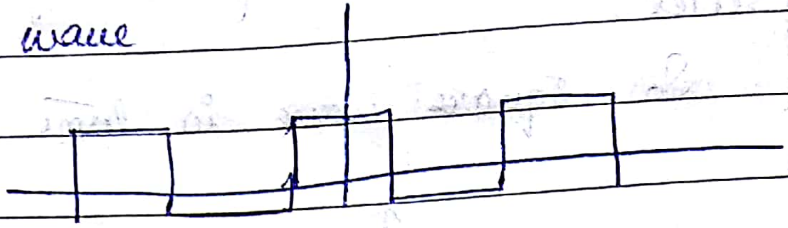
FOURIER TRANSFORM

Take a square wave

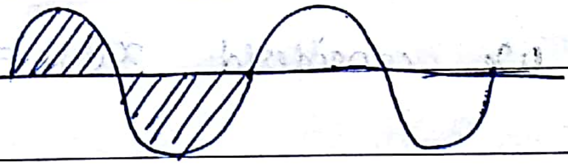


$$x(t) = \sum_{n=-\infty}^{\infty} |C_n| \, e^{jn\omega_0 t}$$

$$|C_n| = \begin{cases} \frac{2A}{\pi n} , & n = odd. \\ 0 , & n = even. \end{cases}$$

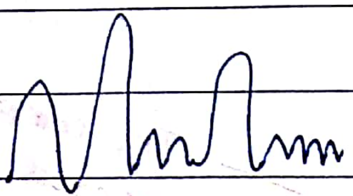
Frequency / spectrum

Time period :- Duration of time ~~is~~ of one cycle in a repeating unit.



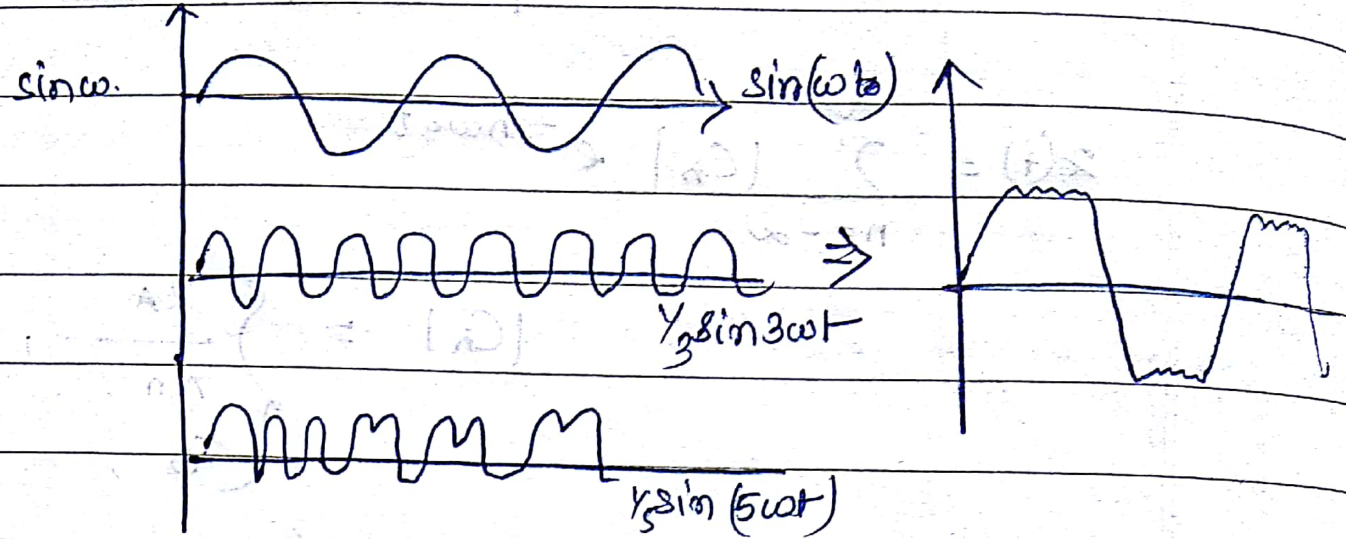
Frequency :- No. of cycles per unit time.

$$F = \frac{1}{T}$$



→ this s/g does not have time period but it has a frequency range.

