m_X(t)+\lambda\overline{X}$$

Solutions are given by:

$$m_R(t) = \ln(R_0) + \left((\sigma_1 - \sigma_2)\overline{X} - \frac{1}{2}\left(\sigma_1^2 - \sigma_2^2\right)\right)t + \frac{(\sigma_1 - \sigma_2)}{\lambda}\left(1 - e^{-\lambda t}\right)\left(X_0 - \overline{X}\right) \tag{A.1}$$

$$m_X(t) = \overline{X} + e^{-\lambda t}\left(X_0 - \overline{X}\right) \tag{A.2}$$

Similarly we obtain the following system of ordinary differential equations for the variance-covariance matrix:

$$\frac{dV_{XX}(t)}{dt} = -2\lambda V_{XX}(t) + \sigma_X^2$$

$$\frac{dV_{RX}(t)}{dt} = (\sigma_1 - \sigma_2)\lambda V_{XX}(t) - \lambda V_{RX}(t) + \sigma_X\left(\sigma_1\rho_{1,X} - \sigma_2\rho_{2,X}\right)$$

$$\frac{dV_{RR}(t)}{dt} = 2(\sigma_1 - \sigma_2)V_{XX}(t) + \left(\sigma_1\rho_{1,X} - \sigma_2\rho_{2,X}\right)^2 + \sigma_1^2\left(1 - \rho_{1,X}^2\right) + \sigma_2^2\left(1 - \rho_{2,X}^2\right)$$

Solutions are given by:

$$V_{XX}(t) = \frac{\sigma_X^2}{\lambda}\left(1 - e^{-2\lambda t}\right) \tag{A.3}$$

$$V_{RX}(t) = \frac{(\sigma_1 - \sigma_2)\sigma_X^2}{2\lambda^2}\left(1 - e^{-\lambda t}\right)^2 + \frac{\left(\sigma_1\rho_{1,X} - \sigma_2\rho_{2,X}\right)\sigma_X}{\lambda}\left(1 - e^{-\lambda t}\right) \tag{A.4}$$

$$V_{RR}(t) = 2(\sigma_1 - \sigma_2)\int_0^t V_{RX}(s)ds + \left(\sigma_1\rho_{1,X} - \sigma_2\rho_{2,X}\right)^2 t + \sigma_1^2\left(1 - \rho_{1,X}^2\right)t + \sigma_2^2\left(1 - \rho_{2,X}^2\right)t \tag{A.5}$$

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Table I

Summary Statistics 1973Q1-2007Q4

KO and *PEP* are the ticker symbols for the stocks of Coca-Cola Co. and Pepsico Inc. respectively. Log-Pr denotes log prices. ln (R_t) is the difference between the log prices of *KO* and *PEP*. E/P denotes the earnings-price ratio, and D/P the dividend-price ratio. $\boldsymbol{X_t}$ stands for the differences between the state variable of each stock.

Correlation Matrix

	Log-Pr KO_t	Log-Pr PEP_t	E/P KO_t	E/P PEP_t	D/P KO_t	D/P PEP_t	ln (R_t)	X_t E/P_t	X_t D/P_t
Log-Pr KO_t	1.00								
Log-Pr PEP_t	0.98	1.00							
E/P KO_t	-0.76	-0.67	1.00						
E/P PEP_t	-0.81	-0.76	0.92	1.00					
D/P KO_t	-0.77	-0.68	0.97	0.90	1.00				
D/P PEP_t	-0.78	-0.74	0.91	0.92	0.94	1.00			
ln (R_t)	0.05	-0.14	-0.47	-0.25	-0.43	-0.20	1.00		
$X_t = E/P_t$	0.06	0.17	0.27	-0.13	0.23	0.03	-0.58	1.00	
$X_t = D/P_t$	-0.59	-0.45	0.84	0.68	0.86	0.64	-0.68	0.47	1.00

Univariate Summary Statistics

Variable	Mean	Standard Deviation	Min	Max	Skewness	Kurtosis	Jarque-Bera Test
Log-Pr KO_t	2.29	1.48	0.19	4.38	-0.06	1.33	16.44
Log-Pr PEP_t	2.15	1.50	-0.30	4.30	-0.11	1.43	14.74
E/P KO_t	0.05	0.02	0.01	0.11	0.82	3.04	15.60
E/P PEP_t	0.03	0.02	0.03	0.13	0.98	3.34	23.23
D/P KO_t	0.03	0.02	0.01	0.07	1.04	3.24	25.77
D/P PEP_t	0.02	0.01	0.01	0.05	1.04	2.81	25.58
ln (R_t)	0.14	0.29	-0.37	0.84	0.48	2.56	6.50
$X_t = E/P_t$	-0.01	0.01	-0.04	0.02	0.08	3.23	0.44
$X_t = D/P_t$	0.00	0.01	-0.01	0.03	1.36	6.08	98.98

