

## Bivariate Distribution

Joint Probability

$$P_{xy}(x,y) \text{ or } f_{xy}(x,y)$$



Joint Prob. of two random variables  
x & y

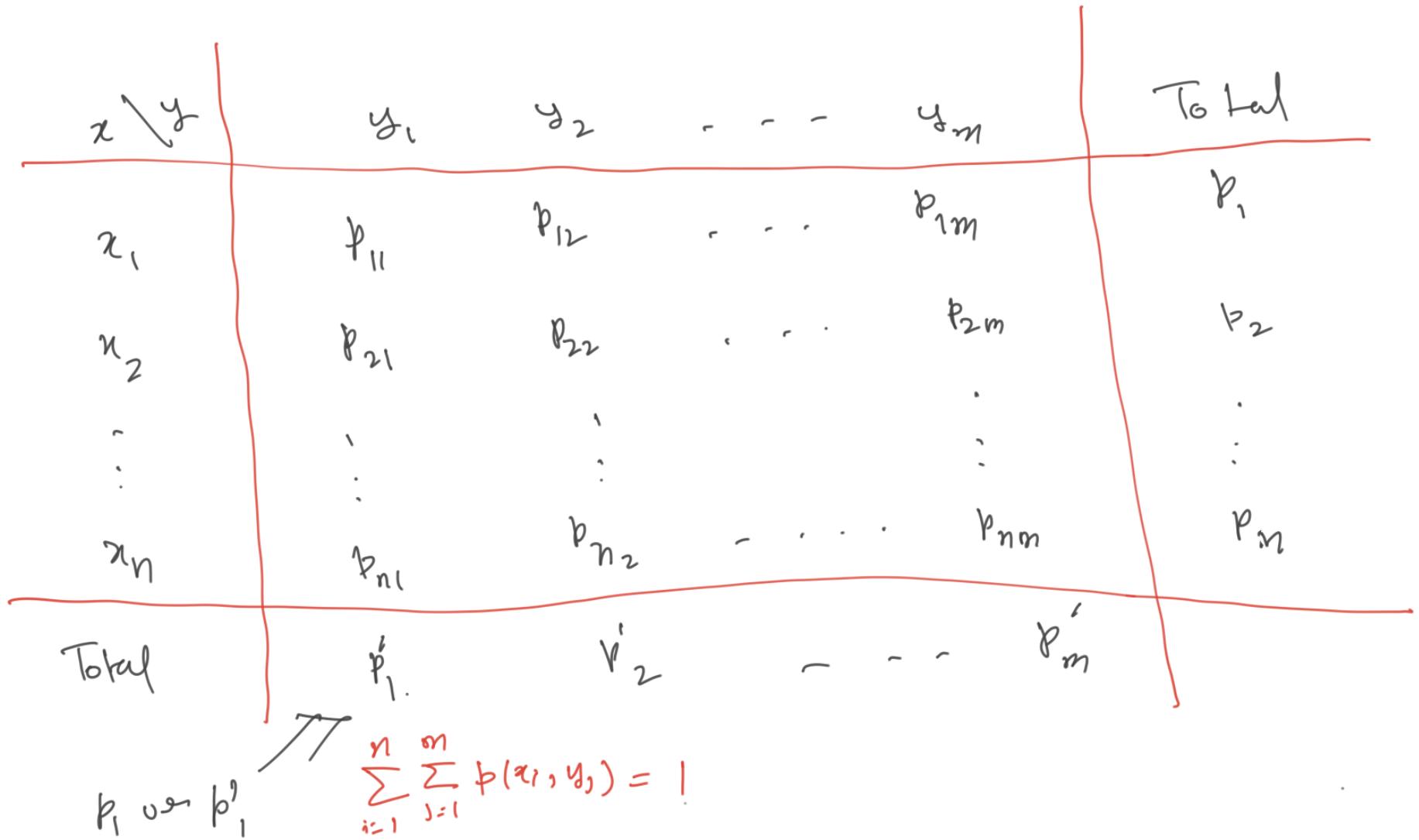
## Joint Probability Mass Function

$X$  and  $Y$  be two R.V. on a sample space  $S$  with respect to image sets  $X(S) = \{x_1, x_2, \dots, x_n\}$  and  $Y(S) = \{y_1, y_2, \dots, y_m\}$

function  $p$  on  $X(S) \times Y(S)$  defined as

$$p_{ij} = P(X=x_i \cap Y=y_j) = p(x_i, y_j)$$

is called joint probability mass fn of  $X$  &  $Y$  where  
 $X(S) \times Y(S) = \{x_1, x_2, \dots, x_n\} \times \{y_1, y_2, \dots, y_m\}$



