A and B throw alternatively with a single die. A having the first throw. The person who first throws an *ace* is to win. What are their respective chances of winning ?

$$\text{Probability of getting an ace in throwing of a die} = \frac{1}{6}$$

$$\text{Probability of not getting an ace in throwing of a die} = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\text{Case I : If } A \text{ is to win then he/she must throw an ace in } 1^{st} \text{ or } 3^{rd} \text{ or } 5^{th} \ldots \text{ throws}$$

$$\text{Probability that } A \text{ gets an ace in } 1^{st} \text{ throw} = \frac{1}{6}$$

$$\text{Probability that } A \text{ gets an ace in } 3^{rd} \text{ throw} = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$\text{Probability that } A \text{ gets an ace in } 5^{th} \text{ throw} = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$\text{Then Probability that A will win} = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \ldots = \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11}$$

Case 2 : If B is to win then he/ she must throw an ace in 2^{nd} or 4^{th} or 6^{th} ... throws

$$\text{Probability that B gets an ace in } 2^{nd} \text{ throw} = \frac{5}{6} \times \frac{1}{6}$$

$$\text{Probability that B gets an ace in } 4^{th} \text{ throw} = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$\text{Probability that B gets an ace in } 6^{th} \text{ throw} = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$\text{Then Probability that B will win} = \left(\frac{5}{6}\right) \frac{1}{6} + \left(\frac{5}{6}\right)^3 \frac{1}{6} + \left(\frac{5}{6}\right)^5 \frac{1}{6} + \ldots = \frac{\frac{5}{6} \times \frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{5}{11}$$

Alternate method to calculate B : $P(B) = 1 - P(A)$

A bag contains 10 balls, two of which are red, three blue and five black. Three balls are drawn at random from the bag. What is the probability that

(i) three balls are of different colours

(ii) two balls are of same colour

(iii)the balls are of different colour

$$\text{Total number of balls} = 10$$

$$\text{Number of red balls} = 2$$

$$\text{Number of blue balls} = 3$$

$$\text{Number of black balls} = 5$$

$$(i)\ \text{Probability that all the three balls are of different colours} = \frac{{}^{2}C_{1} \times {}^{3}C_{1} \times {}^{5}C_{1}}{{}^{10}C_{3}}$$

$$(ii)\ \text{Case } 1 : \text{If the same balls are white balls}$$

$$2 \text{ balls white}, 1 \text{ ball blue} = \frac{{}^{2}C_{2} \times {}^{3}C_{1}}{{}^{10}C_{3}}$$

$$2 \text{ balls white}, 1 \text{ ball black} = \frac{{}^{2}C_{2} \times {}^{5}C_{1}}{{}^{10}C_{3}}$$

$$\text{Probability that 2 balls white} = \frac{{}^{2}C_{2} \times {}^{3}C_{1}}{{}^{10}C_{3}} + \frac{{}^{2}C_{2} \times {}^{5}C_{1}}{{}^{10}C_{3}}$$

Case 2 : If the same balls are blue balls

$$2\ balls\ blue\ ,\ 1\ ball\ \ white\ = \frac{{}^{3}c_{2} \times {}^{2}c_{1}}{{}^{10}c_{3}}$$

$$2\ balls\ blue\ ,\ 1\ ball\ \ black\ = \frac{{}^{3}c_{2} \times {}^{5}c_{1}}{{}^{10}c_{3}}$$

$$Probability\ that\ 2\ balls\ blue\ = \frac{{}^{3}c_{2} \times {}^{2}c_{1}}{{}^{10}c_{3}} + \frac{{}^{3}c_{2} \times {}^{5}c_{1}}{{}^{10}c_{3}}$$

Case 3 : If the same balls are black balls

$$2\ balls\ black\ ,\ 1\ ball\ \ white\ = \frac{{}^{5}c_{2} \times {}^{2}c_{1}}{{}^{10}c_{3}}$$

$$2\ balls\ black\ ,\ 1\ ball\ \ blue\ = \frac{{}^{5}c_{2} \times {}^{3}c_{1}}{{}^{10}c_{3}}$$

$$Probability\ that\ 2\ balls\ black\ = \frac{{}^{5}c_{2} \times {}^{2}c_{1}}{{}^{10}c_{3}} + \frac{{}^{5}c_{2} \times {}^{3}c_{1}}{{}^{10}c_{3}}$$

$$\textit{Required Probablity} = \left(\frac{3}{{}^{10}C_3} + \frac{5}{{}^{10}C_3} \right) + \left(\frac{6}{{}^{10}C_3} + \frac{15}{{}^{10}C_3} \right) + \left(\frac{20}{{}^{10}C_3} + \frac{30}{{}^{10}C_3} \right) = \frac{79}{120}$$

If A and B are events such that $P(A)=0.3,\ P(B)=p,\ P(A\cup B)=0.6$ then

(i) Find p so the A and B are independent events

(ii) For what value of p, A and B are mutually exclusive

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow P(A\cap B)=P(A)+P(B)-\ P(A\cup B)=0.3+p-0.6$$

$$\Rightarrow P(A\cap B)=p-0.3$$

(*i*) *If A and B are independent then* $P(A\cap B)=P(A)\,P(B)$

$$\Rightarrow p-0.3=0.3p$$

$$\Rightarrow 0.7p=0.3$$

$$\Rightarrow p=\frac{3}{7}$$

(*ii*) *If A and B are mutually excluisive then* $P(A\cap B)=0$

$$\Rightarrow P(A\cap B)=p-0.3$$

$$\Rightarrow 0=p-0.3$$

$$\Rightarrow p=0.3$$

Q. An integer is chosen at random from first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8 ?

Q. *A, B* and *C* in order, toss a coin. The first one to throw a head winds. If *A* starts first find their respective probability of winning.

Q. Find the chance of throwing 5 or 6 at least once in four throws of die.

Q. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled out of a random from one of the two purse, what is the probability that it is silver coins ?