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1 Hochschule Bonn-Rhein-Sieg

2 Neural Networks, WS17/18

3 Assignment 03 (23-October-2017)

3.1 Debaraj Barua, Naresh Kumar Gurulingan

```
In [1]: import numpy as np
        from IPython.display import Image
```

3.2 Question 2: (*Problem 2.1*)

The delta rule described in Eq. (2.3) and Hebb's rule Eq. (2.9) represent two different methods of learning. List the features that distinguish these two rules from each other.

$$\Delta w_{kj}(n) = \eta e_k(n) x_j(n) \tag{2.3}$$

$$\Delta w_{kj}(n) = \eta y_k(n) x_j(n) \tag{2.9}$$

Where,
η is a positive constant that determines the rate of learning,
w_{kj} is the synaptic weight of neuron k excited by the element $x_j(n)$,
$\Delta w_{kj}(n)$ is the adjustment applied to synaptic weight w_{kj} at time step n,
$e_k(n)$ is the error signal,
y_k is the output signal (postsynaptic activity).

```
In [2]: Image("images/problem_2.1.png",width=1200)
```

```
Out[2]:
```

Answer:

Delta rule: $\Delta\omega_{kj}(n) = \eta \cdot e_k(n) \cdot x_j(n)$	Hebb's rule: $\Delta\omega_{kj}(n) = \eta \cdot y_k(n) \cdot x_j(n)$
In the delta rule, the error, $e_k(n) = d_k(n) - y_k(n)$ depends upon the desired output. Therefore, this rule can be used in a supervised setting (learning with a teacher).	In Hebb's rule, no desired output is required. Therefore, this rule can be used in a unsupervised setting(learning without a teacher)
Delta rule is based on steepest gradient error minimization on the weight space.	Hebb's rule is based on local covariance between adjacent neurons.
The delta rule cannot be used in networks with hidden neurons becuase in most cases the desired output of only the output neurons is known.	The Hebb's rule also applies to hidden neurons.

3.3 Question 3: (*Problem 2.10*)

Formulate the expression for the output y_i of neuron j in the network of Fig 2.4. You may use the following:

x_i = ith input signal
w_{ji} = synaptic weight from input i to neuron j
c_{kj} = weight of lateral connection from neuron k to neuron j
v_j = induced local field of neuron j
$y_j = \varphi(v_j)$
What is the condition that would have to be satisfied for neuron j to be the winning neuron?

```
In [3]: Image("images/Fig_2.4.png") #screenshot from Haykins book, Chapter 2
```

Out[3]:

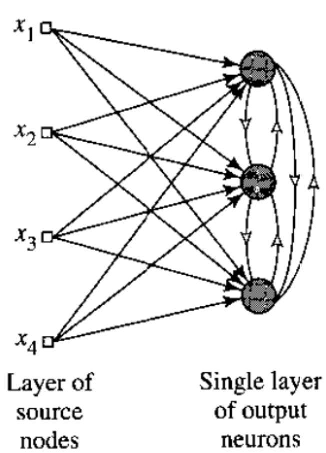


FIGURE 2.4 Architectural graph of a simple competitive learning network with feedforward (excitatory) connections from the source nodes to the neurons, and lateral (inhibitory) connections among the neurons; the lateral connections are signified by open arrows.

3.3.1 Answer:

The expression of output neuron y_j:

$$y_j = \varphi(\Sigma_{i=1}^{4} x_i \cdot \omega_{ji} + \Sigma_{k=1}^{3} c_{kj} \cdot \varphi(v_k)), \quad k \neq j$$

A neuron j is winning if:

$$y_j > y_k, \quad j \neq k, \quad k = 1, 2, 3$$

3.4 Question 4: (*Problem 2.21*)

Figure P2.21 shows the block diagram of an adaptive system. The input signal to the *predictive model* is defined by past values of a process, as shown by

$$x(n-1) = [x(n-1), x(n-2), ..., x(n-m)] \tag{1}$$

The model output, $\hat{x}(n)$, represents an *estimate* of the present value, $x(n)$, of the process. The *comparator* computes the error singal

$$e(n) = x(n) - \hat{x}(n) \tag{2}$$

which in turn applies a correction to the adjustable parameters of the model. It also supplies an output singal for transfer to the next level of neural processing of interpretation. By repeating this operation on a level-by-level basis, the information processed by the system tends to be progressively higher quality (Mead, 1990).

