Probability Function or Probability Mass Function (pmf)

Probability Function or Probability Mass Function (pmf) of a random variable X is a function is a function *p(x)* which gives the probabilities corresponding to different possible discrete set of values say $x_1, x_2, \dots, x_n$ of variable *x*.

$p(x_i) = p(x = x_i)$= Probability that on variable *x* assumes value x_i

The function *p(x)* satisfies the condition

$(i) p(x_i) \geq 0$

$(ii) \sum p(x_i)=1$

Cumulative Distribution Function (Distribution Function)

If X is a random variable then $P(X \leq x)$ is called the cumulative distribution function (cdf) or distribution function and is denoted by *F(x)*.

So, $F(x) = P(X \leq x)$

Expectation of a Discrete Random Variable

If x is a discrete random variable which assumes the discrete set of values $x_1, x_2, \dots, x_n$ with the respective probabilities $p_1, p_2, \dots, p_n$ then the expression or expected value of x is denoted by $E(X)$ and defined as

$$E(X) = p_1x_1 + p_2x_2 + \cdots + p_nx_n = \sum_{i=1}^{n} p_ix_i$$

Similarly, the expected value of X^2 is defined as $E(X^2) = \sum_{i=1}^{n} p_ix_i^2$

Properties of Expectation

1. If X is a random variable and a be constant then

   i. $E(a) = a$

   ii. $E(aX) = aE(X)$

   iii. $E(X - \mu) = 0$

2. If x and y are two random variables then $E(X \pm Y) = E(X) \pm E(Y)$

3. $E(XY) = E(X)E(Y)$ if X and Y are two independent random variables.

4. If $y = ax + b$ where a and b are constants then $E(Y) = E(aX + b) = aE(X) + b$

A pair of coin is tossed , what is the expected value of getting head ?

Let X = number of heads

X = 0, 1, 2

Probability Distribution is given by

X	**0**	**1**	**2**
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$E(X) = (\frac{1}{4} \times 0) + (\frac{1}{2} \times 1) + (\frac{1}{4} \times 2) = 1$$