

Myopic Loss Aversion in Investment Behavior

A Thesis

Presented to

The Established Interdisciplinary Committee for Mathematics and Economics

Reed College

In Partial Fulfillment

of the Requirements for the Degree

Bachelor of Arts

Dean K. Young

May 2016

Approved for the Committee
(Mathematics and Economics)

Jonathan Rork

Andrew Bray

Acknowledgments

This study was supported by the Richter Funds, the Carney Economics Fund, and the Reed College Undergraduate Research Initiative Grant.

First and foremost, I would like to acknowledge God, for I can do all this through Him who gives me strength.

I would like to express my loving appreciation for my mom and my step-dad Scott, for being the most supportive pair I have in my life. Thank you making Reed possible for me.

I would like to express my appreciation for my two thesis advisors, Jon Rork and Andrew Bray, who have given me significant support in both my academic and professional life.

I would like to acknowledge Albert Kim, Rich Majerus, and Chester Ismay for their valuable assistance in creating the investment simulator app used for this thesis. Thank you for making this thesis possible.

I would also like to express sincere gratitude and appreciation for Oh For Christ's Sake (OFCS), Reed's InterVarsity chapter. The people that I have met in this group have brought me so much support, love, and friendship. Most importantly, because of this group, I have not only kept my faith but have grown in it. My time with OFCS has shined in my four years at Reed and is a memory I will cherish for a life-time.

Ahyan Panjwani and Xian Ng, thank you for being the best departmental colleagues and friends I could ever have. Katie MacDougall, Sarah Gross, and Rikki Liu, thank you for becoming some of the closest and caring friends I have ever had in such a short time. It is such a shame that we were only able to spend one year together at Reed. Amelea Ng, Henrik de Savy, and Daniel Hong, thank you for not just being awesome fellow leaders of OFCS, but also important friends and support for me. Kayla Pollard and Brandi Miller, thank you both for being great mentors and supervisors. I admire you both for overcoming significant adversities while leading OFCS at Reed. Andrea Emerson, thank you for being the wise older Christian mentor I needed during my time at Reed.

Preface

This thesis is heavily influenced by the work of Thaler, Tversky, Kahneman, and Schwartz on their paper “The Effect of Myopia and Loss Aversion on Risk Taking: An Experimental Test” (1997).

Table of Contents

Introduction.....	1
Chapter 1: Literature Review.....	3
Chapter 2: Experimental Methods and Design.....	9
2.1 Overview of Implications	9
2.2 Overview of Experimental Design	10
2.3 Hypothesis	13
Chapter 3: Results and Analysis.....	15
3.1 Initial Analysis	15
3.2 Extended Analysis	27
Chapter 4: Conclusion and Discussion	29
4.1 Conclusion	29
4.2 Discussion.....	31
Appendix A: Analytic Solution to Probability of Underperformance	35
Appendix B: Simulated Solution to Probability of Underperformance	39
Appendix C: Post Investment Simulator Survey	41
Appendix D: Source Code.....	43
Bibliography	45

List of Tables

Table 1. Percent of portfolio allocated to the Bond Fund for final choice	17
Table 2A. Percent of portfolio allocated to the Bond Fund for final 40 periods (averaged by subject)	17
Table 2B. Percent of portfolio allocated to the Bond Fund for final 40 periods (cluster- robust standard errors)	17
Table 3. Regressions predicting allocation to Bond Fund (%) from trial number.....	25
Table 4. Regressions of final allocation to Bond Fund (%) on various measurements of finance knowledge and recollection of stock value loss	26
Table 5. Comparison of probability of underperformance of Stock Fund relative to Bond Fund for various time horizons	40

List of Figures

Figure 1. Distribution of final allocation choices for each condition	16
Figure 2A. Bootstrapped differences in average final choice allocation to the Bond Fund for the monthly and yearly conditions	18
Figure 2B. Bootstrapped differences in average final choice allocation to the Bond Fund for the monthly and inflated-monthly conditions	19
Figure 3A. Monthly condition subjects' autocorrelation functions (ACF) of allocation choices.....	21
Figure 3B. Yearly condition subjects' ACFs of allocation choices.....	22
Figure 3C. Inflated monthly condition subjects' ACFs of allocation choices	22
Figure 4A. Comparing average allocations over time (monthly vs. yearly conditions) ...	24
Figure 4B. Comparing average allocations over time (monthly vs. inflated monthly conditions).....	24
Figure 5. Simulated distribution of difference in geometric mean return between the Stock Fund and the Bond Fund over various sample sizes.....	39

Abstract

Loss aversion asserts that humans weigh losses significantly more than gains of equal magnitude. This is in contrast to traditional decision-making theory in neoclassical economics, which only incorporates risk aversion, that is, the variation from the mean result. It was not long after Daniel Kahneman and Amos Tversky coined loss aversion in economics literature before it began to be widely applied in other disciplines. In finance, it was noted that stocks traded at a high premium relative to lower risk securities such as bonds or treasury notes. This premium exceeded what traditional risk aversion models produced based on available information. Joined with Richard Thaler, Kahneman and Tversky investigated the effect of loss aversion on investment behavior. Their findings showed that loss aversion induced myopic behavior from experimental subjects which made them favor bonds over stocks. This thesis replicates this study with a few modifications.

To Anastasia Joy Hansen, for redefining the way I see friendship and showing me that it extends beyond just reason or logic. You completed my Reed experience.

Introduction

It is well known in the investment world that equities (stocks) carry a large risk premium over low-risk investments such as bonds or treasury bills. That is, the risk premium entails that equities must have a higher average return than low-risk investments to compensate for the high volatility (risk) of the investment. In any given moment, equities are expected to give a higher return than low-risk investments but have a much higher variance of returns. It's entirely possible that equities lose all of their value in a given moment whereas the possibility of a U.S. Treasury Bill doing the same is next to zero.

This relationship between expected return and variance of return, leads to an interesting application of the *Weak Law of Large Numbers* (WLLN). Equities are at high risk of underperforming low-risk investments in a short time period, but are nearly guaranteed to outperform them in the long-run (how long is not well defined) due to equities' higher expected returns. The long time period erodes the effect of high volatility in returns. This is why professional investment advisers generally advise younger investors to allocate more of their portfolio in equities and older investors to start shifting to lower-risk investments such as bonds or treasury bills. Younger investors have a longer time horizon and therefore have the advantage of the WLLN. Older investors do not have such a luxury.

A simple example of this would be this: Would you accept this bet? I flip a coin. Heads, you win \$200. Tails, you lose \$100. The expected value of this bet is positive \$50. But due to the high risk of the bet, many people would not take this bet since they are averse to the possible major loss. What if I offered you the chance to play this game a hundred or a thousand times? Most people probably would (and should) take the bet since the WLLN is now in their favor.

This recognition of the advantage that a large time period or large sample size gives to a risk-averse decision maker seems to be lost in the amateur investment world. Most amateur investors are unable to think with a long time horizon when it comes to managing their investments. It has been theorized that major losses induce myopic

decision making which causes investors to behave this way. This phenomenon is known as myopic loss aversion. Myopic loss aversion stems from the ideas of *prospect theory*, a theory that states that losses carry much greater disutility than the utility from a gain of equal magnitude. As an example, a large percentage of Millennials are very afraid of the stock market and prefer to hold cash in low yielding but very safe investments such as bank accounts (Egan, 2015). This is likely due to seeing both the 2000 Dotcom bubble crash and the 2008 Financial Crisis in which stocks tanked heavily, incurring huge losses to portfolios. These investors are only thinking in the short term however. Had Millennials decided to take the long term view rather than the myopic view, they could have ridden the six year bull market in equities after 2009.

Chapter 1: Literature Review

The root of this thesis begins with Thaler, Tversky, Kahneman, and Schwartz (Thaler et al., 1997) investigating the *equity premium puzzle*. The equity premium is a specific example of a risk premium and is the difference between the return on equity investments (stocks) and the return on a risk-free investment such as treasury bills. In “A Random Walk Down Wall Street”, Burton Malkiel talks about how risk and return are related as determined by the Capital-Asset Pricing Model (Malkiel, 2007). Given an increase in the systematic risk of an investment, the return from the investment must also increase. This relationship is derived from *expected utility theory (EUT)* which proposed that all decision makers have a concave utility function and are therefore *risk-averse*. This implied that decision makers are willing to reduce expected return from a gamble for a more certain outcome (lowering risk). If two gambles have the same expected return but one gamble has significantly higher variance in return, the gamble with the lower variance actually has higher expected utility (despite the same expected *return*). This meant that equities require a higher expected return to compensate for their higher risk in order to match the expected utility of a risk-free investment.

Historically, over the ninety-year period 1889-1978 the average real annual yield on the Standard and Poor 500 Index was seven percent, while the average yield on risk-free investments was less than one percent. However, the coefficient of risk-aversion from expected utility theory needed to produce this average historical equity premium was over 30 whereas most other estimates of risk-aversion was around 1 (Mehra & Prescott, 1985). To put this coefficient into perspective, Mankiw and Zeldes (1991) provide this calculation. Suppose that you are offered a gamble where there is a 50% chance of consumption of \$100,000 and a 50% chance of consumption of \$50,000. If you had a coefficient of risk-aversion of 30, you would accept \$51,209 with certainty instead of the gamble. It is unlikely that people are this risk-averse. The inability of the neoclassical economic model of EUT to explain this large equity premium was dubbed by Mehra and Prescott as the equity premium puzzle.

Additionally, Siegel and Thaler (1997) argued that although a large equity premium generated by a large coefficient of risk-aversion could be rational in the short-term, it is certainly unreasonable in the long-term. Benartzi and Thaler (1995) note that from 1876 to 1990, when looking at forty-year periods, a portfolio of bonds has *never* outperformed a portfolio of stocks. They proposed an explanation for the equity premium puzzle, citing the now Nobel-Prize winning work done by Tversky and Kahneman (1992) on *cumulative prospect theory (CPT)* which was an update on their work done on “non-cumulative” prospect theory in 1979. Benartzi and Thaler ultimately concluded that the equity premium emerged from a phenomenon known as *myopic loss aversion* in behavior from investors.

Tversky and Kahneman’s CPT significantly improved on how we modeled risk with just EUT. EUT evaluates risk simply defined as deviation from an expected return and did not place importance on whether or not the deviation involved losses. Additionally, utility is defined relative to wealth rather than relative to gains or losses about the status quo (Baddeley, 2013). In an analysis of evidence from a risk-taking game show, *Deal or No Deal*, CPT was significantly more accurate in predicting contestant behavior than EUT (Post, Assem, J, Baltussen, & Thaler, 2012). As noted above, Mehra and Prescott concluded that EUT was not sufficient to explain the large equity premium phenomenon.

