Validation of empirical formulas to derive model parameters for sands

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ABSTRACT: The right selection of soil model parameters is essential to make good predictions in geo-engineering projects where the Finite Element Method (FEM) is used. In order to support geotechnical engineers with soil model parameter selection, empirical formulas have been developed to derive the model parameters of the Plaxis Hardening Soil model with small-strain stiffness (HSsmall), on the basis of a characteristic property (relative density for sands and plasticity index for clays). This paper shows a validation of formulas for sands which have been derived from published soil testing data. The main goal of the empirical formulas is to give a reasonable first order approximation of soil behaviour in FEM calculations, covering a wide range of sands. In a case study it is demonstrated that the empirical formulas work reasonably well to get a first estimate of deformations and stress developments for a real project.

1 INTRODUCTION

In the last decades, many researchers have investigated the properties of sands and clays, and they have published formulas, charts and tables as a general reference to support the design of geotechnical structures (e.g. Kulhawy & Mayne (1990)). Most general data refers to soil strength properties, such as a friction angle for sand or undrained shear strength for clay, which can be primarily used for stability analysis and ultimate limit state design (ULS). In contrast to ULS, serviceability state design (SLS) requires stiffness properties to be known. Several researchers have published correlations between stiffness and strength, index properties and/or state parameters. For sands, many correlations exist with the relative density, whereas for clay many correlations exist with the plasticity index.

Over the last twenty years, the Finite Element Method (FEM) has gained much popularity for geo-engineering and design. In FEM, the mechanical behaviour of soils is simulated by means of constitutive models, in which model parameters are used to quantify particular features of soil behaviour. Constitutive models may range from simple to very advanced models. In general, simple models require only a limited number of parameters to be selected, but they may also lack some essential features of soil behaviour. More advanced models may include more features of soil behaviour, but also require more parameters to be selected on the basis of soil investigation data. The latter often discourages FEM users to use advanced models for daily geo-engineering projects.

Some authors have published data sets with predefined model parameters for particular types of soils (e.g. Duncan et al. (1980) containing data sets for the Duncan-Chang model). It is challenging but probably unrealistic to extend this idea and cover all existing soil types on a world-wide scale. As an alternative, the authors of this paper are developing empirical formulas to derive all parameters of the Plaxis HSsmall model on the basis of very limited geotechnical data in an attempt to stimulate the use of advanced models for soil in general. This seems contradictive, but since most soil properties are sort of correlated, it is believed that a first approximation could be obtained with reasonable accuracy. The authors are convinced that such a first approximation using an advanced model gives better results than a first approximation with a simple model. It is definitely not the idea to abandon detailed site-specific soil investigation, but the use of the formulas can be very helpful, especially in the early design stage of a project when very limited soil data from the site are available.

In the next chapter formulas are presented for sand to derive the model parameters of the HSsmall model on the basis of the relative density. Even if the relative density is not precisely known, it could be estimated on the basis of very preliminary soil data. The formulas have been derived by regression analysis on a collection of soil data (general soil data, triaxial test data, oedometer test data, etc.) from

Jeffries & Been (2006) and others. The third chapter describes a validation of the formulas by comparing the results of model simulations with real lab test data for different types of sands. Chapter four describes a benchmark example (Schweiger, 2000 and 2002), in which the formulas have been applied to Berlin sand to predict deformations and structural forces due to the excavation. Finally, some conclusions are drawn.

2 FORMULAS FOR SAND

The relative density (RD) is defined as (e_{max}–e) / (e_{max}–e_{min}), where e is the current void ratio, e_{max} is the maximum void ratio (loosest packing) and e_{min} is the minimum void ratio (densest packing). The relative density is usually presented as a percentage, as also used in this paper.

Before considering the parameters of the HSsmall model, the relative density can already be used to estimate the unit weights of sand for practical applications by means of the following formulas:

$$\gamma_{unsat} = 15 + 4.0\,RD/100 \quad [kN/m^3] \tag{1}$$

$$\gamma_{sat} = 19 + 1.6\,RD/100 \quad [kN/m^3] \tag{2}$$

The HSsmall model contains four different stiffness parameters, each of them quantifying the reference stiffness in a particular stress path for a given reference stress level, p^{ref}. For a detailed description of the HSsmall model and the meaning of its parameters, reference is made to Benz (2007) and Brinkgreve et al. (2008).

For (quartz) sand, stiffness is supposed to vary linearly with RD. The following formulas are proposed for the reference stiffness parameters, considering p^{ref} = 100 kN/m^2:

$$E_{50}^{ref} = 60000\,RD/100 \quad [kN/m^2] \tag{3}$$

$$E_{oed}^{ref} = 60000\,RD/100 \quad [kN/m^2] \tag{4}$$

$$E_{ur}^{ref} = 180000\,RD/100 \quad [kN/m^2] \tag{5}$$

$$G_0^{ref} = 60000 + 68000\,RD/100 \quad [kN/m^2] \tag{6}$$

Figure 1 shows the variation of G_0^{ref} with RD for different sands, in comparison with Equation 6.

The actual stiffness is stress-dependent. The rate of stress dependency, m, is observed to be negatively correlated with the density. The following formula is proposed for m:

$$m = 0.7 - RD/320 \quad [-] \tag{7}$$

Poisson's ratio for unloading and reloading, ν_{ur}, is taken 0.2. The parameter relating the modulus reduction curve to the cyclic shear strain level is $\gamma_{0.7}$, for which the following formula is proposed:

$$\gamma_{0.7} = (2 - RD/100)\cdot 10^{-4} \quad [-] \tag{8}$$

The following formulas are proposed for the strength-related properties:

$$\varphi' = 28 + 12.5\,RD/100 \quad [°] \tag{9}$$

$$\psi = -2 + 12.5\,RD/100 \quad [°] \tag{10}$$

$$R_f = 1 - RD/800 \quad [-] \tag{11}$$

These values should be used for drained conditions.

Table 1 gives an example of parameter values for loose, medium, dense and very dense sand using the above formulas.

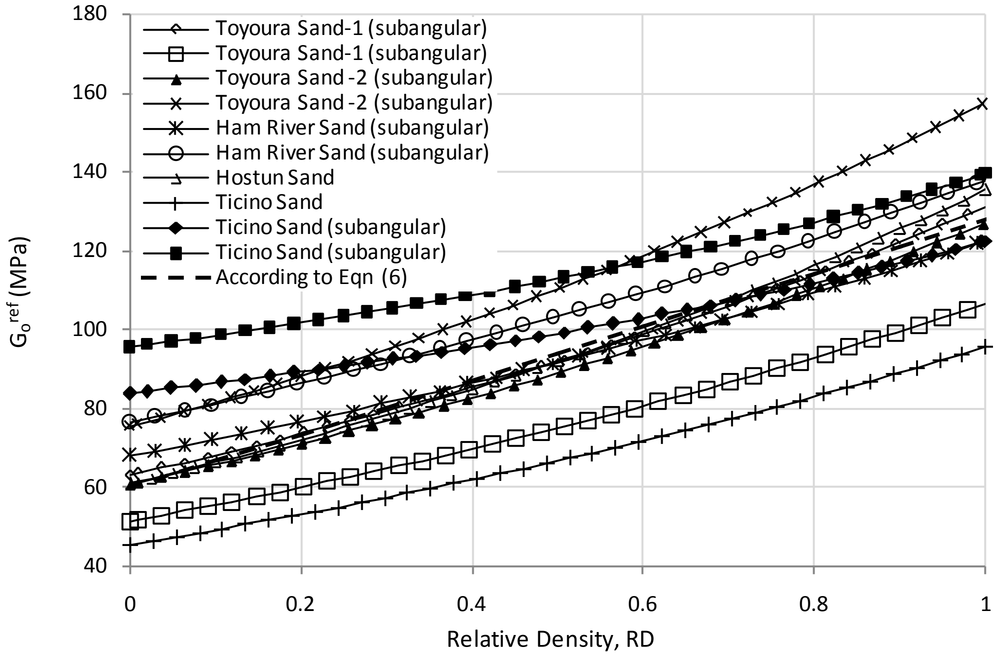


Figure 1. Comparison of formula for small-strain stiffness for different sands at different densities

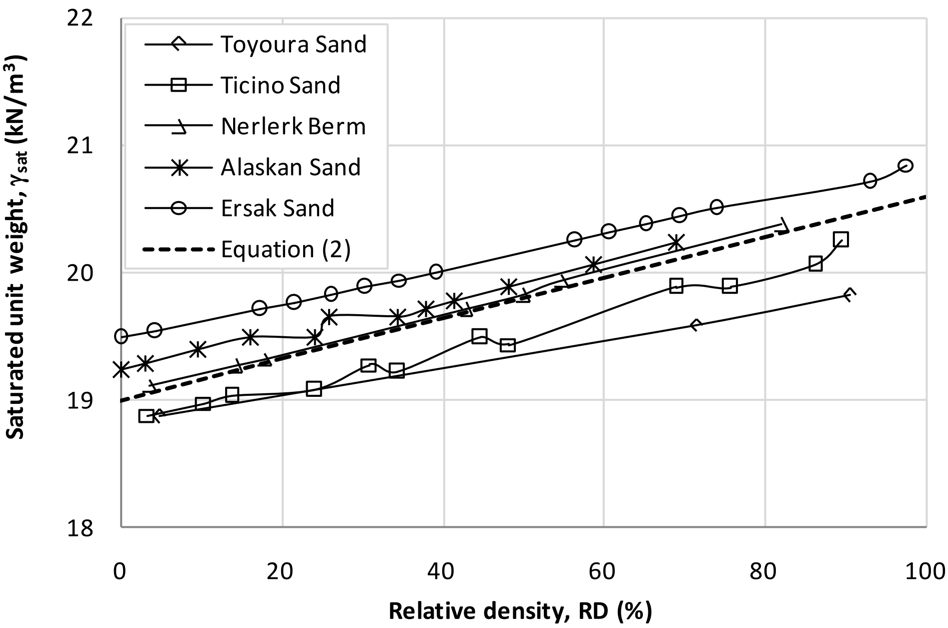


Figure 2. Comparison of formula for saturated unit weight for different sands at different densities

Table 1. Examples of model parameters for sands with different relative densities

RD	γ_{unsat}	γ_{sat}	E_{50}^{ref}	E_{oed}^{ref}	E_{ur}^{ref}	G_0^{ref}	m
-	kN/m^3		kN/m^2	kN/m^2	kN/m^2	kN/m^2	-
25	16.0	19.4	15000	15000	45000	77000	0.622
50	17.0	19.8	30000	30000	90000	94000	0.544
80	18.2	20.3	48000	48000	144000	114000	0.450
100	19.0	20.6	60000	60000	180000	128000	0.388

RD	$\gamma_{0.7}$	φ'	ψ	R_f
-	-	°	°	-
25	$1.8 \cdot 10^{-4}$	31.1	1.1	0.969
50	$1.5 \cdot 10^{-4}$	34.3	4.3	0.938
80	$1.2 \cdot 10^{-4}$	38.0	8.0	0.900
100	$1.0 \cdot 10^{-4}$	40.5	10.5	0.875

3 VALIDATION OF FORMULAS

The formulas for unit weight have been validated for different sands at different densities with data from Jeffries & Been (2006). Both the saturated unit weight γ_{sat} and the dry unit weight γ_{dry} were reported. Figure 2 shows the variation of γ_{sat} with RD for different sands, in comparison with Equation 2. The unsaturated unit weight γ_{unsat}, as proposed in formula 1, is a realistic practical value in between γ_{dry} and γ_{sat}.

To validate the formulas for the HSsmall stiffness parameters, a comparison has been made between data from real drained triaxial tests on different types of sand with the results from numerical simulations with the HSsmall model. Figure 3 shows the results for triaxial tests on Karlsruhe sand of different densities at 100 kN/m^2 cell pressure, based on data by Wu (1990).

Another series of drained triaxial tests have been analysed for loose and very dense Sacramento River sand at different cell pressures, based on data by Lee (1965). Some of these results are shown in Figure 4.

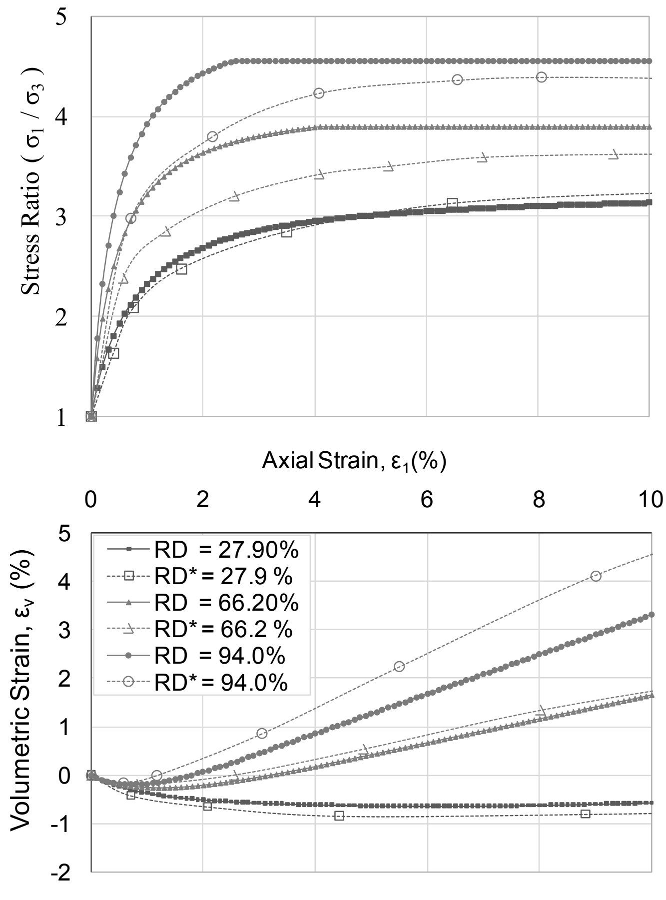


Figure 3. Comparison of drained triaxial tests on Karlsruhe sand with HSsmall model for different relative densities (* indicates experimental data, after Wu, 1990)

Another comparison has been made for dense Hokksund sand, based on a drained triaxial test and an oedometer test as reported by Yang (2004). The results are shown in Figures 5 and 6.

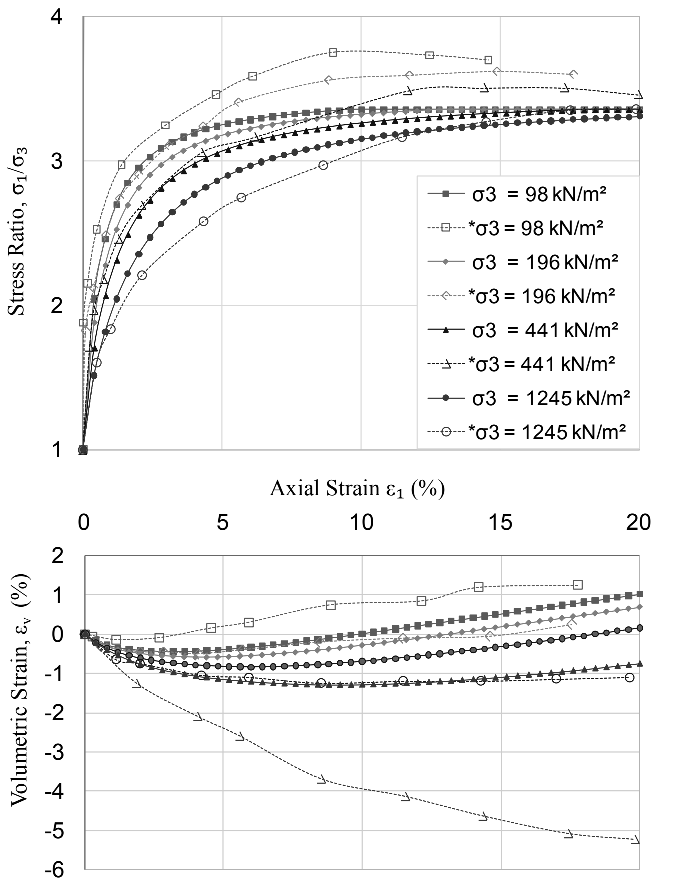


Figure 4. Comparison of drained triaxial tests on loose Sacramento river sand (*RD*=38%) with HSsmall model for different cell pressures (* indicates experimental data, after Lee, 1965)

From the different test results the following can be concluded:

- In most cases, the formulas show a (small) over-estimation of the stiffness in triaxial loading.
- Strength and dilatancy are generally under-estimated. High friction and dilatancy may reduce in reality as a result of shearing and softening, which is not included in the HSsmall model. Hence, a small under-estimation of the peak strength might even be desirable.
- The stiffness in oedometer loading is difficult to match with soil testing data. The data presented in Figure 6 seems reasonable, but other tests were less successful. Reason for this is that in oedometer tests the stress range is usually quite large and the density changes significantly, so one *RD*-value cannot cover the full test.
- Hostun sand seems to be rather soft. The formulas tend to significantly over-estimate the stiffness of Hostun sand (not presented herein).

It should be noted that these tests primarily validate the formulas for loading stiffnesses, friction and dilatancy (Equations 3, 4, 9 and 10) and to a lesser extend the formulas for the other model parameters, but some of these will be considered in the next chapter.

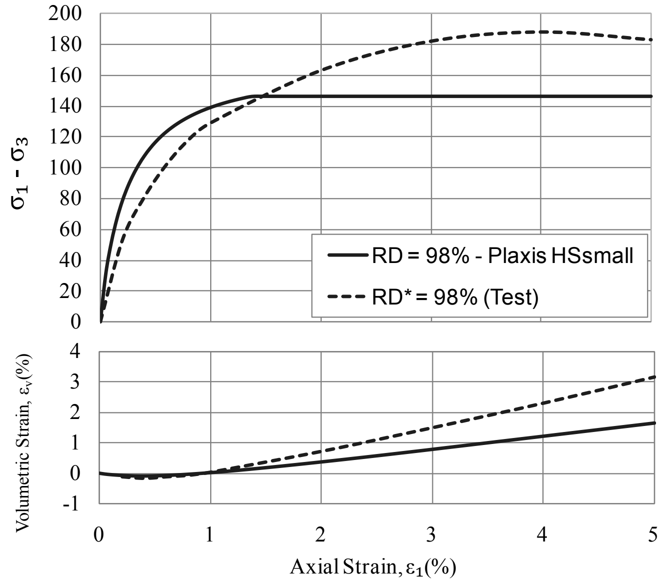


Figure 5. Comparison of drained triaxial test on dense Hokksund sand with HSsmall model

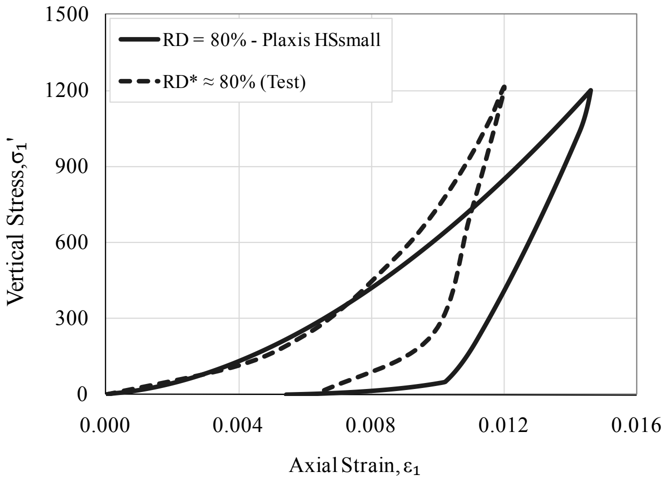
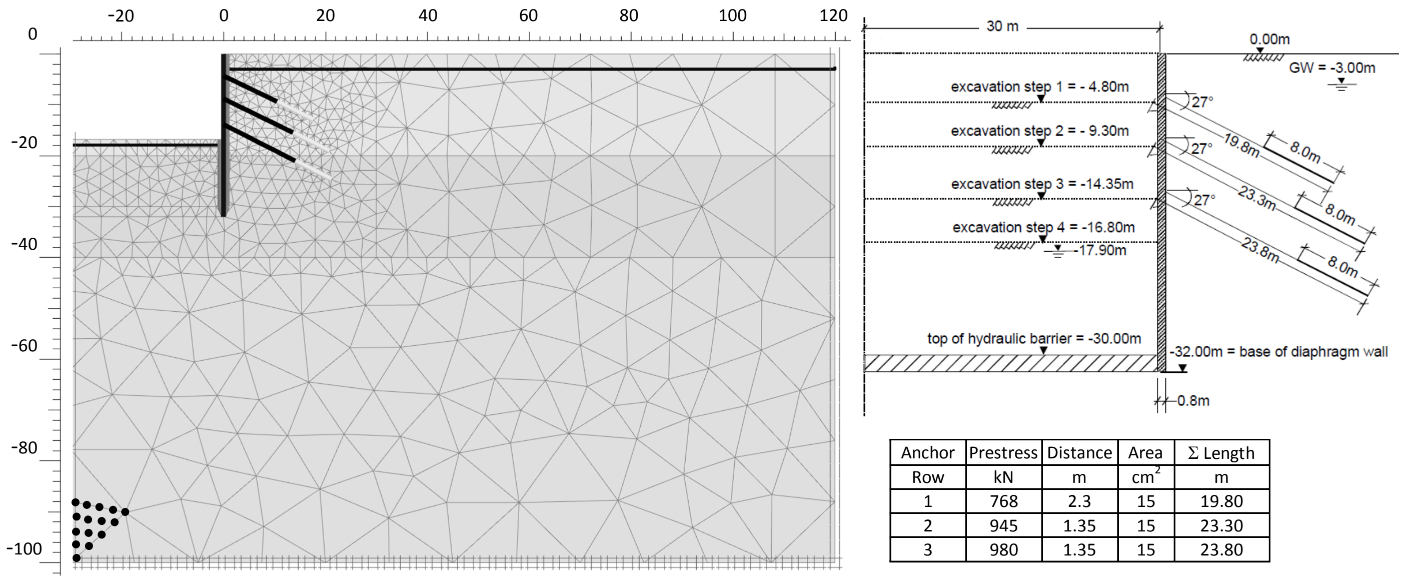


Figure 6. Comparison of oedometer loading and unloading test on dense Hokksund sand with HSsmall model

4 CASE STUDY

In this chapter a case study is presented, based on the benchmark example as described by Schweiger (2002). It concerns an excavation in Berlin sand, supported by a triple anchored retaining wall. The excavation is 16.8m deep and the wall is 32m long. Three rows of anchors are used, starting just above the intermediate excavation levels of 4.8m, 9.3m and 14.4m depth. The wall has been modelled by Mindlin beam elements; the wall-soil interaction by interface elements; the anchors by a combination of membrane elements (grouted body) and two-node spring elements (anchor rod). Figure 7 shows the used 2D finite element mesh composed of 15-node (cubic strain) elements, and some model details.

The soil was reported to be medium dense Berlin sand (from an undisturbed sample at 8 m depth). Although more soil data was provided, only the information ‘medium dense’ was used here and inter-



Anchor Row	Prestress kN	Distance m	Area cm^2	Σ Length m
1	768	2.3	15	19.80
2	945	1.35	15	23.30
3	980	1.35	15	23.80

Figure 7. Geometry and mesh of triple anchored excavation in Berlin sand (with details from Schweiger, 2002)

preted as *RD* = 50%, at least for the upper 20 m of sand. The next 20 m was considered to be denser, with an assumed *RD* of 80%, whereas the lower 60 m was assumed to be very dense with an assumed *RD* of 100%. The formulas presented in Chapter 2 were used to estimate the model parameters (see Table 1 for *RD* = 50%, 80% and 100% respectively). In this case study the unloading stiffness and small-strain stiffness parameters are more relevant than the loading stiffnesses.

The interface strength was related to the strength in the surrounding soil, and reduced by a factor 0.8, as in the original benchmark. The structural properties were taken from Schweiger (2002), i.e. $E_{steel} = 2.1 \cdot 10^8$ kN/m^2 and $E_{concrete} = 3.0 \cdot 10^7$ kN/m^2. The excavation process was simulated in 8 different stages, starting with the installation of the wall, followed by the four excavation stages (including lowering of the water table), and in between separate stages to install and pre-stress the next anchor row.

