Intraday S&P 500 Index Predictability and Options Trading Profitability

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Abstract

In this paper we study the intraday dynamics of E-mini S&P 500 index futures and the option trading strategies employing the weekly E-mini S&P 500 index futures options. We make a number of contributions to the literature in the area of intra-day equity index futures return predictability and trading profitability. As far as we know till at present, ours is one of the first studies on intraday implied moments of S&P 500 index futures return using intraday option prices. We use intra-day E-mini S&P500 European-style weekly options data from August 2009 to December 2012 and improve on existing techniques to extract the first four moments of the risk-neutral futures return distribution. Secondly we perform intraday out-of-sample forecasting or prediction, and document the intraday dynamics of the risk-neutral moments. We introduce a novel local autoregression method that allows variable windows in estimating the autoregressive parameters. This is particularly useful in situations when there may be intraday news that cause structural breaks in the otherwise smooth process. It also distinguishes itself from the conventional autoregressive model with predetermined sample lengths. Thirdly, we show profitability in the options trading strategies involving the various risk-neutral moment forecasts, particularly that involving skewness. The positive profitability after transaction costs in skewness trading indicates that the market is not as efficient as thought to be. We also use a novel technique in kurtosis trading that resulted in positive profits before cost, something new in the literature where negative profits were found in kurtosis trading. These results may explain the persistence of intraday trading activities in the market. Our intraday risk-neutral moments also suggest that forecast increases in volatility and skewness lead to an average increase in subsequent return over the next 10 minutes. On the other hand, intraday forecast increase in risk-neutral kurtosis leads to an average decrease in subsequent return. These intraday results appear to be contrary to existing studies using risk-neutral moments over daily intervals. This suggests that intraday price dynamics is different from daily price dynamics.

JEL Classification: G13 Options Trading; G17 Financial Forecasting

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1 Introduction

Prediction of returns moments for the purposes of financial trading, hedging, and pricing of derivatives, is prevalent in finance. The most common forecast is that of predictive mean usually obtained from a regression model. More general frameworks for setting up predictability ranges from the modeling of stochastic models to time series modeling such as GARCH. For forecasting realized volatility, Andersen, Bollerslev, Diebold, and Labys (2003) is a fundamental paper on the employment of high-frequency intraday data. Ghysels, Harvey, and Renault (1996) and Barndorff-Nielsen, Nicolato, and Shephard (2002) provide reviews on stochastic volatility models. Poon and Granger (2003) provide a comprehensive review of forecasting volatility in financial markets including detailed discussion of the pioneering dynamic time series models of Engle (1982) and Bollerslev (1986). Harvey and Siddique (1999, 2000) were some pioneering studies of skewness in financial markets. Brooks, Burke, Heravi, and Persand (2005) study autoregressive conditional kurtosis. There is a vast literature on empirical measures of asset returns moments, particularly on volatility and skewness, but a relative scarcity of studies on risk-neutral measures of similar moments.

The seminal paper by Breeden and Litzenberger (1978) connected option prices with no-arbitrage state prices, the equivalence of discounted risk-neutral densities. Later papers of the same genre include Rubinstein (1996) and Jackwerth and Rubinstein (1996). Practical approaches of extracting or implying the risk-neutral moments using option prices were developed in Jiang and Tian (2005) and in Bakshi, Kapadia, and Madan (2003). Due to the more general framework and higher moments inferable via the Bakshi, Kapadia, and Madan (2003) method (hereafter referred to as BKM), there has been a surge of interest in using BKM method to study risk-neutral moments. Unless otherwise stated, the research discussed below employ risk-neutral higher moments that are computed using the BKM method.

Conrad, Dittmar, and Ghysels (2013) use daily individual option prices to infer their underlying stock return risk-neutral moments over horizons of 1-month to 1-year. By forming portfolios ranked by the risk-neutral moments, they made several key conclusions in their study. To be clear, Conrad, Dittmar, and Ghysels (2013) state in their paper: "Specifically, we find that high (low) volatility firms are associated with lower (higher) returns over the next month.... We also find that skewness has a strong negative relation with subsequent returns; firms with less negative or positive skewness earn lower returns. That is, investors seem to prefer positive skewness.... We also find a positive relation between kurtosis and returns ..... However, when we control for interactions between volatility, skewness, and kurtosis, we find that the evidence for an independent relation between kurtosis and returns is relatively weak."

While comparisons of portfolio-based ex-post returns with the averaged portfolio ex-ante risk-neutral moments would indicate more of the effect of systematic (risk-neutral) moment effect in explaining ex-post returns, reflecting aggregate market preferences of the moments, the results are intuitive only for skewness. The indicated preference for systematic risk-neutral volatility, and thus a lower ex-post average portfolio return is not entirely convincing by quoting Ang et. al. (2006) result since the latter are with respect to empirical measures. Besides the issue of the pricing of idiosyncratic risk is still an open topic for debate. For daily, monthly, or even longer periods of related analyses, the well-known excessive variance swap premium indicating much higher risk-neutral volatility than realized empirical volatility may help to explain if indeed investors prefer volatility risk excluding the idiosyncratic component. As it turns out, to give a heads-up, one advantage of intraday moments is that the difference between risk-neutral or equivalent martingale measure moments and empirical moments would be too small to make substantive impact or qualifications on the connections between the moments and returns as the near-instantaneous Radon-Nikodỳm derivative of the two measures would be distributed very close to one with trivial variation.

Bali and Murray (2013) use monthly risk-neutral skewness of stocks to form forward 1-month portfolio of skewness assets, and find a strong negative relation between risk-neutral skewness and the skewness asset returns. As in Conrad, Dittmar, and Ghysels (2013), this is consistent with a positive skewness preference or negative skewness premium in asset pricing theory. Chang, Christoffersen, and Jacobs (2013) estimate market return moments from daily S&P 500 index option data. They find that risk-neutral market-wide skewness and kurtosis are important risk factors in explaining the cross section of stock returns. Other studies such as Bali, Hu, and Murray (2013), Cremers, Halling, and Weinbaum (2013), Dennis and Mayhew (2002), and Friesen, Zhang, and Zorn (2012) attempt to explain the existence and significance of risk premia related to systematic risk-neutral moments.

Neumann and Skiadopoulos (2013) use daily S&P 500 index options to extract risk-neutral moments for forecasting and for trading purposes over 1-day, 1-week, and 1-month horizons. They find that all the risk-neutral moments can generally be predicted better out-of-sample relative to the random walk benchmark. Using one-day ahead forecasts of the risk-neutral moments to pre-determine option trades, they find that except for one-day ahead skewness forecast, the other moment forecasts do not support profitable trading. After considering transaction cost in the form of the bid-ask spread, they report that skewness forecasts also did not deliver positive profitability. The results are similar for forecasts involving longer horizons of a week or longer. They thus conclude, "Hence, the hypothesis of the efficiency of the S&P 500 index options market cannot be rejected. This extends the results in Gonçalves and Guidolin (2006) who find that the economic significance of implied volatility trading strategies in the S&P 500 options market over a one-day horizon vanishes as soon as transaction costs are incorporated."

Yet on another hand, Amaya, Christoffersen, Jacobs, and Vasquez (2012) using intraday data to compute weekly realized variance, skewness, and kurtosis for equity returns, find a very strong negative relationship between realized skewness and the subsequent week's stock returns. They estimate a trading strategy using skewness information to generate an average weekly return of 24 basis points before transaction costs. This may be a positive return after transaction cost for major traders with very small commission costs. However, unlike the other studies, they did not find any robust and significant relationship between ex ante volatility or ex ante kurtosis and equity returns.

The differences in the above results on profitability after transaction costs could be due to the longer horizon of at least a day if not a week in time series forecasting before deploying the associated options trading strategies. Another reason could be due to the different methods of forecasting over at least a one-day interval. Yet amongst practitioners, it is widely known that trading strategies relying on such microscopic forecasts of moments or of other variables such as news would best be executed on an intraday basis and not after a whole day or longer. It is thus critical to complement existing studies with a focus on what goes on in intraday dynamics and options trading – what this paper is about.

There are other important pricing issues yet to be resolved or confirmed as existing studies also have some differences regarding the significance of the ex-ante risk-neutral volatility and kurtosis on future returns. However, a necessary condition for efficient asset pricing is that persistent speculative profits should not avail. Therefore it is important also to consider the issue of predictability and trading profitability involving options. The risk-neutral probability distribution embodies a large amount of information on market expectations as well as its risk preferences. In a recent 2013 NYU Stern–Federal Reserve Conference on Risk Neutral Probability Density, Figlewski (2013) suggested that searching for profitable trading strategies is a good question for research. This is indeed a clever insight as trading profitability not only has obvious attractions for the finance industry, but it also has deep implications on theory.

Bakshi, Kapadia, and Madan (2003) and Taylor, Yadav, and Zhang (2009) find that risk-neutral skewness implied from individual stock options are less negative than that implied from stock index option. Gârleanu, Pedersen, and Poteshman (2009) find that recent option-pricing puzzles may be explained by the fact that there is a significant difference between index option prices and the prices of single-stock options due to differences in end-user demands. Figlewski (2008) suggests that the estimation of individual stock risk-neutral density is especially hampered by two serious problems, as stock options trade a relatively small number of strikes, and also face significant microstructural noise. Due to the above, we consider the study of S&P 500 index options to be appropriate for the purpose of examining predictability and trading profitability. The index options are not only more liquid, but also possess more negative skewness moments whereby skewness trading strategy could be effected.

In this paper we study the intraday dynamics of index options and trading strategies using actual traded prices incorporating bid-ask spreads and imposing exogenous transaction costs in the form of additional commission costs which were not mentioned in many studies. This is refreshing as the published studies consider trading at daily, weekly or even monthly intervals. Such long intervals are not common amongst actual speculative traders in the market whose horizons are much shorter.

We make a number of contributions to the literature in the area of intra-day equity index returns predictability and options trading profitability. As far as we know till at present, ours is one of the first studies on intraday implied moments of S&P 500 index returns using intraday option prices. We use intra-day E-mini S&P500 European-style weekly options data from August 2009 to December 2012 and improve on existing techniques to extract the first four moments of the risk-neutral return distribution. Secondly we perform intraday out-of-sample forecasting or prediction, and document the intraday dynamics of the risk-neutral moments. We introduce a novel local autoregression method that allows variable window in fitting the autoregressive parameters. This is particularly useful in situations when there may be intraday news that cause structural breaks in the otherwise smooth process. It also distinguishes itself from the conventional autoregressive model with predetermined sample lengths. Thirdly, we show profitability in the trading strategies involving the various risk-neutral moment forecasts, particularly that involving skewness. The positive profitability after transaction costs in skewness trading indicates that the market is not as efficient as thought to be. This may explain the persistence of intraday trading activities in the market.

Our intraday risk-neutral moments suggest that forecast increases in volatility and skewness lead to an average increase in subsequent return over the next 10 minutes.

But intraday risk-neutral kurtosis suggests that forecast increase in kurtosis leads to an average decrease in subsequent return over the next 10 minutes. These results are contrary to existing studies using risk-neutral moments over daily intervals. This suggests that intraday price dynamics is different from daily price dynamics.

In section 2, we briefly discuss the method for extracting the risk-neutral moments. The data and implementation procedures are then explained. Section 3 provides a discussion of the forecasting models used in the forecast of intraday 10-minute ahead risk-neutral moments. The local autoregressive model is also explained. Section 4 contains the empirical results showing the forecasting performance of the various models. Section 5 provides the results based on different option trading strategies involving the risk-neutral moment forecasts. Section 6 reports results involving regressions of intraday 10-minute ahead returns on ex-ante risk-neutral moments. Section 7 contains the conclusions.

2 Implied Risk-Neutral Moments

In this section we briefly discuss the method of implying the first 4 risk-neutral moments by Bakshi, Kapadia, and Madan (2003). We shall refer to this as the BKM method. We then discuss the data and explain the implementation procedures for calculating the risk-neutral moments by the BKM method. We improve on existing techniques to extract the first four moments of the risk-neutral return distribution.

