* LTI System :—

$\delta(t)$

$x(t) \longrightarrow \boxed{\text{LTI System}} \longrightarrow y(t)$

↳ any delay in i/p is reflected in o/p.

→ **Impulse response:—** o/p of the system when i/p is an impulse. All the input, output, impulse response is in time domain. But if we want to get the relation between i/p, o/p and then it is difficult to get in time domain. So we go to freq. domain using LT or F.T.

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LTI system

$\downarrow$

$h(t)$

F.T./L.T. $\downarrow\uparrow$

$H(s)$ or $H(\omega)$.

which is the Transfer function.

& now if we want to go to the time domain we use IFT or ILT and we get the impulse response.

TRANSFER FUNCTION :-

Defn :- Defined as the ratio of L.T. of o/p to the L.T. of i/p when all initial conditions are assumed to be zero.

$H(s)$ → complex; Complex fn

$H(s)$ $\longrightarrow$ obtained by taking the L.T. of impulse response $h(t)$.

$$H(s) = \left.\frac{Y(s)}{X(s)}\right|_{\text{zero initial cond}^n}$$

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We know $h(t)$ & $H(s)$ are used only for LTI system i.e. they are linear & TIV; & if we see the linearity prop,

if i/p $\Rightarrow 0$, o/p $= 0$

Now if you dont take the initial condⁿ as 0, the property is violated.

Total o/p = zero i/p response + zero state response

zero i/p response ↓ i/p = 0
o/p of the system due to initial condⁿ.

zero state response ↓ calculated when i/p applied
ZSR = 0 as i/p = 0

if i/p = 0
& initial cond $\neq 0$
i/p $\rightarrow 0$
But o/p $\neq 0$ as initial cond $\neq 0$
So, due to this we have to consider all initial condition as zero.

TRANSFER FUNCTION CALCULATION:→

$y(t) = x(t-1) + 5$

$H(S) = ?$

LT

RELATION BETWEEN LT & FT:→

$$L[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt, \quad s = \sigma + j\omega$$

$$\Rightarrow X(s) = \int_{-\infty}^{\infty} x(t) e^{-(\sigma + j\omega)t} dt$$

$$= \int_{-\infty}^{\infty} x(t) e^{-\sigma t} \cdot e^{-j\omega t} dt \quad \text{—(I)}$$

Now, $F[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ —(II)

Comparing (I) & (II),

$$L.T.[x(t)] = F.T.[x(t) e^{-\sigma t}]$$

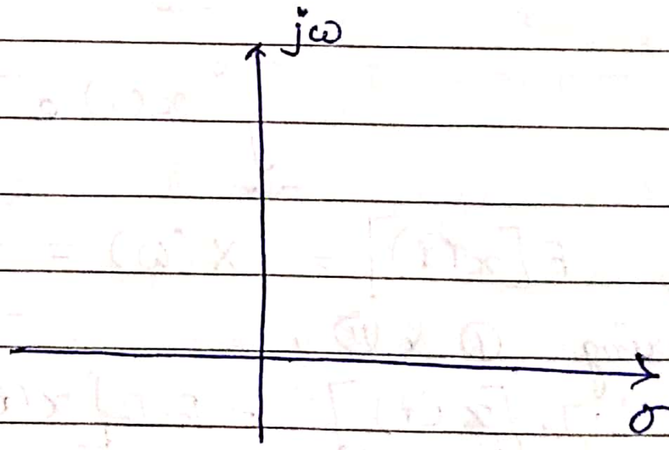
when $s = j\omega$, $\sigma = 0$, $X(s) = X(\omega)\Big|_{s = j\omega, \sigma = 0}$.

ROC AND STABILITY :→

POLES AND ZEROES :→

Poles : If we equate the denominator of T.F. to zero, we get values of 's' i.e. poles.

Zeroes : If we equate the ~~denominator~~ numerator of T.F. to zero, we get value of 's' i.e. zeroes.



$$X(s) = \frac{5s}{(s+1)(s^2+4s+8)}$$

$$= \frac{5s}{(s+1)(s+2-2j)(s+2+2j)}$$

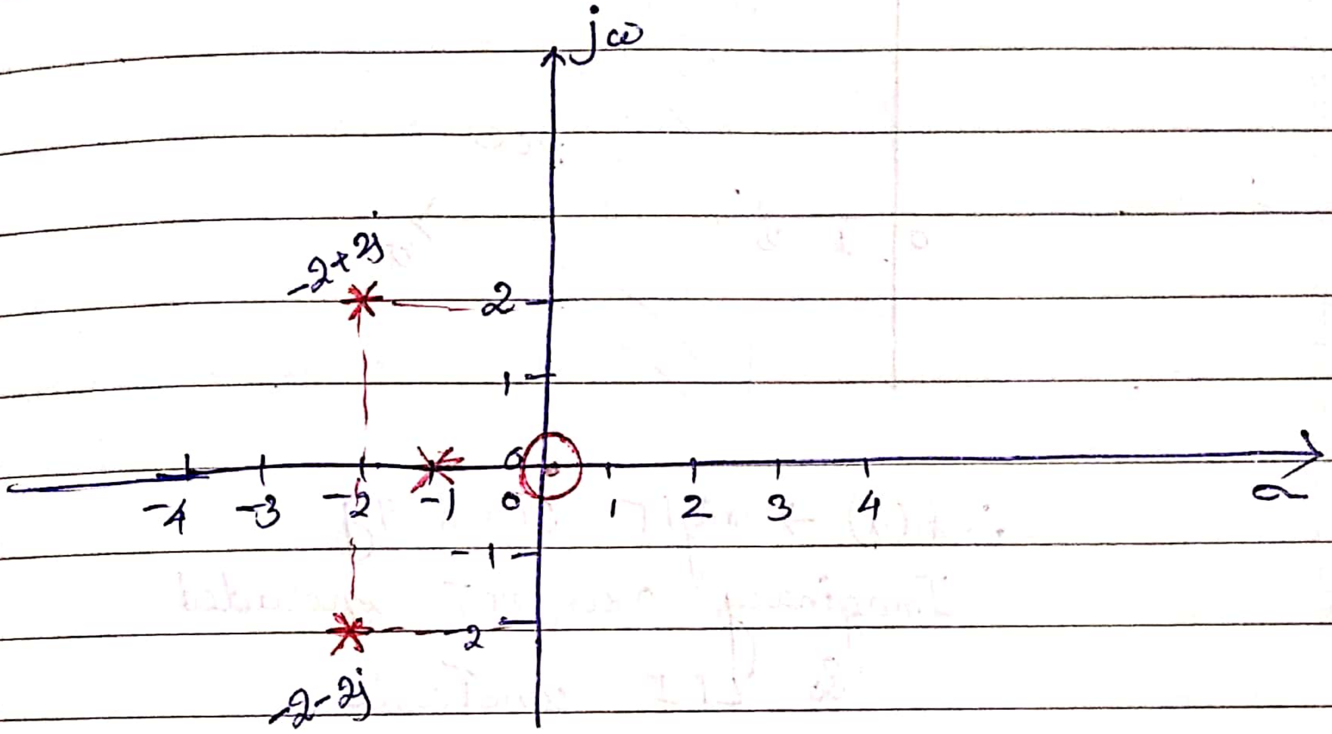
Poles: $D = 0 \Rightarrow (s+1)(s+2-2j)(s+2j+2) = 0$

$s = -1, \; (-2+2j), \; (-2-2j)$ poles.

