Linear Statistical Models

SYS 6021

Project 3

Design Improvements for the University of Virginia Transplant Center

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Honor Pledge: On my honor, I pledge that I am the sole author of this paper and I have accurately cited all help and references used in its completion.

Summary

The organ transplant and procurement network in United States is roughly divided into 11 regions. And in this project the goal is to improve University of Virginia's liver and kidney transplant processes so as to increase the number of transplants. In this project a variety of features have been explored which includes how UVa would perform if they model the best performing center in region 11 which is MCV for kidney transplants and how many more transplants they would be able to perform per year if another additional center is opened for liver transplants. The evidence suggests that if UVa models MCV by 50% then the corresponding confidence interval is (67.26517, 101.77588) and if they model MCV by 80% the confidence interval increases to (72.15496, 108.45433). For liver transplants even though with another additional center increases the confidence interval to (55.2408 , 142.854) from (58.16192 , 81.67621), this might not be economically feasible since the cost of building an additional center is significantly high and the confidence interval is significantly wider thus a huge amount of uncertainty is involved with such a decision.

1. Problem Description

1.1 Situation

The organ transplant and procurement network in United States is roughly divided into 11 regions. In this project[1] using the template [2] we are concentrating on the 11th region which includes the states Kentucky, North Carolina, South Carolina, Tennessee and Virginia. So this is essentially a comparative study of how University of Virginia is performing in terms of transplants against the other medical schools in the same region 11 namely, UNC, MCV and Duke.

Organ transplantation replaces diseased or damaged organs with functioning organs from either deceased or living donors. The complexity of these procedures requires s highly skilled teams of physicians, nurses, and support staff as well as facilities for the surgery and recovery. The University of Virginia has conducted organ transplantation for more than 30 years and now provides services for kidney, pancreas, liver, islet, heart and lung transplantation [3].

"The UVA Health System is consistently ranked as one of the Top 100 Hospitals in America" [3]. The availability of first-rate transplantation services is a component of these rankings. The Transplant Center desires to continue to increase the number of transplants in all categories but needs guidance on how to achieve this goal [4].

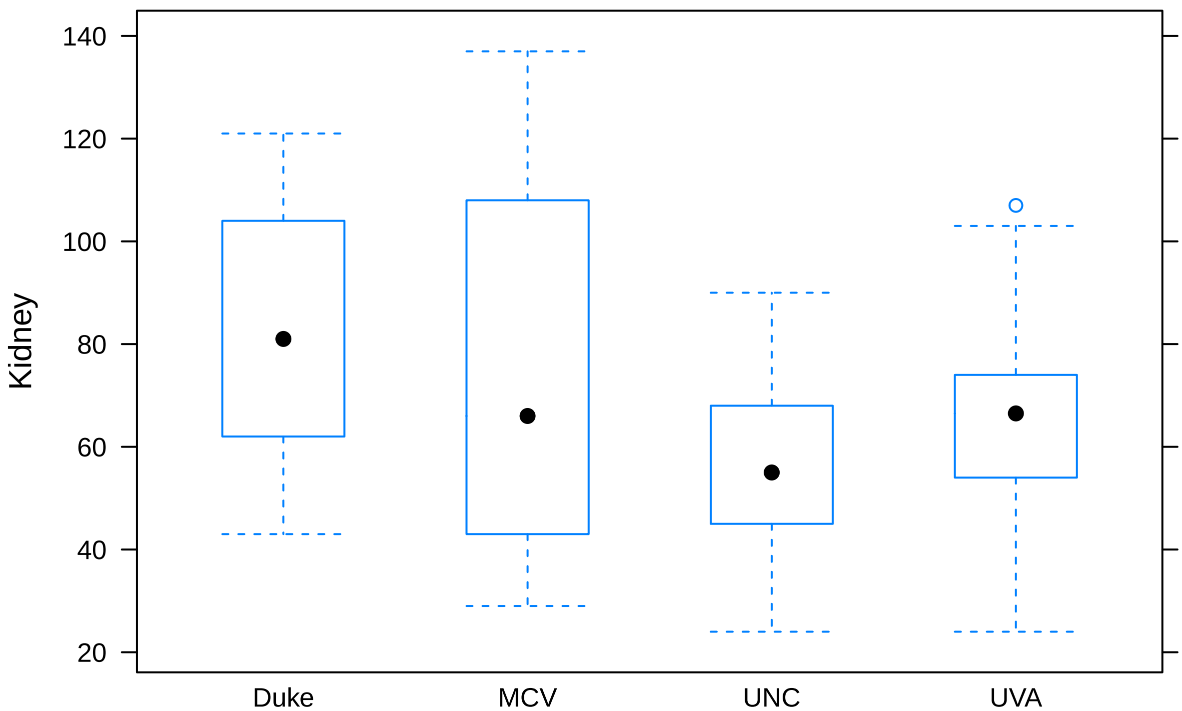
Organ transplantation processes have five primary steps [5]

1. Referral from a primary care physician;
2. Determination of eligibility and placement on a waiting list;
3. Matching of donor organ with the patient;
4. Acceptance of the organ by the transplant center;
5. Transplantation surgery and recovery.

Now since kidney and liver are the parts that we are focussing on, in this project following are two graphs that show how UVa performs in comaprison to the other schools.

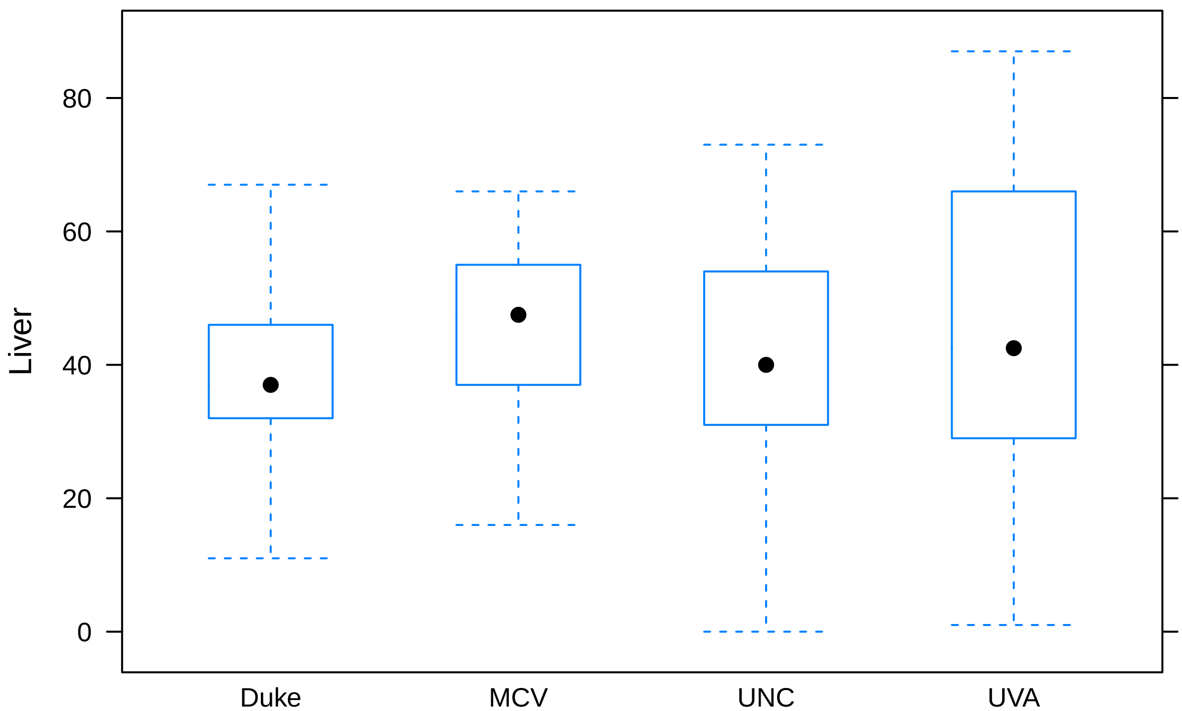
In terms of kidney transplants Duke performs significantly better than UVa but overall UVa is not doing that bad.

Fig 1: Kidney Plot by schools



In terms of liver transplants UVa is doing quite good and MCV is just marginally better than UVa in terms of the number of transplants.

Fig 2: Liver Plot by schools

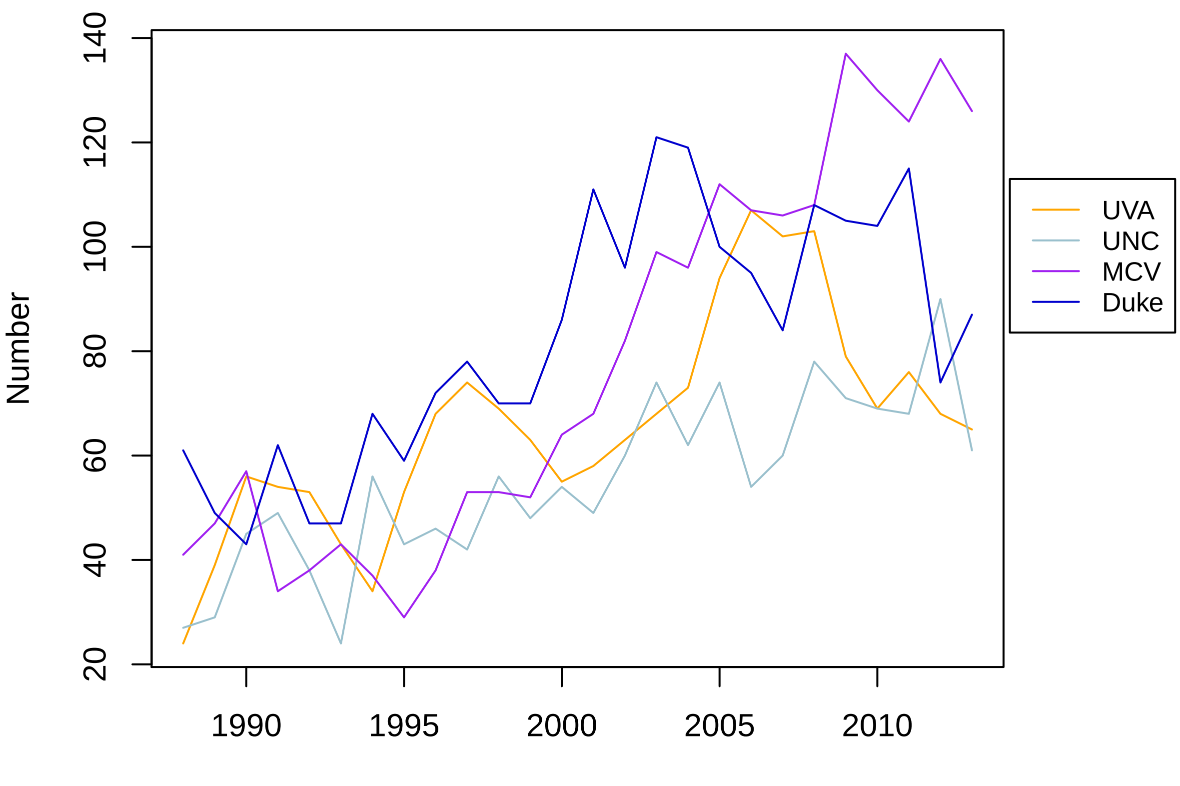


But as could be seen in the graphs in the situation section, the trends vary widely.

A plot of the trends of how each of the schools are performing in the recent years will be more informative in terms of the prediction for the forthcoming years.

As can be seen in the plot that MCV's performance in Kidney transplants have improved a lot since 2006, whereas UVa's performance have decreased sharply.

Fig 3: Kidney transplant trend



In liver transplants however, UVa is doing quite well and the task would be to continue doing this and perform better in the future.

Also in terms of the liver transplant scenario it can be seen that there was a sharp increase in UVa's performance at around 2006 and that is because a new center was opened dedicated to liver transplants and thus in this project the possibility of opening another center is being explored so as to see how better UVa can perform with an additional center.

In fact the Kidney transplant situation at UVa can be summarized in the following table here:

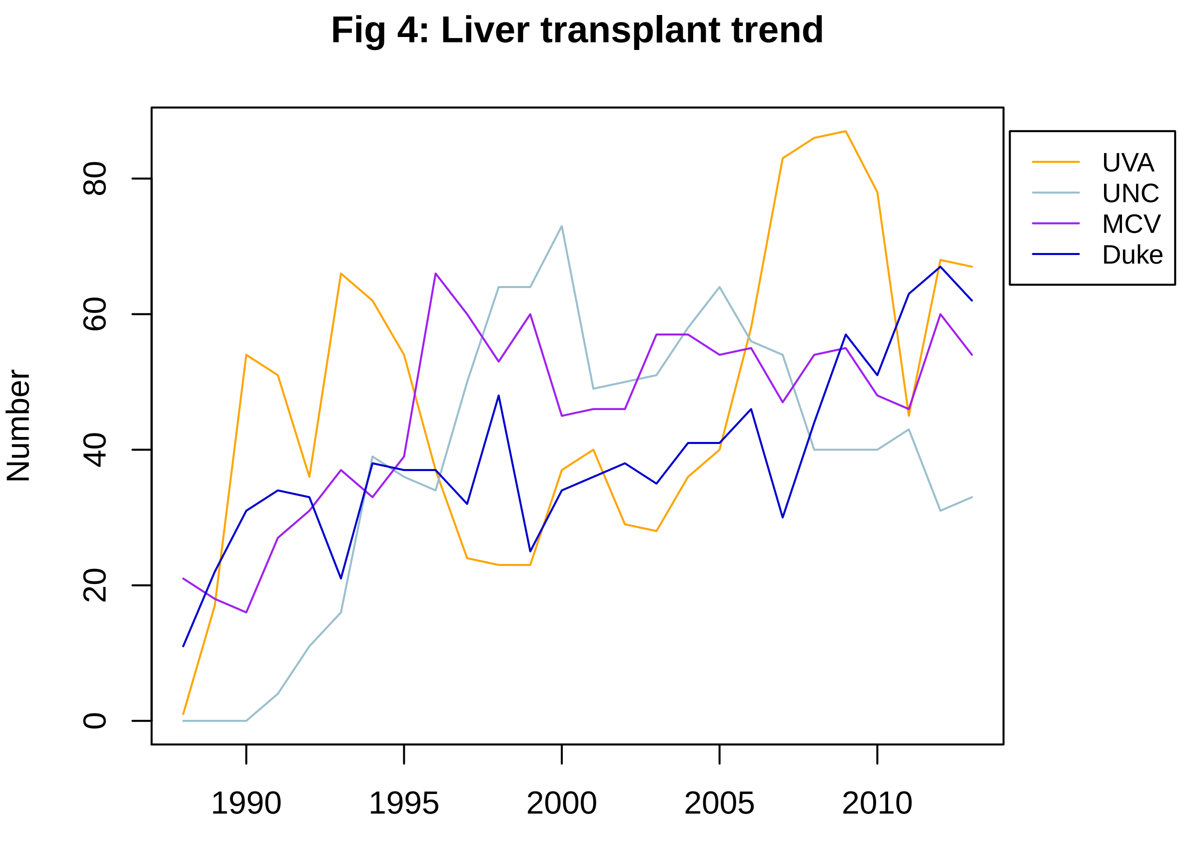
Table 1: Kidney Transplant Scenario at UVa

	x
Min.	24.00
1st Qu.	53.50
Median	65.00
Mean	64.96
3rd Qu.	73.50
Max.	107.00

The liver transplant situation can be seen in this table here:

Table 2: Liver Transplant Scenario at UVa

	x
Min.	1.00
1st Qu.	32.50
Median	40.00
Mean	46.89
3rd Qu.	64.00
Max.	87.00



1.2 Goal

The goal of the project is to verify if the University of Virginia's medical school is doing worse than the other medical schools in Region 11 and the steps necessary to improve the condition so that UVa's Transplant center can perform as good as the other medical schools in region 11.

1.3 Metrics

- The AIC or the Akaike Information Criterion for the evaluation of the model.
- The number of transplants performed by University of Virginia's medical school compared to the other medical schools in the region 11.

The final model has a AIC of 798.7430 for kidney transplants which is better than the base case of 989.786 and the final model for Liver transplants has an AIC of 881.39 which is significantly better than the base case of 1063.243.

