LAPLACE TRANSFORM

Introduction
Region of Convergence (ROC)
Properties of Laplace Transform.
Inverse Laplace Tx.

Introduction to Laplace Tx:-

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further impreavement to Fourier Transform and Fourier Series tech-niques.

- F.T. do not exist for many functions.

- F.T. do not exist for non absolutely integrable s/g but L.T. exists for non absolutely integrable s/g.

- F.T. cant be used to analyse unstable systems.

- An alternate approach is to redefine the transform and include an exponential convergence factor 'σ' along with $j\omega$. That is the s/g $x(t)$ is represented as a sum of complex exponentials of the form e^{st} where S is complex frequency denoted by $s = \sigma + j\omega$.

$$x(t) = G_1 . e^{st}$$

$S \rightarrow$ S-plane

$= \sigma + j\omega$.

$$\boxed{L[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-st} dt.}$$ $\boxed{s = \sigma + j\omega}$

$S \rightarrow$ complex variable/ complex freq.

$\sigma \rightarrow$ damping factor (tells us about stability)

$j\omega \rightarrow$ angular freq (rad/s).

Inverse LT:$\rightarrow$ $f(t) \leftrightarrow F(s)$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} F(s) e^{st} ds$$

BILATERAL L.T. :→

which we integrate from $-\infty$ to ∞

ie. $$\boxed{F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt.}$$

UNILATERAL L.T. :→

if we have a time domain s/g $f(t)$ defined by $t \geq 0$, then the L.T.

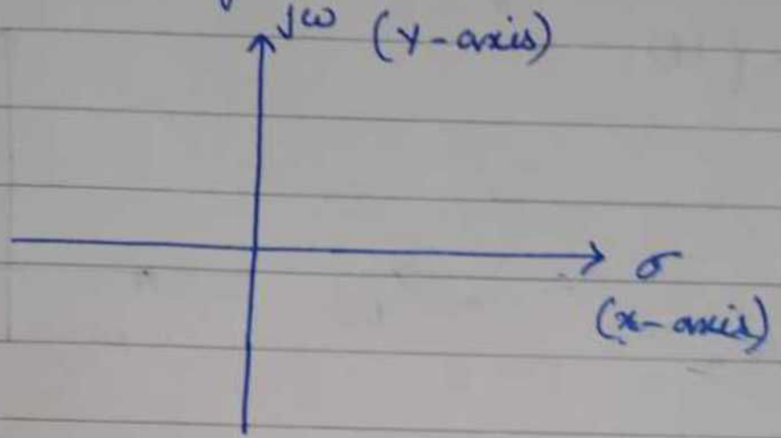
$$\boxed{F(s) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt}$$

REGION OF CONVERGENCE (ROC) :→

ROC is the region in the S-plane.
'S' is having two variables 'σ' & 'ω'. and when we include all the possible values of σ and $j\omega$, we get a plane which is the S-plane. And, the Region in which the Laplace transform is finite is called ROC. So, outside ROC, LT is infinite.

ROC & its properties :→

It is the range of complex variable 's' in s-plane for which Laplace Transform is finite or convergent.



Properties of ROC :→

① ROC does not include any poles.

eg:- $F(s) = \frac{1}{s+2}$

To calculate pole we will equate $s+2$ to '0'.

$\therefore s+2 = 0 \Rightarrow s = -2$

-2 is the pole of the system with transfer f^n $\frac{1}{s+2}$

Now lets include $s=-2$ in ROC,

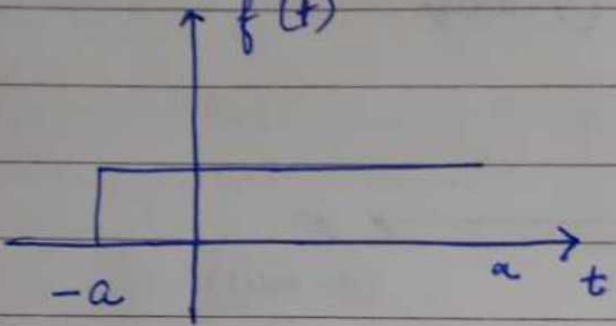
$\therefore F(-2) = \frac{1}{-2+2} = \frac{1}{0} = \infty$

