1. $0 \leq P(E) \leq 1$

2. $\sum_i P(E_i) = 1$

①

X	1	2	3
$P(X)$	$1/4$	$2/4$	$1/4$

It is a Prob. Dist.

②

X	1	2	3
$P(X)$	$-1/4$	$3/4$	$2/4$

Not a Prob. Dist.

③

X	1	2	3
$P(X)$	$1/4$	$1/4$	$1/4$

Not a prob Dist.

Probability Function or Probability Mass Function (pmf)

Probability function or pmf of a random variable (R.V.) X is a mathematical function $p(x)$ which gives the probability corresponding to different possible discrete set of values say $x_1, x_2, \ldots, x_n$ of variable x.

i.e. $p(x_i) = p(x = x_i)$

The f^n $p(x)$ must satisfy the conditions

① $p(x_i) \geq 0$

② $\sum p(x_i) = 1$

<u>Cumulative Distribution Function</u> (Distribution Function)

If X is a R.V. the $P(X \leq x)$ is called the cumulative distribution function (cdf) and is denoted $F(x)$.

$$F(x) = P(X \leq x)$$

Expectation of a Discrete R.V.

If X is a R.V. which assumes the discrete set of values $x_1, x_2, \ldots, x_n$ with respective probabilities $p_1, p_2, \ldots, p_n$ then the expectation or expected value of X is denoted by $E[X]$ and is defined as

$$E[X] = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$$

$$E[X^2] = \sum_{i=1}^{n} x_i^2 p_i$$

$$E[X^m] = \sum_i x_i^m p_i$$

Properties

$X, Y \leftarrow$ R.V. $\qquad a, b \leftarrow$ constants. $\qquad \mu \leftarrow$ mean

$$\left.\begin{array}{l} ① \; E[a] = a \\ ② \; E[aX] = aE[X] \\ ③ \; E[X - \mu] = 0 \end{array}\right\} \text{Try to prove it.}$$

④ $E[X \pm Y] = E[X] \pm E[Y]$

* ⑤ $E[XY] = E[X]E[Y]$ if X & Y are independent R.V.

⑥ if $Z = ax + b$ then $E[Z] = E[aX + b]$

or

$Z = aX + b$

$= a\,E[X] + b$

Variance and Standard Deviation

The variable of discrete R.V. X is expected value of

$(X-\mu)^2$ where μ is mean of the variable X.

σ^2

$$\mathrm{Var}(X) = E[(X-\mu)^2]$$

Also denoted by σ^2

$$SD(X) = \sqrt{\mathrm{Var}(X)} = \sqrt{E[X^2] - (E[X])^2}$$

σ

Q. Prove that $\mathrm{Var}(X) = E[X^2] - (E[X])^2$

Q. Find the expected value of getting head when a pair of coins is tossed.

Soln:

$S = \{HH, HT, TH, TT\}$

let, X = no. of heads

Possible values of X are — $X = 0, 1, 2$

Probability distribution table will be given by

x	0	1	2
$P(x)$	$1/4$	$2/4$	$1/4$

Expected value of getting head, $E[X] = \left(0 \times \frac{1}{4}\right) + \left(1 \times \frac{2}{4}\right) + \left(2 \times \frac{1}{4}\right)$

$= 1$

Alternate

$$E[X] = \sum_i x_i p_i = x_1 p_1 + x_2 p_2 + x_3 p_3$$

$$= \left(0 \cdot \frac{1}{4}\right) + \left(1 \cdot \frac{2}{4}\right) + \left(2 \cdot \frac{1}{4}\right) = 1.$$

Way ahead $\longrightarrow$

$$Var(X) = E[X^2] - (E[X])^2$$

$$= \left(\sum_i x_i^2 p_i\right) - (1)^2$$

$$= \left(x_1^2 p_1 + x_2^2 p_2 + x_3^2 p_3\right) - 1$$

$$= \left[\left(0^2 \cdot \frac{1}{4}\right) + \left(1^2 \cdot \frac{2}{4}\right) + \left(2^2 \cdot \frac{1}{4}\right)\right] - 1$$

$$= \frac{1}{2} + 1 - 1$$

$$= \frac{1}{2}$$

(positive)

$$SD(X) = \sqrt{Var(X)} = \sqrt{1/2}$$

$$= 0.7071$$