## Marginal and Conditional probability function

Marginal Probability fn of X :

$$f_X(x_i) = P_X(x_i) = P(X=x_i) = p_{i1} + p_{i2} + \dots + p_{im}$$
$$= \sum_{j=1}^m p_{ij}$$

$$= p_i$$

Marginal Probability fn of Y :

$$f_Y(y_j) = P_Y(y_j) = P(Y=y_j) = \sum_{i=1}^n p_{ij} = p_j$$

Conditional Probability  $P^n$  of  $X$  when  $Y = y_j$  is given

$$t_{x/y}(x_i/y) = P(X=x_i / Y=y_j) = \frac{P(X=x_i \cap Y=y_j)}{P(Y=y_j)}$$

$$= \frac{p(x_i, y_j)}{p(y_j)}$$

$$P(X=x_1 / Y=y_3)$$

$$= \frac{p_{13}}{p_3}$$

$$= \frac{p_{ij}}{p_j}$$

Conditional prob. fn of  $Y$  when  $X = x_i$  is given

$$f_{Y|X}(y|x) = P(Y=y_j | X=x_i) = \frac{p(x_i, y_j)}{p(x_i)}$$

$$= \frac{p_{ij}}{p_i}$$

Independent :-  $P(X=x_i, Y=y_j) = P(X=x_i) P(Y=y_j)$

## Joint Probability Distribution Function

$(X, Y)$  be two dimensional R.V. then their joint probability dist.  $F^n$  is denoted by  $F_{x,y}(x, y)$  and it is defined as

$$F_{x,y}(x, y) = P(X \leq x, Y \leq y) \quad \forall x, y \in \mathbb{R}$$

where,  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{x,y}(x, y) = 1 \quad \leftarrow \text{cont. dist.}$

$$\sum_x \sum_y f_{x,y}(x, y) = 1 \quad \leftarrow \text{discrete dist.}$$

## Properties

① If  $x_1 < x_2$  and  $y_1 < y_2$

$$P[x_1 < X \leq x_2, y_1 < Y \leq y_2] = F(x_2, y_2) - F(x_2, y_1) - F(x_1, y_2) + F(x_1, y_1) \geq 0$$

② a)  $F(-\infty, y) = \lim_{x \rightarrow -\infty} F(x, y) = 0 \quad \forall y \in \mathbb{R}$

b)  $F(x, -\infty) = \lim_{y \rightarrow -\infty} F(x, y) = 0 \quad \forall x \in \mathbb{R}$

c)  $\lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} F(x, y) = F(\infty, \infty) = 1$

③  $f(x,y)$  is right continuous in each argument

$$\lim_{h \rightarrow 0^+} F(x+h, y) = \lim_{h \rightarrow 0^+} F(x, y+h) = F(x, y)$$

④ If the density  $f^n$   $f(x,y)$  is continuous at  $(x,y)$  then

$$\frac{\partial^2 F}{\partial x \partial y} = f(x, y)$$

## Expectation

$(X, Y)$  be two dimensional R.V. with joint density  $f_{x,y}(x, y)$ . The expectation of  $g(x, y)$  is denoted by  $E[g(x, y)]$  and defined by

$$E[g(x, y)] = \sum_x \sum_y g(x, y) f_{x,y}(x, y)$$

## Note

$$\textcircled{1} E[X] = \sum x f_x(x, y) \quad \textcircled{2} E[Y] = \sum y f_y(x, y)$$

$$\textcircled{3} E[XY] = \sum_x \sum_y xy f_{x,y}(x, y)$$

## Covariance

$X$  and  $Y$  be two R.V. then covariance of  $X$  and  $Y$  is defined as

$$\begin{aligned}\text{cov}(X, Y) &= E[X - E[X]] E[Y - E[Y]] \\ &= E[XY] - E[X]E[Y]\end{aligned}$$

