

Using and Interpreting Logistic Regression: A Guide for Teachers and Students

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USING AND INTERPRETING LOGISTIC REGRESSION: A GUIDE FOR TEACHERS AND STUDENTS

Despite the frequent use of logistic regression in the social sciences, considerable confusion exists about its use and interpretation. This confusion is attributed to a lack of adequate teaching materials and to unfamiliarity with logistic regression by many statistics instructors. The purpose of this paper is to define and illustrate basic concepts of dichotomous logistic regression (DLR) and to present strategies for teaching these concepts. Strategies include 1) using analogies between ordinary least squares (OLS) regression and logistic regression, 2) illustrating concepts with contingency tables, 3) focusing initially on the bivariate case for ease of understanding, and 4) linking logistic regression concepts to interpretation of statistics in computer outputs. After discussing several examples of logistic regression, we present and illustrate statistics for evaluating the goodness of fit and predictive efficacy of a DLR model.

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WALSH (1987:178) WRITES "THE TEACHING OF logistic regression analysis is much neglected in statistics courses within sociology." He blames this neglect by instructors on their lack of understanding of this statistical technique and on the omission of this topic from basic statistical texts. Although logistic regression has gained popularity in recent years, there remains considerable confusion about its use and interpretation (DeMaris 1993a; Morgan and Teachman 1988). Roncek (1991) attributes this confusion to inconsistent treatment of the interpretation of logistic models in the sociological methodological literature. The incorrect use of terms and formulas associated with logistic regression aggravates the problem (DeMaris 1990, 1993a, 1993b; Petersen 1985; Roncek 1993). In addition, Hosmer, Taber, and Lemeshow (1991) emphasize that many users of logistic regression fail to report measures that assess the fit of a model to data, and that such omission makes it impossible to determine whether the stated inferences from the model are correct.

Liao (1994) lists nearly two dozen books, monographs, and articles on probability models, including logistic regression, which have been published over the last decade. A few of the more recent social statistics texts (e.g., Agresti and Finlay 1986; Knoke and Bohrnstedt 1994; Lunneborg 1994) now

include a chapter on logistic regression. The literature, however, offers little advice on how to interpret probability models and their parameters; as a result, some social scientists do not use logistic regression even when it might be more appropriate than other approaches (Liao 1994). With few exceptions, we could not find teaching materials with a clear explanatory focus understandable by the typical sociology graduate student and the nonmathematically trained statistics instructor. In short, the literature seems to cover theoretical and mathematical issues related to logistic regression more thoroughly than the practical and applied aspects needed to put this technique to use.

In this paper we define and illustrate dichotomous logistic regression (DLR) concepts, provide examples in the text to facilitate the understanding and interpretation of these concepts, and present strategies for teaching DLR. The teaching strategies include 1) using analogies between ordinary least squares (OLS) regression and DLR; 2) illustrating concepts with contingency tables; 3) focusing first on the "one-independent-variable case," as is usual in teaching OLS; and 4) linking logistic regression concepts to the interpretation of statistics appearing on computer outputs. Since Walsh's (1987) article appeared, changes have been made in logistic

regression outputs of SPSS and SAS, the two statistical packages most often used by sociologists. We will update some of the links between conceptual and statistical interpretation by using SPSS logistic regression outputs. If basic concepts are understood thoroughly and are linked to outputs, many of the problems encountered in using and interpreting logistic regression results may be eliminated.

We assume that readers have a basic knowledge of the linear regression model. In addition, we concentrate on the case of a dichotomous response because of its common use in the social sciences. Furthermore, a knowledge of DLR sets the stage for understanding polytomous and ordered logit modeling as well as discrete-time survival analysis. Thus DLR makes a wide range of tools available to researchers. The examples used here are based on a comparative study of attitudes and behaviors related to sexuality among university women in Sweden and the United States; they examine the effect of various independent variables on experiences involving sexual coercion.

PROBABILITY, ODDS, AND ODDS RATIOS

DeMaris (1990, 1992, 1993a), Liao (1994), Morgan and Teachman (1988), and Roncek (1991, 1993) have demonstrated how DLR can be interpreted with respect to probabilities, odds, and odds ratios. To introduce and illustrate these terms, we use a 2 x 2 contingency table (Table 1, Part A). The dependent variable, or event of interest, is "Have you ever been physically forced by a male to engage in sexual activity?"¹ This variable, called *Physical Coercion*, indicates physical sexual coercion and is dichotomous: either a "yes" (EVENT, coded 1) or a "no" (NON-EVENT, coded 0). The independent variable is *Country* of respondent. Previous research leads us to expect that Swedish women will be less likely to experience sexual coercion

than American women because women in Sweden have more institutional power, rates of general violence are lower, and the double standard of sexuality is weaker (Deley 1988; Grauerholz and Koralewski 1991; Kutchinsky 1991; Pirog-Good and Stets 1989; Weinberg, Lottes and Shaver 1995). Probability and odds of the EVENT are denoted respectively by Prob (EVENT) and ODDS(EVENT). Probability and odds of the NON-EVENT are denoted respectively by Prob(NON-EVENT) and ODDS(NON-EVENT). Probability and ODDS are based on number of events (#EVENTS) plus number of non-events (#NON-EVENTS), totaling #TOTAL (total sample size, N):

$$\text{Prob (EVENT)} = \# \text{EVENTS} / \# \text{TOTAL}$$

$$\text{Prob (NON-EVENT)} = \# \text{NON-EVENTS} / \# \text{TOTAL}$$

$$\text{Prob (EVENT)} + \text{Prob (NON-EVENT)} = 1$$

$$\text{ODDS (EVENT)} = \# \text{EVENTS} / \# \text{NON-EVENTS} = \text{Prob (EVENT)} / \text{Prob (NON-EVENT)}$$

$$\text{ODDS (NON-EVENT)} = \# \text{NON-EVENTS} / \# \text{EVENTS} = \text{Prob (NON-EVENT)} / \text{Prob (EVENT)}$$

In Example 1 (see Table 1) we calculate the following sample probabilities and ODDS:

$$\text{For U.S. women, Prob(COER)} = 122/273 = .447 \text{ and Prob(NO COER)} = 151/273 = .553$$

$$\text{For Swedish women, Prob(COER)} = 80/348 = .230 \text{ and Prob(NO COER)} = 268/348 = .770$$

$$\text{For U.S. women, ODDS(COER)} = 122/151 = .808 \text{ and ODDS(NO COER)} = 151/122 = 1.24$$

$$\text{For Swedish women, ODDS(COER)} = 80/268 = .299 \text{ and ODDS(NO COER)} = 268/80 = 3.35$$

Probabilities can range from 0 to 1. ODDS, which is a ratio of two probabilities, can range from 0 to infinity. In that the ODDS is a ratio of probabilities, an ODDS of 2, for example, means that one is twice as likely to experience the EVENT as *not* to experience it. In addition, ODDS(EVENT) multiplied by ODDS(NON-EVENT) equals 1; that is, the two are reciprocals of each other.