Tversky and Kahneman’s work on CPT took into account decision makers’ reaction to losses. They proposed that decision makers are not just risk-averse, but loss-averse. Potential losses are weighted much more heavily than potential gains for decision makers. In addition, these losses and gains are weighted relative to the status quo rather than to total wealth (a 10% loss at \$100 carries the same negative utility as a 10% loss at \$1 million). In fact, CPT states that decision makers heavily overweigh extreme losses even though they have a very small probability of occurring. The original “non-cumulative” prospect theory stated that *all* small probability events are over-weighted, not just extreme ones (Baddeley, 2013). This aspect of CPT runs parallel to investors’ fear of major crashes in the stock market. Since 1928, only 3 years have had an annualized loss of greater than 30% for the S&P 500. This means that each year in that period had only a 3% chance of losing more than 30%. Only 1 year (1931) had an

annualized loss greater than 40%. No year had a loss greater than 50% (Damodaran, 2015). However, the fear that you “will lose all of your money if it is placed in stocks” still exists.

Despite several rigorous papers proposing myopic loss aversion as the explanation for such a large equity premium, no empirical evidence was available until Thaler et. al. performed an experiment investigating the phenomenon. They recruited eighty undergraduate students from U.C. Berkeley to participate in their experiment. The premise of the experiment was to have the subjects partake in an investment simulator that simulated returns to a stock fund and a bond fund. The returns for the stock fund and bond fund were generated from a normal distribution and a truncated normal distribution (to eliminate possibility of loss) respectively. The mean and variance of the distributions were obtained from historical data. The distribution of returns for the stock fund had a higher expected value but much larger variance, including possibility of loss, than the bond fund. Subjects were asked to allocate between the two funds over two-hundred periods. After each allocation, a bar chart displayed the returns from both funds for that relevant choice. At the end of the simulation, subjects were asked to lock in an allocation that would persist for four-hundred periods ($2/3$ of the total period length of the simulation). Subjects were paid based on their returns from their allocations.

Thaler et. al. divided the subjects into four groups. The first group was the control group and was called the monthly condition. The subjects in this condition saw the returns for every single period of the two-hundred periods in which they made an allocation. The subjects were also allowed to adjust their allocation in every single period. This meant that subjects in this condition made two-hundred choices. The second group was called the yearly condition. The subjects in this condition only saw the *average* return over eight periods. In addition, they could only adjust their allocation every eight periods. This meant that subjects in this condition made twenty-five choices. The third group was called the five-yearly condition. The subjects in this condition had similar information and adjustment options as the yearly condition except that they were over twenty-five periods. This meant that subjects in this condition made only eight choices. The final group was the inflated-monthly condition. The subjects in this

condition saw returns and were allowed to adjust allocations every period just like the monthly condition except that the returns for both funds were increased by a flat 10% each period. This inflation factor shifts the distribution of returns 10% to the right but maintains the shape of the distributions, meaning that the variance (or risk) of both funds has not changed.

Thaler et. al. found that subjects in the yearly, five-yearly, and inflated-monthly condition allocated more of their portfolios (both during the initial two-hundred periods and for the final allocation) to the stock fund on average than the subjects in the monthly condition did. Since the stock fund performed better on average, the subjects in the yearly, five-yearly, and inflated-monthly conditions outperformed subjects in the monthly condition.

Thaler et. al. concluded that forcing investors to think and act in the long term would remedy this behavior and thus improve investment returns for these individuals. This can be done by not revealing short-term fluctuations in investment returns and also forcing investors to lock-in investment allocations for longer periods of time. This remedy is counter to certain axioms of neoclassical economics. In neoclassical economics, more information is always preferred to less information. There are several neoclassical economics theories that explain market inefficiencies by citing the occurrence of *information asymmetry* and they claim that if there were *perfect information*, these inefficiencies would disappear. By revealing only an average return over a period of time rather than the highly fluctuating individual returns, you are restricting information from the investor. In addition, more choices are always preferred to fewer choices. A choice set of {A, B, C} is always preferred to a choice set of {A, B} because there is always a possibility that one may prefer choice C the most. Even if choice C is not the best choice, choice set {A, B, C} is no worse than choice set {A, B}. Given that the remedy proposed by Thaler et. al. is counter-theoretical to neoclassical economics, an experiment would be appropriate to test the remedy.

In addition, Thaler et. al. showed that risk-aversion is not the only phenomenon that factors into investment behavior. As noted before, inflating the returns do not change the variance of the distribution of returns. The returns shown in the monthly condition had the same relative risk difference between the two funds as the returns shown in the

inflated-monthly condition. According to expected utility theory, there should be no significant difference in portfolio allocation between the two conditions. However, because the inflated-monthly condition had all of its returns inflated, subjects did not see very many losses from the stock fund. This resulted in subjects allocating more of their portfolio to the stock fund in the inflated-monthly condition.

Evidence that restricting information from amateur investors is actually beneficial is found in many other places in addition to Thaler et. al.'s research. Benjamin Graham (2006) remarked in his book "The Intelligent Investor" that during the Great Depression the value of all investments likely tanked significantly, and yet private mortgage investments were much more likely to be retained by investors than corporation bonds of even greater quality than the mortgages. Graham stated that this was due to the fact that the private mortgages did not trade on an exchange and therefore had no quoted value. However, the corporation bonds were listed on an exchange and therefore investors saw the large declines in value during the Great Depression and decided to sell them. It would have been very ignorant to assume that the mortgages had not declined significantly in underlying value as well during the recession simply because they did not have a quoted value. However, that very ignorance that stemmed from a lack of information actually helped investors stay the course with their mortgage investments.

Chapter 2: Experimental Methods and Design

2.1 Overview of Implications

This experiment is a close replication of one performed by Thaler et. al. for their paper. A successful replication of Thaler et. al.'s results would help increase confidence in the existence of myopic loss aversion and the remedies that address that tendency. In addition, Thaler et. al.'s paper was published in 1997, before the major downturns in 2000-2001 (dotcom bubble) and 2008-2009 (financial crisis). It's possible that investors have learned from these downturns that rebounds eventually happen and so myopic behavior is no longer as prevalent as in 1997. Alternatively, investors may be even more wary of losses and so myopic behavior is actually even more prevalent than in 1997.

Another question that surfaces when studying myopic loss aversion is whether the myopic behavior is induced by *loss* aversion or rather by general *risk* aversion. Investors may be fleeing from stocks when there are large losses not because of the losses but because during periods of loss, there is a large deviation from the expected return. In this case, that general deviation from the expected return is interpreted as risk. A way to test for the distinction between loss and risk aversion would be to shift the distribution of returns for the investments a fixed amount to the right. This is done by adding a fixed amount to any return the distribution generates. The deviation from the expected return has not changed since every point in the distribution moved the same distance to the right. However, if the distribution is shifted substantially to the right, almost all possibilities of it returning a loss are eliminated. If investors stop exhibiting myopic behavior in this case, then that behavior should be attributed to loss aversion rather than general risk aversion.

2.2 Overview of Experimental Design

Subjects were informed that they would be participating in an investment simulator in which they had to decide how to allocate their virtual portfolio between two funds, Fund A (the stock fund) and Fund B (the bond fund). Subjects did not know the characteristics of either fund beforehand; they had to learn them through experience. Each period (or ten periods depending on assigned condition), subjects selected from a menu what percentage allocation they wanted for the next period (ten periods). The choices ranged from 100-0% to 0-100% in increments of ten percent. Each subject began with \$100 in their virtual portfolio. The simulator lasted for two-hundred periods. After the two-hundredth period, subjects were asked to lock-in an allocation that would last for the next four-hundred periods. Subjects were told that their rewards will be based on their final portfolio value after the six-hundred total periods. The top 20% of portfolios would receive \$25 as compensation, next 20%: \$20, next 20%: \$15, and so on until the minimum reward of \$5.

The distribution used to generate returns for each period for Fund A and Fund B were the same distributions used by Thaler et. al. for their stock and bond fund respectively. Henceforth, Fund A will be known as the “Stock Fund” and Fund B will be known as the “Bond Fund” to avoid confusion. The Stock Fund’s returns were generated from a normal distribution with a mean return of 1% and a standard deviation of 3.54%. The Bond Fund’s returns were generated from a truncated normal distribution with a mean return of 0.25% and a standard deviation of 0.177% with truncation at 0% to prevent losses. It is not clear whether Thaler et. al. intended these returns to be *continuously compounded* returns or *simple* returns. Based on a best-guess interpretation of their description of their experimental design, this replication assumes those returns are simple returns. The distinction between the two types of returns are elaborated more in Appendix A and B. Whether the returns are considered continuously compounded or simple is not likely to strongly affect experimental results. The relative mean and variance between the two funds are still preserved.

The subjects were split into three experimental groups, the monthly condition, the yearly condition, and the inflated-monthly condition. The monthly condition was the

control group in which subjects saw every period's returns and were able to change their investment allocation every period. The yearly condition only allowed subjects to make decisions every ten periods, meaning each allocation was locked in for ten periods. They also only saw the average returns for both funds over ten periods rather than individual period's returns. Subjects in the yearly condition would be seen as at a disadvantage according to neoclassical economic theory. Subjects in the monthly condition could make every choice subjects from the yearly condition chose (by simply choosing the same allocation for ten straight periods) but they could also adjust their allocation every period. This was a flexibility that subjects in the yearly condition did not have. Subjects in the monthly condition also saw fund returns for every period which means they knew the average return over ten periods as well. Subjects in the yearly condition *only* knew the average return over ten periods. The inflated-monthly condition is just like the monthly condition except that both funds had every period's return increased by 5% (the inflation factor) over the course of the experiment (this was readjusted back down to regular levels when determining reward payouts as to not give the inflated-monthly group an unfair advantage) .

There are a few key differences between this replication and the original experiment. Note that this replication's monthly condition was the same as Thaler et. al.'s monthly condition. However, this replication's yearly condition was over ten periods which is a hybrid of Thaler et. al.'s yearly (eight periods) and five-yearly (forty periods) conditions. In Thaler et. al.'s results, there was no statistical difference in the average proportion of portfolios allocated to stocks between the two conditions. For this replication, the two conditions were merged to allow for greater sample size per condition (which increases statistical power) without a loss in variation in conditions, since Thaler et. al. found statistically similar results for the two conditions.

