Electricity & Magnetism in Light of Relativity

(MONSOON 2024)

ASSIGNMENT-2

Instructor: SUSMITA SAHA | **Due: 28th Nov** (in class)

GA: DEBADYUITI GHOSH | Total Marks: 15

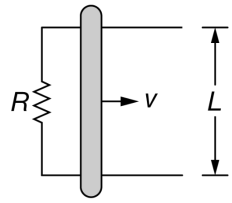
Your submission reflects your commitment to quality - keep it neat and readable.
Please use fresh A4 sheets that seem professional, avoiding torn notebook pages

Problem 1 : Rotating charged sphere [2]

A uniformly charged sphere of radius R, rotates about its diameter with some constant angular velocity ω. If ρ is the charge density on the sphere and μ_0 is permeability of vacuum, show that the magnetic field at the centre is $B = \frac{1}{3}\mu_0\rho\omega R^2$

Problem 2 : Motional emf [2]

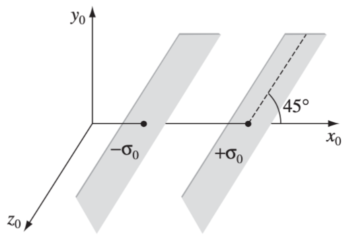
A rod PQ of mass m and length l slides on two conducting rails connected to a resistance R (as shown). A uniform magnetic field acts perpendicular to the plane of the rails and the friction coefficient between the rod and the rails is μ. If the rod is given an initial outward velocity v_0 then find the velocity at a later time t.



Problem 3 : Transformation of EM fields [8]

The transformation relations for the field components are as follows:

$$E'_x = E_x\,, \qquad B_x{}' = B_x\,,$$
$$E'_y = \gamma\left(E_y - vB_z\right)\,, \qquad B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right)\,,$$
$$E'_z = \gamma\left(E_z + vB_y\right)\,, \qquad B'_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)\,,$$



(a) Using the above transformation equations, prove that the $\vec{E}\cdot\vec{B}$ is Lorentz invariant [2]
(Thus, if $\vec{E}$ & $\vec{B}$ are perpendicular in one frame, they remain perpendicular in all inertial frames)

(b) A slanted parallel plate capacitor (see fig.) is at rest *wrt* S-frame and tilted at 45^o with the x-axis. It carries surface charge densities $\pm\sigma_0$ on the two plates. If S'-frame is moving away with velocity v along the x-axis, find the electric field $\vec{E}$ and $\vec{E}'$ *wrt* both the frames. [3]
Is the field perpendicular to the plates in the moving frame? [3]

Problem 4 : What if magnetic charge exists?! [2+1]

If magnetic charge is assumed to exist and if ρ_m be the associated magnetic charge density, the relation : $\vec{\nabla}\cdot\vec{B} = \mu_0\rho_m$ holds true. Show that the usual form of Faraday's law is then inconsistent with the presence of a magnetic charge density that is a function of position and time.

Modify Faraday's law so that whole the set of Maxwell's laws makes sense.