UMC-202 Problems Sheet-2 Fall 2024

1. Let f be a cubic polynomial such that $f(-1) \geq 0$, $f(0) = 0$ and $f(1) \geq 0$. Then show that $\int_{-1}^{1} f(x)\,dx \geq 0$. If $f(0) = -1$, does the integral $\int_{-1}^{1} f(x)\,dx$ remain non-negative? Justify your answer (may be a counter example).

2. Let $f(x) = |1 + x| + |x|$ for $x \in [-2, 1]$. For the numerical integration of f over the interval $[-2, 1]$, determine the minimum value of k so that the composite trapezoidal rule using the break points $\{x_j\}_{j=0}^{k}$ in the interval $[-2, 1]$ gives the exact value of $\int_{-2}^{1} f(x)\,dx$. Justify your answer.

3. Let $p_3(x)$ be the interpolating polynomial of degree less than or equal to 3 interpolating a smooth function $f(x)$ at the points $x_0 = a - 2h$, $x_1 = a - h$, $x_2 = a + h$, and $x_3 = a + 2h$, for $h > 0$. If $f'(a)$ is approximated by $p_3'(a)$, then find the formula for the numerical differentiation (i.e. find $p_3'(a)$) and determine the order of convergence $O(h^k)$ by expressing the error in terms of h^k and the derivatives of f.