So square wave in terms of sine →

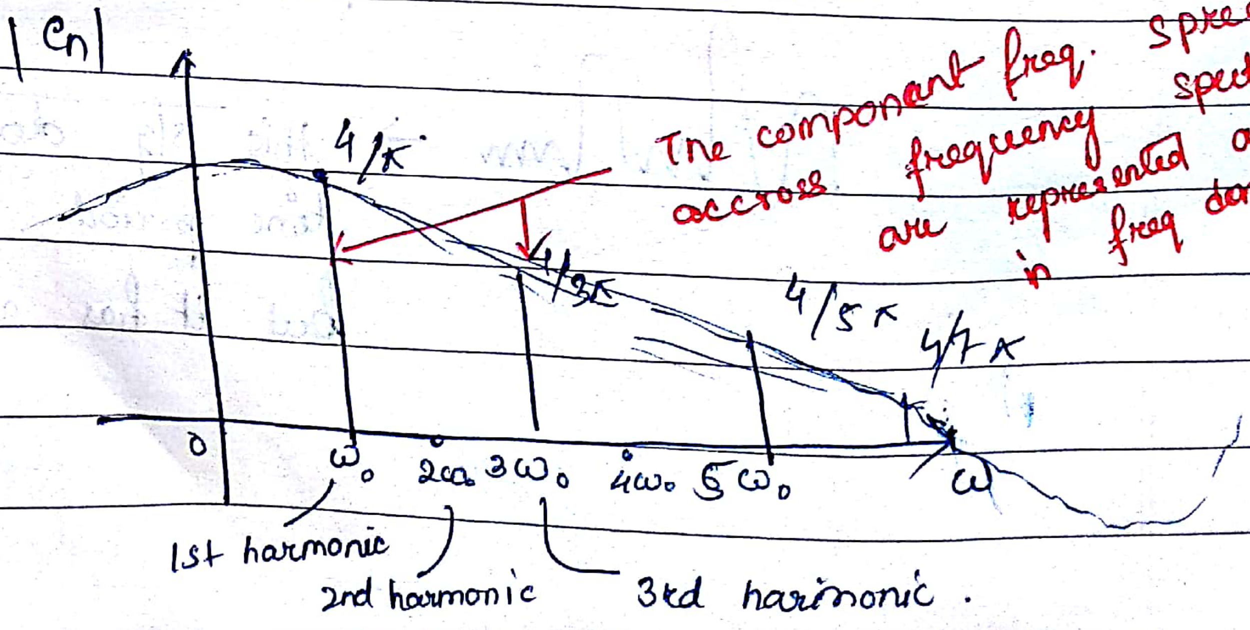


ultimately we will get a square wave [after seeing / observing the values of sin wave for diff. values of n & changing values of time period.]

We considerld $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_0 n t}$

$|C_n| = \propto \frac{4}{\pi n}$, n = odd

0 , n = even.

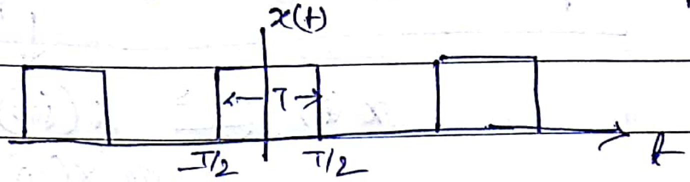


The component freq. spread accross frequency spectrum are represented as peaks in freq domain.

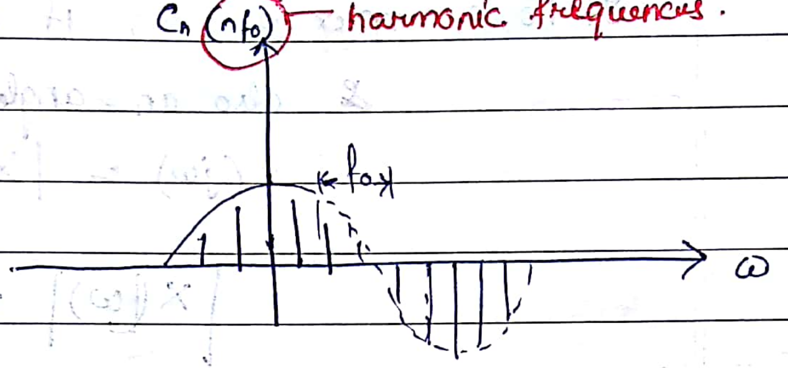
Frequency spectrum is the graphical display of the harmonic content of the s/g. $|C_n|$ is the mag. of the harmonics & plotted against ω.

