Table 2 : SIGNIFICANT VALUES t_v (α) OF t-DISTRIBUTION (TWO TAIL AREAS) $[\,|\,t\,| > t_v(\alpha)] = \alpha$

d.f. (ν)	Probability (Level of Significance)					
	0.50	*0.10*	*0.05*	*0.02*	*0.01*	*0.001*
1	1.00	6.31	12.71	31.82	63.66	636.62
2	0.82	0.92	4.30	6.97	6.93	31.60
3	0.77	2.32	3.18	4.54	5.84	12.94
4	0.74	2.13	2.78	3.75	4.60	8.61
5	0.73	2.02	2.57	3.37	4.03	6.86
6	0.72	1.94	2.45	3.14	3.71	5.96
7	0.71	1.90	2.37	3.00	3.50	5.41
8	0.71	1.80	2.31	2.90	3.36	5.04
9	0.70	1.83	2.26	2.82	3.25	4.78
10	0.70	1.81	2.23	2.76	3.17	4.59
11	0.70	1.80	2.20	2.72	3.11	4.44
12	0.70	1.78	2.18	2.68	3.06	4.32
13	0.69	1.77	2.16	2.05	3.01	4.22
14	0.69	1.76	2.15	2.62	2.98	4.14
15	0.69	1.75	2.13	2.60	2.95	4.07
16	0.69	1.75	2.12	2.58	2.92	4.02
17	0.69	1.74	2.11	2.57	2.90	3.97
18	0.69	1.73	2.10	2.55	2.88	3.92
19	0.69	1.73	2.09	2.54	2.86	3.88
20	0.69	1.73	2.09	2.53	2.85	3.85
21	0.69	1.72	2.08	2.52	2.83	3.83
22	0.69	1.72	2.07	2.51	2.42	3.79
23	0.69	1.71	2.07	2.50	2.81	3.77
24	0.69	1.71	2.06	2.49	2.80	3.75
25	0.68	1.71	2.06	2.49	2.79	3.73
26	0.68	1.71	2.06	2.48	2.78	3.71
27	0.68	1.70	2.05	2.47	2.77	3.69
28	0.68	1.70	2.05	2.47	2.76	3.67
29	0.68	1.70	2.05	2.46	2.76	3.66
30	0.68	1.70	2.04	2.46	2.75	3.65
∞	0.67	1.65	1.96	2.33	2.58	3.29

t – Test

Student's t- Distribution

t - distribution is used for a sample size (≤ 30) and when the population standard deviation is unknown

t - statistic is defined as $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} \sim t\,(n-1 \text{ d.f.})$

where $s = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n-1}}$

Test 1: t - Test of Significance of the mean of a random sample.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$s^2 = \dfrac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$ with d.f. $(n-1)$

Note

(i) 95% confidence limits (level of significance 5%) are

$$\bar{x} \pm t_{0.05}\left(\frac{s}{\sqrt{n}}\right)$$

(ii) 99% confidence limits (level of significance 1%) are

$$\bar{x} \pm t_{0.01}\left(\frac{s}{\sqrt{n}}\right)$$

Q. A random sample of size 16 has 53 as mean. The sum of squares of ~~derivation~~ deviation from mean is 135. Can this sample be regarded to be taken from the population having 56 as mean? Obtain 95% and 99% confidence limits of the mean of the population.

Soln H_0: There is no significant difference between the sample mean and hypothetical population mean.

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t\,(n-1\ d.f.)$$

Given,

$\bar{x} = 53$

$\mu = 56$

$n = 16$

sum of square of deviation from mean $\Sigma (x - \bar{x})^2$

$\Sigma (x - \bar{x})^2 = 135$

$$s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}} = \sqrt{\frac{135}{15}} = 3$$

$$t = \frac{53 - 56}{3/\sqrt{16}} = -4$$

$\Rightarrow |t| = 4$

d.f. $= n - 1 = 16 - 1 = 15$

<u>Conclusion</u> :

$t_{0.05} = \cancel{1.753}$ 2.13

$\because |t| = 4 > t_{0.05} = \cancel{1.753}$ 2.13 (Use t-Test chart)

$\Rightarrow H_0$ is rejected.

