Realized Volatility and Option Time Value Decay Patterns

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Abstract

Options have time value that decays with the passage of time. Whereas the Black-Schole model assumes constant volatility in the underlying, the realized volatility in the underlying can be nonconstant. Option time value decay has a pattern that is driven by market activity and market events, which matches the observed pattern in realized volatility.

Introduction

Option pricing remains a significant challenge in quantitative finance due to its uncertainty. There have been numerous efforts in academia and industry to develop models for modern-day markets. One of the parameters that remains uncertain is the time decay. Options are financial instruments that provide right but not obligation to purchase or sell an underlying asset before a certain expiry. The value of an option contract origins from the optionality factor - that the investor can choose to opt in or out based on market information. This optionality is correlated with time to expiry, since there would be less uncertainty in the underlying, therefore less profitability to hold such optionality.

However, the time value is not linearly correlated with time. The common practice is to use the Black-Scholes formula for pricing, which takes in parameters such as the underlying volatility and time to expiry. The model assumes constant volatility and in such, the time value can be seen as a function of time, holding other parameters constant. However, the real financial market has

nonconstant volatility, and the time value of an option in the formula becomes a function of the product of time and variance, which this paper aims to build on.

In fact, the change of volatility is related with the rate of price discovery in the market on the underlying. However, there is no simple way to represent such value. Since the effect of time decay has received little academic interest. However, in high frequency trading industry, where high precision is required, there are proprietary models developed by firms, but the details on them remain a secret. Some traders remove weekends from the price decay, believing that optionality only changes during trading hours. However, price discovery is effectively still ongoing in during non-trading hours, but is not necessarily reflected on the market. As exchanges created afterhours and pre-open trading, we gain another way to look into the price discovery process during non-regular trading times.

First of all, the time value of an option represents the time left for the option until expiry, in which the option may fluctuate (Hull 2008). According to Hull, the time decay is usually measured by the rate of loss in value in the passage of a day. Theta is usually negative, as the option becomes less valuable as time passes due to decreased volatility and optionality. Although there is a prescribed formula to decay an option's time value, there are restrictions in the industry that require modification to the approach. For example, Harwood talked about the different approaches to option time decay on weekends, where some traders decay by calendar day, while others by trading days (2015). Harwood mentioned that under most situations there is no decay when the market is closed, unless the option is close to expiration. He also mentioned one subtlety that he did not elaborate on, which is how market makers close out their positions prior to close on Friday.

The role of market makers in financial markets is to provide bid and ask prices for assets (Hull 2009). They hold volumes of assets to quote on markets, and will provide liquidity when investors wish to buy or sell. The additional liquidity facilitates faster price discovery (Das, 2008). The price discovery process is the process of determining the price of an asset through interactions between buyers and sellers. There are researches on if the price discovery process in the options market affects the underlying market, but seemingly none in the opposite direction, at least publicly available (Muravyev et al., 2012, Holowczak et al., 2007).

Therefore an important question is how price discovery and the amount of trading activity are related. This project mainly focuses on the discrepancy of trading activity across different times of the day. The fluctuation of trading activities across different trading hours is a widely observed and acknowledged phenomenon. The most common example is after hour trading. After hour trading refers buying and selling from an exchange outside of normal trading hours, which, although becomes more accessible to investors, consists of very small volume, at a mere 4% on Nasdaq, according to Barclay and Hendershott (2003). Moreover, according to Gerety and Mulherin, there exists an intraday pattern in trading volume in the equities market (1992). Admati and Pfleiderer studied the role of liquidity traders and informed traders in forming an intraday trading pattern on stock market, but only provided a model and still left questions unanswered. There has been observations that different amount of trading activities and announcements can cause shift in volatility, as indicated by Andersen and Bollerslev(1997), but there is no existing study on a detailed model on equity and crude oil.

Previous studies often used volume as an indicator as trading activity, however, this study uses realized volatility in the underlying as the indicator, as it gives more indication to the lifting of

quotes and less responsive to partly hitting quotes. Zhang stated that as high frequency trading becomes commonplace practice in recent years, it gives rise to increase volatility in the underlying market (2010). Zhang also concluded that HFT hindered price discovery, but only in the time scope of months out, whereas this study focuses on intraday. Simaan and Wu used trades that are in and out of the National Best Bid and Offer (NBBO) as a measurement of price discovery (2007).

Compared to option time decay, there is much more groundwork done in the research on intraday volatility. Almgren proposed intraday volatility estimators that correct for factors that can arise in data collection in industry (2009). Zhou also proposed estimators that can reduce structural noise. Andersen and Bollerslev proposed a framework to integrate microstructure variables into standard volatility models for investigation (1997). This paper uses method proposed by Andersen as explained in the methodology section.

In conclusion, this study will attempt to use existing measures in studying the dynamics of financial markets to apply to option time decay, a topic rarely touched in academia. Using the existing framework in volatility measurement in combination with the previous studies on price discovery, we can test our hypothesis on the data to see if our observation matches our assumption that realized volatility is a good measure of option time decay.

Data

The methodology proposed by this paper is independent of the underlying. In order to model the price discovery in the market, this paper chooses financial instruments that have large trading

volumes and accessible underlying data, as well as an active options market. The two markets this paper will demonstrate the methodology on are the S&P 500 index and WTI crude oil markets. To calculate the realized volatility on the S&P 500 index, tick data from CBOE S&P 500 index (SPX), SPDR S&P 500 US equity ETF (SPY), and CBOE E-mini futures (ES) were collected from Bloomberg. ES futures that expire in February 2017 and March 2017 were used. For crude oil futures, CBOE WTI May 2017 futures CLK7 was used, collected from Bloomberg. To verify the model from observation, CBOE S&P 500 Volatility Index (VIX) data and observed implied at-the-money volatility for CL options were obtained from Bloomberg.

Raw data contained both bid and ask on the minute tick. The following timestamps are central time. SPX and VIX data are tracked from 8:30 AM to 3:20 PM. SPY trades from 3:00 AM to 7:00 PM, with 8:30 AM to 3:30 PM being normal hours. ES and CL futures trades all day except between 4:15 PM - 4:30 PM, and 5:00 PM to 6:00 PM. The tick data were interpolated to the five minute mark from 0:00 AM to 11:55 PM each day, if the tick is not present in raw data, previous observation is used.

The time range SPX-related data is from January 3, 2017 to March 15, 2017. The time range for crude oil futures data is from January 20, 2017 to April 20, 2017. The week of March 6th was removed due to an abnormal, non-recurring market movement skewing the data. The data for crude oil options implied volatility is gathered for CLM7, CLJ7 and CLK7. The daily implied volatility is collected in weeks that are 1-3 months prior to expiry. Weeks with non-trading weekdays and non-Wednesday EPA announcements were removed.

Methodology

The choice of granularity of tick size is five minutes to avoid microstructure noise and provide a detailed look at the volatility level at specific time of the day. First the 5-minute bid and ask price were taken the logarithmic average to find the 5-minute mid price:

$$P_t = (ln(P_{bid,t}) + ln(P_{ask,t}))/2$$

In order to eliminate the effect of numeric value of the price on the pattern, the price is first converted to log difference:

$$X_t = ln(P_{t+1}) - ln(P_t)$$

Where p is the traded price and x is the log difference. Now we can consider x as a log price process that follows a Brownian motion and has zero yield, considering we are looking at intraday movement. According to the methodology proposed by Almgren, we can express this relation as

$$dX(t) = \sigma(t)dB(t)$$

Where B is a Brownian motion. We cannot precisely determine σ at each t, but we can find the sum of variance over a certain timeframe, which is

$$Q_{j,k} = \int_{t_j}^{t_k} \sigma(t)^2 dt$$

And average volatility

$$q_{j,k} = Q_{j,k}/(t_k - t_j)$$

And if the volatility is independent of the time interval, we can take the average of volatility with equal weight, especially since we have constant time interval:

$$\widehat{\sigma}^2 = \frac{1}{N} \sum_{j=1}^{k} \frac{(X_j - X_{j-1})^2}{t_j - t_{j-1}}$$

\We can define the realized variance over one interval as

$$RV = \sum_{j=1}^{k} (x_i - x_{i-1})^2 = Q + \xi, \text{ where } E(\xi) = 0$$

Calculation of realized volatility followed one of the methodology proposed by Andersen and Bollerslev (Andersen and Bollerslev 1998), where squared log return is used as a measurement of realized volatility:

$$R_t^2 = (Log(P_t) - Log(P_{t-1}))^2 = X_t$$

Furthermore, we can calculate cumulative squared returns for a period to find the total realized volatility of a period:

$$\sum_{n=1}^{N} R_{t,n}^2$$

This cumulative volatility can be calculated for any period of time to find out the total volatility on the underlying for that given time period. From these definitions, we can compare the realized volatility of any given time of day, or the realized volatility over a period of time.

To verify our assumption on realized volatility, we can observed the option time value decay by looking at implied volatility of options at different time to expiration, holding other variables constant. Due to lack of intraday data, we can only calculate day-to-day time value decay. The Black-Scholes model states that the value of an option should be a monotonic function positively correlated with squared volatility multiplied by time, holding all other factors constant. We can then define the time value of the option as a function of the time volatility:

$$TV = f(\sigma^2 t)$$

Where σ is the volatility of the underlying and t is the time to expiry. Note that volatility is also a function of time and may change until expiry. Therefore, we can express the time value on a certain option as a function of t, holding moneyness constant:

$$TV(t) = \sigma(t)^2(t_{exp} - t)$$

Where t is the date of observation, $\sigma(t)$ is the observed implied volatility on the option on day t, and t_{exp} is the date of expiry. Then, we can calculate time value decay by finding $TV(t) - TV(t-1)$ for each t. The difference between every consecutive weekday is calculated, and detrended through subtracting the average time value decay of that week.

Analysis

Realized volatility is calculated for each 5-minute mark of the day, 288 for each day, and cumulative volatility is calculated day of the week. The realized volatility at each corresponding 5-minute mark on each day is summed and taken average to find the expected realized volatility of that point of time in the day to find the intraday pattern. We first construct the expected realized volatility for ESM7 futures, which tracks the SPX index:
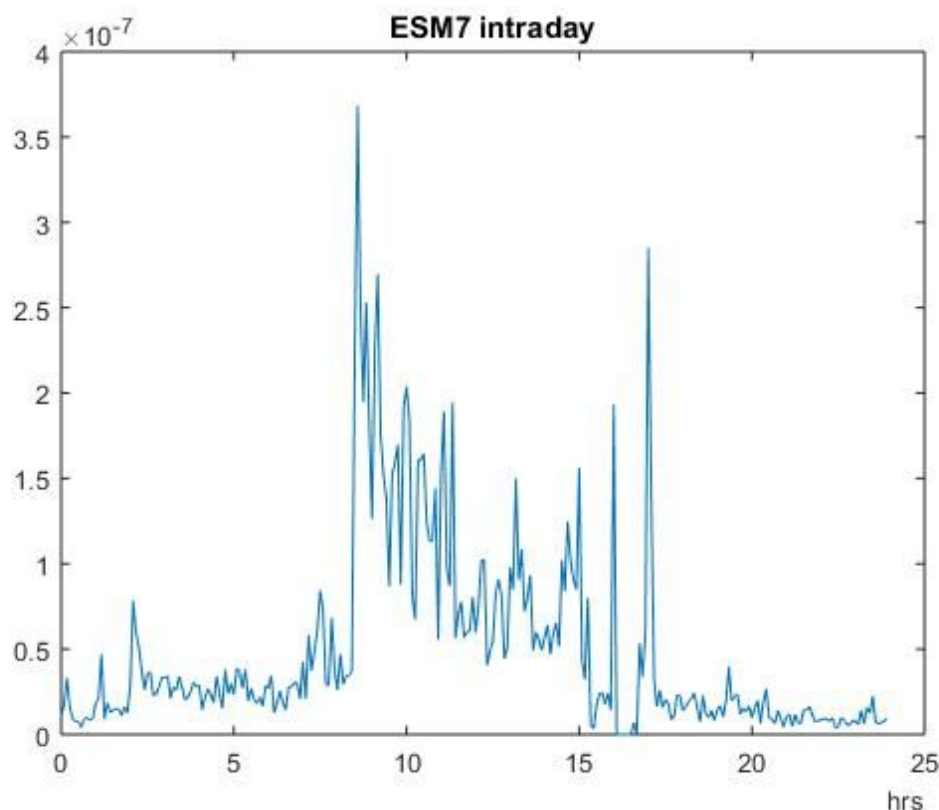
Figure 1., realized variance distribution on ESM7 throughout the trading day.

We can see the realized volatility for ESM7 shows a spike at market open and market close, and is lower during around noon. We can group the observation taken at 8:30-9:00 as a representative of a time period of more frequent trading, and 12:00-12:30 as a time period of less frequent trading. For each trading day in the dataset, the average volatility is calculated for each of the two periods, and results in 51 observations in each group. A t-test is ran for the two groups and shows that the two periods have statistically significant difference in realized volatility:

```
Paired t test

Variable         Obs         Mean      Std. Err.     Std. Dev.    [95% Conf. Interval]

market~n          51     3.19e-07      3.54e-08      2.53e-07     2.48e-07     3.90e-07
    noon          51     1.01e-07      1.25e-08      8.90e-08     7.57e-08     1.26e-07

    diff          51     2.19e-07      3.72e-08      2.66e-07     1.44e-07     2.93e-07

       mean(diff) = mean(market_open - noon)                          t =    5.8767
Ho: mean(diff) = 0                                 degrees of freedom =        50

Ha: mean(diff) < 0              Ha: mean(diff) != 0              Ha: mean(diff) > 0
Pr(T < t) = 1.0000          Pr(|T| > |t|) = 0.0000              Pr(T > t) = 0.0000
```

Figure 2., t-test on realized volatility during market open and noon.

The same pattern is observed on the SPY fund, which also tracks the SPX index:
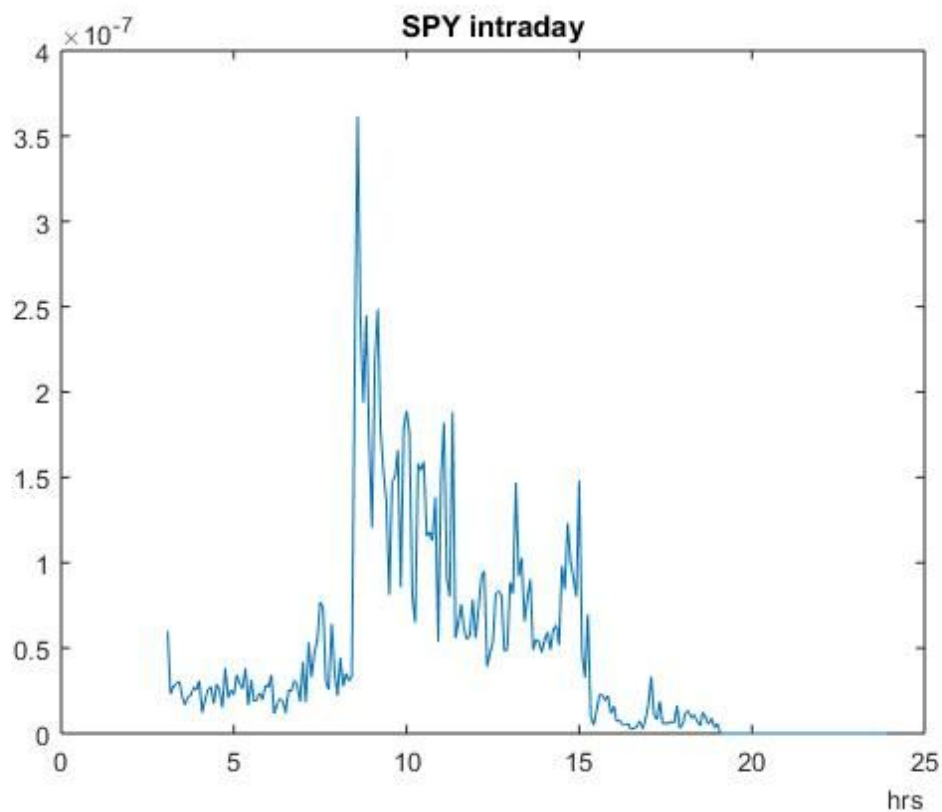
Figure 3. realized variance distribution on SPY throughout the trading day.

And finally for SPX, which does not have data before 8:30AM, and has an artificially high realized volatility at close at 3PM that is removed:
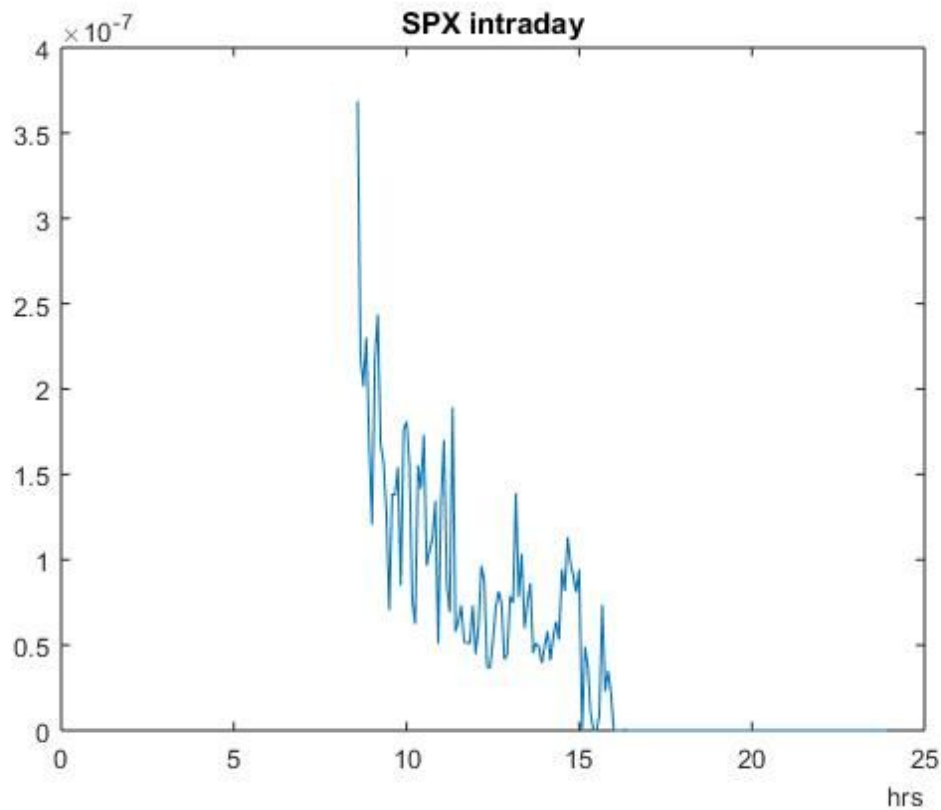

Figure 4. realized variance distribution on SPX throughout the trading day.

From Fig. 1-4, we can see a clear time-of-the-day pattern on realized volatility on SPX index and related products. From this observation, we can also divide one day of price decay into different amounts for each hour, directly proportional to the amount of squared realized return it contributes. The resulting decay model is shown below.
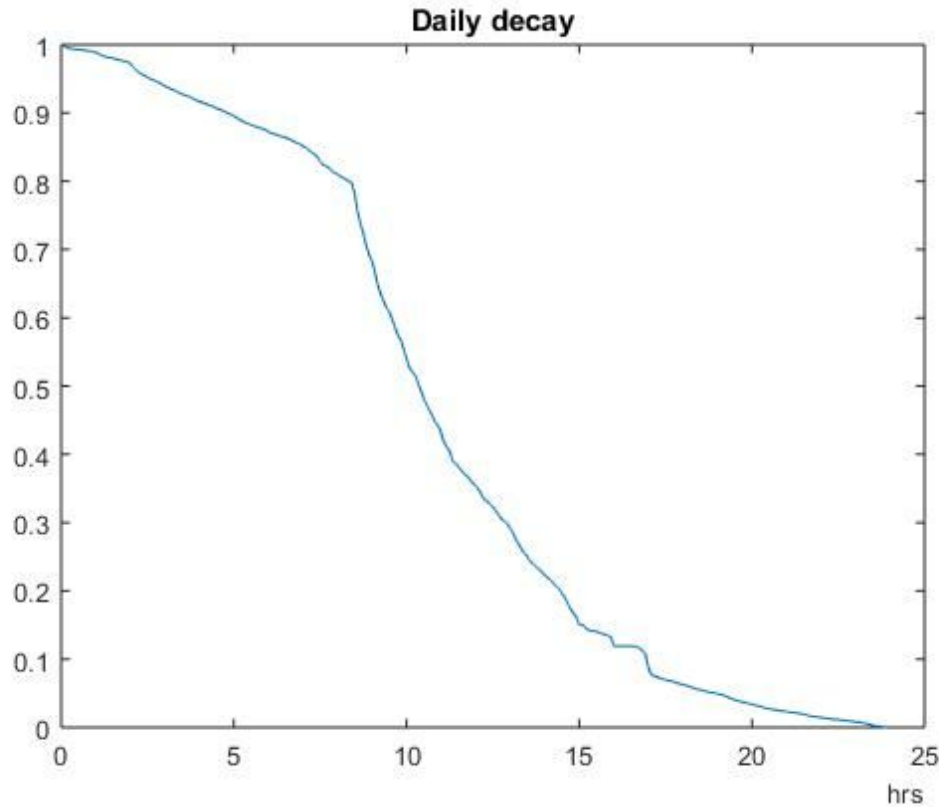
Figure 5., the option time decay model throughout trading day

The model shows that the periods prior to open and after close have flatter slopes, which means less time volatility is lost per unit time during these hours. On the other hand, the most volatility time is lost during the hours right after open as the slope after 8:30AM is the steepest. Here we can propose a model that take in account of the variable volatility on the underlying based on observed patterns. Due to lack of intraday option data, we cannot test whether this model matches the actual option time value decay. However, we can check our intuition through the VIX index, which is a CBOE-constructed index that reflects the market volatility. The expected change on VIX is constructed by calculated the difference between 3AM on each day and every

following 5-minute mark on that day. The differences are averaged over the sample to find the expected difference:
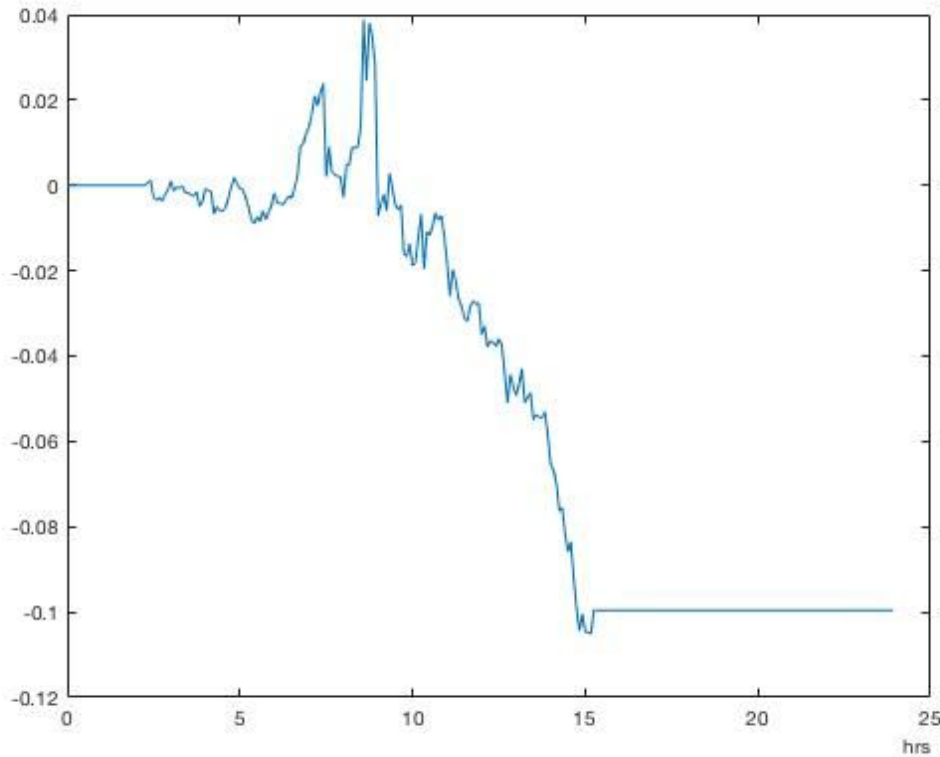

