Cyclic routing of trucks for mid-mile delivery

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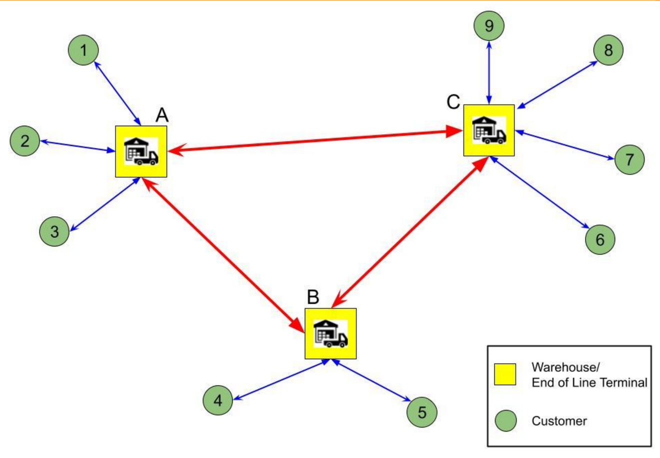
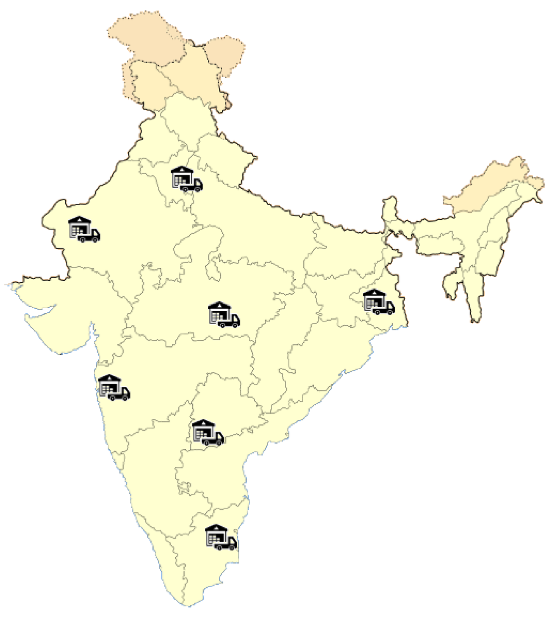
Middle-mile logistics

- It is the process of moving goods between warehouses or distribution centres (usually through trucks or other modes of transport, e.g. rail, airplane, ship)
- The centres are usually far apart (e.g. different cities)
- The movement of goods is done in bulk, to take the advantage of economy of scale.
- The red arrows in the diagram constitute the 'middle-mile' of a hub-and-spoke logistic network.



Hub-and-spoke network diagram showing the distribution centres



Few of the distribution centres across the country, represented on a map of India.

- The distribution centres perform 3 main functions:
  1. Receive goods (unloading) which are to be delivered to customers near it.
  2. Send goods (loading), which originated from customers near it, to other distribution centers.
  3. Transfer goods (transshipment) from one vehicle to another, if required.
- The mode of transport for our problem is trucks, considered to be of a single type, i.e. all the trucks have the same load carrying capacity (different types of trucks and trailers can also be added later).

Problem inputs

- A logistic company has an *n* number of distribution centers (DC_1, DC_2, . . . , DC_n), located in various parts of the country. The quantity of goods that need to be moved daily from an originating DC-i to a destination DC-j, measured in full truckloads, is known. **(Demand is considered deterministic).** These values ($b_{i,j}$) are available as a matrix of demands in the following format, with originating DCs in rows and destinations in columns.

	DC_1	DC_2	. . .	DC_n
DC_1	--	$b_{1,2}$		$b_{1,n}$
DC_2	$b_{2,1}$	--		$b_{2,n}$
. . .				
DC_n	$b_{n,1}$	$b_{n,2}$		--

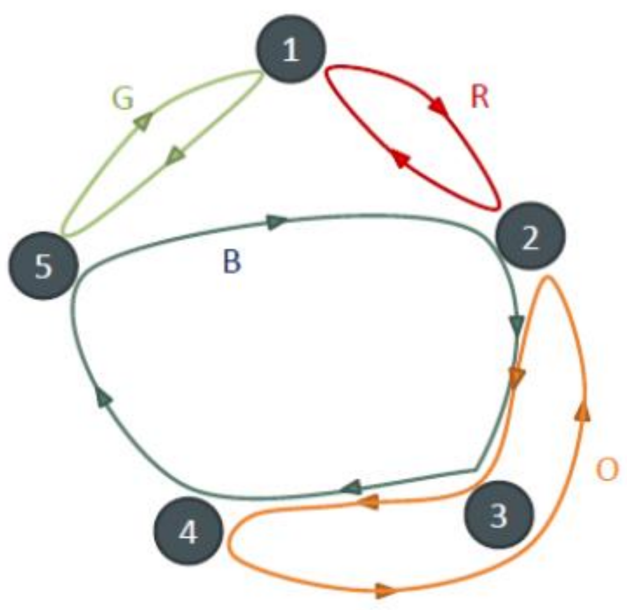
	DC_1	DC_2	. . .	DC_n
DC_1	–	$T_{1,2}$		$T_{1,n}$
DC_2	$T_{2,1}$	–		$T_{2,n}$
. . .				
DC_n	$T_{n,1}$	$T_{n,2}$		–

