An Empirical Model Comparison for Valuing Crack Spread Options*

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Abstract

In this paper, we investigate the pricing of crack spread options. Particular emphasis is placed on the question of whether univariate modeling of the crack spread or explicit modeling of the two underlyings is preferable. Therefore, we contrast a bivariate GARCH volatility model for cointegrated underlyings with the alternative of modeling the crack spread directly. Conducting an empirical analysis of crude oil/heating oil and crude oil/gasoline crack spread options traded on the New York Mercantile Exchange, the more simplistic univariate approach is found to be superior with respect to option pricing performance.

JEL classification: G13, C50, Q40

Keywords: Crack Spread Options, Option Valuation, Cointegrated Underlyings

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1. Introduction

Crack spread options are contracts written on the price differential, or spread, of crude oil futures and refined product futures traded on the New York Mercantile Exchange (NYMEX).[1] When executing a heating oil (gasoline) crack spread call option, one enters into a long position in a heating oil (gasoline) future and, simultaneously, into a short position in a crude oil future. The execution of a put option results in the reversed position, i.e., long in the crude oil and short in the refined product.[2]

Crack spread contracts are primarily used for hedging and speculation purposes. In the process of buying crude oil and converting it into heating oil and gasoline (and some other minor derivatives), petroleum refineries become exposed to price risks on both sides of their business. Thus, the profitability of the refinery is directly linked to the price spread between these two commodity markets, which basically represents the gross margin earned. An unexpected narrowing of this spread might have substantial consequences. Therefore, the trading of crack spread futures and options provides a very suitable tool for hedging against this risk.

Typically, the gasoline output of a refinery is approximately double that of heating oil. Therefore, it is common to focus on 3:2:1 crack spreads, describing a position in three crude oil futures contracts versus two gasoline and one heating oil contracts. For illustration,

[1] The main refined products are gasoline and heating oil. The term "crack" is derived from the engineering term "cracking", denoting the process of converting crude oil into its constituent products.

[2] The payoff profile of the call contract with exercise price K is given by $max\{(0.42 \cdot F_{prod} - F_{crude}) - K, 0\}$ and analogously for the put contract. The multiplier (0.42) is a consequence of different trading conventions in the crude oil and refined product markets. Crude oil futures F_{crude} are traded in \$/Barrel, whereas heating oil and gasoline futures F_{prod} are traded in \$-Cents/Gallon. For additional institutional details, refer to the NYMEX Crack Spread Handbook, NYMEX (2000).

Figure 1 displays the evolution of the crack spread over time. Another prominent ratio is the 5:3:2 combination. To yield a flexible framework covering both these ratios, NYMEX offers two basic option contracts: a 1:1:0 and a 1:0:1 option. These two building blocks allow the hedger (or speculator) to gain the desired exposure.[3] Whereas crack spread futures are sufficient to allow the refinery to "lock in" the gross profit margin, options add additional flexibility for modern risk management strategies, e.g., buying a margin floor while preserving the upside potential. Furthermore, one can argue that a refinery that does not operate at its maximum capacity is naturally long a real option and, thus, writing an option on the spread provides a natural hedge.[4]

The pricing of crack spread options is an interesting field for both practitioners and researchers, serving as the most prominent example of an input-output spread whose respective legs are cointegrated (see Paschke and Prokopczuk (2009), Westgaard et al. (2011) or Nakajima and Ohashi (2012)). This feature is different to many other traded spread options. As a direct consequence, the crack spread itself must be mean-reverting, which is also dictated by standard microeconomic reasoning. If the spread were below its long-term equilibrium for an extended period of time, refineries would operate at a deficit, leading to the closing of refinery capacities, yielding a shortage of refined products, and thus, a rising spread. Contrarily, a spread above the equilibrium would attract new market entries, increasing the heating oil and gasoline supply, and lowering the spread.

As well as cointegration of the price level, the three underlying futures reveal, as many other financial assets do, significantly time-varying volatility and volatility clustering.

[3]Note that although true for futures, this does not entirely hold true for option contracts. As an option on a portfolio is clearly different to a portfolio of options, the positions differ in this respect.

[4]See Shimko (1994).

These effects are even stronger when considering the spread itself. Additionally, the pricing of NYMEX crack spread options are made more complicated by the fact that they are of the American style. Thus, one has to identify an optimal early exercise policy when pricing these contracts.

A fundamental question when pricing spread options is whether to follow a bivariate approach, modeling the two underlying futures explicitly, or, to use a univariate approach, modeling the spread directly. This is of special interest in the case of underlyings with a complex dependence structure, such as the crack spread considered in this paper. Duan and Pliska (2004) and Duan and Theriault (2007) argued that a bivariate approach is to be favored. They illustrated that, in the case of stochastic volatility, the cointegrating relationship should be considered for option pricing. Casassus et al. (2013) extend the theory of storage to the situation of multiple commoditiesand also argue in favour of a bivariate approach. This is opposed to Dempster et al. (2008), who suggest modeling the spread directly, especially when pricing long-term contracts.

In this paper, we empirically analyze the pricing of crack spread options. Particular emphasis is placed on the question, of whether univariate modeling of the crack spread or explicit modeling of the two underlyings is preferable. Furthermore, we study the benefits of explicitly considering the cointegration of crude oil and its refined products in the context of crack spread options pricing. Motivated by the empirical characteristics of the underlyings, we follow a GARCH-based stochastic volatility approach and empirically analyze the pricing performance of univariate vs. bivariate models.

Our paper contributes to the literature in various ways. Firstly, to the best of our knowledge, we are the first to compare the empirical option pricing performance of univariate vs. bivariate models for cointegrated underlyings. This issue is of high relevance, as both approaches have their merits, and it is not obvious which is to be preferred. Secondly there exists, to the best of our knowledge, except of the work of Duan and Theriault (2007), no other study considering the empirical pricing of crack spread options. In contrast to Duan and Theriault (2007), who estimated their model using only historical asset prices, we estimate our models implicitly using option and asset prices. The latter approach has been proven to be superior when valuing options in a GARCH volatility framework.[5] Also, they do not consider the possibility to model the crack spread directly via univariate approaches, hence leaving open the question as to which modeling approach is generally to be favoured; instead, the "horse race" in the empirical part of this paper tries to find an answer to this question. Thirdly, we analyze the importance of the inclusion of maturity and rollover effects for both the univariate and bivariate approaches, thereby taking up important characteristics prevalent both in the dynamics of the underlyings as well as in commodity markets generally.

Our results show that the more simplistic approach of modeling the crack spread directly yields lower pricing errors than the more complicated bivariate model. This holds for the in-sample and, most importantly, for the out-of-sample analysis and is notable as the univariate approach is also much faster computationally. Moreover, the inclusion of maturity and rollover effects clearly improves the pricing accuracy of both models substantially.

[5]See Barone-Adesi et al. (2008).

The rest of this paper is organized as follows. Section 2 provides a brief literature review. Section 3 discusses the advantages and disadvantages of the univariate vs. bivariate modeling approach, outlines the models used in the empirical study, and describes the valuation and estimation approach. Section 4 describes the crack spread options data set. Section 5 contains the empirical results of our study. Section 6 concludes.

2. Literature Review

Previous studies have mainly considered the pricing of spread options in general.[6] An initial point frequently cited is the work of Margrabe (1978), providing a closed-form solution for the special case of constant volatility and zero exercise prices. Shimko (1994) and Poitras (1998) both considered the pricing of spread options in a constant volatility framework. The former suggested using two correlated geometric Brownian motions, which only allows for an approximate solution. Poitras (1998) proposed to consider arithmetic Brownian motions and is thus able to derive closed-form solutions. However, the possibility of negative prices results in a significant shortcoming of this approach. Alexander and Scourse (2004) proposed modeling the underlying spot prices by mixtures of normal distributions, allowing for heavy tails and the capability of fitting the volatility smile and correlation frown. Kirk (1995), and more recently, Alexander and Venkatramanan (2011), proposed different analytical approximations of the spread option value, again in a constant volatility framework.

Studies considering the pricing of crack spread options are few. Mbanefo (1997)

[6]See Carmona and Durrleman (2003) for an extensive overview.

discussed the necessity of considering the complex dependence structure of the underlyings, resulting in the mean-reversion of the crack spread, without explicitly addressing the issue of cointegration. Alexander and Venkatramanan (2011) illustrated their pricing approximation using a small sample of NYMEX crack spread options. Dempster et al. (2008) proposed modeling the crack spread directly in a continuous time framework with constant volatility. Although deriving (European) option pricing formulas, an empirical study was not performed. As such, the only empirical study on crack spread option pricing we are aware of is Duan and Theriault (2007). The bivariate GARCH model with cointegrated underlyings of Duan and Pliska (2004) was adopted to value crack spread options. Estimating the model parameters from historical asset prices, the authors concluded that pricing errors remain relatively large.

