

Date

22/10/21

Q. 1) Check whether the following systems are :-

1. Static or dynamic
2. Linear or non-linear
3. Casual or non-casual
4. Time invariant or time variant.

(a) $y(n) = a^n u(n)$

Solⁿ (i)

$$y(-1) = a^{-1} u(-1)$$

$$y(0) = a^0 u(0)$$

$$y(1) = a^1 u(1)$$

∴ Present y gives present u

∴ It is static.

(ii) ∴ $u(n)$ exists for all $n > 0$

∴ It is $y(n)$ is non-linear

(iii) $y(-1) = a^{-1} u(-1)$

$$y(0) = a^0 u(0)$$

$$y(1) = a^1 u(1)$$

∴ $y(n)$ is casual.

(iv) $y(n-T) = a^{n-T} u(n-T)$

$$y(n, T) = a^n u(n-T)$$

$$∴ y(n-T) \neq y(n, T)$$

∴ It is time variant.

③ $y(t) = ax(t) + bt^2x(t-2)$

Solⁿ:

(i) Given,

$$y(t) = atx(t) + bt^2x(t-2)$$

As there is time-shifting.

∴ It is dynamic.

(ii) For i/p, $x_1(t)$, $y_1(t) = atx_1(t) + bt^2x_1(t-2)$

For i/p, $x_2(t)$, $y_2(t) = atx_2(t) + bt^2x_2(t-2)$

∴ $ay_1(t) + by_2(t) = [a^2tx_1(t) + abt^2x_1(t-2)] + [abt_1x_1(t) + b^2t^2x_1(t-2)]$

The o/p due to weighted sum of i/p's is:-

$$\begin{aligned} y_3(t) &= T[ax_1(t) + bx_2(t)] \\ &= at[ax_1(t) + bx_2(t)] + bt^2[ax_1(t-2) + bx_2(t-2)] \\ &= [a^2tx_1(t) + abt^2x_1(t-2)] + [abt_1x_1(t) + b^2t^2x_1(t-2)] \end{aligned}$$

∴ $y_3(t) = ay_1(t) + by_2(t)$

∴ It is a linear

④ $y(-1) = -ax(-1) + 6x(-3)$

$y(0) = 0$

$y(1) = ax(1) + bx(-1)$

As the o/p depends on present and past values only

∴ It is casual.

$y(t) = atx(t) + bt^2x(t-2)$

$y(t-T) = a(t-T)x(t-T) + b(t-T)^2x(t-T-2)$

$y(t, T) = atx(t-T) + bt^2x(t-T-2)$

∴ $y(t-T) \neq y(t, T)$

∴ It is time variant.