Date
07/12/2021

Q. Find inverse Laplace transform of :

(i) $X(s) = \frac{1}{s(s+2)}$

Solⁿ: Let,

$$X_1(s) = \frac{1}{s+2}$$

$$X(s) = \frac{X_1(s)}{s}$$

$$\therefore x_1(t) = L^{-1}[X_1(s)] = L^{-1}\left(\frac{1}{s+2}\right)$$

$$\Rightarrow x_1(t) = e^{-2t}u(t)$$

Using time integral property,

$$L\left(\int_{-\infty}^{t} x(\tau)d\tau\right) = \frac{X(s)}{s}$$

$$\Rightarrow L^{-1}\left[\frac{X(s)}{s}\right] = \int_{-\infty}^{t} x(\tau)d\tau$$

$$\Rightarrow L^{-1}\left[\frac{1}{s(s+2)}\right] = \int_{-\infty}^{t} e^{-2\tau}u(\tau)d\tau$$

$$= \int_{0}^{t} e^{-2\tau}d\tau$$

$$= \left[\frac{e^{-2\tau}}{-2}\right]_0^t$$

$$= \frac{e^{-2t}-1}{-2}$$

$$= \frac{1-e^{-2t}}{2}$$

(ii) $X(s) = \frac{1+e^{-2s}}{3s^2+2s}$

Solⁿ:
$$L^{-1}\left(\frac{1}{3s^2+2s}\right) = L^{-1}\left\{\frac{1}{3s[s+2/3]}\right\}$$

$$= L^{-1}\left\{-\frac{1}{s} + \frac{1}{s+2/3}\right\}$$

$$= e^{-2/3 t}\cdot u(t) - u(t)$$

$$L^{-1}\left(\frac{e^{-2s}}{3s^2+2s}\right) = L^{-1}\left(\frac{1}{3s^2+2s}\right)_{t=t-2}$$

$$= e^{-2/3(t-2)}\cdot u(t-2) - u(t-2)$$

$$\therefore x(t) = e^{-2/3 t}u(t) - u(t) + e^{-(2/3)(t-2)}u(t-2) - u(t-2)$$

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Q. Find the initial & final values of the following transform:

(i) $X(s) = \dfrac{s+5}{s^2+3s+2}$

Sol^n:

Initial value $= \underset{s\to\infty}{Lt}\; sX(s)$

$= \underset{s\to\infty}{Lt}\; \dfrac{s(s+5)}{s^2+3s+2}$

$= \underset{s\to\infty}{Lt}\; \dfrac{s^2+5s}{s^2+3s+2}$

$= \underset{s\to\infty}{Lt}\; \dfrac{\cancel{s^2}(1+5/s)}{\cancel{s^2}(1+3/s+2/s^2)}$

$= 1$

Final value $= \underset{s\to 0}{Lt}\; s\,X(s)$

$= \underset{s\to 0}{Lt}\; \dfrac{s(s+5)}{s^2+3s+2}$

$= \underset{s\to 0}{Lt}\; \dfrac{s^2+5s}{s^2+3s+2}$

$= 0$ //

(ii) $X(s) = \dfrac{s^2+5s+7}{s^2+3s+2}$

Sol^n: Initial value $= \underset{s\to\infty}{Lt}\; s\,X(s)$

$= \underset{s\to\infty}{Lt}\; \dfrac{s(s^2+5s+6)}{s^2+3s+2}$

$= \underset{s\to\infty}{Lt}\; \dfrac{s^3+5s^2+6s}{s^2+3s+2}$

$= \underset{s\to\infty}{Lt}\; \dfrac{\cancel{s^2}(s+5+6/s)}{\cancel{s^2}(1+3/s+2/s^2)}$

$= \infty$

Final value $= \underset{s\to 0}{Lt}\; sX(s)$

$= \underset{s\to 0}{Lt}\; \dfrac{s(s^2+5s+6)}{s^2+3s+2}$

$= 0$ //

Q. Find Laplace Tr of :-

(i) $x(t) = e^{-2t} \sin 2t \, u(t)$

Solⁿ:- We know,

$$\mathcal{L}[\sin 2t \, u(t)] = \frac{2}{s^2 + (2)^2} = \frac{2}{s^2+4}$$

Using frequency shift,

$$\mathcal{L}[e^{-2t} \sin 2t \, u(t)] = \mathcal{L}[\sin 2t \, u(t)] \Big|_{s=s+2}$$

$$= \frac{2}{s^2+4} \Big|_{s=s+2}$$

$$= \frac{2}{(s+2)^2+4}$$

$$= \frac{2}{s^2+4s+8}$$

(ii) $x(t) = e^{-5t}[u(t) - u(t-2)]$ & its ROC.

Solⁿ:- $$x(s) = \int_{-\infty}^{\infty} e^{-5t}[u(t) - u(t-5)] e^{-st} dt$$

$$= \int_{-\infty}^{\infty} e^{-5t} \cdot e^{-st} \cdot u(t) dt - \int_{-\infty}^{\infty} e^{-5t} \cdot e^{-st} \cdot u(t-5) dt$$

$$= \int_{0}^{\infty} e^{-5t} \cdot e^{-st} dt - \int_{5}^{\infty} e^{-5t} \cdot e^{-st} dt$$

$$= \int_{0}^{\infty} e^{-(5+s)t} dt - \int_{5}^{\infty} e^{-(5+s)t} dt$$

$$= \left[\frac{1}{-(5+s)} e^{-(5+s)t}\right]_0^{\infty} - \left[\frac{1}{-(5+s)} \cdot e^{-(5+s)t}\right]_5^{\infty}$$

$$= \left[0 - \left(-\frac{1}{5+s}\right)\right] - \left[0 - \left(-\frac{1}{5+s}\right) e^{-(5+s)5}\right]$$

$$= \frac{1}{(s+5)} + \frac{1}{(s+5)} e^{-(25+5s)}$$

$$= \frac{1}{(s+5)} \left[1 + e^{-(25+5s)}\right]$$