Q. If $\mu = 50$ and $\sigma = 10$ then find

(i) $P(50 \le x \le 80)$

(ii) $P(60 \le x \le 70)$

(iii) $P(30 \le x \le 40)$

(iv) $P(40 \le x \le 60)$

Normal variate

$$Z = \frac{x - \mu}{\sigma}$$

$$\boxed{z = \frac{x - \mu}{\sigma}}$$

Sol:

Sol: Standard Normal variate, $Z = \dfrac{x - \mu}{\sigma}$

$$\Rightarrow Z = \frac{x - 50}{10} \quad \text{--- (A)} \quad (\because \mu = 50,\ \sigma = 10)$$

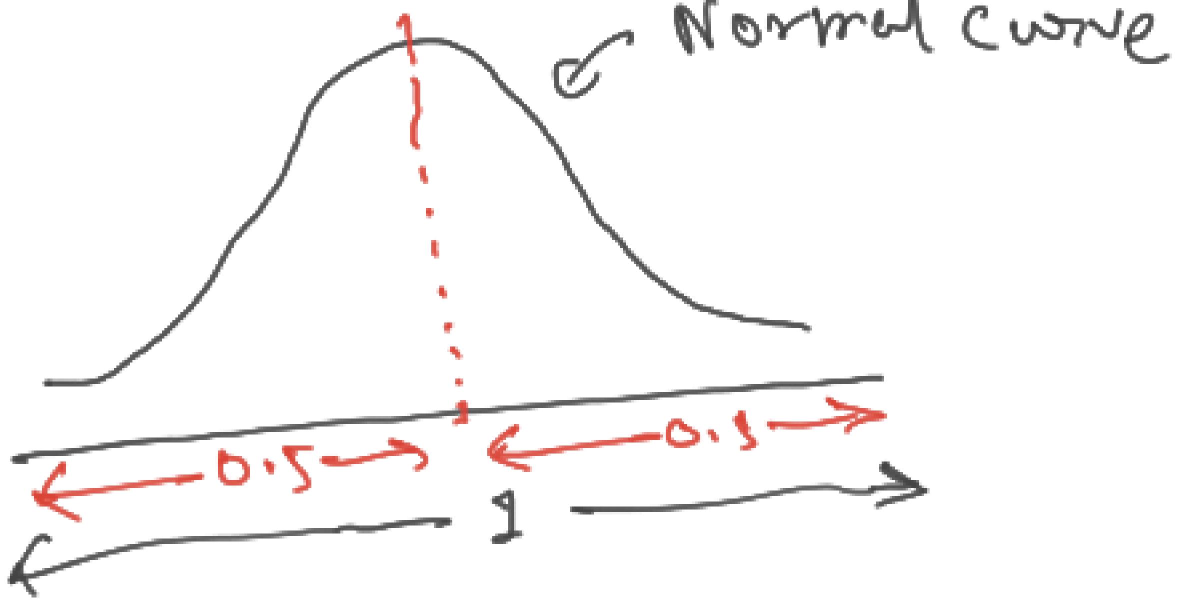
① $P(50 \le X \le 80)$

$$= P\left(\frac{50-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{80-\mu}{\sigma}\right)$$