## Correlation Coefficient

The correlation coefficient of two R.V.  $X$  and  $Y$  is defined as

$$\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var} X} \sqrt{\text{var} Y}} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y}$$

where  $\sigma_X > 0$ ,  $\sigma_Y > 0$

Note :  $-1 \leq \rho(X, Y) \leq 1$

## Conditional Expectation

$(X, Y)$  be Joint Discrete R.V. then conditional expectation of  $g(X, Y)$  given  $X = x$  is defined as

$$E[g(x, y) / X=x] = \sum_j g(x_i, y_j) f_{Y|X}(y_j | x)$$

$$\begin{aligned} \text{i.e., } E[Y | X=x] &= \sum_j y_j f_{Y|X}(y_j | x) \\ &= \sum_j y_j P(Y=y_j | X=x) \end{aligned}$$

Q. For the following bivariate probability distribution  
of  $X$  and  $Y$  find

$$\textcircled{i} \quad P(X \leq 2, Y = 3)$$

$$\textcircled{ii} \quad P(X \leq 1)$$

$$\textcircled{iii} \quad P(Y = 4)$$

$$\textcircled{iv} \quad P(Y \leq 5)$$

$$\textcircled{v} \quad P(X < 2, Y \leq 3)$$

$X$	$Y$	1	2	3	4	5	6
0		0	0	$y_{32}$	$2/32$	$2/32$	$3/32$
1		$y_{16}$	$y_{16}$	$y_8$	$y_8$	$y_8$	$y_8$
2		$y_{32}$	$y_{32}$	$y_{64}$	$y_{64}$	0	$2/64$

Marginal distribution is given by

$x \setminus y$	1	2	3	4	5	6	$P_X(x)$
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	$\frac{8}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{10}{16}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	$\frac{8}{64}$
$P_Y(y)$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$	1

$$\textcircled{1} P(X \leq 2, Y=3) = P(X=0, Y=3) + P(X=1, Y=3) + P(X=2, Y=3)$$

$$= \frac{1}{32} + \frac{1}{8} + \frac{1}{64}$$

$$= \frac{1}{64}$$

$$\textcircled{3} \quad P(X \leq 1) = P(X=0) + P(X=1) = \frac{8}{32} + \frac{10}{16} = \frac{7}{8}$$

$$\textcircled{3} \quad P(Y=4) = 13/64$$

$$\textcircled{4} \quad P(Y \leq 5) = P(Y=1) + P(Y=2) + P(Y=3) + P(Y=4) + P(Y=5)$$

$$1 - P(Y=6) \swarrow \\ = \frac{3}{32} + \frac{3}{32} + \frac{11}{64} + \frac{13}{64} + \frac{6}{32} \\ = \frac{48}{64}$$

$$\textcircled{5} \quad P(X \leq 2, Y \leq 3) = P(X=0, Y=1) + P(X=0, Y=2) + P(X=0, Y=3) + \\ P(X=1, Y=1) + P(X=1, Y=2) + P(X=2, Y=3)$$

Q. Let  $X$  and  $Y$  have joint p.d.f.

		-1	0	1
0	-1	$b$	$2b$	$b$
	0	$3b$	$2b$	$b$
2	-1	$2b$	$b$	$2b$

Find marginal dist. of  $X$  and  $Y$ . Also find conditional distribution of  $X$  given  $Y=1$ .

Q. The joint probability distribution of  $X$  and  $Y$  is given in the following table

$X \setminus Y$	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

- ① Find the marginal probability distribution of  $Y$ .
- ② Find the conditional distribution of  $Y$  when  $X=4$
- ③ Find covariance of  $X$  and  $Y$
- ④ Are  $X$  and  $Y$  independent?

