Q. Probability of a man hitting a target is $\frac{1}{3}$.

(a) If he fires 6 times, what is the probability of hitting (i) atmost 5 times.

(ii) atleast 5 times.

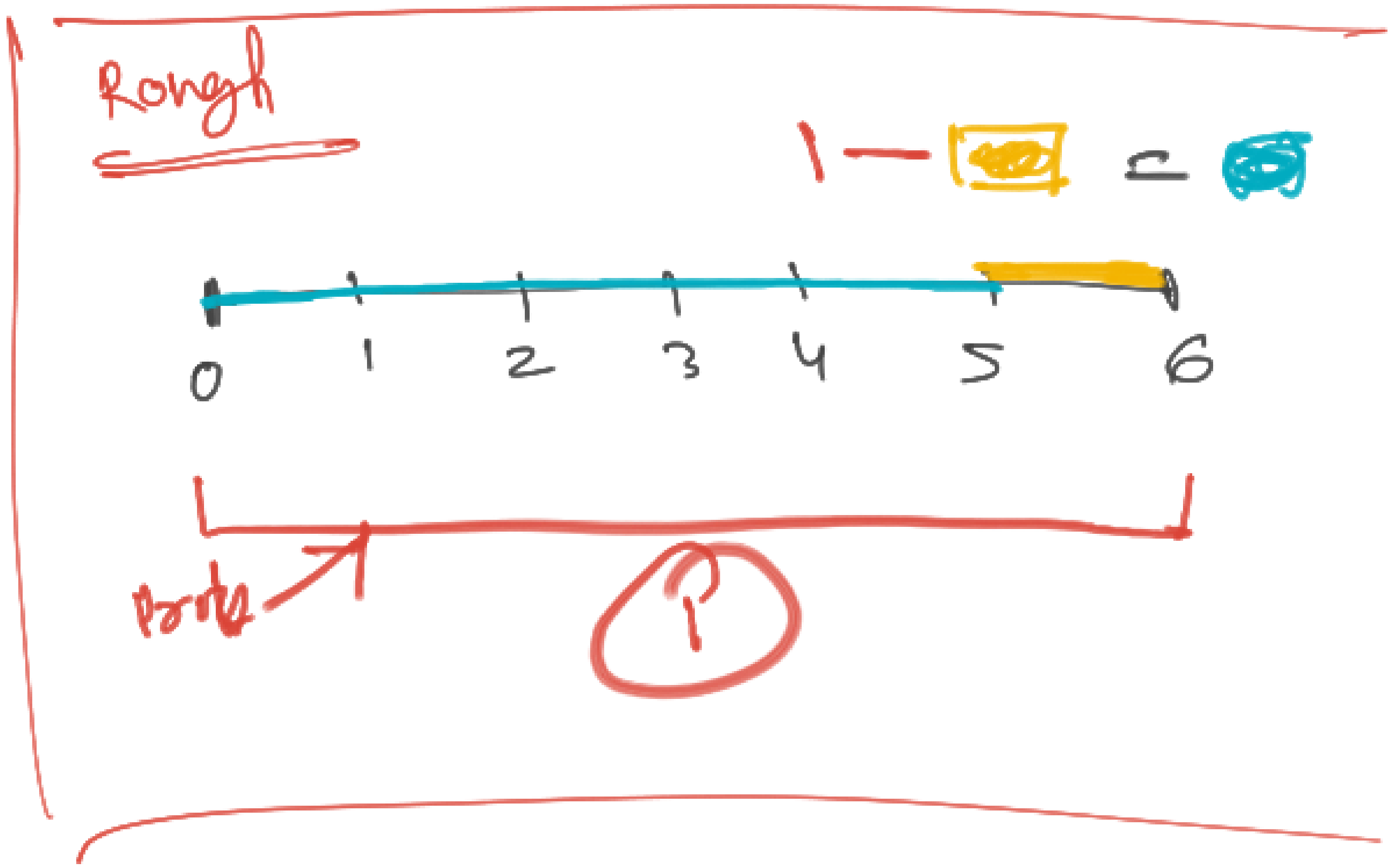
(iii) exactly once.

(b) If he fires so that the probability of his hitting the target atleast once is greater than $\frac{3}{4}$, find n.

