Bss

[redacted]

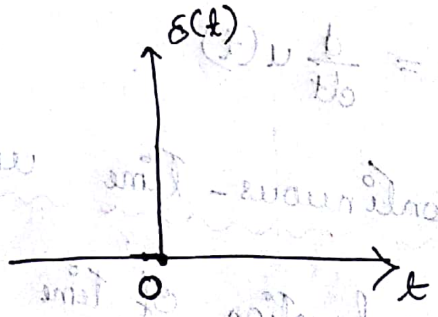
④ Unit Impulse function :-

- Most widely used elementary function.
- The continuous-time unit impulse function, $\delta(t)$, also called Dirac delta function is defined as,

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

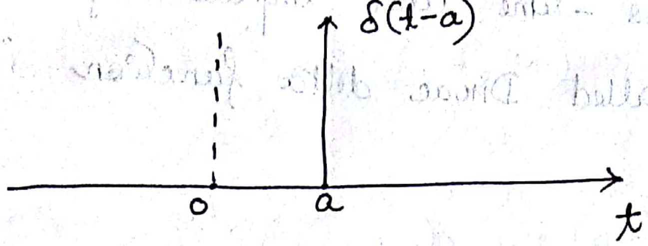
and, $\delta(t) = 0$, for $t \neq 0$

i.e. $\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$



- Delayed unit impulse function $\delta(t-a)$ is defined as,

$$\delta(t-a) = \begin{cases} 1, & t = a \\ 0, & t \neq a \end{cases}$$



- Integral of unit impulse function is a unit step function and derivative of unit step function is a unit impulse function.

$$u(t) = \int_{-\infty}^{\tau} \delta(t)\, dt$$

$$\delta(t) = \frac{d}{dt} u(t)$$

Properties of continuous-time unit impulse fn:-

1. It is an even function of time t, i.e $\delta(t) = \delta(-t)$

2. $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$; $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

3. $\delta(at) = \frac{1}{|a|}\delta(t)$

4. $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0) = x(t_0)$
   $x(t)\,\delta(t) = x(0)\,\delta(t) = x(0)$

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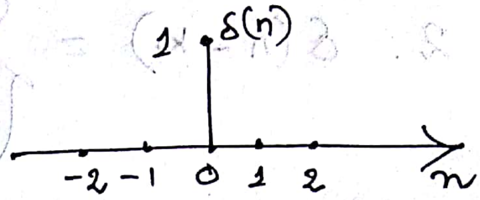
⑤ $x(t) = \int_{-\infty}^{\tau} x(\tau)\,\delta(t-\tau)\,d\tau$

• Discrete time unit impulse function :-

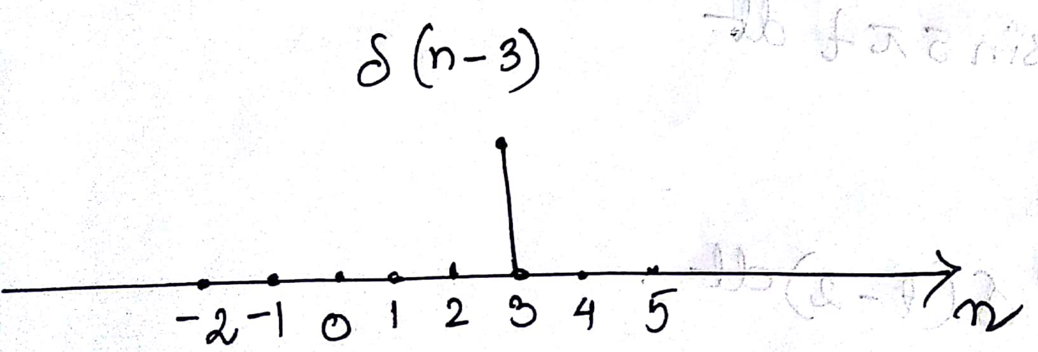
$$\delta(n) = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$



• Shifted discrete time unit impulse function:-

$$\delta(n-k) = \begin{cases} 1, & n=k \\ 0, & n \neq k \end{cases}$$

$\delta(n-3) = ?$


δ (n-3)
-2 -1 0 1 2 3 4 5
n

Properties of discrete - Time unit sample sequence:

1. $\delta(n) = u(n) - u(n-1)$

2. $\delta(n-k) = \begin{cases} 1, & n = k \\ 0, & n \neq k \end{cases}$

3. $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

4. $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

Q. <u>Evaluate the following integrals:-</u>

(a) $\int_{-\infty}^{\infty} e^{-at^2}\,\delta(t-5)\,dt$

(b) $\int_{0}^{\infty} t^2\,\delta(t-6)\,dt$

(c) $\int_{0}^{3} \delta(t)\,\sin 5\pi t\,dt$

(d) $\int_{-\infty}^{\infty} (t-2)^3\,\delta(t-2)\,dt$

(a) given,

$$\int_{-\alpha}^{\alpha} e^{-at^2}\,\delta(t-5)\,dt$$

we know, $\delta(t-5) = \begin{cases} 1, & t=5 \\ 0, & \text{elsewhere} \end{cases}$

$$\therefore \int_{-\alpha}^{\alpha} e^{-at^2}\,\delta(t-5)\,dt$$

$$= \left[e^{-at^2}\right]_{t=5}$$

$$= e^{-25a}$$

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(b) Given, $\int_{0}^{\infty} t^2 \delta(t-6)\, dt$

We know, $\delta(t-6) = \begin{cases} 1, & t=6 \\ 0, & \text{elsewhere} \end{cases}$

$$\therefore \int_0^{\infty} t^2 \delta(t-6)\, dt = \left[t^2\right]_{t=6}$$

$$= 36$$

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(c) Given,

$$\int_0^3 \delta(t) \sin 5\pi t \, dt$$

We know that,

$$\delta(t) = \begin{cases} 1, & t=0 \\ 0, & \text{elsewhere.} \end{cases}$$

$$\therefore \int_0^3 \delta(t) \sin 5\pi t \, dt$$

$$= \left[\sin 5\pi t\right]_{t=0}$$

$$= 0.$$

(d) Given,

$$\int_{-\alpha}^{\alpha} (t-2)^3 \, \delta(t-2) \, dt$$

We know that

$$\delta(t-2) = \begin{cases} 2, & t = 2 \\ 0, & \text{elsewhere} \end{cases}.$$

$$\therefore \int_{-\alpha}^{\alpha} (t-2) \, \delta(t-2) \, dt$$

$$= \left[(t-2)^3 \right]_{t=2}$$

$$= 0 .$$