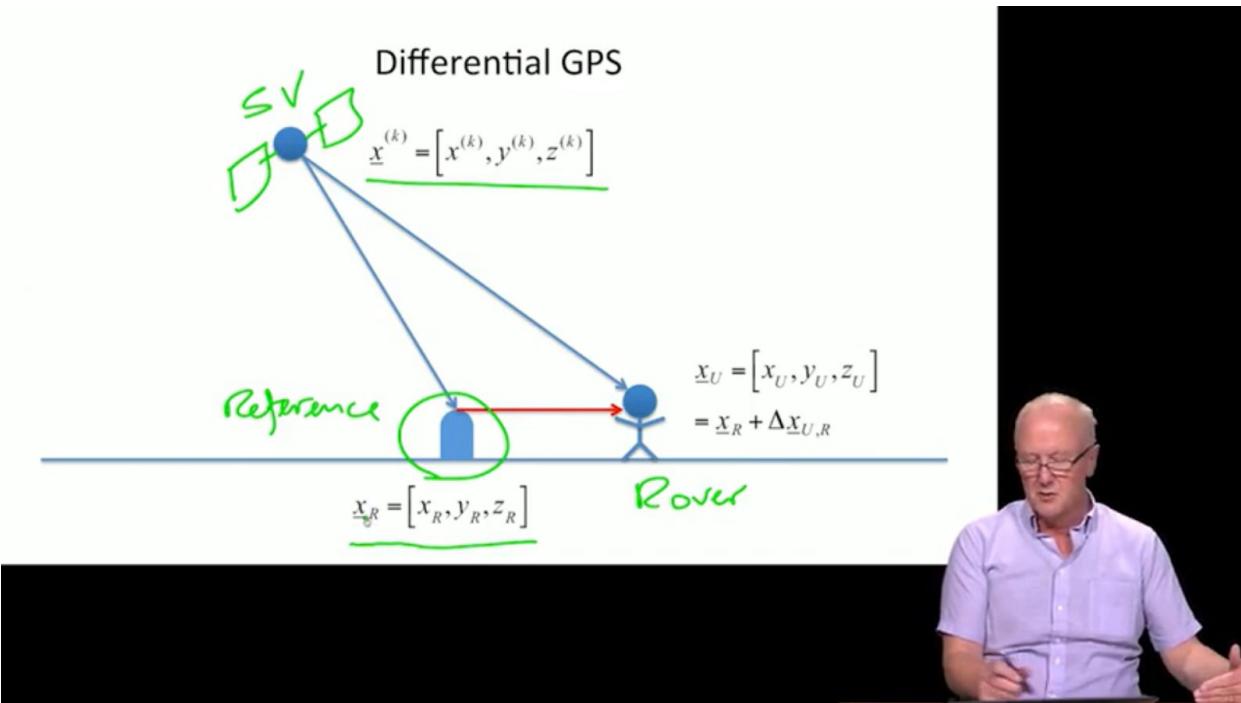


- 2.6 Differential GPS (DGPS)
- 2.7 DGPS error budget
- 2.8 Ionospheric effects on DGPS
- 2.9 Dual Frequency GPS & GNSS
- 2.10 Navigation in Our Lives: Airplanes



$$\frac{\tau_U = \left| \underline{x}^{(k)} - \underline{x}_U \right| + I + T + b_U - B^{(k)} + v}{\tau_0 = \left| \underline{x}^{(k,B)} - \underline{x}_{U,0} \right| + \hat{I} + \hat{T} + b_{U,0} - B^{(k,B)}}$$


---


$$\delta \tau = -\Delta \underline{x}_{U,R} \bullet \underline{l}_U^{(k)} + \delta \underline{x}^{(k)} \bullet \underline{l}_U^{(k)} + \delta b_U - \delta B^{(k)} + \delta I + \delta T + v$$

$$\frac{\tau_U = \left| \underline{x}^{(k)} - \underline{x}_U \right| + I_U + T_U + b_U - B^{(k)} + v_U}{\tau_R = \left| \underline{x}^{(k)} - \underline{x}_R \right| + I_R + T_R + b_R - B^{(k)} + v_R}$$


---


$$\tau_U - \tau_R = \left| \underline{x}^{(k)} - \underline{x}_U \right| - \left| \underline{x}^{(k)} - \underline{x}_R \right| + \Delta I + \Delta T + b_U - b_R + v_U - v_R$$



$$\frac{\tau_U = \left| \underline{x}^{(k)} - \underline{x}_U \right| + I + T + b_U - B^{(k)} + v}{\tau_0 = \left| \underline{x}^{(k,B)} - \underline{x}_{U,0} \right| + \hat{I} + \hat{T} + b_{U,0} - B^{(k,B)}}$$


---


$$\delta \tau = -\Delta \underline{x}_{U,R} \bullet \underline{l}_U^{(k)} + \delta \underline{x}^{(k)} \bullet \underline{l}_U^{(k)} + \delta b_U - \delta B^{(k)} + \delta I + \delta T + v$$

*Stand  
Alone*

$$\frac{\tau_U = \left| \underline{x}^{(k)} - \underline{x}_U \right| + I_U + T_U + b_U - B^{(k)} + v_U}{\tau_R = \left| \underline{x}^{(k)} - \underline{x}_R \right| + I_R + T_R + b_R - B^{(k)} + v_R}$$


---


$$\tau_U - \tau_R = \left| \underline{x}^{(k)} - \underline{x}_U \right| - \left| \underline{x}^{(k)} - \underline{x}_R \right| + \Delta I + \Delta T + b_U - b_R + v_U - v_R$$

*DGPS*



$$\begin{aligned} \tau_U^{(k)} &= \left| \underline{x}^{(k)} - \underline{x}_U \right| + I + T + b_U - B^{(k)} + v \quad PR \\ \tau_0 &= \left| \underline{x}^{(k,B)} - \underline{x}_{U,0} \right| + \hat{I} + \hat{T} + b_{U,0} - B^{(k,B)} \quad \text{Two range} \\ \delta\tau &= \boxed{\Delta \underline{x}_{U,R}} \bullet \underline{l}_U^{(k)} + \delta \underline{x}^{(k)} \bullet \underline{l}_U^{(k)} + \boxed{\delta b_U} - \delta B^{(k)} + \delta I + \delta T + v \end{aligned}$$


---


$$\begin{aligned} \tau_U &= \left| \underline{x}^{(k)} - \underline{x}_U \right| + I_U + T_U + b_U - B^{(k)} + v_U \quad DGPS \\ \tau_R &= \left| \underline{x}^{(k)} - \underline{x}_R \right| + I_R + T_R + b_R - B^{(k)} + v_R \end{aligned}$$


---


$$\tau_U - \tau_R = \left| \underline{x}^{(k)} - \underline{x}_U \right| - \left| \underline{x}^{(k)} - \underline{x}_R \right| + \Delta I + \Delta T + b_U - b_R + v_U - v_R$$



$$\begin{aligned} \tau_U^{(k)} &= \left| \underline{x}^{(k)} - \underline{x}_U \right| + I + T + b_U - B^{(k)} + v \quad PR \\ \tau_0 &= \left| \underline{x}^{(k,B)} - \underline{x}_{U,0} \right| + \hat{I} + \hat{T} + b_{U,0} - B^{(k,B)} \quad \text{Two range} \\ \delta\tau &= \boxed{\Delta \underline{x}_{U,R}} \bullet \underline{l}_U^{(k)} + \delta \underline{x}^{(k)} \bullet \underline{l}_U^{(k)} + \boxed{\delta b_U} - \delta B^{(k)} + \delta I + \delta T + v \end{aligned}$$


---


$$\begin{aligned} \tau_U^{(k)} &= \left| \underline{x}^{(k)} - \underline{x}_U \right| + I_U + T_U + b_U - B^{(k)} + v_U \quad PR_U \quad DGPS \\ \tau_R^{(k)} &= \left| \underline{x}^{(k)} - \underline{x}_R \right| + I_R + T_R + b_R - B^{(k)} + v_R \quad PR_R \end{aligned}$$


---


$$\tau_U - \tau_R = \left| \underline{x}^{(k)} - \underline{x}_U \right| - \left| \underline{x}^{(k)} - \underline{x}_R \right| + \Delta I + \Delta T + b_U - b_R + v_U - v_R$$



$$\tau_U^{(k)} = \left| \underline{x}^{(k)} - \underline{x}_U \right| + I + T + b_U - B^{(k)} + v \quad PR$$

$$\tau_0 = \left| \underline{x}^{(k,B)} - \underline{x}_{U,0} \right| + \hat{I} + \hat{T} + b_{U,0} - B^{(k,B)} \quad \text{Two range}$$


---


$$\delta\tau = \Delta \underline{x}_{U,R} \bullet \underline{l}_U^{(k)} + \delta \underline{x}^{(k)} \bullet \underline{l}_U^{(k)} + \delta b_U - \delta B^{(k)} + \delta I + \delta T + v$$

Stand Alone

$$\tau_U^{(k)} = \left| \underline{x}^{(k)} - \underline{x}_U \right| + I_U + T_U + b_U - B^{(k)} + v_U \quad PR_u$$

$$\tau_R^{(k)} = \left| \underline{x}^{(k)} - \underline{x}_R \right| + I_R + T_R + b_R - B^{(k)} + v_R \quad PR_R$$


---


$$\tau_U^{(k)} - \tau_R^{(k)} = \left| \underline{x}^{(k)} - \underline{x}_U \right| - \left| \underline{x}^{(k)} - \underline{x}_R \right| + \Delta I + \Delta T + b_U - b_R + v_U - v_R$$



common to both measurement the one at the user and the one

$$\left| \underline{x}^{(k)} - \underline{x}_U \right| - \left| \underline{x}^{(k)} - \underline{x}_R \right| = -\Delta \underline{x}_{U,R} \bullet \underline{l}_U^{(k)}$$

