LEVERAGED ETFs:
All You Wanted to Know but Were Afraid to Ask

Leveraged exchange-traded funds present a variety or risks, particularly for buy-and-hold investors. To manage these risks effectively, investors must understand the impact of different factors – including compounding, daily balancing and volatility – on returns.

By Marco Avellaneda and Stanley Jian Zhang

Leveragedexchange-traded-funds(LETFs) are garnering considerable amount of interest from investors, active traders and professional portfolio managers. From the point of view of traders subject Reg T margin, these instruments provide a simple way of doubling or tripling exposure to an index while using the same amount of capital.[1]

Active traders can use reverse LETFs as a substitute for short-selling the underlying asset when the latter is hard to borrow. For instance, many traders took long positions in SKF, a financial bearish fund, toward the end of 2008, when financial stocks were difficult or even impossible to short. Long-short hedge-fund managers often use LETFs as a way of hedging their portfolios.

Classical ETFs track an index or basket in a one-for-one fashion; they are essentially passively managed. In contrast, LETFs require active management: this involves borrowing funds to purchase additional shares (bullish LETFs), or short-selling (bearish LETFs) and rebalancing the position on a daily basis. Managers sometimes simplify the hedging of LETFs by entering into daily resetting of total-return swaps with qualified counterparties. Figures 1 and 2 describe graphically the management of leveraged ETFs.

Figure 1: A "Bullish" Leveraged ETF*

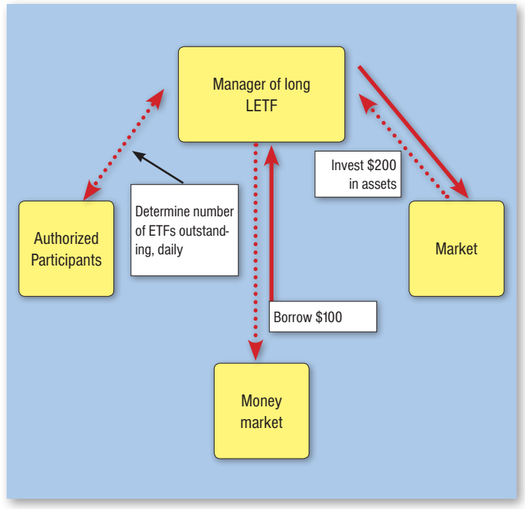

*Figure 1 is a schematic representation of the management of a bullish double-leveraged ETF. The manager finds out how many new shares have been created or redeemed. Based on this, the manager adjusts daily the exposure to the underlying index using leverage.

Figure 2: A "Bearish" Leveraged ETF*

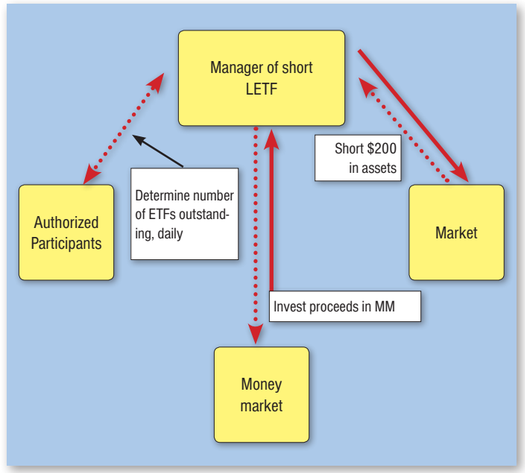

*Figure 2 is a schematic representation of the management of a bearish leveraged ETF for a double-short fund. In this case, the manager adjusts his position daily by shorting the underlying asset with double leverage.

Let's consider the following example: ProShares' Ultra Financial ETF (ticker symbol: UYG) offers investors an exposure to twice the total daily return of the Dow Jones Financial Index. The same index is also tracked (without leverage) by I-Shares Financial Sector ETF, which has symbol IYF. Thus, UYG offers investors a daily return equal to twice the daily return of IYF.

