



**Master Thesis:**

# **Cointegration and Pairs Trading in Commodities Markets**

**- Based on Crude Oil and Gold Spot and Futures markets**

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**October 2017**

### Abstract

The paper examines the cointegration relationship between gold and crude oil price series in both spot and futures markets using 34 years data from 1983 to 2017. With Engle-Granger cointegration and a rolling regression method, a dynamic cointegration is found for the two commodity price series. A pairs trading strategy is developed later based on the cointegration relationship. Sensitivity analysis and robust test is conducted to the return and Sharpe ratio of the strategy.

The paper is consisted of eight sections. The first section is a brief introduction and background of the topic; the second section summarized the current literature on cointegration and pairs trading; the third section described the main methodology and the major hypothesis for the cointegration test, pairs trading triggering mechanism and performance measurement; the fourth and fifth sections include the data descriptions and empirical results; the sixth section consists of the conclusion of the study and some further discussion on the topic; the seventh and eighth sections are the references and appendices.

**Key Words:** Cointegration, Commodity, Gold, Crude oil, Engle-Granger cointegration test, ADF test, Pairs trading

## Table of Contents

1	Introduction .....	<b>Error! Bookmark not defined.</b>
2	Literature review .....	3
2.1	Studies of co-movement in commodity markets .....	3
2.2	Studies of gold and crude oil price relationship .....	3
2.3	Literature of pairs trading .....	5
3	Hypothesis and methodology.....	7
3.1	Unit root analysis .....	7
3.2	Engle-Granger test for cointegration.....	7
3.3	Rolling regression based on Engle-Granger test.....	9
3.4	Pairs trading strategy.....	9
4	Data and statistic descriptions.....	14
4.1	Data Source.....	14
4.2	Statistic descriptions .....	15
5	Empirical results .....	18
5.1	Engle-Granger Cointegration test.....	18
5.2	Rolling regression.....	21
5.3	Pairs Trading.....	23
6	Conclusions and Discussion.....	28
6.1	Conclusions.....	28
6.2	The limitation of the study .....	29
7	References .....	30
8	Appendices .....	35
A.	Statistic descriptions.....	35
B.	Rolling window regression for cointegration test.....	37
C.	Pairs Trading Result .....	39
D.	Matlab Code .....	41

## 1 Introduction

Commodities, especially commodity futures, are quite appealing in asset allocation traditionally based on several characteristics (Kazemi et al. 2016) including: 1) Low correlation with stocks and bonds, improving the risk-return profiles of portfolios; 2) Inflation, business cycle, and event risk hedging abilities; 3) Mean reversion and diversification return, leading to improved performance through rebalancing; 4) Positive risk premium and roll return earned by investors in commodity futures; 5) Return distributions that may be positively skewed. Due to these characteristics, commodity market is rather appealing in unstable time and sometimes even performed as a “safe haven” (Baur and Lucey, 2010; Hood and Malik, 2013). It has attracted lots of attention from academic analysts that whether a significant co-movement of various commodity prices exists or not. One interesting topic is whether a cointegration relationship exists between different commodity prices. If the cointegration relationship exists, then the relationship between the two commodities in question can be modelled as a vector error correction model, which can be used for forecasting, discovering price and developing trading strategies.

Among all commodities, gold and crude oil, as the most traded precious metal commodity and most traded energy commodity, play a rather critical role in any economy. In the paper, the potential cointegration relationship between gold and crude oil prices will be tested by Engle-Granger cointegration test using recent data from both spot and futures markets. The data span for 34 years from 1 July 1983 until 30 June 2017 with three different frequencies. To look into a longer-term relationship between gold and oil, monthly spot prices in the past 49 years are also considered. We aimed to answer the following questions in the paper: 1) Does cointegration exist during the whole sample period? If not does it exist in some subperiods in the whole sample period? 2) How is the relationship developing throughout the sample period? Does evidence of a stronger or weaker cointegration exist in recent years? 3) How is the impact of the 2008 global financial crisis on the co-movement of the two commodities?

In addition to price cointegration, the paper also aims to discover the interactive mechanism between these two commodity markets from the perspective of pairs trading strategy determination. Pairs trading is a very popular market neutral trading strategy by creating a long-short position with securities moving together. It will be a supportive tool for risk managers to reduce the commodity market risk, make trading decisions and forecast market dynamics in the future. There are several different kinds of pairs trading strategies. In the paper, we intend to focus on the statistical arbitrage pairs trading, which will make use of the cointegration relationship between the two commodity prices. The sensitivity analysis and robust test will be conducted to test the profitability of the strategy. Since limited academic studies are available about pairs trading strategy on gold and crude oil currently, we hope our study could add value to the area of research.

## 2 Literature review

### 2.1 Studies of co-movement in commodity markets

Plenty of studies has been conducted on the topic of the cointegration relationship exists between different commodity prices. Some representative studies include Cuddington (1992), Chaudhuri (2001), Bachmeier and Griffin (2006), Nazlioglu and Soytas (2012) and Zhu et al (2016).

By considering 26 individual commodity prices over 84 years in the 20th century, Cuddington (1992) rejected Prebisch-Singer hypothesis and concluded that there is no universal phenomenon of cointegration among commodity prices in terms of manufactured goods. Chaudhuri (2001) compared 29 real commodity prices with real oil prices and showed a co-movement between crude oil price and other major commodity prices. Using daily price data for five different crude oils and coal, Bachmeier and Griffin (2006) conclude that the world oil market is a single, highly integrated economic market. They also found proves of a weak cointegration among the spot prices of oil, coal and natural gas. In the same year, Lucy and Tully (2006) investigated the cointegration relationship between gold and silver based on data from 1978 to 2002 and concluded that gold and silver prices are cointegrated in a long run while in some period only a weak relationship could be found. Nazlioglu and Soytas (2012) found a strong cointegration between oil price and 24 major agricultural commodity prices based on a panel data analysis. A study of long-run quantile cointegration relationship between silver and gold prices was conducted by Zhu et al (2016).

### 2.2 Studies of gold and crude oil price relationship

There are lots of studies about co-movements between gold and oil prices. Some of them applied ordinary regression models and correlation analysis to study the cointegration relationship between the two commodities. Among these studies, Pindyck

and Rotemberg (1990) and Cashin et al. (2002) observed cointegration relationship between the time series of international gold and crude oil prices, while Basit (2013) found no evidence of cointegration between oil and gold prices in Pakistan. Samanta and Zadeh (2012) conducted a cointegration test for gold, crude oil, stocks and real US exchange rate and found significant evidence of co-movements.

Some other studies considered more sophisticated models either with multivariates or non-linear relationships. Narayan et al. (2010) tested the cointegration relationship between spot contract and different kind of future contracts of gold and crude oil. Their conclusion is that the two commodity markets are highly correlated so that the prediction for gold (crude oil) prices could be based on the crude oil (gold) prices. At the same year, by examining the linear and nonlinear relationship between gold and crude oil prices from January 2000 to March 2008, Zhang and Wei (2010) found a significant long-term co-movement between the two markets. Different with the result from Narayan et.al. (2010), they found an irreversible Granger causality relationship. Šimaková (2011) conducted both qualitative and quantitative analysis of oil and gold prices. Using Granger causality test, Johansen cointegration test and Vector Error Correction model on monthly data from 1970 to 2010, Šimaková revealed the existence of a long-term relationship between the two commodities. Lee and Chang (2011) examined the cointegration relationship through the inflation channel and their interaction with the US dollar index using monthly data from January 1986 to April 2011. They used different oil price proxies and found that the impact of oil price on gold price is not asymmetric but non-linear and oil price can be used to forecast gold price. Lee et al. (2012) implemented the momentum threshold error-correction model and find the existence of an asymmetric cointegration and causal relationships between the two futures markets with data from 1 May 1994 to 20 November 2008.

Although strong evidences of cointegration are found in the previous papers, some other papers received less conclusive results. Sujit and Kumar (2011) discovered a weak long-

term dynamic relationship among the daily gold prices, oil prices by testing two models. Bampinas and Panagiotidis (2015) used sub-period data to examine the causal relationship between crude oil and gold spot prices before and after the recent financial crisis. The result is that the significance level of causal linkage from gold to oil is time-dependent. The probability of gold Granger causing oil increased significantly during the post-crisis period. A recent study by Gil-Alana et al (2017) showed a fractionally cointegrated relationship with an order of integration of about 0.46 in the long run gold-oil relationship.

### 2.3 Literature of pairs trading

Pairs trading is a widely-used market neutral long-short strategy which could date back to 1980s. Since the strategy is simple and easy to be understood, it has been widely implemented by professional investors and institutional investors like hedge funds. However, on the contrary, published researches are quite rare since most of the strategies are confidential and restricted to the strategy developers only. Among the academic literature, three main categories of the methodology are considered in construction of pairs trading strategies: 1) the minimum distance approach, which generates trade signals by using the empirical quantiles of the historical spread levels; 2) Mean reversion (stationarity, cointegration, etc.), which is most widely utilized apart from the mean reversion method. It will be adopted in the paper; 3) Combined forecasts and Multi-Criteria Decision Methods (MSCDM) (Huck and Afawubo, 2015). Some of the most cited researches on the topic are listed below.

Gatev et al. (2006) used distance method with a sample of about 2300 stocks, and showed pairs trading after costs can be profitable using a simple standard deviation strategy. Do and Faff (2010) replicated the methodology in the paper of Gatev et al (2006) with more recent data and report a declining significance result. Bowen and Hutchinson (2014) conducted researches with a similar approach.



The second groups of papers involved the most popular methodology so far - using statistical and econometric techniques including stationarity and cointegration to build a mean reversion model. Vidyamurthy (2004) provided a detailed pairs trading framework based on cointegration, but they didn't conduct any empirical research. Elliott et al. (2005) applied a stochastic spread approach by using a mean reverting Gaussian Markov chain model with a Kalman filter to estimate a parametric model of the spread, while Do et al. (2006) conducted a stochastic spread approach by analyzing the behavior of the return series. Goncu (2015) developed a statistical arbitrage in the Black – Scholes framework without providing any empirical results. Zeng and Lee (2014) provided an in-sample back-testing example with the pairs trading model that follows the framework of Avellaneda and Lee (2010) and Bertram (2010). A recent study by Huck and Afawubo (2015), compared the performance of pairs trading based on various pairs selection methods, including the first and second methods discussed above. They used the prices of 500 stocks with S&P 500 index components from August 2000 to September 2011, and showed an insignificant excess return by the distance method and a high, stable and robust return by cointegration method.

The last group of papers, including Huck (2009,2010), Figueria et al. (2005) and etc, forecasted and selected pairs by combining neural networks and multi-criteria decision aids. This method has not been tested by a large group of securities because of its rare reference to any equilibrium model.

