A card is drawn from a pack of well – shuffled palying cards. What is the probability that it is either a spade or an ace ?

Let A be the event of drawing a sapde

B be the event of drawing an ace

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \ldots\ldots\ldots\ldots(i)$$

Now,

$$n(A) = 13 \qquad n(B) = 4 \qquad n(A \cap B) = 1 \qquad n(S) = 52$$

$$P(A) = \frac{13}{52} \qquad P(B) = \frac{4}{52} \qquad P(A \cap B) = \frac{1}{52}$$

Substituting the above values in (i) we get

$$P(A \cup B) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Multiplicative Law of Probability

The probability of simultaneous occurrence of two events is equal to the probability of one multiplied by the conditional probability of the other *i.e.* if **A** and **B** be two events then probability of simultaneous occurrence of both is given by

$$P(A \cap B) = P(A)\,.P\left({}^{B}/_{A}\right)$$

Note

$P(A \cap B)$ *is also written as* $P(AB)$

If A and B are independent events then $P(A \cap B) = P(A)\,P(B)$

A problem in mechanics is given to three students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem will be solved?

$P(A)=\frac{1}{2}$ $\quad$ $P(B)=\frac{1}{3}$ $\quad$ $P(C)=\frac{1}{4}$

$P(A')=1-\frac{1}{2}=\frac{1}{2}$

$P(B')=1-\frac{1}{3}=\frac{2}{3}$

$P(C')=1-\frac{1}{4}=\frac{3}{4}$

Probability that A, B, C cannot solve the problem *i.e.* probability that the problem will not be solved $=\mathrm{P}(\mathrm{A}').\mathrm{P}(\mathrm{B}').\mathrm{P}(\mathrm{C}')=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{1}{4}$

Probability that atleast one of them will solve problem *i.e.* probability that the problem will be solved $=1-\frac{1}{4}=\frac{3}{4}$

A can hit a target 4 times in 5 shots, B 3 times in 4 shots, C twice in 3 shots. They fire a volley. What is the probability that atleast two shots hit?

$$\text{Probability that } A \text{ hit the target } = P(A) = \frac{4}{5}$$

$$\text{Probability that } B \text{ hit the target } = P(B) = \frac{3}{4}$$

$$\text{Probability that } C \text{ hit the target } = P(C) = \frac{2}{3}$$

$Case$ 1 : A, B, C all hit the target then

$$P(A \cap B \cap C) = P(A)\, P(B)\, P(C) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$$

Case 2 : *A, B* hit the target but *C* misses it, then

$$P(A \cap B \cap C') = P(A)\,P(B)\,P(C') = \frac{4}{5} \times \frac{3}{4} \times \left(1 - \frac{2}{3}\right) = \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{12}{60}$$

Case 3 : *A, C* hit the target but *B* misses it, then

$$P(A \cap B' \cap C) = P(A)\,P(B')\,P(C) = \frac{4}{5} \times \left(1 - \frac{3}{4}\right) \times \frac{2}{3} = \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} = \frac{8}{60}$$

Case 4 : B, C hit the target but A misses it, then

$$P(A' \cap B \cap C) = P(A')\,P(B)\,P(C) = \left(1 - \frac{4}{5}\right) \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{6}{60}$$

$$\textit{So the required probability} = \frac{24}{60} + \frac{12}{60} + \frac{8}{60} + \frac{6}{60} = \frac{50}{60} = \frac{5}{6}$$