Problem Set # 2: Demand Estimation

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Course: Empirical IO

September 28, 2022

Estimating demand is the corner stone of modern empirical IO. This problem set is designed to help you further understand how to estimate discrete choice demand models, using individual-level data. We will use simulated fake data, which allows to control the environment. The data is generated from using the following environment.

Demand. Consumers' preference for product $j = 1, \ldots, 9$ and market m is assumed to take the following form:

$$\begin{aligned} u_{ijm} &= \delta_j - \alpha_i p_{jm} + \epsilon_{ijm} \\ \alpha_i &= \alpha + \sigma v_i \\ u_{i0} &= 0 \end{aligned}$$

where the consumer taste shock is i.i.d. with distributions:

- ν_i, where is drawn from a normal distribution
- ϵ_{ijm} is drawn from type I extreme value distribution.

Parameters. In the remainder of this problem set, you will estimate the demand parameters $\theta = (\beta, \alpha, \sigma)$. The true parameters are

- $\delta = (3; 2; 1; 1; 0.5; 2; 1; 1; 0.5)$
- $\alpha = -3$ and $\sigma = 2$

The results that you will obtain for the NML and MNL will differ from these true parameters. The NML coefficients are

- $\delta = (1.42; 0.63; -0.33; -0.26; -0.63; 0.55; -0.21; -0.25; -0.63)$
- $\alpha = -1.21$.

[As a bonus point: try to answer the question: Why is this the case?]

Problem – Multinomial Logit. The idea of the following exercice is to understand the differences between a MNL a and Mix-logit.

1 Start by understanding the dataset. How many individuals are in the dataset? How many choice occasions is each individual making? What it the most often made choice?

2 You need to create the following choice probability for product j for each individual i at choice occasion t:

$$P_{ijt} = \frac{e^{\delta_j - \alpha' p_{jt}}}{1 + \sum_{k=1}^{9} e^{\delta_k - \alpha' p_{kt}}}$$

For this you will need to

- create a matrix containing your "exogenous" data. Here you have the dummy variables identifying the δ's and the price/expenditure on each of the goods. Also, D (or D_{mix}) identifies the choices that each individual did on each choice occasion. Choice occasions are identified with the cdid variable (on the matlab code).
- create a function named "mlogit" that is going to host your log-likelihood for the logit.
- within the "mlogit" function, make sure that you create both numerators and denominator choice each individual-choice occasion.

3 After you have created a probability measure for each choice occasion, you can proceed to create the likelihood. Let y_{ijt} be equal to one if individual i choose product j at choice occasion t and zero otherwise, then the likelihood for individual i yields:

$$L(y_i|\beta) = \prod_{t=1}^{T} \prod_{j=0}^{J} (P_{ij})^{y_{ij}}.$$

The likelihood for the I persons is then:

$$L(y_1, \ldots, y_I|\beta) = \prod_{i=1}^{I} L(y_n) = \prod_{i=1}^{I} \prod_{t=1}^{T} \prod_{j=0}^{J} (P_{ij})^{y_{ij}}$$

4 Find the "maximum" using fminunc [Hint: remember that fminunc minimizes the function, so you need to put a negative sign prior to the minimization object.]

Problem – Multinomial Logit using NFP. The following exercise requires you to estimate the MNL using a two-step process. The first step estimates δ's using a Nested Fixed Point algorithm, and the second step estimates the α dis-utility coefficient.

1 Explain what is a Nested Fixed Point algorithm with your own words.

2 Solve the first order condition of the log-likelihood for the I persons for each δ and show that, it is equal to $\sum_t \sum_i P_{ijt} = 1$.

3 Create a function named "mlogitfe" that is going to host your two step procedure. Also, take a random value for each of the δ's.

4 Create both numerators and denominator choice each individual-choice occasion. Within both the numerator and denominator, make sure to include separately the δ's and αp_{jt}.

5 Create the first step of the process by doing a "while" loop using the following contraction mapping:

$$\delta_j^{s+1} = \delta_j^s - \ln(\sum_i \sum_t P_{ijt})$$

You should use a tight enough tolerance (i.e., stopping rule such as $1e^{-12}$ is usually good practice). Depending on how your code performs, it may take more or less time. Nevertheless, you should find that the convergence is monotone. Make sure this is the case.

6 Step 4 and 5 should provide you with a P_{ijt}. As in the first problem (question [3]), maximize the log-likelihood and finds the coefficients.

7 Using your logit estimates, compute the welfare loss of eliminating product #1.