Table II

Unit Root Test Results I

This table reports the results of several unit root tests for all the variables in levels used in the analysis from 1973Q1 to 2007Q4. We include results from the augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Bierens' higher-order sample autocorrelation (HOAC), and Breitung's nonparametric unit root tests, with null hypothesis H(0): x is a unit root process, and alternative hypothesis H(1): x is a stationary process around some constant. The results from a KPSS test with null H(0): x is a stationary process around some constant against the alternative H(1): x is a unit root process is also included for robustness purposes. The power of some tests is corrected running 1,000 simulations of the p-values using an AR(q) specification, and a wild bootstrapping procedure. p-values at the 10% significance level are shown in parenthesis below the corresponding test statistic value.

	ADF	ADF Corrected	PP	PP corrected	HOAC (1,1)	HOAC corrected	Breitung	Breitung corrected	KPSS
Log-Pr KO_t	-0.01		-0.33		-0.26		0.10		0.80
	(0.76)	(0.67)	(0.94)	(0.96)	(>0.10)	(0.97)	(>0.10)	(0.99)	(<0.10)
Log-Pr PEP_t	-0.004		0.01		-0.40		0.10		0.86
	(0.85)	(0.75)	(0.96)	(0.99)	(>0.10)	(0.97)	(>0.10)	(0.99)	(<0.10)
E/P KO_t	-0.04		-7.76		-5.45		0.05		0.44
	(0.52)	(0.67)	(0.24)	(0.48)	(>0.10)	(0.60)	(>0.10)	(0.43)	(<0.10)
E/P PEP_t	-0.04		-9.15		-11.20		0.06		0.56
	(0.62)	(0.75)	(0.18)	(0.40)	(<0.10)	(0.37)	(>0.10)	(0.50)	(<0.10)
D/P KO_t	-0.03		-5.68		-2.24		0.05		0.43
	(0.42)	(0.67)	(0.37)	(0.64)	(>0.10)	(0.87)	(>0.10)	(0.46)	(<0.10)
D/P PEP_t	-0.04		-6.69		-5.47		0.05		0.49
	(0.51)	(0.63)	(0.30)	(0.61)	(>0.10)	(0.64)	(>0.10)	(0.56)	(<0.10)

Table III

VAR in levels
State Variable: E/P ratio

This table reports the estimation results of the VAR in levels using a conditional full information maximum likelihood (FIML) procedure. t-stats and p-values are shown in parenthesis and brackets below the corresponding predictor/correlation respectively. Black highlighted values denote statistical significance. The last column shows the adjusted-R^2 as well as the sum of squares (s.s.), and the residual estimation error (s.e.) for each regression.

Dependent Variable	Constant (t-stat) [p-value]	Log-Pr KO_t (t-stat) [p-value]	E/P KO_t (t-stat) [p-value]	Log-Pr PEP_t (t-stat) [p-value]	E/P PEP_t (t-stat) [p-value]	σ **(s.e.)**	$\overline{R}^2$ (s.s.) [residual s.e.]
A. *VAR estimates – Log-likelihood =1,787.81*							
Log-Pr KO_{t+1}	**-0.12**	**1.02**	**1.82**	-	-	0.11	0.99
	(-2.00)	(92.41)	(2.72)	-	-	(0.01)	(1.61)
	[0.05]	[0.00]	[0.01]	-	-		[0.11]
E/P KO_{t+1}	**0.01**	**-0.002**	**0.86**	-	-	0.01	0.92
	(3.12)	(-2.77)	(17.97)	-	-	(0.00)	(0.01)
	[0.00]	[0.01]	[0.00]	-	-		[0.01]
Log-Pr PEP_{t+1}	**-0.12**	-	-	**1.02**	**1.83**	0.10	0.99
	(-2.07)	-	-	(100.40)	(2.92)	(0.01)	(1.46)
	[0.04]	-	-	[0.00]	[0.00]		[0.10]
E/P PEP_{t+1}	**0.02**	-	-	**-0.002**	**0.81**	0.01	0.89
	(3.49)	-	-	(-3.11)	(15.10)	(0.00)	(0.01)
	[0.00]	-	-	[0.00]	[0.00]		[0.01]

B. *Correlation Matrix*	Log-Pr *KO*	E/P *KO*	Log-Pr *PEP*	E/P *PEP*
Log-Pr *KO*	1.00			
E/P *KO*	**-0.81**	1.00		
	(-5.18)	-		
	[0.00]	-		
Log-Pr *PEP*	**0.68**	**-0.64**	1.00	
	(4.35)	(-4.68)	-	
	[0.00]	[0.00]	-	
E/P *PEP*	**-0.61**	**0.67**	**-0.81**	1.00
	(-4.51)	(4.00)	(-4.96)	-
	[0.00]	[0.00]	[0.00]	-

Table IV

Johansen's Cointegration Rank Tests
State Variable: E/P ratio

This table reports the results of Johansen's cointegration rank tests for the sample period 1973Q1:2007Q4 assuming no drift in the cointegration vector. Highlighted values in black denote statistical significance at the 10% level. p-values are shown in parenthesis below the corresponding test values.

r_i	Trace test [H0: rank $= r_i$ vs. H1:rank$> r_i$] (p-value)	Lambda Max [H0: rank$= r_i$ vs.H1:rank$= r_{i+1}$] (p-value)
Without drift		
0	63.98	31.26
	(<0.01)	(<0.02)
1	**32.71**	**18.30**
	(<0.10)	(<0.20)
2	14.41	9.73
	(<0.50)	(<0.50)
3	4.68	4.68
	(<0.50)	(<0.50)

Table V
Cointegrating OLS Regression
State Variable: E/P ratio

This table reports the results of the cointegrating regression without drift between the four variables under analysis for the sample period 1973Q1:2007Q4. Highlighted values in black denote statistical significance. t-stats and p-values are shown in parenthesis and brackets below the corresponding predictor values respectively.

Dependent Variable	E/P KO_t (t-stat) [p-value]	Log-Pr PEP_t (t-stat) [p-value]	E/P PEP_t (t-stat) [p-value]	$\overline{R}^2$ (s.s.) [residual s.e.]
Log-Pr KO_t	**-20.40**	**1.01**	**19.38**	0.98
	(-8.75)	(95.59)	(9.13)	(7.52)
	[0.00]	[0.00]	[0.00]	[0.23]

Table VI

Unit Root Test Results II

This table reports the results of several unit root tests for the difference between log-prices, i.e., the log-price ratio, and the difference between the E/P ratios of both stocks from 1973Q1 to 2007Q4. ln (R_t) is the difference between the log prices of KO and PEP. E/P denotes the earnings-price ratio, and D/P the dividend-price ratio. X_t stands for the differences between the state variable of each stock. We include results from the augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Bierens' higher-order sample autocorrelation (HOAC), and Breitung's nonparametric unit root tests, with null hypothesis H(0): x is a unit root process, and alternative hypothesis H(1): x is a stationary process around some constant The results from a KPSS test with null H(0): x is a stationary process around some constant against the alternative H(1): x is a unit root process are also included for robustness purposes. The power of some tests is corrected running 1000 simulations of the p-values using an AR(q) specification, and a wild bootstrapping procedure. p-values at the 10% significance level are shown in parenthesis below the corresponding test statistic value.

	ADF	ADF Corrected	PP	PP corrected	HOAC (1,1)	HOAC corrected	Breitung	Breitung corrected	KPSS
ln (R_t)	-0.07		-8.89		-11.16		0.01		0.13
	(0.42)	(0.33)	(0.19)	(0.18)	(<0.10)	(0.27)	(<0.10)	(0.05)	(>0.10)
X_t =E/P_t	-0.23		-26.24		-19321.00		0.01		0.17
	(0.22)	(0.34)	(0.00)	(0.00)	(<0.10)	(0.00)	(<0.10)	(0.03)	(>0.10)
X_t =D/P_t	-0.06		-11.89		-13897.64		0.02		0.30
	(0.56)	(0.70)	(0.10)	(0.11)	(<0.10)	(0.02)	(>0.10)	(0.23)	(>0.10)

Table VII

VECM Results

State Variable: E/P ratio

This table reports the estimation results of the VECM using a conditional full information maximum likelihood (FIML) procedure. e denotes the error-correction term from the long-run equilibrium (cointegrating) relation between ln (R_t) and X_t=E/P. T-stats and p-values are shown in parenthesis and brackets below the corresponding predictor/correlation respectively. Black highlighted values denote statistical significance. The last column shows the adjusted-R^2 as well as the sum of squares (s.s.), and the residual estimation error (s.e.) for each regression.

