

**A Revisit to the Dependence Structure between Stock and Foreign Exchange
Markets: A Dependence-Switching Copula Approach**

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Abstract

This paper develops a dependence-switching copula model to examine dependence and tail dependence for four different market statuses, namely, rising-stocks/appreciating-currency, falling-stocks/depreciating-currency, rising-stocks/depreciating-currency, and falling-stocks/appreciating-currency. The model is then applied to daily stock and foreign-exchange returns for six major industrial countries over the period 1990–2010. It is found that the dependence and tail dependence among the above four market statuses are asymmetric, for most countries, in the negative correlation regime, but symmetric in the positive correlation regime. These results enrich findings in existing literature and suggest that analyzing cross-market linkages within a time-invariant copula framework may not be appropriate.

JEL code: C32, C51, G15, F30.

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1 Introduction

Measuring dependence and tail dependence is important since it help international investors to manage risk in their portfolios. As to the direction of the dependence between the stock and foreign exchange markets, plausible arguments can be made for either a negative or positive correlation, owing to either the return chasing effect or the portfolio rebalancing and exchange-rate exposure effects, respectively (Hau and Rey, 2006).¹

The conventional Pearson correlation is not appropriate when it comes to measuring the dependence across financial markets since it gives an equal weight to both positive and negative returns as well as large and small realizations.² It may also lead to a significant underestimation of the risk from joint extreme events (Poon et al., 2004; Tastan, 2006). To address the above-mentioned concerns, some researchers have employed multivariate-GARCH models (Ang and Chen, 2002; Tastan, 2006; Dungey and Martin, 2007), or Hamiltonian regime-switching models (Ang and Bekaert, 2002; Ang and Chen, 2002) in modeling the joint dynamics of returns. These articles, however, fail to examine asymmetric tail dependence between two markets since they assume that innovations follow a multivariate normal or student-t distribution which is a symmetric distribution (Patton, 2006; Garcia and Tsafack, 2011). Another approach is based on extreme-value theory, and focuses on the asymptotic value of the exceedance correlation (Longin and Solnik, 2001; Bae, et al.,

¹ Return chasing effects occur when a booming domestic stock market attracts international investors, which in turn leads to an appreciation of the booming country's currency (i.e., a decrease in the nominal exchange rate which is defined as the number of national currency per US dollar). In such a case, the high stock price coexists with a low exchange rate, thus producing a negative correlation between the stock and foreign exchange markets. By contrast, the portfolio rebalancing effect occurs when the boom in domestic stock prices encourages investors to cash in their holdings and to move their capital to other countries whose markets offer better bargains. This will depreciate the domestic currency and hence increase its exposure to exchange-rate risks. High stock prices can, in this way, coexist with domestic currency depreciation (i.e., an increase in the exchange rate), which then gives rise to a positive correlation between the two markets.

² The Pearson correlation coefficient is computed as an average of the deviations from the mean. It assumes a linear relationship.

2003; Poon et al., 2004; Hartmann et al., 2004; Cumperayot et al., 2006). Extreme value theory, however, assumes an asymptotic dependence which may lead to a serious over-estimation of financial risks (Poon et al., 2004). This approach also requires discretion in defining extreme observations (Diebold et al., 2000; Rodriguez, 2007).

A number of previous studies have applied copulas to model the co-movements across international financial markets (Cherubini et al., 2004; Patton, 2006; Hu, 2006; Ning, 2010). A copula is a multivariate cumulative distribution function whose marginal distributions are uniform on the interval $[0,1]$. The copula allows one to measure the dependence structure of multivariate random variables. Several articles argue that a time-invariant copula is not appropriate and hence allow the parameters in a copula function to change over time (Van den Goorbergha et al., 2005; Patton, 2006; Harvey, 2010; Buseti and Harvey, 2011) or allow the copula function to change over time (Okimoto, 2008; Rodriguez, 2007). To model dependence switching between financial markets, the latter approach is better than the former approach since time-varying parameters in a copula function do not imply the time-varying switching of dependence between positive and negative regimes.

Rodriguez (2007) examines the contagious effect in stock markets during the period of a financial crisis. Okimoto (2008) investigates the co-movement of stock returns across countries. These two papers allow Markov-switching on the intercept and the variance of residuals in a return function as well as the dependence and tail dependence of a copula function.³ The current approach, however, has two unavoidable limitations. First, the assumption of the same regime classification in two different markets is imposed. In other words, if a domestic country is in a good state

³ The two states considered in Rodriguez (2007) and Okimoto (2008) are a normal regime and an extreme regime, respectively.

then the foreign country must also be in a good state. This assumption is likely to hold if we discuss the co-movements between stock markets or between foreign exchange markets. However, it is not likely to hold if our focus is to analyze the dependence between stock and foreign exchange markets. Secondly, the technical sophistication of this approach lies in its allowance of the state variable to influence parameters in both marginal distributions and a copula function. The large number of parameters makes it difficult to find the numerical maximum of the likelihood function (Patton, 2004, 2006). To minimize the number of parameters to be estimated, Rodriguez (2007) and Okimoto (2008) simplified their specification of marginal models.

This paper points out that the conditional correlation between the foreign exchange and stock markets switches between positive and negative correlation regimes depending on the dominance of the portfolio rebalancing and return chasing effects. To characterize the above dependence-switching phenomenon, this article adopts a dependence-switching copula model to investigate the dependence structure between the two markets. The switching between these positive and negative correlation regimes depends on an unobserved state variable. A positive correlation between the stock and foreign exchange markets indicates that both stock returns and exchange rate changes move in the same direction (increasing and decreasing in tandem, so that a bull market is associated with the domestic currency's depreciation, and a bear market is associated with its appreciation). Conversely, a negative correlation regime refers to a period where a bull (bear) market synchronizes with the domestic currency's appreciation (depreciation). Instead of emphasizing the discrete changes in the dependence and tail dependence of a single copula, however, this paper places emphasis on the switch in copula functions.

The advantages of our model are manifold. First, by mixing Clayton and

survival Clayton copulas (where the former captures the left-side tail dependence, and the latter captures the right-side tail dependence), this paper allows for asymmetric instead of symmetric tail dependence. Second, by allowing the dependence structure across stock and foreign exchange markets to switch between positive and negative correlation regimes during the sample period, this article more closely matches the real world, where the dependencies may change. By contrast, conventional copulas capture the conditional correlation between the two markets over the whole period and hence fail to allow for dependence switching. Third, our model is able to measure tail dependence among different market statuses such as rising-stocks/appreciating-currency, falling-stocks/depreciating-currency, rising-stocks/depreciating-currency, and falling-stocks/appreciating-currency. However, conventional single copula model, extreme-value models and markov-switching models are not able to provide the above-mentioned measures of tail dependence. In short, this paper proposes a new copula model that allows for a state-varying dependence, and then applies the model to shed light on the dependence structure between the stock and foreign exchange markets. To the best of our knowledge, this is the first paper that applies a dependence-switching copula model to investigate the dependence structure between the stock and foreign exchange markets.

The estimation of the model is complicated since the unobserved state variable affects both marginal models and copula functions. To avoid the previous problem of parameter dimensionality in estimation, we follow the suggestion of Li (2005) to proxy the unobservable state with an instrument, and then estimate marginal models with the quasi-maximum likelihood estimation method proposed by Bollerslev and Wooldridge (1992). The dependence switching copula is then fitted to the residuals from marginal models.

By applying daily stock and foreign-exchange returns for six major industrialized countries over the period of 1990-2010, this paper estimates the dependence and tail dependence for four different market statuses. These dependences are asymmetric, for most countries, in the negative correlation regime, but symmetric in the positive correlation regime. These results suggest that analyzing cross-market linkages within a time-invariant copula framework may not be appropriate. They also have important implications for cross-market risk management and international asset pricing.

The remainder of this article proceeds as follows. In Section 2 we develop the dependence-switching copula model and describe its estimation strategy. In Section 3 we discuss the empirical results. Conclusions are given in the last section.

2. The dependence-switching copula model

2.1 Copula Specification

The purpose of this paper is to develop a dependence-switching copula model. A copula is a multivariate cumulative distribution function whose marginal distribution is uniform on the interval $[0,1]$. It captures the dependence structure of a multivariate distribution. According to Sklar's (1959) theorem, a bivariate joint cumulative distribution function (F) of exchange rate changes ($R_{1,t}$) and stock returns ($R_{2,t}$) can be decomposed into two marginal cumulative distribution functions (F_1 and F_2) and a copula cumulative distribution function (C) that completely describes the dependence structure between the two series:

$$F(R_{1,t}, R_{2,t}; \delta_1, \delta_2, \theta^c) = C(F_1(R_{1,t}; \delta_1), F_2(R_{2,t}; \delta_2); \theta^c), \quad (1)$$

where $F_k(R_{k,t}; \delta_k)$, $k=1,2$, is the marginal cumulative distribution function of $R_{k,t}$ and δ_k and θ^c are the parameter sets of $F_k(R_{k,t}; \delta_k)$ and C , respectively.

Assuming that all cumulative distribution functions are differentiable, the bivariate joint density is then given by

$$f(R_{1,t}, R_{2,t}; \delta_1, \delta_2, \theta^c) = c(u_{1,t}, u_{2,t}; \theta^c) \cdot \prod_{k=1}^2 f_k(R_{k,t}; \delta_k), \quad (2)$$

where $f(R_{1,t}, R_{2,t}; \delta_1, \delta_2, \theta^c) = \partial^2 F(R_{1,t}, R_{2,t}; \delta_1, \delta_2, \theta^c) / \partial R_{1,t} \partial R_{2,t}$ is the joint density of $R_{1,t}$ and $R_{2,t}$; $u_{k,t}$ is the “probability integral transforms” of $R_{k,t}$ based on $F_k(R_{k,t}; \delta_k)$, $k=1,2$; $c(u_{1,t}, u_{2,t}; \theta^c) = \partial^2 C(u_{1,t}, u_{2,t}; \theta^c) / \partial u_{1,t} \partial u_{2,t}$ is the copula density function; and $f_k(R_{k,t}; \delta_k)$ is the marginal density of $R_{k,t}$, $k=1,2$. Thus, the bivariate joint density of $R_{1,t}$ and $R_{2,t}$ is the product of the copula density and the two marginal densities.

Theoretically, the co-movements between the stock and foreign exchange markets could be positive or negative depending on the strength of two different effects: the return chasing effect (negative dependence) and the portfolio rebalancing effect (positive dependence). The former dominates for some periods, but the latter dominates for others. In such a case, the co-movement between the stock and foreign exchange markets switches between positive and negative dependence regimes. To capture the above dependence switching, this paper proposes a Markov-switching copula model in which the unobserved state variable affects both copula functions and marginal models (Rodriguez, 2007; Okimoto, 2008).

Consider the following state-varying copula:

$$C_{S_t}(u_{1,t}, u_{2,t}; \theta_1^c, \theta_0^c) = \begin{cases} C_1(u_{1,t}, u_{2,t}; \theta_1^c), & \text{if } S_t = 1 \\ C_0(u_{1,t}, u_{2,t}; \theta_0^c), & \text{if } S_t = 0 \end{cases},$$

where $C_1(u_{1,t}, u_{2,t}; \theta_1^c)$ and $C_0(u_{1,t}, u_{2,t}; \theta_0^c)$ are two mixed copulas with positive and negative dependence structures, respectively, and S_t is an unobserved state

variable. The above two mixed copula functions comprise a Clayton copula (C^C) and a survival Clayton copula (C^{SC}):⁴

$$C_1(u_{1,t}, u_{2,t}; \theta_1^c) = 0.5C^C(u_{1,t}, u_{2,t}; \alpha_1) + 0.5C^{SC}(u_{1,t}, u_{2,t}; \alpha_2),$$

$$C_0(u_{1,t}, u_{2,t}; \theta_0^c) = 0.5C^C(1-u_{1,t}, u_{2,t}; \alpha_3) + 0.5C^{SC}(1-u_{1,t}, u_{2,t}; \alpha_4),$$

where $\theta_1^c = (\alpha_1, \alpha_2)'$, $\theta_0^c = (\alpha_3, \alpha_4)'$, $C^C(u, v; \alpha) = (u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}$,

$C^{SC}(u, v; \alpha) = u + v - 1 + C^C(1-u, 1-v; \alpha)^{-1/\alpha}$, and $\alpha \in (0, \infty)$. After estimating the

shape parameter, α_i , we can transform it to Kendall's τ_i , correlation coefficient (ρ_i)

and tail dependence (φ_i) with $\tau_i = \alpha_i / (2 + \alpha_i)$, $\rho_i = \sin(\pi * \tau_i / 2)$ and $\varphi_i = 2^{-1/\alpha_i}$

for $i=1, 2, 3, 4$. Therefore, the mixed copula $C_1(u_{1,t}, u_{2,t}; \theta_1^c)$ describes the cases

where the domestic currency's appreciation (depreciation) is associated with low

(high) stock prices, and hence a positive correlation between stock and foreign

exchange markets. On the other hand, the mixed copula $C_0(u_{1,t}, u_{2,t}; \theta_0^c)$ depicts the

cases where the domestic currency's appreciation (depreciation) synchronizes with

high (low) stock prices, and hence a negative correlation between the stock and

foreign exchange markets.

Das and Uppal (2004) define systemic risk as the risk from infrequent events that are highly correlated across a large number of assets. The tail dependence denotes the probability of the joint occurrence of extreme statuses in both markets and hence is a true measure of the systemic joint risks in stock-foreign exchange markets during market extremes (Poon et al., 2004).⁵ An advantage of the dependence-switching

⁴ An unequal weight instead of an equal weight implies that the importance of two different cases in the positive (negative) correlation regime is not the same. Since there is no prior information about the relative importance of these two different cases in the positive (negative) correlation regime, this paper assumes that they are equally important and hence sets the weight to 0.5 (Patton, 2006). The Gumbel copula could alternatively be employed, but it does not fit well according to the model selection criteria such as the AIC and the log-likelihood function values.

⁵ Following Joe (1997), Nelsen (1999) and Patton (2006), the four different tail dependences are

model is that it allows us to capture systemic risk in four different extreme-return statuses.

The unobserved state variable, S_t , follows a two-state Markov chain with a transition probability matrix:

$$P = \begin{bmatrix} P_{00} & 1 - P_{00} \\ 1 - P_{11} & P_{11} \end{bmatrix},$$

where $P_{ij} = \Pr[S_t = j | S_{t-1} = i]$ for $i, j = 0, 1$. The state variable, S_t , switches between a positive dependence regime with dominating portfolio rebalancing effects ($S_t = 1$) and a negative dependence regime with dominating return chasing effects ($S_t = 0$).

The bivariate density function of the above model is expressed as:

$$\begin{aligned} & f(R_{1,t}, R_{2,t}; \theta_1^c, \theta_0^c, \delta_1^0, \delta_1^1, \delta_2^0, \delta_2^1) \\ &= \left\{ \sum_{j=0}^1 \Pr(S_t = j) c_j(u_{1,t}, u_{2,t}; \theta_j^c) \right\} \prod_{k=1}^2 \left\{ \sum_{j=0}^1 \Pr(S_t = j) f_k(R_{k,t}; \delta_k^j, S_t = j) \right\} \end{aligned} \quad (3)$$

where $c_j(\cdot)$ is the copula under regime j and θ_j^c is its parameter set. δ_k^j is the parameter set of the marginal distribution under regime j . It is worth noting that a single copula model can only model either a positive or a negative dependence regime, but not both. However, the model with dependence switching allows us to discuss the dependence structure for four different market statuses: both markets are either booming or crashing, and one market is booming while the other one is crashing. The

defined as follows:

$$\begin{aligned} \phi_1 &= \lim_{\varepsilon \rightarrow 0} \Pr[u_{1,t} \leq \varepsilon | u_{2,t} \leq \varepsilon] = \lim_{\varepsilon \rightarrow 0} \Pr[u_{2,t} \leq \varepsilon | u_{1,t} \leq \varepsilon], \\ \phi_2 &= \lim_{\delta \rightarrow 1} \Pr[u_{1,t} > \delta | u_{2,t} > \delta] = \lim_{\delta \rightarrow 1} \Pr[u_{2,t} > \delta | u_{1,t} > \delta], \\ \phi_3 &= \lim_{\varepsilon \rightarrow 0 \text{ and } \delta \rightarrow 1} \Pr[u_{1,t} > \delta | u_{2,t} \leq \varepsilon] = \lim_{\varepsilon \rightarrow 0 \text{ and } \delta \rightarrow 1} \Pr[u_{2,t} \leq \varepsilon | u_{1,t} > \delta], \\ \phi_4 &= \lim_{\varepsilon \rightarrow 0 \text{ and } \delta \rightarrow 1} \Pr[u_{1,t} \leq \varepsilon | u_{2,t} > \delta] = \lim_{\varepsilon \rightarrow 0 \text{ and } \delta \rightarrow 1} \Pr[u_{2,t} > \delta | u_{1,t} \leq \varepsilon], \end{aligned}$$

where $u_{1,t}$ and $u_{2,t}$ are defined in equation (2). ϕ_1 , ϕ_2 , ϕ_3 and ϕ_4 are the tail dependences of falling-stocks/appreciating-currency, rising-stocks/depreciating-currency, falling-stocks/ depreciating-currency and rising-stocks/appreciating-currency, respectively.

above model covers the model of Patton (2006) and Ning (2010) as a special case by setting $S_t = 0$.⁶

The log-likelihood function of (3) is :

$$L(\Theta) = L_c(\psi_1) + \sum_{k=1}^2 L_k(\psi_{2,k}), \quad (4)$$

where, $\Theta = ((\theta_1^c)', (\theta_0^c)', (\delta_1^0)', (\delta_1^1)', (\delta_2^0)', (\delta_2^1)', P_{00}, P_{11})'$; $L_c(\psi_1)$ and $L_k(\psi_{2,k})$ are the log of the copula density and the marginal density of $R_{k,t}$, respectively. These two densities are given as follows:

$$L_c(\psi_1) = \log[\Pr(S_t = 1)c_1(u_{1,t}, u_{2,t}; \theta_1^c) + (1 - \Pr(S_t = 1))c_0(u_{1,t}, u_{2,t}; \theta_0^c)],$$

$$L_k(\psi_{2,k}) = \log[\Pr(S_t = 1)f_k(R_{k,t}; \delta_k^1, S_t = 1) + (1 - \Pr(S_t = 1))f_k(R_{k,t}; \delta_k^0, S_t = 0)],$$

where, $\psi_1 = ((\theta_1^c)', (\theta_0^c)', P_{11}, P_{00})'$; $\psi_{2,k} = ((\delta_k^0)', (\delta_k^1)', P_{11}, P_{00})'$.

