Time-Frequency Analysis of EEG Signals and Estimating Gamma Burst Duration using Matching Pursuit Algorithm

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ABSTRACT

The brain is the most amazing and complicated organ of the animal body. The signals from the brain can be visualized/measured through different methods. Some of these methods are electroencephalography (EEG), magnetoencephalography (MEG), electrocorticography (ECoG) and functional magnetic resonance imaging (fMRI). In addition, the Local Field Potential (LFP) is yet another way to measure brain signals where the signal is obtained after filtering the raw signal from a single neuronal recording recorded typically using micro-electrodes. However among all these techniques, EEG remains the main functional brain scanning modality as it is cheap, portable, non-invasive and widely available. The signals measured from the brain, irrespective of the method used, can be analysed to understand the underlying phenomena of the brain. The signal obtained from the brain is pre-processed to remove the bad or artifact ridden data viz. muscle artifact, eye movement-eye blinks artifact, head movement artifact, line noise, DC noise etc. In this work, an attempt has been made to study and understand the various time-frequency analyses of brain EEG signal from human subject. An attempt has also been made to understand the Matching Pursuit algorithm, which is reported to have been used by earlier researchers [1] as a method of time frequency analysis of brain signals and found to improve the temporal and spectral resolution.

1. INTRODUCTION

Electroencephalography (EEG) is a method to record an electrogram of the electrical activity on the scalp that has been shown to represent the macroscopic activity of the surface layer of the brain underneath. With the electrodes being positioned along the scalp, it is often non-invasive. Though the brain signal consists of a number of frequencies corresponding to different activities, however, just by visual inspection of the recorded EEG signal, it is not possible to interpret the different kinds of frequencies present in the signal. Typically, the frequency ranges used to describe brain signals are gamma (between 30 Hz – 80 Hz), beta (between 13 Hz – 30 Hz), alpha (between 8 Hz – 12 Hz), theta (between 4 Hz – 8 Hz), and delta (less than 4 Hz). All of these frequency bands have been associated with some or other activity performed in the brain. For example, alpha waves are present when a person is relaxed and quietly resting. The gamma frequency band is linked to activity that is important for regular cognitive processes like learning, memory and attention. In mental diseases such as autism, schizophrenia and Alzheimer’s disease, gamma rhythms are abnormal and are useful as bio marker of such diseases. Therefore, in order to comprehend which frequencies are present in what amounts in EEG signals from the different regions of our brain, it is crucial to visualise the signals in the frequency domain. Different ways to represent the EEG data are as follows:

1.1 FREQUENCY ANALYSIS

In frequency analysis, the time series data is divided into a sum of sinusoids of various frequencies. A mathematical technique called the Fourier transform (FT) converts a function in time domain into a frequency domain function. The Fourier transform of a time-domain signal, *f (t)* is given by

$$f(\omega) = \frac{1}{\sqrt{2\pi}} \int f(t) e^{-j\omega t} dt \qquad (1)$$

where ω is the angular frequency expressed in radians per second.

Fourier Transform of a signal is a complex number that denotes the phase and corresponding scaling factor at each frequency (ω). One limitation of such representation is that FT does not show how this amplitude or phase may be altering with time because it computes the overall amplitude and phase at a frequency by integrating over the full signal length. Therefore, other methods like time frequency analysis are often used. Some of these methods are discussed below.

1.2 TIME-FREQUENCY ANALYSIS

Whereas time-locked EEG activity or event related potentials (ERPs) reveal a fraction of the multidimensional space of EEG data, the time -frequency analyses reveal more although not all of the space. Since oscillations appear to be a common and fundamental neural mechanism that supports myriad aspects of synaptic, cellular and systems- level brain functions across multiple spatial and temporal scales, many results from the time-frequency based analysis can be interpreted in terms of neurophysiological mechanisms of neural oscillations [2].

1.3 TIME-FREQUENCY UNCERTAINITY PRINCIPLE

To get a high resolution of the frequency data of a signal, it is required to observe the signal in time domain for a long interval. If one observes the signal in the time domain for only a short interval, then the Fourier transform of the signal (frequency data) is more spread out. For example, in the discrete Fourier transform (DFT) computation if one wishes to have closely spaced frequency components i.e. high resolution of samples (n) in frequency domain, then one would require an equal number of samples (n) in the time domain for analysis. This is equivalent to saying that one should observe the signal for a longer interval. However this was not a problem as long as one was restricted to analysis in a single domain (i.e. time domain or frequency domain). But a problem arises when one starts to analyse signals in composite domains. Composite domain analysis is studying a given signal in two or more domains simultaneously. The need for such analysis arises when it is required to extract the frequency components at different instants of time.