1.4 Hypothesis

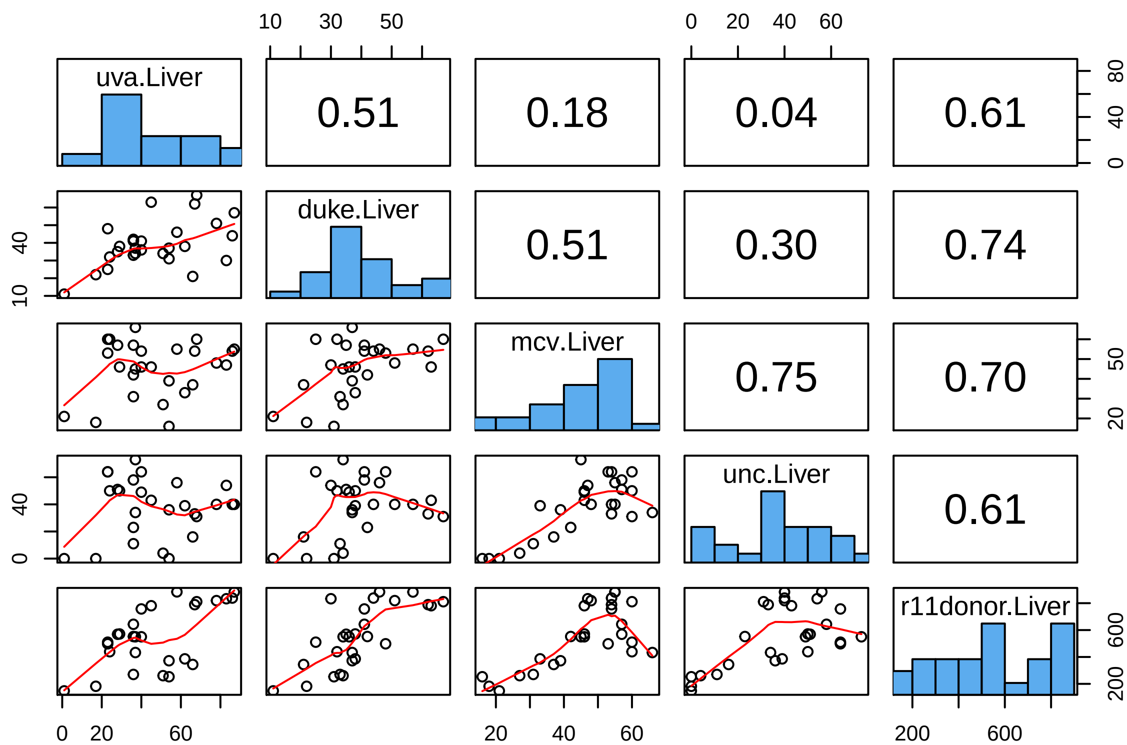
University of Virginia is not performing worse than the other medical schools in terms of the number of transplants in region 11.

- UVa's performance with respect to the number of kidney transplants will not significantly improve if they model MCV's behavior by 80%. This has been evaluated at the 95% confidence interval. The results have been highlighted in the evidence section.
- With respect to the number of liver transplants, UVa's performance will not improve significantly if they open another new centre for liver transplants. This has been evaluated at the 95% confidence interval. The results have been highlighted in the evidence section.

2. Approach

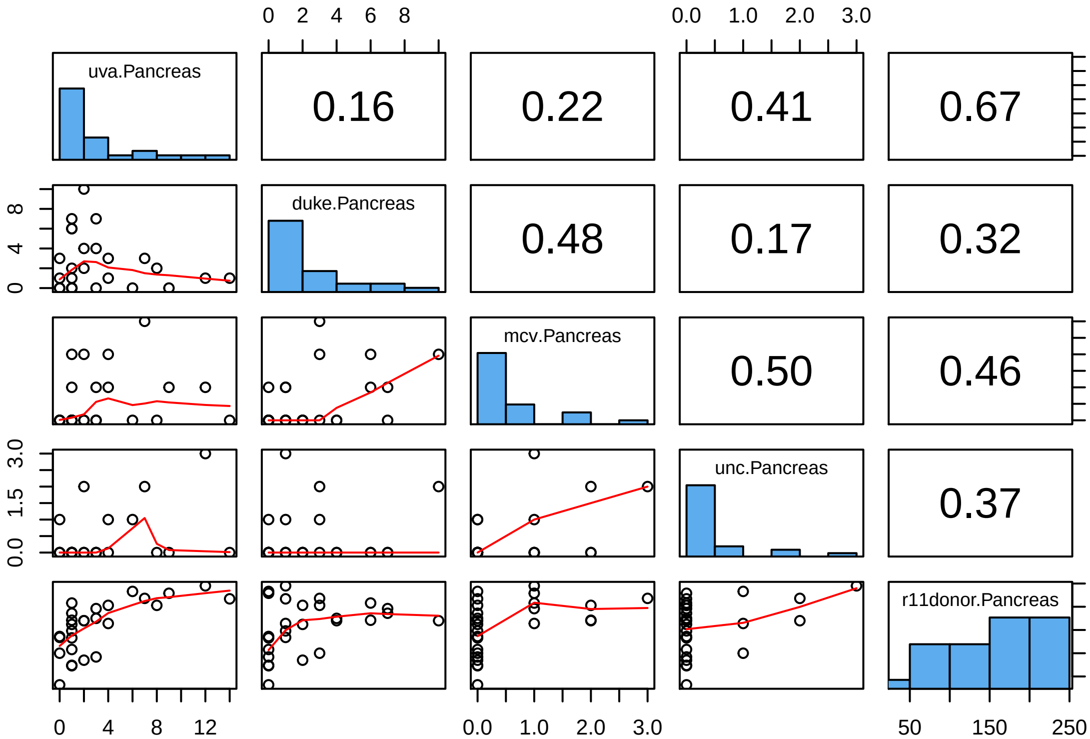
2.1 Data

The following diagram shows the distribution of data [6] for liver transplants between UVA, Duke, MCV, UNC and along with the entire group of region 11 donors.



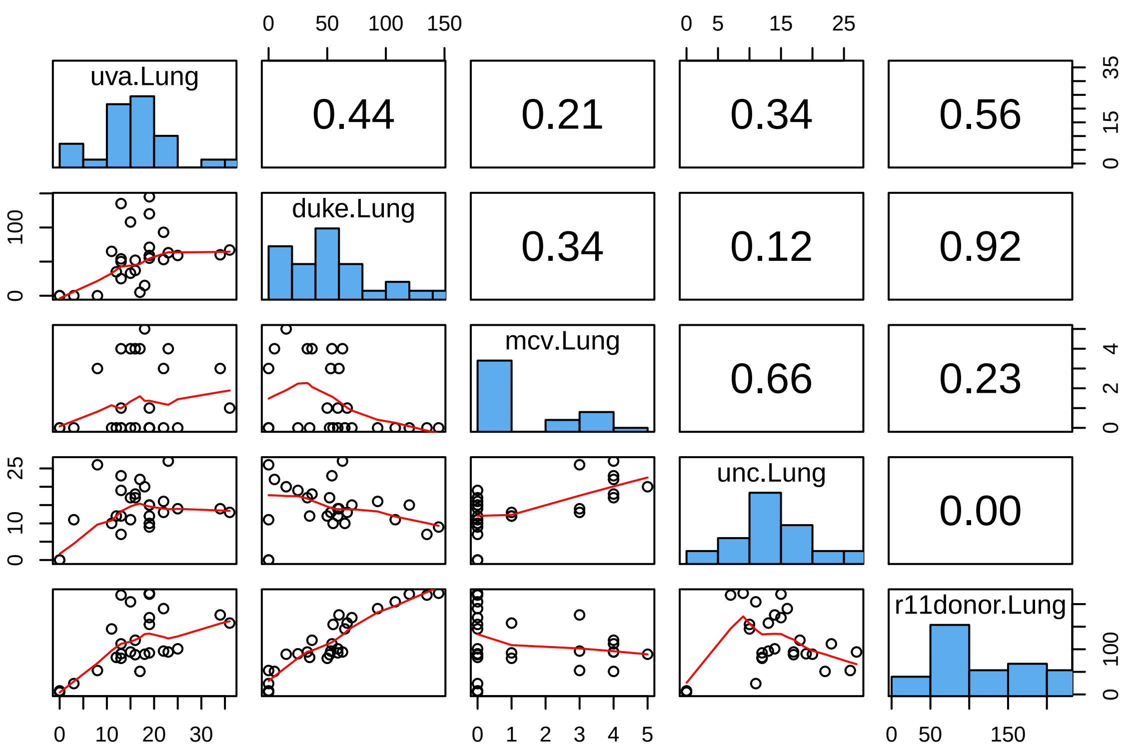
From the above diagram it is clear that Duke is performing quite well since the distribution of Duke is symmetric and also it performs the highest number of transplants compared to the other medical schools in region 11. The high correlation value between Duke and the total number of region 11 donor shows how Duke performs a high number of transplants when compared against the number of donors available in the region.

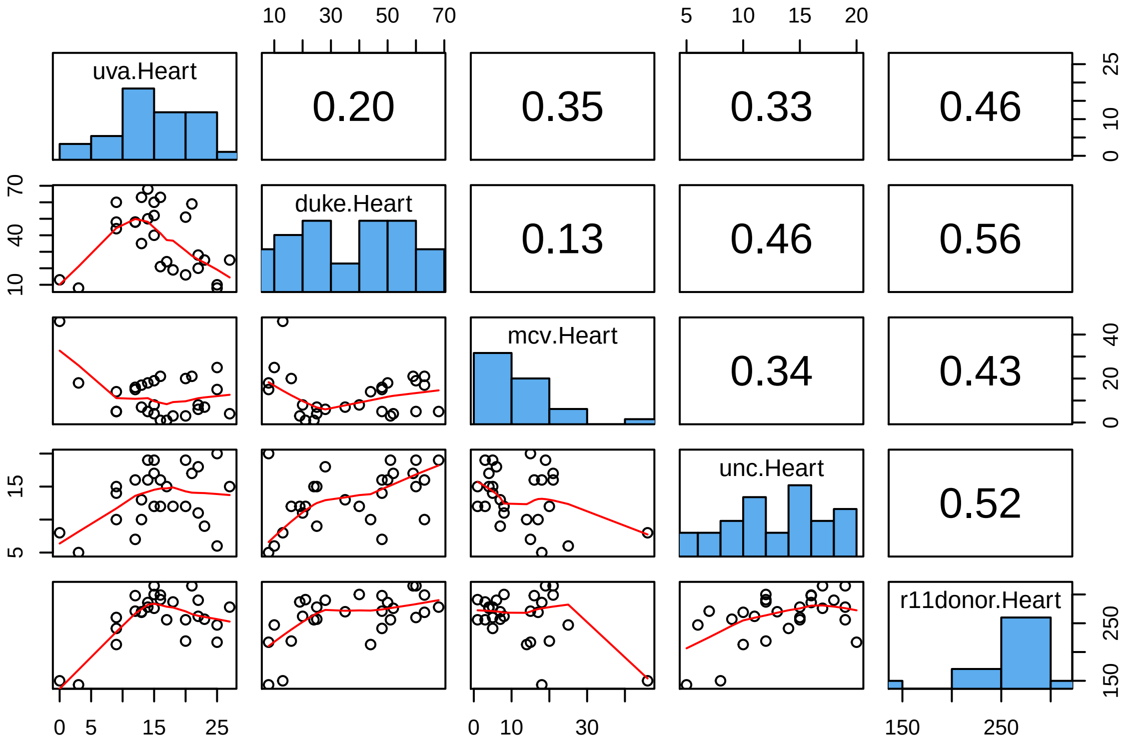
Similarly for the number of pancreas transplants the following graph shows the performance of the various medical schools in region 11 compared to the number of donors in region 11.



As can be seen in the above diagram, University of Virginia's medical school performs quite well with respect to pancreas transplants and it has a high correlation of 0.67 with the number of region 11 donors.

Similarly for heart and lung transplants, we get the following details about the performance of the medical schools compared to the number of donors in region 11.





In terms of lung transplants, Duke performs really well with a correlation of 0.92 with the total number of region 11 donors, compared to the other medical schools in this region and in terms of heart transplants all the medical schools in region 11 are somewhat close to each other with no major performance difference between all these hospitals.

The following diagrams show the performance of each of the medical establishments compared to the donors available in each of the regions and the corresponding transplants.

