Investment Strategies with VIX and VSTOXX Futures

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Abstract

This study examines historical data on S&P500 and EURO STOXX 50, VIX and VSTOXX, VIX and VSTOXX futures, to reveal linkages between these important series that can be used by equity investors to generate alpha and protect their investments during turbulent times. A comparative portfolio performance analysis in the U.S. and the E.U. zone reveals that over time the best investment strategy for a stock investor is to add both bonds and volatility futures to their portfolio. We also reveal a long-short cross border statistical arbitrage strategy pairing volatility index futures that can generate profits using forecasts produced by suitable GARCH models.

Keywords: *volatility indexes, volatility derivatives, GARCH models, statistical arbitrage*

JEL: G15, G11, C58

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1. Introduction

"The CBOE Volatility Index (VIX) is a key measure of market expectations of near-term volatility conveyed by S&P500 stock index option prices. Since its introduction in 1993, VIX has been considered by many to be the world's premier barometer of investor sentiment and market volatility." – Website of CBOE

Including volatility positions in an investment portfolio is done in general for portfolio diversification and for hedging purposes. The latter is true for portfolio managers that are tracking index equity portfolios and who are short volatility. When equity markets become highly volatile then the portfolio tracking error and the rebalancing costs increase but using volatility futures helps to hedge against these frictional costs. At the other extreme, the volatility futures contracts offer a direct exposure to vega with no delta involved. Hence, speculative directional positions can be taken via VIX and VSTOXX futures. An interesting trading strategy that will be explored later on in this paper is based on the correlation between the VSTOXX and VIX. A fund manager may buy be long VSTOXX volatility and short VIX volatility. A similar idea is to trade on the basis between VIX and VSTOXX, given the historical evolution between the two.

The VIX index has been introduced by Whaley (1993) and the methodology was further revised by CBOE in 2003. This index measures the market's implied view of future volatility of the equity S&P500 index, given by the current S&P 500 stock index option prices. When constructing the VIX, the put and call options are near- and next-term, usually in the first and second S&P500 contract months. "Near-term" options must have at least one week to maturity. This condition is imposed in order to minimize pricing anomalies that might appear close to expiration. When this condition is violated VIX "rolls" to the second and third S&P500 contract months. The open interest and trading volume of VIX futures have increased rapidly over the years. As reported in Shu and Zhang (2012), shortly after launch the average open interest and trading volume was 7,000 contracts and 460 contracts, respectively. During the subprime crisis over the period August 2008–November 2008, the average daily trading volume was 4,800 contracts per day, and the average VIX futures price was $19.20. This implies an average daily market value of about $92 million dollars. Currently the VIX futures market is one of the most active markets on the CBOE, with an average daily open interest of almost 70,000 contracts.

The EURO STOXX 50 Index is constructed from Blue-chip companies of sector leaders in the Eurozone: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. The EURO STOXX 50 Volatility Index (VSTOXX) index is relatively new and provides the implied volatility given by the prices of the options with corresponding maturity, on EURO STOXX 50 Index. By design the VSTOXX index is based on the square root of implied variance and it calibrates the volatility skew from OTM puts and calls. The VSTOXX does not measure implied volatilities of at-the-money EURO STOXX 50 options, but the implied variance across all options of a given time to expiry. This model has been jointly developed by Goldman Sachs and Deutsche Börse such that using linear interpolation of the two nearest sub-indices, a rolling index of 30 days to expiration is calculated every 5 seconds using real-time EURO STOXX50 option bid/ask quotes. The VSTOXX is calculated on the basis of eight expiry months with a maximum time to expiry of two years[2]. If there are no such surrounding sub-indices, nearest to the time to expiry of 30 days, the VSTOXX is calculated using extrapolation, using the two

[2] Apart from the VSTOXX main index (which is irrespective of a specific time to expiry), sub-indices for each time to expiry of the EURO STOXX 50 options, ranging from one month to two years, are calculated and distributed. For options with longer time to expire, no such sub-indices are currently available.

nearest available indices which are as close to the time to expiry of 30 calendar days as possible. In the situation that there are no two such indices VSTOXX is calculated by extrapolation based on the nearest available indices, which are as close to 30 calendar days as possible. The payoff of VSTOXX futures resembles more the payoff of a volatility swap, being determined by the difference between the realized 30 day implied volatility and the expected 30 day implied volatility at trade initiation, times the number of contracts and the monetary size of the index multiplied (€100).

An important market change occurred on June 2009 when Eurex introduced the Mini-Futures contracts for VSTOXX, followed shortly by the ending in July 2009 of the VSTOXX futures. This important change has been motivated by the erratic volume of trading in VSTOXX futures. Hence, after this change, investors could use a volatility futures contract trading at 100 EUR per index point rather than 1000 EUR, previously. Thus the Mini-Futures contracts are size comparable with the options on VSTOXX. The minimum price change is 0.05 points which is equivalent to 5 EUR, and the contract months are the eight nearest successive calendar months and the next quarterly month of the February, May, August, and November cycle. The evolution of the futures and mini-futures contract volume are depicted in Figure 1.

Szado (2009) and Rhoads (2011) highlighted the potential benefits of adding volatility derivatives to equity index portfolios. Changing the equity portfolio mix by adding a 10% VIX futures to the base S&P 500 portfolio would have reduced losses over the period August to December 2008 by 80% and also decreased the portfolio standard deviation by one-third. While this improvement should be expected from the design of the volatility derivatives, the impact of changing the portfolio mix by including volatility futures is unclear. Ex ante, an investor should expect an erosion of returns for portfolios including VIX futures contracts during normal time in equity markets. The opposite is expected in turbulent times.

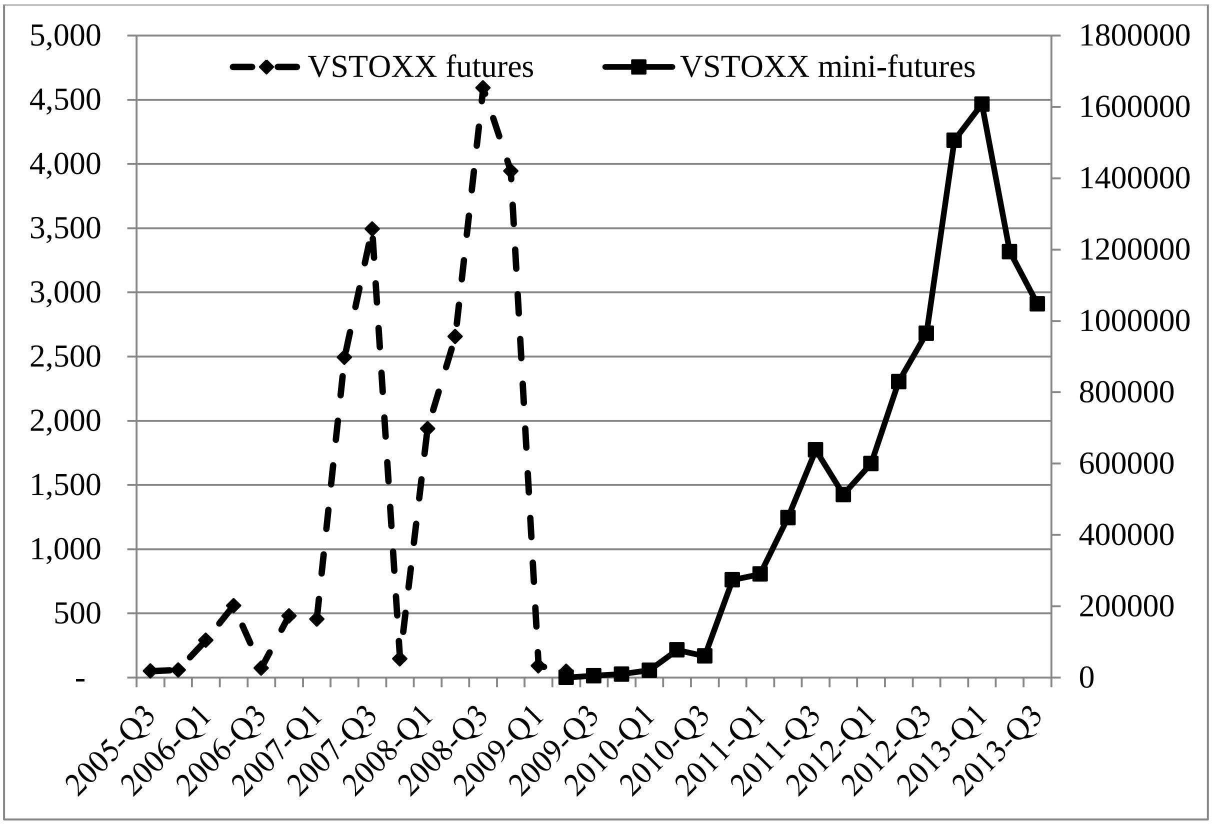

Fig. 1. Evolution of contract volume for VSTOXX futures, aggregated for all maturities and all trading days within each quarter

Hence, it is unclear what an investor should expect for longer time periods. Furthermore, in the aftermath of the subprime liquidity crisis, other crises such as the sovereign bond crisis appeared in the European Union. It would be therefore of great interest to equity investors to see not only an updated analysis of the protective impact of VIX futures on US equity portfolios but also the impact of VSTOXX futures on EU equity portfolios represented by STOXX50. This study presents not only an updated analysis for U.S. but also a new analysis for E.U. for the period from March 2004 to February 2012. Furthermore, the second part of this paper is dedicated to a statistical arbitrage cross-border strategy using the difference between VIX and VSTOXX. The empirical evidence for this second part of the analysis goes to December 2012. To this end a battery of GARCH models is applied to harness the information flow contained by this difference. The suitable GARCH models are then used to signal trading entering and also exiting trading opportunities.

The remaining of the paper is structured as follows. Section 2 reviews the existing literature on volatility indices, while Sections 3 and 4 focus on data, methodology and empirical results. In Section 5 a number of investment strategies are implemented and discussed. The final section puts forth a series of recommendations and conclusions.

2. Previous Research on Investment Analysis using Volatility Futures

In a study covering stock performance in USA over more than two hundred years Schwert (2011) pointed out that the implied volatility extracted from option prices indicated that the market did not expect volatility to remain high for long after the 2008 crisis, a prediction that was confirmed later on. The spikes in volatility, mainly driven by the financial sectors in USA, United Kingdom and Japan, sparked a buoyant market and the nascence of a new asset class, volatility derivatives. The question of how well the implied volatility forecasts future realized volatility has received a great deal of attention in the financial literature, the general conclusion being that implied volatility outperforms the well-known volatility measures based on historical data: see Blair et al. (2001), Corrado and Miller (2005) as well as Carr and Wu (2006) who showed that VIX outperforms GARCH volatility estimated from the S&P 500 index returns. However, Becker et al. (2006) found that VIX is not an efficient forecaster of future realized volatility and other volatility estimates based on historical data can be superior to VIX alone.

Whaley (2000) was among the first to point out that there is a negative statistically significant relationship between the returns of stock and associated implied volatility indexes and moreover, positive stock index returns correspond to declining implied volatility levels, while negative returns correspond to increasing implied volatility levels. For the S&P 100 index, the relationship is asymmetric, negative stock index returns are triggered by greater proportional changes in implied volatility measures than are positive returns. The scatterplots for the two indexes, VIX and VSTOXX, and their corresponding equity indexes are illustrated in Appendix A. Clearly there is a negative correlation between stock index returns and changes in volatility. Using high-frequency data on DAX between 1995 and 2005, Masset and Wallmeier (2010) found that index returns Granger cause volatility changes.

Daigler and Rossi (2006) investigated the diversification benefits from adding a long VIX position to an S&P 500 portfolio. Their results indicate significant diversification benefits. On the other hand, Alexander and Korovilas (2013) were more sceptical about the diversification benefit that can be achieved due to a lack of fundamental relationships between main variables in this space and the excessive reliance on forecasting models.

One example of a discovered useful linkage is described in Cipollini and Manzini (2007), who used the same methodology as in Giot (2005) and Campbell and Shiller (1998) and identified a significant relationship between the VIX levels and the 3-months S&P 500 returns. This linkage seems stronger following spikes in VIX while it is weaker at lower levels of VIX. Their trading strategy to invest in the S&P5 00 index based on the VIX signal outperforms the simple strategy of holding long the S&P 500 index, confirming wide spread belief in investment banking.