Composite domains in general, do not pose a difficulty in analysis. This happens only when the two domains are inversely related to each other as in the case of time (*t*) and frequency (*f*) domains where they are related as $t = 1/f$. In this case it is seen that if a signal has compact support in time then its frequency equivalent does not have it, and vice versa.

1.4 SHORT TIME FOURIER TRANSFORM

The short time Fourier transform (STFT) is a simple extension of the Fourier transform that addresses the two main limitations of the Fourier transform[2] viz.

(i) the Fourier transform obscures time varying changes in the frequency structure of the data and

(ii) the Fourier transform assumes that the data are stationary for the duration of the time series.

In short time Fourier transform, the signals are broken into short time segments of equal size and Fourier transform is then carried out on each signal separately. The signals are broken down using a method known as windowing. The STFT of a signal *f (t)* is computed as

$$f_h(\tau,\omega) = \int f(t)h(t-\tau)e^{-j\omega t}dt \qquad (2)$$

where $h(t-\tau)e^{-j\omega t}$ represent the windowed sinusoids.

The time–frequency energy density spectrum, also called the spectrogram, is obtained as the magnitude-square of the STFT as $|f(\tau,\omega)|^2$. In MATLAB, the built in function *spectrogram* is used to calculate the STFT of a signal.

1.5 MULTI TAPER METHOD

In multi taper method (MTM), multiple spectrograms are created from the signal using a set of orthogonal window functions. The average of these spectra gives the multitaper time-frequency energy spectrum. The multitaper spectral estimate ($S_{MT}(\tau,\omega)$) of a signal *f*(*t*) is defined as

$$S_{MT}(\tau,\omega) = \frac{1}{K}\sum_{m=1}^{K}\left|\int f(t)h_m(t-\tau)e^{-j\omega t}dt\right|^2 \qquad (3)$$

where, K is the number of windows used in the analysis and

h_m is the m^{th} window.

The window functions are chosen to maximize the spectral concentration within the chosen bandwidth, which yields discrete prolate spheroidal sequences (DPSS) or slepian windows (or tapers). The zeroth-order slepian taper looks like a Gaussian window function, but as the order increases, the tapers become more oscillatory. Since the slepian tapers are orthogonal to each other, MTM averages mutually independent spectral estimates, which in turn reduces the variance of the spectrum. The number of tapers (K) that have good spectral concentration and can be used in the MTM analysis is

$$K = 2 \times TB - 1 \qquad (4)$$

where TB is the time band width product.

In MATLAB, the function from the chronux toolbox *mtspegramc* is used to carry out the multi taper time frequency process.

1.6 WAVELET TRANSFORM

The main problem associated with STFT is that it does not give multi-resolution information of the signals as it always has constant size.

In mathematics, a wavelet series is a representation of square integrable which can be real or complex valued function. A wavelet is a wave like oscillation with amplitude that begins at zero, increases and then decreases back to zero. Wavelets are generally crafted to have specific properties that make them useful for signal processing.

The primary benefit of the wavelet transform is its variable window size, which is wide at low frequencies and narrow at high frequencies, resulting in the best temporal frequency resolution throughout all frequency ranges (i.e., it retains the multiresolution features).

1.7 MATCHING PURSUIT

The Matching Pursuit algorithm was introduced by Mallat and Zhang in 1932 [3]. Matching Pursuit is a greedy algorithm that decomposes a signal into a linear combination of waveforms called atoms that are well localized in time and frequency. An example of such waveform is the Gabor function (Gaussian modulated sinusoidal function), which has the least time frequency bandwidth product. A dictionary of time frequency atoms is created by shifting, scaling and modulating a single atom. One can thus form an over complete or a redundant dictionary and MP then decomposes the signal using atoms picked iteratively from this redundant dictionary. This is done as follows:

- First, the 'residue' is set equal to the signal itself and then the inner product between the residue and all the atoms in the dictionary are computed. From those, the atom with the largest inner product is selected.
- Next, the signal is approximated as the chosen atom multiplied by the inner product between the atom and the residue.
- In the next iteration, the difference between the signal and the approximation is taken as the 'residue' and the procedure is repeated.
- The residue after the n^{th} iteration is the difference between the signal and the decomposed iteration till the n^{th} iteration.

The most common approach to the construction of time frequency dictionaries relies on Gabor functions, ie. Gaussian envelopes modulated by sinusoidal oscillations. By multiplying these two functions, a wide variety of shapes could be obtained depending on their parameters. All such Gabor functions can be described using only four numbers per waveform viz. time width, position of the centre of the Gaussian envelope, frequency and phase of the modulating sine. Amplitudes are adjusted such that each function has equal (unit) energy, since the product of a waveform of unit energy with the signal will directly measure the contribution of the structure to the energy of the signal.

