# Being Right is not Enough: Buying Options to Bet on Higher Realized Volatility\*

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Speculators who wish to bet on higher future volatility often purchase options to "go long volatility." Should investors who buy options expect to profit when realized volatility increases? If so, under what conditions? To answer these questions, we conduct an analysis of the relationship between long volatility performance (buying options) and contemporaneous changes in volatility. We find that buying one-month S&P 500 options is only consistently profitable in the highest decile of changes in one-month volatility. Buying options consistently loses money in the lowest seven deciles of changes in volatility. We then study the trade entry and exit timing required to retain the profits from long option positions during significant volatility increases. We find similar results in global equity option markets.

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During most of the mid-2010s, the S&P 500 Index's volatility hovered near its historical lows. This calm environment has led many investors to ask whether low volatility presents an opportunity to obtain "cheap" portfolio downside protection by purchasing index options. Israelov and Nielsen (2015) show that the answer to this question is no. They show that the Volatility Risk Premium (VRP) also exists in low volatility environments, and therefore that portfolio protection is still expensive even in calm markets.

Portfolio protection, however, is not the only rationale for purchasing options. A long option position allows a speculator to bet on rising volatility. Many proponents of long volatility exposure often advocate this trade in a low implied volatility market environment. Heightened levels of uncertainty, geopolitical risk, or merely a reversion to long-term volatility are some of the arguments put forth by those who expect volatility to spike.

A recommendation to buy options to capture a predicted increase in volatility is predicated on the assumption that investors profit from long options (or long volatility) positions when volatility increases. This paper tests that assumption.

Although it may seem unintuitive, we show that long volatility positions can lose money even when realized volatility rises. For example, consider a long position in a one-month volatility swap in which an investor pays implied volatility and receives realized volatility. The investor entered into the trade at 10% implied volatility when historical realized volatility was 6%. After a month passed, realized volatility jumped from 6% to 9%, a 50% increase. Yet the investor still lost 1 volatility point or 10% (of implied volatility) on the trade. Because of the increase in realized volatility, the implied volatility for a newly issued swap then reset higher from 10% to 13%, a 30% increase. In this illustrative example, realized volatility spiked by 50%, implied volatility jumped by 30%, and the long volatility investor lost 10% during the one-month holding period. This process can repeat itself for many months, with realized and implied volatility trending up to significantly higher levels, and the long volatility investor may slowly deplete her capital even as the scenario she was betting on pans out.

This example illustrates that for long volatility investors, increases in volatility may be insufficient to generate profits. How can this be true? Because, long volatility investors must overcome the well-documented VRP (Bakshi and Kapadia, 2003; Fallon et. al., 2015; Garleanu et. al., 2009) – the difference between implied and realized volatility – just to break even. On average this VRP spread is around 3% in the S&P 500. Therefore, the VRP spread typically accrues to the volatility seller, paid by the long volatility investor. As such, long volatility investors enter their trade facing a significant head wind.<sup>3</sup>

How strong is the head wind? Assume that you had a crystal ball that showed you future equity volatility. If you purchased one-month options only when realized volatility increased over the next month (i.e., realized volatility during the option's life was higher than trailing historical volatility), you would have lost money roughly half of the time. Buying options was, on average, profitable in this perfect foresight case, but only barely so.

<sup>2</sup> Long option buyers benefit from a rise in either implied volatility or realized volatility. Through option selection, an investor can adjust exposure to these measures because short-dated and long-dated options have different levels of sensitivity to implied volatility and to realized volatility. In general, implied and realized volatility tend to be highly correlated. As an approximation, implied volatility is equal to trailing realized volatility plus a time-varying volatility risk premium. Most equity investors buy options to hedge against market declines because realized volatility tends to increase when markets decline. On the other hand, we do not think hedgers buy options because they have a view on the time-varying volatility risk premium. Therefore, we chose to focus this paper on changes in realized volatility. We find that the paper's primary results are similar for changes in implied volatility.

<sup>&</sup>lt;sup>1</sup> As measured by the VIX index, which starts in 1990.

<sup>&</sup>lt;sup>3</sup> This dynamic is not unique to volatility markets. For example, long bond positions are not automatically unprofitable over periods when bond yields rise. As long as bond yields do not rise above the break-even forward yield, long bond positions remain profitable. Koijen et. al. (2018) examines many asset contexts where capital market moves need to offset the initial carry cushion (analogous to the VRP) for an investment to be unprofitable.

Extending this analysis further, we sort long option returns into deciles by one-month changes in realized volatility, again with perfect foresight. The lowest deciles contain the largest declines in realized volatility, and the highest decile contains the largest increases. We find that only the highest two deciles are profitable (the second highest decile only marginally so). That the long volatility strategy is only successful 20% of the time makes clear the challenge of buying options to profit from higher realized volatility. Identifying these fleeting opportunities may require an unrealistic level of skill.

Even if long volatility investors could correctly predict a big spike in volatility sometime in the not-to-distant future, timing matters. A long volatility strategy bleeds capital while waiting for the higher volatility forecast to materialize, which erodes any potential profits. Let's say the crystal ball told you that there would be a tenth-decile change in volatility during the next month, and you liquidated your one-month long option position after this volatility increase. You still would have lost money on average if you entered this long option position more than four months prior to the start of the volatility increase.

Nailing the timing of the entry is not enough, because the exit matters too. Even more so than the entry, because realized volatility spikes typically lead to even larger increases in implied volatility and a potentially higher than average *ex ante* VRP. If investors hold on to their long option position too long, their profits can disappear. In fact, without trading out it took just three months on average to give back long option profits after a tenth-decile change in realized volatility.

This analysis paints a bleak picture. Buying monthly equity index options<sup>4</sup> on the basis of forecasting higher volatility has tough odds to overcome on the road to profitability. You have to be exceptional at forecasting both the magnitude and the timing of large volatility increases, to simply not lose money on average.

Our paper begins by defining the variables used in our analysis: change in volatility, VRP, and delta-hedged long option returns. We then report unconditional long option return properties, as well as return properties conditional on volatility increasing. Next we sort the VRP, as well as long option returns, by changes in volatility and analyze their properties in different regimes. To understand the timing required to profit from long option positions, we report the results of an event study that examines long option returns in the tenth decile of volatility changes. Last, we extend our results to global indices.

## **Data Description**

We obtain implied volatilities from Bloomberg. We use the following volatility index for each equity index: VIX for the S&P 500, V2X for the Eurostoxx 50, VFTSE for the FTSE 100, and VNKY for the Nikkei 225. We also source underlying equity index values, which are used to compute realized volatility, from Bloomberg.

The OptionMetrics IVY database provides daily closing prices and option deltas for the S&P 500 options we analyze in this paper. For Eurostoxx 50 and FTSE 100 option data, we use the OptionMetrics International database. We use the OptionMetrics Asia database for Nikkei 225 option data. To calculate excess of cash returns, we use USD LIBOR from Bloomberg. Equity index futures prices and returns, which are used to calculate option hedge sizing and returns, are also derived from Bloomberg data.

## **Changes in Volatility and the Volatility Risk Premium**

Our goal is to understand the extent to which delta-hedged long option returns benefit from a spike in realized volatility and to explore the relationship between the VRP (spread in implied and realized volatility) and changes in

<sup>&</sup>lt;sup>4</sup> Our analysis focuses on delta-hedged option returns because delta exposure can simply be obtained through instruments such as futures.

realized volatility. We begin by describing each of these variables in the next three sub-sections. We focus our attention on the S&P 500 Index. Our sample starts in January 1996 and ends in December 2016.

### **Changes in Volatility**

Throughout this paper, we sort the VRP and delta-hedged option returns by changes in volatility. Specifically, we define change in volatility to be the difference between future realized volatility, measured over the next 30 calendar days<sup>5</sup>, and historical realized volatility, measured over the trailing 90 calendar days:

$$\psi_t = \ln \sigma_{t,t+30} - \ln \sigma_{t-90,t}$$

**Exhibit 1** sorts changes in volatility into deciles and reports the average within each decile. The markers in **Exhibit 1** indicate each decile's average within the historical sample. Realized volatility increased approximately 43% of the time for the S&P 500 Index. The largest increases in volatility were comparable to the biggest reductions in volatility: 63% for the tenth decile versus -64% for the first decile.

To evaluate statistical significance, we construct 95% confidence intervals via bootstrap (10,000 samples with replacement) by re-sampling the log percentage changes in equity volatility, bucketing into deciles, and computing the corresponding averages within each decile. The shaded region depicts the bootstrapped confidence interval of each decile's average. The confidence intervals in **Exhibit 1**, as well as in upcoming exhibits, indicate that historical means within each decile are estimated with little statistical error. <sup>6</sup>

We find that the large drops in volatility tend to occur *after* volatility spikes. For example, December 16, 2008 is an observation from the first decile during the Global Financial Crisis (GFC). On this date, trailing 90 calendar-day volatility was 72%. Future realized volatility over the next 30 calendar days was 27%. In percentage terms, the change in volatility was -62%, and the change in log volatility was -96%. So, as volatility begins to increase, the long volatility investor faces a dilemma: hold onto the position to bet on volatility increasing further or potentially liquidate the position too early in order to avoid a significant decline in volatility.

This paper focuses much of its analysis on the tenth decile of realized volatility changes. <sup>7</sup> For brevity, we will refer to this decile as the **TDVC**.

#### **Volatility Risk Premium**

A good rule of thumb is that buying an option is profitable if the underlier's realized volatility over the life of the option purchased is greater than its implied volatility. There is some basis risk and path dependence in option portfolios, but the *ex post* VRP<sup>8</sup> is a good starting point for evaluating the profitability of a long (or short) option portfolio. In fact, a variance swap's profit and loss (PL) is directly tied to the difference between implied and realized variance. We define the *ex post* VRP as follows:

$$vrp_t = VIX_t - \sigma_{t,t+30}$$

<sup>&</sup>lt;sup>5</sup> Future realized volatility over the next 30 calendar days is calculated using the close-to-close returns of the underlying market.

<sup>&</sup>lt;sup>6</sup> The confidence intervals shown are for the average within each decile. Because decile sorting ensures that similar values are grouped together before the average is computed, the size of the confidence interval for each decile's average is tighter than we would estimate for the entire sample's average.

<sup>&</sup>lt;sup>7</sup> We also sorted by log percentage changes in implied volatility (using the VIX). Similar to the measure used for realized volatility, the largest increases in volatility tend to be comparable to the largest declines in volatility: 42% for the tenth decile versus -33% for the first decile. We find that the primary results of our paper are similar even if we were to sort by changes in implied volatility. This is unsurprising given the high correlation between implied and realized volatility.

<sup>&</sup>lt;sup>8</sup> The *ex post* VRP measures current implied volatility versus future realized volatility, and provides a measure of option profitability (after the fact). This is in contrast to an *ex ante* VRP measure, which evaluates current implied volatility versus trailing realized volatility. Such a measure could potentially be used in a volatility timing strategy if an investor believes that trailing realized volatility can be used to forecast future realized volatility.

**Exhibit 2** sorts the *ex post* VRP into deciles and reports the average <sup>9</sup> within each decile. On average, equity index options have been richly priced, with an average VRP of 3.0%. The VRP was positive around 85% of the time. Correspondingly, **Exhibit 2** shows that, on average, the VRP is positive in eight of the ten deciles, approximately flat in one decile, and negative in one decile. This result implies that long volatility positions in one-month equity index options have been profitable infrequently.

#### **Delta-Hedged Option Returns**

While evaluating the *ex post VRP* provides useful analysis, the most liquid, implementable method to gain volatility exposure is with equity index option contracts. An investor seeking to profit from being long volatility may wish to isolate the volatility exposure of these options, absent any exposure to equity risk. Such an investor can remain equity-neutral by delta-hedging their option positions.

