NN_DebarajBarua_NareshKumarGurulingan_271117

December 2, 2017

1 Hochschule Bonn-Rhein-Sieg

2 Neural Networks, WS17/18

3 Assignment 08 (27-November-2017)

3.1 Debaraj Barua, Naresh Kumar Gurulingan

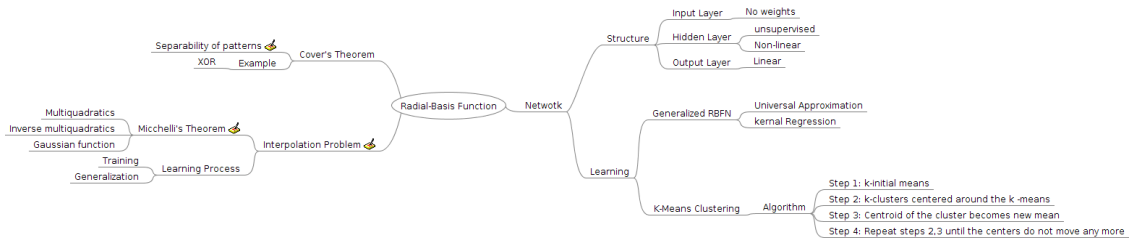
```
In [1]: import numpy as np
        import IPython
        import matplotlib.pyplot as plt
        from sklearn.cluster import KMeans
        from sklearn.model_selection import train_test_split
        from scipy.spatial.distance import cdist
        from scipy.linalg import pinv
```

3.2 Question 1:

Read chapter 5 from Haykin's book (third edition, v.3). Summarize or sketch your insights in mind-map or an outline or a summary.

```
In [2]: IPython.display.Image('images/mindmap.png')
```

Out[2]:



Note: Higher resolution and detailed mindmap is available at: *"MindMap/Radial-Basis Function.mm"*

3.3 Question 2: (Exercise 5.10)

2) The purpose of this computer experiment is to investigate the clustering process performed by the K-means algorithm. To provide insight into the experiment, we fix the number of clusters at K = 6, but vary the vertical separation between the two moons in Fig. 1.8. Specifically, the requirement is to do the following, using an unlabeled training sample of 1,000 data points picked randomly from the two regions of the double-moon pictured in Fig. 1.8:

- (a) Experimentally, determine the mean $\hat{\mu}_j$ and variance $\hat{\sigma}_j^2$, j = 1,2,3,...,6, for the sequence of eight uniformly spaced vertical separations starting at d = 1 and reducing them by one till the final separation d = - 6 is reached.
- (b) In light of the results obtained in part (a), comment on how the mean $\hat{\mu}_j$ of cluster j is affected by reducing the separation d for j = 1, 2, and 3.
- (c) Plot the variance $\hat{\sigma}_j^2$ versus the separation d for j = 1, 2, ..., 6.
- (d) Compare the common σ^2 computed in accordance with the empirical formula of Eq. (5.49) with the trends exhibited in the plots obtained in part (c).

Eq. (5.49) $\implies \sigma = \frac{d_{max}}{\sqrt{2K}}$

```
In [3]: IPython.display.Image('images/fig_1.8.png', embed= True)
```

Out[3]:

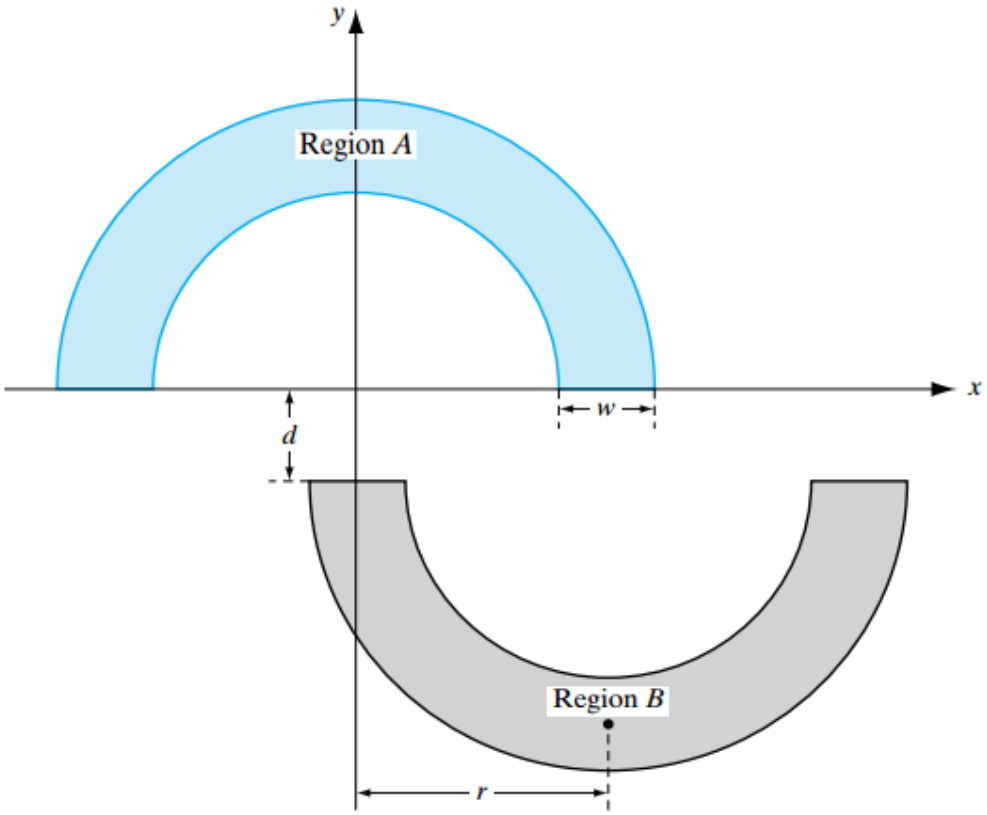


FIGURE 1.8 The double-moon classification problem.

