① Muons detection

$$\left.\begin{array}{l} \mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu \\ \mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu \end{array}\right\} \quad \tau = 2\mu s$$

Even if they moved with speed c, distance $= c\tau \approx 600$ m

Event	S (Earth frame)	S' (Muon's frame)
creation	(x_A, t_A)	(x_A', t_A')
decay	$(x_A + 10\,km, t_B)$	$(x_A', t_A' + 2\mu s)$

$\Delta x' = 0$, $\Delta t' = \tau = 2\mu s$

$\Delta x = ?$ $\quad\Rightarrow\quad \Delta x = \gamma(\Delta x' + v\Delta t') = \gamma v \tau$

Earth-distance would be contracted. How much?

distance $= \Delta x/\gamma = \dfrac{10\,km}{\gamma}$ $\longrightarrow$ muon covers this in $2\mu s$

④ Lightning bolts striking

$\Delta x = 500$ m, $\Delta t = 1$ sec

$\Delta t' = 0$, $\gamma = ?$

Event	S-frame	S'-frame
1st	(x_A, t_A)	(x_A', t_A')
2nd	$(x_A + 500m, t_A + 1s)$	(x_B', t_A')

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$$

$$\Rightarrow 1\mu s = \frac{v}{c^2} \times 500 \text{ m}$$

$$\Rightarrow v = \frac{10^{-6} c^2}{500} = \frac{9 \times 10^{10}}{500} \simeq 1.8 \times 10^8 \text{ m/s}$$

2 Tachyons ($u \to$ Tachyonic speed)

(a) $$t_{\text{round trip}} = \frac{2L}{u}$$

(b) For the return journey, $\frac{\Delta x'}{\Delta t'} = u$

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$
$$\Delta t' = \gamma(\Delta t - v\Delta x)$$

$$\Rightarrow \quad u = \frac{\Delta x - v\Delta t}{-\Delta t + v\Delta x} = \frac{(\Delta x/\Delta t) - v}{v\left(\frac{\Delta x}{\Delta t}\right) - 1}$$

$$\Rightarrow \quad \frac{\Delta x}{\Delta t}(uv - 1) = u - v$$

$$\Rightarrow \quad \boxed{\left.\frac{\Delta x}{\Delta t}\right|_{\text{return}} = \frac{u-v}{uv-1}}$$

So, total time, $$t_{\text{total}} = t_{\text{out going}} + t_{\text{return}} = \frac{L}{u} + L\left(\frac{1-uv}{u-v}\right)$$

$$= L\left[\frac{1}{u} + \left(\frac{1-uv}{u-v}\right)\right]$$

$$-\frac{\Delta x}{\Delta t}$$

For $u > \frac{1+\sqrt{1-v^2}}{v}$, $\Rightarrow uv > 1 + \sqrt{1-v^2}$

$$\Rightarrow 1 - uv < -\sqrt{1-v^2} \qquad \text{(i)}$$

and, $$u - v > \frac{1+\sqrt{1-v^2}}{v} - v = \sqrt{1-v^2}\left(\frac{1+\sqrt{1-v^2}}{v}\right) \qquad \text{(ii)}$$

From (i) & (ii) $$\Longrightarrow \frac{1-uv}{u-v} < \left(\frac{-v}{1+\sqrt{1-v^2}}\right) \qquad \text{(iii)}$$

Also, $$\frac{1}{u} < \frac{v}{1+\sqrt{1-v^2}} \qquad \text{(iv)}$$

$$\therefore \quad t_{tot} = L\left(\frac{1}{u} + \frac{1-uv}{u-v}\right) < 0$$

5 Matrix formulation & Rapidity

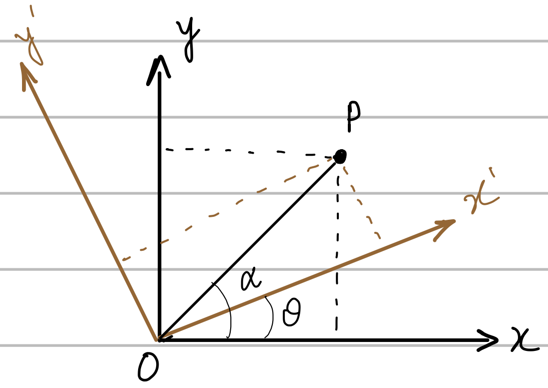
$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \qquad \begin{pmatrix} x' \\ ct' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\frac{v}{c} \\ -v/c & 1 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

GALILEAN $\qquad\qquad$ LORENTZ

Define $\tanh \phi = \frac{v}{c} = \beta$

$$\left\{ \begin{array}{l} \cosh^2\phi - \sinh^2\phi = 1 \\ \Rightarrow \cosh\phi = \gamma \\ \text{and, } \sinh\phi = \beta\gamma \end{array} \right.$$

Then $\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$



* Rotation Matrix

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

As, $x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

$x = OP \cos\alpha$

$y = OP \sin\alpha$

$$\left[\begin{array}{l} x' = OP\cos(\alpha-\theta) = OP(\cos\alpha\cos\theta + \sin\alpha\sin\theta) \\ \quad = x\cos\theta + y\sin\theta \\ y' = OP\sin(\alpha-\theta) = OP(\sin\alpha\cos\theta - \sin\theta\cos\alpha) \\ \quad = y\cos\theta - x\sin\theta \end{array} \right.$$

Velocity addition law :

$$w = \frac{u+v}{1+uv/c^2}$$

↪ dividing by c $\Rightarrow \beta_w = \frac{w}{c} = \frac{u/c + v/c}{1 + uv/c^2} = \frac{\beta_u + \beta_v}{1+\beta_u\beta_v}$

↪ using $\tanh\phi_i = \beta_i$

$$\Rightarrow \tanh\phi_w = \frac{\tanh\phi_u + \tanh\phi_v}{1+\tanh\phi_u \tanh\phi_v} = \tanh(\phi_u + \phi_v)$$