FOURIER SERIES (FS)

Fourier Series Representation :-

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}.}$$

This is the FS representation for fourier s/g $x(t)$.

$$x(t) = x(t+\overset{\text{fundamental period of the s/g.}}{T})$$

Fundamental frequency, $\omega_0 = \frac{2\pi}{T} = 2\pi f$.

We will consider the complex s/g, $x(t)=e^{j\omega_0 t}$, to represent the FS because then we can also solve s/ simple s/g using the complex s/g.

Fundamental period of $e^{j\omega_0 t}$ is T i.e. $e^{j\omega_0 t}$ keeps on repeating for every 'T', i.e., the value of exponential s/g may ~~be~~ change i.e. the value can be $e^{j\omega_0 t}$, $e^{j2\omega_0 t}$, $e^{j3\omega_0 t}$ .... etc.

So if we want to represent the set of exponentials, then,

$$\phi_k = \left\{ e^{j\omega_0 t}, e^{j2\omega_0 t}, e^{j3\omega_0 t} \ldots e^{jk\omega_0 t} \right\}_{k=-\infty \text{ to } \infty}$$

If we want to represent the linear combination of the set, then,

$$\phi_k(t) = \sum_{k=-\infty}^{\infty} e^{jk\omega_0 t}$$

If we want to represent this linear combination in the form of s/g then,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where, a_k is the coefficient of approximation.

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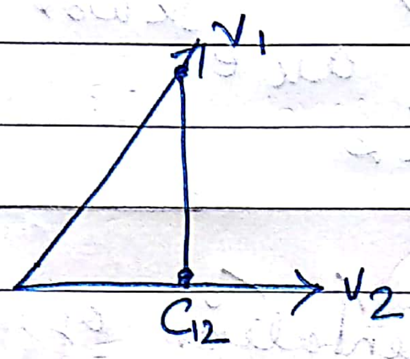
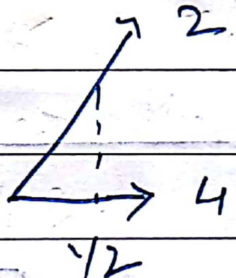
* Vector Analogy

If you want to represent any vectors v_1 in terms of v_2, then draw a perpendicular bisector & you need to trace the component from v_1 to v_2.



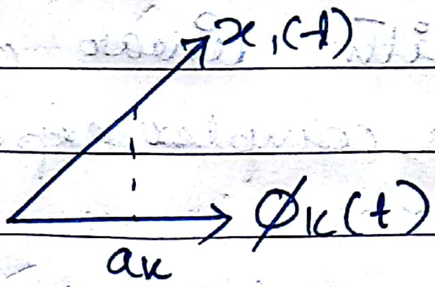
If you want to write v_1 in terms of v_2 then $v_1 = C_{12} V_2$.

* If you want to write 2 in terms of 4 then



$2 = \frac{1}{2} \times 4 = 2$

Here



Represent $x(t)$ in terms of set $\phi_k(t)$

$\therefore\ x(t) = a_k \phi_k(t)$.

$x(t) = a_k \phi_k(t)$

$\Rightarrow x(t) = a_k \sum_{k=-\infty}^{\infty} \phi\, e^{jk\omega_0 t}$

$\Rightarrow \boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.}$, This is the FS representation of the s/g $x(t)$.

Replace 'k' by 'n',

$$\boxed{x(n) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}.}$$

$\therefore$ fourier series is the representation of a s/g with linear combination of continuous complex exponential set.

Deriving the coefficient :—

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

We are going to derive the formula for a_k.

Multiply $e^{-jn\omega_0 t}$ on both sides.

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$$\therefore x(t)\, e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \cdot e^{-jn\omega_0 t}.$$

$$\Rightarrow x(t)\, e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t}$$

Integrating $\int_0^T$ both sides,

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$$\Rightarrow \int_0^T x(t)\, e^{-jn\omega_0 t} = \int_0^T \sum_{k=-\infty}^{\infty} a_k\, e^{j(k-n)\omega_0 t}$$

$$\Rightarrow \int_0^T x(t)\, e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\omega_0 t}$$

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RHS using Euler's formula,

$$\int_0^T e^{j(k-n)\omega_0 t} dt = \int_0^T \cos(k-n)\omega_0 t \, dt$$
$$+ j\int_0^T \sin(k-n)\omega_0 t \, dt$$

$$\int_0^T e^{j(k-n)\omega_0 t} dt = \begin{cases} T, & \text{when } k=n \\ 0, & \text{when } k \neq n. \end{cases}$$

* If $k=n$, $k-n=0$

$\cos 0 = 1$

$\sin 0 = 0$

$$\therefore \int_0^T \cos(k-n)\omega_0 t \, dt$$
$$= (t_0)_0^T = T.$$

If you take diff. values of k and n you will get 0.

At one point of k, we are getting a value i.e. at $k=n$. So, we will consider that.

$$\therefore \int_0^T x(t)\, e^{-jn\omega_0 t} = a_n T \quad \rightarrow k=n.$$

$$\boxed{\therefore a_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt}$$