### 3 Hypothesis and methodology

#### 3.1 Unit root analysis

The first step of the empirical analysis is to determine the stationarity properties of the variables. By using the unit root tests, the order of integration of the variables is determined so as to avoid the spurious regressions by using the conventional OLS estimation with non-stationary variables. To be specific, AR (p) model and augmented Dickey-Fuller (ADF) test, developed by Dickey and Fuller (1979), is used here for the logarithmic historical price ( $y_t$ : *oil<sub>t</sub>*, *gld<sub>t</sub>*) of the two commodities separately. The testing equation:

$$\Delta y_t = \mu + \emptyset y_{t-1} + \beta_1 \Delta y_{t-1} + \dots + \beta_p \Delta y_{t-p} + u_t \quad (3.1.1)$$

Where  $\Delta$  is the differencing operator, such that  $\Delta y_t = y_t - y_{t-1}$ . Using the ADF test to test the null hypothesis of non-stationary, i.e.  $\emptyset = 1$ , against stationary, i.e.  $\emptyset < 1$ . Lag length p should be chosen such that the error term  $u_t$  does not display any autocorrelation. Based on Akaike information criterion (AIC) and Schwarz criterion (SC, also known as Bayesian information criterion), we choose the lag length p with the smallest AIC or SC.

#### 3.2 Engle-Granger test for cointegration

For simplification, assume the lag length p=1 in the previous unit root test for both oil and gold price. After *oil<sub>t</sub>* and *gld<sub>t</sub>* are proved as I(1) processes, the two-step procedure provided by Engle and Granger (1987) will be used to check the cointegration between the commodity prices.

Step 1: Run the following cointegrating regression using OLS and estimate the parameter values.

$$oil_t = \mu_1 + \phi_1 gld_t + u_{1t} \quad (3.2.1)$$

$$gld_t = \mu_2 + \phi_2 oil_t + u_{2t} \quad (3.2.2)$$

Use ADF test for residuals  $\hat{u}_t$ , if the null hypothesis is rejected, the residuals are  $I(0)$ , which means the two series are cointegrated; then proceed to the second step. If the null hypothesis of a unit root in the test regression residuals is not rejected, the two series are not cointegrated, that is they have no long-run relationship. The most appropriate form of the model to estimate the relationship between the two series would be one containing only first differences.

Note that the modified critical values are required for this ADF test, since the test is based on the residuals of an estimated model rather than on raw data. The new set of critical values were tabulated by Engle and Granger (1987), so sometimes the test is also known as the Engle-Granger (EG) test. In theory, with a big sample, the result for the cointegrating test will be the same for the above two equations.

Step 2: Use the residuals  $\hat{u}_t$  as regressor and estimate the error correction model (ECM), also called vector error correction model (VECM):

$$\Delta oil_t = \alpha_1 \hat{u}_{t-1} + e_{1t} \quad (3.2.3)$$

$$\Delta gld_t = \alpha_2 \hat{u}_{t-1} + e_{2t} \quad (3.2.4)$$

Where  $e_{1t}$  and  $e_{2t}$  are both white noise errors. Estimate  $\alpha_1$  and  $\alpha_2$  by OLS. If  $\alpha_2 = 0$ , we can treat the gold price  $gld_t$  as exogenous, vice versa. And consider the equation

$$\Delta oil_t = \alpha_1 \hat{u}_{t-1} + \beta \Delta gld_t + e_t \quad (3.2.5)$$

To increase the power of the hypothesis test, daily data will be the major data source for

the latter part of the study. Both cointegration relationships with either oil or gold as independent variables will be conducted by the permutation of the variables.

### 3.3 Rolling regression based on Engle-Granger test

Apart from testing the potential cointegration relationship during the whole sample period and three sub-periods, a rolling regression will be conducted for the daily data. To eliminate the influence on test result by different sample size, a pair of a predefined M-day rolling window and a N-day rolling interval is used for Engle-Granger test, such that each regression will have a sample size of M observations. In other words, every N days, a cointegrating test is estimated with the previous M-day worth of information. For example, with a sample of 8573-day observations, 252-day (approximately one year) rolling window and 21-day rolling interval,  $397 \left( \lfloor (8573 - 252) / 21 \rfloor + 1 \right)^1$  cointegration tests will be conducted. The first cointegration test is conducted with the 1<sup>st</sup> to 252 observations from the total 8573 observations, the second cointegration test with the 22<sup>nd</sup> to 273<sup>rd</sup> observations, the third cointegration test with 43<sup>rd</sup> to 294<sup>th</sup> observations, and so on. The Matlab code attached in the Appendix D will conduct the rolling regression and output the p-value of the tau-statistics of the test result.

### 3.4 Pairs trading strategy

Pairs trading strategies are developed based on two different approaches in the paper. One is a simple pairs trading strategy using the same parameters and assuming the existence of the long-term cointegration relationship for the whole period, the other is based on cointegration relationship found in the rolling regression. Two self-developed Matlab functions *pairs\_coint* and *pairs\_imp* for the two approaches are utilized for the calculation. The codes are attached in the Appendix D -Matlab Codes.

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<sup>1</sup> symbol " $\lfloor \rfloor$ " refers to round down to the closest integer.

### 3.4.1 Pairs trading with imposed cointegration

Define the sum of the estimated intercept  $\hat{\mu}_1$  and residual  $\hat{u}_t$  in equation (3.2.1),  $\hat{\mu}_1 + \hat{u}_t$ , as the spread between the crude oil and gold prices. We can put on a trade when  $\hat{u}_t$  deviates substantially from 0. To determine what is a substantial deviation, we should first define a targeted deviation  $\Delta$ , which would usually be set as a multiple of the standard error of the regression (3.2.1). And unwind the trade when the next equilibrium is reached. In other words, unwind the trade at the next time point  $t$  when  $\hat{u}_t$  reaches 0 or the sign of  $\hat{u}_t$  changes.

To be specific, the simple trading strategy could be as follows:

- Long one unit of crude oil, and short  $\phi_1$  units of gold at time  $t$  if  $\hat{u}_t > \Delta$ ;  
unwind the position at time  $t+i$  ( $i>0$ ) if  $\hat{u}_{t+i} \leq 0$ , or
- Short one unit of crude oil, and long  $\phi_1$  units of gold at time  $t$  if  $\hat{u}_t < -\Delta$ ;  
unwind the position at time  $t+i$  ( $i>0$ ) if  $\hat{u}_{t+i} \geq 0$ .

In order to get a profitable trading, the targeted deviation  $\Delta$ , should be bigger than the trading cost  $\eta$ . The net profit of the pairs trading strategy above would be  $\Delta - \eta$ . When deciding the targeted deviation, the tradeoff between lower trading costs and shorter holding period should be considered.

In case of no profitable pairs trading opportunity, the “missing” positions will be filled by a long position in the money market, in this case, the 3-month T-bill secondary market. This switch from pairs trading positions to long exposure to the money market is similar to the approach in the paper by Huck, N., & Afawubo, K. (2015), where “missing” positions of pairs of stocks were filled by a long position of the market index.

As a preliminary analysis, we assume the existence of a cointegration relationship

between the two series during the whole sample period without actually testing for it and use the parameters got from the cointegration regression to develop the pairs trading strategy for the same period.

#### 3.4.2 Pairs trading based on rolling regression

Based on the theoretical framework developed by Vidyamurthy (2004), a pairs trading strategy is conducted coordinate with cointegration test. Basically, pairs trading is engaged only when cointegration relationship exist. With the rolling regression method stated in the previous session, the previous M-day worth of information is used to estimate the cointegrating relationship in the following N days. The M-day period could be named as a formation period, and the N-day period could be named as a trading period. For instance, we first set M as 252 days (approximately one year), and N as 126 days (approximately half a year). This means if in the 252 days period, a cointegration relationship was found between the two series, then we adopt the above pairs trading strategy in the next 126 days based on the cointegration relationship found. If not enough evidences of cointegration are suggested in the previous 252 days period, however, no pairs trading position would be entered during the next 126 days. As in the previous approach, we assume a long position on the risk-free rate during the period with no pairs trading positions.

#### 3.4.3 Calculate the return

Regarding the cost of the trading, referring to Do and Faff (2012), the transaction costs of the pairs trading strategy on NYSE stocks consist of commissions, market impact and short-selling constraints. The average commission fee during 1983 to 2017 is around 0.2%, and it keeps reducing during these years. Since gold and crude oil markets are highly liquid, the short-selling cost is not significant. To simplify the model, in the paper a default average total cost of 0.5% per transaction is assumed.

In the empirical study, the pairs trading positions are marked-to-market every day as the approach used in most of the literature. If a pairs trading position is opened but the prices do not reach the equivalent (i.e.,  $\hat{u}_t$  does not change sign or reach zero) at the end of the trading period, the returns are calculated based on the price at the last trading day of the period. Consider the above pairs trading strategy and daily data, if a profitable pairs trading occurs, the daily return of the portfolio at time  $t$  would be:

$$r_{p,t} = \hat{u}_t - \hat{u}_{t-1} - \eta \quad (3.4.1)$$

The holding period return with  $n$  days  $R_{p,n}$  would be the summation of the following equations.

$$\sum_{t=1}^n r_{p,t} \quad \text{for days when pairs trading occurs} \quad (3.4.2)$$

$$\sum_{t=1}^n r_{f,t} \quad \text{for days when pairs trading does not occur} \quad (3.4.3)$$

The annualized holding period return would be  $R_{p,n}/252$  with the assumption of 252 trading days per year.

#### 3.4.4 Profitability and robust check

After the pairs trading strategy is established, the feasibility and profitability of our pairs trading strategy will be discussed. We use Sharpe ratio to measure the profitability. The calculations are as follow:

$$R_{excess} = r_p - r_f \quad (3.4.4)$$

$$\mathit{sharp} = \frac{\overline{R_{exc}}}{\sigma(R_{exc})} \quad (3.4.5)$$

where  $\overline{R_{exc}}$  is the sample mean of the excess return over the whole period;  $r_p$  is the portfolio return;  $r_f$  is the risk-free rate;  $\sigma(R_{exc})$  is the sample standard deviation of the excess return. Using daily data here we will get a daily Sharpe ratio. We get an annualized Sharpe ratio by multiplying daily estimates by  $\sqrt{252}$ , assuming no serial correlation of the excess return series,

The sensitivity analysis on the sharp ratio would be conducted based on different cost  $\eta$ , targeted deviation  $\Delta$ , and significance level  $\alpha$  of the cointegration test in the rolling regression.

A robust check would be conducted for the Sharpe ratios of our pairs trading strategy. Considering possible heavier tails of the returns, a robust inference method will be applied based on the HAC inference in the study of Ledoit and Wolf (2008). The Matlab function *sharpeHAC* for heteroskedasticity and autocorrelation robust (HAC) kernel estimation provided by Ledoit and Wolf (2008) is used in the paper for the robust test. Adjust the codes for a single Sharpe ratio test according to the formulas in the study by Lo, A. W. (2002) and Opdyke (2007). We set the kernel type as the default Parzen-Gallant kernel. The test has a null hypothesis of a negative or zero Sharpe ratio, against a positive Sharpe ratio. HAC p-value without pre-whitened will be used in the empirical results.



## 4 Data and statistic descriptions

### 4.1 Data Source

The data in the paper are time series of gold and crude oil natural logarithmic prices. Oil price is quoted in US dollars per barrel; gold price is quoted in US dollars per troy ounce. Both spot prices and future prices are considered. The future price is the continuous future price, which is constructed by series of individual futures contracts prices into one continuous series of prices with certain rollover methodology <sup>2</sup>. The data span 34 years from 1 July 1983 to 30 June 2017. In the cointegration test, all daily, weekly and monthly data are used to compare the impact of the data frequency. Weekly and monthly data are based on the value at the end of the period. To investigate the cointegration relationship in a longer period, monthly spot prices are also considered. All series are retrieved from FactSet terminal. The four securities, which log price are denoted as *gld\_f*, *oil\_f*, *gld\_s* and *oil\_s* respectively, are as follow:

- Gold – Future: New York Mercantile Exchange (NYM \$/ozt) Continuous
- Crude Oil – Future: West Texas Intermediate (WTI) (NYM \$/bbl) Continuous
- Gold – Spot: New York Mercantile Exchange (NYM \$/ozt)
- Crude Oil – Spot: West Texas Intermediate (WTI) (NYM \$/bbl) Spot

To find out whether cointegration is depended on economic cycle or influenced by global financial crisis, besides considering the whole period, the analysis is also conducted for the following three subperiod:

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<sup>2</sup> The current underlying contract for a continuous future will rollover based on open interest and previous cumulative volume. If a new contract has the highest open interest and previous cumulative volume, it will become the continuous contract, otherwise the current contract does not change until it expires. (FactSet Online Assistant)

- *Subperiod 1* Pre-crisis Period: 1 July 1983 - 9 August 2007<sup>3</sup>.
- *Subperiod 2* During-crisis Period: 10 August 2007 – 31 December 2009
- *Subperiod 3* Post-crisis Period: 1 January 2010 – 30 June 2017

As mentioned in session 3, the risk-free rate for pairs trading strategy performance analysis is the daily 3-month Treasury Bill secondary market rate, which is retrieved from FactSet terminal as well.

#### 4.2 Statistic descriptions

The basic statistic descriptions for the daily future and spot log returns are shown in table 1. The statistic descriptions for weekly and monthly data are attached in the Appendix A.

In general, crude oil has a higher standard deviation of return but not necessarily a higher mean return than gold in both spot and futures markets. During the whole period, both gold and oil return series are slightly negatively skewed. In Subperiod 2, all series have the highest volatility and mean return comparing to the other two subperiods. Comparing the statistic descriptions data across different data frequency, we can find that the kurtosis of the return and the annualized standard deviation increase with the data frequency. It meets the expectation of a smoother weekly and monthly data comparing to the daily counterparty.

The statistic descriptions of future and spot prices are quite similar. In all subperiods and

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<sup>3</sup> On 9 August 2007, BNP Paribas freeze three of their funds due to the risk of exposure to collateralized debt obligations(CDOs). It symbolized the start of the active phase of the crisis.

the whole sample period, crude oil experienced a smaller standard deviation in futures market compared to the spot market, while gold returns appear to have a higher volatility in futures market.

**Table 1a. Statistic descriptions for daily spot log returns**

	Whole Period		Subperiod 1		Subperiod 2		Subperiod 3	
	<i>r_oil_sd</i>	<i>r_gld_sd</i>	<i>r_oil_sd1</i>	<i>r_gld_sd1</i>	<i>r_oil_sd2</i>	<i>r_gld_sd2</i>	<i>r_oil_sd3</i>	<i>r_gld_sd3</i>
Mean	0.0000	0.0001	0.0001	0.0001	0.0002	0.0008	-0.0003	0.0001
Median	0.0000	0.0001	0.0000	0.0001	0.0000	0.0006	-0.0001	0.0000
Maximum	0.2128	0.0771	0.1813	0.0771	0.2128	0.0684	0.1213	0.0484
Minimum	-0.4000	-0.0960	-0.4000	-0.0625	-0.1307	-0.0797	-0.1106	-0.0960
Std. Dev.	0.0247	0.0101	0.0244	0.0090	0.0351	0.0162	0.0216	0.0107
Skewness	-0.6401	-0.1406	-1.1102	0.0330	0.3946	-0.1407	0.2309	-0.5324
Kurtosis	17.5794	9.4722	21.9991	9.5359	6.9642	5.5687	6.3927	8.9063
Mean (yearly)	0.0113	0.0321	0.0342	0.0193	0.0438	0.2034	-0.0764	0.0137
Std.Dev. (yearly)	0.3924	0.1598	0.3871	0.1428	0.5576	0.2571	0.3425	0.1706
Observations	8572	8572	6081	6081	603	603	1886	1886

**Table 1b. Statistic descriptions for future spot log returns**

	Whole Period		Subperiod 1		Subperiod 2		Subperiod 3	
	<i>r_oil_fd</i>	<i>r_gld_fd</i>	<i>r_oil_fd1</i>	<i>r_gld_fd1</i>	<i>r_oil_fd2</i>	<i>r_gld_fd2</i>	<i>r_oil_fd3</i>	<i>r_gld_fd3</i>
Mean	0.0000	0.0001	0.0001	0.0001	0.0002	0.0008	-0.0003	0.0001
Median	0.0003	0.0000	0.0005	0.0000	0.0008	0.0013	-0.0001	0.0001
Maximum	0.1844	0.0883	0.1403	0.0883	0.1844	0.0859	0.1162	0.0461
Minimum	-0.3841	-0.0981	-0.3841	-0.0773	-0.1307	-0.0605	-0.1079	-0.0981
Std. Dev.	0.0228	0.0104	0.0222	0.0095	0.0328	0.0160	0.0209	0.0108
Skewness	-0.6910	-0.1412	-1.1222	-0.0478	0.1509	0.2094	0.0461	-0.7606
Kurtosis	16.9442	10.0035	22.5083	10.6106	6.0682	5.7442	5.6032	9.3898
Mean(yearly)	0.0113	0.0318	0.0342	0.0194	0.0438	0.1986	-0.0763	0.0141
Std.Dev.(yearly)	0.3625	0.1650	0.3525	0.1508	0.5203	0.2546	0.3316	0.1718
Observations	8572	8572	6081	6081	603	603	1886	1886

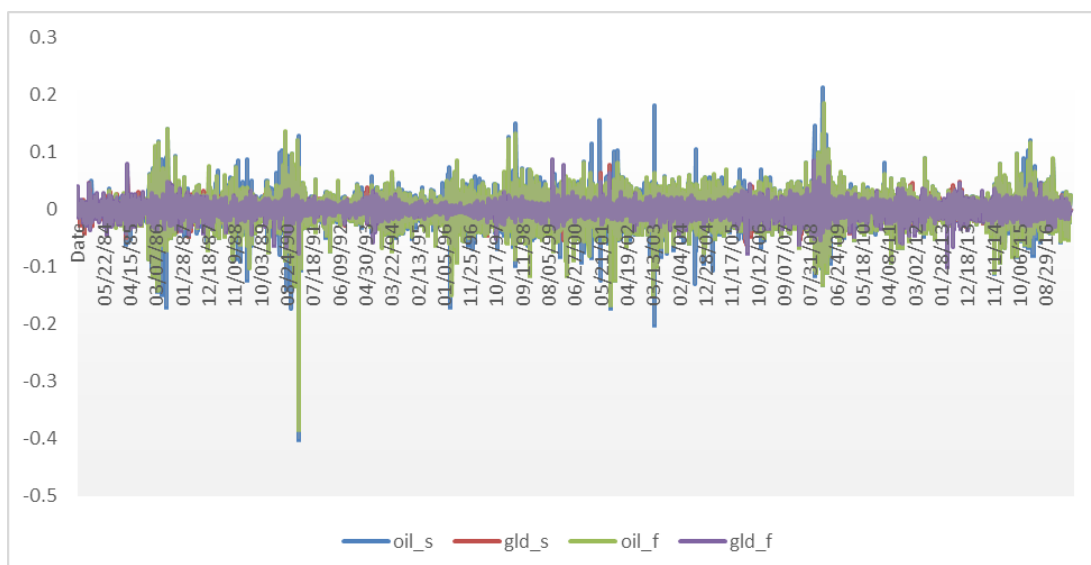
Notation: ‘d’ for daily data, ‘w’ for weekly data, ‘m’ for monthly data, ‘1,2,3’ for subperiod 1,2 and 3. The same notation rules apply for the entire paper. For annualization, assume 252 days, 52 weeks per year.

An intuitional observation of the log price series and log return series could be found in graph 1a and 1b. In graph 1a, for the log price series, the future line and spot line overlap with each other in the long run.

**Graph 1a Daily log price for gold and crude oil from July 1983 to June 2017**



**Graph 1b Daily log returns for gold and crude oil from July 1983 to June 2017**



## 5 Empirical results

### 5.1 Engle-Granger Cointegration test

Firstly, ADF test is conducted to all the series for the whole period and the three subperiods. An intercept is included in the test equation. The optimized lag length based on Akaike Information Criterion(AIC) for all level series is one, for first difference series, two. All time-series are integrated of level 1. P-value of ADF test for each series and their first difference can be found in the first part of table 2. Then, Engle-Granger cointegration test is conducted to estimate the cointegrating regression (3.2.1) and (3.2.2). The automatic lags specification based on AIC Criterion with max-lag is 16. With the help of Eviews, we can find the optimal lag length for all the cointegration test is 1. Conduct Engle-Granger test for each pair of time series using Matlab function *egcitest*. The specific code to generate the following table is attached in the Appendix D. The test results with all the p-values are in table 2. Both Engle-Granger tau-test statistics (t-statistic) and normalized autocorrelation coefficient z-test statistics are reported here.

**Table 2: P-value for unit root test and Engle-Granger cointegration test**

Dependent variable		<i>oil_sd</i>	<i>gld_sd</i>	<i>oil_fd</i>	<i>gld_fd</i>	<i>oil_sw</i>	<i>gld_sw</i>	<i>oil_fw</i>	<i>gld_fw</i>	<i>oil_sm</i>	<i>gld_sm</i>	<i>oil_fm</i>	<i>gld_fm</i>	<i>oil_s</i>	<i>gld_s</i>
Unit root test (ADF test)	Level	0.4052	0.9413	0.4675	0.9350	0.4637	0.9669	0.5126	0.9391	0.5352	0.9728	0.5430	0.9434	0.2695	0.2551
	First Diff.	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
Whole period	<i>tau-test</i>	0.1631	0.4319	0.218	0.482	0.2296	0.4884	0.2663	0.5061	0.0895	0.3126	0.1148	0.39	0.0173	0.0336
	<i>z-test</i>	0.1121	0.3483	0.1527	0.4094	0.1576	0.412	0.1839	0.4325	0.0503	0.1833	0.0666	0.2725	0.008	0.0147
Sub-period 1	<i>tau-test</i>	0.3725	0.6636	0.4245	0.6504	0.5133	0.6626	0.5221	0.6486	0.4027	0.6898	0.4548	0.7953	0.1109	0.0958
	<i>z-test</i>	0.3239	0.6073	0.3910	0.5910	0.4834	0.6086	0.4955	0.5880	0.3264	0.6351	0.3966	0.7289	0.0586	0.0645
Sub-period2	<i>tau-test</i>	0.8658	0.4853	0.8847	0.4967	0.8844	0.5161	0.9024	0.5631	0.5594	0.6610	0.5638	0.4772	0.5984	0.8364
	<i>z-test</i>	0.8940	0.6142	0.9151	0.5918	0.9126	0.6130	0.9316	0.6591	0.4817	0.6865	0.4871	0.5758	0.5329	0.6992
Sub-period3	<i>tau-test</i>	0.6165	0.3304	0.6353	0.3310	0.5155	0.2710	0.5388	0.2791	0.4852	0.2566	0.5022	0.3118	0.5003	0.3006
	<i>z-test</i>	0.6542	0.4330	0.6732	0.4372	0.5454	0.3343	0.5652	0.3321	0.4709	0.3096	0.4799	0.3676	0.4956	0.3622

Note: The table contains p-value for the ADF test (row 2-3) and Engle-Granger cointegration test (row 4-11). As stated in the previous paragraphs, the notations for the variables are: 's' for spot, 'f' for future, 'd' for daily data, 'w' for weekly data and 'm' for monthly data. For example, the third column (*oil\_sd*) contains the unit root test p-value for daily spot oil log price and cointegration test p-value when it is taken as a dependent variable, while the tenth column (*gld\_fm*) is for weekly future gold log price. The last two columns (*oil\_s*, *gld\_s*) contain data for the longer period from March 1968 to June 2017. Both Engle-Granger tau-test statistics (t-statistic) and normalized autocorrelation coefficient z-test statistics are reported in the table. The cell highlighted in red is the one with a p-value below 0.05 and the number in red represent a p-value below 0.1.

The result with data back to March 1968 in the last 2 columns shows cointegration relationship at a significance level of 5% for the whole period with either spot gold or spot oil as dependent variables, while a weaker cointegration is found in the pre-crisis period with a significance level of around 10%. There is not enough evidence for a cointegration in the during-crisis period and post-crisis period. The p-values during the crisis period are larger than the other two sub-periods, which means less evidence of cointegration during the crisis period.

The estimated cointegrating relationship of  $oil_s$  and  $gld_s$  in the last two columns of table 2 for the 49-year longer period as follow:

$$oil_{s_t} = -2.21 + 0.92 gld_{s_t} \quad (5.1.1)$$

$$gld_{s_t} = 2.85 + 0.95 oil_{s_t} \quad (5.1.2)$$

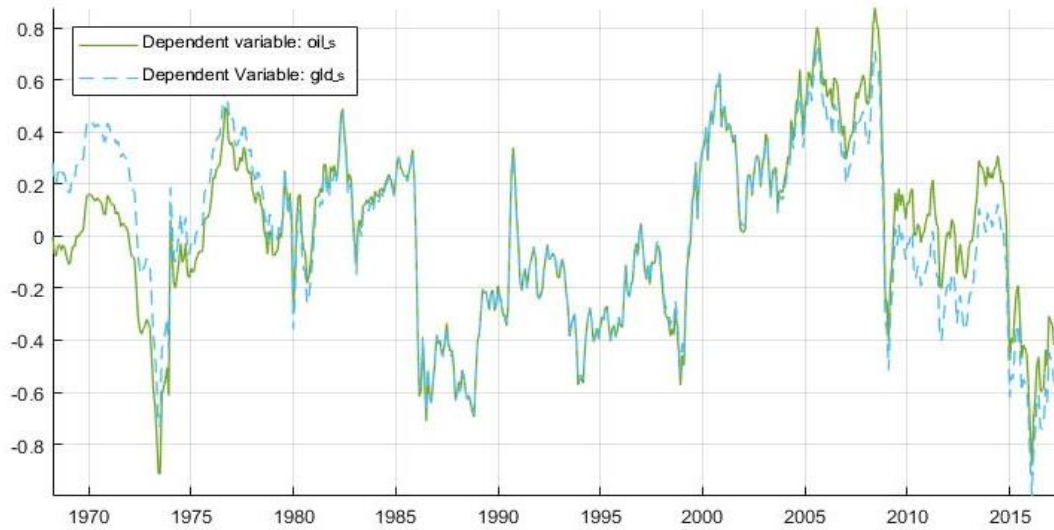
P-values of t-statistics for all the coefficients in both equations are 0.00. Equation (5.1.1) means that a one-percent increase in gold price will lead to 0.92% increase in oil price, while equation (5.1.2) means that a one-percent increase in oil price will lead to 0.95% increase in gold price. The graph of cointegration regression error term could be found in Graph 2. The error term is reverting to a mean around zero in both regressions. However, the volatility seems to change across time. From 2000 to 2009, when internet bubble and the global financial crisis took place, the residual series haven't returned to zero for a long time. After the financial crisis, the cointegration relationship seems to be stronger than before.

Now, use the residual and estimate the error correction model (ECM) for the 49 years data. Estimate  $\alpha_1$  and  $\alpha_2$  in (3.2.3) (3.2.4),

$$\Delta oil_t = \alpha_1 \hat{u}_{t-1} + e_{1t} \quad (3.2.3)$$

$$\Delta gld_t = \alpha_2 \hat{u}_{t-1} + e_{2t} \quad (3.2.4)$$

Graph 2 Cointegrating Relation from March 1968 to June 2017



Note: The graph contains two lines of cointegration residuals in equation (5.1.1) and (5.1.2). The green line represents the residuals of equation (5.1.1), when log oil price is taken as dependent variable. The blue line represents the reversed residuals of equation (5.1.2) when log gold price is taken as dependent variable.

The result shows that  $\alpha_1 = 0.037$  with t-statistic p-value 0.0113, while  $\alpha_2 = -0.009$  with t-statistic p-value 0.2027. The null hypothesis of  $\alpha_2 = 0$  could not be rejected, i.e.,  $\alpha_2$  is not statistically significant. As a result, we can treat the gold price as exogenous. The equation (3.2.5) can be considered. By estimation, we get

$$\widehat{\Delta oil}_t = 0.03\hat{u}_{t-1} + 0.3\Delta gld_t \quad (5.1.3)$$

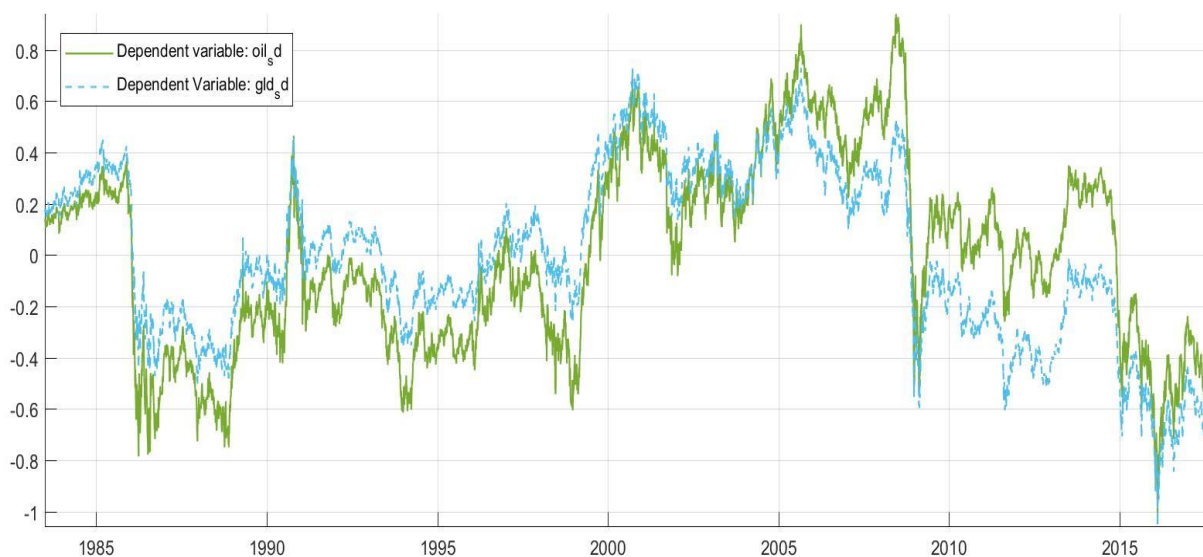
The p-value of t-statistic for both coefficients is 0.00. It means both error correction term and first difference term of gold are statistically significant in the model.

For the sample from 1983 to 2017, it's obvious from the table that with neither tau-test statistics nor z-test statistics, we could reject the null hypothesis of no cointegration at a significance level of 5%. With a 10% significance level, only spot series monthly sample for the whole period show some evidence of cointegration when log oil prices are taken as dependent variable. When the oil is taken as the dependent variable, more evidence of cointegration is found compared to the case when gold is the dependent variable. The p-

values for the whole period are always smaller than the ones for any corresponding sub-periods. Most of the p-values for data during the crisis are bigger than any other periods within the same series. It might indicate a weaker co-movement of oil and gold markets during the financial crisis. The result could also be derived from a smaller sample size in sub-period 2 compared to the other two sub-periods.

Though the cointegration of the two commodity price series is not strong, we can still assume a weak cointegration with the significance level of 20%, which will be considered the default significance level in the pairs trading part. The graph of cointegration regression error term for daily spot series could be found in Graph 3.

**Graph 3 Cointegrating Relation from March 1983 to June 2017**



Note: The green line represents the residuals when log oil price is taken as dependent variable. The blue line represents the reversed residuals when log gold price is taken as dependent variable.

