129

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1 Introduction

First I took the Central Difference Formula for approximating derivatives, that is $f'(x_j) \approx \frac{f(x_{j+1})-f(x_{j-1})}{2\Delta x}$ and $f''(x_j) \approx \frac{f(x_{j+1})-2f(x_j)+f(x_{j-1})}{\Delta x^2}$ (just imagine replacing each f term with ψ).Then I examined the Time Dependent Schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

Ignoring the constants, we can rewrite this as:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

Based upon what we know from quantum mechanics (this is from intro QM classes) using separation of variables, we get the time dependent term as $e^{\frac{-iEt}{\hbar}}$ and we are left with the Time Independent SE:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi = E\psi$$

Where I then plugged in the derivative approximations:

$$-\frac{\hbar^2}{2m}\frac{f(x_{j+1})-2f(x_j)+f(x_{j-1})}{\Delta x^2}+V(x)\psi = E\psi$$

Which when we separate into like terms equates to:

$$-\frac{\hbar^2}{2m}(\frac{\psi(x_{j+1})+\psi(x_{j-1})}{\Delta x^2})+\psi(\frac{\hbar^2}{2m}+V(x)) = E\psi$$