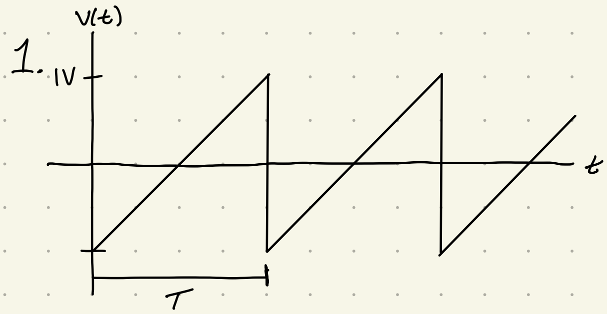
Physics 111A Lab 2

Pre-Lab Questions:

1.



for $0 \le t \le T$: $V(t) = \frac{2}{T}t - 1$

$$b_0 = \frac{1}{T}\int_0^T \frac{2}{T}t - 1\,dt = \frac{1}{T}\left[\frac{1}{T}t^2 - t\right]_0^T = \left[\left(1-\frac{1}{T}\right) - \left(-\frac{1}{T}\right)\right] = 0$$

$$b_n = \frac{2}{T}\int_0^T\left(\frac{2}{T}t-1\right)\cos\left(\frac{2\pi n t}{T}\right)dt = \frac{2}{T}\int_0^T \frac{2}{T}t\cos\left(\frac{2\pi n t}{T}\right) - \cos\left(\frac{2\pi n t}{T}\right)dt$$

$$= \frac{2}{T}\left[\int_0^T \frac{2}{T}t\cos\left(\frac{2\pi n t}{T}\right) - \int_0^T \cos\left(\frac{2\pi n t}{T}\right)dt\right]$$

$$= \frac{2}{T}\left\{\left[\frac{2}{T}\frac{T}{2\pi n}t\sin\left(\frac{2\pi n t}{T}\right)\right]_0^T - \frac{1}{\pi n}\int_0^T \sin\left(\frac{2\pi n t}{T}\right)dt - \cancel{\left[\frac{T}{2\pi n}\sin\left(\frac{2\pi n t}{T}\right)\right]_0^T}\right\}$$

$$= \frac{2}{T}\left\{\frac{1}{\pi n}\left[T\sin(2\pi n) - 0\right] - \frac{1}{\pi n}\left[-\frac{T}{2\pi n}\cos\left(\frac{2\pi n t}{T}\right)\right]_0^T\right\}$$

$$= \frac{2}{T}\frac{T}{2\pi^2 n^2}\left(\cos(2\pi n) - 1\right) = \frac{1}{\pi^2 n^2}(1-1) = 0$$

$$a_n = \frac{2}{T}\int_0^T\left(\frac{2}{T}t-1\right)\sin\left(\frac{2\pi n t}{T}\right)dt$$

$$= \frac{2}{T}\left[\frac{2}{T}\int_0^T t\sin\left(\frac{2\pi n t}{T}\right)dt - \underbrace{\cancel{\int_0^T \sin\left(\frac{2\pi n t}{T}\right)dt}}_{0}\right]$$

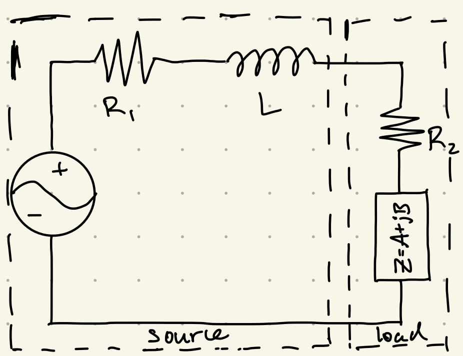
$$= \frac{2}{T}\left\{\frac{2}{T}\left[-\frac{T}{2\pi n}t\cos\left(\frac{2\pi n t}{T}\right)\right]_0^T + \frac{1}{\pi n}\int_0^T\cos\left(\frac{2\pi n t}{T}\right)dt\right\}$$

$$= \frac{2}{T}\left\{-\frac{1}{\pi n}\left[T\cos(2\pi n)\right] + \underbrace{\cancel{\frac{T}{2\pi^2 n^2}\left[\sin\left(\frac{2\pi n t}{T}\right)\right]_0^T}}_{0}\right\}$$

$$= \frac{2}{T}\left(-\frac{T}{\pi n}\right) = -\frac{2}{\pi n}$$

$b_0 = 0,\ b_n = 0,\ a_n = -\frac{2}{\pi n}$

2.



$R_1 > R_2$

a) $Z_L = \frac{V(t)}{I(t)} = j\omega L$, $L = \frac{\mu_H K N^2 A}{\ell}$, μ_K permeability of core

$H(\omega) = \frac{1 - j\omega\frac{L}{R}}{1+(\omega\frac{L}{R})^2}$, $G(\omega) = \frac{1}{\sqrt{1+(\omega\frac{L}{R})^2}}$ $\phi(\omega) = -\tan^{-1}(\omega\frac{L}{R})$

$$Z_{tot} = Z_{R_1} + Z_L + Z_{R_2} + Z = R_1 + j\omega L + R_2 + Z$$

$$P_L = I_{rms}^2 R_L = \frac{1}{2}\left(\frac{|V_0|}{|Z_{tot}|}\right)^2 R_L$$

P_L is maximized when $Z_{source} = Z^*_{load}$ and the complex parts cancel

b) since $Z_{source} = Z^*_{load}$ when power is max, $P_{source} = P_{load}$ so then $P_{load} = \frac{1}{2}P_{tot}$

$$Z_{source} = Z_{R_1} + Z_L = R_1 + j\omega_0 L, \quad Z^*_{load} = Z^*_{R_2} + Z^* = R_2 + A - jB$$

$$R_1 + j\omega L = R_2 + Z^* \Rightarrow Z^* = R_1 - R_2 + j\omega_0 L \Rightarrow Z = R_1 - R_2 - j\omega_0 L$$

$$R_1 + j\omega_0 L = R_2 + A - jB \Rightarrow R_1 = R_2 + A \Rightarrow A = R_1 - R_2$$

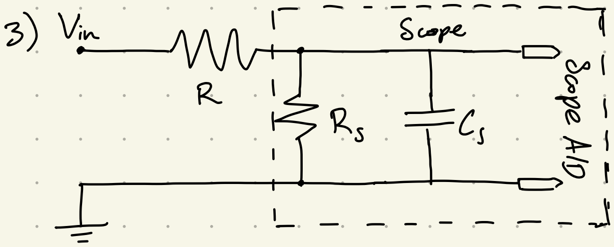
$$\omega_0 L = -B \Rightarrow B = -\omega_0 L$$

$$Z = R_1 - R_2 - j\omega_0 L$$

$$R = R_1 - R_2, \quad \frac{1}{j\omega_0 C} = -j\omega_0 L \Rightarrow -\frac{j}{\omega_0 C} = -j\omega_0 L$$

$$\Rightarrow \frac{1}{\omega_0 C} = \omega_0 L \Rightarrow C = \frac{1}{\omega_0^2 L}$$

You would insert a resistor with $R = R_1 - R_2$ and capacitor with $C = \frac{1}{\omega_0^2 L}$ in series.