Another key difference is the inclusion of a portfolio value. Thaler et. al. based their simulator only on investment percentage returns and not linked to a portfolio value. This replication starts subjects off with a portfolio value of \$100. Each allocation choice divides their portfolio into the two funds. The interim feedback between choices also includes the increase/decrease in portfolio value from the allocation choice for both

funds. However, this begets a problem not found in Thaler et. al.'s sans-portfolio version. Under Thaler et al.'s experiment parameters, the stock fund would return an average of 11% in the inflated condition. Over 200 periods, a portfolio starting with \$100 could end up at an unreasonably high value of over \$100 billion, more than the net worth of Bill Gates. If an investor was particularly lucky, by choosing the bond fund when the stock fund does poorly, they could have a portfolio value much larger than that. This could potentially bias the results of the experiment, given that a portfolio value of over \$100 billion is extraordinarily excessive and could prime subjects' decisions away from reality. Although not explicitly stated in their paper, Thaler et. al. would likely have cited that CPT states that evaluating losses and gains are relative to the status quo rather than wealth. This means that the extraordinary portfolio value would not affect subjects' reaction and consequent decision making from seeing the losses and gains.

In fact, this element of CPT is likely the reason Thaler et. al. did not include a portfolio for their subjects. They likely deemed it unnecessary for subjects to frame their decisions based on how the investment returns would affect their wealth. However, Thaler et. al.'s experiment was not designed to test for that particular element of CPT, nor is any empirical evidence presented in the paper supporting that element. In reality, when actual investors are participating in investment markets, they are putting their wealth on the line. Investors are investing with the intent of improving their wealth as the end goal. This replication will include that element by introducing portfolio values for each subject and using final portfolio value as the measure by which to compare investment performance. However, for this replication, the inflation factor was lowered to 5% for the inflated condition to address the issue of reaching unreasonably large portfolio values. With an average return of 6%, an all-stock portfolio would have just under \$10 million at the end of 200 periods. The maximum possible portfolio value (picking the higher returning fund every period) was about \$66 million. These portfolio values are much more realistic and would reduce likelihood of bias from a framing effect. With an inflation factor of 5%, only 6 of 200 returns were negative for the stock fund (versus 75 of 200 in the monthly condition), which still achieved the original intent of the inflated condition.

At the end of the experiment, all subjects filled out a short exit survey asking them about their familiarity with investing and markets. They were asked if they are or are potentially an economics major. They were asked if they are a part of the Reed College Investment Club or any other investment related entity. They were asked if they own and actively make investments privately. Finally, they were asked about the Dotcom bubble and the Financial Crisis and what they thought was the worst percentage loss in value for stocks in those periods. The information from the exit survey was used to assist in the final data analysis for this experiment by controlling for investment knowledge factors and behaviors. Participants that were already familiar with investing and/or markets may not exhibit myopic loss aversion as strongly. Participants with particularly strong or exaggerated perceptions of recent major downturns may exhibit myopic loss aversion more strongly.

2.3 Hypothesis

It is hypothesized that the subjects from both the yearly condition and the inflated-monthly condition will outperform subjects in the monthly condition. The yearly condition involves averaging returns over a ten period which means subjects are more likely to recognize that the Stock Fund outperforms the Bond Fund over the long-term. In addition, it is hypothesized that the subjects from the inflated-monthly condition will also outperform subjects in the monthly condition. If subjects' myopic behavior is induced by *loss*-aversion and not *risk*-aversion then by inflating away all losses in fund returns subjects are expected to invest more in the Stock Fund due to its higher returns.

Positive results from this experiment would have some significant implications for remedying myopic behavior. They would certainly dampen the proponents of the information-age. Amateur investors today have practically unlimited access to information about investments due to the rise of the internet and smartphones. Positive results from this experiment would imply that in fact, amateur investors should stay away from all of this information, lest a major down-turn occurs and investors panic and think more myopically. The results would also dampen the proponents of day-trading and

market-timing that encourage amateur investors to act myopically in order to beat the market. In a market that is very difficult to predict in the short-term, myopic behavior is usually deadly for one's financial safety. Instead, it would be better to prevent amateur investors from shuffling their portfolio often. Both of these remedies are firmly counter-theoretical to neoclassical economics. However, the counter-theoretical nature to neoclassical economics is what constitutes the essence of behavioral economics.

Chapter 3: Results and Analysis

3.1 Initial Analysis

This section will present tables and figures with an alpha level of both 0.01 and 0.05. The analysis will be presented with an alpha level of 0.01 to mimic Thaler et. al.'s standard. In section 3.2, statistical conclusions that change when a more conventional alpha level of 0.05 is used will be discussed.

Figure 1 displays the dispersion of final allocations across subjects. A few subjects were able to discern that it was in their best interest to put all of their money in the Stock Fund (none in the Bond Fund) in all three conditions. Figure 1 also shows that statistical aggregation of returns (yearly condition) and distribution shifting to mostly eliminate losses (inflated monthly condition) were very effective in causing subjects to allocate towards the Stock Fund instead of the Bond Fund. In the monthly condition, twelve of eighteen subjects chose to allocate at least 70 percent of their portfolio to the Bond Fund as their final allocation. Only two of sixteen and six of nineteen did so for the yearly and inflated monthly conditions respectively. Finally, although the yearly and inflated monthly conditions had relatively normally distributed data, the monthly condition had what appears to be a bimodal distribution.

A likely explanation for this case is that the monthly condition had the strongest effect in inducing myopic loss aversion which clumped most subjects' allocation choice to the right of the distribution. However, there were enough subjects shrewd enough to realize that the Stock Fund was most likely going to outperform the Bond Fund in the long run and chose to allocate their entire portfolio to the Stock Fund. These few subjects clumped to the left of the distribution, completing the bimodal distribution. All of these results are extremely similar to Thaler et. al.'s findings in their experiment.

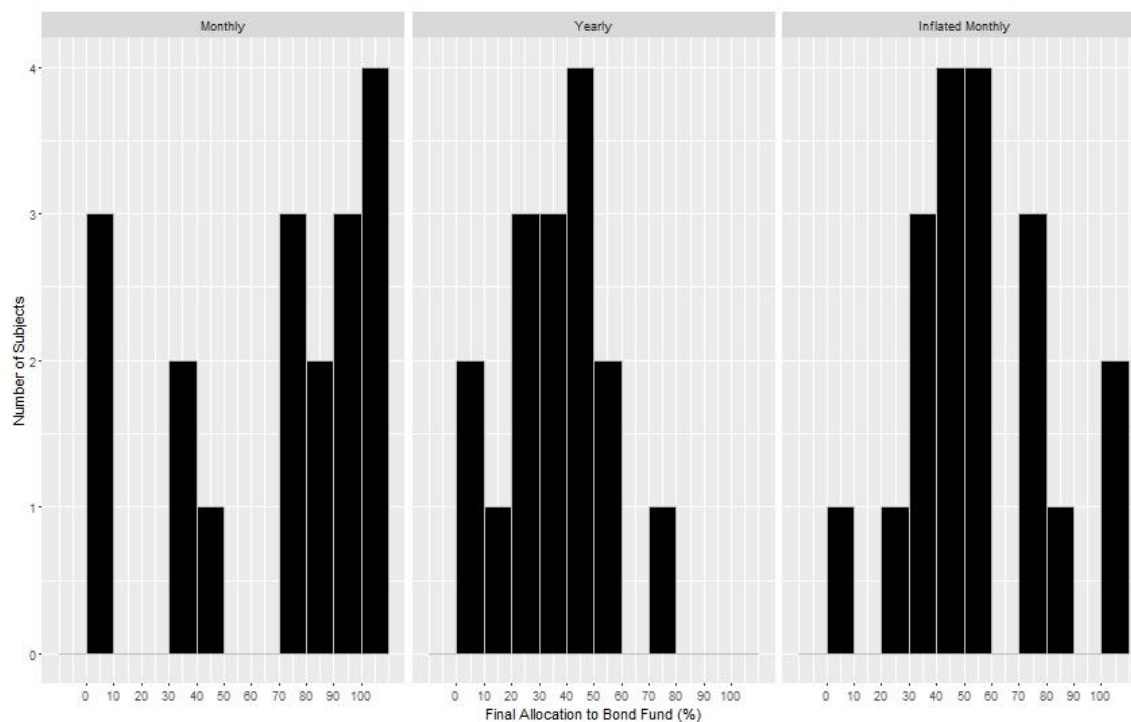


Figure 1. Distribution of final allocation choices for each condition

Statistics for the average allocation to the Bond Fund for the final choice and the last forty periods (this is the last forty choices for the monthly and inflated monthly conditions and the last four choices for the yearly condition; not counting the final choice that locked for four-hundred periods) are presented in Table 1 through Table 2B respectively. The means for each condition in both tables are nearly identical to the means in Thaler et. al.'s experiment. The one exception is that the average final choice to the Bond Fund for the inflated monthly condition here is much higher than what Thaler et. al. found (50.53% to 27.6%).

Welch's t-tests are applied to find any differences in average allocation to the Bond Fund between conditions. The results for the final choice are very similar to the findings Thaler et. al. found. For this replication, there is a statistical difference between the monthly and yearly conditions ($p=0.003$) but not between the yearly and inflated-monthly conditions ($p=0.013$). Thaler et. al. find a statistical difference between the monthly and inflated monthly condition whereas there is no statistical difference here ($p=0.233$). Although the experimental design randomly assigned students into conditions, there remained the possibility that one condition contained a significantly higher

proportion of subjects with more investment experience. Questions 3 through 5 of the post-simulator survey (Appendix C) were used as controls for investment knowledge. Adding these controls did not change any statistical significance conclusions but did reduce the p-value when comparing the monthly and inflated monthly conditions from 0.233 to 0.071. This may indicate that the effect of the inflated condition on subjects was masked by unlucky grouping of subjects with investment knowledge into one condition.

Condition	N	Mean	SD	SE
Monthly ¹	18	63.33	36.78	8.67
Yearly ^a	16	30.63	18.79	4.70
Inflated Monthly ^{a1}	19	50.53	25.92	5.95

Table 1. Percent of portfolio allocated to the Bond Fund for final choice

Conditions with common letter (number) subscripts denote no statistical difference according to a Welch's t-test with $\alpha=0.01$ ($\alpha=0.05$).

Condition	N	Mean	SD	SE
Monthly ^a	18	57.57	28.10	6.62
Yearly ^b	16	30.00	12.91	3.23
Inflated Monthly ^{ab}	19	41.51	13.06	3.00

Table 2A. Percent of portfolio allocated to the Bond Fund for final 40 periods (averaged by subject)

Conditions with common letter (number) subscripts denote no statistical difference according to a Welch's t-test with $\alpha=0.01$ ($\alpha=0.05$).

