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DISPERSION TRADING: AN EMPIRICAL ANALYSIS ON THE S&P 100 OPTIONS

Abstract

This study provides an empirical analysis back-testing the implementation of a dispersion trading strategy to verify its profitability. Dispersion trading is an arbitrage-like technique based on the exploitation of the overpricing of index options, especially index puts, relative to individual stock options. The reasons behind this phenomenon have been traced in literature to the correlation risk premium hypothesis (i.e., the hedge of correlations drifts during market crises) and the market inefficiency hypothesis. This study is aimed at evaluating whether dispersion trading can be implemented with success, with a focus on the Standard & Poor's 100 options. The risk adjusted return of the strategy used in this empirical analysis has beaten a buy-and-hold alternative on the S&P 100 index, providing a significant over-performance and a low correlation with the stock market. The findings, therefore, provide an evidence of inefficiency in the US options market and the presence of a form of "free lunch" available to traders focusing on options mispricing.

INTRODUCTION

Traditionally, volatility has been regarded as a measure of risk in portfolio management; however, since the seminal works by Black and Scholes (1973) and Merton (1973), investment strategies focused on volatility have been the object of study both by scholars and practitioners. The presence of liquid derivatives markets and the availability of seemingly unlimited computing power have made possible the practical implementation and testing of complex quantitative approaches and the development of a whole new category of investment techniques: volatility trading (Sinclair, 2013). These techniques aim at taking profit, regardless of price movements, from the increment or reduction of volatility of listed securities by making use of derivatives. One of these techniques is dispersion trading, defined as the practice of selling index volatility while buying the volatility of its constituents at the same time (Ren, 2010). Plain vanilla options are the instruments most widely used by dispersion traders, but more complex and exotic derivatives, such as variance swaps, can be used for this trading strategy (Hilpisch, 2017).

Due to the absence of empirical analyses of the practical viability of dispersion trading during the last decade, it is an open issue whether it can be still used with success. As a consequence, the implementation of dispersion trading requires a deeper investigation and this article is aimed at evaluating it in a realistic framework, using more recent data than the ones available in literature. In particular, this empirical analysis is focused on the US derivatives market.

The article is organized as follows. The first section describes dispersion trading, taking into account the factors underlying its rationale and the relevant literature. Its first sub-section is devoted to the theoretical foundations of this technique, while the second sub-section illustrates the technical problems encountered in the practical implementation of this trading strategy according to the relevant literature. The second section provides an empirical analysis and is divided into three sub-sections: the first one defines the time series of daily market data; the second provides a step-by-step description of the methodology followed in order to implement a simulation of dispersion trading in a realistic environment; the third discusses the results. Conclusions are outlined in the last section.

1. LITERATURE REVIEW

An in-depth analysis of dispersion trading requires a review of the available literature, taking into account both its theoretical and technical aspects.

1.1. Theoretical foundations

Dispersion trading is an arbitrage-like technique, which is based on the exploitation of the overpricing of index options, especially index puts, relative to individual stock options. This mispricing in the options market has been empirically proven in several past studies, among which we recall Bakshi and Kapadia (2003), Bollen and Whaley (2004), Dennis et al. (2006), and Driessen et al. (2009), and its causes have been traced back to an overestimation of index volatility with respect to the volatilities of its constituents.

These analyses compare the differences between implied volatility and sample volatility of indices and stocks to verify if there is incoherence in the pricing of index and stock options. Despite their different methodological approaches and samples (the Standard & Poor's 500 or the Standard & Poor's 100), these studies reach uniform conclusions. They show that the difference between implied and sample variances is larger for index options than for stock options. Driessen et al. (2009) have measured the implied volatility on the S&P 100 options to be higher than the sample variance by about 3.89% annually between 1996 and 2003, and by 2.47% on single stocks. In comparison, Bakshi and Kapadia (2003) covered the period between 1991 and 1995 and found an implied volatility on stock options in excess of about 1% per year compared to the sample one, significantly lower than the 3.3% measured on the S&P 500. Bollen

and Whaley (2004) confirmed these findings and reported a more pronounced volatility skew for the S&P 500 options when compared to stock options; therefore, implied volatility is less stable across maturities for index options than for the options written on its constituents.

