

Date
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Q.1) a) Solⁿ Let,

$$x(t) = \cos 6\pi t$$

Here,

$$\omega = 6\pi$$

$$\Rightarrow 2\pi f = 6\pi$$

$$\Rightarrow f = 3$$

$$\therefore T = \frac{1}{3} \text{ sec}$$

We know,

$x(t)$ will be periodic if $x(t) = x(t+T)$

Now,

$$x(t + \frac{1}{3}) = \cos 6\pi (t + \frac{1}{3})$$

$$= \cos(6\pi t + 2\pi)$$

$$= \cos 6\pi t \cos 2\pi - \sin 6\pi t \sin 2\pi$$

$$= \cos 6\pi t \cdot 1 - 0$$

$$= \cos 6\pi t$$

$$= x(t)$$

$\therefore \cos 6\pi t$ is periodic

1. (b) Solⁿ

$$x(t) = 2u(t) + 3\cos 2\pi t$$

$$\text{Let, } x(t) = x_1(t) + x_2(t)$$

$$\therefore x_1(t) = 2u(t)$$

$u(t)$ is not periodic

$$x_2(t) = 3\cos 2\pi t$$

$$\text{Here, } \omega = 2\pi$$

$$\therefore T = 1$$

x_2 is periodic

$\therefore 2u(t) + 3\cos 2\pi t$ is not periodic

Q.2) Solⁿ:

$$\begin{aligned}
 x(t) &= e^{j5t} \cos 2t \\
 &= (\cos 5t + j \sin 5t) \cos 2t \\
 &= \cos 5t \cos 2t + j \sin 5t \cos 2t \\
 &= \frac{1}{2} (\cos 7t + \cos 3t) + \frac{j}{2} (\sin 7t - \sin 3t)
 \end{aligned}$$

$$\therefore A_{\text{net}} = \sqrt{\left(\frac{1}{2}\right)^2 \times 4} = 1$$

$$\rho = \frac{A^2}{2} = \frac{1}{2}$$

$$\text{r.m.s.} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

3) Solⁿ:

$$x(n) = \left(\frac{1}{3}\right)^n u(n)$$

We have,

$$\begin{aligned}
 E &= \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x(n)|^2 \\
 &= \lim_{N \rightarrow \infty} \sum_{n=0}^N \left[\left(\frac{1}{3}\right)^n\right]^2 \cdot u(n) \\
 &= \lim_{N \rightarrow \infty} \sum_{n=0}^N \left(\frac{1}{9}\right)^n \\
 &= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n \\
 &= \frac{1}{1 - \frac{1}{9}} \\
 &= \frac{9}{9-1} \\
 &= \frac{9}{8} \text{ Joules}
 \end{aligned}$$

$$\begin{aligned}
 \rho &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2 \\
 &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N \left(\frac{1}{9}\right)^n \\
 &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \left[\frac{1 - \left(\frac{1}{9}\right)^{N+1}}{1 - \left(\frac{1}{9}\right)} \right] \\
 &= 0
 \end{aligned}$$

\therefore It is an energy signal

4. (a) $u(-2\pi)$

→ Anti-casual

(b) $e^{3t} u(t-2)$

→ Casual.

5. (a) Soln:

$$x(t) = \sin(2t + \pi/2)$$

$$= \sin(\pi/2 + 2t)$$

$$= \cos 2t$$

$$x(-t) = \sin(\pi/2 - 2t) = \cos 2t$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \cos 2t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2} = 0$$

5. (b) Soln:

$$x(n) = \{-2, 4, \underset{n}{1}, 3, 6\}$$

$$x(-n) = \{6, 3, \underset{n}{1}, 4, -2\}$$

$$x_e(n) = \frac{1}{2} [x(n) + x(-n)]$$

$$= \frac{1}{2} [4, 7, 2, 7, 4]$$

$$= \{2, 3.5, 1, 3.5, 2\}$$

$$= \{2, 1, 2\}$$

$$x_o(n) = \frac{1}{2} [x(n) - x(-n)]$$

$$= \frac{1}{2} [-8, 1, 0, -1, 0]$$

$$= \{-4, 0, 4\}$$