TUTORIAL - 4

1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1,\ b \geq 1$

On comparing

$a = 3,\ b = 2,\ f(n) = n^2$

Now, $c = \cancel{\ } \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c \Rightarrow T(n) = \Theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$a \geq 1,\ b > 1$

$a = 4,\ b = 2,\ f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

3) $T(n) = T(n/2) + 2^n$

$a = 1$

$b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

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4) $T(n) = 2^n T(n/2) + n^n$

$\Rightarrow a = 2^n$, $b = 2$, $f(n) = n^2$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$f(n) = n^c \Rightarrow f(n) = \Theta(n^2 \log_2 n)$

5) $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$f(n) = n$

$c = \log_4 16 = 2$; $n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

6) $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2$, $f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n \Rightarrow n\log n > n$

$f(n) > n^c$

$T(n) = \Theta(n \log n)$

7) $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2$, $f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

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8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5 \Rightarrow n^c = n^{0.5}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

9) $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5, b = 2, f(n) = 1/n$

$a \geq 1$ but here a is 0.5 so we cannot apply Master's Theorem.

10) $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2 \Rightarrow T(n) = \Theta(n!)$

11) $4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c \Rightarrow \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\therefore \log n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^2)$

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12) $T(n) = \sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}, b = 2, f(n) = \log n$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log n$

$\therefore f(n) > n^c$

$T(n) = \Theta(f(n)) \Rightarrow T(n) = \Theta(\log n)$

13) $a = 3$; $b = 2$; $f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5489}$

$n < n^{1.5489}$

$f(n) < n^c \Rightarrow T(n) = \Theta(n^{1.5849})$

14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, b = 3, f(n) = \sqrt{n}$

$c = \log_b a = 1$

$n^c = n^1$

$n^1 > n^{\frac{1}{2}}$

$T(n) = \Theta(n)$

15) $T(n) = 4T(n/2) + n$

$a = 4, b = 2, f(n) = n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

$T(n) = \Theta(n^2)$

16) $T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4, f(n) = n\log n$

$c = \log_b a \Rightarrow \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$T(n) = \theta(n\log n)$

17) $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3, f(n) = n/2$

$c = \log_b a = 1$

$n^c = n^1$

$n^1 > \frac{n}{2} \Rightarrow T(n) = \theta(n)$

18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6, b = 3, n^2\log n = f(n)$

$c = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$n^c < f(n)$

$T(n) = \theta(n^2\log n)$

19) $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2, f(n) = n/\log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \theta(n^2)$

20) $T(n) = 64T(n/8) - n^2 \log n$

$a = 64, b = 8, f(n) = n^2 \log n$

$c = \log_b a = \log_8 64 = 2$

$n^c = n^2$

$f(n) > n^c$

$T(n) = \theta(n^2 \log n)$

21) $T(n) = 7T(n/3) + n^2$

$a = 7; b = 3; f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < \cancel{\log} n^2$

$T(n) = \theta(n^2)$

22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \theta(n(2 - \cos n))$

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