Physics 111A Lab 1

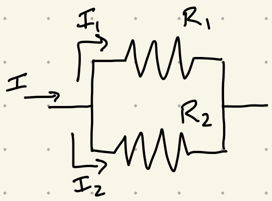
Pre-Lab Questions:

1.

$$V_1 = IR_1$$
$$V_2 = IR_2$$

$$V = V_1 + V_2 = IR_1 + IR_2 = I(R_1 + R_2) = IR_{tot}$$

$$\Rightarrow R_{tot} = R_1 + R_2$$

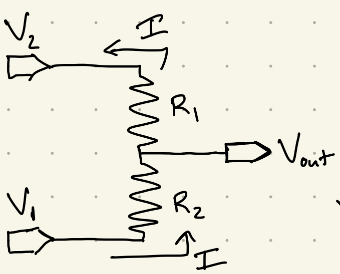


$$V = I_1 R_1 \Rightarrow I_1 = \frac{V}{R_1}$$
$$V = I_2 R_2 \Rightarrow I_2 = \frac{V}{R_2}$$

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V = \frac{V}{R_{tot}}$$

$$\Rightarrow \frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

2.



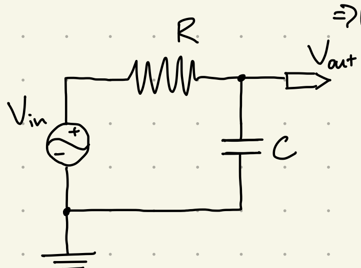
$$V_{out} - V_1 = IR_2 \Rightarrow I = \frac{V_{out} - V_1}{R_2}$$

$$V_2 - V_{out} = IR_1$$

$$V_2 - V_{out} = \frac{R_1}{R_2}(V_{out} - V_1)$$

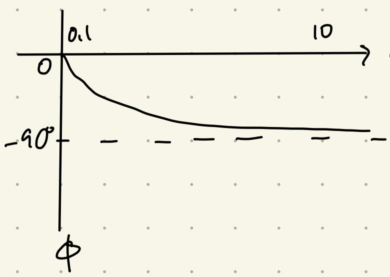
$$\Rightarrow \left(\frac{R_1 + R_2}{R_2}\right)V_{out} = \frac{R_1}{R_2}V_1 + V_2 \Rightarrow V_{out} = \frac{R_1 V_1 + R_2 V_2}{R_1 + R_2}$$

3.

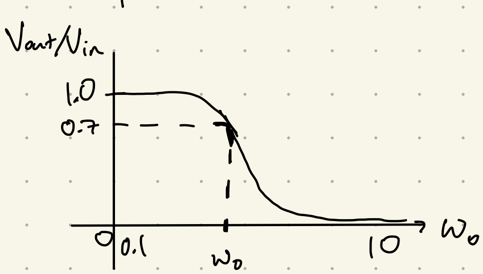


$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_c}{Z_R + Z_c} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1 - j\omega RC}{1 + (\omega RC)^2}$$

$$G(\omega) = |H(\omega)| = \left(\frac{1 - j\omega RC}{1 + (\omega RC)^2} \cdot \frac{1 + j\omega RC}{1 + (\omega RC)^2}\right)^{1/2} = \left(\frac{1 + (\omega RC)^2}{(1 + (\omega RC)^2)^2}\right)^{1/2} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$



$$\phi(\omega) = \tan^{-1}\left(\frac{Im[H(\omega)]}{Re[H(\omega)]}\right) = \tan^{-1}\left(\frac{-\omega RC}{1}\right) = \tan^{-1}(-\omega RC)$$



4. ideal voltmeter : $Z = \infty \, \Omega$, the voltmeter is used in parallel to the component being measured and to minimize changing the circuit, it needs a high impedence to draw the least current

