Process Reward Models for LLM Agents: Practical Framework and Directions

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Abstract

We introduce Agent Process Reward Models (`AgentPRM`), a simple and scalable framework for training LLM agents to continually improve through interactions. `AgentPRM` follows a lightweight actor-critic paradigm, using Monte Carlo rollouts to compute reward targets and optimize policies. It requires minimal modifications to existing RLHF pipelines, making it easy to integrate at scale. Beyond `AgentPRM`, we propose `InversePRM`, which learns process rewards directly from demonstrations without explicit outcome supervision. We also explore key challenges and opportunities, including exploration, process reward shaping, and model-predictive reasoning. We evaluate on ALFWorld benchmark, show that small 3B models trained with `AgentPRM` and `InversePRM` outperform strong GPT-4o baselines, and analyze test-time scaling, reward hacking, and more. Our code is available at: https://github.com/sanjibanc/agent_prm.

1 Introduction

Large language model (LLM) agents excel in decision-making tasks such as web navigation [1], robotics [2], and interactive code generation [3]. However, they rely heavily on prompting [4, 5] or supervised fine-tuning (SFT [6]. Prompting demands extensive manual effort [1, 7] and does not enable autonomous improvement. SFT, while effective, is constrained by demonstration quality and lacks mechanisms for self-correction at test time.

This raises a fundamental question: *How can LLM agents improve through interaction without extensive human supervision?* Reinforcement learning (RL) naturally enables policy refinement through experience, but applying RL to LLM agents presents key challenges: (1) *Long-horizon decision-making:* LLM agents must reason over multiple steps, producing structured multi-token outputs that blend reasoning and actions. (2) *Sparse rewards:* Feedback is often delayed until the end of long interactions, complicating credit assignment. While large-scale RL approaches have been explored [8], they remain impractical due to high sample complexity.

Instead of large-scale RL, we propose a more tractable alternative: *Agent Process Reward Models* (`AgentPRM`). PRMs provide fine-grained supervision at each step, akin to critic [9] or value functions in RL. By evaluating intermediate actions rather than relying on sparse outcome rewards, PRMs improve sample efficiency. While PRMs have been explored in multi-step reasoning tasks [10–12], they are underexplored in agentic settings where actions impact an external environment. Our work addresses this gap.

We propose a simple and scalable framework for training `AgentPRMs`. It has two key aspects:

1. *Automatic PRM annotation:* PRM targets are computed using asynchronous Monte Carlo rollouts, enabling agents to learn without manually labeled rewards.
2. *Iterative training:* PRMs and policies are jointly trained in an iterative process, where each refines the other to improve overall performance.
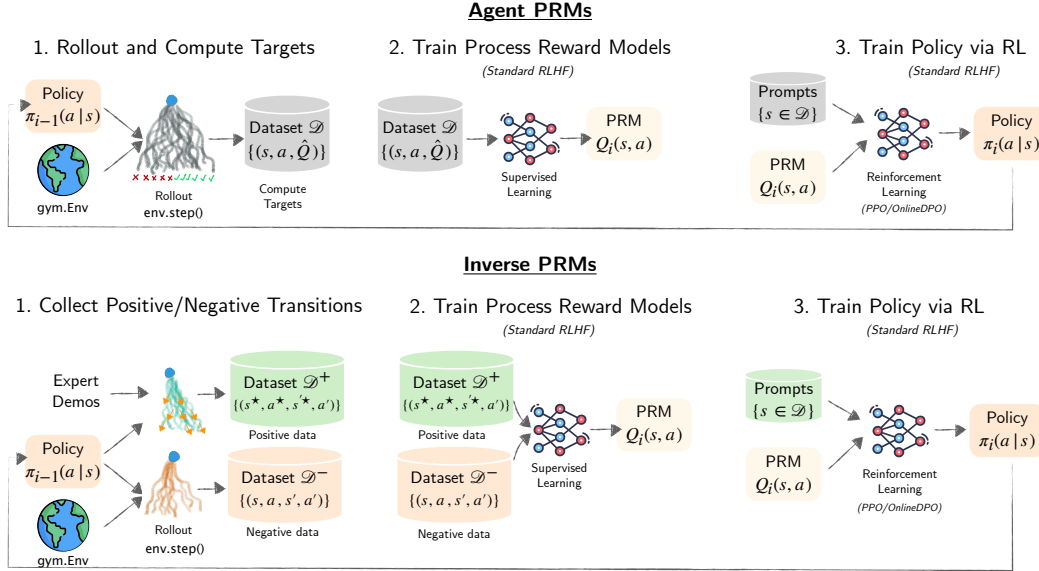
Figure 1: **Overview** (a) `AgentPRM`: Trains an LLM policy π using outcome rewards through three iterative stages. Stage 1: Roll out the current policy π_{i-1} and compute the PRM target dataset $\mathcal{D}$. Stage 2: Train PRM Q_i on $\mathcal{D}$ via supervised learning. Stage 3: Update policy π_i using RL with PRM Q_i. (b) `InversePRM`: Trains π using expert demonstrations in three stages. Stage 1: Roll out π_{i-1} to generate positive $\mathcal{D}^+$ and negative $\mathcal{D}^-$ transition datasets. Stage 2: Train PRM Q_i to distinguish between $\mathcal{D}^+$ and $\mathcal{D}^-$. Stage 3: Optimize π_i via RL with PRM Q_i. Note: Stages 2 and 3 align with standard RLHF pipelines; only Stage 1 is newly introduced.

The framework is *simple*: it follows the actor-critic paradigm, a well-established RL algorithm with strong theoretical foundations and practical flexibility. The framework is *scalable*: it seamlessly integrates into existing RLHF infrastructure [13, 14] with only one additional component—automatic reward annotation.

This simple framework opens up new questions, algorithms, and research directions. We introduce `InversePRM`, which learns PRMs directly from demonstrations without explicit outcome rewards. `InversePRM` achieves higher sample efficiency than `AgentPRM` without added complexity. We also examine challenges in scaling `AgentPRM`, including exploration, sample efficiency, and model-predictive reasoning. To address these, we explore a combination of established RL techniques—such as reset distribution and reward shaping—with LLM-driven strategies like steered exploration and model-predictive reasoning.

Our key contributions are:

1. **Algorithms and Code.** We introduce `AgentPRM` (Sec. 2), a scalable method for training process reward models, and `InversePRM` (Sec. 3), which learns PRMs directly from demonstrations. Our implementation is a light-weight Gym wrapper around OpenInstruct[1], making it easy to integrate with existing RLHF pipelines.

2. **Evaluation and Analysis.** We evaluate on a text game benchmark ALFWorld [15] and find:
   - `AgentPRM` enables small (3B) models to outperform strong GPT-4o baselines. We analyze training curves, test-time scaling, reward hacking, and absolute v/s relative loss (Sec. 2.3).
   - `InversePRM` achieves near expert performance in a single iteration, significantly outperforming SFT and being more sample-efficient than `AgentPRM` (Sec. 3.3).

3. **Challenges and Opportunities.** We discuss challenges and new research opportunities in:
   - *Exploration*: We explore resets and steered exploration to accelerate training (Sec. 4.1).
   - *Process Reward Shaping*: We use reference policies to shape process rewards to stabilize training in low sample regimes (Sec. 4.2).
   - *Model-Predictive Reasoning*: We discuss reasoning as model-predictive planning to practically enable large-scale RL to apply to agent settings (Sec. 4.3).