2.1 BKM Method

The BKM method can be modified for the case of options on futures. Since the S&P 500 futures price converges to the underlying spot at maturity, and present futures price is equal to the current spot index price plus cost of carry net of aggregate dividends from the stocks, the log futures price relative or continuously compounded rate of change in futures price or simply futures return is equal to the log stock price relative minus the net cost of carry, i.e. $\ln F_T/F_0 = \ln S_T/S_0 - (r-\delta)\tau$ where F_t and S_t denote the futures price and spot price respectively, r is the continuously compounded risk-free rate and δ is the continuous aggregate dividend yield from stocks of the index. If this net cost of carry $(r-\delta)\tau$ is negligibly small for very short horizon τ of up to 10 days, then the moments of the futures price distribution and stock index price distribution are basically the same for our computational purpose.

Define the τ-period random log return (or rate of change) of the underlying futures contract at time t as

$$R(t,\tau) \equiv \ln\left(\frac{F(t+\tau)}{F(t)}\right),$$

where $F(t)$ is the index futures price at t. To obtain the risk-neutral mean, variance, skewness and kurtosis of $R(t,\tau)$, it is sufficient to obtain the first 4 risk-neutral moments of $\mathbb{E}^Q[R(t,\tau)]$, $\mathbb{E}^Q[R(t,\tau)^2]$, $\mathbb{E}^Q[R(t,\tau)^3]$, $\mathbb{E}^Q[R(t,\tau)^4]$ under risk-neutral probability measure Q.

Each of the moments above can be viewed as a payoff at maturity $t+\tau$ and is a function of the underlying portfolio. Here we rely on a well-known result in Carr and Madan (2001) that any payoff as a function of underlying can be spanned and priced using a traded set of options across different strike prices. For example, a forward can be decomposed as a long call and a short put with same strike. A call spread can be decomposed as a long call with lower strike and a short call with higher strike. For a more complicated payoff, we need more options with different strikes to replicate the payoff. This can be done assuming that the payoff function is continuously differentiable in the underlying price.

Suppose there are securities with payoffs at maturity τ of quadratic, cubic, and quartic returns respectively, i.e. $R(t,\tau)^2$, $R(t,\tau)^3$, and $R(t,\tau)^4$. Then the no-arbitrage prices of these securities can be expressed as

$$\begin{aligned} V_t(\tau) &= \int_{F_t}^{\infty} \frac{2(1-\ln(K/F_t))}{K^2} C_t(\tau;K)dK \\ &\quad + \int_0^{F_t} \frac{2(1+\ln(K/F_t))}{K^2} P_t(\tau;K)dK \end{aligned}$$

$$\begin{aligned} W_t(\tau) &= \int_{F_t}^{\infty} \frac{6\ln(K/F_t) - 3(\ln(K/F_t))^2}{K^2} C_t(\tau;K)dK \\ &\quad + \int_0^{F_t} \frac{6\ln(K/F_t) + 3(\ln(K/F_t))^2}{K^2} P_t(\tau;K)dK \end{aligned}$$

$$\begin{aligned} X_t(\tau) &= \int_{F_t}^{\infty} \frac{12(\ln(K/F_t))^2 - 4(\ln(K/F_t))^3}{K^2} C_t(\tau;K)dK \\ &\quad + \int_0^{F_t} \frac{12(\ln(K/F_t))^2 + 4(\ln(K/F_t))^3}{K^2} P_t(\tau;K)dK \end{aligned}$$

where $V_t(\tau)$, $W_t(\tau)$, and $X_t(\tau)$ are the time t prices of τ-maturity quadratic, cubic, and quartic contracts respectively. $C_t(\tau;K)$ and $P_t(\tau;K)$ are the time t prices of European calls and puts written on the underlying stock index futures with strike price K and expiration τ periods from time t. These contract prices are also the respective risk-neutral moments over period interval τ. The equations involve weighted sums of out-of-the-money options across varying strike prices, and provide the procedure for finding risk-neutral moments of the log return.

Using the prices of these contracts, $V_t(\tau)$, $W_t(\tau)$, and $X_t(\tau)$, standard moment definitions suggest that the risk-neutral moments can be calculated as

$$\begin{aligned} VAR_t^Q(\tau) &= V_t(\tau) - \mu_t(\tau)^2 \\ SKEW_t^Q(\tau) &= \frac{W_t(\tau) - 3\mu_t(\tau)^2 V_t(\tau) + 2\mu_t(\tau)^3}{[V_t(\tau) - \mu_t(\tau)^2]^{3/2}} \\ KURT_t^Q(\tau) &= \frac{X_t(\tau) - 4\mu_t(\tau) W_t(\tau) + 6\mu_t(\tau)^2 V_t(\tau) - \mu_t(\tau)^4}{[V_t(\tau) - \mu_t(\tau)^2]^2}. \end{aligned}$$

where $\mu_t(\tau) \equiv \mathbb{E}^Q[R(t,\tau)] \approx e^{r\tau}\left[1 - \frac{1}{2}V_t(\tau) - \frac{1}{6}W_t(\tau) - \frac{1}{24}X_t(\tau)\right] - 1$.

2.2 Data and Implementations

We use intra-day E-mini S&P500 European-style options time-stamped traded price data on weekly series (EW1, EW2 and EW4) from August 2009 to December 2012 obtained from the Chicago Mercantile Exchange (CME). These options typically trade actively two weeks before their expiration dates. A recent 2013 World Federation of Exchanges report shows that the E-mini option is one of the most actively traded options in the world and has an annual trading volume of 57 million contracts compared to 7 million for the standard S&P 500 index options. We only consider options traded between 0830hrs and 1500hrs that correspond to regular trading hours, and ignore options traded on the expiration date itself. This is because options are generally illiquid during irregular trading hours, and abnormal option prices tend to occur more frequently on the day of expiration. We incorporate all options, including the in-the-money (ITM) options, to capture the information contained in these options. As the underlying asset is the S&P 500 index futures, we also obtain the E-mini S&P 500 Futures intra-day price data from CME. The options data are cleaned by removing a small percentage of prices that violated no-arbitrage bounds discussed in Merton (1973). The risk-free rate used in our computation is the yield on the secondary market 4-weeks Treasury bills reported in the Federal Reserve Report H.15. Since the risk-free rates in our sample period are close to zero, we can safely assume the risk-free rate is constant throughout any given trading day without incurring non-trivial approximation error.

In order to calculate the risk-neutral moments via the BKM method, we need to estimate equations $V(t,\tau)$, $W(t,\tau)$, and $X(t,\tau)$ that require OTM call and put option prices across a continuum of strike prices. However, market option prices are available for only a finite range of discrete strike prices and this will incur a bias in the calculation of the risk-neutral moments. See Jiang and Tian (2005) for a discussion of this issue. To tackle the problem caused by this limitation, we consider in-the-money (ITM) options as well to augment mere use of OTM options in standard practice. We believe that these options do carry information about their underlying asset returns and should not be unnecessarily ignored.

To incorporate these ITM options, once we apply the data filters, we use the put-call parity equation to transform ITM call option prices into OTM put option prices, and ITM put option prices into OTM call option prices. In this way we are able to increase the number of option price observations in our sample space by including these synthetic OTM option prices and thereby reducing the bias considerably. Since the put-call parity equation is a model-free no-arbitrage formula, by using this transformation, we are able to utilize the information content in these ITM options and at the same time use it to calculate the risk- neutral moments without imposing any model bias. We think this is a good and reasonable technique, and we have not seen this done in some recent related studies. We should mention that in our empirical results we did try the smaller data sample without the above augmentation, and the results are similar though weaker in the case with the smaller sample size.

For each 10-minute time interval within the regular trading hours, we use as many strike prices as are possible where options were traded on those strike prices. For traded calls (puts) having the same strike price within each 10-minute band, we select only the call (put) price that was transacted closest to the end of the 10-minute interval. These call and put prices are then used for computing the risk-neutral moments at every intraday 10-minute interval starting at 8:40 am, 8:50 am, $\ldots$, 2:40 pm, up to 2:50 pm. For moment extraction in the intraday intervals in our sample, we use at least two OTM calls and two OTM puts. After that, we employ the numerical method of piece-wise cubic Hermite interpolation to evaluate the integrals for finding the risk-neutral moments. Piece-wise cubic hermite interpolation has a local smoothing property, and therefore produces more stable estimates as compared to cubic splines. The extracted 1 day to 10 day constant maturity risk-neutral moments are used subsequently in our analysis. We do not use options with maturities longer than 10 days because trading for longer E-mini options are less liquid and the price data are inadequate for the purpose of constructing the risk-neutral moments.

The results of extracting these risk-neutral moments are reported in Table I below.

Table I about here

Table I reports the descriptive statistics of the extracted S&P 500 risk-neutral moments including the risk-neutral volatility, risk-neutral skewness, and risk-neutral kurtosis. E-mini S& P 500 options with different time-to-maturity of 1 day to 10 days are used to produce the risk-neutral moments corresponding to the different time-to-maturity of τ days. Altogether 19,859 moments are used. For the risk-neutral volatility, each τ-day volatility is scaled by $\sqrt{252/\tau}$ so that they are easily compared on an annual basis. The volatility are reported in %. The risk-neutral skewness and kurtosis, however, are reported without any scaling as these quantities do not have simple distribution-free aggregation properties. Except for volatility which is reported in %, the other moments are reported in decimals.

The averages of moments across all intervals on all dates where their maturities are the same τ-day are reflected as the mean for the τ-day. Table I shows that the mean volatility is quite stable across all maturities. The mean and median skewness are all negative for all maturities. This is consistent with similar results reported in the literature. Kurtosis appears to decrease as maturities increase. Most of the kurtosis measures are in excess of 3, indicating large deviations from the normal distribution. While the skewness measures are left-skewed, the volatility and kurtosis measures are right-skewed in their frequency distributions over time.

Our results indicate that the risk-neutral distributions on average have more negative skewness and much higher kurtosis than that of normal distributions. For each horizon τ, the risk-neutral moments computed for each 10-minute intervals are highly variable as can be seen by their standard deviations and maximum-minimum ranges. The annualized standard deviations of the various 10-minute interval risk-neutral moments show that the risk-neutral volatility (Panel A) is the most variable and the intraday time series process is not smooth. On the other hand, the risk-neutral skewness (Panel B) shows a smooth process as its risk-neutral moments evolve very slowly through any day. In our sample, 97.9% of all intraday risk-neutral skewness measures are negative. The risk-neutral kurtosis (Panel C) is also relatively smooth given its annualized standard deviations are also smaller than those of the risk-neutral volatility.

When we compare the descriptive statistics with the 1-month, 2-month, and 3-month risk-neutral moments reported in Neumann and Skiadopoulos (2013) Table 1, it is seen that annualized volatilities for different horizons remain rather constant for even up to 3 months. Similarly, risk-neutral skewness remains on average negative and does not appear to change much. However, it is clearly the case that our 1 to 10 day risk-neutral kurtosis measures are larger than the longer horizon 30-day to 90-day kurtosis measures.

3 Forecasting Models

In this section we describe regression models that we use to fit the time series of each of the risk-neutral moments. For each of the 3 different moments, the regressions are performed also on different time series belonging to the different constant maturities. For ease of exposition, we use the notation $RNM_t(\tau)$ to generically represent any of the 3 risk-neutral moments at time interval t with maturity τ. Also $RNV_t(\tau)$, $RNS_t(\tau)$, and $RNK_t(\tau)$ denote risk-neutral volatility, risk-neutral skewness, and risk-neutral kurtosis respectively. Each time series $RNM_t(\tau)$ is associated with a particular trading day. Thus for each trading day, each τ, and each moment, we run one regression under a particular model.

We consider 4 competing models in this paper: a benchmark Random Walk Model (RW), a Local Autoregressive Model of lag 1 order (LAR(1)), an Autoregressive Model of lag 1 order (AR(1)), and a Vector Autoregressive Model of lag 1 order (VAR(1)). Experimenting with higher lag-orders generally does not yield any clearer results or improvement in analyses. As the lag-order is understood, we shall not clutter the notation and leave this out. In what follows, each interval $[t, t+1)$ is 10-minute within a trading day.

