

## Question 1:

Here the Binomial distribution is applicable.

As we know the 3 conditions pertaining to the binomial distribution are :

1. All trials be independent of each other.
2. The number of total trials is fixed (here it is 10).
3. We can get only two outcomes from each trial (for example like while tossing a coin we can only get heads or tails and nothing else).

Calculating the probability.

If  $A$  be the event that the drug produces better results &  $B$  be the event that the drug produces not as good results.

$$\therefore 4P(B) = P(A) \quad [P(B) + P(A) = 1]$$

we get,

$$P(B) = \frac{1}{5} \text{ and } P(A) = \frac{4}{5}$$

So, out of 10 trials if we pick  $x$  amount of drugs that do not produce better results than the other drug.

$$\therefore P(x) = \sum_{x=0}^3 \binom{10}{x} \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{10-x}$$

$$\approx \text{0.88}$$

## Question 2 :

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

- a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.
- b.) Find the required range.

Let  $\bar{x}$  be the sample mean.

Let  $y$  be the sample standard deviation. Standard deviation is usually denoted by  $\sigma$ .

The population standard deviation is unknown. But since the sample size is more than 30, we will use the  $z$  distribution.

In the second question, you used the  $t$ -table to find the value of  $Z_c$  for sample size = 32 and a significance level of 5%. If you use the  $z$ -table for the same, you would get the same value of  $Z_c$ , since, for sample size  $\geq 30$ , the  $t$ -distribution is the same as the  $z$ -distribution.

So for 95% confidence level, we know that the value for  $Z_c$  will be 1.96.

Confidence Level	z*-value
80%	1.28
90%	1.645 (by convention)
95%	1.96
98%	2.33
99%	2.58

$Z_c = 1.96$ ,  $x = 207$ ,  $y = 65$ .

Standard error for 95% CI =  $Z_c \times (\text{standard deviation} / \text{square root of sample size})$ .

Handwritten calculation showing the standard error (SE) and the resulting confidence interval for the population mean.

$$\begin{aligned} SE &= Z_c \times \left( \frac{y}{\sqrt{n}} \right) \\ &= 1.96 \times \left( \frac{65}{\sqrt{100}} \right) \\ &= 1.96 \times \left( \frac{65}{10} \right) \\ &= 12.74 \end{aligned}$$

Population mean =  $x \pm SE$

$$\begin{aligned} &= 207 \pm 12.74 \\ &= 194.26, 219.74 \end{aligned}$$

So with 95% confidence we can estimate that the mean time of effect will be between 194.26 seconds to 219.74 seconds.

### Question 3:

The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job.

So the null hypothesis( $H_0$ ) :

$H_0$  : time  $\leq$  200 seconds.

Alternate hypothesis

$H_1$ : time  $>$  200 seconds.

Sample size ( $n$ ) = 100

Sample mean( $\bar{x}$ ) = 207 seconds.

Standard deviation ( $y$ ) = 65 seconds.

Significance = 5%

Using the P value method, we fail to reject the null hypothesis.

The null and alternate hypothesis are the same as above.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{65}{\sqrt{100}} = 65/10 = 6.5 \quad \left[ \begin{array}{l} \sigma = \text{standard deviation} \\ n = \text{sample size} \end{array} \right]$$

$$Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{207 - 200}{6.5} = 1.07. \quad \left[ \begin{array}{l} \bar{x} = \text{sample mean} \\ \mu_{\bar{x}} = \text{mean as given in hypothesis} \end{array} \right]$$

Calculating p-value.

Looking for 1.07 in Z-table.

$$Z_{1.07} = 0.8577$$

$$p\text{-value} = 1 - 0.8577 = 0.1423 ; \alpha = 0.05$$

$$\therefore 14.23\% > 5\%$$

$\therefore$  we fail to reject the null hypothesis.

Using Critical value method we fail to reject the null hypothesis.

Critical value method:

$$\bar{\sigma}_x = \frac{\sigma}{\sqrt{n}} = \frac{65}{\sqrt{100}} = 6.5 \quad [\bar{\sigma}_x \rightarrow \text{standard error}]$$
$$\alpha = 0.05$$
$$Z_{\text{score}} = 1 - 0.025 = 0.975$$
$$Z_c = 1.96 \text{ (from Z table)}$$
$$CV = \mu + (Z_c \times \bar{\sigma}_x)$$
$$UCV = 200.7 + (1.96 \times 6.5) = 212.74$$
$$LCV = 187.26$$

$\therefore 200$  falls in the range, we fail to reject the null hypothesis

The probability of committing a Type-I error(that is rejecting the null hypothesis when it is actually true is called alpha).

Here  $\alpha = 0.05$ .

The probability of committing a Type-II error(that is failing to reject the null hypothesis when it is actually false is called beta).

Here  $b = 0.45$ .

So if the value of  $\alpha$  and  $b$  both are maintained at 0.15. That means that the value of  $\alpha$  increases from 0.05 to 0.15. So we reject the true null hypothesis 15 times out of 100.

Similarly the value for  $b$  is decreased from 0.45 to 0.15. So now we fail to reject the false null hypothesis 15 times out of 100 in contrast to the 45 times out of 100.

So for the above example if we reject the true null hypothesis, it is safer because we cannot take any risks with medicines.

But if we fail to reject the false null hypothesis, then it can affect the health of the patient. So here by reducing the value of  $b$  we are doing the right thing.

But if we consider the AC sales per month in an electronic store, then it will be best if we keep the values of  $\alpha$  and  $\beta$  at 0.15.

## Question 4:

Now once the medicine is ready for the market, the marketing team has two different tag lines. So to know which tag line is better we need to do an AB test.

In an AB test, both the tag lines are launched into market. After a stipulated amount of time we find out which tag line attracted more customers.

The tag line which attracted more number of customers is retained and used in all future business purposes. The other tag line is simply dissolved and not used any more.

So the steps of AB testing are:

Pick one variable to test (here for us it is the tag line).

Identify the goal (for us it is which tagline attracts more customers).

Create a “Control” and “Challenger”.

Split the sample size equally and randomly.

Decide how significant the result should be.