For the discrete probability distribution

x	0	1	2	3	4	5	6	7
p	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine

(i) k (ii) mean (iii) variance

(iv) smallest value of k s.t. $P(X \le x) > 1/2$

We know that,

$$\sum f(x) = 1$$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 0$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$\Rightarrow k = -1 \text{ or } 1/10$$

$k = -1$ is not possible

$$\therefore \quad k = 1/10$$

Mean $= \sum x f(x)$ | $p(x) = f(x)$

$= 3.66$

Variance $= E[x^2] - (E[x])^2$

$= \left[0 + \left(1 \times \frac{1}{10}\right) + \left(2^2 \times \frac{2}{10}\right) + \left(3^2 \times \frac{2}{10}\right) + \left(4^2 \times \frac{3}{10}\right) + \left(5^2 \times \frac{1}{100}\right) + \left(6^2 \times \frac{2}{100}\right) + \left(7^2 \times \frac{77}{100}\right)\right] - (3.66)^2$

$= 37.7$

$$P(x \le 0) = f(0) = 0$$

$$P(x \le 1) = f(0) + f(1) = 0 + \frac{1}{10} = 0.1$$

$$P(X \le 2) = f(0) + f(1) + f(2) = 0 + \frac{1}{10} + \frac{2}{10} = 0.03$$

$$P(X \le 3) = f(0) + f(1) + f(2) + f(3) = 0.5$$

$$P(X \le 4) = f(0) + f(1) + f(2) + f(3) + f(4)$$

$$= 0.8 \qquad \left(\because \underline{x = 4} \right.$$

$\therefore$ Smallest value of x s.t. $P(X \le x) > \frac{1}{2}$ is 4

A fair coin is tossed until head or five tails occurs. Find expected no. of tosses of the coin.

Let, X = no. of tosses

X can take values $1, 2, 3, 4, 5, 6$

X	1	2	3	4	5	6
Outcome	H	TH	TTH	TTTH	TTTTH	TTTTT
P(X)	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{2}\right)^4$	$\left(\frac{1}{2}\right)^5$	$\left(\frac{1}{2}\right)^5$

$\therefore E[X] = \sum x\, p(x) = 1.96$

$\Rightarrow$ Expected no. of tosses ≈ 2

If X and Y are discrete random variables and K is a constant then prove that

(i) $E[X+k] = E[X] + k$

(ii) $E[X+Y] = E[X] + E[Y]$

X & Y are discrete R.V.
and K is constant.

$$E[X] = \sum_{i=1}^{n} x_i p_i(x_i)$$

$$\sum_{i=1}^{n} p_i = 1$$

$$E[X] = \sum x\, p(x)$$

$$E[X] = \sum x p$$

$$\sum p = 1 \quad \text{or} \quad \sum p(x) = 1$$

$$E[X+k] = \sum (x+k)\, p = \sum x p + \sum k p$$

$$= E[X] + k \sum p = E[X] + k$$

$$E[X+Y] = \sum (x+y)p$$

$$= \sum xp + \sum yp$$

$$= E[X] + E[Y]$$

A random variables X has the following probability distribution where K is some number

$$P(x) = \begin{cases} K & , x = 0 \\ 2K & , x = 1 \\ 3K & , x = 2 \\ 0 & , \text{otherwise} \end{cases}$$

(a) Determine value of K

(b) Find $P(x < 2)$, $P(x \leq 2)$, $P(x \geq 2)$

3 Men $\xrightarrow{\text{Hats}}$ Centre of a room

$\downarrow$

mixed

$\downarrow$ each person selects one hat

What is prob. none of the three selects their own hat?

decoorangement principle