## 5.2 Rolling regression

Though not enough evidence is found from above results to support a strong cointegration of oil and gold prices during 1983 to 2017, some strong cointegration relationship might appear in some periods during the 34 years. A rolling regression as



described in the methodology part of the paper will be conducted to find potential strong cointegration relationship during the sample period. A rolling window of 252 days and rolling interval of 126 days are considered here. The complete regression result is attached in the Appendix B. With the significance level of 20% and tau test statistics, cointegration is found in the following period in table 3. From the table, we can find that the cointegration test result depends on the chosen regression period.

**Table 3. Rolling regression of Engle-Granger cointegration test**

Start Date	End Date	<i>oil_sd</i>	<i>gld_sd</i>	<i>oil_fd</i>	<i>gld_fd</i>
1983/07	1984/07	0.145412	0.133602	0.158298	0.160937
1984/01	1985/01	0.022272	0.016137	0.026791	0.021306
1984/07	1985/07	0.161152	0.097845	0.130103	0.082356
1985/07	1986/07	0.094081	0.007336	0.16795	0.007051
1986/01	1987/01	0.076821	0.797098	0.070209	0.743101
1988/01	1989/01	0.082063	0.071244	0.108877	0.072192
1989/01	1990/01	0.151922	0.826036	0.383747	0.709356
1989/07	1990/07	0.009656	0.040634	0.062735	0.090719
1990/07	1991/07	0.241392	0.09784	0.449432	0.215283
1991/01	1992/01	0.029359	0.407252	0.113225	0.374247
1994/01	1995/01	0.531545	0.087083	0.558601	0.122643
1995/01	1996/01	0.293512	0.109428	0.338275	0.10603
1996/01	1997/01	0.135669	0.480117	0.045065	0.193104
1998/01	1999/01	0.628498	0.145529	0.558778	0.091266
2000/07	2001/07	0.385442	0.175553	0.208299	0.128191
2001/01	2002/01	0.46602	0.141422	0.57281	0.201156
2002/01	2003/01	0.177882	0.229766	0.257673	0.277461
2002/07	2003/07	0.173983	0.517846	0.226337	0.619023
2012/07	2013/07	0.029643	0.974953	0.027233	0.979833
2013/07	2014/07	0.220826	0.104166	0.238026	0.124209
2014/01	2015/01	0.853069	0.132325	0.793395	0.121153
2016/07	2017/07	0.191069	0.356961	0.247243	0.373976

Note: The table contains p-value of tau-test statistics for Engle-Granger test. The rolling window (formation period in pairs trading part) is 1 year (252 days) and rolling interval (trading period) is half a year (126 days). The first two columns show the regression period with start and end date when at least one of the regression has a p-value smaller than 0.2. For the right four columns, the odd columns (*oil\_sd*, *oil\_fd*) contain the test result of equation (3.2.1), where oil is taken as a dependent variable, while the even columns (*gld\_sd*, *gld\_fd*) contain the test result of equation (3.2.2), where gold is taken as dependent variable. The whole table is attached in Appendix B.

### 5.3 Pairs Trading

Use the self-developed Matlab functions *pairs\_imp* and *pairs\_coint*. The input variables for the functions include two log price series, formation period  $M$  and trading period  $N$  (only for cointegration method *pairs\_coint*), the targeted deviation  $\Delta$ , trading cost  $\eta$ , cointegration test significance level  $\alpha$  (only for *pairs\_coint*), date range (for plotting purpose) and risk-free rate series. Sensitivity analysis will be conducted for the targeted deviation  $\Delta$ , trading cost  $\eta$  and significance level  $\alpha$ .

As stated in the previous paragraphs, the default parameters are set as follow:

- Formation period  $M$ : 252 days; Trading period  $N$ : 126 days
- Targeted Deviation  $\Delta$ : one historical standard deviation
- Trading cost  $\eta$ : 0.5%
- Significance level  $\alpha$ : 0.2

Based on the result of the cointegration test, we would consider log oil price as the dependent variable and log gold price as endogenetic variable. Take equation (3.2.1) as the basic regression equation for both methods. Sensitivity analysis will be conducted for the interested parameters.

#### 5.3.1 Pairs trading with imposed cointegration

Though no evidence of cointegration is found for any sample series during the whole period from 1983 to 2017 at a significance level of 10%, we can still assume the existence of cointegration as a preliminary analysis. With the default input, for spot series, we find a Sharpe ratio of -0.2778, an annualized return of -7.05%, and the number of opening pairs trading position is 8 times. For futures series, we find a Sharpe ratio of -0.2870, an

annualized return of -6.10%, and the number of opening pairs trading position is 9 times. The pairs trading positions in oil and gold during the whole period is shown in the graphs attached in Appendix C.

### 5.3.2 Pairs trading based on rolling regression

By rolling regressions shown in the previous session, we only conduct pairs trading when cointegration is observed in the forming period of the previous one year, or approximately 252 days, pairs trading would be engaged in the following six months, or approximately 126 days.

With the above default input parameters, we find the Sharpe ratios of 0.0375, 0.0959, the annualized returns of 3.52%, 4.88% and the numbers of opening pairs in the whole period are 36 and 25 times respectively for spot and futures series. According to the Engle-Granger cointegration result in Table 2, we only consider crude oil as the dependent variable. The futures portfolio has the higher Sharpe ratio and returns. The pairs trading positions in oil and gold for both spot and futures series during the whole period is shown in the graphs attached in Appendix C.

### 5.3.1 Sensitivity and robust analysis

The sensitivity analysis is based on the rolling regression model for both spot series and futures series. We checked the influence on the annualized returns and Sharpe ratios by the trading cost  $\eta$  and the pairs trading triggering condition - targeted deviation  $\Delta$  and significance level  $\alpha$ . The results are shown in Table 4.

It could be found in Table 4a that for spot series the best performance is reached with one standard error as targeted deviation  $\Delta$  and 0.45 significance level. In Table 4b, we find the best performance for futures series is found with one standard error as targeted deviation  $\Delta$  and 0.5 significance level.

Table 4a. Sensitivity Analysis

Targeted Deviation  $\Delta$  and Significance level  $\alpha$  – Spot Series

Targeted Deviation $\Delta$	1 Standard Error			2 Standard Error			3 Standard Error		
Significant Level ( $\alpha$ )	Return	Sharpe	No.	Return	Sharpe	No.	Return	Sharpe	No.
0.05	2.57%	-0.0001	10	2.33%	-0.0273	5	2.13%	-0.0498	4
0.10	4.63%	0.0886	15	3.51%	0.0442	7	3.24%	0.0315	6
0.15	4.39%	0.0780	18	3.29%	0.0335	9	2.93%	0.0171	8
0.20	3.52%	0.0375	36	2.85%	0.0120	18	2.47%	-0.0043	13
0.25	3.99%	0.0545	43	3.21%	0.0271	22	3.23%	0.0291	15
0.30	5.09%	0.0887	45	5.12%	0.0995	24	4.94%	0.0949	17
0.35	5.51%	0.0954	59	4.17%	0.0580	33	3.75%	0.0446	22
0.40	6.90%	0.1341	70	5.25%	0.0927	37	4.02%	0.0529	25
0.45	7.09%	0.1394	71	5.25%	0.0927	37	4.02%	0.0529	25
0.50	6.91%	0.1313	73	4.77%	0.0746	39	3.89%	0.0479	26

Table 4b. Sensitivity Analysis

Targeted Deviation  $\Delta$  and Significance level  $\alpha$  – Futures series

Targeted Deviation $\Delta$	1 Standard Error			2 Standard Error			3 Standard Error		
Significant Level ( $\alpha$ )	Return	Sharpe	No.	Return	Sharpe	No.	Return	Sharpe	No.
0.05	2.08%	-0.1824	7	1.84%	-0.2901	4	1.84%	-0.3514	3
0.10	4.62%	0.1890	14	3.41%	0.0815	10	3.81%	0.1271	6
0.15	4.02%	0.1294	17	2.83%	0.0249	12	3.15%	0.0579	8
0.20	4.88%	0.0959	25	3.13%	0.0248	17	3.17%	0.0280	9
0.25	6.55%	0.1530	35	4.09%	0.0635	23	3.89%	0.0597	13
0.30	6.25%	0.1404	37	3.93%	0.0563	24	3.50%	0.0418	14
0.35	7.35%	0.1736	42	4.04%	0.0576	29	2.49%	-0.0036	19
0.40	6.69%	0.1462	52	3.14%	0.0221	33	2.03%	-0.0228	21
0.45	7.84%	0.1762	59	4.89%	0.0840	36	3.31%	0.0292	23
0.50	7.91%	0.1706	63	5.59%	0.1063	37	3.62%	0.0399	24

Note: Table 4a and 4b show the annualized returns, Sharpe Ratios and the number of trading during the whole period with different Targeted Deviation  $\Delta$  and significance level  $\alpha$  of cointegration chosen for triggering the pairs trading. Other input parameters are set as default. The highest return and Sharpe Ratio are reached at  $\alpha=0.45$ ,  $\Delta=1$  standard error for spot series and  $\alpha=0.50$ ,  $\Delta=1$  standard error for futures series.

The table 4c shows the impact of trading cost on both spot and futures series. It could be easily found that the lower the trading cost, the better the profitability of the strategy.

**Table 4c. Sensitivity Analysis - Trading Cost  $\eta$**

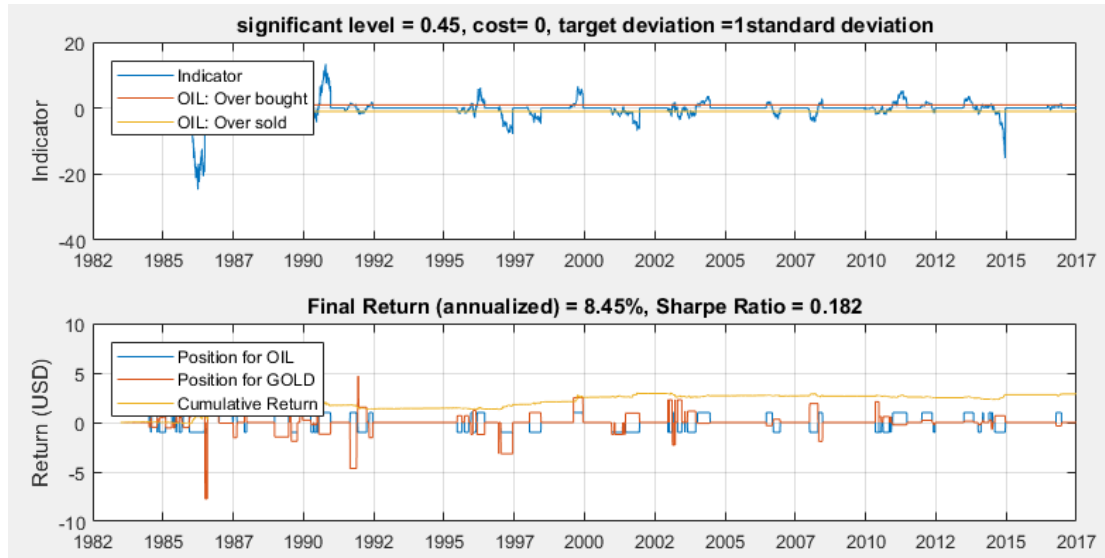
Spot			Futures		
Cost	Annualized Return	Sharpe Ratio	Cost	Annualized Return	Sharpe Ratio
0.0%	4.27%	0.0673	0.0%	5.33%	0.1148
0.1%	4.12%	0.0614	0.1%	5.24%	0.1110
0.2%	3.97%	0.0554	0.2%	5.15%	0.1073
0.3%	3.82%	0.0494	0.3%	5.06%	0.1035
0.4%	3.67%	0.0434	0.4%	4.97%	0.0997
0.5%	3.52%	0.0375	0.5%	4.88%	0.0959
0.6%	3.37%	0.0315	0.6%	4.79%	0.0921
0.7%	3.22%	0.0255	0.7%	4.70%	0.0884
0.8%	3.07%	0.0196	0.8%	4.61%	0.0846
0.9%	2.92%	0.0136	0.9%	4.52%	0.0808
1.0%	2.77%	0.0077	1.0%	4.43%	0.0770

Note: The table shows the annualized returns and Sharpe Ratios with different estimated cost. Other input parameters are set as default.

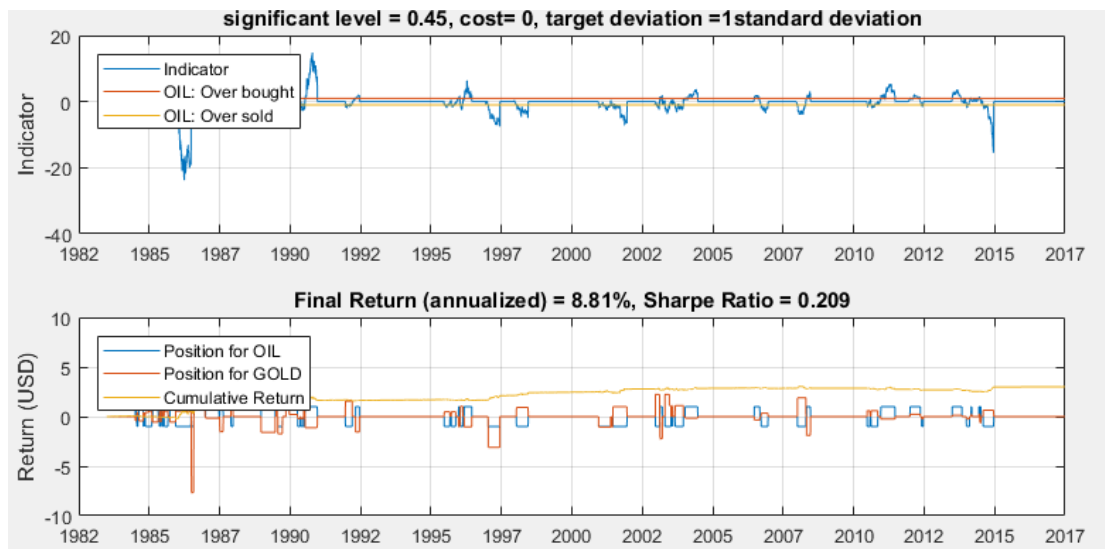
Using the codes provided by Ledoit and Wolf (2008) to test the Sharpe ratios, we find that for the strategy with imposed cointegration, for both future and spot series we have HAC inference p-value of over 0.6, so we cannot reject the null hypothesis of negative or zero Sharpe ratio. It means the preliminary method do not have robust risk-adjusted performance. For pairs trading strategy with rolling regression, with default input parameters, we reject the null hypothesis and conclude of a positive Sharpe ratio with a significance level of 5%. It indicates that the strategy has a robust and profitable risk-adjusted performance.

To get a more intuitionistic idea of the opening trading positions during the sample period, sample plots with  $\alpha=0.45$ ,  $\Delta = 1$  standard error and cost = 0 is provided in Graph 4a and 4b.

Graph 4a Pairs Trading on Spot Series



Graph 4b Pairs Trading on Futures series



Note: The graph shows the opening positions throughout the whole sample period. The upper subplot indicates the period when either oil or gold is over-bought; the second subplot shows the opening positions for both commodities as well as the cumulative return in USD, assuming one USD investment at beginning.

## 6 Conclusions and Discussion

### 6.1 Conclusions

Contrary to a few studies mentioned in the literature review, with the results shown in section 5.1, we cannot find a single cointegration relationship with a significance level of 5% during the whole sample period for all spot and futures series with three different data frequencies. The result might be caused by a lack of power for the Engle-Granger test, which will lead to an increase of type II error. However, with rolling regression, the cointegration relationship between gold and crude oil can be found during some subperiods within the sample period. Stronger cointegration is found before 2000. From 2000 to 2009, when internet bubble and the global financial crisis took place, the gold and crude oil price series diverse from each other for quite a long time, so that the co-movement mechanism disappeared. Evidence from the cointegration test in section 5.2 shows the two commodities have a tendency to move together again after 2010.

Use the same rolling regression method, we developed a pairs trading strategy. The strategy has a decent profitability and robust Sharpe ratio with a loose triggering condition of larger significance level and smaller targeted deviation. Yet, with the default input parameters of a 20% significance level, a targeted deviation of one standard error and 0.5% cost per trading, most of the pairs positions opened in the early period before year 2000 (36 and 25 times trading happened during the whole sample period for spot and futures series respectively, within which only 5 and 3 times trading appeared after year 2000). It might indicate a vanishing profit for pairs trading during recent years. With the table 4c, we can see that the profitability of the strategy is sensitive to the trading cost. Trading in futures market earns a slightly higher return compared to trading in spot market with otherwise identical conditions.

## 6.2 The limitation of the study

In the paper, we used the Engle–Granger 2-step method for cointegration test and a foundation of the pairs trading strategy. The method is simple and easy to implement; however, the method suffers from a number of problems (Brooks 2014):

- A problem of lack of power in unit root and cointegration tests may be caused by limited sample size.
- The researcher is forced to treat the two variables asymmetrically, i.e. to specify one variable as the dependent variable and the others as independent variables. Though in most of the times, no theoretical support could be found to the specific choice.

Though in the model of two variables here it won't cause a big problem for the second point above, we still need to bear in mind the potential problems of the test. Another limitation of the test comes from the robust test for the Sharpe ratio of the pairs trading strategy. According to Ledoit and Wolf (2008), "With the HAC inference, the hypothesis tests tend to reject a true null hypothesis too often compared to the nominal significance level and confidence intervals tend to undercover." It means that our pairs trading strategy might not be as appealing as it is indicated by the test results.



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## 8 Appendices

### A. Statistic descriptions

#### a. Statistic descriptions for weekly spot log Returns

	Whole Period		Subperiod 1		Subperiod 2		Subperiod 3	
	oil_sw	gld_sw	oil_sw1	gld_sw1	oil_sw2	gld_sw2	oil_sw3	gld_sw3
Mean	0.0002	0.0006	0.0007	0.0004	0.0004	0.0040	-0.0014	0.0003
Median	0.0021	0.0006	0.0020	0.0003	0.0087	0.0054	-0.0002	0.0014
Maximum	0.3594	0.1313	0.2513	0.1313	0.3594	0.0875	0.1272	0.0615
Minimum	-0.3494	-0.1166	-0.3494	-0.0696	-0.3122	-0.0905	-0.1590	-0.1166
Std. Dev.	0.0504	0.0184	0.0487	0.0166	0.0795	0.0298	0.0432	0.0190
Skewness	-0.4804	-0.0568	-0.6088	0.3816	-0.2049	-0.3092	-0.2853	-0.9491
Kurtosis	8.8287	7.5013	7.7584	8.1636	8.1644	3.8553	4.0738	7.2730
Mean(yearly)	0.0113	0.0321	0.0364	0.0194	0.0206	0.2059	-0.0725	0.0169
Std.Dev.(yearly)	0.3632	0.1326	0.3510	0.1196	0.5736	0.2152	0.3116	0.1370
Observations	1774	1774	1257	1257	126	126	391	391

#### b. Statistic descriptions for weekly future log Returns

	Whole Period		Subperiod 1		Subperiod 2		Subperiod 3	
	oil_fw	gld_fw	oil_fw1	gld_fw1	oil_fw2	gld_fw2	oil_fw3	gld_fw3
Mean	0.0002	0.0006	0.0007	0.0004	0.0004	0.0037	-0.0014	0.0003
Median	0.0025	0.0007	0.0022	0.0002	0.0147	0.0064	0.0006	0.0016
Maximum	0.2500	0.1236	0.2500	0.1236	0.2061	0.1232	0.1151	0.0682
Minimum	-0.3626	-0.1013	-0.3626	-0.0838	-0.2880	-0.0878	-0.1590	-0.1013
Std. Dev.	0.0471	0.0223	0.0462	0.0203	0.0675	0.0350	0.0419	0.0231
Skewness	-0.7218	-0.0441	-0.7449	0.1438	-0.7873	-0.2563	-0.4311	-0.4014
Kurtosis	7.8360	5.5525	8.7641	5.7911	5.2935	3.9832	4.0132	4.1446
Mean(yearly)	0.0113	0.0317	0.0363	0.0201	0.0207	0.1944	-0.0724	0.0166
Std.Dev.(yearly)	0.3399	0.1608	0.3333	0.1467	0.4870	0.2521	0.3023	0.1667
Observations	1774	1774	1257	1257	126	126	391	391

### c. Statistic descriptions for monthly spot log Returns

	Whole Period		Subperiod 1		Subperiod 2		Subperiod 3	
	oil_sm	gld_sm	oil_sm1	gld_sm1	oil_sm2	gld_sm2	oil_sm3	gld_sm3
Mean	0.0009	0.0027	0.0031	0.0016	0.0005	0.0179	-0.0061	0.0011
Median	0.0059	0.0002	0.0059	-0.0009	0.0092	0.0182	-0.0049	0.0014
Maximum	0.3694	0.1601	0.3694	0.1601	0.2602	0.1022	0.2226	0.1101
Minimum	-0.3948	-0.1248	-0.3525	-0.1248	-0.3948	-0.1134	-0.2332	-0.0709
Std. Dev.	0.0958	0.0355	0.0953	0.0334	0.1302	0.0511	0.0849	0.0355
Skewness	-0.2066	0.3687	-0.0286	0.5368	-0.9401	-0.5019	-0.2361	0.2302
Kurtosis	5.1663	4.3625	5.3155	5.3677	4.4878	2.8057	3.3958	3.0360
Mean(yearly)	0.0108	0.0321	0.0373	0.0195	0.0060	0.2147	-0.0727	0.0134
Std.Dev.(yearly)	0.3319	0.1231	0.3300	0.1157	0.4512	0.1772	0.2940	0.1230
Observations	407	407	288	288	29	29	90	90

### d. Statistic descriptions for monthly future log Returns

	Whole Period		Subperiod 1		Subperiod 2		Subperiod 3	
	oil_fm	gld_fm	oil_fm1	gld_fm1	oil_fm2	gld_fm2	oil_fm3	gld_fm3
Mean	0.0027	0.0009	0.0016	0.0031	0.0179	0.0005	0.0011	-0.0060
Median	0.0002	0.0063	-0.0009	0.0070	0.0182	0.0092	0.0014	-0.0003
Maximum	0.1601	0.3705	0.1601	0.3705	0.1022	0.2602	0.1101	0.2253
Minimum	-0.1248	-0.3948	-0.1248	-0.3507	-0.1134	-0.3948	-0.0709	-0.2328
Std. Dev.	0.0355	0.0948	0.0334	0.0939	0.0511	0.1302	0.0355	0.0844
Skewness	0.3687	-0.2054	0.5368	-0.0108	-0.5019	-0.9389	0.2302	-0.2694
Kurtosis	4.3625	5.2390	5.3677	5.3967	2.8057	4.4945	3.0360	3.4408
Mean(yearly)	0.0321	0.0107	0.0195	0.0372	0.2147	0.0060	0.0134	-0.0726
Std.Dev.(yearly)	0.1231	0.3282	0.1157	0.3253	0.1772	0.4509	0.1230	0.2922
Observations	407	407	288	288	29	29	90	90

## B. Rolling window regression for cointegration test

### a. Rolling regression of Engle-Granger cointegration test

Start Date	End Date	<i>oil_sd</i>	<i>gld_sd</i>	<i>oil_fd</i>	<i>gld_fd</i>
1983/07	1984/07	0.145412	0.133602	0.158298	0.160937
1984/01	1985/01	0.022272	0.016137	0.026791	0.021306
1984/07	1985/07	0.161152	0.097845	0.130103	0.082356
1985/01	1986/01	0.316841	0.259158	0.447987	0.391172
1985/07	1986/07	0.094081	0.007336	0.16795	0.007051
1986/01	1987/01	0.076821	0.797098	0.070209	0.743101
1986/07	1987/07	0.350527	0.318855	0.39251	0.284807
1987/01	1988/01	0.902987	0.893898	0.883675	0.896256
1987/07	1988/07	0.75786	0.563046	0.718882	0.387933
1988/01	1989/01	0.082063	0.071244	0.108877	0.072192
1988/07	1989/07	0.308911	0.223216	0.431173	0.347584
1989/01	1990/01	0.151922	0.826036	0.383747	0.709356
1989/07	1990/07	0.009656	0.040634	0.062735	0.090719
1990/01	1991/01	0.82486	0.495399	0.856712	0.502688
1990/07	1991/07	0.241392	0.09784	0.449432	0.215283
1991/01	1992/01	0.029359	0.407252	0.113225	0.374247
1991/07	1992/07	0.790395	0.804931	0.802667	0.714583
1992/01	1993/01	0.679067	0.469734	0.657616	0.442962
1992/07	1993/07	0.724821	0.979438	0.647718	0.972761
1993/01	1994/01	0.957286	0.679211	0.928424	0.601572
1993/07	1994/07	0.919437	0.434255	0.822882	0.314422
1994/01	1995/01	0.531545	0.087083	0.558601	0.122643
1994/07	1995/07	0.364007	0.330758	0.416242	0.338653
1995/01	1996/01	0.293512	0.109428	0.338275	0.10603
1995/07	1996/07	0.639691	0.639395	0.723505	0.595213
1996/01	1997/01	0.135669	0.480117	0.045065	0.193104
1996/07	1997/07	0.586311	0.80183	0.463988	0.662035
1997/01	1998/01	0.33851	0.924981	0.314593	0.918171
1997/07	1998/07	0.929699	0.341233	0.904196	0.317741
1998/01	1999/01	0.628498	0.145529	0.558778	0.091266
1998/07	1999/07	0.393512	0.775627	0.475346	0.708838
1999/01	2000/01	0.878692	0.704248	0.904599	0.617963
1999/07	2000/07	0.684112	0.574898	0.556671	0.401007
2000/01	2001/01	0.283016	0.509278	0.233783	0.377351
2000/07	2001/07	0.385442	0.175553	0.208299	0.128191

(Continuing)

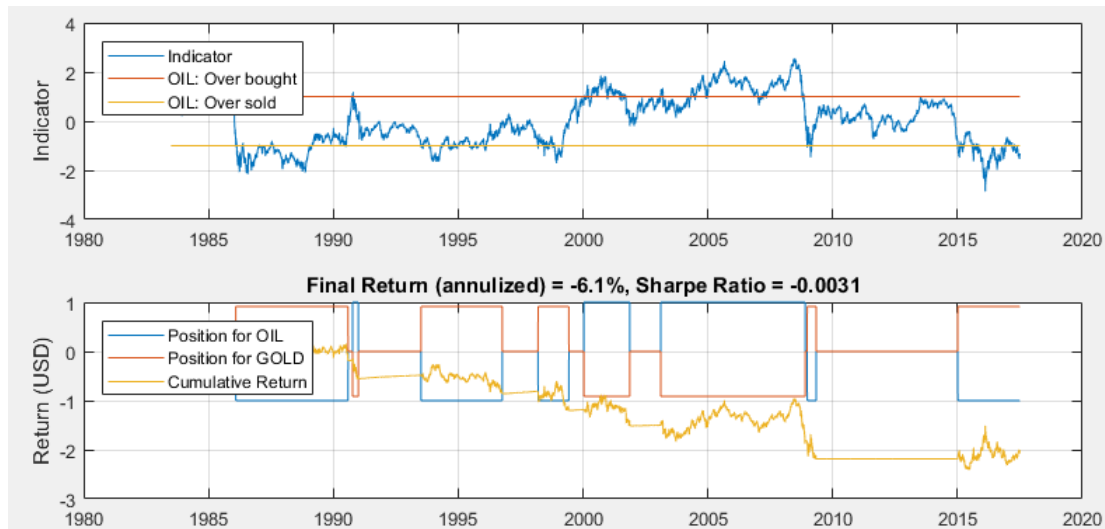


Start Date	End Date	<i>oil_sd</i>	<i>gld_sd</i>	<i>oil_fd</i>	<i>gld_fd</i>
2001/01	2002/01	0.46602	0.141422	0.57281	0.201156
2001/07	2002/07	0.482993	0.948608	0.473209	0.937872
2002/01	2003/01	0.177882	0.229766	0.257673	0.277461
2002/07	2003/07	0.173983	0.517846	0.226337	0.619023
2003/01	2004/01	0.346634	0.932527	0.35453	0.955619
2003/07	2004/07	0.759213	0.563119	0.81438	0.597525
2004/01	2005/01	0.637695	0.779548	0.646061	0.728067
2004/07	2005/07	0.532716	0.59172	0.584328	0.488318
2005/01	2006/01	0.558818	0.978621	0.544	0.955129
2005/07	2006/07	0.260757	0.653756	0.234357	0.620477
2006/01	2007/01	0.711518	0.466305	0.751474	0.425989
2006/07	2007/07	0.725465	0.373696	0.747804	0.280973
2007/01	2008/01	0.334254	0.265945	0.312394	0.236747
2007/07	2008/07	0.969131	0.894119	0.983919	0.91943
2008/01	2009/01	0.999	0.487632	0.989093	0.353656
2008/07	2009/07	0.656022	0.562497	0.630773	0.493876
2009/01	2010/01	0.342079	0.745845	0.591758	0.643422
2009/07	2010/07	0.264374	0.770073	0.207575	0.695797
2010/01	2011/01	0.288398	0.89313	0.35315	0.855604
2010/07	2011/07	0.581063	0.810821	0.583913	0.817852
2011/01	2012/01	0.37251	0.682144	0.354979	0.636224
2011/07	2012/07	0.873829	0.237829	0.888265	0.258863
2012/01	2013/01	0.832052	0.431681	0.828264	0.506193
2012/07	2013/07	0.029643	0.974953	0.027233	0.979833
2013/01	2014/01	0.38122	0.806433	0.396642	0.824582
2013/07	2014/07	0.220826	0.104166	0.238026	0.124209
2014/01	2015/01	0.853069	0.132325	0.793395	0.121153
2014/07	2015/07	0.792595	0.421067	0.770359	0.416287
2015/01	2016/01	0.468047	0.478986	0.48712	0.512795
2015/07	2016/07	0.397406	0.950143	0.424521	0.955669
2016/01	2017/01	0.858441	0.659352	0.909843	0.727256
2016/07	2017/07	0.191069	0.356961	0.247243	0.373976

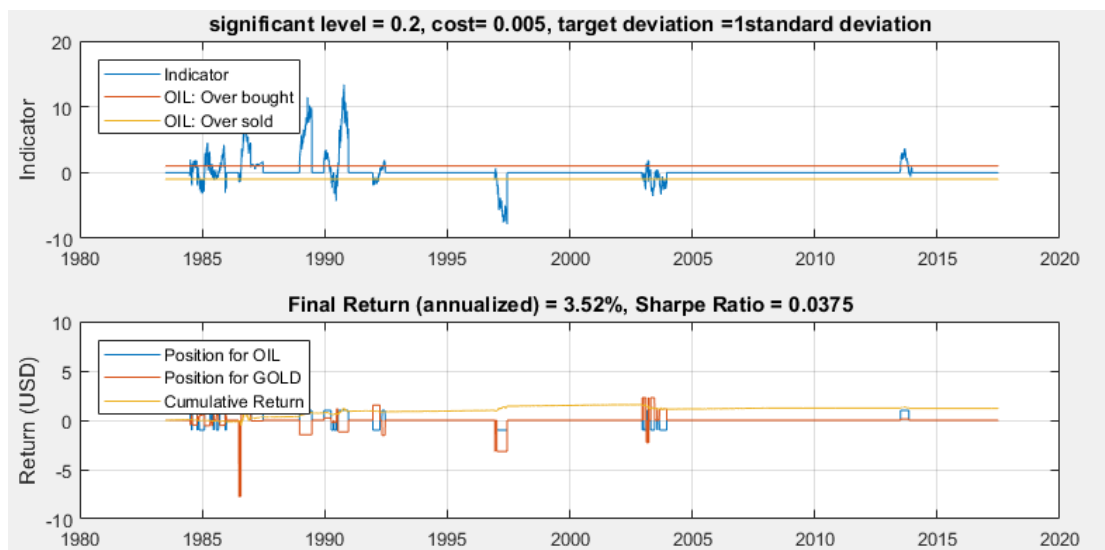
Note: The table is an extension of Table 3. It contains p-value of tau-test statistics for Engle-Granger test. The rolling window (formation period in pairs trading part) is 1 year (252 days) and rolling interval (trading period) is half a year (126 days). For the right four columns, the odd columns (*oil\_sd*, *oil\_fd*) contain the test result of equation (3.2.1), where oil is taken as dependent variable, while the even columns (*gld\_sd*, *gld\_fd*) contain the test result of equation (3.2.2), where gold is taken as dependent variable.