3.3.1 Answer 5.10 (a)

```
In [4]: RADIUS = 10
        WIDTH = 6

        R_INNER = RADIUS - WIDTH/2
        R_OUTER = RADIUS + WIDTH/2
        THETA = np.linspace(0, 2*np.pi, 200)

        def generate_points(d, size= None):
            """
                This function generates random points from
                the moons...
            """
            np.random.seed(0)
            size_desired=size
            size=size*10
```

```
    r_red = np.random.uniform(R_INNER, R_OUTER, size= size)
    theta_red = np.random.uniform(0, 2*np.pi, size= size)
    r_black = np.random.uniform(R_INNER, R_OUTER, size= size)
    theta_black = np.random.uniform(0, 2*np.pi, size= size)

    x_r = r_red * np.cos(theta_red)
    y_r = np.abs(r_red * np.sin(theta_red))
    x_b = r_black * np.cos(theta_black)
    y_b = np.abs(r_black * np.sin(theta_black))
    red_moon = np.array([x_r, y_r]).T
    black_moon = np.array([x_b + RADIUS, -y_b - d]).T

    uniform_datapts=np.vstack((red_moon, black_moon))
    random_idx = np.random.randint(uniform_datapts.shape[0], size=size_desired)
    random_pts=uniform_datapts[random_idx,:]
    return random_pts
```

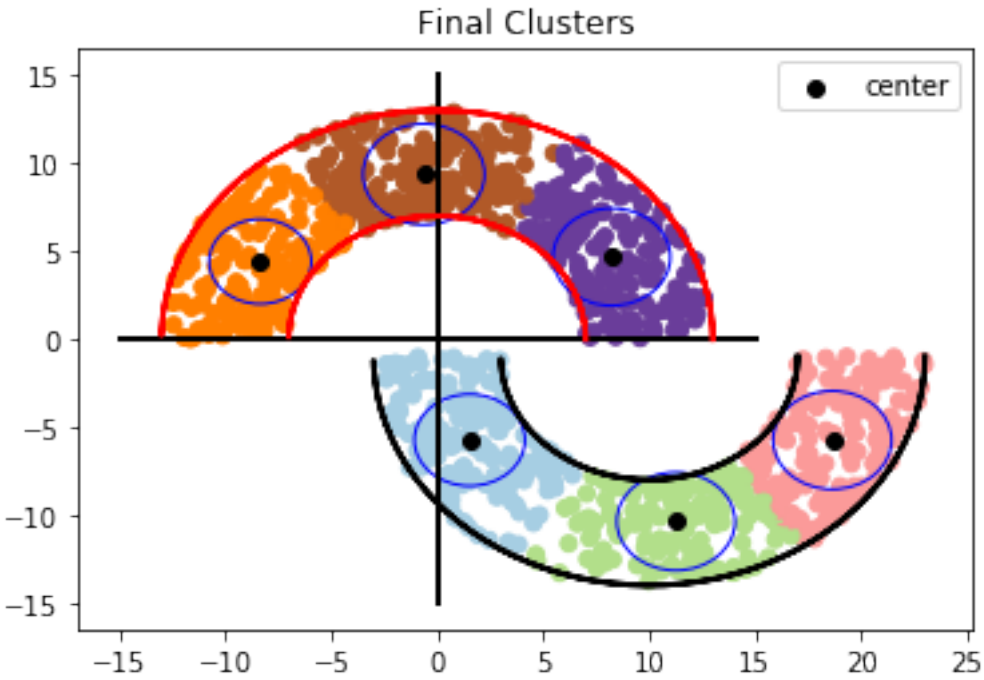
In [5]:
```
def plot_data(d, data, title= None, plt_moon=True,centers=None,std_deviation=None):
    """
        Plots the  labelled data
    """
    x_red_inner = R_INNER * np.cos(THETA)
    y_red_inner = np.abs(R_INNER * np.sin(THETA))
    x_red_outer = R_OUTER * np.cos(THETA)
    y_red_outer = np.abs(R_OUTER * np.sin(THETA))

    x_black_inner = x_red_inner + RADIUS
    y_black_inner = -y_red_inner - d
    x_black_outer = x_red_outer + RADIUS
    y_black_outer = -y_red_outer - d
    fig = plt.figure()
    ax = fig.add_subplot(1, 1, 1)
    plt.title(title)
    plt.plot([0, 0], [-15, 15], color= 'k', linewidth= 2)
    plt.plot([-15, 15], [0, 0], color= 'k', linewidth= 2)
    if (plt_moon):
        plt.plot(x_red_inner, y_red_inner, color= 'r', linewidth= 2)
        plt.plot(x_red_outer, y_red_outer, color= 'r', linewidth= 2)
        plt.plot(x_black_inner, y_black_inner, color= 'k', linewidth= 2)
        plt.plot(x_black_outer, y_black_outer, color= 'k', linewidth= 2)
    if data.shape[1]<3:
        plt.scatter(data[:,0], data[:,1], c='k')
    else:
        plt.scatter(data[:,0], data[:,1], c= data[:,2],cmap=plt.cm.Paired)
    if centers is not None:
        plt.scatter(centers[:,0],centers[:,1],c='k',label='center')
        for i in xrange(6):
            circle = plt.Circle(centers[i,:], np.ndarray.min(std_deviation[i]), color='b
```

```
                ax.add_artist(circle)
            plt.legend()
        plt.show()

In [6]: def k_mean(K,d,train_data):
        print"--------------------"
        print"d: ",d
        np.random.seed(0)
        center_idx = np.random.randint(train_data.shape[0], size=K)
        centers=train_data[center_idx,:]
        centers=centers[:,:2]
        MAX_ITERATION=100
        means=centers.copy()
        variance=np.zeros((means.shape))
        clustered_data=train_data
        clustered_data=np.insert(clustered_data, 2, -1, axis= 1)
        cont=True
        iteration=0
        while(cont):
            iteration+=1
            for idx,data in enumerate(train_data):
                dist=1000
                label=-1
                for i,meand in enumerate(means):
                    if np.linalg.norm(data-meand)<dist:
                        dist=np.linalg.norm(data-meand)
                        label=i+1
                clustered_data[idx,2]=label
            old_means=means.copy()
            for i,cluster in enumerate(np.unique(clustered_data[:,2])):
                data_in_cluster=clustered_data[np.where(clustered_data[:,2]==cluster)]
                means[i]=np.mean(data_in_cluster[:,:2],axis=0)
                #variance of each cluster is calculated
                variance[i]=np.var(data_in_cluster[:,:2],axis=0)
            if iteration>MAX_ITERATION or np.allclose(old_means,means):
                cont=False

        print "Number of Iterations:",iteration

        ######
        ## Select the maximum variance in either x or y direction
        ## Use the standard deviation from this variance to draw the circles in each cluster
        ######

        max_var=np.array(np.ndarray.max(variance,axis=1))[np.newaxis].T
        std_deviation=np.fabs(np.sqrt(max_var))
        plot_data(d,clustered_data,title="Final Clusters",centers=means,std_deviation=std_de
```

```
            return means,max_var
```

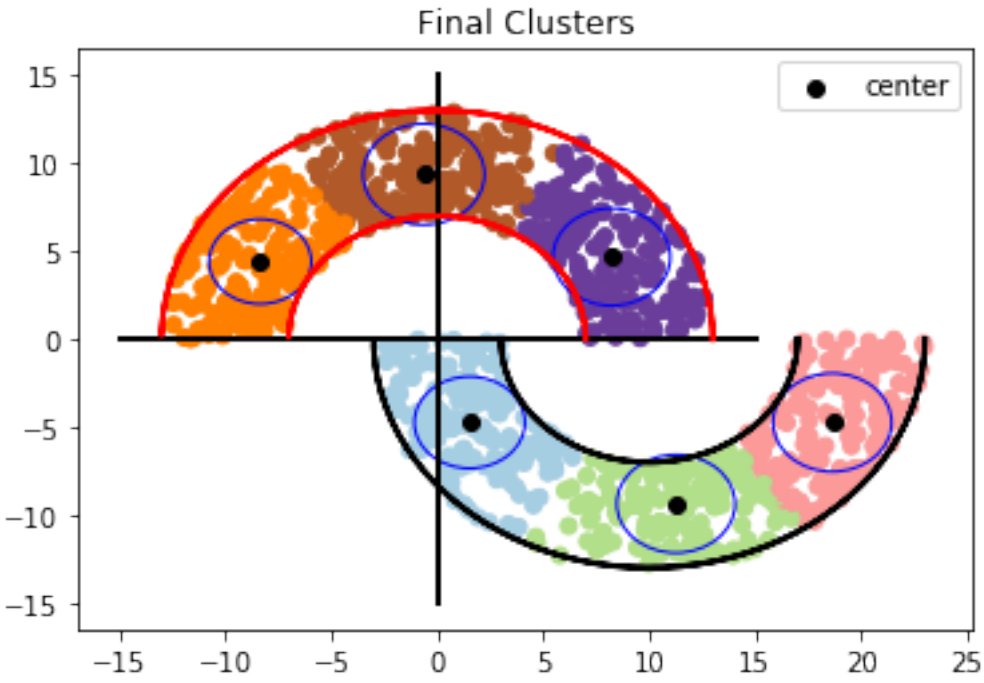
```
In [7]: K=6
        mean_arr=[]
        var_arr=[]
        for d in xrange(1,-7,-1):
            train_data=generate_points(d,size=1000)
            means,variance=k_mean(K,d,train_data.copy())
            #variance_min=np.array(np.ndarray.min(variance,axis=1))[np.newaxis].T
            print "d: ",d, ", \n mean:\n ", means, "\n variance: \n",variance[:,0]
            mean_arr.append(means)
            var_arr.append(variance[:,0])
```

```
--------------------
d:  1
Number of Iterations: 12
```

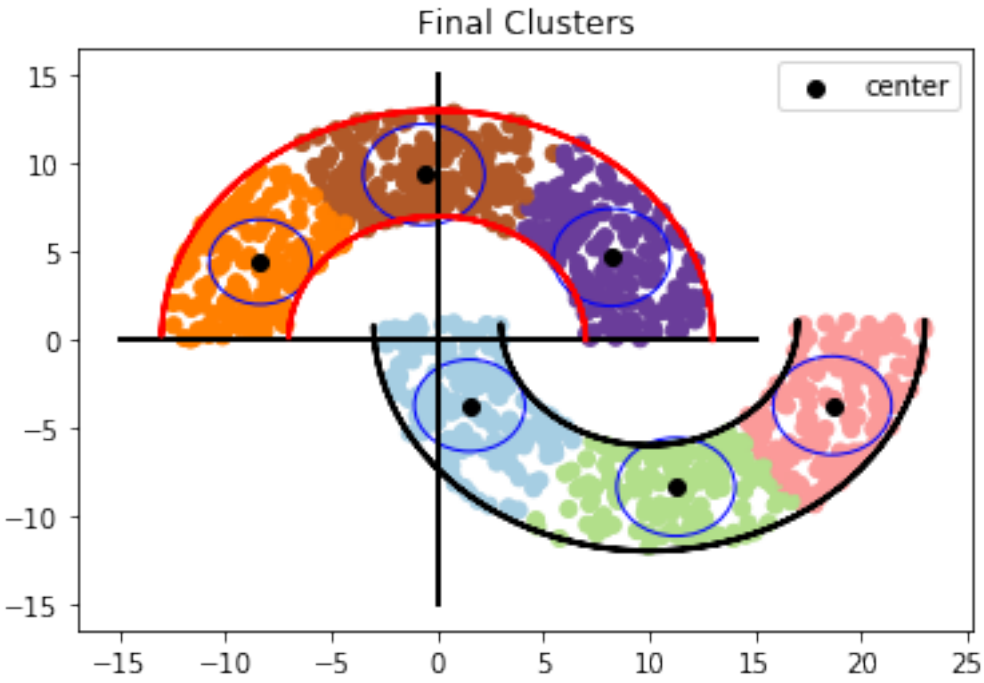



```
d:  1 ,
 mean:
  [[  1.53384649  -5.74065645]
 [ 11.25995174 -10.36174452]
 [ 18.63630696  -5.74250146]
 [ -8.34342405   4.4016335 ]
 [  8.23108037   4.66379749]
```

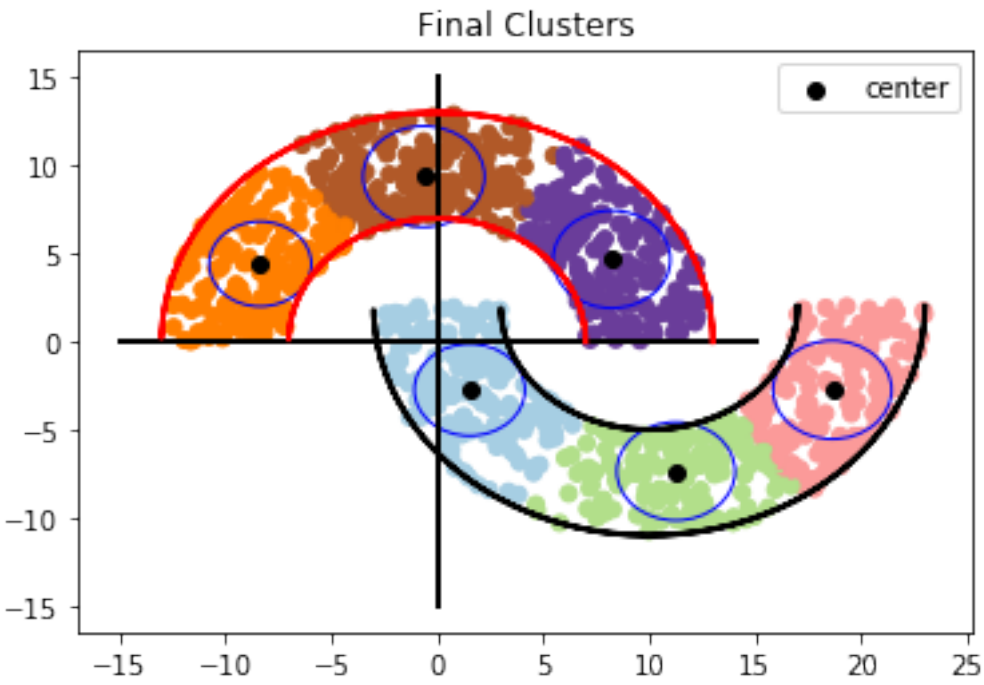
```
 [ -0.62948235   9.36226209]]
 variance:
[ 6.79128468  7.76869246  7.79908737  5.80099227  7.64537411  8.21272432]
--------------------
d:  0
Number of Iterations: 12
```