- All goods leaving from their origin DC have a target time (in number of days), within which the package must reach the destination DC. These times ($T_{i,j}$) are given in the following format (with origins in rows and destinations in columns).
- has a large fleet of identical trucks, each with a capacity *K* (where *K* is the full truck load).
- The time (number of days) required by a truck to move from a DC-i to DC-j directly without stopping in between at any other DC is also known ($t_{i,j}$). This is represented as the travel-time matrix, as shown.

	DC_1	DC_2	. . .	DC_n
DC_1	--	$t_{1,2}$		$t_{1,n}$
DC_2	$t_{2,1}$	--		$t_{2,n}$
. . .				
DC_n	$t_{n,1}$	$t_{n,2}$		--

Cyclic routes

- The company wants to design cyclic routes consisting of two or more DCs, so that empty movements of trucks are avoided. There can be multiple identical routes, each being used by a single truck.
- The following diagram shows four circular routes (where four trucks are moving) on a network of five DCs.



The DCs are numbered 1 to 5, and one possible solution is:

Route 1: 1 -> 5 -> 1 (route G)
Route 2: 1 -> 2 -> 1 (route R)
Route 3: 2 -> 3 -> 4 -> 5 -> 2 (route B)
Route 4: 2 -> 3 -> 4 -> 3 -> 2 (route O)

In the above solution, goods originating at DC-5 and destined for DC-3, can travel in one of the following four ways:

5 -> 2 , 2 -> 3 (one stopover enroute)
5 -> 2 , 2 -> 3 (one transshipment enroute)
5 -> 1 , 1 -> 2 , 2 -> 3 (two transshipments enroute)
5 -> 1 , 1 -> 2 , 2 -> 3 (two transshipments enroute)

Time delays

- A **stopover** (without a change of routes) incurs a time T_1 (1 day) for all goods on that route.
- A **transshipment** (change of trucks) incurs a time of T_2 (2 days).
- These times are also known and fixed for all locations.

Costs

The company wants to select the routes to ensure that the overall cost of the operations is minimized. The cost of a design has two components:

1. **Transportation cost** - refers to the distance travelled by all the trucks moving in all the routes. It is the sum of the transportation cost of each route in the network. For a selected route, its transportation cost is defined as the number of days required to complete it.
2. **Penalty cost** - only applicable if goods do not reach their destination within the respective target times. It is calculated as the product of three quantities - the quantity of goods (in truckloads), the time by which they are late (in days) and some factor for penalty.

Example problem

For a network of 4 distribution centres, the following inputs are available.

Demand matrix (in units of truckload)

	DC_1	DC_2	DC_3	DC_4
DC_1	–	0	0	1.3
DC_2	0	–	0	0.1
DC_3	0	0	–	0.5
DC_4	1.8	0	0	–

Target-time matrix (in days)

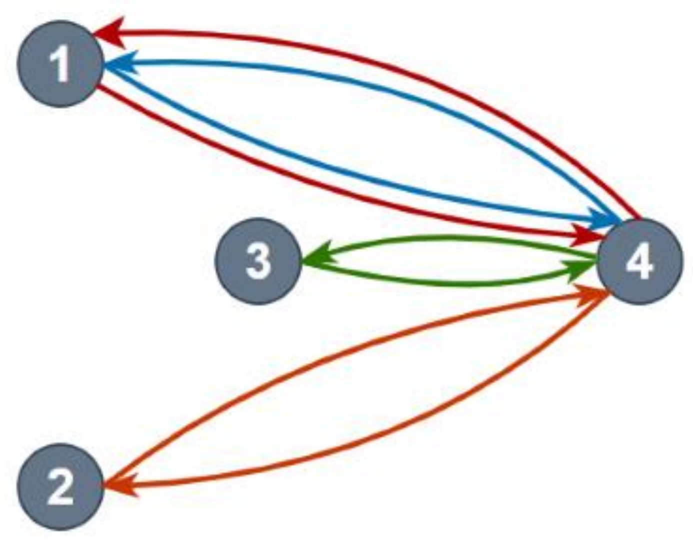
	DC_1	DC_2	DC_3	DC_4
DC_1	–	–	–	6
DC_2	–	–	–	6
DC_3	–	–	–	5
DC_4	6	–	–	–

Travel-time matrix (in days)

	DC_1	DC_2	DC_3	DC_4
DC_1	–	2	2	5
DC_2	2	–	2	5
DC_3	2	2	–	4
DC_4	5	5	4	–

- The company values timely delivery of packages. So a high Penalty Cost Factor of 30 is used.
- Two model solutions, not necessarily optimal, are shown below:

Solution 1



This solution has four routes (red, blue, green and orange). Two of them are identical (red and blue), in order to cater to demand higher than one truckload.

