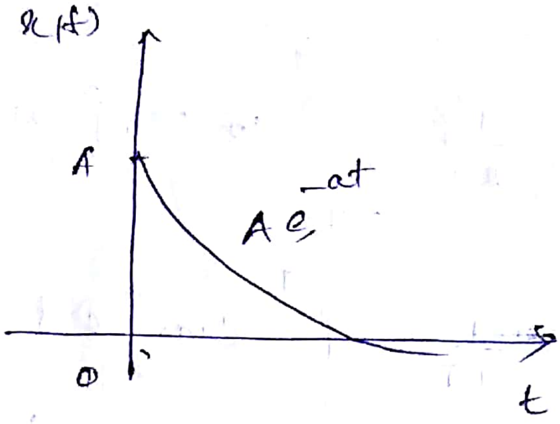
(b) $x(t) = A e^{-at} u(t), \ a > 0$



$x(t) = 1$ for $0 < t < \infty$

$\therefore$ s/g is non periodic & of finite duration.

So energy s/g.

$$x(t) = A e^{-at} \times u(t) = \begin{cases} A e^{-at}, & t \geq 0 \\ 0, & t < 0. \end{cases}$$

$$\therefore |x(t)|^2 = A^2 e^{-2at} \text{ for } t \geq 0.$$

$$\therefore E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{0}^{\infty} |A e^{-at}|^2 \, dt$$

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$] dt$

$$= A^2 \int_0^{\infty} e^{-2at}\, dt.$$

$$= A^2 \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty}$$

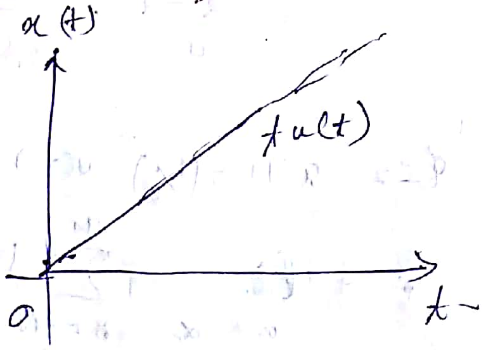
$$= \frac{A^2}{2a}.$$

$$P = \underset{T \to \infty}{lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt$$

$$= \underset{T \to \infty}{lt} \frac{A^2}{2T} \left[e^{-2at} \right]_0^T$$

$$= 0.$$

(d) $x(t) = t\,u(t).$



$t = 0$ to ∞.

$$x(t) = t\,u(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$|x(t)|^2 = t^2 \text{ for } t \geq 0$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= \underset{T \to \infty}{lt} \int_0^T (t)^2\, dt = \lim_{T \to \infty} \left[\frac{t^3}{3} \right]_0^T$$

$$= \lim_{T \to \infty} \left[\frac{T^3}{3} \right] = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} (x(t))^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} t^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\frac{t^3}{3} \right]_0^T = \infty$$

$\therefore P = \infty$.

neither P s/g nor E signal.

Q. Find whether energy / power s/g / neither energy / power

① $\left(\frac{1}{2}\right)^n u(n)$

② $u(n) - u(n-6)$

③ $x(t) = \begin{cases} t-2 &, -2 \le t \le 0 \\ 2-t &, 0 \le t \le 2 \\ 0 &, \text{otherwise} \end{cases}$

Sol^n ① $x(t) = \left(\frac{1}{2}\right)^n u(n)$

$$E = \lim_{n \to \infty} \sum_{n=-N}^{N} (x(n))^2$$

$$= \lim_{n \to \infty} \sum_{n=-N}^{N} \left[\left(\frac{1}{2}\right)^n\right]^2 u(n)$$

$$= \lim_{N \to \infty} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-(1/4)} = \frac{4}{3} \text{ Joules}$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[\frac{1-\left(\frac{1}{4}\right)^{N+1}}{1-\left(\frac{1}{4}\right)} \right]$$

$$= 0$$

$\therefore$ energy s/g.