Another example is the ProShares UltraShort Financial ETF (ticker symbol: SKF). This product offers investors a daily return of equal to –2 times the daily return of the Dow Jones Financial Index, or-2 times the return of IYF. SKF is an example of a reverse LETF.

Buy-and-Holders Beware!

Throughout 2009, issues have been raised in the marketplace regarding the suitability of leveraged ETFs for long-term investors seeking to replicate a multiple of an index performance. UBS AG announced in July 2009 that it would cease marketing LETFs "because such products do not conform to its emphasis on long-term investing."[2]

This was soon followed by actions by other major U.S. brokers, like Morgan Stanley. FINRA then issued a notice in June 2009 requiring its members to disclose fully the risks inherent in these products.[3] Subsequently, in August, the Securities and Exchange Commission issued an Alert notice.

To better understand the suitability issues, let's consider, for instance, UYG and its underlying ETF, the IYF. Figure 3 (below) compares the price history of UYG and that of a static leverage-2 position in IYF, with both funds starting with the same investment in August 2008.

Figure 3: UYG vs. 2X IYF, 1 Year*

*Figure 3 depicts the difference between the performance of UYG and a static double-leveraged position in IYF. Notice, in particular, that UYG does not grow like the double-leveraged fund in the March 2009 rally.

Clearly, the two charts (above) do not coincide. In particular, UYG clearly underperforms the twice leveraged IYF during the rally of the Spring 2009. The corresponding chart for the reverse double-leveraged fund (SKF versus -2 times IYF), shown in Figure 4 (below), is even more dramatic.

Figure 4: SKF vs. –2X IYF*

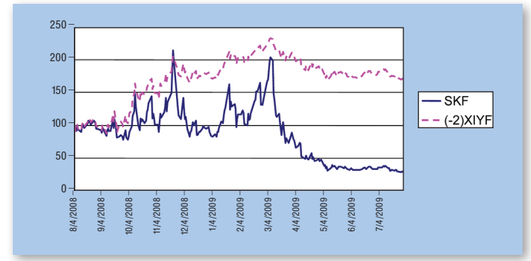

*Figure 4 shows a comparison between the performances of SKF and a double-leveraged short position in IYF. The discrepancy is remarkable.

Another interesting observation can be made by considering the price history of a pair consisting of a leveraged ETF and the corresponding reverse product with same leverage (e.g., UYG and SKF). Typically, the price charts should be "mirror images" of each other, at least over short periods of time.

However, as we increase the time horizon, we see clearly that the graphs are no longer mirror images, and the correlation between the two products breaks down as time passes (see figures 5, 6 and 7). Mathematically speaking, the returns have correlation -1 for short periods of time, but the correlation diminishes as the horizon increases.

Figure 5: SKF/UYG (Three-Month Period)*

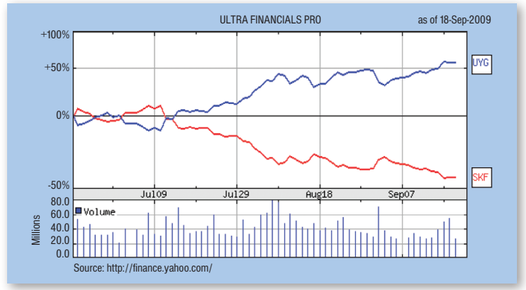

*Figure 5 depicts a comparison of the charts of UYG and SKF for a period of three months, ending on September 18, 2009. Notice that the two charts are "mirror images" of one another, with a slight overperformance by UYG.