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[1] https://github.com/allenai/open-instruct [13]

2 Agent Process Reward Models: A Simple Framework

2.1 Formulation

Consider an agent interacting with an environment over multiple turns to solve a task. We model this interaction as a *turn-level* Markov Decision Process (MDP). At turn t, the state s_t is the history of observations and actions, $s_t = \{o_0, a_0, \ldots, o_{t-1}\}$. The agent selects an action a_t and transitions to a new state s_{t+1} according to the environment dynamics. The agent receives a reward $r(s_t, a_t) \in [0, 1]$, typically provided at terminal states and referred to as the *outcome reward*, which evaluates the overall success of the task. The agent's behavior is determined by a policy $\pi(a_t \mid s_t)$, which maps states to a distribution over actions. The objective of the policy is to maximize the expected return, defined as the sum of discounted rewards $\mathbb{E}_\pi \left[\sum_{t=0}^{T-1} \gamma^t r(s_t, a_t) \right]$, where γ is the discount factor.

For LLM agents, each action a_t consists of a sequence of tokens, encoding both reasoning and an environment action. This induces a two-level decision hierarchy:

1. *Turn-level MDP*: Models the sequence of agent-environment interactions over multiple turns.
2. *Token-level MDP*: Models the sequence of tokens within each turn, each token is an action.

Typically, RLHF frameworks are single-turn and hence perform RL only at token-level MDP. We next look at how to lift these frameworks to solve turn-level MDPs.

Agent Process Reward Models. A *process reward model (PRM)* [10] assigns turn-wise scores in a multi-turn response, providing structured feedback to guide policy learning. In turn-level MDPs, a PRM functions as a state-action value function, analogous to a Q-function in RL. Formally, the PRM is $Q^\pi(s_t, a_t) = \mathbb{E}_\pi \left[\sum_{k=t}^{T} \gamma^{k-t} r(s_k, a_k) \mid s_t, a_t \right]$. Maximizing PRM $Q^\pi(s_t, a_t)$ enables the policy to improve task performance through intermediate feedback rather than relying on outcome rewards.

Distinction from Reasoning Tasks. PRMs have primarily been studied in multi-step math reasoning tasks [10, 16] where transitions are deterministic and known. In these settings, test-time search methods like beam search [17] can be used to optimize reasoning sequences. In contrast, LLM agents operate in external environments with unknown, stochastic transitions, where actions have uncertain effects. This makes beam search impractical, as future states cannot be enumerated in advance. We focus on training PRMs and policies under these complex settings.

2.2 Approach

We adopt a policy iteration framework to jointly train the process reward model $Q^\pi(s, a)$ and the agent policy $\pi(a|s)$. Algorithm 1 describes the three-stage process:

1. Rolling out the current policy π_θ to collect data and compute Q targets
2. Train the PRM $Q_\phi(s, a)$ given Q targets (*standard RLHF*)
3. Train the policy π_θ using reinforcement learning on the trained RM (*standard RLHF*)

This follows standard RLHF pipelines, with the key difference being *Stage 1*, where PRM targets are computed from rollouts rather than preference labels. We describe each stage below.

Stage 1: Rollout and Compute Target. At iteration i, we roll out the policy π_{i-1} in the environment to generate trajectories of states, actions, and rewards $\mathcal{D}_{\text{rollout}} = \{(s_0, a_0, r_0, \ldots, s_{T-1}, a_{T-1}, r_{T-1})\}$. To scale up data collection, we run environments in parallel and step through them in batched mode. Each batch of states is sent to the model, which returns a corresponding batch of actions. We leverage fast inference libraries such as SG-Lang [18] and VLLM [19]. To improve state coverage, we roll out π_{i-1} multiple times on the same task, ensuring repeated state visits. Rollouts are stored in a dictionary $\mathcal{G}(s, a)$, which maps each hashed state-action pair to the set of trajectories passing through (s, a). We compute PRM targets as

$$\hat{Q}(s, a) = \frac{1}{|\mathcal{G}(s, a)|} \sum_{(s_t, a_t) \in \mathcal{D}(s, a)} \sum_{k=t}^{T-1} \gamma^{k-t} r_k \tag{1}$$

Finally we normalize the targets $\hat{Q}(s, a)$ to be between $[0, 1]$. The final dataset is then $\mathcal{D} = \{(s, a, \hat{Q})\}$ which is used to train the PRM. Note that we found this approach to be significantly simpler than

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Algorithm 1 Training Agent PRMs

1: Initialize with agent policy π_0
2: **for** iteration $i = 1, \ldots, K$ **do**
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ▷ **Stage 1: Rollout and Compute Targets**
3: $\quad$ Collect rollout $\{(\ldots, s_t, a_t, r_t, \ldots)\}$ using π_{i-1} and store in dictionary $\mathcal{G}(s, a)$
4: $\quad$ Compute PRM targets $\hat{Q}(s, a) = \frac{1}{|\mathcal{G}(s,a)|} \sum_{(s_t, a_t) \in \mathcal{D}(s,a)} \sum_{k=t}^{T-1} \gamma^{k-t} r_k$
5: $\quad$ Aggregate data into dataset $\mathcal{D} = \{(s, a, \hat{Q})\}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ▷ **Stage 2: Train Process Reward Model**
6: $\quad$ Train PRM $Q_i = \arg\min \mathcal{L}(Q_\phi)$ by minimizing the soft binary cross-entropy loss:

$$\mathcal{L}(Q_\phi) = -\mathbb{E}_{(s,a,\hat{Q}) \sim \mathcal{D}} \left[\hat{Q} \log Q_\phi(s, a) + (1 - \hat{Q}) \log(1 - Q_\phi(s, a)) \right]. \qquad (2)$$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ▷ **Stage 3: Train Policy via RL**
7: $\quad$ Update policy π_i to maximize Q_i while regularizing to π_{i-1}:

$$\pi_i = \arg\max_{\pi_\theta} \mathbb{E}_{s \sim \mathcal{D}, a \sim \pi_\theta(a|s)} [Q_\phi(s, a)] - \beta \mathbb{D}_{\mathrm{KL}} \left[\pi_\theta(a|s) || \pi^{i-1}(a|s) \right] \qquad (3)$$

8: **end for**
9: **return** Best $\pi \in \{\pi_1, \ldots, \pi_K\}$ on validation dataset

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doing a Monte-Carlo Tree Search (MCTS) which requires synchronous exploration and is difficult to scale. In contrast, we collect our rollouts asynchronously.

Stage 2: Train Process Reward Model. At iteration i, the PRM Q_i is trained via supervised learning on dataset $\mathcal{D}$. We use a soft binary cross-entropy (BCE) loss, treating $\hat{Q}(s, a)$ as a soft label:

$$\mathcal{L}(Q_\phi) = -\mathbb{E}_{(s,a,\hat{Q}) \sim \mathcal{D}} \left[\hat{Q} \log Q_\phi(s, a) + (1 - \hat{Q}) \log(1 - Q_\phi(s, a)) \right]. \qquad (4)$$

The PRM is updated by minimizing this loss $Q^i = \arg\min_{Q_\phi} \mathcal{L}(Q_\phi)$. Note that this stage is similar to training a reward model in RLHF, where the loss function is a Bradley-Terry (BT) loss on preference data. We too explore using a BT loss as an ablation in Sec. 2.3.

