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1 Camera Calibration

In this assignment, we implement Zhang's camera caibration technique to find camera's intrinsic and extrinsic parameters given a set of images of calibration pattern taken from different poses of the camera. The steps include, corners extraction, homography estimation between each image and world points, zhang's calibration procedure, refining the parameters using Levenberg-Marquadt optimization and verification of the calibrated parameters.

1.1 Corners Extraction

To find the salient features in each of the image of calibration pattern, we extract corners of the square blocks of the pattern. These can be extracted through the following steps:

- Apply the canny edge detector to the image with appropriate thresholds so that the spurious edges are removed. Parameters used are Low Threshold=210 and High Threshold=240
- Use Hough Transform technique to find the straight edges within the edge-image. These straight lines are ideally the edges of the square blocks. However, multiple lines may appear for a single edge. This is eliminated throught the following technique
  - (i) Find intersections between all possible lines
  - (ii) Within a radius of 20 pixels, retain only one corner and mark their parent lines as true lines
  - (iii) Furthermore, only the lines which have more than 30 degrees of angle between them are used to find the corners.

  The parameters used for hough transform in openCV are, Votes threshold=30, Minimum Line length =30, Maximum Line Gap = 600
- Once the lines and their corresponding corners are found, the corners have to be sorted. This is done through following steps:
  - (i) For all the lines, find their angle with respect to x-axis. If the angle is less than 45 degrees, assign line to "horizontal lines" class, else assign line to "vertical lines" class.
  - (ii) Find x-intercept for vertical lines and y-intercept for horizontal lines. Sort both class of lines based on their intercept values from smaller to larger
  - (iii) Use the sorted horizontal and vertical lines combinations to find all 80 corners of the image in sorted manner.

1.2 Homography Estimation between Each Image and World Plane

Once the corners are found within each image, we need to establish correspondence between the image corner points and world coordinates of the corner points.

The world-ordinates are found by measuring the size of the square block in the calibration pattern. It was found to be 2.44 cm. The top-left corner has been chosen as the world-origin (0,0) cm and all other corners are measured relative to the origin corner. The coordinates of the 80th corner is then (21.96, 17.08) cm. (Please note that the height is x-coordinate and width is y-coordinate)

Based on these measured wprld coordinates and image points, we calculate the homographies using linear least squares method similar to homework 2 and 3. Here, all the 80 corners of each image have been used to estimate the homographies.

1.3 Zhang's Calibration Procedure

The zhang's method involves solving for intrinsic and extrinsic parameters of the camera. The intrinsic parameters are calculated based on the theory of absolute conic which intersects the image plane at two specific points.

The relevant entities are given below:

- $K = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$ is the intrinsic parameters matrix.
- $[R|t]$ is the extrinsic parameter matrix where R is the rotation matrix and t is the translation matrix for the camera with respect to world coordinate system.
- $H = K \begin{bmatrix} \vec{r1} & \vec{r2} & \vec{t} \end{bmatrix} = \begin{bmatrix} \vec{h1} & \vec{h2} & \vec{h3} \end{bmatrix}$ is the homography between $Z = 0$ plane in world-3D and the image plane.
- $P = K[R|t]$ is the 3x4 Camera Projection Matrix which is the final-mapping between image plane and world-3D
- The goal is to estimate 5 parameters of the intrinsic matrix, and 6 parameters which represents the rotation and translation quantities through R and t.
- By absolute conic theory, the image of the obsolute conic is given by $\omega = K^{-T}K^{-1}$
- ω can be found by solving the following equations:

  $$\vec{h1}^T \omega \vec{h1} - \vec{h2}^T \omega \vec{h2} = 0$$

  and

  $$\vec{h1}^T \omega \vec{h2} = 0$$

- We use the homography H found in Previous step and rearrange the equations to the form

  $$Vb = 0$$

where, b is the vector of 6 elements of ω to be estimated. V is a matrix containing the known H parameters arranged accordindly. This is solved by taking SVD of V and then assiging b as the eigen vector corresponding to the smallest eigen value of V

- Once ω is found, we use the Zhang's formulae to find the 5 intrinsic parameters α_x, α_y, x_0, y_0, s and hence K
- Extrinsic parameters are found using the equations below:

$$K^{-1}\begin{bmatrix}\vec{h1} & \vec{h2} & \vec{h3}\end{bmatrix} = \begin{bmatrix}\vec{r1} & \vec{r2} & \vec{t}\end{bmatrix}$$

$$\xi = \frac{1}{||K^{-1}\vec{h1}||}$$

$$\vec{r1} = \xi K^{-1}\vec{h1}$$

$$\vec{r2} = \xi K^{-1}\vec{h2}$$

$$\vec{r3} = \vec{r1} \times \vec{r2}$$

$$\vec{t} = \xi K^{-1}\vec{h3}$$

- Now, we condition the Rotation matrix R so that it is orthonormal. This is done by finding SVD of R as $R = UDV^T$ and then setting $R = UV^T$. This sets the eigen values to 1 and retains the orthonormal property of the rotation matrix.

At the end of zhang's procedure, we will have K and $[R|t]$ matrices which characterizes a camera.

1.4 Refining the calibration parameters using Levenberg-Marquadt