For the RW Model:

$$RNM_{t+1}(\tau) = RNM_t(\tau) + \epsilon_{t+1} \ ,$$

where ϵ_{t+1} is an i.i.d. noise.

For the AR Model:

$$RNM_{t+1}(\tau) = b_0 + b_1 RNM_t(\tau) + \epsilon_{t+1} \ ,$$

where b_0 and b_1 are constants and ϵ_{t+1} is i.i.d.

For the LAR Model:

$$RNM_{t+1}(\tau) = b_{0,I_t} + b_{1,I_t} RNM_t(\tau) + \epsilon_{t+1} \ ,$$

where $t \in I_t$ and I_t denotes a subset of the sample set used in the AR model containing sample data from the latest time point before forecasting to a lagged time point such that the set I_t of consecutive data points are smooth and do not indicate any structural breaks. Thus LAR basically selects an optimal local window to perform the regression fitting where structural breaks do not occur. While it has the advantage of providing a better fit and possibly better forecast in time series that are not smooth and that may have breaks, the disadvantage is that if the time series is smooth, the shorter sampling window may yield forecasts and estimates with larger standard errors.

For the VAR Model:

$$\begin{pmatrix} RNV_{t+1}(\tau) \\ RNS_{t+1}(\tau) \\ RNK_{t+1}(\tau) \end{pmatrix} = B_0 + B_1 \begin{pmatrix} RNV_t(\tau) \\ RNS_t(\tau) \\ RNK_t(\tau) \end{pmatrix} + e_{t+1} \ ,$$

where B_0 is a 3×1 vector of constants, B_1 is a 3×3 matrix of constants, and e_{t+1} is a 3×1 vector of i.i.d. disturbance terms.

Least squares regression method is utilized, except that in the LAR case, the selection of window adds to the regression procedures. In Table II we report summary results of regressions involving the AR(1) Model.[1]

<u>Table II about here</u>

Table II reports the mean and variance of the estimated $\hat{b}_1$ in the AR(1) model for regressions of

$$RNM_{t+1}(\tau) = b_0 + b_1 RNM_t(\tau) + \epsilon_{t+1}.$$

Since the options expire with different time-to-maturity, risk-neutral moments are computed for each of the time-to-maturity where option prices are adequately available. The time-to-maturity, τ, takes values 1 day, 2 days, and so on till 10 days. "% sig" reports the percentage of estimated $\hat{b}_1$'s that are significantly different from zero at the 10% significance level. For all moments, clearly the autoregressive or slope coefficients are mostly estimated to be positive. Their standard deviations are small as seen in the small variance compared with the mean values of the risk-neutral moments. This is particularly significant for the case of volatility, where a strong positive lag effect is seen. While the lag effects do not appear to diminish with maturity for skewness and kurtosis, there is clearly a reduction in the magnitude of the lag effect for volatility as maturity lengthens.

The significance of the regression coefficient estimates indicates the plausibility of using lagged moments to forecast next 10-minute moments within the day. The way the

[1] Results of the other regressions are available from the authors. They are not shown here in order to economize on print-space.

forecasts of future moments is done is to split the data within a trading day into a first part for estimating the coefficients, and then using these coefficients to compute the forecasts in the next part. Unlike daily or weekly methods we do not use rolling windows over the 10-minute intervals. This is largely because we would employ bootstrapping techniques to estimate the sampling errors of trading profits based on the small sample of forecasts. To ensure appropriate bootstrap sampling, we condition on the set of estimated parameters in the model fitting, rather than allow the fitted parameters to change for every point of our profit forecast. In the latter, it would not be reliable to use a bootstrapping technique for measuring the sampling errors.

As we saw in Table I, the time series of risk-neutral volatility is less smooth with indication of plausible breaks in the stationarity at some points within the trading day. One method to address this issue with intraday study is to employ local autoregression (LAR) model, a technique that is gaining popularity in statistical analyses but not often recognized as of practical relevance in finance time series application. We use this method in our context for the forecasts. In the following subsections, we explain and discuss the implementation of the LAR Model.

3.1 Local autoregressive model

We provide in this subsection a concise discussion of the local autoregressive (AR) model used in our forecast. Some of the computational details are presented in the Appendix. For each one-dimensional risk neutral moment RNM_t representing any of the 3 risk-neutral moments (risk-neutral volatility RNV_t, risk-neutral skewness RNS_t, and risk-neutral kurtosis RNK_t), the idea is to consider all the past information available till the current time interval t but only employ a subsample over which the chance of having structural breaks is minimized. In other words, there do not exist significant parameter changes in the estimation window, and the dynamics of the subsample can be well represented by a stationary AR model with constant parameters. The stationarity with constant parameters however only holds over the subsample, and hence the constancy only applies in a local sense and named local homogeneity. Over the sub-sample with local homogeneity, we can safely forecast the RNMs based on the fitted local autoregressive (LAR) model.

The question is of course how to identify the subsample of local homogeneity. At any particular time interval of forecast origin, we seek the longest subsample, beyond which there is a high possibility of structural changes occurring. This longest subsample selection utilizes as much information as possible. It is able to reach the highest efficiency under local homogeneity. A sequential testing procedure is proposed to detect the optimal sub-sample among a number of candidates with increasing sample lengths. For different time interval of forecast origins, the subsample lengths vary depending on the impact of intraday news. The time dependent estimation windows distinguish the LAR model from the conventional AR model with predetermined sample length.

Among many possible modeling candidates, we focus on an AR(1) model, motivated by its simplicity, parsimony and further good out-of-sample forecasting ability. It is also motivated by the comparison with the other benchmark models of RW, AR(1) and VAR(1) where only the latest lagged moments are used to forecast next 10-minute moments within the day.

The LAR model is particularly useful when the series of RNMs for estimation and forecasting may not be stationary and contains structural breaks at some time points – which we loosely referred to earlier as being non-smooth series relative to a smooth or stationary series with low volatility. Due to the possible structural breaks during intraday trading as is common with breaking news or disturbances due to swift changes in high-frequency trading volumes and directions, and also the switches between dark pools and exchange trading orders, it is reasonable to make the above assumption for intraday 10-minute returns time series. The RNM process is then specified as the following.

$$RNM_{t+1} = b_{0,I_t} + b_{1,I_t} RNM_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim (0, \sigma^2_{I_t})$$

where I_t denotes a continuous time interval of the local sub-sample selected from the original full data set that is available for the estimation. The innovation ε_{t+1} has a mean of zero and a population variance of $\sigma^2_{I_t}$ that may also change depending on the specific time set I_t chosen. In this sense, as we vary the time-sets I_t, the parameters may change due to structural breaks occurring in particular time sets. The idea of LAR is to avoid estimation in time sets with such structural breaks so as to obtain estimates that behave with desirable properties according to classical results under stationarity.

We let $\theta_s = (b_{0,I_t}, b_{1,I_t}, \sigma_{I_t})^\top$. The model parameters are time-period dependent and can be obtained by the (quasi) maximum likelihood estimation, once the local sub-sample is specified. Suppose the sub-sample is known for time point t, corresponding to sampling interval $I_t = [t - m_t, t]$, over which the process can be safely described by an autoregressive AR(1) model with constant parameter θ_t. The local maximum likelihood estimator (MLE) $\tilde{\theta}_t = (\tilde{b}_{0,I_t}, \tilde{b}_{1,I_t}, \tilde{\sigma}_{I_t})^\top$ are:

$$\begin{aligned}
\tilde{\theta}_t &= \arg\max_{\theta\in\Theta} L(RNM; I_t, \theta) \\
&= \arg\max_{\theta\in\Theta} \left\{ -(m_t+1)\log\sigma_t - \frac{1}{2\sigma_t^2} \sum_{j=t-m_t}^{t} (RNM_{t+1} - b_{0,I_t} - b_{1,I_t} RNM_t)^2 \right\}
\end{aligned}$$

where Θ is the parameter space and $L(RNM; I_t, \theta)$ is the local log-likelihood function. To avoid confusion, we clarify that we use as follows generic notations of $\tilde{\theta}_t = (\tilde{b}_{0,I_t}, \tilde{b}_{1,I_t}, \tilde{\sigma}_{I_t})^\top$ for the MLE estimated with any given time-set I_t, and we keep $\hat{\theta}_t = (\hat{b}_{0,I_t}, \hat{b}_{1,I_t}, \hat{\sigma}_{I_t})^\top$ to denote the final estimates of the LAR model with the identified optimal subsample in our context of intraday risk-neutral moment estimation.

The task is to estimate the model parameters at time point of forecast origin t in order to use the estimated parameters for safely forecasting RNM changes thereafter. In practice, the longest sub-sample of local homogeneity is unknown and the number of possible candidates is large, e.g. as many sub-samples as there are past sample periods. In our context, for each start of trading day defined as at 8:40 am, we have 10-minute data points till 12:00 pm noon for the estimation. Suppose there is a total of K number of such data points. We then consider choice of an optimal I_t from the candidates

$$I_t^{(1)}, \cdots, I_t^{(K)} \text{ with } I_t^{(1)} \subset \cdots \subset I_t^{(K)}.$$

We start from the shortest reasonable sub-sample, $I_t^{(1)}$, where the AR(1) model with constant parameters should provide a reasonable fit, typically with m_s of window length of about at least 10 data points. The search for the optimal subsample, denoted as $\hat{I}_t$, begins by assuming the local homogeneity for $I_t^{(1)}$, the shortest window going backward in time from the point of forecasting. We use $\hat{\theta}_t^{(k)} = (\hat{b}_{0,I_t^{(k)}}, \hat{b}_{1,I_t^{(k)}}, \hat{\sigma}_{I_t^{(k)}})^\top$ to denote the adaptive estimator in the k-th subsample.

For the first subsample $I_t^{(1)}$, we have $\hat{\theta}_s^{(1)} = \tilde{\theta}_s^{(1)}$, where the ML estimator is accepted under the local homogeneity assumption. The selection procedure then iteratively extends the sub-sample with more periods and sequentially tests for possible structural breaks in the next longer sub-sample. The significance of structural breaks is measured by a sequential of log-likelihood ratio tests. The test statistic is defined in each following subsample $I_t^{(k)}$ as

$$T_t^{(k)} = \left| L(I_t^{(k)}, \tilde{\theta}_t^{(k)}) - L(I_t^{(k)}, \hat{\theta}_t^{(k-1)}) \right|^{1/2}, \quad k = 2, \cdots, K \tag{1}$$

where $L(I_t^{(k)}, \tilde{\theta}_t^{(k)}) = \max_{\theta \in \Theta} L(RNM; I_t^{(k)}, \theta)$ denotes the fitted log-likelihood under hypothetical homogeneity and $L(I_t^{(k)}, \hat{\theta}_t^{(k-1)}) = L(RNM; I_t^{(k)}, \hat{\theta}_t^{(k-1)})$ refers to an alternative fitted log-likelihood with the accepted estimate $\hat{\theta}_t^{(k-1)})$ from previous test procedure. If there do not exist significant structural breaks in the extended time-set, the MLE $\tilde{\theta}_t^{(k)}$ is not far from the estimate $\hat{\theta}_t^{(k-1)})$ obtained within an accepted sub-sample of local homogeneity. The two fitted log-likelihoods should be close and the test statistic is small. In this case, the extended sub-sample with more information $I_t^{(k)}$ is accepted and the corresponding MLE replaces $\hat{\theta}_t^{(k)} = \tilde{\theta}_t^{(k)}$ for an improved estimation accuracy. On the other hand, the test statistic becomes significant, indicating that parameter changes more than what would be expected possibly caused by an occurrence of unstable dynamics over the extended sub-sample. In this case, the selection procedure terminates and the latest accepted sub-sample $I_t^{(k-1)}$ is the final choice. We continue this way, until either structural breaks are detected or the longest sub-sample, $I_t^{(K)}$, is reached.

A set of critical values $\zeta_1, \cdots, \zeta_K$ is calibrated in Monte Carlo experiments and used to control the significance level. Although the critical values are important in the testing procedure, the technical details are standard. They are not shown here in order to economize on print-space. The details are contained in the Appendix to this study.

