Why Are Those Options Smiling?

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ABSTRACT

The implied volatility "smile" in option prices has generally been attributed to errors in the Black-Scholes model, specifically the assumption of constant volatility or the assumption of log-normal returns. In other words, the presumption is that, if the implied volatilities were calculated correctly, the smile would disappear. Using stock index options data, we test and reject the hypothesis that the smile in stock index option prices is wholly due to inappropriate distributional assumptions by the Black-Scholes option pricing model. If the true smile is flat, then a trading strategy in which one buys options at the bottom of the incorrect Black-Scholes smile and sells options at the top(s) should not be profitable *even on a pre-transaction-cost basis.* However, we find that such a strategy yields substantial pre-transaction-cost profits. Moreover these profits vary in line with the Black-Scholes model's predictions while they should not if the true smile is flat. Our calculations suggest that roughly half of the observed smile in the stock index options market is due to a smile in the true implied volatilities with the remainder apparently due to a difference between the Black-Scholes implied volatilities and the true implied volatilities. We argue that the true smile persists despite these substantial pre-transaction-cost profits, because maintaining the trading portfolio's original low risk profile requires frequent re-balancing which quickly eats away the profits. Consequently, the smile is not evidence of market inefficiency.

Why Are Those Options Smiling?

One of the most persistent and well-documented financial market anomalies is the cross-sectional implied volatility "smile" or "sneer" reflected in option prices. Since they supposedly represent the market's expectation of likely volatility over the identical remaining life of the option, implied volatilities should be the same for all options with the same maturity date observed at the same time regardless of the strike price. However, it is well-known that in many markets Black-Scholes (B-S) implied volatilities for options with the same expiration date but different strike prices consistently differ cross-sectionally often displaying a persistent "smile" or "sneer" pattern.

To date, virtually all discussions of the smile or sneer have presumed that it reflects erroneous assumptions by the Black-Scholes model about the distribution of the underlying asset's returns - usually either the B-S assumption of constant volatility over the life of the option or the assumption that the underlying asset price follows a geometric Brownian motion. This presumption has lead to the development of a plethora of "smile consistent" models, i.e., models generating options prices which when plugged into the (erroneous) Black-Scholes model yield B-S implied volatilities that follow a smile pattern even though the true implied volatilities (i.e., implied volatilities using the "correct" model) for that expiration date are identical.[1] Most of these models assume that returns follow a jump-diffusion process or that volatility varies (either stochastically or deterministically).[2] According to these models, if the true (or correctly calculated) implied volatilities are flat or "expressionless", the B-S implied volatilities will "smile." However, other studies, such as Das and Sundaram (1999), have concluded that these corrected models can fully explain the smile arguing that either the required distributional assumptions are too extreme or that the models cannot explain the smile term structure.[3]

An alternative explanation of the smile is that it is primarily due not to the failure of the B-S model's assumptions regarding the distribution of returns but to the failure of the no-arbitrage assumption. In other words, It could be that even correctly calculated implied volatilities display a smile pattern and that for some reason arbitrage fails to flatten it.

In this paper, we test whether the smile or sneer in equity index options (probably the most studied smile in the literature) can be attributed to errors in the Black-Scholes formula. Put another way, we test whether the correctly calculated smile is flat. While several recent studies have tested whether specific alternative models can explain the smile and conclude they cannot, this leaves open the question of whether there is a true but undiscovered model which could. For instance, while Das and Sundaram (1999) show that jump-diffusion models cannot explain the smile at long maturities and stochastic volatility models cannot explain the smile at short maturities, a model which combined both a jump process and stochastic volatility conceivably could. An important advantage of our test is that it is not conditional on a particular assumed model. If the observed smile is wholly due to erroneous distributional assumptions in the B-S model, i.e., if the true smile is flat, then a strategy of buying options at or near the bottom of the B-S smile and selling options at or near the top of the smile should not yield excess profits*even on a pre-transaction-cost basis*. However, we find that such a trading portfolios based on the B-S model yields substantial pre-transaction-cost profits. Moreover, these profits vary in line with the B-S model's predictions while they should not if the true smile is flat. On the other hand, the profits are not as large as the B-S formula predicts, suggesting that part of the smile could be due to errors in Black-Scholes.

Our test was presaged by a somewhat tongue-in-cheek suggestion by Rubinstein in his 1994 AFA Presidential Address (Rubinstein, 1994) in which he developed a binomial tree model consistent with the smile (but a model which Dumas, Fleming, and Whaley (1998) have found forecasts less well than an ad hoc B-S model). Rubinstein wrote,

> This [Rubinstein's] discussion overlooks one possibility: the Black-Scholes formula is true but the market for options is inefficient. This would imply that investors using the Black-Scholes formula and simply following a strategy of selling low striking price index options and buying high striking price index options during the 1988-1992 period should have made considerable profits. I have not tested for this possibility, given my priors concerning market inefficiency and in the face of the large profits that would have been possible under this hypothesis, I will suppose that it would be soundly rejected and not pursue the matter further, or leave it to skeptics whose priors would justify a different research strategy.

Our priors are apparently more diverse than Rubinstein's. We examine Rubinstein's trading strategy for the equity index market and find that traders could make very large profits indeed.

Our finding that a trading strategy based on the smile yields large profits raises two questions: (1) if the smile is not wholly due to erroneous distributional assumptions in the B-S formula, what does cause it? and (2) why isn't the smile eliminated by traders employing the profitable strategy we document? Regarding the former, we hypothesize that the equity market smile or smirk is largely caused by hedging pressures - in particular, hedgers buying out-of-the-money puts to protect their portfolios against a possible stock market crash. We posit that these hedge-driven purchases drive up the option prices, and therefore the implied volatilities (IV), of low-strike-price options.[4] This interpretation is supported (1) by trading volume patterns, (2) by the fact that IV's calculated from low strike prices tend to grossly exceed actual volatilities, and (3) by the fact that IV's from high strike price options have higher predictive ability. Also, our calculations suggest that <u>part</u> of the smile is due to errors in the B-S formula, i,e, that B-S implied volatilities do smile more than the true volatilities.

Why doesn't arbitrage based on our documented trading profits flatten the true smile? We posit that it is due to a combination of risk and transaction costs. While our trading portfolios are initially hedged against changes in the price of the underlying asset, the S&P 500 index, we show that maintaining this delta neutrality requires quite frequent re-balancing entailing substantial transaction costs which quickly eat away the profits. Sizable profits net of transaction costs are only possible with infrequent re-balancing requiring acceptance of substantial risk. Thus, our finding that the smile is not entirely due to errors in Black-Scholes does not mean that the market is inefficient. It should be emphasized that the result that post-transaction-cost profits with are small does not rescue the conventional wisdom that the smile is due to the B-S model's assumption of log-normal returns with constant volatility. That hypothesis implies that, since the true smile is flat, a trading strategy based on the incorrect smile should not be profitable *even on a pre-transaction-cost basis.*

The paper is organized as follows. In the next section, we describe our data and document the volatility smile or sneer. In section II, we test the hypothesis that the smile is due to inappropriate distributional assumptions in the Black-Scholes option pricing formula and examine the profitability of delta and gamma neutral trading portfolios in which we simultaneously buy options with low implied B-S volatilities and sell options with high B-S implied volatilities. In III, we explore why trading based on the smile fails to flatten it. In IV, we present our conjecture on the main source of the smile. Section V concludes the paper.

I. Implied Volatility Patterns in the Equity Index Market

A. Data

Our data set consists of daily closing prices of options on S&P 500 futures traded on the Chicago Mercantile Exchange from January 1, 1988 through April 30, 1998 and closing prices of the S&P 500 futures themselves. While options on the index and options on the nearby futures are virtually indistinguishable, the latter have several advantages: (1) implied volatility calculations are not dependent on unknown future dividends, (2) since the options and futures markets close at the same time, non-synchronous prices not a serious problem, and (3) arbitrage between options and the futures contract is easier than between options and all 500 stocks. Since it is well-documented that a shift in the smile occurred following the 1987 crash, our data set begins after that date.[5]

Our data set consists of daily observations on the nearest-to-expiration options with at least 10 market days to maturity. We generally observe implied volatilities calculated from the closest-to-maturity options. However, on the ninth day before expiration, we switch to the next contract since (1) the time value of very short-lived options relative to their bid-ask spreads is small and (2) the set of traded strike prices shrinks as expiration approaches.[6] We only collect data for one expiry each day.