An ODDS ratio, as the name indicates, is a ratio (quotient) of two ODDS and is a means of comparing the two ODDS. An ODDS ratio greater than 1.0 indicates an

¹ For more precise definitions of variable measures used in the examples given here, contact the senior author.

increased likelihood of the EVENT; an ODDS ratio less than 1.0 indicates a decreased likelihood of the EVENT (Morgan and Teachman 1988). In Example 1, the *Country* ODDS ratio measures how the ODDS of suffering physical sexual coercion changes when *Country* changes. "ODDS ratio of coercion for U.S. women compared to Swedish women" equals "ODDS of coercion for U.S. women" divided by "ODDS of coercion for Swedish women":

$$\text{Country ODDS ratio} = (122/151) / (80/268) \\ = .808/.299 = 2.7066.$$

Thus, for American women, the ODDS of being a victim of physical sexual coercion is about 2.7 times greater than the ODDS for Swedish women, in keeping with our expectations about differences between the two countries.

OLS VERSUS DLR

Ordinary least squares (OLS) regression and dichotomous logistic regression (DLR) are methods for modeling a dependent variable as a function of a set of independent variables. For both techniques, the independent variables may be interval or ratio measures, dichotomies, ordinal variables in dummy-variable format, interaction terms, quadratic terms, or other transformations of the predictors. Either technique can be used in modeling a dichotomous dependent variable. As long as the distribution of the dichotomous response is not excessively skewed or lopsided (i.e., the percentage falling into either category is not less than about 10), the use of OLS and of DLR should not produce dramatically different substantive conclusions. DLR, however, is superior for a number of reasons.

First, let us consider the OLS model and its assumptions. The model with K independent variables is

$$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \dots + \beta_K X_{Ki} + \epsilon_i.$$

Alternatively, we can write the model in terms of the average value of Y , or $E(Y)$:

$$E(Y) = \alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_K X_K.$$

When the dependent variable, Y , is dichotomous (coded 0, 1), its "average" value becomes the proportion of cases experiencing the EVENT, denoted by π . The OLS model then becomes

$$\pi = \alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_K X_K. \quad (1)$$

This equation makes it clear that in using OLS, we are modeling the probability of the EVENT (or the proportion of cases in the population who experience the EVENT) as a linear equation. This formulation presents several problems (see Aldrich and Nelson 1984; Bollen 1989; Fox 1984; Hanushek and Jackson 1977; Maddala 1983). First, one of the assumptions of OLS is that the error term, ϵ_i , has the same variance, σ^2 , at every combination of predictor values, or every "covariate pattern." When Y is dichotomous, this assumption is violated. Consequently the parameter estimates a , b_1 , b_2 , ..., b_K computed from the sample will no longer be the *best* linear unbiased estimators" (or BLUE). Instead they will tend to have larger-than-normal variances and therefore will provide less sensitive tests of model effects (see, for example, DeMaris and Longmore 1996).

Perhaps even more serious are the consequences of supposing that the probability of an EVENT is related in a linear fashion to a set of predictors. For one thing, because the left-hand side of Eq.(1) is a probability, it must lie between 0 and 1. The right-hand side has no such limitation, however. Therefore it is possible for the model to generate predicted probabilities of the EVENT that do not fall between 0 and 1, and thus have no meaning.

More important, an additive model—exemplified by Eq.(1)—implies constant slopes for the predictors. This means that the effect on $E(Y)$, or π , of a unit increase in a given predictor is expected to be the same across the entire range of that predictor. To see that this makes little sense when the dependent variable is a probability, consider trying to model the probability of homeownership as a function of annual income, measured in thousands of dollars. A \$1,000 increase in income would likely have a greater effect on homeownership among middle-income families than

among either low- or high-income families. An additional \$1,000 would not change this probability for poor families (they are still unable to own a home) or for rich families (generally they already own a home). Hence we need a model for the probability of an EVENT that constrains predicted probabilities to lie between 0 and 1, allows predictors to have a diminishing effect whenever the dependent variable is at the extremes of its range (i.e., the probability is near 0 or 1), and uses parameter estimators that have optimum properties (e.g., lack of bias, minimum variances). DLR provides such a model.

In DLR, the probability of an EVENT's occurring is expressed by the equation

$$\pi = \frac{\text{Exp}(\alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_K X_K)}{[1 + \text{Exp}(\alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_K X_K)]} \quad (2)$$

Because the exponential function (e or EXP) is always nonnegative, the right-hand side of Eq.(2) always falls between 0 and 1. This rather complex function is quite unwieldy; hence we will simplify it by transforming the dependent variable. Before we do so, however, let us say that when the response is written as the probability of event occurrence, as in Eq.(2), the partial slope for any predictor, say X_k , is $\beta_k(\pi)(1-\pi)$. This partial slope is no longer a constant, but varies with the value of π (which in turn depends on the levels of all variables in the model). Moreover, at the extreme values of the response—when π is close to either 0 or 1—the predictor has its smallest possible effect because $\pi(1-\pi)$ is near 0. When $\pi = .01$, for example, $1-\pi = .99$ and $\beta_k \pi(1-\pi) = .0099\beta_k$. On the other hand, predictors have their maximum effect of $(.25)\beta_k$ whenever π is $.5$. In this fashion, the DLR model reflects the fact that probabilities are much harder to change when the EVENT is virtually certain either to happen or not to happen than when it has a 50-50 chance of happening.

