Report: Portfolio Optimization and Performance vs NIFTY 50

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1 Introduction

This study applies Markowitz Modern Portfolio Theory (MPT) to a selection of 10 NSE-listed stocks. The objective is to:

Construct the Minimum Variance Portfolio (MVP) using historical data,
Allocate an initial investment of Rs 100,000,
Backtest performance against the NIFTY 50 index,
Evaluate whether MPT yields superior risk-adjusted performance.

2 Data Collection

Stocks: HDFC Bank, ITC, TCS, Reliance, Coal India, Infosys, Bajaj Finance, Asian Paints, L&T, BEL.

Data Source: Yahoo Finance (yfinance).
Period: 30 Aug 2023 – 30 Aug 2025.
Dataset length: 494 daily observations.
Data was split:
First half (248 days) $\rightarrow$ used to estimate expected returns ($\boldsymbol{\mu}$) and covariance ($\boldsymbol{\Sigma}$).
Second half (246 days) $\rightarrow$ used for out-of-sample testing.

3 Methodology

3.1 Returns

Daily percentage returns were computed:

$$r_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$

3.2 Estimation

Expected returns ($\boldsymbol{\mu}$): vector of daily mean returns.

Covariance matrix ($\boldsymbol{\Sigma}$): captures variance and correlation between assets.

Inverse covariance ($\boldsymbol{\Sigma}^{-1}$): used in optimization.

3.3 Portfolio Weights

MVP weights were derived as:

$$\mathbf{w}_{MVP} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}}$$

Investment: Rs 100,000 allocated proportionally to weights.

3.4 Mean-Variance Frontier

For any target return y, the portfolio variance is:

$$\sigma^2(y) = \frac{C}{D}\left(y - \frac{B}{C}\right)^2 + \frac{1}{C}$$

where

$$A = \boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}, \quad B = \boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}, \quad C = \mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}, \quad D = AC - B^2$$

$\boldsymbol{\mu}$ = expected returns vector

$\boldsymbol{\Sigma}$ = covariance matrix of returns

$\mathbf{1}$ = vector of ones

This formula gives the parabolic frontier in mean-variance space.

3.5 Mean-Std Dev Frontier

Replacing variance σ^2 with standard deviation σ, we get the hyperbolic frontier:

$$\sigma(y) = \sqrt{\frac{C}{D}\left(y - \frac{B}{C}\right)^2 + \frac{1}{C}}$$

This is more intuitive to investors, since risk is measured in standard deviation rather than variance.

3.6 Zero-Covariance Portfolio Relationship

If portfolio p has expected return b_p, then the return of portfolio q, which is uncorrelated with p, is:

$$b_q = \frac{B}{C} - \frac{D}{C^2(b_p - B/C)}$$

This illustrates how two portfolios on the frontier can be constructed to be mutually uncorrelated.

4 Results

4.1 Efficient Frontier

Theoretical risk–return trade-off, with the MVP marked.

