Classical Mechanics

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July 2024

Important results

1 Common 2D coordinate systems

1.1 Cartesian coordinate system

- Position, $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$
- Velocity, $\vec{\boldsymbol{v}} = \dot{x}\hat{\boldsymbol{i}} + \dot{y}\hat{\boldsymbol{j}}$
- Accelaration, $\vec{\boldsymbol{a}} = \ddot{x}\hat{\boldsymbol{i}} + \ddot{y}\hat{\boldsymbol{j}}$
- Kinetic energy $= \frac{1}{2}m(\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}) = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

1.2 Polar coordinate system

- Position, $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{r}}$
  where, $\hat{\boldsymbol{r}} = cos\theta\hat{\boldsymbol{i}} + sin\theta\hat{\boldsymbol{j}}$,
- Velocity, $\vec{\boldsymbol{v}} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$
  where, $\hat{\boldsymbol{\theta}} = -sin\theta\hat{\boldsymbol{i}} + cos\theta\hat{\boldsymbol{j}}$
- Accelaration, $\vec{\boldsymbol{a}} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}$
- Kinetic energy $= \frac{1}{2}m(\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$

Constrained motion

Constraints are restraints imposed on the motion or location, or both of a system of particles.Constrained motion occurs when an object is forced to move in a specific manner.

Constraint can be divided in two categories:

Holonomic constraint: Holonomic constraints can be expressed as an equation that involves only the spatial coordinates q_i of the system and the time t.

Non-holonomic constraint: Non-holonomic constraints cannot be written as an equation between coordinates.

Generalized coordinates

Generalized coordinates are a set of parameters used to represent the configuration of a system. Instead of using a particular set of coordinates, we use generalized coordinates (q_i) which may be cartesian, polar, angles, or various combinations of them.