3) 

a) $Z_{sc} = \left(\frac{1}{R_s} + j\omega C_s\right)^{-1} = \frac{R_s}{1+j\omega C_s R_s}$

$$H(\omega) = \frac{Z_{sc}}{Z_{sc}+R} = \left(\frac{R_s}{1+j\omega C_s R_s}\right)\cdot\left(\frac{R_s}{1+j\omega C_s R_s} + R\right)^{-1} = \left(\frac{R}{1+j\omega C_s R_s}\right)\left(\frac{R_s+R+j\omega C_s R_s R}{1+j\omega C_s R_s}\right)^{-1}$$

$$= \frac{R}{R_s+R+j\omega C_s R_s R} = \frac{R(R_s+R-j\omega C_s R R_s)}{(R_s+R)^2+\omega^2 C_s^2 R^2 R_s^2} = \frac{R(R_s+R) - j\omega C_s R^2 R_s}{(R_s+R)^2+\omega^2 C_s^2 R^2 R_s^2}$$

$$G(\omega) = |H(\omega)| = \left(\frac{R^2(R_s+R)^2}{((R_s+R)^2+\omega^2 C_s^2 R^2 R_s^2)^2} + \frac{\omega^2 C_s^2 R^4 R_s^2}{((R_s+R)^2+\omega^2 C_s^2 R^2 R_s^2)^2}\right)^{1/2}$$

$$= \left(\frac{R^2((R_s+R)^2+\omega^2 C_s^2 R^2 R_s^2)}{((R_s+R)^2+\omega^2 C_s^2 R^2 R_s^2)^2}\right)^{1/2} = \frac{R}{\sqrt{(R_s+R)^2+\omega^2 C_s^2 R^2 R_s^2}}$$

$R_s = 1\,M\Omega$

b) $R = R_s$, $G(\omega) = \frac{R_s}{\sqrt{4R_s^2 + \omega^2 C_s^2 R_s^4}}$

$$\frac{R_s}{R_s\sqrt{4+\omega^2 C_s^2 R_s^2}} = \frac{1}{2}\frac{1}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{4+\omega^2 C_s^2 R_s^2}} = \frac{1}{2\sqrt{2}} \Rightarrow 4+\omega^2 C_s^2 R_s^2 = 8 \Rightarrow \omega^2 = \frac{4}{C_s^2 R_s^2} \Rightarrow \omega = \frac{2}{R_s C_s}$$

4) $k=1$, $N_{12} = \frac{N_1}{N_2}$, L_1 drives load $Z_L = R$

$$Z_{in} = Z_L\left(1+\frac{j\omega L_2(1-k^2)}{Z_L}\right)\cdot\left(N_{21}^2\left(1+\frac{Z_L}{j\omega L_2}\right)\right)^{-1} = R\left(1+\frac{\cancel{j\omega L_2(1-1)}^{\,0}}{R}\right)\cdot\left(\frac{N_2^2}{N_1^2}\left(1+\frac{R}{j\omega L_2}\right)\right)^{-1}$$

$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = N_{12} \Rightarrow L_2 = \frac{L_1}{N_{12}^2}$

$$= R\cdot\left(\frac{N_2^2}{N_1^2}\left(1+\frac{R}{j\omega\frac{L_1}{N_{12}^2}}\right)\right)^{-1} = R\frac{N_1^2}{N_2^2}\left(1+\frac{N_{12}^2 R}{j\omega L_1}\right)^{-1}$$

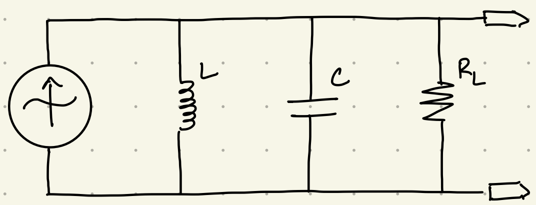
$$= N_{12}^2 R\left(1+\frac{1}{j\omega\frac{L_1}{N_{12}^2 R}}\right)^{-1} = N_{12}^2 R\left(1+\frac{1}{j\omega\tau}\right)^{-1}$$

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega\frac{L_1}{N_{12}^2}}{R+j\omega\frac{L_1}{N_{12}^2}} = \frac{j\omega\frac{L_1}{N_{12}^2 R}}{1+j\omega\frac{L_1}{N_{12}^2 R}} = \left(1+\frac{1}{j\omega\frac{L_1}{N_{12}^2 R}}\right)^{-1} = \left(1+\frac{1}{j\omega\tau}\right)^{-1}$$

therefore $Z_{in} = N_{12}^2 R\, H(\omega)$ where $H(\omega) = \left(1+\frac{1}{j\omega\tau}\right)^{-1}$

$\omega \gg \frac{2\pi}{\tau}$, then $H(\omega)$ would approach 1 and Z_{in} would simplify to $Z_{in} = N_{12}^2 R$

$\omega \ll \frac{2\pi}{\tau}$, then $H(\omega) = \left(1+\frac{1}{j\omega\tau}\right)^{-1} \approx j\omega\tau$ and $Z_{in} = j\omega\tau N_{12}^2 R$

5) 

