

# On the Predictive Power of the Implied Correlation Index

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## Abstract

This paper explores the forecasting power of the ICJ index, i.e. the S&P 500 Implied Correlation Index, for the S&P 500 Index returns. We implement different regressions in which future S&P 500 Index returns are regressed on the current information set of ICJ index changes. The result shows that the ICJ changes, i.e. current weekly change and changes in the past, are strongly linked to the S&P 500 Index returns in the future and our model consistently outperforms the random walk model using the Superior Predictive Ability testing procedure. Specifically, we find that the current information set of ICJ changes can be used for predicting the S&P 500 Index returns 7 to 10 months from now. Finally, we investigate the role for this index in predicting the S&P index weekly return volatilities. These results provide useful information for market trading strategies.

**JEL Codes:** G10, C01, C14, G14.

**Keywords:** implied correlation, S&P 500 index, return volatility.

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# 1 Introduction

In July 2009, CBOE (Chicago Board Options Exchange) began disseminating daily values for the CBOE S&P 500 Implied Correlation Index. There are two publicly known Implied Correlation Indexes tied to two different maturities: ICJ for January 2010 and JCJ for January 2011. The CBOE S&P 500 Implied Correlation Index is regarded as a measure of expected average correlation of price returns of S&P 500 Index components, implied through S&P 500 Index option prices and prices of single-stock options on the 50 largest components of the S&P 500 Index. Therefore, it can be used to monitor the market's overall systematic risk.<sup>1</sup> In the literature of modern portfolio management, systematic risk analysis is the most important element because it cannot be diversified away by any portfolio optimization methods. Also it is unknown *ex ante* and cannot be accurately foreseen by the market. How to monitor and control this type of systematic risk has become a very important issue for both regulatory authority and market participants.

Theoretically, the systematic risk is the major component of stock pricing factors and therefore should be reflected in market stock returns. Since the new index can be used to measure the systematic risk, it can provide useful information on returns of the S&P 500 standard index. For instance, many studies have found that return correlations among those major stocks are strengthened during periods when the market is down substantially.<sup>2</sup> This asymmetric response of the market correlation to changes in the market major indexes reflects the different trading strategies taken by market participants and provides useful information on stock market future trends. A number of empirical findings have confirmed the role of the implied correlations of stock returns as a priced risk. Driessen et al. (2005) shows that the substantial gap between average implied and realized correlations is evidence of a large correlation risk premium and indicates that the entire index variance risk premium can be attributed to the correlation risk price. Krishnan et al. (2008) also finds that correlation carries a significantly negative risk price after controlling for asset volatility and other risk factors.

There are a number of studies which explore the autocorrelation structure of

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<sup>1</sup>For a single stock, the systematic risk can be measured by the 'beta' according to the standard CAPM model. It is the non-diversifiable portion of the stock return.

<sup>2</sup>For example, Schwert (1989), Conrad et al. (1991), Cho and Engle (2000) etc. document asymmetric covariances, volatilities and betas of stock returns in their studies.

correlation in order to forecast future stock market movements. They model the conditional correlation based on a historical information set. One of the most general specifications is the one proposed in Engle and Kroner (1995). Engle (2002) develops a multivariate GARCH model with time-varying correlations.<sup>3</sup> However, one limitation of this type of approach is that it forecasts correlations using historical data and the past information set is not so relevant to future market prediction. This motivates some researchers to focus their attention on using the market forecast of future correlation implied by option prices.<sup>4</sup> Skintzi and Refenes (2003) proposes a new methodology for constructing an implied correlation index from option prices and applies it to the Dow Jones Industrial Average (DJIA) index option prices. They find evidence of the existence of a long-run dependence in correlation and contemporaneous relationship between the correlation index daily changes and the DJIA index returns. However, the relationship becomes insignificant for future DJIA returns, which means the implied correlation index they calculated has no predictive power for the DJIA index returns.

In this paper, we fill this gap by exploring the predictive power of the ICJ index for S&P 500 index returns.<sup>5</sup> As Sim (1984) has indicated, asset prices should follow a martingale process over a short time interval and therefore cannot be predicted with their own historical information. We estimate a regression model in which the SPX<sup>6</sup> index returns are linked to the information on the weekly changes in ICJ and SPX in the past. The data we use are the ICJ index data set provided by CBOE covering a time span from November 17th, 2007 to November 23rd, 2009. Although the Implied Correlation Index was launched in July 2009 by CBOE, historical information dating back to 2007 is available. ICJ is the name of CBOE Implied Correlation Index maturing in January 2010. This data set also contains SPX for that period. We split the whole sample by three different proportions, i.e. 2:1, 3:1 and 4:1, for in-sample estimation and out-of-sample testing. With the Superior Predictive Ability testing procedure<sup>7</sup>, we compare our model with

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<sup>3</sup>This model is also known as the Dynamic Conditional Correlation (DCC) model.

<sup>4</sup>Option prices have been used to obtain implied volatility, i.e. the market short-term forecast of the underlying asset volatility. Latane and Rendleman (1976) and Fleming et al. (1995) have good discussions on the predictive power of the implied volatility index.

<sup>5</sup>The reason why we use ICJ in this paper is because it covers the entire period of the latest financial crisis. ICJ only covers the second half of this financial crisis and therefore suffers from the sample selection bias.

<sup>6</sup>SPX is the market quote of the S&P 500 Index.

<sup>7</sup>This is known as the SPA test proposed by Hansen (2005). It is based on the ‘Reality Check’

the random walk model in terms of MSFE (Mean Squared Forecasting Errors) to see if information on weekly changes in those indexes can be used to improve the forecasting of short-term or medium-term S&P 500 index returns. We find that ICJ and SPX weekly returns have some predictive power for the SPX index return 7 to 10 months from now. The SPA testing results indicate that our model consistently outperforms the random walk model.

We also investigate whether we can use the lagged values of those indexes' returns to improve the forecasting of the short-term future S&P 500 index return volatility. For weekly data, we find some evidence that the lag 6 ICJ weekly return volatility has a significant impact on the SPX weekly return volatility. For the SPX daily return series, we see some predictive power of the lag 1 and 10 ICJ daily return volatility for the current SPX daily return volatility.

The remainder of this paper proceeds as follows. Section 2 provides the background for the ICJ index. Section 3 discusses data issues and model specifications. Section 4 presents the estimation procedures and results. Section 5 summarizes this paper and concludes.

## 2 Background

The ICJ index, as a measure of average price return correlation of the major S&P 500 Index stocks, is derived from S&P 500 Index option prices and prices of single-stock options on the 50 largest index components. Generally, the variance of an index is given by the following formula:

$$\sigma_{index}^2 = \sum_{i=1}^N w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j>i}^N w_i w_j \sigma_i \sigma_j \rho_{ij} \quad (1)$$

where  $\sigma_i$  and  $\sigma_j$  are the volatility of  $i$ th and  $j$ th index components,  $w_i$  and  $w_j$  are the weight of  $i$ th and  $j$ th index components.  $\rho_{ij}$  is the pair-wise correlation of index components. In order to calculate the CBOE S&P 500 Implied Correlation Index, we determine the weight of each index component as follows:

$$w_i = \frac{p_i s_i}{\sum_{i=1}^{50} p_i s_i} \quad (2)$$

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testing procedure by White (2000).

where  $p_i$  is the price of the  $i$ th index component and  $s_i$  is the float-adjusted shares outstanding of the  $i$ th index component. As the ICJ index is designed to measure the average correlation of S&P 500 Index components in which case all the pair-wise correlations are assumed to be equal, we can solve for this index from equation (1) as follows:

$$\rho_{average} = \frac{\sigma_{index}^2 - \sum_{i=1}^N w_i^2 \sigma_i^2}{2 \sum_{i=1}^{N-1} \sum_{j>i}^N w_i w_j \sigma_i \sigma_j} \quad (3)$$

The CBOE S&P 500 Implied Correlation Index is calculated using the index option implied volatilities and the implied volatilities of options on the 50 largest stocks comprising the S&P 500 Index.