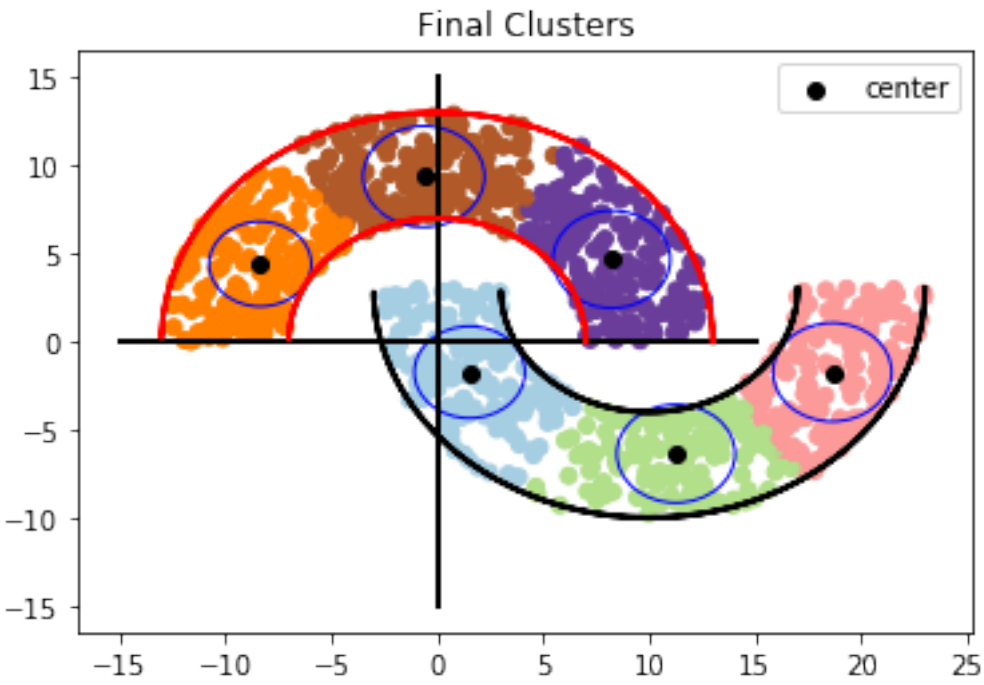

```
d:  0 ,
 mean:
  [[  1.53384649  -4.74065645]
 [ 11.25995174  -9.36174452]
 [ 18.63630696  -4.74250146]
 [ -8.34342405   4.4016335 ]
 [  8.23108037   4.66379749]
 [ -0.62948235   9.36226209]]
 variance:
[ 6.79128468  7.76869246  7.79908737  5.80099227  7.64537411  8.21272432]
--------------------
d:  -1
Number of Iterations: 10
```



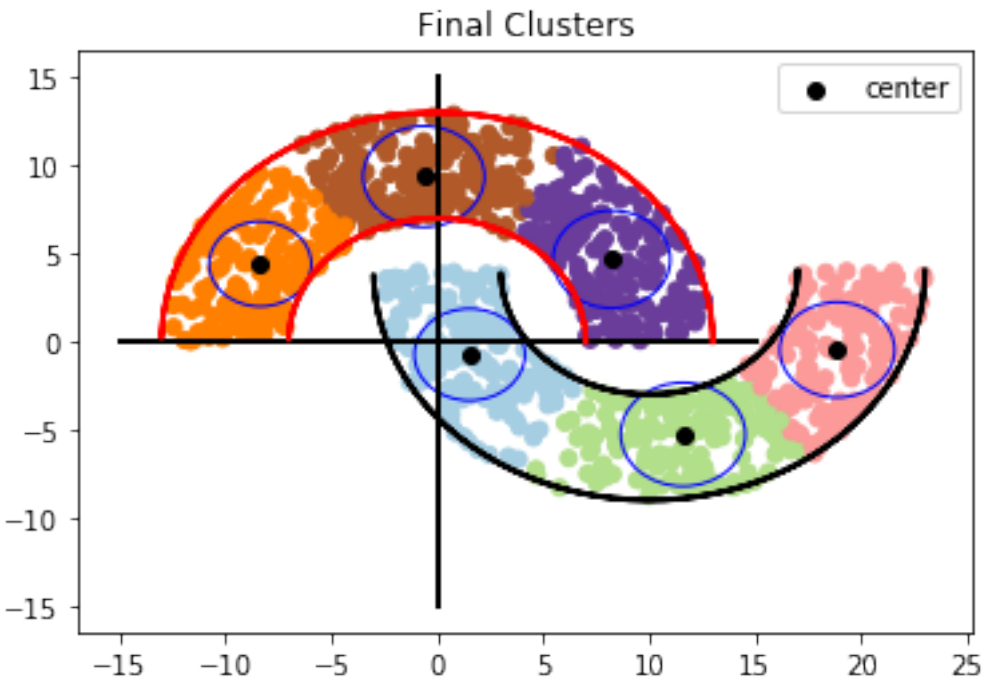

```
d:  -1 ,
 mean:
  [[  1.53384649  -3.74065645]
 [ 11.25995174  -8.36174452]
 [ 18.63630696  -3.74250146]
 [ -8.34342405   4.4016335 ]
 [  8.23108037   4.66379749]
 [ -0.62948235   9.36226209]]
 variance:
[ 6.79128468  7.76869246  7.79908737  5.80099227  7.64537411  8.21272432]
--------------------
d:  -2
Number of Iterations: 9
```