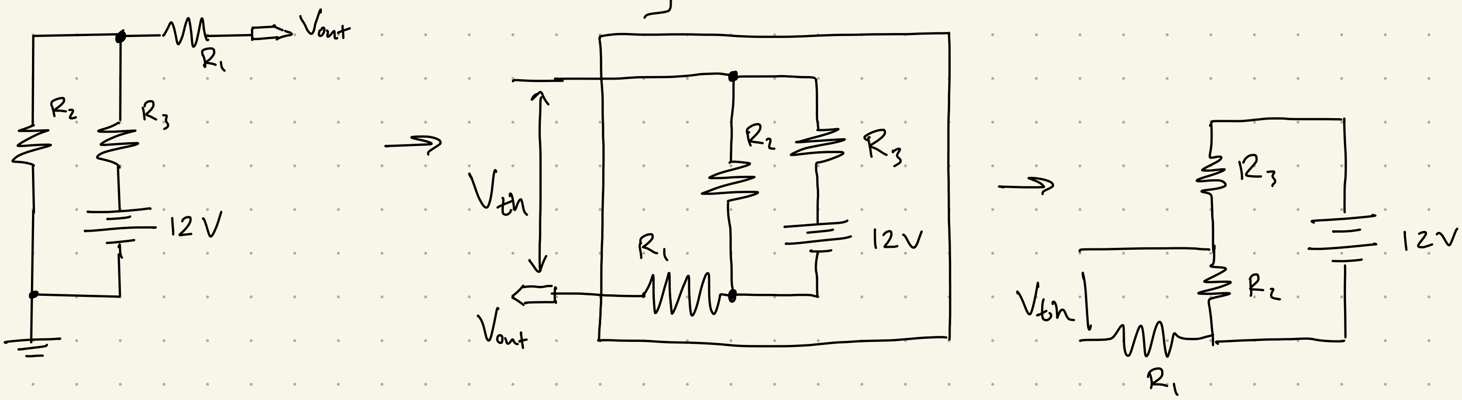
power supply : $Z = 0 \, \Omega$, since if the power supply has an internal resistance it is unable to supply the full voltage it is rated for to the rest of the circuit.

current meter : $Z = 0 \, \Omega$, the current meter is used in series with the component whose current is being measured. To minimize its effect to the circuit, it needs as little resistance as possible due to being in series.

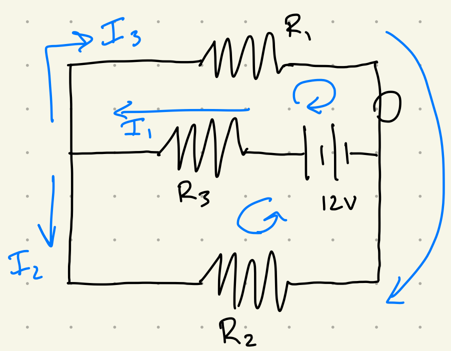
current source : $Z = \infty \, \Omega$, that way the current source's internal resistance draws the least current away from its supply and the rest of the circuit gets the full rated current.

actual power supply : $Z_{out} = 50 \, m\Omega$
actual oscillascope : $Z_{in} = 1 \, M\Omega$

5.



V_{th} is a voltage divider. $V_{th} = \frac{R_2}{R_3 + R_2} V_0$ where $V_0 = 12\,V$



$I_1 = I_2 + I_3$

$12 - I_1 R_3 - I_2 R_2 = 0 \Rightarrow 12 - I_2 R_3 - I_3 R_3 - I_2 R_2 = 0$

$I_2 R_2 - I_3 R_1 = 0 \Rightarrow I_2 R_2 = I_3 R_1 \Rightarrow 12 - I_2 R_3 - I_3 R_3 - I_3 R_1 = 0$

$12 - I_1 R_3 - I_3 R_1 = 0 \Rightarrow 12 - I_3 \frac{R_1 R_3}{R_2} - I_3 R_3 - I_3 R_1 = 0$

$\Rightarrow I_3 \left(\frac{R_1 R_3}{R_2} + R_3 + R_1 \right) = 12$

$\Rightarrow I_3 = \frac{12 R_2}{R_1 R_3 + R_2 R_3 + R_1 R_2} = I_{sc}$

$$Z_{th} = \frac{V_{th}}{I_{sc}} = \left(\frac{12 R_2}{R_3 + R_2} \right) \left(\frac{R_1 R_3 + R_2 R_3 + R_1 R_2}{12 R_2} \right)$$

$$= \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

when $R_2 \to \infty$: $Z_{th} = \frac{R_1 + R_3 + \cancel{\frac{R_1 R_3}{R_2}} \to 0}{1 + \cancel{\frac{R_3}{R_2}} \to 0} = \underline{R_1 + R_3}$

when $R_3 \to 0$: $Z_{th} = \underline{\frac{R_1 R_2}{R_2} = R_1}$

$\underline{V_{th} = \frac{12 R_2}{R_2 + R_3}, \quad I_{sc} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{12 R_2}, \quad Z_{th} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2 + R_3}}$

Lab Exercises:

Problem R1.1:

10 kΩ resistors

#	measured R (kΩ)
1	10.05
2	10.00
3	9.97
4	10.05
5	9.97

A tolerance of ±1% for 10kΩ resistors creates a range of 9.90 kΩ – 10.10 kΩ. All measured resistors fell within this range.