Since the first application to EEG in 1995 [4], matching pursuit algorithm (MP) has been shown to significantly improve the EEG/MEG analysis in a variety of paradigms. This includes

- pharmaco-EEG assessment of propagation dynamics
- signal complexity in epileptic seizures and detection of seizures
- analysis of somatosensory evoked potentials in humans and rats
- detection of sleep spindles in Obstructive Sleep Apnea and investigation of their chirping properties
- studies of high gamma in humans and monkeys
- investigation of brain's pain processing parameterization of vibrotactile driving responses and event-related desynchronization and synchronization .

Some of the recent and important works reported in the direction of time frequency analysis in general and gamma oscillation in particular are discussed in the next section.

2. LITERATURE REVIEW

Ray *et al* [5], in their work showed the distinction between the broad band high gamma activity and gamma rhythms as an easily obtained and reliable electrophysiological index of neuronal firing near the microelectrode. Further, they also highlighted the importance of making a careful dissociation between gamma rhythms and spike related transients that could be incorrectly decomposed as rhythms using traditional signal processing methods.

Fries *et al* [6] in their work reported that for a group of neurons sending a message to be maximally effective, each neuron in the group should bundle its spikes into bursts and all neurons of the group should synchronize those bursts with each other.

According to some theories [7], gamma oscillations may act as a 'clock' signal for exact temporal storing of information and 'binding' of stimulus properties across brain areas. However, a study [8] of the strength and phase of the gamma rhythm in the LFP recorded with microelectrodes revealed that gamma occurs in bursts of extremely short length (l20ms) and displays features of filtered noise rather than a 'clock' signal in the primary visual cortex. All of the prior studies estimated the burst duration in LFP data by using methods like the continuous Gabor transform, wavelet transform and Hilbert transform. In a recent work [9], it has been shown that using the Matching Pursuit (MP) algorithm provides a robust estimate of the gamma duration and that the median gamma duration is much greater than the previous estimates.

3. OBJECTIVES OF RESEARCH

The main objective of the research was to understand why time frequency analysis is crucial to study the EEG data and how Matching Pursuit (MP) proves to be a better method of time frequency analysis. To this effect the objectives of this project has been

- To understand the correlation between different frequency bands and their correlation with activities in EEG data.
- To study the different methods of time frequency analysis commonly used for EEG analysis to understand what does a time-frequency analysis method tries to achieve and how is the resolution in both the domains decided.
- To study Matching Pursuit algorithm and understand how it is applied on the EEG data, its advantages.
- To implement both short time Fourier transform and matching pursuit on the specific EEG data to understand the advantage of MP over the other method in detecting gamma burst duration.

4. METHODOLOGY

The recorded EEG data used in the present work was taken from the work of [10]. The EEG signal used was recorded using Open BCI Cyton Biosensing Board where the protocol corresponding to SF-Ori (data collected when stimulus was shown to the subject) was used. The data was recorded from eight electrodes: O1, O2, P7, P3, Pz, P4, P8, CPz; using the internationally recognized 10-20 system. The data from each electrode for a subject in the protocol was a 2D matrix (301 x 1024), where 301 corresponds to the number of trials and 1024 corresponds to the number of time points.

4.1 VISUALISING THE DATA IN TIME DOMAIN

Figure 1 shows the event related potential (ERP) calculated for the EEG data collected from a single electrode (O1) from a subject by averaging over all the trials.

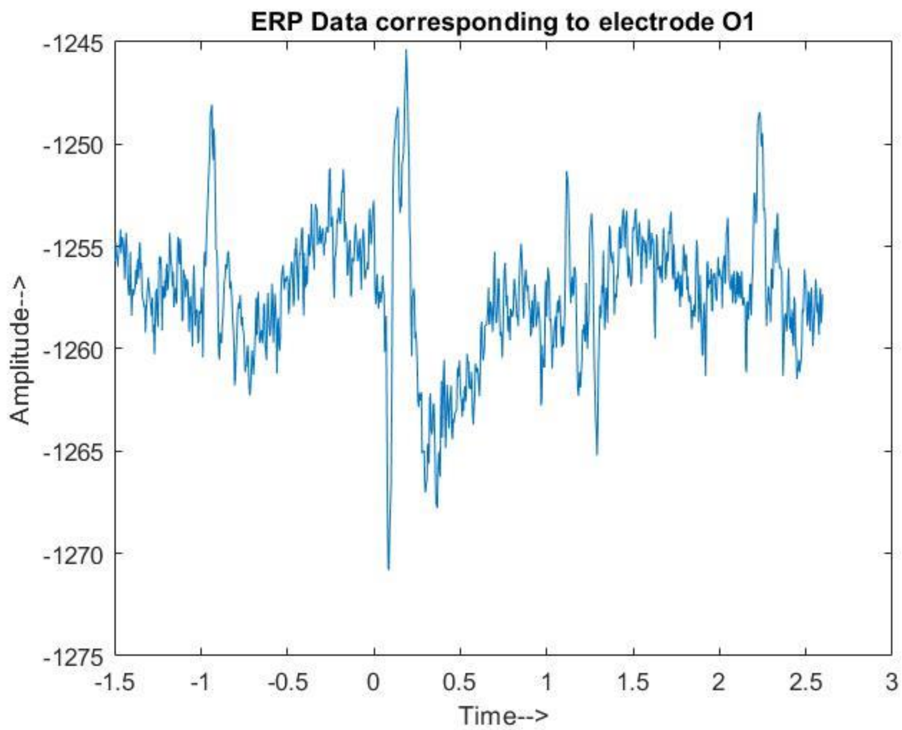

Fig. 1 ERP of the EEG signal

4.2 VISUALISING IN FREQUENCY DOAMAIN—FOURIER TRANSFORM

Data recorded for each subject was available as raw data and noise filtered data. The Fourier Transform was computed for both the raw data and noise filtered data to visualise the data in frequency domain. The MATLAB function ***fft*** (fast Fourier transform) was used to compute the Fourier transform.

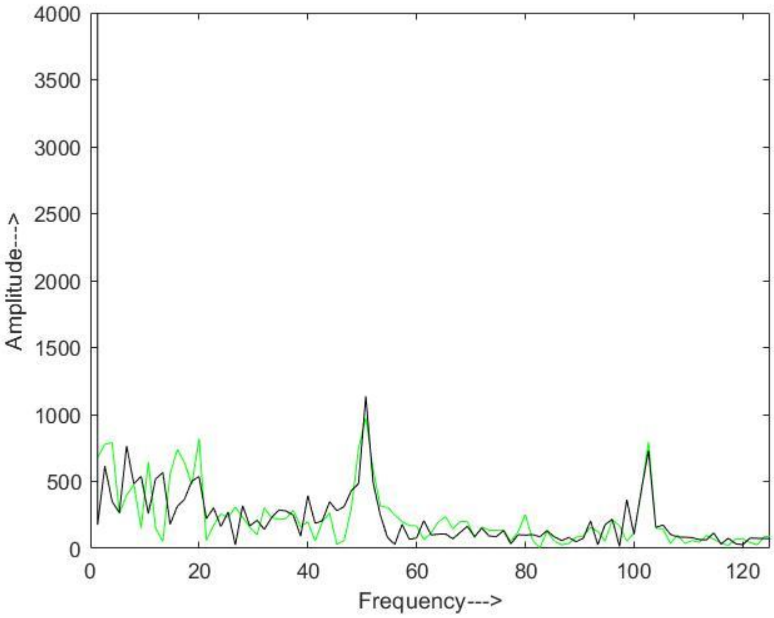

Fig 2. Frequency Vs amplitude for raw data

Figure 2, Fig. 3, and Fig. 4 are the plots for frequency vs amplitude, frequency vs normalised amplitude and frequency Vs amplitude (in log domain) respectively for raw data obtained using MATLAB. The green curve represents the baseline period and black curve represents the stimulus period. In all these figures, a distinct peak is observed at 50 Hz which corresponds to the AC line noise. Therefore the data was noise filtered and the corresponding frequency domain plots for frequency Vs amplitude, frequency Vs normalised amplitude and frequency Vs amplitude (in log domain) are shown in Fig 4, Fig. 5 and Fig. 6 respectively, where the 50 Hz peak is absent.

