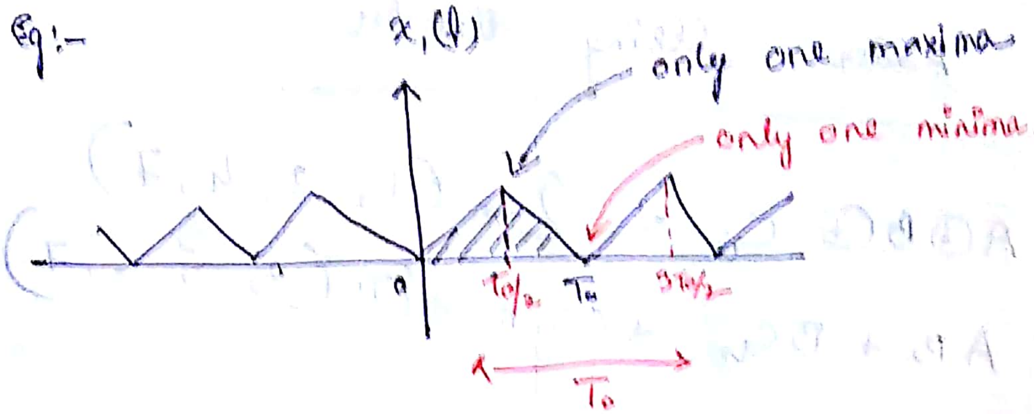
German mathematician Dirichlet.

Dirichlets Conditions

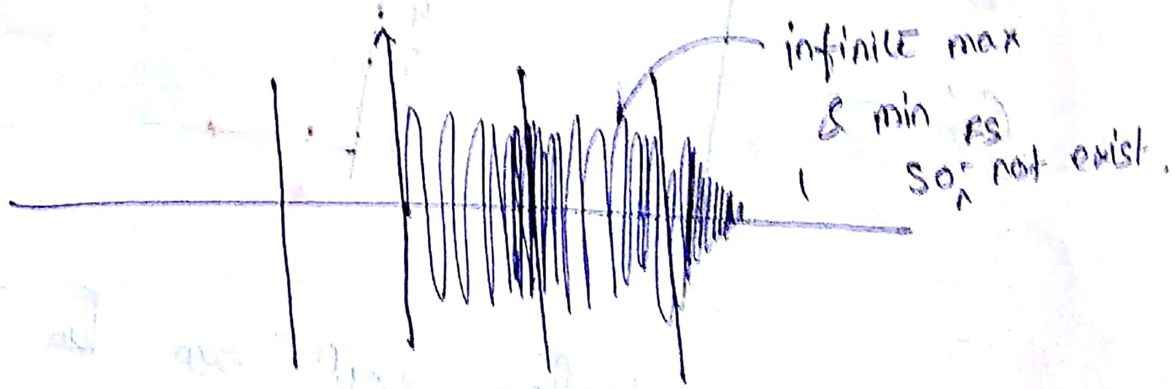
(Conditions for Existance of Fourier Series)

Conditions 1 s/g should have finite no. of maxima & minima over the range of time period.

