The Skewness Premium and The Asymmetric Volatility Puzzle

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Abstract

This paper uses a general equilibrium model to study the source and reward of asymmetric volatility or skewness of market returns in an exchange economy. In particular, the dividend growth rate is modeled as a stochastic volatility process and the representative agent is characterized by Epstein-Zin preferences. The equilibrium equity premium, risk-free rate, and asymmetric volatility (measured by the negative correlation between the market return and its volatility) are derived endogenously. It is shown that the equity premium has three components: the first two components parallel those in the Intertemporal CAPM, while the last one is "new." It reflects the part of excess returns required by investors to take on the asymmetric volatility or negative skewness risk. The paper then uses the Efficient Method of Moments to estimate the stochastic volatility model of the dividend growth rate and uses the estimated process to study the equity premium, the skewness premium, the risk-free rate, and asymmetric volatility under various values of the risk aversion coefficient and elasticity of intertemporal substitution. It is shown that the skewness premium can be as high as 1.2% annually in real terms. However, under conventional levels of risk aversion and elasticity of intertemporal substitution, the asymmetric volatility generated by the model is much smaller than that observed in the data and hence results in the asymmetric volatility "puzzle."

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1 Introduction

There are three distinct but related "puzzles" in the aggregate stock market: (1) returns are too volatile to be explained by changing expected future dividends [Shiller (1981), Campbell and Shiller (1988)]; (2) the equity premium (6% annually) is too large relative to its risk and reasonable risk aversion [Mehra and Prescott (1985)] (3) the (real) risk-free rate (1% annually) is too small to be consistent with the average consumption growth rate (1.6%) [Weil (1989)]. It is also now generally agreed that the volatility of the aggregate stock market return is not only very volatile but also predictable. It seems plausible and interesting to see whether such predictable time-varying/business-cycle risks can help to explain these aggregate stock market behaviors through their effects on required stock returns and price level.[1] Barsky (1989), Gennotte and Marsh (1993), and Abel (1988) first studied the effect of time-varying risk on equity prices and risk-free rates in a general equilibrium framework with power utility preferences. In this paper, we generalize their results to Epstein and Zin (1989) preferences and thus allow for different roles of risk aversion and intertemporal substitution on stock returns and interest rates.[2] In addition, we estimate our model on the dividend growth rate process and use it to calibrate the equity premium, the risk-free rate, and asymmetric volatility (as measured by the negative correlation between shocks to the returns and their volatility) to the data and find not only that time-varying risk fails to explain the equity premium puzzle and the risk-free rate puzzle, but it also highlights another puzzle-the asymmetric volatility "puzzle"-the negative correlation between the shocks to the market return and the shocks to its volatility seems to be too strong to be consist with any equilibrium model with conventional parameters.

In principle, both the leverage effect and the volatility feedback effect can explain the asymmetric volatility of market returns, i.e. the empirical evidence that positive innovations to volatility are correlated with negative innovations to returns. The leverage effect and volatility feedback effect, however, have different implications for the causal relation between returns and volatility. Under the leverage effect, negative shocks to returns cause an increase in volatility because of the higher leverage of the

[1]Our paper is a risk-based explanation along with Kandel and Stambaugh (1990, 1991) in contrast to the risk aversion or preference based explanations - habit type models of Abel (1990), Constantinides (1990), Campbell and Cochrane (1999).

[2]It's known that in the power utility framework, the coefficient of relative risk aversion and the elasticity of intertemporal substitution are the inverse of each other. But there is no theoretical justification for this fact. Risk aversion describes the investor's reluctance to substitute consumption across states of the world whereas the elasticity of intertemporal substitution captures his willingness to substitute consumption over time.

firm. In a volatility feedback model, however, positive shocks to volatility cause a decrease in returns due to the requirement of higher expected future returns induced by higher volatility. The “leverage effect” was first proposed in Black (1976) while the “volatility feedback” effect was first emphasized by Pindyck (1984) and French, Schwert, and Stambaugh (1987). In practice they are likely to work together and reinforce each other. Suppose a negative news shock comes into the market and prices drop. This will drive up volatility due to the leverage effect, which in turn further drives down the price through the volatility feedback effect, which in turn drives up volatility again through leverage effect, and so on. Of course, these effects become smaller and smaller and eventually a “steady state” is reached. Empirically, there have been many reduced form econometric models developed to capture asymmetric volatility. For example, in the GARCH literature, the EGARCH model of Nelson (1991), the Threshold-GARCH model of Glossten, Jagannathan, and Runkle (1993), and the Quadratic GARCH model of Engle (1990) have all been developed to study the relation between returns and volatility. In the stochastic volatility literature, these effects are modeled through a negative correlation between the two random shocks driving the stock return and its volatility as in Bakshi, Cao, and Chen (1997) and Bates (2001).

Empirically, it is found that the leverage effect alone can not explain asymmetric volatility [Christie (1982), Schwert (1989)]. On the other hand, the debate on whether volatility feedback can account for much of the observed variation in asset prices goes back to Pindyck (1984), French, Stambaugh, and Schwert (1987) and Poterba and Summers (1986). For example, French, Schwert, and Stamgbaugh (1987) regress stock returns on innovations in volatility and find a significant and negative coefficient and attribute it to the volatility feedback, while Poterba and Summers show that shocks to volatility are not persistent enough to have any significant impact on prices. Recently, Whaley (2000) uses the Chicago Board Options Exchange’s market volatility index data as an investor fear gauge, and Bekaert and Wu (2000) study the asymmetric volatility in the Japanese equity market and argue that volatility feedback is its main source.

Asymmetric volatility is inherently related to the negative skewness of market returns. Understanding the sources and magnitude of asymmetric volatility may aid our understanding of the sources of negative skewness and the equity premium it commands. Understanding the sources of asymmetric volatility also has important implications for asset pricing and portfolio risk management.[3] Harvey and Siddique (2000) develop an asset pricing model in which individual asset returns have systematic skewness and their expected returns are rewarded for this risk. They show that conditional skewness helps explain the cross-sectional variation in expected returns across assets. Our paper is complementary to theirs in that it investigates the time series property of market return skewness and its premium.

Campbell and Hentschel (1992) is the first structural volatility feedback model, in which they assume a linear relationship between the market return and volatility of the dividend growth rate. Campbell and Hentschel (1992) model volatility with a Quadratic GARCH model and find that volatility feedback has important effects on returns only during periods of high volatility and that it can not explain the finding that the returns are much more variable than revisions of dividend forecasts. In this paper, we use a stochastic volatility model to model the dividend growth rate, and the relationship between market returns and volatilities of the dividend growth rate is derived endogenously in a general equilibrium framework. Wu (2001) uses a similar model to study the determinant of asymmetric volatility. The difference between this paper and Wu (2001) is that this paper uses preferences and an endowment process to study asymmetric volatility while Wu (2001) utilizes a pricing kernel. While Wu (2001) focuses on the determinant of asymmetric volatility, this paper focuses on the premium that negative skewness commands, and the restrictions on preferences required to match the observed equity premium, risk-free rate, and asymmetric volatility.

This paper makes three contributions to the finance literature: (1) theoretically, this paper generalizes the results in Barsky (1989), Gennotte and Marsh (1993), and Abel (1988) about the relative role of risk aversion and intertemporal substitution on asset prices in the presence of time-varying risk (volatility); (2) in particular it shows how risk aversion and intertemporal substitution interact with the endowment

[3]Bakshi, Kapadia, and Madan (2001) study the economic sources of skewness in the risk-neutral distribution and show how risk aversion plus a fat-tailed physical distribution introduces skewness in the risk neutral distribution.

process to produce negative skewness;[4] (3) empirically, it demonstrates what kind of restrictions on preference parameters are needed to generate the high negative correlation between the market return and its volatility (-0.6 in the monthly data, -0.65 in the weekly data, -0.75 in the daily data).

The remainder of the paper is organized as follows. In Section 2, we develop an equilibrium model of asymmetric volatility and characterize the market return, risk-free rate, asymmetric volatility, and the price dividend ratio in terms of parameters of the underlying preference and endowment process. In Section 3, we use the Efficient Method of Moments to estimate our model of the dividend growth rate and use the estimated process to study the equity premium, the risk-free rate, and asymmetric volatility under various values of the risk aversion coefficient and elasticity of intertemporal substitution. Section 4 concludes and provides directions for future research.

2 An Equilibrium Model of Asymmetric Volatility

Consider an exchange economy with the following exogenous stochastic volatility model for the dividends D_t:

$$g_{t+1} = \omega_d + \beta_d g_t + \sigma_{d,t}\varepsilon_{t+1} \tag{1}$$

$$\sigma^2_{d,t+1} = \omega + \beta\sigma^2_{d,t} + \alpha\sigma_{d,t}\eta_{t+1} \tag{2}$$

$$Corr(\varepsilon_{t+1}, \eta_{t+1}) = \pi$$

where $g_{t+1} \equiv \Delta\log(D_{t+1}) = \Delta d_{t+1}$ is the log dividend growth rate.

We choose this model for dividends because similar models are used by Cox, Ingersoll, and Ross (1985), Gennotte and Marsh (1993), and Bansal and Yaron (2001), among others, and there is some empirical evidence that the dividend growth rate is slightly auto-correlated, and has time-varying volatilities [see the empirical part for details]. Moreover, the square root process for volatility allows us to have a simple expression for multi-period volatility forecasts. It also ensures that volatility

[4]Garcia, Luger, and Renault (2001a, 2001b) study how skewness is related to risk aversion and intertemporal substitution from an option-pricing perspective.

is positive with probability 1 in the continuous time limit. We use this time-varying volatility model to capture business cycle effects in dividend growth rate; and the (negative) correlation π between shocks to the dividend growth rate and its volatility is used to capture the fact that recessions are typically characterized by low dividend growth rate and high uncertainty about the future growth outlook. We refer to this (negative) π as "leverage effect" to differentiate it from the volatility feedback effect[5], though it is more or less a story on the individual firm level. Note that unlike in GARCH volatility models, the volatility process now has a separate source of uncertainty, and the volatility of volatility[6] is now proportional to volatility rather than proportional to the square of volatility.[7]

The representative agent in this economy choose his consumption C_t to maximize his utility, which is characterized by the following Epstein and Zin (1989)-Weil (1989) recursive preferences:

$$U_t = \left\{(1-\delta)C_t^{(1-\gamma)/\theta} + \delta(E_t U_{t+1}^{1-\gamma})^{1/\theta}\right\}^{\theta/(1-\gamma)}$$

subject to the budget constraint

$$W_{t+1} = R_{m,t+1}(W_t - C_t) \tag{3}$$

where $\theta = \frac{1-\gamma}{1-1/\psi}$, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, $0 < \delta < 1$ and $\delta^{-1} - 1$ is the rate of time preference, W_t is the total wealth in the economy, C_t is the aggregate consumption at t, and $R_{m,t+1}$ is the *gross* simple return on wealth invested from t to $t+1$. In this economy, the total wealth equals the total market value so the return on invested wealth will, in equilibrium, be the return on the market portfolio. When $\gamma = \frac{1}{\psi}$, this preference collapses to the common expected utility function. Epstein and Zin (1989) show that early resolution of uncertainty is preferred if and only if $\gamma\psi > 1$. In the case of

[5]It is shown in Section (2.5) that for a log utility investor who is myopic, the volatility feedback effect vanishes and the correlation between shocks to market returns and volatilities is equal to π.

[6]In this paper, we use volatility to denote variance as opposed to standard deviation unless noted otherwise.

[7]In GARCH volatility models, for example, the standard GARCH(1,1), $\sigma^2_{d,t+1} = \omega + \beta\sigma^2_{d,t} + \alpha(\sigma_{d,t}\varepsilon_{t+1})^2 = \omega + (\alpha+\beta)\sigma^2_{t,d} + \alpha\sigma^2_{d,t}(\varepsilon^2_{t+1} - 1)$, the volatility of volatility is $\alpha^2\sigma^4_{d,t}$, while the volatility in our model is $\alpha^2\sigma^2_{d,t}$.

expected utility, there is indifference in the way uncertainty about consumption is resolved over time.