The variance of returns of an index can be estimated as follows:

$$\sigma_I^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j>i}^{n} w_i w_j \sigma_i \sigma_j \rho_{i,j} , \qquad (1)$$

where w_i and σ_i are the weight of the i-th stock in the index and its standard deviation, respectively, and $\rho_{i,j}$ is the correlation between the i-th and j-th constituents.

As a consequence, the variance of the index depends not only upon the variances of its constituents, but also their correlations. The same relationship in equation (1) theoretically applies not only for the sample estimates, but also for the variances implied in index options and the options written on its constituents. As noted, this is not empirically proven and, on the contrary, the overestimation of the implied variance of index options can be traced back to an excess implied correlation among its constituents. Implied correlation, which is in a way a mean of all the possible correlations between couples of stocks in an index, can be calculated as follows:

$$\rho_{imp} = \frac{\sigma_I^2 - \sum_{i=1}^{n} w_i^2 \sigma_i^2}{2\sum_{i=1}^{n}\sum_{j>i}^{n} w_i w_j \sigma_i \sigma_j}, \qquad (2)$$

where σ_I and σ_i are the standard deviation implied in the index options and in the options written on its i-th constituent, respectively. Despite the fact that Pearson's correlation index is bounded in

the [−1, +1] interval, this formulation of the index's implied correlation can, and often does, reach values above +1, stressing the evidence of its overestimation by the options market.

Dispersion trading exploits this mispricing by opening trades when the difference between the implied volatility of the index options and its theoretical value (i.e. the actual volatility of the index returns) is at its maximum, aiming at a convergence between these two measures before the expiry of the derivative contracts used for this strategy. This convergence can also be the result of a reversal of the implied correlation to its long-term mean. Usually, a dispersion trade is made of a short position on the index's implied volatility by using a short straddle on its options and a long position on the implied volatility of its constituents, achieved by opening a long straddle on the constituents' options. In other cases, the implied volatility of index options can be lower than its theoretical value and, therefore, dispersion trading is implemented the opposite way, that is, by going long on an index straddle and shorting straddles on its constituents.

Deng (2008) has empirically measured a systematic and significant profitability of dispersion trading on the US stock market until 2000, but also a relevant decrease in the trading results after this year. Meanwhile, Marshall (2009) has shown that a profitable use of dispersion trading was also possible after the year 2000 by making use of some simple indicators.

The factors underlying this phenomenon have been traced in literature to the risk premium and market inefficiency hypotheses.

The first hypothesis was put forward by Bakshi and Kapadia (2003), Driessen et al. (2009), and Bollerslev and Todorov (2011). Based on this hypothesis, the index options' overpricing in relation to the options on the index's constituent stocks is caused by the presence of a risk premium for the increase of correlation among these constituents. Investors diversify their portfolio to reduce risk. Diversification benefits result from the imperfect correlation of assets and therefore, portfolio variance is lower than the sum of the variances of its constituents (subadditivity property).

However, contextual increases in both the volatility and correlations among stocks during market downturns are a known and proven phenomenon (Ang & Chen, 2001). A possible explanation may be identified in the so-called "panic-selling" that is derived from a sudden increase in investors' risk aversion, which causes a phase-locking and increase of correlations (correlations drift) and conversely, a reduction of diversification benefits and an increase in market volatility.

If we apply the usual portfolio variance formula by using the implied volatilities of index constituents as inputs as shown below:

$$\sigma^2{}_I = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_i \sigma_j \rho_{i,j}, \qquad (3)$$

it becomes apparent that implied volatilities and prices of index options will be affected by both the increase of variance in single stocks and the phase-locking of correlations. As noted by Driessen et al. (2009), the index options' overvaluation is due to an overestimation of correlations among constituents and this excess of implied correlation includes the risk premium for correlation risk, that is, the risk of increases in correlations among the index constituents. Said differently, index options can be regarded as a form of hedge on this source of risk and, therefore, dispersion traders are protection sellers and their exposure to correlation risk is remunerated by the premium paid by hedgers.

The second hypothesis regarding the source of index options' mispricing is based on the assumption of market inefficiency. If the assumptions of the Black-Scholes-Merton model were empirically verified, option prices should not be influenced by demand, but only by factors linked to the underlying security such as money market rates and time and therefore, the no-arbitrage condition should hold (Hull, 2018). However, Shleifer and Vishny (1997) and Liu and Longstaff (2004) have discovered the presence of serious limits in the practical implementation of arbitrages. For example, losses incurred during arbitrages can force the closing of the operation with a loss before the convergence of prices to their equilibrium level happens.

