Problem 3:

Here, we are to implement example 6.6 from the book Sutton and Barto. The environment is similar to the gridworld problem but now it is a 4 x 12 grid with start state as (4,1) and goal state as (4,12). The states (4,2) to (4,11) are cliffs. Every transition but the transitions to a cliff state yields a reward of -1. If a transition happens to a cliff state, we get a reward of -100 and we go back to the start state. We compare the performance of Q-Learning and SARSA algorithms in such an environment.

In [1]:

#importing libraries
import numpy as np
import matplotlib.pyplot as plt
from tqdm import tqdm
import random

In [2]:

```
#building the environment
class Cliff_Environment:
    def __init__(self, M,N, holes, start_state, goal_state): #initializing the environm
ent states, holes, terminals and rewards
        self.states = set()
        self.start_state = start_state
        self.grid_shape = (M,N)
        self.holes = holes
        self.terminal_state = goal_state
        for i in range(1,M+1):
            for j in range(1, N+1):
                if (i,j) not in holes:
                    self.states.add((i,j))
        self.rewards = self.initialize rewards()
    def initialize_rewards(self): #function to initialize the rewards for each state of
the environment
        r = \{\}
        for state in self.states:
            if state in self.holes:
                r[state] = -100
            else:
                r[state] = -1
        return r
    def agent_move(self, s, a): #function to update the state of the agent given an act
ion a and current state s
        x, y = s
        if a=='U':
            x = x-1
        elif a=='D':
            x = x + 1
        elif a=='R':
            y = y + 1
        elif a=='L':
            y = y - 1
        stay same = self.check corner((x,y))
        if stay same:
            return s
        return (x,y)
    def move_clockwise90(self, a): #function to return the action which is a 90 degree
 rotation to current action a
        if a=='U':
            return 'R'
        elif a=='R':
            return 'D'
        elif a=='D':
            return 'L'
        elif a=='L':
            return 'U'
    def move_anti_clockwise90(self, a): #function to return the action which is a 90 de
```

```
gree rotation to current action a
        if a=='U':
            return 'L'
        elif a=='L':
            return 'D'
        elif a=='D':
            return 'R'
        elif a=='R':
            return 'U'
    def check_corner(self, s):
        #function to check if the updates state goes out of the gridworld.
        #If so, it returns a True value to address that the update should not take plac
e and agent should remain in current state.
        x1, y1 = s
        stay_same = False
        if x1<1 or x1>self.grid_shape[0]:
            stay_same = True
        if y1<1 or y1>self.grid_shape[1]:
            stay same = True
        return stay_same
    def get_s_r(self, s_new):
        #function to return the new state and reward
        if s_new in env.holes:
            r = -100
            s_new = env.start_state
            r = -1
        return (s_new,r)
    def get_new_state(self, s, a):
        #this is the function to take the agent to an update state given the agent's ch
oice of action and current state.
        #This encapsulates the dynamics of the environment and is not known to the agen
t. The agent only
        #produces the current state s and his choice of action a, to which the environm
ent returns his new state s1.
        x, y = s
        #t = random.random()
        s_new = self.agent_move(s,a)
        if s new in env.holes:
            r = -100
            s_new = env.start_state
        else:
            r = -1
        return (s_new,r)
        '''if t<=0.8: #agent's action succeeds
            s_new = self.agent_move(s,a)
        elif 0.8<t<=0.9:
            #agent stays in same state
            s_new = s
```

```
elif 0.9<t<=0.95: #move in a 90 degree clockwise direction
    a1 = self.move_clockwise90(a)
    s_new = self.agent_move(s, a1)
else: #move in a -90 degree clockwise direction
    a1 = self.move_anti_clockwise90(a)
    s_new = self.agent_move(s, a1)

s_new, r = self.get_s_r(s_new)'''
#return (s_new, r)</pre>
```

In [3]:

```
M = 4
N = 12
holes = []
for i in range(2, N):
    holes.append((4,i))

env = Cliff_Environment(M = M,N = N, holes = holes, start_state = (4,1) , goal_state = (4,12))
```

In [11]:

```
#building the SARSA Agent
class SARSA_Agent:
    def __init__(self, alpha, gamma, epsilon, env):
        #initializing the parameters of the agent
        self.actions = ['L','R','U','D'] #possible actions
        self.gamma = gamma #discount parameter
        self.Q = self.initialize_value_states(env)
        self.alpha = alpha
        self.initial_epsilon = epsilon
        self.epsilon = self.initial epsilon
    def initialize value states(self, env):
        #function to initialize the value of the state action pairs
        q s = \{\}
        for state in env.states:
            q s[state] = \{\}
            for action in self.actions:
                q_s[state][action] = 0
        return q_s
    def find_a_stars(self, s):
        #function to find the best action(s) based on Q value
        best q val = -np.inf
        greedy_actions = []
        for a in self.actions:
            if self.Q[s][a] > best_q_val:
                best_q_val = self.Q[s][a]
                greedy_actions = [a]
            elif self.Q[s][a] == best_q_val:
                greedy_actions.append(a)
        return greedy_actions
    def reset(self, env):
        #function to reset the agent
        self.Q = self.initialize_value_states(env)
    def choose_eps_greedy_action(self, s):
        #function to choose an action in epsilon greedy manner
        best values = []
        p = random.random()
        if p<=self.epsilon:</pre>
            #choose a random action
            a = np.random.choice(self.actions)
        else:
            #choose greedy action based on Q values
            a_stars = self.find_a_stars(s)
            if (len(a_stars)==1): #one best action
                a = a stars[0]
            else: #multiple optimal action. choose one randomly among them.
                a = np.random.choice(a stars)
        return a
    def SARSA_update(self, env, start_state = (4,1)):
        #function for SARSA on policy update of q values for an episode
        s = start state
```

```
rewards = 0
   while s!= env.terminal state:
        #choose a using epsilon greedy policy using Q values
        a = self.choose eps greedy action(s)
        #observe new state and reward
        s1,r = env.get_new_state(s,a)
        rewards+=r
        #choose action a1 based on epsilon greedy policy and state s1
        a1 = self.choose_eps_greedy_action(s1)
        #Q update
        delta t = (r + (self.gamma*self.0[s1][a1])) - self.0[s][a]
        self.Q[s][a] = self.Q[s][a] + (self.alpha*delta_t)
        s = s1
        a = a1
    return rewards
def play(self, env, episodes = 500):
    #function to play the agent for 500 episodes
    returns = []
    for episode no in range(episodes):
        G = self.SARSA_update(env)
        returns.append(G)
        #decay epsilon
        #self.epsilon = self.initial_epsilon/np.sqrt((episode_no+1))
    return returns
def plot_optimal_policy(self, env):
    #function to plot the optimal policy learnt by the agent
   M,N = env.grid_shape
    optimal_moves = []
    for i in range(M):
        row = []
        for j in range(N):
            s = (i+1, j+1)
            if s==env.start_state:
                value = "(S)"
                move = self.find a stars(s)
                value = ','.join(move) + value
                #print (value, end = "\t")
                row.append(value)
            elif s==env.terminal state:
                value = "G"
                #print (value, end = " ")
                row.append(value)
            elif s not in env.holes:
                #value=str(s)
                move = self.find_a_stars(s)
                #value = value + ": " + ','.join(move)
                value = ','.join(move)
                \#print (value, end = "\t\t")
                row.append(value)
            else:
                value = "C"
```

```
#print (value, end = "\t")
    row.append(value)

optimal_moves.append(row)

for row in optimal_moves:
    print ('\t'.join(row))
```

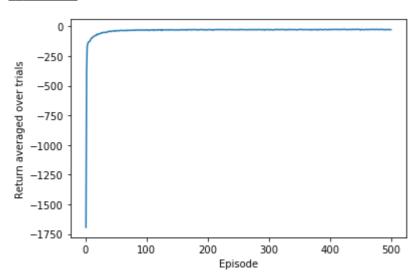
In [9]:

```
sarsa_agent = SARSA_Agent(alpha=0.5, gamma=1, epsilon=0.1, env=env)
trials = 1000
episodes = 500
returns_episodes = []
for trial in tqdm(range(trials)):
    returns = sarsa_agent.play(env, episodes = episodes)
    returns_episodes.append(returns)
    sarsa_agent.reset(env)

returns_episodes = np.array(returns_episodes)
returns_episodes = np.mean(returns_episodes, axis = 0)

t = np.arange(1, episodes + 1)
plt.plot(t, returns_episodes)
plt.xlabel("Episode")
plt.ylabel("Return averaged over trials")
plt.show()
```





Optimal Policy learnt by SARSA

In [14]:

```
sarsa agent = SARSA Agent(alpha=0.5, gamma=1, epsilon=0.1, env=env)
_ = sarsa_agent.play(env, episodes = 500)
sarsa_agent.plot_optimal_policy(env)
        R
                 R
                          R
R
                                            R
                                                     R
                                                             R
                                                                      R
                                                                               R
R
        D
U
        R
                 R
                          U
                                            R
                                                     R
                                                             U
                                                                      U
                                                                               R
D
        D
U
        R
                 U
                          U
                                   U
                                                     U
                                                             U
                                                                      L
                                                                               L
                                            L
        D
R
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                                                                               C
U(S)
        C
                 C
                          C
                                   C
                                                     C
                                                             C
                                                                      C
C
        G
```

We can see here clearly that SARSA learns a conservative policy. Since, the policy is epsilon greedy, the agent will sometimes pick a non optimal action and that may lead to a cliff state incurring a heavy negative reward. So, the SARSA agent learns a policy that keeps the agent as far as possible to the cliff states. It tries to force the agent from (4,1) to the states along (1,j) where j is from 1 to 12. Finally, from (1,12) it starts coming down towards the goal state. So, although it does not learn the optimal shortest path but it learns a safer path where there is very low chance of getting the heavy -100 reward.

It learns the path: $(4,1) \rightarrow (3,1) \rightarrow (2,1) \rightarrow (1,1) \rightarrow (1,2) \rightarrow (1,3) \rightarrow \dots \rightarrow (1,12) \rightarrow (2,12) \rightarrow (3,12) \rightarrow (4,12)$

In [15]:

```
#building the Q Learning Agent
class Q_Agent:
    def __init__(self, alpha, gamma, epsilon, env):
        #initializing the parameters of the agent
        self.actions = ['L','R','U','D'] #possible actions
        self.gamma = gamma #discount parameter
        self.Q = self.initialize_value_states(env)
        self.alpha = alpha
        self.epsilon = epsilon
    def initialize_value_states(self, env):
        #function to initialize the value of the state action pairs
        q_s = \{\}
        for state in env.states:
            q_s[state] = \{\}
            for action in self.actions:
                q_s[state][action] = 0
        return q_s
    def find_a_stars(self, s):
        best_q_val = -np.inf
        greedy_actions = []
        for a in self.actions:
            if self.Q[s][a] > best_q_val:
                best_q_val = self.Q[s][a]
                greedy_actions = [a]
            elif self.Q[s][a] == best_q_val:
                greedy actions.append(a)
        return greedy_actions
    def reset(self, env):
        self.Q = self.initialize value states(env)
    def choose_eps_greedy_action(self, s):
        best values = []
        p = random.random()
        if p<=self.epsilon:</pre>
            #choose a random action
            a = np.random.choice(self.actions)
        else:
            #choose greedy action based on Q values
            a_stars = self.find_a_stars(s)
            if (len(a stars)==1): #one best action
                a = a stars[0]
            else: #multiple optimal action. choose one randomly among them.
                a = np.random.choice(a stars)
        return a
    def Q_update(self, env, start_state = (4,1)):
        #function for SARSA on policy update of q values for an episode
        s = start state
        rewards = 0
        while s!= env.terminal state:
            #choose a using epsilon greedy policy using Q values
```

```
a = self.choose_eps_greedy_action(s)
        s1, r = env.get_new_state(s,a)
        rewards+=r
        a_stars = self.find_a_stars(s1)
        a star = a stars[0]
        delta_t = (r + (self.gamma*self.Q[s1][a_star])) - self.Q[s][a]
        self.Q[s][a] = self.Q[s][a] + (self.alpha*delta_t)
        s = s1
    return rewards
def play(self, env, episodes = 200):
    returns = []
    for episode no in range(episodes):
        G = self.Q update(env)
        returns.append(G)
        #decay epsilon
        #self.epsilon = self.initial_epsilon/np.sqrt((episode_no+1))
    return returns
def plot_optimal_policy(self, env):
    M,N = env.grid_shape
    optimal_moves = []
    for i in range(M):
        row = []
        for j in range(N):
            s = (i+1, j+1)
            if s==env.start_state:
                value = "(S)"
                move = self.find_a_stars(s)
                value = ','.join(move) + value
                #print (value, end = "\t")
                row.append(value)
            elif s==env.terminal state:
                value = "G"
                #print (value, end = " ")
                row.append(value)
            elif s not in env.holes:
                #value=str(s)
                move = self.find_a_stars(s)
                #value = value + ": " + ','.join(move)
                value = ','.join(move)
                #print (value, end = "\t\t")
                row.append(value)
            else:
                value = "C"
                #print (value, end = "\t")
                row.append(value)
        optimal moves.append(row)
    for row in optimal moves:
        print ('\t'.join(row))
```