The Euler equation for the market portfolio is:

$$1 = E_t[M_{t+1}R_{m,t+1}] = E_t\left[\left\{\delta(\frac{C_{t+1}}{C_t})^{-1/\psi}R_{m,t+1}\right\}^{\theta}\right]$$

where $M_{t+1} = \delta^{\theta}(\frac{C_{t+1}}{C_t})^{-\theta/\psi}R_{m,t+1}{}^{-(1-\theta)}$ is the pricing kernel. Because we assume a pure exchange economy, in equilibrium we have $C_t = D_t$, and we can write the log pricing kernel as[8]

$$y_{t+1} \equiv \log(M_{t+1}) = \theta\log\delta - \frac{\theta}{\psi}g_{t+1} - (1-\theta)r_{m,t+1}. \tag{4}$$

Taking log of the Euler Equation gives

$$0 = \theta\log\delta - \frac{\theta}{\psi}E_t\Delta c_{t+1} + \theta E_t r_{m,t+1} + \frac{1}{2}\left[(\frac{\theta}{\psi})^2 V_{cc,t} + \theta^2 V_{mm,t} - \frac{2\theta^2}{\psi}V_{cm,t}\right] \tag{5}$$

where $c_{t+1} = \log C_{t+1}$, $r_{m,+1} = \log R_{m,t+1}$ is the log market return, and $V_{cc,t} \equiv Var_t(\Delta c_{t+1})$, $V_{cm,t} \equiv Cov_t(\Delta c_{t+1}, r_{m,t+1})$. This expression can be viewed as a second-order Taylor approximation around the conditional mean of $\{r_{m,t},\ \Delta c_{t+1}\}$ and the approximation $\log(1+x) \approx x$ for small x. It holds exactly if consumption growth and market returns have jointly conditionally normal distribution.

Defining $\alpha_{m,t} \equiv \psi\log\delta + \frac{\theta}{2}\left[\frac{1}{\psi}V_{cc,t} + \psi V_{mm,t} - 2V_{cm,t}\right]$, we can view $\alpha_{m,t}$ as a proxy for changing expected variance in the investment opportunity set and we can rewrite the (log) Euler equation (5) as

$$E_t\Delta c_{t+1} = \alpha_{m,t} + \psi E_t r_{m,t+1}. \tag{6}$$

Note that if the consumption growth rate and market returns are jointly homoscedastic, $\alpha_{m,t+1}$ would be a constant. Another special case is $\psi = 1$ and $\theta = \infty$. To make the expected consumption growth rate finite, the variance term of $\alpha_{m,t}$ must

[8]The assumption that consumption is equal to dividends seems strong. In the empirical part, we use dividends, not aggregate consumption, as the endowment process, because the stock market is mainly held by the wealthy households. See Aït-Sahalia, Parker, and Yogo (2001) for a similar argument.

be zero to cancel out θ, which means that $\alpha_{m,t}$ is a constant. It can be shown that consumption is a constant fraction of wealth in this case.

The following equation [Equation 10 in Campbell (93)] can be derived from the log-linearized budget constraint:[9]

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j r_{m,t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j \Delta c_{t+1+j} \tag{7}$$

where ρ is a number related to the average log consumption-wealth ratio. Empirically this number is very close to 1.

Substituting equation (6) into equation (7), we can write the innovations in consumption growth as [equation (38) in Campbell (1993)]:

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= r_{m,t+1} - E_t r_{m,t+1} + (1-\psi)(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{m,t+1+j} \\ &\quad -(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j \alpha_{m,t} \\ &\equiv \eta_{m,t+1} + (1-\psi)\eta_{h,t+1} - \eta_{\alpha,t+1}. \end{aligned} \tag{8}$$

Equation (8) says the innovations in consumption have three components: the first term measures the effect of unexpected market returns on invested wealth, the second term measures the effect of the revision of expected future returns, and the third term reflects the influence of changing risks on saving. It is proportional to the conditional variance of consumption growth less ψ times the return on invested wealth. Since the last term still makes reference to consumption growth, in general it is not possible to substitute consumption growth out of the intertemporal asset pricing model without further assumptions. We can view the first term as resulting from the market risk of investment and the last two terms as results of hedging demand due to changing expected returns and expected volatility, respectively.

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[9] The log-linearized budget constraint is

$$\Delta w_{t+1} = r_{m,t+1} + (1 - \frac{1}{\rho})(c_t - w_t) + k$$

where $\rho = 1 - \exp\{\overline{c_t - w_t}\}$ and k is a constant related to ρ.

To solve for the equilibrium market return, we decompose the market return into an unexpected part and an expected part. We conjecture the unexpected part and verify that it is consistent with the expected part.

2.1 The Unexpected Market Return

From Campbell (1991), we can write the unexpected market return as[10]

$$\begin{aligned}\eta_{m,t+1} &\equiv r_{m,t+1} - E_t r_{m,t+1} \\ &= (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{m,t+1+j} \\ &\equiv \eta_{d,t+1} - \eta_{h,t+1} \end{aligned} \tag{9}$$

where $\rho \equiv 1/(1+\exp(\overline{d-p}))$, $\overline{d-p}$ is the average log dividend price ratio.[11] Empirically, this number is about 0.99 for monthly data. This is an equation describing the unexpected part of the market return and can be thought of as a consistency condition for expectations. We can view the first term $\eta_{d,t+1}$ as news about cash flows and the second term $\eta_{h,t+1}$ as news about future expected returns. It says that if the unexpected market return is negative, then either the expected future dividend growth rate must be lower, or the discount rates (i.e., the expected future market returns) at which they are discounted must be higher, or both.

The specification of the dividend growth rate g_t in equation (1) enables us to write news about future dividends $\eta_{d,t+1}$ in a simple expression related to the uncertainty

[10]This follows from the dynamic Gordon growth model [Campbell and Shiller (1988)]:

$$p_t = \frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty}\rho^j[(1-\rho)d_{t+1+j} - r_{m,t+1+j}\right].$$

This equation says that stock prices are high when dividends are expected to grow rapidly or when dividends are discounted at a low rate.

[11]Note we are also using the notation ρ in equation (7) in which it is related to average log consumption-wealth ratio. Thus we are making the assumption that these two definitions of ρ give very close values. In the empirical analysis, we confirm that this is indeed the case. In fact, if we define the stock price P_t as including dividends, the gross simple market returns are $R_{m,t+1} = \frac{W_{t+1}}{W_t - C_t} = \frac{P_{t+1}}{P_t - D_t}$, market prices are the same as the total wealth, and the two definitions of ρ would be the same in equilibrium.

of the next period's dividend growth:

$$\eta_{d,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j g_{t+1+j} = \frac{1}{1-\rho\beta_d} \sigma_{d,t} \varepsilon_{t+1}$$

$$V_{dd,t} = \frac{1}{(1-\rho\beta_d)^2} \sigma_{d,t}^2 \equiv \lambda_{dd} \sigma_{d,t}^2.$$

Since $\eta_{h,t+1}$ can be viewed as a proxy for uncertainty about the future investment opportunity set, we conjecture and verify in the appendix that $\eta_{h,t+1}$ is proportional to the innovation in g_{t+1} and $\sigma_{d,t+1}^2$. The results are summarized in the following proposition.

Proposition 1 *The news about expected future returns is*

$$\eta_{h,t+1} = c_d \sigma_{d,t} \varepsilon_{t+1} + c_h \sigma_{d,t} \eta_{t+1} \tag{10}$$

with

$$c_d = \frac{1}{\psi} \frac{\rho\beta_d}{1-\rho\beta_d}$$

and

$$c_h = \left\{ \begin{array}{c} A_1 \pi - \frac{B}{\theta} + \frac{\sqrt{(B - A_1\theta\pi)^2 - (A_1\theta)^2}}{\theta} \ \textit{if } \alpha \neq 0 \textit{ and } \gamma \neq 1 \textit{ and } \theta \neq \infty \\ 0 \textit{ otherwise} \end{array} \right\}$$

where $A_1 = \frac{1-\frac{1}{\psi}}{1-\rho\beta_d}$, $B = \frac{1-\rho\beta}{\rho\alpha}$.

The innovations in the market return are

$$\eta_{m,t+1} = c_b \sigma_{d,t} \varepsilon_{t+1} - c_h \sigma_{d,t} \eta_{t+1} \tag{11}$$

where

$$c_b = A_1 + \frac{1}{\psi} = \frac{1 - \frac{\rho\beta_d}{\psi}}{1-\rho\beta_d}.$$

P roof. (See Appendix) ■

Note if the expected dividend growth is a constant ($\beta_d = 0$), we would have $c_d = 0$ and $c_b = 1$. In this case, $\eta_{h,t+1}$ is only proportional to innovations in $\sigma_{d,t+1}^2$. In the case of expected utility preference with log utility ($\gamma = \frac{1}{\psi} = 1$), we have $A_1 = 0$,

$c_h = 0$ and $c_b = 1$, so there are no revisions in expected future returns. In general, however, the revisions in future expected returns η_h are related to innovations in both the dividend growth rate and its volatility. Only in the case of a constant dividend growth rate ($\beta_d = 0$) do we obtain Campbell and Hentschel's (1992) results: $\eta_{h,t+1}$ is only related to innovations in the volatility of the dividend growth rate. Also notice under $\beta_d = 0$, $\eta_{d,t+1} = \sigma_d \varepsilon_{t+1}$, so our model reduces to Campbell and Hentschel (1992), except in our model the volatility follows a square root stochastic volatility process while in their model it follows a Quadratic GARCH process.

Define

$$V_{ij,t} \equiv Cov_t(\eta_{i,t+1}, \eta_{j,t+1}), \qquad i, j \in \{m, d, h\},$$

so $V_{mm,t}$ is the conditional variance of market returns $r_{m,t+1}$, $V_{mh,t}$ is the covariance between the unexpected market return $\eta_{m,t+1}$ and revisions in future expected return $\eta_{h,t+1}$, and so on. Equation (9) then implies

$$\begin{aligned} V_{mh,t} &= V_{dh,t} - V_{hh,t} \\ V_{mm,t} &= V_{dd,t} - 2V_{dh,t} + V_{hh,t}. \end{aligned} \tag{12}$$

Given our solution of η_h, we can write the covariance between news about future dividends and news about future expected returns as

$$V_{dh,t} = \frac{1}{1 - \rho\beta_d}(c_d + c_h\pi)\sigma_{d,t}^2$$

and the conditional variance of the market as

$$V_{mm,t} = [c_b^2 - 2\pi c_b c_h + c_h^2]\sigma_{d,t}^2 \equiv \lambda_{mm}\sigma_{d,t}^2. \tag{13}$$

Thus in our model, the volatility of market returns is proportional to and has the same dynamics as the volatility of the dividend growth rate. Likewise $V_{dd,t}$, $V_{dh,t}$, and $V_{hh,t}$ are all proportional to $\sigma_{d,t}^2$ and of similar order and thus have the potential to explain the empirical evidence documented in Campbell (1991) that $V_{dd,t}$, $V_{dh,t}$, $V_{hh,t}$ are of the same order. In contrast, in Campbell and Hentschel (1992), $V_{dd,t}$ is proportional to $\sigma_{d,t}^2$ but $V_{dh,t}$ and $V_{hh,t}$ are proportional to $\sigma_{d,t}^4$ and thus are smaller-order than $V_{dd,t}$. The fact that consumption growth or dividend growth is smoother than market returns requires $\lambda_{mm} > 1$. Campbell and Shiller

(1988) and Campbell (1991) find that stock returns are considerably more variable than revisions of dividend forecasts. This requires $\lambda_{mm} >> \lambda_{dd}$ in our model. In Campbell and Hentschel's (1992) volatility feedback model, they find $\lambda_{mm} \approx \lambda_{dd}$ and claim volatility feedback can not explain these findings.

An implication of equation (13) is that shocks to market volatility $V_{mm,t}$ are proportional to shocks to the volatility of dividend growth (i.e. η_{t+1}). Given equation (11), a positive c_h would capture the volatility feedback effect: positive shocks to market volatilities imply negative shocks to market returns, i.e., positive shocks to market volatilities drive down market returns. Another implication is that a large volatility feedback effect can make the market return much more volatile than the dividend growth rate. This can be seen from the special case of a constant expected dividend growth rate, in which $c_b = 1$ and $V_{mm,t} = [1-2\pi c_h + c_h^2]\sigma_{d,t}^2$ is thus positively related to c_h given that $c_h > 0$ and $\pi < 0$.

The covariance between current returns and news about future returns is

$$V_{mh} = [c_b c_d + c_b c_h \pi - c_d c_h \pi - c_h^2]\sigma_{d,t}^2 \equiv \lambda_{mh}\sigma_{d,t}^2.$$

There is some empirical evidence that innovations in current market returns are negatively correlated with revisions in expected future returns [Campbell 1991], which means $V_{mh} < 0$. This can be generated easily in our model: if the expected dividend growth rate is constant, we would have $c_d = 0$ and $c_b = 1$, and $V_{mh} = [c_h \pi - c_h^2] < 0$ if $c_h > 0$ and $\pi < 0$.