10nF capacitors

#	measured C (nF)
1	9.38
2	9.34
3	8.96
4	9.23
5	9.28

A tolerance of ±10% for 10nF resistors creates a range of 9nF – 11nF. Out of the 5 measured capacitors, only one fell outside this range.

Problem R1.2:

a) DMM Power Supply Measurements

Voltage (V)	measured Voltage (V)
3.3	3.30 ±0.5%
5	5.02 ±0.5%
+12	12.02 ±0.5%
-12	-12.09 ±0.5%

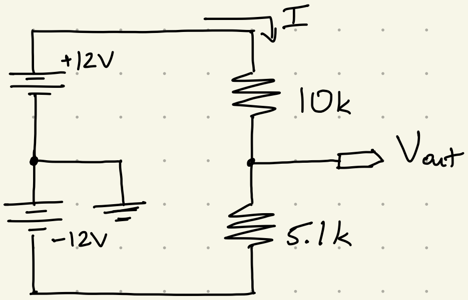
±0.5% uncertainty from DMM specifications

b) ADS Power Supply Measurements

Voltage (V)	measured voltage (V)
3.3	3.316 ±0.5%
5	5.034 ±0.5%
+12	12 ±0.5%
-12	-12.06 ±0.5%

±0.5% uncertainty from ADS specifications

Problem R1.3



$R_1 = 10k\Omega$, $R_2 = 5.1k\Omega$

$V_1 = IR_1 \Rightarrow 12V - V_{out} = 10k\,I \Rightarrow I = \frac{12 - V_{out}}{10k}$

$V_2 = IR_2 \Rightarrow V_{out} + 12V = 5.1k\,I \Rightarrow V_{out} + 12 = \frac{5.1}{10}(12 - V_{out})$

$\Rightarrow \left(\frac{5.1}{10} + 1\right)V_{out} = -12 + \frac{5.1}{10}(12)$

$\Rightarrow \frac{15.1}{10} V_{out} = -\frac{4.9}{10}(12)$

$\Rightarrow V_{out} = -\frac{4.9}{15.1}(12) = -3.89V$

$I = \frac{-3.89 - 12}{10k} = \frac{-15.89}{10k} = -1.59\,mA$

$V_{out} = -3.89\,V$

$V_1 = 12V - V_{out} = 15.89V$ drop accross $10k\Omega$ resistor

$V_2 = V_{out} + 12V = 8.11\,V$ drop accross $5.1k\Omega$ resistor

$I = -1.59\,mA$

Problem R1.4

Nominal Measurement	Actual Measurement
$5.1\,k\Omega$	$5.11\,k\Omega$
$10\,k\Omega$	$9.96\,k\Omega$
$+12\,V$	$+12.02\,V$
$-12\,V$	$-12.09\,V$

$I = \frac{V_{out} - 12.02}{9.96k}$, $-12.09 - V_{out} = 5.11k\,I \Rightarrow -12.09 - V_{out} = \frac{5.11}{9.96}(V_{out} - 12.02)$

$\Rightarrow \left(\frac{5.11}{9.96} + 1\right)V_{out} = -12.09 + \frac{5.11}{9.96}(12.02)$

$V_{out} = -3.91\,V$

$V_1 = V_{out} - 12.02 = -15.93\,V$

$V_2 = -12.09 - V_{out} = -8.18\,V$

$I = \frac{V_{out} - 12.02}{9.96k} = -1.59\,mA$

The difference in the results is negligible.

Problem R1.5

DMM Measurements

a)

Resistor ($k\Omega$)	Voltage (V)
5.1	8.18
10	15.94

measured $V_{out} = -3.9V$

b) measured current : $I = -1.59 \text{ mA}$

c) calculated resistances:

$$V_1 = IR_1 \Rightarrow R_1 = \frac{V_1}{I} = \frac{8.18}{1.59 \times 10^{-3}} = 5.145 \text{ k}\Omega$$

$$R_2 = V_2/I = 15.94/1.59 \times 10^{-3} = 10.03 \text{ k}\Omega$$

Both resistor measurements fall within their respective tolerance ranges.