To write the right-hand side of Eq.(2) as an additive function of the predictors, we use a logit transformation of the probability, π (Hanushek and Jackson 1977). This logit transformation is $\text{LOG}[\pi/(1-\pi)]$,

or LOG [ODDS], where “LOG” refers, in this paper, to the natural logarithm. The term $[\pi/(1-\pi)]$ is an ODDS, the ratio of two probabilities. The LOG[ODDS] can be any number—negative, positive, or 0. Thus it can be expressed as a linear combination of predictor variables. When we write the logistic regression model in terms of the LOG of the ODDS, we obtain

$$\text{LOG}[\pi/(1-\pi)] = \alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_K X_K. \quad (3)$$

This equation is analogous to that for OLS except that the dependent variable is a LOG[ODDS]. As in the additive OLS model, a DLR-model partial slope of a given predictor is interpreted as the change in LOG[ODDS] for a one-unit change in that predictor, controlling for all other predictors in the model. Because most people find it difficult to think in terms of LOG[ODDS], DLR coefficients are generally interpreted with respect to ODDS ratios or probabilities, as we will explain below.

DLR produces predicted EVENT probabilities, which we denote as \hat{p} , for every combination of predictor values, or covariate pattern.

$$\hat{p} = \text{Prob}(\text{EVENT}) = \text{Prob}(Y=1) \quad (4)$$

If we let $z = \text{log}[\frac{\hat{p}}{1-\hat{p}}]$, then

$$z = b_0 + b_1 X_1 + b_2 X_2 + \dots + b_K X_K, \quad (5)$$

where b_i 's are the coefficients computed from the sample. Whereas OLS model parameters are estimated by minimizing the sum of squared deviations between observed and predicted values of the dependent variable (least squares method), DLR model parameters are estimated by maximizing a function relating observed responses to predicted probabilities (maximum likelihood method). The equations for the predicted EVENT probability and predicted ODDS are given by

$$\hat{p} = e^z / (1 + e^z) \quad (6)$$

$$\text{ODDS} = \hat{p} / (1-\hat{p}) = e^z = \text{EXP}(z). \quad (7)$$

In this notation, z is a LOG[ODDS]; a LOG[ODDS] is referred to as a logit.

Table 1. Example 1: Predicting *Physical Sexual Coercion* from *Country*

Part A. Statistics from CROSSTABS Computer Output						
Physical Coercion	Country				Total	
	0 (Sweden)		1 (U.S.)			
0 (No)	268	(77.0%)	151	(55.3%)	419	(67.5%)
1 (Yes)	80	(23.0%)	122	(44.7%)	202	(32.5%)
Total	348	(56.0%)	273	(44.0%)	621	(100.0%)
Chi-Square	Value	df	Sig			
Pearson	32.8241	1	.0000			
Likelihood Ratio	32.8256	1	.0000			
Part B. Statistics from LOGISTIC REGRESSION Computer Output						
-2 Log Likelihood	Value	df	Sig			
INITIAL ^a	783.4372 ^b	620	-			
FINAL	750.612	619	.0002			
MODEL Chi-Square	32.826	1	.0000			
Goodness of Fit	620.999	619	.4698			
Variables in Model 1	b	S.E.	Wald	df	Sig	EXP (b)
Country ^c	.9957	.1762	31.9290	1	.0000	2.7066
Constant	-1.2090	.1274	90.0466	1	.0000	

^a The -2 Log Likelihood INITIAL (-2LL₀) is defined below, where *N*₀ and *N*₁ are observed frequencies for the dichotomous dependent variable. *N*₀+*N*₁=*N*, total sample size.

$$-2LL_0 = -2 \text{ Log Likelihood INITIAL} = -2[N_0 \times \text{LN}(N_0/N) + N_1 \times \text{LN}(N_1/N)]$$

^b -2LL₀ = -2[419 × LN(419/621) + 202 × LN(202/621)] = -2[-391.7186] = 783.4372

^c The dichotomous variable *Country* contrasts the United States (coded 1) with Sweden (coded 0).

EXAMPLE 1: DICHOTOMOUS INDEPENDENT VARIABLE

Table 1, Part A summarizes an SPSS CROSSTABS output, including chi-squares, to test the independence of the variables *Physical Coercion* and *Country*. The significant chi-squares indicate that *Physical Coercion* and *Country* are statistically dependent.²

For DLR computer runs using SPSS, the dichotomous dependent variable EVENT of interest should be assigned the larger value. SPSS codes the values of the dependent variable as 1 for the EVENT and 0 for the NON-EVENT (in SAS the coding is reversed). For all examples in this paper, the EVENT is a "yes" to the variable *Physical Coercion*; that is, $\hat{p} = \text{Prob}(\text{EVENT}) = \text{Prob}(\text{COER})$. The sample size varies across the examples because of listwise deletion of missing values particular to each analysis.

² A significance level of .0000 in SPSS means $p < .00005$.

The DLR computer output is presented in Table 1, Part B. The summary shown includes the -2 log likelihood, model and goodness of fit chi-squares, and two model coefficients plus associated statistics. The "-2 Log Likelihood INITIAL" equals the "-2 Log Likelihood FINAL" plus the "MODEL Chi-Square" ($-2LL_0 = -2LL_F + \text{Model } \chi^2$). The MODEL chi-square, analogous to the OLS *F*-test for the overall model, tests the null hypothesis that all predictor variable β -coefficients in the current model are 0.³ Because this chi-square is significant, *Country* alone contributes significantly to the prediction of *Physical Coercion*. The use and interpretation of the INITIAL and FINAL log likelihoods and of the goodness of fit chi-square will be included in the section discussing DLR model fit. The Wald chi-square—calculated by

³ For the bivariate case with a categorical independent variable, the MODEL chi-square from a DLR output equals the likelihood ratio chi-square from a CROSSTABS output.