2.2 Marginal models

Next, we discuss the specifications of the marginal models. This paper applies an AR(k)-GARCH(1,1) model to describe the marginal densities (Bollerslev, 1987; Ning, 2010; Patton, 2004, 2006). It is important to allow the respective marginal distributions of the stock and foreign-exchange returns to change with states. However, this increases the cost of computation in estimation since the number of parameters to be estimated increases significantly. An alternative strategy, suggested by Li (2005), to avoid the above difficulty in estimation is to proxy the unobservable state, S_t , with instruments, and then estimate the marginal models accordingly.

This paper therefore uses interest differentials to serve as the instrument of the unobservable state variable. The reason for this is that interest differentials are indeed

⁶ Patton (2006) mixed Joe Clayton with Survival Joe Clayton to obtain a symmetric Joe Clayton (SJC). However, we use a Clayton and a Survival Clayton instead. We also estimate SJC but the significance of dependence and tail dependence is worse than that from the mixed Clayton copula. The results of the SJC estimation are not reported here but are available from the authors upon request.

affected by states. If return chasing effects dominate portfolio re-balancing effects, then a booming stock market results in an appreciation of the domestic currency, which in turn results in a negative interest differential according to the uncovered interest parity. Hence, a negative correlation between the two markets is associated with a negative interest differential. On the other hand, the domination of the portfolio rebalancing effects results in a positive correlation between the stock and currency markets and hence a positive interest differential. In short, the unobservable state variable, S_t , is closely related to the observable interest differential and hence the interest differential is a good proxy for S_t . This paper allows interest differentials to affect the mean and conditional variance of marginal models. The marginal models for both returns are therefore given as follows:

$$\begin{aligned}
R_{k,t} &= a_{k,0} + \sum_{i=1}^{p_k} a_{k,i} R_{k,t-i} + c_{k,1} id_t + \varepsilon_{k,t}, \quad k=1, 2, \\
h_{k,t} &= \beta_{k,0} + \beta_{k,1} \varepsilon_{k,t-1}^2 + \beta_{k,2} h_{k,t-1} + c_{k,2} id_t, \\
\eta_{k,t} &= \varepsilon_{k,t} / \sqrt{h_{k,t}}; \quad \eta_{k,t} | \Omega_{t-1} \sim t(v_k),
\end{aligned} \tag{5}$$

where $R_{1,t}$ and $R_{2,t}$ are exchange-rate and stock returns, respectively; $\varepsilon_{k,t}$ is the error term; $h_{k,t}$ is the conditional variance of asset returns; $\eta_{k,t}$ is the standardized residual following a Student-t distribution with v_k degrees of freedom; id_t is the interest differential; and Ω_{t-1} is the past information set. With the above specification, the mean and conditional variance of stock returns and exchange rates are both affected by states.

2.3 Estimation methods

With the above mentioned strategy, the marginal densities and the copula density in our model can be individually estimated. This paper applies the inference of

the margins (IFM) and hence adopts a two-step estimation method (Joe and Xu, 1996). In the first step of the IFM method, the parameters of the marginal AR(k)-GARCH(1,1) models are estimated with the lag order (p_k) being set to 12. After removing those lag returns with insignificant estimates of $a_{k,i}$, we then re-estimate the model for both exchange-rate changes and stock returns. Theoretically, interest differentials affect stock and exchange-rate returns and are also affected by both returns. Therefore, id_t in the GARCH model should be treated as an endogenous variable instead of a pre-determined variable.⁷ To take into account the endogeneity of interest differentials, we adopt a two-stage method to estimate marginal models. In the first stage, we estimate interest differentials with instruments including 1-6 lag variables of stock returns, exchange rate returns and interest differentials. In the second stage, we plug in the estimated interest differentials from the first step and then estimate coefficients in the GARCH model. It is worth noting that replacing id_t by \hat{id}_t results in the disturbance being $\tilde{\varepsilon}_{k,t} = (\varepsilon_{k,t} + c_{k,l}v_t)$ instead of $\varepsilon_{k,t}$ since $id_t = \hat{id}_t + v_t$. The distribution of $\tilde{\varepsilon}_{k,t}$ is unknown and is likely to be non-Gaussian. This paper therefore adopts the quasi-maximum likelihood method provided by Bollerslev and Wooldridge (1992) to estimate the marginal models and then adjusts the var-cov matrix of the estimated coefficients accordingly.

The above estimation also provides estimates of the standardized residuals, $\hat{\eta}_{k,t}$.

⁷ We also adopt the Hausman's (1978) test to examine the endogeneity of interest differentials. Under the exogenous hypothesis, we estimate the AR(k)-GARCH(1,1) model and then obtain coefficient estimates and their covariance matrix. Under the endogenous hypothesis, we estimate the model with the instrumental variable method as described in the paper. After having estimates and covariance matrices from the above two methods, we then construct the Wald statistic of the Hausman test. It is worth noting that there is no guarantee that the Hausman's Wald statistic is positive in empirical applications. Empirical results reveal that the Wald statistic ends up with negative values for two out of twelve cases. For the rest ten cases, the exogenous hypothesis is rejected for seven out of ten cases. Therefore, we assume that id_t is endogenous. However, the empirical results in Tables 2 - 5 and Figures 1 - 4 are not qualitatively affected if id_t is treated as exogenous. The above results are not reported here but are available upon request from authors.

If the marginal models are mis-specified, however, these estimates may then be inconsistent. Theoretically, if we know the true distribution of standardized residuals, we can then use that distribution to transform the standardized residuals to a uniform distribution. However, since the exact distribution of the standardized residuals is empirically unknowable, it follows that applying a specific distribution to transform standardized residuals may not result in a uniform distribution for transformed residuals.

The Canonical Maximum Likelihood (CML) approach points out that transforming standardized residuals based on an empirical CDF will always result in a uniform distribution asymptotically, regardless of the specifications of the marginal model. To mitigate the problem of misspecification in the marginal models, this paper follows the CML approach to transform standardized residuals into a uniform distribution with the following empirical marginal cumulative distribution function:

$$\hat{F}_k(\omega) = \frac{1}{T+1} \sum_{t=1}^T I(\hat{\eta}_{k,t} \leq \omega), \quad (6)$$

where $I(\cdot)$ is an indicator function that is one if $\hat{\eta}_{k,t} \leq \omega$ and is zero otherwise. This paper then obtains the cumulative probability for each observation of $\hat{\eta}_{k,t}$ by $\hat{u}_{k,j} = \hat{F}_k(\hat{\eta}_{k,j})$, $k=1,2$, $j=1,2,\dots,T$.

The second step of IFM is to estimate the copula parameters ψ by maximizing the log-likelihood function $L_c(\psi_1)$. Since the dependence structure follows a Markov-switching process, this paper uses Hamilton's filtered system to transform the log-likelihood function of the model as follows:

$$L_c(\psi_1) = \log \left(\hat{\xi}'_{t|t-1} \eta_t \right),$$

$$\hat{\xi}_{t|t} = \left(\hat{\xi}'_{t|t-1} \eta_t \right)^{-1} \left(\hat{\xi}'_{t|t-1} \circ \eta_t \right),$$

$$\hat{\xi}_{t+1|t} = P' \hat{\xi}_{t|t},$$

$$\eta_t = \begin{bmatrix} c_1(\hat{u}_{1,t}, \hat{u}_{2,t}; \theta_1^c) \\ c_0(\hat{u}_{1,t}, \hat{u}_{2,t}; \theta_0^c) \end{bmatrix},$$

where “ \circ ” is the Hadamard product and c_{S_t} is the density function of C_{S_t} for

$S_t = 0, 1$. The copula parameters $\psi_1 = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, P_{11}, P_{00})'$ can then be estimated by

maximizing $L_c(\psi_1)$:⁸

$$\psi_1 = \arg \max_{\psi_1} \sum_{t=1}^T L_c(\psi_1). \quad (7)$$

3. Empirical Investigation

3.1 Data description

Daily data of stock indices, nominal exchange rates and 3-month London interbank offered rates (LIBOR) over the period from January 2, 1990 to June 8, 2010 are downloaded from the Global Financial Database (GFD). The countries under investigation are Canada (CAN), France (FRN), Germany (GER), Italy (ITA), Japan (JAP) and the United Kingdom (UK). The nominal exchange rates are expressed in national currency per US dollar and the stock price indices are expressed in the national currency of each country.⁹ Because of the launch of the euro in 1999, nominal exchange rates between the local currencies of the euro-zone countries (e.g. French Frank, Mark and Lira) and dollar are not available after December 1998. By using the fixed rate established by the European Central Bank between the Euro and

⁸ To avoid an arbitrary initial value, we first use the simplex search method of Lagarias et al. (1998) to obtain the estimate of $\psi_1(\hat{\psi}_1^o)$ which is then used as the initial value to obtain the MLE estimate of $\psi_1(\hat{\psi}_1)$.

⁹ Ning (2010) expresses exchange rates and stock price indices in terms of US dollars. With this definition, it is not straightforward to compare returns from stock markets with those from foreign exchange markets, since returns from stock markets combine returns from foreign exchange markets. This paper therefore expresses exchange rates and stock price indices in terms of the local currency of each country.

the local currency, GFD provides the dollar-based exchange rates for the local currency of the euro-zone countries after 1999. These rates are constructed by using the euro-dollar rates and the official conversion rates. Alba and Papell (2007) adopted the same way to construct the nominal (dollar) exchange rates of European-Union member countries after 1998.

The 3-month LIBOR rates for FRA, GER and ITA are not available after January 3, 1999 because of the introduction of the euro.¹⁰ Therefore, the 3-month LIBOR denominated in euro (Euro LIBOR) is adopted for those three countries from January 4, 1999 onwards. The stock indices used are Canada's S&P/TSX 300 composite, France's SBF-250 Index, Germany's CDAX composite index, Italy's BCI Index, Japan's Nikkei 225 Stock Average, and the UK's FTSE All-Share Index. Stock returns are the log-difference of the stock index multiplied by 100 (i.e., $R_{2,t} = (\log(P_t) - \log(P_{t-1})) \times 100$). The exchange-rate returns ($R_{1,t}$) are the log-differences of nominal exchange rates multiplied by 100.

Table 1 presents summary statistics of the data and points out that the mean of the returns for a country is smaller than the standard deviation of the returns, indicating high risks in both markets. For each country, the volatility of stock returns is higher than that of exchange rate changes. All series exhibit excess kurtosis and the normality of the series is strongly rejected as indicated by the Jarque-Bera test. Conventional linear correlations between the stock and foreign exchange markets over the whole period indicate that the pair-wise correlation between the two markets is either positive (FRN, GER) or negative (CAN, ITA) or insignificant (JAP, UK).

< Table 1 Here >

¹⁰ The purpose of this paper is to investigate the dependence structure between stock and foreign exchange markets of a country which involves capital mobility across countries and hence short-term interest rates are adopted in our empirical analysis.

3.2 Empirical results from the GARCH estimation of marginal models

Table 2 reports the estimation results of the AR(k)-GARCH(1,1) model for both exchange-rate changes and stock returns. The mean function generally follows an AR process with long lags. The slope parameters ($\beta_{k,1}$ and $\beta_{k,2}$) in the conditional variance equation are all significant at conventional levels. The interest differential is the instrument of the unobserved state variable and its impact is significant on either the conditional mean or the conditional variance for both return series.

< Table2 here >

Several diagnostic tests are adopted to examine the hypotheses of no serial correlation and autoregressive conditional heteroscedasticity in estimated residuals. These tests statistics include the Q-statistic, the Q^2 -statistic and the ARCH-LM statistic.¹¹ Both the Q and the Q^2 -statistics fail to reject the hypothesis of no serial correlation in the residuals at conventional levels. The ARCH-LM statistic also fails to reject the hypothesis of no autoregressive conditional heteroscedasticity in the residuals. In short, the results from Table 2 indicate that the marginal models are well specified by the AR(k)-GARCH(1,1) models. Having well-specified marginal models is crucial because the estimated copula model would otherwise fails to correctly capture the dependence structure of the two return series.

3.3 Results from copula estimation

Before estimating a copula-switching model, this paper first examines the results from a single-copula model. Table 3 reports the parameter estimates and the maximum likelihood value (LV) for the Gaussian copula, the Student-t copula and four different versions of the Clayton copula.¹² The copula parameter estimates ($\hat{\rho}_i$)

¹¹ The Q^2 statistic is the Q statistic of the squared standardized residuals.

¹² These four copulas are: the Clayton, the survive Clayton, the Rotated Clayton, and the Rotated

are significant in four (CAN, FRN, GER and UK) out of six countries, when a normal or a Student-t copula is applied. However, at least half of the copula parameter estimates are insignificant when a single Clayton copula is applied. Among the above six different copula functions, the Student-t copula yields the largest LV in five out of six countries. These results are similar to those in Ning (2010).

< Table 3 here >

A Student-t copula assumes symmetric tail dependences, meaning that the tail dependence between the stock and foreign exchange markets is the same during both booms and crashes. This assumption may, however, be restrictive empirically. To allow for asymmetric tail dependence, this paper therefore examines the dependence structure between the stock and foreign exchange markets based on a dependence-switching copula model.

The results from Table 4 indicate that the copula parameter estimates ($\hat{\alpha}_i$) under different regimes are all significant, except for $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in CAN. In addition, the estimated LV from the dependence-switching copula model for a specific country is larger than that from a single copula model, justifying the appropriateness of adopting a dependence-switching copula model. The estimated transition probabilities (\hat{P}_{11} , \hat{P}_{00}) are all high. This indicates that the duration in each regime is generally long and hence the switching between regimes is not frequent. An advantage of our model is that it allows us to discuss the dependence (ρ_i) and tail dependence (φ_i) of two financial markets for four different market statuses: bull or bear markets associated with the appreciation or depreciation of the domestic currency. Billio and Pelizzon (2000) points out that the Value-at-Risk (VaR) for a portfolio is simply an estimate of a specified percentile of the probability distribution of the portfolio's change in value

survive-Clayton copulas.

over a given holding period. A significant tail dependence indicates a higher probability of extreme events and hence a larger estimate of VaR than that implied by a bivariate normal distribution. Ignoring the significance of tail dependences may lead to an under-estimation of risks. Tail dependence estimates are therefore crucial for risk management and for estimating the true VaR.

< Table 4 Here >

A negative correlation regime includes two cases which are a bear market associated with domestic currency depreciation (both markets are crashing) and a bull market coexisting with domestic currency appreciation (both markets are booming). The right (left) tail dependence, ϕ_3 (ϕ_4), denotes the probability of a simultaneous large loss (profit) in both markets and hence is a true measure of systemic risk in the period of a financial crisis (boom). If ϕ_3 (ϕ_4) is large, then investors holding a long (short) position in both markets are likely to suffer huge losses. Investors in Roy's (1952) safety-first portfolio theory aim to minimize the probability of ruin. Therefore, tail dependences are extremely important for a safety-first investor's portfolio management (Susmel, 2001).

The estimated right dependence ($\hat{\rho}_3$) and tail dependence ($\hat{\phi}_3$) varies from 0.09 to 0.45 and from 0.00 to 0.44, respectively, when both markets are crashing. However, the estimated left dependence ($\hat{\rho}_4$) and tail dependence ($\hat{\phi}_4$) varies from 0.10 to 0.21 and from 0.01 to 0.10, respectively, when both markets are booming. It is worth noting that dependence estimates under a negative correlation regime are positive in Table 4, but they denote a negative correlation relationship between the two markets.¹³ The estimated dependences, $\hat{\rho}_3$ and $\hat{\rho}_4$, are significant in all countries.

¹³ The tail dependence denotes the conditional probability of the joint occurrence of extreme status in both markets, and hence it is positive regardless of the correlation regimes.

The estimated tail dependence is significant for φ_3 in four out of six countries but is insignificant for φ_4 in all countries. The finding of significant right tail dependences is consistent with our observation of the simultaneous crash of stock prices and the serious depreciation of the local currency during the period of a financial crisis.¹⁴ The left tail dependence estimates ($\hat{\varphi}_4$) are, in general and across countries, small and insignificant when both markets are booming. This result indicates that it is very unlikely for global investors with long positions in both markets to simultaneously obtain an extreme profit from the two markets.

A positive correlation regime describes the remaining two cases which are a bear market associated with domestic currency appreciation and a bull market that coexists with domestic currency depreciation. The left (right) tail dependence φ_1 (φ_2) in this regime reflects the probability of simultaneously suffering extreme losses (profits) from stock markets, but having extreme profits (losses) from foreign exchange markets. A large value of φ_1 (φ_2) indicates that an investor is likely to suffer huge losses if the investor holds a long (short) position in the stock market and a short (long) position in the foreign exchange market. The results from the first and second panels of Table 4 indicate that the left dependence ($\hat{\rho}_1$) and tail dependence estimates ($\hat{\varphi}_1$) vary from 0.00 to 0.29 and from 0.00 to 0.23, respectively. The right dependence ($\hat{\rho}_2$) and tail dependence ($\hat{\varphi}_2$) estimates vary from 0.05 to 0.22 and from 0.00 to 0.12, respectively. The estimated dependences ($\hat{\rho}_1$ and $\hat{\rho}_2$) are significant in five out of six countries and the estimated tail dependences are significant in four ($\hat{\varphi}_1$) and three ($\hat{\varphi}_2$) out of six countries. In general, the difference between the estimates of ρ_1 and ρ_2 (φ_1 and φ_2) is small for all countries except JAP (FRN).

¹⁴ It is worth noting that the significance of the left and right tail dependences should not be interpreted as transmitting crises via currency markets.

Tail dependences are, in general and across countries, low when both markets are booming, but they are generally high (except for JAP and UK) when both markets are crashing. These results indicate that the probability of simultaneously suffering extreme losses (profits) in the stock and foreign exchange markets is generally high (low) for an investor with long positions in both markets. Among the four different extreme-return statuses, FRN and GER (JAP and UK) have the lowest tail dependence when a bull (bear) market is associated with domestic currency appreciation (depreciation). This indicates that the systemic risk of both markets that are booming (crashing) is the lowest, among the four cases, for FRN and GER (JAP and UK). Hence, an investor with short (long) positions in both markets of FRN and GER (JAP and UK) suffers the minimum systemic risk. By the same token, CAN (ITA) has the lowest tail dependence in the case where a bear (bull) market is associated with domestic currency appreciation (depreciation). An investor with a long (short) position in the stock market, but a short (long) position in the foreign exchange market for CAN (ITA) has the minimum systemic risk. To sum up, the tail dependences reported in Table 4 are true systemic risks resulting from different extreme-return statuses, which are crucial for safety-first agents investing globally and for correctly estimating VaR.

By adopting a mixture of survival Joe Clayton and Joe Clayton copulas, Ning (2010) finds support for symmetric tail dependence when stock and foreign exchange markets are both booming or both crashing. We therefore examine whether the hypotheses of symmetric dependence and tail dependence are supported by testing the validity of $\rho_3 = \rho_4$ and $\varphi_3 = \varphi_4$, respectively, under the cases where both the stock and foreign exchange markets are in the negative correlation regime (both booming and both crashing). This paper applies a Wald test to examine if these hypotheses do,

in fact, hold for our data. The results from the first panel of Table 5 indicate that the symmetric hypotheses of $\rho_3 = \rho_4$ and $\varphi_3 = \varphi_4$ are rejected, respectively, for CAN, FRN, GER and ITA at the 5% level of significance. Our results are in sharp contrast to those in Ning (2010). In short, by adopting a generalized copula model that nests a conventional state-invariant copula model as a special case, this study rejects the hypothesis of symmetric dependence and tail dependence for most countries. Several papers have found asymmetric tail dependence across foreign exchange markets and across stock markets (Hartmann, 2004; Cappiello, 2006). Our results of asymmetric tail dependence under a negative correlation regime between stocks and foreign exchange markets enrich findings in existing literature.

< Table 5 here >

Furthermore, this paper examines the hypothesis of symmetric dependence and tail dependence, respectively, under a positive correlation regime (one market that is booming is associated with another one that is crashing), a scenario that cannot be investigated by a single regime copula model such as Patton (2006) and Ning (2010). The results from the second panel of Table 5 point out that neither of the hypotheses of $\rho_1 = \rho_2$ and $\varphi_1 = \varphi_2$ are rejected for all countries at conventional levels of significance.

Figures 1 and 2 depict the smoothing probability of the positive correlation regime and point out that most countries stay in the positive correlation regime for most of the time except for CAN and JAP. The above figures also indicate that the portfolio rebalancing effect dominates for FRN, GER, ITA and UK most of the time. Several articles point out that the dynamic rebalancing of the equity portfolio is an important source of exchange rate dynamics (Bohn and Tesar, 1996; Hau and Rey, 2004, 2006; Siouroun, 2004). Our results support their findings. Most of the

literature examining the existence of a portfolio rebalancing effect considers only a sub-sample (Jorion, 1990; Chow et al., 1997; Dominguez and Tesar, 2001; Hau and Rey, 2006), and hence results tend to be subject to sample selections. The advantage of our approach is that it examines the effect that dominates the correlation between the stock and foreign exchange markets without restricting the sample period.

< Figures 1, 2 here >

During the recent global financial crisis (2008-2009), we have witnessed a simultaneous crash in the stock and foreign exchange markets for most countries. Figures 1 and 2 also indicate that there exists a negative correlation regime for all countries over that period except JAP.¹⁵ In addition, during the period of the currency crisis in Europe (1992-1993), the currencies of the euro-zone countries and the UK in our sample reveal a depreciating trend but their stock market indices appear to exhibit a smooth rising trend and hence Figures 1 and 2 reflect a positive correlation regime over that period. By contrast, CAN and JAP reflect a negative correlation regime during the period of the Asian financial crisis over the 1997-1998 period, since the decrease in their stock indices is associated with the depreciation of their currencies. In short, our dependence-switching copula model correctly describes the dependence of the stock and foreign exchange markets for several financial crisis periods in the sample.

Figure 3 plots the smoothing correlation for euro-countries in our sample and shows that the correlation structure is apparently different in the pre-euro period but is similar in the post-euro period.¹⁶ A plausible explanation for the above result is that

¹⁵ Japan's stock index goes down in the period of the recent global financial crisis but the Japanese Yen appreciates rather than depreciates. The major reason is that many countries increase their holdings of Japan's government bonds for portfolio diversification and hence foreign capital flows into Japan causing the Japanese Yen to appreciate.

¹⁶ We estimate the smoothing correlation by using $\rho_{\text{smoothing}} = p_{1,s}(0.5\rho_1 + 0.5\rho_2) - p_{0,s}(0.5\rho_3 + 0.5\rho_4)$,

their common currency brings consistency to their stock and foreign exchange markets. In addition, Figure 3 also indicates that the dominance of the portfolio rebalancing effects is revealed in the euro countries at roughly the same time. The dependence of the stock and foreign exchange markets for CAN and JAP differs from those of the euro-countries as indicated in Figure 4. The dependence between the two markets is negative over the periods 1997-2001 and 2005-2007 for CAN and is negative over the periods 1990-1993, 1997-2002 and 2003-2005 for JAP. As for the UK, Figure 4 reveals that its dependence is most similar to that for France, with the portfolio rebalancing effect usually dominating. To sum up, the above results justify our assumption that the dependence between the stock and foreign exchange markets switches between positive and negative correlation regimes. This in turn supports the appropriateness of adopting a dependence-switching copula model to depict the dependence-switching characteristic between the stock and foreign exchange markets.

< Figures 3, 4 here >

4. Conclusions

This paper provides a dependence-switching copula model to describe the dependence structure between the stock and foreign exchange markets. The advantage of our model is that it allows us to discuss dependence and tail dependence for four different market statuses. They are bear and bull markets associated with domestic currency depreciation and appreciation. Several interesting results are obtained after estimating the model with daily stock and foreign-exchange returns for six major industrial countries over the period 1990–2010. First, dependence and tail dependence are asymmetric for most countries in a negative correlation regime, but are symmetric for most countries in a positive correlation regime. These tail dependence estimates

with $p_{j,s}$ denoting the smoothing probability in regime j , $j = 0, 1$.

are crucial for appropriately estimating VaR, for measuring true systemic risk in a financial crisis, and for the safety-first investor's portfolio management. Second, exchange-rate exposure effects or portfolio rebalancing effects dominate for most countries on most occasions. Third, the smoothing correlation between the above two markets is similar for the euro countries and the UK during the post-euro period. These results suggest that analyzing cross-market linkages within a time-invariant copula framework may not be appropriate. They also have important implications for cross-market risk management and international asset pricing.

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Table 1. Descriptive statistics

	Mean	Max	Min	SD	Skew	Kurtosis	J-B Test	Corr
CAN								
R ₁	-0.0021	3.2987	-3.7665	0.4877	0.1242	8.6065	6532.6060	-0.3095
R ₂	0.0249	9.3707	-9.7881	1.0653	-0.7487	13.9719	25434.4200	
FRN								
R ₁	-0.0012	4.1323	-5.0161	0.6649	-0.0952	5.2632	1086.8610	0.0510
R ₂	0.0120	10.2211	-9.2647	1.2835	-0.0728	8.9192	7386.9930	
GER								
R ₁	-0.0009	3.4897	-4.2466	0.6860	-0.0870	5.0076	862.4230	0.0292
R ₂	0.0111	10.6398	-7.5520	1.3071	-0.1737	8.5857	6651.6610	
ITA								
R ₁	0.0047	6.6893	-3.7188	0.6829	0.4108	7.8937	5081.6840	-0.0499
R ₂	0.0065	7.9138	-9.7226	1.2787	-0.3933	7.1093	3612.5940	
JAP								
R ₁	-0.0091	6.2151	-6.9526	0.7391	-0.3988	8.2172	5702.1670	0.0102
R ₂	-0.0285	13.2346	-12.1110	1.5889	-0.0178	8.1344	5396.7960	
UK								
R ₁	0.0021	4.3904	-3.2994	0.6203	-0.2619	6.1366	2172.8720	0.0083
R ₂	0.0148	8.8107	-8.7099	1.0663	-0.1645	10.1374	10969.5700	

Note: $R_{1,t} = (\log(E_t) - \log(E_{t-1})) \times 100$ and $R_{2,t} = (\log(P_t) - \log(P_{t-1})) \times 100$, where P and E are stock price indices and nominal exchange rates. “SD” and “Skew” indicate the standard deviation and the skewness of a variable, respectively. “J-B” is the Jarque-Bera normality test and “Corr” is the Pearson correlation coefficient. Bold-faced numbers indicate significance at the 5% level.