Condition	N*	Mean	SD	SE**
Monthly	720	57.57	36.51	1.36
Yearly ^a	64	30.00	19.44	2.43
Inflated Monthly ^a	760	41.51	26.82	0.97

Table 2B. Percent of portfolio allocated to the Bond Fund for final 40 periods (cluster-robust standard errors)

Conditions with common letter (number) subscripts denote no statistical difference according to a Welch's t-test with $\alpha=0.01$ ($\alpha=0.05$). *Not fully independent

**Unclustered standard errors

The fact that the monthly condition final choice distribution does not appear to be normal puts the validity of the t-test results in jeopardy. T-tests assume data is drawn from a normal distribution. To address this, non-parametric bootstrap resampling was performed to compare the differences of means involving the monthly condition data. The statistical significance conclusions from the resampling are not different from the statistical significance conclusions of the t-tests. The distributions of the resampled differences and 99 percent confidence intervals for the test-statistics are presented in Figure 2A-B.

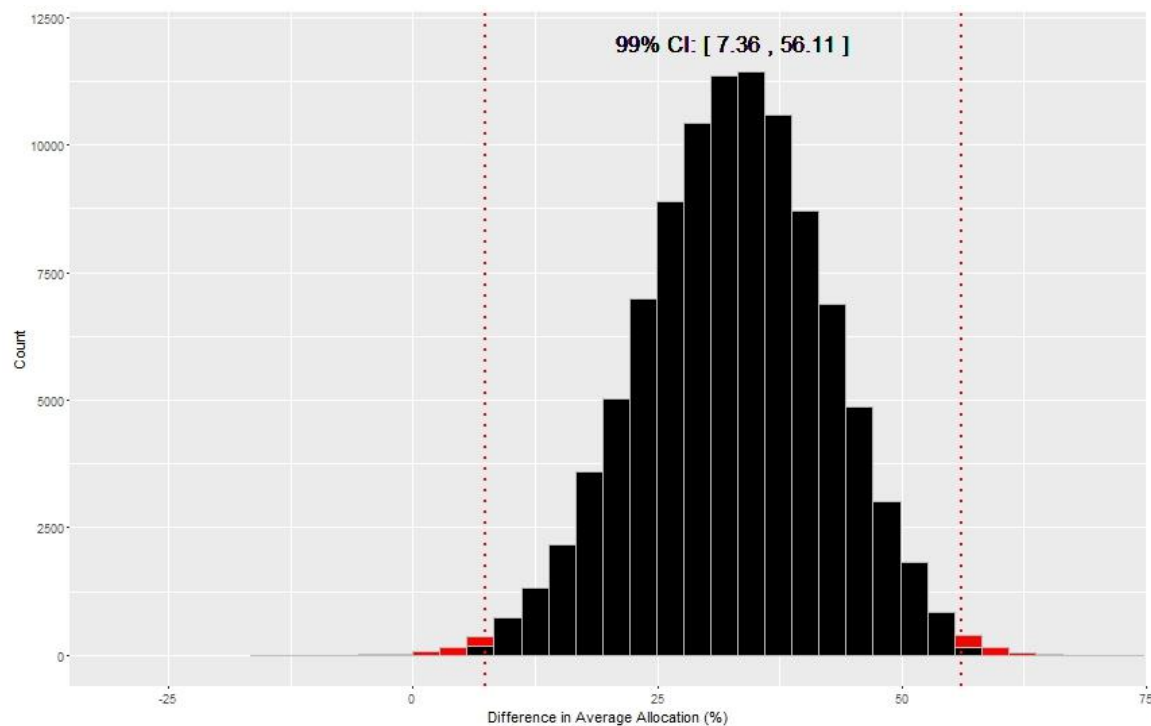


Figure 2A. Bootstrapped differences in average final choice allocation to the Bond Fund for the monthly and yearly conditions
Red bars represent values outside of the 99% confidence interval. 95% confidence interval: [13.47, 50.90]

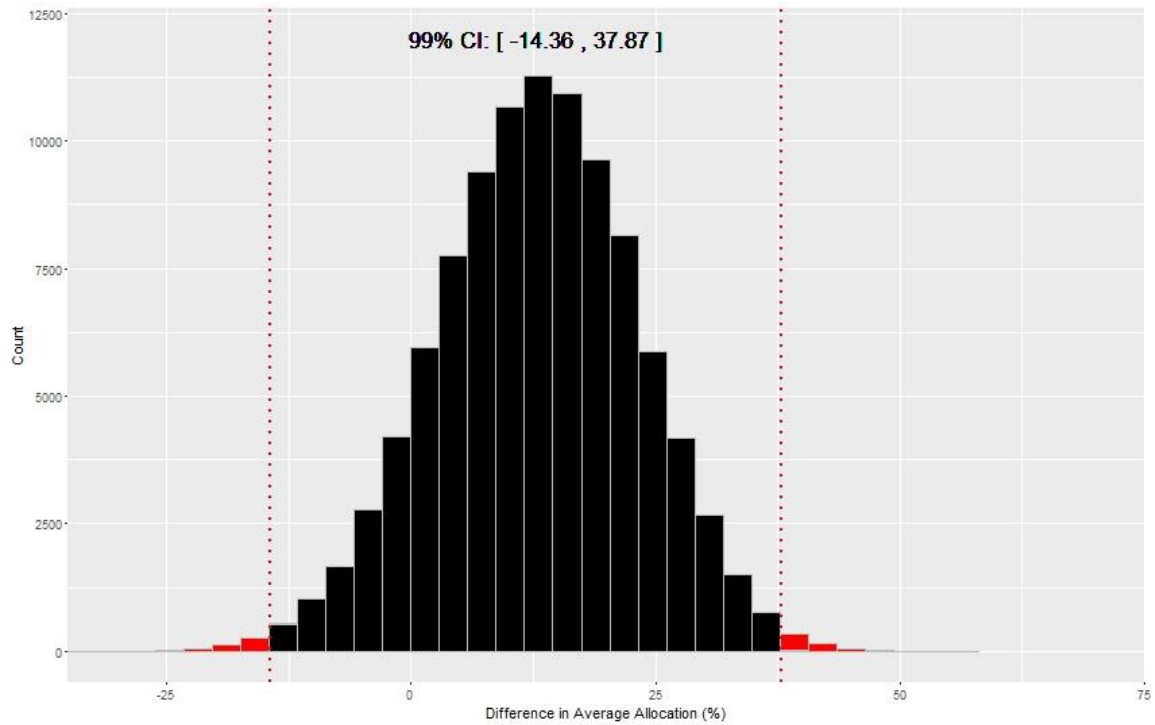


Figure 2B. Bootstrapped differences in average final choice allocation to the Bond Fund for the monthly and inflated-monthly conditions
Red bars represent values outside of the 99% confidence interval. 95% confidence interval: [-7.66, 32.31]

It is reasonable to suspect correlation of choices within subjects over time. Indeed, empirical autocorrelation functions (Figure 3 A-C) confirmed this suspicion for many subjects, with strong correlation between choices with short (1-2) lags. There is less evidence of autocorrelation for subjects in the yearly condition but this is likely a result of lack of statistical power due to lower sample size than the other two conditions. Correlation of choices within subjects violates the basic independence assumptions in the t-tests. Thaler et. al. addressed this issue by using the *average* allocation choice over the last forty periods for each subject (aggregation method). This method is applied to the data and presented in Table 2A. There is a statistical difference between the monthly and yearly conditions ($p < 0.001$) but no difference between the monthly and inflated-monthly conditions ($p = 0.037$) nor between the yearly and inflated-monthly conditions ($p = 0.014$).

An issue with the aggregation method is that it throws away information on the dispersion of choices for each subject and treats the data as if each subject only made one

choice. Table 2B displays the results of t-tests when using cluster-robust standard errors to account for autocorrelation within each subject (Hill, Griffiths, & Lim, 2011). This changes the p-values of the above tests to $p < 0.001$, $p = 0.003$, and $p = 0.012$ respectively. Whereas the aggregation method couldn't reject the null hypothesis that subjects in the monthly condition allocated differently from subjects in the inflated-monthly condition, the cluster-robust method could. These cluster-robust tests also provided intraclass correlation factors that were quite significant (monthly/yearly: 0.559, monthly/inflated-monthly: 0.445, yearly/inflated-monthly: 0.215) which further confirmed the need to adjust regular standard errors.

Paired sample t-tests were performed to compare the final choice with the average of the last forty periods for each condition. None of the three conditions had a statistical difference between the two choices (for either alpha level), an identical result to what Thaler et. al. found. This observed behavior is irrational. In the last forty periods, subjects were allocating assets for only one or ten periods at a time, with a very limited time horizon (less than forty periods). For the final choice, subjects were explicitly told that the time horizon for their final choice would be four-hundred periods, which when applying the WLLN, subjects should have taken the opportunity to accept more risk than they did in the last forty periods. This irrational behavior further strengthens the argument that these subjects were suffering from myopic loss aversion. The subjects were so myopic that they did not distinguish between an allocation for one or ten periods and four-hundred periods.

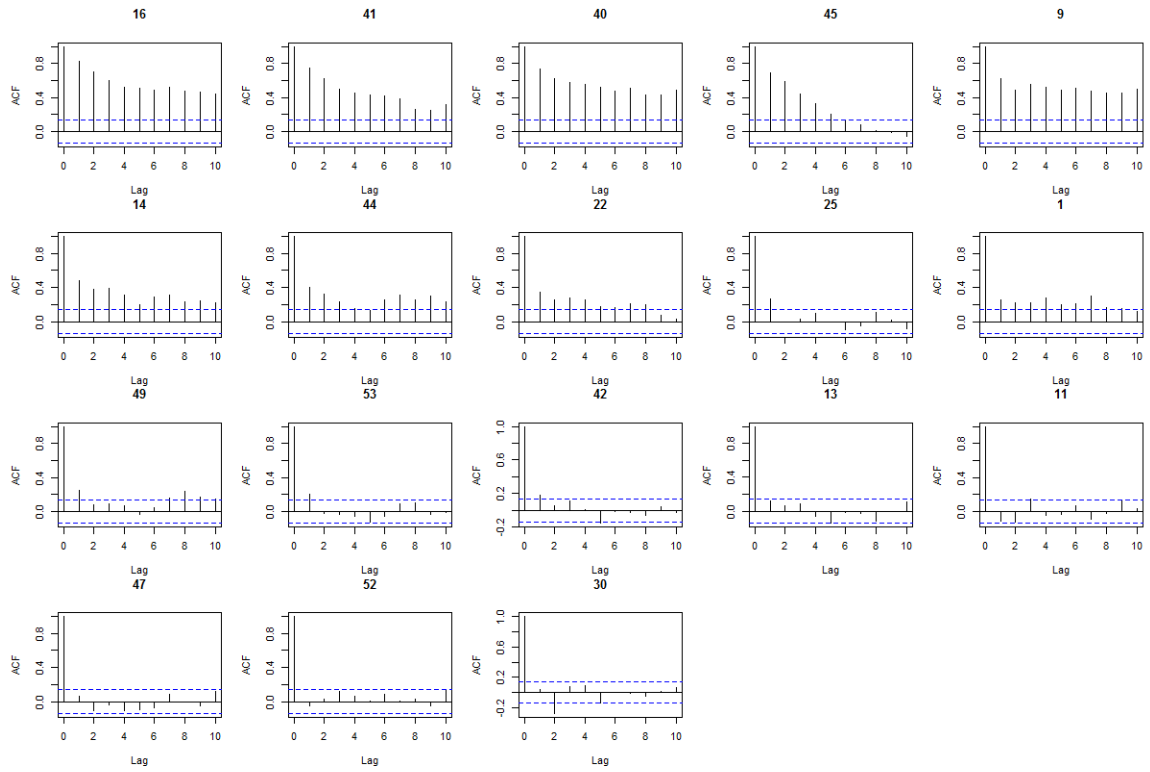


Figure 3A. Monthly condition subjects' autocorrelation functions (ACF) of allocation choices

ACFs that extended above or below the dotted lines indicate that they are statistically significantly different from zero. Numbers above each graph are the unique ID numbers for each subject. Graphs are organized by highest absolute ACF for lag 1.

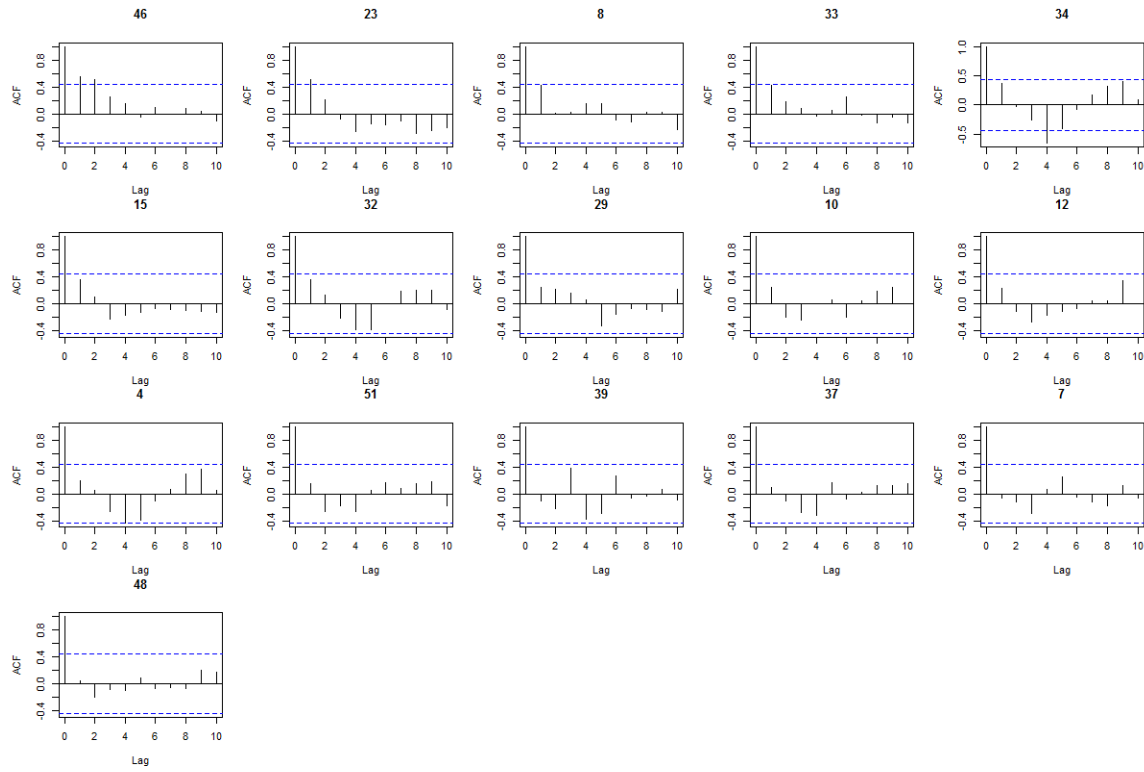


Figure 3B. Yearly condition subjects' ACFs of allocation choices

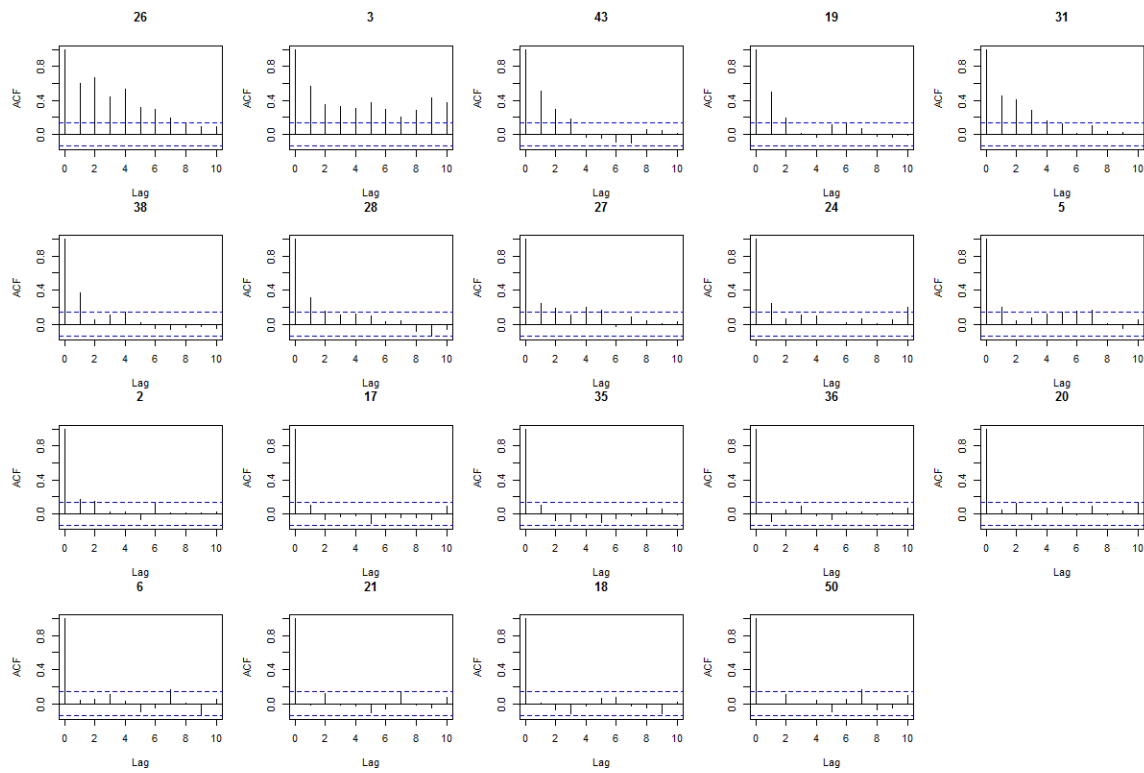


Figure 3C. Inflated monthly condition subjects' ACFs of allocation choices

Next, portfolio allocations over time for each condition were examined. Studying average portfolio allocations over time allows us to examine how subjects learned and adapted to the simulator over time. Figure 4A compares the monthly and yearly conditions. Figure 4B compares the monthly and inflated monthly conditions. At nearly every period, subjects in the monthly condition allocated more of their portfolio to the Bond Fund on average than the subjects in the yearly condition. The same could be said when comparing subjects in the monthly condition with subjects in the inflated monthly condition. Subjects in the monthly condition appear to have their allocations drift towards the Bond Fund over time. Subjects in the yearly and inflated monthly condition appear to have their allocations drift in the opposite direction, with the yearly condition drifting significantly more than the inflated monthly condition.

The coefficient estimates for a pooled-data regression for allocation to the Bond Fund on the trial number (allocation decision) in Table 3 confirm the drifts. Cluster-robust standard errors by subject were used. Only the yearly condition has a statistically significant drift. The yearly condition also has a large estimated drift, a decrease of nearly one percent per trial (every ten periods) compared to the other two conditions with relatively small drifts. This statistically significant negative drift indicates that subjects in the yearly condition became less and less averse to the Bond Fund over time. The same could not be said for the subjects in either the monthly or inflated monthly condition. These results are again nearly identical to Thaler et. al.'s findings except that they find a statistically significant negative drift for their inflated monthly condition as well. However, Thaler et. al. do not use cluster-robust standard errors in their regression which means their standard errors are invalid due to autocorrelation within each subject.

A fixed effects model (varying intercepts only) with regular standard errors and a fixed effects with cluster-robust standard errors model were applied to the data as well. In the fixed effects model with regular standard errors, the trends for all three conditions are statistically significant. However, once cluster-robust standard errors are included, the standard errors return to virtually the same values they were in the first model that did not include fixed effects. This indicates that the fixed effects were unable to pick up the autocorrelation within each subject.

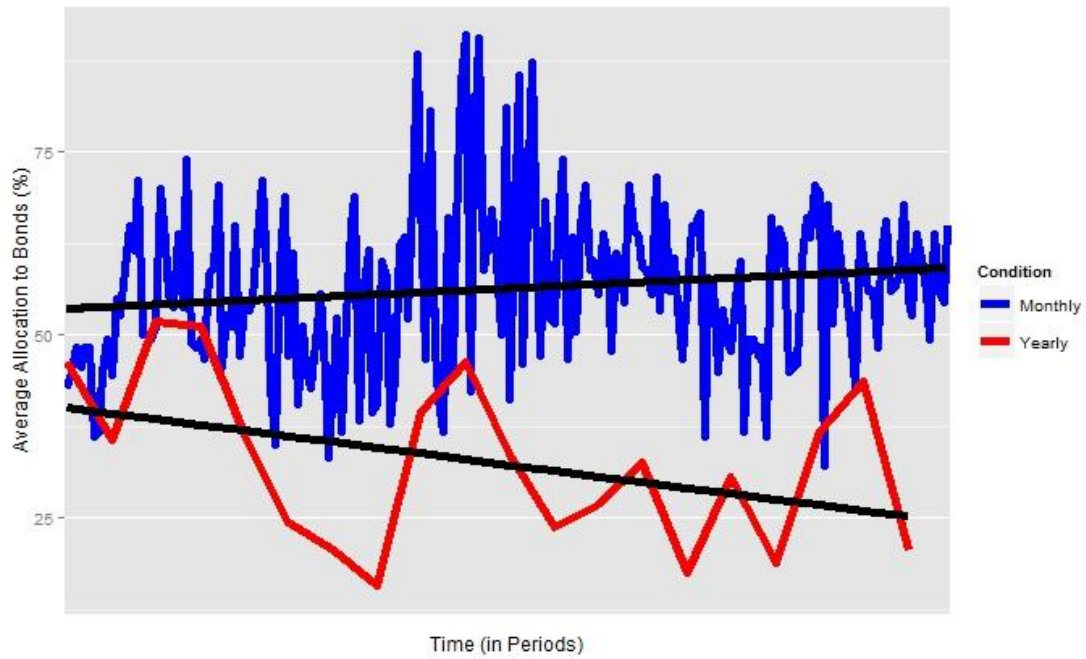


Figure 4A. Comparing average allocations over time (monthly vs. yearly conditions)

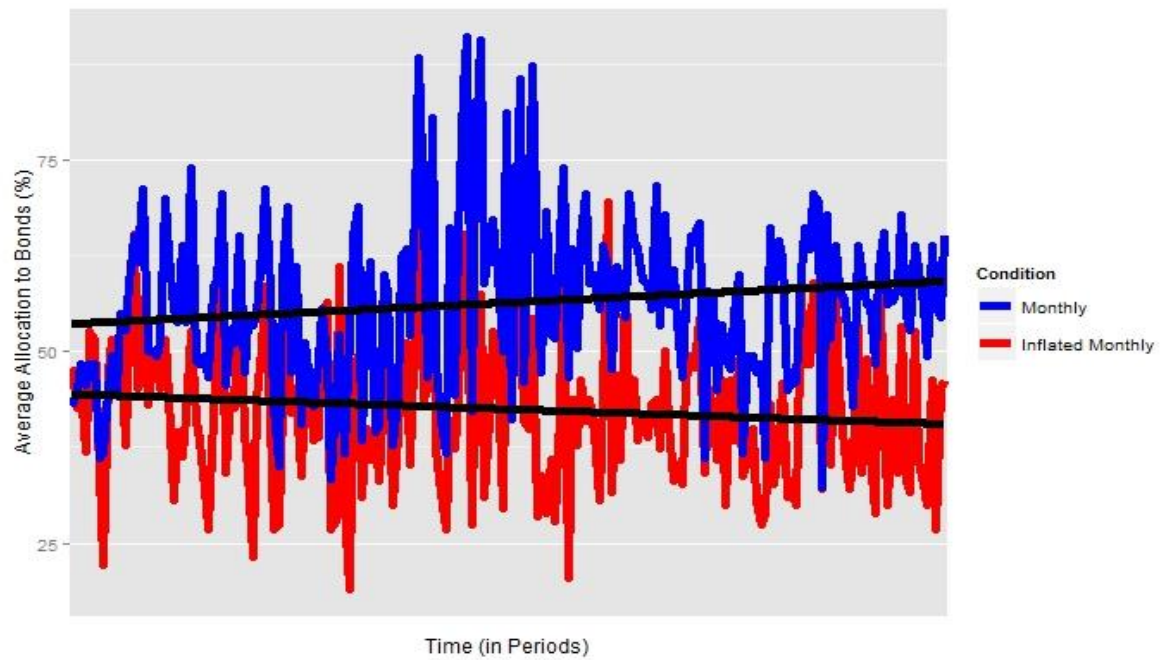


Figure 4B. Comparing average allocations over time (monthly vs. inflated monthly conditions)

Condition	Trial number	Intercept
Monthly	0.028	53.598
<i>Cluster-robust</i>	(0.032) p=0.381	(4.365) p<0.001**
<i>Fixed effects</i>	(0.008) p<0.001**	n/a
<i>Cluster-robust+fixed effects</i>	(0.032) p=0.382	n/a
Yearly	-0.782	40.836
<i>Cluster-robust</i>	(0.160) p<0.001**	(2.385) p<0.001**
<i>Fixed effects</i>	(0.163) p<0.001**	n/a
<i>Cluster-robust+fixed effects</i>	(0.164) p<0.001**	n/a
Inflated Monthly	-0.020	44.458
<i>Cluster-robust</i>	(0.016) p=0.225	(1.987) p<0.001**
<i>Fixed effects</i>	(0.007) p=0.005**	n/a
<i>Cluster-robust+fixed effects</i>	(0.016) p=0.226	n/a

Table 3. Regressions predicting allocation to Bond Fund (%) from trial number

Final allocation choice is excluded. Three standard errors of the trial number coefficient are reported in parenthesis for each of the three different models. * p<0.05 ** p<0.01

Since Thaler et. al.'s experiment performed in 1997, there have been two major stock market crashes. In the post-simulator survey, participants were also asked if they knew what the “dotcom bubble of 2000-2001” and the “financial crisis of 2008-2009” were and if so, name their best guess as to how much value stocks lost over the crashes. Only about half of the subjects (31 out of 53) knew what the Dotcom bubble was, likely due to the young age of the subjects during the time of the crash. Most subjects (47 out of 53) knew what the Financial Crisis was. Final choice allocation was regressed on dummy variables indicating knowledge of the Dotcom bubble and the Financial Crisis. Additionally, for subjects who recalled the crashes, final choice allocation was regressed on best guesses of stock value loss in the Dotcom bubble and also the Financial Crisis to see if subjects who recalled high losses in stocks tended to favor bonds in the simulator. There were no statistically significant results in any of the regressions as shown in Table 4.

Regression	Dotcom dummy	Fin. Crisis dummy	Stock value loss (Dotcom)	Stock value loss (Fin. Crisis)
Knowledge of crashes (N=53)	-0.343 (9.685) p=0.972	2.708 (15.062) p=0.858		
Recollection of stock value loss in dotcom bubble (N=31)			-0.124 (0.130) p=0.349	
Recollection of stock value loss in financial crisis (N=47)				0.057 (0.078) p=0.470

Table 4. Regressions of final allocation to Bond Fund (%) on various measurements of finance knowledge and recollection of stock value loss
Standard errors in parenthesis

3.2 Extended Analysis

Thaler et. al. chose an alpha level of 0.01 for all of their statistical tests, a much stricter standard than what is conventional. Although using a more relaxed standard of 0.05 as an alpha level does not change most of my statistical conclusions, there are a few tests that are affected. In Tables 1-2B, comparisons between yearly and inflated-monthly conditions are affected across the board. At an alpha level of 0.01, there is no statistical difference between the two groups in any test, at an alpha level of 0.05, there *is* a statistical difference in *every* test involving the two groups. This would imply that the effectiveness of aggregating returns and forcing subjects to make long term choices is stronger than simply shifting the distribution of returns to avoid losses in addressing myopic loss aversion. This conclusion is supported by the fact that evidence of aggregation and long-term commitment being effective is much stronger than distribution shifting when comparing to the control condition (the monthly condition).

This statistical conclusion diverges from what Thaler et. al. found in any of their tests. There could be a few different explanations for this diversion. One is that it is simply due to the difference in alpha level chosen. However, since Thaler et. al. do not provide information on results when the alpha level is 0.05 for their data, this explanation is only speculated. Another explanation is the fact that the inflation factor used in this replication to shift the return distributions was significantly smaller than the inflation factor Thaler et. al. used. This point will be elaborated more in section 4.1.

Chapter 4: Conclusion and Discussion

4.1 Conclusion

Overall, the results found in this replication were very similar to the results found in Thaler et. al.'s experiment. As measured by several methods, subjects in the monthly condition were overall more likely to allocate a higher percentage their portfolios to the Bond Fund in comparison to subjects in the yearly and inflated monthly conditions. This phenomenon was most likely caused by elimination of perceived losses either by statistical aggregation or distribution shifting. Given the similarity in results, Thaler et. al.'s conclusion that myopic loss aversion exists still stands. Their proposed remedy of forcing people to think and observe long term trends and restricting information on short term trends stands as effective since losses are minimized through statistical aggregation in the yearly condition. However, the evidence is weaker in this replication for the effectiveness of shifting distributions in reducing myopic loss aversion. Thaler et. al. found much stronger evidence that the inflated monthly condition is effective in reducing myopic loss aversion in their experiment, nearly on par with the yearly condition, than in this replication.

This difference in results could be attributed to several explanations. It could simply be a lack of statistical power since the sample size here was not large. The point estimates for the final choice indicate that subjects in the monthly condition tended to favor the Bond Fund significantly more than subjects in the inflated monthly condition (63.33% to 50.53%). There just appeared to be a lack of statistical confidence in rejecting the null hypothesis that there is no difference between the two conditions. Another explanation is that this replication used an inflation factor of 5% as opposed to 10% used by Thaler et. al. This resulted in 3% (6 of 200) of the inflated monthly returns for the Stock Fund still being negative. Thaler et. al. managed to inflate away all losses for their Stock Fund with the higher inflation factor. It is possible that the subjects were so loss-averse that even seeing a few negative returns from the Stock Fund caused them to shy away from it and towards the Bond Fund. In this case, this lowered magnitude of effect

coupled with lowered significant evidence of reducing allocation to the Bond Fund serves to bolster the evidence for the existence of myopic loss aversion. In order for the shifting of distributions to be fully effective, one must eliminate all possibility of loss rather than just most. However, further testing is required to fully examine this hypothesis.

If the true effect of the shifting of distributions is indeed strong as found in Thaler et. al.'s experiment, it bodes for an interesting implication. In periods of high inflation, stocks would have to increase their nominal return in order to maintain their real return. This effect is very similar to the shifting of distribution done in the inflated-monthly condition. Though the real return would not have changed for stocks, the nominal returns would include much fewer losses than before. If the inflation is unexpected and investors are subject to the money illusion (inability to distinguish nominal versus real) as mentioned by Thaler et. al., this could result in a temporary increase in the attractiveness of stocks for investors due to the decrease in losses.

Given the resounding effectiveness of the yearly condition in helping subjects make better investment choices, it certainly seems appropriate to consider some practical policy changes. Although it would be unlikely for brokerage companies to completely restrict information and trading ability (that's their entire business model) to amateur investors, they could reduce exposure in order to help their clients make better long-run decisions rather than to encourage day-trading. Instead of presenting daily or even hourly information on how their investments are doing, brokerages could have long-term (ten or more years) aggregated returns displayed initially on their web page or mobile app. Also, they could limit the amount of trades their clients can do on their own per month. To activate additional trading ability beyond that limit, the clients must call the brokerage and speak to an investment professional before receiving that privilege. Indeed, the observation that giving more information and allowing more trading flexibility actually hurts investors is not only found in Thaler et. al.'s experiment and this replication. Malkiel (2007) centers the theme of his book "A Random Walk Down Wall Street" around the fact that it is extremely difficult to defeat a long-run passive investing strategy of holding index funds with an active investing (such as day-trading) strategy. This holds for all investors, not just amateurs.

