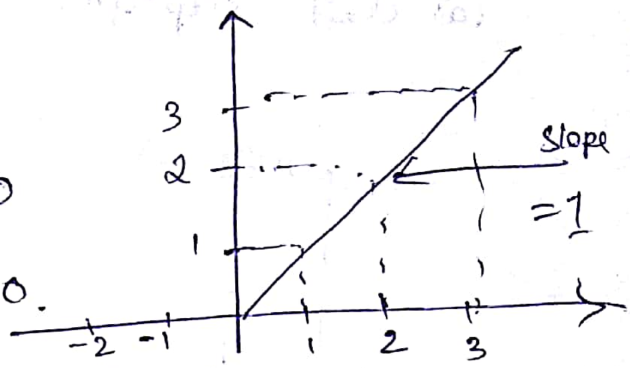
② Unit Ramp function :-

Continuous Unit Ramp f^n, $r(t)$:- That function that starts at $t=0$ and increases ~~later~~ linearly with time and defined as :



$$r(t) = \begin{cases} t, & \text{for } t \geq 0 \\ 0, & \text{for } t < 0. \end{cases}$$

$\therefore u(t) = 1.$

or $r(t) = t\, u(t)$.

→ just explain step-wise.

how?

[Unit ramp f^n has unit slope. It can be obtained by integrating the ~~at~~ unit step f^n which means that a unit step f^n can be obtained by differentiating the unit ramp f^n.]

↓

$$r(t) = \int_0 u(t)\,dt$$

what is slope?

slope :- steepness of a line. change in y for a unit change in x along a line.

$$\Downarrow$$

$$r(t) = \int u(t)\,dt$$

$$= \int dt$$

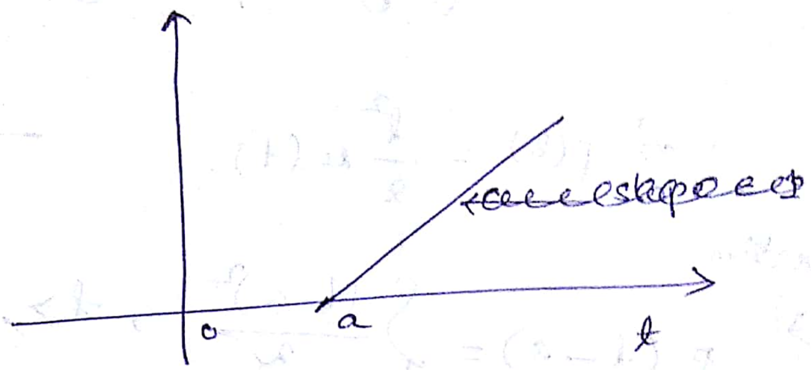
$$= dt \quad \text{for } t \geq 0$$

$$u(t) = \frac{d}{dt} r(t).$$

Delayed unit ramp signal $r(t-a)$ is given by

$$r(t-a) = \begin{cases} t-a, & \text{for } t \geq a \\ 0, & \text{for } t < a \end{cases}$$

