Total Probability

$E_1, E_2, \ldots, E_n \leftarrow$ n - multually exclusive and exhaustive events

$A \leftarrow$ any arbitrary event associated with one/more of the above events

$P(E_i) \neq 0 \ (i=1,2,\ldots,n)$; $P(A) > 0$

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + \ldots + P(E_n)\,P(A/E_n)$$

Bayes' Theorem

$E_1, E_2, \ldots, E_n \leftarrow$ n - mutually exclusive and exhaustive events.

$A \leftarrow$ any arbitrary event associated with one/more of the above events

$P(E_i) \neq 0 \ (i=1,2,\ldots,n)$; $P(A) > 0$.

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + \ldots + P(E_n)\,P(A/E_n)}$$

Q. A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that the drawn ball is from bag Y.

Soln

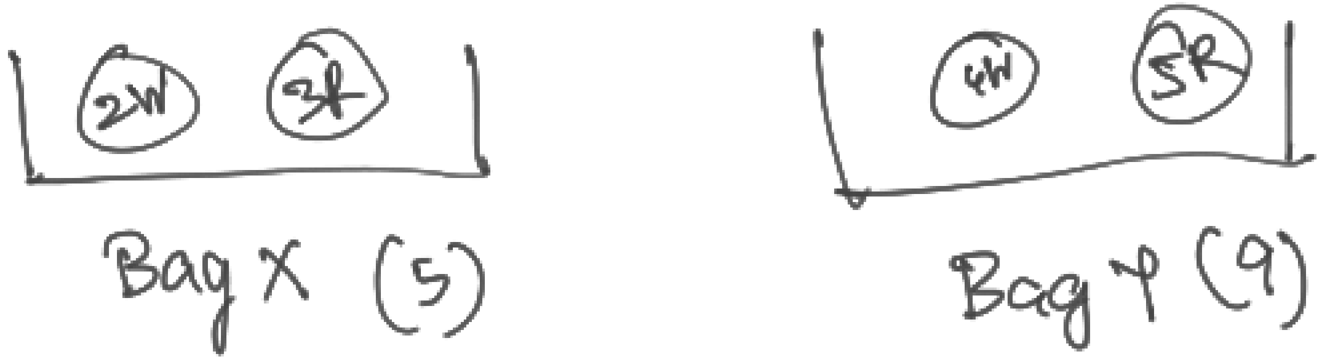
Let,

A = the ball is red

E_1 = the ball is drawn from bag X

E_2 = the ball is drawn from bag Y.

By Bayes' Theorem,



$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)} \tag{1}$$

Here,

$P(E_1)$ = Probability that the ball is drawn from bag X

$= \frac{1}{2}$

$P(A/E_1)$ = Probability that the ball is red given that the ball is drawn from bag X $= \frac{3}{5}$

$P(E_2)$ = Probability that the ball is drawn from bag Y

$= \frac{1}{2}$

$P(A|E_2)$ = Probability that the ball drawn is red given that it is drawn from bag Y

$= \frac{5}{9}$

From ①, $P(E_2|A) = \dfrac{\frac{1}{2} \times \frac{5}{9}}{\left(\frac{1}{2} \times \frac{3}{5}\right) + \left(\frac{1}{2} \times \frac{5}{9}\right)} = ?$

Q. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accident is 0.01, 0.03 and 0.15 respectively. One of the insured person meets an accident. What is the probability that he/she is a scooter driver?

Q. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Sol: Let, A = the man reports it is a six.

E_1 = a six occurs

E_2 = a six doesn't occur.

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)\,P(A/E_2)} \quad \text{--- (1)}$$

Here,

$P(E_1)$ = Probability that a six occurs $= \frac{1}{6}$

$P(A/E_1)$ = Probability that man reports it is a six given that six occurs

= Probability that the man speaks the truth.

$= \frac{3}{4}$

$P(E_2)$ = Probability that six doesn't occur $= \frac{5}{6}$

$P(A/E_2)$ = Probability that the man reports it is a six given that six doesn't occur.

$$\boxed{\left(1-\frac{3}{4}=\frac{1}{4}\right)}$$

$$= \frac{1}{4}$$

From (1), we have

$$P(E_1/A) = \frac{\frac{1}{6}\times\frac{3}{4}}{\left(\frac{1}{6}\times\frac{3}{4}\right)+\left(\frac{5}{6}\times\frac{1}{4}\right)} = ?$$

Q. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she is threw 1, 2, 3 or 4 with the die ?

Q. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the setups are done correctly. If after a certain setup, the machine produces 2 acceptable items, find the probability that the machine is set up correctly.

Soln. Let, A = machine produces 2 acceptable items.

E_1 = machine set up is correct.

E_2 = machine set up is incorrect.

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)} \longrightarrow (1)$$

Here,

$P(E_1)$ = Prob - - - $= \frac{80}{100} = 0.8$

$P(E_2)$ = - - - - $= \frac{20}{100} = 0.2$

$P(A|E_1)$ = Probability that the machine produces two acceptable items given that the machine set up is correct

$$= \frac{90}{100} \times \frac{90}{100}$$

$$= 0.81$$

$$P(A|E_2) = \ldots = \frac{40}{100} \times \frac{40}{100} = 0.16$$

From (1), we have

$$P(E_1/A) = \frac{0.8 \times 0.81}{(0.8 \times 0.81) + (0.2 \times 0.16)} \Rightarrow ?$$