Q. The average height of soldiers of a country is given as 68.22 inches with variance 10.8 sq inch. How many soldiers out of 1000 would you expect to be over 72 inches tall? Given that the area under the normal curve between $z=0$ to $z=0.35$ is 0.1368 and between $z=0$ to $z=1.15$ is 0.3746.

Soln: Given,

$\mu = 68.22$

$\sigma^2 = 10.8 \Rightarrow \sigma = \sqrt{10.8}$

Let X = height of soldier

$$P(X > 72) = P\left(\frac{X-\mu}{\sigma} > \frac{72-\mu}{\sigma}\right)$$

$$= P\left(Z > \frac{72-68.22}{\sqrt{10.8}}\right)$$

$$= P(Z > 1.15)$$

$$= 0.5 - P(Z < 1.15)$$

$$= 0.5 - 0.3746$$

$$= 0.1254$$

∴ No. of soldiers out of 1000 whose height is over 72 inches

$= 1000 \times 0.1254$

$= 125$

Q. Students of a class were given a mathematics aptitude test. These marks were found to be normally distributed with mean 60 and standard deviation 5. What percent of students scored :

(i) more than 60 marks

(ii) less than 56 marks

(iii) between 45 and 65 marks.

Q. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

(i) how many students score between 12 and 15?

(ii) how many score above 18?

(iii) How many below 8?

(iv) How many score 16?

Sol^n: Here, $\mu = 14$

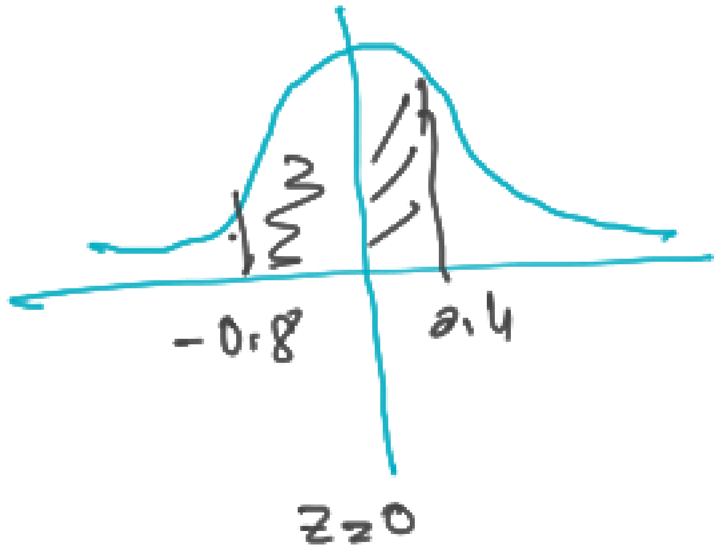
$\sigma = 2.5$

let X = no. of students getting a score

(i) $P(12 < X < 15) = P\left(\frac{12-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{15-\mu}{\sigma}\right)$

