# NN_DebarajBarua_161017

October 21, 2017

# 1 Hochschule Bonn-Rhein-Sieg

# 2 Neural Networks, WS17/18

# 3 Assignment 02 (16-October-2017)

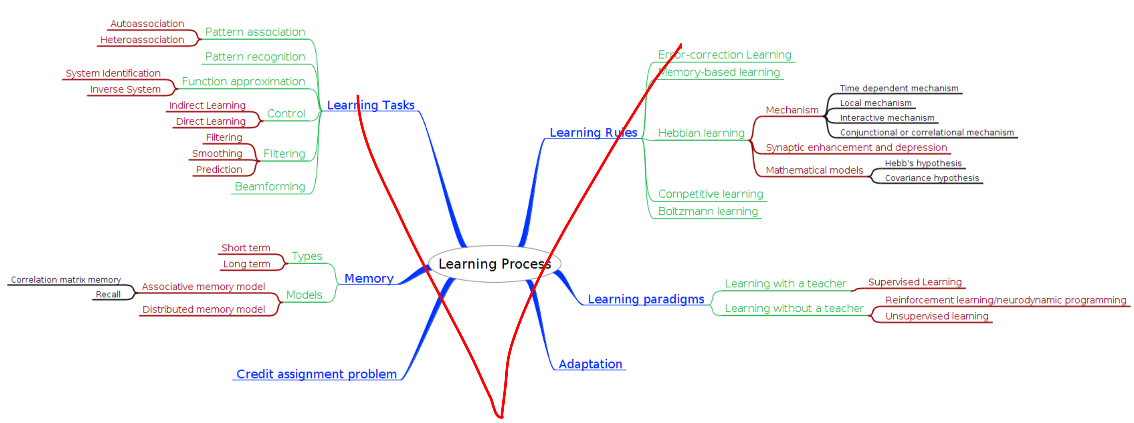
In [1]:
```
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
sp.init_printing(use_latex=True)
from IPython.display import Image
```

---

## 3.1 Question 1: Read chapter 2 from Haykin's book until 2.13 (leaving out Statistical learning theory to end of chapter) and summarize or sketch your insights in mind-map or an outline or a summary.

In [2]:
```
Image("Images/HaykinChapter2_mindmap.png")
```

Out[2]:

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### 3.2 Question 3: Do the problem 1.13 (Network architecture) from the previous week's assignment. This time use Python's (sympy) symbolic toolbox. Finally assume the network presented in fig P1.13 is a binary-classifier, please depict how the input space (R2) is classified on a 2D graph using different colors.

**1.13 (a): Figure P1.13 shows the signal-flow graph of a 2-2-2-1 feedforward network. The function $\varphi(\cdot)$ denotes a logistic function. Write the input/output mapping defined by this network.**

In [3]:
```
Image("Images/ScreenshotP113.png") #screenshot from Haykins book, Chapter 1b
```

Out[3]:

Chapter 1 Introduction

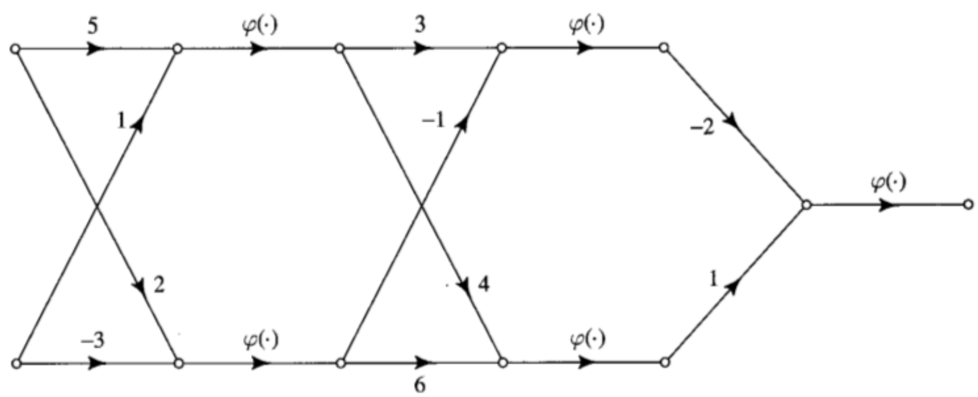



FIGURE P1.13

In [4]:
```
#defining inputs (x1,x2) and logistic function (phi)
x1,x2,v=sp.symbols('x1,x2,v')
phi = 1/(1+sp.exp(-1*v)) #Using logistic function from 1.1

#defining hidden layers
hidden_layer_11 = phi.subs(v,5*x1+x2)
hidden_layer_12 = phi.subs(v,2*x1-3*x2)
hidden_layer_21 = phi.subs(v,3*hidden_layer_11-hidden_layer_12)
hidden_layer_22 = phi.subs(v,4*hidden_layer_11+6*hidden_layer_12)
```

In [5]:
```
output_layer=phi.subs(v,-2*hidden_layer_21+hidden_layer_22)
output_layer
```

Out[5]:

$$\frac{1}{e^{\frac{2}{e^{\frac{1}{e^{-2x_1+3x_2}+1}-\frac{3}{e^{-5x_1-x_2}+1}}+1}-\frac{1}{e^{-\frac{6}{e^{-2x_1+3x_2}+1}-\frac{4}{e^{-5x_1-x_2}+1}}+1}}+1}$$

**1.13 (b): Suppose the output neuron in the signal-flow graph for Figure P1.13 operates in the linear region. Write the linear input-output mapping defined by this new network.**

In [6]:
```
#Slope of the logistic function is
phi_prime=sp.diff(phi,v)
phi_prime_0=phi_prime.subs(v,0) #slope at origin ---slope
phi_0=phi.subs(v,0) #function value at 0 ---intercept

#For a linear signal, output=slope*x+intercept
#So, using this for the output layer

output_layer=phi_prime_0*(-2*hidden_layer_21+hidden_layer_22)+phi_0
output_layer
```

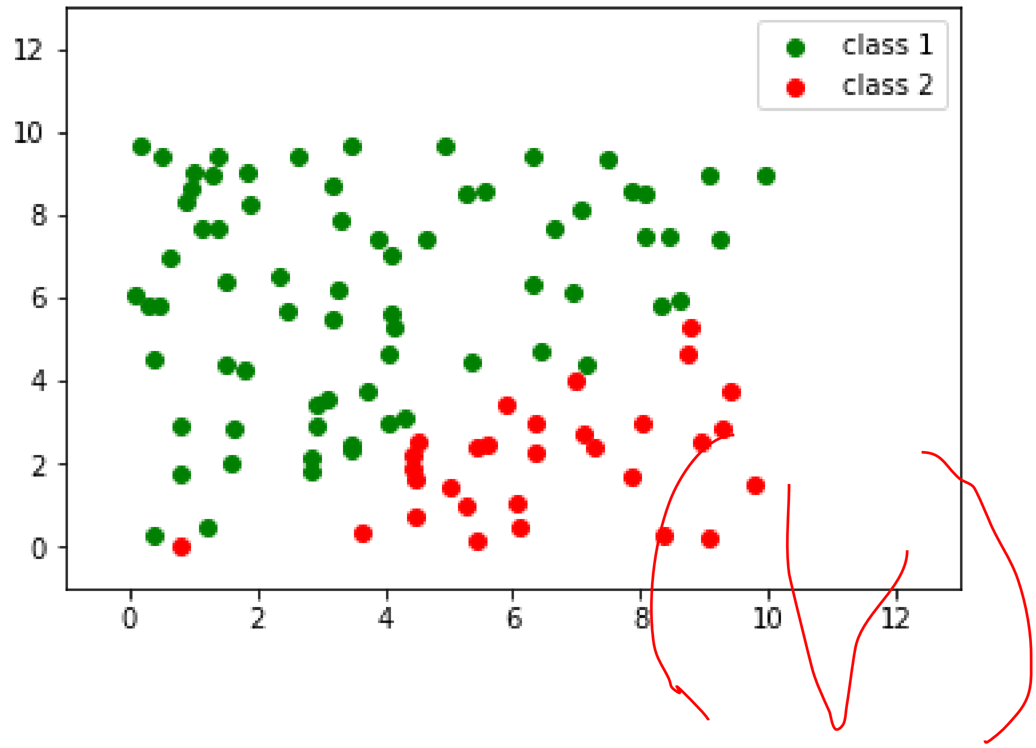
Out[6]:

$$\frac{1}{2} - \frac{1}{2e^{\frac{1}{e^{-2x_1+3x_2}+1} - \frac{3}{e^{-5x_1-x_2}+1}} + 2} + \frac{1}{4e^{-\frac{6}{e^{-2x_1+3x_2}+1} - \frac{4}{e^{-5x_1-x_2}+1}} + 4}$$

**Assume the network presented in fig P1.13 is a binary-classifier, please depict how the input space (R2) is classified on a 2D graph using different colors.**

In [7]:
```
"""
Taking 100 random values X1 and X2 each.

We will use the binary classifier from above to
divide the points (X1,X2) into two classes.
"""

x_1=np.random.rand(100)*10
x_2=np.random.rand(100)*10


y=np.zeros(100)
c1=[]
c2=[]

for i in xrange(100):
    y[i]=output_layer.subs({x1:x_1[i],x2:x_2[i]})

    #classify to class1 if y is<0.3, else class2
    if y[i]<0.3:
        c1.append([x_1[i],x_2[i]])
    else:
        c2.append([x_1[i],x_2[i]])
c1=np.array(c1)
c2=np.array(c2)

plt.scatter(c1[:,0],c1[:,1],color='g', label='class 1')
```

```
plt.scatter(c2[:,0],c2[:,1],color='r', label='class 2')
plt.xlim(-1,13)
plt.ylim(-1,13)
plt.legend()
plt.show()
```