$x$	$y$	1	3	9	$b_x(x)$
$t_y(y)$		$\frac{4}{8}$	$\frac{8}{24}$	$\frac{2}{12}$	1
2		$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{6}{24}$
4		$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{2}{4}$
6		$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{6}{24}$

① Marginal Probability distribution of  $Y$

$$P(Y=1) = \frac{4}{8} = \frac{1}{2}, \quad P(Y=3) = \frac{8}{24} = \frac{1}{3}, \quad P(Y=9) = \frac{2}{12} = \frac{1}{6}$$

② conditional dist- of  $Y$  when  $X = 4$

$$P(Y=1 \mid X=4) = \frac{P(Y=1 \cap X=4)}{P(X=4)}$$

$$= \frac{1}{2}$$

$$= \frac{1}{2}$$

$$P(Y=3 \mid X=4) = 1/2$$

$$P(Y=9 \mid X=4) = 0$$

③

$$E[X] = \sum x f_X(x) = (2 \times \frac{6}{24}) + (4 \times \frac{2}{4}) + (6 \times \frac{6}{24}) \\ = 4$$

$$\text{cov}(X, Y)$$

$$= E[XY] - E[X]E[Y]$$

$$E[Y] = \sum y f_Y(y) = (1 \times \frac{4}{8}) + (3 \times \frac{8}{24}) + (9 \times \frac{2}{12}) \\ = 3$$

$$E[XY] = \sum xy f_{X,Y}(x,y) = (2 \times \frac{1}{8} + 6 \times \frac{1}{24} + 18 \times \frac{1}{12}) + \\ (4 \times \frac{1}{4} + 12 \times \frac{1}{4} + 36 \times 0) + \\ (6 \times \frac{1}{8} + 18 \times \frac{1}{24} + 54 \times \frac{1}{12}) = 12$$

$$\text{cov}(X, Y) = E[XY] - E[X]E[Y]$$

$$= 12 - (4)(3)$$

$$= 0$$

Ⓐ

$$P(X=2 \cap Y=1) = \frac{1}{8}$$

check for  
all the remaining ones

$$P(X=2) = \frac{6}{24} = \frac{1}{4}$$
$$P(Y=1) = \frac{4}{8} = \frac{1}{2}$$

$$\Rightarrow P(X=2)P(Y=1) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

X & Y are indp

$$P(X=x \cap Y=y)$$

$$= P(X=x) P(Y=y)$$

$$X=4, Y=3$$

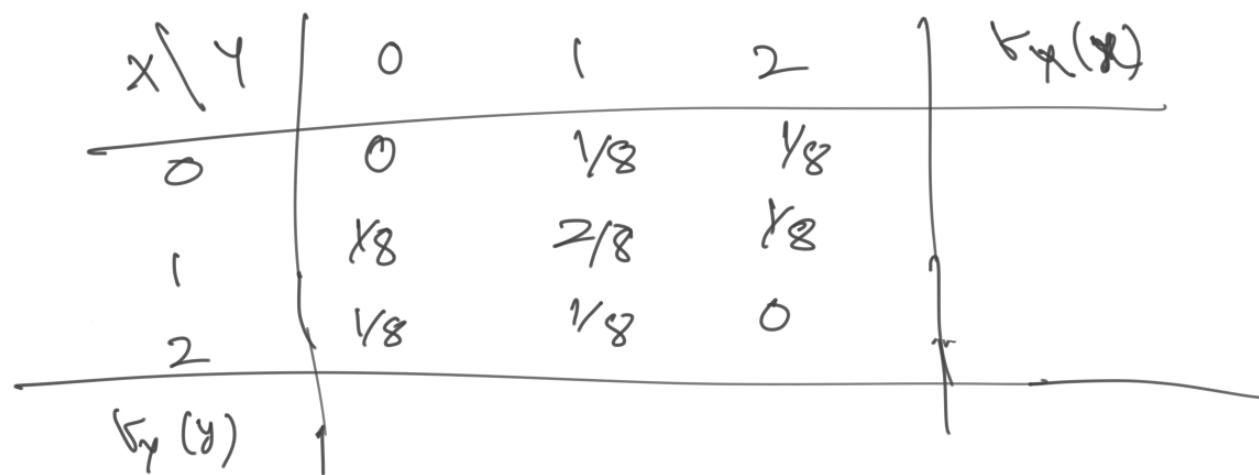
X & Y  
not ind.

Three fair coins are tossed.  $X$  denotes the number of heads on the 1st two coins and  $Y$  denotes the number of tails on the last two coins.

