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Models for warehouse management: Classification and examples

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Abstract

In this paper we discuss warehousing systems and present a classification of warehouse management problems. We start with a typology and a brief description of several types of warehousing systems. Next, we present a hierarchy of decision problems encountered in setting up warehousing systems, including justification, design, planning and control issues. In addition, examples of models supporting decision making at each of these levels are discussed, such as distribution system design, warehouse design, inventory management under space restrictions, storage allocation, and assignment and scheduling of warehouse operations.

Keywords: Inventory systems; Warehouse management; Storage allocation and assignment; Forward/reserve problem; Logistics.

1. Introduction

According to the principles of supply chain management, modern companies attempt to achieve high-volume production and distribution using minimal inventories throughout the logistic chain that are to be delivered within short response times.

The changes outlined above have had a dramatic impact on warehouse management. Low volumes have to be delivered more frequently with shorter response times from a significantly wider variety of *stock keeping units* (SKUs). In a further attempt to decrease total inventory, many companies replaced several relatively small *distribution centers* (DCs) by a small number of large DCs with an extensive distribution network. Often, an entire continent, like North America or Europe, is serviced by a small number of DCs at strategic positions.

These developments have significantly influenced the existing paradigms in inventory research. Unfortunately, the attention paid by researchers in inventory theory to the management of *storage systems* such as warehouses has been relatively limited. Often, it was considered to be a mainly technical issue and therefore belonging to a different field, i.e., material handling research. The goal of this paper is to show that, apart from the close relationship between inventory and warehouse management problems, the latter often lend themselves to a profound and elegant quantitative analysis.

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The new market forces, together with the fast technological developments in material handling, have affected the operation within warehouses tremendously. Shorter product life cycles impose a financial risk on high inventories and, consequently, on the purchase of capital intensive high-performance warehousing systems. Centralized inventory management, on the other hand, requires an increased productivity and short response times of the warehousing systems. The aim of this paper is to show that sophisticated models and decision support systems for the planning and control of warehousing systems may significantly contribute to the overall research in inventory management.

The developments have been made possible due to recent advances in information technology and the introduction of business information systems. Business information systems support the administrative processes of enterprises. For instance enterprise resources planning (ERP) systems are MRP-based business information systems that registrate all processes concerning finances, human resources, production planning and inventory management. Other functions that often are supported by ERP-systems are, e.g., transportation planning, warehouse management, production scheduling and order-entry/order processing. Besides ERP-systems there are specialized systems that support these functions in complex operations. These various systems are linked together using electronic data interchange (EDI). Examples of such specialized systems are *warehouse management systems* that facilitate the registration, planning and control of warehouse processes, and inventory management systems. The models that are presented in this paper may be implemented in inventory management and warehouse management systems and thereby provide significant performance improvements in warehouse operations in comparison with the methods and models that are currently used.

This paper is organized as follows. In Section 2, we present a typology and a short review of warehousing systems. Next, we discuss warehouse planning problems in Section 3. Section 4 is devoted to examples of models for decision support for warehouse planning decisions. In Section 5, we conclude the paper and discuss opportunities for further research.

2. Warehousing systems: A typology and a review

Material Handling is defined as the movement of materials (raw materials, scrap, emballage, semi-finished and finished products) to, through, and from productive processes; in warehouses and storage; and in receiving and shipping areas [1]. Material handling concerns *material flow* and *warehousing*. Typical material flow devices are: conveyors, fork lifts, *automated guided vehicles* (AGVs), shuttles, overhead cranes and power-and-free conveyors. Warehousing concerns those material handling activities that take place within the warehouse, receiving and shipping areas, i.e., receiving of goods, storage, order-picking, accumulation and sorting and shipping.

Basically, we may distinguish three types of warehouses:

- Distribution warehouses,
- Production warehouses,
- Contract warehouses.

A *distribution warehouse* is a warehouse in which products from different suppliers are collected (and sometimes assembled) for delivery to a number of customers. A *production warehouse* is used for the storage of raw materials, semi-finished products and finished products in a production facility. A *contract warehouse* is a facility that performs the warehousing operation on behalf of one or more customers.

2.1. Warehousing activities

In this section we consider the flow of materials in a warehouse. Goods are delivered by trucks, which are unloaded at the *receiving docks*. Here quantities are verified and random quality checks are performed on the delivered loads. Subsequently, the loads are prepared for transportation to the storage area. This means that a label is attached to the load, e.g., a bar code or a magnetic label. If the *storage modules* (e.g., pallets, totes or cartons) for internal use differ from the incoming storage modules, then the loads must be reassembled. After this, the loads are transported to a location within the *storage area*.

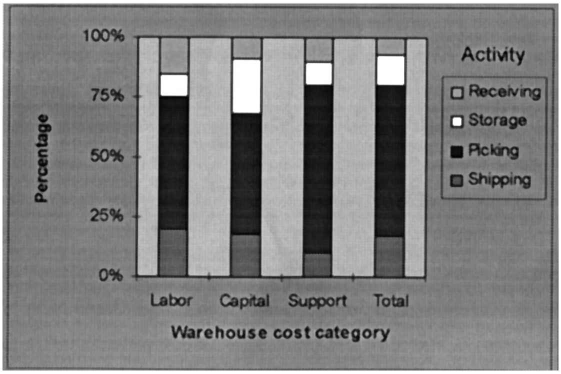

Fig. 1. Warehousing cost by activity.

Subsequently, whenever a product is requested, it must be retrieved from storage. This process is called *order picking*. An *order* lists the products and quantities requested by a customer or by a production/assembly workstation, in the case of a distribution center or a production warehouse, respectively. When an order contains multiple SKUs, these must be accumulated and sorted before being transported to the *shipping area* or to the production floor. Accumulation and sorting may either be performed during or after the order-picking process.

Hence, we may subdivide the activities in a warehouse into four categories: receiving, storage, order-picking and shipping. A study in the United Kingdom [2] revealed that order-picking is the most costly among these activities. More than 60% of all operating costs in a typical warehouse can be attributed to order-picking (Fig. 1).

2.2. A typology of warehousing systems

An *item picking operation* is an operation in which single items are picked from storage positions (*less-than-case picking*), as opposed to a *pallet-picking operation* in which pallet loads are moved in and out. A *warehousing system* refers to the combination of equipment and operating policies used in an item picking or storage/retrieval environment. With respect to the level of automation, we may distinguish three types of warehousing systems:

1. Manual warehousing systems (picker-to-product systems),
2. Automated warehousing systems (product-to-picker systems),
3. Automatic warehousing systems.

We will discuss the three types of warehousing systems in the above sequence.

2.3. A short review of warehousing systems

A warehouse generally consists of a number of parallel aisles with products stored alongsides. A large variety of storage equipment and methods are in use. The most simple storage method is *block stacking* as is used, e.g., for the stacking of crates of beer or soft drinks. *Bin shelving* and *modular storage drawers* are often used for the storage of small items. For larger items, stored on pallets, *pallet racks*, *gravity flow racks* or *mobile storage racks* are often used. For a more elaborate discussion on storage methods we refer to [3].

In the preceding section we distinguished between manual, automated and automatic warehousing systems. Below, we describe each of these in some more detail.

2.3.1. Manual warehousing systems

In a *manual warehousing system* or *picker-to-product system*, the order picker rides a vehicle along pick locations. A wide variety of vehicles is available: we mention *pick carts* or *container carts* for manual horizontal item picking and *man aboard storage/retrieval* (*S/R*) *machines* for both horizontal and vertical item picking (often, but not necessarily, restricted to a specific aisle). For storage/retrieval operations (of complete pallet loads, see Section 2.2), *fork lift trucks* and a variety of *reachtrucks* are often used.

