The mode of the binomial distribution is equal to that value X which has the largest frequency.

Conditions for application of Binomial Dist.

1. Variable should be discrete
2. No. of trials n should be finite & small
3. Trials or events must be repeated under identical condit^n
4. There are two alternatives : success or failure

(5) The trials or events are independent.

Recursion Formula / Recurrence Relⁿ

Binomial Distribution

$$P(X=r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q} \, P(X=r)$$

$r = 1, 2, 3, \ldots$

Ten coins are thrown simultaneously.
Find the prob. of getting atleast 7 heads

No. of coins, $n = 10$

Prob. of getting a head, $p = \frac{1}{2}$

Prob. of not getting a head, $q = 1 - p = \frac{1}{2}$

Let X denotes no. of head

Now,

Prob. of getting atleast 7 heads

$$= P(X \geq 7)$$

$$= P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_7 \left(\tfrac{1}{2}\right)^7 \left(\tfrac{1}{2}\right)^3 + {}^{10}C_8 \left(\tfrac{1}{2}\right)^8 \left(\tfrac{1}{2}\right)^2 + {}^{10}C_9 \left(\tfrac{1}{2}\right)^9 \left(\tfrac{1}{2}\right)^1 + {}^{10}C_{10} \left(\tfrac{1}{2}\right)^{10}$$

$P(X=r)$

$= {}^nC_r \, p^r q^{n-r}$

If on an average, one ship out of 10 is wrecked, find the prob. that out of 5 ships expected to arrive on a port, atleast 4 will arrive safely.

Total no. of ships, $n = 5$

Prob. of surviving, $p = 9/10$

Prob. of getting wrecked, $q = 1/10$

let X denotes no. of ships arriving safely

Prob that atleast 4 ships will arrive safely

$$= P(X \geq 4) = P(X=4) + P(X=5)$$

$$= {}^5C_4 \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right)^1 + {}^5C_5 \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^0$$

$$= 0.91854 \longrightarrow \frac{91854}{10^5}$$

If the sum of the mean and the variance of binomial distribution of 5 trials in 4.8, find the distribution

Let, the reqd binomial distribution be

$$^nC_r \, p^r q^{n-r}$$

Here, $n =$ total no. of trials $= 5$

p = prob. of success

q = prob. of failure

We known,

Mean $= np$

Variance $= npq$

Given, $np + npq = 4.8$

$$5p + 5pq = 4.8$$

$$\Rightarrow 5p(1+q) = 4.8$$

$$\Rightarrow 5(1-q)(1+q) = 4.8 \qquad (\because p = 1-q)$$

$$\Rightarrow 5(1-q^2) = 4.8$$

$$\Rightarrow 50(1-q^2) = 48$$

$$\Rightarrow q = 1/5 \qquad \therefore p = 1 - 1/5 = 4/5$$

$\therefore$ The required dist. is ${}^5C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{5-r}$

Q. The prob. that a bomb dropped from a placed will strike the target is $1/5$. If 6 bombs are dropped, find the prob that

(a) exactly two will hit the target

(b) atleast two will hit the target

Total no. of bombs dropped, $n = 6$

Prob. that a bomb will hit the target, $p = 1/5$

Prob. that a bomb will not hit the target, $q = 1 - p = 1 - 1/5 = 4/5$

Let X denotes the no. of bombs dropped

Prob. that exactly two bombs will the target

$$= P(X = 2)$$

$$= {}^6C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4$$

Prob. that atleast two bombs will strike

$$= P(X \geq 2)$$

$$P(X=2) + P(X=3) + \dots + P(X=6)$$

$$1 - P(X < 2) = 1 - [P(X=0) + P(X=1)]$$

A student obtained the following answer to a certain problem. Mean = 2.4, Variance = 3.2 for a binomial distribution. Comment on the result.

$$\text{Mean} = 2.4 \Rightarrow np = 2.4$$

q cannot greater than 1

So the result is inconsistent

$$\text{Variance} = 3.2$$

$$\Rightarrow npq = 3.2 \Rightarrow 2.4\,q = 3.2 \Rightarrow q = 1.33$$

A die is thrown 5 times. Getting an even number greater than 2 is considered a success. Calculate $P(X=r)$ for $r = 1, 2, 3, 4, 5$ from recurrence formula.

Here, $n = 5$

p = prob. of getting an even no. greater than 2

$= 2/6 = 1/3$, $q = 1 - p = 2/3$

We know that,

$$P(X=r+1) = \frac{n-r}{r+1} \frac{p}{q} P(X=r) \quad \text{———} (1)$$

let X denotes no. of success i.e. getting an even no. greater than 2

$P(X=0)=$ Prob. of getting zero success

$$= {}^5C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{5-0} = 0.1317$$

From 1,

$$P(X=1) = \frac{5-0}{0+1}\left(\frac{1/3}{2/3}\right) P(X=0) = \frac{5}{2} P(X=0) = 0.3292$$

$$P(X=2) = \frac{5-1}{1+1}\left(\frac{1/3}{2/3}\right) P(X=1) = P(X=1) = 0.3292$$

$$P(X=3) = \ldots = 0.1646$$

$$P(X=4) = \ldots = 0.0412$$

$$P(X=5) = \ldots = 0.0041$$

Probability that a man hits a target is $\frac{1}{3}$

(I) If he fires 6 times then what is the prob. of hitting (a) atleast 5 times (b) atmost 5 times (c) exactly once.

(II) If he fires so that the prob. of hitting target atleast once is greater than $3/4$ then find n

Here, p = prob. of success

= prob. that a man hits a target

$= \frac{1}{3}$

$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Let X denotes the no. of success

1 $n=6$

(a) $P(X \geq 5) = P(X=5) + P(X=6)$

$$= {}^6C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^{6-5} + {}^6C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^{6-6}$$

$$= \frac{13}{729}$$

(b) $P(X \leq 5)$ → $P(X=0) + P(X=1) + \ldots + P(X=5)$

→ $1 - P(X > 5)$ ✓

$= 1 - P(X=6)$

$= 728/729$

(i) $P(X=1) = {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{6-1} = \frac{192}{729}$

(ii) $P(X \geq 1) > 3/4$

$\Rightarrow 1 - P(X < 1) > 3/4$

$\Rightarrow 1 - P(X=0) > 3/4 \Rightarrow 1 - \overbrace{{}^nC_0}^{1} \overbrace{\left(\frac{1}{3}\right)^0}^{1} \overbrace{\left(\frac{2}{3}\right)^{n-0}}^{(2/3)^n} > 3/4$

$$1-(2/3)^n > 3/4$$

$$\Rightarrow \left(\frac{2}{3}\right)^n < 1-3/4$$

$$\Rightarrow \left(\frac{2}{3}\right)^n < \frac{1}{4}$$

$$\Rightarrow n = 4$$