NN_DebarajBarua_091017

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1 Hochschule Bonn-Rhein-Sieg

2 Neural Networks, WS17/18

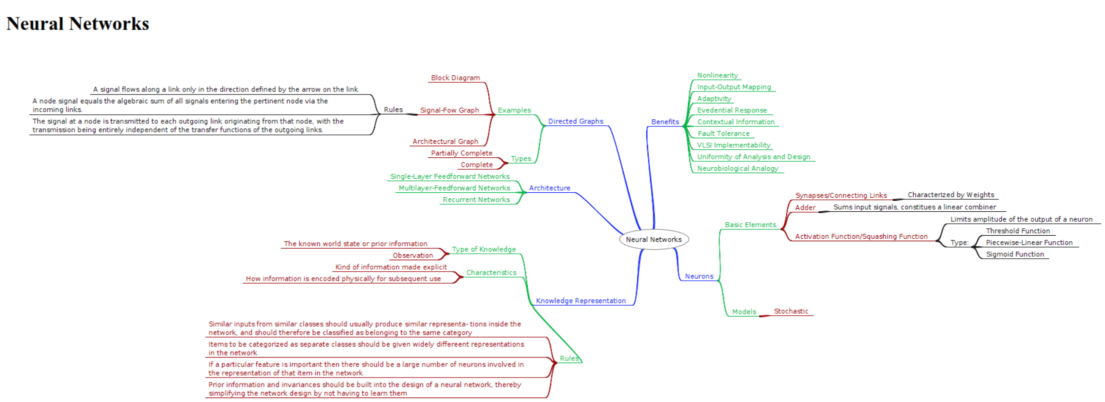
3 Assignment 01 (09-October-2017)

```
In [1]: import sympy as sp
        import numpy as np
        sp.init_printing(use_latex=True)
        from IPython.display import Image
```

3.0.1 Question 2: Read chapter 1 from Haykin's book; summarize or sketch your insights in mind-map or an outline or a summary.

```
In [2]: Image("Images/MindMap.png")
```

```
Out[2]:
```



Note:

Please click here (navigation possible from Jupyter Notebok) to view the mind map in higher resolution as a clickable html file.

3.0.2 Question 4: From Haykin's book, Chapter 1 problems – "Models of a neuron", solve any 2 out of 11 (1.1 to 1.11). use sympy packege to solve them.

3.0.3 1.1

An example of a logistic function is defined by

$$\varphi(v) = \frac{1}{1 + e^{(-av)}}$$

whose limiting values are 0 and 1. Show that the derivative of $\varphi(v)$ with respect to v is given by

$$\frac{d\varphi}{dv} = a\varphi(v)[1 - \varphi(v)]$$

What is the value of this derivative at the origin?

```
In [3]: a,v = sp.symbols('a,v')
        phi = 1/(1+sp.exp(-a*v))
        phi_prime=sp.diff(phi,v)
        print "derivative of logistic function is: "
        phi_prime
```

```
derivative of logistic function is:
```

Out[3]:

$$\frac{ae^{-av}}{(1 + e^{-av})^2}$$

So, we see that the derivative of the function is:

$$\begin{aligned}
\frac{d\varphi}{dv} &= \frac{ae^{-av}}{(1 + e^{-av})^2} \\
&= a \cdot \frac{1}{1 + e^{-av}} \cdot \frac{e^{-av}}{(1 + e^{-av})} \\
&= a\varphi(v)[\frac{1 + e^{-av} - 1}{(1 + e^{-av})}] \\
&= a\varphi(v)[1 - \varphi(v)]
\end{aligned}$$

At origin, $\varphi(0) = \frac{1}{2}$
So, $\frac{d\varphi}{dv}\Big|_{v=0} = \frac{a}{4}$

```
In [4]: print "Value of derivative of logistic function at v=0, is:"
        phi_prime.subs(v,0)
```

```
Value of derivative of logistic function at v=0, is:
```

Out[4]:

$$\frac{a}{4}$$

3.0.4 1.3

Yet another sigmoid function is the algebraic sigmoid:

$$\varphi(v) = \frac{v}{\sqrt{1+v^2}}$$

whose limiting values are -1 and +1. Show that derivative of $\varphi(v)$ with respect to v is

$$\frac{d\varphi}{dv} = \frac{\varphi^3(v)}{v^3}$$

What is the value of this derivative at the origin?

```
In [5]: v = sp.symbols('v')
        phi = v/sp.sqrt(1+v**2)
        print "derivative of algebraic sigmoid function is: "
        phi_prime=sp.diff(phi,v)
        phi_prime
```

```
derivative of algebraic sigmoid function is:
```

```
Out[5]:
```

$$-\frac{v^2}{(v^2+1)^{\frac{3}{2}}} + \frac{1}{\sqrt{v^2+1}}$$

So, we see that the derivative of the function is:

$$\begin{aligned}
\frac{d\varphi}{dv} &= \frac{1}{\sqrt{v^2+1}} - \frac{v^2}{(v^2+1)^{\frac{3}{2}}} \\
&= \frac{v}{\sqrt{v^2+1}} \cdot \frac{1}{v} - \frac{v}{\sqrt{(v^2+1)}} \cdot \frac{v}{(v^2+1)} \\
&= \varphi \cdot [\frac{1}{v} - \frac{v}{1+v^2}] \\
&= \varphi \cdot [\frac{1+v^2-v^2}{v(1+v^2)}] \\
&= \frac{\varphi}{v} \cdot \frac{v^2}{1+v^2} \cdot \frac{1}{v^2} \\
&= \frac{\varphi}{v^3} \cdot \varphi^2 \\
&= \frac{\varphi^3(v)}{v^3}
\end{aligned}$$

At $v = 0$,
So, $\frac{d\varphi}{dv}\Big|_{v=0} = 1$

```
In [6]: print "Value of derivative of algebraic sigmoid at v=0, is:"
        phi_prime.subs(v,0)
```

Value of derivative of algebraic sigmoid at v=0, is:

Out[6]:

$$1$$

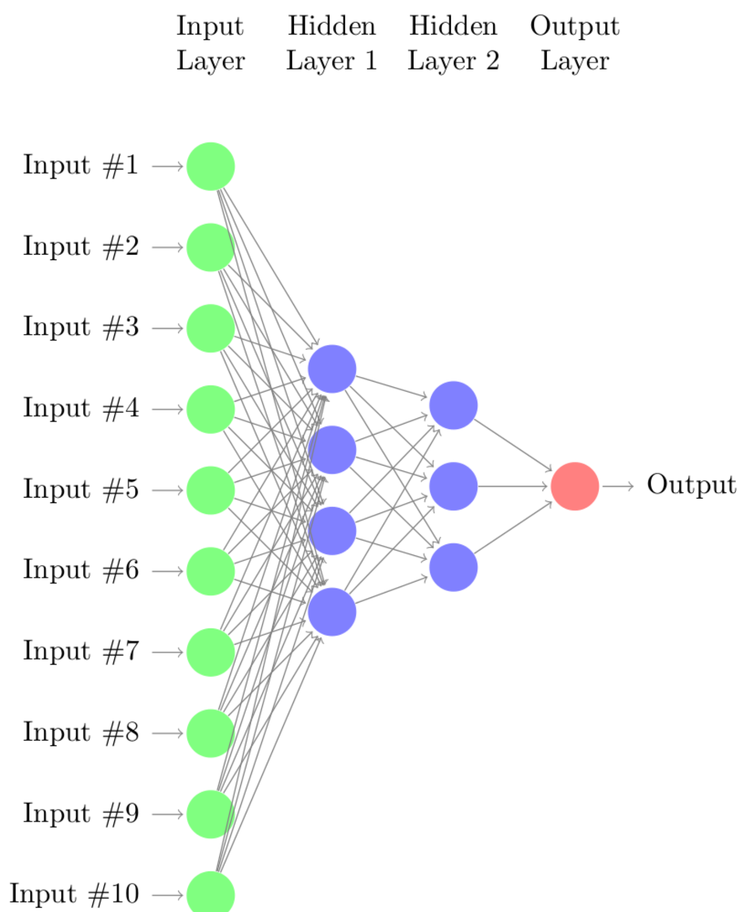
3.0.5 Question 5: From Haykin's book, Chapter 1 problems – "Network architectures", solve any 2 out of 7 (1.12 to 1.19) including 1.13.

1.12

A fully connected feedforward network has 10 source nodes. 2 hidden layers, one with 4 neurons a

```
In [7]: Image("Images/Network.png")
```

Out[7]:



1.13

(a) Figure P1.13 shows the signal-flow graph of a 2-2-2-1 feedforward network. The function $\varphi(\cdot)$ denotes a logistic function. Write the input/output mapping defined by this network.

```
In [8]: Image("Images/ScreenshotP113.png") #screenshot from Haykins book, Chapter 1b
```

Out[8]:

Chapter 1 Introduction

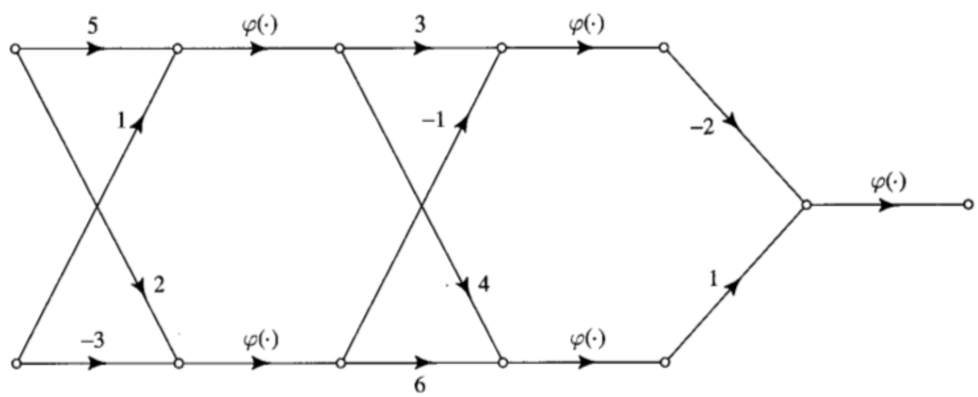


FIGURE P1.13

```
In [9]: #defining inputs (x1,x2) and logistic function (phi)
        x1,x2,a,v=sp.symbols('x1,x2,a,v')
        phi = 1/(1+sp.exp(-a*v)) #Using logistic function from 1.1

        #defining hidden layers
        hidden_layer_11 = phi.subs(v,5*x1+x2)
        hidden_layer_12 = phi.subs(v,2*x1-3*x2)
        hidden_layer_21 = phi.subs(v,3*hidden_layer_11-hidden_layer_12)
        hidden_layer_22 = phi.subs(v,4*hidden_layer_11+6*hidden_layer_12)
```

```
In [10]: output_layer=phi.subs(v,-2*hidden_layer_21+hidden_layer_22)
         output_layer
```

Out[10]:

$$\frac{1}{1+e^{-a\left(\frac{1}{1+e^{-a\left(\frac{4}{1+e^{-a(5x_1+x_2)}}+\frac{6}{1+e^{-a(2x_1-3x_2)}}\right)}}-\frac{2}{1+e^{-a\left(\frac{3}{1+e^{-a(5x_1+x_2)}}-\frac{1}{1+e^{-a(2x_1-3x_2)}}\right)}}\right)}}$$

(b) Suppose the output neuron in the signal-flow graph for Figure P1.13 operates in the linear region. Write the linear input-output mapping defined by this new network.

In [11]:
```
#Slope of the logistic function is
phi_prime=sp.diff(phi,v)

phi_prime_0=phi_prime.subs(v,0) #slope at origin ---slope

phi_0=phi.subs(v,0) #function value at 0 ---intercept

#For a linear signal, output=slope*x+intercept
#So, using this for the output layer

output_layer=phi_prime_0*(-2*hidden_layer_21+hidden_layer_22)+phi_0
output_layer
```

Out[11]:

$$\frac{a}{4}\left(\frac{1}{1+e^{-a\left(\frac{4}{1+e^{-a(5x_1+x_2)}}+\frac{6}{1+e^{-a(2x_1-3x_2)}}\right)}}-\frac{2}{1+e^{-a\left(\frac{3}{1+e^{-a(5x_1+x_2)}}-\frac{1}{1+e^{-a(2x_1-3x_2)}}\right)}}\right)+\frac{1}{2}$$

3.0.6 Question 6: Solve 1.20 or 1.21 in section "Knowledge representation"