Dependent Variable	Constant (t-stat) [p-value]	ln (R_t) (t-stat) [p-value]	X_t =E/P $_t$ (t-stat) [p-value]	e $_t$ (t-stat) [p-value]	σ **(s.e.)**	$\bar{R}^2$ (s.s.) [residual s.e.]
A. *VECM estimates – Log-likelihood =695.30*						
ln (R_{t+1})	0.11	-	**-9.74**	**0.83**	0.14	0.76
	(0.94)	-	(-6.32)	(11.37)	(0.01)	(2.55)
	[0.35]	-	[0.00]	[0.00]		[0.14]
X_t =E/P$_{t+1}$	0.00	**-0.04**	**-**	**0.04**	0.01	0.44
	(0.16)	(-8.16)	-	(4.72)	(0.00)	(0.01)
	[0.87]	[0.00]	-	[0.00]		[0.01]

B. *Correlation Matrix*	ln *(R)*	*X*=E/P
ln *(R)*	1.00	
X =E/P	-0.00	1.00
	(-0.02)	-
	[0.99]	-

Table VIII

VAR in levels

State Variable: D/P ratio

This table reports the estimation results of the VAR in levels using a conditional full information maximum likelihood (FIML) procedure. t-stats and p-values are shown in parenthesis and brackets below the corresponding predictor/correlation respectively. Black highlighted values denote statistical significance. The last column shows the adjusted-R^2 as well as the sum of squares (s.s.), and the residual estimation error (s.e.) for each regression.

Dependent Variable	Constant (t-stat) [p-value]	Log-Pr KO_t (t-stat) [p-value]	D/P KO_t (t-stat) [p-value]	Log-Pr PEP_t (t-stat) [p-value]	D/P PEP_t (t-stat) [p-value]	σ **(s.e.)**	$\overline{R}^2$ (s.s.) [residual s.e.]
A. *VAR estimates – Log-likelihood =2,255.04*							
Log-Pr KO_{t+1}	**0.64**	**0.89**	**-13.81**	-	-	0.19	0.98
	(7.06)	(50.30)	(-6.20)	-	-	(0.03)	(5.17)
	[0.05]	[0.00]	[0.01]	-	-		[0.20]
D/P KO_{t+1}	**-0.01**	**0.002**	**1.33**	-	-	0.01	0.93
	(-4.91)	(4.68)	(18.06)	-	-	(0.00)	(0.00)
	[0.00]	[0.01]	[0.00]	-	-		[0.01]
Log-Pr PEP_{t+1}	**0.83**	-	-	**0.86**	**-18.16**	0.25	0.97
	(8.06)	-	-	(41.52)	(-6.96)	(0.04)	(8.75)
	[0.04]	-	-	[0.00]	[0.00]		[0.26]
D/P PEP_{t+1}	**-0.02**	-	-	**0.002**	**1.50**	0.01	0.86
	(-7.94)	-	-	(6.50)	(15.10)	(0.00)	(0.00)
	[0.00]	-	-	[0.00]	[0.00]		[0.00]

B. *Correlation Matrix*	Log-Pr *KO*	D/P *KO*	Log-Pr *PEP*	D/P *PEP*
Log-Pr *KO*	1.00			
D/P *KO*	**-0.92**	1.00		
	(-13.38)	-		
	[0.00]	-		
Log-Pr *PEP*	**0.91**	**-0.84**	1.00	
	(12.13)	(-5.69)	-	
	[0.00]	[0.00]	-	
D/P *PEP*	**-0.77**	**0.91**	**-0.84**	1.00
	(-3.52)	(12.13)	(-5.38)	-
	[0.00]	[0.00]	[0.00]	-

Table IX

Johansen's Cointegration Rank Tests
State Variable: E/P ratio

This table reports the results of Johansen's cointegration rank tests for the sample period 1973Q1:2007Q4 assuming no drift in the cointegration vector. Highlighted values in black denote statistical significance at the 5% level. p-values are shown in parenthesis below the corresponding test values.

r_i	Trace test [H0: rank = r_i vs. H1:rank> r_i] (p-value)	Lambda Max [H0: rank= r_i vs.H1:rank= r_{i+1}] (p-value)
Without drift		
0	74.82	39.70
	(<0.01)	(<0.01)
1	**35.12**	23.53
	(<0.05)	(<0.05)
2	11.58	8.19
	(<0.50)	(<1)
3	3.39	3.39
	(<1)	(<1)

Table X

Cointegrating OLS Regression

State Variable: D/P ratio

This table reports the results of the cointegrating regression without drift between the four variables under analysis for the sample period 1973Q1:2007Q4. Highlighted values in black denote statistical significance. t-stats and p-values are shown in parenthesis and brackets below the corresponding predictor values respectively.