Konstantinidi et al. (2008) discussed several models for implied volatility indexes including the VIX showing that the directional change can be forecasted using point and interval forecasts. The directional forecast accuracy can be improved by using GARCH models as demonstrated in Ahoniemi (2008). Compared with various standard time series models, an ARIMA(1,1,1) model with GARCH errors fits the historical VIX data well in this study, the directional accuracy of forecasts being close to 60% over a five year out-of-sample period. One major point made by Ahoniemi (2008) is that the addition of GARCH errors contributes significantly to forecast performance while the inclusion of S&P 500 returns in the model does not improve the directional forecasts. This is in line with Christoffersen and Diebold (2006), who demonstrate that it is possible to predict the direction of change of returns in the presence of conditional heteroskedasticity, even if it is not possible to predict the returns themselves.

Banerjee et al.(2007) and Giot (2005) develop models that use the VIX to predict stock market returns. The latter investigates the link between contemporaneous relative changes in VIX and contemporaneous S&P500 returns, but also the relationship between the current VIX levels and the future stock index returns. Denoting VIX_t the value of VIX index and by OEX_t the value of S&P100 index at time *t*, then $r_{VIX,t} = ln(VIX_t / VIX_{t-1})$ and $r_{OEX,t} = ln(OEX_t / OEX_{t-1})$ are the logarithmic returns of the two indexes, then Giot (2005) fitted the regression

$$r_{VIX,t} = \beta_0^+ D_t^+ + \beta_0^- D_t^- + \beta_1^+ r_{OEX,t} D_t^+ + \beta_1^- r_{OEX,t} D_t^- + \varepsilon_t \tag{1}$$

where D_t^- is a dummy variable that is equal to 1 (0) when $r_{OEX,t}$ is negative (positive) and $D_t^+ = 1 - D_t^-$. Based on this regression Giot concluded that negative returns for the stock index are associated with much greater relative changes in the implied volatility index than are positive returns.

Whaley (2009) discussed the observed VIX spikes during market unrest. He noted that when market volatility increases or decreases, respectively, the stock prices will fall, or rise respectively. The relationship between the rate of change on VIX and the rate of return on the corresponding S&P500 index (SPX) is more than one of proportionality and he argues that the change in VIX should rise quicker when the market falls than when the market rises, in line with the leverage argument proposed by Black. This hypothesis is tested using the following regression model

$$r_{VIX,t} = \beta_0 + \beta_1 r_{SPX,t} + \beta_2 r_{SPX,t} D_t^- + \varepsilon_t \tag{2}$$

Szado (2009) showed that adding VIX futures during the 2008 financial crisis to three base portfolios resulted in increased returns and reduced standard deviations. It was shown in the paper that when adding ATM VIX calls to the three base portfolios will increase portfolio returns but the effect on standard deviation was mixed, with more extreme results, not surprisingly given the extra leverage. Using VIX call options increased the profits during market drops but correspondingly also increased the standard deviation. The comparative analysis of buying S&P500 puts with the three base portfolios did not produced better results than when adding VIX Call options. Similarly, Chen et al. (2011) demonstrated that adding VIX futures contracts can improve the mean-variance investment frontier so hedge fund managers for example may be able to enhance their equity portfolio performance, as measured by the Sharpe ratio.

Derivatives on equity indexes can be used to extract useful information for volatility indexes. Jian and Tian (2007) employed a model-free approach for calibrating volatility from option prices and then compare that with the VIX index results. Volatility indexes are a considered by many an asset class of its own and derivatives products on these indexes have been traded recently. Brenner et.al (2007) showed that the term structure of VIX futures price is upward sloping while the term structure of VIX futures volatility is downward sloping. Dash and Moran (2005) discussed the advantages of using VIX as a companion for hedge fund portfolios.

An important question debated in the current literature is whether the volatility futures are predictable. A positive answer will allow some statistical arbitrage strategies to be exploited.

Unexpectedly, Konstantinidi et al. (2008), and Konstantinidi and Skiadopoulos (2011) detected evidence that VIX future prices may be predictable. Nevertheless, they also pointed out that such forecasts do not lead to significant arbitrage profits. Shu and Zhang (2012) used cointegration tests to analyse the lead–lag dynamics between VIX and VIX futures prices. They found empirical evidence suggesting that historical VIX futures prices are useful in forecasting the next period's VIX futures price. They also point out that the VIX futures market contain more information than VIX and historically the VIX values are much higher than VIX futures prices during turbulent times for the stock market. Using more advanced tests Shu and Zhang (2012) found evidence for a bi-directional causality between VIX and VIX futures prices, implying that both VIX and VIX futures prices may react simultaneously to new information. However, the predictability highlighted for the VIX futures market is unstable and not always appropriate to use for prediction of future VIX levels.

In spite of the impressive growth of volatility derivatives as a market, particularly the volatility futures, pricing this new derivative asset class is by no means straightforward. This is an incomplete market not because of the models proposed but because VIX is not a traded asset. Therefore one cannot establish a cost of carry linkage between spot VIX and VIX futures. Other models also focused on the variance. Dupoyet et al. (2010) proposed a stochastic diffusion process with jumps, a similar approach being followed by Psychoyios et.al. (2010). Carr and Wu (2009) analysed the variance risk premiums for individual stock options rather than the entire index. The difficulty in selecting a reliable model for pricing volatility derivatives has been highlighted by Mencia and Sentana (2013). In their study they show that existing models yield large distortions during the crisis because of their restrictive volatility mean reverting and heteroskedasticity assumptions. The solution proposed by them is to combine central tendency and stochastic volatility, which then will produce improved pricing of volatility derivatives across bull and bear markets.

3. Portfolio Diversification with VIX and VSTOXX

For the first part of the analysis presented in this paper, for our US study, daily data on VIX futures, S&P500 and Barclays US Aggregated total return bond index between March 2004 and February 2012 was used, and for the EU study, daily data on VSTOXX futures, STOXX50 and Barclays EUROPE Aggregated total return bond index between May 2009 and February 2012. As a proxy for the risk-free rate the 3-month Treasury Bill rates (secondary markets) is used for the US, and, for Europe, the 3-month EURO LIBOR.

Following Szado (2009), for each of the volatility indices (i.e. VIX and VSTOXX) the following portfolios are considered which will be compared in terms of performance over periods covering volatility spikes due to turbulent events:

1. 100% equity – it is assume that the investor holds a portfolio that tracks the S&P 500 or the EURO STOXX 50 indices, respectively.
2. 60% equity + 40% bonds, where the bond exposure will be represented by a portfolio that resembles the Barclays US or Barclays EURO Total Return Indices, respectively

A set of summary statistics for all the components of these portfolios as well as for the hedge instruments proposed below (i.e. VIX and VSTOXX futures) is given in Tables 1 and 2 below. For the US Sample, the data ranges from March 2004 (when the VIX futures were introduced) to February 2012. By contrast, the European sample is shorter, since VSTOXX futures were only introduced at the end of April 2009. The US sample is split into two sub-periods: a pre-crisis period (2004-2007) and a post-crisis period (2008-2012). The returns of 2008 are also analyzed separately, as this is the period in which markets saw the most dramatic movements. As expected the volatility of the volatility-related assets, namely VIX and VSTOXX futures, is highest and the volatility of the bond indices is lowest; this is true for both samples (US and Europe) and for all sub-periods considered (in the US case). The range of returns is also widest for the volatility related assets, which exhibit both the highest maximums and the lowest minimums, again across both samples and all sub-periods. By contrast, bonds have the narrowest ranges of returns.

Table 1

This table contains the summary statistics are of the daily returns on the S&P 500 equity index, Barclays US Aggregated total return bond index from 26h March 2009 to 17th February 2012. The standard errors are approximately $(6/T)^{1/2}$ and $(24/T)^{1/2}$ for the sample skewness and excess kurtosis, respectively, where T is the sample size. The values of the t statistic for both the sample skewness and excess kurtosis indicate that returns for most of the assets considered follow non-normal distributions, generally leptokurtic.

	S&P500	**Bond Index**	**VIX futures first maturity**	**VIX futures second maturity**
Annualized mean return	2.63%	5.16%	1.36%	2.65%
Volatility	22.27%	3.99%	79.61%	53.79%
Min	-9.47%	-1.26%	-29.48%	-18.57%
Max	2.13%	0.91%	36.02%	13.04%
Skewness	-0.2859	-0.0516	0.9363	0.6234
Excess Kurtosis	9.7162	1.7630	5.6505	3.3574
subperiod 1: 2004 - 2007				
Annualized mean return	7.57%	4.05%	3.47%	4.90%
Volatility	12.10%	3.27%	70.54%	45.31%
Min	-3.53%	-0.98%	-29.48%	-15.38%
Max	2.88%	0.91%	36.02%	14.45%
Skewness	-0.3205	-0.0393	1.4064	0.8376
Excess Kurtosis	1.9553	1.5829	11.9821	5.0127
subperiod 2: 2008-2012				
Annualized mean return	-1.85%	6.17%	-0.56%	0.61%
Volatility	28.53%	4.55%	87.08%	60.51%
Min	-9.47%	-1.26%	-23.13%	-18.57%
Max	10.96%	1.33%	23.57%	17.00%
Skewness	-0.2133	-0.0736	0.6899	0.5207
Excess Kurtosis	5.8829	1.2839	2.7090	2.3143
crisis subperiod: 2008				
Annualized mean return	-50.80%	5.47%	65.80%	63.25%
Volatility	41.41%	5.93%	94.17%	60.86%
Min	-9.47%	-1.26%	-23.13%	-18.57%
Max	10.96%	1.24%	23.57%	12.82%
Skewness	-0.021	-0.1278	0.0069	0.0323
Excess Kurtosis	3.6484	0.4911	2.8567	2.3229

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Table 2

This table contains the summary statistics are of the daily returns on the EURO STOXX 50 equity index, Barclays EURO Aggregated total return bond index from 30th April 2009 to 9th February 2012. The standard errors are approximately $(6/T)^{1/2}$ and $(24/T)^{1/2}$ for the sample skewness and excess kurtosis, respectively, where T is the sample size. The values of the t statistic for both the sample skewness and excess kurtosis indicate that returns for all the assets considered follow non-normal distributions, all of them leptokurtic.

	Euro STOXX 50	**Bond Index (EUR)**	**VSTOXX Futures first maturity**	**VSTOXX Futures second maturity**
Annualized mean return	2.17%	4.65%	-12.85%	-9.41%
Volatility (annualized st dev)	25.49%	3.22%	77.76%	51.73%
Min	-6.54%	-0.78%	-17.38%	-12.57%
Max	9.85%	1.08%	21.22%	12.35%
Skewness	0.0375	0.4037	0.7393	0.3868
t-statistic Skewness	0.4036	4.3480	7.9620	4.1663
Excess Kurtosis	3.0642	3.1098	2.5495	1.4323
t-statistic Kurtosis	16.5014	16.7469	13.7297	7.7130

The returns distributions are generally non-normal: with the exception of US bonds in the 2008 sub-period, all the other returns distributions exhibit positive and highly significant (t-statistics higher than 7) values of the excess kurtosis. As expected, equity index returns are generally negatively skewed, while volatility futures returns exhibit positive skewness. One research question still debated in the current academic literature is whether the introduction of volatility indices helps with diversification and whether it improves the Sharpe ratio when added to a relevant equity portfolio. Consider the position of an equity investor analysing a mixed portfolio comprising a position in the EURO STOXX 50 stock index and a second position in the VSTOXX volatility index. Assume for simplicity that the transaction costs are neglectable for the investor. The value of the hypothetical portfolio is normalized to equal 100% at the investment date and it is rebalanced on a daily basis in such a way that the weight of the VSTOXX position is kept constant over the investment period.

Table 3

This table shows the determination of optimal weight portfolio investment in VSTOXX. The calculations are based on the daily returns on the EURO STOXX 50 equity index and VSTOXX index from 30th April 2009 to 9th February 2012.

	Absolute Returns	**Annualised Returns**	**Annualised Volatility**	**Annualised Sharpe ratio**
STOXX 50	6.19%	2.12%	25.08%	-0.05
STOXX + VSTOXX (5%)	14.67%	4.83%	19.98%	0.08
STOXX + VSTOXX (10%)	22.51%	7.17%	15.67%	0.25
STOXX + VSTOXX (15%)	29.51%	9.13%	12.92%	0.45
STOXX + VSTOXX (20%)	35.48%	10.72%	12.74%	0.59
STOXX + VSTOXX (25%)	40.26%	11.94%	15.19%	0.57
STOXX + VSTOXX (30%)	43.72%	12.8%	19.27%	0.5
STOXX + VSTOXX (40%)	46.31%	13.45%	29.45%	0.35
STOXX + VSTOXX (50%)	43.03%	12.63%	40.56%	0.23

The results illustrated in Table 3 indicate that for the period of investigation the optimal weight of VSTOXX investment is 20%, leading to a Sharpe ratio of almost 60%, driven by the lowest annualised volatility of 12.74% and an annualised return of 10.72%. If the investor is seeking more risky opportunities then a weight of 40% in VSTOXX gives the largest annualised return of 13.45%. One important shortcoming of this investment strategy is that investing in VSTOXX can be realised only by trading a portfolio of options contingent on STOXX 50, and rebalancing this daily may prove costly. Therefore, a more suitable strategy would be to construct portfolios mixing STOXX 50 with the associated VSTOXX futures contracts.

Following Szado (2009), the portfolio weights for the volatility futures are pre-set to 2.5% and then 10%. Tables 4 and 5 summarize the performance of the volatility-diversified portfolios. We assume that the portfolios are rebalanced weekly. The Sharpe ratios are commonly used in portfolio analysis to differentiate between competing portfolios. Since this performance measure is not applicable for negative excess returns, the adjusted Sharpe ratios are also reported. The latter are calculated using excess returns over the mean return obtained for the plain equity index portfolio as a benchmark, for the turbulent periods.[3] In addition, in order to gauge the protective cover obtained from adding volatility futures, the historical value-at-risk measures at 95% and 99% confidence levels is also calculated for each portfolio.

The results in Table 4 demonstrate that adding VIX futures has a beneficial effect on portfolio performance, improving mean return but most importantly reducing the volatility. Comparing the performance of the six portfolios under investigation it is also clear that, in normal times such as the period 2004-2007, adding VIX futures contract improves the mean return and produces excellent Sharpe ratios and of course improves VaR risk measures[4]. Moreover, during turbulent times such as 2008-2012, there is a great benefit in having VIX futures in the investment portfolio, the mean return staying positive and Sharpe ratios being the best for the portfolios containing VIX futures positions. Looking at the event risk of 2008 it can also be remarked that extreme losses can be avoided if VIX futures positions are added. The comparison of portfolio performance for the eventful year of 2008 may lead to wrong conclusions if the standard Sharpe ratio is used as a performance yardstick. In Table 4, when

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[3] The robustness of the results to changing the assumptions – such as daily rather than weekly rebalancing, and whether the notional amount of the futures is held in cash – has also been investigated. An alternative would be to invest this amount at the risk free asset and post this as margin. This case is referred to as the 'collateralized futures' case. Although the results are not reported here for lack of space, we note that whether or not the collateralization is taken into consideration for marking to market the futures contracts does not have an impact on the final conclusions.

[4] Interestingly, when using daily rebalancing, during this period adding only 2.5% VIX futures leads to a better performance than adding 10% VIX futures.

comparing the portfolio comprising 60% equity and 40% bond with the portfolio comprising 58.5% equity, 39% bond and 2.5% VIX futures, for the year 2008 only, the Sharpe ratio is better for the former portfolio. However, the latter portfolio has relatively better mean return and less volatility. The adjusted Sharpe ratio corrects for this type of anomaly.

Table 4

Performance of volatility-diversified US portfolios The performance statistics are of the daily relative returns on the different portfolios. The portfolios are weekly rebalanced, and the notional of the futures contracts is assumed to be held in cash (no collateralization of the futures).

	SPX	**97.5% SPX 2.5% VIX Futures**	**90% SPX 10% VIX Futures**	**60% SPX 40% Bonds**	**58.5% SPX 39% Bonds 2.5% VIX Futures**	**54 % SPX 36% Bonds 10% VIX Futures**
All sample (2004- 2012)						
Annualized Mean return	5.11%	5.50%	6.76%	4.91%	5.36%	6.78%
Volatility	22.25%	20.35%	15.80%	12.92%	11.34%	8.90%
Min	-9.03%	-8.43%	-6.90%	-5.46%	-4.86%	-3.60%
Max	11.58%	10.75%	8.39%	6.72%	6.10%	4.30%
Skew	-0.0390	0.0000	0.2670	-0.1179	-0.0409	0.8430
Excess Kurtosis	9.9619	10.7259	12.3318	9.9720	11.3121	11.3938
Annual Sharpe ratio	17.05%	20.53%	34.44%	27.77%	35.58%	**61.32%**
Adjusted Sharpe ratio	NA	1.92%	10.44%	-1.55%	2.20%	**18.76%**
VaR 1%(historical)	4.43%	4.04%	2.91%	2.50%	2.19%	**1.55%**
VaR 5% (historical)	2.13%	1.94%	1.37%	1.22%	1.03%	**0.64%**
subperiod 1: 2004 - 2007						
Annualized Mean return	8.30%	8.70%	9.91%	6.57%	7.02%	8.38%
Volatility	12.09%	10.84%	8.94%	7.22%	6.22%	6.52%
Min	-3.47%	-2.64%	-1.90%	-1.88%	-1.40%	-1.59%
Max	2.92%	2.59%	3.64%	1.85%	1.41%	3.79%
Skew	-0.2767	-0.2321	0.7687	-0.2049	-0.1105	2.3308
XS Kurt	1.9129	1.5462	3.7727	1.4816	0.8776	14.8792
Annual Sharpe ratio	48.37%	57.59%	83.47%	57.04%	73.40%	**90.84%**
VaR 1%(historical)	2.22%	2.00%	1.37%	1.22%	1.00%	**0.77%**
VaR 5% (historical)	1.27%	1.10%	0.78%	0.76%	0.65%	**0.49%**
subperiod 2: 2008-2012						
Annualized Mean return	2.21%	2.59%	3.90%	3.39%	3.85%	5.32%
Volatility	28.50%	26.15%	20.10%	16.47%	14.51%	10.61%
Min	-9.03%	-8.43%	-6.90%	-5.46%	-4.86%	-3.60%
Max	11.58%	10.75%	8.39%	6.72%	6.10%	4.30%
Skew	-0.0005	0.0324	0.2081	-0.0790	-0.0134	0.4754
XS Kurt	6.0670	6.5080	7.9647	6.2421	7.1060	8.3145

Annual Sharpe ratio	6.75%	8.81%	17.95%	18.85%	24.52%	**47.45%**
Adjusted Sharpe ratio	NA	1.45%	8.41%	7.16%	11.30%	**29.31%**
VaR 1%(historical)	5.24%	4.79%	3.60%	2.98%	2.68%	**1.77%**
VaR 5% (historical)	2.90%	2.62%	1.88%	1.61%	1.35%	**0.94%**
short crisis subperiod: 2008						
Annualized Mean return	-42.23%	-39.74%	-31.95%	-24.35%	-22.07%	-14.99%
Volatility	41.37%	38.43%	30.36%	24.03%	21.61%	15.75%
Min	-9.03%	-8.43%	-6.90%	-5.46%	-4.86%	-3.60%
Max	11.58%	10.75%	8.39%	6.72%	6.10%	4.30%
Skew	0.1999	0.2116	0.2942	0.0925	0.1310	0.3818
XS Kurt	3.8773	4.0208	4.4947	3.8998	4.1829	4.9141
Annual Sharpe ratio	-104.37%	-105.89%	-108.39%	-105.29%	-106.49%	-101.22%
Adjusted Sharpe ratio	NA	6.48%	33.86%	74.41%	93.29%	**172.95%**
VaR 1%(historical)	8.24%	7.66%	6.22%	4.97%	4.47%	**3.20%**
VaR 5% (historical)	4.52%	4.11%	2.90%	2.59%	2.23%	**1.45%**

Table 5

This table summarizes the performance of volatility-diversified European portfolios. The performance statistics are of the daily relative returns on the different portfolios. The portfolios are weekly rebalancing, and the notional of the futures contracts is assumed to be held in cash (no collateralization of the futures)**.**

	STOXX	**97.5% STOXX 2.5% VSTOXX Futures**	**90% STOXX 10% VSTOXX Futures**	**60% STOXX 40% Bonds**	**58.5% STOXX 39% Bonds 2.5% VSTOXX Futures**	**54 % STOXX 36% Bonds 10% VSTOXX Futures**
Annualized Mean return	5.42%	5.97%	7.68%	5.02%	5.58%	**7.31%**
Volatility	25.51%	23.42%	17.92%	15.05%	13.28%	**9.51%**
Min	-6.33%	-5.81%	-5.21%	-3.77%	-3.22%	-3.10%
Max	10.35%	9.43%	6.78%	6.48%	5.72%	3.54%
Skewness	0.1616	0.1765	0.1912	0.2552	0.3047	0.4004
Excess Kurtosis	3.3844	3.3979	3.4526	3.9618	4.1059	3.9719
Annualized Sharpe ratio	10.66%	13.98%	27.82%	15.45%	21.73%	**48.45%**
Adjusted Sharpe ratio	NA	2.35%	12.61%	-2.66%	1.20%	**19.87%**
VaR 1%(historical)	4.28%	3.83%	2.69%	2.65%	2.15%	**1.42%**
VaR 5% (historical)	2.55%	2.32%	1.78%	1.57%	1.36%	**0.88%**

A similar story follows from the results of Table 5, although this analysis covers only most recent period due to the availability of VSTOXX futures contracts introduced by EUREX.

For the European case, the results show that, for the period under analysis (i.e. May 2009 – February 2012), adding volatility exposure to an equity portfolio that tracks the EURO STOXX 50 indeed provides risk diversification benefits: the volatility decreases from over 25% to under 18% (i.e. a reduction of around 30%) for a 10% exposure to VSTOXX futures (nearest maturity). Downside risk, as measured by Value-at-Risk, computed using the historical methodology for two different significance levels, 1% and 5%, also decreases. Moreover, the average return also increases, from a (annualized daily) value of 5.42% to 7.68% (an increase of 40%), resulting in a very significant increase in the annualized Sharpe ratio, from less than 0.06 to over 0.21, an almost 4-fold increase. A reduction in volatility coupled with an increase in returns is also obtained by investing as little of 2.5% of the portfolio value in VSTOXX futures, only that improvements are more moderate in this case.

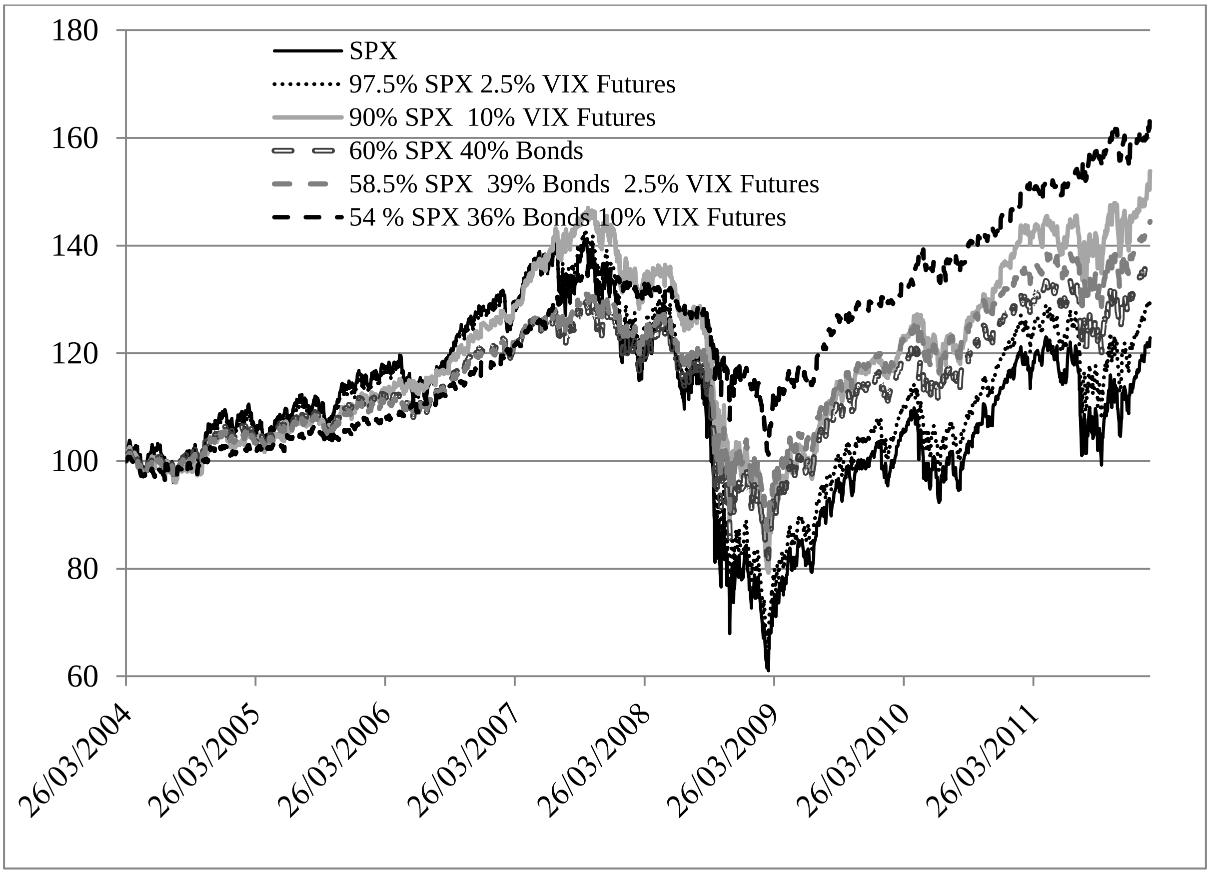

Fig. 2. Comparative Performance of various portfolios based on S&P 500

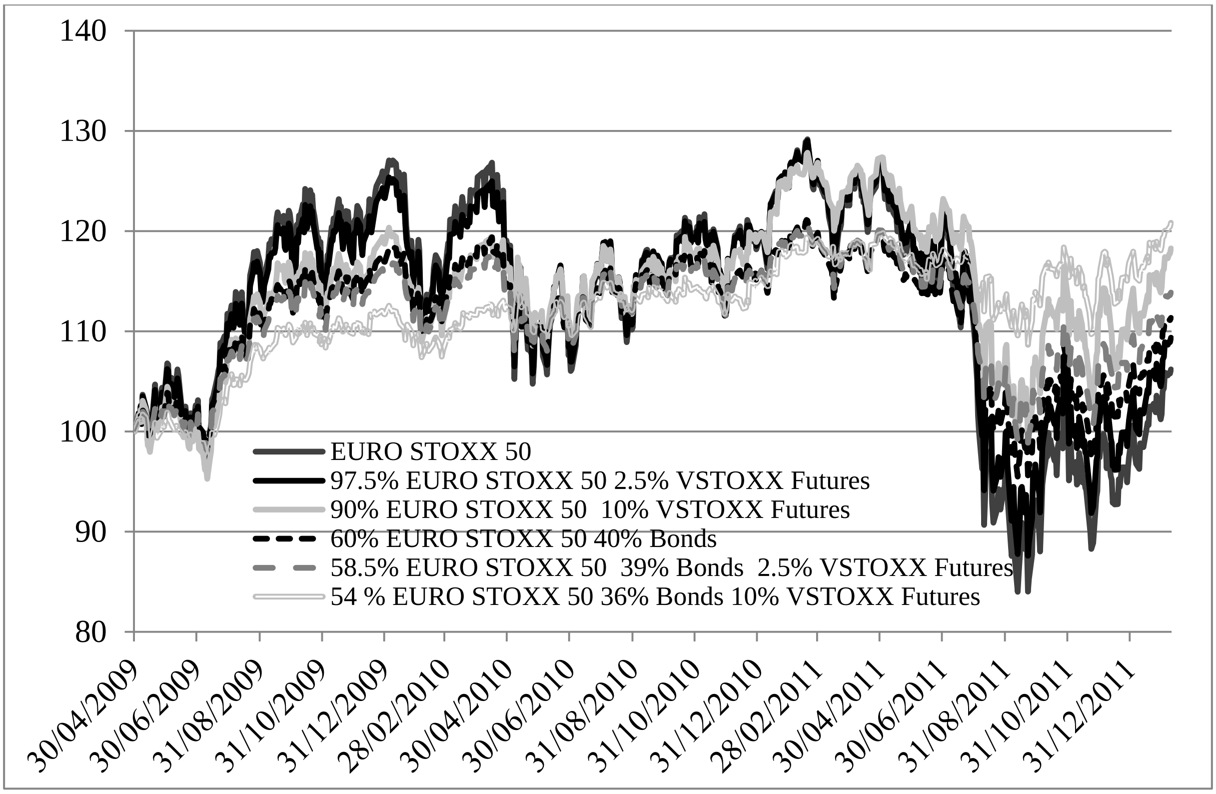

Fig. 3. Comparative Performance of various portfolios based on EURO STOXX 50

In Figures 2 and 3 various portfolios combining equity positions, bond positions and volatility index positions are compared. Overall it can be seen that VSTOXX and VIX futures contracts can help investors to preserve positive returns after unexpected shocks in the equity markets. On the other hand, over periods of market calmness, the futures contracts are more of a break, confirming similar analyses in Szado (2009) and Rhoads (2011).

4. Modelling the VIX-VSTOXX difference

This section investigates the nature of the difference between the VIX and VSTOXX volatility indices. If significant, it is shown how this difference can be exploited in a trading strategy, hence futures prices on the two volatility indices rather than with their respective spot levels are considered here, since futures, unlike their respective underlying volatility indices, are tradeable. Since these are the most actively traded contracts, the nearest maturity futures contracts both for the VIX as well as for the VSTOXX are employed in the analysis.

For the validity of the trading strategies suggested below, VIX and VSTOXX nearest maturity futures prices must be synchronous. The VIX prices are opening prices, and taking into account that, according to CBOE's website[5], trading in VIX futures starts at 7am Chicago time and also considering the 7 hour time difference between Chicago and Frankfurt, the VSTOXX prices utilised below are 2pm CET prices[6].

We start by testing whether this difference is statistically significant and we then proceed to modelling the stochastic behaviour of the difference by means of discrete-time GARCH modelling.

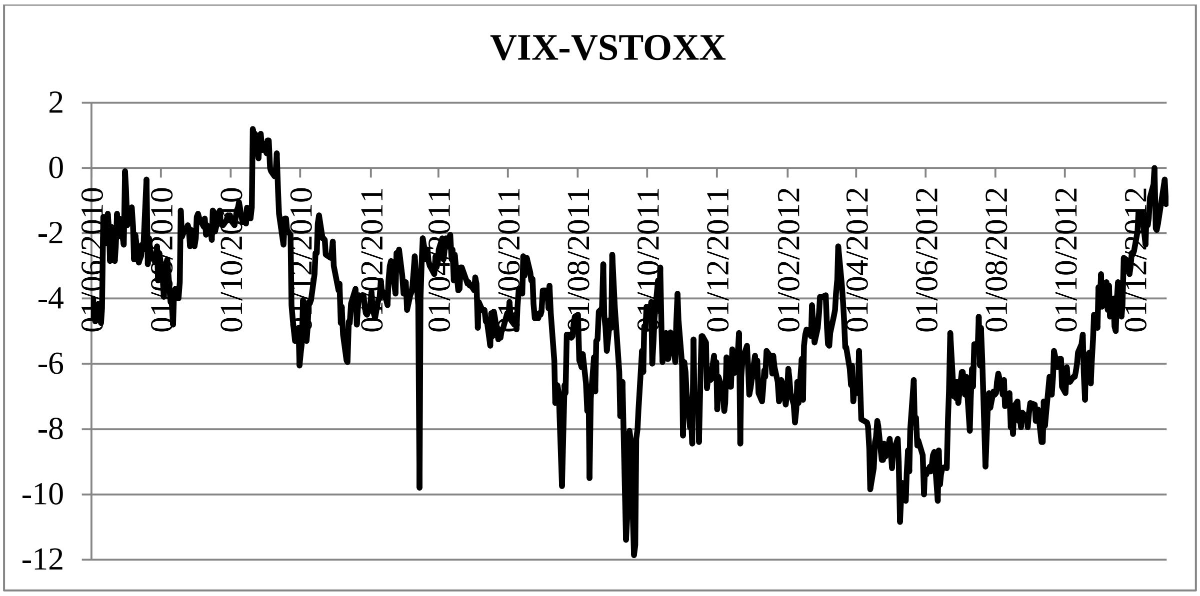

Fig. 4. VIX-VSTOXX Futures Historical Difference

Figure 4 plots the daily series of differences between the VIX and the VSTOXX nearest maturity futures prices, for a period of 2.5 years, ranging from 1st June 2010 to the 28th

[5] http://cfe.cboe.com/Products/Spec_VIX.aspx

[6] VSTOXX futures are traded on between 8:50 and 17:30 CET: http://www.eurexchange.com/exchange-en/products/vol/vol/14566/

December 2012, while Table 6 summarizes the main statistics for this series. From Figure 4, it can be inferred that the VIX-VSTOXX futures prices difference series appears to be stationary and also characterized by ARCH effects. Both features are confirmed by the ADF and ARCH test results, respectively (see Table 6). According to the Augmented Dickey Fuller (ADF) test, the difference series is stationary, but only if the test is performed at significance levels higher than 6% level. However, using the Phillips-Peron unit root test, the series appears to be stationary even at the 1% significance level. Furthermore, the series exhibits volatility clustering, ARCH test results being significant at the 1% level.

Table 6

This table displays the summary statistics for the difference between the VIX and VSTOXX nearest maturity futures synchronous prices, from 1st June 2010 to 28th December 2012. Asterisks denote significance at 10% (*), 5% (**) and 1% (***). The standard error of the sample mean is equal to the sample standard deviation, divided by the square root of the sample size, while the standard errors are approximately $(6/T)^{1/2}$ and $(24/T)^{1/2}$ for the sample skewness and excess kurtosis, respectively, where T is the sample size.

Mean	-4.8269***
t stat mean	-50.4781
Std dev	2.4304
Min	-11.8700
Max	1.2000
Skewness	-0.0624
t stat skew	-0.2643
Excess Kurtosis	-0.4060*
t stat kurt	-1.7197
ARCH test	396.3328***
ADF test	-2.781030*

The difference between the nearest futures prices of the two volatility indices appears significant and negative, which means that the volatility implied by the EURO STOXX 50 options was (expected to be) significantly higher than that of S&P 500 options, at least for the period under consideration. The series also exhibits mild non-normality features in the higher moments – namely significant negative excess kurtosis – further advocating the use of GARCH modelling which can (at least partially) also explain these features.

A number of models from the GARCH family are estimated below in order to see which one captures best the dynamics of the difference series; furthermore, as models from the GARCH family also lend themselves to forecasting applications, the forecasts implied by these models shall also be considered. A very general specification of a GARCH model is given by:

$$\begin{aligned} y_t &= E(y_t \mid \Omega_{t-1}) + \varepsilon_t \\ \varepsilon_t &= z_t \sigma_t \\ z_t &\sim D(0,1) \\ \sigma_t &= f(\{\varepsilon_{t-i}\}, \{\sigma_{t-j}\}, \{X_{t-1}\} \forall i \geq 1, j \geq 1) \end{aligned} \tag{3}$$

In the above set of equations, y_t denotes financial time series under analysis, in our case this will be the difference series described above; $E(y_t \mid \Omega_{t-1})$ denotes the conditional mean of this difference, while ε_t is a disturbance process. $\{z_t\}$ is a sequence of *i.i.d* random variables with (zero mean and unit variance) probability distribution *D*. The last equation provides an expression for the conditional standard deviation; X_t is a vector of predetermined variables included in the information set Ω_t, available at time *t*.

A plethora of models have been developed in the literature following Engle and Bollerslev's seminal papers, many of them listed in a recent and very useful glossary compiled by Bollerselv (2008). In order to find the most appropriate GARCH model to explain the VIX-VSTOXX difference (which was shown above to have ARCH effects), the mean equation is first fitted and alternative error distributions and conditional variance specifications to forecast the VIX – VSTOXX difference are subsequently considered.

