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1 Elimination of Projective and Affine Distortion using 2-step method

If x_{world} represents the point in world co-ordinates and x_{image} represents the same point in image co-ordinates, then the homography which maps world to image plane is given by,

$$x_{image} = Hx_{world}$$

The homography H may contain projective and pure affine distortions. The task is to remove those in two steps.

1.1 Step-1: Removing Projective Distortin

Projective distortion is removed by finding the vanishing line in the distorted image and sending it back to infinity. To do this, we first find the vanishing line in the distorted image. The vanishing line is the line which passes through the intersection points of two sets of world-parallel lines. By taking four corners points of a distorted rectangular/square frame, we can find the vanishing line as shown below using seven cross products.

If x_1, x_2, x_3, x_4 are the four corner points, then two sets of world-parallel lines are given by
$L_1 = x_1 \times x_2$
$L_2 = x_2 \times x_3$
$L_3 = x_3 \times x_4$
$L_4 = x_4 \times x_1$

Intersection points P, Q of the two sets of world-parallel lines L_1, L_3 and L_2, L_4 are
$P = L_1 \times L_3$
$Q = L_2 \times L_4$

The line joining P and Q is the vanishing line and is given by,
$L_v = P \times Q = [l_1, l_2, l_3]^T$

Now, the homography which removes the pure projective distortion should be,

$H_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$ because, if points get transformed by H_p, then the lines get transformed by H_p^{-T} which means, the vanishing line goes to line at infinity as shown below.

$$L_{world} = H_p^{-T} L_v = \begin{bmatrix} 1 & 0 & -l_1/l_3 \\ 0 & 1 & -l_2/l_3 \\ 0 & 0 & 1/l_3 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = L_\infty$$

To summarize, we should apply homography H_p to the image plane to get the projective distortion corrected world-plane.

The vanishing lines in two images of the same scene are different because each image is distorted to a different extent. If the vanishing lines are different, we cannot correct projective distortion of an image of a scene using the vanishing line from another image of the same scene.

1.2 Step-2: Removing Affine Distortin

In this step, we remove affine distortion using the property that the cosine of angle between two orthogonal lines is zero. Let H_a represents the affine homography which maps world plane to image plane. If l and m are two orthogonal lines in world-plane, then $l' = H_a^{-T} l$ and $m' = H_a^{-T} m$ are their corresponding lines in image frame.

Noting that $cos(\theta) = 0 \implies l^T C_\infty^\star m = 0$(because of orthogonality), we can claim that

$$l'^T H_a C_\infty^\star H_a^T m' = 0$$

Where, $C_\infty^\star = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $H_a = \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$ Simplifying yields,

$$\begin{bmatrix} l_1' & l_2' & l_3' \end{bmatrix} \begin{bmatrix} AA^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_1' \\ m_2' \\ m_3' \end{bmatrix} = 0$$

If we allow $S = AA^T = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$ then,

$$\begin{bmatrix} l_1' & l_2' \end{bmatrix} \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} m_1' \\ m_2' \end{bmatrix} = 0$$

Since S is symmetric, $s_{12} = s_{21}$ and all the information is in ratios, we can set $s_{22} = 1$. This gives us,

$$l_1' m_1' s_{11} + (l_1' m_2' + l_2' m_1') s_{12} = -l_2' m_2'$$

We have two unknows in the above equation and we can solve for S if two sets of distorted orthogonal lines are known. So, we get two distroted-orthogonal lines from image plane to find S. Then A can be found using SVD decomposition of S. We then apply the inverse of H_a to the image plane to correct the affine distortion.

2 Elimination of Projective and Affine Distortion in one step

If x_{world} represents the point in world co-ordinates and x_{image} represents the same point in image co-ordinates, then the homography which maps world to image plane is given by,

$$x_{image} = Hx_{world}$$

where, $H = \begin{bmatrix} A & 0 \\ \vec{v} & 1 \end{bmatrix}$

Using the concept that the removal of projective and affine distortion together will retrieve the original angle between the lines back, we can say that two world-orthogonal lines are also orthogonal in image when both the distortions are removed. Hence, if l and m are two orthogonal lines in world-plane, then $l' = H^{-T}l$ and $m' = H^{-T}m$ are their corresponding lines in image frame.