### 3 Data and Model Specification

#### 3.1 Model Specification

Our model takes the following regression form:

$$r_{spx}^{t,t+k} = \alpha_0 + \alpha_1 r_{spx}^{t-1,t} + \alpha_2 r_{icj}^{t-1,t} + \alpha_3 r_{spx}^{t-1,t} r_{icj}^{t-1,t} + \alpha_4 r_{icj}^{t-k,t-1} + \epsilon_t \quad (4)$$

where  $r_{spx}$  and  $r_{icj}$  are returns of SPX and ICJ.<sup>8</sup> Superscript  $t-k$ ,  $t-1$ ,  $t$  and  $t+k$  denote different time points. Thus,  $r_{spx}^{t,t+k}$  is the return of SPX over the time period from  $t$  to  $t+k$ .  $r_{spx}^{t-1,t}$  and  $r_{icj}^{t-1,t}$  are one-period return of SPX and ICJ.  $r_{icj}^{t-k,t-1}$  is added in the regression because the time series of the dependent variable contain overlapping components. Theoretically, if the market is efficient, the price simply follows a random walk process<sup>9</sup>

$$p_t = p_{t-1} + \epsilon_t \quad (5)$$

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<sup>8</sup>As always, returns are calculated as log-difference of index prices.

<sup>9</sup>Stock market indexes, formed by a basket of individual stocks, follow a random walk process under the hypothesis of efficient markets. A number of studies, e.g. Kendall (1953) and Fama (1965), have been concentrated on statistical approaches to testing the serial independence of stock or index prices. In general, most studies on this topic tend to uphold the theory of random walk.

where  $\epsilon_t$  is independently distributed.<sup>10</sup> In this case, returns are martingale difference sequences

$$E(r_t|F_{t-1}) = 0 \quad (6)$$

where  $F_{t-1}$  is the information set at time  $t - 1$ . In order to justify our model setting, we first do a unit root test for both the level of SPX and ICJ and the daily return of these two indexes. The results are summarized in the following table:

Table 1: unit root test for the level and daily return of SPX and ICJ (2007-2009)

p-value	Level	Return
SPX	0.98	0.01
ICJ	0.71	0.01

From the table, we cannot reject the null hypothesis that the level of these two indexes is a unit root process. However, the test statistic for the daily return series of SPX and ICJ strongly suggests that the daily return is stationary for both indexes. We show the level and daily return time series for both indexes graphically:

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<sup>10</sup>Residuals can be homogeneous or heterogeneous.

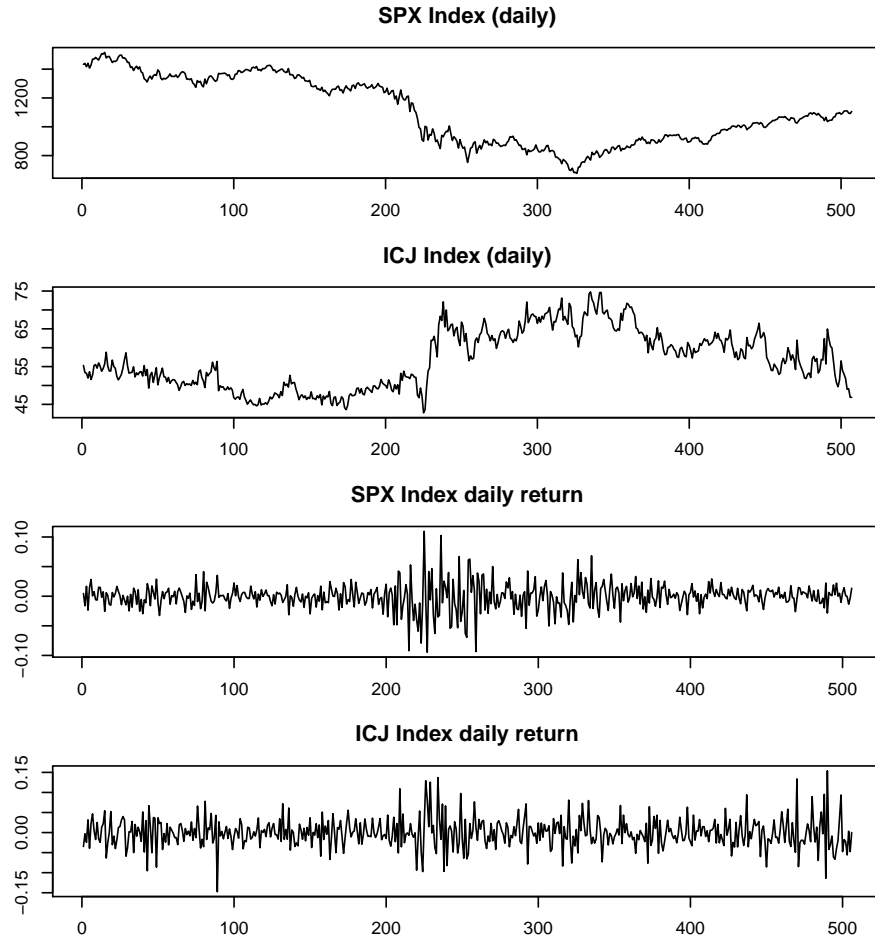


Figure 1 - Time Series for SPX and ICJ Daily Closing Prices and their Daily Returns

The first two graphs in Figure 1 show the level of SPX and ICJ. Clearly, they are not stationary and most likely follow a random walk process. The last two graphs show the SPX and ICJ daily return series, which appear to be stationary with a zero mean. We show the autocorrelation function of SPX and ICJ in the following Figure:

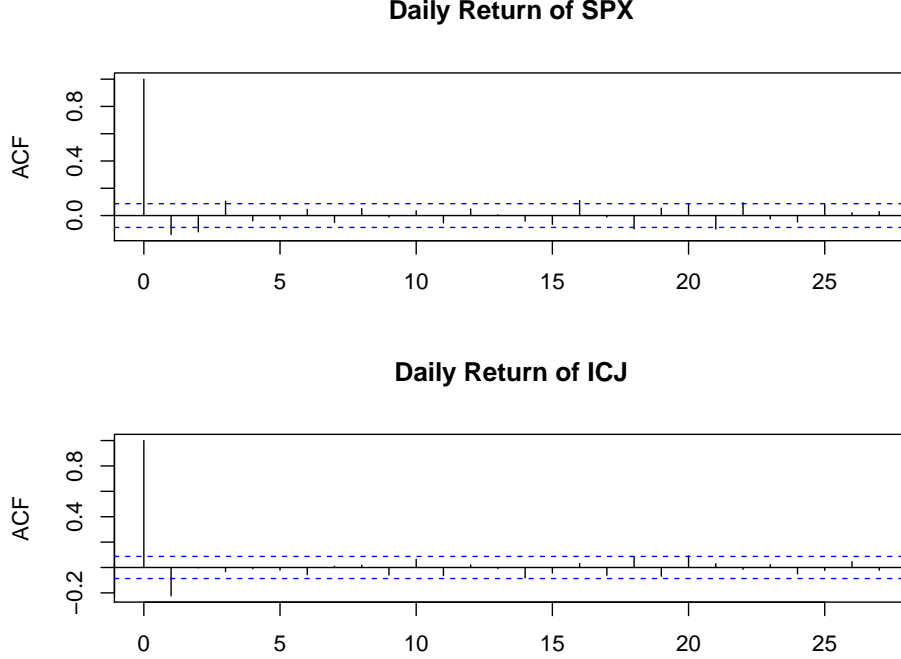


Figure 1 - Autocorrelation Function for SPX and ICJ Daily Returns

There is no evidence that the current SPX return is correlated with any of its past values. Therefore, we cannot reject the market efficiency hypothesis that SPX return series are a martingale difference sequence. This leads to a natural conclusion that SPX may follow a random walk process and the expectation of its multi-period return, e.g. weekly or monthly return etc., would be zero conditional on today's information.<sup>11</sup> We take this as the benchmark model and it can be expressed as