From the ACF and PACF analysis[7] as well as from a detailed mean equation model selection based on various (information) criteria,[8] it is found that a constrained AR(4) model (with the coefficient on the third lag constrained to be equal to zero) is the most parsimonious model that eliminates the autocorrelation. We therefore proceed to GARCH estimation, based on a constrained AR(4) mean equation.

The GARCH model in (3) now becomes:

$$\begin{aligned} y_t &= \mu + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_4 y_{t-4} + \varepsilon_t \\ \varepsilon_t &= z_t \sigma_t \\ z_t &\sim D(0,1) \\ \sigma_t &= f(\{\varepsilon_{t-i}\},\{\sigma_{t-j}\},\{X_{t-1}\} \forall i \geq 1, j \geq 1) \end{aligned} \tag{4}$$

where the error distribution D will be either the normal or the (standardized) Student-t with ν degrees of freedom.

We now turn our attention to the final equation in (4), the conditional variance equation, where the focus of a GARCH model lies. Three different variance specifications are considered in this paper: the classical symmetric GARCH(1,1) of Bollerslev (1986) and two asymmetric specifications, the exponential GARCH (EGARCH) model of Nelson (1991) and the GJR model, first introduced by Glosten, Jagannathan and Runkle (1993). The choice of these particular three versions out of the great variety of GARCH models available is not random. The basic GARCH(1,1) model offers the advantage of having a simple specification of the conditional variance equation. This is especially important in a forecasting exercise. Even if more elaborate models tend to fit better in sample, parsimonious models are preferred

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[7] See Figure A3 from Appendix A.

[8] See Table B1 from Appendix B.

in prediction because they have more degrees of freedom. Moreover, previous empirical studies have proved that no more than a GARCH(1,1) is needed to account for volatility clustering.[9] However, in equity markets, volatility tends to increase more following unexpectedly large negative returns than following unexpected positive returns of the same magnitude. To capture this asymmetry in volatility, often attributed to the “leverage effect” (i.e. a fall in the market value of a firm will increase its degree of leverage), more than a GARCH(1,1) is needed. Both the GJR and the EGARCH models allow for asymmetric responses of volatility to positive and negative shocks respectively. Hence, the final equation in (2) will, in turn, take one of the following forms:

$$
\begin{aligned}
&GARCH(1,1): \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \\
&GJR: \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \lambda\varepsilon_{t-1}^2 1(\varepsilon_{t-1} < 0) \\
&EGARCH: ln(\sigma_t^2) = \omega + \alpha\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - E\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}}\right]\right] + \beta\, ln(\sigma_{t-1}^2) + \lambda\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}
\end{aligned}
\tag{5}
$$

where $1(\varepsilon_t < 0) = \begin{cases} 1, \text{ if } \varepsilon_t < 0 \\ 0, \text{ otherwise} \end{cases}$.

Since the variance is always a positive quantity, non-negativity constraints apply for GARCH(1,1) and GJR: in both models $\omega>0$, α, $\beta \geq 0$; for the latter model, $\alpha + \gamma \geq 0$ is also sufficient for non-negativity.[10] One advantage of the EGARCH model is that it does not

[9] For example, Berkowitz and O’Brien (2002) show that VaR forecasts based on a simple ARMA(1, 1)-GARCH(1,1) model were at least as accurate as those produced by the complicated structural models employed by six large commercial banks.

[10] Parameter conditions that ensure that the conditional variance converges to a finite unconditional variance are given in Table B.2 from Appendix B. We note that, for all 7 models considered, the parameter estimates reported in Table 7 satisfy these convergence conditions.

necessitate any non-negativity constraints. Moreover, for the leverage effect to hold one would need $\gamma>0$ for the GJR and $\gamma<0$ for the EGARCH.[11] In the interest of clarity, the full details of the estimated GARCH models are summarized in Table B2 from Appendix B, while the estimation results obtained for alternative GARCH models are reported in Table 7 below.

The results in Table 7 show that all GARCH models considered fit very well in sample, with the Student-t EGARCH model maximizing the value of the log-likelihood. For the symmetric models (i.e. the normal and Student-t GARCH(1,1) models) the estimated parameters are significant, with the sole exception of the constant from the conditional mean equation of the GARCH(1,1) with normally distributed innovations. With perhaps the exception of the Student-t EGARCH, the asymmetric models do not offer a particularly good fit for the VIX-VSTOXX difference series. The asymmetry coefficient, λ, is statistically insignificant for three out of the four asymmetric GARCH models estimated, being marginally significant (only at 10% significance level) for the case of the normal EGARCH. A negative estimated value of the λ coefficient in the context of the EGARCH model would signify that the conditional variance and consequently the volatility of the difference series would respond more to an unexpected negative change in the VIX-VSTOXX series than to a positive unexpected change of the same magnitude.

[11] The coefficients of the GARCH models are estimated using the technique of Maximum Likelihood (ML). Note that for the EGARCH models a slightly restricted versions of the specification given in (2) is actually estimated, namely: $\ln(\sigma_t^2) = \omega_0 + \alpha\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}}\right] + \beta\ln(\sigma_{t-1}^2) + \lambda\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$; this restriction however has no impact on the parameter estimates α, β and λ and $\omega_0 = \omega + \alpha E\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}}\right]$.

Table 7

This table shows the results on GARCH model estimation using Bollerslev-Wooldridge robust standard errors. Asterisks denote significance at 10% (*), 5% (**) and 1%(***).

	AR(4)-N-GARCH(1,1)	**AR(4)-T-GARCH(1,1)**	**AR(4)-N-GJR**	**AR(4)-T-GJR**	**AR(4)-N-EGARCH**	**AR(4)-T-EGARCH**	**AR(4)-in-mean-N-EGARCH**
μ	-0.0874	-0.1042*	-0.1079*	-0.1088*	-0.2090***	-0.0998*	-0.1846*
φ_1	0.7480***	0.6993***	0.7455***	0.6975***	0.7110***	0.6992***	0.7635***
φ_2	0.1622***	0.1900***	0.1704***	0.1927***	0.1115*	0.1892***	0.1500***
φ_4	0.0708**	0.0835***	0.0640*	0.0822***	0.1311***	0.0856***	0.074402**
GARCH-in-mean	-	-	-	-	-	-	0.1961*
ω	0.0947*	0.1055**	0.0984**	0.0842**	-0.7136***	-0.2024***	-0.2594***
α	0.2030*	0.1687***	0.0649	0.0957	0.0509	0.2331***	0.2822***
β	0.6920***	0.7033***	0.7089***	0.7544***	-0.9663***	0.9242***	0.8764***
λ	-	-	0.2091	0.0797	-0.0596*	-0.0548	-0.1010***
ν	-	4.0185***	-	4.1217***	-	4.2557***	-
Log Likelihood	-781.8187	-731.4942	-777.3889	-731.1060	-800.7735	-727.1336	-771.7436

Moreover, the response parameter, α, which quantifies the response of the conditional variance of the difference series to squared unexpected shocks in the series is statistically insignificant for all asymmetric GARCH models estimated above apart from the Student-t EGARCH model. It is therefore apparent that most of the asymmetric models, at most with the exception of the Student-t EGARCH, provide a poorer fit than their symmetric counterparts, with the normal EGARCH even having a wrong (counter-intuitive) negative sign on the persistence parameter β. Also apparent from Table 7 is the fact that the models with (standardized) Student-t distributed innovations fit comparatively better than the corresponding normal models.

Although not reported in this table because of lack of space, GARCH-in-mean versions for all the models in Table 7 were also estimated (i.e. an additional regressor was added to the conditional mean equation, which was either the conditional variance, or its square root or its natural logarithm). However, the GARCH-in-mean terms were insignificant for all 17 out of the 18 specifications estimated and hence results are not reported here. Table 7 includes the results obtained for the only model for which the GARCH-in-mean (variance) term was significant (as well as all the other parameters), namely the normal (restricted) AR(4)-EGARCH-in-mean.

5. Statistical Arbitrage Strategies using GARCH Forecasting

Knowing that the difference between the VIX and VSTOXX is significant and negative the following 'naïve' trading strategy is first investigated. A cross-country spread is entered into, short 100 VIX futures and long a number of VSTOXX futures, adjusted daily, such that the size (i.e. point value) of the short and long positions is the same. For example, given that the size of one VSTOXX mini-futures contract is 100 EUR per point and the size of one VIX futures is 1000 USD per point, if the exchange rate is 1.3 USD for 1 EUR, one would go long: $\frac{1000\ \mathrm{USD}/100\ \mathrm{EUR}}{1.3\,\mathrm{USD}/\ \mathrm{EUR}} \cong 7.69$ VSTOXX futures contracts for each VIX contract, which gives a total position of 100 short VIX futures contracts to 769 long VSTOXX contracts. While the number of VIX contracts stays fixed at 100, the number of VSTOXX contracts is adjusted daily such that the size of both legs of the transaction matches at 100, 000 USD per point.[12]

Fig. 5. Number of (long) VSTOXX contracts for each 100 (short) VIX contracts

Figure 5 illustrates the evolution of the number of VSTOXX contracts from 1st June 2010 to 28th December 2012, while Figure 6a plots the cumulative profits (given in US dollars) that could have been generated by this strategy. It is note that, if put in place at the beginning of June 2010, the strategy would have been profitable for roughly half of horizons up 2.5 years. To make the performance of this strategy directly comparable to that of the GARCH strategy presented below, in Figures 6c and 6d the cumulative and daily P&L are computed assuming the strategy commences at the start of June 2012 (rather than June 2010). This will correspond to the out-of-sample period employed for the evaluation of the GARCH strategy.

[12] To compute the number of VSTOXX contracts, it is considered that the closing FX rate on day *t-1* is equal to the opening FX rate on date *t*, i.e. 'aligned' (as it is not exactly synchronous) with the opening of the VIX and the 2pm VSTOXX futures prices, which are synchronous.