V_{out}

a) $Z_{tot} = \left(\frac{1}{j\omega L} + j\omega C + \frac{1}{R_L}\right)^{-1}$

$$V_{out} = Z_{tot} I_0 = I_0\left(\frac{1}{j\omega L} + j\omega C + \frac{1}{R_L}\right)^{-1}$$

$$H(\omega) = \frac{V_{out}}{I_0} = \left(\frac{1}{j\omega L} + j\omega C + \frac{1}{R_L}\right)^{-1} = \left(\frac{R_L + j\omega C(j\omega L)R_L + j\omega L}{j\omega L R}\right)^{-1}$$

$$= \left(\frac{R_L - \omega^2 LCR_L + j\omega L}{j\omega LR}\right)^{-1} = \frac{j\omega LR}{R_L - \omega^2 LCR_L + j\omega L}$$

$$= \frac{j\omega LR(R_L-\omega^2 LCR_L - j\omega L)}{(R_L-\omega^2 LCR_L + j\omega L)(R_L - \omega^2 LCR_L - j\omega L)} = \frac{\omega^2 L^2 R + j\omega LR(R_L - \omega^2 LCR_L)}{(R_L-\omega^2 LCR_L)^2 + \omega^2 L^2}$$

$$G(\omega) = |H(\omega)| = \left(\frac{(\omega^2 L^2 R)^2}{((R_L-\omega^2 LCR_L)^2+\omega^2 L^2)^2} + \frac{(\omega LR)^2(R_L-\omega^2 LCR_L)^2}{((R_L-\omega^2 LCR_L)^2+\omega^2 L^2)^2}\right)^{1/2}$$

$$= \left(\frac{(\omega LR)^2(\omega^2 L^2 + (R_L-\omega^2 LCR_L)^2)}{(\omega^2 L^2 + (R_L-\omega^2 LCR_L)^2)^2}\right)^{1/2} = \frac{\omega LR}{\sqrt{\omega^2 L^2 + (R_L-\omega^2 LCR_L)^2}}$$

$$V_0 = I_0\left(\frac{\omega^2 L^2 R_L + j\omega LR_L(R_L-\omega^2 LCR_L)}{(R_L-\omega^2 LCR_L)^2 + \omega^2 L^2}\right),\quad \omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$= I_0\left(\frac{\frac{LR_L}{C} + jR_L\sqrt{\frac{L}{C}}(R_L - R_L)}{(R_L-R_L)^2 + \frac{L}{C}}\right) = I_0\left(\frac{LR_L}{C}\cdot\frac{C}{L}\right) = I_0 R_L$$

b) $\frac{G(\omega)}{G(\omega_0)} = \frac{1}{\sqrt{2}} \Rightarrow G(\omega) = \frac{1}{\sqrt{2}} G(\omega_0)$

$G(\omega) = \frac{\omega L R_L}{\sqrt{\omega^2 L^2 + (R_L - \omega^2 LCR_L)^2}}$, $G(\omega_0) = R_L$, $BW = 2\Delta\omega = \omega_+ - \omega_-$

$\Rightarrow \frac{\omega L R_L}{\sqrt{\omega^2 L^2 + (R_L - \omega^2 LCR_L)^2}} = \frac{R_L}{\sqrt{2}} \Rightarrow \sqrt{\omega^2 L^2 + (R_L - \omega^2 LCR_L)^2} = \sqrt{2}\,\omega L$

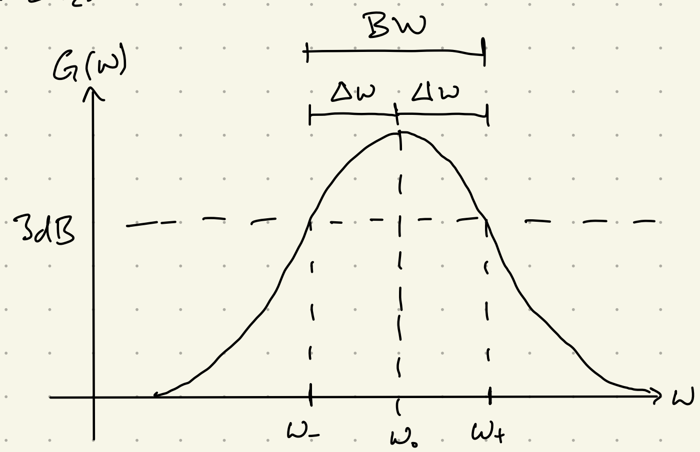
$\omega^2 L^2 + (R_L - \omega^2 LCR_L)^2 = 2\omega^2 L^2$

$(R_L - \omega^2 LCR_L)^2 = \omega^2 L^2$

$R_L - \omega^2 LCR_L = \pm \omega L$

$\Rightarrow \omega^2 LCR_L \pm \omega L - R_L = 0$

$\Rightarrow \omega^2 \pm \omega \frac{1}{CR_L} - \frac{1}{LC} = 0$



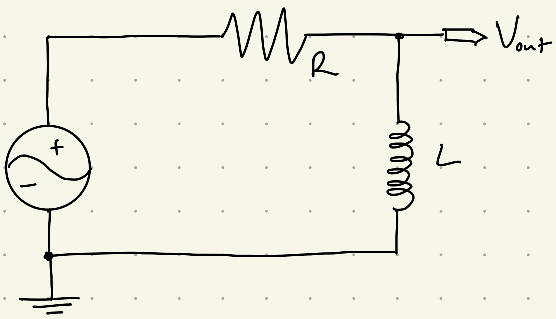
$\Rightarrow \omega_\pm = \frac{\pm \frac{1}{CR_L} + \sqrt{\frac{1}{C^2R_L^2} + \frac{4}{LC}}}{2} = \pm \frac{1}{2CR_L} + \frac{1}{2}\sqrt{\frac{1}{C^2R_L^2} + \frac{4}{LC}} = \pm \frac{1}{2CR_L} + \frac{1}{2CR_L}\sqrt{1 - \frac{4CR_L^2}{L}}$

$= \frac{1}{2CR_L}\left(\sqrt{1 - \frac{4CR_L^2}{L}} \pm 1\right)$

$BW = \omega_+ - \omega_- = \frac{1}{2CR_L}\left(\sqrt{1 - \frac{4CR_L^2}{L}} + 1 - \sqrt{1 - \frac{4CR_L^2}{L}} + 1\right) = \frac{1}{CR_L}$

$Q = \frac{\omega_0}{BW} = \omega R_L C = \frac{1}{\sqrt{LC}} R_L C = R_L\sqrt{\frac{C}{L}}$

6)