Problem R1.6

a) $P = IV = I^2R = V^2/R$

$10k\Omega$ resistor : $V = 15.89 V$, $P = \frac{V^2}{R} = (15.89)^2/10k = 25.25 \text{ mW}$

$5.1 k\Omega$ resistor: $V = 8.11 V$, $P = \frac{V^2}{R} = (8.11)^2/5.1k = 12.896 \text{ mW}$

b) $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$, $P_{max} = 0.25W$, $V = 12 - (-12) = 24$

$R = \frac{24^2}{0.25} = 2304 \Omega \approx 2.3 k\Omega$

c) $\frac{10}{5.1} = 1.96$

$P_{10} = I^2 R_{10}$, $P_{5.1} = I^2 R_{5.1}$

$\frac{P_{10}}{P_{5.1}} = \frac{R_{10}}{R_{5.1}} = 1.96 \Rightarrow P_{10} = 1.96 P_{5.1}$

The power of the resistor corresponding to the $10k\Omega$ will exceed the maximum power rating first since it grows at rate of 1.96x that of the $5.1k\Omega$ one.

$P_{10} = V^2/R_{10} \Rightarrow R_{10} = \frac{V^2}{P_{10}} \Rightarrow R_{10} = \frac{15.94^2}{0.25} = 1016.3 \Omega = 1.02 k\Omega$

d) $P_{10} = 0.25W$, $\frac{P_{10}}{P_{5.1}} = 1.96 \Rightarrow P_{5.1} = \frac{P_{10}}{1.96} = \frac{0.25}{1.96} = 0.1276 W$

$P_{tot} = 0.3776 W$

Problem R1.7

$$V_{RMS}^2 = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} [V(t)]^2\,dt$$

a) $V(t) = V_0 \sin(\omega t)$, $T_1 = 0$, $T_2 = \frac{2\pi}{\omega}$

$$V_{rms}^2 = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} V_0^2 \sin^2(\omega t)dt = \frac{\omega}{2\pi} V_0^2 \int_0^{2\pi/\omega} \sin^2(\omega t)dt = \frac{-\omega}{4\pi} V_0^2 \int_0^{2\pi/\omega} \cos(2\omega t) - 1\, dt$$

$$= -\frac{\omega}{4\pi} V_0^2\left[\frac{1}{2\omega}\sin(2\omega t) - t\right]_0^{2\pi/\omega} = -\frac{\omega}{4\pi} V_0^2\left[-\frac{2\pi}{\omega} + 0\right] = \frac{1}{2}V_0^2$$

$$V_{rms} = \frac{1}{\sqrt{2}} V_0$$

$$V_{pp} = V_0\,(-V_0) = 2V_0$$

b) $V(t) = \frac{2}{\pi} V_0 \omega t$ for $0 \le t \le \frac{\pi}{2\omega}$ where $\omega = \frac{2\pi}{T}$

$$V_{\frac{1}{4}} = \int_0^{\frac{\pi}{2\omega}} \frac{4}{\pi^2} V_0^2 \omega^2 t^2\, dt = \frac{4}{\pi^2} V_0^2\,\omega^2 \int_0^{\frac{\pi}{2\omega}} t^2\, dt = \frac{4}{\pi^2}\frac{V_0^2}{3}\omega^2 t^3\Big|_0^{\pi/2\omega} = \frac{4}{\pi^2}\frac{V_0^2\omega^2}{3}\;\frac{\pi^3}{8\omega^3} = \frac{V_0^2\pi}{6\omega}$$

for full period, $V_{rms}^2 = \frac{\omega}{2\pi}\, 4V_{\frac{1}{4}} = \frac{\omega}{2\pi}\,\frac{2V_0^2\pi}{3\omega} = \frac{1}{3}V_0^2$

$$V_{rms} = \frac{1}{\sqrt{3}} V_0$$

$$V_{pp} = V_0 - (-V_0) = 2V_0$$

c) $V(t) = V_0$ for $0 \le t \le \frac{\pi}{\omega}$ where $\omega = \frac{2\pi}{T}$

$$V_{1/2} = \int_0^{\pi/\omega} V_0^2\, dt = V_0^2\, t\Big|_0^{\pi/\omega} = \frac{\pi}{\omega}V_0^2$$

for full period, $V_{rms}^2 = \frac{\omega}{2\pi}\, 2V_{1/2} = \frac{\omega}{2\pi}\,\frac{2\pi}{\omega} V_0^2 = V_0^2$

$$V_{rms} = V_0$$

$$V_{pp} = V - (-V_0) = 2V_0$$

sine wave

	V_0	V_{rms}	V_{pp}
V_0	1	$\sqrt{2}$	$\frac{1}{2}$
V_{rms}	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{2\sqrt{2}}$
V_{pp}	2	$2\sqrt{2}$	1

