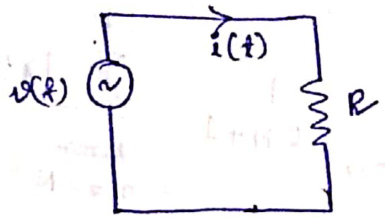
Energy & Power signal

In electrical systems, s/g may represent voltage or current. Consider voltage $v(t)$ across resistance R producing current $i(t)$



Instantaneous power developed, $P = v(t)\, i(t)$

$$= v(t)\frac{v(t)}{R}$$

$$= \frac{v^2(t)}{R}$$

$$= i(t)\, R \cdot i(t)$$

$$= i^2(t)\, R.$$

When $R = 1\,\Omega$, power dissipated is normalized power $p(t)$

$$p(t) = v^2 t \text{ or } i^2(t).$$

So, if $v(t)$ or $i(t)$ is denoted by a s/g $x(t)$ then instantaneous power is ~~eqal~~ equal to square of the amplitude of the s/g.

$$p(t) = |x(t)|^2$$

Total Energy of a signal, $\boxed{E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt \text{ Joules}}$

Avg power, $\boxed{P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \text{ Watts}}$

For discrete s/g.

$$* \quad E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$* \quad P = \cancel{\frac{1}{T}} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

** Notes :-

1. A s/g is energy s/g if and only if E is finite ($0 < E < \infty$). For energy s/g $P = 0$.
   eg:- Non periodic s/g.

2. A s/g is power s/g if P is finite ($0 < P < \infty$)
   For power s/g, $E = \infty$.
   eg:- Periodic s/g.

3. No s/g can be both Energy & power s/g at the same time.

Q. Determine the power & rms value of s/g,

$$x(t) = A \sin(\omega_0 t + \theta).$$

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Q. Determine the power & rms value of s/g,

$$x(t) = A\sin(\omega_0 t + \theta)$$

→ $x(t) = A\sin(\omega_0 t + \theta)$

Avg power, $P = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$

$= \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |A\sin(\omega_0 t + \theta)|^2 \, dt$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T}^{T} \left[\frac{1 - \cos(2\omega_0 t + 2\theta)}{2} \right] dt$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} dt - \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} \cancel{\cos(2\omega_0 t + 2\theta)}\, dt \rightarrow 0$$

↳ Integration of cosine f^n over one full cycle is always 0

$$= \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} dt - 0$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} (T + T)$$

$$= \frac{A^2}{2}$$

rms value $= \sqrt{A^2/2} = \dfrac{A}{\sqrt{2}}$

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$$E = \int_{-\alpha}^{\alpha} \left| A \sin(\omega_0 t + \theta) \right|^2 dt$$

$$= \int_{-\alpha}^{\alpha} \frac{A^2 \left[1 - \cos 2(\omega_0 t + \theta)\right] dt}{2}$$

$$= \frac{A^2}{2} \int_{-\alpha}^{\alpha} dt - \frac{A^2}{2} \int_{-\alpha}^{\alpha} \left[\cos 2\omega_0 t + 2\theta\right] dt$$

$$= \frac{A^2}{2} \left[t\right]_{-\alpha}^{\alpha} - 0$$

$$= \infty$$

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Q. Prove the following :

(a) The power of an energy s/g is zero over infinite time.

(b) The energy of the power s/g is infinite over infinite time.

⑥ Power of the energy s/g :-

Let, $x(t)$ be an energy s/g.

$$\therefore E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \quad \text{is finite.}$$

$$\text{Power of } x(t),\ P = \left\{ \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \right.$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt \right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\int_{-\infty}^{\infty} |x(t)|^2 \, dt \right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} E$$

$$= 0 \times E = 0 \qquad \left|\ \because \lim_{T\to\infty} \frac{1}{2T} = 0.\right.$$

⑤ Energy of the power s/g :—

Let $x(t)$ be a power s/g.

$\therefore P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$ is finite.

Energy of the s/g — $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$.

Let the limits of integration be changed from $-T$ to T & take limit $T \to \infty$.

(This will not change the meaning of above eqn)

$$\therefore E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T \to \infty} \left[2T \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \right]$$

$$= \lim_{T \to \infty} 2T \left[\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \right]$$

$$= \lim_{T \to \infty} 2TP \qquad \because P$$

$$= \infty$$

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Q. Determine power & rms value :-

(a) $x(t) = 7\cos\left(20t + \pi/2\right)$

(b) $x(t) = 12\cos\left(20t + \frac{\pi}{3}\right) + 16\sin\left(30t + \pi/2\right)$

(c) $x(t) = Ae^{j5t}$.

(d) $x(t) = e^{j2t}\cos 10t$.

Soln :-

(a) $x(t) = 7\cos\left(20t + \pi/2\right)$ | $A\cos(\omega t + \theta)$

It is in the form

$$A\cos(\omega t + \theta)$$

$\therefore$ power of s/g, $P = \frac{A^2}{2}$

$$= \frac{7^2}{2} = 24.5W.$$

$\text{rms} = \sqrt{24.5}$

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(b) ~~x(t)~~

$P = \frac{12^2}{2} + \frac{16^2}{2} = 200\,W$ $\quad$ rms $= \sqrt{200}$

(c) $x(t) = A e^{j5t}$

$= A\cos 5t + jA\sin 5t$.

$P = \frac{A^2}{2} + \frac{A^2}{2} = A^2$

$P_{rms} = \sqrt{A^2} = A$

(d) $x(t) = e^{j2t} \cos 10t$.

$= (\cos 2t + j\sin 2t)\cos 10t$.

$= \cos 2t \cos 10t + j\sin 2t \cos 10t$.

$= \frac{\cos 12t + \cos 8t}{2} + \frac{j(\sin 12t - \sin 8t)}{2}$

$\therefore P = \frac{(1/2)^2}{2} + \frac{(1/2)^2}{2} + \frac{(1/2)^2}{2} + \frac{(1/2)^2}{2}$

$= 1/2$.

rms $= \sqrt{1/2}$

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Q. Determine whether the following s/g are energy s/g or power s/g & calculate their energy & power.

Soln

(a) $x(t) = \sin^2 \omega_0 t$.

(b) $x(t) = A e^{-at} u(t)$, $a > 0$.

(c) $x(t) = t\, u(t)$

1 * if s/g is periodic

* if s/g is periodic & of infinite duration ~~en~~ it is power s/g

* if s/g is periodic only over a finite duration or not periodic at all then it is energy s/g.

Soln :-

Given, $x(t) = \sin^2 \omega_0 t$.

$\downarrow$

squared sine wave.

So, It is periodic.

So, it is a power s/g.

* if s/g is periodic only over a finite duration or not periodic at all then it is energy s/g.

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt.$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \sin^4 \omega_0 t \, dt$$

$[\because (\sin^2(\omega_0 t))^2$ has some period T & it is real)

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{8} [3 - 4\cos 2\omega_0 t + \cos 4\omega_0 t] \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{3}{8} - \lim_{T \to \infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} \frac{4}{8} \cos 2\omega_0 t \, dt}_{0}$$

$$+ \lim_{T \to \infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} \frac{1}{8} \cos 4\omega_0 t \, dt}_{0}$$

$$= \lim_{T \to \infty} \frac{1}{2T} \frac{3}{8} [t]_{-T}^{T}$$

$$= \lim_{T \to \infty} \frac{1}{\cancel{2T}} \frac{3}{8} [\cancel{2T}] = \frac{3}{8} \, W$$ //

