Name :- Zubayer Ahmed Zidhan Laskar , Roll :- 20071000 7062

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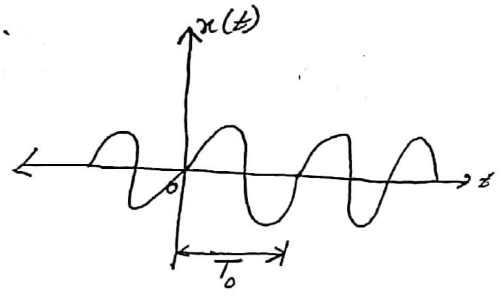
Subject :- Basics of Signal and Systems.

CIE-2

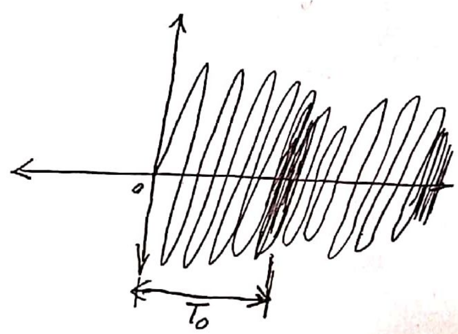
1. Ans:- The conditions under which any periodic waveform can be expressed using fourier series are as follows :-

(i) The signal should have a finite no. of maxima and minima over the range of the time period.

e.g.



(a)

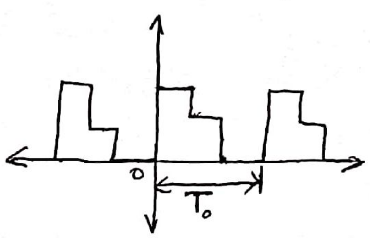


(b)

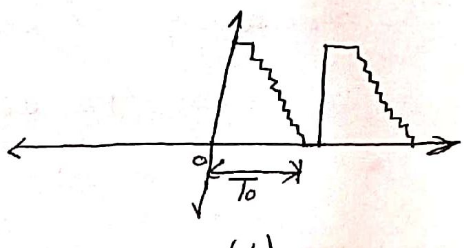
In (a) there is one maxima and one minima in the time-period T_0 , so a can be expressed using fourier series.

Whereas, in (b) there are infinite no. of maxima and minima , so the s/g cannot be expressed using fourier series.

(ii) The s/g should should have a finite no. of discontinuities over the range of the time period.



(a)



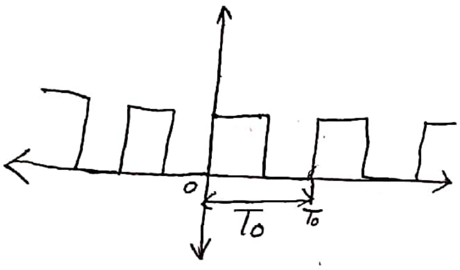
(b)

from the above diagrams,

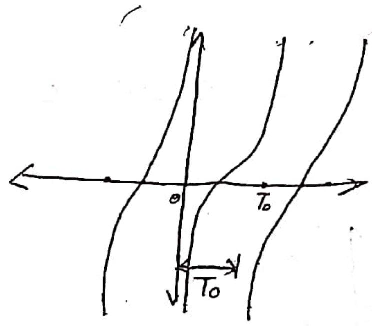
(a) has a finite no. of discontinuities in its time-period, whereas (b) has infinite no. of discontinuities in its time-period.

∴ (a) can be expressed in fourier series, while b cannot.

(iii) <u>The signal should be absolutely integrable over the range of its time period.</u>



(a)



(b)

In (b) as the graph approaches "0", or, "T_0", its value tends to infinity and so it is not integrable over the range of its time period.

Whereas, in (a), the s/g is absolutely integrable over the range of its time period. So, s/g. (a) can be expressed in fourier series and (b) cannot.

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3) <u>Ans</u>:- The different types of symmetries present in wave forms are :-

(i) <u>Even symmetry</u>:-

In case ~~ease~~ of even symmetery,

$x(t) = \text{even}$

i.e. $x(-t) = x(t)$

~~For~~ a A fourier series expression having even symmetry, it will only have even ~~term~~ terms, i.e. a fourier series of even s/g will not contain the sine term.