Figure 6. Change of VIX throughout the trading day

The steepest regions are near market open and close, which match the two spikes on realized volatility. However, VIX should not be taken as a direct measurement of time value since VIX is an artificial construction and we don't have access to the implementation of the index. Although intraday data is not available for us to verify the validity of the intraday pattern, we can calculate the calendar/announce effect on crude oil. Crude oil has a weekly scheduled announcement on crude oil production and stock on every Wednesday, which reveals a lot of information of crude oil supply and demand, which is an important price discovery event. Under

our assumption, this should reduce the time value of the option. The cumulative volatility is calculated for each day, of the week and deranged by subtracting the mean volatility of that week. The result data points were plotted:
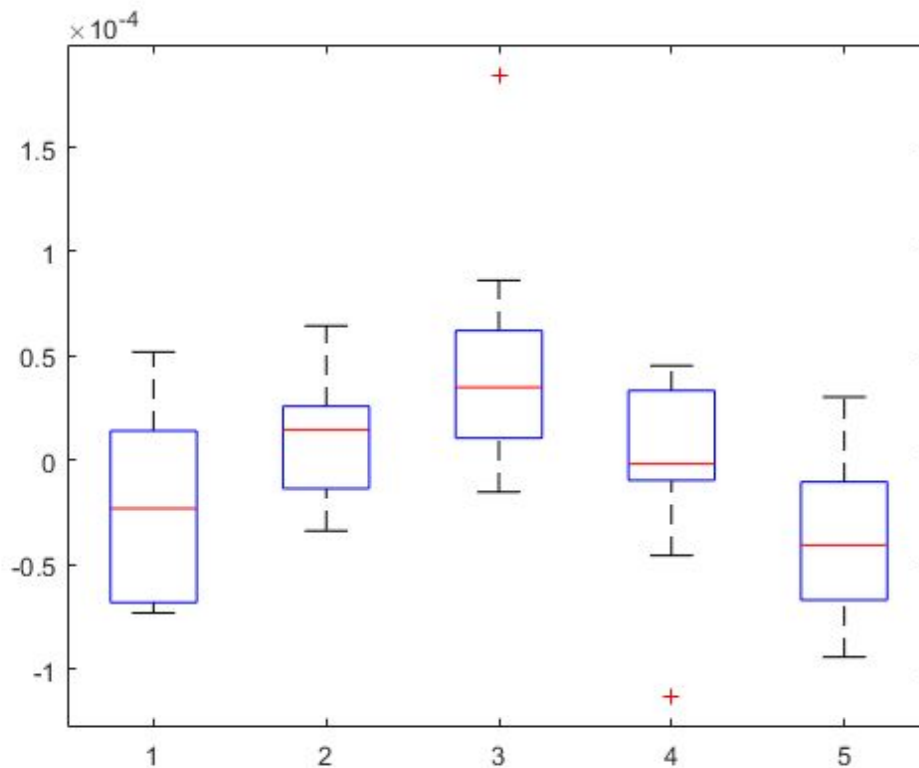
Figure 7. Amount of cumulative realized return on CLK7 for each weekday of the week, compared to the average, 1 being Monday. Box plot over eight weeks of data.

In the graph, on x-axis are the days of the week, 1 being Monday, 2 being Tuesday, etc. We can see that on average the weekdays have different realized volatility levels, with Wednesday being the highest compared to the average, as we expected. We can group the observations into five groups representing the five weekdays, assuming constant volatility in each group, and run an ANOVA test to see if they have the same means:

```
                        Analysis of Variance
    Source              SS          df       MS              F        Prob > F
-------------------------------------------------------------------------------
Between groups      4.2384e-08       4    1.0596e-08       5.17       0.0016
 Within groups      9.2285e-08      45    2.0508e-09
-------------------------------------------------------------------------------
    Total           1.3467e-07      49    2.7483e-09

Bartlett's test for equal variances:  chi2(4) =    2.6214  Prob>chi2 = 0.623
```

Figure 8. ANOVA on cumulative realized volatility for crude oil on weekdays

We can reject the null hypothesis that they have different means. Furthermore, we can perform a pairwise comparison to see which days have significant differences:

```
Pairwise comparisons of means with equal variances

over         : v2

                    | Number of
                    | Comparisons
         _____|_____
              v2    |         10
```

```
                    |                    Tukey               Tukey
           v1       | Contrast  Std. Err.    t    P>|t|   [95% Conf. Interval]
         _____|_____
              v2    |
        2 vs 1      | .000031   .0000203   1.53   0.549   -.0000265    .0000885
        3 vs 1      | .0000655  .0000203   3.23   0.018    7.95e-06    .000123
        4 vs 1      | .0000133  .0000203   0.66   0.965   -.0000443    .0000708
        5 vs 1      | -.0000203 .0000203  -1.00   0.853   -.0000778    .0000372
        3 vs 2      | .0000345  .0000203   1.70   0.442    -.000023    .000092
        4 vs 2      | -.0000177 .0000203  -0.88   0.905   -.0000753    .0000398
        5 vs 2      | -.0000513 .0000203  -2.53   0.101   -.0001088    6.25e-06
        4 vs 3      | -.0000522 .0000203  -2.58   0.092   -.0001098    5.32e-06
        5 vs 3      | -.0000858 .0000203  -4.24   0.001   -.0001433   -.0000283
        5 vs 4      | -.0000336 .0000203  -1.66   0.470   -.0000911    .000024
```

Figure 9. Pariwise test for cumulative realized volatility for crude oil on weekdays

We see significant difference between Monday and Wednesday, and Friday and Wednesday. We expect a lower cumulative volatility on Friday because trading ceases on Friday evening. However, we can conclude that we can see a significant realized volatility difference between Monday and Wednesday, and conclude that these days have different amount of market volatility, and according to our assumption, being the expectation that crude futures to demonstrate a stronger calendar effect due to EPA announcements every Wednesday, different level of price discovery. However, we do not yet have an explanation on why Monday as a lower level of activity.

To verify our observation, we can find out the amount of time value decay in crude oil options. As introduced in the methodology section, we calculated the remaining time value in the observed option implied volatility. Again, we detrend each observation by subtracting the average time value decay of that week. This is calculated for data over 10 weeks, and the result is the following:
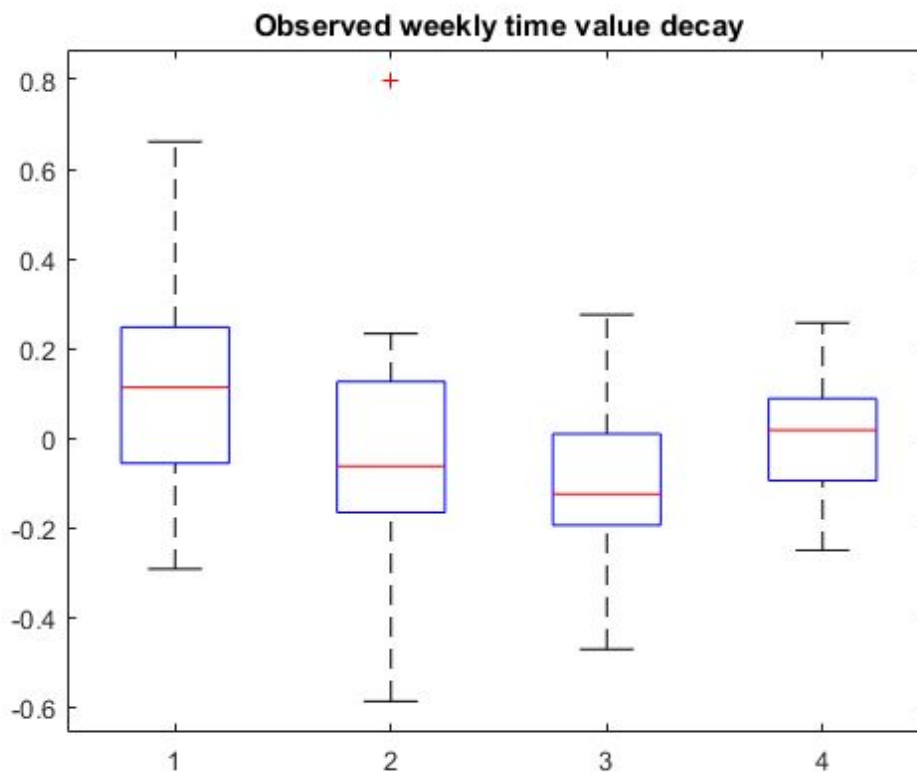

Figure 10. Observed time value decay in crude options.

The graph shows the amount of expected time value change on weekdays in options, compared to the average decay of the week, where 1 is the amount decayed from monday to tuesday, etc. We can see that the change from Wednesday to Thursday there is the most negative compared to weekly average, implying the largest decay on Wednesday, and the change from Monday to

Tuesday there is the most positive, implying the smallest decay on Monday. We can also do an

ANOVA analysis to see if they indeed have different means, and pairwise comparison to see

which pairs are different:

```
                    Analysis of Variance
    Source              SS        df      MS               F      Prob > F

Between groups     .473174405      3   .157724802         3.66     0.0154
 Within groups     3.79008447     88   .043069142

    Total          4.26325887     91   .046848999

Bartlett's test for equal variances:   chi2(3) =   13.2768   Prob>chi2 = 0.004
```

Figure 11. ANOVA on observed time value decay for crude oil on weekdays

```
Pairwise comparisons of means with equal variances

over          : v2

            |  Number of
            |  Comparisons

       v2   |       6
```

v1	Contrast	Std. Err.	Tukey t	Tukey P>|t|	Tukey [95% Conf. Interval]	
v2						
2 vs 1	-.1292841	.0611976	-2.11	0.157	-.2895488	.0309806
3 vs 1	-.1996296	.0611976	-3.26	0.008	-.3598943	-.0393649
4 vs 1	-.1001262	.0611976	-1.64	0.364	-.260391	.0601385
3 vs 2	-.0703455	.0611976	-1.15	0.660	-.2306102	.0899193
4 vs 2	.0291579	.0611976	0.48	0.964	-.1311068	.1894226
4 vs 3	.0995033	.0611976	1.63	0.370	-.0607614	.2597681

Figure 11. Pairwise test on observed time value decay for crude oil on weekdays

We can see that they indeed have different means, and the only pair that has a significant difference is between the decay from Monday to Tuesday and the decay from Wednesday to Thursday. Therefore, we can conclude that we do see significant difference between the option time value decay on Monday and Wednesday, with Wednesday decaying much more. This also matches our observation that Wednesday has a significantly higher realized volatility on Monday.

