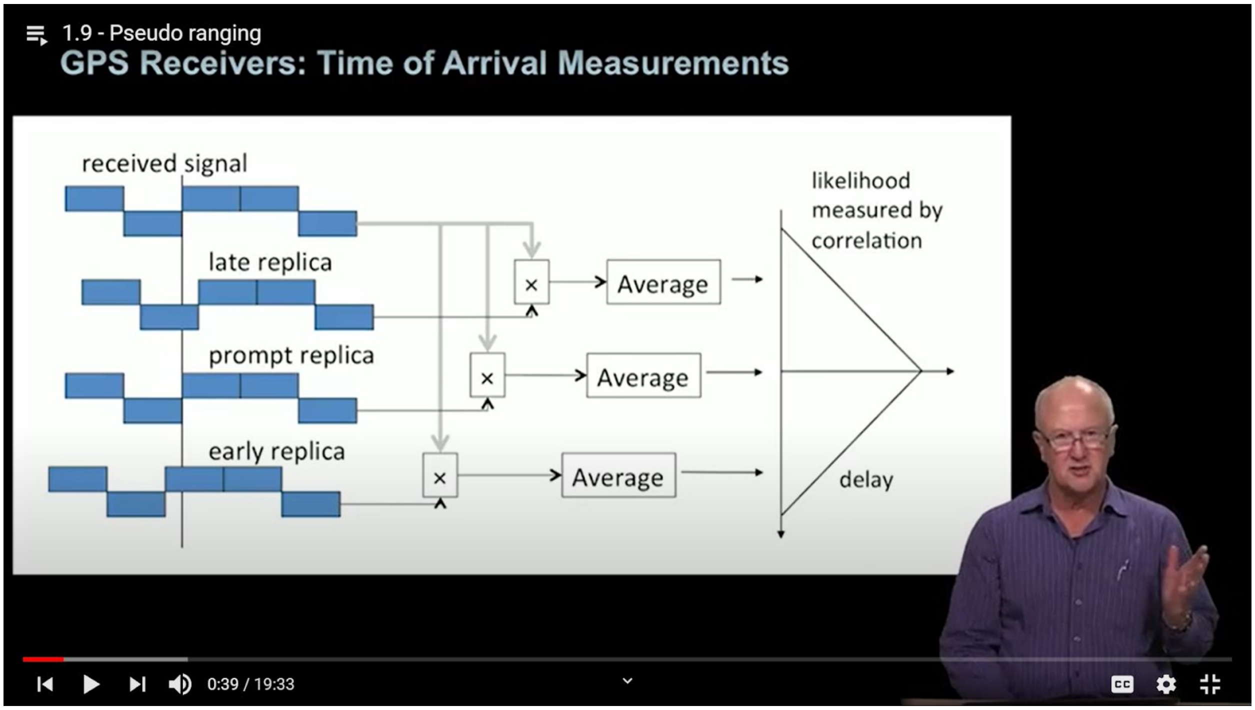
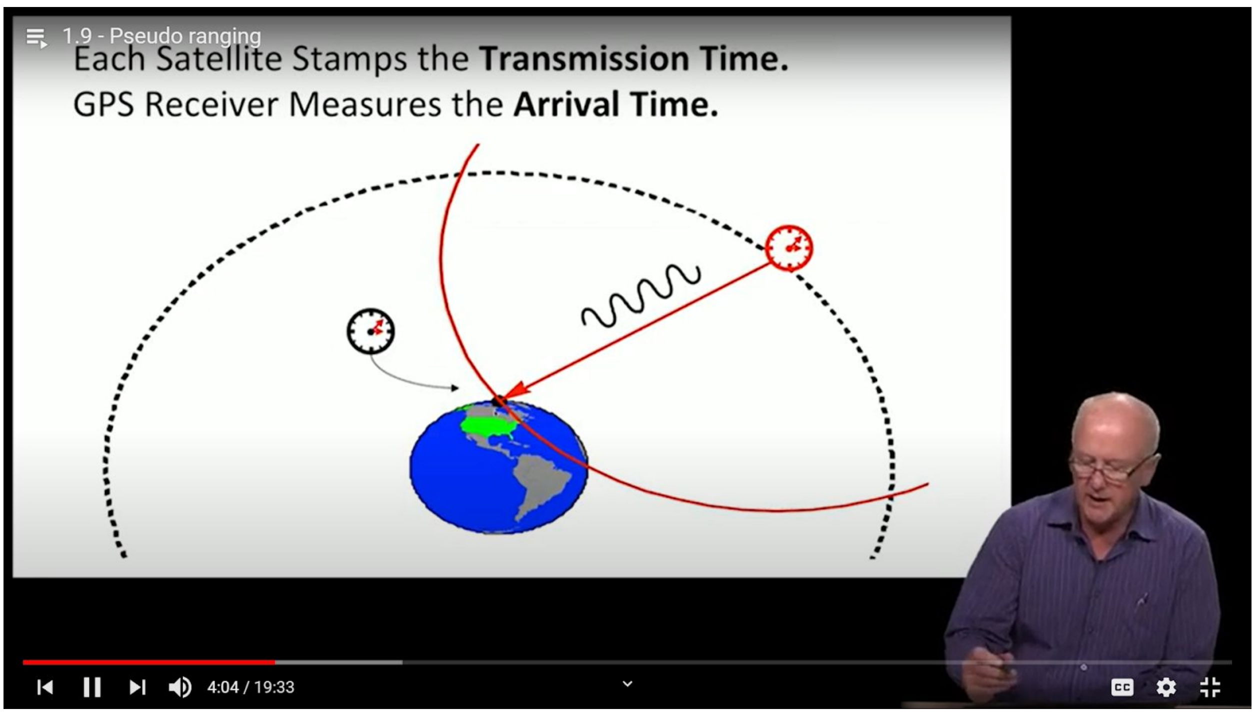
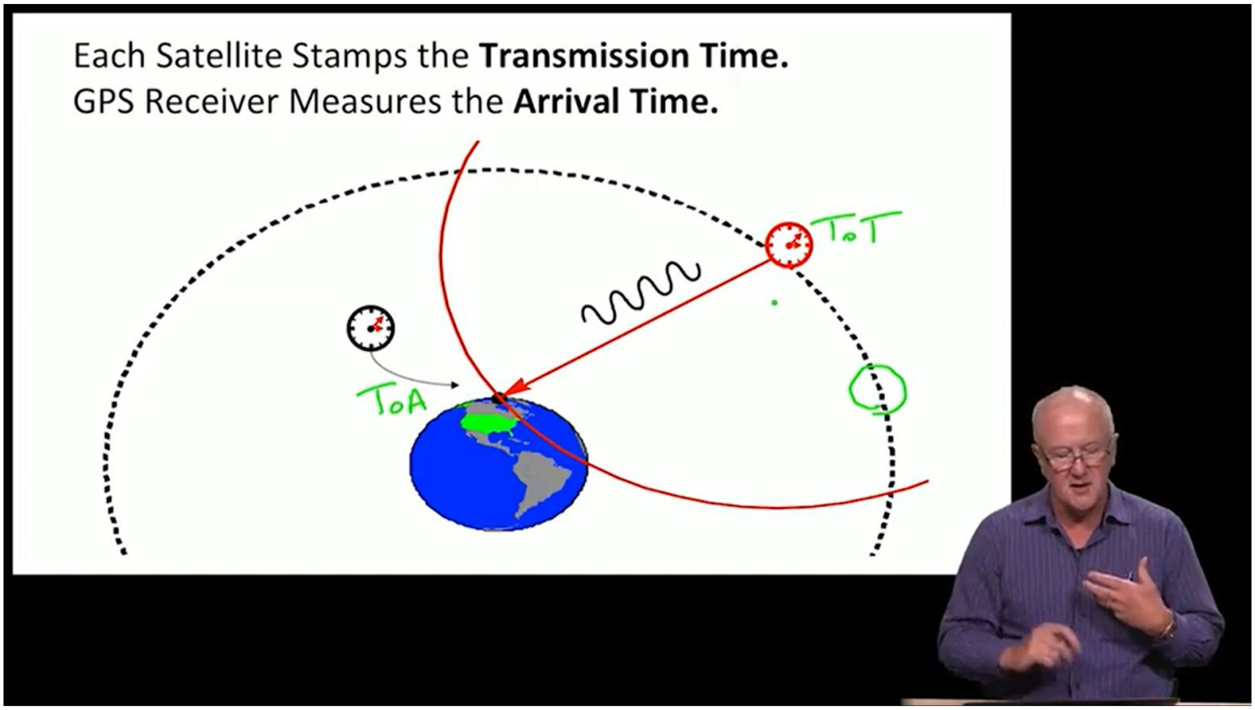


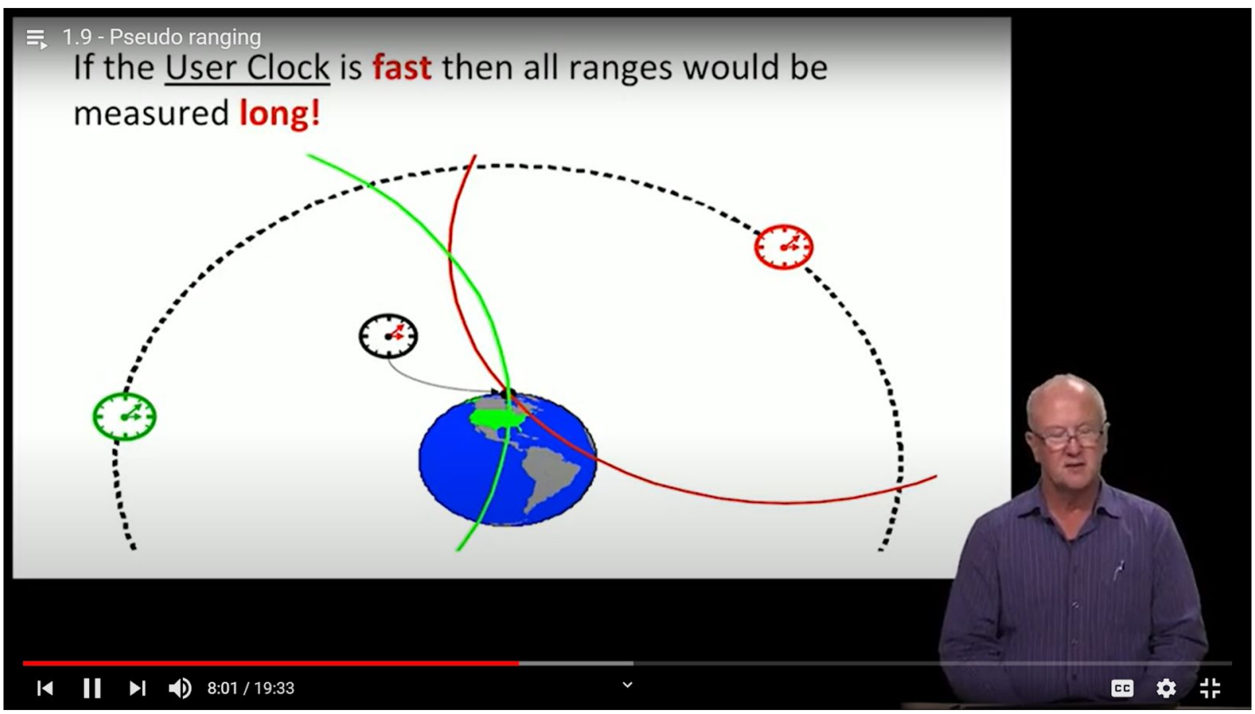
1.9 - Pseudo ranging
GPS Receivers: Time of Arrival Measurements
received signal
late replica
prompt replica
early replica
×
Average
likelihood
measured by
correlation
delay
0:39 / 19:33



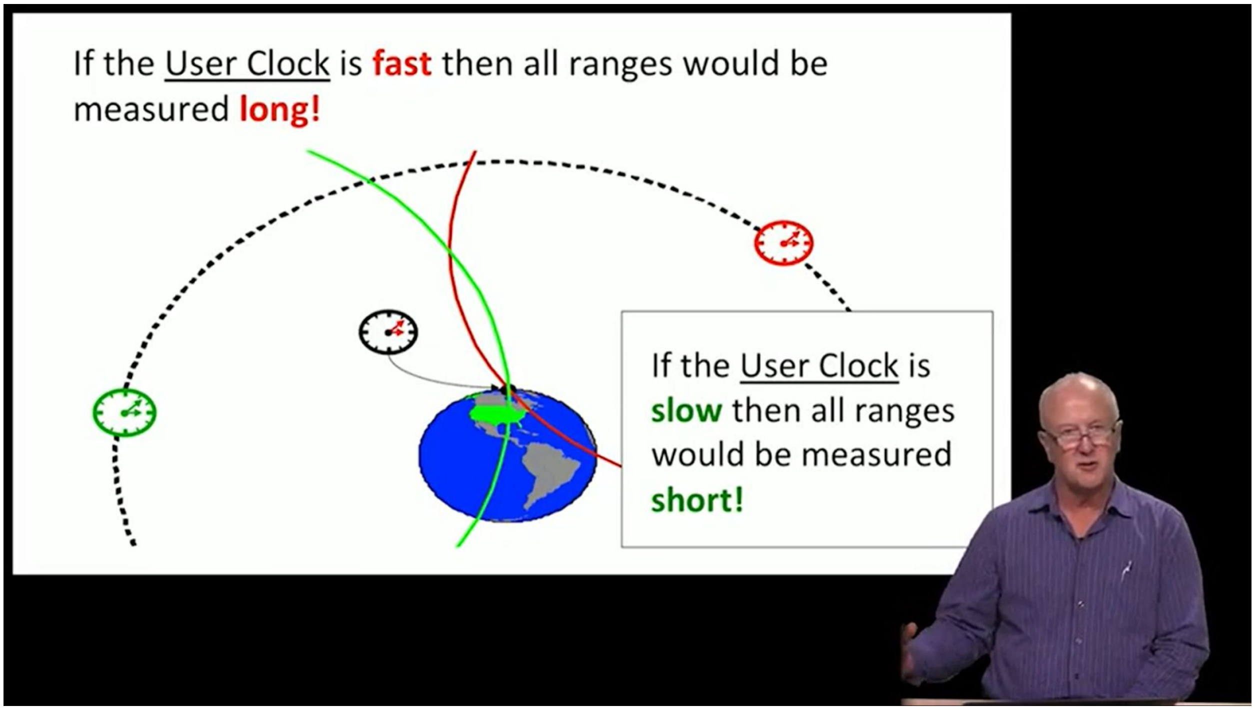
1.9 - Pseudo ranging
Each Satellite Stamps the Transmission Time.
GPS Receiver Measures the Arrival Time.
4:04 / 19:33

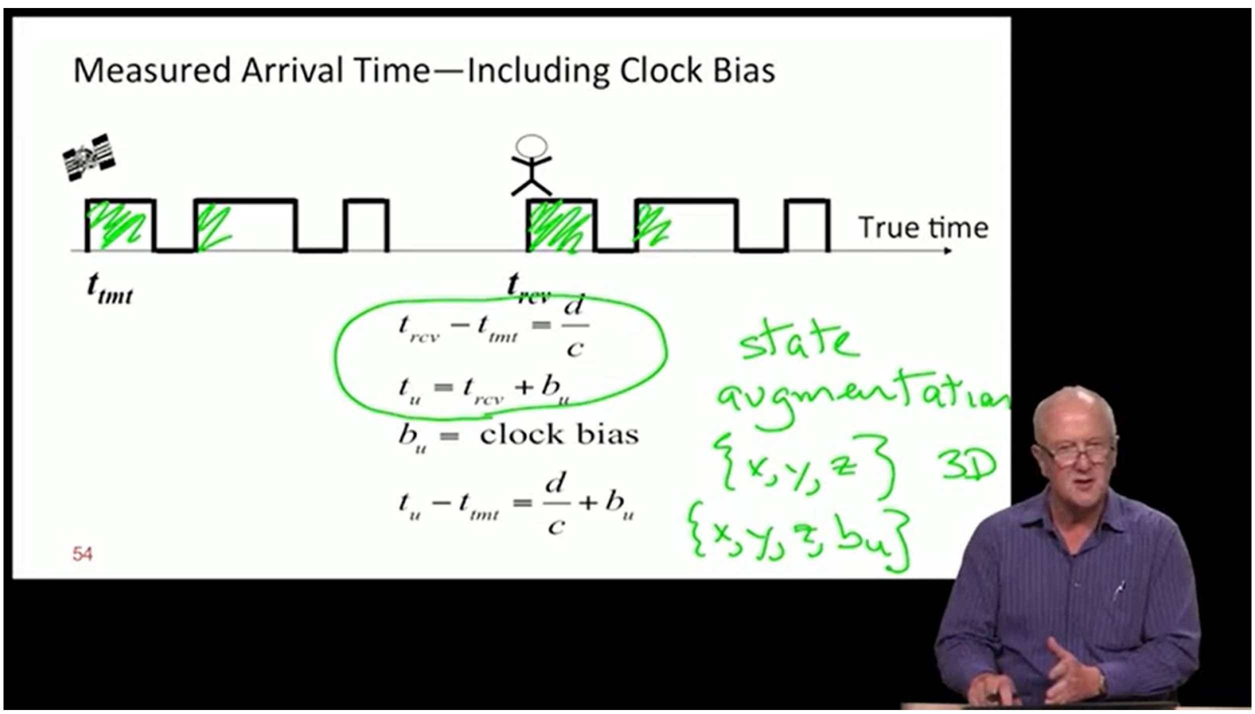


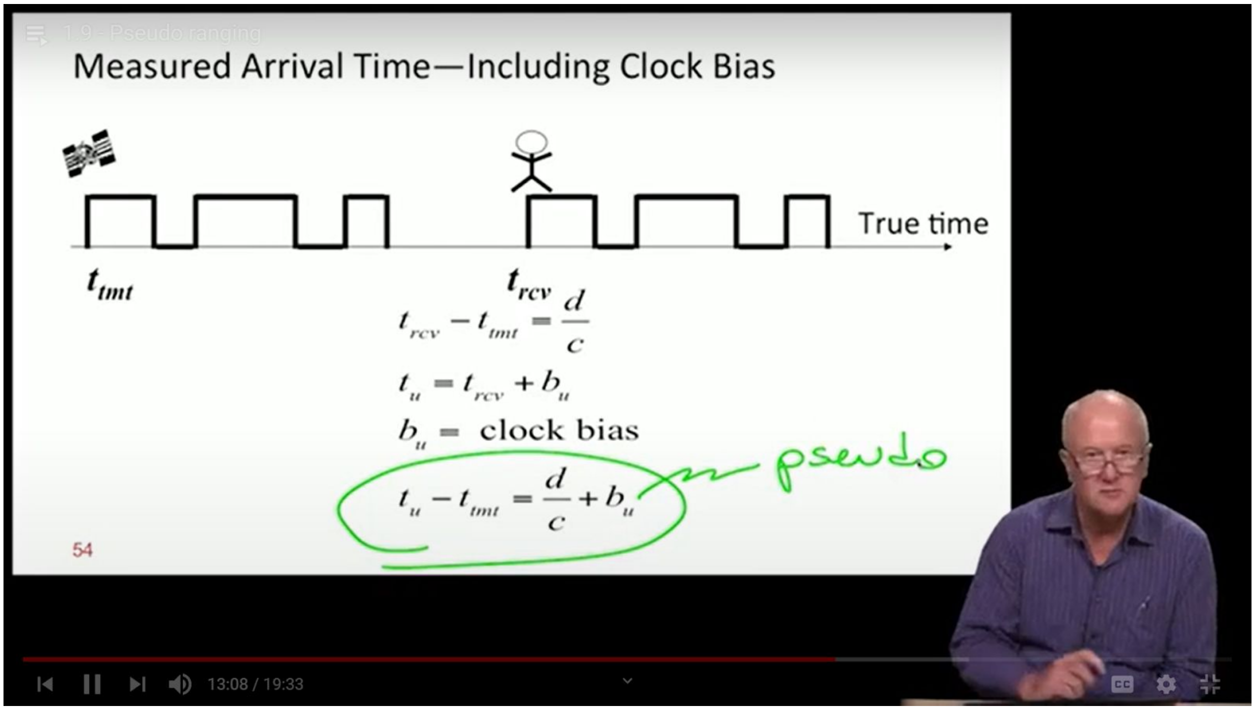
Each Satellite Stamps the Transmission Time.
GPS Receiver Measures the Arrival Time.
ToT
ToA