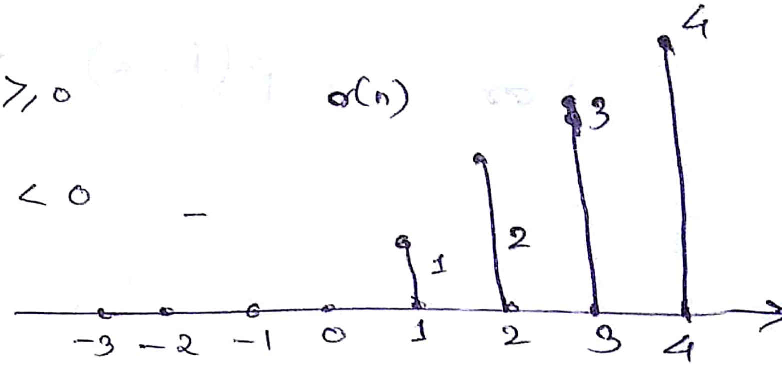
or $$r(t-a) = (t-a)\,u(t-a)$$



Discrete-time unit ramp signal :-

$$r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

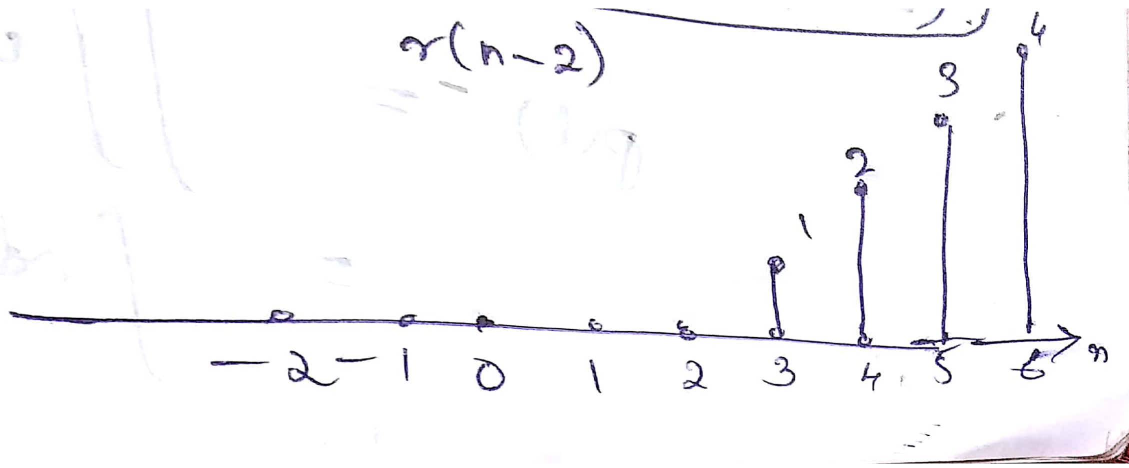
or $r(n) = n\,u(n)$.



shifted version →

$$r(n-k) = \begin{cases} n-k, & \text{for } n \geq k \\ 0, & \text{for } n < k \end{cases}$$

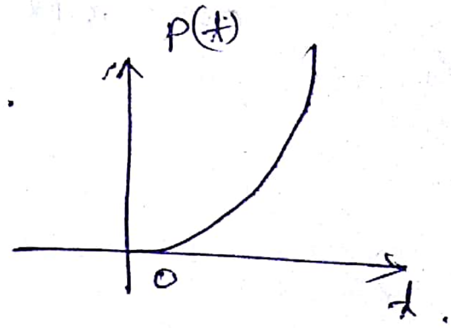
'or' $\boxed{r(n) = \cancel{r\,u(n)}\,(n-k)\,u(n-k)}$.


r(n−2)
1
2
3
4
−2 −1 0 1 2 3 4 5 6
n

③ Unit Parabolic function :—

Continuous-time unit parabolic function p(t) also called unit acceleration s/g starts at t = 0 & defined as,

$$p(t) = \begin{cases} \frac{t^2}{2}, & t \geq 0 \\ 0, & t < 0. \end{cases}$$

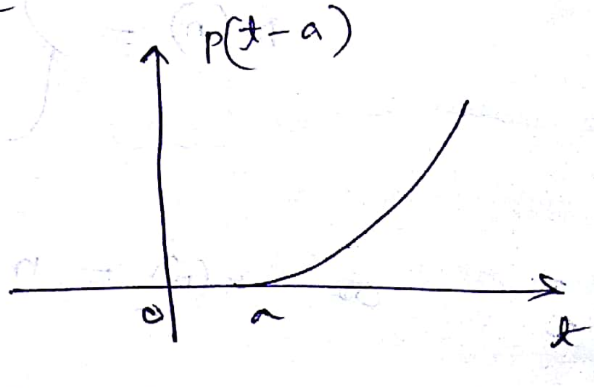


'or' $p(t) = \frac{t^2}{2} u(t)$.

shifted version →

$$p(t-a) = \begin{cases} \frac{(t-a)^2}{2}, & t \geq a \\ 0, & t < a \end{cases}$$

'or' $p(t-a) = \frac{(t-a)^2}{2} u(t-a)$.



Unit parabolic fn can be obtained by integrating the unit ramp fn or double integrating the unit step fn.