Stage 3: Train Policy via RL. Finally, we update the policy π_i to maximize the PRM while staying close to the previous policy.

$$\pi_i = \arg\max_{\pi_\theta} \mathbb{E}_{s \sim \mathcal{D}, a \sim \pi_\theta(a|s)} [Q_\phi(s, a)] - \beta \mathbb{D}_{\mathrm{KL}} \left[\pi_\theta(a|s) || \pi^{i-1}(a|s) \right] \qquad (5)$$

The above can be solved via standard RLHF frameworks that employ PPO [20], Online DPO [21], or Rejection Sampling [22]. We use Online DPO in our experiments.

Notably, the policy is regularized to stay close to π_{i-1} rather than the initial SFT policy. Since the PRM is trained on rollouts generated by π_{i-1}, straying too far from this reference can degrade PRM accuracy. This aligns with the principle of conservative policy iteration [23], where policies are updated within a restricted distributional shift to maintain validity of learned reward estimates. This approach is also consistent with best practices in online DPO [21].

Inference. At test time, we can improve policy execution using a Best-of-N strategy, denoted as $\mathrm{BoN}(\pi, Q)$. At each turn, we sample N candidate responses from π and select one with the highest PRM score $Q(s, a)$. This provides a simple yet effective way to leverage the process reward model for inference. Test-time scaling is controlled via N: increasing N allows the agent to explore a wider set of responses while still relying on Q for selection.

2.3 Experiments

Setup. We evaluate our approach on ALFWorld [24], a standard text-based game benchmark for language agents. Each task specifies a high-level goal, e.g., "heat mug and put it in cabinet," which the agent must accomplish by issuing text commands (e.g., "go to shelf 1," "pick up mug 2"). Solving

Method	All tasks		PICK tasks	CLEAN tasks	HEAT tasks	COOL tasks	LOOK tasks	PICK 2 tasks
	%suc↑	#act↓	%suc↑	%suc↑	%suc↑	%suc↑	%suc↑	%suc↑
BUTLER [1]	35.0	-	50.0	74.0	83.0	91.0	39.0	65.0
ReAct few-shot [2]	57.0	-	65.0	39.0	83.0	76.0	55.0	24.0
Autogen gpt-3.5 [3]	77.0	-	-	-	-	-	-	-
ExpeL gpt-3.5 [4]	59.0	-	-	-	-	-	-	-
Reflexion gpt-3 [5]	88.0	-	75.0	90.3	91.3	90.5	88.9	94.1
AdaPlanner gpt-3 [6]	91.7	-	100.0	96.7	95.6	100.0	100.0	47.0
ReAct gpt-4o	65.7	20.2	91.7	35.5	56.5	52.4	100.0	76.5
ReAct gpt-4o-mini	29.9	25.5	33.3	25.8	17.4	14.3	66.7	29.4
ReAct claude-3.5-sonnet	76.1	19.0	95.8	61.3	60.9	81.0	88.9	76.5
ReAct claude-3.5-haiku	16.4	27.2	33.3	9.7	8.7	9.5	38.9	0.0
ReAct gemini-1.5-flash	19.4	26.3	41.7	12.9	13.0	19.0	16.7	11.8
Llama3.2-3B π_0	64.9	14.9	62.5	74.2	69.6	71.4	66.7	35.3
Llama3.2-3B BoN(π_0, Q_0)	67.9	15.1	66.7	74.2	69.6	71.4	66.7	52.9
Llama3.2-3B π_1	73.9	14.0	58.3	80.6	73.9	71.4	100.0	58.8
Llama3.2-3B BoN(π_1, Q_0)	84.3	13.5	75.0	90.3	95.7	76.2	100.0	64.7
Llama3.2-3B π_2	85.8	12.6	75.0	87.1	91.3	100.0	100.0	58.8
Llama3.2-3B BoN(π_2, Q_1)	88.8	12.0	79.2	87.1	91.3	100.0	100.0	76.5
Llama3.2-3B π_3	88.1	12.7	79.2	90.3	91.3	100.0	100.0	64.7
Llama3.2-3B BoN(π_3, Q_2)	91.0	12.5	87.5	87.1	91.3	100.0	100.0	82.4

Table 1: `AgentPRM` **Evaluation on Alfworld** on 136 out-of-distribution games (max 30 actions). Baseline comparisons include [1] BUTLER [15], [2] ReAct few-shot [4], [3] Autogen [25], [4] ExpeL [26]. Note [5] Reflexion [5] and [6] AdaPlanner [27] make multiple attempts on the same test task, while we do not. We also add our own REACT instruction prompt with different models. `AgentPRM` with a 3B model across iterations (π_1, π_2, π_3) outperforms the stronger models like `claude-3.5-sonnet`.

these tasks requires subgoal planning, progress tracking, and efficient object search (e.g., mugs are likely on shelves or in cabinets). Each task consists of 30 timesteps. The dataset contains 6 task categories, a training set of 3257 games, and two evaluation sets: 139 in-distribution tasks and 134 out-of-distribution tasks. Performance is measured by task success rate (%suc↑) and average number of actions (#act↓).

We compare against a prior work BUTLER [24] and a number of prompting baselines ReAct [4], Autogen gpt-3.5 [25], ExpeL gpt-3.5 [26], Reflexion gpt-3 [5], AdaPlanner gpt-3 [27]. The prompting baselines all use larger gpt models along with few-shot examples. Adaplanner and reflexion get multiple attempts on the same task at test time, which significantly boosts performance. We also add ReAct baselines using the exact same prompt that our fine-tuned agent uses, with stronger models such as gpt-4o[2], claude[3], and gemini[4].

For `AgentPRM`, we fine-tune Llama3.2-3B [22] for both PRM and policy models, and run the process for 3 iterations. The policy π_0 is initialized using SFT data. At each iteration, we collect $10k$ rollout trajectories (parallelized) which are used to train the PRM and the generator. See code for hyperparameters and prompts for the agent. There are two modes of inference: using the policy π_i directly or doing Best-of-N BoN(π, Q) with policy π and PRM Q and $N = 16$.

Overall Results. Table 1 shows the performance of `AgentPRM` against all baselines. `AgentPRM` outperforms all baselines, with the best policy achieving 88.1% success rate and 91.0% success rate in best-of-N mode.[5] Iteration 2 has the biggest performance gain ($73.9\% \rightarrow 85.8\%$) **leading to a policy π_2 that surpasses the strongest model `claude-3.5-sonnet`** with higher success rate ($85.8\% > 76.1\%$) and lower actions ($12.0 < 19.0$). Best-of-N always leads to a higher performance gain, the iteration 1 having the largest gain ($73.9\% \rightarrow 84.3\%$), eventually plateuing for iteration 3 with ($88.1\% \rightarrow 91.0\%$).