Hence, the sample mean has not come from a populn having mean 56

95% confidence limits of the population mean

$$= \bar{X} \pm \frac{s}{\sqrt{n}} t_{0.05}$$

$$= 53 \pm \frac{3}{\sqrt{16}} (\cancel{1.725}) \quad 2.13$$

$$= \cancel{51.706 \text{ \& } 54.293}$$

99% confidence limits of the population mean

$$= \bar{X} \pm \frac{s}{\sqrt{n}} t_{0.01}$$

$$= 53 \pm \frac{3}{\sqrt{16}} (\cancel{2.602}) \quad 2.95$$

$$= \cancel{51.048 \text{ \& } 54.951}$$

Q. The following results are obtained from a sample of 10 boxes is biscuit.

Mean weight content = 490 gm

S.D. of the weight = 9 gm. Could the sample come from a population having a mean of 500 gm?

Q. A random sample of 10 boys had the I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 160?

Q. Ten individuals are chosen at random from a normal population of students and their grades are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 72. In the light of these data discuss the suggestion that the mean grade of the students is 66.

Soln: H_0 : There is no significant difference betn $\bar{x}$ and μ i.e. $\bar{x} = \mu$

H_1 : There is a significant difference betn $\bar{x}$ and μ i.e. $\bar{x} \neq \mu$.

Here,

$$\bar{x} = \frac{63+63+66+67+68+69+70+70+71+72}{10}$$

$$= 67.9$$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
63	−4.9	24.01
63	−4.9	24.01
66	−1.9	3.61
67	−0.9	0.81
68	0.1	0.01
69	1.1	1.21
70	2.1	4.41
70	2.1	4.41
71	3.1	9.61
72	4.1	16.81
		$\Sigma = 88.9$

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = \frac{88.9}{9} = 9.87$$

$$\Rightarrow s = \sqrt{9.87} = 3.142$$

Now,

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t\,(10-1 \text{ d.f})$$

$$= \frac{67.9 - 66}{3.142} \sqrt{10}$$

$\therefore\ t = 1.912$

$\Rightarrow |t| = 1.912$

Again, $t_{0.05} = 2.26$

$\therefore\ |t| = 1.912 < 2.26$

$\Rightarrow H_0$ is accepted

TEST 2 : t – Test for difference of means of two small samples

$$t = \frac{\bar{X} - \bar{Y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t\,(n_1 + n_2 - 2 \text{ d.f})$$

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$; $s_1, s_2 \leftarrow$ sample S.D.

Q. Two samples of sodium vapour bulbs were tested for length of life and the following results were obtained

	Size	Sample Mean	Sample S.D.
Type I	8	1234 hrs	36 hrs
Type II	7	1036 hrs	40 hrs

Is the difference in the means significant to generate that type I is superior to type II regarding the life.

<u>Soln</u>: H_0 : $\mu_1 = \mu_2$ i.e. two types of bulbs have same lifetime.

H_1 : $\mu_1 > \mu_2$ i.e. type I is superior to type II.

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{8(36)^2 + 7(40)^2}{8+7-2} = 1659.076$$

$$\Rightarrow s = 40.7317$$

Now,

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{1234 - 1036}{40.7317\sqrt{\frac{1}{8} + \frac{1}{7}}}$$

$$= 18.1480$$

Again,

$$n_1 + n_2 - 2 = 15 - 2 = 13 \text{ d.f.}$$

$t_{0.05}$ at 13 d.f. $= 1.77$ (one failed test)

Conculsion :

$$|t| = 18.1480 > t_{0.05}$$

$\Rightarrow$ H_0 is rejected and H_1 is accepted

Q. The mean life of 10 electric motors was found to be 1450 hrs with S.D. of 423hrs. A second sample of 17 motors chosen from a different batch showed a mean life of 1280 hrs with a S.D. of 398 hrs. Is there a significant difference between means of two samples.