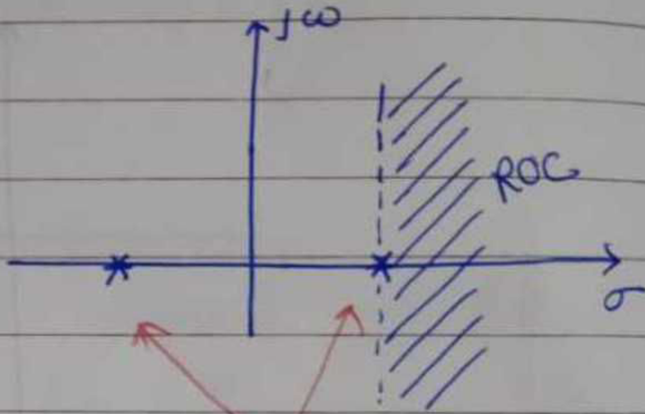
But acc. to def^n, L.T. is finite for ROC.

$\therefore$ we cant include poles is ROC.

② For right sided s/g, ROC is right side to the rightmost pole.

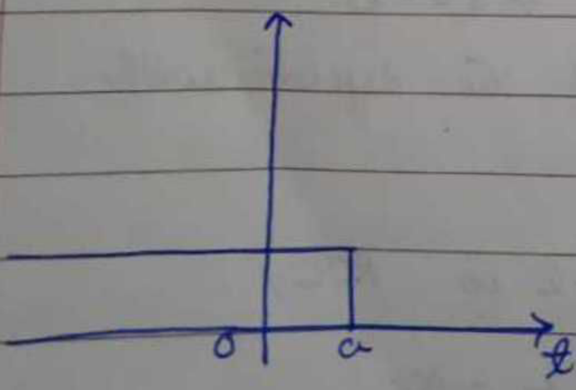
which extend from a finite value of time to infinite.

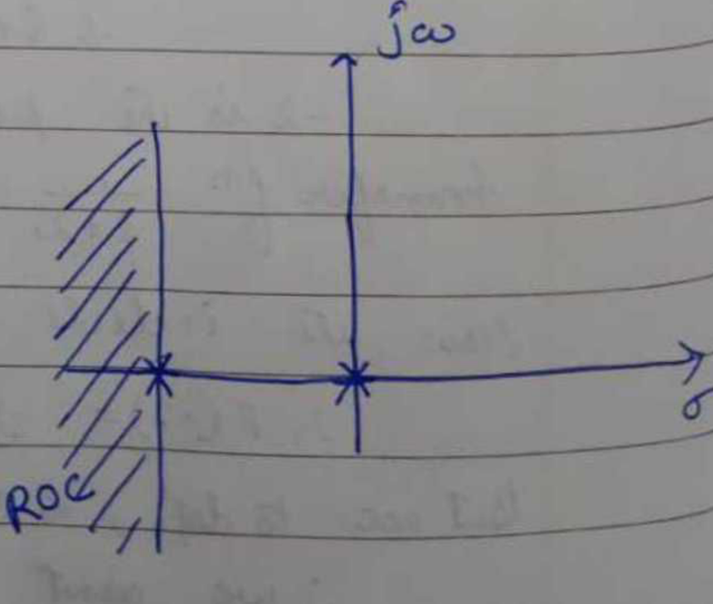




Acc. to the t.f. if we get two poles, among the poles ROC is the right side of the rightmost pole.

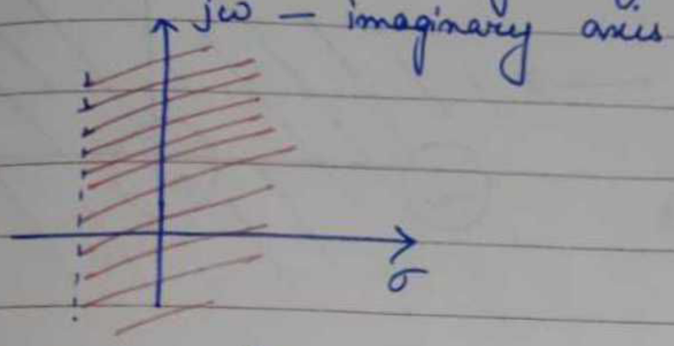
③ For left sided s/g ROC is the left side to the leftmost pole.