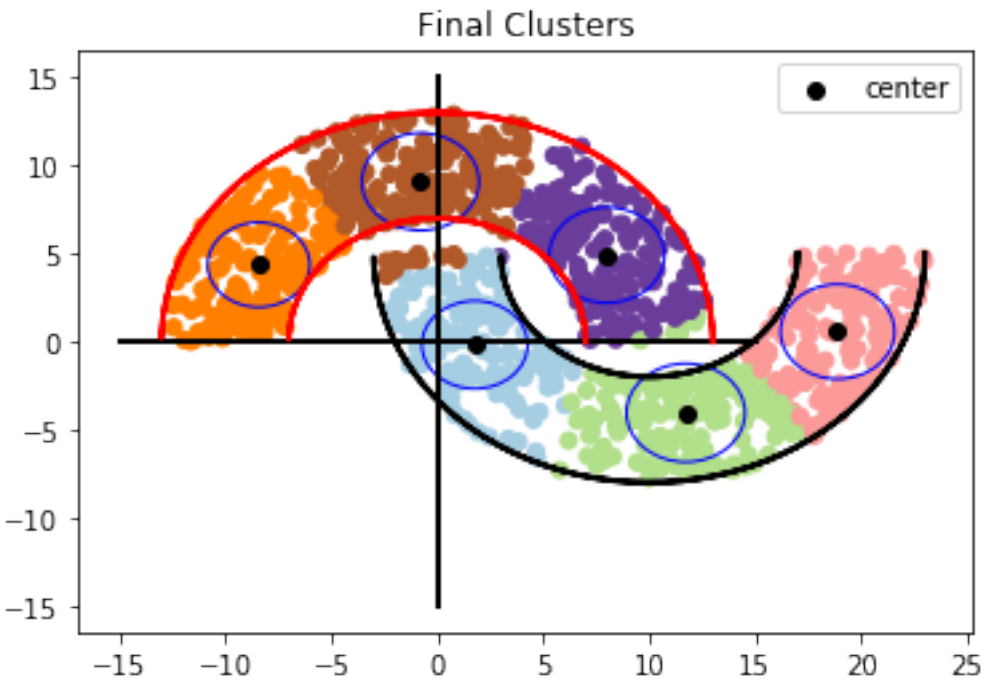

```
d:  -2 ,
 mean:
  [[  1.53384649  -2.74065645]
 [ 11.25995174  -7.36174452]
 [ 18.63630696  -2.74250146]
 [ -8.34342405   4.4016335 ]
 [  8.23108037   4.66379749]
 [ -0.62948235   9.36226209]]
 variance:
[ 6.79128468  7.76869246  7.79908737  5.80099227  7.64537411  8.21272432]
--------------------
d:  -3
Number of Iterations: 8
```



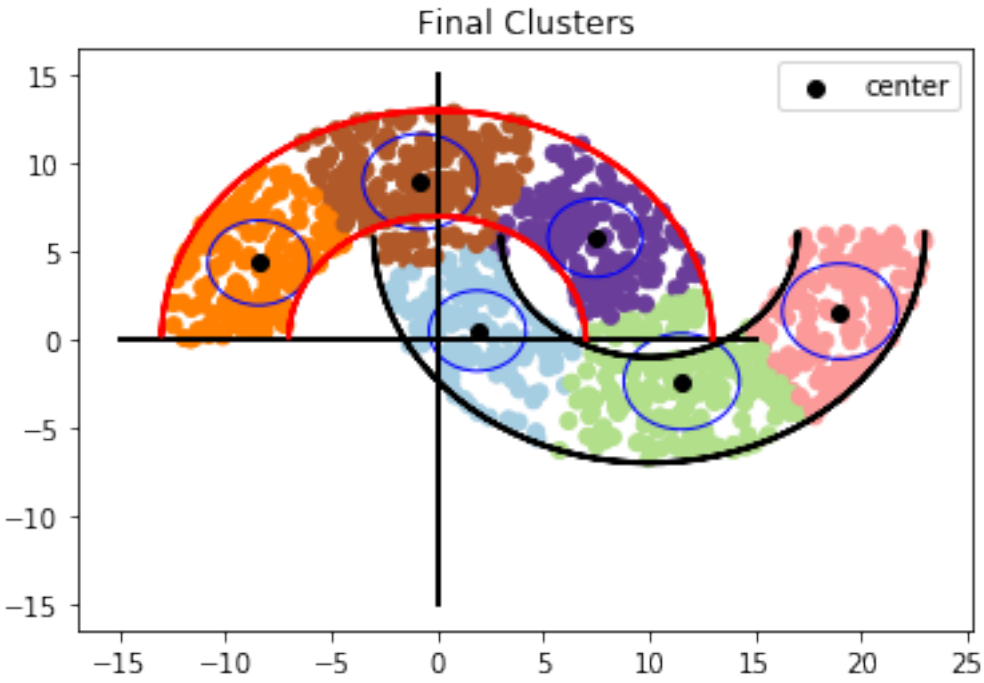

```
d:  -3 ,
 mean:
  [[  1.53384649  -1.74065645]
 [ 11.25995174  -6.36174452]
 [ 18.63630696  -1.74250146]
 [ -8.34342405   4.4016335 ]
 [  8.23108037   4.66379749]
 [ -0.62948235   9.36226209]]
 variance:
[ 6.79128468  7.76869246  7.79908737  5.80099227  7.64537411  8.21272432]
--------------------
d:  -4
Number of Iterations: 11
```




```
d:  -4 ,
 mean:
  [[  1.53384649  -0.74065645]
 [ 11.59547637  -5.27132635]
 [ 18.86557725  -0.46878889]
 [ -8.34342405   4.4016335 ]
 [  8.23108037   4.66379749]
 [ -0.62948235   9.36226209]]
 variance:
[ 6.79128468  8.64273356  7.31852224  5.80099227  7.64537411  8.21272432]
--------------------
d:  -5
Number of Iterations: 11
```




```
d:  -5 ,
 mean:
  [[  1.77476016  -0.18347103]
 [ 11.72524703  -4.04161889]
 [ 18.8801906    0.56248351]
 [ -8.41275389   4.35620142]
 [  8.00782483   4.90639428]
 [ -0.78836102   9.07054869]]
 variance:
[ 6.2130706   7.80778294  7.22225516  5.78999409  7.3697661   7.64605868]
--------------------
d:  -6
Number of Iterations: 8
```




```
d:  -6 ,
 mean:
  [[  1.89840369   0.4986249 ]
 [ 11.52871362  -2.37466708]
 [ 18.99119638   1.60078547]
 [ -8.41275389   4.35620142]
 [  7.44457906   5.76675746]
 [ -0.78789073   8.96938824]]
 variance:
[ 5.20751386  7.44748017  7.3628316   5.78999409  4.93099869  7.34677749]
```

3.3.2 Answer 5.10 (b)

The mean and variance does not change significantly, *relative to each moon*, as the separation between them is reduced.

