* Representation of Fourier Series in Exponential form

Last class we got,

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}} \quad \text{——— (1)}$$

where $\omega_0 t = \frac{2\pi}{T}$

$$a_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt.$$

or if we integrate over one time period $\int_{t_0}^{t_0+T}$

$$\boxed{a_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, e^{-jn\omega_0 t}\, dt} \quad \text{——— (2)}$$

$$\therefore \quad \boxed{a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, dt.} \quad \text{——— (3)}$$

* Representation in Trigonometric form :—

let us consider a sinusoidal wave, with

$x(t) = A \sin\omega_0 t.$ with period $T = \frac{2\pi}{\omega_0}$

The sum of two ~~periodics~~ sinusoids is periodic provided that their frequencies are integral multiples of a fundamental freq, ω_0.

We can ... function ... periodi... L...

We can show that a signal $x(t)$, a sum of sine & cosine functions whose functions are integral multiples of ω_0, is a periodic signal.

Let the s/g $x(t)$ be

$$x(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \dots + a_K \cos K\omega_0 t$$
$$+ b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + \dots + b_K \sin K\omega_0 t$$

$$\Rightarrow x(t) = a_0 + \sum_{n=1}^{K} a_n \cancel{\cos n\omega_0 t} \cos\omega_0 n t$$
$$+ \sum_{n=1} b_n \sin \omega_0 n t .$$

where, $a_0, a_1, a_2 \dots a_K$

& $b_0, b_1, b_2 \dots b_K$ are constants

& ω_0 is the fundamental frequency.

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Now,

$$x(t + T)$$

Now,

$$x(t+T) = a_0 + \sum_{n=1}^{k} a_n \cos \omega_0 n (t+T) + b_n \sin \omega_0 n (t+T)$$

$$= a_0 + \sum_{n=1}^{k} a_n \cos \omega_0 n \left(t + \frac{2\pi}{\omega_0}\right) + b_n \sin \omega_0 n \left(t + \frac{2\pi}{\omega_0}\right)$$

$$= a_0 + \sum_{n=1}^{k} a_n \cos (\omega_0 n t + 2\pi n) + b_n \sin (\omega_0 n t + 2\pi n)$$

$$= a_0 + \sum_{n=1}^{k} a_n \cos \omega_0 n t + b_n \sin \omega_0 n t$$

$$= x(t).$$

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This proves that s/g $x(t)$, which is summation of sine & cosine functions of freq. $0, \omega_0, 2\omega_0 \dots k\omega_0$ is a periodic s/g with period T.

If $k \rightarrow \infty$ in the exp^n for $x(t)$, we obtain the Fourier series representation of any periodic s/g $x(t)$

This proves that s/g $x(t)$, which is summation of sine & cosine functions of freq. $0, \omega_0, 2\omega_0 \dots k\omega_0$ is a periodic s/g with period T.

If $k \to \infty$ in The exp^n for $x(t)$, we obtain the Fourier series representation of any periodic s/g $x(t)$

$$x(t) = \sum_{n=0}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t .$$

$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t} \quad \text{—④}$$

$a_n, b_n \to$ constants.

$a_0 \to$ also called dc component.

$a_1 \cos \omega_0 t + b_1 \sin \omega_0 t \to$ 1st harmonic

$a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t \to$ 2nd harmonic.

.... so on.

* Evaluation of Fourier coefficients

$$\left.\begin{matrix} a_0, & a_1 & \cdots & a_n \\ b_0, & b_1 & \cdots & b_n \end{matrix}\right\}$$ Fourier coefficients.

Now $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$.

Integrating both sides $\int_{t_0}^{t_0+T}$ (over 1 period)

$$\Rightarrow \int_{t_0}^{t_0+T} x(t)\,dt = a_0 \int_{t_0}^{t_0+T} dt + \int_{t_0}^{t_0+T} \left\{ \left[\sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \cos n\omega_0 t \right] dt \right\}$$

$$= a_0 T + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T} \cancel{\cos n\omega_0 t \, dt}^{\,0} + \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T} \cancel{\sin n\omega_0 t}^{\,0} \, dt$$

[$\therefore$ net areas of sinusoides are zero over complete periods for any non-zero integer n & any time t_0].

$$\therefore \int_{t_0}^{t_0+T} x(t)\,dt = a_0 T$$

$$\therefore \boxed{a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) \, dt} \quad \text{—— (5)}$$

$$\boxed{a_n = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \cos n\omega_0 t \, dt} \quad \longleftarrow \text{(6)}$$

$$\boxed{b_n = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \sin n\omega_0 t \, dt.} \quad \text{—— (7)}$$

[To find a_n, multiply the eq^n (4)

by $\cos m\omega_0 t$ & integrate over 1 period.

To find b_n, multiply the eq^n (4)

~~wt~~ by $\sin m\omega_0 t$ & integrate over 1 period.