Symmetricities in Fourier Series (Clear

① Even Symmetry :-

$$x(t) = \underbrace{a_0}_{\text{dc value}} + \underbrace{\sum_{n=-\infty}^{\infty} a_n \cos n\omega_0 t}_{\text{cosine terms.}} + \underbrace{\cancel{\sum_{n=-\infty}^{\infty} b_n \sin n\omega_0 t}}_{\text{Sine terms}} \rightarrow 0$$

∴

$x(t) \rightarrow$ even means F.S. Expansion will have harmonics of even sig.

[when we perform time reversal of $\cos t$, i.e $\cos(-t)$ we get $\cos t$ itself i.e. it is even

but $\sin(-t) = -\sin t \rightarrow$ i.e. it is odd]

∴ $\boxed{b_n = 0}$

∴ The Fourier series expansion of an even sig does not contain sine terms. $\Rightarrow$ $b_n = 0$.

② Odd Symmetry :-

$x(t) =$ odd in nature.

F.S. expansion won't have even terms.

$\Rightarrow \boxed{a_n = 0}$

also avg. values $= 0$ i.e. $\boxed{a_0 = 0}$

& $\boxed{b_n \neq 0}$

& ∮.

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③ Half Wave Symmetry :- (HWS)

Fourier series expansion of HWS s/g contains only "odd harmonics."

$$x(t) = -x(t + T_0/2)$$

Condition for HWS

If there is s/g $x(t)$ & $x(t) = -x(t + T_0/2)$ half time period

& we perform time shifting by half time period & reversal & after performing it if

$$\underset{\downarrow}{x(t)} = -x\underset{\downarrow}{(t + T/2)}$$

C_{n_1} $\qquad$ C_{n_2}

∵ the two s/g are same

So their fourier coefficients are also same.

Now we have to find C_{n_2} in terms of C_{n_1}.

$$x(t) \rightleftharpoons C_{n_1}$$

$$x(t - t_0) \rightleftharpoons C_{n_1} e^{-jn\omega_0 t_0}$$

to get $T/2$

$$t_0 = -T/2$$

$$x(t + T/2) \rightleftharpoons C_{n_1} e^{jn\omega_0 T/2}$$

$$-x(t + T_0/2) \rightleftharpoons -C_{n_1} e^{jn\omega_0 T/2}$$

$$= C_{n_2}$$

Now, $C_{n_1} = C_{n_2}$

$$\cancel{C_{n_1}} = -\cancel{C_{n_1}} e^{jn\omega_0 T/2} \qquad \omega_0 = \frac{2\pi}{T}$$

$$1 = -e^{jn\omega_0 T/2} \qquad \frac{\omega_0 T}{2} = \pi$$

$$1 = -e^{jn\pi}$$

$$\Rightarrow 1 + e^{jn\pi} = 0 \qquad \Rightarrow 1 + (-1)^n = 0$$

$$\Rightarrow 1 + e^{(j\pi)n} = 0$$

$\Rightarrow$ n is an odd integer

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freq is $n\omega_0$ and it is odd.

$\Rightarrow$ only odd harmonics.