(ii) Odd symmetry:-

for odd symmetry,

$$x(-t) = -x(t)$$

$x(t)$ is odd in nature.

$\therefore$ fourier series of odd s/g will only contain the sine term.

(iii) Half Wave Symmetry:-

Here, $x(t) = -x(t + T/2)$, wher T= time period of the s/g.

~~Half~~ fourier series of half-wave s/g only has odd harmonics i.e. the sine term.

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2) Solⁿ:- Given, $x(t) = 2\sin 3\omega_0 t$

Comparing the above expression with, $x(t) = A\cos\omega t$, we get,

$$\omega = 3\omega_0$$

Now, the complex fourier series representation gives,

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega t}$$

$$\therefore x(t) = 2\sin 3\omega_0 t = \sum_{n=-\infty}^{\infty} a_n e^{jn3\omega_0 t} \quad \text{——— (1)}$$

Now,

$$\cancel{x(t)} \; 2\sin 3\omega_0 t = 2\left[\frac{e^{j3\omega_0 t} - e^{-j3\omega_0 t}}{2j}\right]$$

$$= \left(e^{j3\omega_0 t} - e^{-j3\omega_0 t}\right)\frac{1}{j} \quad \text{--- (11)}$$

$$\therefore \sum_{n=-\infty}^{\infty} a_n e^{jn3\omega_0 t} = \frac{1}{j}\left(e^{j3\omega_0 t} - e^{-j3\omega_0 t}\right)$$

$\therefore$ The complex fourier series coefficients are:-

$$a_1 = \frac{1}{j}, \quad a_n = 0$$

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Q. 5) Sol^n. Given, $X(\omega) = \dfrac{j\omega}{(3+j\omega)^2}$

$$= \frac{j\omega + 3 - 3}{(3+j\omega)^2}$$

$$= \frac{3+j\omega}{(3+j\omega)^2} - \frac{3}{(3+j\omega)^2}$$

$$= \frac{1}{3+j\omega} - \frac{3}{(3+j\omega)^2}$$

Now,

$$x(t) = F^{-1}(X(\omega))$$

$$= F^{-1}\left[\frac{1}{3+j\omega} - \frac{3}{(3+j\omega)^2}\right]$$

$$= e^{-3t}u(t) - 3te^{-3t}u(t)$$

$$= e^{-3t}u(t)[1 - 3t]$$

$$= (1-3t)e^{-3t}u(t)$$

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4) (a) Sol^n: We know,

$$x(t-t_0) \xleftrightarrow{F.T.} e^{-j\omega t_0} X(\omega)$$

$$\therefore x(t-4) \xleftrightarrow{F.T.} e^{-j\omega 4} X(\omega)$$

$$\& \ x(t+4) \xleftrightarrow{F.T.} e^{j\omega 4} X(\omega)$$

Now,

$$x(t-4) + x(t) \longleftrightarrow e^{-j\omega 4} X(\omega) + e^{j\omega 4} X(\omega)$$

$$\longleftrightarrow \left[\frac{e^{-j\omega 4} + e^{j\omega 4}}{2} \right] \times 2(\omega)$$

$$\Leftrightarrow \cos 4\omega \times 2(\omega)$$

$$\Leftrightarrow 2\cos 4\omega (\omega)$$

(b) Sol^n: ~~We know,~~

Given,

$$x(t) = \sin(4t + 0.2\pi)$$

$$\therefore x_1(t) = \sin(t + 0.2\pi) \qquad \therefore x(t) = x_1(4t)$$

We know,

$$F(\sin t) = j\pi [\delta(\omega+1) - \delta(\omega-1)]$$

Using time-shifting property we get,

$$F[\sin(t+0.2\pi)] = e^{j\omega(0.2\pi)} F(\sin t)$$

$$= e^{j\omega(0.2\pi)} \times j\pi [\delta(\omega+1) - \delta(\omega-1)]$$

Now, using time-scaling property,

$$F[\sin(4t+0.2\pi)] = \frac{1}{|4|} F[\sin(t+0.2\pi)]$$

$$\Rightarrow F[\sin(4t + 0.2\pi)] = \frac{1}{|4|}\left[e^{j\omega(0.2\pi)} \times j\pi[\delta(\omega+1) - \delta(\omega-1)]\right]$$