From equation (11), we can easily compute the covariance between shocks to the market return and shocks to its volatility as

$$V_{m\sigma_m^2,t} \equiv COV_t(r_{m,t+1}, \sigma_{m,t+1}^2) = \lambda_{mm}(c_b\pi - c_h)\alpha\sigma_{d,t}^2.$$

2.2 The Equity Premium, Skewness Premium, and Risk-Free Rate

We now give the main result of the paper, which relate equity premium to volatility of the market return $V_{mm,t}$, the covariance between the current market return and news about future returns $V_{mh,t}$, and the covariance between the market return and its volatility $V_{m\sigma_m^2,t}$:

Proposition 2 *The equity premium is given by*

$$E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} = \gamma V_{mm,t} + (\gamma - 1)V_{mh,t} - \kappa V_{m\sigma_m^2,t} \tag{14}$$

where

$$\kappa = \left\{ \begin{array}{c} \frac{1}{2}(\frac{\gamma-1}{\psi-1})^2 \frac{Var_t(\Delta c_{t+1} - \psi r_{m,t+1})}{V_{mm,t}} \frac{\rho}{(1-\rho\beta)} > 0 \ if \ \psi \neq 1 \\ 0 \ otherwise \end{array} \right\}$$

and the log risk-free rate is given by

$$r_{f,t+1} = -\log\delta + [\frac{\theta - 1}{2}\lambda_{mm} - \frac{\theta}{2\psi^2}]\sigma_{d,t}^2 + \frac{1}{\psi}E_t g_{t+1}. \tag{15}$$

P roof. (See Appendix) ■

Equation (14) says that the equity premium has three components: (1) the first is the same as in the conditional CAPM and is always positive since investors don't like volatility and need a positive reward for bearing it. The relationship is directly related to the risk aversion coefficient γ; (2) the second one is due to the hedging demand against changes in expected future returns and can have either a positive sign or a negative sign depending on whether the investor is more or less risk averse than the log utility investor; (3) the third one is due to hedging demand against changes in expected future volatility as the volatility is correlated with expected returns (i.e., the volatility risk is priced) and it says that all risk-averse investors like positive skewness no matter what his preference is [see Section 2.5 for a discussion on the relationship between asymmetric volatility and skewness]. Campbell (1993) assumes market returns and the consumption growth rate are jointly homoskedastic and thus his formula does not have the last term. Note if volatility is time-varying but not correlated to returns [e.g., a standard GARCH volatility process], the last term would still be zero, which in turn implies symmetric stock returns and zero skewness. Thus this last term is closely related to the skewness and we can also view it as a reward for the negative skewness generated by asymmetric volatility. Since empirically the asymmetric volatility implies $V_{m\sigma_m^2,t} < 0$, it has the potential to help explain the equity premium puzzle as Harvey and Siddique (2000) suggested.[12]

[12]Campbell and Viceira (1999) and Chacko and Viceira (2000) study the opposite question of time varying risk's effect on portfolio choice, taking asset prices as given while Bates (2001) and Pan (2000) study the magnitude of volatility risk from option data.

Note for an investor with a unit risk aversion coefficient, the last two terms would be zero, and a logarithmic version of the static CAPM holds. In this case the investor is myopic in that he doesn't care about the future and has no hedging demand. However, for an investor with unit elasticity of intertemporal substitution, only the last term is zero and there is still a hedging demand against changing expected returns. Note in this case ($\psi = 1$), the consumption wealth ratio is a constant and $Var_t(\Delta c_{t+1} - \psi r_{m,t+1}) = 0$, so that the expression for κ doesn't jump to ∞ when $\psi = 1$.

If the preference is the standard time-separable one ($\gamma = \frac{1}{\psi}$), the equity premium becomes

$$E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} = \gamma(c_b - c_h\pi)\sigma^2_{d,t}$$

where $c_b = \frac{1-\gamma\rho\beta_d}{1-\rho\beta_d}$. In the case of log utility ($\gamma = 1$), $c_b = 1$, $c_h = 0$ and we have the usual formula $E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} = \sigma^2_{d,t}$.

Another special case is with a constant investment opportunity set, which gives us $c_d = 0$, $c_b = 1$ and $c_h = 0$, and the equity premium can be simplified to $E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} = \gamma V_{mm,t}$, which is independent of the elasticity of intertemporal substitution ψ . The reason is that under i.i.d. consumption there is nothing that an investor can do to smooth his consumption no matter what his desire is.[13]

As in most standard models, equation (15) says that a rise in the expected dividend growth rate increases the risk-free rate. For an increase in expected dividend volatility, the effect on the risk-free rate is not clear. However, under the condition $\beta_d \geq 0$, $\psi > 1$, $\gamma > 1$, $\pi < 0$, we would have $\theta < 0$, $c_h > 0$ [c_h has the opposite sign of θ from Proposition 4] and

$$\lambda_{mm} = c_b^2 - 2c_b c_h \pi + c_h^2 > c_b^2 \geq 1 > \frac{1}{\psi^2}$$

$$\gamma_f \equiv \frac{\theta - 1}{2}\lambda_{mm} - \frac{\theta}{2\psi^2} = \frac{1}{2}[(\theta - 1)(\lambda_{mm} - \frac{1}{\psi^2}) - \frac{1}{\psi^2}] < 0.$$

In this case, an increase in expected dividend volatility would reduce the risk-free rate and increase bond prices. Another case is that the preference is the standard

[13]Weil (1989) and Kocherlakota (1990) first demonstrate that under i.i.d. consumption, disentangling the coefficient of relative risk aversion from the elasticity of intertemporal substitution can not help explain the equity premium.

time-separable one ($\gamma = \frac{1}{\psi}$), and the risk-free rate becomes

$$r_{f,t+1} = -\log\delta - \frac{1}{2}\gamma^2\sigma_{d,t}^2 + \gamma E_t g_{t+1}.$$

It is obvious that an increase in expected dividend volatility would reduce the risk-free rate and increase bond prices.

Note that under i.i.d. dividend/consumption growth rate ($c_b = 1$, $c_h = 1$), $\gamma_f = -\frac{1}{2}(1-\theta+\frac{\theta}{\psi^2}) = -\frac{1}{2}[1-(1-\gamma)(1+\frac{1}{\psi})] = -\frac{1}{2}\frac{\gamma(1+\psi)-1}{\psi}$ and the risk free-rate is a constant:

$$r_{f,t+1} = -\log\delta + \frac{1}{\psi}E(g_{t+1}) - \frac{1}{2}\frac{\gamma+\gamma\psi-1}{\psi}Var(g_{t+1}).$$

A similar expression is derived in Kandel and Stambaugh (1991). When $\gamma > 1$ (more risk averse than the log utility) or $\gamma\psi > 1$ (early resolution of uncertainty is preferred), the relation between the interest rate and consumption growth uncertainty would be negative and the negative variance effect (precautionary savings effect) is increasing in γ and decreasing in ψ.

If preferences are time-separable ($\frac{1}{\psi} = \gamma$), the risk-free rate in equation (15) simplifies to

$$r_{f,t+1} = -\log\delta - \frac{1}{2}\gamma^2\sigma_{d,t}^2 + \gamma E_t g_{t+1} \tag{16}$$

and the log pricing kernel in equation (4) becomes

$$y_{t+1} = \log\delta - \gamma g_{t+1} = -r_{f,t+1} - \frac{1}{2}\gamma^2\sigma_{d,t}^2 - \gamma\sigma_{d,t}\varepsilon_{t+1}.$$

Wu (2001) uses a similar dividend growth rate process and a log pricing kernel of the form

$$\begin{aligned} y_{t+1} &= -r_f - \frac{1}{2}\sigma_{y,t}^2 + \varepsilon_{y,t+1} \\ \varepsilon_{y,t+1} &\sim N(0, \sigma_{y,t}^2) \\ Cov_t(\varepsilon_{y,t+1}, \varepsilon_{t+1}) &= \rho_y\sigma_{d,t}^2 \end{aligned}$$

and is thus a "special case" of our model with $\theta = 1$, $\varepsilon_{y,t+1} = -\gamma\varepsilon_{t+1}$, and $\rho_y = -\gamma$. However his assumption of constant interest rates is not consistent with a representative agent economy with power utility.

Note that if expected dividend growth $E_t g_{t+1}$ is not very variable, the risk-free rate would be highly correlated with $\sigma^2_{d,t}$. This may explain why the results of Campbell (1987) on market returns are sensitive to the instruments used. In particular, he finds that whether the instruments include the risk-free rate has a large effect on the results.

2.3 The Expected Market Return

Putting the equity premium and the risk-free rate [equations (14)] and (15)] together, we have the following proposition for expected market returns:

Proposition 3 *The expected simple market return is*

$$E_t r_{m,t+1} + \frac{1}{2} V_{mm,t} = -\log\delta + [\gamma_m + \frac{1}{2}\lambda_{mm} - \frac{1}{2\psi}]\sigma^2_{d,t} + \frac{1}{\psi} E_t[g_{t+1} + \frac{1}{2}\sigma^2_{d,t}]$$

and the expected log market return is

$$E_t r_{m,t+1} = -\log\delta + \gamma_m \sigma^2_{d,t} + \frac{1}{\psi} E_t g_{t+1} \tag{17}$$

where

$$\gamma_m = \left\{ \begin{array}{c} -\frac{\theta}{2}[A_1^2 - 2A_1 c_h \pi + c_h^2] \ \mathit{if}\ \theta \neq \infty \\ = 0 \ \mathit{otherwise} \end{array} \right\}$$

$$sign(\gamma_m) = -sign(\theta).$$

P roof. This follows from the definition of equity premium. ■

Campbell and Hentschel (1992) assume expected market returns are proportional to the volatility of the dividend growth rate. In our model, this relationship is derived endogenously. Note γ_m has the opposite sign of θ. This means that under time-separable preferences ($\theta = 1$), expected *log* stock returns are always a non-increasing function of the expected variance of the dividend growth rate. It looks like our results are not consistent with those of Abel (1988), Barsky(1989), and Gennotte and Marsh (1999), who show that expected (simple) stock return is a decreasing function of expected dividend volatility if and only if $\gamma > 1$ under time-separable preferences. But the difference is that they consider *simple* dividend growth rate and *simple* market returns. We analyze their case in the appendix and show that the results are reconcilable.

To see the intuition behind this proposition, consider the special case of time-separable preferences, in which the log equity premium becomes $E_t r_{m,t+1} - r_{f,t+1} = \{\gamma(c_b - c_h\pi) - \frac{1}{2}[c_b^2 - 2\pi c_b c_h + c_h^2]\}\sigma_{d,t}^2$ and the risk-free rate becomes $r_{f,t+1} = -\log\delta - \frac{1}{2}\gamma^2\sigma_{d,t}^2 + \gamma E_t g_{t+1}$. Note that higher dividend growth volatilities drive up the equity premium and drive down the risk-free rate simultaneously. It happens to be the case that the effect on the risk-free rate is larger than on the equity premium. As a result, the net effect on expected log returns is negative. If we further assume a unit risk aversion coefficient: the log equity premium is now $E_t r_{m,t+1} - r_{f,t+1} = \frac{1}{2}\sigma_{d,t}^2$, and the risk-free rate is $r_{f,t+1} = -\log\delta - \frac{1}{2}\sigma_{d,t}^2 + E_t g_{t+1}$. The expected log returns become $E_t r_{m,t+1} = -\log\delta + E_t g_{t+1}$, which is independent of dividend growth volatilities.

2.4 The Price Dividend Ratio

Given the expressions for expected and unexpected returns, we can easily get the expression for market returns as follows:

$$r_{m,t+1} = E_t r_{m,t+1} + \eta_{m,t+1} = -\log\delta + \gamma_m\sigma_{d,t}^2 + \frac{1}{\psi}E_t g_{t+1} + c_b\sigma_{d,t}\varepsilon_{t+1} - c_h\sigma_{d,t}\eta_{t+1}.$$

If the expected dividend growth rate is a constant, market returns simplifies to

$$r_{m,t+1} = E_t r_{m,t+1} + \eta_{m,t+1} = c + \gamma_m\sigma_{d,t}^2 + \sigma_{d,t}\varepsilon_{t+1} - c_h\sigma_{d,t}\eta_{t+1}$$

where c is a constant. This is the same setup as in Campbell and Hentschel (1992). Thus we can view their model as a special case of ours, except that the relationship between market returns and volatilities of dividend growth rate γ_m is derived endogenously in our model while it is assumed in their paper. Another minor difference is that volatilities of dividend growth rate are modeled as a stochastic volatility process in our model while they are modeled as a Quadratic GARCH process in Campbell and Hentschel (1992).[14]

[14] Campbell and Hentschel (1992) model revisions in dividend growth rate $\eta_{d,t+1}$ instead of innovations in current dividend growth rate $\eta_{t,+1}$ as having time-varying volatilities. Under our model they are proportional and the two modelings are equivalent.