Fig 1: Heart

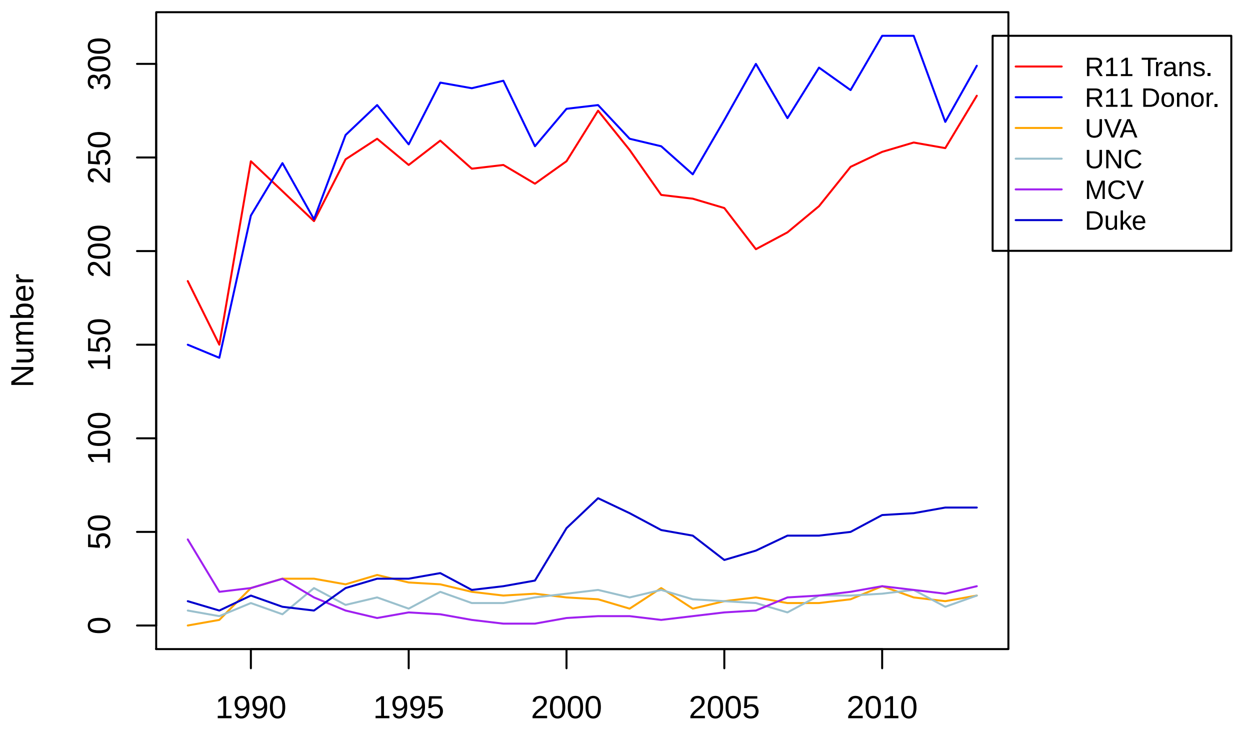


Fig 2: Liver

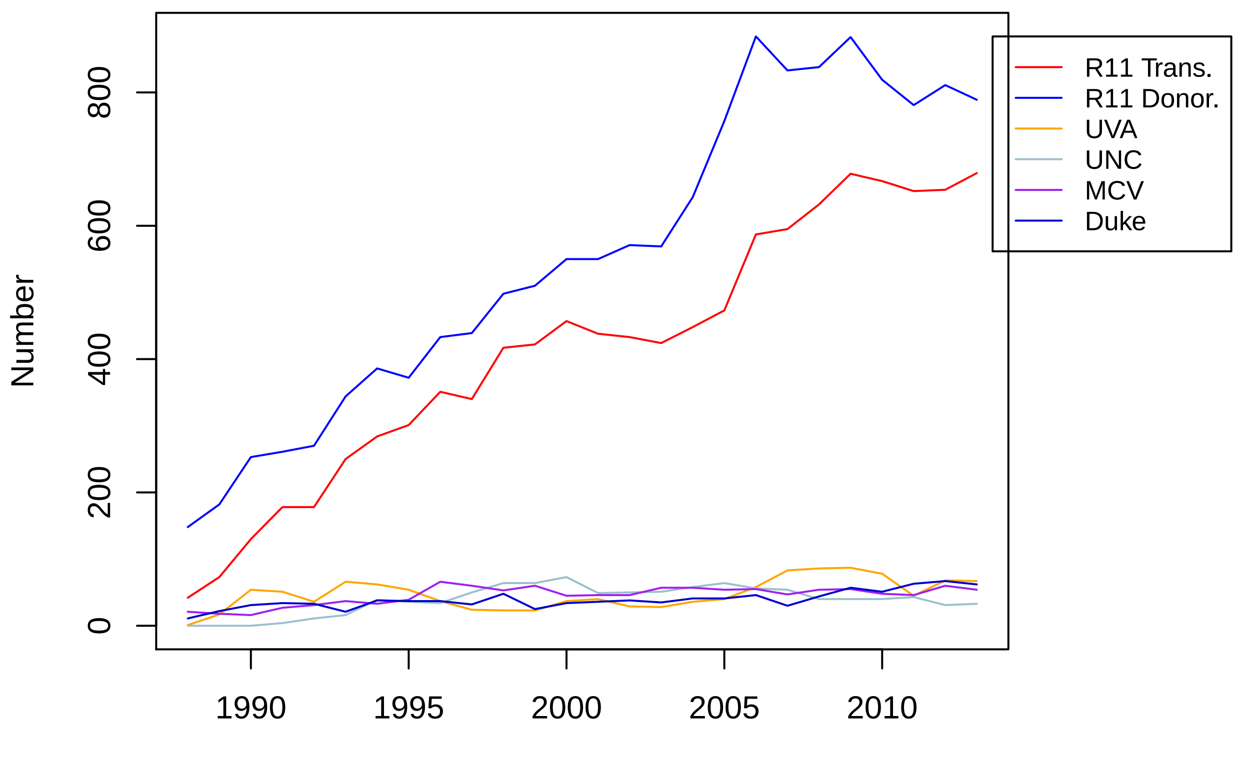


Fig 3: Kidney

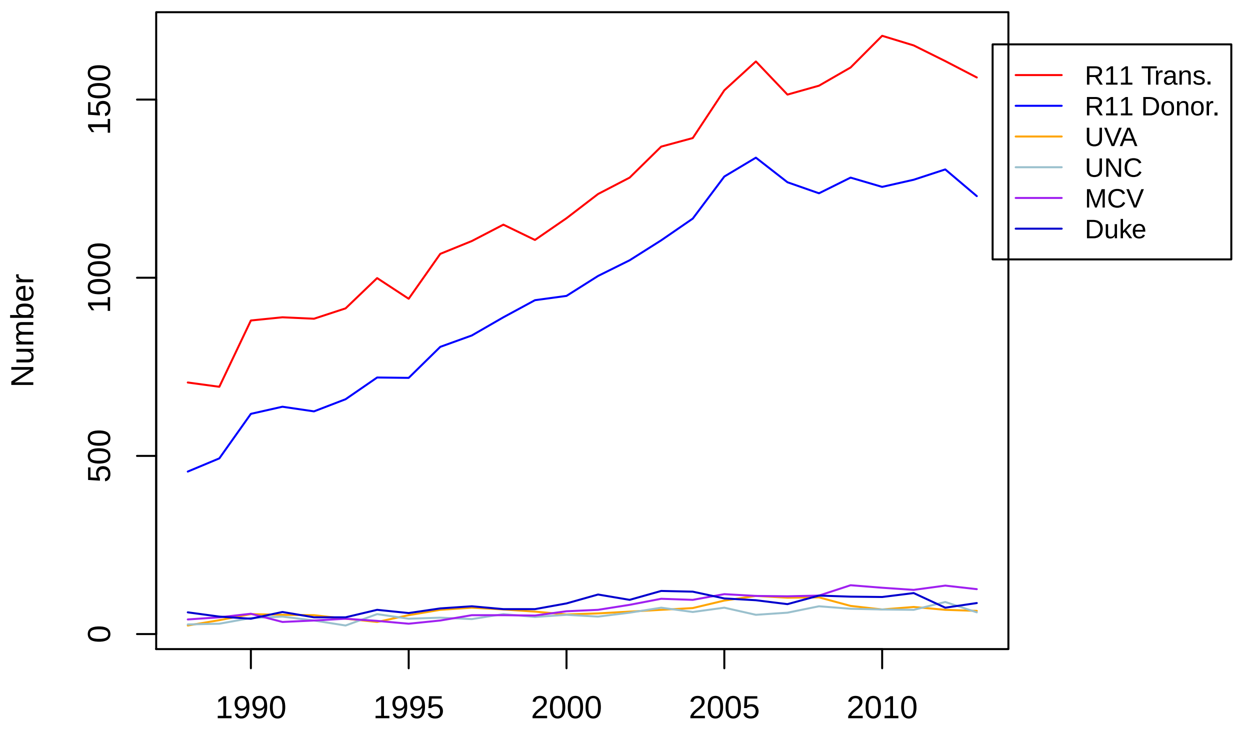
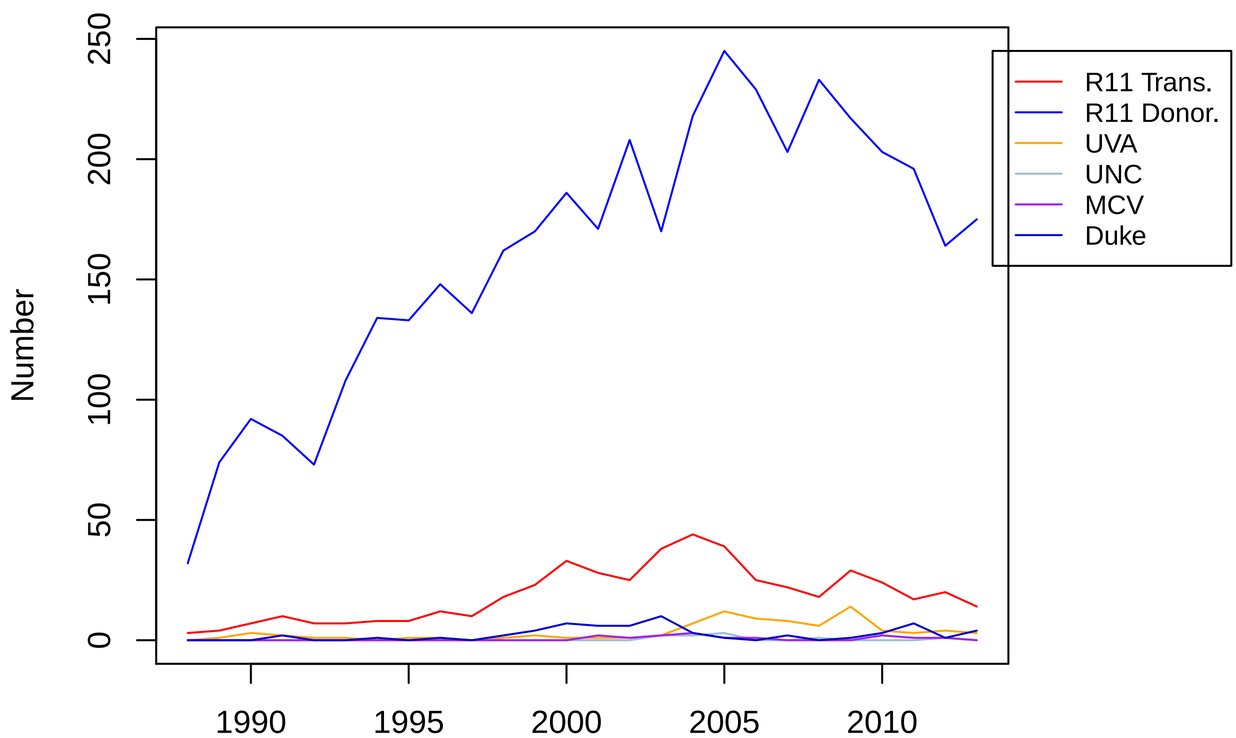


Fig 4: Pancreas



The above graph validates the previous arguments. As can be seen Duke performs really well when it comes to heart transplants especially since 2000. Another interesting observation is that there are more number of pancreas donors than transplants.

The data does not have any missing values nor does it have any noticeable bias.

Now in this project we will mainly consider the liver and kidney transplant senario at UVa compared to the other schools.

The summary for the data for UVa's transplant is as follows:

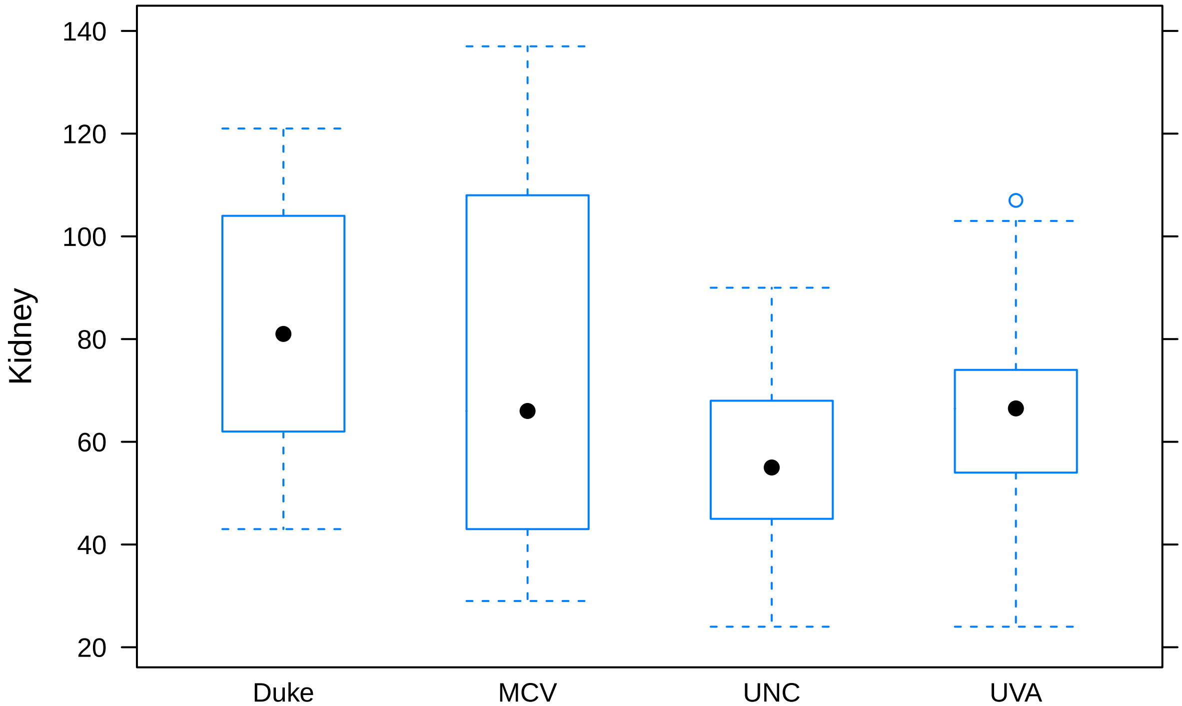
	Category.Variable	Minimum	X1st.Qu.	Median	Mean	X3rd.Quantile	Max
1	Year	1988.00	1994.00	2001.00	2001.00	2008.00	2014.00
2	All_Organs	943.00	1732.00	2160.00	2104.00	2613.00	2917.00
3	Kidney	694.00	970.00	1167.00	1228.00	1532.00	1679.00
4	Liver	42.00	292.50	433.00	416.80	591.00	679.00
5	Pancreas	3.00	8.00	18.00	18.56	25.00	44.00
6	Kidney.Pancreas	2.00	55.50	80.00	73.93	95.00	112.00
7	Heart	150.00	223.50	245.00	234.40	253.50	283.00
8	Lung	0.00	106.00	135.00	126.30	157.50	239.00
9	Heart.Lung	0.00	1.00	3.00	3.89	6.00	13.00
10	Intestine	0.00	0.00	1.00	1.37	2.00	6.00
11	All.Organs_DD	775.00	1465.00	1705.00	1725.00	2144.00	2518.00
12	Kidney_DD	508.00	689.00	773.00	864.00	1094.00	1286.00
13	Liver_DD	42.00	292.50	409.00	403.80	574.00	667.00
14	Pancreas_DD	3.00	8.00	18.00	18.56	25.00	44.00
15	Kidney.Pancreas_DD	2.00	55.50	80.00	73.93	95.00	112.00
16	Heart_DD	150.00	223.50	245.00	234.30	253.50	283.00
17	Lung_DD	0.00	104.00	133.00	125.50	157.50	239.00
18	Heart.Lung_DD	0.00	1.00	3.00	3.89	6.00	13.00
19	Intestine_DD	0.00	0.00	1.00	1.37	2.00	6.00
20	All.Organs_LD	156.00	263.00	404.00	378.30	464.50	582.00
21	Kidney_LD	156.00	257.50	393.00	364.40	436.00	568.00
22	Liver_LD	0.00	0.00	11.00	13.00	17.50	48.00
23	Heart_LD	0.00	0.00	0.00	0.11	0.00	3.00
24	Lung_LD	0.00	0.00	0.00	0.78	1.00	3.00

2.2 Analysis

Now since kidney and liver are the parts that we are focussing on, in this project following are two graphs that show how UVa performs in comaprison to the other schools.

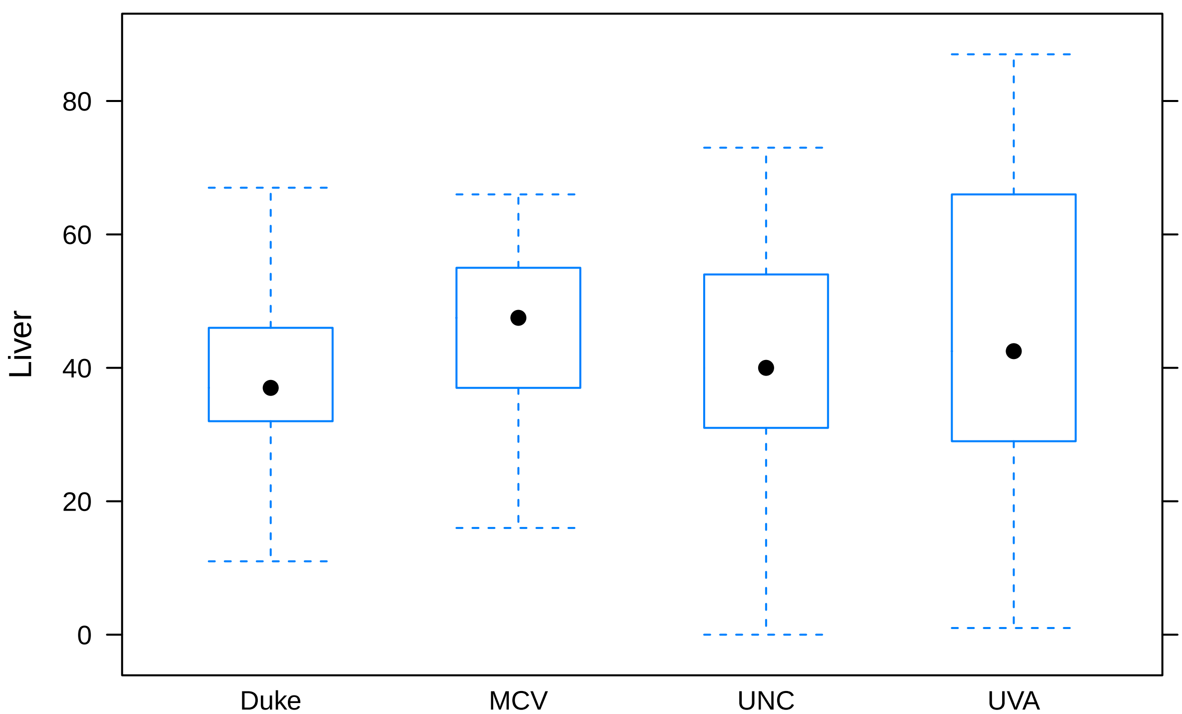
In terms of kidney transplants Duke performs significantly better than UVa but overall UVa is not doing that bad.

Fig 5: Kidney Plot by schools



In terms of liver transplants UVa is doing quite good and MCV is just marginally better than UVa in terms of the number of transplants.

Fig 6: Liver Plot by schools

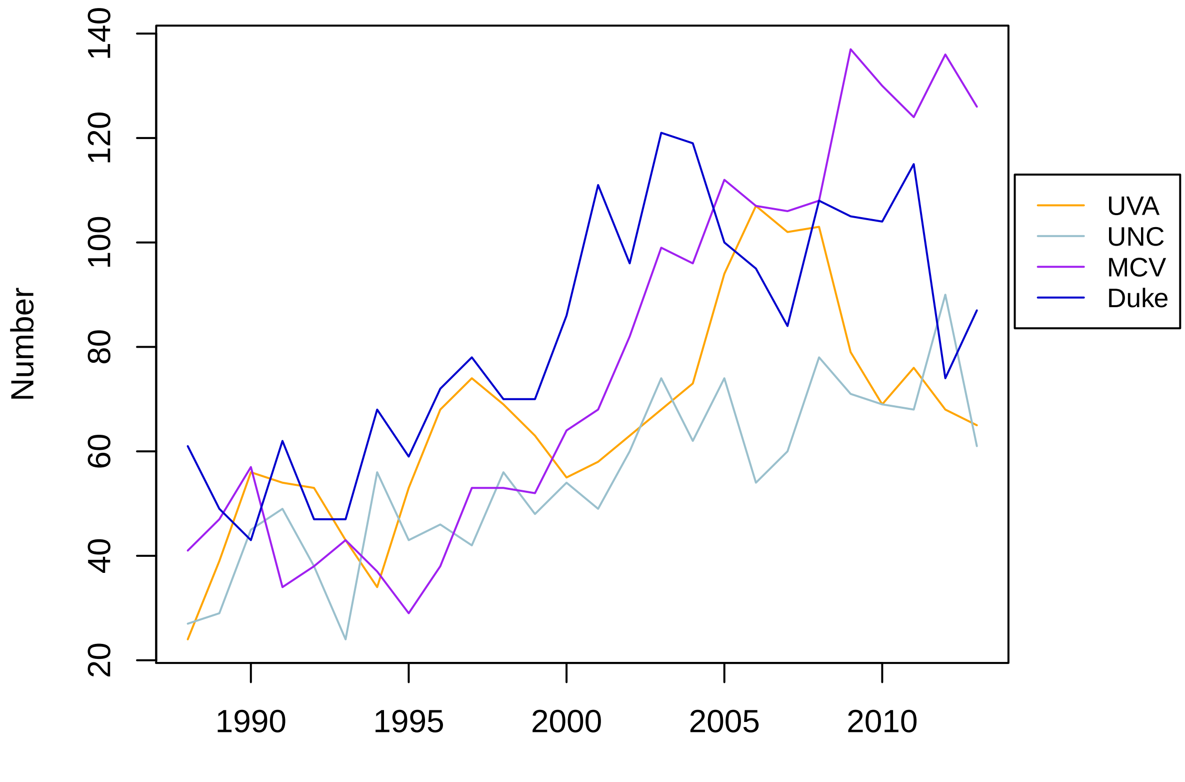


But as could be seen in the graphs in the situation section, the trends vary widely.

A plot of the trends of how each of the schools are performing in the recent years will be more informative in terms of the prediction for the forthcoming years.