We construct our illustrative long volatility portfolio as follows. Each month on the standard monthly option expiration date (third Friday of the month), we buy one-third of a -25 delta put option, one-third of a +25 delta call option, and one-sixth each of an at-the-money call option and put option. This options portfolio is sized such that the option notional is equivalent to the NAV, and places one-third weight at each of the three strikes. The purchased options initially have one month until expiration. These cash-settled options are held to expiration, at which point we purchase a new portfolio of options using the same criteria. Each day, the long option portfolio is delta-hedged using equity index futures. Option deltas are computed using Black-Scholes. We define option returns in the following way:

$$R_{i,t} = \frac{(P_{opt,i,t} - P_{opt,i,t-1}) - \Delta_{opt,i,t-1}(P_{fut,t} - P_{fut,t-1}) - R_f P_{opt,i,t-1}}{SPX_{t-1}}$$

**Exhibit 3** sorts the returns of the long option portfolios into deciles, and reports their average annualized return within each decile. Similar to **Exhibit 2**, **Exhibit 3** shows that long option portfolios were infrequently profitable (28% of the time). Average returns were negative in seven of the ten deciles, flat in one decile, and profitable in two. Sorting on actual profitability reveals that there are relatively few profit opportunities. Sorting long option returns on any other variable (or predictive signal) can only reduce the likelihood of profit.

## **Betting on Increasing Volatility**

Volatility mean-reverts over the long run (Engle and Patton, 2007). With equity volatility at historical lows in the mid-2010s, many market commentators suggested long volatility positions in order to bet on increases in volatility. But are bets on increasing volatility actually profitable when volatility increases?

Using our change in volatility measure, we document that over 30-day periods volatility increased 43% of the time in the S&P 500 between 1996 and 2016. **Exhibit 4** reports the long option portfolio's return properties conditional on only holding options during those 30-day periods in which volatility actually increased. For a more complete picture, the exhibit also reports return properties of the unconditional long option portfolio in which the investor does not attempt to time, as well as return properties conditional on periods in which volatility decreased.

Having perfect foresight into whether equity markets become more volatile is profitable, but not reliably so. The conditional *ex post* VRP is -0.9% during periods in which volatility increased versus the 3.0% unconditional VRP. During periods when volatility increased, the average conditional delta-hedged long option annualized return is 1.1%, with a 0.6 Sharpe ratio. This is certainly better than the unconditional long option annualized return of -1.7%, with a -1.1 Sharpe ratio, and considerably better than the conditional long option annualized return of -3.9% and

<sup>&</sup>lt;sup>9</sup> The average volatility risk premium is computed as the square root of the average implied variance (VIX squared) minus the square root of the average coinciding realized variance. If computed as the average VIX minus average realized volatility, the estimate has a positive bias.

corresponding -3.9 Sharpe ratio during periods when volatility decreased. However, even when realized volatility increases, long option returns are only profitable about half the time. A small set of out-sized volatility increases is what led to positive conditional average long option returns.

This suggests that even knowing with certainty that volatility will increase is not enough to reliably profit from a long volatility position. The investor is left with a coin flip in terms of the hit rate of holding a long option position in this scenario. Then what condition is sufficient to reliably profit? We sort the *ex post* VRP into deciles by change in volatility and report the average in each decile in **Exhibit 5**. The *ex post* VRP is only negative in the tenth decile. It is also interesting that the first decile's average VRP is more positive than the tenth decile's is negative. <sup>10</sup>

**Exhibit 6** shows the distribution of the *ex post* VRP within each decile. The *ex post* VRP was positive more than 90% of the time in seven of the ten deciles, and positive more than 65% of the time in the eighth and ninth decile. The only decile in which the *ex post* VRP was consistently negative was the tenth decile. Correspondingly, large increases in volatility are the only time that we expect long option positions to be profitable.

Realized option returns are likely more interesting to option investors than the difference between implied and realized volatility. In that regard, we next sort long option returns into deciles by change in volatility, and report the average annualized return in each decile in **Exhibit 7**. Average long option returns were negative in eight of the ten deciles. **Exhibit 8** shows the distribution of long option returns in each decile. Long option returns were consistently negative in most deciles, positive slightly more than half the time in the ninth decile, and consistently positive (85% of the time) in the tenth decile.

Overall, the results for long option returns are similar to those found for the *ex post* VRP. To consistently profit from a long volatility position on the basis of forecasting changes in volatility, you would have had to predict a 10% probability outcome. The average, annualized long option return in this decile was 6.6% and the average return across the other nine deciles was -2.7%. Clearly, getting this prediction right takes considerable skill.

#### (Mis) Timing the Entry and Exit

We have shown that there's not much room for error when buying options to profit from increases in volatility. Long option positions are consistently profitable only when volatility increases significantly, which means successful long option investors need to be extremely prescient in order to profitably time entry and exit. As perfectly timing the entry and exit points around these spikes in realized volatility is unrealistic, we next investigate the impact on long option profitability when entries and exits are mistimed. Given that owning options has negative return drag unconditionally (**Table 4** shows us that the daily unconditional return drag was approximately 0.67 basis points per day) due to the VRP, we are specifically interested in determining how long it takes either before or after the TDVC event for this return drag to wipe out the speculator's profits.

We conduct an event study to evaluate the returns to buying options before, during, and after the TDVC. Our sample includes 547 such observations. **Exhibit 9** plots average cumulative returns over the 240-business day period that is centered on the first day of the TDVC event, which we call the 'reference date'. The shaded region indicates the 80% confidence interval, estimated via bootstrap.

The average return in the case study peaked at 54 basis points 24 business days after the reference date. <sup>11</sup> We find that if a long volatility investor entered their position approximately 76 business days (3.6 months) prior to the

<sup>&</sup>lt;sup>10</sup> For robustness, we repeated the same analysis after defining change in volatility as the difference in realized volatility instead of the log percentage change. The general findings were unchanged.