We can now solve for the market price from the definition of market returns $r_{m,t+1} \equiv \log(P_{t+1} + D_{t+1}) - p_t$ as given in the following proposition:

Proposition 4 *The log price dividend ratio is*

$$z_t \equiv p_t - d_t = A_0 + A_1 E_t g_{t+1} + A_2 \sigma_{d,t}^2 \tag{18}$$

$$= A_0 + \frac{1 - \rho\beta + \rho\frac{1}{\psi}(\beta - \beta_d)}{1 - \rho\beta_d} E_t g_{t+1} - \frac{1}{1 - \rho\beta} E_t r_{m,t+1} \tag{19}$$

where

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \rho\beta_d} \tag{20}$$

$$A_2 = -\frac{\gamma_m}{1 - \rho\beta} \tag{21}$$

$$sign(A_2) = -sign(\gamma_m) = sign(\theta) = -sign(c_h).$$

Therefore the volatility feedback effect ($A_2 < 0$) requires $c_h > 0$.

P roof. (See Appendix) ■

Similar results are derived in Bansal and Yaron (2001). It says variations in the price-dividend ratio are due to either variations in expected dividend growth rates or expected discount rates [Campbell and Shiller (1988)], and their contributions depend on their persistence[15] (β_d and β). For example, Bansal and Yaron (2001) assume a persistent expected dividend growth rate process and attribute most of the variations in price-dividend ratio to variations in expected growth rate, while Grossman and Shiller (1981) assume a constant expected dividend growth rate and ascribe much of the variations in price to variations in expected discount rates. Campbell and Shiller (1988) allow both time-varying expected dividend growth rates and time-varying expected stock returns to explain the movements in the price-dividend ratio and find various measures of expected returns are unhelpful in explaining the movements, but their specifications of expected returns do not come from the equilibrium analysis. In particular, they assume either a constant interest rate or a constant risk premium

[15]Poterba and Summers (1986) first point out that the magnitude of the effect on stock prices of increased dividend volatility is an increasing function of the persistence in volatility (β). Campbell (1990) first show that when expected returns follow a persistent process, then their movements will have a large impact on asset prices.

to get $E_t r_{m,t+1}$. In our model, both of these are endogenous and time-varying in an internally consistent way.

Equation (20) implies that when investors have a strong preference for temporally smooth consumption ($\psi < 1$), equity prices are higher when expected consumption growth is low ($A_1 < 0$). This is because investors desiring smooth consumption will attempt to invest more when expected consumption growth is low and thus bid up equity prices at those states/times.

Note that A_2 has the same sign as θ. It says that under time-separable preference ($\theta = 1$), higher expected volatility in the dividend growth rate will always be reflected in higher current stock prices, while the results of Abel (1988), Barsky(1989), and Gennotte and Marsh (1999) say the current stock price is positively related with expected dividend volatility if and only if $\gamma > 1$. Again the difference is that they consider simple dividend growth rate and hold its mean constant when considering the effect of its volatility. For the more general time-non-separable preference, if we believe that higher expected dividend growth raises the price-dividend ratio, then we need a positive A_1, and equation (20) implies $\psi > 1$. If $A_2 < 0$, then an increase in economic uncertainty $\sigma^2_{d,t}$ would lead to a drop in asset price. In this case, $\gamma_m > 0$ by equation (21), which in turn implies an increase in expected market return by equation (17).

Equations (17), (15) and (18) together also shed some light on the signs of predictive regression of returns on dividend price ratio and interest rate [tables 7.1 and 7.2 of Campbell, Lo, and MacKinlay (1997)]: if $A_1 E_t g_{t+1}$ is a constant or relatively smooth compared to $A_2\sigma^2_{d,t}$, then given a positive γ_m, a high $d_t - p_t$ (i.e., a low z_t) denotes a high $\sigma^2_{d,t}$ and thus predicts a high expected return. On the other hand, a high $r_{f,t+1}$ means a low $\sigma^2_{d,t}$ and thus predicts a low expected return. Furthermore, since the expected return is linearly related to $\sigma^2_{d,t}$, a persistent process, it is also persistent. The analysis of Campbell, Lo, and MacKinlay (1997) (pp 271-271) then shows that the regression R^2 will have the specific pattern given in those two tables.

2.5 The Asymmetric Volatility

The asymmetric volatility measured by the negative correlation between the market return and its volatility can be computed from equation (11) as

$$Corr_t(r_{m,t+1}, \sigma^2_{m,t+1}) = \frac{c_b \pi - c_h}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}. \tag{22}$$

Similar expressions are derived in Campbell and Hentschel (1992) and Wu (2001). Note the expression for the conditional correlation is a constant as in Wu (2001), while it is dependent on market volatilities $\sigma^2_{m,t}$ and thus is time-varying in Campbell and Hentschel (1992). This is because in Campbell and Hentschel (1992) the volatility feedback effect is stronger during periods of higher volatilities, while it is independent of the level of volatilities in our model and in Wu (2001).

It is easy to see that asymmetric volatility can be generated from two sources-a negative π and a positive c_h. The first one is due to the leverage effect. To see this, consider the special case of time-separable preference ($\theta = 1$) with log utility ($\gamma = 1$). In this case we have a myopic investor with $c_h = 0$ $c_b = 1$, for whom there should be no volatility feedback effect and asymmetric volatility can only come from the leverage effect. As to the second source, we already know from previous sections that a positive c_h means volatility feedback effect. Note if the expected dividend growth rate is constant, we would have $c_b = 1$ and $Corr_t(r_{m,t+1}, \sigma^2_{m,t+1}) = \frac{\pi - c_h}{\sqrt{1 - 2\pi c_h + c_h^2}}$. It is easy to see that as c_h becomes larger, the correlation approaches -1 (to see this, consider setting $\pi = 0$).

The asymmetric volatility is closely related to the distributional skewness. In fact, for the standard continuous time square root stochastic volatility model, Das and Sundaram (1997) show that the (conditional) skewness is proportional to the asymmetric volatility. Since our model can be viewed as a discrete time version of their model, their results can be used in our model with little modification. The distributional skewness is in turn closely related to the "volatility smile" in the option pricing literature [Bakshi, Cao, and Chen (1997), Bates (2001)]. Thus our model can also help us understand the source of the "volatility smile" and how preferences parameters affect it [see also Garcia, Luger, and Renault (2001a, 2001b)].

Given equation (17), we can compute revisions in expected future returns $\eta_{h,t+1}$

as

$$\eta_{h,t+1} = \frac{1}{\psi}\frac{\rho\beta_d}{1-\rho\beta_d}\sigma_{d,t}\varepsilon_{t+1} + \frac{\gamma_m\rho\alpha}{1-\rho\beta}\sigma_{d,t}\eta_{t+1}$$

and rewrite c_h as

$$c_h = \frac{\gamma_m\rho\alpha}{1-\rho\beta},$$

so the strength of the volatility feedback effect as captured by the magnitude of c_h depends on three factors: the volatility of volatility (α), the persistence of volatility (β) in the dividend growth rate process or the market return (since their volatilities are proportional), and the relationship between the expected return and its volatility (γ_m): the more volatile the volatility, the higher the persistence of the volatility or the stronger the relationship between expected returns and volatility, the stronger the volatility feedback effect.

3 Empirical Results

The data used in this paper are from the CRSP Database and the CBOE website. Specifically, from CRSP, we get data on value-weighted returns including and excluding dividends (VWRETD and VWRETX), CPI growth rate (CPIRET), and the thirty days TBill rate (T30RET) from 1958:1-1999:12, and from the CBOE website, we download data on the S&P 100 option implied standard deviation[16] (Vix) from 1988-1999:12, matched by the S&P 100 data from CRSP.[17] The monthly series of real dividend growth rates are then constructed as in Hodrick (1992)]: the real dividend level is first computed from CRSP value-weighted returns including and excluding dividends, deflated by the price level implied by the inflation rate, then deseasonalized by a trailing twelve months moving average; from the real dividend level we then construct the real dividend growth rate. Real market returns and real risk-free rates

[16]The CBOE market volatility index (Vix) is a measure of expected average volatility in the following month. It's constructed from the weighted average of implied volatilities of eight near-the-money, nearby, and second-nearby S&P 100 index (OEX) option prices. The weights are applied in such a manner that the Vix represents the implied volatility of a thirty calendar day at-the-money OEX option.

[17]In this version of the paper, we use the S&P 500 index as a proxy for the S&P 100 index since CRSP doesn't have data on the latter. Presumably, the correlation between those two indexes would be very high.

are derived by subtracting a trailing twelve months moving average of the inflation rate from the nominal ones. See Table 1 for the summary statistics. The stylized facts discussed in the introduction are evident. For example, the average equity premium and real risk-free rate in the whole sample period are about 6% and 1.4% annually. The stock returns are much more volatile than the dividend growth rate [a standard deviation of 14.6% vs. 3.6% in the whole sample period]. The option-implied conditional variance (Vix^2) is persistent with a first order autocorrelation of 0.7. Note the dividend growth rate is slightly negatively correlated in this particular sample period [also negatively correlated in Wu (2001)] while it is positively correlated in Bansal and Yaron (2001) whose sample period goes back to 1935 and includes the Second World War period. We confirm that if we enlarge our sample to the same size as in Bansal and Yaron (2001), we also get a positively correlated dividend growth rate process. However, we feel more comfortable with the post-war data and will use our sample 1958:1 to 1999:12 for all analysis. Nevertheless, in our calibration exercises, we try three different values for the autocorrelation coefficient: a positive number, zero, and a negative number, and show our results are not very sensitive to this parameter. Note also that the average dividend growth rate is about 1.2% with a standard deviation of 3.6% annually, which is close to calibration parameters of the consumption process in Campbell and Cochrane (1999).[18]

Listed in Table 1 are also summary statistics for two sub-periods. One is the pre-'87 crash period (1958:1-1997:8) and the other one is the post-crash period (1988:1-1999:12). The four months surrounding the October crash are excluded as outliers. It is interesting to note that the average equity premium in the second period is about twice that in the first period (10.36% vs. 5.28%), and the standard deviation of equity returns is actually smaller in the second period. However, the negative skewness in the second period is about three times that in the first period. In light of our findings in Section 2 that investors demand higher premiums for more negatively skewed returns, this is probably one reason why the average equity premium is much higher in the second period. Apparently, market returns became much more negatively skewed

[18]Campbell and Cochrane (1999) use quarterly consumption data from 1947:1 to 1993:4 with a mean of 0.44% and a standard deviation of 0.56%, not annualized. This implies a monthly consumption growth rate with a mean of 0.11% and a standard deviation of 0.19%, not annualized, or 1.32% and 2.3% annually.

after the '87 market crash possibly due to changes in the endowment process and/or preferences.

We first use S&P 100 data and its option-implied volatility to get a feel for asymmetric volatility. Let P_t denote the S&P 100 index level and lower case p_t denote $\log P_t$. The correlation between S&P 100 price change $\Delta p_t \equiv p_t - p_{t-1}$ and implied volatility change $\Delta Vix_t^2 \equiv Vix_t^2 - Vix_{t-1}^2$ given by the data is

$$Corr\{\Delta p_t, \Delta Vix_t^2\} = -0.59$$

which is high for a monthly dataset. This strong negative correlation can also be seen from the following two regressions. The first is the regression of the index return on changes in its implied volatility and the second is the regression of changes in implied volatility on the index return. Both regressions have high R^2.

$$\begin{aligned}
\text{Regression 1} &: \quad \Delta p_t = 0.012 - 12.14\Delta Vix_t^2 \\
& \quad\; (4.618)\ (-8.643) \\
R^2 &= 0.345 \qquad DW = 2.133
\end{aligned}$$

$$\begin{aligned}
\text{Regression 2} &: \quad \Delta Vix_t^2 = 0.0003 - 0.028\Delta p_t \\
& \quad\; (2.330)\ (-8.6433) \\
R^2 &= 0.345 \qquad DW = 2.063
\end{aligned}$$

Recall that the leverage effect and volatility feedback effect work in opposite directions: for the leverage effect, it is the negative shock to returns that causes higher volatility, while for the volatility feedback effect it is higher volatility that drives down returns. Thus an informal test of whether either effect exists is the Granger Causality test in which we estimate a bivariate VAR on $(\Delta p_t,\ \Delta Vix_t\}$ for various lags and test for zero restrictions on the VAR coefficients. The results are given in Table 2. As expected, the test results support the hypothesis that both effects exist in the data, though the leverage effect appears to be stronger.