- a) Find the joint distribution of  $X$  and  $Y$
- b) Find the conditional distribution of  $Y$  given  $X=1$
- c) Find  $\text{cov}(X, Y)$
- d) Find  $f(x, y)$

	HHH	HHT	HTH	THH	THT	TTH	HTT	TTT
X :	2	2	1	1	1	0	1	0
Y :	0	1	1	0	1	1	2	2

① Joint distribution of  $X$  &  $Y$  is



$X \setminus Y$	0	1	2	$f_{X,Y}(x,y)$
0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
1	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{2}$
2	$\frac{1}{8}$	$\frac{1}{8}$	0	$\frac{1}{4}$
$f_Y(y)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1

①

Joint  
distribution

of  $X \& Y$



② conditional dist. of  $Y$  when  $x=1$

$$P(Y=0|x=1) = \frac{P(Y=0 \cap X=1)}{P(X=1)}$$

$$= \frac{1/8}{1/2}$$

$$= \frac{1}{4}$$

$$P(Y=1|x=1) = \frac{P(Y=1 \cap X=1)}{P(X=1)} = \frac{2/8}{1/2} = \frac{1}{4}$$

$$P(Y=2/X=1) = \frac{P(Y=2 \cap X=1)}{P(X=1)}$$

$$= \frac{\gamma_2}{\gamma_2}$$

$$= \gamma_4$$

∴ Conditional dist. of  $Y$  given  $X=1$

$$f_{Y|X}(y|x=1) = \begin{cases} \frac{1}{4}, & y=0 \\ \frac{1}{2}, & y=1 \\ \frac{1}{4}, & y=2 \end{cases}$$

③

$$\text{Cov}(x, y) = E[xy] - E[x]E[y]$$

$$E[x] = \sum x f_x(x)$$

$$= (0 \times \frac{1}{4}) + (1 \times \frac{1}{2}) + (2 \times \frac{1}{4})$$

$$= 1$$

$$E[y] = \sum y f_y(y)$$

$$= (0 \times \frac{1}{4}) + (1 \times \frac{1}{2}) + (2 \times \frac{1}{4})$$

$$= 1$$

$$E[XY] = \sum xy f_{X,Y}(x,y)$$

$$= 0 + (1 \times \frac{2}{8}) + (2 \times \frac{1}{8}) + (2 \times \frac{1}{8}) + (4 \times 0)$$

$$= \frac{2}{8} + \frac{2}{4}$$

$$= \frac{1}{4} + \frac{1}{2}$$

$$= \frac{3}{4}$$

$$\therefore \text{Cov}(X,Y) = E[XY] - E[X]E[Y] = \frac{3}{4} - 1 = -\frac{1}{4}$$

$$(d) \quad \rho(x, y) = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$$

$$\text{Var}(x) = E[x^2] - (E[x])^2 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\Rightarrow \sigma_x = \sqrt{\text{Var}(x)} = \frac{1}{\sqrt{2}}$$

$$\text{Var}(y) = E[y^2] - (E[y])^2 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\Rightarrow \sigma_y = \sqrt{\text{Var}(y)} = \frac{1}{\sqrt{2}}$$

$$\therefore \rho(x, y) = \frac{-y_4}{\sqrt{2} \cdot \sqrt{2}} = \frac{-y_4}{2} = -\frac{1}{2}$$

Q. Consider a sample of size 2 drawn without replacement from an urn containing three balls numbered 1, 2 and 3. Assume  $X$  is the smaller of the two numbers drawn and  $Y$  the larger.

- (a) Find the joint discrete density  $f^{xy}$  of  $X$  &  $Y$
- (b) Find the conditional dist. of  $Y$  when  $X=1$
- (c) Find  $f(x,y)$

## Joint Continuous Density Function

A two dimensional R.V.  $(X, Y)$  is said to be continuous iff  $\exists$  a  $f^n$   $f_{x,y}(x,y) \geq 0$  s.t.

$$F_{x,y}(u,y) = \int_{-\infty}^u \int_{-\infty}^y f(u,v) du dv$$

$f_{x,y}(x,y)$  or  $f(x,y)$  is called Joint Probability density  $f^n$ .