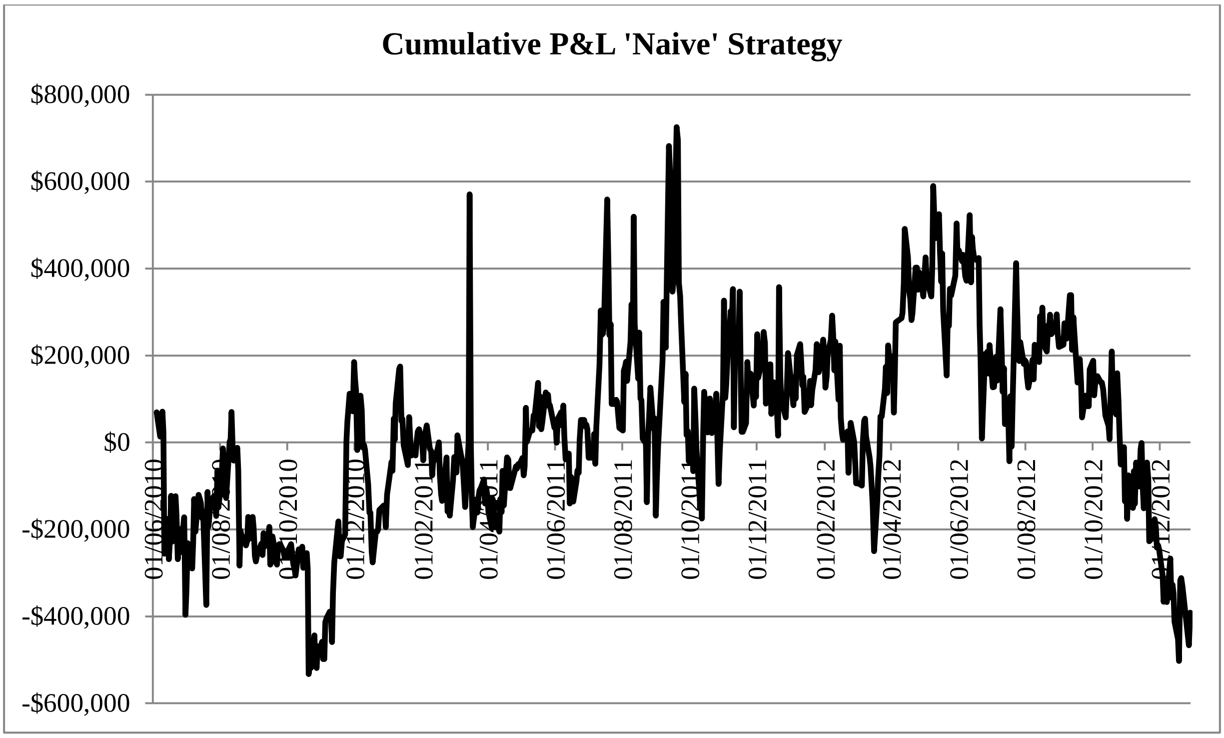

Fig. 6a. Cumulative P&L – 'Naïve' Long-Short Strategy

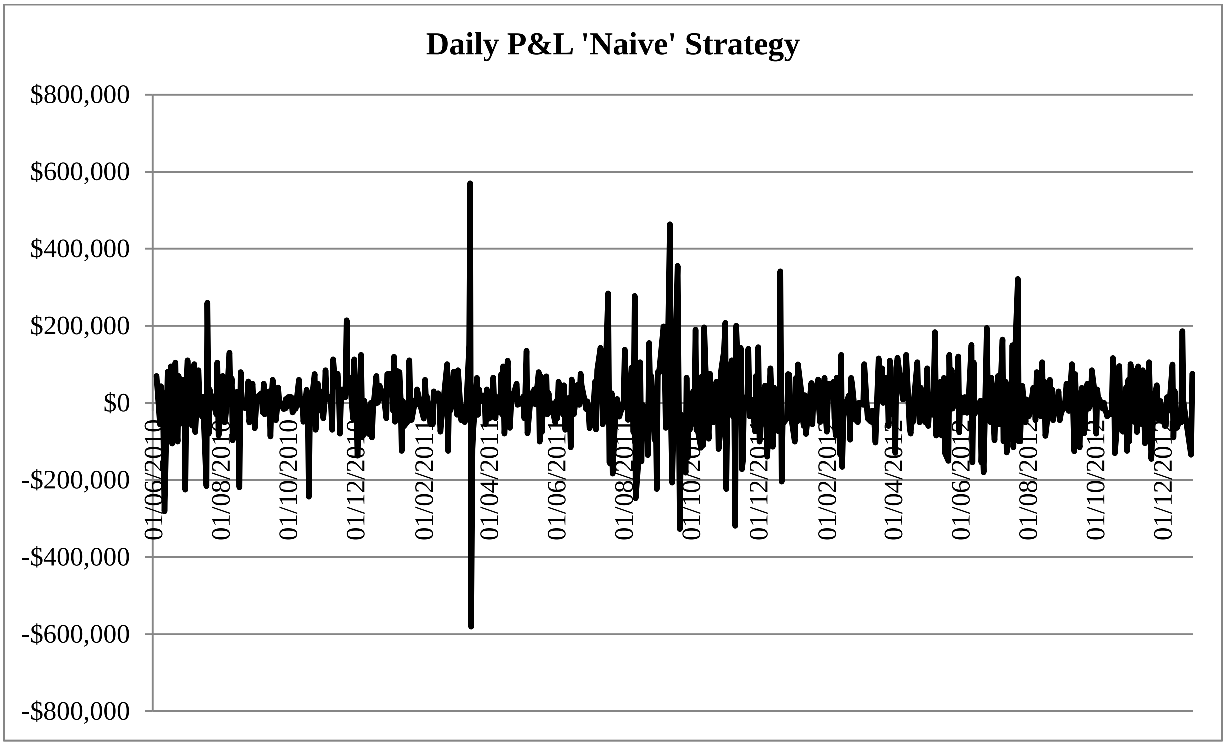

Fig. 6b. Daily P&L – 'Naïve' Long-Short Strategy

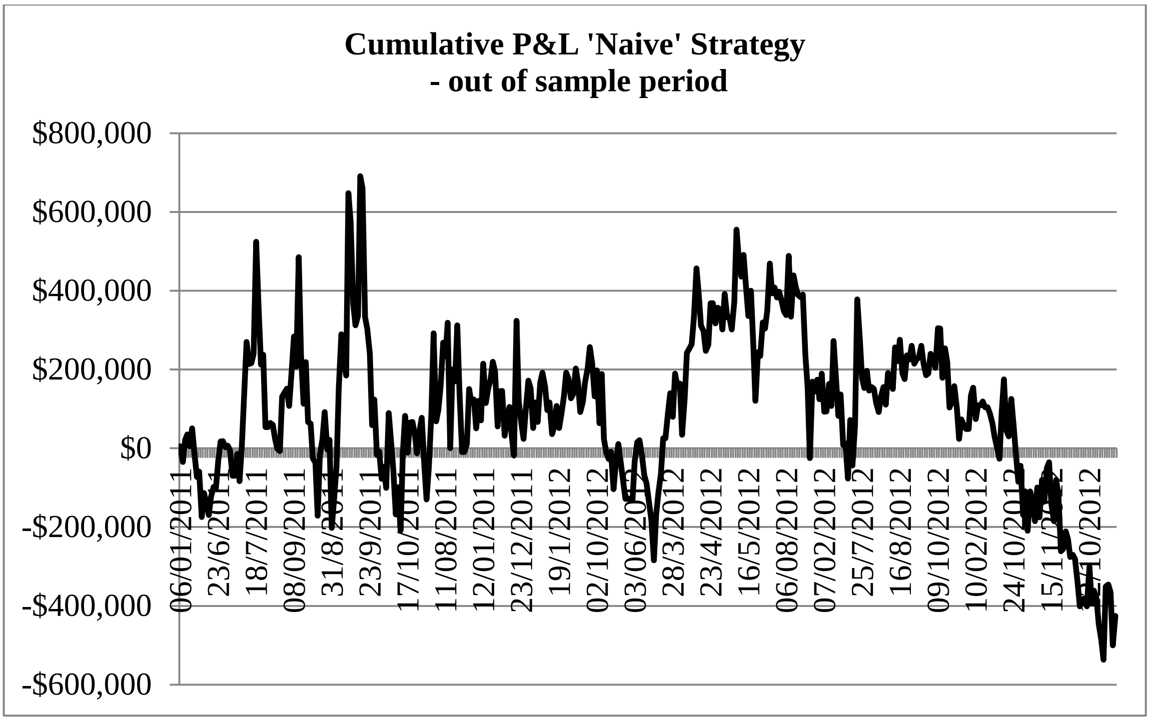

Fig. 6c. Cumulative P&L – 'Naïve' Long-Short Strategy, out-of-sample period

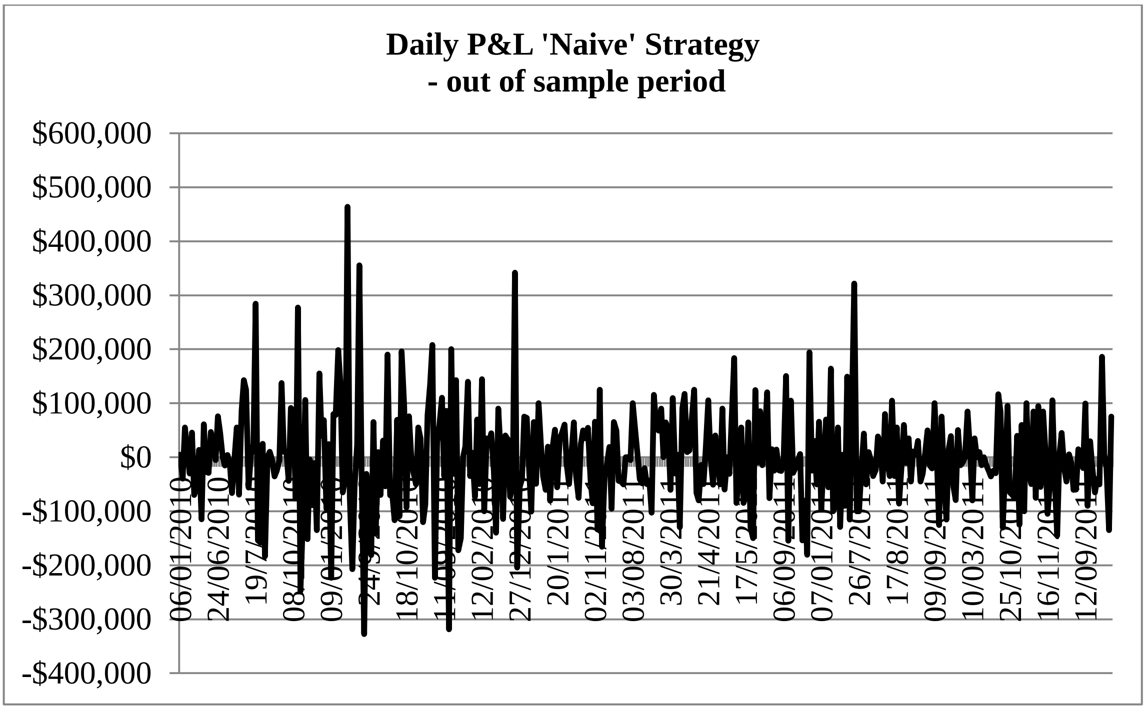

Fig. 6d. Daily P&L – 'Naïve' Long-Short Strategy, out-of-sample period

A potential application of the GARCH modelling results is for the forecasting of the VIX-VSTOXX (nearest futures price) difference which in turn can be used to inform trading strategies. Figure 7 plots the series of one-step ahead forecasts obtained from a AR(4)-T - GARCH(1,1) (see Appendix B, Table B.2 for the exact model specification). The model parameters are re-estimated daily, using a rolling sample of 500 observations, with 146 observations used for out-of-sample forecasting. The results depicted in Figure 7 show that the VIX-VSTOXX (actual) futures difference remains negative for the entire forecasting period (i.e. June 2012-December 2012). This is not surprising given that during this period the European markets have been affected by the recent European sovereign debt crisis, which had a much lesser impact on the US market. Remarkably, the proposed model correctly forecasts the sign of the difference throughout the observation period.

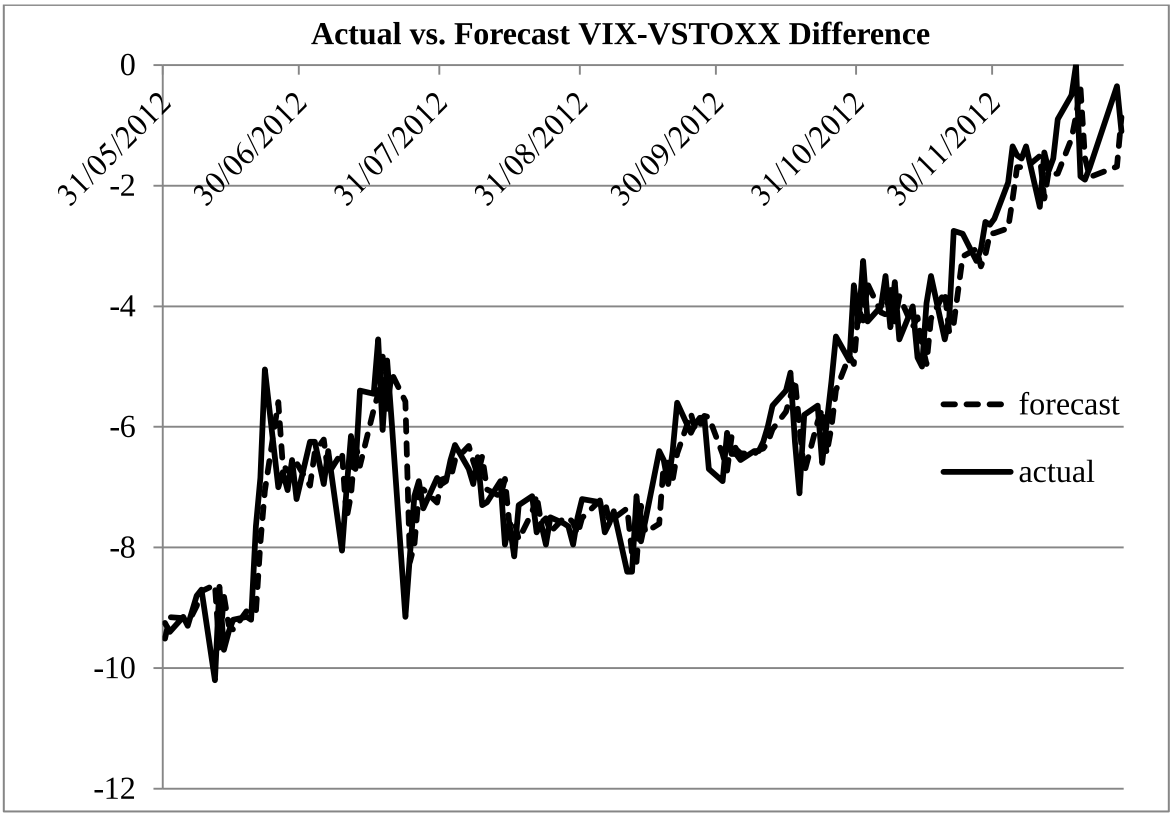

Fig. 7. One-step ahead forecasts of the VIX-VSTOXX nearest Futures Price Difference

Given that the VIX-VSTOXX futures price difference is negative throughout the forecasting evaluation period (and has a significant negative mean throughout the entire sample), the following (GARCH) strategy is proposed: We start the strategy by going long the nearest maturity VSTOXX futures, with a size equivalent to \$100, 000 per point and simultaneously going short 100 contracts (i.e. size \$100,000 per point) of the nearest maturity VIX futures the first time our AR(4)-T-GARCH(1,1) model forecasts an increase of the spread in absolute value and unwind when the model signals a reduction in spread. Figure 8 plots the forecasted change in the VIX-VSTOXX futures price difference, for our chosen out of sample period (beginning of June to end December 2012). Since the difference is negative throughout, a positive change will signify a decrease in the VIX-VSTOXX nearest futures price difference. Therefore, the strategy will be activated each time the forecasted change in the VIX-VSTOXX futures difference is negative and unwind when it is positive. More generally, one could also choose to activate or deactivate (unwind) the positions depending on whether the difference is above or below a certain threshold, which could, but need not, be equal to zero.