Table 2. Example 2: Predicting *Physical Sexual Coercion* from *Sex Frequency*

Part A. Statistics from CROSSTABS Computer Output						
Physical Coercion	Sex Frequency					Total
	Low (1)		Medium (2)		High (3)	
No (0)	137	(64.6%)	160	(66.4%)	117 (72.7%)	414 (67.4%)
Yes (1)	75	(35.4%)	81	(33.6%)	44 (27.3%)	200 (32.6%)
Total	212	(34.5%)	241	(39.3%)	161 (26.2%)	614 (100.0%)
Chi-Square	Value	df	Sig			
Pearson	2.8928	2	.2354			
Likelihood Ratio	2.9447	2	.2294			
Part B. Statistics from LOGISTIC REGRESSION Computer Output						
-2 Log Likelihood	Value	df	Sig			
INITIAL	775.0098	613	-			
FINAL	772.065	611	.0000			
MODEL Chi-Square	2.945	2	.2294			
Goodness of Fit	614.000	611	.4583			
Variables in Model 2	b	S.E.	Wald	df	Sig	EXP(b)
Sex Frequency ^a						
Low	.3755	.2278	2.7163	1	.0993	1.4557
Medium	.2973	.2233	1.7719	1	.1832	1.3462
Constant	-.9780	.1768	30.5827	1	.0000	

^aThe trichotomous variable *Sex Frequency* is converted into two dummy variables, *Low* and *Medium*. *Low* contrasts Low (coded 1) and High (coded 0) *Sex Frequency* categories. *Medium* contrasts Medium (coded 1) and High (coded 0) *Sex Frequency* categories.

squaring the quotient of the *b*-coefficient divided by its standard error—is used to test the null hypothesis that a specific *B*-coefficient is 0. Thus a significant Wald also tells us that *Country* is a significant predictor of *Physical Coercion*.

In general, for a multivariate DLR model without interaction terms, the statistic EXP(b) is the predicted EVENT ODDS ratio for those who are a unit apart on a given predictor variable, controlling for all other predictor variables in the model. With this notation, the predicted ODDS ratio of a variable is equal to *e* raised to the power of its corresponding *b*-coefficient—that is, EXP(*b*) in an SPSS DLR output. For Model 1 in Table 1, EXP(.9957) = *e*^{.9957} = 2.7066.⁴ The value 2.7066 is the ODDS ratio for those who are a unit apart on *Country*. Thus, as

⁴To evaluate *e*^{.9957} on most calculators, enter .9957 and then press the buttons labeled INV and LN. For calculators with an *e*^x button, enter .9957 and then press the *e*^x button.

Country changes from 0 (Sweden) to 1 (U.S.), the ODDS of being physically coerced differ by a factor of 2.7066, as was calculated earlier from the contingency table.

Next we illustrate how to calculate predicted probabilities of *Physical Coercion* for U.S. and Swedish women by using statistics in the DLR output together with Eqs.(5) and (6) with *K* = 1 predictor variables (bivariate case). For Model 1 in Table 1, Part B,

Model 1: $z = -1.2090 + .9957 \times Country$

For Sweden (*Country* = 0), $z = -1.2090$ and $Prob(COER) = e^z / 1 + e^z = .298 / 1.298 = .230$

For the U.S. (*Country* = 1), $z = -.2133$ and $Prob(COER) = e^z / 1 + e^z = .808 / 1.808 = .447.$

These two probabilities predicted by DLR are the same as the two sample probabilities we calculated earlier from the contingency table. Yet, in a bivariate analysis, probabilities and ODDS ratios calculated from a contingency table and those pre-

Table 3. Example 3: Predicting *Physical Sexual Coercion* from *Sex Partners*

Statistics from LOGISTIC REGRESSION Computer Output						
-2 Log Likelihood	Value	df	Sig			
INITIAL	761.9711	602	-			
FINAL	722.914	601	.0004			
MODEL Chi-Square	39.057	1	.0000			
Goodness of Fit	600.162	601	.5020			
Variables in Model 3	b	S.E	Wald	df	Sig	EXP(b)
<i>Sex Partners</i>	.1103	.0196	31.5317	1	.0000	1.1166
Constant	-1.2066	.1238	95.0057	1	.0000	

dicted from a DLR output will be equal only when the independent variable is *categorical*.

EXAMPLE 2: TRICHOTOMOUS INDEPENDENT VARIABLE

The variable *Sex Frequency* categorizes the frequency of respondent's partner-related sexual activities within the last year. The six response categories, ranging from never to more than four times a week, were collapsed into three frequency levels: low, medium, and high. We used *Low* and *Medium* as dummy variables, with the high frequency category as the reference. Studies in the United States have found that more sexually experienced women (i.e., women who report a higher frequency of sexual activity or more sexual partners) are at greater risk for sexual coercion (Koss and Dinero 1989; Lottes 1991), mainly because some men consider them legitimate targets of sexual aggression. Although our results show that *Sex Frequency* alone is not a significant predictor (see Table 2), this example illustrates the use of DLR for an ordinal variable in dummy format.

First we show how probabilities of *Physical Coercion* for the three sex-frequency categories can be calculated from either a contingency table or a DLR output. From the contingency table (Table 2, Part A) we calculate probabilities:

For women with Low *Sex Frequency*, Prob(COER) = 75/212 = .354

For women with Medium *Sex Frequency*, Prob(COER) = 81/241 = .336

For women with High *Sex Frequency*, Prob(COER) = 44/161 = .273.

To calculate these probabilities from the DLR output, we use Eqs.(5) and (6) with *K* = 2 dummy predictor variables. For Model 2 in Table 2, Part B,

Model 2: $z = -.9780 + .3755 \times Low + .2973 \times Medium$

For Low *Sex Frequency* (*Low* = 1; *Med* = 0):
 $z = -.6025$ and Prob(COER) = $e^{-.6025}/1 + e^{-.6025} = .547/1.547 = .354$

For Medium *Sex Frequency* (*Low* = 0; *Med* = 1): $z = -.6807$ and Prob(COER) = $e^{-.6807}/1 + e^{-.6807} = .506/1.506 = .336$

For High *Sex Frequency* (*Low* = 0; *Med* = 0): $z = -.9780$ and Prob(COER) = $e^{-.9780}/1 + e^{-.9780} = .376/1.376 = .273$.

