

Note

① $P(\phi) = 0$

② If E be any event that probability of non-happening / non-occurrence is denoted by $P(\bar{E})$ or $P(E')$. It is given

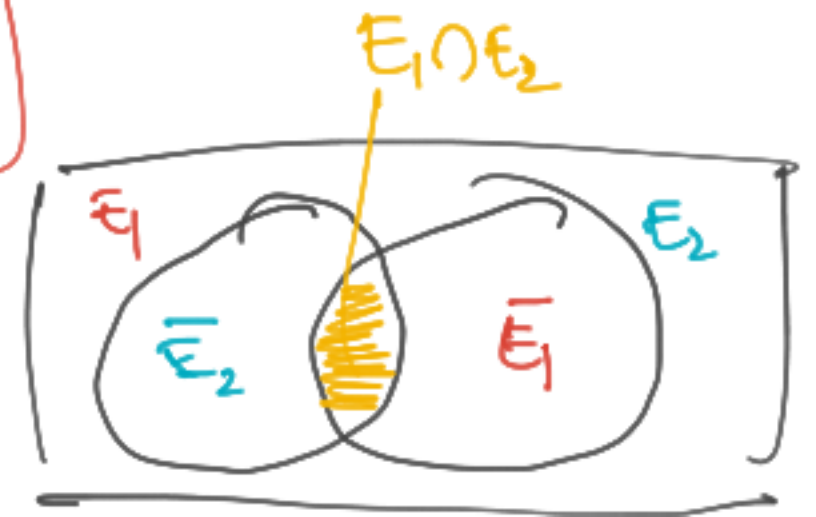
by

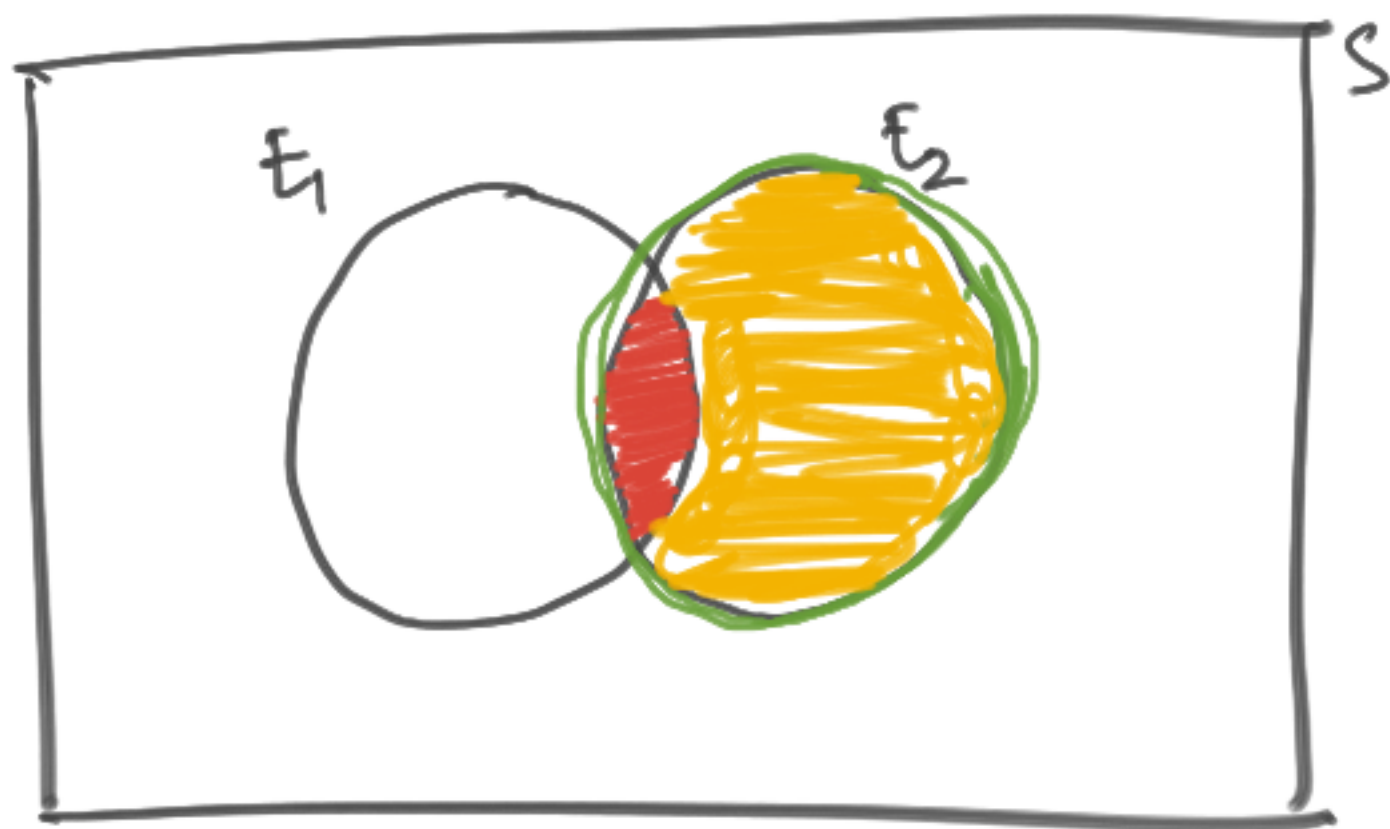
$$P(\bar{E}) = 1 - P(E)$$

$$\sum_i P(E_i) = 1$$

Prob. (occurrence) + Prob. (non-occurrence) = 1

③ $P(\bar{E}_1 \cap E_2) = P(E_2) - P(E_1 \cap E_2)$





$$E_1 \cap E_2$$

$$E_2 - (E_1 \cap E_2) = \overline{E_1} \cap E_2$$

$$\textcircled{4} \quad P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

(M.E = mutually
exclusive

If E_1 and E_2 are M.E. then $E_1 \cap E_2 = \phi$ i.e. $P(E_1 \cap E_2) = 0$

$$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$$\textcircled{5} \quad P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3)$$

If E_1, E_2, E_3 are M.E. then ?

Conditional Probability

Let E_1 and E_2 be two events of a random experiment.

Then probability of occurrence of E_1 given that E_2 has already occurred is denoted $P(E_1/E_2)$ and is defined as

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}, \quad P(E_2) \neq 0.$$

Note

$$\textcircled{1} P(S/E) = P(E/E) = 1$$

$$\begin{aligned} P(S/E) &= \frac{P(S \cap E)}{P(E)} \\ &= \frac{P(E)}{P(E)} \\ &= 1 \end{aligned}$$

$$\textcircled{2} P(E_1 \cup E_2 / F) = P(E_1 / F) + P(E_2 / F) - P(E_1 \cap E_2 / F)$$

$$\textcircled{3} P(\bar{E} / F) = 1 - P(E / F)$$

Multiplicative Law of Probability

The probability of simultaneous occurrence of two events is equal to the probability of one multiplied by the conditional probability of the other.

