Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

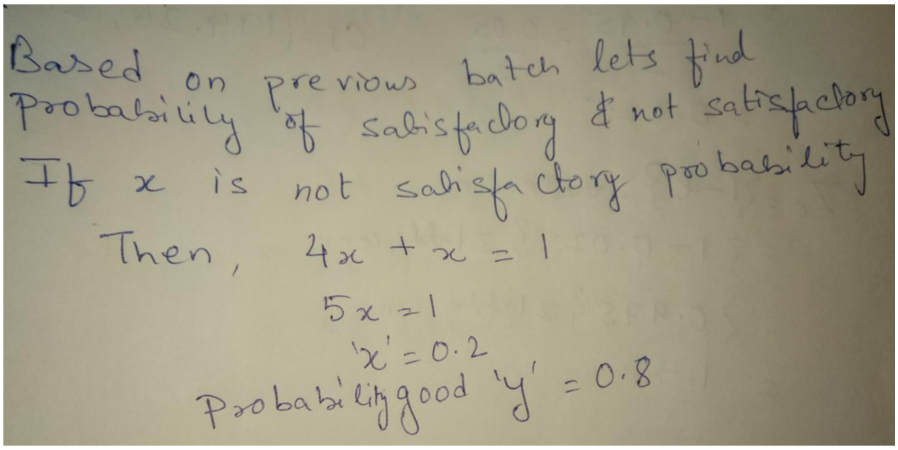
Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

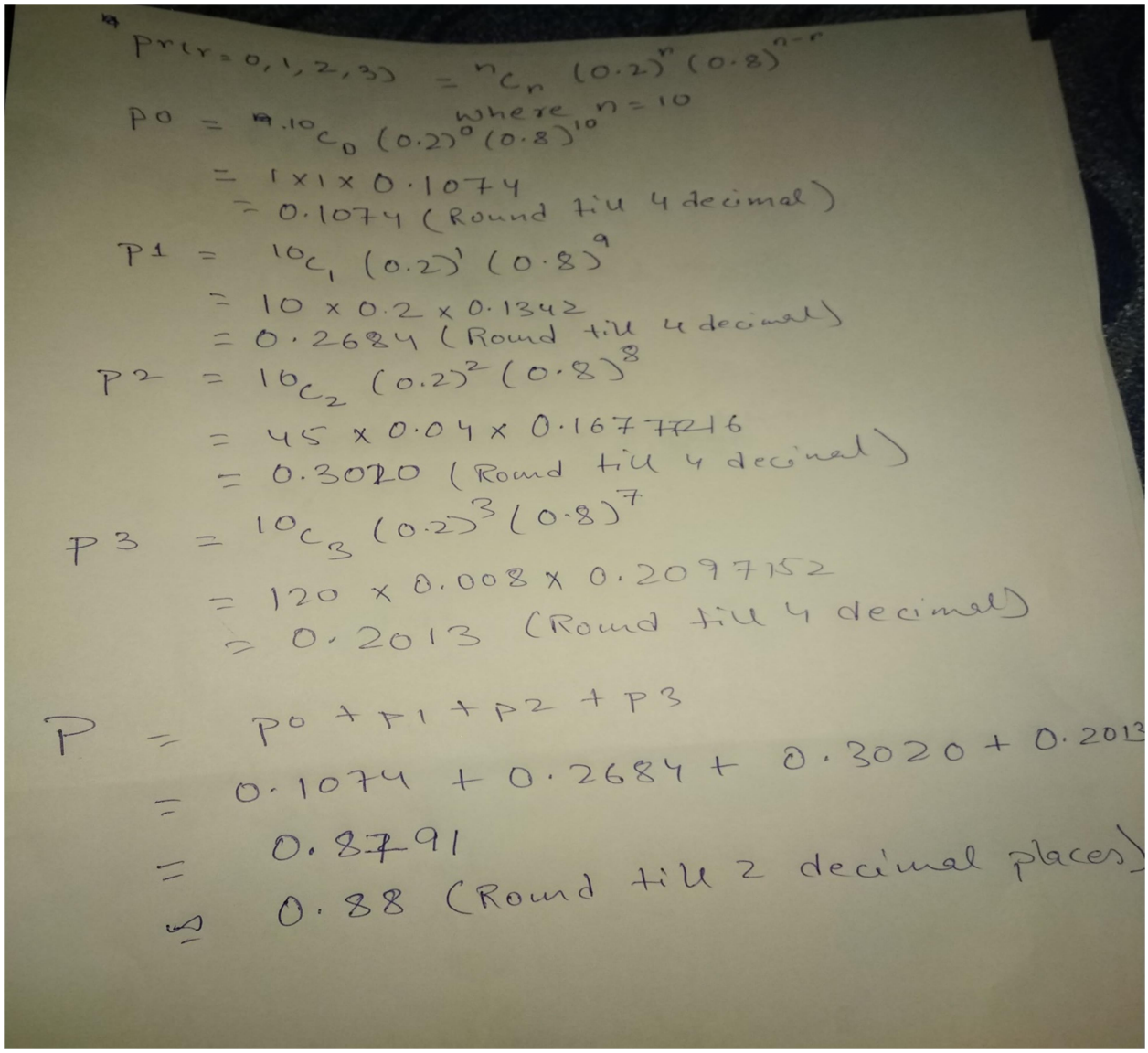
b.) Calculate the required probability.