$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega \frac{L}{R}}{1 + j\omega \frac{L}{R}}, \quad \tau = \frac{L}{R}$$

$$= \frac{j\omega\tau}{1 + j\omega\tau} = \frac{j\omega\tau(1 - j\omega\tau)}{1 + \omega^2\tau^2} = \frac{j\omega\tau + \omega^2\tau^2}{1 + \omega^2\tau^2}$$

$$G(\omega) = |H(\omega)| = \left| \frac{\omega^2\tau^2 + j\omega\tau}{1 + \omega^2\tau^2} \right| = \left(\left(\frac{\omega^2\tau^2}{1+\omega^2\tau^2} \right)^2 + \left(\frac{\omega\tau}{1+\omega^2\tau^2} \right)^2 \right)^{\frac{1}{2}}$$

$$= \left(\frac{(\omega\tau)^2 (1 + \omega^2\tau^2)}{(1 + \omega^2\tau^2)^2} \right)^{1/2} = \frac{\omega\tau}{\sqrt{1 + \omega^2\tau^2}}$$

$$\varphi(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}[H(\omega)]}{\mathrm{Re}[H(\omega)]} \right) = \tan^{-1}\left(\frac{\omega\tau}{1+\omega^2\tau^2} \cdot \frac{1+\omega^2\tau^2}{\omega^2\tau^2} \right) = \tan^{-1}\left(\frac{1}{\omega\tau} \right)$$

for RC high-pass filter: $\varphi(\omega) = \tan^{-1}\left(\frac{1}{\omega\tau}\right)$ and $G(\omega) = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}$ where $\tau = RC$

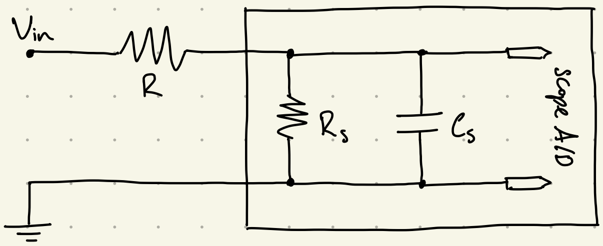
RL high-pass filter: $\varphi(\omega) = \tan^{-1}\left(\frac{1}{\omega\tau}\right)$ and $G(\omega) = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}$ where $\tau = \frac{L}{R}$

Therefore the RL and RC high pass have the exact same functional form.

$$G(\omega) = \frac{\omega\tau}{\sqrt{1 + \omega^2\tau^2}} = \frac{1}{\sqrt{2}} \Rightarrow 2\omega^2\tau^2 = 1 + \omega^2\tau^2 \Rightarrow \omega^2\tau^2 = 1 \Rightarrow \omega = \frac{1}{\tau}$$

Lab Exercises:

Problem R2.1:

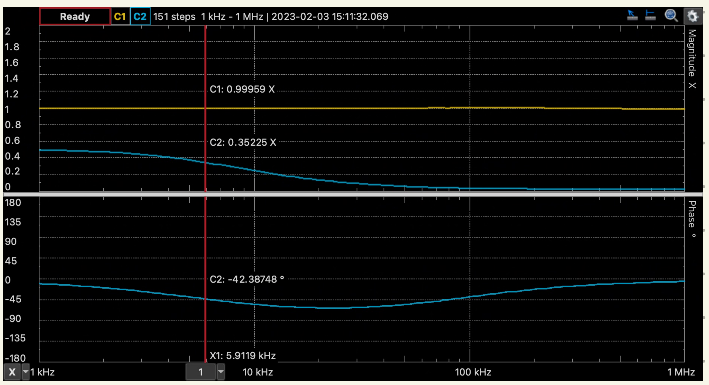


a) $\omega_0 = \dfrac{2}{R_S C_S} \Rightarrow C_S = \dfrac{2}{\omega_0 R_S} = \dfrac{1}{\pi f_0 R_S}$

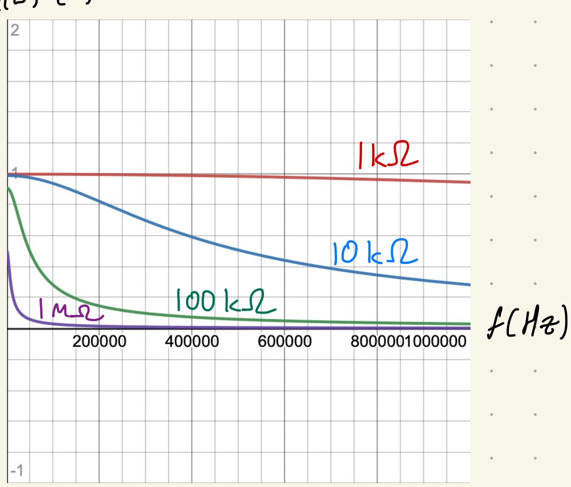
$R = R_S = 1\,M\Omega$

$f_0 = 5.9\ kHz$

$$C_S = \frac{1}{\pi(5.9\times10^3)(1\times10^6)} = 5.39508\times10^{-11}\ F = 53.95\ pF$$

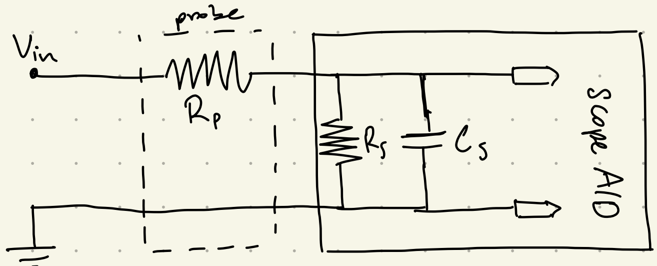


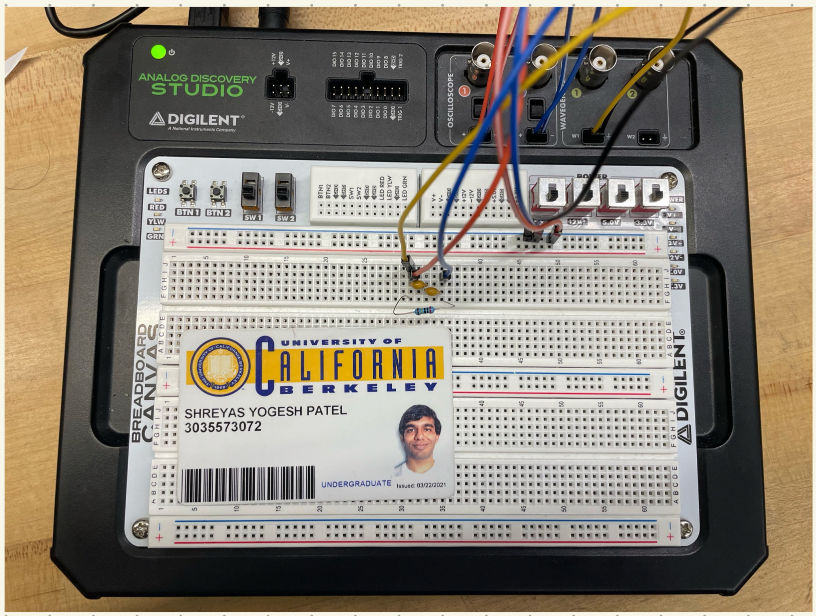
b) $G(\omega)$ (x)