Zeroes : N = 0

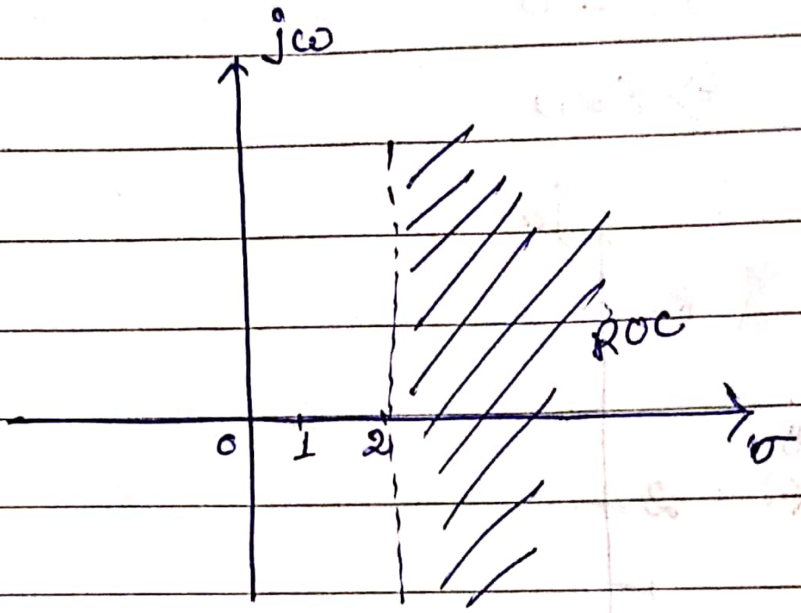
$5s = 0$

$\Rightarrow s = 0$



* Poles and zeroes of a transfer f^n are the frequencies for which the value of the denominator & numerator of transfer f^n becomes zero respectively. The values of the poles and the zeroes determine whether the system is stable, & how well the system performs.

* $h(t) \rightleftharpoons H(s)$; ROC : $\sigma > 2$



$\therefore h(t) \rightarrow$ right sided sig

Imaginary axis not included

So LTI unstable.

$h(t) \rightarrow$ not absolutely integrable

* $h(t) \rightleftharpoons H(s)$; ROC : $\sigma < 2$

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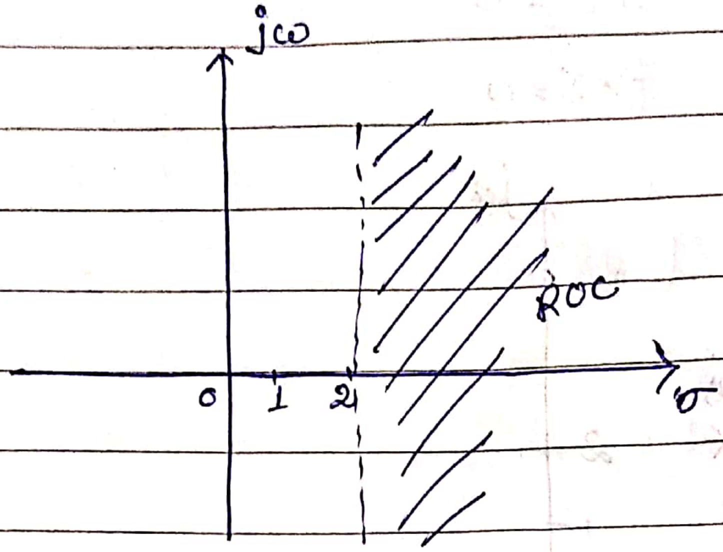
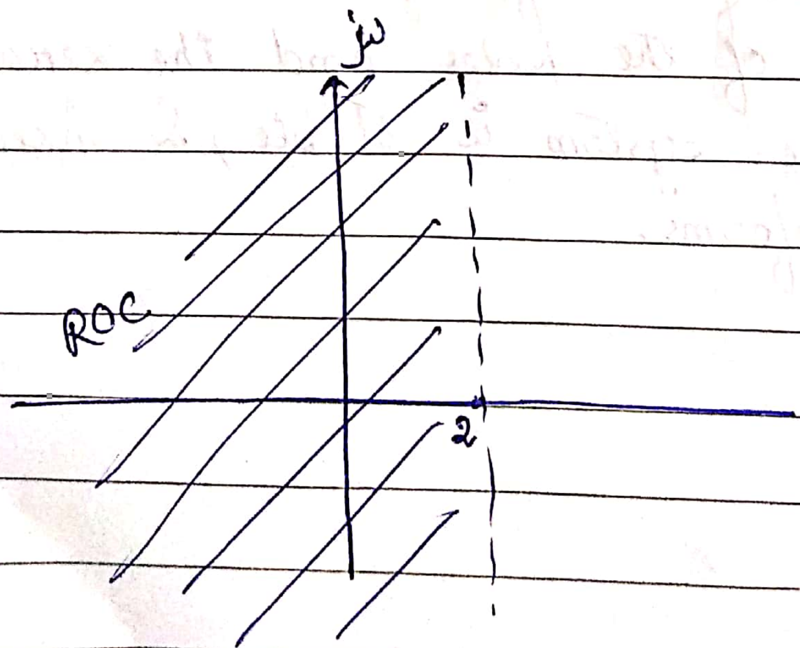