As the above calculations show, probabilities predicted by DLR are equal to those computed from contingency table frequencies.

Second, we calculate *Sex Frequency* ODDS ratios from a contingency table and show how they are equal to EXP(*b*) (i.e., *Sex Frequency* ODDS ratios) from a DLR output. From the contingency table (Table 2, Part A), we calculate ODDS and ODDS ratios:

For women with Low *Sex Frequency*, ODDS(COER) = 75/137 = .547

For women with Medium *Sex Frequency*, ODDS(COER) = 81/160 = .506

For women with High *Sex Frequency*, ODDS(COER) = 44/117 = .376.

When women with Low *Sex Frequency* are compared to women with High *Sex Frequency* and round-off error is excluded,

$$\text{ODDS ratio (Low/High)} = \text{ODDS}(\text{COER})_L / \text{ODDS}(\text{COER})_H = .547/.376 = 1.4557.$$

Thus the ODDS of being a physical sexual coercion victim for women with Low *Sex Frequency* are about 1.5 times greater than for women with High *Sex Frequency*. Similarly, we can compare women with Medium *Sex Frequency* to women with High *Sex Frequency*:

$$\text{ODDS ratio (Medium/High)} = \text{ODDS}(\text{COER})_M / \text{ODDS}(\text{COER})_H = .506/.376 = 1.3462.$$

These two ODDS ratios are equal to the EXP(*b*s) reported for the Low and Medium dummy predictor variables in the DLR output (Table 2, Part B). We can also compute an ODDS ratio to compare the Low and the Medium frequency categories:

$$\text{ODDS ratio (Low/Medium)} = \text{ODDS}(\text{COER})_L / \text{ODDS}(\text{COER})_M = .547/.506 = 1.0814.$$

To compute this ODDS ratio from the DLR output, compute the quotient of the respective ratios: $1.4557/1.3462 = 1.0814$. One could also subtract the two coefficients and raise *e* to this power [EXP (*b*₁–*b*₂) = $e^{.3755-.2973} = e^{.0782} = 1.0813$]. Because this ODDS ratio is so close to 1, the ODDS of being a physical sexual coercion victim is essentially the same for women in the Low and the Medium *Sex Frequency* categories.

EXAMPLE 3: RATIO-LEVEL INDEPENDENT VARIABLE

In Example 3 (see Table 3) *Physical Coercion* is predicted from the ratio-level independent variable *Sex Partners*. As stated earlier, we expect that women with more sex partners will be more likely to experience sexual coercion. First we investigate the bivariate relationship between *Sex Partners* and *Physical Coercion*. The Pearson correlation coefficient of *Sex Partners* with *Physical Coercion* is significant ($r = .25, p < .001$). Therefore those who have been sexually coerced tend to have more sex partners than those who have not experi-

enced sexual coercion. The significant MODEL chi-square and the Wald statistic also show that *Sex Partners* contributes significantly to the prediction of *Physical Coercion*.

Next we show how to interpret the DLR model, using both probabilities and ODDS ratios. We can calculate the predicted probabilities for women with two, four, and five *Sex Partners* from the DLR output by using Eqs. (5) and (6) with *K* = 1 predictor variables. For Model 3 in Table 3,

$$\text{Model 3: } z = -1.2066 + .1103 \times \text{SexPartners}$$

$$\text{For 2 Sex Partners, } z = -1.2066 + .1103(2) = -.9860 \text{ and Prob(COER)} = e^z / 1 + e^z = .373/1.373 = .272$$

$$\text{For 4 Sex Partners, } z = -1.2066 + .1103(4) = -.7654 \text{ and Prob(COER)} = e^z / 1 + e^z = .465/1.465 = .317$$

$$\text{For 5 Sex Partners, } z = -1.2066 + .1103(5) = -.6551 \text{ and Prob(COER)} = e^z / 1 + e^z = .519/1.519 = .342.$$

For a noncategorical variable, DLR probabilities no longer equal contingency-table probabilities because of the imposition of linearity. That is, in DLR we force the LOG[ODDS] of coercion to have a linear relationship with the number of *Sex Partners*. The probabilities would be equal if the variable *Sex Partners* were dummied before entry in the DLR model.

To interpret Model 3 in terms of ODDS and ODDS ratios, we use Eq.(7):

$$\text{Model 3: ODDS} = \text{EXP}[-1.2066 + .1103 \times \text{SexPartners}]$$

$$\text{For 2 Sex Partners, ODDS(COER)} = \text{EXP}[-1.2066 + .1103(2)] = e^{-.9860} = .373$$

$$\text{For 4 Sex Partners, ODDS(COER)} = \text{EXP}[-1.2066 + .1103(4)] = e^{-.7654} = .465$$

$$\text{For 5 Sex Partners, ODDS(COER)} = \text{EXP}[-1.2066 + .1103(5)] = e^{-.6551} = .519.$$

We now look at the ODDS change for a one-unit change in the predictor variable. For a one-unit change from four to five *Sex Partners*, ODDS(COER) changes from .465 to .519, which is equivalent to an ODDS ratio or factor of $.519/.465 = 1.1166$. This is the same ODDS ratio as we obtain from the expression $\text{EXP} [.1103 \times (5 - 4)] = e^{.1103} = 1.1166$. In general, for the additive DLR

Table 4. Example 4: Predicting *Physical Sexual Coercion* from *Country* and *Sex Frequency*

Part A. Statistics from CROSSTABS Computer Output

Physical Coercion	Sex Frequency							
	Low (1)		Medium (2)		High (3)		Total	
Sweden								
No (0)	81	(68.6%)	106	(79.1%)	77	(84.6%)	264	(77.0%)
Yes (1)	37	(31.4%)	28	(20.9%)	14	(15.4%)	79	(23.0%)
Total	118	(34.4%)	134	(39.1%)	91	(26.5%)	343	(100.0%)
U.S.								
No (0)	56	(59.6%)	54	(50.5%)	40	(57.1%)	150	(55.4%)
Yes (1)	38	(40.4%)	53	(49.5%)	30	(42.9%)	121	(44.6%)
Total	94	(34.7%)	107	(39.5%)	70	(25.8%)	271	(100.0%)
Total Sample	212	(34.5%)	241	(39.3%)	161	(26.2%)	614	