Dependent Variable	D/P KO_t (t-stat) [p-value]	Log-Pr PEP_t (t-stat) [p-value]	D/P PEP_t (t-stat) [p-value]	$\overline{R}^2$ (s.s.) [residual s.e.]
Log-Pr KO_t	**-39.91**	**0.98**	**54.66**	0.98
	(-9.68)	(124.79)	(10.29)	(4.90)
	[0.00]	[0.00]	[0.00]	[0.19]

Table XI

VECM Results

State Variable: D/P ratio

This table reports the estimation results of the VECM using a conditional full information maximum likelihood (FIML) procedure. e denotes the error-correction term from the long-run equilibrium (cointegrating) relation between ln (R_t) and X_t=E/P. t-stats and p-values are shown in parenthesis and brackets below the corresponding predictor/correlation respectively. Black highlighted values denote statistical significance. The last column shows the adjusted-R^2 as well as the sum of squares (s.s.), and the residual estimation error (s.e.) for each regression.

Dependent Variable	Constant (t-stat) [p-value]	ln (R_t) (t-stat) [p-value]	X_t=D/P $_t$ (t-stat) [p-value]	e $_t$ (t-stat) [p-value]	σ **(s.e.)**	$\bar{R}^2$ (s.s.) [residual s.e.]
VECM estimates – Log-likelihood =695.30						
ln (R_{t+1})	0.09	-	**-27.55**	**0.70**	0.13	0.80
	(1.34)	-	(-8.44)	(10.51)	(0.01)	(2.18)
	[0.18]	-	[0.00]	[0.00]		[0.13]
X_t=D/P$_{t+1}$	0.00	**-0.03**	**-**	**0.02**	0.003	0.80
	(1.14)	(-9.54)	-	(7.10)	(0.00)	(0.00)
	[0.26]	[0.00]	-	[0.00]		[0.00]

B. *Correlation Matrix*	ln *(R)*	*X*=D/P
ln *(R)*	1.00	
X=D/P	-0.24	1.00
	(-1.90)	-
	[0.06]	-

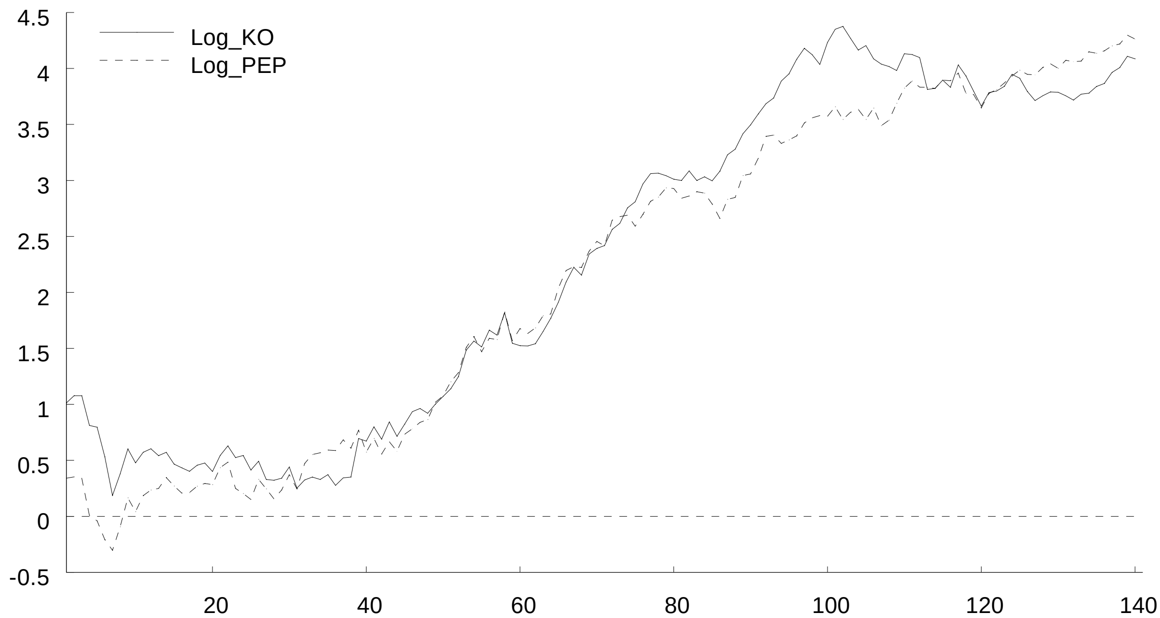

Figure 1a. Time-series behavior of log-prices of *KO* and *PEP*.