Fill in the details of the level of signal processing next to that described in P2.21

```
In [4]: Image("images/Fig_P2.21.png") #screenshot from Haykins book, Chapter 2
```

```
Out[4]:
```

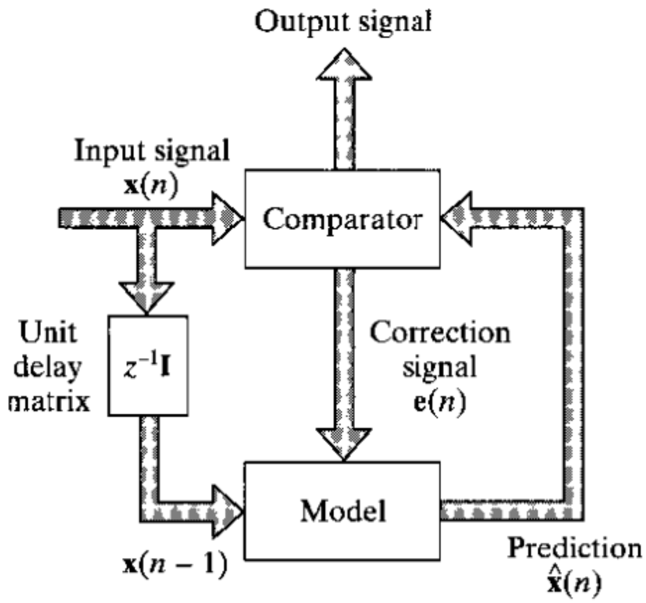


FIGURE P2.21

3.4.1 Answer:

- The prediction signal, $\hat{x}(n)$, is generated by the Model using a delayed input ($x(n-1)$) and the coorection signal $e(n)$.
- The correction signal $e(n)$ is generated by comparing the current signal to the predicted signal
- The input signal $x(n)$ is passed through a Unit delay matrix to get $x(n-1)$.
- The correction signal $e(n)$ is used to update the model.
- On repeating the above, we can imporove the model prediction, thus get better estimates.

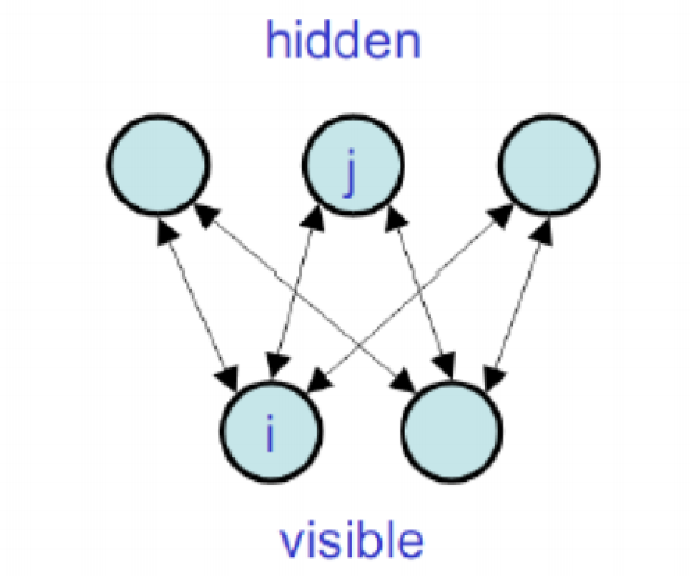
3.5 Question 5:

A simple network is given below (from lecture slides). You have to update the weights once using Boltzmann learning for this network. Please do calculations by hand or by using Python.

Use random numbers to initialise the weights. For training use a training set as $[(0,1),(1,0)]$ or any training set of your choice.

```
In [5]: Image("images/Fig_Q5.png") #screenshot from Haykins book, Chapter 2
```

```
Out[5]:
```



3.5.1 Answer:

The code is based on the github code in this link: GitHub
Also, the following blog has been referred to: Introduction to Restricted Boltzmann Machines