## Properties

$$\textcircled{1} \quad f(x,y) \geq 0 \quad \forall (x,y) \in \mathbb{R}^2$$

$$\textcircled{2} \quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dx dy = 1.$$

$$\text{Note : } F_{x,y}(x,y) = P(X \leq x, Y \leq y)$$

Marginal and conditional Probability Density Function

Marginal —

$$f_x(x) = \int_{-\infty}^{\infty} f(x,y) dy$$

$$f_y(y) = \int_{-\infty}^{\infty} f(x,y) dx$$

$X, Y$  ↗  
Joint Continuous  
Random Variable

Conditional —

$$f_{Y|X}(y|x) = \frac{f_{x,y}(x,y)}{f_x(x)}, \quad f_x(x) > 0$$

$$f_{X|Y}(x|y) = \frac{f_{x,y}(x,y)}{f_y(y)}, \quad f_y(y) > 0$$

## Conditional Cumulative Distribution

$$F_{Y|X}(y|x) = \int_{-\infty}^y f_{Y|X}(z|x) dz$$

$$F_{X|Y}(x|y) = \int_{-\infty}^x f_{X|Y}(z|y) dz$$

## Expectation

$$E[g(x,y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y) dx dy$$

$$E[x] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f(x,y) dx dy$$

$$E[y] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f(x,y) dx dy$$

$$E[x^y] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f(x,y) dx dy$$

## Joint Moment Generating $F^n$

The joint moment generating  $f^n$  of a two dimensional random variable  $(X, Y)$  denoted by

$m_{x,y}(t_1, t_2)$  is defined as

$$m_{x,y}(t_1, t_2) = \mathbb{E}[e^{t_1 X + t_2 Y}]$$

where  $t_1$  &  $t_2$  are real parameters

Q. Find  $k$  so that  $f(x,y) = kxy$ ,  $1 \leq x \leq y \leq 2$   
 will be a joint probability density  $f^n$ .

For  $f(x,y)$  to be joint prob. density  $f^n$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) = 1$$

$$\Rightarrow \int_1^2 \int_x^2 kxy \, dy \, dx = 1 \quad \rightarrow k \int_1^2 x \left( \int_x^2 y \, dy \right) \, dx = 1.$$

$$\rightarrow k \int_1^2 x \left[ \frac{y^2}{2} \right]_x^2 \, dx = 1$$

$$k \int_1^2 \left( \frac{x}{2} - \frac{x^2}{2} \right) dx = 1$$

$$\Rightarrow k \int_1^2 \left( \frac{x^3}{2} - \frac{x^3}{2} \right) dx = 1$$

$$\Rightarrow k \left\{ 2 \left[ \frac{x^2}{2} \right]_1^2 - \frac{1}{2} \left[ \frac{x^4}{4} \right]_1^2 \right\} = 1$$

$$\Rightarrow \frac{9k}{8} = 1$$

$$\Rightarrow k = \frac{8}{9}$$

Q. Find K so that  $f(x, y) = K(x+y)$ ,  $0 < x < 1$  and  $0 < y < 1$   
 is a joint probability density f.

$\therefore f(x, y)$  is a joint probability density f

$$\therefore \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$$

$$\Rightarrow \int_0^1 \int_0^1 K(x+y) dx dy = 1$$

$$\Rightarrow K \int_0^1 \left[ xy + \frac{y^2}{2} \right]_0^1 dx = 1$$

$$\Rightarrow k \int_0^1 (x + \frac{1}{2} - 0) dx = 1$$

$$\Rightarrow k \int_0^1 (x + \frac{1}{2}) dx = 1$$

$$\Rightarrow k \left[ \frac{x^2}{2} + \frac{1}{2}x \right]_0^1 = 1$$

$$\Rightarrow k \left[ \frac{1}{2} + \frac{1}{2} - 0 \right] = 1$$

$$\Rightarrow k = 1$$

Q. Let the joint p.d.f. of  $X$  and  $Y$  be

$$f(x,y) = \begin{cases} (x+y), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