Training Curves. Fig. 2 (a) shows how success rate evolves during policy training via RL (Stage 3). Success improves across iterations (π_0: 64.9%, π_1: 73.9%, π_2: 85.8%, π_3: 88.1%), with each policy achieving higher success than its predecessor. At each iteration, success rate increases over training steps but eventually plateaus due to over-optimization—i.e., the policy exploits the PRM beyond its training distribution. Re-training with the updated PRM mitigates this issue and enables further improvements, though performance saturates at π_3, likely due to model capacity limits. The largest improvement occurs between π_1 (73.9%) and π_2 (85.8%), with gains appearing early in training

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[2] https://platform.openai.com/docs/models   [3] https://docs.anthropic.com/en/docs/about-claude/models
[4] https://ai.google.dev/gemini-api/docs/models/gemini   [5] Adaplanner with gpt-3 has a higher success rate, but gets multiple attempts at test time rendering comparison unfair.
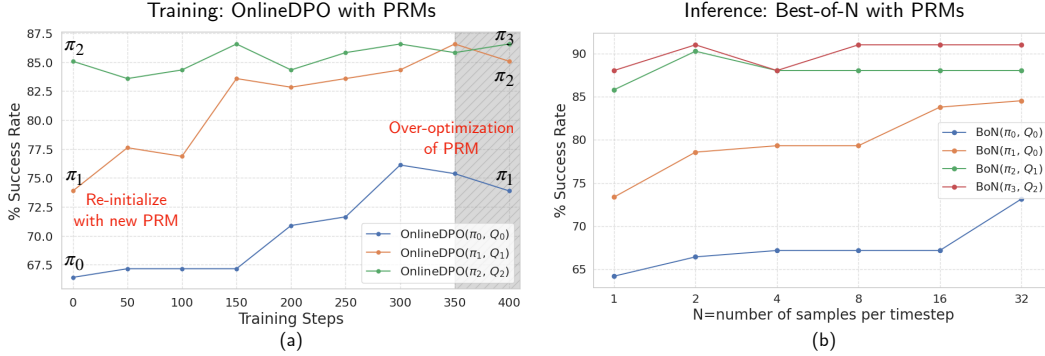
Figure 2: **Training and Inference.** (a) Success rate vs training steps during online DPO with PRMs for 3 iterations of `AgentPRM`. π_0 is initialized with SFT. PRM Q_0 is trained on π_0 rollouts. OnlineDPO(π_0, Q_0) is run for 400 training steps, during which the success rate goes up till it plateaus. The final checkpoint π_1 is taken and the process repeated to get π_2, π_3 till success rate limit is reached. (b) Inference with Best-of-N with varying $N = 1, 2, \ldots, 32$. For earlier policies π_0, π_1 success rate increases significantly, but scaling gains are limited for later policies π_2, π_3.

(within 150 steps). In contrast, $\pi_0 \rightarrow \pi_1$ gains emerge later (after 150 steps). This suggests that Q_1 is trained on more successful trajectories than Q_0, providing a better optimization landscape for policy improvement.

Test-time Scaling. Fig. 2 (b) shows success rates in Best-of-N mode as N varies from 1 to 32. For earlier policies (π_0, π_1), performance improves significantly as N increases, with the largest gains for $N > 16$. However, for later policies (π_2, π_3), scaling gains diminish. This is both due to the limited head-room, but also due to reward over-optimization which we discuss next.

Question: Can we measure and mitigate reward hacking? A common issue in RLHF-style training is *reward hacking* [28, 29], where the policy optimizes the learned reward model rather than achieving true task success. This occurs when:

1. The policy drifts too far from the distribution on which the PRM was trained.
2. The PRM is trained on insufficient rollouts, leading to poor generalization.

We control for (1) and investigate (2) by training PRMs on $10k$ vs. $70k$ rollouts.

Fig. 3 shows how both success rates (outcome rewards) and process rewards vary over training steps when the PRM is trained over $10k$ rollouts. After 400 steps, the success rate begins to fall from 82% to 70%. In contrast, the reward on the
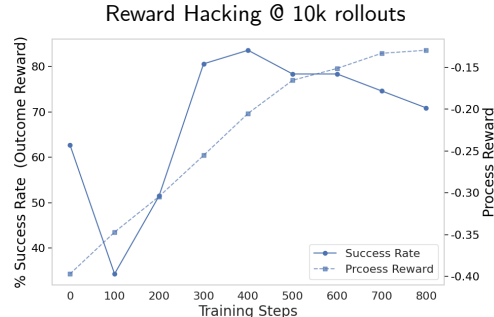

Figure 3: **Process Reward Hacking.** Success rate (outcome reward) and process reward over training step for a PRM trained with 10k rollouts. Process reward on validation data keeps increasing while outcome reward peaks and then degrades.

validation set keeps increasing. This shows clear signs of reward hacking. An open question remains how to reliably detect over-optimization without evaluating the success rate (which is difficult to scale). We tried an ensemble technique, training multiple reward models on different partitions of the data, but they all increased over training steps.

Question: Can we train PRMs on relative vs absolute losses? While we train PRMs using an absolute fashion, i.e., predict $Q(s, a)$, we use them in a relative fashion: (1) During training (online DPO), the PRM ranks two different responses by the policy. (2) During inference, the PRM ranks different responses generated by the policy. This raises the question: should PRMs predict absolute values ($Q^\pi(s, a)$) or relative values ($A^\pi(s, a)$)?

From an RL perspective, advantage functions $A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$ often exhibit lower variance, improving stability during training. Prior work in mathematical reasoning [12] has also

made similar arguments for training PRMs as advantage estimators. Intuitively, it might be difficult to judge how good an action in a globally normalized manner, but much easier to judge the action locally among other functions.

To train PRMs in a relative manner, we use the following procedure:

1. **(Stage 1)** Collect rollouts and construct a dictionary $\mathcal{G}(s)$ that maps each state to its sampled actions and corresponding Q values.

2. **(Stage 1)** Construct a preference dataset consisting of ranked action pairs $(s, a_1 \geq a_2)$, where $Q(s, a_1) - Q(s, a_2) \geq \delta$. Here, δ is a hyperparameter that defines a minimum margin for preference.

3. **(Stage 2)** Train Q using a Bradley-Terry loss [30]: $-\mathbb{E}_{(s,a_1,a_2)\sim\mathcal{D}}[\log \sigma(Q_\phi(s, a_1) - Q_\phi(s, a_2))]$ where $\sigma()$ is the sigmoid function.

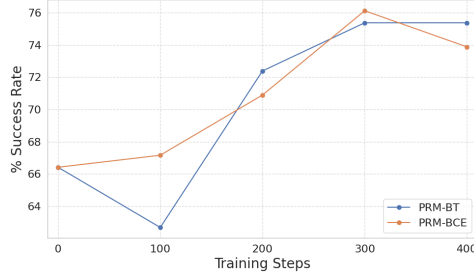Bradley-Terry (BT) vs Binary Cross Entropy (BCE)

Figure 4: **Absolute vs Relative Loss for PRM**. Success rate over training steps for PRM trained with $70k$ rollouts. Both losses lead to similar performance.

Fig. 4 compares PRMs trained with absolute vs. relative losses. Surprisingly, both approaches yield similar performance. One explanation is that the dataset size for absolute vs relative is not equal. If a state isn't visited multiple times, it is discarded for the relative loss. There are far fewer states that are visited multiple times, leading to a small dataset and hence higher error for the PRM.

