Independent Random Variable

$E[xy] = E[x]\,E[y]$

$$E[XY] = E[X]\,E[Y]$$

where X & Y are independent R.V.

Covariance

If x & y are two R.V. with mean $\bar{x}$ & $\bar{y}$ respectively then covariance between x & y is defined as

$$\operatorname{cov}(x, y) = E\left[(x-\bar{x})(y-\bar{y})\right]$$

$\Rightarrow$ The covariance of two independent random variable is zero.

Let x & y be two independent R.V.

$$\therefore E[xy] = E[x]\,E[y] \quad \text{—} \ (1)$$

$$\mathrm{Cov}(x,y) = E\left[(x-\bar{x})(y-\bar{y})\right]$$

$$= E\left[(xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y})\right]$$

$$= E[xy] - E[x\bar{y}] - E[\bar{x}y] + E[\bar{x}\bar{y}]$$

$$= E[x]E[y] - \bar{y}E[x] - \bar{x}E[y] + \bar{x}\bar{y}$$

(using ①)

$$= \bar{x}\bar{y} - \bar{y}\bar{x} - \bar{x}\bar{y} + \bar{x}\bar{y}$$

$$= 0$$

$E[x+y]$

$= E[x] + E[y]$

$\bar{x}, \bar{y}$

const

Correlation Coefficient

$$\rho = \frac{\text{Cov}(x, y)}{\sqrt{\text{Var}(x)}\,\sqrt{\text{Var}(y)}}$$

Properties of covariance

1. $\text{Cov}(X, X) = \text{Var}(X)$
2. If X & Y are two independent R.V. $\text{Cov}(X, Y) = 0$

③ $\operatorname{cov}(X, Y) = \operatorname{cov}(Y, X)$

④ $\operatorname{cov}(aX, Y) = a \operatorname{cov}(X, Y)$, $a = \text{const}$

⑤ $\operatorname{cov}(X+c, Y) = \operatorname{cov}(X, Y)$, $c = \text{const}$

⑥ $\operatorname{cov}(X+Y, Z) = \operatorname{cov}(X, Z) + \operatorname{cov}(Y, Z)$

Properties of correlation

① $-1 \leq \rho(X, Y) \leq 1$

(2) If $\rho(X, Y) = 1$ then $Y = aX + b$ where $a > 0$

(3) If $\rho(X, Y) = -1$ then $Y = aX + b$ where $a < 0$

(4) $\rho(aX + b, cY + d) = \rho(X, Y)$ for $a, c > 0$

Binomial Distribution

$$P(x) = C(n, r)\, p^{r}\, q^{n-r}$$

$C(n, r) = {}^{n}C_{r}$

p = prob. of success

q = prob. of failure

s.t. $p + q = 1$

n = total no. of outcome

x = fav. " " "

Properties

① parameters : p or q & n

② The distribution is symmetrical if $p = q$

③ Mean $= np$, Variance $= npq$,

S.D. $= \sqrt{npq}$