find ①  $P[0 < X < \frac{1}{2}, 0 < Y < \frac{1}{4}]$

②  $E[X]$

③  $E[X, Y]$

④  $E[Y]$

⑤  $E[XY]$

⑥  $E[X+Y]$

$$= \int \int x f(x,y) dx dy$$

$$P[0 < X < \frac{1}{2}, 0 < Y < \frac{1}{4}] = \int_0^{1/2} \left[ \int_0^{1/4} (x+y) dy \right] dx$$

$$= \int_0^{1/2} \left[ xy + \frac{y^2}{2} \right]_0^{1/4} dx$$

$$= \int_0^{1/2} \left[ \frac{x}{4} + \frac{1}{32} - 0 \right] dx$$

$$= \int_0^{1/2} \left( \frac{x}{4} + \frac{1}{32} \right) dx$$

$$= \left[ \frac{x^2}{8} + \frac{x}{32} \right]_0^{1/2} = \left[ \frac{1/4}{8} + \frac{1/2}{32} - 0 \right] = \frac{3}{64}$$

$$E[x] = \int_0^1 \int_0^1 x f(x,y) dndy$$

$$= \int_0^1 \int_0^1 x(x+y) dndy$$

$$= \int_0^1 \int_0^1 (x^2 + xy) dndy$$

$$= \int_0^1 \left\{ \int_0^1 (x^2 + xy) dy \right\} dx$$

$$= \int_0^1 \left[ x^2 y + \frac{xy^2}{2} \right]_0^1 dx$$

$$E[X] = \int_0^1 \left( x + \frac{x}{2} - 0 \right) dx$$

$$= \left[ \frac{x^3}{3} + \frac{x^2}{4} \right]_0^1$$

$$= \frac{1}{3} + \frac{1}{4} - 0$$

$$= \frac{7}{12}$$

$$E[Y] = \int_0^1 \int_0^1 y f(x, y) dx dy = ?$$

$$E[XY] = \int_0^1 \int_0^1 xy (x+y) dx dy$$

$$= \int_0^1 \int_0^1 (x^2 y + xy^2) dx dy$$

$$= \int_0^1 \left\{ \int_0^1 (x^2 y + ny^2) dy \right\} dx$$

$$= \int_0^1 \left[ \frac{nx^2 y^2}{2} + \frac{ny^3}{3} \right]_0^1 dx$$

$$E[X^4] = \int_0^1 \left( \frac{x^2}{2} + \frac{y}{3} - 0 \right) dx$$

$$= \left[ \frac{x^3}{6} + \frac{y^2}{2} \right]_0^1$$

$$= \frac{1}{6} + \frac{1}{6} - 0$$

$$= \frac{1}{6}$$

$$= \frac{1}{3}$$

$$E[X+Y] = \int_0^1 \int_0^1 (x+y)(x+y) dx dy$$

$$= \int_0^1 \int_0^1 (x+y)^2 dx dy$$

= solve this

or

$$E[X+Y] = E[X] + E[Y] = \frac{7}{12} + \frac{7}{12} - \frac{7}{6}$$

$$\rho(x, y) = \frac{\text{cov}(x, y)}{\sqrt{\text{var} x} \sqrt{\text{var} y}}$$

$$\text{cov}(x, y) = E[xy] - E[x]E[y]$$

$$= \frac{1}{3} - \frac{7}{12} \cdot \frac{7}{12}$$

$$= \frac{1}{3} - \frac{49}{144}$$

$$= -\frac{1}{144}$$

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

$$E[X^2] = \iint_0^1 x^2(x+y) dx dy = \frac{5}{12}$$

$$E[Y^2] = \frac{5}{12} \quad \uparrow$$

calculation

$$\text{Var}(X) = \frac{5}{12} - \left(\frac{7}{12}\right)^2 = \frac{11}{144}$$

$$\text{Var}(Y) = \frac{11}{144}$$

$$f(x,y) = \frac{-\chi_{yy}}{\sqrt{\frac{11}{144}}} \sqrt{\frac{11}{144}} = -\frac{1}{11}$$

Q. The probability density function of a continuous bivariate distribution is given

$$f(x,y) = x+y, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1 \\ = 0, \quad \text{otherwise.}$$

Find correlation coefficient of  $x$  &  $y$ .