$$= P\left(\frac{50-50}{10} \le Z \le \frac{80-50}{10}\right)$$

$$= P(0 \le Z \le 3)$$

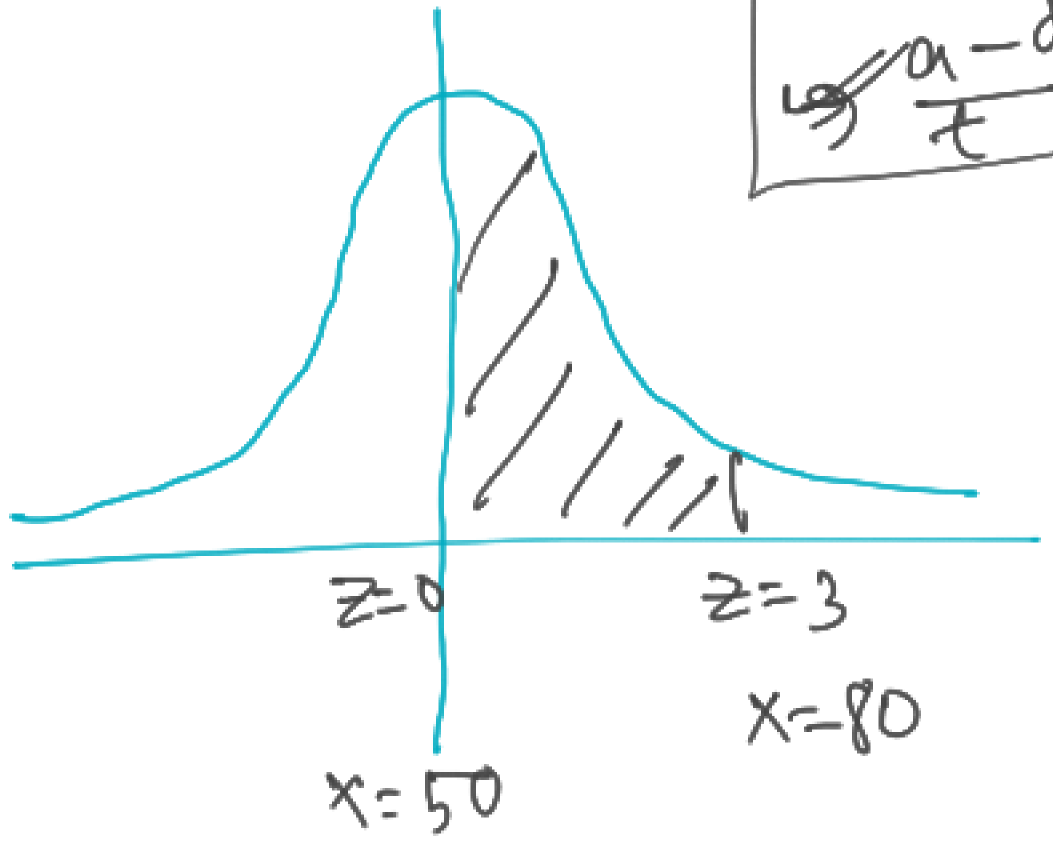
$$= 0.4987$$



$$a \le x \le b$$

$$a-d \le x-d \le b-d$$

$$\frac{a-d}{t} \le \frac{x-d}{t} \le \frac{b-d}{t}$$



(ii) $P(60 \le X \le 70) = P\left(\frac{60-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{70-\mu}{\sigma}\right)$

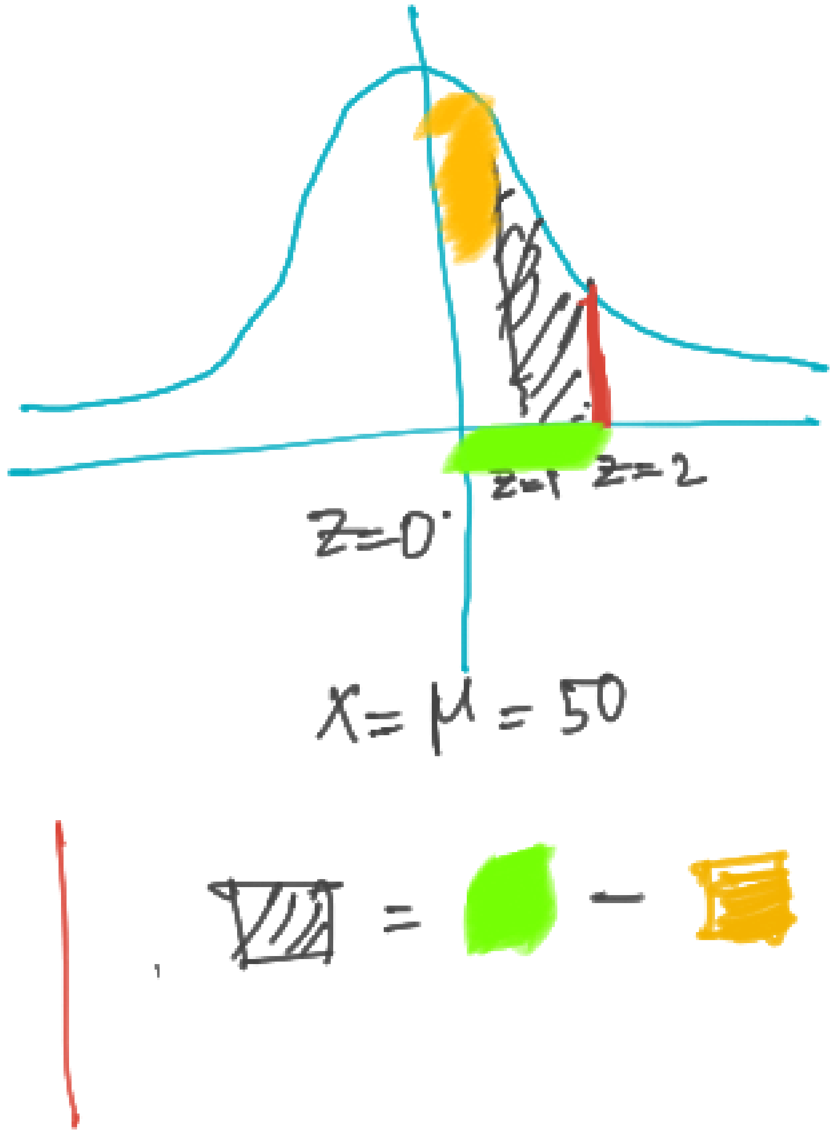
$$= P\left(\frac{60-50}{10} \le Z \le \frac{70-50}{10}\right)$$

$$= P(1 \le Z \le 2)$$

$$= P(0 \le Z \le 2) - P(0 \le Z \le 1)$$

$$= 0.4772 - 0.3413$$

$$= 0.1359$$

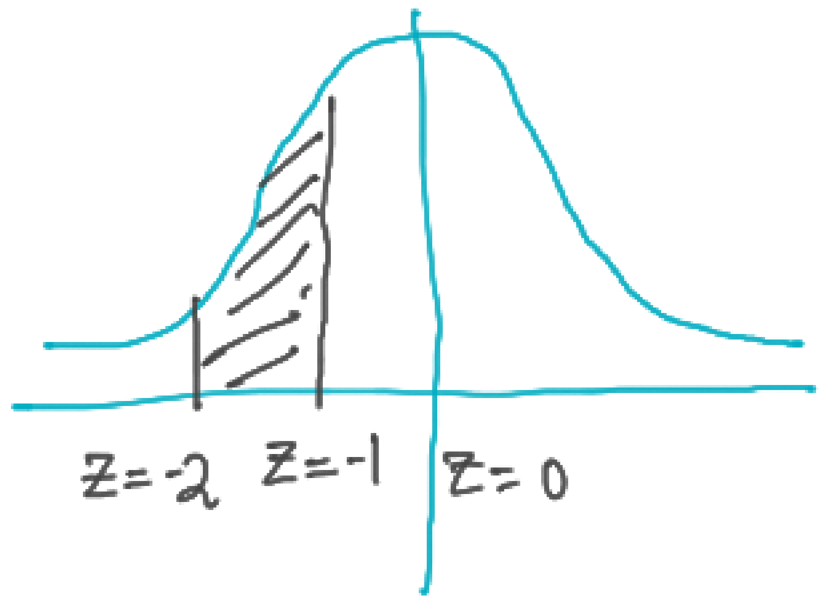


(iii) $P(30 \le X \le 40) = P\left(\frac{30-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{40-\mu}{\sigma}\right)$

$$= P\left(\frac{30-50}{10} \le Z \le \frac{40-50}{10}\right)$$

$$= P(-2 \le Z \le -1)$$

$$= P(1 \le Z \le 2) \quad (\because \text{The curve is symmetric})$$



$$= 0.1359$$

Due to Symmetricity

$P(-2 \le Z \le -1)$

$= P(1 \le Z \le 2)$

(iv) $P(40 \le X \le 60) = P\left(\frac{40-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{60-\mu}{\sigma}\right)$

