Classical Mechanics

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Important results

1 Common 2D coordinate systems

1.1 Cartesian coordinate system

- Position, $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$
- Velocity, $\vec{\boldsymbol{v}} = \dot{x}\hat{\boldsymbol{\imath}} + \dot{y}\hat{\boldsymbol{\jmath}}$
- Accelaration, $\vec{\boldsymbol{a}} = \ddot{x}\hat{\boldsymbol{\imath}} + \ddot{y}\hat{\boldsymbol{\jmath}}$
- Kinetic energy $= \frac{1}{2}m(\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}) = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

1.2 Polar coordinate system

- Position, $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{r}}$
  where, $\hat{\boldsymbol{r}} = cos\theta\hat{\boldsymbol{\imath}} + sin\theta\hat{\boldsymbol{\jmath}}$
- Velocity, $\vec{\boldsymbol{v}} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$
  where, $\hat{\boldsymbol{\theta}} = -sin\theta\hat{\boldsymbol{\imath}} + cos\theta\hat{\boldsymbol{\jmath}}$
- Accelaration, $\vec{\boldsymbol{a}} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}$
- Kinetic energy $= \frac{1}{2}m(\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$

2 Common 3D coordinate systems

2.1 Cartesian coordinate system

- Position, $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$
- Velocity, $\vec{\boldsymbol{v}} = \dot{x}\hat{\boldsymbol{\imath}} + \dot{y}\hat{\boldsymbol{\jmath}} + \dot{z}\hat{\boldsymbol{k}}$
- Accelaration, $\vec{\boldsymbol{a}} = \ddot{x}\hat{\boldsymbol{\imath}} + \ddot{y}\hat{\boldsymbol{\jmath}} + \ddot{z}\hat{\boldsymbol{k}}$
- Kinetic energy $= \frac{1}{2}m(\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}) = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$

2.2 Cylindrical coordinate system

- Cartesian coordinates in terms of Cylindrical coordinates,

$$x = rcos\phi,$$
$$y = rsin\phi,$$
$$z = z$$

- Transformation table for unit vectors:

	$\hat{\boldsymbol{r}}$	$\hat{\boldsymbol{\phi}}$	$\hat{\boldsymbol{z}}$
$\hat{\boldsymbol{i}}$	$cos\phi$	$-sin\phi$	0
$\hat{\boldsymbol{j}}$	$sin\phi$	$cos\phi$	0
$\hat{\boldsymbol{k}}$	0	0	1

- Position, $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{r}} + z\hat{\boldsymbol{z}}$
- Velocity, $\vec{\boldsymbol{v}} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\phi}\hat{\boldsymbol{\phi}} + \dot{z}\hat{\boldsymbol{z}}$
- Accelaration, $\vec{\boldsymbol{a}} = (\ddot{r} - r\dot{\phi}^2)\hat{\boldsymbol{r}} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{\boldsymbol{\phi}} + \ddot{z}\hat{\boldsymbol{z}}$
- Kinetic energy $= \frac{1}{2}m(\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2 + \dot{z}^2)$

2.3 Spherical coordinate system

- Cartesian coordinates in terms of Spherical coordinates,

$$x = rsin\theta cos\phi,$$
$$y = rsin\theta sin\phi,$$
$$z = rcos\theta$$

- Transformation table for unit vectors:

	$\hat{\boldsymbol{r}}$	$\hat{\boldsymbol{\theta}}$	$\hat{\boldsymbol{\phi}}$
$\hat{\boldsymbol{i}}$	$sin\theta cos\phi$	$cos\theta cos\phi$	$-sin\phi$
$\hat{\boldsymbol{j}}$	$sin\theta sin\phi$	$cos\theta sin\phi$	$cos\phi$
$\hat{\boldsymbol{k}}$	$cos\theta$	$-sin\theta$	0

- Position, $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{r}}$
- Velocity, $\vec{\boldsymbol{v}} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} + rsin\theta\dot{\phi}\hat{\boldsymbol{\phi}}$
- Accelaration, $\vec{\boldsymbol{a}} = (\ddot{r} - r\dot{\theta}^2 - rsin^2\theta\dot{\phi}^2)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - rsin\theta cos\theta\dot{\phi}^2)\hat{\boldsymbol{\theta}} + (\dot{r}sin\theta\dot{\phi} + 2rcos\theta\dot{\theta}\dot{\phi} + rsin\theta\ddot{\phi})\hat{\boldsymbol{\phi}}$
- Kinetic energy $= \frac{1}{2}m(\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2sin^2\theta\dot{\phi}^2)$

Constrained motion

Constraints are restraints imposed on the motion or location, or both of a system of particles.Constrained motion occurs when an object is forced to move in a specific manner.

Constraint can be divided in two categories:

Holonomic constraint: Holonomic constraints can be expressed as an equation that involves only the spatial coordinates q_i of the system and the time t.

Non-holonomic constraint: Non-holonomic constraints cannot be written as an equation between coordinates.

Generalized coordinates

Generalized coordinates are a set of parameters used to represent the configuration of a system. Instead of using a particular set of coordinates, we use generalized coordinates (q_i) which may be cartesian, polar, angles, or various combinations of them.

Calculus of variation

In calculus of variation we find a function or a curve, for which the given functional has a stationary value.

Euler equation

$$S = \int_{x_1}^{x_2} f(y, \dot{y}, x) dx$$