Noting that $cos(\theta) = 0 \implies l^T C^{\star}_{\infty} m = 0$(because of orthogonality), we can claim that

$$l'^T H C^{\star}_{\infty} H^T m' = 0$$

which implies,

$$l'^T C^{\star\prime}_{\infty} m' = 0$$

where, $C^{\star\prime}_{\infty} = \begin{bmatrix} AA^T & A\vec{v} \\ \vec{v}^T A^T & \vec{v}\vec{v}^T \end{bmatrix}$ is called the dual conic.

The general form of any conic is $C^{\star\prime}_{\infty} = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$. This gives us,

$$\begin{bmatrix} l'_1 & l'_2 & l'_3 \end{bmatrix} \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} \begin{bmatrix} m'_1 \\ m'_2 \\ m'_3 \end{bmatrix} = 0$$

Since the information is only is ratios, we can substitute, $f = 1$. So we have,

$$l'_1 m'_1 a + ((l'_1 m'_2 + l'_2 m'_1)/2)b + l'_2 m'_2 c + ((l'_1 m'_3 + l'_3 m'_1)/2)d + ((l'_2 m'_3 + l'_3 m'_2)/2)e = -l'_3 m'_3$$

This implies we need 5 set of orthogonal lines l and m so that the above equation is solved to obtain the values from a through e.

Once dual conic is estimated, the value of A is found by SVD decomposition of top 2x2 matrix of the dual conic. $\vec{v}$ can be found by solving $A\vec{v} = \begin{bmatrix} d/2 \\ e/2 \end{bmatrix}$.

The homography H is then formed using A and $\vec{v}$. The inverse of this homography is used to correct both projective and affine distortions.

3 Comparison of two methods

- Single step method requires more orthogonal features from the image and may fail if such features are less. Careful selection of lines is required to get good results. However, single step method requires less time to program and also execution is faster since we apply a single correction homography. Generally, single step method gave better results given that lines are selected carefully.
- Two step method require less amount of information from the image and is based on two simple concepts. However, my results made me infer that two step method doesn't produce good results. May be the reason is that, it is a multi-step process and the error carries forward.
- Both the methods are sensitive to the selection of orthogonal lines.

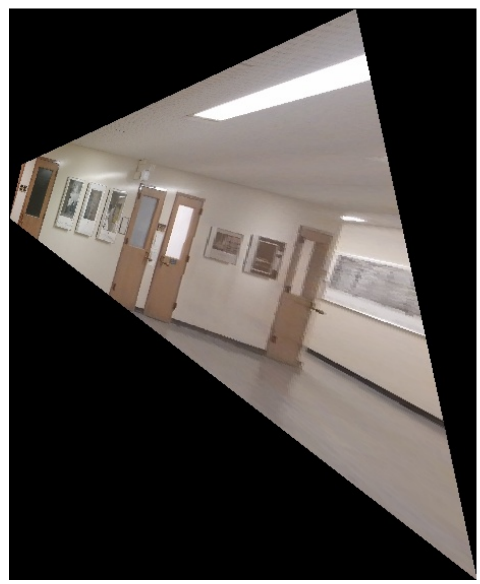
Note for Results section: Blue lines shown are used for 2-step method and red lines are used for single step method along with 2 sets of blue lines. This applies to all the images except set 4, where four sets of orthogonal lines are from same rectangle.

4 Results

Original Image - Set 1, Image 1

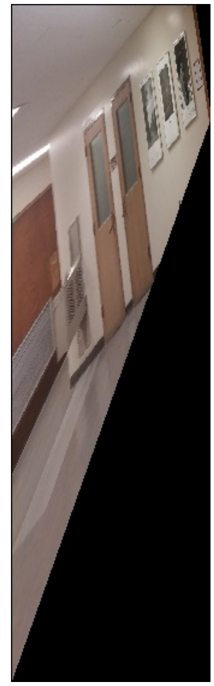
Projective distortion removed - Set 1, Image 1