3. Option Valuation and Estimation

3.1. Theoretical Considerations

Before describing the univariate and bivariate GARCH pricing models used in the empirical study, we briefly discuss the advantages and disadvantages of these two opposing approaches for pricing crack spread options. Initially, the univariate modeling of the crack spread as a (conditionally) log-normal asset might seem too simplistic for assets with a complex dependence structure. Moreover, several criticisms exist regarding the univariate modeling of spread processes. These include:

- Most importantly, univariate modeling approaches neglect any influence of the underlying assets' correlation on the distribution of the spread (Alexander and Scourse

(2004)). Additionally, the assets might be cointegrated (Duan and Pliska (2004)).

- If one chooses to model log prices, the univariate modeling approach is also unable to capture negative spreads. Whether this is a severe limitation or can be ignored in practice, depends very much on the kind of spread that is modelled.

- Garman (1992) pointed out that postulating a log-normally distributed spread automatically assumes the magnitude of spread fluctuations to be proportional to prevailing spread levels. This is an implicit assumption that has yet to be empirically confirmed.

- Moreover, the effect of a negative *vega*, often mentioned as one remarkable peculiarity of spread options (Duan and Pliska (2004)), cannot be reproduced by univariate log-normal modeling at all.

- In the case of options on (spot) commodity spreads, it is important to note that little is currently known about the inherent concept of the net convenience yield of the spread, which is rather difficult to model, let alone empirically validate (Mbanefo (1997) or Garman (1992)). Even though we need not explicitly model the net convenience yield of the crack spread in our case of options on futures spreads, we implicitly incorporate assumptions on the nature of the spread convenience yield.

- Carmona and Durrleman (2003) argued that, from a risk management perspective, the univariate modeling of the crack spread comes at the cost of a considerable loss of informational content in terms of the respective *Greeks*: while one variable may suffice for pricing purposes, the second and higher order derivatives thereof cannot reflect the same accuracy as in the case of bivariate modeling.

The first argument is the most important point with respect to option valuation. At first sight, it might seem obvious that both the correlation and the cointegration structure should be considered explicitly. However, one should keep in mind that both dependence concepts will also have an influence in a univariate context. As the crack spread can be considered as a portfolio of the two underlying futures, its volatility will be implicitly determined by the volatility of the two underlyings and their correlation. Furthermore, the cointegration relationship will also manifest itself implicitly in the spread volatility as it affects the underlying futures.

Although being more explicit about modelling the correlation and cointegration structure, the bivariate models' richer parameterization imposes further burdens on the practicability, since more computational time is needed for both, parameter estimation, especially when estimating using option data, and pricing simulations. Finally, we do not focus on the hedging-related criticisms as the focus of our study is on the models' pricing performance, rather than on hedging performance. That being said, modelling the crack spread as a conditionally log-normally distributed asset may seem justified in the context of options pricing.

3.2. Univariate Modeling

In their paper, Dempster et al. (2008) propose to model the spread process directly by means of a two-factor mean-reverting model with constant volatility, which offers a number of advantages: (i) an adequate reflection of the mean-reverting nature of the crack spread in the long-run as well as (ii) the availability of analytical formulae for European options. In the context of our empirical study, however, different requirements had to be met. First, as will be seen in the next section, the majority of traded crack spread contracts in our sample have short- to mid-term maturities so that our univariate modeling approach of choice should also be adapted to shorter timeframes during which the mean-reversion property of the spread may be less distinct than in the long-term. Second, the crack spread options in our sample are of American-type, allowing for the convenient combination of simulation-based pricing approaches that at the same time offer the possibility for modeling stochastic volatility, which is more important for shorter timeframes than the long-run case examined by Dempster et al. (2008).

The typical GARCH option pricing model was first proposed by Duan (1995) in its original form employing the standard GARCH dynamics. Numerous papers have proposed various extensions of the basic model. Empirical studies such as Christoffersen and Jacobs (2004b) and Stentoft (2005) demonstrated that the asymmetric NGARCH-type volatility dynamics developed by Engle and Ng (1993) performs well in pricing a variety of options. We therefore decided to use this type of volatility dynamics when modeling the crack spread directly.[7]

Let $F^s_{t,T}$ be the price of the crack spread with maturity T formed by the value of the corresponding input and output futures at time t. We assume the spread to obey the

[7] The bivariate model of Duan and Pliska (2004) also follows an NGARCH approach. This yields another motivation for using this type of volatility dynamics, as we wish to keep things comparable.

following dynamics under the risk-neutral measure $\mathbb{Q}$:

$$\ln(F^s_{t,T}) - \ln(F^s_{t-1,T}) = -\frac{1}{2}\sigma_t^2 + \sigma_t \xi_t, \quad (1)$$

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \left(\xi_{t-1} - \lambda - \theta\right)^2, \quad (2)$$

where σ_t is the volatility of the spread at time t; λ is the constant unit risk premium; θ captures the asymmetric reaction of the variance process to negative and positive shocks.[8] ξ_t is a conditionally standard normally distributed random variable. Furthermore, we assume $\beta_0 > 0$, $\beta_1 \geq 0$ and $\beta_2 \geq 0$. To ensure stationarity of the volatility process, we additionally require $1 - \beta_1 - \beta_2 \left[1 + (\lambda + \theta)^2\right] > 0$. Within the class of univariate modeling approaches considered in this study, this model will henceforth be referred to as **U-NG**.

Note that we directly model the dynamics under the risk-neutral measure $\mathbb{Q}$ and do not explicitly specify the dynamics under the physical measure $\mathbb{P}$, making a change of measure dispensable. This approach follows the spirit of Barone-Adesi et al. (2008), who argued that the approach of specifying and estimating the dynamics under $\mathbb{P}$ and performing a change of measure to recover the risk-neutral dynamics usually yields inferior empirical results compared to calibrating the pricing dynamics directly under $\mathbb{Q}$.

Since one of the special characteristics of commodity futures markets is what has become known as the Samuelson or maturity effect (see Samuelson (1965)), we also consider a slight modification of the **U-NG** model, labeled **U-NG-M** (M for maturity effect). The maturity effect describes the effect of rising volatility levels of futures returns with the

[8]Note that since we will work only under the risk-neutral measure, we can not identify λ and θ separately, but only the sum of the two. We formulate the model in this way, to keep in general and also applicable to the physical measure for different practical applications.

respective contracts approaching their maturity. Among the various explanations for this inverse relationship between variance and days to maturity, a stronger increase in the news flow around maturity furthering price discovery and less supplier flexibility with respect to shocks in demand are usually brought forward.

Another less prominent effect on commodity futures volatility around maturity relates to investors rolling over their positions into longer-term contracts with these periods of contract-switching adding to the already increased levels of volatility due to the maturity effect. To our knowledge, the phenomenon of volatility increases due to contract switching has only been examined by Chatrath and Christie-David (2004), who identified switching periods for several commodities by means of correlation analysis between maturity and several measures of trading volume. Theriault (2008), in turn, identified these periods for the commodity futures considered in this study, namely crude oil, heating oil, and gasoline. In order to take these effects into account, the $\mathbb{Q}$-dynamics for the volatility of the **U-NG-M** model is specified as

$$\sigma_t^2 \;=\; \beta_0 (T-t)^\gamma + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \left(\xi_{t-1} - \lambda - \theta\right)^2 + \eta SW_t, \tag{3}$$

where the term $(T-t)^\gamma$ (with $\gamma < 0$) captures the maturity effect and SW_t, a dummy variable, is set to one during the switching period and zero otherwise. Following Theriault (2008), the switching period for crude oil (heating oil respectively gasoline) is assumed to be between 24 and 32 (25 and 39 respectively 25 and 38) days to maturity. For the spread as a whole, we assume a combined switching period between 24 and 39 days to maturity.[9]

3.3. Bivariate Modeling

The family of nested bivariate models implemented in this study have their origins in a direct application of the option pricing model for cointegrated underlyings of Duan and Pliska (2004). These models were adapted for the case of crack spread options by Duan and Theriault (2007).

Starting with the general model of Duan and Pliska (2004), and assuming that the relationship between the spot prices S_t and corresponding futures prices F_t with net convenience yield y, risk-free rate r, and maturity T at time t can be described as $S_t e^{(r-y)(T-t)} = F_{t,T}$. We can now derive the pricing model for two cointegrated underlying futures forming the crack spreads between crude and heating oil and crude oil and gasoline, respectively.