As frequency increases, Harmonic magnitude decreases.

So what happens in Fourier Tx is that suppose a s/g $x(t)$



In frequency domain we can draw / plot the s/g as



$\frac{2\pi}{T/2}$

$2\frac{T}{2}$

$\frac{2\pi}{~~T~~}$ T

If we further increase the time period the freq will decrease the space betⁿ the spectral lines will become lesser & lesser and at one point infinity. At the same time,

→ Fourier T_x is used for frequency analysis of s/g.

→ Representation:–

$$x(t) \rightleftharpoons \overset{\text{rad/s}}{X(j\omega)} \text{ or } \overset{\text{Hz}}{X(f)}$$

$$\text{or } X(\omega)$$

It is a complex no. So, it will have a magnitude & also an angle.

$$X(j\omega) = |X(j\omega)| \angle X(j\omega).$$

$$|X(\omega)| =$$

$$\& \quad \angle X(\omega) = \tan^{-1}$$

Conditions for Existence of Fourier Tx (Dirichlet Cond^n):–

*1. S/g should have finite no. of maxima & minima over any finite interval.

*2. S/g should have finite no. of discontinuities over any finite interval.

*3. S/g should be absolutely integrable.

These are sufficient but not necessary cond^n.

eg:- $x(t) = u(t)$ → Power s/g which is not absolutely integrable.

Sol^n $$X(j\omega) = \int_{-\alpha}^{\alpha} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\alpha}^{\alpha} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\alpha}^{0} 0 . e^{-j\omega t} dt + \int_{0}^{\alpha} 1 . e^{-j\omega t}\, dt$$

$$= 0 + \frac{\left(e^{-j\omega t}\right)_0^{\alpha}}{-j\omega}$$

$$= \frac{e^{-j\omega\alpha} - e^{-j\omega t 0}}{-j\omega}$$

$$= \frac{e^{-j\alpha} - 1}{-j\omega}$$

$$= \frac{\underbrace{\cos\alpha - j\sin\alpha}_{\text{not defined}} - 1}{-j\omega}$$

∴ $X(j\omega)$ also not defined for using the formula we will use diff^n properties to find its F.T.

Formulae :-

F.T. pair

① $x(t) \longrightarrow X(j\omega)$

$$X(j\omega) = \int_{-\alpha}^{\alpha} x(t).e^{-j\omega t}\, dt \quad — ①$$

← F.T.

② $X(j\omega) \longrightarrow x(t)$

$$x(t) = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} X(j\omega).e^{j\omega t}\, d\omega \quad — ⑪$$

← Inverse F.T

* Properties

① Linearity :-

$$x_1(t) \overset{F.T.}{\rightleftharpoons} X_1(j\omega)$$

$$x_2(t) \rightleftharpoons X_2(j\omega)$$

$$\alpha x_1(t) \rightleftharpoons \alpha X_1(j\omega)$$

$$\beta x_2(t) \rightleftharpoons \beta X_2(j\omega)$$

If we add then the transformed s/g also get added

$$\alpha x_1(t) + \beta x_2(t) \rightleftharpoons \alpha X_1(j\omega) + \beta X_2(j\omega).$$

Pf :- $x(t) \rightleftharpoons X(j\omega)$

$$X(j\omega) = \int_{-\alpha}^{\alpha} x(t) e^{-j\omega t}\, dt$$

If $\alpha x_1(t) + \beta x_2(t) = x(t)$

$$\therefore x(j\omega) = \int_{-\alpha}^{\alpha} \left[\alpha x_1(t) + \beta x_2(t)\right] e^{-j\omega t}\, dt$$

$$= \alpha \int_{-\alpha}^{\alpha} x_1(t)\, e^{-j\omega t}\, dt + \beta \int_{-\alpha}^{\alpha} x_2(t)\, e^{-j\omega t}\, dt$$

$$= \alpha X_1(j\omega) + \beta X_2(j\omega).$$

$$\rightleftharpoons x(t)$$

$$= \alpha x_1(t) + \beta x_2(t)$$