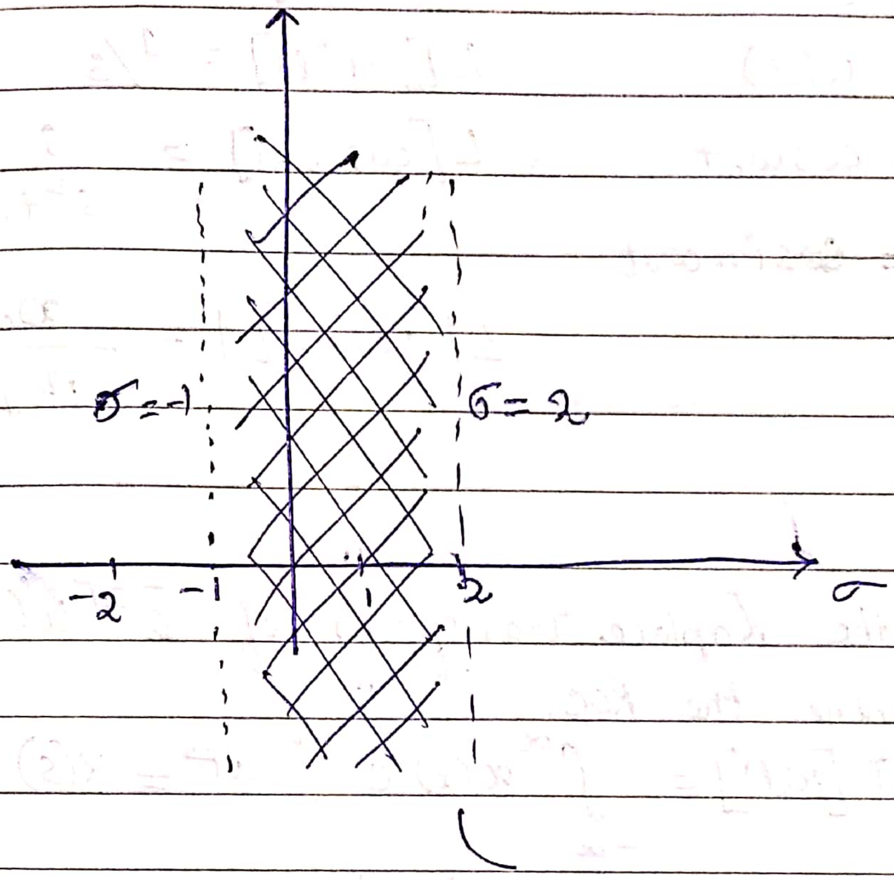
$\therefore h(t) \rightarrow$ right sided s/g
Imaginary axis not included
So LTI unstable.
$h(t) \rightarrow$ not absolutely integrable.

* $h(t) \rightleftharpoons H(s)$; ROC : $\sigma < 2$



left sided s/g, stable, absolutely integrable.

* $h(t) \rightleftharpoons H(s)$; ROC: $-1 < \sigma < 2$.