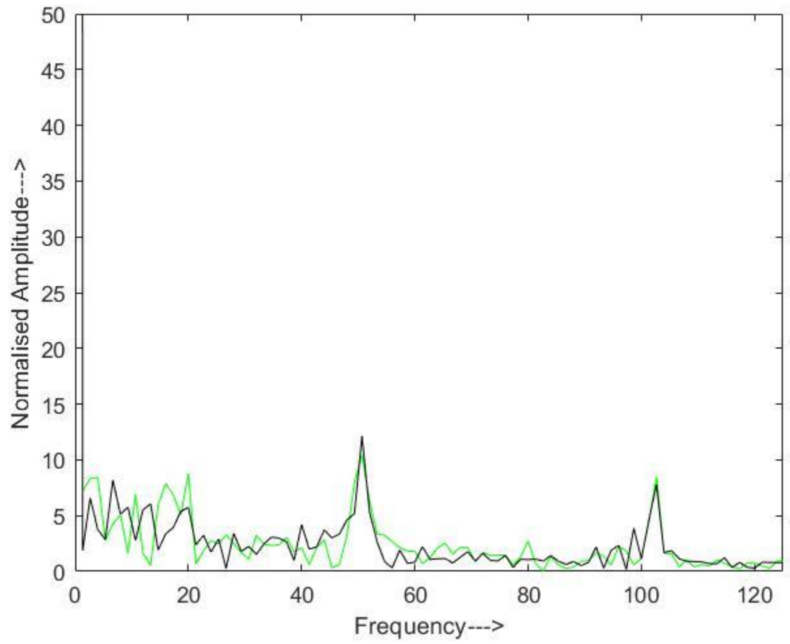

Fig 3. Frequency Vs normalised amplitude for raw data

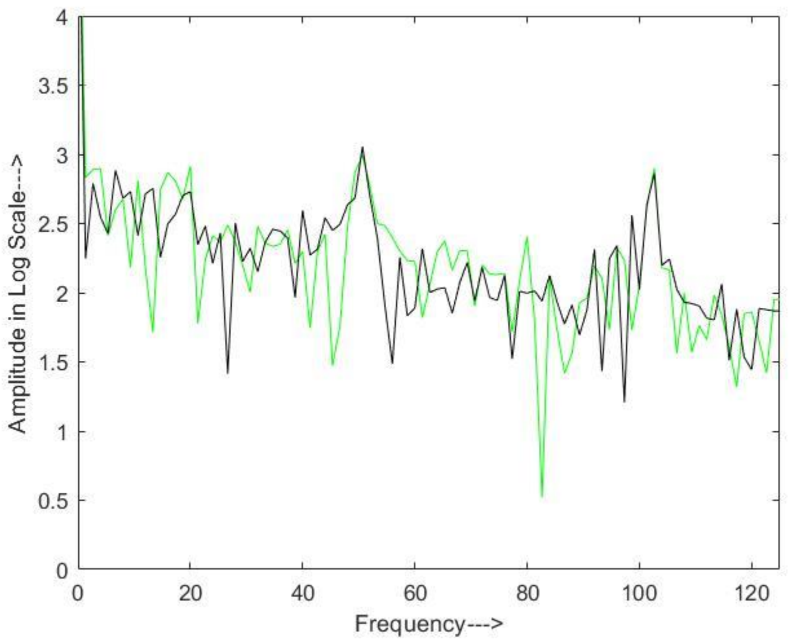

Fig 4. Frequency Vs amplitude (in log scale) for raw data

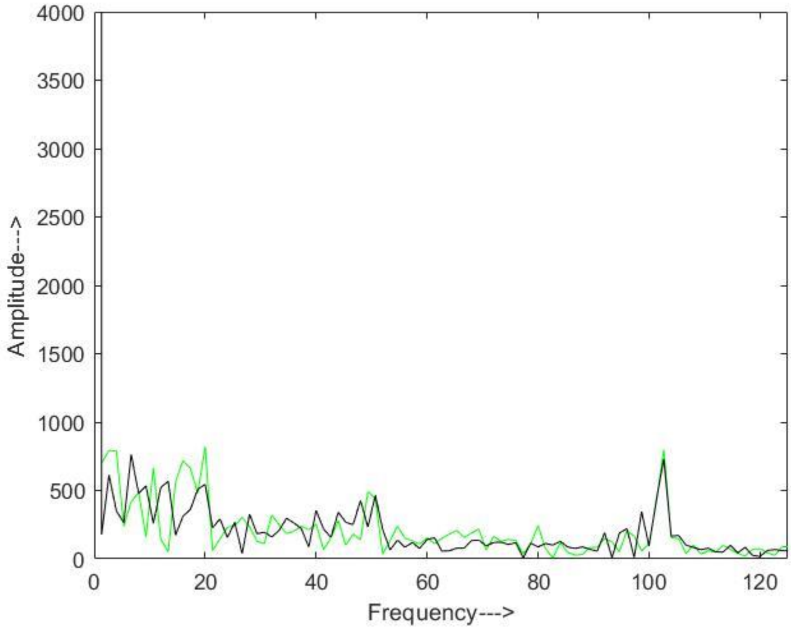

Fig 5. Frequency vs Amplitude for filtered data

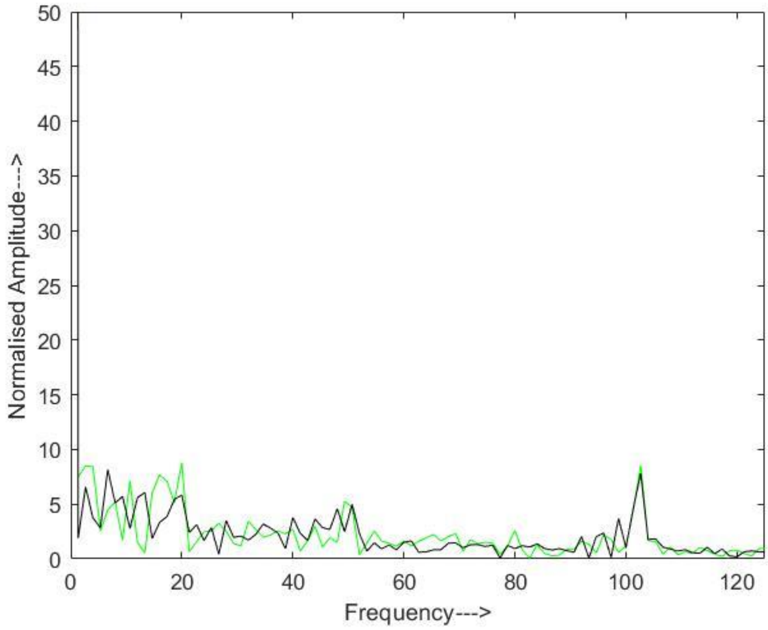

Fig 6. Frequency vs Normalised Amplitude for filtered data

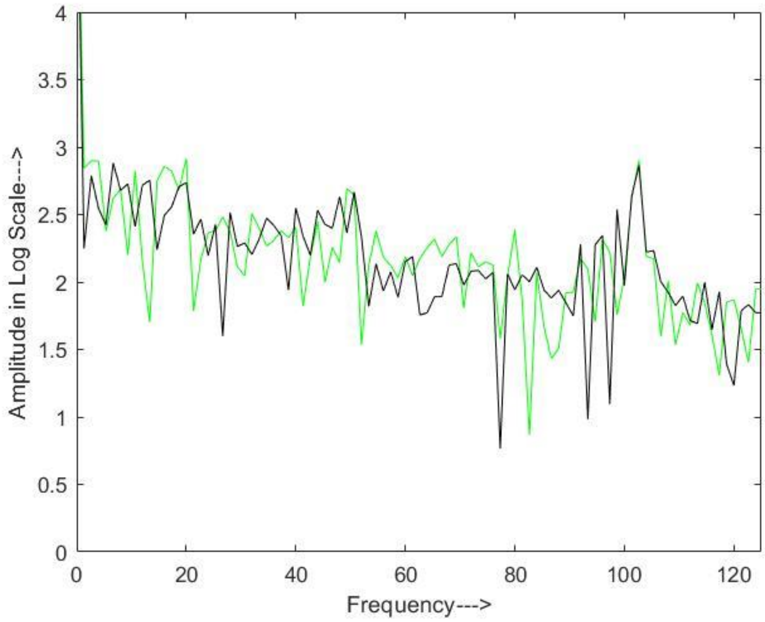

Fig 7. Frequency Vs amplitude (in log scale) for filtered data

4.3 SHORT TIME FOURIER TRANSFORM

In order to extract the time frequency response of the EEG signal, the short time Fourier transformation of the noise filtered EEG data was performed in MATLAB using the function ***spectrogram***. Figure 8 shows the spectrogram of the EEG signal corresponding to a single trial (trial 7). It could be noticed that the spectrogram of this particular trial does not reveal any distinct dominant frequency except the constant 50 Hz almost throughout the entire period. Therefore, short time Fourier transform was performed to obtain the average spectrogram plots over all the trials with an objective to eliminate the random noise and to get a clearer dominant frequency during attention/stimulus. Figure 9 shows the average spectrogram plots over all the trials.