Discussion

Using realized volatility as a proxy, we demonstrated that the SPX equity index and related products that track the index have different market activity levels, which is low at afterhours and noon, and highest after market opens. This is matches a pattern that arises from trading practice, being that traders are most active during morning and less active around lunch time. Andersen and Bollerslev (1998) discusses a similar pattern, the "Japanese lunch time", being that the FX market has an artificially lower realized volatility at Japanese noon time when Japanese traders go to lunch.

Since this pattern is expected, there is an argument that when applying the Black-Scholes model, we need to take in consideration of non-constant volatility in the underlying. Furthermore, the pattern in the activity level is recurring and matches our expectation based on assumption of price discovery process. For those reasons, it is reasonable to produce a model to take in account of volatility and apply different amount of time decay at different time of the day, rather than

using the naive method of reducing the option price by considering time alone. As time passes, the usual method of calculating loss of time value in Black-Scholes formula, assuming constant volatility, is $\theta \cdot \Delta t$, but we can argue it should be multiplied by a factor that takes underlying volatility as an input. One way to do so is to argue that different time of the day contributes to a different amount of price discovery, therefore we can multiply the change of option value due to time, as described in the Black-Scholes formula, $\theta \cdot \Delta t$, by a coefficient, which can be the slope on the curve in Figure 5. We can test if this matches market behavior better. Or we can use the observed intraday data to demonstrate that options lose value more quickly during the hours that have higher market activity, as the realized volatility model suggests. However, we do not have observations on the intraday option prices, and cannot develop a model or verify its validity in this paper. This remains an important question to be resolved in future research.

On the other hand, there is an observable market pattern in the crude oil market that is observed outside of intraday data. As the EPA announcement on crude oil stock and production takes places on Wednesdays, we expect a higher realized volatility along with a higher loss of option time value. Through calculation on the realized volatility on each weekday, we do see a higher expected value for Wednesday. ANOVA and pairwise comparison suggests that the weekdays have different level of realized volatility, with significant difference between Monday and Wednesday, and Wednesday and Friday.

The difference between Wednesday and Friday is not conclusive because market ceases trading on Friday evening, therefore the realized volatility after market closes on Friday is zero, where the other weekdays have non-zero realized volatility for these hours. As a result, the cumulative realized volatility is expected to be lower, therefore does not suggest Friday has lower market

activity. This reveals a flaw in our definition of each day being 12:00 AM to 11:59 PM. In future works, we can either adjust each trading day as 5PM on previous day to 4PM, or find an average cumulative realized volatility per hour on that day.

However, this does not apply to Monday since we do have a full day of data, therefore the significant difference between Monday and Wednesday becomes interesting. Although we have a theory on Wednesday having a higher realized volatility, being the effect of EPA announcement, we do not have an explanation on why Monday has a lower realized volatility. A possible reason is that trading on crude oil begins on Sunday evening, but traders and trading algorithms, due to corporate limitations, may only start trading on Monday. Therefore it could be that they take a more cautious stance and trade lower volumes or less aggressively. However, there is no evidence supporting this claim. Finding an explanation on why Monday has a lower realized volatility remains an important future work.

We can verify our claim that higher realized volatility is related with higher option time value decay by looking into option time value decay through implied volatility. Implied volatility is a value derived from the Black-Scholes equation, given option price, stock price, time to expiry, interest rate, and moneyness as inputs. This is a value that represents the market belief on how much the optionality on the uncertainty, or time value of the option, is worth. If the underlying has constant volatility, or if volatility doesn't play a role in the time value of the option, we should see that the option time decay should be the same for days that have different level of realized volatility. However, we showed that options experience more time value decay on Wednesdays than Mondays, matching our observation that Wednesdays have higher realized volatility than Mondays. We can assume that Wednesdays have higher decay due to price

discovery in the EPA announcement, which reduces the uncertainty in the underlying, the crude futures, but we still lack an explanation for the lower decay on Monday. There could be a factor that ties into the calendar effect in market activity in addition to the announcement effect. Some future work include gathering more data on these financial instruments. Aside from the important piece of intraday options data, we can gather a larger dataset to improve the power of the statistical tests. Currently we have around 2 months of data on equity index and crude oil. They provide around 10 weeks of data, meaning for the week-of-the-day analysis, we only have around 10 samples in each treatment group. A larger sample size can lead us to a higher sensitivity and higher predictive power.

Another future area of work involves looking into the effect of announcements and calendar effect on equities. Equity have FOMC announcements that are less frequent than EPA announcements, therefore we can separate the two effects. We may find that financial markets have an activity pattern that is driven by calendar effect independent from announcements, or that there is only the announcement effect. We can also look into the weeks in crude oil which have announcements on Thursdays instead of Wednesdays to see if such pattern persist or disappear. This will allow us to assess the cause of market activity pattern better.

In summary, the realized volatility pattern, both intraday and in day-of-the-week, provide valuable information for calculating option time decay. With the addition of more data and a more detailed model, we may use these information to re-evaluate mispricing in options, or construct a predictive model to calculate the loss of option value due to the passage of time based on the expected realized volatility level of that time. However, both of these remain to be work to be done in the future.

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