Sol^n : $\because$ Probability of the man hitting a target is $\frac{1}{3}$

$\therefore$ $p = \frac{1}{3}$

$q = 1 - p = \frac{2}{3}$



(a) Given, $n = 6$

Let X = no. of times he hit the target

(i) $P(X \le 5) = 1 - P(X > 5) = 1 - P(X = 6) = 1 - \left[{}^6C_6 \, (p)^6 (q)^{6-6}\right]$

$= 1 - \left[1 \cdot \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^0\right] = 1 - \left(\frac{1}{3}\right)^6 = ?$

(ii) $P(x \geq 5) = P(X=5) + P(X=6)$

$$= \left[{}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{6-5}\right] + \left[{}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^6\right]$$

$= ?$

(iii) $P(X=1) = {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{6-1} = ?$

(b)

A|q

$$P(X \geq 1) > \frac{3}{4}$$

$$\Rightarrow 1 - P(X < 1) > \frac{3}{4}$$

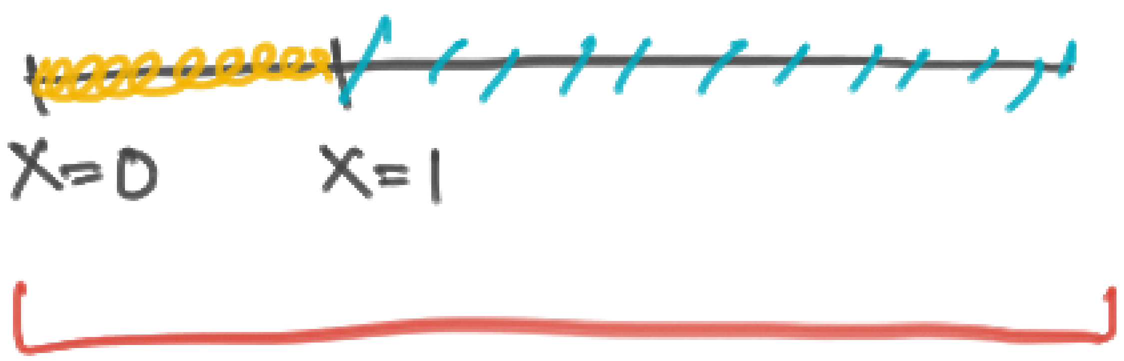
$$\Rightarrow 1 - P(X = 0) > \frac{3}{4}$$

$$\Rightarrow 1 - {}^{n}C_{0} \left(\frac{1}{3}\right)^{0} \left(\frac{2}{3}\right)^{n-0} > \frac{3}{4}$$

$$\Rightarrow 1 - \left(\frac{2}{3}\right)^{n} > \frac{3}{4}$$

$$\Rightarrow 1 - \frac{3}{4} > \left(\frac{2}{3}\right)^{n} \Rightarrow \left(\frac{2}{3}\right)^{n} < \frac{1}{4} \Rightarrow 2^{n} \cdot 4 < 3^{n}$$

$$\Rightarrow 2^{n} \cdot 2^{2} < 3^{n}$$



$$\Sigma P(X) = 1$$

$$P[x \geq 1] = 1 - P[x < 1]$$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$\boxed{P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}}$$

$\Rightarrow 2^{n+2} < 3^n$

The above inequality holds for $n = 4$

Hence, the man must fire 4 times so that the probability of hitting the target atleast once is greater than $\frac{3}{4}$

$n=1$, $2^{1+2} = 2^3 = 8$

$8 \nless 3$ $\quad 3^1 = 3$

not possible

$n=2$, $2^{2+2} = 2^4 = 16$

$16 \nless 9$ $\quad 3^2 = 9$

not possible

$n=3$,

$2^{3+2} = 2^5 = 32$

$3^3 = 27$

$32 \nless 27$

not possible

$n=4$, $\quad 2^{4+2} = 64$

$3^4 = 81$

$2^{4+2} < 3^4$ $\quad$ possible

POISSON DISTRIBUTION

French Mathematician — S.D. Poisson

Poisson Distribution — Discrete Probability Distribution

Characteristics

1. $n \to \infty$ i.e. sufficiently large (infinitely large
   $p \to 0$ i.e. sufficiently small (infinitely small

parameter, $\lambda = np$ $\leftarrow$ finite no.

2. It consists of a single parameter i.e. λ.