Again, we consider the respective dynamics directly under the pricing measure $\mathbb{Q}$ (see the previous section for a discussion on this issue). The model proposed by Duan and Pliska (2004) and Duan and Theriault (2007) can be considered a multivariate NGARCH specification. More precisely, we assume the following dynamics under the

[9] Theriault (2008) identifies contract switching behavior by investigating open interest, trading volume and the ratio of these metrics, which is also applied by Chatrath and Christie-David (2004).

pricing measure $\mathbb{Q}$:

$$\ln\left(\frac{F_{i,t,T_i}}{F_{i,t-1,T_i}}\right) = -\frac{1}{2}\sigma_{i,t}^2 + \sigma_{i,t}\xi_{i,t} \tag{4}$$

$$\sigma_{i,t}^2 = \beta_{i,0} + \beta_{i,1}\sigma_{i,t-1}^2 + \beta_{i,2}\sigma_{i,t-1}^2\left(\xi_{i,t-1} - \lambda_i - \theta_i - \delta_i\frac{z_{t-2}}{\sigma_{i,t-1}}\right)^2 \tag{5}$$

$$z_t = a + bt + c\ln(F_{2,t,T_2}) + \ln(F_{1,t,T_1})\ , \tag{6}$$

where F_{i,t,T_i}, $i = 1, 2$, is the futures price at time t with maturity T_i; δ_i can be interpreted as the ith asset's co-integration premium (Duan and Pliska (2004)); z_{t-2} captures the cointegrating relationship between the futures and $\xi_{i,t} = \varepsilon_{i,t} + \omega_{i,t}$ is distributed as follows:[10]

$$\begin{bmatrix}\xi_{1,t}\\ \xi_{2,t}\end{bmatrix} \overset{\mathbb{Q}}{\sim} N\left(0, \begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}\right) \quad \text{conditional on } \mathcal{F}_{t-1}. \tag{7}$$

Over the course of this paper, the bivariate cointegrated GARCH model is referred to as **B-NGC**. As an extension, and similar to the univariate case, the bivariate model **B-NGC-M** captures the maturity and switching effects by modifying the $\mathbb{Q}$-volatility dynamics in Equation (5) to:

$$\sigma_{i,t}^2 = \beta_{i,0}(T_i - t)^{\gamma_i} + \beta_{i,1}\sigma_{i,t-1}^2 + \beta_{i,2}\sigma_{i,t-1}^2\left(\xi_{i,t-1} - \lambda_i - \theta_i - \delta_i\frac{z_{t-2}}{\sigma_{i,t-1}}\right)^2 + \eta_i SW_{i,t},$$

with the additional parameters and switching periods for the respective futures contracts defined as in Equation (3).

As there exists a whole battery of benchmark models nested in **B-NGC**, this additional

[10] $\omega_{i,t}$ is $\mathcal{F}_{t-1}$-measurable, and thus, a locally deterministic process with $\omega_{i,t} = \lambda_i + \frac{\delta_i z_{t-1}}{\sigma_{i,t}}$.

model offers an interesting opportunity to examine the implications of the modeling choices made so far in detail: setting all parameters but $\beta_{i,0}$ and ρ to zero yields the standard bivariate constant volatility model (**B-CV**), whereas, the simple bivariate GARCH model **B-NG** only restricts the parameters δ_i, γ_i and η_i to zero. This family of models allows for a comparison of pricing performance between the cases of constant vs. GARCH volatility, the inclusion of error correction terms, and the consideration of volatility effects of futures contracts when approaching maturity.

Another interesting point that can be made in this respect directly draws upon Duan and Pliska (2004) who stated that using a GARCH model without the error correction terms to price options with cointegrated underlyings clearly implies using a misspecified model. Due to the nature of the GARCH option pricing model and following its inherent principle of local risk-neutralization, the cointegration premium of the mean under $\mathbb{P}$ must always enter the volatility dynamics when volatility is stochastic. Ignoring this (technical) effect when pricing options on cointegrated assets is clearly inconsistent with the implications of this misspecification on pricing errors being examined in Section 5.

3.4. Valuation Algorithm

When dealing with option pricing models in a GARCH framework, it is common practice to rely on simulation based valuation approaches. This is the case because, on the one hand, the final asset distribution is not known in closed form, but, on the other hand, the discrete nature of the GARCH framework makes simulation-based approaches straightforward to implement. However, as crack spread options are of the American-type, a simple simulation-based approach is not sufficient, as one needs to simultaneously determine the optimal exercise policy during the option's life by deciding upon the optimal stopping time:

$$\sup_{\tau \in \mathcal{T}} E^{\mathbb{Q}} \left[e^{-r\tau} h(X_\tau) \right], \tag{8}$$

where h is a payoff function, r the risk-free rate, $\mathcal{T}$ denotes the class of admissible stopping times with values in $[0; T^{option}]$ and X_τ is the state vector comprising the state variables at time τ. The state vector in Equation (8) includes the spread and its conditional volatility in the univariate model and two futures prices and their conditional volatilities in the bivariate model.

In this study, we apply the least squares monte carlo algorithm (LSM) developed by Longstaff and Schwartz (2001) to price American options by simulation, as it is especially well suited to value contracts that depend upon the realizations of several stochastic factors, i.e., if X_τ has many elements. The basic idea behind this approach is to compare the value of immediate exercise with the conditional expected continuation value which, in turn, is obtained by the fitted value of a least squares regression of the discounted payoffs obtained during the simulation, following an optimal exercise policy at all future dates on a set of basis functions of X_τ.[11] We combine the LSM algorithm with additional techniques developed for simulation-based valuation methods to enhance the pricing performance. First, we employ the method of empirical martingale simulation proposed by

[11]For more details, see Longstaff and Schwartz (2001) or Stentoft (2005).

Duan and Simonato (1998), ensuring that the simulated sample paths of the futures prices retain the martingale property. Second, we also apply antithetic variates as a variance reduction technique in combination with a simple batching technique of the paths to be simulated.[12] Altogether, these modifications allow us to simulate 50,000 paths when valuing options in a reasonable computation time, yielding a sufficient numerical precision.[13]

3.5. Estimation Approach

Estimating parameters for GARCH option pricing models can basically be performed in two different ways. First, it is possible to estimate all parameters (including the market price of risk) from a historical time series of the underlying under the physical measure, and, afterwards, transform these parameters to the risk-neutral measure. This approach is frequently applied in the literature, especially when dealing with American options as it is quite simple and computationally fast.[14] However, as argued by Barone-Adesi et al. (2008), it has been shown empirically that this approach leads to a rather poor pricing performance. This is not surprising, as one has to employ a rather lengthy time series and thus, use information dating far back in the past.

The second, and "dominating" approach (Barone-Adesi et al. (2008)), is to use option data and implicitly estimate the parameters directly under the risk-neutral pricing measure by minimizing a pre-specified loss function capturing the discrepancies between a sample

[12] This helps to avoid exceeding memory space by allocating the simulation paths into batches that are handled separately by the valuation algorithm.

[13] We also considered applying a computationally faster, but less precise way to deal with the American feature of the options considered: we first estimated the early exercise premium using the approximation of Barone-Adesi and Whaley (1987) and then transformed the entire data set into pseudo European observations as suggested by Trolle and Schwartz (2009). This approach, however, yielded substantially inferior results when compared to the LSM algorithm.

[14] See, e.g., Stentoft (2005) or Duan and Theriault (2007).

of observed option prices used for estimation and the corresponding prices, as computed by the valuation model. We follow this second methodology and estimate the models' parameters implicitly employing a framework similar to the one considered by Lehar et al. (2002). The only exception are the cointegrating parameters a, b, and c. As these represent a long-run relationship they cannot be estimated implicitly from short term options. We therefore follow Duan and Pliska (2004) and estimate the cointegrating parameters in a first step using OLS and ten years of historical data preceding our study period. The estimated values are then hold constant throughout the rest of the analysis.

As the number of options available on each day is not sufficient to estimate all model parameters (especially for the bivariate models), we employ a sliding window approach and minimize the pricing errors in the sample interval. The length of the sample window is chosen to be one week, i.e., five business days. We decided to use the relative root mean squared error (RRMSE) as loss function, to give out of the money options sufficient weight. In summary, our estimation approach uses the following criterion function:

$$\min\ RRMSE\left(\boldsymbol{\vartheta}\right) = \sqrt{\frac{1}{\sum_{t=1}^{5} n_t} \sum_{t=1}^{5} \sum_{i=1}^{n_t} \left(\frac{P_{t,i}^{Mod} - P_{t,i}}{P_{t,i}} \right)^2}, \tag{9}$$

where $\boldsymbol{\vartheta}$ denotes the vector of parameters to be estimated, $t = 1, ..., 5$, are the trading days of the estimation (in-sample) period, and n_t is the number of option prices used for estimation at day t; $P_{t,i}^{Mod}$ represents the i-th crack spread option price given by the pricing model and $P_{t,i}$ is the i-th actually observed price. Since the option prices used for the parameter estimation are observed on more than one trading day, the volatility for every futures price process considered at the respective dates needs to be linked using the time series of the contract's returns. The locally deterministic nature of the GARCH processes enables us to implement a simple volatility updating rule, as outlined by Christoffersen and Jacobs (2004b).

The minimization problem is solved using numerical optimization procedures and therefore represents in-sample a mere fitting exercise. Yet, for the entire sample of heating oil and gasoline crack spread options, we use the estimated model parameters to test the out-of-sample pricing performance of the models for the subsequent five days, thus, only employing historical information. Finally, the time window is shifted to cover the next five days for which again option prices are collected to re-estimate the models.

It is worth mentioning that the estimation of the option implied parameters is carried out using directly the least squares Monte Carlo algorithm as pricing procedure, as described in the previous section, however, with a reduced number of 5,000 sample paths. While it has often been considered infeasible to estimate implied parameters directly from American options (see, for example, Broadie and Detemple (2004)), increasing computational efficiency is making it possible that this procedure is successfully carried out.[15] One should note that this approach is not suitable if pricing has to be conducted in real time, as in daily practice. However, as our main objective is to compare the general suitability of different pricing models for pricing crack spread options, we choose to employ this superior estimation algorithm to reduce pricing errors resulting from inferior parameter estimates. By using 5,000 paths for the simulations in the estimation procedure,

[15]Hsieh and Ritchken (2005) proceeded in a similar way, although for European options.

the estimation of one parameter set and one model specification takes approximately 1-2 hours for the univariate case and 3-6 hours for the bivariate case.

We would also like to mention that we experimented with estimating the parameters using the historical time series of the underlying under $\mathbb{P}$ (the first method), as was conducted by Duan and Theriault (2007). However, it became abundantly clear that the latter approach of implicit estimation is by far superior, supporting the statements of Barone-Adesi et al. (2008). We refrain from reporting the results of the historical estimation approach to maintain the focus on the most interesting aspects of the study.

4. Option Data

The NYMEX heating oil and gasoline crack spread options are the most liquidly exchange traded crack spread options available. We obtained end-of-day prices for heating oil and gasoline crack spread options directly from the NYMEX. Prices for these options are recorded at the end of the open-outcry session of each trading day.[16] Trading of the underlying futures contracts, however, is supported by an electronic platform enabling trades to be conducted nearly 24 hours a day.

The sample of heating oil crack spread call and put option prices used in our empirical study comprises price observations, covering the period from July 2004 to November 2006. The sample of crack spread contracts related to gasoline is based on price observations recorded between December 2003 and January 2006. The uneven and only partially overlapping periods were selected given the significant fluctuations in trading volume.

[16]The data can be purchased from NYMEX. For details see www.nymex.com.

The chosen windows cover the most active years, which is especially important for our estimation methodology using option implied information, which generally necessitates a sufficiently large number of traded contracts.[17]

We applied a number of exclusion criteria to our option price sample to obtain prices of options with sufficient liquidity and open interest: we only considered option contracts in the moneyness interval $0.85 \leq m \leq 1.15$, with moneyness m defined as $m = F^s/K$ for crack spread futures price F^s and strike price K. As a matter of definition, we classify contracts with $0.98 \leq m \leq 1.02$ as at-the-money (ATM) contracts, all calls (puts) with $m < 0.98$ ($m > 1.02$) as out-of-the money (OTM) and those with $m > 1.02$ ($m < 0.98$) as in-the-money (ITM), respectively. Concerning the days to maturity (DTM) of the options, we define $DTM \leq 21$ as being short-term, $21 < DTM \leq 60$ as being mid-term and $DTM > 60$ as being long-term crack spread options. Note that we only count business days.

We also excluded all option contracts violating elementary no-arbitrage boundaries.[18] Finally, following the exclusion criteria of Bakshi et al. (1997), we excluded all options with a market price below \$ 3/8 and with $DTM < 6$ or $DTM > 120$ from the sample to reduce liquidity and price discreteness related biases. The proposed moneyness and maturity classification yields nine categories for which we will present the empirical results. After applying these liquidity filters the data sets comprise 4,100 heating oil crack spread options and 5,400 gasoline crack spread options.

[17]The assessment that the chosen sample periods have been periods of active trading is based on private conversations with traders.

[18]In order to avoid stale data, contracts trading below intrinsic value, for example, were excluded from our sample.

Table 1 and Table 2 provide a summarizing overview of the two crack spread options samples. For the heating oil crack spread options, the number of calls and puts are of equal size, whereas the number of calls are higher for the gasoline crack spread sample. The number of observations is the highest for mid-term maturities. Regarding the moneyness classification, one can observe the largest number of option prices in the out-of-the-money bracket, followed by the in-the-money bin. Not surprisingly, the average prices increase with maturity and moneyness in every case. The average price of all heating oil crack spread options is \$1.87, and \$1.32 for the gasoline crack spread options.

5. Empirical Results

5.1. Preliminary Data Analysis

Before conducting the option pricing study we have analyzed the time series properties of the underlyings. As these properties are generally well established in the literature, we only provide a brief summary.[19] The returns of crude oil, heating oil, and gasoline exhibit a volatility which is similar to equity returns, between 20 % and 30 %. Crude oil and heating oil returns are weakly skewed to the left, whereas gasoline is skewed to the right. All returns exhibit pronounced excess kurtosis, and thus, in every instance, the Jarque-Bera test rejects the null of normality. Analyzing the squared residuals documents strong non-zero autocorrelation, i.e. GARCH effects. Applying the augmented Dickey-Fuller test provides evidence that the return series are stationary, whereas the price series are not. Both the Johansen and the Philips-Oularis test strongly support the hypothesis

[19]Detailed results are available upon request.

of cointegration between the three markets.

Since the main objective of this study is to investigate whether univariate modeling or bivariate modeling of the crack spread is preferable with respect to options pricing, we also analyzed the crack spread as a univariate asset. The volatility of the spread is considerably higher than the volatility of its components, with annualized values between 80 % and 140 %. Kurtosis is again high, yielding to the conclusion that normality must be rejected. Furthermore, strong GARCH effects are detected. Finally, the augmented Dickey-Fuller provides evidence for the stationarity of the crack spread series, which is in agreement with the cointegration between the individual series.

5.2. Parameter Estimates

All model parameters are re-estimated on a weekly basis, always considering all options available during the considered week (after applying the data filters discussed in the data section). Instead of listing the implied parameter estimates for all weekly re-estimations for both heating oil and gasoline crack spread options, we present a summarizing overview of some descriptive statistics. This information is presented in Table 3 for the case of crude oil/heating oil options and Table 4 for the case of crude oil/gasoline options. The parameters for the cointegration relationship are not included in the tables. For heating oil (gasoline) they were estimated as follows: $a = 0.7776$ (1.2074), $b = -3.1 \cdot 10^{-5}$ ($-1.2 \cdot 10^{-5}$), and $c = -0.9336$ (-1.0262), all significant at the 1 % level.

Although the combinations of these statistics only provide a limited picture of the estimates, one can observe the point that their means and standard errors indicate that there does exist some significant weekly fluctuations among the estimates that cannot be captured if these parameters are held constant, like in the case of an historical estimation approach. Notably, we found a significantly lower persistence of the resulting GARCH processes for any model class, which contrasts the relatively high levels of persistence implied by the historical estimates (not reported). Without elaborating on this point in more detail, we therein may see some evidence for the argument of Lamoureux and Lastrapes (1990), who claimed that relatively high levels of persistence for historically estimated GARCH processes could be attributed to non-constant parameter levels and, ultimately, to structural breaks in the data generating process that is not adequately reflected in the historical estimation methodology.

5.3. Option Pricing Performance

Before analyzing the pricing errors of the models, we briefly introduce the error measures that we will be focusing on. They include:

- the mean percentage error $MPE = \frac{1}{N}\sum_{i=1}^{N}\frac{P_i^{Mod}-P_i}{P_i}$ and the median percentage error (MdPE),
- the mean absolute percentage error $MAPE = \frac{1}{N}\sum_{i=1}^{N}\frac{|P_i^{Mod}-P_i|}{P_i}$ and the median absolute percentage error (MdAPE),
- the root mean squared error $RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(P_i^{Mod}-P_i\right)^2}$ and
- the relative root mean squared error $RRMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{P_i^{Mod}-P_i}{P_i}\right)^2}$.

One should note that, following the arguments of Christoffersen and Jacobs (2004a), the relative root mean squared error is the most appropriate metric to measure the models' pricing performance as it has been used for the parameter estimation. However, especially for the out-of-sample pricing performance, we also consider the other error measures to analyze the robustness of the results.