For two events E_1 and E_2

$$P(E_1 \cap E_2) = P(E_1)P(E_2/E_1) \quad ; \quad P(E_1) \neq 0$$

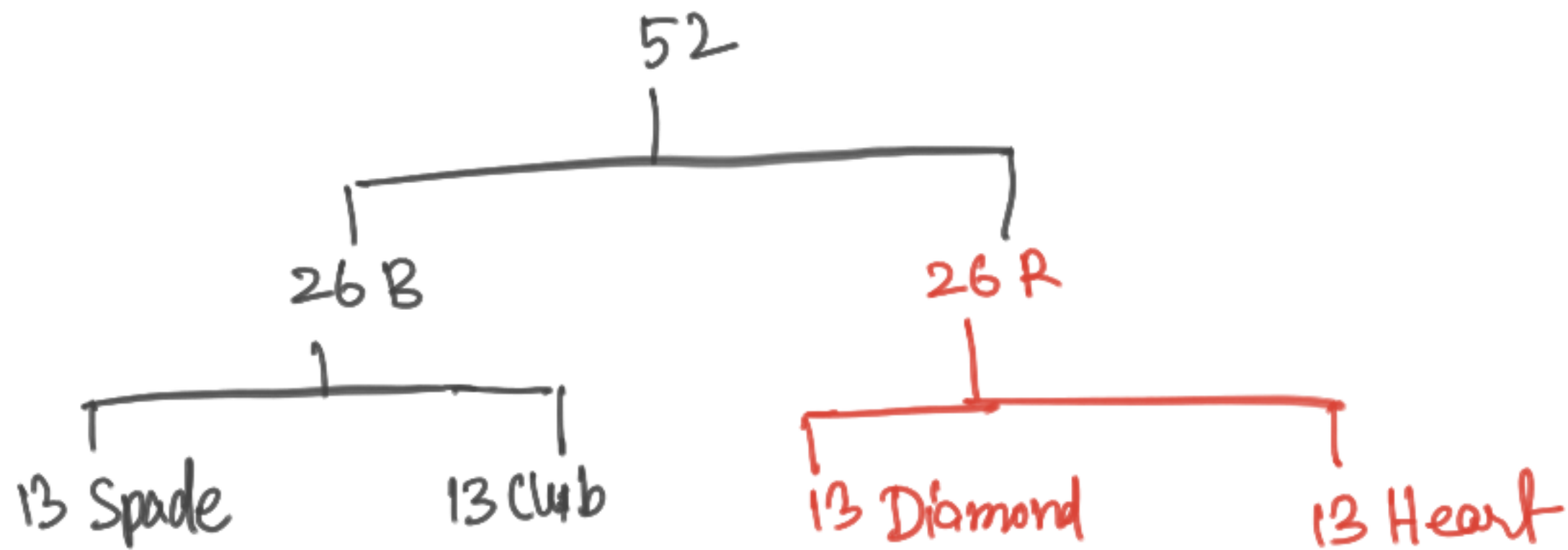
$$\text{or } P(E_1 \cap E_2) = P(E_2)P(E_1/E_2) \quad ; \quad P(E_2) \neq 0$$

Note

① $P(E_1 \cap E_2)$ is also written as $P(E_1 E_2)$

② If E_1 and E_2 are independent events then

$$P(E_1 \cap E_2) = P(E_1) P(E_2)$$



↓

FACE CARDS {
1 King
1 Queen
1 Jack
1 Ace (A)
2-10 numbers

Q. A card is drawn from a well shuffled deck of 52 cards and then a 2nd card is drawn. Find the probability that the first card is a spade and second card is a club if the first card is not replaced.

and $\rightarrow \cap \rightarrow \times$
or $\rightarrow \cup \rightarrow +$

Soln:

Let, $S =$ getting the first card a spade
 $C =$ getting the 2nd card a club

Now,

$$P(S \cap C) = P(S) P(C|S) \xleftarrow{\quad} \textcircled{1}$$

Here,

$$P(S) = \text{Probability that the 1st card is spade} = \frac{{}^{13}C_1}{{}^{52}C_1}$$
$$= \frac{13}{52}$$

$P(C/S)$ = Probability that 2nd card is club given that 1st card was spade

$$= \frac{{}^{13}C_1}{{}^{51}C_1}$$

$$= \frac{13}{51}$$

(\because Replacement doesn't occur)

From ①

$$P(S \cap C) = \left(\frac{13}{52} \right) \times \left(\frac{13}{51} \right) = \frac{?}{?}$$

Q. A problem in physics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?