triangular wave

	V_0	V_{rms}	V_{pp}
V_0	1	$\sqrt{3}$	$\frac{1}{2}$
V_{rms}	$\frac{1}{\sqrt{3}}$	1	$\frac{1}{2\sqrt{3}}$
V_{pp}	2	$2\sqrt{3}$	1

square wave

	V_0	V_{rms}	V_{pp}
V_0	1	1	$\frac{1}{2}$
V_{rms}	1	1	$\frac{1}{2}$
V_{pp}	2	2	1

Problem R 1.8

a) DMM Measurement : 0.7 V

b) AC RMS from voltmeter : 708 mV
update rate of 1s

c) Scope Measurement : 0.708 V
Amplitude : 0.999 V

d)

	Vrms (V)	Vamp (V)	Vrms/Vamp	$\frac{V_{rms}}{V_{amp}}$ from 1.7
DMM	0.7		0.7007	$\sqrt{\frac{1}{2}} = 0.707$
voltmeter	0.708		0.708	
scope	0.707	0.999	0.707	

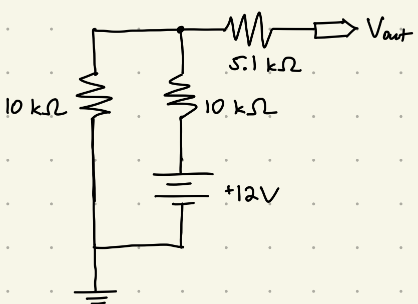
e)

	10Hz	30 Hz	100 Hz	300 Hz	1kHz	10 kHz
DMM	0.718	0.700	0.698	0.700	0.696	0.681
voltmeter	0.708	0.708	0.708	0.706	0.690	0.120
scope	0.707	0.707	0.708	0.708	0.707	0.705

← discrepency likely due to 1s update time with range 4Hz – 2.048kHz since 10kHz > 2.048 kHz

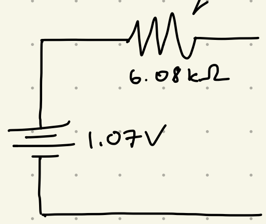
Greatest difference in DMM measurements occurs at the extreme cases

Problem R1.9



a) $V_{out} = 1.07\,V$ $R_{th} = \frac{1.07}{0.176 \times 10^{-3}} = 6.08\,k\Omega$
$I_{sc} = 0.176\,mA$

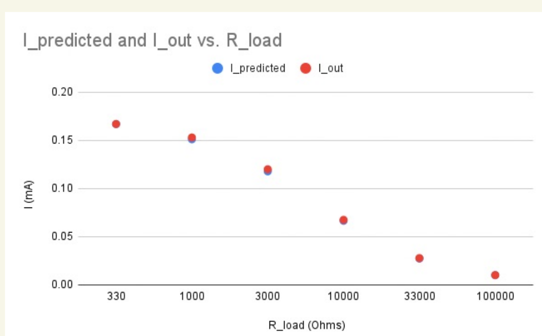
Thevenin Equivalent Circuit:



b)

R_{load} Ω	V_{out} (V)	I_{out} (mA)
330	0.055	0.167
1k	0.153	0.153
3k	0.359	0.120
10k	0.675	0.0675
33k	0.916	0.0278
100k	1.02	0.0102

$I_{out} = \frac{V_{out}}{R_{load}}$

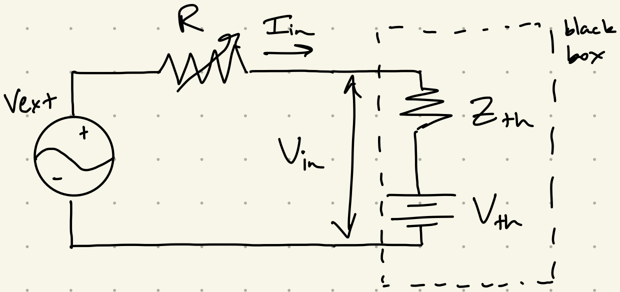


c) 6.01 kΩ

$\left(1 - \frac{6.01}{6.08}\right) \times 100 = 1.15\%$ difference

The resistance is close to the Thevenin resistance.