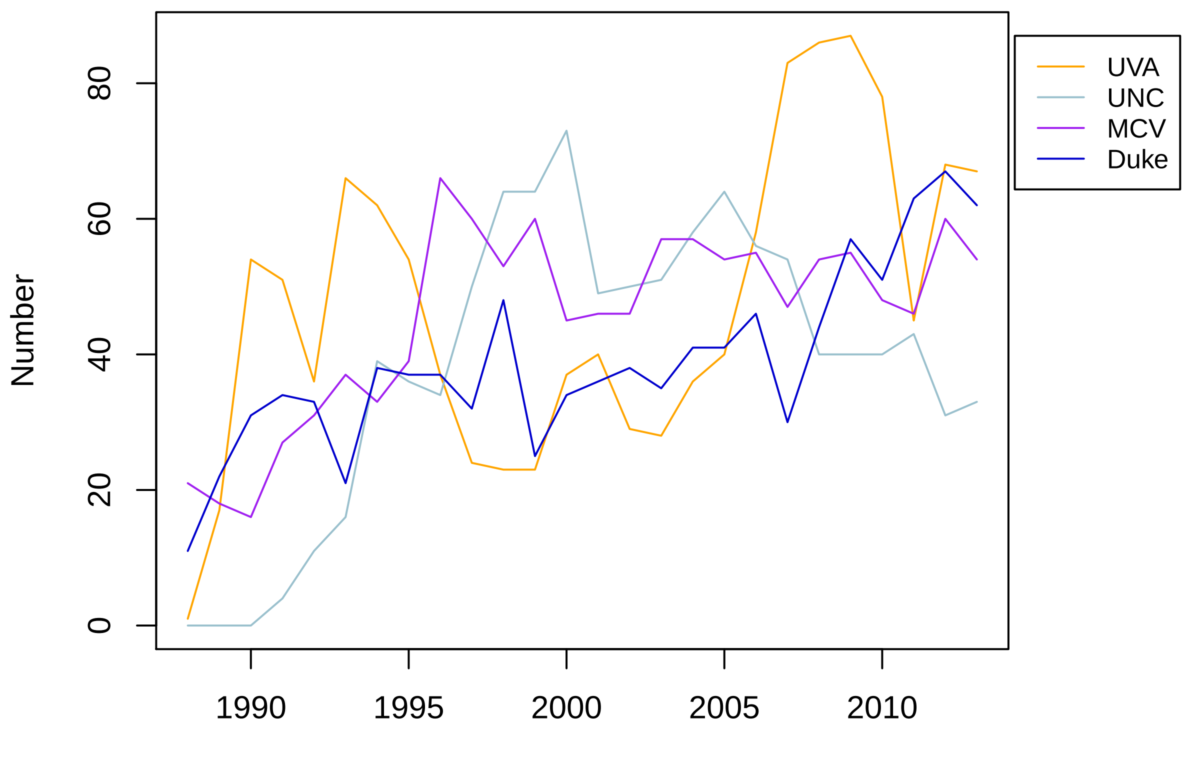
As can be seen in the plot that MCV's performance in Kidney transplants have improved a lot since 2006, whereas UVa's performance have decreased sharply.

Fig 7: Kidney transplant trend



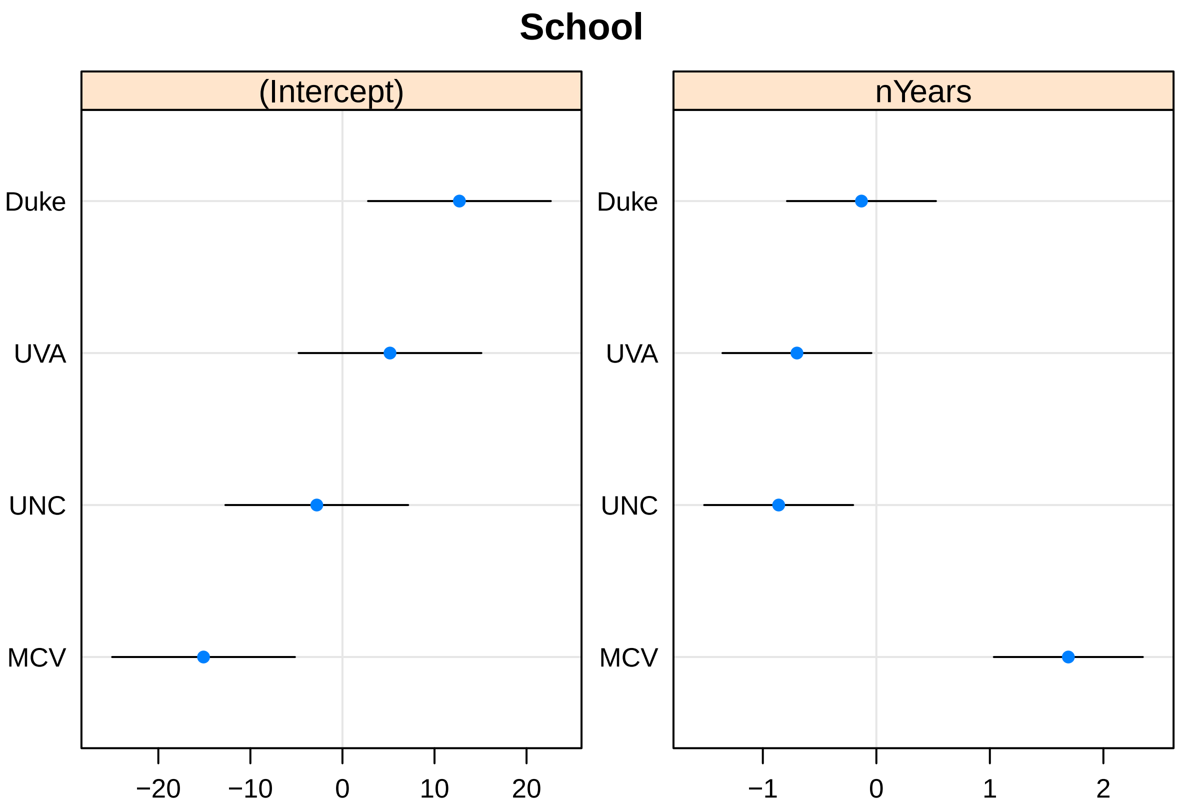
In liver transplants however, UVa is doing quite well and the task would be to continue doing this and perform better in the future.

Fig 8: Liver transplant trend



A linear mixed effects model takes into account the various aspects of the model and it is conditioned on the school and takes into account the number of years and the performance of the school.

Now the dot plot below shows the effect of the random effects part of the mixed effects model and it can be seen according to the intercepts of the model Duke performs the best, however MCV performs the best when the trend component is taken into account. It is important to take the time factor into account for predicting the future since it matters how good a school is performing in the recent years than how it performed 10 years ago. This time factor greatly determines how it will perform in the future.



The following is the residuals vs fitted plot the linear model and it shows that the variance of the residuals are more or less constant. This plot shows very little amount of heteroscedasticity. The qqplot is descent and this is a good model that takes into account the effect of the various schools and even the time component.

Fig 10: Residual vs Fitted plot

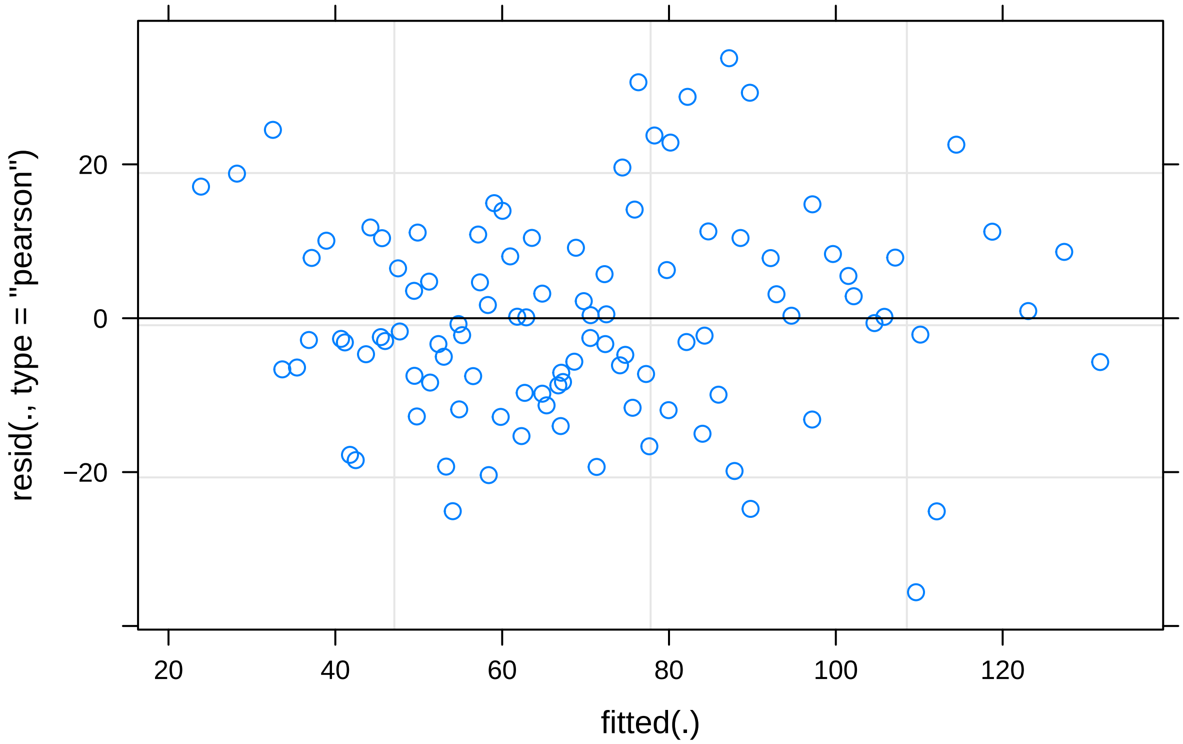
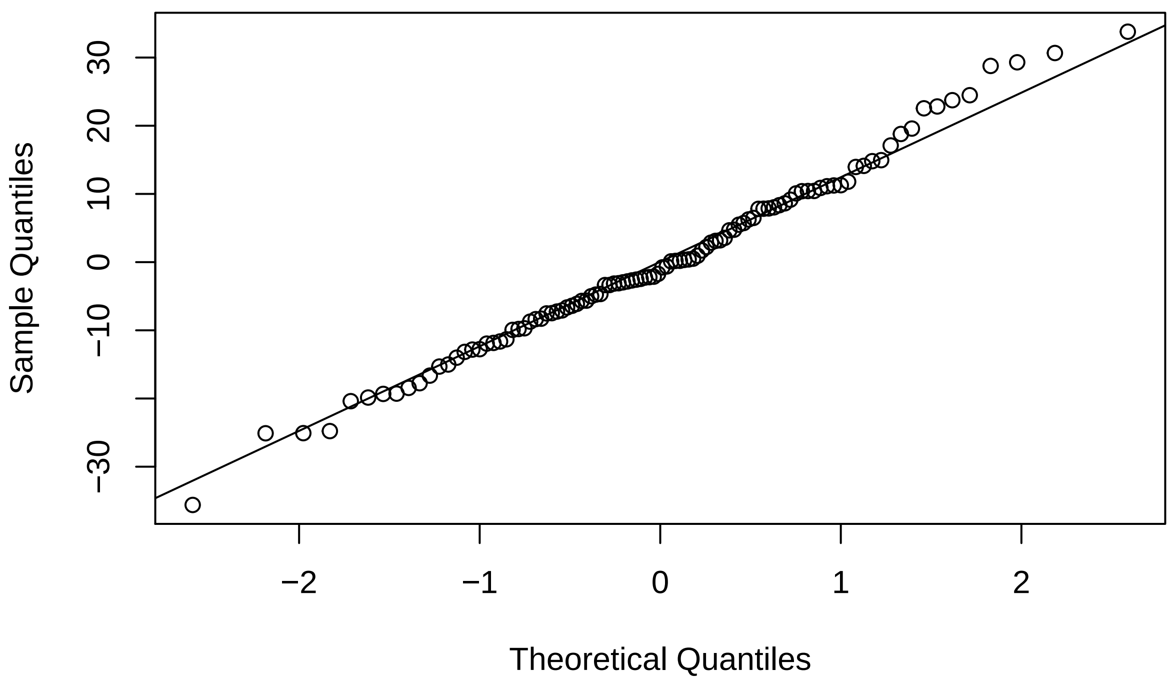
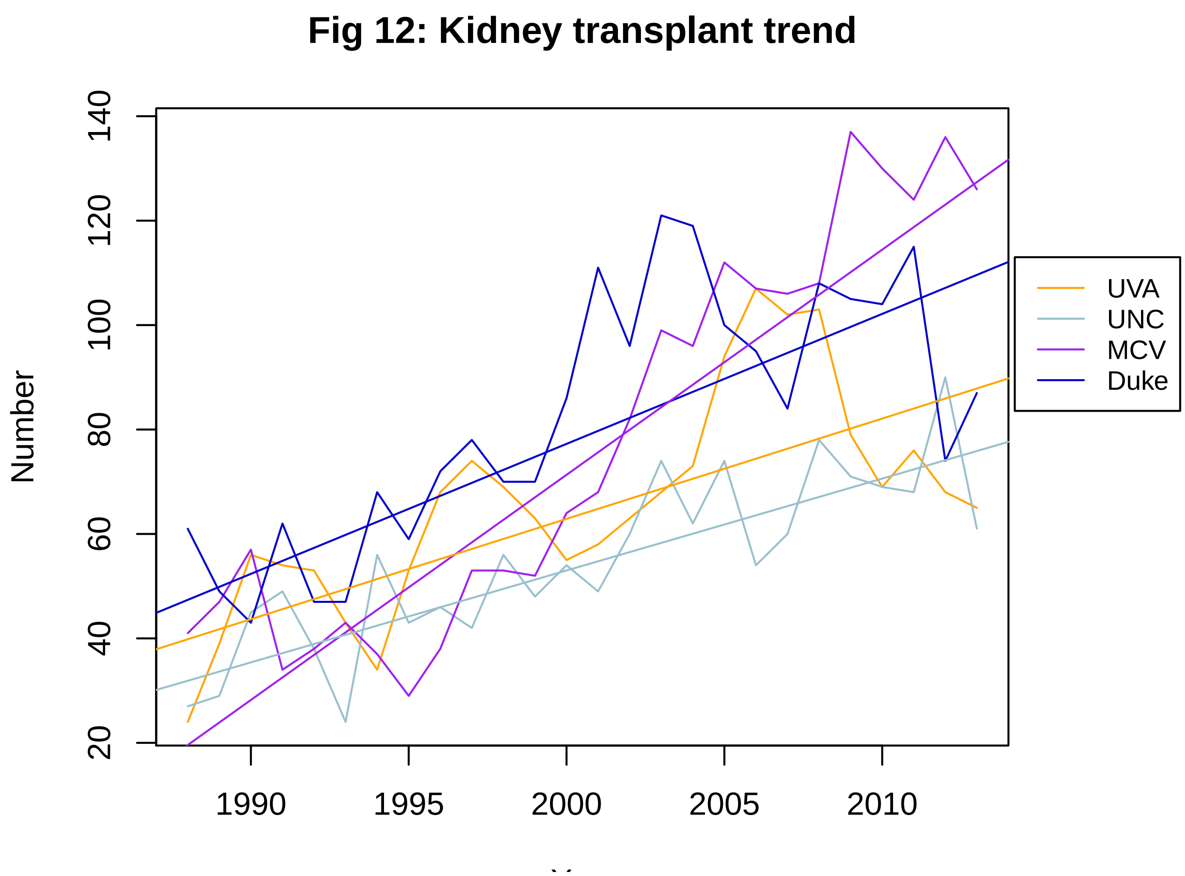


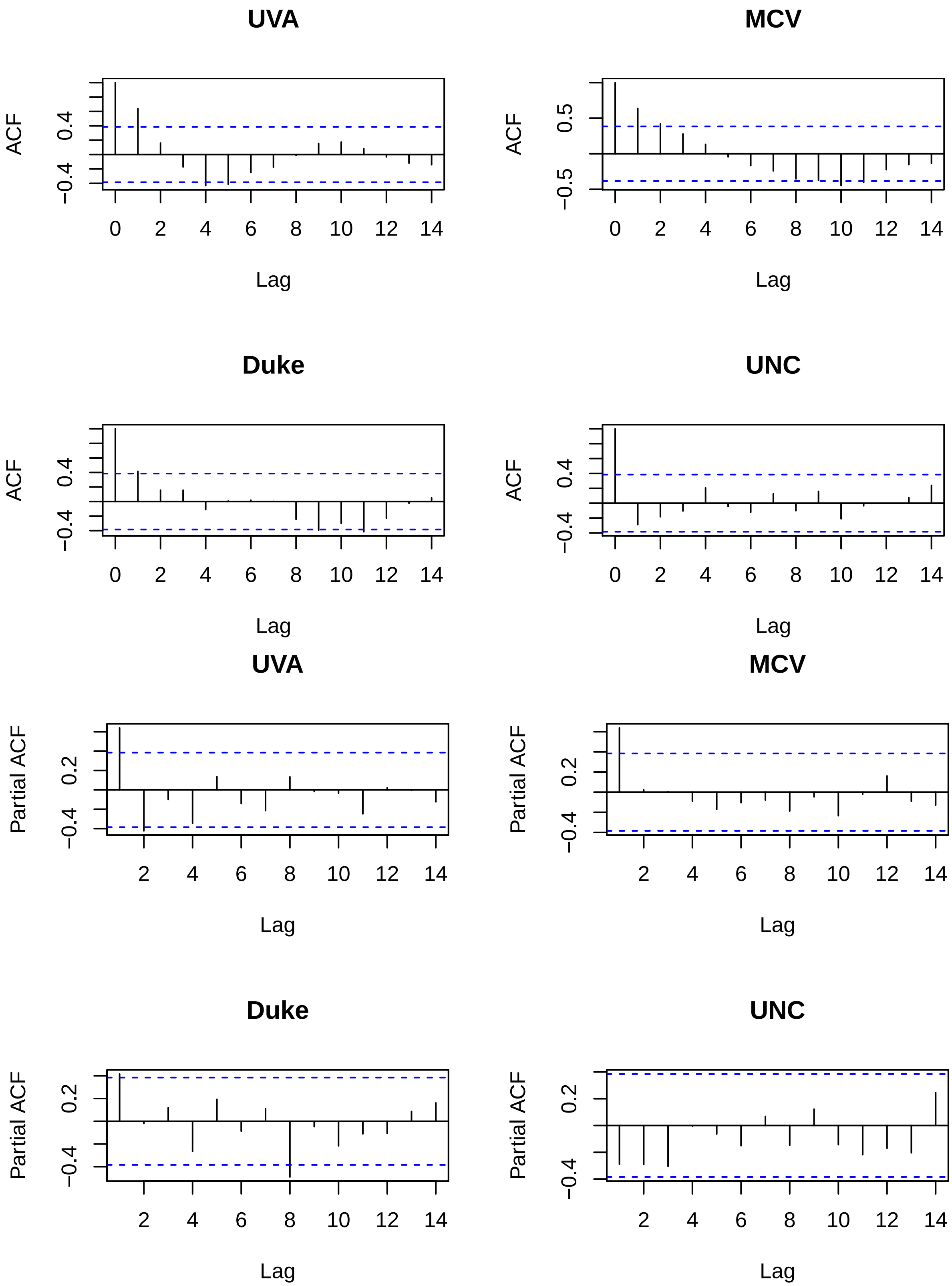
Fig 11: QQ Plot



In fact when the trend lines are plotted in the graph to see the effects we see that this model performs quite well compared to the others.



