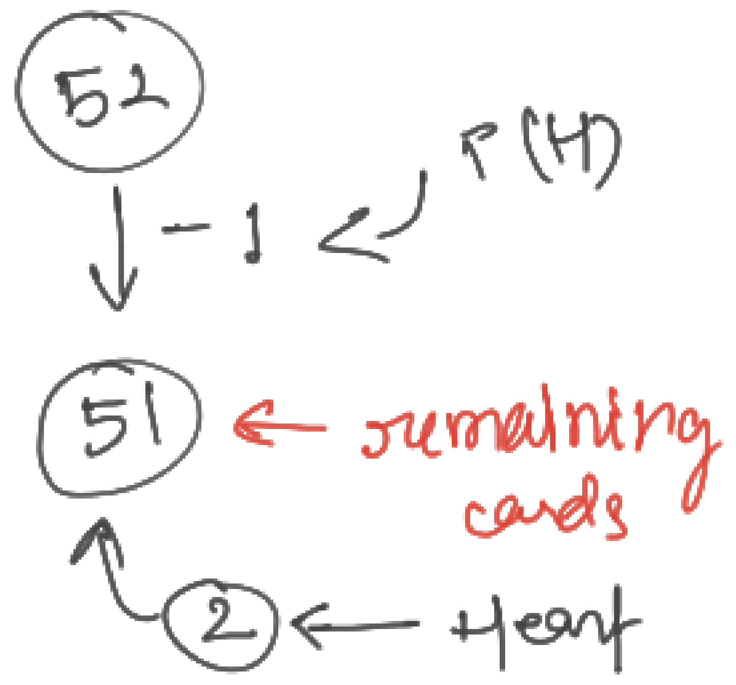
Q. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be heart. Find the probability of the missing card to be a heart.



Solⁿ: let A = two cards drawn from the remaining cards are heart.

E_1 = the missing card is heart

E_2 = " " " " diamond

E_3 = the missing card is club

E_4 = the missing card is spade.

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + P(E_4)P(A/E_4)} \quad \text{--- (1)}$$

Here,

$P(E_1)$ = Probability that the missing card is heart $= \frac{^{13}C_1}{^{52}C_1}$

$= \frac{13}{52} = \frac{1}{4}$

$P(E_2) = \ldots \quad = \ldots \quad = \frac{1}{4}$

$P(E_3) = \ldots \quad = \ldots \quad = \frac{1}{4}$

$P(E_4) = \ldots \quad = \ldots \quad = \frac{1}{4}$

$P(A/E_1)$ = Probability that two cards drawn are heart given that the missing card is heart

$$= \frac{^{12}C_2}{^{51}C_2}$$

$P(A/E_2)$ = Prob. that two cards drawn are heart given that the missing card is diamond

$$= \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$P(A/E_3) = \dots = \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$P(A/E_4) = \dots = \frac{{}^{13}C_2}{{}^{51}C_2}$$

from ①

$$P(E_1/A) = \frac{\frac{1}{4} \cdot \frac{{}^{12}C_2}{{}^{51}C_2}}{\frac{1}{4} \cdot \frac{{}^{12}C_2}{{}^{51}C_2} + \frac{1}{4} \frac{{}^{13}C_2}{{}^{51}C_2} + \frac{1}{4} \frac{{}^{13}C_2}{{}^{51}C_2} + \frac{1}{4} \cdot \frac{{}^{13}C_2}{{}^{51}C_2}} = ?$$

Random Variable

A random variable is a rule that assigns a real number to each outcome of a random experiment. It is usually a f^n which is denoted by X.

eg: Tossing of a pair of coins

Define X = no. of tails

$$S = \{HH, HT, TH, TT\}$$

$\therefore$ X takes the value $0, 1, 2$ i.e. $X = 0, 1, 2$

<u>Note</u> : Random variable is also called stochastic variable or variate.

- Random Variable (R.V.)
  - Discrete R.V.
  - Continuous R.V.

Discrete Probability Distribution

If a random variable X have discrete set of values say $x_1, x_2, \ldots, x_n$ with respect to the probabilities $p_1, p_2, \ldots, p_n$ s.t. $\sum_i p_i = 1$ then occurrences of values x_i with respective probabilities p_i is called discrete probability distribution of X.

eg: If X denotes no. of tails in tossing of a pair of coins then probability distribution is given by

$S = \{HH, HT, TH, TT\}$

$X = 0, 1, 2$

$P(X=0)$ = Prob. that no. of tails is zero $= \frac{1}{4}$

$P(X=1)$ = " " " " " " one $= \frac{2}{4} = \frac{1}{2}$

$P(X=2)$ = " " " " " " two $= \frac{1}{4}$

X	0	1	2
$P(X)$	$1/4$	$1/2$	$1/4$

← Discrete Prob. Distribution