Problem R1.10



$$Z_{in} = R\left(\frac{V_{in}}{V_{ext} - V_{in}}\right)$$

Resistance (Ω)	V_{in} (V)	Z (MΩ)
0	1.00	—
10k	0.99	0.99
33k	0.97	1.067
100k	0.91	1.011
1 M	0.51	1.04
2M	0.34	1.03
5.6 M	0.15	0.99

$V_{ext} = 1V$

$10k\left(\frac{0.99}{0.01}\right)$

$33k\left(\frac{0.97}{0.03}\right)$

$100k\left(\frac{0.91}{0.09}\right)$

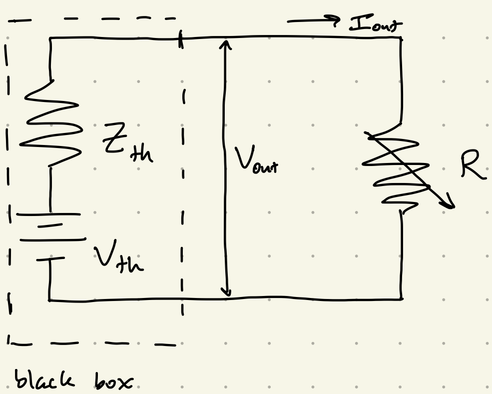
$1M\left(\frac{0.51}{0.49}\right)$

$2M\left(\frac{0.34}{0.66}\right)$

$5.6M\left(\frac{0.15}{0.85}\right)$

$Z_{avg} = 1.021\ M\Omega$

Problem R1.11



$$Z_{out} = R\left(\frac{V_{th} - V_{out}}{V_{out}}\right)$$

a)

R (Ω)	V_{out} (mV)	Z_{out} (Ω)
0	999	—
47	484	50.01
100	665	50.23
1k	951	50.47

$V_{th} = 0.999\ V$

$Z_{out} = 47\left(\frac{999-484}{484}\right)$

$Z_{out} = 100\left(\frac{999-665}{665}\right)$

$Z_{out} = 1k\left(\frac{999-951}{951}\right)$

b) $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$, $P_{max} = 0.25W$, $V = 12V$

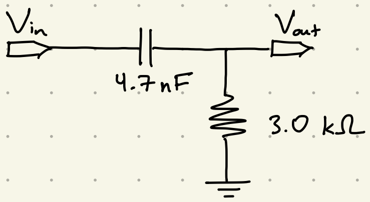
$R_{min} = \frac{144}{0.25} = 576\ \Omega$

$R = 0\Omega$, $V = 11.998\ V = V_{th}$

$R = 680\Omega$, $V = 11.992\ V$

$$Z_{out} = 680\left(\frac{11.998 - 11.992}{11.992}\right) = 0.34\ \Omega$$

Problem R1.12



$C_{act} = 4.31\,nF$

$R_{act} = 2.98\,k\Omega$

$G(\omega) = \left(1 + (\omega RC)^2\right)^{-1/2}$

$\tau = RC = 12.8 \times 10^{-6}\,s$

a)

Frequency (kHz)	V_{amp} (mV)	V_{RMS} (mV)	Δt (μs)	$\Delta\phi$ (rad)	$G(\omega)_{exp}$
1	3.4	2.7	2.70	1.7	0.99962
10	29	21	14.607	0.918	0.992
100	47	34	0.225	0.141	0.616