$$E(r_{spx}^{t,t+k} | F_t) = 0 \quad (7)$$

Given this fact, it is reasonable to have the following regression equation:

$$r_{spx}^{t,t+k} = \alpha_0 + \alpha_1 r_{icj}^{t-1,t} + \alpha_2 r_{spx}^{t-1,t} r_{icj}^{t-1,t} + \alpha_3 r_{icj}^{t-k,t-1} + \epsilon_t \quad (8)$$

Also, in this paper, we estimate a regression equation in which we only keep the interacted term of SPX and ICJ as the independent variable for the current time

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<sup>11</sup>The Unit root test for weekly data does not reject the market efficiency hypothesis either.



period:

$$r_{spx}^{t,t+k} = \alpha_0 + \alpha_1 r_{spx}^{t-1,t} r_{icj}^{t-1,t} + \alpha_2 r_{icj}^{t-k,t-1} + \epsilon_t \quad (9)$$

We compare our model, i.e. equation (4), (8) and (9), with the benchmark random walk model for SPX and ICJ weekly returns.

## 3.2 Data

We use the data provided by CBOE. The ICJ data set contains daily closing prices of SPX and ICJ for a two-year time period. We take log-difference of those index prices to transform them into weekly return series.

## 3.3 Test of Superior Predictive Ability

Following common practice, we divide the whole sample of weekly returns into two parts. The first part is for model estimation and the second is for forecasting evaluation. We make three different sample splits 2:1, 3:1 and 4:1 for weekly return series in order to make our results more robust. The out-of-sample performance evaluation is carried out by implementing the Superior Predictive Ability test proposed by Hansen (2005). This test is intended for testing whether a benchmark forecasting model is outperformed by alternative forecasting models. In this paper, we use this testing procedure to compare our model with the random walk model in terms of MSFE (Mean Square Forecasting Error).

# 4 Model Estimation and Test

## 4.1 Estimation and tests for the SPX weekly returns

We consider using equations (4), (8) and (9) as our model to see if additional information on current and past SPX and ICJ returns would be useful to forecast the S&P 500 Index returns in the next  $k$  periods.<sup>12</sup> We look at the weekly return data and try different values of  $k$ . The results for different sample splits are shown in Tables in the Appendix.

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<sup>12</sup>For weekly return data,  $k$  denotes the number of weeks for which the SPX return is forecasted.

It can be seen from those tables that the model coefficients are statistically significant and relatively stable. We only report the results of model estimation for SPX multi-period returns of 28 to 39 weeks (7 to 10 months) because the Superior Predictive Ability test indicates that, within this time range, the out-of-sample forecasting performance of our model is better than the random walk model. We show the SPA test results of 23 to 40 weeks (5 to 10 months) to make comparison in the following tables:

Table 2: SPA Test of S&P 500 Index Weekly Return for Equation (4) against Random Walk as Benchmark

weeks	23	24	25	26	27	28	29	30	31
2:1	0.10	0.07	0.08	0.01	0.00	0.00	0.00	0.00	0.00
3:1	0.85	0.70	0.46	0.20	0.05	0.00	0.00	0.00	0.00
4:1	1.00	0.96	0.95	0.58	0.55	0.11	0.00	0.00	0.00
weeks	32	33	34	35	36	37	38	39	40
2:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03

Table 3: SPA Test of S&P 500 Index Weekly Return for Equation (8) against Random Walk as Benchmark

weeks	23	24	25	26	27	28	29	30	31
2:1	0.10	0.08	0.06	0.00	0.00	0.00	0.00	0.00	0.00
3:1	0.81	0.66	0.39	0.14	0.03	0.00	0.00	0.00	0.00
4:1	1.00	0.98	0.93	0.52	0.46	0.02	0.00	0.00	0.00
weeks	32	33	34	35	36	37	38	39	40
2:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
4:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.09

From the above, we see that the p-value is zero for most of SPA tests for equations (4), (8) and (9). All three models outperform the benchmark Random Walk model no matter what splits we try. In order to figure out whether model (4), (8) or (9) is the best model to be selected, we compare model (4) and (8)

Table 4: SPA Test of S&P 500 Index Weekly Return for Equation (9) against Random Walk as Benchmark

weeks	23	24	25	26	27	28	29	30	31
2:1	0.07	0.05	0.03	0.00	0.00	0.00	0.00	0.00	0.00
3:1	0.77	0.60	0.32	0.08	0.01	0.00	0.00	0.00	0.00
4:1	1.00	0.91	0.85	0.42	0.36	0.00	0.00	0.00	0.00
weeks	32	33	34	35	36	37	38	39	40
2:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4:1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03

against (9) as benchmark and and report the SPA results of 28 to 39 weeks in the following table:<sup>13</sup>

Table 5: SPA Test of S&P 500 Index Weekly Return for Equation (4) against (9) as Benchmark

weeks	28	29	30	31	32	33	34	35	36	37	38	39
2:1	0.89	0.90	0.35	0.09	0.48	0.07	0.03	0.13	0.31	0.31	0.99	0.66
3:1	0.94	0.68	0.40	0.20	0.85	0.07	0.10	0.83	0.04	0.39	0.32	0.10
4:1	0.96	0.63	0.74	0.46	0.59	0.05	0.17	0.78	0.03	0.38	0.13	0.05

While model (4) and (8) appear to be a little better than model (9) in terms of the SPA results, there is no strong evidence that model (9) can be dominated by either of them at the significance level of 5% or 10%. Parsimony consideration would prefer model (9) as the best one for predicting the SPX future returns.

It is worth noting that except for the lagged SPX term in model (4), all the model coefficients are positive. It makes a lot of sense, particularly for the interacted term of SPX and ICJ. It suggests that if the market sees a slump and a strengthened average stock correlation on average in a particular week, the S&P 500 Index return 7 to 10 months from now would likely be negative. This implies that the current downward trend is likely to continue in the near future. The opposite is true when the market witnesses a big rally and an enlarged average stock

<sup>13</sup>Those are cases where the p value is zero or a small number.

Table 6: SPA Test of S&P 500 Index Weekly Return for Equation (8) against (9) as Benchmark

weeks	28	29	30	31	32	33	34	35	36	37	38	39
2:1	0.72	0.49	0.15	0.11	0.35	0.07	0.03	0.16	0.41	0.34	0.98	0.40
3:1	0.78	0.33	0.23	0.19	0.52	0.08	0.10	0.16	0.04	0.42	0.33	0.08
4:1	0.72	0.08	0.15	0.17	0.25	0.07	0.11	0.19	0.03	0.40	0.24	0.03

correlation in a week.

