Reduced density matrix

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Introduction

I was confused about what is the reduced density matrix and why it can be used to describe the state of focused system. Here is the explaination. For simplicity, ρ_A is the density matrix of studied system and ρ_B is another system which might be the bath or the other thing which are all asked to be normalized.

$$\rho_A = \frac{1}{4}\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, \rho_B = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}. \tag{1}$$

The total system is obtained by the direct product between ρ_A and ρ_B.

$$\rho_T \equiv \rho_A \otimes \rho_B = \frac{1}{8}\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \otimes \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{8}\left(\begin{array}{cc|cc} 1 & -1 & 2 & -2 \\ 1 & 1 & 2 & 2 \\ \hline 2 & -2 & 3 & -3 \\ 2 & 2 & 3 & 3 \end{array}\right) \tag{2}$$

The reduced density matrix of total system for A is taken form

$$\rho_A = \mathrm{Tr}_B[\rho_T] = \frac{1}{8}\begin{pmatrix} 1+1 & 2+2 \\ 2+2 & 3+3 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \rho_A. \tag{3}$$

Similarily, the reduced density matrix for B system is read as

$$\rho_B = \mathrm{Tr}_A[\rho_T] = \frac{1}{8}\begin{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} & \\ & \begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} = \rho_B. \tag{4}$$