5.10.20

Linear Algebra.

Geometrical of Linear Equations –

→ Row picture Method:. * Individual point satisfying eqⁿ.

l_1 $2x - y = 1$ * find plot it.

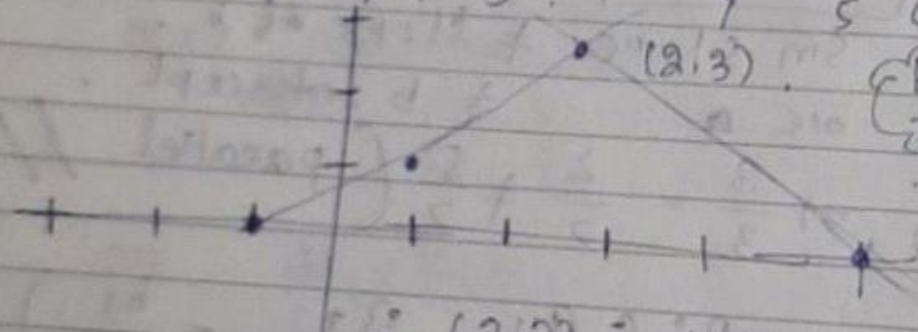
l_2 $x + y = 5$ * If intersect (unique), parallel (no), coinciding (infinite)

For l_1: $2x - y = 1$

x	0	1	2
y	-1	1	3

For l_2 $x + y = 5$

x	0	5	2
y	5	0	3



For point of intersection

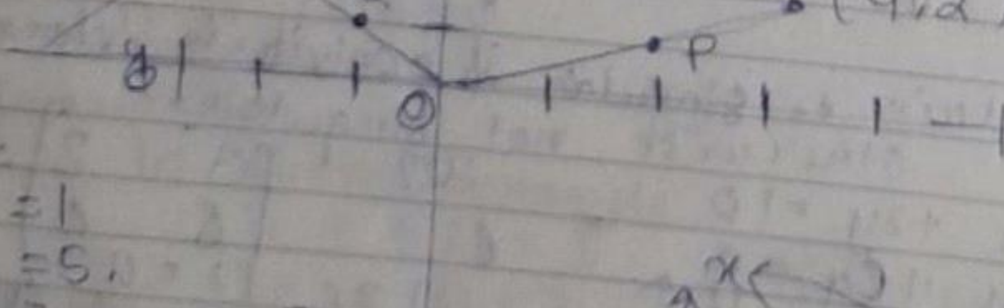
$$2x - y = 1$$
$$x + y = 5$$
$$3x = 6 \Rightarrow x = 2$$
$$y = 3$$

∴ (2,3) is the point of intersection or is the unique solⁿ for the system of eqⁿ. (Ans)

→ Column picture method:

$$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} x = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} y = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$



$2x - y = 1$

$x + y = 5$.

$$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} x + \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} y = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

P Q B

* $\underset{P}{\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}} x + \underset{Q}{\begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}} y = \underset{B}{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}$. Plot P, Q, B. Join OP, OQ.

* Draw parallel to OP, OQ from B.
* BP intersect at (h_1, h_2), BQ at (k_1, k_2).

coordinates of the obtained parallelogram.
(1,5), (4,2), (0,0), (-3,3)

* Now, $\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} x = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix} x = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

on comparing $x = 2$.

$\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} y = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix} y = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$.

on comparing $y = 3$.

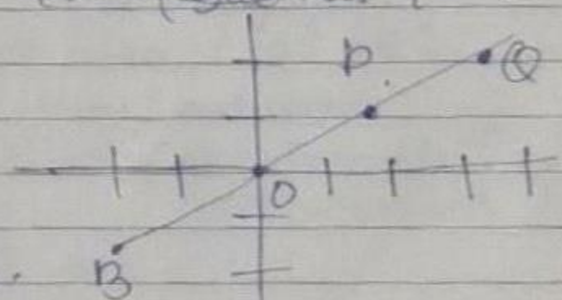
So, ~~the~~ soln is $x = 2, y = 3$ (2,3). (Ans)

Now, 3 condns:

(a) If $\boxed{O, P, Q, B}$ lie on one str line (such as $\frac{2}{1} = \frac{4}{2} = \frac{-2}{-1}$

Ex: $\begin{bmatrix} 2 \\ 1 \end{bmatrix} x + \underset{Q}{\begin{bmatrix} 4 \\ 2 \end{bmatrix}} y = \underset{B}{\begin{bmatrix} -2 \\ -1 \end{bmatrix}}$

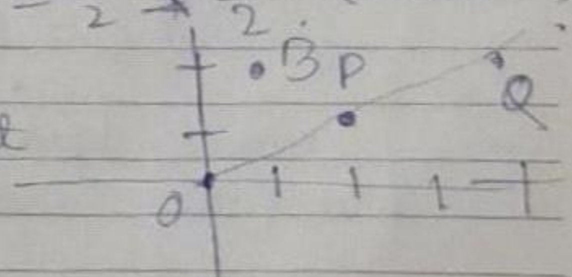
So, infinite many soln. Every point on d line is a soln of d system of linear eqn.



(b) If

Ex: $\underset{O}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} x + \underset{Q}{\begin{bmatrix} 4 \\ 2 \end{bmatrix}} y = \underset{B}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$. $\frac{2}{1} = \frac{4}{2} \neq \frac{1}{2}$ (no soln)

If $\boxed{O, P, Q}$ lie on one str line but B doesn't lie on that, then system of eqn has no soln.



(c) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} y = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. $\frac{2}{1} \neq \frac{-1}{1}$ (Ans)

$x+y=3$, $2x+y=3 \longrightarrow$ System of eq^n.

$$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} x + \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix} y = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

P Q B.

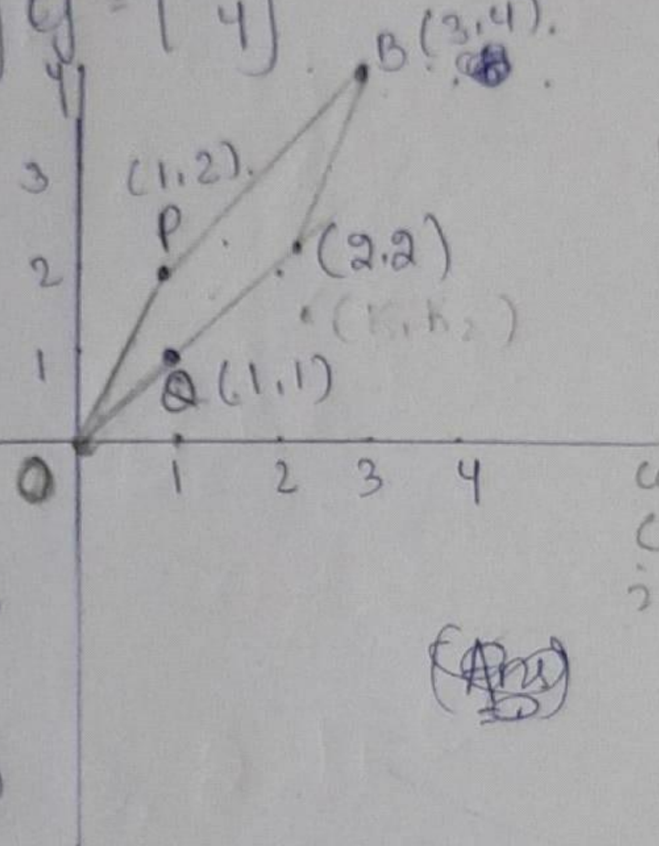
$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\boxed{x=1}$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$\boxed{y=2}$.



Here,
$(h_1, h_2) = (1,2)$
$(k_1, k_2) = (2,2)$.

$\therefore$ The system of eq^n has unique sol^n (1,2).

(Ans)

(1,2).
unique sol^n.
(1,2).
i.e. $\boxed{\begin{matrix} x=1 \\ y=2 \end{matrix}}$

(Ans)