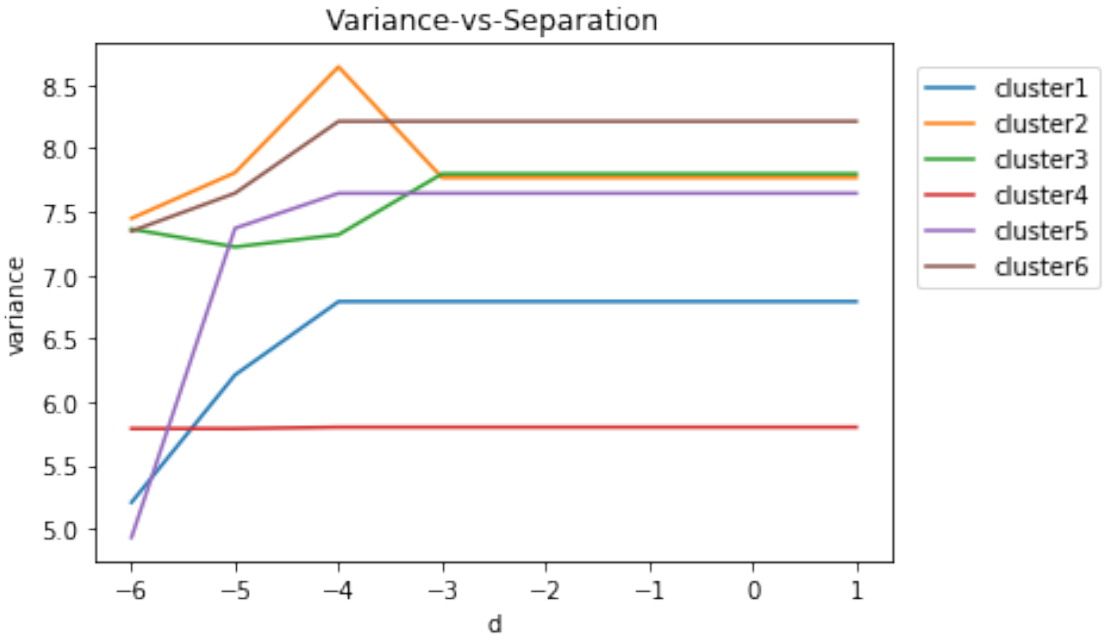
However, as the separation is decreased below -3, the clusters in the two moons seems to merge and the extent of this can be seen as d approaches -6, where the variances of each cluster drops significantly, relative to earlier results.

This can be visualized in the plot between variance and separation given below.

3.3.3 Answer 5.10 (c)

```
In [8]: clustered_var= np.flip(np.array(var_arr),axis=0).T
        d = np.array([-6,-5,-4,-3,-2,-1,0,1])
```

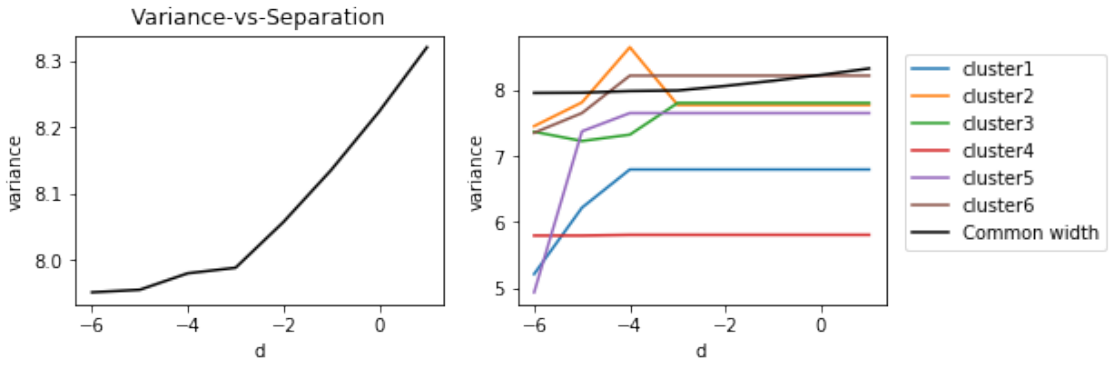
```
for i,var in enumerate(clustered_var):
    plt.plot(d,var,label='cluster'+str(i+1))
plt.xlabel('d')
plt.ylabel('variance')
plt.legend(bbox_to_anchor=(1,1), borderaxespad=1.)
plt.title("Variance-vs-Separation")
plt.show()
```



3.3.4 Answer 5.10 (d)

```
In [9]: clustered_mean= np.flip(np.array(mean_arr),axis=0)
        d = np.array([-6,-5,-4,-3,-2,-1,0,1])

        d_max=[]
        for i, mean in enumerate(clustered_mean):
            d_max.append(np.max(cdist(mean, mean)))

        d_max=np.array(d_max)
        emperical_var=d_max/np.sqrt(2*K)

        plt.figure(figsize=(8, 6))
        plt.subplot(221)
        plt.plot (d,emperical_var,label='Common width',color='k')
        plt.xlabel('d')
        plt.ylabel('variance')
```

```
#plt.legend(bbox_to_anchor=(1,1), borderaxespad=1.)
plt.title("Variance-vs-Separation")
plt.subplot(222)
for i,var in enumerate(clustered_var):
    plt.plot(d,var,label='cluster'+str(i+1))

plt.plot (d,emperical_var,label='Common width',color='k')
plt.xlabel('d')
plt.ylabel('variance')
plt.legend(bbox_to_anchor=(1,1),borderaxespad=1.)
plt.show()
```



From the above, it can be seen that the common width calculated from the emperical formula follows a similar trend that is observed in the other clusters. That is, as the separatin distance decreases, the variance starts decreasing.

However, the clusters do not reflect the smooth decrease that can be seen from the common width, the vaariance values in the clusters changes rapidly as the separation distance goes below -3

3.4 Question 3:

3) Repeat the Ex3 from the MPL assignment (topic :: BP) with the Radial Basis Functions

Investigate the use of Radial Basis Functions to achieve one-to-one mappings, as described here:

1. F(x) = 1/x 1<=x<=100
2. F(x) = $log_{10}x$ 1<=x<=10
3. F(x) = exp(-x) 1<=x<=10
4. F(x) = sin(x) 0<=x<=$\frac{\pi}{2}$

For each mapping, do the following:

- (a) Set up two sets of data, one for network training, and the other for testing.
- (b) Use the training data set to compute the synaptic weights of the network, assumed to have a single hidden layer.
- (c) Evaluate the computation accuracy of the network by using the test data. Use a single hidden layer but with a variable number of hidden neurones. Investigate how the network performance is affected by varying the size of the hidden layer.

For solving this problem you have to use your own implementation of RBF but you do not need to implement K-means algorithm yourself.

```
In [9]: class Radial_Basis_Function_Network():

            def __init__(self, K= 5):
                self.K = K
                self.weights = np.random.rand(self.K, 1)

            def unsupervised_params(self, data):

                cluster = KMeans(n_clusters= self.K)
                cluster.fit(data)
                centers = cluster.cluster_centers_
                d_max = np.max(cdist(centers, centers))
                sigma = d_max/np.sqrt(2*self.K)
                return centers, sigma

            def rbf(self, data, c, sig):

                return np.exp(-(np.linalg.norm(data - c, axis= 1, keepdims= True)**2)
                              /(2 * sig**2))

            def train(self, data, t):

                self.centers, self.sigma = self.unsupervised_params(data)
                activations = np.zeros((data.shape[0], 1))
                for center in self.centers:
                    activations = np.hstack((activations, self.rbf(data, center, self.sigma)))

                activations = activations[:,-self.K:]
                self.weights = np.dot(pinv(activations), t)

            def predict(self, data, y):

                activations = np.zeros((data.shape[0], 1))
                for center in self.centers:
                    activations = np.hstack((activations, self.rbf(data, center, self.sigma)))

                activations = activations[:,-self.K:]
```

```
                predictions = np.dot(activations, self.weights)
                return 1 - np.mean(np.abs(predictions - y))
```