Eg:-

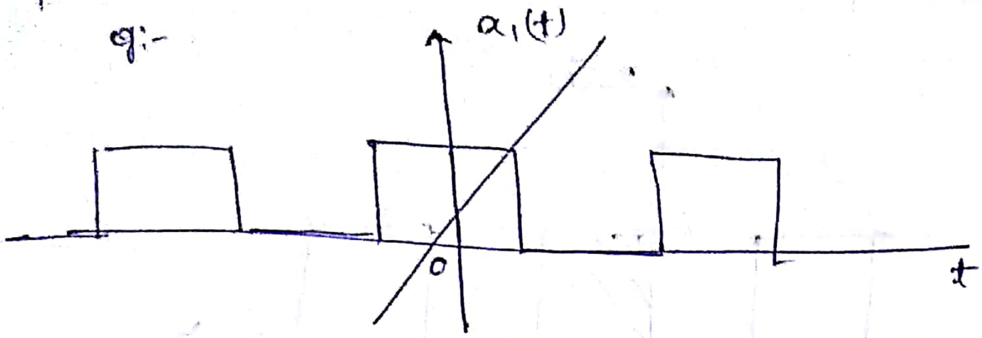


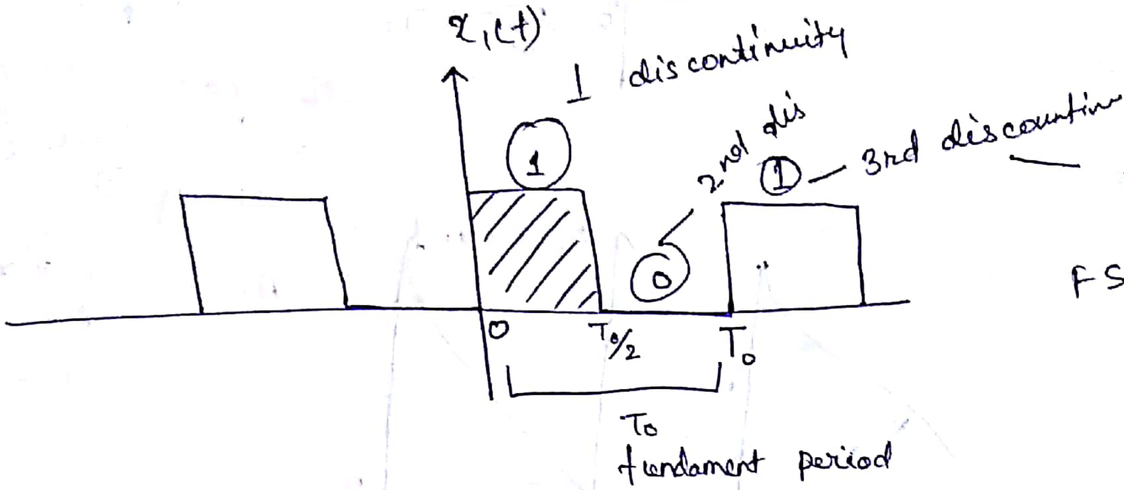
over a range of a time period T_0 we have only ~~only~~ one max. and one min.



Condition 2 S/g should have finite no. of discontinuities over the range of time period.

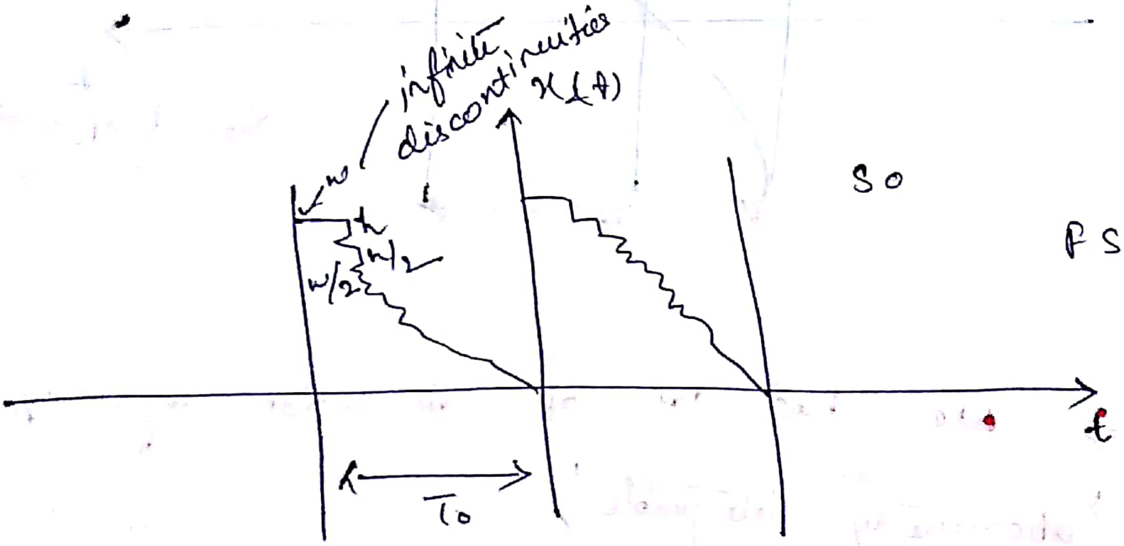
eg:-





finite .

