Coordinate System

Debanjan Koley

16 July, 2024

Important results

1 Common 2D coordinate systems

1.1 Cartesian coordinate system

- Position, $\vec{r} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$

- Velocity, $\vec{v} = \dot{x}\hat{\boldsymbol{i}} + \dot{y}\hat{\boldsymbol{j}}$

- Accelaration, $\vec{a} = \ddot{x}\hat{\boldsymbol{i}} + \ddot{y}\hat{\boldsymbol{j}}$

- Kinetic energy $= \frac{1}{2}m(\vec{v} \cdot \vec{v}) = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

1.2 Polar coordinate system

- Position, $\vec{r} = r\hat{\boldsymbol{r}}$
  where, $\hat{\boldsymbol{r}} = cos\theta\hat{\boldsymbol{i}} + sin\theta\hat{\boldsymbol{j}}$,

- Velocity, $\vec{v} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$
  where, $\hat{\boldsymbol{\theta}} = -sin\theta\hat{\boldsymbol{i}} + cos\theta\hat{\boldsymbol{j}}$

- Accelaration, $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}$

- Kinetic energy $= \frac{1}{2}m(\vec{v} \cdot \vec{v}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$