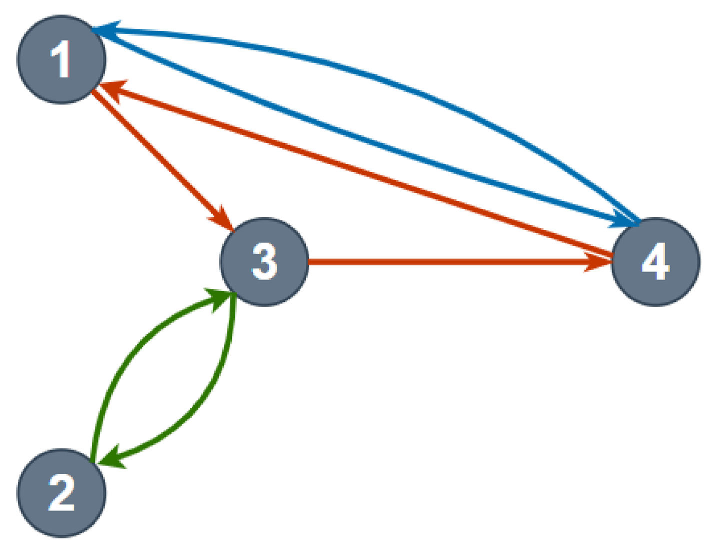
Transportation Cost = (5+5) + (5+5) + (4+4) + (5+5) = 38

Penalty Cost

- **DC-1 -> DC-4** A total demand of 1.3 truckloads is routed as 1 truckload through the blue route, and 0.3 truckload through the red route. The paths from DC-1 to DC-4 through both the routes take 5 days (which is less than the target
- **DC-2 -> DC-4** Demand of 0.1 truckload is routed entirely through the green route, and takes 5 days, which is less than the target time of 6 days. Hence, no penalty cost.
- **DC-3 -> DC-4** Demand of 0.5 truckload is routed entirely through the orange route, and takes 4 days, which is less than the target time of 5 days. Hence, no penalty cost.
- **DC-4 -> DC-1** A total demand of 1.8 truckloads is routed as 1 truckload through the blue route and 0.8 truckload through the red route. The paths from DC-4 to DC-1 through both the routes take 5 days (which is less than the target time of 6 days). Hence, no penalty cost.

- All the packages reach in time in this solution.
- Therefore, the total **Penalty Cost = 0**
- So, the total cost of the solution is 38 + 0 = **38**.

Solution 2



This solution has three routes (blue, orange and green).

Transportation cost = (2+4+5) + (5+5) + (2+2) = 25

Penalty Cost

- **DC-1 -> DC-4** A total demand of 1.3 truckloads is routed as 1 truckload through the blue route and 0.3 truckload through the orange route. The blue route takes 5 days(which is less than the target time of 6 days), while the orange route takes 2 + 1(stopover time) + 4 = 7 days (which is more than the target time by 1 day). Hence, the penalty cost for this path is 0.3 x 1 x 30 = 9.
- **DC-2 -> DC-4** A demand of 0.1 truckload is routed through the green and orange route with one transhipment in between. Hence, for the packages to reach DC-4 from DC-2, it takes 2 + 2(transhipment time) + 4 = 8 days (which is 2 days more than the target time of 6 days). Therefore, the penalty cost for this path is 0.1 x 2 x 30 = 6.
- **DC-3 -> DC-4** Entire demand of 0.5 truckload is routed through the orange route, which takes 4 days (that is less than the promised time of 5 days) to reach from DC-3 to DC-4. So, no penalty cost for this path.
- **DC-4 -> DC-1** The total demand of 1.8 truckloads is routed as 1 truckload through the blue route and 0.8 truckload through the orange route. The blue route, to go from DC-4 to DC-1, takes 5 days (which is less than the target time of 6 days). Hence, no penalty for this path. Also, the orange route, to go from DC-4 to DC-1, takes 5 days (which is less than the target time). Hence, no penalty for this path as well.

- Therefore, the total **penalty cost = 9 + 6 = 15**.
- So, the total cost of this solution is 25 + 15 = **40**.

Thus, it can be said that solution 1 is better than solution 2.

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