Unit parabolic fn can be obtained by integrating the unit ramp fn or double integrating the unit step fn.

$$p(t) = \iint u(t)\, dt$$

$$= \int r(t)\, dt = \int t\, dt = \frac{t^2}{2} \text{ for } t \geq 0$$

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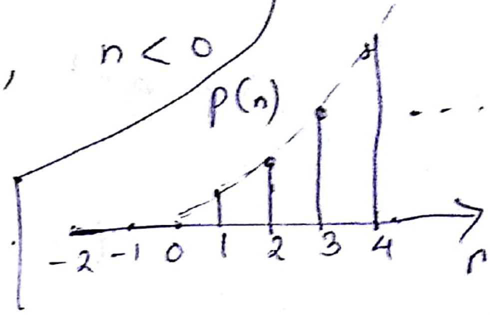
The ramp f^n is derivative of parabolic f^n and step f^n is double derivative of parabolic f^n.

$$r(t) = \frac{d}{dt} p(t)$$

$$u(t) = \frac{d^2}{dt^2} p(t).$$

Discrete-time unit parabolic sequence $p(n)$,

$$p(n) = \begin{cases} \frac{n^2}{2}, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

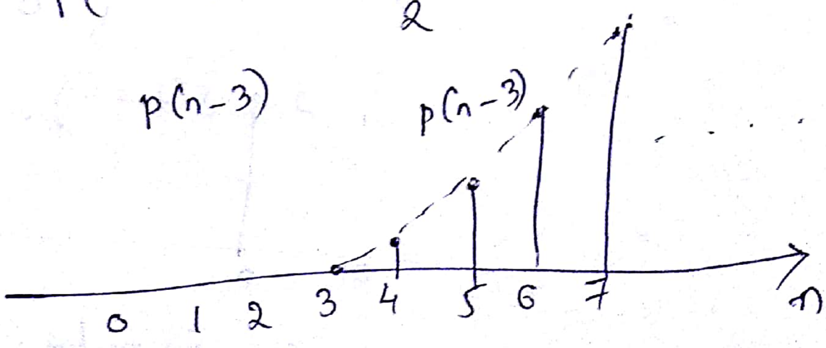


$$\Rightarrow p(n) = \frac{n^2}{2} u(n).$$

$$p(n-k) = \begin{cases} \frac{(n-k)^2}{2}, & n \geq k \\ 0, & n < k. \end{cases}$$

$$\Rightarrow p(n-k) = \frac{(n-k)^2}{2} u(n-k).$$

$p(n-3)$



Bss

[redacted]

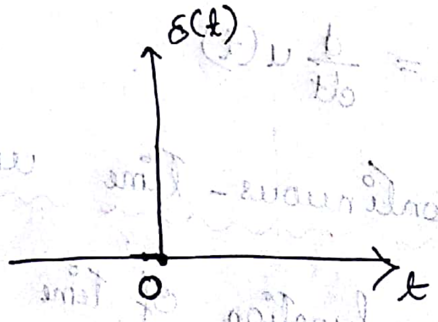
(4) Unit Impulse function :-

- Most widely used elementary function.
- The continuous-time unit impulse function, $\delta(t)$, also called Dirac delta function is defined as,

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

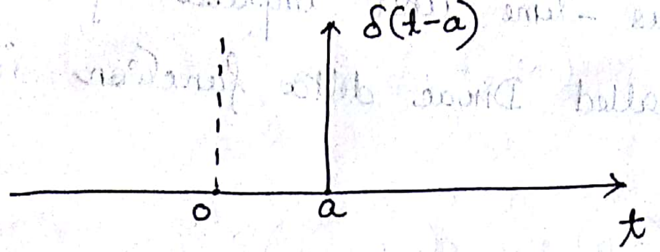
and, $\delta(t) = 0$, for $t \neq 0$

i.e. $$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



- Delayed unit impulse function $\delta(t-a)$ is defined as,

$$\delta(t-a) = \begin{cases} 1, & t = a \\ 0, & t \neq a \end{cases}$$



- Integral of unit impulse function is a unit step function and derivative of unit step function is a unit impulse function.