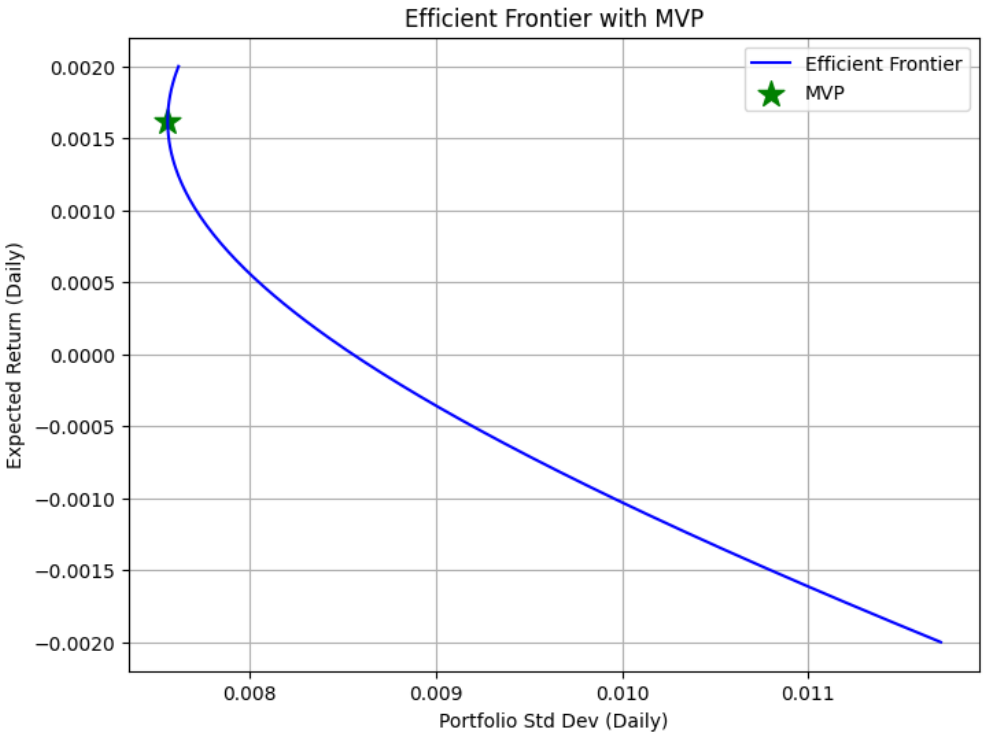

Figure 1: Efficient Frontier with MVP

4.2 Mean-Variance and Mean-Std Dev Frontier

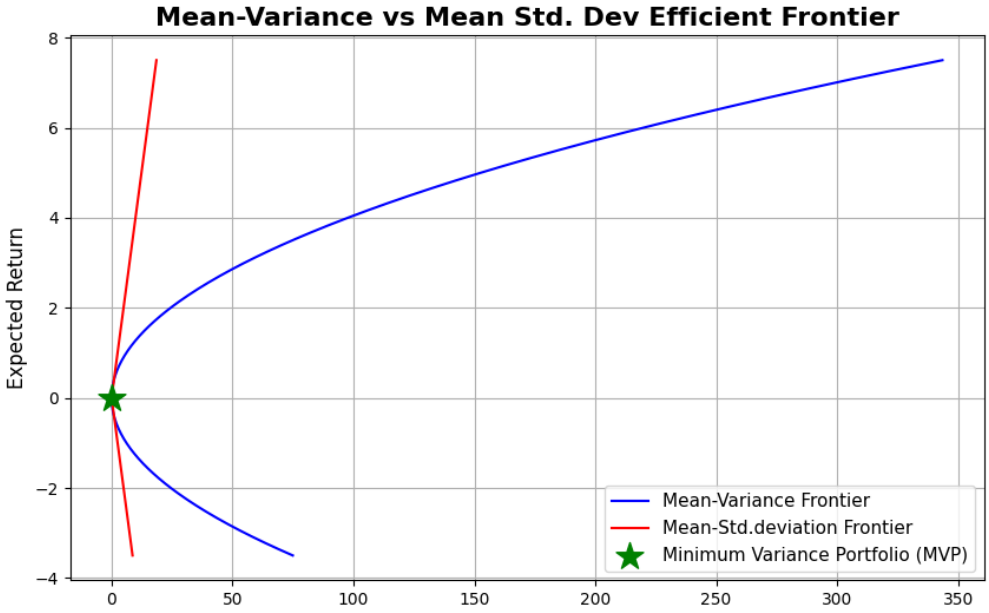

Figure 2: Mean-Variance vs Mean-Std Dev Efficient Frontier

This figure shows both:

The parabolic frontier (Variance),

The hyperbolic frontier (Std Dev),

The Minimum Variance Portfolio (MVP) marked with a green star.