For the same reason, Bollen and Whaley (2004) stated that market makers are not willing to sell any amount of the same option at a constant price. In actuality, when the amount sold increases, the costs of hedging the exposure to volatility force the market maker to ask for a higher price for the option. From this it can be inferred that the supply curve is positively sloped (Constantinides & Lian, 2018), that is, demand has a direct relationship with option prices. Gârleanu et al. (2009) have measured a net positive demand of index options and a net negative demand for individual stock options. The causes of this phenomenon are probably rooted in common investment funds' policies. Their passive or semi-passive management styles require the use of index options for their hedging strategies, not of derivatives written on individual stocks. Given this pressure on demand and the positive slope of the supply curve, index options are systematically overpriced when compared to stock options.

Deng (2008) provides an in-depth empirical analysis of dispersion trading on the S&P 500 index and of its underlying factors, that is, risk premium and market inefficiency. The reform of the US options markets in 1999 (after an intervention by the SEC aimed at limiting anti-competitive practices) provides a "natural experiment" useful in distinguishing the inefficiency factor in options pricing, given the dramatic contraction of transaction costs and of bid-ask spreads that occurred after its adoption (De Fontnouvelle et al., 2003). If overpricing of index options were a consequence of risk premium alone, the impact of the 1999 reform on the profits of dispersion trading should be negligible. Deng (2008) has measured a mean return for this strategy (implemented by buying at-the-money straddles on the S&P 500 and selling straddles on each of its constituents) of 24% between 1996 and 2000 and –0.03% between 2001 and 2005. This outcome provides strong evidence of the role of market inefficiency, while it is still not entirely possible to rule out the presence of a limited risk premium.

1.2. Implementation and technical issues

Non-directional trades, such as dispersion trading, require a delta-neutral position on the returns of the underlying index and stocks. Delta hedg-ing, while appealing on a theoretical perspective, poses serious issues when applied in practice. First of all, options' delta is not constant and rebalancing is subject to transaction costs. This requires a reduction of its frequency which, as a consequence, leads to an imperfect hedge. As mentioned, dispersion trading can be implemented by buying and selling straddles and thus its initial delta is zero. However, in order to maximize the exposure to volatility, the options involved in this operation are at the money, which are characterized by the highest gamma and, at the same time, the highest volatility of the delta. Therefore, the efficacy of delta hedging is limited given the changes of delta before a new rebalancing, making it a suboptimal solution.

As previously noted, dispersion trading derives profit from the overpricing of index options compared to stock options due to an overestimation of the index's implied volatility. What makes it possible to isolate the effects of implied volatility on option prices is the greek "vega", that is, the first partial derivative of the price function with respect to volatility. Therefore, in order to maximize the return of the strategy, it is necessary to maximize the exposure of option prices to implied volatility which is the absolute value of their vega. Moreover, in the presence of a vega close to zero, the realignment of implied volatilities with their true value would not have any significant impact on option prices and thus on the profitability of dispersion trading. Like delta, vega also varies with the price of the underlying asset: it is maximum for at-the-money options and minimum for deep in- or out-of-the-money options. Therefore, in order to keep vega as distant from zero as possible, it is necessary to rebalance the portfolio towards the at-the-money options.

Vega plays a crucial role in dispersion trading, because it is the input for the calculation of the number of options bought for each index constituent in order to keep the positive vega on the portfolio of long straddles on stocks as close as possible to the absolute value of the negative vega on short straddles on the index. This way, the trade is hedged from changes in the implied volatilities of the individual constituents and the investment is exposed only to the changes in the implied volatility of the index resulting

from an increase or reduction of correlation as implied in the price of index options.

Theta is seldom taken into account in dispersion trading because of its marginal, albeit not irrelevant, role. Given that the passing of time negatively affects the price of options, the strategy is best kept as close as possible to a zero theta, taking into account the positive theta of the long straddles on stocks and the negative theta of the short straddles on the index. Therefore, the calculation of the number of options to be included in the strategy should take the form of a joint minimization of the algebraic sums both of the vegas and the thetas of the straddles involved.

