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Q. Find the complex exponential Fourier Series representation of the following signals —

(a) $x(t) = 4\cos 2\omega_0 t$

Soln:- Comparing with $x(t) = A\cos\omega_0 t$ we get —

fundamental ang. freq., $\omega = 2\omega_0$

$\therefore x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$ [Complex Fourier Series representation]

Now,

$$x(t) = 4\cos 2\omega_0 t = \sum_{n=-\infty}^{\infty} a_n e^{jn 2\omega_0 t}$$

$$\therefore 4\cos 2\omega_0 t = 2[\cos 2\omega_0 t + j\sin 2\omega_0 t + \cos 2\omega_0 t - j\sin 2\omega_0 t]$$
$$= 2[e^{j2\omega_0 t} + e^{-j2\omega_0 t}]$$
$$= 2e^{j2\omega_0 t} + 2e^{-j2\omega_0 t}$$

$\therefore$ The complex fourier coefficients for $4\cos 2\omega_0 t$ are —

$a_{-1} = 2$ and $a_1 = 2$, $a_n = 0$, $|n| \neq 1$ //

(b) $x(t) = \cos^2 t$

Soln:- Given,

$$x(t) = \cos^2 t$$
$$\Rightarrow x(t) = \frac{1+\cos 2t}{2}$$
$$\Rightarrow x(t) = \frac{1}{2} + \frac{\cos 2t}{2}$$

$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}$

Here, fundamental angular frequency of $\cos 2t$ is $\omega_0 = 2$.

$$\therefore x(t) = \cos^2 t = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} a_n e^{jn2t}$$

By Euler's formula,

$$x(t) = \cos^2 t$$
$$\Rightarrow x(t) = \left(\frac{e^{jt} + e^{-jt}}{2}\right)^2$$

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$$\Rightarrow x(t) = \frac{e^{2jt} + e^{-2jt} + 2e^{2jt} \cdot e^{-2jt}}{4}$$

$$\Rightarrow x(t) = \frac{1}{2} + \frac{1}{4} \cdot e^{2jt} + \frac{1}{4} \cdot e^{-2jt}$$

$$\Rightarrow x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn2t}$$

The complex fourier coefficient of $\cos^2 t$ are —

$a_0 = 1/2$, $a_{-1} = 1/4$, $a_1 = 1/4$ and $a_n = 0$, for $|n| \neq 1$

(c) $x(t) = \sin(2t + \pi/4)$

Solⁿ:- Given,

$$x(t) = \sin(2t + \pi/4)$$

Comparing with $\sin(\omega_0 t + \theta)$ we get —

fundamental angular freq., $\omega_0 = 2$

$$\therefore x(t) = \sin(2t + \pi/4) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} a_n e^{jn2t}$$

By Euler's formula —

$$x(t) = \sin(2t + \pi/4)$$

$$\Rightarrow x(t) = \frac{e^{j(2t+\pi/4)} - e^{-j(2t+\pi/4)}}{2j}$$

$$\Rightarrow x(t) = -\frac{1}{2j} \cdot e^{-j\pi/4} \cdot e^{-j2t} + \frac{1}{2j} \cdot e^{j\pi/4}$$

$$\Rightarrow x(t) = \sum_{n=-\infty}^{\infty} a_n e^{j2nt}$$

$\therefore$ The complex fourier coefficient for $\sin(2t + \pi/4)$ are —

$$a_{-1} = -\frac{1}{2j} e^{-j\pi/4}, \qquad a_1 = \frac{1}{2j} e^{j\pi/4}$$

$$= -\frac{1}{2j}[\cos \pi/4 - j\sin \pi/4] \qquad = \frac{1}{2j}[\cos \pi/4 + j\sin \pi/4]$$

$$= -\frac{1}{2j}[1/\sqrt{2} - j/\sqrt{2}] \qquad = \frac{1}{2j}\left[\frac{1}{\sqrt{2}} + j/\sqrt{2}\right]$$

$$= -\frac{1}{2j}\left[\frac{1-j}{\sqrt{2}}\right] \qquad = \frac{1}{2j}\left[\frac{1+j}{\sqrt{2}}\right]$$

$$= \frac{\sqrt{2}}{4j}(j-1) \qquad = \frac{\sqrt{2}}{4j}(j+1)$$

$a_n = 0$, for $|n| = 1$

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