We now use the Efficient Method of Moments (EMM) [Gallant and Tauchen (1996)] to estimate the following structural model on the log dividend growth rate.

$$g_{t+1} = \omega_d + \beta_d g_t + \sigma_{d,t}\varepsilon_{t+1}$$

$$\sigma^2_{d,t+1} = \omega + \beta\sigma^2_{d,t} + \alpha\sigma_{d,t}\eta_{t+1}$$

$$corr(\varepsilon_{t+1}, \eta_{t+1}) = \pi$$

The score generator or auxiliary model is a non-linear, non-parametric model expanded around an AR(1)-GARCH(1,1) model. The EMM is a simulated method of moments estimator [Duffie and Singleton (1993)]. Instead of choosing a few low-order moments on an ad hoc basis, it uses the score from an auxiliary model to generate the moment conditions. If the auxiliary model approximates the actual distribution of the data well, the EMM estimator is close to the true maximum likelihood estimator and is nearly as efficient.

The EMM estimates of the structural model parameters $\{\omega_d, \beta_d, \omega, \beta, \alpha, \pi\}$ are given in Table 3 along with their standard errors and t-statistics. All parameters are significant at 95% confidence level except for π, the correlation between shocks to the dividend growth rate and shocks to its volatility. This is probably due to the fact that our auxiliary model is an expansion around an AR(1)-GARCH(1,1) model which has no asymmetry in the GARCH part. We are in the process of implementing an Exponential GARCH auxiliary model. In the calibration exercises below, we try two values of π: one is the estimated one, the other is twice the estimated value. The fitted score for the auxiliary model at these parameters is given in Table 4. None of the fitted scores is significant, which means the structural model adequately captures the dynamics in the data as summarized by the auxiliary model. The chi-square test statistics of the over-identifying restrictions is also low and supports this claim.

Givens the estimates of the structural model on dividend growth rates, we study the equity premium, the risk-free rate and asymmetric volatility given by equations (14), (15), and (22), respectively, under different values of the investor's attitude toward risk and intertemporal substitution. The results are given in Tables 5 and 6. Table 6 also gives the decomposition of asymmetric volatility in two parts-the leverage effect part and the volatility feedback effect part. The results are as distressing as in Mehra and Prescott (1985) and Weil (1989). We still need a large risk aversion coefficient to generate a reasonable value of equity premium. For example, with $\gamma = 30$ and $\psi = 0.1$, we get an equity premium of 3.4% and risk-free rate of 0.93%. As to the asymmetric volatility, we also need a high value of ψ. In fact, from Proposition

4, we know that if the risk aversion coefficient γ is greater than 1, then the elasticity of intertemporal substitution ψ must be also greater than 1 in order to generate volatility feedback effect.

Table 7 gives the decomposition of the equity premium into three parts as per equation (14). The first two parts are the same as in Campbell (1993) while the last part, due to the asymmetric volatility or the skewness, is new. If this last part is large, it would help explain the equity premium puzzle. However, given that fact the asymmetric volatility generated by the model is very small, the premium attributed to it is also very small.

We know that from Section (2.1) that the volatility feedback effect is positively related to the volatility of the market return. Therefore another way to see that the model-generated volatility feedback effect is too small is through the volatilities of market returns. Table 8 presents the standard deviations of market returns for selected values of γ and ψ. If we choose a large γ and ψ to match the asymmetric volatility and equity premium, the model generated standard deviation of market return would be much smaller than the one given by the data [See Table 1]. Therefore we have a paradox. We know from the results of Section (2.5) that if we can generate a highly volatile dividend growth rate with a highly persistent volatility, the volatility feedback will be stronger and produce a much higher asymmetric volatility, which in turn can generate a larger equity premium. In fact, Wu (2001) reports a very large volatility feedback effect. This is due to the fact that in his paper, the annualized stock return volatility (standard deviation) of 20% is almost entirely driven by the extremely high dividend growth rate volatility (standard deviation) of 18%, and the estimated volatility process of the dividend growth rate is very persistent (a first order autocorrelation of 0.95 in their estimates vs. 0.86 in our estimates).

In tables 9-12, we set the value of π to be -0.4, about twice as much as the one estimated in the data, and repeat the exercises in tables 5-8. The asymmetric volatility is bigger now, and closer to the valued implied from the option-implied volatility data. However, the stock returns are still not volatile enough.

Tables 13 and 14 give the results of the equity premium, the risk-free rate, and asymmetric volatility when we set the autocorrelation coefficient of dividend growth rate at zero. The value of ω_d is adjusted to match the observed average dividend growth rate in Table 1. For large values of γ and ψ, e.g., $\gamma = 30$ and $\psi = 2$, the model-generated equity premium, risk-free rate, and asymmetric volatility can match those observed in the data [see table 1]. Table 15 gives the decomposition of the equity premium. It can be seen that the skewness premium can be very important, especially when the model generated asymmetric volatility matched that in the data. However, the standard deviations of market return as shown in table 16 are still too small. This is also the main reason why large risk aversion coefficients are need to match the observed equity premium as it's mainly determined by the first component ($\gamma V_{mm,t}$).

4 Conclusion

This paper uses a general equilibrium model to study asymmetric volatility in an exchange economy. In particular, the dividend growth rate is modeled as a stochastic volatility process and the representative agent is endowed with Epstein-Zin preferences. The asymmetric volatility as measured by the negative correlation between market return and its volatility is generated through the leverage effect and the volatility feedback effect. The equilibrium equity premium, risk-free rate, and asymmetric volatility are derived endogenously. It is shown that the equity premium has three components: the first two components are the same as in Intertemporal CAPM, while the last one is "new." It reflects the part of excess returns required by investors for the asymmetric volatility risk or negative skewness risk. It is also shown that the magnitude of volatility feedback effect depends on (1) the positive relationship between expected return and its volatility, (2) the persistence of its volatility, and (3) the volatility of its volatility. The paper then uses the Efficient Method of Moments to estimate the stochastic volatility model on the dividend growth rate and uses the estimated process to study the equity premium, risk-free rate and asymmetric volatility under various values of the risk aversion coefficient and elasticity of intertemporal substitution. It is shown that the premium commanded by the negative skewness can be as high as 1.2% annually in real terms. However under conventional values of the risk aversion coefficient and elasticity of intertemporal substitution, the asymmetric volatility generated by the model is much smaller than that observed in the data and hence results in an asymmetric volatility "puzzle."

The "puzzle" could be due to our specification of the volatility of the underlying dividend growth rate process as a square root process. For example, Anderson, Benzoni, and Lund (2001) show that the square root process can not explain the tail fatness of the stock return distribution. Since the magnitude of volatility feedback is positively related to the volatility of volatility which also governs the tail fatness, it could also be that the square root process can not generate enough volatility feedback.[19] Some extension would be to introduce jumps in volatility as well as a regime switching process. Other possible extensions include separating the consumption from the dividend, introducing two factors in the volatility process (one persistent and one temporary), and introducing time-varying risk aversion and/or human capital.

[19] Das and Sundaram (1997) show that the conditional skewness and kurtosis of the square root stochastic volatility process are positively related to the volatility of volatility.

Appendix

4.1 Proof of Propositions 1 and 2

We start from equation (8):

$$c_{t+1} - E_t c_{t+1} = \eta_{m,t+1} + (1-\psi)\eta_{h,t+1} - \eta_{\alpha,t+1}$$

This implies that

$$V_{mc,t} = V_{mm,t} + (1-\psi)V_{mh,t} - V_{m\alpha,t}. \tag{23}$$

The log risk free rate in this economy is given by

$$r_{f,t+1} \equiv \log(R_{f,t+1}) = -\log E_t(M_{t+1}) = -E_t(y_{t+1}) - \frac{1}{2}Var_t(y_{t+1})$$

where the first equality is from the Euler equation for the *gross* simple risk-free rate $R_{f,t+1} = 1/E_t(M_{t+1})$ and second equality follows from the log-normality of M_{t+1}, and $y_{t+1} \equiv \log(M_{t+1})$ is the log pricing kennel. Now substituting in the expression for the log pricing kernel: $y_{t+1} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} - (1-\theta)r_{m,t+1}$, we have the following results for the log risk-free rate:

$$\begin{aligned} r_{f,t+1} &= -E_t(y_{t+1}) - \frac{1}{2}Var_t(y_{t+1}) \\ &= -\theta\log\delta + \frac{\theta}{\psi}E_t\Delta c_{t+1} + (1-\theta)E_t r_{m+1} - \frac{1}{2}Var_t[\frac{\theta}{\psi}\Delta c_{t+1} + (1-\theta)r_{m,t+1}] \qquad (24) \\ &= -\theta\log\delta + \frac{\theta}{\psi}\alpha_{m,t} + E_t r_{m+1} - \frac{1}{2}[(\frac{\theta}{\psi})^2 V_{cc,t} + 2\frac{\theta(1-\theta)}{\psi}V_{mc,t} + (1-\theta)^2 V_{mm,t}] \end{aligned}$$

where the second equality follows from equation (6).

Substituting in the definition of $\alpha_{m,t} \equiv \psi \log\delta + \frac{\theta}{2}\left[\frac{1}{\psi}V_{cc,t} + \psi V_{mm,t} - 2V_{cm,t}\right]$, we can rewrite the above equation as

$$r_{f,t+1} = E_t r_{m,t+1} + \theta V_{mm,t} - \frac{1}{2}V_{mm,t} - \frac{\theta}{\psi}V_{mc,t}. \tag{25}$$

Rearranging it gives us the fundamental pricing equation:

$$\begin{aligned} E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} &= (1-\theta)V_{mm,t} + \frac{\theta}{\psi}V_{mc,t} \\ &= (1-\theta+\frac{\theta}{\psi})V_{mm,t} + \frac{\theta}{\psi}(1-\psi)V_{mh,t} - \frac{\theta}{\psi}V_{m\alpha,t} \qquad (26) \end{aligned}$$

where the second equality follows from equation (23). Using the definition of θ, the above equation can be simplified as:

$$E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} = \gamma V_{mm,t} + (\gamma - 1)V_{mh,t} + \frac{\gamma - 1}{\psi - 1}V_{m\alpha,t}. \tag{27}$$

We conjecture that $\eta_{h,t+1}$ is proportional to the innovation in g_{t+1} and $\sigma^2_{d,t+1}$:

$$\eta_{h,t+1} = c_d \sigma_{d,t} \varepsilon_{t+1} + c_h \sigma_{d,t} \eta_{t+1}. \tag{28}$$

This implies

$$V_{mm,t} = [\frac{1}{(1-\rho\beta_d)^2} - 2\frac{c_d + c_h\pi}{1-\rho\beta_d} + (c_d^2 + 2c_d c_h \pi + c_h^2)]\sigma^2_{d,t} \equiv \lambda_{mm}\sigma^2_{d,t}$$

$$V_{mc,t} = [\frac{1}{1-\rho\beta_d} - c_d - c_h\pi]\sigma^2_{d,t} \equiv \lambda_{mc}\sigma^2_{d,t}.$$

Substituting those expressions into equation (26) gives us the risk premium as

$$E_t r_{m,t+1} - r_{f,t+1} = [(1 - \theta - \frac{1}{2})\lambda_{mm} + \frac{\theta}{\psi}\lambda_{mc}]\sigma^2_{d,t} \equiv \gamma_e \sigma^2_{d,t}.$$

For the risk-free rate, assuming $\theta \neq 0$, subtract $(1-\theta)r_{f,t+1}$ from both sides of equation (24) and divide by θ, and we get the following expression:

$$\begin{aligned} r_{f,t+1} \quad &= \quad -\log\delta + \frac{1}{\psi}E_t g_{t+1} + \frac{(1-\theta)}{\theta}E_t(r_{m+1} - r_{f,t+1}) \\ &\quad -\frac{1}{2\theta}[\frac{\theta^2}{\psi^2}V_{cc,t} + (1-\theta)^2 V_{mm,t} + 2\frac{\theta(1-\theta)}{\psi}V_{cm,t}]. \end{aligned}$$

