Q. Ten coins are thrown simultaneously. Find the probability of getting atleast seven heads.

$P(X = r) = {}^nC_r \, p^r q^{n-r}$

$n \leftarrow$ total outcome

$p \leftarrow$ prob. of success

$q \leftarrow$ prob. of failure

Soln let, X = no. of heads

p = prob. of getting a head $= \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

$p = \frac{1}{2}$, $q = \frac{1}{2}$

Now,

Probability of getting atleast seven heads $= P(X \geq 7)$

$$= P(X=\overset{r}{7}) + P(X=\overset{r}{8}) + P(X=\overset{r}{9}) + P(X=\overset{r}{10})$$

$$= {}^{10}C_7 \left(\tfrac{1}{2}\right)^7 \left(\tfrac{1}{2}\right)^{10-7} + {}^{10}C_8 \left(\tfrac{1}{2}\right)^8 \left(\tfrac{1}{2}\right)^{10-8} + {}^{10}C_9 \left(\tfrac{1}{2}\right)^9 \left(\tfrac{1}{2}\right)^{10-9} + {}^{10}C_{10} \left(\tfrac{1}{2}\right)^{10} \left(\tfrac{1}{2}\right)^{10-10}$$

$$= ?$$

Q. $\overset{n}{\text{Six}}$ dice are thrown together at a time, the process is repeated 729 times. $\leftarrow N$ How many times do you expect atleast three dice to have 4 or 6.

Soln: let X = no. of die (having) giving 4 or 6.

p = prob. of getting 4 or 6 $= \frac{2}{6} = \frac{1}{3}$

1, 2, 3, 4, 5, 6

die's outcome

$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

At a single time, six dice are thrown simultaneously

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6).$$

$$= {}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 + {}^6C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^1 + {}^6C_6 \left(\frac{1}{3}\right)^6$$

$$= ?$$

The process is repeated 729 times so

Required no. of times atleast 3 dice will have 4 or 6

$= 729\, P(x \geq 3)$

$= 729\,(?)$

$= ?$

Q. If the sum of the mean and variance of binomial distribution of 5 trials is 4.8, find the distribution.

Soln: let, the binomial distribution be $^{n}C_{r}\, p^{r} q^{n-r}$

$n =$ no. of trials $= 5$

Mean of binomial distribution $= np$

variance of binomial distribution $= npq$

A/q

$np + npq = 4.8$

$np(1+q) = 4.8$

$\Rightarrow 5p(1+q) = 4.8$

$\Rightarrow 5(1-q)(1+q) = 4.8$ $\quad (\because p = 1-q)$

$\Rightarrow 5(1-q^2) = 4.8$

$\Rightarrow 1-q^2 = \frac{4.8}{5}$

$\Rightarrow 1-q^2 = \frac{48}{50}$

$\Rightarrow 50 - 50q^2 = 48$

$$50q^2 = 2$$

$$\Rightarrow q^2 = \frac{2}{50}$$

$$\Rightarrow q^2 = \frac{1}{25}$$

$$\Rightarrow q = \frac{1}{5}$$ ($\because$ Prob. cannot be negative).

$$\therefore \quad p = 1 - q = \frac{4}{5}$$

Hence, the required binomial distribution $= {}^5C_x \left(\frac{4}{5}\right)^x \left(\frac{1}{5}\right)^{5-x}$

Q. A student obtained the following answer to a certain problem given to him. Mean = 2.4, Variance = 3.2 for a binomial distribution. Comment on the result.

Soln: We know that,

Mean of Binomial distribution $= np$

Variance " " " $= npq$

Given,

$np = 2.4$

$npq = 3.2$

$\Rightarrow (2.4)q = 3.2$

$\Rightarrow q = \dfrac{3.2}{2.4} = 1.33 > 1$ which is not possible

$\therefore$ The given results are inconsistent.

q

Q. If on an average one ship in every ten is wrecked, find the probability that out of 5 ships expected to arrive, 4 atleast will arrive safely.

n

Soln Let, X = no. of ships arrived safely.

q = Prob. of ship getting wrecked $= \frac{1}{10}$

p = Prob. of ship arriving safely $= 1 - q = 1 - \frac{1}{10}$

$= \frac{9}{10}$

Here, $n = 5$

Required Prob. $= P(X \geq 4) = P(X = 4) + P(X = 5)$

$= ?$