$= P\left(\frac{12-14}{2.5} < Z < \frac{15-14}{2.5}\right)$

$= P(-0.8 < Z < 0.4)$



$= P(-0.8 < Z < 0) + P(0 < Z < 0.4)$

$= P(0 < Z < 0.8) + P(0 < Z < 0.4)$

$= 0.2881 + 0.1554$

$= 0.4435$

$\therefore$ No. of students scoring in both 12 and 15

$= 1000 \times 0.4435 = 443.5 = 444$

(a) $P(X>18) = P\left(\frac{X-\mu}{\sigma} > \frac{18-\mu}{\sigma}\right)$

$$= P\left(Z > \frac{18-14}{2.5}\right)$$

$$= P(Z > 1.6)$$

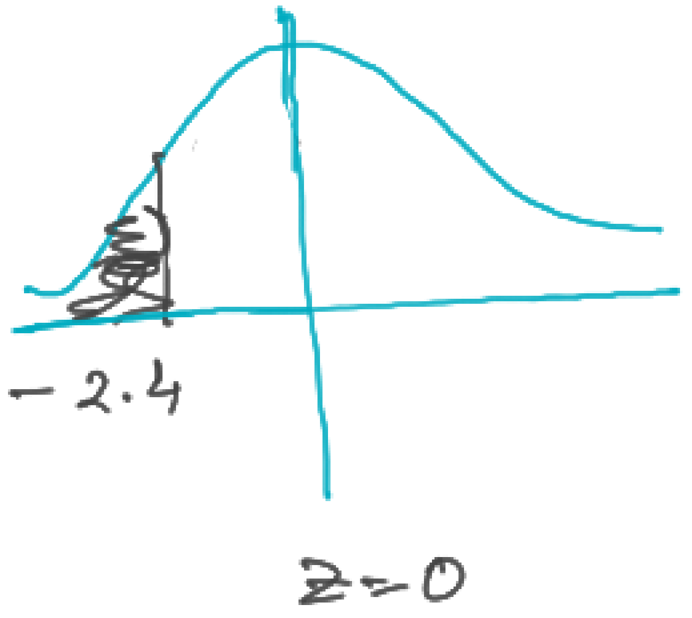
$$= 0.5 - P(Z < 1.6)$$

$$= 0.5 - 0.4452$$

$$= 0.0548$$

$\therefore$ No. of students getting a score above 18 $= 1000 \times 0.0548 = 54.8$
$= 55$

(iii) $P(X<8) = P\left(\frac{X-\mu}{\sigma} < \frac{8-\mu}{\sigma}\right)$



$= P\left(Z < \frac{8-14}{2.5}\right)$

$= P(Z < -2.4)$

$= P(Z > 2.4)$

$= 0.5 - P(Z < 2.4)$

$= 0.5 - 0.4918$

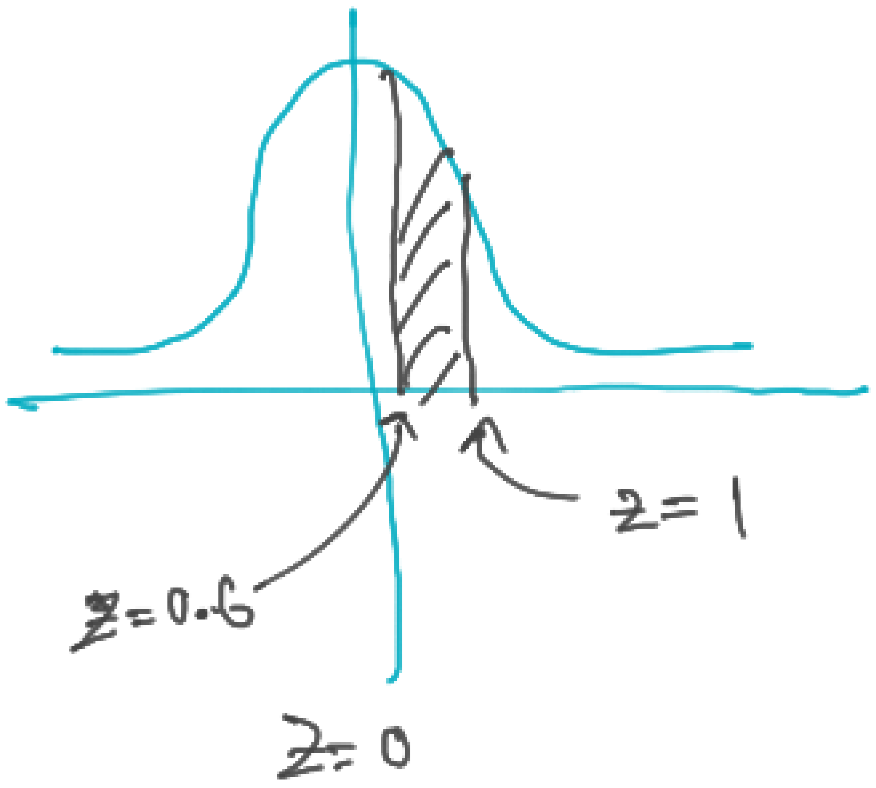
$= 0.0082$

$\therefore$ No. of students scoring below $8 = 1000 \times 0.0082 = 8.2$
$= 8$

(2) $P(X=16) = P(15.5 < X < 16.5)$

$$= P\left(\frac{15.5-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{16.5-\mu}{\sigma}\right)$$

$$= P\left(\frac{15.5-14}{2.5} < Z < \frac{16.5-14}{2.5}\right)$$



$$= P(0.6 < Z < 1)$$

$$= P(0 < Z < 1) - P(0 < Z < 0.6)$$

$$= 0.3413 - 0.2258$$

$$= 0.1155$$

$\therefore$ No. of students getting a score 16

$= 1000 \times 0.1155$

$= 115.5$

$= 116$

Q. The distribution of a random variable is given by

$$f(x) = c e^{-\frac{1}{50}(9x^2 - 30x)} \quad ; \quad -\infty < x < \infty$$

Find the constant c, the mean and the variance of the random variable. Find also the upper 5% value of the R.V.