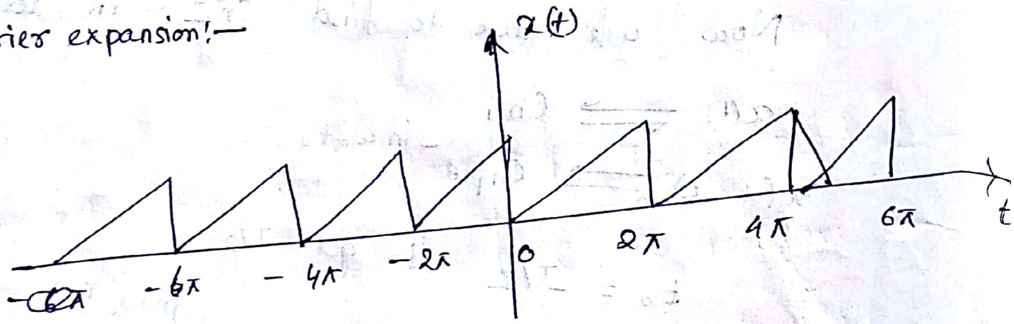
Odd + HWS

Even + HWS

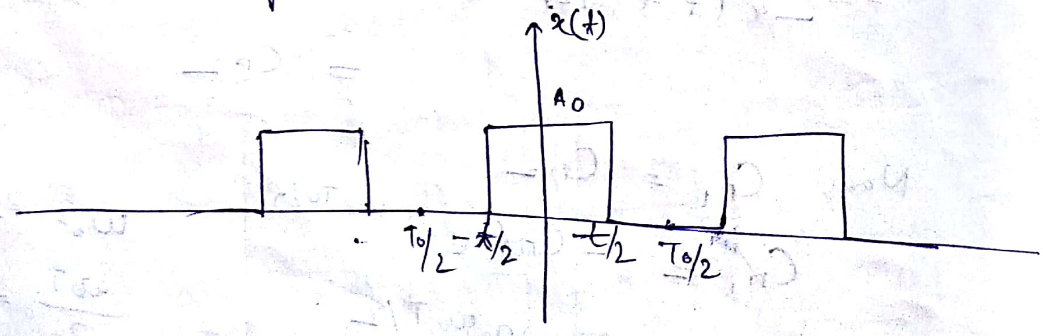
Hidden Symmetry

Questions :-

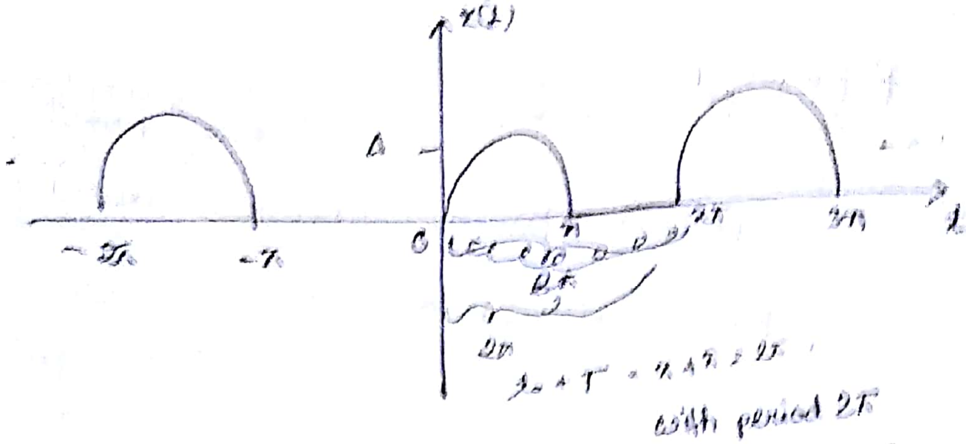
① Find the fourier expansion :-



② Find C_n for the s/g $x(t)$:-



Q Find the Fourier Series expansion of the half wave rectified sine wave —



Soln :- The periodic waveform, shown in the figure is half of a sine wave with period 2π.

$$\omega = \frac{2\pi}{T}$$

$$x(t) = \begin{cases} A \sin \omega t = A \sin \frac{2\pi}{2\pi} t = A \sin t & , 0 \le t \le \pi \\ 0 & , \pi \le t \le 2\pi \end{cases}$$

[Now the fundamental period T = 2π.

" freq. of $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

Let, $t_0 = 0$, $t_0 + T = T = 2\pi$.]

$$a_0 = \frac{1}{T} \int_0^T x(t)\,dt = \frac{1}{2\pi} \int_0^{2\pi} x(t)\,dt = \frac{1}{2\pi} \int_0^{\pi} A \sin t\,dt$$

$$= \frac{A}{2\pi} \left[-\cos t\right]_0^{\pi}$$

$$= \frac{A}{2\pi} \left[(-\cos\pi - \cos 0)\right]$$

$$= \frac{2A}{2\pi} = \frac{A}{\pi}$$

$$\therefore \boxed{a_0 = \frac{A}{\pi}}$$

a_n

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\, dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} x(t)\cos nt\, dt \qquad \because \omega_0 = 1$$

$$= \frac{1}{\pi}\int_0^{\pi} A\sin t\cos nt\, dt$$

$$= \frac{A}{\pi}\int_0^{\pi} \sin t\cos nt\, dt$$

$$= \frac{A}{2\pi}\int_0^{\pi} \sin(1+n)t + \sin(1-n)t$$

$$= \frac{A}{2\pi}\left[-\frac{\cos(1+n)t}{1+n} - \frac{\cos(1-n)t}{1-n}\right]_0^{\pi}$$

$$= -\frac{A}{2\pi}\left[\frac{\cos(1+n)\pi - \cos 0}{1+n} + \frac{\cos(1-n)\pi - \cos 0}{1-n}\right]$$

$$= -\frac{A}{2\pi}\left\{\left[\frac{(-1)^{n+1} - 1}{1+n}\right] + \frac{(-1)^{n-1} - 1}{1-n}\right\}$$

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For odd n, $a_n = -\frac{A}{2\pi}\left[\frac{1-1}{1+n} + \frac{1-1}{1-n}\right] = 0$

For even n, $a_n = \frac{-A}{2\pi}\left[\frac{-1-1}{1+n} + \frac{-1-1}{1-n}\right]$

$$= \frac{A}{2\pi}\left[\frac{2}{n+1} - \frac{2}{n-1}\right]$$

$$= -\frac{2A}{\pi(n^2-1)}$$

$\therefore$ $a_n = \frac{-2A}{\pi(n^2-1)}$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\, dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} x(t)\sin nt\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} A\sin t\sin nt\, dt = \frac{A}{\pi}\int_0^{\pi}\sin t\sin nt\, dt$$

$n=1$

$\frac{A}{\pi}\int_0^{\pi}\sin t\sin t\, dt$

$n=1$

$\frac{A}{\pi}\int_0^{\pi}\sin^2 t\, dt$

$= \frac{A}{2\pi}\int_0^{\pi}(1-\cos 2t)\, dt$

$= \frac{A}{2\pi}\left[t - \frac{\sin 2t}{2}\right]_0^{\pi}$

$= \frac{A}{2\pi}\left[\left(\pi - \frac{\sin 2\pi}{2}\right) - 0 - \frac{\sin 0}{2}\right]$

π