Chi-Square	Sweden			U.S.		
	Value	df	Sig	Value	df	Sig
Pearson	7.9592	2	.0187	1.8020	2	.4062
Likelihood Ratio	7.9310	2	.0190	1.8017	2	.4062

Part B. Summary Statistics from LOGISTIC REGRESSION Computer Output

-2Log Likelihood	Model 4A			Model 4B		
	Sweden and U.S. with Interactions			Sweden and U.S.		
	Value	df	Sig	Value	df	Sig
INITIAL	775.0098	613	–	775.0098	610	–
FINAL	733.053	608	.0004	739.833	610	.0002
MODEL Chi-Square	41.957	5	.0000	35.177	3	.000
Goodness of Fit	613.979	608	.4247	611.422	610	.4762

Part C. Coefficient Statistics from LOGISTIC REGRESSION Computer Output

Variable	b (S.E.)	EXP (b)	b (S.E.)	EXP (b)
Constant	-1.7044*** (.2905)	–	-1.467*** (.2054)	–
Sex Frequency				
Low	.9209** (.3518)	2.5116	.3869 (.2339)	1.4724
Medium	.3732 (.3599)	1.4524	.3036 (.2292)	1.3547
Country	1.4168*** (.3778)	4.1237	.9943*** (.1776)	2.7028
Country Interactions with:				
Sex Frequency				
Low	-1.0210* (.4757)	.3602		
Medium	-.1042 (.4746)	.9010		

*p < .05; **p < .01; ***p < .001

model, the factor by which ODDS change for a one-unit change in the predictor variable x_k is the ODDS ratio $EXP(b_k)$ —that is, the ratio of ODDS at two values of x_k whose difference is 1, controlling for all other predictor vari-

ables in the model. Hence the ODDS ratio is the multiplicative analog of the partial slope in OLS.

To assess the ODDS change for more than a one-unit change in a ratio-level vari-

able, consider the change in ODDS of coercion for women with five *Sex Partners* compared to women with two *Sex Partners*. To calculate this change in ODDS, raise $\text{EXP}(b)$ to the third power (three units difference): the ODDS ratio of five compared to two *Sex Partners* equals $\text{EXP}[\.1103 \times (5 - 2)] = (e^{\.1103})^3 = (1.1166)^3 = 1.3922$. Thus the ODDS of coercion differ by a factor of 1.3922 for women with five *Sex Partners*, compared to women with two *Sex Partners*. To check, we can multiply the $\text{ODDS}(\text{COER}) = .373$ for two *Sex Partners* by the factor 1.3922, which results in $\text{ODDS}(\text{COER}) = .519$ for five *Sex Partners*. In general, for an m -unit change in x_k , ODDS at $x_k + m$ equals ODDS at x_k multiplied by $\text{EXP}[(b_k)m]$. Alternatively we can say that when the predictor variable x_k changes by m units, the ODDS change by a factor of $\text{EXP}(b_k)$ raised to the power m .

EXAMPLE 4: TWO INDEPENDENT VARIABLES AND THEIR INTERACTION

In Example 4 we consider the effects of *Country* and *Sex Frequency* and their interaction on *Physical Coercion*. Because of important societal differences between Sweden and the United States, we must determine whether the effect of *Sex Frequency* on *Physical Coercion* varies by *Country*. In general, when interactions are included in a DLR model, the ODDS ratio for a given predictor variable may no longer be independent of the other predictors in the model, as we illustrate below. Table 4, Part A contains CROSSTABS outputs for the United States and Sweden. The chi-squares indicate that *Physical Coercion* and *Sex Frequency* are statistically dependent for Sweden but not for the United States. Furthermore, for Sweden, the difference between the percentages of women who have experienced coercion in the Low and the High *Sex Frequency* categories (31.4% minus 15.4%) is greater than the difference for the United States (40.4% minus 42.9%). Thus, from bivariate analyses, we already know that the relationship between *Physical Coercion* and *Sex Frequency* is different

in the two countries, and that we should look for a potentially significant *Country-by-Sex Frequency* interaction in the DLR output.

We compare Model 4A, which contains the country interactions, with Model 4B, in which the interactions are absent. The test to determine the significance of any interaction terms added to the model is the difference between the MODEL chi-squares of the models with and without the country interactions ($41.957 - 35.177 = 6.78$; $df = 2$; $p < .05$). This global test indicates that at least one of the interaction terms is statistically significant. The output of Model 4A displayed in Table 4, Part C, shows a significant interaction between *Country* and *Low*. This means that the ODDS ratios comparing the ODDS of coercion for women in the low and the high *Sex Frequency* categories is different for Swedish and American women.

From the contingency table, we calculate the Low *Sex Frequency* ODDS ratios for women in each *Country*.

In Sweden, ODDS ratio for Low *Sex Frequency* = $(37/81)/(14/77) = .457/.182 = 2.5123$

In the U.S., ODDS ratio for Low *Sex Frequency* = $(38/56)/(30/40) = .679/.750 = .9048$.

Thus, for those with Low *Sex Frequency*, the ODDS ratio is not a constant but depends on *Country*. In Sweden, the ODDS of physical coercion for women with Low *Sex Frequency* are about 2.5 times higher than for women with High *Sex Frequency*. In the United States, the corresponding ODDS ratio is close to 1, indicating nearly equal ODDS of physical coercion for women in the Low and the High *Sex Frequency* categories. This information is conveyed in Part C of Table 4. The equation for Model 4A, which contains main effects for *Sex Frequency* and *Country* and the terms representing the interaction of *Sex Frequency* with *Country*, is

Model 4A: $z = -1.7044 + .9209 \times \text{Low} + .3732 \times \text{Medium} + 1.4168 \times \text{Country} - 1.0210 \times [\text{Low} \times \text{Country}] - .1042 \times [\text{Medium} \times \text{Country}]$.

Table 5. Example 5: Two Models Predicting *Physical Sexual Coercion*

Statistics from LOGISTIC REGRESSION Computer Output

	Model 5A		Model 5B			
Variable	<i>b</i> (S.E.)	EXP (<i>b</i>)	<i>b</i> (S.E.)	EXP (<i>b</i>)		
Constant	-3.1470*** (.4346)		-3.1067*** (.3267)			
<i>Sex Partners</i>	.1198*** (.0298)	1.1272	.1118*** (.0224)	1.1183		
<i>Nonphysical Sexual Coercion Victim</i>	.8572** (.3124)	2.3565	.9967*** (.2260)	2.7093		
<i>Physical Violence Victim</i>	1.1739*** (.3547)	3.2345	.9531*** (.2340)	2.5937		
<i>Sex Frequency</i>						
<i>Low</i>	1.2128** (.4230)	3.3630	.9758*** (.2918)	2.6533		
<i>Medium</i>	.4560 (.4192)	1.5778	.6395* (.2728)	1.8955		
<i>Country</i>	.9196 (.6263)	2.5083	.8115*** (.2145)	2.2512		
<i>Country Interactions with:</i>						
<i>Sex Partners</i>	-.0168 (.0459)	.9834				
<i>Nonphysical Sexual Coercion Victim</i>	.2304 (.4588)	1.2591				
<i>Physical Violence Victim</i>	-.3782 (.4737)	.6851				
<i>Sex Frequency</i>						
<i>Low</i>	-.5030 (.5870)	.6047				
<i>Medium</i>	.2664 (.5576)	1.3053				
		Model 5A		Model 5B		
-2Log Likelihood	Value	df	Sig	Value	df	Sig
INITIAL	690.3269	550	—	690.3269	550	—
FINAL	563.375	539	.2261	566.569	544	.2435
MODEL Chi-Square	126.952	11	.0000	123.757	6	.0000
Goodness of Fit	547.917	539	.3858	548.209	544	.4414

* $p < .05$; ** $p < .01$; *** $p < .001$

To obtain the *Low* ODDS ratio, we generally exponentiate the coefficient associated with *Low* in this model. In contrast to the additive Model 4B, however, this coefficient depends on *Country*. First we factor out the common multipliers of *Low* in the equation. The coefficient for *Low* becomes $(.9209 - 1.0210 \times \textit{Country})$. For Swedish women, *Country* equals 0, so this coefficient simply becomes .9209. The ODDS ratio for *Low* sex frequency then is $\exp(.9209) = 2.5116$. For U.S. women, *Country* equals 1; this coefficient then becomes $.9209 - 1.0210 = -.1001$. The ODDS ratio for U.S. women then is $\exp(-.1001) =$

.9047. Thus, within round-off and DLR convergence errors, the ODDS ratios calculated from an interaction DLR model are equal to those calculated from a contingency table.

EXAMPLE 5: MULTIPLE INDEPENDENT VARIABLES

Our final example contains five independent variables thought to explain and predict *Physical Coercion*. These variables are *Country*, *Sex Frequency*, *Sex Partners*, and two dichotomous variables, *Physical Violence Victim* and *Nonphysical Sexual Co-*

ercion Victim (coded 0 if never a victim and 1 if ever a victim). A physical violence victim was a woman who reported ever having been roughly pushed, shoved, slapped, kicked, punched, or bitten by a partner or injured by a thrown object. Nonphysical sexual coercion relates to women who reported ever being pressured verbally (by lying, continuous arguments, or inducement to consume alcohol or drugs) to have intercourse against their will. Theoretical and empirical research suggests that factors which influence men to commit acts of nonphysical coercion and physical violence also influence men to rape women (see Grauerholz and Koralewski 1991; Pirog-Good and Stets 1989). Thus women who experience physical abuse and/or verbal sexual coercion should be more likely to suffer physical sexual coercion than those who do not.

We compare Models 5A (with country interactions) and 5B (without interactions). The DLR summary in Table 5 shows that none of the *Country* interactions is statistically significant at $p < .05$. The IMPROVEMENT chi-square due to the five *Country* interaction terms as a group, determined by the difference between the MODEL chi-squares for Model 5A and Model 5B, is not statistically significant (IMPROVEMENT chi-square = 126.952 - 123.757 = 3.195; df = 5; $p < .05$). Thus Model 5B is preferred because it is more parsimonious, contains only significant predictors, and allows interpretation of the EXP(b_k)s as ODDS ratios.

Probabilities of the EVENT of interest may be calculated for specific covariate patterns. Such calculations are appropriate when the model is being developed as a predictive tool to identify categories of individuals at risk of the EVENT. For example, consider Model 5B with the following logit:

$$\text{Model 5B: } z = -3.1067 + .8115 \times \text{Country} + .1118 \times \text{SexPart} + .9967 \times \text{NPhyCoer} + .9531 \times \text{PhyViol} + .9758 \times \text{Low} + .6395 \times \text{Medium}.$$

We calculate logits for women in Sweden and the United States with the following "covariate pattern": 1) the median

number of *Sex Partners* (*SexPart* = 2), 2) not a *NonPhysical Sexual Coercion Victim* (*NPhyCoer* = 0), 3) not a *Physical Violence Victim* (*PhyViol* = 0), and 4) *High Sex Frequency* (*Low* = 0, *Medium* = 0). Substituting these values into Eqs. (5), (6) and (7), we obtain the following:

$$\text{Logit for Sweden is } z_{\text{SW}} = -3.1067 + .1118(2) = -2.8831$$

$$\text{Logit for the U.S. is } z_{\text{US}} = -3.1067 + .8115(1) + .1118(2) = -2.0716$$

$$\text{Corresponding predicted ODDS are } \text{ODDS}_{\text{SW}} = e^{-2.8831} = .056 \text{ and } \text{ODDS}_{\text{US}} = e^{-2.0716} = .126$$

$$\text{Corresponding predicted probabilities are } \hat{p}_{\text{SW}} = .056/1.056 = .053 \text{ and } \hat{p}_{\text{US}} = .126/1.126 = .112.$$

If we take our covariate pattern for each country and add the risk factor with the largest positive b -coefficient, *Nonphysical Sexual Coercion Victim* ($b = .9967$ and ODDS ratio = 2.7093), we can predict the following results for Sweden and the United States:

$$\text{Logits } z_{\text{SW}} = -2.8831 + (.9967) = -1.8864 \text{ and } z_{\text{US}} = -2.0716 + (.9967) = -1.0749$$

$$\text{ODDS}_{\text{SW}} = .056(2.7093) = .152 \text{ and } \text{ODDS}_{\text{US}} = .126(2.7093) = .341$$

$$\hat{p}_{\text{SW}} = .152/1.152 = .132 \text{ and } \hat{p}_{\text{US}} = .341/1.341 = .254.$$

By adding the risk factor *Nonphysical Sexual Coercion*, we add $b = .9967$ to each logit and multiply each ODDS by the factor ODDS ratio = EXP(.9967) = 2.7093. The net effect of this procedure is an increase in Swedish and American *predicted probabilities* by different amounts: .079 (.132 - .053) and .142 (.254 - .112) respectively. We could examine other risk factor patterns in a similar fashion.

ASSESSING THE FIT AND THE PREDICTIVE EFFICACY OF A DLR MODEL

We present only a few of the numerous measures that have been proposed to evaluate the fit and the predictive efficacy of DLR models in order to illustrate their use with Model 5B. In assessing model fit, we examine how well the model describes

(fits) the observed data. One way to assess goodness of fit (GOF) is to examine how likely the sample results are, given the parameter estimates. A good model results in a high likelihood of the observed results (likelihood = 1 for a "perfect" model). Two chi-square statistics, the -2 log likelihood FINAL and the Pearson GOF, can be used to test the null hypothesis that the observed likelihood does not differ from 1 (Knoke and Bohrnstedt 1994; Norusis and SPSS Inc. 1990). Both chi-squares have $N - K$ degrees of freedom, where N is the sample size and K the number of predictors; these values are routinely printed in SPSS DLR outputs (see Tables 1–5). For these chi-squares a good model fit corresponds to a *nonsignificant* p -value because the null hypothesis that the observed likelihood does not differ from 1 cannot be *rejected*. For Model 5B, both of these chi-squares are nonsignificant. Thus the model appears to fit the data well.

Predictive efficacy refers to the degree to which prediction error is reduced when the predictor variables are included rather than excluded. Whereas R^2 assesses predictive efficacy in OLS, no unanimously accepted analog exists in DLR. Nevertheless, several measures have been proposed, which 1) fall within the range between 0 and 1, and 2) increase as the model fit improves. One such measure is defined as follows by Hosmer and Lemeshow (1989:148):

$R^2_L = \text{MODEL chi-square} / -2 \log \text{likelihood INITIAL}$, or

$$R^2_L = [(-2LL_0) - (-2LL_K)] / (-2LL_0). \quad (8)$$

The -2 log likelihood INITIAL is analogous to the total sum of squares in OLS, and -2LL_K is the -2 log likelihood FINAL, which is analogous to the residual sum of squares in OLS. Nevertheless, because the log likelihood is not a sum of squares, R^2_L does not permit a "variation accounted for" interpretation. For Model 5B, $R^2_L = 123.757/690.327 = .179$.

A measure proposed by Aldrich and Nelson (1984:57) is a "pseudo R^2 " defined as

$$R^2_{\text{PSEUDO}} = \text{MODEL chi-square} / (N + \text{MODEL chi-square}), \quad (9)$$

where N is the sample size and MODEL chi-square = $(-2LL_0) - (-2LL_K)$. For Model 5B, $R^2_{\text{PSEUDO}} = (123.757)/(551 + 123.757) = .183$. Because both R^2_L and R^2_{PSEUDO} tend to be lower than analogous R^2 s in OLS, Hagle and Mitchell (1992) recommend using a corrected version of the pseudo R^2 for DLR. This measure, the corrected Aldrich and Nelson R^2 , is defined as

$$R^2_{\text{CAN}} = R^2_{\text{PSEUDO}} / [\max(R^2_{\text{PSEUDO}} | \hat{p})], \quad (10)$$

where $\max(R^2_{\text{PSEUDO}} | \hat{p})$ is the maximum value attainable by R^2_{PSEUDO} , given \hat{p} , the sample proportion for which $Y = 1$ (the proportion of the sample experiencing the EVENT). Fortunately we can calculate the denominator of R^2_{CAN} using DLR computer output:

$$\text{Denominator} = -2 \log \text{likelihood INITIAL} / [N + (-2 \log \text{likelihood INITIAL})]. \quad (11)$$

For Model 5B (see Table 5), this denominator is equal to $(690.3269)/(551 + 690.3269) = .556$. Thus $R^2_{\text{CAN}} = .183/.556 = .329$, which suggests that the predictive efficacy of the model is moderately strong.

The logistic regression procedures of SPSS and SAS also produce a classification table for a model in which predicted (EVENT vs. NON-EVENT) and observed (EVENT vs. NON-EVENT) categories are compared. For Model 5B, the classification table showed that 75 of 176 (43%) of $Y = 1$ responses are classified correctly, 334 of 375 (89%) of $Y = 0$ responses are classified correctly, and 409 of 551 (74%) of all responses are classified correctly. In the absence of a model (all cases are classified in the modal category), these numbers are 0 of 176 (0%), 375 of 375 (100%), and 375 of 551 (68%). The net result is 142 misclassified cases when our model is used, versus 176 misclassified cases based on chance alone. Thus Model 5B has reduced the misclassification by .193—that is, a proportional reduction of error equal to $(176 - 142)/176$ (about 19%).

In this paper we have discussed two research issues addressed by DLR—one related to the determination of significant predictors and the other to overall model fit and predictive efficacy. We have also tried to provide a clear explanation of the

basic concepts of DLR, and to synthesize aspects of the literature that focus on interpretation or are useful as teaching strategies. We hope this article will serve as a resource for statistics instructors and will also help others learn how to use DLR.⁵

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⁵ We are preparing a teaching packet that complements and reinforces the concepts illustrated in this paper. It contains additional explanations, graphs, an SAS output illustration, and four student worksheets based on the examples given here, together with worksheet answers. This packet will be available on request from the senior author. For an additional source, see DeMaris (1995), in which the logistic regression discussion is expanded to include polytomous logistic regression and ordered logit models.

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