Independent Random Variable

$$E[XY] = E[X]E[Y] \longrightarrow X \text{ \& } Y \text{ are independent R.V.}$$

Covariance

| X or x

If X and Y are two random variables with respective means $\bar{X}$ and $\bar{Y}$, then the covariance between X and Y is denoted by $\text{cov}(X, Y)$ and defined as

$$\text{cov}(X, Y) = E[(X - \bar{X})(Y - \bar{Y})]$$

⇒ The expected value of the deviations of the two variables from their means is called their covariance.

* The covariance of two independent variables is equal to zero

Proof

if X and Y are two R.V. then

$\bar{X}, \bar{Y} \in$ mean & constants

$$Cov(X, Y) = E[(X-\bar{X})(Y-\bar{Y})]$$

$$= E[XY - X\bar{Y} - \bar{X}Y + \bar{X}\bar{Y}]$$

$$\mathrm{cov}(X,Y) = E[XY] - E[\bar{X}Y] - E[X\bar{Y}] + E[\bar{X}\bar{Y}]$$

$$= \underline{E[X]E[Y]} - \bar{X}E[Y] - \bar{Y}E[X] + \bar{X}\bar{Y}$$

($\because$ X & Y are independent)

$$= \bar{X}\bar{Y} - \bar{X}(\bar{Y}) - \bar{Y}(\bar{X}) + \bar{X}\bar{Y}$$

($\because E[X] = \bar{X}$, $E[Y] = \bar{Y}$)

$$= \bar{X}\bar{Y} - \bar{X}\bar{Y} - \bar{Y}\bar{X} + \bar{X}\bar{Y}$$

$$= 0$$

* Correlation coefficient

$$\rho = \frac{\operatorname{cov}(X, Y)}{\sqrt{\operatorname{var}(X)}\,\sqrt{\operatorname{var}(Y)}}$$

Properties (covariance)

① $\operatorname{cov}(X, X) = \operatorname{var}(X)$

② If X and Y are independent then $\operatorname{cov}(X, Y) = 0$

3. $\operatorname{cov}(X, Y) = \operatorname{cov}(Y, X)$

4. $\operatorname{cov}(aX, Y) = a \operatorname{cov}(X, Y)$ $\qquad a, c \leftarrow$ constants

5. $\operatorname{cov}(X + c, Y) = \operatorname{cov}(X, Y)$

6. $\operatorname{cov}(X + Y, Z) = \operatorname{cov}(X, Z) + \operatorname{cov}(Y, Z)$.

Properties (correlation)

1. $-1 \leq \rho(X, Y) \leq 1$

2. If $\rho(X, Y) = 1$ then $Y = aX + b$ where $a > 0$

3. If $\rho(X, Y) = -1$ then $Y = aX + b$ where $a < 0$.

4. $\rho(ax+b, cY+d) = \rho(X, Y)$ for $a, c > 0$

Binomial Distribution

$n \leftarrow$ trials

$p \leftarrow$ probability of success

$q \leftarrow$ " " failure

$$\boxed{p + q = 1}$$

Bernoulli's Trial

$p \leftarrow$ prob. of success

$q \leftarrow$ prob. of failure

If there are 'r' success then $(n-r)$ failures in n trials then

$$\underline{P(X=r) = P(r) = {}^nC_r\, p^r q^{n-r}} \quad ; \quad r = 0, 1, 2, \ldots, n$$

→ Binomial Distribution

$$\underline{f(r) = N\,P(r) = N\left[{}^nC_r\, p^r q^{n-r}\right]}$$

N = no. of times the experiment is repeated (consisting of n trials)

→ Binomial Frequency Distribution

Properties

1. It is a discrete distribution.

2. It depends on two parameters p or q and n.

3. It is symmetrical if $p = q$.

4. Mean $= np$

   Variance $= npq$

   S.D. $= \sqrt{npq}$

5. Mode of Binomial Distribution = value of X that has the largest frequency.

$$\boxed{X \sim B(n, p)}$$

eg: $X \sim B\left(10, \frac{1}{2}\right)$

$\Rightarrow n = 10$

$p = \frac{1}{2}$

$q = 1 - p = \frac{1}{2}$

Q. Prove that in case of Binomial distribution

$$\text{Mean } (\mu) = np$$

$$\text{Variance } (\sigma^2) = npq$$

Conditions for application of Binomial Distribution

1. The variable should be discrete.
2. A dichotomy must exists, i.e. there should be two alternatives either success or failure.
3. n must be finite and small.
4. Trials or events must be independent, i.e. happening of one event must not affect happening of other.

5. The Trials or Events must be repeated under identical conditions.

Recursion formula or Recurrence relation for Binomial Dist.

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$\boxed{P(X=r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q}\, P(X=r)} \quad ; \quad r = 1, 2, 3, \ldots$$