Electricity & Magnetism in Light of Relativity

DS : Magnetism & stuff

(MONSOON 2024)

DEBADYUITI GHOSH

Problem 1 : Crossed EM fields - Nothing relativistic here! ⋆

Suppose uniform electric and magnetic fields act perpendicular to each other, say, $\vec{B} = B_0\hat{z}$ and $\vec{E} = B_0\hat{y}$. Then consider the following questions (in a sequential fashion):

(a) Write the equation of motion for a charge q kept at the origin, which is initially at rest $u = 0$. *(Do not panic if you find solving coupled differential equations difficult!)*

(b) Consider another inertial frame of reference (let's call it S'-frame), moving relative to the above system with a velocity $v_0\,\hat{x}$. Analyse the system as you did in part(a) *wrt* this frame. There's a specific value that might help, $v_0 = E/B$ [1]. Can you solve it easily, now?

(c) Now, if the charge had an initial velocity $u \neq 0$ along the x-direction, how will the trajectory look like from the original frame of reference ? Discuss separately - three different cases, $v_0 < E/B$, $v_0 = E/B$ and $v_0 > E/B$.

Problem 2 : Electric & Magnetic forces - together, there be light!

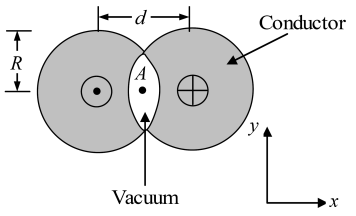
Two infinite line charges, with the same charge density λ, are kept along the x-axis, d distance apart from each other. If the lines are now moved together at a velocity v along the x-direction, what should be the numerical value of v so that the electrical and magnetic forces cancel?

Problem 3 : Magnetic dipoles

A small magnetic dipole is kept at the origin in the xy *plane*. One wire L_1 is located at $z = -a$ in the xz *plane* with a current I flowing in the positive x-direction. Another wire L_2 is at $z = +a$ in yz *plane* with the same current I as in L_1, flowing in the positive y direction. Find the angle made by the magnetic dipole with respect to the x-axis, at equilibrium.

Problem 4: Overlapping cylindrical current carrying wires ⋆

Two long conductors forms overlapping cylinders, each of radius R, whose centres are separated by a distance $d << R$. A current with current density J flows into the plane of the page along the shaded part of one conductor and an equal current flows out of the plane for the other, as shown. Find the magnetic field at point A.
(Assume that the vacuum region in the overlapping centre is small compared to the area of the conductors.)



[1] This need not be relativistic, so assume: $v_0 << c$