4 Forecasting Performance

After the regression models are estimated, the estimated coefficients are used to provide a fitted model for the purpose of predicting the next period or future risk-neutral moments. The forecast for the various models are shown as follows. Forecasts are made for risk-neutral moments pertaining to different horizons τ of one up to ten days as described earlier.

For the RW Model:

$$E_{t+1}\left(RNM_{t+2}(\tau)\right) = RNM_{t+1}(\tau)\,,$$

where the subscript to the expectation operator denotes a condition on the information of $RNM_{t+1}(\tau)$ at $t+1$.

For the LAR Model:

$$E_{t+1}\left(RNM_{t+2}(\tau)\right) = \hat{b}_{0,I_{t+1}} + \hat{b}_{1,I_{t+1}} RNM_{t+1}(\tau),$$

where $\hat{b}_{0,I_{t+1}}$ and $\hat{b}_{1,I_{t+1}}$ are the estimated parameters in I_{t+1}.

For the AR Model:

$$E_{t+1}\left(RNM_{t+2}(\tau)\right) = \widehat{b_0} + \widehat{b_1} RNM_{t+1}(\tau),$$

where $\widehat{b_0}$ and $\widehat{b_1}$ are the estimated parameters.

For the VAR Model:

$$E_{t+1}\begin{pmatrix} RNV_{t+2}(\tau) \\ RNS_{t+2}(\tau) \\ RNK_{t+2}(\tau) \end{pmatrix} = \hat{B}_0 + \hat{B}_1 \begin{pmatrix} RNV_{t+1}(\tau) \\ RNS_{t+1}(\tau) \\ RNK_{t+1}(\tau) \end{pmatrix},$$

where $\hat{B}_0$ and $\hat{B}_1$ are the estimated matrices.

4.1 Error Metrics

To measure the forecasting performances of these models, we employ 3 error metrics or loss functions.

The Root Mean Square Error (RMSE) is defined as

$$RMSE = \sqrt{\frac{1}{T-1}\sum_{t=0}^{T-2}\Big(RNM_{t+2}(\tau) - E_{t+1}\left(RNM_{t+2}(\tau)\right)\Big)^2} \quad ,$$

where T is the number of periods of forecasts, each period being a 10-minute interval.

The Mean Absolute Deviation (MAD) is defined as

$$MAD = \frac{1}{T-1}\sum_{t=0}^{T-2}\Big|RNM_{t+2}(\tau) - E_{t+1}\left(RNM_{t+2}(\tau)\right)\Big|.$$

The Mean Correct Prediction (MCP) percentage is defined as

$$MCP = \frac{1}{T-1}\sum_{t=0}^{T-2} J_{t+2} \ \times 100,$$

where indicator $J_{t+2} = 1$ if $\Big(RNM_{t+2}(\tau) - RNM_{t+1}(\tau)\Big)\Big(E_{t+1}\left(RNM_{t+2}(\tau)\right) - RNM_{t+1}(\tau)\Big) > 0$, $J_{t+2} = 0$ if $\Big(RNM_{t+2}(\tau) - RNM_{t+1}(\tau)\Big)\Big(E_{t+1}\left(RNM_{t+2}(\tau)\right) - RNM_{t+1}(\tau)\Big) < 0$, and $J_{t+2} = \frac{1}{2}$ if $\Big(RNM_{t+2}(\tau) - RNM_{t+1}(\tau)\Big)\Big(E_{t+1}\left(RNM_{t+2}(\tau)\right) - RNM_{t+1}(\tau)\Big) = 0$. The last case is to provide for the RW process.

<u>Table III about here</u>

Table III reports the out-of-sample performances of the Random Walk (RW), the Local Autoregressive (LAR), the Autoregressive (AR), and the Vector Autoregressive (VAR) models in the forecasting of next period risk-neutral moments. The respective autoregressive models are lag-one models. Results are reported for each of the risk-neutral moments of volatility, skewness, and kurtosis. For each risk-neutral moment category, moments of all maturities are pooled in the regression. There is a total of 8534 observations for each regression. For every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-minute intervals from 8:40 am to 12:00 pm (noon) are used to estimate the parameters of each model. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-minute interval from 12:10 pm to 14:50 pm. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) % are shown in the table. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct.

For the forecasting of volatility which is a smoother process than the other moment time series, it is seen that LAR performs the best in terms of the lowest RMSE, lowest MAD, and highest MCP. For skewness and kurtosis, LAR performs about as well as AR in terms of RMSE and MAD. Both are superior to RW and VAR. They certainly indicate consistently superior performance against random walk. Although LAR performs a bit weaker than AR in RNS and RNK perhaps due to the smoother processes of the latter two RNMs versus that of RNV – which we had indicated in an earlier section, LAR is better in terms of out-of-sample MCP. It is significant to note that for LAR, its prediction of change in the RNM is correct in about 70% of the times for all the moments.

It is not surprising that the results of forecasting based on LAR and AR are quite close since both are autoregressive methods albeit using slightly different sampling windows. Both methods outperform the benchmark RW and VAR. The poorer performance of VAR could be due to multi-correlation of the RNM's in a finite sample setting. In terms of within-sample fitting which we do not report here due to economy of space, clearly all models LAR, AR, and VAR outperform the random walk benchmark. In summary, there is statistical evidence of a lot of intraday information that can be utilized to successfully make better assessments of intraday forward momentum and price movements as we shall see.

Next we consider the forecasts of the individual risk-neutral moments, and also consider the forecasts in specific time series corresponding to different maturities. The results are reported in tables IV to VI.

<ins>Tables IV to VI about here</ins>

Table IV reports the statistical performances of the Random Walk (RW), the Local Autoregressive (LAR), the Autoregressive (AR), and the Vector Autoregressive (VAR) models in the forecasting of next period risk-neutral volatility. A separate regression is performed for moments data corresponding to each time-to-maturity τ. Clearly LAR outperforms all the other methods most of the times. The closest competitor is the AR model. Both beat the benchmark RW model consistently and by wide margins. It is observed that forecast performance deteriorates for moments with longer maturities. This also indicates that moments with longer maturities have longer periods in the future and may therefore be more susceptible to informational impact and thus resulting in larger changes and deviations from current forecasts.

In Table V, for skewness, AR outperforms with LAR coming in together with VAR as close seconds. All models beat the RW model in skewness forecasting. However, there is no apparent deterioration in forecasting for skewness with longer maturities. In Table VI, the LAR model performs marginally better than AR. LAR is clearly superior in terms of highest MCP and generally higher MAD, though AR generally has the lowest RMSE. Both LAR and AR models perform better than the others. VAR is a close third in this case. All these models beat the RW model. Thus it appears that forecasting modeling is always better than throwing a fair coin as in random walk. On the whole, LAR is seen as superior with AR being a close second.

It is observed that forecast performance deteriorates for volatility with longer maturities – in our context over horizons of up till 10 days. However, this deterioration does not occur for skewness and kurtosis. For RNK, in fact the forecast performance appears to improve by a bit. The latter phenomena could be due to reversions over time and higher stability of such moments over a slightly longer horizon, thus enabling slightly better forecasts.

We also compare our out-of-sample intraday risk-neutral moments forecasts with the 30-day out-of-sample forecasts reported in Neumann and Skiadopoulos (2013)'s Table 8. As the statistics are measured on different horizons, the error metrics are not readily comparable. However the MCP are measured on exactly the same basis, so are perfectly comparable. In our LAR/AR method which is generally similar in genre to the ARIMA they used in their study, our 10-day forecasts on RNV, RNS, and RNK have MCP's of 0.68 (68%), 0.72, and 0.70 respectively compared to lower 0.53, 0.60, and 0.63 in their study. One implication is that the longer the horizon, the less accurate would be the forecast. This is evident in the statistics above. For our VAR method, we compare with the closest method of VECM(1) in their study. Our 10-day forecasts on RNV, RNS, and RNK have MCP's of 0.63 (63%), 0.73, and 0.69 respectively compared to lower 0.55, 0.64, and 0.67 in their study. Even across studies, there is corroborative evidence that kurtosis and skewness provide for more accurate forecasts in longer horizons than is volatility.

5 Options Trading Strategies

Using the forecasts generated by the 3 methods of LAR, AR, and VAR, we attempt to construct a trading strategy to benefit from accurate forecast of future moment changes. We also add the benchmark case of perfect knowledge forecast (PK) whereby prediction of moment increase or decrease is 100% correct. We construct 3 different trading strategies corresponding to the forecasts of the 3 different moments. Although the direction and not the magnitude of the risk-neutral moment prediction is key to the options portfolio strategy, we also incorporate some information of the magnitude of the forecast changes by setting thresholds such that the strategies are only executed if the predicted moment changes are larger than the thresholds. These thresholds serve as cautionary checks to prevent unnecessary trading due to sampling forecast errors. In practice, using thresholds based on expected forecast changes is much more parsimonious and clearer to the trader than trying to use statistical significance levels on the parameters of each regressions or using other forecast metrics as decision thresholds to trade or not.

For forecast on risk-neutral volatility, the trading strategy involves creating a volatility portfolio each 10-minute interval as follows: long an OTM call and short delta amount of underlying asset, together with long an OTM put and short a delta amount of underlying asset. The respective deltas are based on the strike prices of the call and the put. Since the delta of a put is negative, shorting delta related to a put amounts to buying the underlying asset. Each trading interval is 10-minutes intraday. At the end of the interval the positions are liquidated at market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral volatility is higher than the current risk-neutral volatility by at least the threshold percentage, the above portfolio of long call and long put is executed. The execution and liquidation next interval constitute one round-trip trade. There can be more than one round-trip trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval.

The options and underlying futures portfolio has zero cost as the net balance of the cost in the call, put, and underlying asset is invested in a risk-free bond. In the case where the net balance is negative, risk-free loan is taken. Since the interval is intraday, the effective risk-free rate is about zero. In the opposite case when predicted risk-neutral volatility is lower than the current risk-neutral volatility by at least the threshold percentage, the portfolio is short. In this case, a call and a put are sold together with associated positions in the underlying asset and risk-free bond. The overall cost of the portfolio, π^{Vola} can be expressed as:

$$\pi_t^{Vola} = C_{t,OTM} - \Delta_{C_{t,OTM}} F_t + P_{t,OTM} - \Delta_{P_{t,OTM}} F_t - B_t.$$

B_t is chosen such that $\pi_t^{Vola} = 0$. In the above, since the underlying is futures, the cost reflected as price F_t is zero unless it is fully fiduciary. However, the profit measure can be evaluated as a change in the above cost.

Note that since the underlying is a futures contract with no initial outlay, the delta of the futures call $\Delta_{C_{t,OTM}}$ is in effect equal to e^{-rT} times the delta of an underlying stock index call, where r is the continuously compounded risk-free rate. The thresholds are chosen to be 5%, 7.5%, and 10%. Too high a threshold means that we will have fewer trades. Too low a threshold means that some weak signals or forecasts may be included. We impute a trading cost of 22.5 cents per option contract or 45 cents per round trade of a single contract position. This is an average figure obtained from brokers dealing with large orders or familiar clients.[2] For the larger part of the trading cost involving bid-ask spread, our trading strategy is based on market orders, so we would buy at ask and sell at bid, assuming we hit the transacted prices in the first instance after each 10-minute interval past 12:00 pm noon. Trading takes place from 12:10 pm till 15:00 pm each trading day based on the forecasts.

Table VII reports the average trading profit in $ per round-trip trade according to the different forecasting methods on RNV and the various threshold signals. As the theoretical probability distribution of the average profit is not known, and the sample size is small, we employ the bootstrap method suggested by Efron (1979) to compute the standard errors for the statistics. The standard errors are used to compute $t-$ values reported within the brackets in all the tables VII, VIII, and IX. The bootstrap is carried out over 2000 iterations of the sample size indicated by the total number of trades.

