Physics 111A Lab 7

Pre-Lab Questions

1) When connected to an impedence of Z, a negative impedence converter has an impedence of $-Z$, which can be useful since the load can be seen as supplying a voltage instead of consuming it.

$$V_- = I_- Z, \quad V_{out} - V_- = I_- R, \quad V_+ - V_{out} = I_+ R$$

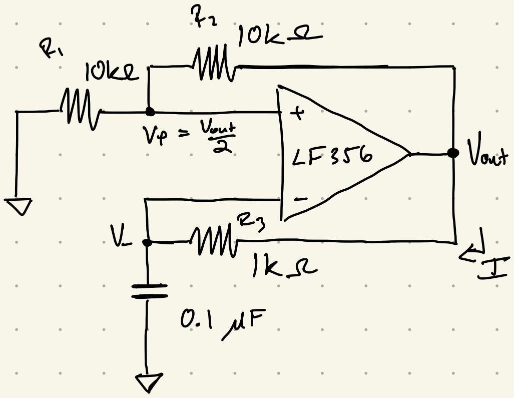
$$\Rightarrow \frac{I_+}{I_-} = \frac{V_+ - V_{out}}{V_{out} - V_-}, \quad V_+ = V_-$$

$$\Rightarrow \frac{I_+}{I_-} = \frac{V_- - V_{out}}{V_{out} - V_-} = -1$$

$$V_+ = I_+ Z_{in}$$

$$\Rightarrow I_+ Z_{in} = I_- Z \Rightarrow Z = \frac{I_+}{I_-} Z_{in} \Rightarrow Z = -Z_{in}$$

2)



$$-\frac{V_{out}}{2} \le V_- \le \frac{V_{out}}{2}$$

$$\frac{V_{out} - V_-}{R_3} = I, \quad C = \frac{q}{V_-} \Rightarrow CV_- = q \Rightarrow C\frac{dV_-}{dt} = I$$

$$\frac{V_{out} - V_-}{R_3} = C\frac{dV_-}{dt} \Rightarrow \frac{dV_-}{dt} + \frac{V_-}{R_3 C} = \frac{V_{out}}{R_3 C}$$

$$\frac{dV_-}{dt} + \frac{V_-}{R_3 C} = 0 \Rightarrow \frac{1}{V_-} dV_- = -\frac{1}{R_3 C} dt \Rightarrow \ln V_- = -\frac{1}{R_3 C}(t + k)$$

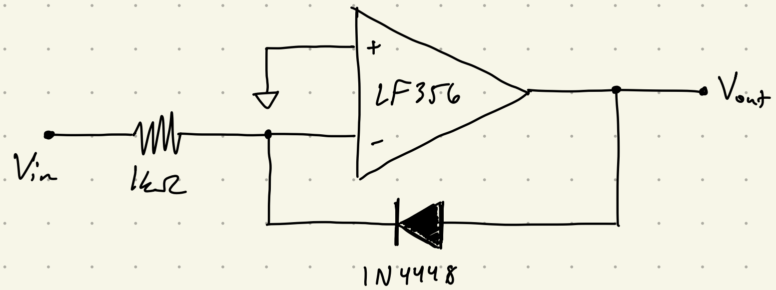
$$V_- = k e^{-t/R_3 C}, \quad k = \frac{V_{out}}{2}$$

the capacitor discharges for $t \approx R_3 C$ then switches back to charging.

so $\frac{1}{2}T \approx \tau = R_3 C \Rightarrow T = 2R_3 C \Rightarrow f = \frac{1}{2R_3 C}$

$$f = \frac{1}{2(1k\Omega)(0.1\,\mu F)} = 5\ kHz$$

3) $V_- = V_+ = 0$



$I_+ = -\frac{V_{in}}{R}$, $I_D = I_s \exp\left(\frac{eV_{out}}{nkT}\right)$

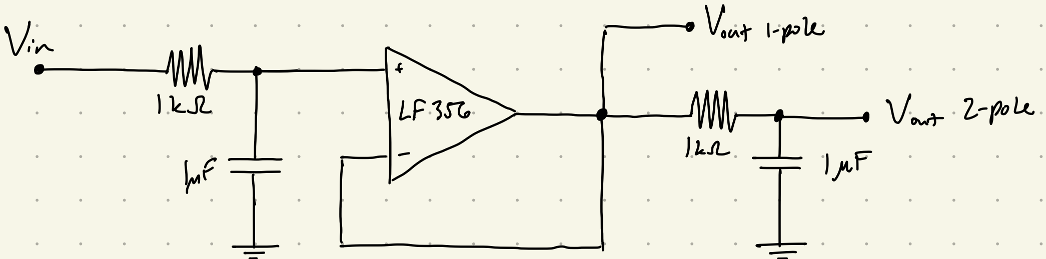
$I_+ = I_D \Rightarrow -\frac{V_{in}}{R} = I_s \exp\left(\frac{eV_{out}}{nkT}\right)$

$\exp\left(\frac{eV_{out}}{nkT}\right) = -\frac{V_{in}}{I_s R}$

$V_{out} = \frac{nkT}{e} \ln\left(-\frac{V_{in}}{I_s R}\right)$

for $V_{in} = -1V$, $R = 1k\Omega$, $I_s = 5nA$, $e = 1.6 \times 10^{-19}$, $k = 1.38 \times 10^{-23}$, $T = 293.15K$, $n = 2$

$V_{out} = \frac{2(1.38 \times 10^{-23})(293.15)}{1.6 \times 10^{-19}} \ln\left(\frac{1}{(5 \times 10^{-9})(1000)}\right) = \underline{0.616\ V}$

