19 JANUARY 3.10.20

28 Wk 05 • 028 Day
MONDAY

APPLIED LINEAR ALGEBRA.

* System of eq^{n}s:

$$3x_1 + 2x_2 = 5$$
$$x_1 + x_2 = 2$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

* Linear Eq^n: An eq^n in which unknown variables are of degree 1.

Here, x_1, x_2, are of degree 1.

Here $a_{11} x_1 + a_{12} x_2 = b_1 (5)$

$a_{21} x_1 + a_{22} x_2 = b_2 (2)$.

or $a_1 x_1 + a_2 x_2 = b$, $b = 0$ (homogeneous)
$b \neq 0$ (non homogeneous)

* Coefficient matrix: (A).

In above example, $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$A' = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \Big| \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ ← join = Augmented matrix.

↓ such a system is called augumentation.

A' join b.

* In above example:

$$3x_1 + 2x_2 = 5$$
$$x_1 + x_2 = 2.$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

on solving $x_1, x_2 \doteq (1, 1)$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \neq \frac{b_1}{b_2}$ (No soln)

$= \frac{b_1}{b_2}$ (infinately many soln)

(a). Unique soln: $\left(\frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}}\right)$ intersecting → unique.

$\frac{3}{1} \neq \frac{2}{1}$.

(b) No soln: $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \neq \frac{b_1}{b_2}$

Sm slope ≠ slope of b.

or ≠ b intercept.

Ex:
$3x_1 + 2x_2 = 5$.
$3x_1 + 2x_2 = 2$. $\rightarrow \frac{3}{3} = \frac{2}{2} \neq \frac{5}{2}$ (parallel //)

(c). Infinately many soln: $\left(\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}\right)$.

$a_1 x + b_1 y = c_1$
or $a_2 x + b_2 y = c_2$. $\rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

For what value, below eqn is singular & has infinately many soln.

$3x + 2y = 10$. $\rightarrow (0,5), \left(3, \frac{1}{2}\right)$ soln.
$6x + cy = g$. ← putting here $c = 4$, $g = 20$.

* Square matrix is singular if & only if its determinant is 0, otherwise not singular.

$(0,5)$, $(3, \frac{1}{2})$ soln. $3x + 2y = 10$
$6x + cy = g$

$= \begin{bmatrix} 3 & 2 \\ 6 & c \end{bmatrix}$ → Determinant.

$\Rightarrow 3c - 12 = 0$
$\Rightarrow c = 4$.

So $g = 20$.

$\frac{1}{2} = \frac{1}{2} = \frac{10}{20}\left(\frac{1}{2}\right)$

(Singular, infinately many soln.