Задачи для Клуба теории вероятностей ФЭН ВШЭ

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Задача 1

Prove **Chebyshev's inequality** - if $a > 0$ then

$$\mathbb{P}(|X| \geqslant a|\mathcal{F}) \leqslant a^{-2}\mathbb{E}(X^2|\mathcal{F})$$

Задача 2

Show that if X and Y are random variables with $\mathbb{E}(Y|\mathcal{F}) = X$ and $\mathbb{E}Y^2 = \mathbb{E}X^2 < \infty$, then $X = Y$ *a.s.*

Задача 3†

The result in the last exercise implies that if $\mathbb{E}Y^2 < \infty$ and $\mathbb{E}(Y|\mathcal{F})$ has the same distribution as Y, then $\mathbb{E}(Y|\mathcal{F}) = Y$ *a.s.* Prove this under the assumption $\mathbb{E}Y < \infty$.

Задача 4

A coin shows heads with probability p. Let X_n be the number of flips required to obtain a run of n consecutive heads. Show that

$$\mathbb{E}X_n = \sum_{k=1}^{n} p^{-k}$$

Задача 5

An urn contains initially b blue balls and r red balls, where b, $r \geqslant 2$. Balls are drawn one by one without replacement. Show that the mean number of draws until the first colour drawn is first repeated equals 3.