V_{in} is set to 50mV

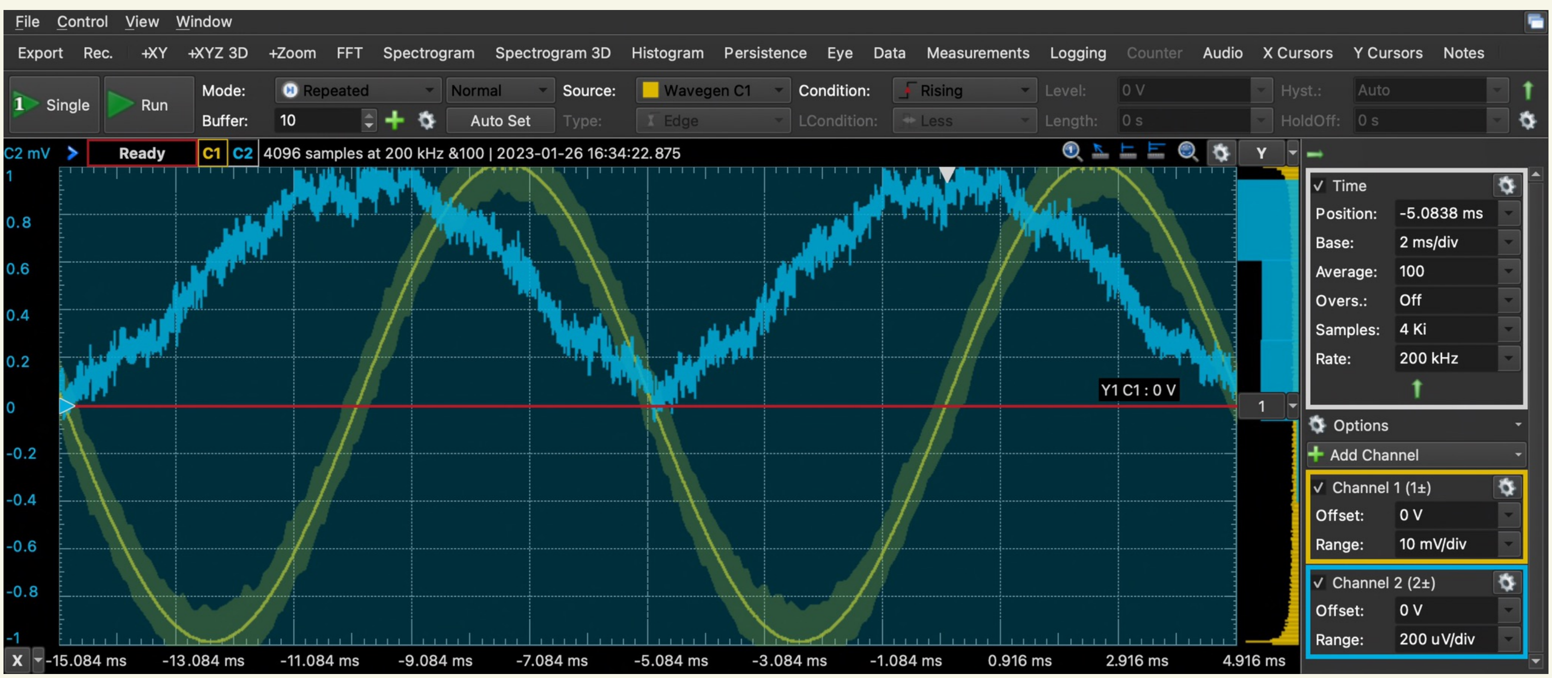
$G(\omega) = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$ $\quad \phi(\omega) = \frac{1}{\omega RC}$

$G(1\,kHz) = \frac{1000(2.98)(4.31\times10^{-9})}{\sqrt{1+(1000(2.98)(4.31\times10^{-9}))^2}} = 1.28\times10^{-5}$ $\quad \phi(1\,kHz) = 7.79\times10^{4}$

$G(10\,kHz) = 1.28\times10^{-4}$ $\quad \phi(10\,kHz) = 7.79\times10^{3}$

$G(100\,kHz) = 1.28\times10^{-3}$ $\quad \phi(100\,kHz) = 779$

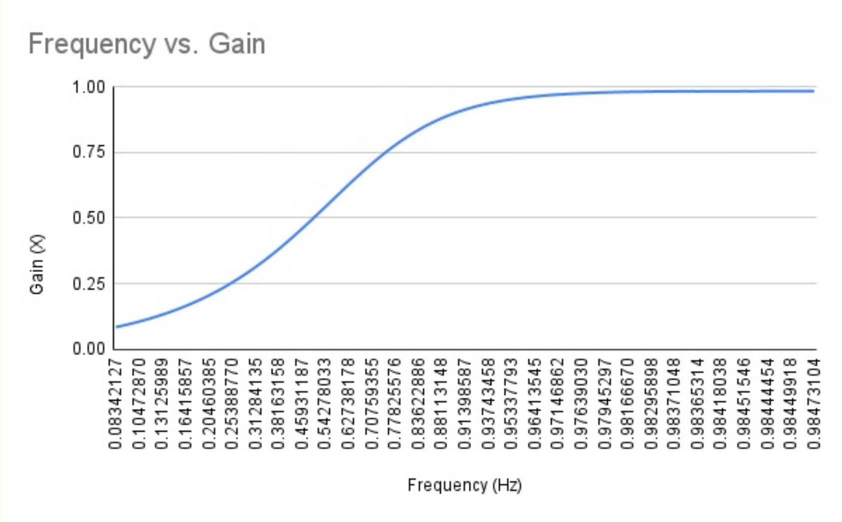
b) There is an unexpected offset in the V_{out} reading from oscilloscope channel 2.