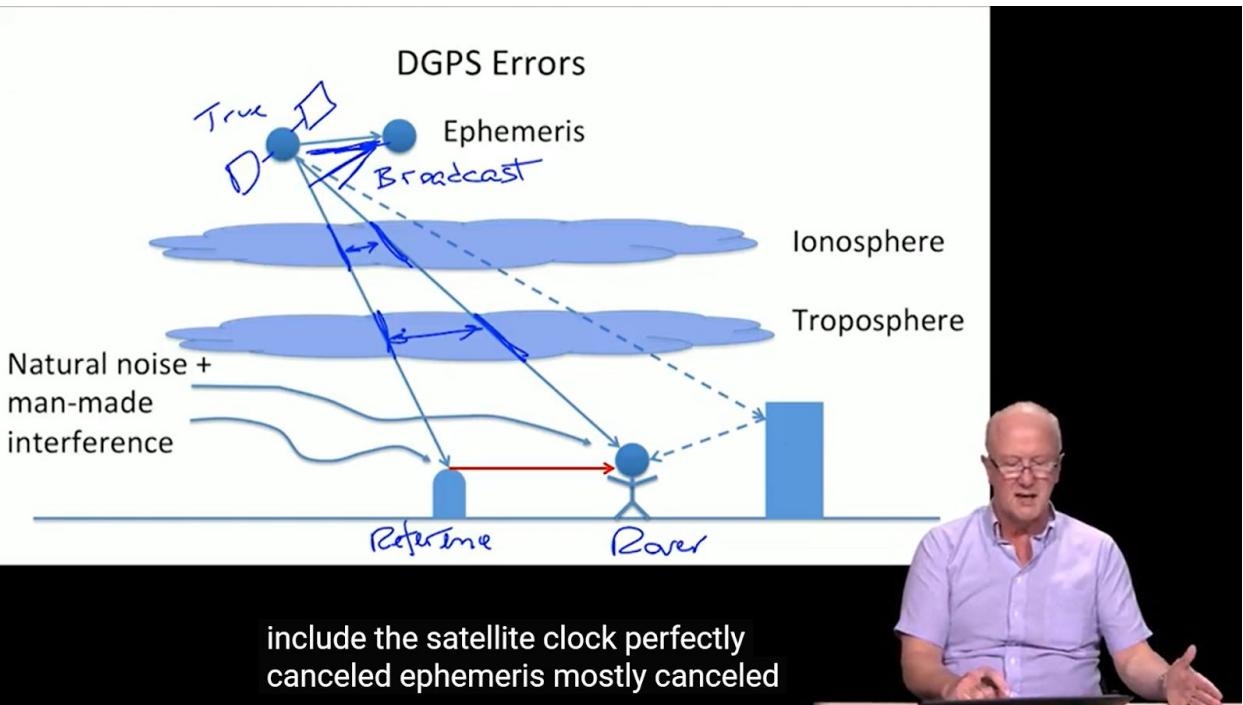
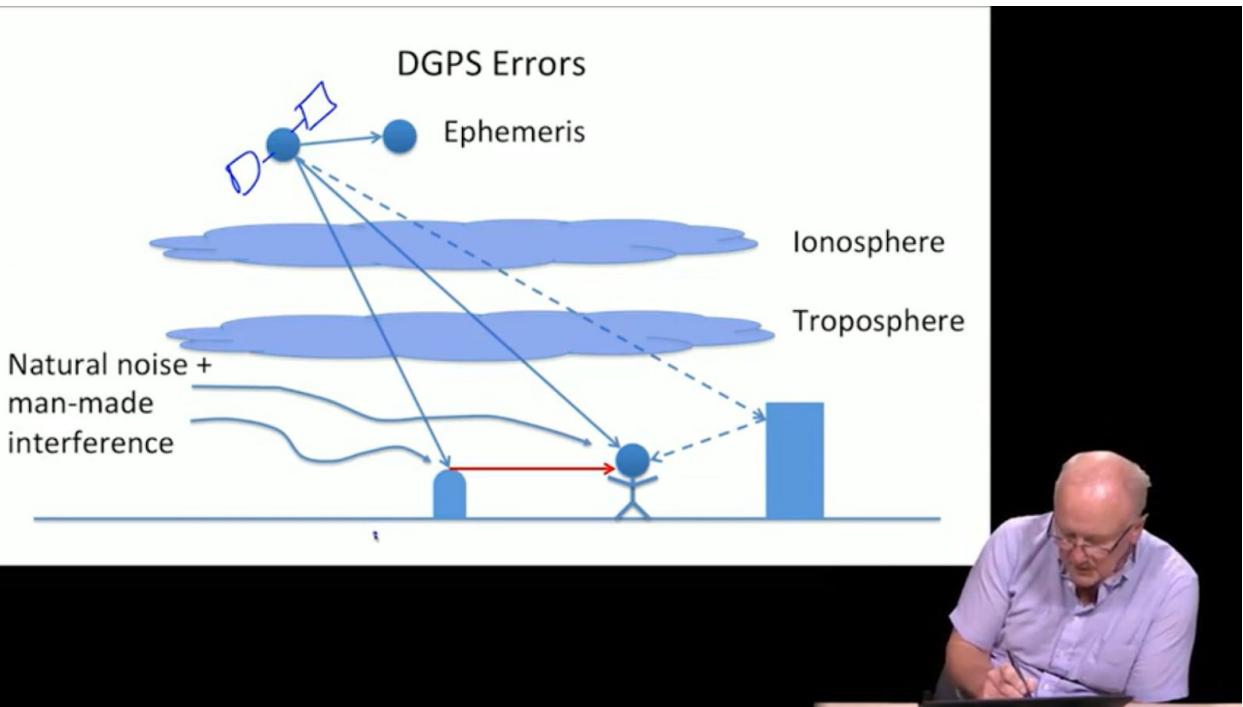
$$\tau_U - \tau_R = \left| \underline{x}^{(k)} - \underline{x}_U \right| - \left| \underline{x}^{(k)} - \underline{x}_R \right| + \Delta I + \Delta T + b_U - b_R + v_U - v_R$$

$$\tau_U - \tau_R = -\Delta \underline{x}_{U,R} \bullet \underline{l}_U^{(k)} + \Delta I + \Delta T + \Delta b_{U,R} + \Delta v_{U,R}$$

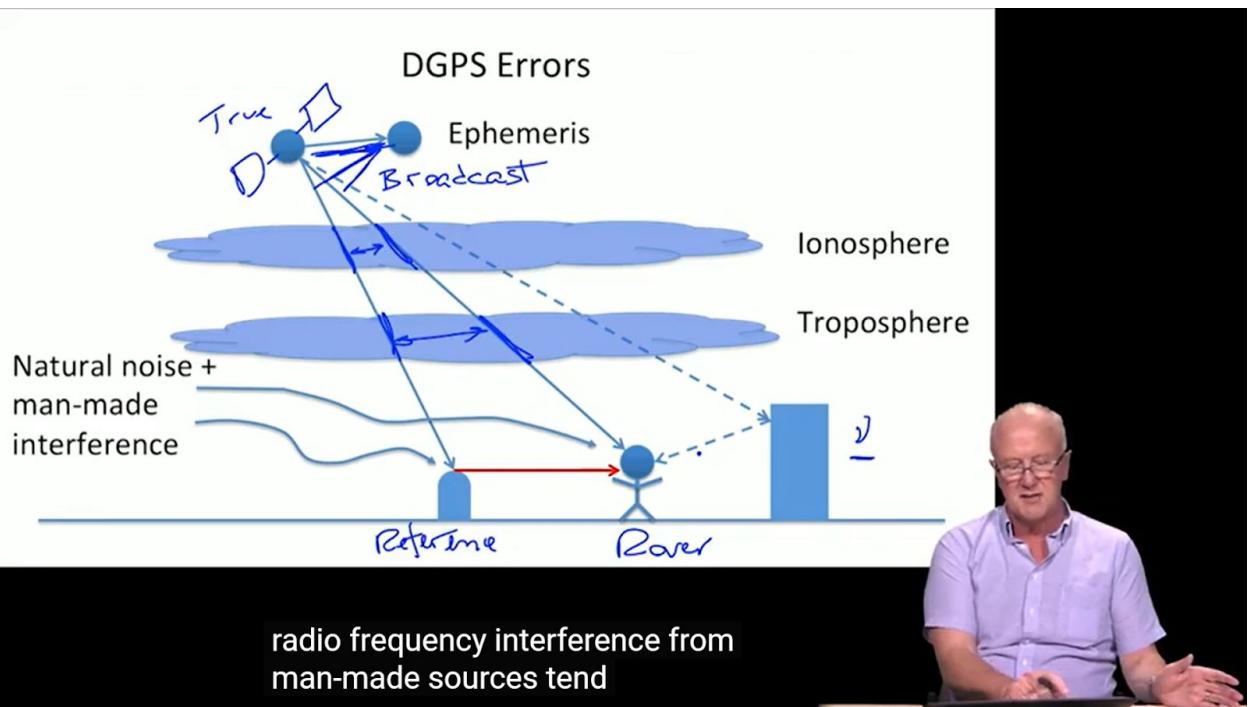
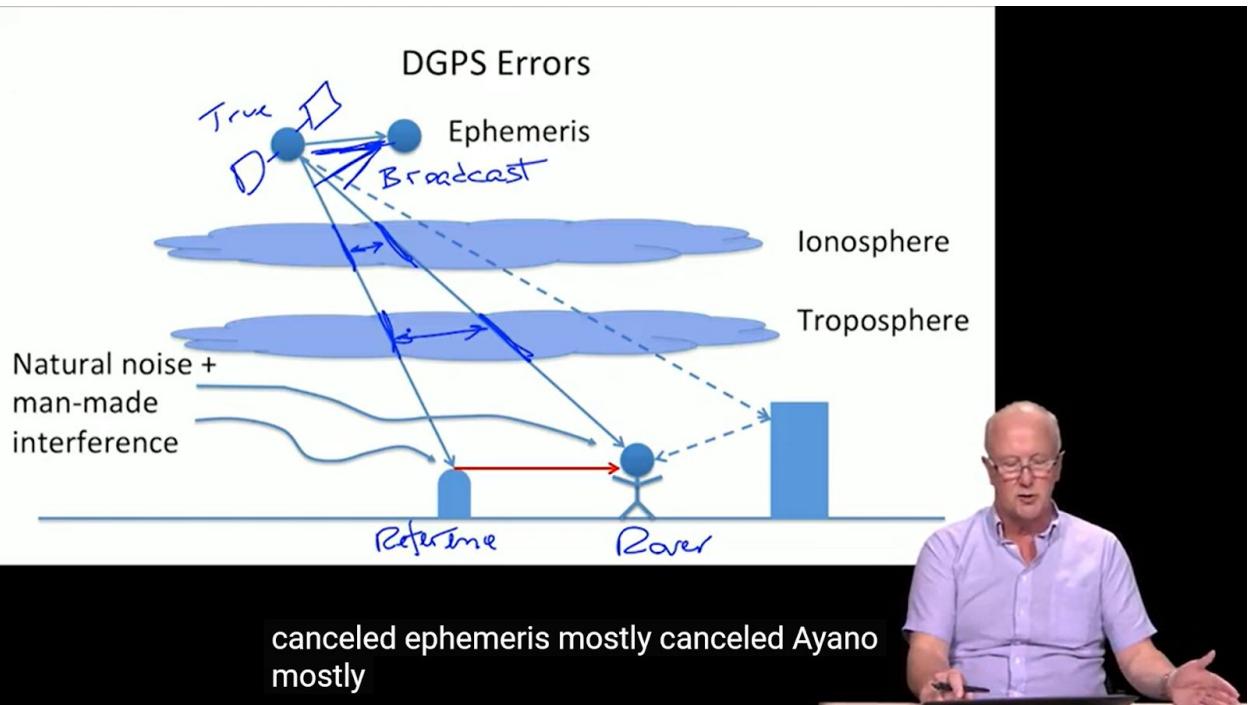
$$\begin{bmatrix} \tau_U^{(1)} - \tau_R^{(1)} \\ \vdots \\ \tau_U^{(K)} - \tau_R^{(K)} \end{bmatrix} = G \begin{bmatrix} \Delta x_{U,R} \\ \Delta y_{U,R} \\ \Delta z_{U,R} \\ \Delta b_{U,R} \end{bmatrix} + \Delta I + \Delta T + \Delta v_{U,R}$$



for the difference in the



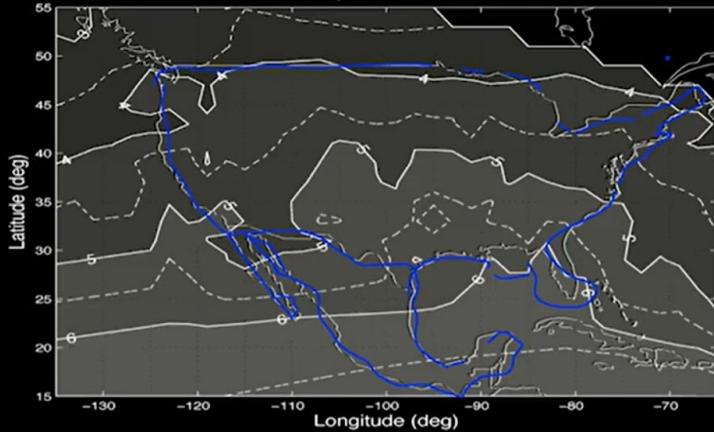
include the satellite clock perfectly  
canceled ephemeris mostly canceled



## Ionosphere on a “Calm Day”

November 19, 2003 19:00:00 UTC

ΔI



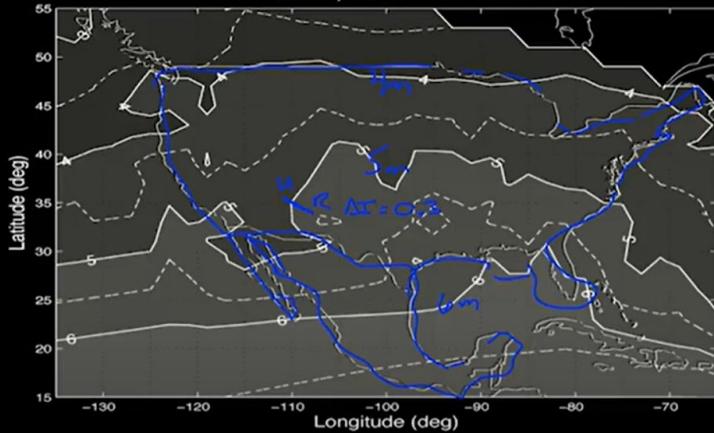
Zenith and in other words directly  
overhead as that

2.7 - DGPS Error Budget

## Ionosphere on a “Calm Day”

November 19, 2003 19:00:00 UTC

ΔI



it's ok  
it means that differential

### A Third Error Budget (DGPS)

Error Source	Random	Bias	Total
Ephemeris	0.0	0.1	0.1
Clock	0.2	0.0	0.2
Ionosphere (depending on space weather)	0.0	0.3	0.3
Troposphere	0.0	0.3	0.3
Multipath (better receivers)	0.3	0.3	0.4
Receiver Noise (better receivers)	0.2	0.0	0.2
Raw pseudorange	0.4	0.5	0.6
Smoothed pseudorange	0.1	0.5	0.5
Horizontal (HDOP=2)			1.0
Vertical (VDOP=2.5)			1.25
Ionosphere & multipath are still wild cards!			

what happens on a storm day but for now  
let's stick to the calm day in this case



### A Third Error Budget (DGPS) *calm day*

Error Source	Random	Bias	Total
Ephemeris	0.0	0.1	0.1
Clock	0.2	0.0	0.2
Ionosphere (depending on space weather)	0.0	0.3	0.3
Troposphere	0.0	0.3	0.3
Multipath (better receivers)	0.3	0.3	0.4
Receiver Noise (better receivers)	0.2	0.0	0.2
Raw pseudorange	0.4	0.5	0.6
Smoothed pseudorange	0.1	0.5	0.5
Horizontal (HDOP=2)			1.0
Vertical (VDOP=2.5)			1.25
Ionosphere & multipath are still wild cards!			

like this remember we put together a few  
of these we had a first error budget



### A Third Error Budget (DGPS) *Calm day*

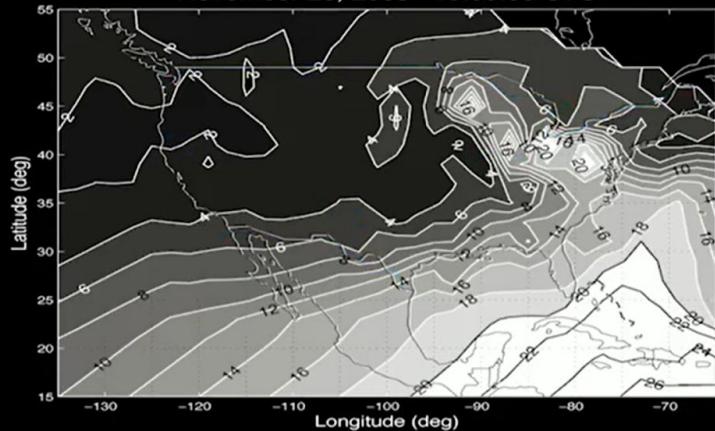
Error Source	Random	Bias	Total
Ephemeris	0.0	0.1	0.1
Clock	0.2	0.0	0.2
Ionosphere (depending on space weather)	0.0	0.3	0.3 $= \sqrt{R^2 + 0^2}$
Troposphere	0.0	0.3	0.3
Multipath (better receivers)	0.3	0.3	0.4
Receiver Noise (better receivers)	0.2	0.0	0.2
Raw pseudorange	0.4	0.5	0.6
Smoothed pseudorange	0.1	0.5	0.5
Horizontal (HDOP=2)			1.0
Vertical (VDOP=2.5)			1.25
Ionosphere & multipath are still wild cards!			

simply doesn't an error budget like this  
he's a wild



### Ionosphere During on a Storm Day

November 20, 2003 19:00:00 UTC



### A Third Error Budget (DGPS)

Error Source	Random	Bias	Total
Ephemeris	0.0	0.1	0.1
Clock	0.2	0.0	0.2
Ionosphere (depending on space weather)	0.0	0.3	0.3
Troposphere	0.0	0.3	0.3
Multipath (better receivers)	0.3	0.3	0.4
Receiver Noise (better receivers)	0.2	0.0	0.2
Raw pseudorange	0.4	0.5	0.6
Smoothed pseudorange	0.1	0.5	0.5
Horizontal (HDOP=2)			1.0
Vertical (VDOP=2.5)			1.25
Ionosphere & multipath are still wild cards!			

what happens on a storm day but for now  
let's stick to the calm day in this case



### A Third Error Budget (DGPS) *calm day*

Error Source	Random	Bias	Total
Ephemeris	0.0	0.1	0.1
Clock	0.2	0.0	0.2
Ionosphere (depending on space weather)	0.0	0.3	0.3
Troposphere	0.0	0.3	0.3
Multipath (better receivers)	0.3	0.3	0.4
Receiver Noise (better receivers)	0.2	0.0	0.2
Raw pseudorange	0.4	0.5	0.6
Smoothed pseudorange	0.1	0.5	0.5
Horizontal (HDOP=2)			1.0
Vertical (VDOP=2.5)			1.25
Ionosphere & multipath are still wild cards!			

like this remember we put together a few  
of these we had a first error budget



### A Third Error Budget (DGPS) *Calm day*

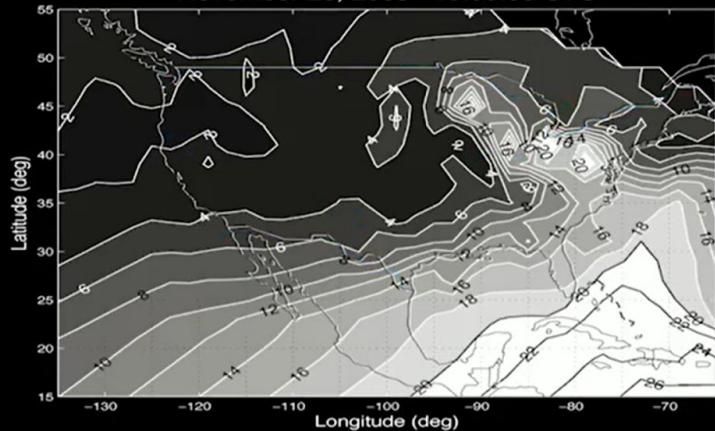
Error Source	Random	Bias	Total
Ephemeris	0.0	0.1	0.1
Clock	0.2	0.0	0.2
Ionosphere (depending on space weather)	0.0	0.3	0.3 $= \sqrt{R^2 + 0^2}$
Troposphere	0.0	0.3	0.3
Multipath (better receivers)	0.3	0.3	0.4
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Smoothed pseudorange	0.1	0.5	0.5
Horizontal (HDOP=2)			1.0
Vertical (VDOP=2.5)			1.25
Ionosphere & multipath are still wild cards!			

simply doesn't an error budget like this  
he's a wild



### Ionosphere During on a Storm Day

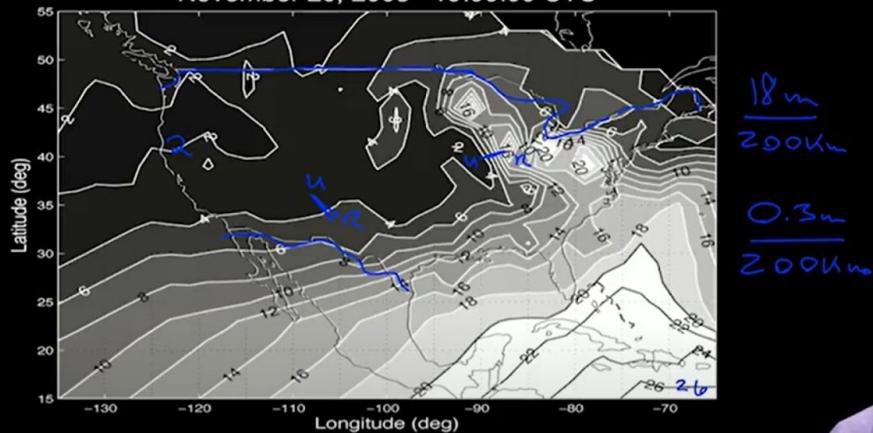
November 20, 2003 19:00:00 UTC



2.8 - Ionospheric effects on DGPS

## Ionosphere During on a Storm Day

November 20, 2003 19:00:00 UTC



◀ ▶ 3:55 / 8:14

CC

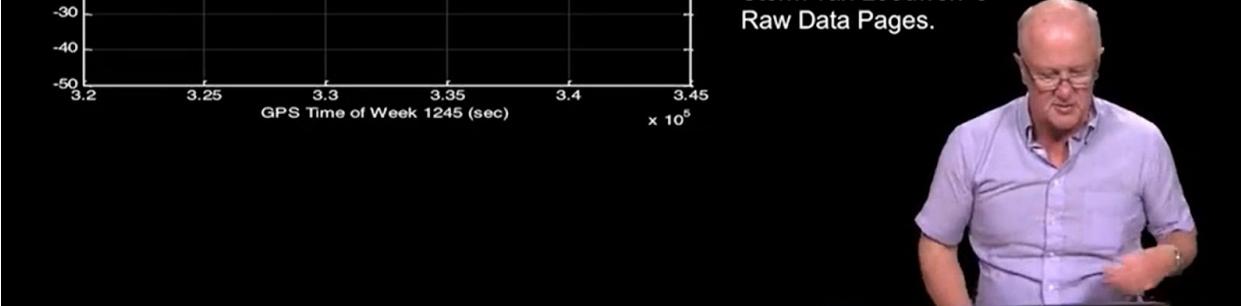
## Stand Alone Position Errors for a Non-Storm Day

Nov. 19, 2003 17:30 – 23:30



Ionospheric delay  
is corrected by  
Klobuchar model.

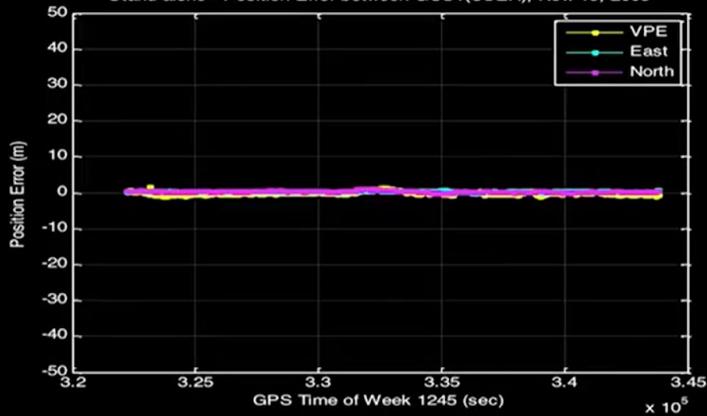
Tropospheric delay  
is corrected by the  
strategy from Sam  
Storm van Leeuwen's  
Raw Data Pages.



## DGPS Position Errors for the Non-Storm Day Nov.

19 2003 17:30 – 23:30

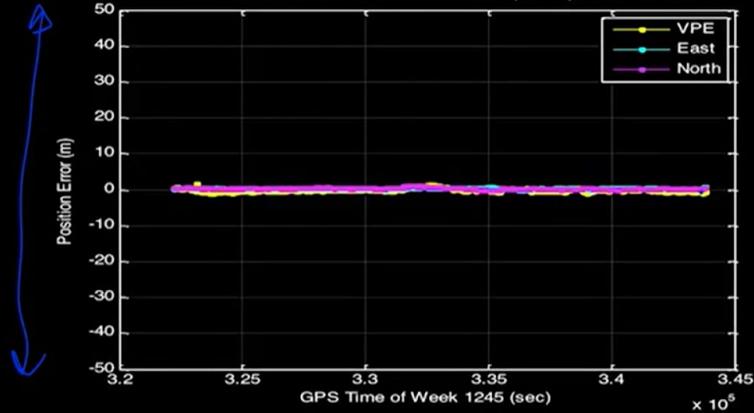
Stand alone - Position Error between GUST(USER), Nov. 19, 2003

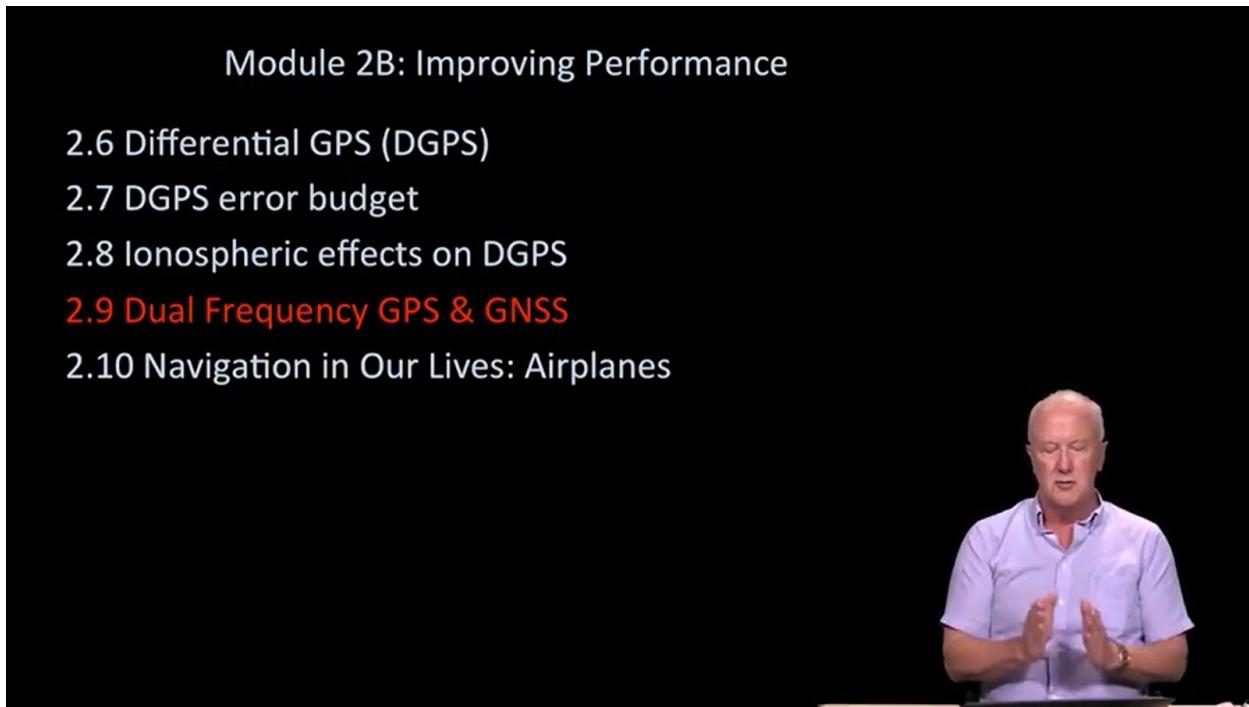
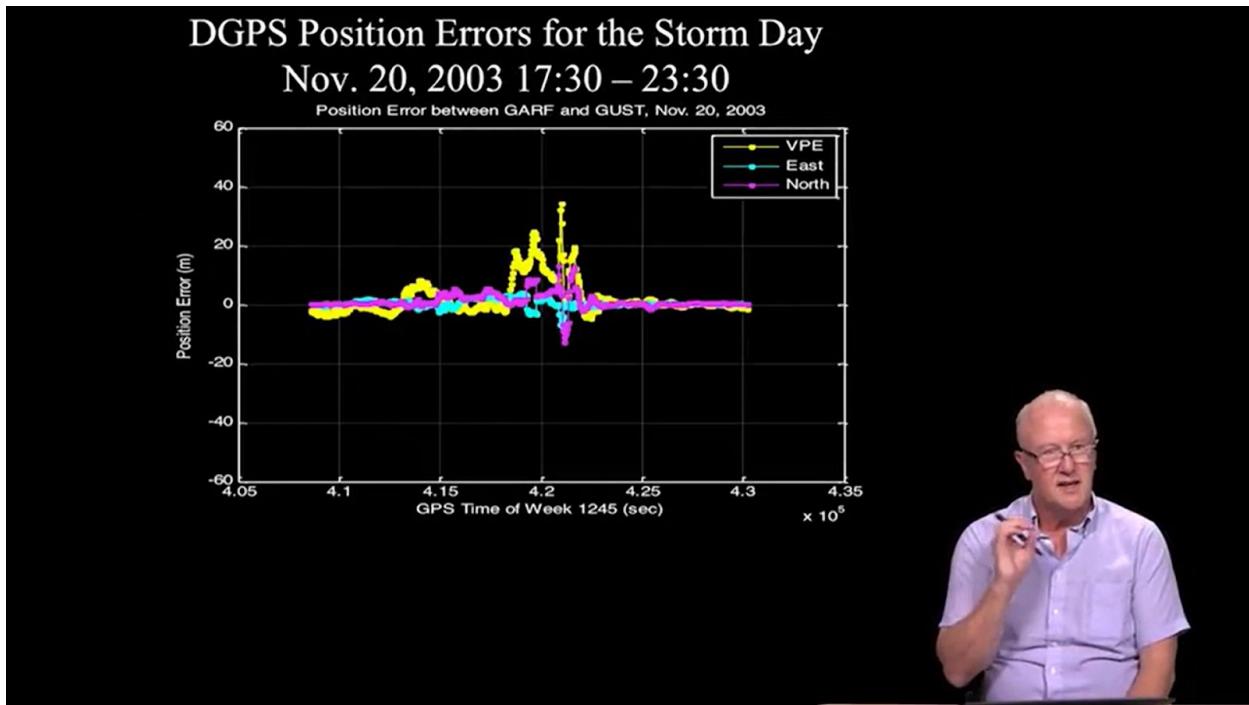