Answer 1:

a) The probability distribution in my opinion for the above scenario can be described by Binomial Distribution. Binomial Distribution has following 3 properties:
   i. Number of observations are fixed
      In above scenario the sample size is 10 (which is fixed).
   ii. Each observation/ trial is independent
      The outcome of testing one drug sample (trial) does not impact the testing of next drug.
   iii. Probability of success or failure is exactly same in each trial.
      In above testing, the consideration is satisfactory (success) and not satisfactory (failure) is same or fixed. In our case the probability not satisfactory based on previous batch test is 0.2 and probability of satisfactory is 0.8



b) Let's calculate the probability that at most, 3 drugs are not able to do a satisfactory job.
   It is given by P = p0 + p1 + p2 +p3 (where p0 is probability that 0 drugs are not satisfactory, p1 is probability that 1 drug is not satisfactory,….so on p3 is probability that 3 drugs are not satisfactory)



Theoretical probability that at most 3 drugs are not able to do a satisfactory job is ~ 0.88 (round to 2 places of decimal). For detailed calculation refer above.

Question 2:

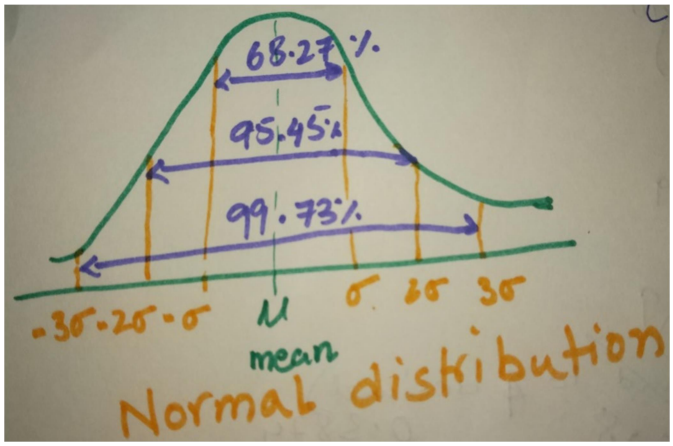
For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the populations mean might lie — with a 95% confidence level.

a.)Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

b.)Find the required range.

Answer 2:

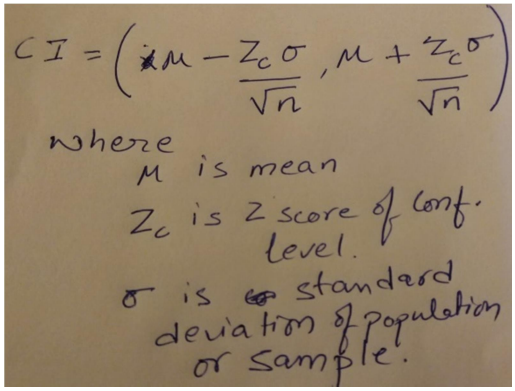
a) The population of drugs is 80,000 and sample of 100 drugs are picked up. So we will use Central limit theorem for this and since n > 30. The spread of the means of the sample distribution tends towards a normal distribution. The normal distribution and its properties look like below.