3 Inverse Process Reward Models

The agent PRM framework in Sec. 2 assumes access to outcome rewards, which may not always be available. Designing rewards manually is labor-intensive and susceptible to misspecification [28, 31], as it requires explicitly capturing every success and failure condition. Instead, consider a setting where the agent has access only to expert demonstrations—sequences of successful actions performed by a human, rule-based agent, or promoted LLM agent. The key challenge is: *How can we learn process reward models solely from demonstrations, without access to explicit outcome rewards?*

3.1 Formulation

Given a set of expert demonstrations $\mathcal{D}^* = \{(s^\star, a^\star)\}$, the goal is to infer a reward function $r(s, a)$[6] that explains expert behavior. We formulate this as inverse reinforcement learning (IRL), which learns a reward that maximizes the expert's expected return relative to any other policy. Formally, IRL can be posed as a min-max adversarial game between a reward player $r(s, a)$ (discriminator) and a policy player π (generator):

$$\min_\pi \max_r \mathbb{E}_{\pi^*}[r(s^\star, a^\star)] - \mathbb{E}_\pi[r(s, a)]. \tag{6}$$

This game is solved iteratively. At each iteration i, the reward function $r_i(s, a)$ is updated to distinguish expert demonstrations from *all past* learner policies (no-regret update). The policy player $\pi_i(a|s)$ then optimizes against the updated reward function (best response update):

$$r_i = \arg\max_r \mathbb{E}_{\pi^*}\left[r(s^\star, a^\star)\right] - \mathbb{E}_{\pi_{0:i-1}}\left[r(s, a)\right] \qquad \pi_i = \arg\max_\pi \mathbb{E}_\pi\left[r_i(s, a)\right]$$

$$\tag{7}$$

where sampling from $\pi_{0:i-1}$ amounts to aggregating (s, a) data from all past policies and sampling uniformly from that.

IRL via PRMs. A naive IRL implementation would require an outer optimization loop around the agent PRM framework, making it computationally impractical. Instead, we use a telescoping

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[6] Note this is a one-step reward, different from process rewards which are Q-values, i.e., cumulative rewards.

identity to express the one-step reward in terms of Q-values, allowing direct estimation of the PRM. Specifically, we rewrite the reward function as: [7]

$$r(s,a) = Q^\pi(s,a) - \gamma \mathbb{E}_{a' \sim \pi} Q^\pi(s',a'). \tag{8}$$

Writing the reward in terms of Q, or the verifier in terms of a generator, is an age-old trick that has been used effectively in various imitation learning [32] and reinforcement learning formulations [33].

We revisit the IRL update (7) but replace the one-step reward with the PRM parameterization in (8). At iteration i, the update for PRM Q_i^π is:

$$\begin{aligned}
Q_i^\pi = \arg\max_Q \quad & \mathbb{E}_{\substack{(s^\star, a^\star, s'^\star) \sim \pi^\star \\ a' \sim \pi_{i-1}(.|s'^\star)}} [Q(s^\star, a^\star) - \gamma Q(s'^\star, a')] - \\
& \mathbb{E}_{\substack{(s,a,s') \sim \pi_{0:i-1} \\ a' \sim \pi_{i-1}(.|s')}} [Q(s,a) - \gamma Q(s',a')]
\end{aligned} \tag{9}$$

Here, the difference in Q-values increases along expert trajectories $(s^\star, a^\star, s'^\star)$ and decreases along all past learner trajectories (s,a,s'). Since Q_i^π estimates the Q-values for the current policy π_{i-1}, the action a' is always sampled from π_{i-1}.

The policy update remains an RL step, where π_i is trained to maximize the learned PRM, following the same procedure as in Sec. 2:

$$\pi_i = \arg\max_\pi \mathbb{E}_\pi[Q_i^\pi(s,a)] - \beta \mathbb{D}_{\mathrm{KL}}\left[\pi(a|s) \,||\, \pi^{i-1}(a|s)\right] \tag{10}$$

3.2 Approach

Algorithm 2 describes `InversePRM`: a simple three-stage iterative process to learn and refine PRMs and policies given expert demonstration.

1. Create positive $\mathcal{D}^+$ and negative $\mathcal{D}^-$ transitions using expert demos and rollouts from π_{i-1}.
2. Train the PRM $Q_i(s,a)$ to discriminate between $\mathcal{D}^+$ and $\mathcal{D}^-$ (*similar to RLHF*)
3. Train the policy π_i using reinforcement learning on the trained RM (*similar to RLHF*)

The framework is very similar to the three stage process in `AgentPRM` (Algorithm 1) with the difference being no outcome reward and instead expert demonstrations. Stage 1 and 2 differ to accommodate this, while Stage 3 remains the same. Just like `AgentPRM`, the algorithm for `InversePRM` builds on existing RLHF frameworks making it easy to implement and use. We describe each stage in detail below:

Stage 1: Create Positive / Negative Transitions. We initialize with an positive dataset $\mathcal{D}^+ = \{(s^*, a^*, s'^*)\}$ containing state, action, next-state transitions from expert demonstrations. At iteration i, we rollout policy π_{i-1} in the environment to collect $\mathcal{D}_i = \{(s,a,s',a')\}$ to get state, action, next-state, next-action transitions. These rollouts are then aggregated with an existing negative dataset $\mathcal{D}^- \leftarrow \mathcal{D}^- \cup \mathcal{D}_i$. Finally, the next-action in both $\mathcal{D}^+$ and $\mathcal{D}^-$ are relabeled by calling $a' \sim \pi_{i-1}(s')$. We end up with a positive dataset $\mathcal{D}^+ = \{(s^*, a^*, s'^*, a')\}$ where the transitions are from expert demonstrations, and negative dataset $\mathcal{D}^- = \{(s,a,s',a')\}$ where the transitions are from all previous learner policies.

Stage 2: Training Process Reward Model. At iteration i, the PRM $Q_i(s,a)$ is trained to distinguish expert transitions $\mathcal{D}^+$ from learner transitions $\mathcal{D}^-$. We frame this as a binary classification problem, where expert transitions are labeled as positive (1) and learner transitions as negative (0).

A key distinction from standard reward modeling is that the classifier operates on the *difference of PRM values*, $Q_\phi(s,a) - \gamma Q_\phi(s',a')$, capturing the relative advantage of one transition over another. The loss function is:

$$\begin{aligned}
\mathcal{L}(\phi) = -\ & \mathbb{E}_{(s^*, a^*, s'^*, a') \sim \mathcal{D}^+}\left[\log \sigma(Q_\phi(s^*, a^*) - \gamma Q_\phi(s'^*, a'))\right] \\
+\ & \mathbb{E}_{(s,a,s',a') \sim \mathcal{D}^-}\left[\log(1 - \sigma(Q_\phi(s,a) - \gamma Q_\phi(s',a')))\right]
\end{aligned}$$

---

[7] This identity holds for any Q-function, but we use Q^π since we can sample on-policy.

