Lecture - 4

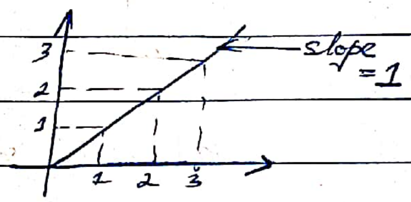
(2) Unit Ramp function :-

Continous Unit Ramp f^n, $r(t)$:-

It is that f^n that starts at $t=0$ and increases linearly with time and is defined as,

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

or, $r(t) = t\,u(t)$ [$\because u(t)=1$]



[Unit ramp f^n has unit slope. It can be obtained by integrating the unit step f^n, which means that a unit step f^n can be obtained by differentiating the unit ramp f^n]

$$r(t) = \int u(t)\,dt$$

$$\Rightarrow r(t) = \int dt \quad [\because u(t)=1]$$

$$\Rightarrow r(t) = dt \quad \text{for } t \geq 0$$

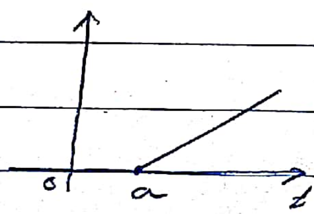
$$\text{or, } u(t) = \frac{d}{dt} r(t)$$

Slope :- steepness of a line. Change in y for unit change in x along a line.

Delayed unit ramp signal $r(t-a)$ is given by,

$$r(t-a) = \begin{cases} t-a, & t \geq a \\ 0, & t < a \end{cases}$$

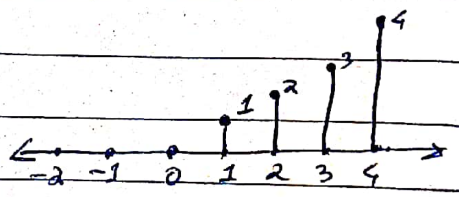
Or, $r(t-a) = (t-a)\,u(t-a)$



Discrete time unit ramp signal :-

$$r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

or, $r(n) = n\,u(n)$

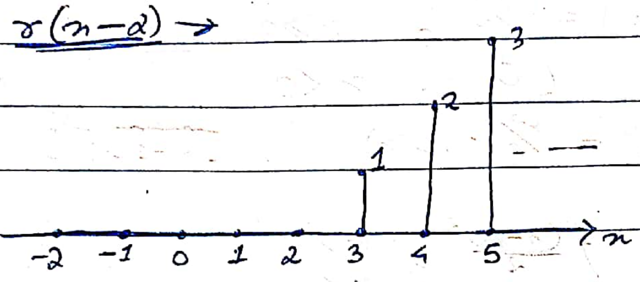


Shifted version →

$$r(n-k) = \begin{cases} n-k, & n \geq k \\ 0, & n < k \end{cases}$$

or, $r(n-k) = \cancel{(n-k)\,u(n)}\ (n-k)\,u(n-k)$

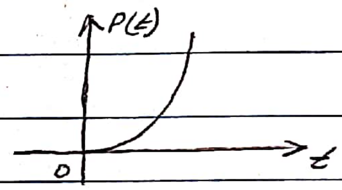
e.g. $r(n-2) \rightarrow$



③ Unit Parabolic Function :-

Continous-time unit parabolic f^n $p(t)$ also called unit acceleration s/g starts at $t=0$ & defined as,

$$p(t) = \begin{cases} t^2/2, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

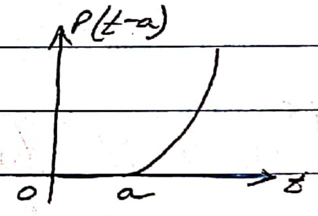


or, $p(t) = t^2/2 \cdot u(t)$

Shifted version,

$$p(t-a) = \begin{cases} \frac{(t-a)^2}{2}, & t \geq a \\ 0, & t < a \end{cases}$$

or, $p(t-a) = \frac{(t-a)^2}{2}\, u(t-a)$



Unit parabolic f^n can be obtained by integrating the unit ramp f^n, or, double integrating the unit step f^n.

i.e.
$$\begin{aligned} p(t) &= \iint u(t)\,dt \\ &= \int r(t)\,dt \\ &= \int t\,dt \\ &= t^2/2 \quad \text{for } t \geq 0 \end{aligned}$$

The ramp f^n is a derivative, and, the step f^n is a double-derivative, of the parabolic function.

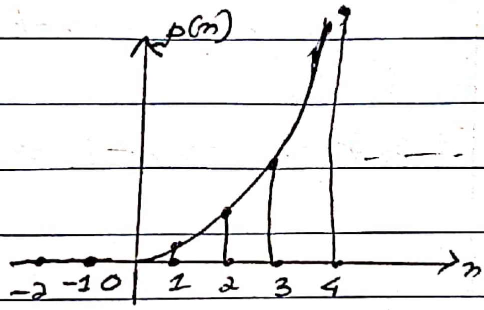
i.e. $r(t) = \frac{d}{dt} p(t)$

& $u(t) = \frac{d^2}{dt^2} p(t)$

Discrete time parabolic f^n $p(n)$,

$$p(n) = \begin{cases} n^2/2 & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$