Substituting the expressions for $V_{mm,t}$ and $V_{mc,t}$, we can write the risk-free rate as

$$r_{f,t+1} = -\log\delta + [\frac{\theta - 1}{2}\lambda_{mm} - \frac{\theta}{2\psi^2}]\sigma^2_{d,t} + \frac{1}{\psi}E_t g_{t+1} \equiv -\log\delta + \gamma_f \sigma^2_{d,t} + \frac{1}{\psi}E_t g_{t+1}.$$

Putting the equity premium and risk-free rate together gives the expected return:

$$E_t r_{m,t+1} = -\log\delta + (\frac{\theta-1}{2}\lambda_{mm} - \frac{\theta}{2\psi^2} + \gamma_e)\sigma^2_{d,t} + \frac{1}{\psi}E_t g_{t+1} \equiv -\log\delta + \gamma_m \sigma^2_{d,t} + \frac{1}{\psi}E_t g_{t+1} \tag{29}$$

which in turn implies the revision in future expected returns is

$$\eta_{h,t+1} \equiv (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} = \frac{1}{\psi} \frac{\rho \beta_d}{1 - \rho \beta_d} \sigma_{d,t} \varepsilon_{t+1} + \frac{\gamma_m \rho \alpha}{1 - \rho \beta} \sigma_{d,t} \eta_{t+1}.$$

This confirms our conjecture (28) with

$$c_h = \frac{\gamma_m \rho \alpha}{1 - \rho \beta} \tag{30}$$

$$c_d = \frac{1}{\psi} \frac{\rho \beta_d}{1 - \rho \beta_d}.$$

Note that c_h has the same sign as γ_m. If $\gamma = 1$, then $\theta = 0$, which implies $\gamma_m = 0$ by equation (29) and $c_h = 0$. Another special case is $\theta = \infty$, which implies also $c_h = 0$. For the case of $\gamma \neq 1$ and $\theta \neq \infty$, what remains is to find the equation that determines the value of c_h. Consider the log-linearized Euler equation:

$$0 = \theta \log \delta + E_t(-\frac{\theta}{\psi} \Delta c_{t+1} + \theta r_{m,t+1}) + \frac{1}{2} Var_t \left[-\frac{\theta}{\psi} \Delta c_{t+1} + \theta r_{m,t+1} \right]$$

$$\gamma_m = -\frac{\theta}{2} [A_1^2 - 2 A_1 c_h \pi + c_h^2].$$

If $\alpha = 0$, then it's easy to see $\gamma_m = -\frac{\theta}{2} A_1^2$. If $\alpha \neq 0$, then equation (30) implies we can also write γ_m as

$$\gamma_m = \frac{c_h (1 - \rho \beta)}{\rho \alpha} \equiv c_h B.$$

Combining them gives us the equation for c_h (if $\theta \neq 0$):

$$c_h^2 + 2 c_h (\frac{B}{\theta} - A_1 \pi) + A_1^2 = 0$$

whose solutions are

$$c_h = A_1 \pi - \frac{B}{\theta} \pm \frac{\sqrt{(B - A_1 \theta \pi)^2 - (A_1 \theta)^2}}{\theta}.$$

In the case of time-separable preference with log utility ($\gamma = \frac{1}{\psi} = 1$, $\theta = 1$), we would like a constant price dividend ratio, $A_1 = 0$, $A_2 = 0$, and no volatility feedback effect $c_h = 0$. This gives the unique solution of c_h

$$c_h = A_1 \pi - \frac{B}{\theta} + \frac{\sqrt{(B - A_1 \theta \pi)^2 - (A_1 \theta)^2}}{\theta}$$

and concludes the proof of Proposition 1.

Proposition 2 then follows from equation (27) and expressions for $V_{m\sigma_m^2}$ and $V_{m\alpha}$:

$$V_{m\sigma_m^2,t} = COV_t(r_{m,t+1}, \sigma_{m,t+1}^2) = \lambda_{mm}(c_b\pi - c_h)\alpha\sigma_{d,t}^2$$

$$V_{m\alpha,t} = \frac{\lambda_\alpha}{B}(c_b\pi - c_h)\sigma_{d,t}^2 = \frac{\theta}{2}[\frac{1}{\psi} + \psi\lambda_{mm} - 2\lambda_{mc}]\frac{1}{B}\frac{V_{m\sigma_m^2,t}}{\lambda_{mm}\alpha}.$$

4.2 Proof of Proposition 4

From Campbell, Lo, and MacKinlay (1997) [equation (7.1.24) p. 264], the log price dividend ratio in the economy is:[20]

$$\begin{aligned} z_t &\equiv p_t - d_t = \frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty}\rho^j[g_{t+1+j} - r_{m,t+1+j}]\right] \\ &= \frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty}\rho^j\{\frac{\psi-1}{\psi}E_{t+j}[g_{t+1+j}] + \log\beta - \gamma_m\sigma_{d,t+j}^2\}\right] \\ &= A_0 + A_1E_tg_{t+1} + A_2\sigma_{d,t}^2 \end{aligned}$$

where $A_1 = \frac{1-\frac{1}{\psi}}{1-\rho\beta_d}$ and $A_2 = -\gamma_m\frac{1}{1-\rho\beta}$.

4.3 Relation to Previous Work

To relate our results to Abel (1988), Barsky (1989) and Gennotte and Marsh (1993), we impose the same restrictions as in their papers: the expected value of *gross* simple dividend growth rate, $G_{t+1} = D_{t+1}/D_t$, is a constant: $log(E_tG_{t+1}) = E_tg_{t+1} + \frac{1}{2}\sigma_{d,t}^2 = m$. Then the log dividend growth rate process becomes

$$g_{t+1} = m - \frac{1}{2}\sigma_{d,t}^2 + \sigma_{d,t}\varepsilon_{t+1}$$

where $\sigma_{d,t}^2$ follows the same process as in (2), and the revisions in the expected future dividend growth rate becomes

$$\eta_{d,t+1} = \sigma_{d,t}\varepsilon_{t+1} - \frac{1}{2}\frac{\rho\alpha}{1-\rho\beta}\sigma_{d,t}\eta_{t+1}.$$

[20] This comes from the Cambell and Shiller (1988) approximation $r_{m,t+1} \equiv \log(P_{t+1}+D_{t+1}) - p_t \approx k + \rho p_{t+1} + (1-\rho)d_{t+1} - p_t$ where ρ and k are defined as in Section (2.1).

It is easy to verify that the revisions in expected future returns are proportional only to the innovations in expected dividend growth variance $\eta_{h,t+1} = c_h\sigma_{d,t}\eta_{t+1}$. The unexpected market returns are

$$\begin{aligned}\eta_{m,t+1} &= \eta_{d,t+1} - \eta_{h,t+1} = \sigma_{d,t}\varepsilon_{t+1} - (c_h + \frac{1}{2}\frac{\rho\alpha}{1-\rho\beta})\sigma_{d,t}\eta_{t+1} \\ &\equiv \sigma_{d,t}\varepsilon_{t+1} - c_k\sigma_{d,t}\eta_{t+1}\end{aligned}$$

where

$$c_k = \left\{ \begin{array}{c} (1-\frac{1}{\psi})\pi - \frac{B}{\theta} + \frac{\sqrt{[B-(1-\gamma)\pi]^2+\gamma(1-\gamma)}}{\theta} \text{ if } \alpha \neq 0 \text{ and } \theta \neq \infty \\ 0 \text{ otherwise} \end{array} \right\}$$

with $B = \frac{1-\rho\beta}{\rho\alpha}$.

The equity premium is

$$E_t r_{m,t+1} - r_{f,t+1} = [(1-\theta-\frac{1}{2})\lambda_{mm} + \frac{\theta}{\psi}\lambda_{mc}]\sigma_{d,t}^2$$

with $\lambda_{mm} = 1 - 2c_k\pi + c_k^2$, $\lambda_{mc} = 1 - c_k\pi$.

The risk-free rate is given by

$$r_{f,t+1} = -\log\delta + \frac{m}{\psi} + (\frac{\theta-1}{2}\lambda_{mm} - \frac{\theta}{2\psi^2} - \frac{1}{2\psi})\sigma_{d,t}^2.$$

Note that under time-separable preference ($\theta = 1$), the risk-free rate becomes

$$r_{f,t+1} = -\log\delta + m\gamma - \frac{1}{2}\gamma(\gamma+1)\sigma_{d,t}^2$$

which is the same formula given in Gennotte and Marsh (1993).

The expression for expected returns is then given by

$$E_t r_{m,t+1} = -\log\delta + \frac{m}{\psi} + (\gamma_m - \frac{1}{2\psi})\sigma_{d,t}^2 \tag{31}$$

where

$$\gamma_m = -\frac{\theta}{2}[(1-\frac{1}{\psi})^2 - 2(1-\frac{1}{\psi})c_k\pi + c_k^2].$$

The expected simple return $E_t R_{m,t+1}$ is

$$\begin{aligned}
\log(E_t R_{m,t+1}) &= E_t r_{m,t+1} + \frac{1}{2} V_{mm,t} \\
&= -\log\delta + \frac{m}{\psi} + \frac{\gamma}{2(1-\frac{1}{\psi})}[(1-\frac{1}{\psi} - c_k\pi)^2 + c_k^2(1-\pi^2-\frac{1}{\gamma\psi})]\sigma_{d,t}^2 \\
&\equiv -\log\delta + \frac{1}{\psi} m + \Gamma_m \sigma_{d,t}^2.
\end{aligned}$$

In general the sign of Γ_m depends on both γ and ψ and it is impossible to sign Γ_m. A special case is that when $\psi > 1$ and $\frac{1}{\gamma\psi} < 1-\pi^2$, we have $\Gamma_m > 0$.

Given equation (31), we can compute revisions in future expected returns $\eta_{h,t+1}$ as

$$\eta_{h,t+1} = \frac{(\gamma_m - \frac{1}{2\psi})\rho\alpha}{1-\rho\beta}\sigma_{d,t}\eta_{t+1} = c_h\sigma_{d,t}\eta_{t+1}$$

where $c_h = \frac{(\gamma_m - \frac{1}{2\psi})\rho\alpha}{1-\rho\beta}$, and the log price dividend ratio as

$$z_t = A_0 + A_1 m + A_2 \sigma_{d,t}^2$$

where $A_1 = \frac{1-\frac{1}{\psi}}{1-\rho}$ and $A_2 = -\frac{\gamma_m + \frac{1}{2}(1-\frac{1}{\psi})}{1-\rho\beta}$.

The asymmetric volatility is now:

$$Corr_t(r_{m,t+1}, \sigma_{m,t+1}^2) = \frac{\pi - c_k}{\sqrt{1 - 2\pi c_k + c_k^2}}.$$

Note $c_k = c_h + \frac{1}{2}\frac{\rho\alpha}{1-\rho\beta} = \frac{\rho\alpha}{1-\rho\beta}[\gamma_m + \frac{1}{2}(1-\frac{1}{\psi})] = -\rho\alpha A_2$, so A_2 has the opposite sign of c_k. Since the sign of c_k in general depends on both γ and ψ, we would expect that the response of today's stock price to expected dividend volatility depends also both on γ and ψ. Under certain conditions, it may depend only on γ or ψ. For example, under time-separable preference, the expected simple return becomes

$$\log(E_t R_{m,t+1}) = -\log\delta + \frac{m}{\psi} + \frac{\gamma}{2}[1-\gamma-2c_k\pi]\sigma_{d,t}^2$$

and the expression of c_k becomes

$$c_k = (1-\gamma)\pi - B + \sqrt{[B-(1-\gamma)\pi]^2 + \gamma(1-\gamma)}$$

If $\pi = 0$, then it is obvious that c_k has the sign as $(1-\gamma)$. This is also true if $B - (1-\gamma)\pi > 0$ for reasonable values of γ. Under such circumstances, we reach the results of Abel (1988) and Gennotte and Marsh (1993): today's stock price is negatively related to dividend volatility if and only if $\gamma < 1$.