Figure 6: UYG/SKF (One-Year Period)*

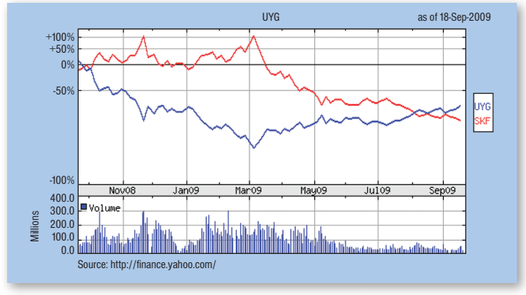

*Figure 6 compares the charts of UYG and SKF for a period of one year, prior to September 18, 2009. Notice that the two charts are no longer mirror images. Both funds actually have negative returns over the one-year period.

Figure 7: UYG/SKF (Since Inception)*

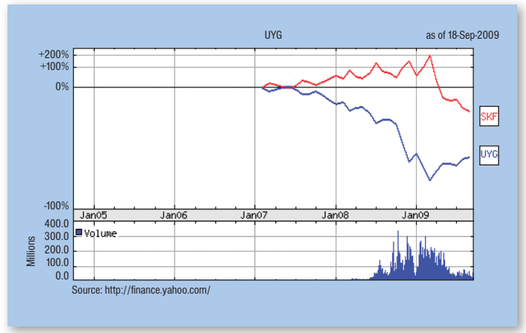

*Figure 7 compares the charts of UYG and SKF from their inception to September 18, 2009. There is little correlation between long-term returns.

Tracking daily returns is not the same as tracking long-term returns. As we shall see, there are two primary differences. First, there is the issue of compounding.

We all know the difference between daily compounded and annually compounded interest rates. This effect is sometimes referred to as convexity. In fixed income, compounding is tantamount to investing at higher returns more frequently, which implies a faster growth of capital.

In the case of LETFs, there is a second important reason for the discrepancies seen in the charts: volatility. The more volatile the assets are, the larger the tracking error, as we shall see in the next section.

Daily Rebalancing and Path-dependence due to Volatility

We consider a leveraged ETF with leverage ratio β (β =2 for UYG, a β = -2 for SKF), and denote its net asset value (NAV) at time t by L_t. We also consider an ETF that tracks the underlying index and denote its price by S_t.

For simplicity, we assume that all prices and NAVs are dividend-adjusted. If we assume daily rebalancing, which is quite frequent, we can approximate the dynamics of the NAV by the following stochastic differential equation:

$$\frac{dL_t}{L_t} = \beta \frac{dS_t}{S_t} + (1 - \beta)\, r dt - f dt, \qquad (1)$$

where r is the funding rate and f is the expense ratio of the LETF. We model the underlying ETF price as an Itô process, as below:

$$\frac{dS_t}{S_t} = \sigma_t dZ_t + \mu_t dt, \qquad (2)$$

where Z_t is a Brownian motion and σ_t and μ_t are stochastic processes that are non-anticipative with respect to Z_t.

Cheng and Madhavan (2009) and Avellaneda and Zhang (2009) showed that this leads to an explicit formula that relates the prices of LETFs to their underlying index or ETF, as follows:

$$\frac{L_t}{L_0} = \left(\frac{S_t}{S_0}\right)^{\beta} \exp\left[(1-\beta)\, rt - ft - \frac{\beta^2 - \beta}{2} \int_0^t \sigma_s^2 ds\right] \qquad (3)$$

The interpretation of the formula is simple: the factor $\left(\frac{S_t}{S_0}\right)^{\beta}$ corresponds to the "fast" compounding of returns alluded to earlier, which will produce a divergence from simply-compounded returns over long periods of time. We note that if $|\beta| > 1$ or $\beta = -1$, the function $f(s) = s^{\beta}$ is convex.

The tendency would be thus to *overperform* simply compounded returns over the same time horizon. However, this is not the case empirically: several LETFs underperformed the simply compounded leveraged return since their inception. This has to do with the fact that there is also frequent *rebalancing*, not just compounding.

The effect of daily rebalancing for a bullish LETF is to buy stocks when the NAV goes up and to sell them when it goes down. This means that the manager is "short Gamma."

