Theoretical Physics T2
Quantum Mechanics

Course of Lectures by
Reinhold A. Bertlmann

Script written by
Reinhold A. Bertlmann and Nicolai Friis

T2–Script of Sommersemester 2008

E-mail: Reinhold.Bertlmann@univie.ac.at
Homepage: http://homepage.univie.ac.at/Reinhold.Bertlmann/

Contents

1 Wave–Particle Duality **1**

1.1 Planck's Law of Black Body Radiation 1
1.1.1 Quantization of Energy 1
1.1.2 Black Body Radiation 2
1.1.3 Derivation of Planck's Law 5
1.2 The Photoelectric Effect 7
1.2.1 Facts about the Photoelectric Effect 7
1.2.2 Einstein's Explanation for the Photoelectric Effect 8
1.2.3 The Millikan Experiment 9
1.3 The Compton Effect 11
1.3.1 The Compton Shift Formula 12
1.3.2 The Experiment of Compton 13
1.4 Bohr's Theses 13
1.5 Wave Properties of Matter 16
1.5.1 Louis de Broglie's Hypothesis 16
1.5.2 Electron Diffraction from a Crystal 17
1.6 Heisenberg's Uncertainty Principle 19
1.6.1 Heisenberg's Microscope 19
1.6.2 Energy–Time Uncertainty Principle 20
1.7 Double–Slit Experiment 20
1.7.1 Comparison of Classical and Quantum Mechanical Results 20
1.7.2 Interpretation of Quantum Mechanical Results 22
1.7.3 Interferometry with $C_{60}-$Molecules 24

2 Time–Dependent Schrödinger Equation **27**

2.1 Wave Function and Time–Dependent Schrödinger Equation 27
2.2 Continuity Equation 30
2.3 Observables 33
2.3.1 Operators 33
2.3.2 Scalar Product 34
2.4 Expectation Values 36
2.4.1 Expectation Values of Operators 36
2.4.2 Uncertainty of Observables 39
2.5 Commutator of Operators 41
2.6 Uncertainty Principle 43

2.6.1 Uncertainty Relation for Observables . . . 43
2.6.2 Energy–Time Uncertainty . . . 45
2.7 Ehrenfest Theorem and Virial Theorem . . . 47
2.7.1 Ehrenfest Theorem . . . 47
2.7.2 Virial Theorem . . . 48
2.8 Time Evolution of the Wave Packet . . . 49
2.8.1 Motion of Plane Waves and Wave Packets . . . 49
2.8.2 Spreading of the Free Wave Packet . . . 52

3 Mathematical Formalism of Quantum Mechanics 55
3.1 Hilbert Space . . . 55
3.1.1 Finite–Dimensional Hilbert Spaces . . . 55
3.1.2 Infinite–Dimensional Hilbert Spaces . . . 56
3.2 Dirac Notation . . . 58
3.3 Projection Operators . . . 59
3.3.1 Projectors for Discrete Spectra . . . 59
3.3.2 Projectors for Continuous Spectra . . . 61
3.4 Eigenvectors and Spectral Theorem . . . 63
3.4.1 Eigenvalue Equation . . . 63
3.4.2 Spectral Theorem . . . 64
3.5 Résumé: Axioms and Physical Interpretation . . . 66

4 Time–Independent Schrödinger Equation 69
4.1 Stationary States . . . 69
4.2 Schrödinger Equation as Eigenvalue Equation . . . 72
4.3 Expansion into Stationary States . . . 73
4.4 Infinite Potential Well . . . 74
4.5 Finite Potential Well . . . 76
4.5.1 Bound States . . . 77
4.5.2 Scattered States . . . 84
4.6 Tunnel Effect . . . 91
4.6.1 Finite Potential Barrier . . . 91
4.6.2 Tunneling – Transmission Coefficient . . . 92
4.6.3 Physical Examples of Tunneling . . . 93

5 Harmonic Oscillator and Coherent States 95
5.1 Harmonic Oscillator . . . 95
5.1.1 Algebraic Method . . . 95
5.1.2 Zero Point Energy . . . 100
5.1.3 Comparison with the Classical Oscillator . . . 102
5.1.4 Matrix Representation of the Harmonic Oscillator . . . 104
5.1.5 Analytic Method . . . 106
5.2 Coherent States . . . 107
5.2.1 Definition and Properties of Coherent States . . . 108
5.2.2 Coordinate Representation of Coherent States . . . 115

6 The 3–dimensional Schrödinger Equation 121
6.1 Angular Momentum . . . 121
6.2 Angular Momentum Eigenvalues . . . 124
6.3 Angular Momentum Eigenfunctions . . . 127
6.4 The 3–dimensional Schrödinger Equation . . . 131
6.5 Coulomb Potential and H–Atom . . . 135

7 Spin and Spin–Addition 139
7.1 Stern-Gerlach Experiment – Electron Spin . . . 139
7.1.1 Electron in a Magnetic Field . . . 139
7.1.2 Stern-Gerlach Experiment . . . 140
7.2 Mathematical Formulation of Spin . . . 143
7.2.1 The Pauli–Matrices . . . 143
7.2.2 Spin Algebra . . . 144
7.2.3 Spin Measurements . . . 146
7.3 Two Particles with Spin $\frac{1}{2}$ . . . 147

8 Perturbation Theory, Zeeman Effect, Stark Effect 151
8.1 Time–Independent Perturbation Theory . . . 151
8.1.1 First Order Corrections . . . 152
8.1.2 Second Order Corrections . . . 154
8.2 Zeeman Effect . . . 154
8.3 Stark Effect . . . 155

9 Density Matrices 159
9.1 General Properties of Density Matrices . . . 159
9.2 Pure and Mixed States . . . 160
9.2.1 Pure States . . . 161
9.2.2 Mixed States . . . 161
9.3 Time Evolution of Density Matrices . . . 162

10 Entanglement of Quantum Systems 165
10.1 Entanglement and Separability . . . 165
10.2 EPR Paradox . . . 166
10.3 Bell Inequalities . . . 167
10.4 Quantum Teleportation . . . 171
10.4.1 Qubits . . . 171
10.4.2 Quantum Teleportation Set-Up . . . 172