Now consider the special case where consumption growth is i.i.d.: $\beta = 1$ $\alpha = 0$ $\sigma^2_{d,t} = \bar{\sigma}^2_{d,t}$. Then we would have $c_h = c_k = \pi = 0$, $\lambda_{mm} = \lambda_{mc} = 1$ and the following simplified expressions:

Equity premium:

$$\log(E_t R_{m,t+1}) - \log(R_{f,t+1}) = [1 - \theta + \frac{\theta}{\psi}]\bar{\sigma}^2_d = \gamma\bar{\sigma}^2_d$$

Risk-free rate:

$$r_{f,t+1} = -\log\delta + \frac{1}{\psi}m - \frac{1}{2}\gamma(1 + \frac{1}{\psi})\bar{\sigma}^2_d$$

Expected market returns:

$$\log E_t R_{m,t+1} = -\log\delta + \frac{1}{\psi}m + \frac{1}{2\psi}(1 - \frac{1}{\psi})\bar{\sigma}^2_d \tag{32}$$

Log price dividend ratio:

$$z_t = p_t - d_t = A_0 + \frac{1 - \frac{1}{\psi}}{1-\rho}m - \frac{1}{2}\frac{\frac{1}{\psi}(1 - \frac{1}{\psi})}{1-\rho}\bar{\sigma}^2_d. \tag{33}$$

Note now the expression for expected returns and price-dividend ratio depend only on the intertemporal elasticity of substitution ψ, not on the coefficient of risk aversion γ. Under time-separable preference, those expressions become the same as Barsky (1989). Equations (32) and (33) say increased risk reduces today's stock price and raises required (simple) stock returns if and only if ψ is bigger than 1.[21] However, as pointed out in Abel (1988), this analysis is appropriate only for comparing prices in two economies that are identical in every respect except that the variance of dividends is higher in one economy than in the other. This analysis is not appropriate for

[21] Under time separable preference, the results would be that increased risk raises required stock returns only if the investor is not very risk-averse ($\gamma < 1$), which is counter-intuitive. Thus it's very important to separate the coefficient of risk aversion from the intertemporal elasticity of substitution.

analyzing the time series behavior of stock prices in an economy with time-varying dividend volatility. In general, the response of stock prices to dividend volatility depends on both ψ and γ, as analyzed above.

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Table 1

Summary Statistics

(a). 1958:1-1999:12

	g	r_m	r_f
Mean (%)	1.17	7.38	1.40
Std. Dev. (%)	3.60	14.59	0.61
Skewness	0.38	−0.79	−0.27
First Order AutoCorr.	−0.17	0.06	0.89

(b). 1958:1-1987:8

	g	r_m	r_f
Mean (%)	1.06	6.47	1.19
Std. Dev. (%)	3.58	14.67	0.69
Skewness	0.22	−0.28	−0.06
First Order AutoCorr.	−0.21	0.06	0.89

(c). 1988:1-1999:12

	g	r_m	r_f	Vix	Vix^2
Mean (%)	1.48	12.28	1.92	19.44	0.35
Std. Dev. (%)	3.69	12.44	0.34	6.23	0.25
Skewness	0.73	−0.89	0.86	1.22	2.84
First Order AutoCorr.	−0.09	−0.06	−0.42	0.79	0.70

This table gives the summary statistics of real dividend growth rate g, real market return r_m, real risk-free rate r_f, and S&P 100 option-implied conditional one-month ahead standard deviation Vix and variance Vix^2. Mean and standard deviation are annualized and in percentage. The real dividend level is computed from CRSP value-weighted returns including and excluding dividends (VWRETD and VWRETX), deflated by price level implied by the inflation rate, then deseasonalized using a trailing twelve months moving average. From it we construct the real dividend growth rate. The inflation rate is the CPI growth rate (CPIRET) from CRSP. Risk-free rate is the thirty days TBill rate (T30RET) from CRSP. Market return is the CRSP value-weighted return including dividends (VWRETD). The Vix data is from the CBOE website.

Table 2

P-Values of the Granger Causality Test

	Lag Length					
	1	2	3	4	5	6
$H_0 : \Delta p_t$ doesn't Cause ΔVix_t^2	0.038	0.074	0.011	0.079	0.008	0.002
$H_0 : \Delta Vix_t^2$ doesn't Cause Δp_t	0.107	0.286	0.524	0.234	0.169	0.123

This table gives the results of the Granger Causality Test of shocks to return and its volatility, in which we estimate a bivariate VAR on (Δp_t, ΔVix_t^2) for lags 1 to 6 and test for zero restrictions on the VAR coefficients.

Table 3

EMM Model Estimation Results

Parameters	Estimates	Standard Errors	t-Statistics
$\omega_d \times 10^3$	1.364	0.412	3.310
β_d	-0.216	0.047	-4.625
$\omega \times 10^5$	1.374	0.378	3.637
β	0.866	0.028	31.314
$\alpha \times 10^3$	4.200	0.703	5.970
π	-0.191	0.372	-0.512

This table presents the Efficient Method of Moments (EMM) estimation results of the following stochastic volatility model on dividend growth rate (g_t) 1958:1-1999:12:

$$g_{t+1} = \omega_d + \beta_d g_t + \sigma_{d,t}\varepsilon_{t+1}$$

$$\sigma^2_{d,t+1} = \omega + \beta\sigma^2_{d,t} + \alpha\sigma_{d,t}\eta_{t+1}$$

$$corr(\varepsilon_{t+1}, \eta_{t+1}) = \pi.$$

Table 4

EMM Fitted Score and χ^2 Statistics

Parameters	Score	t-Statistics
A_2	-26.111	-0.857
A_3	-98.451	-1.495
A_4	28.237	0.511
A_5	8.622	0.032
A_6	-158.461	-1.297
ψ_1	0.367	1.022
ψ_2	0.558	1.243
τ_1	0.790	0.234
τ_2	-0.699	-0.269
τ_3	1.005	0.244
$\chi^2(4) = 9.071$		$Z = 1.793$

This table lists the fitted score for parameter estimates given in Table 3. The score generator is [11112010] as described in the SNP User Guide. The As are parameters in the polynomial expansion part, the ψs are the parameters in the mean dynamics, and the τs are the parameters in the GARCH part. Insignificant t-statistics denote a good fit at that particular dimension.

Table 5

Equity Premium and Risk-Free Rate ($\pi = -0.19$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	0.05	0.10	0.47	0.94	1.99	3.32
	5.47	**5.43**	**5.15**	**4.78**	**3.99**	**3.04**
1	0.07	0.12	0.52	1.03	2.09	3.24
	6.15	**6.11**	**5.70**	**5.19**	**4.13**	**2.97**
0.2	0.27	0.35	1.01	1.79	3.03	2.94
	11.51	**11.35**	**10.02**	**8.36**	**5.08**	**2.25**
0.1	0.68	0.80	1.77	2.92	4.52	3.40
	18.14	**17.83**	**15.35**	**12.23**	**6.12**	**0.93**
0.05	2.07	2.27	3.85	5.77	8.62	7.05
	31.11	**30.52**	**25.74**	**19.68**	**7.65**	**-3.08**
1/30	4.22	4.48	6.65	9.42	14.17	14.38
	43.71	**42.83**	**35.76**	**26.74**	**8.45**	**-8.93**

This table presents the equity premium given by equation (14) and the risk-free rate given by (15) for selected values of risk aversion coefficient (γ) and elasticity of intertemporal substitution (ψ). The values of other parameters are as given in table 3. The bold numbers are the risk free-rate.

Table 6

Correlation between Shocks to Market Return and Its Variance ($\pi = -0.19$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	-0.19	-0.19	-0.21	-0.24	-0.3	-0.38
	-0.19	**-0.19**	**-0.19**	**-0.19**	**-0.18**	**-0.18**
	0	*0*	*-0.02*	*-0.05*	*-0.11*	*-0.2*
1	-0.19	-0.19	-0.19	-0.19	-0.19	-0.19
	-0.19	**-0.19**	**-0.19**	**-0.19**	**-0.19**	**-0.19**
	0	*0*	*0*	*0*	*0*	*0*
0.2	-0.2	-0.19	-0.1	0.02	0.29	0.59
	-0.19	**-0.19**	**-0.19**	**-0.19**	**-0.19**	**-0.16**
	-0.01	*0*	*0.09*	*0.22*	*0.48*	*0.74*
0.1	-0.21	-0.19	-0.06	0.12	0.48	0.76
	-0.19	**-0.19**	**-0.19**	**-0.19**	**-0.17**	**-0.13**
	-0.02	*0*	*0.14*	*0.32*	*0.65*	*0.89*
0.05	-0.21	-0.19	-0.02	0.2	0.59	0.83
	-0.19	**-0.19**	**-0.19**	**-0.19**	**-0.16**	**-0.11**
	-0.02	*0*	*0.17*	*0.39*	*0.75*	*0.94*
1/30	-0.21	-0.19	-0.01	0.24	0.63	0.86
	-0.19	**-0.19**	**-0.19**	**-0.19**	**-0.15**	**-0.1**
	-0.02	*0*	*0.19*	*0.42*	*0.78*	*0.96*

This table lists the correlation between shocks to market return and its variance as given by equation (22):

$$Corr_t(r_{m,t+1}, \sigma^2_{m,t+1}) = \frac{c_b\pi - c_h}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$$

and the decomposition of it into two parts: (1) leverage effect (bold numbers) $\frac{c_b\pi}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$ and (2) volatility feedback effect (italic numbers) $\frac{-c_h}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$. The values of the parameters used are taken from Table 3.

Table 7

Equity Premium Decomposition ($\pi = -0.19$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	0.05	0.10	0.50	1.02	2.11	3.37
	0.00	**0.00**	**-0.04**	**-0.10**	**-0.25**	**-0.53**
	0.00	*0.00*	*0.00*	*0.02*	*0.13*	*0.49*
1	0.06	0.12	0.60	1.20	2.41	3.61
	0.01	**0.00**	**-0.09**	**-0.19**	**-0.41**	**-0.62**
	0.00	*0.00*	*0.00*	*0.02*	*0.09*	*0.25*
0.2	0.18	0.35	1.70	3.36	7.36	15.37
	0.09	**0.00**	**-0.70**	**-1.57**	**-4.09**	**-10.81**
	0.00	*0.00*	*0.00*	*0.00*	*-0.25*	*-1.62*
0.1	0.40	0.80	3.88	7.85	20.13	54.83
	0.28	**0.00**	**-2.11**	**-4.91**	**-14.94**	**-47.47**
	0.00	*0.00*	*0.00*	*-0.03*	*-0.67*	*-3.96*
0.05	1.14	2.27	10.93	22.80	67.61	216.23
	0.93	**0.00**	**-7.08**	**-16.94**	**-57.48**	**-200.53**
	0.00	*0.00*	*0.00*	*-0.09*	*-1.51*	*-8.64*
1/30	2.26	4.48	21.58	45.69	144.32	487.60
	1.96	**0.00**	**-14.94**	**-36.13**	**-127.79**	**-459.90**
	0.00	*0.00*	*0.00*	*-0.14*	*-2.35*	*-13.33*

This table decomposes the equity premium into three components according to equation (14):

$$E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} = \gamma V_{mm,t} + (\gamma - 1)V_{mh,t} - \kappa V_{m\sigma_m^2,t}$$

The first term (normal number) is the same as the traditional conditional CAPM, the second term (bold number) is the same as in Intertemporal CAPM and is due to the hedge demand against changes in expected future returns, and the last term (italic number) is due to asymmetric volatility or the skewness of market returns. The values of the parameters used are taken from Table 3.

Table 8

Standard Deviation of Market Return ($\pi = -0.19$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	3.18	3.18	3.20	3.22	3.27	3.38
1	3.56	3.56	3.56	3.55	3.55	3.55
0.2	7.05	7.04	6.97	6.94	7.18	8.20
0.1	11.81	11.79	11.66	11.69	12.66	15.77
0.05	21.54	21.49	21.26	21.47	24.03	31.45
1/30	31.32	31.25	30.93	31.32	35.52	47.26

This table lists the annualized volatility (standard deviation) of market returns for selected values of γ and ψ. The annualized volatility (standard deviation) of the dividend growth rate is 3.46% (see Table 1). The values of the parameters used are taken from Table 3.

Table 9

Equity Premium and Risk-Free Rate ($\pi = -0.4$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	0.05	0.10	0.47	0.97	2.17	4.22
	5.47	**5.43**	**5.14**	**4.76**	**3.87**	**2.47**
1	0.07	0.12	0.52	1.05	2.22	3.72
	6.15	**6.11**	**5.70**	**5.17**	**4.00**	**2.48**
0.2	0.27	0.35	0.98	1.72	2.65	0.29
	11.51	**11.35**	**10.02**	**8.34**	**4.96**	**2.35**
0.1	0.68	0.80	1.71	2.69	3.48	-2.71
	18.13	**17.83**	**15.36**	**12.23**	**6.02**	**1.54**
0.05	2.09	2.27	3.64	5.13	6.11	-4.39
	31.10	**30.52**	**25.80**	**19.78**	**7.65**	**-2.25**
1/30	4.26	4.48	6.22	8.20	10.04	-0.29
	43.68	**42.83**	**35.92**	**27.01**	**8.64**	**-8.93**

This table presents the equity premium given by equation (14) and the risk-free rate given by (15) for selected values of risk aversion coefficient (γ) and elasticity of intertemporal substitution (ψ) when $\pi = -0.4$. The value of other parameters are as given in Table 3. The bold numbers are the risk-free rates.

Table 10

Correlation between Shocks to Market Return and Its Variance ($\pi = -0.4$)

EIS (ψ)	CRRA (γ) 0.5	1	5	10	20	30
2	-0.40	-0.40	-0.42	-0.44	-0.50	-0.60
	-0.40	**-0.40**	**-0.40**	**-0.39**	**-0.38**	**-0.35**
	0.00	*0.00*	*-0.02*	*-0.05*	*-0.12*	*-0.25*
1	-0.40	-0.40	-0.40	-0.40	-0.40	-0.40
	-0.40	**-0.40**	**-0.40**	**-0.40**	**-0.40**	**-0.40**
	0.00	*0.00*	*0.00*	*0.00*	*0.00*	*0.01*
0.2	-0.41	-0.40	-0.32	-0.19	0.17	0.68
	-0.40	**-0.40**	**-0.41**	**-0.43**	**-0.43**	**-0.32**
	-0.01	*0.00*	*0.10*	*0.24*	*0.60*	*1.00*
0.1	-0.41	-0.40	-0.28	-0.08	0.42	0.85
	-0.40	**-0.40**	**-0.42**	**-0.44**	**-0.40**	**-0.23**
	-0.02	*0.00*	*0.14*	*0.36*	*0.82*	*1.08*
0.05	-0.42	-0.40	-0.24	0.01	0.57	0.90
	-0.40	**-0.40**	**-0.42**	**-0.44**	**-0.36**	**-0.19**
	-0.02	*0.00*	*0.18*	*0.45*	*0.93*	*1.09*
1/30	-0.42	-0.40	-0.23	0.05	0.62	0.92
	-0.40	**-0.40**	**-0.43**	**-0.44**	**-0.34**	**-0.17**
	-0.02	*0.00*	*0.20*	*0.49*	*0.96*	*1.09*

This table lists the correlation between shocks to the market return and to its variance as given by equation (22):

$$Corr_t(r_{m,t+1}, \sigma^2_{m,t+1}) = \frac{c_b\pi - c_h}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$$

and the decomposition of it into two parts: (1) leverage effect (bold numbers) $\frac{c_b\pi}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$ and (2) volatility feedback effect (italic numbers) $\frac{-c_h}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$. The values of the parameters used are taken from Table 3 except for π, which is -0.4.

Table 11

Equity Premium Decomposition ($\pi = -0.4$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	0.05	0.10	0.51	1.04	2.24	3.95
	0.00	**0.00**	**-0.04**	**-0.11**	**-0.32**	**-0.90**
	0.00	*0.00*	*0.01*	*0.04*	*0.26*	*1.17*
1	0.06	0.12	0.60	1.20	2.40	3.60
	0.01	**0.00**	**-0.09**	**-0.19**	**-0.40**	**-0.61**
	0.00	*0.00*	*0.01*	*0.04*	*0.22*	*0.73*
0.2	0.18	0.35	1.63	3.04	6.03	16.38
	0.09	**0.00**	**-0.65**	**-1.35**	**-3.24**	**-13.39**
	0.00	*0.00*	*0.01*	*0.03*	*-0.14*	*-2.70*
0.1	0.41	0.80	3.65	6.78	16.37	71.31
	0.28	**0.00**	**-1.95**	**-4.10**	**-12.29**	**-67.03**
	0.00	*0.00*	*0.01*	*0.02*	*-0.59*	*-6.99*
0.05	1.15	2.27	10.11	19.05	56.39	311.03
	0.49	**0.00**	**-6.49**	**-13.92**	**-48.78**	**-299.83**
	0.00	*0.00*	*0.02*	*-0.01*	*-1.49*	*-15.59*
1/30	2.28	4.48	19.82	37.72	122.20	723.90
	1.98	**0.00**	**-13.62**	**-29.49**	**-109.78**	**-700.00**
	0.00	*0.00*	*0.02*	*-0.03*	*-2.39*	*-24.19*

This table decomposes the equity premium into three components according to equation (14):

$$E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} = \gamma V_{mm,t} + (\gamma - 1)V_{mh,t} - \kappa V_{m\sigma_m^2,t}$$

The first term (normal number) is the same as the traditional conditional CAPM, the second term (bold number) is the same as in Intertemporal CAPM and is due to the hedge demand against changes in expected future returns, and the last term (italic number) is due to asymmetric volatility or the skewness of market returns. The values of the parameters used are taken from Table 3 except for π, which is -0.4.

Table 12

Standard Deviation of Market Return ($\pi = -0.4$)

EIS (ψ)	CRRA (γ) 0.5	1	5	10	20	30
2	3.18	3.18	3.21	3.25	3.37	3.66
1	3.56	3.56	3.56	3.55	3.55	3.55
0.2	7.06	7.04	6.86	6.68	6.65	8.41
0.1	11.84	11.79	11.43	11.16	11.77	17.48
0.05	21.59	21.49	20.81	20.44	22.58	36.27
1/30	31.40	31.25	30.26	29.80	33.53	55.19

This table lists the annualized volatility (standard deviation) of market returns for selected values of γ and ψ. The annualized volatility (standard deviation) of the dividend growth rate is 3.46% (see Table 1). The values of the parameters used are taken from Table 3 except for π, which is -0.4.

Table 13

Equity Premium and Risk-Free Rate ($\pi = -0.4$, $\beta_d = 0$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	0.08	0.17	0.86	1.78	3.88	6.71
	5.67	**5.61**	**5.09**	**4.41**	**2.88**	**0.91**
1	0.08	0.17	0.85	1.74	3.64	5.83
	6.59	**6.50**	**5.82**	**4.93**	**3.01**	**0.80**
0.2	0.09	0.17	0.79	1.43	1.89	-0.51
	13.78	**13.53**	**11.51**	**8.97**	**3.97**	**-0.34**
0.1	0.09	0.17	0.72	1.12	0.08	-7.16
	22.78	**22.33**	**18.64**	**14.01**	**4.99**	**-2.42**
0.05	0.09	0.17	0.62	0.71	-2.31	-16.10
	40.80	**39.93**	**32.87**	**23.97**	**6.41**	**-8.75**
1/30	0.10	0.17	0.58	0.59	-3.04	-19.25
	58.82	**57.53**	**47.08**	**33.80**	**7.00**	**-17.99**

This table presents the equity premium given by equation (14) and the risk-free rate given by (15) for selected values of risk aversion coefficient (γ) and elasticity of intertemporal substitution (ψ). The bold numbers are the risk-free rates. The values of the parameters used are taken from Table 3 except for $\pi = -0.4$, and $\beta_d = 0$.

Table 14

Correlation between Shocks to Market Return and Its Variance ($\pi = -0.4$, $\beta_d = 0$)

EIS (ψ)	CRRA (γ) 0.5	1	5	10	20	30
2	-0.40	-0.40	-0.42	-0.44	-0.49	-0.55
	-0.40	**-0.40**	**-0.40**	**-0.39**	**-0.38**	**-0.36**
	0.00	*0.00*	*-0.02*	*-0.05*	*-0.11*	*-0.19*
1	-0.40	-0.40	-0.40	-0.40	-0.40	-0.40
	-0.40	**-0.40**	**-0.40**	**-0.40**	**-0.40**	**-0.40**
	0.00	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*
0.2	-0.42	-0.40	-0.26	-0.03	0.47	0.81
	-0.40	**-0.40**	**-0.42**	**-0.44**	**-0.39**	**-0.26**
	-0.02	*0.00*	*0.17*	*0.40*	*0.85*	*1.07*
0.1	-0.43	-0.40	-0.05	0.43	0.87	0.96
	-0.39	**-0.40**	**-0.44**	**-0.39**	**-0.22**	**-0.12**
	-0.04	*0.00*	*0.39*	*0.82*	*1.08*	*1.08*
0.05	-0.47	-0.40	0.35	0.83	0.97	0.99
	-0.38	**-0.40**	**-0.41**	**-0.24**	**-0.10**	**-0.05**
	-0.09	*0.00*	*0.76*	*1.07*	*1.07*	*1.05*
1/30	-0.51	-0.40	0.63	0.93	0.99	1.00
	-0.38	**-0.40**	**-0.34**	**-0.16**	**-0.07**	**-0.03**
	-0.13	*0.00*	*0.97*	*1.09*	*1.05*	*1.03*

This table lists the correlation between shocks to market return and its variance as given by equation (22):

$$Corr_t(r_{m,t+1}, \sigma^2_{m,t+1}) = \frac{c_b\pi - c_h}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$$

and the decomposition of it into two parts: (1) leverage effect (bold numbers) $\frac{c_b\pi}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$ and (2) volatility feedback effect (italic numbers) $\frac{-c_h}{\sqrt{c_b^2 - 2\pi c_b c_h + c_h^2}}$. The values of the parameters used are taken from Table 3 except for $\pi = -0.4$, and $\beta_d = 0$.

Table 15

Equity Premium Decomposition ($\pi = -0.4$, $\beta_d = 0$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	0.08	0.17	0.85	1.75	3.70	6.10
	0.00	**0.00**	**-0.01**	**-0.03**	**-0.18**	**-0.62**
	0.00	*0.00*	*0.01*	*0.06*	*0.36*	*1.23*
1	0.08	0.17	0.84	1.68	3.35	5.02
	0.00	**0.00**	**0.00**	**0.00**	**0.00**	**0.01**
	0.00	*0.00*	*0.01*	*0.06*	*0.28*	*0.80*
0.2	0.09	0.17	0.76	1.41	3.62	12.26
	0.00	**0.00**	**0.03**	**0.02**	**-1.38**	**-10.22**
	0.00	*0.00*	*0.01*	*0.00*	*-0.35*	*-2.55*
0.1	0.09	0.17	0.71	1.73	11.39	60.43
	0.00	**0.00**	**0.01**	**-0.55**	**-10.18**	**-60.83**
	0.00	0.00	0.00	-0.06	-1.13	-6.76
0.05	0.09	0.17	0.81	4.53	51.74	287.26
	0.00	**0.00**	**-0.17**	**-3.63**	**-51.34**	**-288.20**
	0.00	*0.00*	*-0.01*	*-0.19*	*-2.70*	*-15.17*
1/30	0.09	0.17	1.17	10.22	125.17	688.08
	0.01	**0.00**	**-0.57**	**-9.32**	**-123.93**	**-683.75**
	0.00	*0.00*	*-0.02*	*-0.32*	*-4.27*	*-23.58*

This table decomposes the equity premium into three components according to equation (14):

$$E_t r_{m,t+1} - r_{f,t+1} + \frac{1}{2}V_{mm,t} = \gamma V_{mm,t} + (\gamma - 1)V_{mh,t} - \kappa V_{m\sigma_m^2,t}$$

The first term (normal number) is the same as the traditional conditional CAPM, the second term (bold number) is the same as in Intertemporal CAPM and is due to the hedge demand against changes in expected future returns, and the last term (italic number) is due to the skewness risk of market returns. The values of the parameters used are taken from Table 3 except for $\pi = -0.4$, and $\beta_d = 0$.

Table 16

Standard Deviation of Market Return ($\pi = -0.4$, $\beta_d = 0$)

EIS	CRRA (γ)					
(ψ)	0.5	1	5	10	20	30
2	4.09	4.10	4.13	4.18	4.30	4.52
1	4.10	4.10	4.10	4.10	4.10	4.09
0.2	4.13	4.10	3.89	3.80	4.44	6.82
0.1	4.17	4.10	3.80	4.33	8.07	15.17
0.05	4.26	4.10	4.16	7.19	17.18	32.95
1/30	4.36	4.10	5.10	10.82	26.67	50.92

This table lists the annualized volatility (standard deviation) of market returns for selected values of γ and ψ. The annualized volatility (standard deviation) of dividend growth rate is 3.46% (see Table 1). The values of the parameters used are taken from Table 3 except for $\pi = -0.4$, and $\beta_d = 0$.