④ For ~~separate~~ the absolute integrability of a s/g or the stability of a system, ROC should include imaginary axis.

(i)



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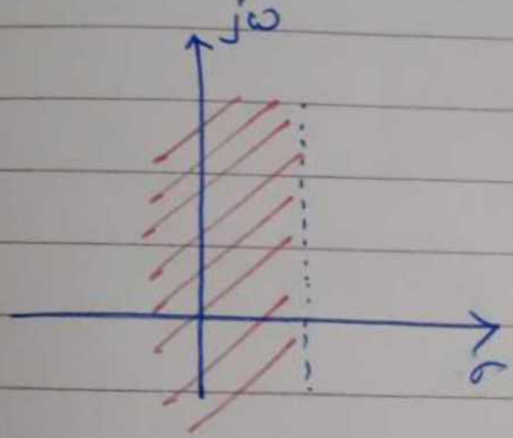

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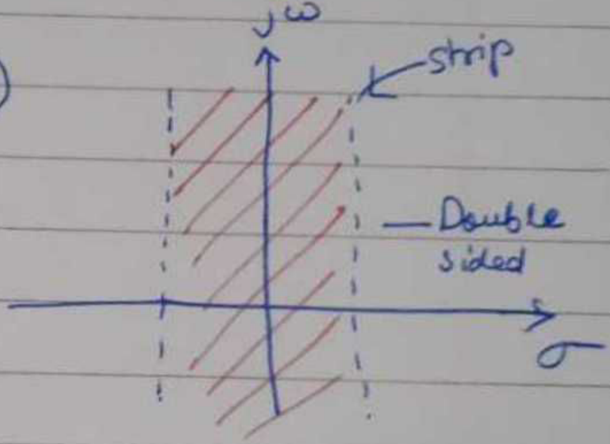
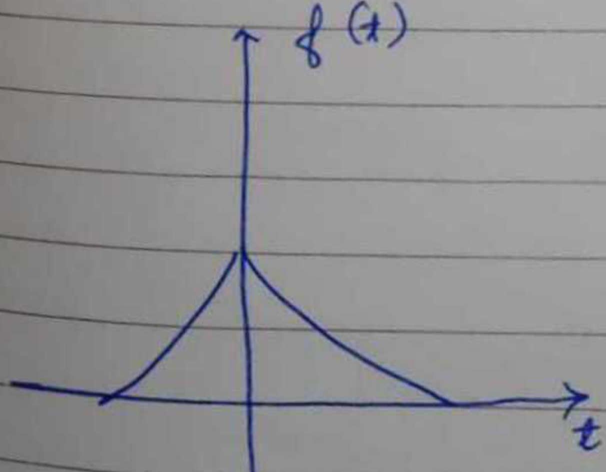
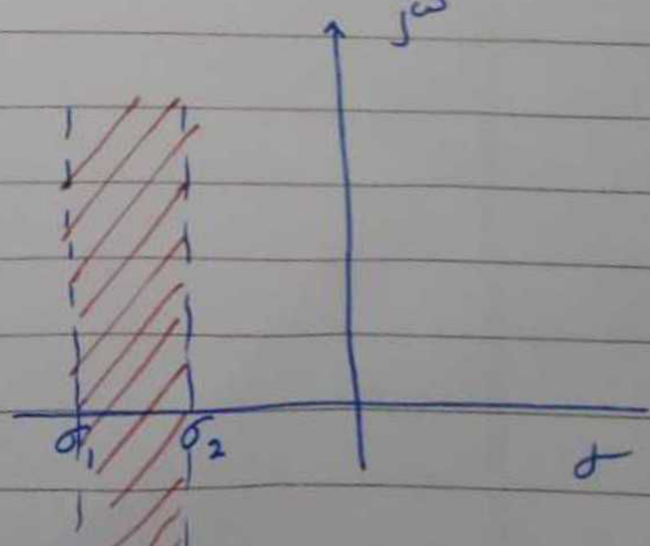


⑤ For both sided s/g, ROC is a strip in the s-plane.





6. For finite duration s/g, ROC is the entire S-plane excluding $s=0$ &/or $+\infty$ &/or $-\infty$.