4) 

$G_{tot} = G_1 \cdot G_2$, at 3dB point , $G_1 = G_2 = \frac{1}{\sqrt{2}}$

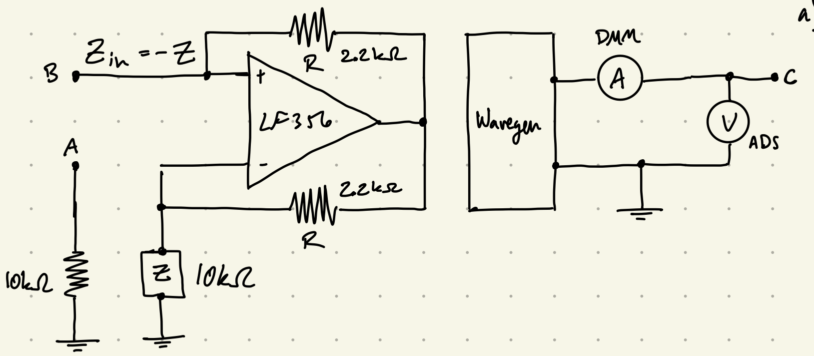
$\underline{G_{tot} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}}$

$\phi_{tot} = \phi_1 + \phi_2$, $\phi_1 = \phi_2 = -45°$ for a low-pass filter at 3dB point

$\underline{\phi_{tot} = -45° + (-45°) = -90°}$

Lab Exercises:

Problem R7.1



a)

Wavegen (V)	DMM (mA)	ADS (V)	R(kΩ)
-5	-0.496	-4.934	9.95
-2.5	-0.248	-2.472	9.97
0	0	0	——
2.5	0.248	2.464	9.94
5	0.496	4.976	10.03

The resulting resistance through Ohm's Law is very close to 10kΩ.

b)

Wavegen (V)	DMM (mA)	ADS (V)	Z_{eff} (kΩ)
-5	0.501	-5.08	-10.14
-2.5	0.250	-2.54	-10.16
1	-0.100	1.018	-10.18
3	-0.301	3.05	-10.13
5	-0.502	5.082	-10.12

The impedence of the circuit calculated through Ohm's Law gives us an impedence of ≈ -10 kΩ

c)

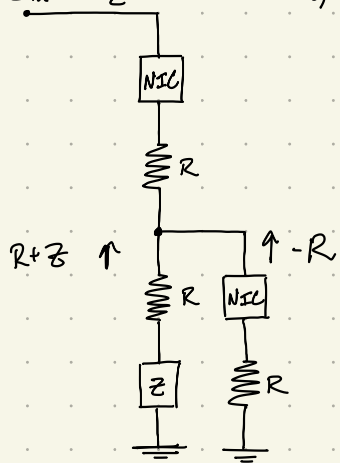
Wavegen (V)	DMM (mA)	ADS (V)	Z_{eff} (kΩ)
-5	-0.005	-5.002	1000.4
-2.5	-0.003	-2.502	834
1	0	1.002	——
3	0.002	3.004	1502
5	0.004	5.006	1251.5

$$Z_{eq} = \left(\frac{1}{Z_A} + \frac{1}{Z_B}\right)^{-1} = \left(\frac{1}{Z_A} - \frac{1}{Z_A}\right)^{-1} = 0^{-1} = \infty$$

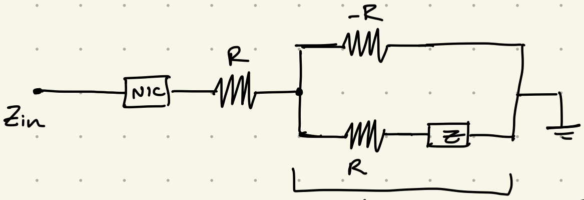
the impedence would be very high which is supported by the very small current measurements.

Problem R7.2

$Z_{in} = R^2/Z$



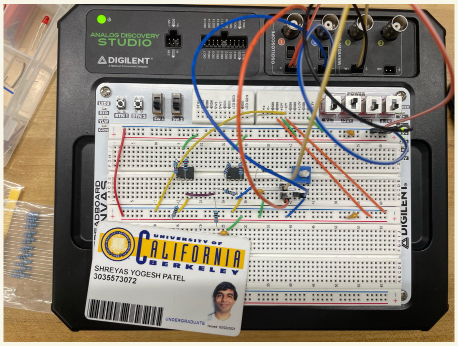
a)



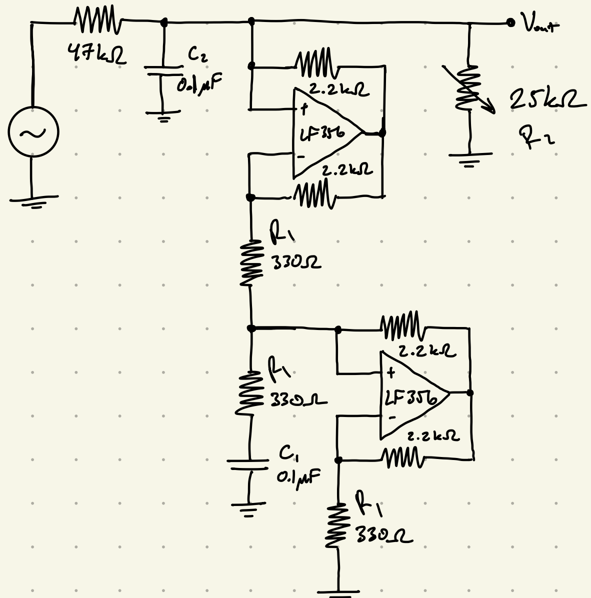
$$Z_p = \left(-\frac{1}{R} + \frac{1}{R+Z}\right)^{-1} = \left(\frac{R - R - Z}{R(R+Z)}\right)^{-1}$$
$$= -\frac{R}{Z}(R+Z)$$

$$Z_s = R - \frac{R}{Z}(R+Z) = R - \frac{1}{Z}R^2 - R = -\frac{1}{Z}R^2$$

$$Z_{in} = \frac{1}{Z}R^2, \text{ for } Z = \frac{1}{j\omega C} \Rightarrow Z_{in} = j\omega C R^2$$



b)



$f_o = 5.4099$ kHz

$BW = |5.2743\text{ kHz} - 5.6916\text{ kHz}| = 0.4173$ kHz

$Q = \frac{f_o}{BW} = 12.964$

$R = 24.5\text{ k}\Omega$

$f_{o_{calc}} = \frac{1}{2\pi\sqrt{LC}}, \quad L = CR^2, \quad C = C_2$

$f_o = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi(330\Omega)(0.1\mu F)} = 4.822$ kHz

$Q = R_2\sqrt{\frac{C}{L}}, \quad R_2 = 24.5\text{k}\Omega, \; C = 0.1\mu F, \; L = CR^2$

$Q = R_2\sqrt{\frac{C_2}{C R_1^2}}, \quad C_1 = C_2 \Rightarrow Q = \frac{R_2}{R_1} = \frac{24.5\text{k}\Omega}{330\Omega} = 74.2$

184.73µs

20ms

The calculated Q is higher than the measured.

c) $V_{peak} = 15$ mV, $5\%\ V_{peak} = 0.75$ mV

$Q = \frac{20\text{ ms}}{184.73\mu s} = 108.27$ the value is far greater than what was measured by the network analysis

d) $f_o = 1.7118$ kHz

$f_{o_{calc}} = \frac{1}{2\pi R_1\sqrt{C_1 C_2}} = \frac{1}{2\pi(330\Omega)\sqrt{(0.1\mu F)(1\mu F)}} = 1.53$ kHz

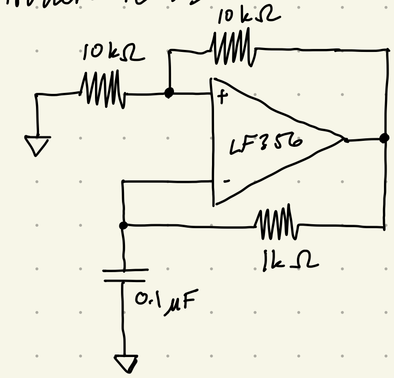
$BW = |1.6711 - 1.7747|$ kHz

$Q_{calc} = R_2\sqrt{\frac{C_2}{C_1 R_1^2}} = 23.4$

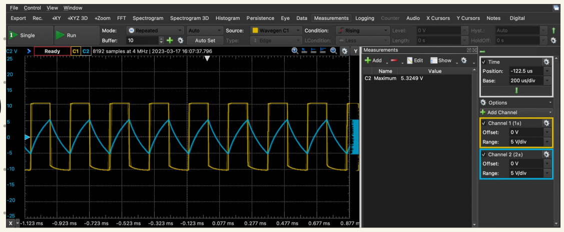
$Q = 16.5$

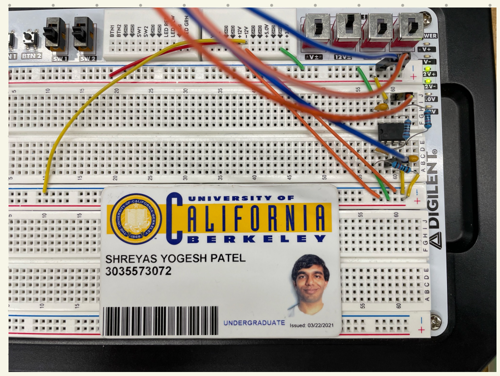
The expected f_o and Q are very close the measured values.

Problem R 7.3



a)





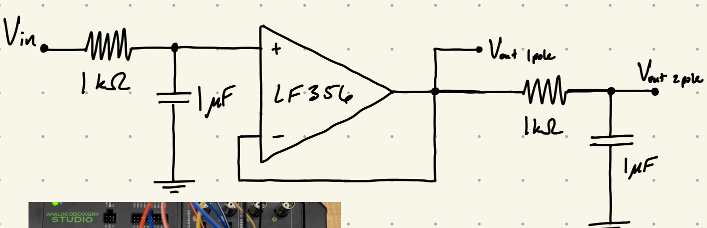
$T = 242.28\ \mu s$

$f_{out} = 4.13\ kHz$

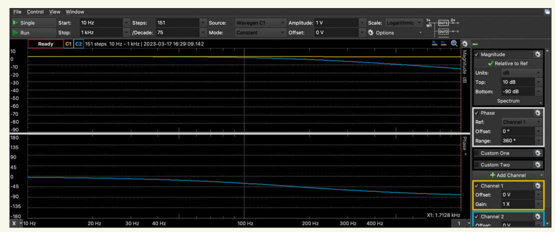
b) from pre-lab Q2, $f_{out} \approx \frac{1}{2RC} = 5\ kHz$

the op-amp is ensuring that V_+ and V_- are equal. V_+ is always $\frac{1}{2}V_{out}$, but V_- changes due to the capacitor charging. The op amp then has to switch V_{out} so the capacitor can discharge and this oscillation continues back and forth to regulate V_-.