Affine distortion removed - Set 1, Image 1

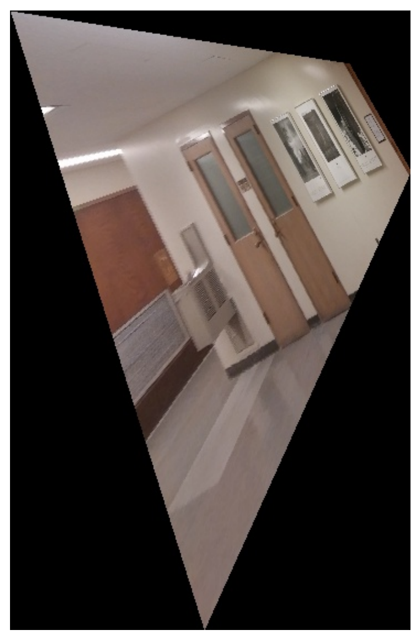


Projective and Affine removed together - Set 1, Image 1

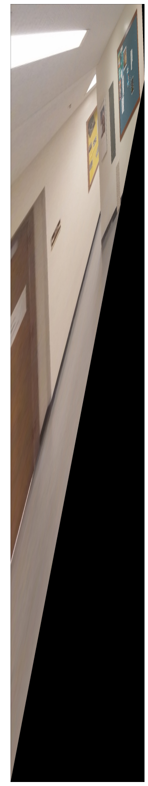
Original Image - Set 1, Image 2


Projective distortion removed - Set 1, Image 2

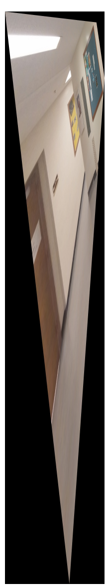

Affine distortion removed - Set 1, Image 2


Projective and Affine removed together - Set 1, Image 2

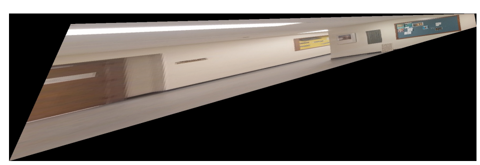
Original Image - Set 2, Image 1



Projective distortion removed - Set 2, Image 1



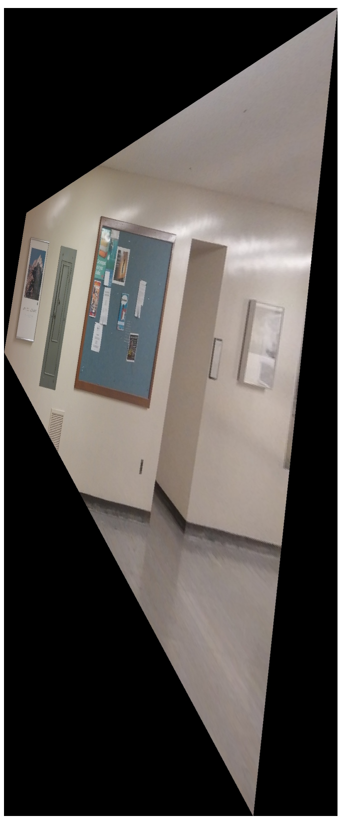
Affine distortion removed - Set 2, Image 1



Projective and Affine removed together - Set 2, Image 1

Original Image - Set 2, Image 2

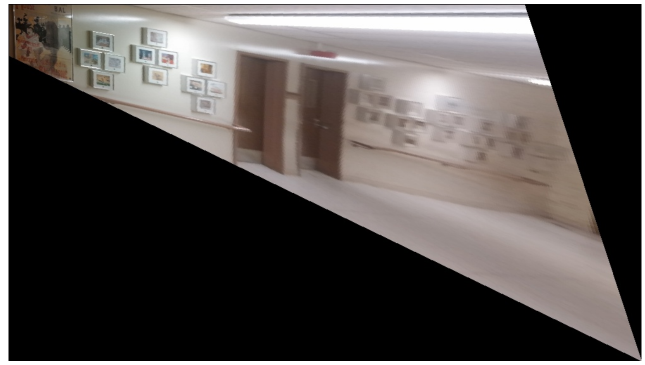
Projective distortion removed - Set 2, Image 2



Affine distortion removed - Set 2, Image 2

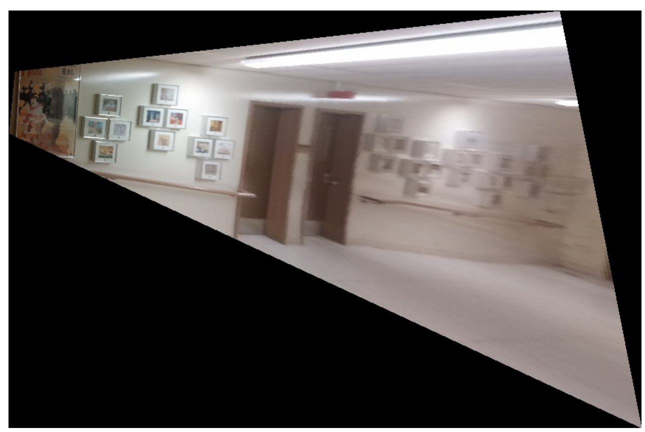
Projective and Affine removed together - Set 2, Image 2

Original Image - Set 3, Image 1



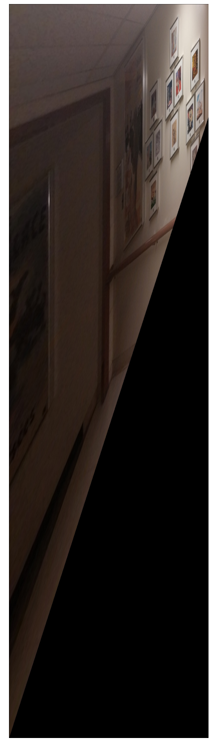
Projective distortion removed - Set 3, Image 1

Affine distortion removed - Set 3, Image 1



Projective and Affine removed together - Set 3, Image 1

Original Image - Set 3, Image 2



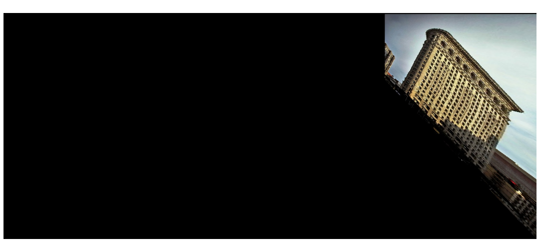
Projective distortion removed - Set 3, Image 2

Affine distortion removed - Set 3, Image 2

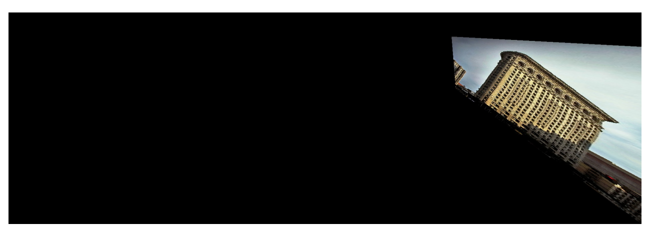
Projective and Affine removed together - Set 3, Image 2

Original Image - Set 4, Image 1

Projective distortion removed - Set 4, Image 1

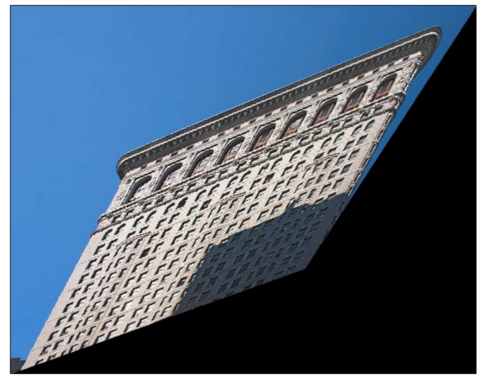


Affine distortion removed - Set 4, Image 1



Projective and Affine removed together - Set 4, Image 1

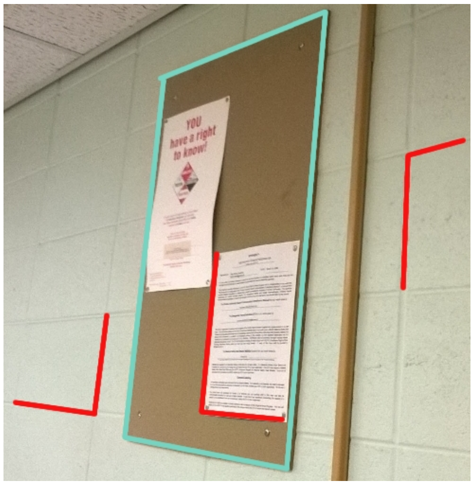
Original Image - Set 4, Image 2



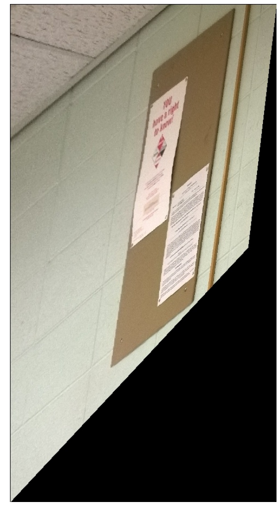
Projective distortion removed - Set 4, Image 2