In [6]:
```
# boltzmann learning

hidden_neurons = 3
visible_neurons = 2

# random seed to get same random values everytime..
np.random.seed(0)
input_data = np.array([[0., 1.], [1., 0.]])

# number of data points...
number_data = input_data.shape[0]
learning_rate = 0.1

# zero centered weights generated from normal distribution..
weights = np.random.normal(0, 1, (visible_neurons, hidden_neurons))
# clamp visible states to input..
visible_state = input_data
print 'Initial state of visible neurons: '
print visible_state
print '----------------------------------------'
# random hidden states..
hidden_state = np.random.randint(0, 1, (number_data, hidden_neurons))
```

```
        print 'Initial state of hidden neurons: '
        print hidden_state
        print '---------------------------------------'
        print 'Initial random weights: '
        print weights
```

```
Initial state of visible neurons:
[[ 0.  1.]
 [ 1.  0.]]
---------------------------------------
Initial state of hidden neurons:
[[0 0 0]
 [0 0 0]]
---------------------------------------
Initial random weights:
[[ 1.76405235  0.40015721  0.97873798]
 [ 2.2408932   1.86755799 -0.97727788]]
```

```
In [7]: def logistic_function(v):
            return 1/(1+np.exp(-v))
```

```
In [8]: # local filed of hidden neurons in clamped condition..
        clamped_hid_v = input_data.dot(weights)
        # flipping probability of hidden neurons..
        flip_prob_clamped_hidden = logistic_function(clamped_hid_v)
        print 'Hidden state probabilities: '
        print flip_prob_clamped_hidden
        print '---------------------------------------'

        # clamped condition..
        clamped_correlation = input_data.T.dot(flip_prob_clamped_hidden)
        print 'Positive correlation in clamped condition: '
        print clamped_correlation
        print '---------------------------------------'
        # flip hidden states based on flipping probability...
        np.random.seed() # randomize flip_hidden..
        flip_hidden = flip_prob_clamped_hidden > (
            np.random.rand(number_data, hidden_neurons))
        hidden_state = np.abs(hidden_state - flip_hidden)
        print 'Flipped hidden states: '
        print hidden_state
        print '---------------------------------------'

        # network now runs free without effect of external input...
        # local field of visible neurons in free running state..
        free_run_vis_v = hidden_state.dot(weights.T)
        # flipping probability of visible neurons..
```

```
        flip_prob_free_visible = logistic_function(free_run_vis_v)
        # flip visible neuron states based on flipping probability...
        np.random.seed() # randomize flip_visible..
        flip_visible = flip_prob_free_visible > (
            np.random.rand(number_data, visible_neurons))
        visible_state = np.abs(visible_state - flip_visible)
        print 'Flipped visible states: '
        print visible_state
        print '---------------------------------------'

        # local filed of hidden neurons in free running state..
        free_run_hid_v = visible_state.dot(weights)
        # flipping probability of hidden neurons..
        flip_prob_free_hidden = logistic_function(free_run_hid_v)
        print 'Updated hidden probabilites in free running condition: '
        print flip_prob_free_hidden
        print '---------------------------------------'

        # free running condition..visible and hidden neurons used..
        free_run_correlation = visible_state.T.dot(flip_prob_free_hidden)
        print 'Negative correlation in free running condition: '
        print free_run_correlation
        print '---------------------------------------'

        # weight updated once...normalized over number of data points..
        weights += learning_rate * ((clamped_correlation -
                                     free_run_correlation) / number_data)
        print 'Updated weights: '
        print weights
```

```
Hidden state probabilities:
[[ 0.9038621   0.86617546  0.27343225]
 [ 0.85371646  0.59872543  0.72685773]]
---------------------------------------
Positive correlation in clamped condition:
[[ 0.85371646  0.59872543  0.72685773]
 [ 0.9038621   0.86617546  0.27343225]]
---------------------------------------
Flipped hidden states:
[[1 0 1]
 [1 0 0]]
---------------------------------------
Flipped visible states:
[[ 1.  0.]
 [ 0.  1.]]
---------------------------------------
Updated hidden probabilites in free running condition:
[[ 0.85371646  0.59872543  0.72685773]
```

```
 [ 0.9038621   0.86617546  0.27343225]]
---------------------------------------
Negative correlation in free running condition:
[[ 0.85371646  0.59872543  0.72685773]
 [ 0.9038621   0.86617546  0.27343225]]
---------------------------------------
Updated weights:
[[ 1.76405235  0.40015721  0.97873798]
 [ 2.2408932   1.86755799 -0.97727788]]
```