The same happens for a bearish LETF: if the NAV goes up (stocks go down), the manager must short more stock, and so forth. The result is that the NAV of the leveraged fund experiences a "decay" rate approximately equal to

$$r\ decay,\ t = \frac{\beta^2 - \beta}{2} \sigma_t^2 - (1 - \beta)\, r + f \qquad (4)$$

Notice, in particular, that bearish LETFs have more exposure to volatility (a higher decay rate) than bullish ETFs with the same leverage ratio.

In summary, if static leveraged ETFs existed, they would be able to replicate (tautologically) the multiple of the returns of the ETFs over any time horizon, and both returns would be perfectly correlated. Leveraged ETFs are dynamically hedged to replicate daily returns (and therefore do not replicate over long-term horizons), and correlations with the reference ETF tend to zero across time.

Bearish LETFs have more exposure to volatility than bullish LETFs with the same leverage ratio.

Empirical Validation

We analyzed the returns of a broad class of double-and triple-leveraged ETFs (bullish and bearish). For double-leveraged ETFs (issued by ProShares), we considered the period from February 2008 to March 2009. In the case of tripe-leveraged ETFs (issued by Direxion), we took the histories of these products since their inception until March 2009.

In all cases, we calculated the difference between the right-hand and left-hand sides of equation (3) and plotted the average error, as well as the standard deviation of the standard error.

Tables 1 through 4 display the results. The agreement between theory and observation is striking.[4] Notice that the average and standard deviations of tracking errors are inferior to the daily volatility of the price of the ETFs, which shows that the formula matches very well the observations.

Table 1: Double-Leverage Ultra Long ETFs

Underlying ETF	Tracking Error (avg., %)	Standard Deviation (%)	Leveraged ETF
QQQQ	0.04	0.47	QLD
DIA	0.00	0.78	DDM
SPY	-0.06	0.40	SSO
IJH	-0.06	0.38	MVV
IJR	1.26	0.71	SAA
IWM	1.26	0.88	UWM
IWD	1.00	0.98	UVG
IWF	0.50	0.59	UKF
IWS	-0.33	1.20	UVU
IWP	-0.02	0.61	UKW
IWN	2.15	1.29	UVT
IWO	0.50	0.74	UKK
IYM	1.44	1.21	UYM
IYK	1.20	0.75	UGE
IYC	1.56	1.04	UCC
IYF	-0.22	0.74	UYG
IYH	0.40	0.42	RXL
IYJ	1.05	0.74	UXI
IYE	-0.73	1.71	DIG
IYR	1.64	1.86	URE
IYW	0.51	0.55	ROM
IDU	0.25	0.55	UPW

The effect of daily rebalancing for a bullish LETF is to buy stocks when the NAV goes up and to sell them when it goes down. This means that the manager is "short Gamma."

In Table 1 (previous page), the average tracking error and standard deviation are obtained by applying equation (3) to the Proshares long LETFs from January 2, 2008, to March 20, 2009. Notice that that the average tracking error is for the most part below 100bps, and that the standard deviation is comparable.

In particular, the standard deviation is inferior to the daily volatility of these assets, which often exceeds 100 basis points as well. This suggests that equation (3) gives the correct relation between the NAV of the LETFs and their underlying ETFs.

Table 2: Double-Leveraged Ultra Short ETFs

Underlying ETF	Tracking Error (avg., %)	Standard Deviation (%)	Leveraged ETF	Underlying ETF	Tracking Error (avg., %)	Standard Deviation (%)	Leveraged ETF
QQQQ	0.22	0.80	QID	IWO	-0.19	0.79	SKK
DIA	-2.01	3.24	DXD	IYM	1.82	0.99	SMN
SPY	-1.40	2.66	SDS	IYK	-0.76	1.98	SZK
IJH	0.69	0.64	MZZ	IYC	0.79	0.92	SCC
IJR	-0.55	0.86	SDD	IYF	3.30	3.03	SKF
IWM	0.94	0.91	TWM	IYH	1.04	0.91	RXD
IWD	0.32	1.40	SJF	IYJ	0.32	0.74	SIJ
IWF	-0.30	1.34	SFK	IYE	0.43	3.09	DUG
IWS	-2.06	3.03	SJL	IYR	2.00	2.07	SRS
IWP	0.93	0.92	SDK	IYW	0.01	0.80	REW
IWN	-2.21	1.80	SJH	IDU	1.75	1.06	SDP