A method to keep a perfect delta hedge and a high vega regardless of an option's remaining time to expiry and of the price of its underlying asset is to make use of derivative contracts known as variance swaps (Nelken, 2006; Härdle & Silyakova, 2012). A variance swap is a forward contract that pays the difference between the realized variance (floating leg) of the underlying asset and a predefined strike variance (fixed leg) multiplied by the notional value at maturity. Unlike options, variance swaps do not require the payment of a premium and unlike plain vanilla swaps they contemplate only one payment at expiry. Given their technical features, variance swaps are comparable to forward contracts and are also known as realized volatility forwards (Demeterfi et al., 1999).

Formally, the payoff of a variance swap at expiry is equal to:

$$Payoff = \left(\sigma_R^2 - K_{var} \right) \cdot N, \qquad (4)$$

where σ_R^2 is the variance of the returns of the underlying on an annual basis, K_{var} is the so-called "delivery price" (strike variance), N is the notional value of the variance swap contract.

Given the no-arbitrage condition, the delivery price of variance swaps can be replicated by an options portfolio, but unlike these latter derivatives, variance swaps are not subject to factors different from volatility and time to expiry. Therefore, the first and clear advantage of using variance swaps

is the absence of the practical problems typical of delta hedging. Moreover, as noted by Nelken (2006), variance swaps allow keeping a constant vega without the need for rebalancing.

Despite these apparent advantages, the use of variance swaps in dispersion trading poses some issues. First, the 2008 crisis has caused serious losses to banks selling this type of derivatives resulting in a sudden implosion of their market, especially with regard to contracts written on single stocks (Carr & Lee, 2009; Martin, 2013) that are necessary for dispersion trading. Second, variance swaps are over-the-counter contracts and thus their market is subject to serious inefficiencies caused by mispricing, transaction costs, and information asymmetry.

A last alternative technique for the implementation of dispersion trading exploits the so-called volatility skew to enhance the return on this strategy. Implied volatility depends upon the moneyness of the option as documented in literature (e.g. Derman & Miller, 2016), and it decreases with an inverse relationship to strike prices. This phenomenon implies that market operators do not follow the Black-Scholes-Merton assumption that stock returns are lognormally distributed; on the contrary, volatility skew is compatible with a negatively skewed and leptokurtic distribution. Consequently, dispersion trading techniques can take into account the presence of this deviation from the standard pricing model and sell strangles instead of straddles on the index. Specifically, the trader should sell short at-the-money calls and out of the money puts which, coherent to volatility skews, have a higher premium and thus are more profitable. The trade on the constituent stocks would be unaltered. This change in the strategy's implementation enhances the return on diversification trading by exploiting the higher implied volatility of out of the money index puts.

2. EMPIRICAL ANALYSIS

The following analysis provides a back-testing of the implementation of a dispersion trading strategy for the period from January 2010 to

December 2015 to verify its profitability. The index selected for this study is the Standard & Poor's 100 composed of 100 companies selected from the S&P 500. Companies included in this index are among the largest and most stable companies in the S&P 500 and have listed options (S&P Dow Jones Indices, 2018). The availability of listed stock and index options is the primary reason for the choice of the S&P 100 in this study. The options involved are listed on the most efficient derivative markets worldwide, causing two contrasting effects. On the one hand, transaction costs are expected to be limited; on the other hand, mispricing of index options is likely to be negligible, negatively affecting the return of dispersion trading.

The strategy is implemented by selling at-the-money straddles on the S&P 100 and buying at-the-money straddles on its constituents, but some enhancements will be applied. Particularly, a principal components analysis is used to limit the number of stocks underlying the straddles. Moreover, an indicator is calculated to signal the best timing for opening and closing the trades and, if necessary, to invert the strategy, that is, to buy straddles on the index and sell straddles on the constituents. Finally, delta hedging is implemented to guarantee portfolio neutrality in terms of delta.

2.1. Data sample

An empirical analysis of diversification trading requires the time series of the options written both on the S&P 100 and on its constituents and of the stocks included in the index. As such, for each listed option with a residual life between one and 31 days, the following data were extracted from the OptionMetrics database with a daily frequency:

- identification number;
- closing bid and ask prices;
- expiry date;
- strike price;
- delta;
- vega;
- 30-day implied volatility;
- daily trading volume;
- open interest.

Daily time series data for stocks came from the Center for Research in Security Prices as follows:

- closing bid and ask prices;
- floating stock;
- dividends;
- splits and reverse splits.

The one-month US Dollar Libor was selected as the proxy for the risk-free rate and was obtained from the Federal Reserve Economic Research database.

