UMC-202 Problems Sheet Fall 2024

1. For each of the following Adams-Moulton implicit methods, derive the formula for the method and the form of their local truncation errors. Furthermore, determine the stability with the help of the characteristic polynomial.

   - two step Adams-Moulton implicit method.
   - three step Adams-Moulton implicit method.
   - four step Adams-Moulton implicit method.

2. Using the form of the Runge-Kutta method of order 2 (RK-2) for a single IVP, derive the RK-2 method for the following system:

$$\begin{aligned} u_1' &= f_1(t, u_1, u_2), \quad a \le t \le b, \\ u_2' &= f_2(t, u_1, u_2), \quad a \le t \le b, \\ u_1(a) &= \alpha_1, \\ u_2(a) &= \alpha_2. \end{aligned}$$

3. Consider the initial value problem(IVP):

$$y''(t) = f(t, y') + y, \;\; t \in [a, b], \quad y(a) = \alpha, \; y'(a) = \beta,$$

   where f is a smooth functions of its variables. Convert the IVP into an initial value problem for a system of first order differential equations and derive the Adams-Bashforth two step method for the resulting system. Express the local truncation error in terms of the mesh size and the derivatives of y.

4. Consider the two point boundary value problem (BVP):

$$y''(x) + a(x)y'(x) - y(x) = f(x), \;\; x \in [0, 1], y(0) = \alpha, \; y(1) = \beta.$$

   For BVP, define the linear shooting method using initial value problems(IVPs) and Euler's method (for IVPs) to deduce a numerical algorithm to determine numerical solution of BVP.