Problem R 7.4

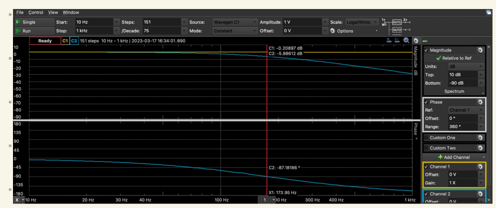


a) 1-pole filter:

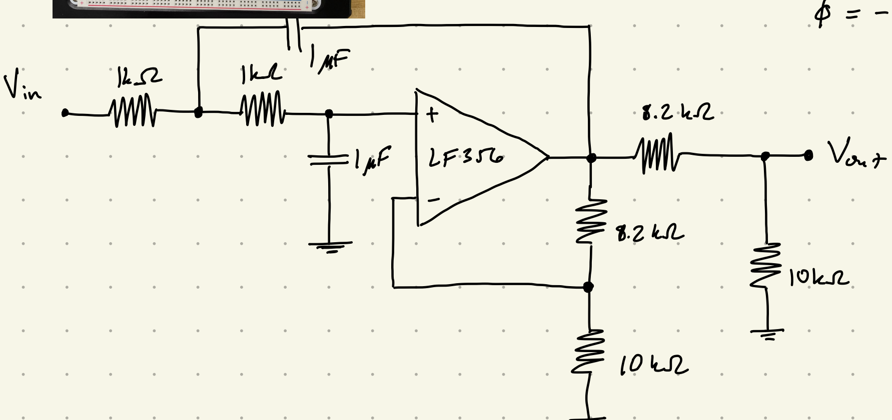


$f_{3dB} = 173.95\ Hz$
$\phi = -43.47344°$

2-pole filter:



@ $f = 173.95\ Hz$
$G = -5.98612\ dB$
$\phi = -87.18186°$

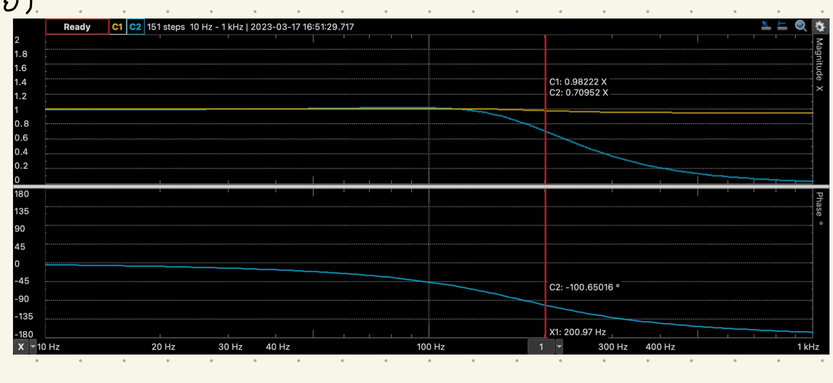


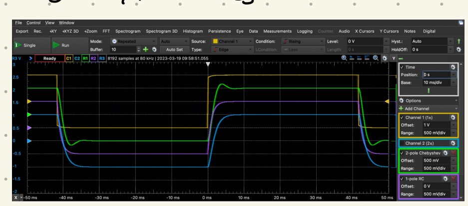
c) 2-pole Chebyshev
time to settle: $t = 2.1467\ ms$
$t = 866.7\ \mu s$
overshoot: $0.1722\ V$, $t = 3.5\ ms$

1-pole RC:
time to settle: $t = 1.85\ ms$
$t = 330\ \mu s$
no overshoot

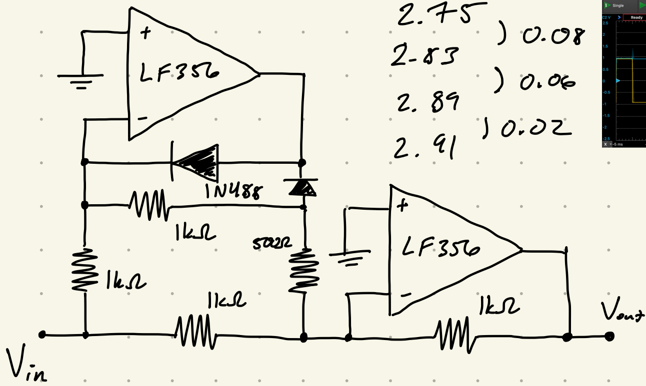
2-pole RC:
time to settle: $t = 3.46\ ms$
$t = 1.05\ ms$
no overshoot

b)



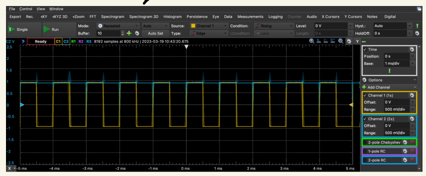


Problem R7.5



2.75
) 0.08
2.83
) 0.06
2.89
) 0.02
2.91

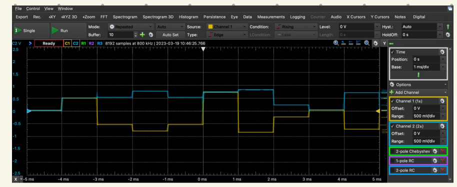
square wave



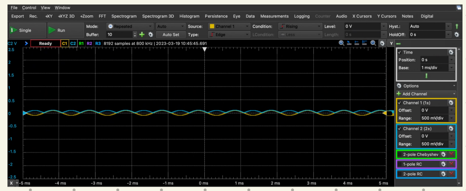
sine +0.5V

sine wave

noise



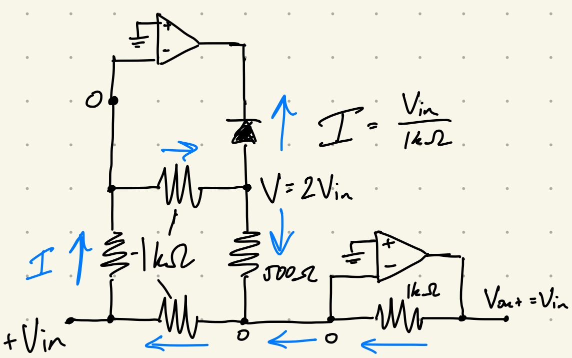
The circuit works very well for all ranges. There is no distortion in the rectified wave at very high and very low amplitudes

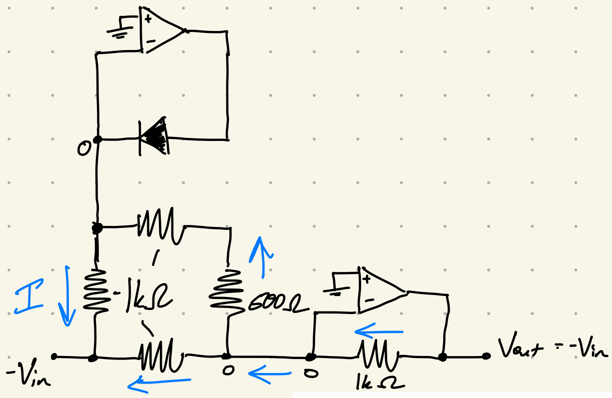


0.1 sine wave

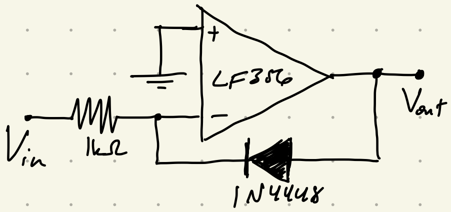
This circuit takes the absolute value of the input, so any negative voltage input becomes a positive output, while a passive diode rectifier only allows positive voltage input and cuts off any negative input. The smallest signal a passive diode rectifier could rectify was 0.4V

Problem R7.6

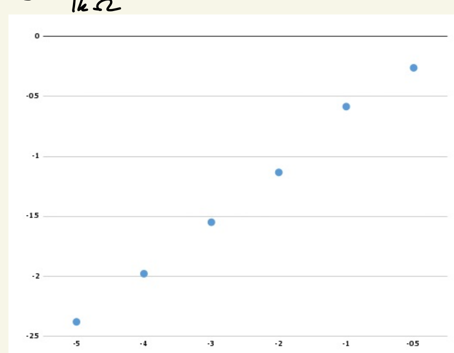




Problem R7.7

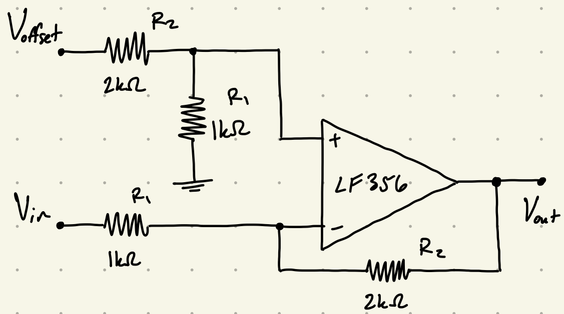


Vin (V)	Vout (V)
-5	-2.378
-4	-1.977
-3	-1.548
-2	-1.133
-1	-0.585
-0.5	-0.262



The Vout is offset by -0.585 V and scaled by a factor of -1.63 when compared to just the value of ln(-Vin).

Problem R7.8



$$\frac{V_{out}-V_-}{R_2} = I = \frac{V_- - V_{in}}{R_1} \Rightarrow \frac{R_1}{R_2}V_{out} + V_{in} = V_- + \frac{R_1}{R_2}V_- \Rightarrow V_- = \frac{R_2}{R_1+R_2}\left(\frac{R_1}{R_2}V_{out} + V_{in}\right)$$

$$V_- = \frac{R_1 V_{out} + R_2 V_{in}}{R_1+R_2}$$

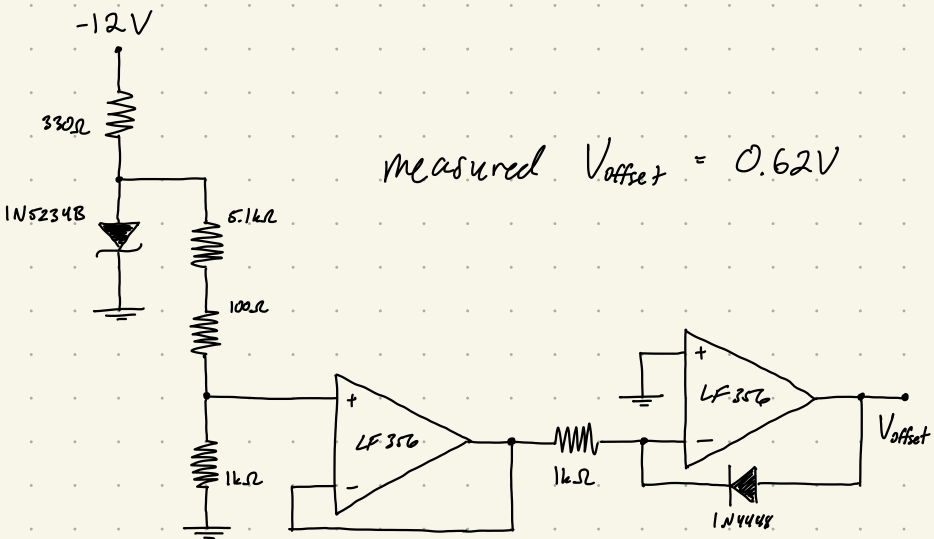
$$\frac{V_+ - V_{offset}}{R_2} = I = \frac{0 - V_+}{R_1} \Rightarrow \frac{R_1}{R_2}V_+ + V_+ = \frac{R_1}{R_2}V_{offset} \Rightarrow V_+ = \frac{R_2}{R_1+R_2}\left(\frac{R_1}{R_2}V_{offset}\right)$$

$$V_+ = \frac{R_1 V_{offset}}{R_1+R_2}$$

$$V_- = V_+ \Rightarrow \frac{R_1 V_{out} + R_2 V_{in}}{R_1+R_2} = \frac{R_1 V_{offset}}{R_1+R_2} \Rightarrow V_{out} = V_{offset} - \frac{R_2}{R_1}V_{in}, \quad R_1 = 1k\Omega, \; R_2 = 2k\Omega$$

$$V_{out} = V_{offset} - 2V_{in}$$

V_{in} (V)	V_{offset} (V)	V_{out} (V)	expected V_{out}
0	3.3	3.317	3.3
3.3	5	-1.57	-1.6
5	0	-10.07	-10
3.3	3.3	-3.31	-3.3



measured $V_{offset} = 0.62V$

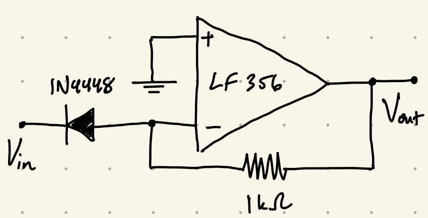
$$\frac{-1-0}{R} = I = I_{sat}\exp\left(\frac{eV_{off}}{nkT}\right)$$

$$-\frac{1}{RI_{sat}} = \exp\left(\frac{eV_{off}}{nkT}\right)$$

$$V_{offset} = -\frac{nkT}{e}\ln\left(\frac{1}{RI_{sat}}\right)$$

expected $V_{offset} = 0.6$ V

Problem R7.9



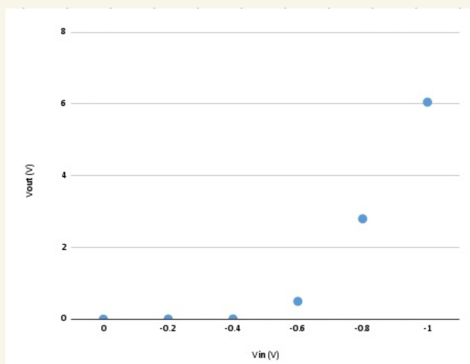
$$\frac{V_{out}}{R} = I_{diode} = I_{sat}\exp\left(-\frac{eV_{in}}{nkT}\right)$$

$$V_{out} = RI_{sat}\exp\left(-\frac{eV_{in}}{nkT}\right)$$

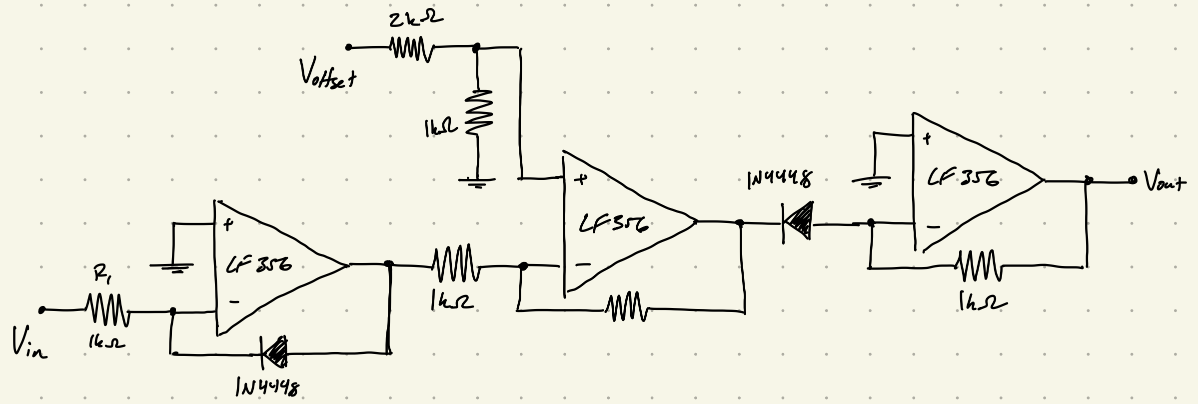
V_{in} (V)	V_{out} (V)
0	0.0043
-0.2	0.00805
-0.4	0.0095
-0.6	0.501
-0.8	2.8
-1	6.05

The data is very close to following the behavior of a true exponential.

The minimum input voltage before the output saturates is -0.4V.



Problem R 7.10



from logarithm component: $V_{out_1} = \frac{nkT}{e}\left[\ln(-V_{in}) - \ln(R_1 I_{sat})\right]$

from multiplier & shifter: $V_{out_2} = V_{offset} - 2V_{in}$, $V_{in} = V_{out_1}$

$\Rightarrow V_{out_2} = V_{offset} - 2V_{out_1}$, $V_{offset} = -\frac{nkT}{e}\ln(RI_{sat})$

$\Rightarrow V_{out_2} = -\frac{nkT}{e}\ln(RI_{sat}) - 2\frac{nkT}{e}\left[\ln(-V_{in}) - \ln(R_1 I_{sat})\right]$

$\Rightarrow V_{out_2} = -\frac{2nkT}{e}\ln(-V_{in}) + \frac{nkT}{e}\ln(RI_{sat}) = -\frac{nkT}{e}\ln\left(\frac{V_{in}^2}{RI_{sat}}\right)$

from exponentiation: $V_{out} = R_1 I_{sat}\exp\left(\frac{-eV_{in}}{nkT}\right)$, $V_{in} = V_{out_2}$

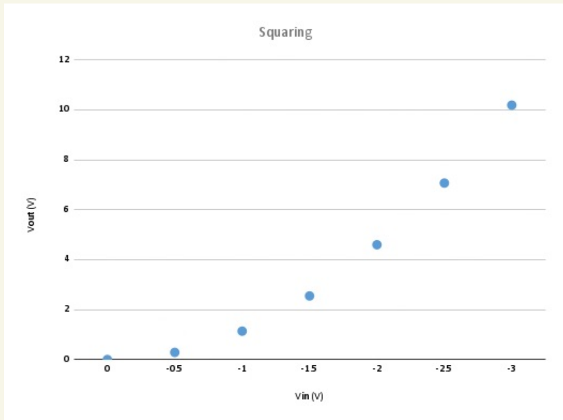
$V_{out} = R_1 I_{sat}\exp\left(-\frac{e}{nkT}\left(-\frac{nkT}{e}\ln\left(\frac{V_{in}^2}{RI_{sat}}\right)\right)\right)$

$\Rightarrow V_{out} = RI_{sat}\exp\left(\ln\left(\frac{V_{in}^2}{R_1 I_{sat}}\right)\right) = R_1 I_{sat}\left(\frac{V_{in}^2}{R_1 I_{sat}}\right)$

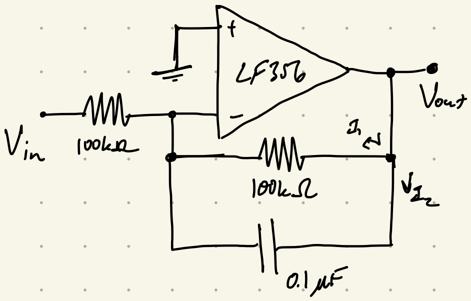
$\Rightarrow V_{out} = V_{in}^2$

V_{in} (V)	V_{out} (V)	expected V_{out}	% diff
0	0.004	0	—
-0.5	0.29	0.25	16%
-1	1.14	1	14%
-1.5	2.55	2.25	13.8%
-2	4.63	4	15%
-2.5	7.07	6.25	13.12%
-3	10.2	9	13.2%
-4	10.2	16	—
-5	10.2	25	—

The max V_{in} is -3V before the output saturates.



Problem R7.11



$$\langle H\rangle = \frac{1}{T_f - T_i}\int_{T_i}^{T_f} H(t)\,dt$$

$$\langle H\rangle(t) = \frac{1}{\tau}\int_{-\infty}^{t} H(t')\exp\left(\frac{t'-t}{\tau}\right)dt'$$

$$V_+ = 0, \quad \frac{V_{in}-V_-}{R_1} = I \Rightarrow \frac{V_{in}}{R_1} = I$$

$$V_- - V_{out} = R_1 I_1 = \frac{1}{C} I_2 \Rightarrow I_1 = \frac{1}{R_1 C} I_2$$

$$I = I_1 + I_2 = \frac{1}{R_1 C} I_2 + I_2 = \left(\frac{1}{RC}+1\right) I_2$$

$$Z_{tot} = \left(\frac{1}{R_1} + j\omega C\right)^{-1} = \left(\frac{1+j\omega CR}{R}\right)^{-1} = \frac{R_1}{1+j\omega CR_1}$$

$$\frac{V_- - V_{out}}{Z} = I = -\frac{V_{out}}{Z} = \frac{V_{in}}{R_1}$$

$$V_{out} = -V_{in}\frac{Z}{R_1} = -V_{in}\frac{1}{1+j\omega CR_1}$$

$$I = C\frac{dV_c}{dt} + \frac{V_c}{R_1}, \quad V_c = RI_0\left(1-e^{-t/RC}\right)$$

$$I = C\left(\frac{1}{RC}\right)RI_0 e^{-t/RC} + I_0\left(1-e^{-t/RC}\right)$$

$V_{in} = 1V$, 1kHz sine wave $\quad \langle H\rangle = \frac{1}{T_f-T_i}\int_{T_i}^{T_f} H(t)dt = (1000)\int_0^{1ms}\sin\left(\frac{2\pi t}{1ms}\right)dt = 0$

$V_{out} = 18.3\,mV$

$V_{in} = 1V$, 1kHz sine wave, offset +1V, $\langle H\rangle = (1000)\left[\int_0^{1ms}\sin\left(\frac{2\pi t}{1ms}\right)dt + \int_0^{1ms}dt\right] = 1000(0.001) = 1$

$V_{out} = -1\,V$

$V_{in} = 1\,V$, 1kHz pulse wave, offset +1V

$V_{avg} = 1.5V$

$V_{out} = -1.5V$

$V_{in} = 1V$, DC

$V_{avg} = 1V$

$V_{out} = -1V$

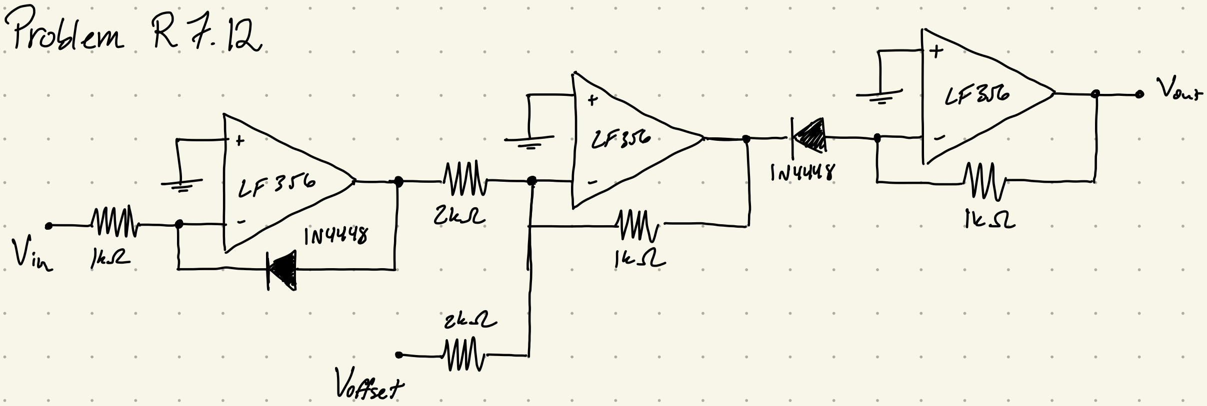
$V_{in} = 1V$, 10kHz sine wave, $V_{avg} = -1.2\,mV$, $V_{out} = 3\,mV$

$V_{in} = 1V$, 1μHz sine wave, offset +1V, $V_{avg} = 0.999\,V$, $V_{out} = -0.987\,V$

$V_{in} = 1V$, 25 MHz sine wave offset +1V, $V_{avg} = 1.0008\,V$, $V_{out} = -0.9867\,V$

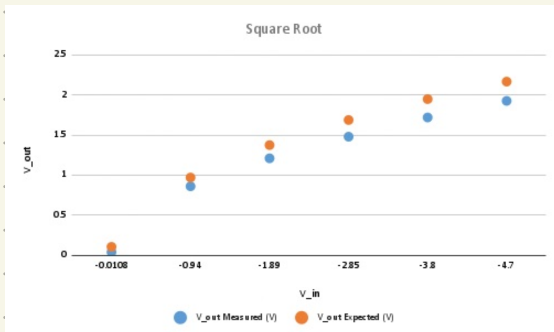
The circuit seems to work within reasonable expectation over the entire frequency range of the ADS.

Problem R7.12



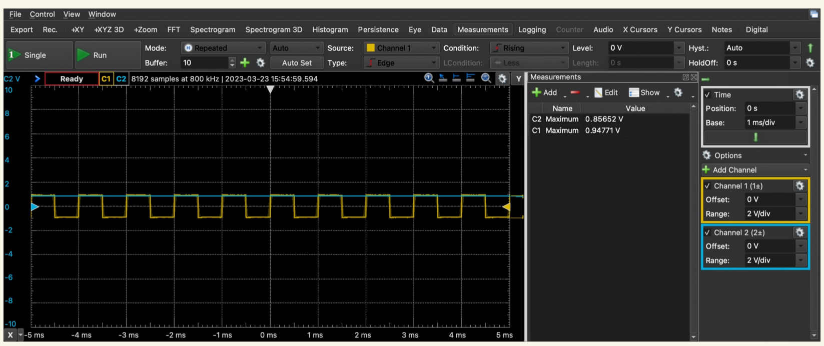
V_{in} (V)	V_{out} (V)	$\sqrt{-V_{in}}$
-4.7	1.92	2.17
-3.8	1.72	1.95
-2.85	1.48	1.69
-1.89	1.21	1.37
-0.94	0.86	0.97
0.011	0.038	0.104

The circuit is more accurate the closer it is to 0, and diverges a little at larger input values.



Problem R7.13

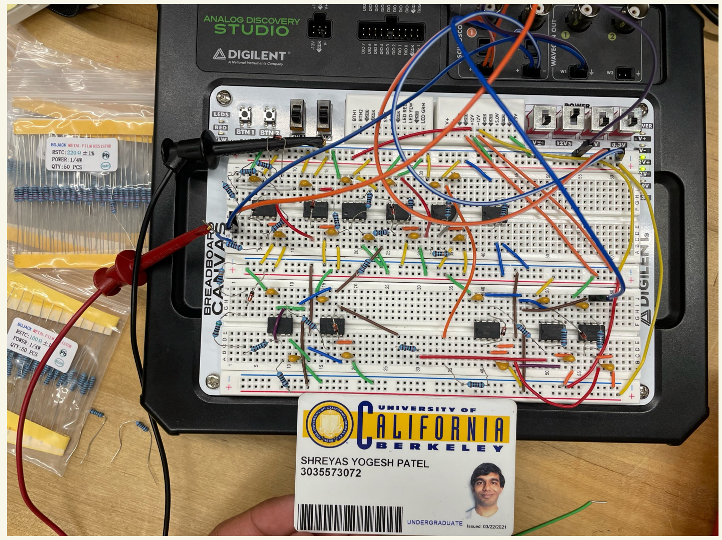
An input of a 1V amplitude square wave into the non-averaging circuit should output a 1V DC voltage



Problem R7.14

The output becomes less noisy

	f (kHz)	V_{in} (V)	V_{out} (V)	DMM (V)	ADS (V)
Sine	1	0.91	0.63	0.637	0.64
		1.82	1.18	1.28	1.29
	10	0.94	0.60	0.624	0.66
	100	0.91	0.60	0.141	0.64
Square	1	0.95	0.83	0.998	0.911
Triangle	1	0.92	0.482	0.500	0.525



The circuit works very well, with its V_{out} being very close to the RMS readings from both the DMM and ADS.