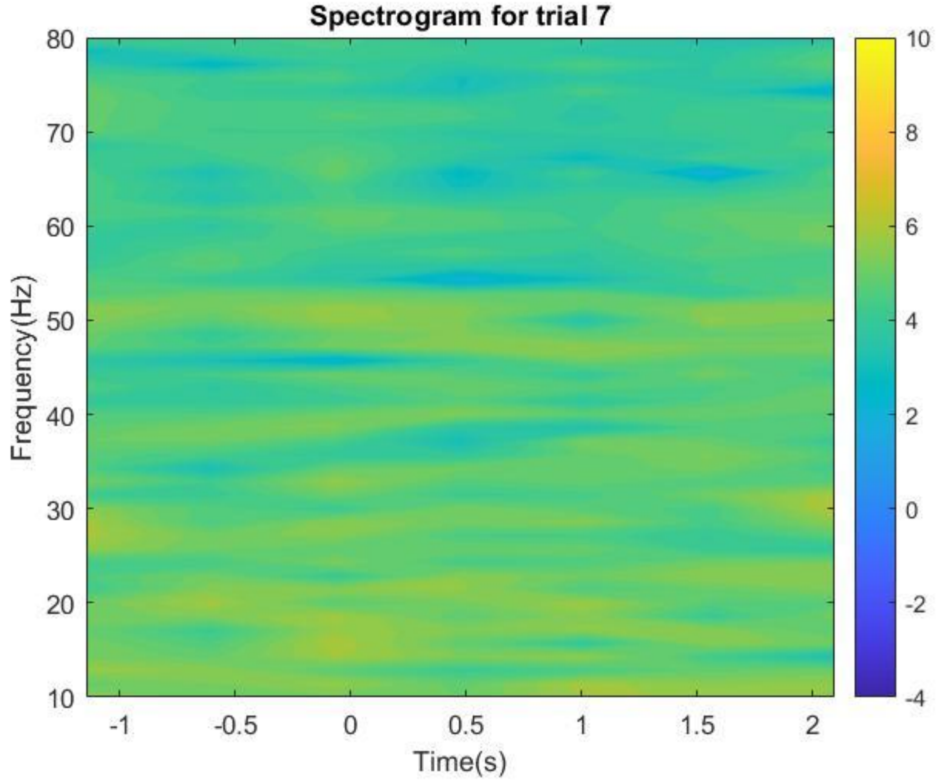

Fig. 8 Spectrogram for a single trial (trial 7)

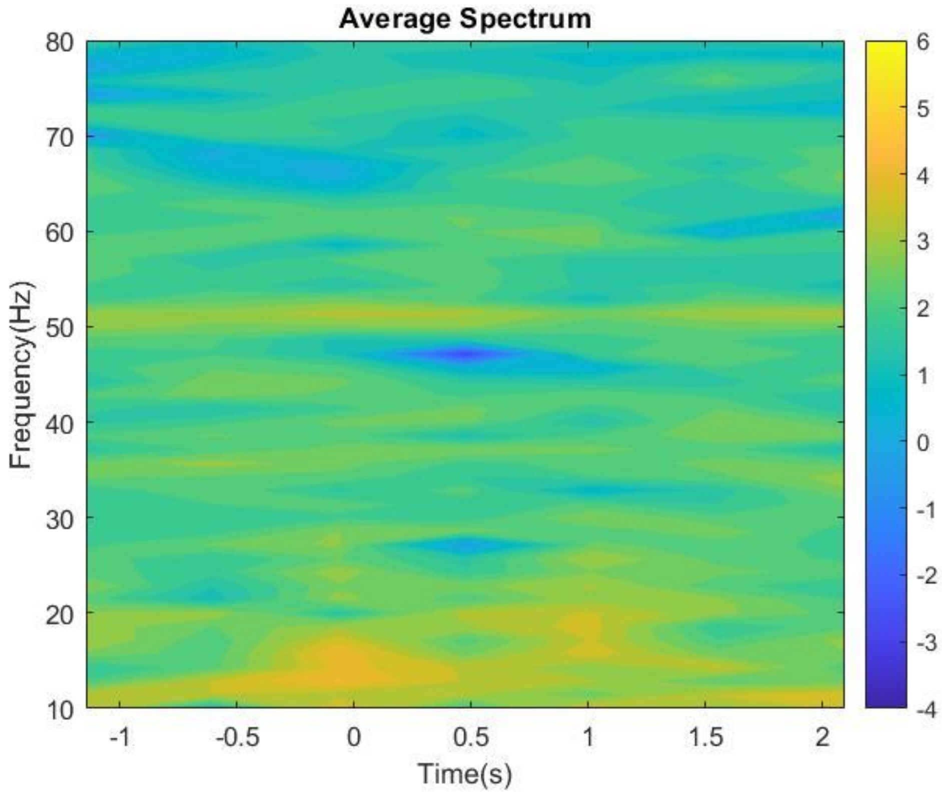

Fig. 9 Average spectrogram ove all the trials

Though Fig. 9 gives a better plot compared to Fig. 8 and shows some dominant frequency (around 50 Hz) in and around the stimulus point, but this is still not a very clear picture of the expected gamma burst, may be some more fine tuning of parameters are required to get a clearer picture which is being studied.

4.4 MATCHING PURSUIT ALGORITHM

As reported in earlier work [1], matching pursuit algorithm performs better compared to short time Fourier transform in detecting gamma bursts in LFP data. Therefore, in the present work attempts to use matching pursuit algorithm on the EEG data set to see whether MP works better. In this direction, first the steps involved in implementing matching pursuit are understood by developing a simple MATLAB code. Following the steps described in section 1.7, a simple MATLAB code was written to extract the best $\boldsymbol{x}$ given $\boldsymbol{A}$ (dictionary) and $\boldsymbol{b}$ (signal) in the Equation $\boldsymbol{Ax} = \boldsymbol{b}$. The columns of matrix $\boldsymbol{A}$ are known as atoms and $\boldsymbol{b}$ can be written as the weighted sum of the columns of $\boldsymbol{A}$. If we could use only column of $\boldsymbol{A}$ to represent $\boldsymbol{b}$, we will choose the column which $\boldsymbol{b}$ has maximum projection on.

