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1 Image Mosaicing

In Image mosaicing, given a sequence of images of a scene which can't be captured completely in one image, the images are stitched together using the concepts of interest points, robust homography estimation using RANSAC and non-linear least squares refinement of the homographies.

1.1 SURF for interest points matching

Speeded Up Robust Feature (SURF) is used to find the interest points in an image. Each interest point if described by a 64x1 vector. The interest points in two sequential images are said to match when the euclidean distance between their respective feature vectors is below a pre-defined threshold. Such matched interest points are called correspondences.

SURF is applied to all the 7 input images and the correspondences between consecutive images are stored.

1.2 Robust Homography Estimation using RANSAC

The correspondences found using SURF may not always be true correspondences. There may be multiple interest points in an image which are all mapped to a single interest point in the other image. Such false correspondences shouldn't be considered while estimating the homography between the images.

To eliminate such outliers, we use a well known Random Sample Consensus (RANSAC) algorithm. The steps involved in RANSAC are as follows:

- Linear least squares minimization using minimal-set of correspondences

  The Homography (H_{12}) that maps a point on the image1 to image2 is given by,

  $$x_{image2} = H_{12} x_{image1}$$

  where, $x_{image1} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ and $x_{image2} = \begin{bmatrix} \acute{x} \\ \acute{y} \\ 1 \end{bmatrix}$

  If we know the correspondences between image 1 and 2, one can calculate H_{12} using,

  $$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -x\acute{x} - y\acute{x} \\ 0 & 0 & 0 & x & y & 1 & -x\acute{y} - y\acute{y} \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = \begin{bmatrix} \acute{x} \\ \acute{y} \end{bmatrix}$$

We need 4 correspondences to calculate H. This set of correspondences is called minimal set.

- Random sampling of correspondences

  The set of correspondences contain some outliers and hence we randomly select 8 correspondences out of all the available correspondences in each iteration to find a homography. We use this homography to find out set of outliers and set of inliers. Inliers are the correspondences which supports the calculated homography. Naturally, we retain the homography which gives us the largest inlier set and least sum of squared errors.

  In this method, the parameter N (Number of iterations) is chosen based on the fact that we want the probability that at least one of the N trials will be free of outliers is $p = 0.99$. Then,

$$N = \frac{\log(1-p)}{\log[1-(1-\epsilon)^n]}$$

  where, ϵ is the probability that a correspondence is an outlier and n is the number of correspondences in the minimal set. The parameter M (Minimum number of inliers for homography to be considered valid) is then decided based on the percentage of inliers in the complete set of correspondences. So, $M = (1-\epsilon)n_{total}$ where, n_{total} is the total number of correspondences.

- Finding the best homography

  Once the minimal set of correspondences which provides the maximum number of inliers is known, we use the minimal set along with the best inlier set to find the homography in linear least squares fashion.

1.3 Homography refinement using Dog-Leg method

Dog-Leg is an iterative non-linear least squares minimization method which can be used to find an optimal homography given that there is a good estimate of homography to start with.

In this assignment we use the homography found using RANSAC to start the Dog-Leg optimization procedure. The procedure combines the best of gradient descent and gauss newton optimization methods.

1.4 Image Stitching

From the Homographies for every pair of successive images obtained above, the cummulative homographies, to project each image onto the centre image (Image 4), are calculated as follows:

$$H_{14} = H_{12}H_{23}H_{34}$$

$$H_{24} = H_{23}H_{34}$$

$$H_{34} = H_{34}$$
$$H_{44} = I$$
$$H_{54} = H_{45}^{-1}$$
$$H_{64} = H_{56}^{-1} H_{45}^{-1}$$
$$H_{74} = H_{67}^{-1} H_{56}^{-1} H_{45}^{-1}$$

The boundaries of each image in the plane of image 4 are calculated. The size of the final image is hence obtained and the boundaries of each image are converted so that the image 1 starts with origin of final image. We then fill each pixel of the final image with the pixel values of one of the seven images using inverse homographies.

1.5 Parameters

Parameters used in the procedure are:

- H_{th} = Threshold on Hessian value of feature point in SURF
- SURFscore = Threshold on the SURF euclidean distance value between two descriptors.
- SURFratio = Threshold on the ratio of minimum score and second minimum sore for SURF feature correspondence
- δ = Inlier distance threshold to determine whether a correspondence is an inlier or not.
- ϵ = Probability that a correspondence is an outlier
- p= Probability that at least one of the trials of RANSAC will be free of outlier.
- M = Minumum number of inliers required to treat the homography as acceptable.
- N= Number of iterations of RANSAC
- r= Trust region for dog-leg method
- ϵ_{dl} = Norm of the δ_{dl}, the step in dog-leg method. Used as a stopping criterion

Image Set	H_{th}	SURFscore	SURFratio	δ	ϵ	p	M	N	r	ϵ_{dl}
Fountain	1500	0.25	0.7	9	0.2	0.99	85	26	1.0	0.05
Northwestern	2500	0.15	0.7	15	0.3	0.99	91	78	1.0	0.05

2 Results

2.1 Fountain Images

7 input images are shown below. The inlier set(in Green) and outliers (in Red) are shown for two sets of consecutive images. Finally, the stitched image is also shown.

Figure 1: Input Images

Figure 2: RANSAC at work - Outliers detected and eliminated

Figure 3: Fountain Stitched Image

2.2 Northwestern Images

7 input images are shown below. The inlier set(in Green) and outliers (in Red) are shown for two sets of consecutive images. Finally, the stitched image is also shown.

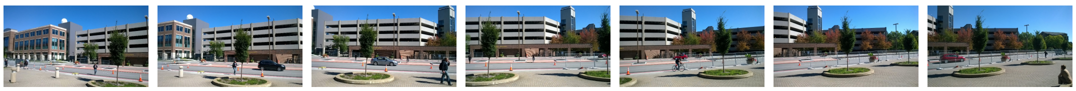

Figure 4: Input Images

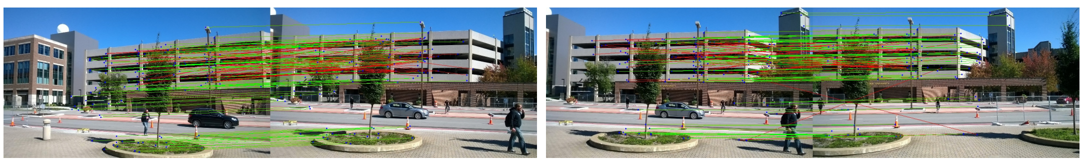

Figure 5: RANSAC at work - Outliers detected and eliminated

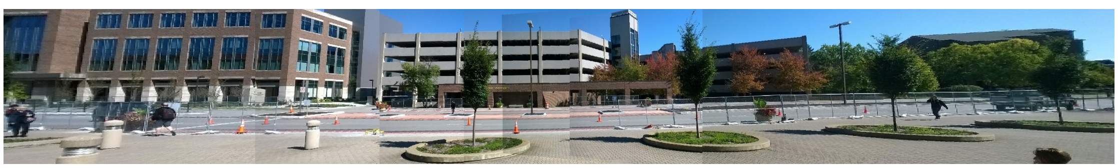

Figure 6: Northwestern Stitched Image