Affine distortion removed - Set 4, Image 2

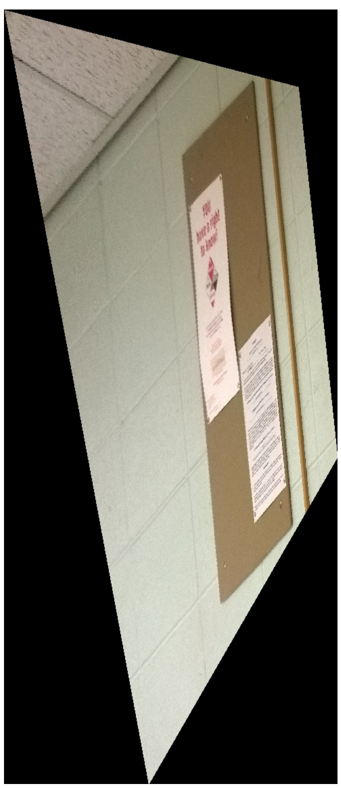
Projective and Affine removed together - Set 4, Image 2



Original Image - My Set, Image 1



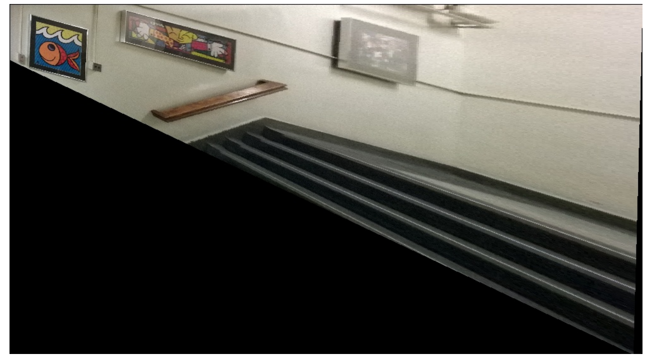
Projective distortion removed - My Set, Image 1



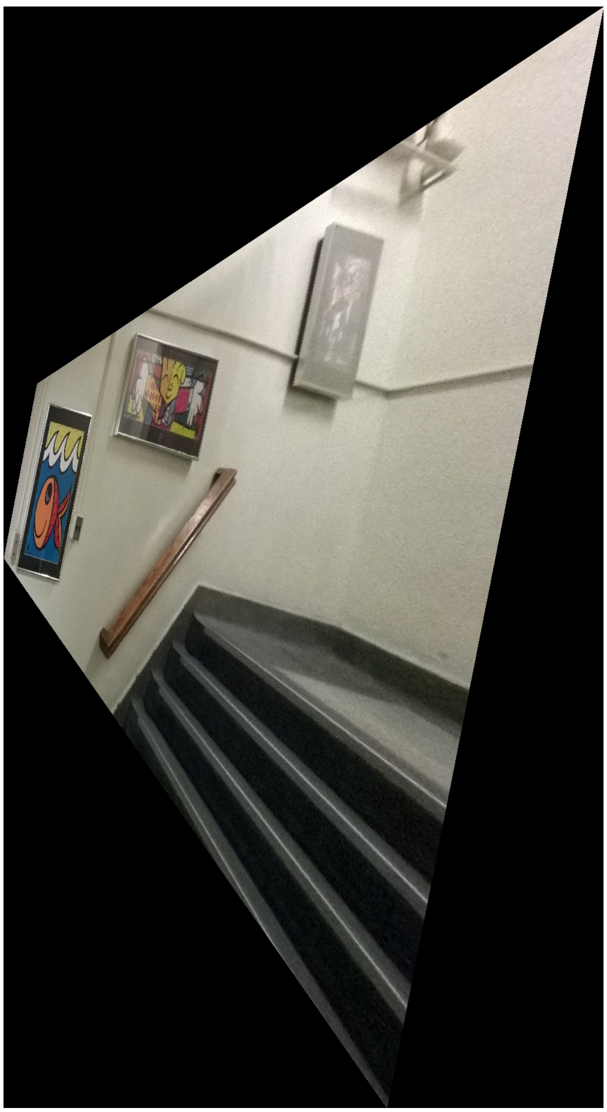
Affine distortion removed - My Set, Image 1