Following are properties

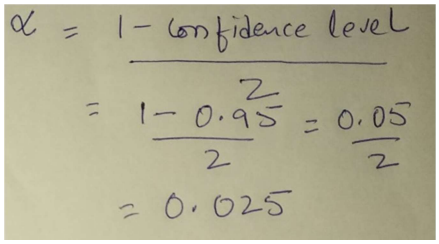
1. Percentage area under curve
   Between $\mu + \sigma$ and $\mu - \sigma$ is 68.27%
   Between $\mu + 2\sigma$ and $\mu - 2\sigma$ is 95.45%
   Between $\mu + 3\sigma$ and $\mu - 3\sigma$ is 99.73 %
2. The curve is symmetric about μ
3. Area under the curve sum up to 1.

Given the confidence level, we need to calculate confidence interval (CI), which is given by following equation.

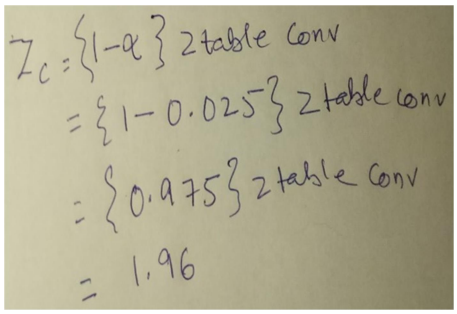


Let's assume $H_{0\ (\text{null hypothesis})}$ = 207 s time of effect for whole population and $H_{1\ (\text{alternate hypothesis})} \neq$ 207s time of effect.
So we will proceed for the Z score and CI calculation for 2 tailed distributions assuming the above alternate hypothesis

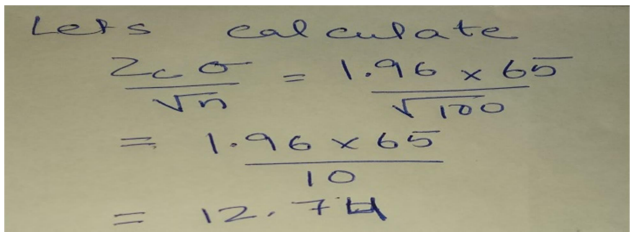
b) Let's proceed with the calculation for 2 tailed



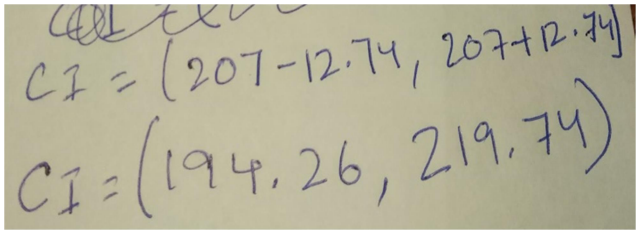
Z score for confidence level is:



Evaluate the right hand side of u in the CI equation:



CI calculation is as follows:



Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure(with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice(Assume that only the values of α and β as mentioned above are provided to you and no other information is available).

Answer 3:

a) For the scenario mentioned in Question 3a.
Null Hypothesis $H_{0:} \leq 200s$
Alternate Hypothesis H_1: > 200s

