02_Exercise1_MaxL

April 22, 2018

1 Assignment 02

1.1 Exercise 01

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from matplotlib import cm
        import scipy as sp
        import seaborn as sns
        from mpl_toolkits.mplot3d import Axes3D
        from scipy.stats import multivariate_normal
        import pandas as pd
```

Let's suppose we have a set of observations $x = (x_1, , x_N)^T$, that are drawn independent and identically distributed (i.i.d) from a Gaussian distribution with unknown mean μ and variance σ^2

For this example, we are going to assume that the unknown parameters are μ=2 and σ^2=25 and the number of samples N=100.

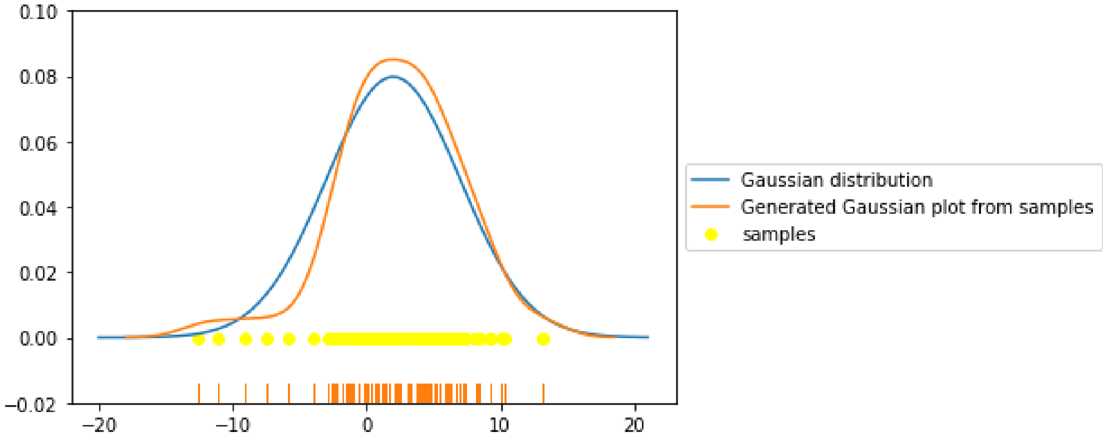
```
In [2]: mean = 2
        variance = 25
        std_dev = np.sqrt(25)
        N = 100
```

1.1.1 Task1:

Plot this (unknown) distribution together with the samples in the range $[-20, 20]$.

```
In [3]: distribution = np.random.normal(loc = 2, scale = std_dev, size=N)

        data= np.hstack(((distribution[:,np.newaxis]),
                         np.zeros(distribution[:,np.newaxis].shape)))

        plt.plot(np.linspace(-20,21,100),
                 plt.mlab.normpdf(np.linspace(-20,21,100),
                                  mean, std_dev),
                 label = "Gaussian distribution")

        sns.distplot(distribution,hist=False, rug=True,
```

```
                    label = "Generated Gaussian plot from samples")

        plt.scatter(data[:,0],data[:,1],color='yellow', label = 'samples')
        plt.ylim(-0.02,0.1)
        plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
        plt.show()
```



1.1.2 Task2:

- Implement the likelihood function in python (you can simply use the existing python implementations)
- Use a general optimization method to find the values for μ and σ^2.

```
In [4]: mu_calculated, std_dev_calulated = sp.stats.norm.fit(distribution)

        print ("Calculated Mean: ", mu_calculated,"; Calculated Variance (sigma^2): ", std_dev_c
```

```
Calculated Mean:  2.432037837648402 ; Calculated Variance (sigma^2):  20.61522552087914
```

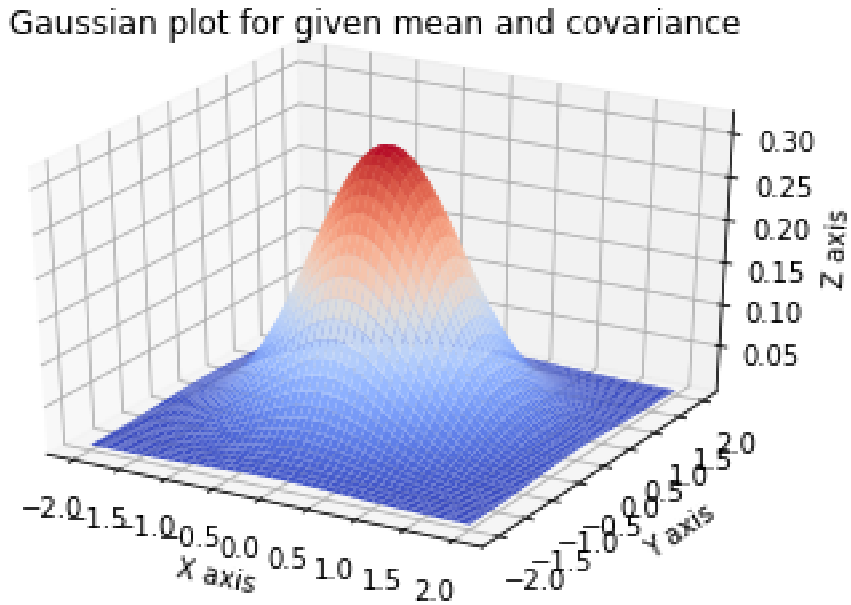
```
In [5]: x_i = data[:,0]
        N_samples = (x_i.size)

        sample_mean = 1/N_samples*np.sum(x_i)
        sample_variance = (1/(N_samples-1))*np.sum((x_i-sample_mean)**2)

        print ("Sample Mean: ", sample_mean,"; Sample Variance: ", sample_variance)
```

```
Sample Mean:  2.432037837648402 ; Sample Variance:  20.823460122100144
```

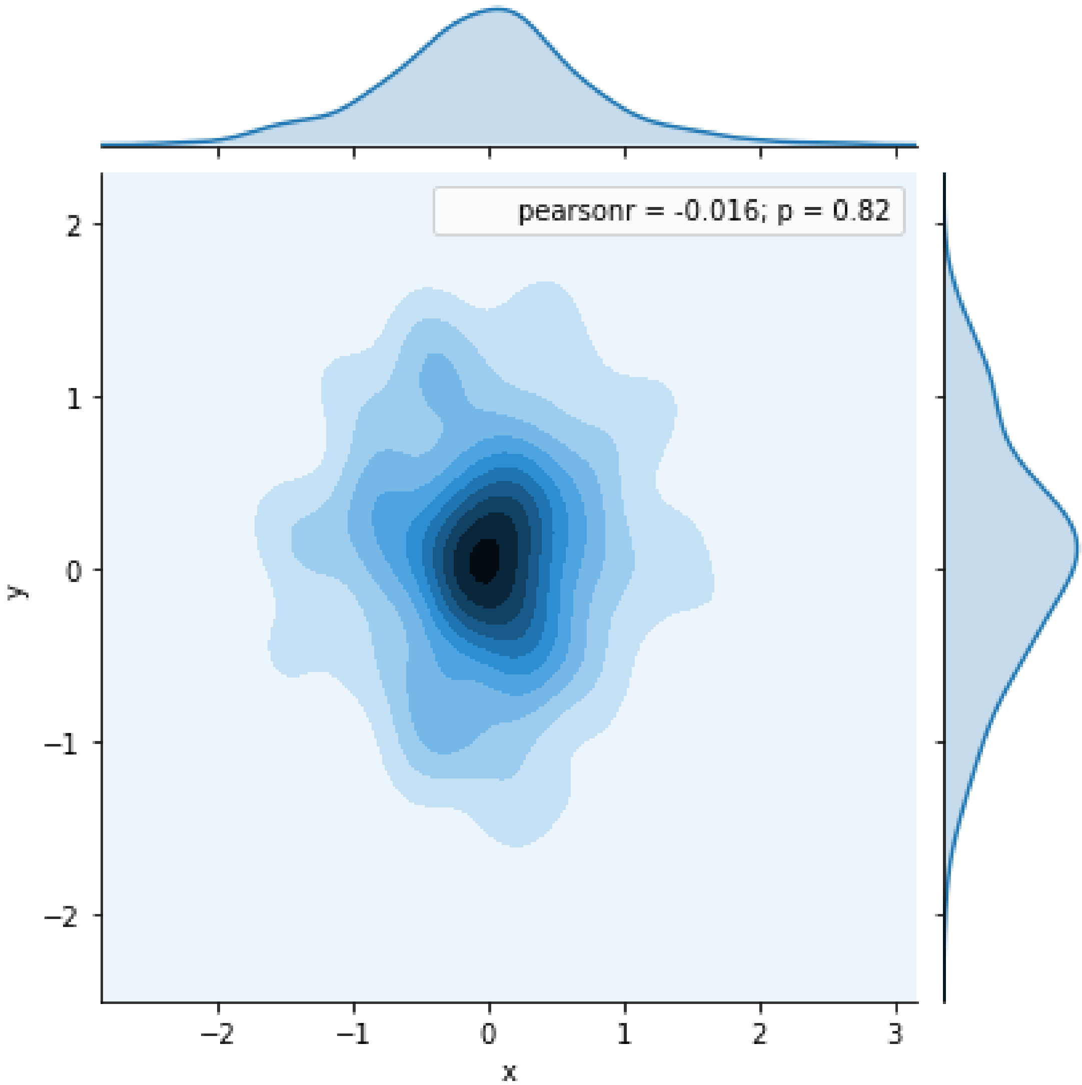
1.1.3 Task3:

Given: $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ 1. Visualise a Gaussian with the given parameters. 2. Visualise a marginal Gaussian. 3. Visualise a slice of Gaussian.

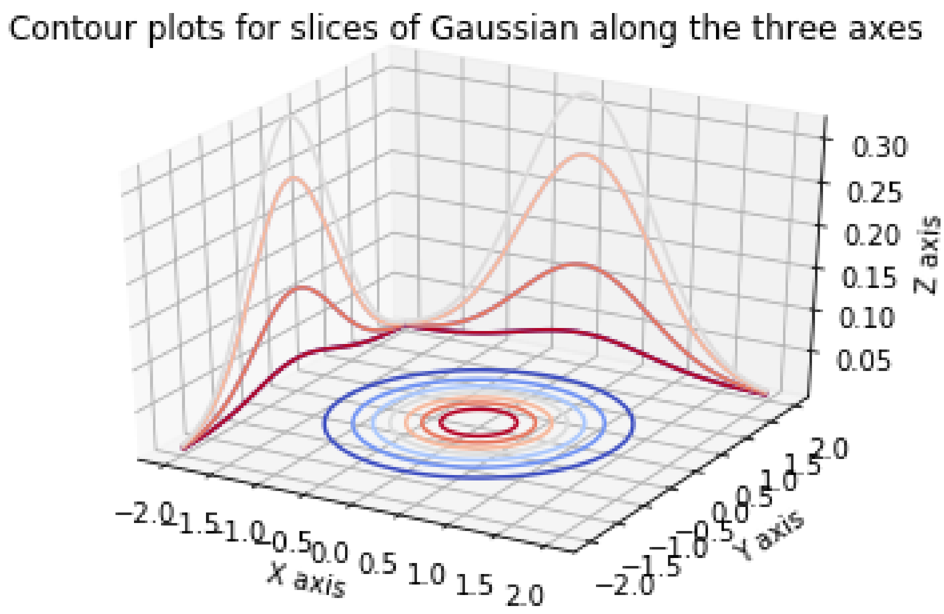
In [6]:
```
#https://stackoverflow.com/questions/38698277/plot-normal-distribution-in-3d

mu = np.array([0,0])
sigma = np.array([[0.5, 0],[0, 0.5]])
data = np.random.multivariate_normal(mu, sigma, 200)
df = pd.DataFrame(data, columns=["x", "y"])


N = 100
X = np.linspace(-2, 2, N)
Y = np.linspace(-2, 2, N)
X, Y = np.meshgrid(X,Y)
xy = np.empty(X.shape + (2,))
xy[:, :, 0] = X; xy[:, :, 1] = Y

Z = multivariate_normal.pdf(xy,mu,sigma)


fig = plt.figure()
ax = fig.gca(projection='3d')
ax.plot_surface(X, Y, Z ,cmap=cm.coolwarm)
plt.title('Gaussian plot for given mean and covariance')
ax.set_xlabel('X axis')
ax.set_ylabel('Y axis')
ax.set_zlabel('Z axis')
plt.show()
```



In [7]:
```
# Marginal Gaussians against the two variables

sns.jointplot(x="x", y="y", data=df, kind="kde");
```




```
In [8]: #Plots for slices of the Gaussian along x, y and z axes
        fig = plt.figure()
        ax = fig.gca(projection='3d')
        ax.contour(X, Y, Z, zdir='z', offset=0, cmap=cm.coolwarm)
        ax.contour(X, Y, Z, zdir='y', offset=2, cmap=cm.coolwarm)
        ax.contour(X, Y, Z, zdir='x', offset=-2, cmap=cm.coolwarm)
        plt.title('Contour plots for slices of Gaussian along the three axes')
        ax.set_xlabel('X axis')
        ax.set_ylabel('Y axis')
        ax.set_zlabel('Z axis')
        plt.show()
```



1.1.4 Task4:

Given:

Number of samples is 1000 from them 330 samples are labeled as class A and 670 samples are labeled as class B. There are 2 features $X1$ and $X2$. It is observed that $p(A, X1)$=248, $p(A, X2)$=82, $p(B, X1)$=168, $p(B, X2)$=502

Compute:

- Prior $p(A)$, $p(B)$
- Likelihood $p(X1|A)$, $p(X1|B)$
- Posterior $p(A|X1)$

In [9]:
```
N = 1000
N_A = 330
N_B = 670

P_AX1 =  0.248
P_AX2 = 0.082
P_BX1 = 0.168
p_BX2 = 0.502

P_A = N_A/N
P_B = N_B/N

P_X1_A = P_AX1 / P_A
P_X1_B = P_BX1 / P_B
```

```
    P_A_X1 = P_AX1/((P_X1_A * P_A) + (P_X1_B * P_B))

    print ("Prior P(A)=",P_A,",  P(B)=",P_B)
    print ("Likelihood P(X1|A)=",P_X1_A,",  P(X1|B)=",P_X1_B)
    print ("Posterior P(A|X1)=",P_A_X1)
```

```
Prior P(A)= 0.33 ,  P(B)= 0.67
Likelihood P(X1|A)= 0.7515151515151515 ,  P(X1|B)= 0.2507462686567164
Posterior P(A|X1)= 0.5961538461538461
```