Problem R2.2

a)





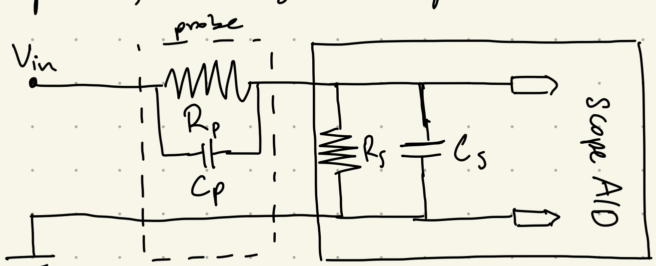
$R_p = 2M\Omega$

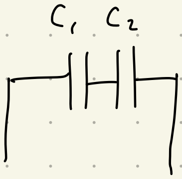
$Z_{in} = R_p = 2M\Omega$

$\omega_{3dB} = 4.4318\ kHz$

b) No, there is an observed distortion due to the charging and discharging of the capacitor, causing the scope to behave like a low-pass filter.

c)





$$C_{tot} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

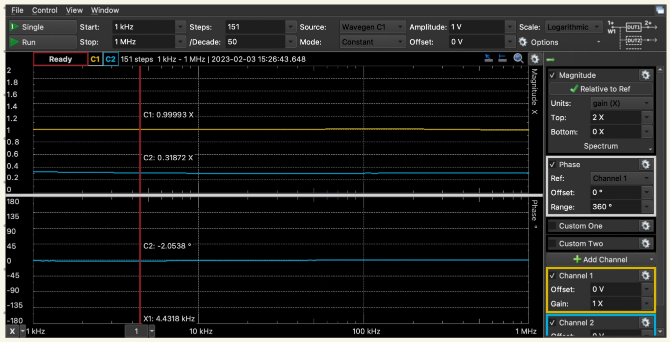
$C_1 = C_2 = 47\ pF$

$C_{tot} = 23.5\ pF = C_p$

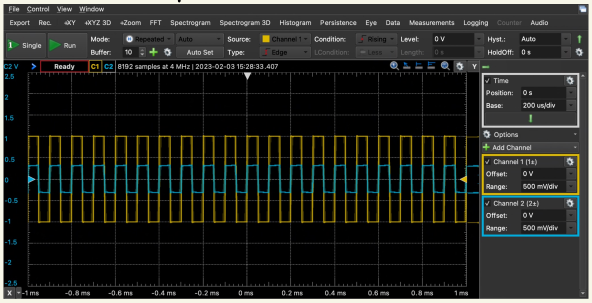
$R_s = 1M\Omega,\ C_s = 54\ pF$

$$R \propto \frac{1}{C} \Rightarrow \frac{R_p}{C_s} = \frac{R_s}{C_p} \Rightarrow C_p = \frac{R_s}{R_p} C_s = \frac{1}{2}(54\ pF) \approx 27\ pF$$

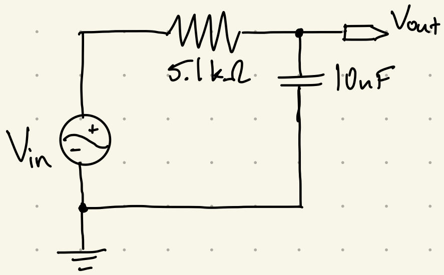
used $C_p = 23.5\ pF$



d) The compensated probe recovers the original shape of the square input wave.



Problem R2.3



a)

f (kHz)	Vpeak before (V)	Vpeak after (V)
1	1.273	1.2077
3	0.43693	0.32396
5	0.28041	0.16245
7	0.22219	0.1088
9	0.20071	0.09048

b)

n	Fourier Coefficient	V_{peak} (V)
1	1.273	1.273
3	0.424	0.437
5	0.255	0.280
7	0.182	0.222
9	0.141	0.201

The recorded peak voltages are close to their corresponding Fourier coefficients, but begin to diverge at higher frequencies/resonances.

c) Filtered coefficients are attenuated by a factor $A_n H(\omega)$ where A_n comes from the Fourier decomposition and $H(\omega)$ from the filter.

$$H(\omega) = \frac{1}{1+j\omega\tau} = \frac{1-j\omega\tau}{1+\omega^2\tau^2} = \frac{1}{1+\omega^2\tau^2} - j\frac{\omega\tau}{1+\omega^2\tau^2}, \quad \tau = 51\,\mu s$$

n	Actual Attenuation	$H(\omega_n)$	$\lvert H(\omega_n)\rvert$
1	0.9487	0.91 − 0.286j	0.954
3	0.7414	0.53 − 0.499j	0.728
5	0.5793	0.288 − 0.453j	0.5368
7	0.4897	0.171 − 0.377j	0.414
9	0.4508	0.111 − 0.314j	0.333

d)

n	Pre-Lab Calc	Unfiltered Fourier Coefficients	
1	0.5	0.637	0.8079
2	0.398	0.322	0.0885
3	0.265	0.218	0.0312
4	0.0997	0.169	0.0154
5	0.0639	0.141	0.00887
		ramp	triangle

The harmonics for the triangle wave were at odd integer values while they were at even harmonics for the ramp wave.

