Date
30/11/2021

Q. Find the Laplace Transform & region of convergence for the following signals :-

ⓐ $x(t) = e^{2t}u(-t) + e^{3t}u(-t)$

Solⁿ:- We have,

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{2t}u(-t)e^{-st}\, dt + \int_{-\infty}^{\infty} e^{3t}u(-t)e^{-st}\, dt$$

$$= \int_{-\infty}^{0} e^{(2-s)t}\, dt + \int_{-\infty}^{0} e^{(3-s)t}\, dt$$

$$= \left[\frac{e^{-(s-2)t}}{-(s-2)}\right]_{-\infty}^{0} + \left[\frac{e^{-(s-3)t}}{-(s-3)}\right]_{-\infty}^{0}$$

$$= -\frac{1}{s-2} - 0 + \frac{1}{-(s-3)} - 0$$

$$= -\frac{1}{s-2} - \frac{1}{s-3}$$

(b) $x(t) = t \cdot e^{-2|t|}$

Solⁿ :- We have,

$$X(S) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$= \int_{-\infty}^{\infty} t \cdot e^{-2|t|} \cdot e^{-st} dt$$

$$= \int_{-\infty}^{0} t \cdot e^{2t} \cdot e^{-st} dt + \int_{0}^{\infty} t \cdot e^{-2t} \cdot e^{-st} dt$$

$$= -\frac{1}{(s-2)^2} + \frac{1}{(s+2)^2}$$

$\therefore$ R.O.C : $-2 < Re(s) < 2$

Q. Find the inverse Laplace Transform of :-

(a) $X(s) = \frac{s^2}{(s-4)^2}$

Solⁿ :-

$$x(t) = L^{-1}[X(s)]$$

$$= L^{-1}\left[\frac{s^2}{(s-4)^2}\right]$$

$$= \frac{d}{dt}\left\{L^{-1}\left[\frac{s}{(s-4)^2}\right]\right\}$$

$$= \frac{d}{dt}\left\{\frac{d}{dt}\left[e^{4t} L^{-1}\left(\frac{1}{s^2}\right)\right]\right\}$$

$$= \frac{d}{dt}\left[\frac{d}{dt}\left[e^{4t} \cdot t\right]\right]$$

$$= \frac{d}{dt}\left[e^{4t} \cdot 1 + 4e^{4t} \cdot t\right]$$

$$= 4e^{4t} + e^{4t} + 16te^{4t}$$

$$= e^{4t}(5 + 16t)$$