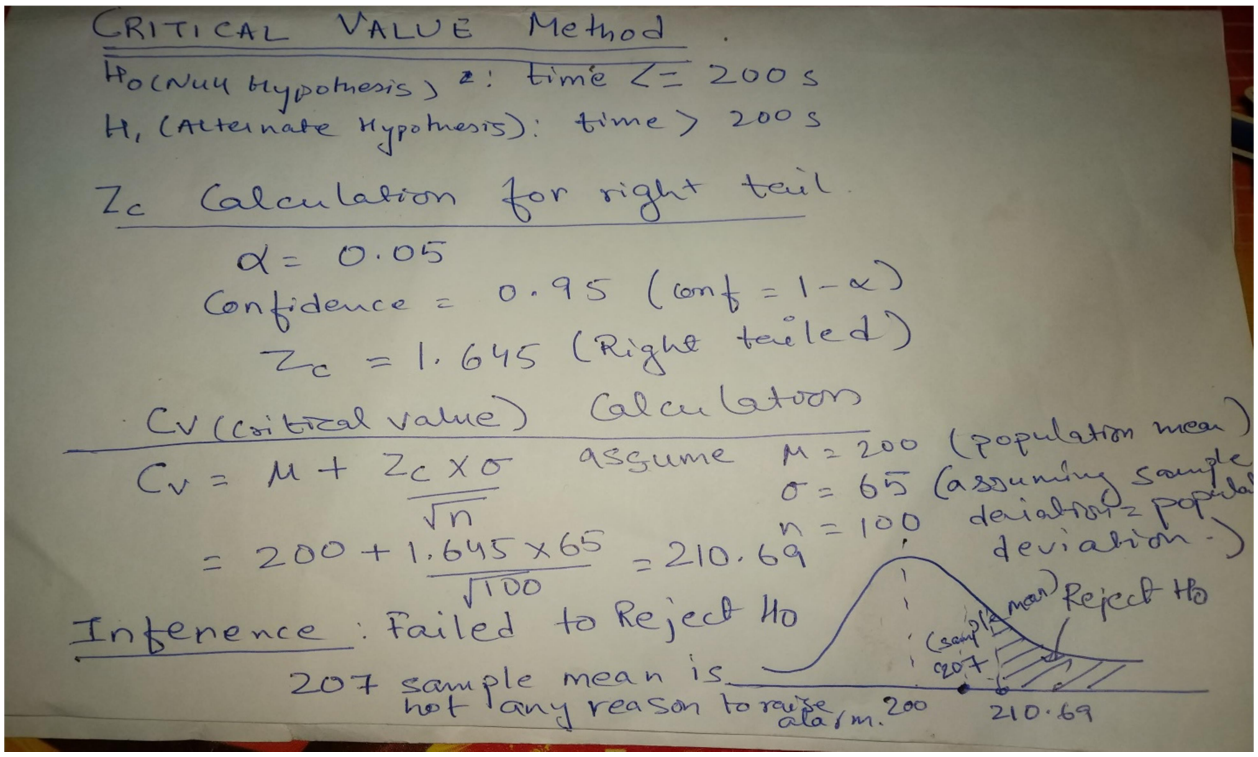
From above alternate hypothesis test is **right tailed** test.

