Financial math problems solutions

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1 Binomial model

2 Ito Integral

$h(\cdot)$ – is a harmonic function if:

$$\sum_{i=1}^{n} \frac{\partial^2 h}{\partial x_i^2} = 0.$$

$h(\cdot)$ – is a subharmonic function if:

$$\sum_{i=1}^{n} \frac{\partial^2 h}{\partial x_i^2} \geq 0.$$

Prove that for independent Wiener processes $W_1, \ldots, W_n$ and a processes X is defined by the formula: $X(t) = h(W_1(t), \ldots, W_n(t))$. Show that if h is harmonic (subharmonic) $\Rightarrow X$ is a martingale (submartingale).

3 Stochastic differential equations

4 Black-Scholes