<u>Table VII about here</u>

Table VII shows that for the case of 7.5% threshold, using LAR for RNV forecast yields an average profit per round-trip trade of $1.33 on the E-mini options and E-mini S&P 500

[2] We find that most studies do not indicate the size of their trading commission costs, being part of transactions costs. Therefore we venture that what we have is an estimate. For example, the internet trading firm "TradeStation" offers flat-fee trading of $4.99 + $0.20 per option trade for trading volumes of 200 or more contracts per month. This works out to about $ 0.20 + $4.99/200 = $ 0.225 per option trade. For a large institutional broker with a seat on the relevant exchange, it is plausible for this level of low commission cost. The futures position costs would even be smaller and are close to zero for very large contract sizes.

index futures portfolio. The other cases are not as distinct. Though statistically the high standard deviation makes for a t-value of only 1.56, economically it implies steady profit if the trading is carried out over a long period of time with the average profit converging to a long-term positive mean.

For forecast on skewness, the trading strategy involves creating a skewness portfolio each 10-minute interval as follows: long an OTM call and short a number of OTM puts equal to the ratio of the call vega to put vega. Also short a number of underlying assets equal to the call delta less the same vega ratio times put delta. The respective vegas and deltas are based on the strike prices and other features of the call and the put. Each trading interval is 10-minutes intraday. At the end of the interval the positions are liquidated at market prices. If the predicted next interval risk-neutral skewness is higher than the current risk-neutral skewness by at least the threshold percentage, the above portfolio of long call and short puts is executed. The execution and liquidation next interval constitute one round-trip trade. The portfolio has zero cost as the net balance of the cost in the call, puts, and underlying asset is invested in a risk-free bond. In the case where the net balance is negative, risk-free loan is taken. Since the interval is intraday, the effective risk-free rate is about zero. In the opposite case when predicted risk-neutral skewness is lower than the current risk-neutral skewness by at least the threshold percentage, the portfolio is short. In this case, there is a short call and a long put together with associated positions in the underlying asset and risk-free bond. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Skew} = C_{t,OTM} - (\frac{\upsilon_{C_{t,OTM}}}{\upsilon_{P_{t,OTM}}})P_{t,OTM} - (\Delta_{C_{t,OTM}} - (\frac{\upsilon_{C_{t,OTM}}}{\upsilon_{P_{t,OTM}}})\Delta_{P_{t,OTM}})F_t - B_t.$$

B_t is chosen such that $\pi_t^{Skew} = 0$.

<u>Table VIII about here</u>

In the skewness case as shown in Table VIII, the thresholds are set higher as the skewness variance is high. For the cases of 10% and 22.5% thresholds, using any of the forecasting methods produces significantly positive profits. The VAR method yields the highest profit for the lower threshold of 10%, while LAR produces the highest profit at a higher threshold of 22.5%. (We lower this from the round number of 25% as many trades were missed in the interval of 22.5% to 25%.) Using small sample simulations to obtain the standard error estimates of these ex-post average per round-trip trade \$ profits, they are seen to be highly significantly positive at the 1% level.

For forecast of kurtosis, the trading strategy involves creating a kurtosis portfolio each 10-minute interval as follows: long an ATM call and an ATM OTM put and simultaneously short X number of OTM calls and OTM puts, where $X = (C_{t,OTM} + P_{t,OTM})/(C_{t,ATM} + P_{t,ATM})$. Each trading interval is 10-minutes intraday. At the end of the interval the positions are liquidated at market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral kurtosis is higher than the current risk-neutral kurtosis by at least the threshold percentage, the above portfolio of long ATM calls and puts, and short OTM call and put is executed. The execution and liquidation next interval constitute one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval. The portfolio has zero cost. In the opposite case when predicted risk-neutral kurtosis is lower than the current risk-neutral kurtosis by at least the threshold percentage, the portfolio is short. In this case, there is a short ATM calls and puts, and long OTM call and put. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Kurt} = X(C_{t,ATM} + P_{t,ATM}) - (C_{t,OTM} + P_{t,OTM}).$$

<u>Table IX about here</u>

The average per round-trip trade profits after transaction costs are reported in Table IX. For a predicted increase in kurtosis, Aït-Sahalia, Wang, and Yared (2001)'s study on S&P 500 options indicated that the risk-neutral probabilities would increase in prices closer to the ATM strike while they would decrease in prices far OTM. Therefore the trading strategy as indicated above would be to buy ATM or near ATM options and sell OTM options. Table IX shows that there are far fewer trades possible under this kurtosis strategy. Although not statistically significant at 5% significance level, with some p-values close to 0.05, the strategy appears to yield high per trade profits that are positive for all forecasting methods. For the kurtosis case, the AR forecasting model outperforms the other models.

We also attempt an approximate comparison of our trading results with those reported in the Neumann and Skiadopoulos (2013) study. In our study there are 7137 trades for the skewness trading strategy and 682 trades for the kurtosis trading strategy. We compute the Sharpe ratio S as follows. Let P_t be the value of our trading portfolio at time t. In the next interval, it is liquidated at value P_{t+1} with a profit of $\pi_{t+1} = P_{t+1} - P_t$. $S = \hat{E}(\pi_{t+1})/\sqrt{\hat{\mathrm{var}}(\pi_{t+1})}$. This Sharpe ratio is computed using the sample mean and variance of all the 10-minute interval profits from the trading. To compare with Neumann and Skiadopoulos (2013) that did not use any threshold signal, we also re-compute our trading results with zero threshold. We also perform the same bootstrap method and report the 95% confidence in the pair of numbers in the brackets. One important point of difference is that while Neumann and Skiadopoulos' results are based on daily profits, ours are based on profits from intraday 10-minute intervals. Therefore, to fix things on equal footings, we also annualize the Sharpe ratio by a multiplicative factor of $\sqrt{252 \times 18}$ since there are 18 trading intervals each trading day during 12:10 pm to 14:50 pm in our empirical study. The comparison results are reported in Table X.

<u>Table X about here</u>

In the case of skewness, as in the the Neumann and Skiadopoulos study, all the Sharpe ratios are significantly greater than zero at 2.5% significance level. Our study yields higher Sharpe ratios for all forecasting methods, though we are more conservative in not using other external variables to increase the fit. For the case of kurtosis, our forcasting methods on intraday trading yields positive profits and positive Sharpe ratios whereas in Neumann and Skiadopoulos (2013), there are cases of losses.

We do not compare trading outcomes in risk-neutral volatility as our study shows positive average though not statistically significant intraday profits whereas in other studies such as Neumann and Skiadopoulos, predictability of the risk-neutral moments do not lead to profitable outcomes in volatility trading. They state in their conclusions that, "... we provide evidence for the efficiency of the S&P 500 option market given that no economically significant profits can be attained from the higher RNM strategies once transaction costs are included." The sharp difference of extant studies with ours is that we are testing for intraday options trading whereas almost all significant published studies so far had concentrated on trading over at least one day interval if not weeks or months. These kinds of trading intervals are not typical of a trader with a smart strategy and higher trading frequencies in a highly liquid market such as S&P 500 instruments. Also, there is a marked difference in our options trading strategies compared to those for example in the Neumann et. al. study. Again while they used forecast of 60-day and 90-day implied moments compared to our intraday moments, their study also employed delta and vega hedged portfolios to attempt to extract profits from kurtosis changes. For trading intervals longer than a day, such hedging ratios are not accurate and contain high sampling risks, so their use are not particularly effective in capturing advantages from a change in kurtosis. On the other hand, we rely on a keen observation reported in Aït-Sahalia, Wang, and Yared (2001) and develop a much cleaner and effective kurtosis strategy as is evident in our comparative trading results.

6 Ex-Ante Moments and Subsequent Returns

In this section we provide comparisons of intraday moment-return relationships with those reported in various other earlier published studies such as Conrad, Dittmar, and Ghysels (2013) that uses daily moments or moments with longer horizons. While the debate between ex-ante total risk and ex-post return, usually interpreted as expected return in the context of equilibrium asset pricing, is still open with specific reference to the role of idiosyncratic risk, there is conundrum in the relationship between ex-ante kurtosis and ex-post returns. Part of this could be due to the difficulty of isolating the effects of other moments that interact with kurtosis. The case for the negative relationship between ex-post return and ex-ante skewness is more strongly documented as in Conrad, Dittmar, and Ghysels (2013) and Bali and Murray (2013) for daily and monthly intervals. However, to-date there is scarcely any documented evidence of intraday relationship between ex-post returns and ex-ante skewness. Our study attempts to provide some insights into such intraday moment issues.

We run an in-sample regression as follows.

$$\ln(\frac{F_{t+1}}{F_t}) * 100 = \alpha + \beta_0 \ln(\frac{F_t}{F_{t-1}}) * 100 + \beta_1 RNM_t + \epsilon_{t+1} \ ,$$

where $t = 1, ..., T$ and T is the number of 10-minutes interval in any given day. T is about 36 to 38 depending on the trading day. For each trading day the regression is performed and the coefficient estimates are collected. Over the total sample of 635 trading days the estimated coefficients are averaged to obtain their means. Their variances and also their t-statistics reflecting if their average deviates significantly from zero, are also evaluated. These results are shown in Table XI.

A separate in-sample intraday regression involving all three RNM's as explanatory variables is also run:

$$\ln(\frac{F_{t+1}}{F_t}) * 100 = \alpha + \beta_0 \ln(\frac{F_t}{F_{t-1}}) * 100 + \beta_1 RNV_t + \beta_2 RNS_t + \beta_3 RNK_t + \epsilon_{t+1} \ ,$$

where $t = 1, ..., T$ and T is the number of 10-minutes interval in any given day. T is about 36 to 38 depending on the trading day. Similarly, the average estimated coefficients and their distributional statistics are shown in the same Table XI in a separate column.

<u>Table XI about here</u>

The results in Table XI indicate that forecast increases in volatility and skewness lead to an average increase in subsequent return over the next 10 minutes. This is evidenced by the highly significant $\hat{\beta}_1$ statistics not only as a collection but also individually in trade day regressions. On the other hand, intraday forecast increase in risk-neutral kurtosis leads to an average decrease in subsequent return. These intraday results appear to be contrary to existing studies using risk-neutral moments over daily or longer intervals. Although we are dealing with the S&P 500 futures prices and not individual components of the S&P 500 stocks, the results are overwhelmingly different particularly for the case of volatility and also for skewness. This suggests that intraday price dynamics is different from daily price dynamics, and there should be more research done in linking intraday dynamics to longer horizon dynamics. Given that we utilize market prices incorporating bid-ask spread in this intraday highly liquid market of S&P 500 futures and the liquid weekly European-style options, the micro-structural agency cost impacts may not necessarily be the explanation.

In the regression equations we incorporated a lagged 10-minute return variable so as to be consistent with the idea that the RNM's variables themselves are predictable. Table XI shows that estimates β_0 are on average significantly negative, indicating an important possibility of reversions in returns that may significant over a day. While skewness may not change as much during the day in terms of its direction, there could be possibly an explanation of the difference in ex-ante skewness and ex-post return results due to the different rates of reversions in the two entities. However, we leave such a wider topic for future research.

7 Conclusions

Two recent papers Conrad, Dittmar, and Ghysels (2013) and Neumann and Skiadopoulos (2013) appear to suggest that though risk-neutral moments are cross-correlated and also correlated over time, the market has absorbed this knowledge and the resulting option and stock prices are efficient. As a result, there appears to have no profitable strategy. However, these studies employ daily or end of the day option prices for extracting the market information. As profitable information is absorbed very quickly, what appears to be an efficient market may not well be during intraday trades.

In this paper we study the intraday dynamics of E-mini S&P 500 index weekly options and their option and futures trading strategies using actual traded prices and imposing transaction costs. Using intraday options data and the implied intraday risk-neutral moments, we are able to show a substantial degree of ability to forecast accurately movements in the moments over 10-minute intervals. In particular, the local autoregressive model we utilized appears to be most effective in such a prediction as it uses a variable window to estimate the parameters, avoiding fixed window lengths and avoiding periods when there is a structural break in the data. We contribute to the literature by showing that intraday moment estimation and prediction could lead to profitable option trading strategies. Our trading results included consideration of costs, and provides more evidence indicating a less than efficient intraday options and futures market. This result is new to the literature as all previous results, as far as we know, use daily and weekly or monthly trading. In the kurtosis trading which proves to be a challenge in past studies in extracting positive profits, we employ an interesting observation from Aït-Sahalia, Wang, and Yared (2001)'s study on S&P 500 options indicating that risk-neutral probabilities would increase in prices closer to the ATM strike while they would decrease in prices for OTM options, to construct a parsimonious portfolio of ATM and OTM options for trading.