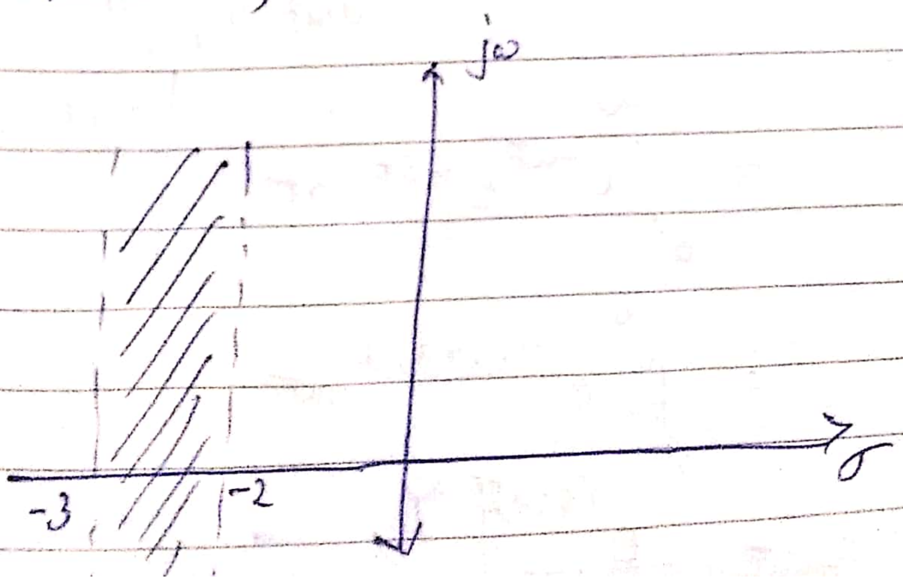


strip.

both sided s/g, stable, AI.

* $h(t) \rightleftharpoons H(s)$; ROC: $-3 < \sigma < -2$

* $h(t) \rightleftharpoons H(s)$; ROC : $-3 < \sigma < -2$



both sided
unstable
NAI.

Laplace Tx of standard s/g :→

$x(t) = \delta(t)$ $\quad$ $L[\delta(t)] = 1.$

$x(t) = u(t)$ $\quad$ $L[u(t)] = 1/s$

$x(t) = \cos\omega_0 t$ $\quad$ $L[\cos\omega_0 t] = \dfrac{s}{s^2+\omega_0^2}$

$x(t) = \sin\omega_0 t$

$$L[\sin\omega_0 t] = \frac{\omega_0}{s^2+\omega_0^2}$$

Q.1. Find the Laplace transform of $e^{-at}u(t)$ and determine the ROC.

Solⁿ:-

$$LT[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = X(s)$$

$$LT[e^{-at}u(t)] = \int_{-\infty}^{\infty} e^{-at}\, u(t)\, e^{-st}\, dt$$

↓ unit step f^n

$$= \int_0^{\infty} e^{-at} e^{-st}\, dt$$

$u(t) = 1,\ t \geq 0$
$\quad = 0,\ t < 0$

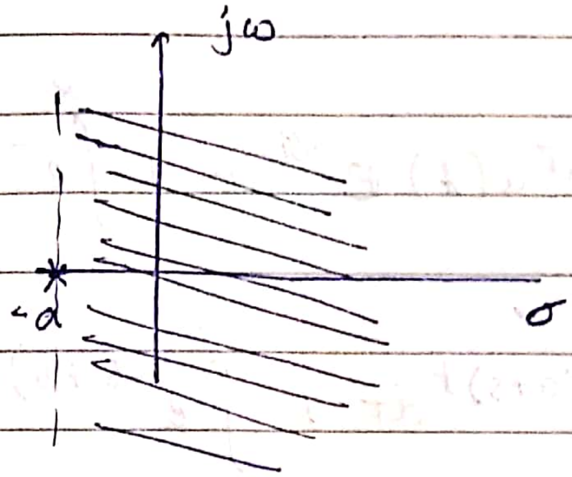
$$= \int_0^{\infty} e^{-(s+a)t}\, dt$$

$$= \left.\frac{e^{-(s+a)t}}{s+a}\right|_0^{\infty}$$

$$= \left[0 - \frac{1}{-(s+a)}\right]$$

$$= \frac{1}{s+a}$$

ROC: $Re(s) > -a$



Numerator $= 0$

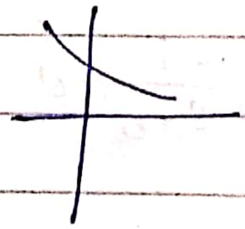
$s + a = 0$

$s = -a$

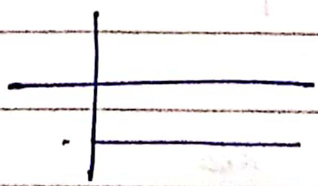
$>$ or $<$ is determined by the s/g itself.