Problem R2.4

a) low-pass filter: $G(\omega) = \frac{1}{\sqrt{1+\omega^2\tau^2}} = \frac{V_{out}}{V_{in}} \Rightarrow \left(\frac{V_{in}}{V_{out}}\right)^2 = 1+\omega^2\tau^2 \Rightarrow \tau = \frac{1}{\omega}\left(\left(\frac{V_{in}}{V_{out}}\right)^2 - 1\right)^{1/2}$

$V_{in} = 0.99256\ V$
$V_{out} = 46.414\ mV$

$RC = \tau = \frac{1}{2\pi f}\left(\frac{V_{in}}{V_{out}} - 1\right) = \frac{1}{2\pi(50\ kHz)}\left(\left(\frac{0.99256\ V}{46.414\ mV}\right)^2 - 1\right)^{1/2} = 68\ \mu s$

b) high-pass filter: $G(\omega) = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}} = \frac{V_{out}}{V_{in}} \Rightarrow \omega\tau^2 = \left(\frac{V_{out}}{V_{in}}\right)^2(1+\omega^2\tau^2) \Rightarrow \omega^2\tau^2\left(1-\left(\frac{V_{out}}{V_{in}}\right)^2\right) = \left(\frac{V_{out}}{V_{in}}\right)^2$

$\Rightarrow \tau = \frac{1}{\omega}\left(\frac{V_{out}}{V_{in}}\right)\left(1-\left(\frac{V_{out}}{V_{in}}\right)^2\right)^{-1/2}$

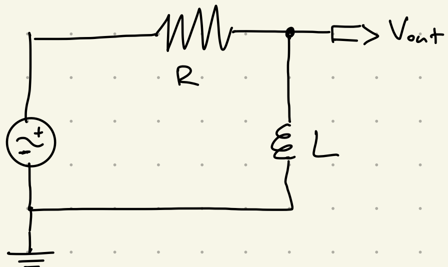
$V_{in} = 0.99239\ V$
$V_{out} = 0.98176\ V$

$RC = \tau = \frac{1}{2\pi f}\left(\frac{V_{out}}{V_{in}}\right)\left(1-\left(\frac{V_{out}}{V_{in}}\right)^2\right)^{-1/2} = 21.57\ \mu s$

Problem R2.5

a) $f = 10\,kHz$, $L = 2.577\,mH$

b)



$$G(\omega) = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}} = \frac{1}{\sqrt{2}}, \quad \tau = \frac{L}{R}$$

$$2\omega^2\tau^2 = 1 + \omega^2\tau^2$$

$$\omega^2\tau^2 = 1$$

$\omega = 2\pi f$

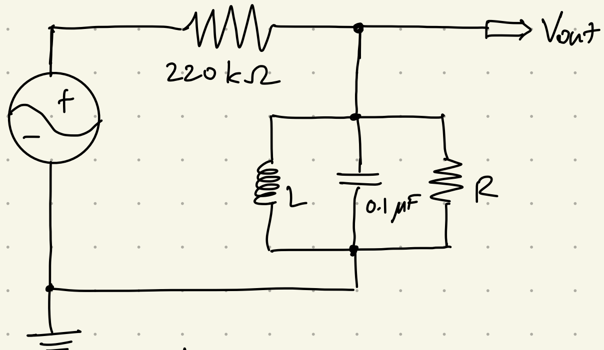
$$\omega = \frac{1}{\tau} = \frac{R}{L} \Rightarrow R = \omega L = 2\pi f L$$

$$R = 2\pi(11\,kHz)(2.577\,mH) = 178.109\,\Omega$$

we used $100\Omega, 68\Omega, 10\Omega$ resistors in series

the measured 3dB is 10.835 kHz which agrees with our design.

Problem R2.6



a) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2.577mH)(0.1\mu F)}} = 9.914\,kHz$

b) $R = 10k\Omega$

$f = 10.285\,kHz$

This measurement is within 4% of our expectations

c) $Q = \frac{f_0}{BW}$, $f_{3DB} = 10.493\,kHz \Rightarrow BW = 2(10.493 - 10.285) = 0.416\,kHz$ (2×0.208)

$$Q = \frac{10.285}{0.416} = 24.72$$

d) $R = 1k\Omega$, $f_0 = 10.326\,kHz$, $f_{3DB} = 11.370\,kHz \Rightarrow BW = 2.088 \Rightarrow Q = 5.05$

$R = 3k\Omega$, $f_0 = 10.326\,kHz$, $f_{3DB} = 10.706\,kHz \Rightarrow BW = 0.760 \Rightarrow Q = 13.59$

e) $Q = R\sqrt{\frac{C}{L}} = R\sqrt{\frac{(0.1\mu F)}{(2.577\,mH)}} = 0.00623\,R$

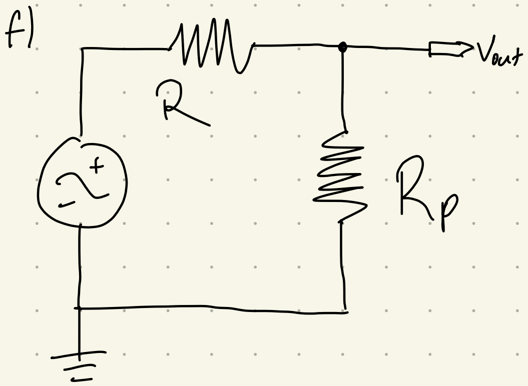
$R = 10k\Omega$, $Q = 62.29$

$R = 1k\Omega$, $Q = 6.23$

$R = 3k\Omega$, $Q = 18.69$

at higher resistances, the Q factor diverges, where the ideal is higher than actual.

f)



$$V_{out} = V_o \frac{R_p}{R_s + R_p} \Rightarrow (R_s + R_p)\frac{V_{out}}{V_o} = R_p$$

$$\Rightarrow \frac{V_{out}}{V_o} R_s = \left(1 - \frac{V_{out}}{V}\right) R_p$$

$$\Rightarrow R_p = \left(1 - \frac{V_{out}}{V}\right)^{-1} \frac{V_{out}}{V} R_s$$

