Coding Exercise 5

1. Write a code to compute the root of the following functions using bisection method:

   (a) $f(x) = \cos(x) - x^3$.

   (b) $f(x) = 3\cos(x) - e^x$.

   (c) $f(x) = x^{\frac{1}{3}} - e^{-x^2}$.

   (d) $f(x) = x^4 - 3x^3 + 4x^2 + 5x - 2$.

   (e) $f(x) = 8 - 12x + 86x^2 - 121x^3 + 60x^4 - 10x^5$.

   Use a tolerance of 10^{-6}. Print n, the number of iterations for convergence. Plot the trajectories of both the left hand limits $x_{l0}, x_{l1}, \ldots, x_{ln}$ and the right hand limits $x_{r0}, x_{r1}, \ldots, x_{rn}$ in the same plot.

2. Write a code to compute the root of the above functions using Newton-Raphson method. Use a tolerance of 10^{-6}. Print n, the number of iterations for convergence. Plot the trajectory of the points $x_0, x_1, \ldots, x_n$.

3. Generate $X_1, X_2, \ldots, X_{1000}$ i.i.d. $\sim f(x)$, where $f(x) = xe^{-x}$, $x > 0$. Use either the bisection or the Newton-Raphson method to compute F^{-1}. Verify with a histogram.

4. For a given positive integer n, generate $X_1, X_2, \ldots, X_{1000}$ i.i.d. $\sim f(x)$, where $f(x) = \frac{n+1}{2n}(1 - |x|^n)$, $x \in [-1, 1]$. Again, use any of the numerical methods to compute F^{-1}. Verify with a histogram.

5. (a) Consider solving the nonlinear equations $x^{\frac{1}{5}} + y^{\frac{1}{5}} = 2$ and $x^{\frac{1}{10}} + y^{\frac{2}{5}} = 2$ simultaneously. Apply Newton's method starting from $(3, 3)$. What do you observe?

   (b) Now consider the equations $(x^{\frac{1}{5}} + y^{\frac{1}{5}})^5 = 32$ and $(x^{\frac{1}{10}} + y^{\frac{2}{5}})^4 = 16$. Apply Newton's method starting from $(3, 3)$. What do you observe?

   (c) Apply Newton's method starting from $(0.5, 0.5)$ for both cases. In which case is the convergence faster? Why?

6. Consider the Cournot's oligopoly setting with three firms. Compute the quantities produced at equilibrium if

   (a) $P(X) = (1 - X)_+$, $C_i(x) = x\log(x)$.

   (b) $P(X) = (1 - X^2)_+$, $C_i(x) = \frac{x}{2}$.

   (c) $P(X) = (1 - X)_+$, $C_i(x) = x^i$.