Unfortunately, brokerages make significant profit from trading fees. They would not be interested in policies that encourage patience and passiveness. Myopic loss aversion in amateur investors actually benefits the brokerages, who don't mind major losses in the market because it induces investors to use their services. These policies proposals may be better suited for brokerages such as Vanguard and Fidelity that focus on managing retirement accounts for their clients, which are generally long-run prospects.

4.2 Discussion

It is quite interesting that the field of economics only considered risk-aversion when evaluating decision making for an extremely long time until prospect theory was proposed in 1979 and then expanded into *cumulative* prospect theory in 1992. In fact, the proposal was so groundbreaking to the field of economics it warranted a Nobel Prize to the authors. Though the underlying equations of prospect theory are extremely complex, the basic idea of the theory was already intuitive to many. Malkiel notes in his book, which was first published in 1973, that most investment professionals considered variance an inadequate measure of the riskiness of an investment. Surely, they thought, no one would consider the possibility of earning a return *greater* than expected as a *risk*. However, this is exactly what risk-aversion theory states. CPT alleviates this issue by using loss aversion to weigh losses more appropriately relative to gains.

Though CPT and loss aversion seem intuitive, some criticism emerges when examining the experimental design of Thaler et. al.'s work and this replication. Thaler et. al.'s claim that the yearly condition contains less information than the monthly condition certainly holds for a machine with the ability to record information, but what about humans? Humans on average can only remember about seven numbers at once (Miller, 1956). How then could a subject remember twenty returns (ten per fund) and also be able to compute the average return for both funds all from memory? In the yearly condition, the simulator already does that work for the subject. In this case, subjects in the yearly condition have the critical piece of information (long term trends) necessary for success

in the simulator. Subjects in the monthly condition lack this information due to the weakness of human memory and computation abilities. To account for this issue, the simulator could display information about the trailing ten period average returns for both funds in the monthly condition. This modification to the experimental design would be the next step in researching myopic loss aversion in investment behavior.

Benoit and West (2010) performed some interesting investigation into the equity premium a few decades after Thaler et. al.'s performed their experiment that linked myopic loss aversion to the equity premium. With more extended data, Benoit and West noted that the more recent period of 1963-2009 had an average equity premium of about 4 percent, a much lower figure than in the period (1889-1978) that Mehra and Prescott had observed. This lower figure was heavily influenced by the tumultuous times for equities in the most recent decade and a half. The equity premium from 1997-2008 was actually negative (risky investments actually returned less on average than risk-free investments).

Benoit and West also proposed an alternative explanation to the equity premium puzzle. They state that using ex-post data on past equity premiums is not an accurate indicator of investors' perceived future equity premium and are poor predictors of future returns. An indicator of this is derived from the negative equity premium from 1997-2008. No rational investor in their right mind would invest in a riskier asset that returned less on average than a risk-free asset, and yet investors in the twelve year period did just that. Investors during that time period must have expected at the very least a positive equity premium but the ex-post equity premium was not reflective of that expectation. Given the unreliability of ex-post equity premiums to proxy for expectations, it is speculated investors have been underestimating the equity premium on average. Most importantly, this implies that expected utility theory may still be valid in explaining expected equity premium, because the large historical equity premium that was put into question by Mehra and Prescott does not actually reflect investors' future expectations.

Additionally, since the equity premium is measured using the S&P 500 Index, there is a possibility of survivorship bias. Only the surviving companies remain on the index, and thus inflate the equity premium, since failed or failing companies and their tanked equity returns are not counted. Actual expected equity premium would be much

lower than the ex-post equity premium, since investors would take into account the failed or failing companies. These explanations, along with myopic loss aversion, are all strong contenders as explanations for the equity premium. It remains to be seen which explanation, if any, come out on top in the expanding field of behavioral economics and finance.

Appendix A: Analytic Solution to Probability of Underperformance

The following derivation attempts to quantify the probability of the Stock Fund underperforming versus the Bond Fund over the long run. Intuitively, we would expect the probability to decrease as the amount of periods (n) involved increases.

To begin, we define the returns to the Stock Fund and the Bond Fund for as r_A and r_B respectively. r_A is distributed Normal(mean= μ_A , s.d.= σ_A) and r_B is distributed Truncated Normal(mean= μ_B , s.d.= σ_B , min.=0). The possibility of analytically solving for the geometric mean of simple returns from these distributions depends on whether the returns are continuously compounded returns or simple returns. The distribution is analytically solvable if these returns are seen as continuously compounded returns (r) rather than simple returns (R). Appendix B will address the second case. Note that converting a continuously compounded return to a simple return *simply* requires exponentiation with base e.

A simple return (and adding 1) for the Stock Fund in period i will be denoted $e^{r_A^i}$ and similarly for the Bond Fund. Equation (1) states the definition of the geometric mean as applied to the simple return for the Stock Fund and then using the laws of exponentiation to arrive at the right-hand side. The number of periods or time horizon of the investment life is indicated by n. The Bond Fund's geometric mean return is derived similarly using its own simple return and is shown in equation (2).

$$(1) \left(\prod_{i=1}^n e^{r_A^i} \right)^{\frac{1}{n}} = e^{\frac{\sum_{i=1}^n r_A^i}{n}}$$

$$(2) \left(\prod_{i=1}^n e^{r_B^i} \right)^{\frac{1}{n}} = e^{\frac{\sum_{i=1}^n r_B^i}{n}}$$

After establishing equations (1) and (2), we take the ratio of the two equations to find the equation for the ratio of the geometric mean of returns for the Stock Fund and the Bond Fund in equation (3) and apply more laws of exponentiation.

$$(3) \frac{e^{\frac{\sum_{i=1}^n r_A^i}{n}}}{e^{\frac{\sum_{i=1}^n r_B^i}{n}}} = e^{\left(\frac{\sum_{i=1}^n r_A^i}{n} - \frac{\sum_{i=1}^n r_B^i}{n} \right)}$$

Recognize that the arithmetic mean of the continuously compounded returns for the Stock Fund is distributed Normal(mean= μ_A , s.d.= $\frac{\sigma_A}{\sqrt{n}}$). Also recognize that the arithmetic mean of the continuously compounded returns for the Bond Fund converges in distribution to a Normal(mean= μ_B , s.d.= $\frac{\sigma_B}{\sqrt{n}}$) distribution by applying the Central Limit Theorem. Given this, we know that the exponent of the right-hand side of equation (3) converges to the difference between two normal distributions which is also distributed normally. The asymptotic distribution of the exponent is then Normal(mean= $\mu_A - \mu_B$, s.d.= $\sqrt{\frac{\sigma_A^2 + \sigma_B^2}{n}}$). This leads us to equation (4) which recognizes that the right-hand side of equation (3) is the exponentiation of a normal distribution with base e. This means that the ratio of the geometric mean of returns of the Stock Fund to the Bond Fund converges in distribution to a lognormal distribution.

$$(4) e^{\left(\frac{\sum_{i=1}^n r_A^i}{n} - \frac{\sum_{i=1}^n r_B^i}{n} \right)} = Ratio_{ab} \sim \text{LnNormal}(\mu_R = \mu_A - \mu_B, \sigma_R(n) = \sqrt{\frac{\sigma_A^2 + \sigma_B^2}{n}})$$

We are interested in the probability that the ratio is less than 1 meaning that the Stock Fund has underperformed the Bond Fund. Interestingly and perhaps also intuitively, the cumulative distribution function as a function of x for a lognormal distribution is the cumulative distribution function of the underlying normal distribution as a function of $\text{Ln}(x)$.

$$(5) C.D.F. Ratio_{ab}(x, \mu_R, \sigma_R(n)) = \Phi\left(\frac{\ln(x) - \mu_R}{\sigma_R(n)}\right)$$

Where $\Phi(\cdot)$ is the cumulative distribution function for a standard normal distribution

That is to say, the cumulative distribution function of a lognormal distribution is fully characterized by the underlying normal distribution. In application to this specific instance, this means that the cumulative distribution of the *ratio* of the *geometric mean of simple returns* from two funds is fully characterized by the cumulative distribution of the *difference* of the *arithmetic mean of continuously compounded returns* from the two funds.

Take the perspective of an experimental subject who has completed the initial two-hundred periods of investment simulation and now must make a choice that locks in for four-hundred periods. Clearly, one would want to invest their money in the fund with the higher expected return but also understand the risks of having that fund underperform. Given the computation of the parameters for $C.D.F. Ratio_{ab}$, we can establish the probability of the Stock Fund underperforming the Bond Fund over four-hundred periods as $C.D.F. Ratio_{ab}(1, 0.0075, 0.001772211)$. This will evaluate to a 0.001% probability that the Stock Fund underperforms the Bond Fund over four-hundred periods. It would take an enormous amount of risk-aversion, or an enormous amount of irrationality, for someone to put any amount of their portfolio into Fund B.

Given $x = 1$, the argument inside of $\Phi(\cdot)$ is $\frac{-\mu_R}{\sigma_R(n)}$. As $n \rightarrow \infty$, $\sigma_R \rightarrow 0$, which means $\frac{-\mu_R}{\sigma_R(n)} \rightarrow -\infty$. In this case, $\Phi\left(\frac{-\mu_R}{\sigma_R(n)}\right) \rightarrow 0$. This means that as the number periods over which we are evaluating fund returns approaches infinity, the probability of the Stock Fund underperforming the Bond Fund approaches zero, otherwise speaking, impossible. Indeed, given that $\Phi(\cdot)$ is monotonically increasing and that $\frac{-\mu_R}{\sigma_R}$ is monotonically decreasing as a function of n , the probability of the Stock Fund underperforming the Bond Fund decreases monotonically as n increases.

Thaler et. al. does not make fully clear whether the returns generated from the normal and truncated normal distributions are continuously compounded returns or simple returns. In finance theory, returns that come from normal distributions are generally continuously compounded returns. This allows for the simple returns, and therefore asset price, to be distributed lognormally. It is unreasonable to allow for the possibility of greater than 100% loss from an investment, since assets cannot be valued below zero. The lognormal distribution sets that limitation whereas the normal distribution does not (Zucchi, 2014). However, based on a best guess of Thaler et. al.'s work, it appears that they chose (incorrectly) to use a normal distribution (and truncated normal distribution) to model simple returns and asset prices. This experimental replication has assumed the same. Estimating the probability distribution of the ratio of

the two funds when simple returns are distributed normally (or truncated normally) can only be done via simulation, which will be discussed in Appendix B.

Appendix B: Simulated Solution to Probability of Underperformance

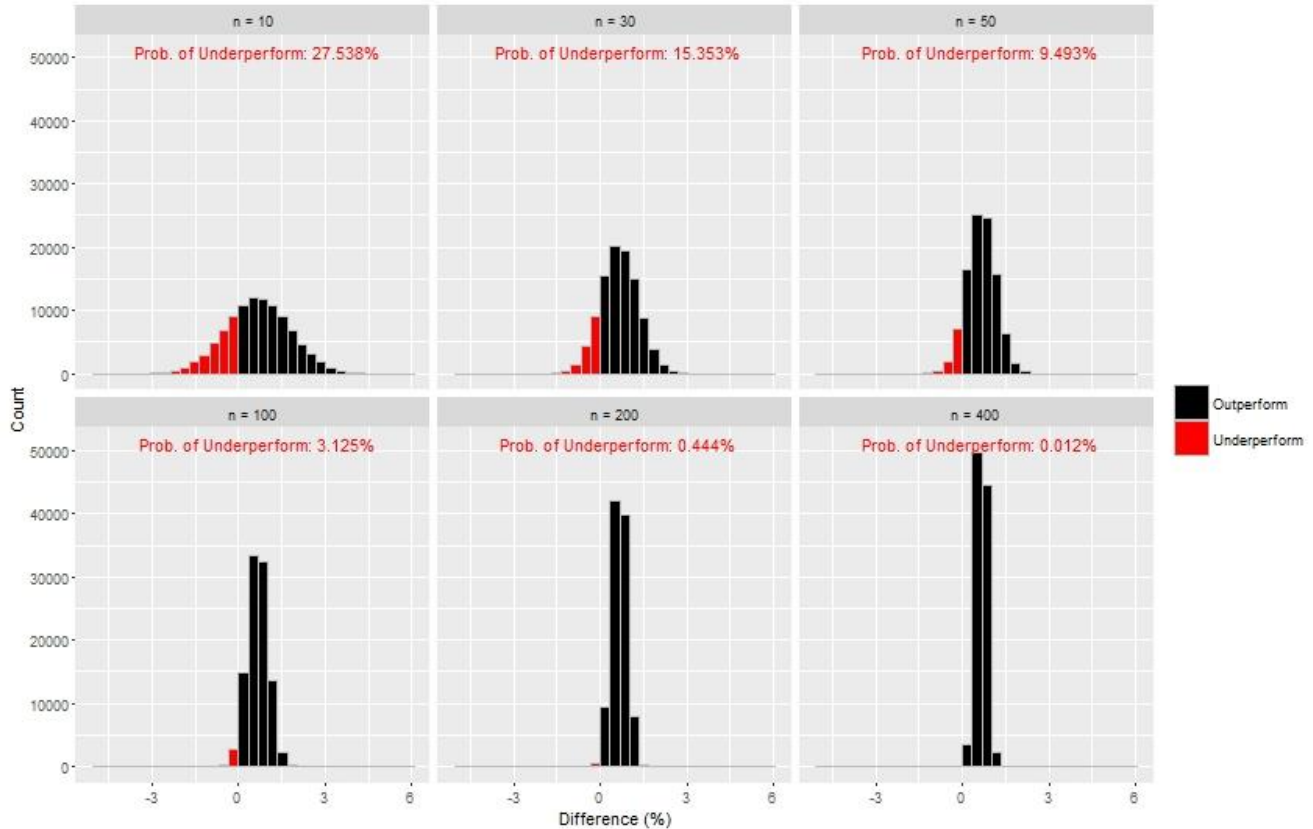


Figure 5. Simulated distribution of difference in geometric mean return between the Stock Fund and the Bond Fund over various sample sizes

Each distribution has 100,000 simulated observations. Red bars indicate underperformance (difference is negative) from the Stock Fund relative to the Bond Fund. On average, the Stock Fund outperformed the Bond Fund by 0.66% per period.

To simulate the distribution of relative performance between the Stock Fund and Bond Fund when assuming Thaler et. al.'s returns are simple returns, n observations (time horizon) are drawn from the distributions for both funds. The geometric mean of those n observations are computed for both funds and the difference between the two (Stock Fund – Bond Fund) is saved as one observation in the distribution. A ratio was used in Appendix A to ensure that the distribution could be solved analytically. Given

that constraint no longer applies, the difference is used here which is a more common metric of comparing returns¹. This process is repeated 100,000 times for $n = 10, 30, 50, 100$, and 400 for a total of 600,000 times. The resulting distributions are presented in Figure 4.

The resulting distributions appear to be normally distributed. Kolmogorov-Smirnov tests could not reject the hypothesis that the distribution is normal for all six distributions. Notably, the WLLN seemed to be in effect here. As n increases, the variance of the distribution decreases, clustering the distribution around the mean (roughly 0.66%). Due to this effect, the probability of the Stock Fund underperforming the Bond Fund ($x < 0$) decreases as n increases. This is very similar to the findings in Appendix A where the returns are assumed to be continuously compounded. With a four-hundred period time horizon, the probability of the Stock Fund underperforming the Bond Fund is only 0.012%, again showing that it would take an enormous amount of risk-aversion or irrationality for a subject to allocate to the Bond Fund in the final choice.

N (time horizon)	Cont. Compounded (Appendix A)	Simple (Appendix B)
10	25.170%*	27.538%
30	12.323%	15.353%
50	6.730%	9.493%
100	1.717%	3.125%
200	0.138%	0.444%
400	0.001%	0.012%

Table 5. Comparison of probability of underperformance of Stock Fund relative to Bond Fund for various time horizons

*Analytically solving for the distribution in appendix A required the application of the Central Limit Theorem which is only valid with large sample sizes. A sample size of 10 may not be sufficient.

¹ Note that if the difference between two returns is negative, it implies that the ratio is also less than one. Thus the probability of underperformance remains the same whether difference or ratio is used.

Appendix C: Post Investment Simulator Survey

Post Investment Simulator Survey

Name: _____ Student ID #: _____

Box #: _____ Session #: _____

- 1) What was your portfolio value before your final allocation choice?

- 2) What was your final portfolio allocation choice? (Fund A – Fund B)

- 3) Are you a current/aspiring economics major? (Yes, No, Unsure)

- 4) Are you affiliated with Reed College Investment Club (RCIC) or any investment related entity? (Yes, No, Unsure)

- 5) Do you own investments (Stocks, Bonds, CDs, etc.) in which you actively chose to invest in? (Yes, No, Unsure)

- 6) In the dotcom bubble of 2000-2001, what is your best guess of the total loss in value for stocks during that time? (Write % loss or “Don’t know what dotcom bubble is”)

- 7) In the financial crisis of 2008-2009, what is your best guess of the total loss in value for stocks during that time? (Write % loss or “Don’t know what financial crisis is”)

Appendix D: Source Code

To view and interact with the investment simulator used in this thesis, visit:

https://youngde.shinyapps.io/Inv_Sim_Demo

To view the source code for the investment simulator, visit:

https://github.com/deanyoung/Thesis_Base

To view the source code for the data analysis (Section 3) and distribution simulation (Appendix B), visit:

https://github.com/deanyoung/Thesis_Data_Analysis

Bibliography

Baddeley, M. (2013). *Behavioural Economics and Finance*. Routledge.

Benartzi, S., & Thaler, R. H. (1995). Myopic Loss Aversion and the Equity Premium Puzzle. *The Quarterly Journal of Economics*, 110(1), 73–92.

Benoit, B., & West, T. (2010). The real-time equity risk premium: An introduction. Grant Thornton. Retrieved from
<http://www.grantthornton.com/staticfiles/GTCom/Advisory/Valuation/Files/risk%20premiums%20-%20white%20paper%20-%20FINAL.pdf>

Damodaran, A. (2015, January 5). Historical Returns. Retrieved September 24, 2015, from
http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html

Egan, M. (2015, March 11). Crisis hangover: Millennials are scared to invest. Retrieved September 22, 2015, from <http://money.cnn.com/2015/03/11/investing/investing-millennials-stocks-markets/index.html>

Graham, B. (2006). *The Intelligent Investor: The Definitive Book on Value Investing. A Book of Practical Counsel* (Rev Sub edition). New York: HarperBusiness.

Hill, R. C., Griffiths, W. E., & Lim, G. C. (2011). *Principles of Econometrics*. Wiley.

Malkiel, B. G. (2007). *A Random Walk Down Wall Street: The Time-Tested Strategy for Successful Investing* (Ninth Edition edition). New York: W. W. Norton & Company.

- Mankiw, N. G., & Zeldes, S. P. (1991). The consumption of stockholders and nonstockholders. *Journal of Financial Economics*, 29(1), 97–112.
[http://doi.org/10.1016/0304-405X\(91\)90015-C](http://doi.org/10.1016/0304-405X(91)90015-C)
- Mehra, R., & Prescott, E. C. (1985). The equity premium: A puzzle. *Journal of Monetary Economics*, 15(2), 145–161. [http://doi.org/10.1016/0304-3932\(85\)90061-3](http://doi.org/10.1016/0304-3932(85)90061-3)
- Miller, G. A. (1956). The magical number seven plus or minus two: some limits on our capacity for processing information. *Psychological Review*, 63(2), 81–97.
- Post, T., Assem, V. den, J. M., Baltussen, G., & Thaler, R. H. (2012). *Deal or No Deal? Decision Making under Risk in a Large-Payoff Game Show* (SSRN Scholarly Paper No. ID 636508). Rochester, NY: Social Science Research Network.
Retrieved from <http://papers.ssrn.com/abstract=636508>
- Siegel, J. J., & Thaler, R. H. (1997). Anomalies: The Equity Premium Puzzle. *The Journal of Economic Perspectives*, 11(1), 191–200.
- Thaler, R. H., Tversky, A., Kahneman, D., & Schwartz, A. (1997). The Effect of Myopia and Loss Aversion on Risk Taking: An Experimental Test. *The Quarterly Journal of Economics*, 112(2), 647–661. <http://doi.org/10.1162/003355397555226>
- Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5(4), 297–323.
<http://doi.org/10.1007/BF00122574>
- Zucchi, K. (2014, October 20). Lognormal and Normal Distribution. Retrieved January 10, 2016, <http://www.investopedia.com/articles/investing/102014/lognormal-and-normal-distribution.asp>