Figure 7 shows the horizontal displacements of the wall after full excavation, and a comparison with the reference solution by Schweiger (2000). The calculated maximum displacement of the wall is 51 mm. This is about 1.5 times higher than the (corrected) measurements. The overall deformation shape is quite similar, but shows a 15 mm 'shift' compared to the reference solution. This indicates that the soil is not stiff enough, which could indicate that the small-strain stiffness is too low.

The distribution of bending moments is quite similar to the reference solution; the maximum value (735 kNm/m) is almost equal. Anchor forces are in the right order, but show more variations in the phases after installation than in the reference solution, due to the lower soil stiffness.

Considering that the reference solution is based on more detailed soil data, the results presented herein are quite reasonable for a first approximation.

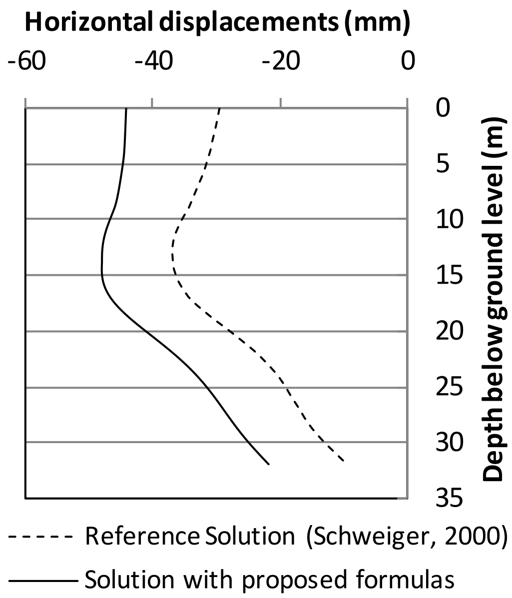


Figure 8. Comparison of horizontal displacement profile of the wall (after Schweiger, 2000)

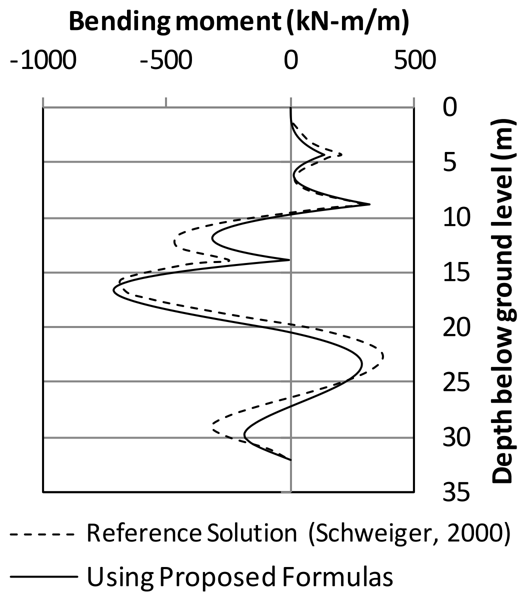


Figure 9. Comparison of bending moment diagrams of the wall with triple anchors (after Schweiger, 2000)

5 CONCLUSIONS

Empirical formulas have been presented for sands to select all model parameters of the Plaxis Hardening Soil model with small-strain stiffness, on the basis of the relative density. The formulas have been validated against lab test data for different sands at different densities and different pressures. Moreover, the formulas have been used in a benchmark example involving a triple anchored excavation in Berlin sand.

Although not all formulas or model parameters have been validated in sufficient detail, it can be concluded from the current results that the formulas may give a reasonable first approximation of the drained behaviour of (quartz) sands in geo-engineering applications. Some formulas may be reconsidered or improved, but it should be realized that a high accuracy can never be achieved unless additional information (such as grain size distribution, grain shape, etc.) is taken into account. Nevertheless, the formulas can be quite useful in the beginning of a project when only limited soil data is available.

By using general formulas for model parameter selection it is not the idea to abandon detailed soil investigation. Since the formulas cannot provide sufficient accuracy for a final design, more detailed soil investigation remains definitely required. A first analysis based on these formulas may actually help to define a detailed soil investigation plan, because it can give insight in dominant stress paths and critical locations in the project.

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