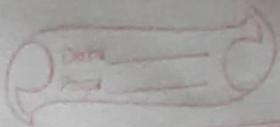


Probability & Statistics



Basics

L-1

$$\begin{aligned} n! &= 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n \\ &= (n-1)! \cdot n \\ &= (n-2)! \cdot (n-1) \cdot n \\ &\dots = (n-r)! \cdot (n-r+1) \cdots (n-1) \cdot n. \end{aligned}$$

$$\Rightarrow \frac{n!}{(n-r)!} = (n-r+1) \cdots (n-1)(n)$$

When $r=n$ ← when I put $r=n$

$$\frac{n!}{(n-n)!} = 1 \cdot 2 \cdots (n-1)(n)$$

cancel out

$$\Rightarrow n! = n!$$

Permutation \rightarrow It means arrangement.

The arrangement of n distinct objects taken r at a time is denoted as $P(n, r)$ or ${}^n P_r$ or nPr ($r \leq n$) and defined as ${}^n P_r = {}^n P_n = \frac{n!}{(n-r)!}$

The arrangement of n different objects taken all at a time $= {}^n P_n = \frac{n!}{(n-n)!} = n!$

Formula of circular permutation $\rightarrow (n-1)!$

" " " " where all the

objects are alike $= \frac{(n-1)!}{2}$ (like necklace with diamonds)

L-2

(Q) How many numbers can be formed less than 400 using digits 1, 2, 3, 4, 5, 6

$$\text{No. of 1-digit numbers} \rightarrow \boxed{} \\ \downarrow \\ 6 = 6$$

$$\text{No. of 2-digit numbers} \rightarrow \boxed{} \boxed{} + \\ \downarrow \quad \downarrow \\ 6 \times 6 = 36$$

$$\text{No. of 3-digit numbers} \rightarrow \boxed{} \boxed{} \boxed{} - \frac{1}{100} \\ \downarrow \quad \downarrow \quad \downarrow \\ 3 \times 6 \times 6 = 108 \\ 150.$$

Combinations

* Combination means group of selection.

* The combination of n -distinct objects taken r at a time where $r \leq n$ is ${}^n C_r$ or $C(n, r)$ and defined as ${}^n C_r = \frac{n!}{r!(n-r)!}$ $\leftarrow = P(n, r)$.

$$P(n, r) = r! \cdot C(n, r).$$

$${}^5 C_2 = \frac{5!}{2! 3!} = \frac{5 \times 4 \times 3!}{2! 3!} = 10.$$

$${}^{100} C_3 = \frac{100!}{3! 97!} = \frac{100 \times 99 \times 98}{3 \times 2} = 3300 \times 49.$$

$$C(n, r) = C(n, n-r).$$

$${}^n C_r = {}^n C_{n-r}.$$

$${}^{15} C_{12} = {}^{15} C_3 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3}$$

$${}^5 C_3 = {}^5 C_2.$$



$${}^n C_0 = 1$$

$${}^n C_1 = n$$

$${}^n C_2 = \frac{n(n-1)}{2}$$

$${}^n C_n = 1.$$

$${}^n C_0 + {}^n C_1 + {}^n C_2 + {}^n C_3 + \dots + {}^n C_n = 2^n$$

If we put $x=1$ in $(1+x)^n$

(Q) In how many ways can you invite 2 or more friends out of 10 friends?

$$\begin{aligned} (\text{A}) \quad {}^{10} C_2 + {}^{10} C_3 + {}^{10} C_4 + {}^{10} C_5 + \dots + {}^{10} C_{10} \\ = 2^{10} - ({}^{10} C_0 + {}^{10} C_1) \end{aligned}$$

(Q) There are 3 vowels & 5 consonants how many 5 letter words can be formed from 2V & 3C

$$\begin{aligned} (\text{A}) \quad (3C_2 \times 5C_3) \times 5! \\ = 30 \times 120 = 3600 \end{aligned}$$

Probability

Sample Space

The collection of all possible outcomes of a statistical experiment is called sample space (S).

Ex:- → a coin is tossed once -

$$S = \{H, T\}$$

→ a coin is tossed twice.

$$S = \{HH, HT, TH, TT\}$$

→ a coin is tossed n times → 2^n

→ a dice is rolled n times then → $|S| = 6^n$

Sample Point → Each of the samples from the sample space is sample point.

Event → Any subset of a sample space is called an event.