Following data from last question:
σ = 65

Sample size n = 100

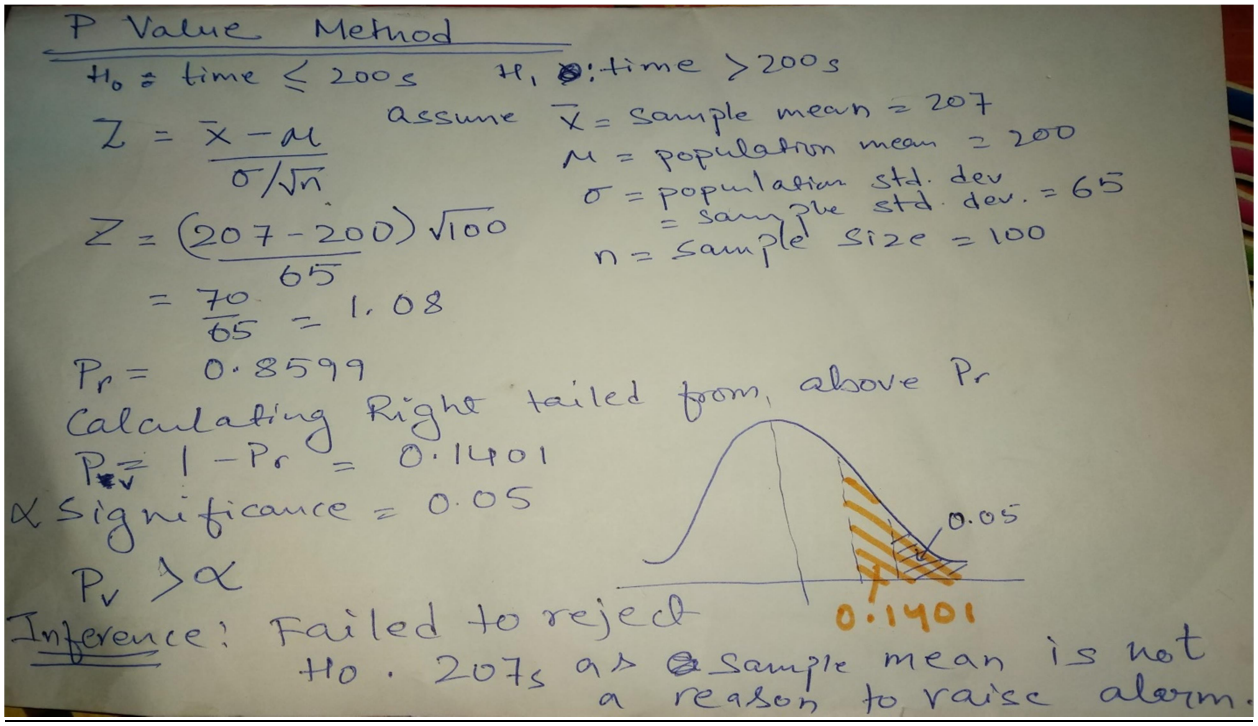
Sample mean μ = 207 s

Using critical value hypothesis testing



Our sample mean 207 < critical value 210.69. So not able to Reject H0 (null hypothesis). No need to raise alarm about the quality.

Using P value hypothesis testing:



Our P value 0.14 > significance value 0.05. So not able to Reject H0 (null hypothesis). No need to raise alarm about the quality.

b) Let's start with setting up Null (H_0) and Alternate (H_1) Hypothesis.
H_0 = Drug usage is satisfactory (or Time of effect is <= 200s)
H_1 = Drug usage is not satisfactory (or Time of effect is > 200s)

Type-I error probability α is when in reality H_0 is true but by test H_0 is rejected. So test proves drug is not satisfactory and in reality it is satisfactory.
Consequences:
Good drug will be wasted and Company has to manufacture again. It is loss for the company.
Customer will not get this batch. But they will get other batch. No impact on customers.

Type-II error probability β is when in reality H0 should be rejected but test proves H_0 is true. So test proves that drug is satisfactory while in reality it is not (takes longer time to effect).
Consequences:
Customer will be unsatisfied. They will be in pain for longer duration.
Company will lose customer in longer run to rival company with more effective product.

Error which is more dangerous:
In my opinion, Type-II is more dangerous as it affects both customer and company.
Let's say Sampling method -1 has α=0.5 and β=0.45 and Sampling method -2 has α=0.15 and β=0.15. Since type – II(β) is more dangerous so company should choose lower β value method.

So company should use Sampling method -2.

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer 4:

A/B test lets us decide the better performance

Following are steps using which we can perform the A/B test:

1. Design 2 campaigns with just 1 difference (Tagline only should be different, voices , music, price etc should be same)

2. Introduce the campaigns to 2 different groups but which should be approAfetximately same in every other aspect such as age of candidates, gender, occupation, sample size..etc.
3. Decide what should be winning criteria (purchase of drugs or appeal,..) , what should be the percentage when it is considered a win (90% increase in sale, or 50% positive feedback)
4. Fix the time and duration for which both the campaigns should be run to 2 different groups.

After running the campaigns with above criteria collect the data from 2 different groups collect the data and decide which one is the winning campaign.

Run the A/B test again with another 2 different groups and check for the consistency. IF consistent then the winning “Tagline” can be selected and used.