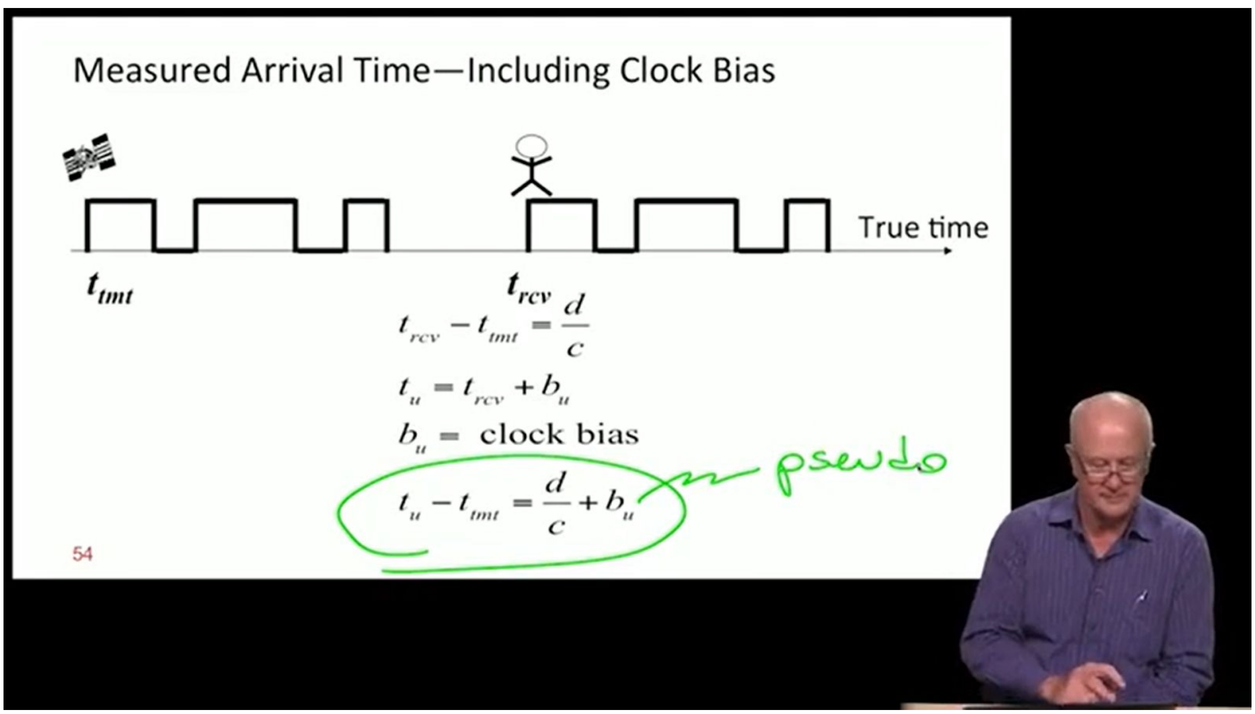


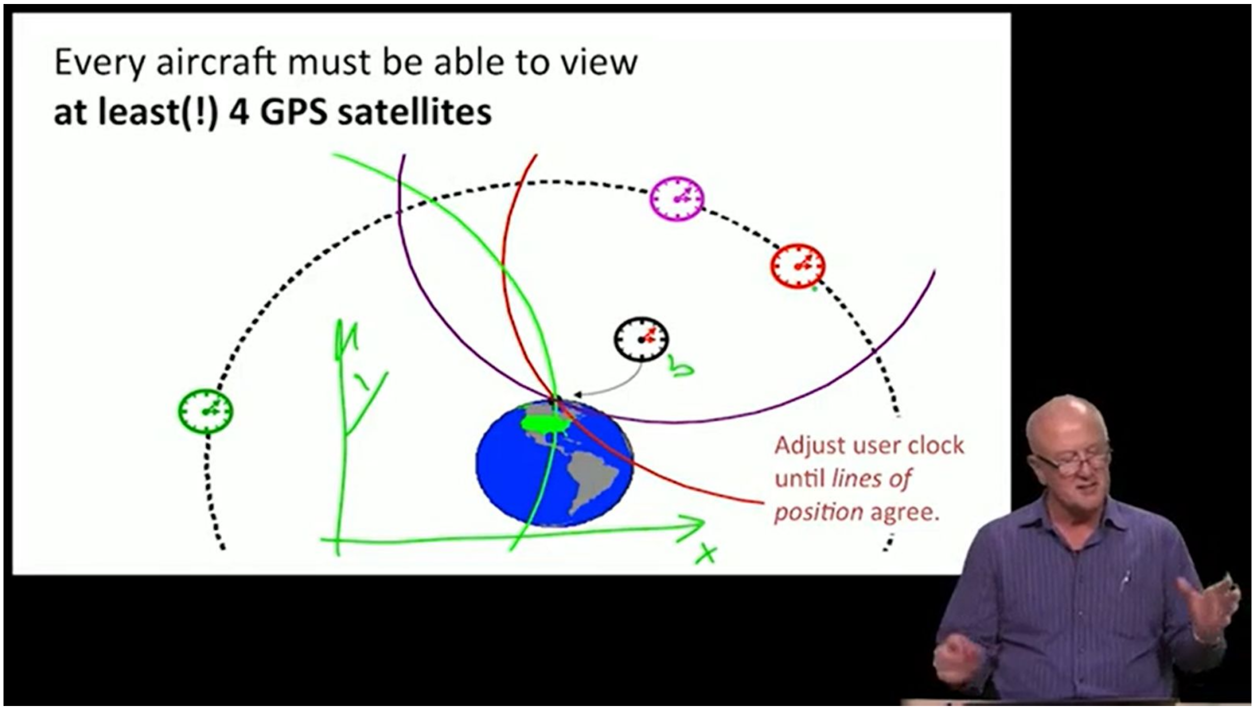
1.9 - Pseudo ranging
If the User Clock is fast then all ranges would be measured long!
8:01 / 19:33

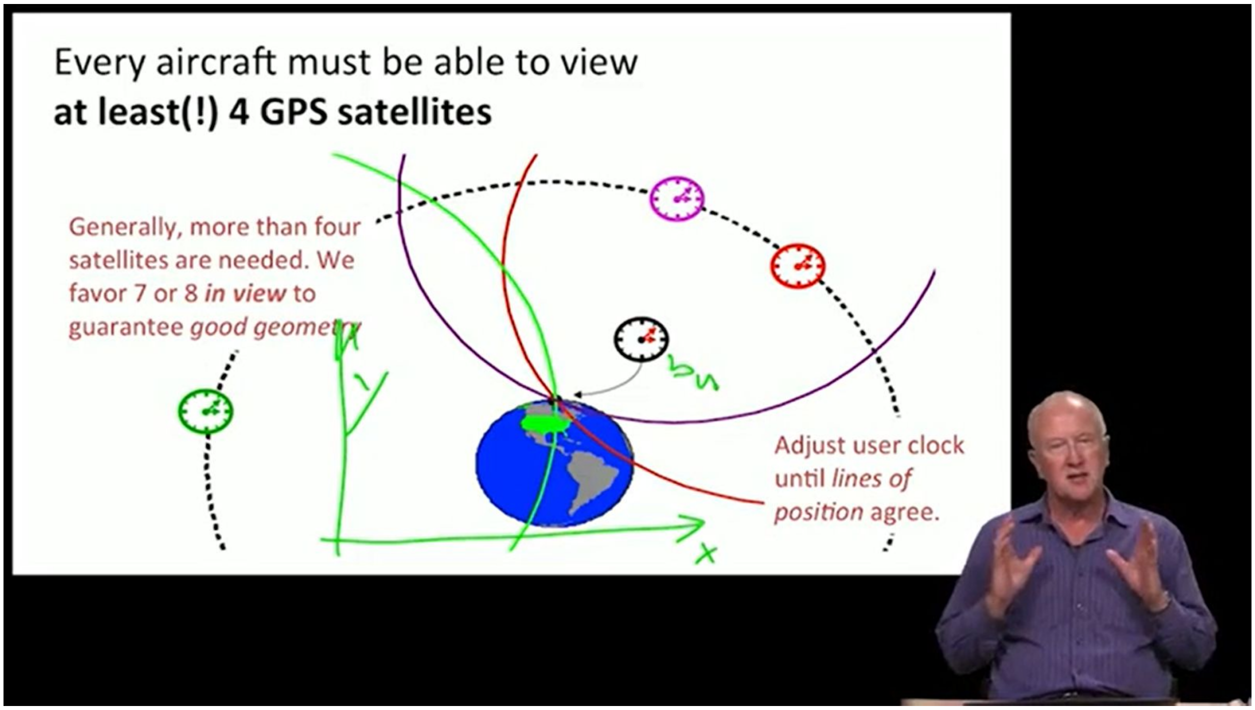

If the User Clock is fast then all ranges would be measured long!
If the User Clock is slow then all ranges would be measured short!

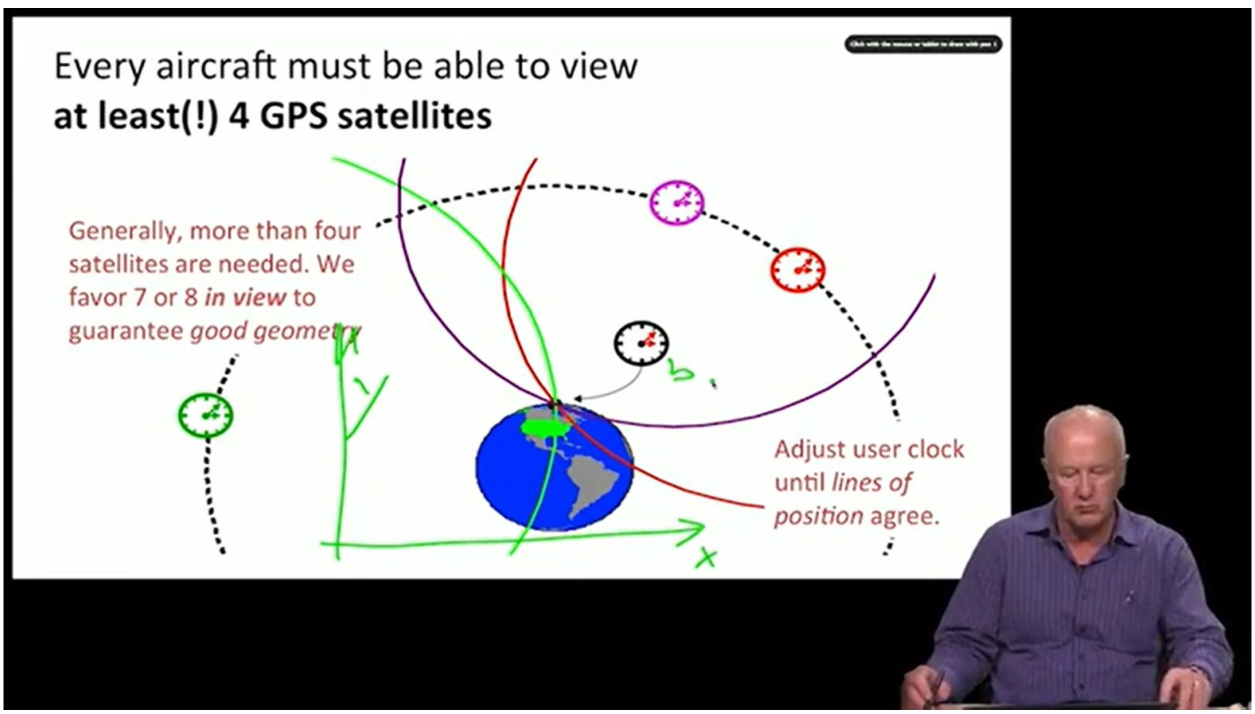

Measured Arrival Time—Including Clock Bias
True time
t_{tmt}
t_{rcv}
$t_{rcv} - t_{tmt} = \frac{d}{c}$
$t_u = t_{rcv} + b_u$
$b_u =$ clock bias
$t_u - t_{tmt} = \frac{d}{c} + b_u$
54
state augmentation
{x, y, z} 3D
{x, y, z, bu}


1.9 - Pseudo ranging
Measured Arrival Time—Including Clock Bias
True time
t_{tmt}
t_{rcv}
$t_{rcv} - t_{tmt} = \frac{d}{c}$
$t_u = t_{rcv} + b_u$
b_u = clock bias
$t_u - t_{tmt} = \frac{d}{c} + b_u$
pseudo
54
13:08 / 19:33


Measured Arrival Time—Including Clock Bias
True time
t_{tmt}
t_{rcv}
$t_{rcv} - t_{tmt} = \frac{d}{c}$
$t_u = t_{rcv} + b_u$
b_u = clock bias
$t_u - t_{tmt} = \frac{d}{c} + b_u$
pseudo
54


Every aircraft must be able to view
at least(!) 4 GPS satellites
y
x
b
Adjust user clock
until *lines of*
position agree.


Every aircraft must be able to view
at least(!) 4 GPS satellites
Generally, more than four satellites are needed. We favor 7 or 8 in view to guarantee good geometry
Adjust user clock until lines of position agree.
y
x


Every aircraft must be able to view
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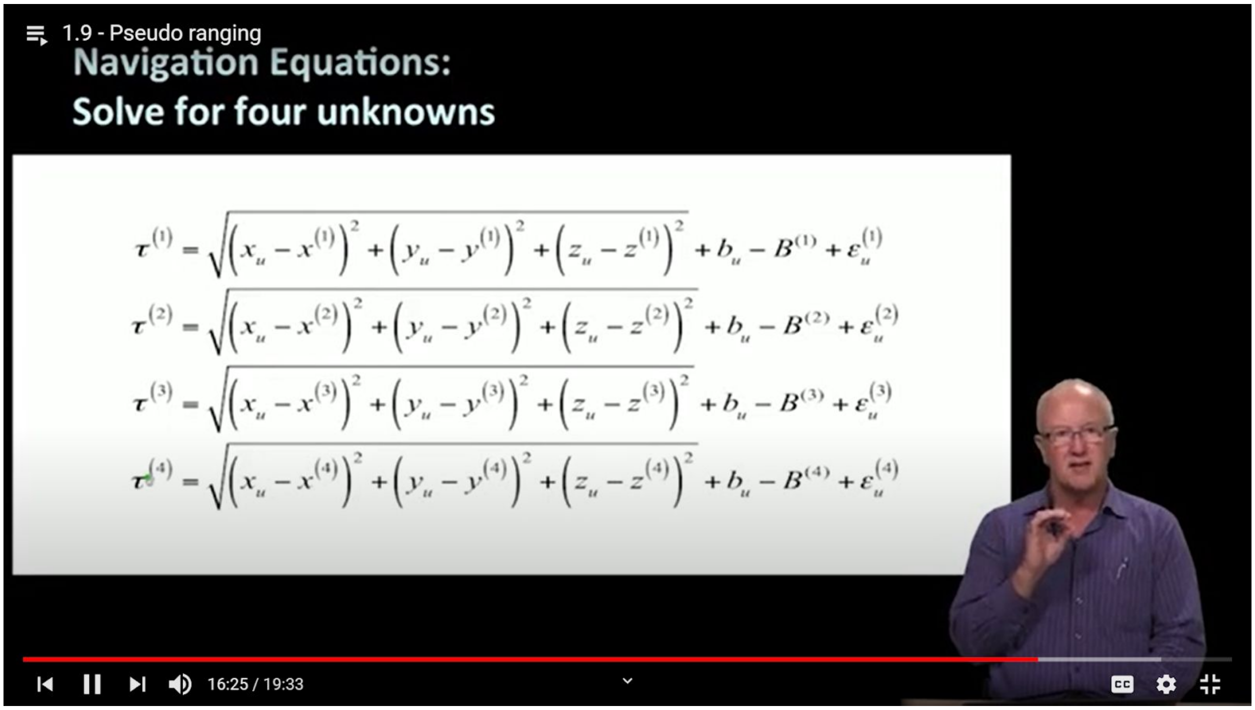
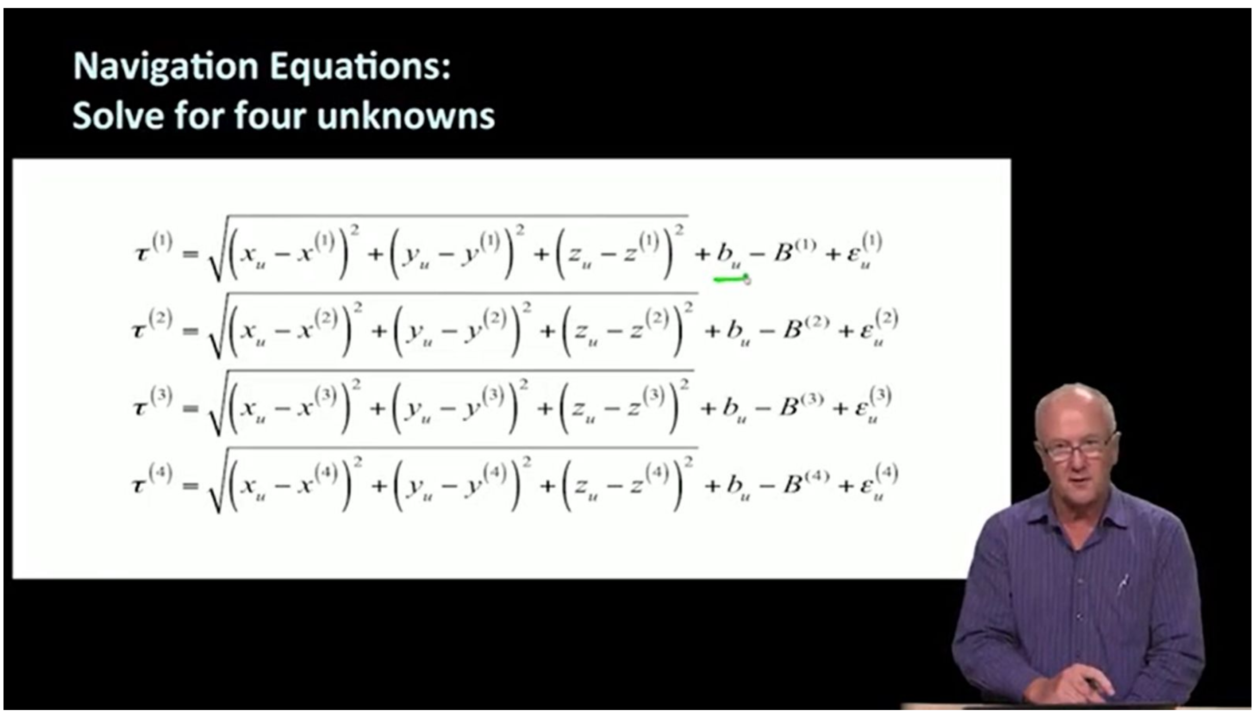
Generally, more than four satellites are needed. We favor 7 or 8 in view to guarantee good geometry
Adjust user clock until lines of position agree.
y
x