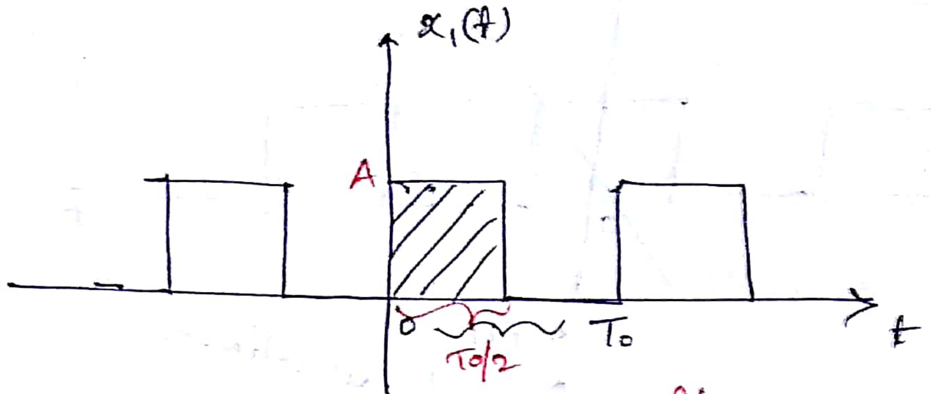
FS exist .



So

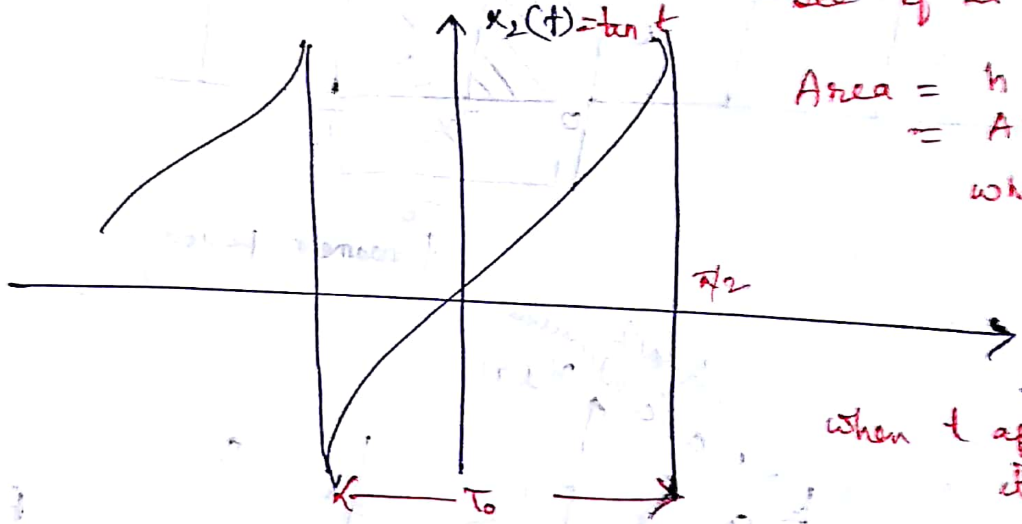
FS does not exist

Condition 3 S/g should be absolutely integrable over the range of time period.



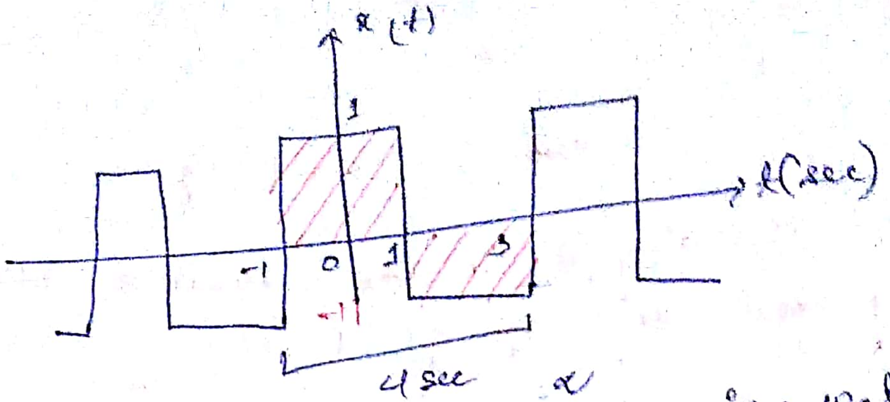
If you integrate it for 1 time period then see if it is finite.