The average tracking errors and standard deviations depicted in Table 2 are relatively small, but there are a few funds where the tracking error is superior to 200 basis points. We attribute these errors to the fact that many ETFs (as well as the stocks in their holdings) were hard-to-borrow from July to November 2008. This particularly applied to ETFs in the financial and energy sectors.

Table 3: Triple-Leveraged Bullish ETFs*

Underlying ETF/Index	Tracking Error (avg., %)	Standard Deviation (%)	3x Bullish LETF
IWB	0.44	0.55	BGU
IWM	0.81	0.75	TNA
RIFIN.X	3.67	2.08	FAS
RIENG.X	2.57	0.70	ERX
EFA	1.26	2.32	DZK
EEM	1.41	1.21	EDC

*This table analyzes the average tracking errors and standard deviations for triple-leveraged long ETFs, from their inception in November 2008.

Table 4: Triple-Leveraged Bearish ETFs*

Underlying ETF/Index	Tracking Error (avg., %)	Standard Deviation (%)	3x Bearish LETF
IWB	-0.08	0.64	BGZ
IWM	0.65	0.76	TZA
RIFIN.X	-1.63	4.04	FAZ
RIENG.X	-1.41	1.01	ERY
EFA	-1.54	1.86	DPK
EEM	0.49	1.43	EDZ

*This table analyzes the average tracking errors and standard deviations for triple-leveraged short ETFs, from their inception in November 2008. Notice again that the errors for ETFs in the financial and energy sectors are slightly higher than the rest.

Closing Thoughts

We have seen that the apparently erratic behavior of prices of leveraged ETFs can be explained completely via a simple formula. The main element that we retain from the study is that the realized volatility (standard deviation of price moves), which builds up across time, affects the NAV of LETFs negatively.

Accordingly, if the volatility is relatively high, the value of LETFs should decline, all other things being equal. The inter-

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play between the "convexity" provided by daily compounding and the "time-decay" associated with realized volatility gives the correct general picture for understanding LETF returns.

What does this means for retail and institutional investors? Clearly, investors that expect to, say, recover from losses incurred in the subprime/credit crunch period by investing in LETFs may be surprised — even if the market rallies — for not realizing their expected multiple of the market return.

Regarding future regulations in this market, we do not believe that these products should be unnecessarily regulated. The reason is that LETFs are mostly used by professionals who have shorter-term trading horizons and, increasingly, by institutional investors with strong investment skills.

The market should be able to benefit from the advantages of leveraged funds, once their price behavior and dependence on volatility are understood and risk management practices pertaining to leverage and loss of correlation for long investment horizons are put in place.

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FOOTNOTES

1. We note, however, that FINRA recently modified rules on the margining of LETFs, presumably to close such "regulatory loophole" and to reduce risk to retail investors.

2. Bloomberg News, July 2009.

3. FINRA regulatory notice, June 31, 2009. "Due to the effects of compounding, their performance over longer periods of time can differ significantly from their stated daily objective. Therefore, inverse and leveraged ETFs that are reset daily typically are unsuitable for retail investors who plan to hold them longer than one trading session, particularly in volatile markets."

4. This brings up the interesting point of whether equation (3) is actually dependent on the special assumption about the ETF prices made in equation (2). We believe that equation (3) applies under much less stringent assumptions, as argued in Avellaneda and Zhang (2009).

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