In order to explore the serial correlation, the autocorrelation and partial autocorrelation plots were drawn. It can be seen that there is serial autocorrelation present from the acf and the pacf graphs.



Another model is being created to take into account the diversity ratio to see if that is a better predictor than the present model which takes into account the number of years condtioned on the respective schools. Here the ethnic ratio is defined as the number of non-white people who underwent kidney transplants over the total number of people who underwent kidney transplant operations.

Now when a comparison is done with the model that takes into account the ethnic ratio and the one that does not, it turns out that the model that does not take the ethnic ratio into account performs better than the one that takes that into account. But it is not showing up as a significant predictor. Because it is correlated with years.

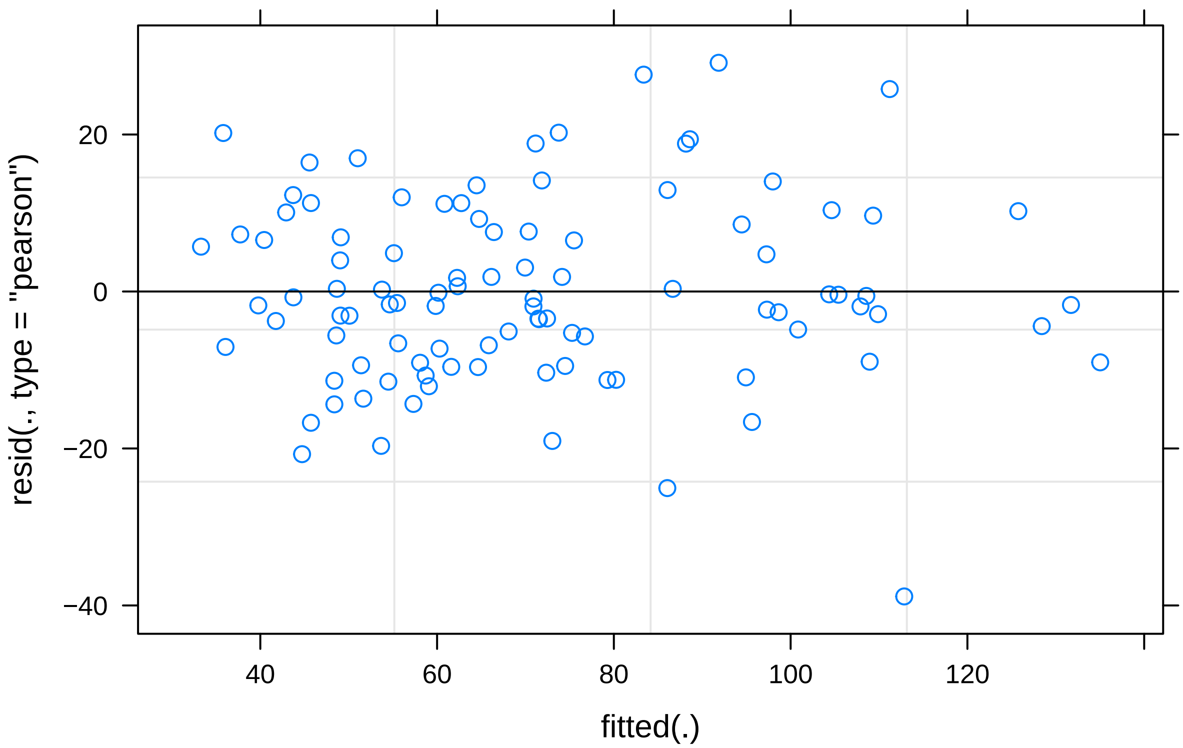
Years is a much bigger driver than diversity ratio. But the two of them are so correlated that the effect is captured to a much larger amount through the years.

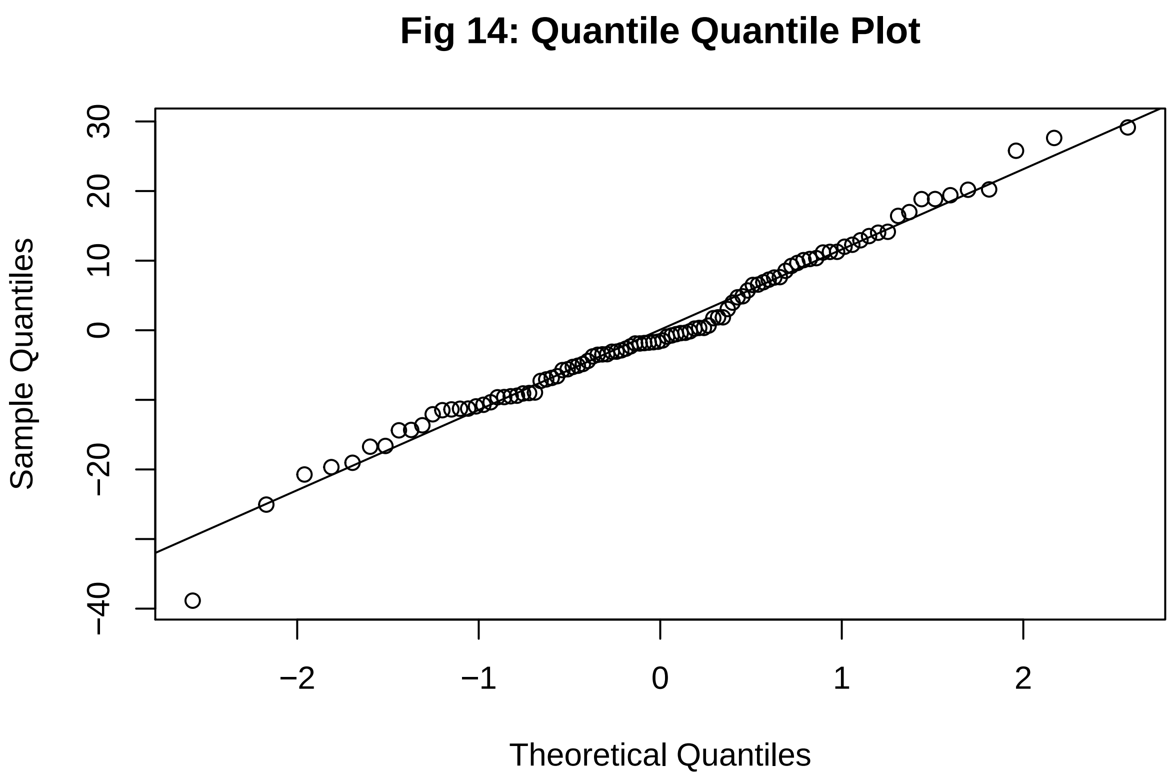
Since we explored the acf and pacf graphs and found that there was serial correlation of 1, a new model is being constructed to take into account the effects of the serial correlation with a lag of 1.

Now diagnostic plots are drawn for the mix effects model and the plot shows fairly good distribution of the points along the mean of zero and seems like a good plot.

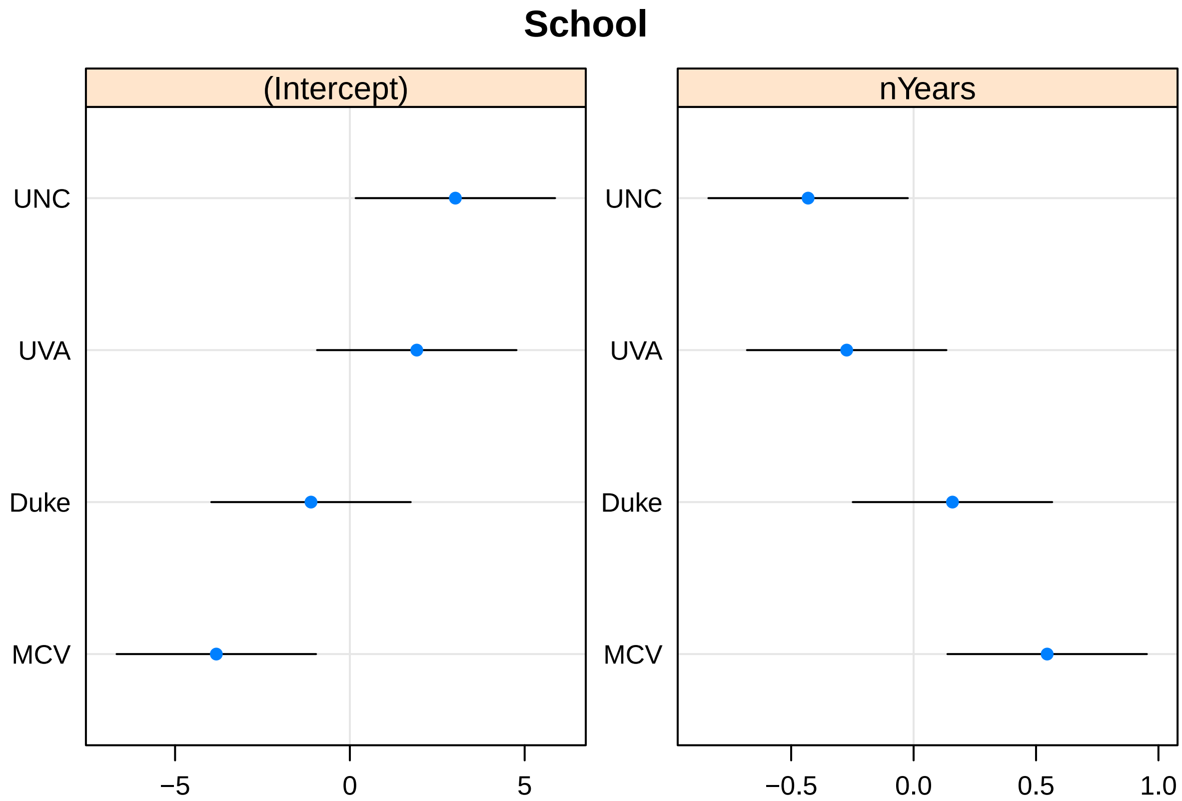
Quantile-Quantile plots are also drawn and the distribution of the points show an almost normal distribution while the tails diverge.

Fig 13: Residuals vs Fitted plot

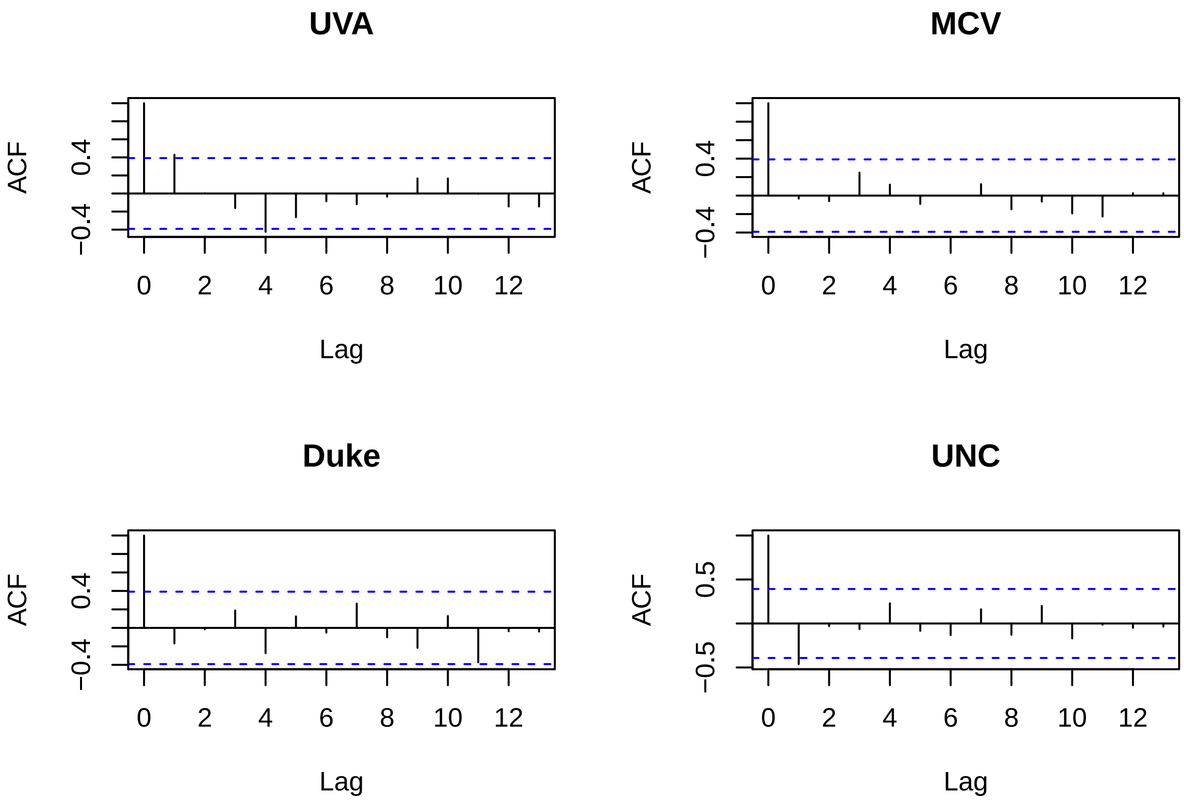


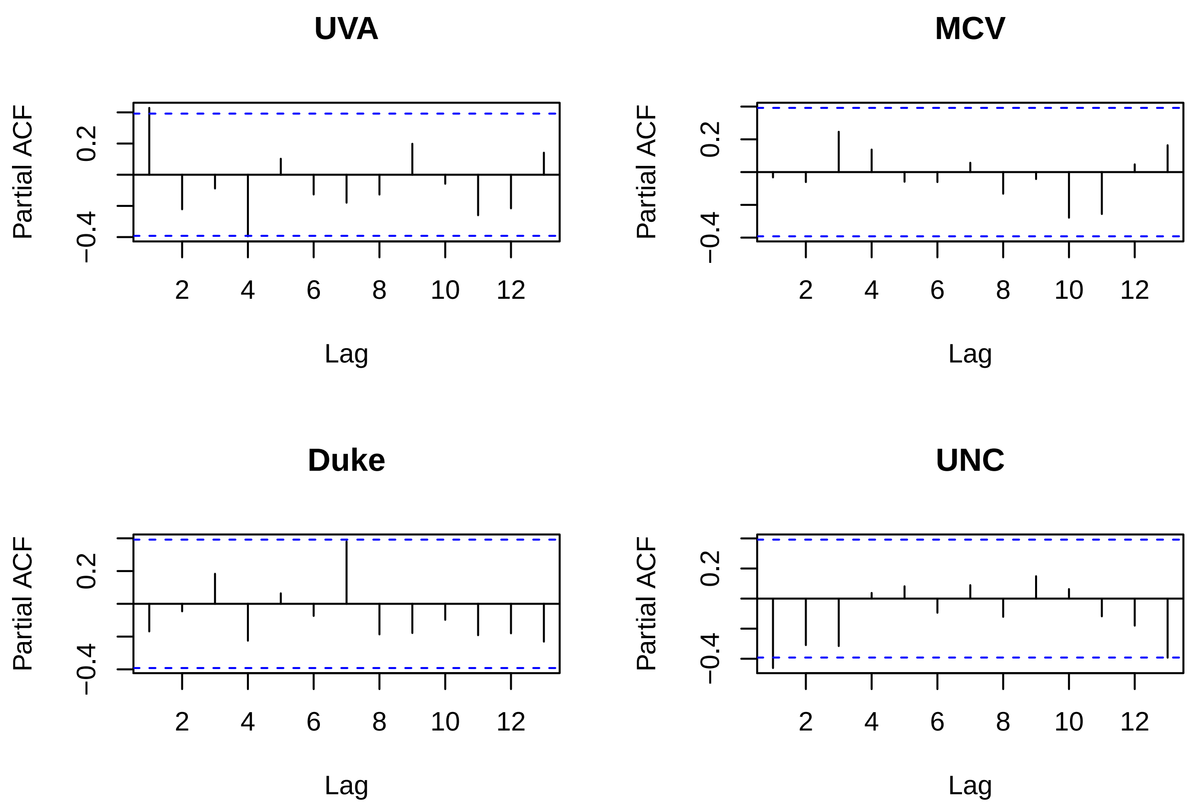


The random effects model shows that according to the intercepts UNC is doing really well but rightly takes into account MCV's spike towards the end and the fact that it has been doing well recently.



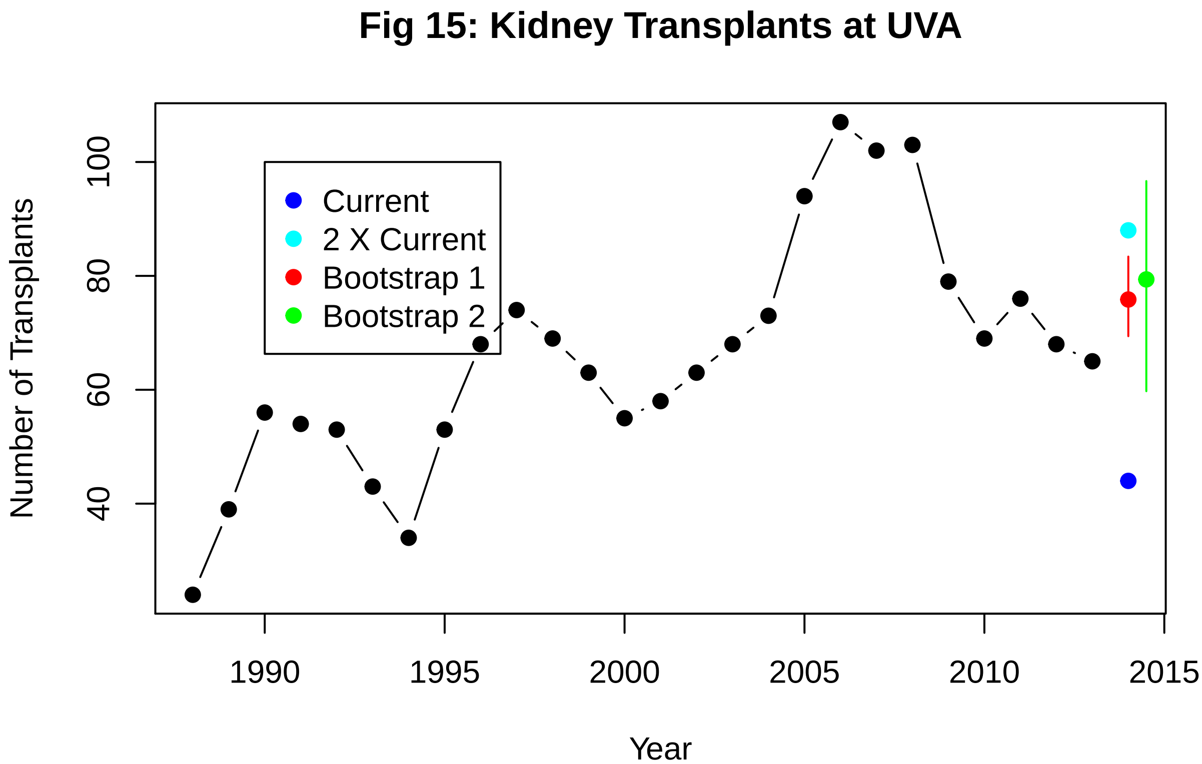
Also the auto correlation and partial auto correlation plots show that serial correlation have been taken into account.





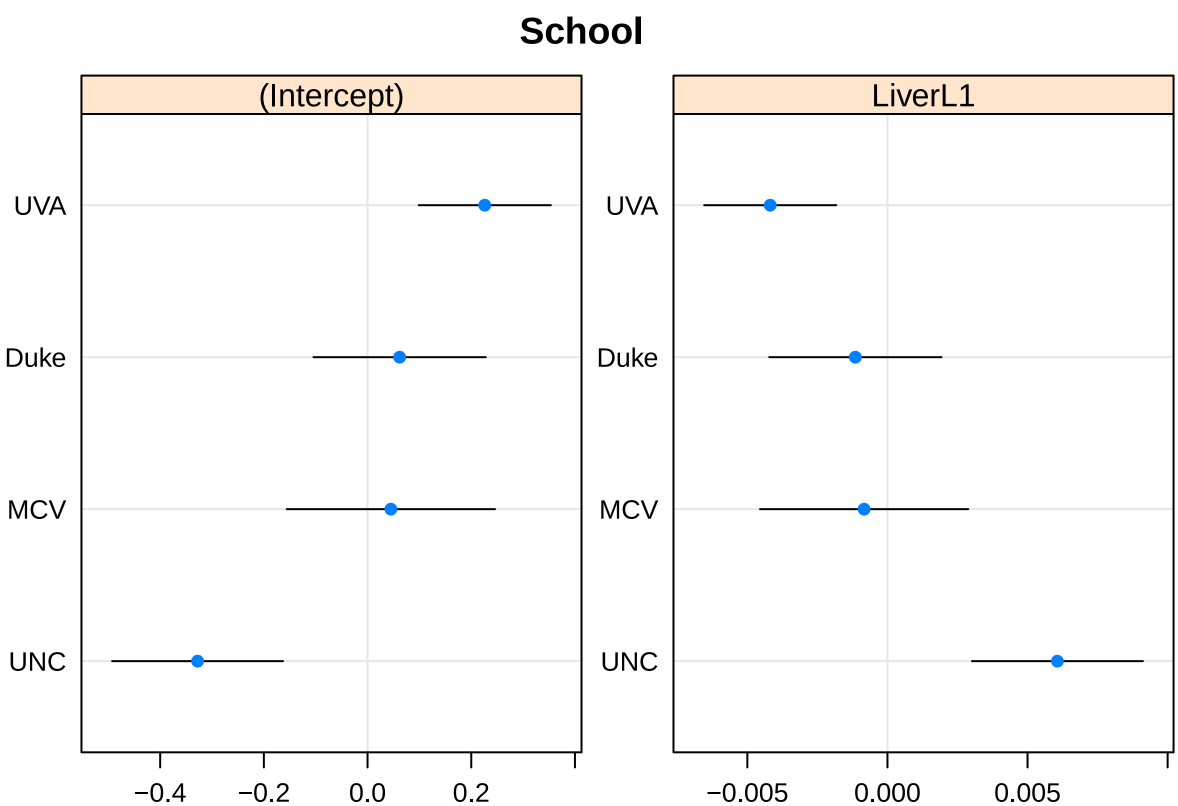
So the final model is the one that takes into account the lag of 1 and now the same model is being used for test set prediction.

Now the following shows the performance of the model for the first half of 2014. Also 2 times the prediction is shown to account for the entire year of 2014.



Now for the liver again a similar analysis is done.

Now a dotplot shows the how the schools vary in their performance based on the intercept and the LiverL1 variable.



The diagnostic plots show that the data is a really bad fit. The residuals vs fitted plot shows some amount of heteroscedasticity. The qqplot however is a very bad fit to the gaussian assumption and the tails widely diverge.

Fig 16: Residuals vs Fitted Plot

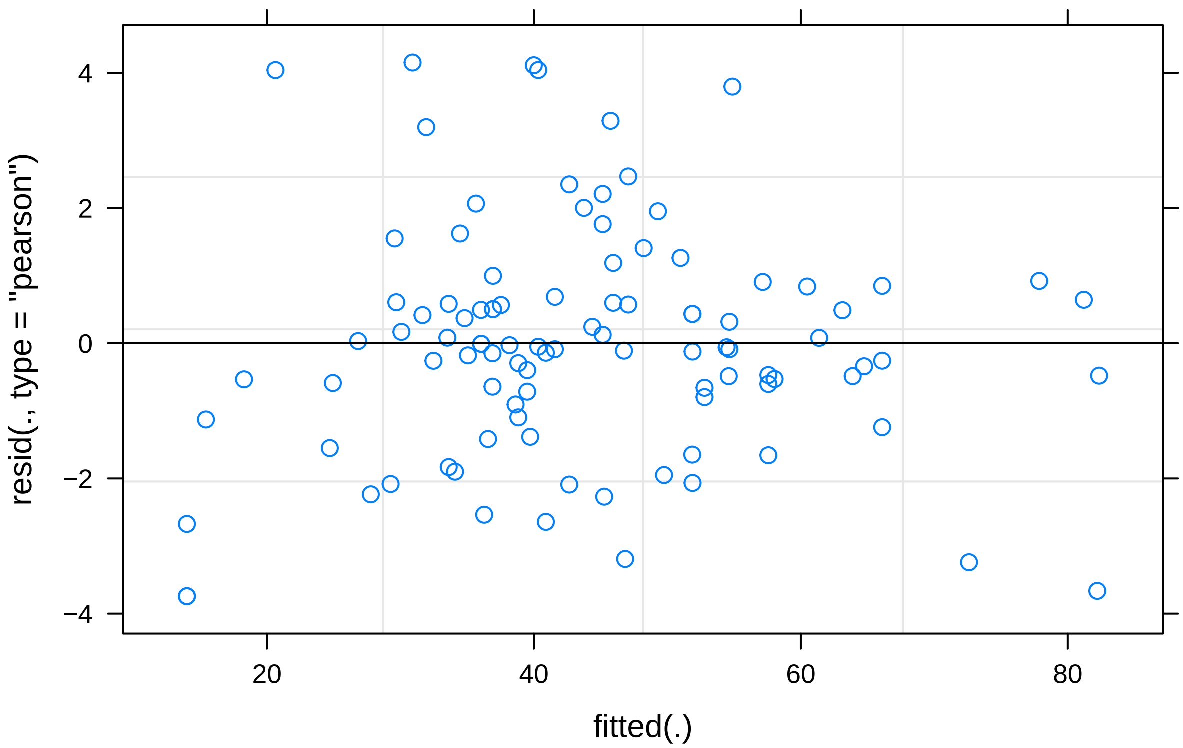
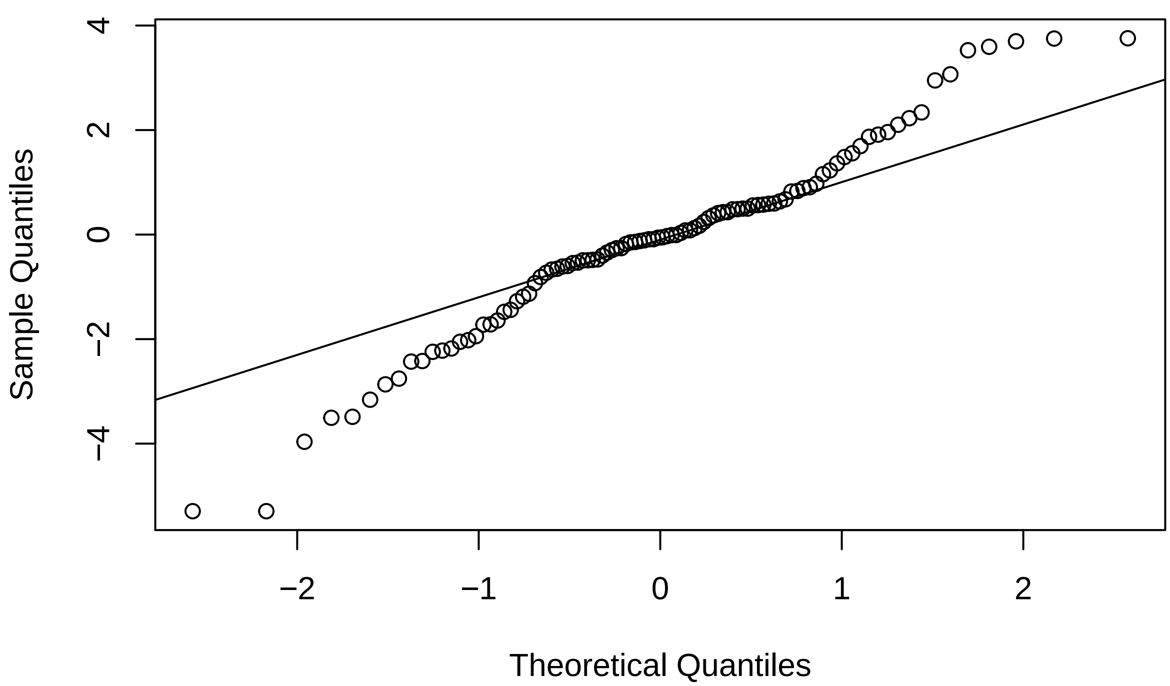
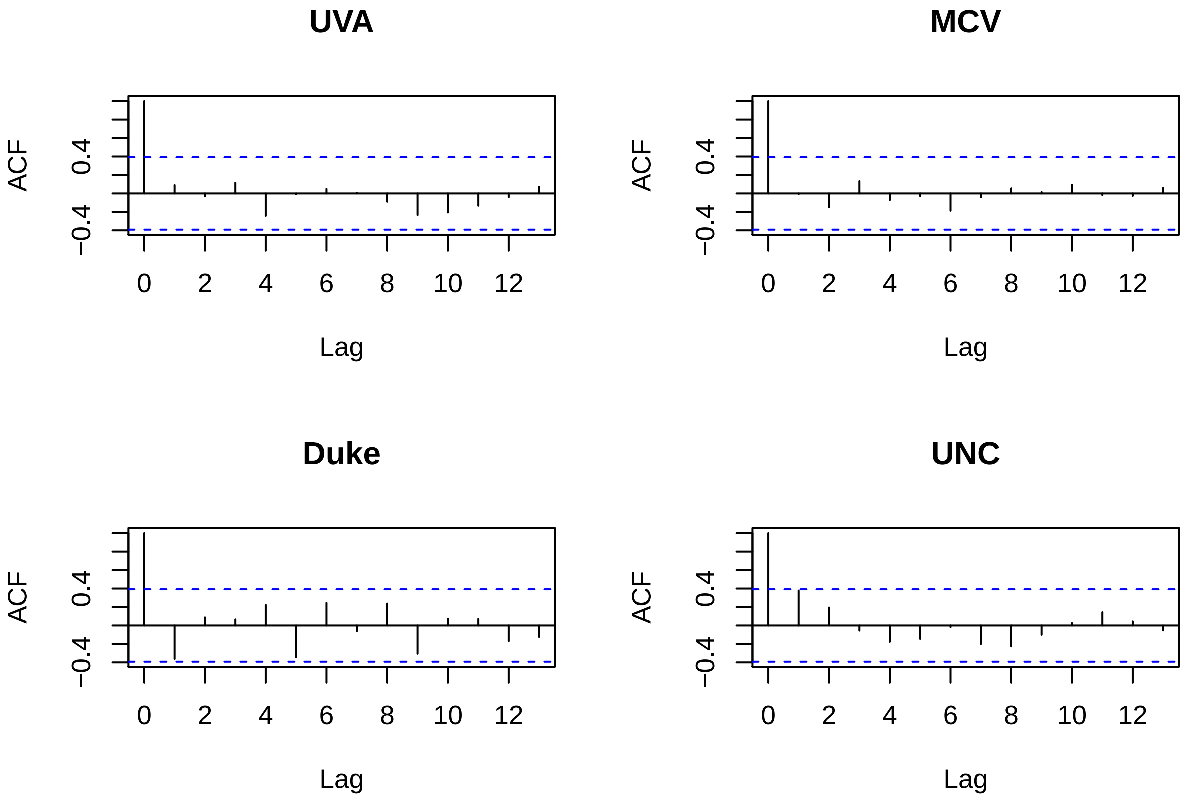
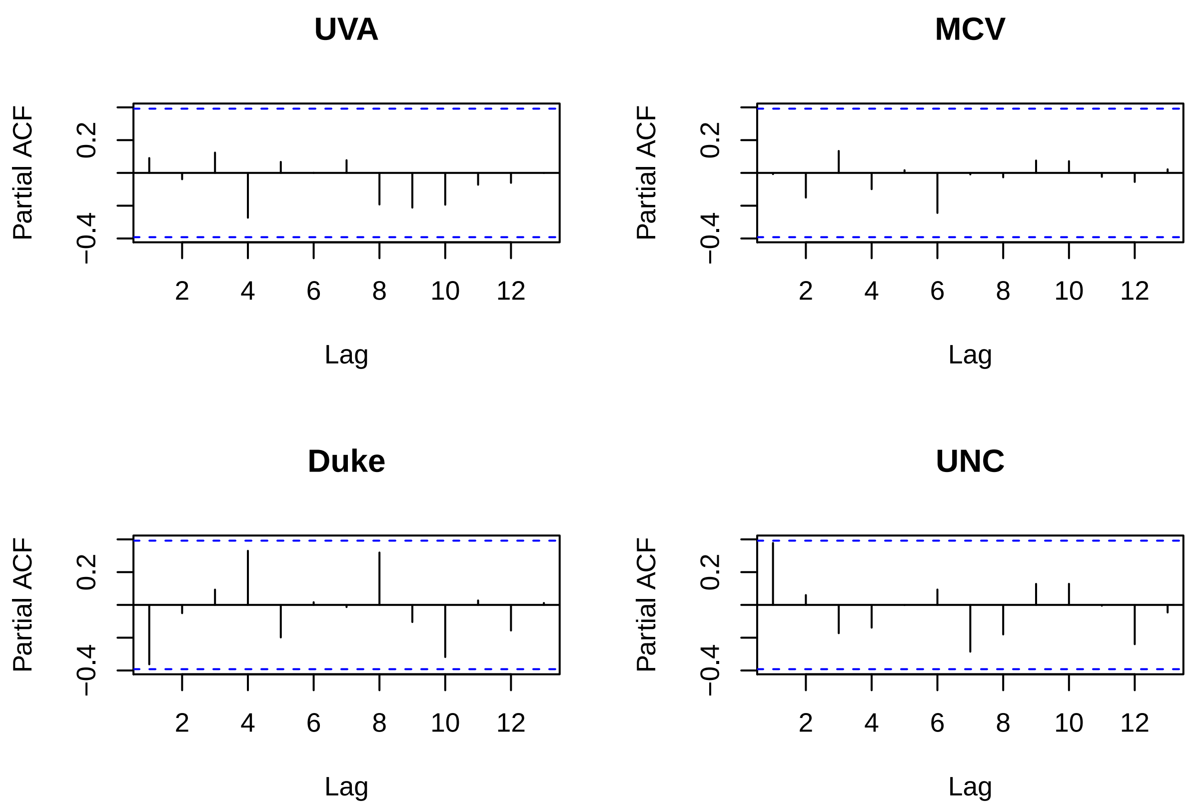


Fig 17: QQ Plot



There is however no serial correlation observed from the auto correlation and the partial autocorrelation plots.



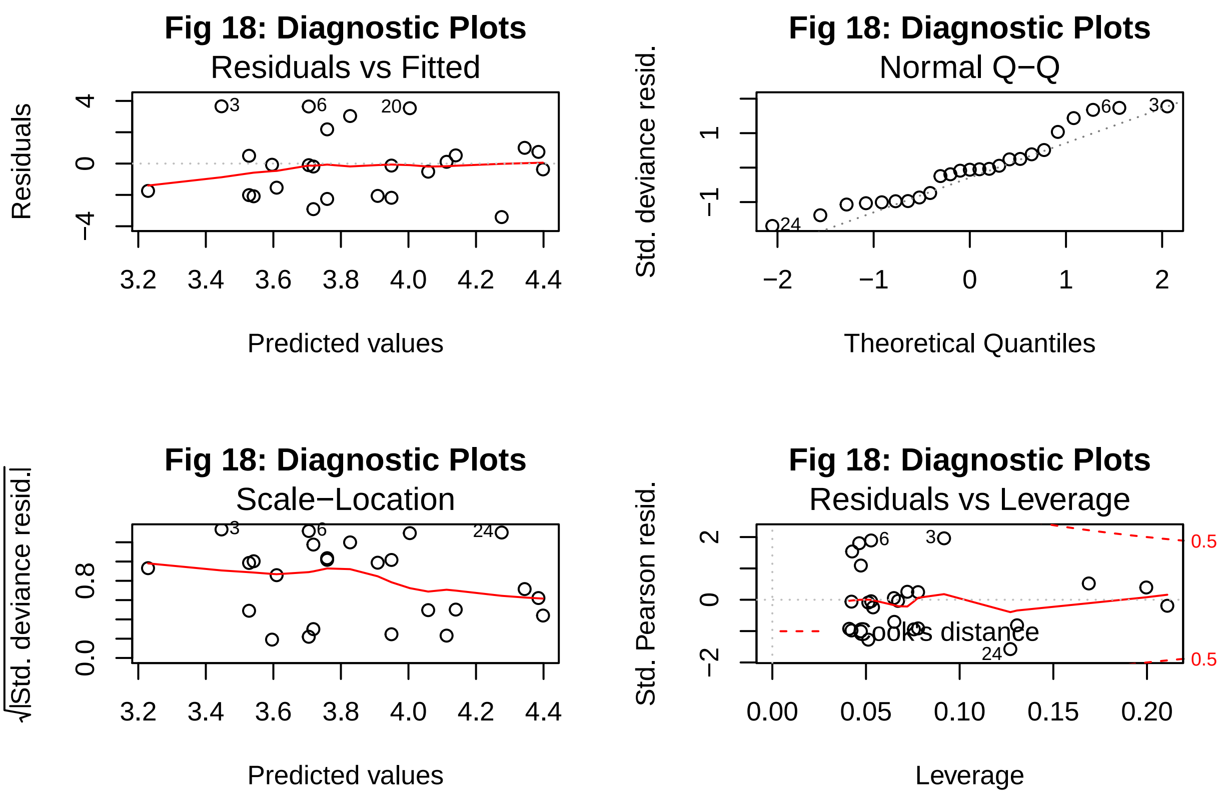


Now the maximum likelihood model is compared against the varying intercepts model and it can be seen that the maximum likelihood model performs significantly better than the varying intercepts model. In fact the maximum likelihood model as a chi-squared value of 3.871e-05, when compared to the model with varying intercepts model.

Now when the data is checked for overdispersion it was seen that it has significant amount of overdispersion. A quasi-poisson model accounts for overdispersion. Thus in order to account for the same, an unpooled model is used.

After accounting for over dispersion unpooled models are constructed. The generalized linear model when compared to the null model which is essentially the regression of liver against constant 1, it can be seen that the unpooled model is highly significant with the chi-squared value of 5.688e-07.

The diagnostic plots are not extremely bad even though it appears to be so. This is because the number of data points in consideration here is quite few.



This model is also checked for autocorrelation and partial autocorrelation. It could be seen from the following diagrams that there is no serial correlation that is observed.

Fig 19: ACF plots

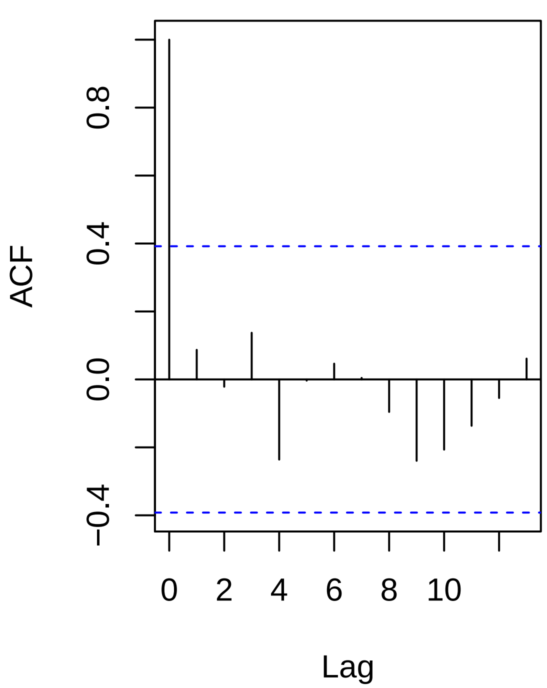
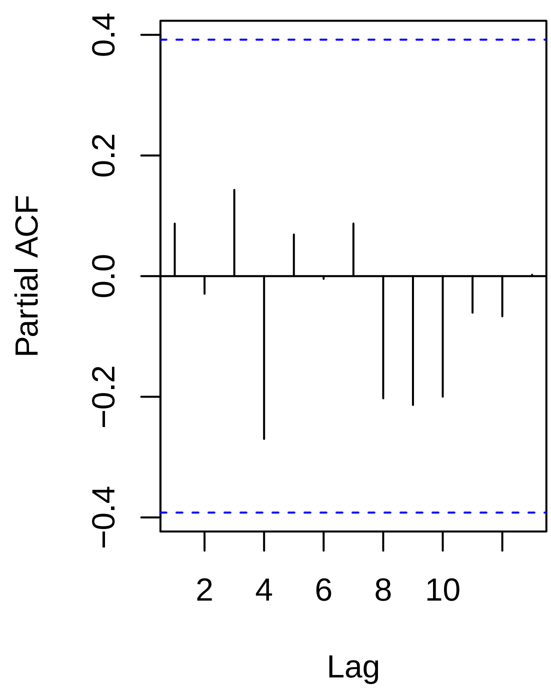
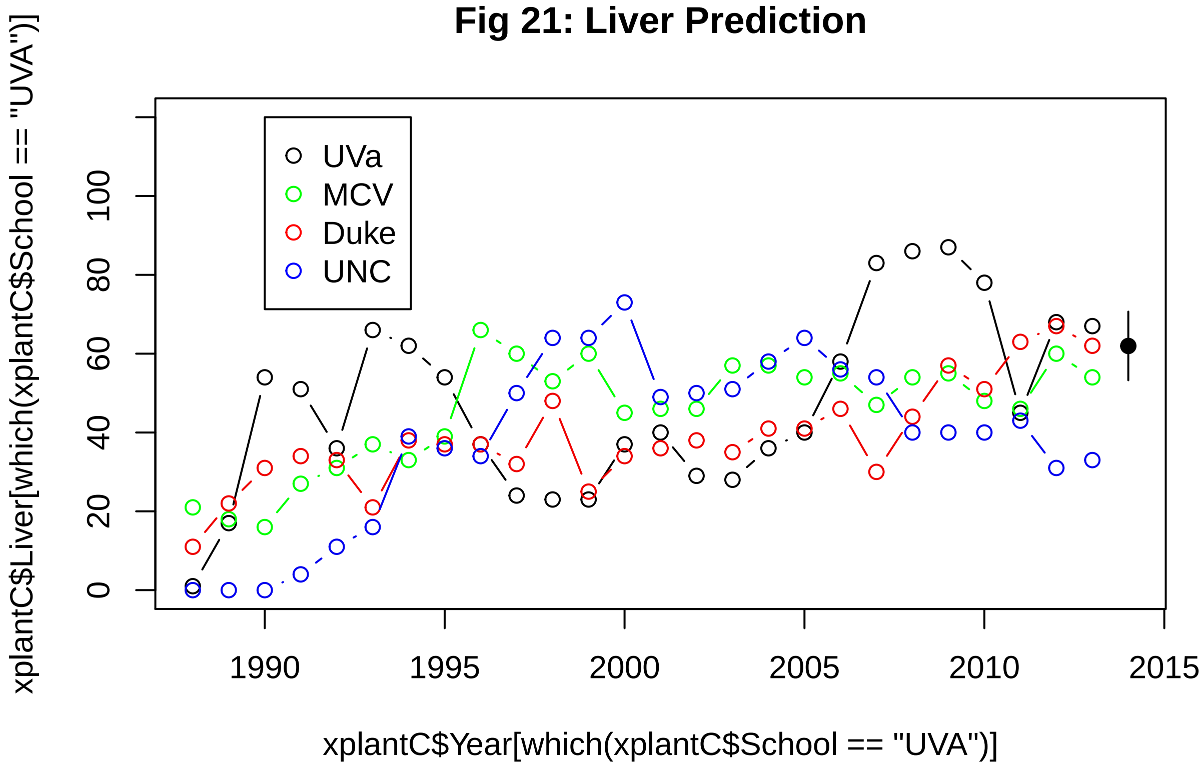


Fig 20: PACF plots



Thus this model is then used for prediction on the new dataset. The black line is UVA, green MCV, red is DUKE and blue is UNC. The prediction of UVa's performance is shown shown using a black dot and the confidence interval is also show.

Fig 21: Liver Prediction



3. Evidence

In fact a comparison between the model that takes into account the lag components and the one that does not shows that the model that takes the lag components into account has a chi-squared test value of 4.442e-10 and it shows it is much more significant than the other model.

The confidence interval of the final model that takes into account the serial correlation component is as follows:

Table 3: Confidence interval using percentile method

2.5%	97.5%
69.32650	83.58704

Table 4: Confidence interval using BCa method

2.5%	97.5%
60.56142	93.57852

The 95% confidence interval for the final model for liver prediction is

Table 5: Confidence interval using Percentile for liver prediction

2.5%	97.5%
53.2449	70.67192

Now it was shown in the analysis section that MCV will perform better than UVa in terms of the kidney transplants. Thus since MCV is performing better the goal would be to model MCV to improve UVa's performance.

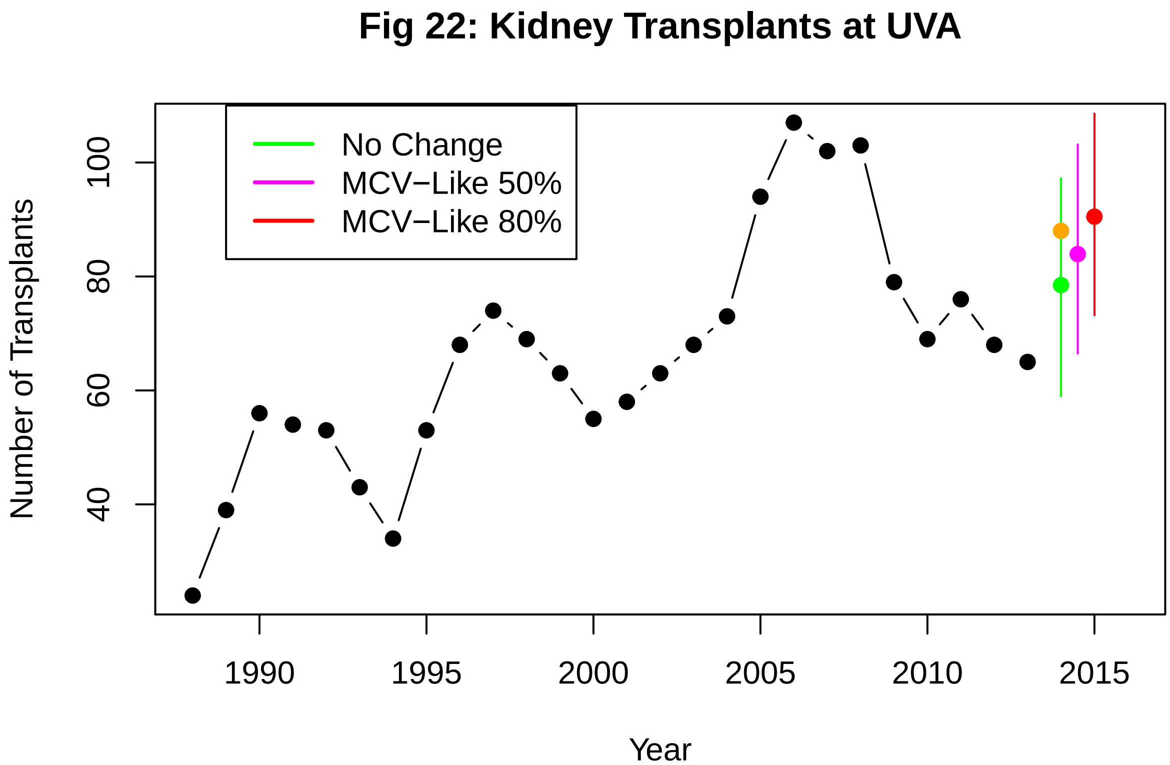
Now since MCV was doing the best the goal would be to emulate MCV to improve UVa's performance. Now for the final model, a bootstrap simulation is run 200 times to get a better estimate of the confidence interval.

The 97.5% confidence interval is as follows after running the bootstrap simulation is as follows:

The prediction is shown in the diagram below. As can be seen that the half year prediction is descent for UVa and the full year prediction can be seen in the orange dot in the diagram below. Also if we model MCV like performance by 50%, that is shown in pink and if we model MCV like performance by 80% that is shown in red. Now it can easily be seen that the performance improves significantly for UVa if they model MCV by 50 % and improves even more if they model MCV by 80%.

Table 6: Confidence Interval after running bootstrap simulation

2.5%	50%	97.5%
60.45367	77.58414	93.38923

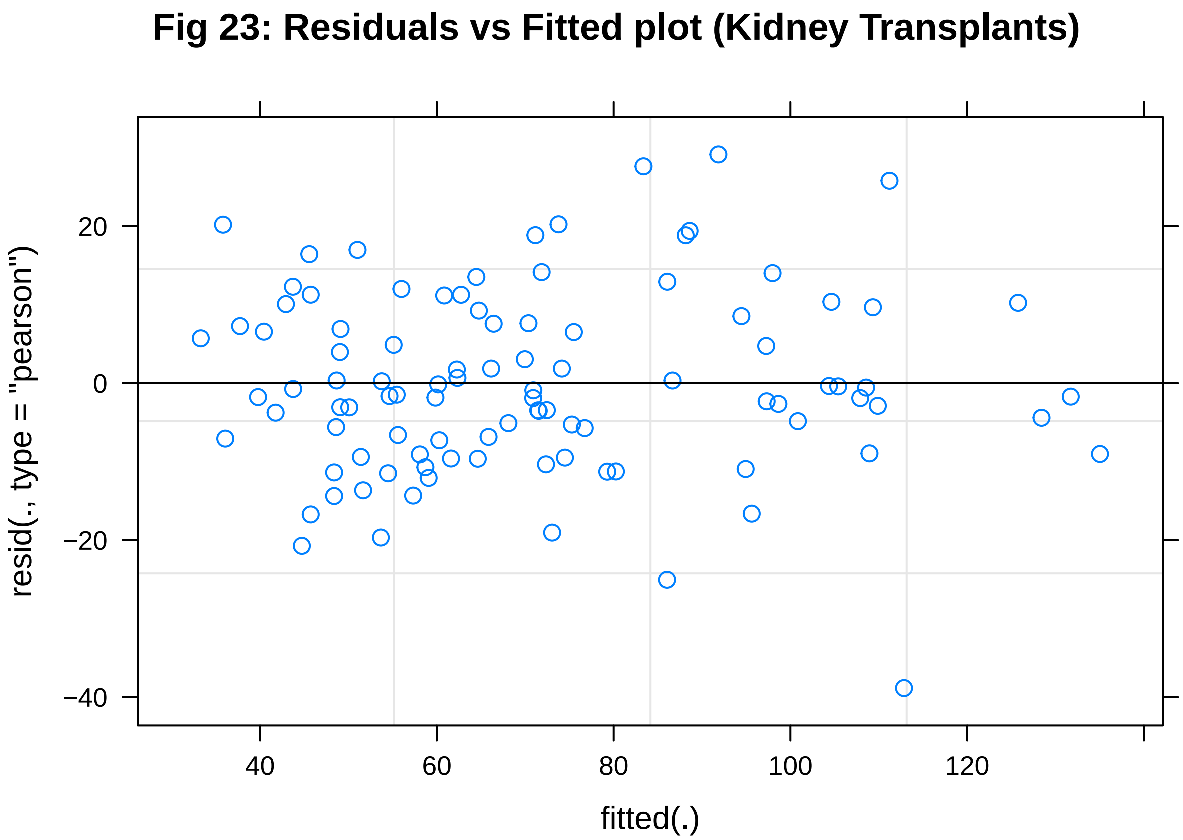


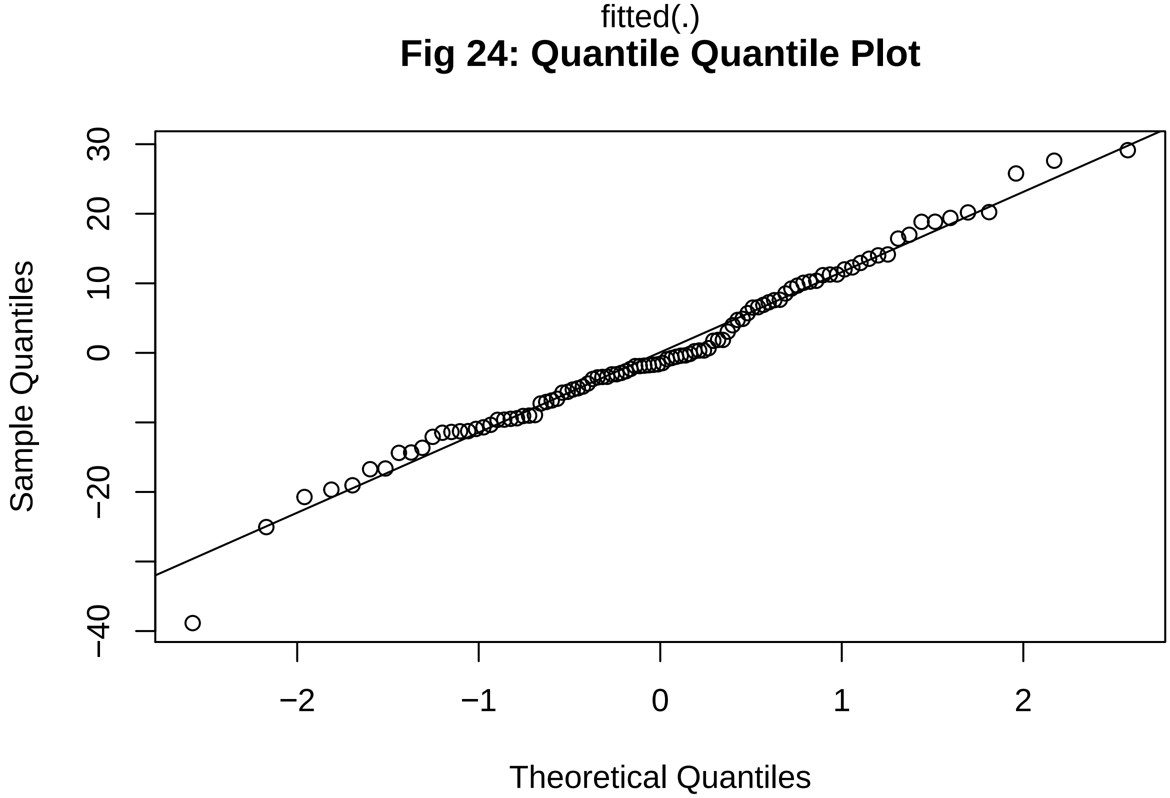
The confidence interval for modelling MCV by 50% and 80% is as follows:

Table 7: Modelling MCV's performance

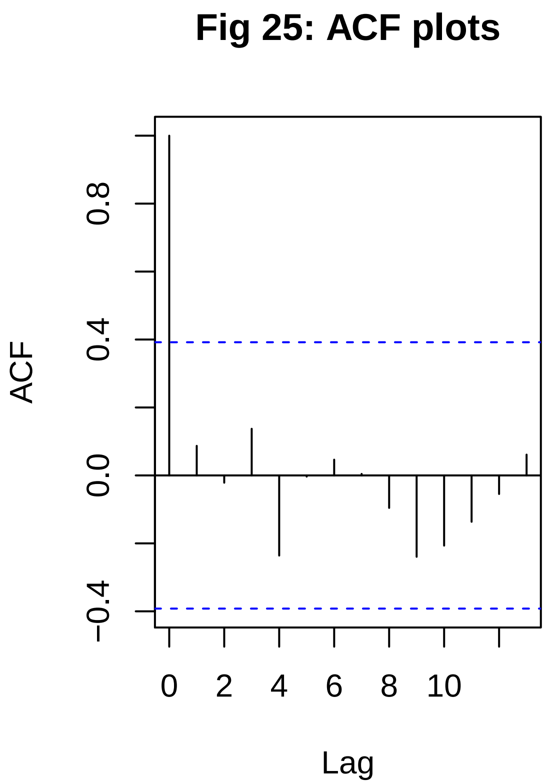
	50% like MCV	80% like MCV
2.5%	67.26517	72.15496
97.5%	101.77588	108.45433

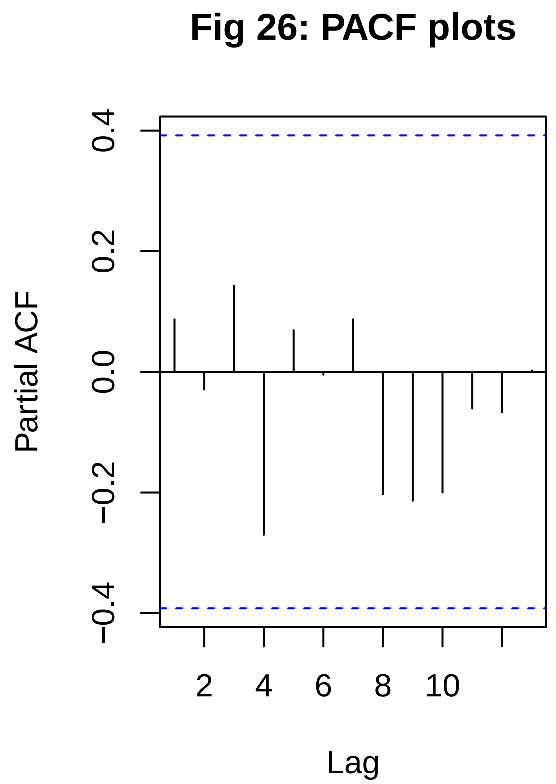
Now the diagnostic plots for the model also seem quite good and the qqplot shows a distribution that is quite close to the normal distribution.



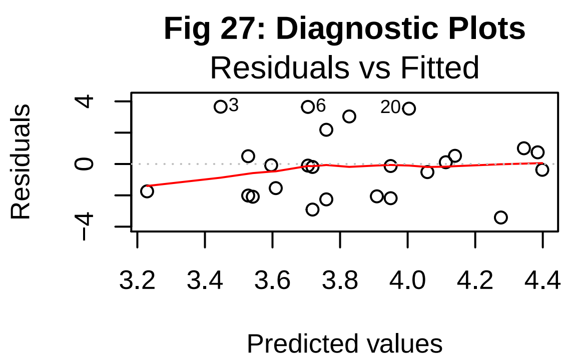


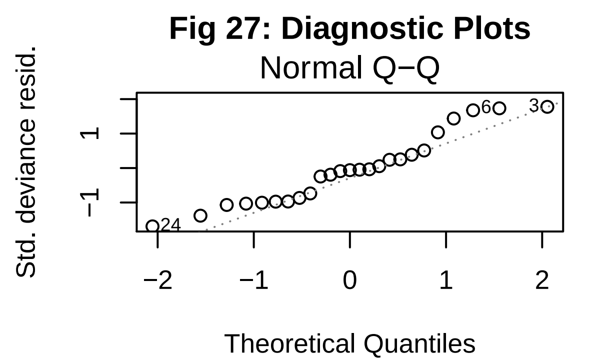
The diagnostic plots for the final model for liver transplants also show that all serial auto correlation and partial auto correlation have been taken into account and thus in the final model there is no more serial auto correlation and partial auto correlation present.

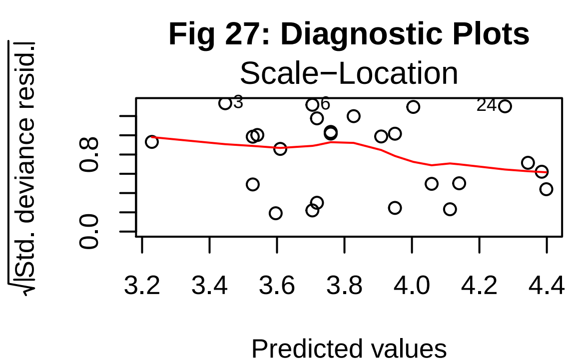


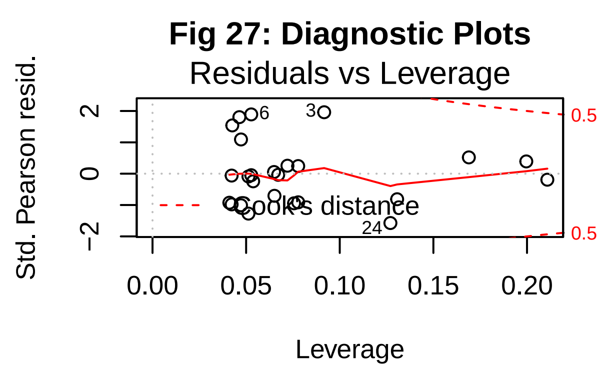


The diagnostic plots for the final model seem quite bad since the number of points are really small, but the prediction performances and the corresponding confidence intervals have been shown above.

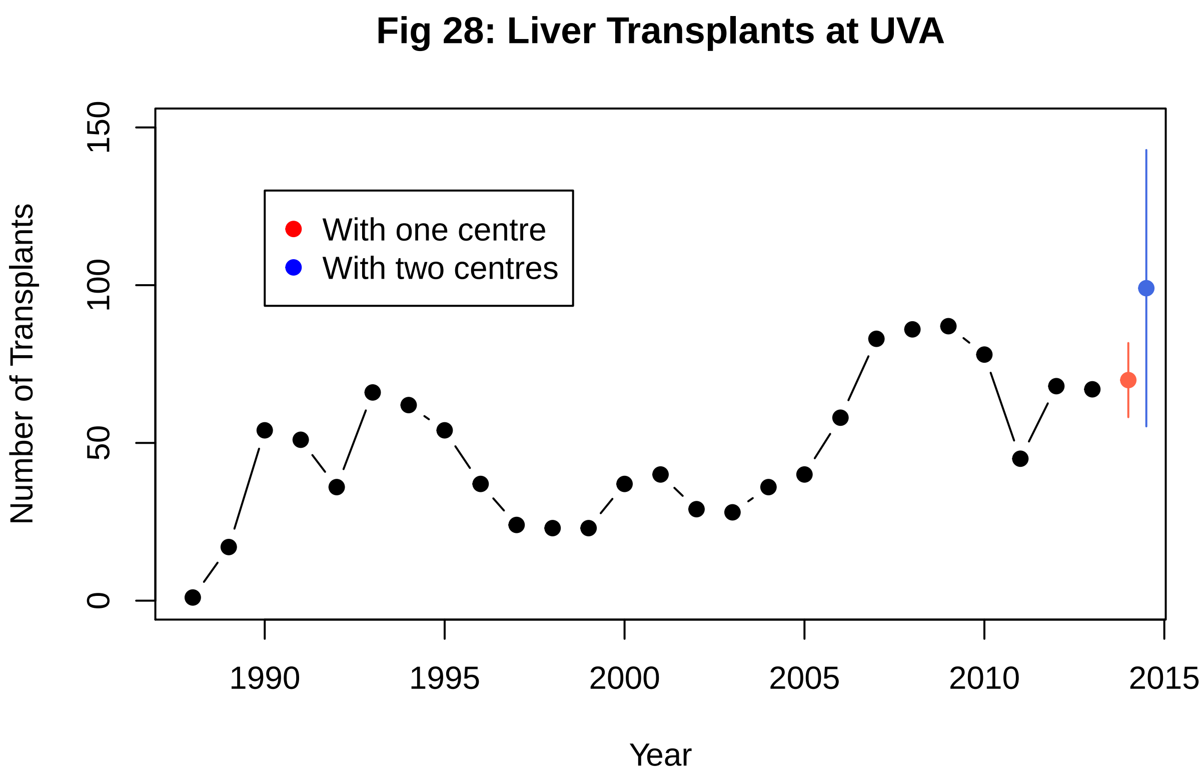








Now a new centre was added at Raonok in Virginia. In order to explore the effect of adding the new centre in Virginia the following plot is drawn.



Also the confidence interval of UVa's performance next year based on having one center and also in the case when another center is established is below

Table 8: UVa's performance based on the number of centers

	With one center	With two centers
2.5%	58.16192	55.2408
97.5%	81.67621	142.854

Now it can be seen that the confidence interval is really wide with the second centre added and it thus may not be economically feasible to actually construct a new centre in this case.

The final model has a AIC of 798.7430 for kidney transplants which is better than the base case of 989.786 and the final model for Liver transplants has an AIC of 881.39 which is significantly better than the base case of 1063.243.

4. Recommendation

Summarize your design recommendations for the UVA Transplant Center and your evidence for these recommendation. Be sure to include measures of confidence or significance.

The recommendation for UVa for liver and kidney transplant is as follows:

Liver Transplant:

A liver ransplant centre was established in Raonoke, Virginia and that improved the number of transplants immediately after the establishment of the centre. However the prediction of the performance with this centre has a 95 % confidence interval of (58.16192 , 81.67621). The establishment of another centre even though according to the prediction model the number of transplants is increasing but it might not be benefical because of primarily two reasons. Economically establishment of another centre might not be feasible and the 95% confidence interval for the second centre is much wider (55.2408, 142.854)

Kidney Transplant:

For kidney transplant MCV was doing significantly better than UVa. Now in this case for imporving UVa's performance UVa should model MCVs performance. If UVa can model MCV's performance then UVa's performance will be in the range. Now the confidence interval for the 50% like MCV, then the 95% confidence interval will be (67.26517, 101.77588). However if UVa performs better and models MCV upto 80% then UVa's performance will significantly improve and the confidence interval will be (72.15496, 108.45433).

References

[1] L. E. Barnes and D. E. Brown, *Project 3: Design Improvements for the UVA Transplant Center, Class project in SYS 6021, 2014.*

[2] Project 3 template, Class template in SYS 6021, 2014.

[3] (2014) University of Virginia Transplant Services. http://uvahealth.com/services/ transplant-services/transplant-services.

[4] R. Teaster, "Operations of the University of Virginia Transplant Center," Personal Communication, 2010.

[5] United States Government Accountability Office, "Organ Transplant Programs: Federal Agencies Have Acted to Improve Oversight, but Implementation Issues Remain ," GAO-08-412, 2008

[6] *OPTN: Organ Procurement and Transplantation Network. http://optn.transplant. hrsa.gov*

A Optional Appendix

Coefficient of final model for Kidney Transplants:

Table 9: Coefficient for Final Model for Kidney Transplants

	(Intercept)	LiverL1
Duke	3.03	0.02
MCV	3.01	0.02
UNC	2.64	0.02
UVA	3.19	0.01

Random Effects of final model for kidney transplants:

Table 10: Random Effects for Kidney Transplants

Groups	Name	Std.Dev	Corr
School	Intercept	0.220334	
	LiverL1	0.004077	-1.00

Fixed Effects of final model for kidney transplants:

Table 11: Fixed Effects for Kidney Transplants

Fixed Effects	
Intercept	LiverL1
2.96714	0.01818

GLMs describe the dependence of a scalar variable y_i $(i = 1, \dots, n)$ on a vector of regressors x_i. The conditional distribution of $y_i|x_i$ is a linear exponential family with probability density function

$$f(y; \lambda, \phi) \quad = \quad \exp\left(\frac{y \cdot \lambda - b(\lambda)}{\phi} + c(y, \phi)\right), \tag{1}$$

where λ is the canonical parameter that depends on the regressors via a linear predictor and ϕ is a dispersion parameter that is often known. The functions $b(\cdot)$ and $c(\cdot)$ are known and determine which member of the family is used, e.g., the normal, binomial or Poisson distribution. Conditional mean and variance of y_i are given by $\mathsf{E}[y_i \,|\, x_i] = \mu_i = b'(\lambda_i)$ and $\mathsf{VAR}[y_i \,|\, x_i] = \phi \cdot b''(\lambda_i)$. Thus, up to a scale or dispersion parameter ϕ, the distribution of y_i is determined by its mean. Its variance is proportional to $V(\mu) = b''(\lambda(\mu))$, also called variance function.