Algorithm 2 Inverse PRM

1: **Initialize:** Policy π_0, expert demonstrations $\mathcal{D}^+ = \{(s^*, a^*, s'^*)\}$, negative dataset $\mathcal{D}^- = \{\}$
2: **for** iteration $i = 1, \ldots, K$ **do**
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ▷ **Stage 1: Construct Positive and Negative Transitions**
3: $\qquad$ Collect rollouts $\mathcal{D}_i = \{(s, a, s', a')\}$ using policy π_{i-1}
4: $\qquad$ Aggregate into the negative dataset: $\mathcal{D}^- \leftarrow \mathcal{D}^- \cup \mathcal{D}_i$
5: $\qquad$ Relabel next actions: $a' \sim \pi_{i-1}(s')$ for all $(s, a, s', a') \in \mathcal{D}^- \cup \mathcal{D}^+$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ▷ **Stage 2: Train Process Reward Model**
6: $\qquad$ Train PRM Q_i by minimizing the classification loss:

$$
\begin{aligned}
\mathcal{L}(\phi) = - \; & \mathbb{E}_{(s^*, a^*, s'^*, a') \sim \mathcal{D}^+} \left[\log \sigma(Q_\phi(s^*, a^*) - \gamma Q_\phi(s'^*, a')) \right] \\
+ \; & \mathbb{E}_{(s, a, s', a') \sim \mathcal{D}^-} \left[\log(1 - \sigma(Q_\phi(s, a) - \gamma Q_\phi(s', a'))) \right]
\end{aligned}
$$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ▷ **Stage 3: Train Policy via RL**
7: $\qquad$ Update policy π_i to maximize Q_i while regularizing to π_{i-1}:

$$
\pi_i = \arg\max_{\pi_\theta} \mathbb{E}_{s \sim \mathcal{D}_i, a \sim \pi_\theta(a|s)} \left[Q_\phi(s, a) \right] - \beta \mathbb{D}_{\mathrm{KL}} \left[\pi_\theta(a|s) || \pi^{i-1}(a|s) \right] \qquad (11)
$$

8: **end for**
9: Best $\pi \in \{\pi_1, \ldots, \pi_K\}$ on validation dataset

---

Stage 3: Train Policy via RL. The policy update follows the same procedure as in `AgentPRM`: the policy π_i is optimized to maximize the PRM Q_i while remaining close to the previous iteration's policy π_{i-1}. Formally, we solve:

$$
\pi_i = \arg\max_{\pi_\theta} \mathbb{E}_{s \sim \mathcal{D}, a \sim \pi_\theta(a|s)} \left[Q_\phi(s, a) \right] - \beta \mathbb{D}_{\mathrm{KL}} \left[\pi_\theta(a|s) || \pi^{i-1}(a|s) \right] \qquad (12)
$$

As in `AgentPRM`, the KL regularization ensures stability by preventing π_i from straying too far from the reference policy, mitigating distribution shift and reward hacking risks.

3.3 Experiments

Setup. We evaluate `InversePRM` using an expert policy from our prior work, `LEAP` [34], a Llama-3-8B model trained via privileged feedback from `gpt-4o`. We sample $10k$ expert demonstrations and train `InversePRM` for 2 iterations. The policy π_0 is initialized identically to `AgentPRM`. At each iteration, we collect rollouts to ensure the aggregated negative dataset contains $10k$ trajectories. As in `AgentPRM`, inference can be performed directly using the trained policy or via Best-of-N selection $\mathrm{BoN}(\pi, Q)$. See code for hyperparameters and agent prompts.

We compare `InversePRM` against two baselines: (1) SFT: A policy trained directly on expert demonstrations. (2) `AgentPRM`: A policy trained using only outcome rewards, without expert demonstrations, but with increased rollouts ($70k$ rollouts).

Overall Results. Table 2 compares `InversePRM` with SFT and `AgentPRM`. `InversePRM` outperforms both baselines, with its final policy π_2 approaching expert performance ($86.6\% < 91.0\%$). `InversePRM` significantly outperforms SFT on the same expert demonstrations ($86.6\% > 63.4\%$). The key reason is that SFT policies struggle to recover once they deviate from expert trajectories, whereas `InversePRM` actively interacts with the environment to correct mistakes. Compared to `AgentPRM` trained with $70k$ rollouts, `InversePRM` achieves *substantial gains in just one iteration* ($82.8\% > 73.9\%$). This highlights that leveraging dense expert demonstrations enables far greater sample efficiency than training purely with outcome rewards.

Training Curves. Fig. 2 (a) shows the success rate evolution during policy training (Stage 3). The success rate improves dramatically in the first iteration ($64.9\% \rightarrow 82.8\%$), whereas `AgentPRM` required multiple iterations to reach similar performance. This difference arises from the *exploration challenge* [35]: `AgentPRM` must discover high-reward actions through trial-and-error, whereas `InversePRM` benefits from expert demonstrations that implicitly capture successful strategies. We further analyze these exploration advantages in later sections.

Method	All tasks		PICK tasks	CLEAN tasks	HEAT tasks	COOL tasks	LOOK tasks	PICK 2 tasks
	%suc↑	#act↓	%suc↑	%suc↑	%suc↑	%suc↑	%suc↑	%suc↑
Expert Policy*	91.0	11.9	83.3	90.3	91.3	95.2	94.4	94.1
SFT	63.4	13.9	79.2	80.6	69.6	52.4	50.0	29.4
AgentPRM π_0	64.9	14.9	62.5	74.2	69.6	71.4	66.7	35.3
AgentPRM π_1	73.9	14.0	58.3	80.6	73.9	71.4	100.0	58.8
AgentPRM π_2	85.8	12.6	75.0	87.1	91.3	100.0	100.0	58.8
InversePRM π_0	64.9	14.9	62.5	74.2	69.6	71.4	66.7	35.3
InversePRM π_1	82.8	13.1	83.3	96.8	73.9	95.2	100.0	35.3
InversePRM π_2	86.6	12.5	79.2	90.3	91.3	100.0	94.4	64.7

Table 2: **Evaluation of** `InversePRM` **on ALFWorld.** Success rates (%) on 136 out-of-distribution tasks (max 30 actions). `InversePRM` is trained on 10K expert demonstrations over 2 iterations. It outperforms SFT on expert demonstrations (86.6% vs. 63.4%). Compared to `AgentPRM` trained with 70K rollouts, `InversePRM` achieves a significantly higher success rate in iteration 1 (82.8% vs. 73.9%) and approaches expert-level performance (86.6% vs. 91.0%). By leveraging dense expert demonstrations, `InversePRM` achieves greater sample efficiency than `AgentPRM`.
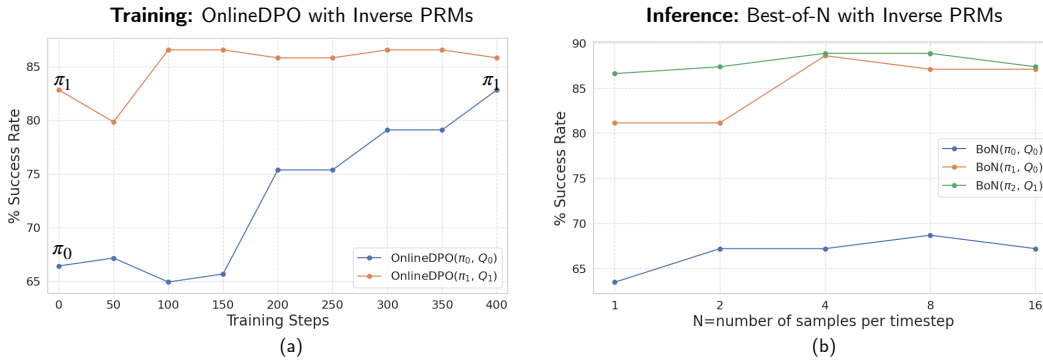