1.9 - Pseudo ranging
Navigation Equations:
Solve for four unknowns
$\tau^{(1)} = \sqrt{\left(x_u - x^{(1)}\right)^2 + \left(y_u - y^{(1)}\right)^2 + \left(z_u - z^{(1)}\right)^2} + b_u - B^{(1)} + \varepsilon_u^{(1)}$
$\tau^{(2)} = \sqrt{\left(x_u - x^{(2)}\right)^2 + \left(y_u - y^{(2)}\right)^2 + \left(z_u - z^{(2)}\right)^2} + b_u - B^{(2)} + \varepsilon_u^{(2)}$
$\tau^{(3)} = \sqrt{\left(x_u - x^{(3)}\right)^2 + \left(y_u - y^{(3)}\right)^2 + \left(z_u - z^{(3)}\right)^2} + b_u - B^{(3)} + \varepsilon_u^{(3)}$
$\tau^{(4)} = \sqrt{\left(x_u - x^{(4)}\right)^2 + \left(y_u - y^{(4)}\right)^2 + \left(z_u - z^{(4)}\right)^2} + b_u - B^{(4)} + \varepsilon_u^{(4)}$
16:25 / 19:33


Navigation Equations:
Solve for four unknowns
$\tau^{(1)} = \sqrt{\left(x_u - x^{(1)}\right)^2 + \left(y_u - y^{(1)}\right)^2 + \left(z_u - z^{(1)}\right)^2} + b_u - B^{(1)} + \varepsilon_u^{(1)}$
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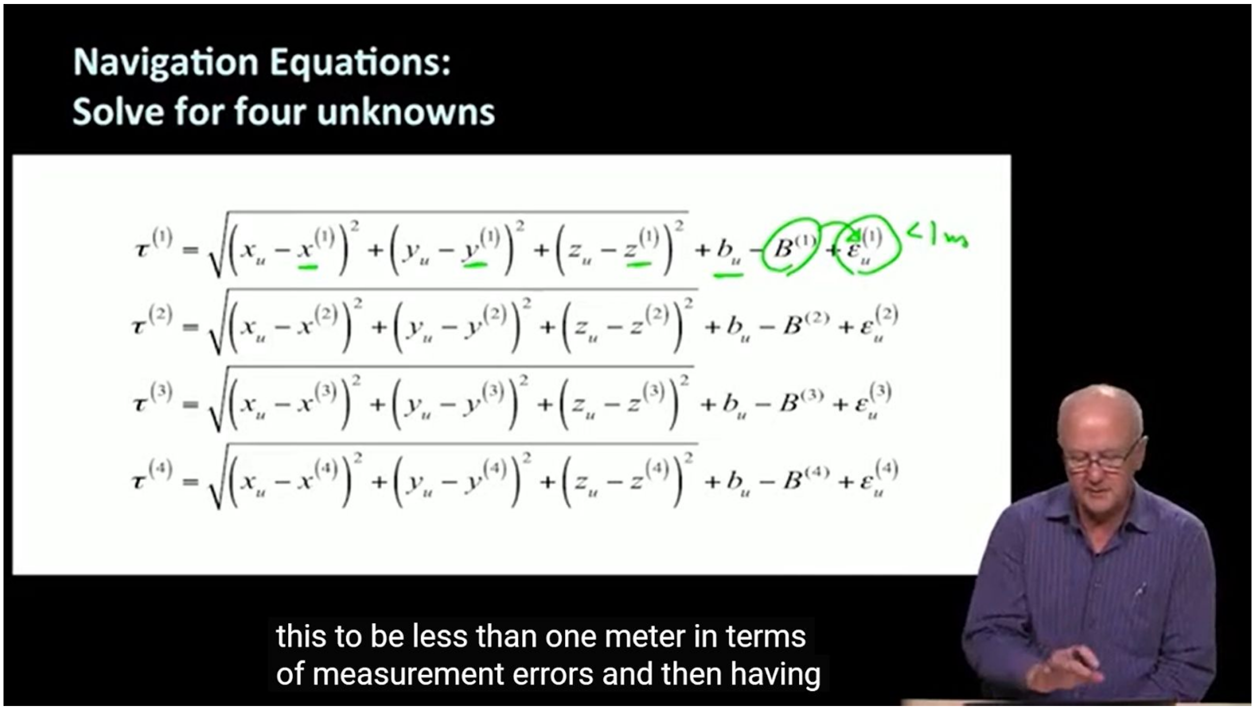
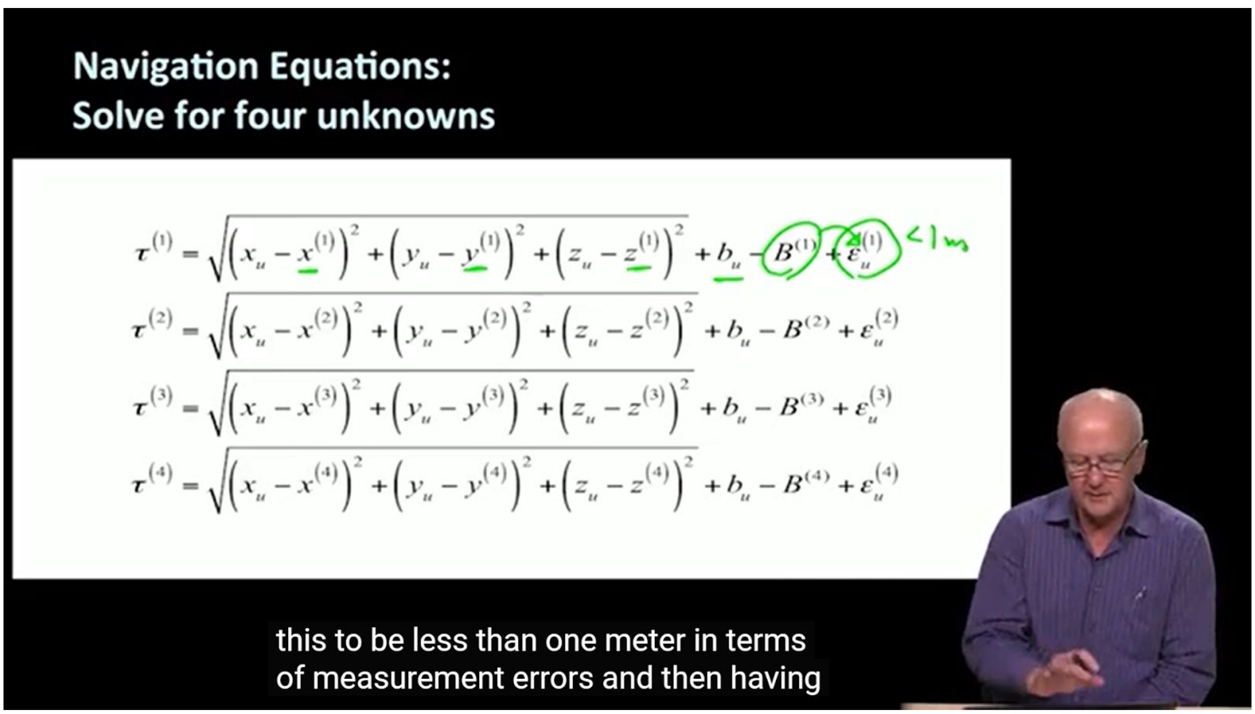
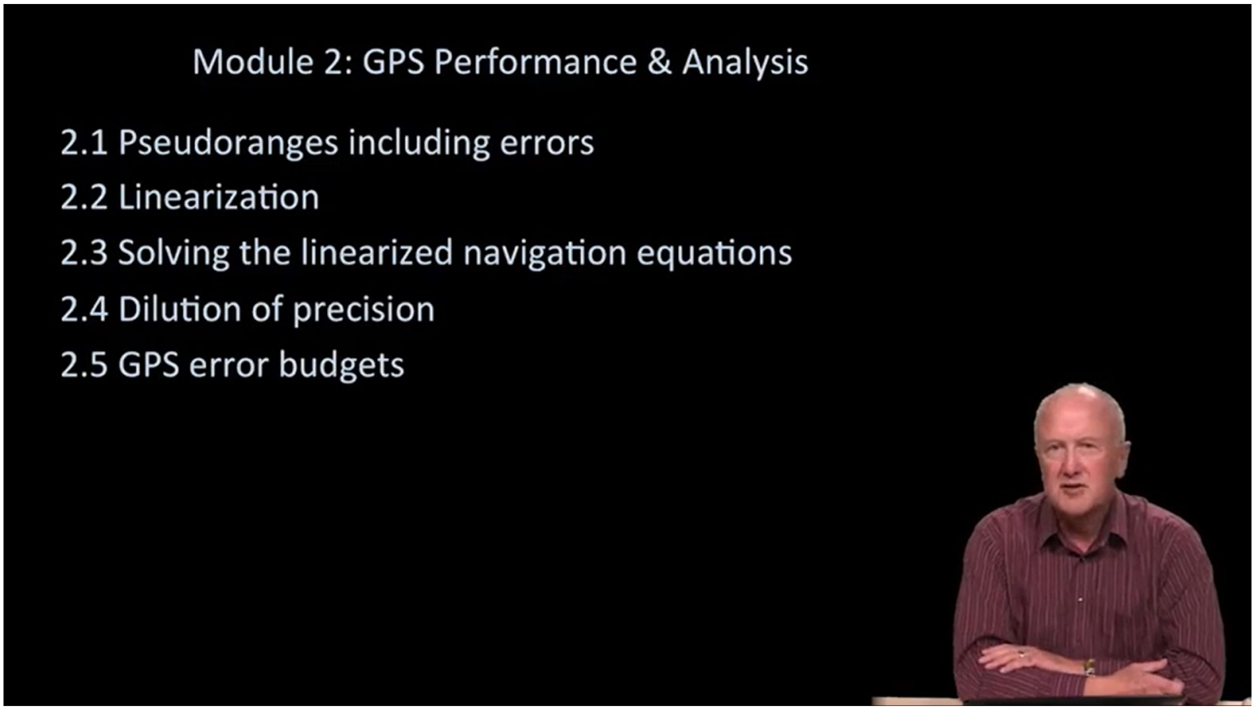
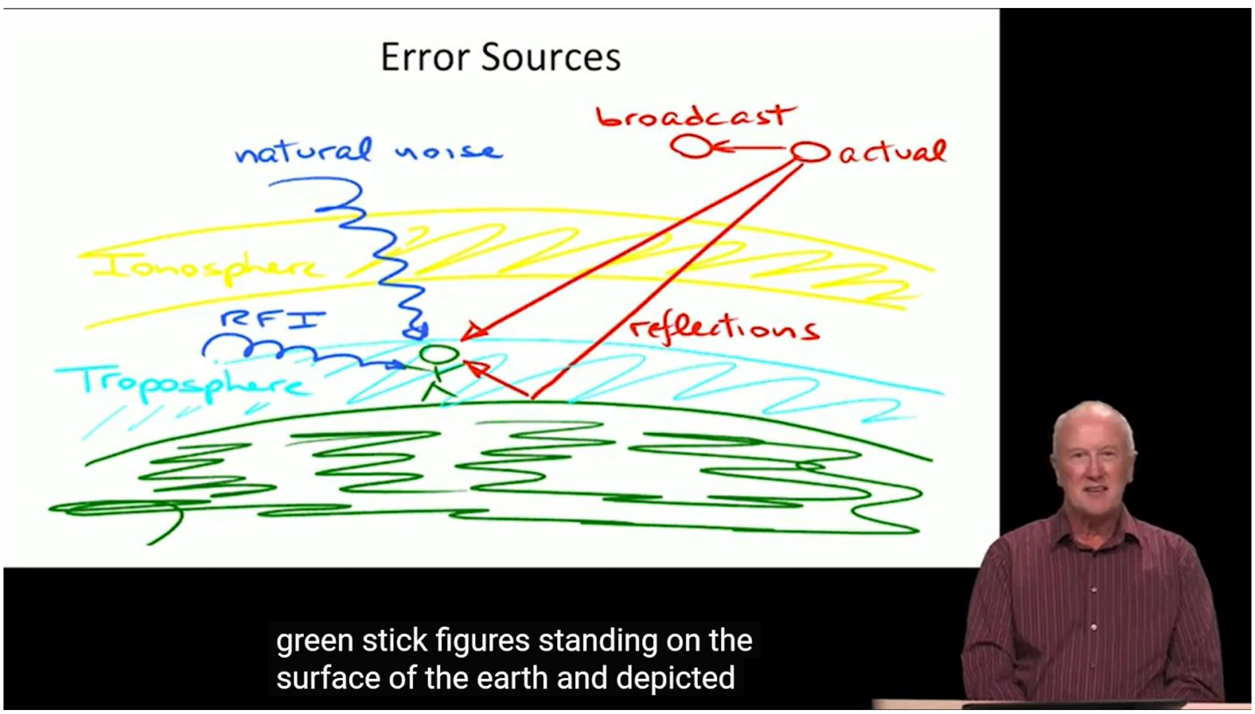
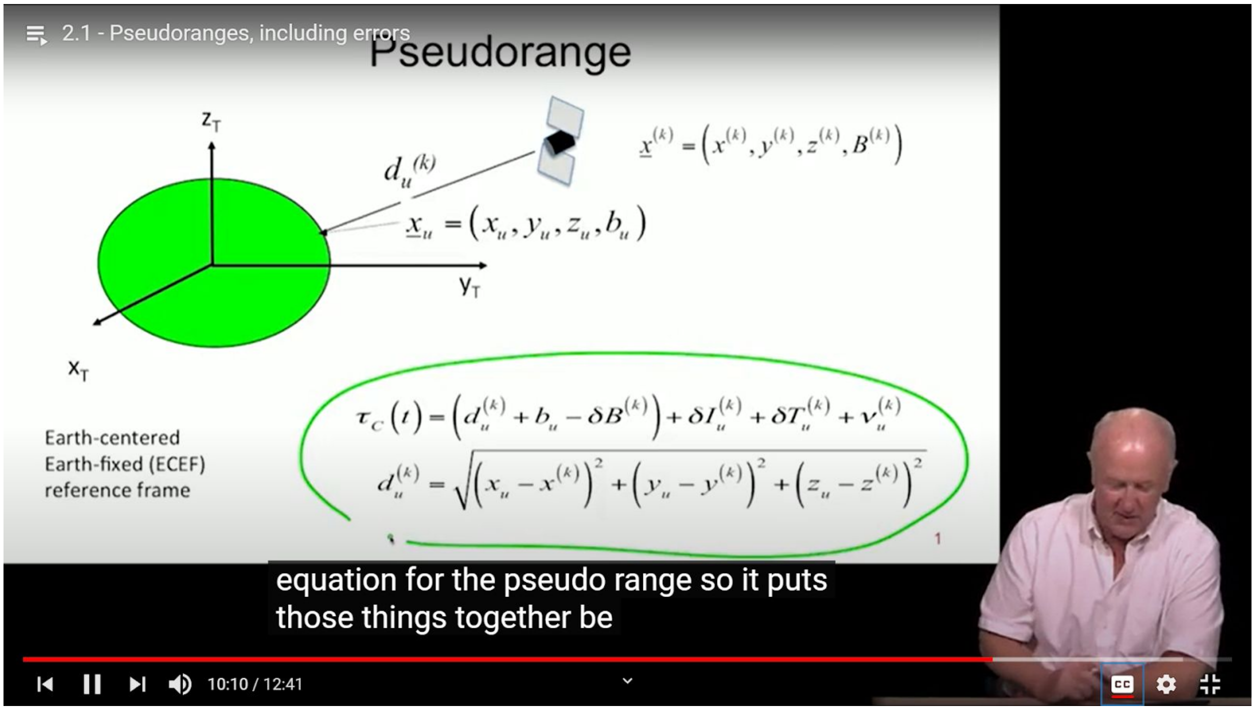
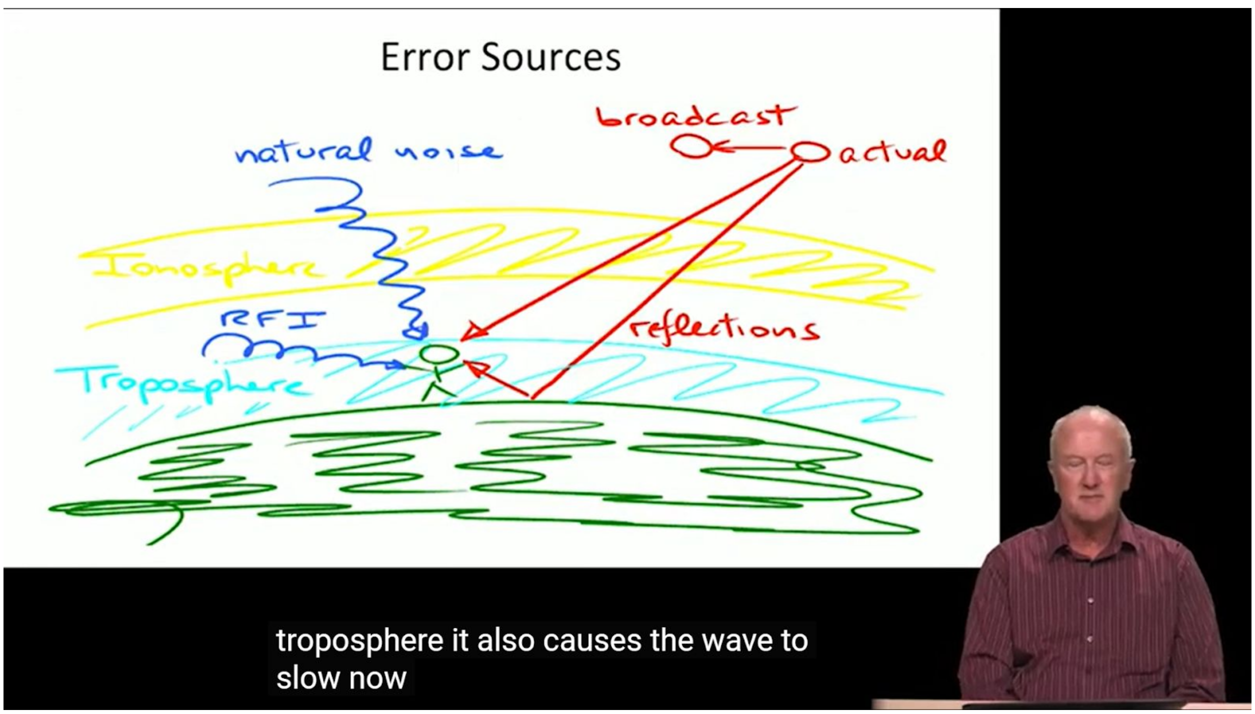

Navigation Equations:
Solve for four unknowns
$$\tau^{(1)} = \sqrt{\left(x_u - x^{(1)}\right)^2 + \left(y_u - y^{(1)}\right)^2 + \left(z_u - z^{(1)}\right)^2} + b_u - B^{(1)} + \varepsilon_u^{(1)} < 1\,m$$
$$\tau^{(2)} = \sqrt{\left(x_u - x^{(2)}\right)^2 + \left(y_u - y^{(2)}\right)^2 + \left(z_u - z^{(2)}\right)^2} + b_u - B^{(2)} + \varepsilon_u^{(2)}$$
$$\tau^{(3)} = \sqrt{\left(x_u - x^{(3)}\right)^2 + \left(y_u - y^{(3)}\right)^2 + \left(z_u - z^{(3)}\right)^2} + b_u - B^{(3)} + \varepsilon_u^{(3)}$$
$$\tau^{(4)} = \sqrt{\left(x_u - x^{(4)}\right)^2 + \left(y_u - y^{(4)}\right)^2 + \left(z_u - z^{(4)}\right)^2} + b_u - B^{(4)} + \varepsilon_u^{(4)}$$
this to be less than one meter in terms
of measurement errors and then having

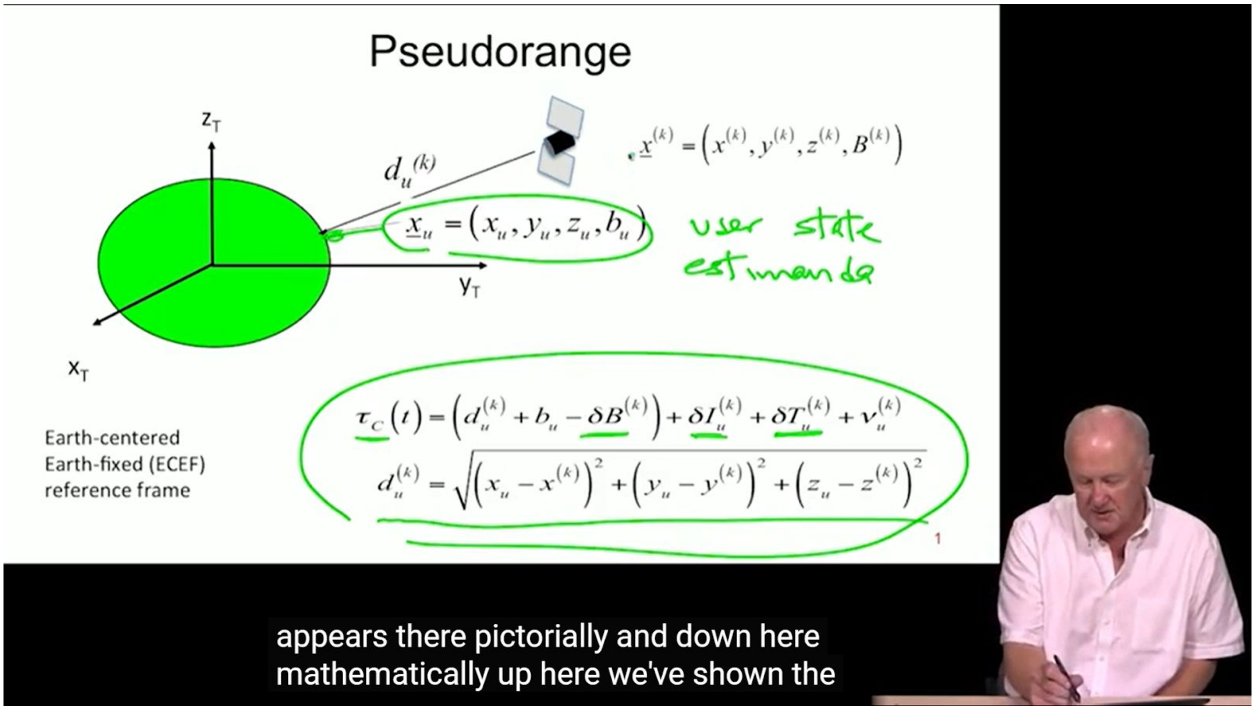

Navigation Equations:
Solve for four unknowns
$$\tau^{(1)} = \sqrt{\left(x_u - x^{(1)}\right)^2 + \left(y_u - y^{(1)}\right)^2 + \left(z_u - z^{(1)}\right)^2} + b_u - B^{(1)} + \varepsilon_u^{(1)} < 1\,m$$
$$\tau^{(2)} = \sqrt{\left(x_u - x^{(2)}\right)^2 + \left(y_u - y^{(2)}\right)^2 + \left(z_u - z^{(2)}\right)^2} + b_u - B^{(2)} + \varepsilon_u^{(2)}$$
$$\tau^{(3)} = \sqrt{\left(x_u - x^{(3)}\right)^2 + \left(y_u - y^{(3)}\right)^2 + \left(z_u - z^{(3)}\right)^2} + b_u - B^{(3)} + \varepsilon_u^{(3)}$$
$$\tau^{(4)} = \sqrt{\left(x_u - x^{(4)}\right)^2 + \left(y_u - y^{(4)}\right)^2 + \left(z_u - z^{(4)}\right)^2} + b_u - B^{(4)} + \varepsilon_u^{(4)}$$
this to be less than one meter in terms
of measurement errors and then having

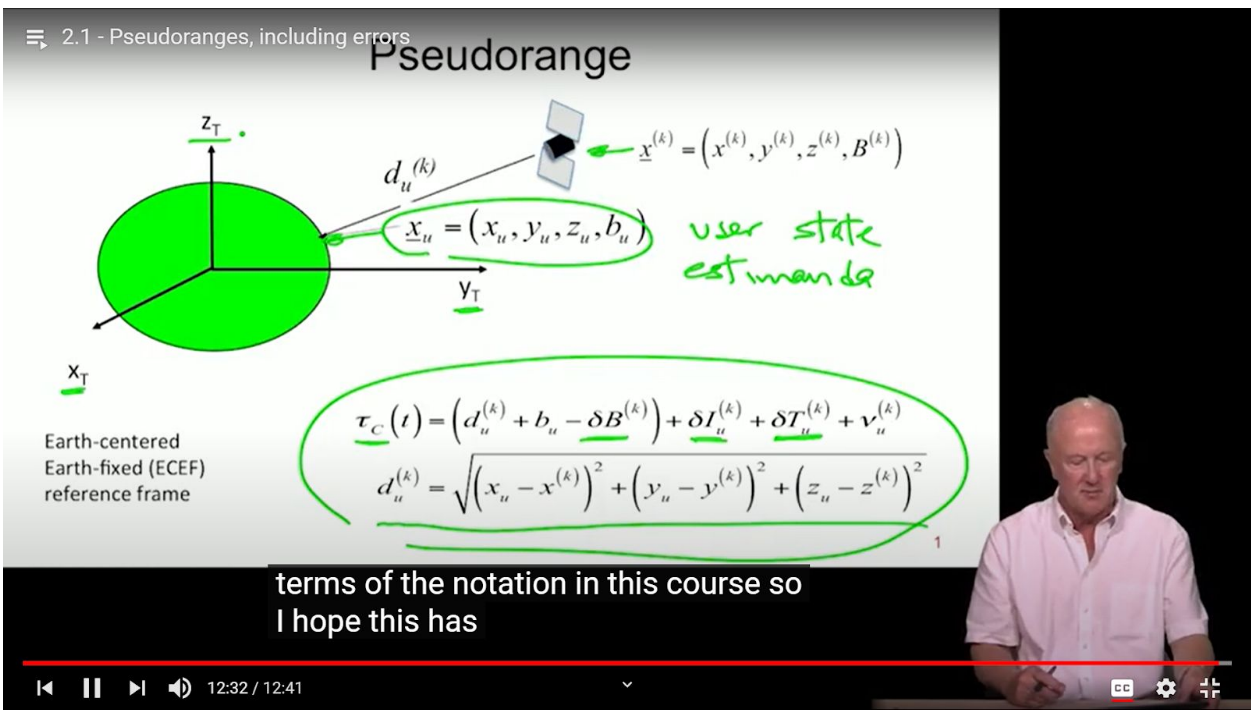

Module 2: GPS Performance & Analysis
2.1 Pseudoranges including errors
2.2 Linearization
2.3 Solving the linearized navigation equations
2.4 Dilution of precision
2.5 GPS error budgets

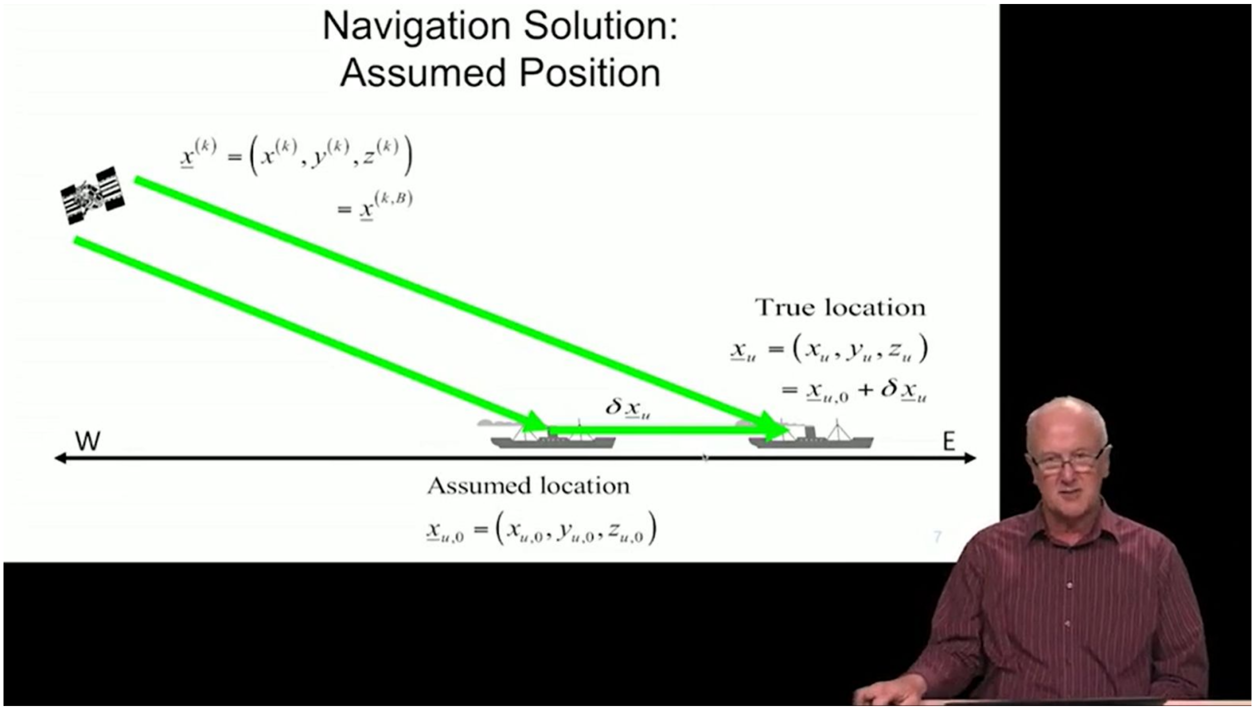

Error Sources
broadcast
actual
natural noise
Ionosphere
RFI
reflections
Troposphere
green stick figures standing on the
surface of the earth and depicted

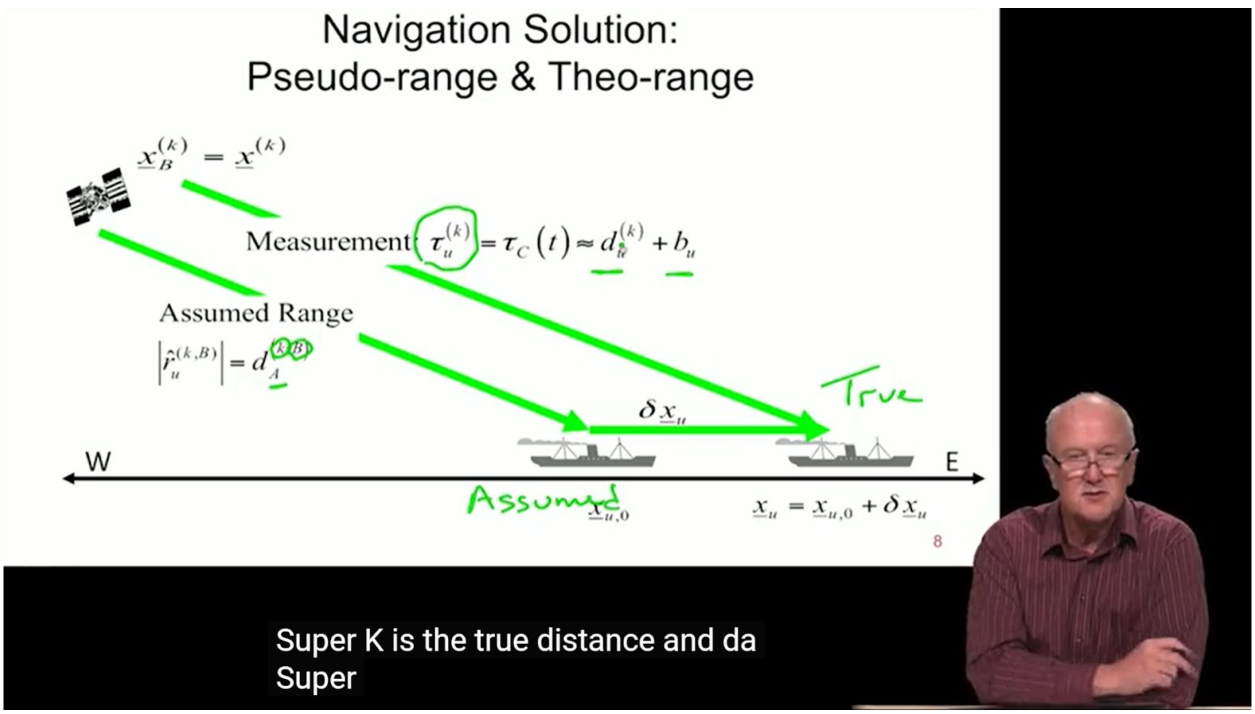

2.1 - Pseudoranges, including errors
Pseudorange
z_T
y_T
x_T
$d_u^{(k)}$
$\underline{x}^{(k)} = \left(x^{(k)}, y^{(k)}, z^{(k)}, B^{(k)}\right)$
$\underline{x}_u = \left(x_u, y_u, z_u, b_u\right)$
Earth-centered
Earth-fixed (ECEF)
reference frame
$\tau_C(t) = \left(d_u^{(k)} + b_u - \delta B^{(k)}\right) + \delta I_u^{(k)} + \delta T_u^{(k)} + \nu_u^{(k)}$
$d_u^{(k)} = \sqrt{\left(x_u - x^{(k)}\right)^2 + \left(y_u - y^{(k)}\right)^2 + \left(z_u - z^{(k)}\right)^2}$
1
equation for the pseudo range so it puts
those things together be
10:10 / 12:41

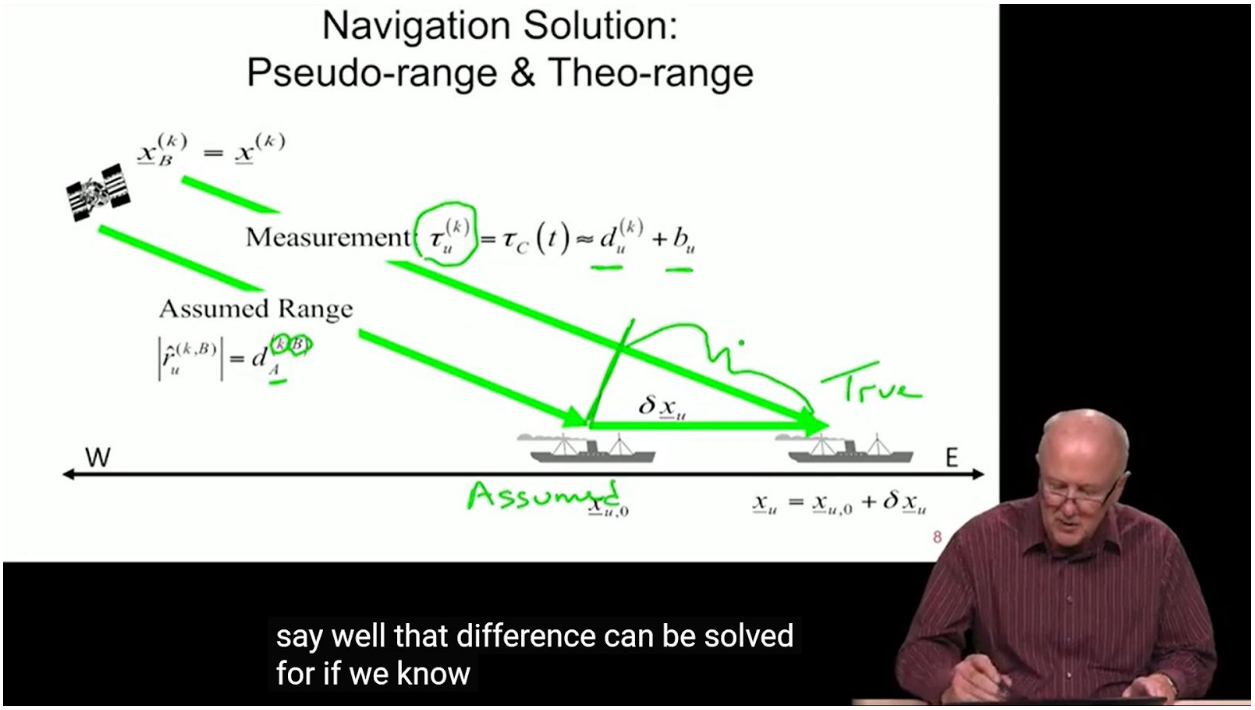

Error Sources
broadcast
actual
natural noise
Ionosphere
RFI
reflections
Troposphere
troposphere it also causes the wave to
slow now

