BAYES' THEOREM

If $E_1, E_2, \ldots, E_n$ *are mutually exclusive and exhaustive events with* $P(E) \neq 0,\ (i = 1, 2, \ldots, n)$ *of a random experiment then for any arbitrary event A of the sample space of the above experiment with* $P(A) > 0$, *we have*

$$P\left(E_i / A\right) = \frac{P(E_i)\, P\left(A / E_i\right)}{\sum_{i=1}^{n} P(E_i)\, P\left(A / E_i\right)}$$

A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

$$Let,\ E_1 = the\ ball\ is\ drawn\ from\ bag\ X$$

$$E_2 = the\ ball\ is\ drawn\ from\ bag\ Y$$

$$A = the\ ball\ is\ red$$

$$By\ Bayes'\ Theorem,$$

$$P\left(E_2/A\right) = \frac{P\left(E_2\right)P\left(A/E_2\right)}{P\left(E_1\right)P\left(A/E_1\right) + P\left(E_2\right)P\left(A/E_2\right)}$$

$$P(E_1) = \text{Probability that the ball is drawn from bag } X = \frac{1}{2}$$

$$P(E_2) = \text{Probability that the ball is drawn from bag } Y = \frac{1}{2}$$

$$P(A/E_1) = \text{Probability that a red ball drawn from bag } X = \frac{3}{5}$$

$$P(A/E_2) = \text{Probability that a red ball drawn from bag } Y = \frac{5}{9}$$

$$P(E_2/A) = \frac{\frac{1}{2} \times \frac{5}{9}}{\left(\frac{1}{2} \times \frac{3}{5}\right) + \left(\frac{1}{2} \times \frac{5}{9}\right)} = \frac{25}{52}$$

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accident is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets an accident. What is the probability that he is a scooter driver?

$$\text{Let } E_1 = \text{insured person is scooter driver}$$

$$E_2 = \text{insured person is car driver}$$

$$E_3 = \text{insured person is truck driver}$$

$$A = \text{insured person meets accident}$$

$$\text{By Bayes' Theorem,}$$

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$P(E_1) = Probabilty\ that\ the\ insured\ \ person\ is\ scooter\ driver = \frac{2000}{12000} = \frac{1}{6}$$

$$P(E_2) = Probabilty\ that\ the\ insured\ \ person\ is\ car\ driver = \frac{4000}{12000} = \frac{1}{3}$$

$$P(E_3) = Probabilty\ that\ the\ insured\ \ person\ is\ truck\ driver = \frac{6000}{12000} = \frac{1}{2}$$

$$P\left(A/E_1\right) = Probability\ that\ insured\ \ person\ meets\ accident\ given\ he\ is\ scooter\ driver = 0.01$$

$$P\left(A/E_2\right) = Probability\ that\ insured\ \ person\ meets\ accident\ given\ he\ is\ car\ driver = 0.03$$

$$P\left(A/E_3\right) = Probability\ that\ insured\ \ person\ meets\ accident\ given\ he\ is\ truck\ driver = 0.15$$

$$P\left(E_1/A\right)=\frac{P\left(E_1\right)\,P\left(A/E_1\right)}{P\left(E_1\right)\,P\left(A/E_1\right)+P\left(E_2\right)\,P\left(A/E_2\right)+P\left(E_3\right)\,P\left(A/E_3\right)}$$

$$=\frac{\frac{1}{6}\times 0.01}{\left(\frac{1}{6}\times 0.01\right)+\left(\frac{1}{3}\times 0.03\right)+\left(\frac{1}{2}\times 0.15\right)}=\frac{1}{52}$$