In Poisson Distribution, Probability of 'r' success in a random experiment with total 'n' trials is given by

$$P(r) = \frac{e^{-\lambda} \lambda^{r}}{r!} \quad \text{where} \quad \lambda = np$$

parameter

p = probability of success

Conditions

① The R.V. must be discrete.

② A dichotomy exists i.e. the happening of the event must be of two alternatives — success and failure.

3. It is applicable in those cases where the number of trials (n) is very large and the probability of success (p) is very small, but mean (λ) must be finite.

Parameter

$\lambda = np$

↳ mean

Q. Prove that in case of Poisson Distribution

Mean $= \lambda$

Variance $= \lambda$.

Recurrence Formula

$$P(r) = \frac{e^{-\lambda} \lambda^{r}}{r!}$$

$$P(r+1) = \frac{\lambda}{r+1} P(r) \quad ; \quad r = 0, 1, 2, \ldots$$

Q. Suppose a book of 585 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random, will be free from errors? (Use $e^{-0.735} = 0.4795$)

↳ zero error

Soln:

Let X = no. of errors in 10 pages

$n = 10$

$p = \frac{43}{585} = 0.0735$

$\therefore \lambda = np = 0.0735 \times 10 = 0.735$

Here, n is very large in comparison to p

So we can use Poisson Distribution

We know that,

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

Probability of getting no error i.e. free from errors.

$$P(X=0) = \frac{e^{-\lambda}\lambda^{0}}{0!}$$

$$= \frac{e^{-0.735}\ 1}{1} = 0.4795$$

Q. If a random variable X follows a Poisson Distribution such $P(X=2) = 9\,P(X=4) + 90\,P(X=6)$, find the mean and variance of X.

<u>Soln</u>: For Poisson Distribution,

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

A/q

$$P(X=2) = 9\,P(X=4) + 90\,P(X=6)$$

$$\Rightarrow \frac{e^{-\lambda}\lambda^{2}}{2!} = 9\left[\frac{e^{-\lambda}\lambda^{4}}{4!}\right] + 90\left[\frac{e^{-\lambda}\lambda^{6}}{6!}\right]$$

$$\Rightarrow \frac{1}{2} = \frac{9\lambda^2}{4.3.2} + \frac{90\lambda^4}{6.5.4.3.2}$$

$$\Rightarrow 1 = \frac{3\lambda^2}{4} + \frac{\lambda^4}{4}$$

$$\Rightarrow \lambda^4 + 3\lambda^2 - 4 = 0$$

$$\Rightarrow \lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0$$

$$\Rightarrow \lambda^2(\lambda^2+4) - 1(\lambda^2+4) = 0$$

$$\Rightarrow (\lambda^2-1)(\lambda^2+4) = 0 \qquad \Rightarrow \lambda^2 - 1 = 0 \qquad (\because \lambda > 0$$
$$\Rightarrow \lambda = 1 \qquad \lambda^2+4 \neq 0)$$

for Poisson Distribution,

$$\text{Mean} = \lambda = 1$$

$$\text{Variance} = \lambda = 1.$$

Q. If a random variable has a Poisson distribution s.t. $P(1) = P(2)$, find

(a) mean of the distribution

(b) $P(4)$

Soln. For Poisson distribution

$$P(X=r) = \frac{e^{-\lambda} \lambda^{r}}{r!}$$

A/q $P(1) = P(2)$

$\Rightarrow P(X=1) = P(X=2)$

$$\Rightarrow \frac{e^{-\lambda} \lambda^{1}}{1!} = \frac{e^{-\lambda} \lambda^{2}}{2!}$$

$\Rightarrow 1 = \frac{\lambda}{2} \quad \Rightarrow \lambda = 2$

(a) Mean $= \lambda = 2$

(b) $P(4) = P(X = 4) = \dfrac{e^{-\lambda} \lambda^4}{4!}$

$$= \frac{e^{-2} (2)^4}{4!}$$

$$= ?$$

Q. A certain screw making machine on average produces 2 defective screws out of 100, and packs them in boxes of 500. Find the probability of getting 15 defective screws.

Soln

let x = no. of defective screws

$n = 500$, $p = \frac{2}{100} = 0.02$

$\therefore \lambda = np = ?$

Probability of getting 15 defective screws $= P(X=15) = \frac{e^{-\lambda} \lambda^{15}}{15!} = ?$