Lecture – 5

Q. Evaluate :-

① $\int_{-3}^{\infty}(t+1)\,\delta t\,dt$

Sol^n: $\int_{-3}^{\infty}(t+1)\,\delta t\,dt$

$= |t+1|_{t=0}$

$= 1$

② $\int_{-3}^{-5}(t^2+1)\,\delta(t)\,dt$

Sol^n:- $\int_{-3}^{-5}(t^2+1)\,\delta(t)\,dt$

$= 0$

* When you are having unit impulse function, the integration limits must contain zero.

③ $\int_{-\infty}^{-3} e^{-at}\,u(t)\,dt$

$= 0$ [$u(t)$ has value from 0 to ∞]

④ $\int_{-\infty}^{\infty} e^{(2-t)}\,\delta(t-2)\,dt$

$= |e^{2-t}|_{t=2}$

$= e^0$

$= 1$

Q. Find the following summations :-

ⓐ $\sum_{n=-\infty}^{\infty} e^{3n}\,\delta(n-3)$

Sol^n: We know, $\delta(n-3) = \begin{cases} 1, & n=3 \\ 0, & n \neq 3 \end{cases}$

$\therefore \sum_{n=-\infty}^{\infty} e^{3n}\,\delta(n-3) = [e^{3n}]_{n=3} = e^9$

(14)

(b) $\sum_{n=-\infty}^{\infty} \delta(n-2) \cos 3n$

$= [\cos 3n]_{n=2}$

$= \cos 6$

(c) $\sum_{n=-\infty}^{\infty} n^2 \delta(n+4)$

$= [n^2]_{n=-4}$

$= 16$

(d) $\sum_{n=-\infty}^{\infty} \delta(n-2) e^{n^2}$

$= [e^{n^2}]_{n=2}$

$= e^4$

(e) $\sum_{n=0}^{\infty} \delta(n+1) 4^n$

$= 0$ $\quad$ [$\because \delta(n+1) = 1$ for $n=-1$ & $\delta(n+1) = 0$ for $n \neq -1$]

Q. Evaluate :-

(a) $\int_{-\infty}^{\infty} (t-1)^2 \delta(t-1)\, dt$

$= [(t-1)^2]_{t=1}$

$= 0$

(b) $\sum_{n=-\infty}^{\infty} \delta(n) \sin 2n$

$= [\sin 2n]_{n=0}$

$= 0$

(c) $\sum_{n=-\infty}^{\infty} n^2 \delta(n-3)$

$= [n^2]_{n=3}$

$= 9$

(d) $\int_{0}^{\infty} t^3 \delta(t-2)\, dt$

$= [t^3]_{t=2}$

$= 2^3$

$= 8$

(e) $\int_{-\infty}^{\infty} \delta(t+3) e^{-2t}\, dt$

$= [e^{-2t}]_{t=-3}$

$= e^{-2\times(-3)}$

$= e^6$