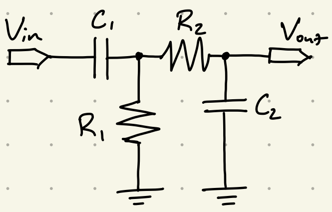
Problem 1.13

a) $\phi = 43.31°$

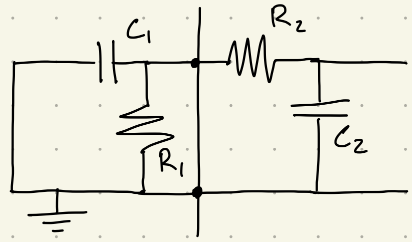
b)



Problem 1.14



$\Rightarrow$



$f_{low} = 1\,kHz$, $f_{high} = 10\,kHz$

a) scope : $Z_{scope} = 1\,M\Omega$
gen : $Z_{gen} = 50\,\Omega$

From the point between the two filters the first filter can be thought of as having the capacitor and resistor in parallel to each other.

$$Z_{out,1} = \left(\frac{1}{R_1} + j\omega C_1\right)^{-1}$$

The second filter can be seen as having the resistor and capacitor in series.

$$Z_{in,2} = R_2 + \frac{1}{j\omega C_2}$$

$$f_\ell = \frac{1}{2\pi R_1 C_1} \Rightarrow C_1 = \frac{1}{2\pi R_1 f_\ell} \quad ; \quad f_h = \frac{1}{2\pi R_2 C_2} \Rightarrow C_2 = \frac{1}{2\pi R_2 f_h}$$

$$Z_{out,1} = \left(\frac{1}{R_1} + \frac{j\omega}{2\pi R_1 f_\ell}\right)^{-1} = \left(\frac{2\pi f_\ell + j\omega}{2\pi R_1 f_\ell}\right)^{-1} = \frac{2\pi f_\ell}{2\pi f_\ell + j\omega} R_1$$

$$Z_{in,2} = R_2 + \frac{2\pi R_2 f_h}{j\omega} = \left(\frac{j\omega + 2\pi f_h}{j\omega}\right) R_2$$

$$\Rightarrow |Z_{gen}| \ll |Z_{out,1}| \ll |Z_{in,2}| \ll |Z_{scope}|$$

b) $C_1 = 0.1\,\mu F$, $C_2 = 680\,pF$

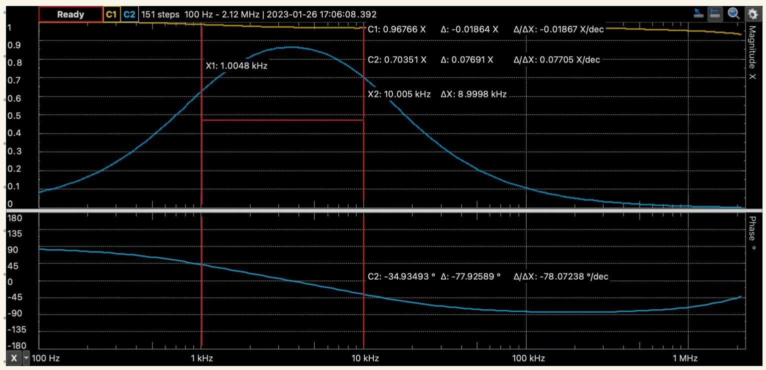
$$C_1 = \frac{1}{2\pi f_\ell R_1} \Rightarrow R_1 = \frac{1}{2\pi f_\ell C_1} = \frac{1}{2\pi (1000)(0.1\times10^{-6})} = 1591.5\,\Omega \approx 1.6\,k\Omega$$

$$C_2 = \frac{1}{2\pi f_h R_2} \Rightarrow R_2 = \frac{1}{2\pi f_h C_2} = \frac{1}{2\pi (10\,000)(680\times10^{-12})} = 23405\,\Omega \approx 23.4\,k\Omega$$

The closest matching resistors are $1.5\,k\Omega$ and $22\,k\Omega$

c) $f = 1\,kHz$, $G = 0.63$, $\phi = 43°$

$f = 10\,kHz$, $G = 0.704$, $\phi = -35°$



Problem R1.15

for 3.3V, $V_{rms} = 334\,\mu V$, $f = 1.13\,MHz$, $V_{peak} = 0.04\,mV$

for 12V, $V_{rms} = 7.7\,mV$, f and V_{peak} are too noisy to obtain

e) The 12V signal is much noisier because the offset is set to a value greater than 5V, so the incoming 12V signal gets divided by 10 so in comparison to the 3.3V signal it is much smaller.

f) Using the high pass filter, $f_h = 1\,kHz$ and $V_{peak} = 0.75\,mV$