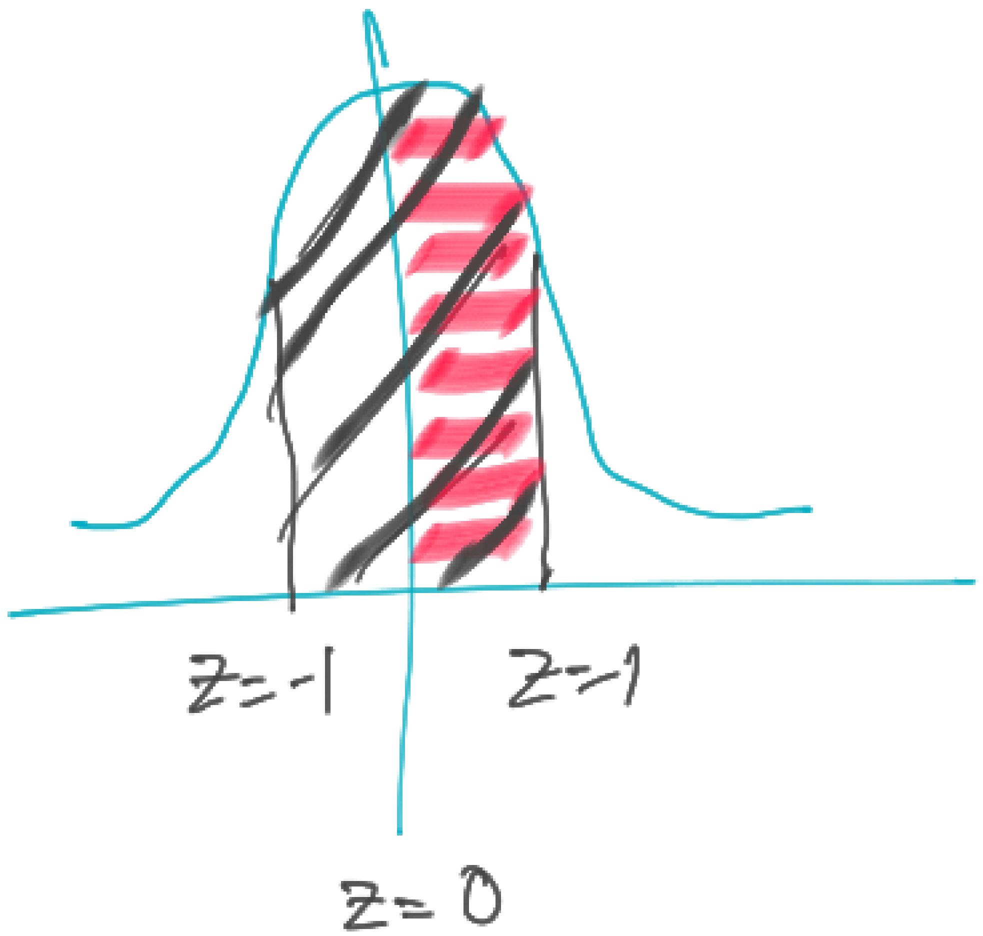
$= P\left(\frac{40-50}{10} \le Z \le \frac{60-50}{10}\right)$

$= P(-1 \le Z \le 1)$

$= 2\,P(0 \le Z \le 1)$ ($\because$ The curve is symmetric)

$= 2\,(0.3413)$

$= 0.6826$



Q. A sample of 100 dry battery cells tested to find the length of life produced the following results :

$$\mu = 12 \text{ hrs} \quad , \quad \sigma = 3 \text{ hrs}$$

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life

(a) more than 15 hrs

(b) less than 6 hrs

(c) betn 10 and 14 hrs.