Initially the residue ($\boldsymbol{r}$) is set equal to the signal. This program choses one column vector at a time so that the residual error decreases the most at every iteration. Thus at each iterative step the residual error is reduced the most by finding the largest projection of the residual into the column vectors. The column vector which is selected in the first iteration is not considered further for the next iteration. The iteration continues for the number of columns of A.

MATLAB code for Matching Pursuit Algorithm

```
%Ax=b,
A = readmatrix('A.xlsx'); % dictionary
b = readmatrix('b.xlsx'); % signal

% predict x------------
x = zeros(size(A,2),1);       % Initializing solution vector as all zeros.
index = 1:size(A,2);
r = b;                        % Initializing residue as b vector.

for i=1:size(A,2)
 DotP_own(i) = dot(A(:,i),A(:,i)); % Dotproduct of 'A'column with itself
end

while size(A,2)>0
   for i=1:size(A,2)
      DotP1(i) = dot(r,A(:,i));   % Dotproduct of residue with 'A' columns.
        XST(i) = DotP1(i)/DotP_own(i);
    end
   [~,xst_n] = max(abs(XST));
   xst = XST(xst_n);              % x star value.
   r = r - xst*A(:,xst_n);
   x(index(xst_n)) = xst;
   A(:,xst_n) = [];
   DotP1(xst_n) = [];
   DotP_own(xst_n) = [];
   XST(xst_n) = [];
   index(xst_n) = [];
end
x
```

The code is validated by running a few $\boldsymbol{Ax} = \boldsymbol{b}$ by providing $\boldsymbol{A}$ as an $m \times n$ matrix and $\boldsymbol{b}$ as a $m \times 1$ vector as inputs to obtain approximate $\boldsymbol{x}$ ($n \times 1$) vector.

For example considering the following inputs to the code

$$[A] = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 1 & 4 \end{bmatrix} \text{ and } \{b\} = \begin{Bmatrix} -2 \\ 11 \end{Bmatrix}$$

The code returns the $$\{x\} = \begin{Bmatrix} 0.3507 \\ -1.2294 \\ 2.4706 \end{Bmatrix}.$$

Having understood the basic working principle of the matching pursuit and its implementation in the EEG data file, the codes made available by Prof Ray corresponding to the work [1] entitled "*Comparison of Matching Pursuit Algorithm with Other Signal Processing Techniques for Computation of the Time-Frequency Power Spectrum of Brain Signals*" was studied to implement the Matching Pursuit program on the EEG data set. The program named figure5.m computed the time-frequency spectrum using MP algorithm for which the work flow is as follows.

1. Signal to be taken as input is decided.
2. Matching Pursuit is performed using the function runGabor.m
3. After the decomposition is performed using Matching Pursuit, some matlab functions are provided for the retrieval and visualization of gabor atoms. getGaborData.m is used to retrieve the information about gabor atoms.
4. The signal can be reconstructed from the atoms with the program reconstructSignalFromAtomsMPP. The time-frequency wigner-ville spectrum can be generated using the program reconstructEnergyFromAtomsMPP.

However, while trying to use the said matching pursuit algorithm on the EEG data set, some errors were encountered especially while trying to generate the time-frequency Wigner-Ville spectrum. Possible reasons could be incomplete understanding of some of the parameters used in the code which requires further in depth study.

5. CONCLUDING REMARKS

In the present summer project, a study was carried out to understand the complete process of analysis of EEG data right from recording of the EEG data from a subject, removing the artefacts from the data and then performing different analyses like ERP, FFT, STFT, MTM for extracting important time frequency features for correlating with important neural activities. Frequency bands presents in EEG signals obtained from brain is studied in details for understanding different frequencies attached with important activities. The advantages and limitations of different time frequency analysis methods have been studied from literatures with a special emphasis on implementing matching pursuit algorithm for detecting gamma burst. Though STFT could be used on the EEG data but the matching pursuit algorithm could not be implemented on the said EEG data set. This requires a better and in depth understanding of both the data set as well as the existing matching pursuit code and could be undertaken as a future extension.

6. ACKNOWLEDGEMENT

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I would also like to thank Mr. Ankan Biswas, Research Scholar, Centre for Neuroscience, IISc Bengaluru who has been always supportive whenever his help is sought during the period of internship.

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