DEPARTMENT OF MATHEMATICS
JORHAT ENGINEERING COLLEGE
JORHAT-785007, ASSAM

Continuous Internal Evaluation (CIE) II **(I/II/III)**

Programme : B. Tech.

Semester : III

Branch/Department : Computer Science & Engineering

Course Code : MA181301B

Course Name : Mathematics III-B

Roll No. : 200710007062

Date of Examination : 03/12/2021

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Q.1) Soln: Let, x = no. of cars hired out per day

Given,

mean = 1.5

For Poisson distribution, mean = λ

$\therefore \lambda = 1.5$

We know, $P(x=r) = \dfrac{e^{-\lambda} \cdot \lambda^r}{r!}$

$\Rightarrow P(x=r) = \dfrac{e^{-1.5} \cdot (1.5)^r}{r!}$

(i) $P(\text{neither car is used}) = P(x=0)$

$= \dfrac{e^{-1.5} \cdot (1.5)^0}{0!}$

$= e^{-1.5}$

$= 0.2231$

$\therefore$ Proportion of days on which neither car is used

$= 0.2231 \times 100$

$= \underline{\underline{22.31\%}}$

(ii) $P(\text{some demand is refused}) = P(x>2)$

$= 1 - P(x \leq 2)$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - \left[\dfrac{e^{-1.5} \cdot (1.5)^0}{0!} + \dfrac{e^{-1.5} \cdot (1.5)^1}{1!} + \dfrac{e^{-1.5} \cdot (1.5)^2}{2!} \right]$

$= 1 - \left[e^{-1.5} + e^{-1.5} \times (1.5) + \frac{1}{2} \cdot e^{-1.5} \cdot 2.25 \right]$

$= 1 - [0.2231 + 1.5 \times 0.2231 + \frac{1}{2} \times 2.25 \times 0.2231]$

$= 1 - [0.2231 + 0.33465 + 0.2509875]$

$= 1 - 0.8087$

$= 0.1913$

$\therefore$ The proportion of days on which some demand is refused

$= 0.1913 \times 100$

$= \underline{\underline{19.13\ \%}}$

Q.2) Sol^n:- Given, $f(x) = \begin{cases} kx & , \text{ for } 0 \leq x < 2 \\ 2x & , \text{ for } 2 \leq x < 4 \\ -kx+6k & , \text{ for } 4 \leq x < 6 \\ 0 & , \text{ otherwise} \end{cases}$

$\because f(x)$ is a density function.

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx + \int_{4}^{6} f(x)\,dx + \int_{6}^{\infty} f(x)\,dx = 1$

$\Rightarrow 0 + \int_{0}^{2} kx\,dx + \int_{2}^{4} 2x\,dx + \int_{4}^{6} (-kx+6k)\,dx + 0 = 1$

$\Rightarrow k\left[\frac{x^2}{2}\right]_0^2 + \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_2^4 + \left[-\frac{kx^2}{2} + 6kx\right]_4^6 = 1$

$\Rightarrow \frac{k}{2}\left[2^2 - 0\right] + \left[4^2 - 2^2\right] + \left[-\frac{k}{2}(6^2-4^2) + 6k(6-4)\right] = 1$

$\Rightarrow 2k + (16-4) + \left[-\frac{k}{2}(36-16) + 6k \cdot 2\right] = 1$

$\Rightarrow 2k + 12 + \left[-k \cdot 10 + 12k\right] = 1$

$\Rightarrow 2k + 12 + 2k = 1$

$\Rightarrow 4k = -11$

$\Rightarrow \boxed{k = -11/4}$

$\therefore f(x) = \begin{cases} -11/4\, x & , \text{ for } 0 \leq x < 2 \\ 2 \cdot x & , \text{ for } 2 \leq x < 4 \\ -(-11/4)x + 6(-11/4) & , \text{ for } 4 \leq x < 6 \\ 0 & , \text{ otherwise.} \end{cases}$

$$\Rightarrow f(x) = \begin{cases} -\frac{11}{4}x & , \text{for } 0 \leq x < 2 \\ 2x & , \text{for } 2 \leq x < 4 \\ \frac{11}{4}x - \frac{33}{2} & , \text{for } 4 \leq x < 6 \\ 0 & , \text{otherwise.} \end{cases}$$

Now,

$$\text{Mean} = E[x]$$

$$= \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_{-\infty}^{0} x f(x) dx + \int_{0}^{2} x f(x) dx + \int_{2}^{4} x f(x) dx + \int_{4}^{6} x f(x) dx + \int_{6}^{\infty} x f(x) dx$$

$$= 0 + \int_{0}^{2} x \cdot \left(-\frac{11}{4}x\right) dx + \int_{2}^{4} x \cdot 2x \cdot dx + \int_{4}^{6} x\left(\frac{11}{4}x - \frac{33}{2}\right) dx + 0$$

$$= -\frac{11}{4}\int_{0}^{2} x^2 dx + 2\int_{2}^{4} x^2 dx + \int_{4}^{6}\left(\frac{11x^2}{4} - \frac{33x}{2}\right) dx$$

$$= -\frac{11}{4}\left[\frac{x^3}{3}\right]_0^2 + 2\left[\frac{x^3}{3}\right]_2^4 + \left[\frac{11}{4}\cdot\frac{x^3}{3} - \frac{33}{2}\cdot\frac{x^2}{2}\right]_4^6$$

