Probability & Statistics

Basics L-1

$n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$

$= (n-1)! \, n$

$= (n-2)! (n-1) n$

$= \cdots = (n-r)! (n-r+1) \cdots (n-1) n.$

$$\Rightarrow \frac{n!}{(n-r)!} = (n-r+1) \cdots (n-1)(n)$$

When $r = n$

$$\frac{n!}{(n-n)!} = 1 \cdot 2 \cdots (n-1)(n)$$

$$\Rightarrow \frac{n!}{0!} = n!$$

$$\Rightarrow 0! = 1.$$

Permutation -> It means arrangement.

The arrangement of n distinct objects taken r at a time is denoted as $P(n,r)$ or ${}^nP_r$ or nPr $(r \leq n)$ and defined as $nPr = {}^nP_r = \frac{n!}{(n-r)!}$

The arrangement of n different objects taken all at a time $= {}^nP_n = \frac{n!}{(n-n)!} = n!$

Formula of circular permutation -> $(n-1)!$

" " " " where all the objects are alike $= \frac{(n-1)!}{2}$ (like necklace with diamonds)

L-2

(Q) How many numbers can be formed less than 400 using digits 1, 2, 3, 4, 5, 6

No. of 1-digit numbers → [ ]
↓
6. = 6

No. of 2-digit numbers → [ ][ ] +
↓ ↓
6 × 6 = 36 +

No. of 3-digit numbers → [ ][ ][ ] = 108
↓ ↓ ↓
3 × 6 × 6 150.

Combinations

* Combination means group of selection.
* The combination of n-distinct objects taken r at a time where $r <= n$ is ${}^nC_r$ or $C(n,r)$ and defined as ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{1}{r!} P(n,r)$.

$P(n,r) = r! \, C(n,r)$.

$${}^5C_2 = \frac{5!}{2!\,3!} = \frac{5 \times 4 \times 3!}{2!\,3!} = 10.$$

$${}^{100}C_3 = \frac{100!}{3!\,97!} = \frac{100 \times 99 \times 98}{3 \times 2.} = 3300 \times 49.$$

or $C(n,r) = C(n, n-r)$

${}^nC_r = {}^nC_{n-r}$.

$${}^{15}C_{12} = {}^{15}C_3 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3}$$

${}^5C_3 = {}^5C_2$.

$^nC_0 = 1$	$^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n = 2^n$
$^nC_1 = n$	
$^nC_2 = \frac{n(n-1)}{2}$	If we put $x=1$ in $(1+x)^n$ ↗
$^nC_n = 1.$	

(Q) In how many ways can you invite 2 or more friends out of 10 friends. ?

(A) $^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + \cdots + {}^{10}C_{10}$

$= 2^{10} - ({}^{10}C_0 + {}^{10}C_1)$

(Q) There are 3 vowels & 5 consonants how many 5 letter words can be from 2V & 3C

(A) $({}^3C_2 \times {}^5C_3) \times 5!$

$= 30 \times 120 = 3600.$

Probability

Sample Space

The collection of all possible outcomes of a statistical experiment is called sample space (S).

Ex:- → a coin is tossed once.

$S = \{H, T\}$

→ a coin tossed twice.

$S = \{HH, HT, TH, TT\}$

→ a coin is tossed n times → 2^n

→ a dice is rolled n times then → $|S| = 6^n$

Sample Point → Each of the samples from the sample space is sample point.

Event → Any subset of a sample space is called an event.