Each day we observe both calls and puts at a variety of strike prices for options with the same expiration date. In the S&P 500 futures options market, these strikes are in increments of 5 points, e.g., 825, 830, 835 etc. Since trading is light in some far-from-the-money contracts, we restrict our sample to the first eight out-of-the-money and first eight in-the-money contracts relative to the underlying S&P 500 futures price. When, for instance, the S&P 500 futures index is 1001, we collect prices for options with strike prices from 965 to 1040. In summary, each day we observe and calculate implied volatilities for up to sixteen calls and up to sixteen puts. However, since all 32 options do not trade each day, we do not always have 32 observations. In Figure 1 we report average daily trading volumes at each relative strike price. Note that trading is higher in out-of-the-money than in in-the-money-options and is generally higher the closer the strike is to the underlying index. Note that among far-from-the-money strikes, trading is heaviest in out-of-the-money puts, which is consistent with the argument that hedgers demand these to insure their portfolios against stock market declines. In Figure 2, we report the number of daily observations at each relative strike price.

Using Black's (1976) model for options on futures,[7] day t closing prices for both the option and S&P 500 futures, and 3-month T-bill rates, we solve for the implied standard deviation, $ISD_{j,t}$, on each of the (32 or less) options j observed on day t. For easier interpretation, these are annualized by multiplying by the square root of 252, the approximate number of trading days each year.

B. The Smile

The smile in our data set is reported in Table 1 and in Figure 3. For each option j on every day t, we calculate both the implied standard deviation, $ISD_{j,t}$, and the relative percentage "moneyness" of option j's strike price measured as $(K_{j,t}/F_t)$-1 where $K_{j,t}$ is option j's strike price and F_t is the underlying futures price on day t. Time series means of both ISD and (K/F)-1 are reported in Table 1 and the former is graphed against the latter in Figure 3. The following nomenclature is used in Table 1 to identify calls and puts and strike price groups j. The first letter, "C" or "P," indicates **c**all or **p**ut, the second, "I" or "O", indicates whether the option is **i**n or **o**ut of the money, and the last digit, "1" through "8", reports the strike price position relative to the underlying futures price where "1" is the closest to the money and "8" is the furthest in- or out-of-the-money. For example, CI3 indicates an in-the-money call option whose strike price is the third strike below the futures price.

As shown in Figure 3, there is a strong "smile," or "smirk" pattern in this market with the implied standard deviation consistently falling as the strike price increases until moderately high strike prices are reached at which point the ISDs start to rise. The cross-sectional differences in implied volatilities are striking and sizable in that implied volatilities at the peak of the smile are over 50% higher than those at the trough. Clearly, the null hypothesis that the ISDs do not differ cross-sectionally is rejected.

In Table 1 and Figure 3, we also include a measure of actual or “realized” volatility. For each day t, RLZ_t, is calculated as the standard deviation of returns over the period from day t through the option’s expiration date:

$$RLZ_t = \sqrt{252 \times \left[\frac{1}{N-t-1} \sum_{s=t+1}^{N} (R_s - \bar{R})^2 \right]} \tag{1}$$

where $R_s = \ln(F_s / F_{s-1})$ and F_s is the final futures price on day s. As noted above, 10#N-t#35. Since all options j observed on day t expire on the same day, RLZ only has a t subscript.

The mean of RLZ_t, is shown in both Table 1 and Figure 3. As shown there, both put and call ISDs at <u>all sixteen</u> strike prices tend to overestimate actual realized volatility over the life of the options and this over-estimation is substantial for many options. For all 32 options, the null that $ISD_{j,t}$ is an unbiased predictor of RLZ_t is rejected at the .01 level.[8] As further evidence, in columns 6 and 13 of Table 1, we report the frequency with which the implied standard deviation overestimates actual volatility over the life of the option. As shown there, for both puts and calls, the ISDs calculated from the five lowest strike prices overestimated RLZ_t more than 90% of the time. Even at-the-money ISDs overestimated RLZ_t more than 75% of the time. Clearly, the tendency for implied volatilities to over-estimate actual subsequent volatility is strong. One caveat is in order however. Since our data set begins in 1988, it is possible that the ISDs tended to exceed realized volatility partially because the former reflected the possibility of another 1987 style market crash which did not in fact reoccur . If the data set is extended backward through the 1987 crash, then the mean ISDs at the bottom of the smile are insignificantly different from the mean RLZ.

As previously noted, we do not observe option prices at all sixteen strike prices every day. If far-from-the-money option prices tend to be observed more when expected volatility is high or low, then the smile in Figure 3 could be a biased representative of the true cross-sectional smile. Accordingly, we recalculate the smile as follows. First, on each day t, we calculate the average ISD of the four nearest-the-money options: $CI1_t$, $CO1_t$, $PO1_t$, and $PI1_t$ which we label A_t. For each of options observed on day t, we then calculate the ratio $R_{j,t} = ISD_{j,t}/A_t$. Group j means of this ratio are reported in columns 5 and 11 of Table 1 and graphed in Figure 4. As shown there, the smile now resembles a sneer which levels off rather than increasing at high strike prices and most of the differences between call and put ISDs disappear.[9]

II. Trading on the Smile

A. The Trading Portfolios

Presuming that the smile is due to errors in the B-S formula, researchers have focused on developing pricing models based on alternative assumptions which could explain it, such as models based on stochastic volatility, models in which volatility rises if prices fall, or models based on alternative return distributions (e.g. jump-diffusion processes). The presumption is that, if measured correctly, there would be no smile, i.e., that the true implied volatilities are identical for all options with the same expiry. Consider a trading strategy of buying options which the B-S formula identifies as underpriced (that is options near the bottom of the smile) and selling the options which are overpriced according to B-S (those near the top of the smile). If indeed the true smile is flat, that is if the true cross-sectional ISD's are all equal and a smile is only observed because of inappropriate distributional assumptions in the B-S formula, then this strategy should not return abnormal profits even on a pre-transaction-cost basis since the options which B-S identifies as overvalued are not in fact overpriced and those B-S identifies as undervalued are not truly undervalued. However, if B-S is substantially correct, this strategy should be profitable.

To test whether the true smile is flat, we implement a trading strategy in which we buy puts and calls near the bottom of the B-S smile and simultaneously sell puts and calls near the top of the B-S smile in proportions such that net investment is zero and the portfolio is hedged against changes in the underlying S&P 500 index. After one, five, or ten days, we close our position. To implement this trading strategy we proceed as follows:

1. For each day t in our data set, we restrict the set of tradable options to those whose time value is at least $1.00 and whose strike price is within five percent of the underlying S&P 500 index futures price. In order to maximize potential profits, we should normally sell the options with the highest ISDs, which according to Table 1 and Figures 3 and 4 would normally be CI8 and PO8. However, as illustrated in Figures 1 and 2, trading in deep-in-the-money calls, like CI8, is relatively light so it is possible that traders would have difficulty unwinding positions involving these more extreme options. Also, since S&P 500 options are quoted in increments of $.05, ISDs calculated from very low price options may not be reliable. Hence, we place this restriction on our choices.[10]

2. From among the remaining options on day t, we choose the call, CH, and put, PH, with the highest implied volatilities subject to the condition that the strike price, $K_{j,t}$ is less than the futures price, F_t. Likewise, we find the call, CL, and put, PL, with the lowest ISDs subject to the condition $K_{j,t}>F_t$. We form a trading portfolio from these four options on day t iff $ISD_{CH,t}-ISD_{CL,t}>.03$ and $ISD_{PH,t}-ISD_{PL,t}>.03$. Since the intention of the trading strategy is to exploit ISD differences, we require that a difference of at least .03 exist (which is small relative to the differences documented in Table 1). One or both of these conditions is not met on 618 of our 2611 trading days leaving us with a sample of 1993. Descriptive statistics on these options: their

implied standard deviations, their relative strike prices, and their sensitivity to the price of the underlying asset (both delta and gamma) are reported in Table 2. As shown there, the difference between $ISD_{CH,t}$ and $ISD_{CL,t}$ averages .051 while the difference between $ISD_{PH,t}$ and $ISD_{PL,t}$ averages .052.

3. We form trading portfolios which are hedged against changes in the S&P 500 index. Let $P_{CL,t}$, $\Delta_{CL,t}$, and $\gamma_{CL,t}$ represent the price, delta, and gamma[11] respectively of the low ISD call on day t and similarly for CH, PH, and PL. Let $N_{CL,t}$ and $N_{PL,t}$ represent the number of contracts in options CL and PL which are purchased on day t and let $N_{CH,t}$ and $N_{PH,t}$ represent the number of contracts in CH and PH which are sold or written on day t. $N_{CL,t}$, $N_{PL,t}$, $N_{CH,t}$, and $N_{PH,t}$ are chosen to satisfy the following requirements:

$$N_{CL,t}P_{CL,t} + N_{PL,t}P_{PL,t} - N_{CH,t}P_{CH,t} - N_{PH,t}P_{PH,t} = 0 \tag{2}$$

$$N_{CL,t}P_{CL,t} + N_{PL,t}P_{PL,t} = N_{CH,t}P_{CH,t} + N_{PH,t}P_{PH,t} = \$100 \tag{3}$$

$$N_{CL,t}\Delta_{CL,t} + N_{PL,t}\Delta_{PL,t} - N_{CH,t}\Delta_{CH,t} - N_{PH,t}\Delta_{PH,t} = 0 \tag{4}$$

$$N_{CL,t}\gamma_{CL,t} + N_{PL,t}\gamma_{PL,t} - N_{CH,t}\gamma_{CH,t} - N_{PH,t}\gamma_{PH,t} = 0 \tag{5}$$

Equation 2 imposes the condition that the trading portfolio is costless (ignoring transaction costs), i.e., that sale of the high ISD options finances the purchase of the low ISD options. To make the portfolios comparable, equation 3 standardizes all portfolios to a nominal gross value of $100. In other words, $100 is raised by selling the high ISD options and these funds are used to purchase $100 of low ISD options. As is conventional, all prices are quoted in terms of one S&P 500 index unit.[12]

Equation 4 ensures that the trading portfolio is delta neutral so that the value of the portfolio is unaffected by small changes in the underlying S&P 500 index (assuming the B-S pricing formula is correct). Of course, since option prices are convex, an option's delta changes as the underlying asset's price changes so a portfolio which was originally delta neutral may not be after the underlying index futures price changes. Equation 5 imposes the condition that the portfolio also be gamma neutral, i.e., that small changes in the underlying index leave the portfolio delta unchanged at zero. If the B-S formula is correct, equation 5 should increase the range over which the portfolio is immunized against changes in the underlying asset price.[13]

For each day in our data set, we form the trading portfolios by solving equations 2-5 for the four N's. Descriptive statistics on the composition and characteristics of the resulting portfolios are provided in Table 3. As shown there, on an average day we write 2.39 high-ISD call contracts raising $48.83 and 24.57 high-ISD put contracts raising $51.17. We use this $100 to buy an average of 5.73 low-ISD put contracts at an average cost of $81.12 and 11.06 low-ISD call contracts at an average cost of $18.88.

In Table 3, we also report the portfolios' time t "Greeks". Of course delta and gamma are zero by construction. In all of our portfolios, theta is positive implying that the portfolio value should increase as time passes.[14] In all portfolios, vega is negative implying that an equal increase in <u>implied</u> volatility across all four options would lower the portfolio's value.

B. Predicted Trading Profits.

Before examining the results, consider the expected profitability of this strategy according to the Black-Scholes formula. The greatest and quickest profit opportunity would occur if the options are only temporarily mis-priced, i.e., if soon after the trading portfolio is formed,.the ISD's of the sold options, CH_t and PH_t, decline while those of the bought options, CL_t and PL_t, rise so that the smile flattens. This does not happen in our data. Previous studies have observed that the smile is more pronounced at shorter maturities than at longer horizons and indeed in our data, $ISD_{CH,t}$ and $ISD_{PH,t}$ rise slightly on average over our holding periods while $ISD_{CL,t}$ and $ISD_{PL,t}$ are basically unchanged. Given the negative vegas, this tends to cause small losses for our trading strategy.

However, a flattening of the smile is not necessary in order for our strategy to be profitable. All other things equal, all options tend to lose value as time passes and the time-to-expiration becomes shorter. Consequently, a given ISD difference between two options translates into a smaller price difference as time passes. The greater an option's time value, the further it falls as expiration approaches so, all other things equal, prices of options with high ISDs (which we sold) should fall more than prices of low ISD options (which we buy) over time. All other things equal, the expected trading profits according to the B-S formula are equal to the portfolio's theta times the length of the holding period. As shown in Table 3, the mean theta is 506.7315 so the mean predicted one-day holding period profit according to the Black-Scholes formula is 506.7315/252=$2.011 - a substantial one-day profit on a $100 nominal position with $0 net investment.[15]

C. Results: Pre-transaction Cost Trading Profits

In Panel A of Table 4, we report profits and losses for 1, 5, and 10 (market) day holding periods without re-balancing and without transaction costs. For the 5 and 10 day holding periods, we restrict the data set to options which will have at least 10 days to expiration at the end of the holding period since low trading volume in far-from-the-money-close-to-expiration options may make closing these positions close to expiration difficult.[16]

As reported in Table 4, average profits are \$0.86, \$4.35, and \$ 8.11 for the 1, 5, and 10 day holding periods respectively. These are substantial short-term profits for costless positions with a nominal value of \$100, and all are significant at the .0001 level.[17] Over a one-day holding period 63.1% of the trades are profitable and 65.1% are profitable over a five day horizon. Clearly the data reject the hypothesis that the smile is totally due to erroneous distributional assumptions by the Black-Scholes model which would disappear if the correct pricing formula were used. Instead, it is clear that B-S correctly identifies mis-priced options allowing one to profit from these mis-pricings.

The profits in Panel A of Table 3 are ex-post in the sense that prices and implied volatilities at time t are used to choose which options will be bought or sold and how many, N, and it is assumed that the trader can trade at these same time t prices. In reality, once the trader has observed the prices, determined the composition of his portfolio, and placed his order, the prices will have changed. Consequently, in Panel B we present ex-ante profits. Again the options to be traded are determined on day t and the proportions, $N_{CL,t}$, $N_{CH,t}$, $N_{PL,t}$, and $N_{PH,t}$ are determined using the prices, deltas, and gammas on day t. However, the portfolio's are formed on day t+1 based on the prices at that time and unwound on day t+2 (for the 1-day holding period), day t+6 (for the 5-day holding period) or day t+11 (for the 10-day holding period). Again, we observe significant positive holding period returns at all three holding periods. Indeed, the profits in Panel B actually slightly exceed those in Panel A for the one and five-day holding periods. Clearly, there is ample time to run the calculations, determine the composition of the trading portfolio, place one's order, and still make money. The standard deviation of profits for the one-day holding period increase sharply from \$3.77 to \$5.69. This increase was expected since the portfolios which were delta and gamma neutral on day t need not be on day t+1. Since it makes little difference and one day is much longer than necessary to determine the portfolio's composition and execute the trades, we henceforth work with the ex-post profits in Panel A.

To further test whether the smile represents errors in the B-S model, we next examine whether these profits behave as the B-S equation predicts. As explained above, according to the B-S formula, profits on these portfolios should be greater the greater their time t thetas. If the true smile is flat, then the profits should be unrelated to the B-S formula's predictions. To test whether trading profits vary with theta as the B-S model predicts, we split our sample of trading portfolios into two subsamples: (1) portfolios whose estimated theta exceeds the median theta value of 464.3 and (2) portfolios with thetas below 464.3. One-day holding period profits average $1.28 for the first subsample versus $.44 for the second. Average 5-day trading profits are $5.46 when theta is above its median versus $3.25 when theta is below its median. Both differences are significant at the .0001 level. Clearly profits behave as the B-S formula predicts and our trader could use this information to increase her average profits.

While it is clear that the smile cannot be entirely attributed to the B-S model's assumption of constant volatility or log-normal returns, there is also evidence that the correctly calculated smile is somewhat flatter than that calculated from B-S implied volatilities. As noted above, according to the figures in Table 3, the B-S model predicts an average one-day profit of $2.011. Actual one-day ex-post profits average 43% of that or $.859 while average ex-ante profits are $1.031 or 51% of the B-S estimate. A small part of this difference can be attributed to the fact discussed above that the smile tends to become more pronounced as expiration approaches but this effect is minor. Hence, it appears that the true smile is somewhat flatter than the observed B-S smile as the stochastic volatility and jump-diffusion models predict.

In summary, if the smile were entirely due to incorrect distributional assumptions by the Black-Scholes formula as is commonly assumed, then a trading strategy based on the smile and B-

S should not be profitable. In other words, if the true smile were flat, then a trading strategy based on buying options at the bottom of the false smile and selling options at the top should not be profitable even on a pre-transaction-cost basis. This hypothesis is overwhelmingly rejected. Instead, we find that a trading strategy based on the Black-Scholes smile is quite profitable indeed. Moreover, trading profits vary across portfolios as the B-S model predicts.   Clearly B-S correctly identifies over- and under-priced options. However, there is also evidence that part of the smile may be due to errors in the B-S model since trading profits are somewhat less than the B-S model predicts.

III. Risk and Transaction Costs

These results raise the obvious question: "If such large trading profits are possible, why is the smile not flattened by traders following this profitable strategy"? As traders buy options at the bottom of the smile and sell options at the top of the smile, ISDs on the former should rise and ISDs on the latter should fall flattening the smile. We think the answer involves a combination of risk and transaction costs. Although our trading portfolios are both delta and gamma neutral initially, they quickly lose this immunization against changes in the S&P 500 index. Keeping the positions low-risk requires frequent re-balancing which would eat away all our pre-transaction cost profits.

While the mean five and ten day profits documented in Table 4 exceed liberal estimates of transaction costs, the trades are quite risky. In Table 4, we report the standard deviation of profits and upper and lower decile values. The one-day and five-day profit standard deviations of ex-post profits are $3.77 and $10.97 respectively. In 10% of the cases, one would have lost $2.69 or more in one day and $6.15 or more in five days following our strategy. This high profit variability is particularly surprising for the one-day holding period, since our portfolios are initially delta neutral and changes in the S&P 500 index over a single day should normally be small. Clearly, the trading strategy is not riskless despite the fact that our portfolios are constructed so that they are initially both delta and gamma neutral.[18]

However, the standard deviations in Table 4 overstate the risk somewhat since, as we have just seen, the trading profits vary with the portfolio's theta, which traders would know a priori. To remove the predictable part of the profit variation in Table 4, we regress actual profits on predicted profits, i.e., on the product of theta times the length of the holding period. For instance, for the 1-day holding period we estimate the regression:

$$P_t = \alpha_0 + \alpha_1 [\theta_t (1/252)] + U_t \qquad (6)$$

where P_t is the 1-day holding period profit or loss for the portfolio formed on day t and θ_t is its theta. The residual, U_t measures the unpredictable profits. U's standard deviation, which gives a better measure of the true risk faced by a trader following our strategy is $3.716, $10.858, and $19.117 for the 1, 5, and 10 day holding periods respectively. While slightly less than the standard deviations in Panel A of Table 4, these are still sizable. Clearly, substantial risk remains.

What causes this profit variation across our trading portfolios? According to B-S, profits on an option portfolio over a given holding period should depend on changes in (1) the price of the underlying asset, (2) expected volatility, and (3) the interest rate. Since our trading portfolios are delta-gamma neutral by construction and since the impact of changes in the interest rate over our holding periods is negligible, we first examine how much of the variation is due to changes in the market's estimate of likely future volatility. As shown in Table 3, our portfolios have negative vegas so they rise(fall) in value if expected volatility over the life of the option falls(rises). While each of the four options in our portfolio has a separate ISD, to relate portfolio profits to changes in each of these would be spurious. Since the ISDs are calculated from the option's price, they would reflect price fluctuations from any source. However, we can estimate how much of the profit variation can be attributed to overall changes in the market's volatility expectation. To do this, we calculate the average time t ISD on the four nearest-the-money options, CO1, CI1, PO1, and PI1, which we label $ISD4_t$. We also calculate $ISD4_{t+1}$ (or t+5 or t+10). We then estimate the regression:

$$U_t = \alpha_0 + \alpha_1 [V_t (ISD4_{t+1} - ISD4_t)] + \epsilon_t \tag{7}$$

where U_t represents the unpredictable profits from equation 6 above and V_t is the portfolio's time t vega. While $\hat{\alpha}_1$ is positive and significant at the .001 level for all three holding periods, the R^2 is only .061, .001, and .019 for the 1, 5, and 10 day holding periods respectively. Clearly little of the profit variation can be attributed to changes in expected volatility.

Although our trading portfolios are delta and gamma neutral initially by construction, most of the unexpected profits and losses are apparently due to changes in the S&P 500 index. Despite the fact that the portfolios have a zero gamma initially, many of our portfolios quickly lose their delta neutrality. When we calculate deltas on day t+1, the mean is still approximately zero but the standard deviation is .589. Clearly, the portfolios are only immunized against very small changes in the underlying S&P 500 index.

To explore how much of the unexpected profits or losses can be attributed to changes in the underlying index over the holding period, we proceed as follows. While the time t delta, $\Delta_{t,}$ is zero, Δ_{t+1} is not and an average of the two, i.e. $.5\Delta_{t+1}$, gives us a measure of the average delta over the one-day holding period. Accordingly, we regress the unexpected trading profits, U_t, on: (1) this average delta times the change in the underlying S&P 500 index, e.g., $.5\Delta_{t+1}(F_{t+1} - F_t)$,and (2) .5 times the average of the time t and t+1 gammas times the squared change in the underlying index,[19] e.g., $.25\gamma_{t+1}(F_{t+1} - F_t)^2$:

$$U_t = \beta_0 + \beta_1 [.5\Delta_{t+1}(F_t - F_{t+1})] + \beta_2 [.25\gamma_{t+1}(F_t - F_{t+1})^2] + \epsilon_t \tag{8}$$

For the 5 and 10 day holding periods, t+1 in these regressions is replaced by t+5 and t+10 respectively and U_t is measured accordingly as the profit surprise over these holding periods. In all three regressions, both coefficients are significant at the .0001 level. More important, the R^2s are .429, .667, and .785 for the 1, 5, and 10 day holding periods respectively. It is clear from these regressions that most of the profit variation - particularly at the longer horizons - is due to the fact that our portfolios do not remain delta neutral over our holding periods.

One way to reduce this risk would be to re-balance the portfolios frequently which raises the issue of transaction costs, which in this market consist of brokerage commissions, and bid-ask spreads. The impact of brokerage commissions without rebalancing is shown in panel A of Table 5 where we subtract estimated commissions from the profits reported in Panel A of Table 4. Commissions at on-line and discount brokerages range from \$1.50 to \$2.25 per contract and are higher at full-service brokerages. In Panel B, we assume a one-way commission of \$2.00 per contract. Prior to Nov 1, 1997, one S&P 500 futures contract called for payment of 500 times the index. On 11/1/97, this was changed to 250 times the index. Consequently, a \$2.00 commission translates into a transaction cost of \$.004 per index unit, which has been our unit of measurement in all the tables, prior to 11/1/97 and \$.008 thereafter. Profits net of these charges are reported in Panel A of Table 5. As shown there, these commission charges reduce average profit figures about \$.35 from those in Panel A - leaving substantial profits at all holding periods.

Except for floor traders, traders following our trading strategy would also face bid-ask spreads. Since there are no specialists, bid-ask spreads cannot be directly observed in this market but it is commonly accepted is that the bid-ask spread in the S&P 500 futures options market is generally one tick or \$.05. Since some of our options are fairly low in price (C_L and P_H average \$2.23 and \$3.22) respectively, this translates into a fairly major expense in percentage terms. In Panel C we report profits assuming both a bid-ask spread of \$.05 and a commission of \$2.00. As shown there, the average 1-day profit of $.86 turns into an average loss after transaction costs of $1.69 and average 5-day profits are reduced from $4.35 to $1.94.

In summary, it is possible to make profits by trading on the smile but only by accepting substantial risk. Over a 1-day holding period, transaction costs completely eliminate all profits. Over five or ten day holding periods, positive profits can be earned on average after transactions costs but only without re-balancing which means accepting substantial risk. Frequent re-balancing to reduce this risk would entail additional transaction costs which would quickly eliminate the positive expected profits.[20] Consequently, while we reject the hypothesis that the smile is attributable to errors in the Black-Scholes formula, it does not follow that the market is inefficient. We have shown that the true smile is indeed a smile, not flat as alternative models assume. However, it is impossible to profit from this phenomenon without accepting substantial risk.

IV. An Alternative Explanation of the Smile

If the smile is not wholly caused by errors in the B-S formula, what is responsible? Although we can offer only circumstantial evidence, we conjecture that the smile is at least partially due to hedging pressures. Holders of common stock portfolios can hedge against declines in the market value of their portfolios by buying out-of-the-money S&P 500 puts. This is a commonly cited use of the market and as shown in Figure 1, trading is relatively high in out-of-the-money puts. Such purchases for hedging purposes would tend to drive up the price of these puts and therefore their implied standard deviations. As the ISD's on out-of-the-money puts rise relative to calls with the same strike prices, this would set off put-call-parity arbitrage, which (unlike the trading strategy documented here) is relatively riskless since the two strike prices are the same. Such arbitrage would equalize ISDs on puts and calls with the same strike prices (and as shown in Figure 4, they are virtually identical) while moderating the rise in the put ISDs.

This explanation is consistent with both the high trading volume in out-of-the-money puts and the fact that generally ISD's on low strike price options far exceed actual ex-post volatility in the S&P 500 index. Consider another piece of evidence. If the smile is caused by hedgers' strong demand for far-out-of-the-money puts, then prices of these low strike price options should be less closely related to the market's volatility expectation than the prices of higher strike price options. In other words, prices and ISD's on low strike price options should be largely determined by hedging demand while prices and ISD's on at-the-money and high strike price options should be largely determined by volatility expectations. If our hypothesis is correct, we would expect ISD's on low-strike-price options to be relatively unrelated to actual ex post volatility than higher-strike-price options - as well as being upward biased. In a separate paper, the authors test whether this is in fact the case and find, as our theory predicts, that ISD's at the bottom of the smile have considerable predictive ability while those at the top do not.

V. Conclusions

We have shown that the smile in equity index options cannot be wholly attributed to deficiencies in the Black-Scholes formula, such as the presumption of constant volatility or the presumption of a log-normal return distribution. While the true, or correctly calculated, smile appears somewhat flatter than that calculated using the B-S model, it is clearly far from flat. Despite its supposed deficiencies, the Black-Scholes formula performs very well in correctly identifying mis-priced options.

As reflected in the quote from Rubinstein's Presidential address at the beginning of this paper, the profession's reluctance to accept the proposition that a correctly calculated smile could truly exist stems from the notion that, if a true smile did exist, it should be quickly eliminated by trading or arbitrage. We have shown that while such a trading strategy exists and is quite profitable, it is not so simple. In particular, despite the fact that it is initially gamma neutral, keeping the trading portfolio delta neutral would require quite frequent re-balancing which would eat up the profits. Consequently, the existence of a smile is not necessarily incompatible with market efficiency. The fact that the trading strategy is no longer profitable once re-balancing cost are considered does not mean that the B-S formula is wrong and that the correctly calculated smile is flat. If the correctly calculated smile is flat, then the trading strategy should not be profitable even on a pre-transaction-cost basis and it clearly is. Moreover the profits behave as the B-S model predicts while they should not if the correctly calculated smile is flat.

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Table 1
The Implied Volatility Smile: Implied Volatilities and Moneyness by Strike Price

Calls							Puts						
Strike Price	Implied Standard Deviation		Mean K/F -1	Mean ISD Ratio	% of obs > RLZ	Obs	Strike Price	Implied Standard Deviation		Mean K/F -1	Mean ISD Ratio	% of obs > RLZ	Obs
	Mean	Std. Dev.						Mean	Std. Dev.				
CI8	0.2200	0.0507	-0.074	1.4840	0.9403	1,306	PO8	0.2269	0.0568	-0.085	1.5390	0.9602	2,487
CI7	0.2110	0.0498	-0.066	1.4186	0.9424	1,475	PO7	0.2184	0.0566	-0.075	1.4598	0.9557	2,574
CI6	0.2008	0.0473	-0.059	1.3542	0.9370	1,729	PO6	0.2071	0.0539	-0.064	1.3802	0.9464	2,595
CI5	0.1913	0.0467	-0.049	1.2857	0.9203	1,956	PO5	0.1964	0.0530	-0.052	1.3007	0.9301	2,603
CI4	0.1816	0.0471	-0.039	1.2199	0.9084	2,248	PO4	0.1859	0.0515	-0.040	1.2245	0.9115	2,609
CI3	0.1723	0.0468	-0.028	1.1499	0.8799	2,447	PO3	0.1755	0.0502	-0.029	1.1516	0.8834	2,608
CI2	0.1645	0.0473	-0.017	1.0838	0.8452	2,564	PO2	0.1657	0.0489	-0.017	1.0841	0.8461	2,605
CI1	0.1568	0.0469	-0.006	1.0247	0.7989	2,596	PO1	0.1568	0.0474	-0.006	1.0242	0.8010	2,598
CO1	0.1497	0.0460	0.0056	0.9752	0.7509	2,605	PI1	0.1493	0.0460	0.0056	0.9756	0.7538	2,547
CO2	0.1443	0.0448	0.0173	0.9386	0.7083	2,605	PI2	0.1433	0.0449	0.0170	0.9369	0.7089	2,442
CO3	0.1404	0.0435	0.0289	0.9139	0.6746	2,603	PI3	0.1405	0.0432	0.0281	0.9081	0.6695	2,085
CO4	0.1382	0.0419	0.0403	0.9022	0.6579	2,575	PI4	0.1410	0.0413	0.0380	0.8908	0.6511	1,625
CO5	0.1380	0.0404	0.0507	0.9038	0.6466	2,414	PI5	0.1448	0.0404	0.0474	0.8909	0.6545	1,230
CO6	0.1411	0.0402	0.0596	0.9082	0.6637	2,052	PI6	0.1519	0.0403	0.0584	0.9131	0.7119	972
CO7	0.1459	0.0409	0.0671	0.9092	0.6696	1,592	PI7	0.1649	0.0413	0.0679	0.9509	0.7176	694
CO8	0.1519	0.0409	0.0720	0.9052	0.6593	1,168	PI8	0.1764	0.0429	0.0749	1.0002	0.7338	556
RLZ	0.1299	0.0564	--------	--------	--------	2,611							

Based on daily observations from Jan. 1, 1988 through April 30, 1998 of options on S&P 500 futures maturing in 10 to 35 trading days.
In the “Strike Price” column, the first letter (C or P) stands for a **C**all or a **P**ut; the second letter (I or O) refers to **I**n-the-money or **O**ut-of-the-money; and the last digit indicates the relative position of an option from the money where 1 indicates that the option is the nearest-to-the-money and 2 indicates that the option is the second nearest-to-the-money etc.
RLZ stands for the realized volatility over the remaining life of the options.
(K/F -1) is a measure of how far in or out of the money an option is. K = strike price. F=underlying S&P 500 index futures price.
The “ ISD Ratio” in columns 5 and 11 measures the ratio of the implied standard deviation at that strike price to the average ISD of the four at-the-money options: CI1, CO1, PI1, and PO1.

Table 2
Characteristics of Options Included in the Trading Portfolios

On each of 1993 trading days between 1/1/1988 and 4/30/1998, we form costless delta-gamma neutral trading portfolios consisting of four options subject to the restrictions outlined in the text: the call, CH, with the highest implied standard deviation (ISD) that day, the call, CL, with the lowest ISD that day and the puts, PH and PL, with the highest and lowest ISDs respectively- all subject to the restrictions outlined in the text designed to restrict the choice set to actively traded options. Characteristics of those options are reported..

	Mean	Std. Dev.	10%	Median	90%
Price					
CL	$2.2299	$1.9137	$1.10	$1.60	$4.00
CH	$24.1104	$10.8425	$13.80	$20.00	$42.70
PL	$18.1288	$10.5401	$8.90	$14.25	$37.75
PH	$3.2182	$3.0568	$1.15	$2.15	$6.60
Implied Standard Deviations (ISD)					
CL	0.1382	0.0425	0.0918	0.1299	0.1988
CH	0.1887	0.0490	0.1346	0.1776	0.2539
PL	0.1391	0.0427	0.0928	0.1306	0.2000
PH	0.1910	0.0492	0.1375	0.1802	0.2573
Delta					
CL	0.2022	0.0550	0.1336	0.1971	0.2766
CH	0.7987	0.0640	0.7101	0.8137	0.8693
PL	-0.7684	0.0687	-0.8481	-0.7786	-0.6778
PH	-0.1871	0.0556	-0.2710	-0.1738	-0.1247
Gamma					
CL	0.0176	0.0077	0.0066	0.0178	0.0281
CH	0.0120	0.0044	0.0059	0.0123	0.0177
PL	0.0185	0.0077	0.0069	0.0188	0.0288
PH	0.0114	0.0041	0.0058	0.0118	0.0168
ISD Difference					
CH - CL	0.0506	0.0135	0.0351	0.0484	0.0688
PH - PL	0.0520	0.0138	0.0357	0.0503	0.0707

Table 3
Composition and Characteristics of the Trading Portfolios

On each of 1993 trading days between 1/1/1988 and 4/30/1998, we form trading portfolios by shorting N_{CH} option contracts in the high ISD call, CH, and shorting N_{PH} units of the high ISD put, PH. We use these funds to buy N_{CL} contracts in the low ISD call, CL, and N_{Pl} contracts in the low ISD put. The portfolios are constructed to be both costless and delta-gamma neutral. Characteristics of the resulting trading portfolios are reported.

	Mean	**Std. Dev.**	**10%**	**Median**	**90%**
Number of options (N) bought or sold					
CL	11.0570	4.0904	4.8879	11.4914	15.9619
CH	2.3898	0.9643	1.1578	2.3520	3.6459
PL	5.7298	2.4796	2.1654	5.6519	8.9831
PH	24.5722	12.5072	7.8858	23.7050	41.9247
Dollar Amount (N*Price)					
CL	$18.8827	$3.6389	$14.4925	$18.6178	$23.4538
CH	$48.8280	$7.5501	$38.5236	$49.6867	$57.5486
PL	$81.1174	$3.6389	$76.5462	$81.3822	$85.5075
PH	$51.1720	$7.5501	$42.4514	$50.3133	$61.4764
Portfolio Greeks					
Delta	0.0000	0.0000	0.0000	0.0000	0.0000
Gamma	0.0000	0.0000	0.0000	0.0000	0.0000
Theta	506.7315	238.2896	249.7466	464.2617	832.1887
Vega	-242.2520	115.3106	-413.7370	-218.7530	-108.2500

Table 4
Trading Profits

We report pre-transaction-cost trading profits and losses for the zero cost trading portfolios described in Table 3 in which we long puts and calls with low Black-Scholes implied standard deviations and short puts and calls with high B-S ISDs. The portfolios are initially both delta and gamma neutral. The trading portfolios are held one, five, or ten market days without re-balancing. The one, five, and ten day holding periods end at least ten days before the options expire. In Panel A we report ex post profits in which the trading portfolios are formed at the same day t prices used todetermine the composition of the portfolios. In Panel B, we report ex ante profits for portfolios established using the prices on day t+1.

Holding Period	Profits						Obs
	Mean	t	Std. Dev.	10%	Median	90%	
Panel A: Ex - Post Results							
1 Day	$0.859	10.17	$3.769	-$2.690	$0.750	$4.607	1,992
5 Days	$4.354	9.95	$10.972	-$6.145	$2.850	$17.222	1,574
10 Days	$8.110	6.19	$19.292	-$10.914	$4.842	$32.347	1,084
Panel B: Ex-Ante Results							
1 Day	$1.031	8.08	$5.689	-$3.260	$0.764	$5.814	1,989
5 Days	$4.739	8.14	$14.590	-$7.690	$3.074	$20.668	1,571
10 Days	$8.076	5.52	$22.141	-$10.754	$4.831	$32.220	1,079

Table 5
Post-transaction-cost Trading Profits

We report trading profits and losses after deducting measures of transaction costs for the zero net cost trading portfolios described in Table 3 in which we long puts and calls with low Black-Scholes implied standard deviations and short puts and calls with high B-S ISDs. The portfolios are initially both delta and gamma neutral and the portfolios are formed at the same prices used to determine the composition of the portfolio.. The trading portfolios are held one, five, or ten market days without re-balancing. The one, five, and ten day holding periods end at least ten days before the options expire. In Panel A, we assume a brokerage commission of $2.00 per contract. In Panel B we assume a $2.00 commission and a bid/ask spread of $.05.

	Profits						
Holding Period	**Mean**	**t**	**Std. Dev.**	**10%**	**Median**	**90%**	**Obs**
Panel A: With Brokerage Commission Costs							
1 Day	$0.503	5.96	$3.763	-$3.000	$0.447	$4.207	1,992
5 Days	$4.017	9.19	$10.964	-$6.432	$2.521	$16.845	1,574
10 Days	$7.794	5.95	$19.290	-$11.205	$4.568	$31.975	1,084
Panel B: With Commission Costs and a Bid-Ask Spread							
1 Day	-$1.685	-19.56	$3.844	-$5.479	-$1.504	$2.001	1,992
5 Days	$1.942	4.45	$10.948	-$8.710	$0.659	$14.634	1,574
10 Days	$5.841	4.46	$19.285	-$12.912	$2.771	$29.903	1,084

Figure 1
Average Daily Trading Volumes for Options on S&P 500 Futures With 10 - 35 Market Days to Expiration Arranged by Strike Price (1988 - 1998)

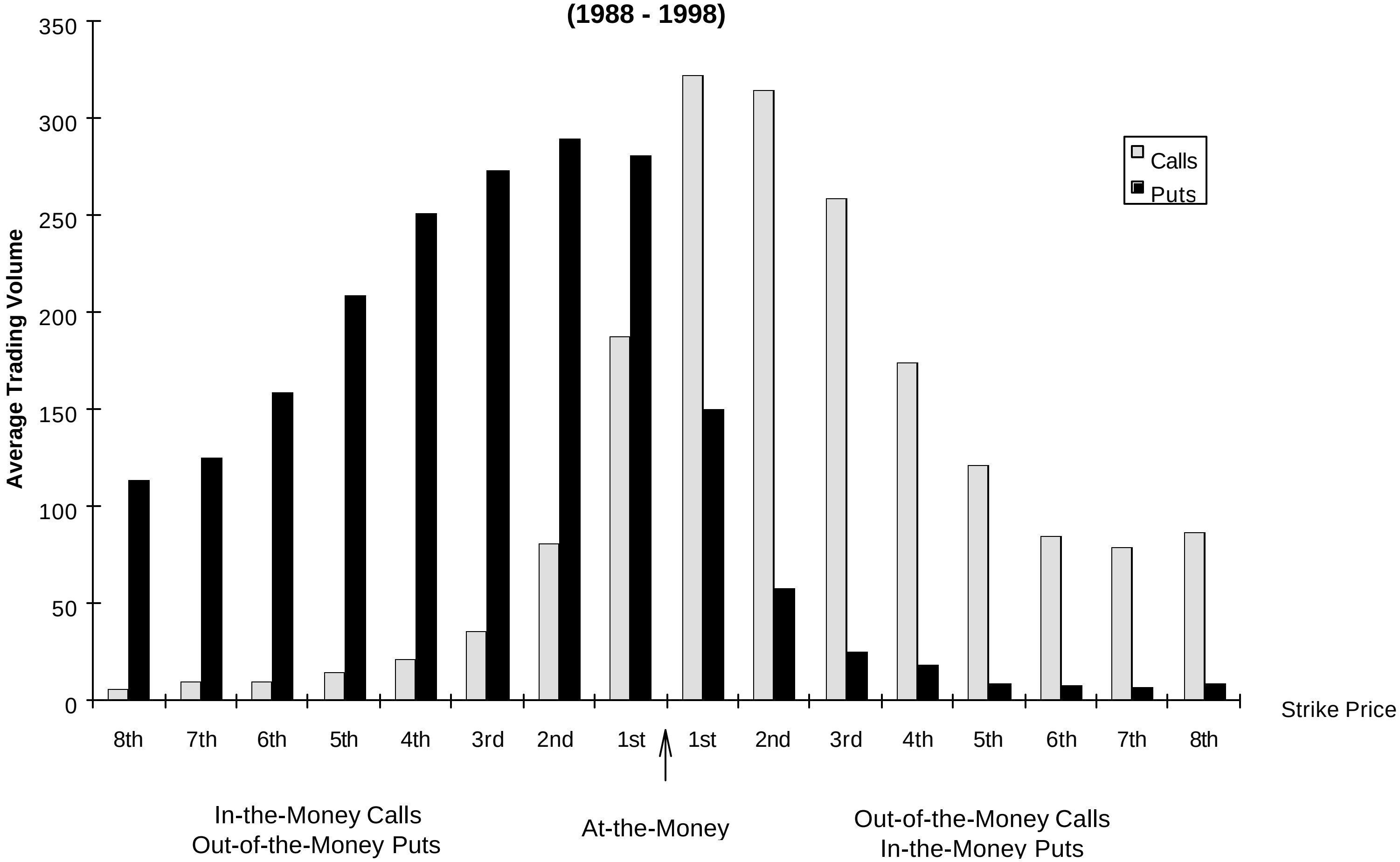


Figure 2
Number of Trading Days With Price Observations for Options on S&P 500 Futures
With 10 - 35 Market Days to Expiration Arranged by Strike Price
(1988 - 1998)

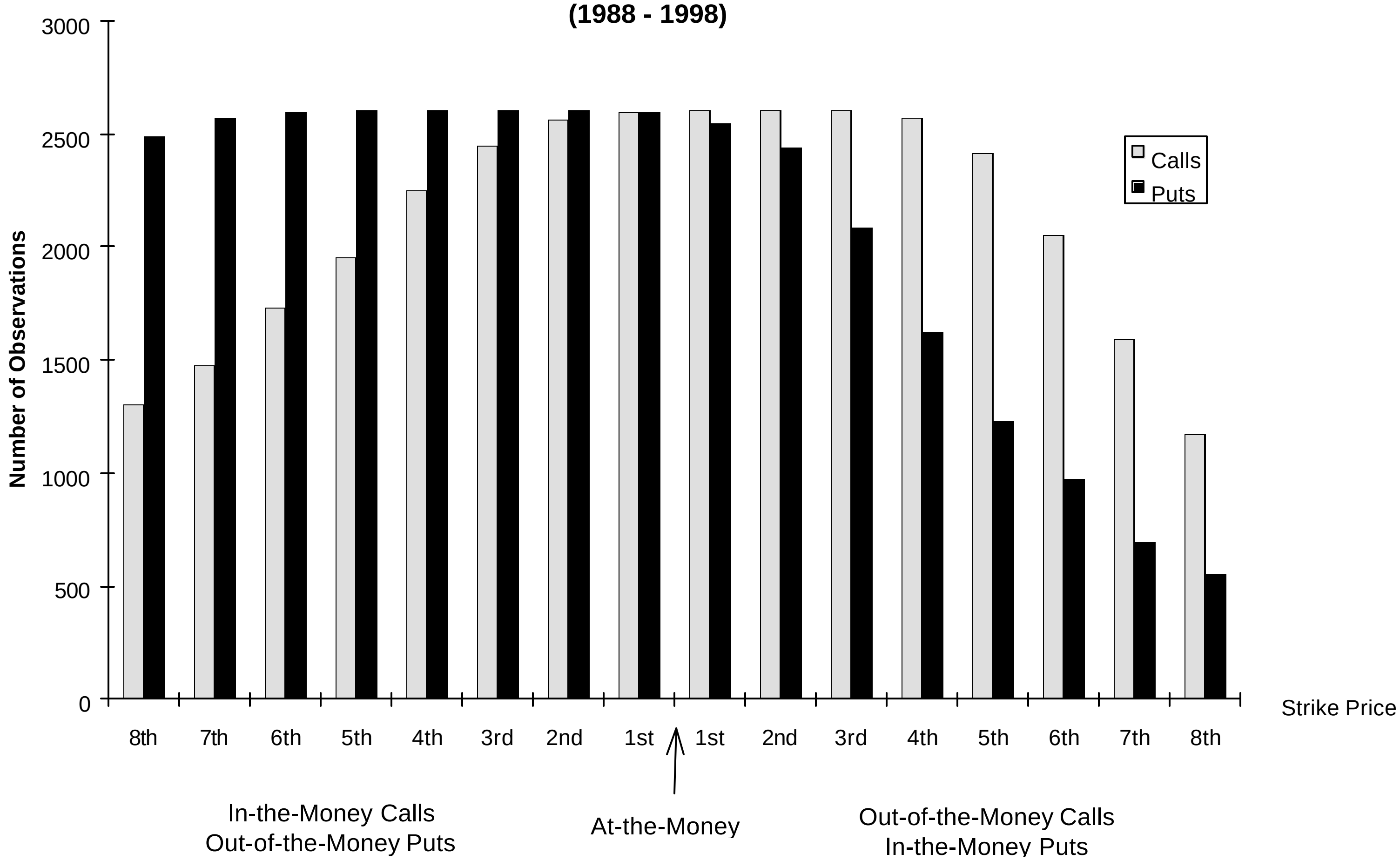
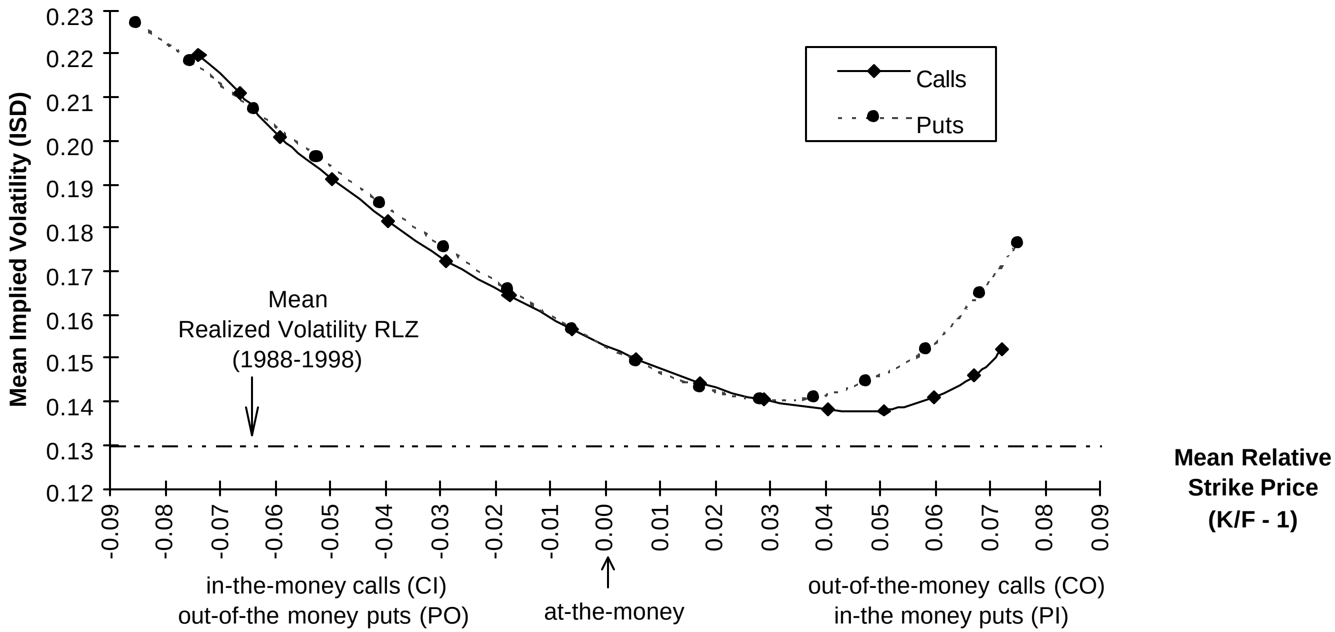
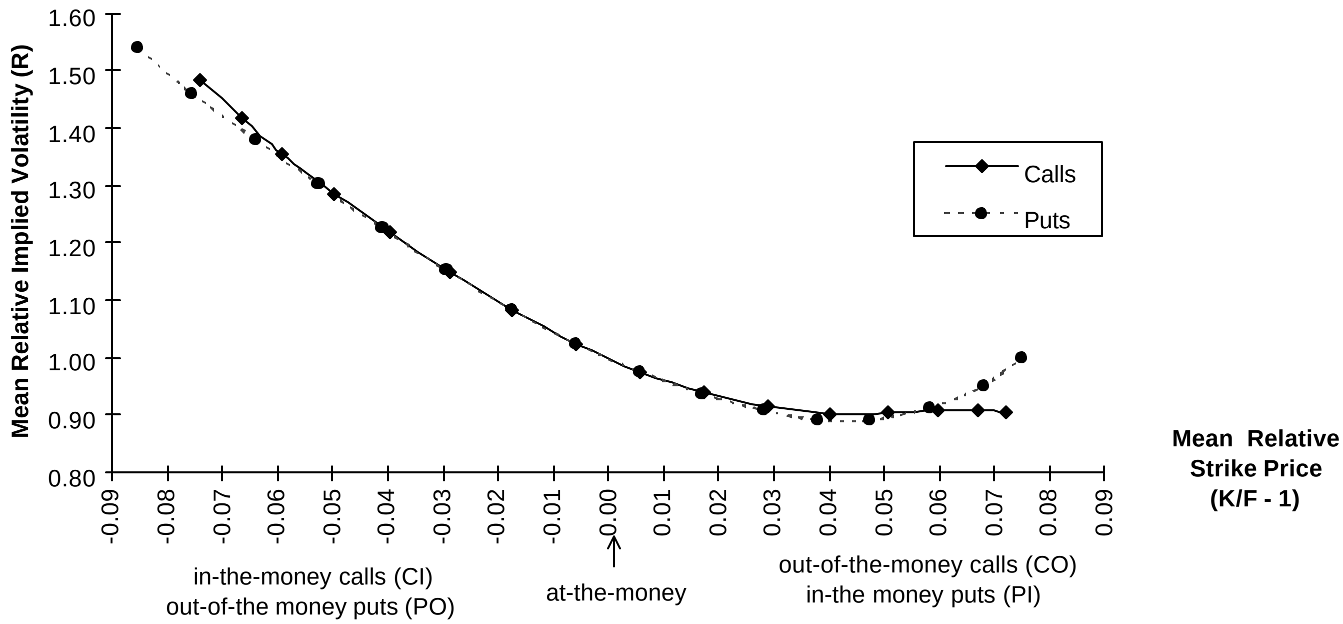


Figure 3
The Implied Volatility Smile - S&P 500 Futures Options



Note: The mean implied standard deviation (ISD) over the period 1/1/1988-4/30/1998 is graphed against the mean relative strike price for 16 call option groupings and 16 put option groupings. These groupings differ by strike price. Each day we observe the first eight in-the-money calls and first eight out-of-the-money calls and the same for puts. For example, the farthest right diamond shows the mean relative strike price and the mean implied volatility for the eighth from-the-money, ou of-the-money call. The relative strike price is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. For comparison, the mean realized volatility, calculated as the actual standard deviation of returns over the life of the options in the data set is also shown.

Figure 4
The Implied Volatility Smile in Relative Terms



Note: The mean relative implied standard deviation (R) over the period 1/1/1988-4/30/1998 is graphed against the mean relative strike price for 16 call option groupings and 16 put option groupings which differ by strike price. Each day we observe the first eight in-the-money calls and first eight out-of-the-money calls and the same for puts. For example, the farthest right diamond shows the mean relative strike price and the mean relative implied volatility for the eighth from-the-money, out-of-the-money call. The relative strike price is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the mean ratio of the implied standard deviation for that strike price relative to the average of the two nearest-the-money calls and the two nearest-the-money puts. The diamonds represent calls and the dots represent puts.

ENDNOTES

1. For a survey of these "smile consistent models" see Skiadopoulos (1999).

2. Examples of smile-consistent jump-diffusion models are: Merton (1976), Ball and Torous (1985), Jarrow and Rosenfeld (1984), Amin (1993), and Bates (1996), which augment the B-S return distribution with a Poisson-driven jump process. Examples of smile-consistent stochastic volatility models include: Hull and White (1987a), Wiggins (1987), Amin and Ng (1993), and Heston (1993). Smile consistent deterministic volatility models include: Derman and Kani (1994), Rubinstein (1985, 1994), and Jackwerth (1997).

3. For instance, Heynen (1994) finds that the observed smile pattern is inconsistent with various stochastic volatility models. Jorion (1988) concludes that jump processes cannot explain the smile while Bates (1996) concludes the same for stochastic volatility models. Das and Sundaram (1999) find that the implied volatility smiles implied by stochastic volatility models are too shallow and that jump-diffusion models imply a smile only at short maturities. Dumas, Fleming, and Whaley (1998) conclude that Rubinstein's deterministic tree model forecasts less well than the naive B-S model.

4. As explained below, while most hedger interest in this market is concentrated in out-of-the-money puts, we posit that put-call parity arbitrage pulls up implied volatilities on calls at the same strike prices as well.

5. Beginning in 1988 has other advantages as well. Trading was light in the first years (1983-1987) of the market so it may not have been as efficient. Also, until serial options were introduced in August 1987, only options maturing in March, June, September, and December were traded. After that date we have a continuous monthly series. In a separate paper, one of the authors has examined the 1983-1987 period. Our results are not sensitive to this choice.

6. One of the authors has looked at longer maturity options with results similar to those presented below.

7. Black's model is a simplified version of B-S adjusted for the facts that (1) futures pay no dividends, and (2) futures entail no investment at time t. While Black's model is for European options, S&P 500 futures options are American. While use of a European option model introduces a small upward bias in implied volatility, Jorion (1995) shows that this bias is small, e.g., a 12% volatility is measured as 12.02%. In any case, our tests test whether the smile is due to inappropriate assumptions in the B-S model including the European options assumption.

8. Note that the observations of RLZ_t are not independent. For example, the realized volatilities calculated for an option with 20 days to expiration and the same option one day latter with 19 days to expiration only differ in that one day is dropped from the sample over which RLZ is calculated for the option with 19 days to expiration. The differences are significant even after adjusting for this dependence.

9. The small differences between the implied volatilities for puts and calls observed at relatively high strike prices could represent the possibility of early exercise on deep-in-the-money puts.

10. With these restrictions, there were still one to three (depending on the holding period) observations (out of 1992 to 1084 total observations) in which at least one of the four options was not traded when the positions were unwound. As explained below, since the time value of these options is nil, their impact on the results is nil

11. Delta and gamma are the option price's first and second derivatives which respect to the underlying S&P 500 index according to Black's options on futures model.

12. Until 11/1/97 when it was changed to 250, each S&P 500 futures and option contract called for paying 500 times the quoted price so this trading portfolio would actually represent a $50,000 gross value before 11/1/97 and $25,000 afterwards.

13. Comparing the relative performance of various hedging schemes, Hull and White (1987b) find that delta-gamma neutral hedging performs well when the traded option has a short maturity as here and a relatively constant implied volatility. We have also examined portfolios in which equation 5 is replaced by a requirement that the portfolios be vega neutral, i.e., that an equal across-the-board change in implied volatility leave the portfolio value unchanged. The overall profitability and risk of these delta-vega neutral portfolios are similar to the results presented for the delta-gamma neutral portfolios - although (as expected) they are more sensitive to changes in the underlying asset price and less to overall changes in implied volatility.

13. Theta is not measured consistently in the literature. Some express theta as the partial derivative with respect to time so that it is negative for all options. Others express it as the partial with respect to the time to expiration so that it is positive. We use the former convention.

15. This assumes that there are 252 trading days in an average year.

16. We have one less observation in Table 4 than in Tables 2 and 3 since we cannot calculate profits for the very last observation, 4/30/98, in our data set. In one of our 1-day, two of our 5-day, and three of our 10-day holding periods, one of the four options did not trade on the day the position was closed so we do not have a price quote. All were far-from-the-money. If the unobserved option was far out-of-the money we assigned it a value of zero and if far in-the-money a value equal to its intrinsic value. These have virtually no impact on the profit figures in Table 4.

17. Since successive 5 and10 day holding periods overlap, the observations are not fully independent. The reported t-values are adjusted for this overlap.

18. As explained in section II.C, these figures somewhat overstate the true risk.

19. This is the format implied by a second order Taylor series expansion.

20. It is also possible that traders trying to follow this strategy in substantial magnitudes would face price pressures, that is their buying would drive prices up and their selling would drive prices down reducing profits even further (and flattening the smile).