Table 5 contains the in-sample pricing errors for both heating oil (upper part) and gasoline (lower part) crack spread options for the two univariate and four bivariate model specifications and all of the six considered error metrics. As discussed above the last column containing the RRMSE contains the most important information.

The in-sample results indicate that the pricing errors are of similar magnitude for both contract types considered. It ranges between 11.69 % and 19.74 % for the case of heating oil, and 10.34 % and 24.74 % for the case of gasoline. For both underlyings, the highest in-sample error is observed for the bivariate constant volatility model, the lowest in-sample error is observed for the univariate **U-NG-M** model. Whereas the first result is not surprising, the latter is, to some extent, unexpected, as the bivariate models include more parameters. However, one should keep in mind that the univariate models are not nested in the bivariate ones and thus, more parameters do not need to yield a superior fit.

Table 6 reports the out-of-sample pricing errors. In most instances, the out-of-sample performance is slightly worse than the in-sample performance, which is as expected. It is rather interesting that the differences are quite small, however. Out-of-sample errors exceed their in-sample counterparts by only 2-3 percentage points in most instances as can be seen for MAPE, MdAPE, and RRMSE.

Comparing the univariate and bivariate models illustrates that the univariate models perform better in most cases. The out-of-sample RRMSE of the model **U-NG-M** yields 13.71 % for heating oil and 13.17 % for gasoline contracts, compared to 16.30 % and 14.55 % for the bivariate model including cointegration and maturity effects **B-NGC-M**. In terms of RMSE, this relates to \$0.23 and \$0.17 compared to \$0.29 and \$0.19. Thus, our results illustrate that the more simple approach of modelling the crack spread directly empirically outperforms the more complex approach to model the two underlyings explicitly (when including maturity effects).

For both model variants, the effect of allowing for the explicit modeling of maturity effects seems to notably pay off in terms of pricing performance: those models with maturity effects dominate the respective models without these additional parameters. Within the bivariate model specifications, one can observe a clear ordering of model performance with increasing complexity. The constant volatility model **B-CV** performs worst, followed by the model neglecting cointegration **B-NG**. Thus, one can conclude that if one decides to follow a bivariate approach, although the univariate model (including maturity effects) has shown to outperform for our data set, one should clearly consider the cointegration relationship of crude oil, heating oil, and gasoline.

In the following we examine the previous findings in more detail. A breakdown of errors by moneyness and maturity is contained in Table 7 for heating oil crack spread options and in Table 8 for the gasoline version of the contract. Altogether, these tables allow us to thoroughly analyze the implications of integrating maturity effects into the models: in both uni- and bivariate cases, a comparison of the respective models with and without maturity effects reveals that short-term options are less underpriced and long-term options are less overpriced (or even slightly underpriced, too) if these effects are integrated. This is attributed to the term $\beta_{i,0}\,(T_i - t)^{\gamma_i}$ that not only effects a gradual increase in volatility for shorter maturities but, correspondingly, also leads to a decrease in volatility levels for longer maturities, whereas for the models **B-NGC** and **U-NG**, higher maturities (monotonously) have underpricings turn into overpricing.

As to the performance of the candidates with the best aggregate pricing performance, the breakdown by moneyness and maturity reveals that in most categories, the **U-NG-M** model performs best. Yet, there does not seem to be a general pattern/ranking indicating for which maturities or levels of moneyness univariate modeling might not be preferable to bivariate approaches; instead, we observe that, over all categories, the differences in pricing errors between the two models remain roughly at the same level. Moreover, and somewhat counterintuitive, this is not found for short-term options where **U-NG-M** seems to outdo **B-NGC-M** more than in the other categories.

5.4. Analysis of Pricing Errors

We conduct an error regression to identify and explore the potential sources for and possible structures of systematic pricing errors to which the implemented models might be susceptible. To accomplish this, let $PE_i(t)$ denote the i-th option's percentage pricing error on day t. For each of the considered pricing models, we run the following regression:

$$\begin{aligned} PE_i(t) &= \beta_1 + \beta_2 DTM_i + \beta_3 DTM_i^2 + \beta_4 m_i(t) + \beta_5 m_i(t)^2 + \beta_6 OI_i(t) \\ &\quad + \beta_7 \sigma_{CL}(t) + \beta_8 \sigma_{CL}^2(t) + \beta_9 \sigma_{prod}(t) + \beta_{10} \sigma_{prod}^2(t) + \beta_{11} \rho(t) \\ &\quad + \beta_{12} PUT_i(t) + \beta_{13} BACKWARD(t) + u_i(t) \end{aligned} \tag{10}$$

where DTM_i is the maturity of option i (in days), $m_i(t)$ is the moneyness of option i, $OI_i(t)$ is the open interest in option i, $\sigma_{CL}(t)$ and $\sigma_{prod}(t)$ are the 20-day standard deviations of the log returns for the respective crude oil or output product futures, and $\rho(t)$ is the correlation between these futures. $PUT_i(t)$ and $BACKWARD(t)$ are dummy variables taking on the value of one (zero) if puts (calls) are priced and for distinguishing those pricing errors for which the crack spread futures term structure is in backwardation or contango. As a matter of simplification, we consider the difference between the 1-months and those crack spread futures with the longest maturity available to determine the values for the variable $BACKWARD_t$.

Table 9 reports the regression results. The t-statistics reported in brackets for each coefficient estimate are based on the White heteroscedasticity-consistent adjustment of standard errors. First, one can note that regardless of the model and the underlying, almost all independent variables have statistically significant explanatory power for the percentage pricing errors. Consequently, the pricing errors of each model have some biases related to moneyness, days to maturity, and the other considered variables. However, the sign of the bias and its magnitude differ substantially across the models.

The coefficients of days to maturity (DTM) and moneyness (m) are nearly always significant. Interestingly, the significance of the volatility coefficients diminishes for the **B-NGC-M** model, indicating that the individual underlying volatilities are sufficiently well modeled. The correlation coefficients, although, remain significant. The PUT dummy variable coefficients is primarily negative, indicating a smaller pricing error for put options than for calls. The magnitude is, however, low. The backwardation dummy coefficients also remain significant, with the changing signs between models and underlyings not leading to any clear conclusions.

Even though the coefficients for most models' relative pricing errors are mostly statistically significant, the collective explanatory power varies substantially across models. In the case of crude oil/heating oil options, the adjusted R^2 is with 29 % the highest for the **U-NG**, and the lowest for the **B-NGC-M** and **U-NG-M** models, yielding only 3 % and 7 %, respectively. A similar picture is painted for the crude oil/gasoline options. The pricing error regressions for the **U-NG** and **B-CV** model yield a high explanatory power of 39 %, with drops to 5 % and 6 % for the **B-NGC-M** and **U-NG-M** models. From these observations, one can clearly draw the conclusion that the inclusion of maturity (and switching) effects substantially reduces the systematic pricing errors.

The results of the previous section provided evidence for the superior pricing performance of the simple **U-NG-M** model. In terms of systematic pricing errors, the highly parameterized model **B-NGC-M** consistently outperforms. Thus, the **B-NGC-M** option pricing model seems to be either biased by a factor not included in the regressions, or, what we consider more likely, prone to a higher unsystematic pricing error. It seems quite likely that this unsystematic error is induced by the high number of parameters of the **B-NGC-M** and thus, estimation errors.

6. Conclusions

In this paper, we considered the pricing of crack spread options. Two different approaches to this problem are distinguished. First, it is possible to assume a dynamics for the spread itself, avoiding specifying the dependence structure of the underlyings explicitly. Second, one can model the two underlying assets separately. In a GARCH-type volatility framework, the pricing performance of these two competing approaches was compared using a data set of crack spread option prices from NYMEX. The empirical results revealed that, when including maturity effects, the univariate and more simplistic approach yielded a better pricing performance, mainly due to a lower unsystematic pricing error.

We conclude the paper by outlining some future research ideas. As briefly discussed, it is unclear, whether a univariate approach is sufficiently suited when the model is employed for risk management purposes, not only option pricing. Thus, it would be interesting to compare the two approaches in this respect, i.e., their hedging performance. A second idea for future research is to extend the empirical evidence to other spread options of (potentially) cointegrated underlyings.[20]. The clean spark spread would be one such case. Here the role of carbon emission certificates would be crucial (see Benz and Trueck (2009) and Carmona et al. (2012)). In the case of the clean or also dark spark spread, the underlyings are less correlated than they are in the case of the crack spread. Thus, there are

[20]In this context we would also like to mention that the superiority of the univariate model might disappear, once one considers a spread that becomes frequently negative, as in the univariate model, the spread cannot take on negative values (see discussion in Section 3.1.)

reasons to suspect that the bivariate model might work better than the univariate model, as other factors might dominate the influence of the correlation.[21] The CBOE crush spread, a portfolio of soybean futures on the one side, and soybean oil and soybean meal futures on the other side is another interesting candidate. In contrast to the NYMEX crack spread, the CBOE crush spread is written on all three futures at the same time. Thus, one has to consider a trivariate system of underlyings.