Specifically, in all the cases, the local autoregression model of forecasting 10-minutes ahead risk-neutral moments and adopting the appropriate options and underlying futures portfolio to capture the moment implications for their very short-run price changes, consistently return average positive economic profits. In particular, the profits for trading based on skewness is highly significant even after accounting for transaction costs. This is a clear case of intraday market inefficiency whereby superior technology such as moment prediction and an appropriate trading strategy can lead to consistent profits, at least during the sample period from August 2009 till December 2012.

Our intraday risk-neutral moments also suggest that forecast increases in volatility and skewness lead to an average increase in subsequent return over the next 10 minutes, whereas intraday forecast increase in risk-neutral kurtosis leads to an average decrease in subsequent return. These intraday results are contrary to those in some studies employing inter-day or time series with longer horizons. Thus intraday price dynamics is different from daily price dynamics as we document in this study. Using intraday moments and study of intraday price dynamics is promising as an area for future research in options and futures pricing.

Table I

Descriptive Statistics of S&P 500 Risk-Neutral Moments

This table reports the descriptive statistics of the extracted S&P 500 risk-neutral moments including the risk-neutral volatility, risk-neutral skewness, and risk-neutral kurtosis in different panels A, B, and C respectively. E-mini S& P 500 options with different time-to-maturities of 1 day to 10 days are used to produce the risk-neutral moments corresponding to the different time-to-maturities of τ days. The options data are collected over the sample period of August 24, 2009 to December 31, 2012. Altogether 19,859 such observations are used. For the risk-neutral volatility, each τ-day volatility is scaled by $\sqrt{252/\tau}$ so that they are easily compared on an annual basis. The risk-neutral volatility numbers are computed in %. The risk-neutral skewness and kurtosis, however, are reported without any scaling as these quantities do not have simple distribution-free aggregation properties.

Panel A: Risk-Neutral Volatility (%)										
Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	2628	2769	2594	2277	1920	1987	1784	1471	1205	1224
Mean	21.05	19.85	18.97	19.07	20.49	20.31	20.35	20.89	20.43	21.41
Median	18.47	17.30	17.02	17.10	17.64	17.60	18.29	18.14	17.60	17.73
Max	67.44	85.90	54.26	84.68	81.59	84.20	72.56	79.24	83.25	77.49
Min	9.70	9.35	8.84	8.83	8.61	8.92	9.42	9.61	9.65	7.93
Std Dev.(%)	8.21	7.68	6.90	7.32	7.91	8.18	7.34	8.25	8.43	8.97
Skewness	1.85	2.144	1.62	2.329	1.98	1.83	1.41	1.38	1.61	1.32
Kurtosis	7.29	10.19	5.78	12.27	10.12	8.51	6.31	6.86	6.10	4.62

Panel B: Risk-Neutral Skewness										
Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	2628	2769	2594	2277	1920	1987	1784	1471	1205	1224
Mean	-1.01	-1.03	-1.07	-1.17	-1.26	-1.12	-1.08	-1.05	-1.03	-1.02
Median	-0.95	-0.97	-1.00	-1.10	-1.21	-1.08	-1.05	-1.04	-0.97	-1.02
Max	2.34	2.07	1.03	1.80	2.26	0.87	0.88	1.02	0.88	1.18
Min	-3.65	-3.81	-3.53	-3.17	-3.45	-3.14	-3.00	-3.64	-3.68	-3.27
Std Dev.(%)	0.62	0.58	0.61	0.61	0.62	0.52	0.48	0.51	0.52	0.56
Skewness	-0.41	-0.59	-0.60	-0.46	-0.31	-0.23	-0.22	-0.04	-0.61	0.15
Kurtosis	3.89	4.79	4.00	3.92	4.36	4.01	4.21	4.16	5.19	4.61

Panel C: Risk-Neutral Kurtosis										
Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs	2628	2769	2594	2277	1920	1987	1784	1471	1205	1224
Mean	6.28	5.72	5.76	6.28	6.53	5.16	4.68	4.49	4.37	4.14
Median	4.90	4.56	4.48	4.98	5.38	4.34	4.06	3.81	3.58	3.59
Max	30.11	29.60	28.97	30.11	29.70	24.70	19.47	21.25	28.91	26.40
Min	0.88	0.01	0.03	0.02	0.02	0.02	0.03	0.03	0.03	0.03
Std Dev.(%)	4.41	3.94	4.00	4.20	4.30	3.09	2.60	2.62	2.97	2.64
Skewness	2.11	2.32	2.09	1.77	1.89	1.90	1.87	1.83	2.89	2.50
Kurtosis	8.73	10.32	8.54	6.69	7.41	8.30	8.09	7.95	16.21	14.17

Table II
Statistics of AR(1) Slope Estimates

This table reports the mean and variance of the estimated $\hat{b}_1$ in the AR(1) model:

$$RNM_{t+1}(\tau) = b_0 + b_1 RNM_t(\tau) + \epsilon_{t+1} ,$$

where $RNM_t(\tau)$ denotes the respective risk-neutral moments at time interval t with maturity τ. ϵ_{t+1} is assumed to be i.i.d. Since the options expire with different time-to-maturity, risk-neutral moments are also computed for each of the time-to-maturity where option prices are adequately available. The time-to-maturities are 1 day, 2 days, till 10 days. For each time-to-maturity, the regression is performed on the corresponding risk-neutral moment. "% sig" reports the percentage of estimated b_1's that are significantly different from zero at the 10% significance level. "RNV", "RNS", and "RNK" denote risk-neutral volatility, skewness, and kurtosis respectively.

	1 Day			2 Days			3 Days			4 Days			5 Days		
	Mean	Var	%sig	Mean	Var	%sig	Mean	Var	%sig	Mean	Var	%sig	Mean	Var	%sig
RNV	0.60	0.08	81.60	0.52	0.09	73.71	0.45	0.09	66.14	0.42	0.13	61.13	0.35	0.11	52.83
RNS	0.13	0.06	20.49	0.11	0.06	17.22	0.08	0.07	18.90	0.16	0.07	23.97	0.12	0.08	26.90
RNK	0.16	0.08	24.05	0.16	0.08	26.12	0.14	0.09	26.51	0.15	0.08	27.69	0.16	0.10	31.57

	6 Days			7 Days			8 Days			9 Days			10 Days		
	Mean	Var	%sig	Mean	Var	%sig	Mean	Var	%sig	Mean	Var	%sig	Mean	Var	%sig
RNV	0.32	0.10	45.38	0.27	0.10	38.21	0.27	0.08	36.57	0.24	0.09	32.53	0.18	0.08	25.57
RNS	0.06	0.05	10.07	0.07	0.07	15.94	0.09	0.07	18.12	0.15	0.06	17.27	0.12	0.07	23.70
RNK	0.11	0.07	17.77	0.12	0.07	18.18	0.19	0.07	25.44	0.23	0.06	27.91	0.15	0.08	27.23

Table III

Out-of-Sample Error Metrics for Forecasting Models

This table reports the statistical performances of the Random Walk (RW), the Local Autoregressive (LAR), the Autoregressive (AR), and the Vector Autoregressive (VAR) models in the forecasting of next period risk-neutral moment. The respective autoregressive models are lag-one models. Results are reported for each of the risk-neutral moments of volatility, skewness, and kurtosis. For each risk-neutral moment category, moments of all maturities are pooled in the regression. There is a total of 8534 observations for each regression. Every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-minute intervals from 8:40 am to 12:00 pm (noon) are used to estimate the parameters of each model. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-minute interval from 12:10 pm to 14:50 pm. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) % are shown in the table. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct.

	Risk-Neutral volatility			Risk-Neutral Skewness			Risk-Neutral Kurtosis		
	RMSE	MAD	MCP	RMSE	MAD	MCP	RMSE	MAD	MCP
RW	0.6622	0.1743	50%	0.6467	0.4633	50%	4.001	2.490	50%
LAR	0.5256	0.1589	68.67%	0.5353	0.3742	71.01%	3.391	2.110	70.23%
AR	0.5800	0.1745	58.15%	0.5106	0.3740	70.75%	3.252	2.174	68.98%
VAR	0.6543	0.2101	58.35%	0.5410	0.3866	69.87%	3.465	2.255	68.40%

Table IV

Out-of-Sample Error Metrics for Forecasting Risk-Neutral Volatility

This table reports the statistical performances of the Random Walk (RW), the Local Autoregressive (LAR), the Autoregressive (AR), and the Vector Autoregressive (VAR) models in the forecasting of next period risk-neutral volatility. The respective autoregressive models are lag-one models. A separate regression is performed for moments data corresponding to each time-to-maturity. Every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-minute intervals from 8:40 am to 12:00 pm (noon) are used to estimate the parameters in each regression. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-minute interval from 12:10 pm to 14:50 pm. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) % are shown in panels A, B, and C respectively. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct. Note: The risk-neutral volatility in our study is expressed in %. If expressed as decimals, the magnitudes of the various error metrics would be reduced by a multiple of 10^{-2}.

Panel A: Root-Mean-Square Error (RMSE)

Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	0.0786	0.2964	0.2583	0.8272	0.9959	0.4987	0.7791	0.9035	0.8995	0.9980
LAR	0.0646	0.2060	0.1762	0.6522	0.7851	0.4984	0.5405	0.5203	0.9330	0.7984
AR	0.0779	0.2861	0.1930	0.6794	0.9109	0.4544	0.6798	0.7705	0.9199	0.7470
VAR	0.1382	0.2941	0.2134	0.6596	1.222	0.4657	0.6495	0.9624	0.9251	0.8086

Panel B: Mean Absolute Deviation (MAD)

Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	0.0560	0.0885	0.0936	0.1945	0.2442	0.1687	0.2395	0.2740	0.2730	0.3821
LAR	0.0450	0.0783	0.0793	0.1730	0.2257	0.1927	0.2273	0.2316	0.2645	0.3125
AR	0.0565	0.0969	0.0923	0.1872	0.2513	0.2004	0.2434	0.2695	0.2789	0.3073
VAR	0.1046	0.1342	0.1261	0.2190	0.3105	0.2250	0.2595	0.3006	0.3220	0.3293

Panel C: Mean Correct Prediction (MCP%)

Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
LAR	71.33	68.93	69.13	67.55	66.96	67.67	66.71	70.16	68.38	68.95
AR	52.80	52.98	58.58	56.84	61.49	57.33	59.97	64.03	62.22	66.81
VAR	54.28	53.64	57.61	57.55	59.13	59.50	61.73	65.00	61.40	62.96

Table V

Out-of-Sample Error Metrics for Forecasting Risk-Neutral Skewness

This table reports the statistical performances of the Random Walk (RW), the Local Autoregressive (LAR), the Autoregressive (AR), and the Vector Autoregressive (VAR) models in the forecasting of next period risk-neutral skewness. The respective autoregressive models are lag-one models. A separate regression is performed for moments data corresponding to each time-to-maturity. Every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-minute intervals from 8:40 am to 12:00 pm (noon) are used to estimate the parameters in each regression. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-minute interval from 12:10 pm to 14:50 pm. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) % are shown in panels A, B, and C respectively. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct.

Panel A: Root-Mean-Square Error (RMSE)

Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	0.7197	0.6474	0.6282	0.6271	0.7044	0.5938	0.5729	0.6058	0.6410	0.6958
LAR	0.6102	0.5294	0.5404	0.5146	0.5918	0.4887	0.4754	0.4918	0.4940	0.5504
AR	0.5663	0.5081	0.5002	0.5079	0.5508	0.4725	0.4609	0.4794	0.4959	0.5297
VAR	0.5786	0.5187	0.5157	0.5907	0.6291	0.4765	0.4828	0.5033	0.5343	0.5414

Panel B: Mean Absolute Deviation (MAD)

Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	0.5391	0.4765	0.4556	0.4441	0.4862	0.4213	0.4156	0.4321	0.4189	0.5029
LAR	0.4347	0.3758	0.3750	0.3637	0.3949	0.3476	0.3406	0.3480	0.3334	0.3863
AR	0.4221	0.3769	0.3693	0.3772	0.3946	0.3462	0.3390	0.3545	0.3480	0.3786
VAR	0.4303	0.3842	0.3790	0.4024	0.4177	0.3480	0.3534	0.3641	0.3692	0.3879

Panel C: Mean Correct Prediction (MCP%)

Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
LAR	70.98	71.16	71.06	71.22	72.05	68.15	72.24	70.32	72.07	71.31
AR	71.24	70.41	70.80	69.29	72.55	69.35	71.56	71.29	69.40	72.16
VAR	70.80	69.17	69.92	68.67	71.43	69.47	69.68	70.16	66.74	73.02

Table VI

Out-of-Sample Error Metrics for Forecasting Risk-Neutral Kurtosis

This table reports the statistical performances of the Random Walk (RW), the Local Autoregressive (LAR), the Autoregressive (AR), and the Vector Autoregressive (VAR) models in the forecasting of next period risk-neutral kurtosis. The respective autoregressive models are lag-one models. A separate regression is performed for moments data corresponding to each time-to-maturity. Every trade day from August 24, 2009 to December 31, 2012, the risk-neutral moments computed in each 10-minute intervals from 8:40 am to 12:00 pm (noon) are used to estimate the parameters in each regression. The estimated or fitted model is then used to forecast the risk-neutral moments for each 10-minute interval from 12:10 pm to 14:50 pm. The error metrics or loss functions of root mean square error (RMSE), mean absolute deviation (MAD), and mean correct prediction (MCP) % are shown in panels A, B, and C respectively. The MCP is the % of times that the forecast of directional change in the risk-neutral moment is correct.

Panel A: Root-Mean-Square Error (RMSE)										
Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	5.2235	4.2445	3.9871	4.0847	4.4714	3.4320	2.9385	2.9682	3.5206	2.8665
LAR	4.3267	3.7312	3.3826	3.7868	3.6898	2.7722	2.4943	2.4049	2.6815	2.3835
AR	4.2609	3.4443	3.2926	3.5348	3.5941	2.6667	2.3454	2.3995	2.6268	2.1935
VAR	4.4064	3.5412	3.4268	4.0681	3.9968	2.6961	2.5530	2.5531	2.9152	2.2783

Panel B: Mean Absolute Deviation (MAD)										
Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	3.3088	2.6771	2.5601	2.6161	2.8126	2.1831	1.9016	1.9256	1.9443	1.8539
LAR	2.7387	2.2512	2.1673	2.4027	2.3237	1.8264	1.6557	1.5957	1.6303	1.5307
AR	2.8174	2.3429	2.2707	2.4308	2.3654	1.8910	1.6718	1.6994	1.7005	1.5113
VAR	2.8557	2.4202	2.3478	2.5943	2.5242	1.9201	1.7520	1.7565	1.7839	1.5419

Panel C: Mean Correct Prediction (MCP%)										
Time-to-Maturity τ	1 Day	2 Days	3 Days	4 Days	5 Days	6 Days	7 Days	8 Days	9 Days	10 Days
No. of Obs.	1151	1225	1141	993	814	844	759	629	498	480
RW	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
LAR	72.03	70.74	69.39	69.59	70.68	68.75	70.49	71.45	67.76	70.24
AR	69.76	70.33	68.60	67.65	70.43	67.79	69.00	69.52	66.32	68.95
VAR	68.44	68.51	67.72	67.76	69.32	68.39	69.54	68.87	66.53	68.95

Table VII

Profitability of Trading Strategy using Risk-Neutral Volatility Prediction

The table reports the average trading profit per trade in $ according to the different forecasting methods and threshold signals. Trading cost per option contract is 22.5 cents, and this has been deducted to arrive at the net trading profit. The trading strategy involves creating a volatility portfolio each 10-minute interval as follows: long an OTM call and short delta amount of underlying asset, together with long an OTM put and short a delta amount of underlying asset. The respective deltas are based on the strike prices of the call and the put. Since the delta of a put is negative, shorting delta related to a put amounts to buying the underlying asset. Each trading interval is 10-minutes intraday. At the end of the interval the positions are liquidated at market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral volatility is higher than the current risk-neutral volatility by at least the threshold percentage, the above portfolio of long call and long put is executed. The execution and liquidation next interval constitute one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval. The portfolio has zero cost as the net balance of the cost in the call, put, and underlying asset is invested in a risk-free bond. In the case where the net balance is negative, risk-free loan is taken. Since the interval is intraday, the effective risk-free rate is about zero. In the opposite case when predicted risk-neutral volatility is lower than the current risk-neutral volatility by at least the threshold percentage, the portfolio is short. In this case, a call and a put are sold together with associated positions in the underlying asset and risk-free bond. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Vola} = C_{t,OTM} - \Delta_{C_{t,OTM}} F_t + P_{t,OTM} - \Delta_{P_{t,OTM}} F_t - B_t.$$

B_t is chosen such that $\pi_t^{Vola} = 0$. Actual outlay for F_t may be close to zero for the initial futures position. The numbers in the brackets indicate the t-statistics based on bootstrapped variances calculated for the average profit. The bootstrap is carried out over 2000 iterations of the sample size indicated by the total number of trades. * denotes significance at the 1% significance level.

	Threshold Signal 5.0%		Threshold Signal 7.5%		Threshold Signal 10.0%	
	Profit	No. of Trades	Profit	No. of Trades	Profit	No. of Trades
PK	2.56*	2152	3.39*	1264	4.31*	773
	(4.56)		(4.06)		(4.48)	
LAR	0.09	2050	1.33	1235	0.84	884
	(0.15)		(1.56)		(0.98)	
AR	-0.32	1403	0.46	921	0.87	664
	(-0.51)		(0.56)		(0.89)	
VAR	-0.61	3121	-0.05	1962	0.59	1296
	(-1.64)		(-0.09)		(0.93)	

Table VIII

Profitability of Trading Strategy using Risk-Neutral Skewness Prediction

The table reports the average trading profit per trade $ according to the different forecasting methods and threshold signals. Trading cost per option contract is 22.5 cents, and this has been deducted to arrive at the net trading profit. The trading strategy involves creating a skewness portfolio each 10-minute interval as follows: long an OTM call and short a number of OTM puts equal to the ratio of the call vega to put vega. Also short a number of underlying assets equal to the call delta less the same vega ratio times put delta. The respective vegas and deltas are based on the strike prices and other features of the call and the put. Each trading interval is 10-minutes intraday. At the end of the interval the positions are liquidated at market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral skewness is higher than the current risk-neutral skewness by at least the threshold percentage, the above portfolio of long call and short puts is executed. The execution and liquidation next interval constitute one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval. The portfolio has zero cost as the net balance of the cost in the call, puts, and underlying asset is invested in a risk-free bond. In the case where the net balance is negative, risk-free loan is taken. Since the interval is intraday, the effective risk-free rate is about zero. In the opposite case when predicted risk-neutral skewness is lower than the current risk-neutral skewness by at least the threshold percentage, the portfolio is short. In this case, there is a short call and a long put together with associated positions in the underlying asset and risk-free bond. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Skew} = C_{t,OTM} - \Big(\frac{\upsilon_{C_{t,OTM}}}{\upsilon_{P_{t,OTM}}}\Big) P_{t,OTM} - \left(\Delta_{C_{t,OTM}} - \Big(\frac{\upsilon_{C_{t,OTM}}}{\upsilon_{P_{t,OTM}}}\Big)\Delta_{P_{t,OTM}}\right) F_t - B_t.$$

B_t is chosen such that $\pi_t^{Skew} = 0$. Actual outlay for F_t may be close to zero for the futures position. The numbers in the brackets indicate the t-statistics based on bootstrapped variances calculated for the average profit. The bootstrap is carried out over 2000 iterations of the sample size indicated by the total number of trades. * denotes significance at the 1% significance level.

	Threshold Signal 10.0%		Threshold Signal 22.5%		Threshold Signal 50.0%	
	Profit	No. of Trades	Profit	No. of Trades	Profit	No. of Trades
PK	2.97*	5657	2.79*	4064	2.68*	2406
	(8.94)		(7.33)		(5.59)	
LAR	1.62*	5458	1.97*	3369	1.06	1622
	(4.75)		(4.57)		(1.7)	
AR	1.74*	5178	1.67*	2969	1.16	1507
	(5.06)		(3.63)		(1.81)	
VAR	1.94*	5246	1.55*	3055	1.02	1562
	(5.65)		(3.51)		(1.69)	

Table IX

Profitability of Trading Strategy using Risk-Neutral Kurtosis Prediction

The table reports the average trading profit per trade $ according to the different forecasting methods and threshold signals. Trading cost per option contract is 22.5 cents, and this has been deducted to arrive at the net trading profit. The trading strategy involves creating a kurtosis portfolio each 10-minute interval as follows: long an ATM call and an ATM OTM put and simultaneously short X number of OTM calls and OTM puts, where $X = (C_{t,OTM} + P_{t,OTM})/(C_{t,ATM} + P_{t,ATM})$. Each trading interval is 10-minutes intraday. At the end of the interval the positions are liquidated at market prices. Prediction is done on moments with the same maturity. If the predicted next interval risk-neutral kurtosis is higher than the current risk-neutral kurtosis by at least the threshold percentage, the above portfolio of long ATM calls and puts, and short OTM call and put is executed. The execution and liquidation next interval constitute one trade. There can be more than one trade per interval if different maturity moment forecasts exceed the threshold, or there may be no trade in a particular interval. The portfolio has zero cost. In the opposite case when predicted risk-neutral kurtosis is lower than the current risk-neutral kurtosis by at least the threshold percentage, the portfolio is short. In this case, there is a short ATM calls and puts, and long OTM call and put. The overall cost of the portfolio can be expressed as:

$$\pi_t^{Kurt} = X(C_{t,ATM} + P_{t,ATM}) - (C_{t,OTM} + P_{t,OTM}).$$

The numbers in the brackets indicate the t-statistics based on bootstrapped variances calculated for the average profit. The bootstrap is carried out over 2000 iterations of the sample size indicated by the total number of trades. * denotes significance at the 1% significance level.

	Threshold Signal 10.0%		Threshold Signal 22.5%		Threshold Signal 50.0%	
	Profit	No. of Trades	Profit	No. of Trades	Profit	No. of Trades
PK	2.70*	377	4.79	208	4.35	87
	(2.01)		(1.87)		(1.15)	
LAR	1.64	374	4.02	163	2.25	45
	(1.18)		(1.69)		(0.38)	
AR	2.82	319	5.14	123	4.00	33
	(1.70)		(1.72)		(0.62)	
VAR	2.16	328	4.88	123	4.18	38
	(1.33)		(1.55)		(0.74)	

Table X
Comparison of Trading Strategies

Comparison of our skewness and kurtosis trading strategies with those reported in Neumann and Skiadopoulos (2013). There are 7137 trades for the skewness trading strategy and 682 trades for the kurtosis trading strategy. We compute the Sharpe ratio S as follows. Let P_t be the value of our trading portfolio at time t. In the next interval, it is liquidated at value P_{t+1} with a profit of $\pi_{t+1} = P_{t+1} - P_t$. $S = \hat{E}(\pi_{t+1})/\sqrt{\hat{\text{var}}(\pi_{t+1})}$ where the sample moments are used in place of the population moments. The sample mean and sample variance of all the 10-minute interval profits from the trading are utilized. To compare with Neumann and Skiadopoulos (2013) that did not use any threshold signal, we also re-compute our trading results with zero threshold. Following Neumann and Skiadopoulos, we perform the same bootstrap method and report the 95% confidence in the pair of numbers in the brackets. * denotes that the Sharpe ratio is significantly greater than zero at 2.5% significance level. One important point to note is that while Neumann and Skiadopoulos' results are based on daily profits, ours are based on profits from intraday 10-minute intervals. Therefore, to fix things on equal footings, we also annualize the Sharpe ratio by a multiplicative factor of $\sqrt{252 \times 18}$ since there are 18 trading intervals each trading day during 12:10 pm to 14:50 pm in our empirical study.

