Electricity & Magnetism in Light of Relativity

DS : Vector Calculus-2

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Problem 1 : A paradox - or is it? ⋆

(i) For a constant vector $\mathbf{A}$, first find the $\mathbf{A} \times \mathbf{r}$, where $\mathbf{r}$ is the position vector in terms of the components of $\mathbf{A}$ and $\mathbf{r}$. Then evaluate $\boldsymbol{\nabla} \times (\mathbf{A} \times \mathbf{r})$ in terms of the components.

(ii) We know the relation, $\boxed{\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}\,(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}\,(\mathbf{A} \cdot \mathbf{B})}$ *(If not, REMEMBER now!)*
we might say that $\boldsymbol{\nabla} \times (\mathbf{A} \times \mathbf{r}) = \mathbf{A}\,(\boldsymbol{\nabla} \cdot \mathbf{r}) - (\boldsymbol{\nabla} \cdot \mathbf{A})\,\mathbf{r} = 3\mathbf{A}$, which is clearly wrong!
What is the issue here, then?

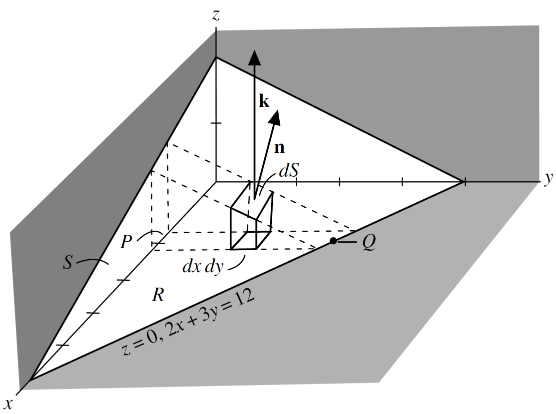
Problem 2 : Spherical & Cylindrical Polar Coordinates ⋆⋆

1. Express the unit vectors of a Spherical polar coordinate system (*i.e.* $\hat{r}, \hat{\theta}, \hat{\phi}$) in terms of the Cartesian unit vectors $(\hat{x}, \hat{y}, \hat{z})$. Also find the inverse relations between the unit vectors.
2. Do the same for the Cylindrical polar coordinate system (*i.e.* $\hat{\rho}, \hat{\phi}, \hat{z}$).
3. **Classical Mechanics :** Armed with all the previous relations, try finding the velocity and acceleration vector components in the spherical (or cylindrical) coordinate system.

Problem 3 : Surface integrals of vectors ⋆⋆

(a) **Starting with a simple one :** Take a simple vector function say, $\mathbf{r} = x\hat{x} + y\hat{y} + z\hat{z}$ and find the surface integral through the faces of a unit cube (sitting in the positive octant and having the origin as one of its vertex).

(b) Evaluate $\iint_S \mathbf{A} \cdot d\mathbf{S}$, where $\mathbf{A} = 18z\hat{x} - 12\hat{y} + 3y\hat{z}$ and S is part of the plane $2x + 3y + 6z = 12$, located in the first octant.



(c) **One more, for goodwill :** Suppose the velocity vector, $\mathbf{v} = 2x^2y\hat{x} - y^2\hat{y} + 4xz^2\hat{z}$ m/sec. Show that the flux of the fluid through the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$, along with the plane $x = 2$ amounts to 180 m^3/sec.