Electricity & Magnetism in Light of Relativity

(Monsoon 2024)

ASSIGNMENT-1

Instructor: Susmita Saha | **Due: 27th Sept, 2024** (EoD)

GA: Debadyuiti Ghosh | Total Marks: 20

Problem 1 : Charge Conservation [2]

The Maxwell's equations *(with some source charge density ρ and current density* **J***)*:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

These four equations contain all of electromagnetism, along with the *Lorentz force law*.

The conservation of any quantity α means that the density of that quantity ρ_α and its current density $\mathbf{j}_\alpha$ must always satisfy the continuity equation :

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot \mathbf{j}_\alpha = 0 \tag{1}$$

Show that charge conservation is implied in the Maxwell's equation, using vector identities.

(**Hint:** You might find some vector identities in the *DS Prob Set*)

Problem 2 : Variable charged cylinder [3]

Consider a very long cylindrical shell of height h and radius R, with non-uniform volume charge distribution, $\rho(r) = \alpha\, r^n \quad (n \geq 0)$. Find the electric field inside and outside this cylinder.

Plot different graphs corresponding to the variation in charge density, electric field and potential for $n = 1/2$, with distance r from the cylinder axis. (You may use *computational tools* for plotting).

Problem 3 : Cosmic rays - Potential consideration [3]

Cosmic rays are high-energy particles arriving from outer space that are almost entirely (about 90%) protons. The average energy of the bombarding protons is of the order of $1\,GeV$ and the intensity of the protons hitting the Earth is about 1 proton per cm^2 per sec.

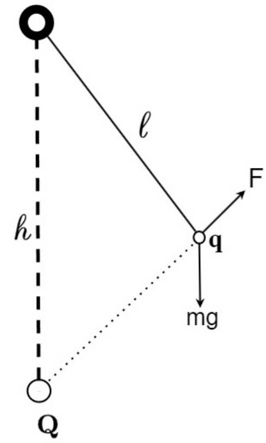
Assuming this over simplified, non-relativistic model with only electrostatic considerations, estimate how long it takes before the potential of the Earth becomes high enough to reject anymore protons coming from the outer space.

(The age of the Earth is ~ 5 billion yrs. Surprised?! Any guesses about how to resolve this?)

Problem 4 : Charged Pendulum **[6]**

Consider a charged point particle with charge q and mass m is suspended by a string of length l.

A second charge with $+Q$ charge is kept vertically below the point of suspension at a distance $h > l$. Assume that the pendulum is confined in the XY-plane.



(a) Calculate the value of the angle θ_0 for which the pendulum is at equilibrium with the effect of gravity. [3]

(b) Now suppose the pendulum is displaced by a small angle θ from its equilibrium position, then find the period of oscillation. [3]

Use the equation of motion expression directly from what you have learnt in *Classical Mechanics* and compare with the uncharged case.

Problem 5 : Charge enclosed from Flux calculation **[3]**

Find the total charge enclosed by the cube depicted in Fig.1, where the electric field is $\mathbf{E} = ky^2\hat{j}$.

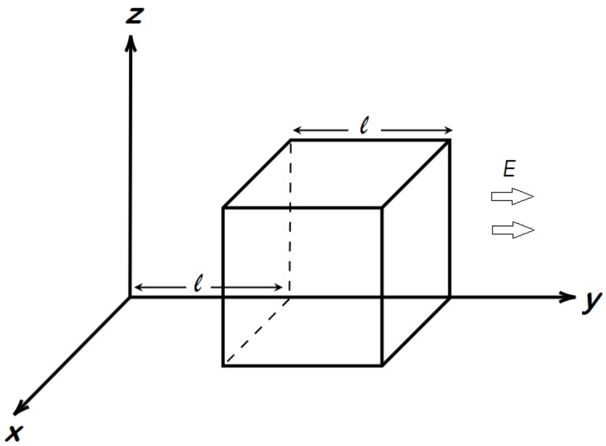


Figure 1: For Problem-5

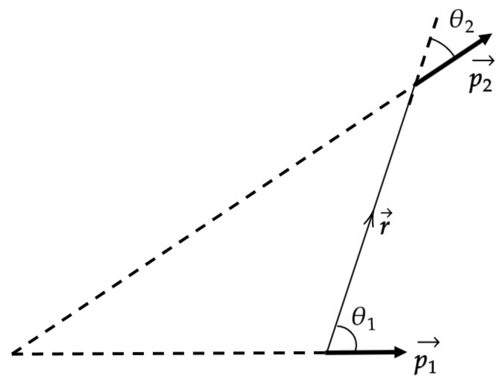


Figure 2: For Problem-6

Problem 6 : Molecular dipole moments **[3]**

One dipole (with dipole moment $\mathbf{p}_1$) is fixed at the origin while another (with dipole moment $\mathbf{p}_2$) is free to rotate at position $\mathbf{r}$, as shown in the Fig.2. Show that at equilibrium condition, the relation between the angles is given by,

$$\tan\theta_1 = -2\,\tan\theta_2$$