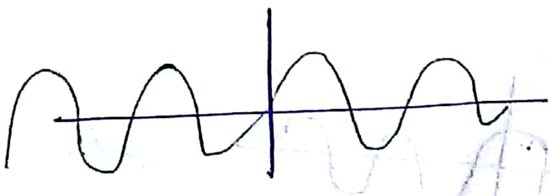
Classification of signals

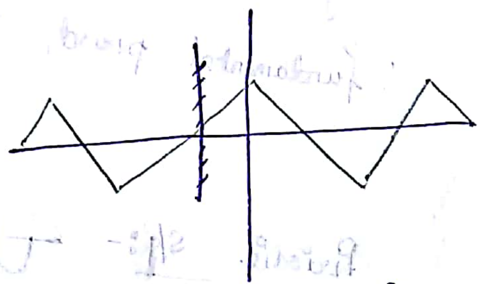
- Continuous-time s/g
- Discrete time s/g
  - (1) Deterministic & Random s/g
  - (2) Periodic & Non-Periodic s/g
  - (3) Energy & Power s/g
  - (4) Causal & Non Causal s/g
  - (5) Even & Odd s/g.

Deterministic & Random s/gs :—

A signal exhibiting no uncertainity of its magnitude and phase at any given instant of time is called deterministic s/g. It has a regular pattern and can be completely represented by a mathematical eqn at any time.

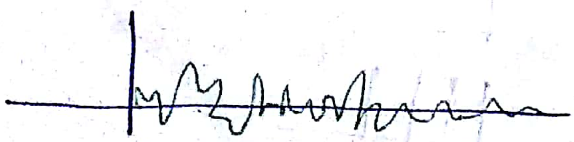
eg: Sine wave, $x(t) = \cos \omega t$, Exponential s/g, square wave, triangular wave etc.





The signal characterised by uncertainity of its occerrance is called Random s/g. It cannot be represented by any mathematical eqn.

eg:- Thermal noise generated in an electric circuit.



Periodic & Non Periodic s/g :-

A signal which has a definite pattern & repeats itself at regular intervals of time is called periodic s/g, & a s/g which does not repeat at regular intervals of time is called a non-periodic or aperiodic s/g.

Mathematically, a cont. - time s/g $x(t)$ is called periodic if & only if $x(t+T) = x(t)$ for all t, i.e. for $-\infty < t < \infty$

where t = time

T = constant representing the period.

T is the fundamental period

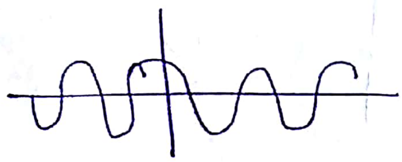
Its reciprocal is the fundamental freq i.e 'f'

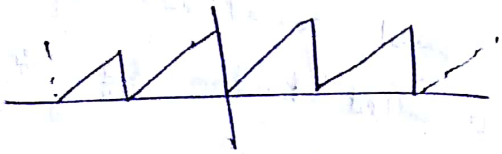
$$f = \frac{1}{T}$$

angular freq, $\omega = 2\pi f = \frac{2\pi}{T}$

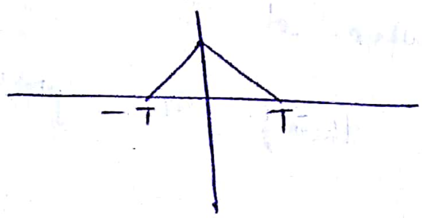
$\therefore$ fundamental period, $T = \frac{2\pi}{\omega}$

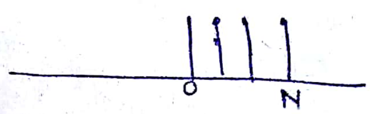
Periodic s/g :-





Aperiodic :-





NB

1. The sum of two continuous-time periodic s/g $x_1(t)$ & $x_2(t)$ with periods T_1 & T_2 may or may not be periodic depending on T_1 & T_2

2. Sum of two periodic s/g is periodic if T_1/T_2 is rational no. or ratio of two integers.

3. T is the LCM of T_1 & T_2

4. Sum of two discrete-time periodic sequences is always periodic.

* For a discrete-time s/g to be periodic

$\omega_0 = 2\pi\left(\frac{m}{N}\right)$ — rational no.

Q.1. Determine whether $x(t) = \cos^2(2\pi t)$ is periodic or not.

→ $x(t) = \cos^2(2\pi t)$

$= \frac{1}{2}[1 + \underbrace{\cos 4\pi t}]$

compare it with $\cos \omega t$.

$\omega = 4\pi$

$\Rightarrow 2\pi f = 4\pi$

$\Rightarrow f = 2 \text{ Hz}$

$\Rightarrow T = \frac{1}{2} = 0.5$ sec.