Area = h × b
= A × $T_0/2$
which is finite



when t approaches $\pi/2$ it gives infinity

Ex :-



$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

a_0 : The given gr s/g is symmetrical about Time axis

i.e. the area in 1 Time period is $= 0$

The positive area will cancel out the negative area

$\therefore$ When you divide the total area by total Time period then it is 0.

$\therefore \boxed{a_0 = 0}$

b_n

Again, $x(-t) = x(t)$

$\therefore$ It is even s/g

When there is even s/g no need to calculate b_n as there will be no sine terms.

a_n

$$\therefore x(t) = \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t \, dt$$

$=$

$T_0 = 4$ sec

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/sec.

$\therefore a_n = \frac{2}{4} \int_{-1}^{3} x(t) . \cos \frac{n\pi}{2} t \, dt$

$= \frac{1}{2} \left[\int_{-1}^{1} (1) \cos \frac{n\pi}{2} t \, dt + \int_{1}^{3} (-1) . \cos \frac{n\pi}{2} t \, dt \right]$

$= \frac{1}{2} \left[\int_{-1}^{1} \cos \frac{n\pi}{2} t \, dt - \int_{1}^{3} \cos \frac{n\pi}{2} t \, dt \right]$

Assume $\frac{n\pi}{2} t = \theta$

$\frac{n\pi}{2} dt = d\theta$

$\Rightarrow dt = \frac{2}{n\pi} d\theta$

when $t = -1$

$\Rightarrow \theta = -\frac{n\pi}{2}$

$t = 1, \theta = \frac{n\pi}{2}$

$t = 3 = \theta = \frac{3n\pi}{2}$

$= \frac{1}{2} \left[\int_{-n\pi/2}^{n\pi/2} \cos \theta \frac{2}{n\pi} d\theta - \int_{n\pi/2}^{3n\pi/2} \cos \theta \frac{2}{n\pi} d\theta \right]$

$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{n\pi} \left[\int_{-n\pi/2}^{n\pi/2} \cos \theta \, d\theta - \int_{n\pi/2}^{3n\pi/2} \cos \theta \, d\theta \right]$

$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{n\pi} \left[(\sin \theta)_{-n\pi/2}^{n\pi/2} - (\sin \theta)_{n\pi/2}^{3n\pi/2} \right]$

$= \frac{1}{n\pi} \left[\sin n\pi/2 - \sin(-n\pi/2) - \sin 3n\pi/2 + \sin(n\pi/2) \right]$

$= \frac{1}{n\pi} \Big[$

S | A

T | TC

Case I $n =$ even,

$$a_n = \frac{1}{n\pi}\left[\sin\frac{n\pi}{2} + \cancel{\sin\frac{n\pi}{2}} - \cancel{\sin\frac{n\pi}{2}} + \sin\frac{n\pi}{2}\right]$$

$$a_n = \frac{2}{n\pi}\sin\frac{n\pi}{2}$$

$\boxed{a_n = 0}$ when n is even. (put values of n)

Case II $n =$ odd

Case a $n = 1, 5, 9, 13, \ldots$

a_n Case b $n = 3, 7, 11, 15, \ldots$

Case a $a_n = \frac{1}{n\pi}\left[\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2} + \sin\frac{n\pi}{2} + \sin\frac{n\pi}{2}\right]$

$\sin\frac{1\pi}{2}$ or $\sin 5\pi/2 = +1$

$$= \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

but $\sin 3\pi/2$ $\sin 7\pi/2 = -1$

Case b $a_n = -\frac{4}{n\pi}$

Parseval's Power Theorem

fourier coefficient

$$x(t) \rightleftharpoons C_n$$

& the time period $= T_0$

then Average Power

$$P_{x(t)} = \sum_{n=-\infty}^{\infty} |C_n|^2$$

Using this theorem we can calculate the power of the signal if we know its complex exponential fourier coeff.

Proof

$$x(t) = \sum_{n=-\infty}^{\infty} C_n . e^{jn\omega_0 t}$$

$z = a+ib$

$z^* = a-ib$

$|z| = \sqrt{a^2+b^2}$

$|z|^2 = a^2+b^2$

$z . z^* = a^2 - i^2 b^2$

$= a^2+b^2$

$= |z|^2$

Taking conjugates on both sides,

$$x^*(t) = \sum_{n=-\infty}^{\infty} C_n^* . e^{-jn\omega_0 t}$$

Now,

$$x(t)\, x^*(t) = |x(t)|^2$$

Again

$$P_{x(t)} = \frac{1}{T}\int_0^T |x(t)|^2 \, dt$$

$$= \frac{1}{T}\int_0^T x(t) . x^*(t)\, dt$$

$$= \frac{1}{T}\int_0^T x(t) \sum_{n=-\infty}^{\infty} C_n^* e^{-jn\omega_0 t} \, dt$$

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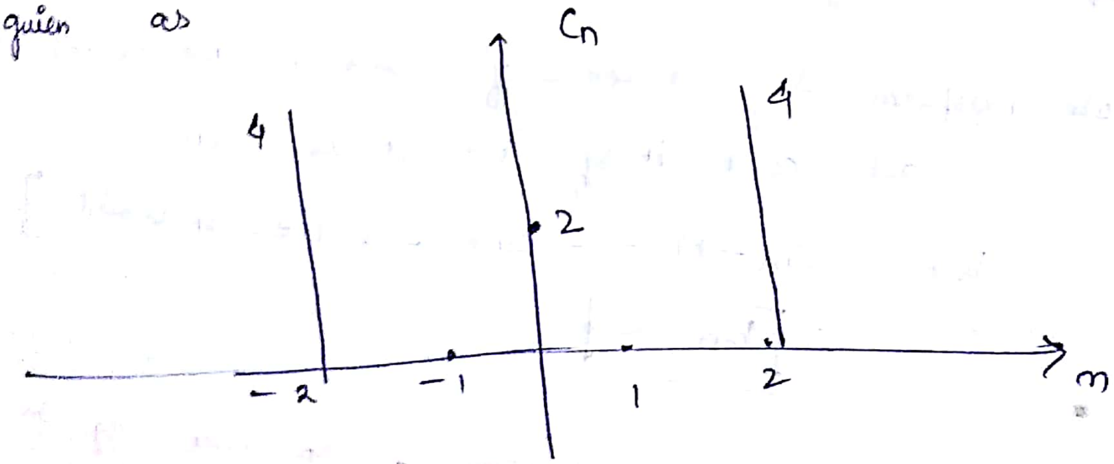
$$= \sum_{n=-\infty}^{\infty} C_n^* \underbrace{\frac{1}{T_0} \int_0^{T_0} x(t) e^{-jn\omega_0 t} dt}_{C_n}$$

$$= \sum_{n=-\infty}^{\infty} C_n^* . C_n$$

$$= \sum_{n=-\infty}^{\infty} |C_n|^2$$

Q. Find the average power of s/g $x(t)$, when C_n is given as



Sol^n

$$P_{x(t)} = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$-\infty$ to -3, $C_n = 0$

$C_{-2} = 4$

$\underline{\underline{Sol^n}}$

$$P_x(t) = \sum_{n=-\alpha}^{\alpha} |C_n|^2$$

$-\alpha$ to -3, $C_n = 0$

$C_{-2} = 4$

$C_{-1} = 0$

$C_0 = 2$

$C_1 = 0$

$C_2 = 4$

3 to $+\alpha$, $C_n = 0$

$$= |C_{-2}|^2 + |C_0|^2 + |C_2|^2$$

$$= |4|^2 + |2|^2 + |4|^2$$

$$= 16 + 4 + 16$$

$$= 36 \text{ Watts}$$