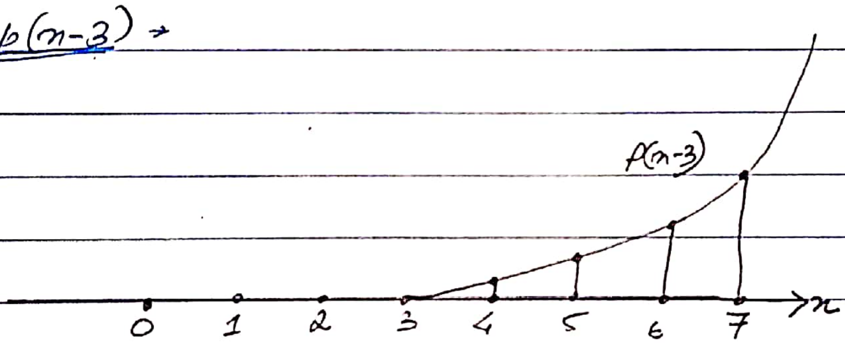
$$\Rightarrow p(n) = n^2/2 \cdot u(n)$$



And, $p(n-k) = \begin{cases} \frac{(n-k)^2}{2} & , n \geq k \\ 0 & , n < k \end{cases}$

or, $p(n-k) = \frac{(n-k)^2}{2} \cdot u(n-k)$

Ex:- $p(n-3) \rightarrow$



④ Unit Impulse Function :-

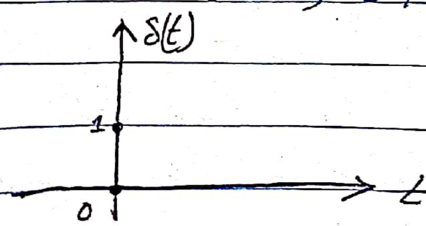
- Most widely used elementary function.
- The continous - time unit impulse f^n $\delta(t)$, also called "Dirac delta function" is defined as,

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

And, $\delta(t) = 0$, for $t \neq 0$

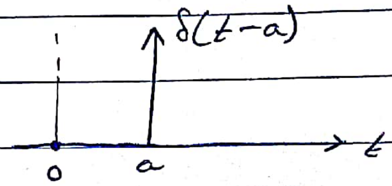
i.e.,

$$\delta(t) = \begin{cases} 1 & , t = 0 \\ 0 & , t \neq 0 \end{cases}$$



• Delayed unit impulse f^n $\delta(t-a)$ is defined as,

$$\delta(t-a) = \begin{cases} 1, & t = a \\ 0, & t \neq a \end{cases}$$



Integral of unit impulse f^n is a unit step f^n and, the derivative of unit step f^n is a unit impulse f^n.

$$\therefore \quad u(t) = \int_{-\infty}^{\infty} \delta(t)\,dt$$

$$\& \quad \delta(t) = \frac{d}{dt} u(t)$$

Properties of continous-time unit impulse function:-

① It is an even f^n of time i.e. $\delta(t) = \delta(-t)$

② $\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0)$; $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)\,dt = x(t_0)$

③ $\delta(at) = \frac{1}{|a|}\delta(t)$

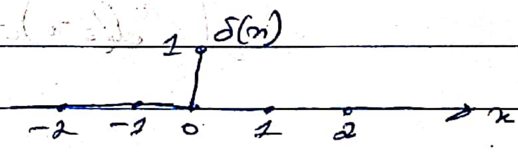
④ $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0) = x(t_0)$

$x(t)\,\delta(t) = x(0)\,\delta(t) = x(0)$

⑤ $x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$

• Discrete-time unit impulse f^n:-

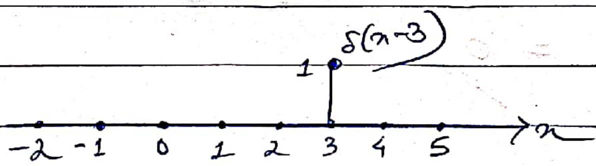
$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & ; n \neq 0 \end{cases}$$



Shifted discrete time unit impulse function,

$$\delta(n-k) = \begin{cases} 1, & n = k \\ 0, & n \neq k \end{cases}$$

Ex:- $\delta(n-3)$



Properties of discrete-time unit sample sequence :-

1. $\delta(n) = u(n) - u(n-1)$
2. $\delta(n-k) = \begin{cases} 1, & n=k \\ 0, & n \neq k \end{cases}$
3. $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$
4. $\sum_{k=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

—x—

Q. Evaluate the following integrals :-

ⓐ $\int_{-\infty}^{\infty} e^{-at^2}\,\delta(t-5)\,dt$

Solⁿ:- We know,

$$\delta(t-5) = \begin{cases} 1, & t=5 \\ 0, & \text{elsewhere} \end{cases}$$

$$\therefore \int_{-\infty}^{\infty} e^{-at^2}\,\delta(t-5)\,dt = \left[e^{-at^2}\right]_{t=5}$$
$$= e^{-a\cdot 25}$$
$$= e^{-25a}$$

ⓑ $\int_{0}^{\infty} t^2\,\delta(t-6)\,dt$

Solⁿ:- We know,

$$\delta(t-6) = \begin{cases} 1, & t=6 \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore \int_{0}^{\infty} t^2\,\delta(t-6)\,dt = \left[t^2\right]_{t=6}$$
$$= 36$$

ⓒ $\int_{0}^{3} \delta(t)\sin 5\pi t\,dt$

Solⁿ:- We know,

$$\delta(t) = \begin{cases} 1, & t=0 \\ 0, & \text{elsewhere} \end{cases}$$

$$\therefore \int_{0}^{3} \delta(t)\sin 5\pi t\,dt = \left[\sin 5\pi t\right]_{t=0}$$
$$= 0$$

(d) $\int_{-\alpha}^{\alpha} (t-2)^3 \, \delta(t-2) \, dt$

Soln:- We know,

$$\delta(t-2) = \begin{cases} 1, & t=2 \\ 0, & \text{elsewhere} \end{cases}$$

$$\therefore \int_{-\alpha}^{\alpha} (t-2)^3 \delta(t-2) \, dt = \left[(t-2)^3\right]_{t=2}$$

$$= 0$$

— x —