Fourier Transform of standard signals:-

Impulse fn $\delta(t)$

Signal	F.T.
$\delta(t)$	1
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
Constant Amplitude (1)	$2\pi\delta(\omega)$
$\text{sgn}(t)$	$\frac{2}{j\omega}$
Unit step fn $u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
Rectangular pulse (Gate pulse) $\Pi(t/\tau)$ or $\text{rect}(t/\tau)$	$\tau\,\text{sinc}\frac{\omega\tau}{2}$
Triangular pulse $\Delta(t/\tau)$	$\tau/2\,\text{sinc}^2\frac{\omega\tau}{2}$

~~cosw~~ $\cos \omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
$\sin \omega_0 t$	$-j\pi[\delta(\omega+\omega_0)-\delta(\omega+\omega_0)]$

Properties of Continuous-Time Fourier Tx:

① Linearity Property	If $x_1(t) \xleftrightarrow{FT} X_1(\omega)$, $x_2(t) \xleftrightarrow{} X_2(\omega)$ Then $ax_1(t)+bx_2(t) \xleftrightarrow{FT} aX_1(\omega)+bX_2(\omega)$
② Time Shifting Property	If $x(t) \xleftrightarrow{F.T.} X(\omega)$ Then $x(t-t_0) \xleftrightarrow{F.T.} e^{-j\omega t_0}X(\omega)$
③ Frequency shifting Property	If $x(t) \xleftrightarrow{F.T.} X(\omega)$ $e^{j\omega_0 t}x(t) \xleftrightarrow{} X(\omega-\omega_0)$
④ Time Reversal Property	If $x(t) \xleftrightarrow{F.T.} X(\omega)$ Then $x(-t) \xleftrightarrow{F.T.} X(-\omega)$.
⑤ Time Scaling Property	If $x(t) \xleftrightarrow{FT} X(\omega)$ $x(at) \xleftrightarrow{F.T.} \frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$ $a>1 \rightarrow x(at)$ is compressed $a<1 \rightarrow x(at)$ is expanded version of $x(t)$.

⑥ Differentiation in Time Domain Property

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

$\frac{d}{dt}x(t) \xleftrightarrow{F.T.} j\omega X(\omega)$

⑦ Differentiation in Frequency Domain Prop.

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

then $-jt\,x(t) \xleftrightarrow{F.T.} \frac{d}{d\omega}X(\omega)$

⑧ Time Integration Property

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

then $\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{F.T.} \frac{1}{j\omega}X(\omega)$

if $X(0)=0$

⑨ Convolution Property

If $x_1(t) \xleftrightarrow{F.T.} X_1(\omega)$

& $x_2(t) \xleftrightarrow{F.T.} X_2(\omega)$

then $x_1(t) * x_2(t) \xleftrightarrow{F.T.} X_1(\omega)X_2(\omega)$

⑩ Multiplication Property

If $x_1(t) \xleftrightarrow{F.T.} X_1(\omega)$

& $x_2(t) \xleftrightarrow{F.T.} X_2(\omega)$

then $x_1(t)\,x_2(t) \xleftrightarrow{F.T.} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$

⑪ Duality Property (Symmetry)

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

$X(t) \xleftrightarrow{F.T.} 2\pi x(-\omega)$

(12) Modulation Property

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

then $x(t)\cos\omega_c t \xleftrightarrow{F.T.} \frac{1}{2}[X(\omega-\omega_c)+X(\omega+\omega_c)]$

(13) Conjugation Property

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

then $x^*(t) \xleftrightarrow{F.T.} X^*(-\omega)$

(14) Autocorrelation Property

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

then $R(\tau) \xleftrightarrow{F.T.} |X(\omega)|^2$

(15) Parseval's Relation/ Property

If $x_1(t) \xleftrightarrow{F.T.} X_1(\omega)$

& $x_2(t) \xleftrightarrow{F.T.} X_2(\omega)$

then $$\int_{-\infty}^{\infty} x_1(t)\, x_2^*(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega) X_2^*(\omega)\, d\omega$$

for complex $x_1(t)$ & $x_2(t)$.

(16) Area under the curve

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

$$\int_{-\infty}^{\infty} x(t)\, dt = \frac{1}{2\pi} X(0), \text{ for } \omega = 0$$

$$\int_{-\infty}^{\infty} X(\omega)\, d\omega = 2\pi x(0), \text{ for } t = 0$$

(17) (i) If $x(t)$ is real then $X_I(t) = 0$ & $X(-\omega) = X^*(\omega)$

$$X_R(\omega) = \int_{-\infty}^{\infty} x(t) \cos \omega t \, dt$$

then

$$X_I(\omega) = -\int_{-\infty}^{\infty} x(t) \sin \omega t \, dt$$

(ii) If $x(t)$ is even & real,

$$X_R(\omega) = 2\int_{0}^{\infty} x_e(t) \cos \omega t \, dt$$

$$X_I(\omega) = 0$$

(iii) When $x(t)$ is odd & real,

~~$x(t) = 2o$~~

$$X_1(\omega) = jX(\omega) = -j2\int_{0}^{\infty} x_o(t) \sin \omega t \, dt$$

(iv) For non-symmetrical f^n -

$$X(\omega) = X_e(\omega) + X_o(\omega).$$

$\cos\omega_0 t$

$$X(\omega) = F\left[\tfrac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)\right]$$

$$= \frac{1}{2}\left[F\left(e^{j\omega_0 t}\right) + F\left(e^{-j\omega_0 t}\right)\right]$$

$$= \frac{1}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$$

$$= \pi\left[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right]$$