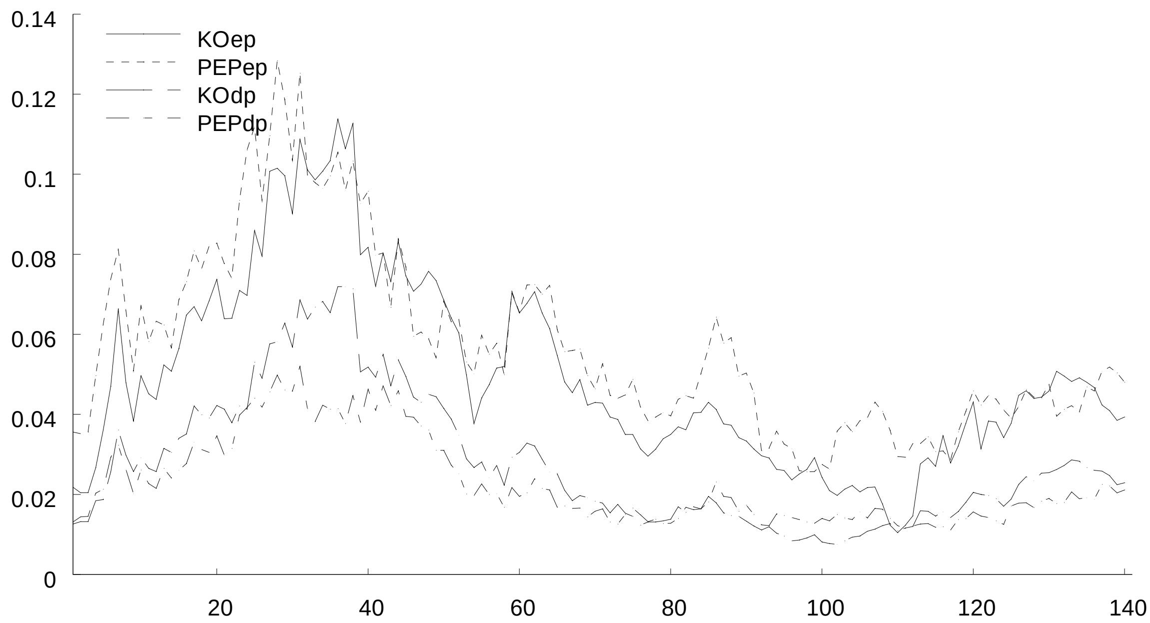

Figure 1b. Time-series behavior of E/P and D/P for *KO* and *PEP*.