$x(t)$ will be periodic if $x(t) = x(t+T)$

$x(t+0.5) = \cos^2(2\pi(t+0.5))$

$= \frac{1}{2}[1 + \cos 4\pi(t+0.5)]$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$4\pi \times 0.5\pi$ $\frac{1}{2}\pi$

$\therefore x(t+0.5) = \frac{1}{2}[1 + (\cos 4\pi t \cos 2\pi - \sin 4\pi t . \sin 2\pi)]$

$= \frac{1}{2}[1 + \cos 4\pi t]$

$= \cos^2(2\pi t)$.

$\therefore x(t) = x(t+T)$

$\therefore$ periodic

Q2. $x[n] = (-1)^n$ – Determine if periodic or not.

Q3. $\cancel{x(t)} = e^{j\pi t}$ – "

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Q3. $x(t) = e^{j\pi t}$ "

$\Rightarrow$

Compare with $e^{j\omega t}$

$$\omega = \pi$$

$$\Rightarrow 2\pi f = \pi$$

$$\Rightarrow f = 1/2$$

$$\Rightarrow T = 2 \text{ sec.}$$

To be periodic,

$$x(t) = x(t+T)$$

$$\therefore x(t+T) = e^{j\pi(t+2)}$$

$$= e^{j\pi t} \cdot e^{j2\pi}$$

$$= e^{j\pi t}\left[\cancelto{1}{\cos 2\pi} + j\cancelto{0}{\sin 2\pi}\right]$$

$$= e^{j\pi t}$$

$$= x(t).$$

$\cos \omega n$

$\omega =$

$= e^{j\pi t}\left[\cos 2\pi\right.$

$= e^{j\pi t}$

$= x(t)$.

Q4. $x[n] = \cos(0.01\pi n)$.

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Q4. $x[n] = \cos(0.01\pi n)$.

$\rightarrow$ Compare with ~~cos ωA~~ $\cos\omega n$ $\qquad \omega = \frac{2\pi m}{N}$

$$\omega = 0.01\pi$$

$$= \frac{1}{100}\pi$$

$$= \frac{2\pi}{200}$$

$$N = 200 \text{ sec.}$$

So we can express ω as a rational multiple of 2π

$\therefore$ periodic.

Q5. $x[n] = \cos(2\pi n)$

Q6. $x[n] = \cos 2n$

Q7. $x[n] = \sin[0.2n + \pi]$

Q8. $x(t) = \cos(t + \pi/4)$

Q9. $x(t) = \cos 2t + \sin\sqrt{3}t$.

Q5 $x[n] = \cos(2\pi n)$

$\omega = 2\pi$

$\omega = 2\pi \frac{m}{N}$

$= 2\pi \cdot \frac{1}{1}$

$\therefore$ Periodic ($\because$ ω can represented as rational multiple of 2π)

$N = 1$ s.

Q6. $x[n] = \cos 2n$. $x[n] = \cos 2n$

$\cos \omega n$

$\rightarrow$ $\omega = 2$

$\omega = \frac{2\pi m}{N}$

$N \rightarrow$ integer

Not periodic.

π is not integer

$\therefore 2\pi \cdot \frac{1}{\pi} \rightarrow$ Not possible.

Q7. $x[n] = \sin[0.2n + \pi]$.

Compare with $\sin(\omega n + \theta)$

$\omega = 0.2$

$\omega = 2\pi \frac{m}{N}$

Not periodic.

Q8. $x(t) = \cos(t + \pi/4)$

Compare with

$x(t) = \cos(\omega t + \phi)$ phase angle.

Here, $\omega = 1$

$2\pi f = 1$

$f = \frac{1}{2\pi}$

$T = \frac{1}{f} = 2\pi$

Acc to trigonometry

$\cos(2\pi + \theta) = \cos\theta$

$x(t+T) = \cos\left(t + 2\pi + \frac{\pi}{4}\right) = \cos\left(2\pi + t + \frac{\pi}{4}\right)$

$= \cos\left(t + \frac{\pi}{4}\right)$

$= x(t)$ $\therefore$ periodic.

Q. $x(t) = \cos 2t + \sin\sqrt{3}\,t$.

→ ~~x+t~~ Let, $x(t) = x_1(t) + x_2(t)$

$x_1(t) = \cos 2t$

$\omega_1 = 2$

$2\pi f_1 = 2$

$f_1 = \frac{1}{\pi}$

$T_1 = \pi$ sec

$x_2(t) = \sin\sqrt{3}\,t$.

$\omega_2 = \sqrt{3}$

$2\pi f_2 = \sqrt{3}$

$f_2 = \frac{\sqrt{3}}{2\pi}$

$T_2 = \frac{2\pi}{\sqrt{3}}$ sec.

$$\frac{T_1}{T_2} = \frac{\cancel{\pi} \times \sqrt{3}}{2\cancel{\pi}} = \frac{\sqrt{3}}{2} \neq \text{rational no.}$$

∴ non periodic.