Recall that an order may contain a list of quantities of different SKU's (each SKU in an order corresponds to a unique item of supply). Two fundamental approaches may be distinguished in manual order picking: *single-order-picking* and *batch-picking*. The former approach indicates that the order-picker is responsible for the picking of a complete order. The latter approach indicates that multiple orders are picked simultaneously by one order-picker, who is typically restricted to a certain zone in the warehouse (*zoning*). Batch picking reduces the mean travel time per pick. However, it requires that orders are to be sorted afterwards. The order-picker may either sort the orders while traversing the warehouse (*sort-while-pick*) or the items may be lumped together and sorted afterwards (*pick-and-sort*). To apply the sort-while-pick strategy, the order-picking vehicle must be equipped with separate containers for individual orders. *Wave picking* is a popular strategy if batching and zoning are both applied. This strategy implies that all order-pickers start picking in their respective zones at the same time. Only after all pickers have completed their tour, the next wave starts.

Instead of a vehicle we may also use a conveyor for the transportation of the picked products. The order-picker directly deposits the picked items on a conveyor that is positioned within the aisle. Such an operation is referred to as *pick-to-belt*.

2.3.2. Automated warehousing systems

The systems that we discussed so far, were *picker-to-product* systems. A *carousel* is an example of a *product-to-picker* system. A carousel is a computer-controlled warehousing system that is used for storage and order picking of small-to medium-sized products. A carousel may hold many different products stored in bins or drawers that rotate around a closed loop. The order picker occupies a fixed position at the front of the carousel. Upon request, the carousel automatically rotates the container with the requested product to the position of the order picker. The order picker may effectively use the rotation time of the carousel for activities such as sorting, packaging and labeling of the retrieved goods.

In some situations the order picker serves two to four carousels in parallel. The advantage of this configuration is that while the order picker is extracting items from one carousel, the other carousels are rotating. This reduces the waiting time of the order-picker. The *rotary rack* is a more expensive version of the horizontal carousel, with the extra feature that every storage level can rotate independently, thus reducing the waiting time of the order picker significantly.

The *automated storage/retrieval system* (AS/RS) is also a product-to-picker system. The AS/RS consists of one or multiple parallel aisles with two high bay pallet racks alongside each aisle. Within the aisle travels a *storage/retrieval* (S/R) *machine* or automated stacker crane. The S/R machine travels on rails that are mounted to the floor and the ceiling. In a typical configuration, the S/R machine may carry at most one pallet at the same time. Pallets for storage arrive at the *input station* and wait at an accumulator conveyor until the S/R machine transports them to a storage location in the racks. Consequently, storages are performed according to a *first come first served* (FCFS) routine. The S/R machine deposits retrieved loads at the *output station*, after which a transportation system routes them to their destination. The S/R machine has three independent drives for horizontal, vertical and *shuttle* movement. Due to the independent horizontal and vertical travel, the travel time of the S/R machine is measured by the maximum of the isolated horizontal and vertical travel times. In many applications the S/R machine is confined to one aisle. We may enable movement of the S/R machines between aisles by providing curves in the rails that connect the aisles. To maintain stability in the giant construction, the cranes have to assume creep speed in the curves. Another possibility that enables the S/R machine to enter multiple aisles, is to use a shuttle device that transfers the S/R machine between the aisles.

Due to its unit-load capacity, the operational characteristics of the S/R machine are limited to *single-command cycles* and *dual-command cycles*. In a single-command cycle either a storage or a retrieval is performed between two consecutive visits of the input and output station. In a dual-command cycle the S/R machine consecutively performs a storage, travels empty to a retrieval location and performs a retrieval. The empty travel between the storage and retrieval location is referred to as *interleaving travel*.