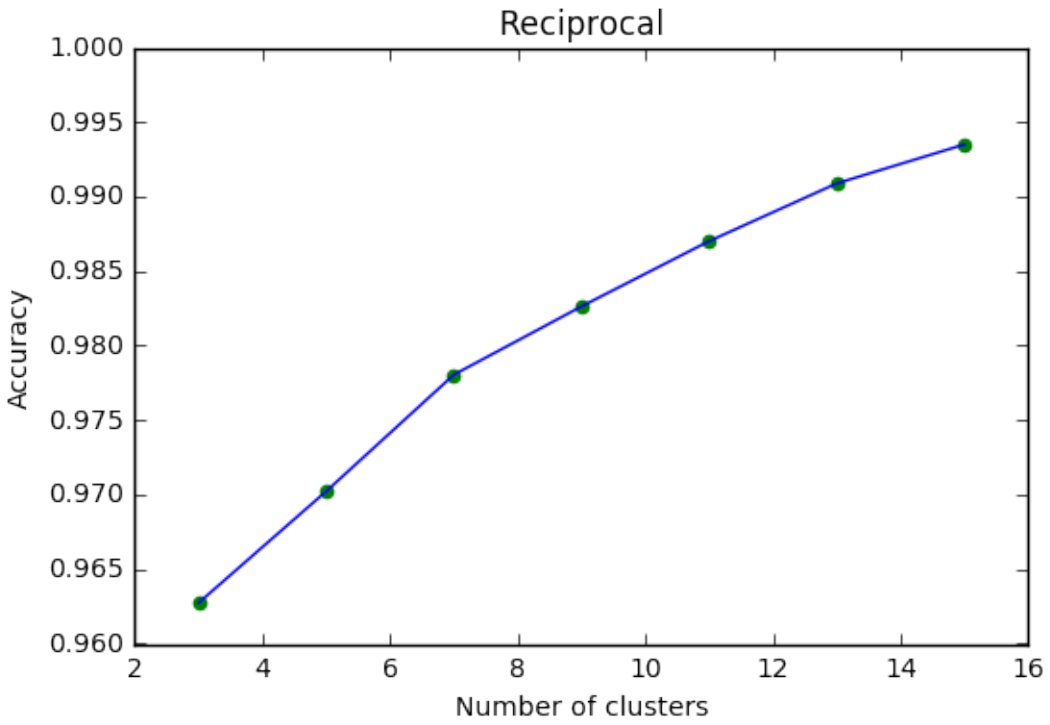
```
In [10]: K_list = np.array([3, 5, 7, 9, 11, 13, 15])

         def obtain_performance(data_x, data_y):
             data = np.hstack((data_x, data_y))
             train, test = train_test_split(data)

             accuracy = []
             for K in K_list:
                 nn = Radial_Basis_Function_Network(K= K)
                 nn.train(train[:,0].reshape(-1, 1),
                          train[:,1].reshape(-1, 1))
                 accuracy.append(nn.predict(test[:,0].reshape(-1, 1),
                                            test[:,1].reshape(-1, 1)))

             return np.array(accuracy)

         def plot_results(result, title):
             plt.plot(result[:,0], result[:,1], color= 'b')
             plt.scatter(result[:,0], result[:,1], color= 'g')
             plt.xlabel('Number of clusters')
             plt.ylabel('Accuracy')
             plt.title(title)
             plt.show()
```

3.4.1 1. Reciprocal:

```
In [11]: # generating x...
         reciprocal_x = np.linspace(1, 101, 1000).reshape(-1, 1)
         # generating y...
         reciprocal_y = 1/reciprocal_x

         accuracy =  obtain_performance(reciprocal_x, reciprocal_y)
         result = np.hstack((K_list.reshape(-1, 1), accuracy.reshape(-1, 1)))

         print ['number of clusters', 'accuracy']
         print result

         plot_results(result, 'Reciprocal')
```

```
['number of clusters', 'accuracy']
[[  3.          0.96274917]
 [  5.          0.9702349 ]
 [  7.          0.97805657]
 [  9.          0.98264011]
 [ 11.          0.98702036]
```

```
 [ 13.           0.99089268]
 [ 15.           0.99350616]]
```

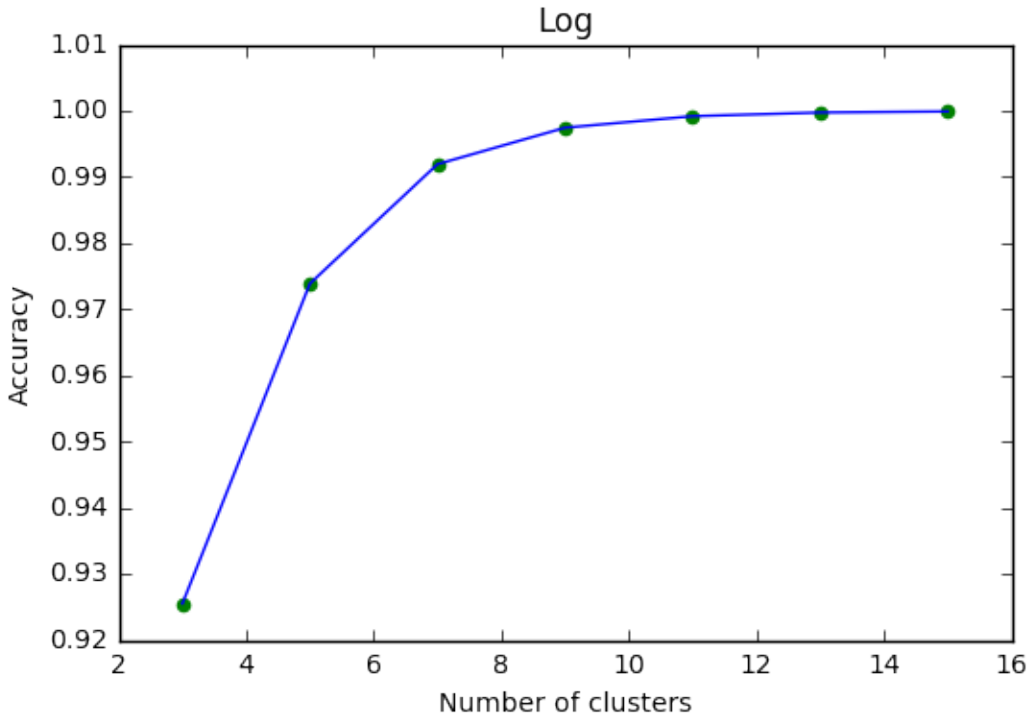


3.4.2 2. Log:

```
In [12]: # generating x...
         log_x = np.linspace(1, 11, 1000).reshape(-1, 1)
         # generating y...
         log_y = np.log10(log_x)

         accuracy =  obtain_performance(log_x, log_y)
         result = np.hstack((K_list.reshape(-1, 1), accuracy.reshape(-1, 1)))

         print ['number of clusters', 'accuracy']
         print result

         plot_results(result, 'Log')
```

```
['number of clusters', 'accuracy']
[[  3.           0.92549352]
 [  5.           0.97387425]
 [  7.           0.99192108]
 [  9.           0.99743685]
```

```
 [ 11.           0.99917476]
 [ 13.           0.99972966]
 [ 15.           0.99989323]]
```

