DS : Relativity & EM

(MONSOON 2024)

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Problem : Detection of Muons on Earth's surface

Muons are highly unstable particles created in the upper atmosphere of the Earth (about $\sim 10km$ above the surface). Due to energy constraints, muons can't be created anywhere near the surface of the Earth.

(a) What is the farthest that muons could travel, given that they decay in about $\tau = 2\mu s$? Is it possible to detect them on Earth's surface if this was the case?

(b) It's known that these muons generally have very high kinetic energies, with relativistic speeds $v \approx 0.99c$. Find the distance that the muons can now travel before "dying".

(c) Can you explain the above phenomenon using some other consequence of Special Relativity? (*Think - Is this result due to length contraction or time dilation?*)

Problem : The hypothetical particle - Tachyons ⋆⋆

Tachyons always moves at a velocity faster than the speed of light. If two persons are communicating via tachyonic signals (with transmission velocity $u > c$), answer the following questions:

(a) When the second person is at rest at a distance L, how much time will elapse before the sender can expect a reply?

(b) What happens if the person is now moving with a velocity v away from the sender and is at a distance L when the message is received and replied back?

(c) Check that for a certain velocity v such that $u > [1 + \sqrt{(1 - v^2)}]/v$, the reply is received **before the signal is sent!**

Problem : 19th century physics ⋆

A flash of light is emitted from A and passes through a glass slab which itslef moves to the right with speed $v << c$, to arrive at B at a distance L away from A. In the rest frame of the glass, it has thickness D and the speed of light in the glass is c/n, where n is the refractive index.

(a) If you were a 19th century physicist, who didn't know relativity but did know about the index of refraction and Galilean velocity addition, how long would you expect it to take the light to go from A to B? Keep everything to the lowest order term in v/c.

(b) How long does it actually take the light to go from A to B, again to lowest order in v/c?

This kind of setup could be part of an interference experiment, which would allow the tiny time difference to be effectively measured. Before the advent of special relativity, experiments like these which require relativistic velocity addition were very puzzling.

Problem : Lightning bolts striking

Consider the lightning example that we saw visually in one of the DSs. As seen by an observer at rest, lightning bolt strikes at a point A on the x axis and then $1\,sec$ later, another lightening bolt strikes at point B, $500\,m$ away from A.

(a) A second observer claims that he has seen the two lightning bolts striking simultaneously. Is this physically viable?

(b) What is the magnitude and the direction of the velocity v with which the second observer is moving relative to the first one?

Problem : Matrix Formulation of Lorentz Transformations ⋆

For this question, suppress the y and z coordinates, since they are unchanged due to being perpendicular to the direction of the "boost" (*i.e.*, the relative motion between two inertial frames). The transformation equations thus, should have only two coordinates (ct, x).

(a) Write the *Galilean* and the *Lorentz* transformation equations in matrix form.

(b) For a $2D$ system, write the rotation matrix $\mathcal{R}$ which transforms $(x, y) \rightarrow (x', y')$ where the primed coordinate system is rotated by an angle θ.

(c) The Lorentz transformations can be thought of as some sort of "generalized rotations" which mixes up time and space, just as ordinary rotations mix different spatial axes.

Define the generalized angle, called *rapidity* $\phi = \tanh^{-1}(v/c)$. Using this, express the Lorentz transformation matrix so that its comparable to the rotation matrix.

(d) Express the *velocity addition law* in terms of rapidity.

> **Note :** In some sense, *rapidity* is a more natural way of describing motion than velocity. Rapidities add linearly unlike normal velocities. Also the range of rapidity $\phi \in (-\infty, \infty)$ makes more sense than that of velocity $v \in (-c, c)$.