A *miniload* AS/RS is an AS/RS that is designed for the storage and order picking of small items. The items are stored in modular storage drawers or in bins. These containers may be subdivided into multiple compartments each containing a specific SKU. In a typical miniload AS/RS operation, the order-picker resides at the end of the aisle at a *pick station*. The pick station contains at least two container positions. While the order-picker extracts items from the container in one pick position, the S/R machine stores the container from the other pick position at its location in the rack and retrieves the next container. Also miniload AS/RS's with more than two pick positions per pick station do exist, as well as systems with a conveyor delivery system to transport containers to remote order pickers. A miniload AS/RS is generally referred to as an *end-of-aisle* order-picking system, as opposed to *in-the-aisle* order-picking systems such as the manual order-picking systems discussed in Section 2.3.1.

2.3.3. Automatic warehousing systems

Automatic order-picking systems perform high-speed picking of small- or medium-sized non-fragile items of uniform size and shape, e.g., compact disks or pharmaceuticals. If we replace the order picker of a carousel system or rotary rack by a robot, then we obtain an automatic order-picking system.

An *A-frame automatic dispenser machine* is another order-picking device without order-pickers. The A-frame consists of a conveyor belt with magazines arranged in A-frame style on either side of the belt. Each magazine contains a powered mechanism that automatically dispenses items onto the belt. Each order is assigned a certain section on the conveyor (a *cell*). When the cell passes a magazine that contains an item requested by the corresponding order, the item is automatically dispensed upon the passing cell. At the end of the belt the items belonging to the same order fall down into a bin or carton.

2.3.4. Order accumulation and sorting systems

Order accumulation and sorting systems (OASSs) are used to establish order integrity when orders are not picked in a single-order fashion. OASSs exist in various types, ranging from manual staging using a kitting matrix to high volume automatic systems. An automatic OASS usually consists of a closed-loop conveyor with automatic divert mechanisms and accumulation lanes. A sensor scans SKUs that enter the loop. SKUs corresponding to the same order are then automatically diverted into one lane. Also carousels and rotary racks are used for the accumulation and sorting of orders.

3. Warehouse management

Typical planning issues in warehouses are inventory management and storage location assignment. Intelligent inventory management may result in a reduction of the warehousing costs. For example, by applying sophisticated production planning and ordering policies we may reduce the total inventory, while guaranteeing a satisfactory *service level*. The service level specifies the percentage of the orders to be supplied directly from stock. Reduced inventory levels not only reduce inventory costs, but also improve the efficiency of the order-picking operation within the warehouse. Clearly, in a smaller warehouse, the travel times for order-picking are smaller.

Furthermore, an effective storage location assignment policy may reduce the mean travel times for storage/retrieval and order-picking. Also, by distributing the activities evenly over the warehouse subsystems, congestion may be reduced and activities may be balanced better among subsystems, thus increasing the throughput capacity.

The planning policies define a framework for the control of the warehouse processes. Inventory management and storage location assignment policies determine which products arrive and where these should be stored. Control problems typically deal with the sequencing of order picking and storage/retrieval operations, and hence with the routing of manual order pickers or S/R machines, the allocation of products to storage positions in a class-based or random location system, the internal movement of items to more attractive retrieval positions, the dwell point of S/R machines, etc.

4. Warehousing models

In this section, we discuss examples of models that have been presented in the literature or have been developed recently, to illustrate the application of operations research techniques for the planning of warehousing operations.

Inventory management/production planning decide which products are to be stored in the warehouse and in what quantities. Storage location assignment decides where the products are to be stored. Here we may distinguish between a forward and a reserve area while also the basic storage policy in S/R systems is determined (e.g., dedicated, class-based or random storage). First, we discuss inventory management.

4.1. Reduction of inventory levels

Intelligent inventory management/production planning may reduce the inventory levels and thereby the operational costs for storage/retrieval and order picking. Inventory reductions may be established by having smaller ordering quantities delivered more frequently. However, the total storage space needed may still be considerable if all deliveries occur at the same time. Hence, we may further reduce the need for storage space by carefully scheduling the deliveries. Ultimately, products from incoming trucks are immediately transferred to outgoing trucks, a phenomenon known as *cross docking*.

Classical inventory management and production planning models determine ordering and production policies for a single product. Hadley and Whitin [4] consider inventory models for multiple products with a constraint on the total storage space. They determine ordering policies for all products which minimize the long-run inventory holding and ordering costs per unit time by solving the following problem:

$$\text{Min} \quad \sum C_jD_j + A_jD_j/Q_j + rC_jQ_j/2 \tag{1}$$

s.t.

$$\sum f_jQ_j = F, \tag{2}$$

where D_j is the demand rate in units per year for product j, A_j the fixed ordering costs for product j, C_j the unit variable purchase costs for product j, r the annual inventory carrying cost rate, Q_j the order quantity for product j, f_j the amount of space occupied by one unit-load of product j, and F the available storage space.

If the unconstrained solution exceeds the available storage space, then a *Lagrangian multiplier technique* is used to find the optimal ordering policies. Here, the storage space estimation is based on the possibility of receiving all deliveries at the same epoch. However, by properly staggering the deliveries in time, the peak demand for warehouse space may be moderated. The combined problem of order sizing and delivery staggering is known as the *Economic Warehouse Lot Scheduling Problem* (EWLSP). For a survey on the EWLSP we refer to [5].

All models discussed so far assume fixed cost parameters, a constant demand rate, no delivery leadtimes and no backlogging. Clearly, the problem of order sizing and staggering deliveries becomes much more complicated in a stochastic setting. Suppose for example that pallet loads for each SKU are ordered according to a (continuous review) (s, Q)-policy (cf. [6]). Under certain conditions, the number of pallets per SKU is uniformly distributed at an arbitrary point in time. Assuming stochastic independence of the demands for different SKUs, the total number of pallets can then be approximated by a normal distribution. Hence, under a random storage policy, the necessary storage space is determined by specifying a probability on stock overflow (cf. [7]). However, under rigid space restrictions, the orders for the different SKUs are no longer independent. Besides, many warehouse managers follow a *can-order policy* (cf. [6,8]) for groups of products to be delivered by the same supplier, thereby taking advantage of shared fixed costs or combined transport facilities. Hence, in such a situation, various orders of different SKUs arrive at the same time.

4.2. Storage allocation and assignment

A popular approach to reduce the amount of work associated with order picking is to divide the warehouse into a *forward area* and a *reserve area*. The forward area is used for efficient order picking. The reserve area holds the bulk storage and is used for replenishing the forward area and for picking the products that are not assigned to the forward area. The forward and reserve area may be distinct areas within the warehouse or the forward and reserve area may be located in the same (pallet) rack. In the latter case, the lower levels represent the forward area, the higher levels represent the reserve area. In some facilities the reserve area is once again subdivided into two separate areas: one for order-picking and one for replenishing.

The *forward-reserve problem* (FRP) is the problem of deciding which products should be stored in the forward area and in what quantities. If a product is not assigned to the forward area, then it is picked from the reserve area. Hackman and Rosenblatt [9] describe a heuristic for the FRP that attempts to minimize the total costs for picking and replenishing. Frazelle et al. [10] incorporate the heuristic into a framework for determining the size of the forward area together with the allocated products. The costs in the model for picking in the forward area and for replenishing depend on the size of the forward area.

Van den Berg and Sharp [11] focus on operations that observe *busy* and *idle* periods. In these operations, it is possible to reduce the number of replenishments in busy periods, by performing replenishments in the preceding idle periods. This not only increases the throughput during the busy periods, it also reduces possible congestion and accidents. A typical example is a distribution center in which trucks are loaded during the afternoon, so that the workforce is available in the morning hours for replenishing the forward area. The authors consider a picking period during which the order-picking operation takes place. Prior to the picking period, the forward area is replenished in advance. Their objective is to find an allocation of product quantities to the forward area, which minimizes the expected labor time during the picking period.

The authors consider a situation observed in many operations (e.g. pallet storage), where unit loads are replenished one at the time. They use the following notation:

S set of products assigned to the forward area,
P_i random variable representing the number of picks for product i during the picking period, $i = 1, \dots, N$,
R_{ij} random variable representing the number of concurrent replenishments for product i, if the forward area contains j unit-loads of product i at the beginning of the picking period, $i = 1, \dots, N$, $j = 1, \dots, m_i$,
U_i random variable representing the number of unit-loads of product i that is needed to fulfil demand during the picking period.

The expected number of picks from the forward area and the reserve area are given by expressions (3) and (4), respectively.

$$\sum_{i \in S} E(P_i), \tag{3}$$

$$\sum_{i \notin S} E(P_i). \tag{4}$$

Let z_i denote the number of unit-loads of product i that is stored in the forward area at the beginning of the picking period. Accordingly, the expected number of concurrent replenishments is given by expression (5).

$$\sum_{i \in S} E(R_{iz_i}). \tag{5}$$

We derive an expression for $E(R_{iz})$.

$$\begin{aligned} E(R_{iz}) &= \sum_{k=z+1}^{\infty} (k - z) \cdot P(U_i = k) \\ &= \sum_{k=z+1}^{\infty} P(U_i \geqslant k) \\ &= E(U_i) - \sum_{k=1}^{z} P(U_i \geqslant k). \end{aligned} \tag{6}$$

Subsequently, they formulate the FRP as the binary programming problem (B-FRP), using the following notation:

m_i number of unit-loads available of product i, $i = 1, \dots, N$,
p_i $E(P_i)$,
u_i $E(U_i) - P(U_i \geqslant 1)$,
u_{ij} $P(U_i \geqslant j)$, $i = 1, \dots, N$, $j = 2, \dots, m_i$,
V available storage space in the forward area,
T^{pf} average time for performing one pick from the forward area,
T^{pr} average time for performing one pick from the reserve area ($T^{\mathrm{pr}} > T^{\mathrm{pf}}$),

T^{cr} average time for performing one concurrent replenishment.

They define decision variables x_i for $i = 1, \ldots, N$, and y_{ij} for $i = 1, \ldots, N$, $j = 2, \ldots, m_i$.

$$x_i = \begin{cases} 1 & \text{if product } i \text{ is assigned to the forward area,} \\ 0 & \text{otherwise,} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if the } j\text{th unit-load of product } i \text{ is replenished in advance,} \\ 0 & \text{otherwise.} \end{cases}$$

(B-FRP)

$$\mathrm{Min} \sum_{i=1}^{N} \left\{ T^{pf} p_i x_i + T^{pr} p_i (1 - x_i) + T^{cr} \left(u_i x_i - \sum_{j=2}^{m_i} u_{ij} y_{ij} \right) \right\}, \tag{7}$$

s.t.

$$\sum_{i=1}^{N} v_i \left(x_i + \sum_{j=2}^{m_i} y_{ij}\right) \leqslant V, \tag{8}$$

$$y_{i2} \leqslant x_i, \qquad i = 1, \ldots, N, \tag{9}$$

$$y_{ij} \leqslant y_{i(j-1)}, \qquad i = 1, \ldots, N,\ j = 3, \ldots, m_i, \tag{10}$$

$$x_i \in \{0, 1\}, \qquad i = 1, \ldots, N, \tag{11}$$

$$y_{ij} \in \{0, 1\}, \qquad i = 1, \ldots, N,\ j = 2, \ldots, m_i. \tag{12}$$

The objective function follows from expressions (3)–(6) after substituting p_i, u_i and u_{ij} and multiplying each term with the corresponding labor-time average. Constraint (8) stresses that the space occupied by the unit-loads allocated to the forward area may not exceed the available space. The remaining set of constraints (9) and (10) allows the jth unit-load of product i to be stored in advance, only if unit-loads $1, \ldots, (j-1)$ of product i are assigned to the forward area, for $i = 1, \ldots, N$.