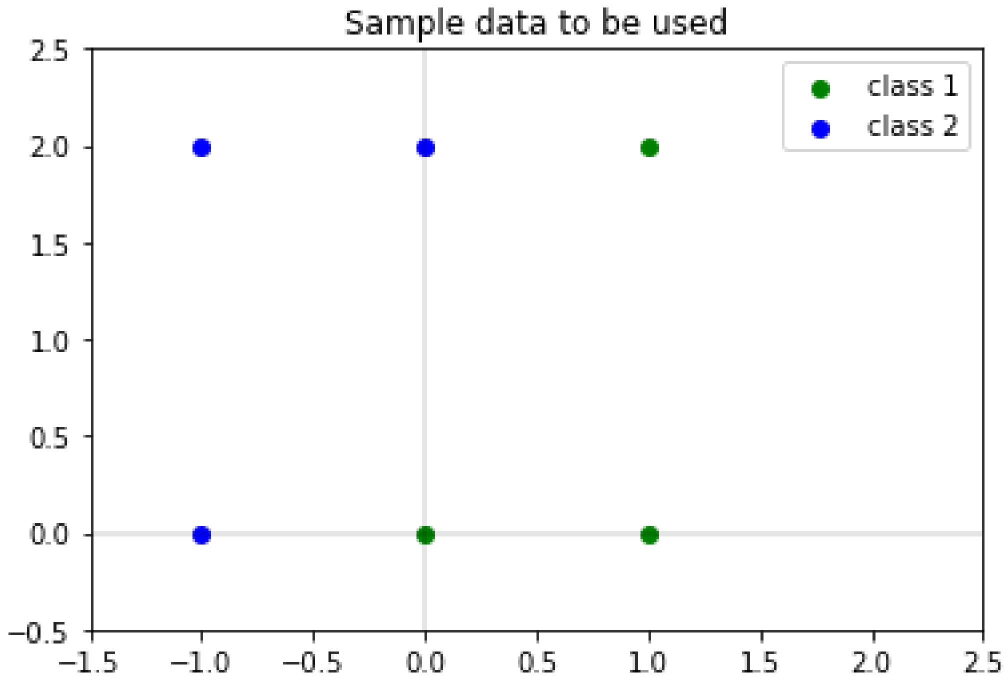



---

### 3.3 Question 4: Adjust the data at the "New Classification Example (now *with* bias)" slide, such that a bais becomes necessary (not 0). Validate the perceptron learning algorithm.

In [8]:
```
"""
Setting up sample data
"""
C1=np.array(([1,2],[1,0],[0,0]))       #class 1
C2=np.array(([-1,0],[-1,2],[0,2]))    #class -1

plt.scatter(C1[:,0], C1[:,1],label='class 1', color='g')
plt.scatter(C2[:,0], C2[:,1],label='class 2', color='b')
plt.plot([0,0],[-3,3],color='black', linewidth=0.2)
plt.plot([-3,3],[0,0],color='black', linewidth=0.2)
plt.xlim(-1.5,2.5)
plt.ylim(-0.5,2.5)
plt.title('Sample data to be used')
```

```
plt.legend()
plt.show()
```




In [9]:
```
"""
Using perceptron learning algorithm to classify
the above data.
"""
C1_dash=np.array(([1,1,2],[1,1,0],[1,0,0]))
C2_dash=np.array(([-1,1,0],[-1,1,-2],[-1,0,-2]))
C=np.vstack((C1_dash,C2_dash))

w=np.array(([1,0,0]))[np.newaxis].T #initial weights
eta=1                                #learning rate

flag=True
count=0

print "----------"
while(flag):
    count+=1
    flag=False
    for i,val in enumerate(C):
        val=val[np.newaxis].T
        print "w: ",w.T
        if w.T.dot(val)<=0:
```

```
                w=w+eta*(val)

                """
                since there was an update of the weights,
                so, proceed to next epoch.
                set "flag" to "True"
                """
                flag=True

        print "Ending Epoch ", count
        print "----------"

    w=w.T
    print ""
    print ""
    print "Final weight vector: ",w
    print "Total number of iterations needed: ",count
    print"-----------"
    x1,x2=sp.symbols('x1,x2')
    eq=w[0,1]*x1+w[0,2]*x2+ w[0,0]
    print "Equation of Decision hyperplane is: \n"
    eq
```

```
----------
w:  [[1 0 0]]
w:  [[1 0 0]]
w:  [[1 0 0]]
w:  [[1 0 0]]
w:  [[0 1 0]]
w:  [[0 1 0]]
Ending Epoch  1
----------
w:  [[-1  1 -2]]
w:  [[0 2 0]]
w:  [[0 2 0]]
w:  [[1 2 0]]
w:  [[1 2 0]]
w:  [[1 2 0]]
Ending Epoch  2
----------
w:  [[ 0  2 -2]]
w:  [[1 3 0]]
w:  [[1 3 0]]
w:  [[1 3 0]]
w:  [[1 3 0]]
w:  [[1 3 0]]
Ending Epoch  3
----------
```

```
w:  [[ 0  3 -2]]
w:  [[1 4 0]]
w:  [[1 4 0]]
w:  [[1 4 0]]
w:  [[1 4 0]]
w:  [[1 4 0]]
Ending Epoch  4
----------
w:  [[ 0  4 -2]]
w:  [[1 5 0]]
w:  [[1 5 0]]
w:  [[1 5 0]]
w:  [[1 5 0]]
w:  [[1 5 0]]
Ending Epoch  5
----------
w:  [[ 0  5 -2]]
w:  [[ 0  5 -2]]
w:  [[ 0  5 -2]]
w:  [[ 1  5 -2]]
w:  [[ 1  5 -2]]
w:  [[ 1  5 -2]]
Ending Epoch  6
----------
w:  [[ 1  5 -2]]
w:  [[ 1  5 -2]]
w:  [[ 1  5 -2]]
w:  [[ 1  5 -2]]
w:  [[ 1  5 -2]]
w:  [[ 1  5 -2]]
Ending Epoch  7
----------


Final weight vector:  [[ 1  5 -2]]
Total number of iterations needed:  7
-----------
Equation of Decision hyperplane is:
```

