8) Find the complex exponential Fourier Series representation of the following signals -

a) $x(t) = 4\cos 2\omega_o t$

Sol) Comparing with $x(t) = A\cos\omega_o t$, we get -

Fundamental angular frequency, $\omega = 2\omega_o$

$\therefore x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_o t}$ [Complex Fourier series ~~exp~~ representation]

Now,

$x(t) = 4\cos 2\omega_o t = \sum_{n=-\infty}^{\infty} a_n e^{jn2\omega_o t}$

$\therefore 4\cos 2\omega_o t = 2[\cos 2\omega_o t + j\sin 2\omega_o t + \cos 2\omega_o t - j\sin 2\omega_o t]$

$= 2[e^{j2\omega_o t} + e^{-j2\omega_o t}]$

$\Rightarrow x(t) = 2e^{j2\omega_o t} + 2e^{-j2\omega_o t} = \sum_{n=\infty}^{\infty} a_n e^{jn2\omega_o t}$

$\therefore$ The complex fourier coefficients for $4\cos 2\omega_o t$ are -

$a_{-1} = 2$ and $a_1 = 2$, $a_n = 0$ $|n| \neq 1$.

$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$

$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2\theta}$

b) $x(t) = \cos^2 t$

$\Rightarrow x(t) = \frac{1+\cos 2t}{2} = \frac{1}{2} + \frac{\cos 2t}{2}$

Here, fundamental angular frequency of $\cos 2t$ is $\omega_0 = 2$.

$\therefore x(t) = \cos^2 t = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} a_n e^{jn2t}$

By Euler's formula :-

$x(t) = \cos^2 t$

$\Rightarrow x(t) = \left(\frac{e^{jt}+e^{-jt}}{2}\right)^2$

$= \frac{e^{2jt}+e^{-2jt}+2e^{2jt}\cdot e^{-2jt}}{4}$

$\Rightarrow x(t) = \frac{1}{2} + \frac{1}{4}e^{2jt} + \frac{1}{4}e^{-2jt} = \sum_{n=-\infty}^{\infty} a_n e^{jn2t}$

The complex Fourier coefficients of $\cos^2 t$ are -

$a_0 = \frac{1}{2}$, $a_{-1} = \frac{1}{4}$, $a_1 = \frac{1}{4}$, and $a_n = 0$, for $|n| \neq 1$.

6) $x(t) = \sin(2t + \pi/4)$

Comparing with $\sin(\omega_0 t + \theta)$ we get -

Fundamental angular frequency, $\omega_0 = 2$.

$$\therefore x(t) = \sin\left(2t + \frac{\pi}{4}\right) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} a_n e^{jn2t}$$

By Euler's formula -

$$x(t) = \sin(2t + \pi/4)$$

$$\Rightarrow x(t) = \frac{e^{j(2t+\pi/4)} - e^{-j(2t+\pi/4)}}{2j} = -\frac{1}{2j} e^{-j\pi/4} \cdot e^{-j2t} + \frac{1}{2j} e^{j\pi/4}$$

$$\Rightarrow x(t) = \sum_{n=-\infty}^{\infty} a_n e^{j2nt}$$

$\therefore$ The complex fourier coefficient for $\sin(2t + \pi/4)$ are -

$$a_{-1} = -\frac{1}{2j} e^{-j\pi/4} = -\frac{1}{2j}\left[\cos\pi/4 - j\sin\pi/4\right]$$

$$= -\frac{1}{2j}\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$$

$$\Rightarrow a_{-1} = -\frac{1}{2j}\left[\frac{1-j}{\sqrt{2}}\right] = \frac{\sqrt{2}}{4j}(j-1)$$

$$a_1 = \frac{1}{2j} e^{j\pi/4} = \frac{1}{2j}\left[\cos\pi/4 + j\sin\pi/4\right] = \frac{1}{2j}\left[\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right]$$

$$\Rightarrow a_1 = \frac{1}{2j}\left[\frac{1+j}{\sqrt{2}}\right] = \frac{\sqrt{2}}{4j}(j+1)$$

$a_n = 0$, for $|n| \neq 1$.