$$= -\frac{11}{4}\cdot\frac{1}{3}\left[2^3 - 0\right] + \frac{2}{3}\left[4^3 - 2^3\right] + \left[\frac{11}{12}(6^3 - 4^3) - \frac{33}{4}(6^2 - 4^2)\right]$$

$$= -\frac{11}{12}\cdot 8 + \frac{2}{3}\cdot(64-8) + \frac{11}{12}(216 - 64) - \frac{33}{4}(36 - 16)$$

$$= -\frac{88}{12} + \frac{2}{3}\cdot 56 + \frac{11}{12}\cdot 152 - \frac{33}{4}\cdot 20$$

$$= -7.33 + 37.33 + 139.33 - 165$$

$$= 4.33$$

$$\therefore \boxed{k = -\frac{11}{4}, \quad \text{Mean} = 4.33}$$

Q. 3) Sol^n:- Marginal distribution table is given by,

X \ Y	1	3	9	$f_X(x)$
2	$1/8$	$1/24$	$1/12$	$6/24$
4	$1/4$	$1/4$	0	$2/4$
6	$1/8$	$1/24$	$1/12$	$6/24$
$f_Y(y)$	$4/8$	$8/24$	$2/12$	1

(a) Marginal distribution of Y is,

$$P(Y=1) = \frac{4}{8} = 1/2$$

$$P(Y=3) = 8/24 = 1/3$$

$$P(Y=9) = 2/12 = 1/6$$

i.e. $P(Y=y) = \begin{cases} 1/2, & y=1 \\ 1/3, & y=3 \\ 1/6, & y=9 \end{cases}$

(b) The conditional distribution of Y given X=4 is,

$$P(Y=y / X=4) = \frac{P(Y=y \cap X=4)}{P(X=4)}$$

Now,

$$P(Y=1 / X=4) = \frac{P(Y=1 \cap X=4)}{P(X=4)} = \frac{1/4}{2/4} = 1/2$$

$$P(Y=3 / X=4) = \frac{P(Y=3 \cap X=4)}{P(X=4)} = \frac{1/4}{2/4} = 1/2$$

$$P(Y=9 / X=4) = \frac{P(Y=9 \cap X=4)}{P(X=4)} = \frac{0}{2/4} = 0$$

$\therefore P(Y=y/X=4) = \begin{cases} 1/2, & Y=1, X=4 \\ 1/2, & Y=3, X=4 \\ 0, & Y=9, X=4 \end{cases}$

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ⓒ $\text{Cov}(X,Y) = E[XY] - E[X]E[Y]$ ——— ①

$\therefore E[X] = \sum x f_X(x) = [(2\times \frac{6}{24}) + (4\times \frac{2}{4}) + (6\times \frac{6}{24})]$

$= [(2\times \frac{1}{4}) + 2 + 6\times \frac{1}{4}]$

$= \frac{1}{2} + 2 + 3/2$

$= 4$

$E[Y] = \sum y f_Y(y) = [(1\times \frac{4}{8}) + (3\times \frac{8}{24}) + (9\times \frac{2}{12})]$

$= \frac{1}{2}\times 1 + 9\times \frac{1}{6}$

$= \frac{1}{2} + 1 + 3/2$

$= 2+1$

$= 3$

$E[XY] = \sum xy f_{X,Y}(x,y)$

$= [(2\times 1\times \frac{1}{8}) + (2\times 3\times \frac{1}{24}) + (2\times 9\times \frac{1}{12})] + [(4\times 1\times \frac{1}{4}) + (4\times 3\times \frac{1}{4}) + (4\times 9\times 0)]$
$+ [(6\times 1\times \frac{1}{8}) + (6\times 3\times \frac{1}{24}) + (6\times 9\times \frac{1}{12})]$

$= \frac{1}{4} + \frac{1}{4} + 3/2 + 1 + 3 + 0 + 3/4 + 3/4 + 9/2$

$= 12$

Now, from ①,

$\text{cov}(X,Y) = 12 - 4\cdot 3$

$\Rightarrow \text{cov}(X,Y) = 12 - 12$

$\Rightarrow \boxed{\text{cov}(X,Y) = 0}$

ⓓ We know,

if $f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y)$, then X & Y are independent.

Now, from the distribution table,

$f_{X,Y}(2,3) = 1/24$

$f_X(2) = 6/24$

$f_Y(3) = 8/24$

& $f_X(2) \cdot f_Y(3) = \frac{6}{24} \cdot \frac{8}{24} = \frac{1}{12} \neq f_{X,Y}(2,3)$

Hence, X & Y are not independent.

Q.4) Solⁿ: Let the age and intelligence be denoted by x & y resp.

Mid value	y \ x	18	19	20	21	f	u	fu	fu^2	fuv
15	10-20	4	2	2		8	-3	24	72	30
25	20-30	5	4	6	4	19	-2	-38	76	20
35	30-40	6	8	10	11	35	-1	-35	35	9
45	40-50	4	4	6	8	22	0	0	0	0
55	50-60		2	4	4	10	1	10	10	2
65	60-70		2	3	1	6	2	12	24	-2
	f	19	22	31	28	100	Totals	-75	217	59
	v									
	fv									
	fv^2									
	fuv									