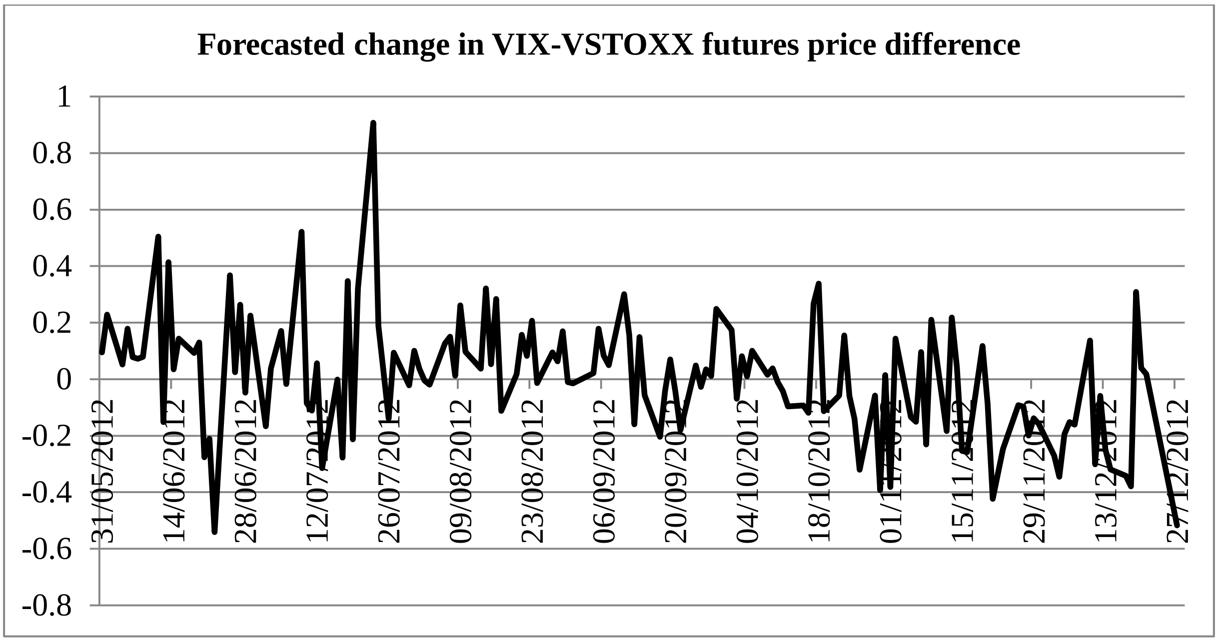

Fig. 8. Daily forecasted change in the VIX-VSTOXX nearest futures price difference

The performance of the trading strategy is illustrated in Figure 9, with panel a) of this figure depicting the cumulative P&L that the strategy could have generated, while the plot in panel b) is of the daily P&Ls. The results in Figure 9 are directly comparable to those in Figure 6, panels c) and d). This strategy seems to work much better, taking advantage of the excellent forecast of the spread. Moreover, this latter (GARCH) strategy should involve lower transaction costs as well, as the naïve strategy requires daily rebalancing of the VSTOXX leg (so that sizes of the two legs always match), whereas with the GARCH strategy one would only trade on certain days rather than every day, namely when the models signals a widening of the VIX-VSTOXX futures spread.

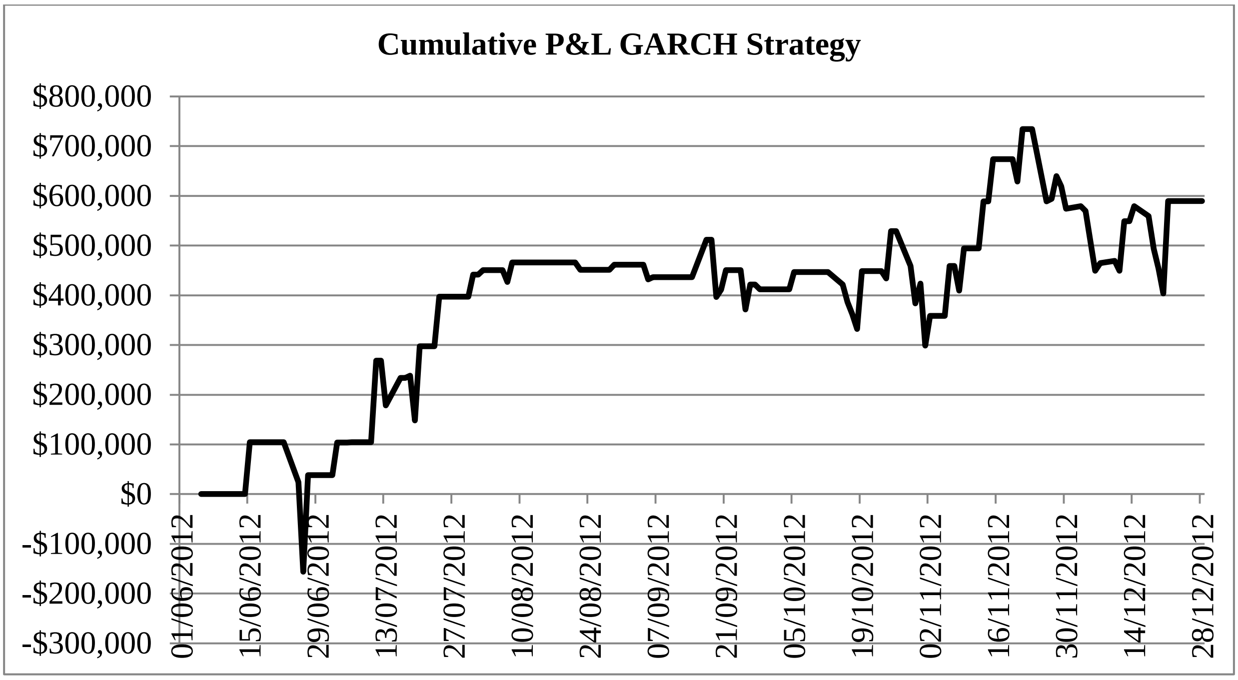

Fig. 9a. Cumulative P&L for the GARCH strategy

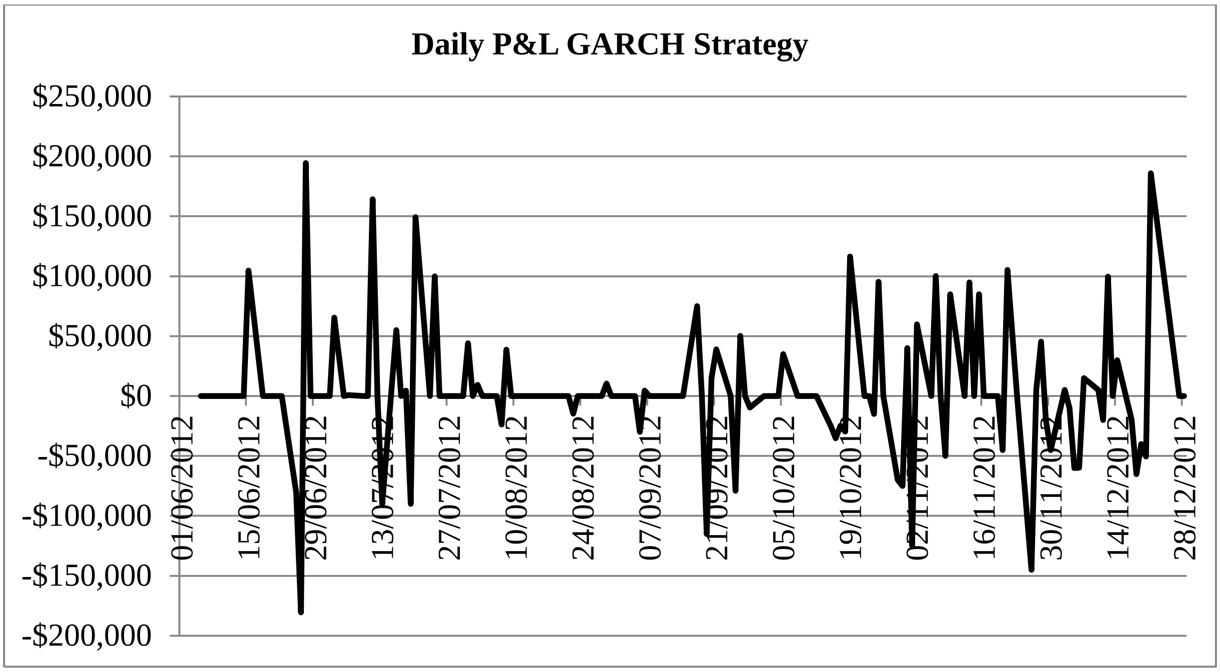

Fig. 9b. Daily P&L for the GARCH strategy

6. **Conclusions**

In this paper empirical evidence that adding volatility index futures to an equity and portfolio improves overall performance is brought forward. This is true for both VIX and VSTOXX. The best composition would have equity, bonds and volatility derivatives futures with first or second maturity. While the current literature is divided on the usefulness of volatility index futures for portfolio diversification, the first major contribution of the paper is to use the methodology described in Szado (2009) and demonstrate that using VIX and VSTOXX futures improves the return-risk profile of investment portfolios, particularly during turbulent times. The benefits seem to be larger for VSTOXX, although there is less historical data involving futures contracts.

The second major contribution of this paper is to tackle the data for U.S. and Europe with a battery of state-of-the art GARCH models. Since the difference in VIX versus VSTOXX futures prices seems to be stationary, there is a clear incentive to identify suitable models for statistical arbitrage. Identifying a GARCH model that works well with data allows investors to engage in directional trading given by the signal produced by the GARCH model. GARCH models can be used to forecast the spread between VIX and VSTOXX futures on a daily basis and signal entering and exiting the trades. The analysis in this paper shows that the statistical arbitrage approach could have provided substantial gains over recent periods.

