DIGITAL SYSTEMS

BOOLEAN ALGEBRA

- Switching circuits are also called logic circuits, gate circuits, and digital circuits. Switching algebra is also called Boolean algebra. Hence, the terms switching expressions and Boolean expressions mean the same thing.

- Boolean algebra is a system of mathematical logic. It is an algebraic system consisting of the set of elements (0, 1), two binary operators called OR and AND and one unary operator called NOT. It is the basic mathematical tool in the analysis ~~of~~ and synthesis of switching circuits. It is a way to express logic functions algebraically.

- In Boolean algebra,

$$A + A = A$$

$$\& \quad A \cdot A = A.$$

because the variable A has only a logical value.

In ordinary Algebra $A + A = 2A$

$$A \cdot A = A^2$$

In Boolean algebra,

$$1+1=1$$

In Binary no. system,

$$1+1=10$$

In ordinary algebra,

$$1+1=2$$

There is nothing like subtraction or division in Boolean algebra. Also, there are no negative or fractional no. in Boolean algebra. Multiplication & addition of the variables and functions are also only logical.

In Boolean algebra,

if $A = 1$, then $A \neq 0$

if $A = 0$, then $A \neq 1$.

LOGIC OPERATIONS :

→ The AND, OR & NOT are the three basic operations or function that are performed in Boolean algebra.

→ In addition, NAND, NOR, X-OR & X-NOR are also performed.

↳ AND operation
↳ OR operation
↳ NOT operation
↳ NAND operation
↳ NOR operation
↳ X-OR & X-NOR operation

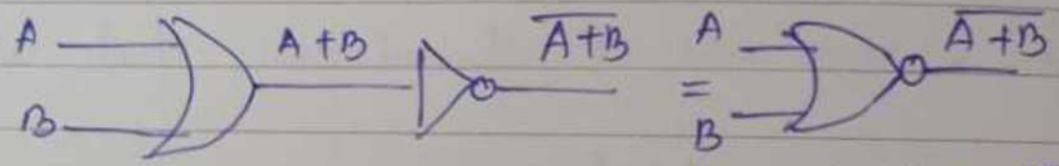
AXIOMS AND LAWS OF BOOLEAN ALGEBRA

De Morgan's Theorem :-

Law 1 : $\overline{A+B} = \bar{A}\,\bar{B}$

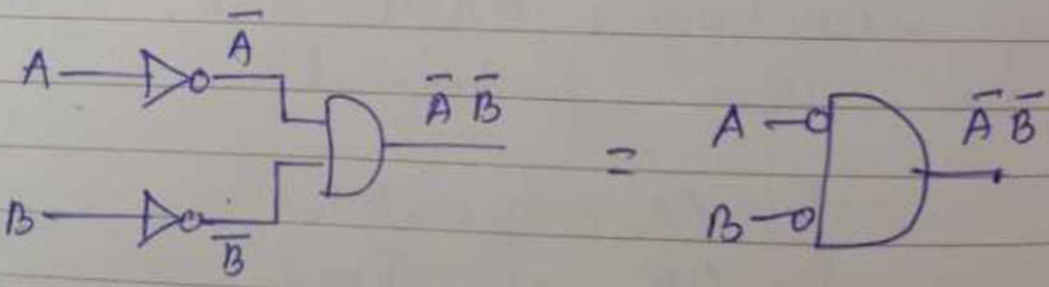
This law states that the complement of a sum of variables is equal to the product of their individual complements.

LHS



NOR gate

RHS



A	B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

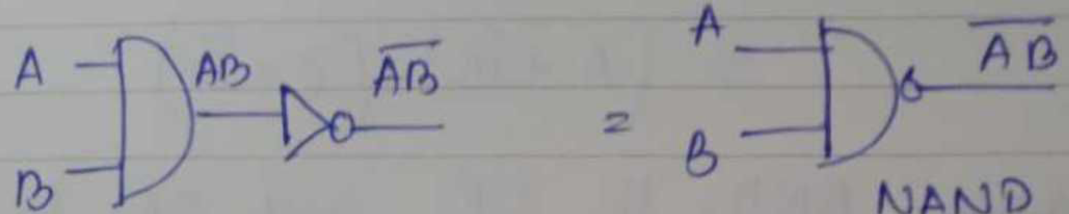
=

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\cdot\bar{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

Law 2 : $\overline{AB} = \bar{A} + \bar{B}$

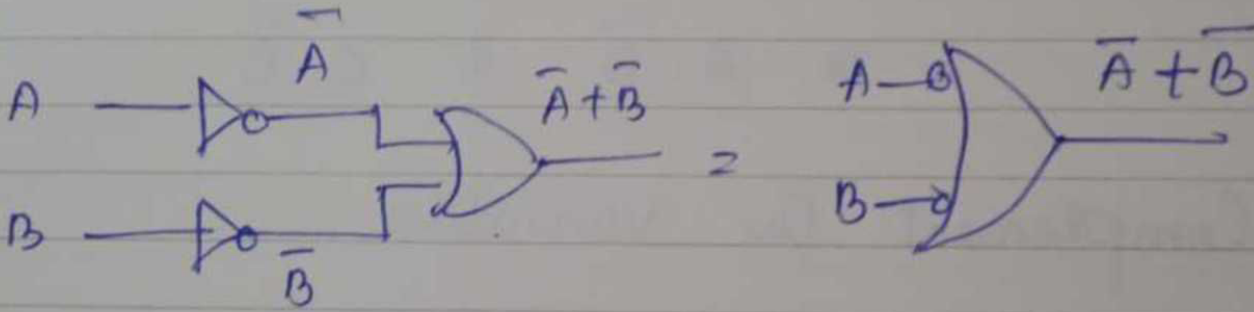
This law states that the complement of the product of variables is equal to the sum of their individual ~~components~~ complements.

LHS



NAND gate

RHS



A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

Ex:-1. Demorganize $f = \overline{(A+\overline{B})(C+\overline{D})}$

$$f = \overline{(A+\overline{B})(C+\overline{D})}$$

Complement the entire function

$$= (A+\overline{B})(C+\overline{D})$$

Change ANDs to ORs and ORs to ANDs

$$= A.\overline{B} + C.\overline{D}$$

Complement the variables

$$= \overline{A}.B + \overline{C}.D$$

Ex:-2. Apply Demorgan's theorem to the expression $f = \overline{\overline{A}B(CD+\overline{E}F)(\overline{AB}+\overline{CD})}$