$$\frac{V_{out}}{V} = 0.06261$$

$$R_p = (1 - 0.06261)^{-1}(0.06261)(220\,k\Omega)$$

$$= 14.69\,k\Omega$$

$$(R_s + R_p)\frac{V_{out}}{V_o} = R_s$$

$$\frac{V_{out}}{V_o} R_p = \left(1 - \frac{V_{out}}{V_o}\right) R_s$$

$$R_p = \left(1 - \frac{V_{out}}{V_o}\right)\frac{V_o}{V_o} R_s$$

3.3 MΩ

g) $f_o = 10.574\,kHz$, $f_{3dB} = 10.648\,kHz \Rightarrow BW = 0.148$

$\Rightarrow Q = 71.45$

$Q = R_p\sqrt{\frac{C}{L}} = 91.52$

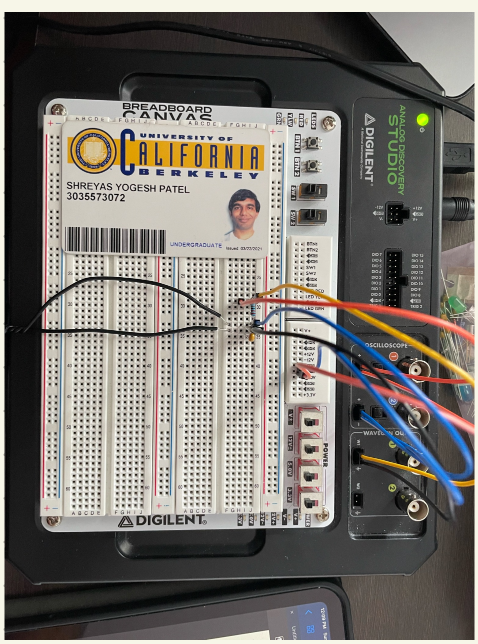
The measured Q is same order of magnitude as the theoretical Q, but the value is much lower

h) $R = 1\,k\Omega$, $R_{tot} = 936.27\,\Omega$, $Q = 5.83$

$R = 3\,k\Omega$, $R_{tot} = 2.491\,k\Omega$, $Q = 15.52$

$R = 10\,k\Omega$, $R_{tot} = 5.95\,k\Omega$, $Q = 37.07$

These values are closer to the measured values, but still greater and diverge at higher values of R.



The inductor is just barely offscreen, but the black wires are its leads. It can be seen in full in the picture on the next page.

Problem R2.7

a) $f = 10\,kHz$, $L = 104.8\ \mu H$

b) $L = \mu_H K N^2 \frac{A}{l} \Rightarrow \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2} = \frac{25^2}{5^2} = 25$

$\frac{L_1}{L_2} = \frac{2.577\,mH}{104.8\ \mu H} = 24.59$

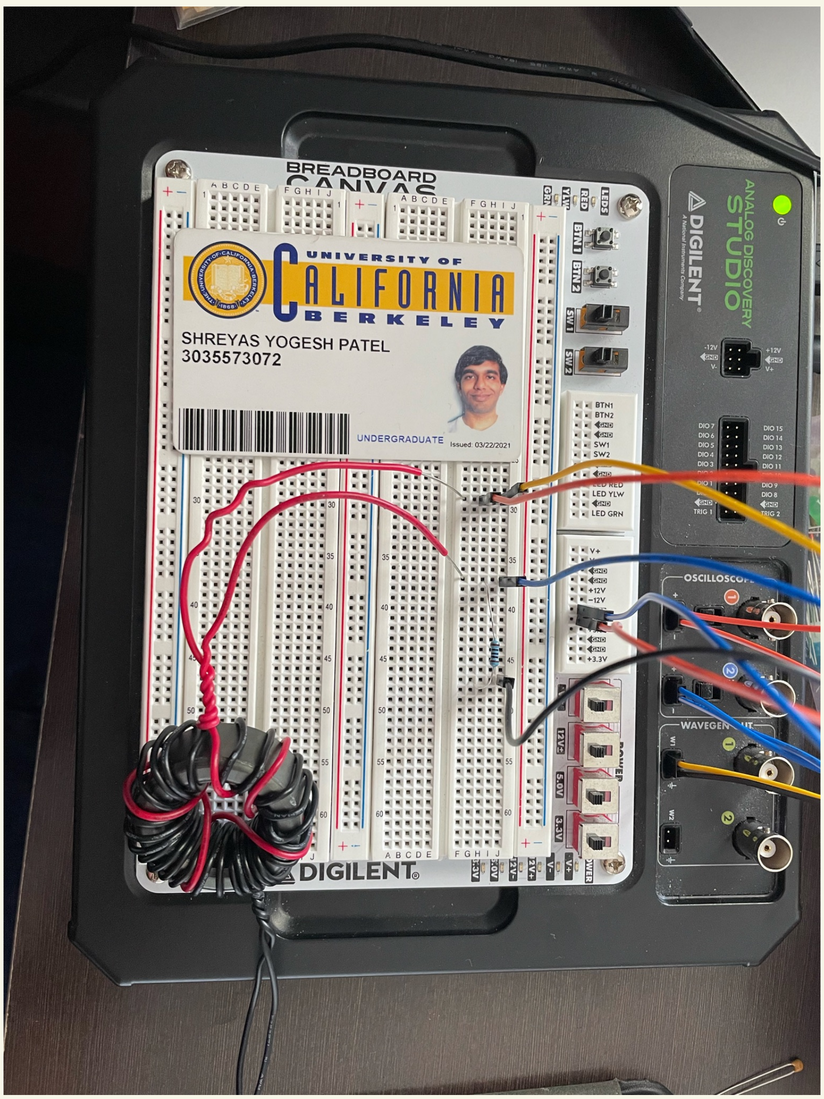
The ratio agrees with our theoretical expectations.

c) $Z_L = j\omega L$, $\omega = 2\pi f \Rightarrow Z_L = 2\pi j f L$

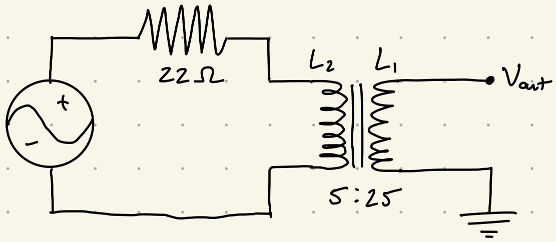
$f = 1\,kHz$, $Z_L = 0.658j$

$f = 10\,kHz$, $Z_L = 6.58j$

$f = 100\,kHz$, $Z_L = 65.8j$



Problem R2.8

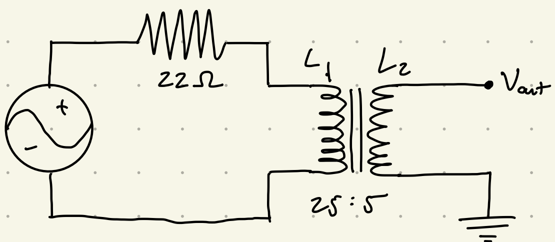


step-up transformer

a) theoretical: $\frac{V_1}{V_2} = \frac{25}{5} = 5$

measured: $\frac{V_1}{V_2} = \frac{2.7551}{0.705} = 3.9$

The values are close but have a 28% difference



step-down transformer

b) theoretical: $\frac{V_2}{V_1} = \frac{5}{25} = 0.2$

measured: $\frac{V_2}{V_1} = \frac{200.79 \times 10^{-3}}{1.0058} = 0.1996$

The measured ratio is almost the same as the theoretical.