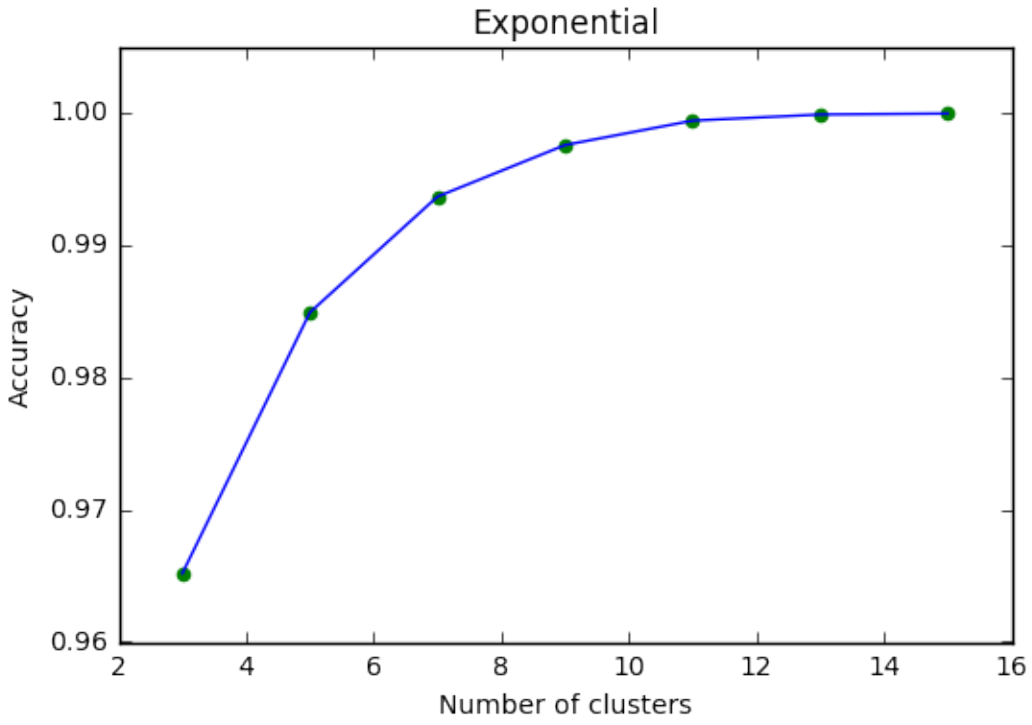


3.4.3 3. Exponential:

```
In [13]: # generating x...
         exp_x = np.linspace(1, 11, 1000).reshape(-1, 1)
         # generating y...
         exp_y = np.exp(-exp_x)

         accuracy =  obtain_performance(exp_x, exp_y)
         result = np.hstack((K_list.reshape(-1, 1), accuracy.reshape(-1, 1)))

         print ['number of clusters', 'accuracy']
         print result

         plot_results(result, 'Exponential')
```

```
['number of clusters', 'accuracy']
[[  3.           0.96518958]
 [  5.           0.98491113]
 [  7.           0.99366049]
```

```
 [  9.           0.99755883]
 [ 11.           0.99940331]
 [ 13.           0.99986519]
 [ 15.           0.99994021]]
```

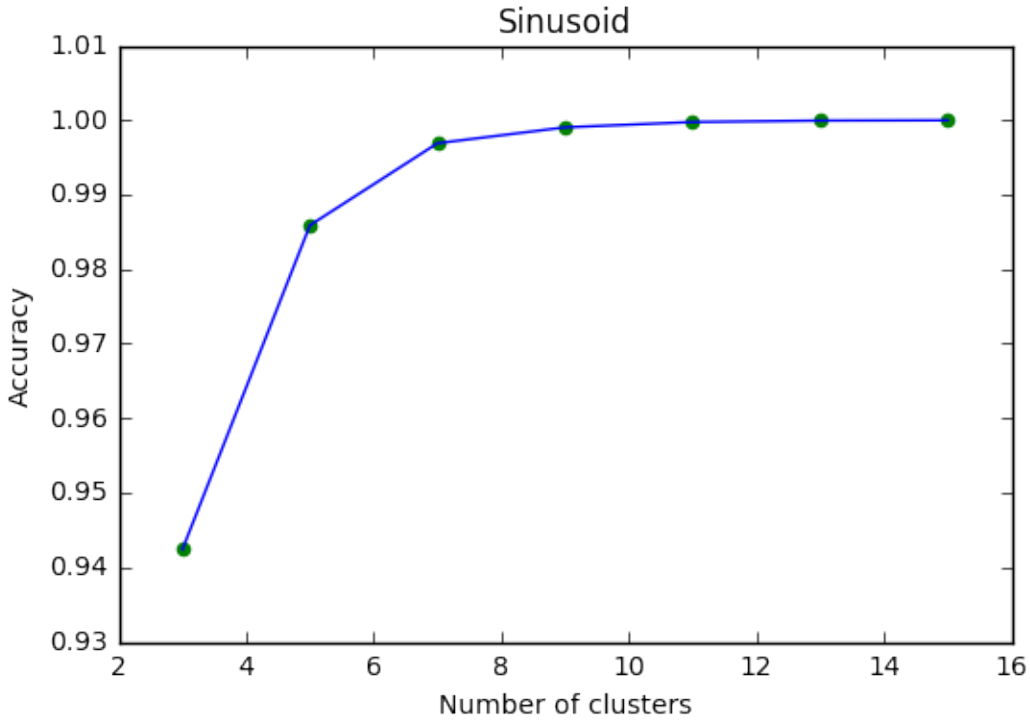


3.4.4 4. Sinusoid:

```
In [14]: # generating x...
         sin_x = np.linspace(0, np.pi/2, 1000).reshape(-1, 1)
         # generating y...
         sin_y = np.sin(sin_x)

         accuracy =  obtain_performance(sin_x, sin_y)
         result = np.hstack((K_list.reshape(-1, 1), accuracy.reshape(-1, 1)))

         print ['number of clusters', 'accuracy']
         print result

         plot_results(result, 'Sinusoid')
```

```
['number of clusters', 'accuracy']
[[  3.           0.94239183]
 [  5.           0.98589547]
```

```
[  7.          0.99690691]
[  9.          0.99904038]
[ 11.          0.99975268]
[ 13.          0.99994482]
[ 15.          0.9999851 ]]
```



In [ ]: