1.19. MEAN OF BINOMIAL DISTRIBUTION

For a binomial distribution the probability function is

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

The discrete probability distribution for the binomial distribution can be displayed as follows:

X	0	1	2	...	r	...	n
$P(X)$	${}^nC_0 q^n$	${}^nC_1 pq^{n-1}$	${}^nC_2 p^2 q^{n-2}$	...	${}^nC_r p^r q^{n-r}$	...	${}^nCp^n$

$\therefore$

$$\text{Mean } (\mu) = E(X) = \sum_{r-0}^{n} rP(X = r)$$

$$= {}^nC_0\, q^n \times 0 + {}^nC_1\, pq^{n-1} \times 1 + {}^nC_2\, p^2\, q^{n-2} \times 2 + \ldots + {}^nC_r\, pr\, q^{n-r} \times + \ldots + {}^nC_n\, p^n \times n$$

$$= 0 + npq^{n-1} + \frac{n(n-1)}{2!} p^2 q^{n-2} \times 2 + \ldots + np^n$$

$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2} p^2 q^{n-3} + \ldots + p^{n-1}\right]$$

$$= np\,(q + p)^{n-1} = np \quad (\because q + n)$$

$\therefore$ **Mean = np**

1.20. VARIANCE OF BINOMIAL DISTRIBUTION

Since $\quad$ Variance $= \Sigma px^2 - \mu^2$

Now

$$\Sigma px^2 = {}^nC_0\, q^n \times (0)^2 + {}^nC_1\, pq^{\,n-1} \times (1)^2 + {}^nC_2\, p^2\, q^{n-2} \times (2)^2 + {}^nC_3\, \text{p}^3\, q^{n-3} \times (3)^2 + \ldots + {}^nC_n\, p^n \times n^2$$

$$= 0 + n.pq^{n-1} + \frac{n(n-1)}{2!} p^2 q^{n-2} \times 4 + \frac{n(n-1)(n-2)}{3!} p^3 q^{n-3} \times 9 + \ldots + p^n \times n^2$$

Breaking second, third and following terms into parts, we get

$$\Sigma px^2 = np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!} p^2.q^{n-3} + \ldots + p^{n-1}\right] + n(n-1)\, p^2\left[q^{n-2} + (n-2)pq^{n-3} + \frac{(n-2)(n-3)}{2!} p^2.q^{n-4} + \ldots + p^{n-2}\right]$$

$$= np\,(q + n)^{n-1} + n\,(n-1)\,p^2\,(q + p)^{n-2}$$

$$= np + n\,(n-1)\,p^2 = np\,[1 + (n-1)\,p] = np\,[q + np]\; npq + n^2 p^2$$

$\therefore \quad$ Variance $= npq + n^2p^2 - (np)^2 = npq.$