Figure 5: **Training and Inference of** `InversePRM`**.** (a) Success rate (%) vs. training steps for 2 iterations of `InversePRM` using online DPO with PRMs. The initial policy π_0 is initialized identically to `AgentPRM`. PRM Q_0 is trained on π_0 rollouts. OnlineDPO(π_0, Q_0) runs for 400 training steps, where success rate increases to near peak performance before saturating in iteration 2. (b) Best-of-N inference results for varying $N = \{1, 2, \ldots, 32\}$. Policy quality has a greater impact than the PRM or N: BoN(π_0, Q_0) provides only modest improvement (64.9% → 69.0%), whereas BoN(π_1, Q_0) reaches 88.0%. Performance saturates in iteration 2 (BoN(π_2, Q_1)).

Test-time Scaling. Fig. 2 (b) shows the effect of Best-of-N sampling on success rate as N varies from 1 to 32. The policy quality has a greater impact than scaling N. For instance, increasing N provides only moderate gains for BoN(π_0, Q_0) (64.9% → 69.0%), but has a much larger effect for BoN(π_1, Q_0) (88.0%). Performance saturates with BoN(π_2, Q_1).

4 Challenges and Opportunities

Reinforcement learning presents several challenges, some well-known in RL (e.g., exploration) and others specific to LLM agents (e.g., model-predictive reasoning). Addressing these challenges requires both established RL/IL techniques—such as reset distributions and reward shaping—and novel strategies leveraging LLM-specific capabilities, such as steered exploration.

4.1 Exploration

Exploration remains a fundamental challenge in RL, requiring agents to explore effectively at both the *turn level* (solving multi-step tasks) and the *token level* (generating improved reasoning and actions). Fig. 6 shows that the first iteration of `AgentPRM` progresses slowly, requiring over 500 training steps before ramping up and plateauing at 73.9% success rate.

Traditional exploration strategies include stochastic action selection methods such as ϵ-*greedy*, entropy bonuses, or adjusting sampling temperature. However, these approaches do not scale well to high-

dimensional, long-horizon tasks where reasoning quality is crucial. Instead, we explore structured strategies that leverage LLM-specific capabilities to guide exploration.

Strategy 1: Reset Distribution. A simple yet effective exploration strategy is to reset the agent to a *good* distribution of states $\rho(s)$ that an optimal policy is likely to visit. A good distribution is one that covers optimal state distribution[8]. Practitioners often use a $50\% - 50\%$ reset distribution [35], where 50% of initial states are sampled from successful expert demonstrations—such as human demonstrations or rule-based policies—while the remaining 50% come from the agent's on-policy rollouts. Intuitively, this approach helps bootstrap learning by exposing the agent to good states early, making it easier to recover from errors. We call this strategy `Reset-50-50`.

Figure 6: **Different exploration strategies.** Success rate vs training steps with $\mathrm{OnlineDPO}(\pi_0, Q_0)$. Both `Reset-50-50` and `SteeredExploration` learn faster and reach higher performance.

Fig. 6 shows `Reset-50-50` for $\mathrm{OnlineDPO}(\pi_0, Q_0)$ where the distribution of states (prompts) is a mixture of 50% states visited by π_0 and 50% states visited by the expert policy from Sec. 3.3. Note the only change is the set of prompts used in Stage 3, everything else including the starting policy and PRM remains the same. We observe `Reset-50-50` learns much faster and reaches a higher peak $82\% > 73.9\%$. By simply exposing the policy to good states and optimizing the same PRM, the policy learns to generate improved reason-action that helps the policy recover from other states.

Strategy 2: Steered Exploration. Unlike conventional RL policies, LLMs can be explicitly *prompted* to explore, rather than relying on stochastic action selection. We call this strategy `Steered Exploration`. Concretely, during RL (stage 3), we inject a small addition to the agent prompt:

```
Use the following strategy for generating actions:
*   In your REASON, try to come up with a strategy for how you want
    to solve the task. This strategy could be a hypothesis of where
    the object might be based on your history of observations. Then
    base your ACTION on the REASON.
* Try to explore possible strategies
```

Listing 1: Injected prompt snippet in `Steered Exploration`

We remove this addition while training the agent, i.e., the agent still trains on the original prompt. This results in the generation of reason-actions that are more diverse than sampling reason-actions, but are of a much higher quality than simply increasing the temperature.

Fig. 6 shows the `Steered Exploration` strategy for $\mathrm{OnlineDPO}(\pi_0, Q_0)$. Again, the only thing that changed is how we are sampling reason-actions in Stage 3 (online DPO). We see that learning is much faster and reaches a much higher peak of $84\% > 73.9\%$. An explanation for why this works as well can be tied to Posterior Sampling for RL [37]; the LLM samples diverse "models" of how the world works (consistent with the history of observations) in its reason and proposes actions according to that model, while the PRM selects for the correct actions and consequently the correct model.

Strategy 3: Post-hoc Rationalization. The connection to posterior sampling yields another interesting way to do exploration. Suppose the agent had access to some privileged information, e.g., the future trajectory or hidden information about the MDP (hidden location of objects). Conditioned on that information, the agent can generate post-hoc rationalization for good actions to take. We explore training agents in this fashion in our prior work `LEAP` [34]. However, one challenge we faced is that not all post-hoc rationalizations are good, some are better than others.
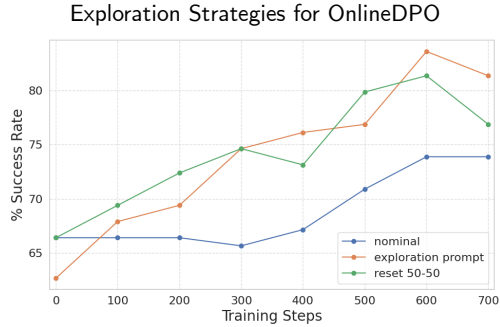
---

[8] Formally, a bounded density ratio $|\frac{\rho(s)}{d^{\pi^\star}(s)}| \leq C$, see [36].

Instead, we could imagine using this post-hoc rationalizer as an exploration policy. We call this strategy `PosteriorExplorer`. `PosteriorExplorer` suggests a diverse set of reason-actions that are then selected by the PRM based on which rationalization leads to good actions. The theory behind LEAP [38, 39] shows that the rationalizer learns a posterior over possible MDPs consistent with the POMDP the agent is solving, which is then refined by the RL procedure to select actions that lead to success.

4.2 Process Reward Shaping

Reinforcement learning from scratch is slow and sample inefficient. Practitioners often try to bootstrap RL using existing policies that have reasonable performance. We study a setting where only **10K rollout trajectories** can be collected, but a **reference policy** with moderate performance (65.0%) is available. We look at two such strategies: (1) initializing the agent via imitation learning and then doing RL, and (2) using *process reward shaping*, where the reference policy provides structured guidance during RL training.

Figure 7: **Process Reward Shaping**. Success rate vs training steps of $\text{OnlineDPO}(\pi_0, Q_0)$ when training with shaped rewards vs non-shaped rewards with $10k$ rollouts. Non-shaped rewards are noisy at low sample regimes with unstable performance. Shaped rewards lead to much more stable performance.

Strategy 1: Initialize with IL, then do RL. The simplest approach is to initialize the agent via SFT on trajectories generated by the reference agent. This ensures the initial policy is not random.

Fig. 7 shows $\text{OnlineDPO}(\pi_0, Q_0)$ for $10k$ rollouts where π_0 is initialized via SFT and then used for RL. We see that though π_0 begins at 64%, the training curve is unstable dropping to 32% before climbing back up. Hence, even though the initialization is good, the policy unlearns some of that good behavior due to noise in the PRM. This would be true for more sophisticated imitation learning methods like DAGGER [40] because the reference policy is not used at all during RL.

Strategy 2: Process Reward Shaping. We next look at involving the reference policy in the RL process itself. We look at *process reward shaping*, where instead of relying solely on sparse rewards, we shape the process reward using the advantage function of the reference policy.

Given a reference policy μ, we add a shaping term to the PRM target:

$$Q(s,a) \leftarrow (1-\alpha)Q^\pi(s,a) + \alpha A^\mu(s,a) \tag{13}$$

where $A^\mu(s,a)$ is the advantage w.r.t the reference policy μ, i.e., $A^\mu(s,a) = r(s,a) + \gamma V^\mu(s') - V^\mu(s)$.

α controls the power of the reference policy. Setting $\alpha = 0$ recovers the original PRM. Setting $\alpha = 1$ amounts to doing imitation learning, notable the AGGREVATE [41, 42] algorithm. Our procedure is:

1. Fit a value $V^\mu(s)$ using trajectories from the reference policy.
2. In Stage 1, modify the PRM target to be $(1-\alpha)Q^\pi(s,a) + \alpha A^\mu(s,a)$
3. Stage 2 and 3 remain unchanged.

Fig. 7 shows the shaped PRM training curves for $\alpha = 0.5$. The learning is much more stable and continues to steadily rise to 700 steps. This is because the $A^\mu(s,a)$, trained on much more rollouts from the reference policy ($70k$), counters the noisy PRM targets. Note that the learned policy significantly outperforms the reference policy ($82.0\% > 65.0\%$), which IL alone would not have ensured.

4.3 Model-Predictive Reasoning

Recent large-scale RL advances have demonstrated promising results in multi-step reasoning tasks [8]. However, applying RL to agentic settings remains challenging because each interaction requires
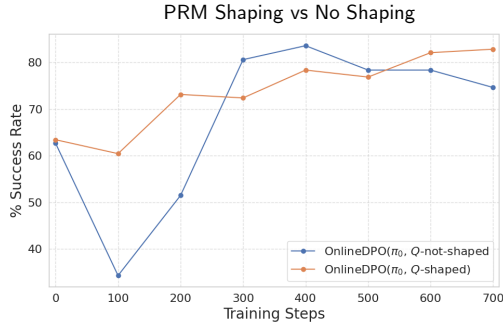
querying the environment, significantly slowing down learning. This raises a key question: *How can we reduce costly interactions while enabling agents to reason and plan effectively?*

One approach is to leverage learned world models. Instead of relying solely on trial-and-error, an LLM agent can simulate future trajectories using an internal model of the environment. This paradigm has been central in robotics, where real-world interactions are expensive and risky [43]. Model-based RL strategies, such as training policies in simulation before real-world deployment [44], have proven effective. Theoretically, generative models can provide mini-max optimal policies in model-based RL [45]. We extend this perspective to LLM agents: *Can we train them to plan (or deliberatively reason) with their internal models to improve decision-making?*

Strategy: Deliberative Reasoning with a Learned World Model. Instead of treating reasoning as a single-step process that immediately outputs an action, we propose a structured multi-stage approach where the agent explicitly predicts *future consequences* before committing to an action. This decomposes the learning problem into two components:

1. **Learning a world model**: Train an internal reasoning model to predict future states given an action, using rollouts from the current agent.
2. **Multi-turn planning and RL**: Optimize the agent's reasoning process via reinforcement learning to maximize outcome rewards.
3. **Plan-and-execute policy**: Structure the agent's reasoning to first generate a complete plan, select the initial action, execute it, and then replan iteratively.

This approach naturally connects to *model-predictive control* (MPC), where agents reason over predicted trajectories before taking actions, rather than relying purely on reactive decision-making.

5 Related Work

Fine-tuning agents. Most of the work on LLM agents rely on prompting LLMs, e.g. ReAct [4], Reflexion [5], AdaPlanner [27]. However, prompting alone is insufficient to correct errors encountered at test-time [1, 46]. A simple way to improve LLMs is to fine-tune on successful trajectories generated manually or via a prompted LLM [6, 47, 48]. However, manually collecting demonstrations of reason and actions is challenging and hard to scale.

Recent work `LEAP` has looked at leveraging privileged AI feedback [34] to design critics that distill the information into student agents, showing strong performance in text-based games, web navigation and interactive coding. However, the privileged correction in `LEAP` can be unrealizable for the agent, leading to poor success rates. Hence, we look at training agents directly using RL to maximize the outcome reward.

Finally, ARCHER [49] proposes a very similar framework to train LLM agents using hierarchical RL. The Q-value is trained using temporal difference, while the policy is trained using REINFORCE. However, the results are limited to small models (GPT2). We simplify the framework so it connects with existing RLHF pipelines, do RL with Llama 3B models, propose novel algorithms like `InversePRM`, and provide practical recipes like using reset distribution and reward shaping to improve efficiency.

Process Reward Models. PRMs have mostly been looked at in the context of multi-stage math reasoning problems [50], where they were trained in human annotation data to provide fine-grained supervision [10, 11]. Recent works look at automatically computing PRMs as Q value estimates [16, 51]. PRMs have been used to train generators [52] and used for test-time scaling with beam search [17], heuristic search [53] or tree search [54].

There are interesting similarities and differences between PRMs used for math reasoning and the agent setting we look at here. Many works [11, 52, 55] report small gains from optimizing PRMs rather than the outcome reward. In contrast, we see pretty strong gains with PRMs, where outcome reward is infeasible given long-horizons and limited access to the external environment. Some works have noted the reward-hacking / value-estimation issues with PRMs that we also analyze in Sec. 2.3. To counter such issues, recent works [12] propose reward shaping PRMs using reference policies, which we also explore in Sec. 4.2.

6 Conclusion

We introduced `AgentPRM`, a simple and scalable framework for training LLM agents using process reward models, and `InversePRM`, which learns PRMs directly from demonstrations without explicit outcome rewards. Our results on ALFWorld show that small models trained with `AgentPRM`

outperform strong GPT-4o baselines, and `InversePRM` achieves near-expert performance with significantly fewer rollouts. We outlined key challenges—exploration, process reward shaping, and model-predictive reasoning—and proposed methods that leverage both RL techniques and LLM-specific capabilities. Future work includes extending PRMs to richer agentic environments and exploring large-scale RL via model-predictive reasoning.

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