<sup>&</sup>lt;sup>11</sup> Average long option returns start increasing several business days *before* the reference date. This is because our event study uses overlapping observations. For some observations in the TDVC, volatility has already started increasing prior to the measured 30-calendar day realized volatility window. For these observations, long option returns are positive ahead of the measured future realized volatility window. For robustness, we also carried out the event study using non-overlapping observations. The general findings were comparable, but noisier given

reference date, the return drag from the mistimed entry fully wiped out their TDVC profits on average. <sup>12</sup> With respect to exiting the long option position, we find that TDVC profits would have been fully conceded on average if positions were held an extra 58 business days (2.8 months) beyond the optimal 24-business-day holding period. Because option prices react to increases in realized volatility, the *ex ante* VRP tends to be heightened relative to its long-term average after a volatility event. A larger VRP increases the return drag of holding a long option position, thereby reducing the buffer speculators had to exit their profitable positions.

For each observation in the event study, we also calculate the historical frequency that long option positions were profitable across different holding periods and report our findings in **Exhibit 10**. We begin by measuring the long option position return for the 21-business day holding period that starts on the reference date. This holding period approximately corresponds to the duration of the TDVC event. We then separately consider holdings periods that are extended by entering the position prior to the reference date or exiting the trade after the TDVC event. Approximately 82% of the long option returns were profitable at the end of the TDVC event. The hit rate drops to 50% for positions that were entered 34 business days (7 weeks) prior to the TDVC event. On the other end, the hit rate drops to 50% for positions that were exited 31 business days (6 weeks) after the TDVC event. Not only does a long volatility investor require exceptional skill in predicting a volatility increase, monetizing the gains of this volatility increase requires nearly impeccable timing.

#### **International Evidence**

Our analysis shows the difficulty in profiting from S&P 500 long volatility positions by betting on increasing volatility. We now test whether these results hold across a larger universe of international equity indexes. Specifically, we analyze changes in volatility, the *ex post* volatility risk premium, and long option returns over a matched sample for the S&P 500, Euro Stoxx 50, FTSE 100, and Nikkei 225.

We start by reporting percentage changes in realized volatility across international indices in **Exhibit 11**. Specifically, we sort changes in volatility into deciles and report the average within each decile. The results are notably consistent across the indices. Similar to the finding in the S&P 500, the largest increases in volatility tend to be comparable to the largest declines: the cross-sectional average is +68% in the tenth decile and -64% in the first decile.

We then sort the *ex post* VRP into deciles by changes in volatility and report the results across international indices in **Exhibit 12**. The top panel shows the average in each decile, and the bottom panel shows the percentage of time the *ex post* VRP was positive. Again, the results are fairly consistent across international indices. The *ex post* VRP is positive on average in eight of the ten deciles, flat in the ninth decile, and negative in the tenth decile. Likewise, the *ex post* VRP is almost always positive across indices in the first eight deciles, positive around half of the time in the ninth decile, and rarely positive in the tenth decile. As seen in the S&P 500, large volatility increases are the only time that we expect long option positions to be profitable.

**Exhibit 13** reports the same analysis as **Exhibit 12** for long option positions on international equity indices. On average, across international indices, long option positions were unprofitable in eight of the ten deciles. Similar to the S&P 500 results, long option returns were consistently negative in eight of the ten deciles, positive around two-thirds of the time in the ninth decile, and consistently positive (cross-sectional average is 84% of the time) in the

the reduced amount of data.

<sup>&</sup>lt;sup>12</sup> The 76-business day estimate is close to the 81 business days (54 bps / 0.67 bps per day = 81 days) expected using the full-sample unconditional return when holding options.

<sup>&</sup>lt;sup>13</sup> A 21-business day period most closely matches the 30-calendar day period of the TDVC event.

tenth decile. The international long option results are consistent across international indices<sup>14</sup>, and are also consistent with the findings from the international *ex post* VRP analysis.

We then repeat the event study of long option returns before, during, and after the TDVC for international indices. The matched sample period includes 327 observations for each equity index. **Exhibit 14** plots cumulative returns centered on the beginning of the identified TDVC events. The solid lines report average cumulative returns across all event observations for each index. As was the case for the S&P 500 event study, long option positions were profitable on average during volatility spikes. However, across the four indices, the profits earned during the TDVC events would have quickly been overcome by the return drag from entering positions too early or exiting them too late.

As an example, buying Euro Stoxx 50 options 78 business days (3.7 months) too early was sufficient to lose money for the average TDVC path. Nikkei 225 provided the largest buffer of 136 business days (6.5 months) for entering the trade too early and FTSE 100 had the tightest buffer of just 45 business days (2.1 months). With respect to exiting long volatility positions, the S&P 500 had the largest cushion of 75 business days (3.6 months) for the average TDVC path. The Nikkei 225 was close behind with 67 business days (3.2 months). With the Euro Stoxx 50 and FTSE 100, speculators had only 43 and 47 business days (2.0 and 2.2 months), respectively, before profits were wiped out. Mirroring the conclusion from the S&P 500 event study, we find that timing with near pinpoint precision is required to profit from long global option positions by betting on increasing volatility.

## **Conclusion**

The odds of profiting from a long volatility exposure using equity index options are low, even if you *know* for a fact that equity volatility will increase. Due to the volatility risk premium, profiting from a long volatility position requires successfully predicting an unusually large increase in volatility. Even if an investor is able to forecast such an event, timing the entry and exit points of the trade are critical. Otherwise, gains are quickly paid away because short-dated equity index options tend to be expensively priced on average.

Even though outright profiting from long volatility exposure by predicting volatility increases is quite difficult, it is feasible that long volatility investors can use this predictive signal to generate *less* negative returns. Long volatility exposure with *less* negative carry can certainly be desirable for investors seeking to protect their exposure to the underlying market. If less negative carry is indeed the objective of a long volatility manager attempting to forecast volatility increases, this should be stated clearly—rather than setting an expectation of overall positive returns.

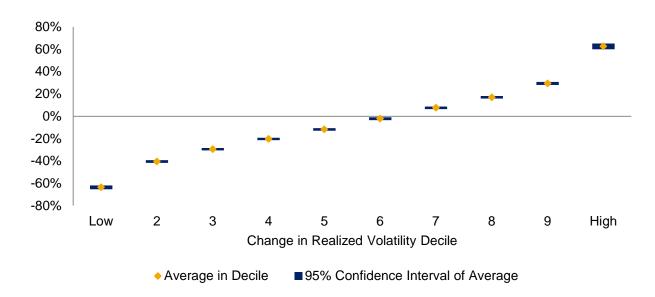
Due to these considerations, tactical long volatility traders face a steep uphill battle. Managers who are able to successfully implement this strategy should be given credit for demonstrating exceptional skill (or congratulated for their good luck). However, some amount of healthy skepticism is likely warranted for those who claim to consistently have such exceptional skill.

<sup>14</sup> In this section, we look at global index option markets to show that the paper's results are robust across different markets. Using 3-month S&P 500 option data, we also evaluated longer maturity options. The paper's overall findings hold when evaluating three-month changes in S&P 500 volatility along with the corresponding three-month S&P 500 *ex post* VRP, as well as three-month S&P 500 option returns.

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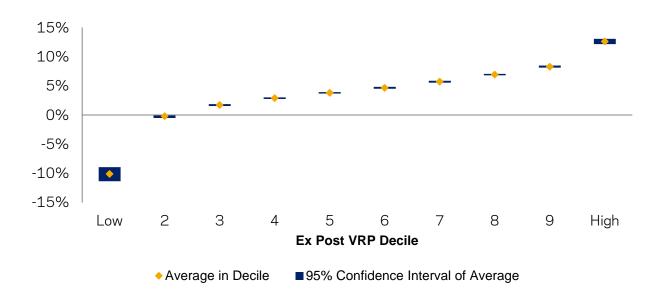
Exhibit 1: Percentage Change in S&P 500 Volatility (Bucketed by Percentage Change in S&P 500 Volatility) 1996 - 2016



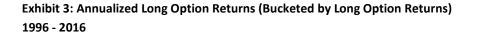
Source: AQR, Bloomberg. Average change in S&P 500 volatility is computed after sorting by change in S&P 500 volatility into deciles. We define percentage change in volatility as the log percentage change between historical and future S&P 500 volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. The marker indicates averages in each decile for the historical sample. The shaded region indicates the 95% confidence interval of the average in each decile based on sampling the change in realized volatility and repeating the entire analysis with 10,000 bootstraps.

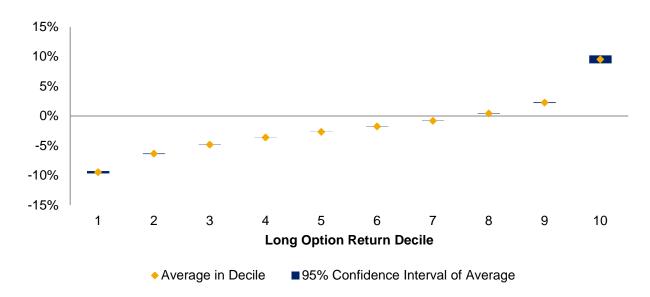
Exhibit 2: Ex Post Volatility Risk Premium (Bucketed by Ex Post VRP)

1996 - 2016



Source: AQR, Bloomberg. Average *ex post* volatility risk premium is computed after sorting by *ex post* volatility risk premium into deciles. Volatility risk premium defined as VIX Index minus the S&P 500 Index's subsequent annualized volatility. The marker indicates averages in each decile for the historical sample. The shaded region indicates the 95% confidence interval of the average in each decile based on jointly sampling implied and subsequent realized volatility, and repeating the entire analysis with 10,000 bootstraps.



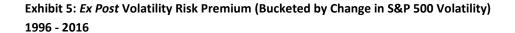


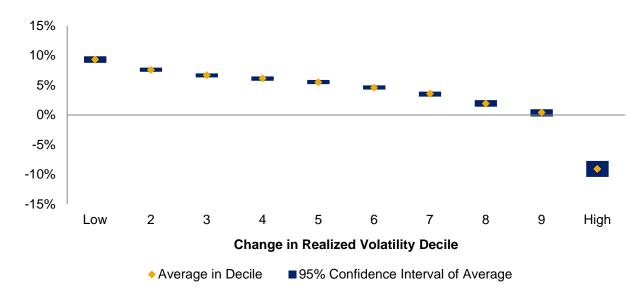
Source: AQR, OptionMetrics, Bloomberg. Average, annualized long option returns are computed after sorting by long option returns into deciles. Long option returns are calculated using long one-month S&P 500 option positions with strikes at -25, 50, and 25 delta, with an equal number of options held at each strike. Options were sized to have unit notional. Options were delta-hedged daily and held until expiration. Upon expiration, the same one-month option structure was initiated for the next expiration date. The marker indicates averages in each decile for the historical sample. The shaded region indicates the 95% confidence interval of the average in each decile based on sampling long option returns and repeating the entire analysis with 10,000 bootstraps.

