

9. Probability of a man hitting a target is $\frac{1}{3}$.

(a) If he fires 6 times, what is the probability of hitting

(i) atmost 5 times.

(ii) at least 5 times.

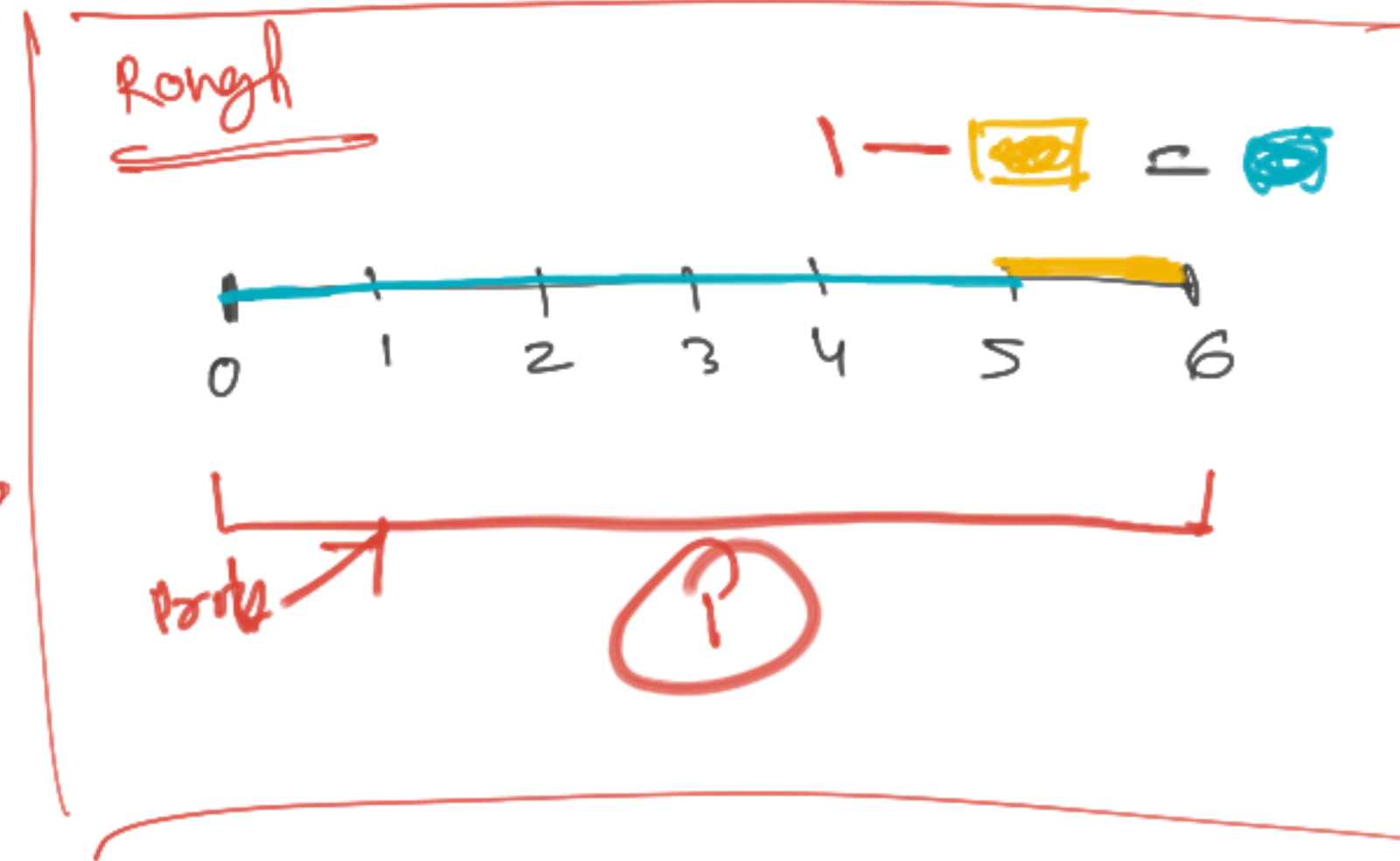
(iii) exactly once.

(b) If he fires so that the probability of his hitting the target at least once is greater than $\frac{3}{4}$, find n .

Soln: \therefore Probability of the man hitting a target is $\frac{1}{3}$

$\therefore p = \frac{1}{3}$

$q = 1 - p = \frac{2}{3}$



(a) Given, $n = 6$

Let $X =$ no. of times he hit the target

(i) $P(X \leq 5) = 1 - P(X > 5) = 1 - P(X = 6) = 1 - \left[{}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{6-6} \right]$
 $= 1 - \left[1 \cdot \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^0 \right] = 1 - \left(\frac{1}{3}\right)^6 = ?$

$$\begin{aligned}
 \textcircled{ii} \quad P(X \geq 5) &= P(X=5) + P(X=6) \\
 &= \left[{}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{6-5} \right] + \left[{}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{6-6} \right] \\
 &= ?
 \end{aligned}$$

$$\textcircled{iii} \quad P(X=1) = {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{6-1} = ?$$

⑥

$$A|_Z$$

$$P(X \geq 1) > \frac{3}{4}$$

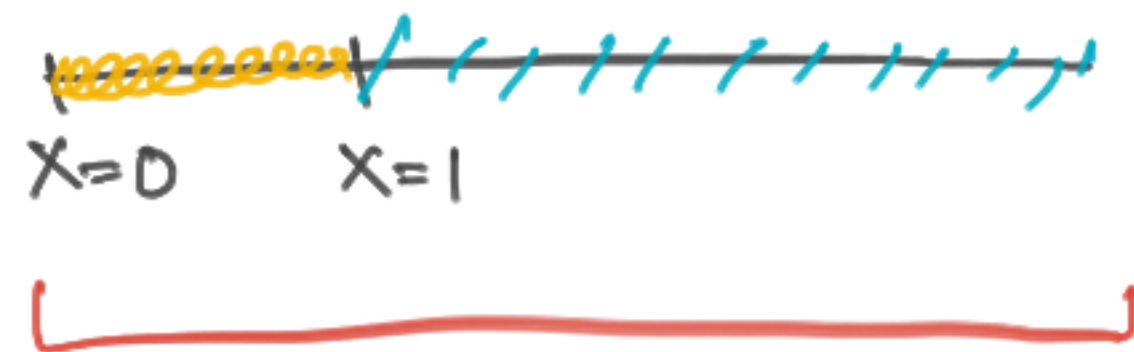
$$\Rightarrow 1 - P(X < 1) > \frac{3}{4}$$

$$\Rightarrow 1 - P(X=0) > \frac{3}{4}$$

$$\Rightarrow 1 - n_{c_0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{n-0} > \frac{3}{4}$$

$$\Rightarrow 1 - \left(\frac{2}{3}\right)^n > \frac{3}{4}$$

$$\Rightarrow 1 - \frac{3}{4} > \left(\frac{2}{3}\right)^n \Rightarrow \left(\frac{2}{3}\right)^n < \frac{1}{4} \Rightarrow 2^n \cdot 4 < 3^n$$



$$\sum p(x) = 1$$

$$P[0n] = 1 - P[1n]$$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$\left(P(X=x) = {}^nC_x p^x q^{n-x} \right)$$

$$\Rightarrow 2^{n+2} < 3^n$$

The above inequality holds for $n = 4$

Hence, the man must fire 4 times
so that the probability of hitting
the target atleast once is greater than $\frac{3}{4}$

$$\begin{array}{ll} n=4, & 2^{4+2} = 64 \\ & 3^4 = 81 \\ 2^{4+2} < 3^4 & \text{possible} \end{array}$$

$$\begin{array}{ll} n=1, & 2^{1+2} = 2^3 = 8 \\ 8 < 3 & 3^1 = 3 \\ & \text{not possible} \\ n=2, & 2^{2+2} = 2^4 = 16 \\ 16 < 9 & 3^2 = 9 \\ & \text{not possible} \end{array}$$

$$\begin{array}{ll} n=3, & 2^{3+2} = 2^5 = 32 \\ & 3^3 = 27 \\ 32 < 27 & \\ & \text{not possible} \end{array}$$

POISSON DISTRIBUTION

French Mathematician — S.D. Poisson

Poisson Distribution — Discrete Probability Distribution

Characteristics

1. $n \rightarrow \infty$ i.e. sufficiently large (infinitely large

$p \rightarrow 0$ i.e. sufficiently small (infinitely small

parameter, $\lambda = np \leftarrow$ finite no.

2. It consists of a single parameter i.e. λ .