Acc to the property of ROC, for rt. sided s/g ~~ROC is given by~~, $Re(s) > \sigma_{max}$ or $Re(s) >$ real part.

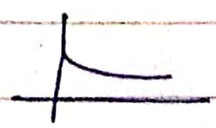
$e^{-at} \rightarrow$ decaying exp.



$u(t) \rightarrow$



$e^{-at} \cdot u(t)$



$\therefore$ rt. sided s/g.

Q2. Find the Laplace Transform of the s/g $x(t) = e^{-at}u(t) + e^{-bt}u(-t)$ and find the ROC. What are pole locations?

Solⁿ:- $X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$

$= \int_{-\infty}^{\infty} e^{-at} u(t) e^{-st} dt + \int_{-\infty}^{\infty} e^{-bt} u(-t) e^{-st} dt$

$= \underbrace{\int_{0}^{\infty} e^{-(a+s)t} dt}_{\substack{\text{converges if} \\ Re(s) > -a}} + \underbrace{\int_{-\infty}^{0} e^{-(s+b)t} dt}_{\substack{\text{converges if} \\ Re(s) < -b}}$

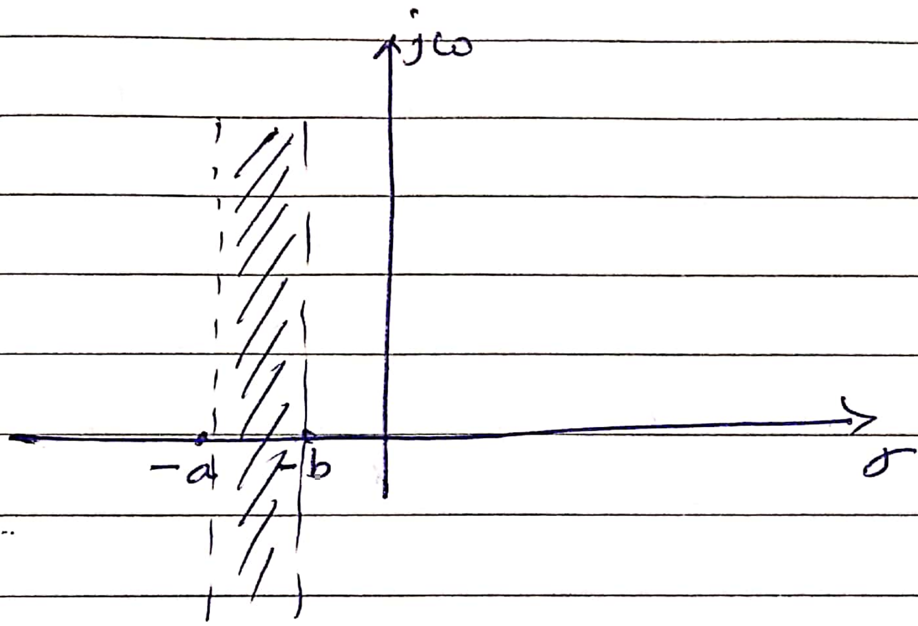
converges if $-a < Re(s) < -b$.

$= \frac{-1}{s+b} e^{-(s+b)t} \Big|_{-\infty}^{0} + \frac{1}{-(s+a)} e^{-(s+a)t} \Big|_{0}^{\infty}$

$= \frac{1}{s+b} + \frac{1}{s+a}$

ROC: $-a < Re(s) < -b$

Poles are at $s = -a$ & $s = -b$



Q3. Find LT. of (i) $x(t) = e^{-3t}u(t) + e^{-2t}u(t)$ & find ROC.

(ii) $x(t) = e^{-b|t|}$ & find ROC.