4.3 Zero-Covariance Relationship

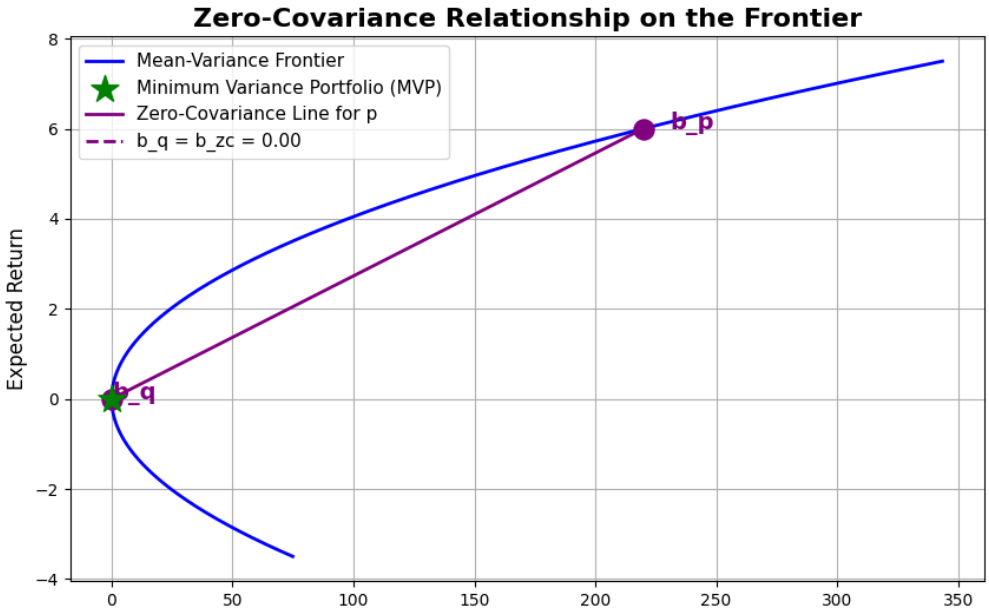

Figure 3: Zero-Covariance Relationship on the Frontier

This figure highlights:
The MVP,
A chosen portfolio p with return b_p,
Its zero-covariance counterpart q with return b_q.
Together, these illustrate the geometric properties of the frontier.

4.4 Portfolio Allocation

Final MVP allocation (weights):
HDFCBANK.NS: 21.2%
ITC.NS: 0.6%
TCS.NS: –1.0%
RELIANCE.NS: –0.4%
COALINDIA.NS: 17.1%
INFY.NS: 6.5%
BAJFINANCE.NS: 26.1%
ASIANPAINT.NS: ≈0%
LT.NS: 12.4%
BEL.NS: 17.5%