(2) $u(n) - u(n-6)$

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} \underbrace{\left[u(n) - u(n-6)\right]^2}_{n=0\text{ to }5}$$

$$= \lim_{N \to \infty} \sum_{n=0}^{5} 1 = 6\ \text{J}.$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[u(n) - u(n-6)\right]^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{5} 1 = 0.$$

(3) $x(t) = \begin{cases} t-2 , & -2 \le t \le 0 \\ 2-t , & 0 \le t \le 2 \\ 0 , & \text{otherwise} . \end{cases}$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \cancel{\int_{-\infty}^{\infty}} \left[\int_{-2}^{0} (t-2)^2 \, dt + \int_{0}^{2} (2-t)^2 \, dt \right]$$

$$= \int_{-2}^{0} (t^2 - 4t + 4) \, dt + \int_{0}^{2} (4 + t^2 - 4t) \, dt$$

$$= \left[\frac{t^3}{3} - \frac{4t^2}{2} + 4t \right]_{-2}^{0}$$

$$+ \left[4t + \frac{t^3}{3} - \frac{4t^2}{2} \right]_{0}^{2}$$

$$= 64/3 \text{ Joules} .$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt .$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\int_{-2}^{0} (t-2)^2 \, dt + \int_{0}^{2} (2-t) \, dt \right]$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[64/3 \right] = 0 .$$

$\therefore$ energy s/g.

Causal & Non-Causal s/g :-

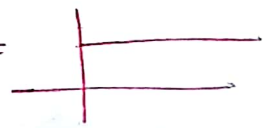
Causal :-> A continuous-time signal $x(t)$ is said to be causal if $x(t) = 0$ for $t < 0$, otherwise the s/g is non causal. A continuous time s/g $x(t)$ is said to be anticausal if $x(t) = 0$ for $t > 0$.

(A causal s/g does not exist for $-ve$ time and an anti-causal s/g does not exist for $+ve$ time).

A signal which exists in positive as well as $-ve$ time is called non-causal.

eg:- $u(t)$ is causal.

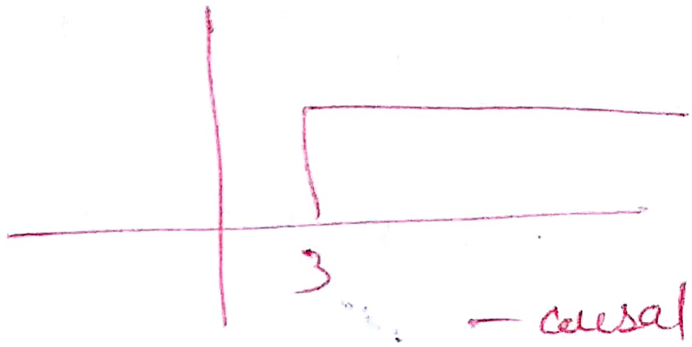
$u(-t)$ is anti-causal.

$u(t) =$  → causal as it exist for posit

$\sin(t)$  → non causal

Lly, in discrete-time s/g, $x(n)$ is said to be causal if $x(n) = 0$ for $n < 0$, otherwise non-causal. A discrete-time $x(n)$ is said to be anticausal if $x(n) = 0$ for $n > 0$.