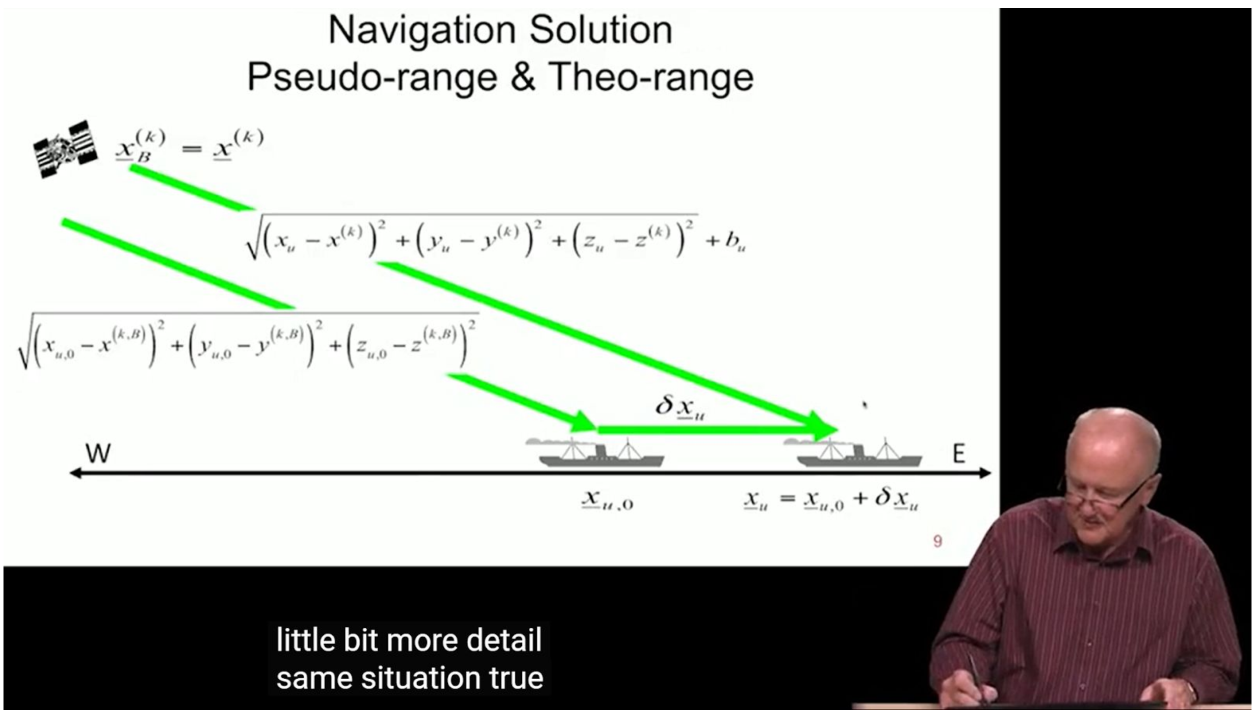

Pseudorange
z_T
$d_u^{(k)}$
$\underline{x}^{(k)} = \left(x^{(k)}, y^{(k)}, z^{(k)}, B^{(k)}\right)$
$\underline{x}_u = \left(x_u, y_u, z_u, b_u\right)$
user state
estimanda
y_T
x_T
Earth-centered
Earth-fixed (ECEF)
reference frame
$\tau_C(t) = \left(d_u^{(k)} + b_u - \delta B^{(k)}\right) + \delta I_u^{(k)} + \delta T_u^{(k)} + \nu_u^{(k)}$
$d_u^{(k)} = \sqrt{\left(x_u - x^{(k)}\right)^2 + \left(y_u - y^{(k)}\right)^2 + \left(z_u - z^{(k)}\right)^2}$
1
appears there pictorially and down here
mathematically up here we've shown the

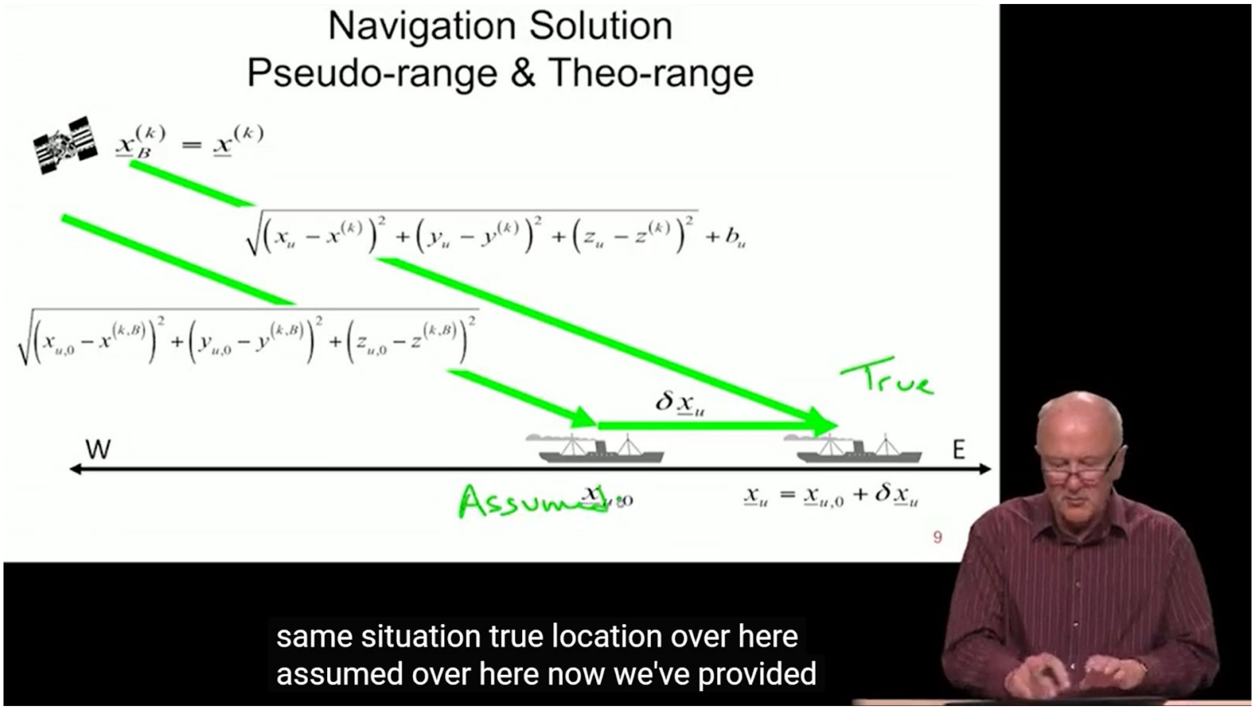

2.1 - Pseudoranges, including errors
Pseudorange
z_T
$d_u^{(k)}$
$\underline{x}^{(k)} = \left(x^{(k)}, y^{(k)}, z^{(k)}, B^{(k)}\right)$
$\underline{x}_u = \left(x_u, y_u, z_u, b_u\right)$
user state
estimanda
y_T
x_T
Earth-centered
Earth-fixed (ECEF)
reference frame
$\tau_C(t) = \left(d_u^{(k)} + b_u - \delta B^{(k)}\right) + \delta I_u^{(k)} + \delta T_u^{(k)} + \nu_u^{(k)}$
$d_u^{(k)} = \sqrt{\left(x_u - x^{(k)}\right)^2 + \left(y_u - y^{(k)}\right)^2 + \left(z_u - z^{(k)}\right)^2}$
1
terms of the notation in this course so
I hope this has
12:32 / 12:41

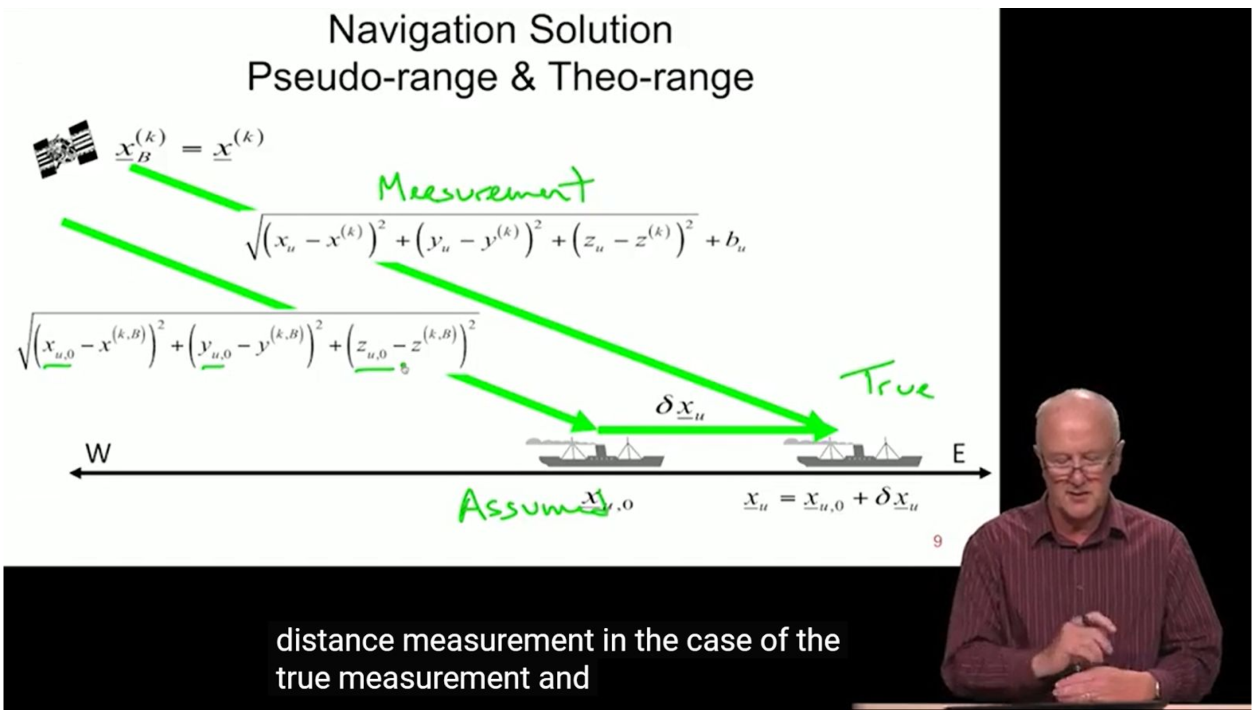

Navigation Solution:
Assumed Position
$\underline{x}^{(k)} = \left(x^{(k)}, y^{(k)}, z^{(k)}\right) = \underline{x}^{(k,B)}$
True location
$\underline{x}_u = \left(x_u, y_u, z_u\right) = \underline{x}_{u,0} + \delta\underline{x}_u$
$\delta\underline{x}_u$
W
E
Assumed location
$\underline{x}_{u,0} = \left(x_{u,0}, y_{u,0}, z_{u,0}\right)$
7

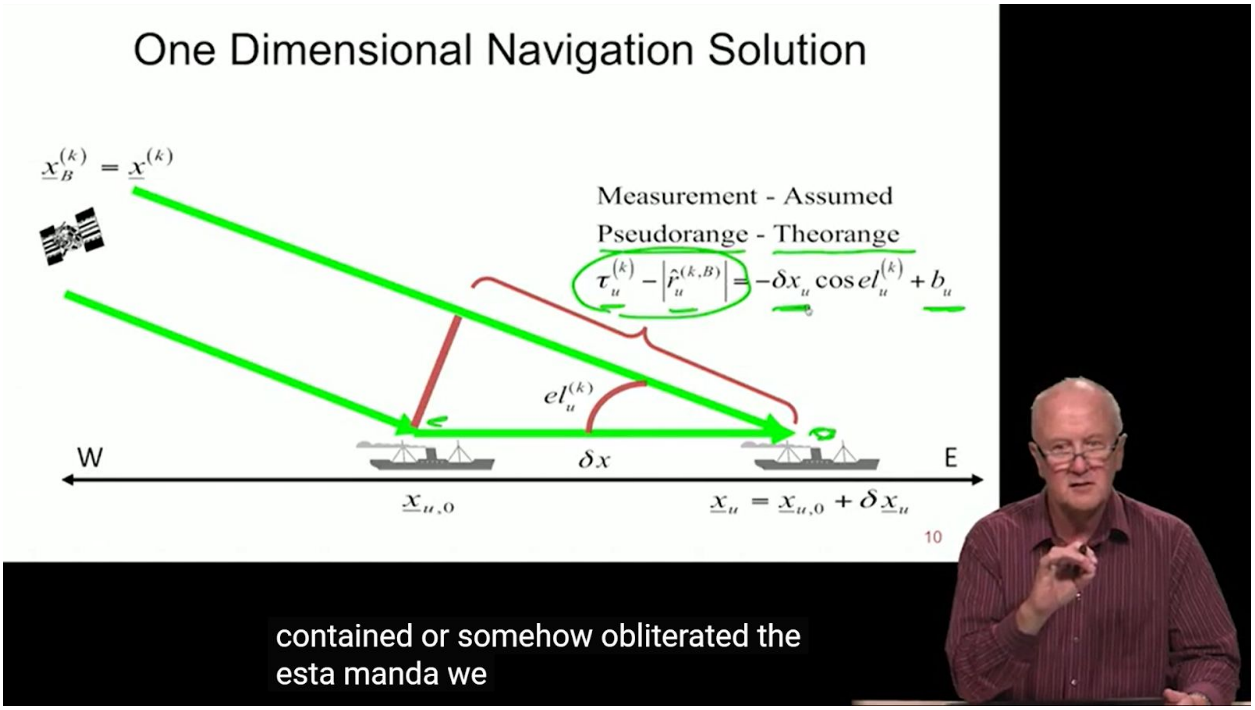

Navigation Solution:
Pseudo-range & Theo-range
$\underline{x}_B^{(k)} = \underline{x}^{(k)}$
Measurement $\tau_u^{(k)} = \tau_C(t) \approx d_u^{(k)} + b_u$
Assumed Range
$\left|\hat{r}_u^{(k,B)}\right| = d_A^{(k,B)}$
True
$\delta\underline{x}_u$
W
E
Assumed
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta\underline{x}_u$
8
Super K is the true distance and da
Super

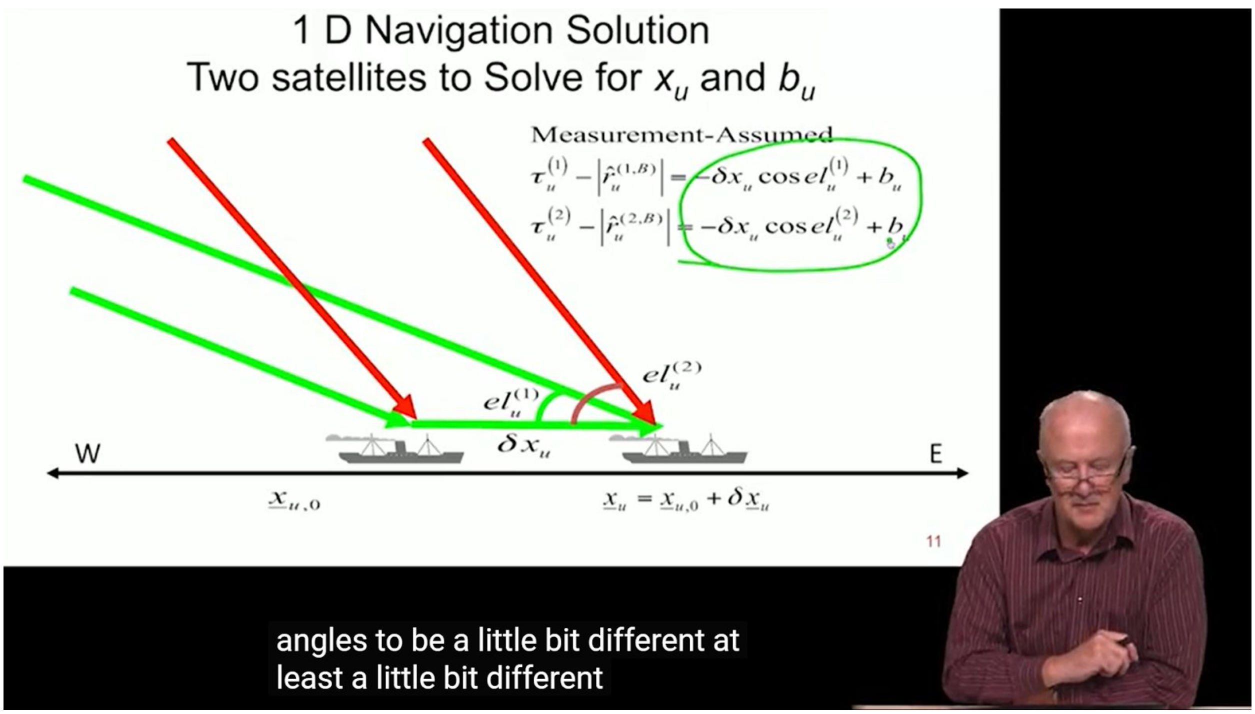

Navigation Solution:
Pseudo-range & Theo-range
$\underline{x}_B^{(k)} = \underline{x}^{(k)}$
Measurement $\tau_u^{(k)} = \tau_C(t) \approx d_u^{(k)} + b_u$
Assumed Range
$\left|\hat{r}_u^{(k,B)}\right| = d_A^{(k,B)}$
$\delta \underline{x}_u$
True
W
E
Assumed
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
8
say well that difference can be solved
for if we know

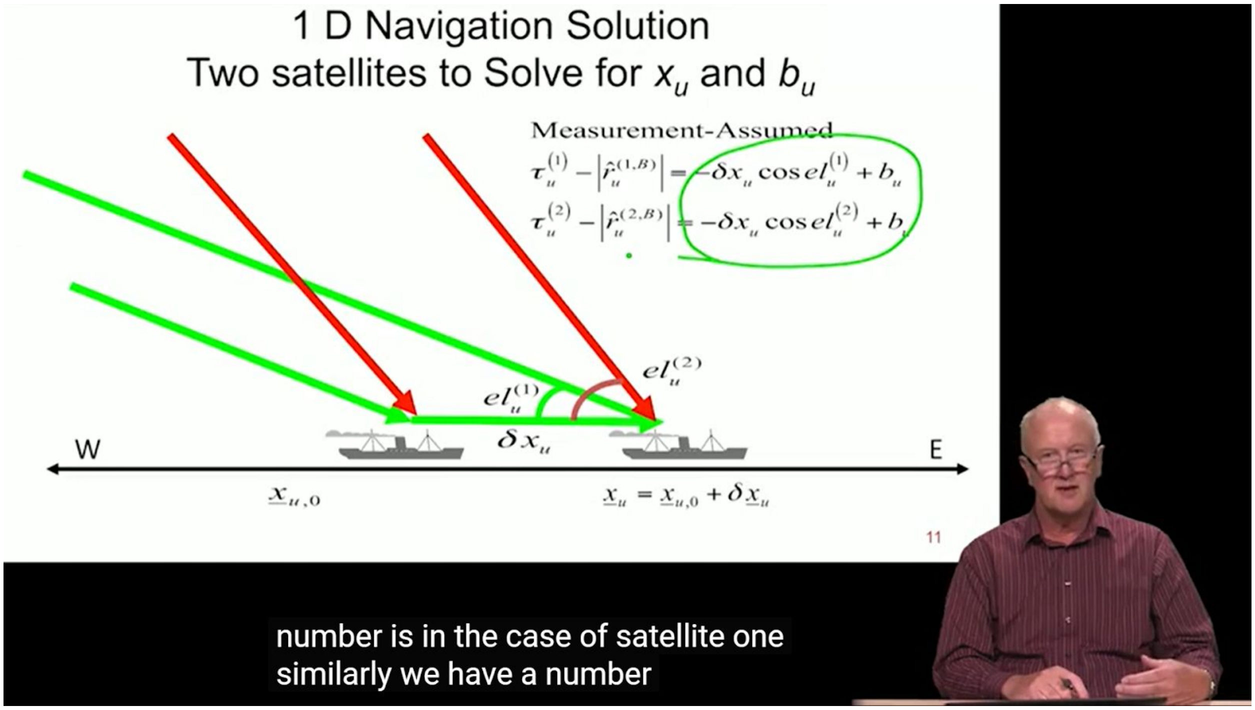

Navigation Solution
Pseudo-range & Theo-range
$\underline{x}_B^{(k)} = \underline{x}^{(k)}$
$\sqrt{\left(x_u - x^{(k)}\right)^2 + \left(y_u - y^{(k)}\right)^2 + \left(z_u - z^{(k)}\right)^2} + b_u$
$\sqrt{\left(x_{u,0} - x^{(k,B)}\right)^2 + \left(y_{u,0} - y^{(k,B)}\right)^2 + \left(z_{u,0} - z^{(k,B)}\right)^2}$
$\delta \underline{x}_u$
W
E
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
9
little bit more detail
same situation true