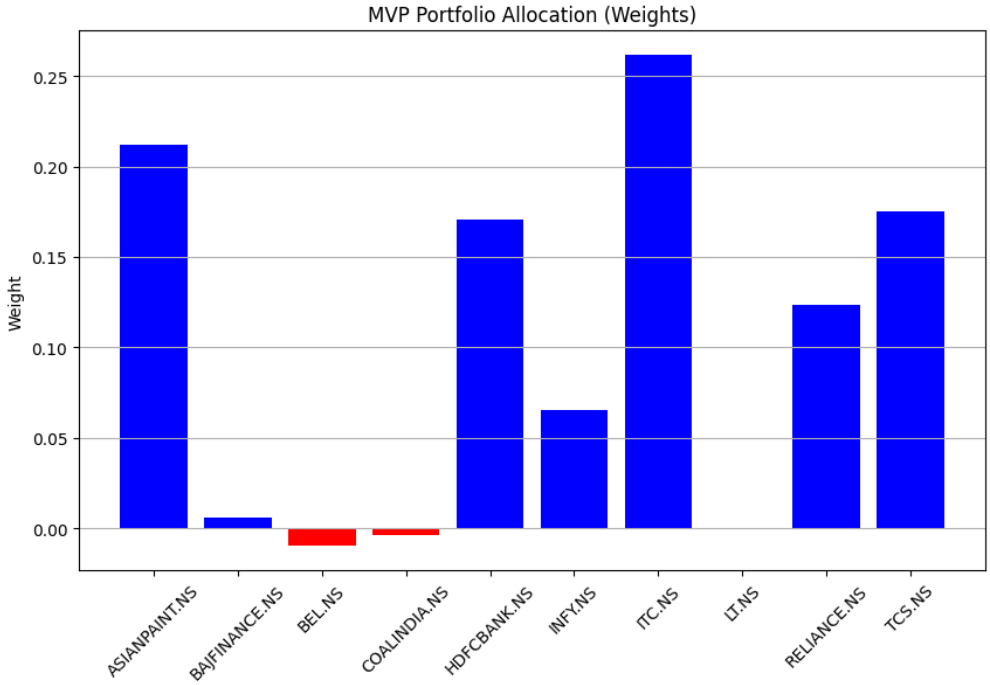

Figure 4: Bar Chart of MVP Portfolio Weights

4.5 Performance (Out-of-Sample, 2024-2025)

Comparison of MVP vs NIFTY 50:

Metric	Your MVP	NIFTY 50
Total Return (%)	–13.6%	–1.4%
Annualized Return (%)	–13.3%	–0.6%
Annualized Volatility (%)	13.1%	13.3%
Sharpe Ratio	–1.09	–0.05
Max Drawdown (%)	–20.7%	–15.8%
Final Value (Rs 100k)	Rs 86,446	Rs 98,587

Table 1: Performance Comparison

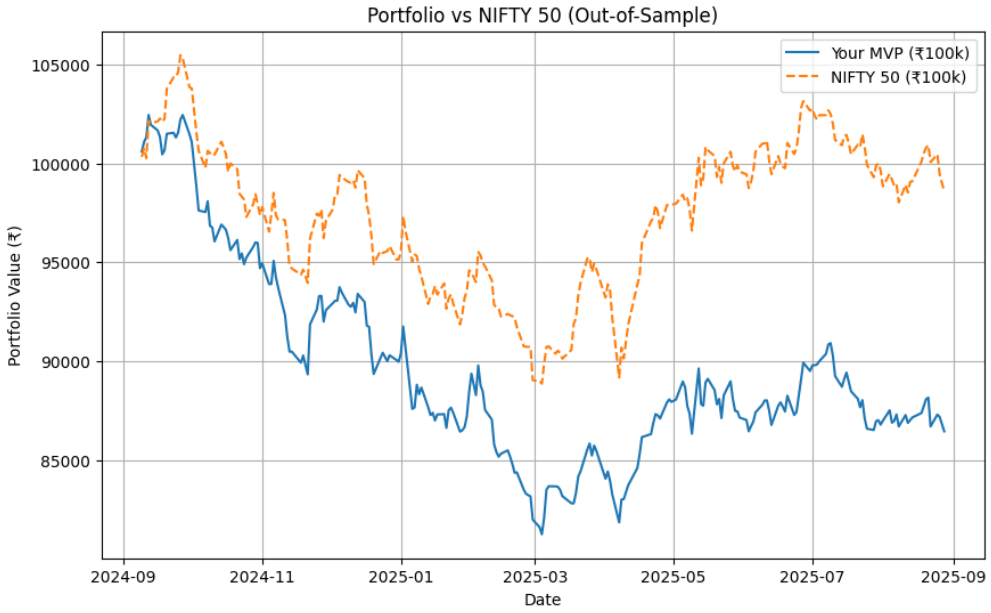

Figure 5: Cumulative Value Curve: MVP vs NIFTY 50

5 Discussion

MVP underperformed relative to NIFTY 50 in the test period.

Despite having similar volatility, MVP produced significantly lower returns.

The Sharpe ratio was highly negative, showing poor risk-adjusted performance.

Reason:

MVP focuses only on variance minimization, not return maximization.

Estimated returns/covariance are unstable with limited data.

Short positions (TCS, Reliance) worsened performance.

6 Conclusion

We successfully applied MPT and constructed the MVP.

Out-of-sample results showed MVP lost ∼13.6% vs NIFTY's –1.4%.

The study highlights the gap between theory and practice:

Optimization improves understanding,

But naive application may yield worse results than benchmarks.