## 4.2 Volatility estimation of SPX weekly returns

We first test the null hypothesis that weekly returns of SPX are martingale difference sequences using the standard GARCH(1,1) model:

$$r_t = \sigma_t \epsilon_t \quad (10)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (11)$$

Under the null hypothesis, the return series are independently distributed but the residuals are heterogeneous as the variance at each time depends on values of return and variance in the previous time period. The result is shown in the following table:

Table 7: GARCH(1,1) for Weekly Returns of S&P 500 Index (2007-2009)

	Estimate	Std. Error	Pr(>  t )
$\alpha_0$	0.0005	0.0002	0.0240*
$\alpha_1$	0.5709	0.1810	0.0016**
$\beta_1$	0.1308	0.2080	0.5296

The Jarque Bera test and Box-Ljung test statistics are 0.9928 and 0.3051, respectively, which says that the hypothesis of residuals being normally distributed can not be rejected and the standard GARCH model is not capable of fitting in the weekly returns of S&P 500 Index. This implies that the past information on the SPX index cannot be used to forecast the current or future index return volatility.

In this section, we try to use information on the ICJ index to see if it can help explain some variations of the SPX weekly return volatility.

The SPX weekly return volatility forecasting model takes this following simple regression form:

$$\sigma_{spx,t}^2 = \alpha_0 + \alpha_1 \sigma_{icj,t-k}^2 + \epsilon_t \quad (k = 1, 2, 3, \dots) \quad (12)$$

We summarize the results for different lags of ICJ in the following tables:

Table 8: Regression of Volatility for SPX Weekly Returns

	coefficient	estimate	Std. Error	Pr(>  t )
lag 1	$\alpha_0$	0.0017	0.0005	0.0004
	$\alpha_1$	0.0199	0.0288	0.4890
lag 2	$\alpha_0$	0.0016	0.0005	0.0010
	$\alpha_1$	0.0445	0.0288	0.1255
lag 3	$\alpha_0$	0.0017	0.0005	0.0004***
	$\alpha_1$	0.0178	0.0292	0.5444
lag 4	$\alpha_0$	0.0018	0.0005	0.0002***
	$\alpha_1$	0.0002	0.0303	0.9949
lag 5	$\alpha_0$	0.0017	0.0005	0.0006***
	$\alpha_1$	0.0320	0.0309	0.3025
lag 6	$\alpha_0$	0.0015	0.0005	0.0017**
	$\alpha_1$	0.0743	0.0302	0.0158*

Table 9: Regression Fit of SPX Weekly Returns

	Adj. R-squared	F-statistic	p-value
lag 1	-0.0050	0.482	0.4890
lag 2	0.0133	2.387	0.1255
lag 3	-0.0062	0.370	0.5444
lag 4	-0.0100	0.000	0.9949
lag 5	0.0007	1.074	0.3025
lag 6	0.0484	6.037	0.0158

From these tables, we see that the lag 6 return volatility of ICJ has some predictive value for the current SPX weekly return volatility. The adjusted R-

squared and F-statistic also indicate that the lag 6 ICJ return volatility regression is not a bad fit.

### 4.3 Volatility estimation for the SPX daily returns

In this section, we investigate the predictive power of ICJ for the SPX daily returns. Instead of using ICJ to forecast the SPX daily returns, we try to figure out whether this kind of information can help explain some variations of the SPX daily return volatility.

The stock market index daily returns are normally serially independent as we mentioned above. Their distributions are martingale difference sequences. However, residuals can be heterogeneous for some return series and the variances are often found to be clustered or serially correlated. The GARCH models are usually used to deal with the return volatility dynamics. For example, Equation 8 and 9 in the last section are the standard GARCH(1,1) model.

We fit the SPX daily return data with the standard GARCH(1,1) model and the results are summarized as follows:

Table 10: GARCH(1,1) for Daily Returns of SPX Index (2007-2009)

	Estimate	Std. Error	Pr(>  t )
$\alpha_0$	4.834e-006	0.0000	0.0882
$\alpha_1$	9.880e-002	0.0243	0.0001***
$\beta_1$	8.897e-001	0.0261	0.0000***

The p-value of Jarque Bera and Box-Ljung test statistic (squared residuals) are 0.00695 and 0.1428, respectively. This implies that residuals are not normally distributed and the daily return data cannot be appropriately modeled by the standard GARCH(1,1) model.

There are a number of ways to use the lagged ICJ values in forecasting the SPX daily returns. In our model, we add different squared lagged values of ICJ in the SPX return volatility equation

$$\sigma_{spx,t}^2 = \alpha_0 + \sum_{s=1}^k \alpha_s \sigma_{icj,t-s}^2 + \epsilon_t \quad (13)$$

where  $k$  is the number of lags. We try 12 lagged values of ICJ and show the results in the following table:

Table 11: Regression for SPX Daily Returns (2007-2009)

	Estimate	Std. Error	Pr(>  t )
$\alpha_0$	0.0002	0.0001	0.0575
$\alpha_1$	0.0752	0.0199	0.0002***
$\alpha_2$	0.0250	0.0201	0.2131
$\alpha_3$	0.0283	0.0201	0.1592
$\alpha_4$	0.0022	0.0199	0.9134
$\alpha_5$	0.0140	0.0200	0.4839
$\alpha_6$	0.0301	0.0201	0.1349
$\alpha_7$	0.0392	0.0201	0.0514
$\alpha_8$	0.0064	0.0202	0.7526
$\alpha_9$	0.0004	0.0202	0.9830
$\alpha_{10}$	0.0560	0.0202	0.0059**
$\alpha_{11}$	-0.0496	0.0203	0.0147*
$\alpha_{12}$	0.0209	0.0201	0.2983

Table 12: Regression Fit of SPX Daily Returns

	Adj. R-squared	F-statistic	p-value
	0.0787	4.5080	0.0000

From Table 10 and 11, we see that the lagged 1, 10 and 11 ICJ return volatilities are the most influential factors on the current SPX daily return volatility. This indicates that today's ICJ return volatility can be used to forecast tomorrow's SPX return volatility and volatility exactly two weeks from now. The model fit has an adjusted R-squared of approximately 8% and the F-statistic indicates a rejection of the null hypothesis that lagged values of ICJ daily return volatility have no explanatory power for the current SPX daily return volatility.

## 5 Conclusion

Stock or index return and volatility are two central quantities to financial markets. They are fundamental to a variety of financial applications, such as asset pricing, portfolio optimization, risk management etc. In most of the financial literature, the stock or index price change is treated as a random walk process, which implies that the return series are virtually the martingale difference sequences. Although the GARCH models can be used to model the second moment dynamics of return series in some cases, information on past return series generally cannot be used to forecast future returns.

In this paper, we focus on the S&P 500 Index return series and study whether ICJ, i.e. the newly launched Implied Correlation Index, can provide useful information about the forecasting of the future S&P 500 Index returns. We turn our attention to the SPX and ICJ weekly return series and regress the future SPX multi-period return on the current information set of weekly changes in SPX and ICJ. The estimation result suggests a big role for information on the SPX and ICJ weekly changes and our new model outperforms the benchmark random walk model in terms of MSFE with the Superior Predictive Ability test. We find a close relationship between the SPX return in the next 7 to 10 months and the current information set of SPX and ICJ weekly returns. We check the robustness of this result by resorting to 3 different sample splits and our model consistently beats the benchmark random walk model.

Another interesting finding is that the coefficient sign of the interacted term in our model is consistently positive for three different models. Intuitively, this indicates that when we observe a stock market rally or slump with a strengthened average correlation, the current market trend is most likely to continue in the future 7 to 10 months.

