# Technical Report about how to define fibre distribution through LDDMM

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## 1 Geometric and Fibre Construction

For a neonatal porcine heart (Fig. 1(a)), a 3D FE bi-ventricular model is reconstructed from 3D computed tomography (CT) data. Details of the data acquisition can be found in [1]. The 3D CT data is first segmented using Seg3D[1], then the boundary contours are exported into SolidWorks (Dassault Systemes, MA USA) for 3D geometry reconstruction, and then meshed with ICEM (ANSYS, Inc. PA USA). Finally, explicit Abaqus (Dassault Systemes, MA USA) is used for the FE simulation. User-subroutines are implemented for different strain energy functions. The fibre distribution is introduced from the available datasets of Cardiovascular Research Grid[2], which have been collected DT-MRI data using ex-vivo samples of canine and human hearts. We choose the data set of canine that was collected on April 26th, 2004, construct corresponding bi-ventricular geometry through the above way and fit primary eigenvector of diffusion tensor as fibre direction to geometry according to image pixel position (Fig. 1(b)).

To map the fibre from canine LV (Fig. 1(b)) to neonatal porcine LV (Fig. 1(c)), Large deformation diffeomorphic metric mapping (LDDMM) [6, 4, 5, 2] is introduced, which can provide displacement, $\mathbf{u}$, of each node from initial to target geometric template. Therefore, it is easy to compute the deformation gradient tensor, $\mathbf{F}$, for each element of initial geometry by

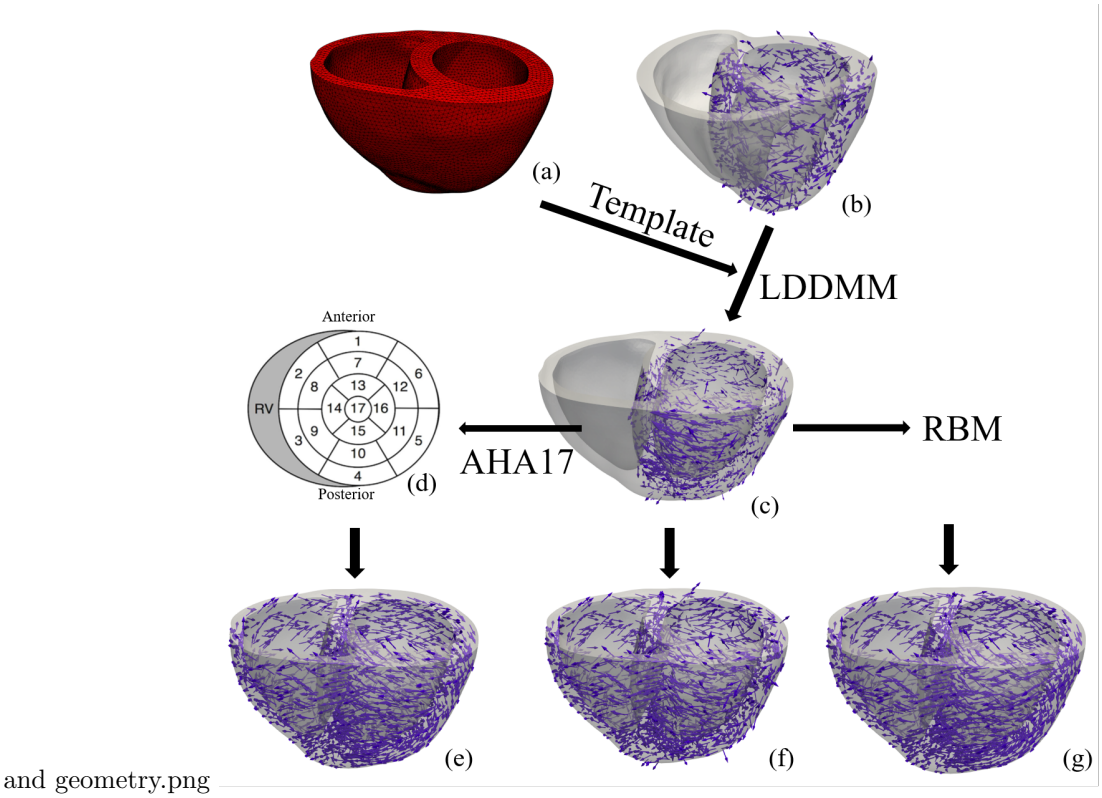
$$\mathbf{F} = (\nabla \mathbf{u})^T + \mathbf{I}, \tag{1}$$

and then reorienting fibre through

$$\mathbf{f}_{\text{target}} = \mathbf{F}\,\mathbf{f}_{\text{initial}}. \tag{2}$$

where $\mathbf{f}_{\text{target}}$ and $\mathbf{f}_{\text{initial}}$ are the fibre direction after and before deformation. Finally, FFM from initial to target mesh can be realized according to position of element, i.e. $\min |\mathbf{X}_{\text{initial}} - \mathbf{X}_{\text{target}}|$, where $X$ is the central coordinate of element.

[1] `http://www.sci.utah.edu/cibc-software/seg3d.html`
[2] `http://cvrgrid.org/data/ex-vivo`




and geometry.png

Figure 1: Fibre distribution of neonatal porcine heart according to canine heart with LV fibre data. (a) is the meshed geometry of neonatal porcine heart. (b) is geometry of canine heart with only fibre distribution in LV. (c) reorients fibre distribution through LDDMM method while (a) as a target geometric template. Then by statistics, summarizing mean fibre rotation at each segment in (d) and mean fibre rotation in whole LV to use AHA17 and RBM respectively to define fibre distribution. (e-f) are corresponding final fibre distribution when combing same right ventricular fibre definition by RBM.

However, there is only fibre data for LV. Here, we assume right ventricle has same mean fibre rotation as LV. We intend to design three types of fibre architecture for neonatal porcine heart, which have different fibre distributions that are determined respectively by FFM, AHA17 and RBM in LV while have same fibre structure in RV defined by RBM as shown in Fig. 1(e-f). Basing on the fibre distribution in neonatal LV (Fig. 1(c)), mean fibre rotation in respect to local circumferential direction for each segment in Fig. 1(d) is able to be estimated by analyzing statistical fibre direction from epicaridum to endocardium. Similarly, mean fibre rotation for whole LV can be estimated. Sheet direction of collagen fibre is perpendicular to local surface.

## 1.1 Geometric Construction

### 1.1.1 Geometry of neonatal porcine heart

For a neonatal porcine heart (Fig. 1(a)), a 3D FE bi-ventricular model is reconstructed from 3D computed tomography (CT) data. Details of the data acquisition can be found in [1]. The 3D CT data is first segmented using Seg3D[3], then the boundary contours are exported into SolidWorks (Dassault Systemes, MA USA) for 3D geometry reconstruction, and then meshed with ICEM (ANSYS, Inc. PA USA). Finally, explicit Abaqus (Dassault Systemes, MA USA) is used for the FE simulation.

### 1.1.2 Geometry and fibre of canine heart

The geometric data is introduced from the available datasets of Cardiovascular Research Grid[4], which have been collected DT-MRI data using ex-vivo samples of canine and human hearts. We choose the data set of canine that was collected on April 26th, 2004, and construct corresponding bi-ventricular geometry through Geomagic basing on counters of peicardium and endocardium.

The fibre data is provided by primary eigenvector of diffusion tensor, which is proved to be as fibre direction. According to DT-MRI pixel position, we can map the fibre to FE geometry.

## 1.2 Large deformation diffeomorphic metric mapping (LDDMM)

### 1.2.1 Logic process

- Step 1: Extract primary eigenvector of DT-MRI diffusion tensor as fibre direction ($\mathbf{f}_0$) from datasets of Cardiovascular Research Grid, and map it to geometric element through comparing image pixel coordinate and central coordinate of element. (MapFibreFromTableToMesh.m)

- Step 2: Make surface mesh according to the mesh of geometry, because LDDMM is only can deal with surface mesh format. (SurfaceMake.m)

[3] `http://www.sci.utah.edu/cibc-software/seg3d.html`
[4] `http://cvrgrid.org/data/ex-vivo`

- Step 3: Run LDDMM python code in "deformetrica" conda environment when setting surface mesh of canine heart as initial template and surface mesh of neonatal porcine heart as target template. (data-set.xml, model.xml, optimization-parameters.xml)
- Step 4: Compute the displacement of every node through comparing the coordinate before and after deformation. (ReadDxDyDz.m)
- Step 5: To get displacement of every node in geometry, run Poisson Function code in "fenics" conda environment when setting the displacement of surface mesh node as boundary condition. (ReadVTU.m, demo-bcs-dx.py,demo-bcs-dy.py,demo-bcs-dz.py)
- Step 6: Compute the deformation gradient tensor $\mathbf{F}$ basing on the displacement of every node in the geometry and then compute fibre direction after deformation by $\mathbf{f} = \mathbf{F}\mathbf{f}_0$. (ComputeDeformationGradient.m, IsoTet4ShapeFunDer.m)
- Step 7: Map the deformed fibre from mesh of deformed geometry to mesh of target geometry also through comparing the central coordinate of element, i.e., $\min|X_{\text{inital}} - X_{\text{target}}|$, where $X$ is the central coordinate of element. (adjustposition-template-target-model.m, MapFibreFromTemplateToTarget.m)
- Step 8: Sheet direction $\mathbf{s}$ is defined to be perpendicular to local surface, which is achieved by computing Poisson function when setting epicardium as 1 and endocardium as 0. (surfacenode-poissonBC.m, demo-bcs.py)
- Step 9: Finally, we can build local coordinate according to $\mathbf{f}$ and $\mathbf{s}$. (adjust-fibre-coordinate.m, CombineRVLVfibre.m)

### 1.2.2 LDDMM package

The LDDMM framework proposes to compare shapes (meshes or images) using diffeomorphic transformations of the 2D or 3D ambient space between these objects. To use it, we need to be able to parametrize a large family of transformations of the 2D or 3D space as well as to be able to compute distances between the objects. Bône et al [3] provides a concise reference summarizing the theory behind Deformetrica applications. For more details, see Durrleman et al [5].

The Deformetrica applications are called with command lines of the form:

- deformetrica model.xml data-set.xml optimization-parameters.xml
- deformetrica compute model.xml [-p optimization-parameters.xml]

where:

- deformetrica is an alias for python deformetrica.py: the main script of the software, in the src folder, automatically created during the conda installation.

- model.xml is an xml file containing the description of the task that is to be performed (such as estimating a registration, an atlas, ...) as well as some hyper-parameters for the objects and the deformations used.
- data-set.xml is an xml file containing the paths to the input objects from which a statistical model will be estimated. The shooting and parallel transport applications are direct computations, and do not need this xml file.
- optimization-parameters.xml contains optional details about the optimization method.

More details can be seen in registration method[5]

### 1.2.3 Fenics package

The Poisson equation is the canonical elliptic partial differential equation. For a domain $\Omega \subset \mathbb{R}^n$ with boundary $\partial\Omega = \Gamma_D \cup \Gamma_N$, the Poisson equation with particular boundary conditions reads:

$$\begin{aligned} -\nabla^2 u &= f \quad \text{in} \quad \Omega \\ u &= 0 \quad \text{on} \quad \Gamma_D \\ \nabla u \cdot n &= g \quad \text{on} \quad \Gamma_N \end{aligned} \tag{3}$$

Here, $f$ and $g$ are input data and n denotes the outward directed boundary normal. The most standard variational form of Poisson equation reads: find $u \in V$ such that

$$a(u, v) = L(v) \quad \forall v \in V, \tag{4}$$

where $V$ is a suitable function space and

$$\begin{aligned} a(u, v) &= \int_\Omega \nabla u \cdot \nabla v \, dx, \\ L(v) &= \int_\Omega f v \, dx + \int_{\Gamma_N} g v \, ds \end{aligned} \tag{5}$$

The expression $a(u, v)$ is the bilinear form and $L(v)$ is the linear form. It is assumed that all functions in $V$ satisfy the Dirichlet boundary conditions ($u = 0$ on $\Gamma_D$).

For a simple demo, the following definitions of the input functions, the domain, and the boundaries:

- $\Omega = [0, 1] \times [0, 1]$ (a unit square)
- $\Gamma_D = \{(0, y) \cup (1, y) \subset \partial\Omega\}$ (Dirichlet boundary)
- $\Gamma_N = \{(x, 0) \cup (x, 1) \subset \partial\Omega\}$ (Neumann boundary)

[5] `https://gitlab.icm-institute.org/aramislab/deformetrica/wikis/2_tutorials/2.1_registration`

- $g = \sin(5x)$ (normal derivative)
- $f = 10\exp(-((x-0.5)^2 + (y-0.5)^2)/0.02)$ (source term)

In our model, we need to set boundary conditions for meshes nodes $N$, so we firstly define boundary indicators in boundary file and then solve the poisson equation with Dirichlet boundary values.

We will use the Poisson equation as a model problem when we demonstrate how to set boundary conditions for an imported mesh that includes boundary indicators. The Poisson equation is the canonical elliptic partial differential equation. For a domain $\Omega \subset \mathbb{R}^3$ with boundary $\partial\Omega = \bigcup_{i=1}^{N} \Gamma_{D,i}$, the Poisson equation with particular boundary conditions reads:

$$\begin{aligned} -\nabla^2 u &= f \quad \text{in} \quad \Omega \\ u &= u_1 \quad \text{on} \quad \Gamma_{D,1} \\ u &= u_2 \quad \text{on} \quad \Gamma_{D,2} \\ &\dots \\ u &= u_N \quad \text{on} \quad \Gamma_{D,N} \end{aligned} \tag{6}$$

Here, $f$ is some given input data and $u_1 \dots u_N$ are the prescribed values of u at the boundaries(each node). The variational form of the Poisson equation reads: find $u \in V$ such that

$$a(u, v) = L(v) \quad \forall v \in V, \tag{7}$$

where $V$ is a suitable function space and

$$\begin{aligned} a(u, v) &= \int_\Omega \nabla u \cdot \nabla v \, dx, \\ L(v) &= \int_\Omega f v \, dx \end{aligned} \tag{8}$$

The expression $a(u, v)$ is the bilinear form and $L(v)$ is the linear form. It is assumed that all functions in $V$ satisfy the Dirichlet boundary conditions.

In this demo we shall consider the domain $\Omega$ to be a model of bi-ventricle heart. We define noslip boundary conditions on each mesh node that is $u_1 \dots u_N$. In summary, we have:

- $u = u_1$ on $\Gamma_{D,1}$
- $u = u_2$ on $\Gamma_{D,2}$
- $\dots$
- $u = u_N$ on $\Gamma_{D,N}$
- $f = 0.0$ (source term)

# References


[1] Faizan Ahmad, Jun Liao, Shwe Soe, Michael D Jones, Jonathan Miller, Parker Berthelson, Daniel Enge, Katherine M Copeland, Samar Shaabeth, Richard Johnston, et al. Biomechanical properties and microstructure of neonatal porcine ventricles. *Journal of the mechanical behavior of biomedical materials*, 88:18–28, 2018.

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