Exhibit 4: Long Option Returns and *Ex Post* Volatility Risk Premium Conditional on S&P 500 Volatility Increasing 1996 - 2016

	Unconditional	Realized Volatility Higher	Realized Volatility Lower
Annualized Excess Return	-1.7%	1.1%	-3.9%
Annualized Volatility	1.6%	1.9%	1.0%
Sharpe Ratio	-1.1	0.6	-3.9
Ex Post Volatility Risk Premium	3.0%	-0.9%	6.7%
Percentage of Time Long Option Return is Positive	28%	52%	10%

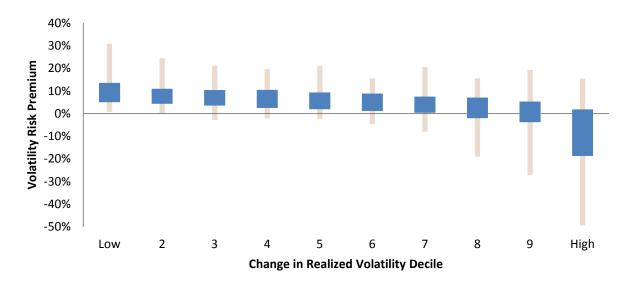
Source: AQR, Bloomberg, OptionMetrics. one-month long option return statistics are computed after determining whether S&P 500 volatility increased. Long option returns are calculated using long one-month S&P 500 option positions with strikes at -25, 50, and 25 delta, with an equal number of options held at each strike. Options were sized to have unit notional. Options were delta-hedged daily and held until expiration. Upon expiration, the same one-month option structure was initiated for the next expiration date. We define change in volatility as the log percentage change between historical and future S&P 500 volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. *Ex post* volatility risk premium is defined as VIX Index minus the S&P 500 Index's subsequent annualized volatility.

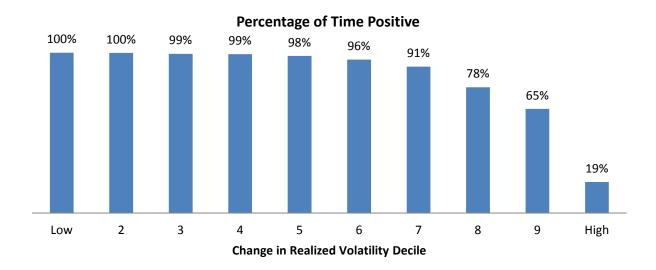




Source: AQR, Bloomberg. Average *ex post* volatility risk premium is computed after sorting by change in S&P 500 volatility into deciles. *Ex post* volatility risk premium is defined as VIX Index minus the S&P 500 Index's subsequent annualized volatility. We define change in volatility as the log percentage change between historical and future S&P 500 volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. The marker indicates averages in each decile for the historical sample. The shaded region indicates the 95% confidence interval of the average in each decile based on jointly sampling implied volatility, subsequent realized volatility, and the change in realized volatility, and repeating the entire analysis with 10,000 bootstraps.

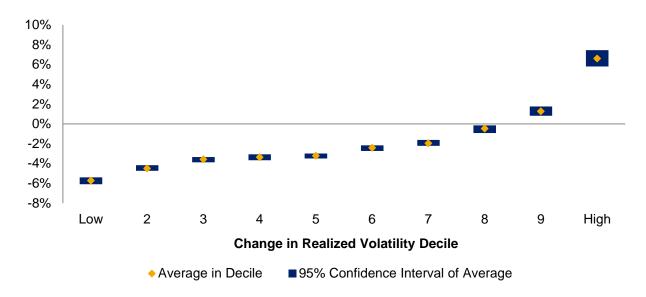
Exhibit 6: Ex Post Volatility Risk Premium Distribution (Bucketed by Change in S&P 500 Volatility) 1996 - 2016





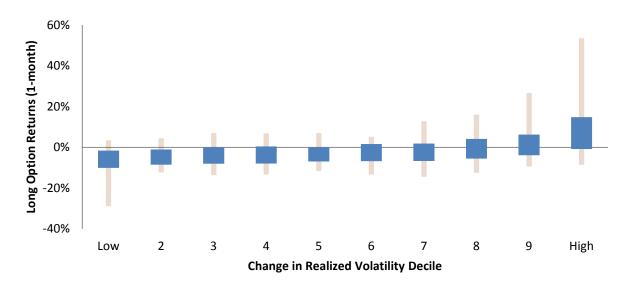
Source: AQR, Bloomberg. Ex post volatility risk premium is computed after sorting by change in S&P 500 volatility into deciles. Ex post volatility risk premium is defined as VIX Index minus the S&P 500 Index's subsequent annualized volatility. We define change in volatility as the log percentage change between historical and future S&P 500 volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. In the top panel, the box depicts the 80% confidence interval and the whiskers are the minimum and maximum values within each bucket. The bottom panel reports the percentage of observations in each decile that were positive.

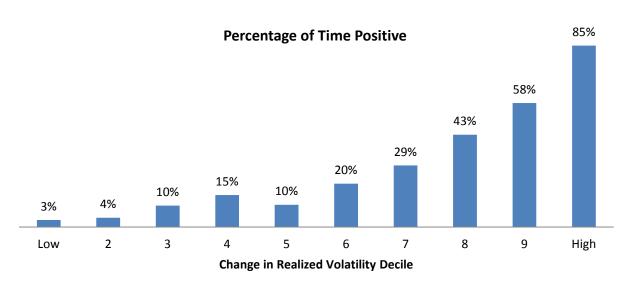
Exhibit 7: Annualized Long Option Returns (Bucketed by Change in S&P 500 Volatility) 1996 - 2016



Source: AQR, Bloomberg, OptionMetrics. Average, annualized one-month long option returns are computed after sorting by change in S&P 500 volatility into deciles. Long option returns are calculated using long one-month S&P 500 option positions with strikes at -25, 50, and 25 delta, with an equal number of options held at each strike. Options were sized to have unit notional. Options were delta-hedged daily and held until expiration. Upon expiration, the same one-month option structure was initiated for the next expiration date. We define change in volatility as the log percentage change between historical and future S&P 500 volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. The marker indicates averages in each decile for the historical sample. The shaded region indicates the 95% confidence interval of the average in each decile based on jointly sampling long option returns and the change in realized volatility, and repeating the entire analysis with 10,000 bootstraps.

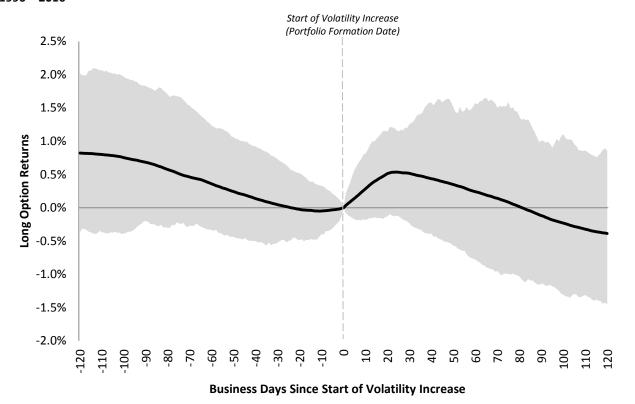
Exhibit 8: Annualized Long Option Return Distribution (Bucketed by Change in S&P 500 Volatility) 1996 - 2016





Source: AQR, Bloomberg, OptionMetrics. Average, annualized one-month long option returns are computed after sorting by change in S&P 500 volatility into deciles. Long option returns are calculated using long one-month S&P 500 option positions with strikes at -25, 50, and 25 delta, with an equal number of options held at each strike. Options were sized to have unit notional. Options were delta-hedged daily and held until expiration. Upon expiration, the same one-month option structure was initiated for the next expiration date. We define change in volatility as the log percentage change between historical and future S&P 500 volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. In the top panel, the box depicts the 80% confidence interval and the whiskers are the minimum and maximum values within each bucket. The bottom panel reports the percentage of observations in each decile that were positive.

Exhibit 9: Long Option Returns Before, During, and After 10<sup>th</sup> Decile of Changes in S&P 500 Volatility 1996 – 2016

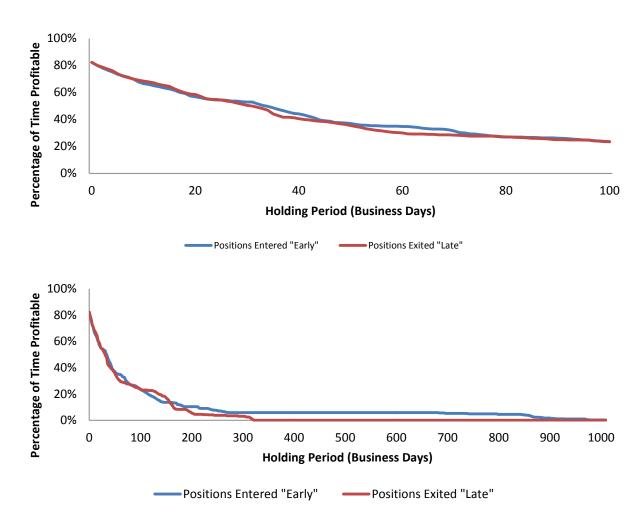


Source: AQR, Bloomberg, OptionMetrics. Long option returns are calculated using long one-month S&P 500 option positions with strikes at -25, 50, and 25 delta, with an equal number of options held at each strike. Options were sized to have unit notional. Options were delta-hedged daily and held until expiration. Upon expiration, the same one-month option structure was initiated for the next expiration date. We then sort long option returns into deciles by changes in volatility. We define change in volatility as the log percentage change between historical and future S&P 500 volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility.

We select all observations in the 10<sup>th</sup> decile, and then calculate compounded returns. Day zero is at the very beginning of the 30 calendar-day future realized volatility interval, and represents the date of option portfolio formation. Therefore, the portfolio's cumulative return starts at zero on this formation date. Negative business days represent time periods before the portfolio's formation. The shaded region indicates the 80% confidence interval and the solid line indicates the average for each business day offset.

 $\label{performance} \mbox{ Past performance is not a guarantee of future performance.}$ 

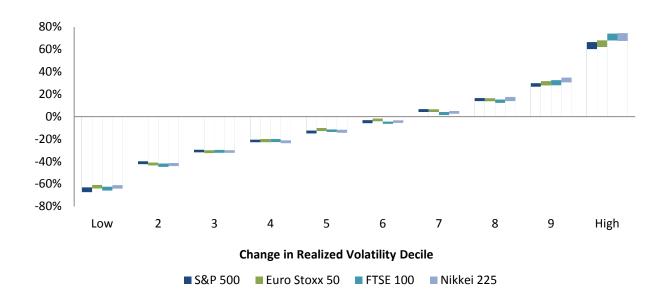
Exhibit 10: Hit Rate for Holding Periods Earlier and Later than 10<sup>th</sup> Decile of Changes in S&P 500 Volatility 1996 – 2016



Source: AQR, Bloomberg, OptionMetrics. Long option returns are calculated using long one-month S&P 500 option positions with strikes at -25, 50, and 25 delta, with an equal number of options held at each strike. Options were sized to have unit notional. Options were delta-hedged daily and held until expiration. Upon expiration, the same one-month option structure was initiated for the next expiration date. We then sort long option returns into deciles by changes in volatility. We define change in volatility as the log percentage change between historical and future S&P 500 volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility.

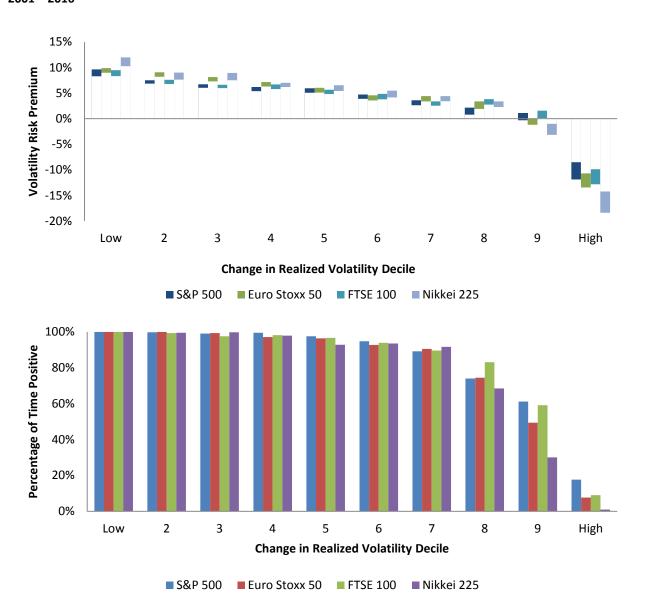
We select all observations in the 10<sup>th</sup> decile, and then calculate compounded returns. Day zero is at the very beginning of the 30 calendar-day future realized volatility interval. For each observation, we then calculate the long option return at the end of 21 business days, which roughly corresponds to 30 calendar days. The charts above plot the percentage of observations that are still profitable within N business days, relative to the gains accrued during the increase in volatility. For Positions Entered "Early", we evaluate N business days prior to the start of the volatility increase. For Positions Entered "Late", we evaluate N business days after the increase in volatility. This analysis ignores transactions costs. The top panel shows the results for the first 100 business days and the bottom panel shows the results for 1000 business days.

