A man is known to speak truth 3 out 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

$$Let\ \ A = the\ man\ reports\ it\ is\ a\ six$$

$$E_1 = six\ occurs$$

$$E_2 = six\ doesn't\ occur$$

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$Now,$$

$$P(E_1) = Probability\ that\ six\ occurs\ = \frac{1}{6}$$

$$P(E_2) = Probability\ that\ six\ doesn't\ occur\ = 1 - \frac{1}{6} = \frac{5}{6}$$

$P(A/E_1) =$ *Probability that the man reprots that it is six given that six occurs*

$$= \textit{Probability that the man speaks truth } = \frac{3}{4}$$

$P(A/E_2) = =$ *Probability that the man reprots that it is six given that six doesn't occur*

$$= \textit{Probability that the man doesn't speak the truth } = 1 - \frac{3}{4} = \frac{1}{4}$$

So,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{(\frac{1}{6} \times \frac{3}{4}) + (\frac{5}{6} \times \frac{1}{4})} = \frac{3}{8}$$

Suppose that 5% of men and 0.25% of women have a grey hair. A grey haired person is selected at random. What is the probability of this person being a male ? Assume that there are equal number of males and females.

$$\text{Let } A = \text{a grey haired person is chosen}$$

$$E_1 = \text{a male is chosen}$$

$$E_2 = \text{a female is chosen}$$

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$Now,$$

$$P(E_1) = \text{Probability that a male is chosen} = \frac{1}{2}$$

$$P(E_2) = \text{Probability that a female is chosen} = \frac{1}{2}$$

$$P\left(A/E_1\right)=Probability\ that\ a\ grey\ haired\ person\ is\ chosen\ when\ it\ known\ that\ the\ person\ is\ male\ =\frac{5}{100}=0.05$$

$$P\left(A/E_2\right)=Probability\ that\ a\ grey\ haired\ person\ is\ chosen\ when\ it\ known\ that\ the\ person\ isf\ emale=\frac{0.25}{100}=0.0025$$

$So,$

$$P\left(E_1/A\right)=\frac{P\left(E_1\right)P\left(A/E_1\right)}{P\left(E_1\right)P\left(A/E_1\right)+P\left(E_2\right)P\left(A/E_2\right)}$$

$$=\frac{\frac{1}{2}\times 0.05}{\left(\frac{1}{2}\times 0.05\right)+\left(\frac{1}{2}\times 0.0025\right)}=\frac{20}{21}$$

Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die ?

$$\text{Let } A = \textit{getting exactly one head}$$

$$E_1 = \textit{getting } 5 \textit{ or } 6 \textit{ in a single throw of a die}$$

$$E_2 = \textit{getting } 1,\ 2,\ 3 \textit{ or } 4 \textit{ in a single throw of a die}$$

$$P(E_1) = \frac{2}{6} = \frac{1}{3}$$

$$P(E_2) = \frac{4}{6} = \frac{2}{3}$$

$$\textit{By Bayes' Theorem},$$

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$P\left(A/E_1\right)=Probability\ of\ \ getting\ exactly\ one\ head\ given\ that\ a\ coin\ is\ tossed\ three\ times$$

$$= {}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{3-1} = \frac{3}{8}$$

$$P\left(A/E_2\right)=Probability\ of\ \ getting\ exactly\ one\ head\ given\ that\ a\ coin\ is\ tossed\ once$$

$$=\frac{1}{2}$$

$So,$

$$P\left(E_2/A\right)=\frac{P\left(E_2\right)P\left(A/E_2\right)}{P\left(E_1\right)P\left(A/E_1\right)+P\left(E_2\right)P\left(A/E_2\right)}$$

$$=\frac{\frac{2}{3}\times\frac{1}{2}}{\left(\frac{1}{3}\times\frac{1}{2}\right)+\left(\frac{2}{3}\times\frac{1}{2}\right)}=\frac{8}{11}$$