Projective and Affine removed together - My Set, Image 1

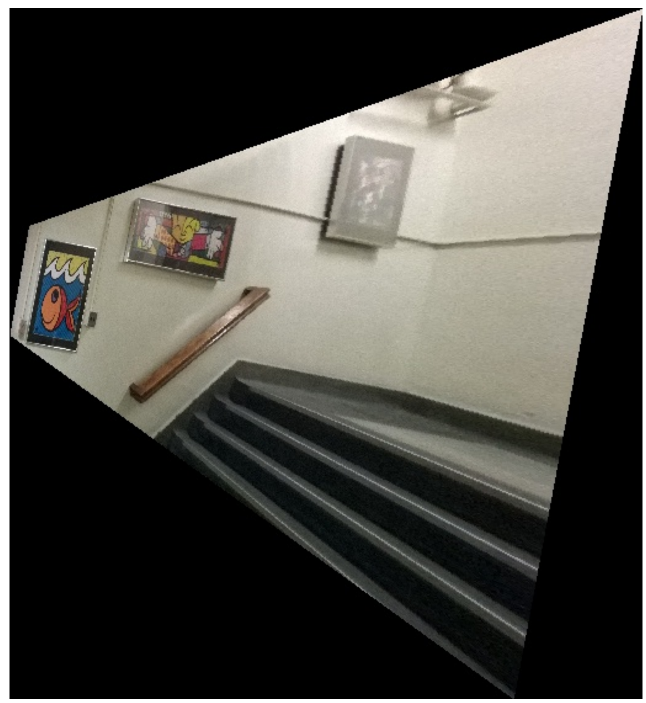
Original Image - My Set, Image 2



Projective distortion removed - My Set, Image 2

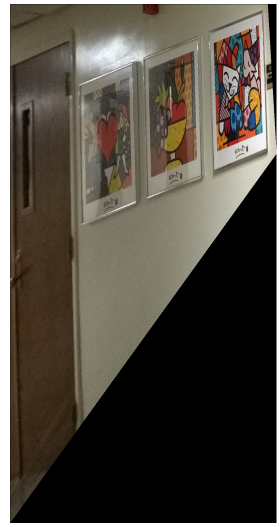


Affine distortion removed - My Set, Image 2

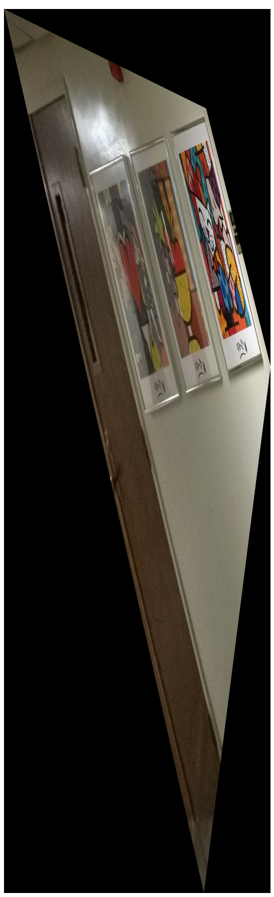


Projective and Affine removed together - My Set, Image 2

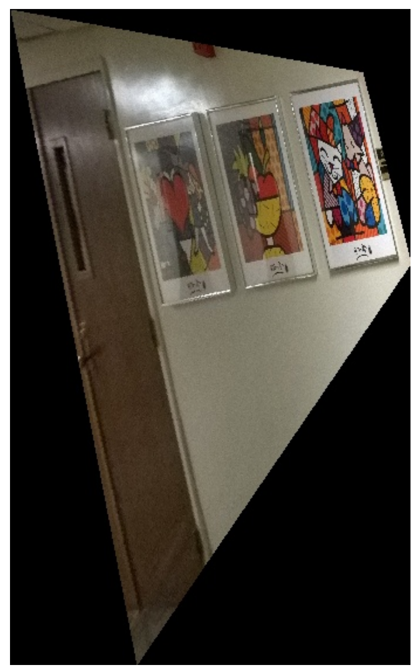
Original Image - My Set, Image 3



Projective distortion removed - My Set, Image 3



Affine distortion removed - My Set, Image 3



Projective and Affine removed together - My Set, Image 3