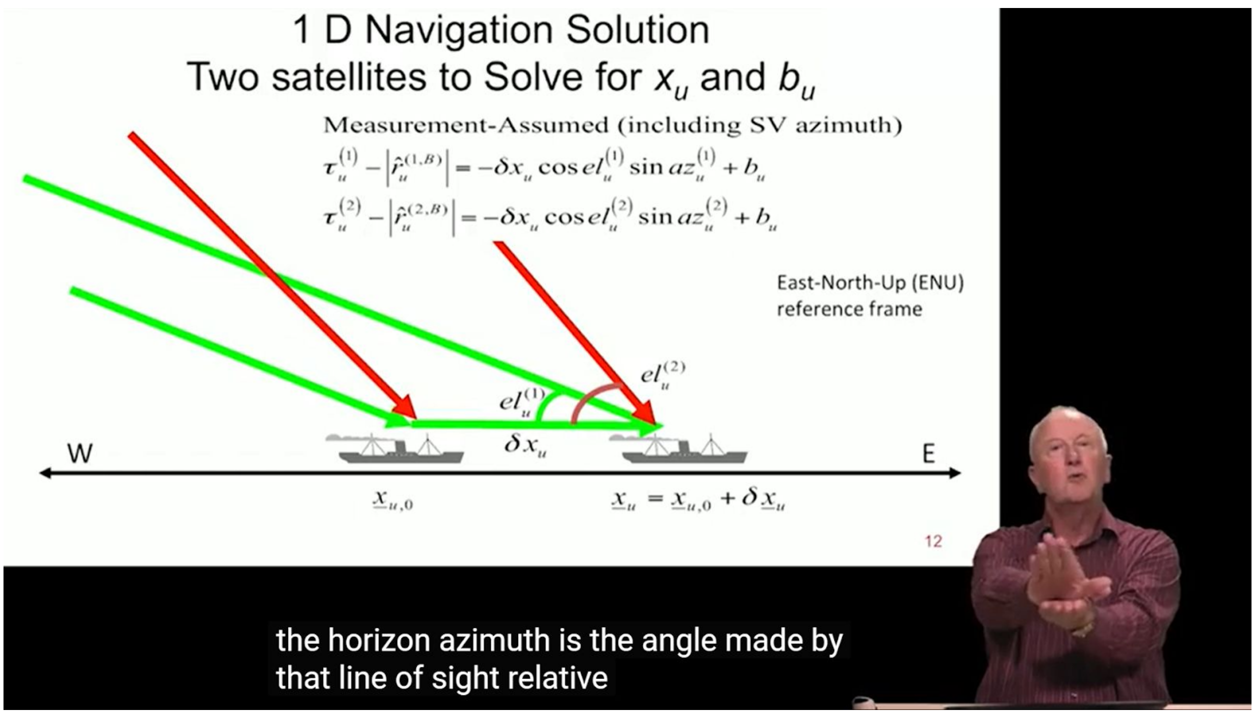

Navigation Solution
Pseudo-range & Theo-range
$\underline{x}_B^{(k)} = \underline{x}^{(k)}$
$\sqrt{\left(x_u - x^{(k)}\right)^2 + \left(y_u - y^{(k)}\right)^2 + \left(z_u - z^{(k)}\right)^2} + b_u$
$\sqrt{\left(x_{u,0} - x^{(k,B)}\right)^2 + \left(y_{u,0} - y^{(k,B)}\right)^2 + \left(z_{u,0} - z^{(k,B)}\right)^2}$
True
$\delta \underline{x}_u$
W
E
Assumed
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
9
same situation true location over here
assumed over here now we've provided

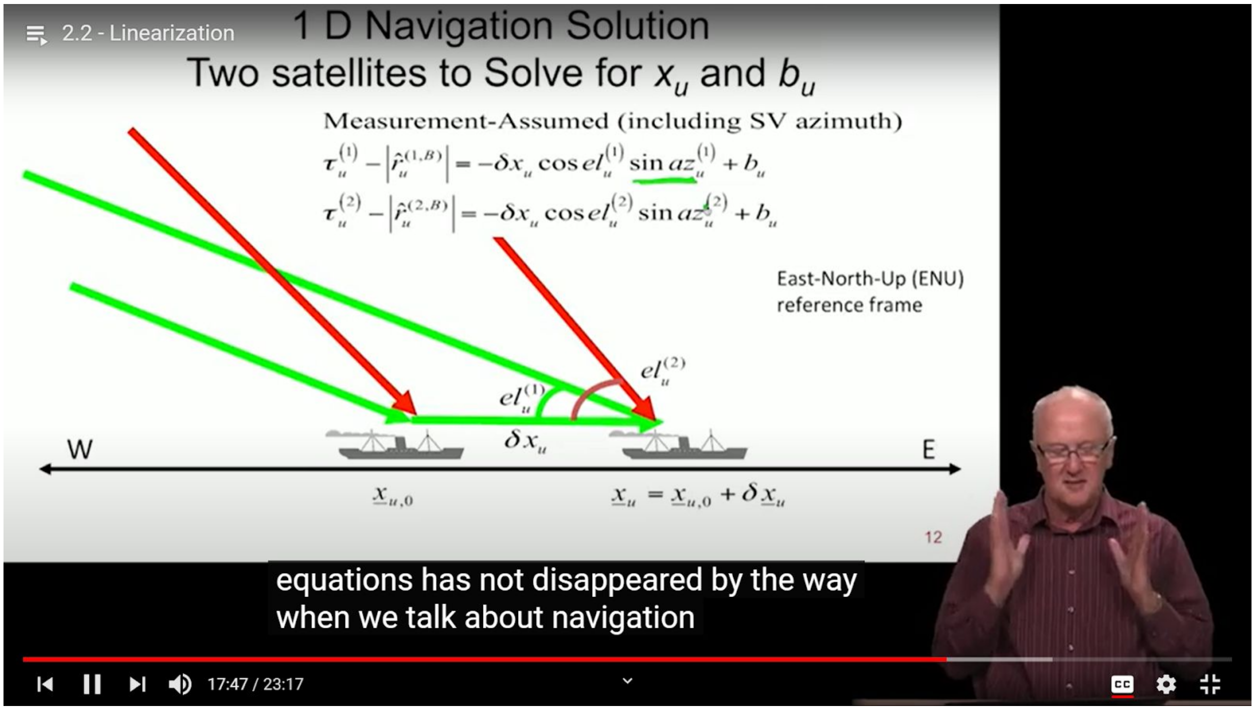

Navigation Solution
Pseudo-range & Theo-range
$\underline{x}_B^{(k)} = \underline{x}^{(k)}$
Measurement
$\sqrt{\left(x_u - x^{(k)}\right)^2 + \left(y_u - y^{(k)}\right)^2 + \left(z_u - z^{(k)}\right)^2} + b_u$
$\sqrt{\left(x_{u,0} - x^{(k,B)}\right)^2 + \left(y_{u,0} - y^{(k,B)}\right)^2 + \left(z_{u,0} - z^{(k,B)}\right)^2}$
True
$\delta \underline{x}_u$
W
E
Assumed
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
9
distance measurement in the case of the
true measurement and

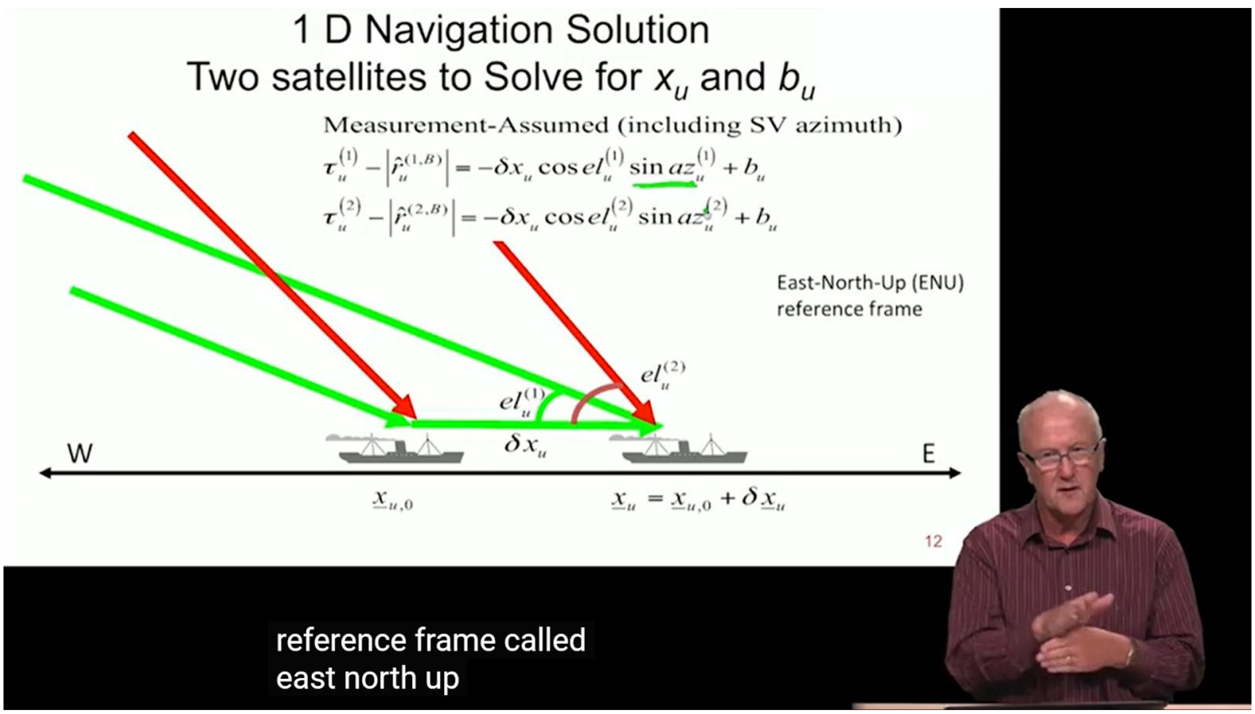

One Dimensional Navigation Solution
$\underline{x}_B^{(k)} = \underline{x}^{(k)}$
Measurement - Assumed
Pseudorange - Theorange
$\tau_u^{(k)} - \left|\hat{r}_u^{(k,B)}\right| = -\delta x_u \cos el_u^{(k)} + b_u$
$el_u^{(k)}$
W
δx
E
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
10
contained or somehow obliterated the
esta manda we

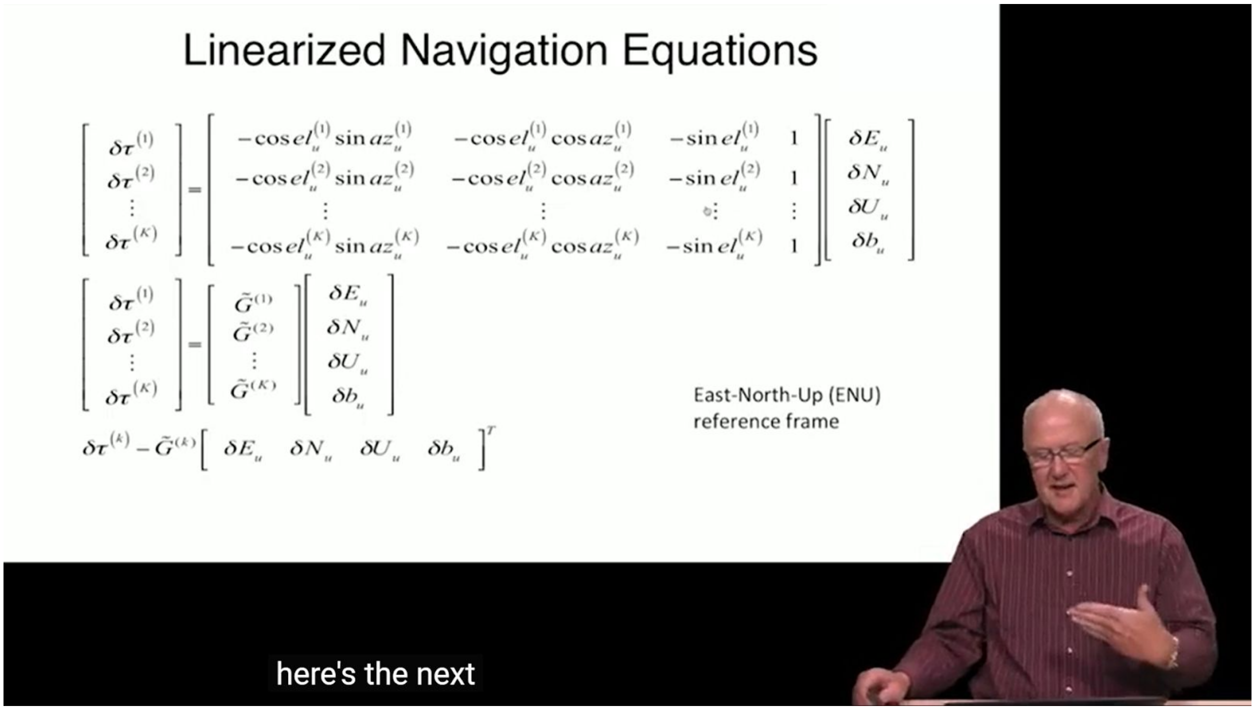

1 D Navigation Solution
Two satellites to Solve for x_u and b_u
Measurement-Assumed
$\tau_u^{(1)} - \left|\hat{r}_u^{(1,B)}\right| = -\delta x_u \cos el_u^{(1)} + b_u$
$\tau_u^{(2)} - \left|\hat{r}_u^{(2,B)}\right| = -\delta x_u \cos el_u^{(2)} + b_u$
$el_u^{(1)}$
$el_u^{(2)}$
W
δx_u
E
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
11
angles to be a little bit different at
least a little bit different

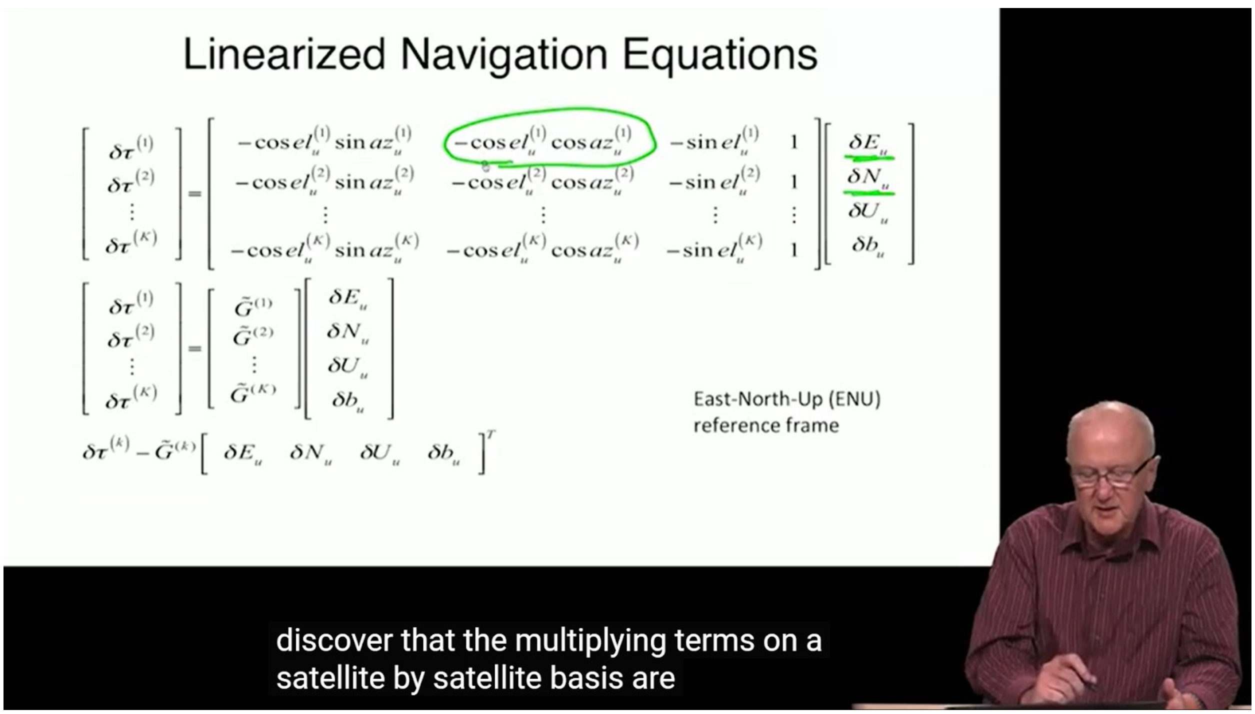

1 D Navigation Solution
Two satellites to Solve for x_u and b_u
Measurement-Assumed
$\tau_u^{(1)} - \left|\hat{r}_u^{(1,B)}\right| = -\delta x_u \cos el_u^{(1)} + b_u$
$\tau_u^{(2)} - \left|\hat{r}_u^{(2,B)}\right| = -\delta x_u \cos el_u^{(2)} + b_u$
$el_u^{(1)}$
$el_u^{(2)}$
δx_u
W
E
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
11
number is in the case of satellite one
similarly we have a number

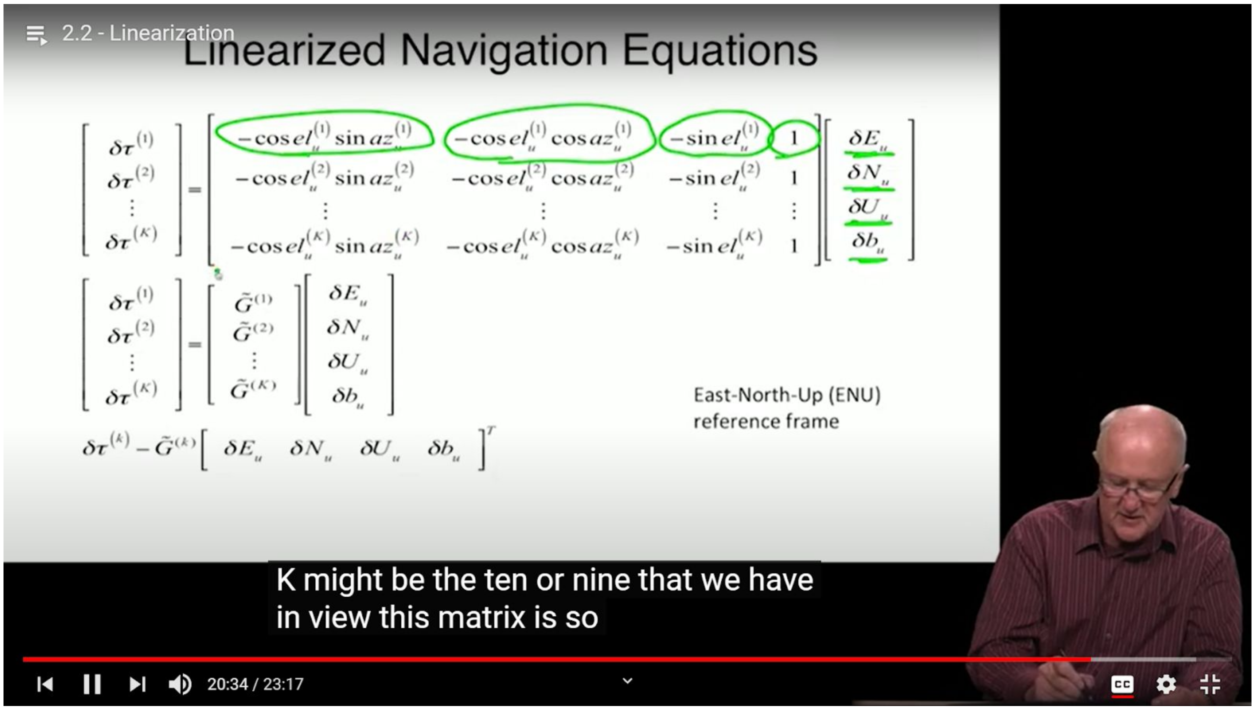

1 D Navigation Solution
Two satellites to Solve for x_u and b_u
Measurement-Assumed (including SV azimuth)
$\tau_u^{(1)} - \left|\hat{r}_u^{(1,B)}\right| = -\delta x_u \cos el_u^{(1)} \sin az_u^{(1)} + b_u$
$\tau_u^{(2)} - \left|\hat{r}_u^{(2,B)}\right| = -\delta x_u \cos el_u^{(2)} \sin az_u^{(2)} + b_u$
East-North-Up (ENU)
reference frame
$el_u^{(1)}$
$el_u^{(2)}$
δx_u
W
E
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
12
the horizon azimuth is the angle made by
that line of sight relative


2.2 - Linearization
1 D Navigation Solution
Two satellites to Solve for x_u and b_u
Measurement-Assumed (including SV azimuth)
$\tau_u^{(1)} - \left|\hat{r}_u^{(1,B)}\right| = -\delta x_u \cos el_u^{(1)} \sin az_u^{(1)} + b_u$
$\tau_u^{(2)} - \left|\hat{r}_u^{(2,B)}\right| = -\delta x_u \cos el_u^{(2)} \sin az_u^{(2)} + b_u$
East-North-Up (ENU)
reference frame
$el_u^{(2)}$
$el_u^{(1)}$
δx_u
W
E
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
12
equations has not disappeared by the way
when we talk about navigation
17:47 / 23:17

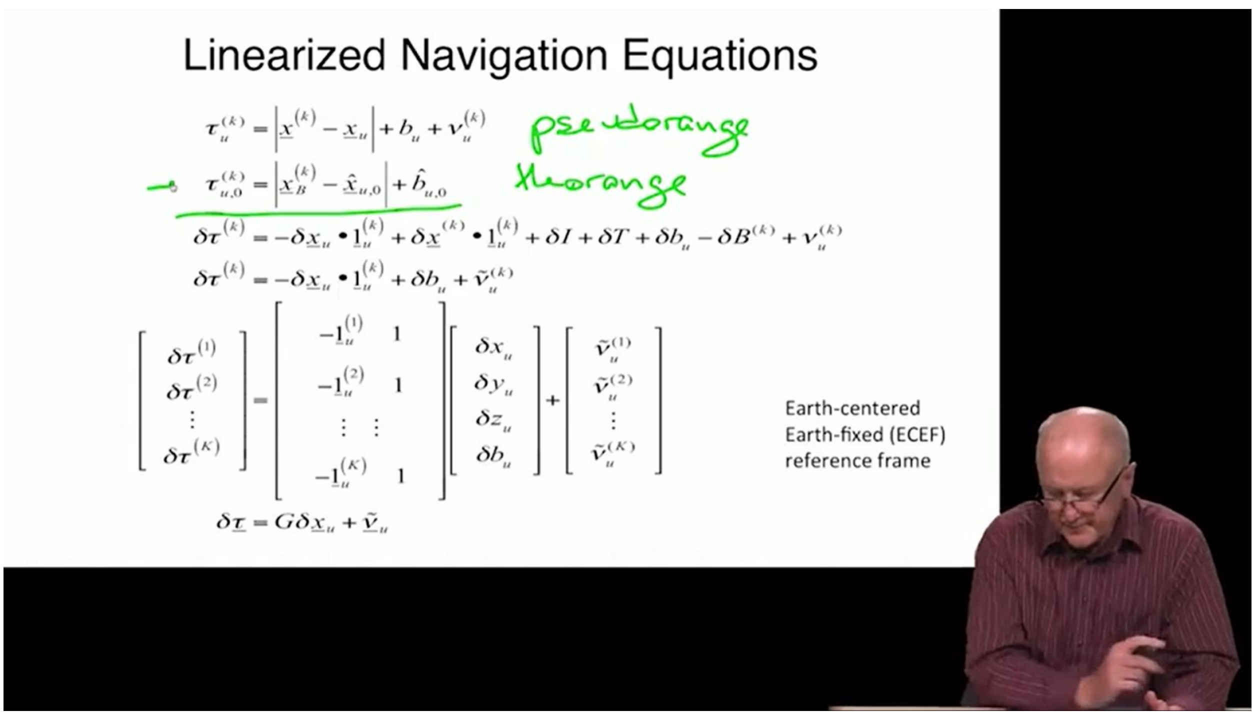

1 D Navigation Solution
Two satellites to Solve for x_u and b_u
Measurement-Assumed (including SV azimuth)
$\tau_u^{(1)} - \left|\hat{r}_u^{(1,B)}\right| = -\delta x_u \cos el_u^{(1)} \sin az_u^{(1)} + b_u$
$\tau_u^{(2)} - \left|\hat{r}_u^{(2,B)}\right| = -\delta x_u \cos el_u^{(2)} \sin az_u^{(2)} + b_u$
East-North-Up (ENU)
reference frame
$el_u^{(2)}$
$el_u^{(1)}$
δx_u
W
E
$\underline{x}_{u,0}$
$\underline{x}_u = \underline{x}_{u,0} + \delta \underline{x}_u$
12
reference frame called
east north up


Linearized Navigation Equations
$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} -\cos el_u^{(1)} \sin az_u^{(1)} & -\cos el_u^{(1)} \cos az_u^{(1)} & -\sin el_u^{(1)} & 1 \\ -\cos el_u^{(2)} \sin az_u^{(2)} & -\cos el_u^{(2)} \cos az_u^{(2)} & -\sin el_u^{(2)} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\cos el_u^{(K)} \sin az_u^{(K)} & -\cos el_u^{(K)} \cos az_u^{(K)} & -\sin el_u^{(K)} & 1 \end{bmatrix} \begin{bmatrix} \delta E_u \\ \delta N_u \\ \delta U_u \\ \delta b_u \end{bmatrix}$
$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} \tilde{G}^{(1)} \\ \tilde{G}^{(2)} \\ \vdots \\ \tilde{G}^{(K)} \end{bmatrix} \begin{bmatrix} \delta E_u \\ \delta N_u \\ \delta U_u \\ \delta b_u \end{bmatrix}$
$\delta\tau^{(k)} - \tilde{G}^{(k)} \begin{bmatrix} \delta E_u & \delta N_u & \delta U_u & \delta b_u \end{bmatrix}^T$
East-North-Up (ENU)
reference frame
here's the next

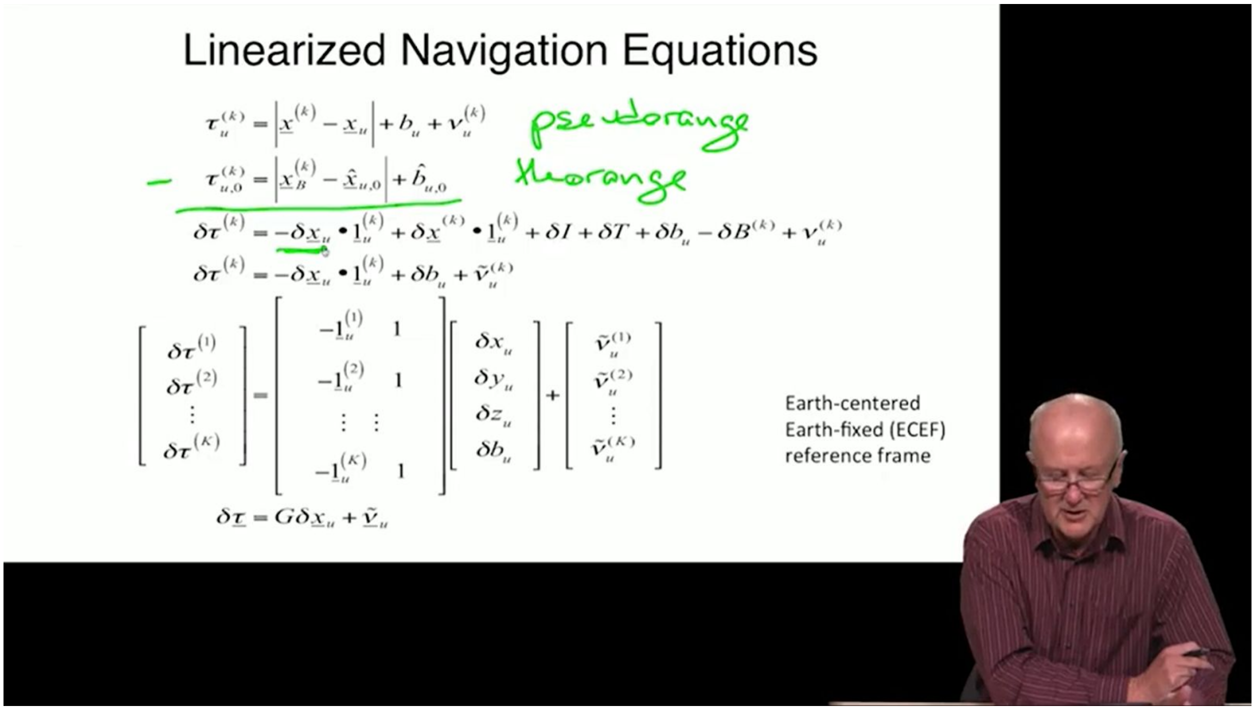

Linearized Navigation Equations
$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} -\cos el_u^{(1)} \sin az_u^{(1)} & -\cos el_u^{(1)} \cos az_u^{(1)} & -\sin el_u^{(1)} & 1 \\ -\cos el_u^{(2)} \sin az_u^{(2)} & -\cos el_u^{(2)} \cos az_u^{(2)} & -\sin el_u^{(2)} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\cos el_u^{(K)} \sin az_u^{(K)} & -\cos el_u^{(K)} \cos az_u^{(K)} & -\sin el_u^{(K)} & 1 \end{bmatrix} \begin{bmatrix} \delta E_u \\ \delta N_u \\ \delta U_u \\ \delta b_u \end{bmatrix}$
$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} \tilde{G}^{(1)} \\ \tilde{G}^{(2)} \\ \vdots \\ \tilde{G}^{(K)} \end{bmatrix} \begin{bmatrix} \delta E_u \\ \delta N_u \\ \delta U_u \\ \delta b_u \end{bmatrix}$
$\delta\tau^{(k)} - \tilde{G}^{(k)} \begin{bmatrix} \delta E_u & \delta N_u & \delta U_u & \delta b_u \end{bmatrix}^T$
East-North-Up (ENU)
reference frame
discover that the multiplying terms on a
satellite by satellite basis are

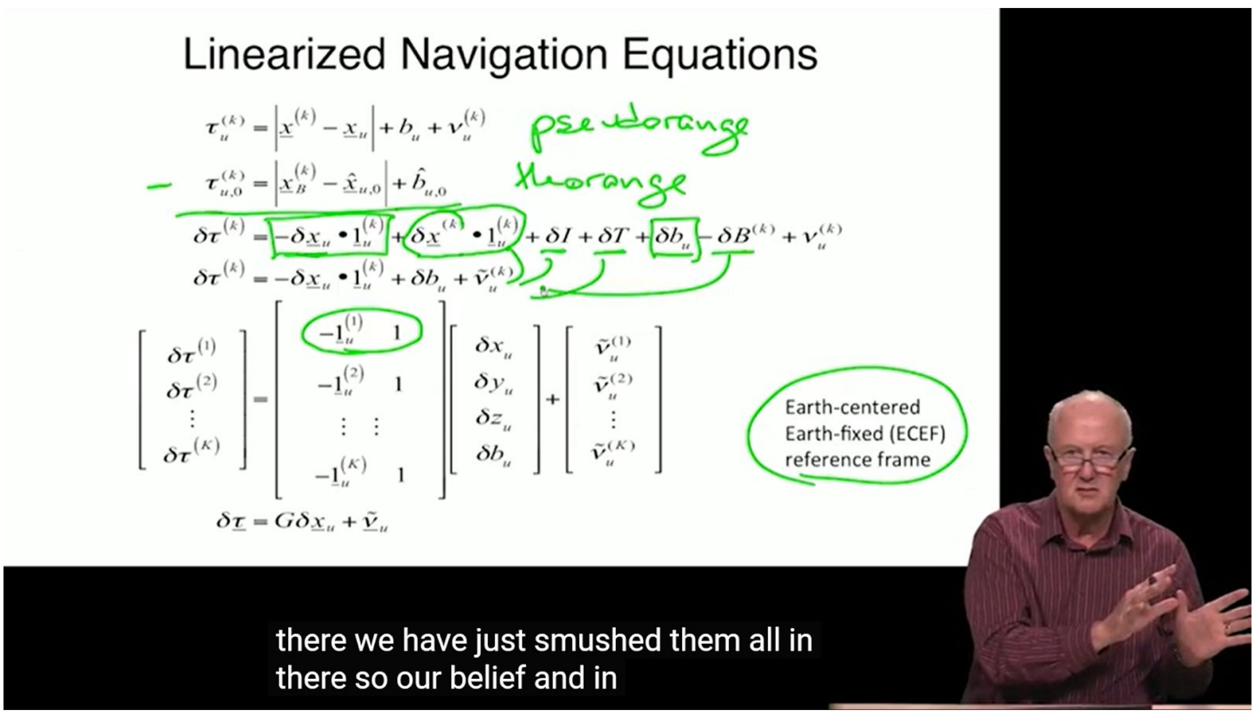


2.2 - Linearization
Linearized Navigation Equations
$$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} -\cos el_u^{(1)} \sin az_u^{(1)} & -\cos el_u^{(1)} \cos az_u^{(1)} & -\sin el_u^{(1)} & 1 \\ -\cos el_u^{(2)} \sin az_u^{(2)} & -\cos el_u^{(2)} \cos az_u^{(2)} & -\sin el_u^{(2)} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\cos el_u^{(K)} \sin az_u^{(K)} & -\cos el_u^{(K)} \cos az_u^{(K)} & -\sin el_u^{(K)} & 1 \end{bmatrix} \begin{bmatrix} \underline{\delta E_u} \\ \underline{\delta N_u} \\ \underline{\delta U_u} \\ \underline{\delta b_u} \end{bmatrix}$$
$$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} \tilde{G}^{(1)} \\ \tilde{G}^{(2)} \\ \vdots \\ \tilde{G}^{(K)} \end{bmatrix} \begin{bmatrix} \delta E_u \\ \delta N_u \\ \delta U_u \\ \delta b_u \end{bmatrix}$$
$$\delta\tau^{(k)} - \tilde{G}^{(k)} \begin{bmatrix} \delta E_u & \delta N_u & \delta U_u & \delta b_u \end{bmatrix}^T$$
East-North-Up (ENU) reference frame
K might be the ten or nine that we have
in view this matrix is so
20:34 / 23:17



Linearized Navigation Equations
$$\tau_u^{(k)} = \left| \underline{x}^{(k)} - \underline{x}_u \right| + b_u + v_u^{(k)}$$
pseudorange
$$\tau_{u,0}^{(k)} = \left| \underline{x}_B^{(k)} - \hat{\underline{x}}_{u,0} \right| + \hat{b}_{u,0}$$
theorange
$$\delta\tau^{(k)} = -\delta\underline{x}_u \bullet \underline{1}_u^{(k)} + \delta\underline{x}^{(k)} \bullet \underline{1}_u^{(k)} + \delta I + \delta T + \delta b_u - \delta B^{(k)} + v_u^{(k)}$$
$$\delta\tau^{(k)} = -\delta\underline{x}_u \bullet \underline{1}_u^{(k)} + \delta b_u + \tilde{v}_u^{(k)}$$
$$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} -\underline{1}_u^{(1)} & 1 \\ -\underline{1}_u^{(2)} & 1 \\ \vdots & \vdots \\ -\underline{1}_u^{(K)} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix} + \begin{bmatrix} \tilde{v}_u^{(1)} \\ \tilde{v}_u^{(2)} \\ \vdots \\ \tilde{v}_u^{(K)} \end{bmatrix}$$
$$\delta\underline{\tau} = G\delta\underline{x}_u + \tilde{\underline{v}}_u$$
Earth-centered
Earth-fixed (ECEF)
reference frame


Linearized Navigation Equations
$\tau_u^{(k)} = \left|\underline{x}^{(k)} - \underline{x}_u\right| + b_u + v_u^{(k)}$
pseudorange
$- \quad \tau_{u,0}^{(k)} = \left|\underline{x}_B^{(k)} - \hat{\underline{x}}_{u,0}\right| + \hat{b}_{u,0}$
theorange
$\delta\tau^{(k)} = -\delta\underline{x}_u \bullet \underline{1}_u^{(k)} + \delta\underline{x}^{(k)} \bullet \underline{1}_u^{(k)} + \delta I + \delta T + \delta b_u - \delta B^{(k)} + v_u^{(k)}$
$\delta\tau^{(k)} = -\delta\underline{x}_u \bullet \underline{1}_u^{(k)} + \delta b_u + \tilde{v}_u^{(k)}$
$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} -\underline{1}_u^{(1)} & 1 \\ -\underline{1}_u^{(2)} & 1 \\ \vdots & \vdots \\ -\underline{1}_u^{(K)} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix} + \begin{bmatrix} \tilde{v}_u^{(1)} \\ \tilde{v}_u^{(2)} \\ \vdots \\ \tilde{v}_u^{(K)} \end{bmatrix}$
Earth-centered
Earth-fixed (ECEF)
reference frame
$\delta\underline{\tau} = G\delta\underline{x}_u + \tilde{\underline{v}}_u$


Linearized Navigation Equations
$\tau_u^{(k)} = \left|\underline{x}^{(k)} - \underline{x}_u\right| + b_u + v_u^{(k)}$
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$\delta\tau^{(k)} = \boxed{-\delta\underline{x}_u \bullet \underline{1}_u^{(k)}} + \delta\underline{x}^{(k)} \bullet \underline{1}_u^{(k)} + \delta I + \delta T + \boxed{\delta b_u} - \delta B^{(k)} + v_u^{(k)}$
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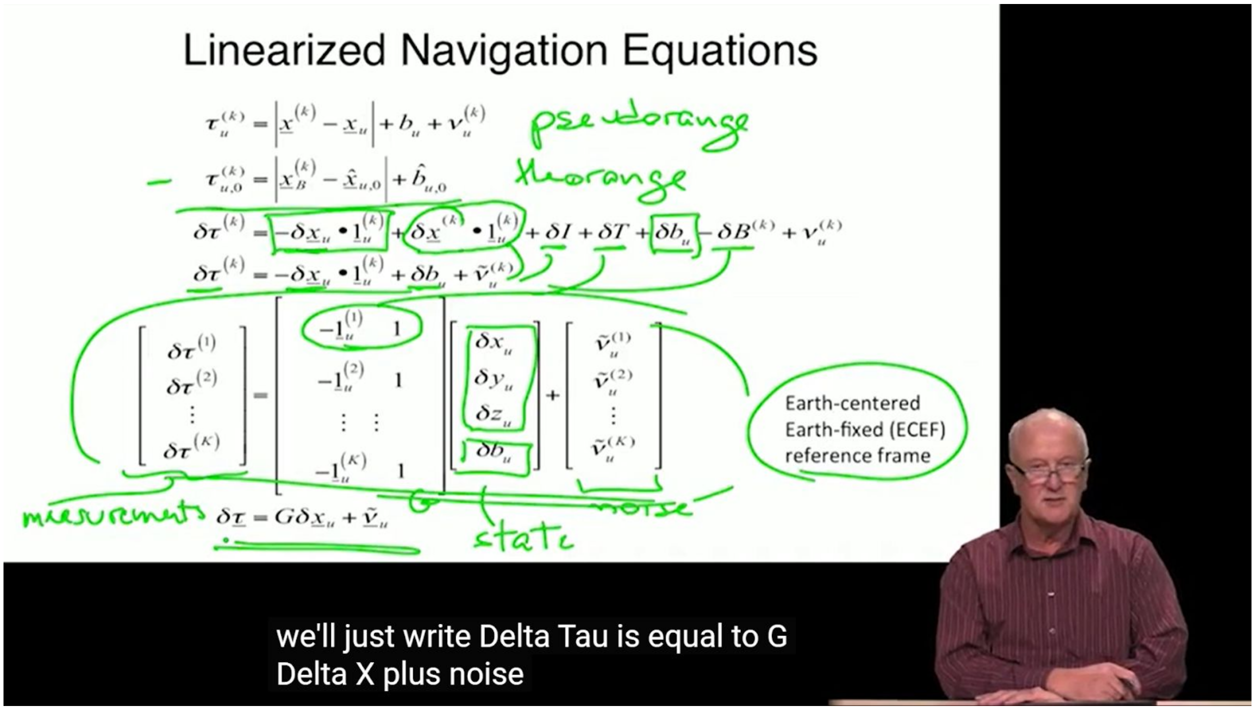
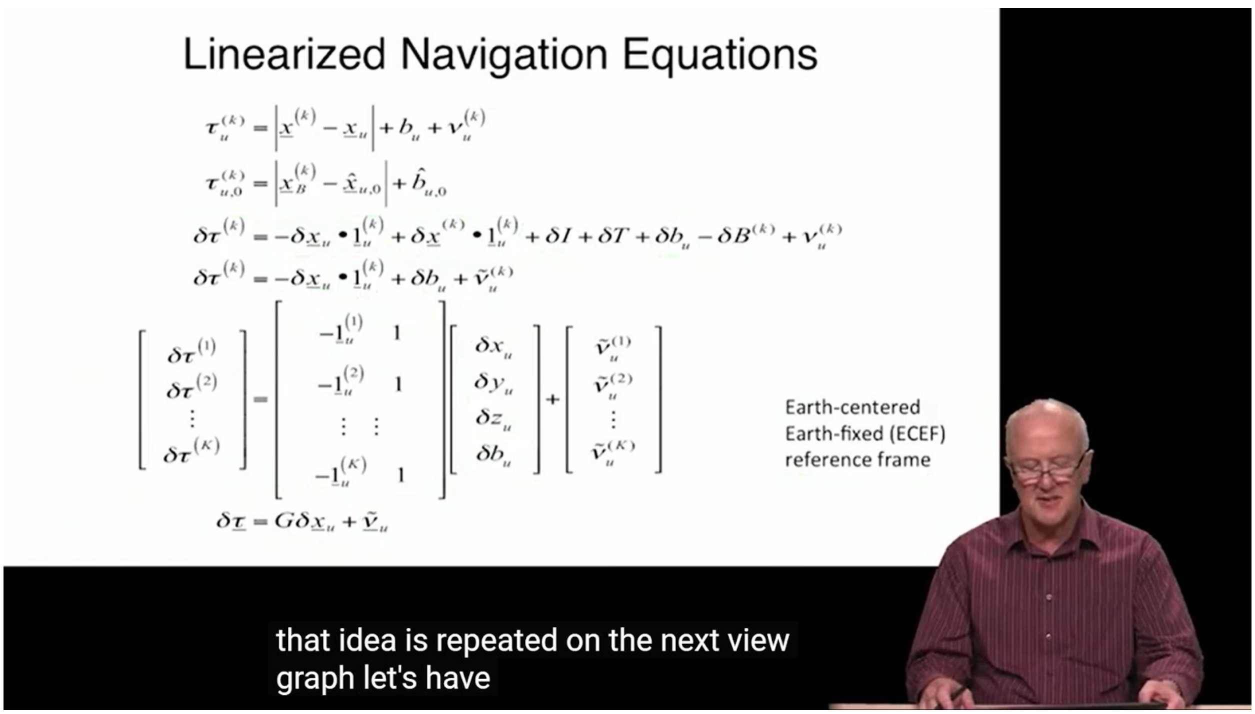
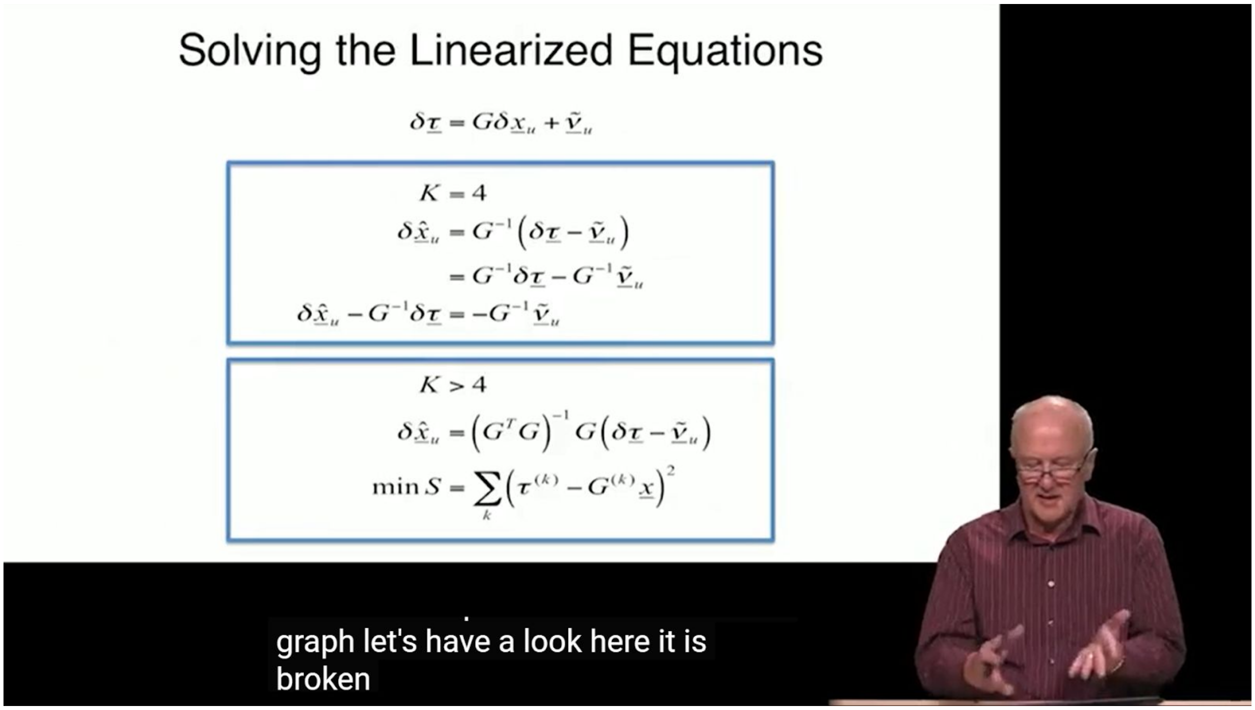
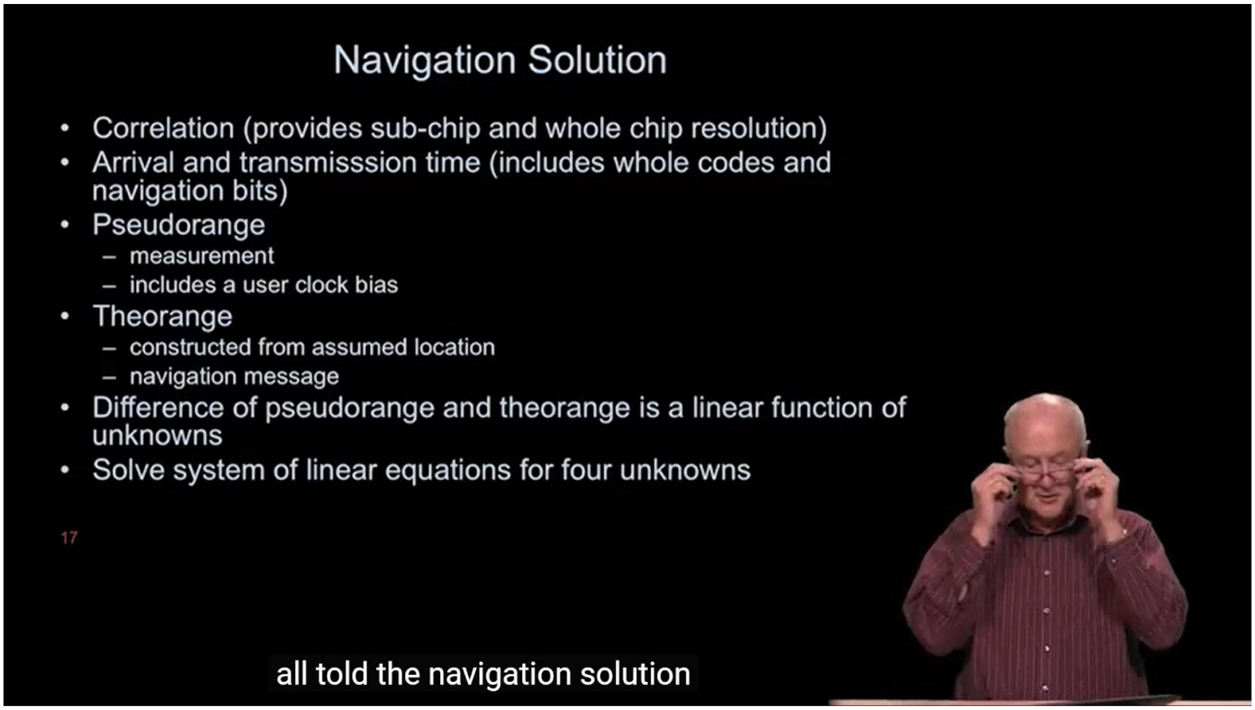
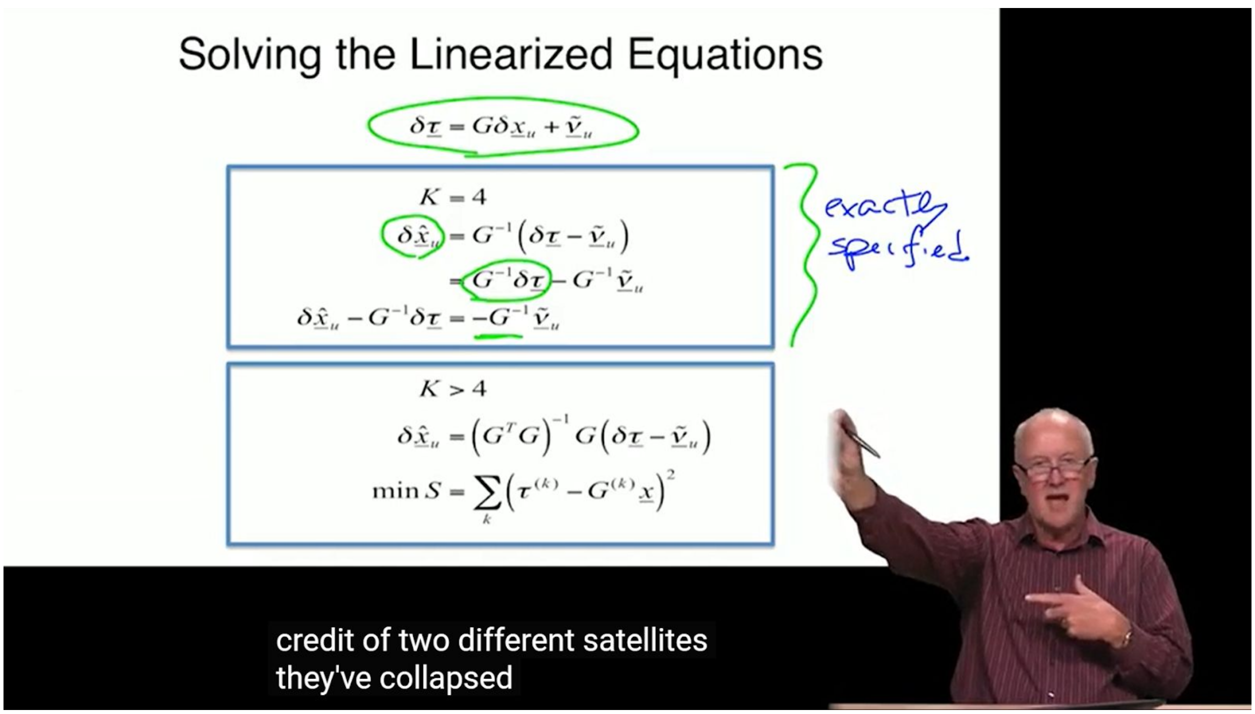
$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} -\underline{1}_u^{(1)} & 1 \\ -\underline{1}_u^{(2)} & 1 \\ \vdots & \vdots \\ -\underline{1}_u^{(K)} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix} + \begin{bmatrix} \tilde{v}_u^{(1)} \\ \tilde{v}_u^{(2)} \\ \vdots \\ \tilde{v}_u^{(K)} \end{bmatrix}$
Earth-centered
Earth-fixed (ECEF)
reference frame
$\delta\underline{\tau} = G\delta\underline{x}_u + \tilde{\underline{v}}_u$
there we have just smushed them all in
there so our belief and in


Linearized Navigation Equations
$\tau_u^{(k)} = \left| \underline{x}^{(k)} - \underline{x}_u \right| + b_u + v_u^{(k)}$ pseudorange
$- \tau_{u,0}^{(k)} = \left| \underline{x}_B^{(k)} - \hat{\underline{x}}_{u,0} \right| + \hat{b}_{u,0}$ theorange
$\delta\tau^{(k)} = \boxed{-\delta\underline{x}_u \bullet \underline{1}_u^{(k)}} + \delta\underline{x}^{(k)} \bullet \underline{1}_u^{(k)} + \delta I + \delta T + \boxed{\delta b_u} - \delta B^{(k)} + v_u^{(k)}$
$\delta\tau^{(k)} = -\delta\underline{x}_u \bullet \underline{1}_u^{(k)} + \delta b_u + \tilde{v}_u^{(k)}$
$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} -\underline{1}_u^{(1)} & 1 \\ -\underline{1}_u^{(2)} & 1 \\ \vdots & \vdots \\ -\underline{1}_u^{(K)} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix} + \begin{bmatrix} \tilde{v}_u^{(1)} \\ \tilde{v}_u^{(2)} \\ \vdots \\ \tilde{v}_u^{(K)} \end{bmatrix}$
Earth-centered
Earth-fixed (ECEF)
reference frame
measurements $\delta\underline{\tau} = G\delta\underline{x}_u + \tilde{\underline{v}}_u$
G
state
noise
we'll just write Delta Tau is equal to G
Delta X plus noise


Linearized Navigation Equations
$\tau_u^{(k)} = \left| \underline{x}^{(k)} - \underline{x}_u \right| + b_u + v_u^{(k)}$
$\tau_{u,0}^{(k)} = \left| \underline{x}_B^{(k)} - \hat{\underline{x}}_{u,0} \right| + \hat{b}_{u,0}$
$\delta\tau^{(k)} = -\delta\underline{x}_u \bullet \underline{1}_u^{(k)} + \delta\underline{x}^{(k)} \bullet \underline{1}_u^{(k)} + \delta I + \delta T + \delta b_u - \delta B^{(k)} + v_u^{(k)}$
$\delta\tau^{(k)} = -\delta\underline{x}_u \bullet \underline{1}_u^{(k)} + \delta b_u + \tilde{v}_u^{(k)}$
$\begin{bmatrix} \delta\tau^{(1)} \\ \delta\tau^{(2)} \\ \vdots \\ \delta\tau^{(K)} \end{bmatrix} = \begin{bmatrix} -\underline{1}_u^{(1)} & 1 \\ -\underline{1}_u^{(2)} & 1 \\ \vdots & \vdots \\ -\underline{1}_u^{(K)} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix} + \begin{bmatrix} \tilde{v}_u^{(1)} \\ \tilde{v}_u^{(2)} \\ \vdots \\ \tilde{v}_u^{(K)} \end{bmatrix}$
Earth-centered
Earth-fixed (ECEF)
reference frame
$\delta\underline{\tau} = G\delta\underline{x}_u + \tilde{\underline{v}}_u$
that idea is repeated on the next view
graph let's have


Solving the Linearized Equations
$\delta\underline{\tau} = G\delta\underline{x}_u + \underline{\tilde{v}}_u$
$K = 4$
$\delta\underline{\hat{x}}_u = G^{-1}\left(\delta\underline{\tau} - \underline{\tilde{v}}_u\right)$
$= G^{-1}\delta\underline{\tau} - G^{-1}\underline{\tilde{v}}_u$
$\delta\underline{\hat{x}}_u - G^{-1}\delta\underline{\tau} = -G^{-1}\underline{\tilde{v}}_u$
$K > 4$
$\delta\underline{\hat{x}}_u = \left(G^T G\right)^{-1} G\left(\delta\underline{\tau} - \underline{\tilde{v}}_u\right)$
$\min S = \sum_k \left(\tau^{(k)} - G^{(k)}\underline{x}\right)^2$
graph let's have a look here it is
broken


Navigation Solution
• Correlation (provides sub-chip and whole chip resolution)
• Arrival and transmisssion time (includes whole codes and navigation bits)
• Pseudorange
– measurement
– includes a user clock bias
• Theorange
– constructed from assumed location
– navigation message
• Difference of pseudorange and theorange is a linear function of unknowns
• Solve system of linear equations for four unknowns
17
all told the navigation solution


Solving the Linearized Equations
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$= G^{-1}\delta\underline{\tau} - G^{-1}\underline{\tilde{v}}_u$
$\delta\underline{\hat{x}}_u - G^{-1}\delta\underline{\tau} = -G^{-1}\underline{\tilde{v}}_u$
exactly specified
$K > 4$
$\delta\underline{\hat{x}}_u = \left(G^T G\right)^{-1} G\left(\delta\underline{\tau} - \underline{\tilde{v}}_u\right)$
$\min S = \sum_k \left(\tau^{(k)} - G^{(k)}\underline{x}\right)^2$
credit of two different satellites
they've collapsed