Sample Survey Class Notes (Day 1)

Solution to Probability Problem 1

(i) Since we are drawing without replacement so

$P(a=b)=0$

Now, $P(a>b)+P(a=b)+P(a<b)=1$

$\Rightarrow \quad 2\cdot P(a>b)=1$ (symmetry)

$\Rightarrow \quad P(a>b)=\frac{1}{2}$ (Ans.)

(ii) Let a, b, c denote the selected numbers

a → 1st no.
b → 2nd no.
c → 3rd no.

Total no. of possible arrangements $= 3! = 6$

No. of arrangements we are interested in $= 2$

$a<b<c$

$c<a<b$

$\therefore$ Required probability $= \frac{2}{6}$ (Ans.)

Solution to Exercise 1

SRSWR

$\pi_i = P($ ith unit belongs to the sample$)$

$= P(U_i \in s)$

$= 1 - P(U_i \notin s)$

$= 1 - \frac{(N-1)^n}{N^n}$

SRSWOR

$P(U_i \in s)$

$= 1 - P(U_i \notin s)$

$= 1 - \frac{{}^{N-1}P_n}{{}^{N}P_n} = 1 - \frac{1}{N}(N-n)$

$= 1 - \left(1-\frac{n}{N}\right) = \frac{n}{N}$

Solution to Exercise 2

SRSWR

$$\pi_{ij} = P(\underbrace{U_i \in s}_{A} \text{ and } \underbrace{U_j \in s}_{B})$$

$$= P(A \cap B)$$

$$= 1 - P((A \cap B)^c)$$

$$= 1 - P(A^c \cup B^c)$$

$$= 1 - P(A^c) - P(B^c) + P(A^c \cap B^c)$$

$$= 1 - P(U_i \notin s) - P(U_j \notin s) + P(U_i \notin s \text{ and } U_j \notin s)$$

$$= 1 - 2\left(\frac{N-1}{N}\right)^n + \left(\frac{N-2}{N}\right)^n$$

SRSWOR

$$\pi_{ij} = 1 - P(U_i \notin s) - P(U_j \notin s) + P(U_i \notin s, U_j \notin s)$$

$$= 1 - 2\left(1 - \frac{n}{N}\right) + \frac{{}^{N-2}P_n}{{}^{N}P_n}$$

$$= 1 - 2 + \frac{2n}{N} + \frac{(N-n)(N-n-1)}{N(N-1)}$$

$$= -1 + \frac{2n}{N} + \left(1 - \frac{n}{N}\right)\left(1 - \frac{n}{N-1}\right)$$

$$= \cancel{-1} + \frac{2n}{N} + \cancel{1} - \frac{n}{N-1} - \frac{n}{N} + \frac{n^2}{N(N-1)}$$

$$= \frac{n}{N} - \frac{n}{N-1} + \frac{n^2}{N(N-1)}$$

$$= \frac{-n}{N(N-1)} + \frac{n^2}{N(N-1)} = \frac{n(n-1)}{N(N-1)}$$

Solution to Probability Problem 2

$Z_i = 1$ if U_i appears atleast once in the sample

$= 0$ otherwise

Note $\{Z_i\}_{i=1}^{n}$ are identical but not independent

$$m = \sum_{i=1}^{N} Z_i$$

$$E(m) = E\left(\sum_{i=1}^{N} Z_i\right)$$

$$= N \cdot E(Z_i) = N \cdot \pi_i \quad (\text{for SRSWR})$$

$$= N\left[1 - \left(\frac{N-1}{n}\right)^n\right]$$

$$V(m) = V\left(\sum_{i=1}^{N} Z_i\right)$$

$$= \sum_{i=1}^{N} V(Z_i) + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \operatorname{Cov}(Z_i, Z_j)$$

$$V(Z_i) = \pi_i - \pi_i^2 = \pi_i(1 - \pi_i)$$

$$\operatorname{Cov}(Z_i, Z_j) = \pi_{ij} - \pi_i \pi_j$$

We know the values of these inclusion probabilities for SRSWR → just plug in.

Solution to Probability Problem 3

(a) SRSWR

$X \equiv$ RV denoting the highest no. written on the tickets

$$P(X = M)$$

$$= P(X \leq M) - P(X \leq M-1)$$

$$= \frac{M^n}{N^n} - \frac{(M-1)^n}{N^n} = \frac{M^n - (M-1)^n}{M^n} \quad \text{(Ans)}$$

↑ Can do by this approach too

(b) SRSWOR

P(M is the highest no drawn)

$$= \binom{M-1}{n-1} \Big/ \binom{N}{n} \quad \text{(Ans)}$$