

In [1]:

```
#importing necessary libraries  
import numpy as np  
import matplotlib.pyplot as plt  
import random  
from tqdm import tqdm
```

In [2]:

```

#building the environment
class Environment:
    def __init__(self, M,N, holes, terminal_state): #initializing the environment state
s, holes, terminals and rewards
        self.states = set()
        self.shape = (M,N)
        self.holes = holes
        self.terminal_state = terminal_state

        for i in range(1,M+1):
            for j in range(1, N+1):
                if (i,j) not in holes:
                    self.states.add((i,j))

        self.rewards = self.initialize_rewards()
        self.prob_agent_action = [0.8, 0.1, 0.05, 0.05]
    def initialize_rewards(self): #function to initialize the rewards for each state of
the environment
        r = {}
        for state in self.states:
            if state == (6,3):
                r[state] = -15
            elif state == (6,6):
                r[state] = 15
            else:
                r[state] = 0

        return r

    def agent_move(self, s, a): #function to update the state of the agent given an act
ion a and current state s
        x, y = s
        if a=='U':
            x = x-1
        elif a=='D':
            x = x + 1
        elif a=='R':
            y = y + 1
        elif a=='L':
            y = y - 1

        stay_same = self.check_corner_and_hole((x,y))

        if stay_same:
            return s

        return (x,y)

    def move_clockwise90(self, a): #function to return the action which is a 90 degree
rotation to current action a
        if a=='U':
            return 'R'
        elif a=='R':
            return 'D'
        elif a=='D':
            return 'L'
        elif a=='L':
            return 'U'

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def move_anti_clockwise90(self, a): #function to return the action which is a 90 de  
gree rotation to current action a  
    if a=='U':  
        return 'L'  
    elif a=='L':  
        return 'D'  
    elif a=='D':  
        return 'R'  
    elif a=='R':  
        return 'U'  
  
def check_corner_and_hole(self, s):  
    #function to check if the updates state goes out of the gridworld or goes into  
holes.  
    #If so, it returns a True value to address that the update should not take plac  
e and agent should remain in current state.  
    x1, y1 = s  
    stay_same = False  
    for hole in self.holes:  
        if (x1,y1) == hole:  
            stay_same = True  
  
    if x1<1 or x1>6:  
        stay_same = True  
    if y1<1 or y1>6:  
        stay_same = True  
  
    return stay_same
```