Out[9]:

$$5x_1 - 2x_2 + 1$$

Equation of the hyperplane is given by:

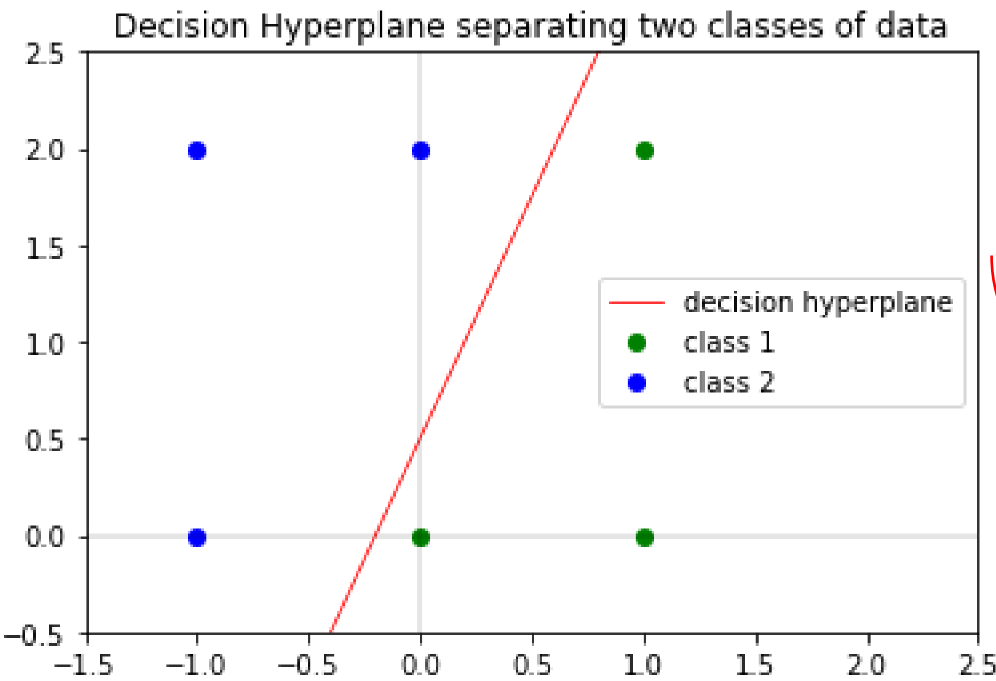
$$w_1 \cdot x_1 + w_2 \cdot x_2 + c = 0$$

So,

$$x_2 = \frac{-w_1 \cdot x_1 - c}{w_2}$$

Here,
$w_1 = w[0,1] = 5$,
$w_2 = w[0,2] = -2$,
$c = w[0,0] = 1$

In [10]:
```
x1=np.linspace(-1,1,3)
x2=(-w[0,1]*x1- w[0,0])/w[0,2]

fig, ax = plt.subplots()
plt.plot(x1,x2,label='decision hyperplane',color='r', linewidth=0.8)
plt.scatter(C1[:,0], C1[:,1],label='class 1', color='g')
plt.scatter(C2[:,0], C2[:,1],label='class 2', color='b')
plt.plot([0,0],[-3,3],color='black', linewidth=0.2)
plt.plot([-3,3],[0,0],color='black', linewidth=0.2)
plt.xlim(-1.5,2.5)
plt.ylim(-0.5,2.5)
plt.title('Decision Hyperplane separating two classes of data')
plt.legend()
plt.show()
```




From the above graph, we see that the decision hyperplane divides the two classes.