$u(t-3)$



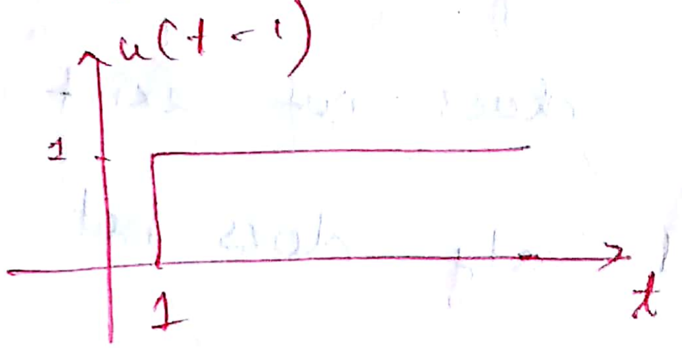
← causal

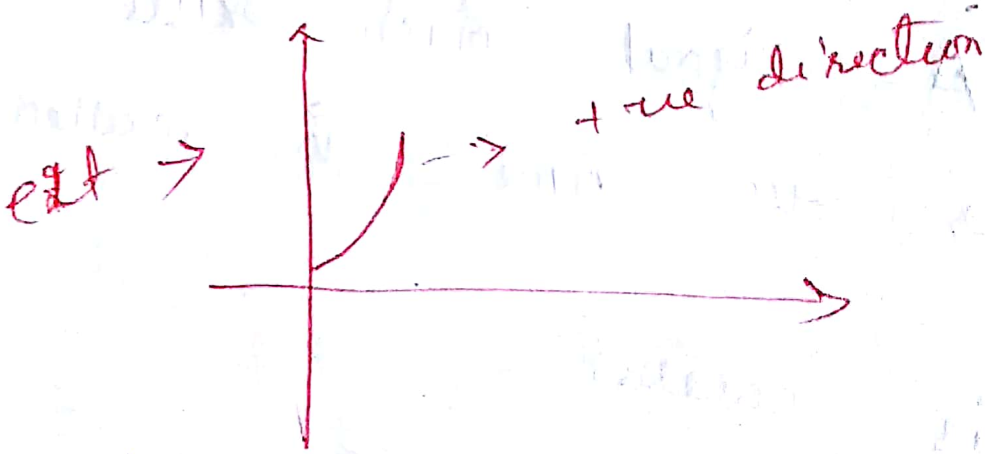
$u(t+3$

← non-causal

Q. Find the causal or non causal :-

ⓐ $x(t) = e^{2t} \underbrace{u(t-1)}$





∴ causal.

(5) $x(n) = 2u(-n)$

$\therefore$ causal.

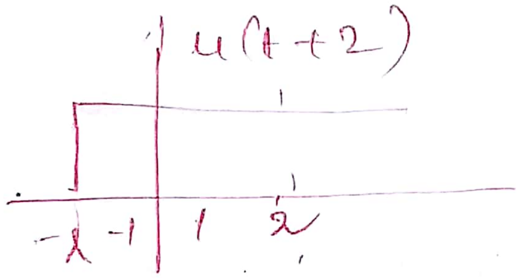
(b) $x(n) = \cancel{2t}\ 2u(-n)$

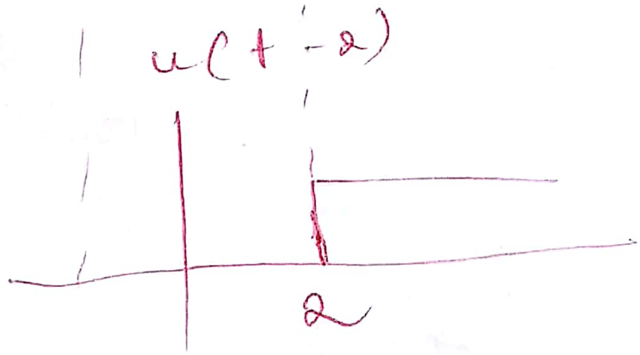
Anti causal.

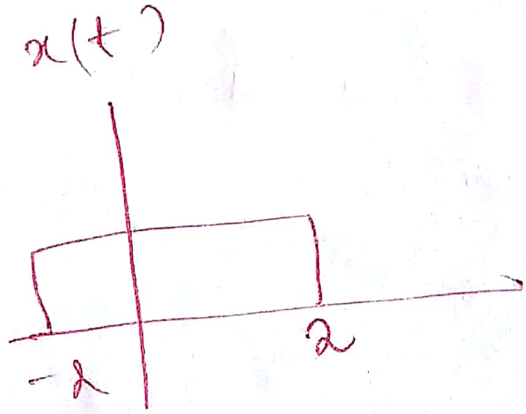
(c) ~~$x(t) = 3\sin c\ 2t$~~.

(d) $x(t) = u(t+2) - u(t-2)$

(d) $x(t) = u(t+2) - u(t-2)$







– N.C

~~Alter~~

* Even & Odd signals :-

* A signal is said to be even when it satisfies the condn,

$$x(t) = x(-t).$$

eg :- ① $x(t) = \cos t$

$x(-t) = \cos(-t)$

$= \cos t = x(t)$

② $x(t) = t^2$

$x(-t) = (-t)^2 = t^2 = x(t).$

A signal is said to be odd s/g when it satisfies the condn,

$$x(t) = -x(-t).$$

Eg :- $\sin t$ [$x(t) = \sin(t)$

$x(-t) = \sin(-t)$

$= -\sin t.$

$= -x(t)$]

$\therefore x(t) = -x(-t)$

t^3

$x(t) = t^3$

$x(-t) = (-t^3)$

$= -t^3$

$-x(-t) = x(t)$

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Note

* Sum of two or more even f^n
  Product of two or more even f^n
  product of even no. of odd f^n results in even f^n.

$x(t) = t^2 + t^4 + \cos t$

$x(t) = t^2 \cdot \cos t$.

$x(t) = t^3 \cdot \sin t$.

* Sum of two or more odd f^n or
  product of ~~two or more~~ odd no. of odd f^n results odd f^n.

$x(t) = t^3 + t^7 \sin t$

$t^3 \sin t \sin 2t$

Even & odd components

$$x(t) = x_e(t) + x_o(t) \quad \text{--- (1)}$$

$x_e(t)$ → even component of the s/g

$x_o(t)$ → odd component of the s/g.

(+)

$$x(-t) = x_e(t) - x_o(t) \quad \text{--- (2)}$$

Adding $\quad x(t) + x(-t) = 2x_e(t)$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \quad \text{--- (3)}$$

This eq^n is the even comp. of any s/g.

Subtraction ② from ①,

$$x(t) - x(-t) = 2x_o(t)$$

$$\Rightarrow x_o(t) = \frac{x(t) - x(-t)}{2} \quad \text{--- (4)}$$

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Q. Find the even & odd comp of

(a) $x(t) = e^{j2t}$

(b) $x(t) = 1 + 2t + 3t^2 + 4t^3$.

(c) $x(n) = \{5, 4, 3, 2, \underset{\uparrow}{1}\}$

(a) $x(t) = e^{j2t}$

$x(-t) = e^{-j2t}$

$x_e(t) = \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right) = \cos 2t$.

$x_o(t) = \frac{1}{2}\left(e^{j2t} - e^{-j2t}\right) = j\sin 2t$.

(5) $x(t) = \{ 1 + 2t + 3t^2 + 4t^3$

$x(-t) = 1 - 2t + 3t^2 - 4t^3$

$$x_e(t) = \frac{1}{2}\left[2 + 6t^2\right]$$

$$= \frac{2}{2}\left[1 + 3t^2\right]$$

$$= 1 + 3t^2.$$

$$x_o(t) = \frac{1}{2}\left[4 + 8t^3\right]$$

$$= 2\left[1 + 2t^3\right]$$

$$= 2 + 4t^3.$$

(c) $x(n) = \{5, 4, 3, 2, \underset{\uparrow}{1}\}$.

$n = -4, -3, -2, -1, 0$

$x(n) = 5, 4, 3, 2, \underset{\uparrow}{1}$

$x(-n) = \underset{\uparrow}{1}, 2, 3, 4, 5.$

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$= \frac{1}{2}[5, 4, 3, 2, 2, 2, 3, 4, 5]$

$= [2.5, 2, 1.5, 1, 1, 1, 1.5, 2, 2.5]$

$x_o(n) = \frac{1}{2}[5, 4, 3, 2, 0, -2, -3, -4, -5]$