In [3]:

```
gridworld = Environment(6,6, [(4,3),(5,3)], (6,6))
```

In [8]:

```

#building the agent
class Agent:
    def __init__(self, gamma, env, terminal_state_value = 0):
        #initializing the agent parameters
        self.actions = ['L','R','U','D'] #possible actions
        self.gamma = gamma #discount parameter
        self.terminal_state_value = terminal_state_value
        self.pi = self.initialize_policy(env)
        self.V = self.initialize_value_states(env)
        self.Q = self.initialize_Qvalue(env)

    def initialize_policy(self,env):
        #initializing the agent policy
        pi = {}
        for s in env.states:
            pi[s] = {}
            for a in self.actions:
                pi[s][a] = 0.25

        return pi

    def initialize_Qvalue(self, env):
        #initializing q values for the agent
        Q = {}
        for s in env.states:
            Q[s] = {}
            for a in self.actions:
                Q[s][a] = 0

        return Q

    def initialize_value_states(self, env):
        #initializing value of states
        v_s = {}
        for state in env.states:
            if state == env.terminal_state:
                #0 for part 1.a, 15 for for part 1.b
                v_s[state] = self.terminal_state_value
            else:
                v_s[state] = random.random()
        return v_s

    def compute_value(self, env, epsilon = 1e-8):
        #function for policy evaluation to compute value of states and state-action pairs
        Q values
        delta = np.inf

        while delta > epsilon:
            max_diff = -np.inf
            for s in env.states:
                if s!=env.terminal_state:
                    v = 0
                    for a in self.pi[s]:
                        p = self.pi[s][a]
                        #with prob 0.8 take action a to get to state s1 with reward r1
                        s1 = env.agent_move(s,a)
                        r1 = env.rewards[s1]

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        v1 = 0.8 * (r1 + self.gamma*self.V[s1])

        #with prob 0.1 stay in same state s
        r2 = env.rewards[s]
        v2 = 0.1 * (r2 + self.gamma * self.V[s])

        #with prob 0.05 take action in direction +90 degree clockwise d
        a3 = env.move_clockwise90(a)
        s3 = env.agent_move(s, a3)
        r3 = env.rewards[s3]
        v3 = 0.05 * (r3 + self.gamma * self.V[s3])

        #with prob 0.05 take action in direction -90 degree clockwise d
        a4 = env.move_anti_clockwise90(a)
        s4 = env.agent_move(s, a4)
        r4 = env.rewards[s4]
        v4 = 0.05 * (r4 + self.gamma * self.V[s4])

        v = v + (p * (v1+v2+v3+v4))
        self.Q[s][a] = (v1 + v2 + v3 + v4)

        diff = abs(self.V[s] - v)
        self.V[s] = v
        max_diff = max(max_diff, diff)

    delta = max_diff

def policy_improve(self, env):
    #function for policy improvement where the policy is improved greedily

    for s in env.states:
        if s!=env.terminal_state:
            greedy_actions = []
            max_q = -np.inf

            for a in self.pi[s]:
                #if self.pi[s][a]>0:
                s1 = env.agent_move(s,a)
                r1 = env.rewards[s1]

                v1 = 0.8 * (r1 + self.gamma*self.V[s1])

                #with prob 0.1 stay in same state s
                r2 = env.rewards[s]
                v2 = 0.1 * (r2 + self.gamma * self.V[s])

                #with prob 0.05 take action in direction +90 degree clockwise direc
                a3 = env.move_clockwise90(a)
                s3 = env.agent_move(s, a3)
                r3 = env.rewards[s3]
                v3 = 0.05 * (r3 + self.gamma * self.V[s3])

                #with prob 0.05 take action in direction -90 degree clockwise direc
                a4 = env.move_anti_clockwise90(a)
                s4 = env.agent_move(s, a4)

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        r4 = env.rewards[s4]
        v4 = 0.05 * (r4 + self.gamma * self.V[s4])

        v = (v1 + v2 + v3 + v4)

        if v > max_q:
            greedy_actions = [a]
            max_q = v
        elif v == max_q:
            greedy_actions.append(a)

        #update policy greedily
        n = len(greedy_actions)
        for a in self.pi[s]:
            if a in greedy_actions:
                self.pi[s][a] = 1.0/n
            else:
                self.pi[s][a] = 0

def policy_iterate(self, env, iterations = 100):
    #function for policy iteration
    v_11_across_ep = []
    for iteration_number in range(iterations):
        self.compute_value(env)
        v_11_across_ep.append(self.V[(1,1)])
        self.policy_improve(env)

    return v_11_across_ep

def find_optimal_move(self, s):
    #function to find the optimal move at a given state s
    optimal_moves = []
    max_v = 0
    for a in self.pi[s]:
        if self.pi[s][a] > max_v:
            optimal_moves = [a]
            max_v = self.pi[s][a]
        elif self.pi[s][a] == max_v:
            optimal_moves.append(a)

    return optimal_moves

def plot_state_and_policy(self, env, plot_state_values = False, label = None):
    #function to plot state values and policy
    plt.figure(figsize = (20, 10))
    Grid_plot=plt.subplot()
    M,N = env.shape
    for i in range(M):
        for j in range(N):
            s = (i+1, j+1)
            if s==env.terminal_state:
                if plot_state_values:
                    t = round(self.V[s], 3)
                    value = str(s) + "\n\n" + "TERMINAL"
                else:
                    value = "TERMINAL"
            elif s not in env.holes:
                value=str(s)
                move = self.find_optimal_move(s)

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        if plot_state_values==False:
            value = value + '\n\n' + ','.join(move)
        else:
            t = round(self.V[s],3)
            value = value + '\n\n' + str(t)
    else:
        value = "HOLE"

    Grid_plot.text(j+0.5,N-i-0.5,value,ha='center',va='center')

Grid_plot.grid(color='k')
Grid_plot.axis('scaled')
Grid_plot.axis([0, M, 0, N])

Grid_plot.set_yticklabels([])
Grid_plot.set_xticklabels([])
if label is not None:
    plt.savefig(label)
```

## 1.a)

i) Value of states for random policy of taking each action with probability 0.25

In [9]:

```
agent = Agent(0.9, gridworld)
agent.compute_value(gridworld)
agent.plot_state_and_policy(gridworld, plot_state_values = True, label = 'plots/1_a.png'
)
```