c) $k = \sqrt{V_{12} V_{21}}$, $N_{21} = \sqrt{\frac{V_{21}}{V_{12}}} \Rightarrow N_{12} = \sqrt{\frac{V_{12}}{V_{21}}}$

$N_{12} = \left(\frac{V_{12}}{V_{21}}\right)^{1/2} = \left(\frac{3.9}{0.1996}\right)^{1/2} = \underline{4.42}$

$k = ((3.9)(0.1996))^{1/2} = \underline{0.882}$

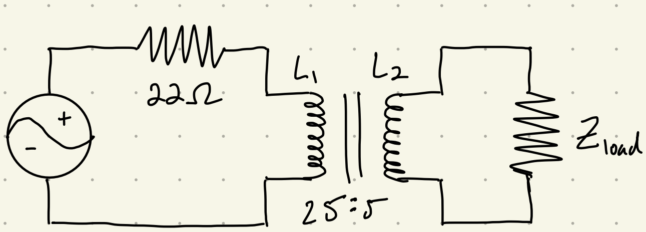
d) $I = \frac{V}{R} = \frac{13.9\,mV}{22\,\Omega} = 0.632\,mA$

$Z_{primary} = \frac{V}{I} = \frac{1.0051\,V}{0.632\,mA} = 1.59\,k\Omega$

$Z_{exp} = 2\pi(100\,kHz)(2.577\,mH)j = 1.62j\,k\Omega$

The value agrees with the expectation value.

e)



$Z_{L_1} \gg Z_{load}$

$2\pi f L \gg Z_{load} \Rightarrow (100\,kHz)(104.8\,\mu H) = 65.85\,\Omega \gg Z_{load}$

$\underline{Z_{load} \ll 65.85\,\Omega}$

f) $\frac{V_2}{V_1} = \frac{125.45\,mV}{0.63280\,V} = 0.198$

The ratio is not different

g) $I_1 = \frac{V}{R} = \frac{113.17\,mV}{22\,\Omega} = 5.14\,mA$

$Z_{primary} = \frac{V}{I} = \frac{0.63449\,V}{5.14\,mA} = 123\,\Omega$

Z_{load} has a voltage drop accross it, and the transformer maintains the step-down ratio, so there is less of a drop accross L_1 since L_2 also has a lower potential drop accross it.

h) with 4.7Ω resistor: $\Delta t = 164.16\,ns \Rightarrow \Delta w = 2\pi \frac{\Delta t}{T} = 2\pi \frac{164.16\,ns}{9972.6\,ns} = 0.103$

floating leads: $\Delta t = 1.4364\,\mu s \Rightarrow T = 10.096\,\mu s \Rightarrow \Delta w = 2\pi \frac{1.4364}{10.096} = 0.894$

$0.33\,\mu F$ capacitor: $\Delta t = 2.4213\,\mu s \Rightarrow T = 9.9316\,\mu s \Rightarrow \Delta w = 2\pi \frac{2.4213}{9.9316} = 1.532$

The greater the impedence of the load, the smaller the phase difference becomes

i) $I_2 = \frac{124.11\,mV}{4.7\Omega} = 26.41\,mA$

$$\frac{I_2}{I_1} = \frac{26.41\,mA}{5.14\,mA} = 5.14$$

$$0 = -jwMI_1 + jwL_2I_2 + Z_2I_2$$

$$jwM\left(\frac{I_1}{I_2}\right) = jwL_2 + Z_2$$

$$\frac{I_1}{I_2} = \frac{jwL_2 + Z_L}{jwM} \Rightarrow \frac{jwk\sqrt{L_1L_2}}{jwL_2 + Z_2} = \frac{I_2}{I_1} = \frac{2\pi(100\,000)(0.882)\sqrt{(104.8\mu H)(2.577\,mH)}\,j}{2\pi(100\,000)(104.8\mu H)j + 4.7} = 4.08$$

The current ratio is comparable to the expected ratio

Problem R2.9

a) $R = 10\,k\Omega$
$f_0 = 9.9189\,kHz$, $f_{3DB} = 9.9950\,kHz \Rightarrow BW = 0.1522 \Rightarrow Q = 65.17$

$R = 100\,k\Omega$
$f_0 = 9.9141\,kHz$, $f_{3DB} = 9.9223\,kHz \Rightarrow BW = 0.0164 \Rightarrow Q = 604.52$

b) $R = 10\,k\Omega$
$f_0 = 9.9189\,kHz$, $f_{3DB} = 9.9970\,kHz \Rightarrow BW = 0.1562 \Rightarrow Q = 63.5$
$Q_{exp} = R\sqrt{\frac{C}{L}} = (10k\Omega)(0.1\mu F / 2.577\,mH)^{1/2} = 62.29$

$R = 100k\Omega$
$f_0 = 9.9149\,kHz$, $f_{3DB} = 9.9262\,kHz \Rightarrow BW = 0.0226 \Rightarrow Q = 438.7$
$Q_{exp} = 622.94$

The actual Q is close to expected at low R_{load}, but diverges to lower values at higher R_{load} values.

c) $R = 1k\Omega$
$f_0 = 9.9189\,kHz$, $f_{3DB} = 10.811\,kHz \Rightarrow BW = 1.7842\,kHz \Rightarrow Q = 5.56$

$R = 3k\Omega$
$f_0 = 9.9189\,kHz$, $f_{3DB} = 10.243\,kHz \Rightarrow BW = 0.6482\,kHz \Rightarrow Q = 15.3$

$R = 10\,k\Omega$
$f_0 = 9.9149\,kHz$, $f_{3DB} = 10.053\,kHz \Rightarrow BW = 0.2762\,kHz \Rightarrow Q = 35.898$

These Q values are close, but still greater then the recorded values, diverging at higher values of R.