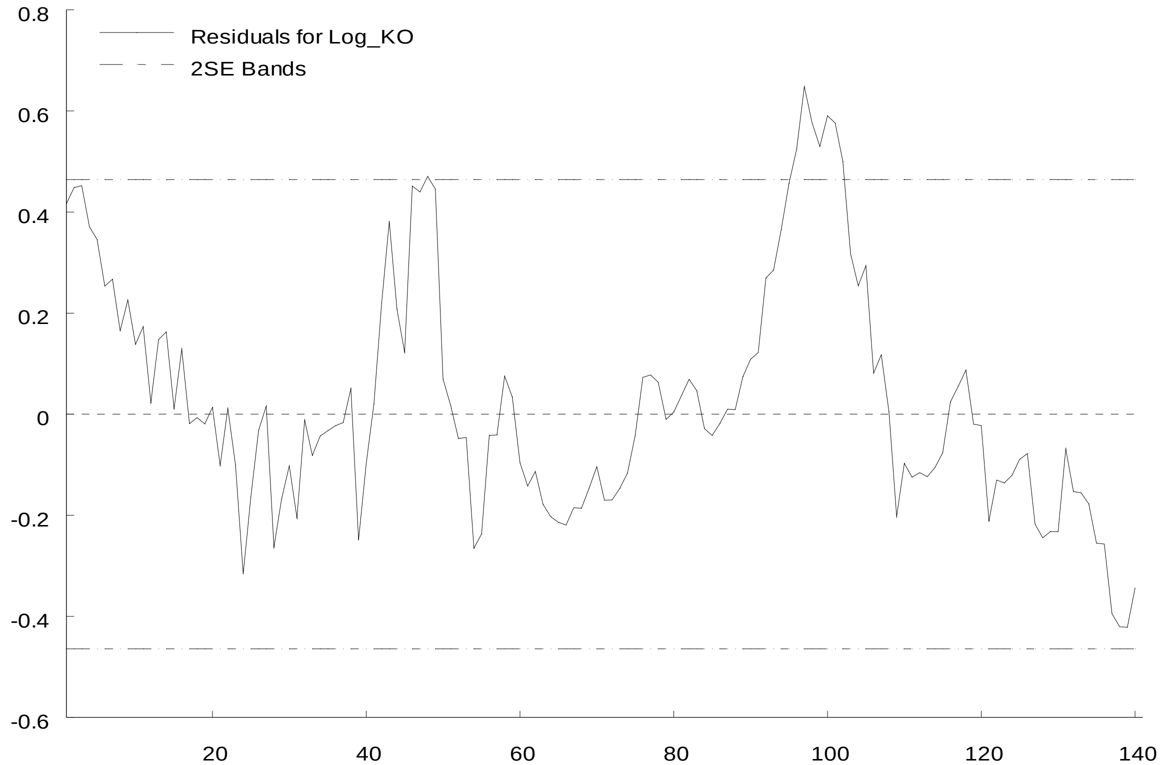

Figure 2a. Error correction term with 2-standard error bounds using E/P ratios as state variables.

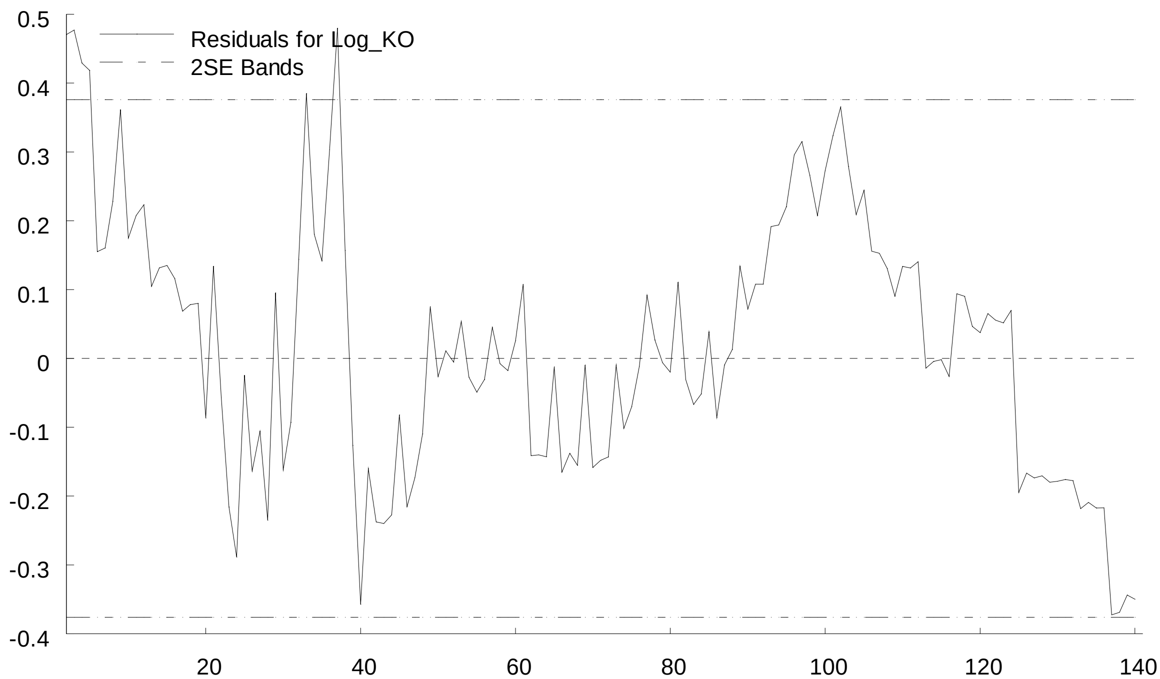

Figure 2b. Error correction term with 2-standard error bounds using D/P ratios as state variables.