[21] We thank a referee for pointing this out to us.

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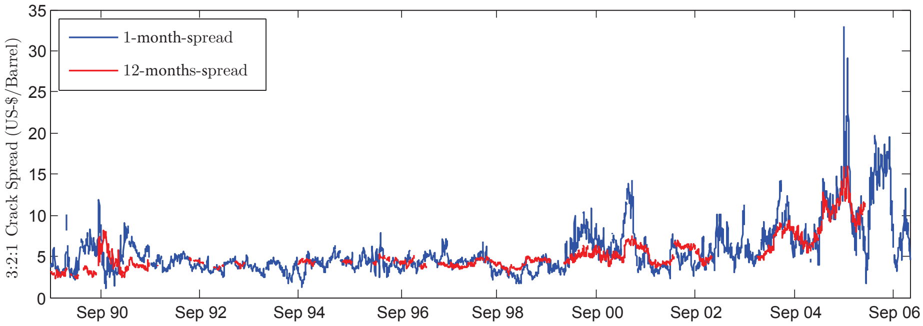

Figure 1: **Crack Spread**

This figure displays the 3:2:1 crack spread with one month maturity and 12 months maturity over time.

Table 1: **Sample of Heating Oil Crack Spread Option Prices**

This table reports the summary statistics for the sample of heating oil crack spread option prices covering the period from July 28, 2004 until November 7, 2006. # denotes the number of observed prices and ∅ the average end-of-day price of the options.

		out-of-the-money		at-the-money		in-the-money		all	
	Maturity	#	∅	#	∅	#	∅	#	∅
Calls	short	393	0.86	107	1.26	229	1.73	729	1.19
	mid	457	1.44	122	1.98	292	2.58	871	1.90
	long	273	2.03	58	2.60	223	3.21	554	2.57
	all	1,123	1.38	287	1.84	744	2.51	2,154	1.83
Puts	short	247	0.90	90	1.22	184	1.76	521	1.26
	mid	426	1.47	114	2.01	284	2.35	824	1.85
	long	386	2.13	68	2.52	178	3.49	632	2.56
	all	1,059	1.58	272	1.88	646	2.50	1,977	1.92
	TOTAL	**2,182**	**1.48**	**559**	**1.86**	**1,390**	**2.50**	**4,131**	**1.87**

Table 2: **Sample of Gasoline Crack Spread Option Prices**

This table reports the summary statistics for the sample of gasoline crack spread option prices covering the period from December 3, 2003 until January 3, 2006. # denotes the number of observed prices and ∅ the average end-of-day price of the options.

		out-of-the-money		at-the-money		in-the-money		all	
	Maturity	#	∅	#	∅	#	∅	#	∅
Calls	short	502	0.75	154	1.09	346	1.53	1,002	1.07
	mid	740	1.07	217	1.43	505	1.98	1,462	1.43
	long	323	1.26	107	1.52	345	1.94	775	1.60
	all	1,565	1.00	478	1.34	1,196	1.84	3,239	1.36
Puts	short	315	0.71	86	0.97	258	1.30	659	0.98
	mid	526	1.05	147	1.27	322	1.55	995	1.24
	long	346	1.52	79	1.58	123	1.86	548	1.60
	all	1,187	1.10	312	1.26	703	1.51	2,202	1.25
	TOTAL	**2,752**	**1.04**	**790**	**1.31**	**1,899**	**1.72**	**5,441**	**1.32**

Table 3: **Heating Oil Crack Spread Option-Implied Model Parameters**

This table reports the mean and standard errors [in brackets] of weekly re-estimated crude oil/heating oil crack spread option-implied model parameters. The sample period spans July 28, 2004 to November 7, 2006.

		β_0	β_1	β_2	$\lambda + \theta$	δ	γ	η	ρ
U-NG	HO-CL	0.0017	0.5183	0.0241	-1.1683	-	-	-	-
		[0.0002]	[0.0257]	[0.0034]	[0.3971]				
U-NG-M	HO-CL	0.0055	0.3528	0.0758	1.5732	-	-0.4596	0.0014	-
		[0.0003]	[0.0225]	[0.0059]	[0.2574]		[0.0316]	[0.0002]	
B-CV	CL	0.0016	-	-	-	-	-	-	0.7772
		[0.0002]							[0.0157]
	HO	0.0014	-	-	-	-	-	-	0.7772
		[0.0002]							[0.0157]
B-NG	CL	0.0001	0.2805	0.1466	0.3154	-	-	-	0.7600
		[0.0000]	[0.0138]	[0.0009]	[0.0076]				[0.0008]
	HO	0.0001	0.5346	0.1423	-0.1206	-	-	-	0.7600
		[0.0000]	[0.0188]	[0.0089]	[0.0846]				[0.0088]
B-NGC	CL	0.0002	0.4616	0.0927	-0.5764	-0.0651	-	-	0.9087
		[0.0000]	[0.0184]	[0.0075]	[0.1590]	[0.0082]			[0.0085]
	HO	0.0002	0.5289	0.0384	0.2359	0.0119	-	-	0.9087
		[0.0000]	[0.0224]	[0.0056]	[0.2089]	[0.0093]			[0.0085]
B-NGC-M	CL	0.0005	0.4282	0.1129	0.2469	-0.0910	-0.1934	0.0048	0.8393
		[0.0000]	[0.0228]	[0.0078	[0.1560]	[0.0078]	[0.0141]	[0.0003]	[0.0121]
	HO	0.0013	0.5441	0.0694	0.3309	-0.0063	-0.2115	0.0020	0.8393
		[0.0001]	[0.0241]	[0.0051]	[0.1716]	[0.0096]	[0.0158]	[0.0002]	[0.0121]

Table 4: **Gasoline Crack Spread Option-Implied Model Parameters**

This table reports the mean and standard errors [in brackets] of weekly re-estimated crude oil/gasoline crack spread option-implied model parameters. The sample period spans December 3, 2003 to January 3, 2006.

		β_0	β_1	β_2	$\lambda+\theta$	δ	γ	η	ρ
U-NG	HU-CL	0.0018	0.4412	0.0250	0.6941	-	-	-	-
		[0.0006]	[0.0319]	[0.0005]	[0.4224]				
U-NG-M	HU-CL	0.0064	0.4373	0.0438	2.0057	-	-0.4873	0.0015	-
		[0.0003]	[0.0268]	[0.0070]	[0.3211]		[0.0861]	[0.0002]	
B-CV	CL	0.0010	-	-	-	-	-	-	0.6607
		[0.0002]							[0.0469]
	HU	0.0008	-	-	-	-	-	-	0.6607
		[0.0002]							[0.0141]
B-NG	CL	0.0001	0.3472	0.1229	0.6953	-	-	-	0.6967
		[0.0000]	[0.0146]	[0.0081]	[0.0924]				[0.0130]
	HU	0.0000	0.6113	0.0786	-1.3688	-	-	-	0.6967
		[0.0000]	[0.0155]	[0.0295]	[0.2281]				[0.0130]
B-NGC	CL	0.0003	0.4032	0.1006	0.3063	-0.0772	-	-	0.9520
		[0.0000]	[0.0206]	[0.0059]	[0.1636]	[0.0098]			[0.0044]
	HU	0.0003	0.4188	0.0827	0.4749	-0.0064	-	-	0.9520
		[0.0000]	[0.0217]	[0.0072]	[0.1355]	[0.0119]			[0.0044]
B-NGC-M	CL	0.0008	0.3232	0.1149	0.7667	-0.1007	-0.5671	0.0055	0.8847
		[0.0001]	[0.0194]	[0.0064]	[0.1265]	[0.0063]	[0.2216]	[0.0004]	[0.0085]
	HU	0.0027	0.4559	0.0832	-0.3128	0.0097	-0.6114	0.0038	0.8847
		[0.0003]	[0.0242]	[0.0078]	[0.1593]	[0.0072]	[0.0930]	[0.0003]	[0.0085]

Table 5: **In-Sample Pricing Errors**

This table reports the in-sample pricing errors for the six considered models: U-NG, U-NG-M, B-CV, B-NG, B-NGC, and B-NGC-M; all option prices are determined by the algorithm of Longstaff and Schwartz (2001) using 50,000 paths in the Monte Carlo simulation for all option prices with moneyness $m = (0,42 * F_{HO} - F_{CL})/K \in (0,85;1,15)$ *and maturity* $6 \leq DTM \leq 120$*. Furthermore, the exclusion criteria of Bakshi et al. (1997) were applied.*

	MPE	MdPE	MAPE	MdAPE	RMSE	RRMSE
Crude Oil / Heating Oil						
U-NG	-3.66%	-3.60%	9.71%	6.57%	0.21$	14.43%
U-NG-M	-5.32%	-5.00%	7.66%	5.58%	0.19$	11.69%
B-CV	-5.92%	-5.49%	13.97%	9.72%	0.35$	19.74%
B-NG	-7.76%	-6.99%	12.14%	8.67%	0.30$	17.03%
B-NGC	-3.97%	-3.95%	9.91%	6.70%	0.25$	14.74%
B-NGC-M	-3.87%	-3.84%	7.57%	6.04%	0.18$	12.21%

	MPE	MdPE	MAPE	MdAPE	RMSE	RRMSE
Crude Oil / Gasoline						
U-NG	-6.56%	-5.83%	12.59%	9.43%	0.19$	17.10%
U-NG-M	-4.93%	-4.50%	7.74%	6.12%	0.13$	10.34%
B-CV	-10.93%	-7.52%	17.40%	10.98%	0.29$	24.74%
B-NG	-7.93%	-7.06%	13.20%	9.01%	0.22$	18.92%
B-NGC	-8.04%	-7.35%	11.13%	8.52%	0.17$	15.03%
B-NGC-M	-5.74%	-5.72%	8.38%	6.52%	0.16$	11.51%

Table 6: **Out-of-Sample Pricing Errors**

This table reports the out-of-sample pricing errors for the six considered models: U-NG, U-NG-M, B-CV, B-NG, B-NGC, and B-NGC-M; all option prices are determined by the algorithm of Longstaff and Schwartz (2001) using 50,000 paths in the Monte Carlo simulation for all option prices with moneyness $m = (0,42 * F_{HO} - F_{CL})/K \in (0,85;1,15)$ *and maturity* $6 \leq DTM \leq 120$*. Furthermore, the exclusion criteria of Bakshi et al. (1997) were applied.*

	MPE	MdPE	MAPE	MdAPE	RMSE	RRMSE
Crude Oil / Heating Oil						
U-NG	-3.53%	-3.82%	11.09%	7.78%	0.24$	15.94%
U-NG-M	-5.03%	-4.91%	9.51%	6.99%	0.23$	13.71%
B-CV	-5.97%	-6.27%	15.72%	10.82%	0.41$	22.27%
B-NG	-8.46%	-9.42%	14.22%	10.47%	0.34$	19.47%
B-NGC	-4.92%	-5.00%	11.88%	8.33%	0.29$	17.09%
B-NGC-M	-5.39%	-5.46%	10.92%	7.45%	0.29$	16.30%

	MPE	MdPE	MAPE	MdAPE	RMSE	RRMSE
Crude Oil / Gasoline						
U-NG	-7.35%	-7.52%	13.84%	10.91%	0.20$	18.02%
U-NG-M	-5.66%	-5.72%	10.17%	8.29%	0.17$	13.17%
B-CV	-10.51%	-8.36%	18.54%	12.56%	0.31$	25.91%
B-NG	-7.35%	-7.59%	15.55%	10.74%	0.26$	22.24%
B-NGC	-9.10%	-8.44%	13.16%	10.31%	0.21$	17.47%
B-NGC-M	-5.68%	-5.44%	10.43%	7.68%	0.19$	14.55%

Table 7: **Breakdown of Out-of-Sample Pricing Errors by Moneyness and Maturity (Crude Oil / Heating Oil)**

This table reports the pricing errors by moneyness and maturity of crude oil/heating oil spread options. All option prices determined by the algorithm of Longstaff and Schwartz (2001) using 50,000 paths in the Monte-Carlo simulation for all option prices with moneyness $m = (0.42 * F_{HO} - F_{CL})/K \in (0.85, 1.15)$ *and* $6 \leq DTM \leq 120$. *Furthermore, the exclusion criteria of Bakshi et al. (1997) were applied.*

	out-of-the-money				at-the-money				in-the-money			
	MPE	MAPE	RMSE	RRMSE	MPE	MAPE	RMSE	RRMSE	MPE	MAPE	RMSE	RRMSE
short-term												
U-NG	-16.95%	18.79%	0.23$	22.79%	-7.81%	11.21%	0.16$	13.47%	-4.58%	9.82%	0.20$	14.66%
U-NG-M	-8.14%	12.78%	0.20$	16.91%	-3.99%	8.23%	0.13$	10.23%	-2.34%	8.03%	0.17$	12.96%
B-CV	-17.80%	26.51%	0.35$	31.77%	-10.94%	16.78%	0.31$	20.50%	-6.19%	12.12%	0.28$	15.42%
B-NG	-15.26%	22.18%	0.29$	28.09%	-8.95%	13.74%	0.23$	16.99%	-5.21%	11.16%	0.24$	16.12%
B-NGC	-17.35%	20.15%	0.26$	25.38%	-8.93%	12.26%	0.18$	15.02%	-5.69%	9.86%	0.21$	13.76%
B-NGC-M	-13.16%	17.00%	0.24$	21.77%	-5.14%	10.65%	0.17$	13.86%	-3.43%	8.67%	0.18$	12.82%
middle-term												
U-NG	-4.77%	9.41%	0.20$	12.24%	-2.29%	7.36%	0.17$	11.12%	-0.55%	7.60%	0.23$	13.89%
U-NG-M	-5.30%	9.92%	0.21$	13.14%	-3.46%	7.56%	0.19$	11.15%	-1.20%	7.75%	0.24$	13.05%
B-CV	-10.22%	15.31%	0.37$	21.04%	-5.65%	12.51%	0.41$	18.37%	-3.61%	11.09%	0.39$	15.85%
B-NG	-13.16%	15.21%	0.34$	19.93%	-8.62%	12.03%	0.32$	15.44%	-6.29%	11.67%	0.39$	16.06%
B-NGC	-7.62%	11.25%	0.26$	15.27%	-3.82%	7.88%	0.21$	11.85%	-1.65%	8.13%	0.27$	13.24%
B-NGC-M	-4.49%	10.19%	0.23$	13.56%	-2.99%	8.50%	0.27$	12.79%	-0.81%	7.76%	0.25$	12.32%
long-term												
U-NG	5.50%	12.70%	0.33$	17.15%	5.06%	10.58%	0.33$	13.45%	3.80%	7.49%	0.28$	14.32%
U-NG-M	-8.51%	10.89%	0.28$	14.48%	-7.12%	9.02%	0.30$	11.99%	-3.26%	7.54%	0.28$	12.91%
B-CV	5.32%	16.29%	0.52$	26.14%	4.28%	13.06%	0.53$	18.38%	2.47%	10.53%	0.50$	16.47%
B-NG	-2.29%	12.87%	0.37$	17.78%	-2.67%	11.86%	0.46$	15.45%	-1.95%	9.48%	0.40$	13.79%
B-NGC	4.05%	11.89%	0.33$	17.54%	4.22%	11.19%	0.40$	14.99%	2.46%	9.53%	0.42$	15.08%
B-NGC-M	-6.21%	12.15%	0.41$	20.52%	-7.76%	11.49%	0.49$	17.19%	-3.52%	8.80%	0.41$	14.35%

Table 8: **Breakdown of Out-of-Sample Pricing Errors by Moneyness and Maturity (Crude Oil / Gasoline)**

This table reports the pricing errors by moneyness and maturity of crude oil/gasoline spread options. All option prices determined by the algorithm of Longstaff and Schwartz (2001) using 50,000 paths in the Monte-Carlo simulation for all option prices with moneyness $m = (0.42 * F_{HO} - F_{CL})/K \in (0.85, 1.15)$ *and* $6 \leq DTM \leq 120$. *Furthermore, the exclusion criteria of Bakshi et al. (1997) were applied.*