2.2. Methodology

Options, even on efficient markets, are subject to transaction costs. One way to limit their impact is to reduce the number of options bought or sold by selecting a subsample of constituents that is sufficient to explain the volatility of the index (i.e., representative of the factor structure of the options market). Our study makes use of principal components analysis, in order to reduce the dimensionality of the problem (Christoffersen et al., 2018). This statistical procedure uses an orthogonal transformation to convert a sample of correlated variables into a set of linearly uncorrelated ones called principal components. In this analysis we adopted the procedure described in Su (2006), followed also by Deng (2008).

The first step of this procedure requires the calculation of the daily continuously compounded return as follows:

$$r_{i,t} = \ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right), \tag{5}$$

where $S_{i,t}$ is the price of the i-th stock at time t.

The covariance matrix of these returns is estimated as follows:

$$V(R) = \frac{R^{*'}R^{*}}{n} = \begin{bmatrix} \sigma_{1,1}^2 & \sigma_{1,2} & \cdots & \sigma_{1,k} \\ \sigma_{2,1} & \sigma_{2,2}^2 & \cdots & \sigma_{2,k} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{k,1} & \sigma_{k,2} & \cdots & \sigma_{k,k}^2 \end{bmatrix}, \tag{6}$$

where R^{*} is the matrix of the centered returns calculated by subtracting the mean return of each constituent from the corresponding column of the $n \times k$ matrix of the constituents returns, n is the length,

measured in days, of the time series of returns for each stock, and k is the number of stocks.

After defining the inputs, we apply the eigenvalue decomposition, ordering the eigenvectors of the covariance matrix, that is, the principal components, according to the share of variance explained. In our sample, the first seven principal components jointly explain 90.52% of the sample variance, with the first and the seventh components explaining 33.27% and 1.44% of the total, respectively.

The next step necessary for the practical implementation of the model is the selection of the stocks that mimic the seven principal components, based on the following procedure for each constituent stock:

1) estimate the Pearson correlation coefficient of each of the seven principal components;

2) calculate the weighted arithmetic mean of the squared correlations using the ratio between the percentage of variance explained by each component and the sum of the seven selected components (for example, for the first one, the weight is 33.27%/90.52% = 36.75%)[1];

3) order the stocks based on their mean calculated in step 2 and select the first 20 stocks in the ranking (Table 1);

4) regress, without intercept, the daily returns of the index on those of the 20 selected stocks (Table 2);

5) discard the stocks, seven in this case, with a significance of at least 1%;

6) repeat step 4 for the remaining stocks, 13 in this case (Table 3).

By applying this procedure, it is possible to identify the stocks that best explain index volatility and as a consequence, dispersion trading can be implemented by buying or selling straddles written only on them and not on the 100 constituents of the selected index.

Table 1. Ranking of stocks

No	Ticker	Constituent name	Weighted mean correlation
1	HON	Honeywell International	0.353839326
2	MET	Metlife	0.348578960
3	WFC	Wells Fargo	0.333608035
4	JPM	JPMorgan Chase	0.329813444
5	USB	US Bancorp	0.324702226
6	BK	Bank of New York Mellon	0.321710049
7	CVX	Chevron	0.318882532
8	SLB	Schlumberger	0.316688635
9	EMR	Emerson Electric	0.316600419
10	BLK	Blackrock	0.314560849
11	XOM	Exxon	0.314519303
12	C	Citigroup	0.314214965
13	UTX	United Technologies	0.310955303
14	CAT	Caterpillar	0.309998038
15	MMM	3M	0.308558810
16	OXY	Occidental Petroleum	0.307202009
17	GE	General Electrics	0.305588500
18	BAC	Bank of America	0.292363556
19	RF	Regions Financial	0.284197114
20	AXP	American Express	0.280715103

Table 2. Daily returns for 20 stocks

Residuals				
Min	**1Q**	**Median**	**3Q**	**Max**
−0.0109757	−0.0014596	0.0001633	0.0016881	0.0120707

Coefficients				
	Estimate	**Std. error**	**t-value**	**Pr(> |t|)**
HON$return	0.090480	0.009609	9.416	< 2e-16***
MET$return	0.025418	0.006572	3.868	0.000115***
WFC$return	0.042460	0.009078	4.677	3.17e-06***
JPM$return	0.034257	0.008279	4.138	3.70e-05***
USB$return	0.035095	0.009796	3.582	0.000351***
BK$return	0.011631	0.007444	1.563	0.118373
CVX$return	0.045261	0.009928	4.559	5.55e-06***
SLB$return	0.013548	0.006135	2.208	0.027386*
EMR$return	0.014589	0.007825	1.864	0.062468.
BLK$return	0.053779	0.006315	8.516	< 2e-16***
XOM$return	0.119519	0.010833	11.033	< 2e-16***
C$return	0.018132	0.006103	2.971	0.003016**
UTX$return	0.062428	0.009637	6.478	1.26e-10***
CAT$return	0.010254	0.006640	1.544	0.122735
MMM$return	0.084748	0.009544	8.879	< 2e-16***
OXY$return	0.008785	0.006718	1.308	0.191225
GE$return	0.064843	0.007763	8.352	< 2e-16***
BAC$return	0.004717	0.005811	0.812	0.417023
RF$return	−0.011055	0.004850	−2.279	0.022783*
AXP$return	0.061899	0.006739	9.185	< 2e-16***

Notes: Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1; residual standard error: 0.002603 on 1,512 degrees of freedom (1 observation deleted due to missingness); multiple R-squared: 0.9292; adjusted R-squared: 0.9283; F-statistic: 992.1 on 20 and 1,512 DF, p-value: < 2.2e-16.

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[1] With respect to this step, we depart from Su (2005) wherein the arithmetic mean correlations were not weighted. Our decision is based on the assumption that it is more useful to select stocks that are highly correlated with the most relevant principal components given the large dispersion in explained variances.

Table 3. Daily returns for 13 stocks

Residuals				
Min	**1Q**	**Median**	**3Q**	**Max**
−0.0106540	−0.0014540	0.0001440	0.0016900	0.0103740

Coefficients				
	Estimate	**Std. error**	**t-value**	**Pr(> |t|)**
HON$return	0.102563	0.009296	11.033	< 2e-16***
MET$return	0.028279	0.006402	4.417	1.07e-05***
WFC$return	0.042243	0.008918	4.737	2.38e-06***
JPM$return	0.036541	0.007994	4.571	5.25e-06***
USB$return	0.029818	0.009602	3.105	0.001935**
CVX$return	0.058902	0.009407	6.262	4.95e-10***
BLK$return	0.056575	0.006229	9.082	< 2e-16***
XOM$return	0.127405	0.010695	11.913	< 2e-16***
C$return	0.020944	0.005662	3.699	0.000224***
UTX$return	0.067742	0.009499	7.132	1.53e-12***
MMM$return	0.089705	0.009478	9.464	< 2e-16***
GE$return	0.069196	0.007741	8.939	< 2e-16***
AXP$return	0.062760	0.006750	9.298	< 2e-16***

Notes: Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1; residual standard error: 0.002621 on 1,519 degrees of freedom (1 observation deleted due to missingness); multiple *R*-squared: 0.9279; adjusted *R*-squared: 0.9273; *F*-statistic: 1,503 on 13 and 1,519 DF, *p*-value: < 2.2e-16.

An additional requirement for an optimal implementation of dispersion trading is the construction of a timing indicator that provides entry signals for this strategy. As previously discussed, Deng (2008) has measured a sharp decrease in performance since the year 2000, but, as noted by Marshall (2009), profit opportunities have also lasted after that date provided that trades are opened and closed following correct timing.

This empirical analysis is primarily based on the premise that dispersion trading is founded upon the difference between the correlation among constituents implicit in index options and the actual correlation. Therefore, the timing indicator is calculated, on a daily basis, as the difference between the implicit correlation of S&P 100 options and the sample correlation of the 13 stocks selected, measured on 30-day-rolling windows. Specifically, the implied volatility is calculated as the mean between the implied volatilities of the at-the-money call and put on each stock. Thereafter, to identify a

proxy of the mean level of correlations among constituents implied by option prices, equation (2) is modified as follows:

$$\rho_t^{impl} = \frac{\sigma_{I,t}^2 - \sum_{i=1}^n w_i^2 \sigma_{i,t}^2}{2\sum_{i=1}^n \sum_{j>i}^n w_i w_j \sigma_{i,t} \sigma_{j,t}}, \quad (7)$$

where $\sigma_{I,t}$ is the 30-day implied volatility of index options at time t, $\sigma_{i,t}$ is the 30-day implied volatility of stock options on the i-th constituent at time t, w_i is the weight in the index of the i-th stock, calculated as in Table 3.

Then, for each day t, the 30-day correlation between the returns of each couple of the 13 selected constituents is estimated and, following equation (7), the aggregated measure of correlation is calculated as follows:

$$\rho_t^{sam} = \frac{\sum_{i=1}^n \sum_{j>i}^n w_i w_j \sigma_{i,t} \sigma_{j,t} \rho_{i,j,t}}{\sum_{i=1}^n \sum_{j>i}^n w_i w_j \sigma_{i,t} \sigma_{j,t}}, \quad (8)$$

where $\rho_{i,j,t}$ is the correlation coefficient between the returns of the i-th and j-th stocks estimated on the preceding 30 days.

Finally, the timing indicator is calculated as the difference between (7) and (8):

$$Indicator_t = \rho_t^{impl} - \rho_t^{sam}. \quad (9)$$

When the indicator is high, a long position in dispersion trading is opened, selling a straddle on the S&P 100 and buying a straddle on the subsample of 13 constituents, because, this way, we invest in a reduction of the implied correlation, converging toward the sample one. An opposite position is taken when the indicator is low, opening a short dispersion trade.

The identification of "high" and "low" levels of the timing indicator requires the use of Bollinger bands, calculated as the 30-day moving average of the indicator plus and minus 1.5 times its 30-day standard deviation (Figure 1). Long and short dispersion trades are opened when the indicator breaks above the upper band or breaks under the lower band, respectively. The list of the opening dates is provided in Table 4.
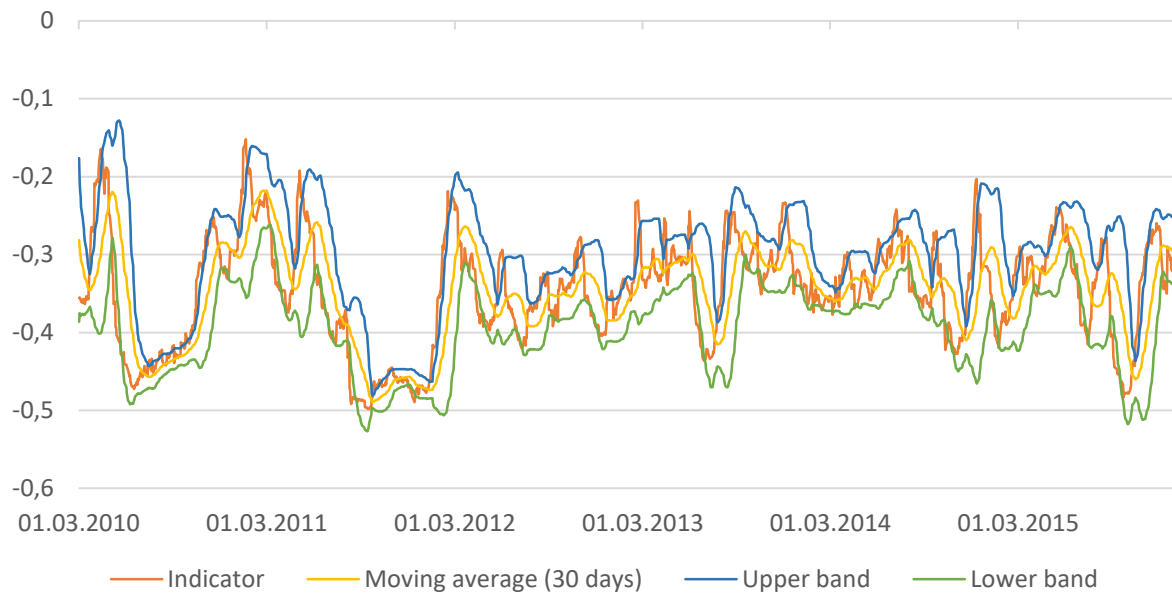
Figure 1. Timing indicator and Bollinger bands

Table 4. List of opening dates

No	Date	Position
1	22/03/2010	Long
2	05/05/2010	Short
3	13/07/2010	Long
4	03/12/2010	Short
5	06/01/2011	Long
6	09/03/2011	Short
7	25/04/2011	Long
8	02/06/2011	Short
9	23/09/2011	Long
10	01/12/2011	Short
11	12/01/2012	Long
12	27/03/2012	Short
13	22/05/2012	Long
14	26/06/2012	Short
15	26/07/2012	Long
16	07/09/2012	Short
17	19/10/2012	Long
19	24/12/2012	Long
20	06/06/2013	Short
21	24/07/2013	Long
22	16/09/2013	Short
23	10/10/2013	Long
24	11/10/2013	Short
25	20/11/2013	Long
26	19/12/2013	Short
27	11/03/2014	Long
28	17/04/2014	Short
29	27/05/2014	Long
30	01/08/2014	Short
31	15/09/2014	Long
32	09/10/2014	Short
33	19/11/2014	Long
34	09/01/2015	Short
35	19/02/2015	Long
36	01/04/2015	Short
37	29/04/2015	Long
38	11/06/2015	Short
39	03/08/2015	Long
40	27/08/2015	Short
41	15/10/2015	Long
42	07/12/2015	Short

The first step before the actual implementation of the back-testing is to verify that every option, bought or sold, both on the S&P 100 and on its subsample of constituents, has the same expiry date. The second step is the calculation of the number of stock options required to be bought or sold for each index option.

Therefore, for each opening date shown in Table 4, a dispersion trade is opened only if there are couples of put and call options with the same expiry and with residual life not less than 10 days. This additional constraint is included because of the erratic trend typical of options close to expiry.

The number of contracts is calculated to minimize the difference between the vega of the S&P 100 options and the vega of the portfolio of options on the 13 selected constituents in order to hedge an increase or decrease in the volatilities of the constituents themselves. The practical implementation requires the calculation of the vega on S&P 100 options and then the theoretical vega of each option according to the weight of the i-th stock in the index w_i as follows:

$$Vega\ th_{i,t} = w_i \cdot Vega\ S\&P100_t \qquad (10)$$

Finally, the number of options bought or sold for each i-th stock is the ratio between the theoretical vega and the measured vega on the i-th constituent:

$$Number\,of\,options_{i,t} = \frac{Vega\,th_{i,t}}{Vega_{i,t}} \qquad (11)$$

Dispersion trades are then closed when the options are at seven days from expiry, or when an opposite trade should be opened according to the timing indicator.

Moreover, dispersion trading is tested by taking into account delta hedging given the deltas calculated at market close. The trade is rebalanced once daily. The delta for the index and each of its 13 selected constituents is the algebraic sum of the deltas of calls and puts, taking into account the long or short position of the trade. The profit and loss of the delta hedging is calculated as the summation of the products of the daily variation of the delta and the price (value) of the corresponding stock (index).

3. RESULTS

The return of the strategy of dispersion trading implemented in this empirical analysis is 23.51% per year compared to the 9.71% return of the S&P 100 index in the same time span. Given an annual standard deviation of 9.42% and a risk-free rate of 0.21%, the Sharpe ratio of the strategy is significant and equal to 2.47.

Another interesting feature is the extremely low correlation coefficient between the returns of this strategy and those of the S&P 100 equal to a mere 0.0372. It is clear that dispersion trading is able to provide a performance completely independent from the stock market; therefore, with a very limited systematic risk.

These outcomes are coherent with the findings on dispersion trading provided by earlier literature, as reported in section 1, in particular with Deng (2008) and Marshall (2009).

CONCLUSION

This article has provided a review of the theoretical basis for dispersion trading and has clarified its interpretation as an arbitrage on the mispricing of index options with regard to the overestimation of the implied correlations among its constituents. The empirical analysis showed how a simple version of dispersion trading, implemented using at-the-money plain vanilla straddles on the S&P 100 and a representative subsample of constituents, has significantly over performed the stock market, showing almost no correlation to the chosen index.

While the empirical results show a strong predominance of dispersion trading if compared to a simple buy-and-hold strategy, a limitation of this analysis is the assumption of the absence of transaction costs. If we ignore slippage (the difference between the expected price of a trade and the price at which it is executed), only a market maker could have replicated the performance of our back-testing. Another limitation of the present analysis is the rather simplified delta hedging technique based upon a simple daily rebalancing. An optimized hedge would have gained higher returns, therefore compensating, at least partially, transaction costs. As a consequence, these two limitations of this study, while present, have opposing effects, and their combined impact is expected to be negligible, especially given the low number of qualified trades (just 29 in six years) and their brief mean length (only 16.31 days).

Therefore, our analysis provides a strong evidence of inefficiencies in the US options market and the presence of a form of "free lunch" available to traders focusing on options mispricing.

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