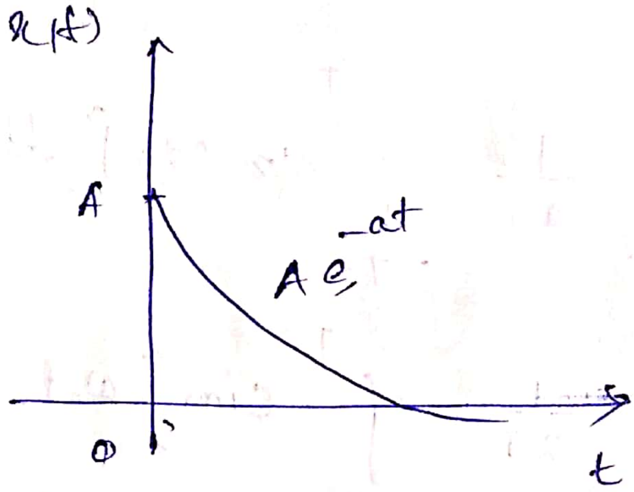
$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \cancel{\int_{-\infty}^{\infty}} \lim_{T \to \infty} \int_{-T}^{T} (\sin^2 \omega_0 t)^2 \, dt$$

$$= \lim_{T \to \infty} \int_{-T}^{T} \frac{1}{8} \left[3 - 4 \cos 2\omega_0 t + \cos 4 \omega_0 t \right] dt$$

$$= \lim_{T \to \infty} \frac{3}{8} \left[t \right]_{-T}^{T} = \lim_{T \to \infty} \frac{3}{8} 2T = \infty$$

(b) $x(t) = A e^{-at} u(t)$, $a > 0$



$x(t) = 1$ for $0 < t < \infty$

$\therefore$ s/g is non periodic & of finite duration.

So energy s/g.

$x(t) = A e^{-at} \times u(t) = \begin{cases} A e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$\therefore |x(t)|^2 = A^2 e^{-2at}$ for $t \geq 0$.

$\therefore E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_{0}^{\infty} |A e^{-at}|^2 dt$

$$= A^2 \int_0^{\infty} e^{-2at} \, dt .$$

$$= A^2 \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty}$$

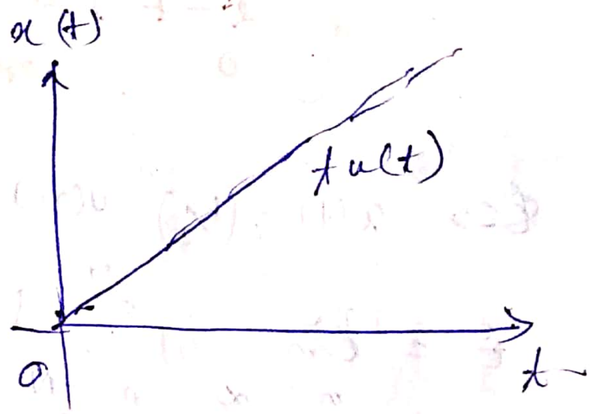
$$= \frac{A^2}{2a} .$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{A^2}{2T} \left[e^{-2at} \right]_0^T$$

$$= 0 .$$

(a) $x(t) = t\,u(t)$.



$t = 0$ to ∞.

$$x(t) = t\,u(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$|x(t)|^2 = t^2 \text{ for } t \geq 0$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$= \lim_{T \to \infty} \int_0^T (t)^2\,dt = \lim_{T \to \infty} \left[\frac{t^3}{3}\right]_0^T$$

$$= \lim_{T \to \infty} \left[\frac{T^3}{3}\right] = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} (x(t))^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} t^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\frac{t^3}{3} \right]_0^T = \infty$$

$\therefore P = \infty$.

neither P s/g nor E signal.

Q. Find whether energy / power s/g / neither energy nor power s/g

① $\left(\frac{1}{2}\right)^n u(n)$

② $u(n) - u(n-6)$

③ $x(t) = \begin{cases} t-2, & -2 \le t \le 0 \\ 2-t, & 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$