4.3. Storage location assignment

The storage location assignment problem (SLAP) concerns the assignment of incoming stock to storage locations. For automated storage/retrieval systems, Hausman et al. [12] present three storage location assignment policies: class-based storage, randomized storage and dedicated storage. The *class-based storage* policy distributes the products, based on their demand rates, among a number of classes and reserves a region within the storage area for each class. Accordingly, an incoming load is stored at an arbitrary open location within its class. The class-based storage policy and the dedicated storage policy attempt to reduce the mean travel times for storage/retrieval by storing products with high demand at locations that are easily accessible.

Van den Berg [13] presents a polynomial time dynamic programming algorithm that partitions products and locations into classes such that the mean single command cycle time is minimized. The algorithm works under any demand curve, any travel time metric, any warehouse layout and any positions of the input station and output station.

We use the following notation:

Q_i independent random variables representing the number of unit-loads present of product i at an arbitrary epoch,

P_k set of products in class $k = 1, \ldots, K$.

Due to the demand and supply processes the inventory level fluctuates. We estimate the storage space requirement such that the storage space in every class suffices for at least a fraction $0 < \alpha < 1$ of the time. In other words, the probability of a stock overflow is less than $1 - \alpha$. Let Q^k be a random variable representing the inventory level of class k at an arbitrary epoch, i.e., $Q^k = \sum_{i \in P_k} Q_i$. Now, we want to find the smallest size S^k for the class-region of class k such that

$$P(Q^k \leqslant S^k) \geqslant \alpha. \tag{13}$$

Let t_j^{in} denote the travel time between the input station and location j and let t_j^{out} denote the travel time between the output station and location j. Every stored unit-load is retrieved some time later, so that over a long time period half of the single command cycles are storages and half are retrievals. Accordingly, the mean single command cycle time to location $j \in L$ equals: $\frac{1}{2}(2t_j^{in} + 2t_j^{out}) = (t_j^{in} + t_j^{out})$.

The single command cycle time, $E(\mathrm{SC})$, is defined as

$$E(\mathrm{SC}) = \sum_{k=1}^{K} \frac{\sum_{i \in P_k} E(D_i)}{\sum_{i \in P} E(D_i)} \cdot \sum_{j \in L_k} \frac{(t_j^{\mathrm{in}} + t_j^{\mathrm{out}})}{|L_k|}, \qquad (14)$$

where L_k denotes the set of storage locations of class k.

The first factor represents the probability that a request concerns class k. The second factor represents the mean travel time to a location in class k. In order to minimize the expected single command cycle time, we assign the products i that constitute the largest demand per reserved space and the locations j with the smallest $(t_j^{\mathrm{in}} + t_j^{\mathrm{out}})$ to the first class and we assign the products i that constitute the next largest demand per reserved space and the locations j with the next smallest $(t_j^{\mathrm{in}} + t_j^{\mathrm{out}})$ to the second class, and so on. Accordingly, the locations are ranked according to non-decreasing $(t_j^{\mathrm{in}} + t_j^{\mathrm{out}})$ and the products are ranked according to non-increasing demand per reserved space. We define $g_k(p,l)$ as the contribution of classes $1,\dots,k$ to Eq. (14), when products $1,\dots,p$ and storage locations $1,\dots,l$ are distributed among these classes such that $g_k(p,l)$ is minimal. Then $g_k(p,l)$ satisfies

$$g_k(p,l) = \min_{1 \leqslant i \leqslant p, 1 \leqslant j \leqslant l} \{ h_{i+1,p}^{j+1,l} + g_{k-1}(i,j) \}, \qquad (15)$$

where $h_{i+1,p}^{j+1,l}$ denotes the contribution to Eq. (14) if the products $i+1,\dots,p$ and the locations $j+1,\dots,l$ form one class k. Recalling that the number of locations required in each class is determined by Eq. (13), the values $g_k(p,l)$ are found by iteratively solving the dynamic programming equation (15). Each $g_k(p,l)$ corresponds to an optimal solution of the subproblem with k classes and the first p products and the first l storage locations when ranked as indicated before.

