An investigation of the lead-lag relationship between the VIX index and the VIX futures on the S&P500

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Abstract
This study investigates the lead-lag relationship between the price movements of VIX futures and VIX index levels. As a proxy for the futures, the front month VIX futures contract is used. A Johansen cointegration approach with a vector error correction model and Granger causality analysis are applied. The results suggest that VIX futures lead spot VIX index, which implies that VIX futures market seems to play a more important role in price discovery.

Keywords: *Lead-Lag Relationship, Causality, Price Discovery, Futures, VIX*

1. INTRODUCTION

The lead-lag relationship between spot and futures markets has been an important subject of intense investigation in recent years. Numerous papers have dealt with this issue by looking in different aspects such as the price discovery ability of futures markets or the existence of market efficiency and stability. If the markets are frictionless, the spots and their derivatives must simultaneously reflect new information. Since spots and futures are prices for the same derivative at different points in time, they will be affected in very similar ways (Brooks, 2002) [1]. Otherwise, we may have arbitrage profits.

Most of the literature review shows that the futures lead the spot market (Silvapulle & Moosa, 1999; Gwilym & Buckle, 2001) [2] [3]. A reason for this is market frictions, such as trading costs or capital market microstructure effects. Chan (1992) [4] states that when futures lead the spot market it might be because of non-synchronous trading. On the other hand, there are also several studies showing that cash market have a mild positive impact on futures (Stoll & Whaley, 1990) [5] or even a bi-directional causality (Turkington & Walsh, 1999) [6].

With the enormous increase in derivatives trading and the focus on volatility, especially after the crash in 1987, came the realization that stochastic volatility is an important risk factor affecting pricing and hedging. Academics and practitioners have started to examine it. The need to hedge potential volatility changes required a reference index. It was also suggested that such an index should be the reference index for volatility derivatives that will be used to cope with stochastic volatility and its effect on portfolio returns.

Whaley created the Volatility Index (VIX), or "fear index," in 1993 while working as a consultant for Chicago Board Options Exchange (CBOE). The same year, the CBOE introduced the VIX, the first product on market volatility to be listed on a Security and Exchange Commission (SEC)-regulated securities exchange. Moreover, the dramatic volatility changes during the recent financial crisis, starting in September 2008, serve as a reminder of the need of volatility derivatives.

Shu and Zhang (2012) [7] is the first paper that examined causality in the VIX futures market. They found evidence that VIX futures prices lead the spot VIX index. However, they found bi-directional causality with a modified Baek and Brock nonlinear Granger test.

We provide a few studies that focus on VIX from another perspective. In general, it has been supported that VIX has a predictive ability with regard to stock returns. Chung et al. (2011) [8] investigate the informational role of S&P 500 options and VIX options on returns, volatility and density predictions in the S&P 500 index. They find that the information contents are similar but not identical and that the information recovered from VIX options improves all of the predictions on the S&P 500 index. Zhang et al. (2010) [9] use market data to establish the relationship between VIX futures prices and the index itself and they observe

that VIX futures and VIX are highly correlated; the term structure of average VIX futures prices is upward sloping, whereas the term structure of VIX futures volatility is downward sloping. After, they model the instantaneous variance using a simple square root mean-reverting process with a stochastic long-term mean level, in order to establish a theoretical relationship between VIX futures and VIX. Using daily calibrated long-term mean and VIX, the model gives good predictions of VIX futures prices under normal market situation. These parameter estimates could be used to price VIX options. Degiannakis (2008) [10] show that the VIX index is hard to forecast and does not seem to be very closely connected to the volatility of the underlying index. Also, it seems to have little connection to observable behavior of the actual S&P500 volatility. In the case of trading VIX futures instead of VIX itself, there is no economic gain from forecasting the VIX index. Zhang & Zhu (2006) [11] work in modeling in order to develop an expression for VIX Futures.
The rest part of the study is organized as follows: Section 2 provides some background of the VIX and data description. Section 3 introduces the methodology. Section 4 presents the empirical evidence and Section 5 concludes.

2. Background & Data Description

2.1 What is VIX?

The VIX was originally designed to measure the market's expectation of 30-day volatility implied by at-the-money S&P 100 index option prices [12]. However, the original VIX has a slightly different calculation than the current VIX and differs in two main respects: it was based on the S&P 100 instead of the S&P 500 (SPX), which is the core index for U.S. equities and it targets at-the-money options instead of the estimation of expected volatility by averaging the weighted prices of SPX puts and calls over a wide range of strike prices. The number that is generated is actually an annualized percentage rate.
The formulation[1] of the new method took place in September of 2003, by CBOE together with Goldman Sachs. In attempting to understand the new VIX, it is important to emphasize that it is a forward-looking, measuring volatility that the investors expect to see. More specifically, it is supposed to be a measure of what the option markets are anticipating in terms of volatility for the next 30 days.

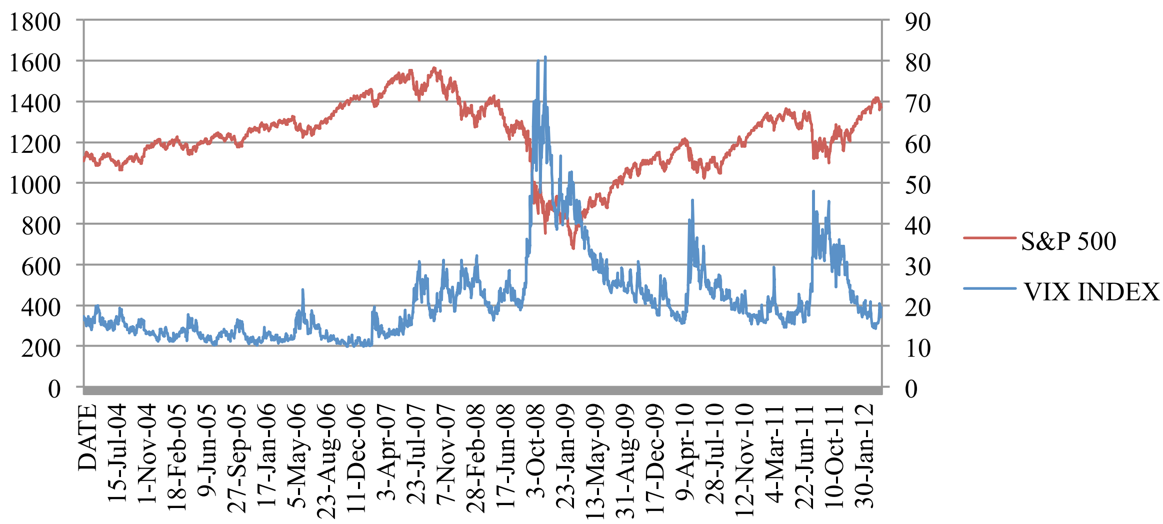

Fig. 1 – VIX vs S&P500

In Figure 1, VIX and implied volatility in general tend to have an inverse relationship to the value of the S&P or any stock or index for that matter. When stocks drop, volatility (measured by the VIX) tends to rise and vice versa. However, Delisle et al (2011) [13] highlight the importance of an asymmetric relationship and note that this negative relation is

[1] More information on how VIX is calculated and a sample calculation are available in VIX White Paper of CBOE at: http://www.cboe.com/micro/vix/vixwhite.pdf

more complex than originally thought. Investors require compensation for market volatility risk when rises, but when market falls there is no differential return requirement. Whaley (2009) [14] published a very good paper in explaining the VIX and why VIX is a useful "market fear gauge".

There are several reasons to pay attention to the VIX. Most investors monitor VIX because it provides important information about investor sentiment that can be helpful in evaluating potential market turning points. It can be considered a very powerful, flexible trading risk management tool in a broad equity portfolio because of this strong negative correlation to the SPX and is generally about four times more volatile. However, other investors use VIX financial instruments in order to speculate on the future direction of the market.

The first exchange-traded VIX futures contract was introduced on the 24th of March 2004 and VIX options followed almost two years later, on the 24th of February 2006. VIX options and VIX futures are among the most actively traded contracts at CBOE[2]. As already mentioned, the high interest in these products is mainly due to their ability to hedge the risks of investments in the S&P 500 index or to speculate.

2.2 Data Description

Regarding daily VIX levels, including open high, low, and close levels are available from the 2nd of January 1990, as well as VIX futures data, including open, high, low, close, settlement prices, trading volume and open interest are available from the 26th of March 2004. All data can be downloaded from the CBOE website[3]. In this study, the starting date of the sample is the 26th of March 2004 and the ending date is the 16th of April 2012. This sample consists of 2029 trading days. As a proxy for the futures price, the price of the front month VIX futures is used. We note that the software EViews version 7 was used for all statistical tests.

3. Methodology

VIX future is a contract written on the VIX index. It is cash settled with the VIX. As VIX is not a traded asset, one cannot replicate a VIX futures contract using the VIX and a risk-free asset. Thus, a cost-of-carry relationship between VIX futures and VIX cannot be established, such as that seen between the S&P 500 index and index futures. Following Shu and Zhang (2012) [7], the long-run equilibrium relationship between the VIX index levels and VIX futures prices is given by the following equation:

$$F_t = \beta_0 + \beta_1 S_t + \varepsilon_t \qquad (3.1)$$

where F_t is VIX futures prices at time t and similarly S_t is the spot VIX index level at time t; β_0 and β_1 the parameters that have to be estimated and ε_t the error term. We can justify Ordinary Least Squares (OLS), if F_t, or S_t or both series are stationary. A series is said to be stationary, if the mean and auto-covariances of the series do not depend on time. However, most prices series seem to be non-stationary. Given the time-series nature of the data, co-integration analysis is therefore the preferred tool.

Hence, we apply an Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1981) [15], in order to determine the order of integration of each series. ADF method is used to test for unit roots. If a non-stationary series y_t, must be differenced d times before it becomes stationary, then it said to be integrated of order d and denoted $y_t \sim I(d)$. Brooks (2002) [1] states that "*The efficient market hypothesis together with rational expectations suggest that asset prices should follow a random walk or a random walk with a drift, so that their differences are unpredictable*", thus we decided to use an intercept in this test. The null hypothesis is that the series contains a unit root and the alternative is that the series is stationary.

[2] Since then, several other volatility indices were introduced such as: NASDAQ-100 (VXN), Crude Oil (OVX), EuroCurrency (EVZ), Gold (GVZ), Russell 2000 small cap index (RVX) etc.

[3] http://www.cboe.com/micro/VIX/vixintro.aspx

An information criterion can be used in order to decide the lag length of our Vector Autoregressive Model (VAR) (Hayashi, 2000) [16]. The mathematical representation of a VAR with k-lags is:

$$\underbrace{y_t}_{g\times 1} = \underbrace{\beta_1}_{g\times g}\underbrace{y_{t-1}}_{g\times 1} + \underbrace{\beta_2}_{g\times g}\underbrace{y_{t-2}}_{g\times 1} + \ldots + \underbrace{\beta_k}_{g\times g}\underbrace{y_{t-k}}_{g\times 1} + \underbrace{u_t}_{g\times 1} \quad (3.2)$$

There are several information criteria such as Akaike Information Criterion (AIC), Hannan-Quinn Criterion (HQIC) or Schwartz's Bayesian Information Criterion (SBIC). Algebraically, the multivariate version of SBIC is expressed as:

$$SBIC = \ln(\hat{\Sigma}) + \frac{k'}{T}\ln T \quad (3.3)$$

where $\hat{\Sigma}$ is the variance-covariance matrix of the residuals, k is the total number of repressors in all equations and T is the sample size.

We also do an analysis on the residuals regarding serial correlation and normality. We perform a multivariate LM test statistic for residual serial correlation up to the specified order. Under the null hypothesis of no serial correlation of order m, the LM statistic is asymptotically distributed χ^2 with k^2 degrees of freedom. Regarding normality of the residuals is an important property in terms of justifying the whole inference procedures. A typical test is the Jarque-Bera test [17]. We apply a multivariate extension of the Jarque-Bera residual normality test, which compares the third and fourth moments of the residuals to those from the normal distribution. The Jarque-Bera test is given by:

$$JB = \left[\frac{T}{6}SK^2 + \frac{T}{24}(EK-3)^2\right] \sim \chi^2 \quad (3.4)$$

where SK is skewness, EK is excess kurtosis and T the number of observations. In case normality is rejected, this fact can be partially attributed to intertemporal dependencies in the moments of the series.

When there is more than one linearly independent cointegrating relationship, Johansen's multivariate VAR approach is the preferred approach [18], because of two important advantages: i) testing for number of co-integrating vectors $N \geq 2$ and allowing each endogenous variable to appear on the left hand side of the estimated equations in the multivariate model, and ii) joint procedure: testing and maximum likelihood estimation of the Vector Error Correction Model (VECM) and long run equilibrium relations are being permitted. The Johansen's technique based on a VAR model needs to be turned into a VECM, which is the resultant model of the pricing relationship between VIX index and VIX futures. The VECM is a restricted VAR designed for use with non-stationary series that are known to be co-integrated and it can is represented as follows:

$$\Delta Y_t = c + \Gamma_1 \Delta Y_{t-1} + \ldots + \Gamma_{k-1}\Delta Y_{t-(k-1)} + \Pi Y_{t-1} + e_t \quad (3.5)$$

where Y_t is the 2x1 vector (F_t, S_t) of VIX futures and VIX spots respectively, Δ denotes the first difference operator, e_t is also a 2x1 vector of the residuals $(e_{F,t}, e_{S,t})$ which are contemporaneous correlated, and the term Π involving Y_{t-1} is the Error Correction Term (ECT). The ECT captures the possible effects of deviations from the estimated long-run relationship.

The cointegration rank test determines the linearly independent columns of the matrix Π [19] [20] [21]. Johansen (1988) [22] and Johansen and Juselius (1990) [23] proposed the cointegration rank test using the reduced rank regression. So, the basis of the Johansen test is the examination of this Π matrix. Johansen and Juselius showed that the coefficient matrix Π contains the essential information about the relationship between F_t and S_t. Thus, it can be interpreted as a long run coefficient matrix. The test is calculated by looking at the rank of Π via its eigenvalues denoted as λ_t. The rank of Π is equal to the number of its eigenvalues that are significantly different from zero. The eigenvalues of an nxn matrix, must satisfy the equation $|\Pi-\lambda I_n|=0$, where I_n is a nxn identity matrix. We obtain the number of cointegrating vectors by estimating the number of these eigenvalues. There are the following two test statistics for cointegration under the Johansen approach, which are formulated as follows:

$$\lambda_{trace}(r) = -T\sum_{i=r+1}^{n}\ln(1-\hat{\lambda}_i) \text{ and } \lambda_{\max}(r,r+1) = -T\sum_{i=r+1}^{n}\ln(1-\hat{\lambda}_{r+1}) \quad (3.6)$$

where $\hat{\lambda}$ are the eigenvalues obtained from the estimate of the Π matrix and T is the number of observations. The λ_{trace} test the null hypothesis that the number of cointegrating vectors is less or equal to r, while the λ_{max} test the null hypothesis the number of cointegrating vectors is r, against an alternative of $r+1$.

Weak exogeneity is also needed for estimation. By definition, X is weakly exogenous if it is a function of only lagged Y and the parameters which determine Y are independent of those determining X

The Granger Causality tests, studied by Granger (1969) [24], are a useful way to describe the relationship between two or more variables when one is causing the other(s). The null hypothesis of this test is that the series X_t does not Granger causes Y_t, and thus lags of X_t should be insignificant in the equation of Y_t. This simply means that lags of X_t do not explain Y_t, or even better, it is only a correlation between the current value of one variable and the past values of the other. In other words, knowledge of past values of X_t does not help in the prediction of current and future values of Y_t. In our case, when VIX index and VIX futures are cointegrated, the ECM will capture the dynamic correlations and causalities between them. Consider the form of the VECM, which can be written as follows:

$$\begin{aligned}\Delta F_t &= c_F + a_F\hat{u}_{t-1} + \sum_{i=1}^{k-1} a_{F,i}\Delta S_{t-i} + \sum_{i=1}^{k-1}\beta_{F,i}\Delta F_{t-i} + e_{F,t} \\ \Delta S_t &= c_S + a_S\hat{u}_{t-1} + \sum_{i=1}^{k-1} a_{S,i}\Delta F_{t-i} + \sum_{i=1}^{k-1}\beta_{S,i}\Delta S_{t-i} + e_{S,t}\end{aligned} \quad (3.7)$$

where Δ is the first difference operator, $a_{F,i}$, $\beta_{F,i}$, $a_{S,i}$, $\beta_{S,i}$ are the short run coefficients, $\hat{u}_{t-1} = F_{t-1} - a_0 - a_1 S_{t-1}$ is the ECT and e are the residuals. The coefficients of the ECT, a_F and a_S, are the speed of adjustment coefficients, which are very important in a VECM. At least one of them must be nonzero for the model to be a VECM. In addition, the relationship of Granger causality between the two series is determined by the coefficients $a_{S,i}$ and $a_{F,i}$. For example, if some of the $a_{S,i}$ are non zero, then F_t Granger causes S_t. Therefore, to test the information causality, the Wald test (Wald, 1943) [25] with an asymptotical χ^2 distribution and a degree of freedom equal to the number of restrictions is used, in order to test the null hypothesis that all lagged variables are zero.

4. Empirical results

The descriptive statistics of the VIX Index and VIX futures are presented in Table 1.

Table 1 – Descriptive Statistics

	VIX Index	VIX Futures
Mean	21.22831	21.55379
Median	18.19000	19.15000
Maximum	80.86000	67.95000
Minimum	9.890000	9.954000
Std. Dev.	10.67795	9.811476
Skewness	2.033704	1.721954
Kurtosis	8.251653	6.498180
Jarque-Bera	3730.288	2037.265
Probability	0.000000	0.000000
Sum	43072.25	43732.63

Sum Sq. Dev.	231229.8	195225.6
Observations	2029	2029

In Table 2, the results of ADF with an intercept are presented for spots and futures as well as for their first differences.

Table 2 – Unit Root Tests

Null Hypothesis: Spot (S) has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on SIC, maxlag=25)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.827415	0.0546
Test critical values:	1% level	-3.433382	
	5% level	-2.862766	
	10% level	-2.567469	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: Futures 1 month (F1) has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=25)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.527438	0.1090
Test critical values:	1% level	-3.433379	
	5% level	-2.862764	
	10% level	-2.567468	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(S) has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=25)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-27.79898	0.0000
Test critical values:	1% level	-3.433382	
	5% level	-2.862766	
	10% level	-2.567469	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(F1) has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=25)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-38.14815	0.0000
Test critical values:	1% level	-3.433379	
	5% level	-2.862764	
	10% level	-2.567468	

*MacKinnon (1996) one-sided p-values.

Clearly, S_t and F_t are both non-stationary since their t-statistics are below 5% critical values. Thus, we don't reject the null hypothesis for a unit root. However, their first differences are stationary. Therefore, VIX index and VIX futures are *I(1)* processes. Nevertheless, we must note that applying the ADF test in this study with an intercept, a trend and an intercept or none, they all give the same results.
The lag length is presented in Table 3 by using the Schwartz Information Criterion. So, we have a VAR with 3 lags.

Table 3 – Lag Length Selection

VAR Lag Order Selection Criteria
Endogenous variables: F1 S
Exogenous variables: C
Sample: 3/26/2004 4/16/2012
Included observations: 2014

Lag	LogL	LR	FPE	AIC	SBIC	HQ
0	-11601.47	NA	346.2142	11.52281	11.52838	11.52486
1	-6571.919	10044.12	2.354761	6.532193	6.548900	6.538326
2	-6549.563	44.60130	2.312226	6.513965	6.541809	6.524185
3	-6525.493	47.97266	2.266599	6.494034	$\boxed{6.533017*}$	6.508343
4	-6517.730	15.45622	2.258145	6.490298	6.540418	6.508694
5	-6503.769	27.76934	2.235919	6.480406	6.541664	6.502891
6	-6499.302	8.876017	2.234883	6.479943	6.552338	6.506515
7	-6487.124	24.17550	2.216806	6.471821	6.555354	6.502482
8	-6475.363	23.32200	2.199789	6.464115	6.558786	6.498864
9	-6470.699	9.240462	2.198339	6.463455	6.569264	6.502292
10	-6463.939	13.37914	2.192323	6.460714	6.577661	6.503639
11	-6443.446	40.51812	2.156709	6.444336	6.572420	6.491349*
12	-6436.687	13.35075	2.150809	6.441596	6.580818	6.492697
13	-6434.224	4.860212	2.154095	6.443122	6.593482	6.498311
14	-6428.386	11.50633	2.150171	6.441297	6.602795	6.500575
15	-6412.744	30.80345*	2.125456*	$\boxed{6.429736*}$	6.602372	6.493102

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

In Table 4 and Table 5, we present the results on autocorrelation and normality tests respectively on the residuals.

Table 4 – Autocorrelation Test on Residuals

VAR Residual Serial Correlation LM Tests
Null Hypothesis: no serial correlation at lag order h
Sample: 3/26/2004 4/16/2012
Included observations: 2026

Lags	LM-Stat	Prob
1	14.79743	0.0051
2	11.27159	0.0237
3	21.69720	0.0002
4	11.20198	0.0244
5	10.38195	0.0345
6	16.29439	0.0026

7	9.293675	0.0542
8	5.649535	$\boxed{0.2269}$
9	22.05588	0.0002
10	55.04823	0.0000
11	5.501699	0.2396
12	7.513971	0.1111

Probs from chi-square with 4 df.

Table 5 – Normality Test on Residuals

VAR Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: residuals are multivariate normal
Sample: 3/26/2004 4/16/2012
Included observations: 2026

Component	Skewness	Chi-sq	df	Prob.
1	0.580112	113.6348	1	0.0000
2	2.527846	2157.691	1	0.0000
Joint		2271.326	2	0.0000

Component	Kurtosis	Chi-sq	df	Prob.
1	14.50004	11164.19	1	0.0000
2	36.47663	94604.45	1	0.0000
Joint		105768.6	2	0.0000

Component	Jarque-Bera	df	Prob.
1	11277.82	2	0.0000
2	96762.14	2	0.0000
Joint	108040.0	4	0.0000

As we suspected, the residuals are serially correlated and non-normal. A solution could be to add more lags in our model. We have chosen SBIC in order to find the optimal number of lags since it is highly consistent. However, many authors suggest that the AIC have theoretical advantages over SBIC and provide better results. Thus, we could add more lags based on AIC. We could also check for outliers and then introduce a dummy variable and check for normality again. In Figure 2, we can see that during the period of global financial crisis after 2008, the market was very volatile and thus, the presence of outliers is high. We could also split the data before and after the crisis and check these issues again. It seems that the relationship between spots and futures changes after the financial crisis.

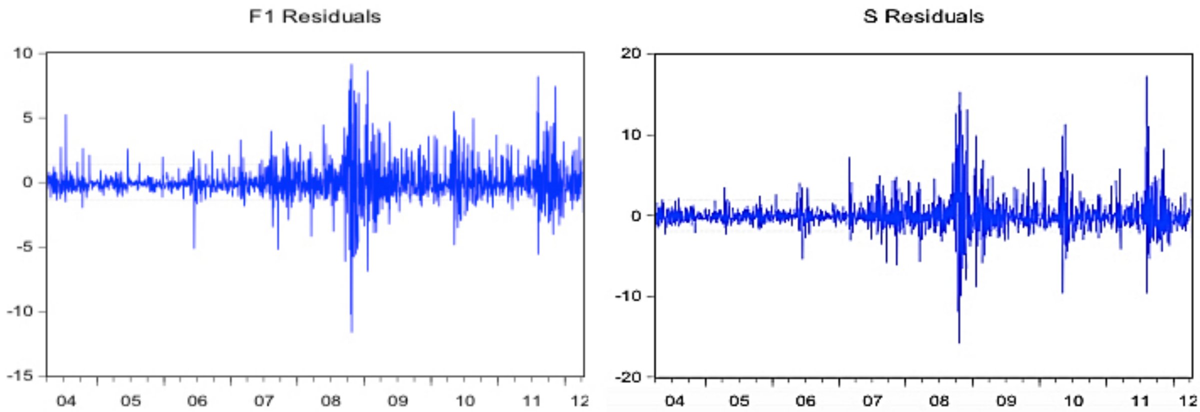

Figure 2 – Residuals

The results of the Johansen approach are presented in Table 6. Both the maximum eigenvalue and trace statistics agree that there is only one co-integration relationship between the VIX index levels and VIX futures. Therefore, there is a stable long-run equilibrium relationship between the two series.

Table 6 – Johansen Cointegration

Sample (adjusted): 4/01/2004 4/16/2012
Included observations: 2025 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: F1 S
Lags interval (in first differences): 1 to 3

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.068071	151.0857	20.26184	0.0001
At most 1	0.004103	8.325577	9.164546	0.0720

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.068071	142.7601	15.89210	0.0001
At most 1	0.004103	8.325577	9.164546	0.0720

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

In Table 7, we present the results of the Normalized VECM. The cointegration relationship is $C_1=1*F(-1) - 0.936*S(-1) - 1.654$ or the normalized cointegrating vector is [1, -0.936]. An illustration of the full error correction model is:

$$\begin{bmatrix} DF_t \\ DS_t \end{bmatrix} = \begin{bmatrix} -0.097 \\ 0.061 \end{bmatrix} \begin{bmatrix} 1 & -0.936 \end{bmatrix} \begin{bmatrix} F_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} -0.111DF_{t-1} \\ -0.072DS_{t-1} \end{bmatrix} + \begin{bmatrix} -0.145DF_{t-2} \\ -0.070DS_{t-2} \end{bmatrix} + \begin{bmatrix} -0.053DF_{t-3} \\ -0.152DS_{t-3} \end{bmatrix}$$

Table 7 – Normalized Cointegrating Vector

Vector Error Correction Estimates
Sample (adjusted): 4/01/2004 4/16/2012
Included observations: 2025 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
F1(-1)	1.000000	
S(-1)	-0.936868 (0.01199) [-78.1330]	
C	-1.654020 (0.28437) [-5.81640]	
Error Correction:	D(F1)	D(S)
CointEq1	-0.097043 (0.01967) [-4.93437]	0.061059 (0.02815) [2.16887]
D(F1(-1))	-0.111384 (0.03890) [-2.86334]	-0.071952 (0.05568) [-1.29214]
D(F1(-2))	-0.145555 (0.03850) [-3.78065]	-0.070595 (0.05511) [-1.28093]
D(F1(-3))	0.053392 (0.03791) [1.40833]	0.152602 (0.05427) [2.81192]
D(S(-1))	-0.017678 (0.02877) [-0.61447]	-0.115995 (0.04118) [-2.81661]
D(S(-2))	0.001222 (0.02830) [0.04317]	-0.054656 (0.04051) [-1.34918]
D(S(-3))	-0.036006 (0.02720) [-1.32392]	-0.122666 (0.03893) [-3.15087]
R-squared	0.042034	0.045999
Adj. R-squared	0.039186	0.043162
Sum sq. resids	3735.346	7654.239
S.E. equation	1.360520	1.947558
F-statistic	14.75774	16.21684
Log likelihood	-3493.275	-4219.662
Akaike AIC	3.457061	4.174481
Schwarz SC	3.476466	4.193885
Mean dependent	4.94E-05	0.001388
S.D. dependent	1.387986	1.991000

Determinant resid covariance (dof adj.)	2.226264
Determinant resid covariance	2.210899
Log likelihood	-6550.018
Akaike information criterion	6.485944
Schwarz criterion	6.533068

Since the t-statistic is 2.81192 > 1.96, we reject the null hypothesis that the coefficient of D(F1(-3)) is zero, so D(F1(-3)) enters the equation determining S. Hence in the short run VIX futures cause the spot market. From the same table, all coefficients of S do not enter the equation determining F, since they are statistically insignificant and therefore, the spot market does not causes the futures market.

The output of Eviews for this representation is given by (4.1):

```
Estimation Proc:                                                                    (4.1)
===============================
EC(C,1) 1 3 F1 S

VAR Model:
===============================
D(F1) = A(1,1)*(B(1,1)*F1(-1) + B(1,2)*S(-1) + B(1,3)) + C(1,1)*D(F1(-1)) + C(1,2)*D(F1(-2)) +
C(1,3)*D(F1(-3)) + C(1,4)*D(S(-1)) + C(1,5)*D(S(-2)) + C(1,6)*D(S(-3)) + C(1,7)

D(S) = A(2,1)*(B(1,1)*F1(-1) + B(1,2)*S(-1) + B(1,3)) + C(2,1)*D(F1(-1)) + C(2,2)*D(F1(-2)) +
C(2,3)*D(F1(-3)) + C(2,4)*D(S(-1)) + C(2,5)*D(S(-2)) + C(2,6)*D(S(-3)) + C(2,7)

VAR Model - Substituted Coefficients:
===============================
D(F1) = - 0.0970465226742*( F1(-1) - 0.936867112757*S(-1) - 1.66189069908 ) -
0.111380962094*D(F1(-1)) - 0.145552414776*D(F1(-2)) + 0.0533937958753*D(F1(-3)) -
0.0176806821704*D(S(-1)) + 0.00121910228992*D(S(-2)) - 0.0360076941821*D(S(-3)) +
1.8471786598e-05

D(S) = 0.0610543188162*( F1(-1) - 0.936867112757*S(-1) - 1.66189069908 ) -
0.0719480155971*D(F1(-1)) - 0.0705907791989*D(F1(-2)) + 0.152604683693*D(F1(-3)) -
0.115999334132*D(S(-1)) - 0.0546599595899*D(S(-2)) - 0.122669549361*D(S(-3)) +
0.00168413269172
```

In Table 8, the results of the weak exogeneity test are presented. It is a test on the coefficients of the α matrix. In particular, which coefficient of this matrix is equal to zero. The two coefficients from (4.1) are *A(1,1)* and *A(2,1)*. If the lags of *F* do not enter the equation that determines *S*, then *F* does not causes *S* and vice versa. It is obvious that the coefficient *A(1,1)* is statistically significant different from zero. So, we can say that *F* is not weakly exogenous. Using the same interpretation, *S* looks weakly exogenous since the coefficient *A(2,1)* is not zero at 1% significance level.

Table 8 – Weak Exogeneity Test

Vector Error Correction Estimates
Sample (adjusted): 4/01/2004 4/16/2012
Included observations: 2025 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
    A(1,1)=0
Convergence achieved after 3 iterations.
Not all cointegrating vectors are identified
LR test for binding restrictions (rank = 1):

Chi-square(1)	22.90863
Probability	0.000002
Cointegrating Eq:	CointEq1
F1(-1)	-0.638113
S(-1)	0.607995
C	0.838407

Vector Error Correction Estimates
Sample (adjusted): 4/01/2004 4/16/2012
Included observations: 2025 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
A(2,1)=0
Convergence achieved after 3 iterations.
Not all cointegrating vectors are identified
LR test for binding restrictions (rank = 1):

Chi-square(1)	4.448608
Probability	$\boxed{0.034930}$
Cointegrating Eq:	CointEq1
F1(-1)	-0.655279
S(-1)	0.609758
C	1.172484

In Tables 9A and 9B, we present the results of the Granger causality in the VECM. This are the Wald tests, where we put restrictions to the lagged coefficients on each equation presented in (4.1). Clearly, from Table 9A, VIX Index does not Granger cause VIX futures, while from Table 9B VIX futures Granger causes VIX Index.

Table 9 – Granger Causality

VEC Granger Causality/Block Exogeneity Wald Tests
Sample: 3/26/2004 4/16/2012
Included observations: 2025

A) Dependent variable: D(F1)

Excluded	Chi-sq	df	Prob.
D(S)	2.216907	3	0.5286
All	2.216907	3	0.5286

B) Dependent variable: D(S)

Excluded	Chi-sq	df	Prob.
D(F1)	12.98765	3	0.0047
All	12.98765	3	0.0047

5. Conclusion

In a perfect market, price discrepancies should not exist. The spot and futures markets should reflect the same information simultaneously. More specifically, futures markets should neither lead nor lag spot markets. However, in real markets, only short leads occur. We might expect futures, with greater leverage, to lead spot prices, if information trading is important (Chan et al., 1993 [26].

Unit root tests, cointegration analysis, VECM and causality analysis are utilized in order to identify the price discovery process of the VIX spots and VIX futures. The unit root tests indicate that each series contains a unit root and they are stationary after first differencing. The Johansen approach suggests that the two series are cointegrated. Thus a long-run equilibrium relationship exists. After applying Wald tests for Granger causality in the VECM, we show that the VIX futures lead the VIX spot market. Hence, VIX futures contain more and useful information about spots, and therefore, they can be used as price discovery vehicles, since such information may be used for decision-making. In summary, empirical evidence suggests that the futures market leads the spot market, reflecting news more quickly. Shu & Zang (2012) [7] state that: *"The relative stable VIX futures prices combined with high liquidity makes VIX futures market very attractive. If more informed traders are attracted by the VIX futures market, it is reasonable that VIX futures prices lead spot VIX"*.

A possible extension for future research could be the examination of transmission of volatility (volatility spillovers) across the two markets and how information flows in those markets. Moreover, the informational role of options volume in the equity options market is a major point in many papers, so it may be feasible to extract useful information from trading volume of VIX options. Finally, no causality tests have been done on the similar as VIX European index, the VSTOXX by EUREX.

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