Table 2. The estimation of marginal distribution models

$$R_{k,t} = a_{k,0} + \sum_{i=1}^{p_k} a_{k,i} R_{1,t-i} + c_{k,1} id_t + \varepsilon_{k,t}, \quad k=1, 2,$$

$$h_{k,t} = \beta_{k,0} + \beta_{k,1} \varepsilon_{k,t-1}^2 + \beta_{k,2} h_{k,t-1} + c_{k,2} id_t,$$

$$\eta_{k,t} = \varepsilon_{k,t} / \sqrt{h_{k,t}}; \quad \eta_{k,t} | \Omega_{t-1} \sim t(v_k)$$

CAN			FRN			GER		
	R ₁	R ₂		R ₁	R ₂		R ₁	R ₂
a _{k,0}	-0.1202 (0.0425)	a _{k,0} -0.2653 (0.1128)	a _{k,0}	0.0171 (0.0291)	a _{k,0} 0.0739 (0.0549)	a _{k,0}	-0.0089 (0.0088)	a _{k,0} 0.0460 (0.0136)
a _{k,9}	-0.0386 (0.0148)	a _{k,1} 0.1467 (0.0168)			a _{k,5} -0.0385 (0.0157)	a _{k,8}	0.0323 (0.0148)	a _{k,1} 0.0411 (0.0150)
a _{k,10}	0.0353 (0.0150)							a _{k,5} -0.0329 (0.0151)
c _{k,1}	0.2801 (0.0975)	0.6653 (0.2602)		-0.0609 (0.0592)	-0.0837 (0.1139)		-0.0046 (0.0043)	-0.0155 (0.0053)
β _{k,0}	-0.0034 (0.0031)	-0.0236 (0.0314)		-0.0054 (0.0035)	-0.0571 (0.0177)		0.0038 (0.0013)	0.0281 (0.0113)
β _{k,1}	0.0434 (0.0067)	0.1026 (0.0165)		0.0334 (0.0052)	0.0794 (0.0121)		0.0316 (0.0051)	0.1034 (0.0129)
β _{k,2}	0.9542 (0.0067)	0.8837 (0.0178)		0.9623 (0.0058)	0.9013 (0.0113)		0.9604 (0.0065)	0.8782 (0.0132)
c _{k,2}	0.0091 (0.0070)	0.0811 (0.0706)		0.0161 (0.0077)	0.1802 (0.0452)		0.0004# (0.0002)	0.0002 (0.0019)
Q (10)	12.699 [0.241]	14.371 [0.157]		4.6668 [0.912]	14.830 [0.138]		11.447 [0.324]	15.127 [0.128]
Q(20)	19.411 [0.495]	22.809 [0.298]		9.1266 [0.981]	28.759 [0.093]		16.686 [0.673]	25.232 [0.193]
Q ² (5)	3.3596 [0.645]	1.7577 [0.882]		2.5919 [0.763]	1.9532 [0.856]		5.2603 [0.385]	2.9664 [0.705]
Q ² (10)	10.326 [0.412]	3.2417 [0.975]		8.8912 [0.542]	2.8294 [0.985]		13.178 [0.214]	4.0268 [0.946]
Arch(5)	3.3503 [0.6461]	1.7356 [0.8844]		2.6022 [0.7610]	1.9791 [0.8520]		5.3157 [0.3786]	2.9738 [0.7040]
Arch(10)	10.5035 [0.3975]	3.2637 [0.9745]		8.8652 [0.5449]	2.9227 [0.9832]		13.0039 [0.2234]	4.0595 [0.9446]

Notes: $R_{1,t} = (\log(E_t) - \log(E_{t-1})) \times 100$ and $R_{2,t} = (\log(P_t) - \log(P_{t-1})) \times 100$, where P and E are stock price indices and nominal exchange rates; id_t is interest differentials. The instruments of id_t are lags of stock returns, exchange rate changes and interest differentials. After replacing id_t by \widehat{id}_t , the AR(p)-GARCH (1,1) model is estimated by quasi-maximum likelihood method. Bold-faced numbers indicate significance at the 5% level. ‘#’ indicates significance at the 10% level. Numbers in brackets are p-values and numbers in parentheses are standard deviations. Q(p) and Q²(p) are Q-statistics for testing the hypothesis of no serial correlation in residuals and in squared residuals, respectively. Arch(p) is the LM test for no autoregressive conditional heteroscedasticity in residuals. These statistics each has a chi-square distribution, with p degrees of freedom.

Table 2. Continued

	ITA		JAP		UK	
	R ₁	R ₂	R ₁	R ₂	R ₁	R ₂
a _{k,0}	0.3180 (0.1108)	a _{k,0} -0.2295 (0.2040)	a _{k,0} -10.1039 (3.6247)	a _{k,0} -1.7140 (0.3945)	a _{k,0} -0.0042 (0.0101)	a _{k,0} 0.0593 (0.0144)
a _{k,9}	-0.0334 (0.0147)	a _{k,1} 0.1641 (0.0164)	a _{k,1} -0.3373 (0.1132)	a _{k,10} 0.0326 (0.0153)	a _{k,7} 0.0327 (0.0147)	a _{k,5} -0.0279# (0.0148)
a _{k,12}	0.0314 (0.0151)	a _{k,3} 0.0287# (0.0167)	a _{k,2} -0.3165 (0.1119)			
			a _{k,3} -0.3077 (0.1084)			
			a _{k,4} -0.2968 (0.1074)			
			a _{k,5} -0.3145 (0.1093)			
			a _{k,6} -0.3014 (0.1080)			
c _{k,1}	-0.1978 (0.0673)	-0.1914 (0.1242)	-3.6941 (1.3259)	-0.6351 (0.1439)	-0.0032 (0.0044)	-0.0092# (0.0055)
β _{k,0}	-0.0312# (0.0184)	0.0919 (0.0893)	0.0360 (0.0484)	0.2583 (0.2596)	0.0025 (0.0009)	0.0103 (0.0026)
β _{k,1}	0.0426 (0.0078)	0.8997 (0.0167)	0.0439 (0.0080)	0.1075 (0.0129)	0.0379 (0.0066)	0.0980 (0.0110)
β _{k,2}	0.9514 (0.0084)	0.1281 (0.0147)	0.9429 (0.0100)	0.8738 (0.0135)	0.9524 (0.0083)	0.8922 (0.0106)
c _{k,2}	0.0209# (0.0113)	-0.1914 (0.0559)	0.0105 (0.0176)	0.0752 (0.0924)	0.0006 (0.0003)	0.0008 (0.0009)
Q(10)	14.275 [0.161]	11.387 [0.328]	11.169 [0.344]	4.0732 [0.944]	6.4737 [0.774]	16.789 [0.079]
Q(20)	28.120 [0.107]	20.101 [0.452]	20.085 [0.453]	19.687 [0.478]	16.548 [0.682]	28.697 [0.094]
Q ² (5)	6.9640 [0.223]	2.3191 [0.803]	5.2730 [0.383]	7.8238 [0.166]	0.5759 [0.989]	1.9933 [0.850]
Q ² (10)	15.603 [0.112]	4.0807 [0.944]	7.8718 [0.641]	8.8422 [0.547]	2.2950 [0.994]	6.5460 [0.767]
Arch(5)	7.0577 [0.2164]	2.3519 [0.7986]	5.2377 [0.3876]	7.7829 [0.1686]	0.5769 [0.9890]	1.9792 [0.8520]
Arch(10)	15.7974 [0.1056]	4.0040 [0.9472]	8.0648 [0.6225]	8.8283 [0.5485]	2.2283 [0.9943]	6.4777 [0.7737]

Table 3. Single copula models

	CAN	FRN	GER	ITA	JAP	UK
Normal Copula						
ρ	-0.1548 (0.0138)	0.1195 (0.0139)	0.0726 (0.0138)	-0.0074 (0.0143)	-0.0083 (0.0143)	0.0965 (0.0137)
LV	-14120	-14298	-14435	-14115	-13916	-14593
Student's t Copula						
ρ	-0.1538 (0.0148)	0.1241 (0.0129)	0.0711 (0.0146)	-0.0065 (0.0154)	-0.0136 (0.0159)	0.0998 (0.0147)
dof	12.1085 (2.4747)	7.1308 (0.8419)	8.4190 (1.1834)	7.9627 (1.0431)	10.3870 (1.8241)	12.7312 (2.6261)
LV	-14105	-14253	-14403	-14080	-13896	-14578
Clayton(u,v)						
α	0.0001 (0.0184)	0.1572 (0.0179)	0.0934 (0.0162)	0.0200 (0.0144)	0.0138 (0.0141)	0.1067 (0.0170)
LV	-14180	-14285	-14429	-14114	-13915	-14593
Clayton(1-u,1-v)						
α	0.0001 (0.0180)	0.1115 (0.0177)	0.0733 (0.0162)	0.0090 (0.0143)	0.0097 (0.0139)	0.0939 (0.0170)
LV	-14180	-14310	-14436	-14115	-13916	-14599
Clayton(1-u, v)						
α	0.2088 (0.0189)	0.0001 (0.0158)	0.0001 (0.0159)	0.0328 (0.0149)	0.0193 (0.0151)	0.0001 (0.0164)
LV	-14100	-14334	-14448	-14113	-13915	-14617
Clayton(u,1-v)						
α	0.1194 (0.0180)	0.0001 (0.0171)	0.0001 (0.0166)	0.0237# (0.0144)	0.0209 (0.0150)	0.0001 (0.0169)
LV	-14154	-14334	-14448	-14114	-13915	-14617

Notes: LV indicates the estimated log likelihood value. dof is the degree of freedom of the Student-t distribution. α is the shape parameter of the Clayton copula and ρ is the correlation coefficient of two returns in the normal or Student-t copula. Numbers in parentheses are standard deviations. Bold-faced numbers indicate significance at the 5% level. '#' indicates significance at the 10% level.

Table 4. The dependence-switching copula model

	CAN	FRN	GER	ITA	JAP	UK
A positive correlation regime						
Bear markets associate with domestic currency appreciation:						
α_1	0.0001 (0.0918)	0.4700 (0.0636)	0.3763 (0.0630)	0.2548 (0.0693)	0.3107 (0.0773)	0.3107 (0.0626)
ρ_1	0.0001 (0.0721)	0.2945 (0.0313)	0.2462 (0.0340)	0.1766 (0.0421)	0.2097 (0.0445)	0.2096 (0.0360)
φ_1	0.0000 (0.0000)	0.2288 (0.0457)	0.1585 (0.0489)	0.0659 (0.0487)	0.1075# (0.0596)	0.1074 (0.0483)
Bull markets associate with domestic currency depreciation:						
α_2	0.0705 (0.0829)	0.3233 (0.0643)	0.3068 (0.0637)	0.1319 (0.0615)	0.2019 (0.0684)	0.2707 (0.0610)
ρ_2	0.0534 (0.0607)	0.2169 (0.0365)	0.2074 (0.0368)	0.0970 (0.0423)	0.1436 (0.0439)	0.1862 (0.0365)
φ_2	0.0001 (0.0002)	0.1172 (0.0449)	0.1044 (0.0490)	0.0052 (0.0128)	0.0323 (0.0376)	0.0772# (0.0446)
A negative correlation regime						
Bear markets associate with domestic currency depreciation:						
α_3	0.8436 (0.1934)	0.5449 (0.1016)	0.5581 (0.1020)	0.4355 (0.0897)	0.1175 (0.0578)	0.2576 (0.0868)
ρ_3	0.4493 (0.0671)	0.3300 (0.0465)	0.3360 (0.0461)	0.2772 (0.0457)	0.0871 (0.0404)	0.1783 (0.0526)
φ_3	0.4397 (0.0828)	0.2802 (0.0665)	0.2888 (0.0656)	0.2036 (0.0668)	0.0027 (0.0080)	0.0678 (0.0615)
Bull markets associate with domestic currency appreciation:						
α_4	0.3071 (0.1449)	0.1810 (0.0883)	0.1484 (0.0677)	0.1786 (0.0713)	0.2581 (0.0730)	0.2756 (0.1019)
ρ_4	0.2076 (0.0837)	0.1300 (0.0578)	0.1083 (0.0458)	0.1284 (0.0468)	0.1786 (0.0442)	0.1891 (0.0607)
φ_4	0.1046 (0.1115)	0.0217 (0.0406)	0.0094 (0.0200)	0.0206 (0.0320)	0.0682 (0.0518)	0.0808 (0.0752)
Regime Switching						
P_{11}	0.9911 (0.0037)	0.9971 (0.0015)	0.9962 (0.0017)	0.9977 (0.0016)	0.9957 (0.0023)	0.9975 (0.0020)
P_{00}	0.9890 (0.0079)	0.9922 (0.0041)	0.9926 (0.0031)	0.9971 (0.0023)	0.9968 (0.0017)	0.9926 (0.0051)
LV	-14057	-14180	-14332	-14054	-13873	-14535

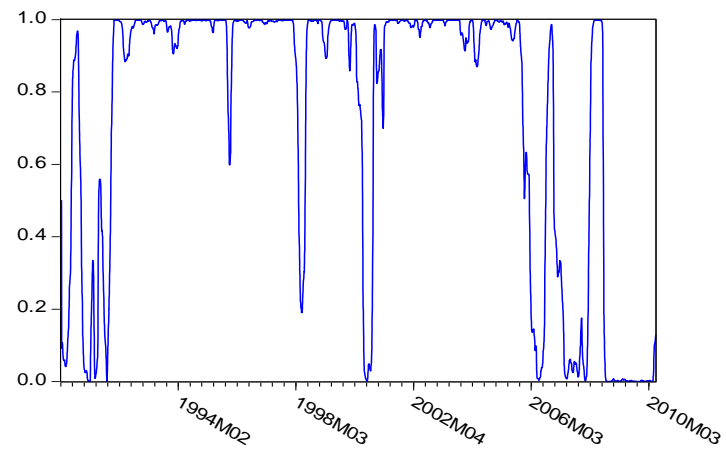
Note: α_i is the shape parameter of the dependence-switching copula, ρ_i and φ_i are the measures of dependence and of tail dependence. Although ρ_3 and ρ_4 are positive in the table, they indicate a negative correlation between stock returns and exchange rate changes. Numbers in parentheses are standard deviations. Bold-faced numbers indicate significance at the 5% level. '#' indicates significance at the 10% level. LV indicates the estimated value of the likelihood function. P_{11} and P_{00} are two transition probabilities.

Table 5. Symmetric tests of dependence and tail dependence

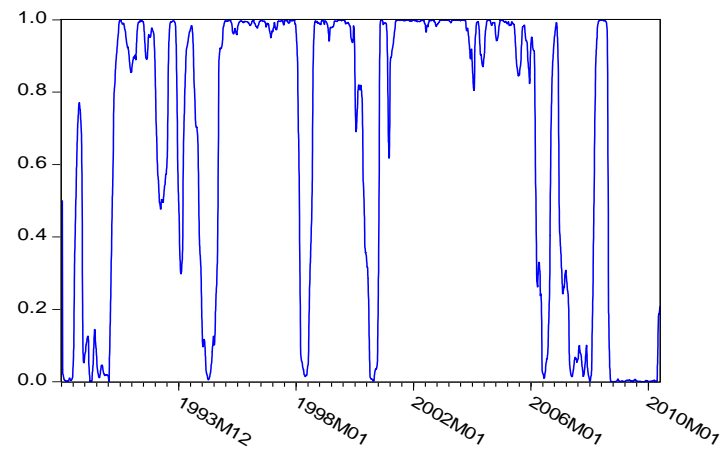
Wald Test	CAN	FRN	GER	ITA	JAP	UK
Symmetric dependence under a negative correlation regime (both markets are booming and crashing)						
$\rho_3 = \rho_4$	22.4711 [0.0000]	5.5534 [0.0184]	9.6560 [0.0019]	3.7351# [0.0533]	1.6579 [0.1979]	0.0152 [0.9019]
$\varphi_3 = \varphi_4$	24.1657 [0.0000]	8.6023 [0.0034]	14.4339 [0.0001]	4.4691 [0.0302]	1.3895 [0.2385]	0.0151 [0.9022]
Symmetric dependence under a positive correlation regime (one market in booming associates with the other one in crashing)						
$\rho_1 = \rho_2$	0.5589 [0.4547]	1.8302 [0.1761]	0.4549 [0.5000]	1.1838 [0.2766]	0.8567 [0.3547]	0.1760 [0.6749]
$\varphi_1 = \varphi_2$	0.0075 [0.9311]	1.9076 [0.1672]	0.4625 [0.4965]	1.1657 [0.2805]	0.8903 [0.3454]	0.1771 [0.6739]

Note: Bold-faced numbers indicate significance at the 5% level. ‘#’ indicates significance at the 10% level. Numbers in brackets are p-values. ρ_i and φ_i measure dependence and tail dependence of stock and foreign exchange markets under different market statuses.

FRN



GER



ITA

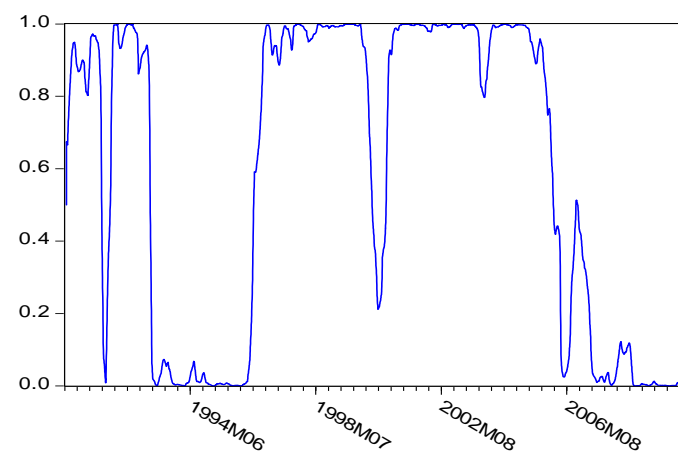
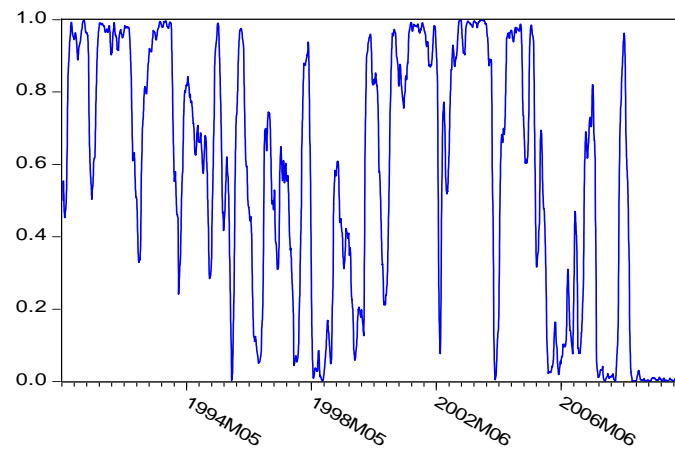
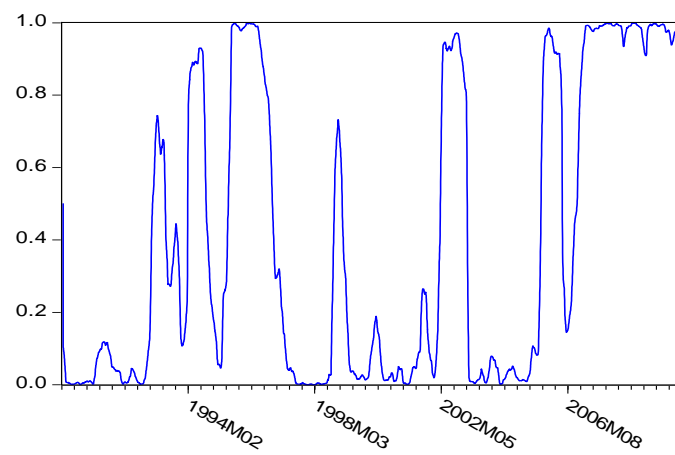


Figure 1. Smoothing probability in positive regime for euro countries

CAN



JAP



UK

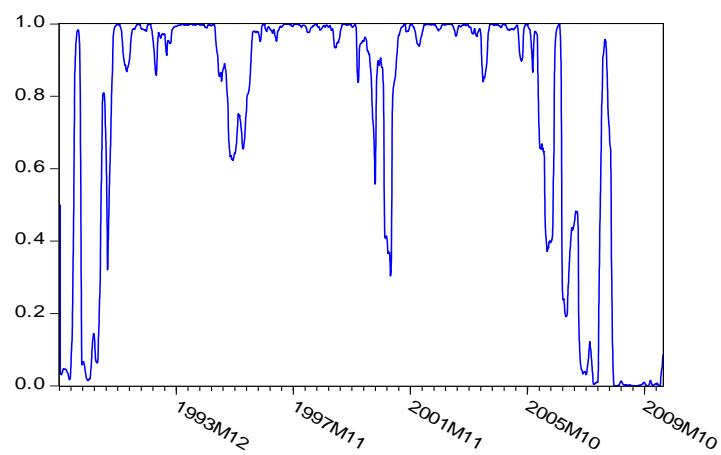
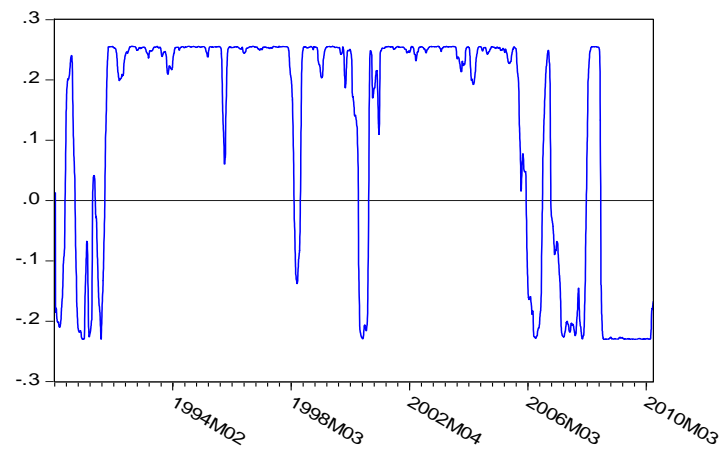
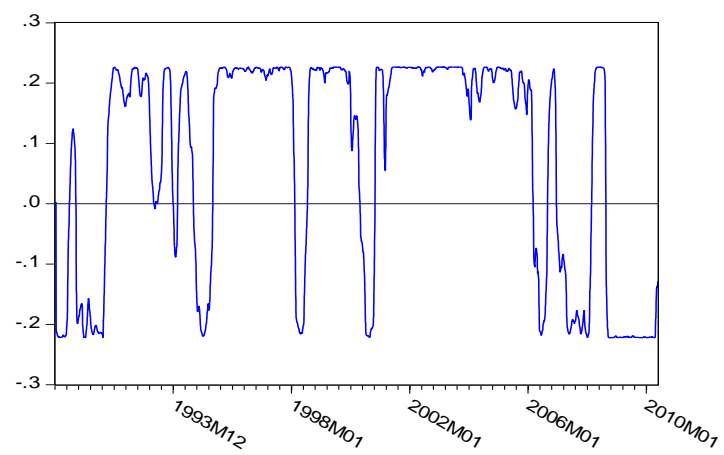


Figure 2. Smoothing probability in positive regime for non-euro countries

FRN



GER



ITA

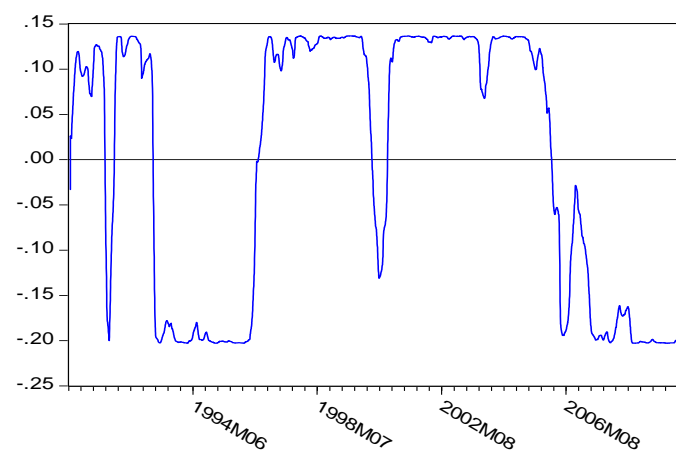
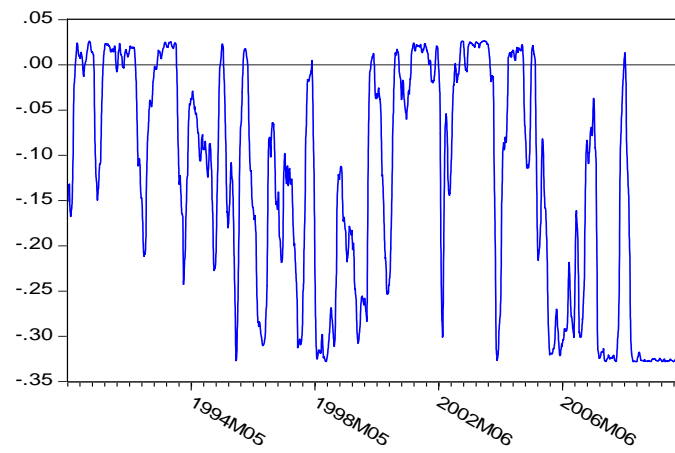
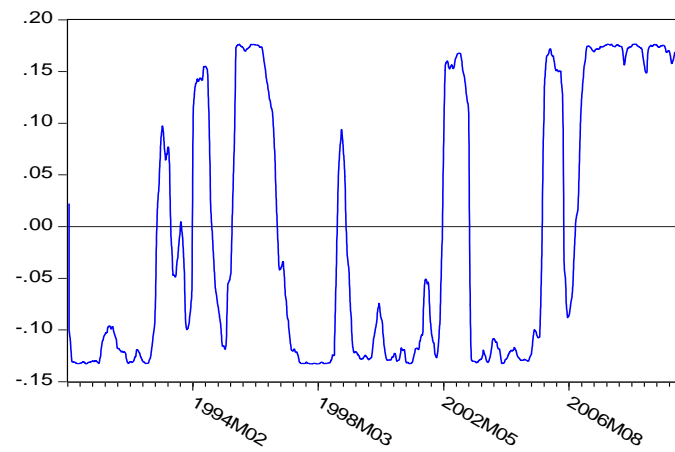


Figure 3. Estimated smoothing correlation for euro countries

CAN



JAP



UK

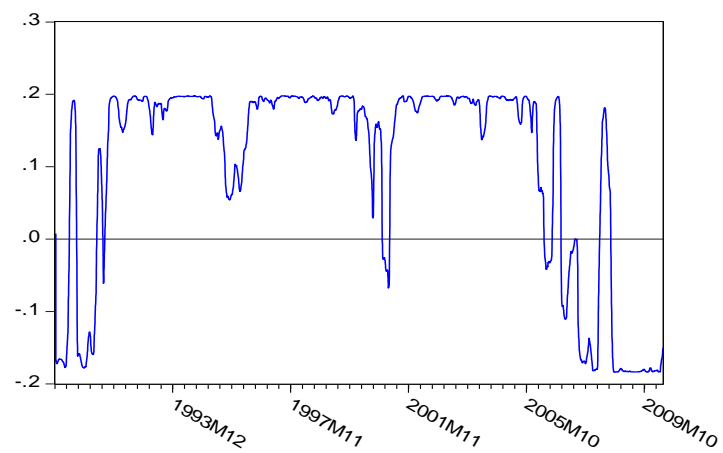


Figure 4. Estimated smoothing correlation for non-euro countries