We may use the algorithm to determine the optimal class-partition for $1,\dots,K$ classes. Subsequently, the number of classes among $1,\dots,K$ may be selected that constitutes an acceptable mean travel time and space requirement.

5. Conclusions, trends and further developments

In this paper, we have presented a review of warehouse management systems and subsequently discussed examples of models in some specific areas that in particular highlight the relation between inventory control decisions and product allocation and assignment problems. Other fields of interest, not discussed here, include warehouse justification and design problems, as well as operational short-term routing problems. For instance, Gross et al. [14] outline the relation between multi-echelon inventory control policies and the choice of warehouse locations on a strategic level. Many authors concentrate on the development of smart order-picking strategies (both for manual orderpickers and automatic storage and retrieval machines). Indeed, also the examples discussed here focus on a maximum reduction of retrieval time, e.g., the forward/reserve policy discussed in Section 4.2 has led to a reduction of the orderpick time of more than 40% in a warehouse with 200 products and 800 storage locations. In one particular case study carried out at a distribution center of Yamaha Motor Co. at Amsterdam Airport, the class-allocation method discussed in Section 4.3 led to a 10% travel time reduction compared with the current four class-based strategy while the algorithm also compared favorably with other recent procedures (see e.g. [15]). In addition, a sophisticated class-allocation leads to a higher overall service level, since storage space is better used (i.e., for the right products). For a more detailed discussion of these results, as well as for an extensive literature review, the reader is referred to [16].

It will be clear that a higher warehouse service level and shorter response times may lead to additional savings downstream the logistic chain as well. For instance, in the case of a production warehouse supplying a two-bin operating assembly line, shorter response times may significantly reduce the total amount of stock placed along the line. In the food and retail sector, where many stores have moved towards just-in-time delivery, there is a constant pressure to improve response times of the warehouses. Wall Mart, a major retail chain in the U.S., has adopted cross docking (i.e., receive, sort and regroup, and ship) as the leading principle in their supply chain, as opposed to conventional storage in distribution warehouses. As a result, the interest in new, sophisticated sorting techniques is rapidly growing. ICA, the leading supermarket chain in Sweden, operates with order-picking robots that can handle a large variety of different cases and boxes, again in an attempt to move towards just-in-time delivery.

And that is still the beginning. The introduction of electronic shopping and ordering will radically change the logistics of the supply chain and lead to a drastic change in inventory management. An order to delivery cycle of 2.5 days, which is expected for the best consumer products, leaves less than 24 h for manufacturing, assembly, expedition and loading of the shelves of the retail store, after removing the average transportation time. Such a future places a tremendous pressure on the organization, planning and control of the production warehouse, as well as on materials handling, manufacturing and assembly. Indeed, some companies are completely re-engineering their manufacturing systems by introducing a so-called Use Point Manager concept in which warehousing, material handling, assembly and packing are completely integrated in independent cells within the factory (for an interesting account, the reader is referred to [16]). Trends such as cross-docking and electronic shopping are expected to remove some intermediate stages in the supply chain and lead to an, already observable, renewed interest in production warehouses as opposed to distribution warehouses.

The above observations clearly indicate the need for research that focuses on the mutual relations between warehousing and inventory management. Unfortunately, as once was the case with set-up times in manufacturing, many inventory researchers assume the storage and material handling infrastructure as given. A better insight in warehousing systems and in the key factors for improving both their design and control, may lead to significant further reductions of inventory levels and improvement of response times. Facing future market trends, in particular the increased use of electronic media such as Internet in shopping and ordering, the integration of inventory and warehouse management issues may prove to be a prom: ising research area.

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