|                  |                   |                   |                   |                  |                    |
|------------------|-------------------|-------------------|-------------------|------------------|--------------------|
| (1, 1)<br>-0.471 | (1, 2)<br>-0.406  | (1, 3)<br>-0.276  | (1, 4)<br>-0.15   | (1, 5)<br>-0.022 | (1, 6)<br>0.067    |
| (2, 1)<br>-0.768 | (2, 2)<br>-0.673  | (2, 3)<br>-0.409  | (2, 4)<br>-0.227  | (2, 5)<br>0.006  | (2, 6)<br>0.19     |
| (3, 1)<br>-1.54  | (3, 2)<br>-1.44   | (3, 3)<br>-0.662  | (3, 4)<br>-0.465  | (3, 5)<br>0.084  | (3, 6)<br>0.592    |
| (4, 1)<br>-3.174 | (4, 2)<br>-3.594  | HOLE              | (4, 4)<br>-1.284  | (4, 5)<br>0.245  | (4, 6)<br>1.793    |
| (5, 1)<br>-5.954 | (5, 2)<br>-7.943  | HOLE              | (5, 4)<br>-4.266  | (5, 5)<br>0.507  | (5, 6)<br>5.429    |
| (6, 1)<br>-9.687 | (6, 2)<br>-18.203 | (6, 3)<br>-29.301 | (6, 4)<br>-14.128 | (6, 5)<br>0.872  | (6, 6)<br>TERMINAL |

In [10]:

```
def plot_q_values(agent, env, label = None):
    plt.figure(figsize = (20, 10))
    Grid_plot=plt.subplot()
    M,N = env.shape
    for i in range(M):
        for j in range(N):
            s = (i+1, j+1)
            if s==env.terminal_state:
                value = str(s) + "\n\n" + "TERMINAL"
            elif s not in env.holes:
                value=str(s)
                qs = {}
                for a in agent.actions:
                    qs[a] = round(agent.Q[s][a],3)

                value = value + "\n\n" + "L : {} \nR : {} \nU : {} \nD : {}".format(qs
['L'],qs['R'],qs['U'],qs['D'])
            else:
                value = "HOLE"

            Grid_plot.text(j+0.5,N-i-0.5,value,ha='center',va='center')

    Grid_plot.grid(color='k')
    Grid_plot.axis('scaled')
    Grid_plot.axis([0, M, 0, N])

    Grid_plot.set_yticklabels([])
    Grid_plot.set_xticklabels([])
    if label is not None:
        plt.savefig(label)
```

ii) Q-Values of each state action pair for the random policy of choosing each action with probability 0.25

In [11]:

```
plot_q_values(agent, gridworld, label = "plots/1_a_q_values.png")
```

|  |  |   |  |   |   |
|--|--|---|--|---|---|
| (1, 1)<br>L: -0.437<br>R: -0.391<br>U: -0.421<br>D: -0.635 | (1, 2)<br>L: -0.424<br>R: -0.284<br>U: -0.363<br>D: -0.555   | (1, 3)<br>L: -0.348<br>R: -0.164<br>U: -0.249<br>D: -0.344    | (1, 4)<br>L: -0.23<br>R: -0.046<br>U: -0.135<br>D: -0.19     | (1, 5)<br>L: -0.111<br>R: 0.046<br>U: -0.022<br>D: -0.002 | (1, 6)<br>L: 0.002<br>R: 0.066<br>U: 0.057<br>D: 0.145  |
| (2, 1)<br>L: -0.713<br>R: -0.644<br>U: -0.473<br>D: -1.243 | (2, 2)<br>L: -0.697<br>R: -0.438<br>U: -0.406<br>D: -1.15    | (2, 3)<br>L: -0.563<br>R: -0.242<br>U: -0.276<br>D: -0.554    | (2, 4)<br>L: -0.343<br>R: -0.044<br>U: -0.147<br>D: -0.373   | (2, 5)<br>L: -0.16<br>R: 0.14<br>U: -0.017<br>D: 0.059    | (2, 6)<br>L: 0.051<br>R: 0.184<br>U: 0.075<br>D: 0.452  |
| (3, 1)<br>L: -1.425<br>R: -1.352<br>U: -0.826<br>D: -2.558 | (3, 2)<br>L: -1.431<br>R: -0.798<br>U: -0.713<br>D: -2.816   | (3, 3)<br>L: -1.144<br>R: -0.442<br>U: -0.44<br>D: -0.622     | (3, 4)<br>L: -0.587<br>R: -0.049<br>U: -0.231<br>D: -0.992   | (3, 5)<br>L: -0.316<br>R: 0.445<br>U: 0.017<br>D: 0.19    | (3, 6)<br>L: 0.203<br>R: 0.569<br>U: 0.221<br>D: 1.375  |
| (4, 1)<br>L: -2.908<br>R: -3.21<br>U: -1.699<br>D: -4.877  | (4, 2)<br>L: -3.031<br>R: -3.333<br>U: -1.664<br>D: -6.347   | HOLE  | (4, 4)<br>L: -1.253<br>R: -0.152<br>U: -0.497<br>D: -3.234   | (4, 5)<br>L: -0.876<br>R: 1.34<br>U: 0.105<br>D: 0.41     | (4, 6)<br>L: 0.609<br>R: 1.723<br>U: 0.679<br>D: 4.162  |
| (5, 1)<br>L: -5.402<br>R: -6.833<br>U: -3.446<br>D: -8.136 | (5, 2)<br>L: -5.983<br>R: -7.415<br>U: -3.928<br>D: -14.446  | HOLE  | (5, 4)<br>L: -4.149<br>R: -0.712<br>U: -1.478<br>D: -10.726  | (5, 5)<br>L: -2.976<br>R: 4.005<br>U: 0.274<br>D: 0.726   | (5, 6)<br>L: 1.684<br>R: 5.228<br>U: 2.047<br>D: 12.756 |
| (6, 1)<br>L: -8.55<br>R: -14.682<br>U: -6.414<br>D: -9.101 | (6, 2)<br>L: -9.789<br>R: -35.912<br>U: -9.862<br>D: -17.249 | (6, 3)<br>L: -21.38<br>R: -18.447<br>U: -38.689<br>D: -38.689 | (6, 4)<br>L: -35.196<br>R: -1.472<br>U: -6.373<br>D: -13.473 | (6, 5)<br>L: -10.032<br>R: 12.14<br>U: 0.558<br>D: 0.82   | (6, 6)<br>TERMINAL                                      |

## 1.b) Policy Iteration

In [12]:

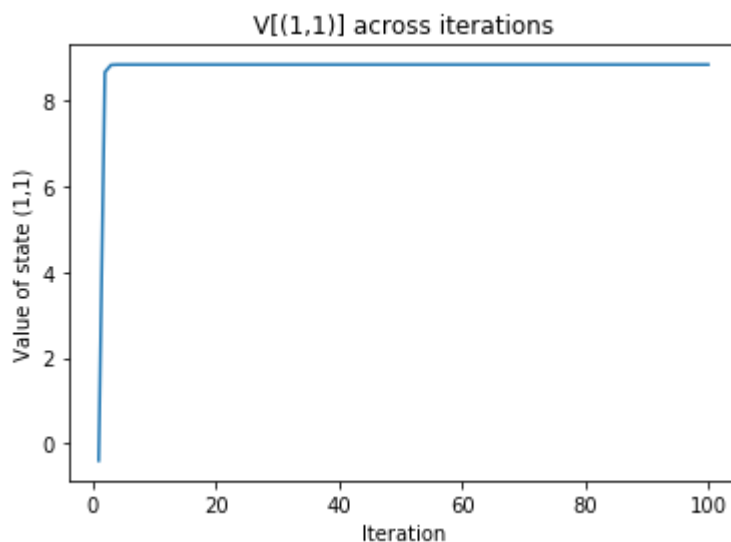
```
agent = Agent(0.9, gridworld, terminal_state_value = 15)
values = agent.policy_iterate(gridworld)
```

Plot of values of state (1,1) across iterations

In [13]:

```
def plot_values(values, label = None):
    ts = np.arange(1, (len(values)+1))
    plt.plot(ts, values)
    plt.xlabel("Iteration")
    plt.ylabel("Value of state (1,1)")
    plt.title("V[(1,1)] across iterations")
    if label is not None:
        plt.savefig(label)
    plt.show()

plot_values(values, label = 'plots/1_b_value_state_across_iterations.png')
```



Plot of optimal policy for each state

In [14]:

```
agent.plot_state_and_policy(gridworld, label = 'plots/1_b_opt_policy.png')
```

|             |             |             |             |               |             |
|-------------|-------------|-------------|-------------|---------------|-------------|
| (1, 1)<br>R | (1, 2)<br>R | (1, 3)<br>R | (1, 4)<br>D | (1, 5)<br>D   | (1, 6)<br>D |
| (2, 1)<br>R | (2, 2)<br>R | (2, 3)<br>R | (2, 4)<br>D | (2, 5)<br>D   | (2, 6)<br>D |
| (3, 1)<br>R | (3, 2)<br>R | (3, 3)<br>R | (3, 4)<br>D | (3, 5)<br>D   | (3, 6)<br>D |
| (4, 1)<br>U | (4, 2)<br>U | HOLE        | (4, 4)<br>D | (4, 5)<br>D   | (4, 6)<br>D |
| (5, 1)<br>U | (5, 2)<br>U | HOLE        | (5, 4)<br>R | (5, 5)<br>R,D | (5, 6)<br>D |
| (6, 1)<br>U | (6, 2)<br>U | (6, 3)<br>R | (6, 4)<br>R | (6, 5)<br>R   | TERMINAL    |

In [ ]: