LINEAR & NON-LINEAR SYSTEMS:-

A system which supports/obeys the principle of homogeneity and principle of superposition is called a linear system, and a system which does not obey the principle of superposition and homogeneity is called a non-linear system.

Homogeneity property means a system that produces an output $y(t)$ for an input $x(t)$ must produce an output $ay(t)$ for an input $ax(t)$.

Superposition property means a system which produces an output $y_1(t)$ for input $x_1(t)$ and an output $y_2(t)$ for input $x_2(t)$ must produce an output $y_1(t) + y_2(t)$ for input $x_1(t)+x_2(t)$.

Combining them we can say that a system is linear if,

an arbitrary i/p $x_1(t) \longrightarrow y_1(t)$

and, " $x_2(t) \longrightarrow y_2(t)$,

then the weighted sum of the inputs,

$ax_1(t) + bx_2(t)$, where a & b are const.

produces output,

$ay_1(t) + by_2(t)$, which is the sum of the weighted o/ps.

i.e. $$\boxed{T[ax_1(t) + bx_2(t)] = aT[x_1(t)] + bT[x_2(t)]}$$

For discrete-time linear system,

$$\boxed{T[ax_1(n) + bx_2(n)] = aT[x_1(n)] + bT[x_2(n)]}$$

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Q. Check whether the following systems are linear or not :—

(a) $$\frac{d^2 y(t)}{dt^2} + 2t \, y(t) = t^2 x(t)$$

(b) $$y(t) = x(t^2)$$

(c) $$y(t) = 2x^2(t)$$

(d) $$y(n) = 2x(n) + 4.$$

(e) $$y(n) = x(n) + \frac{1}{2x(n-2)}$$

(f) $$y(t) = e^{x(t)}$$

Solutions :-

(a) $$\frac{d^2 y(t)}{dt^2} + 2t\, y(t) = t^2 x(t)$$

Let, an input $x_1(t)$ produces o/p $y_1(t)$.

$$\therefore \frac{d^2 y_1(t)}{dt^2} + 2t\, y_1(t) = t^2 x_1(t).$$

Let, an i/p $x_2(t)$ produces o/p $y_2(t)$.

$$\therefore \frac{d^2 y_2(t)}{dt^2} + 2t\, y_2(t) = t^2 x_2(t)$$

$\therefore$ Linear combinations of the above eqn's gives,

$$a\frac{d^2 y_1(t)}{dt^2} + a\,2t\, y_1(t) + b\frac{d^2 y_2(t)}{dt^2} + b\,2t\, y_2(t)$$

$$= a t^2 x_1(t) + b t^2 x_2(t)$$

i.e. $$\frac{d^2}{dt^2}\left[a y_1(t) + b y_2(t)\right] + 2t\left[a y_1(t) + b y_2(t)\right] = t^2\left[a x_1(t) + b x_2(t)\right]$$

$\therefore$ Linear system

(b) $y(t) = x(t^2)$

Let, $x_1(t) \longrightarrow y_1(t) \Rightarrow y_1(t) = x_1(t^2)$

$x_2(t) \longrightarrow y_2(t) \Rightarrow y_2(t) = x_2(t^2)$

$\therefore \underbrace{a y_1(t) + b\, y_2(t)}_{\text{weighted sum of O/Ps}} = \underbrace{a x_1(t^2) + b x_2(t^2)}_{\text{weighted sum of i/ps.}}$

$\therefore$ Linear

$y(t) = T[x(t^2)]$

$= x(t^2)$

$y_3(t) = T[a x_1(t) + B x_2(t)]$

$= a x_1(t^2) + b x_2(t^2)$

(c) $y(t) = 2x^2(t)$

$y(t) = T[x(t)] = 2x^2(t)$

For an i/p $x_1(t)$,

$$y_1(t) = 2x_1^2(t)$$

For i/p $x_2(t)$

$$y_2(t) = 2x_2^2(t)$$

$$ay_1(t) + by_2(t) = a[2x_1^2(t)] + b[2x_2^2(t)]$$
$$= 2[ax_1^2(t) + bx_2^2(t)]$$

The output due to weighted sum of i/ps is

$$y_3(t) = T[ax_1(t) + bx_2(t)]$$
$$= 2[ax_1(t) + bx_2(t)]^2$$

$$y_3(t) \neq ay_1(t) + by_2(t)$$

$\therefore$ Non-linear

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(d) $y(n) = 2x(n) + 4$

$y(n) = T[x(n)] = 2x(n) + 4$

For i/p $x_1(n)$, $y_1(n) = T[x_1(n)] = 2x_1(n) + 4$

" $x_2(n)$, $y_2(n) = T[x_2(n)] = 2x_2(n) + 4$

weighted sum of o/p,

$ay_1(n) + by_2(n) = a[2x_1(n) + 4] + b[2x_2(n) + 4]$

$= 2[ax_1(n) + bx_2(n)]$

$+ 4(a + b)$

The o/p due to weighted sum of i/ps is,

$y_3(n) = T[ax_1(n) + bx_2(n)]$

$= 2[ax_1(n) + bx_2(n)] + 4$

$y_3(n) \neq ay_1(n) + by_2(n)$

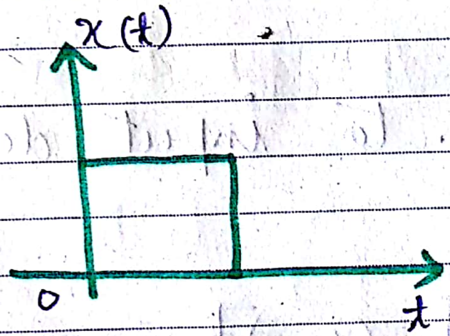
$\therefore$ Non-linear.

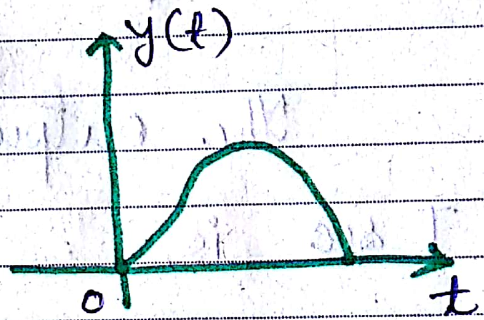
TIME INVARIANT & TIME-VARYING SYSTEMS:

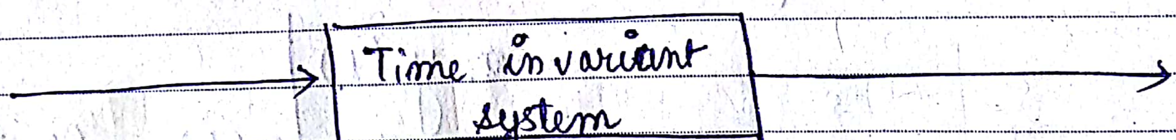
A system is said to be time-~~vese~~ invariant if its i/p / o/p characteristics do not change with time i.e. if a time shift in the input results in corresponding time shift in the output, i.e.,

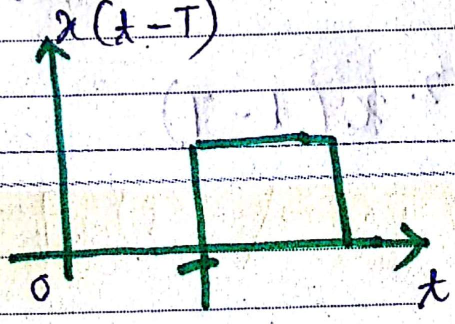
If $\quad x(t) \longrightarrow y(t)$

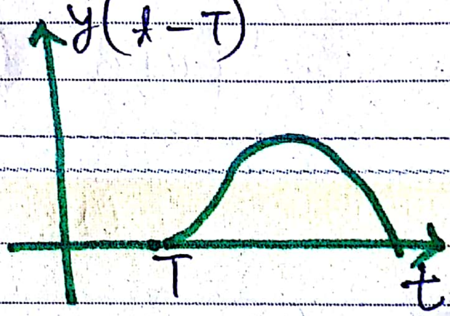
then $\quad x(t-T) \rightarrow y(t-T)$











A system not satisfying the above requirements is called time - varying system.

Q. Determine whether the following systems are time-invariant or not :—

(a) $y(t) = t^2 x(t)$

$$y(t) = T[x(t)] = t^2 x(t)$$

The output due to input delayed by T sec is,

$$y(t, T) = T[x(t-T)]$$

$$= y(t)\Big|_{x(t) = x(t-T)}$$

$$= t^2 x(t-T)$$

The output delayed by T sec is,

$$y(t-T) = y(t)\Big|_{t=t-T}$$

$$= (t-T)^2 x(t-T)$$

$$y(t, T) \neq y(t-T)$$

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Testing time invariance property :-

Let, $x(t)$ be the input and let $x(t-T)$ be the input delayed by T units.

$y(t) = T[x(t)]$ be the output of i/p $x(t)$

$$y(t,T) = T[x(t-T)]$$

$$= y(t)\Big|_{x(t)=x(t-T)}$$

be the o/p for the delayed i/p $x(t-T)$.

$y(t-T) = y(t)\Big|_{t=t-T}$ be the o/p delayed by T units.

If $y(t,T) = y(t-T) \rightarrow$ Time invariant.

If $y(t,T) \neq y(t-T) \rightarrow$ Time variant.