- Parseval's Theorem.
- F.T. of periodic s/g.
- Limitations & advantages of F.T.
- Numericals.

Q1. Find the F.T. of $x(t) = \delta(t+2) + \delta(t+1) + \delta(t-1) + \delta(t-2)$

$$\Rightarrow X(\omega) = \int_{-\infty}^{\infty} [\delta(t+2) + \delta(t+1) + \delta(t-1) + \delta(t-2)] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \delta(t+2) e^{-j\omega t} dt + \int_{-\infty}^{\infty} \delta(t+1) e^{-j\omega t} dt$$

$$+ \int_{-\infty}^{\infty} \delta(t-1) e^{-j\omega t} dt$$

$$+ \int_{-\infty}^{\infty} \delta(t-2) e^{-j\omega t} dt$$

$$= e^{2j\omega t} + e^{j\omega} + e^{-j\omega} + e^{-2j\omega}$$

$$= 2\left[\frac{e^{j\omega} + e^{-j\omega}}{2} + \frac{e^{j2\omega} + e^{-j2\omega}}{2}\right]$$

$$= 2[\cos\omega + \cos 2\omega]$$

Q2. Find F.T. of $x(t) = e^{-|t|}, \quad -2 \le t \le 2$

$= 0, \quad$ otherwise.

$$\Rightarrow X(\omega) = \int_{-\infty}^{\infty} e^{-|t|} e^{-j\omega t} dt$$

$$= \int_{-2}^{0} e^{-(-t)} e^{-j\omega t} dt + \int_{0}^{2} e^{-(t)} e^{-j\omega t} dt$$

$$= \int_{-2}^{0} e^{t} e^{-j\omega t} dt + \int_{0}^{2} e^{-t} e^{-j\omega t} dt$$

$$= \int_{-2}^{0} e^{-(1-j\omega)t} dt + \int_{0}^{2} e^{-(1+j\omega)t} dt$$

$$= \left[\frac{e^{-(1-j\omega)}}{-(1-j\omega)}\right]_{0}^{2} + \left[\frac{e^{-(1+j\omega)}}{-(1+j\omega)}\right]_{0}^{2}$$

$$= \frac{e^{-2(1-j\omega)} - 1}{-(1-j\omega)} + \frac{e^{-2(1+j\omega)} - 1}{-(1+j\omega)}$$

8. Find F.T. of the fgs:—

(1) $e^{-2t} \cos 5t \, u(t)$

② $t e^{-at} u(t)$.

Q. Find F.T. of the f/gs:—

① $e^{-2t} \cos 5t\, u(t)$

$\Rightarrow x(t) = e^{-2t} \cos 5t\, u(t)$

$= e^{-2t}\left[\frac{e^{j5t} + e^{-j5t}}{2}\right] u(t)$

$\therefore X(\omega) = F\left[e^{-2t} \cos 5t\, u(t)\right]$

$= \int_{-\infty}^{\infty} e^{-2t}\left[\frac{e^{j5t} + e^{-j5t}}{2}\right] e^{-j\omega t}\, u(t)\, dt$

$= \frac{1}{2}\left[\int_{0}^{\infty} e^{-2t} e^{j5t} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-2t} e^{-j5t} e^{-j\omega t}\, dt\right]$

$= \frac{1}{2}\left[\int_{0}^{\infty} e^{-[2+j(\omega-5)]t}\, dt \cdot \int_{0}^{\infty} e^{-[2+j(\omega+5)]}\, dt\right]$

$$= \frac{1}{2}\left[\frac{e^{-[2+j(\omega-5)]t}}{-[2+j(\omega-5)]} + \frac{e^{-[2+j(\omega+5)]t}}{-[2+j(\omega+5)]}\right]_0^{\infty}$$

$$= \frac{1}{2}\left[\frac{e^{-\infty} - e^{0}}{-[2+j(\omega-5)]} + \frac{e^{-\infty} - e^{0}}{-[2+j(\omega+5)]}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(2+j\omega)-j5} + \frac{1}{(2+j\omega)+j5}\right]$$

$$= \frac{1}{2}\left[\frac{2+j\omega}{(2+j\omega)^2+5^2}\right]$$

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② $t e^{-at} u(t)$.

$$X(\omega) = F[t e^{-at} u(t)]$$

$$= \int_{-\infty}^{\infty} t e^{-at} u(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} t e^{-(a+j\omega)t} dt$$

$$= \left[t \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$= -\int_{0}^{\infty} \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} dt$$

$$= \left[t \frac{e^{-a(+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty} - \left[\frac{e^{-(a+j\omega)}}{-(a+j\omega)^2} \right]_0^{\infty}$$

$$= 0 - 0 - 0 + \frac{1}{(a+j\omega)^2}$$

$$= \frac{1}{(a+j\omega)^2}$$