Exhibit 11: International Change in Volatility (Bucketed by Change in Volatility) 2001 - 2016



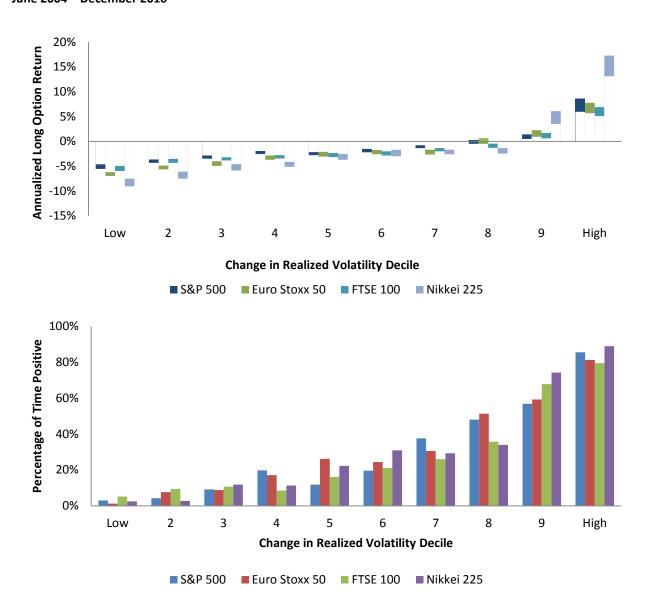
Source: AQR, Bloomberg. Average change in volatility is computed for each index after sorting by change in volatility into deciles. We define change in volatility as the log percentage change between historical and future realized volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. The shaded region for each index indicates the 95% confidence interval of the average in each decile based on sampling the change in realized volatility and repeating the entire analysis with 10,000 bootstraps.

Exhibit 12: International *Ex Post* Volatility Risk Premium (Bucketed by Change in Volatility) 2001 – 2016

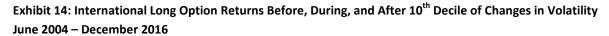


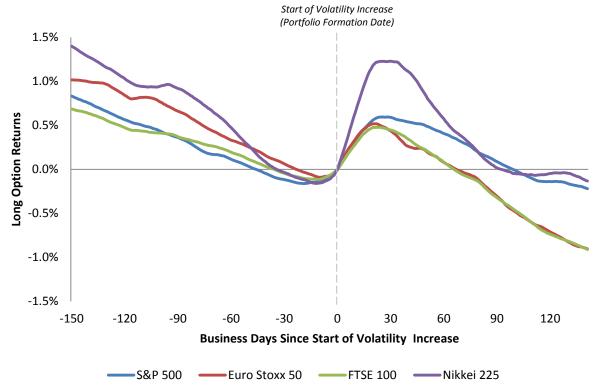
Source: AQR, Bloomberg. Average *ex post* volatility risk premium is computed for each index after sorting by change in volatility into deciles. *Ex post* volatility risk premium is defined as each respective index's volatility index (for example, VIX for the S&P 500 and V2X for the Euro Stoxx 50) minus that index's subsequent annualized volatility. We define change in volatility as the log percentage change between historical and future realized volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. The top panel reports the average *ex post* volatility risk premium in each decile. The shaded region for each index indicates the 95% confidence interval of the average in each decile based on jointly sampling implied volatility, subsequent realized volatility, and the change in volatility, and repeating the entire analysis with 10,000 bootstraps. The bottom panel reports the percentage of observations in each decile that were positive.

Exhibit 13: International Annualized Long Option Returns (Bucketed by Change in Volatility)
June 2004 – December 2016



Source: AQR, Bloomberg, OptionMetrics. Average, annualized one-month long option returns are computed for each index after sorting by change in volatility into deciles. Long option returns are calculated using long one-month option positions with strikes at -25, 50, and 25 delta, with an equal number of options held at each strike. Options were sized to have unit notional. Options were delta-hedged daily and held until expiration. Upon expiration, the same one-month option structure was initiated for the next expiration date. We define change in volatility as the log percentage change between historical and future realized volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility. The top panel reports the average annualized option return in each decile. The shaded region for each index indicates the 95% confidence interval of the average in each decile based on jointly sampling long option returns and the change in volatility, and repeating the entire analysis with 10,000 bootstraps. The bottom panel reports the percentage of observations in each decile that were positive.





Source: AQR, Bloomberg, OptionMetrics. Long option returns are calculated for each index using long one-month option positions with strikes at -25, 50, and 25 delta, with an equal number of options held at each strike. Options were sized to have unit notional. Options were delta-hedged daily and held until expiration. Upon expiration, the same one-month option structure was initiated for the next expiration date. We then sort long option returns into deciles by changes in volatility. We define change in volatility as the log percentage change between historical and future realized volatility. Historical volatility is measured as 90 calendar-day realized volatility and future volatility is measured as 30 calendar-day future realized volatility.

For each index, we select all observations in the 10<sup>th</sup> decile, and then calculate compounded returns. Day zero is at the very beginning of the 30 calendar-day future realized volatility interval, and represents the date of option portfolio formation. Therefore, the portfolio's cumulative return starts at zero on this formation date. Negative business days represent time periods before the portfolio's formation. The lines indicate the average for each business day offset.