We continue our investigation on the role played by ICJ for the SPX weekly return volatility. Similar to the case of the SPX weekly returns, we bring different lags of the ICJ volatilities into the regression and estimate their impacts on the SPX weekly return volatility. The result shows that the lag 6 ICJ volatility has a significant impact on the current SPX weekly return volatility, which means that the current information on ICJ volatility would be useful to forecast the SPX weekly return volatility six weeks later. There is no evidence that other lagged



values of ICJ play such a big role.

We finally look into the forecasting issue of the SPX daily return volatility. Again, we try different lags of ICJ and add them all together in our regression to figure out which lags of ICJ are most informative for SPX daily return volatility forecasting. The result demonstrates that the ICJ daily return volatility can be used to predict the SPX return volatility next day and the day two weeks later.

All the results we obtain from this paper demonstrate that ICJ plays a big role in judging the stock market directions in the future. In this sense, it is a leading indicator and should merit special attention from market participants.

## Appendix

Table 13: Regression of S&P 500 Index Weekly Returns for Equation (4)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=28	2:1	$\alpha_0$	-0.3319	0.0282	0.00***
		$\alpha_1$	-0.9226	0.2904	0.00**
		$\alpha_2$	1.0806	0.1432	0.00***
		$\alpha_3$	8.2775	1.1511	0.00***
		$\alpha_4$	1.1468	0.0939	0.00***
	3:1	$\alpha_0$	-0.3283	0.0296	0.00***
		$\alpha_1$	-0.9730	0.2847	0.00**
		$\alpha_2$	1.1245	0.1499	0.00***
		$\alpha_3$	7.9909	1.2138	0.00***
		$\alpha_4$	1.2186	0.0916	0.00***
	4:1	$\alpha_0$	-0.3271	0.0277	0.00***
		$\alpha_1$	-1.0800	0.3163	0.00**
		$\alpha_2$	1.1066	0.1675	0.00***
		$\alpha_3$	7.9618	1.3382	0.00***
		$\alpha_4$	1.2632	0.1020	0.00***
k=29	2:1	$\alpha_0$	-0.3257	0.0254	0.00***
		$\alpha_1$	-0.9337	0.3726	0.02*
		$\alpha_2$	1.0307	0.2177	0.00***
		$\alpha_3$	8.1731	1.2980	0.00***
		$\alpha_4$	1.1526	0.0861	0.00***
	3:1	$\alpha_0$	-0.3208	0.0268	0.00***
		$\alpha_1$	-0.8701	0.3742	0.03*
		$\alpha_2$	1.0304	0.2253	0.00***
		$\alpha_3$	8.1446	1.3746	0.00***
		$\alpha_4$	1.2115	0.0800	0.00***
	4:1	$\alpha_0$	-0.3207	0.0263	0.00***
		$\alpha_1$	-0.9949	0.3628	0.01**
		$\alpha_2$	1.0582	0.2425	0.00***
		$\alpha_3$	7.7599	1.4711	0.00***
		$\alpha_4$	1.2594	0.0853	0.00***

Table 14: Regression of S&P 500 Index Weekly Returns for Equation (4)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=30	2:1	$\alpha_0$	-0.3270	0.0201	0.00***
		$\alpha_1$	-0.9212	0.2200	0.00***
		$\alpha_2$	1.0970	0.1230	0.00***
		$\alpha_3$	8.4332	0.8134	0.00***
		$\alpha_4$	1.2502	0.0853	0.00***
	3:1	$\alpha_0$	-0.3248	0.0206	0.00***
		$\alpha_1$	-0.9101	0.1942	0.00***
		$\alpha_2$	1.1122	0.1198	0.00***
		$\alpha_3$	8.3539	0.8008	0.00***
		$\alpha_4$	1.2868	0.0771	0.00***
	4:1	$\alpha_0$	-0.3245	0.0215	0.00***
		$\alpha_1$	-1.0517	0.1895	0.00***
		$\alpha_2$	1.1569	0.1248	0.00***
		$\alpha_3$	7.9556	0.7983	0.00***
		$\alpha_4$	1.3409	0.0775	0.00***
k=31	2:1	$\alpha_0$	-0.3159	0.0140	0.00***
		$\alpha_1$	-0.7748	0.1790	0.00***
		$\alpha_2$	0.9352	0.1167	0.00***
		$\alpha_3$	7.6628	0.7831	0.00***
		$\alpha_4$	1.2942	0.0675	0.00***
	3:1	$\alpha_0$	-0.3130	0.0146	0.00***
		$\alpha_1$	-0.8434	0.1828	0.00***
		$\alpha_2$	0.9857	0.1091	0.00***
		$\alpha_3$	7.2923	0.8123	0.00***
		$\alpha_4$	1.3347	0.0563	0.00***
	4:1	$\alpha_0$	-0.3129	0.0149	0.00***
		$\alpha_1$	-0.9347	0.1750	0.00***
		$\alpha_2$	1.0307	0.1023	0.00***
		$\alpha_3$	6.9898	0.7771	0.00***
		$\alpha_4$	1.3571	0.0539	0.00***

Table 15: Regression of S&P 500 Index Weekly Returns for Equation (4)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=32	2:1	$\alpha_0$	-0.3089	0.0165	0.00***
		$\alpha_1$	-0.8186	0.1839	0.00***
		$\alpha_2$	1.0557	0.1245	0.00***
		$\alpha_3$	6.7368	0.8879	0.00***
		$\alpha_4$	1.3364	0.0641	0.00***
	3:1	$\alpha_0$	-0.3056	0.0173	0.00***
		$\alpha_1$	-0.8354	0.1703	0.00***
		$\alpha_2$	1.0902	0.1152	0.00***
		$\alpha_3$	6.5253	0.8328	0.00***
		$\alpha_4$	1.3811	0.0633	0.00***
	4:1	$\alpha_0$	-0.3052	0.0178	0.00***
		$\alpha_1$	-0.8278	0.1519	0.00***
		$\alpha_2$	1.0941	0.1034	0.00***
		$\alpha_3$	6.5360	0.7942	0.00***
		$\alpha_4$	1.3747	0.0585	0.00***
k=33	2:1	$\alpha_0$	-0.2949	0.0172	0.00***
		$\alpha_1$	-0.7628	0.3967	0.07
		$\alpha_2$	0.9905	0.1020	0.00***
		$\alpha_3$	8.9469	1.4977	0.00***
		$\alpha_4$	1.3331	0.0648	0.00***
	3:1	$\alpha_0$	-0.2911	0.0230	0.00***
		$\alpha_1$	-0.7185	0.5822	0.23
		$\alpha_2$	0.9908	0.1259	0.00***
		$\alpha_3$	9.0216	1.8563	0.00***
		$\alpha_4$	1.3556	0.0592	0.00***
	4:1	$\alpha_0$	-0.2875	0.0132	0.00***
		$\alpha_1$	-0.6138	0.2477	0.02*
		$\alpha_2$	0.9700	0.0765	0.00***
		$\alpha_3$	9.2766	1.0306	0.00***
		$\alpha_4$	1.3498	0.0414	0.00***

Table 16: Regression of S&P 500 Index Weekly Returns for Equation (4)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=34	2:1	$\alpha_0$	-0.2582	0.0156	0.00***
		$\alpha_1$	0.1192	0.3702	0.75
		$\alpha_2$	0.8313	0.1225	0.00***
		$\alpha_3$	12.9459	1.2930	0.00***
		$\alpha_4$	1.3776	0.1315	0.00***
	3:1	$\alpha_0$	-0.2575	0.4088	0.00***
		$\alpha_1$	0.1571	0.4088	0.70
		$\alpha_2$	0.8173	0.1421	0.00***
		$\alpha_3$	13.0418	1.3629	0.00***
		$\alpha_4$	1.3798	0.1168	0.00***
	4:1	$\alpha_0$	-0.2558	0.0134	0.00***
		$\alpha_1$	0.2155	0.3011	0.48
		$\alpha_2$	0.8027	0.1139	0.00***
		$\alpha_3$	13.1839	1.0707	0.00***
		$\alpha_4$	1.3874	0.0857	0.00***
k=35	2:1	$\alpha_0$	-0.2274	0.0163	0.00***
		$\alpha_1$	-0.2407	0.3985	0.55
		$\alpha_2$	0.8984	0.1253	0.00***
		$\alpha_3$	9.7932	1.6020	0.00***
		$\alpha_4$	1.3692	0.1315	0.00***
	3:1	$\alpha_0$	-0.2436	0.0133	0.00***
		$\alpha_1$	-1.2515	0.2480	0.00***
		$\alpha_2$	0.0224	0.5389	0.97
		$\alpha_3$	0.0183	3.2251	1.00
		$\alpha_4$	1.1971	0.0644	0.00***
	4:1	$\alpha_0$	-0.2327	0.0148	0.00***
		$\alpha_1$	-0.2681	0.3735	0.48
		$\alpha_2$	0.8563	0.1236	0.00***
		$\alpha_3$	9.6887	1.9644	0.00***
		$\alpha_4$	1.2634	0.0706	0.00***

Table 17: Regression of S&P 500 Index Weekly Returns for Equation (4)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=36	2:1	$\alpha_0$	-0.2050	0.0325	0.00***
		$\alpha_1$	-0.1882	0.8858	0.83
		$\alpha_2$	0.7076	0.1776	0.00***
		$\alpha_3$	9.0795	2.9390	0.01**
		$\alpha_4$	1.3480	0.2615	0.00***
	3:1	$\alpha_0$	-0.2118	0.0240	0.00***
		$\alpha_1$	-0.3297	0.7209	0.65
		$\alpha_2$	0.6829	0.1555	0.00***
		$\alpha_3$	8.4992	2.6370	0.00**
		$\alpha_4$	1.2067	0.1301	0.00***
	4:1	$\alpha_0$	-0.2114	0.0213	0.00***
		$\alpha_1$	-0.2970	0.5801	0.61
		$\alpha_2$	0.6779	0.1465	0.00***
		$\alpha_3$	8.6405	2.1653	0.00***
		$\alpha_4$	1.2083	0.0892	0.00***
k=37	2:1	$\alpha_0$	-0.2060	0.0243	0.00***
		$\alpha_1$	-0.2801	0.5521	0.62
		$\alpha_2$	0.8330	0.1625	0.00***
		$\alpha_3$	9.0221	1.8637	0.00***
		$\alpha_4$	1.2074	0.1842	0.00***
	3:1	$\alpha_0$	-0.2023	0.0200	0.00***
		$\alpha_1$	-0.1129	0.4288	0.80
		$\alpha_2$	0.7578	0.1349	0.00***
		$\alpha_3$	9.3253	1.6725	0.00***
		$\alpha_4$	1.2890	0.1065	0.00***
	4:1	$\alpha_0$	-0.2032	0.0220	0.00***
		$\alpha_1$	-0.1122	0.4140	0.79
		$\alpha_2$	0.7337	0.1412	0.00***
		$\alpha_3$	9.3135	1.8703	0.00***
		$\alpha_4$	1.2328	0.0791	0.00***

Table 18: Regression of S&P 500 Index Weekly Returns for Equation (4)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=38	2:1	$\alpha_0$	-0.1719	0.0171	0.00***
		$\alpha_1$	-0.0739	0.1349	0.59
		$\alpha_2$	0.7750	0.1111	0.00***
		$\alpha_3$	7.5427	0.7155	0.00***
		$\alpha_4$	1.4559	0.0961	0.00***
	3:1	$\alpha_0$	-0.1775	0.0145	0.00***
		$\alpha_1$	-0.1341	0.1254	0.30
		$\alpha_2$	0.7832	0.0799	0.00***
		$\alpha_3$	7.5693	0.5953	0.00***
		$\alpha_4$	1.4123	0.0760	0.00***
	4:1	$\alpha_0$	-0.1957	0.0119	0.00***
		$\alpha_1$	-0.6907	0.1609	0.00***
		$\alpha_2$	-0.0872	0.3414	0.80
		$\alpha_3$	-7.5920	2.9252	0.02*
		$\alpha_4$	1.3121	0.0532	0.00***
k=39	2:1	$\alpha_0$	-0.1835	0.0297	0.00***
		$\alpha_1$	-0.6794	0.2976	0.04*
		$\alpha_2$	0.9800	0.1757	0.00***
		$\alpha_3$	6.8614	1.0957	0.00***
		$\alpha_4$	1.3802	0.1231	0.00***
	3:1	$\alpha_0$	-0.1704	0.0275	0.00***
		$\alpha_1$	-0.4925	0.2951	0.11
		$\alpha_2$	0.8983	0.1814	0.00***
		$\alpha_3$	6.8345	1.0724	0.00***
		$\alpha_4$	1.3938	0.1140	0.00***
	4:1	$\alpha_0$	-0.1683	0.0303	0.00***
		$\alpha_1$	-0.4779	0.3367	0.17
		$\alpha_2$	0.8947	0.2102	0.00***
		$\alpha_3$	6.8712	1.0853	0.00***
		$\alpha_4$	1.4085	0.1097	0.00***



Table 19: Regression of S&amp;P 500 Index Weekly Returns for Equation (8)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=28	2:1	$\alpha_0$	-0.3161	0.0279	0.00***
		$\alpha_1$	0.8476	0.1391	0.00***
		$\alpha_2$	10.0192	1.4749	0.00***
		$\alpha_3$	1.1357	0.1009	0.00***
	3:1	$\alpha_0$	-0.3111	0.0288	0.00***
		$\alpha_1$	0.8886	0.1464	0.00***
		$\alpha_2$	9.8056	1.7078	0.00***
		$\alpha_3$	1.2101	0.0959	0.00***
	4:1	$\alpha_0$	-0.3080	0.0267	0.00***
		$\alpha_1$	0.8446	0.1687	0.00***
		$\alpha_2$	9.9693	1.6188	0.00***
		$\alpha_3$	1.2551	0.1092	0.00***
k=29	2:1	$\alpha_0$	-0.3109	0.0232	0.00***
		$\alpha_1$	0.7704	0.1970	0.00***
		$\alpha_2$	9.9540	1.6441	0.00***
		$\alpha_3$	1.1545	0.0990	0.00***
	3:1	$\alpha_0$	-0.3065	0.0245	0.00***
		$\alpha_1$	0.7909	0.2050	0.00***
		$\alpha_2$	9.8945	1.7273	0.00***
		$\alpha_3$	1.2092	0.0821	0.00***
	4:1	$\alpha_0$	-0.3075	0.0210	0.00***
		$\alpha_1$	0.0722	0.3788	0.85
		$\alpha_2$	-5.4505	5.3658	0.32
		$\alpha_3$	1.3345	0.0879	0.00***
k=30	2:1	$\alpha_0$	-0.3131	0.0199	0.00***
		$\alpha_1$	0.8171	0.1133	0.00***
		$\alpha_2$	10.1398	1.4657	0.00***
		$\alpha_3$	1.2241	0.0962	0.00***
	3:1	$\alpha_0$	-0.3111	0.0200	0.00***
		$\alpha_1$	0.8310	0.1099	0.00***
		$\alpha_2$	10.0290	1.4994	0.00***
		$\alpha_3$	1.2526	0.0777	0.00***
	4:1	$\alpha_0$	-0.3070	0.0201	0.00***
		$\alpha_1$	0.8427	0.1296	0.00***
		$\alpha_2$	9.7477	1.6604	0.00***
		$\alpha_3$	1.3262	0.0827	0.00***

Table 20: Regression of S&P 500 Index Weekly Returns for Equation (8)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=31	2:1	$\alpha_0$	-0.3067	0.0156	0.00***
		$\alpha_1$	0.6913	0.1256	0.00***
		$\alpha_2$	9.3480	1.0115	0.00***
		$\alpha_3$	1.2749	0.0793	0.00***
	3:1	$\alpha_0$	-0.3034	0.0164	0.00***
		$\alpha_1$	0.7178	0.1140	0.00***
		$\alpha_2$	9.1548	1.1549	0.00***
		$\alpha_3$	1.3059	0.0634	0.00***
	4:1	$\alpha_0$	-0.3011	0.0164	0.00***
		$\alpha_1$	0.7474	0.1178	0.00***
		$\alpha_2$	8.9540	1.3021	0.00***
		$\alpha_3$	1.3447	0.0673	0.00***
k=32	2:1	$\alpha_0$	-0.2991	0.0156	0.00***
		$\alpha_1$	0.8019	0.1309	0.00***
		$\alpha_2$	8.5169	0.9869	0.00***
		$\alpha_3$	1.3137	0.0688	0.00***
	3:1	$\alpha_0$	-0.2941	0.0156	0.00***
		$\alpha_1$	0.8410	0.1213	0.00***
		$\alpha_2$	8.1880	1.0148	0.00***
		$\alpha_3$	1.3652	0.0635	0.00***
	4:1	$\alpha_0$	-0.2927	0.0159	0.00***
		$\alpha_1$	0.8620	0.1159	0.00***
		$\alpha_2$	8.1124	1.0322	0.00***
		$\alpha_3$	1.3801	0.0594	0.00***
k=33	2:1	$\alpha_0$	-0.2899	0.0142	0.00***
		$\alpha_1$	1.2239	0.0983	0.00***
		$\alpha_2$	0.8627	2.8613	0.77
		$\alpha_3$	1.3300	0.0695	0.00***
	3:1	$\alpha_0$	-0.2871	0.0132	0.00***
		$\alpha_1$	1.2289	0.0864	0.00***
		$\alpha_2$	1.0335	2.0980	0.63
		$\alpha_3$	1.3533	0.0500	0.00***
	4:1	$\alpha_0$	-0.2864	0.0127	0.00***
		$\alpha_1$	1.2111	0.0805	0.00***
		$\alpha_2$	1.3829	1.5977	0.39
		$\alpha_3$	1.3518	0.0429	0.00***

Table 21: Regression of S&P 500 Index Weekly Returns for Equation (8)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=34	2:1	$\alpha_0$	-0.2602	0.0130	0.00***
		$\alpha_1$	0.8621	0.0776	0.00***
		$\alpha_2$	12.6426	0.8347	0.00***
		$\alpha_3$	1.3669	0.1193	0.00***
	3:1	$\alpha_0$	-0.2601	0.0125	0.00***
		$\alpha_1$	0.8567	0.0889	0.00***
		$\alpha_2$	12.6495	0.8371	0.00***
		$\alpha_3$	1.3658	0.0849	0.00***
	4:1	$\alpha_0$	-0.2591	0.0123	0.00***
		$\alpha_1$	0.8599	0.0828	0.00***
		$\alpha_2$	12.6613	0.8394	0.00***
		$\alpha_3$	1.3733	0.0679	0.00***
k=35	2:1	$\alpha_0$	-0.2249	0.0163	0.00***
		$\alpha_1$	0.8382	0.0842	0.00***
		$\alpha_2$	10.3559	0.8505	0.00***
		$\alpha_3$	1.3700	0.1359	0.00***
	3:1	$\alpha_0$	-0.2286	0.0160	0.00***
		$\alpha_1$	0.8035	0.0913	0.00***
		$\alpha_2$	10.3393	1.0572	0.00***
		$\alpha_3$	1.2855	0.0923	0.00***
	4:1	$\alpha_0$	-0.2295	0.0161	0.00***
		$\alpha_1$	0.7956	0.0929	0.00***
		$\alpha_2$	10.3646	0.8980	0.00***
		$\alpha_3$	1.2714	0.0618	0.00***
k=36	2:1	$\alpha_0$	-0.2009	0.0189	0.00***
		$\alpha_1$	0.6668	0.0678	0.00***
		$\alpha_2$	9.6068	0.6428	0.00***
		$\alpha_3$	1.3761	0.1749	0.00***
	3:1	$\alpha_0$	-0.2055	0.0161	0.00***
		$\alpha_1$	0.6127	0.0913	0.00***
		$\alpha_2$	9.4109	0.8448	0.00***
		$\alpha_3$	1.2357	0.1042	0.00***
	4:1	$\alpha_0$	-0.2058	0.0159	0.00***
		$\alpha_1$	0.6123	0.0907	0.00***
		$\alpha_2$	9.4173	0.8346	0.00***
		$\alpha_3$	1.2249	0.0812	0.00***

Table 22: Regression of S&P 500 Index Weekly Returns for Equation (8)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=37	2:1	$\alpha_0$	-0.2000	0.0189	0.00***
		$\alpha_1$	0.7520	0.1350	0.00***
		$\alpha_2$	9.5567	0.9927	0.00***
		$\alpha_3$	1.3151	0.1631	0.00***
	3:1	$\alpha_0$	-0.2005	0.0182	0.00***
		$\alpha_1$	0.7353	0.1094	0.00***
		$\alpha_2$	9.6298	1.0192	0.00***
		$\alpha_3$	1.2977	0.0978	0.00***
	4:1	$\alpha_0$	-0.2011	0.0194	0.00***
		$\alpha_1$	0.7061	0.1091	0.00***
		$\alpha_2$	9.5677	1.0186	0.00***
		$\alpha_3$	1.2367	0.0723	0.00***
k=38	2:1	$\alpha_0$	-0.1694	0.0154	0.00***
		$\alpha_1$	0.7329	0.1000	0.00***
		$\alpha_2$	7.7350	0.7398	0.00***
		$\alpha_3$	1.4606	0.0970	0.00***
	3:1	$\alpha_0$	-0.1746	0.0129	0.00***
		$\alpha_1$	0.7388	0.0596	0.00***
		$\alpha_2$	7.8366	0.6579	0.00***
		$\alpha_3$	1.4070	0.0750	0.00***
	4:1	$\alpha_0$	-0.1757	0.0128	0.00***
		$\alpha_1$	0.7046	0.0562	0.00***
		$\alpha_2$	7.8176	0.5893	0.00***
		$\alpha_3$	1.3408	0.0667	0.00***
k=39	2:1	$\alpha_0$	-0.1640	0.0228	0.00***
		$\alpha_1$	0.7431	0.1255	0.00***
		$\alpha_2$	7.7892	1.1087	0.00***
		$\alpha_3$	1.3239	0.1515	0.00***
	3:1	$\alpha_0$	-0.1575	0.0218	0.00***
		$\alpha_1$	0.7469	0.1213	0.00***
		$\alpha_2$	7.6948	1.1114	0.00***
		$\alpha_3$	1.3706	0.1466	0.00***
	4:1	$\alpha_0$	-0.1559	0.0217	0.00***
		$\alpha_1$	0.7645	0.1163	0.00***
		$\alpha_2$	7.7377	1.1020	0.00***
		$\alpha_3$	1.4015	0.1336	0.00***

Table 23: Regression of S&P 500 Index Weekly Returns for Equation (9)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=28	2:1	$\alpha_0$	-0.3023	0.0246	0.00***
		$\alpha_1$	6.0191	2.1118	0.01**
		$\alpha_2$	1.0870	0.1151	0.00***
	3:1	$\alpha_0$	-0.2988	0.0256	0.00***
		$\alpha_1$	4.9381	2.2722	0.04*
		$\alpha_2$	1.1816	0.0940	0.00***
	4:1	$\alpha_0$	-0.2946	0.0247	0.00***
		$\alpha_1$	5.6600	2.1761	0.01*
		$\alpha_2$	1.2259	0.1044	0.00***
k=29	2:1	$\alpha_0$	-0.3094	0.0213	0.00***
		$\alpha_1$	5.5026	2.2737	0.02*
		$\alpha_2$	1.1680	0.1115	0.00***
	3:1	$\alpha_0$	-0.3087	0.0204	0.00***
		$\alpha_1$	-4.8041	4.7815	0.32
		$\alpha_2$	1.2768	0.0795	0.00***
	4:1	$\alpha_0$	-0.3065	0.0208	0.00***
		$\alpha_1$	-5.9074	4.5416	0.20
		$\alpha_2$	1.3342	0.0871	0.00***
k=30	2:1	$\alpha_0$	-0.3005	0.0227	0.00***
		$\alpha_1$	6.5997	1.7744	0.00***
		$\alpha_2$	1.1770	0.1152	0.00***
	3:1	$\alpha_0$	-0.2985	0.0208	0.00***
		$\alpha_1$	6.1823	2.1138	0.00**
		$\alpha_2$	1.2227	0.1022	0.00***
	4:1	$\alpha_0$	-0.2948	0.0207	0.00***
		$\alpha_1$	5.6318	2.4216	0.03*
		$\alpha_2$	1.2958	0.1003	0.00***

Table 24: Regression of S&P 500 Index Weekly Returns for Equation (9)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=31	2:1	$\alpha_0$	-0.3012	0.0163	0.00***
		$\alpha_1$	5.3084	1.0084	0.00***
		$\alpha_2$	1.2597	0.0760	0.00***
	3:1	$\alpha_0$	-0.2965	0.0165	0.00***
		$\alpha_1$	4.8692	1.2270	0.00***
		$\alpha_2$	1.2948	0.0646	0.00***
	4:1	$\alpha_0$	-0.2937	0.0167	0.00***
		$\alpha_1$	4.4471	1.5410	0.01**
		$\alpha_2$	1.3288	0.0660	0.00***
k=32	2:1	$\alpha_0$	-0.2946	0.0144	0.00***
		$\alpha_1$	3.8698	1.2198	0.00**
		$\alpha_2$	1.3065	0.0757	0.00***
	3:1	$\alpha_0$	-0.2876	0.0141	0.00***
		$\alpha_1$	3.2444	1.4105	0.03*
		$\alpha_2$	1.3653	0.0675	0.00***
	4:1	$\alpha_0$	-0.2860	0.0133	0.00***
		$\alpha_1$	-3.8036	4.3007	0.38
		$\alpha_2$	1.3949	0.0604	0.00***
k=33	2:1	$\alpha_0$	-0.2846	0.0162	0.00***
		$\alpha_1$	-21.0270	5.0467	0.00***
		$\alpha_2$	1.2534	0.0636	0.00***
	3:1	$\alpha_0$	-0.2793	0.0163	0.00***
		$\alpha_1$	-22.2313	2.5694	0.00***
		$\alpha_2$	1.3026	0.0669	0.00***
	4:1	$\alpha_0$	-0.2773	0.0159	0.00***
		$\alpha_1$	-21.6490	2.7622	0.00***
		$\alpha_2$	1.3108	0.0577	0.00***

Table 25: Regression of S&P 500 Index Weekly Returns for Equation (9)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=34	2:1	$\alpha_0$	-0.2534	0.0149	0.00***
		$\alpha_1$	7.3237	0.5468	0.00***
		$\alpha_2$	1.2991	0.1559	0.00***
	3:1	$\alpha_0$	-0.2511	0.0129	0.00***
		$\alpha_1$	7.2443	0.5249	0.00***
		$\alpha_2$	1.3358	0.1080	0.00***
	4:1	$\alpha_0$	-0.2495	0.0127	0.00***
		$\alpha_1$	7.2586	0.5111	0.00***
		$\alpha_2$	1.3454	0.0824	0.00***
k=35	2:1	$\alpha_0$	-0.2148	0.0212	0.00***
		$\alpha_1$	5.9314	1.7671	0.00**
		$\alpha_2$	1.2413	0.3543	0.00**
	3:1	$\alpha_0$	-0.2161	0.0212	0.00***
		$\alpha_1$	5.8728	1.0704	0.00***
		$\alpha_2$	1.2357	0.1554	0.00***
	4:1	$\alpha_0$	-0.2184	0.0196	0.00***
		$\alpha_1$	5.8101	0.8048	0.00***
		$\alpha_2$	1.2380	0.0998	0.00***
k=36	2:1	$\alpha_0$	-0.1880	0.0212	0.00***
		$\alpha_1$	6.5264	1.3525	0.00***
		$\alpha_2$	1.2358	0.3615	0.00**
	3:1	$\alpha_0$	-0.2138	0.0141	0.00***
		$\alpha_1$	-27.1086	5.6332	0.00***
		$\alpha_2$	1.1343	0.0640	0.00***
	4:1	$\alpha_0$	-0.2136	0.0137	0.00***
		$\alpha_1$	-27.0404	5.7863	0.00***
		$\alpha_2$	1.1404	0.0543	0.00***

Table 26: Regression of S&P 500 Index Weekly Returns for Equation (9)

	split	coefficient	estimate	Std. Error	Pr(>  t )
k=37	2:1	$\alpha_0$	-0.1745	0.0264	0.00***
		$\alpha_1$	6.4998	2.0575	0.01**
		$\alpha_2$	1.1288	0.1869	0.00***
	3:1	$\alpha_0$	-0.1758	0.0281	0.00***
		$\alpha_1$	6.1295	1.6479	0.00**
		$\alpha_2$	1.1696	0.1536	0.00***
	4:1	$\alpha_0$	-0.1766	0.0282	0.00***
		$\alpha_1$	6.1246	1.5752	0.00***
		$\alpha_2$	1.1395	0.1150	0.00***
k=38	2:1	$\alpha_0$	-0.1625	0.0218	0.00***
		$\alpha_1$	3.5165	1.6136	0.04*
		$\alpha_2$	1.3930	0.4586	0.00**
	3:1	$\alpha_0$	-0.1856	0.0126	0.00***
		$\alpha_1$	-24.5050	4.1307	0.00***
		$\alpha_2$	1.3065	0.0714	0.00***
	4:1	$\alpha_0$	-0.1894	0.0120	0.00***
		$\alpha_1$	-22.9968	2.4012	0.00***
		$\alpha_2$	1.2601	0.0493	0.00***
k=39	2:1	$\alpha_0$	-0.1486	0.0186	0.00***
		$\alpha_1$	3.4585	1.1580	0.01**
		$\alpha_2$	1.3107	0.2617	0.00***
	3:1	$\alpha_0$	-0.1466	0.0177	0.00***
		$\alpha_1$	3.2664	0.8134	0.00***
		$\alpha_2$	1.3773	0.1484	0.00***
	4:1	$\alpha_0$	-0.1439	0.0167	0.00***
		$\alpha_1$	3.1588	0.8660	0.00**
		$\alpha_2$	1.4201	0.1333	0.00***



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