

Q. A random variable X has the following distribution

X	-2	-1	0	1	2	3
$P(X)$	0.1	k	0.2	$2k$	0.3	k

Determine

① the value of k

② $P(X < 1)$

③ $P(X \geq 1)$

④ $P(X > 1)$

⑤ μ

⑥ σ^2

Soln:

\therefore It is a probability distribution

$$\therefore \sum P(X) = 1$$

$$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + k = 1$$

$$\Rightarrow 0.6 + 4k = 1$$

$$\Rightarrow 4k = 0.4$$

$$\Rightarrow k = 0.1$$

The Probability Distribution is given by

X	-2	-1	0	1	2	3
P(X)	0.1	0.1	0.2	0.2	0.3	0.1

$$\textcircled{2} \quad P(X < 1) = P(X = -2) + P(X = -1) + P(X = 0)$$

$$= 0.1 + 0.1 + 0.2$$

$$= 0.4$$

$$\textcircled{3} \quad P(X \geq 1) = P(X=1) + P(X=2) + P(X=3)$$

$$= 0.2 + 0.3 + 0.1$$

$$= 0.6$$

$$\textcircled{4} \quad P(X > 1) = P(X=2) + P(X=3)$$

$$= 0.3 + 0.1$$

$$= 0.4$$

$$(5) \text{ Mean, } \mu = \sum x p(x)$$

$$= [(-2) \cdot (0.1)] + [(-1) \cdot (0.1)] + [0 \cdot (0.2)] + \\ [1 \cdot (0.2)] + [(2) \cdot (0.3)] + [(3) \cdot (0.1)]$$

$$= 0.8$$

$$(6) \text{ Variance, } \sigma^2 = \sum x^2 p(x) - [\sum x p(x)]^2$$

$$= [(-2)^2 \cdot (0.1)] + [(-1)^2 \cdot (0.1)] + [0^2 \cdot (0.2)] + [1^2 \cdot (0.2)] + [2^2 \cdot (0.3)] + [3^2 \cdot (0.1)] \\ - [0.8]^2$$

Q. A player tossed two coins. If he gets two heads^{then} he wins ₹4. If he gets one head then he wins ₹2 but if he gets two tails then he pays a penalty of ₹3. Calculate the expected value of the game to him.

HH	HT	TH
	TT	

Soln Let $X = \text{no. of heads}$

X can take the values 0, 1, 2

$$P_0 = P(X=0) = \text{Prob. of getting no heads} = \frac{1}{4}$$

$$p_1 = P(X=1) = \text{probability of getting one head} = \frac{2}{4}$$

$$p_2 = P(X=2) = \text{probability of getting two heads} = \frac{1}{4}$$

Now,

$$X=0 \Rightarrow x_0 = -3$$

(negative sign denotes penalty)

$$X=1 \Rightarrow x_1 = 2$$

| x_i denotes money.

$$X=2 \Rightarrow x_2 = 4$$

$$\therefore E[X] = \sum_i x_i p_i = x_0 p_0 + x_1 p_1 + x_2 p_2 = \left[(-3) \left(\frac{1}{4} \right) \right] + \left[2 \cdot \left(\frac{2}{4} \right) \right] + \left[4 \cdot \left(\frac{1}{4} \right) \right] = 1.5$$

∴ Expected value of the game = ₹ 1.25

Q. A fair coin is tossed until head or five tails occurs.

Find expected number of tosses of the coins.

Soln:

$p = \text{prob. of success}$
 $q = \text{prob. of failure}$

$p = \text{prob. of getting head} = \frac{1}{2}$

$q = \text{prob. of getting tail} = \frac{1}{2}$

X	1	2	3	4	5	6
outcome	H	TH	TTH	TTTH	TTTTH	TTTTT
probability	p	$q^1 p$	$q^2 p$	$q^3 p$	$q^4 p$	q^5

Expected no. of tosses is given by

$$E(X) = \sum x_i p_i$$

$$= 1 \cdot p + 2 \cdot q^1 p + 3 \cdot q^2 p + 4 \cdot q^3 p + 5 \cdot q^4 p + 6 \cdot q^5$$