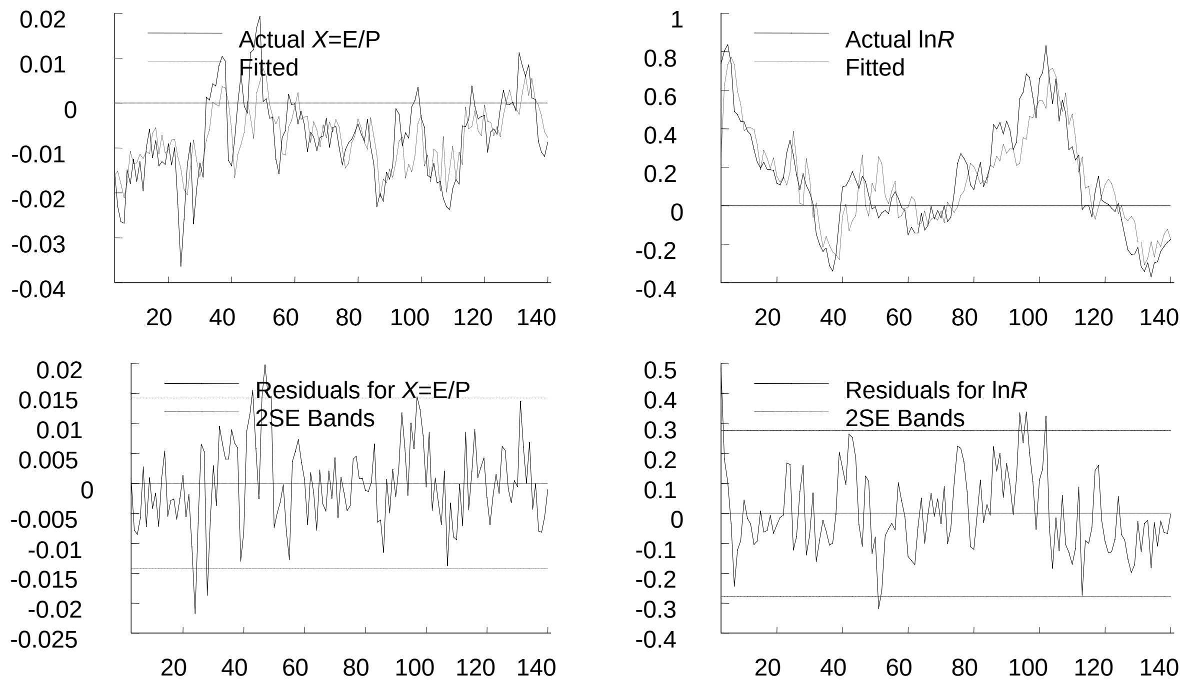

Figure 3a. VECM results using X=E/P as state variable.

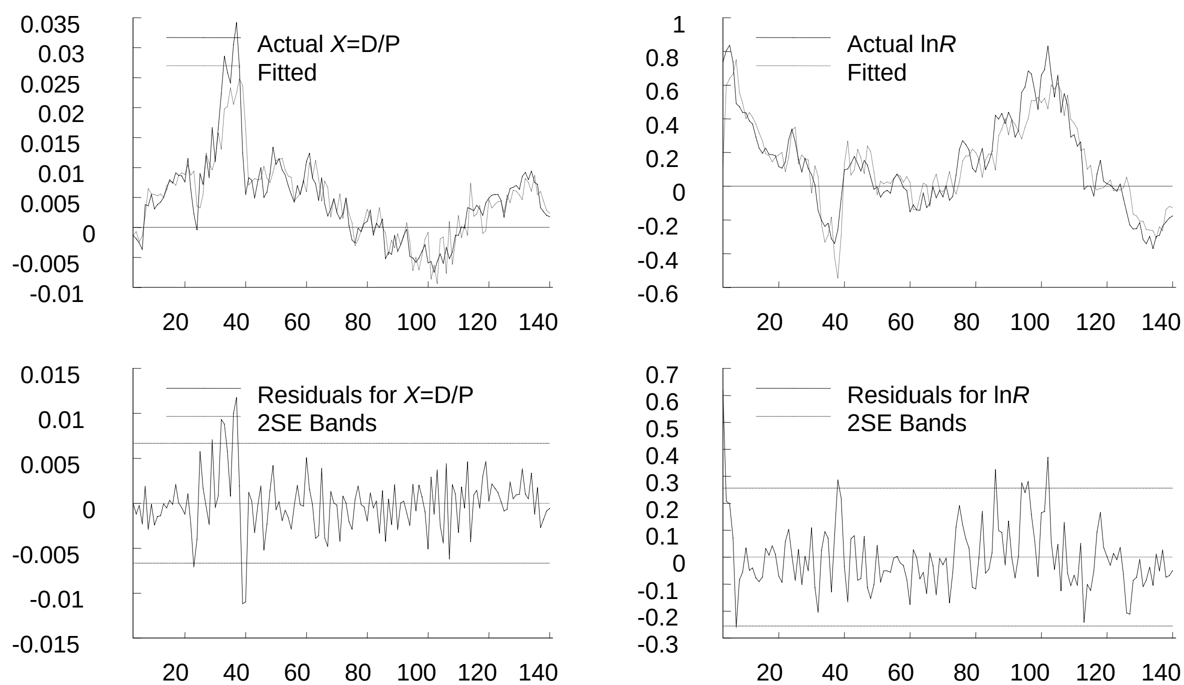

Figure 3b. VECM results using X=D/P ratio as state variable.