Introduction to Machine Learning (CS771)

HW Assignment - 3 (Bonus)

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Problem :- 1

Given, $S = \frac{1}{N} X^T X$ ...(1) and let, $B = \frac{1}{N} X X^T$ ..... (2)

For this problem we need to show, how using the eigen vector $v \in R^N$ of B we can derive the eigen vector of S ($u \in R^D$)

We know that,

$\lambda v = Bv$ ...(3), where λ is the corresponding eigen value

$\therefore \lambda v = \frac{1}{N} X X^T v$ .... (4) (Using equation (2))

Now, multiplying both sides by X^T on the left, we get.

$\Rightarrow \lambda X^T v = \frac{1}{N} X^T X X^T v$ .... (5)

$\Rightarrow \lambda (X^T v) = \frac{1}{N} (X^T X)(X^T v)$ .... (6)

$\Rightarrow \lambda (X^T v) = S (X^T v)$ ..... (7)

$\therefore X^T v$ is the corresponding eigen vector of S for eigen value λ

$\therefore u = X^T v$ ...... (8)

Since it is specified $D > N$, the dimension of S will be $D \times D$ and dimension of B will be $N \times N$. Now, as N is smaller than D, solving the eigen decomposition problem for B will be faster than solving it for S. Thus, it will be advantageous for this case.