Soln: Given,

$$f(x) = c e^{-\frac{1}{50}(9x^2 - 30x)} \quad ; \quad -\infty < x < \infty$$

We know that

$$P(-\infty < X < \infty) = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow c \int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx = 1$$

$\therefore \mu = \frac{5}{3}$, $2\sigma^2 = \frac{50}{9} \Rightarrow \sigma^2 = \frac{25}{9}$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$-\frac{1}{50}(9x^2 - 30x)$$

$$= -\frac{9}{50}\left(x^2 - \frac{30}{9}x\right)$$

$$= -\frac{9}{50}\left(x^2 - \frac{10}{3}x\right)$$

$$= -\frac{9}{50}\left[x^2 - 2.x.\frac{5}{3} + \left(\frac{5}{3}\right)^2\right.$$

$$-\frac{9}{50}\left[\left(x - \frac{5}{3}\right)^2 - \left(\frac{5}{3}\right)^2\right) - (5/3)^2$$

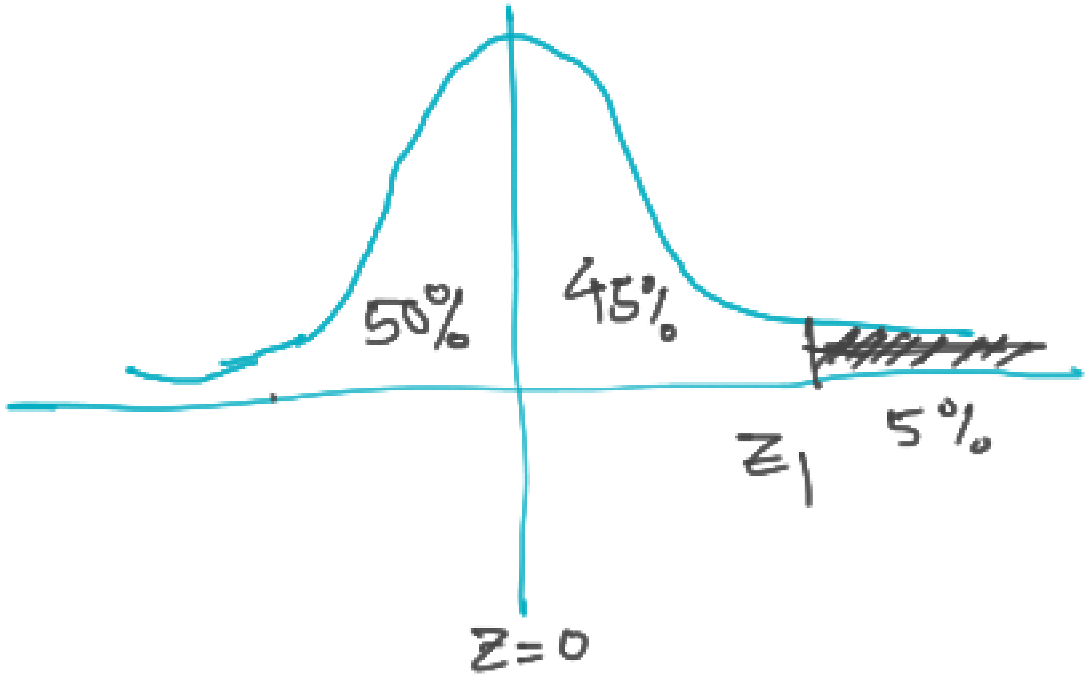
$\therefore \sigma = 5/3$

$\mu = 5/3$

$$C = \frac{1}{\sigma\sqrt{2\pi}}$$

$$\Rightarrow C = \frac{1}{(5/3)\sqrt{2\pi}} = \frac{3}{5\sqrt{2\pi}} = 0.239 = 0.24$$

let, $z = z_1$ be the co-ordinate of z at 45% mark



$\swarrow$ 5% area

$$P(0 < z < z_1) = 0.5 - 0.05$$

$$= 0.45$$

Value of z corresponding to this area (From the area table)

$$z_2 = 1.66$$

Now,

standard normal variate, $z = \frac{x - M}{\sigma}$

$$\therefore \quad z_2 = \frac{x - M}{\sigma}$$

$$\Rightarrow 1.66 = \frac{x - 5/3}{5/3}$$

$$\Rightarrow x = 1.66 \times \frac{5}{3} + \frac{5}{3}$$

$$= 4.44$$