Properties of Laplace Tx :→

① Linearity Property :

If $x_1(t)$ & $x_2(t)$ are two signals with Laplace Tx $X_1(s)$ & $X_2(s)$, then

$$L\left[a x_1(t) + b x_2(t)\right] = a X_1(s) + b X_2(s)$$

② Conjugation Property :

$$x(t) \rightleftharpoons X(s)$$
$$x^*(t) \rightleftharpoons F^*(s^*)$$

③ Time Reversal :

$$x(t) \rightleftharpoons X(s), \quad ROC = R$$
$$x(-t) \rightleftharpoons F(-s), \quad ROC = -R$$

④ Time Scaling :

$$x(t) \rightleftharpoons X(s), \quad ROC = R$$
$$x(at),\ a \neq 0 \rightleftharpoons \frac{1}{|a|} F(s/a), \quad ROC = |a| R.$$

⑤ Time Shifting :–

$$x(t) \rightleftharpoons X(s), \quad ROC = R$$
$$x(t \pm t_0) \rightleftharpoons X(s) \cdot e^{\pm s t_0}, \quad ROC = R$$

⑥ Frequency Shifting : (Shifting in s-domain)

$$x(t) \rightleftharpoons X(s) \ , \ ROC = R$$

$$e^{+s_0 t} \cdot x(t) \rightleftharpoons X(s - s_0) \ , \ ROC = R + Re(s_0)$$

⑦ Convolution in Time :

$$x_1(t) \rightleftharpoons X_1(s) \ , \ ROC = R_1$$

$$x_2(t) \rightleftharpoons X_2(s) \ , \ ROC = R_2$$

$$x_1(t) * x_2(t) \rightleftharpoons X_1(s) \cdot X_2(s) \ , \ ROC = R_1 \cap R_2 .$$

⑧ Multiplication in Time :

$$x_1(t) \cdot x_2(t) \rightleftharpoons \frac{1}{2\pi j} \left[X_1(s) * X_2(s) \right] , \ ROC = R_1 \cap R_2$$

⑨ Differentiation in time :

$$\left. \begin{aligned} & x(t) \rightleftharpoons X(s) \ , \ ROC = R \\ & \frac{dx(t)}{dt} \rightleftharpoons sX(s) \ , \ ROC = R . \end{aligned} \right\} \text{Bilateral L.T.}$$

$$\left[\frac{d^n x(t)}{dt^n} \rightleftharpoons s^n X(s) \right]$$

(10) Integration in Time:—

$$x(t) \rightleftharpoons X(s), \quad ROC = R$$

$-\alpha$ to $t \longrightarrow$ range

$t \rightarrow \tau$ (dummy variable)

$$x'(t) = \int_{-\infty}^{\alpha} x(\tau)\,d\tau \rightleftharpoons \frac{X(s)}{s}, \quad ROC = R \cap Re\{s\} > 0$$

(11) Differentiation in frequency:— (or Diff in s-domain)

$$x(t) \rightleftharpoons X(s), \quad ROC = R$$

$$t \cdot x(t) \rightleftharpoons (-1)\frac{dX(s)}{ds}, \quad ROC = R$$

$$t^n \cdot x(t) \rightleftharpoons (-1)^n \frac{d^n X(s)}{ds}, \quad ROC = R.$$

(12) Integration in Frequency:-

$$x(t) \rightleftharpoons X(s)$$

$$\frac{x(t)}{t} \rightleftharpoons \int_{s}^{\infty} F(s)\,ds.$$

INITIAL VALUE THEOREM : (HW → Proof)

If the transform $X(s)$ of an unknown function $x(t)$ is known, then it is possible to determine the initial value of $x(t)$; i.e., the value of $x(t)$ at $t=0$. If $x(t)$ and its first derivative are Laplace transformable then initial value of $x(t)$ is,

$$\boxed{x(0^+) = \lim_{s \to \infty} s X(s)}$$

eg: $F(s) = \frac{1}{s+1}$, find the initial value.

$\rightarrow \quad f(0^+) = \lim_{s \to \infty} s F(s)$

$$= \lim_{s \to \infty} s . \frac{1}{s+1}$$

$$= \lim_{s \to \infty} . \cancel{s} \frac{1}{\cancel{s}(1+1/s)}$$

$$= \lim \frac{1}{1+\frac{1}{\infty}}$$

$$= 1/1 = 1.$$

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FINAL VALUE THEOREM :→ (HW → Proof)

If the transform $X(s)$ of an unknown function $x(t)$ is known then it is possible to determine the final value of $x(t)$, i.e. the value of $x(t)$ at $t = \infty$.
If $x(t)$ & its first derivative are Laplace transformable, then the final value of $x(t)$ is,

$$\boxed{x(\infty) = \lim_{s \to 0} s\,X(s)}$$

eg: ① $X(s) = 3s / 2s^2 + s$

② $X(s) = 2s+1 / 3s^2 + s$

find the final value.

$\underline{\underline{Sol^n}}$:- ① $x(\infty) = \lim_{s \to 0} s \cdot 3s / 2s^2 + s$

$= \lim_{s \to 0} s^2 \, 3 / s^2 (2 + 1/s) \rightarrow 1/0 = \infty$

$= 0$

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2) → $x(\infty) = \lim_{s \to 0} s \cdot \frac{2s+1}{s(3s+1)} = 1.$

LAPLACE TRANSFORM OF STD. S/Gs: →

$x(t)$	$X(s)$
$\delta(t)$	1
1	$1/s$
t	$1/s^2$
t^n	$n!/s^{n+1}$
$e^{-at}u(t)$	$1/s+a$
$t\,e^{-at}u(t)$	$1/(s+a)^2$
$t^n e^{-at}u(t)$	$n!/(s+a)^{n+1}$
$\sin\omega_0 t\; u(t)$	$\omega_0/s^2+\omega_0^2$
$\cos\omega_0 t\; u(t)$	$s/s^2+\omega_0^2$
$e^{-at}\sin\omega_0 t$	$\omega_0/(s+a)^2+\omega_0^2$
$e^{-at}\cos\omega_0 t$	$s+a/(s+a)^2+\omega_0^2$

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NUMERICALS:—

Q1. $x(t) = u(t-2)$ } Find L.T.

Q2. $x(t) = t^2 e^{-2t} u(t)$

Q3. $X(s) = \frac{1}{s(s+2)}$ } Find I.L.T.

Q4. $X(s) = \frac{1+e^{-2s}}{3s^2+2s}$

Solⁿ

Q1. ⇒ $X(s) = \int_0^\infty x(t)e^{-st}\,dt$

$= \int_0^\infty u(t-2)\, e^{-st}\,dt$

$= \int_2^\infty e^{-st}\,dt$

$= \frac{1}{-s} e^{-st} \Big|_2^\infty = e^{-2s}/s$

Q.(2) $\Rightarrow$ $x(t) = t^2 e^{-2t} u(t)$

We know

$$L\left[e^{-2t}\, u(t)\right] = \frac{1}{s+2}$$

Using diff. in s-domain property,

$$L\left[(-t)^n\, x(t)\right] = \frac{d^n X(s)}{ds^n}$$

for $n = 2$

$$L\left[t^2\, x(t)\right] = \frac{d^2 X(s)}{ds^2}$$

$$\Rightarrow L\left[t^2 e^{-2t} u(t)\right] = \frac{d^2}{ds^2}\left[\frac{1}{s+2}\right]$$

$$= \frac{d}{ds}\left[-\frac{1}{(s+2)^2}\right]$$

$$= \frac{2}{(s+2)^3}$$

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Q.③ ⇒ $X(s) = \frac{1}{s(s+2)}$

Let, $X_1(s) = \frac{1}{s+2}$

$\therefore X(s) = \frac{X_1(s)}{s}$

$\left(= \frac{1}{s(s+2)} \right)$

$x_1(t) = L^{-1}[X_1(s)]$

$= L^{-1}\left[\frac{1}{s+2}\right]$

$= e^{-2t}u(t)$

Q5. Find the s/g $x(t)$ for which L.T.

$$X(s) = \frac{s+1}{s^2+3s+2}$$

Q5. Find the s/g $x(t)$ for which L.T.

$$X(s) = \frac{s+1}{s^2+3s+2}$$

Solⁿ:- $X(s) = \frac{s+1}{s^2+3s+2}$

$$= \frac{s+1}{s^2+2s+s+2}$$

$$= \frac{s+1}{s(s+2)+1(s+2)}$$

$$= \frac{\cancel{s+1}}{(s+2)\cancel{(s+1)}} = \frac{1}{s+2}$$

$$\therefore L^{-1}\left(\frac{1}{s+2}\right) = e^{-2t}u(t).$$

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Q6 Determine the s/g $x(t)$, if $X(s) = \frac{s+2}{s+2s+10}$

$X(s) = s+2$

Q6 Determine the s/g $x(t)$, if $X(s) = \dfrac{s+2}{S+2s+10}$

Sol:-

$$X(s) = \frac{S+2}{S^2+2s+10}$$

$$= \frac{S+2}{s^2+2s+1+9}$$

$$= \frac{S+2}{(S+1)^2+3^2}$$

$$= \frac{(S+1)+1}{(S+1)^2+3^2}$$

$$= \frac{S+1}{(S+1)^2+3^2} + \frac{1}{(S+1)^2+3^2}$$

$$x(t) = L^{-1}[X(s)] = L^{-1}\left[\frac{S+1}{(S+1)^2+3^2}\right] + L^{-1}\left[\frac{1}{(S+1)^2+3^2}\right]$$

We know,

$$e^{-at}\cos\omega t\, u(t) \longleftrightarrow \frac{S+a}{(S+a)^2+\omega^2}$$

$$e^{-at}\sin\omega t\, u(t) \longleftrightarrow \frac{\omega}{(S+a)^2+\omega^2}$$

$$\therefore L^{-1}\left[\frac{S+1}{(S+1)^2+3^2}\right] = e^{-t}\cos 3t\, u(t)$$

$$L^{-1}\left[\frac{1}{3}\cdot\frac{3}{(S+1)^2+3^2}\right] = \frac{1}{3}e^{-3t}\sin 3t\, u(t)$$

$$\therefore x(t) = e^{-t}\cos 3t\, u(t) + \frac{1}{3}e^{-3t}\sin 3t\, u(t).$$

Q7. Find I.L.T. of $X(s) = \dfrac{2s+4}{s^2+4s+3}$