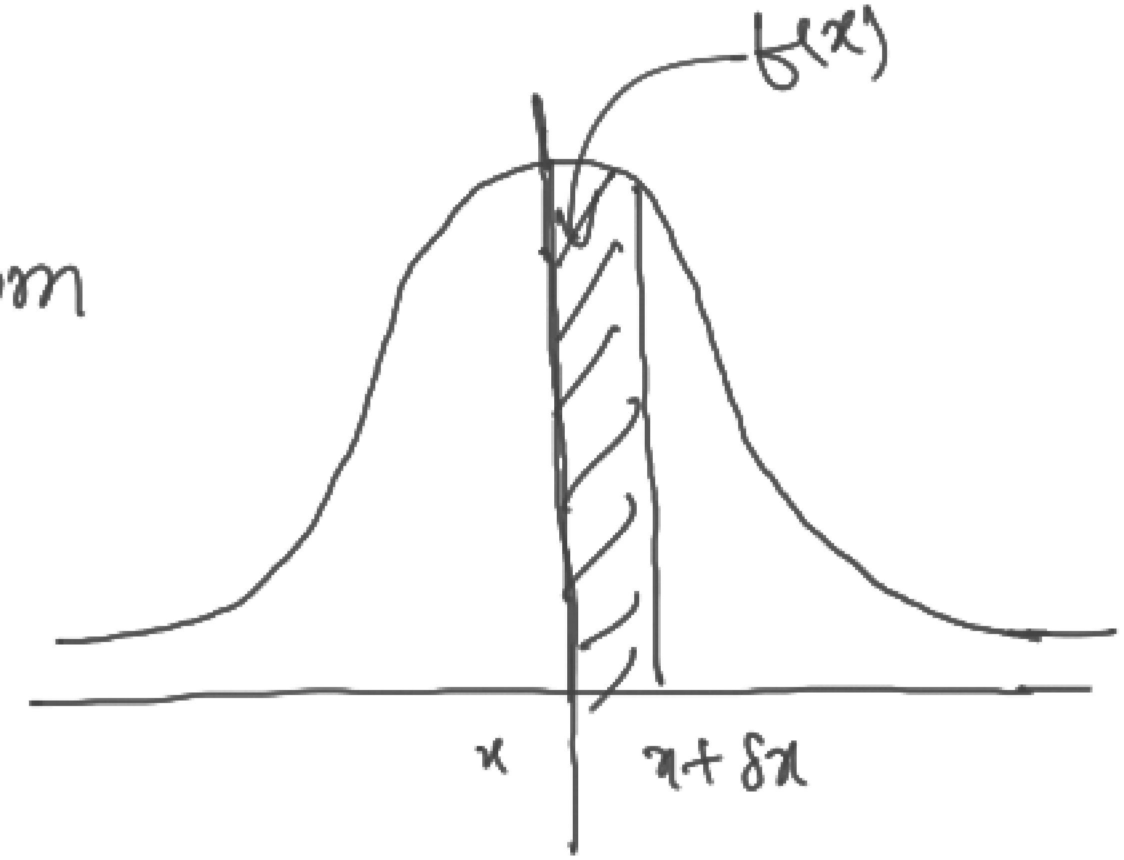
CONTINUOUS RANDOM VARIABLE

Probability Density Function

The Probability density function of random variable X is defined as



$$f_X(x) = P\left(x \le X \le x+\delta x\right)/\delta x$$

for small interval $[x, x+\delta x]$ of length dx around the point x.

$$\boxed{P(a \le X \le b) = \int_a^b f(x)\,dx}$$

Properties

1. $\int_{-\infty}^{\infty} f(x)\,dx = 1$ i.e. $P(-\infty < X < \infty) = 1$.

2. $f(x) \geq 0$, $-\infty < x < \infty$

$f_X(x)$ or $f(x)$

↳ p.d.f.

probability density f^n

Cumulative Distribution (Distribution Function)

$X \leftarrow$ R.V.

c.d.f. (cumulative distribution or Distribution F^n) is denoted by $F(x)$ and is given by

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

Expectation

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$E[X^n] = \int_{-\infty}^{\infty} x^n f(x)\,dx$$

Properties of $E[X]$ is same as discussed earlier.

$$\mathrm{var}(X) = E\{X - \bar{X}\}^2 = E\{X^2\} - [E(X)]^2$$

$$S.D.(x) = \sigma = \sqrt{\mathrm{var}\,X} = +\sqrt{E[X^2] - \{E(X)\}^2}$$

Q. A continuous random variable X has probability density f^n defined by

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2, & -3 \le x < -1 \\ \frac{1}{16}(6-2x^2), & -1 \le x < 1 \\ \frac{1}{16}(3-x)^2, & 1 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

Verify that $f(x)$ is density f^n and also find the mean of the random variable X.

Soln:
$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{-3} f(x)\,dx + \int_{-3}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{-3} 0.\,dx + \int_{-3}^{-1} \frac{1}{16}(3+x)^2\,dx + \int_{-1}^{1} \frac{1}{16}(6-2x^2)\,dx + \int_{1}^{3} \frac{1}{16}(3-x)^2\,dx + \int_{3}^{\infty} 0.\,dx$$

$$= \frac{1}{16}\left\{ \int_{-3}^{-1} (9+x^2+6x)\,dx + \left[6x - \frac{2x^3}{3}\right]_{-1}^{1} + \int_{1}^{3} (9+x^2-6x)\,dx \right\}$$

$$= \frac{1}{16}\left\{ \left[9x + \frac{x^3}{3} + \frac{6x^2}{2}\right]_{-3}^{-1} + \left[6(1+1) - \frac{2}{3}(1^3-(-1)^3)\right] + \left[9x + \frac{x^3}{3} - \frac{6x^2}{2}\right]_{1}^{3} \right\}$$

$$\int_{-3}^{-1} (3+x)^2\,dx$$

$$\left[\frac{(3+x)^3}{3}\right]_{-3}^{-1}$$

Short cut

$$= \frac{1}{16}\left[9(-1+3) + \frac{1}{3}(-1+27) + 3(1-9) + 12 - \frac{4}{3} + 9(3-1) + \frac{1}{3}(27-1) - 3(9-1) \right]$$

$$= \frac{1}{16}(16)$$

$$= 1$$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$\therefore$ $f(x)$ is a density f^n.

Mean of the random variable X is

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{-3} x f(x)\, dx + \int_{-3}^{-1} x f(x)\, dx + \int_{-1}^{1} x f(x)\, dx + \int_{1}^{3} x f(x)\, dx + \int_{3}^{\infty} x . f(x) dx$$

$$= \int_{-\infty}^{-3} x . 0 . dx + \int_{-3}^{-1} x . \frac{1}{16}(3+x)^2 dx + \int_{-1}^{1} x . \frac{1}{16}(6-2x^2)\, dx + \int_{1}^{3} x . \frac{1}{16}(3-x)^2 dx$$

$$+ \int_{3}^{\infty} x . 0 . dx$$

$$= ?$$