$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt$$

$$\delta(t) = \frac{d}{dt} u(t)$$

Properties of continuous-time unit impulse f^n:-

① It is an even function of time t, i.e $\delta(t) = \delta(-t)$

② $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$; $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

③ $\delta(at) = \frac{1}{|a|}\delta(t)$

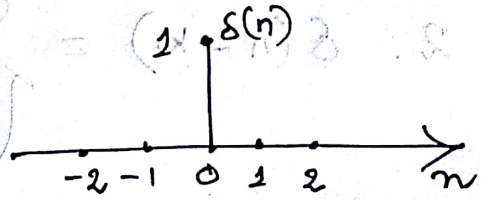
④ $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0) = x(t_0)$

$x(t)\,\delta(t) = x(0)\,\delta(t) = x(0)$

⑤ $$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

- **Discrete time unit impulse function :-**

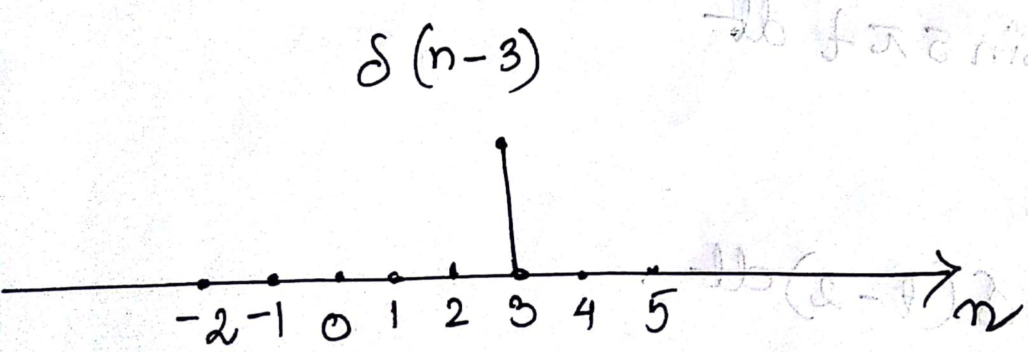
$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



- **Shifted discrete time unit impulse function:-**

$$\delta(n-k) = \begin{cases} 1, & n = k \\ 0, & n \neq k \end{cases}$$

$$\delta(n-3) = ?$$


δ (n-3)
-2 -1 0 1 2 3 4 5
n

Properties of discrete - Time unit sample sequence:

1. $\delta(n) = u(n) - u(n-1)$

2. $\delta(n-k) = \begin{cases} 1, & n = k \\ 0, & n \neq k \end{cases}$

3. $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

4. $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

Q. Evaluate the following integrals:-

ⓐ $\int_{-\infty}^{\infty} e^{-at^2} \delta(t-5)\,dt$

ⓑ $\int_{0}^{\infty} t^2 \delta(t-6)\,dt$

ⓒ $\int_{0}^{3} \delta(t) \sin 5\pi t\,dt$

ⓓ $\int_{-\infty}^{\infty} (t-2)^3 \delta(t-2)\,dt$.

ⓐ Given,

$$\int_{-\infty}^{\infty} e^{-at^2}\,\delta(t-5)\,dt$$

We know, $\delta(t-5) = \begin{cases} 1, & t=5 \\ 0, & \text{elsewhere} \end{cases}$

$$\therefore \int_{-\infty}^{\infty} e^{-at^2}\,\delta(t-5)\,dt$$

$$= \left[e^{-at^2}\right]_{t=5}$$

$$= e^{-25a}$$

(b) Given, $\int_{0}^{\infty} t^2 \delta(t-6)\, dt$

We know, $\delta(t-6) = \begin{cases} 1, & t=6 \\ 0, & \text{elsewhere} \end{cases}$

$\therefore \int_{0}^{\infty} t^2 \delta(t-6)\, dt = \left[t^2\right]_{t=6}$

$= 36$

(c) Given,

$$\int_0^3 \delta(t) \sin 5\pi t \, dt$$

We know that,

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{elsewhere.} \end{cases}$$

$$\therefore \int_0^3 \delta(t) \sin 5\pi t \, dt$$

$$= \left[\sin 5\pi t \right]_{t=0}$$

$$= 0.$$

(d) Given,

$$\int_{-\infty}^{\infty} (t-2)^3 \, \delta(t-2) \, dt$$

We know that

$$\delta(t-2) = \begin{cases} 2, & t=2 \\ 0, & \text{elsewhere} \end{cases}.$$

$$\therefore \int_{-\infty}^{\infty} (t-2)\, \delta(t-2)\, dt$$

$$= \Big[(t-2)^3 \Big]_{t=2}$$

$$= 0 .$$