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Appendix A

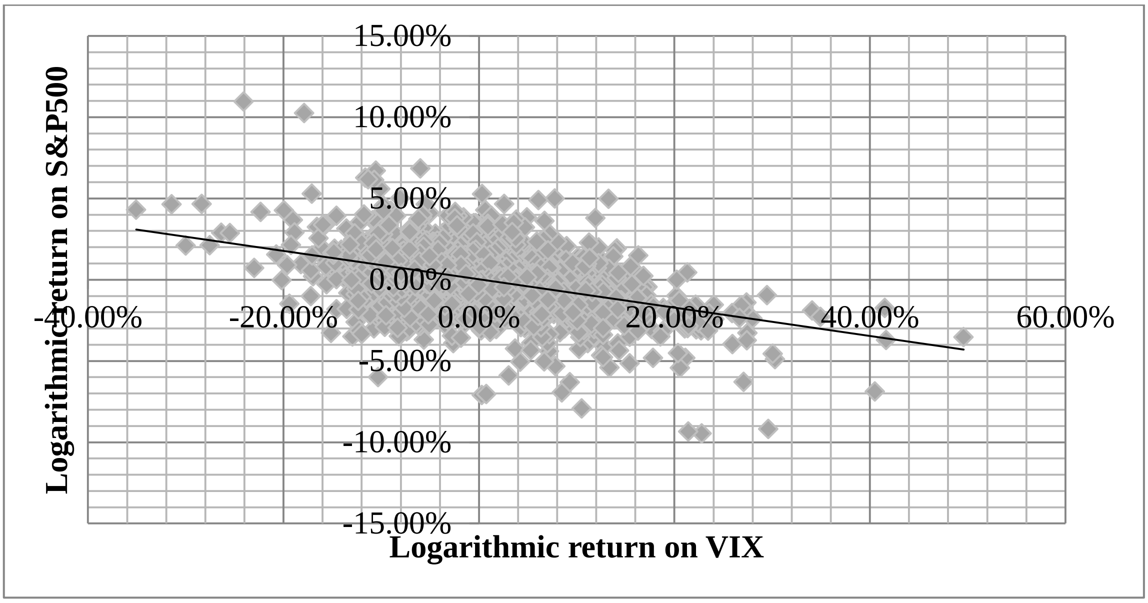

FIG. A1 Scatter plot of pairs of logarithmic returns for VIX and S&P500 between 02-01-1990 and 01-03-2012

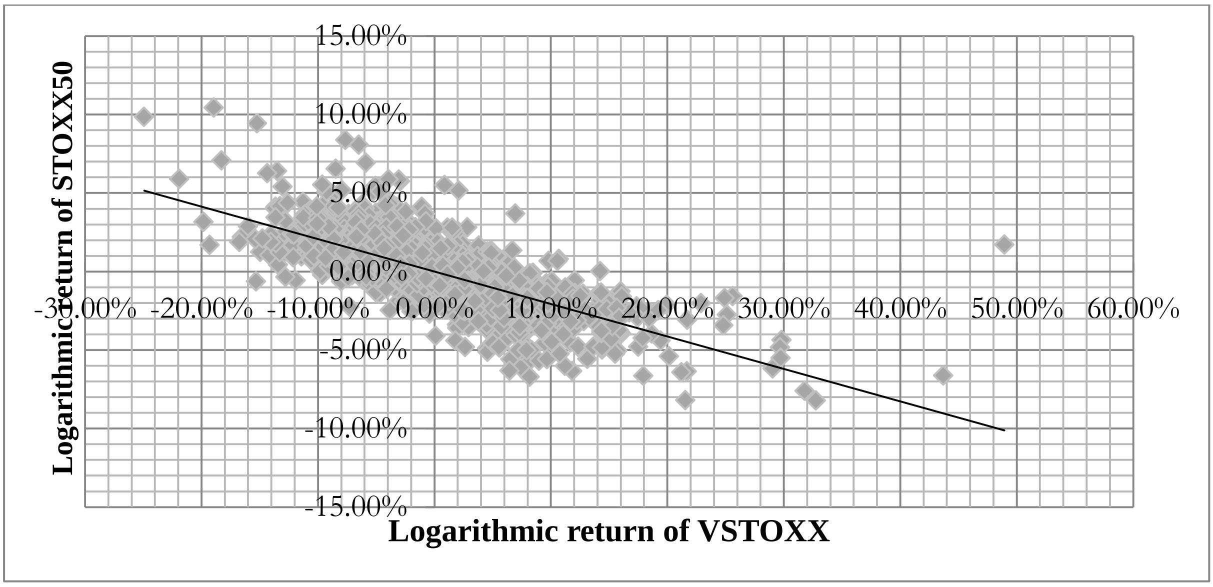

FIG. A2 Scatter plot of pairs of logarithmic returns for VSTOXX and EURO STOXX 50 between 04-01-1999 and 24-02-2012

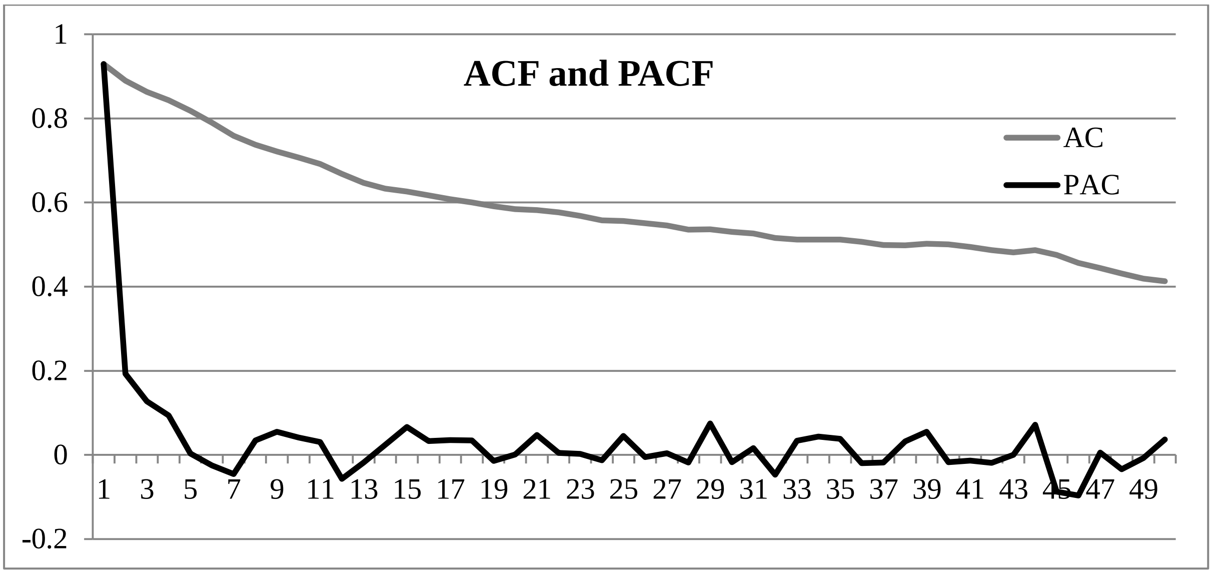

FIG. A3 Autocorrelation function (ACF) and the partial autocorrelation function (PACF)

The graph illustrates the autocorrelation functions used in deciding how many lags should be included in the autoregressive part of the mean for the GARCH models, for the VIX-VSTOXX Nearest Futures Difference time series

Appendix B

Table B.1

This table reports ARMA model selection results. AIC, BIC, HQIC stand for the Akaike, Bayesian and Hannan-Quinn information criteria. The optimal model, according to a particular information criterion, should minimize the respective information criterion. Asterisks denote significance at 10% (*), 5% (**) and 1%(***).

	AR(1)	AR(2)	AR(3)	AR(4)	AR(4) constrain AR(3)=0	ARMA(1,1)	ARMA(2,1)	ARMA(1,2)	ARMA(2,2)	MA(1)	MA (2)	MA(3)	MA(4)
AIC	2.6064	2.5672	2.5550	2.5500	2.5492	2.5518	2.5482	2.5467	2.5508	3.7704	3.3405	3.1265	2.9896
BIC	2.6202	2.5880	2.5828	2.5847	2.5770	2.5726	2.5759	2.5744	2.5855	3.7843	3.3612	3.1542	3.0242
HQIC	2.6118	2.5753	2.5658	2.5634	2.5600	2.5599	2.5589	2.5574	2.5643	3.7758	3.3485	3.1372	3.0031
AR(1)	***	***	***	***	***	***	***	***	**	-	-	-	-
AR(2)	-	***	**	**	***	-	**	-	not signif	-	-	-	-
AR(3)	-	-	***	not signif	-	-f	-	-	-	-	-	-	-
AR(4)	-	-	-	**	***	-	-	-	-	-	-	-	-
MA(1)	-	-	-	-	-	***	***	***	not signif	***	***	***	***
MA(2)	-	-	-	-	-	-	-	**	not signif	-	***	***	***
MA(3)	-	-	-	-	-	-	-	-	-	-	-	***	***
MA(4)	-	-	-	-	-	-	-	-	-	-	-	-	***
Ljung-Box	Autocorr	No autocorr at lag 1, but lag 2 signif	No autocorr	No autocorr	No autocorr	No autocorr at 1%, but yes at higher signif levels	No autocorr	No autocorr	No autocorr	Autocorr	Autocorr	Autocorr	Autocorr

Table B.2.

This table shows the selected GARCH in mean models fitted to the time series of VIX-VSTOXX futures differences, daily between 1st June 2010 to the 28th December 2012.

Model Name	Variance Model Specification	Condition for finite unconditional variance
AR(4)-N-GARCH(1,1)	$y_t=\mu+\varphi_1 y_{t-1}+\varphi_2 y_{t-2}+\varphi_4 y_{t-4}+\varepsilon_t$ $\varepsilon_t = z_t\sigma_t$ $z_t \sim N(0,1)$ $\sigma_t^2=\omega+\alpha\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2$	$\alpha+\beta<1$
AR(4)-T-GARCH(1,1)	$y_t=\mu+\varphi_1 y_{t-1}+\varphi_2 y_{t-2}+\varphi_4 y_{t-4}+\varepsilon_t$ $\varepsilon_t = z_t\sigma_t$ $z_t \sim Student-t(0,1)$ $\sigma_t^2=\omega+\alpha\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2$	$\alpha+\beta<1$
AR(4)-N-GJR	$y_t=\mu+\varphi_1 y_{t-1}+\varphi_2 y_{t-2}+\varphi_4 y_{t-4}+\varepsilon_t$ $\varepsilon_t = z_t\sigma_t$ $z_t \sim N(0,1)$ $\sigma_t^2=\omega+\alpha\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2+\lambda\varepsilon_{t-1}^2 1(\varepsilon_{t-1}<0)$ $1(\varepsilon_t<0)=\begin{cases}1, \text{ if } \varepsilon_t<0\\ 0, \text{ otherwise}\end{cases}.$	$\alpha+\beta+\frac{\lambda}{2}<1$
AR(4)-T-GJR	$y_t=\mu+\varphi_1 y_{t-1}+\varphi_2 y_{t-2}+\varphi_4 y_{t-4}+\varepsilon_t$ $\varepsilon_t = z_t\sigma_t$ $z_t \sim Student-t(0,1)$ $\sigma_t^2=\omega+\alpha\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2+\lambda\varepsilon_{t-1}^2 1(\varepsilon_{t-1}<0)$ $1(\varepsilon_t<0)=\begin{cases}1, \text{ if } \varepsilon_t<0\\ 0, \text{ otherwise}\end{cases}.$	$\alpha+\beta+\frac{\lambda}{2}<1$
AR(4)-N-EGARCH	$y_t=\mu+\varphi_1 y_{t-1}+\varphi_2 y_{t-2}+\varphi_4 y_{t-4}+\varepsilon_t$ $\varepsilon_t = z_t\sigma_t$ $z_t \sim N(0,1)$ $ln(\sigma_t^2)=\omega+\alpha\left[\frac{\lvert\varepsilon_{t-1}\rvert}{\sqrt{\sigma_{t-1}^2}}-\sqrt{\frac{2}{\pi}}\right]+\beta\, ln\left(\sigma_{t-1}^2\right)+\lambda\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$	Not applicable (variance always converges to a finite long term mean)
AR(4)-T-EGARCH	$y_t=\mu+\varphi_1 y_{t-1}+\varphi_2 y_{t-2}+\varphi_4 y_{t-4}+\varepsilon_t$ $\varepsilon_t = z_t\sigma_t$ $z_t \sim Student-t(0,1)$ $ln(\sigma_t^2)=\omega+\alpha\left[\frac{\lvert\varepsilon_{t-1}\rvert}{\sqrt{\sigma_{t-1}^2}}-E\left[\frac{\lvert\varepsilon_{t-1}\rvert}{\sqrt{\sigma_{t-1}^2}}\right]\right]+\beta\, ln\left(\sigma_{t-1}^2\right)+\lambda\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$	$\alpha+\lvert\gamma\rvert\geq 0$

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