## C. Pairs Trading Result

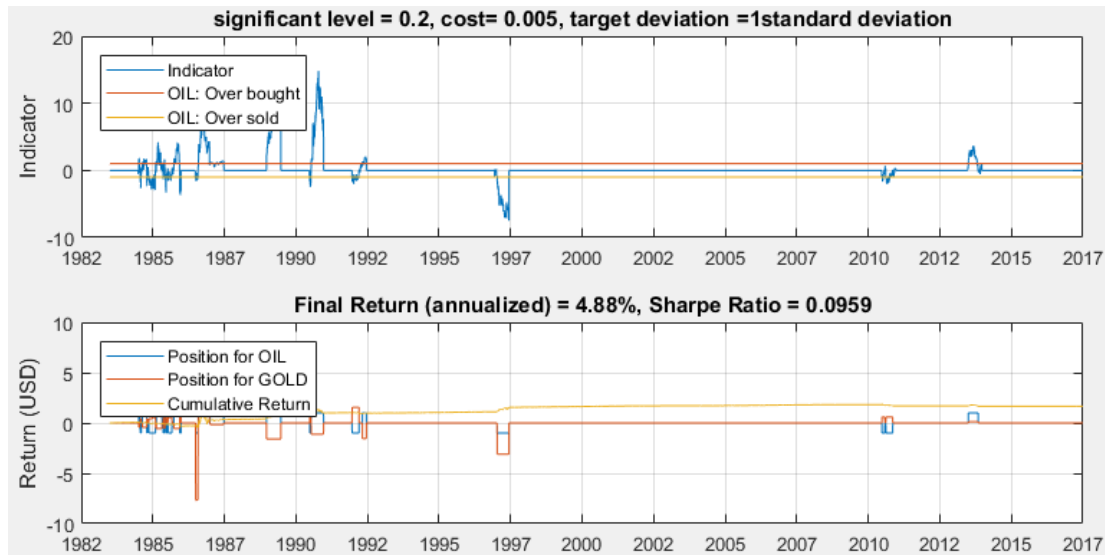
### a. Pairs Trading with imposed cointegration



### b. Pairs Trading on Spot Series with Default Input Parameters



### c. Pairs Trading on Futures series with Default Input Parameters



## D. Matlab Code

## a. Code for rolling regression

```
Y_d = Y(:,1:4);
M=252; N=126;% window = 1 y
PEG_dt = zeros(8573-M,4);
i=1; c=1; lags = 1;
%daily spot
for j=1:N:8573-M
    % oil as dependent var.
    [~,PEG_dt(c,i)] =
egcitest(Y_d(j:j+M,i:i+1),'test','t1','lags',lags);
    % gold as dependent var.
    X_d=[Y_d(j:j+M,i+1),Y_d(j:j+M,i)];
    [~,PEG_dt(c,i+1)] = egcitest(X_d,'test','t1','lags',lags);
    c=c+1;
end
%daily futures
i=3; c=1;
for j=1:N:8573-M
    [~,PEG_dt(c,i)] =
egcitest(Y_d(j:j+M,i:i+1),'test','t1','lags',lags);
    X_d=[Y_d(j:j+M,i+1),Y_d(j:j+M,i)];
    [~,PEG_dt(c,i+1)] = egcitest(X_d,'test','t1','lags',lags);
    c=c+1;
end
```

## b. Function pairs\_coint

```
function [sh,sumr,r,trn] = pairs_coint(series, M, N, delta,  
cost,rf,alpha,Date)  
%[sh,sumr] = pairs_coint(series, M, N, delta=1, scaling=252,  
cost=0,rf=0,alpha=0.05,Date)  
% This function returns a annualized return and sharp ratio for  
% pairs trading strategy based cointegration relationship.  
% (trade only there's evidence of cointegration.  
%Every N periods, we use the previous M periods' worth of information  
to  
%estimate the cointegrating relationship.)  
  
%% Process input args  
  
if ~exist('cost','var')  
    cost = 0.005;  
end  
  
if ~exist('delta', 'var')  
    delta = 1;  
    % delta defined the targeted deviation,  
    %the default targeted deviation is one standard deviation  
end  
  
if ~exist('Date','var')  
    Date = 1:size(series(:,1));  
end  
  
if ~exist('rf','var')  
    rf =0;  
end  
  
if ~exist('alpha','var')  
    alpha =0.2;  
end  
  
%Suppress the warnings  
warning('off', 'econ:egcitest:LeftTailStatTooSmall')  
warning('off', 'econ:egcitest:LeftTailStatTooBig')
```

```
%% test for cointegration

s = zeros(size(series));
indicate = zeros(length(series),1);

for i = M: N : length(s)-N
    [h,~,~,~,reg1] = egcitest(series(i-
M+1:i, :), 'lags',1, 'alpha',alpha);
    if h == 1

        % 1. Compute residuals over next N days
        res = series(i:i+N-1,:)*[1;-reg1.coeff(2)]- reg1.coeff(1);

        % 2. enter and unwind the position
        indicate(i:i+N-1) = res/reg1.RMSE; % for plot
        if reg1.RMSE *delta > cost
            j=1;
            s(i, 1)=(res(j)/reg1.RMSE > delta)- (res(j)/reg1.RMSE < -
delta);
            i=i+1;
            for j=2:N
                if (s(i-1,1)~=0)*(res(j)*res(j-1)>=0)
                    s(i, 1)=s(i-1,1);
                else
                    s(i, 1)=(res(j)/reg1.RMSE > delta)- (res(j)/reg1.RMSE
< -delta);
                end
                i=i+1;
            end
            i=i-N;
        end
        s(i:i+N-1, 2) = - reg1.coeff(2).* s(i:i+N-1, 1);
    end
end

%% Calculate performance statistics

r = sum([0 0; s(1:end-1, :) .* diff(series) -
abs(diff(s))*cost/2] ,2);
for i=1:length(s(:,1))
    if s(i,1)==0
```

```
        r(i)=rf(i);
    end
end
sh = sqrt(252)*sharpe_lml(r,rf);
sumr=sum(r)*252/length(series(:,1));

%count the numbers of tradings
trn=0;
for i=M:length(series)
    if (s(i,1)~=0)*(s(i-1,1)~=s(i,1))
        trn=trn+1;
    end
end

%% Plot results
if nargout == 0

    ax(1) = subplot(2,1,1);
    plot(Date,[indicate,delta*ones(size(indicate)),-
delta*ones(size(indicate))])
    grid on
    legend(['Indicator'],'OIL: Over bought','OIL: Over sold',...
        'Location','NorthWest')
    title(['significant level = ', num2str(alpha,3), ', cost= ',
num2str(cost,3) ', target deviation =',num2str(delta),'standard
deviation'])
    ylabel('Indicator')
    dateFormat = 10;
    datetick('x',dateFormat);

    ax(2) = subplot(2,1,2);
    plot(Date,[s,cumsum(r)]), grid on
    legend('Position for OIL','Position for GOLD','Cumulative
Return',...
        'Location','NorthWest')
    title(['Final Return (annulized) = ',num2str(sumr*100,3),'%,
Sharpe Ratio = ',num2str(sh,3)])
    ylabel('Return (USD)')
    xlabel('Serial time number')
    linkaxes(ax,'x')
    dateFormat = 10;
    datetick('x',dateFormat);
end
```

## c. Function pairs\_imp

```
function [sh,sumr] = pairs_imp(series,delta, cost,rf,alpha,Date)
%[sh,sumr,r,trn] = pairs_imp(series,delta, cost,rf,Date)

%% Process input args

if ~exist('cost','var')
    cost = 0;
end

if ~exist('delta', 'var')
    delta = 1;
    % delta defined the targeted deviation,
    %the default targeted deviation is one standard deviation
end

if ~exist('Date','var')
    Date = 1:size(series(:,1));
end

if ~exist('rf','var')
    rf =0;
end

%Suppress the warnings
warning('off', 'econ:egcitest:LeftTailStatTooSmall')
warning('off', 'econ:egcitest:LeftTailStatTooBig')

%% test for cointegration

[~,~,~,~,reg1] = egcitest(series,'lags',1);
res = series*[1;-reg1.coeff(2)]- reg1.coeff(1);
s = zeros(size(series));
indicate = res/reg1.RMSE;

if reg1.RMSE *delta > cost
    s(1, 1)=(indicate(1) > delta)- (indicate(1)< -delta);
    for i =2:length(s)
        if (s(i-1,1)~=0)*(res(i)*res(i-1)>=0)
            s(i, 1)=s(i-1,1);
        end
    end
end
```



```
else
    s(i, 1)=(indicate(i) > delta)- (indicate(i)< -delta);
end
end
end

s(:, 2) = - reg1.coeff(2).* s(:, 1);

%% Calculate performance statistics

r = sum([0 0; s(1:end-1, :) .* diff(series) -
abs(diff(s))*cost/2] ,2);
for i=1:length(s(:,1))
    if s(i,1)==0
        r(i)=rf(i);
    end
end
end

sh = sqrt(252)*sharpe_lml(r,rf);
sumr=sum(r)*252/length(series(:,1));

%count the numbers of trading
trn=0;
for i=1:length(series)
    if (s(i,1)~=0)*(s(i-1,1)~=s(i,1))
        trn=trn+1;
    end
end
end

%% Plot results
if nargout == 0

    ax(1) = subplot(2,1,1);
    plot(Date,[indicate,delta*ones(size(indicate)), -
delta*ones(size(indicate))])
    grid on
    legend(['Indicator'], 'OIL: Over bought', 'OIL: Over sold', ...
        'Location', 'NorthWest')
    ylabel('Indicator')
    dateFormat = 10;
    datetick('x',dateFormat);
```

```
ax(2) = subplot(2,1,2);
plot(Date,[s,cumsum(r)], grid on
legend('Position for OIL','Position for GOLD','Cumulative
Return',...
       'Location', 'NorthWest')
title(['Final Return (annulized) = ',num2str(sumr*100,3),'%',
Sharpe Ratio = ',num2str(sh,3)])
ylabel('Return (USD)')
xlabel('Serial time number')
linkaxes(ax,'x')
dateFormat = 10;
datetick('x',dateFormat);

end
```