Skewness Trading Strategies

Our Forecasting Models	Sharpe Ratio	N&S Forecasting Models	Sharpe Ratio
PK	7.32*	ARIMA(1,1,1)	2.58*
	(5.80,8.85)		(2.03,3.15)
LAR	3.63*	ARIMAX(1,1,1)	2.81*
	(2.10,5.16)		(2.24,3.39)
AR(1)	3.66*	VECM(1)	1.94*
	(2.13,5.20)		(1.35,2.50)
VAR(1)	4.11*	VECM-X(1)	2.16*
	(2.61,5.61)		(1.59,2.77)

Kurtosis Trading Strategies

Our Forecasting Models	Sharpe Ratio	N&S Forecasting Models	Sharpe Ratio
PK	7.42*	ARIMA(1,1,1)	-0.22
	(2.62,12.08)		(-0.64,0.32)
LAR	2.23	ARIMAX(1,1,1)	-0.05
	(-7.26,2.66)		(-0.52,0.62)
AR(1)	1.79	VECM(1)	0.01
	(-3.17,6.56)		(-0.52,0.69)
VAR(1)	3.46	VECM-X(1)	0.52
	(-1.42,8.09)		(-0.14,1.31)

Table XI

Intraday Ex-Ante Risk-Neutral Moments and Subsequent Returns

In-sample linear regression of the following is run:

$$\ln(\frac{F_{t+1}}{F_t}) * 100 = \alpha + \beta_0 \ln(\frac{F_t}{F_{t-1}}) * 100 + \beta_1 RNM_t + \epsilon_{t+1} ,$$

where $t = 1, ..., T$ and T is the number of 10-minutes interval in any given day. T is about 36 to 38 depending on the trading day. For each trading day the regression is performed and the coefficient estimates are collected. Over the total sample of 635 trading days the estimated coefficients are averaged to obtain their means. Their variances and also their t-statistics reflecting if their average deviates significantly from zero, are also evaluated and reported in this table. These results for the above regressions are shown in columns A, B, and C.

A separate in-sample intraday regression involving all three RNM's as explanatory variables is also run:

$$\ln(\frac{F_{t+1}}{F_t}) * 100 = \alpha + \beta_0 \ln(\frac{F_t}{F_{t-1}}) * 100 + \beta_1 RNV_t + \beta_2 RNS_t + \beta_3 RNK_t + \epsilon_{t+1} ,$$

where $t = 1, ..., T$ and T is the number of 10-minutes interval in any given day. T is about 36 to 38 depending on the trading day. For each trading day the regression is performed and the coefficient estimates are collected. Over the total sample of 635 trading days the estimated coefficients are averaged to obtain their means. Their variances and also their t-statistics reflecting if their average deviates significantly from zero, are also evaluated and reported in this table. The results for the above regression are shown in column D.

Estimates	A	B	C	D
$\hat{\alpha}$	RNV(%)	RNS	RNK	(RNV,RNS,RNK)
Mean	-0.2111	0.0165	0.0182	-0.3481
Variance	0.0006	0.0000	0.0000	0.0011
t-statistic	-8.8278	4.2190	4.5678	-10.5504
$\hat{\beta}_0$				
Mean	-0.0545	-4.5273	-4.8124	-0.0484
Variance	0.0001	0.5460	0.5286	0.0001
t-statistic	-7.5492	-6.1272	-6.6189	-6.1812
$\hat{\beta}_1$				
Mean	0.0742	0.0164	-0.0064	0.1417
Variance	0.0001	0.0000	0.0000	0.0001
t-statistic	9.4631	4.3312	-4.0597	12.8319
$\hat{\beta}_2$				
Mean				0.0127
Variance				0.0001
t-statistic				1.3294
$\hat{\beta}_3$				
Mean				-0.0101
Variance				0.0000
t-statistic				-3.7464

Appendix

Critical value calibration

In the Monte Carlo simulation for critical value calibration on the LAR model estimation, we mimic a situation of ideal time homogeneity in data generation. In this case that the null hypothesis of time homogeneity holds everywhere, the critical values should be able to accept the prescribed homogeneity in the testing procedure.

In particular, we generate a globally homogeneous AR(1) time series, where the modeling parameters are constant:

$$RNM_t = \theta_0^* + \theta_1^* RNM_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^{*2})$$

where $\theta^* = (\theta_0^*, \theta_1^*, \sigma^*)$ for $t = 1, \cdots, T$. Under time homogeneity, the ML estimate $\tilde{\theta}_t^{(k)}$ can be safely obtained in every subsample $I_t^{(k)}$, $k = 1, \cdots, K$. The estimation error is measured by the fitted log-likelihood ratio:

$$R_k = E_{\theta^*} \left| L\left(I_t^{(k)}, \tilde{\theta}_t^{(k)}\right) - L\left(I_t^{(k)}, \theta^*\right) \right|^{1/2}, \qquad (A.1)$$

where R_k can be computed numerically, with the knowledge of θ^*.

Alternatively, one can conduct the adaptive estimation, where local homogeneity also holds for the generated time homogenous samples. Given a set of critical values, the adaptive estimator $\hat{\theta}_t^{(k)}$ is obtained. The temporal difference between the ML estimator and the adaptive estimator, denoted as $D_t^{(k)}$, can be measured by the log likelihood ratio:

$$D_t^{(k)} = \left| L\left(I_t^{(k)}, \tilde{\theta}_t^{(k)}\right) - L\left(I_t^{(k)}, \hat{\theta}_t^{(k)}\right) \right|^{1/2}.$$

The rational is to choose a set of critical values that leads to a prescribed performance as good as the true underlying characteristics under the null of time homogeneity. The stochastic distance $D_t^{(k)}$ is request to be bounded by the estimation error R_k in Eq. (A.1):

$$E_{\theta^*}\left(D_t^{(k)}\right) = E_{\theta^*} \left| L\left(I_t^{(k)}, \tilde{\theta}_t^{(k)}\right) - L\left(I_t^{(k)}, \hat{\theta}_t^{(k)}\right) \right|^{1/2} \leq R_k. \qquad (A.2)$$

In the above risk bound Eq.(A.2), critical values become the only unknown parameters, which are to be calibrated. With too large a critical value, there is a higher probability of accepting subsamples everywhere, and hence it is insensitive to changing parameters. On the contrary, too small a critical value implies a more stringent test that unnecessarily favors shorter subsamples, discarding useful past observations and resulting in higher parameter uncertainty. Thus the optimal critical values are the minimum values that just cross the risk bound at each sub-sample.

The computation of critical values relies on a set of hyperparameters (K, θ^*). In our procedure, at each point of time, we consider three subsamples for the test procedure, with length of 60 mins (including past 6 observations), 120 mins (including past 12 observations) and then all the past information (equivalent to recursive estimation). Ideally, θ^* should be close to the true parameter θ_t^* underlying the real data series at each point of time, which is actually the target of our estimation. In the following numerical analysis, we consider θ^* as the ML estimate using information from the available real sample observations till 12:00 pm (noon) before the forecast exercise starts. We use θ^* to simulate data, and calibrate the set of critical values as described above. The same set of calibrated critical values are adopted for every day throughout the real-time estimation and forecast.

References

Aït-Sahalia, Y., Wang, Y., Yared, F., 2001. Do option markets correctly price the probabilities of movement of the underlying asset? Journal of Econometrics 102, 67–110.

Amaya, D., Christoffersen, P., Jacobs, K., Vasquez, A., 2012. Does realized skewness predict the cross-section of expected returns? Working Paper .

Andersen, T. G., Bollerslev, T., Diebold, F. X., Labys, P., 2003. Modeling and forecasting realized volatility. Econometrica 71, 579–625.

Bakshi, G., Kapadia, N., Madan, D., 2003. Stock return characteristics, skew laws and the differential pricing of individual equity options. Review of Financial Studies 16, 101–143.

Bali, T. G., Hu, J., Murray, S., 2013. Option implied volatility, skewness, and kurtosis and the cross-section of expected stock returns. Working Paper .

Bali, T. G., Murray, S., 2013. Does risk-neutral skewness predict the cross section of equity option portfolio returns? Journal of Financial and Quantitative Analysis 48, 1145–1171.

Barndorff-Nielsen, O., Nicolato, E., Shephard, N., 2002. Some recent developments in stochastic volatility modelling. Quantitative Finance 2, 11–23.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307–328.

Breeden, D. T., Litzenberger, R. H., 1978. Prices of state-contingent claims implicit in option prices. The Journal of Business .

Brooks, C., Burke, S. P., Heravi, S., Persand, G., 2005. Autoregressive conditional kurtosis. Journal of Financial Econometrics 3, 399–421.

Carr, P., Madan, D., 2001. Optimal positioning in derivative securities. Quantitative Finance 1, 19–37.

Chang, B., Christoffersen, P., Jacobs, K., 2013. Market skewness risk and the cross section of stock returns. Journal of Financial Economics 107, 46–68.

Conrad, J., Dittmar, R. F., Ghysels, E., 2013. Ex ante skewness and expected stock returns. The Journal of Finance 68, 85–124.

Cremers, M., Halling, M., Weinbaum, D., 2013. Aggregate jump and volatility risk in the cross-section of stock returns. Working Paper .

Dennis, P., Mayhew, S., 2002. Risk-neutral skewness: Evidence from stock options. Journal of Financial and Quantitative Analysis 37, 471–493.

Efron, B., 1979. Bootstrap methods: Another look at the jackknife. The Annals of Statistics 7, 1–26.

Engle, R. F., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of united kingdom inflation. Econometrica 50, 987–1007.

Figlewski, S., 2008. Estimating the implied risk neutral density for the U.S. market portfolio. In: Bollerslev, T., Russell, J. R., Watson, M. (eds.), *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle*, Oxford University Press.

Figlewski, S., 2013. Some results on extracting and understanding the risk neutral returns distribution for the U.S. stock market. In: *NYU Stern–Federal Reserve Conference on Risk Neutral Densities*.

Friesen, G. C., Zhang, Y., Zorn, T. S., 2012. Heterogeneous beliefs and risk-neutral skewness. Journal of Financial and Quantitative Analysis 47, 851–872.

Gârleanu, N., Pedersen, L. H., Poteshman, A. M., 2009. Demand-based option pricing. The Review of Financial Studies 22, 4259–4299.

Ghysels, E., Harvey, A., Renault, E., 1996. Stochastic volatility. In: Maddala, G., Rao, C. (eds.), *Statistical Methods in Finance*, North Holland Amsterdam, pp. 119–191.

Gonçalves, S., Guidolin, M., 2006. Predictable dynamics in the s&p 500 index options implied volatility surface. Journal of Business 79, 1591–1635.

Harvey, C., Siddique, A., 1999. Autoregressive conditional skewness. Journal of Financial and Quantitative Analysis 34, 465–487.

Harvey, C., Siddique, A., 2000. Conditional skewness in asset pricing tests. Journal of Finance 55, 1263–1296.

Jackwerth, J., Rubinstein, M., 1996. Recovering probability distributions from option prices. Journal of Finance 51, 1611–1631.

Jiang, G. J., Tian, Y. S., 2005. The model-free implied volatility and its information content. Review of Financial Studies 18, 1305–1342.

Merton, R. C., 1973. Theory of rational option pricing. Journal of Economics and Management Science 4(1), 141–183.

Neumann, M., Skiadopoulos, G., 2013. Predictable dynamics in higher-order risk-neutral moments: Evidence from the s&p 500 options. Journal of Financial and Quantitative Analysis 48, 947–977.

Poon, S.-H., Granger, C. W., 2003. Forecasting volatility in financial markets: A review. Journal of Economic Literature 41, 478–539.

Rubinstein, M., 1996. Implied binomial trees. Journal of Finance 49, 771–818.

Taylor, S., Yadav, P. K., Zhang, Y., 2009. Cross-sectional analysis of risk-neutral skewness. Journal of Derivatives 16, 38–52.