Soln : Let X = length of life of the battery cells.

Standard Normal Variate, $z = \dfrac{X - \mu}{\sigma}$ —— (1)

Here, $\mu = 12$

$\sigma = 3$

$\therefore \quad z = \dfrac{X - 12}{3}$ —— (11)

(a) Probability of battery cells having life more than 15 hrs

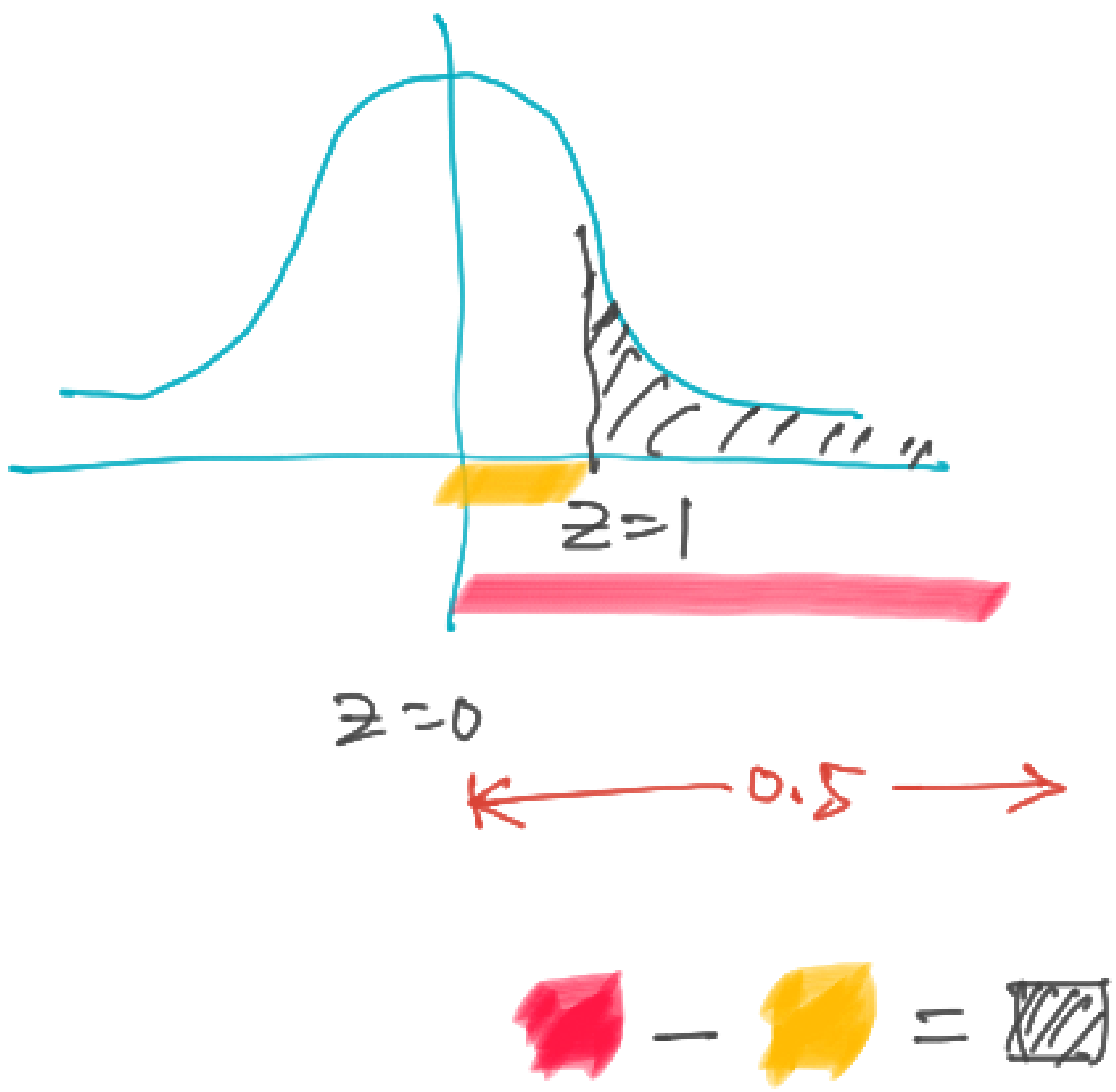
$= P(X > 15)$

$$= P\left(\frac{X-\mu}{\sigma} > \frac{15-\mu}{\sigma}\right)$$

$$= P\left(Z > \frac{15-12}{3}\right)$$

$$= P(Z > 1)$$

$$= 0.5 - P(0 \le Z \le 1)$$



$$= 0.5 - 0.3413$$

$$= 0.1587$$

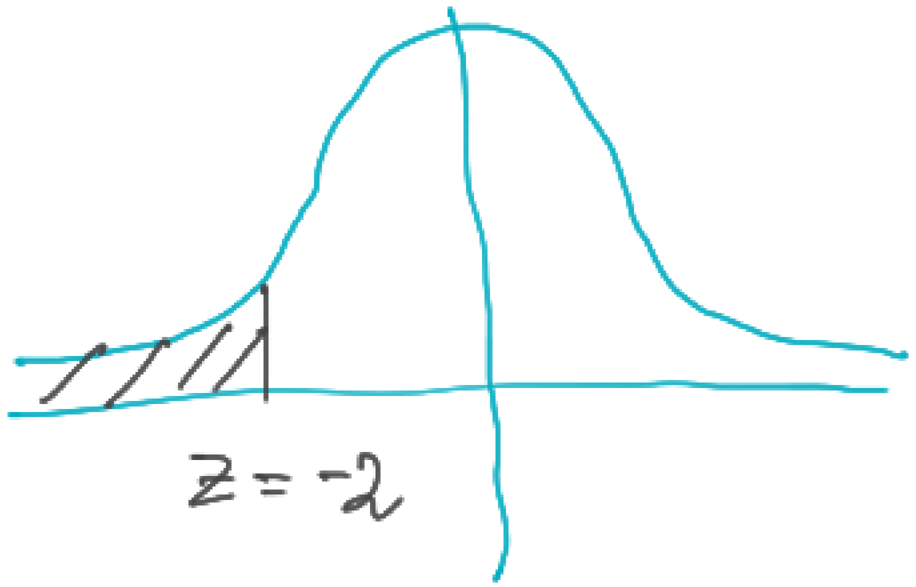
$\therefore$ Percentage of battery cells having life more than 15 hours $= 0.1587 \times 100 = 15.87\%$

(b) Probability of battery cells having life less than

6 hrs $= P(X < 6)$

$= P\left(\frac{X-\mu}{\sigma} < \frac{6-\mu}{\sigma}\right)$

$= P\left(Z < \frac{6-12}{3}\right)$



$= P(Z < -2)$

$= P(Z > 2)$ (∵ The curve is symmetric)

$= 0.5 - P(Z < 2)$ $= 0.5 - 0.4772$

$= 0.0228$

Percentage of battery cells having life less than 6 hrs

$= 0.0228 \times 100$

$= 2.28\ \%$

ⓒ Prob. of battery cells having life in betn 10 to 14 hrs

$= P(10 < X < 14)$

$= P\left(\frac{10-12}{3} < Z < \frac{14-12}{3}\right)$

$= P(-0.67 < Z < 0.67)$

$= 2P(0 < Z < 0.67)$

$= 2(0.2486)$

$= 0.4972$

$\therefore$ Percentage of battery cells having life in between 10 hrs and 14 hrs $= 0.4972 \times 100$

$= 49.72\%$