	out-of-the-money				at-the-money				in-the-money			
	MPE	MAPE	RMSE	RRMSE	MPE	MAPE	RMSE	RRMSE	MPE	MAPE	RMSE	RRMSE
short-term												
U-NG	-23.15%	24.28%	0.20$	28.66%	-15.26%	16.31%	0.22$	19.62%	-11.94%	12.36%	0.19$	14.59%
U-NG-M	-9.58%	14.88%	0.14$	18.34%	-4.63%	10.61%	0.17$	13.36%	-4.46%	7.44%	0.13$	9.22%
B-CV	-34.09%	35.15%	0.35$	42.40%	-23.04%	23.67%	0.38$	28.90%	-13.99%	14.86%	0.29$	18.40%
B-NG	-17.09%	25.05%	0.24$	31.72%	-10.66%	16.35%	0.28$	20.58%	-6.11%	10.25%	0.21$	13.11%
B-NGC	-20.34%	22.29%	0.21$	26.22%	-12.95%	14.43%	0.23$	17.49%	-9.72%	10.76%	0.20$	12.84%
B-NGC-M	-9.20%	16.25%	0.17$	21.48%	-5.16%	10.83%	0.20$	14.41%	-3.96%	7.90%	0.17$	10.56%
middle-term												
U-NG	-8.31%	13.66%	0.17$	17.34%	-5.61%	10.75%	0.18$	13.72%	-5.61%	8.29%	0.18$	10.56%
U-NG-M	-5.31%	11.57%	0.15$	14.88%	-2.93%	8.65%	0.15$	11.07%	-3.78%	7.30%	0.17$	9.18%
B-CV	-13.06%	18.40%	0.29$	24.51%	-7.90%	12.96%	0.28$	17.20%	-7.34%	10.44%	0.29$	14.07%
B-NG	-11.71%	16.50%	0.24$	21.56%	-8.04%	11.51%	0.23$	14.92%	-6.69%	9.40%	0.24$	11.96%
B-NGC	-11.35%	14.35%	0.19$	18.10%	-7.27%	10.62%	0.19$	13.27%	-7.42%	9.07%	0.21$	11.42%
B-NGC-M	-6.46%	10.73%	0.16$	13.94%	-3.94%	7.63%	0.15$	9.78%	-4.01%	6.49%	0.17$	8.58%
long-term												
U-NG	7.76%	14.18%	0.27$	18.21%	5.43%	10.33%	0.20$	12.72%	3.01%	8.90%	0.25$	11.43%
U-NG-M	-6.84%	10.47%	0.19$	13.04%	-5.42%	8.38%	0.16$	10.45%	-6.44%	8.63%	0.23$	10.90%
B-CV	6.26%	16.14%	0.31$	24.63%	12.01%	17.22%	0.41$	26.79%	9.50%	14.09%	0.35$	19.87%
B-NG	1.24%	19.00%	0.34$	30.20%	4.33%	14.16%	0.30$	22.40%	4.49%	12.63%	0.32$	19.74%
B-NGC	-1.54%	11.94%	0.23$	17.38%	0.13%	9.56%	0.18$	12.64%	0.51%	9.03%	0.26$	14.69%
B-NGC-M	-7.39%	13.08%	0.25$	18.18%	-2.97%	9.82%	0.21$	13.41%	-3.23%	8.18%	0.21$	11.09%

Table 9: **Regression Analysis of Pricing Errors**

This table reports the regression coefficient estimates, as well as heteroscedasticity consistent t-statistics [in brackets] for the following regression model: $PE_i(t) = \beta_1 + \beta_2 DTM_i + \beta_3 DTM_i^2 + \beta_4 m_i(t) + \beta_5 m_i(t)^2 + \beta_6 OI_i(t) + \beta_7 \sigma_{CL}(t) + \beta_8 \sigma_{CL}^2(t) + \beta_9 \sigma_{prod}(t) + \beta_{10} \sigma_{prod}^2(t) + \beta_{11} \rho(t) + \beta_{12} PUT_i(t) + \beta_{13} BACKWARD(t) + u_i(t)$*, where* DTM_i *is the maturity of option* i *(in days),* $m_i(t)$ *is the moneyness of option* i*,* $OI_i(t)$ *the open interest in option* i*,* $\sigma_{CL}(t)$ *and* $\sigma_{prod}(t)$ *is the 20-day standard deviation of the log returns for the respective crude oil or output product futures, and* $\rho(t)$ *is the correlation between these futures.* $PUT_i(t)$ *and* $BACKWARD(t)$ *are dummy variables taking the value of one (zero) if puts (calls) are priced and distinguishing those pricing errors for which the crack spread futures term structure is in backwardation or contango. As a matter of simplification, we consider the difference between the 1-months and those crack spread futures with the longest maturity available in order to determine the values for the variable* $BACKWARD_t$*.*

	Crude Oil / Heating Oil						Crude Oil / Gasoline					
	U-NG	U-NG-M	B-CV	B-NG	B-NGC	B-NGC-M	U-NG	U-NG-M	B-CV	B-NG	B-NGC	B-NGC-M
Constant	-1.43	-0.45	-2.00	-2.05	-2.07	-1.54	-1.59	-1.71	-3.04	-2.21	-1.56	-1.72
	[-4.57]	[-1.48]	[-4.23]	[-5.07]	[-5.51]	[-4.19]	[-5.42]	[-6.60]	[-7.04]	[-5.11]	[-5.20]	[-5.91]
DTM	0.96	0.34	0.54	-0.24	0.77	0.63	1.07	0.00	0.84	-0.75	0.34	-0.26
	[11.30]	[4.55]	[4.31]	[-2.17]	[8.36]	[5.24]	[15.95]	[-0.06]	[7.70]	[-6.48]	[4.30]	[-3.44]
DTM^2	-0.74	-0.91	0.21	1.23	-0.55	-1.43	-0.64	-0.07	0.45	3.05	0.33	0.58
	[-3.86]	[-5.61]	[0.68]	[4.85]	[-2.64]	[-4.89]	[-4.18]	[-0.56]	[1.71]	[10.21]	[1.69]	[3.21]
m	2.12	0.92	3.53	2.82	3.06	2.43	2.21	2.49	4.58	3.74	2.76	2.28
	[3.39]	[1.52]	[3.78]	[3.53]	[4.11]	[3.35]	[3.84]	[4.85]	[5.42]	[4.38]	[4.71]	[3.99]
m^2	-1.23	-0.56	-1.77	-1.37	-1.57	-1.18	-1.13	-1.25	-2.18	-1.83	-1.31	-1.09
	[-3.96]	[-1.87]	[-3.81]	[-3.43]	[-4.25]	[-3.26]	[-3.96]	[-4.90]	[-5.16]	[-4.31]	[-4.48]	[-3.84]
OI	0.00	-0.01	0.03	0.02	0.01	0.02	0.03	0.02	0.00	-0.01	0.00	-0.02
	[0.45]	[-3.47]	[4.74]	[4.27]	[3.13]	[3.27]	[9.48]	[5.17]	[0.35]	[-1.52]	[0.55]	[-4.48]
σ_{CL}	0.11	0.29	0.27	0.19	0.04	0.04	0.48	0.10	-0.43	-0.46	-0.12	-0.05
	[2.31]	[6.41]	[3.66]	[3.06]	[0.68]	[0.86]	[11.19]	[2.76]	[-6.22]	[-8.97]	[-2.89]	[-1.30]
σ_{CL}^2	-0.03	-0.07	-0.06	-0.04	-0.02	-0.01	-0.11	-0.01	0.07	0.10	0.03	0.01
	[-2.65]	[-6.09]	[-3.10]	[-2.25]	[-1.30]	[-0.69]	[-11.44]	[-1.70]	[4.03]	[8.62]	[3.12]	[1.40]
σ_{prod}	0.01	-0.13	-0.08	-0.13	0.16	0.05	-0.34	-0.07	-0.14	-0.20	-0.09	0.04
	[0.24]	[-3.42]	[-1.32]	[-2.51]	[3.41]	[1.22]	[-7.11]	[-1.89]	[-1.44]	[-3.46]	[-1.78]	[1.14]
σ_{prod}^2	-0.01	0.03	-0.01	0.01	-0.04	-0.01	0.06	0.01	0.05	0.06	0.01	-0.01
	[-0.68]	[3.24]	[-0.51]	[1.16]	[-3.97]	[-1.39]	[6.35]	[1.03]	[2.48]	[4.95]	[1.23]	[-1.33]
ρ	0.32	-0.08	-0.03	0.54	0.30	0.12	0.20	0.42	1.12	0.99	0.22	0.56
	[6.91]	[-1.78]	[-0.49]	[7.91]	[5.50]	[1.92]	[3.38]	[7.26]	[11.81]	[11.70]	[3.95]	[9.10]
PUT	-0.04	-0.03	-0.01	0.00	-0.01	0.00	-0.04	-0.03	0.01	-0.03	-0.01	0.02
	[-9.15]	[-6.65]	[-1.27]	[0.52]	[-2.46]	[-0.43]	[-10.39]	[-8.12]	[2.11]	[-5.45]	[-3.08]	[4.74]
$BACKWARD$	0.05	-0.01	0.03	-0.01	0.00	-0.01	0.05	0.03	-0.07	-0.04	-0.01	-0.03
	[10.59]	[-2.45]	[3.59]	[-2.16]	[0.43]	[-2.00]	[13.96]	[10.06]	[-13.72]	[-8.23]	[-3.13]	[-8.42]
$Adj.\ R^2$	0.29	0.07	0.19	0.11	0.22	0.03	0.39	0.06	0.39	0.20	0.18	0.05