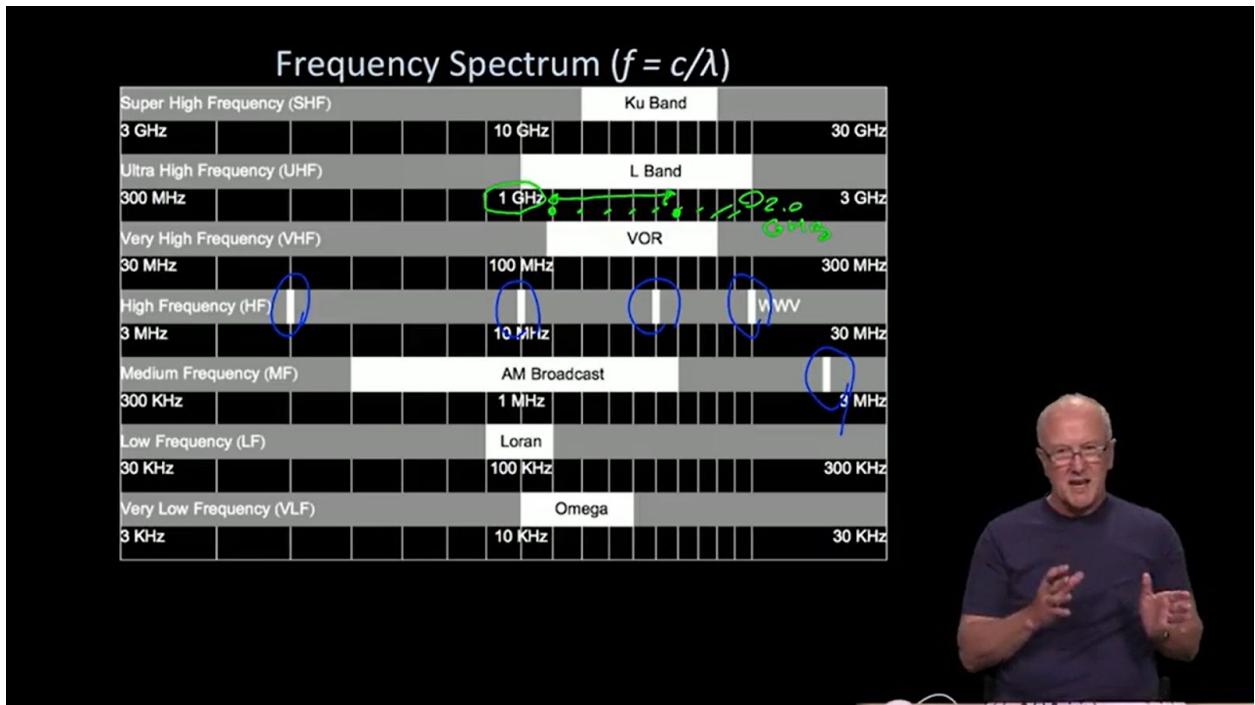
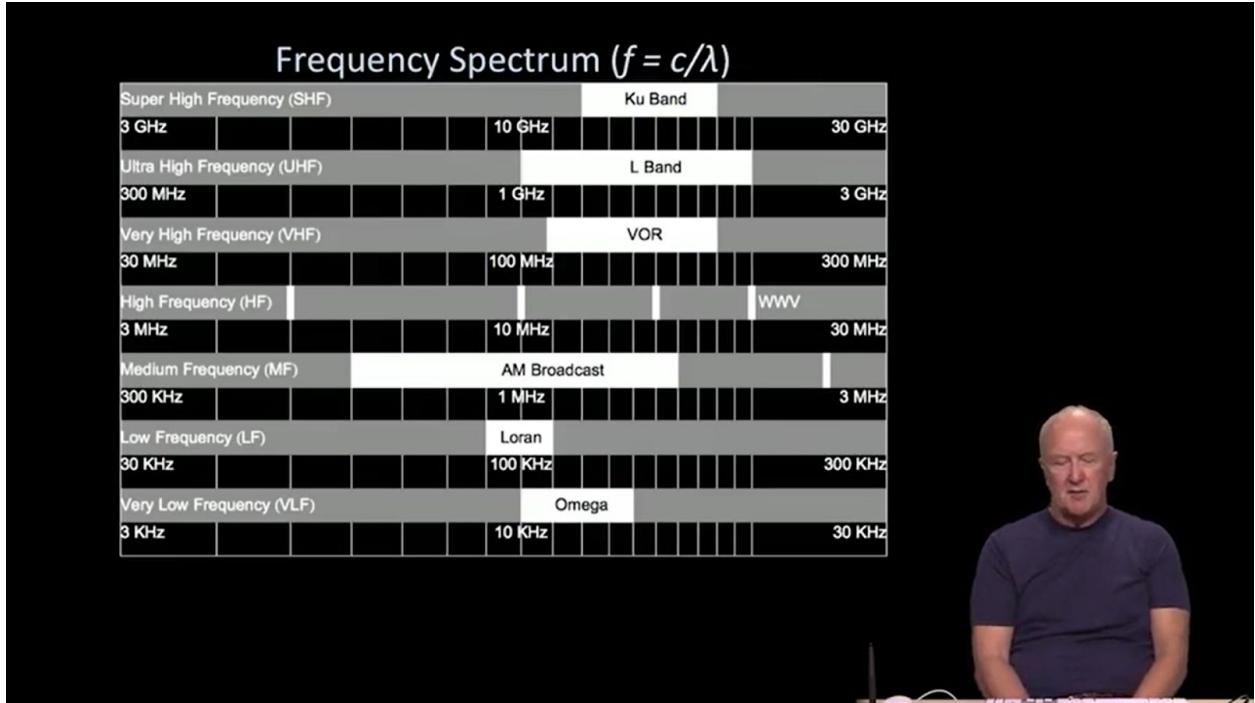
## DGPS Position Errors for the Non-Storm Day Nov.

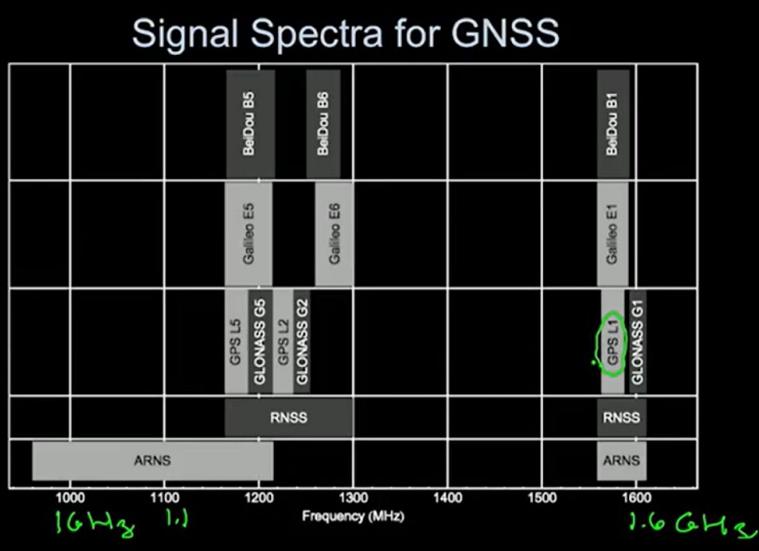
19 2003 17:30 – 23:30

Stand alone - Position Error between GUST(USER), Nov. 19, 2003



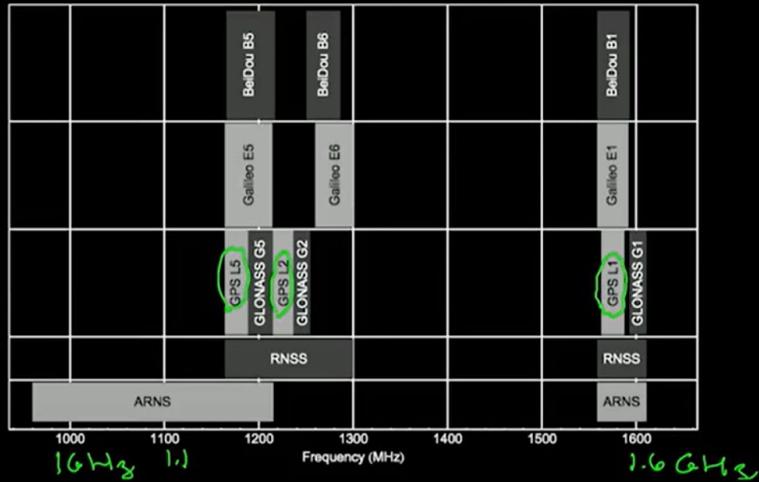


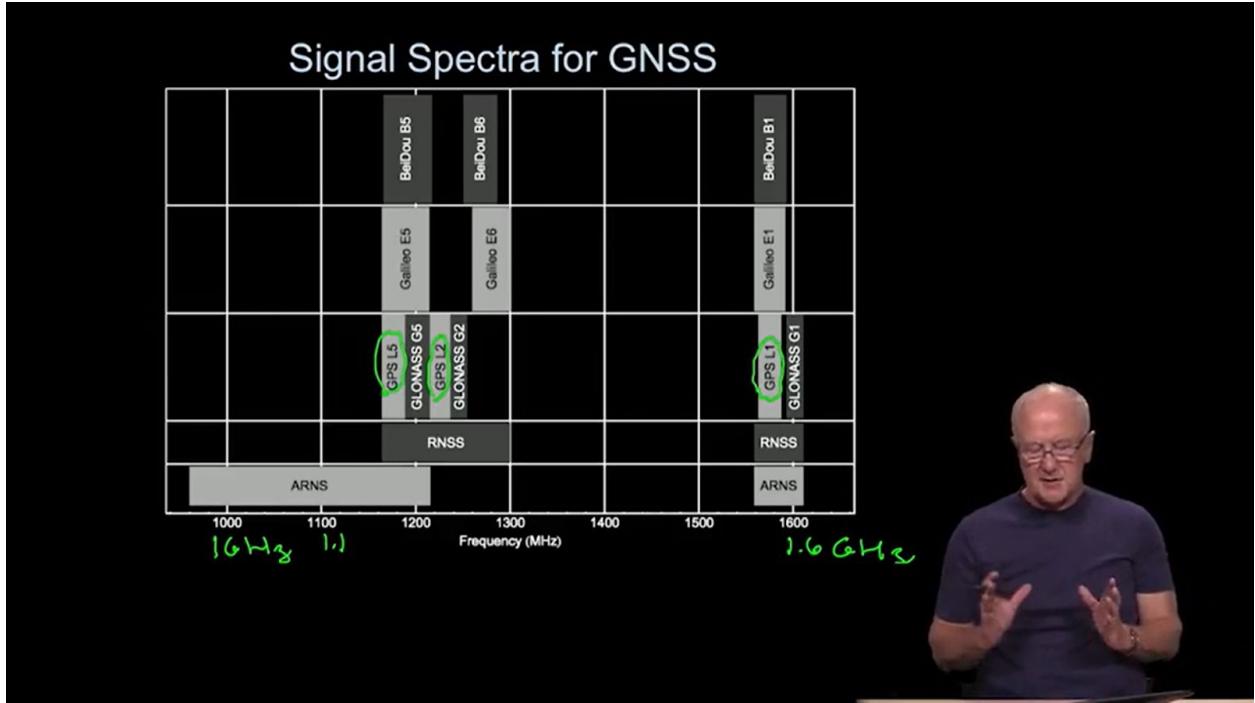




2.9 - Dual Frequency GPS & GNSS

# Signal Spectra for GNSS





## Dual Frequency for Ionospheric Measurement

Ionospheric errors are around 10 - 20 meters

Single frequency users use a model to cut the error in half.

Dual frequency users can measure the ionosphere in real time.

$$\tau_{L1} = \rho + \frac{40.3TEC}{f_{L1}^2} \text{ meters and } \tau_{L2} = \rho + \frac{40.3TEC}{f_{L2}^2} \text{ meters}$$

$$\tau_{\text{iono-free}} = \alpha\tau_{L1} + \beta\tau_{L2}$$

$$\text{where } \alpha + \beta = 1 \text{ and } \alpha \frac{40.3TEC}{f_{L1}^2} + \beta \frac{40.3TEC}{f_{L2}^2} = 0$$

$$\alpha = 2.54573 \text{ and } \beta = -1.54573$$



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$$\tau_{\text{iono-free}} = \alpha\tau_{L1} + \beta\tau_{L2}$$

where  $\alpha + \beta = 1$  and  $\alpha \frac{40.3TEC}{f_{L1}^2} + \beta \frac{40.3TEC}{f_{L2}^2} = 0$

$$\alpha = 2.54573 \text{ and } \beta = -1.54573$$

$1575.42 \times 10^6$

24



## Dual Frequency for Ionospheric Measurement

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$$\tau_{\text{iono-free}} = \alpha\tau_{L1} + \beta\tau_{L2}$$

where  $\alpha + \beta = 1$  and  $\alpha \frac{40.3TEC}{f_{L1}^2} + \beta \frac{40.3TEC}{f_{L2}^2} = 0$

$$\alpha = 2.54573 \text{ and } \beta = -1.54573$$

$1575.42 \times 10^6$

24



## Dual Frequency for Ionospheric Measurement

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$$\tau_{L1} = \rho + \frac{40.3TEC}{f_{L1}^2} \text{ meters and } \tau_{L2} = \rho + \frac{40.3TEC}{f_{L2}^2} \text{ meters}$$
$$\tau_{\text{iono-free}} = \alpha\tau_{L1} + \beta\tau_{L2}$$

where  $\alpha + \beta = 1$  and  $\alpha \frac{40.3TEC}{f_{L1}^2} + \beta \frac{40.3TEC}{f_{L2}^2} = 0$

$$\alpha = 2.54573 \text{ and } \beta = -1.54573$$

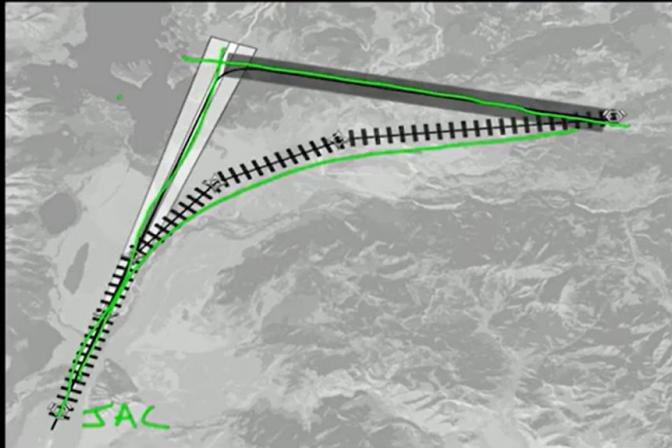
$1227.40 \times 10^6$

$1575.42 \times 10^6$

24



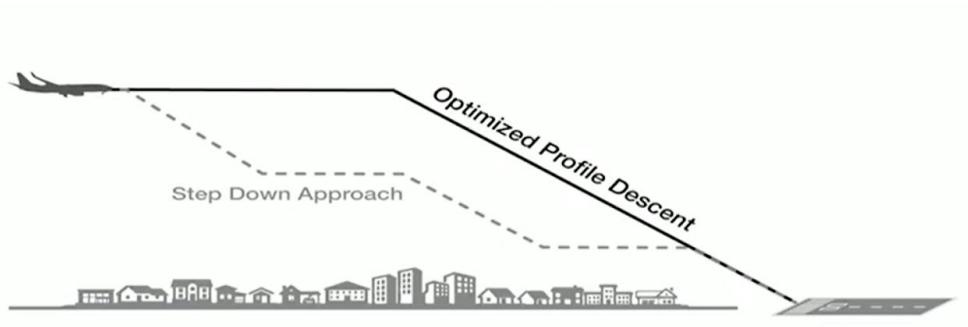
## Approach to Jackson Hole, Wyoming



### Gastineau Approach to Juneau Alaska



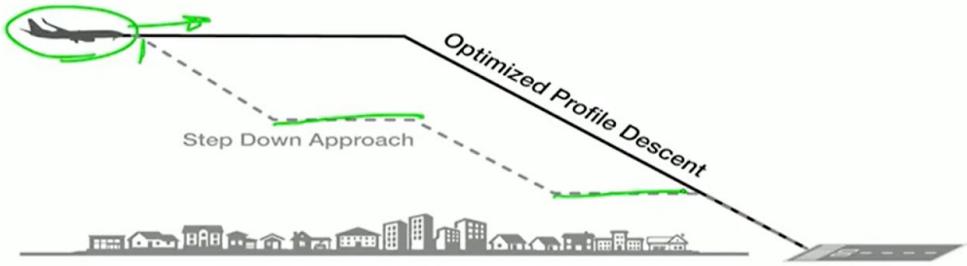
### Optimized Profile Descent



Louisville OPD Trials: 200 kg/flight fuel savings  
and 7% smaller 50 dB contour if fully adopted



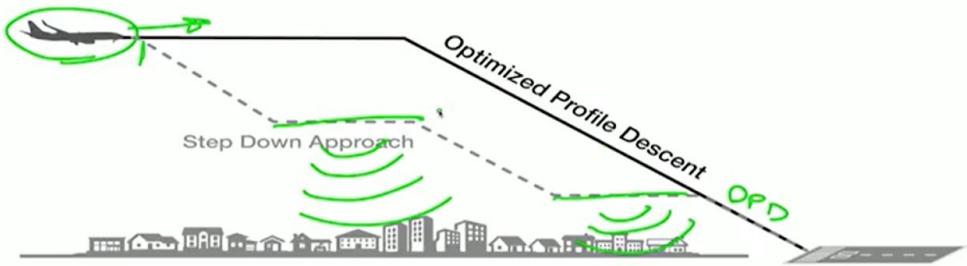
## Optimized Profile Descent



Louisville OPD Trials: 200 kg/flight fuel savings  
and 7% smaller 50 dB contour if fully adopted



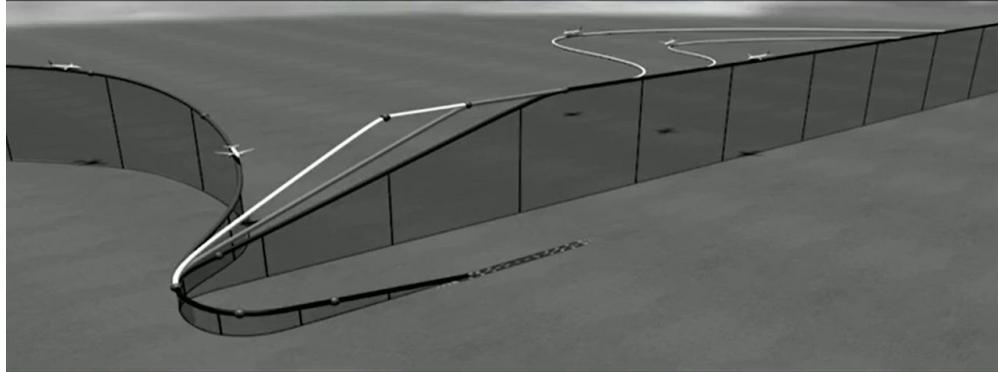
## Optimized Profile Descent



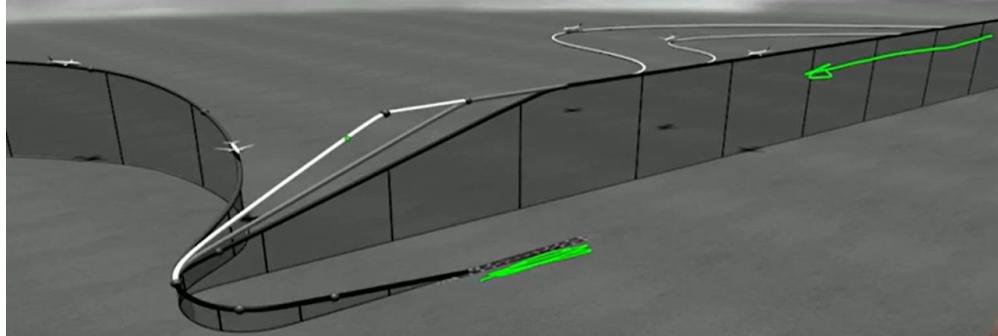
Louisville OPD Trials: 200 kg/flight fuel savings  
and 7% smaller 50 dB contour if fully adopted



Tailored Arrivals Saved 730 kg Per Flight in  
UAL Honolulu to SFO Trials



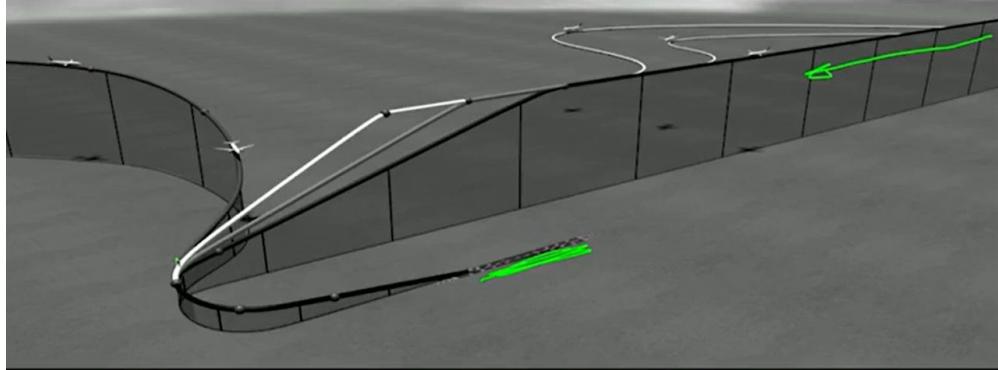
2.10 - Navigation in Our Lives: Landing Airplanes Using GPS  
Tailored Arrivals Saved 730 kg Per Flight in  
UAL Honolulu to SFO Trials



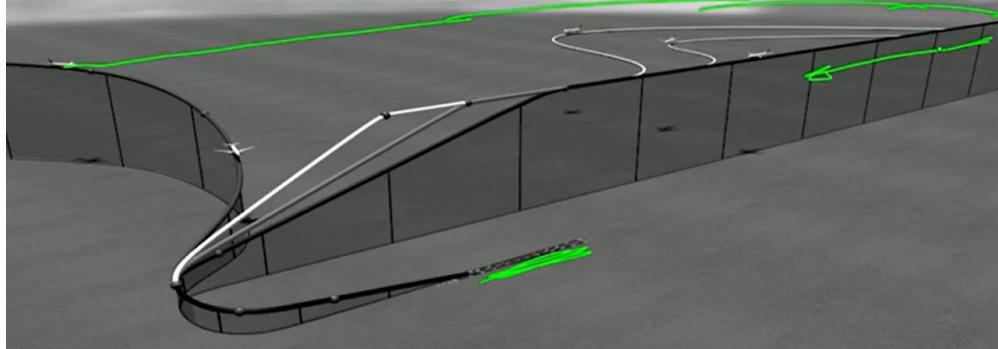
◀ ▶ ⏪ ⏩ 12:20 / 23:27

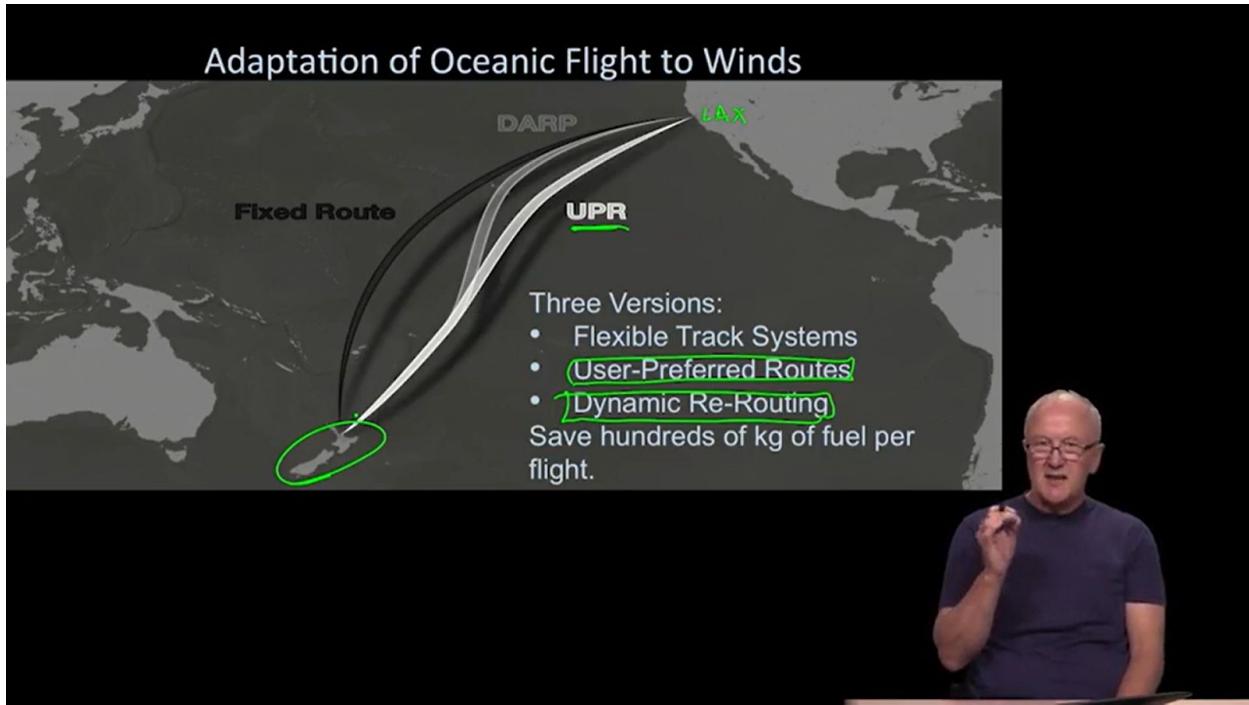
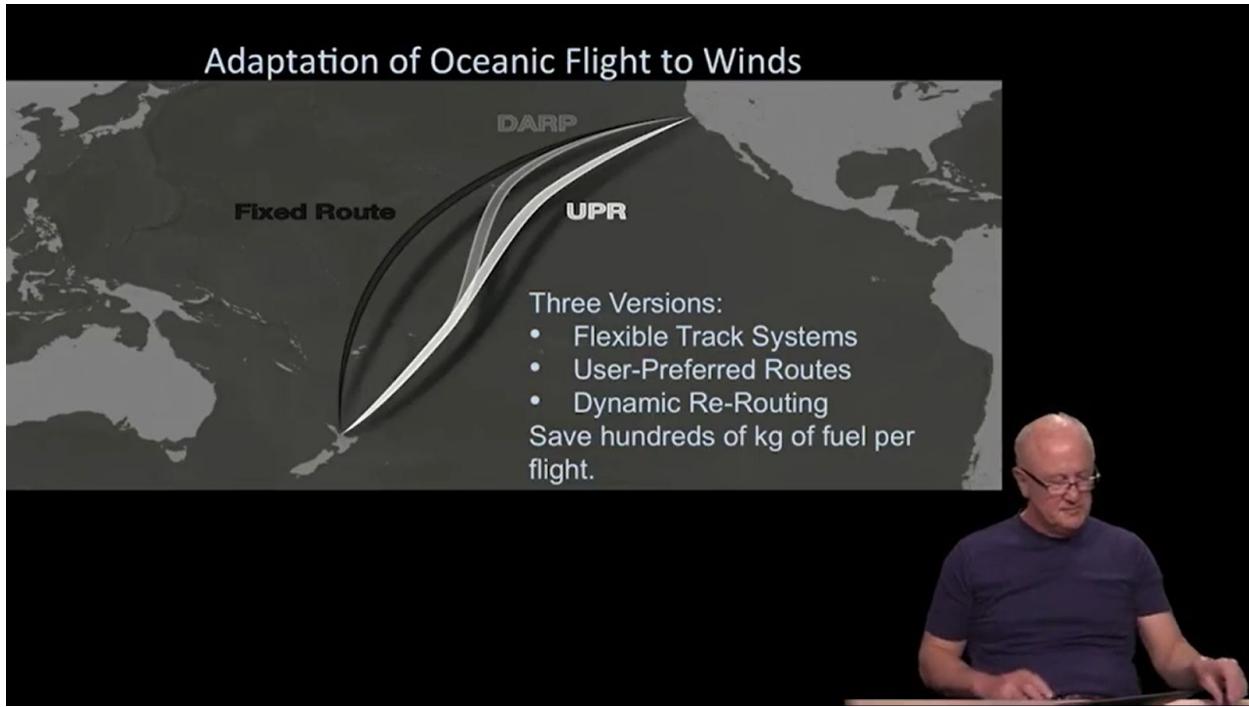


Tailored Arrivals Saved 730 kg Per Flight in  
UAL Honolulu to SFO Trials



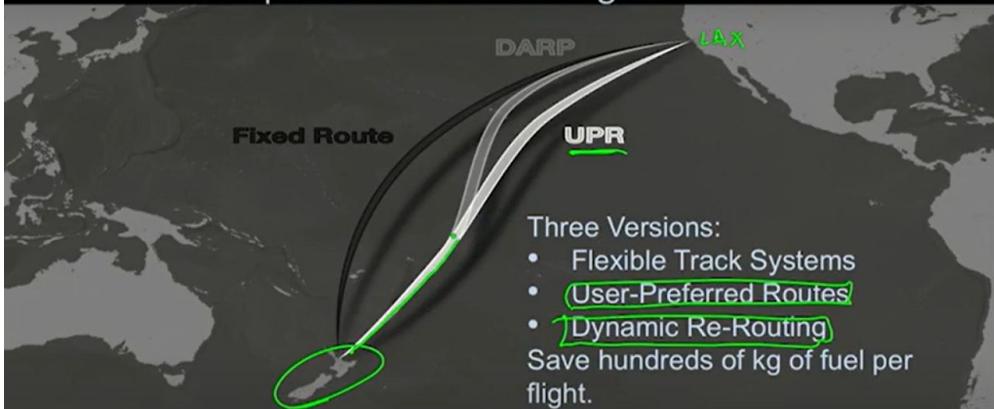
Tailored Arrivals Saved 730 kg Per Flight in  
UAL Honolulu to SFO Trials





2.10 - Navigation in Our Lives: Landing Airplanes Using GPS

### Adaptation of Oceanic Flight to Winds

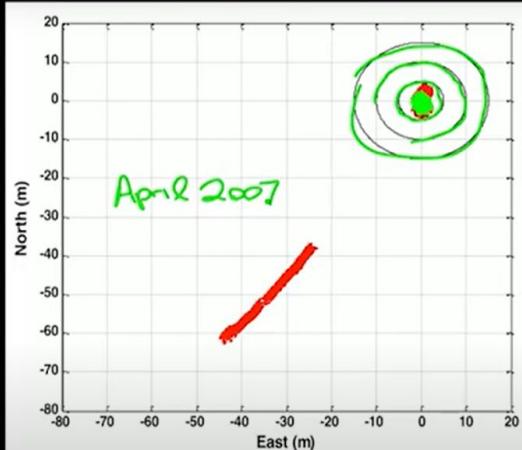


◀ ▶ 15:43 / 23:27

CC

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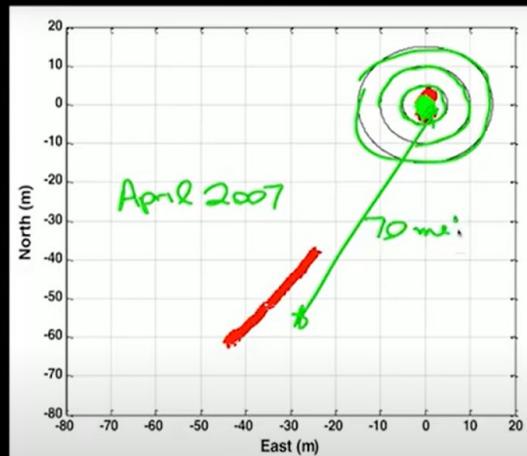
### A GPS Blunder



◀ ▶ 18:08 / 23:27

CC

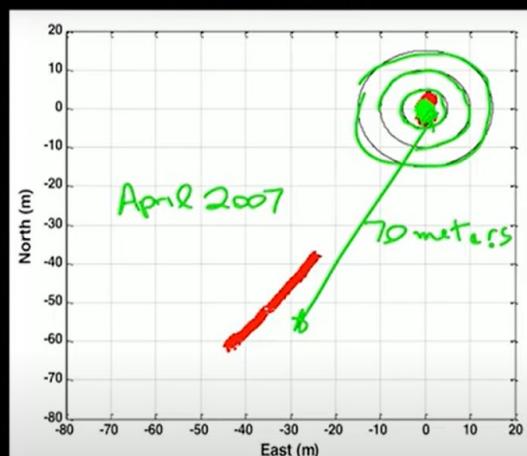
2.10 - Navigation in Our Lives: Landing Airplanes Using GPS  
A GPS Blunder



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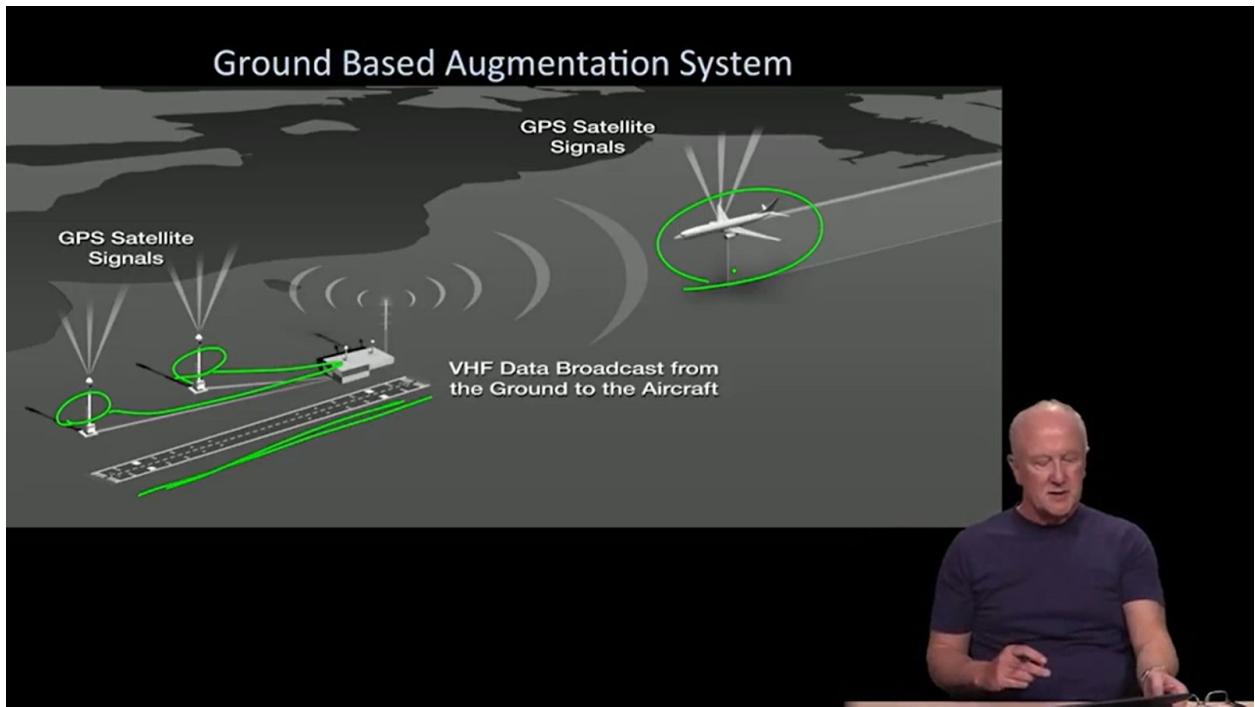
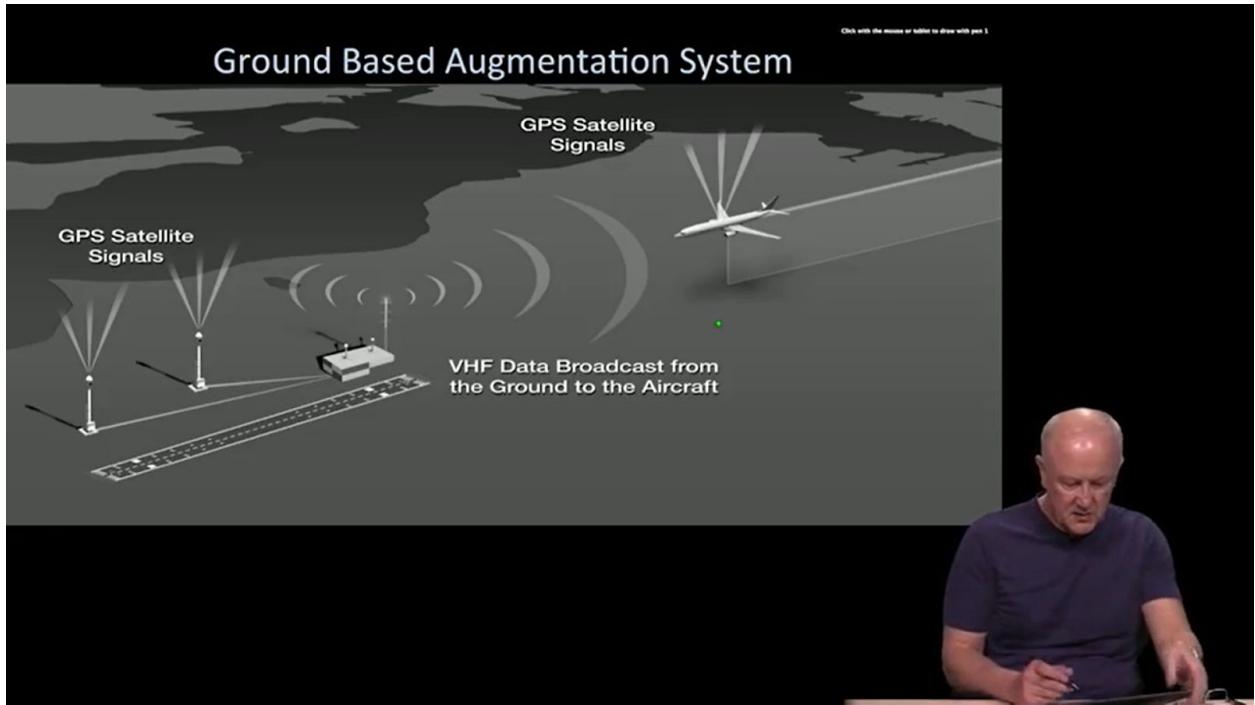
CC ⚙

2.10 - Navigation in Our Lives: Landing Airplanes Using GPS  
A GPS Blunder



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CC ⚙



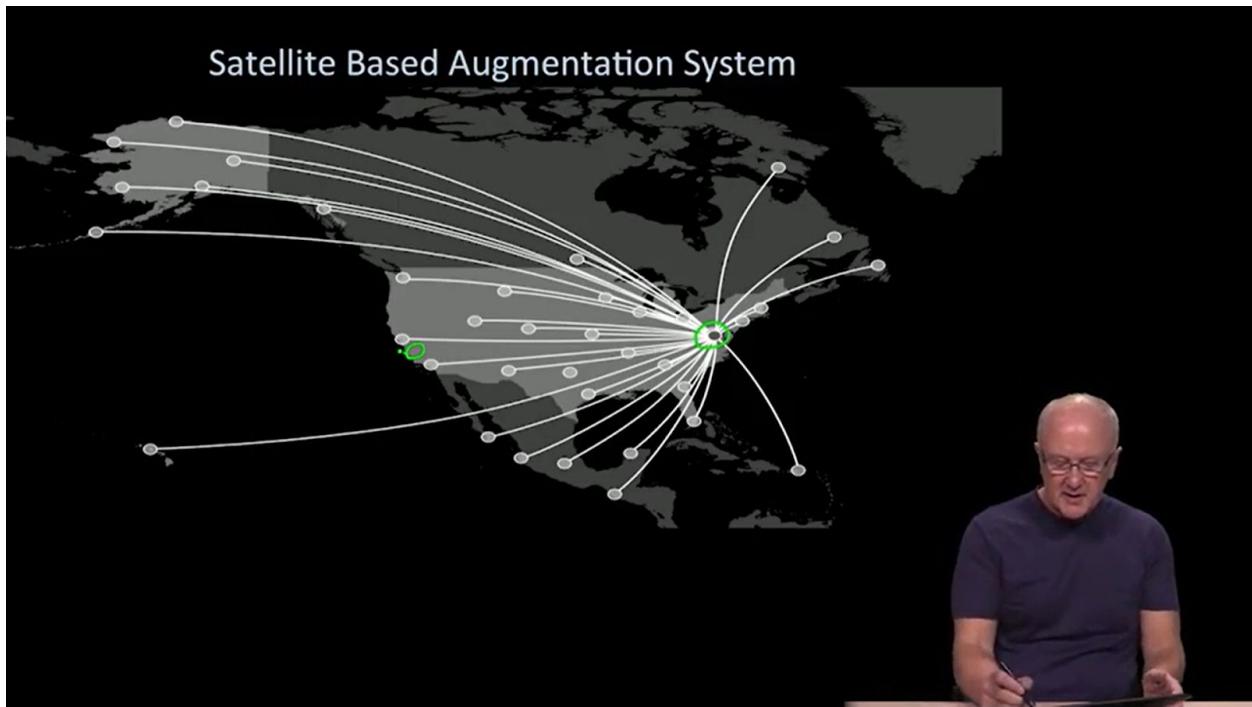
2.10 - Navigation in Our Lives: Landing Airplanes Using GPS

## Satellite Based Augmentation System



◀ ▶ 21:51 / 23:27

CC ⚙



# Module 3: Orbits & Signals

GNSS Orbits

3.1 Brahe, Kepler & Newton

3.2 Coordinate frames

3.3 Transforming from Keplerian parameters to ECEF

3.4 Decoding the GPS navigation message

GNSS Signals

3.5 L band

3.6 Frequency domain

3.7 Amplitude spectra of GNSS signals

3.8 Auto-correlation & Cross-correlation

3.9 New GNSS signals