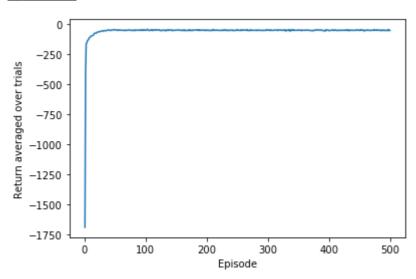
In [16]:

```
q_agent = Q_Agent(alpha=0.5, gamma=1, epsilon=0.1, env=env)
trials = 1000
episodes = 500
returns_episodes_q = []
for trial in tqdm(range(trials)):
    returns = q_agent.play(env, episodes = episodes)
    returns_episodes_q.append(returns)
    q_agent.reset(env)

returns_episodes_q = np.array(returns_episodes_q)
returns_episodes_q = np.mean(returns_episodes_q, axis = 0)

t = np.arange(1, episodes + 1)
plt.plot(t, returns_episodes_q)
plt.xlabel("Episode")
plt.ylabel("Return averaged over trials")
plt.show()
```

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Optimal Policy learnt by Q-Learning

In [18]:

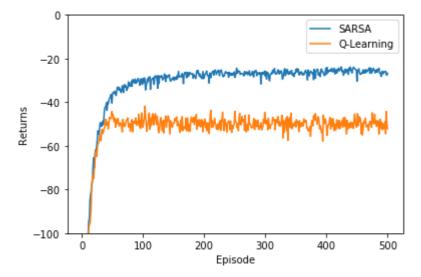
<pre>q_agent = Q_Agent(alpha=0.5, gamma=1, epsilon=0.1, env=env) q_agent.play(env, episodes = 500) q_agent.plot_optimal_policy(env) R</pre>		1.									
R D U D R R R R D R D R R D R <th colspan="11"><pre>q_agent.play(env, episodes = 500)</pre></th>	<pre>q_agent.play(env, episodes = 500)</pre>										
U D R R R R D R D R R D			R	R	R	R	R	R	R	R	
R R R R R R R R R	U		R	R	R	R	D	R	D	R	
		_	R	R	R	R	R	R	R	R	
	R U(S)	D C	C	C	C	C	C	C	C	C	

Here, we can clearly see that Q-Learning learns the optimal policy itself in spite of the fact that actions will be selected in a epsilon greedy fashion. It learns to reach the goal state in the shortest path from $(4,1) \rightarrow (3,1) \rightarrow (3,2) \rightarrow \dots \rightarrow (3,12) \rightarrow (4,12)$.

Comparison of learning curves for SARSA and Q-Learning

In [19]:

```
ts = np.arange(1, 501)
plt.plot(ts, returns_episodes, label = "SARSA")
plt.plot(ts, returns_episodes_q, label = "Q-Learning")
plt.xlabel("Episode")
plt.ylabel("Returns")
plt.ylim(-100,0)
plt.legend()
plt.show()
```



As we can see from the learning curve, SARSA yields a better cumulative reward than Q-learning. This is due to the policies learnt by the two different algorithms. SARSA learns a more conservative policy, traversing a longer path ensuring that the heavy negative reward is not incurred while Q-Learning learns the optimal shortest path but due to the randomess of action selection, will sometime incur the heavier negative reward bringing down its average return.

In []: